DECAY CONSTANT OF PSEUDO-SCALAR MESONS IN BETHE-SALPETER FRAMEWORK

By

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Abstract

We have employed the frame work of the Bethe-Salpeter equation under Covariant Instantaneous Ansatz to calculate the leptonic decay constant of pion and have also reviewed calculation of leptonic decay constants of unequal mass pseudoscalar mesons. In the Dirac structure of BS wave function, the covariants are incorporated in accordance with a recently proposed power counting rule. The decay constants are calculated with incorporation of both Leading-Orders (LO) as well as Next-to-Leading-Order (NLO) covariants. The main contribution of this thesis is an alternative derivation of pion leptonic decay constant which hopefully should raise the contribution of LO covariants in comparision to NLO covariants and hence validating the power counting rue for pion.

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Chapter 1

Introduction

Quantum Chromodynamics QCD is a successful theory of the strong interactions which has been tested by confronting theory and experiment in many different ways in both perturbative and nonperturbative regimes [1].Year after year, QCD continues to succeed in explaining the physics of strong interactions, and no contradictions between this theory and experiment have been found yet [2]. QCD is unique within the standard model of particle physics in that it is a theory which must exist and be internally consistent over all possible energy scales, both in perturbative and nonperturbative domains. Thus QCD is a theory which deserves a most intense scrutiny in a diverse set of circumstances, and such a scrutiny will certainly be rewarded by exciting surprises and a deeper and more satisfying understanding of this profound theory of matter. The growth of the QCD coupling at long distances suggested that free quarks would have infinite energy and so could only appear in color singlet combinations with other quarks and gluons. That is, quarks are confined. It was quickly realized that quark confinement could only emerge from essentially nonperturbative aspects of QCD. It can be also shown that (QCD) is the correct theory to describe strong interactions at high energy, however, strong gauge coupling at low energies destroys the perturbative expansion. As a result, many non-perturbative approaches have been proposed to deal with the long distance properties of QCD such as QCD sum rules, Lattice QCD, dynamical equation based approaches such as Schwinger-Dyson equation (SDE) and Bethe-Salpeter equation (BSE) and potential models. All the above approaches have both advantages as well as shortcomings. Since the task of calculating hadron structures from QCD itself is very difficult, (as can be seen from various Lattice QCD methods), one relies on specific models to gain some understanding of it at low energies (long distances). The dynamical equation based approach such as BSE is a conventional approach in dealing with two-body relativistic bound state problems. From solutions of BSE we can obtain useful information about the inner structure of hadrons. This equation which is firmly rooted in field theory provides a realistic description for analyzing mesons as composite objects. However, one drawback while solving BSE can be traced to the fact that one has to use input kernel which is model dependent. However, calculations have shown that BSE using phenomenological potentials can give satisfactory results. In this thesis we study leptonic decays of unequal mass pseudoscalar mesons such as K, D, DS and B, as well as equal mass meson such as pion which proceed through the coupling of quark-antiquark loop to the axial vector current. A realistic description of pseudoscalar mesons at quark level of compositeness would be an important element in our understanding of hadron dynamics at scales where QCD degrees of freedom are relevant. These studies have become an interesting topic in recent years as more and more data are being accumulated. Such studies offer direct probe of hadronic structure. We employ QCD motivated BSE [3] under Covariant Instantaneous Ansatz (CIA) in this thesis. CIA is a Lorentz-invariant generalization of Instantaneous Ansatz. For a $q\bar{q}$ system, the CIA formulation ensures an exact interconnection between 3D and 4D forms of BSE [3]. The 3D form of BSE serves for making contact with the mass spectrum, whereas the 4D form provides the Hadronquark vertex function for evaluation of various hadronic transition amplitudes through quark loop diagrams. In these studies the main ingredient is the BS wave function. Recent studies [4-6] have revealed that various mesons have many different covariant structures in their wave function whose inclusion is necessary to obtain quantitatively accurate observables. And it was further noticed that all covariants do not contribute equally and only some are relevant for calculation of meson observables. Such a copious Dirac structure of BS wave function was already indicated by Llewllyn Smith [7] much earlier. However, in this thesis, we calculate leptonic decay constants f_P for both pion and unequal mass pseudoscalar mesons [8] using both LO as well as NLO Dirac covariants and try to see the relative importance of various covariants in calculation of f_P. We wish to argue that our power counting scheme [3, 8] provides a practical means of incorporating various Dirac structures from their complete set into the BS wave function. We have employed equal/unequal mass kinematics in this calculation. The thesis is organized as follows: In chapter 2, we discuss gauge theories: QED and QCD. In chapter 3, we discuss the structure and derivation of Bethe-Salpeter integral equation. In chapter 4, we discuss the quark level calculation of pseudoscalar meson decays as well as we discuss in detail an alternative calculation for pion f_P. And finally I present conclusion in chapter 5.

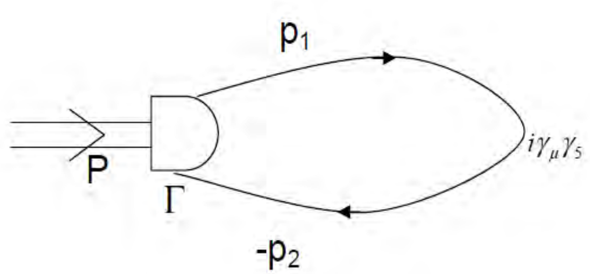

Figure 1.1: Quark loop diagram for leptonic decay of P-meson

Chapter 2

Gauge Theories: QED and QCD

2.1 Maxwell equation

Photon is the most common and important spin-one particle [9]. Emission and absorption of photon by matter is an important phenomenon in many fields of physics. So let's begin by considering the basic laws of classical electrodynamics, which can be explained by a set of four Maxwell equation [10]:

$$\nabla.\vec{E} = \rho, \tag{2.1.1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{2.1.2}$$

$$\nabla.\vec{B} = 0, \tag{2.1.3}$$

$$\nabla \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t}, \tag{2.1.4}$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, ρ is the charge density and $\vec{j}$ is the current density. We have also continuity equation

$$\nabla.\vec{j} + \frac{\partial \rho}{\partial t} = 0 \tag{2.1.5}$$

it implies that the rate of decrease of charge in any arbitrary volume Ω is due precisely and only to the flux of the current out of its surface, that is no net charge can be created nor destroyed in Ω. Since Ω can be made as arbitrary as we please, this means that electric charge must be locally conserved: a process in which charge is created at one point and destroyed at a distant one is not allowed, despite the fact that it conserves charge globally. In classical electrodynamics, and especially in quantum mechanics it is convenient to introduce the vector potentials $\vec{A}$ and scalar potential ϕ in place of $\vec{E}$ and $\vec{B}$. we take the point of view that $\vec{E}$ and $\vec{B}$ are the only quantity of physical significance and can be extracted from the potentials $\vec{A}$ and ϕ i.e,

$$\vec{B} = \nabla \times \vec{A}, \tag{2.1.6}$$

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}, \tag{2.1.7}$$

In the next section we discuss gauge theories.

2.2 Gauge Theory

The notion of local gauge symmetry is turned out to be a cornerstone of modern field theory since so-called gauge theories describe fundamental interactions very successfully. Our present understanding is that all fundamental interactions; strong, electromagnetic, weak and gravitational are described by some form of gauge theory. The origin of gauge invariance [10-12] in classical electrodynamics lies in the fact that the potentials $\vec{A}$ and ϕ are not unique for given physical fields $\vec{E}$ and $\vec{B}$. The transformation which $\vec{A}$ and ϕ may undergo while preserving $\vec{E}$ and $\vec{B}$ (and hence the Maxwell's equations) unchanged are called gauge transformations, and the associated invariance of the Maxwell's equations is termed as gauge invariance. It is observed that $\vec{E}$ and $\vec{B}$ as well as Maxwell's equation are invariant under the following transformation on $\vec{A}$ and ϕ :

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla\chi(x,t), \tag{2.2.1}$$

$$\phi \rightarrow \phi' = \phi - \frac{\partial \chi}{\partial t}(x,t), \tag{2.2.2}$$

where $\chi(x,t)$ is an arbitrary function of x and t. These transformations can be combined into a single compact equation by introducing the 4-vector potential $A_\mu(x) = (\vec{A}, i\phi)$ as,

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \chi(x). \tag{2.2.3}$$

and we have the field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.2.4}$$

It is clear to see that $F_{\mu\nu}$ is obviously invariant under the gauge transformation.That is,

$$F'_{\mu\nu} = F_{\mu\nu}. \tag{2.2.5}$$

This automatically implies that Maxwell's equation

$$\partial_\mu F_{\mu\nu} = j_\nu. \tag{2.2.6}$$

is invariant under the above transformations eqn(2.2.3). In the next section we discuss the gauge theory of quantum electromagnetism (QED).

2.3 Quantum Electrodynamics (QED)

QED, is the theory of photons interacting with the electrons and positrons of the Dirac field [9, 11-15]. The Lagrangian of free Dirac field ψ can be defined as:

$$\pounds(\psi, \bar{\psi}, \partial_\mu \psi) = -\bar{\psi}(\not{\partial}_\mu + m)\psi. \tag{2.3.1}$$

Let's make global gauge transformations on the Dirac's field $\psi(x)$ and the adjoint Dirac's field $\bar{\psi}(x)$

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha}\psi(x). \tag{2.3.2}$$

$$\bar{\psi}(x) \rightarrow \psi'(x) = \bar{\psi}(x)e^{-i\alpha}. \tag{2.3.3}$$

where α is constant in all space time points in the universe. We can easily verify that the Lagrangian is invariant under this global gauge transformation. that is.

$$\pounds' = \pounds. \tag{2.3.4}$$

It's well known that the best understood particle-field interaction, 'Quantum Electrodynamics,' is based on the concept of local invariance [16]. To see that, if fore example a fermion (denoted by ψ)carries electric charge, it's electromagnetic interactions can be derived by demanding that the Lagrangian for this particle be invariant under local symmetry; so let's check whether this Lagrangian is invariant under local gauge transformation on $\psi(x)$ and $\bar{\psi}(x)$ or not.

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x). \tag{2.3.5}$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)e^{-i\alpha(x)}. \tag{2.3.6}$$

where α=$\alpha(x)$ i.e α varies from point to point in space time. Under this transformation, let

$$\pounds_{dirac} \rightarrow \pounds'_{dirac} = -\bar{\psi}'(x)(\not{\partial}_\mu + m)\psi'(x), \tag{2.3.7}$$

$$\pounds'_{dirac} = -\bar{\psi}(x)e^{-i\alpha(x)}(\not{\partial}_\mu + m)e^{i\alpha(x)}\psi(x), \tag{2.3.8}$$

$$\pounds'_{dirac} = -\bar{\psi}(x)(\not{\partial}_\mu + m)\psi(x) - i\bar{\psi}(x)\not{\partial}_\mu\alpha(x)\psi(x). \tag{2.3.9}$$

We thus see that Lagrangian for a free Dirac particle is not invariant under local gauge transformation on $\psi(x)$ due to a symmetry breaking term $-i\bar{\psi}(x)\not{\partial}_\mu\alpha(x)\psi(x)$. So to make the Lagrangian invariant under local gauge transformation we should add a new term in addition to $\pounds_{dirac}$, that is we are forced to introduce a new carrier gauge field $A_\mu(x)$ which transforms under local gauge transformations as:

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\chi(x). \tag{2.3.10}$$

so that,

$$\pounds_{Dirac} = -\bar{\psi}(\not{\partial}_\mu + m)\psi + ie\bar{\psi}\gamma_\mu\psi A_\mu(x). \tag{2.3.11}$$

This new Lagrangian is now invariant under local gauge transformations with the substitutions of $\alpha(x) = e\chi(x)$. Impositions of local gauge invariance on $\pounds_{dirac}$ automatically leads to the introduction of a new carrier gauge field $A_\mu(x)$, which should couple to matter fields ψ and $\bar{\psi}$ through a term $ie\bar{\psi}\gamma_\mu\psi A_\mu$ in the Lagrangian so that the total $\pounds_{modified}$ is invariant under local gauge transformation on both the matter fields and gauge fields. Invariance of the Lagrangian under gauge transformation leads to conservation of charge, i.e to conserve total charge of the universe, the electromagnetic field should couple to matter field in a mathematically self consistent manner. The charge e plays the role of coupling constant between matter field and gauge field, or charge of electron e can tell us the strength how matter fields interact with electromagnetic field. The full QED Lagrangian must include free electromagnetic field Lagrangian i.e $-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}$ corresponding to kinetic part. The complete Lagrangian $\pounds_{QED}$ under combined local gauge transformations on $\psi(x)$ and $A_\mu(x)$ fields should remains the same. This can be easily checked. Thus we write the full QED Lagrangian as:

$$\pounds = -\bar{\psi}(\not{\partial}_\mu + m)\psi + ie\bar{\psi}\gamma_\mu\psi A_\mu(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{2.3.12}$$

which is gauge invariant as shown above. That is the complete Lagrangian have gotten to be invariant under local gauge transformations. On the other hand it can be checked that a simple device for making Lagrangian locally gauge invariant is to make use of the prescription:

$$\partial_\mu \rightarrow D_\mu - ieA_\mu. \tag{2.3.13}$$

which is also called the minimal coupling prescription. Hence the locally invariant QED Lagrangian can written as:

$$\pounds = -\bar{\psi}(\not{D}_\mu + m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{2.3.14}$$

The local gauge transformations on $\psi(x)$ and $\bar{\psi}(x)$ can be written as

$$\psi(x) \rightarrow \psi'(x) = U\psi(x). \tag{2.3.15}$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x)U^{\dagger}. \tag{2.3.16}$$

where, $U = e^{i\alpha(x)}$ and $U^{\dagger} = e^{-i\alpha(x)}$. The set of transformations $\{U = e^{i\alpha(x)}\}$ form a U(1) group, whose elements commute with one another. Hence QED is an abelian U(1) [9] gauge theory. In the next section we discuss SU(3) non-abelian gauge theory QCD.

2.4 Quantum Chromodynamics (QCD)

We will begin by arguing that the most natural candidate for a model of the strong interaction is the non abelian gauge theory with gauge group SU(3), coupled to quarks in the fundamental representations. This theory is termed as Quantum Chromodynamics. In 1963 Gell-Mann and Zweig proposed a model that explained the spectrum of strongly interacting particles in terms of the elementary constituents called quarks. Mesons were expected to be quark anti-quark bound states. To explain electric charge and other quantum numbers of hadrons, Gell-Mann and Zweig needed to assume three species of quarks, namely, up (u), down (d) and strange (s). In addition to the three species of quarks there is also another three species of quarks, charm(c), bottom(b) top(t). These attributes u, d, t, s, c, b are called quark flavors. The quark model has great success in predicting the hadronic states and in explaining the strength of weak and electromagnetic-interaction transitions among different hadrons. In particular the quark model, naturally incorporates the most important symmetry relations among strongly interacting particles. Despite the phenomenological success of the original quark model, it had two serious problems. First free particles, with fractional charge couldn't be found. Second, the spectrum of baryons showed that, the wave function of the three quarks be totally symmetric under the interchange of the quark spin number and flavor quantum numbers contradicting the expectation that quarks, which must have spin half, should obey Fermi-Dirac statistics. Quarks have additional 'unobserved' color quantum number [13]. The quanta of the SU(3) gauge field are called 'gluons' and the theory is termed as quantum-chromodynamics

[9, 13]. The group U(1) of electromagnetism is abelian [9]: group elements commute, which makes group multiplications equivalent to multiplications of real numbers. Here we see that $u(\theta_1)u(\theta_2) = u(\theta_1 + \theta_2)$ i.e linearity of this addition is related directly to the linearity of the field equation for electromagnetism without matter. On the other hand, the non-linearity of non-abelian group causes the corresponding particles to interact with themselves. The Yang-Mills theory is then obtained as a straight forward generalization of electromagnetism, the only difference being that the gauge transformation, and therefore the covariant derivatives, now depends on the generators of some non-abelian group. QCD is based on the extension of the idea in QED, but with the U(1) gauge group replaced by the SU(3) gauge group of phase transformations on the quark color field [9, 11-16]. In the color quark model, each flavor of quark comes in three colors - Red, Blue and Green. Although the various flavors carry different masses, the three colors of a given flavor are all supposed to have the same mass. To construct the Lagrangian which is invariant under this new group of transformation, we must define a covariant derivative. Thus the three Lagrangian for a particular flavor reads.

$$\pounds_{Dirac} = -\bar{\psi}_R(\partial\!\!\!/_\mu + m)\psi_R - \bar{\psi}_B(\partial\!\!\!/_\mu + m)\psi_B - \bar{\psi}_G(\partial\!\!\!/_\mu + m)\psi_G. \tag{2.4.1}$$

Euler-Lagrange equation gives us:

$$\frac{\partial \pounds}{\partial \psi_R} - \partial_\nu\left(\frac{\partial \pounds}{\partial(\partial_\mu \psi_R)}\right) = 0. \tag{2.4.2}$$

$$\frac{\partial \pounds}{\partial \psi_B} - \partial_\nu\left(\frac{\partial \pounds}{\partial(\partial_\mu \psi_B)}\right) = 0. \tag{2.4.3}$$

$$\frac{\partial \pounds}{\partial \psi_G} - \partial_\nu\left(\frac{\partial \pounds}{\partial(\partial_\mu \psi_G)}\right) = 0. \tag{2.4.4}$$

We can simplify the notation by introducing a three component object, i.e 3×1 matrix as:

$$\psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix} \tag{2.4.5}$$

and the adjoint Dirac spinor as:

$$\bar{\psi} = \begin{pmatrix} \bar{\psi}_R & \bar{\psi}_B & \bar{\psi}_G \end{pmatrix} \tag{2.4.6}$$

The Dirac Lagrangian can compactly written as:

$$\pounds_{Dirac} = -\bar{\psi}(\not{\partial}_\mu + m)\psi. \tag{2.4.7}$$

where

$$m = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix} \tag{2.4.8}$$

This looks like a single particle Dirac Lagrangian, but ψ is now a 3 x 1 matrix. This Lagrangian should be governed by SU(3) symmetry. That is to say, it is invariant under transformation of the form:

$$\psi \rightarrow \psi' = U\psi. \tag{2.4.9}$$

where U is any unitary 3 x 3 matrix, ie $\mid U \mid = \mathbf{1}$ and $U^\dagger U = UU^\dagger = \mathbf{1}$. Any unitary matrix can be written in the form:

$$U = e^{iH} \tag{2.4.10}$$

where H is 3 x 3 Hermitian matrix, $H^\dagger = H$. The hermitian 3 x 3 matrix can be expressed as superposition of $\mathbf{1}$ and $\lambda^1, \lambda^2, \lambda^3, ...\lambda^8$ as shown below.

$$H = \alpha^a 1 + \lambda^a \alpha^a(x) \tag{2.4.11}$$

where a=1,2...8 and $\lambda^1, \lambda^2....\lambda^8$ are Gell-Man matrices and are the generators of SU(3) group. Thus the unitary 3 x 3 matrix can be expressed as

$$U = e^{i\alpha^a} e^{i\lambda^a \alpha^a}. \tag{2.4.12}$$

Let's first consider an SU(3) global gauge transformation on $\psi(x)$ and $\bar{\psi}(x)$

$$\psi \rightarrow \psi' = e^{i\lambda^a \alpha^a}\psi. \tag{2.4.13}$$

Where α^a is constant and is the same at all space time points. Thus

$$\pounds'_{Dirac} = -\bar{\psi}e^{-i\lambda^a\alpha^a}(\not{\partial}_\mu + m)e^{i\lambda^a\alpha^a}\psi = -\bar{\psi}(\not{\partial}_\mu + m)\psi. \tag{2.4.14}$$

which implies that

$$\pounds'_{Dirac} = \pounds_{Dirac}. \tag{2.4.15}$$

We can see that $\pounds_{Dirac}$ is invariant under global SU(3) group transformation on ψ. Our next objective is to check invariance of $\pounds_{Dirac}$ under SU(3) local gauge transformations by considering the following local SU(3) transformation.

$$\psi(x) \rightarrow \psi'(x) = e^{i\lambda^a\alpha^a(x)}\psi(x). \tag{2.4.16}$$

In this case $\alpha^a(x)$, the eight independent phase angles, vary from one space time point to another.

$$\pounds \rightarrow \pounds'(x) = -\bar{\psi}(x)U^\dagger(\alpha)(\not{\partial}_\mu + m)U(\alpha)\psi(x),$$

which is equal to

$$-\bar{\psi}(x)(\not{\partial}_\mu + m)\psi(x) - \bar{\psi}U^\dagger(\alpha)(\not{\partial}_\mu U(\alpha))\psi(x). \tag{2.4.17}$$

Where $U = e^{i\lambda^a\alpha^a}$ and eqn (2.4.17) tell us $\pounds$ is not invariant under such a transformation, by virtue of a'gauge breaking'term $-\bar{\psi}U^\dagger(\not{\partial}_\mu U)\psi$. Now let us replace the derivative in $\pounds$ by a 'covariant derivative'

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig\lambda^a A^a(x). \tag{2.4.18}$$

where g is a coupling constant, extracted from α, then above equation becomes:

$$\pounds'_{Dirac} = -\bar{\psi}(\not{\partial}_\mu + m)\psi - ig\bar{\psi}\gamma_\mu\psi\partial_\mu\lambda(x). \tag{2.4.19}$$

The minimal prescription for transformation of $D_\mu\psi$ is

$$D_\mu\psi \rightarrow D'_\mu\psi' = U(\alpha)D_\mu\psi. \tag{2.4.20}$$

with this,

$$(\partial_\mu - ig\lambda^a A'^a_\mu)U\psi(x) = U(\partial_\mu - ig\lambda^a A^a_\mu)\psi(x), \tag{2.4.21}$$

$$(\partial_\mu U - ig\lambda \cdot A'U)\psi(x) = -igU\lambda \cdot A\psi(x). \tag{2.4.22}$$

$$\lambda \cdot A'(x) = U\lambda \cdot AU^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.4.23}$$

Let $\lambda \cdot A = A_\mu$ where A_μ is a 3 x 3 matrix

$$A'_\mu(x) = UA_\mu(x)U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1} \tag{2.4.24}$$

This is how the gauge field transforms in $SU(3)$ transformation group. U and U^{-1} in the first term of equation (2.4.24) can not brought together, because they do not commute with $A_\mu(x)$. If we add the term ig$\bar{\psi}\lambda^a A^a_\mu \psi(x)$ to the initial Lagrangian, then the modified Lagrangian will have the form:

$$\pounds_{Dirac} = -\bar{\psi}(\partial\!\!\!/_\mu + m)\psi + ig\bar{\psi}\gamma_\mu \lambda^a A^a_\mu \psi. \tag{2.4.25}$$

$\pounds$ is now invariant under local SU(3) gauge transformations at the expense of introduction of A_μ. In this gauge transformation group $F_{\mu\nu}$, the free gluon field tensor, must be modified as,

$$F_{\mu\nu} \equiv \frac{i}{g}[D_\mu, D_\nu]. \tag{2.4.26}$$

where $D_\mu = \partial_\mu - igA_\mu(x)$ this implies that

$$F_{\mu\nu} = \frac{i}{g}(\partial_\mu - igA_\mu)(\partial_\nu - igA_\nu) - (\partial_\nu - igA_\nu)(\partial_\mu - igA_\mu) \tag{2.4.27}$$

$$F_{\mu\nu} = \frac{i}{g}(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu) - ig(A_\mu\partial_\nu - A_\nu\partial_\mu) - ig(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2(A_\mu A_\nu - A_\nu A_\mu). \tag{2.4.28}$$

which is equal to,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu(x), A_\nu(x)]. \tag{2.4.29}$$

To complete our task it is important to say something about the free gluon Lagrangian

$$\pounds_{gluon} = -\frac{1}{4}Tr(F_{\mu\nu}F_{\mu\nu}). \tag{2.4.30}$$

Thus we write the full QCD Lagrangian as

$$\pounds_{QCD} = -\bar{\psi}(\partial\!\!\!/_{\mu} + m)\psi + ig\bar{\psi}\gamma_{\mu}\lambda^{a}A^{a}_{\mu}\psi - \frac{1}{4}Tr(F^{a}_{\mu\nu}F^{a}_{\mu\nu}). \tag{2.4.31}$$

$\pounds_{QCD}$ is invariant under local gauge transformation provided that $(-\frac{1}{4}F^{a}_{\mu\nu}F^{a}_{\mu\nu})$ is invariant; in order to verify the invariance of the gluon Lagrangian, let us consider local SU(3) transformation: $F^{a}_{\mu\nu} \rightarrow F'^{a}_{\mu\nu}$

$$F'_{\mu\nu} = \partial_{\mu}A'_{\nu} - \partial_{\nu}A'_{\mu} - ig[A'_{\mu}, A'_{\nu}]. \tag{2.4.32}$$

$$\begin{aligned} F'_{\mu\nu} &= \partial_{\mu}(UA_{\nu}U^{-1} - \frac{i}{g}(\partial_{\nu}U)U^{-1}) - \partial_{\nu}(UA_{\mu}U^{-1} - \frac{i}{g}(\partial_{\mu}U)U^{-1}) \\ &-ig[(UA_{\mu}U^{-1} - \frac{i}{g}(\partial_{\mu}U)U^{-1}), (UA_{\nu}U^{-1} - \frac{i}{g}(\partial_{\nu}U)U^{-1})]. \end{aligned} \tag{2.4.33}$$

Using the identity $\partial_{\mu}U = U(\partial_{\mu}U^{-1})U$ in the above equation, we get,

$$F'_{\mu\nu} = UF_{\mu\nu}U^{-1}. \tag{2.4.34}$$

$$\pounds'_{gluon} = -\frac{1}{4}Tr(F'_{\mu\nu}F'_{\mu\nu}) = -\frac{1}{4}(UF_{\mu\nu}U^{-1}UF_{\mu\nu}U^{-1}) \tag{2.4.35}$$

which automatically lead us to conclude that

$$\pounds'_{gluon} = -\frac{1}{4}(F_{\mu\nu}F_{\mu\nu}) = \pounds_{gluon}. \tag{2.4.36}$$

Thus

$$\pounds_{QCD} = -\bar{\psi}(\partial\!\!\!/_{\mu} + m)\psi + ig\bar{\psi}\gamma_{\mu}\psi A_{\mu} - \frac{1}{4}F^{a}_{\mu\nu}F^{a}_{\mu\nu}, \tag{2.4.37}$$

is invariant under local SU(3) gauge transformation. We now derive Feynman rules for QCD in the next section.

2.5 The Feynman Rules for Non Abelian SU(3) Gauge Theory

Let's begin by considering non-abelian gauge theory without any spinor fields; i.e let's concentrate on free gluon field alone.

$$\pounds_{gluon} = -\frac{1}{4}Tr(F_{\mu\nu}F_{\mu\nu}), \tag{2.5.1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{2.5.2}$$

and

$$A_\mu = \lambda^a A^a_\mu \tag{2.5.3}$$

substitution reveals that

$$F_{\mu\nu} = (\partial_\mu A^a_\nu)\lambda^a - (\partial_\nu A^a_\mu)\lambda^a - igA^b_\mu A^c_\nu[\lambda^b, \lambda^c], \tag{2.5.4}$$

$$F_{\mu\nu} = (\partial_\mu A^a_\nu)\lambda^a - (\partial_\nu A^a_\mu)\lambda^a + gf_{abc}\Lambda^a A^b_\mu A^c_\nu. \tag{2.5.5}$$

where f_{abc} are structure constants of SU(3) defined as, $f_{abc} = 1$ for even permutation of 'abc' and $f_{abc} = -1$ for odd permutation of 'abc'. thus we get,

$$\pounds_{gluon} = -\frac{1}{4}Tr[(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)] - \frac{1}{4}Tr[gf_{abc}A^b_\mu A^c_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)]$$
$$-\frac{1}{4}Tr[gf_{ade}A^d_\mu A^e_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g^2 f_{abc}f_{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu] \tag{2.5.6}$$

From this we can see that, with out matter fields, the gauge has none trivial interactions and is not a free theory like QED. In the equation (2.5.6) the first term is for free A_μ field propagation, the second and third terms are three gluon vertex ('g' is linear in this gluon-gluon interaction), and the last term is for a four-gluon vertex. The Dirac fields constitute eight color currents $J^{\mu a} = ig\bar{\psi}\gamma_\mu\lambda^a$ which act as sources for the color fields(A_μ) in the same way that electric currents act as source for the electromagnetic field. Next, we see interaction among three gauge fields in the second term of eqn (2.5.6), the Lagrangian for the interaction of 3-gauge fields is given by:

$$\pounds_{3-g} = -\frac{1}{2} g f_{abc} A^b_\mu A^c_\nu (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu). \tag{2.5.7}$$

Using all possibilities of a, b, c we can rewrite $\pounds_{3-g}$ as

$$\pounds_{3-g} = -g f_{abc}(\partial_\mu A^a_\nu) A^b_\mu A^c_\nu - g f_{acb}(\partial_\mu A^a_\nu) A^c_\mu A^b_\nu - g f_{bca}(\partial_\mu A^b_\nu) A^c_\mu A^a_\nu$$

$$-g f_{bac}(\partial_\mu A^b_\nu) A^a_\mu A^c_\nu - g f_{cab}(\partial_\mu A^c_\nu) A^b_\mu A^a_\nu - g f_{cab}(\partial_\mu A^c_\nu) A^a_\mu A^b_\nu. \tag{2.5.8}$$

Using the fourier transform of A^a_ν, A^b_ν, A^c_ν from eqn (2.5.6) we again write $\pounds_{3-g}$ as

$$\pounds_{3-g} = -ig f_{abc}[(k_3 - k_1) A^b_\nu A^a_\nu A^c_\nu + (k_1 - k_2) A^c_\nu A^a_\nu A^b_\nu + (k_2 - k_3) A^a_\nu A^b_\nu A^c_\nu] \tag{2.5.9}$$

The 3-gluon vertex function in covariant form is then given by:

$$-ig f_{abc}[(k_\gamma - k_\alpha)_\beta \delta_{\alpha\gamma} + (k_\alpha - k_\beta)_\gamma \delta_{\alpha\beta} + (k_\beta - k_\gamma)_\alpha \delta_{\beta\gamma}]. \tag{2.5.10}$$

Similarly we see the interaction among 4-gluon fields from the fourth term of eqn (2.5.6)

$$\pounds_{4-g} = g^2 f_{abc} f_{ade} A^b_\mu A^c_\nu A^d_\mu A^e_\nu. \tag{2.5.11}$$

we can rewrite $\pounds_{4-g}$ by taking all possible permutation indices b, c, d and e as,

$$\pounds_{4-g} = -\frac{1}{4} g^2 [f_{nab} f_{ncd}((A^a A^c)(A^b A^d) - (A^a A^d)(A^b A^c)) +$$

$$f_{nac} f_{nbd}((A^a A^b)(A^c A^d) - (A^a A^d)(A^b A^c)) + f_{nad} f_{nbc}((A^a A^b)(A^d A^c) - (A^d A^b)(A^a A^c)] \tag{2.5.12}$$

Finally the 4-gluon vertex function in covariant form is:

$$-\frac{1}{4} g^2 [f_{nab} f_{ncd}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) + f_{nac} f_{nbd}(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}) + f_{nad} f_{nbc}(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\nu\sigma}\delta_{\rho\mu})]. \tag{2.5.13}$$

We can have the graph for $3 - gluon$ and $4 - gluon$ interaction vertex as shown below.

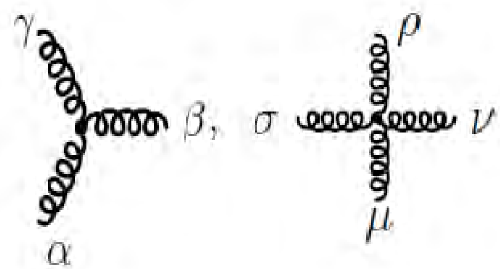

Figure 2.1: 3-gluon interaction vertex and 4-gluon interaction vertex

Let's also trace out the gluon propagator from free gluon Lagrangian and making use of Euler-Lagrange equation of motion, that is given by

$$\frac{\partial \pounds_{fgf}}{\partial A_\mu^a} - \frac{\partial \pounds_{fgf}}{\partial(\partial_\nu A_\mu^a)} = 0. \tag{2.5.14}$$

where the Lagrangian is defined as,

$$\pounds_{freegluonfield} = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a). \tag{2.5.15}$$

So the free gluon propagator, after substituting eqn(2.5.15) in eqn(2.5.14) will be,

$$\Box^2 A_\nu^a \delta_{\mu\nu} - \partial_\mu \partial_\nu A_\nu^a = 0. \tag{2.5.16}$$

Imposing Lorentz gauge condition ($\partial_\nu A_\nu^a = 0$) we have

$$\Box^2 A_\nu^a \delta_{\mu\nu} = 0. \tag{2.5.17}$$

Now we take the fourier transform of A_ν^a:

$$A_\nu^a(x) = \frac{1}{(2\pi)^4} \int d^4 k e^{ikx} A_\nu^a(k). \tag{2.5.18}$$

Using eqn(2.5.18), eqn (2.5.17) becomes

$$-k^2 A^a_\nu \delta_{\mu\nu} = 0. \tag{2.5.19}$$

Now taking the inverse of the above equation, leaving the field terms, the form of the propagator for free gluon field will be:

$$D_{\mu\nu} = -\frac{\delta_{\mu\nu}}{k^2}. \tag{2.5.20}$$

Chapter 3

Derivation of Bethe-Salpeter Equation

3.1 One Particle Propagator in an External Field

We recall the Schrodinger's equation as:

$$(i\frac{\partial}{\partial t} - H)\psi(x) = 0. \tag{3.1.1}$$

The form of the full propagator for forward propagation in time(i.e t > t') satisfies the equation,

$$(i\frac{\partial}{\partial t} - H)K(\vec{x}, t; \vec{x'}, t') = i\delta^3(\vec{x} - \vec{x'})\delta(t - t'). \tag{3.1.2}$$

The above equation can be termed as propagator equation for Schrodinger particles which moves only forward in time. If replacement is conducted between the Schrodinger hamiltonian and the Dirac hamiltonian, then eqn(3.1.1) will be free Dirac's equation i.e, it will have a form of Dirac equation in the absence of electromagnetic field:

$$(i\frac{\partial}{\partial t} - \vec{\alpha}.\vec{p} - \beta m)\psi(x) = 0. \tag{3.1.3}$$

where α and β are the Dirac 4 x 4 matrices.

And in the presence of electromagnetic field, $A_\mu(x)$, with the well known substitution

$$p_{\mu \rightarrow} p_\mu - eA_\mu,$$

$$\nabla \to \nabla - ie\vec{A}, \tag{3.1.4}$$

and

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + ie\phi.$$

we get,

$$(i\frac{\partial}{\partial t} - e\phi + \vec{\alpha}.\vec{\nabla} - e\vec{A}.\vec{\alpha} - \beta m)\psi(x) = 0. \tag{3.1.5}$$

Let us consider the simpler unit source problem,

$$(i\frac{\partial}{\partial t} - e\phi + i\vec{\alpha}.\vec{\nabla} - e\vec{A}.\vec{\alpha} - \beta m)\psi(x) = i\delta^4(x - x'). \tag{3.1.6}$$

To make it covariant let me multiply by$-\beta$ from the left on both sides of the above equation. A relativistic Dirac particle of charge e and mass m which is placed in an external electromagnetic field A_μ, will have the form of the equation,

$$(\not{\partial}_\mu - ie\not{A}_\mu + m)K(x, x') = -i\delta^4(x - x'). \tag{3.1.7}$$

where K(x,x'), is the full Dirac propagator.

The above equation is termed as full propagator equation, where as

$$(\not{\partial}_\mu + m)K_0(x, x') = -i\delta^4(x - x') \tag{3.1.8}$$

is termed as propagator for free Dirac particle.

Eqn (3.1.7) can be rewritten as

$$(\not{\partial}_\mu + m)K(x, x') = -i\delta^4(x - x') + ie\not{A}_\mu(x)K(x, x'). \tag{3.1.9}$$

which equals to

$$(\not{\partial}_\mu + m)K(x, x') = -i\int d^4x''\delta^4(x - x'')\delta^4(x'' - x') + ie\int d^4x''\not{A}_\mu(x'')K(x'', x')\delta^4(x'' - x). \tag{3.1.10}$$

But we have also $(\partial\!\!\!/_\mu + m)K_0(x, x'') = -i\delta^4(x - x'')$

Hence, the above equation can be written as:

$$(\partial\!\!\!/_\mu + m)K(x, x') = \int d^4x''(\partial\!\!\!/_\mu + m)K_0(x, x'')\delta^4(x'' - x')$$

$$-e \int d^4x'' A_\mu(x'')K(x'', x')\gamma_\mu(\partial\!\!\!/_\mu + m)K_0(x, x'') \tag{3.1.11}$$

In more compact form we can write,

$$K(x, x') = K_0(x, x') - e \int d^4x'' K_0(x, x'')A\!\!\!/\mu(x'')K(x'', x'). \tag{3.1.12}$$

3.2 Two Particle Bound State Propagators

First let me define the two particle free propagator for simultaneous propagation of two free particles. We now generalize to two-particle Dirac propagator with particles in mutual interaction. The free propagator for simultaneous propagation of two particles from $(3, 4) \rightarrow (1, 2)$ is

$$K_{0a}(1, 3)K_{0b}(2, 4) = K_0(12; 34). \tag{3.2.1}$$

Lets consider the two particles a and b interact through the exchange of a single photon; let the interaction for simplicity be Coulomb interaction.

$$\frac{e^2}{r_{ab}} \tag{3.2.2}$$

However this Coulomb interaction is non-covariant. To write the mutual interaction between these two particles in a covariant form let's take the fourier transform of this equation in momentum space,

$$\frac{e^2}{r_{ab}} \rightarrow \frac{e^2}{(p_1 - p_2)^2}\gamma_4^a\gamma_4^b \tag{3.2.3}$$

we can write this equation in covariant form [17] as:

$$\frac{e^2}{r_{ab}} \Rightarrow -e^2\gamma_\mu^a[\frac{-1}{q^2}]\gamma_\mu^b. \tag{3.2.4}$$

where $q = p_1 - p_2$ is the momentum transfer between the two electrons.

$$\frac{e^2}{r_{ab}} \Rightarrow -e^2\gamma_\mu^a(D_F(x-x'))\gamma_\mu^b = I(5,6). \tag{3.2.5}$$

where the RHS in the above equation is interaction potential and $D_F(x-x')$ is the photon propagator in momentum space and is expressed as:

$$D_F(x-x') = \frac{1}{(2\pi)^4}\int d^4q[\frac{-1}{q^2}]e^{iq(x-x')}. \tag{3.2.6}$$

To generalize a single particle propagator to two particles propagator, consider simultaneous propagation of two particles for two charged particles propagating from space-time points $(1,2)$ to $(3,4)$ and interacting through a photon exchange connecting them at space-time vertices 5 and 6 , we can write the first order correction to the propagator

$$K_1(12;34) = \int d^45\int d^46K_0(12;56)I(5,6)K_0(56;34). \tag{3.2.7}$$

This is first order correction to two particle propagator due to simultaneous interaction of both Dirac particles through exchange of a single photon. Similarly we can write

$$K_2(12;34) = \int d^45\int d^46\int d^47\int d^48K_0(12;56)I(5,6)K_0(56;78)I(7,8)K_0(78;34). \tag{3.2.8}$$

which is second order correction to two particle propagator. If we iterate one quantum exchange between two particles , we can write the two particle propagator as an infinite series ;

$$K(12;34) = K_0(12;34) + \int d^45\int d^46K_0(12;56)I(5,6)K_0(56;34)+$$

$$\int d^45\int d^46\int d^47\int d^48K_0(12;56)I(5,6)K_0(56;78)I(7,8)K_0(78;34) + ... \tag{3.2.9}$$

which can in turn be written as

$$K(12;34) = K_0(12;34) + \int d^45\int d^46K_0(12;56)I(5,6)K(56;34). \tag{3.2.10}$$

where $K(56,34)$ is the full two-particle propagator for simultaneous propagation of two-particles from $(3,4)$ to $(5,6)$, thus we arrive exactly to an integral equation written symbolically as

$$K = K_0 + K_0 I K. \tag{3.2.11}$$

which is complete Green's function for two particles as a sequential action of Møller interaction in ladder approximation. In next section we discuss two particle wave function.

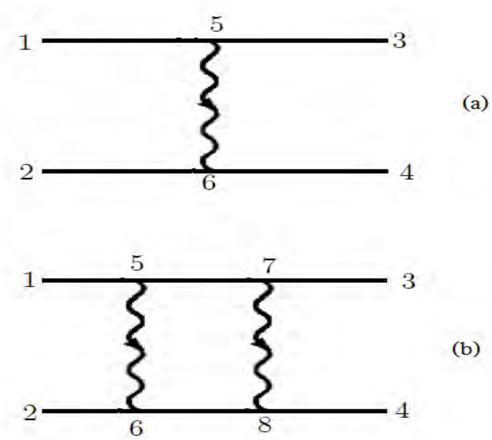

Figure 3.1: Interaction of two particles through exchange of (a) single photon (b) two photons.

3.3 Two Particle Wave Function

To begin with, let's start from the wave function for a single particle case:

$$\psi(\vec{x}, t) = \int d^3\vec{x'} K(\vec{x}, t; \vec{x'}, t)\psi(\vec{x'}, t'). \tag{3.3.1}$$

In more compact form the above single particle wave function can be expressed as:

$$\psi(1) = \int d^3 2 K(1, 2)\gamma_4\psi_2. \tag{3.3.2}$$

where γ_4 is introduced to make the equation covariant. Now let's generalize the above single particle wave function to two particle wave function

$$\psi(1, 2) = \int d^4 3 \int d^4 4 K(12; 34)\beta^a\beta^b\psi(3, 4). \tag{3.3.3}$$

with the above substitution one can have:

$$\psi(1, 2) = \int d^4 3 \int d^4 4[K_0(12; 34) + \int d^4 5 \int d^4 6 k_0(12; 56)I(56)K(56; 34)]\beta^a\beta^b\psi(3, 4). \tag{3.3.4}$$

This implies that

$$\psi(1, 2) = \psi_0(1, 2) + \int d^4 3 \int d^4 4 \int d^4 5 \int d^4 6 \beta^a\beta^b k_0(12; 56)I(56)K(56; 34)\psi(3, 4). \tag{3.3.5}$$

But $\psi(5, 6) = \int d^4 3 \int d^4 4 \beta^a K(56; 34)\beta^b\psi(3, 4)$
with this substitution we will get

$$\psi(1, 2) = \psi_0(1, 2) + \int d^4 5 \int d^4 6 k_0(12; 56)I(56)\psi(5, 6). \tag{3.3.6}$$

3.4 Derivation of Bethe-Salpeter Equation

BSE is a relativistic equation for bound-state problems. The Bethe-Salpeter equation is a direct application of the Feynman rules, and a resummation of the infinite set of diagrams by using integral equation. If both the irreducible kernel, and the n-point Green functions are understood as the full expressions, then the Bethe-Salpeter result for any n-point Green function is the exact result of field theory, as far as the full expression for a given Green function is considered, and as far as the existence of a fixed value of the mass of bound system is taken into account. To obtain BSE we use

$$\psi(1,2) = \psi_0(1,2) + \int d^4 5 \int d^4 6 K_0(12;56) I(5,6)\psi(5,6). \tag{3.4.1}$$

Let's operate this equation both side by Dirac operator

$$(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)$$

Thus,we obtain

$$(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\psi(1,2) = (\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\psi_0(1,2)$$

$$+(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\int d^4 5 \int d^4 6 K_0(12;56) I(5,6)\psi(5,6) \tag{3.4.2}$$

If two particles after interaction form a bound state, then the free particle wave function,

$$\psi_0(1,2) = 0 \tag{3.4.3}$$

Equation(3.4.2) is then reduced to

$$(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\psi(1,2) = (\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\int d^4 5 \int d^4 6 K_o(12;56) I(5,6)\psi(5,6), \tag{3.4.4}$$

which gives

$$(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\psi(1,2) = \int d^4 5 \int d^4 6 (-i\delta^4(1,5))(-i\delta^4(2,6)) I(5,6)\psi(5,6). \tag{3.4.5}$$

Thus we get two particle mutual interaction equation written down in covariant form as:

$$(\not{\partial}^a_\mu + m_a)(\not{\partial}^b_\mu + m_b)\psi(1,2) = -I(1,2)\psi(1,2). \tag{3.4.6}$$

where $I(1,2)$ is interaction potential. Let's consider, to make life simple, cases where one can consider their center of mass coordinates and the relative coordinates of the two particles as,

$$X = \frac{x_1 + x_2}{2} \qquad and \qquad x = x_1 - x_2 \tag{3.4.7}$$

respectively

Using equation (3.4.7) we can write $\partial_{\mu 1}$ and $\partial_{\mu 2}$ in the following way

$$\partial^a_\mu = \frac{1}{2}\partial_{X\mu} + \partial_{x\mu} \qquad and \qquad \partial^b_\mu = \frac{1}{2}\partial_{X\mu} - \partial_{x\mu}. \tag{3.4.8}$$

After substituting eqn(3.4.8) into eqn(3.4.6) we obtain

$$(\frac{1}{2}\partial_{X\mu} + \partial_{x\mu} + m_a)(\frac{1}{2}\partial_{X\mu} - \partial_{x\mu} + m_b)\psi(X,x) = -I(x)\psi(X,x). \tag{3.4.9}$$

where $\psi(X,x) = \psi_1(X)\psi_2(x)$

Now we take the fourier transform of $\psi(X)$, $I(x)$ and $\psi(x)$ as follows :

$$\begin{aligned} \psi(X) &= \frac{1}{(2\pi)^4}\int d^4P e^{iPX}\psi(P), \\ \psi(x) &= \frac{1}{(2\pi)^4}\int d^4q e^{iqx}\psi(q), \\ I(x) &= \frac{1}{(2\pi)^4}\int d^4k e^{ikx} I(k). \end{aligned} \tag{3.4.10}$$

Inserting eqn(3.4.10) into eqn (3.4.9) we can get:

$$\begin{aligned} \int d^4q(\frac{1}{2}i\not{P}_\mu + i\not{q} + m_a)(\frac{1}{2}i\not{P}_\mu - i\not{q} + m_b)\psi(P)\psi(q)e^{iqx} = \\ -\frac{1}{(2\pi)^4}\int d^4q^{'}\int d^4k e^{i(k+\hat{q}')} I(k)\psi(P)\psi(\hat{q}') \end{aligned} \tag{3.4.11}$$

Now to simplify the above equation let's take $q = k + \hat{q}'$ and $\hat{q}' = constant$ so we see in the right hand side of the above equation $d^4k = d^4q$.

Then eqn (3.4.11) becomes

$$(\frac{1}{2}i\not{P}_\mu + i\not{q} + m_a)(\frac{1}{2}i\not{P}_\mu - i\not{q} + m_b)\psi(P)\psi(q)e^{iqx}$$

$$= -\frac{1}{(2\pi)^4}\int d^4q' I(q,q')\psi(P)\psi(q)e^{iqx} \tag{3.4.12}$$

For a meson which is the bound state of quark and anti-quark [18], the total momentum of the hadron is $P = p_1 + p_2$ and the relative momentum of $q\bar{q}$ is given by $q = \frac{1}{2}(p_1 - p_2)$, we can take the momenta of the constituents as; $p_1 = \frac{1}{2}P + q$ and $p_2 = \frac{1}{2}P - q$.

And $\psi(P)\psi(q)$ also written as $\psi(P,q)$.

Then we rewrite eqn (3.4.12) as:

$$(m_1 + i\not{p}_1)(m_2 - i\not{p}_2)\psi(P,q) = -\frac{1}{(2\pi)^4}\int d^4q^{'} K(q,q^{'})\psi(P,q). \tag{3.4.13}$$

where $m_a = m_1, m_b = m_2$. Lets define

$$\psi(P,q) = (m_1 + i\not{p}_1)(m_2 - i\not{p}_2)\Phi(P,q). \tag{3.4.14}$$

where $\Phi(P,q)$ is the $4D$ wave function.

Substituting eqn (3.4.14) into eqn (3.4.13) we obtain after simplification

$$(m_1^2 + p_1^2)(m_2^2 + p_2^2)\Phi(P,q) = \frac{i}{(2\pi)^4}\int d^4q^{'}\bar{K}(q,q^{'})\Phi(P,q) \tag{3.4.15}$$

where $\bar{K}(q,q^{'}) = K(q,q')(m_1 + i\not{p}_1)(m_2 - i\not{p}_2)$ is the $q\bar{q}$ scattering kernel. Now we also define the inverse propagators for two constituent quarks $\triangle_1 = (m_1^2 + p_1^2)$ and $\triangle_2 = (m_2^2 + p_2^2)$.

The full Bethe-Salpeter Equation for spinless quarks is given by

$$\triangle_1\triangle_2\Phi(P,q) = \frac{i}{(2\pi)^4}\int d^4q^{'}\bar{K}(q,q^{'})\Phi(P,q). \tag{3.4.16}$$

In the next section we see generalized Hadron-quark vertex function for both equal and unequal mass pseudoscalar mesons.

3.5 Structure of BS wave function and Hadron-quark vertex function

We first outline the BSE framework under Covariant Instantaneous Ansatz (CIA) that we have employed for the case of scalar quarks for simplicity. For a $q\bar{q}$ system with an effective kernel K and 4D wave function $\Phi(P,q)$, the 4D BSE takes the form [1, 8, 19, 20].

$$i(2\pi)^4\Delta_1\Delta_2\Phi(P,q) = \int d^4q' K(q,q')\Phi(P,q') \tag{3.5.1}$$

where $\Delta_{1,2}$ are the inverse propagators of two scalar quarks,

$$\Delta_{1,2} = m_{1,2}^2 + p_{1,2}^2 \tag{3.5.2}$$

Here $m_{1,2}$ are (effective) constituent masses of quarks. The 4-momenta of the quark and antiquark, $p_{1,2}$, are related to the internal 4-momentum q_μ and total momentumP_μ of hadron of mass M as

$$p_{1,2\mu} = \hat{m}_{1,2}P_\mu \pm q_\mu \tag{3.5.3}$$

where $\hat{m}_{1,2} = \frac{[M^2 \pm (m1^2 - m2^2)]}{2M^2}$ are the Wightman-Garding (WG) definitions of masses of individual quarks. Now it is convenient to express the internal momentum of the hadron q as the sum of two parts, $\hat{q}_\mu$ and σP_μ where $\hat{q}_\mu$ is orthogonal to total hadron momentum P, where as σP_μ (where the longitudinal component,$\sigma = q.P/P^2$) is parallel to P. We now use an Ansatz on the BS kernel K in Eqn (3.5.1) which is assumed to depend on the 3D variables $\hat{q}_\mu, \hat{q'}_\mu$ [19, 21] i.e

$$K(q,q') = K(\hat{q},\hat{q}') \tag{3.5.4}$$

where

$$\hat{q}_\mu = q_\mu - \frac{q.P}{P^2}P_\mu. \tag{3.5.5}$$

is observed to be orthogonal to the total 4-momentum P (i.e., $\hat{q}.P = 0$), irrespective of whether the individual quarks are on-shell or off-shell (a similar form of the BS kernel was also suggested earlier in [22]. Hence, the longitudinal component of q_μ,

$$M\sigma = M\frac{q.P}{P^2}. \tag{3.5.6}$$

does not appear in the form $K(\hat{q},\hat{q}')$ of the kernel. For reducing Eqn (3.5.1) to the 3D form, we define a 3D wave function $\phi(\hat{q})$ as

$$\phi(\hat{q}) = \int M d\sigma \Phi(P,q). \tag{3.5.7}$$

Substituting Eqn(3.5.7) in Eqn (3.5.1), with definition of kernel in Eqn (3.5.4), we get a covariant version of Salpeter equation,

$$(2\pi)^3 D(\hat{q})\phi(\hat{q}) = \int d^3\hat{q}' K(\hat{q},\hat{q}')\phi(\hat{q}'). \tag{3.5.8}$$

where$D(\hat{q})$is the 3D denominator function defined by

$$\frac{1}{D(\hat{q})} = \frac{1}{2\pi i}\int \frac{M d\sigma}{\Delta_1\Delta_2}. \tag{3.5.9}$$

whose value can easily be worked out by contour integration by noting positions of poles in the complex $\sigma-$plane [23] as,

$$D(\hat{q}) = \frac{(\omega_1+\omega_2)^2 - M^2}{\frac{\hat{m_1}}{\omega_1}+\frac{\hat{m_2}}{\omega_2}}. \tag{3.5.10}$$

Where

$$\omega_{1,2}^2 = m_{1,2}^2 + \hat{q}^2.$$

We can see that RHS of Eqn (3.5.8) is identical to RHS of Eqn (3.5.1) by virtue of Equations (3.5.4) and (3.5.7). We thus have an exact interconnection between 3D wave function $\phi(\hat{q})$ and 4D wave function $\Phi(P,q)$

$$\Delta_1\Delta_2\Phi(P,q) = \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \equiv \Gamma(\hat{q}). \tag{3.5.11}$$

We thus also get the $Hq\bar{q}$ vertex function$\Gamma(\hat{q})$ under CIA for case of scalar quarks. Further in the process, an exact interconnection between 3D and 4D BSE [21] is thus brought out where the 3D form serves for making contact with the mass spectrum of hadrons, whereas the 4D form provides the vertex $Hq\bar{q}$ function $\Gamma(\hat{q})$ which satisfies a 4D BSE with a natural off-shell extension over the entire 4D space (due to the positive definiteness of the quantity $\hat{q}^2 = q^2 - \frac{(q.P)^2}{P^2}$ throughout the entire 4D space) and thus provides a fully Lorentz-invariant basis for evaluation of various transition amplitudes through various quark loop diagrams. Due to these properties the framework [1] can be profitably employed not only for mass spectral predictions but also for transition amplitudes all the way from low energies to high energies [21]. To obtain the form of Hadron-quark vertex function for the case of fermionic quarks constituting a particular meson, we first replace the scalar propagators Δ_i^{-1} in Eq. (3.5.2) by the proper fermionic propagators S_F . Then for incorporation of relevant Dirac structures in vertex function $\Gamma(\hat{q})$ we make use of the power counting rule developed in [3, 8] for incorporating various Dirac covariants in the structure of $Hq\bar{q}$ vertex function for a particular pseudoscalar meson, order-by-order in powers of inverse of meson mass [3, 8]. As far as a pseudoscalar meson is concerned, its hadron-quark vertex function which has a certain dimensionality of mass can be expressed as a linear combination of four Dirac covariants [7], each multiplying a Lorentz scalar amplitude. For writing the structure of $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ for a particular meson, we re-express this function by making these scalar amplitudes dimensionless by weighing each covariant by an appropriate power M, the meson mass. Thus each term in the expansion of $\Gamma(\hat{q})$ is associated with a certain power of M and hence in detail we can express the hadron-quark vertex,$\Gamma(\hat{q})$ as a polynomial in various powers of $1/M$ [8]:

$$\Gamma_P(\hat{q}) = \Omega_P(\hat{q})\frac{1}{2\pi i}N_P D(\hat{q})\phi(\hat{q}). \tag{3.5.12}$$

with

$$\Omega_P = \gamma_5 B_0 - i\gamma_5 \not{P}\frac{B_1}{M} - i\gamma_5 \not{q}\frac{B_2}{M} - \gamma_5[\not{P}\not{q} - \not{q}\not{P}]\frac{B_3}{M^2}. \tag{3.5.13}$$

where $B_i(i = 0, ...3)$ are four dimensionless and constant coefficients (which are taken to be constant on lines of [3] [8]) to be determined. Now since we use constituent quark masses, where quark mass m is approximately half of the hadron mass M, we can use the ansatz $q << P \sim M$ in the rest frame of the hadron Then each of the four terms in Eqn (3.5.13) would again receive suppression by different powers of 1/M [8]. Thus we can arrange these terms as an expansion in powers of O(1/M). We can then see in the expansion of Ω_P , that the structures associated with the coefficients B_0, B_1 have magnitudes $O(1/M^0)$ and are of leading order, while those with B_2, B_3 are $O(1/M^1)$ and are next-to-leading-order. This naive power counting rule [1, 3, 8] suggests that the maximum contribution to the calculation of any pseudoscalar meson observable should come from the Dirac structures γ_5 and $i\gamma_5(\gamma.p)/M$ associated with the constant coefficients B_0 and B_1 respectively, followed by the other two NLO covariants associated with coefficients B_2 and B_3. Nevertheless we have to see the results of employing the full vertex function in Eqn (3.5.12) which incorporates both LO as well as NLO covariants, and in the process see the relative importance of various covariants. We give this detailed calculation in chapter 4. Before presenting this calculation, we first give the description of the BSE kernel. As far as the interaction Kernel is concerned, it has the form as in [23-24] and has the general form

$$K(q, \hat{q'}) = (\frac{1}{2}\lambda^{(1)}).(\frac{1}{2}\lambda^{(2)})V_\mu^{(1)}V_\mu^{(2)}V(q-\bar{q}). \tag{3.5.14}$$

where $(\frac{1}{2}\lambda^{(1)}).(\frac{1}{2}\lambda^{(2)})$ is the color part of the one gluon exchange interaction between two quarks, $\gamma_\mu^{(1)}\gamma_\mu^{(2)}$ is the spin dependent part of the interaction, while V(q-q')is the spatial part of the interaction. For details on the Kernel see [24]. The ground state wave function $\phi(\hat{q})$ satisfies the 3D BSE on the surface P.q=0, which is appropriate for making contact with mass spectrum [24, 25]. The ground state wave function $\phi(\hat{q})$ has a gaussian structure [8, 22, 24] and is expressed as

$$\phi(\hat{q}) \sim e^{\frac{-\hat{q}^2}{2\beta^2}} \tag{3.5.15}$$

In the structure of $\phi(\hat{q})$ in the above eqn (3.5.15), the parameter β is the inverse range parameter which incorporates the content of BS dynamics and is dependent on the input kernel $K(q, q')$. We present the derivation of decay constant f_P for unequal mass pseudoscalar meson (including an alternative calculation for pion) in the next chapter).

Chapter 4

Leptonic decays of pseudoscalar mesons

4.1 Determination of decay constant f_P

Decay constant f_P can be evaluated through the loop diagram which gives the coupling of the two-quark loop to the axial vector current and can be evaluated as [1]:

$$f_P P_\mu =< 0|\bar{Q} i\gamma_\mu\gamma_5 Q|P(P) > . \tag{4.1.1}$$

which can, in turn, be expressed as a loop integral [1] [8],

$$f_P P_\mu = \sqrt{3}\int d^4q Tr[\Psi_P(P,q) i\gamma_\mu\gamma_5]. \tag{4.1.2}$$

A Bethe-Salpeter wave function $\Psi(P,q)$ for a P-meson [8] is expressed as,

$$\Psi(P,q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2). \tag{4.1.3}$$

with

$$S_F(p_1) = -i\frac{(m_1 - i\not{p}_1)}{\Delta_1}. \tag{4.1.4}$$

$$S_F(-p_2) = -i\frac{(m_2 + i\not{p}_2)}{\Delta_2}. \tag{4.1.5}$$

which is expressed as the quark and anti-quark propagators flanking the Hadron-quark vertex $\Gamma(\hat{q})$ function. Let's first evaluate the trace over the gamma matrices, by using

$\Psi(P,q)$ from Eqn(4.1.3) and incorporating $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ from Eqn (3.5.12)

$$Tr\{\Psi(P,q)i\gamma_\mu\gamma_5\} = Tr\{[-i\frac{m_1 - i\not{p}_1}{\Delta_1}][\gamma_5 B_0 - i\gamma_5\frac{\not{P}}{M}B_1 - i\gamma_5\frac{\not{q}}{M}B_2 - \gamma_5(\not{P}\not{q} - \not{q}\not{P})\frac{B_3}{M^2}]$$
$$[-i\frac{(m_2 + i\not{p}_2)}{\Delta_2}i\gamma_\mu\gamma_5]\}. \tag{4.1.6}$$

This can in turn be expressed as

$$Tr\{\Psi(P,q)i\gamma_\mu\gamma_5\} = \frac{-1}{M^2\Delta_1\Delta_2}Tr\{[m_1 - i\not{p}_1][\gamma_5 B_0 M^2 - i\gamma_5\not{P}B_1 M - i\gamma_5\not{q}B_2 M]$$
$$[-\gamma_5 B_3(\not{P}\not{q} - \not{q}\not{P})][m_2 + i\not{p}_2]i\gamma_\mu\gamma_5\}. \tag{4.1.7}$$

After expanding the above expression, the individual terms are:

$$Tr\{-m_1 B_0 M^2\gamma_5\not{p}_2\gamma_\mu\gamma_5\} = -m_1 B_0 M^2 Tr\{\not{p}_2\gamma_\mu\} = -4m_1 B_0 M^2 p_{2\mu},$$

$$Tr\{m_1 m_2 M B_1\gamma_5\not{P}\gamma_\mu\gamma_5\} = m_1 m_2 M B_1 Tr\{\not{p}\gamma_\mu\} = 4m_1 m_2 M B_1 P_\mu,$$

$$Tr\{m_1 m_2 M B_2\gamma_5\not{q}\gamma_\mu\gamma_5\} = m_1 m_2 M B_2 Tr\{\not{q}\gamma_\mu\} = 4m_1 m_2 M B_2 q_\mu,$$

$$Tr\{m_2 M^2 B_0\not{p}_1\gamma_5\gamma_\mu\gamma_5\} = m_2 M^2 B_0 Tr\{\not{p}_1\gamma_\mu\} = -4m_2 M^2 B_0 p_{1\mu},$$

$$Tr\{-m_2 B_3\not{p}_1\gamma_5\not{P}\not{q}\gamma_\mu\gamma_5\} = 4m_2 B_3\{(p_1.P)q_\mu - (p_1.q)P_\mu + (P.q)p_{1\mu}\},$$

$$Tr\{-m_2 B_3\not{p}_1\not{q}\not{P}\gamma_\mu\} = -4m_2 B_3\{(p_1.q)P\mu - (p_1.P)q_\mu + (q.P)p_{1\mu}\},$$

$$Tr\{m_1 B_3\gamma_5\not{P}\not{q}\not{p}_2\gamma_\mu\gamma_5\} = 4m_1 B_3\{(P.q)p_{2\mu} - (P.p_2)q_\mu + (q.p_2)P_\mu\},$$

$$Tr\{-m_1 B_3\gamma_5\not{q}\not{P}\not{p}_{2\mu}\gamma_\mu\gamma_5\} = -4m_1 B_3\{(q.P)p_{2\mu} - (q.p_2)P_\mu + (P.p_2)q_\mu\},$$

$$Tr\{B_1 M\not{p}_1\gamma_5\not{P}\not{p}_2\gamma_\mu\gamma_5\} = 4B_1 M\{(-p_1.p)p_{2\mu} + (p_1.p_2)P_\mu - (P.p_2)p_{1\mu}\},$$

$$Tr\{B_2 M\not{p}_1\gamma_5\not{q}\not{p}_2\gamma_\mu\gamma_5\} = 4B_2 M\{(-p_1.q)p_{2\mu} + (p_1.p_2)q_\mu - (q.p_2)p_{1\mu}\}. \tag{4.1.8}$$

Collecting the above results systematically with respect to LO and NLO Dirac covariants we have,

$$\frac{-1}{M^2\Delta_1\Delta_2}Tr(LO) = \frac{-1}{M^2\Delta_1\Delta_2}\{4M^2 B_0[-m_2 p_{1\mu} - m_1 p_{2\mu}]+$$

$$4MB_1[m_1m_2P_\mu - [(p_1.P)p_{2\mu} - (p_1.p_2)P_\mu + (P.p_2)p_{1\mu}]]\}, \tag{4.1.9}$$

and

$$\frac{-1}{M^2\Delta_1\Delta_2}Tr(NLO) = \frac{-1}{M^2\Delta_1\Delta_2}\{4MB_2[m_1m_2q_\mu - [(p_1.q)p_{2\mu} - (p_1.p_2)q_\mu + (q.p_2)p_{1\mu}]]$$

$$+8B_3[m_2(p_1.P)q_\mu - m_2(p_1.q)P_\mu + m_1(q.p_2)P_\mu - m_1(P.p_2)q_\mu]\}. \tag{4.1.10}$$

We have seen in chapter 3, that the four dimensional volume element will have the form,

$$d^4q = d^3\hat{q}Md\sigma. \tag{4.1.11}$$

We can express the leptonic decay constant f_P as:

$$f_P = \sqrt{3}N_P\int d^3\hat{q}\phi(\hat{q})I = f_P^{(0)} + f_P^{(1)} + f_P^{(2)} + f_p^{(3)}. \tag{4.1.12}$$

where $f_P^{(0)}, f_P^{(1)}, f_P^{(2)}, f_P^{(3)}$, are the contributions to f_P from the four Dirac covariants associated with coefficients $B_i(i = 0,1,2,3)$, and are expressed as:

$$f_P^{(0)} = \sqrt{3}N_PB_0\int d^3\hat{q}D(\hat{q})\phi(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{2\pi i\Delta_1\Delta_2}[-2m_1 + 2\frac{m_1^3}{M^2} - 2m_2 - 2\frac{m_1^2m_2}{M^2} -$$

$$2\frac{m_1m_2^2}{M^2} + 2\frac{m_2^3}{M^2} + 4(m_1 - m_2)\sigma], \tag{4.1.13}$$

$$f_P^{(1)} = \sqrt{3}N_PB_1\int d^3\hat{q}D(\hat{q})\phi(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{2\pi i\Delta_1\Delta_2}[M - \frac{m_1^4}{M^3} + 4\frac{m_1m_2}{M} + 2\frac{m_1^2m_2^2}{M^3} - \frac{m_2^4}{M^3} -$$

$$4\frac{\hat{q}^2}{M} + \frac{4}{M}(m_2^2 - m_1^2)\sigma - 4M\sigma^2], \tag{4.1.14}$$

$$f_P^{(2)} = \sqrt{3}N_PB_2\int d^3\hat{q}D(\hat{q})\phi(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{2\pi i\Delta_1\Delta_2}[\frac{4}{M^3}(m_1^2 - m_2^2)\hat{q^2} + (M - \frac{m_1^4}{M^3} + 4\frac{m_2m_1}{M}$$

$$+2\frac{m_1^2m_2^2}{M^3} - \frac{m_2^4}{M^3})\sigma + 4\frac{\hat{q}^2}{M}\sigma + \frac{4}{M}(m_2^2 - m_1^2)\sigma^2 - 4M\sigma^3], \tag{4.1.15}$$

$$f_P^{(3)} = \sqrt{3}N_PB_3\int d^3\hat{q}D(\hat{q})\phi(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{2\pi i\Delta_1\Delta_2}(-8\frac{m_1 + m_2}{M^2}\hat{q^2}). \tag{4.1.16}$$

in deriving the above expressions, we make use of the scalar products of various momenta expressed as,

$$\begin{aligned}
p_1.P &= -M^2(\hat{m}_1+\sigma), \\
p_2.P &= -M^2(\hat{m}_2-\sigma), \\
P.q &= -M^2\sigma + \frac{1}{2}(m_2^2-m_1^2), \\
p_1^2 &= -M^2(\hat{m}_1+\sigma)^2+\hat{q}^2, \\
p_2^2 &= -M^2(\hat{m}_2-\sigma)^2+\hat{q}^2, \\
p_1.p_2 &= -M^2(\hat{m}_1+\sigma)(\hat{m}_2-\sigma)-\hat{q}^2, \\
p_1.q &= \frac{1}{2}\{2\hat{q}^2-\sigma[m_1^2-m_2^2+M^2(1+2\sigma)]\}, \\
p_2.q &= \frac{1}{2}\{-2\hat{q}^2+\sigma[m_1^2-m_2^2+M^2(-1+2\sigma)]\}.
\end{aligned} \tag{4.1.17}$$

and multiply both sides of Eqn(4.1.2) by $\frac{P_\mu}{(-M^2)}$. We carry out integration over $d\sigma$ by method of contour integration by noting the poles positions in the complex σ plane

$$\Delta_1 = 0 \Rightarrow \sigma_1^{\pm} = \pm\frac{\omega_1}{M} - \hat{m}_1 \mp i\varepsilon; \omega_1^2 = m_1^2 + \hat{q}^2 \tag{4.1.18}$$

$$\Delta_2 = 0 \Rightarrow \sigma_2^{\mp} = \mp\frac{\omega_2}{M} + \hat{m}_2 \pm i\varepsilon; \omega_2^2 = m_2^2 + \hat{q}^2 \tag{4.1.19}$$

where $\Delta_1 = \omega_1^2 - M^2(\hat{m}_1+\sigma)^2$ and $\Delta_2 = \omega_2^2 - M^2(\hat{m}_2-\sigma)^2$ Having this all in mind let's integrate by considering semi-circular contour of large radius R with center at the origin. The various integrals are calculated as follows:

$$R_1 = \int_{-\infty}^{\infty}\frac{M\sigma d\sigma}{2\pi i\Delta_1\Delta_2} = \lim_{R\to\infty}\int_{-R}^{R}\frac{M\sigma d\sigma}{2\pi i\Delta_1\Delta_2} + \lim_{R\to 0}\int_{\Gamma}\frac{M\sigma d\sigma}{2\pi i\Delta_1\Delta_2}$$

$$R_1 = \lim_{R\to\infty}\int_{-R}^{R}\frac{M\sigma d\sigma}{2\pi i\Delta_1\Delta_2} = \Sigma a_{-1}$$

where $\Sigma\ a_{-1}$ is the sum of the residues of the complex function at it's poles in the region of integration. In our problems the poles lie on the real σ axis. So, we should shift the

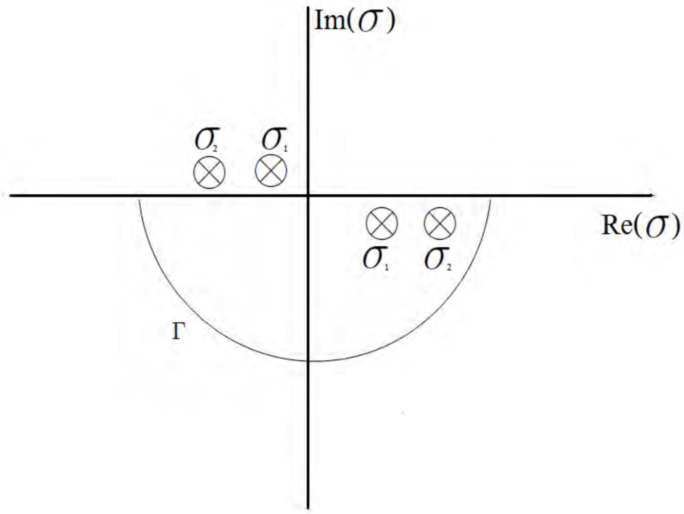

Figure 4.1: pole position for integration in complex $\sigma - plane$

poles infinitesimally away from the real axis. The poles lying within the contour are: $\sigma_1 = \frac{\omega_1}{M} - \hat{m}_1$ and $\sigma_2 = \frac{\omega_2}{M} + \hat{m}_2$. Now at the poles $\sigma_1 = \frac{\omega_1}{M} - \hat{m}_1 - i\varepsilon$ the residue will (taking $\lim_{\varepsilon \to 0}$) be:

$$a_{-1} = \lim_{\sigma_1 \to \frac{\omega_1}{M} - \hat{m}_1} \{ \frac{\sigma - \frac{\omega_1}{M} + \hat{m}_1}{[\omega_1^2 - M^2(\hat{m}_1 + \sigma)^2][\omega_2^2 - M^2(\hat{m}_2 - \sigma)^2]} \} \{\sigma\}$$

and at the pole $\sigma_2 = \frac{\omega_{12}}{M} + \hat{m}_2$ the residue will be:

$$a_{-1} = \lim_{\sigma_2 \to \frac{\omega_2}{M} + \hat{m}_2} \{ \frac{\sigma - \frac{\omega_2}{M} - \hat{m}_2}{[\omega_1^2 - M^2(\hat{m}_1 + \sigma)^2][\omega_2^2 - M^2(\hat{m}_2 - \sigma)^2]} \} \{\sigma\}$$

After simplifying and substituting the above results we can find immediately:

$$R_1 = \int_{-\infty}^{\infty} \frac{M\sigma d\sigma}{2\pi i \Delta_1 \Delta_2} = \frac{M^2(\omega_2 - \omega_1) + (m_1^2 - m_2^2)(\omega_1 + \omega_2)}{4M^2\omega_1\omega_2(M^2 - (\omega_1 + \omega_2)^2)}. \tag{4.1.20}$$

With the same procedure we will have:

$$R_2 = \int_{-\infty}^{\infty} \frac{M\sigma^2 d\sigma}{2\pi i \Delta_1 \Delta_2} = \frac{[4M^2\omega_1\omega_2 - M^4 - m_{12}^2\delta m^2][\omega_1 + \omega_2] + 2M^2 m_{12}\delta m[\omega_2 - \omega_1]}{8M^4\omega_1\omega_2[M^2 - (\omega_1 + \omega_2)^2]}. \tag{4.1.21}$$

$$R_3 = \int_{-\infty}^{\infty} \frac{M\sigma^3 d\sigma}{2\pi i \Delta_1 \Delta_2} = \lim_{R\to\infty} \int_{-R}^{R} \frac{M\sigma^3 d\sigma}{2\pi i \Delta_1 \Delta_2} + \lim_{R\to 0} \int_{\Gamma} \frac{M\sigma^3 d\sigma}{2\pi i \Delta_1 \Delta_2} = \lim_{R\to\infty} \int_{-R}^{R} \frac{M\sigma^3 d\sigma}{2\pi i \Delta_1 \Delta_2}$$
$$= \Sigma\ a_{-1}$$

the residue at the pole $\sigma = \frac{\omega_1}{M} - \hat{m}_1$ will be:

$$a_{-1} = \lim_{\sigma \to \frac{\omega_1}{M} - \hat{m}_1} \frac{-M[\sigma - \frac{\omega_1}{M} + \hat{m}_1]^2}{M^2[\sigma + \frac{\omega_1}{M} + \hat{m}_1][\Delta_2]} = 0,$$

and the residue at the pole $\sigma = \frac{\omega_2}{M} + \hat{m}_2$ will be:

$$a_{-1} = \lim_{\sigma \to \frac{\omega_2}{M} + \hat{m}_2} \frac{-M[\sigma - \frac{\omega_2}{M} - \hat{m}_2]^2}{M^2[\sigma + \frac{\omega_2}{M} - \hat{m}_2][\Delta_1]} = 0,$$

The above two residue results leads us to conclude that

$$R_3 = 0. \tag{4.1.22}$$

With the above substitutions the decay constant of pseudoscalar meson can equally be expressed as:

$$f_P^{(0)} = \sqrt{3} N_P B_0 \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) [(-2m_1 + 2\frac{m_1^3}{M^2} - 2m_2 - 2\frac{m_1^2 m_2}{M^2} - 2\frac{m_1 m_2^2}{M^2} + 2\frac{m_2^3}{M^2})\frac{1}{D(\hat{q})} + 4(m_1 - m_2)R_1], \tag{4.1.23}$$

$$f_P^{(1)} = \sqrt{3} N_P B_1 \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) [(M - \frac{m_1^4}{M^3} + 4\frac{m_1 m_2}{M} + 2\frac{m_1^2 m_2^2}{M^3} - \frac{m_2^4}{M^3})\frac{1}{D(\hat{q})} - 4\frac{\hat{q}^2}{M}\frac{1}{D(\hat{q})} + \frac{4}{M}(m_2^2 - m_1^2)R_1 - 4MR_2], \tag{4.1.24}$$

$$f_P^{(2)} = \sqrt{3} N_P B_2 \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) [(\frac{4}{M^3}(m_1^2 - m_2^2)\hat{q}^2 \frac{1}{D(\hat{q})} + (M - \frac{m_1^4}{M^3} + 4\frac{m_2 m_1}{M} + 2\frac{m_1^2 m_2^2}{M^3} - \frac{m_2^4}{M^3})R_1 + 4\frac{\hat{q}^2}{M}R_1 + \frac{4}{M}(m_2^2 - m_1^2)R_2], \tag{4.1.25}$$

$$f_P^{(3)} = \sqrt{3} N_P B_3 \int d^3\hat{q} D(\hat{q}) \phi(\hat{q}) (-8\frac{m_1 + m_2}{M^2}\hat{q}^2)\frac{1}{D(\hat{q})}. \tag{4.1.26}$$

where $\phi(\hat{q}) = e^{\frac{-\hat{q}^2}{2\beta^2}}$ is the ground state 3D BS wave function [20, 21] obtained by solving the 3D spectral equation. and

$$D(\hat{q}) = \frac{D_0(q)}{[\frac{\hat{m_1}}{\omega_1} + \frac{\hat{m_2}}{\omega_1}]}. \tag{4.1.27}$$

where $D_0(q) = (\omega_1 + \omega_2)^2 - M^2$; We have thus evaluated the general expressions for f_P. Our next objective lies on the determination of Bethe-Salpeter normalizer, N_P

4.1.1 Determination of Bethe-Salpeter normalizer N_P

To calculate BS normalizer N_P for a pseudoscalar meson in the expression for f_P we use the current conservation condition [8],

$$2iP_\mu = (2\pi)^4 \int d^4q Tr[\bar{\Psi}(P,q)(\frac{\partial}{\partial P_\mu} S_F^{-1}(p_1))\Psi(P,q)S_F^{-1}(-p_2)] + (1 \leftrightarrow 2). \tag{4.1.28}$$

where

$$\bar{\Psi}(P,q) = \gamma_4 \Psi^\dagger(P,q)\gamma_4. \tag{4.1.29}$$

Substituting the value of $\Psi(P,q)$, $\bar{\Psi}(P,q)$ and

$$S_F^{-1}(p_1) = i(m_1 + i\not{p}_1) \tag{4.1.30}$$

$$S_F^{-1}(-p_2) = i(m_2 - i\not{p}_2) \tag{4.1.31}$$

we have:

$$2iP_\mu = (2\pi)^4 \int d^4q TR; \tag{4.1.32}$$

$$TR = Tr[(m_2 + ip_2)(\gamma_5 B_0 + i\not{P}\gamma_5 \frac{B_1}{M} + i\not{q}\gamma_5 \frac{B_2}{M} + \not{q}\not{P}\gamma_5 \frac{B_3}{M^2} - \not{P}\not{q}\gamma_5 \frac{B_3}{M^2})]$$
$$[(m_1 - ip_1)\gamma_\mu(m_1 - ip_1)][\gamma_5 B_0 - i\gamma_5 \not{P}\frac{B_1}{M} - i\gamma_5 \not{q}\frac{B_2}{M} - \gamma_5 \not{P}\not{q}\frac{B_3}{M^2} + \gamma_5 \not{q}\not{P}\frac{B_3}{M^2}]. \tag{4.1.33}$$

After expanding this expression, it's difficult if not impossible to put all the trace results here, but we can take few examples, where traces over four gamma matrices are involved such as:

$$Tr(\gamma_\mu \not{p}_1 \not{P} \not{q}) = 4[p_{1\mu}(P.q) - P_\mu(p_1.q) + q_\mu(p_1.P)],$$

$$Tr(\not{p}_2 \not{q} \not{P} \gamma_\mu) = 4(p_2.q)P_\mu - 4(p_2.P)q + 4(q.P)p_{2\mu},$$

$$Tr(\not{p}_2 \not{P} \not{q} \gamma_\mu) = 4(p_2.P)q_\mu - 4(p_2.q)P_\mu + 4(q.P)p_{2\mu},$$

There are 7 terms with traces over products of two gamma matrices, 52 terms with traces over products of four gamma matrices, 32 terms with traces over products of six gamma matrices and four terms with traces over products of eight gamma matrices. Hence the expression for full BS normalizer is extremely complicated. Evaluating trace over the γ matrices we can again express the expression for TR in terms of the products of constituent quark momenta, the internal momentum and the bound state momentum as:

$$TR = B_0^2[4m_1^2(p_2.P) - 8(p_1.p_2)p_1.P) + 8m_1m_2(p1.P) + 4p_1^2(p_2.P)] + \frac{B_0B_1}{M^3}[16m_1(p_1.P)(p_2.P]$$

$$+\frac{B_1^2}{M^4}[-4m_1^2P^2(p_2.P) + 8m_1m_2P^2(p_1.P) + 4P^2(p_2.P)p_1^2 + 32(p_2.P)(p_1.P)^2 - 16(p_1.p_2)(p_1.P)P^2]$$

$$-16p_1^2P^2] + \frac{B_0B_2}{M^3}[-8m_1^2m_2(q.P) + 16m_1(p_2.q)(p_1.P) + 16(q.p_1)(p_1.P) - 8m_2(q.P)p_1^2]$$

$$+\frac{B_1B_2}{M^4}[-8m_1^2(p_2.q)P^2 + 8m_1^2(q.P)(p_2.P) - 8m_1(q.P)(p_2.P) - 16m_1m_2(q.P)(p_1.P)16(p_2.q)$$

$$+(p_1.P)^2 + 16(p_2.P)(p_1.q)(p_1.P) - 16(p_1.p_2)(q.P)(p_1.P) - 8(p_2.q)p_1^2P^2] + \frac{B_2^2}{M^4}[-8m_1^2(p_2.q)(q.P)$$

$$+4m_1^2q^2(p_2.P) - 8m_1m_2q^2(p_1.P) + 16(p_1.q)(p_2.q)(p_1.P) - 8p_1^2(p_2.q)(q.P) - 8q^2(p_1.p_2)(p_1.P)$$

$$+4q^2p_1^2(p_2.P)] + \frac{B_0B_3}{M^4}[16m_1^2(p_2.P)(q.P) - 16m_1^2P^2(p_2.q) + 32(p_2.q)(p_1.P)^2 - 32(p_2.P)(p_1.q)$$

$$(p_1.P) - 16(p_2.q)p_1^2P^2 + \frac{B_1B_3}{M^5}[32m_1(p_2.P)(q.P)(p_1.P) - 32m_1(p_2.q)(p_1.P)P^2 - 32m_2(p_1.P)^2$$

$$(q.P) + 32m_2(p_1.q)(p_1.P)P^2] + 16(P.q)(p_2.P)p_1^2] + \frac{B_2B_3}{M^5}[-16m_1^2m_2(q.P)^2 + 16m_1^2m_2q^2P^2$$

$$-32m_1(p_2.q)(q.P)(p_1.P) + 32m_1(p_2.P)(p_1.P)q^2 + 32m_2(q.P)(p_1.q)(p_1.P) - 32m_2q^2(p_1.P)^2$$

$$-16m_2(q.P)^2p_1^2 + 16m_2p_1^2P^2q^2] + \frac{B_3^2}{M^6}[-16m_1^2(p_2.P)q^2P^2 + 16m_1^2(q.P)^2(p_2.P)$$

$$-32m_1m_2(q.P)^2(p_1.P) + 32m_1m_2(p_1.P)q^2P^2 + 64(p_2.q)(p_1.q)(p_1.P)P^2 - 64(p_2.q)(P.q)$$

$$(p_1.P)^2 + 16(p_1.p_2)(p_1.P)(q.P)^2 - 32(p_1.p_2)(p_1.P)q^2P^2 + 64(p_2.P)(p_1.P)^2q^2$$

$$-64(p_2.P)(q.P)(q.p_1)(p_1.P)+16(p_2.P)(q.P)^2p_1^2-16(p_2.P)p_1^2q^2P^2]. \qquad (4.1.34)$$

After substitutions of the values of the dot products of various momenta we can express the trace as a function of kinematical variables and the four dimensionless and constant coefficients alone, and therefore the normalizer can be expressed as:

$$N_p^{-2}=(2\pi)^2 i\int d^3\hat{q}D^2(\hat{q})\phi^2(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}(\hat{m_1}+\sigma)(T_1+T_2+T_3+\cdots+T_{10})+(1\leftrightarrow 2). \qquad (4.1.35)$$

where

$$T_1=\frac{B_0^2}{M^2}\{-\frac{M^4}{2}-\frac{5M^2m_1^2}{2}+\frac{5m_1^4}{2}+\frac{m_1^6}{2M^2}-4M^2m_1m_2-4m_1^3m_2+\frac{M^2m_2^2}{2}-3m_1^2m_2^2-\frac{3m_1^4m_2^2}{2M^2}$$
$$+\frac{m_2^4}{2}+4m_1m_2^3+\frac{3m_1^2m_2^2}{2M^2}-\frac{m_2^6}{2M^2}-6M^2\hat{q}^2-2m_1^2\hat{q}^2+2m_2^2\hat{q}^2+[6M^2m_1^2-M^4+3m_1^2m_2^2-8M^2m_1m_2$$
$$-2M^2m_2^2-6m_1^2m_2^2+3m_2^4-4M^2\hat{q}^2]\sigma+[2M^4+6M^2m_1^2-6M^2m_2^2]\sigma^2+4M^4\sigma^3\},$$

$$T_2=\frac{B_0B_1}{M^3}\{4M^4m_1-4m_1^5+8m_1^3m_2^2-4m_1m_2^4+[16M^2m_1m_2^2-16M^2m_1^3]\sigma-16M^4m_1\sigma^2\},$$

$$T_3=\frac{B_0B_2}{M^3}\{M^2m_1^2m_2+6m_1^4m_2+\frac{m_1^6m_2}{M^2}-M^2m_2^3-8m_1^2m_2^3-\frac{3m_1^4m_2^3}{M^2}+2m_2^5+\frac{3m_1^2m_2^5}{M^2}$$
$$-\frac{m_2^7}{M^2}+4M^2m_1\hat{q}^2-M^2m_2\hat{q}^2-4m_1^2m_2\hat{q}^2+4m_2^3\hat{q}^2+[2M^4m_2+2M^2m_1^2m2+6m_1^4m_2$$
$$-M^2m_2^3-12m_1^2m2^3+6m2^5]\sigma-8M^2m_2\hat{q}^2\sigma+[8M^4m_2+12M^2m_1^2m_2^2-12M^2m_2^3]\sigma^2+8M^4m_2\sigma^3\},$$

$$T_4=\frac{B_0B_3}{M^2}\{-12M^4\hat{q}^2-24M^2m_1^2\hat{q}^2+4m_1^4\hat{q}^2+8M^2m_2^2\hat{q}^2-8m_1^2m_2^2\hat{q}^2+4m_2^4\hat{q}^2-16M^2\hat{q}^4$$
$$+[16M^2m_1^2\hat{q}^2-16M^4\hat{q}^2-16M^2m_2^2\hat{q}^2]\sigma+16M^4\hat{q}^2\sigma^2\},$$

$$T_5=\frac{B_1B_1}{M^2}\{m_1^6-M^6-3M^4m_1^2+3M^2m_1^4+4M^4m_1m_2+4M^2m_1^3m_2+M^4m_2^2-4M^2m_1^2m_2^2$$
$$-3m_1^4m_2^2-4M^2m_1m_2^3+M^2m_2^4+3m_1^2m_2^4-m2^6+4M^4\hat{q}^2+12M^2m_1^2\hat{q}^2-12M^2m_2^2\hat{q}^2$$
$$+[8M^4m_1^2-2M^6+6M^2m_1^4+8M^4m_1m_2-4M^4m_2^2-12M^2m_1^2m_2^2+6M^2m_2^4+24M^4\hat{q}^2]\sigma$$
$$+[4M^6+12M^4m_1^2-12M^4m_2^2]\sigma^2+8M^6\sigma^3\},$$

$$T_6=\frac{B_1B_2}{M^2}\{\frac{m_1^6}{2}-\frac{M^4m_1^2}{2}-2M^2m_1^3-\frac{M^2m_1^4}{2}+2m_1^5m_2+\frac{m_1^8}{2M^2}-4M^2m_1^3m2-4m_1^5m_2$$

$$+\frac{M^4m_2^2}{2}+2M^2m_1m_2^2+M^2m_1^2m_2^2-4m_1^3m_2^2-\frac{24m_1^4m_2^2}{2}-2m_1^6m_2^2M^2+4M^2m_1m_2^3+8m_1^3m_2^3$$
$$-\frac{M^2m_2^4}{2}+2m_1m_2^4+\frac{3m_1^2m_2^4}{2}+3m_1^4m_2^4M^2-4m_1m_2^5-\frac{m_2^6}{2}-2m_1^2m_2^6M^2+\frac{m_2^8M^2}{2}+2M^4\hat{q}^2$$
$$-10M^2m_1^2\hat{q}^2+2M^2m_2^2\hat{q}^2-M^2\hat{q}^4+[3M^2m_1^4-M^6-4M^4m_1-2M^4m_1^2+8M^2m_1^3$$
$$+4m_1^6-8M^4m_1m_2-16M^2m_1^3m_2+2M^4m_2^2-8M^2m_1m_2^2-6M^2m_1^2m_2^2-12m_1^4m2^2+16M^2$$
$$m_1m_2^3+3M^2m_2^4+12m_1^2m_2^4\sigma-\frac{8m_2^6}{2}]\sigma-4M^4\hat{q}^2\sigma+[8M^4m_1-2M^6+6M^4m_1^2+12M^2m_1^4$$
$$-16M^4m_1m_2-6M^4m_2^2-24M^2m_1^2m_2^2+12M^2m_2^4]\sigma^2+[4M^6+16M^4m_1^2-16M^4m_2^2]\sigma^3\},$$

$$T_7=\frac{B_1B_3}{M^3}\{16M^4m_1\hat{q}^2+16M^2m_1^3\hat{q}^2+16M^4m_2\hat{q}^2+16M^2m_1^2m_2\hat{q}^2-16M^2m_1m_2^2\hat{q}^2$$
$$-16M^2m_2^3\hat{q}^2+[32M^4m_1\hat{q}^2+32M^4m_2\hat{q}^2]\sigma\}$$

$$T_8=\frac{B_2B_2}{M^2}\{-\frac{1}{8}M^2m_1^4-\frac{5m_1^6}{8}+\frac{5m_1^8}{8M^2}+\frac{m_1^{10}}{8M^4}-m_1^5m2-\frac{m_1^7m_2}{M^2}+\frac{M^2m_1^2m_2^2}{4}+\frac{11m_1^4m_2^2}{8}$$
$$-\frac{2m_1^6m_2^2}{M^2}-\frac{5m_1^8m_2^2}{8M^2}+2m_1^3m_2^3+\frac{3m_1^5m_2^3}{M^2}-\frac{M^2m_2^4}{8}-\frac{7m_1^2m_2^4}{8}+\frac{9m_1^4m_2^4M^2}{4}+\frac{5m_1^6m_2^4}{4M^4}$$
$$-m_1m_2^5-\frac{3m_1^3m_2^5}{M^2}+\frac{m_2^6}{8}-\frac{m_1^2m_2^6}{M^2}-\frac{5m_1^4m_2^6}{4M^4}+\frac{m_1m_2^7}{M^2}+\frac{m_2^8}{8M^2}+\frac{5M^4m_1^2m_2^8}{8}-\frac{m_2^{10}}{8M^4}-$$
$$\frac{M^4\hat{q}^2}{2}-\frac{3M^2m_1^2\hat{q}^2}{2}-3m_1^4\hat{q}^2-\frac{m_1^6\hat{q}^2}{M^2}+4M^2m_1m_2\hat{q}^2+4m_1^3m_2\hat{q}^2-\frac{M^2m_2^2\hat{q}^2}{2}$$
$$+4m_1^2m_2^2\hat{q}^2+\frac{3m_1^4m_2^2\hat{q}^2}{M^2}-4m_1m_2^3\hat{q}^2-m_2^4\hat{q}^2-\frac{3m_1^2m_2^4\hat{q}^2}{M^2}+\frac{m_2^6\hat{q}^2}{M^2}+2M^2\hat{q}^4+2m_1^2\hat{q}^4$$
$$-2m_2^2\hat{q}^4+[4m_1^6-\frac{1}{2}M^4m_1^2-\frac{11}{4}M^2m_1^4+\frac{5}{4M^2}m_1^8-4M^2m_1^3m_2-6m_1^5m_2+\frac{M^4m_2^2}{2}$$
$$+\frac{7M^2m_1^2m_2^2}{2}-9m_1^4m_2^2-\frac{5m_1^6m_2^2}{M^2}+4M^2m_1m_2^3+12m_1^3m_2^3-\frac{3M^2m_2^4}{4}+6m_1^2m_2^4+\frac{15m_1^4m_2^4}{2M^2}$$
$$-6m_1m_2^5-m_2^6-\frac{5m_1^2m_2^6}{M^2}+\frac{5m_2^8}{4M^2}]\sigma+[M^4\hat{q}^2-8M^2m_1^2\hat{q}^2-6m_1^4\hat{q}^2+8M^2m_1m_2\hat{q}^2+4M^2m_2^2\hat{q}^2+$$
$$12m_1^2m_2^2\hat{q}^2-6m_2^4\hat{q}^2]\sigma+4M^2\hat{q}^4\sigma+[9M^2m_1^4-\frac{1}{2}M^6-\frac{7}{2}M^4m_1^2+5m_1^6-4M^4m_1m_2$$
$$-12M^2m_1^3m_2+\frac{3M^4m_2^2}{2}-12M^2m_1^2m_2^2-15m_1^4m_2^2+12M^2m_1m_2^3+3M^2m_2^4+15m_1^2m_2^4-5m_2^6]\sigma^2$$
$$+[-4M^4\hat{q}^2-12M^2m_1^2\hat{q}^2+12M^2m_2^2\hat{q}^2]\sigma^2+[-M^6+8M^4m_1^2+10M^2m_1^4-8M^4m_1m_2$$
$$-4M^4m_2^2-20M^2m_1^2m_2^2+10M^2m_2^4]\sigma^3-8M^4\hat{q}^2\sigma^3+[2M^6+10M^4m_1^2-10M^4m_2^2]\sigma^4\},$$

$$T_9 = \frac{B_2B_3}{M^3}\{8m_1^6m_2 - 16m_1^4m_2^3 + 8m_1^2m_2^5 + 8M^4m_1\hat{q}^2 + 8M^2m_1^3\hat{q}^2 - 4M^4m_2\hat{q}^2 - 16M^2m_1^2m_2\hat{q}^2$$
$$+4m_1^4m_2\hat{q}^2 - 8M^2m_1m_2^2\hat{q}^2 - 8m_1^2m_2^3\hat{q}^2 + 4m_2^5\hat{q}^2 - 16M^2m_2\hat{q}^4 + [32M^2m_1^4m_2 - 32M^2m_1^2m_2^3]\sigma$$
$$+ [16M^4m_1\hat{q}^2 + 16M^2m_1^2m_2\hat{q}^2 - 16M^2m_2^3\hat{q}^2]\sigma + 32M^4m_1^2m_2\sigma^2 + 16M^4m_2\hat{q}^2\sigma^2\},$$

$$T_{10} = \frac{B_3B_3}{M^4}\{-\frac{M^4m_1^4}{2} - \frac{M^2m_1^6}{2} + \frac{m_1^8}{2} + \frac{m_1^{10}}{2M^2} + M^4m_1^2m_2^2 + \frac{3M^2m_1^4m_2^2}{2} - 2m_1^6m_2^2$$
$$-\frac{5m_1^8m_2^2}{2M^2} - \frac{M^4m_2^4}{2} - \frac{3M^2m_1^2m_2^4}{2} + 3m_1^4m_2^4 + \frac{5m_1^6m_2^4}{M^2} + \frac{M^2m_2^6}{2} - 2m_1^2m_2^6$$
$$-\frac{5m_1^4m_2^6}{M^2} + \frac{m_2^8}{2} + \frac{5m_1^2m_2^8}{2M^2} - \frac{m_2^{10}}{2M^2} - 2M^6\hat{q}^2 - 10M^4m_1^2\hat{q}^2 + 8M^2m_1^4\hat{q}^2 + 16M^4m_1m_2\hat{q}^2$$
$$+ 16M^2m_1^3m_2\hat{q}^2 + 2M^4m_2^2\hat{q}^2 - 8M^2m_1^2m_2^2\hat{q}^2 - 16M^2m_1m_2^3\hat{q}^2 - 24M^4\hat{q}^4 - 8M^2m_1^2\hat{q}^4$$
$$+8M^2m_2^2\hat{q}^4 + [-2M^6m_1^2 - 3M^4m_1^4 + 4M^2m_1^6 + 5m_1^8 + 2M^6m_2^2 + 6M^4m_1^2m_2^2 - 12M^2m_1^4m2^2$$
$$-20m_1^6m_2^2 - 3M^4m_2^4 + 12M^2m_1^2m_2^4 + 30m_1^4m_2^4 - 4M^2m_2^6 - 20m1^2m2^6 + 5m_2^8]\sigma$$
$$+[-4M^6\hat{q}^2 + 16M^4m_1^2\hat{q}^2 + 32M^4m_1m_2\hat{q}^2]\sigma - 16M^4\hat{q}^4\sigma + [-2M^8 - 6M^6m_1^2 + 12M^4m_1^4$$
$$+20M^2m_1^6 + 6M^6m_2^2 - 24M^4m_1^2m2^2 - 60M^2m_1^4m_2^2 + 12M^4m_2^4 + 12M^2m_1^2m_2^4 - 6m_1^4m_2^2M^2$$
$$+16M^6m_1^2 + 40M^4m_1^4 - 16M^6m_2^2 - 80M^4m_1^2m_2^2 + 40M^4m_2^4]\sigma^3\}.$$

In the above expression many terms would give divergent contributions due to the presence of the factor $(\hat{m}_1 + \sigma)$ in the integrand. To overcome this problem, one can consider, following [20, 26] the charge to be concentrated on one of the quark lines (say p_1). This may amount to taking the derivative with respect to p_1 (instead of P) [20, 26] in the expression for $N_P^{(-2)}$. Thus we can express the above normalizer with out $(\hat{m}_1 + \sigma)$

$$N_p^{-2} = (2\pi)^2 i \int d^3\hat{q} D^2(\hat{q})\phi^2(\hat{q})(T_1 + T2 + T_3 + ... + T_{10}) + (1 \leftrightarrow 2) \qquad (4.1.36)$$

where

$$T_1 = \frac{B_0^2}{M^2}\{[-\frac{M^4}{2} - \frac{5M^2m_1^2}{2} + \frac{5m_1^4}{2} + \frac{M_1^6}{2M^2} - 4M^2m_1m_2 - 4m_1^3m_2 + \frac{M^2m_2^2}{2} - 3m_1^2m_2^2 - \frac{3m_1^4m_2^2}{2M^2} + \frac{m_2^4}{2}$$
$$+4m_1m_2^3 + \frac{3m_1^2m_2^4}{2M^2} - \frac{m_2^6}{2M^2} - 6M^2\hat{q}^2 - 2m_1^2\hat{q}^2 + 2m_2^2\hat{q}^2]I_1 + [6M^2m_1^2 - M^4 + 3m_1^2m_2^2 - 8M^2m_1m_2$$

$$-2M^2m_2^2-6m_1^2m_2^2+3m_2^4-4M^2\hat{q}^2]I_2+[2M^4+6M^2m_1^2-6M^2m_2^2]I_3+4M^4I_4\},$$

$$T_2=\frac{B_0B_1}{M^3}\{[4M^4m_1-4m_1^5+8m_1^3m_2^2-4m_1m_2^4]I_1+[16M^2m_1m_2^2-16M^2m_1^3]I_2-16M^4m_1I_3\},$$

$$T_3=\frac{B_0B_2}{M^3}\{[M^2m_1^2m_2+6m_1^4m_2+\frac{m_1^6m_2}{M^2}-M^2m_2^3-8m_1^2m_2^3-\frac{3m_1^4m_2^3}{M^2}+2m_2^5+\frac{3m_1^2m_2^5}{M^2}-\frac{m_2^7}{M^2}$$

$$+4M^2m_1\hat{q}^2-M^2m_2\hat{q}^2-4m_1^2m_2\hat{q}^2+4m_2^3\hat{q}^2]I_1+[2M^4m_2+2M^2m_1^2m2+6m_1^4m_2-M^2m_2^3$$

$$-12m_1^2m2^3+6m2^5]I_2-8M^2m_2\hat{q}^2I_2+[8M^4m_2+12M^2m_1^2m_2^2-12M^2m_2^3]I_3+8M^4m_2I_4\},$$

$$T_4=\frac{B_0B_3}{M^2}\{[-12M^4\hat{q}^2-24M^2m_1^2\hat{q}^2+4m_1^4\hat{q}^2+8M^2m_2^2\hat{q}^2-8m_1^2m_2^2\hat{q}^2+4m_2^4\hat{q}^2-16M^2\hat{q}^4]I_1$$

$$+[16M^2m_1^2\hat{q}^2-16M^4\hat{q}^2-16M^2m_2^2\hat{q}^2]I_2+16M^4\hat{q}^2I_3\},$$

$$T_5=\frac{B_1B_1}{M^2}\{[m_1^6-M^6-3M^4m_1^2+3M^2m_1^4+4M^4m_1m_2+4M^2m_1^3m_2+M^4m_2^2-4M^2m_1^2m_2^2$$

$$-3m_1^4m_2^2-4M^2m_1m_2^3+M^2m_2^4+3m_1^2m_2^4-m2^6+4M^4\hat{q}^2+12M^2m_1^2\hat{q}^2-12M^2m_2^2\hat{q}^2]I_1$$

$$+[8M^4m_1^2-2M^6+6M^2m_1^4+8M^4m_1m_2-4M^4m_2^2-12M^2m_1^2m_2^2+6M^2m_2^4+24M^4\hat{q}^2]I_2$$

$$+[4M^6+12M^4m_1^2-12M^4m_2^2]I_3+8M^6I_4\},$$

$$T_6=\frac{B_1B_2}{M^2}\{[\frac{m_1^6}{2}-\frac{M^4m_1^2}{2}-2M^2m_1^3-\frac{M^2m_1^4}{2}+2m_1^5m_2+\frac{m_1^8}{2M^2}-4M^2m_1^3m2-4m_1^5m_2$$

$$+\frac{M^4m_2^2}{2}+2M^2m_1m_2^2+M^2m_1^2m_2^2-4m_1^3m_2^2-\frac{24m_1^4m_2^2}{2}-2m_1^6m_2^2M^2+4M^2m_1m_2^3+8m_1^3m_2^3$$

$$-\frac{M^2m_2^4}{2}+2m_1m_2^4+\frac{3m_1^2m_2^4}{2}+3m_1^4m_2^4M^2-4m_1m_2^5-\frac{m_2^6}{2}-2m_1^2m_2^6M^2+\frac{m_2^8M^2}{2}+2M^4\hat{q}^2$$

$$-10M^2m_1^2\hat{q}^2+2M^2m_2^2\hat{q}^2-M^2\hat{q}^4]I_1+[3M^2m_1^4-M^6-4M^4m_1-2M^4m_1^2+8M^2m_1^3$$

$$+4m_1^6-8M^4m_1m_2-16M^2m_1^3m_2+2M^4m_2^2-8M^2m_1m_2^2-6M^2m_1^2m_2^2-12m_1^4m2^2$$

$$+16M^2m_1m_2^3+3M^2m_2^4+12m_1^2m_2^4-\frac{8m_2^6}{2}]I_2-4M^4\hat{q}^2I_2+[8M^4m_1-2M^6+6M^4m_1^2+12M^2$$

$$m_1^4-16M^4m_1m_2-6M^4m_2^2-24M^2m_1^2m_2^2+12M^2m_2^4]I_3+[4M^6+16M^4m_1^2-16M^4m_2^2]I_4\},$$

$$T_7=\frac{B_1B_3}{M^3}\{[16M^4m_1\hat{q}^2+16M^2m_1^3\hat{q}^2+16M^4m_2\hat{q}^2+16M^2m_1^2m_2\hat{q}^2-16M^2m_1m_2^2\hat{q}^2$$

$$-16M^2m_2^3\hat{q}^2]I_1+[32M^4m_1\hat{q}^2+32M^4m_2\hat{q}^2]I_2\}$$

$$T_8=\frac{B_2B_2}{M^2}\{[-\frac{1}{8}M^2m_1^4-\frac{5m_1^6}{8}+\frac{5m_1^8}{8M^2}+\frac{m_1^{10}}{8M^4}-m_1^5m2-\frac{m_1^7m_2}{M^2}+\frac{M^2m_1^2m_2^2}{4}+\frac{11m_1^4m_2^2}{8}$$

$$- \frac{2m_1^6m_2^2}{M^2} - \frac{5m_1^8m_2^2}{8M^2} + 2m_1^3m_2^3 + \frac{3m_1^5m_2^3}{M^2} - \frac{M^2m_2^4}{8} - \frac{7m_1^2m_2^4}{8} + \frac{9m_1^4m_2^4M^2}{4} + \frac{5m_1^6m_2^4}{4M^4}$$
$$- m_1m_2^5 - \frac{3m_1^3m_2^5}{M^2} + \frac{m_2^6}{8} - \frac{m_1^2m_2^6}{M^2} - \frac{5m_1^4m_2^6}{4M^4} + \frac{m_1m_2^7}{M^2} + \frac{m_2^8}{8M^2} + \frac{5M^4m_1^2m_2^8}{8} - \frac{m_2^{10}}{8M^4}$$
$$- \frac{M^4\hat{q}^2}{2} - \frac{3M^2m_1^2\hat{q}^2}{2} - 3m_1^4\hat{q}^2 - \frac{m_1^6\hat{q}^2}{M^2} + 4M^2m_1m_2\hat{q}^2 + 4m_1^3m_2\hat{q}^2 - \frac{M^2m_2^2\hat{q}^2}{2}$$
$$+ 4m_1^2m_2^2\hat{q}^2 + \frac{3m_1^4m_2^2\hat{q}^2}{M^2} - 4m_1m_2^3\hat{q}^2 - m_2^4\hat{q}^2 - \frac{3m_1^2m_2^4\hat{q}^2}{M^2} + \frac{m_2^6\hat{q}^2}{M^2} + 2M^2\hat{q}^4 + 2m_1^2\hat{q}^4$$
$$- 2m_2^2\hat{q}^4]I_1 + [4m_1^6 - \frac{1}{2}M^4m_1^2 - \frac{11}{4}M^2m_1^4 + \frac{5}{4M^2}m_1^8 - 4M^2m_1^3m_2 - 6m_1^5m_2 + \frac{M^4m_2^2}{2}$$
$$+ \frac{7M^2m_1^2m_2^2}{2} - 9m_1^4m_2^2 - \frac{5m_1^6m_2^2}{M^2} + 4M^2m_1m_2^3 + 12m_1^3m_2^3 - \frac{3M^2m_2^4}{4} + 6m_1^2m_2^4 + \frac{15m_1^4m_2^4}{2M^2}$$
$$- 6m_1m_2^5 - m_2^6 - \frac{5m_1^2m_2^6}{M^2} + \frac{5m_2^8}{4M^2}]I_2 + [M^4\hat{q}^2 - 8M^2m_1^2\hat{q}^2 - 6m_1^4\hat{q}^2 + 8M^2m_1m_2\hat{q}^2 + 4M^2m_2^2\hat{q}^2$$
$$+ 12m_1^2m_2^2\hat{q}^2 - 6m_2^4\hat{q}^2]I_2 + 4M^2\hat{q}^4I_2 + [9M^2m_1^4 - \frac{1}{2}M^6 - \frac{7}{2}M^4m_1^2 +$$
$$5m_1^6 - 4M^4m_1m_2 - 12M^2m_1^3m_2 + \frac{3M^4m_2^2}{2} - 12M^2m_1^2m_2^2 - 15m_1^4m_2^2$$
$$+ 12M^2m_1m_2^3 + 3M^2m_2^4 + 15m_1^2m_2^4 - 5m_2^6]I_3 + [-4M^4\hat{q}^2 - 12M^2m_1^2\hat{q}^2 + 12M^2m_2^2\hat{q}^2]I_3$$
$$+ [-M^6 + 8M^4m_1^2 + 10M^2m_1^4 - 8M^4m_1m_2 - 4M^4m_2^2 - 20M^2m_1^2m_2^2 + 10M^2m_2^4]I_4 - 8M^4\hat{q}^2I_4\},$$
$$T_9 = \frac{B_2B_3}{M^3}\{[8m_1^6m_2 - 16m_1^4m_2^3 + 8m_1^2m_2^5 + 8M^4m_1\hat{q}^2 + 8M^2m_1^3\hat{q}^2 - 4M^4m_2\hat{q}^2 - 16M^2m_1^2m_2\hat{q}^2$$
$$+ 4m_1^4m_2\hat{q}^2 - 8M^2m_1m_2^2\hat{q}^2 - 8m_1^2m_2^3\hat{q}^2 + 4m_2^5\hat{q}^2 - 16M^2m_2\hat{q}^4]I_1 + [32M^2m_1^4m_2 - 32M^2m_1^2m_2^3]I_2$$
$$+ [16M^4m_1\hat{q}^2 + 16M^2m_1^2m_2\hat{q}^2 - 16M^2m_2^3\hat{q}^2]I_2 + 32M^4m_1^2m_2I_3 + 16M^4m_2\hat{q}^2I_3\},$$
$$T_{10} = \frac{B_3B_3}{M^4}\{[-\frac{M^4m_1^4}{2} - \frac{M^2m_1^6}{2} + \frac{m_1^8}{2} + \frac{m_1^{10}}{2M^2} + M^4m_1^2m_2^2 + \frac{3M^2m_1^4m_2^2}{2} - 2m_1^6m_2^2$$
$$- \frac{5m_1^8m_2^2}{2M^2} - \frac{M^4m_2^4}{2} - \frac{3M^2m_1^2m_2^4}{2} + 3m_1^4m_2^4 + \frac{5m_1^6m_2^4}{M^2} + \frac{M^2m_2^6}{2} - 2m_1^2m_2^6 -$$
$$\frac{5m_1^4m_2^6}{M^2} + \frac{m_2^8}{2} + \frac{5m_1^2m_2^8}{2M^2} - \frac{m_2^{10}}{2M^2} - 2M^6\hat{q}^2 - 10M^4m_1^2\hat{q}^2 + 8M^2m_1^4\hat{q}^2 + 16M^4m_1m_2\hat{q}^2$$
$$+ 16M^2m_1^3m_2\hat{q}^2 + 2M^4m_2^2\hat{q}^2 - 8M^2m_1^2m_2^2\hat{q}^2 - 16M^2m_1m_2^3\hat{q}^2 - 24M^4\hat{q}^4 - 8M^2m_1^2\hat{q}^4$$
$$+ 8M^2m_2^2\hat{q}^4]I_1 + [-2M^6m_1^2 - 3M^4m_1^4 + 4M^2m_1^6 + 5m_1^8 + 2M^6m_2^2 + 6M^4m_1^2m_2^2 - 12M^2m_1^4m2^2$$
$$- 20m_1^6m_2^2 - 3M^4m_2^4 + 12M^2m_1^2m_2^4 + 30m_1^4m_2^4 - 4M^2m_2^6 - 20m1^2m2^6 + 5m_2^8]I_2 + [-4M^6\hat{q}^2$$

$$+16M^4m_1^2\hat{q}^2+32M^4m_1m_2\hat{q}^2]I_2-16M^4\hat{q}^4I_1+[-2M^8-6M^6m_1^2+12M^4m_1^4+20M^2m_1^6$$

$$+6M^6m_2^2-24M^4m_1^2m2^2-60M^2m_1^4m_2^2+12M^4m_2^4+12M^2m_1^2m_2^4-6m_1^4m_2^2M^2$$

$$+16M^6m_1^2+40M^4m_1^4-16M^6m_2^2-80M^4m_1^2m_2^2+40M^4m_2^4]I_4\}.$$

where $I_1, ...I_4$ are analytic results of pole integration over the off-shell variable σ in the complex σ plane and are evaluated as:

$$\int \frac{M d\sigma}{\Delta_1^2\Delta_2} = 2\pi i \Sigma a_{-1}$$

The residue at the pole $\sigma = \frac{\omega_1}{M} - \hat{m}_1$ will be:

$$a_{-1} = \lim_{\sigma \to \frac{\omega_1}{M} - \hat{m}_1} \frac{d}{d\sigma}\{\frac{[M(\sigma - \frac{\omega_1}{M} + \hat{m}_1)^2]}{[M^4(-\sigma + \frac{\omega_1}{M} - \hat{m}_1)^2)(\frac{\omega_1}{M} + \hat{m}_1 + \sigma)^2][\omega_2^2 - M^2(\hat{m}_2 - \sigma)^2]}\}$$

And at the pole $\sigma = \frac{\omega_2}{M} + \hat{m}_2$

$$\lim_{\sigma \to \frac{\omega_2}{M} + \hat{m}_2}\{\frac{[\sigma - \frac{\omega_2}{M} - \hat{m}_2]M}{[\omega_1^2 - M^2(\hat{m}_1 + \sigma)^2]^2 M^2[\frac{\omega_2}{M} - \hat{m}_2 + \sigma][\frac{\omega_2}{M} + \hat{m}_2 - \sigma]}\}$$

which is equal to

$$\frac{-1}{2\omega_2(\omega_1^2 - \omega_2^2 - M^2 - 2M\omega_2)}$$

hence substitution automatically reveals that:

$$I_1 = \int_{-\infty}^{\infty} \frac{M d\sigma}{\Delta_1^2\Delta_2} = (2\pi)^3[\frac{2\omega_1^3 - M^2\omega_2 + 5\omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2^3}{4\omega_1^3\omega_2(M^2 - (\omega_1 + \omega_2)^2)^2}], \tag{4.1.37}$$

with the same procedure we can have:

$$I_2 = \int_{-\infty}^{\infty} \frac{M d\sigma}{\Delta_1^2\Delta_2}\sigma$$

$$= (2\pi)^3 \frac{-M^4\omega_2 + (m_1^2 - m_2^2)(\omega_1 + \omega_2)^2(2\omega_1 + \omega_2)M^2[6\omega_1^3 + 9\omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2(-m_1^2 + m_2^2 + \omega_2^2)]}{8M^2\omega_1^3\omega_2[M^2 - (\omega_1 + \omega_2)^2]^2} \tag{4.1.38}$$

$$I_3 = \int_{-\infty}^{\infty} \frac{M d\sigma}{\Delta_1^2\Delta_2}\sigma^2$$

$$= (2\pi)^3 \frac{1}{16M^4\omega_1^3\omega_2(-M^2+(\omega_1+\omega_2)^2)^2}\{-M^6\omega_2 + (m_1^2-m_2^2)^2(\omega_1+\omega_2)^2(2\omega_1+\omega_2)\}$$

$$+M^4[2\omega_1^3 - 2m_1^2\omega_2 + 2m_2^2\omega_2 + \omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2^3]$$

$$-M^2[m_1^4\omega_2 + m_2^4\omega_2 + 4\omega_1^2\omega_2(\omega_1+\omega_2)^2 + 2m_2^2(-2\omega_1^3 + \omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2^3)]$$

$$-2m_1^2(-2\omega_1^3 + m_2^2\omega_2 + \omega_1^2\omega_2 + 4\omega_1\omega_2^2 + \omega_2^3)]\} \tag{4.1.39}$$

$$I_4 = \int_{-\infty}^{\infty} \frac{Md\sigma}{\Delta_1^2\Delta_2}\sigma^3 = (2\pi)^3\{\frac{(M^2-m_1^2+m_2^2+2M\omega_2)^3}{8M^6\omega_2(M^2-\omega_1^2+2M\omega_2+\omega_2^2)^2}$$

$$+\frac{(M^2+m_1^2-m_2^2-2m\omega_1)^2[M^4+M^2(m_1^2-m_2^2-\omega_1^2-\omega_2^2)+(m_1^2-m_2^2)(3\omega_1^2-\omega_2^2)]}{16M^6\omega_1^3(M^2-2M\omega_1+\omega_1^2-\omega_2^2)^2}$$

$$+\frac{(M^2+m_1^2-m_2^2-2M\omega_1)^2[-4M\omega_1(m_1^2-m_2^2+\omega_2^2)]}{16M^6\omega_1^3(M^2-2M\omega_1+\omega_1^2-\omega_2^2)^2}\} \tag{4.1.40}$$

After this numerical integration over the 3-D variable $d^3\hat{q}$ in is performed to evaluate N_P. We have thus evaluated the general expressions for f_P and N_P in the framework of BSE under CIA, with Dirac structure in the H$q\bar{q}$ vertex function with the leading order structure as well next to leading order structure according to our power counting rule. We see that so far the results are independent of any model . However, for calculating the numerical values of these decay constants one needs to know the constant coefficients B_0, B_1, B_2 and B_3 which are associated with the Dirac structures. The relative value,B_1 $B_0, B_2, B_3$are a free parameter without any further knowledge of the meson structure in the framework discussed above. The numerical results on various pseudoscalar mesons calculated were[1]; $f_\pi = 0.130Gev(Exp = 0.130Gev), f_K = 0.164Gev(Exp = 0.159Gev), f_D = 0.194Gev(Exp = 0.220Gev), fB = 0.228Gev(Exp = 0.240Gev)$.

However in case of pion though the numerical results coincide with the experimental data, the contribution from LO is lesser than the contribution from NLO which is not in accordance with the power counting rule. To overcome this problems we propose in the next section an alternative way of calculating pion f_P which hopefully would raise the contribution of LO terms in comparison to NLO terms in the Hadron quark vertex. In the next section we discuss about the results.

4.2 Decay constant of pion

Decay constants describe the simplest electroweak transitions, where a meson couples directly to the photon or W boson. They are often used in more complicated calculations. Decay constants for some mesons could be determined from experimental data. What we do is to calculate the decay constants theoretically with specific QCD motivated models and after their theoretical values compare them with experimental data to see the predictive powers of our model. Thus comparison to the experimental data makes it possible to test the meson wave function in different models and help us understand our models better. In this section we present an alternative calculation of the decay constant of the lightest meson pion. As we have seen earlier decay constants f_P can be evaluated through the loop diagram which gives the coupling of the two-quark loop to the axial vector current and can be evaluated as [1]:

$$f_P P_\mu =< 0|\bar{Q} i\gamma_\mu\gamma_5 Q|P(P) > \tag{4.2.1}$$

which can, in turn, be expressed as a loop integral,[1]

$$f_P P_\mu = \sqrt{3}\int d^4q Tr[\Psi_P(P,q) i\gamma_\mu\gamma_5] \tag{4.2.2}$$

A Bethe-Salpeter wave function $\Psi(P,q)$ for a π-meson is expressed as,

$$\Psi(P,q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2), \tag{4.2.3}$$

with

$$S_F(p_1) = -i\frac{(m - i\not{p}_1)}{\Delta_1} \tag{4.2.4}$$

$$S_F(-p_2) = -i\frac{(m + i\not{p}_2)}{\Delta_2} \tag{4.2.5}$$

which is expressed as the quark and anti-quark propagators flanking the Hadron-quark vertex $\Gamma(\hat{q})$ function. Let's first evaluate the trace over the gamma matrices, by using

$\Psi(P,q)$ from Eqn(4.1.3) and incorporating $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ from Eqn (3.5.12)

$$Tr\{\Psi(P,q)i\gamma_\mu\gamma_5\} = Tr\{[-i\frac{(m-i\not{p}_1)}{\Delta_1}][\gamma_5 B_0 - i\gamma_5\frac{\not{P}}{M}B_1 - i\gamma_5\frac{\not{q}}{M}B_2 - \gamma_5(\not{P}\not{q}-\not{q}\not{P})\frac{B_3}{M^2}]$$
$$[-i\frac{(m+i\not{p}_2)}{\Delta_2}i\gamma_\mu\gamma_5]\} \tag{4.2.6}$$

This can in turn be expressed as

$$Tr\{\Psi(P,q)i\gamma_\mu\gamma_5\} = \frac{-1}{M^2\Delta_1\Delta_2}Tr\{(m-i\not{p}_1)[\gamma_5 B_0 M^2 - i\gamma_5\not{P}B_1 M - i\gamma_5\not{q}B_2 M]$$
$$-\gamma_5(\not{P}\not{q}-\not{q}\not{P})B_3](m+i\not{p}_2)i\gamma_\mu\gamma_5\} \tag{4.2.7}$$

After expanding the above expression, we can have the trace results:

$$-4B_0 m\frac{p_{2\mu}}{\Delta_1\Delta_2} - 4B_0 m\frac{p_{1\mu}}{\Delta_1\Delta_2} - 4B_1 m^2\frac{P_\mu}{M\Delta_1\Delta_2} + 4B_1\frac{(p_2.p_1)P_\mu - (p_2.P)p_{1\mu} + (p_1.P)p_{2\mu}}{M\Delta_1\Delta_2}$$
$$-4B_2 m^2\frac{q_\mu}{M\Delta_1\Delta_2} + 4B2\frac{(p_2.p_1)q_\mu - (p_2.q)p_{1\mu} + (p_1.q)p_{2\mu}}{M\Delta_1\Delta_2} + 4B_3 m\frac{(P.q)p_{2\mu} - (p_2.P)q_\mu + (p_2.q)P_\mu}{M^2\Delta_1\Delta_2}$$
$$+4B_3 m\frac{(P.q)p_{1\mu} - (p_1.P)q_\mu + (p_1.q)P_\mu}{M^2\Delta_1\Delta_2} - 4B_3 m\frac{(q.P)p_{2\mu} - (p_2.q)P_\mu + (P.p_2)q_\mu}{M^2\Delta_1\Delta_2} -$$
$$4B_3 m\frac{(q.P)p_{1\mu} - (p_1.q)P_\mu + (P.p_1)q_\mu}{M^2\Delta_1\Delta_2} \tag{4.2.8}$$

To simplify the above results let's make use of the dot products of different momenta,

$$p_1.p_1 = \Delta_1 - m^2$$

$$p_2.p_2 = \Delta_2 - m^2$$
$$P.\hat{q} = \frac{1}{2}(\Delta_2 - \Delta_1)$$
$$p_1.P = \frac{1}{2}(\Delta_1 - \Delta_2 - M^2)$$
$$p_2.P = \frac{1}{2}(\Delta_2 - \Delta_1 - M^2)$$
$$p_1.\hat{q} = \frac{1}{4}(3\Delta_1 + \Delta_2 + M^2) - m^2$$
$$p_2.\hat{q} = m^2 - \frac{1}{4}(\Delta_1 + 3\Delta_2 + M^2)$$

$$p_1.p_2 = m^2 - \frac{1}{2}(\Delta_1 + \Delta_2 + M^2) \tag{4.2.9}$$

The four dimensional volume element will have the form:

$$d^4q = d^3\hat{q}Md\sigma \tag{4.2.10}$$

We can express the leptonic decay constant f_P as:

$$f_P = \sqrt{3}N_P \int d^3\hat{q}\phi(\hat{q})I = f_P^{(0)} + f_P^{(1)} + f_P^{(2)} + f_p^{(3)} \tag{4.2.11}$$

where $f_P^{(0)}, f_P^{(1)}, f_P^{(2)}, f_P^{(3)}$, are the contributions to f_P from the four Dirac covariants associated with coefficients $B_i(i = 0, 1, 2, 3)$, and are expressed as:

$$f_P^{(0)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q}) \int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i}[\frac{4B_0 m}{\Delta_1\Delta_2}] \tag{4.2.12}$$

$$f_P^{(1)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q}) \int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i}[\frac{2B_1}{M\Delta_1} + \frac{2B_1}{M\Delta_2} + \frac{2B_1 M}{\Delta_1\Delta_2}] \tag{4.2.13}$$

$$f_P^{(2)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q}) \int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i}[\frac{2B_2\sigma}{M\Delta_1} + \frac{2B_2\sigma}{M\Delta_2} + \frac{2B_2 M\sigma}{\Delta_1\Delta_2}] \tag{4.2.14}$$

$$f_P^{(3)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q}) \int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i}[-\frac{8B_3 m}{M^2\Delta_1} - \frac{8B_3 m}{M^2\Delta_2} - \frac{4B_3 m}{\Delta_1\Delta_2} + \frac{16B_3 m^3}{M^2\Delta_1\Delta_2}] \tag{4.2.15}$$

Our next task lies on the evaluation of the analytical values of the contour integral:

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_1\Delta_2} &= \frac{1}{D(\hat{q})}; \\
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_1} &= \frac{1}{2\omega_1} \\
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_2} &= \frac{1}{2\omega_2} \\
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_1}\sigma &= \frac{(\frac{\omega_1}{M} - \frac{1}{2})}{2\omega_1 M} \\
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_2}\sigma &= \frac{(\frac{\omega_2}{M} + \frac{1}{2})}{2\omega_2 M} \\
\int_{-\infty}^{\infty} \frac{Md\sigma}{2\pi i\Delta_1\Delta_2}\sigma &= \frac{(\omega_2 - \omega_1)}{4\omega_1\omega_2[M^2 - (\omega_1 + \omega_2)^2]}
\end{aligned} \tag{4.2.16}$$

with the above substitution we can immediately have the contribution of the decay constant

$$f_P = f_P^{(0)} + f_P^{(1)} + f_P^{(2)} + f_P^{(3)} \tag{4.2.17}$$

where

$$f_P^{(0)} = 4\sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q})\frac{B_0 m}{D(\hat{q})}, \tag{4.2.18}$$

$$f_P^{(1)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q})[\frac{B_1}{M\omega_1} + \frac{B_1}{M\omega_2} + \frac{2B_1 M}{D(\hat{q})}], \tag{4.2.19}$$

$$f_P^{(2)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q})[\frac{2B_2}{M}\frac{(\frac{\omega_1}{M} - \frac{1}{2})}{2\omega_1 M} + \frac{2B_2}{M}\frac{(\frac{\omega_2}{M} + \frac{1}{2})}{2\omega_2 M} + \frac{2B_2 M}{D(\hat{q})}], \tag{4.2.20}$$

$$f_P^{(3)} = \sqrt{3}N_P \int d^3\hat{q}D(\hat{q})\phi(\hat{q})[-\frac{8B_3 m}{M^2}\frac{1}{2\omega_1} - \frac{8B_3 m}{M^2}\frac{1}{2\omega_2} - \frac{4B_3 m}{D(\hat{q})} + \frac{16B_3 m^3}{M^2 D(\hat{q})}]. \tag{4.2.21}$$

We have calculated the decay constant of pseudoscalar meson using covariant instantaneous ansatz by making use of power counting rule order by order in powers of the reciprocal mass of the pion. We have shown in the previous sections that in case of the pion the major contribution for the decay constant is resulted from the NLO rather than the LO Dirac covariants in contrast to other heavy mesons. That is why we tried to modify the dot products of the constituent momenta in the manner shownin eqn(4.2.9). This modification i.e expressing the dot products(only in terms of Δ) of the constituent momenta is expected to make LO contribution higher than NLO contribution to f_P for pion. That means major of the contribution for f_P should come from the Leading-Order-Covariants rather than Next-to-Leading-Covariants. It was earlier seen that decay constants of pseudoscalar mesons including pion from our theoretic point successfully coincided with the experimental values for the psudoscalar mesons [1]. However for pion, in the earlier calculation, the Lo contribution was lower than NLO contribution for f_P. With the treatment for pion we now expect the LO contribution to dominate the NLO contribution for pion in line with other mesons calculated earlier. Concerning to the numerical results we have to parametrize the calculation for pion.

Chapter 5

Conclusion

In this thesis we have studied the decay constants f_P [1,8] of unequal mass pseudoscalar mesons in BSE under Covariant Instantaneous Ansatz (CIA) using Hadron-quark vertex function which incorporates various Dirac covariants order-by-order in powers of inverse of meson mass within its structure in accordance with a power counting rule from their complete set. This power counting rule[1,3,8] suggests that the maximum contribution to any meson observable should come from Dirac structures associated with Leading Order (LO) terms alone, followed by Dirac structures associated with Next-to-Leading Order (NLO) terms in the vertex function. Incorporation of all these covariants is found to bring calculated f_P values much closer to results of experimental data [27] and some recent calculations [4-6, 28]. The f_P predicted are within the error bars of experimental data for each one of these five mesons [1]. They were worked out earlier as [1]: $f_\pi = 0.130Gev(Exp = 0.130Gev), f_K = 0.164Gev(Exp = 0.159Gev), f_D = 0.194Gev(Exp = 0.220Gev), f_B = 0.228Gev(Exp = 0.240Gev)$.

It was seen in the earlier calculation[1] that the contribution to f_P from Lo covariants was much higher than the NLO contribution for unequal mass pseudoscalar mesons. This is in conformity with the power counting rule according to which the leading order covariants, should contribute maximum to decay constants followed by the next-to-leading order covariants, γ_5 and $i\gamma_5\frac{\not{P}}{M}$ (associated with coefficients B_0 and B_1) should contribute maximum to decay constants followed by the next-to-leading order covariants, $-i\gamma_5\frac{\not{q}}{M}$ and $-\gamma_5\frac{(\not{P}\not{q}-\not{q}\not{P})}{M^2}$ (associated with coefficients B_2 and B_3) in the BS wave function. The numerical results for f_P obtained in this framework with use of leading order covariants along with the next to leading order covariants demonstrates the validity of the power counting rule, which also provides a practical means of incorporating various Dirac covariants in the BS wave function of a hadron. By this rule, we also get to understand the relative importance of various covariants to calculation of meson observables. This would in turn help in obtaining a better understanding of the hadron structure. However, the situation is different for the lightest meson π which enjoys a unique status due to the fact that mass of a pion, M is unusually small ($q << \Lambda_{QCD}$), and the large difference between the sum of two constituent quark masses and the pion mass shows that the quarks are far off shell and the internal momentum q should be of the same order as the pion mass and the approximation $q << P \sim M$ breaks down for pion. Hence the contribution of NLO covariants in pion case is larger than the contribution of LO covariants owning to the fact that pion is Goldstone boson. To overcome this problem in this thesis we have given an alternative way to calculate the pion f_P where there are no divergences in the integrals to be regularized. Due to this fact we strongly expect the LO contribution to dominate the NLO contribution for pion to make it in line with the power counting rule proposed earlier. This would in turn lead to a better understanding of the Hadron structure.

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DECLARATION

I here under signed declare that the thesis is my original work, has not been presented for a degree in any other university and that all sources of material used for the thesis have been dully acknowledged.

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This thesis has been submitted for examination with my approve as a university advisor.

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