ADDIS ABABA UNIVERSITY
INSTITUTE OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING
POST GRADUATE PROGRAM UNDER
STRUCTURAL ENGINEERING

Investigation of Effective Length Method For Slender Reinforced Concrete Column Having Linearly Varying Cross-Section

A Thesis Submitted to the Graduate School of the Addis Ababa
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Requirements for the Degree of
Master of Science in Structural Engineering
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BY

Tinsae Woldegebreal
September, 2013

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DECLARATION

I declare that this thesis entitled *"Investigation of Effective Length Method for Slender Reinforced Concrete Column Having Linearly Varying Cross-Section"* is my original work. This thesis has not been presented for any other university and is not concurrently submitted in candidature of any other degree, and that all sources of material used for the thesis have been duly acknowledged.

Candidate:

Name: Tinsae Woldegebreal

Signature: ____________________

Dedicated to …

Peace of the World

Acknowledgment

First and at most, greatest thanks from the depth of my heart is to Almighty *GOD* for endowing me with the courage, strength as well as health throughout my school time and the full help provided by him for the successful accomplishment of this final research for the partial fulfillment for the Msc. Degree in Structural Engineering.

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Tinsae Woldegebreal

September 2013

ABSTRACT

Effective length is the length of the equivalent pin ended column, which has the same elastic buckling load as the actual column. The effective length method has been used for many decades as the basis for the design of columns made of reinforced concrete, structural steel or steel-concrete components. The underlying assumption with this method is the sweeping generalization that; because the actual column and the equivalent column have the same elastic buckling load, they also posses the same design resistances in the ultimate limit state. This assumption has been the subject of many investigations and it was found out that, although the method gives reliable results that lie on the safe side in many cases, it could sometimes lie on the unsafe side. The present investigation deals with slender reinforced concrete columns with linearly varying cross-sections. The effective length method for such columns poses an additional complication. That is; unlike prismatic columns, the choice of cross-section for the equivalent column is not straight forward since the cross-section of actual column is not constant. The results of the present investigation which is limited to pin ended columns with equal and opposite end moments, have shown that the effective length method gives reliable results if the cross-section of the equivalent column is chosen as per certain rules and computations indicated in this document. Extensive tables are compiled for the determination of equivalent section and effective length for any pin-ended tapered column which has the same and opposite end moments. Computer programs, which are done using FORTRAN (FTN95) programming language and MICROSOFT EXCEL spread sheets are used as a supporting tools for analysis processes committed in this investigation. The result of this research can be taken as an approach, which replaces the cumbersome and lengthy way of traditional iterative analysis for tapered columns.

KEY PHRASES AND WORDS:

Effective length, Elastic buckling load, Equivalent section, Failure node, Moment-curvature, second order analysis, Slender columns, Stabilized length, Tapered column.

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LIST OF SYMBOLES

:-A:	Area of cross-section
:-A_s:	Total area of reinforcement
:-A_{st}:	Area of reinforcement at the top of section
:-A_{sb}:	Area of reinforcement at the bottom of section
:-a:	Portion length of column, above the node under consideration
:-B:	Breadth of column
: - C_m:	Moment magnification factor
:-Δ:	Flexural deflection
:-d:	Effective depth of the section
:-δ:	Lateral deflection of the column system
:-E:	Modulus of elasticity
:-e:	End eccentricity of axial load from centroid of section
:-ε_c:	Strain of concrete
:-ε_{cm}:	The strain of the most compressed fiber of concrete in a section
:-ε_s:	Strain of reinforcement steel
:-ϕ:	Diameter of reinforcement bars
:-F_{st}:	Concentrated force exerted by the top reinforcement steel
:-F_{sb}:	Concentrated force exerted by the bottom reinforcement steel
:-F_c:	Concentrated force exerted by concrete
:-f_c:	Stress in concrete
:-f_{cd}:	Design compressive strength of concrete
:-f_{yd}:	Design strength of reinforcement steel
:-I:	Moment of inertia
:-I_1:	Moment of inertia at the minimum section
:-I_1:	Moment of inertia at the maximum section
:-K:	Curvature
:-k:	Effective length factor
:-L:	Length of column
:-L_{eff}:	Effective length of column
:-L_s:	Stabilized length
:-$L_{s,\ lim}$:	The limiting value of stabilized length
:-λ:	Length of the divided segment
:-M:	Bending moment
:-M_e:	End moment
:-M_{max}:	The maximum bending moment the column system can support

:-M_o:	First order moment
:-P:	Axial load
:-P_{cr}:	Elastic buckling load of the column
:-P_{max}:	The maximum axial load the column system can support
: - r:	Radius of gyration
:-σ:	Stress in the material under consideration
:-W_o:	Minimum width of column
:-W_{max}:	Maximum width of column
: - X:	Neutral axis depth, from the top
:-X_i:	Distance of section, under consideration, from the top
:-Y:	Distance from centroid of section to the most compressed fiber of that section

CHAPTER 1

BASIC CONCEPTS

1.1 Historical overview of columns

In ancient times, the name column was given to a large circular support with a capital and base, made up of stone, or appearing to be so. Instead of using as structural parts, a column has often been a decorative element. All significant Iron Age civilizations of the Near East and Mediterranean made some use of columns. In Ancient Egyptian architecture as early as 2600 BC the architect Imhotep made use of stone columns whose surface was carved to reflect the organic form of bundled reeds. In later Egyptian architecture faceted cylinders were also common.

The Egyptians, Persians and other civilizations mostly used columns for the practical purpose of holding up the roof inside a building, preferring outside walls to be decorated with reliefs or painting, but the ancient Greeks, followed by the Romans, loved to use them on the outside as well, and the extensive use of columns on the interior and exterior of buildings is one of the most characteristic features of classical architecture, in buildings like the Parthenon.

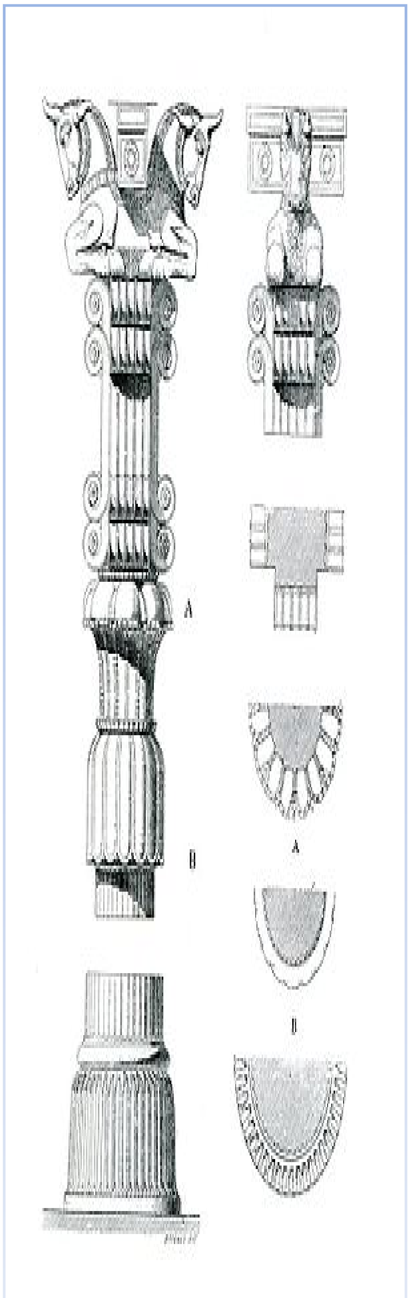


Fig. 1.1 The Parthenon in Greece (left), columns in ancient Egyptian architecture (right)

The Greeks developed the classical orders of architecture, which are most easily distinguished by the form of the column and its various elements. Their Doric, Ionic, and Corinthian orders were expanded by the Romans to include the Tuscan and Composite orders.

Columns, or at least large structural exterior ones, became much less significant in the architecture of the middle ages. The classical forms were abandoned in both Byzantine architecture and the Romanesque and Gothic architecture with capitals often using various types of foliage decoration, and figures carved in relief. Renaissance architecture revived the classical vocabulary and styles, and the informed use and variation of the classical orders remained fundamental to the training of architects throughout Baroque, Rococo and Neo-classical architecture.

Early columns were constructed using stone, some out of a single piece of stone. Monolithic columns are among the heaviest stones used in architecture. Other stone columns are created out of multiple sections of stone, mortared or dry-fit together. In many classical sites, sectioned columns were carved with a centre hole or depression so that they could be pegged together, using stone or metal pins.

The design of most classical columns incorporates the inclusion of a slight outward curve in the sides as well as a reduction in diameter along the height of the column, so that the top is as little as 83% of the bottom diameter (Circular tapered columns). This reduction imitates the parallax effects which the eye expects to see, and tends to make columns look taller and straighter.

Fig. 1.2 Ancient Greece architecture of columns

1.2 Structural reinforced concrete column and its behavior

A column is a vertical structural member supporting axial compressive load, with or without end moment. The cross-sectional dimensions of a column are generally considerably less than its height. Vertical loads from the floors and roof are supported by columns for the safe transmission of the loads to the foundations. In more general terms, compression members and members subjected to combined axial load and bending are sometimes used to refer to columns, walls, and members in concrete trusses or frames. These may be vertical, inclined or horizontal. A column is a special case of compression member that is vertical and limited ratio of its cross-sectional dimension.

Stability effects must be considered in the design of columns. If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender column or a long column. The great majority of concrete columns are sufficiently stocky that slenderness can be ignored. Such columns are refered to as short columns.

Columns can be classified as tied and spiral, as per the transversal reinforcment pattern. *Tied* columns may be square, rectangular, L-shaped, circular or any other required shape. Occasionally, when high strength and/ or high ductility are reqired, the bars are placed in circle, and the ties are replaced by a bar bent in to helix or spral. Such a column are called *spiral* column.

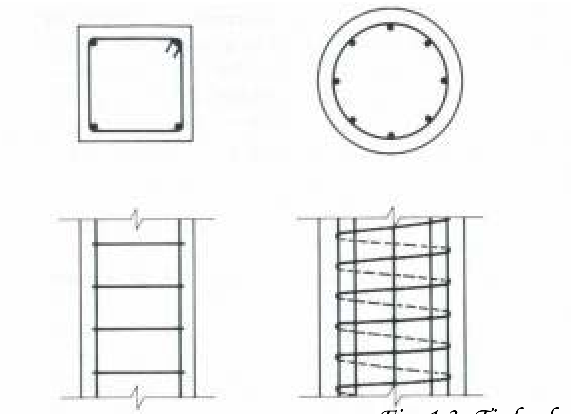


Fig. 1.3 Tied columns (left), spiral column (right)

When a symmetrriacal column is subjected to concentric load ,P, longitudnal strains develop uniformly across the section. Because the steel and concrete are bonded together, the strains in the concrete and steel are equal. For any given strain, it is possible to compute the stress in the concrete and steel using the stress-strain curves for the two materials. The curves are shown below, which are tken from the Ethiopian building code standared, EBCS 2,1995.

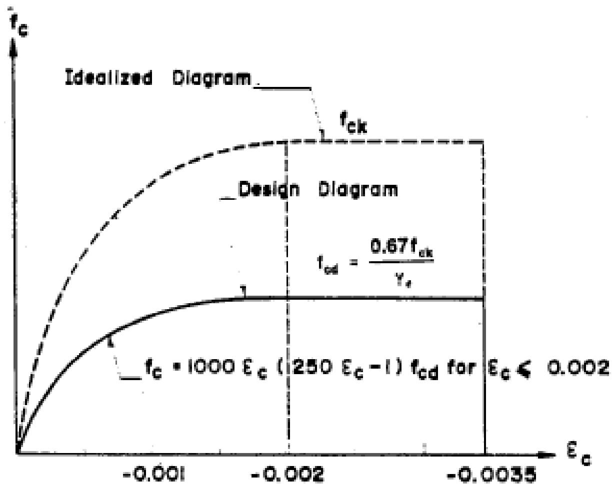


Fig. 1.4 a. Stress-strain curve for structural concrete, taken from EBCS 2, 1995 section 4.2.4

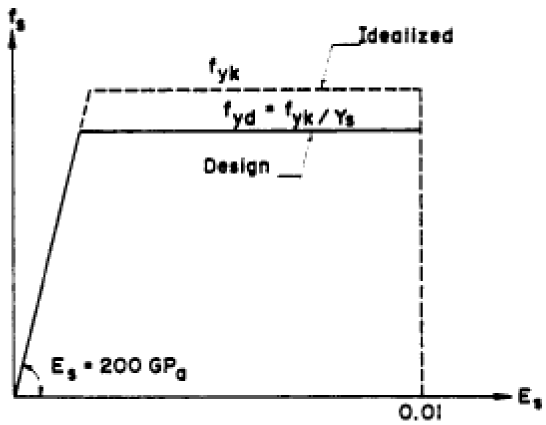


Fig. 1.4b, Stress-strain curve for reinforcement steel, taken from EBCS 2, 1995 section 4.2.4

The forces F_c and F_s in the concrete and steel are equal to stresses multiplied by the corresponding areas. The total load on the column, P, is the sum of the two quantities. Failure occurs when P reaches maximum. For steel with well defined yield strength, this occurs when $F_c=f_{cd}A_c$ and $F_s=f_{yd}A_s$

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to misalignment of the load on the column or may result from the column resisting a portion of the unbalanced moments at the ends of beams/slabs supported by the columns. The distance *e* is referred to as the eccentricity of the load. These two cases are the same, because the eccentric load P can be replaced by a load, P acting the centroidal axis, with the presence of a moment, M=Pe about the centroid. The load P and moment M are calculated with respect to the geometric centroidal axis because the moments and forces obtained from structural analysis normally are referred to this axis. Failure would occur in compression when the maximum stresses reached f_{cd} as given by:

$$\frac{P}{A}+\frac{My}{I}=f_{cd}$$

Or

$$\frac{P}{f_{cd}A}+\frac{My}{f_{cd}I}=1$$

The maximum axial load the column can support occurs when M=0 and P is $P_{max}=f_{cd}A$. Similarly, the maximum moment that can be supported occurs when P=0 and $M=M_{max}$ which is equal to $f_{cd}I/y$. Substituting gives:

$$\frac{P}{P_{max}}+\frac{M}{M_{max}}=1\ldots\textit{(Eq.1.1)}$$

Equation, Eq.1.1 is known as an interaction equation, since it shows the interaction of P and M at failure. The curve plotted P vs M is referred to as an interaction diagram. Points on the lines plotted in interaction diagram represent combinations of P and M corresponding to the resistance of the section. A point inside the diagram represents a combination of P and M that will not cause failure. Combinations of P and M falling on the line outside the line will equal or exceed the resistance of the section hence will cause failure.

An eccentrically loaded, pin-ended column is shown in fig. 1.5; the moments at the ends of the column are:

$$M_e=Pe\ldots\ldots\textit{(Eq. 1.2)}$$

When the load P is applied, the column deflects laterally by an amount δ, as shown. For equilibrium, the internal moment at point of maximum deflection, along column height will be:

$$M_e=P(e+\delta)\ldots\ldots\textit{(Eq. 1.3)}$$

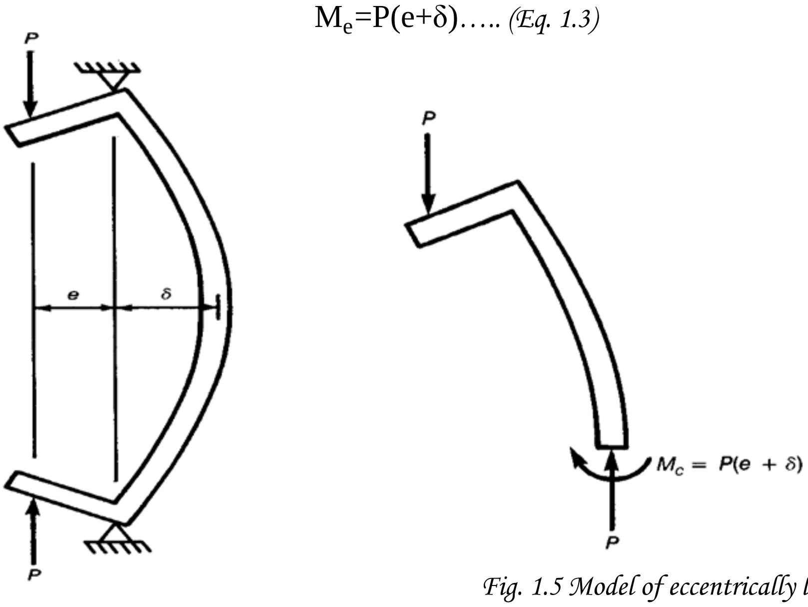


Fig. 1.5 Model of eccentrically loaded, pin ended column

The deflection increases the moments for which the column must be designed. In the symmetrical column, the maximum moment occurs at mid height, where the maximum deflection occurs.

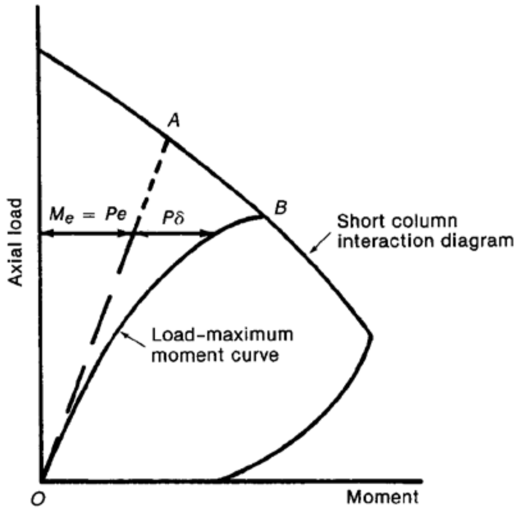


Fig. 1.6 Interaction diagram for reinforced concrete column

Fig. 1.6 shows an interaction diagram for a reinforced concrete column. This diagram gives the combinations of axial load and moment required to cause failure of a column cross-section for a very short length of column. The dashed radial line O-A is a plot of the end moment on the column. Because this load is applied at a constant eccentricity, e, the end moment, M_e, is a linear function of P, previously given as *Eq.1.2:*

$M_e=Pe$

The curved, solid line O-B is the moment M_c at mid height of the column, as previously given in *Eq.1.3*:

$M_c=P(e+\delta)$

At any given load P, the moment at mid height is the sum of the end moment, *Pe*, and the moment due to the deflections, $P\delta$. The line O-A is referred to as a *load-moment curve for the end moment*, while the line O-B is a *load-moment curve for the maximum column moment*.

Failure occurs when the load-moment curve O-B for the point of maximum moment intersects the interaction diagram for the cross section. Thus the load and moment at failure are denoted by point B in *fig 1.6*. Because of the increase in maximum moment due to deflections, the axial-load capacity is reduced from A to B. This reduction in axial-load capacity resulted from what are referred to as slenderness effects.

A *slender column* is defined as a column that has a significant reduction in its axial-load capacity due to moments resulting from lateral deflections of the column. In derivation of the ACI code, "a significant reduction" was arbitrarily taken as anything greater than about 5%.

1.3 Analysis of pin ended column

Lateral deflections of a slender column cause an increase in the column moments, as shown in fig. 1.5, previously. These increased moments cause an increase in the deflections, which in turn lead to a further increase in moments. As a result, the load-moment line O-B in fig. 1.6 is non-linear. If the axial load is below the critical load, the process will converge to a stable position. If the axial load is greater than the critical load, it will not. This is referred to as a second-order process, because it is described by a second order differential equation of *Eq. 1.4*.

$$EI\frac{d^2Y}{dX^2} = -PY \ldots\ldots (Eq.\ 1.4)$$

In a first-order analysis, the equations of equilibrium are derived by assuming that the deflections have a negligible effect on the internal forces in the members. In second-order analysis, the equations of equilibrium consider the deformed shape of the structure. Instability can be investigated only via a second order analysis, because it is the loss of equilibrium of the deformed structure that causes instability. However, because many engineering calculations and computer programs are based on first order analysis, methods have been derived to modify the results of a first order analysis to approximate the second order effects.

1.4 Failure modes

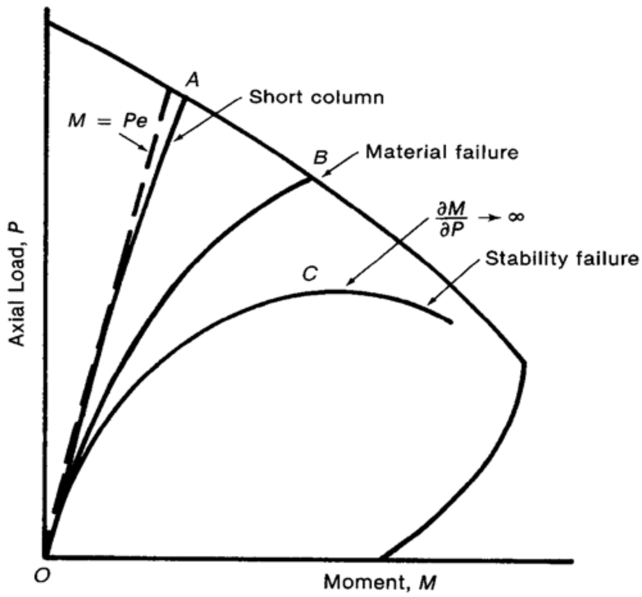


Fig. 1.7 Failure modes in short and slender columns

Fig.1.7 is a load-moment curves for columns of three different lengths, all loaded with the same end eccentricity, e. The load-moment curve O-A for a relatively short column is practically the same as the line M=Pe. For a column of moderate length, line O-B, the deflections become significant, reducing the failure load. This column fails when the load-moment curve intersects the interaction diagram at point B. This is called a *material failure* and is the type of failure expected in most practical columns in braced frames. If a very slender column is loaded with increasing axial load, P, applied at constant end eccentricity, e, it may reach a deflection δ at which the value $\frac{\partial M}{\partial P}$ approaches infinity or becomes negative. When this occurs, the column becomes unstable, because, with further deflections, the axial load capacity will drop. This type of failure is known as *stability failure* and occurs only with very slender braced columns or with slender columns in sway frames.

According to Euro code 2, 2004; section 6.1, the strain diagram of a reinforced concrete column under ultimate limit state shall be assumed to pass through one of the three points A, B or C shown in *Fig. 1.8*.

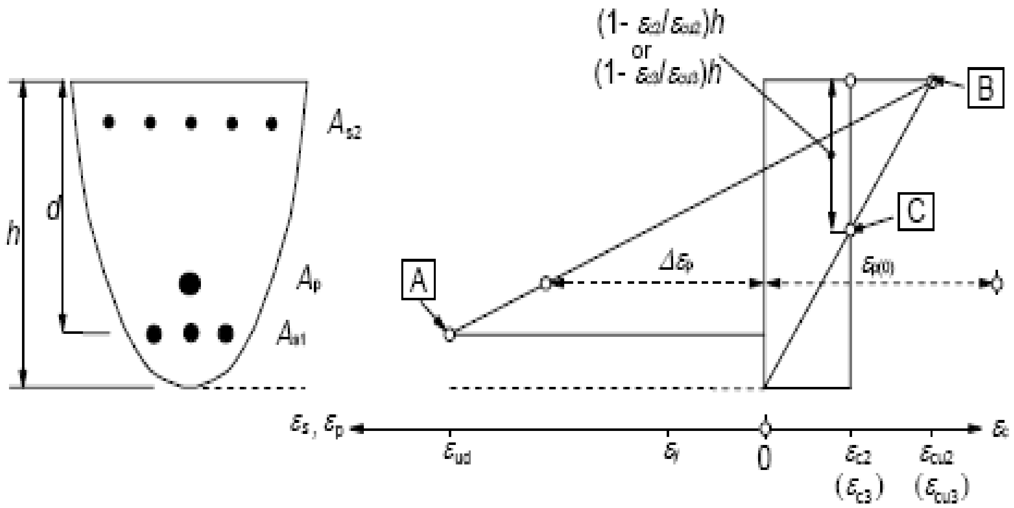


Fig. 1.8 strain diagram in the ultimate limit state

The Ethiopian code, EBCS 2,1995 sets the reinforcing steel tension strain limit (point A) and concrete compression strain limit (point B) to be 0.01 and 0.0035, respectively. The concrete compression strain limit (point C) is also 0.002 which has to be checked at 3h/7.

1.5 Moment curvature relationship

While ultimate strength calculations are of prime importance in practical design, the load-deformation characteristics of the column section also need to be considered when the behavior and load carrying capacity of a slender column are analyzed. A more complete picture of column behavior in the overload and failure range is provided by a family of moment – curvature curves, plotted for various constant values of applied trust P.

The bending moment acting alone in a column section produces tension at one face and compression at the other and these are called the tensile and compressive faces, even though the tension may be cancelled by compression due to the axial force.

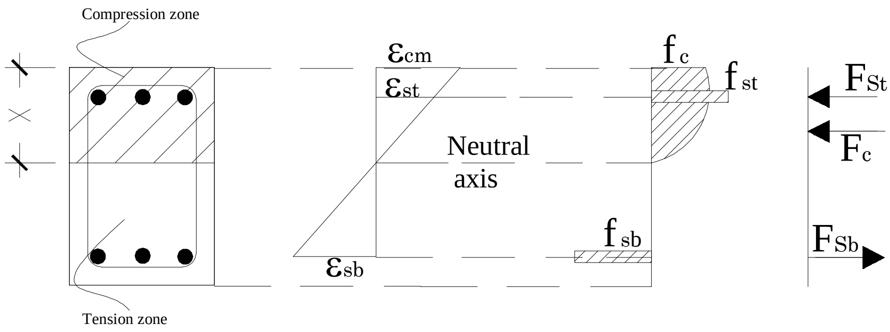


Fig. 1.9 Compression-Tension zones

The stress-strain relationship assumed for concrete in compression is shown previously in *fig.1.4.* This model consists of parabola from zero to the compressive strength of the concrete, f_{cd}, as long as the reinforcements remain to be safe. The strain that corresponds to the peak compressive stress, ε_o, is often assumed to be 0.002. The equation of this parabola is:

$$f_c=1000\varepsilon_c(250\varepsilon_c-1)f_{cd}\ldots\text{for } \varepsilon_c\leq 0.002\ldots.. \quad \textit{(Eq. 1.5)}$$

Consider a section subjected to axial load and positive bending. A_{st} represents area of reinforcement at the top of section, A_{sb} represents area of reinforcement at the bottom of section and *d* represents the effective flexural depth of the section. A complete moment-curvature relationship can be generated for this section by continuously increasing the section curvature (slope of the strain diagram) and using the assumed material stress-strain relationships to determine the resulting section stresses and forces. The major cases of stress and strain distributions are presented in *Fig. 1.10.*

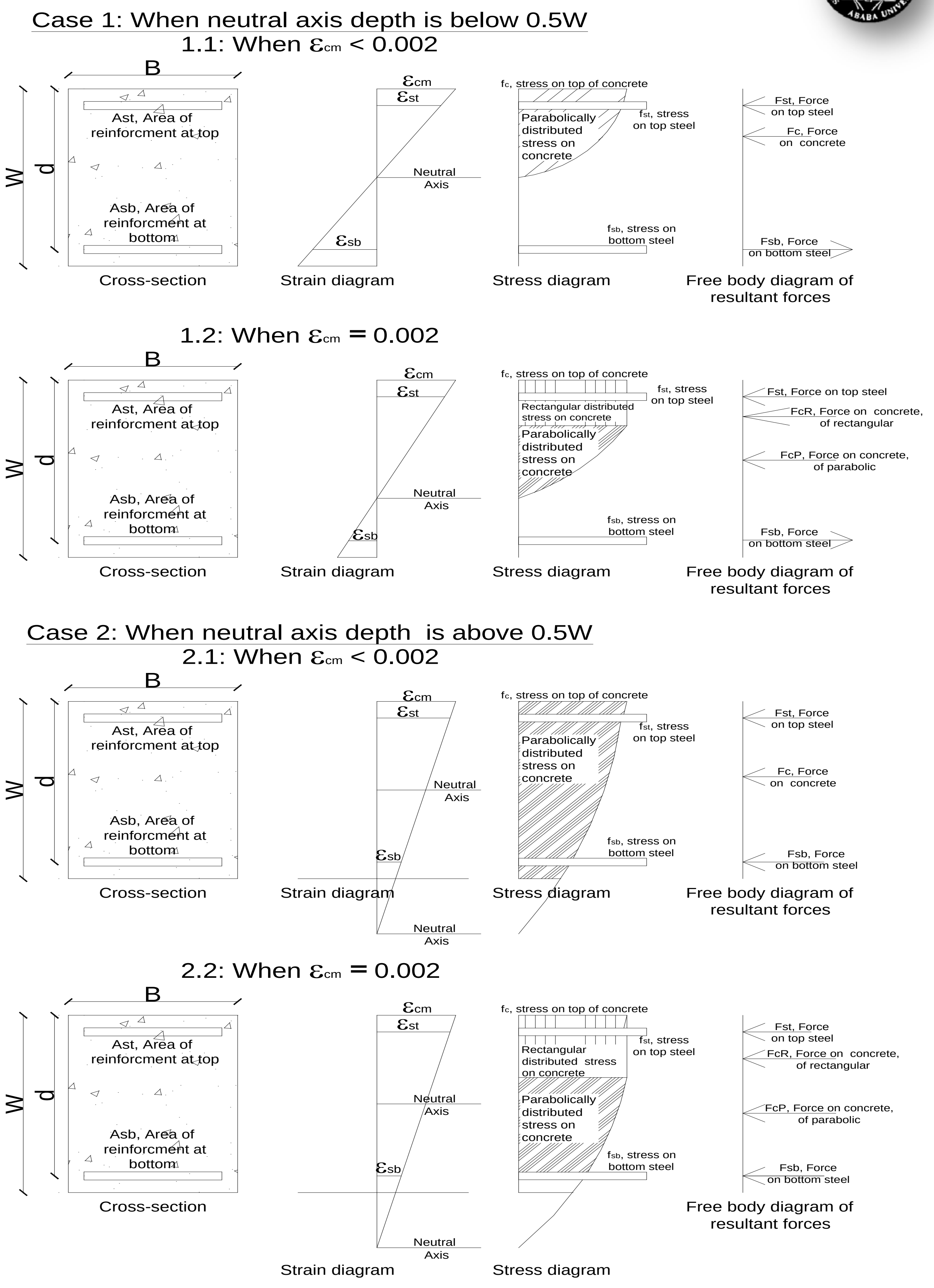


Fig. 1.10 Major cases for stress and strain distributions at failure point

The calculation of specific points on the moment-curvature curve follows the process represented in *Fig. 1.10*. Each point is usually determined by selecting a specific value for the maximum compression strain at the extreme compression fiber of the section, ε_{cm}. Then assumption should be given for the value of neutral axis depth from the extreme top of the section. From the assumption that plane sections before bending remain plane, the strain distribution through the depth of the section is linear. From the strain diagram and the assumed material stress-strain relationships, the distribution of stresses is determined. Finally, by integration, the volume under the stress distributions (i.e. the section forces) and their points of action can be determined. The magnitude of the forces and moment M can be obtained by the following ways.

- ✓ The compressive force from the concrete, F_c, can generally be calculated as:

  $$F_C=\int_{Area} f_c dA$$

- ✓ The point of application of this force, dc, can generally be calculated as:

  $$d_c=\frac{\int_{Area} X\, f_c dA}{\int_{Area} f_c dA}$$

- ✓ Then check equilibrium;

  $$F_c+F_{sb}+F_{sbottom}=P$$

Here, care should be taken to the sign of each force. Compression force can generally be taken as positive.

- ✓ The neutral axis depth should then be revised until the above equilibrium equation is satisfied.
- ✓ Once the exact neutral axis depth is determined, the moment can be taken at any of the locations inside the section. For the case of this paper, take the moment about the bottom fiber of the section.
- ✓ The curvature, K then can be calculated as:

$$K=\frac{\varepsilon_{cm}}{X}$$

The stress distribution of the concrete section is determined by the distribution of strain. As long as the strain in concrete at any point does not exceed 0.002, the stress distribution will remain purely parabolic, following an equation presented in E*q.1.5.* Otherwise, the stress distribution follows the parabolic curve up to the point where strain is 0.002; while the rest part of concrete will have rectangular distribution having a stress of f_{cd}.

The neutral axis depth, X, will determine the range of compression zone of the section under consideration. If the neutral axis depth is below the height of section, both tension and compression zones will exist in the section. The type of forces (compression or tension) exerted by the top and bottom reinforcements will be set accordingly. If the neutral axis depth exceeds the height of the section, the whole section will be in compression, and hence the concrete, the top reinforcement and bottom reinforcements will resist actions by its compression capacities.

For this paper, the moment curvature relationship task is committed by the help of computer programming using FORTRAN language as a supporting material of the research. Please refer the written program with its flow chart on Annex 3.

1.6 Non-linear second order analysis

Elastic behavior of the structural elements is the basis for the ordinary structural design process, which is the so-called first-order analysis. Here in this case it is necessary to verify both the resistance capacity to guarantee the structural safety and the serviceability to ensure the proper function during the working life.

Since the real structural behavior is often complex, the designers usually employ simplified analysis methodologies to verify the two requirements. Basically, these methodologies are based on a simple material structural behavior; most of the times a linear and elastic behavior is used, and the assumption that the structural deformation due to the internal efforts is irrelevant and that it does not influence those efforts. However, another way of analysis is needed for some structures that require a more detailed analysis to guarantee a truthful result. For such types of structures, the linear first-order analysis must be replaced by a more reliable structural analysis methodology that can simulate the proper material behavior and the influence of the structural deformation during the loading procedure.

The real material behavior can be simplified to obtain the desired constitutive law according to the analysis methodology that was selected to study the structural behavior. *Fig.1.10* illustrate some examples of simplified material models that can be obtained from a real constitutive law. As the figures show, it is evident that some material models are simpler than the others. For design purposes elastic and plastic analyses are often chosen because they are based on simple stress strain relationship.

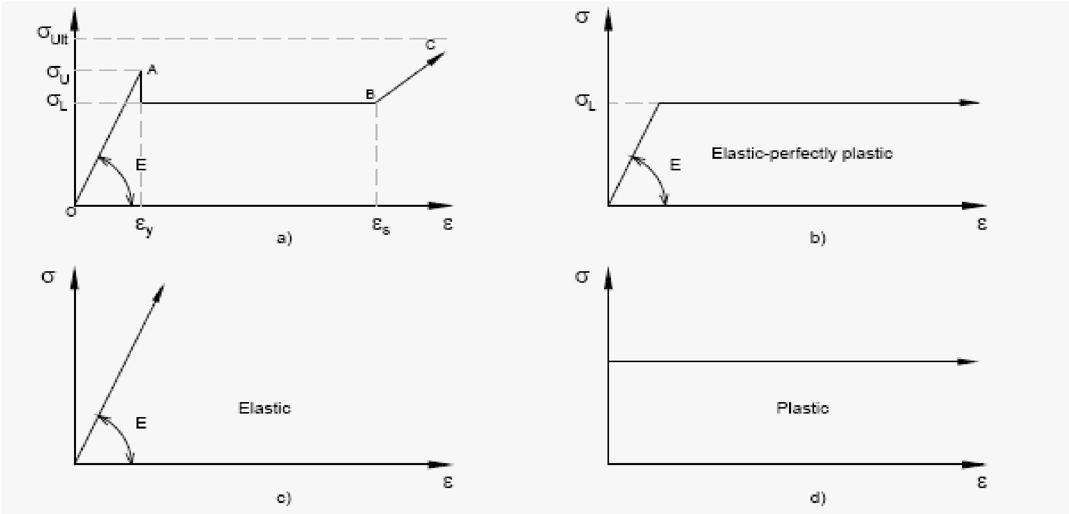


Fig. 1.11 Illustration for simplified stress-strain diagrams

Material nonlinearity is one of the most important sources of nonlinear structural behavior. Besides this, it is important to realize that this nonlinearity is often associated with a nonlinear geometric behavior, namely in slender structures because those can have a significant nonlinear response before reaching the resistant capacity as shown in *Fig.1.12.*

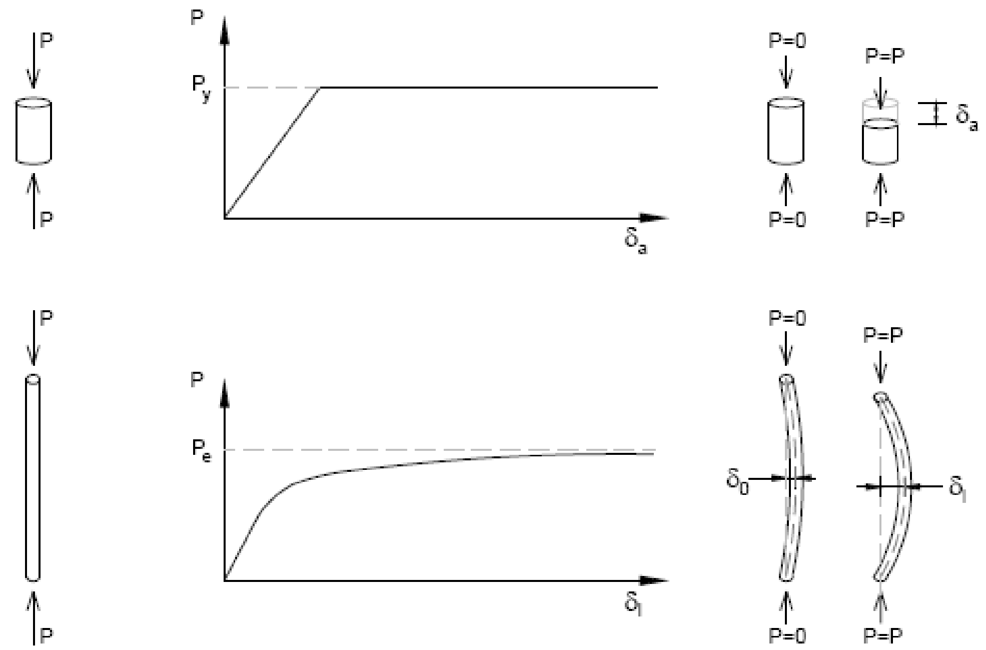


Fig. 1.12: Non-linear behavior of a column

If an elastic behavior is expected when a large loading is applied, then it is possible to exclude the material nonlinearity and in this case only a nonlinear geometric analysis can be done. In this case, the source of the nonlinearity is related with initial imperfections in the structural members, global deformation of the structure (P-Δ effect) or local deformation of the structural members (P-δ effect), as shown in *Fig. 1.13.*

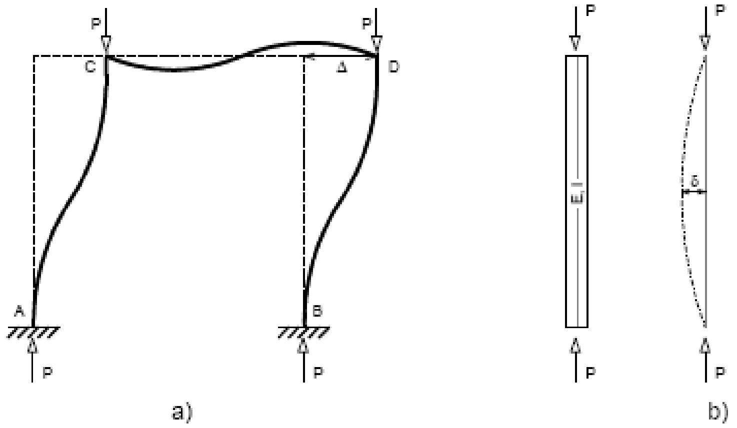


Fig.1:13: The two P-Delta effects: (a) P-Δ ; (b) P-δ

In real life, the structural behavior is always nonlinear and the simplified analysis is only valid for small stress levels and for specific configurations where the linear equilibrium is possible. However, there are structures in which it is impossible to apply these simplifications and their analysis can only be performed with a nonlinear methodology such as the structure shown in *Fig. 1.14.*

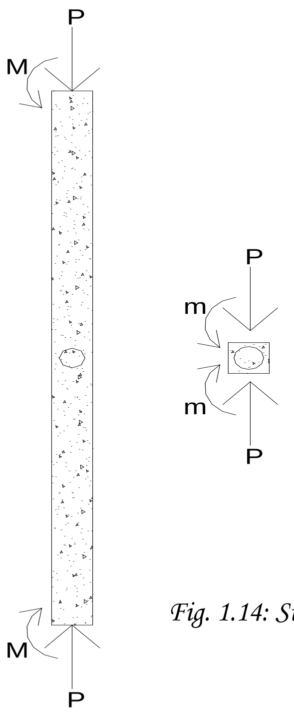


Fig. 1.14: Structural scheme for deformed Vs un-deformed equilibrium.

In this case is easy to verify that a first-order analysis (analysis performed with an un-deformed structural configuration) does not allow computing the axial effort in the structural members since the equilibrium is not possible at the centre node. The solution to these types of structural systems is obtained by performing a large-displacement analysis, i.e. a nonlinear geometric analysis, where the equilibrium is achieved in the deformed structural configuration as shown in *Fig. 1.15.*

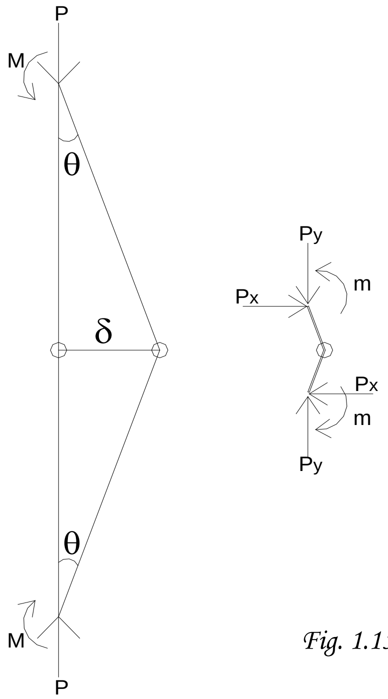


Fig. 1.15: Equilibrium in deformed structural configuration

Thus, a geometric nonlinear analysis is carried out when a structure undergoes large displacements and the change of its geometric shape causes a nonlinear displacement-strain relationship. In this case, the structure exhibits significant change of its shape under applied loads such that the resulting large displacements change the coordinates of the structure or additional loads induced. So, the equilibrium relationships are written with respect to the deformed structural shape and the analysis is referred as a second-order analysis. Unlike the first-order analysis, a second-order analysis requires an iterative procedure to obtain solutions since the deformed shape is not known during the equilibrium formulation.

Second order non linear analysis for slender columns can be made by making use of beam analogy, summarized as follows.

- Divide the column system in to a number of segments, to form nodes at ends of each segment.
- Construct bending moment diagram for the column system by making use of the initial end moments.
- Calculate the curvature values of moments at each node, K_i; by making use of the way discussed in section 1.5 of this document.
- Multiply the curvature value by tributary length of the node.
- Calculate the average curvature, which will be the approximated curvature at the given node. In this paper, the average curvature at the given node is calculated as:

$$K_{av} = \frac{K_{i-1} + 4K_i + K_{i+1}}{6}$$

Where K_i, K_{i-1} and K_{i+1} are curvature values at the given node, at the previous and the next consecutive nodes; respectively.

- Load the average curvature to the column model, as a concentrated force at respective nodes. (the loaded column can then be termed as conjugate system)

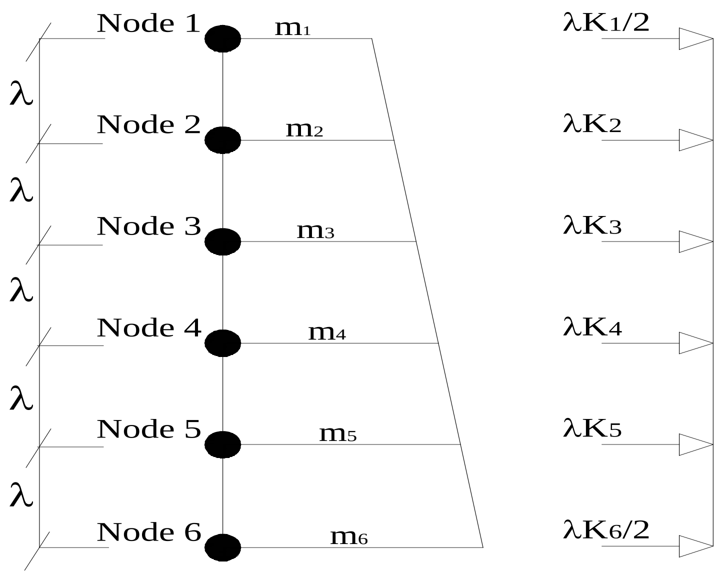


Fig. 1.16: Illustration of conjugate system

- Calculate the bending moment of the conjugate system. According to the beam analogy that is being used, the bending moment of the conjugate system is equal to the deflection of the actual system. As a result, deflection at each node can be known by this step.

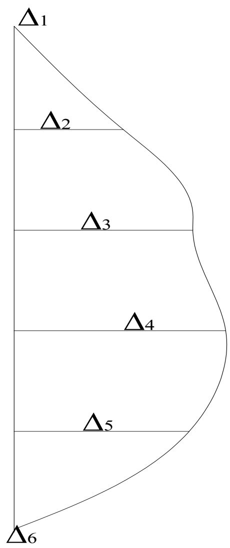


Fig. 1.17: Illustration of second order deflection

- Then calculate new moments at each nodes by using:

$$M_n = M_n{}^{-1} + P\Delta_n{}^{-1}$$

Where

- M_n is the moment at the node under consideration, of the n^{th} iteration.
- M_n^{-1} is the moment at the node under consideration, of the previous iteration.
- Δ_n^{-1} is the calculated deflection at the node under consideration, of the previous iteration

- Continue all the above steps until the change in moment values in consecutive iterations are negligible at each node. The moment values at each nodes of the last iteration will be taken as the second order moment value of that node.

For this study, non-linear second order analysis is committed by the help of linked cells of *Ms. Excel* spreadsheets as a supporting material of the research.

1.7 Buckling of axially loaded elastic columns

Buckling load in column is the limiting load which results the system to lose its stability. If the column load exceeds this value, the equilibrium will not be maintained at any section of the column, under the applied actions.

For axially loaded column in *Fig. 1.18*, if the column returns to its original position after it is displaced laterally and released, the condition is termed as in *stable equilibrium*. If it can't to return to its original position, it will be *unstable equilibrium*. The transition between stable and unstable equilibrium is neutral equilibrium.

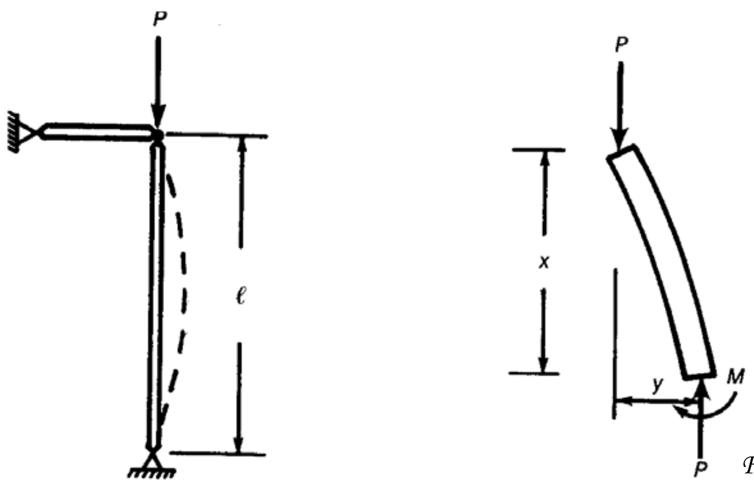


Fig. 1.18: Free-body of neutral equilibrium

Fig. 1.18 shows a portion of a column that is in state of neutral equilibrium. The differential equation for this column is:

$$EI\frac{d^2y}{dX^2} = -Py$$

Or

$$EI\frac{d^2Y}{dx^2}+Py=0$$

1.7.1 Buckling load for prismatic columns

The solution of the *Eq. 1.6* will be observed as follows, for prismatic column system. According to Leonhard Euler method of solution, this is a second order differential equation has the general solution of the form:

$$Y = A\sin\sqrt{\frac{P}{EI}}\,x + B\cos\sqrt{\frac{P}{EI}}\,x$$

Applying the following boundary conditions; the deflection of the column must be zero at each end since it is pinned.

a. At x = 0, y = 0

Here it is noted that B must be zero since COS (0) = 1.

b. At x = L, y = 0.

This implies that either A must be zero (which leaves us with no equation at all) or that:

$$\sin\sqrt{\frac{P}{EI}}\,L = 0$$

Noting that SIN (π) = 0, we can solve for P:

$$\sqrt{\frac{P}{EI}}\,L = \pi$$

$$P_{cr}=\frac{\pi^2 EI}{L^2}$$

: Which is the so called Euler's buckling load.

Or in general

$$P_{cr}=\frac{n^2\pi^2 EI}{L^2}$$

For n=1, 2, 3...etc, which indicates the mode of curvature.

/ Recall that an assumption of P to be constant along the length of column is proofed here in the above expression of P_{cr}./

Another way of looking at this involves the concept of the effective length of the column. The effective length is the length of a pin ended column having the same buckling load. The effective length kL, is equal to L/n. the effective length factor is k=1/n. equation 1.7 can now be written as:

$$P_{cr}=\frac{\pi^2 EI}{(kL)^2} \quad \text{..... (Eq. 1.8)}$$

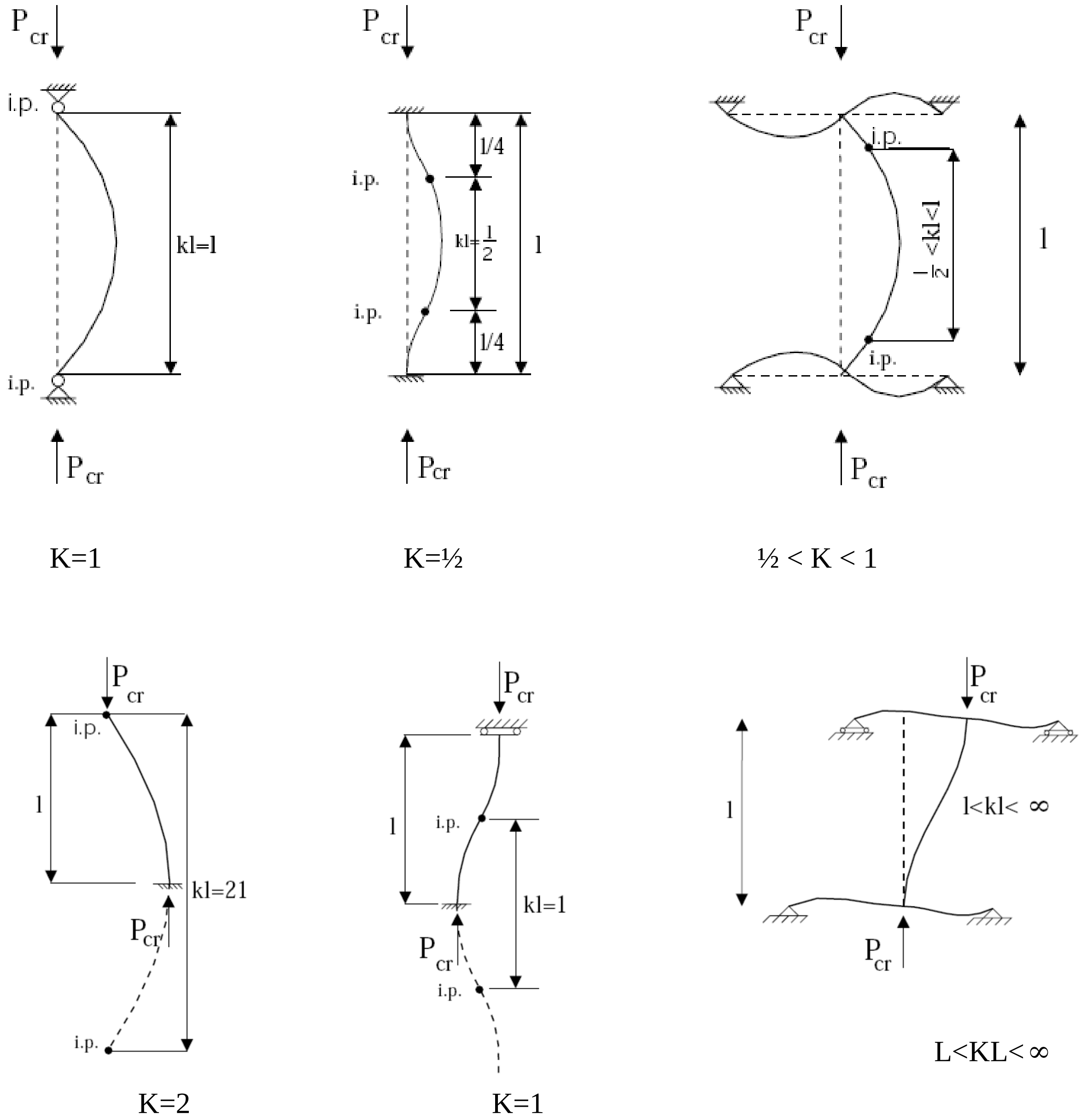


Fig. 1.19: effective length for various column systems

1.7.2 Buckling load for Non-prismatic (tapered) columns

Early studies of the effect of varying cross-sections on buckling loads were done by *Dinnik* and by *Bairstow & Sedman*. *Dinnik* presented a table for columns which tapered symmetrical about a central prismatic section. He based his analytical calculations on the assumption of various power laws for the stiffness in the end portions of the columns.

Bairstow and Stedman presented a numerical method of solving for buckling loads in columns with varying cross-sections. This method was later improved by *Ratzersdorfer*. *Vianello* also presented a graphical method of solution for buckling loads, and *Morley* introduced numerical integration into *Vianello*. *Stodola* (1906) applied a method of successive approximations similar to *Vianello's* to the rotation of turbine shafts where it is called *Stodola's method*.

A remarkable method of solution for tapered columns is provided by *Nathan M. Newmark* who introduced a number of refinements to the early process and summarized the entire procedure. In 1943, *Newmark* published a paper in the Transactions of the American Society of Civil Engineers (ASCE) entitled *"Numerical Procedure for Computing Deflections, Moments and Buckling Loads."* This paper thoroughly outlined a procedure by which the value of a function of a single variable is calculated so long as the second derivative of the function is known. The procedure is an adaptation of the general mathematical method of successive approximations. *Newmark* expounded on this work with a book devoted to numerical solution techniques and published in honor *Hardy Cross*, father of numerical methods in engineering in 1949. *Newmark's* unique contribution to the field was in his treatment of specific problems and generalization of the procedure to a wide range of problems while providing for increased accuracy.

Lately, *Nakahira et al (1992)* used what they called the *Stodola-Newmark* method (a combination of the *Newmark* numerical method and *Stodola's* iterative process) to determine the natural frequencies of cantilever beams of varying cross-section.

A thesis submitted in partial fulfillment of the requirement for the degree of Master of Science in mechanical engineering from Naval post Graduate School is used in this research. The thesis was done by *Thomas A. Branch, Jr. Lieutenant in 1960*. The researcher used the *Vaianello* iteration method refined by *Newmark*. An extensive table of buckling factor is compiled with the aid of digital computer. Table in *Annex 1* is taken from the above mentioned thesis paper which is specifically done for tapered non-symmetric columns having linearly varying cross-section.

CHAPTER 2

INTRODUCTION TO THE RESEARCH

2.1 Background of Effective length method

In general, the effective length of a column, KL, is defined as the length of an equivalent pin-ended column having the same buckling load to an arbitrary given column. The parameter K is termed as effective length factor. Thus the column in fig.2.1(c) has the same buckling load as that in fig. fig.2.1 (b). Consider prismatic columns in fig.2.1. The effective length of the column is L/2 in this case, where L/2 is the length of each of the half sine waves in the deflected shape of the column in fig. 2.1(b). The effective length, KL, is equal to L/n. the effective length factor is K=1/n. the buckling load equation, then, becomes

$$P_c = \frac{\pi^2 EI}{(kl)^2} \ldots\ldots (\text{Eq. } 2.1)$$

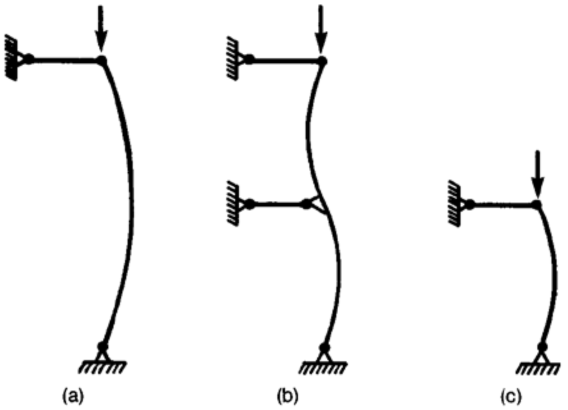


Fig. 2.1: Illustration of effective length of column

The second order design moment of the original and the equivalent column system having a length of KL is therefore expected to be the same, as both have the same elastic buckling load. *Eq. 2.1* is valid only for columns having constant cross-section throughout the length. As a result the equation is valid for the equivalent columns of samples considered in this paper. For the actual tapered type of column, the elastic buckling load can be determined by making use of analytical and numerical methods, as per the discussion in section 1.5.2. For our case, we are using the thesis results by Thomas A. Branch; 1960, in accordance with *Newark's* refinement. Please refer section 1.5.2 and *Appendix A.1* for the methodology and result table that is used in the investigation process of this paper.

2.2 Objective of the research

The cross-sectional dimensions of a column are generally considerably less than its height. Columns usually have constant cross-section, which is determined to resist the maximum factored loads. The usual trend in use of constant cross-section emerges from various reasons. Firstly, construction of column structures having constant section is relatively easier and less costly. On the other side, analysis of non-prismatic sectioned columns is cumbersome.

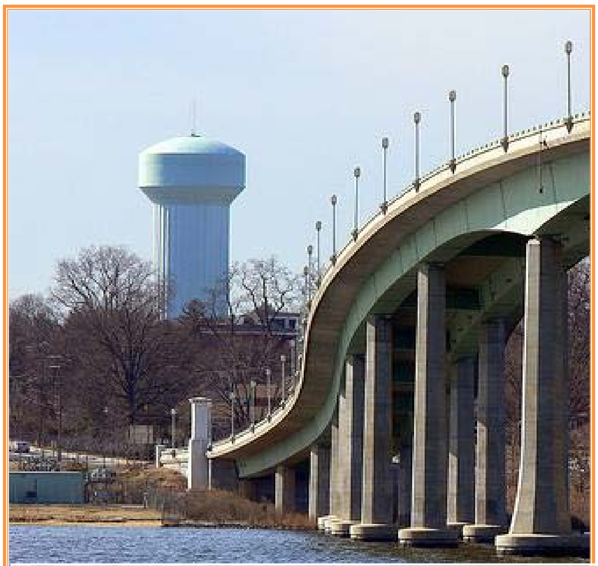


Fig. 2.2: Tapered column in Bridges

In-spite of the above reasons, some circumstances might compel designers to use columns having varying cross-section. For instance; in bridge structures, section of columns are usually smaller at the base level and increases with the height, when larger section is required at deck-column junction. In some instances the cross-section might become smaller along the height of pillars, due to another reason. Likewise, reinforced concrete columns for the support of elevated water tankers often designed to have varying cross-section with its height. In addition, tapered columns do have meaningful architectural value in the building design profession. These types of columns are being common at the entrance of artistic buildings. Structurally, columns with varying cross-section are often preferred when the prismatic column is inadequate for the actions on it. The varying cross-section column is used in many applications in order to decrease weight or provide additional strength.

Fig. 2.3: Tapered column in the American traditional building architecture

Many engineering calculations and computer programs on column structures have two basic limitations. They are based on first-order analysis and used for prismatic cross-section. However there are some approaches for the consideration of second order effects for slender columns. For example, Ethiopian building code standard-2 (EBCS-2) provides a simplified methodology for isolated columns having constant cross-section. Effective length method consists of calculation of buckling load and second order moments by taking the actual cross-section of the columns and effective length. However, this method cannot be used for tapered columns, since the latter do not have constant cross-section.

Fig. 2.4: Tapered column used in Addis Ababa railway systems

Fig. 2.5: Tapered column used in head office of Awash international bank, Addis Ababa

Fig. 2.6: Tapered column used in pedestrians' pass, Addis Ababa

Lack in applicability of most programs, codal provisions and computations on tapered slender columns could here be taken as the problem, which in turn initiate me for this research. Rather than providing unique and rigorous calculation procedure for the analysis of non prismatic columns, it is better to use the same method of prismatic sections, but with the presence of some modifications and rules.

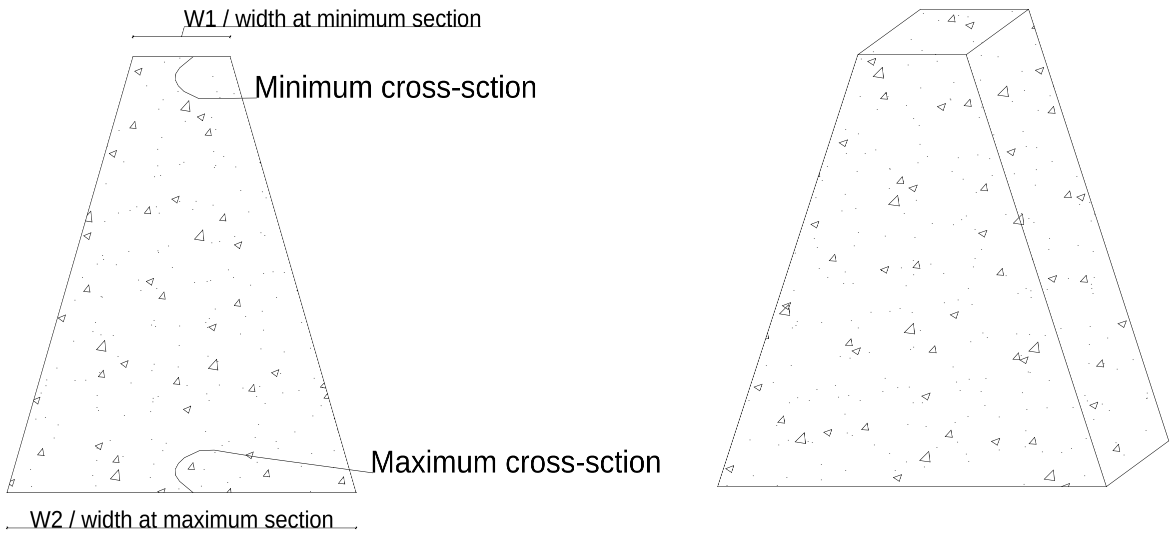
- *General Objective*
    - Investigation of simplified effective length method of analysis for tapered columns is the goal of this research; so as to approximate the second-order effect and elastic buckling load of the system by the result of a modified, first order analysis of

prismatic section. Column with equal end moment will be studied. The output of the research will help designers to represent tapered columns by equivalent column structure having prismatic section.

- *Specific Objectives*
  - General approach for the determination of the equivalent prismatic section will be evaluated.
  - The effective length factor will be determined for the equivalent column.
  - Degree of safety of the methodology will also be studied.

2.3 Typical column system under study

A typical section of column system, to be studied is as illustrated in fig. 2.7. It has to be noted that the minimum section should always be positioned at the top while the maximum section at the bottom. As a result; whenever the reverse condition is desired to be treated by the result of this paper, the column should first be arranged as per the typical column system, illustrated in *Fig. 2.7.*



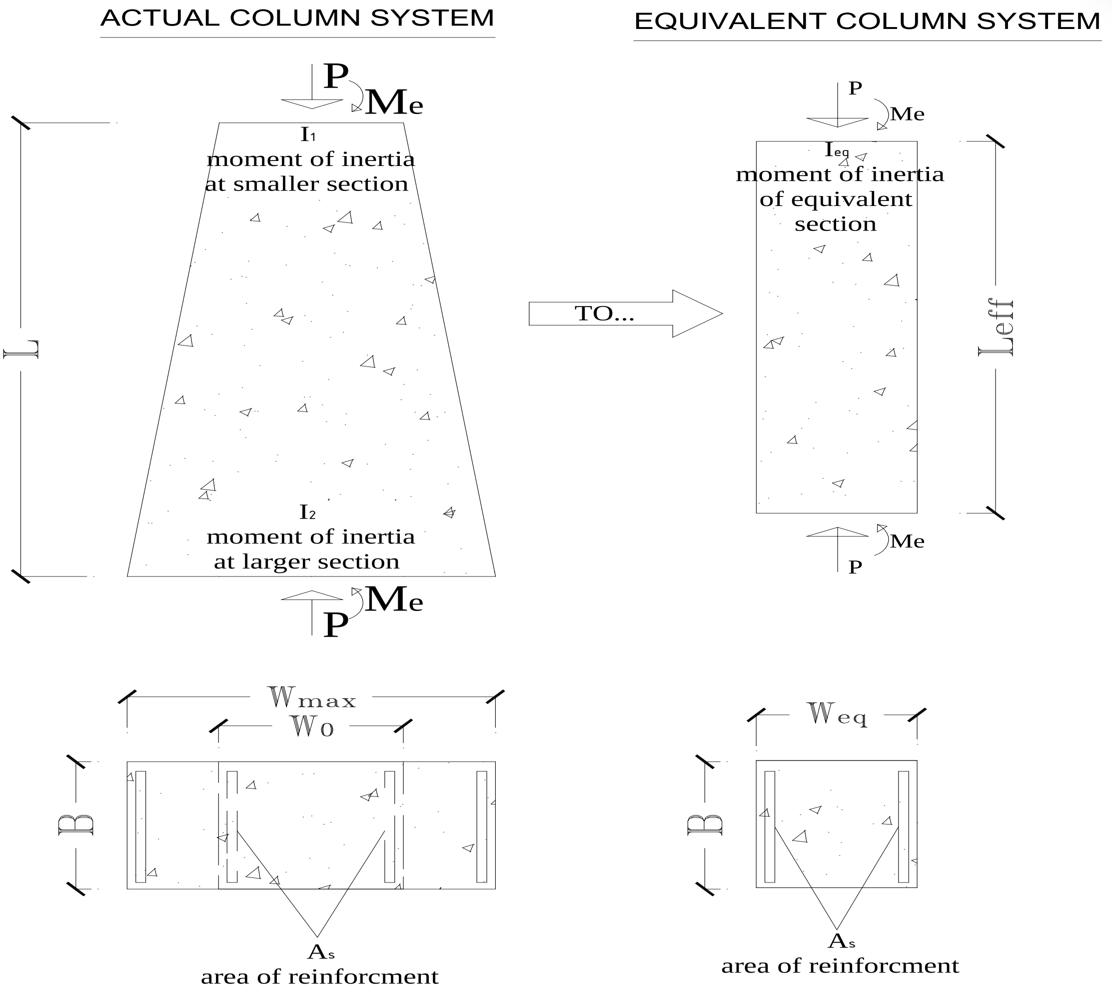


Fig. 2.7: Typical column section under study

2.4 Methodology of investigation

The methodology of investigation is summarized in the following "9 stepped procedure"

Step I

Construction of a program for moment-curvature relationship

As per the procedures indicated in *section 1.5*, a *FORTRAN* program should be prepared for the relation of curvatures for the respective moment values. Here, it is important to note that all possible material failure cases are set on the program. However, checking stability failure of the system will be checked in the following steps.

Step II

Construction of a program for Second order analysis.

As per the procedures indicated in *section 1.6*, a computer program should be prepared for the iterative analysis of second order moment. A linked *Ms. EXCEL* spread sheets are to be used in this research.

Step III

Selection of samples for analysis

Select samples for analysis. The samples should cover the variety of:

- Length of column
- Cross-section of column
- Concrete and reinforcement steel classes
- Loadings
- Reinforcement ratio
- Side slopes and inertial ratio

Step IV

Preparation of moment-curvature relationship data for nodes under each sample

Here the moment vs. curvature values for each node of every sample will be prepared to make the next step easier. The number of nodes should be decided before running the computer program of moment –curvature relationship, indicated in step I.

- Divide the column length into a number of finite segments having length of λ.
- Record the cross-section of every node, accordingly.
- Open the FORTRAN program indicated in step I. Run the program for each node by feeding all the necessary inputs
- Feed the results of the relationship to the datasheet of second order analysis program.

Step V

Non linear second order analysis

As per the detailed illustration in *section 1.6* and using computer program in *step 2*, the non-linear second order analysis will be done according to the following steps.

- Choose the end moment of the column, in such a way that ultimate loading is satisfied. In order to choose the end moment either of the following conditions govern:
  - The material, either concrete or the reinforcement, reaches the ultimate strength.
  - The strain diagram of the section passes through point C. /refer fig. 1.8/

  - Pre-mature stability failure before the materials reaches its ultimate capacity.
- Construct bending moment diagram for the column system by making use of the initial end moments.
- Follow all procedures of *section 1.6* to commit the needed iterations for the determination of the second order moments and deflections.

Step VI

Selection of equivalent prismatic section from the

criteria of second order analysis

- Observe the second order bending moment diagram of the actual column and its failure node. Select the cross-section of the column at failure node as *"The equivalent prismatic section of the column"*.

Step VII

Calculation of elastic buckling load of actual tapered column

- As indicated in section 1.7.2 and Appendix A.1, the extensive tables of buckling load factors are prepared for columns having different parameters. According to the parameters of the sample column, elastic buckling load of the sample column should be calculated based on the procedural indication of the author.

Step VIII

Calculation of effective length factors from the criteria of buckling load

As indicated in step VII, the buckling load of the actual column, $P_{cr,act}$, is already determined. Similarly the buckling load of the equivalent column can be calculated by using Euler's equation:

$$P_{cr}=\frac{\pi^2 EI}{(kL)^2} \ldots\ldots (Eq. 2.1)$$

Since the buckling load of the actual and equivalent columns must be equal, eq.2.1 will be rewritten as:

$$P_{cr,act}=\frac{\pi^2 EI}{(kL)^2}$$

Or the effective length factor can be calculated as:

$$K=\frac{\pi}{L}\sqrt{\frac{EI}{P_{cr,act}}} \ldots\ldots (\text{Eq. 2.2})$$

- The effective length factor, K will then multiplied with the actual length of column, and the result will be set as *"the effective length of the equivalent column"*.

Step IX

Evaluation of degree of safety of the methodology

- Using the selected equivalent section and effective length, make second order analysis using an iterative approach like what is done in the tapered column, previously.
- Evaluate and figure out the degree of safety of the methodology by comparing the capacity of the actual column with the equivalent one. This will be done by evaluating how much additional end moment the equivalent column can carry when it is compared to the actual column.
- The ratio of end moment capacity of equivalent column to the actual column will finally be presented as the safety factor of the methodology (which indeed indicates the degree of safety).

2.5 Scope of the research

Tapered columns can vary to each other on many bases. According to the member end restraint conditions, the columns can be pin-ended, fixed-ended, free-restraint or any combinations of the above. The columns could also be uni-axially loaded or bi-axially loaded according to the axis of application of applied moments. Likewise, the direction of end moments, curvature of deflected member and the ratio of end moments can vary from column to column. Accordingly, columns will structurally behave differently from one to the other on the basis of all the above mentioned parameters.

The column systems to be studied in this paper do have the following unique properties. Accordingly, the output of this research is applicable only those systems which fit the below stated properties.

i. Reinforced concrete column, having linearly varying cross-section
ii. Uni-axial column having end moments in the major axis
iii. Equal moments at the top and bottom end of column
iv. Columns under single curvature

CHAPTER 3

SAMPLE STUDY

3.1 Sampling

Varity of samples are taken according to the needed requirement. A total of 237 samples are selected carefully to have a variety of:

- Length of column
- Cross-section of column
- Concrete and reinforcement steel classes
- Loadings, and
- Reinforcement ratio
- Side slopes and inertial ratio

From these samples, it is planned to get the following general behavioral information about tapered columns;

1. The general location of maximum second order moment
2. Effect of material quality on the location of second order moment
3. Effect of steel ratio on the location of second order moment
4. Effect of side slope of column on the location of second order moment
5. Mode of increment in section capacity along the column height
6. Effect of increment in axial load on the section capacity

Columns having constant cross-section along the column height, generally do have maximum bending moment at the middle of the column system. The middle of the column, hence, is considered to be taken as a failure section (or in general the design section) in which the section and moment for design purpose are located. However, it is generally believed that different tapered columns fail at different locations along the column height. This is true since not only that the maximum moment is located at a different location than the middle of the column, but also moment carrying capacity of sections are variable along the column height. Sampled column systems also are prepared to study the failure point of different tapered columns having linearly varying cross-section. The following two geometrical and loading parameters are here introduced to categorize tapered columns having similar failure points.

a. $\frac{I_1}{I_2}$

b. $\sqrt{\frac{P}{f_{cd}A_{min}}}$

Where, the first parameter is the ratio of moment of inertia of concrete section at minimum section to maximum section, about the major bending axis. In the second parameter; P is the axial load on the actual column, f_{cd} is the design compressive strength of concrete and A_{min} is the gross area of concrete at the minimum section. The parameters are decided to be governing ones after a very cumbersome trials made in this research.

In addition to the six important goals indicated in the previous page, it is planned to get the following additional four vital results and information for the investigation of effective length method for columns having linearly varying cross-section;

7. The parameters which have a power to affect the location of failure section of tapered column systems.
8. Way of categorization of tapered column systems, which have similar point of failure.
9. A general methodology to investigate the constant cross-section of equivalent column
10. Effective length for the equivalent column system and degree of safety of the methodology.

The last four above mentioned points will be the milestone of effective length method under investigation, in a nutshell. Accordingly, these results can be taken as the output of the research. The summary of all samples, with appropriate parameters are indicated in tables 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6. All designations indicated in the tables are as per *fig.2.7* of section 2.3.

Sample No.	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
1	0.600	1.292	0.400	1000.00	429.0	30.0	13.60	0.1	0.553
2	0.600	1.292	0.400	1000.00	500.0	25.0	13.60	0.1	0.553
3	0.600	1.292	0.400	1000.00	528.0	22.0	13.60	0.1	0.553
4	0.600	1.292	0.400	1000.00	550.0	19.0	13.60	0.1	0.553
5	0.600	1.292	0.400	1000.00	171.0	30.0	13.60	0.1	0.553
6	0.600	1.292	0.400	1000.00	237.2	25.0	13.60	0.1	0.553
7	0.600	1.292	0.400	1000.00	270.5	22.0	13.60	0.1	0.553
8	0.600	1.292	0.400	1000.00	297.9	19.0	13.60	0.1	0.553
9	0.600	1.292	0.400	1000.00	315.4	16.0	13.60	0.1	0.553
10	0.400	0.862	0.250	347.140	64.3	28.6	11.33	0.1	0.553
11	0.400	0.862	0.250	347.140	96.8	24.3	11.33	0.1	0.553
12	0.400	0.862	0.250	347.140	113.9	21.8	11.33	0.1	0.553
13	0.400	0.862	0.250	347.140	128.8	19.3	11.33	0.1	0.553
14	0.400	0.862	0.250	347.140	139.0	16.6	11.33	0.1	0.553
15	0.400	0.862	0.250	347.140	144.94	14.1	11.33	0.1	0.553
16	0.300	0.448	0.300	101.970	30.9	20.0	11.33	0.3	0.320
17	0.300	0.448	0.300	101.970	38.0	17.0	11.33	0.3	0.320
18	0.300	0.448	0.300	101.970	42.5	14.0	11.33	0.3	0.320
19	0.300	0.448	0.300	101.970	48.2	11.0	11.33	0.3	0.320
20	0.300	0.448	0.300	101.970	50.4	8.7	11.33	0.3	0.320
21	0.300	0.448	0.300	101.970	51.4	6.3	11.33	0.3	0.320
22	0.600	0.896	0.400	326.400	105.9	30.0	13.60	0.3	0.320
23	0.600	0.896	0.400	326.400	132.2	25.5	13.60	0.3	0.320
24	0.600	0.896	0.400	326.400	155.0	21.0	13.60	0.3	0.320
25	0.600	0.896	0.400	326.400	173.5	16.4	13.60	0.3	0.320
26	0.600	0.896	0.400	326.400	183.6	13.0	13.60	0.3	0.320
27	0.600	0.896	0.400	326.400	191.44	8.99	13.60	0.3	0.320

Table 3.1: summary of samples used for investigation, I_1/I_2 =0.1 & 0.3

Sample No.	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
28	0.300	0.646	0.250	25.500	2.50	85.000	13.60	0.1	0.16
29	0.300	0.646	0.250	25.500	34.00	65.000	13.60	0.1	0.16
30	0.300	0.646	0.250	25.500	46.67	55.000	13.60	0.1	0.16
31	0.300	0.646	0.250	25.500	52.77	49.000	13.60	0.1	0.16
32	0.300	0.646	0.250	25.500	57.18	43.000	13.60	0.1	0.16
33	0.300	0.646	0.250	25.500	59.16	39.000	13.60	0.1	0.16
34	0.300	0.646	0.250	25.500	61.38	31.000	13.60	0.1	0.16
35	0.700	1.508	0.500	396.550	28.00	95.000	11.33	0.1	0.32
36	0.700	1.508	0.500	396.550	59.00	80.000	11.33	0.1	0.32
37	0.700	1.508	0.500	396.550	251.90	45.000	11.33	0.1	0.32
38	0.700	1.508	0.500	396.550	289.35	39.000	11.33	0.1	0.32
39	0.700	1.508	0.500	396.550	308.50	35.000	11.33	0.1	0.32
40	0.700	1.508	0.500	396.550	326.00	30.000	11.33	0.1	0.32
41	0.700	1.508	0.500	396.550	343.20	23.420	11.33	0.1	0.32
42	0.800	1.724	0.500	1224.00	3.00	90.000	13.60	0.1	0.48
43	0.800	1.724	0.500	1224.00	58.00	80.000	13.60	0.1	0.48
44	0.800	1.724	0.500	1224.00	148.00	65.000	13.60	0.1	0.48
45	0.800	1.724	0.500	1224.00	450.00	40.000	13.60	0.1	0.48
46	0.800	1.724	0.500	1224.00	539.59	34.000	13.60	0.1	0.48
47	0.800	1.724	0.500	1224.00	586.00	30.000	13.60	0.1	0.48
48	0.800	1.724	0.500	1224.00	616.00	27.000	13.60	0.1	0.48
49	0.800	1.724	0.500	1224.00	654.18	21.413	13.60	0.1	0.48
50	0.700	1.508	0.700	2220.68	1.00	54.000	11.33	0.1	0.64
51	0.700	1.508	0.700	2220.68	39.00	49.000	11.33	0.1	0.64
52	0.700	1.508	0.700	2220.68	203.00	40.000	11.33	0.1	0.64
53	0.700	1.508	0.700	2220.68	348.00	31.000	11.33	0.1	0.64
54	0.700	1.508	0.700	2220.68	478.00	24.000	11.33	0.1	0.64
55	0.700	1.508	0.700	2220.68	515.00	21.500	11.33	0.1	0.64
56	0.700	1.508	0.700	2220.68	552.29	18.000	11.33	0.1	0.64
57	0.700	1.508	0.450	2230.594	1.00	43.000	11.33	0.1	0.80
58	0.700	1.508	0.450	2230.594	38.00	40.000	11.33	0.1	0.80
59	0.700	1.508	0.450	2230.594	145.00	35.000	11.33	0.1	0.80
60	0.700	1.508	0.450	2230.594	270.00	28.000	11.33	0.1	0.80
61	0.700	1.508	0.450	2230.594	320.00	25.000	11.33	0.1	0.80
62	0.700	1.508	0.450	2230.594	370.00	22.000	11.33	0.1	0.80
63	0.700	1.508	0.450	2230.594	394.66	20.000	11.33	0.1	0.80

Table 3.2: summary of samples used for investigation, I_1/I_2 =0.1

Sample No.	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
64	1.000	1.710	0.800	272.000	526.55	85.000	13.60	0.200	0.16
65	1.000	1.710	0.800	272.000	620.85	75.000	13.60	0.200	0.16
66	1.000	1.710	0.800	272.000	652.50	65.000	13.60	0.200	0.16
67	1.000	1.710	0.800	272.000	683.00	58.000	13.60	0.200	0.16
68	1.000	1.710	0.800	272.000	700.00	51.500	13.60	0.200	0.16
69	1.000	1.710	0.800	272.000	710.00	48.500	13.60	0.200	0.16
70	1.000	1.710	0.800	272.000	734.59	38.500	13.60	0.200	0.16
71	0.800	1.368	0.500	544.000	61.00	80.000	13.60	0.200	0.32
72	0.800	1.368	0.500	544.000	84.00	70.000	13.60	0.200	0.32
73	0.800	1.368	0.500	544.000	225.50	38.000	13.60	0.200	0.32
74	0.800	1.368	0.500	544.000	275.20	29.000	13.60	0.200	0.32
75	0.800	1.368	0.500	544.000	293.80	24.500	13.60	0.200	0.32
76	0.800	1.368	0.500	544.000	299.00	23.000	13.60	0.200	0.32
77	0.800	1.368	0.500	544.000	309.50	20.000	13.60	0.200	0.32
78	0.800	1.368	0.500	544.000	323.79	15.080	13.60	0.200	0.32
79	0.300	0.513	0.250	305.994	1.00	35.000	18.13	0.200	0.48
80	0.300	0.513	0.250	305.994	27.00	20.000	18.13	0.200	0.48
81	0.300	0.513	0.250	305.994	81.60	14.000	18.13	0.200	0.48
82	0.300	0.513	0.250	305.994	92.20	12.300	18.13	0.200	0.48
83	0.300	0.513	0.250	305.994	98.90	11.000	18.13	0.200	0.48
84	0.300	0.513	0.250	305.994	105.00	9.500	18.13	0.200	0.48
85	0.300	0.513	0.250	305.994	107.00	8.800	18.13	0.200	0.48
86	0.300	0.513	0.250	305.994	110.91	7.390	18.13	0.200	0.48
87	0.900	1.539	0.500	2856.060	259.00	47.000	15.87	0.200	0.64
88	0.900	1.539	0.500	2856.060	490.00	37.500	15.87	0.200	0.64
89	0.900	1.539	0.500	2856.060	578.00	34.000	15.87	0.200	0.64
90	0.900	1.539	0.500	2856.060	1000.00	22.500	15.87	0.200	0.64
91	0.900	1.539	0.500	2856.060	1052.00	20.500	15.87	0.200	0.64
92	0.900	1.539	0.500	2856.060	1080.00	19.000	15.87	0.200	0.64
93	0.900	1.539	0.500	2856.060	1120.13	16.570	15.87	0.200	0.64
94	0.200	0.342	0.200	283.250	1.00	08.3	11.33	0.200	0.80
95	0.200	0.342	0.200	283.250	10.00	07.0	11.33	0.200	0.80
96	0.200	0.342	0.200	283.250	20.90	06.2	11.33	0.200	0.80
97	0.200	0.342	0.200	283.250	26.10	05.6	11.33	0.200	0.80
98	0.200	0.342	0.200	283.250	28.00	05.1	11.33	0.200	0.80
99	0.200	0.342	0.200	283.250	28.30	04.8	11.33	0.200	0.80
100	0.200	0.342	0.200	283.250	29.24	04.8	11.33	0.200	0.80

Table 3.3: summary of samples used for investigation, I_1/I_2 =0.2

Sample No.	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
101	0.450	0.672	0.250	25.500	5.00	79.000	09.07	0.300	0.16
102	0.450	0.672	0.250	25.500	33.60	65.000	09.07	0.300	0.16
103	0.450	0.672	0.250	25.500	50.10	50.000	09.07	0.300	0.16
104	0.450	0.672	0.250	25.500	57.79	41.000	09.07	0.300	0.16
105	0.450	0.672	0.250	25.500	59.82	38.000	09.07	0.300	0.16
106	0.450	0.672	0.250	25.500	61.60	35.000	09.07	0.300	0.16
107	0.450	0.672	0.250	25.500	63.90	30.000	09.07	0.300	0.16
108	0.450	0.672	0.250	25.500	65.00	27.000	09.07	0.300	0.16
109	0.450	0.672	0.250	25.500	66.61	18.630	09.07	0.300	0.16
110	0.300	0.448	0.300	101.97	3.00	35.000	11.33	0.300	0.32
111	0.300	0.448	0.300	101.97	6.50	29.000	11.33	0.300	0.32
112	0.300	0.448	0.300	101.97	36.03	18.000	11.33	0.300	0.32
113	0.300	0.448	0.300	101.97	42.35	15.000	11.33	0.300	0.32
114	0.300	0.448	0.300	101.97	45.75	13.000	11.33	0.300	0.32
115	0.300	0.448	0.300	101.97	48.40	11.000	11.33	0.300	0.32
116	0.300	0.448	0.300	101.97	49.35	10.100	11.33	0.300	0.32
117	0.300	0.448	0.300	101.97	49.90	9.500	11.33	0.300	0.32
118	0.300	0.448	0.300	101.97	51.65	6.560	11.33	0.300	0.32
119	0.700	1.046	0.600	1070.685	97.00	43.000	11.33	0.300	0.48
120	0.700	1.046	0.600	1070.685	391.50	22.000	11.33	0.300	0.48
121	0.700	1.046	0.600	1070.685	470.00	17.000	11.33	0.300	0.48
122	0.700	1.046	0.600	1070.685	495.00	15.000	11.33	0.300	0.48
123	0.700	1.046	0.600	1070.685	512.00	13.500	11.33	0.300	0.48
124	0.700	1.046	0.600	1070.685	520.00	12.800	11.33	0.300	0.48
125	0.700	1.046	0.600	1070.685	527.00	12.000	11.33	0.300	0.48
126	0.700	1.046	0.600	1070.685	543.55	9.870	11.33	0.300	0.48
127	0.500	0.747	0.500	1360.000	1.00	30.000	13.60	0.300	0.64
128	0.500	0.747	0.500	1360.000	45.00	25.000	13.60	0.300	0.64
129	0.500	0.747	0.500	1360.000	162.00	16.000	13.60	0.300	0.64
130	0.500	0.747	0.500	1360.000	208.00	13.000	13.60	0.300	0.64
131	0.500	0.747	0.500	1360.000	234.20	11.100	13.60	0.300	0.64
132	0.500	0.747	0.500	1360.000	247.40	10.000	13.60	0.300	0.64
133	0.500	0.747	0.500	1360.000	257.00	9.000	13.60	0.300	0.64
134	0.500	0.747	0.500	1360.000	263.00	8.200	13.60	0.300	0.64
135	0.500	0.747	0.500	1360.000	270.50	7.300	13.60	0.300	0.64
136	0.800	1.195	0.500	3966.680	1.00	36.000	15.87	0.300	0.80
137	0.800	1.195	0.500	3966.680	285.00	27.000	15.87	0.300	0.80
138	0.800	1.195	0.500	3966.680	533.00	20.000	15.87	0.300	0.80
139	0.800	1.195	0.500	3966.680	650.00	17.500	15.87	0.300	0.80
140	0.800	1.195	0.500	3966.680	704.00	15.800	15.87	0.300	0.80
141	0.800	1.195	0.500	3966.680	740.00	14.500	15.87	0.300	0.80
142	0.800	1.195	0.500	3966.680	760.00	13.500	15.87	0.300	0.80
143	0.800	1.195	0.500	3966.680	793.21	12.200	15.87	0.300	0.80

Table 3.4: summary of samples used for investigation, I_1/I_2 =0.3

Sample Name	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
144	0.250	0.339	0.250	14.167	1.00	48.000	09.07	0.400	0.16
145	0.250	0.339	0.250	14.167	116.64	30.000	09.07	0.400	0.16
146	0.250	0.339	0.250	14.167	19.92	25.000	09.07	0.400	0.16
147	0.250	0.339	0.250	14.167	22.44	20.000	09.07	0.400	0.16
148	0.250	0.339	0.250	14.167	23.07	18.500	09.07	0.400	0.16
149	0.250	0.339	0.250	14.167	23.69	16.500	09.07	0.400	0.16
150	0.250	0.339	0.250	14.167	24.02	15.500	09.07	0.400	0.16
151	0.250	0.339	0.250	14.167	24.80	12.000	09.07	0.400	0.16
152	0.250	0.339	0.250	14.167	25.01	10.680	09.07	0.400	0.16
153	0.350	0.475	0.250	99.138	1.00	44.000	11.33	0.400	0.32
154	0.350	0.475	0.250	99.138	2.10	41.000	11.33	0.400	0.32
155	0.350	0.475	0.250	99.138	39.13	21.000	11.33	0.400	0.32
156	0.350	0.475	0.250	99.138	48.00	17.000	11.33	0.400	0.32
157	0.350	0.475	0.250	99.138	55.05	13.000	11.33	0.400	0.32
158	0.350	0.475	0.250	99.138	57.00	11.500	11.33	0.400	0.32
159	0.350	0.475	0.250	99.138	57.86	10.800	11.33	0.400	0.32
160	0.350	0.475	0.250	99.138	59.20	9.500	11.33	0.400	0.32
161	0.350	0.475	0.250	99.138	60.17	8.520	11.33	0.400	0.32
162	0.350	0.475	0.250	99.138	61.48	5.880	11.33	0.400	0.32
163	0.400	0.543	0.250	254.925	1.00	32.600	11.33	0.400	0.48
164	0.400	0.543	0.250	254.925	6.50	30.000	11.33	0.400	0.48
165	0.400	0.543	0.250	254.925	71.60	15.400	11.33	0.400	0.48
166	0.400	0.543	0.250	254.925	85.40	12.800	11.33	0.400	0.48
167	0.400	0.543	0.250	254.925	95.73	10.500	11.33	0.400	0.48
168	0.400	0.543	0.250	254.925	100.85	9.100	11.33	0.400	0.48
169	0.400	0.543	0.250	254.925	105.00	7.800	11.33	0.400	0.48
170	0.400	0.543	0.250	254.925	107.00	7.100	11.33	0.400	0.48
171	0.400	0.543	0.250	254.925	108.00	6.600	11.33	0.400	0.48
172	0.400	0.543	0.250	254.925	109.57	5.829	11.33	0.400	0.48
173	0.500	0.679	0.300	816.000	1.00	29.000	13.60	0.400	0.64
174	0.500	0.679	0.300	816.000	135.00	16.000	13.60	0.400	0.64
175	0.500	0.679	0.300	816.000	183.90	12.800	13.60	0.400	0.64
176	0.500	0.679	0.300	816.000	205.00	11.000	13.60	0.400	0.64
177	0.500	0.679	0.300	816.000	222.30	9.500	13.60	0.400	0.64
178	0.500	0.679	0.300	816.000	230.00	8.600	13.60	0.400	0.64
179	0.500	0.679	0.300	816.000	236.00	7.700	13.60	0.400	0.64
180	0.500	0.679	0.300	816.000	238.30	7.200	13.60	0.400	0.64
181	0.500	0.679	0.300	816.000	241.83	6.415	13.60	0.400	0.64
182	0.600	0.814	0.350	2082.544	1.00	28.000	15.87	0.400	0.80
183	0.600	0.814	0.350	2082.544	134.00	21.000	15.87	0.400	0.80
184	0.600	0.814	0.350	2082.544	236.00	17.000	15.87	0.400	0.80
185	0.600	0.814	0.350	2082.544	311.00	13.800	15.87	0.400	0.80
186	0.600	0.814	0.350	2082.544	282.00	11.000	15.87	0.400	0.80
187	0.600	0.814	0.350	2082.544	409.00	9.500	15.87	0.400	0.80
188	0.600	0.814	0.350	2082.544	417.50	8.750	15.87	0.400	0.80
189	0.600	0.814	0.350	2082.544	421.00	8.500	15.87	0.400	0.80
190	0.600	0.814	0.350	2082.544	424.86	7.964	15.87	0.400	0.80

Table 3.5: summary of samples used for investigation, I_1/I_2 =0.4

Sample Name	W_o (m)	W_{max} (m)	B (m)	P (KN)	M_e (KNm)	L (m)	f_{cd} (Mpa)	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$
191	0.300	0.378	0.200	16.995	1.00	57.000	11.33	0.500	0.16
192	0.300	0.378	0.200	16.995	14.27	40.000	11.33	0.500	0.16
193	0.300	0.378	0.200	16.995	19.50	33.800	11.33	0.500	0.16
194	0.300	0.378	0.200	16.995	26.97	23.000	11.33	0.500	0.16
195	0.300	0.378	0.200	16.995	29.30	18.200	11.33	0.500	0.16
196	0.300	0.378	0.200	16.995	30.00	16.300	11.33	0.500	0.16
197	0.300	0.378	0.200	16.995	30.29	15.400	11.33	0.500	0.16
198	0.300	0.378	0.200	16.995	30.71	14.000	11.33	0.500	0.16
199	0.300	0.378	0.200	16.995	31.10	12.300	11.33	0.500	0.16
200	0.300	0.378	0.200	16.995	31.72	8.556	11.33	0.500	0.16
201	0.500	0.630	0.250	170.000	3.00	55.500	13.60	0.500	0.32
202	0.500	0.630	0.250	170.000	14.50	40.800	13.60	0.500	0.32
203	0.500	0.630	0.250	170.000	78.70	20.000	13.60	0.500	0.32
204	0.500	0.630	0.250	170.000	87.92	16.500	13.60	0.500	0.32
205	0.500	0.630	0.250	170.000	91.40	15.000	13.60	0.500	0.32
206	0.500	0.630	0.250	170.000	96.00	12.500	13.60	0.500	0.32
207	0.500	0.630	0.250	170.000	99.78	10.600	13.60	0.500	0.32
208	0.500	0.630	0.250	170.000	102.80	8.700	13.60	0.500	0.32
209	0.500	0.630	0.250	170.000	104.00	7.800	13.60	0.500	0.32
210	0.500	0.630	0.250	170.000	106.16	6.124	13.60	0.500	0.32
211	0.300	0.380	0.300	321.370	1.00	21.500	15.87	0.500	0.48
212	0.300	0.380	0.300	321.370	13.10	16.000	15.87	0.500	0.48
213	0.300	0.380	0.300	321.370	53.14	7.900	15.87	0.500	0.48
214	0.300	0.380	0.300	321.370	60.05	6.400	15.87	0.500	0.48
215	0.300	0.380	0.300	321.370	62.50	5.800	15.87	0.500	0.48
216	0.300	0.380	0.300	321.370	66.13	4.800	15.87	0.500	0.48
217	0.300	0.380	0.300	321.370	68.60	4.000	15.87	0.500	0.48
218	0.300	0.380	0.300	321.370	69.20	3.800	15.87	0.500	0.48
219	0.300	0.380	0.300	321.370	69.30	3.700	15.87	0.500	0.48
220	0.300	0.380	0.300	321.370	70.98	3.100	15.87	0.500	0.48
221	1.000	1.260	0.500	3626.600	294.00	40.000	18.13	0.500	0.64
222	1.000	1.260	0.500	3626.600	921.00	23.000	18.13	0.500	0.64
223	1.000	1.260	0.500	3626.600	1135.80	18.000	18.13	0.500	0.64
224	1.000	1.260	0.500	3626.600	1230.50	15.500	18.13	0.500	0.64
225	1.000	1.260	0.500	3626.600	1310.00	13.000	18.13	0.500	0.64
226	1.000	1.260	0.500	3626.600	1350.00	11.450	18.13	0.500	0.64
227	1.000	1.260	0.500	3626.600	1370.00	10.500	18.13	0.500	0.64
228	1.000	1.260	0.500	3626.600	1380.00	9.800	18.13	0.500	0.64
229	1.000	1.260	0.500	3626.600	1394.90	9.090	18.13	0.500	0.64
230	0.650	0.819	0.650	2991.800	355.00	18.000	11.33	0.500	0.80
231	0.650	0.819	0.650	2991.800	534.00	13.500	11.33	0.500	0.80
232	0.650	0.819	0.650	2991.800	630.00	11.000	11.33	0.500	0.80
233	0.650	0.819	0.650	2991.800	656.00	10.100	11.33	0.500	0.80
234	0.650	0.819	0.650	2991.800	670.00	9.600	11.33	0.500	0.80
235	0.650	0.819	0.650	2991.800	698.00	8.400	11.33	0.500	0.80
236	0.650	0.819	0.650	2991.800	710.00	7.700	11.33	0.500	0.80
237	0.650	0.819	0.650	2991.800	723.37	7.070	11.33	0.500	0.80

Table 3.6: summary of samples used for investigation; I_1/I_2 =0.5

3.2 Sample Analysis

Sample analysis, as per the context of this section is the construction of second order moment curves for all selected samples from all categories. As indicated in the previous section, analysis results will be used for a several goals. An iterative second order analysis is to be committed for all sample columns. For each column, the entire length is divided in to 20 divisions (segments) to have a total of 21 nodes. The moment values at each node will be calculated as per the methodology indicated in *section 1.5 and 2.4* of this document. Moment values at each node will be connected by a series of lines to form second order moment curve. In addition to moment curves, the effective length of an equivalent column having a constant section, which is equal to the section at failure point of actual column, should also be computed. Effective lengths will then be divided by the actual length of tapered column to get the effective length factor. Computation of effective length factors should be done as per *section 2.4* of this document and *equation 2.2*; which is re-indicated as:

$$K=\frac{\pi}{L}\sqrt{\frac{EI}{P_{cr,act}}} \quad \ldots\ldots (Eq. 2.2)$$

Where the value of $P_{cr,act}$ is going to be computed by making use of research result which is indicated in *Annex A.1*

Finally, the degree of safety of the methodology will be evaluated by comparing the results from analysis of actual column and equivalent column. Buckling load, the design section and failure (design) moment in the actual column should be similar to the formed equivalent column. If we can make this requirement to be satisfied, the selected section will be termed as *"The equivalent prismatic section of the column"*. The effective length computed for the failure section will be set as *"the effective length of the equivalent column"*; accordingly both will be the output of the research.

3.3 Results

An iterative second order analysis is committed for all samples. For illustration purpose, observe *Fig. 3.1* to *fig. 3.12*, the second order moment diagrams of few samples.

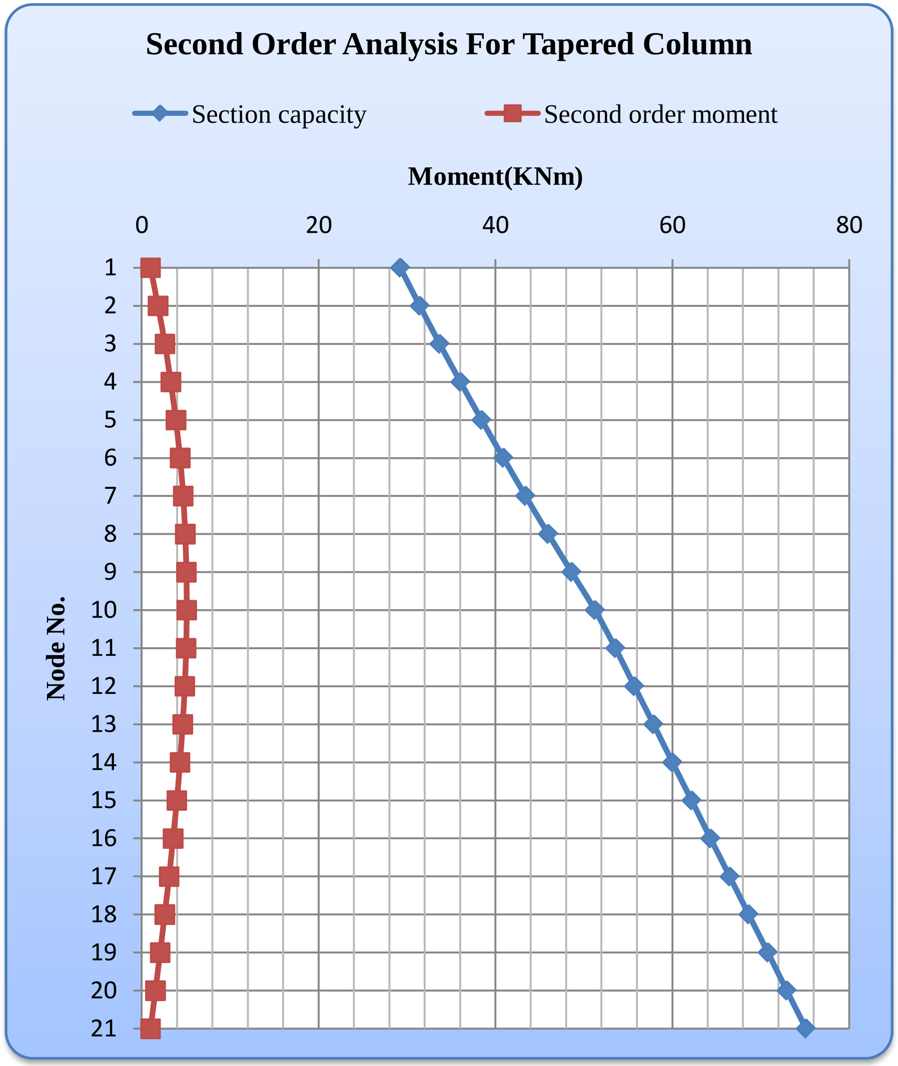


Fig. 3.1: Second order bending moment diagram for sample No. 94

L=8.30m; P=283.25KN; Me=1.00KNm; M_d*=4.693KNm*

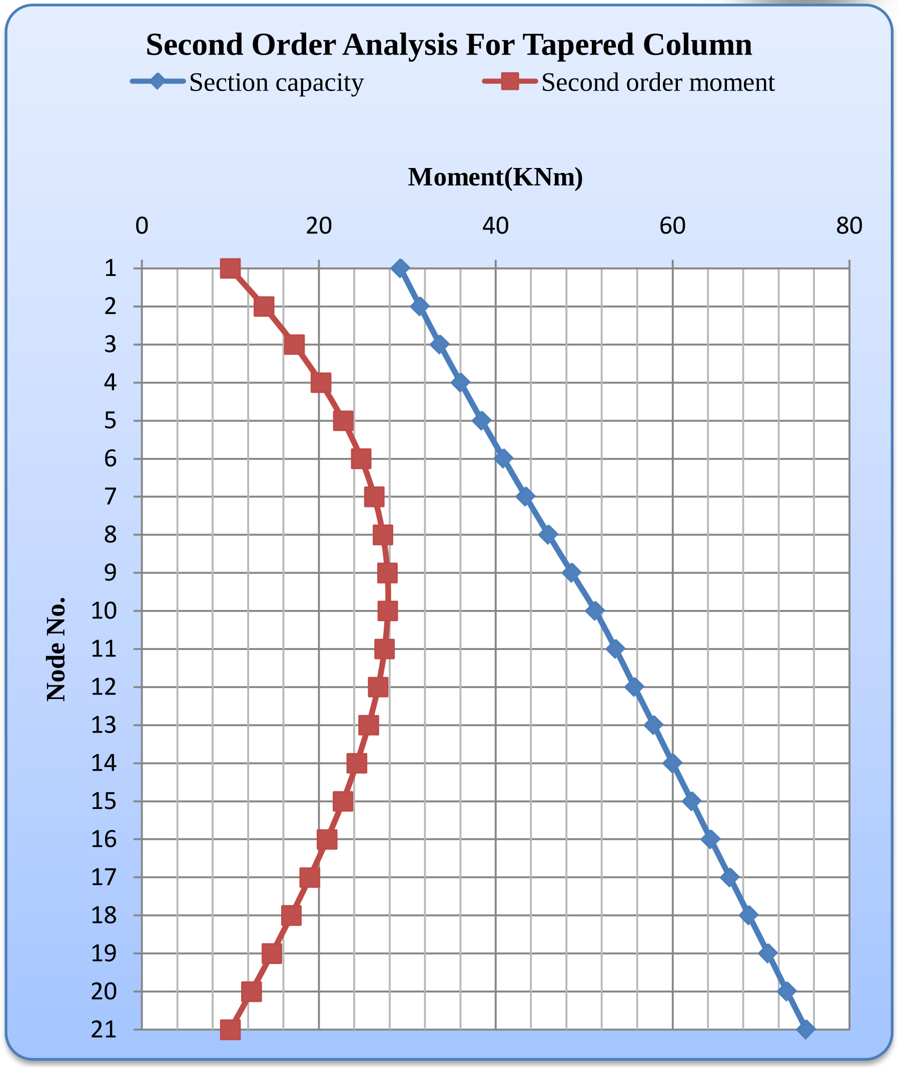


Fig. 3.2: Second order bending moment diagram for sample No. 95

L=7.00m; P=283.25KN; Me=10.00KNm; M_d*=24.783KNm*

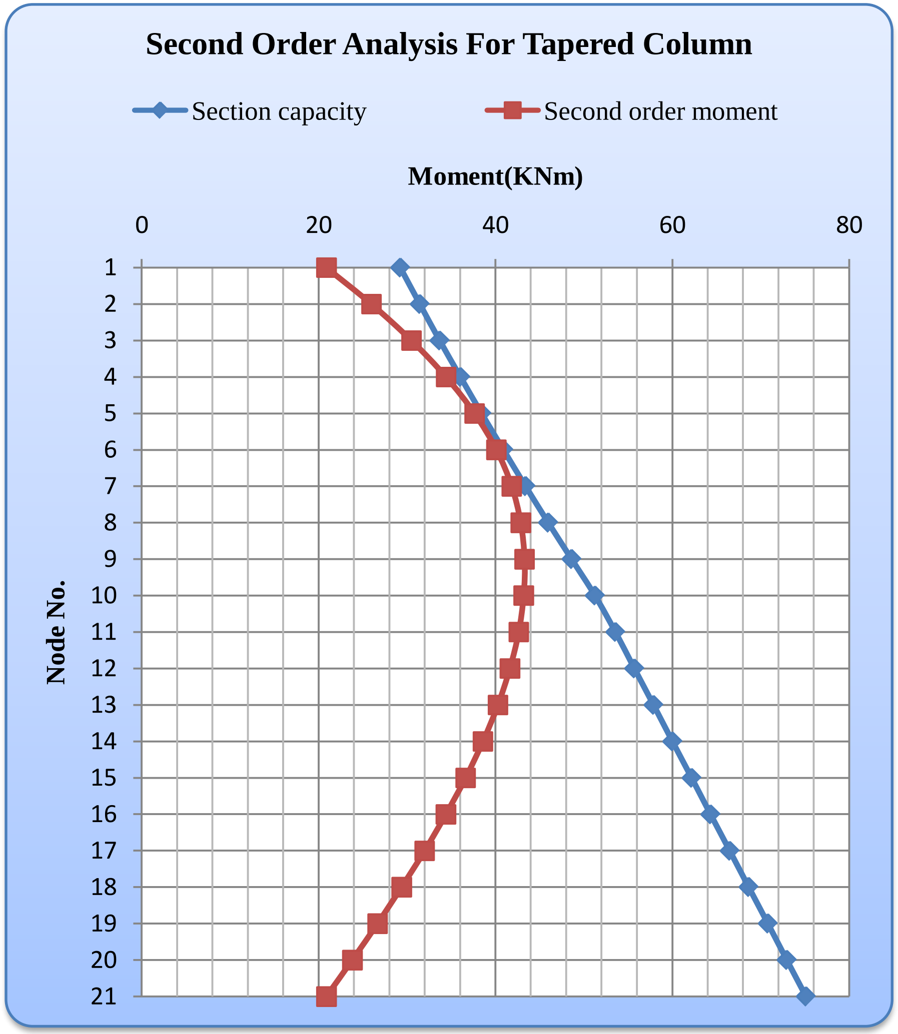


Fig. 3.3: Second order bending moment diagram for sample No. 96

L=6.20m; P=283.25KN; Me=20.90KNm; M_d*=37.643KNm*

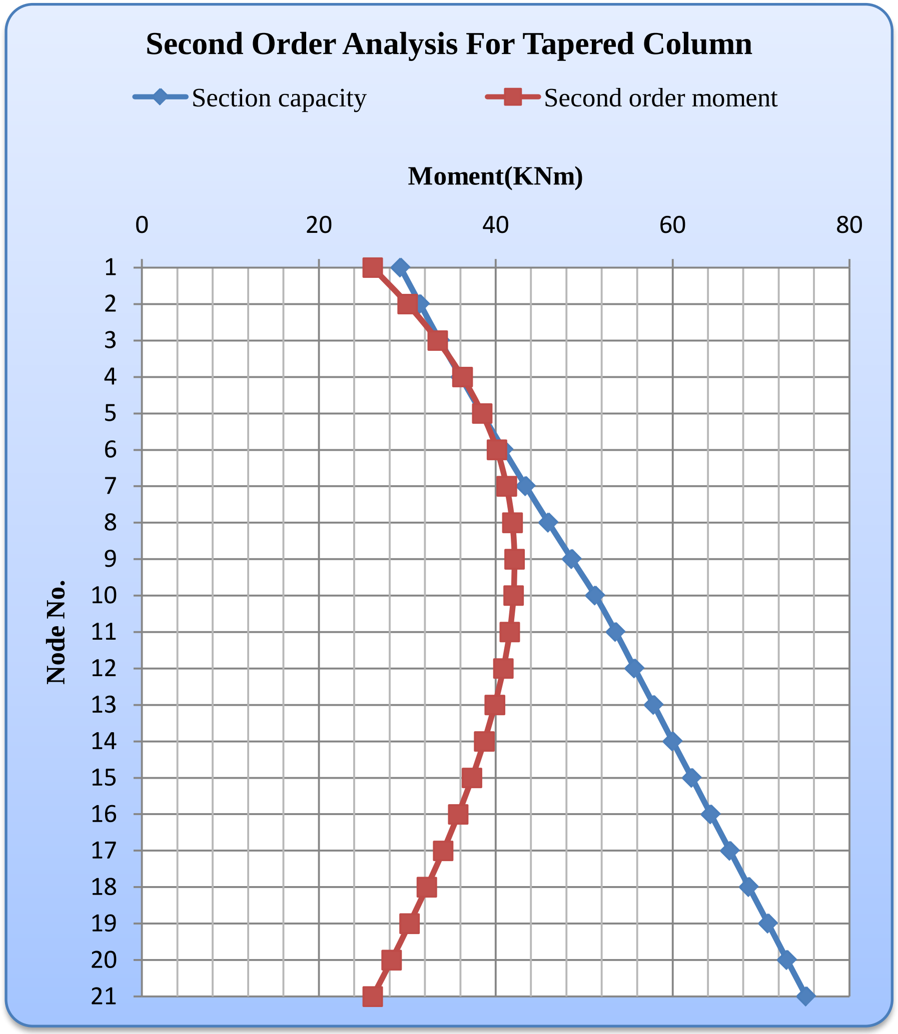


Fig. 3.4: Second order bending moment diagram for sample No. 97

L=5.60m; P=283.25KN; Me=26.10KNm; M_d*=36.254KNm*

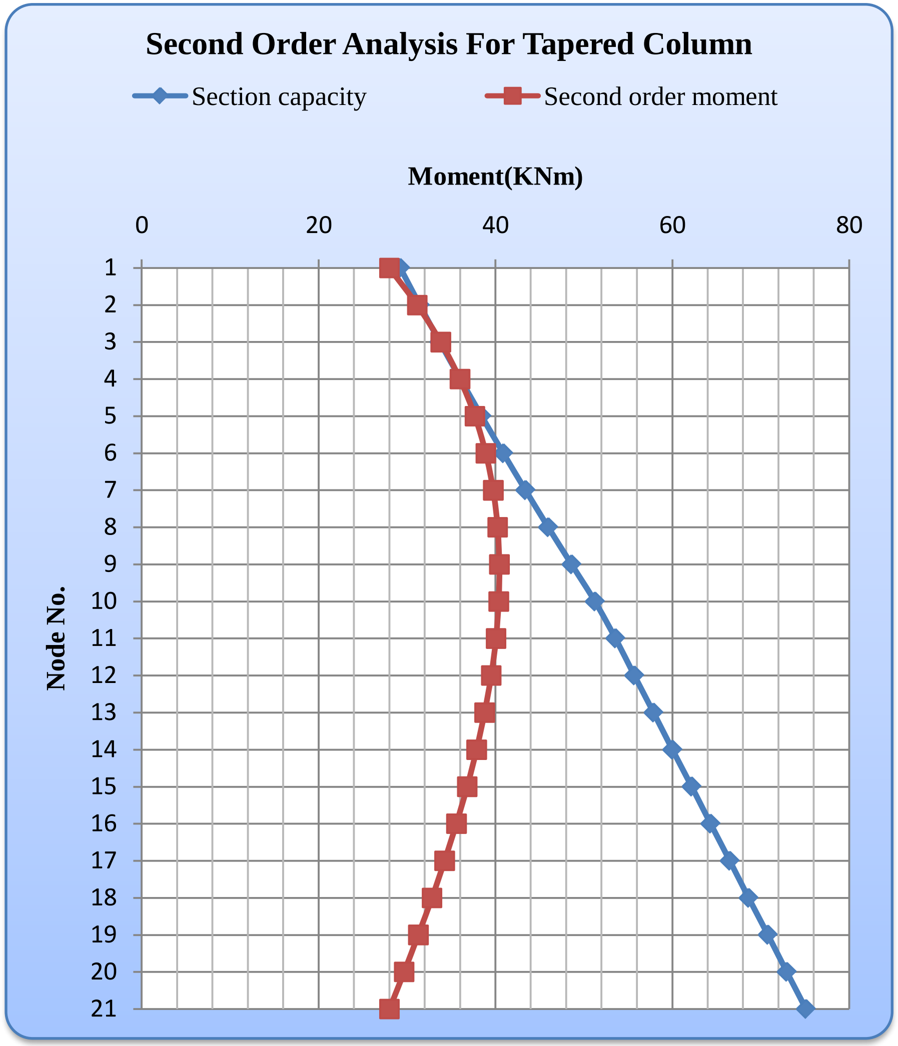


Fig. 3.5: Second order bending moment diagram for sample No. 98

L=5.10m; P=283.25KN; Me=28.00KNm; M_d*=33.824KNm*

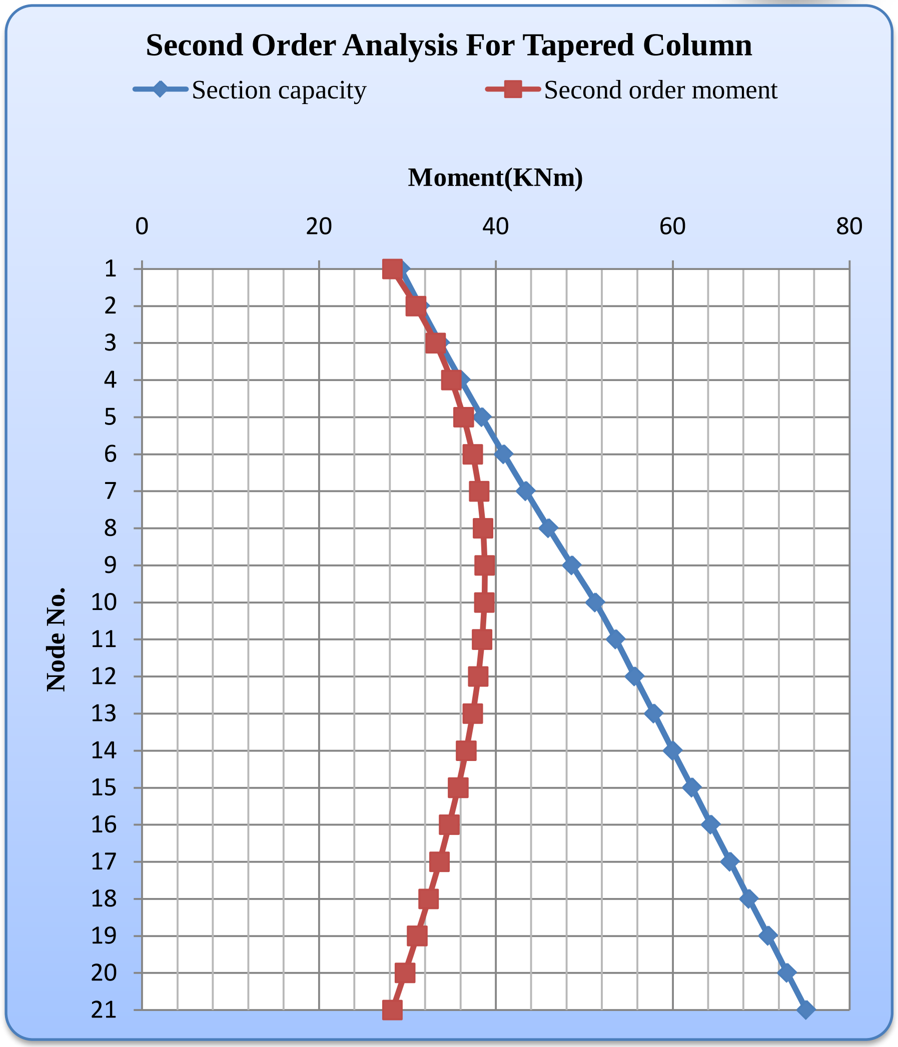


Fig. 3.6: Second order bending moment diagram for sample No. 99

L=4.80m; P=283.25KN; Me=28.30KNm; M_d*=30.959KNm*

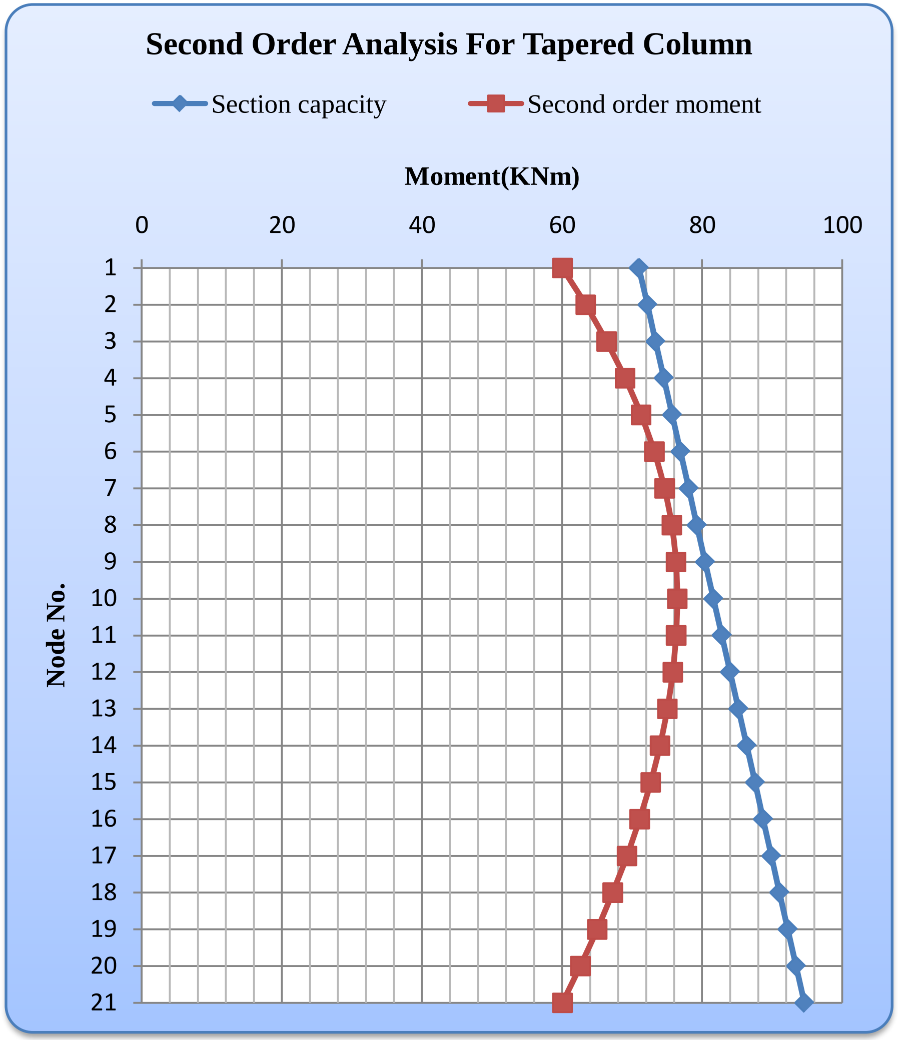


Fig. 3.7: Second order bending moment diagram for sample No. 214
L=6.40m; P=321.368KN; Me=60.05KNm; M_d*=74.631KNm*

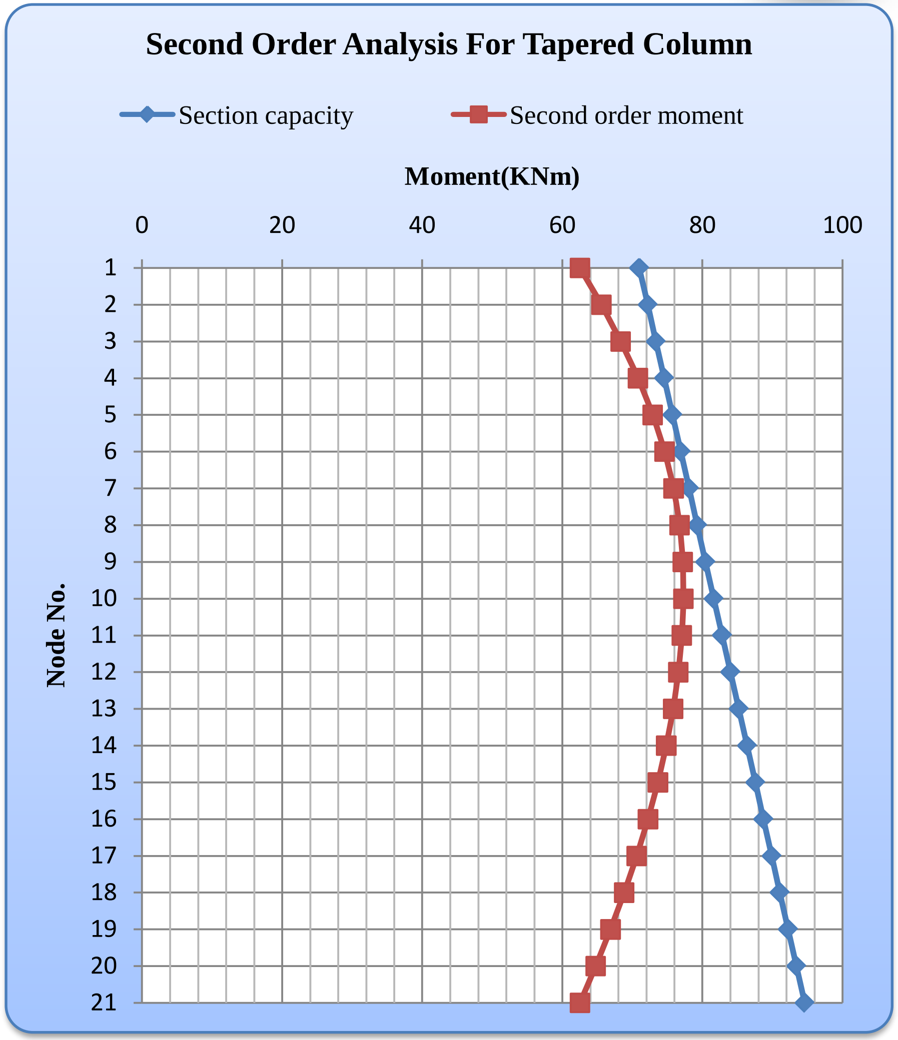


Fig. 3.8: Second order bending moment diagram for sample No. 215
L=5.80m; P=321.368KN; Me=62.50KNm; M_d*=74.569KNm*

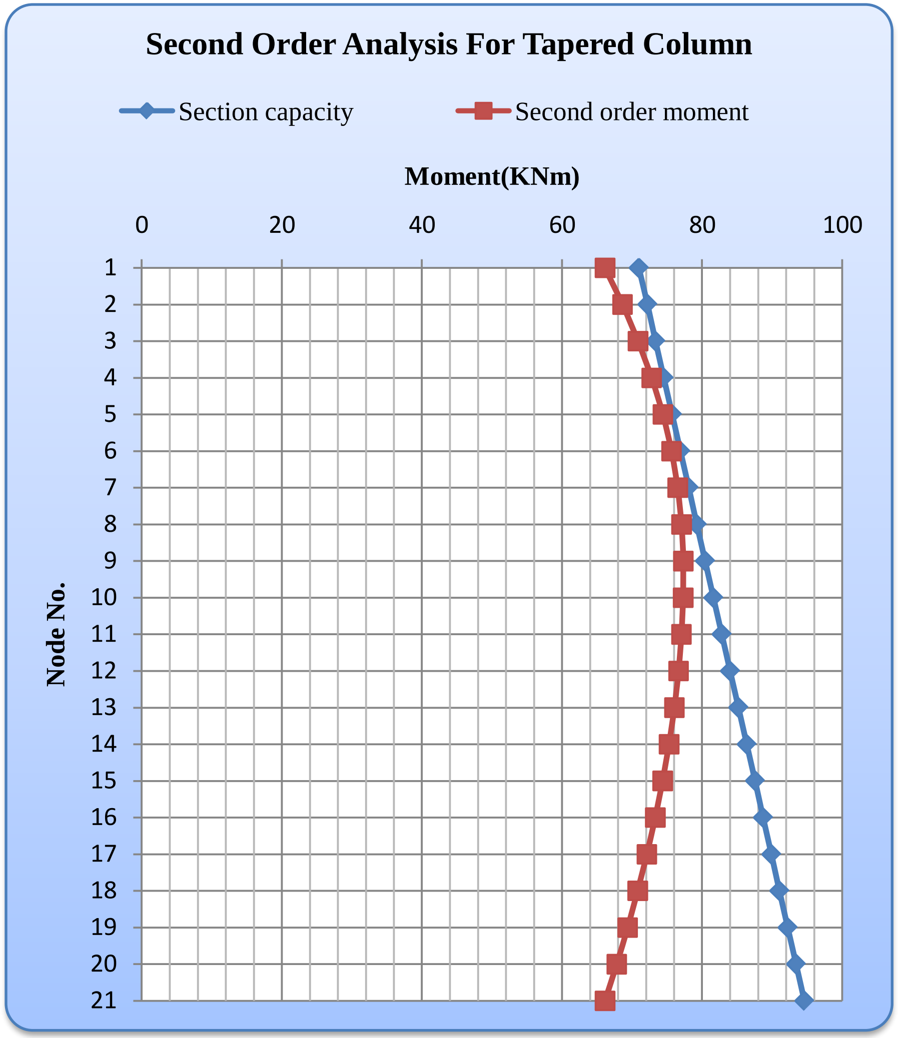


Fig. 3.9: Second order bending moment diagram sample No. 216

L=4.80m; P=321.368KN; Me=66.13KNm; M_d*=74.372KNm*

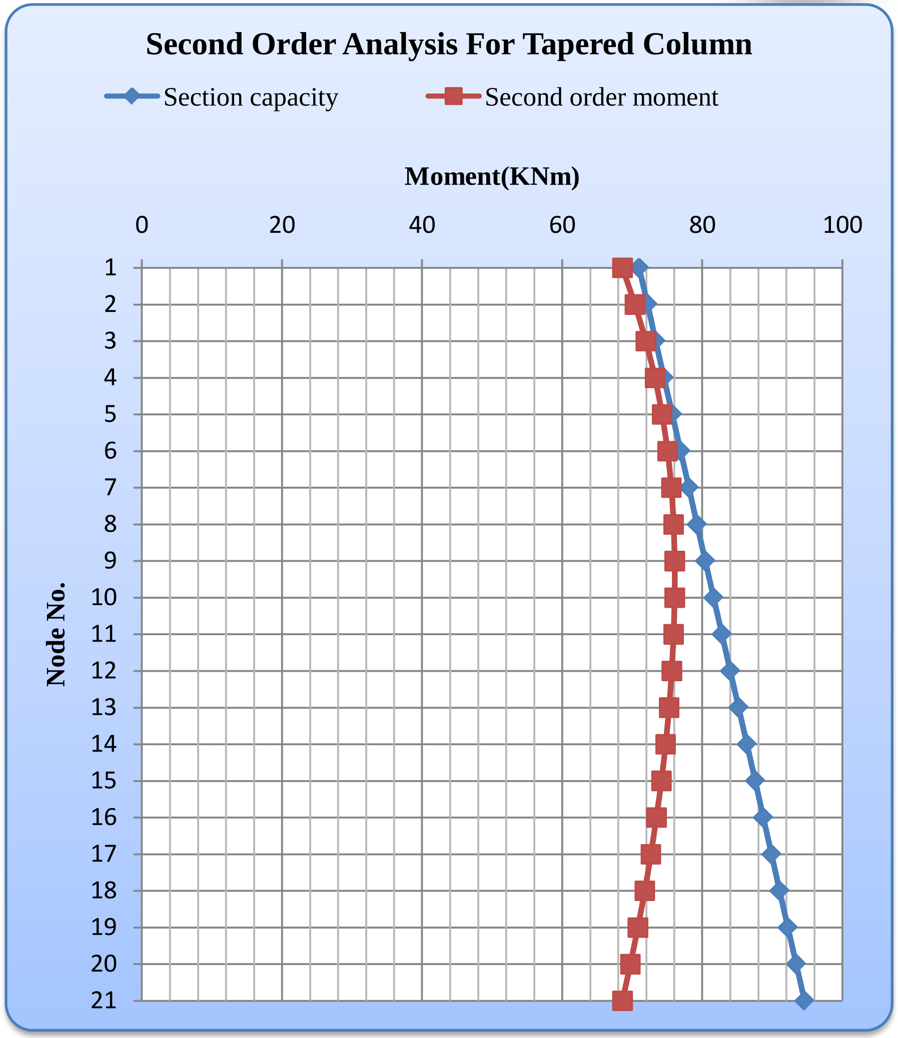


Fig. 3.10: Second order bending moment diagram sample No. 217

L=4.00m; P=321.368KN; Me=68.60KNm; M_d*=73.2261KNm*

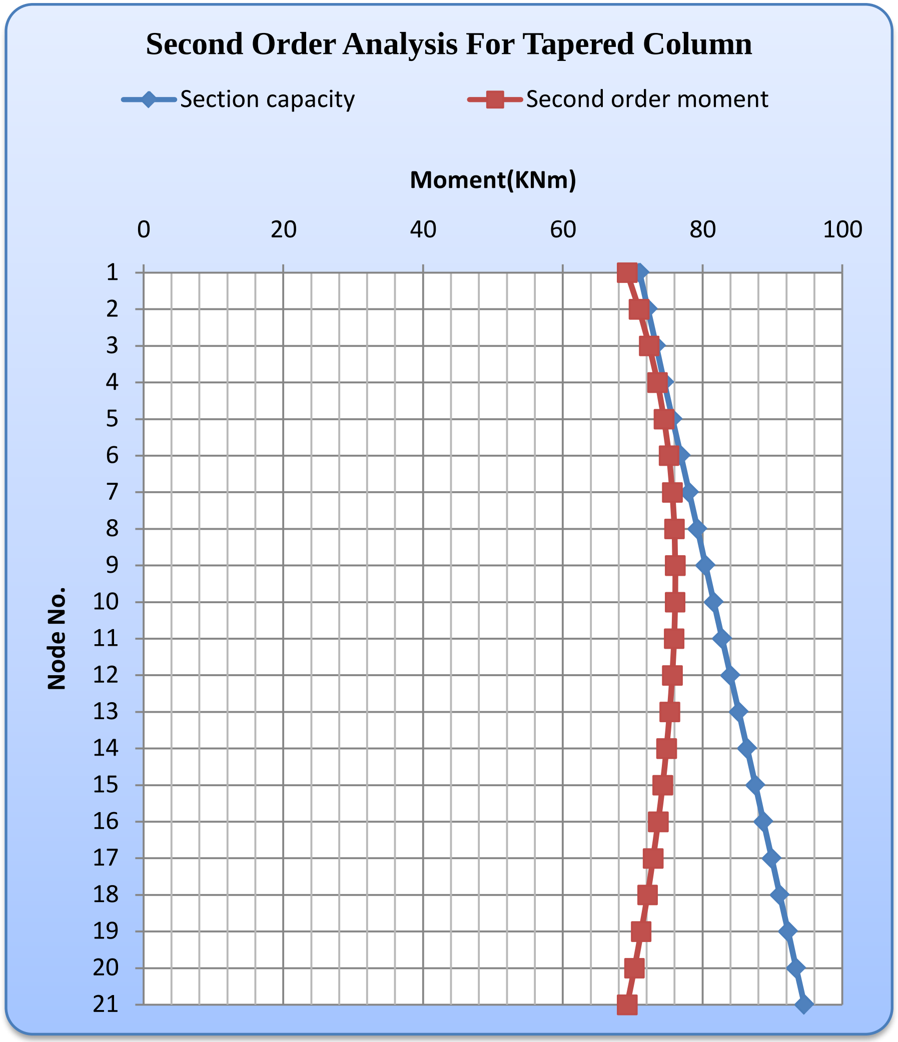


Fig. 3.11: Second order bending moment diagram for sample No. 218

L=3.80m; P=321.368KN; Me=69.20KNm; M_d*=72.338KNm*

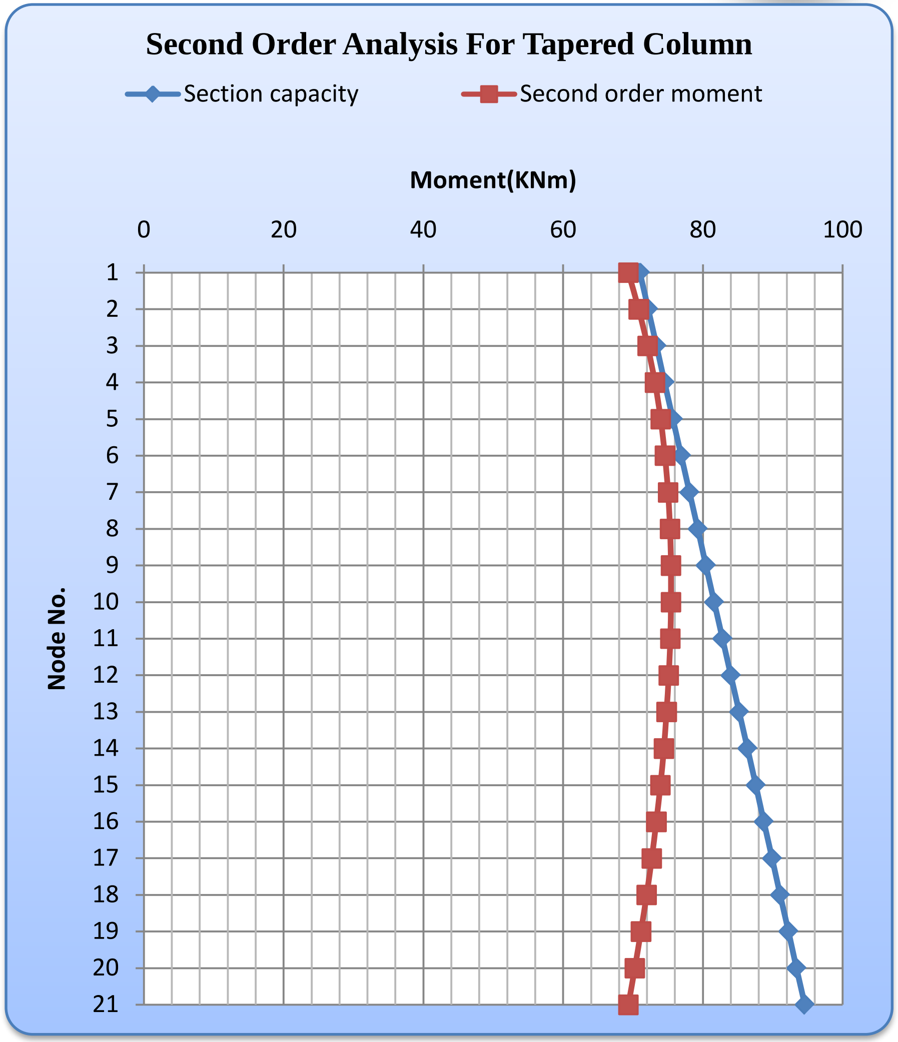


Fig. 3.12: Second order bending moment diagram for sample No. 219

L=3.70m; P=321.368KN; Me=69.30KNm; M_d*=70.797KNm*

Summary of second order analysis results with all other appropriate observations are indicated in tables 3.7, 3.8, 3.9, 3.10, 3.11 and 3.12. A new geometric parameter, $\frac{B}{W_i}L$ which is termed as the *'STABILIZED LENGTH, L_s'* is included in the summary table; where:

B = the constant cross-sectional dimension (the breadth) of actual column.

L = the length (the height) of actual column

W_i = the width at failure section

Please refer the loading, material and geometric data of all the below mentioned samples from table 3.1 to table 3.6.

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
1	0.100	0.553	2X1884.96	347.83	Stability	0.225L	15.520
2	0.100	0.553	2X1884.96	347.83	Stability	0.175L	13.540
3	0.100	0.553	2X1884.96	347.83	Stability	0.125L	12.502
4	0.100	0.553	2X1884.96	347.83	Stability	0.025L	12.502
5	0.100	0.553	2X603.18	347.83	Stability	0.25L	15.520
6	0.100	0.553	2X603.18	347.83	Stability	0.20L	13.540
7	0.100	0.553	2X603.18	347.83	Stability	0.15L	12.502
8	0.100	0.553	2X603.18	347.83	Stability	0.10L	11.356
9	0.100	0.553	2X603.18	347.83	Material	0.05L	10.085
10	0.100	0.553	2X942.48	313.04	Stability	0.25L	15.531
11	0.100	0.553	2X942.48	313.04	Stability	0.20L	13.517
12	0.100	0.553	2X942.48	313.04	Stability	0.15L	12.456
13	0.100	0.553	2X942.48	313.04	Material	0.10L	11.275
14	0.100	0.553	2X942.48	313.04	Material	0.05L	10.065
15	0.100	0.553	2X942.48	313.04	Material	0.00L	8.8125
16	0.300	0.32	2X615.75	260.87	Material	0.35L	8.490
17	0.300	0.32	2X615.75	260.87	Stability	0.30L	10.242
18	0.300	0.32	2X615.75	260.87	Stability	0.25L	12.462
19	0.300	0.32	2X615.75	260.87	Stability	0.15L	14.807
20	0.300	0.32	2X615.75	260.87	Stability	0.05L	17.053
21	0.300	0.32	2X615.75	260.87	Stability	0.00L	6.300
22	0.300	0.32	2X565.50	347.83	Stability	0.35L	8.458
23	0.300	0.32	2X565.50	347.83	Stability	0.30L	10.179
24	0.300	0.32	2X565.50	347.83	Stability	0.25L	12.461
25	0.300	0.32	2X565.50	347.83	Stability	0.15L	14.806
26	0.300	0.32	2X565.50	347.83	Stability	0.05L	17.052
27	0.300	0.32	2X565.50	347.83	Stability	0.00L	6.000

Table 3.7: summary of results of failure sections and stabilized length, I_1/I_2 =0.1 & 0.3

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
28	0.10	0.16	2X942.50	260.87	Stability	0.30L	52.612
29	0.10	0.16	2X942.50	260.87	Material	0.25L	42.035
30	0.10	0.16	2X942.50	260.87	Material	0.20L	37.236
31	0.10	0.16	2X942.50	260.87	Material	0.15L	34.806
32	0.10	0.16	2X942.50	260.87	Material	0.10L	32.125
33	0.10	0.16	2X942.50	260.87	Material	0.05L	30.726
34	0.10	0.16	2X942.50	260.87	Material	0.00L	25.833
35	0.10	0.32	2X1005.3	347.83	Stability	0.30L	50.402
36	0.10	0.32	2X1005.3	347.83	Stability	0.25L	44.345
37	0.10	0.32	2X1005.3	347.83	Stability	0.20L	26.114
38	0.10	0.32	2X1005.3	347.83	Material	0.15L	23.745
39	0.10	0.32	2X1005.3	347.83	Material	0.10L	22.413
40	0.10	0.32	2X1005.3	347.83	Material	0.05L	20.259
41	0.10	0.32	2X1005.3	347.83	Material	0.00L	16.729
42	0.10	0.48	2X1206.37	313.04	Stability	0.35L	40.063
43	0.10	0.48	2X1206.37	313.04	Stability	0.30L	37.138
44	0.10	0.48	2X1206.37	313.04	Stability	0.25L	31.526
45	0.10	0.48	2X1206.37	313.04	Stability	0.20L	20.311
46	0.10	0.48	2X1206.37	313.04	Stability	0.15L	18.113
47	0.10	0.48	2X1206.37	313.04	Stability	0.10L	16.809
48	0.10	0.48	2X1206.37	313.04	Material	0.05L	15.954
49	0.10	0.48	2X1206.37	313.04	Material	0.00L	13.383
50	0.10	0.64	2X656.49	260.87	Stability	0.30L	40.109
51	0.10	0.64	2X656.49	260.87	Stability	0.25L	38.026
52	0.10	0.64	2X656.49	260.87	Stability	0.20L	32.497
53	0.10	0.64	2X656.49	260.87	Stability	0.15L	26.424
54	0.10	0.64	2X656.49	260.87	Stability	0.10L	21.516
55	0.10	0.64	2X656.49	260.87	Material	0.05L	20.327
56	0.10	0.64	2X656.49	260.87	Material	0.00L	18.000
57	0.10	0.80	2X769.70	260.47	Stability	0.30L	20.532
58	0.10	0.80	2X769.70	260.47	Stability	0.25L	19.955
59	0.10	0.80	2X769.70	260.47	Stability	0.20L	18.280
60	0.10	0.80	2X769.70	260.47	Stability	0.15L	15.343
61	0.10	0.80	2X769.70	260.47	Stability	0.10L	14.408
62	0.10	0.80	2X769.70	260.47	Material	0.05L	13.371
63	0.10	0.80	2X769.70	260.47	Material	0.00L	12.857

Table 3.8: summary of results of failure sections and stabilized length; I_1/I_2 =0.1

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
64	0.20	0.16	2X1885.00	347.83	Material	0.30L	56.062
65	0.20	0.16	2X1885.00	347.83	Material	0.25L	48.240
66	0.20	0.16	2X1885.00	347.83	Material	0.20L	45.536
67	0.20	0.16	2X1885.00	347.83	Material	0.15L	41.935
68	0.20	0.16	2X1885.00	347.83	Material	0.10L	39.590
69	0.20	0.16	2X1885.00	347.83	Material	0.05L	37.857
70	0.20	0.16	2X1885.00	347.83	Material	0.00L	30.400
71	0.20	0.32	2X679.70	260.87	Stability	0.35L	40.048
72	0.20	0.32	2X679.70	260.87	Stability	0.30L	36.068
73	0.20	0.32	2X679.70	260.87	Stability	0.25L	20.170
74	0.20	0.32	2X679.70	260.87	Stability	0.20L	15.871
75	0.20	0.32	2X679.70	260.87	Stability	0.15L	13.839
76	0.20	0.32	2X679.70	260.87	Stability	0.10L	13.422
77	0.20	0.32	2X679.70	260.87	Material	0.05L	12.071
78	0.20	0.32	2X679.70	260.87	Material	0.00L	9.425
79	0.20	0.48	2X942.48	347.83	Stability	0.35L	23.362
80	0.20	0.48	2X942.48	347.83	Stability	0.30L	13.740
81	0.20	0.48	2X942.48	347.83	Stability	0.25L	9.908
82	0.20	0.48	2X942.48	347.83	Stability	0.20L	8.976
83	0.20	0.48	2X942.48	347.83	Material	0.15L	8.284
84	0.20	0.48	2X942.48	347.83	Material	0.10L	7.392
85	0.20	0.48	2X942.48	347.83	Material	0.05L	7.082
86	0.20	0.48	2X942.48	347.83	Material	0.00L	6.158
87	0.20	0.64	2X1256.60	347.83	Stability	0.30L	21.526
88	0.20	0.64	2X1256.60	347.83	Stability	0.25L	17.693
89	0.20	0.64	2X1256.60	347.83	Stability	0.20L	16.540
90	0.20	0.64	2X1256.60	347.83	Material	0.15L	11.297
91	0.20	0.64	2X1256.60	347.83	Material	0.10L	10.634
92	0.20	0.64	2X1256.60	347.83	Material	0.05L	10.194
93	0.20	0.64	2X1256.60	347.83	Material	0.00L	9.206
94	0.20	0.80	2X461.80	347.83	Stability	0.30L	9.316
95	0.20	0.80	2X461.80	347.83	Stability	0.25L	7.643
96	0.20	0.80	2X461.80	347.83	Stability	0.20L	6.130
97	0.20	0.80	2X461.80	347.83	Material	0.15L	5.242
98	0.20	0.80	2X461.80	347.83	Material	0.10L	4.902
99	0.20	0.80	2X461.80	347.83	Material	0.05L	4.829
100	0.20	0.80	2X461.80	347.83	Material	0.00L	4.592

Table 3.9: summary of results of failure sections and stabilized length; I_1/I_2 =0.2

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
101	0.30	0.16	2X603.20	260.87	Stability	0.40L	36.650
102	0.30	0.16	2X603.20	260.87	Stability	0.35L	30.790
103	0.30	0.16	2X603.20	260.87	Material	0.30L	24.194
104	0.30	0.16	2X603.20	260.87	Material	0.25L	20.275
105	0.30	0.16	2X603.20	260.87	Material	0.20L	19.214
106	0.30	0.16	2X603.20	260.87	Material	0.15L	18.104
107	0.30	0.16	2X603.20	260.87	Material	0.10L	15.882
108	0.30	0.16	2X603.20	260.87	Material	0.05L	14.639
109	0.30	0.16	2X603.20	260.87	Material	0.00L	10.350
110	0.30	0.32	2X615.75	260.87	Stability	0.40L	31.734
111	0.30	0.32	2X615.75	260.87	Stability	0.35L	24.728
112	0.30	0.32	2X615.75	260.87	Stability	0.30L	15.678
113	0.30	0.32	2X615.75	260.87	Stability	0.25L	13.352
114	0.30	0.32	2X615.75	260.87	Stability	0.20L	11.832
115	0.30	0.32	2X615.75	260.87	Stability	0.15L	10.242
116	0.30	0.32	2X615.75	260.87	Material	0.10L	9.625
117	0.30	0.32	2X615.75	260.87	Material	0.05L	9.271
118	0.30	0.32	2X615.75	260.87	Material	0.00L	6.560
119	0.30	0.48	2X1005.3	400.00	Stability	0.35L	31.426
120	0.30	0.48	2X1005.3	400.00	Stability	0.30L	16.424
121	0.30	0.48	2X1005.3	400.00	Stability	0.25L	12.970
122	0.30	0.48	2X1005.3	400.00	Material	0.20L	11.701
123	0.30	0.48	2X1005.3	400.00	Material	0.15L	10.773
124	0.30	0.48	2X1005.3	400.00	Material	0.10L	10.455
125	0.30	0.48	2X1005.3	400.00	Material	0.05L	10.038
126	0.30	0.48	2X1005.3	400.00	Material	0.00L	8.460
127	0.30	0.64	2X615.75	260.87	Stability	0.40L	25.052
128	0.30	0.64	2X615.75	260.87	Stability	0.35L	21.316
129	0.30	0.64	2X615.75	260.87	Stability	0.30L	13.936
130	0.30	0.64	2X615.75	260.87	Stability	0.25L	11.571
131	0.30	0.64	2X615.75	260.87	Stability	0.20L	10.102
132	0.30	0.64	2X615.75	260.87	Material	0.15L	9.310
133	0.30	0.64	2X615.75	260.87	Material	0.10L	8.576
134	0.30	0.64	2X615.75	260.87	Material	0.05L	8.002
135	0.30	0.64	2X615.75	260.87	Material	0.00L	7.300
136	0.30	0.80	2X923.63	400.00	Stability	0.35L	19.184
137	0.30	0.80	2X923.63	400.00	Stability	0.30L	14.698
138	0.30	0.80	2X923.63	400.00	Stability	0.25L	11.571
139	0.30	0.80	2X923.63	400.00	Stability	0.20L	9.954
140	0.30	0.80	2X923.63	400.00	Material	0.15L	9.194
141	0.30	0.80	2X923.63	400.00	Material	0.10L	8.636
142	0.30	0.80	2X923.63	400.00	Material	0.05L	8.234
143	0.30	0.80	2X923.63	400.00	Material	0.00L	7.625

Table 3.10: summary of results of failure sections and stabilized length; I_1/I_2 =0.3

Sample Name	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
144	0.40	0.16	2X461.81	260.87	Stability	0.40L	41.999
145	0.40	0.16	2X461.81	260.87	Stability	0.35L	26.666
146	0.40	0.16	2X461.81	260.87	Stability	0.30L	22.580
147	0.40	0.16	2X461.81	260.87	Stability	0.25L	18.360
148	0.40	0.16	2X461.81	260.87	Material	0.20L	17.266
149	0.40	0.16	2X461.81	260.87	Material	0.15L	15.661
150	0.40	0.16	2X461.81	260.87	Material	0.10L	14.772
151	0.40	0.16	2X461.81	260.87	Material	0.05L	11.789
152	0.40	0.16	2X461.81	260.87	Material	0.00L	10.680
153	0.40	0.32	2X615.75	260.87	Stability	0.45L	27.076
154	0.40	0.32	2X615.75	260.87	Stability	0.40L	25.624
155	0.40	0.32	2X615.75	260.87	Stability	0.35L	13.333
156	0.40	0.32	2X615.75	260.87	Stability	0.30L	10.968
157	0.40	0.32	2X615.75	260.87	Stability	0.25L	8.524
158	0.40	0.32	2X615.75	260.87	Stability	0.20L	7.667
159	0.40	0.32	2X615.75	260.87	Stability	0.15L	7.322
160	0.40	0.32	2X615.75	260.87	Stability	0.10L	6.552
161	0.40	0.32	2X615.75	260.87	Material	0.05L	5.979
162	0.40	0.32	2X615.75	260.87	Material	0.00L	4.200
163	0.40	0.48	2X603.19	347.83	Stability	0.45L	17.553
164	0.40	0.48	2X603.19	347.83	Stability	0.40L	16.406
165	0.40	0.48	2X603.19	347.83	Stability	0.35L	8.555
166	0.40	0.48	2X603.19	347.83	Stability	0.30L	7.226
167	0.40	0.48	2X603.19	347.83	Stability	0.25L	6.024
168	0.40	0.48	2X603.19	347.83	Stability	0.20L	5.308
169	0.40	0.48	2X603.19	347.83	Material	0.15L	4.627
170	0.40	0.48	2X603.19	347.83	Material	0.10L	4.284
171	0.40	0.48	2X603.19	347.83	Material	0.05L	4.053
172	0.40	0.48	2X603.19	347.83	Material	0.00L	3.643
173	0.40	0.64	2X804.25	347.83	Stability	0.40L	15.225
174	0.40	0.64	2X804.25	347.83	Stability	0.35L	8.533
175	0.40	0.64	2X804.25	347.83	Stability	0.30L	6.937
176	0.40	0.64	2X804.25	347.83	Stability	0.25L	6.059
177	0.40	0.64	2X804.25	347.83	Material	0.20L	5.320
178	0.40	0.64	2X804.25	347.83	Material	0.15L	4.898
179	0.40	0.64	2X804.25	347.83	Material	0.10L	4.461
180	0.40	0.64	2X804.25	347.83	Material	0.05L	4.244
181	0.40	0.64	2X804.25	347.83	Material	0.00L	3.849
182	0.40	0.80	2X1256.64	347.83	Stability	0.40L	14.291
183	0.40	0.80	2X1256.64	347.83	Stability	0.35L	10.889
184	0.40	0.80	2X1256.64	347.83	Stability	0.30L	8.957
185	0.40	0.80	2X1256.64	347.83	Stability	0.25L	7.390
186	0.40	0.80	2X1256.64	347.83	Material	0.20L	5.989
187	0.40	0.80	2X1256.64	347.83	Material	0.15L	5.260
188	0.40	0.80	2X1256.64	347.83	Material	0.10L	4.928
189	0.40	0.80	2X1256.64	347.83	Material	0.05L	4.871
190	0.40	0.80	2X1256.64	347.83	Material	0.00L	4.646

Table 3.11: summary of results of failure sections and stabilized length; I_1/I_2 =0.4

Sample Name	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	A_s (mm^2)	f_{yd} (Mpa)	System failure mode	Failure section at:	$L_s = \frac{B}{W_i}L$ (m)
191	0.50	0.16	2X461.81	260.87	Stability	0.45L	34.021
192	0.50	0.16	2X461.81	260.87	Stability	0.40L	24.155
193	0.50	0.16	2X461.81	260.87	Stability	0.35L	20.654
194	0.50	0.16	2X461.81	260.87	Stability	0.30L	14.224
195	0.50	0.16	2X461.81	260.87	Stability	0.25L	11.393
196	0.50	0.16	2X461.81	260.87	Stability	0.20L	10.330
197	0.50	0.16	2X461.81	260.87	Material	0.15L	9.881
198	0.50	0.16	2X461.81	260.87	Material	0.10L	9.097
199	0.50	0.16	2X461.81	260.87	Material	0.05L	8.095
200	0.50	0.16	2X461.81	260.87	Material	0.00L	5.704
201	0.50	0.32	2X603.20	260.87	Stability	0.45L	24.844
202	0.50	0.32	2X603.20	260.87	Stability	0.40L	18.479
203	0.50	0.32	2X603.20	260.87	Stability	0.35L	9.166
204	0.50	0.32	2X603.20	260.87	Stability	0.30L	7.653
205	0.50	0.32	2X603.20	260.87	Stability	0.25L	7.042
206	0.50	0.32	2X603.20	260.87	Stability	0.20L	5.941
207	0.50	0.32	2X603.20	260.87	Stability	0.15L	5.101
208	0.50	0.32	2X603.20	260.87	Material	0.10L	4.240
209	0.50	0.32	2X603.20	260.87	Material	0.05L	3.850
210	0.50	0.32	2X603.20	260.87	Material	0.00L	3.062
211	0.50	0.48	2X452.39	313.04	Stability	0.45L	19.249
212	0.50	0.48	2X452.39	313.04	Stability	0.40L	14.493
213	0.50	0.48	2X452.39	313.04	Stability	0.35L	7.241
214	0.50	0.48	2X452.39	313.04	Stability	0.30L	5.937
215	0.50	0.48	2X452.39	313.04	Stability	0.25L	5.446
216	0.50	0.48	2X452.39	313.04	Material	0.20L	4.563
217	0.50	0.48	2X452.39	313.04	Material	0.15L	3.850
218	0.50	0.48	2X452.39	313.04	Material	0.10L	3.704
219	0.50	0.48	2X452.39	313.04	Material	0.05L	3.653
220	0.50	0.48	2X452.39	313.04	Material	0.00L	3.100
221	0.50	0.64	2X1005.31	347.83	Stability	0.40L	18.116
222	0.50	0.64	2X1005.31	347.83	Stability	0.35L	10.541
223	0.50	0.64	2X1005.31	347.83	Stability	0.30L	8.349
224	0.50	0.64	2X1005.31	347.83	Stability	0.25L	7.277
225	0.50	0.64	2X1005.31	347.83	Stability	0.20L	6.179
226	0.50	0.64	2X1005.31	347.83	Material	0.15L	5.510
227	0.50	0.64	2X1005.31	347.83	Material	0.10L	5.117
228	0.50	0.64	2X1005.31	347.83	Material	0.05L	4.837
229	0.50	0.64	2X1005.31	347.83	Material	0.00L	4.545
230	0.50	0.80	2X1885.00	400.00	Stability	0.35L	16.499
231	0.50	0.80	2X1885.00	400.00	Stability	0.30L	12.523
232	0.50	0.80	2X1885.00	400.00	Stability	0.25L	10.329
233	0.50	0.80	2X1885.00	400.00	Material	0.20L	9.601
234	0.50	0.80	2X1885.00	400.00	Material	0.15L	9.240
235	0.50	0.80	2X1885.00	400.00	Material	0.10L	8.187
236	0.50	0.80	2X1885.00	400.00	Material	0.05L	7.601
237	0.50	0.80	2X1885.00	400.00	Material	0.00L	7.070

Table 3.12: summary of results of failure sections and stabilized length; I_1/I_2 =0.5

Results of effective length and degree of safety

In this sub section effective length and degree of safety are compiled for all samples under this category for the fulfillment of the effective length methodology under investigation. As we discussed earlier, computation of effective length should follow *equation 2.2,* by taking the equivalent section to be the observed failure section of actual column. End moment capacity of actual column is try to be compared with that of the equivalent column to evaluate the degree of safety of the methodology by taking the requirement of failure moment in actual and equivalent columns. This is done by changing end moment on equivalent column system, having a length equal to the effective length, until the second order failure moment is similar to the original column. The end moment capacity from actual column analysis will then be divided by the end moment capacity from analysis of equivalent column, to get the degree of safety of the methodology.

For illustration purpose, observe the second order bending moment diagrams of few samples, which are done for both actual column and equivalent column for the observation of degree of safety. Discussion on safety of the methodology is widely covered on chapter 4 of this paper.

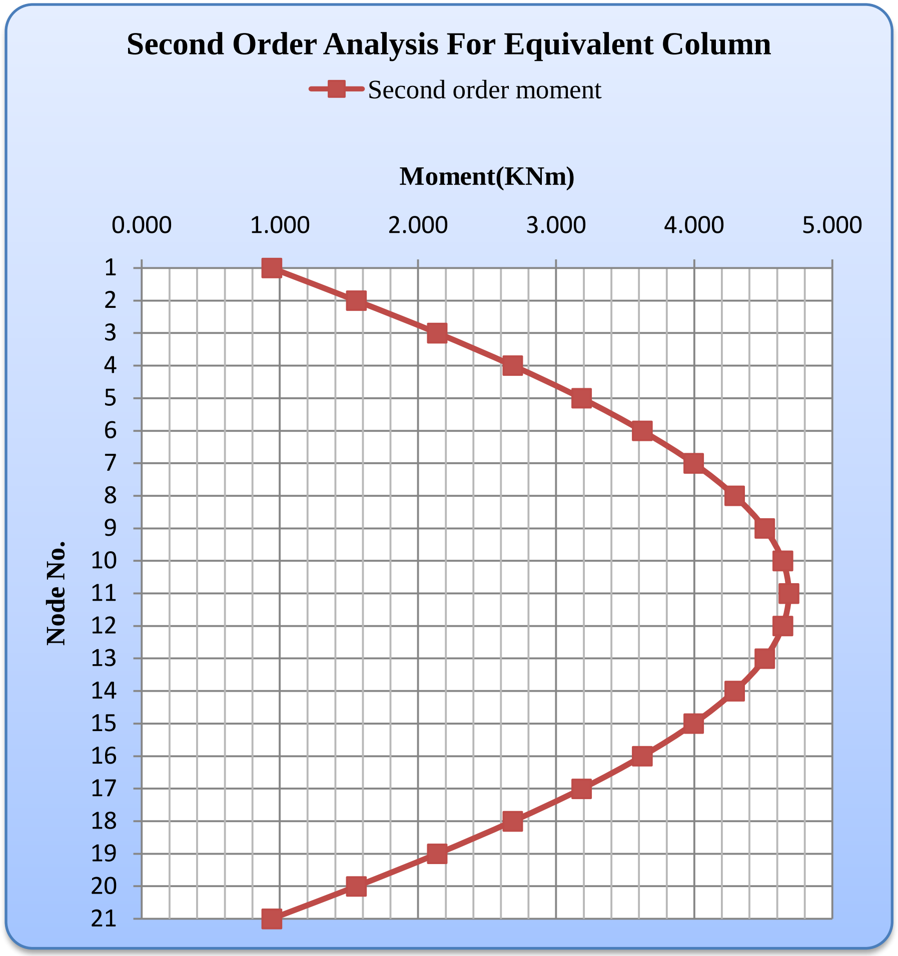


Fig. 3.13: Second order moment diagram of <u>equivalent column</u> for sample No.94
/ Please refer Fig. 3.1 for the analysis of actual column /

L_{eff}=9.9783m
$M_{end,\ equivalent}$=0.9413KNm (Safety factor of 1.062)
Design & equivalent Section at Node 7, (0.2m X 0.242599m)
Design Moment = 4.686KNm

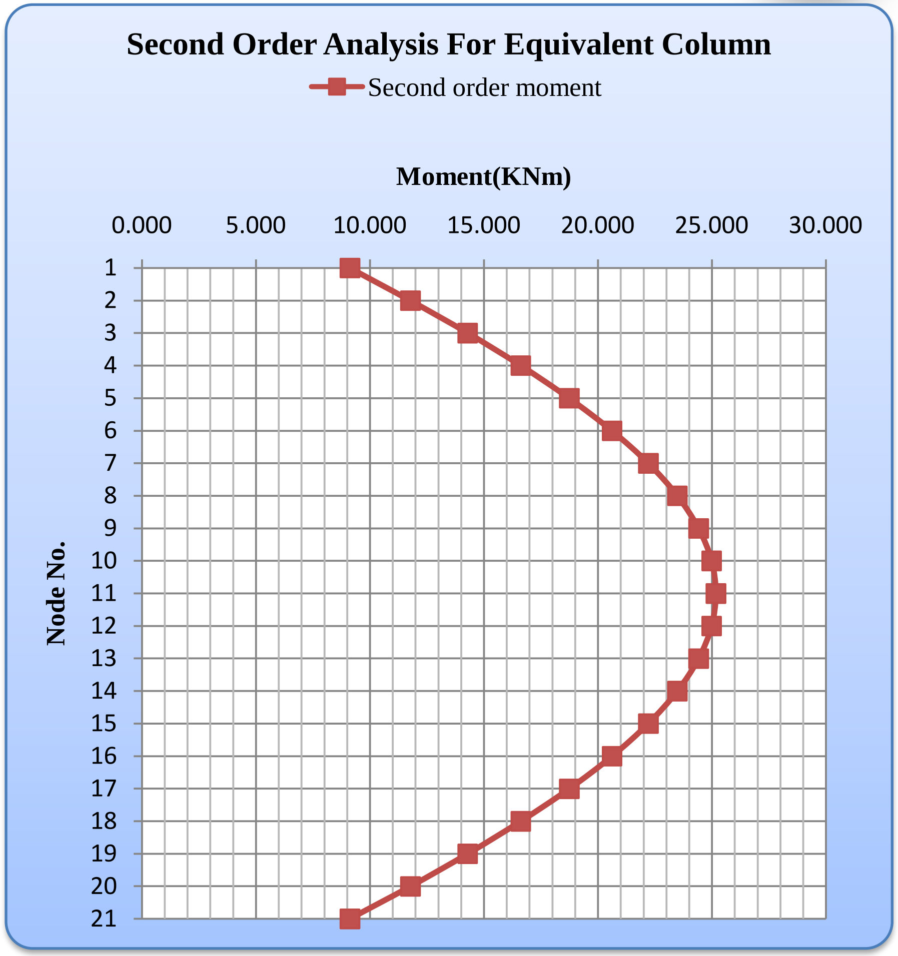


Fig. 3.14: Second order moment diagram of <u>equivalent column</u> for sample No.95
/ Please refer Fig. 3.2 for the analysis of actual column /

L_{eff}=7.6010m
$M_{end,\ equivalent}$=9.1169KNm (Safety factor of 1.097)
Design & equivalent Section at Node 6, (0.2m X 0.235499m)
Design Moment = 25.175KNm

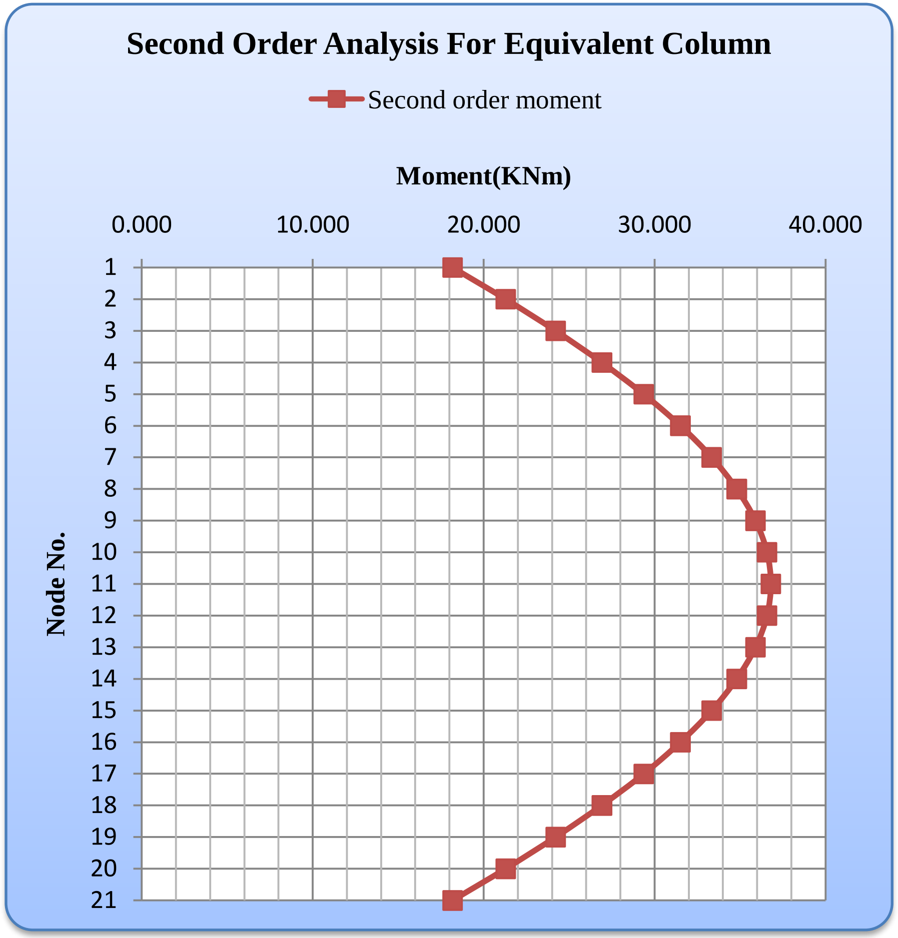


Fig. 3.15: Second order moment diagram of <u>equivalent column</u> for sample No.96
/ Please refer Fig. 3.3 for the analysis of actual column /

L_{eff}=5.6466m
$M_{end,\ equivalent}$=18.1834KNm (Safety factor of 1.149)
Design & equivalent Section at Node 5, (0.2m X 0.228399m)
Design Moment = 36.796KNm

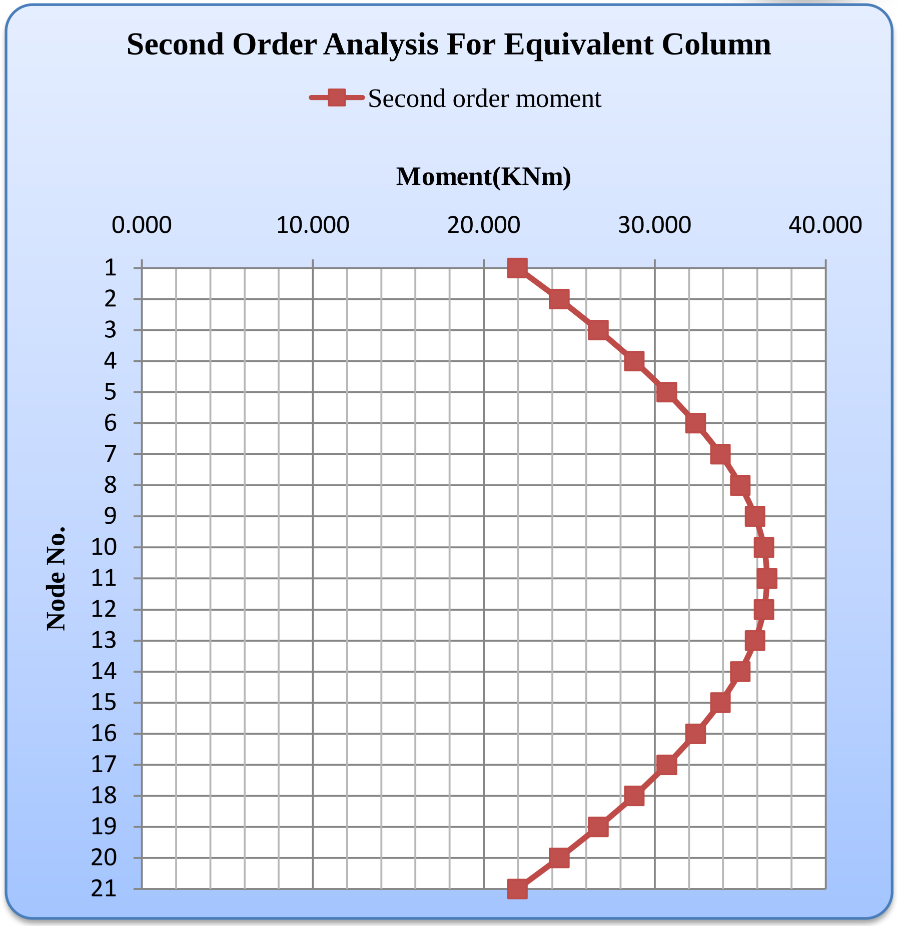


Fig. 3.16: Second order moment diagram of <u>equivalent column</u> for sample No.97
/ Please refer Fig. 3.4 for the analysis of actual column /

L_{eff}=4.4622m
$M_{end,\ equivalent}$=21.9633KNm (Safety factor of 1.188)
Design & equivalent Section at Node 4, (0.2m X 0.221299m)
Design Moment = 36.564KNm

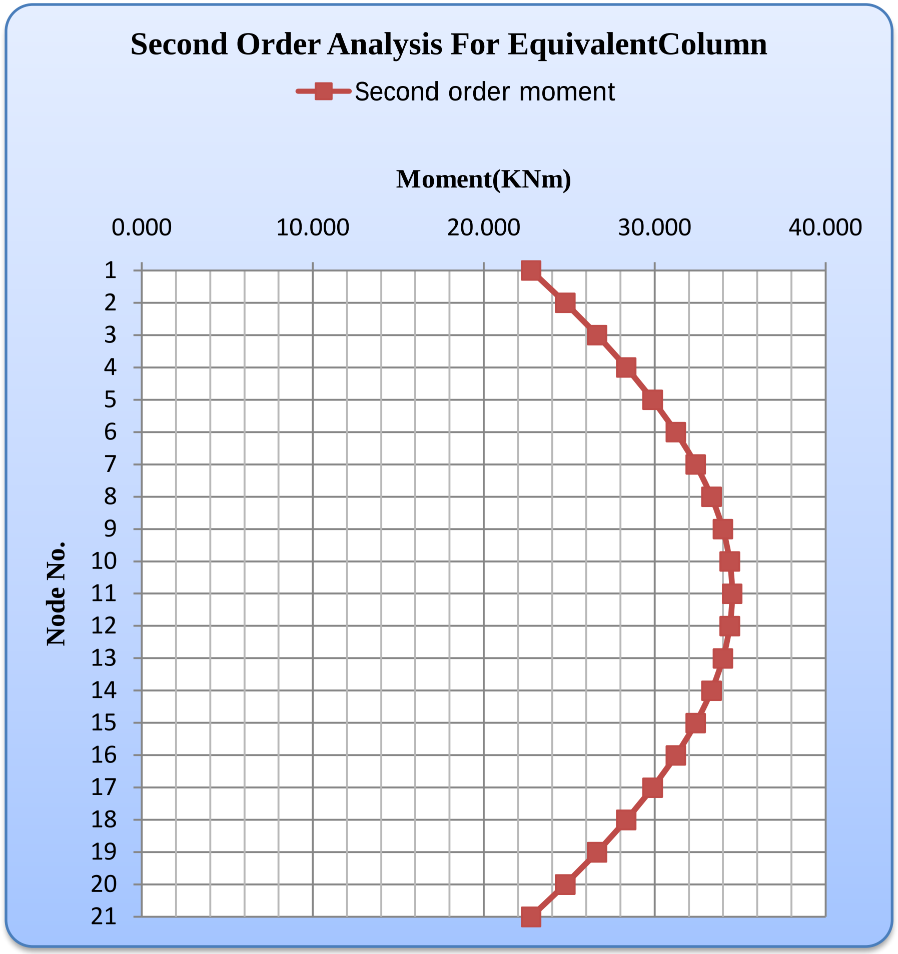


Fig. 3.17: Second order moment diagram of <u>equivalent column</u> for sample No.98
/ Please refer Fig. 3.5 for the analysis of actual column /

L_{eff}=3.8462m
$M_{end,\ equivalent}$=18.1834KNm (Safety factor of 1.2299)
Design & equivalent Section at Node 3, (0.2m X 0.214200m)
Design Moment = 34.526KNm

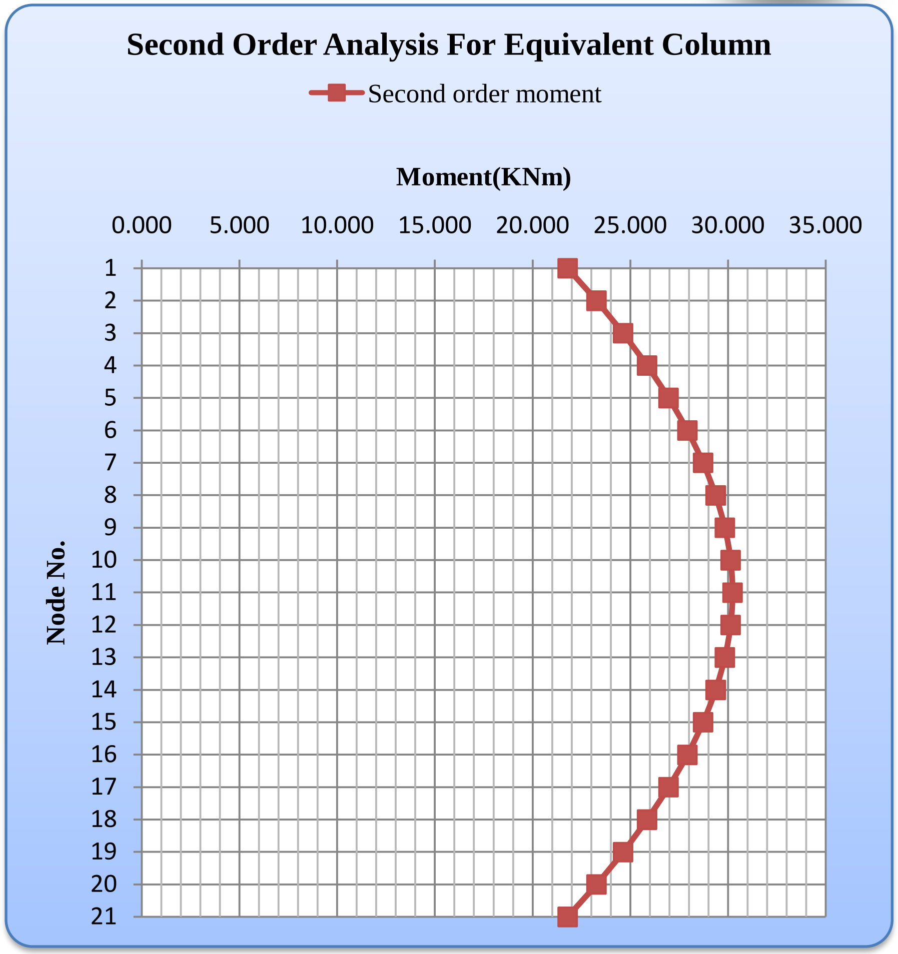


Fig. 3.18: Second order moment diagram of <u>equivalent column</u> for sample No.99
/ Please refer Fig. 3.6 for the analysis of actual column /

L_{eff}=3.4825m
$M_{end,\ equivalent}$=21.781KNm (Safety factor of 1.299)
Design & equivalent Section at Node 2, (0.2m X 0.207100m)
Design Moment = 30.230KNm

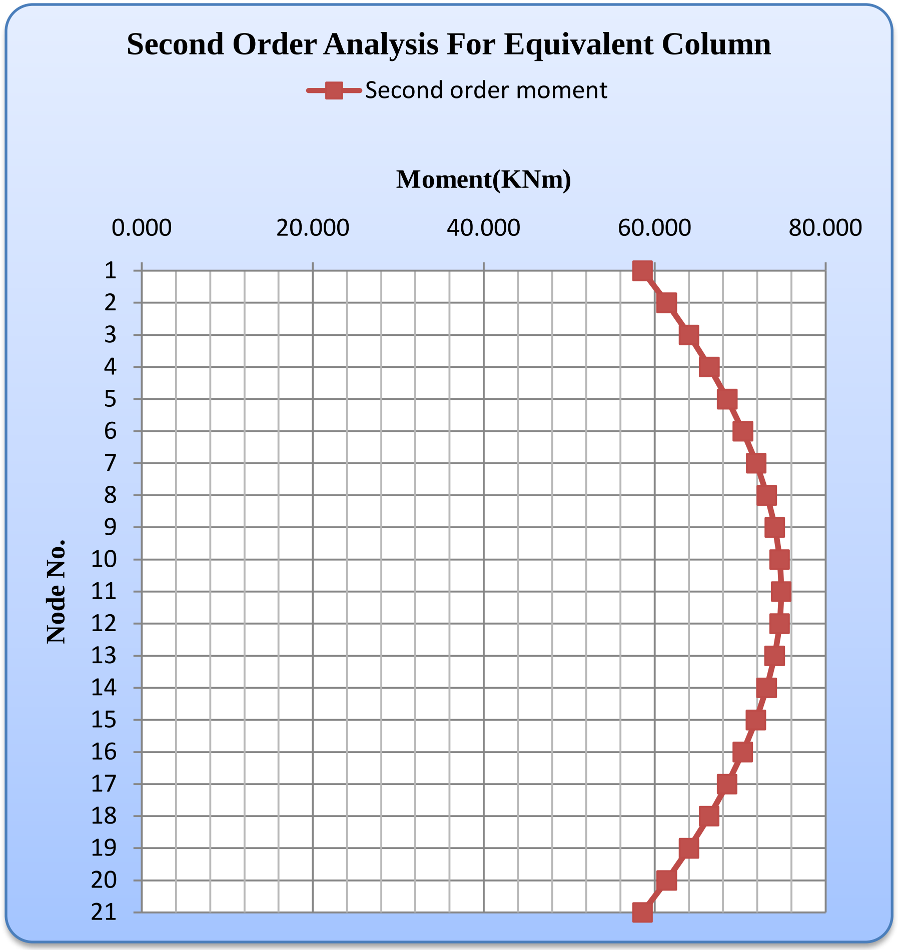


Fig. 3.19: Second order moment diagram of <u>equivalent column</u> for sample No.214
/ Please refer Fig. 3.7 for the analysis of actual column /

L_{eff}=6.0104m
$M_{end,\ equivalent}$=58.5708KNm (Safety factor of 1.0253)
Design & equivalent Section at Node 7, (0.3m X 0.323393m)
Design Moment = 74.809KNm

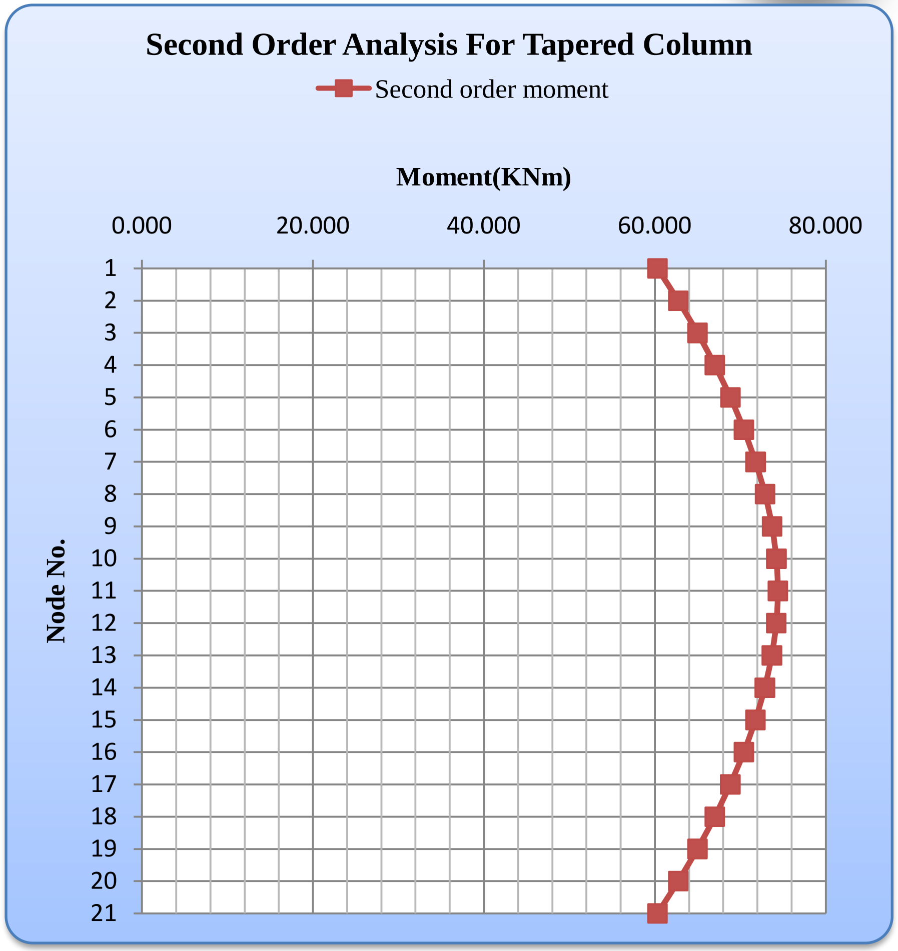


Fig. 3.20: Second order moment diagram of <u>equivalent column</u> for sample No.215
/ Please refer Fig. 3.8 for the analysis of actual column /

L_{eff}=5.348692m
$M_{end,\ equivalent}$=60.3015KNm (Safety factor of 1.0365)
Design & equivalent Section at Node 6, (0.3m X 0.319494m)
Design Moment = 74.809KNm

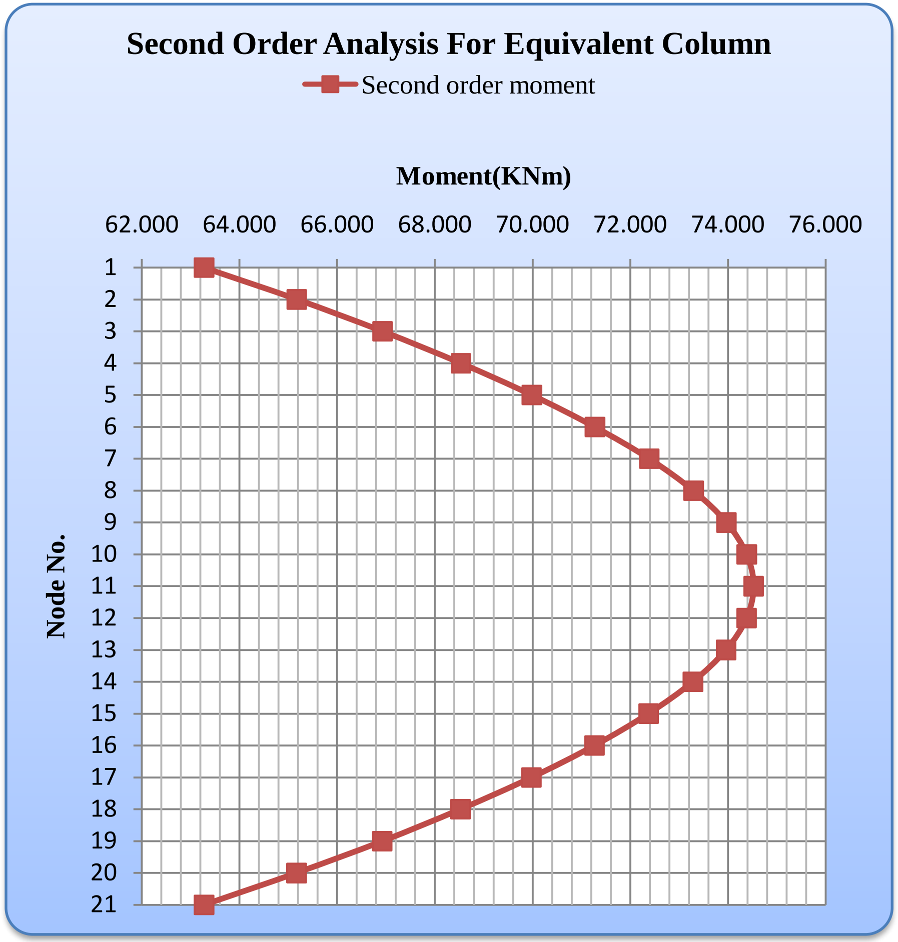


Fig. 3.21: Second order moment diagram of equivalent column for sample No.216
/ Please refer Fig. 3.9 for the analysis of actual column /

L_{eff}=4.3457m
$M_{end, equivalent}$=60.2726KNm (Safety factor of 1.097)
Design & equivalent Section at Node 5, (0.3m X 0.315595m)
Design Moment = 74.7438KNm

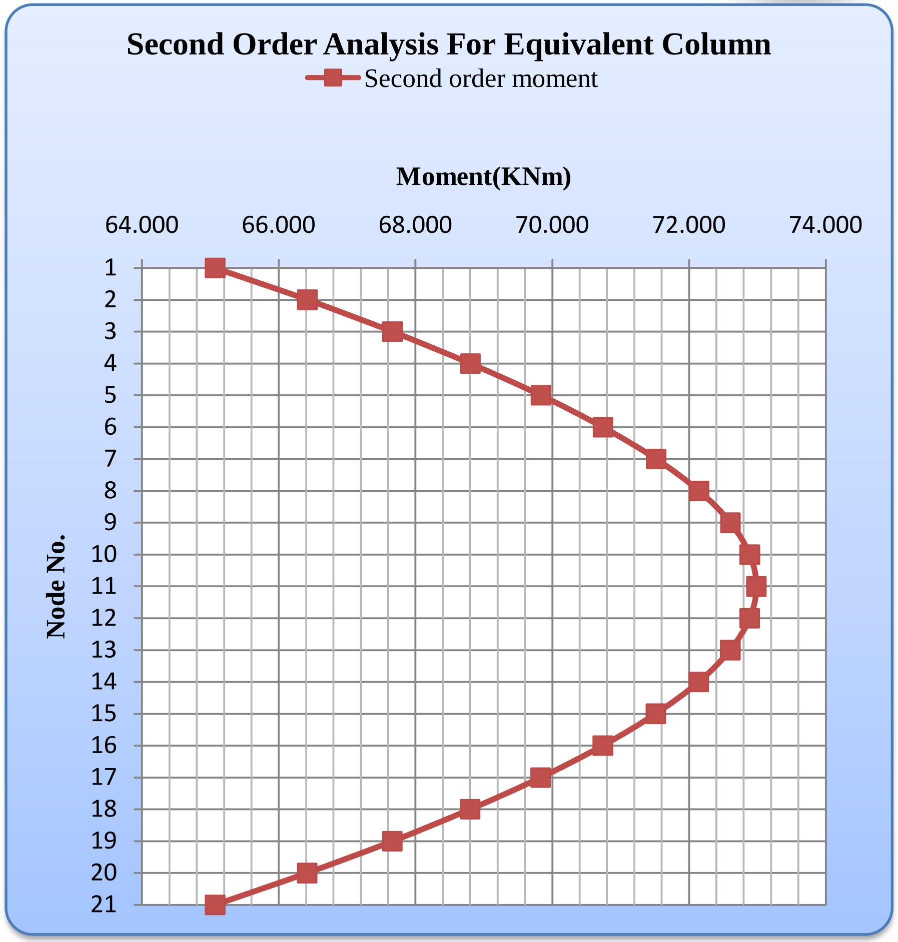


Fig. 3.22: Second order moment diagram of equivalent column for sample No.217
/ Please refer Fig. 3.10 for the analysis of actual column /

L_{eff}=3.5545m
$M_{end, equivalent}$=65.0678KNm (Safety factor of 1.008)
Design & equivalent Section at Node 4, (0.3m X 0.3117m)
Design Moment = 72.983KNm

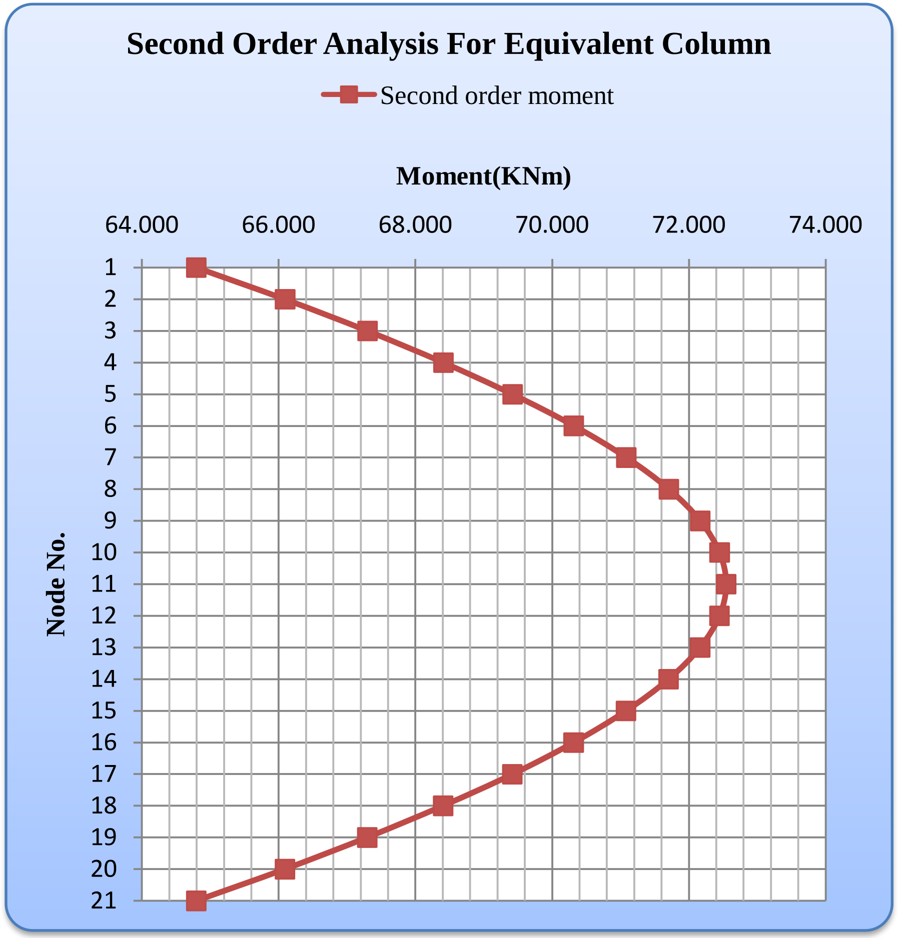


Fig. 3.23: Second order moment diagram of <u>equivalent column</u> for sample No.218
/ Please refer Fig. 3.11 for the analysis of actual column /

L_{eff}=3.3137m
$M_{end,\ equivalent}$=64.7946KNm (Safety factor of 1.0680)
Design & equivalent Section at Node 3, (0.3m X 0.3078m)
Design Moment = 72.543KNm

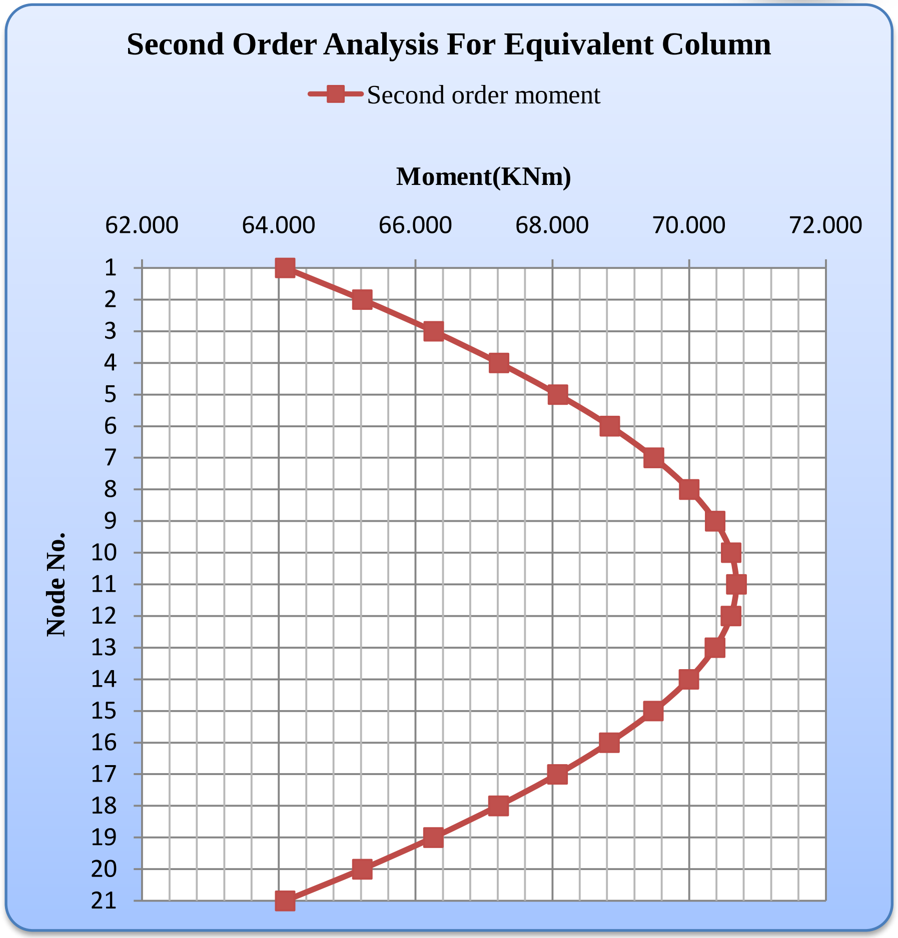


Fig. 3.24: Second order moment diagram of <u>equivalent column</u> for sample No.219
/ Please refer Fig. 3.12 for the analysis of actual column /

L_{eff}=3.1653m
$M_{end,\ equivalent}$=64.0914KNm (Safety factor of 1.0813)
Design & equivalent Section at Node 2, (0.3m X 0.303899m)
Design Moment = 70.691KNm

Results for effective length, effective length factor and safety factor, for each sample are summarized in *tables 3.13, 3.14, 3.15, 3.16 and 3.17*

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	Failure section at:	L_{actual} (m)	L_{eff} (m)	Safety factor
28	0.100	0.16	0.30L	80.000	72.916	0.699
29	0.100	0.16	0.25L	65.000	52.212	1.018
30	0.100	0.16	0.20L	55.000	41.245	1.061
31	0.100	0.16	0.15L	49.000	34.191	1.108
32	0.100	0.16	0.10L	43.000	27.817	1.136
33	0.100	0.16	0.05L	39.000	23.297	1.179
34	0.100	0.16	0.00L	31.000	-	-
35	0.100	0.32	0.30L	95.000	81.494	1.030
36	0.100	0.32	0.25L	80.000	64.261	1.050
37	0.100	0.32	0.20L	45.000	33.746	1.058
38	0.100	0.32	0.15L	39.000	27.213	1.093
39	0.100	0.32	0.10L	35.000	22.642	1.128
40	0.100	0.32	0.05L	30.000	17.921	1.160
41	0.100	0.32	0.00L	23.420	-	-
42	0.100	0.48	0.35L	90.000	82.208	0.446
43	0.100	0.48	0.30L	80.000	68.613	1.010
44	0.100	0.48	0.25L	65.000	52.200	1.067
45	0.100	0.48	0.20L	40.000	29.988	1.085
46	0.100	0.48	0.15L	34.000	23.718	1.119
47	0.100	0.48	0.10L	30.000	19.401	1.155
48	0.100	0.48	0.05L	27.000	16.123	1.201
49	0.100	0.48	0.00L	21.413	-	-
50	0.100	0.64	0.30L	54.000	46.326	0.221
51	0.100	0.64	0.25L	49.000	39.362	1.123
52	0.100	0.64	0.20L	40.000	29.998	1.160
53	0.100	0.64	0.15L	31.000	21.633	1.194
54	0.100	0.64	0.10L	24.000	15.527	1.246
55	0.100	0.64	0.05L	21.500	12.844	1.311
56	0.100	0.64	0.00L	18.000	-	-
57	0.100	0.80	0.30L	43.000	36.887	0.382
58	0.100	0.80	0.25L	40.000	32.131	1.163
59	0.100	0.80	0.20L	35.000	26.247	1.210
60	0.100	0.80	0.15L	28.000	19.538	1.260
61	0.100	0.80	0.10L	0.80	16.173	1.364
62	0.100	0.80	0.05L	22.000	13.142	1.499
63	0.100	0.80	0.00L	20.000	-	-

Table 3.13: summary of effective length and safety factors of column samples; I_1/I_2 =0.1

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	Failure section at:	L_{actual} (m)	L_{eff} (m)	Safety factor
64	0.200	0.16	0.30L	85.000	75.066	1.014
65	0.200	0.16	0.25L	75.000	59.970	1.031
66	0.200	0.16	0.20L	65.000	52.439	1.049
67	0.200	0.16	0.15L	58.000	44.628	1.067
68	0.200	0.16	0.10L	51.500	38.834	1.088
69	0.200	0.16	0.05L	48.500	34.134	1.112
70	0.200	0.16	0.00L	38.500	-	-
71	0.200	0.32	0.35L	80.000	73.767	1.009
72	0.200	0.32	0.30L	70.000	61.813	1.045
73	0.200	0.32	0.25L	38.000	32.093	1.053
74	0.200	0.32	0.20L	29.000	23.393	1.064
75	0.200	0.32	0.15L	24.500	18.849	1.079
76	0.200	0.32	0.10L	23.000	16.850	1.108
77	0.200	0.32	0.05L	20.000	13.930	1.136
78	0.200	0.32	0.00L	15.080	-	-
79	0.200	0.48	0.35L	30.000	27.662	0.959
80	0.200	0.48	0.30L	20.000	17.661	1.038
81	0.200	0.48	0.25L	14.000	11.824	1.055
82	0.200	0.48	0.20L	12.300	9.922	1.084
83	0.200	0.48	0.15L	11.000	8.463	1.104
84	0.200	0.48	0.10L	9.500	6.960	1.123
85	0.200	0.48	0.05L	8.800	6.129	1.154
86	0.200	0.48	0.00L	7.390	-	-
87	0.200	0.64	0.30L	47.000	41.503	1.056
88	0.200	0.64	0.25L	37.500	31.671	1.084
89	0.200	0.64	0.20L	34.000	27.426	1.118
90	0.200	0.64	0.15L	22.500	17.310	1.133
91	0.200	0.64	0.10L	20.500	15.019	1.165
92	0.200	0.64	0.05L	19.000	13.233	1.202
93	0.200	0.64	0.00L	16.570	-	-
94	0.200	0.80	0.30L	08.3	9.978	1.062
95	0.200	0.80	0.25L	07.0	7.601	1.097
96	0.200	0.80	0.20L	06.2	5.647	1.149
97	0.200	0.80	0.15L	05.6	4.462	1.188
98	0.200	0.80	0.10L	05.1	3.846	1.230
99	0.200	0.80	0.05L	04.8	3.482	1.299
100	0.200	0.80	0.00L	04.8	-	-

Table 3.14: summary of effective length and safety factors of column samples; I_1/I_2 =0.2

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	Failure section at:	L_{actual} (m)	L_{eff} (m)	Safety factor
101	0.300	0.16	0.40L	79.000	73.777	1.000
102	0.300	0.16	0.35L	65.000	62.629	1.000
103	0.300	0.16	0.30L	50.000	45.227	1.023
104	0.300	0.16	0.25L	41.000	35.897	1.036
105	0.300	0.16	0.20L	38.000	32.179	1.052
106	0.300	0.16	0.15L	35.000	28.646	1.070
107	0.300	0.16	0.10L	30.000	23.712	1.081
108	0.300	0.16	0.05L	27.000	20.592	1.099
109	0.300	0.16	0.00L	18.630	-	-
110	0.300	0.32	0.40L	35.000	33.726	1.000
111	0.300	0.32	0.35L	29.000	27.085	1.000
112	0.300	0.32	0.30L	18.000	16.283	1.039
113	0.300	0.32	0.25L	15.000	13.134	1.045
114	0.300	0.32	0.20L	13.000	11.010	1.059
115	0.300	0.32	0.15L	11.000	9.004	1.071
116	0.300	0.32	0.10L	10.100	7.984	1.090
117	0.300	0.32	0.05L	9.500	7.246	1.115
118	0.300	0.32	0.00L	6.560	-	-
119	0.300	0.48	0.35L	43.000	40.157	1.019
120	0.300	0.48	0.30L	22.000	19.900	1.032
121	0.300	0.48	0.25L	17.000	14.884	1.041
122	0.300	0.48	0.20L	15.000	12.702	1.056
123	0.300	0.48	0.15L	13.500	11.049	1.074
124	0.300	0.48	0.10L	12.800	10.117	1.099
125	0.300	0.48	0.05L	12.000	9.152	1.121
126	0.300	0.48	0.00L	9.870	-	-
127	0.300	0.64	0.40L	30.000	28.906	0.708
128	0.300	0.64	0.35L	25.000	23.347	1.030
129	0.300	0.64	0.30L	16.000	14.473	1.049
130	0.300	0.64	0.25L	13.000	11.382	1.071
131	0.300	0.64	0.20L	11.100	9.400	1.091
132	0.300	0.64	0.15L	10.000	8.184	1.114
133	0.300	0.64	0.10L	9.000	7.113	1.136
134	0.300	0.64	0.05L	8.200	6.254	1.158
135	0.300	0.64	0.00L	7.300	-	-
136	0.300	0.80	0.35L	36.000	33.620	1.667
137	0.300	0.80	0.30L	27.000	24.423	1.088
138	0.300	0.80	0.25L	20.000	18.211	1.104
139	0.300	0.80	0.20L	17.500	14.819	1.134
140	0.300	0.80	0.15L	15.800	12.931	1.175
141	0.300	0.80	0.10L	14.500	11.461	1.219
142	0.300	0.80	0.05L	13.500	10.296	1.262
143	0.300	0.80	0.00L	12.200	-	-

Table 3.15: summary of effective length and safety factors of column samples; I_1/I_2 =0.3

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	Failure section at:	L_{actual} (m)	L_{eff} (m)	Safety factor
144	0.400	0.16	0.40L	48.000	46.462	1.004
145	0.400	0.16	0.35L	30.000	28.361	1.041
146	0.400	0.16	0.30L	25.000	23.073	1.062
147	0.400	0.16	0.25L	20.000	18.014	1.042
148	0.400	0.16	0.20L	18.500	16.255	1.080
149	0.400	0.16	0.15L	16.500	14.136	1.091
150	0.400	0.16	0.10L	15.500	12.943	1.085
151	0.400	0.16	0.05L	12.000	9.763	1.083
152	0.400	0.16	0.00L	10.680	-	-
153	0.400	0.32	0.45L	44.000	43.593	1.000
154	0.400	0.32	0.40L	41.000	39.686	1.000
155	0.400	0.32	0.35L	21.000	19.853	1.027
156	0.400	0.32	0.30L	17.000	15.690	1.029
157	0.400	0.32	0.25L	13.000	11.709	1.033
158	0.400	0.32	0.20L	11.500	10.104	1.043
159	0.400	0.32	0.15L	10.800	9.253	1.060
160	0.400	0.32	0.10L	9.500	7.933	1.073
161	0.400	0.32	0.05L	8.520	6.931	1.089
162	0.400	0.32	0.00L	5.880	-	-
163	0.400	0.48	0.45L	32.600	32.298	1.000
164	0.400	0.48	0.40L	30.000	29.039	1.000
165	0.400	0.48	0.35L	15.400	14.464	1.010
166	0.400	0.48	0.30L	12.800	11.814	1.031
167	0.400	0.48	0.25L	10.500	9.457	1.039
168	0.400	0.48	0.20L	9.100	7.996	1.050
169	0.400	0.48	0.15L	7.800	6.683	1.062
170	0.400	0.48	0.10L	7.100	5.929	1.079
171	0.400	0.48	0.05L	6.600	5.369	1.093
172	0.400	0.48	0.00L	5.829	-	-
173	0.400	0.64	0.40L	29.000	28.071	0.553
174	0.400	0.64	0.35L	16.000	15.126	1.021
175	0.400	0.64	0.30L	12.800	11.814	1.038
176	0.400	0.64	0.25L	11.000	9.908	1.052
177	0.400	0.64	0.20L	9.500	8.347	1.064
178	0.400	0.64	0.15L	8.600	7.368	1.078
179	0.400	0.64	0.10L	7.700	6.430	1.090
180	0.400	0.64	0.05L	7.200	5.858	1.106
181	0.400	0.64	0.00L	6.415	-	-
182	0.400	0.80	0.40L	28.000	27.103	0.413
183	0.400	0.80	0.35L	21.000	19.853	1.035
184	0.400	0.80	0.30L	17.000	15.690	1.056
185	0.400	0.80	0.25L	13.800	12.430	1.078
186	0.400	0.80	0.20L	11.000	9.665	1.114
187	0.400	0.80	0.15L	9.500	8.139	1.129
188	0.400	0.80	0.10L	8.750	7.307	1.151
189	0.400	0.80	0.05L	8.500	6.915	1.186
190	0.400	0.80	0.00L	7.964	-	-

Table 3.16: summary of effective length and safety factors of column samples; I_1/I_2 =0.4

Sample No.	I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{min}}}$	Failure section at:	L_{actual} (m)	L_{eff} (m)	Safety factor
191	0.500	0.16	0.45L	57.000	56.460	0.901
192	0.500	0.16	0.40L	40.000	38.932	1.010
193	0.500	0.16	0.35L	33.800	32.318	1.017
194	0.500	0.16	0.30L	23.000	21.600	1.041
195	0.500	0.16	0.25L	18.200	16.784	1.033
196	0.500	0.16	0.20L	16.300	14.757	1.038
197	0.500	0.16	0.15L	15.400	13.685	1.056
198	0.500	0.16	0.10L	14.000	12.208	1.061
199	0.500	0.16	0.05L	12.300	10.523	1.070
200	0.500	0.16	0.00L	8.556	-	-
201	0.500	0.32	0.45L	55.500	54.974	0.874
202	0.500	0.32	0.40L	40.800	39.710	1.010
203	0.500	0.32	0.35L	20.000	19.123	1.014
204	0.500	0.32	0.30L	16.500	15.495	1.022
205	0.500	0.32	0.25L	15.000	13.833	1.033
206	0.500	0.32	0.20L	12.500	11.317	1.039
207	0.500	0.32	0.15L	10.600	9.420	1.050
208	0.500	0.32	0.10L	8.700	7.587	1.060
209	0.500	0.32	0.05L	7.800	6.673	1.071
210	0.500	0.32	0.00L	6.124	-	-
211	0.500	0.48	0.45L	21.500	21.296	0.901
212	0.500	0.48	0.40L	16.000	15.573	1.010
213	0.500	0.48	0.35L	7.900	7.554	1.017
214	0.500	0.48	0.30L	6.400	6.010	1.025
215	0.500	0.48	0.25L	5.800	5.349	1.036
216	0.500	0.48	0.20L	4.800	4.346	1.045
217	0.500	0.48	0.15L	4.000	3.555	1.054
218	0.500	0.48	0.10L	3.800	3.314	1.068
219	0.500	0.48	0.05L	3.700	3.165	1.081
220	0.500	0.48	0.00L	3.100	-	-
221	0.500	0.64	0.40L	40.000	38.932	1.021
222	0.500	0.64	0.35L	23.000	21.992	1.116
223	0.500	0.64	0.30L	18.000	16.904	1.036
224	0.500	0.64	0.25L	15.500	14.294	1.048
225	0.500	0.64	0.20L	13.000	11.770	1.056
226	0.500	0.64	0.15L	11.450	10.175	1.066
227	0.500	0.64	0.10L	10.500	9.156	1.078
228	0.500	0.64	0.05L	9.800	8.384	1.090
229	0.500	0.64	0.00L	9.090	-	-
230	0.500	0.80	0.35L	18.000	17.211	1.030
231	0.500	0.80	0.30L	13.500	12.678	1.114
232	0.500	0.80	0.25L	11.000	10.144	1.065
233	0.500	0.80	0.20L	10.100	9.144	1.084
234	0.500	0.80	0.15L	9.600	8.531	1.107
235	0.500	0.80	0.10L	8.400	7.325	1.117
236	0.500	0.80	0.05L	7.700	6.587	1.130
237	0.500	0.80	0.00L	7.070	-	-

Table 3.17: summary of effective length and safety factors of column samples; I_1/I_2 =0.5

CHAPTER 4
DISCUSSION ON RESULTS

In chapter 3, we made an iterative second order analysis for a number of columns having linearly varying section with different geometrical and loading conditions. Conversion of actual column to the equivalent one was another task that has been made. The current chapter is introduced to make a detailed discussion on the observed results together with conclusion and generalization. This chapter can be taken as the one that the output of the research is to be presented. As a result, a general effective length methodology will be introduced for the application of result of the research for any type of column that has linearly varying cross-section and equal end moments with single mode of curvature. In addition, the direction of the future on the research area will be indicated at last.

4.1 General observation

Non-tapered (prismatic) columns do have common and comfortable features that make the analysis easy. Such type of columns do have constant cross-section and hence do have the same section capacity along the column height. Such types of column do have a unique mode of deflection shape, which is a half sine function curve. This helped us to have an easier methodology for computation of the elastic buckling load of a given prismatic column using its geometrical and material features (recall the Euler's buckling load computation on section 1.7.1 of this document). They have symmetric geometrical orientation by its nature; accordingly, the second order moment curve will be symmetrical about the mid height of the column. The maximum deflection as well as bending moment is located at the middle of the column height as long as the applied end moments are similar at top and bottom of the column. Middle of column is where both design moment and design section are located. These and such simplified features make prismatic columns to be preferable and destination of remarkable studies. Investigation of effective length method for prismatic column is one example of research out puts from the past.

Tapered columns, in contrary, do have inter-related features that are not comfortable for the one who make analysis. Such type of columns do not have constant cross-section and hence do have variable section capacity along the column height. Buckling curves of tapered columns of different geometrical pattern are not unique. Their nature of geometrical orientation is not symmetric and hence the analysis is more complicated. Middle of column is not the location of maximum deflection and bending moment, even though the end moments are equal. Design section and design moment are located differently for different tapered columns. These and such features of tapered columns cannot attract researchers to make investigations on the area.

In spite of the above mentioned discouraging features, remarkable and meaningful results are observed in this paper regarding on columns which have linearly varying cross-section and equal end moments with single mode of curvature. Tapered columns of similar pre-determined loading and geometrical parameters (which are to be discussed latter) are found to have similar mode and location of failure.

Results of all columns samples under study indicate practical columns of linearly varying cross-section with equal end moment and single curvature do have a unique range in location of maximum second order moment, and hence maximum deflection (refer *fig. 1 to 12* and *tables 3.7 to 3.12*). That location is found not to exceed 0.45L from the minimum section. However, this can further extend to 0.50L when the system approaches to be prismatic. As try to be discussed in section 3, all samples are carefully selected to cover a variety of material quality, steel ratio, loading and side slope of the study column. Here, it is important to note that the maximum moment is not to mean the failure (design) moment. Hence the location of maximum moment is not necessarily the location of failure moment. The design (failure) moment is the one which do have the lowest gap with the moment capacity of that section. However knowing the location of maximum moment is important to set the general extreme failure point of practical tapered columns. Since the section capacity continuously increases along the column height, it can be understood from the result that any tapered column system can never fail at a distance more than 0.50L from the minimum section.

It can also clearly be observed that the slope of capacity curve is more dependent on the difference in the ratio of moment of inertia of the minimum section to the maximum section. When this ratio is smaller, the capacity curve becomes steeper, so that there will be a significant section capacity difference along the column height. As the ratio of moment of inertia of minimum section to maximum section become greater, the capacity curve will be flatter so that it becomes vertical when the ratio is unity (case of prismatic column).

The other aspects of results of all column samples come with magnificent and new findings to become the core stone of effective length methodology under investigation. Before discussing on these findings, it is equally important to review a previously committed thesis document entitled "buckling loads for tapered slender elastic columns" by *Thomas A.Branch*, since it is one of the reference documents for this research paper. The researcher, finally, investigated that the elastic buckling load of tapered columns should be calculated as:

$$P_{cr} = \frac{mEI_2}{L^2}$$

Where, m is a constant dependent on *the ratio of moment of inertia of the minimum section to that of the maximum (I_1/I_2)*, about major bending axis. The researcher pointed out that the behavior of tapered columns is greatly dependent on its inertial ratio of the minimum section to the maximum.

For the research of this paper, it is by this basis that the column samples are categorized according to its ratio of moment of inertia (I_1/I_2). Varity of columns with different loading, material and geometric data are treated by categorizing according to its value of I_1/I_2 (refer *tables 3.1* to *3.6*). I_1/I_2 values of 0.10, 0.20, 0.30, 0.40 and 0.50 are covered on the study. Tapered columns which have inertial ratio of more than 0.50 are not included in this paper due to practical and technical reasons. Such types of columns are not common since they are closer to prismatic types of columns. Rather than using columns of higher inertial ratio, it is usually preferable to use accustomed prismatic columns, since similarity in sections along the column height neither provides significant structural benefits nor appears to be architecturally unique.

A parameter; $\sqrt{\frac{P}{f_{cd}\, A_{c,min}}}$ is also used under the process of sample categorization. It is the square root of the ratio of axial load to the design axial load capacity of the gross minimum concrete section. This parameter answers a question: how much axial load is applied on the column with the reference to the design axial capacity of the minimum section? For instance if a column has a value of $\sqrt{\frac{P}{f_{cd}\, A_{c,min}}} = 0.8$, it is supposed to mean an axial load of 64% of the design axial capacity of the minimum section is applied. If we try to treat the minimum section, the rest 34% of concrete capacity together with the reinforcements will make the system tough to carry the additional stresses from the end moments as well as the second order moment. However it should be noted that the parameter $\sqrt{\frac{P}{f_{cd}\, A_{c,min}}}$ is indirectly related to the steel ratio of column.

An Author of the book “Reinforced concrete mechanics and design” J.G.MacGregor in his fifth edition states the following article by quoting the prior editions of ACI code.

Prismatic columns that may have P-δ moments between the ends of the column that exceed the P-Δ moments at the ends:

$$\frac{L}{r} > \frac{35}{\sqrt{\frac{P}{f_{cd} A_c}}}$$

Where, r is the radius of gyration of the column section.

If the L/r exceeds the value given in the above equation, there is a chance that the maximum moment on the column will exceed the larger end moment. Absolutely $\sqrt{\frac{P}{f_{cd}\,A_c}}$ is a parameter among few which can determine the location of failure section along the column height.

As per expectation, irrespective all other parameters, the value of $\sqrt{\frac{P}{f_{cd}\,A_{c,min}}}$ is found be an important parameter for the categorization of tapered columns having similar point of failure. In addition the stabilized length $L_s = \frac{B}{W_i} L$, is also found to be another powerful parameter for the categorization purpose, where:

- B is the constant dimension (breadth) of column section.
- L is the actual length (height) of tapered column.
- W_i is the width of column section at the point of failure

Samples from 1 to 27 are presented and analyzed for illustration purpose of this section (refer *table 3.1 and table 3.7*). Consider the following summary of observation from the above mentioned sample columns.

I_1/I_2	$\sqrt{\frac{P}{f_{cd}A_{c,min}}}$	Sample No.	$L_s = \frac{B}{W_i}L$ (m)	Failure section at:
0.10	0.553	1	15.52	0.225L
		5	15.52	0.250L
		10	15.52	0.250L
0.10	0.553	2	13.54	0.175L
		6	13.54	0.200L
		11	13.52	0.200L
0.10	0.553	3	12.502	0.125L
		7	12.502	0.150L
		12	12.456	0.150L
0.10	0.553	4	12.502	0.075L
		8	11.356	0.100L
		13	11.275	0.100L
0.10	0.553	9	10.085	0.050L
		14	10.065	0.050L
0.30	0.32	16	8.490	0.350L
		22	8.458	0.350L
0.30	0.32	17	10.242	0.300L
		23	10.179	0.300L
0.30	0.32	18	12.462	0.250L
		24	12.461	0.250L
0.30	0.32	19	14.807	0.150L
		25	14.806	0.150L
0.30	0.32	20	17.053	0.050L
		26	17.052	0.050L
0.30	0.32	21	6.300	0.000L
		27	6.000	0.000L

Table 4.1: Illustration for comparison of failure nodes

Observe that all the column samples in *table 4.1*, cover variety of material, loading and geometric features, except the computed values of I_1/I_2 and $\sqrt{\frac{P}{f_{cd}A_{c,min}}}$. But failure point of those columns having similar value of I_1/I_2 and $\sqrt{\frac{P}{f_{cd}A_{c,min}}}$ are the same. For example, column samples 1, 5 and 10 fail at in narrow range of 0.225L to 0.25L from the top of column. The computed value of L_s at section is found to be similar, which is 15.52m. The same features can also be observed on the remaining rows of *table 4.1*. From this we can introduce *Rule 1* as follows.

Rule 1: *"Tapered columns having the same values of I_1/I_2 and $\sqrt{\frac{P}{f_{cd}A_{c,min}}}$ do generally have a similar value of L_s at failure section, but with the presence of some boundary conditions"*

Recall that the failure node of tapered column can never exceed 0.5L from the minimum section. This is clearly discussed in the fourth paragraph of this section. From this we can introduce *Rule 2* as follows.

Rule 2: *"For columns having linearly varying cross-section and equal end moment with single curvature, the location of failure node never exceed 0.50L from the smaller cross-section"*

A concentrically loaded column, which lacks end moment, can carry an axial load safely as long as its length is not more than:

$\pi\sqrt{\frac{EI}{P_{cr}}}$ For prismatic columns Eq. 4.1a

$\sqrt{\frac{mEI_2}{P_{cr}}}$ For tapered columns Eq. 4.1b

However slender columns having moment at the end of the column and a length as of *Eq. 4.1* cannot carry the axial load since the presence of end moments reduce the system's capacity. In order to create a safe column system of the same cross-section, either the axial load or the length of column should be reduced if end moments are desired to be considered. In this research, the limit of column length is introduced by considering a minimum eccentricity at the end of columns. The limit of column length then creates a *limit for the stabilized length* which is here designated as $L_{s,lim}$. Rule 1 holds true as long as the value of L_s do not exceed a constant value $L_{s,lim}$. The limiting value is determined by taking a consideration of practical column length and end eccentricity ranges, which are covered in this research. Accordingly *Rule 3* should be written as follows:

Rule 3: *"If L_s at 0.45L from the minimum section exceeds its limiting value, which is also dependent on I_1/I_2 and $\sqrt{\frac{P}{f_{cd}A_{c,min}}}$, the failure section will be at a point where the limiting value of L_s is satisfied"*

Computation of L_s with its limiting values for ranges of tapered column systems are carefully committed and presented in Annex 2 of this document. All the results are presented for the user in tables A, B, C D and E, indicated at the end of this chapter, in user friendly manner. The combination of all the above mentioned Rules is going to form a general methodology, which is presented in the next section. (Refer *section 4.2*)

The evaluation of effective length should be done from the requirement of elastic buckling load. This is as per the detailed discussion made in section 2.4 of this document. The effective length computation involves the following simple derivation.

$$\begin{bmatrix} Elastic\ buckling \\ load\ of\ actual \\ column \end{bmatrix} = \begin{bmatrix} Elastic\ buckling \\ load\ of\ equivalent \\ column \end{bmatrix}$$

$$\left[P_{cr,actual}\right]=\left[\frac{\pi^2 EI_{eq}}{{L_{eff}}^2}\right] \ldots\text{(Eq. 4.1)}$$

Where $P_{cr,actual}$ is the actual elastic buckling load of tapered column, which should to be calculated as;

$$P_{cr,actual}=\frac{mEI_2}{L_{actual}} \ldots\text{(Eq. 4.2)}$$

Where, m is a constant which is presented in *Annex 1*.

Substituting *eq. 4.2* in to *eq. 4.1* and Re-arranging gives:

$$L_{eff}=\pi L_{actual}\sqrt{\frac{I_{eq}}{mI_2}}$$

Where, I_{eq} and I_2 are moment of inertia for the equivalent and maximum sections.

The safety factor of the effective length methodology investigated in this paper should be an important issue and should also be discussed here. The last column of *table 3.13* to *table 3.17* show how further end moment can the equivalent column carry without further requirement of reinforcement and cross-section than the actual column. This is the same to mean as how much further end eccentricity is allowed without affecting its safety, when the column is analyzed and designed using effective length methodology. All values are the safety factors; each is the ratio of column capacity of equivalent column to the actual column. The graphical illustration of computation of the safety factors are indicated in *figures 3.13 to 3.24* for few selected samples under study. *Fig. 4.1* indicates the summary of safety factors observed in all samples; so as to use it to evaluate the general evaluation of safety of the methodology.

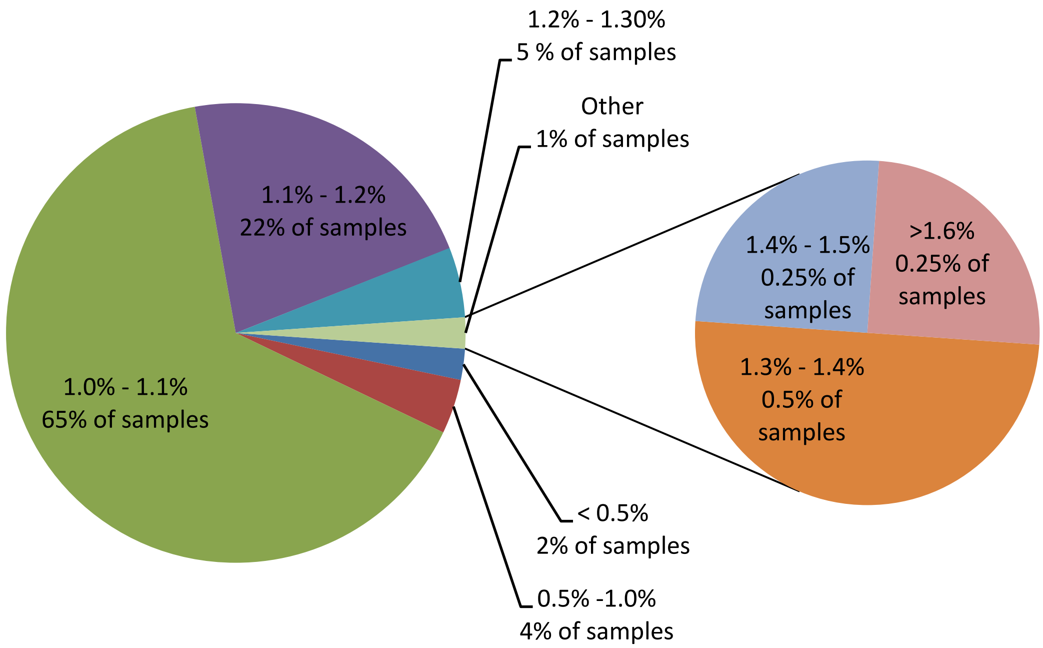


Fig. 4.1: Evaluation of safety factors of effective length methodology

As can be seen in *Fig. 4.1*, 65% of the entire samples exhibit a safety factor of 1.0 to 1.1 when effective length methodology is used. The safety factors of in the range 1.1 to 1.2 are also observed in 22% of samples, which are also not negligible. Higher safety factors than 1.2 is not observed in meaningful manner, since only 6% of the samples are found to have the factors from 1.2 to 1.66.

Apart from the above discussed safe factors, safety factors of below 1.0 are observed, even though it is in negligible quantity. Only 6% of all samples took the methodology adversely, by having safety factor below 1.00; in which one third of these exhibits safety factor below 0.5. *Table 4.2* presents features of column samples, those have lower safety factor.

Sample No.	Safety factor	Axial load (KN)	End moment (KNm)	End eccentricity (mm)
50	0.221	2220.68	1.00	0.45
57	0.382	2230.59	1.00	0.44
182	0.413	2082.54	1.00	0.48
42	0.446	1224.00	3.00	2.45
144	0.553	816.00	1.00	0.12
101	0.699	25.50	2.50	98.03
127	0.708	1360.00	1.00	0.74
191	0.874	170.00	3.00	17.65

Table 4.2: End eccentricity of selected samples

As can be shown in the *table 4.2*, almost all of the unsafe samples have end eccentricity below the lowest permissible value in EBCS, which is 20mm. For a given column having end eccentricity above 20 mm, the methodology can generally be taken as safe.

Therefore, we can conclude that the methodology of effective length approach is conservative; which can hold an additional end moment of up to *20%*, without any structural hazard.

4.2 Conclusion and generalization

Now, we have finished all the necessary steps as planned in our methodology. It is the time to announce that "Effective length method for columns having linearly varying cross-section and equal end moments with single curvature" is established. The following a "Two stepped general procedure" is presented next as a conclusion and generalization.

General Procedure for Analysis of Reinforced Concrete Pin Ended Column, Having Linearly Varying Cross-Section With Equal End Moments In Major Axis Using Effective Length Method

For reinforced concrete pin ended column having a linearly varying cross-section, with single curvature and equal end moments in major axis; the equivalent section and the effective length should be decided by using the following steps.

Step 1, Determination of the equivalent section

- The constant dimension of the actual tapered column section (the breadth of section) will also be the breadth of the equivalent column.
- Table A to E, attached at the end of this section; present the requirement of various points along the column height for the selection of those points to be point of prismatic section of equivalent column. Select one of the tables according to the value of inertial ratio (I_1/I_2).
- In each table, there is a list of values of L_s under each columns, each for a specified value of $\sqrt{\frac{P}{f_{cd} A_{c,min}}}$. Select an appropriate column for the calculated value of $\sqrt{\frac{P}{f_{cd} A_{c,min}}}$.
- Check value of $L_s = \frac{B}{W_i} L$, for each trial points as indicated in the selected table. The trial node which satisfies the requirement stated in the table, unanimously, will be taken as the point of equivalent section.
- If no node along the column height satisfies the requirement of the selected table, the width of equivalent column should be the lesser of:

$$\begin{cases} \text{Width of actual column at 0.50L from the minimum section} \\ \dfrac{BL}{L_{s,lim}} \end{cases}$$

Where $L_{s,lim}$ is the limiting value of stabilized length, which is also indicated in each tables.

Step 2, Determination of the effective length

- Use the equation at top of each table to compute the effective length to be used in equivalent column.

Important Remarks

- If the inertial ratio (I_1/I_2) of a given column is in between the figures found in the tables, it recommended that the same analysis procedures should be done for both the upper and lower boundary values of the inertial ratio. Finally, the governing failure section, which is the larger section, should be selected.
- If the value of $\sqrt{\frac{P}{f_{cd} A_{c,min}}}$ of a given column is in between the figures found in the tables, it recommended that the same analysis procedures should be done for both the upper and lower boundary values of $\sqrt{\frac{P}{f_{cd} A_{c,min}}}$. Finally, the governing failure section, which is the larger section, should be selected.
- If the inertial ratio (I_1/I_2) exceeds 0.50; take failure section at 0.50L from the minimum section, but the design should be compared with a design with minimum section having actual end moment.

TABLE A

$\frac{I_1}{I_2} = 0.10,\ L_{eff} = 0.5527\pi L\sqrt{\frac{I_{eq}}{I_2}}$

Equivalent Section at	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.16$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.32$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.48$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.80$
0.45L	-	-	-	-	-
0.40L	-	-	-	-	-
0.35L	-	-	$40.063 \geq L_s \geq 35.611$	-	-
0.30L	$52.612 \geq L_s \geq 40.233$	$50.402 \geq L_s \geq 42.443$	$37.138 \geq L_s \geq 30.175$	$40.109 \geq L_s \geq 36.395$	$20.532 \geq L_s \geq 19.100$
0.25L	$42.035 \geq L_s \geq 35.568$	$44.345 \geq L_s \geq 24.944$	$31.526 \geq L_s \geq 19.401$	$38.026 \geq L_s \geq 31.041$	$19.955 \geq L_s \geq 17.461$
0.20L	$37.236 \geq L_s \geq 33.174$	$26.114 \geq L_s \geq 22.632$	$20.311 \geq L_s \geq 17.264$	$32.497 \geq L_s \geq 25.185$	$18.280 \geq L_s \geq 14.624$
0.15L	$34.806 \geq L_s \geq 30.544$	$23.745 \geq L_s \geq 21.310$	$18.113 \geq L_s \geq 15.982$	$26.424 \geq L_s \geq 20.457$	$15.343 \geq L_s \geq 13.699$
0.10L	$32.125 \geq L_s \geq 29.136$	$22.413 \geq L_s \geq 19.211$	$16.809 \geq L_s \geq 15.129$	$21.516 \geq L_s \geq 19.275$	$14.408 \geq L_s \geq 12.679$
0.05L	$30.726 \geq L_s \geq 24.424$	$20.259 \geq L_s \geq 15.816$	$15.954 \geq L_s \geq 12.653$	$20.327 \geq L_s \geq 17.018$	$13.371 \geq L_s \geq 12.156$
0.00L	$25.833 \geq L_s \geq 0.00$	$18.571 \geq L_s \geq 00.000$	$13.383 \geq L_s \geq 00.000$	$18.000 \geq L_s \geq 00.000$	$12.857 \geq L_s \geq 00.000$
$L_{s,lim}$	***52.612***	***50.402***	***40.063***	***40.109***	***20.532***

- ***Unit for L_s is meter***

P = Axial load on actual tapered column,
$A_{g,min}$= Area of column section, at the smaller cross-section,
I_2= Moment of inertia of the larger end cross-section,
L_s=BL/W_i,,
Wi=Non-constant width of column at the point of consideration

F_{cd}= Design compressive strength of concrete,
I_1= Moment of inertia of the smaller end cross-section,
I_{eq}= Moment of inertia of the equivalent cross-section,
B=the prismatic breadth of column section,
L=height (length) of the column

TABLE B

$\frac{I_1}{I_2} = 0.20, \; L_{eff} = 0.4705\pi L\sqrt{\frac{I_{eq}}{I_2}}$

Equivalent Section at	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.16$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.32$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.48$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.80$
0.45L	-	-	-	-	-
0.40L	-	-	-	-	-
0.35L	-	$40.048 \geq L_s \geq 35.042$	$23.362 \geq L_s \geq 13.349$	-	-
0.30L	$56.062 \geq L_s \geq 46.829$	$36.068 \geq L_s \geq 19.580$	$13.740 \geq L_s \geq 9.618$	$21.526 \geq L_s \geq 17.175$	$9.316 \geq L_s \geq 7.420$
0.25L	$48.240 \geq L_s \geq 44.163$	$20.170 \geq L_s \geq 15.393$	$9.908 \geq L_s \geq 8.705$	$17.693 \geq L_s \geq 16.042$	$7.643 \geq L_s \geq 5.945$
0.20L	$45.536 \geq L_s \geq 40.632$	$15.871 \geq L_s \geq 13.409$	$8.976 \geq L_s \geq 8.027$	$16.540 \geq L_s \geq 10.946$	$6.130 \geq L_s \geq 5.079$
0.15L	$41.935 \geq L_s \geq 38.320$	$13.839 \geq L_s \geq 12.991$	$8.284 \geq L_s \geq 7.155$	$11.297 \geq L_s \geq 10.293$	$5.242 \geq L_s \geq 4.745$
0.10L	$39.590 \geq L_s \geq 36.602$	$13.422 \geq L_s \geq 11.671$	$7.392 \geq L_s \geq 6.847$	$10.634 \geq L_s \geq 9.856$	$4.902 \geq L_s \geq 4.669$
0.05L	$37.857 \geq L_s \geq 29.358$	$12.071 \geq L_s \geq 9.102$	$7.082 \geq L_s \geq 5.947$	$10.194 \geq L_s \geq 8.890$	$4.829 \geq L_s \geq 4.435$
0.00L	$30.400 \geq L_s \geq 0.00$	$9.425 \geq L_s \geq 00.000$	$6.158 \geq L_s \geq 00.000$	$9.206 \geq L_s \geq 00.000$	$4.592 \geq L_s \geq 00.000$
$L_{s,lim}$	***56.062***	***40.048***	***23.362***	***21.526***	***9.316***

- ***Unit for L_s is meter***

P = Axial load on actual tapered column,
$A_{g,min}$= Area of column section, at the smaller cross-section,
I_2= Moment of inertia of the larger end cross-section,
L_s=BL/W_i,,
Wi=Non-constant width of column at the point of consideration

F_{cd}= Design compressive strength of concrete,
I_1= Moment of inertia of the smaller end cross-section,
I_{eq}= Moment of inertia of the equivalent cross-section,
B=the prismatic breadth of column section,
L=height (length) of the column

TABLE C

$\frac{I_1}{I_2} = 0.30,\ L_{eff} = 0.4273\pi L\sqrt{\frac{I_{eq}}{I_2}}$

Equivalent Section at	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.16$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.32$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.48$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.80$
0.45L	-	-	-	-	-
0.40L	36.650 $\geq L_s \geq$ 30.155	31.734 $\geq L_s \geq$ 24.218	-	-	-
0.35L	30.790 $\geq L_s \geq$ 23.684	24.728 $\geq L_s \geq$ 15.348	31.426 $\geq L_s \geq$ 16.078	21.316 $\geq L_s \geq$ 14.069	19.184 $\geq L_s \geq$ 14.388
0.30L	24.194 $\geq L_s \geq$ 19.839	15.678 $\geq L_s \geq$ 13.065	16.424 $\geq L_s \geq$ 12.691	13.936 $\geq L_s \geq$ 12.194	14.698 $\geq L_s \geq$ 11.323
0.25L	20.275 $\geq L_s \geq$ 18.791	13.352 $\geq L_s \geq$ 11.572	12.970 $\geq L_s \geq$ 11.444	11.571 $\geq L_s \geq$ 10.548	11.571 $\geq L_s \geq$ 9.736
0.20L	19.214 $\geq L_s \geq$ 17.697	11.832 $\geq L_s \geq$ 10.012	11.701 $\geq L_s \geq$ 10.531	10.102 $\geq L_s \geq$ 9.465	9.954 $\geq L_s \geq$ 8.987
0.15L	18.104 $\geq L_s \geq$ 15.517	10.242 $\geq L_s \geq$ 9.404	10.773 $\geq L_s \geq$ 10.215	9.310 $\geq L_s \geq$ 8.752	9.194 $\geq L_s \geq$ 8.438
0.10L	15.882 $\geq L_s \geq$ 14.294	9.625 $\geq L_s \geq$ 9.053	10.455 $\geq L_s \geq$ 9.802	8.576 $\geq L_s \geq$ 8.119	8.636 $\geq L_s \geq$ 8.040
0.05L	14.639 $\geq L_s \geq$ 10.101	9.271 $\geq L_s \geq$ 6.402	10.038 $\geq L_s \geq$ 8.256	8.002 $\geq L_s \geq$ 7.612	8.234 $\geq L_s \geq$ 7.441
0.00L	10.350 $\geq L_s \geq$ 00.000	6.560 $\geq L_s \geq$ 00.000	8.460 $\geq L_s \geq$ 00.000	7.300 $\geq L_s \geq$ 00.000	7.625 $\geq L_s \geq$ 00.000
$L_{s,lim}$	***36.650***	***31.734***	***39.562***	***21.316***	***19.184***

- ***Unit for L_s is meter***

P = Axial load on actual tapered column,
$A_{g,min}$= Area of column section, at the smaller cross-section,
I_2= Moment of inertia of the larger end cross-section,
L_s=BL/W_i,,
Wi=Non-constant width of column at the point of consideration

F_{cd}= Design compressive strength of concrete,
I_1= Moment of inertia of the smaller end cross-section,
I_{eq}= Moment of inertia of the equivalent cross-section,
B=the prismatic breadth of column section,
L=height (length) of the column

TABLE D

$\frac{I_1}{I_2} = 0.40,\ L_{eff} = 0.3987\pi L\sqrt{\frac{I_{eq}}{I_2}}$

Equivalent Section at	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.16$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.32$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.48$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.80$
0.45L	-	$27.076 \geq L_s \geq 25.230$	$17.553 \geq L_s \geq 16.153$	-	-
0.40L	$41.999 \geq L_s \geq 26.249$	$25.624 \geq L_s \geq 13.125$	$16.406 \geq L_s \geq 8.422$	$15.225 \geq L_s \geq 8.400$	$14.291 \geq L_s \geq 10.719$
0.35L	$26.666 \geq L_s \geq 22.222$	$13.333 \geq L_s \geq 10.793$	$8.555 \geq L_s \geq 7.111$	$8.533 \geq L_s \geq 6.827$	$10.889 \geq L_s \geq 8.815$
0.30L	$22.580 \geq L_s \geq 18.064$	$10.968 \geq L_s \geq 8.387$	$7.226 \geq L_s \geq 5.927$	$6.937 \geq L_s \geq 5.961$	$8.957 \geq L_s \geq 7.271$
0.25L	$18.360 \geq L_s \geq 16.983$	$8.524 \geq L_s \geq 7.541$	$6.024 \geq L_s \geq 5.221$	$6.059 \geq L_s \geq 5.233$	$7.390 \geq L_s \geq 5.890$
0.20L	$17.266 \geq L_s \geq 15.400$	$7.667 \geq L_s \geq 7.200$	$5.308 \geq L_s \geq 4.550$	$5.320 \geq L_s \geq 4.816$	$5.989 \geq L_s \geq 5.172$
0.15L	$15.661 \geq L_s \geq 14.522$	$7.322 \geq L_s \geq 6.441$	$4.627 \geq L_s \geq 4.212$	$4.898 \geq L_s \geq 4.385$	$5.260 \geq L_s \geq 4.845$
0.10L	$14.772 \geq L_s \geq 11.586$	$6.552 \geq L_s \geq 5.876$	$4.284 \geq L_s \geq 3.983$	$4.461 \geq L_s \geq 4.171$	$4.928 \geq L_s \geq 4.787$
0.05L	$11.789 \geq L_s \geq 10.493$	$5.979 \geq L_s \geq 4.126$	$4.053 \geq L_s \geq 3.579$	$4.244 \geq L_s \geq 3.781$	$4.871 \geq L_s \geq 4.564$
0.00L	$10.680 \geq L_s \geq 00.000$	$4.200 \geq L_s \geq 00.000$	$3.643 \geq L_s \geq 00.000$	$3.849 \geq L_s \geq 00.000$	$4.646 \geq L_s \geq 00.000$
$L_{s,lim}$	***41.999***	***27.076***	***17.553***	***15.225***	***14.291***

- ***Unit for L_s is meter***

P = Axial load on actual tapered column,
$A_{g,min}$= Area of column section, at the smaller cross-section,
I_2= Moment of inertia of the larger end cross-section,
L_s=BL/W_i,,
Wi=Non-constant width of column at the point of consideration

F_{cd}= Design compressive strength of concrete,
I_1= Moment of inertia of the smaller end cross-section,
I_{eq}= Moment of inertia of the equivalent cross-section,
B=the prismatic breadth of column section,
L=height (length) of the column

TABLE E

$\frac{I_1}{I_2} = 0.50, \; L_{eff} = 0.3777\pi L\sqrt{\frac{I_{eq}}{I_2}}$

Equivalent Section at	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.16$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.32$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.48$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$	$\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.80$
0.45L	$34.021 \geq L_s \geq 23.874$	$24.844 \geq L_s \geq 18.264$	$19.249 \geq L_s \geq 14.325$	-	-
0.40L	$24.155 \geq L_s \geq 20.411$	$18.479 \geq L_s \geq 9.058$	$14.493 \geq L_s \geq 7.156$	$18.116 \geq L_s \geq 10.417$	-
0.35L	$20.654 \geq L_s \geq 14.055$	$9.166 \geq L_s \geq 7.562$	$7.241 \geq L_s \geq 5.866$	$10.541 \geq L_s \geq 8.250$	$16.499 \geq L_s \geq 12.374$
0.30L	$14.224 \geq L_s \geq 11.256$	$7.653 \geq L_s \geq 6.957$	$5.937 \geq L_s \geq 5.380$	$8.349 \geq L_s \geq 7.189$	$12.523 \geq L_s \geq 10.204$
0.25L	$11.393 \geq L_s \geq 10.204$	$7.042 \geq L_s \geq 5.869$	$5.446 \geq L_s \geq 4.507$	$7.277 \geq L_s \geq 6.103$	$10.329 \geq L_s \geq 9.484$
0.20L	$10.330 \geq L_s \geq 9.759$	$5.941 \geq L_s \geq 5.038$	$4.563 \geq L_s \geq 3.802$	$6.179 \geq L_s \geq 5.442$	$9.601 \geq L_s \geq 9.126$
0.15L	$9.881 \geq L_s \geq 8.983$	$5.101 \geq L_s \geq 4.187$	$3.850 \geq L_s \geq 3.657$	$5.510 \geq L_s \geq 5.053$	$9.240 \geq L_s \geq 8.085$
0.10L	$9.097 \geq L_s \geq 7.992$	$4.240 \geq L_s \geq 3.801$	$3.704 \geq L_s \geq 3.606$	$5.117 \geq L_s \geq 4.776$	$8.187 \geq L_s \geq 7.505$
0.05L	$8.095 \geq L_s \geq 5.631$	$3.850 \geq L_s \geq 3.023$	$3.653 \geq L_s \geq 3.060$	$4.837 \geq L_s \geq 4.487$	$7.601 \geq L_s \geq 6.979$
0.00L	$5.704 \geq L_s \geq 00.000$	$3.062 \geq L_s \geq 00.000$	$3.100 \geq L_s \geq 00.000$	$4.545 \geq L_s \geq 00.000$	$7.070 \geq L_s \geq 00.000$
$L_{s,lim}$	***34.021***	***24.844***	***19.249***	***18.116***	***16.499***

- ***Unit for L_s is meter***

P = Axial load on actual tapered column,
$A_{g,min}$= Area of column section, at the smaller cross-section,
I_2= Moment of inertia of the larger end cross-section,
L_s=BL/W_i,,
Wi=Non-constant width of column at the point of consideration
F_{cd}= Design compressive strength of concrete,
I_1= Moment of inertia of the smaller end cross-section,
I_{eq}= Moment of inertia of the equivalent cross-section,
B=the prismatic breadth of column section,
L=height (length) of the column

4.3 Direction of the future

Whenever unequal end moment exist in the system, establishing an equivalent equal end moment is rather useful for the designer to directly apply the accustomed procedures of column design. This step deals with the replacement of the column system by an equivalent column, subjected to equal moments of C_mM_2 at both ends. Where; C_m is a moment magnification factor.

- *C_mM_2 is chosen so that the maximum design moment is the same in the actual and equivalent columns.*

For example, for prismatic column systems:

According to ACI code section 10.10.6.4,

$$C_m=0.6+0.4\frac{M_1}{M_2}$$

Investigation of the moment magnification factor for tapered column, should therefore, be the future aspect of study on this area

In addition, investigations of effective length method for tapered column in double curvature and/or bi-axially loaded will also believed to be somewhat a challenging assignment of researchers, who need to reserve themselves on the area, in the future.

CHAPTER 5
ILLUSTRATIVE EXAMPLES

Example 1

The tapered column shown in *Fig. 5.1* is desired to be reinforced using the ultimate limit state design philosophy. Determine the reinforcement using iterative analysis approach and compare the result with the effective length methodology by determining the equivalent column system with effective length, elastic buckling load, design section and design moment.

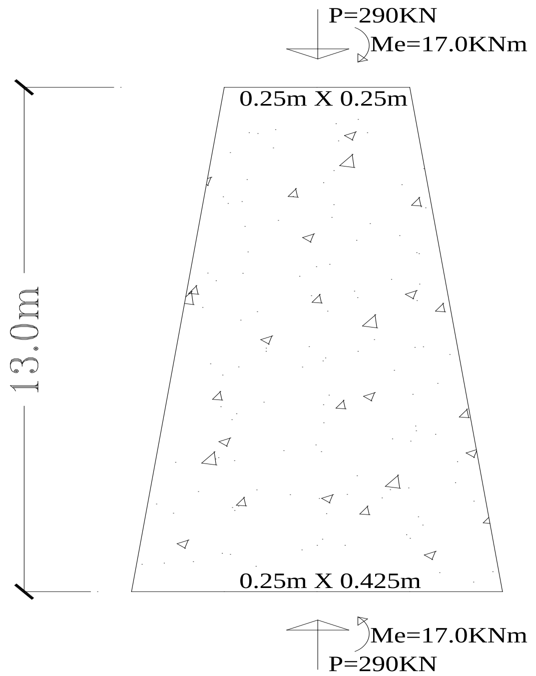


Fig. 5.1: figure for example 1

i.	Top width	=0.25m
ii.	Bottom width	=0.45m
iii.	Breadth	=0.25m
iv.	Concrete class	= C-25, f_{cd}=11.33Mpa
v.	Reinforcement bars class	= S-300, f_{yd}=260.87Mpa
vi.	Axial load	= 290KN
vii.	End moment	= 17.0KNm
viii.	Length	= 13.0m

Solution

a. Using iterative analysis approach

A total reinforcement of 6ϕ16mm bars are provided for the design of column system shown in fig. 5.1, by committing iterative analysis of the system. 3ϕ16mm bars should be arranged in two adjacent faces of the column section as shown in fig. 5.2.

The second order bending moment diagram, shown in fig.5.3, is found after an iterative analysis is committed on the actual column. The column length is made to divide in to 20 equal segments, each spans 0.65m. The system fails by stability before sections reach its capacities. The design section can be taken as section at node 5 or node 6 (located at 0.20L and 0.25L from the minimum section, respectively). The two nodes are selected since moments at these nodes reach its section capacities in closer gap than all other nodes. Second order moment at node 5 is 57.3% of moment capacity of that section and second order moment at node 6 is 59.4% of moment capacity of section at that node. As a result either of the tow nodes can be taken as the node for the selection of design section and moment.

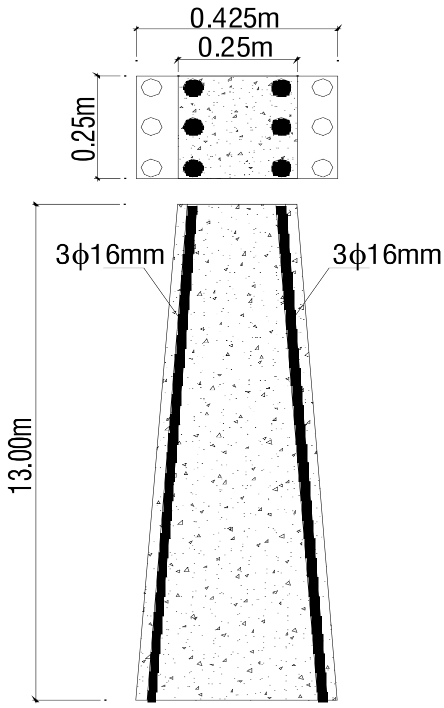


Fig. 5.2: reinforcement of column system for example 1

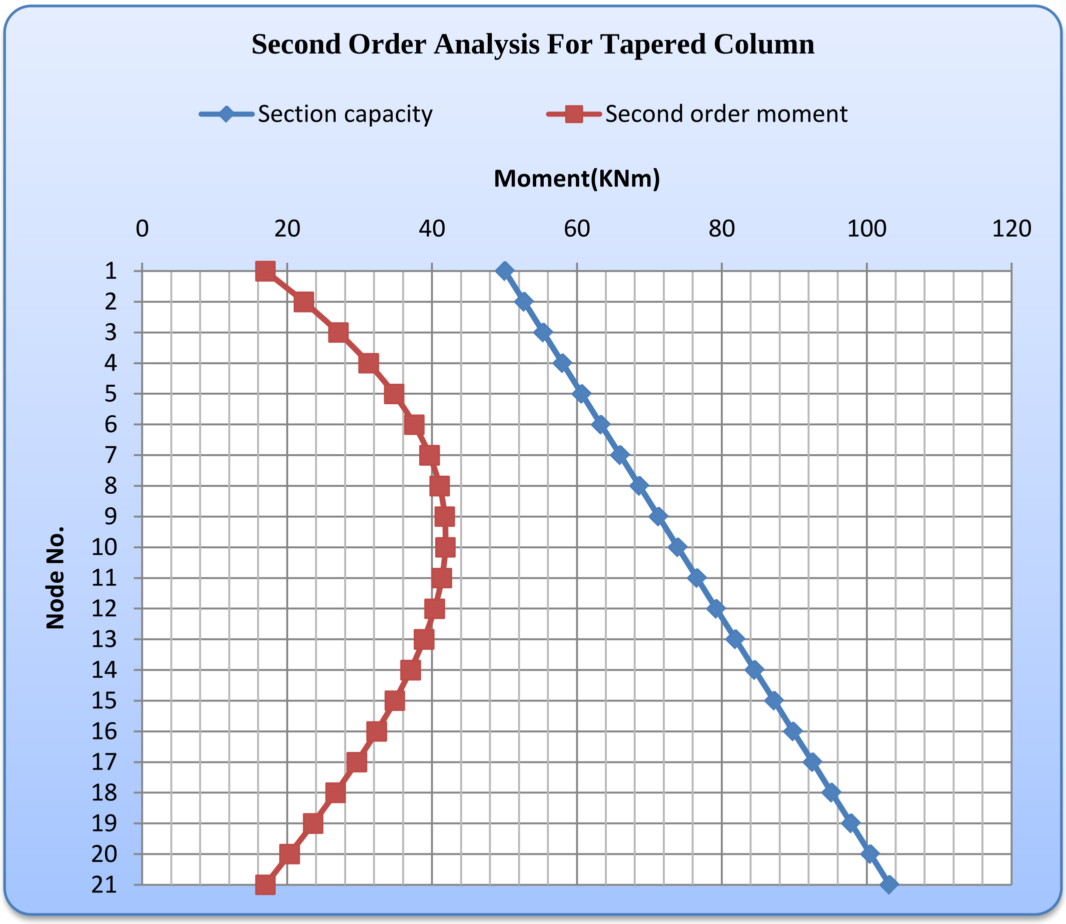


Fig. 5.3: Second order analysis of actual column for example 1

b. ***Using effective length analysis approach***

- $I_1/I_2 = 0.2$
- $\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.64$
  - Therefore use table B, and column No.5
    - o Checking at 0.00L : $L_{s,0.00L} = \frac{B}{W_{0.00L}}L = \frac{0.25m}{0.25\,m}X13.0m = 13.0\text{m}$, ~~Not OK~~
    - o Checking at 0.05L : $L_{s,0.05L} = \frac{B}{W_{0.05L}}L = \frac{0.25m}{0.25875m}X13.0m = 12.6\text{m}$, ~~Not Ok~~
    - o Checking at 0.10L : $L_{s,0.10L} = \frac{B}{W_{0.10L}}L = \frac{0.25m}{0.2675m}X13.0m = 12.2\text{m}$, ~~Not Ok~~
    - o Checking at 0.15L : $L_{s,0.15L} = \frac{B}{W_{0.15L}}L = \frac{0.25m}{0.27626m}X13.0m = 11.8\text{m}$, ~~Not Ok~~
    - o Checking at 0.20L : $L_{s,0.20L} = \frac{B}{W_{0.20L}}L = \frac{0.25m}{0.285m}X13.0m = 11.4\text{m}$

Inside the limit, **OK!**

- ❖ ***Use a section at 0.20L as equivalent section / 0.25mX0.285m/***
- ❖ *Effective length*, $L_{eff} = 0.4273\pi L\sqrt{\frac{I_{eq}}{I_2}} = 9.583\text{m}$

And the analysis of the equivalent column having a length of equal to 9.583m and section 0.25mX0.285m is presented in fig.5.4

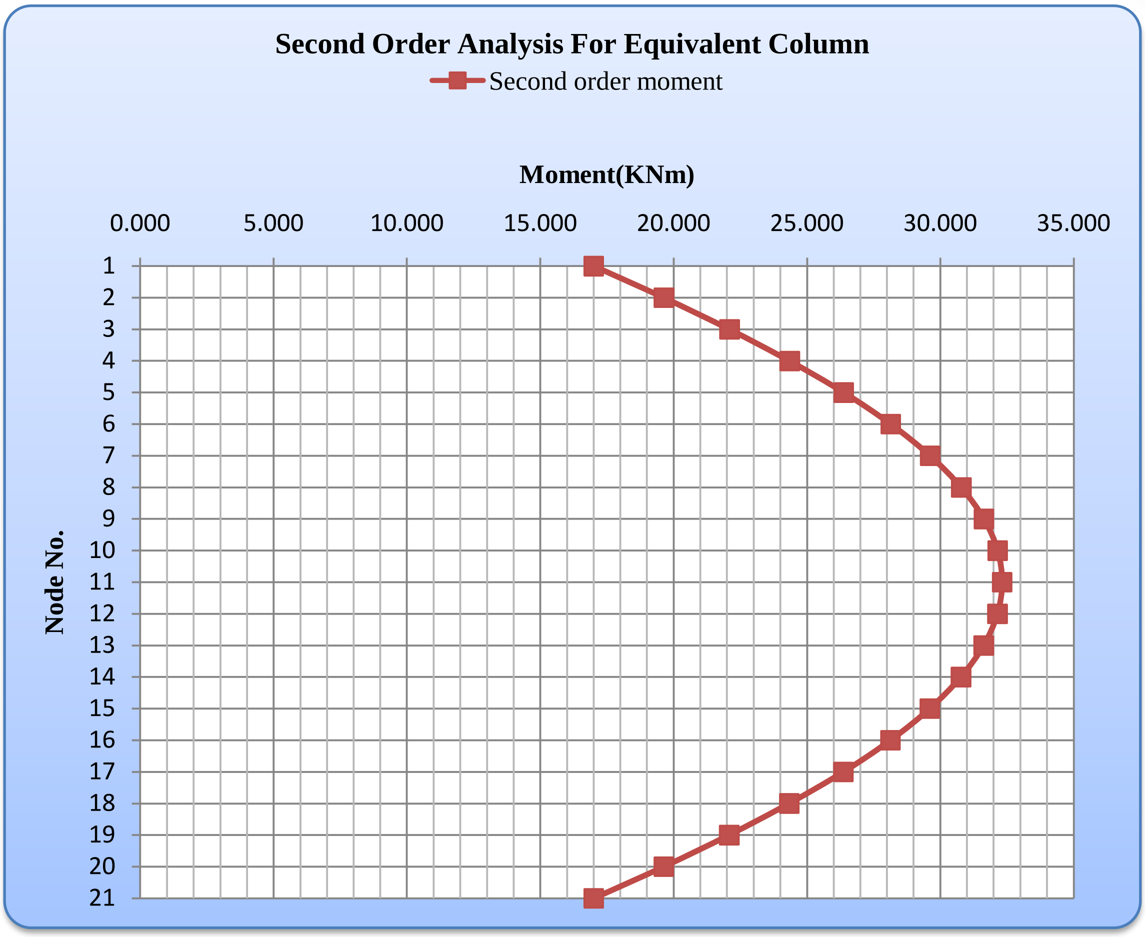


Fig. 5.4: Second order analysis of equivalent column for example 1

Comparison of results

	Iterative Analysis Approach	Effective Length Approach
Design node	Node 5	Node 5
Design Section	0.25m X 0.285m	0.25m X 0.285m
Design moment	34.726KNm	32.315KNm

Table 5.1: Comparison of results for column in example 1

Example 2

The tapered column shown in *Fig. 5.5* is desired to be reinforced using the ultimate limit state design philosophy. Determine the reinforcement using iterative analysis approach and compare the result with the effective length methodology by determining the equivalent column system with effective length, elastic buckling load, design section and design moment.

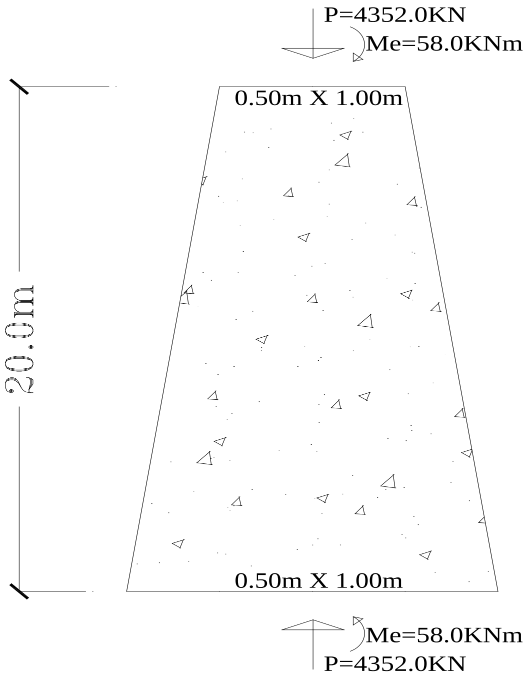


Fig. 5.5: figure for example 2

i.	Top width	=1.00m
ii.	Bottom width	=1.35m
iii.	Breadth	=0.50m
iv.	Concrete class	= C-30, f_{cd}=13.6Mpa
v.	Reinforcement bars class	= S-400, f_{yd}=347.83Mpa
vi.	Axial load	= 4352 KN
vii.	End moment	= 858 KNm
viii.	Length	= 20.0 m

Solution

a. Using iterative analysis approach

A total reinforcement of 8ϕ20mm bars are provided for the design of column system shown in fig. 5.6, by committing iterative analysis of the system. 4ϕ16mm bars should be arranged in two adjacent faces of the column section as shown in fig. 5.6.

The second order bending moment diagram, shown in fig.5.7, is found after an iterative analysis is committed on the actual column. The column length is made to divide in to 20 equal segments, each spans 1.0m. Material type of failure occurs by which section at node 7 reaches its capacity. The design section, hence, will be section at node 7 (located at 0.30L from the minimum section).

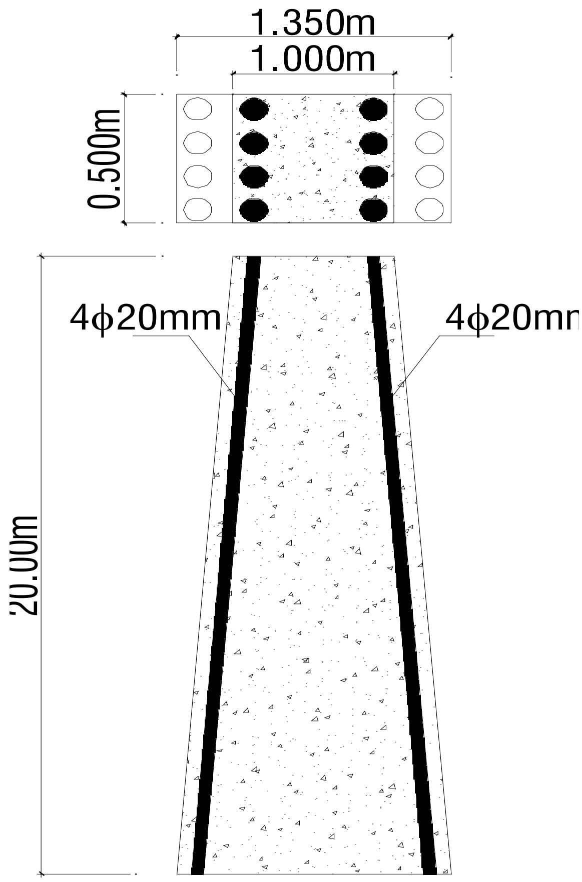


Fig. 5.6: reinforcement of column system for example 2

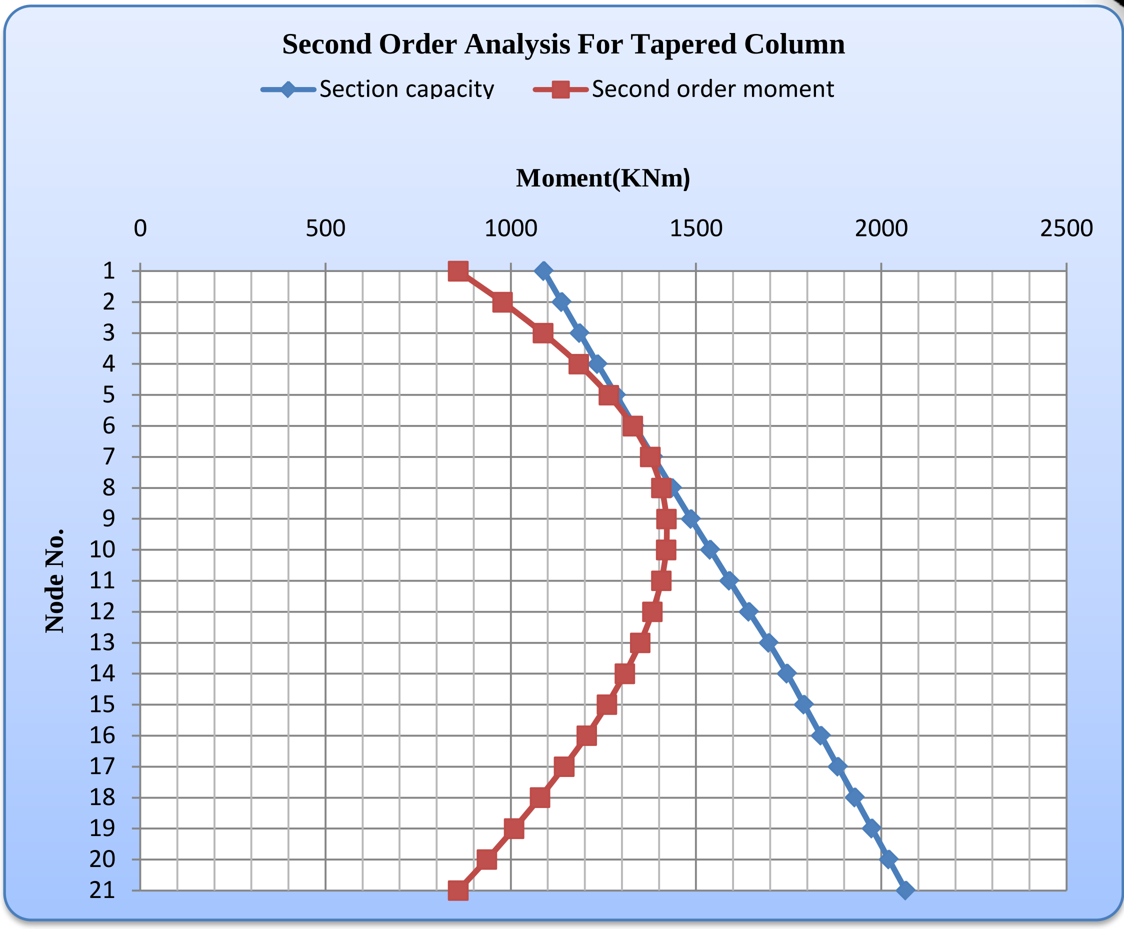


Fig. 5.7: Second order analysis of actual column for example 2

c. *Using effective length analysis approach*

- I_1/I_2 =0.4
- $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.80$

    - Therefore use table D, and column No.6

        - Checking at 0.00L : $L_{s,0.00L} = \frac{B}{W_{0.00L}}L = \frac{0.50m}{1.00\,m}X20m$ = 10.0m, ~~Not OK~~
        - Checking at 0.05L : $L_{s,0.05L} = \frac{B}{W_{0.05L}}L = \frac{0.50m}{1.0175m}X20m$ =9.8m, ~~Not Ok~~
        - Checking at 0.10L : $L_{s,0.10L} = \frac{B}{W_{0.10L}}L = \frac{0.50m}{1.035m}X20m$ =9.7m, ~~Not Ok~~
        - Checking at 0.15L : $L_{s,0.15L} = \frac{B}{W_{0.15L}}L = \frac{0.50m}{1.0525m}X20m$=9.5m, ~~Not Ok~~
        - Checking at 0.20L : $L_{s,0.20L} = \frac{B}{W_{0.20L}}L = \frac{0.50m}{1.070m}X20m$ = 9.3m, ~~Not Ok~~
        - Checking at 0.25L : $L_{s,0.25L} = \frac{B}{W_{0.25L}}L = \frac{0.50m}{1.0875m}X20m$ = 9.2m, ~~Not Ok~~
        - Checking at 0.30L : $L_{s,0.30L} = \frac{B}{W_{0.30L}}L = \frac{0.50m}{1.105m}X20m$ = 9.0m, ~~Not Ok~~
        - Checking at 0.35L : $L_{s,0.35L} = \frac{B}{W_{0.35L}}L = \frac{0.50m}{1.1225m}X20m$ = 8.9m,

Inside the limit, ***OK!***

- ***Use a section at 0.35L as equivalent section / 0.50mX1.1225m/***
- *Effective length,* $L_{eff} = 0.3987\pi L\sqrt{\frac{I_{eq}}{I_2}}$ =18.9935m

And the analysis of the equivalent column having a length of equal to 18.9935m and section 0.50mX1.1225m is presented in fig.5.8

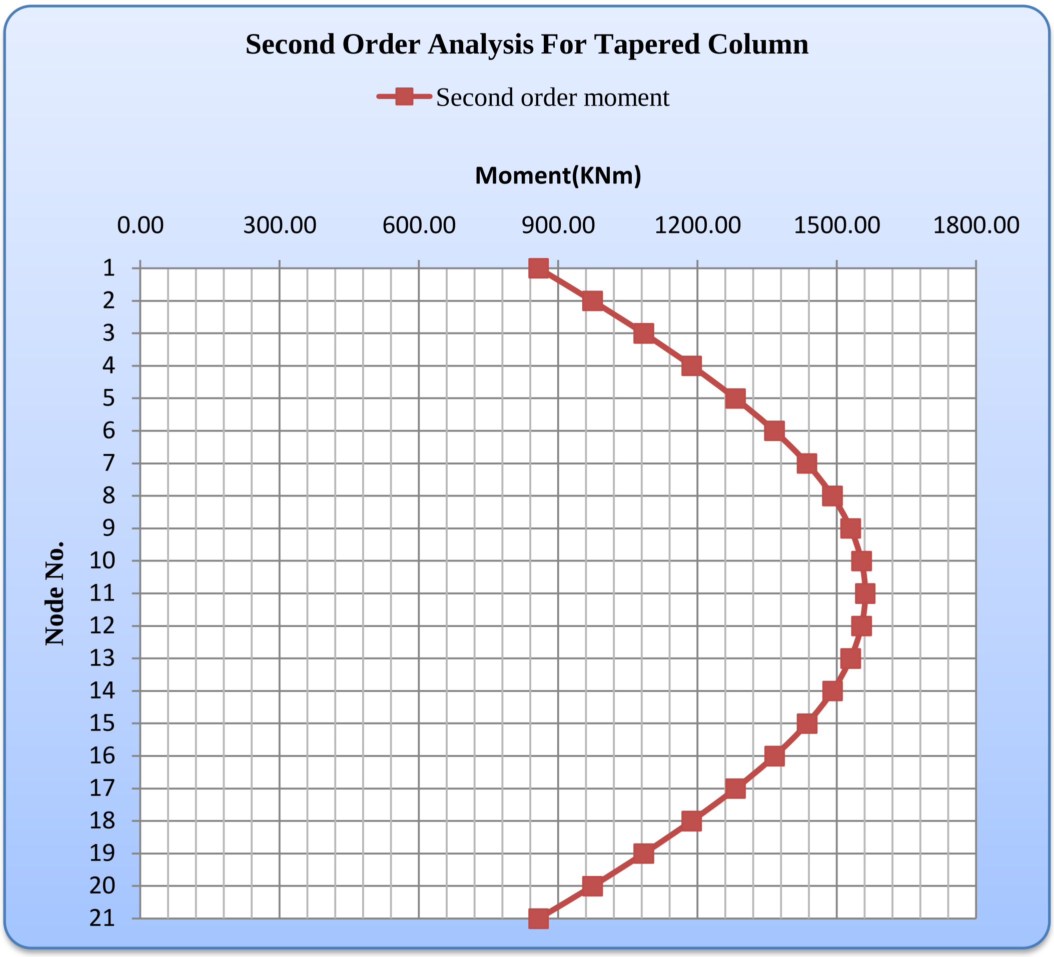


Fig. 5.8: Second order analysis of equivalent column for example 2

Comparison of results

	Iterative Analysis Approach	Effective Length Approach
Design node	Node 7	Node 8
Design Section	0.50m X 1.105m	0.50m X 1.1225m
Design moment	1376.43KNm	1561.38KNm

Table 5.2: Comparison of results for column in example 2

Example 3

The tapered column shown in *Fig. 5.9* is desired to be reinforced using the ultimate limit state design philosophy. Determine the reinforcement using iterative analysis approach and compare the result with the effective length methodology by determining the equivalent column system with effective length, elastic buckling load, design section and design moment.

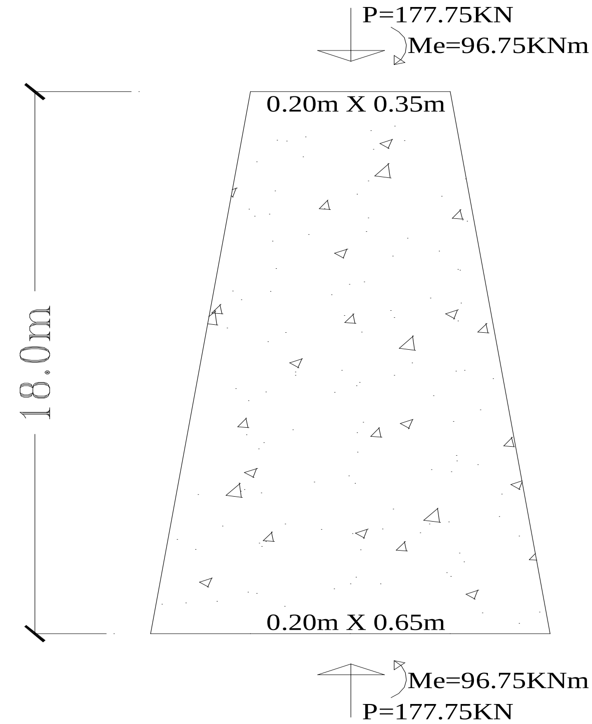


Fig. 5.9: figure for example 3

i.	Top width	=0.35m
ii.	Bottom width	=0.65m
iii.	Breadth	=0.20m
iv.	Concrete class	= C-35, fcd=15.87Mpa
v.	Reinforcement bars class	= S-460, fyd=400Mpa
vi.	Axial load	= 177.75 KN
vii.	End moment	= 96.75 KNm
viii.	Length	= 18.0 m

Solution

a. ***Using iterative analysis approach***

A total reinforcement of 10ϕ14mm bars are provided for the design of column system shown in fig. 5.10, by committing iterative analysis of the system. 4ϕ16mm bars should be arranged in two adjacent faces of the column section as shown in fig. 5.10.

The second order bending moment diagram, shown in fig.5.11, is found after an iterative analysis is committed on the actual column. The column length is made to divide in to 20 equal segments, each spans 0.90m. Material type of failure occurs by which section at node 5 reaches its capacity. The design section, hence, will be section at node 5 (located at 0.20L from the minimum section).

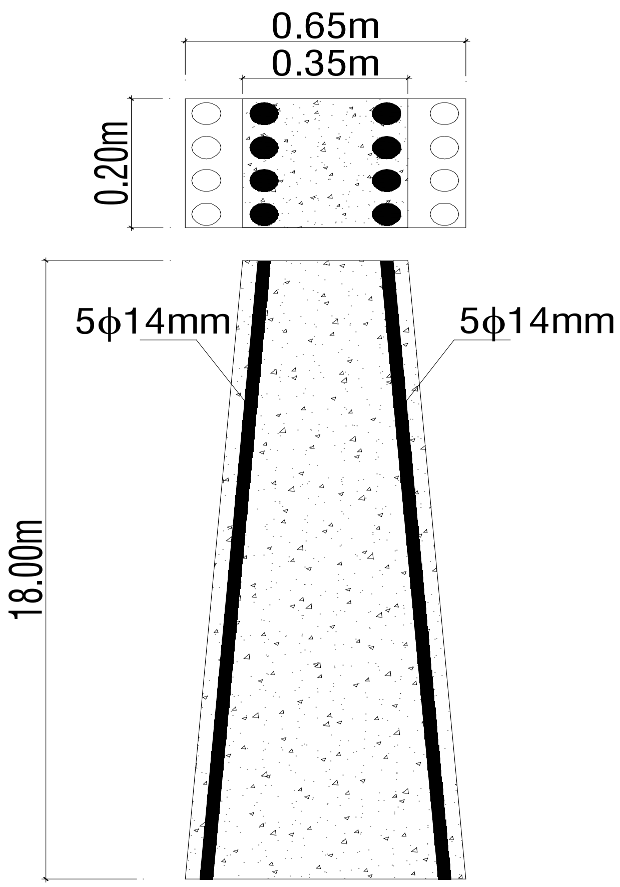


Fig. 5.10: reinforcement of column system for example 3

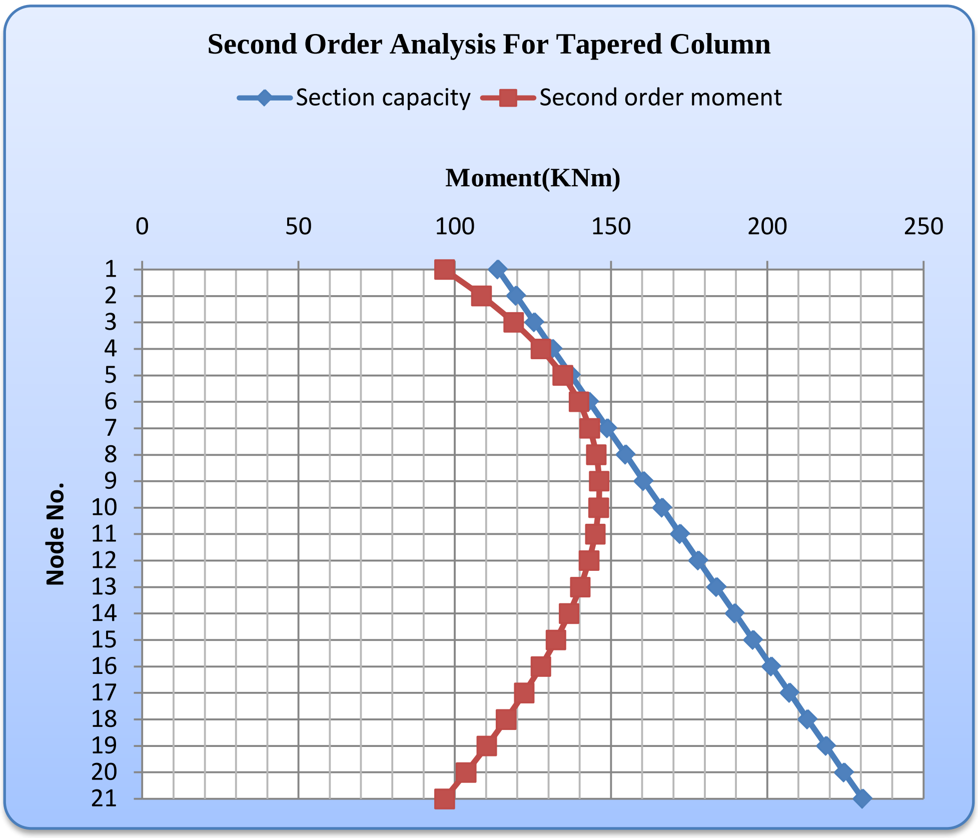


Fig. 5.11: Second order analysis of actual column for example 3

b. *Using effective length analysis approach*

- $I_1/I_2 = 0.15$
- $\sqrt{\frac{P}{f_{cd}A_{g,min}}} = 0.40$

Therefore both inertial ratio of 0.1 and 0.2 should be checked for $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.32$ and $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} =$ 0.48. By taking all the procedures as per the previous examples, the results are summarized as below.

	Design Node	Location of design node
$I_1/I_2 = 0.1$ & $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.32$	1	0.00L
$I_1/I_2 = 0.1$ & $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.48$	1	0.00L
$I_1/I_2 = 0.2$ & $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.32$	1	0.00L
$I_1/I_2 = 0.2$ & $\sqrt{\frac{P}{f_{cd}\,A_{g,min}}} = 0.48$	5	0.20L

Table 5.3: Determination of failure point for column in example 3

- ***Use a section at 0.20L as equivalent section / 0.20mX0.410m/***
- *Effective length*, $L_{eff} = 0.4705\pi L\sqrt{\frac{I_{eq}}{I_2}} = 13.3287\text{m}$

And the analysis of the equivalent column having a length of equal to 13.3287m and section 0.20mX0.410m is presented in fig.5.12

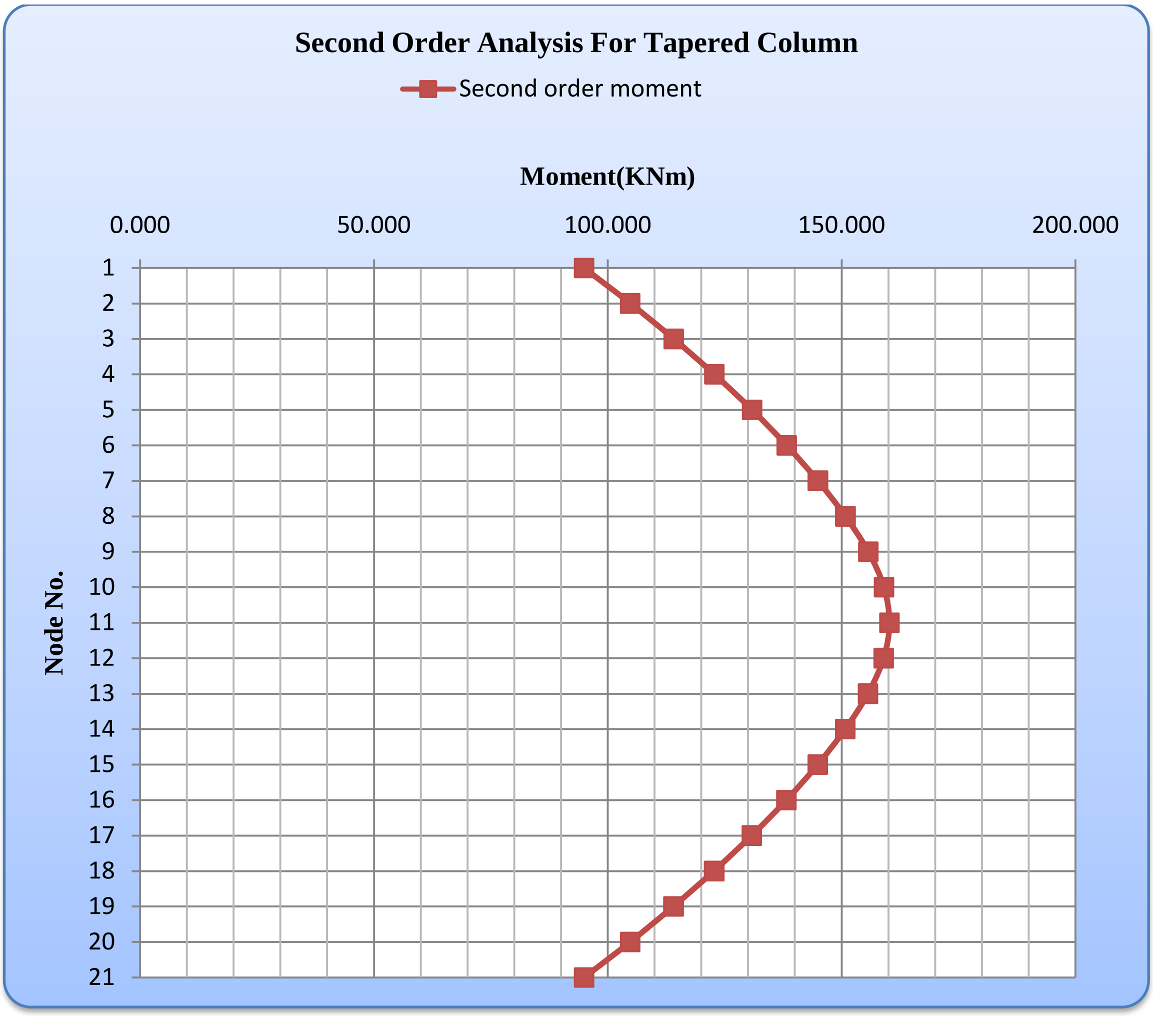


Fig. 5.12: Second order analysis of equivalent column for example 3

Comparison of results

	Iterative Analysis Approach	Effective Length Approach
Design node	Node 5	Node 5
Design Section	0.20m X 0.410m	0.20m X 0.410m
Design moment	134.571KNm	160.23KNm

Table 5.4: Comparison of results for column in example 1

ANNEX 1

ELASTIC BUCKLING LOADS OF COLUMN HAVING LINEARLY VARYING CROSS-SECTION

/ taken from a thesis document entitled "Buckling loads for tapered slender elastic columns" by: Thomas A. Branch /

BUCKLING LOADS FOR TAPERED SLENDER ELASTIC COLUMNS

by

Thomas A. Branch, Jr.
Lieutenant, United States Navy
B.S.M.E., Montana State University, 1960

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1967

Signature of Author Thomas A Branch, Jr.

Approved by R. E. Newton
Thesis Advisor and Chairman, Department of Mechanical Engineering

R. F. Rinehart
Academic Dean

N=3.0 C/L=0.00

I1/I2	A/L 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.05	2.391	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.10	3.273	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.15	3.948	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.20	4.518	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.25	5.021	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.30	5.477	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.35	5.897	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.40	6.290	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.45	6.659	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.50	7.009	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.55	7.343	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.60	7.663	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.65	7.971	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.70	8.267	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.75	8.554	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.80	8.832	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.85	9.102	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.90	9.364	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.95	9.620	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1.00	9.870	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Table A.1.1 buckling load factors for columns having linearly varying cross-section

Notes:

- The critical elastic buckling load should be calculated as:
  $P_{cr} = \frac{mE\,I_2}{L^2}$
- The table in appendix A.4 is prepared for the value of m.
- I_2 is the maximum moment of inertia of tapered column while I_1 is the minimum one.
- The parameter 'N' in the table indicates exponent characterizing the degree of taper. Use N=0 for the system in this research, which is linearly tapered, with non-symmetrical column systems.
- Use C/L = 0, the parameter which is applicable for column systems having the combination of both prismatic and non-prismatic portions.
- Use a/L=0, the parameter which is applicable for column systems having the combination of both prismatic and non-prismatic portions.

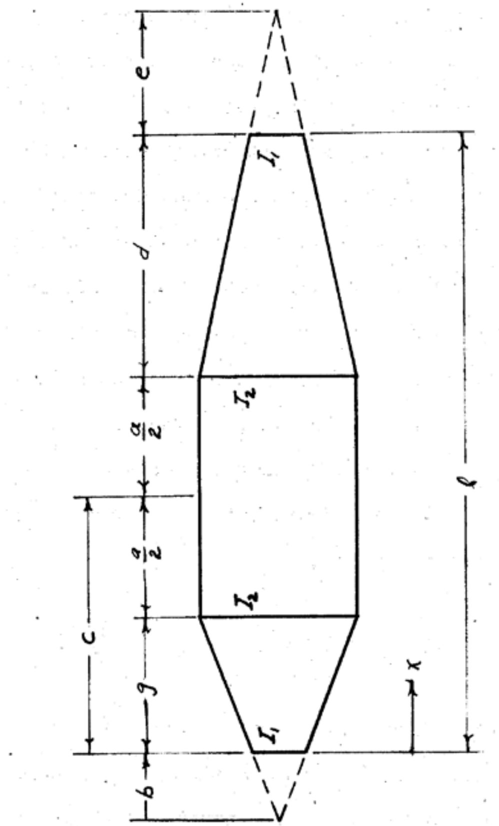


Fig. A.1.1: Illustration of symbols used in table A.1.1

ANNEX 2

COMPUTATION OF RANGES FOR STABILIZED LENGTH

Sample No.	I_1/I_2	L (m)	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
28	0.100	85.00	0.30L	0.16	52.612
		65.00		0.16	40.233
29	0.100	65.00	0.25L	0.16	42.035
		55.00		0.16	35.568
30	0.100	55.00	0.20L	0.16	37.236
		49.00		0.16	33.174
31	0.100	49.00	0.15L	0.16	34.806
		43.00		0.16	30.544
32	0.100	43.00	0.10L	0.16	32.125
		39.00		0.16	29.136
33	0.100	39.00	0.05L	0.16	30.726
		31.00		0.16	24.424
34	0.100	31.00	0.00L	0.16	25.833
		0.00		0.16	0.000

Table A.2.1: Evaluation of range values of L_s for I_1/I_2=0.1 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.16$

Sample No.	I_1/I_2	L (m)	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
35	0.100	95.00	0.30L	0.32	50.402
		80.00		0.32	42.443
36	0.100	80.00	0.25L	0.32	44.345
		45.00		0.32	24.944
37	0.100	45.00	0.20L	0.32	26.114
		39.00		0.32	22.632
38	0.100	39.00	0.15L	0.32	23.745
		35.00		0.32	21.310
39	0.100	35.00	0.10L	0.32	22.413
		30.00		0.32	19.211
40	0.100	30.00	0.05L	0.32	20.259
		23.42		0.32	15.816
41	0.100	23.42	0.00L	0.32	16.729
		0.00		0.32	0.000

Table A.2.2: Evaluation of range values of L_s for I_1/I_2=0.1 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.32$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
42	0.100	90.000	0.35L	0.48	40.063
		80.000		0.48	35.611
43	0.100	80.000	0.30L	0.48	37.138
		65.000		0.48	30.175
44	0.100	65.000	0.25L	0.48	31.526
		40.000		0.48	19.401
45	0.100	40.000	0.20L	0.48	20.311
		34.000		0.48	17.264
46	0.100	34.000	0.15L	0.48	18.113
		30.000		0.48	15.982
47	0.100	30.000	0.10L	0.48	16.809
		27.000		0.48	15.129
48	0.100	27.000	0.05L	0.48	15.954
		21.413		0.48	12.653
49	0.100	21.413	0.00L	0.48	13.383
		0.000		0.48	0.000

Table A.2.3: Evaluation of range values of L_s for I_1/I_2=0.1 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.48$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
50	0.100	54.000	0.30L	0.64	40.109
		49.000		0.64	36.395
51	0.100	49.000	0.25L	0.64	38.026
		40.000		0.64	31.041
52	0.100	40.000	0.20L	0.64	32.497
		31.000		0.64	25.185
53	0.100	31.000	0.15L	0.64	26.424
		24.000		0.64	20.457
54	0.100	24.000	0.10L	0.64	21.516
		21.500		0.64	19.275
55	0.100	21.500	0.05L	0.64	20.327
		18.000		0.64	17.018
56	0.100	18.000	0.00L	0.64	18.000
		0.000		0.64	0.000

Table A.2.4: Evaluation of range values of L_s for I_1/I_2=0.1 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.64$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
57	0.100	43.000	0.30L	0.80	20.532
		40.000		0.80	19.100
58	0.100	40.000	0.25L	0.80	19.955
		35.000		0.80	17.461
59	0.100	35.000	0.20L	0.80	18.280
		28.000		0.80	14.624
60	0.100	28.000	0.15L	0.80	15.343
		25.000		0.80	13.699
61	0.100	25.000	0.10L	0.80	14.408
		22.000		0.80	12.679
62	0.100	22.000	0.05L	0.80	13.371
		20.000		0.80	12.156
63	0.100	20.000	0.00L	0.80	12.857
		0.000		0.80	0.000

Table A.2.5: Evaluation of range values of L_s for I_1/I_2=0.1 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.80$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
64	0.200	85.00	0.30L	0.16	56.062
		71.00		0.16	46.829
65	0.200	71.00	0.25L	0.16	48.240
		65.00		0.16	44.163
66	0.200	65.00	0.20L	0.16	45.536
		58.00		0.16	40.632
67	0.200	58.00	0.15L	0.16	41.935
		53.00		0.16	38.320
68	0.200	53.00	0.10L	0.16	39.590
		49.00		0.16	36.602
69	0.200	49.00	0.05L	0.16	37.857
		38.00		0.16	29.358
70	0.200	38.50	0.00L	0.16	30.400
		0.00		0.16	0.000

Table A.2.6: Evaluation of range values of L_s for I_1/I_2=0.2 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.16$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
71	0.200	80.00	0.35L	0.32	40.048
		70.00		0.32	35.042
72	0.200	70.00	0.30L	0.32	36.068
		38.00		0.32	19.580
73	0.200	38.00	0.25L	0.32	20.170
		29.00		0.32	15.393
74	0.200	29.00	0.20L	0.32	15.871
		24.50		0.32	13.409
75	0.200	24.50	0.15L	0.32	13.839
		23.00		0.32	12.991
76	0.200	23.00	0.10L	0.32	13.422
		20.00		0.32	11.671
77	0.200	20.00	0.05L	0.32	12.071
		15.08		0.32	9.102
78	0.200	15.08	0.00L	0.32	9.425
		0.000		0.32	0.000

Table A.2.7: Evaluation of range values of L_s for I_1/I_2=0.2 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 10$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
79	0.200	35.000	0.35L	0.48	23.362
		20.000		15.000	13.349
80	0.200	20.000	0.30L	0.48	13.740
		14.000		0.48	9.618
81	0.200	14.000	0.25L	0.48	9.908
		12.300		0.48	8.705
82	0.200	12.300	0.20L	0.48	8.976
		11.000		0.48	8.027
83	0.200	11.000	0.15L	0.48	8.284
		9.500		0.48	7.155
84	0.200	9.500	0.10L	0.48	7.392
		8.800		0.48	6.847
85	0.200	8.800	0.05L	0.48	7.082
		7.390		0.48	5.947
86	0.200	7.390	0.00L	0.48	6.158
		0.000		0.48	0.000

Table A.2.8: Evaluation of range values of L_s for I_1/I_2=0.2 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 15$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
87	0.200	47.000	0.30L	0.64	21.526
		37.500		0.64	17.175
88	0.200	37.500	0.25L	0.64	17.693
		34.000		0.64	16.042
89	0.200	34.000	0.20L	0.64	16.540
		22.500		0.64	10.946
90	0.200	22.500	0.15L	0.64	11.297
		20.500		0.64	10.293
91	0.200	20.500	0.10L	0.64	10.634
		19.000		0.64	9.856
92	0.200	19.000	0.05L	0.64	10.194
		16.570		0.64	8.890
93	0.200	16.570	0.00L	0.64	9.206
		0.000		0.64	

Table A.2.9: Evaluation of range values of L_s for I_1/I_2=0.2 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.64$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
94	0.200	11.300	0.30L	0.80	9.316
		9.000		0.80	7.420
95	0.200	9.000	0.25L	0.80	7.643
		7.000		0.80	5.945
96	0.200	7.000	0.20L	0.80	6.130
		5.800		0.80	5.079
97	0.200	5.800	0.15L	0.80	5.242
		5.250		0.80	4.745
98	0.200	5.250	0.10L	0.80	4.902
		5.000		0.80	4.669
99	0.200	5.000	0.05L	0.80	4.829
		4.592		0.80	4.435
100	0.200	4.592	0.00L	0.80	4.592
		0.000		0.80	0.000

Table A.2.10: Evaluation of range values of L_s for I_1/I_2=0.2 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.80$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
101	0.300	79.00	0.40L	0.16	36.650
		65.00		0.16	30.155
012	0.300	65.00	0.35L	0.16	30.790
		50.00		0.16	23.684
103	0.300	50.00	0.30L	0.16	24.194
		41.00		0.16	19.839
104	0.300	41.00	0.25L	0.16	20.275
		38.00		0.16	18.791
105	0.300	38.00	0.20L	0.16	19.214
		35.00		0.16	17.697
106	0.300	35.00	0.15L	0.16	18.104
		30.00		0.16	15.517
107	0.300	30.00	0.10L	0.16	15.882
		27.00		0.16	14.294
108	0.300	27.00	0.05L	0.16	14.639
		18.63		0.16	10.101
109	0.300	18.63	0.00L	0.16	10.350
		0.000		0.16	0.000

Table A.2.11: Evaluation of range values of L_s for I_1/I_2=0.3 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.16$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
110	0.300	38.00	0.40L	0.32	31.734
		29.00		0.32	24.218
111	0.300	29.00	0.35L	0.32	24.728
		18.00		0.32	15.348
112	0.300	18.00	0.30L	0.32	15.678
		15.00		0.32	13.065
113	0.300	15.00	0.25L	0.32	13.352
		13.00		0.32	11.572
114	0.300	13.00	0.20L	0.32	11.832
		11.00		0.32	10.012
115	0.300	11.00	0.15L	0.32	10.242
		10.10		0.32	9.404
116	0.300	10.10	0.10L	0.32	9.625
		9.50		0.32	9.053
117	0.300	9.50	0.05L	0.32	9.271
		6.56		0.32	6.402
118	0.300	6.56	0.00L	0.32	6.560
		0.000		0.32	0.000

Table A.2.12: Evaluation of range values of L_s for I_1/I_2=0.3 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.32$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
119	0.300	43.000	0.35L	0.48	31.426
		22.000		0.48	16.078
120	0.300	22.000	0.30L	0.48	16.424
		17.000		0.48	12.691
121	0.300	17.000	0.25L	0.48	12.970
		15.000		0.48	11.444
122	0.300	15.000	0.20L	0.48	11.701
		13.500		0.48	10.531
123	0.300	13.500	0.15L	0.48	10.773
		12.800		0.48	10.215
124	0.300	12.800	0.10L	0.48	10.455
		12.000		0.48	9.802
125	0.300	12.000	0.05L	0.48	10.038
		9.870		0.48	8.256
126	0.300	9.870	0.00L	0.48	8.460
		0.000		0.48	0.000

Table A.2.13: Evaluation of range values of L_s for I_1/I_2=0.3& $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.48$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
127	0.300	30.000	0.40L	0.64	25.052
		25.000		0.64	20.876
128	0.300	25.000	0.35L	0.64	21.316
		16.500		0.64	14.069
129	0.300	16.000	0.30L	0.64	13.936
		14.000		0.64	12.194
130	0.300	13.000	0.25L	0.64	11.571
		11.850		0.64	10.548
131	0.300	11.100	0.20L	0.64	10.102
		10.400		0.64	9.465
132	0.300	10.000	0.15L	0.64	9.310
		9.400		0.64	8.752
133	0.300	9.000	0.10L	0.64	8.576
		8.520		0.64	8.119
134	0.300	8.200	0.05L	0.64	8.002
		7.800		0.64	7.612
135	0.300	7.300	0.00L	0.64	7.300
		0.000		0.64	0.000

Table A.2.14: Evaluation of range values of L_s for I_1/I_2=0.3& $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.64$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
136	0.300	36.000	0.35L	0.80	19.184
		27.000		0.80	14.388
137	0.300	27.000	0.30L	0.80	14.698
		20.800		0.80	11.323
138	0.300	20.800	0.25L	0.80	11.571
		17.500		0.80	9.736
139	0.300	17.500	0.20L	0.80	9.954
		15.800		0.80	8.987
140	0.300	15.800	0.15L	0.80	9.194
		14.500		0.80	8.438
141	0.300	14.500	0.10L	0.80	8.636
		13.500		0.80	8.040
142	0.300	13.500	0.05L	0.80	8.234
		12.200		0.80	7.441
143	0.300	12.200	0.00L	0.80	7.625
		0.000		0.80	0.000

Table A.2.15: Evaluation of range values of L_s for I_1/I_2=0.3& $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.80$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
144	0.400	48.000	0.40L	0.16	41.999
		30.000		0.16	26.249
145	0.400	30.000	0.35L	0.16	26.666
		25.000		0.16	22.222
146	0.400	25.000	0.30L	0.16	22.580
		20.000		0.16	18.064
147	0.400	20.000	0.25L	0.16	18.360
		18.500		0.16	16.983
148	0.400	18.500	0.20L	0.16	17.266
		16.500		0.16	15.400
149	0.400	16.500	0.15L	0.16	15.661
		15.300		0.16	14.522
150	0.400	15.300	0.10L	0.16	14.772
		12.000		0.16	11.586
151	0.400	12.000	0.05L	0.16	11.789
		10.680		0.16	10.493
152	0.400	10.680	0.00L	0.16	10.680
		0.000		0.16	0.000

Table A.2.16: Evaluation of range values of L_s for I_1/I_2=0.4 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.16$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
153	0.400	44.000	0.45L	0.32	27.076
		41.000		0.32	25.230
154	0.400	41.000	0.40L	0.32	25.624
		21.000		0.32	13.125
155	0.400	21.000	0.35L	0.32	13.333
		17.000		0.32	10.793
156	0.400	17.000	0.30L	0.32	10.968
		13.000		0.32	8.387
157	0.400	13.000	0.25L	0.32	8.524
		11.500		0.32	7.541
158	0.400	11.500	0.20L	0.32	7.667
		10.800		0.32	7.200
159	0.400	10.800	0.15L	0.32	7.322
		9.500		0.32	6.441
160	0.400	9.500	0.10L	0.32	6.552
		8.520		0.32	5.876
161	0.400	8.520	0.05L	0.32	5.979
		5.880		0.32	4.126
162	0.400	5.880	0.00L	0.32	4.200
		0.000		0.32	0.000

Table A.2.17: Evaluation of range values of L_s for I_1/I_2=0.4 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.32$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
163	0.400	32.600	0.45L	0.48	17.553
		30.000		0.48	16.153
164	0.400	30.000	0.40L	0.48	16.406
		15.400		0.48	8.422
165	0.400	15.400	0.35L	0.48	8.555
		12.800		0.48	7.111
166	0.400	12.800	0.30L	0.48	7.226
		10.500		0.48	5.927
167	0.400	10.500	0.25L	0.48	6.024
		9.100		0.48	5.221
168	0.400	9.100	0.20L	0.48	5.308
		7.800		0.48	4.550
169	0.400	7.800	0.15L	0.48	4.627
		7.100		0.48	4.212
170	0.400	7.100	0.10L	0.48	4.284
		6.600		0.48	3.983
171	0.400	6.600	0.05L	0.48	4.053
		0.000		0.48	3.579
172	0.400	5.829	0.00L	0.48	3.643
		0.000		0.48	0.000

Table A.2.18: Evaluation of range values of L_s for I_1/I_2=0.4 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.48$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
173	0.400	29.000	0.40L	0.64	15.225
		16.000		0.64	8.400
174	0.400	16.000	0.35L	0.64	8.533
		12.800		0.64	6.827
175	0.400	12.800	0.30L	0.64	6.937
		11.000		0.64	5.961
176	0.400	11.000	0.25L	0.64	6.059
		9.500		0.64	5.233
177	0.400	9.500	0.20L	0.64	5.320
		8.600		0.64	4.816
178	0.400	8.600	0.15L	0.64	4.898
		7.700		0.64	4.385
179	0.400	7.700	0.10L	0.64	4.461
		7.200		0.64	4.171
180	0.400	7.200	0.05L	0.64	4.244
		6.415		0.64	3.781
181	0.400	6.415	0.00L	0.64	3.849
		0.000		0.64	0.000

Table A.2.19: Evaluation of range values of L_s for I_1/I_2=0.4 & $\sqrt{\frac{P}{f_{cd}\,A_{gmin}}} = 0.64$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
182	0.400	28.000	0.40L	0.80	14.291
		21.000		0.80	10.719
183	0.400	21.000	0.35L	0.80	10.889
		17.000		0.80	8.815
184	0.400	17.000	0.30L	0.80	8.957
		13.800		0.80	7.271
185	0.400	13.800	0.25L	0.80	7.390
		11.000		0.80	5.891
186	0.400	11.000	0.20L	0.80	5.989
		9.500		0.80	5.172
187	0.400	9.500	0.15L	0.80	5.260
		8.750		0.80	4.845
188	0.400	8.750	0.10L	0.80	4.928
		8.500		0.80	4.787
189	0.400	8.500	0.05L	0.80	4.871
		7.964		0.80	4.564
190	0.400	7.964	0.00L	0.80	4.646
		0.000		0.80	0.000

Table A.2.20: Evaluation of range values of L_s for I_1/I_2=0.4 & $\sqrt{\frac{P}{f_{cd}\,A_{gmin}}} = 0.80$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
191	0.500	57.00	0.45L	0.16	34.021
		40.00		0.16	23.874
192	0.500	40.00	0.40L	0.16	24.155
		33.80		0.16	20.411
193	0.500	33.80	0.35L	0.16	20.654
		23.00		0.16	14.055
193	0.500	23.00	0.30L	0.16	14.224
		18.20		0.16	11.256
195	0.500	18.20	0.25L	0.16	11.393
		16.30		0.16	10.204
196	0.500	16.30	0.20L	0.16	10.330
		15.40		0.16	9.759
197	0.500	15.40	0.15L	0.16	9.881
		14.00		0.16	8.983
198	0.500	14.00	0.10L	0.16	9.097
		12.30		0.16	7.992
199	0.500	12.30	0.05L	0.16	8.095
		8.56		0.16	5.631
200	0.500	8.56	0.00L	0.16	5.704
				0.16	

Table A.2.21: Evaluation of range values of L_s for I_1/I_2=0.5 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.16$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
201	0.500	55.50	0.45L	0.32	24.844
		40.80		0.32	18.264
202	0.500	40.80	0.40L	0.32	18.479
		20.00		0.32	9.058
203	0.500	20.00	0.35L	0.32	9.166
		16.50		0.32	7.562
204	0.500	16.50	0.30L	0.32	7.653
		15.00		0.32	6.957
205	0.500	15.00	0.25L	0.32	7.042
		12.50		0.32	5.869
206	0.500	12.50	0.20L	0.32	5.941
		10.60		0.32	5.038
207	0.500	10.60	0.15L	0.32	5.101
		8.70		0.32	4.187
208	0.500	8.70	0.10L	0.32	4.240
		7.80		0.32	3.801
209	0.500	7.80	0.05L	0.32	3.850
		6.12		0.32	3.023
210	0.500	6.12	0.00L	0.32	3.062

Table A.2.22: Evaluation of range values of L_s for I_1/I_2=0.5 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.32$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
211	0.500	21.50	0.45L	0.48	19.249
		16.00		0.48	14.325
212	0.500	16.00	0.40L	0.48	14.493
		7.90		0.48	7.156
213	0.500	7.90	0.35L	0.48	7.241
		6.40		0.48	5.866
214	0.500	6.40	0.30L	0.48	5.937
		5.80		0.48	5.380
215	0.500	5.80	0.25L	0.48	5.446
		4.80		0.48	4.507
216	0.500	4.80	0.20L	0.48	4.563
		4.00		0.48	3.802
217	0.500	4.00	0.15L	0.48	3.850
		3.80		0.48	3.657
218	0.500	3.80	0.10L	0.48	3.704
		3.70		0.48	3.606
219	0.500	3.70	0.05L	0.48	3.653
		3.10		0.48	3.060
220	0.500	3.10	0.00L	0.48	3.100
		0.000		0.48	0.000

Table A.2.23: Evaluation of range values of L_s for I_1/I_2=0.5 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.48$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
221	0.500	40.00	0.40L	0.64	18.116
		23.00		0.64	10.870
222	0.500	23.00	0.35L	0.64	10.999
		18.00		0.64	8.250
223	0.500	18.00	0.30L	0.64	8.349
		15.50		0.64	6.957
224	0.500	15.50	0.25L	0.64	7.042
		13.00		0.64	5.634
225	0.500	13.00	0.20L	0.64	5.704
		11.45		0.64	4.753
226	0.500	11.45	0.15L	0.64	4.812
		10.50		0.64	4.716
227	0.500	10.50	0.10L	0.64	4.776
		9.80		0.64	4.678
228	0.500	9.80	0.05L	0.64	4.738
		9.09		0.64	4.442
229	0.500	9.09	0.00L	0.64	4.500
		0.000		0.64	0.000

Table A.2.24: Evaluation of range values of L_s for I_1/I_2=0.5 & $\sqrt{\frac{P}{f_{cd}A_{gmin}}} = 0.64$

Sample No.	I_1/I_2	L	Location of Equivalent node	$\sqrt{\frac{P}{f_{cd}A_g}}$ at min.	L_s
230	0.500	18.00	0.35L	0.80	16.499
		13.50		0.80	12.374
231	0.500	13.50	0.30L	0.80	12.523
		11.00		0.80	10.204
232	0.500	11.00	0.25L	0.80	10.329
		10.10		0.80	9.484
233	0.500	10.10	0.20L	0.80	9.601
		9.60		0.80	9.126
234	0.500	9.60	0.15L	0.80	9.240
		8.40		0.80	8.085
235	0.500	8.40	0.10L	0.80	8.187
		7.70		0.80	7.505
236	0.500	7.70	0.05L	0.80	7.601
		7.07		0.80	6.979
237	0.500	7.07	0.00L	0.80	7.070
		0.000		0.80	0.000

Table A.2.25: Evaluation of range values of L_s for I_1/I_2=0.5 & $\sqrt{\frac{P}{f_{cd}\,A_{gmin}}} = 0.80$

ANNEX 3

COMPUTER PROGRAM
FOR
MOMENT-CURVATURE RELATIONSHIP OF
COMBINED AXIALLY AND FLEXURAL LOADED SECTIONS
USING
FORTRAN PROGRAMMING LANGUAGE

GENERAL FLOW CHART

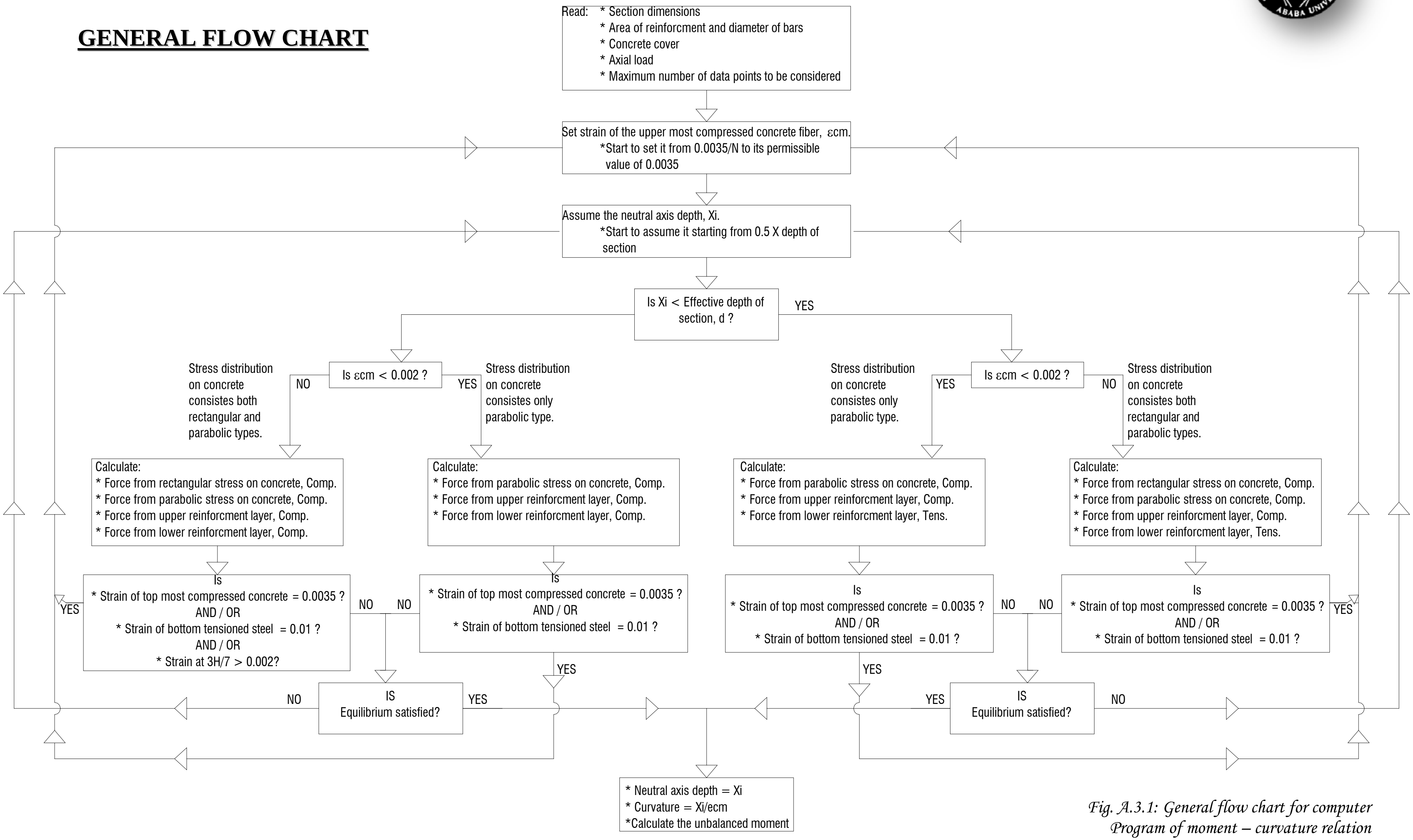


Fig. A.3.1: General flow chart for computer Program of moment – curvature relation

```
!       Moment Curvature Relationship For column sections
!*******************************************************************************************
!       PROGRAM:                Moment curvature relationship
!       PURPOSE:                To establish the moment curvature relationship
!       PROGRAMMED BY:          Tinsae Woldegebreal
!       Target:                 For the supporting document of research paper
!*******************************************************************************************
   Program Moment_curvature
   Implicit none
   INTEGER I, N, k
   Real :: Ems,fc,FSS,FSC,FCC,FCCp,FCCR,ASBOTTOM,ASTOP,t, h, d,COVER,b, fyd,esyd,dia,P,EE
   REAL :: Pcap,STLSTRS,STLSTRC, Maxststrn = 0.01
   REAL :: alpha = 0.85, X, Xcp, A, bb, c, dd, E, centroid1, centroid2,ff,gg,hh,ii,jj,y,yy,z,zz
   REAL, ALLOCATABLE :: ecm(:), NXD(:), es(:), Moment(:), CURVATURE(:),est(:),ecb(:)
   CHARACTER (len = 20) FILENAME
   WRITE (*,*) "     COMPUTER PROGRAM ON MOMENT-CURVATURE RELATION"
   WRITE (*,*) "PREPARED AS SUPPORTING MATERIAL FOR MASTER'S THESES PAPER"
   WRITE (*,*) "ENTER THE NAME OF THIS FILE!"
   READ*, FILENAME
   OPEN (UNIT=100, FILE=FILENAME, Status ='REPLACE', ACTION='WRITE')
! ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
  100 FORMAT (5F10.2)
! ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
   WRITE (*,*) "ENTER THE CYLINDERICAL COMPRESSIVE STRENGTH (N/mm2)"
   READ*, fc
   WRITE (*,*) "ENTER THE DESIGN YIELD STRENGTH OF REINFORCMENT BARS (N/mm2)"
   READ*, fyd
   WRITE (*,*) "  * ENTER THE AREA OF BOTTOM STEEL (mm2)"
   WRITE (*,*) "  * ENTER THE AREA OF TOP STEEL (mm2)"
   WRITE (*,*) "  * ENTER THE TOTAL DEPTH OF THE SECTION (mm)"
   WRITE (*,*) "  * ENTER THE DIAMETER OF BARS (mm)"
   WRITE (*,*) "  * ENTER THE WIDTH OF THE SECTION (mm) "
   WRITE (*,*) "  * ENTER THE CONCRETE COVER  (mm) "
   WRITE (*,*) "  * ENTER THE AXIAL LOAD ( N )  "
   WRITE (*,*) "______________________________________________________________________"
   WRITE (*,*) "  ***** Plese press the 'ENTER' key after each input *******  "
   WRITE (*,*) "______________________________________________________________________"
   READ*, ASBOTTOM,ASTOP,h,dia,b,COVER,p
   WRITE (*,*) "ENTER THE MAXIMUM NUMBER OF DATA POINTS YOU PREFER TO BE CONSIDERED!"
   READ*, N
```

```
WRITE (*,*) "_____________________________________________________________________________"
WRITE (*,*) "KEY: (Column 1)=Strain of at top,(Column 2)=neutral axis depth"
WRITE (*,*) "     (Column 3)=Strain at 3H/7, (Column 4)=moment, (Column5)= Curvature"
WRITE (*,*) "_____________________________________________________________________________"
write (*,*) "STRAIN AT    N.A.DEPTH  STRAIN AT     MOMENT   CURVATURE  "
write (*,*) "TOP(X10-6)   (mm)       3H/7(X10-6)    (KNm)   (X10-6)             "
ALLOCATE (MOMENT(N+1))
ALLOCATE (CURVATURE(N+1))
ALLOCATE (ecm(N+1))
ALLOCATE (NXD(N+1))
ALLOCATE (es(N+1))
ALLOCATE (est(N+1))
Allocate (ecb(N+1))
EMS = 200000                                                        !modulus of elasticity of rebar
d=h-cover-(dia/2)                                                   !effective depth of section
esyd = fyd/EMS                                                      !strain of steel when yielding
X = 0.5*d                                              !neutral axis depth starting point for iteration
Do I = 1, N
ecm(i) = i*0.0035/N                                    !Setting out of strain of the most compressed concrete fiber
Pcap=((alpha*0.8*fc/1.5)*b*h)+(fyd*(AStop+ASbottom))                !section capacity
IF(P.GE.Pcap) then ; Write (*,*) "     "
write (*,*)"SORRY, The section can not carry the applied axial load!"
stop
else
0001  est(i) = ((X-cover-(dia/2))*ecm(i)/X)
!CASE 1, when the neutral axis depth is below effective depth of the section!
If(x.LE.d)then
ecb(i)=0E-06;                                          !strain at 3H/7 is not necessary to be checked
es(i) = ((d-X)/X)*ecm(i);                                           !strain of tensioned steel at bottom
IF (es(i).GE. Maxststrn) EXIT
IF (est(i).GE. Maxststrn) EXIT
!Case 1.1, When stress distribution on concrete is purely parabolic
IF (ecm(i) .LE. 0.002) THEN
FCC = ((500*b*(alpha*0.8*fc/1.5)*(ecm(i))*X)-(250000*b*(alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*X/3))   !Force from concrete,case1.1
IF (es(i).LE. esyd) THEN ; STLSTRS = EMS*es(i); ELSE ; STLSTRS = fyd ; END IF
FSS = STLSTRS*ASBOTTOM                                              ! Force from tensioned steel, case1.1
IF (est(i).GE. Maxststrn) EXIT
IF (est(i).LE. esyd) THEN ; STLSTRC = EMS*est(i) ; ELSE ; STLSTRC = fyd ; END IF
FSC = STLSTRC*ASTOP                                                 ! Force from compressed steel, case1.1
t= FCC+FSC-FSS                                                      ! Equilibrium of forces, case1.1
```

```
If(t>(1000.0+P))Then ; X = X - 0.05 ; GO TO 0001 ;
Else if (t < (-1000.0+P)) Then ; X = X + 0.05 ; GO TO 0001; ELSE ; NXD(i) = X ; END IF
! Case 1.2, combination of parabolic and rectangular
ELSE
Xcp=(X*(1-(0.002/(ecm(i)))))                    !Changing point of depth from rectangular to parabolic stress distribution on concrete,case1.2
A=((X*X*X/3)-(X*X*Xcp)+(X*Xcp*Xcp)-(Xcp*Xcp*Xcp/3))*250*(ecm(i))/X
bb=(X*X/2)-(X*Xcp)+(Xcp*Xcp/2)
FCCp=(bb-A)*1000*b*(ecm(i))*(alpha*0.8*fc/1.5)/X          !Force from Parabolic stress distribution on concrete,case1.2
FCCR = (alpha*0.8*fc/1.5)*b*Xcp                           !Force from rectangular stress distribution on concrete,case1.2
FCC = FCCp + FCCR                                         !Total force from concrete,case1.2
IF (es(i).LE. esyd) THEN ; STLSTRS = EMS*es(i); ELSE ; STLSTRS = fyd ; END IF
FSS = STLSTRS*ASBOTTOM                                              !Force from tensioned steel,case1.2
IF (est(i).GE. Maxststrn) EXIT
IF (est(i).LE. esyd) THEN ; STLSTRC = EMS*est(i); ELSE ; STLSTRC = fyd ; END IF
FSC = STLSTRC*ASTOP                                                 !Force from compressed steel,case1.2
t= FCC+FSC-FSS                                                      !Equilibrium of forces,case1.2
If(t>(1000.0+P))Then ; X = X - 0.05 ; GO TO 0001
Else if (t < (-1000.0+P)) Then ; X = X + 0.05 ; GO TO 0001
ELSE ; NXD(i) = X ; END IF
End if
IF (ecm(i) .LE. 0.002) THEN
dd=(500*b*(alpha*0.8*fc/1.5)*(ecm(i))*X)-(250000*b*(alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*X/3)
E=(1000*b*(alpha*0.8*fc/1.5)*(ecm(i))*X*X/6)-(250000*b*(alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*X*X/12)
If(abs(dd).LE.00)then ; centroid1=0 ; else ; centroid1 =abs(E/dd) ; endif
Moment(i)=((abs(FCC))*(h-centroid1))+((abs(Fsc))*(h-cover-(dia/2)))-(P*h/2)-(FSS*(cover+(dia/2)))      !Unbalanced moment,case1.1
else
c=((X*X*X/3)-(X*X*Xcp)+(X*Xcp*Xcp)-(Xcp*Xcp*Xcp/3))*250*(ecm(i))/X
ff=(X*X/2)-(X*Xcp)+(Xcp*Xcp/2)
gg=(c-ff)*1000*b*(ecm(i))*(alpha*0.8*fc/1.5)/X
hh=((X*X*X*X/12)-(X*X*Xcp*Xcp/2)+(2*X*Xcp*Xcp*Xcp/3)-(Xcp*Xcp*Xcp*Xcp/4))*250*(ecm(i))/X
ii=(X*X*X/6)-(X*Xcp*Xcp/2)+(Xcp*Xcp*Xcp/3)
jj=1000*b*(ecm(i))*(alpha*0.8*fc/1.5)*(ii-hh)/X
if(abs(gg).LE.10)then ; centroid2=0 ; else ; centroid2=abs(jj/gg) ; endif
Moment(i) =(abs(FCCp)*(h-centroid2))+(abs(FCCR)*(h-(Xcp/2)))+((abs(FSC))*d)-(P*h/2)-(FSS*(cover+(dia/2)))      !Unbalanced moment,case1.2
endif
CURVATURE(i) = ecm(i)/NXD(i)*1E06                                    !Curvature, Case1
!CASE 2 , When the neutral axis depth is above the effective depth of the section!
Else
es(i) = ((X-d)/X)*ecm(i)
IF (es(i).GE. Maxststrn) EXIT
```

```
IF (est(i).GE. Maxststrn) EXIT
!Case 2.1, When stress distribution on concrete is purely parabolic
IF (ecm(i) .LE. 0.002) THEN
y=(1000*b*(alpha*0.8*fc/1.5)*(ecm(i))*h*(X-(h/2))/X)
YY=(250000*b*(alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*h*((X*X)-(X*h)+(h*h/3))/(X*X))
FCC = Y-YY                                                        !Force from concrete,case2.1
IF (es(i).LE. esyd) THEN ; STLSTRS = EMS*es(i) ; ELSE ; STLSTRS = fyd ; END IF
FSS = STLSTRS*ASBOTTOM                                            !Force from bottom compressed steel,case2.1
IF (est(i).GE. Maxststrn) EXIT
IF (est(i).LE. esyd) THEN ; STLSTRC = EMS*est(i) ; ELSE ; STLSTRC = fyd ; END IF
FSC = STLSTRC*ASTOP                                               !Force from top compressed steel,case2.1
t= FCC+FSC+FSS                                                    !Equilibrium of forces,case2.1
If(t>(10.0+P))Then
if (X.LE.0)then
Nxd(i)=0
ecb(i)=0                                                     !strain at 3H/7 is not necessary to be cheched,case2.1
Else ; X = X - 0.001 ; GO TO 0001 ; endif
Else if (t < (-10.0+P)) Then
If(X.GE.30000)then ; NXD(i)=0 ; ecb(i)=0 ; else ; X = X + 0.001 ; GO TO 0001 ; endif
ELSE ; NXD(i) = X ; ecb(i)=ecm(i)*(x-(3*h/7))/x ; END IF
If (ecb(i).GE.0.002) exit
ELSE
If((X-(0.002*X/(ecm(i)))).GE.h) then ;FCCP=0 ; FCCR=(alpha*0.8*fc/1.5)*b*h
!Case 2.2, When stress distribution on concrete is a combination of parabolic and rectangular
Else
Xcp=(X-(0.002*X/(ecm(i))))                 !Changing point of depth from rectangular to parabolic stress distribution on concrete,case2.2
A=((X*X*h)-(X*h*h)+(h*h*h/3)-(X*X*Xcp)+(X*Xcp*Xcp)-(Xcp*Xcp*Xcp/3))*250*(ecm(i))/X
bb=(X*h)-(h*h/2)-(X*Xcp)+(Xcp*Xcp/2)
FCCp=(bb-A)*1000*b*(ecm(i))*(alpha*0.8*fc/1.5)/X            !Force from parabolic stress distribution on concrete,case2.2
FCCR = (alpha*0.8*fc/1.5)*b*Xcp                              !Force from rectangular stress distribution on concrete,case2.2
FCC = FCCp + FCCR                                            !Total force from concrete,case2.2
Endif
IF (es(i).LE. esyd) THEN ; STLSTRS = EMS*es(i) ; ELSE ; STLSTRS = fyd ; END IF
FSS = STLSTRS*ASBOTTOM                                       !Force from bottom compressed steel, case 2.2
IF (est(i).GE. Maxststrn) EXIT
IF (est(i).LE. esyd) THEN ; STLSTRC = EMS*est(i) ; ELSE ; STLSTRC = fyd ; END IF
FSC = STLSTRC*ASTOP                                          !Force from top compressed steel case 2.2
t= FCC+FSC+FSS                                               !Equilibrium of forces, case 2.2
If(t>(1000.0+P))Then
If (X.LE.0)then ; Nxd(i)=0 ; ecb(i)=0 ; else ; X = X - 0.05 ; GO TO 0001 ; endif
```

```
Else if (t < (-1000.0+P)) Then
If(X.GE.30000)then ; nxd(i)=0 ; ecb(i)=0 ; else ; X = X + 0.05 ; GO TO 0001; endif
ELSE ; NXD(i) = X ; ecb(i)=ecm(i)*(x-(3*h/7))/x ; END IF
If (ecb(i).GE.0.002) exit
endif
If(nxd(i)>0)then
IF (ecm(i) .LE. 0.002) THEN
z=(1000*b*(alpha*0.8*fc/1.5)*(ecm(i))*h*(X-(h/2))/X)
zz=(250000*b*(alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*h*((X*X)-(X*h)+(h*h/3))/(X*X))
dd = (Z-ZZ)
E=(1000*b*(alpha*0.8*fc/1.5)*(ecm(i))*h*h*((X/2)-(h/3))/X)
EE=250000*b*((alpha*0.8*fc/1.5)*(ecm(i))*(ecm(i))*h*h*((X*X/2)-(2*X*h/3)+(h*h/4))/(X*X) )
If(abs(dd).LE.10)then ; centroid1=0 ; else ; centroid1 =((E-EE)/dd) ; endif
Moment(i) = (((FCC))*(h-centroid1))+(((Fsc))*(h-cover-(dia/2)))-(P*h/2)+(FSS*(cover+(dia/2)))          !Unbalanced moment, case 2.1
else
C=((X*X*h)-(X*h*h)+(h*h*h/3)-(X*X*Xcp)+(X*Xcp*Xcp)-(Xcp*Xcp*Xcp/3))*250*(ecm(i))/X
ff=(X*h)-(h*h/2)-(X*Xcp)+(Xcp*Xcp/2)
gg=((ff-C)*1000*b*(ecm(i))*(alpha*0.8*fc/1.5)/X)
hh=(((X*X*h*h/2)-(2*X*h*h*h/3)+(h*h*h*h/4)-(X*X*Xcp*Xcp/2)+(2*X*Xcp*Xcp*Xcp/3)-(Xcp*Xcp*Xcp*Xcp/4)))
ii=((X*h*h/2)-(h*h*h/3)-(X*Xcp*Xcp/2)+(Xcp*Xcp*Xcp/3))
jj=(1000*b*(ecm(i))*(alpha*0.8*fc/1.5)*(ii-(250*(ecm(i))/X*hh))/X)
If(abs(gg).LE.10)then ; centroid2=0 ; else ; centroid2=(jj/gg) ; endif
Moment(i) =((FCCp)*(h-centroid2))+((FCCR)*(h-(0.5*Xcp)))+(FSC*d)-(P*h/2)+(FSS*(cover+(dia/2)))          !Unbalanced moment, case 2.2
Endif
Else ; Moment(i)=0 ; endif
If(NXD(i).le.0)then ; curvature(i)=0 ; else
CURVATURE(i) = ecm(i)/NXD(i)*1E06          !Curvature, case 2
End if; END IF; endif
END DO
k = i -1
WRITE (1,100)
DO I = 1,k
write (1,100) ecm(i)*1E06, NXD(i), ecb(i)*1E06,Moment(i)*1E-06, curvature(i)
END DO
WRITE (*,*) "______________________________________________________________________"
WRITE (*,*) "                              Thank You!"
WRITE (*,*) "______________________________________________________________________"
CLOSE (UNIT = 100)
End Program Moment_curvature
```

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