DRIFT OF CHARGED PARTICLES IN HIGH FREQUENCY FIELDS

A Thesis

Presented to

The School of Graduate Studies

and

The Faculty of Science

Addis Ababa University

In Partial Fulfillment of the

Requirements for the Degree

Master of Science in Physics

By

Ghebregziabiher Kelati

June, 1985

ADDIS ABABA

ADDIS ABABA UNIVERSITY

School of Graduate Studies

DRIFT OF CHARGED PARTICLES IN HIGH FREQUENCY FIELDS

by

Ghebregziabiher Kelati

Faculty of Science

Approved by the Examining Board:

Dr. Andreas Brauer External Examiner	[signature]
Dr. V. Schepilov Advisor	[signature]
Dr. S.C. Chhajlany Member	[signature]
Dr. A. Spartakov Chairman	[signature]

ACKNOWLEDGEMENT

I am deeply grateful to Dr. Schepilov, my advisor and instructor, for his sound comments and criticisms and for doing his best to help me finish this work in time.

I would also like to express my best thanks to my instructors and colleagues in the Physics Department for their valuable suggestions and constant encouragement they have offered me.

Finally I extend my special thanks to W/O Azeb Beley for typing the final draft of the thesis with great care.

CONTENTS

Page

INTRODUCTION 1

CHAPTER 1

INTERACTION OF CHARGED PARTICLES WITH ELECTROMAGNETIC FIELDS 4

1.1 Motion of Charged Particles in Uniform and Constant Electric and Magnetic Fields 4

1.1.1 Magnetic Field only 6

1.1.2 Electric Field Parallel to the Magnetic Field 7

1.1.3 Electric Field Perpendicular to the Magnetic Field 8

1.1.4 Effects of Constant External Forces 10

1.2 Static Inhomogeneous Magnetic Field 11

1.2.1 Effects of Angular Divergence of $\vec{B}$ Lines 12

1.2.2 Effects of Curvature of $\vec{B}$ Lines 13

1.2.3 Effects of Gradient of B in a Direction Perpendicular to $\vec{B}$ 14

1.2.4 Effect of Shears 16

1.3 Time Dependent Magnetic Field 16

1.4 Time Dependent Electric Field 18

1.4.1 Low Frequency Electric Field 19

1.4.2 High Frequency Electric Field 21

1.4.3 Cyclotron Resonance 21

CHAPTER 2

WAVE PROPAGATION IN PLASMA 23

2.1 The Dielectric Tensor 23

2.2 Anisotropy of Magnetized Plasma 27

2.3 Dispersion Relation 28

Page

2.4 Cut-off and Resonance Frequencies 31

2.5 Wave Propagation in Plasmas without External Magnetic Field 32

2.6 Wave Propagation in Magnetized Plasma 32

2.6.1 Propagation Across $\vec{B}_o$ 32

2.6.2 Propagation Parallel to $\vec{B}_o$ 34

CHAPTER 3

MOTION OF CHARGED PARTICLES IN THE PRESENCE OF WHISTLER WAVE 37

3.1 Equation of Motion of an Electron 37

3.2 Averaged Equations of Motion of Electrons in Cyclotron Resonance 44

3.3 Analysis of the Averaged Equations of Motion of Charged Particles in Cyclotron Resonance 46

CONCLUSION 52

REFERENCES 53

ABSTRACT

The non-relativistic equations of motion for a charged particle, an electron, in a wave of slowly varying amplitude are examined. The wave, whistler wave, is supposed to be propagating at an arbitrary but small angle to a uniform background magnetic field, in an infinite collisionless plasma. Approximate solutions for the equations of motion of particles in cyclotron resonance with the shistler wave are evaluated by method of averaging. An expression for the period of trapped particles in a "potential well" and an integral of motion in the space of phase difference are derived.

INTRODUCTION

All of us are familiar with the classification of matter into three states - solids, liquids, and gases. There is a fourth state of matter also. It is called plasma. Plasma is the most common state of matter in nature. It may be defined as a state of matter at temperature so high that an appreciable fraction of atoms and molecules are dissociated into ions and electrons. Often one also calls the electron gas in metals and semi-conductors a plasma. In this thesis, however, we have in mind only a gaseous plasma, for description of which it is sufficient to restrict ourselves to classical approximations. It is just this kind of plasma which one encounters in most cases in astrophysics, the physics of the ionosphere, and when analyzing the problem of thermonuclear fusion using a rarified plasma.

Although plasma may be regarded as a special form of gas of ions, electrons and neutral particles, there are many important physical properties which make it different from ordinary gas containing only neutral particles. These differences are specially evident in the behavior of plasma in electric and magnetic fields. Plasma exhibits strong interaction with electromagnetic waves.

As in any problem which deals with a large ensemble of individuals, plasma physics uses two complementary modes of description: the analysis of the movement of single particle and fluid model. In order to understand the specific properties, which are exhibited by plasma in its interaction with electric and magnetic fields, the behavior of individual electrons and ions which make up the plasma will be considered. In the presence of external magnetic field, the motion of charged particles in plasma are quite complex and the problem of finding an exact solution of their equation of motion in a wave propagating through magnetized plasma is rather complicated. Thus, in many cases approximation methods of investigation are employed and the most effective one at present time is the method of averaging. The method of averaging can be applied in the derivation of approximate solutions to the equations of charged particles in fast varying electromagnetic fields.

In a few cases, exact solutions for the equation of motion are found. Lutomirski and Sudan [1] solved exactly the nonlinear, relativistic equations of motion for an electron moving in a right circularly polarized wave which propagates along a static uniform magnetic field. In the wave frame where the induction electric field disappears, they found two constants of motion with the help of which particle trajectories were examined and the periods and amplitudes of oscillation in the direction of the magnetostatic field were obtained.

Interaction of spiral monochromatic waves with resonant particles and the resulting non-linear effects were discussed in [2]. Investigations of non-linear effects of whistler waves propagating at an angle to magnetic field is popular at the present time because of great interest connected with experimental investigation in magnetosphere and in laboratory [3]. In the work of P.J. Palmadesso [4] the case of plane monochromatic wave propagating at an angle to a magnetic field in a homogeneous plasma was consider ed. From his work it follows that interchange of energy between wave and particle stops when ergodic distribution in resonance region of velocity space is established.

The main objective of this thesis is the investigation of the behavior of charged particles in non-homogeneous, collisionless magnetized plasma when a whistler wave propagates at an angle to a uniform magnetostatic field. The motion of a charged particle in various electric and magnetic field configurations will be investigated first. This will be followed by a discussion of the nature of wave propagation in both unmagnetized and magnetized plasma. Then the equations of motion of charged particles, especially electrons, in the fields of whistler wave will be derived. Using method of averaging approximate solutions for the equations will be found. Cyclotron resonance interactions of the particles and waves will be considered. Throughout the thesis rationalized mks units will be used.

CHAPTER 1

INTERACTION OF CHARGED PARTICLES WITH ELECTROMAGNETIC FIELDS

The dynamics of charged particles in a plasma is determined by the Lorentz force contributed by the electric and magnetic fields arising from external source and from their own motion as well as those due to the other charged particles. Orbit theory gives insight into physical phenomena that determine the plasma behavior, provided that all interactions between its constituent particles are neglected with the result that the plasma reduces to a free gas of electrons, ions and neutral particles. This assumption is useful in predicting the behavior of highly rarified plasma, since the behavior of such plasma is determined primarily by interactions of charged particles with external magnetic fields rather than with themselves [5].

1.1 Motion of Charged Particles in Uniform and Constant Electric and Magnetic Fields

A charged particle, in the presence of electric field $\vec{E}$ and magnetic field $\vec{B}$, is acted on by two forces; an electrical force which is parallel to the electric field and a magnetic force perpendicular to both the particle velocity and the magnetic field. The motion of the particle is governed by the following Newton's law of motion :

$$m\frac{d\vec{V}}{dt} = q\ (\vec{E} + \vec{V} \times \vec{B}) \tag{1.1}$$

where m, $\vec{V}$, and q are particle's mass, velocity and charge respectively.

If we introduce the quantity

$$\vec{\omega}_c = \frac{-q\vec{B}}{m}$$

which is the cyclotron frequency of the particle about the magnetic field lines then eq.(1.1) becomes

$$\frac{d\vec{V}}{dt} = \frac{q\vec{E}}{m} + (\vec{\omega}_c \times \vec{V}) \qquad (1.2)$$

Assuming $\vec{B}$ to be along the z-axis of the coordinate system, the three components of eq.(1.2) are

$$\frac{dV_x}{dt} = \frac{qE_x}{m} - \omega_c V_y \qquad (1.3a)$$

$$\frac{dV_y}{dt} = \frac{qE_y}{m} + \omega_c V_x \qquad (1.3b)$$

$$\frac{dV_z}{dt} = \frac{qE_z}{m} \qquad (1.3c)$$

Solving eqs.(1.3a) and (1.3b) simultaneously and integrating eq (1.3c), we obtain the velocity components as

$$V_x = (V_{xo} + \frac{qE_y}{m\omega_c}) \cos \omega_c t - V_{yo} \sin \omega_c t - \frac{qE_y}{m\omega_c} \qquad (1.4a)$$

$$V_y = (V_{xo} + \frac{qE_y}{m\omega_c}) \sin \omega_c t + V_{yo} \cos \omega_c t + \frac{qE_x}{m\omega_c} \qquad (1.4b)$$

$$V_z = V_{zo} + \frac{qE_z}{m} t \qquad (1.4c)$$

Integration of these equations yields the components of the position of the particle which are

$$x = x_o + \frac{1}{\omega_c} \{(V_{xo} + \frac{qE_y}{m\omega_c}) \sin \omega_c t + V_{yo}(\cos \omega_c t - 1) - \frac{qE_y}{m} t\} \quad (1.5a)$$

$$y = y_o + \frac{1}{\omega_c} \{(V_{xo} + \frac{qE_y}{m\omega_c})(1 - \cos \omega_c t) + V_{yo} \sin \omega_c t + \frac{qE_y}{m} t\} \quad (1.5b)$$

$$z = z_o + V_{zo} t + \frac{1}{2m} qE_z t^2 \quad (1.5c)$$

where $\vec{r}_o = (x_o, y_o, z_o)$ and $\vec{V}_o = (V_{xo}, V_{yo}, V_{zo})$ are the initial position and velocity of the particle respectively.

1.1.1 Magnetic Field Only

The motion of a charged particle in a uniform and constant magnetic field can be examined by setting $E_x = E_y = E_z = 0$ in eqs.(1.5). The components of particle position are then

$$x = x_o + \frac{V_{yo}}{\omega_c} (\cos \omega_c t - 1) + \frac{V_{xo}}{\omega_c} \sin \omega_c t \quad (1.6a)$$

$$y = y_o + \frac{V_{yo}}{\omega_c} \sin \omega_c t - \frac{V_{xo}}{\omega_c} (\cos \omega_c t - 1) \quad (1.6b)$$

$$z = z_o + V_{zo} t \quad (1.6c)$$

From eqs. (1.6a) and (1.6b) it can be shown that

$$\sqrt{(x - X)^2 + (y - Y)^2} = \frac{V_{\perp o}}{|\omega_c|} = r_L \quad (1.7)$$

where $X = x_o - \frac{V_{yo}}{\omega_c}$

$Y = y_o + \frac{V_{xo}}{\omega_c}$

and $V_{\perp o}^2 = V_{xo}^2 + V_{yo}^2$ = constant

Examining eqs.(1.6) and (1.7) we can conclude that the trajectory of the particle is helix with a) its axis parallel to the magnetic field and passing through the point (X,Y) (b) "Larmor" radius r_L and (c) pitch equal to $2\pi V_{zo}/V_{\perp o}$

If $V_{\perp o} = 0$, the trajectory is a straight line parallel to B and if $V_{zo} = 0$, the trajectory is a circle of the radius r_L and center (X,Y) which is called the guiding center. In general the guiding center moves in the direction of the magnetic field with a constant velocity V_{zo}. We also note that the speed of the particle is unchanged because the magnetic field has no component in the direction of motion of the particle and hence does no work on the particle.

1.1.2 Electric Field Parallel to the Magnetic Field

Consider a charged particle in a combination of electric and magnetic fields which are both in the z-direction. For this case we set $E_x = E_y = 0$ in eqs.(1.5) and obtain

$$x = x_o + \frac{V_{yo}}{\omega_c}(\cos \omega_c t - 1) + \frac{V_{xo}}{\omega_c}\sin \omega_c t \qquad (1.8a)$$

$$y = y_o + \frac{V_{yo}}{\omega_c}\sin \omega_c t - \frac{V_{xo}}{\omega_c}(\cos \omega_c t - 1) \qquad (1.8b)$$

$$z = z_o + V_{zo}\, t + \frac{1}{2}\frac{qE_z}{m}\, t^2 \qquad (1.8c)$$

Comparison of eqs.(1.8) with the corresponding eqs.(1.6) shows that the particle motion in the X-Y plane are identical. The only effect of the imposed electric field is to produce acceleration in the z-direction which depends on the sign and magnitude of the charge on the particle. Thus, the trajectory of the particle will be either a gradually extending or gradually contracting helix (depending on the relative directions of the electric field and the initial longitudinal velocity component).

1.1.3 Electric Field Perpendicular to the Magnetic Field

Consider the motion of a charged particle in a combination of magnetic and electric fields which are perpendicular to each other. Assume the electric field to be in the y-direction and the magnetic field in the z-direction. Let the initial velocity and position be $\vec{V}_o = (V_{xo}, V_{yo}, 0)$ and $\vec{r}_o = (x_o, y_o, 0)$ respectively. For this case eqs.(1.4) and (1.5) become

$$V_x = (V_{xo} + \frac{qE_y}{m\omega_c}) \cos \omega_c t - V_{yo} \sin \omega_c t - \frac{qE_y}{m\omega_c} \tag{1.9a}$$

$$V_y = (V_{xo} + \frac{qE_y}{m\omega_c}) \sin \omega_c t + V_{yo} \text{Cos } \omega_c t \tag{1.9b}$$

$$V_z = 0$$

and
$$x = x_o + \frac{1}{\omega_c} \{V_{yo}(\cos \omega_c t - 1) + (V_{xo} + \frac{qE_y}{m\omega_c}) \sin \omega_c t - \frac{qE_y}{m} t\} \tag{1.10a}$$

$$y = y_o + \frac{1}{\omega_c} \{V_{yo} \sin \omega_c t - (V_{xo} + \frac{qE_y}{m\omega_c}) (\cos \omega_c t - 1)\} \tag{1.10b}$$

$$z = 0 \tag{1.10c}$$

These equations show that the motion is entirely in the x-y plane and consists of circular motion at cyclotron frequency upon which is superimposed a constant drift perpendicular to both the electric and magnetic field as shown in fig. (1.1). The drift velocity for this case is:

$$V_E = \frac{-qE_y}{m\omega_c} = \frac{E_y}{B_z}$$

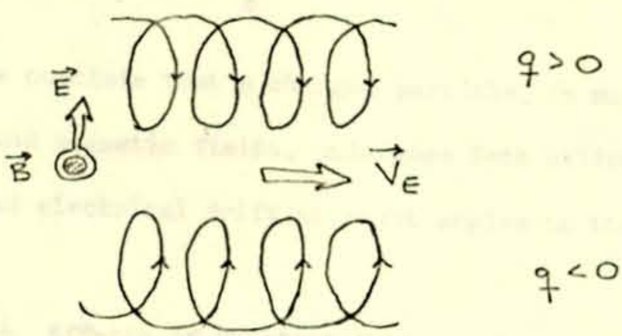


Fig. 1.1: The $\vec{E} \times \vec{B}$ drift

In general it is possible and convenient to separate the electric field and velocity of particle into two parts as follows

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp} \text{ and } \vec{V} = \vec{V}_{\parallel} + \vec{V}_{\perp} \qquad (1.11)$$

where the subscripts ' $\parallel$ ' and ' $\perp$ ' denote the components of a quantity parallel and perpendicular to the direction of the magnetic field respectively. As it was shown in sub-section (1.1.2) the component $E_{\parallel}$ produces a uniform acceleration along the magnetic lines of force. For the perpendicular motion we have

$$\frac{d\vec{V}_{\perp}}{dt} = \frac{q}{m}\left[\vec{E}_{\perp} + \vec{V}_{\perp} \times \vec{B}\right] \qquad (1.12)$$

Let a constant velocity $\vec{V}_E$ be a particular solution of the above equation. Then we obtain.

$$\vec{E} + \vec{V}_E \times \vec{B} = 0 \tag{1.13}$$

and on cross-multiplying this equation by $\vec{B}$, we will get a general expression for the electrical drift velocity

$$\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2} \tag{1.14}$$

we can now conclude that a charged particle, in mutually perpendicular electric and magnetic fields, undergoes both helical motion under the action of $\vec{B}$ and an electrical drift at right angles to the magnetic lines of force.

1.1.4 Effects of Constant External Forces

When an external force field is applied to a plasma in a magnetic field $\vec{B}$, there appears a drift motion of charged particles in the direction perpendicular to both $\vec{F}$ and $\vec{B}$. The equation of motion for the particle is

$$\frac{m d\vec{V}}{dt} = \vec{F} + q\vec{V} \times \vec{B} \tag{1.15}$$

For the components parallel to $\vec{B}$ eq.(1.15) describes a motion of constant acceleration. For the perpendicular component we have

$$\frac{m d\vec{V}_\perp}{dt} = \vec{F}_\perp + q\vec{V}_\perp \times \vec{B} \tag{1.16}$$

Defining $\vec{V}_F = \dfrac{\vec{F}_\perp \times \vec{B}}{B^2}$ (1.17

we substitute $\vec{V}_\perp = \vec{V}_F + \vec{V}_L$ (1.18) into (1.16); it then reduces to

$$m\frac{d\vec{V}_L}{dt} = q\vec{V}_L \times \vec{B} \qquad (1.19)$$

an equation of motion for a simple cyclotron motion: Eq.(1.16) describes a superposition of a uniform drift $\vec{V}_F$ and simple cyclotron motion $\vec{V}_L$.

When the force in particular arises from the electric field $\vec{E}$, we substitute $\vec{F} = q\vec{E}$ in eq.(1.17) to obtain

$$\vec{V}_E = \frac{\vec{E} \times \vec{B}}{B^2} \qquad (1.14)$$

and when the gravitational acceleration $\vec{g}$ is considered we have the gravitational drift.

$$\vec{V}_g = \frac{m}{qB^2}\vec{g} \times \vec{B} \qquad (1.20)$$

Since the direction of the drift $\vec{V}_F$ depends on the sign of the charge of the particle it produces a net current even in a neutral plasma.

1.2 Static Inhomogeneous Magnetic Field

We now discuss the effect of small spatial gradients of the magnetic field on orbits of a charged particle. We choose $\vec{B}$ to be in the z-direction at the origin. The gradient of $\vec{B}$ forms a tensor of nine parameters which may convinіently be grouped into four catagories [5]

a) Divergence terms: $\frac{\partial B_x}{\partial x}$ $\frac{\partial B_y}{\partial y}$ $\frac{\partial B_z}{\partial z}$

b) Gradient terms: $\frac{\partial B_z}{\partial x}$ $\frac{\partial B_z}{\partial y}$

c) Curvature terms: $\frac{\partial B_x}{\partial z}$ $\frac{\partial B_y}{\partial z}$

d) Shear terms: $\frac{\partial B_x}{\partial y}$ $\frac{\partial B_y}{\partial x}$

It is assumed that changes in the components of the magnetic field in a distance of the order of Larmor radius are very small in comparison with B. Because the resultant effect is the sum of the separate effects due to the constituent groups of terms we treat the effect of each group of terms separately.

1.2.1 Effects of Angular Divergence of $\vec{B}$ lines

For simplicity, it is assumed that the magnetic flux lines are cylinderically symetric about the z-axis. Then using cylinderical coordinates we have, from Maxwell's equation,

$$\nabla \cdot \vec{B} = \frac{1}{r} \frac{\partial}{\partial r} (r B_r) + \frac{\partial B_z}{\partial z} = 0 \tag{1.21}$$

Taking $\frac{\partial B_z}{\partial z}$ as constant for the range $0 < r < r_L$, integration of eq.(1.21) over one orbit of Larmor radius yields:

$$B_r = \frac{-r_L}{2} \frac{\partial B_z}{\partial z} = -\frac{r_L}{2} \frac{\partial B}{\partial z} \tag{1.22}$$

The magnetic field is essentially in the axial direction with the result that the orbital velocity $\vec{V}_L$ is in the Θ-direction. Therefore with the help of eq.(1.22) it can be shown that the longitudinal component of force is given by,

$$F_{\parallel} = \frac{-W_{\perp}}{B} \frac{\partial B}{\partial z} = -\mu \frac{\partial B}{\partial z} \qquad (1.23)$$

where $W_{\perp} = \frac{1}{2} m V_{\perp}^2$ is the kinetic energy of the particle associated with its transverse motion and $\mu = \frac{W_{\perp}}{B}$ is the magnitude of the magnetic moment.

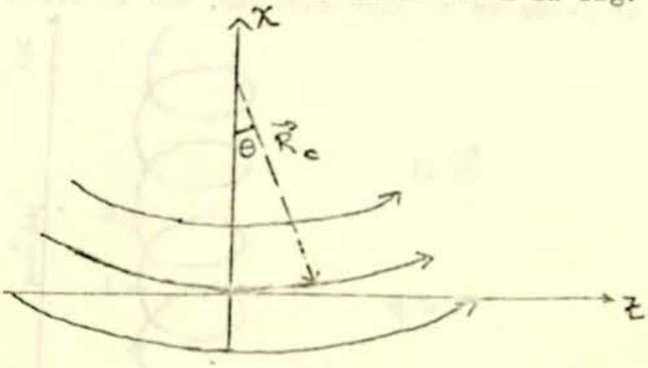
So long as the magnetic field does not change appreciably in space over several Larmor radii and in time over several Larmor periods, one supposes that the magnetic moment of a charged particle is a constant of motion [6]. Because of the constancy of μ, the force $F_{\parallel}$ given by eq.(1.23), can be derived from a potential U

$$F_{\parallel} = \frac{-\partial U}{\partial z} \text{ and } U = \mu B \qquad (1.24)$$

From this it can be seen that if a particle runs into a region in which B becomes large enough so that μB exceeds the total energy of the particle, it will be reflected. The phenomena of reflection of charged particles from a region of sufficiently strongly converging magnetic flux lines is called the "magnetic mirror" effect.

1.2.2 Effect of Curvature of $\vec{B}$ Lines

The terms $\frac{\partial B_x}{\partial z}$ and $\frac{\partial B_y}{\partial z}$ have similar effects. If $\frac{\partial B_x}{\partial z}$, for example is positive, the $\vec{B}$ lines are curved as shown in fig. (1.2);



The guiding center of the particle moves along the lines of force through the origin and experiences the centrifugal force [7].

$$\vec{F}_c = \frac{mV_{\parallel}^2 \vec{R}_c}{R_c^2} \qquad (1.25)$$

where $\vec{R}_c$ is the radius of curvature of $\vec{B}$ lines at some point which is given by

$$\frac{\vec{R}_c}{R_c^2} = -(\vec{n}\cdot\vec{\nabla})\vec{n}, \text{ and } \vec{n} = \frac{\vec{B}}{B} \qquad (1.26)$$

The force given by (1.25) is perpendicular to $\vec{B}$ and can be thought as an extraneous force as discussed in sub-section (1.1.4). By eq.(1.17) we expect it to produce the curvature drift velocity of guiding center.

$$\vec{V}_c = \frac{\vec{F}_c \times \vec{B}}{qB^2} = \frac{2V_{\parallel}}{qB^2R_c^2}\,\vec{R}_c \times \vec{B} \qquad (1.27)$$

The drift can produce an electric current in neutral plasma.

1.2.3 Effect of Gradient of B in a Direction Perpendicular to $\vec{B}$

If the terms $\frac{\partial B_z}{\partial x}$ and $\frac{\partial B_z}{\partial y}$ do not vanish, the magnitude of B_z varies in the direction perpendicular to B_z i.e, $B_z = B_z(x,y)$. When $\frac{\partial B_z}{\partial x} \neq 0$ and $\frac{\partial B_z}{\partial y} = 0$ the orbit of the particle will be as shown in fig. (1.3) for $\frac{\partial B_z}{\partial x} > 0$.

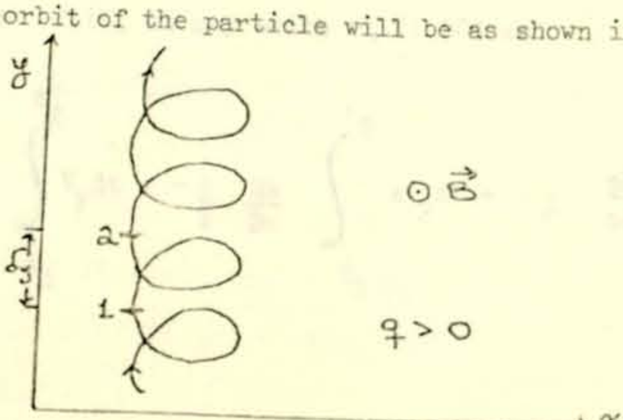


Fig. 1.3 Gyration of particle

The Larmor radius being smaller on the left than on the right a positive particle drifts upward while a negative charge drifts downward. In the x-direction the orbit has periodic variation but no net drift. The x-component of the force acting on the particle is

$$F_x = qV_y B_z(x) = qV_y \left(B_z(0) + x \frac{\partial B_z}{\partial x} + \dots\right) \qquad (1.28)$$

and integrating the equation of motion

$$\frac{m dV_x}{dt} = F_x$$

between the points 1 and 2 (fig, 1.3) where the particle has the same motion, we obtain

$$\int_{t_1}^{t_2} F_x dt = 0 \qquad (1.29)$$

or

$$\int_{t_1}^{t_2} V_y B_z dt = - \int_{t_1}^{t_2} x V_y \frac{\partial B_z}{\partial x} dt \qquad (1.30)$$

Since the field is slowly varying $\frac{\partial B_z}{\partial x} \simeq \frac{\partial B}{\partial x}$ and remains practically constant over one Larmor orbit, we may write eq.(1.30) as

$$\int_{t_1}^{t_2} V_y dt = - \frac{1}{B} \frac{\partial B}{\partial x} \int_{t_1}^{t_2} x v_y dt = - \frac{1}{B} \frac{\partial B}{\partial x} \oint x dy$$

or $$\delta y = \frac{1}{B}\frac{\partial B}{\partial x}\pi r_L^2 \qquad (1.31)$$

This gives for the drift velocity in the y-direction called the gradient drift.

$$V_G = \frac{\delta y}{T} = \frac{1}{B}\frac{\partial B}{\partial x}\pi r_L^2 \cdot \frac{qB}{2\pi m} = \frac{\mu}{qB}\cdot\frac{\partial B}{\partial x} \qquad (1.32)$$

Thus in general we can write for the gradient drift

$$\vec{V}_G = \frac{\mu}{qB^2}\,\vec{B} \times \nabla_\perp B \qquad (1.33)$$

A comparison of (1.33) with eq (1.17) shows that the variation of B in the transverse direction is equivalent to an external force $\vec{F}_G$ as given by

$$F_G = -\mu\,\nabla_\perp B = -\nabla(\mu B) \qquad (1.34)$$

which shows that μB acts as a kind of potential.

1.2.1 Effect of Shears

When the two remaining terms $\frac{\partial B_x}{\partial y}$ and $\frac{\partial B_y}{\partial x}$ are finite the lines of force twist or shear about a central line. This makes a component of $\vec{B}$ in the directions around the Larmor orbit, and when crossed with $V_\parallel$, gives a force. This force slightly changes, the Larmor radius or slightly changes the orbit from a circle, but produces no first order drifts.

1.3 Time Dependent Magnetic Field

In a uniform and constant magnetic field a charged particle gyrates about the field lines in a helical path. Suppose the uniform magnetic fie increases in time. Then by Faraday's law an electric field will be induced along the path of the orbit according to the equation.

$$\oint \vec{E}.\vec{dl} = -\frac{d}{dt}\int \vec{B}.\vec{ds} \qquad (1.35)$$

Assuming the magnetic field to be directed into the page, the induced electric field will be in the direction shown in fig. (1.4). From eq (1.35) we obtain

$$E_\theta = \tfrac{1}{2} r \frac{dB}{dt} \qquad (1.36)$$

We see that the direction of E_θ is such as to accelerate both electrons and positive ions.

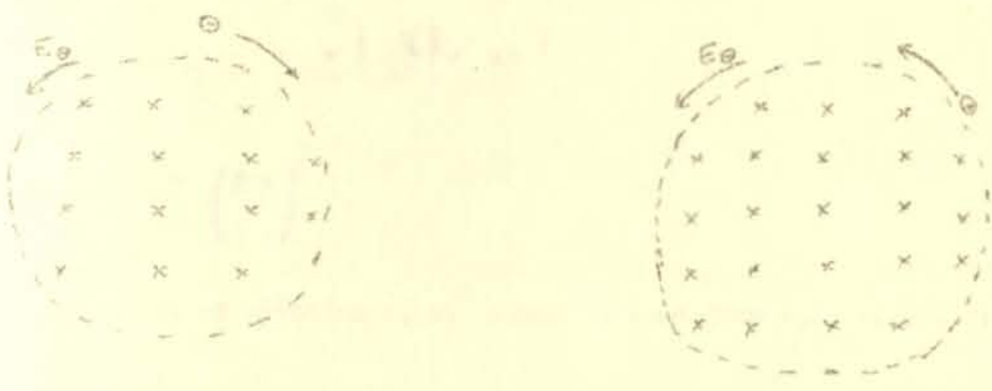


Fig. 1.4: Direction of induced electric field for $\frac{dB}{dt} > 0$.

If we assume that during a single gyration the magnetic field changes sufficiently slow so that $T \frac{dB}{dt} \ll B$, where T is the period of gyration, then we can use E_θ given by eq.(1.36) because the orbit can be considered nearly closed.

The increase in the transverse kinetic energy of the particle during one gyration is

$$\Delta W_\perp = qE_\theta 2\pi r \qquad (1.37)$$

The rate at which the kinetic energy of the particle increases is therefore

$$\frac{dW_\perp}{dt} = \frac{\Delta W_\perp}{T} = \frac{W_\perp}{B}\frac{dB}{dt} \qquad (1.38)$$

from which we obtain

$$\frac{W_\perp}{B} = \frac{\frac{1}{2}mV_\perp^2}{B} = \mu = \text{constant} \qquad (1.39)$$

The radius of gyration is given by

$$r = \left|\frac{mV_\perp}{qB}\right| = kB^{-\frac{1}{2}} \qquad (1.40)$$

where $k = \left(\frac{2\mu m}{q^2}\right)^{\frac{1}{2}}$

The radius of gyration r, as it can be seen from eq(1.40), decreases as B increases.

Time Dependent Electric Field

In this section we will consider the case in which the electric field is time dependent and uniform. The magnetic field is taken to be along the z-axis and the electric field along the y-axis given by

$$E(t) = E_y(t) = E \sin \omega t \qquad (1.41)$$

By substituting this into eqs (1.3) we obtain

$$\frac{dv_x}{dt} = -\omega_c v_y \qquad (1.42a)$$

$$\frac{dv_y}{dt} = a \sin \omega t + \omega_c v_x \quad (1.42b)$$

$$\frac{dV_z}{dt} = 0 \quad (1.42c)$$

where $a = \frac{qE}{m}$.

Assuming $\vec{V}_o = (0,0,0)$ and $\vec{r}_o = (0,0,0)$ eqs (1.42a) and (1.42b) can be solved simultaneously to yield

$$v_x = - \frac{a}{\omega^2 - \omega_c^2} (\omega \sin \omega_c t - \omega_c \sin \omega t) \quad (1.43a)$$

$$v_y = \frac{a}{\omega^2 - \omega_c^2} (\cos \omega_c t - \cos \omega t) \quad (1.43b)$$

integration of which gives

$$x = \frac{a}{\omega^2 - \omega_c^2} \left(\frac{-\omega}{\omega_c} \cos \omega_c t - \frac{\omega_c}{\omega} \cos \omega t\right) \quad (1.44a)$$

$$y = \frac{a}{\omega^2 - \omega_c^2} \left(\frac{\omega}{\omega_c} \sin \omega_c t - \sin \omega t\right) \quad (1.44b)$$

We next consider various ranges for the ratio ω/ω_c.

1.4.1 Low Frequency Electric Field ($\omega \ll \omega_c$)

For this case, eqs.(1.44) can be approximated as

$$x = \frac{a}{\omega\omega_c} \cos \omega t \quad (1.45a)$$

$$y = \frac{a}{\omega_c^2} \sin \omega t \qquad (1.45b)$$

which show that the drift motion is in the form of an ellipse with its major axis along the x-axis and the minor axis along the y-axis. The ratio of the semi-major to the semi-minor axis is $\omega_c/\omega >> 1$.

The equation of motion (1.16), together with (1.14) and (1.18), may be written as

$$\frac{m d\vec{V}_L}{dt} + \frac{m d\vec{V}_E}{dt} = q \vec{V}_L \times \vec{B} \qquad (1.46)$$

If we let

$$\vec{V}_L = \vec{V}_p + \vec{V}_c \qquad (1.47a)$$

with $$\vec{V}_p = \frac{m}{q} \frac{\vec{B}}{B^2} \times \frac{d\vec{V}_E}{dt} = \frac{m}{qB^2} \frac{d\vec{E}}{dt} \qquad (1.47b)$$

eq(1.46) will reduce to

$$\frac{m d\vec{V}_c}{dt} + \frac{m d\vec{V}_p}{dt} = q\vec{V}_c \times \vec{B} \qquad (1.48)$$

For $\omega << \omega_c$ and $V_E << V_c$ eq(1.48) becomes

$$\frac{m d\vec{V}_c}{dt} = q \vec{V}_c \times \vec{B} \qquad (1.49)$$

which shows that the motion is the usual circular motion upon which are superimposed the drifts $\vec{V}_E$ and $\vec{V}_p$. The drift $\vec{V}_p$ is called 'polarization drift'. It gives rise to a current density in neutral plasma.

1.4.2 High Frequency Electric Field ($\omega >> \omega_c$)

In this case eqs.(1.44) can be approximated by

$$x \simeq \frac{a}{\omega\omega_c} \cos \omega_c t \qquad (1.50a)$$

$$y \simeq \frac{a}{\omega\omega_c} \sin \omega_c t \qquad (1.50b)$$

which show a circular motion of radius $\frac{a}{\omega\omega_c}$ at the low cyclotron frequency ω_c.

In the case when $\omega \simeq \omega_c$ the coordinates of the particle are given as [8]

$$x = \frac{2a\omega}{\omega_c(\omega^2-\omega_c^2)} \sin \tfrac{1}{2}(\omega-\omega_c)t \sin \tfrac{1}{2}(\omega+\omega_c t) \qquad (1.51a)$$

$$y = \frac{2a\omega}{\omega_c(\omega^2-\omega_c^2)} \sin \tfrac{1}{2}(\omega-\omega_c)t \cos \tfrac{1}{2}(\omega+\omega_c)t \qquad (1.51b)$$

These equations describe a circular motion of frequency $(\omega+\omega_c)/4\pi$ whose radius of gyration is given by

$$r_L = \frac{2a\omega}{\omega_c(\omega^2-\omega_c^2)} \sin \tfrac{1}{2}(\omega-\omega_c)t \qquad (1.52)$$

which varies sinusoidally at the frequency $\frac{(\omega-\omega_c)}{4\pi}$

1.4.3 Cyclotron Resonance ($\omega = \omega_c$)

In a constant and uniform magnetic field, the charged particles have a natural frequency of oscillation called the cyclotron or gyromagnetic frequency. When a particle gyrating in such a field is also subjected to a time-varying electric field in such a way that both the particle and the field rotate in the same direction in synchronism, that is, with the same frequency

($\omega = \omega_c$) the particle is able to abstract energy from the field and thus has its speed increased continuously and indefinetly with time. This phenomenon is called cyclotron resonance.

In this case eq.(1.42a) and (1.42b) may be written as

$$\frac{dv_x}{dt} = -\omega_c V_y \tag{1.53a}$$

$$\frac{dv_y}{dt} = a \sin \omega_c t + \omega_c V_x \tag{1.53b}$$

and solving them simultaneously, we obtain

$$V_x = V_{xo} \cos \omega_c t - \frac{a}{2\omega_c} (\sin \omega_c t - \omega_c t \cos \omega_c t) \tag{1.54a}$$

$$V_y = V_{yo} \cos \omega_c t + V_{xo} \sin \omega_c t + \frac{a}{2} t \sin \omega_c t \tag{1.54b}$$

Integration of these equations yields

$$x = x_o + \frac{V_{xo}}{\omega_c} \sin \omega_c t + \frac{1}{\omega_c} (V_{yo} + \frac{a}{2\omega_c})(\cos \omega_c t - 1) + \frac{at}{2\omega_c} \sin \omega_c t \tag{1.55a}$$

$$y = y_o - \frac{V_{xo}}{\omega_c} (\cos \omega_c t - 1) + \frac{1}{\omega_c} (V_{yo} + \frac{a}{2\omega_c}) \sin \omega_c t - \frac{at}{2\omega_c} \cos \omega_c t \tag{1.55b}$$

For large t, the last terms of eqs (1.55) are dominant and the charged particles move in circles of ever increasing radii

$$r = \frac{at}{2\omega_c} \tag{1.56}$$

During this spiral motion the velocity of the particle continuously increases, which means that the particle absorbs energy from the electric field.

CHAPTER 2

WAVE PROPAGATION IN PLASMA

In this chapter we shall present the general theory of wave propagation in plasma. The wave amplitudes are taken to be small quantities to permit linearization of the relevant equations. For the sake of simplicity, we shall assume the ions to have a single charge, and use condition of quasi-neutrality. One can get a good physical picture of the various processes in plasma by neglecting the thermal motions of the constituent particles, that is, by considering the case of a cold plasma.

2.1 The Dielectric Tensor

The dielectric tensor contains essentially all information about electromagnetic properties of plasma. Therefore, for analysis of electromagnetic wave propagation in plasma, all the elements of a dielectric tensor must be determined.

The relative influence of electric and magnetic components of the wave fields upon the plasma particles is such that the effects due to electric components exceed those due to the magnetic field components when the particle velocities are small compared to the speed of light. Thus, in the discussion that follows, we neglect the effects of the wave magnetic fields. Furthermore, the magnetic field which results from the current distribution arising due to the motion of the charged particles is also assumed to be negligible compared with the external magnetic field $\vec{H}_0$

The linearized equation of motion for an electron is

$$m_e \frac{\partial \vec{V}}{\partial t} = -e \{ \vec{E} + \vec{V} \times \vec{B}_o \} \qquad (2.1)$$

We shall assume that the particle velocity and the electric field have temporal variations of the form exp ($i\omega t$). Then it may be shown that the phasor amplitudes of the velocity and electric field are related by

$$i\omega \vec{V}(\vec{r}) = \frac{-e}{m_e} \{ \vec{E}(\vec{r}) + \vec{V}(\vec{r}) \times \vec{B}_o \} \qquad (2.2)$$

When $\vec{B}_o$ is assumed to be along the z axis then the components of $\vec{V}(\vec{r})$ are expressed in terms of the components of $\vec{E}(\vec{r})$ as follows

$$\begin{bmatrix} V_x(\vec{r}) \\ V_y(\vec{r}) \\ V_z(\vec{r}) \end{bmatrix} = \frac{-ie}{m_e \omega} \begin{bmatrix} \frac{\omega^2}{\omega^2 - \omega_{ce}^2} & \frac{-i\omega\,\omega_{ce}}{\omega^2 - \omega_{ce}^2} & 0 \\ \frac{i\omega\,\omega_{ce}}{\omega^2 - \omega_{ce}^2} & \frac{\omega^2}{\omega^2 - \omega_{ce}^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_x(\vec{r}) \\ E_y(\vec{r}) \\ E_z(\vec{r}) \end{bmatrix} \qquad (2.3)$$

where $\omega_{ce} = \frac{eB_o}{m_e}$ is the electron cyclotron frequency

The linearized form of the current density $\vec{J}e$ arising from the motion of the electron is

$$\vec{J}_e = -e N_o \vec{V} = \vec{\Gamma}_e \cdot \vec{V} \qquad (2.4)$$

where $\overset{\leftrightarrow}{\Gamma}_e$ is the contribution to the electrical conductivity tensor arising from the electron motion. N_o is the electron or ion density.

Combining eqs (2.3) and (2.4) we obtain the matrix form of $\overset{\leftrightarrow}{\Gamma}_e$ as

$$\overset{\leftrightarrow}{\Gamma}_e = \frac{iN_o e^2}{m_e \omega} \begin{bmatrix} \frac{\omega^2}{\omega^2 - \omega_{ce}^2} & \frac{-i\omega\,\omega_{ce}}{\omega^2 - \omega_{ce}^2} & 0 \\ \frac{i\omega\,\omega_{ce}}{\omega^2 - \omega^2} & \frac{\omega^2}{\omega^2 - \omega_{ce}^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} . \qquad (2.5)$$

Similar calculations for ion motion result in identical form for ion conductivity tensor $\overset{\leftrightarrow}{\Gamma}_i$

$$\overset{\leftrightarrow}{\Gamma}_i = \frac{iN_o e^2}{m_i \omega} \begin{bmatrix} \frac{\omega^2}{\omega^2 - \omega_{ci}^2} & \frac{-i\omega\,\omega_{ci}}{\omega^2 - \omega_{ci}^2} & 0 \\ \frac{-i\omega\,\omega_{ci}}{\omega^2 - \omega_{ci}^2} & \frac{\omega^2}{\omega^2 - \omega_{ci}^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.6)$$

Where $\omega_{ci} = \frac{eB_o}{m_i}$ is the ion cyclotron frequency.

The plasma dielectic tensor $\overset{\leftrightarrow}{\varepsilon}_r$ is related to the electrical conductivity $\overset{\leftrightarrow}{\Gamma}$ by the equation

$$\overset{\leftrightarrow}{\varepsilon}_r = \overset{\leftrightarrow}{I} + \frac{i\overset{\leftrightarrow}{\Gamma}}{\omega\varepsilon_o} = \overset{\leftrightarrow}{I} + \frac{i\,(\overset{\leftrightarrow}{\Gamma}e + \overset{\leftrightarrow}{\Gamma}i)}{\omega\varepsilon_o} \qquad (2.7)$$

Where $\vec{I}$ is the unit tensor.

Substituting eqs (2.5) and (2.6) into (2.7), we obtain

$$\vec{\varepsilon}_r = \begin{bmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \qquad (2.8)$$

where

$$\varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \qquad (2.9a)$$

$$\varepsilon_2 = \frac{\omega_{pi}^2\,\omega_{ci}}{\omega(\omega^2 - \omega_{ci}^2)} - \frac{\omega_{pe}^2\,\omega_{ce}}{\omega(\omega^2 - \omega_{ce}^2)} \qquad (2.9b)$$

$$\varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (2.9c)$$

and $$\omega_{pe} = \left(\frac{Noe^2}{m_e\varepsilon_o}\right)^{\frac{1}{2}} \qquad (2.10a)$$

$$\omega_{pi} = \left(\frac{Noe^2}{m_i\varepsilon_o}\right)^{\frac{1}{2}} \qquad (2.10b)$$

are the electron and ion plasma frequencies respectively.

2.2 Anisotropy of Magnetized Plasma

When plasma is immersed in a uniform magnetostatic field along the z-direction the electromagnetic fields satisfy the time harmonic Maxwell's equation

$$\nabla \times \vec{H}(\vec{r}) = -i\omega\varepsilon_0 \, \vec{\varepsilon}_r \, . \, \vec{E}(\vec{r}) \qquad (2.11)$$

This equation shows that a magnetized plasma behaves like a dielectric characterized by the permittivity tensor $\vec{\varepsilon} = \varepsilon_0 \vec{\varepsilon}_r$

The components of the electric flux density $\vec{D}(\vec{r})$ may be written in terms of the components of the electric field $\vec{E}(\vec{r})$ as

$$D_x(\vec{r}) = \varepsilon_1 E_x(\vec{r}) - i\varepsilon_2 E_y(\vec{r}) \qquad (2.12a)$$

$$D_y(\vec{r}) = i\varepsilon_2 E_x(\vec{r}) + \varepsilon_1 E_y(\vec{r}) \qquad (2.12b)$$

$$D_z(\vec{r}) = \varepsilon_3 E_z(\vec{r}) \qquad (2.12c)$$

These relations show that in the presence of external magnetic field, the electric field and the electric flux density are not in the same direction. This tells us that the medium is anisotropic. In the absence of external magnetic field ε_2 vanishes and the dielectric permitivity will be a scalar equal to ε_3. In such a case $\vec{E}$ and $\vec{D}$ are parallel and the plasma is isotropic. We thus conclude that it is the external magnetic field which makes the plasma anisotropic. Waves propagating in different directions with respect to the external magnetic field have different characteristics.

It can be seen from eqs.(2.8) and (2.9) that the dielectric properties of the plasma depend on the frequency of the wave, hence the phase and group velocities of the wave through the medium are different for different frequencies. Such a medium is said to be dispersive.

2.3 Dispersion Relation

An equation that relates the propagation constant to the wave frequency is called a dispersion relation. Consider a plasma immersed in an external magnetic field along the z-direction and let the propagation vector lie on the x-z plane making an angle θ with the z-axis. Then its components are

$$k_x = k_\perp = k \sin \theta \qquad (2.13a)$$

$$k_y = 0 \qquad (2.13b)$$

$$k_z = k_\parallel = k \cos \theta \qquad (2.13c)$$

From Maxwell's equations the wave equation for a plane monochromatic wave in a plasma is obtained to be

$$\nabla \times (\nabla \times \vec{E}) + \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad (2.14)$$

Such an equation has a wave solution of the form

$$\vec{E} = \vec{E}_0 \exp \{i(\vec{k}.\vec{r} - \omega t)\} \qquad (2.15)$$

Substitution of such a solution back into the wave equation gives the equation

$$\vec{k} \times (\vec{k} \times \vec{E}_0) + \mu_0 \omega^2 \vec{\epsilon}.\vec{E}_0 = 0 \qquad (2.16)$$

which may be expressed in matrix notation as

$$\begin{bmatrix} \omega^2\mu_o\varepsilon_{xx} - k_y^2-k_z^2 & k_xk_y+\omega^2\mu_o\varepsilon_{xy} & k_xk_z+\omega^2\mu_o\varepsilon_{xz} \\ k_yk_y+\omega^2\mu_o\varepsilon_{yx} & \omega^2\mu_o\varepsilon_{yy}-k_x^2-k_z^2 & k_yk_z+\omega^2\mu_o\varepsilon_{yz} \\ k_zk_x+\omega^2\mu_o\varepsilon_{zx} & k_zk_y+\omega^2\mu_o\varepsilon_{zy} & \omega^2\mu_o\varepsilon_{zz}-k_x^2-k_y^2 \end{bmatrix} \begin{bmatrix} E_{ox} \\ E_{oy} \\ E_{oz} \end{bmatrix} = 0 \quad (2.17)$$

with the help of eqs (2.8), (2.13) and the relation $\vec{\varepsilon} = \varepsilon_o\vec{\varepsilon}_r$ eq(2.17) is simplified to

$$\begin{bmatrix} k_o^2\varepsilon_1-k^2\cos^2\theta & -ik_o^2\varepsilon_2 & k^2\cos\theta\sin\theta \\ i\,k_o^2\varepsilon_2 & k_o^2\varepsilon_1-k^2 & 0 \\ k_o^2\sin\theta\cos\theta & 0 & k_o^2\varepsilon_3-k^2\sin^2\theta \end{bmatrix} \begin{bmatrix} E_{ox} \\ E_{oy} \\ E_{oz} \end{bmatrix} = 0 \quad (2.18)$$

where $k_o^2 = \omega^2\mu_o\varepsilon_o = \frac{\omega^2}{c^2}$ is the square of the propagation constant in free space.

The index of refraction of the plasma is

$$n = \frac{k}{k_o} \quad \text{and} \quad n^2 = \frac{k^2c^2}{\omega^2} \quad (2.19)$$

combining eqs (2.18) and (2.19) we get

$$\begin{bmatrix} \epsilon_1 - n^2\cos^2\theta & -i\epsilon_2 & n^2\cos\theta\sin\theta \\ i\epsilon_2 & \epsilon_1 - n^2 & 0 \\ n^2\sin\theta\cos\theta & 0 & \epsilon_3 - n^2\sin^2\theta \end{bmatrix} \begin{bmatrix} E_{ox} \\ E_{oy} \\ E_{oz} \end{bmatrix} = 0 \quad (2.20)$$

In order to have a non-trivial solution for the above equation the determinant of the coefficients must vanish. Evaluation of this determinant gives the dispersion equation.

$$An^4 - Bn^2 + C = 0 \quad (2.21)$$

where $A = \epsilon_1\sin^2\theta + \epsilon_3\cos^2\theta$ (2.22a)

$$B = (\epsilon_1^2 - \epsilon_2^2)\sin^2\theta + \epsilon_1\epsilon_3(1+\cos^2\theta) \quad (2.22b)$$

$$C = \epsilon_3(\epsilon_1^2 - \epsilon_2^2) \quad (2.22c)$$

The coefficients of the dispersion equation are functions of ω, θ, and the plasma parameters, so the dispersion equation is a biquadratic in n. The two roots of n^2 are then

$$n^2 = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} \quad (2.23)$$

and if we solve for θ we obtain

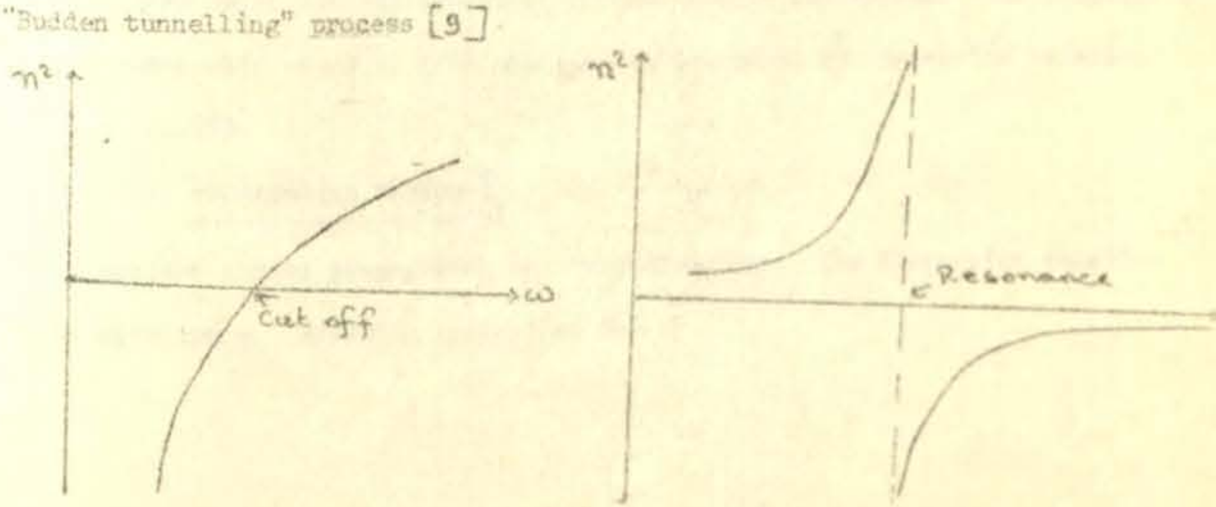
$$\tan^2\theta = -\frac{\epsilon_3(n^2-\epsilon_1-\epsilon_2)(n^2-\epsilon_1+\epsilon_2)}{(\epsilon_1 n^2 - \epsilon_1^2 - \epsilon_2^2)(n^2 - \epsilon_3)} \quad (2.24)$$

2.4 Cut-off and Resonance Frequencies

From the dispersion relation (2.23), which generally has more than one root, we can determine the relative phase velocities of different modes. If for a given frequency , an eigen value of n is real, that is, $n^2 > 0$ then the wave will propagate without attenuation. But if n is imaginary, that is, $n^2 < 0$ then the wave will be attenuated rapidly and thus will not propagate.

A point at which n^2 changes its sign marks a boundary between propagating and non-propagating frequency domains. This happens either at $n^2 = 0$ or $n^2 = \infty$. The frequency at which $n^2 = 0$ is called a cut-off frequency. Those waves in the propagating domain near the cut-off frequency travel with extremely large phase velocities. Dissipation process in the medium becomes insignificant. The electromagnetic waves will be reflected at these places where the cut-off condition is satisfied.

The frequency at which $n^2 = \infty$ is called the resonance frequency. In the vicinity of resonance the waves travel with very small phase velocities; dissipations in the medium thus become substantial. The electromagnetic wave will be strongly absorbed by the medium under these resonant conditions. In case where both a cut-off and resonance exist with an evanescent layer ($n^2 < 0$) between them, the wave can penetrate through the evanescent layer; this process is the so called "Budden tunnelling" process [9]



2.5 Wave Propagation in Plasmas without External Magnetic Field

In section (2.2) we have seen that unmagnetized plasma is an isotropic medium characterized by a scalar dielectric $\varepsilon = \varepsilon_3$. From eqs (2.23), (2.9) and $\omega_{ce} = \omega_{ci} = 0$, we find that the propagation of an electromagnetic wave in the plasma under consideration is described by the dispersion relation

$$n^2 = \left(\frac{kc}{\omega}\right)^2 = 1 - \frac{\omega_p^2}{\omega^2} \qquad (2.25a)$$

$$\text{or} \quad \omega^2 = \omega_p^2 + k^2c^2 \qquad (2.25b)$$

According to these equations, the electromagnetic wave propagates in the plasma only when its frequency is greater than the plasma frequency. Thus the plasma frequency represents the cut-off frequency for electromagnetic waves in the unmagnetized plasma.

2.6 Wave Propagation in Magnetized Plasma.

We proceed to consider the case of plasma immersed in a uniform magnetostatic field. The behavior of magnetized plasma is quite complex. It is anisotropic and there exist several resonance frequencies in the system. The propagation of electromagnetic waves in such plasmas, is described by dispersion relation (2.23) or (2.24).

2.6.1 Propagation Across $\vec{B}_0$

Consider a wave propagating in the x-direction. The dispersion relation (2.24) with $\theta = \frac{\pi}{2}$ gives two solutions for n^2

$$n^2_{ord} = \varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2} \qquad (2.26a)$$

and

$$n^2_{ext} = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1} \simeq 1 - \frac{\omega_p^2}{\omega^2 - \omega_{ce}\omega_{ci} + \dfrac{\omega^2(\omega_{ce} - \omega_{ci})^2}{\omega_p^2 - \omega^2 + \omega_{ce}\omega_{ci}}} \qquad (2.26b)$$

The former mode, eq(2.26a), is known as the ordinary mode. From eq. (2.20) we see that $E_z \neq 0$ but $E_x = E_y = 0$ for this mode. Thus the ordinary mode is a transverse wave with its electric field parallel to the external magnetic field. Its dispersion relation is similar to that discussed in the previous section and thus it is not affected by the presence of the external magnetic field.

The second mode of propagation characterized by the refractive index of eq(2.26b) is called extraordinary mode. From eq.(2.20), we find that $E_z = 0$ and the remaining components satisfy

$$\frac{E_x}{E_y} = \frac{i\,\varepsilon_2}{\varepsilon_1} \qquad (2.27)$$

Since E_x corresponds to the longitudinal component of the electric field while E_y to the transverse one, the extra-ordinary mode represents a hybrid mode. This mode has two resonance frequencies. Setting the denominator of the right hand side of eq(2.26b) equal to zero and expanding the solution for ω in terms of m_e/m_i and retaining only the lowest terms we obtain [10].

$$\omega^2 = \begin{cases} \omega_{h1}^2 \left(\dfrac{\omega_p^2 + \omega_{ce}\,\omega_{ci}}{\omega_p^2 + \omega_{ce}^2}\right) \\ \\ \omega_{h2}^2 \end{cases} \tag{2.28}$$

where ω_{h1} and ω_{h2} are the lower and the upper hybrid frequencies, defined by

$$\omega_{h1}^2 = \omega_{ce}\,\omega_{ci} \tag{2.29a}$$

$$\omega_{h2}^2 = \omega_p^2 \quad \omega_{ce}^2 \tag{2.29b}$$

At the lower hybrid frequency, electrons and ions oscillate together while at the upper hybrid frequency only the electrons oscillate.

2.6.2 Propagation Parallel to $\vec{B}_o$

We consider a wave propagating in the z-direction parallel to $\vec{B}_o$. Setting $\theta = 0$ in eq(2.24) we obtain the solutions

$$\varepsilon_3 = 0 \tag{2.30a}$$

$$n^2 = \begin{cases} n_+^2 = \varepsilon_1 + \varepsilon_2 & (2.30b) \\ \\ n_-^2 = \varepsilon_1 - \varepsilon_2 & (2.30c) \end{cases}$$

When $\varepsilon_3 = 0$ we have $\omega^2 = \omega_{pi}^2 + \omega_{pe}^2$ which corresponds to the non-propagating plasma oscillation.

In order to identify the type of waves corresponding $n^2 = n_+^2$ and $n^2 = n_-^2$ we consider the second equation of eqs(2.20) which is

$$i\varepsilon_2 E_{ox} + (\varepsilon_1 - n^2)E_{oy} = 0 \quad (2.31a)$$

or $$\frac{E_{oy}}{E_{ox}} = \frac{i\varepsilon_2}{\varepsilon_1 - n^2} \quad (2.31b)$$

If we substitute now $n^2 = n_\pm^2 = \varepsilon_1 \pm \varepsilon_2$ into this equation we obtain

$$\frac{E_{oy}}{E_{ox}} = \pm i \quad (2.32)$$

which tells us that the two possible modes of waves that can propagate along the magnetic field are circularly polarized waves; the right circularly polarized wave characterized by n_+^2 and the left circularly polarized wave characterized by n_-^2. If a wave of any other polarization is incident on a plasma with the direction or propagation along the magnetic field it will split into these two characteristic waves which will propagate at different speeds as determined by $n_\pm^2$ [8].

The resonances for the circularly polarized waves, obtained from the resonance condition and eqs(2.30b) and (2.30c), are

$$\omega = \begin{cases} \omega_{ce} - \text{for the right circularly polarized wave} \\ \omega_{ci} - \text{for the left circularly polarized wave.} \end{cases}$$

In a circularly polarized wave the electric field rotates about the propagation direction once each period as the wave proceeds through the medium. Thus the wave electric field can rotate in synchronism with the gyrating partic if the wave frequency equals the particle cyclotron frequency. From the resonance frequencies given above we see that the right hand circularly polari wave is in resonance with the electron and its electric field will rotate in synchronism with the gyrating electrons. The electrons can then absorb energy from the electric field. At a wave frequency equal to the ion cyclotron frequency the left hand polarized wave is in resonance with the ions in which case the ions may extract energy from the wave as they gyrate in synchronism with the wave electric field.

Although propagation and non-propagation of electromagnetic waves may occur over a wide ranges of frequency, only near frequencies that are characteristic of the plasma or of the plasma particles will really interesting effects take place. The left-hand circularly polarized waves, at frequencies somewhat less than the ion cyclotron frequency, are called ion-cyclotron waves and the right hand polarized waves in the vicinity of the electron cyclotron resonance are called the electron cyclotron waves. The electron cyclotron waves are also called "whistler" waves in connection with the propagation of waves along the earth's magnetic field lines in the ionosphere.

In a magnetized plasma the role of ions is usually unimportant provided $\omega \gg \omega_{ci}$, waves which can be considered neglecting the effect of ions are called high-frequency waves. The dispersion relation for high-frequency waves in a cold collisionless plasma is [11]

$$n^2 = 1 - \frac{2\omega_p^2(\omega^2 - \omega_p^2)}{2(\omega^2 - \omega_p^2)\omega^2 - \omega_{ce}^2\omega^2\sin^2\theta \pm \left[\omega^4\omega_{ce}^4\sin^4\theta + 4\omega_{ce}^2\omega^2(\omega^2-\omega_p^2)^2\cos^2\theta\right]^{\frac{1}{2}}} \qquad (2.33)$$

CHAPTER 3

MOTION OF CHARGED PARTICLES IN THE PRESENCE OF WHISTLER WAVE

Let us consider an infinite, non-homogeneous, collisionless plasma immersed in a uniform magnetostatic field. A wave of slowly varying amplitude is assumed to propagate at an arbitrary angle to the magnetostatic field. We restrict ourselves to an electron cyclotron wave, that is, a whistler wave of frequency ω such that $\omega_{ci} << \omega \leq \omega_{ce}$. In this case the ions in the plasma are paractically immobile and constitute a background of neutralizing positive charge. Thus we consider only the motion of electrons.

3.1 The Equation of Motion of an Electron

The motion of any particle depends on the forces which act on it. In the case considered above the electron is subjected to two forces; one of electrical and another of magnetic origin. Thus the non-relativistic motion of the electron is determined by the equations

$$\frac{d\vec{r}}{dt} = \dot{\vec{r}} = \vec{v} \qquad (3.1a)$$

and $$\frac{d\dot{\vec{r}}}{dt} = \dot{\vec{v}} = \frac{-e\vec{E}}{m} - \frac{e\vec{v}\times\vec{B}}{m} \qquad (3.1b)$$

In these equations $\vec{r}$, $\vec{v}$, $-e$ and m are the position, velocity, charge and mass of the electron, respectively. The electric field $\vec{E}$ and magnetic field $\vec{B}$ have spatial and temporal variations. They can be considered as a superposition of quasi-static fields $\vec{E}_o$, $\vec{B}_o$ and fast varying high frequency fields $\vec{E}_\sim$, $\vec{B}_\sim$ as follows

$$\vec{E} = \vec{E}_o + \vec{E}_\sim \qquad (3.2a)$$

$$\vec{B} = \vec{B}_o + \vec{B}_\sim \qquad (3.2b)$$

We consider the whistler wave under the following configuration: the static magnetic field lies in the z-axis, while the direction of propagation of the wave lies in the x-z plane making angle χ with respect to the z-axis.

Karpman and Lundin [12] have shown that the electric field $\vec{E}_\sim$ and magnetic field $\vec{B}_\sim$ of the quasi-monochromatic whistler packet can be described by

$$E_x = -\alpha_x A \cos \phi \qquad (3.3a)$$

$$E_y = -A \sin \phi \qquad (3.3b)$$

$$E_z = -\alpha_z A \cos \phi \qquad (3.3c)$$

and

$$B_x = nA\cos \chi \sin \phi \qquad (3.4a)$$

$$B_y = -nA\alpha_y \cos \phi \qquad (3.4b)$$

$$B_z = 0 \qquad (3.4c)$$

where $A = A(\vec{r},t)$ is the amplitude of the wave whichsatisfies the conditions

$$\frac{1}{A}\left|\frac{\partial A}{\partial t}\right| << \omega \qquad (3.5a)$$

$$\frac{1}{A}\left|\frac{\partial A}{\partial r}\right| << k \qquad (3.5b)$$

n is the refractive index of the plasma and is related to k and ω by the equation.

$$n^2 = \left(\frac{kc}{\omega}\right)^2$$

α_x, α_y and α_z are coefficients of polarization of whistler wave in the linear approximation and are related by

$$\alpha_{y'} = \alpha_x \cos\chi + \alpha_z \sin\chi$$

We assume that $0 \leq \chi \ll \frac{\pi}{2}$. In this case of quasilongitudinal propagation of the wave n, α_x, α_y and α_z are expressed in terms of the wave, cyclotron and plasma frequencies as [13]

$$n^2 = \left(\frac{kc}{\omega}\right)^2 = \frac{\omega_{pe}^2}{\omega(\omega_{ce}\cos\chi - \omega)} \qquad (3.6)$$

$$\alpha_x = \frac{\omega\cos\chi - \omega_{ce}}{\omega_{ce}\cos\chi - \omega} \qquad (3.7a)$$

$$\alpha_z = \frac{\omega\sin\chi}{\omega_{ce}\cos\chi - \omega} \qquad (3.7b)$$

$$\alpha_y = -1 \qquad (3.7c)$$

The above equations are valid in the whistler range under the conditions

$$\frac{\omega_{pe}^2}{\omega^2} \gg 1 \qquad (3.8a)$$

$$\frac{\omega}{\omega_{ce}} < \cos\chi \qquad (3.8b)$$

Let us now introduce a coordinate system in the velocity space as shown in Fig (3.1) in which $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$ are defined by

$$\vec{e}_x = \vec{e}_y \times \vec{e}_z \tag{3.9a}$$

$$\vec{e}_y = \vec{e}_z \times \vec{e}_x \tag{3.9b}$$

$$\vec{e}_z = \vec{e}_x \times \vec{e}_y = \frac{\vec{B}_o}{B_o} \tag{3.9c}$$

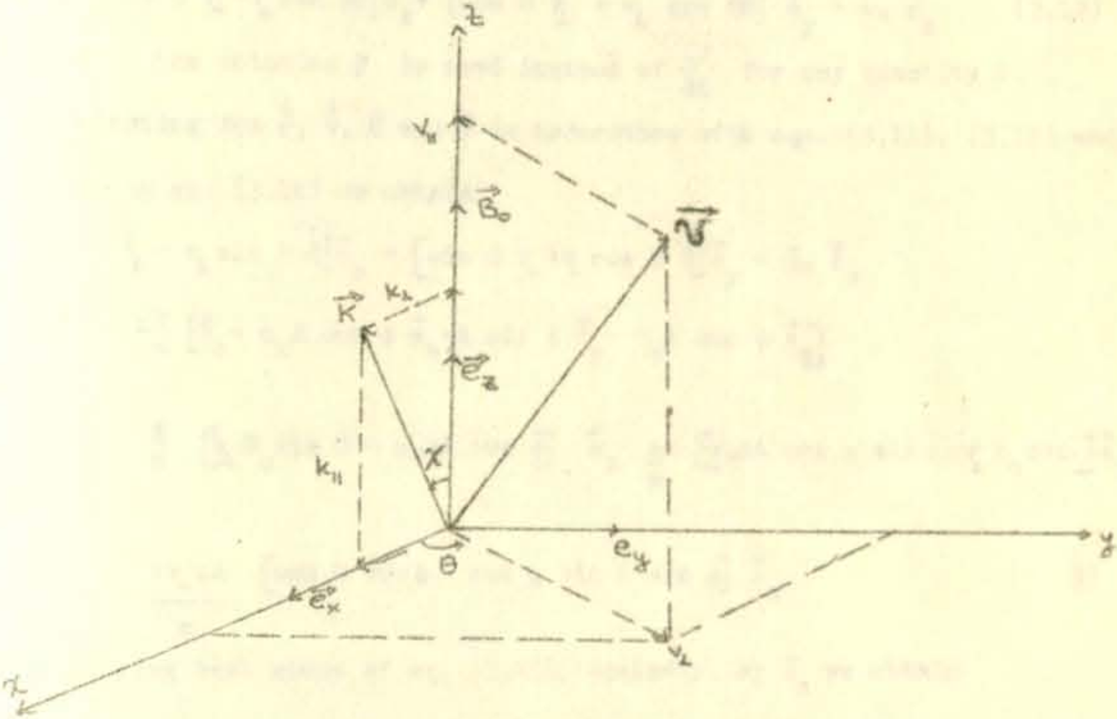


Fig. 3.1 Configuration of Various Vectors

The expressions for $\vec{E}$ and $\vec{B}$ given by eqs.(3.2) can now be written as

$$\vec{E} = \vec{E}_o - \alpha_x A \cos\phi\, \vec{e}_x - A \sin\phi\, \vec{e}_y - \alpha_z A \cos\phi\, \vec{e}_z \tag{3.10a}$$

$$\vec{B} = nA \cos\chi \sin\phi\, \vec{e}_x + nA \cos\phi\, \vec{e}_y + B_o\, \vec{e}_z \tag{3.10b}$$

The equation of motion of the particle in the velocity space is

$$\dot{\vec{r}} = v_\perp(\cos\theta\, \vec{e}_x + \sin\theta\, \vec{e}_y) + v_\parallel\, \vec{e}_z \tag{3.11}$$

where $\Theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ is the phase of the cyclotron motion of the particle;

$v_{\parallel} = v_z$ and $v_{\perp} = (v_x^2 + v_y^2)^{\frac{1}{2}}$ are the longitudinal and transverse components of the particle velocity relative to the magnetostatic field.

Differentiating eq.(3.11) with respect to time we obtain

$$\dot{\vec{v}} = [\cos\Theta\,\dot{v}_{\perp} - v_{\perp}\sin\Theta\dot{\Theta}]\vec{e}_x + [\sin\Theta\,\dot{v}_{\perp} + v_{\perp}\cos\Theta\dot{\Theta}]\,\vec{e}_y + \dot{v}_{\parallel}\,\vec{e}_z \quad (3.12)$$

in which the notation $\dot{P}$ is used instead of $\frac{dP}{dt}$ for any quantity P.

Substituting for $\vec{r}$, $\vec{v}$, $\vec{E}$ and $\vec{B}$ in accordance with eqs. (3.11), (3.12) and (3.10) in eq. (3.1b) we obtain

$$[\cos\Theta\,\dot{v}_{\perp} - v_{\perp}\sin\Theta\,\dot{\Theta}]\vec{e}_x + [\sin\Theta\,\dot{v}_{\perp} + v_{\perp}\cos\Theta\,\dot{\Theta}]\vec{e}_y + \dot{v}_{\parallel}\,\vec{e}_z$$

$$= \frac{-e}{m}\left[\vec{E}_o - \alpha_x A\cos\phi\,\vec{e}_x - A\sin\phi\,\vec{e}_y - \alpha_z A\cos\phi\,\vec{e}_z\right]$$

$$\frac{-e}{m}\left[v_{\perp}B_o\sin\Theta - v_{\parallel}nA\cos\phi\right]\vec{e}_x - \frac{e}{m}\left[v_{\parallel}nA\cos\chi\sin\phi - v_{\perp}B_o\cos\Theta\right]\vec{e}_y -$$

$$\frac{ev_{\perp}nA}{m}\left[\cos\Theta\cos\phi - \cos\chi\sin\Theta\sin\phi\right]\vec{e}_z \quad (3.13)$$

Multiplying both sides of eq. (3.13), scalarly, by $\vec{e}_z$ we obtain

$$\dot{v}_{\parallel} = \frac{-e\vec{E}_o\cdot\vec{e}_z}{m} + \frac{e\alpha_z A\cos\phi}{m} - \frac{ev_{\perp}nA}{m}\left[\cos\Theta\cos\phi - \cos\chi\sin\Theta\sin\phi\right] \quad (3.14)$$

With the help of the identities

$$\sin\Theta\sin\phi = \tfrac{1}{2}\left[\cos(\Theta-\phi) - \cos(\Theta+\phi)\right] \quad (3.15a)$$

and $\cos\Theta\cos\phi = \frac{1}{2}\left[\cos(\Theta-\phi) + \cos(\Theta+\phi)\right]$ (3.15b)

the above equation can be written as

$$\dot{v}_{\parallel} = a_o + a_1 \cos\phi + a_2 \cos(\theta+\phi) + a_3 \cos(\theta-\phi) \qquad (3.16)$$

where $a_o = -\dfrac{e\vec{E}_o}{m} \cdot \vec{e}_z$, $a_1 = \dfrac{e\alpha_z A}{m}$

$$a_2 = -\frac{ev_\perp nA}{2m} (\cos\chi + 1)$$

$$a_3 = \frac{ev_\perp nA}{2m} (\cos\chi - 1)$$

If we next multiply eq.(3.13) by $(\cos\theta\, \vec{e}_x + \sin\theta\, \vec{e}_y)$, we obtain

$$\dot{v}_\perp = \left(\frac{-e\vec{E}_o}{m} \cdot \vec{e}_x \right) \cos\theta + \left(\frac{-e\vec{E}_o}{m} \cdot \vec{e}_y \right) \sin\theta + \frac{eA}{2m} \left[\alpha_x - 1 + v_\parallel n + v_\parallel n \cos\chi\right] \cos(\theta+\phi) + \frac{eA}{2m} \left[\alpha_x + 1 + v_\parallel n - v_\parallel n \text{ cox } \chi\right] \cos(\theta-\phi) \qquad (3.17)$$

$$= b_1 \cos\theta + b_2 \sin\theta + b_3 \cos(\theta+\phi) + b_4 \cos(\theta-\phi) \qquad (3.18)$$

where $b_1 = \vec{F}_o \cdot \vec{e}_x$; $b_2 = \vec{F}_o \cdot \vec{e}_y$, $\vec{F}_o = \dfrac{-e\vec{E}_o}{m}$

$$b_3 = \frac{eA}{2m} (\alpha_x - 1 + v_\parallel n + v_\parallel n \cos\chi)$$

$$b_4 = \frac{eA}{2m} (\alpha_x + 1 + v_\parallel n - v_\parallel n \cos\chi)$$

Similarly, when both sides of eq.(3.13) are scalarly multiplied by $(-\sin\theta\, \vec{e}_x + \cos\theta\, \vec{e}_y)$ and the identities

$$\sin\theta \cos\phi = \tfrac{1}{2}\left[\sin(\theta+\phi) + \sin(\theta-\phi)\right] \qquad (3.19a)$$

$$\cos\theta \sin\phi = \tfrac{1}{2}\left[\sin(\theta+\phi) - \sin(\theta-\phi)\right] \qquad (3.19b)$$

are used we obtain

$$v_\perp \dot{\Theta} = \frac{e\vec{E}_o}{m} \cdot \vec{e}_x \sin\Theta - \frac{e\vec{E}_o}{m} \cdot \vec{e}_y \cos\Theta + \frac{eB_o v_\perp}{m} - \frac{ev_\perp A}{2m}\left[\alpha_x - 1 + v_\parallel n + v_\parallel n \cos\chi\right]\sin(\Theta+\phi) - \frac{ev_\perp A}{2m}\left[\alpha_x + 1 + v_\parallel n - v_\parallel n \cos\chi\right]\sin(\Theta-\phi) \quad (3.20)$$

This can be rewritten as

$$\dot{\Theta} = \omega_o + g_1 \cos\Theta + g_2 \sin\Theta + g_3 \sin(\Theta+\phi) + g_4 \sin(\Theta-\phi)$$
$$= \omega_o + \dot{A}_o \quad (3.21)$$

where $\omega_o = \frac{eB_o}{m}$, $g_1 = \frac{b_2}{v_\perp}$, $g_2 = -\frac{b_1}{v_\perp}$, $g_3 = \frac{-b_3}{v_\perp}$, $g_4 = \frac{-b_4}{v_\perp}$

The phase of the high frequency field ϕ, supposed to be fast varying is described by the equation

$$\dot{\phi} = -\omega + \vec{k} \cdot \dot{\vec{r}} \quad (3.22)$$

where $\omega = -\frac{\partial\phi}{\partial t}$ and $\vec{k} = \vec{\nabla}\phi$

with the help of eq.(3.11), we can write eq.(3.22) as

$$\dot{\phi} = -\omega + k_\parallel v_\parallel + k_\perp v_\perp \cos\Theta$$
$$= \omega_1 + \dot{A}_1 \quad (3.23)$$

where $\omega_1 = -\omega + k_\parallel v_\parallel$ and $\dot{A}_1 = k_\perp v_\perp \cos\Theta$.

The equations of particle motion, eqs. (3.11), (3.16), (3.18) and the evolution of the phases of particle and wave i.e. eqs. (3.21) and (3.23) are expressed in the standard form of multiperiodic system of equations for averaging of which the conditions of drift approximation given below are fulfilled.

$$\frac{a}{L} = \zeta \ll 1, \quad \frac{1}{|\omega_o| T} \sim \zeta, \quad V_E \sim \zeta V$$

where a stands for the Larmor radius, L is the distance in which the changes in the components of the magnetic field become of the order of the magnetic field, T is the time necessary for the particle to drift through this distance and V_E is the electrical drift velocity.

3.2 Averaged Equations of Motion of Electrons in Cyclotron Resonance

Multiperiodic or multi frequency system of differential equations, containing fast rotating phases are generally represented as follows.

$$\dot{\vec{X}} = f(t, \vec{X}, \vec{\theta}, \varepsilon) \quad (3.24a)$$

$$\dot{\vec{\theta}} = \frac{1}{\varepsilon}\,\omega(t, \vec{X}) + A(t, \vec{X}, \vec{\theta}\,;\,\varepsilon) \quad (3.24b)$$

Where $\vec{X} = (x_1, \ldots, x_N)$ is a slowly varying vector,

$\vec{\theta} = (\theta_1, \ldots, \theta_M)$ is a fast varying vector,

ω = is frequency which depends on the slowly varying vector

ε is a small parameter

f, A are periodic vector functions.

A method of averaging of such a system of multiperiodic differential equations in the resonance case was suggested by N.N. Bogliubov and Yu. A Mitropolskii [14]. Applying their method of averaging to the eqs. (3.11), (3.16), (3.18), (3.21) and (3.23), assuming the longitudinal electric field to be weak, we obtain the averaged equations.

$$\dot{\vec{r}} = v_{\parallel}\vec{e}_z + \frac{1}{\omega_b}\,\vec{e}_z \times \vec{F}_o \quad (3.25)$$

$$\dot{v}_{\parallel} = a_o + a_4 \cos \psi \tag{3.26}$$

$$\dot{v}_{\perp} = b_4 \cos \psi \tag{3.27}$$

$$\dot{\psi} = \omega_o - \omega_1 + g_4 \sin \psi \tag{3.28}$$

where $\psi = \theta - \phi$

From these averaged equations we obtain the expressions for the "actual" variables of the particle motion, with the oscillating terms taken into consideration, as follows

$$\vec{r} = \vec{R} + \frac{v_{\perp}}{\omega_o} (\sin \theta \, \vec{e}_x - \cos \theta \, \vec{e}_y) \tag{3.29}$$

$$v_{\parallel} = v_{o\parallel} + \frac{a_1 \sin \phi}{\omega_1} + \frac{a_2 \sin (\theta + \phi)}{\omega_o + \omega_1} \tag{3.30}$$

$$v_{\perp} = v_{o\perp} + \frac{b_1 \sin \theta - b_2 \cos \theta}{\omega_o} + \frac{b_3 \sin(\theta + \phi)}{\omega_o + \omega_1} \tag{3.31}$$

$$\theta = \theta_o + \frac{g_1 \sin \theta - g_2 \cos \theta}{\omega_o} - \frac{g_3 \cos(\theta + \phi)}{\omega_o + \omega_1} \tag{3.32}$$

$$\phi = \phi_o + \frac{k_1 v_{\perp} \sin \theta}{\omega_o} \tag{3.33}$$

where each of the first term on the right side of each equation represents an averaged value in the resonance case.

The ratio $\frac{v}{\omega_o}$ appearing in the oscillating term of the position vect of the particle is the Larmor radius of cyclotron rotation of the particle around the constant magnetic field. The equation for the longitudinal compo of the particles velocity shows that its oscillating term depends on the hig frequency of the wave. The oscillating terms, of the transverse component c velocity and that of the phase of cyclotron motion, are dependent on the hig frequency of the wave and on the quasi-stationary electric field $\vec{E}_o$. Since the wave is propagating at small angle χ, the transverse component of the wa vector is small compared with the longitudinal component, i.e.,

$$k_{\perp} = k \sin \chi << k \cos \chi = k_{\parallel}$$

Thus, in eq. (3.33) the second term is much less than the first term.

3.3 Analysis of the Averaged Equations of Motion of Charged Particles in Cyclotron Resonance

In the absence of magnetic field the motion of the particle in the wave's field would be resonant when phase velocity of the wave is equal to the component of the velocity of the particle along the direction of the wave's propagation. That is,

$$(\vec{k}) = \vec{k}.\ \vec{v} \tag{3.34}$$

This type of resonance is called cherenkov resonance. Under this condition the particle emits cherenkov radiation (waves) (for those $\frac{\omega}{k} = v_{ph} = v \cos \Theta$ where Θ is the angle between $\vec{k}$ and $\vec{v}$). However, each emission process can be reversed and, hence, for the same condition of ω and $\vec{k}$ the wave must cause the inverse effect, cherenkov absorption, connected with the transfer of approperiate energy and momentum from wave to the particle [11].

In the presence of magnetic field we have another type of resonance known as cyclotron resonance. Such resonance occurs when the frequency of the wave is equal to the rotational speed of the particle around the magnetic field. It is to be noted that cyclotron resonance also takes place when the frequency of the wave field is an integral multiple of the cyclotron frequency. Thus, the resonance condition for a particle moving across the magnetic field is given by

$$\omega = n\omega_c \quad n = 1,2,3,\ldots \tag{3.35}$$

If on the other hand a particle has a constant velocity component $v_{\parallel}$ in the direction of the magnetic field, the frequency "felt" by the particle is the Doppler shifted frequency equal to $\omega(\vec{k}) - k_{\parallel}v_{\parallel}$. For such a case the cyclotron resonance condition is

$$\omega(\vec{k}) - k_{\parallel}v_{\parallel} = n\omega_c \quad n = 0, \ 1, \ 2,\ldots \tag{3.36}$$

For n = 0 this condition reduces to the cherenkov resonance. For n = 1,2,3,... eq(3.36) becomes

$$\omega(\vec{k}) = n\,|\omega_c| + k_{\parallel}v_{\parallel} \tag{3.37}$$

Thus, for n > 0 the phase velocity of the wave along the magnetic field is greater than the velocity of the particle $v_{\parallel}$ and we have normal Doppler effect. In this case the particle absorbs energy from the wave and we have damping of waves under normal Doppler effects.

When n = -1, -2, -3, ... condition (3.36) becomes

$$\omega(k) = k_{\parallel} v_{\parallel} - |n\omega_c| \tag{3.38}$$

Thus, for n < 0 the phase velocity of the wave along the magnetic field is less than the particle's velocity $v_{\parallel}$. In this case the wave absorbs energy from the particle and is amplified under anomalous Doppler effect.

The averaged equations of motion of charged particles, eqs. (3.25)-(3.28) are derived under the cyclotron resonance condition (3.36), which corresponds to normal Doppler effect for n=1.

In the exact resonance i.e., when n = 1 we have

$$v_{\parallel} = \frac{\omega(\vec{k}) - \omega_c}{k_{\parallel}} \equiv v_R \tag{3.39}$$

For whistler wave $\omega(\vec{k})$ is smaller than ω_c and from eq(3.39) we conclude that the resonance velocity v_R and $k_{\parallel}$ have opposite signs. This shows that the resonant particles move opposite to the whistler wave.

The investigation of the motion of charged particles, with the help of eqs. (3.25)-(3.28) is a very complicated problem. As stated earlier let the whistler wave be propagating in a weakly ionized non homogeneous plasma. The wave number $\vec{k}$ is changing and can be found from the dispersion equation (3.6).

Differentiation of eq (3.28) with respect to time gives

$$\ddot{\psi} = -v_{\parallel}\dot{k}_{\parallel} - k_{\parallel}\dot{v}_{\parallel} + \sin\psi\,\dot{g}_{k} + g_{k}\cos\psi\,\dot{\psi} \qquad (3.40)$$

Suppose that the high frequency wave is quite weak and the amplitude varies slowly, then the last two terms of eq.(3.40) can be neglected. Since we know that

$$\dot{k}_{\parallel} = \frac{\partial k_{\parallel}}{\partial t} + \vec{v}\cdot\vec{\nabla}k_{\parallel}\,, \quad v_{\parallel} \simeq v_R$$

and in the considered case of monochromatic wave

$$\omega = \text{const} \;; \quad \frac{\partial k_{\parallel}}{\partial t} = 0$$

then eq (3.40) may be written as

$$\ddot{\psi} + v_R^{2}\frac{\partial k_{\parallel}}{\partial z} + k_{\parallel}a_o + \frac{ek_{\parallel}}{2m}\,nAv_{\perp}(\cos\chi - 1)\cos\psi = 0 \qquad (3.41)$$

This equation leads to an integral of motion

$$\tfrac{1}{2}\dot{\psi}^{2} + U(\psi) = W = \text{constant} \qquad (3.42)$$

where $U(\psi) = \left[v_R^{2}\frac{\partial k_{\parallel}}{\partial z} + k_{\parallel}a_o\right]\psi + \frac{ek_{\parallel}nAv_{\perp}}{2m}(\cos\chi - 1)\sin\psi$ (3.43)

is the "potential energy" in the space of phase difference ψ.

Eq. (3.42) fully describes the non-relativistic motion of charged particles in the phase space $(\dot{\psi},\psi)$. As far as the function $U(\psi)$ has minima (potential well), there will be trapped resonant particles whose motion is finite or bounded. There are also untrapped particles, whose motion is not bounded.

By analyzing eq.(3.43) it is possible to find the value of the angle ψ for which the resonant particles are trapped in the potential well. To find the extremum value of $\psi = \psi_{ext}$ we have to have

$$\left.\frac{dU(\psi)}{d\psi}\right|_{\psi=\psi_{ext}} = 0$$

which yields

$$v_R^2 \frac{\partial k_\parallel}{\partial z} + k_\parallel a_o + \frac{ek_\parallel}{2m} nAv \ (1-\cos \chi) \cos \psi_{ext} = 0$$

From this equation follows that

$$\cos \psi_{ext} = \frac{\gamma}{\beta} \leq 1 \tag{3.44}$$

where $\gamma = v_R^2 \frac{\partial k_\parallel}{\partial z} + k_\parallel a_o$

$$\beta = \frac{ek_\parallel nAv_\perp}{2m} (1-\cos \chi)$$

Equation (3.44) is the necessary condition for the particles to be trapped.

In the case of homogenous plasma $\frac{\partial k_\parallel}{\partial z} = 0$ and in the absence of quasi-stationary electric field $\vec{E}_o$, the trapped particles oscillate about the phase $\frac{\pi}{2}$ with frequency

$$\Omega = \sqrt{\frac{ek_\parallel nAv_\perp}{2m} (1-\cos \chi)}$$

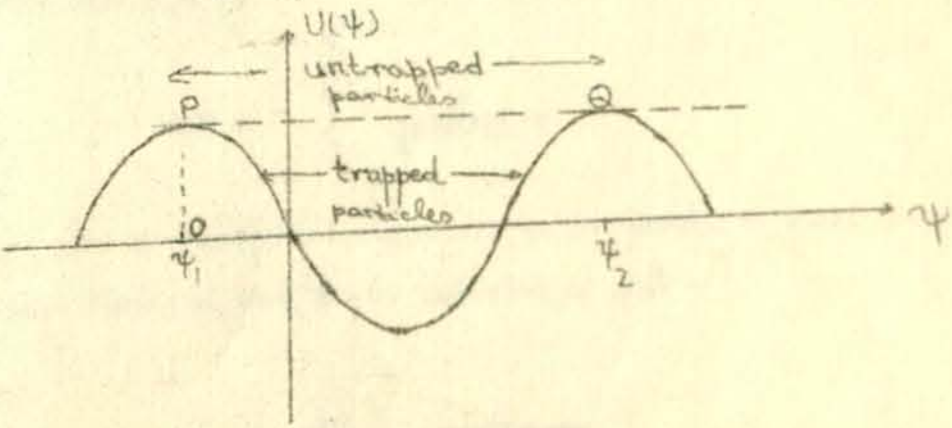


Fig. 3.2 Potential curve for $\gamma = 0$

In the case where $\gamma = 0$, that is, when

$$v_R^2 \frac{\partial k_\parallel}{\partial z} = - k_\parallel a_o \qquad (3.45)$$

resonance interaction of particles with the wave is similar to the interaction in the case of a homogeneous plasma. Expression (3.45) is then the condition of compensation effect of non-homogenifety of plasma by quasi-stationary electr field.

Drawing horizontal line (fig 3.2), corresponding to the given value of the total energy, it is possible to specify the possible range of motion ($\gamma=0$) the particles. Those electrons whose energy, in the system of coordinates whc is less than the amplitude OP are trapped in the potential well. The motion of the particles is limited by two borders as it is shown in potential well PQ between points ψ_1 and ψ_2. These particles remain in the potential well, ir changing energy with the wave, until they interchange energy with other partic and get out of this state.

It can be noted that the one dimensional finite motion in the potenti well PQ is periodic. According to the property of periodic motion the period motion of particle T(W) is equal to twice the time of motion from P to Q. From eq. (3.42) it can be shown that the period of oscillation for resonant trapped particles is

$$T(W) = 2 \int_{\psi_1}^{\psi_2} \frac{d\psi}{\sqrt{2(W-U(\psi))}} \qquad (3.46)$$

In the case of homogeneous plasma and in the absence of quasi-stationary electric field, or when $\gamma = 0$, the period is given as

$$T(W) = \sqrt{\frac{2}{W}} \int_{\psi_1}^{\psi_2} \frac{d\psi}{\sqrt{1+k \sin \psi}}$$

CONCLUSION

In this thesis the motion of charged particles, especially electrons, in a collisionless, non-homogeneous plasma is considered. The equation of motion of an electron, in the magnetized plasma, when a high frequency whistler wave is propagating oblique to the magnetostatic field, are derived. The system of equations are found to be multiperiodic. The equations are then averaged in the case of cyclotron resonance with the help of method of averaging. Expressions for the "real" variables of the particle's motion in the resonance case are derived. It is found that these expressions consist of an averaged term and oscillatory terms which mainly depend either on the wave's frequency or on the quasi-stationary electric field.

By analyzing the averaged equations in the phase difference space, an approximate integral of motion is found. It is shown that, with the help of the integral of motion, it is possible to determine the region of bounded and unbounded particle motion in the case of weak quasi-stationary electric field. It is also shown that there is a condition of compensation effect of the non-homogeniety of plasma by quasi-stationary electric field.

REFERENCES

1. Lutomiriski, R.F. and R.N. Sudan, "Exact Non-linear Electromagnetic Whistler Modes," Physical Review, 147: 156-65, 1966.

2. Palmadesso, P. and G. Schmidt, "Collisionless Damping of a Large-Amplitude Whistler Wave," The Physics of Fluids, 14:1411-18, 1971.

3. Sugai, H., et.al., "Whistler Wave Ducting Caused by Antenna Action," The Physics of Fluids, 21:690-94, 1978.

4. Palmadesso, P.J., "Resonance, Particle Trapping and Landau Damping in Finite Amplitude Obliquely Propagating Waves," The Physics of Fluids, 15:2006-13, 1973.

5. Seshadri, S.R., Fundamentals of Plasma Physics. New York: American Elesevier Comp., 1973.

6. Chandrasekhar, S., Plasma Physics. Chicago: University of Chicago, 1960.

7. Kruskal M., Plasma Physics: Elementary Oribit and Drift Theory, IAEA, Vienna, 1965.

8. Holt, E.H. and Haskell, Foundations of Plasma Dynamics. New York: MacMillan Comp., 1965.

9. Sugai, H., "Refraction and Conversion of Electromagnetic waves in Magnetized Plasma", XVI International conference on Phenomena in Ionized Gases, 266-75, 1983.

10. Spitzer, L., Physics of Fully Ionized Gas. New York: Interscience, 1962.

11. Ginzburg, V.L., Theoretical Physics and Astrophysics. Oxford: Pergamon Press, 1979.

12. Karpman, W.I and B.W Lundin, Soviet Journal of Plasma physics. 3: 486, 1973.

13. Shafranov, W.D., Reviews of Plasma Physics V3, Moscow, Atomizdat, 1963

14. Bogoliubov, N.N., Yu. A. Mitropolskii, Asymptotic Methods in the Theory of Non-linear Oscillations. Moscow: Nauka, 1974.

DECLARATION

I hereby declare that the thesis entitled, "Drift of Charged Particles in High Frequency Fields", being submitted by me in partial fulfillement for Master of Science Degree in Physics, is my original work, done under the supervision and guidance of Dr. V. Schepilov. Sources of relevant findings and equations taken from books and articles are dully acknowledged in the body of the thesis and the references.

Ghebregziabiher Kelati
G/Egziabiher Kelati
The Thesis was submitted to the
Physics Department Graduate
Committee on June, 1985

This Thesis has been submitted for examination with my approval as University advisor.

V. Schepilov, Ph.D