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SCHOOL OF GRADUATE STUDIES

ALTERNATIVE APPROACH FOR THE ANALYSIS OF SUSPENDED SLAB PANELS UNDER PARTITION WALL LOAD

Thomas Seyoum

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A Thesis Submitted to School of Graduate Studies, Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE in CIVIL ENGINEERING

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Description of Symbols

Symbols	**Descriptions**
D	flexural rigidity of plate
E	young's modulus of plate material
ε	strain
γ	support restraint factor
I_a	second moment of inertia of the peripheral beam
L_x	the shorter dimension of the slab panel
L_y	the longer dimension of the slab panel
M_x	moment per meter along the longer side
M_y	moment per meter along the shorter side
M_{xs}	support moment per meter along the longer side
M_{xf}	field moment per meter in shorter strip
M_{ys}	support moment per meter along the shorter side
M_{yf}	field moment per meter in longer strip
M_{xy}	twisting moment along the edge of the panel
q	load per sq. meter
σ_x	normal stress in the x-direction
σ_y	normal stress in the y-direction
σ_z	normal stress in the z-direction
t	thickness of the plate
θ_x	angle of rotation of normal line with respect to x-axis
θ_y	angle of rotation of normal line with respect to y-axis
τ_{xy}	shear stress along the face of the plate
υ	Poisson's ratio of the plate material
V_x	shear force per meter along the longer side
V_y	shear force per meter along the shorter side
u_0	displacement in the x-direction at $z = 0$
v_0	displacement in the y-direction at $z = 0$
w_0	displacement in the z-direction at $z = 0$
u	displacement in the x-direction
u	displacement in the x-direction

v	displacement in the y-direction
w	displacement in the z-direction

Abstract

Analysis techniques for most structural elements with different arrangement of externally applied loads are well developed and sufficiently covered with modern analysis theories. Further more, the application of today's high-speed computers enables the analysis of complex structures with different load arrangements. In addition to these effective analysis tools, building design codes provide table of values and analysis charts for the analysis of different elements of structural systems. As one of the structural element in building structure, different analysis methods were proposed for the analysis of suspended slab panel subjected to non-uniform loads.

Among the possible arrangement of externally applied loads on suspended slab panel a line load is the one which can best represent the weight of partition wall on it. Different simplified analysis methods were proposed to consider the contribution of this load to the design action effects. The current practice of accounting this partition wall loads in the analysis of suspended slab panel is to change it to the 'equivalent' uniformly distributed load. The analysis of regular slab panel subjected to uniformly distributed load may be carried out using computer softwares or using coefficients which are presented in many design codes (including the Ethiopian Building Code of Standards for concrete structures, EBCS 2 – 1995). The Swedish method and Reynolds method can be referred as some of the examples which are used to obtain equivalent uniformly distributed load. These methods analysis need to be investigated whether they can represent the actual load or not.

Therefore, as it is important to address this problem, comparative analysis has been carried out. A new simplified method which considers the actual scenario has been developed. This newly proposed method makes use of coefficients derived from the basic principle of elastic analysis of plate. The results have been verified by comparing it with results of the finite element analysis. It enables us to make elastic analysis of suspended slab panel subjected to the weight of partition wall and avoids the uncertain use of approximate methods.

1

Introduction

1.1 Problem Background

Analysis techniques for most structural elements with different arrangement of externally applied loads are well developed and sufficiently covered with modern analysis theories. When someone is talking about the analysis of structural elements, he/she may refer to the analysis of beams, columns, slab panels, footings, truss elements and etc. Among this wide group of structural elements, in this thesis, it is intended to investigate the response of a slab panel under certain load arrangement. This flat structural element in the form of plate is an important component in the field of structural analysis. It is in most cases the primary structural element liable to externally applied loads in different arrangement. Those different arrangements may include uniformly distributed loads, point loads, line loads and triangularly distributed loads. The uniformly distributed load is due to dead weight of the slab and weight of finishing material imposed on it. That of a point load is may be as a result of leg-supported object on it. A line load may originate from the weight of partition wall and a triangularly distributed load may result from surface load. The arrangement of these externally applied loads leads the analysis of slab panel from relatively simple one to complex partial differential equations.

Among the above possible arrangement of externally applied loads on suspended slab panel a line load is the one which can best represent the weight of partition wall on it. These partition walls are important structural element to make the space functional. Figure 1-1 shows a typical application of partition walls in residence building.

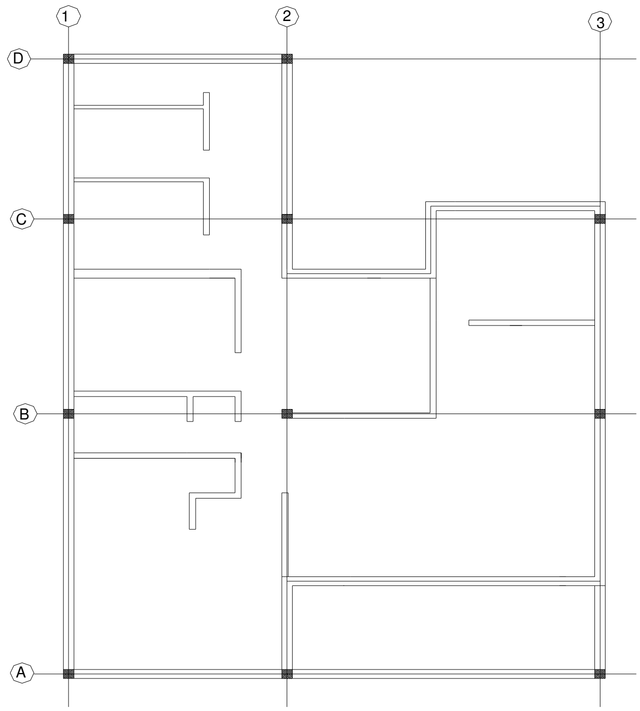


Figure 1-1: Typical application of partition wall in residence building

The analysis of suspended slab panel with this partition wall load hereafter referred as a line load, results in complex partial differential equations. Different simplified analysis methods were proposed to consider the contribution of this load to the internal action effects.

The current practice of accounting this partition wall loads in the analysis of suspended slab panel is to change it to the 'equivalent' uniformly distributed load. The Swedish method and Reynolds method can be referred to as some of the examples which are used to obtain the 'equivalent' uniformly distributed load. After getting the 'equivalent' uniformly distributed load, the analysis of a regular slab panel may be carried out using computer softwares or using coefficients which are presented in many design codes (including the Ethiopian Building Codes of Standards for concrete structures, EBCS 2 – 1995). But these methods of changing the line load to 'equivalent' uniformly distributed load need detail investigation whether they can represent the actual load or not.

Therefore, detailed investigation should be carried out and alternative simplified methods have to be developed in order to account this line loads. Ultimately it might be important to include those methods of accounting partition wall load in building design codes.

1.2 Objectives

The general theme of this thesis work is to device techniques for the analysis of suspended slab panel under partition wall load. The specific objectives of this thesis are, therefore:

- To make comparative analysis of the existing methods
- To analyze slab panels loaded with partition wall loads with application of finite element method.
- To propose alternative methods of accounting partition wall load.
- To provide tables of values or/and analysis chart for the analysis of slab panel under partition wall load.

1.3 Approaches and Methods used for the Study

To achieve the above objectives of the study, some methodologies are followed. The first step is to investigate what is so far done on the specified problem. As a result, literatures on the analysis techniques of suspended slab panel under different load arrangements are reviewed. Different building codes provisions for the problem specified are studied thoroughly. Having the basic concept of plate analysis at hand, a computer program is developed to produce coefficients for the newly proposed method of analysis of slab panel subjected to the weight of partition wall. Using this program tables of coefficients are produced for different arrangements of partition wall. In addition, as verification to these produced coefficients finite element analysis was done to a total of 72 models. To make a comparative analysis of different methods the above 72 models are analyzed using the existing simplified methods and the coefficients developed in this thesis work. Finally conclusions and recommendations are derived from the comparative analysis of different methods.

2

Basic Concept of Plate Theory

2.1 Basic Theory of Plates

The classical theory of plates has been built up by the contribution of many scientists around the world. The term "plate" denotes a body in a shape of a prism or cylinder whose height is small in comparison with the dimensions of its base [10]. Any three dimensional flat body satisfying equation 2.1 can be referred to as a plate.

$$t << L_x, L_y \tag{2.1}$$

where t, L_x, and L_y are representative dimensions in three dimensions. This can be demonstrated as shown in the figure 2-1 below.

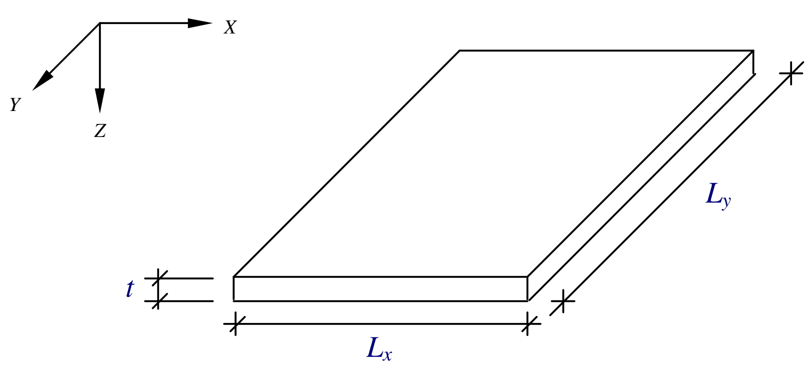


Figure 2-1: Plate.

The classical theory of isotropic plates (sometimes referred to as the Poisson-Kirchhoff theory of thin plates) is based on certain idealized assumptions and limiting conditions. These assumptions and limiting conditions can be stated as follows:

1- The material of which the plate consists is completely elastic satisfying Hooke's law and has the same elastic constant for all kinds of loading;
2- The material of the plate is homogeneous and isotropic.
3- The thickness of the plate is constant and it is small compared to the other dimension of plate.
4- Fibers which were perpendicular to the middle plane of the plate before bending occurs remain perpendicular to the (deformed) middle plane after the occurrence of bending.
5- The normal stress perpendicular to the plane of the plate is negligible. This can be stated in equation form as

$$\sigma_z = 0 \tag{2.2}$$

$$\varepsilon_z = \varepsilon_{xz} = \varepsilon_{yz} = 0 \tag{2.3}$$

6- The deflections of the plate are so small that the curvature in any particular direction is given by the second derivative of the deflection in that direction.

2.1.1 Governing Equations

Consider the infinitesimal element of *dx* by *dy* in figure 2-2, which is extracted from a category of thin plate with small deflection, in which the flexural stresses are dominant over membrane stresses.

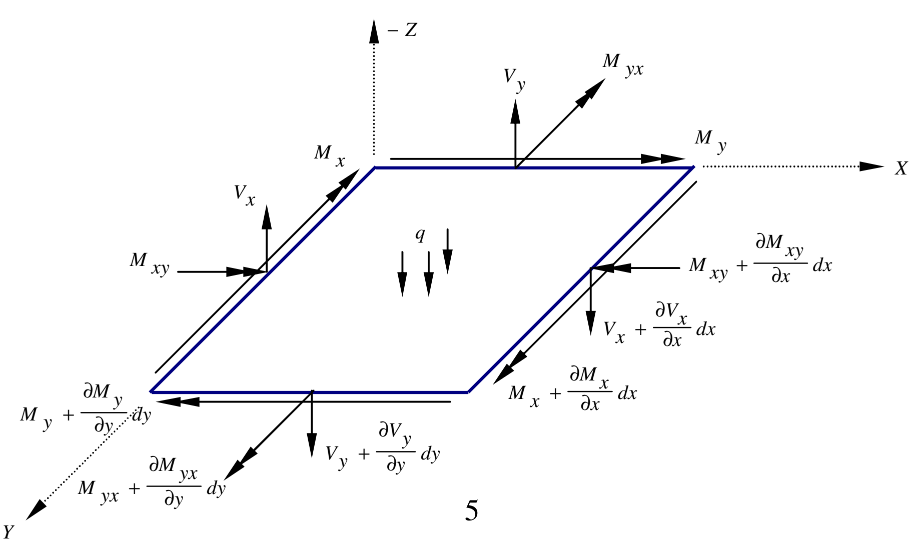


Figure 2-2: Free body diagram of plate element.

Starting, from the fifth assumption in the above paragraph, the displacement at any point of the plate can be written as

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} \\
v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{2.4}
$$

where u, v, and w are displacement components in the directions of x-, y-, and z-axes, respectively. And u_0 and v_0 are displacement components associated with the plane z=0. As it can be realized in equation (2.4) that the straight lines of the plate initially perpendicular to the middle surface remain straight and perpendicular to the deformed middle surface. Having the above displacement field equations we can write the strain-displacement relationship using continuum mechanics as

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} \\
\varepsilon_y &= \frac{\partial u_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2} \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) = -z\frac{\partial^2 w_0}{\partial x \partial y}
\end{aligned}
\tag{2.5}
$$

The equation (2.5) constitutes the strain-displacement relationship for the plate theory.

Equation of equilibrium conditions are preferably derived using resultant forces and moments instead of stresses. Thus to get the complete form of equilibrium equations one needs to integrate those equations over the thickness of the plate.

Assuming a distributed transversal load of magnitude of *qdxdy*, the vertical equilibrium condition yields the equation

$$\frac{\partial V_x}{\partial x}dxdy + \frac{\partial V_y}{\partial y}dydx + qdxdy = 0$$

from which

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + q = 0 \tag{2.6a}$$

Taking moments of all forces acting on the element with respect to the x-axis, we obtain the equation of equilibrium

$$\frac{\partial M_{xy}}{\partial x}dxdy + \frac{\partial M_y}{\partial y}dydx - V_y dxdy = 0$$

after simplification becomes

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - V_y = 0 \tag{2.6b}$$

in a similar manner, by taking moments with respect to the y-axis, we obtain

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - V_x = 0 \tag{2.6c}$$

Equation (2.6) constitutes the equilibrium equations of plate bending theory.

The resultant forces and the moments are defined as

$$V_x = \int_z \tau_{xz} dz \tag{2.7a}$$

$$V_y = \int_z \tau_{yz} dz \tag{2.7b}$$

$$M_x = \int_z \sigma_x z dz \tag{2.7c}$$

$$M_y = \int_z \sigma_y z dz \tag{2.7d}$$

$$M_{xy} = M_{yx} = \int_z \tau_{xy} z dz \tag{2.7e}$$

Since the thickness of a plate is small in comparison with the other dimensions, it is usually accepted that the constitutive relations for a state of plane-stress are acceptable [2]. Hence, the stress-strain relationship for isotropic plate are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{(1-\upsilon)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.8}$$

where E and ν are Young's modulus and Poisson's ratio, respectively.

Using equation 2.5, 2.7, and 2.8, the constitutive relationship can be written in terms of stress resultant and displacement as follows

$$M_x = D\left(\frac{\partial^2 w_0}{\partial x^2} + \upsilon \frac{\partial^2 w_0}{\partial y^2}\right) \tag{2.9a}$$

$$M_y = D\left(\frac{\partial^2 w_0}{\partial y^2} + \upsilon \frac{\partial^2 w_0}{\partial x^2}\right) \tag{2.9b}$$

$$M_{xy} = -M_{yx} = -(1-\upsilon) D \frac{\partial^2 w_0}{\partial x \partial y} \tag{2.9c}$$

where D is the flexural rigidity of the plate defined by

$$D = \frac{Et^3}{12(1-\upsilon^2)}$$

The two groups equations (2.6a-2.6c) and (2.9a-2.9c), which are derived from equilibrium conditions and constitutive relationship of material respectively, construct the governing equations for plate bending theory. These are six equations and six unknowns, i.e. M_x, M_y, M_{xy}, V_x, V_y, and w_0. Rewriting equation (2.6a) and (2.6b) and substituting in to equation (2.6a) gives

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q=0 \tag{2.10}$$

Substituting equation (2.9) in to the above equation and drop the subscript 0 that has been associated for the sake of brevity, we obtain

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=q \tag{2.11}$$

or

$$\nabla^4 w=\frac{q}{D} \tag{2.12}$$

where the operator is defined as

$$\nabla^4=\nabla^2\nabla^2 \tag{2.13}$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{2.14}$$

It is seen that the problem of bending of plates by transverse load q reduces to equation 2.11. For a particular case a solution of this equation is found that satisfies the condition at the boundaries of the plate. The bending and twisting moments can be calculated using equation 2.9 and that of shear forces are obtained from equation 2.15, which is derived by combing equation 2.6b and 2.6c with equation 2.9 as

$$V_x=-D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{2.15a}$$

$$V_y=-D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{2.15b}$$

The other well known plate theory which considers the transverse shear deformation is Mindlin plate theory. It states that a line that is straight and normal to the mid-surface before loading is assumed to remain straight but not necessarily normal to the mid-surface after loading. Here the motion of a point on the mid-surface is not governed by the slopes

$\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ as in Kirchhoff theory, rather its motion depends on rotations θ_x and θ_y of lines that were normal to the mid-surface before loading. Therefore, θ_x and θ_y being small angle of rotation the displacement field equations are

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z\theta_x \\ v(x, y, z) &= v_0(x, y) - z\theta_y \\ w(x, y, z) &= w_0(x, y) \end{aligned} \tag{2.16}$$

Where u, v, and w are displacement components in the directions of x-, y-, and z-axes, respectively. Thus the strain displacement relationship becomes

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial \theta_x}{\partial x} \tag{2.16a}$$

$$\varepsilon_y = \frac{\partial u_0}{\partial y} - z\frac{\partial \theta_y}{\partial y} \tag{2.16b}$$

$$\varepsilon_{xy} = -\frac{1}{2}\left[\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right] \tag{2.16c}$$

$$\varepsilon_{xz} = \frac{\partial w_0}{\partial x} - \theta_x \tag{2.16d}$$

$$\varepsilon_{yz} = \frac{\partial w_0}{\partial y} - \theta_y \tag{2.16e}$$

Equation 2.16a-e are strain-displacement relations of Mindlin plate theory. This theory accounts for transverse shear deformation and is therefore especially suited to the analysis of thick plate and sandwich plates.

2.1.2 Boundary Conditions

Boundary conditions for some of the more common types of edge supports are presented in Table2.1 below.

Table 2.1: Boundary Conditions [3].

Types of edge bearing **x = constant**	**Boundary conditions**
Bearing allowing freedom of rotation on rigid support or simply supported edge.	$w = 0;\qquad \frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2} = 0$
Rigid restraint (full fixity) on rigid support	$w = 0;\qquad \frac{\partial w}{\partial x} = 0$
Unsupported edge (free edge)	$\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2} = 0;\ \frac{\partial^3 w}{\partial x^3} + (2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2} = 0$
Resilient restraint (partial fixity) with restraint factor γ on peripheral beam with stiffness EI_a	$\frac{\partial w}{\partial x} = \gamma D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2}\right);$ $EI_a \frac{\partial^4 w}{\partial y^4} = D\left[\frac{\partial^3 w}{\partial x^3} + (2-\upsilon)\frac{\partial^3 w}{\partial x \partial y^2}\right]$

2.2 Numerical methods

Once the equations and the boundary conditions are defined the solution can be obtained using different numerical methods. It is preferable to set the solution of plate bending equations in generalized dimensionless coefficient format. After producing those coefficients, the internal actions; moments and shear forces can be easily calculated. These coefficients are based on elastic analysis but also can be modified to account for inelastic redistribution. In consequence, the design moment in either direction is smaller by an appropriate amount than the elastic maximum moment in that direction. Thus, the internal actions are calculated using the following formulae:

- Moments for individual panels with edges either simply supported or fully fixed are calculated as:

$$m_i = \alpha_i (g_d + q_d) L_x^{\,2} \qquad (2.17)$$

where α_i is moment coefficient given in table or chart format depending on the simplified method.

- Shear force or the design loads on beams supporting solid slabs spanning in two directions at right angles supporting uniformly distributed loads may be assessed from the following equations:

$$V_i = \beta_i (g_d + q_d) L_x \tag{2.18}$$

where β_i is shear coefficient that may be provided either in chart or tabular form.

These coefficients, α_i and β_i, which are important to determine the internal design action effects, are developed and provided in many designer's handbook.

2.2.1 Navier Method

The solution of the fourth order partial differential plate theory equation,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{2.19}$$

is required, for a plate of dimensions L_x x L_y, and the solution must satisfy the different boundary conditions.

Let us see the procedural development of the Navier method using simply supported slab panel subjected to the distributed load over the surface of the panel given by the expression

$$q = q_o \sin\frac{\pi x}{L_x} \sin\frac{\pi y}{L_y} \tag{2.20}$$

in which q_o represents the intensity of the load at the centre of the plate. The differential equation Eq. 2.19 for the deflection surface in this case becomes

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q_o}{D}\sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y} \tag{2.21}$$

and the solution has to satisfy the boundary conditions:

$$w = 0, \qquad \frac{\partial^2 w}{\partial x^2} = 0 \qquad \textit{for} \qquad x = 0 \quad \textit{and} \qquad x = L_x$$
$$w = 0, \qquad \frac{\partial^2 w}{\partial y^2} = 0 \qquad \textit{for} \qquad y = 0 \quad \textit{and} \qquad y = L_y \tag{2.22}$$

Consider a solution of deflection surface given by the equation,

$$w = C\sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y} \tag{2.23}$$

in which C must be chosen so as to satisfy Eq.2.21. Substituting expression of Eq.2.23 into Eq.2.21, we find

$$C = \frac{q_o}{\pi^4 D\left(\frac{1}{L_x^{\ 2}} + \frac{1}{L_y^{\ 2}}\right)} \tag{2.24}$$

Hence, the deflection surface satisfying Eq.2.21 and boundary conditions Eq.2.22 is

$$w = \frac{q_o}{\pi^4 D\left(\frac{1}{L_x^{\ 2}} + \frac{1}{L_y^{\ 2}}\right)}\sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y} \tag{2.25}$$

If the sinusoidal load distribution is given by the equation

$$q = q_o\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y} \tag{2.26}$$

where m and n are integer numbers, we can proceed as before, and we shall obtain for the deflection surface the following expression

$$w = \frac{q_o}{\pi^4 D\left(\frac{m^2}{L_x{}^2} + \frac{n^2}{L_y{}^2}\right)^2} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y} \tag{2.27}$$

from which the expressions for bending and twisting moments can be readily obtained by differentiation.

This can be extended to a general case of loading represented by the equation

$$q = f(x, y) \tag{2.28}$$

For this purpose we represent the function $f(x, y)$ in the form of double trigonometric series,

$$f(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y} \tag{2.29}$$

for which

$$a_{mn} = \frac{4}{L_x L_y}\int_0^{L_x}\int_0^{L_y} f(x, y)\sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y}\, dydx \tag{2.30}$$

this will represent the given load as a sum of partial sinusoidal loadings. The deflection produced by each partial loading is determined as the above simply supported case, and the total deflection will be obtained by summation of such terms, hence we find

$$w = \frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{L_x{}^2} + \frac{n^2}{L_y{}^2}\right)^2} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y} \tag{2.31}$$

2.2.2 Levy's Method

In solving the partial differential equation of plate bending problem for a plate having two opposite edges $x=0$ and $x=L_x$ both simply supported, Levy suggested that it is advantageous to take the solution in the form

$$w=\sum_{m=1}^{\infty}F_m(y)\sin\frac{m\pi x}{L_x} \qquad (2.32)$$

where $F_m(y)$ is a function of y only; the boundary conditions at $x=0$ and $x=L_x$ are then satisfied automatically. It remains to choose $F_m(y)$ so as to satisfy the boundary conditions on the other two edges. Here the solution is determined in two part, say w_1 and w_2, where $w=w_1+w_2$. The first part of the deflection, w_1, is a solution of the expression

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q}{D} \qquad (2.33)$$

satisfying the boundary condition at $x=0$ and $x=L_x$.

The expression w_2 evidently has to satisfy the equation

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=0 \qquad (2.34)$$

and must be chosen in such a manner as to make the sum, $w=w_1+w_2$ satisfy all the boundary conditions of the plate. Considering w_2 in the form of series Eq.(2.32) in which, from symmetry, $m=1,3,5,....$ and substituting in the equation Eq.(2.34), we obtain

$$\sum_{m=1}^{\infty}\left(\frac{d^4F_m}{dy^4}-2\frac{m^2\pi^2}{L_x^{\,2}}\frac{d^2F_m}{dy^2}+\frac{m^4\pi^4}{L_x^{\,4}}F_m\right)\sin\frac{m\pi x}{L_x}=0 \qquad (2.35)$$

This equation can be satisfied for all values of x only if the function F_m satisfies the equation

$$\frac{d^4 F_m}{dy^4} - 2\frac{m^2\pi^2}{L_x^{\ 2}}\frac{d^2 F_m}{dy^2} + \frac{m^4\pi^4}{L_x^{\ 4}}F_m = 0 \tag{2.36}$$

The general solution of this equation can be taken in the form

$$F_m(y) = \frac{qL_x^{\ 4}}{D}\left(A_m \cosh\frac{m\pi y}{L_x} + B_m\frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x} + C_m\sinh\frac{m\pi y}{L_x} + D_m\frac{m\pi y}{L_x}\cosh\frac{m\pi y}{L_x}\right) \tag{2.37}$$

These are the two numerical methods, Navier method and Levy's method, used to develop almost all of the simplified methods so far developed and implemented.

2.3 Analysis Methods for Suspended Slab Panel under Partition Wall Load

There are many simplified approximate analysis methods for suspended slab panel subjected to the weight of partition wall. These methods varies from the simplest, which accounts the partition wall load by taking additional twenty percent of the slab panel weight, to an empirical formulae given by many researcher around the world. In this study we will consider two of them, Swedish method and Reynolds method, because of their wide application.

2.3.1 The Swedish Method

The type of slab dealt with is one composed of rectangular panels supported at all four edges by walls or beams stiff enough to be treated as unyielding. These methods are intended for slabs with uniformly distributed loads. In some practical cases, the slab panel may be subjected to a concentrated or a line load, in addition to a uniform load. These can generally be treated by considering them as equivalent uniform loads using the rules given below, provided that the sum of the non-uniform loads on a panel does not exceed 20% of the total load.

A line load can in principle be treated as a series of point loads, but may also be assumed to be equivalent to a uniformly distributed load of intensity

$$q_a = \frac{6(2Q_1 + Q_2)}{L_y(3L_x + L_y)} \qquad (2.38)$$

where Q_1 = sum of line loads within the panel area limited by straight lines drawn parallel to the support at distance $\frac{L_x}{4}$ from them (see Figure 2-3)

Q_2 = sum of line loads within the area between the supports and the straight line drawn parallel to the supports at distances $\frac{L_x}{4}$ from them (see Figure 2-3)

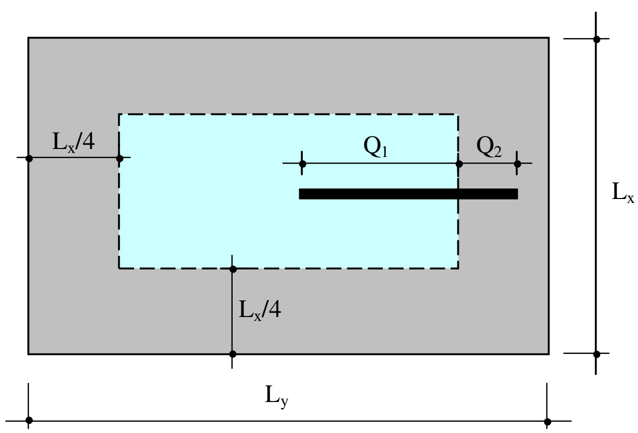


Figure 2-3: Slab panel subjected to line load.

2.3.2 The Reynolds Method

The other well known and wide applicable simplified analysis method for suspended slab panel under partition wall load is Reynolds method. This method follows logical procedure rather than using empirical formula to determine the 'equivalent' uniformly distributed load. The procedure is stated below

- Determine the actual partition load per meter length parallel to L_x and L_y.

- W_{p1} = load per unit length of partition walls parallel to L_x.

- W_{p2} = load per unit length of partition walls parallel to L_y.

- After determining the line loads parallel to L_x and L_y, we read shear distribution coefficient form Table.
- Modified line load which accounts the contribution of the above line load to the internal action in the two orthogonal direction is determined as:

$$W_{px1} = W_{p2} * W_a \qquad W_{py1} = W_{p1} * W_b$$

$$W_{px2} = W_{p1} * W_a \qquad W_{py2} = W_{p2} * W_b$$

where W_a and W_b are shear coefficients.

- Equivalent width and length of slab panel:

- Equivalent width along the shorter direction

$$e_x = g + 0.6L_x$$

- Equivalent length along the longer direction

$$e_y = g + 0.6L_y$$

where

$$g = t + 2d$$

d is the thickness of the slab panel and *t* is the thickness of the partition wall.

- Equivalent load is determined as

- Equivalent Load on L_x

$$W_{ex1} = \frac{C_x(W_{px1})}{L_x}$$

$$W_{ex2} = \frac{W_{px2}}{e_x}$$

$$W_{ex} = W_{ex1} + W_{ex2}$$

- Equivalent Load on L_y

$$W_{ey1} = \frac{C_y(W_{py1})}{L_y}$$

$$W_{ey2} = \frac{W_{py2}}{e_y}$$

$$W_{ey} = W_{ey1} + W_{ey2}$$

where C_x and C_y are support condition factor:

- Support condition factor:
  - C = 2.0 for simply supported case.
  - C = 1.7 for simply supported at one end and fixed (or continuous) at the other end.

- C = 1.5 for continuous over both supports.

▪ Effective equivalent uniformly distributed load:

$$q_a = \max(W_{ex}, W_{ey})$$

2.4. Finite Element Method

The finite element method is a widely accepted numerical procedure for solving the differential equations of engineering and other science fields. It is computational basis of many computer aided design programs. As its applications to solid mechanics problems are extensive, it is important tool in solving partial differential equations of plate bending theory. It has got a primary advantage of the ease with which it can be generalized to solve two dimensional problem of plate bending theory with different irregular boundary conditions.

There are many general purpose finite element programs available for solving two dimensional problems. In this study, SAFE integrated slab analysis software is preferred and used because it has sufficient capability of manipulating plate bending problems. It uses triangular and rectangular slab elements as shown in Fig2-4 and Fig2-5. Each of these slab elements can be isotropic or orthotropic, thin or thick plate bending element. The thin plate element is a three to four-node element and is based upon the classical linear thin plate bending theory, neglecting the effect of out-of-plane shear deformations. The thick plate is also a three to four-node element and accounts for the effect of out-of-plane shear deformations.

Some essential features of the plate elements that provided by SAFE [9]:

- Each of the element nodes has the three degree of freedom, w, θ_x, and θ_y.
- The material properties and thickness within each slab element are constant.
- Optionally, to model orthotropic effects, it is possible to specify three different effective thicknesses: x-direction bending, y-direction bending, and twist.
- The slab system must be planar and exist in xy plane. Changes in slab elevations that cause definite moment discontinuities may be reasonably captured using the release options.
- In-plane action is not allowed in the xy plane; therefore, membrane stresses in the plane of the slab system do not exist.

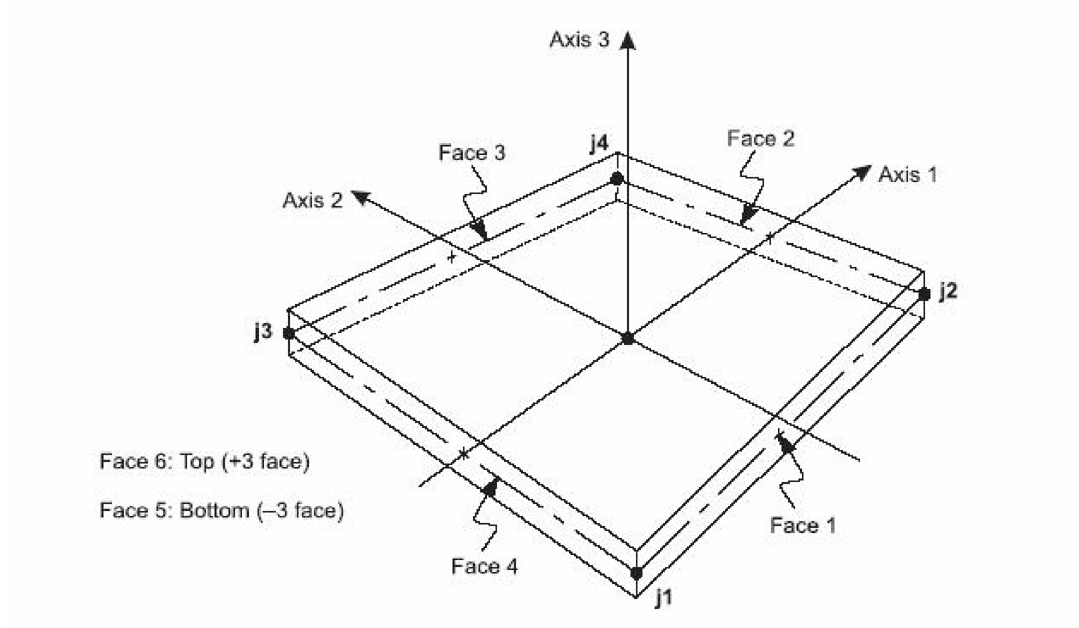


Figure 2-4: Four-node quadrilateral slab element [9].

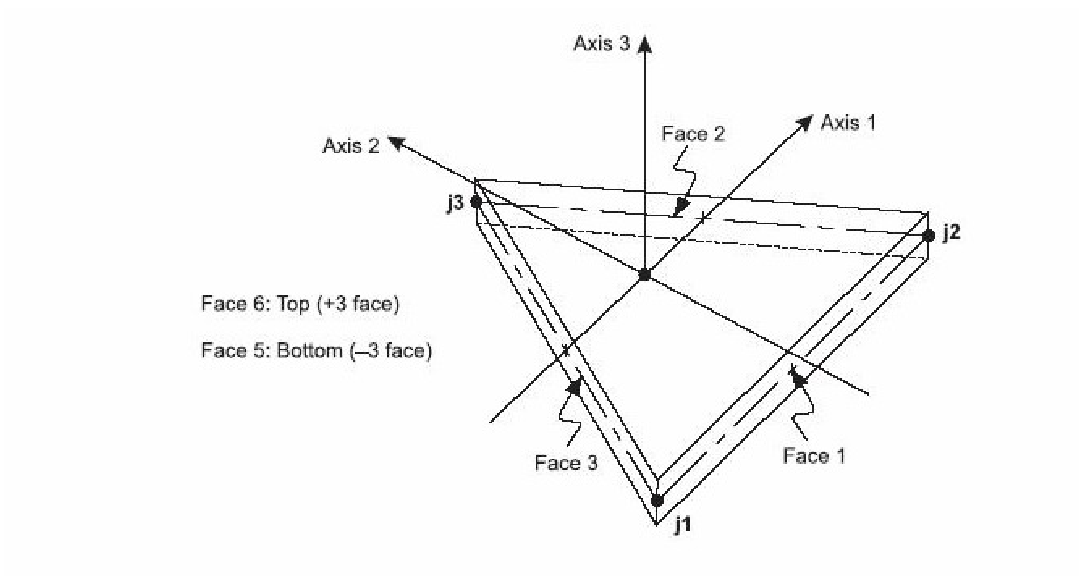


Figure 2-5: Three-node triangular slab element [9].

- The calculation of self-weight of the slab element is based upon the design thickness, the dimensions between mesh points in the x and y directions, and the unit weight of the material. The weight of the slab element is lumped (as concentrated loads) and distributed equally to the mesh points.
- Slab element moments and shear are calculated at the mesh points of the element.

3

Analysis of Suspended Slab under Partition Wall Load

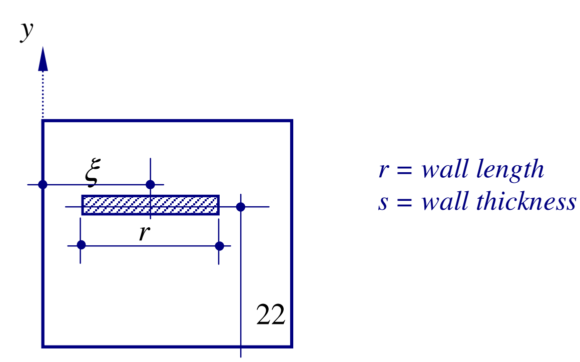
3.1 Alternative approach for analysis of slab under partition wall load

It has been seen in subsection 2.2.1 that the deflection of a simply supported rectangular plate can be represented in the form of a double trigonometric series of equation Eq. 2.31, the coefficients a_{mn} being given by equation Eq. 2.30. This deflection equation can be further applied to simply supported slab panel subjected to a total partition wall load of q as follows. Referring to figure 3-1 the coefficient a_{mn} can be calculated from equation Eq. 2.31 as

$$a_{mn} = \frac{4w}{L_x L_y rs} \int_{\xi - r/2}^{\xi + r/2} \int_{\eta - s/2}^{\eta + s/2} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y} dxdy \qquad (3.1)$$

from which,

$$a_{mn} = \frac{16w}{\pi^2 mnrs} \sin\frac{m\pi\xi}{L_x} \sin\frac{n\pi\eta}{L_y} \sin\frac{m\pi r}{2L_x} \sin\frac{n\pi s}{2L_y} \qquad (3.2)$$



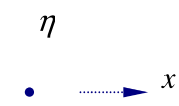


Figure 3-1: Plate subjected to partition wall load.

and, by equation Eq. 2.31, the deflection

$$w = \frac{16q}{\pi^6 Drs} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\frac{m\pi\xi}{L_x} \sin\frac{n\pi\eta}{L_y} \sin\frac{m\pi r}{2L_x} \sin\frac{n\pi s}{2L_y} \sin\frac{m\pi x}{L_x} \sin\frac{n\pi y}{L_y}}{mn\left(\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2}\right)^2} \tag{3.3}$$

Thus, we are able to determine the internal actions, bending moments and shear forces, in the panel using equations Eq. 2.9 and Eq. 2.15. To see the detail derivation of the formulae for internal responses, we will consider the derivation of bending moment about y-axis, M_x. Before applying the equations for internal actions, it is important to define the following relative dimensions of slab panel and partition wall. These relative dimensions make the final equation applicable to the general case of the plate problem under such load.

Relative dimensions:

$$\psi_1 = \frac{\xi}{L_x} \qquad \psi_2 = \frac{\eta}{L_y} \tag{3.4a}$$

$$\psi_3 = \frac{r}{2L_x} \qquad \psi_4 = \frac{s}{2L_y} \tag{3.4b}$$

$$\psi = \frac{L_y}{L_x} \tag{3.4c}$$

where ψ is span ratio.

Referring to Equation Eq. 2.9a, repeated here for convenience,

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \upsilon \frac{\partial^2 w}{\partial y^2}\right) \tag{3.5}$$

The formula for the internal action, M_x, becomes

$$M_x = \frac{16q}{\pi^4 rs}\left[\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m\sin\frac{m\pi\xi}{L_x}\sin\frac{n\pi\eta}{L_y}\sin\frac{m\pi r}{2L_x}\sin\frac{n\pi s}{2L_y}\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y}}{nL_x^2\left(\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}\right)^2}\right)+\right.$$
$$\left.\upsilon\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{n\sin\frac{m\pi\xi}{L_x}\sin\frac{n\pi\eta}{L_y}\sin\frac{m\pi r}{2L_x}\sin\frac{n\pi s}{2L_y}\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y}}{mL_y^2\left(\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}\right)^2}\right)\right] \tag{3.6}$$

using the relative dimensions, M_x, can be rewritten as

$$M_x = \frac{4q}{\pi^4 \psi\psi_3\psi_4}\left[\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m\sin(m\pi\psi_1)\sin(n\pi\psi_2)\sin(m\pi\psi_3)\sin(n\pi\psi_4)\sin(m\pi x)\sin(n\pi y)}{n\left(m^2+\frac{n^2}{\psi^2}\right)^2}\right)+\right.$$
$$\left.\upsilon\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{n\sin(m\pi\psi_1)\sin(n\pi\psi_2)\sin(m\pi\psi_3)\sin(n\pi\psi_4)\sin(m\pi x)\sin(n\pi y)}{m\left(m^2\psi+\frac{n^2}{\psi}\right)^2}\right)\right] \tag{3.7}$$

where all the relative dimensions range from zero to one and q is given by

$$q = q''rs \tag{3.8}$$

where q'' is the wall load per square meter. If we take q' as load per meter length, we obtain

$$q = q'r = q'\psi_3 L_x \tag{3.9}$$

Hence the above moment equation can be written as

$$M_x = \alpha_x q' L_x \tag{3.10}$$

where α_x is bending moment coefficient for M_x, and calculated from equations Eq. 3.7 and Eq. 3.9 as

$$\alpha_x = \frac{8}{\pi^4 \psi \psi_4}\left[\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m\sin(m\pi\psi_1)\sin(n\pi\psi_2)\sin(m\pi\psi_3)\sin(n\pi\psi_4)\sin(m\pi x)\sin(n\pi y)}{n\left(m^2+\frac{n^2}{\psi^2}\right)^2}\right)+\right.$$

$$\left.\upsilon\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{n\sin(m\pi\psi_1)\sin(n\pi\psi_2)\sin(m\pi\psi_3)\sin(n\pi\psi_4)\sin(m\pi x)\sin(n\pi y)}{m\left(m^2\psi+\frac{n^2}{\psi}\right)^2}\right)\right] \tag{3.11}$$

Slab panel with different edge condition can be treated by assuming some moment distribution along their continuous edges. To investigate this, let's consider a slab panel continuous along the edges $\pm L_y/2$ as shown in the figure 3-2. The deflection w must satisfy the homogeneous differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = -\frac{q}{D} \tag{3.12}$$

rewritten as

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = 0 \tag{3.13}$$

and the solution has to satisfy the boundary conditions:

$$w = 0, \qquad \frac{\partial^2 w}{\partial x^2} = 0 \qquad for \qquad x = 0 \quad and \quad x = L_x \tag{3.14a}$$

$$w = 0 \qquad for \qquad y = \pm\frac{L_y}{2} \tag{3.14b}$$

$$-D\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=Ly/2} = f_1(x) \qquad -D\left(\frac{\partial^2 w}{\partial y^2}\right)_{y=-Ly/2} = f_2(x) \tag{3.14c}$$

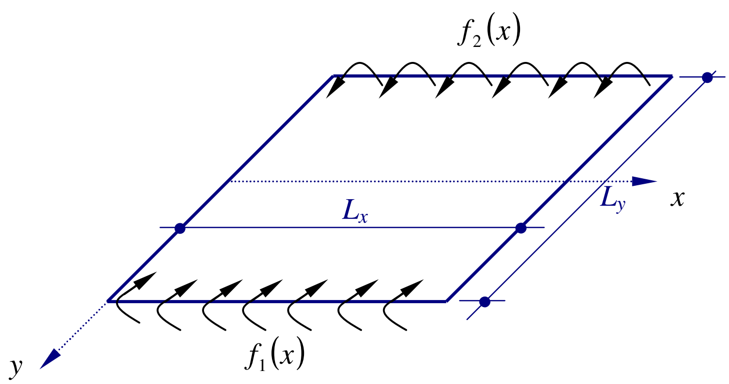


Figure 3-2: Plate subjected distributed moment along the edge.

in which f_1 and f_2 the bending moment distribution along the edges $y = \pm L_y/2$. Considering the solution of equation Eq. 3.13 to be in the form of a series

$$w = \sum_{m=1}^{\infty} Y_m \sin\frac{m\pi x}{L_x} \tag{3.15}$$

The function Y_m will take the form

$$Y_m = A_m \sinh\frac{m\pi y}{L_x} + B_m \cosh\frac{m\pi y}{L_x} + C_m \frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x} + D_m \frac{m\pi y}{L_x}\cosh\frac{m\pi y}{L_x} \tag{3.16}$$

It is possible to write any distribution of bending moment along the edge of the panel by linear combination of symmetric and anti-symmetrical moment distribution. Thus we can investigate each case independently. Assuming the moment distribution to be symmetrical Y_m has to be even function of y, and it is necessary to put $A_m = D_m = 0$ in the expression Eq. 3.16.

Thus the deflection becomes

$$w = \sum_{m=1}^{\infty}\left(B_m \cosh\frac{m\pi y}{L_x} + C_m \frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x}\right)\sin\frac{m\pi x}{L_x} \tag{3.17}$$

Applying the boundary condition of equation Eq. 3.14b and letting $\alpha_m = \frac{m\pi L_y}{2L_x}$, equation 3.17 can be written as

$$w = \sum_{m=1}^{\infty} C_m\left(\frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x} - \alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{L_x}\right)\sin\frac{m\pi x}{L_x} \tag{3.18}$$

We use the boundary condition Eq. 3.14c to determine the constants C_m. Representing the distribution of bending moments along the edges $y = \pm L_y/2$ by a trigonometric series, we have in the case of symmetry;

$$f_1(x) = f_2(x) = \sum_{m=1}^{\infty} E_m \sin\frac{m\pi x}{L_x} \tag{3.19}$$

Substituting expression Eq. 3.17 and Eq. 3.18 in to Eq. 3.14c, we obtain

$$C_m = -\frac{L_x^{\,2} E_m}{2Dm^2\pi^2 \cosh\alpha_m} \tag{3.20}$$

and

$$w = \frac{L_x^{\,2}}{2\pi^2 D}\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{L_x}}{m^2\cosh\alpha_m} E_m\left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x}\right) \tag{3.21}$$

similarly for anti-symmetric case the equation for the deflection will be

$$w = \frac{L_x^{\ 2}}{2\pi^2 D}\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{L_x}}{m^2 \sinh\alpha_m} E_m\left(\alpha_m \coth\alpha_m \sinh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\cosh\frac{m\pi y}{L_x}\right) \qquad (3.22)$$

We can obtain the deflection surface for the general case form the symmetrical and anti-symmetrical cases. The total deflection is obtained by superposing the deflection produced by each of the two moment distribution.

Hence,

$$w = \frac{L_x^{\ 2}}{2\pi^2 D}\sum_{m=1}^{\infty}\frac{\sin\frac{m\pi x}{L_x}}{m^2}\left[\frac{E_m}{\cosh\alpha_m}\left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x}\right) + \frac{E_m^{'}}{\sinh\alpha_m}\left(\alpha_m \coth\alpha_m \sinh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\cosh\frac{m\pi y}{L_x}\right)\right] \qquad (3.23)$$

If the bending moments are distributed only along one edge, say $y = {L_y}/{2}$, the deflection becomes:

$$w = \frac{L_x^{\ 2}}{4\pi^2 D}\sum_{m=1}^{\infty}\frac{E_m \sin\frac{m\pi x}{L_x}}{m^2}\left[\frac{1}{\cosh\alpha_m}\left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\sinh\frac{m\pi y}{L_x}\right) + \frac{1}{\sinh\alpha_m}\left(\alpha_m \coth\alpha_m \sinh\frac{m\pi y}{L_x} - \frac{m\pi y}{L_x}\cosh\frac{m\pi y}{L_x}\right)\right] \qquad (3.24)$$

Thus for a slab panel with continuous edges, the deflection can be obtained by first solving the problem on the assumption that all edges are simply supported and then applying bending moment along the continuous edges. The magnitude of the bending moments is selected so that it eliminates the rotations produced along this edge by the transverse loads. Using this analogy the deflection equation for a slab panel having continuous edges and subjected to the weight of partition wall load is derived and solved using a program written for this purpose.

In similar way, the formulae for the bending moment and shear force α_i and β_i respectively has been found. It is cumbersome and impossible to calculate the internal responses using the above equation, Eq. 3.7. A small program, Plate Analysis Program (PAP), is developed and used to find out the coefficients for internal actions. The flow chart for the algorithm, the program code and the user interface are summarized under Appendix-A. Having the coefficients at hand the bending moment and shear force can be determined from

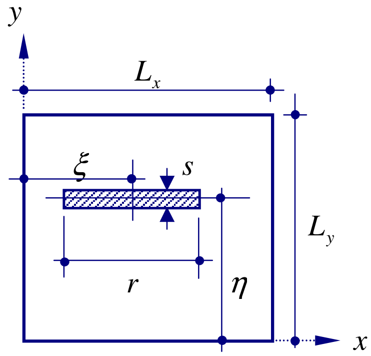
$$M_i = \alpha_i q L_x \tag{3.12a}$$

$$V_i = \beta_i q \tag{3.12b}$$

3.1.1 Tables for the analysis of suspended slab under partition wall load

Using the plate analysis program which is developed for the purpose of this thesis work, tables of coefficient for the analysis are developed. Tables for the analysis of slab panel subject to wall load parallel to L_x and running along full length are presented here as an example and remaining tables are attached under Appendix-B.

Table 3.1: Bending Moment coefficient for a rectangular panel subjected to partition wall loads parallel to L_x.



r = wall length

s = wall thickness

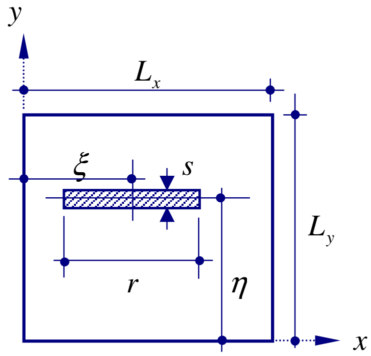
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_x} = 0.5 \qquad \psi_4 = \frac{s}{2L_y} = 0.01$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.093	0.113	0.145	0.168	0.190	0.208	0.214	0.229
	α_{ys}	0.093	0.098	0.110	0.114	0.117	0.130	0.111	0.103
	α_{xf}	0.068	0.080	0.102	0.117	0.132	0.146	0.164	0.175
	α_{yf}	0.068	0.073	0.085	0.091	0.095	0.101	0.101	0.100
2	α_{xs}	0.107	0.124	0.155	0.177	0.197	0.199	0.216	0.229
	α_{ys}	0.099	0.087	0.112	0.115	0.117	0.122	0.111	0.103
	α_{xf}	0.076	0.101	0.109	0.123	0.137	0.154	0.166	0.176
	α_{yf}	0.068	0.072	0.083	0.088	0.092	0.100	0.099	0.098
3	α_{xs}	0.099	0.122	0.161	0.192	0.223	0.234	0.270	0.298
	α_{ys}	0.107	0.115	0.135	0.144	0.152	0.162	0.152	0.144
	α_{xf}	0.068	0.082	0.106	0.125	0.144	0.160	0.188	0.205
	α_{yf}	0.076	0.083	0.099	0.108	0.115	0.125	0.124	0.122
4	α_{xs}	0.118	0.141	0.180	0.210	0.239	0.248	0.278	0.302
	α_{ys}	0.118	0.124	0.142	0.150	0.155	0.164	0.153	0.144
	α_{xf}	0.078	0.092	0.117	0.135	0.153	0.173	0.193	0.208
	α_{yf}	0.078	0.084	0.099	0.106	0.113	0.122	0.120	0.120
5	α_{xs}	0.121	0.136	0.166	0.186	0.204	-0.205	0.218	0.230
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.085	0.095	0.115	0.129	0.141	0.158	0.168	0.176
	α_{yf}	0.068	0.070	0.080	0.085	0.089	0.097	0.096	0.097
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.121	0.137	0.167	0.186	0.203	0.226	0.228	0.226
	α_{xf}	0.068	0.084	0.112	0.136	0.161	0.188	0.229	0.266
	α_{yf}	0.085	0.096	0.117	0.131	0.144	0.159	0.164	0.167
7	α_{xs}	0.141	0.163	0.202	0.229	0.256	0.263	0.297	0.306
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.091	0.105	0.130	0.147	0.163	0.182	0.198	0.211
	α_{yf}	0.082	0.086	0.098	0.105	0.110	0.118	0.117	0.117
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.141	0.099	0.186	0.203	0.218	0.238	0.235	0.229
	α_{xf}	0.082	0.156	0.130	0.155	0.181	0.208	0.245	0.278
	α_{yf}	0.091	0.102	0.122	0.135	0.146	0.159	0.162	0.164
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.078	0.084	0.089	0.093	0.096	0.099	0.103	0.105
	α_{yf}	0.114	0.112	0.111	0.109	0.107	0.106	0.104	0.101

Table 3.2 Shear force coefficient for a rectangular panel subjected to partition wall loads parallel to L_x.



r = wall length
s = wall thickness

$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_x} = 0.5 \qquad \psi_4 = \frac{s}{2L_y} = 0.01$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.667	0.770	0.956	1.080	1.198	1.305	1.070	1.127
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.667	0.664	0.722	0.725	0.724	0.930	0.617	0.559
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.726	0.817	0.996	1.110	1.219	1.011	1.075	1.126
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.684	0.672	0.725	0.725	0.722	0.702	0.616	0.559
	$\beta_{y,Disc}$	0.257	0.251	0.271	0.272	0.280	0.309	0.310	0.308
3	$\beta_{x,Con}$	0.684	0.803	1.016	1.167	1.315	1.122	1.251	1.346
	$\beta_{x,Disc}$	0.257	0.306	0.394	0.460	0.527	0.498	0.562	0.610
	$\beta_{y,Con}$	0.726	0.743	0.830	0.852	0.867	0.872	0.781	0.711
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	0.767	0.879	1.088	1.230	1.367	1.168	1.274	1.357
	$\beta_{x,Disc}$	0.301	0.350	0.439	0.502	0.564	0.525	0.578	0.619
	$\beta_{y,Con}$	0.767	0.773	0.852	0.867	0.876	0.876	0.781	0.710
	$\beta_{y,Disc}$	0.301	0.301	0.330	0.335	0.340	0.316	0.320	0.321
5	$\beta_{x,Con}$	0.791	0.868	1.037	1.140	1.240	1.028	1.079	1.124
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.253	0.246	0.266	0.268	0.282	0.307	0.306	0.304
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.253	0.304	0.397	0.471	0.549	0.540	0.644	0.730
	$\beta_{y,Con}$	0.791	0.836	0.964	1.021	1.071	1.117	1.057	0.993
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	0.869	0.966	1.169	1.297	1.422	1.215	1.296	1.487
	$\beta_{x,Disc}$	0.351	0.396	0.484	0.543	0.600	0.553	0.593	0.626
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.305	0.301	0.328	0.333	0.338	0.323	0.321	0.319
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.305	0.359	0.458	0.575	0.612	0.594	0.683	0.759
	$\beta_{y,Con}$	0.869	0.904	1.029	1.077	1.117	1.154	1.075	1.001
	$\beta_{y,Disc}$	0.351	0.361	0.406	0.422	0.436	0.455	0.388	0.369
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.370	0.425	0.529	0.604	0.679	0.651	0.725	0.788
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.370	0.374	0.416	0.429	0.440	0.416	0.388	0.368

3.2 Analysis using Finite Element Modeling

As it was briefly stated in section 2.4, finite element method is so far the well developed numerical method to deal with the partial differential equations of plate bending problems. Obviously, it is very tedious and difficult to think of the solutions of those partial differential equations using finite element method without applying any computer analysis software. For the purpose of this study, it has been investigated the internal response of a total number of 72 suspended slab panel models with different boundary conditions and span ratios, using SAFE.

To study the internal actions (M_x, M_y, V_x, and V_y) of a suspended slab panel loaded with the weight of partition wall, the following model data are selected. Nine boundary conditions, which are shown in the figure 3-3 below, are selected since they simulate most of practically found slab boundary conditions.

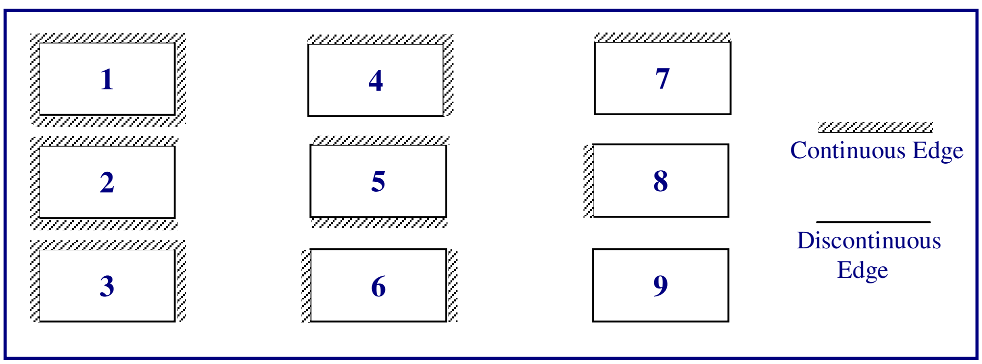


Figure 3-3: Boundary conditions.

To see the effect of span ratio to the internal action resulted from the weight of partition wall eight span ratios are selected. Table 3-3 shows panel dimension and span ratio used for this study.

Table 3.3: Model panel dimensions and span ratio.

Panel dimension, L_x (m)	Panel dimension, L_y (m)	Span ratio, $\frac{L_y}{L_x}$
6	6	1.0
5	5.5	1.1
5	6	1.2
5	6.5	1.3
5	7	1.4
4	6	1.5
4	7	1.75
4	8	2.0

All the other remaining data are taken to be the same for all slab panel models including a partition wall load of 4KN/m at the center of the panel in the two orthogonal directions. Here it is shown only one case and the results of the other cases are shown under subsection 3.3 for comparison purpose. For the purpose of demonstration a slab panel with all edges continuous and span ratio equal to one is selected.

- **Suspended slab panel under partition wall load**

  Case 1: Continuous on four sides with span ratio equal to unity

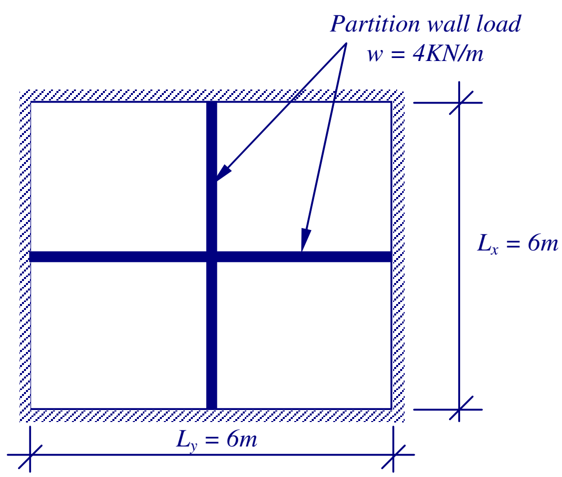


Figure 3-4: Slab Panel.

Data

- Panel size = L_xxL_y = $6m$x$6m$
- Edge beam on four side = 30x$40cm$
- Corner support columns = 30x$30cm$

- Slab panel thickness = *15cm*
- Modulus of elasticity = $29000000 KN/m^2$
- Poisson's ratio = *0.2*

Load case:

- LineL (LL) = *w* = 4*KN/m*

Modeling Procedure

To investigate the internal actions of a suspended slab panel under the weight of partition wall, the problem is analyzed employing a maximum mesh size of 0.5m, as shown in the Figure 3-5. The slab is modeled using thin plate elements in SAFE. The continuous edge is modeled by providing the same additional panel at that edge. Only one load case is considered. Self-weight is not included in these analyses.

To obtain internal actions, the plate is divided into three strips-two edge strips and one middle strip-each way, based on the definition of design strip widths for a two-way slab system as shown in the Figure 3-6.

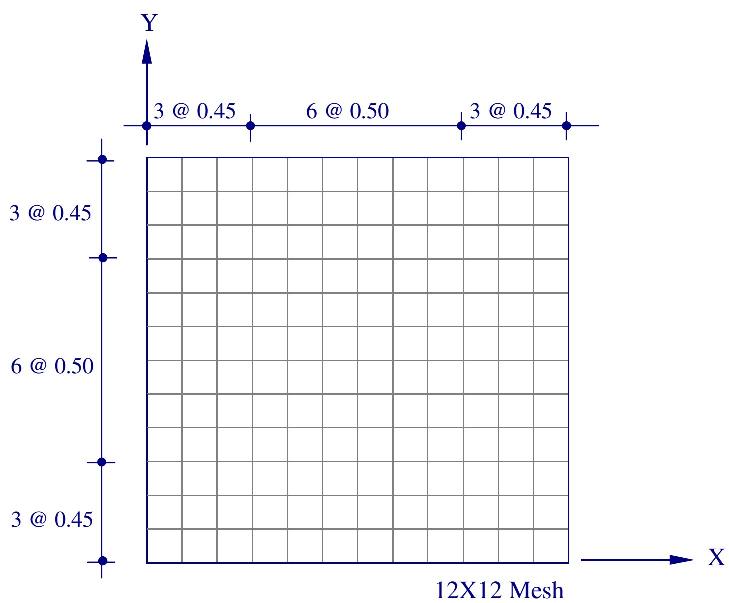


Figure 3-5: SAFE Meshes for Rectangular Plate.

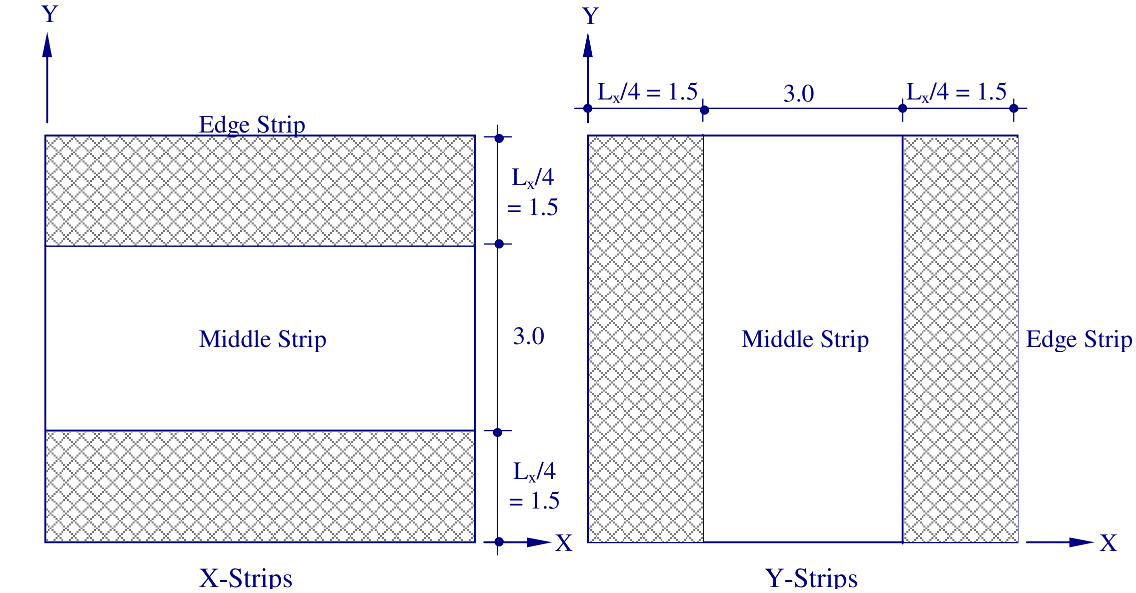


Figure 3-6: SAFE Definition of Design Strips.

Analysis Output

The analysis result of a slab panel under a line load with span ratio equal to unity is tabulated and presented in Table 3:4. The moments and shears are calculated for the nine boundary conditions for edges and center of the panel. The analysis results are shown under subsection 3.3 and can be interpreted using Figure 3-7.

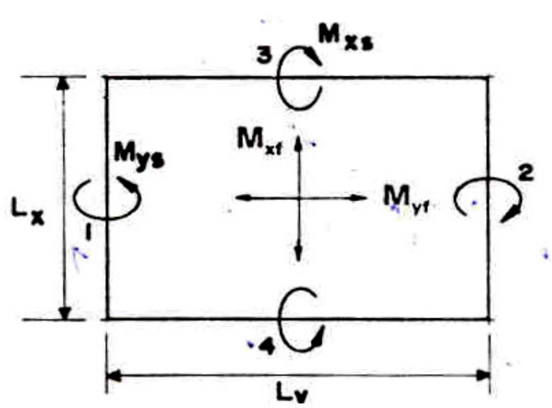
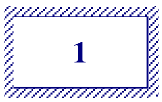
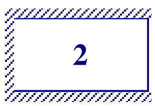
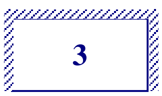
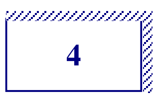
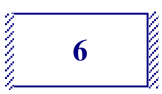
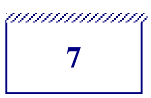
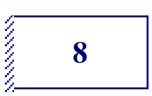
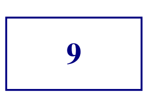


Figure 3-7: Notations for Critical Moments[6].

Table 3.4: Internal Actions using FEM, Panel with Span Ratio equal to Unity.

		M_{xs}	M_{ys}	M_{xf}	M_{yf}	V_x	V_y
 1	*Con* *DisC*	10.88 -	10.88 -	27.77	27.77	17.30 -	17.30 -
 2	*Con* *DisC*	10.77 -	11.71 -	28.63	28.82	17.65 -	17.86 15.36
 3	*Con* *DisC*	11.71 -	10.78 -	28.82	28.62	17.86 15.36	17.65 -
 4	*Con* *DisC*	11.59 -	11.59 -	29.72	29.72	18.22 15.75	18.22 15.75
 5	*Con* *DisC*	10.61 -	- -	29.61	30.19	18.06 -	- 15.81
 6	*Con* *DisC*	- -	10.61 -	30.19	29.61	- 15.81	18.06 -
 7	*Con* *DisC*	11.42 -	- -	30.73	31.13	18.62 16.21	- 16.20
 8	*Con* *DisC*	- -	11.42 -	31.13	30.73	- 16.20	18.62 16.21
 9	*Con* *DisC*	- -	- -	32.18	32.18	- 16.66	- 16.66

3.3 Distribution of internal actions

As verification to the program, the distribution of the internal actions; shear force and moments for both cases, the finite element method and the coefficient from elastic analysis over the middle strip are shown below.

3.3.1 Distribution of Moments

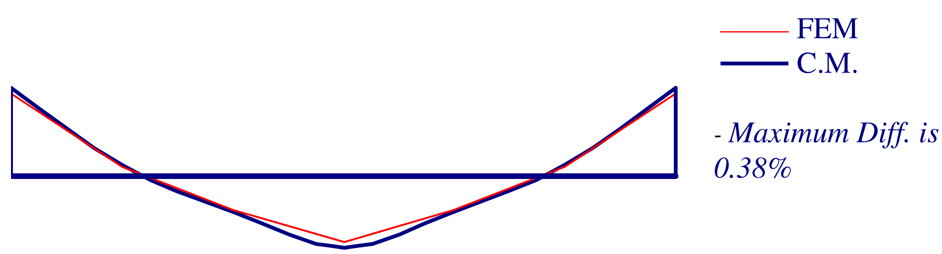


Figure 3-6: Bending Moment Diagram for Span ratio 1.0 and BC = 1.

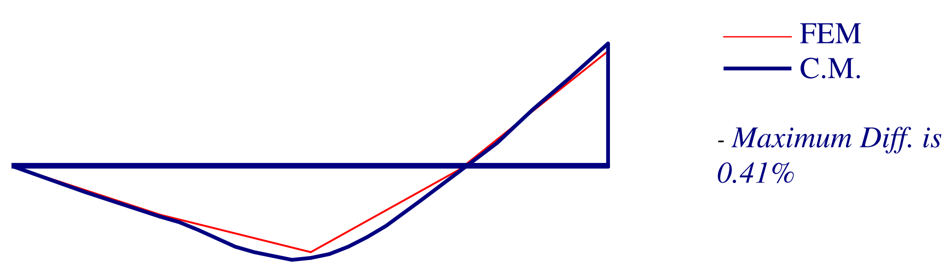


Figure 3-7: Bending Moment Diagram for Span ratio 1.5 and BC = 4.

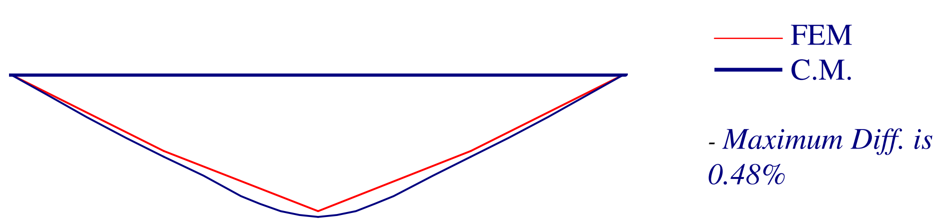


Figure 3-8: Bending Moment Diagram for Span ratio 2.0 and BC = 9.

3.3.2 Distribution of Shear Forces

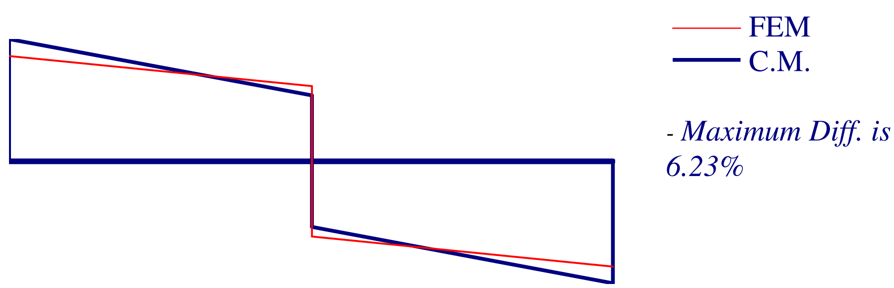


Figure 3-9: Shear Force Diagram for Span ratio 1.0 and BC = 1.

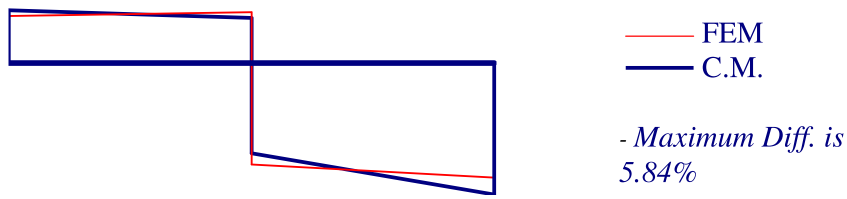


Figure 3-10: Shear Force Diagram for Span ratio 1.5 and BC = 4.

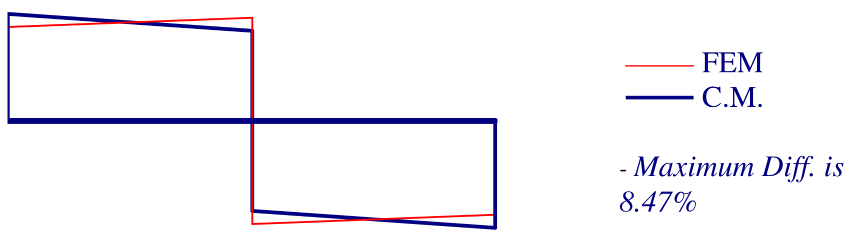


Figure 3-11: Shear Force Diagram for Span ratio 2.0 and BC = 9.

As additional means of verification to the plate analysis program, the coefficient for thickness ratio of $\frac{s}{2L_y} = 0.5$ is checked with the uniformly distributed load case and it gives exact solution.

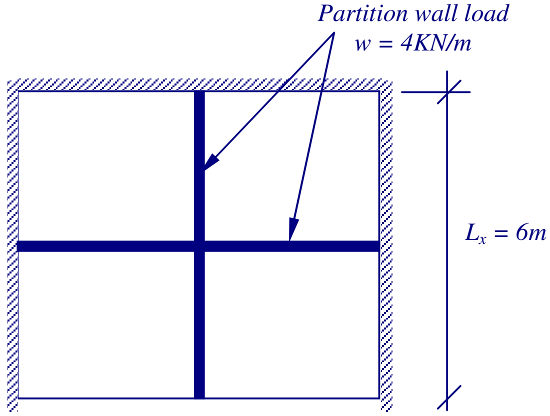
4

Comparison of Analysis Methods

4.1 Comparison of Analysis Methods

To make the comparison complete, the internal actions of the slab panel models of the previous subsection are also calculated using simplified methods. As in finite element method, the detail analysis of the same slab panel using Swedish and Reynolds methods are presented here. The analysis result for the other remaining slab panel with different span ratio and boundary condition are shown in the comparison table, Table 3.6, at the end of this section.

Case 1: Continuous on four sides with span ratio equal to unity



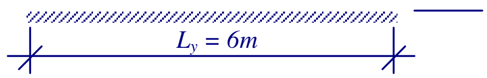


Figure 3-10: Slab Panel.

4.1.1 Swedish Method

Referring to Sec2.3.1

- Assuming the total load including own weight and live load to be 300KN, the sum of partition wall load on the two orthogonal direction doesn't exceed the limit.

Span ratio:

$$\frac{L_y}{L_x} = \frac{6}{6} = 1.00\,; \qquad \frac{L_x}{4} = 1.50$$

Load Q_1 and Q_2:

$$Q_1 = 24.00 \text{ KN}; \qquad Q_2 = 24.00 \text{ KN}$$

Equivalent UDL:

$$q_a = \frac{6(2Q_1 + Q_2)}{L_y(3L_x + L_y)} = 3.00 \text{ KN/m}^2$$

Analysis coefficient for UDL [3]:

$$\alpha_{xs} = 0.046 \qquad \beta_{xv,con} = 0.41$$

$$\alpha_{ys} = 0.046 \qquad \beta_{xv,disc} = -$$

$$\alpha_{xf} = 0.034 \qquad \beta_{yv,con} = 0.41$$

$$\alpha_{yf} = 0.034 \qquad \beta_{yv,disc} = -$$

Internal actions:

$$M_{xs} = 4.94 \text{ KNm/m} \qquad V_{x,con} = 7.43 \text{ KN/m}$$

$$M_{ys} = 4.94 \text{ KNm/m} \qquad V_{x,disc} = -$$

$$M_{xf} = 3.70 \text{ KNm/m} \qquad V_{y,con} = 7.43 \text{ KN/m}$$

$$M_{yf} = 3.70 \text{ KNm/m} \qquad V_{y,disc} = -$$

4.1.2 Reynolds Method

Referring to Sec2.3.2

- Span ratio:

$$\frac{L_y}{L_x} = \frac{6}{6} = 1.00$$

- Shear coefficient: [Refer Table 2.5]

$$W_a = 0.50$$

$$W_b = 0.50$$

- Actual line load from partition wall load:

$$W_{p1} = 4.00 \text{ KN/m}$$

$$W_{p2} = 4.00 \text{ KN/m}$$

- Modified line load:

$$W_{px1} = W_{p2} * W_a = 2.00 \text{ KN/m} \qquad W_{py1} = W_{p1} * W_b = 2.00 \text{ KN/m}$$

$$W_{px2} = W_{p1} * W_a = 2.00 \text{ KN/m} \qquad W_{py2} = W_{p2} * W_b = 2.00 \text{ KN/m}$$

- Equivalent width and length of slab panel:

$$g = t + 2D = 0.45$$

$$e_x = g + 0.6L_x = 4.05$$

$$e_y = g + 0.6L_y = 4.05$$

- Support factor: $C_x = 1.5$; $C_y = 1.5$

- Equivalent load:

- Equivalent Load on L_x

$$W_{ex1} = \frac{C_x (W_{px1})}{L_x} = 0.50 \text{ KN/m}^2$$

- Equivalent Load on L_y

$$W_{ey1} = \frac{C_y (W_{py1})}{L_y} = 0.50 \text{ KN/m}^2$$

$$W_{ex2} = \frac{W_{px2}}{e_x} = 0.49 \text{ KN/m}^2 \qquad W_{ey2} = \frac{W_{py2}}{e_y} = 0.49 \text{ KN/m}^2$$

$$W_{ex} = W_{ex1} + W_{ex2} = 0.99 \text{ KN/m}^2 \qquad W_{ey} = W_{ey1} + W_{ey2} = 0.99 \text{ KN/m}^2$$

- Effective equivalent load UDL:

$$q_a = \max\left(W_{ex}, W_{ey}\right) = 0.99 \text{ KN/m}^2$$

- Analysis coefficient for UDL [3]:

$$\alpha_{xs} = 0.046 \qquad \beta_{xv,con} = 0.41$$

$$\alpha_{ys} = 0.046 \qquad \beta_{xv,disc} = -$$

$$\alpha_{xf} = 0.034 \qquad \beta_{yv,con} = 0.41$$

$$\alpha_{yf} = 0.034 \qquad \beta_{yv,disc} = -$$

- Internal actions:

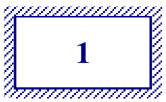
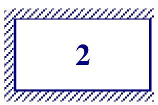
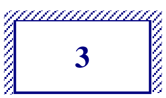
$$M_{xs} = 1.64 \text{ KNm/m} \qquad V_{x,con} = 2.46 \text{ KN/m}$$

$$M_{ys} = 1.64 \text{ KNm/m} \qquad V_{x,disc} = -$$

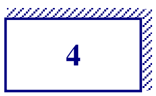
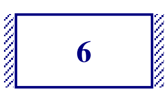
$$M_{xf} = 1.23 \text{ KNm/m} \qquad V_{y,con} = 2.46 \text{ KN/m}$$

$$M_{yf} = 1.23 \text{ KNm/m} \qquad V_{y,disc} = -$$

Table 3.5: Comparison table.

Boundary Conditions	ψ	1.0			1.1			1.2			1.3		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
	M_{xs}	4.937	1.636	4.486	4.500	1.840	4.167	4.857	2.407	4.517	5.078	2.868	4.794
	M_{ys}	4.937	1.636	4.486	3.892	1.591	3.613	3.701	1.834	3.443	3.533	1.995	3.256
	M_{xf}	3.703	1.227	3.272	3.406	1.392	2.979	3.701	1.834	3.186	3.864	2.182	3.351
	M_{yf}	3.703	1.227	3.272	2.919	1.193	2.704	2.776	1.375	2.653	2.650	1.497	2.586
	$V_{x,Con}$	7.425	2.460	5.334	7.663	3.132	5.700	7.893	3.911	5.975	7.921	4.474	6.171
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	7.425	2.460	5.334	7.024	2.871	4.916	6.679	3.309	4.512	6.376	3.601	4.140
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xs}	6.017	2.671	5.117	5.352	2.744	4.595	5.551	3.276	4.851	5.741	3.725	5.046
	M_{ys}	6.017	2.671	4.736	4.744	2.432	3.236	4.510	2.662	3.505	4.306	2.793	3.284
	M_{xf}	4.474	1.986	3.642	4.014	2.058	3.732	4.163	2.457	3.392	4.306	2.793	3.511
	M_{yf}	4.474	1.986	3.267	3.527	1.808	2.657	3.354	1.979	2.583	3.202	2.077	2.506
	$V_{x,Con}$	8.100	3.596	5.808	8.302	4.256	6.054	8.500	5.017	6.225	8.501	5.515	6.342
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	8.100	3.596	5.473	7.663	3.928	4.980	7.286	4.300	4.534	6.955	4.512	4.141
	$V_{y,Disc}$	5.400	2.397	2.059	5.109	2.619	1.858	4.857	2.867	1.691	4.637	3.008	1.555
	M_{xs}	6.017	2.671	4.736	5.960	2.114	4.517	6.476	2.620	5.031	6.845	3.236	5.480
	M_{ys}	6.017	2.671	5.117	4.744	1.683	4.277	4.510	1.825	4.223	4.306	2.035	4.121
	M_{xf}	4.629	2.055	3.267	4.379	1.553	3.037	4.857	1.965	3.321	5.189	2.453	3.571
	M_{yf}	4.629	2.055	3.642	3.649	1.295	3.088	3.469	1.404	3.099	3.312	1.566	3.078
	$V_{x,Con}$	8.100	3.596	5.473	8.514	3.021	5.949	8.905	0.000	6.348	9.081	4.292	6.670
	$V_{x,Disc}$	5.400	2.397	2.059	5.747	2.039	2.269	5.869	3.663	2.462	5.989	2.831	2.631
	$V_{y,Con}$	8.100	3.596	5.808	7.663	2.719	5.505	7.286	0.000	5.187	6.955	3.288	4.869
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-

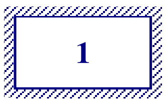
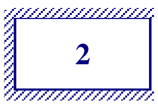
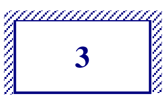
*S. M. = Swedish Method R. M. = Reynolds Method C. M. = Coefficient Method

Boundary Conditions	ψ	1.0			1.1			1.2			1.3		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
	M_{xs}	7.251	2.563	5.643	6.812	2.973	5.206	7.286	3.855	5.636	7.618	4.594	5.994
	M_{ys}	7.251	2.563	5.643	5.717	2.495	4.599	5.435	2.876	4.449	5.189	3.129	4.274
	M_{xf}	5.554	1.963	3.762	5.109	2.230	3.420	5.435	2.876	3.663	5.630	3.396	3.867
	M_{yf}	5.554	1.963	3.762	4.379	1.911	3.126	4.163	2.203	3.089	3.974	2.397	3.035
	$V_{x,Con}$	9.000	3.181	6.136	9.366	4.088	6.508	9.512	5.033	6.802	9.660	5.826	7.027
	$V_{x,Disc}$	5.850	2.068	2.411	6.173	2.694	2.589	6.274	3.319	2.741	6.376	3.845	2.866
	$V_{y,Con}$	9.000	3.181	6.136	8.514	3.716	5.724	8.095	4.283	5.326	7.728	4.661	4.954
	$V_{y,Disc}$	5.850	2.068	2.411	5.534	2.415	2.228	5.262	2.784	2.062	5.023	3.030	1.917
	M_{xs}	7.097	3.903	5.827	6.082	3.708	5.053	6.245	4.141	5.198	6.293	4.467	5.303
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	5.246	2.885	4.059	4.622	2.818	3.510	4.626	3.067	3.605	4.747	3.370	3.673
	M_{yf}	5.246	2.885	3.260	4.136	2.521	2.607	3.932	2.607	2.509	3.754	2.664	2.424
	$V_{x,Con}$	9.000	4.949	6.329	9.153	5.580	6.427	9.107	6.039	6.484	9.081	6.446	6.514
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	5.850	3.217	2.027	5.534	3.374	1.820	5.262	3.489	1.662	5.023	3.566	1.533
	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	6.943	3.818	5.827	5.474	2.670	5.074	5.204	2.236	5.225	4.968	1.895	5.314
	M_{xf}	5.246	2.885	3.260	5.595	2.729	3.106	6.476	2.783	3.495	7.176	2.738	3.874
	M_{yf}	5.246	2.885	4.059	4.136	2.017	3.548	3.932	1.690	3.671	3.754	1.432	3.753
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	5.850	3.217	2.027	6.386	3.115	2.255	6.679	2.870	2.479	6.955	2.653	2.694
	$V_{y,Con}$	9.000	4.949	6.330	8.514	4.153	6.191	8.095	3.478	6.024	7.728	2.948	5.837
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-

[*] S. M. = Swedish Method R. M. = Reynolds Method C. M. = Coefficient Method

Boundary Conditions	ψ	1.0			1.1			1.2			1.3		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
7	M_{xs}	8.794	4.414	6.786	7.906	4.562	6.022	8.211	5.366	6.318	8.390	5.949	6.554
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	6.634	3.330	4.389	5.838	3.369	3.873	6.129	4.006	4.048	6.293	4.462	4.188
	M_{yf}	6.789	3.408	3.917	5.352	3.088	3.171	5.088	3.326	3.077	4.858	3.444	2.986
	$V_{x,Con}$	10.125	5.082	6.951	10.217	5.895	7.158	10.321	6.746	7.307	10.240	7.261	7.413
	$V_{x,Disc}$	6.750	3.388	2.808	6.812	3.930	2.930	6.881	4.497	3.026	6.762	4.795	3.102
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	6.750	3.388	2.439	6.386	3.685	2.228	6.071	3.968	2.051	5.796	4.110	1.902
8	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	8.949	4.492	6.786	7.055	3.005	3.673	6.707	2.629	5.812	6.403	3.029	5.796
	M_{xf}	6.789	3.408	3.917	6.568	2.798	5.769	7.286	2.856	4.062	7.838	3.708	4.425
	M_{yf}	6.789	3.408	4.389	5.352	2.280	3.760	5.088	1.995	3.821	4.858	2.298	3.849
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	6.750	3.388	2.438	7.024	2.993	2.660	7.286	2.856	2.865	7.342	3.473	3053.000
	$V_{y,Con}$	10.125	5.082	6.951	9.579	4.081	6.698	9.107	3.570	6.429	8.694	4.113	6.153
	$V_{y,Disc}$	6.750	3.388	2.808	6.386	2.720	2.672	6.071	2.380	2.539	5.796	2.742	2.413
9	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	8.640	3.342	4.872	7.906	3.779	4.443	8.558	4.959	4.789	8.942	5.907	5.098
	M_{yf}	8.640	3.342	4.872	6.812	3.256	4.049	6.476	3.753	4.014	6.182	4.084	3.965
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	7.425	2.872	2.960	7.663	3.663	3.146	7.893	4.574	3.309	7.921	5.233	3.451
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	7.425	2.872	2.960	7.024	3.358	2.772	6.679	3.870	2.603	6.376	4.212	2.452

*S. M. = Swedish Method R. M. = Reynolds Method C. M. = Coefficient Method

Boundary Conditions	ψ	1.4			1.5			1.75			2.0		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
	M_{xs}	5.288	3.371	5.006	4.307	2.919	4.060	4.380	3.563	3.981	4.320	3.930	4.068
	M_{ys}	3.384	2.157	3.068	2.601	1.763	2.536	2.376	1.933	2.060	2.194	1.996	1.838
	M_{xf}	3.913	2.494	3.480	3.251	2.203	2.842	3.267	2.657	3.056	3.291	2.994	3.116
	M_{yf}	2.538	1.618	2.513	1.950	1.322	1.972	1.782	1.449	1.881	1.646	1.497	1.778
	$V_{x,Con}$	7.958	5.073	6.304	8.000	5.422	6.365	7.795	6.341	4.977	7.500	6.823	5.007
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	6.107	3.893	3.808	5.867	3.976	4.535	5.359	4.360	2.868	4.950	4.503	2.484
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xs}	5.816	4.125	5.193	4.714	3.503	3.892	4.677	3.972	4.023	4.594	4.313	4.078
	M_{ys}	4.124	2.925	3.077	3.170	2.355	2.372	2.895	2.459	2.056	2.674	2.511	1.836
	M_{xf}	4.336	3.075	3.601	3.495	2.597	3.001	3.489	2.963	3.088	3.429	3.219	3.125
	M_{yf}	3.067	2.175	2.432	2.357	1.751	1.950	2.153	1.828	1.835	1.989	1.867	1.750
	$V_{x,Con}$	8.328	5.906	6.416	8.356	6.209	4.932	8.120	6.896	5.002	7.800	7.323	5.005
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	6.662	4.725	3.801	6.400	4.756	3.426	5.847	4.965	2.863	5.400	5.070	2.483
	$V_{y,Disc}$	4.442	3.150	1.474	4.267	3.171	1.509	3.898	3.310	1.442	3.600	3.380	1.368
	M_{xs}	7.191	4.099	5.862	5.933	3.728	4.564	6.088	4.828	5.023	6.103	5.619	5.290
	M_{ys}	4.124	2.351	3.989	3.170	1.992	3.156	2.895	2.296	2.832	2.674	2.462	2.552
	M_{xf}	5.393	3.075	3.786	4.470	2.809	3.124	4.603	3.650	3.489	4.594	4.230	3.652
	M_{yf}	3.173	1.809	3.036	2.438	1.532	2.435	2.227	1.766	2.302	2.057	1.894	2.177
	$V_{x,Con}$	9.068	5.170	6.923	9.067	5.698	5.471	8.932	7.084	5.817	8.850	8.148	5.983
	$V_{x,Disc}$	5.922	3.376	2.774	6.044	3.799	2.428	5.847	4.637	2.614	5.700	5.248	2.710
	$V_{y,Con}$	6.662	3.798	4.564	6.400	4.022	4.254	5.847	4.637	3.632	5.400	4.972	3.158
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-

*S. M. = Swedish Method   R. M. = Reynolds Method   C. M. = Coefficient Method

Boundary Conditions	ψ	1.4			1.5			1.75			2.0		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
4	M_{xs}	7.826	5.327	6.289	6.339	4.592	4.840	6.459	5.616	5.176	6.377	6.201	5.369
	M_{ys}	4.970	3.383	4.088	3.820	2.767	3.200	3.489	3.034	2.843	3.223	3.134	2.553
	M_{xf}	5.816	3.959	4.036	4.795	3.474	3.383	4.826	4.196	3.585	4.800	4.668	3.703
	M_{yf}	3.807	2.591	2.971	2.926	2.120	2.371	2.673	2.324	2.240	2.469	2.401	2.129
	$V_{x,Con}$	9.623	6.550	7.197	9.600	6.955	5.698	9.257	8.049	5.927	9.000	8.752	6.030
	$V_{x,Disc}$	6.292	4.283	2.967	6.222	4.508	2.563	6.171	5.366	2.689	6.000	5.835	2.751
	$V_{y,Con}$	7.403	5.039	4.612	7.111	5.152	4.275	6.496	5.648	3.632	6.000	5.835	3.156
	$V_{y,Disc}$	4.812	3.275	1.792	4.622	3.349	1.540	4.223	3.671	1.490	3.900	3.792	1.425
5	M_{xs}	6.345	4.803	5.379	5.039	3.950	-4.000	4.974	4.358	4.064	4.800	4.599	4.088
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	4.759	3.602	3.723	3.820	2.994	3.077	3.712	3.252	3.119	3.634	3.482	3.134
	M_{yf}	3.596	2.722	2.350	2.763	2.166	1.886	2.524	2.211	1.789	2.331	2.234	1.721
	$V_{x,Con}$	8.883	6.724	6.525	8.711	6.829	5.016	8.445	7.398	5.019	8.100	7.761	4.996
	$V_{x,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	4.812	3.642	1.485	4.622	3.624	1.497	4.223	3.699	1.421	3.900	3.737	1.350
6	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	4.759	1.951	5.347	3.657	1.750	4.401	3.341	2.275	4.251	3.086	2.661	4.016
	M_{xf}	7.614	3.122	4.237	6.339	3.034	3.665	6.756	4.602	4.267	6.857	5.912	4.728
	M_{yf}	3.596	1.474	3.802	2.763	1.323	3.095	2.524	1.719	3.057	2.331	2.010	2.966
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	7.032	2.884	2.892	7.111	3.404	2.635	7.146	4.867	2.995	7.050	6.079	3.246
	$V_{y,Con}$	7.403	3.035	5.635	7.111	3.404	5.451	6.496	4.425	4.917	6.000	5.173	4.415
	$V_{y,Disc}$	-	-	-	-	-	-	-	-	-	-	-	-

*S. M. = Swedish Method R. M. = Reynolds Method C. M. = Coefficient Method

Boundary Conditions	ψ	1.4			1.5			1.75			2.0		
		S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.	S. M.	R. M.	C. M.
7	M_{xs}	8.566	6.632	6.741	6.827	5.482	5.125	6.830	6.265	5.531	6.720	6.743	5.448
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	6.345	4.913	4.300	5.120	4.112	3.558	5.123	4.699	3.683	5.074	5.092	3.754
	M_{yf}	4.653	3.603	2.902	3.576	2.872	2.304	3.267	2.996	2.177	3.017	3.028	2.081
	$V_{x,Con}$	10.179	7.881	7.486	10.133	8.137	5.929	9.744	8.938	6.030	9.450	9.483	6.607
	$V_{x,Disc}$	6.662	5.159	3.160	6.578	5.282	2.698	6.334	5.810	2.760	6.150	6.171	2.784
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	5.552	4.299	1.777	5.333	4.283	1.576	4.872	4.469	1.494	4.500	4.516	1.418
8	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	6.134	3.487	5.736	4.714	2.978	4.637	4.306	3.565	4.371	3.977	3.880	4.073
	M_{xf}	8.249	4.689	4.760	6.827	4.313	4.052	7.127	5.900	4.565	7.200	7.025	4.943
	M_{yf}	4.653	2.645	3.851	3.576	2.259	3.097	3.267	2.704	3.016	3.017	2.944	2.914
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	7.403	4.208	3.220	7.467	4.717	2.899	7.308	6.050	3.179	7.200	7.025	3.375
	$V_{y,Con}$	8.328	4.734	5.878	8.000	5.054	5.627	7.308	6.050	4.998	6.750	6.586	4.450
	$V_{y,Disc}$	5.552	3.156	2.295	5.333	3.369	2.219	4.872	4.033	1.806	4.500	4.391	1.640
9	M_{xs}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{ys}	-	-	-	-	-	-	-	-	-	-	-	-
	M_{xf}	9.200	6.859	5.373	7.477	5.940	4.487	7.647	7.292	4.882	7.611	8.117	5.164
	M_{yf}	5.922	4.415	3.907	4.551	3.616	3.098	4.158	3.964	2.973	3.840	4.095	2.860
	$V_{x,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{x,Disc}$	7.958	5.933	3.574	8.000	6.356	3.178	7.795	7.433	3.373	7.500	7.998	3.500
	$V_{y,Con}$	-	-	-	-	-	-	-	-	-	-	-	-
	$V_{y,Disc}$	6.107	4.553	2.317	5.867	4.661	2.030	5.359	5.110	1.804	4.950	5.279	1.637

*S. M. = Swedish Method R. M. = Reynolds Method C. M. = Coefficient Method

5

Conclusions and Recommendations

5.1 Conclusions

It has been discussed in the previous chapter the various alternatives for analyzing slab panel subjected to the weight of partition wall. As an additional approach a coefficient method similar to the method for which we use for analyzing uniformly distributed load was proposed. This method has an additional advantage of getting more realistic result, since it has based directly on the basic concepts of plate bending theory.

As it is clearly presented in the comparison table at the end of the previous chapter, this coefficient method of analysis gives generally results greater than Reynolds and less than the values of Swedish method. At this point we can conclude that the Swedish method which is proposed by Swedish regulation is some how conservative.

The coefficients of this newly proposed method is presented by taking the critical values without any modification. For example the bending moment coefficient for M_x is taken from the maximum ordinate at the longer edges. This bending moment has got its own distribution along the edges of the panel. It has to be taken some fraction of this moment to be distributed uniformly over the entire length of edge. Considering the simplified methods regarding to this coefficient, the Reynolds method seems make use of about half of the peak value and the Swedish method takes a factor greater than unity.

Comparing the shear force using different methods, it has a wider variation than the bending moment. Since the coefficient method proposed in this thesis takes the peak value of the shear coefficient to be uniformly distributed over the entire length of the beam, it seems more conservative. But when we consider the distribution of the reaction shear force coming from orthogonal strip to the beams at the edges, the effective support length of the beam should be considered. If we take the case of panel loaded with uniformly distributed load as an example, this effective supporting length is determined to be 75% by applying yield line concept[6]. For a panel loaded with partition wall, the shear force coming to the edge beams will have reasonable value if the effective supporting length of the beams is taken to be less than half. But this needs further detail analysis and laboratory investigation.

5.2 Recommendations

Comparing the shear force to the results of the other methods, the Swedish method give conservative result. This is may be due to that they have experiences of using light material for partition wall. When the weight of the partition wall is getting lighter, the Swedish method may be recommended to be used as it gives easier empirical formula to get the equivalent load. But according to the analysis of this thesis it is not recommended to use it here in Ethiopia, where most of the time partitions are made of relatively heavy material.

The thesis is assumed to be continued by some future studies considering the limitations made in the study. Therefore, one can extend this study to include the following subjects:

- This study dealt only for a wall load placed parallel to Lx and Ly, thus one can extend it to consider diagonally placed walls.

- The coefficients produced in this study are derived from pure elastic analysis of plate, so one can make further study to include plastic moment redistribution.

- One can make future study to device some means of taking the effective values of internal actions instead of taking peak values. This may require extensive study and laboratory experiment.

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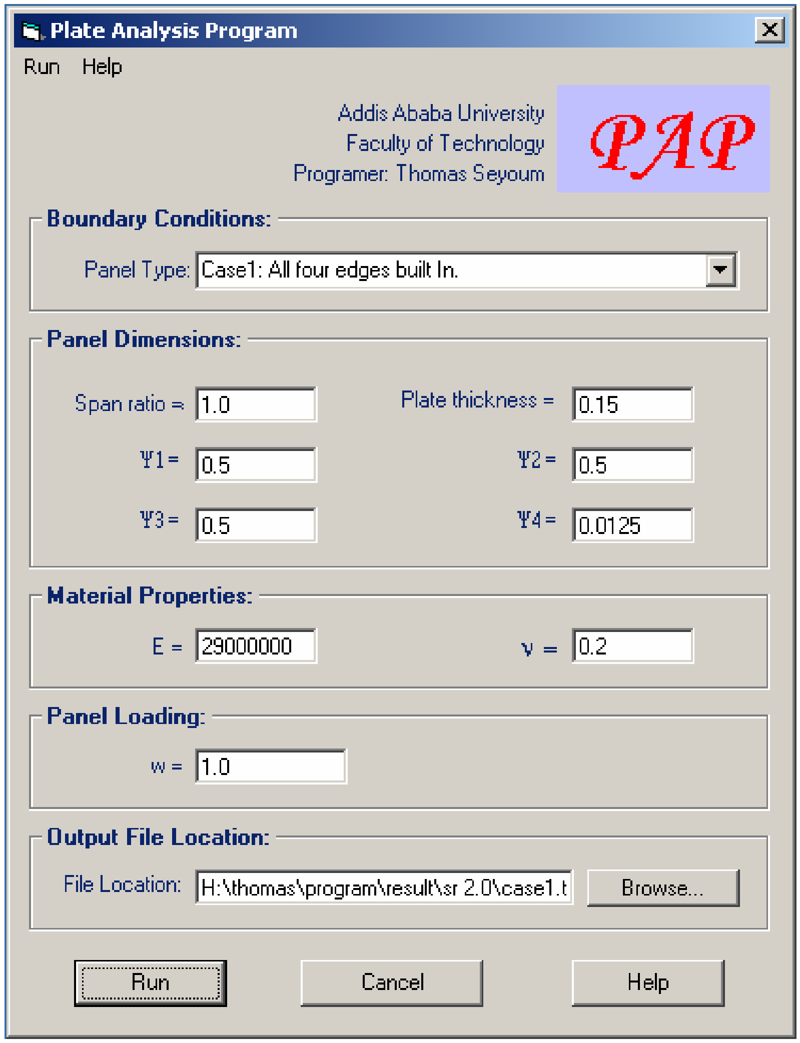
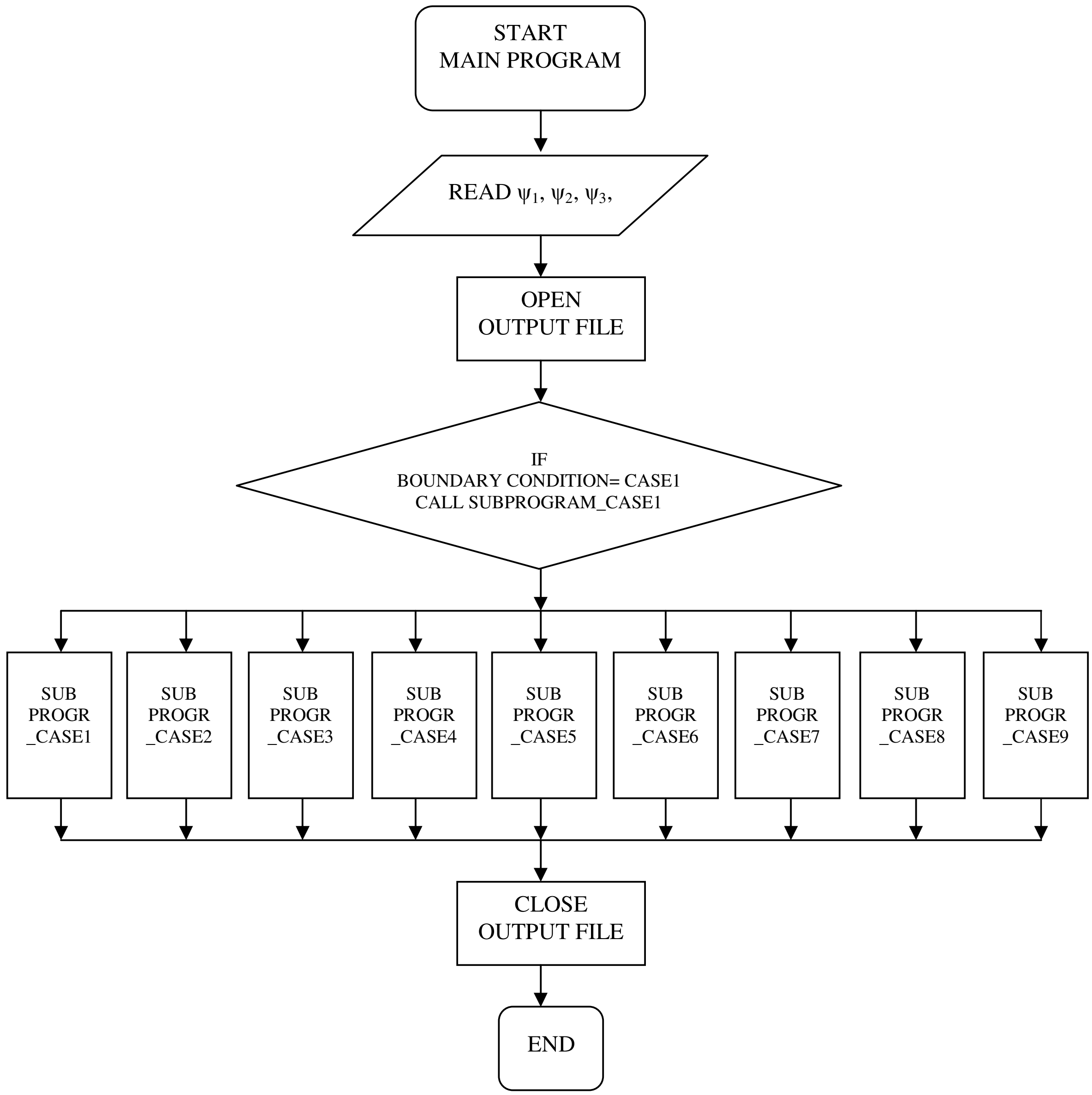
APPENDIX A

Plate Analysis Program



FLOW CHART


START
MAIN PROGRAM
READ ψ1, ψ2, ψ3,
OPEN
OUTPUT FILE
IF
BOUNDARY CONDITION= CASE1
CALL SUBPROGRAM_CASE1
SUB
PROGR
_CASE1
SUB
PROGR
_CASE2
SUB
PROGR
_CASE3
SUB
PROGR
_CASE4
SUB
PROGR
_CASE5
SUB
PROGR
_CASE6
SUB
PROGR
_CASE7
SUB
PROGR
_CASE8
SUB
PROGR
_CASE9
CLOSE
OUTPUT FILE
END

```
Private Sub Main_Click()

'----------------------------------------------------------
'Declaration

Dim Q As Integer
Dim eta As Double
Dim So As Double
Dim r As Double
Dim s As Double
Dim t As Double
Dim filenam As String
Dim E As Double
Dim nu As Double
Dim x As Double
Dim y As Double
.
.
.
.
.

'-------------------------------------------------
'Read Data

pi = 3.14159265358979
Q = Text5.Text
E = Text1.Text
nu = Text6.Text
s = Text2.Text
t = Text8.Text
Lx = Text4.Text
Ly = Text3.Text
filenam = Text7.Text
D = E * t ^ 3 / (12 * (1 - nu ^ 2))

'------------------------------------------------
'Open Result File

Open filenam For Output As #1
```

APPENDIX A

```
'-------------------------------------------------
'Call Subprograms

If Combo1.Text = "Case1: All four edges built In." Then
Call Case1

ElseIf Combo1.Text = "Case2: One shorter edge simply supported and the other three edges
clamped." Then
Call Case2

ElseIf Combo1.Text = "Case3: One longer edge simply supported and the other three edges
clamped." Then
Call Case3

ElseIf Combo1.Text = "Case4: Two adjacent edges simply supported and the other
clamped." Then
Call Case4

ElseIf Combo1.Text = "Case5: Two opposite longer edges simply supported and two edges
clamped." Then
Call Case5

ElseIf Combo1.Text = "Case6: Two opposite shorter edges simply supported and two edges
clamped." Then
Call Case6

ElseIf Combo1.Text = "Case7: One longer edge clamped and the other three edges simply
supported." Then
Call Case7

ElseIf Combo1.Text = "Case8: One shorter edge clamped and the other three edges simply
supported." Then
Call Case8

ElseIf Combo1.Text = "Case9: Simply supported on four edges." Then
Call Case9

End If

Close 1
MsgBox " Analysis Completed"
End Sub
```

APPENDIX A

```
Private Sub Case1()

pi = 3.14159265358979
Q = Text5.Text
E = Text1.Text
nu = Text6.Text
s = Text2.Text
t = Text8.Text
Lx = Text4.Text
Ly = Text3.Text
filenam = Text7.Text

D = E * t ^ 3 / (12 * (1 - nu ^ 2))

For eta = 1 To Lx Step 6
For So = 1 To Ly Step 6
r = 1

Do While (r / 2 + eta <= Lx And r / 2 <= eta)

alp1 = pi * Ly / (2 * Lx)
alp2 = pi * eta / Lx
alp3 = pi * So / Ly
alp4 = pi * r / (2 * Lx)
alp5 = pi * s / (2 * Ly)
alp6 = pi * Lx / (2 * Ly)
tex = "Eta = " & "," & FormatNumber(eta, 2) & "," & "Taw = " & "," & FormatNumber(So,
2) & "," & "r = " & "," & FormatNumber(r, 2)
Print #1, tex

'*********************************************************************
'MOMENT Mx

tex = "Moment per meter (Mx)"
Print #1, tex


cons1 = -16 * Q / ((pi ^ 4) * r * s)
```

```
tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex


For y = 0 To Ly Step 1
tex = FormatNumber(y, 1)
For x = 0 To Lx Step 1

 ddw1x = 0
 ddw1y = 0
 ddw2x = 0
```

APPENDIX A

```
 ddw2y = 0
 '-------------------------------------------------
   m = 1
   Do While (m < 50)
        cosh1 = (Exp(m * alp1) + Exp(-1 * m * alp1)) / 2
        cosh2 = (Exp(m * alp6) + Exp(-1 * m * alp6)) / 2
        tanh1 = (Exp(m * alp1) - Exp(-1 * m * alp1)) / (Exp(m * alp1) + Exp(-1 * m * alp1))
        tanh2 = (Exp(m * alp6) - Exp(-1 * m * alp6)) / (Exp(m * alp6) + Exp(-1 * m * alp6))
        R1 = 0
        i = 1
        Do While (i < 50)
        R1 = R1 + ((4 * Ly ^ 2 / (pi * Lx * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Ly ^ 2 / Lx ^ 2
        + i ^ 2 / m ^   2) ^ 2))
        i = i + 2
        Loop
        R2 = (Ly / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh2 + m * alp6 / (cosh2 ^ 2))
        R3 = 0
        n = 1
        Do While (n < 50)
        R3 = R3 + ((-16 * Q / (pi ^ 4 * r * s * Lx * n)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
        2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2
        + n ^ 2 / Ly ^ 2) ^ 2))
        n = n + 2
        Loop
        R4 = (Lx / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh1 + m * alp1 / (cosh1 ^ 2))
        R5 = 0
        i = 1
        Do While (i < 50)
        R5 = R5 + ((4 * Lx ^ 2 / (pi * Ly * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Lx ^ 2 / Ly ^ 2
        + i ^ 2 / m ^ 2) ^ 2))
        i = i + 2
        Loop
        R6 = 0
        n = 1
        Do While (n < 50)
```

```
R6 = R6 + ((-16 * Q / (pi ^ 4 * r * s * Ly * m)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2
+ n ^ 2 / Ly ^ 2) ^ 2))
n = n + 2
Loop
Em = (R2 * R6 - R3 * R5) / (R2 * R4 - R1 * R5)
Fm = (R3 - R1 * Em) / R2
sinhy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) - Exp(-1 * m * pi * (y - Ly / 2) / Lx))
sinhx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) - Exp(-1 * m * pi * (x - Lx / 2) / Ly))
coshy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) + Exp(-1 * m * pi * (y - Ly / 2) / Lx))
coshx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) + Exp(-1 * m * pi * (x - Lx / 2) / Ly))
ddw1x = ddw1x + (0.5 * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) * (Cos(m * pi * (x -
Lx / 2) / Lx)) * ((m * pi * (y - Ly / 2) / Lx) * sinhy - m * alp1 * tanh1 * coshy))
ddw1y = ddw1y + (-0.5 * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) * (Cos(m * pi * (x -
Lx / 2) / Lx)) * (2 * coshy + (m * pi * (y - Ly / 2) / Lx) * sinhy - m * alp1 * tanh1 *
coshy))
ddw2x = ddw2x + (-0.5 * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) * (Cos(m * pi * (y -
Ly / 2) / Ly)) * (2 * coshx + (m * pi * (x - Lx / 2) / Ly) * sinhx - m * alp6 * tanh2 *
coshx))
 ddw2y = ddw2y + (0.5 * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) * (Cos(m * pi * (y -
Ly / 2) / Ly)) * ((m * pi * (x - Lx / 2) / Ly) * sinhx - m * alp6 * tanh2 * coshx))
```



```
 m = m + 2
 Loop
 '--------------------------------
mx1 = 0
mx2 = 0
m = 1
Do While (m < 50)
n = 1
Do While (n < 50)
c1 = (n * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
mx1 = mx1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r
* 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n * pi * y) /
Ly)) / c1)
c2 = (m * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
mx2 = mx2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n * pi * y) / Ly))
/ c2)
n = n + 1
Loop
m = m + 1
Loop
'------------------------------------------------

mx = cons1 * (mx1 + nu * mx2)
mmx1 = (ddw1x + nu * ddw1y)
mmx2 = (ddw2x + nu * ddw2y)
mmx = mx + mmx1 + mmx2
tex = tex & "," & FormatNumber(mmx, 3)
```

```
 Next
 Print #1, tex
 Next

'************************************************************************
'MOMENT my

tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex

tex = "Moment per meter (My)"
Print #1, tex

tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex
For y = 0 To Ly Step 1
tex = FormatNumber(y, 1)
For x = 0 To Lx Step 1

ddw1x = 0
ddw1y = 0
```

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```
ddw2x = 0
ddw2y = 0
 '-------------------------------------------------
    m = 1
    Do While (m < 50)
      cosh1 = (Exp(m * alp1) + Exp(-1 * m * alp1)) / 2
      cosh2 = (Exp(m * alp6) + Exp(-1 * m * alp6)) / 2
      tanh1 = (Exp(m * alp1) - Exp(-1 * m * alp1)) / (Exp(m * alp1) + Exp(-1 * m * alp1))
       tanh2 = (Exp(m * alp6) - Exp(-1 * m * alp6)) / (Exp(m * alp6) + Exp(-1 * m * alp6))
      R1 = 0
      i = 1
      Do While (i < 50)
      R1 = R1 + ((4 * Ly ^ 2 / (pi * Lx * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Ly ^ 2 / Lx ^ 2 + i
      ^ 2 / m ^ 2) ^ 2))
      i = i + 2
      Loop
      R2 = (Ly / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh2 + m * alp6 / (cosh2 ^ 2))
      R3 = 0
      n = 1
      Do While (n < 50)
```

```
R3 = R3 + ((-16 * Q / (pi ^ 4 * r * s * Lx * n)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
n ^ 2 / Ly ^ 2) ^ 2))
n = n + 2
Loop
R4 = (Lx / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh1 + m * alp1 / (cosh1 ^ 2))
R5 = 0
i = 1
Do While (i < 50)
R5 = R5 + ((4 * Lx ^ 2 / (pi * Ly * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Lx ^ 2 / Ly ^ 2 + i
^ 2 / m ^ 2) ^ 2))
i = i + 2
Loop
R6 = 0
n = 1
Do While (n < 50)
R6 = R6 + ((-16 * Q / (pi ^ 4 * r * s * Ly * m)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
n ^ 2 / Ly ^ 2) ^ 2))
n = n + 2
Loop
Em = (R2 * R6 - R3 * R5) / (R2 * R4 - R1 * R5)
Fm = (R3 - R1 * Em) / R2
sinhy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) - Exp(-1 * m * pi * (y - Ly / 2) / Lx))
sinhx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) - Exp(-1 * m * pi * (x - Lx / 2) / Ly))
coshy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) + Exp(-1 * m * pi * (y - Ly / 2) / Lx))
coshx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) + Exp(-1 * m * pi * (x - Lx / 2) / Ly))
ddw1x = ddw1x + (0.5 * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) * (Cos(m * pi * (x - Lx
/ 2) / Lx)) * ((m * pi * (y - Ly / 2) / Lx) * sinhy - m * alp1 * tanh1 * coshy))
ddw1y = ddw1y + (-0.5 * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) * (Cos(m * pi * (x - Lx
/ 2) / Lx)) * (2 * coshy + (m * pi * (y - Ly / 2) / Lx) * sinhy - m * alp1 * tanh1 * coshy))
ddw2x = ddw2x + (-0.5 * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) * (Cos(m * pi * (y - Ly
/ 2) / Ly)) * (2 * coshx + (m * pi * (x - Lx / 2) / Ly) * sinhx - m * alp6 * tanh2 * coshx))
ddw2y = ddw2y + (0.5 * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) * (Cos(m * pi * (y - Ly
/ 2) / Ly)) * ((m * pi * (x - Lx / 2) / Ly) * sinhx - m * alp6 * tanh2 * coshx))
```



```
m = m + 2
Loop
'--------------------------------
mx1 = 0
mx2 = 0
m = 1
Do While (m < 50)
n = 1
Do While (n < 50)
c1 = (n * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
mx1 = mx1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n * pi * y) / Ly)) /
c1)
c2 = (m * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
```

```
    mx2 = mx2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
    0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Sin((n * pi * y) / Ly)) /
    c2)
    n = n + 1
    Loop
   m = m + 1
  Loop
'-----------------------------------------------

my = cons1 * (nu * mx1 + mx2)
mmy1 = (nu * ddw1x + ddw1y)
mmy2 = (nu * ddw2x + ddw2y)
mmy = my + mmy1 + mmy2
tex = tex & "," & FormatNumber(mmy, 3)
Next
Print #1, tex
Next

'*********************************************************************
'SHEAR Vx
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex
cons3 = -16 * Q / ((pi ^ 3) * r * s)
tex = "Shear Force per meter (Vx)"
Print #1, tex
tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex
For y = 0 To Ly Step 1
tex = FormatNumber(y, 1)
For x = 0 To Lx Step 1
dddw1x = 0
dddw1xy2 = 0
dddw2x = 0
dddw2xy2 = 0
'-----------------------------------------------
```



```
m = 1
  Do While (m < 50)
    cosh1 = (Exp(m * alp1) + Exp(-1 * m * alp1)) / 2
    cosh2 = (Exp(m * alp6) + Exp(-1 * m * alp6)) / 2
    tanh1 = (Exp(m * alp1) - Exp(-1 * m * alp1)) / (Exp(m * alp1) + Exp(-1 * m * alp1))
    tanh2 = (Exp(m * alp6) - Exp(-1 * m * alp6)) / (Exp(m * alp6) + Exp(-1 * m * alp6))
```

```
R1 = 0
 i = 1
Do While (i < 50)
R1 = R1 + ((4 * Ly ^ 2 / (pi * Lx * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Ly ^ 2 / Lx ^ 2 + i
^ 2 / m ^ 2) ^ 2))
i = i + 2
Loop
R2 = (Ly / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh2 + m * alp6 / (cosh2 ^ 2))
R3 = 0
n = 1
Do While (n < 50)
R3 = R3 + ((-16 * Q / (pi ^ 4 * r * s * Lx * n)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
n ^ 2 / Ly ^ 2) ^ 2))
n = n + 2
Loop
R4 = (Lx / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh1 + m * alp1 / (cosh1 ^ 2))
R5 = 0
i = 1
Do While (i < 50)
R5 = R5 + ((4 * Lx ^ 2 / (pi * Ly * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Lx ^ 2 / Ly ^ 2 + i
^ 2 / m ^ 2) ^ 2))
i = i + 2
Loop
R6 = 0
n = 1
Do While (n < 50)
R6 = R6 + ((-16 * Q / (pi ^ 4 * r * s * Ly * m)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
n ^ 2 / Ly ^ 2) ^ 2))
n = n + 2
Loop
Em = (R2 * R6 - R3 * R5) / (R2 * R4 - R1 * R5)
Fm = (R3 - R1 * Em) / R2
sinhy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) - Exp(-1 * m * pi * (y - Ly / 2) / Lx))
sinhx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) - Exp(-1 * m * pi * (x - Lx / 2) / Ly))
coshy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) + Exp(-1 * m * pi * (y - Ly / 2) / Lx))
coshx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) + Exp(-1 * m * pi * (x - Lx / 2) / Ly))
dddw1x = dddw1x + (-0.5 * pi * (1 / Lx) * m * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) *
(Sin(m * pi * (x - Lx / 2) / Lx)) * ((m * pi * (y - Ly / 2) / Lx) * sinhy - m * alp1 * tanh1
* coshy))
dddw1xy2 = dddw1xy2 + (0.5 * pi * (1 / Lx) * m * Em * ((-1) ^ ((m - 1) / 2)) * (1 /
cosh1) * (Sin(m * pi * (x - Lx / 2) / Lx)) * (2 * coshy + (m * pi * (y - Ly / 2) / Lx) *
sinhy - m * alp1 * tanh1 * coshy))
dddw2x = dddw2x + (-0.5 * pi * (1 / Ly) * m * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) *
(Cos(m * pi * (y - Ly / 2) / Ly)) * (3 * sinhx + (m * pi * (x - Lx / 2) / Ly) * coshx - m *
alp6 * tanh2 * sinhx))
```

```
      dddw2xy2 = dddw2xy2 + (0.5 * pi * (1 / Ly) * m * Fm * ((-1) ^ ((m - 1) / 2)) * (1 /
      cosh2) * (Cos(m * pi * (y - Ly / 2) / Ly)) * (sinhx + (m * pi * (x - Lx / 2) / Ly) * coshx -
      m * alp6 * tanh2 * sinhx))
      m = m + 2
      Loop
      '-------------------------------------------------
      vx1 = 0
      vx2 = 0
      n = 1
      Do While (n < 50)
      m = 1
      Do While (m < 50)
      c5 = (n * Lx ^ 3 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
      vx1 = vx1 + ((m ^ 2 * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r
      * 0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Cos((m * pi * x) / Lx) * Sin((n * pi * y) / Ly))
      / c5)
      c6 = (Lx * Ly ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
      vx2 = vx2 + ((n * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
      0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Cos((m * pi * x) / Lx) * Sin((n * pi * y) / Ly)) /
      c6)
      m = m + 1
      DoEvents
     Loop
   n = n + 1
   Loop
 '-------------------------------------------------
 vx = cons3 * (vx1 + vx2)
 vvx1 = (dddw1x + dddw1xy2)
 vvx2 = (dddw2x + dddw2xy2)
 vvx = vx + vvx1 + vvx2
 tex = tex & "," & FormatNumber(vvx, 2)
 Next
 Print #1, tex
Next
'************************************************************************
'SHEAR Vy
tex = ""
Print #1, tex
tex = ""
Print #1, tex
tex = ""
Print #1, tex

cons3 = -16 * Q / ((pi ^ 3) * r * s)
tex = "Shear Force per meter (Vy)"
Print #1, tex
tex = "y\x"
For x = 0 To Lx Step 1
tex = tex & "," & FormatNumber(x, 1)
Next
Print #1, tex
For y = 0 To Ly Step 1
```

```
tex = FormatNumber(y, 1)
For x = 0 To Lx Step 1
dddw1y = 0
dddw1yx2 = 0
```



```
dddw2y = 0
dddw2yx2 = 0
 '-----------------------------------------------
   m = 1
   Do While (m < 50)
     cosh1 = (Exp(m * alp1) + Exp(-1 * m * alp1)) / 2
     cosh2 = (Exp(m * alp6) + Exp(-1 * m * alp6)) / 2
     tanh1 = (Exp(m * alp1) - Exp(-1 * m * alp1)) / (Exp(m * alp1) + Exp(-1 * m * alp1))
     tanh2 = (Exp(m * alp6) - Exp(-1 * m * alp6)) / (Exp(m * alp6) + Exp(-1 * m * alp6))
     R1 = 0
     i = 1
     Do While (i < 50)
     R1 = R1 + ((4 * Ly ^ 2 / (pi * Lx * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Ly ^ 2 / Lx ^ 2 + i
     ^ 2 / m ^ 2) ^ 2))
     i = i + 2
     Loop
     R2 = (Ly / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh2 + m * alp6 / (cosh2 ^ 2))
     R3 = 0
     n = 1
     Do While (n < 50)
     R3 = R3 + ((-16 * Q / (pi ^ 4 * r * s * Lx * n)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
     2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
     n ^ 2 / Ly ^ 2) ^ 2))
     n = n + 2
     Loop
     R4 = (Lx / (2 * m)) * ((-1) ^ ((m - 1) / 2)) * (tanh1 + m * alp1 / (cosh1 ^ 2))
     R5 = 0
     i = 1
     Do While (i < 50)
     R5 = R5 + ((4 * Lx ^ 2 / (pi * Ly * m ^ 3)) * (i * (-1) ^ ((i - 1) / 2)) / ((Lx ^ 2 / Ly ^ 2 + i
     ^ 2 / m ^ 2) ^ 2))
     i = i + 2
     Loop
     R6 = 0
     n = 1
     Do While (n < 50)
     R6 = R6 + ((-16 * Q / (pi ^ 4 * r * s * Ly * m)) * ((-1) ^ ((m - 1) / 2)) * ((-1) ^ ((n - 1) /
     2)) * (Sin(m * alp2) * Sin(n * alp3) * Sin(m * alp4) * Sin(n * alp5)) / ((m ^ 2 / Lx ^ 2 +
     n ^ 2 / Ly ^ 2) ^ 2))
     n = n + 2
     Loop
     Em = (R2 * R6 - R3 * R5) / (R2 * R4 - R1 * R5)
     Fm = (R3 - R1 * Em) / R2
     sinhy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) - Exp(-1 * m * pi * (y - Ly / 2) / Lx))
     sinhx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) - Exp(-1 * m * pi * (x - Lx / 2) / Ly))
```

```
coshy = 0.5 * (Exp(m * pi * (y - Ly / 2) / Lx) + Exp(-1 * m * pi * (y - Ly / 2) / Lx))
coshx = 0.5 * (Exp(m * pi * (x - Lx / 2) / Ly) + Exp(-1 * m * pi * (x - Lx / 2) / Ly))
dddw1y = dddw1y + (-0.5 * pi * (1 / Lx) * m * Em * ((-1) ^ ((m - 1) / 2)) * (1 / cosh1) *
(Cos(m * pi * (x - Lx / 2) / Lx)) * (3 * sinhy + (m * pi * (y - Ly / 2) / Lx) * coshy - m *
alp1 * tanh1 * sinhy))
dddw1yx2 = dddw1yx2 + (0.5 * pi * (1 / Lx) * m * Em * ((-1) ^ ((m - 1) / 2)) * (1 /
cosh1) * (Cos(m * pi * (x - Lx / 2) / Lx)) * (sinhy + (m * pi * (y - Ly / 2) / Lx) * coshy -
m * alp1 * tanh1 * sinhy))
```



```
dddw2y = dddw2y + (-0.5 * pi * (1 / Ly) * m * Fm * ((-1) ^ ((m - 1) / 2)) * (1 / cosh2) *
(Sin(m * pi * (y - Ly / 2) / Ly)) * ((m * pi * (x - Lx / 2) / Ly) * sinhx - m * alp6 * tanh2
* coshx))
dddw2yx2 = dddw2yx2 + (0.5 * pi * (1 / Ly) * m * Fm * ((-1) ^ ((m - 1) / 2)) * (1 /
cosh2) * (Sin(m * pi * (y - Ly / 2) / Ly)) * (2 * coshx + (m * pi * (x - Lx / 2) / Ly) *
sinhx - m * alp6 * tanh2 * coshx))
m = m + 2
Loop
'------------------------------------------------
vy1 = 0
vy2 = 0
n = 1
Do While (n < 50)
m = 1
Do While (m < 50)
c7 = (Ly * Lx ^ 2 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
vy1 = vy1 + ((m * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
0.5) /    Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Cos((n * pi * y) / Ly))
/ c7)
c8 = (m * Ly ^ 3 * (((m ^ 2 / Lx ^ 2) + (n ^ 2 / Ly ^ 2)) ^ 2))
vy2 = vy2 + ((n ^ 2 * Sin((m * pi * eta) / Lx) * Sin((n * pi * So) / Ly) * Sin((m * pi * r *
0.5) / Lx) * Sin((n * pi * s * 0.5) / Ly) * Sin((m * pi * x) / Lx) * Cos((n * pi * y) / Ly)) /
c8)
m = m + 1
DoEvents
Loop
n = n + 1
Loop
'------------------------------------------------
vy = cons3 * (vy1 + vy2)
vvy1 = (dddw1y + dddw1yx2)
vvy2 = (dddw2y + dddw2yx2)
vvy = vy + vvy1 + vvy2
tex = tex & "," & FormatNumber(vvy, 2)
Next
Print #1, tex
Next
r = r + 1
DoEvents
Loop
Next
```

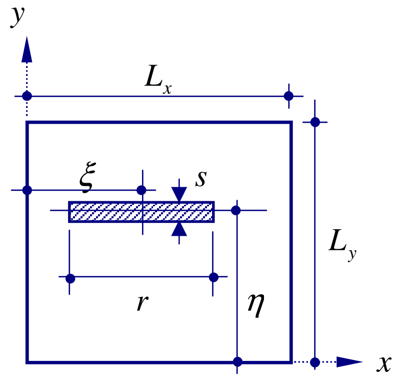
```
Next

End Sub
```

APPENDIX B

Table of Coefficient for Slab Panel Subjected to Partition Wall Load

Table B.1: Bending Moment coefficient for a rectangular panel subjected to partition wall loads parallel to L_x.



r = wall length

s = wall thickness

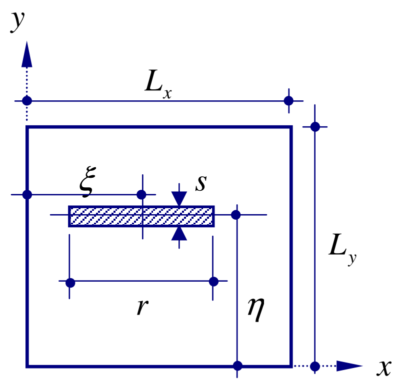
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_x} = 0.5 \qquad \psi_4 = \frac{s}{2L_y} = 0.02$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.082	0.058	0.063	0.067	0.069	0.070	0.072	0.072
	α_{ys}	0.082	0.079	0.076	0.073	0.069	0.066	0.061	0.058
	α_{xf}	0.080	0.043	0.045	0.047	0.048	0.049	0.050	0.050
	α_{yf}	0.080	0.081	0.082	0.082	0.082	0.082	0.083	0.083
2	α_{xs}	0.064	0.057	0.051	0.045	0.039	0.035	0.025	0.018
	α_{ys}	0.012	0.021	0.027	0.032	0.036	0.039	0.042	0.041
	α_{xf}	0.080	0.086	0.091	0.095	0.098	0.101	0.105	0.107
	α_{yf}	0.116	0.114	0.113	0.111	0.109	0.108	0.104	0.102
3	α_{xs}	0.044	0.024	0.006	0.012	0.028	0.043	0.076	0.101
	α_{ys}	0.123	0.131	0.137	0.142	0.146	0.149	0.152	0.151
	α_{xf}	0.080	0.086	0.091	0.095	0.098	0.101	0.105	0.107
	α_{yf}	0.117	0.115	0.114	0.112	0.110	0.109	0.106	0.103
4	α_{xs}	0.172	0.177	0.177	0.175	0.171	0.165	0.147	0.128
	α_{ys}	0.117	0.099	0.080	0.062	0.045	0.031	0.001	0.021
	α_{xf}	0.080	0.086	0.091	0.095	0.098	0.101	0.105	0.107
	α_{yf}	0.116	0.114	0.113	0.111	0.109	0.107	0.104	0.102
5	α_{xs}	0.098	0.075	0.054	0.036	0.023	0.013	0.002	0.000
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.111	0.114	0.116	0.116	0.115	0.113	0.108	0.103
	α_{yf}	0.106	0.112	0.115	0.116	0.115	0.114	0.110	0.107
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.098	0.098	0.096	0.092	0.088	0.082	0.068	0.054
	α_{xf}	0.106	0.113	0.116	0.119	0.119	0.119	0.117	0.115
	α_{yf}	0.111	0.114	0.116	0.116	0.116	0.115	0.111	0.107
7	α_{xs}	0.117	0.087	0.061	0.041	0.026	0.015	0.002	0.000
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.113	0.115	0.115	0.114	0.113	0.111	0.107	0.103
	α_{yf}	0.096	0.101	0.105	0.107	0.108	0.108	0.107	0.107
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.042	0.038	0.031	0.024	0.017	0.011	0.007	0.003
	α_{xf}	0.096	0.101	0.105	0.108	0.110	0.111	0.111	0.111
	α_{yf}	0.113	0.115	0.115	0.114	0.113	0.112	0.109	0.105
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.080	0.086	0.091	0.095	0.098	0.101	0.105	0.107
	α_{yf}	0.117	0.115	0.114	0.112	0.110	0.109	0.106	0.103

Table B.2 Shear force coefficient for a rectangular panel subjected to partition wall loads parallel to L_x.



r = wall length

s = wall thickness

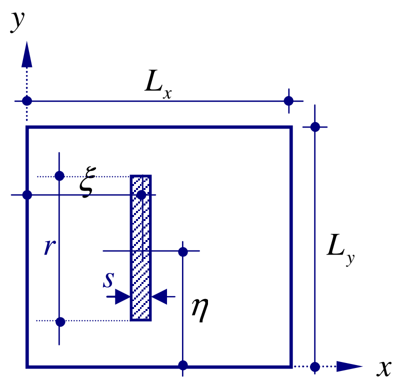
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_x} = 0.5 \qquad \psi_4 = \frac{s}{2L_y} = 0.02$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	1.206	1.238	1.254	1.259	1.257	1.251	1.224	1.192
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	1.206	0.275	0.227	0.184	0.145	0.112	0.047	0.005
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	1.222	1.213	1.202	1.189	1.176	1.162	1.127	1.094
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.289	0.244	0.205	0.171	0.141	0.116	0.069	0.038
	$\beta_{y,Disc}$	0.241	0.208	0.179	0.153	0.130	0.111	0.072	0.046
3	$\beta_{x,Con}$	1.177	1.176	1.172	1.166	1.159	1.150	1.127	1.102
	$\beta_{x,Disc}$	1.177	1.094	1.008	0.921	0.834	0.748	0.654	0.562
	$\beta_{y,Con}$	0.250	0.217	0.188	0.162	0.139	0.119	0.081	0.055
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	1.141	1.138	1.132	1.124	1.115	1.104	1.077	1.049
	$\beta_{x,Disc}$	1.141	1.058	0.974	0.888	0.803	0.718	0.625	0.535
	$\beta_{y,Con}$	0.242	0.208	0.179	0.153	0.130	0.111	0.073	0.046
	$\beta_{y,Disc}$	0.242	0.193	0.154	0.121	0.094	0.072	0.042	0.023
5	$\beta_{x,Con}$	0.618	0.499	0.375	0.265	0.177	0.111	0.024	0.000
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.056	0.064	0.058	0.043	0.021	0.018	0.033	0.031
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	1.220	1.203	1.188	1.177	1.168	1.161	1.146	1.130
	$\beta_{y,Con}$	0.368	0.406	0.426	0.430	0.423	0.408	0.350	0.284
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	1.277	1.270	1.262	1.252	1.240	1.228	1.195	1.162
	$\beta_{x,Disc}$	1.277	1.294	1.288	1.272	1.252	1.232	1.190	1.158
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.250	0.219	0.188	0.160	0.136	0.116	0.079	0.055
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.051	1.247	1.239	1.231	1.222	1.213	1.187	1.159
	$\beta_{y,Con}$	0.096	0.197	0.173	0.151	0.132	0.115	0.080	0.055
	$\beta_{y,Disc}$	0.113	0.209	0.171	0.135	0.103	0.076	0.028	0.003
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	1.277	1.270	1.260	1.250	1.238	1.225	1.193	1.161
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.250	0.217	0.188	0.162	0.139	0.119	0.081	0.055

Table B.3 Bending Moment coefficient for a rectangular panel subjected to partition wall loads parallel to L_y.



r = wall length
s = wall thickness

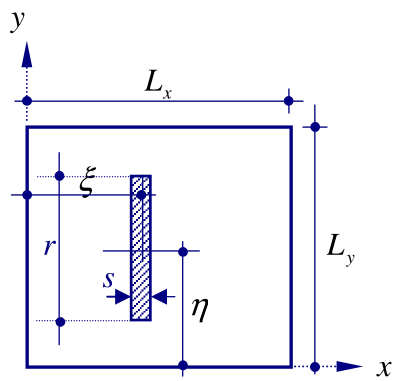
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_y} = 0.5 \qquad \psi_4 = \frac{s}{2L_x} = 0.01$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.068	0.096	0.081	0.072	0.060	0.046	0.035	0.046
	α_{ys}	0.068	0.083	0.062	0.049	0.037	0.029	0.018	0.021
	α_{xf}	0.093	0.069	0.057	0.050	0.042	0.032	0.027	0.035
	α_{yf}	0.093	0.062	0.048	0.039	0.030	0.022	0.016	0.020
2	α_{xs}	0.099	0.106	0.087	0.076	0.062	0.044	0.035	0.046
	α_{ys}	0.107	0.074	0.063	0.049	0.037	0.027	0.018	0.021
	α_{xf}	0.068	0.086	0.061	0.053	0.043	0.034	0.027	0.035
	α_{yf}	0.076	0.061	0.046	0.038	0.029	0.022	0.016	0.020
3	α_{xs}	0.107	0.104	0.091	0.082	0.070	0.051	0.044	0.060
	α_{ys}	0.099	0.098	0.076	0.062	0.048	0.036	0.025	0.029
	α_{xf}	0.076	0.070	0.060	0.054	0.045	0.035	0.031	0.041
	α_{yf}	0.068	0.071	0.056	0.046	0.036	0.027	0.020	0.024
4	α_{xs}	0.118	0.120	0.101	0.090	0.075	0.054	0.045	0.060
	α_{ys}	0.118	0.106	0.080	0.064	0.049	0.036	0.025	0.029
	α_{xf}	0.078	0.079	0.066	0.058	0.048	0.038	0.031	0.042
	α_{yf}	0.078	0.072	0.056	0.046	0.036	0.027	0.020	0.024
5	α_{xs}	0.121	0.116	0.094	0.080	0.065	0.045	0.036	0.046
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.068	0.081	0.065	0.055	0.045	0.035	0.027	0.035
	α_{yf}	0.085	0.060	0.045	0.036	0.028	0.021	0.016	0.019
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.121	0.117	0.094	0.080	0.064	0.050	0.037	0.045
	α_{xf}	0.085	0.071	0.063	0.058	0.051	0.041	0.037	0.053
	α_{yf}	0.068	0.082	0.066	0.056	0.046	0.035	0.027	0.033
7	α_{xs}	0.141	0.139	0.114	0.098	0.081	0.058	0.048	0.061
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.082	0.089	0.073	0.063	0.052	0.040	0.032	0.042
	α_{yf}	0.091	0.073	0.055	0.045	0.035	0.026	0.019	0.023
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.141	0.084	0.105	0.087	0.069	0.052	0.038	0.046
	α_{xf}	0.091	0.133	0.073	0.066	0.057	0.046	0.040	0.056
	α_{yf}	0.082	0.086	0.069	0.058	0.046	0.035	0.026	0.033
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.114	0.110	0.109	0.107	0.105	0.104	0.101	0.098
	α_{yf}	0.078	0.084	0.089	0.093	0.096	0.099	0.103	0.105

Table B.4 Shear force coefficient for a rectangular panel subjected to partition wall loads parallel to L_y.



r = wall length
s = wall thickness

$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_y} = 0.5 \qquad \psi_4 = \frac{s}{2L_x} = 0.01$$

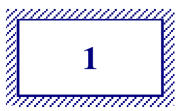
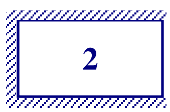
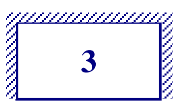
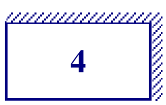
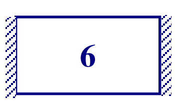
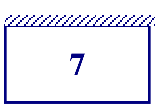
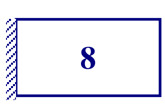
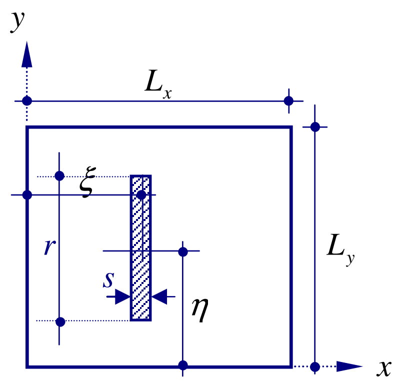
Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
 1	$\beta_{x,Con}$	0.667	0.770	0.956	1.080	1.198	1.305	1.070	1.127
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.667	0.664	0.722	0.725	0.724	0.930	0.617	0.559
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
 2	$\beta_{x,Con}$	0.726	0.817	0.996	1.110	1.219	1.011	1.075	1.126
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.684	0.672	0.725	0.725	0.722	0.702	0.616	0.559
	$\beta_{y,Disc}$	0.257	0.251	0.271	0.272	0.280	0.309	0.310	0.308
 3	$\beta_{x,Con}$	0.684	0.803	1.016	1.167	1.315	1.122	1.251	1.346
	$\beta_{x,Disc}$	0.257	0.306	0.394	0.460	0.527	0.498	0.562	0.610
	$\beta_{y,Con}$	0.726	0.743	0.830	0.852	0.867	0.872	0.781	0.711
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
 4	$\beta_{x,Con}$	0.767	0.879	1.088	1.230	1.367	1.168	1.274	1.357
	$\beta_{x,Disc}$	0.301	0.350	0.439	0.502	0.564	0.525	0.578	0.619
	$\beta_{y,Con}$	0.767	0.773	0.852	0.867	0.876	0.876	0.781	0.710
	$\beta_{y,Disc}$	0.301	0.301	0.330	0.335	0.340	0.316	0.320	0.321
 5	$\beta_{x,Con}$	0.791	0.868	1.037	1.140	1.240	1.028	1.079	1.124
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.253	0.246	0.266	0.268	0.282	0.307	0.306	0.304
 6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.253	0.304	0.397	0.471	0.549	0.540	0.644	0.730
	$\beta_{y,Con}$	0.791	0.836	0.964	1.021	1.071	1.117	1.057	0.993
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
 7	$\beta_{x,Con}$	0.869	0.966	1.169	1.297	1.422	1.215	1.296	1.487
	$\beta_{x,Disc}$	0.351	0.396	0.484	0.493	0.520	0.553	0.593	0.626
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.305	0.301	0.328	0.333	0.338	0.323	0.321	0.319
 8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.305	0.359	0.458	0.534	0.612	0.594	0.683	0.759
	$\beta_{y,Con}$	0.869	0.904	1.029	1.077	1.117	1.154	1.075	1.001
	$\beta_{y,Disc}$	0.351	0.361	0.406	0.422	0.436	0.455	0.388	0.369
 9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.570	0.425	0.529	0.604	0.679	0.651	0.725	0.788
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.370	0.374	0.416	0.429	0.440	0.416	0.388	0.368

Table B.5 Bending Moment coefficient for a rectangular panel subjected to partition wall loads parallel to L_y.



r = wall length
s = wall thickness

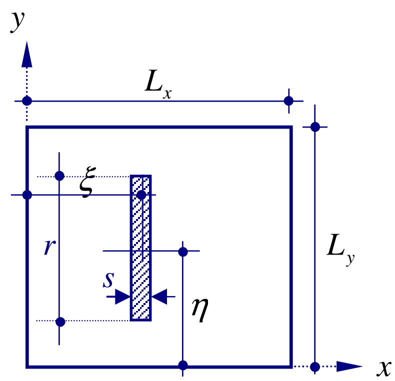
$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_y} = 0.5 \qquad \psi_4 = \frac{s}{2L_x} = 0.02$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	α_{xs}	0.080	0.112	0.095	0.084	0.070	0.054	0.041	0.054
	α_{ys}	0.080	0.097	0.073	0.057	0.043	0.034	0.021	0.025
	α_{xf}	0.109	0.081	0.067	0.059	0.049	0.037	0.032	0.041
	α_{yf}	0.109	0.073	0.056	0.046	0.035	0.026	0.019	0.023
2	α_{xs}	0.116	0.124	0.102	0.089	0.073	0.051	0.041	0.054
	α_{ys}	0.125	0.087	0.074	0.057	0.043	0.032	0.021	0.025
	α_{xf}	0.080	0.101	0.071	0.062	0.050	0.040	0.032	0.041
	α_{yf}	0.089	0.071	0.054	0.044	0.034	0.026	0.019	0.023
3	α_{xs}	0.125	0.122	0.106	0.096	0.082	0.060	0.051	0.070
	α_{ys}	0.116	0.115	0.089	0.073	0.056	0.042	0.029	0.034
	α_{xf}	0.089	0.082	0.070	0.063	0.053	0.041	0.036	0.048
	α_{yf}	0.080	0.083	0.066	0.054	0.042	0.032	0.023	0.028
4	α_{xs}	0.138	0.140	0.118	0.105	0.088	0.063	0.053	0.070
	α_{ys}	0.138	0.124	0.094	0.075	0.057	0.042	0.029	0.034
	α_{xf}	0.091	0.092	0.077	0.068	0.056	0.044	0.036	0.049
	α_{yf}	0.091	0.084	0.066	0.054	0.042	0.032	0.023	0.028
5	α_{xs}	0.142	0.136	0.110	0.094	0.076	0.053	0.042	0.054
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.080	0.095	0.076	0.064	0.053	0.041	0.032	0.041
	α_{yf}	0.099	0.070	0.053	0.042	0.033	0.025	0.019	0.022
6	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.142	0.137	0.110	0.094	0.075	0.059	0.043	0.053
	α_{xf}	0.099	0.083	0.074	0.068	0.060	0.048	0.043	0.062
	α_{yf}	0.080	0.096	0.077	0.066	0.054	0.041	0.032	0.039
7	α_{xs}	0.165	0.163	0.133	0.115	0.095	0.068	0.056	0.071
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.096	0.104	0.085	0.074	0.061	0.047	0.037	0.049
	α_{yf}	0.106	0.085	0.064	0.053	0.041	0.030	0.022	0.027
8	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	0.165	0.098	0.123	0.102	0.081	0.061	0.044	0.054
	α_{xf}	0.106	0.156	0.085	0.077	0.067	0.054	0.047	0.066
	α_{yf}	0.096	0.101	0.081	0.068	0.054	0.041	0.030	0.039
9	α_{xs}	-	-	-	-	-	-	-	-
	α_{ys}	-	-	-	-	-	-	-	-
	α_{xf}	0.133	0.129	0.128	0.125	0.123	0.122	0.118	0.115
	α_{yf}	0.091	0.098	0.104	0.109	0.112	0.116	0.121	0.123

Table B.6 Shear force coefficient for a rectangular panel subjected to partition wall loads parallel to L_y.



r = wall length

s = wall thickness

$$\psi = \frac{L_y}{L_x}$$

$$\psi_1 = \frac{\xi}{L_x} = 0.5 \qquad \psi_2 = \frac{\eta}{L_y} = 0.5$$

$$\psi_3 = \frac{r}{2L_y} = 0.5 \qquad \psi_4 = \frac{s}{2L_x} = 0.02$$

Boundary Conditions	ψ	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0
1	$\beta_{x,Con}$	0.780	0.901	1.119	1.264	1.402	1.527	1.252	1.319
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.780	0.777	0.845	0.848	0.847	1.088	0.722	0.654
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
2	$\beta_{x,Con}$	0.849	0.956	1.165	1.299	1.426	1.183	1.258	1.317
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	0.800	0.786	0.848	0.848	0.845	0.821	0.721	0.654
	$\beta_{y,Disc}$	0.301	0.294	0.317	0.318	0.328	0.362	0.363	0.360
3	$\beta_{x,Con}$	0.800	0.940	1.189	1.365	1.539	1.313	1.464	1.575
	$\beta_{x,Disc}$	0.301	0.358	0.461	0.538	0.617	0.583	0.658	0.714
	$\beta_{y,Con}$	0.849	0.869	0.971	0.997	1.014	1.020	0.914	0.832
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
4	$\beta_{x,Con}$	0.897	1.028	1.273	1.439	1.599	1.367	1.491	1.588
	$\beta_{x,Disc}$	0.352	0.410	0.514	0.587	0.660	0.614	0.676	0.724
	$\beta_{y,Con}$	0.897	0.904	0.997	1.014	1.025	1.025	0.914	0.831
	$\beta_{y,Disc}$	0.352	0.352	0.386	0.392	0.398	0.370	0.374	0.376
5	$\beta_{x,Con}$	0.925	1.016	1.213	1.334	1.451	1.203	1.262	1.315
	$\beta_{x,Disc}$	-	-	-	-	-	-	-	-
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.296	0.288	0.311	0.314	0.330	0.359	0.358	0.356
6	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.296	0.356	0.464	0.551	0.642	0.632	0.753	0.854
	$\beta_{y,Con}$	0.925	0.978	1.128	1.195	1.253	1.307	1.237	1.162
	$\beta_{y,Disc}$	-	-	-	-	-	-	-	-
7	$\beta_{x,Con}$	1.017	1.130	1.368	1.517	1.664	1.422	1.516	1.740
	$\beta_{x,Disc}$	0.411	0.463	0.566	0.635	0.702	0.647	0.694	0.732
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.357	0.352	0.384	0.390	0.395	0.378	0.376	0.373
8	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.357	0.420	0.536	0.625	0.716	0.695	0.799	0.888
	$\beta_{y,Con}$	1.017	1.058	1.204	1.260	1.307	1.350	1.258	1.171
	$\beta_{y,Disc}$	0.411	0.422	0.475	0.494	0.510	0.532	0.454	0.432
9	$\beta_{x,Con}$	-	-	-	-	-	-	-	-
	$\beta_{x,Disc}$	0.383	0.497	0.619	0.707	0.794	0.762	0.848	0.922
	$\beta_{y,Con}$	-	-	-	-	-	-	-	-
	$\beta_{y,Disc}$	0.433	0.438	0.487	0.502	0.515	0.487	0.454	0.431