Investigating Conceptual Change of Students Using Apos Theory in Two Preparatory Schools

Esmael Mohamed

A Thesis Submitted to

The Department of Science and Mathematics Education

Presented in Partial Fulfillment of the Requirements for the Degree of Masters of Mathematics Education

Addis Ababa University
Addis Ababa, Ethiopia

August, 2015

Addis Ababa University

School of Graduate studies

This is to certify that the thesis prepared by Esmael Mohamed ,entitled: Conceptual change Strategies on students' Mathematics Achievement and its Challenge and submitted in partial fulfillment of the requirements for the Degree of Master of Mathematics Education compiles with the regulations of the University and meets the accepted standard with respect to originality and quality.

Signed by the examining committee

Examiner _______________________ Signature ________________ Date ____________________

Examiner _______________________ Signature ________________ Date ____________________

Advisor _______________________ Signature ________________ Date ____________________

____

Chair of the Department or Graduate program coordinator

Acknowledgement

I am so thankful to my advisor Dr. kassa Michael for his intelligible guidance and painstaking evaluation of this work. It is his skillful, sympathetic and considerate support that makes the work to be accomplished.

I am grateful to my uncle Abdella Yussuf, my friend Ato Kedir Emiwedih and my brothers Hamid Mohammed and Bilal Mola for their greatful support.

I want also to express my gratitude from my heart to my wife W/O Habiba Abdu for her priceless support, and tolerance during the study time.

Table of Contents **Page**

Table of Contents i

List of tables iv

List of Acronyms vi

Abstract vii

CHAPTER ONE 1

1. INTRODUCTION.. 1

1.1 Background of the study 1

1.2 Statement of the problem 4

1.3 General objective of the study 5

1.4 Specific objectives of the study 5

1.5 Research questions.. 6

1.6 Significance of the study.. 6

1.7 Delimitation /scope of the study 7

1.8 Operational definitions of related terms.. 7

CHAPTER TWO 11

2.REVIEW OF RELATED LITERATURE 11

2.1 What is Apos theory?.. 14

2.1.1 Theoretical analysis.. 15

2.1.1.1 Mental constructions to learn polynomial and rational equations and functions........... 16

2.1.2 Design and implementation of instruction 17

2.2 Theoretical context.. 19

2.2.1 Learning as a social activity (small group environment) 19

2.2.2. Learning as individual development in a social context; co-construction of knowledge 20

CHAPTER THREE 25

3. RESEARCH DESIGN AND METHODOLOGY 25

3.1 Design 25

3.2 Data sources 25

3.3 Sampling and sampling techniques 26

3.4 Data collection instruments 26

3.4.1 Tests 27

3.4.2 Observation 27

3.5 Pilot test 28

3.6 Methods of data analysis 36

3.7 Procedure of the study 36

3.8 Methodology 38

3.8.1 Pedagogical strategies 38

CHAPTER FOUR 43

4. PRESENTATION, ANALYSIS AND INTERPRETATION OF DATA 43

4.1Data analysis (Before Group Circulation) 43

4.2 Data analysis (After Group Circulation) 73

4.3 Relations (Difference) on Before and After Group Circulation 84

4.4 The researcher’s aggregation on students’ understanding of the concept of polynomial and Rational Equations and Functions 87

4.4.1 Mathematical achievement on polynomial and rational equations and functions before group circulation 87

4.4.2 Mathematical Achievement on polynomial and Rational equations and functions after group circulation 91

4.4.3 Students' Mathematical Achievement on two questions of polynomial and rational equations and functions before Group circulation 95

4.4.4 Students’ mathematical achievement on two questions of polynomial and rational equations and functions after group circulation 96

4.5 Challenges towards the Mathematical Achievement of the students 98

CHAPTER FIVE 100

5. SUMMARY, CONCLUSION AND RECOMMENDATION 100

5.1 Summary 110

5.1.1. Summary on the achievement of polynomial and rational equations and functions before group circulation 103

5.1.2 Summary on the achievement of polynomial and rational equations and functions after group circulation 106

5.1.3 Summary on the achievement of polynomial and rational equations and Functions on the two questions before group circulation 108

5.1.4 Summary on the achievement of polynomial and rational equations and functions on the two questions after group circulation 109

5.2 Conclusion 111

5.3 Recommendation 115

REFERENCE 117

APPENDIX A: Tests 119

APPENDIX B: Tests of the group circulation 121

APPENDIX C: Observation checklist 122

APPENDIX D: Additional given activities and some of students' response 123

List of Table **Page**

Table 3.3.1: The number of students and schools.. 26
Table 3.5.1: Question 1 analysis of student response (N=20) 29
Table 3.5.2: Question 2 analysis of student response (N=20) 30
Table 3.5.3: Question 3 analysis of student response (N=20) 31
Table 3.5.4: Question 4 analysis of student response (N= 20) 32
Table 3.5.5: question 5 analysis of student response (N=20) 34
Table 3.8.1.1: The arrangement of the group for conception 41
Table 4.1.1: Question 1 analysis of student response (N = 80) 43
Table 4.1.2: Question 2 analysis of student response (N = 80) 48
Table 4.1.3: Question 3 analysis of student response (N = 80) 51
Table 4.1.4: Question 4 analysis of student response (N = 80) 53
Table 4.1.5: Question 5 analysis of student response (N = 80) 54
Table 4.1.6: Question 6 analysis of student response (N = 80) 58
Table 4.1.7: Question 7 analysis of student response (N = 80) 60
Table 4.1.8: Question 8 analysis of student response (N=80) 63
Table 4.1.9: Question 9 analysis of student response (N = 80) 66
Table 4.1.10: Question 10 analysis of student response (N = 80) 68
Table 4.1.11: Question 11 analysis of student response (N=80) 70
Table 4.2.1: Question 1 analysis of students response (N=20) 73
Table 4.2.2: Question 2 analysis of student response (N=20) 78
Table 4.2.3: Question 3 analysis of student response (N=20) 82
Table 4.4.1.1: Mathematical Achievement (N=80) 87
Table 4.4.2.1: Mathematical achievement (N=20) 91
Table 4.4.3.1: Mathematical Achievement (N=80) 95
Table 4.4.4.1: Mathematical Achievement (N=20) 96
Table 5.1.1.1: Summary on Mathematical Achievement (N=80) 103

Table 5.1.2.1: Summary on Mathematical Achievement (N=20) ………..................................106

Table 5.1.3.1: Summary on Mathematical Achievement (N=80) ………..................................108

Table5.1.4.1: Summary on Mathematical Achievement (N=20) ………………………………109

ABSTRACT

Conceptual change strategies on students' Mathematics Achievement and its challenge

Esmael Mohammed

Addis Ababa University, 2015

Students are facing problem in understanding of what they are learning in mathematics lesson. There are a lot of students who are suffering from the conceptual change of mathematics. To conduct the study, a group circulation (discussion) based on qualitative method Was employed. This pedagogical system is used to develop students' conception towards polynomial and rational equations and functions.The main objective of this study was to identify problems associated with conceptual change of students and assess conceptual change strategies students use to enhance their mathematics achievement in areas of polynomial and rational equations and functions .To do these Qualitative research method was used employing phenomenology research design where the phenomenon was traced through individual effort and group circulation guided by Apos theory. The study collected data from 80 students that were selected from two schools from Addis Ababa. Narrative and thematic data analysis was employed to classify the levels and practical conceptual change strategies employed by students. From the data analysis the findings from this study include: Before intervention no student reached the higher level-schema. 34.4% of the students were found to be at level 3, and similarly 16.9% and 13.8% were found to be at levels 2 and 1 respectively and 35% did not respond. But after intervention 25% of the students reached the highest level 4, schema. Therefore this study concluded that effective use of conceptual change strategies help students to develop their understanding and achievement. The teaching learning process of mathematics needs serious conceptual change strategies. The mathematics subject needs more strategies or methods in order to understand about it. The data was analyzed through narration before and after group circulation. The results obtained were: conceptual change on this title, making students to do problems by their own, good teaching practices and improving ability. In conclusion, students conceptualized different ideas of their performance through the level of thinking (Apos theory).

CHAPTER ONE

1. INTRODUCTION

Misconceptions influence students' understanding of Mathematics concepts. Among the mathematical contents, it is found that the concept in polynomial and rational equations and functions is considered by many students as a difficult subject. Therefore, it is very crucial to find an appropriate method that can prevent this conception and overcome misconceptions in polynomial and rational equations and functions.

This shows the necessity to conduct further research to improve learning activities in Mathematics education. Several researchers have demonstrated that conceptual change strategies of instruction can be effective at changing students' conceptions related to Mathematical concepts. For example, in the APOS theory, a genetic decomposition of the concepts to be learned by the students in terms of the mental constructions is vital that such learning requires. In the present study, the genetic decomposition focused on the mental constructions required for student success during the conceptual change of polynomial and rational equations and functions for preparatory students is investigated. Among the many possible strategic approaches, conceptual change strategies guided by APOS theory which is intended to deal with conceptual difficulties was used, and its consequential effect underpinned. Students learning with new scientific content are influenced by their current ideas, in ways that may hinder or may help their learning. It therefore is useful to think of learning the desired out comes as the process of conceptual change.

Many of teaching studies in recent years have attempted to take in to account research on student's conceptions of natural phenomena.

1.1 BACKGROUND OF THE STUDY

A conceptual change strategy on student's mathematics achievement and its challenge in preparatory schools has a great value in student's conceptual change and mathematics achievement. The reason is that students who are taking the mathematics course are facing problems in understanding it. Students in different schools complain about the mathematics subject they learn which can be attributed to varying causes, one being causes for misconception and conceptual learning conflicts. They say that during learning of mathematics,'' **it is difficult for us to have conceptual change''**. Due to this, they do not want to join the science field. In addition the achievement of students in fields of science and mathematics, mathematics is the least as compared to others. It is, hence, time to make the mathematics education in a popular style through varying approaches one being the use of conceptual change strategies. In learning, we can attach the importance of conceptual change strategies so that students can hold the basic knowledge. Comparing with the mathematics conceptual change standard in other countries, we should get the development trend of our own standard, which will be useful to guide the level of conceptual ability of students. Supporting this idea Johnson, 1971 stated that:

"The reality of our current educational system is quite the opposite. Students working together are engaged in the learning process instead of passively listening to the teacher. When students work in groups, some members are listening while the others are discussing the question under investigation. All in the groups are developing problem-solving skills by formulating their ideas, discussing them, receiving immediate feedback and responding to questions and comments by their partner".

These days, countries throughout the world are giving due attention to conceptual change through science and mathematics education. The most influential conceptual change models assume that each child comes to school with misconception about natural phenomena. That this misconception needs to be elicited, challenged by explaining or demonstrating contrary examples and corrected by providing a more general concept that the students will accept and assimilate. With this standing, making students to have conceptual change on mathematics through conceptual change strategies so that students' mathematics achievement will be improved. Nelson – LeGall (1992) support this idea by saying that: **"learning and understanding are not merely individual processes supported by the social context; rather they are the result of continuous, dynamic negotiation between the individual and the social setting in which the individual's activity take place. Both the individual and the social context are active and constructive in producing learning and understanding" (P.52).**

This paper focuses on how to promote deep understanding through conceptual change strategies and by making the students to question their inherent conceptual knowledge and review alternative methods of teaching that promote real learning and explain what it entails to provoke deep understanding and conceptual change.

Making meaningful associations and suggesting on how to chunk the new information might help. In cases when students come to class with an erroneous conceptual understanding of the world around them, a more radical approach is needed to change student's prior belief. What is important throughout the learning process of the students is that they really think through all arguments on their ‘’ own’’ and ‘’ construct’’ further knowledge up on already understood Concepts.

The ultimate goal is to promote deep learning in the students own minds through conceptual change strategies so that conceptual change will occur. To do this I would like to describe in this paper APOS theory to guide my research on conceptual change and how it is used in such a program. APOS theory is a constructivist theory of how learning a mathematical concept might take place. APOS theory is an elaboration of the mental constructions of actions, processes, objects and schemas. Studying how students might learn a mathematical concept, I will provide analysis of the concept. The description resulting from this analysis is called a genetic decomposition of the concept taught. For example, the purpose of theoretical analysis is to propose specific mental constructions (the genetic decomposition) through which a student might learn the concept under consideration.

Helping students develop clear, correct concepts in mathematic is a major task for teachers in schools. There is much concern about the mathematics competence of our students. There has been a slight improvement in the mastery of basic facts, but in the area of problem solving and critical thinking there has been little improvement. In mathematics curriculum focusing on problem solving, there need to be relevant, challenging problems for students to solve.

1.2 STATEMENT OF THE PROBLEM

Conceptual change strategies on student's mathematics achievement and its challenge in preparatory schools have a great value in students' conceptual change and mathematics achievement. Students are facing problem in understanding of what they are learning in mathematics lesson. There are a lot of students who are suffering from the conceptual change of mathematics. They say that **"we do not understand mathematics". "Due to this we dislike it." So, we need a good methodology in the teaching learning of mathematics**.

Slavin (1992, p.162) emphasizes that "students will learn from one another because in their discussions of the content, cognitive conflicts will arise, inadequate reasoning will be exposed, disequilibrium will occur, and higher quality understandings will emerge".

A fitting conclusion to this section would be a quote from Williams, (1988):

Tell me mathematics and I will forget; show me mathematics and I may remember; involve me in a tension- free atmosphere in small group work and with manipulative aids in mathematics and I will understand. If I understand mathematics, I will be less likely to have math anxiety, and if I become a teacher of mathematics I can thus begin a cycle that will produce less math anxious students for generation to come (P.100).

Misconceptions hinder learning and understanding of mathematics. Overcoming misconceptions is an important subject. Alternative instructional approaches should be developed to overcome misconceptions and facilitate learning. These instructional approaches are consistent with constructivist view of learning where each learner is actively engaged in constructing knowledge where links between new information and prior knowledge is constructed actively. Since in the teaching-learning process of mathematics students should be changed conceptually, addressing conceptual change strategy and assessing potential challenges is of important value.

1.3 GENERAL OBJECTIVE OF THE STUDY

The study had the following general objectives:

- To identify the basic problem in conceptual change of students' mathematics achievement in a preparatory school. .

1.4 SPECIFIC OBJECTIVES OF THE STUDY

The study had the following specific objectives

a. Examining the overall activities of the preparatory schools in conceptual change strategies to bring the intended conceptual change on mathematics.

a. Identifying the most important factors that promote conceptual change and its challenge.

b. To identify the advantages of using conceptual change strategies in mathematics.

1.5 RESEARCH QUESTIONS

In order to achieve the above objectives, the following basic research questions were raised to be answered in the course of the study.

1. How are students using conceptual change strategies in the learning process to have conceptual change?

2) What is the basic problem in conceptual change of student's mathematics achievements in a preparatory schools?

3) Is there a difference in achievement between groups with and without conceptual change strategies for students of preparatory schools?

1.6 SIGNIFICANCE OF THE STUDY

The result of any investigation may serve different stakeholders who perhaps directly benefited from the process. Evaluation findings have an impact on policy and can provide guidance for improving a program of school or education system. Students are facing a great problem to understand mathematics in the teaching- learning process. So this problem should be removed by improving students' conceptual change. Hence assessing the status of conceptual change strategies in mathematics for preparatory students can provide useful information for the immediate use by students, teachers and schools. The findings of the study are also expected to serve students' academic achievement significantly. It can also serve as a spring board for other researchers to conduct study in similar area.

1.7 DELIMITATION /SCOPE OF THE STUDY

It would have been useful to conduct such a study dealing with new intervention at a wider scale. But, subject to limiting factors this study focused only on conceptual change strategies at preparatory students. It also was delimited to the implementation of APOS with conceptual change strategies.

The focus of the content in mathematics was also delimited in scope to the concepts in polynomial and rational equations and functions.

1.8 OPERATIONAL DEFINITIONS OF RELATED TERMS

The following terms are defined as follows in relation to the study:

Conceptual Change:- a process of learning science [and mathematics] in meaningful way that requires the learner to rearrange or replace existing misconceptions in order to accommodate new ideas (Smith et al. 1993).

Group Circulation:- according to the researcher it is when one group exchanges conception with each of the other group. That is when one group rounds with each of the other groups.

Zone of proximal development:- is the difference between the level of an individual's actual development and more advanced level of potential development that could be observed in the interaction between more or less capable participants (Vygotsky 1992).

Action (Level 1):– An action is characterized specifically by the individual having an external perception of the mathematical concept. That is to say, the individual can only carry out transformations via specific external cues and detailed step by step procedure.

Process (Level 2):– The process of interiorization occurs when the individual begins to reflect upon the action which he or she is performing. An individual who is at the process level of understanding can "reflect on, describe, or even reverse the steps" of a transformation on previously learned objects without actually performing those steps.

Object (Level 3):- The process of encapsulation involves reflection on a particular set of processes until the individual can construct transformations on the mathematical concept. Once this is achieved the concept is said to be at the object level.

Schema (Level 4):- Objects and processes are interconnected in the individual's minds to construct schemas. Schemas are less understood as they vary from person to person since the various connections can differ. Schemas can be thematized into objects so that they can be used to build new mathematical objects and can sometimes be called higher schemas. Development of a schema occurs using a process called reflective abstraction. This process utilizes two mechanisms: projection in to a higher level of abstraction and reflection aimed at reconstruction and reorganization into larger systems. The process of reflective abstraction is the means by which concepts can evolve from actions to processes to objects and finally into schemes. These processes are termed interiorization, encapsulation and thematization, respectively.

Interiorization:- Transformation of an action is the process by which a physical series of actions can be performed in the mind without the need to be prompted or having to perform every individual step. Once achieved, it can be said that a given action has been interiorized into a process.

Encapsulation:– A process is encapsulated when the given mathematical concept exists without the need to perform any specific actions or steps. At this stage the concept gains invariant properties. Once this is achieved, the concept can be transformed and new actions can be learned using the encapsulated mathematics process, now said to be at the object level.

Thematization:– is the process by which multiple objects, processes, and actions, form a coherent body, called a scheme, where concepts can be manipulated and related to one another.

Misconception:- conceptual knowledge that differs from commonly accepted scientific consensus (Sanger, 1996).

Accommodation: - the creation of new schemata or the modification of old schemata which reflects the child's current level of understanding and knowledge of the world (Wodsworth, 1996).

Assimilation: - the cognitive process by which a person integrates new perceptual, motor, or conceptual matter into existing schemata or patterns of behavior (Wodsworth, 1996).

Cognitive Conflict: - providing students with evidence that contradicts with their existing conceptions.

Conception: - means one's particular application or interpretation of a concept (Kaplan, 1964).

Constructivism: - is a theory of learning in which every learner constructs his or her ideas (Rasmussen, 1998).

Discrepant Event: - a phenomenon that occurs in a way that seems to run contrary to initial reasoning (Wright & Govindarajan, 1995).

Equilibration:- a balance between assimilation and accommodation (Yıldırım, Güneri, &Sümer, 2002).

Meta cognition: - knowledge, awareness and control of one's own learning (Baird, 1990).

Exercises – Standard exercises are given to the students to perform, usually in groups, outside of class. These exercises are designed to reinforce the concepts learned in class and to use the mathematics learned in class.

Activities –The students perform activities which are designed to foster specific mental constructions which are suggested by the genetic decomposition.

CHAPTER TWO

2. REVIEW OF RELATED LITERATURE

In the teaching learning process, there are students who do not have conceptual change in mathematics education especially in the given specific object. To develop mathematical maturity for each topic like polynomial and rational equations and functions, students should fit the pedagogical strategy and conceptual change strategies so that their conception will be improved. For this it is better to use individually and group work methodology and stages of APOS theory for the mental development of the students. supporting the conceptual change strategies (especially stages of APOS theory) and pedagogical strategies different scholars researched at different time. For example APOS theory explains, stages or levels which are used to check students conceptual level or the development of their mind. Through APOS theory we can check whether students understand about polynomial and rational equations and functions or not. Both the review literature of pedagogical strategy (group work) and the APOS theory stages are explained in this chapter.

According to APOS theory, an individual deals with a mathematical situation by using certain marital mechanisms to build cognitive structures that are applied to the situation. The main mechanisms are called interiorization and encapsulation and the related structures are actions, process, objects and schemas. The theory postulates that a mathematical concept begins to form as one applies a transformation on objects to obtain other objects. A transformation is first conceived as an action, in that it requires specific instruction as well as the ability to perform each step of the transformation explicitly. "For example, an individual who requires an explicit expression in order to think about the concept of function and can do little more than substitute for the variable in the expression and manipulate it, is considered to have an action understanding of functions.

As an individual repeats and reflects on an action, it may be interiorized in to a mental process. A process is a mental structure that performs the same operation as the action being interiorized, but wholly in the mind of the individual, thus enabling her or him to imagine performing the transformation without having to execute each step explicitly.

Thus, for example, an individual with a process understanding of function will construct a mental process for a given function and think in terms of inputs, possibly unspecified, and transformations of those inputs to produce out puts.

If one becomes aware of a process as a totality, realizes that transformations can act on the totality and can actually construct such transformations (explicitly or in ones imagination), then we say the individual has encapsulated the process in to cognitive object. For the function concept, encapsulation allows one to apply transformations of functions such as forming a set of functions, defining arithmetic operations etc.

While these structures describe how an individual may construct a single transformation, a mathematical topic often involves many actions, processes, and objects that need to be organized and linked in to a coherent frame work, which is called schema. It is coherent in the sense that it provides an individual with a way of deciding which mental structures to use in dealing with a mathematical problem situation. In the case of functions, it is the schema structure that is used, according to APOS theory, to see a function in a given mathematical or “real world” situation.

In recent years, there has been great interest in the encapsulation (or reification) of a process in to a mental object as a fundamental method of constructing mathematical object. Piaget (1985, p.49) focused on the idea, of how actions and operations became thematized objects of thought or assimilation.

The notion of the transformation of a process in to an object took new impetus in the work of Dubinsky (1986, 1991) and Sfard (1988,1989,1991). Sfard hypothesized two approaches to concept development, one operational, focusing on objects. For example Sfard (1992, 64-65) proposed that:

A constant three step pattern can be identified in the successive transitions: first there must be a process performed on the already familiar objects, than the idea of turning this process in to a more compact, self-contained whole should emerge, and finally an ability to view this new entity as a permanent object in its own right must be acquired. These three components of concept development will be called interiorization, condensation, and reification, respectively.

Condensation means a rather technical change of approach, which expresses itself in an ability to deal with a given process in terms of input /output without necessarily considering its component steps.

APOS A (step-by-step) action becomes conceptualized as a total process, is encapsulated as a mental object to later become part of a mental schema. (see chapter four, in the analysis part, in question 1) about APOS stages which are formulated by Cottrill et al., (1996, p.171)

The action becomes a process when the individual can describe or reflect up on all of the steps in the transformation without necessarily performing them." A process becomes an object when" individual becomes aware of the totality of the process, realizes that transformations can act on it, and is able to construct such transformations.

The final part of the APOS structure occurs when" actions, processes, and objects are organized in the structures, which is referred to as schema." When this is achieved, it is also proposed by Cottrill et al., (1996, p.172) that:

> … an individual can reflect on a schema and act up on it. This results in the schema becoming a new object. Thus, we now see that there are at least two ways of constructing objects, from processes and from schema (Cottrill et al., 1996, p.172)

APOS stages which are based on Piagets are one way of evaluating students understanding of a mathematical concept and helps the student in the development. In this study my main concern is with the action, process, object and schema stages of APOS theory. An action is the least powerful stage and the process stage is the next higher stage. An action stage of conception is described by Dubinsky and MC Donald (2001) as a stage in learning where**" a memory, step-by-step interactions on how to perform the operation." A process stage occurs only after an action is repeated and the individual can think of as performing the same kind of action, but without the need to external stimuli".**

2.1 WHAT IS APOS THEORY?

APOS theory is a constructivist theory of how learning a mathematical concept might take place. It is based on the following hypothesis about the nature of mathematical knowledge and how it is developed:

An individual's mathematical knowledge is her or his tendency to respond to perceived mathematical problem situations and their solutions by reflecting on them in a social context and constructing or reconstructing mathematical actions, processes, and objects and organizing these in schemas to use in dealing with the situation.

APOS theory is our elaboration of the mental constructions of actions, processes, objects and schemas. In studying how students might learn a particular mathematical concept, an essential ingredient which the researcher must provide is an analysis of the concept in terms of these specific constructs. The description resulting from this analysis is called a genetic decomposition of the concept. APOS theory is a fame work for instructional research in mathematics education. It proposes to conduct research in three-steps

1. Theoretical analysis of the content to be thought and learned.
2. Design and implementation of instruction and
3. Collection and analysis of data.

2.1.1 THEORETICAL ANALYSIS

Theoretical analysis is one component in a general program of research and curriculum development. The purpose of the theoretical analysis of the concept is to propose a model of cognition that means a description or an explanation of specific mental constructions that students make in order to develop her or his understanding of the conception of polynomial and rational equations and functions. According to APOS theory the result of this analysis is referred to as a genetic decomposition of the concept. That is, a genetic decomposition of a concept is a structured set of mental constructs which describes how the concept can develop in the mind of an individual.

The researchers own understanding of the topic greatly influences the genetic decomposition and therefore many iterations of the framework are required to eventually incorporate the actual learners understanding and background.

According to APOS theory four levels of understanding are assumed, called action, process, object and schema. The process of understanding of a concept begins with actions that the learner is familiar with from the previous studies. At this level the learner must be told which actions to use and in what sequence. At the process level, the learner is already about to choose the actions to perform and is able to decide in which order to perform them. We say that the learner has interiorized the action to form a process as a new object   has "encapsulated" the process in to a mathematical object.

2.1.1.1 MENTAL CONSTRUCTIONS TO LEARN POLYNOMIAL AND RATIONAL EQUATIONS AND FUNCTIONS

Understanding the polynomial and rational equations and functions concept begins with the previous mental or physical objects to form actions; actions are then interiorized to form processes which are then encapsulated to form objects. Objects can be de-encapsulated back to the process from which they are formed. So, actions, processes and objects can be organized in schema.

Therefore just knowing the definition of a mathematical concept is not enough. The learner must be able to call up the concept of polynomial and rational equations and functions (problems) through APOS theory.

2.1.2 DESIGN AND IMPLEMENTATION OF INSTRUCTION

The design and implementation of instruction is the second part of the APOS based research, where the theoretical analysis influences the creation of specific instructional treatments.

The student has pre-existing knowledge. Depending on this pre-existing knowledge the student first develops partial understanding, repeatedly, again and again returns to the same idea, and gradually brings his/her idea or concept together. APOS theory also assumes that learning is dependent on cognitive conflict (between the pre-existing knowledge and the coming new concept or idea) cognitive conflicts may arise when the learners' ideas are verbalized and confronted with contrasting ideas of others. Therefore, in research based on APOS theory, students are organized to do different exercises or questions individually and in group by checking conceptual change through group discussion guided by APOS theory to implement mathematical concepts on polynomial and rational equations and functions through individual and cooperative learning (group work). And these students are encouraged to reflect on what they performed or interiorized. The students should start to contradict each other in their mind. The effort to overcome those contradictions may lead to the formation of new mental constructions. Therefore students in the class room who did not understand what they learned from their teachers and what they get from their text books. The researcher found that conceptual change of these students comes from the cycle of group work and from the individuals more and more activities. That is when students discuss each other (when there are group, G1 circulates, each of the other groups G2, G3 and G4 groups and the same is true for the other (see in chapter 3) the students conceptualized more and more on this polynomial and rational equations and functions.

In general, the students who learned through pedagogical strategies and conceptual change strategies reflected a better conceptual change when it is analyzed by the stages of APOS theory. But the students who learned without conceptual change strategies reflected less conception when it is analyzed through APOS theory.

APOS is a cognitive theory. Objects in this framework are considered as mental objects that individuals construct in order to learn about mathematical objects. APOS defines mathematical knowledge as "an individual's tendency to deal with perceived mathematical problem situations by constructing mental actions, processes, and objects and organizing them in schemas to make sense of the situations and solve the problems" (Dubinsky and Mc Donald, 2001).

APOS is an acronym that stands for those constructions (Action, process, object, and schema). As an action is repeated and the individual reflects up on it, it may be interiorized in to a process. When the individual is able to describe, or reflect up on, the steps of the transformation wholly in her/his mind without actually performing those steps actions have been interiorized in to a process.

Once a process has been constructed, it is possible for an individual to think of it reverse and possibly construct a new process. Different processes can be composed to construct a new one. When an individual is aware of the process as a totality and can apply actions to it, the process in encapsulated and an object is constructed, when necessary, an individual may de-encapsulate an object back to its process.

A schema is an individual's collection of actions, processes, objects and other schema linked, consciously or unconsciously, by some general principles in a framework that the individual uses in situations related to that concept or mathematical topic. A schema can be used as an object; it is thematized, when the individual is aware that transformations can be applied on it. When applying the theory in research, a detailed theoretical analysis of the actions, processes, objects and schemas that a learner may construct in order to learn a specific mathematical concepts or topics are designed. The resulting description is called a genetic decomposition of the concept.

2.2 THEORETICAL CONTEXT

Since the middle of the 20^{th} century, two distinct perspectives on learning have predominated in much of the mathematics research. One is a psychological focus on individual learning and knowledge. Important examples include Piaget's developmental psychology, which had influenced much recent work in mathematics education research (for, example, the APOS theory of mathematical learning described by Asiala et al., 1996).

2.2.1 LEARNING AS A SOCIAL ACTIVITY (SMALL GROUP ENVIRONMENT)

According to (Vygotsky, 1962), **"Students bring pre-existing schemas, from academic and from other life experiences, to small group settings. It is classified that as small group problem solving only those group activities in which two or more individuals are cooperating to ensure their own learning and facilitate the learning of all others in the group. It is believed that small group problem solving occurs when the shared knowledge stays within the individual zones of proximal development of group members. Briefly the zone of proximal development is defined as the difference between the level of an individual's actual development and more advanced level of potential development that could be observed in interaction between more or less capable participants**".

There is no shortage of suggestion for improving the mathematical learning that takes place in properly class rooms since the mid-1980s especially increasing number of mathematician's devoted considerable attention to the nature of mathematics. They have focused on each of the core areas of content, instruction and learning, producing reformed curricular materials and instructional strategies, and developed, tested and revised theories of learning.

Any teacher or tutor of mathematics has experienced the increased understanding of a topic that comes from the act of explaining mathematics to others. Such beliefs and experiences support the idea that effective cooperative group work on appropriate mathematical tasks can be a highly effective instructional strategy.

2.2.2. LEARNING AS INDIVIDUAL DEVELOPMENT IN A SOCIAL CONTEXT; CO-CONSTRUCTION OF KNOWLEDGE

Constructivist view learning as the joint construction (or co-construction) of the psychological system of the developing person by himself or herself, and the "social others" who influence the development of the individual psychological framework through attempts to communicate ideas. Learning is seen as arising from the two-way interplay between individual and social activities. The co-constructivist view blends the complementary constructivist and socio genetic view points in the learning process. **Valsiner (1987) recognized that "there is a collective culture of socially shared meanings and the individual's personal cultures. Thus, culture is partially shared and partially personal".**

Because of personal contributions, individuals are said to co-construct the collective culture. According to Valsiner individuals construct their personal meanings from the collective cultures by way of internalization, where at the same time contributing to the reconstruction of that collective culture by process of externalization. Although there are constructs, his or her own knowledge socially, in Vygotsky's language (1962) Valsiner emphasizes **"that because of**

individual experiences, it is unlikely that two people construct exactly the same understandings".

From stand point of an observer, we could say that the development of a group's understanding of a mathematical idea, both collectively and at the level of individuals in the group, consists of a series of internalization and externalization transformations or representations alternating with one another, the nation of internalization implies a critical transition or transformation from perceived external social experiences to individual inner thinking, invoking new mental functions within the individual.

The formation of new mental functions takes in to account the individuals previous experience, their mental structure, and the dynamic nature of group interactions. In parallel with transformation of external experiences to the internal sphere internalization, the reverse, transformation of internal experience in to the external expression, externalization, takes place completing a cycle and making it possible to study the cognitive development of an individual. We can infer these changes by comparing the original, external expression of ideas with the transformed externalization cycle. According to Valsiner (1993), "**externalization involves constructive transformation of the internalized psychological phenomena in to the social, interpersonal domain".**

Valsiner suggested that : **"the existence of two forms of coordination between internalization and externalization (a) parallel functioning and (b) delayed functioning of the externalization with respect to internalization. Any externalization feeds back some internalization, which is the source of new externalization, and the cycle continues**". For example, when we, as teacher, provide same information to our students we with others we are encouraging the externalization of their thoughts. Social activities such as class discussion may produce a new series of internalization and externalization transformations of ideas. We expect this increased student activity or engagement to facilitate learning that is changes in their psychological frame work.

In teacher-centered class rooms students shouldn't be asked to reflect and share immediately; to internalize then externalize ideas-more commonly, the process of externalization should be left until a subsequent assessment (assignment quiz, or test etc.). It is possible that by delaying a student's externalization he/she misses the opportunity for better are only asking them to internalize. If at the same time, we ask them to reflect and share their thinking internal transformation and consequently further development of understanding relative to the individuals prior knowledge.

Valsiner's view of learning seems to differ from other theories in another important way. Rather than seeing internalization and externalization as inverse, hence reversible, functional operations that are determined by existing conditions, he instead describes an uncertainity principle he calls canalization**: "A set of constraints that direct but do not precisely determine the next state of human conduct."** (1993, p.25).

APOS theory is interested in mental constructions that students make when they are learning a mathematical concept. When using this theory, the researcher should make a description of a model that might explain the way that students would follow in order to make a proposed construction (Vidakovic and Martin, 2004, pp.465-492).

The model is known as genetic decomposition and consists of mental constructions (Actions, processes, objects, schemas) and mental mechanisms (such as assimilation, exteriorization, encapsulation, de-encapsulation, coordination) put together in a way to explain the learning of the concept in question. It should be emphasized that the genetic decomposition is given in terms of cognitive construction, and not in terms of mathematical results. It is now briefly explained some of these notions that are used in the research.

An action conception of a concept is when the individual can perform calculations and transformations of mathematical objects as a result of external stimuli, such as plugging in numbers for variables in a formula he/she can perform multiple step algorithms, where each step is triggered by the previous one. When the individual reflects about these actions, he or she can run through the steps in her/his mind without having to perform them explicitly. In this case we say that the actions have been interiorized and the individual possesses a process conception. Two or more processes can be coordinated to form a new process.

When the need arises to perform transformations on these processes, the individual encapsulates them in to objects and now can apply actions on these newly constructed entities. In this case he/she shows an object conception of of the concept in question. When necessary, the object can be de-encapsulated for the individual to have access to the underlying processes.

Finally, objects, processes and actions related to the concept in question form a coherent structure called a schema which can be involved in order to resolve problem situations. A new object can be assimilated by an existing schema; this way the reach of the schema is expanded in order to include new objects. According to piaget schema development passes through three levels: Intra, Inter and Trans. At the Intra level the newly constructed object is present, together with other objects and processes, but at this stage the individual is not aware of the relationships that might exist between them. At the inter level these relationships start to be present and Trans level is characterized by being aware of the complete structure and being able to decide whether a given situation can be resolved by that particular schema.

CHAPTER THREE

3. RESEARCH DESIGN AND METHODOLOGY

3.1 DESIGN

A qualitative research method is used by employing phenomenology research design where the phenomenon was traced through individual effort and group discussion guided by Apos theory. A test is given to students before intervention to check the level of thinking and it is analyzed through narration. Then there was an intervention through group discussion. Finally, a test is given to students after intervention and it is analyzed through narration by guidance of Apos theory.

In order to get relevant and sufficient information on this research, an appropriate methodology was employed to investigate adequately the current practice of preparatory mathematics education that the intended conceptual change would be achieved in polynomial and rational equations and functions.

The study was carried out at different stages. At the initial stage of investigation, a review of related literature was made to develop an introductory theoretical background. In the second stage, a contact was made with the preparatory schools to secure relevant information for further understandings. In the third stage based on the review of related literature data collection instruments were developed and piloted. Then the reviewed research questions were administered. Besides, observations were conducted. Finally, based on collected data analysis and discussions were made followed by summary, conclusions and recommendations.

3.2 DATA SOURCES

In this study both primary and secondary sources will be used to gather adequate information about the conceptual change strategies on students' mathematics achievement and its challenges on preparatory schools. Primary sources are used to get first hand information concerning the methods of conceptual change strategies on students' mathematics achievement and its challenges.

3.3 POPULATION AND SAMPLING TECHNIQUES

Sampling is used usually to select representatives from the large population because the information to be gained from the sample will be considered as the representative of the total population. The study will be conducted on two preparatory schools. Regarding selection of sample students. Regarding selection of sample students, out of four hundred (Total: T) students in the sample schools around 80 (N) students are selected before intervention and 20(N)

Students are selected after intervention for the study. As noted by Gay et.al (2009:133) if the Population size is beyond a certain point (about N=5,000) the population size is almost irrelevant and a sample size of 400 will be adequate.

Table 3.3.1: The number of students and schools from which the data is selected is given below

School	Name of the school	No of prep. students	School Type
1	Spring of knowledge Academy (prep. and high school).	47	Private
2	Abadir prep. And high school	33	Community

3.4 DATA COLLECTION INSTRUMENTS

Both quantitative and especially more detail in qualitative instrument of data collection is used for the study. The instrument used to gather data is test, and observation. Employing multiple data collection instruments help the researcher to combine, strengthen and amend some of the inadequacies of the data. Accordingly tests are used as the main data gathering tools where as, observation is used to enrich the data obtained through tests.

3.4.1 Tests

Tests are used to collect relevant and first hand information from key informants such as students. The researcher preferred tests as the main data gathering instruments. Besides it allows respondents to respond to questions confidentially and independents without any interference so as to minimize biases because of the presence of other persons. Tests are two types close and open ended but open ended tests are prepared for the concerned bodies of the respondents independently. This is because it is to allow the chance for respondents to give free responses. There are eleven questions before intervention to students' level of thinking by the guidance of Apos theory. After intervention questions are also distributed to check the conceptual change.

3.4.2 OBSERVATION

Observations were used to obtain supplementary data for the study. Direct observation was conducted in the class room. The aim was to draw pertinent data from class room that supplement the tests and could assist comparisons with the standard adopted in the school. Mathematical performances among different approaches to instruction were also observed. The main objective is to determine whether students have made the mental constructions set forth by the genetic decomposition used in a particular study. The idea is to access data that shows a range of mathematical performance on different mathematical tasks in order to compare the thinking of student who had difficulty with the thinking students who succeeded. These differences enable the researchers to determine whether the mental constructions called for by the theoretical analysis account for differences in performance.

In the teaching learning process of polynomial and rational equations and functions in the class room, the researcher observed that there is a great problem in conceptual understanding of the students. In most of the researcher's observations, most of the students do not understand mathematics (in this specific especially on polynomial and rational equations and functions). The main reasons are that teachers are not using pedagogical and conceptual change strategies in the teaching learning process. This may be also due to that even some teachers do not have an understanding of conceptual change strategies and how to apply it. In the class room students need more participation, lesson should be presented coherently step by step from simple to complex, students should be reinforced to think and try individually and work in group. Students need to work constructively individually and in group by the help of their teacher. These are some of the reasons for the lack of conceptual change of the students.

Most of the time students are lectured in this polynomial and rational equations and functions in order to cover the schedule of completing the text book. Teachers worry to be consistent according to their lesson plan rather than doing on the conceptual change of students understanding. I prepared checklist for this observation(see Appendix C).

3.5 PILOT TEST

This pilot test reports on a study which used the APOS (Action ,Process ,Object and Schema) theory to investigate preparatory students' understanding of polynomial and rational equations and functions. This pilot test reports on the analysis of students' responses to some types of questions on polynomial and rational equations and functions. The findings of this pilot test indicates that the polynomial concept is one that students find difficult to understand and suggests that this is the result of more students not having appropriate mental structures at the action, process, object and schema levels.

1. Is the function f(x) = $\frac{x2+1}{x2-1}$ *a polynomial function? Why?*

Table 3.5.1: Question 1 analysis of student response (N=20)

No.	Respondents' response	Number of respondents	Percent	APOS level
1	Totally no response (no answer)	9	45%	_
2	Yes, because the domain is all real number	1	5%	Level 1
3	No, because the domain is not the set of all real number	10	50%	Level 2

Table 3.5.1 indicates that 10 students (about 50%) reflected or described about this function from what they already interiorized. This response shows that each of these 10 students are able to imagine or construct or give solution from their mind by constructing a mental process and thinking that the answer should be as they Stated in table 3.5.1, number “3” above. Here the students responded the solution from their internal conception which is already transformed from the action stage. In the context of the stages of APOS theory (genetic decomposition) this suggests that 50% of the students had mental constructions which are developed up to the process level. But 1 student (about 5%) gave a response for the external stimulus. This response needs step-by-step activities and more interactions to develop the action level of the APOS theory.

Table 3.5.1 also shows that about 45% of the students gave no response there could be a number of reasons for this. One of the reasons could be that some of them had inadequate (no full) conception and for the others it may be that students do not know how to explain or how to give response. Therefore students should be treated more and more through conceptual change strategies so that students' conception will be transformed well from one stage of the APOS theory to the next stage (genetic decomposition).

2. Is the graph of the function f(x) = |x| a polynomial function? Why?

Table 3.5.2: Question 2 analysis of student response (N=20)

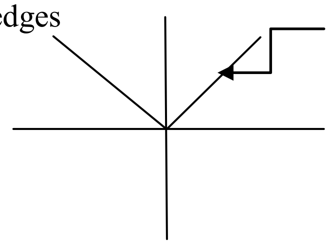
No.	**Respondents' response**	**Number of respondents**	**Percent**	**APOS level**
1.	Totally no response (no answer)	15	75%	—
2.	NO	4	20%	level 1
3.	No, because the graph of a polynomial function does not have sharp edges, it is smooth curve. But f(x) = |x| has sharp edges. The graph is Sharp edges 	1	5%	Level-3

Table 3.5.2 Suggests that 4 students (about 20%) responded for the external stimulus (question 2 about absolute function). But this response still needs step-by-step (repeated) practices by learning through conceptual change strategies in order to be transformed to the next stage since students did not interiorized more concept about this specific question to give a better response or to reason out it. So an analysis of this question reveals that these students possibly had mental constructions which were at the action level.

Table 3.5.2 also shows that these students have acquired the ability to think about this specific question. If we see their answer, it shows that they have responded from their internal thinking or conception since they have already interiorized and encapsulated about this function.

So, the mental structure of these students already transformed in to a process and then encapsulated the process in to the next cognitive development (object level). Therefore these students possibly had been appropriately developing towards the third stage of the APOS theory (object level) for application on absolute functions in relation to polynomial functions.

Finally, table 3.5.2 indicates that 15 students (about 75%) did not have full idea how to work the solution of this question and even did not guess. The reason may be that they did not learn their lesson through conceptual change strategies. So, their conception is limited even to give any response.

3. Can a polynomial function have more zeros than its degree? Why?

Table 3.5.3: Question 3 analysis of student response (N=20)

No.	**Respondents' response**	**Number of respondents**	**Percent**	**APOS level**
1	No	14	70%	Level 1
2	Totally no response (no answer)	2	10%	—
3	No, because the maximum attainable zeros are equal to the number of degrees.	4	20%	Level 3

Table 3.5.3 suggests that 14 students (about 70%) had mental constructions which were developed up to the action level. This is because these students gave a single response for the coming external stimulus (question 3). Here students' understanding is limited to carry out a

transformation to the next level. They responded a single answer that needs activities or repeatedly different exercises on the zeros of polynomial functions. If students develop their mental constructions through practices, they improve their mental development to transform to the next stage.

Table 3.5.3 also shows that 4 students (about 20%) had an internal mental construction because they are enabling to imagine and construct or give solution by explaining the reason. Here the students have interiorized questions and encapsulated this question into cognitive object. Therefore the conception of these students had developed towards the object level.

Table 3.5.3 also shows that 2 students (about 10%) did not give any response to the stimulus. This may be due to that these students couldn't have enough idea about this question or they might have been poor in learning through conceptual change strategies.

4. How many zeros does $x^4 - 1 = 0$ have? Why?

Table 3.5.4: Question 4 analysis of student response (N= 20)

No.	**Respondents' response**	**Number of respondents**	**Percent**	**APOS level**
1.	It has 4 zeros	5	25%	level 1
2.	It has 2 solutions	3	15%	Level 1
3.	Totally no response (no answer)	10	50%	—
4.	It has 2 zeros, because $x^4 = 1$ => $x = \sqrt[4]{1}$ => $x = + 1$	2	10%	Level 3

Table 3.5.4 suggests that 8 students (about 40%) for question 4(zeros of polynomial function)the students could not interpret well since it was difficult for the students to explain more details on

this function. So the mental structure of these students is developed up to the action level of APOS theory.

Their response shows that students had a reaction to the given question which is not developed enough to transform to the next step.

Table 3.5.4 also indicates that 2 students (about 10%) had reflected and described the solution on the zeros of polynomial function from their internal conception because they have already encapsulated without the need of external stimulus. This shows that there was a transformation process from one stage of the APOS theory to the next by developing the mental structure.

Finally 10 students (about 50%) did not give any response on this polynomial function. This shows that these students did not fully understand the concept of this function. This may be due to that students did not learn well, through the pedagogical and conceptual change strategies in order to transform to the next step of the APOS theory stages (genetic decomposition).

5. Do the functions Y= x^2+2 and Y= -x^2-2 have intersection point (a point in common)? Why?

Table 3.5.5: question 5 analysis of student response (N=20)

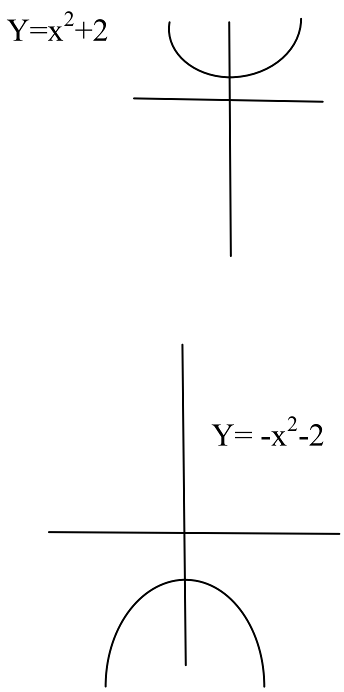
No.	**Respondents' Response**	**Number of respondents**	**Percent**	**APOS level**
1	No intersection	9	45%	Level 1
2	Totally no response (no answer)	8	40%	—
3	No intersection because y= x^2+2 and y = $-x^2$-2 do not have common point i.e one is upward the other is down ward. 	3	15%	Level 4

Table 3.5.5 shows that 9 students (about 45%) responded for the external stimulus (question 5) about the intersection point of these two specific functions. Here the students need more

interactions with this question by doing questions repeatedly by the help of their teacher as a facilitator.

To transform the mental structure of the students in to the next stage of APOS theory, students should be given enough time to practice the activities. In addition teachers should use pedagogical strategies and conceptual change strategies through individual work, group work, group circulation and the stages of APOS theory respectively. Therefore the mental structure of these students is towards the action level of the APOS theory.

Table 3.5.5 also indicates that, 8 students (about 40%) did not give any response about this question. The reason may be due to that they did not have full concept or it may be that they did not know how to do the solution of this question.

Finally table 3.5.5 shows that 3 students (about 15%) explained well from their internal conception which is already developed through interiorization and encapsulation of what they have been discussing about it.

According to their response, these students have realized how to solve this question. This internal thought shows that the mental structure of these students include the stage of APOS theory (action, process, object and schema).

Therefore these students had a mental construction which is developed towards the schema level.

3.6 METHODS OF DATA ANALYSIS

The qualitative and quantitative data from the tests are categorized and tabulated taking in to account the basic items which involve in the test and the basic research questions. After this the analysis and interpretation is made with the help of percentages to analyze the data . The qualitative data from test and observation is organized in to meaningful themes and narrated through narrative description that is qualitatively to support the quantitative data. The data collection and analysis is crucial for APOS-based research, since without empirical evidence, a generic decomposition remains, merely a hypothesis. So the data collected through test organized and analyzed through APOS theory.

3.7 PROCEDURE OF THE STUDY

Before the actual data collection, the tools which are prepared by the researcher are given to the research advisor in order to comment the extent to which the items are appropriate in securing relevant information for the research, and to make amendment when necessary. After developing the data gathering instrument, pilot testing were conducted. This was done to check whether or not the items included in the instrument enable the researcher to gather relevant information. Thus in the pilot study, 20 questionnaires were distributed to preparatory students of mathematics. The questionnaires after having been evaluated and cheeked through pilot test, the final form was prepared. Furthermore, in this process the participants of pilot test were informed about the objective of the pilot test, and how to fill, evaluate and give feedback to the instruments. Based on the suggestion forwarded the instrument were improved and necessary correction were done by avoiding ambiguous words and simplifying some terms to facilitate easy understanding before it is administered to the respondents of the study.

A four week lessen is given to students on polynomial and rational equations and functions. The lesson is given through different continuous assessment (like, class work, home work, worksheets, tests and exams) by teachers through the school systems.

Next a questionnaire or (questions) are prepared by the researcher and given to 80 students to do individually. This questionnaire is to analyze the conceptual change of students' mental development on polynomial and rational equations and functions. There are 11 questions which are given to these 80 students to analyze the conceptual development (change) through the stages of APOS theory.

Then 20 students are taken from the 80 students who already gave response for the questionnaires before group circulation. So a three week lesson is given to these 20 students on polynomial and rational equations and functions through group discussion and circulation or an instructional treatment is given with the lesson on these students by the researcher to analyze the conceptual change of the students through group circulation guided by the stages of APOS theory. The researcher used the group division (group circulation) by constructing table and used the stages of APOS theory to analyze the students' conceptual change through these conceptual change strategies.

These twenty students are arranged in four divisions (groups), G1,G2, G3 and G4, (see in 3.8.1.1).

In each group there are 5 students. So during the lesson time of polynomial and rational equations and functions the researcher used conceptual change strategies using the group circulation and stages of APOS theory to check their conception. (see the table in 3.8.1.1).

First each student is facilitated to do different questions (activities) related to polynomial and rational equations and functions individually. Next each group (G1, G2, G3, and G4) practiced these exercises among themselves. That is the 5 students of each sub-group (small-group) discussed about each different question of activity on polynomial and rational equations and functions among themselves. Then a pair of two group (as indicated in table 3.8.1.1) were made to discuss together on those different questions. In addition three different groups are arranged together and interacted with in themselves by sharing what they understood in the previous activities. Finally all the group (G1, G2, G3 and G4) joined together to have the whole discussion what they did previously and they did additional related questions (activities) together. Moreover they interchanged and discussed their conceptual development on this polynomial and rational equations and functions.

Therefore learning (or conceptual change) is seen as arising from the two way interplay between individual and social activities. In each of the above steps the students' conceptual change and mental transformation from one stage of APOS theory to the next stage is seen.

Finally a questionnaire (question) is distributed to analyze students' conceptual change which is done through group circulation and guided by the stages of APOS theory.

3.8 METHODOLOGY

3.8.1 PEDAGOGICAL STRATEGIES

To bring conceptual change to students, teachers should design different activities and exercises to help students construct actions, interiorize then in to process, encapsulate processes in to objects and coordinate two or more processes to construct new processes.

There are a great variety of student centered active teaching and learning methods to bring conceptual change. But the researcher advices teachers all methods may not be appropriate for specific learning objective. A class room teacher should decide how a particular lesson will be thought, there is no one best way to teach a particular topic or group of students. The teacher should select the method he /she feel most appropriately addresses his/her learning goals for the lesson and is most appropriate for students. The researcher advices teachers to check the APOS level of the students mental constructions through these methodologies.

But the reasons why the researcher focuses on the individual and group work activities to have conceptual change on mathematics will be described next in detail. For example, if students are going to learn about polynomial and rational equations and functions first they have to read more and more and try to understand the concept of this polynomial and rational equations and functions by themselves. This should be the starting point of brain conflict between the pre-existing knowledge and the new concept. This is to mean that first each student should read individually by himself or herself.

After each student analyzed each question by himself or herself, the students should come together and discuss in group what they understood about the questions on the given topic.

These students have given enough time to analyze and describe their conception on polynomial and rational equations and functions. The researcher used the term group work to represent a combination of collaborative and cooperative work. Students bring the- existing schemas, from academic and from other life experiences to a group. In the class room the researcher classified the students in to four groups. In each group activities there are 5 individuals who are cooperating to ensure there own learning and facilitate the learning of all others in their group.

The researcher believes that the group problem solving occurs when the shared knowledge stays within the individual zone of proximal development of group members. Briefly, the zone of proximal development is defined as the difference between the level of an individual's actual development and more advanced level of potential development that could be observed in interaction between more or less capable participants (Vygotsky, 1962). Valsiner emphasizes that because of individual experiences, it is unlikely that two people construct exactly the same understandings.

The following table shows the arrangement of the groups in which each group shared concept in each other group. But this is done after in each group each individual has been given activities to check his/her level of conception. Then each group discussed the questions with in itself. That is the students in group one discussed each other about polynomial and rational equations and functions. The same was true for group two, group three and group four. Then

a) Group one discussed with group two, group three and group four (G1 with G2, G3 and G4)
b) Group two discussed with group three and group four (G2 with G3 and G4)
c) Group three discussed with group four. (G3 with G4)
d) Group one, group two and group three discussed together (G1, G2 and G3)
e) Group one, group two and group four (G1, G2 and G4) discussed together.
f) Group two, group three and group four (G2, G3 and G4) discussed together
g) Group one, group two, group three and group four (G1, G2, and G4) discussed together.

In this method or interaction the individuals conceptual change development or APOS level improvement is checked. This group arrangement is written in the following table.

There is group discussion at least for 90 minutes each day for 30 days only on this lesson.

Table 3.8.1.1: The arrangement of the group for conception.(Group Circulation)

No	Number of groups			
	One group	Two groups	Three groups	Four groups
1	G1	G1, G2	G1,G2,G3	G1, G2, G3, G4
2	G2	G1, G3	G1,G2,G4	__
3	G3	G1, G4	G1,G3,G4	__
4	G4	G2, G3	G2,G3,G4	__
5	__	G2, G4	__	__
6	__	G3, G4	__	__

The main purpose of education is enabling students develop knowledge, skill and attitude. This can be achieved through different methods. Matter are means of conveying ideas and skills to get or acquire a certain subject Mather in more concrete and comprehensive way. Methods are used to achieve desired educational objective. They are tools to educate learners. At different times different methods of learning has appeared and has dominant at certain period of time. Previously the teacher centered method, which mainly puts the teacher at the center of every activity in the instructional process, was dominant. Teacher centered approach is a kind of approach which revolves around the teacher as the center of any activity in the teaching process with the motion that as Farrant (1980,120) cited in Hanna 2007, put it “the teacher knows best”. Similarly Borich (1988) states the teacher’s role is to pass “facts”, rules, or “action sequence” on to the students in the most direct way possible. The main focus of attention in this approach is what is though rather than the child who is thought.

High emphasis is given to content coverage and educating a child is seen as more of working through the syllabus than, as Farrant (1980), cited in Hanna 2007, put it trying each child develop his potential.

CHAPTER FOUR

4. PRESENTATION, ANALYSIS AND INTERPRETATION OF DATA

4.1 DATA ANALYSIS (Before Group Circulation)

An APOS analysis of Students' Understanding of the concept of polynomial and rational equations and functions

1. When do we say that an expression which is written in the form of $\frac{p(x)}{Q(x)}$ is rational? Why?

Table 4.1.1.: Question 1 analysis of student response (N = 80)

NO.	Respondents' Response	Number of respondents	percent	APOS level
1.	When p(x) and Q(x) are polynomial expressions	12	15%	Level 2
2.	When Q(x) ≠ 0 because if Q(x) =0 the function becomes undefined.	37	46.25%	Level 1
3.	When both p(x) and Q(x) are polynomials and q(x) ≠ 0	8	10%	Level 3
4.	When both p(x) and Q(x) are polynomials and q(x) ≠ 0 because if Q(x) =0 the function becomes undefined.	2	2.5%	Level 4
5.	No response (no answer)	21	26.25%	—

Table 4.1.1 suggests that for question 1 (about rational expression) the response of the students indicated that 12 students (about 15%) had an internal construction which is not necessarily

directed by external stimuli. Here these students reflected or described the steps of transformation without actually performing steps. These students have an individual understanding as being internal or under each one's control rather than as something one does in response to the external stimuli. It shows that the action is already repeated and now the students reflect up on it. It is to mean that the students interiorized this conception. That is why they gave output for the given input. Therefore the students who gave this answer are developing towards the process conception. Supporting this idea Dubinsky and Mc Donald stated that: **a process stage occurs only after an action is repeated and the individual has the opportunity to reflect up on the action. Then "he or she can make an internal mental construction called a process, which the individual can think of as performing the same kind of action, but without the need of external stimuli."**

Table 4.1.1 also indicates that 37 students (about 46.25%), have an understanding which is limited to carry out a transformation to the next level only by reacting to this question by giving only few details that does not show more conception. These students are unable to interpret more about question 1 except to evaluate specific points. Here it was difficult for students to explain well about rational expression. According to my theoretical perspective, a major reason for the difficulty is that the learner is not able to go beyond this response because his/ her conception is limited. Here students reacted to the stimulus (question 1) which each student perceives as external. So these students developed their best up to the action level of their mental constructions. In the action stage, a student who is unable to interpret a situation as a function unless he or she has a (single) formula for computing values is restricted to an action concept of function.

In addition table 4.1.1also shows that 8 students (about 10%) had a mental structure that has already interiorized and encapsulated the concept of this specific question, i.e about the rational expression. This response shows that each of these 8 students are enable to imagine and construct or give solution for this question in their mind by constructing a mental process and thinking or realizing that the answer or the solution should be as described in the above table number 3. Since these students are aware of a process as a totality, they realizes that transformations can act on that totality and can actually construct such transformations (in their imagination), then we say that these students have encapsulated the process in to a cognitive object

Moreover table 4.1.1 also indicates that 2 students (about 2.5%) had a mental structure that describe the solution of this question by constructing the solution through detail structures. These structures show that the students have passed through action, process and object that is organized and linked in to a coherent frame work, which is schema. When we say a coherent it is to mean that it provides each of these students with a way of deciding which mental structures to use to deal with this question (about rational expression).

Finally, table 4.1.1 shows that 21 students (about 26.25%) gave responses that did not indicate any understanding of the concept at all. One of the reasons for this may be that they did not have full idea about this specific question or how to give solutions of this question. But according to the researcher, if the students learn through pedagogical strategies so that students will have more and more conceptions about the given lesson or topic. And this pedagogical strategy should be evaluated through APOS stages in order to check students' conceptual change or development. Stressing the importance of APOS stages to check students' conceptual change or development in mental constructions.

Dubinsky (1986, 1991) and Conttrill et at. (1996) formulate the encapsulation of process in to object as three stages of a four- part theory with the acronym APOS. A (step-by-step) action becomes conceptualized as a total process, is encapsulated as a mental object, later to become part of a mental schema. Conttrill et al. (1996;171) explained that:

> An action is any physical or mental transformation of objects to obtain other objects. It occurs as a reaction to stimuli, which the individual perceives as external. It may be a single step response, such as a physical reflex, or an act of recalling some fact from memory. It may also be a multi-step response, by then it has the characteristic that at each step, the next step is triggered by what has come before, rather than by the individual's conscious control of the transformation… when the individual reflects up an action, he or she may begin to establish conscious control over it. We would then say that the action is interiorized, and it becomes a process. Conttrill et al. (1996, p.171)

When we see the students on the **action level** the response of these students on this question (rational expression) which is described in number "2" under table 4.1.1 shows that these students do not develop their conception towards rational expression (i.e unable to define rational expression even with respect to the definition of polynomial functions). But they expressed that the denominator of the rational expression is different from zero. That is why these students are grouped under the action level of the APOS theory. The reason may be that they did not learn through pedagogical strategies or they did not understand through the given methodology. The teachers' deficiency in understanding the meaning of rational expression may be the other reason for the lack of student's concept. Even their pedagogical knowledge could not make up for their ignorance of the concept. Anyhow most of the students are at the action level. Here students were learning through procedural understanding. If they keep on doing through procedure, their conception would be changed to the next stage of the APOS theory. Here in polynomial and rational equations and functions students' conception should be assessed through participation, motivation, group discussion or group circulation to transform to the next stage of APOS theory.

When we see the students at the **process level** on this question these students have a better understanding or concept on rational expression by responding that Q(X) and P(X) should be polynomial expressions. Students at the process level of understanding show that if they are treated with pedagogical strategies (methodologies), their conceptual understanding will be improved. The number of students whose conception is at the process level of this question is less than the students' conception at the same level of questions number 4, 7, 8 and 11. The reason may be that these four questions (problems) are easier than this question or to increase the number of students in the process level of their conception, the students should do more and more activities that should be checked regularly through conceptual change strategies to develop their conception. But these students have a better conception of the process level than questions (problems) number 5, and 9. The reason may be that even though students are learned about polynomial and rational equations and functions to have deep understanding of this topic, students should do more and more activities that should be checked regularly through conceptual change strategies to develop their conception. Due to lack of this method (strategy), it might have been more difficult to find the solution of these two questions. The other reason may be that in the teaching learning process, the teacher may have restricted by teaching a certain subtopic of this topic or the students may not read more reference books to develop their knowledge.

When we see the students at the **object and schema levels** on this polynomial and rational equations and functions the other response for question 1 is that "when both P(x) and Q(x) are polynomials and Q(x) #0". This is a response or conception which is the object level of the APOS theory on an expression which is written in the form of $\frac{P(x)}{Q(X)}$.

The number of respondents in object level (conception) of the APOS theory is less than questions number 3,9 and 11. The reason may be that for these students this question is more difficult than the three questions (number 3, 9 and 11) to explain it. The other reason may be that these students need methodologies to understand easily on this topic. Students should be guided to do more and more activities through different methodologies. The number of students who have the schema level of conception is two on this specific question. When we compare with the other questions, the number of students at the schema level of this question is very low (it is only about 2.5%). This shows that students do not understand even what a rational expression is, what a polynomial function is and what the domain of rational expression is etc.

In addition what I understand from these responses is that students' conceptual understanding towards polynomial and rational equations and functions is limited. The reason is that there are only 2.5% students who responded for the definition of rational expression. They said that "it is a rational expression when both P(x) and Q(x) are polynomials and Q(x)#o.

2. Is the function f(x) = $\sqrt{9-x2}$ a rational function? Why?

Table 4.1.2: Question 2 analysis of student response (N = 80)

NO.	Respondents' Response	Number of respondents	Percent	APOS level
1.	No, because $9-x^2$ is not polynomial	1	1.25%	level 1
2.	No, because it is not written in the form of $\frac{a}{b}$	44	55%	Level 1
3.	No response (no answer)	24	30%	—
4.	No, because it is not polynomial	11	13.75%	Level 4

Question 2 is based on recognizing the given function, whether it is a rational or not. In table 4.1.2, 55 students (about 56.25%) had mental constructions which is developing towards the action level. This is because these students gave a reaction to the given questions but they could not interpret well. It was difficult for the students to explain more detail more than those responses which are given in table 4.1.2 number"1" and "2". An action stage of conception is described by Dubinsky and McDonald (2001):**" as a stage in learning where a transformation of objects is perceived by the individual as essentially external and as requiring, either explicitly, or from memory, step-by-step instructions on how to perform the operation".**

Table 4.1.2 also suggests that 11 students (about 13.75%) had an understanding of the question. These 11 students have concept about this function because they described or interpreted well. They reflected what they already interiorized and encapsulated in to a cognitive object. According to APOS theory, an individual who deals with a mathematical situation by using certain mental mechanisms to build cognitive structures are called interiorization and encapsulation and the related structures are actions, processes, objects develop to the higher level which is called schema. At this schema stage of these questions students responded what they have in their mind and described that this function is not a rational function. Therefore these students are in a good schema level.

Table 4.1.2 also shows that 24 students (about 30%) had no full mental construction or conception towards a rational function of this question. For these students who gave no or little response, there could be a number of reasons for this. For example reasons could be that some of them had inadequate (no full) conception, and for the others it may be that students do not know how to explain or how to give a response.

In general, most of the respondents' conception in question number 2 is found at the action level of the APOS theory. But the process and the object levels of APOS theory are not observed in this question. This shows that the students' conception is not developed to the next stages (process and object levels). The reason may be that to understand such kind of questions students should do more activities on polynomial and rational equations and functions. The other reason is that this question is hard to have conception for students. This is because they did not learn through pedagogical strategies or methodologies to understand easily the given topic. But there are about 11 students who have the schema level or conception on this question. This conception of the students may be from their own activities, discussions and by doing more and more exercises by searching problems.

3. Is the function $f(x) = x^2+1 / x^2-1$ a polynomial function? Why?

Table 4.1.3: Question 3 analysis of student response (N = 80)

No.	Respondents' response	**APOS analysis**	Number of Respondents	Percent
1	No, because the domain has to be the set of all real numbers	Students who gave this answer had mental constructions which is developed up to L3.	20	25%
2	No response(no answer)	—	18	22.5%
3	**a**. when it is simplified it will not be polynomial	Students who gave this answer had mental constructions which is not even at the action level	26	32.5%
	b. It is a rational function	Students who gave this answer did not have full idea to work out the solution of such questions up to the action level.		
4	**a**. yes, because it fulfills all things to be polynomial	Students who gave this answer totally had no full idea about this question.	16	20%
	b. yes because their exponents are equal and have a positive degree.	Students who gave this answer had little idea of the basic technique for identifying a polynomial function. Hence no mental construction for this specific question but not up to the action level.		
	c. yes because both the denominator and numerator are rationals and they have power.	Students who gave this answer had mental constructions which were developing up to the action level.		

An analysis of question 3 reveals that 20 students (about 25%) have acquired the ability to think about the structural way and gave a response. In addition, the students described or reflected up on the given question (question 3) in the transformation without necessarily performing them. Thus the mental structure of these 20 students became aware of the totality of the processes by realizing that transformations can act on it, and is able to construct such transformations. Therefore these students had mental structures or constructions, which appropriately developed up to the object level. Supporting this idea Conttrill et al., (1996, p.172) proposed that**: "We see that there are at least two ways of constructing objects- from processes and from schemas."**Table 4.1.3 also indicates that 18 students (about 22.5%) did not indicate any answer. One of the reasons for this may be that they did not indicate any understanding of the concept on this specific question. In addition the response of the students which is indicated in number 3 and 4 is not related with the question.

To conclude on this question, only 25% of the students are at the object level of the APOS theory. On this question the students at the action level are more than the students at the object level. And the students who did not well understand this question are more than the students who are at the action and object level. In this question the students responded different solutions but most of their answer shows that most of the students do not know the solution of this question. This indicates that these students may not understand the basic concept on polynomial and rational equations and functions. The reason could be that the students do not like or they afraid of mathematics. The reason for this is that students were not learning mathematics through pedagogical strategies. If they learn through methodologies they will understand easily and if they understand, the will be eager to learn, to do more and more activities so that the students conceptual development will be improved.

A well developed conceptual understanding of a topic includes understanding of another dimension of structure of the subject, attitudes toward mathematics.

4. To decompose a rational expression $\frac{p(x)}{Q(x)}$, what should be the degree of p(x) in relation to the degree of Q(x)? Why?

Table 4.1.4: Question 4 analysis of student response (N = 80)

No.	**Respondents' Response**	**Number of respondents**	**Percent**	**APOS level**
1.	The degree of P(x) is less than Q(x)	26	32.5%	Level 2
2.	The degree of P(x) is greater than Q(x)	54	67.5%	level 1

Table 4.1.4 suggests that for question 4 (decomposition of rational expression) 54 students (about 67.5%) showed a single step response that need more practice and activities in order to have mental development or to pass (or transform) to the next stage of the APOS theory. So, their mental structure is towards the action level.

Stressing on this stage, an action stage of conception is described by Dubinsky and McDonald (2001) as a stage in learning: **where "a transformation of objects is perceived, by the individual as essentially external and as requiring, either explicitly, or from memory, step by step instructions on how to perform the operation."** The action level becomes a process when the students describe or reflect what students have in their conception. For example in table 4.1.4, 26 students (about 32.5%) gave response what they already have in their mind. Because the response of these students shows that they can think by performing without the need of external stimuli. Therefore these students had a mental structure which is developed towards the process level. Essentially, Sfard's exteriorization is the catalyst that allows for a student to move from an understanding of function. Similarly, as the student continues to work with functions more and more, he/she begins to condense the process involved in functions. According to APOS theory the challenge in teaching is to help such students to first attain the mental structures required for the process understanding. The challenge is also to help such students develop some sort of effective objective which incorporates an organization framework for using the actions, process, object and schema to evaluate on decomposition of rational expression.

Finally I can say that most of the student's conception on question 4 is found at the action level of the APOS theory. That is about 67.5% of the students are at the action level. And next the rest 32.5% of the students are at the process level of the APOS theory. The students' conception on this question is not transformed to the object and schema level or students' concept is restricted to the action and process levels. The reason may be that students did not practice well on decomposition of rational expression from the polynomial and rational equations and functions. This may be due that students did not learn through conceptual change strategies.

5. How many zeros does x^4-1=0 have? Why?

Table 4.1.5: Question 5 analysis of student response (N = 80)

No.	Respondents' Response	Number of respondents	Percent	APOS Level
1.	It has 2 zeros because $x^4 = 1$, implies That $x = \sqrt[4]{1}$ i.e $x = 1$ or $x = -1$	24	30%	level 3
2.	It has 4 zeros because the degree of leading term is 4	13	16.25%	Level 1
3.	It has one zeros	9	11.25	Level 1
4.	It has two zeros	6	7.5%	Level 2
5.	No response (no answer)	28	35%	—

The polynomial equation in question 5 is based on finding the zeros of the equation. Table 4.1.5 suggests that 22 students (about 27.5%) had mental constructions which is at the action level. This is because these students gave response for the external stimulus (question) and little without thinking deep in to the question. That is their conception on this question is limited to transform to the next stage of APOS theory. It shows that they did not learn in the conceptual change strategy. Because they couldn't even interpret well or they are unable to describe the solution. This shows that they should practice more and more through different exercises rather than using a single formula which restricts the concept of this function.

Table 4.1.5 also suggests that 6 students (about 7.5%) had an internal construction rather than giving an immediate response without an internal construction. So, these students are able to give solution or reflect to the question from their mental structure. Therefore it shows that these students can be transformed in to the next stage of the APOS theory if teachers use more and more conceptual change strategies through checking the conceptual development. In general in table 4.1.5 number 4, shows that the students are developing up to the process level of the APOS theory. A brief note should be made about the use of the word "process". "process" has been used multiple times with in multiple meanings. In particular, "process" is used within the description of interiorization as well as with in the process conception. The meaning of "process" for interiorization places the student inside the process while the meaning of "process" with in the process conception places the student outside of the process. For example when a student is inside the process he/she focuses on the individual components of the process even if he/she sees the whole process. Consider question number "5" in the analysis part of chapter four, the Zeros of the function X^4-1=0. Here the individual component is shown. That is

X^4-1=0

X^4=1

$X^4=\sqrt[4]{1}$ =

X=1 or X= -1

When an individual has towards the point of outside the process, the individual sees the process as one thing, i.e f(±1) is what it makes f true when you put only ±1.In addition question 5 is based on recognizing the zeros of a polynomial equation for which 24 students (about 30%) have interpreted or reflected or described well what they already constructed in their mind. This response shows that these students have already interiorized, encapsulated and developed their mental structure up to the object level. Because a collection of actions, processes and objects are organized in a structured manner to form a schema. Therefore these students developed up to the schema level. Supporting these ideas Piaget and Garcia (1983/1989) introduced the notion of thematization of a schema:

> Abstract mathematical notions have in many cases first been used in an instrumental way, without giving prize to any reflection concerning their general significance or even any conscious awareness of the fact that they were being used. Such consciousness comes about only after a process that may be more or less long, at the end of which the particular notion used becomes an object of reflection, then constitutes itself as a fundamental concept. This change from which usage, or implicit application to consequent use, and conceptualization constitutes what has come to be known under the term thematization (Piaget and Garcia, 1983/1989, p.105).

Table 4.1.5 also shows that 28 students (about 35%) did not indicate any answer. A possible reason may be that they did not fully understand the concept of this question. The reason may be due to poor teaching learning conceptual change strategy and lack of applying the stages of APOS theory to test the conceptual change through conceptual change strategies. When it is compared to the other questions the students who responded on this question have a better understanding on the schema level of APOS theory than any other questions (from the 11 questions) which are given before group circulation. The reason is that students may have spent more time on solving the zeros of polynomial functions through procedural understanding. Or these students may have their own way or method of understanding the given topic or problems. They may not expect everything from their teachers. If these students were learned through conceptual change strategies, more students would have been developed to the highest level of conception. There are only six students who are at the process level of this question because some students' conception is transformed to the schema level and the rest students are at the action level of the APOS theory. To develop the students' conception which is at the action level, there should be follow up whether students are doing activities and teachers should use the pedagogical strategies in the teaching learning process.

6. Can a polynomial function have more zeros than its degree? Why?

Table 4.1.6: Question 6 analysis of student response (N = 80)

No.	**Students' response**	**APOS analysis**	**Number of respondents**	**Percent**
1	Yes, totally 4 students did not give reason.	—	4	5%
2	No, totally 29 students did not give reason.	29 Students who gave this answer had mental constructions up to the action level	29	36.25%
3	No answer (no response) ,totally 10 students did not give any response.	—	10	12.5%
4	**a**. No because we can find the zeros by the formula(n-1)	7 Students (about 8.75% who gave this answer had mental constructions up to the action level. i,e Level 1.	37	46.25%
	b. No, because it cannot have more zeros than its degree since the number of zeros is its maximum attainable degree.	8 Students who gave this answer had mental constructions up to the process level. i,e Level 2.		
	c. No, because the degree should be greater **d.** No, because it has only one zero. **e**. No, because the degree is used to determine the zeros.	14 Students (about 17.5%) who gave this answer had mental constructions up to the action level. i.e Level 1		
	f. yes because their exponents are equal and have a positive degree.	3 Students (about 3.75%) who gave this answer had no idea of the zeros of polynomial functions.		
	g. yes because both the denominator and numerator are rational and they have power.	5 Students who gave this answer had no idea about the zeros of polynomial functions.		

An analysis of question 6 reveals that 22 students (about 27.5%) did not have idea about this question. One of the reasons for this is that they did not fully understand how to give a solution of such questions. This also may be due to lack of conceptual change strategies in the teaching learning process.

Table 4.1.6 also shows that 50 students (about 62.5%) gave a response for the given question. But this response still needs more and more (step-by-step) repeated practices by learning through conceptual change strategies in order to be transformed to the next stage. So, these students had mental structures which are towards the action level. According to (Asiala et al., 1996):

"The act of reflecting, a construct essential to APOS theory, is the learners conscious attention to the operations being performed. This reflection plays an integral role in learning and knowing since it involves reaching beyond contemplation on the particular performance of techniques and algorithms no matter how complicated".

In addition table 4.1.6 indicates that 8 students (about 10%) have mental constructions which is developed up to the process level. Because their response shows that they have already interiorized the concept of this specific question. That is why they reflected that a function cannot have more zeros than its degree. Therefore we can realize that transformations (whether actions or processes) can act on the process and is able to actually construct a transformation.

When we see the students' conception most of the respondents in this question responded on the action level of APOS theory and there are some students whose conception is at the process level. The response of these students shows that most of them do not have a better conception on the zeros of polynomial functions. In addition they responded seven types of answers most of which lost the correct answer of this question. If these students were learned about polynomial and rational equations and functions through conceptual change strategies, a better conception of the students might have been transforming from one level to the next level.

7. ***What is polynomial function?***

Table 4.1.7: Question 7 analysis of student response (N = 80)

No	Students' response	APOS analysis	Number of respondents	Percent
1	A polynomial function can be written in the form of $anx^n+a_{n-1}x^{n-1}+\ldots\ldots+a_1x+a_0$	The students who gave this response (about 31.25%) had mental structures which is developed towards process level.(level 2)	25	31.25%
2	It is a function that can be written in the form of $a_1x_1+a_2x_2+\ldots\ldots+a_nx_n$	About 11.25% of the students had mental structure which were not even at the action level.(level 1)	9	11.25%
3	Polynomial function is a function when it has a leading coefficient ,constant terms and degrees.	The students(about 16.25%) who gave this answer had mental construction which is developed to the action level(level 1)	13	16.25%
4	It is a function which is written in the form of ax^3+bx^2+cx	The students who wrote this answer (about 8.5%) had mental structures towards the action level.(level 1)	7	8.75%
5	No response(no answer)	The students did not write any answer. The reason is that they have no full concept about polynomial functions.	23	28.75%
6	It is a function with domain all real numbers.	The students (about 3.75%) who gave this answer had mental construction which is developed towards the process level.	3	3.75%

An analysis of question 7 shows that 29 students (about 36.25%) had a reaction to the given question which is a single step response or an act of recalling some fact from memory.

These students did not describe any more steps that show their interiorization on this specific question. So the mental structure of these students is limited up to the action level of APOS theory. Table 4.1.7 also indicates that 28 students (about 35%) had mental constructions which are developed up to the process level. This is because these students described or explained what they have in their mind or what they interiorized before. These students established the conscious control over the action stage. So this shows that they are supported more and more by conceptual change strategies through APOS theory. Stressing the importance of APOS theory, Asiala et al. (1996) by building on Piaget's work and constructivist ideas, introduced Action-process-object-schema (APOS theory) that: **"all action conception is a transformation of a mathematical object according to an explicit algorithm seen as externally driven. It may be a manipulation of objects or acting up on a memorized fact. When one reflects up on an action, constructing an internal operation for a transformation, the action begins to be interiorized. A process conception is this internal transformation of an object. Each step may be described or reflected up on without actually performing it. Processes may be transformed through reversal or coordinating with other processes. To construct an object conception, a person may reflect on actions applied to a particular process and become aware of the process as a totality. One realized that the transformations (whether actions or processes) can act on the process, and is able to actually construct such transformation. In this case we say that the process conception has been encapsulated in to an object conception. And mathematical schema is a collection of action, process and object conceptions".** Finally, table 4.1.7 , shows that 23 students (about 28.75%) did not write any answer. A possible reason may be that they did not fully understand the concept of polynomial functions. The reason may be due to poor conceptual change strategies and lack of applying the stages of APOS theory to check the conceptual change through conceptual change strategies. These students could not define even about polynomial functions. Their conception is restricted at the action and process level of the APOS theory. The students' deficiency in understanding the meaning of polynomial function determined or showed their inability to generate conception. So, teachers’ pedagogical knowledge could not make up the students to have more conception. This implies that students need conceptual change strategies so that students’ level of understanding would be developed. One thing that may be true is that the mathematical power of a concept depends on its relationship with other concepts. The closure a concept is to the structure of the subject, the more relationship it may have with other topics. Compared with subject Matter knowledge, other aspects of teaching usually receive more attention, perhaps because they seem to affect students more directly. In thinking of how to teach a topic a major concern would be what approach to use. In general a teacher’s subject matter knowledge may not automatically produce promising teaching methods or new teaching conception. But without solid support from subject matter knowledge, promising methods or new teaching conceptions cannot be successfully realized.

8. What is rational function? Why?

Table 4.1.8: Question 8 analysis of student response (N=80)

No	Students' response	APOS analysis	Number of respondents	Percent
1	A function that can be written in the form $\frac{p(x)}{Q(X)}$ where Q(x)#0 because if Q(x)=0 it will be undefined.	These students (about 47.5%) had conception towards the action level of the APOS theory.(towards level 1)	38	47.5%
2	It is a function that is written in the form of $\frac{p(x)}{Q(X)}$,where p(x) and Q(x) are polynomials and Q(x)#0 because if Q(x)=0 it becomes undefined.	The students who gave this answer had mental constructions which is developed up to the schema level that is level 4.	11	13.75%
3	It is a function written in the form of $\frac{p(x)}{Q(X)}$, where p(x) and Q(x) are polynomials.	About 8.75% of the students had mental construction up to the process level. i,e level 2	17	21.25%
4	No response (no answer)	Question 8 suggests that 17.5% of the students did not indicate any answer. This shows that they have no full idea on the concept of rational function.	14	17.5%

Table 4.1.8 shows that 38 students (about 47.5%) were performing on the given question depending on their pre existing knowledge of rational functions. Here students were unable to interpret about this question. So they are restricted to an action conception. Table 4.1.8 also suggests that 17 students (about 21.25%) had mental constructions which is developed up to the

process conception. This is because these students are able to reflect or describe their internal construction which is not now necessarily directed by external stimuli. The students answered this question from their interiorized conception.

In addition table 4.1.8 indicates that 11 students (about 13.75%) have acquired the ability to think about this specific question. Here these students gave the solution of this question. This shows that the students have already interiorized and encapsulated about rational function. Because if we see their answer, they responded from their internal thinking or conception that is already transformed in to a process (imagination) and then encapsulated the process in to the next cognitive object leading to the schema level. This idea is supported by Sfard (1992, 64-65) by hypothesizing two approaches to concept development, one operational, focusing on processes the other structural, focusing on object. He explained that:

A constant three step pattern can be identified in the successive transitions from operational to structural conceptions: first there must be a process performed on the already familiar objects, then the idea of turning this process in to a more compact, self contained whole should emerge, and finally an ability to view this new entity as a permanent object in its own right must be acquired. These three components of concept development will be called interiorization, condensation, and reification respectively. Condensation means a rather technical change of approach, which expresses itself in an ability to deal with a given process interms of input /output without necessarily considering its component steps.

Table 4.1.8, also indicates that 14 students (about 17.5%) did not give any response. This may be due to lack of the appropriate concept or unable to respond questions by the respondents.

Here students are asked to explain what a rational function is and to reason out. This is to investigate the conception or misconception of the students in polynomial and rational equations and functions. The number of students who are in the process level of conception on question number 7 (about the definition of polynomial function) are more than the number of students on question number 8 (about the definition of rational functions). And there are students at the action level of students' conception on both questions. But there are no students at the object or schema level of conception on question number 7 where as there are 11 students who are at the schema level on question number 8. The teachers should use conceptual change strategies. The focus of these strategies did not completely parallel to the teachers' knowledge. But to bring conceptual change towards the students, teachers should use conceptual change strategies. For example I used the group circulation pedagogical strategy so that students learn through this method to have conception. This method brought a change in conception on polynomial and rational equations and functions. I advise teachers to use this method through the teaching – leaning process of mathematics.

9. For the polynomial function, suppose the degree of the dividend is n and the degree of the divisor is m. If n>m, then what will be the degree of the quotient, q?

Table 4.1.9: Question 9 analysis of student response (N = 80)

No.	Respondents' response	Number of respondents	Percent	APOS level
1.	The degree of q is less than **m**	15	18.75%	Level 1
2.	The degree of q is less than **m** and **n**	7	8.75%	Level 1
3.	The degree of q is greater than or equal to **m**	3	3.75%	Level 2
4.	The degree of q is less than or equal to **m**	2	2.5%	Level 2
5.	The degree of q is **n-m**	17	21.25%	Level 3
6.	No response (no answer)	36	45%	—

Table 4.1.9 indicates that 5 students (about 6.25%) reflected or explained up on the steps in the transformation without performing them. So these 5 students had mental constructions which were developed up to the process level. It shows that the action is already repeated and now the students are giving response up on it. It is to mean that the students interiorized this conception in the second stage of the APOS theory.

Table 4.1.9 also shows that 22 students (about 27.5%) had conception towards the action level. Because students were simply giving action to the coming stimulus before it is interiorized in their mental structure. It is to mean that the students' understanding is limited to an action conception.

In addition 17 students (about 21.25%) had appropriately developed the object level of the APOS theory. This is because when an individual reflects on operations applied to a particular process, it becomes aware of the process as a totality and realizes that transformations can act on it. Then he or she was thinking of this process as an object. In this case, we say that the process has been encapsulated to an object. Supporting this idea, Piaget (1972,70) stated that:

> ---the whole of mathematics may therefore be thought of in terms of the construction of structures,---mathematical entities move from one level to another; an operation on such "entities" becomes in its turn an object of the theory, and this process is repeated until we reach structures that are alternatively structuring or being structured by "stronger" structures. Piaget (1972, p.70)

Table 4.1.9 also shows that 36 students (about 45%) did not try any solution. The reason may be that they did not have full idea how to answer the solution of such questions.

When we compare the responses of the students of this question before and after group circulation, a better conception is seen after group circulation, After group circulation there are students who have conception on each level of APOS theory and the number of students at the process and object level. But the conception of the students before group circulation on this question is seen on the first three stages of APOS theory. And the number of students at the action level is more than the number of students at the process and object levels. Here there are only five students at the process level. In general, it shows that the students who learned through conceptual change strategies have a better conceptual change than the students who learned without these strategies. But for such kind of questions students may need more time to do more and more questions through procedural understating so that their conception will be developed gradually. For example if you see the response of some students on this question after group discussion in 4.2, there are students who responded through giving examples and explaining more detail ideas.

10. ***Is the graph of the function f(x) = |x| a polynomial function? Why?***

Table 4.1.10: Question 10 analysis of student response (N = 80)

No.	Respondents' response	Number of respondents	Percent	APOS level
1.	No, because it is the absolute value function	12	15%	Level 2
2.	No, because the domain is all real number and the range is y> 0	22	27.5%	Level 1
4.	No, because it has sharp corner, the polynomial function graph should be smooth and continues.	14	17.5%	Level 4
5.	No response (no answer)	32	40%	__

An analysis of question 10 reveals that 22 students (about 27.5%) had mental structures which is reacted to the coming stimulus. Students' conception is limited in order to transform to the next stage. So students should react or response more and more or should do more activities so that there will be transformation to the next level. Therefore these students possibly had mental structures which were at the action level.

Table 4.1.10 also indicates that 32 students (about 40%) did not have full idea of how to work the solution of such questions. These students need repeated conceptual change strategies in order to join their mental structure to the APOS theory stages.

According to table 4.1.10, 12 students (about 15%) have an interiorization of this specific question because their response shows that there is internal thinking which is reflected due to this specific question. So these students had mental construction towards the process level. More over table 4.1.10, suggests that 14 students (about 17.5%) possibly had been appropriately developed towards the schema level for applications on absolute functions in relation to polynomial functions. This is because actions, processes and objects can be organized in a structured manner to form a schema. So the response of these students shows that, they have already interiorized and encapsulated the conception about this question. That is they have an understanding of this function with respect to polynomial function.

To answer this question,students should know the definition of polynomial functions. But as described in question number 7 the students' conception towards polynomial function is limited. Anyhow on this question even if the students are responded on the action, process and schema levels of the APOS theory, still they need conceptual change strategies to develop their conception.

To do this the students should be made to do problems by their own through methodologies. This response (the schema level) may be due to that these respondents may have home teacher who help them to practice through procedural understanding. After I collected the questionnaires some students said to me that " unless our home teacher comes it is hard for us to do the mathematical problems like homework, worksheets, assignments etc.". This shows that there are students who depend on another person (home- teacher, families etc) to solve the problems. This is because in the school students are not learning through conceptual change strategies (pedagogical systems). If they would learn through methodologies, they would adapt solving the problem by their own.

11. Do the functions $y=x^2+2$ and $y=-x^2-2$ have intersection point (a point in common)? Why?

Table 4.1.11: Question 11 analysis of student response (N=80)

No.	Respondents' response	Number of respondents	Percent	APOS level
1	No, because they do not meet each other	7	8.75%	level 2
2	No, because the first function is upward parabola and the second is downward parabola.	38	47.5%	Level 3
3	No, because when we intercept they have no common point.	15	18.75%	level 2
4	No response (no answer)	20	25%	__

Table 4.1.11 suggests that 38 students (about 47.5%) had mental constructions which are developed up to the object level. At this stage, students interiorized a concept that need more practice in order to have better mental development up to pass or to transfer to the next stage of APOS theory. Here students showed some fact about these functions by recalling from their memory, so, these students had mental structure which is developed up to the object level of the APOS theory. When a learner can reflect on a process and transform it by some action, then the process is considered to have been encapsulated to become an object. According to (Asiala et al., 19996): **"De-encapsulation enables the learner to use the properties inherent in the object to perform new manipulations up on it. For instance, with respect to functions, a learner must be facile in encapsulating processes into objects and de-encapsulating objects in to processes when considering manipulations such as adding, multiplying, or creating sets of functions".**

Table 4.1.11 also indicates that 22 students (27.5%) gave what students already have in their mind. Because the response of these students shows that they have internal mental development (or interiorization) about these specific functions. In addition from their reflection and description we understand that these students encapsulated ideas about the intersection point of these functions. Therefore the mental construction of these students is developed up to the process stage of the APOS theory.

Moreover, 11 students (about 13.75%) did not fully understand about the intersection point of these functions or they did not know how to explain (how to respond) this external stimulus (question). Therefore these students need more and more interactions with different questions of polynomial and rational equations and functions in order to get the way to the mental development or to transform their conception from one stage of APOS theory to the next stage.

Here most of the students' conception is on the process and object level of the APOS theory. This conceptual change may be due to that these students might have done more exercises on this topic or they might have got the question easier in relation to some other questions which are given to them. But still they need to develop their conception towards the next stages of the APOS theory. To do this, students should learn through pedagogical strategies. The students who learned through pedagogical strategies have a better conceptual development than the students who learned without this methodology. For example those students who learned about polynomial and rational equations and functions through group circulation (see 4.2) have developed a better conception on the same question than the students who learned without methodologies. After group circulation, when students are asked to give response on this question, their conceptual change was at the action process and especially more students at the schema level of the APOS theory. But before group circulation the students' conception is restricted on a certain conceptual change.

The students who did not give response from question 1 up to 11 before group circulation have no full idea or they did not understand what teachers thought. The reason is that teachers are not using pedagogical strategies (methodologies) in the teaching learning process or here there are students who are not ready to learn mathematics, they do not want to do more activities. There are also students who have no background at mathematics. These also influence the students' understanding on the current topic. Due to this they afraid mathematics as if it were impossible to do it. One of the main reasons on the lack of students' concept is that teachers write note on the board and use lecture method and they do not assess students understanding. They do not motivate students by showing that mathematics is an interesting subject which is related with the real life situation and its application. As I observed there are students who do not like to continue their education beyond preparatory. They want to give up their education at grade 12. After I distributed and collected the questionnaires they said that "we do not want to continue our life through mathematics, we want to participate (continue our life) in another business, we want to be merchant like our families. The other reason for misconceptions is that students are not guided to do questions through assignments even outside of the class room. Teachers run to complete the text book. I can say that in Ethiopia there are a lot of students who lack conceptual change in mathematics. For example from the response of the students on polynomial and rational equations and functions students are spending time on learning mathematics without having concept .How a teacher tended to help the students depended heavily on his or her own knowledge of the topic. The teachers' knowledge was limited to procedures that they would simply tell the students to remember instead of presenting conceptually based strategies to help the students to understand the problem.

4.2 Data Analysis After Group Circulation

1. For the polynomial function, suppose the degree of the dividend is n and the degree of the divisor is m. If n > m, then what will be the degree of the quotient, q?

Table 4.2.1: Question 1 analysis of students response (N=20)(After Group Circulation)

No.	**Respondents' Response**	**Number of respondents**	**Percent**	**APOS level**
1	Totally no response (no answer)	4	20%	—
2	Degree of q = degree of **m**	3	15%	Level 1
3	Degree of q $\geq$ degree of **m**	6	30%	Level 2
4	Degree of q = **n-m**	5	25%	Level 3
5	Degree of q= n-m Or d(q) < d(m) d(q) = d(m) d(q)> d(m) but always d(q) <d(n) example: a. $(x^3+1)+(x^2+1)= x+\frac{1-x}{x^2+1}$ => d(q)<d(m) b) (x^4-1) ÷ $x^2+1) = x^2-1$ => d(q) = d(m) c) (x^4-1) ÷ $x+1= x^3-x^2+x-1$ => d(q) >d(m)	2	10%	Level4

Table 4.2.1 suggests that 3 students (about 15%) showed a single step response (reaction) to the given specific question (stimulus).If students are treated more and more with pedagogical and conceptual change strategies this action conception will be transformed in to the next stage.

During group work, students followed procedures to give responses for the given stimulus. As students practiced more and more on activities through procedures it would be as an input for the action stage in order to be transformed to the coming stages. Supporting this idea Davis (1984, 29-30) described that:

"When a procedure is first being learned, one experiences it almost one step at a time; the overall patterns and continuity and flow of the entire activity are not perceived. But as the procedure is practiced, the procedure itself becomes an entity it becomes a thing. It, itself is an input or object of scrutiny. All of the full range of perception, analysis pattern recognition and other information processing capabilities that can be used on any input data can be brought to bear on this particular procedure. Its similarities to other procedure can be noted, and also its key points of difference. The procedure, formerly only a thing to be done a verb has now becomes an object of scrutiny and analysis".

Table 4.2.1 also shows that for question 1, the responses of the students indicated that 6 students (about 30%) have explained or reflected what they already interiorized through this pedagogical (group circulation) strategy and conceptual change strategies of the APOS theory in to their mental structure. It is due to the repetition of the action level that the internal construction is made through different exercises. Therefore these students had the mental structure that is developed towards the process level of the APOS theory there is a transformation (mental development) from one stage to the next stage of the APOS theory. This is due to the strategy designed for the conceptual change of students' mental development. According to (Asiala et. al., 1996), he stated that:

"Even though an action conception appears limited in scope, it is a necessary element in the construction of understanding since actions are done on the mathematical objects with in a learner's realm of experience. As an action becomes interiorized through a sequence of repeating the action and reflecting upon it, the action no longer remains driven by external influenced since it becomes an internal construct called a process. Attainment of this process conception indicates the learner can reflect on the process, describe it, and even reverse the steps of transformation without restoring to external stimuli. In particular, reflection provides the learner with an awareness of how procedures work, a feeling for the results without physically performing the operations, an ability to analyze and manipulate variant algorithms, and a capability to see relationships and organize experience. This reflection is an integral part of reflective abstraction that consists of drawing properties for situations by paying conscious attention to the actions, interiorizing those actions into processes, encapsulating the processes into objects and finally organizing related schemas. In particular, the reflection on a schema with intention to transform it extends a learner's understanding by yielding an additional means of constructing an object". Thus, APOS theory accounts for the construction of objects from two different sources by way of encapsulation of processes and reflections up on schemas. In addition table 4.2.1 also suggests that 4 students (about 20%) did not give response. The reason for this may be that the students did not know how to give response for this question or may be lack of using conceptual change strategies due to ignoring more practices in the action level of the APOS theory. More over table

4.2.1 also shows that 5 students (about 25%) reflected and described what they already interiorized and encapsulated in the action and process stages respectively.

According to their response, these students used the pedagogical strategy (group circulation) by the help of the researcher in this group circulation method and checking their mental development through the stages of APOS theory.(Action, Process, Object, Schema). So since they have practiced different activities in the class room continuously, and encapsulated, they developed the mental structure up to the object level of the APOS theory. This is because these students are enable to imagine and construct or give solution for this question in their mind by constructing a mental process and thinking or realizing that the answer or the solution should be as described in the above table number “4”.

Moreover, table 4.2.1 also suggests that 2 students (about 10%) had an internal conception or thinking or imagination which is reflected or described well from what they already interiorized and encapsulated in their mind. That is this response indicates a coherent collection of actions, processes, objects and schemas. But schemas are formed by a collection of processes and applying objects that are organized in a structure manner. This idea is supported by Cottrell et al, (1996, 172) proposed that:

> ----an individual can reflect on a schema and act up on it. This results in the schema becoming a new object. Thus, we now see that there are at least two ways of constructing objects from process and from schemas. Cottrill et al., (1996; 172)

The number of students who are at the action level is less to that of the process level. This shows that the students’ conception is developing towards better understanding. The reason may be that during learning polynomial and rational equations and functions students are treated through group circulation. Even though students are given different activities to do until they reach to the maximum conception of APOS theory, there are students who did not use this method properly to have a good conception. In addition due to this group circulation method, most of the students' conception is at the process and object levels of the APOS theory. It shows that the procedural understanding of the students is changed to the conceptual understanding. Here in this specific question there are few students who achieved their conception at the schema level of the APOS theory. This question may be more difficult to be solved by the respondents than the next two questions or it may need more additional time during learning polynomial and rational equations and functions to reach the maximum level (schema level) of the APOS theory. There are also students who did not give any response on this question. The reason may be that they might have been ignored to give response, or they may need more time to understand the concept.

2. Do the functions $y=x^2+2$ and $y=-x^2-2$ have intersection point (a point in common)? Why?

Table 4.2.2: Question 2 analysis of student response (N=20) (After Group Circulation)

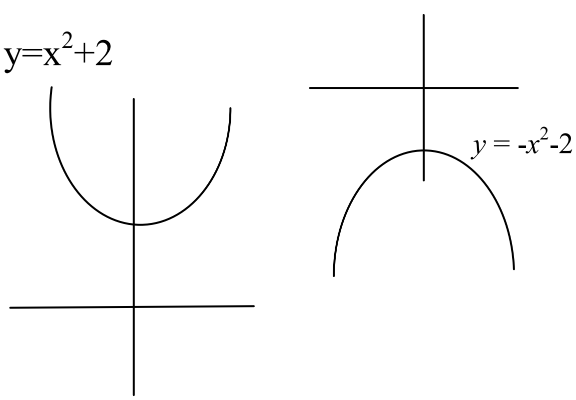
No.	Respondents' response	Number of respondents	Percent	APOS level
1	No, because they never intercept in the graph	3	15%	level 2
2	No	4	20%	Level 1
3	Yes	5	25%	—
4	No, because when we intercept they have no common point. that is $X^2+2=-x^2-2$ $X^2+x^2=-2-2$ $2x^2=-4$ $X^2=-2$ X^2#-2 because when we square any number the result is positive but here it is negative.	3	15%	Level 4
5	No, because the first function is upward parabola and the second function is downward parabola.  They are in opposite direction so they do not intersect	5	25%	Level 4

Table 4.2.2 indicates that 4 students (about 20%) gave a single response by recalling some fact from memory. They responded to the external stimulus. These students need additional more and more activities that make their mental construction ready to be transferred to the next stages of APOS theory, so these students had mental structure which is developed up to the action level of APOS theory. Stressing on this point Clark stated that:

An action equates with any repeatable physical or mental operation that transforms either a physical or mental object in some manner. As a result, the actions tend to be algorithmic in nature and externally driven (Clark et al., 1997).

Table 4.2.2 also shows that 3 students (about 15%) reflected or described about this question from their internal mental constructions or from what they already interiorized and encapsulated in the cognitive structure. These students were doing different questions, through pedagogical strategy and the conceptual change strategies. That is the students were doing activities by group circulation after each group interacted on this question. (See on methodology and research procedure part) since students are treated through individual practice, in pair or in group and in group circulation by checking their concepts through conceptual change strategies, they have an internal construction which is developed up to the process level .As the student repeats carrying out manipulations on numbers, effectively interiorizing mental processes, he/she develops an action conception of function..Table 4.2.2 also indicates that 5 students (about 25%) did not give response. This may be due to that they did not know how to express this question or may be due to lack of full conception to explain it. Finally, table 4.2.2 describes that 8 students (about 40%) have an individual understanding as being internal or under each one's control. It is to mean that these students gave response what they already have conception in their mind. This conception came after students practiced a lot about this topic. Through this practice they interiorized and encapsulated in their mind enabling them to imagine after performing the transformations starting from repeating and reflecting on action, and then interiorizing and encapsulating in to an object and finally internalizing under each one's control. That comes to the schema level. For these transformations, individual work, group work and group circulations have a great application with conceptual change strategies by checking students' conception through the stages of APOS theory. Therefore these 8 students developed a mental construction up to the schema level. Asiala et al. (1996) assert by saying that: **"our tentative understanding suggests that an individual's schema for a concept includes her or his version of the concept that is described by the genetic decomposition, as well as other concepts that are perceived to be linked to the concept in the context of problem situations "(p.12)**. As a result, a learner's schema may or may not represent the whole or even a part of the genetic decomposition. This schema may lack essential elements or contain elements not considered mathematically connected to the concept. However, as Dubinsky (1991) pointed out that:, **"it is not possible to observe directly any of a subject's schemas or their objects and processes. We can only infer them from our observation of individuals who may or may not bring them to bear on problems-situations in which the subject is seeking a solution or trying to understand a phenomenon. But these very acts or recognizing and solving problems, of asking new questions and creating new problems are the means (in our opinion essentially the only means) by which a subject constructs new mathematical knowledge" (p.103**). When we compare the students' conception towards the action level in questions 1 and 2, in this question there are four students who are at the action level when we compare to question one in which three students responded at the action level. These students had the chance of learning through pedagogical strategies (group discussion through circulation). But the students who are at the action level after group circulation shows that these students may not properly participate in the group discussion or activates or they may need additional time to adapt this method or they may need alternative method to have a better conception.

After students did more and more questions procedurally, their conception started to develop conceptual understanding. But the conceptual development is due to the conceptual change strategy (group circulation) that students' conception in polynomial and rational equations and functions is changed to the process level of the APOS theory. Due to group circulation method, the students conception were transforming from one stage of the APOS theory to the next stage of the APOS theory. In question 1 all the stages are observed on different respondents. Even though some conceptual change on polynomial and rational equations and functions was seen in question 1, still students need more activities through this method to have more conception. Even if each student passed through each of the APOS levels, in question "2" and "3" the response of the students shows that, some students' conception is at the action level, some other students' conception is on the process level. But here most of the students' conception is transformed from the object level to the schema level of the APOS theory. This conceptual change (conceptual transformation) from one stage to the next stage of APOS theory is due to conceptual change strategy (group circulation). So if students are guided or learned through the appropriate methods, their conception will be improved. Therefore the students' conception on polynomial and rational equations and functions reached the highest level of (schema level) of the APOS theory due to conceptual change strategies. But to achieve this highest level of understanding, students have been doing different questions and were developing their conception starting from procedural understanding. There are also students who did not give response for these questions after they have been learned through group circulation. Even though most students' conception is developed through this pedagogical strategy, these students may have not used (ignored) this method, or they may not know how to respond or they may not practiced well (procedural understanding) towards new concept through this method.

3. ***What values of a, b and c in the function $f(x) = ax^2 + bx + c$ make the equation to be a quadratic function? Why?*** **(After Group Circulation)**

Table 4.2.3: question 3 analysis of student response (N=20)

No.	**Respondents' Response**	**Number of respondents**	**Percent**	**APOS level**
1.	b=0 and c=0 because it becomes quadratic $ax^2+0+0=0$ i.e ax^2	3	15%	Level 1
2	If a≠0, b≠0 and c≠0 because we find the equation ax^2+bx+c	4	20%	level 2
3	Totally no response (no answer)	6	30%	—
4	All real number but a≠0 i.e a≠0, b E|R,c£lR because if a=0 it will be linear but if a ≠0 it is quadratic.	7	35%	Level 4

Table 4.2.3 suggests that 3 students (about 15%) had mental constructions which are developed up to the action level since these students at this stage responded for the external stimulus. That is these students need more practice on conceptual change strategies to develop their Supporting this idea Asiala stated that: In APOS theory the development of understanding "… begins with manipulating previously constructed mental or physical objects to form actions; actions are then interiorized to form processes which are then encapsulated to form objects. Objects can be de-

encapsulated back to the processes from which they were formed. Finally, actions, processes and objects can be organized in schemas” (Asiala et al., 1996, P.8).

Table 4.2.3 also shows that 4 students (about 20%) gave an explanation and reflected what they already learned. The action is already repeated and now the students reflected up on it. It is to mean that the students interiorized this conception in the second stage of the A pos theory after it has been practiced. Here it shows a process understanding of this question as it includes thinking about these specific questions. These students described what they already have in their mind (or what they already internalized) at the first stage of Apos theory. Therefore these students had mental constructions which are developed up to the process level. Supporting this idea

(Sfard, 1992, p.64) stated that: **"Interiorization may be thought of as an individual’s mental “process performed on already familiar objects”.** In addition, Sfard (1992) also defines **condensation as taking the mental process used in interiorization and creating “a more compact, self-contained whole” (p.64).**

In addition table 4.2.3 shows that 6 students (about 30%) did not give response. The reason may be that these students may need more repeated actions until they come to the conceptual change or it may be that they do not know how to explain this question.

Finally, an analysis of table 4.2.3 reveals that this question is based on finding the coefficient such that the resulting function will be quadratic. The question analysis in table 4.2.3 indicates that 7 students (about 35%) had an internal understanding or thinking (or imagination) which is developed by the conceptual change strategies. Students did different activities, individually, in group and in group circulation by the help of the researcher from the back. Here the students understanding is transformed step-by-step from the action level up to the schema level by interiorization and encapsulation of the concept through this activity. For example an object is constructed through the encapsulation of a process. This encapsulation is achieved when the individual becomes aware of the totality of the process realizes that transformations can act on it, and is able to construct such transformations. Therefore this schema is a collection of actions, processes, objects and other schemas that are linked in some way and brought to bear on a problem situation. Therefore these students had a mental structure which is developed up to the schema level.

4.3 Relations (Difference) on Before and After Group Circulation

One of the reasons for the lack of conceptual change or development of the students on polynomial and rational equations and functions is that teachers did not investigate the current status of their students conception to make their own curriculum or to use conception or to use their own methodologies or strategies to bring conceptual change of their students. Here the result of the questions before group circulation (the students' response) shows that in the teaching learning process teachers did not use their own teaching strategies according to the students' level of understanding instead depends on text books.

As I have been teaching for more than ten years what I observed from my experience is that we should use our own different teaching strategies according to the students' level of understanding rather than being fully dependent on text books.

The other reason for this conceptual change gap is that I would like to indicate that I take the gap in teacher knowledge and conceptual change strategy usage as a factor in the gap in student learning. I do not regard improvement of teachers' knowledge necessarily preceding improvement of students learning. Rather, I believe teachers should use conceptual change strategies to fill the gap in the stages of APOS theory (conceptual difference) on polynomial and rational equations and functions. In addition I believe that both teachers and students should be addressed, and that work on each should support the improvement of the other. Because they are interdependent processes, we shouldn't expect to improve teachers' mathematical knowledge first, and in so doing automatically improve students' mathematics education. Students should be guided through different methods or strategies to have their own conceptual change under the control of their teacher. That is why in these polynomial and rational equations and functions by using group circulation strategy, I found that the conceptual change of the students (the stages of APOS theory) on these students is improved when we compare to those students who were given the questions before group discussion supporting this idea:

As it is cited in LIPING MA, 1999 Putnam and his colleagues (1992) interviewed California teachers and state and district mathematics educators. Some thought the primary focus of the 1985 California Framework was what to teach **" important mathematical content"** ;others thought it was how to teach **"a call to use manipulatives and cooperative groups" (p.24)**. During 1992 and 1993, the Recognizing and Recording Reform in mathematics education project studied schools across the united states. Project member Ferrini- Mundy and Johnson (1994) noted that superficial efforts can pass for change**."Mathematics class rooms can appear to be quite standards oriented, with calculators in evidence, students working in groups, manipulative available, and interesting problems under discussion" (p.191),** but investigators need a deeper understanding of what is happening in these class rooms. The students before group discussion in this study did not perform better than after group discussion. I have observed that mathematics teaching in Ethiopia, especially in the high school and preparatory areas lacks an interaction between the study of the mathematics taught and study of how to teach it. Several factors hinder teachers from careful study of the school mathematics they teach and how to teach it. One of these is that teachers do not need further study of the subject they teach and how to teach. And also hinders teachers' further study of school mathematics. Schifter wrote:

> The notion that even experienced teachers can and should be expected to continue learning in their own class rooms contrasts sharply with the traditional assumption that becoming a teacher marks a sufficient of learning. It is no great exaggeration to say that, according to the conventions of school. Culture, teachers, by definition, already know the content domain they are to teach, the sequence of lessons they must go through to teach it, and the techniques for imposing order on a roomful of students. (1996a, p.163).

The other reason for the difference of conception among students is that the usage of math text books. There are teachers who focus on giving and doing examples on a certain topic for students. Students do not have the chance of doing activities, exercises and assignments. As it is cited in LIPNG MA 1999, text book manual after teachers little guidance (Armstrong and Bezuk, 1995; Schmidt, 1996, p.194), possibly because teachers are not expected to read them. Burkhard (personal communication, may 11, 1998) said:

> The math text book provides a script (with stage directions) for the teacher to use in explaining the topic and guiding the lesson; the students are only expected to read and do the exercises at the end of the chapter. No body reads "teachers' guides" except on master courses.

I had let the students to learn or to do more activities by their own. My focus was on polynomial and rational equations and functions. Students are made to learn a new mathematical concept as well as a new computational skill on this polynomial and rational equations and functions. I had let them discuss the problems after they learn this topic through group circulations which is discussed in chapter three. They have made to have their own idea by the help of the researcher. The point is that they have to think over why, and to explain it. They usually had group discussions, in each group there are 5 students. Each group discussed with each of the other groups. The problem of group discussion is that somehow learners tend to relay on their classmates to explain the issue. The aim of group discussion is that to make students have conceptual change by discussing what they understand from the problems.

4.4 The researcher's aggregation on students' understanding of the concept of polynomial and Rational Equations and Functions.

4.4.1 Mathematical achievement on polynomial and rational equations and functions before group circulation.

Table 4.4.1.1 : Mathematical Achievement (N=80)

No	Questions	L_1	L_2	L_3	L_4	NR
1	Q_1	37	12	8	2	21
2	Q_2	45	-	-	11	24
3	Q_3	26	-	20	-	34
4	Q_4	54	26	-	-	-
5	Q_5	22	6	-	24	28
6	Q_6	50	8	-	-	22
7	Q_7	29	28	-	-	23
8	Q_8	38	17	-	11	14
9	Q_9	22	5	17	-	36
10	Q_{10}	22	12	-	14	32
11	Q_{11}	-	22	38	-	20
Total		345	136	83	62	254
Percent (%)		39.2045	15.4545	9.4318	7.045	28.8636

From my investigation what I observed is that teachers come to the class and teach about a specific topic. They did not assess students' understanding on this topic. They did not use conceptual change strategies (pedagogical strategy) in the teaching learning process. Teachers simply come to the class, teach today's lesson and leave the class, sometimes they give class work, homework and assignments. They simply check these activities and give feedback. They did not assess their understanding or conceptual change on this topic. Even the given activities are done through cheating, copying each other, letting the home teacher and their parents to do these activities and there are some teachers who do this activities by themselves as giving feedback instead of using different methodologies and giving more time for students to do these questions.

Moreover what I observed through this research is that there is a great gap between the students' conception and the mathematics subject that the teacher is teaching. Here the teachers were teaching, the students were learning, about polynomial and rational equations and functions but the students' conception is not fully developed towards this topic.

The students learned about polynomial and rational equations and functions by their teacher. They did different activities , class works, homework assignments, tests and examinations. But as researched the understanding of the students through questionnaires (questions) on this topic, many of the students' conception is limited. The students' mathematical achievement is low even if they did the above activities. What is the reason for this lack of conception?

The reason is that even if there is a teaching learning process, the lack of using conceptual change strategies (pedagogical methods) hinders the student's achievement towards mathematics. That is why in my research even if students were learned by their teachers about polynomial and rational equations and functions, the student's response on these eleven questions shows that, they did not develop concept towards this topic.

In the teaching learning process of mathematics on polynomial and rational equations and functions I observed a mathematical achievement of the students through conceptual change strategies.

From my study, there is a conceptual difference or understanding on this topic after group circulation. For example 39.20% of the students in all eleven questions are found on the first level of conception before group circulation. This shows that there are many students whose understanding on this topic is limited towards the first stage of the APOS theory.

In other words it shows that to develop the concept of the students on polynomial and rational equations and functions, teachers did not use conceptual change strategies so that the mathematical achievement of the students will be improved.

In addition the mathematical achievement of the students before group circulation on this topic is not improved. The students did not show progress in mental development. The student's level of understanding is declining rather than achieving the highest level of thinking. In table 4.4.1.1, we can observe that there are only 7.045% of the students who achieved the highest level of thinking or the schema stage. Only 9.43% of the students are at the object level and 15.45% of them are at the process level of the APOS theory. What we understand from this is that there is insignificant change of conception on this topic. Most of the students' conception is on level one and next on level two. This shows that their conception is limited at a certain stage of understanding or thinking. Students' conception could not be improved to the next higher conception or their mind could not be conceptually changed as we expect.

This is because teachers are not using conceptual change strategies or pedagogical systems. Since they did not use methodologies and strategies in the teaching learning process, it had an impact on the students' mathematical achievement.

According to table 4.4.1.1, 28.86% of the students did not response any answer. This shows that these students did not understand how to give the solution of these questions or they are not confident to give response on these questions since during the instruction time, students are not supported with conceptual change strategies or pedagogical systems.

However there are some students who have showed a better conceptual change on this topic. This is due to that these students use their own methodologies to understand about the topic. They call each other and discuss on the days lesson. There are few students who share ideas each other through discussion but most of the students do not realize how to understand the given lesson. They need support from their teachers, parents, friends and other bodies.

The conceptual development of the students in this circulation is not necessarily fixed to a certain point. Rather it is variable depending on the given methodologies. Sometime it is also dependent on the type of questions. If questions are easy, there will be more number of students that give responses.

When I compare the responses of the students on the first and the second levels of APOS theory, there are more respondents on the first level.

The difference between the object and the schema level is almost insignificant. For those students who have a higher conceptual development or thinking their conceptual change may be due to that these questions may be easy or it may be that students might have studied by their own and through group formation. There are students who study by sharing ideas each other. Some students study through group formation by their own to discuss what each of them did not understand during the lecture time. This may be the reason for the students with a higher conceptual thinking.

In addition there are different internal and external extremes that affect the development of students' conception on the given topic. Some of them affect not to have a good conception and others may help to have a good conception. It is the teachers, students, parents, the schools etc responsibility that should do together to help the mathematical achievement of the students. Especially teachers should be critical in the teaching learning process of the students making them to develop students' conceptual change towards the given lesson.

4.4.2 Mathematical Achievement on polynomial and rational equations and functions after group circulation

Table 4.4.2.1: Mathematical achievement (N=20)

No.	Questions	L_1	L_2	L_3	L_4	NR
1	Q_1	3	6	5	2	4
2	Q_2	4	3	-	8	5
3	Q_3	3	4	-	7	6
Total		10	13	5	17	15
Percent (%)		16.6666%	21.6666%	8.3333%	28.3333%	25%

My investigation shows that the concept development or the conceptual change of students in polynomial and rational equations and functions is improved through conceptual change strategies (pedagogical strategy), specifically by applying different activities through group discussion.

I suggest that in the teaching learning process teachers should not use the lecture method, they should guide the students to change their existing knowledge (or to assimilate) in to new idea (accommodation). To do these, the method is to make the students to think and try to understand about a specific topic and do problems. And then students should be treated to discuss in pair and then in group about this specific topic and do problems. They should be helped from the back until they get the concept of that specific topic. If students are treated in this way repeatedly, their conception towards that topic will be improved.

Table 4.4.2.1 also suggests that the students' understanding is developed towards the higher conception after group discussion. For example 28.33% of the students are at the schema level of conception. Furthermore the number of students who did not give response in the given questions in table 4.4.1.1 is more than table 4.4.2.1. This indicates that there is a difference in conceptual change of the students in polynomial and rational equations and functions before and after group circulation.

In general, in the teaching learning process of these students in polynomial and rational equations and functions I suggest that there is an improvement in the students understanding when they learn through conceptual change strategies. In addition to using group discussion, teachers should follow up the students' conceptual development in each activity to keep the continuous development of their mind.

In general from my study I suggest that teaching should be related with different conceptual change strategies or methodologies (pedagogical systems). The reason is that if there is no teaching methods that make the students active and more participants, their conceptual change will be restricted. But if we teach students through pedagogical systems, they will understand more about the given lesson. They will be more confident and start to do questions by their own.

They need little help from their teacher. Otherwise the flow of knowledge will be one directional, from the teacher to the students but no response from the students. The students will be passive rather than being active. The mathematical achievement on polynomial and rational equations and functions after group circulation is observed in my study. In the teaching learning process on this lesson the students are made to use conceptual change strategies (or pedagogical systems) through group circulation

After group discussion, I prepared questions to check the conceptual change of the students. As table 4.4.2.1 shows 16.67% of the students are at the action level. This indicates that the students' conception is developing or transforming to the higher level of understanding. For example the students' concept before group circulation at the action level of APOS theory is 39.20% and at the process level it is 15.45% whereas after group circulation 21.67% of the students are at the process level. When we see these two levels of conception, it shows that before group circulation most of the student conception is restricted especially to the first level of APOS theory.

In addition table 4.4.2.1 also shows that there is insignificant difference at the object level on before and after group circulation, More number of students have a better conception on this question than those questions which are described under “Before group circulation”. The reason is that the students have been learned about polynomial and rational equations and functions through conceptual change strategies.

The mathematical achievement of the students on this circulation (after group circulation) is better than the first circulation (before group circulation). The main reason may be that in the former circulation students have been learning through different conceptual change strategies. For example the group discussion method which is applied through group circulation is the one which brought conceptual change on polynomial and rational equations and functions. The other reason may be that these questions are easier when we compare to the other questions. The reason is that when students learn through pedagogical strategies they will understand the concept of the given lesson. That is why these students developed a better conceptual change on polynomial and rational equations and functions.

When we see the overall response of these questions there is a conceptual change of the students on this topic. In other words, after these students are treated with the conceptual change strategy (pedagogical strategy), I observed a conceptual development towards polynomial and rational equations and functions. In the teaching Learning process conceptual change strategies should be focused to make education fruitful. I can take my research as an example to show the usage of pedagogical strategies.

As table 4.4.2.1 indicate that the concept of the students towards polynomial and rational equations and functions was developing as I keep on using teaching methodologies. Especially the 28.33% of the students who are at the schema level shows that their pre-existing knowledge on this topic is developed. This mental development is achieved using different activities, exercises, participations etc. through group circulation. In addition, there are more students whose conception is developed towards the process level after discussing a lot in the action level.

In general, these all shows that if students are learned their education consistently through pedagogical strategies the existing concept (knowledge) of their mind will be changed to the higher level of thinking. However if there are students whose conception is not changed through the given methods, it is teachers responsibility to use alternative methods.

In take 4.4.2.1 for those students whose conceptual development is at the lower level, the reason may be that some students may need additional time, some may need alternative method, some students may not concern on this method and there may be internal and external factors that affect some of these students during these conceptual change strategies.

4.4.3 Students' Mathematical Achievement on two questions of polynomial and rational equations and functions before Group circulation

Table 4.4.3.1: Mathematical Achievement (N=80)

Questions	L_1	L_2	L_3	L_4	NR	Total
Q 9	22	5	17	-	36	80
Q 11	-	22	38	-	20	80
Total	22	27	55	0	56	160
Percent	13.75%	16.875%	34.375%	0%	35%	

As table 4.4.3.1 shows that on these two specific questions 13.75% of the students are at the action level, 16.875% of them are at the process level, 34.375% of them are at the object level of conception. From table 4.4.3.1 we can indicate that, the student's conception on these two questions towards the schema level is almost negligible.

This shows that these students need conceptual change strategies or methodologies to develop their level of thinking. Even if 34.375% of the students are at the object level of the APOS theory, they need more work on methodologies to reach the highest level of thinking. There are also students (about 35%) who did not give response on these two questions. Therefore the students' conception will develop if teachers teach through pedagogical strategies.

On these two questions the students' conception is better on the object level but it is minimum at the action and process level respectively. When we compare the result of these two questions with the whole questions before group circulation, there are more students in the action and process levels. But on these two questions the responses on action and process levels are less than the object level. The reason may be that the two questions may be easier than the other questions.

In table 4.4.3.1, 35% of the students did not respond for these two questions. But if they were treated well with conceptual change strategies, the number of students who gave no response would have been less.

As table 4.4.3.1 showed, in these two questions there are no students who gave response for the highest level of thinking in APOS theory (schema level). But this is not mean that students totally missed concept at the schema level.

4.4.4 Students' mathematical achievement on two questions of polynomial and rational equations and functions after group circulation

Table 4.4.4.1: Mathematical Achievement (N=20)

Questions	L_1	L_2	L_3	L_4	NR
Q 1	3	6	5	2	4
Q 2	4	3	-	8	5
Total	7	9	5	10	9
Percent	17.5%	22.5%	12.5%	25%	22.5%

Table 4.4.4.1 shows that 17.5% of the students on these two questions are on the action level. This indicates that the students are participating well to develop their concept on this topic. Table 4.4.4.1 also indicates that 22.5% of the students on these questions are developing their concept to the next higher thinking. And 12.5% of them are at the object level and 25% of these students are at the schema level of thinking. This shows that the students' conception is developing when we use conceptual change strategy. I used a group circulation method in teaching mathematics about polynomial and rational equations and functions. I observed that after group circulation the students' mathematical achievement on this topic is better than before group circulation. For

example, the higher level of thinking (L_3 and L_4) on these two questions (see on table 4.4.3.1) the reason is that in the after group circulation, the students were learning through group discussion. That is students participated in the class, they did questions in group, asked each other, shared ideas what each of them understood and studied through group circulation. From my study I can say that students' mental conception was developing in this circulation. What I found is that when we teach students through methodologies they will be active participant, more confident, can solve problems by their own, their concept will be developed and the teaching – learning process will be easy. For instance, as shown in table 4.4.4.1, the students' conception is developed to the higher levels of thinking than the students in table 4.4.3.1.

There are also students in the after group circulation whose conception is not well developed. The reason is that these students did not practice well through this method, it may be due to behavioral problem, inconsistency on this method, ignoring their friends and teachers in the teaching – learning process and so on.

In table 4.4.4.1, 22.5% of the students did not response any answer. The reason may be that they may not understand the concept through this method. These students might need another alternative method. The other reason might be that I should give more and more time through different pedagogical strategies to make students try their best. In addition each of these students should have been checked every day through different alternative methods.

In general, if the students learn through conceptual change strategies (pedagogical systems) the mathematical achievement of the students will be improved. That is why; I used the group circulation method in the teaching learning process on polynomial and rational equations and functions.

4.5 Challenges towards the Mathematical Achievement of the students

In describing his view of why teachers usually tend not to use active learning, Borrich (1984,163): stated that (cited in Rahel Abraham, June, 2006). Because of scarcity of time… and thinking of covering portions, make teachers use direct teaching methods. …. When the teacher is using direct teaching method, in most cases the teacher informs and students passively listen and take notes. This shows, as a result of barriers such as time limitation and external imposition to cover portions an time, teachers are urged to use direct methods which do not actively involve the students in the teaching – learning process.

Teachers are expected to teach in harmony with the annual plan and finish it all by the end of the year. They do not use pedagogical strategy. Because when they use it and involve the students in activities, it will take much more time to finish the lesson because most of the students have low ability or poor back ground.

The other reason is that the school administration consistently use follow ups to check whether the portions of the text are covered or not. As teachers are evaluated at the end of the year based on that, their focus is merely on the coverage of the portions rather than using conceptual change strategies (pedagogical systems). From the above description, it is evident that these challenges (limited time, content coverage and students' ability) are interrelated and affect the mode of instruction in the class room. It seems that the teachers are influenced by the strict notion the school system has content coverage rather than using pedagogical strategies. Besides, the students' inability to finish the given tasks on time has also aggravated the problem of lack of conceptual change on polynomial and rational equations and functions. This in turn needs the intervention of the teachers, which is time consuming. Thus, it appears that the above challenges have made the teachers to partly refrain from using conceptual change strategies.

Therefore these are some of the challenges that affect not to use the conceptual change strategies. If there is no method, students will not easily understand what they learned. So these challenges lead the students to fail in the mathematical achievement through pedagogical strategies.

The other challenge I faced is that there are also teachers who do not know how to use pedagogical strategies or methodologies in the teaching – learning process. These teachers might not have back ground how to teach or they may not have knowledge of pedagogy (methodologies). It is to mean that there are teachers who do not take even the course about education of teaching – learning methodologies.

Before group circulation in the class room situation, tasks are given to the students and make them discuss and come up with their own ideas, explanations and responses. However I cannot be sure whether the students are doing the assignments properly. The class room is crowded; therefore, they have to form many groups, besides some students lack discipline. I sometimes feel the class room is conductive to carry out the tasks. But this happens only when the difficult points are clarified and ask questions. The above discussion shows the existence of different hindering challenges related with large number of students, disciplinary problems and inconvenient seating arrangement. These challenges among others seem the reason why teachers are unable to implement some pedagogical strategies in their classes.

CHAPTER FIVE

5. SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 SUMMARY

This study is based on APOS theory. APOS theory proposes that an individual has to have appropriate mental structures to make sense of a given mathematical concept. This study is analyzed through the stages of APOS theory. The stages or the mental structures are actions processes, objects and schema required to learn the concept. Research based on this theory requires that for a given concept, mental structures need to be detected, and then suitable learning activities should be designed to support the construction of those mental structures. In this paper the researcher mentioned the conceptual change strategies on students' mathematics achievement through APOS theory. The researcher described how one such perspective, APOS theory is being used, in an organized way, to conduct research and develop curriculum. In this study the researcher found that the mental structure of the students who are learning through conceptual change strategies is more developed to the mental structure without using method. For example the students who learned about polynomial and rational equations and functions through group circulation have more conceptual change than the students who learned without this strategy. The reason is that during the group circulation and conceptual change strategies of APOS theory, students are made to discuss in group about this topic. And their conceptual development is evaluated through conceptual change strategies.

It is important to emphasize that, although our theoretical analysis of a mathematical concept results in models of the mental constructions that an individual might make in order to understand the concept, we are in no way suggesting that this analysis of a mathematical concept results in models of the mental constructions that an individual might make in order to understand the concept, we are in no way suggesting that this analysis is an accurate description of the constructions that are actually made. We believe that it is impossible for one individual to really know what is going on in the mind of another individual.

Because it is seen that knowledge is partially shared and partially personal, it is unlikely that two people construct exactly the same understandings. Therefore in this study the genetic decomposition of the APOS theory focused on the mental constructions required for student success during the initial conceptual of polynomial and rational equations and Functions.The genetic decomposition is used to construct an Action, process, object and schema levels.

I used APOS theory, which gives a description of a possible process by which the function concept can be learned. APOS theory can be used as in many studies has been used, successfully, as a strictly developmental perspective. APOS theory focuses on models of what might be going on in the mind of an individual when he/she is trying to learn a mathematical concept and uses these models to evaluate students' success and failures in dealing with mathematical problem situations. I chose APOS theory for this study because of its effectiveness in previous studies over 28 years. However explanations offered by an APOS analysis are limited to descriptions of the thinking which an individual might be capable. It is not asserted that such analysis describe what " really" happens in an individuals' mind. Since this is probably unknowable. Also the fact that an individual possesses a certain mental structure does not mean that he/she will necessarily apply it in a given situation.

Dubinsky (1991) has developed an epistemological framework referred to as Action-process-object-schema, or APOS. The framework considers the development of a mathematical concept as moving from an action (intraoperational) to a process (interoperational) via a type of reflective abstraction called interiorization. The resulting process can be encapsulated in to an object (transoperational). The framework notes that objects constructed in this manner can be de-encapsulated back to the process when needed. Schemas are constructed by coordinating processes and actions and can also be thematized in to objects (Asiala et al, 1996).

This framework results in descriptions of the mental constructions a student makes to come to understand a concept. These descriptions are called genetic decompositions. Instructional treatments are devised which may bring the student to make the constructions described in the genetic decomposition. These treatments generally involve the use of a mathematical programming language on computers, cooperative learning strategies and alternatives to lecturing (Asiala et al., 1996).

Sfard (1991) also employs a framework which seeks to describe mental constructions in two ways: Structurally (as objects) or operationally (as processes). Her description of the reflective abstractions necessary to move from processes to object is interiorization, condensation and reification. Following is a brief discussion of levels and stages as they appear in Piagets work. A stage can not be skipped. If it is, the students understanding of the concept will lack coherence. Thus, stages are sequential, with each stage necessary for development of successive stages (Piaget 1975, 1974/1976).

A level may or may not be reflected in the data of a specific student. This is because the student may be able to move to the next level or stage rapidly so that the level is skipped, done very quickly, or is not observable in the already acquired higher level or stage (Piaget 1974 (1976).

The stages are invariant over topics and are part of the general theory. Levels will be different for different concepts (Piaget 1974/1976).The role of the level is to analyze and provide mechanisms for building the next level in the stage or the stage itself; this should be reflected in the definition of the level. According to Piaget (1975), stages, together with their levels, are sequential, each contributing to the development of its successor. In particular, every level, contributes to the development of the stage of which it is part (Piaget, 1975).

5.1.1. Summary on the achievement of polynomial and rational equations and functions before group circulation

Table 5.1.1.1: Summary on Mathematical Achievement (N=80)

No	Questions	L_1	L_2	L_3	L_4	NR	Total
1	Q_1	37	12	8	2	21	80
2	Q_2	45	-	-	11	24	80
3	Q_3	26	-	20	-	34	80
4	Q_4	54	26	-	-	-	80
5	Q_5	22	6	-	24	28	80
6	Q_6	50	8	-	-	22	80
7	Q_7	29	28	-	-	23	80
8	Q_8	38	17	-	11	14	80
9	Q_9	22	5	17	-	36	80
10	Q_{10}	22	12	-	14	32	80
11	Q_{11}	-	22	38	-	20	80
Total		345	136	83	62	254	880
Mean		4.3125	1.7	1.0375	0.775	3.175	
Grand mean		$7.825 \div 4 = 1.95625$					

Table 5.1.1.1 shows that the mean of the concept of the 80 students on these questions at the action level is 4.3125.

This means that most of the students' concept on polynomial and rational equations and functions involve around the first stage of the APOS theory. The reason is that students did not learn through conceptual change strategies (pedagogical methods). Or they need more support how to learn the mathematical ideas so that their conception will develop to the higher stage of thinking.

Table 5.1.1.1 also indicates that the mean of the concept of the students at the process level is 1.7. There is a decrement of the development of the concept of the students towards the next higher stages. For example the mean difference between the first two stages of APOS theory is 2.6125. This means that there are more and more number of students who did not understand about this topic in the teaching learning process. To eliminate this difference or gap and develop the students' concept towards the higher level of thinking teachers should teach through different conceptual change techniques or strategies.

If a certain method is not appropriate for students, teachers should use different alternative conceptual change strategies depending on the students status. In table 5.1.1.1, the mean of the concept of the students on this topic is 1.0375 at the object level and 0.775 at the schema level. Even these students may have developed this concept through their own techniques to understand the concept. As I have seen in my study there are some students who call each other and discuss what they learn today. They share ideas, read, do questions, ask each other and do solutions of the problem. This may be the reason for those students who are at the higher level of conception.

In general, the mean of the concept of the students decreases as we go from the lower level of thinking to the higher level of thinking (in table 5.1.1.1, see L1, L2, L3 and L4). This shows that the number of students towards the conceptual development is decreasing on polynomial and rational equations and functions as we go to the higher level of thinking. To eliminate this conceptual decrement, the appropriate strategies should be implemented. The conceptual change strategy is the one which is critical for the development of students' conception. For example group circulation method (through group discussion) is the strategy that should be implemented by teacher to develop the students' conception on mathematics. If this method is given, students will be free to share ideas, have more time to discuss, develop their concept through assimilation and accommodation. In addition table 5.1.1.1 also shows that the mean of the students who did not give any response is 3.175. It means that there are more number of students who did not give answer next to the number of students who are at the action level of conception.

This indicates that even if there are internal and external extremes that affect the student in the teaching learning process, these students either did not learn through conceptual change strategies or they may not give attention to their lesson or they did not understand how to do the solution of these questions.

When we see table 5.1.1.1, it shows that from the given eleven questions seven questions are not responded in the object level and six questions are not answered in the schema level of APOS theory. This indicates that, these students need pedagogical strategies to learn mathematics. And these questions might have been difficult to understand in the higher level of thinking because the students' conception is restricted to a certain level of thinking. This is due to that they are not helped through conceptual change strategies. Even those questions which are responded at the object and schema level may be easy to give solutions.

In general, if teachers use conceptual change strategies in the teaching learning process, through different pedagogical strategies, students' mind will be developed to the higher conception.

5.1.2 Summary on the achievement of polynomial and rational equations and functions after group circulation

Table 5.1.2.1: Summary on Mathematical Achievement (N=20)

No.	Questions	L_1	L_2	L_3	L_4	NR	Total
1	Q_1	3	6	5	2	4	20
	Q_2	4	3	-	8	5	20
	Q_3	3	4	-	7	6	20
Total		10	13	5	17	15	60
Mean		0.5	0.65	0.25	0.85	0.75	
Grand mean		$2.25 \div 4 = 0.5625$					

Table 5.1.2.1 shows that the mean of the students' conceptual development is seen from the action level to the schema level of APOS theory. The mean of the students thinking towards the action level is 0.5 and towards the process level is 0.65. So, there are more number of students towards the process level than the action level. This means that, due to group discussion method, students showed a conceptual change. In other words, due to this pedagogical strategy, students' pre-existing knowledge on polynomial and rational equations and functions is improved. If this method is kept continuously by teachers, students' conceptual change will be achieved to the higher level of thinking.

In table 5.1.2.1, the mean of the concept of the students on this topic is 0.25 at the object level and 0.85 at the schema level. Here after group circulation, we see that most of the mean of the concept of the students is at the schema level or at the higher thinking. Therefore, as table 5.1.2.1 indicated that the students' conceptual understanding in polynomial and rational equations and functions is improved after group circulation. The reason is that to change the pre existing knowledge in to the new one, I used the pedagogical strategy (group discussion). It is the pedagogical system that brought the conceptual change of the students. So if teachers used this or another alternative method in the teaching learning process then they can change the students' knowledge in to the new one.

The achievement of the students on polynomial and rational equations and functions after group circulation is good on the conceptual change of the students. The conceptual improvement of the students shows that there is a mental understanding of this topic through the given conceptual change strategy. This strategy should be kept until the students thinking is developed to the higher level of thinking.

The mean of the students who did not response any answer after group circulation is 0.75. But before group circulation it was 3.175. This shows that due to the conceptual change strategy (pedagogical system), the number of students who did not give response is decreasing after group circulation. For example even if there are varies factors, the mean difference before and after group circulation is 2.425.

This indicates that due to lack of pedagogical strategy there are more and more number of students who did not give response before group circulation than after the discussion is made. There may be also additional reasons for the lack of response. They may not know how to respond and they may need more activities and time to do it.

In general, as table 5.1.2.1 indicates that the conception of the students towards this topic is developing to the higher level of thinking. The reason is that students were learning through conceptual change strategies (pedagogical methods). The students were supported by the researcher through group discussion. During using this method, students were active, more participant, share ideas, discuss in group, develop their understanding on this topic, do more activities.

Teachers should not restrict the pedagogical strategies. To develop this conceptual understanding on polynomial and rational equations and functions, teachers should use different alternative methods.

5.1.3 Summary on the achievement of polynomial and rational equations and Functions on the two questions before group circulation.

Table 5.1.3.1: Summary on Mathematical Achievement (N=80)

Questions	L_1	L_2	L_3	L_4	NR	Total
Q 9	22	5	17	-	36	80
Q 11	-	22	38	-	20	80
Total	22	27	55	0	56	160
Mean	0.275	0.3375	0.6875	0	0.7	
Grand mean	0.325					

Table 5.1.3.1 shows that even if the student's conception is developing towards the next higher level of thinking, there is almost no student who reached at the schema level of thinking. The mean of the student's towards the action level is 0.275 and 0.3375 is towards the process level of understanding. At the object level it is 0.6875 and at the schema level there is no student who responded. This indicates that these students need a conceptual change strategy (pedagogical systems) to score a good conception at the higher level. Table 5.1.3.1 also indicates that there are

students who are studying hard by creating their own methodologies. For example, here the object level of the students is scored due to students' own methodologies.

But still we should know that students should be treated with the conceptual change strategies so that their achievement on polynomial and rational equations and functions will be improved. In addition the mean of the students who did not give response is 0.7. This shows that there is a problem of teaching through conceptual change strategies.

5.1.4 Summary on the achievement of polynomial and rational equations and functions on the two questions after group circulation

Table 5.1.4.1: Summary on Mathematical Achievement (N=20)

Questions	L_1	L_2	L_3	L_4	NR	Total
Q 1	3	6	5	2	4	20
Q 2	4	3	-	8	5	20
Total	7	9	5	10	9	40
Mean	0.35	0.45	0.25	0.5	0.45	
Grand mean	0.3875					

Table 5.1.4.1 indicates that the achievement of the students on polynomial and rational equations and functions showed an improvement in conceptual change. The mean of 0.35 is on the action level, 0.45 is on the process level, 0.25 is on the object level and 0.5 is on the schema level of APOS theory.

The students' conceptual development is seen on table 5.1.4.1. More students are found at the higher level of thinking after group circulation. The mean of the student's conception is more at the schema level of thinking. It is also investigated that there are more number of students at the process level than at the action level. The mean of the students who did not give response is 0.45. When we compare this with the mean in before group circulation, there is more number of students before group circulation who did not respond. In addition in the teaching – learning process of this topic students showed mental development or conceptual change which is done through conceptual change strategies (pedagogical system).

In general the grand mean of the students before group circulation is 0.325 and after group circulation it is 0.3875. The grand mean difference between the two circulations is 0.0625. This suggests that due to the group circulation method, there are more number of students who developed a better mental understanding (conception) after group circulation.

Therefore the mathematical achievement of the students in polynomial and rational equations and functions after group circulation is better than the mathematical achievement before group circulation. The reason is that in the former I used the conceptual change strategies (group circulation) in the teaching – learning process.

5.2 Conclusion

Findings obtained from this research provide evidence to conclude that the research is accepted. The ability to provide of students who have been involved in APOS theory instruction is significantly greater than the ability to provide of students who have involved in only teacher-centered traditional learning.

To implement the new instructional method (APOS) clearly, students need time to adjust with an innovative approach to learning polynomial and rational equations and functions characterized by collaborative learning, thus the effect of APOS theory instruction in providing ability do not immediately occurs.

APOS theory approach to teaching polynomial and rational equations and functions has advantage to students: active involvement (social interaction), opportunities to communicate mathematically, informal class room atmosphere, freedom to ask questions, closer student teacher relationship, opportunities to pursue challenging mathematical situation, and the instructor continuously attends to students' thinking in order to access their individual (and communal) learning capacities with respect to the task at hand, where all of these would influence students ability to prove.

The findings of this study suggest that the use of APOS theory instruction may benefit Ethiopian students in the development of polynomial and rational equations and function concepts in particular and mathematical concepts in general. APOS theory is still relatively new in Ethiopia and more information about the impact of APOS theory on Ethiopian students is very much needed. This study contributes to use APOS theory instruction in Ethiopia and sets the precedence in encouraging Ethiopian educators to focus on the strength and effectiveness of APOS theory in our mission to realize our vision in education at the preparatory level.

Vidakovic described that: APOS is cognitively oriented theory and as such provides a useful tool for modeling student understanding of mathematical concept. It also has a social component that relies on cooperative learning, as the context of group work is more likely to give rise to make explicit questions, doubts, and explanations by students than what would typically transpire in individual contexts (Vidakovic 1993).

It is pointed out that APOS theory was introduced in Cottrill et al. (1996[2]). This theory focuses on an attempt to analyze the internal mental structures and mechanisms constructed and used by an individual as he or she is thinking about a mathematical concept. The model of thinking developed by this analysis is then related to the individuals' apparent understanding of the concept. In the teaching learning process of mathematics there are characteristic difference of APOS theory and Traditional Approach.

The instruction with APOS theory is better than traditional instruction in polynomial and rational equations and functions. Theoretically APOS theory based instruction has some advantages when compared to the traditional one. That is why in this research APOS theory is applied. For example in APOS theory topics are designed regarding the mental constructions that are actions, processes, objects and schemas. Students are involved actively in learning. The role of lecturer or instructor is seen as a facilitator. That is, supports guidance to students, group or the entire class through group discussion. There is also a multi-direction interaction among students, as well as, students and teachers, students' learn from peer through work group, discussion.

But in traditional instruction topics are not designed specifically, generally refers to text books or lectures' class notes. Students receive information passively. Mathematics ideas are given in a ready-made fashion. The role of lecturer or instructor is as a knowledge transformer, i.e the lecturer or instructor directly explains mathematics ideas. There is one or two way interaction involving teacher.

Therefore in the teaching learning process of polynomial and rational equations and functions, the conceptual change strategy had a great application in developing the students understanding or concept. It is seen that group circulation strategy is one of the methods that has shown conceptual development. It is reasonable to conclude that the conceptual change of students about the concept of polynomial and rational equations and functions comes through conceptual change strategies. In order to achieve the proposed mental constructions, it is seen that the researcher should use a model or genetic decomposition (Actions, processes, objects and schemas) and mental mechanisms like interiorization and encapsulation to explain the learning of the concept in this functions. The result of the data analysis in chapter 4 shows that when we use teaching methodologies and conceptual change strategies in the teaching learning process, there is a change in conception of the students. For example see the analysis of the students' response after group circulation in chapters 4 above. And the researcher found that if teachers do not use methods and a model that might explain the way the students follow (genetic decomposition) of the A pos theory, students lack a concept of that specific topic. Regarding this research question, the students at least have begun the development of a level of knowledge and understanding regarding the concept of polynomial and rational equations and functions when we teach through the genetic decomposition of the APOS theory and group circulation. Therefore useful insight in to the relevant mental structures towards which teaching should focus was revealed by the APOS genetic decomposition of the polynomial and rational equations and functions concept. The findings of this study confirmed that the polynomial and rational equations and functions concept is one that students find difficult to understand, and suggest that this is possibly the result of many students not having appropriate mental structures at the process, object and schema levels. It seems that my genetic decomposition was adequate. However my reflections, on the teaching design indicates that more time needs to be devoted to helping students develop the mental structures at the process, object and schema levels.

In general, it seems that these students began to develop an understanding of a polynomial and rational equations and functions concept. Finally it is reasonable to conclude that the students in this study learned a great deal about the concept of polynomial and rational equations and functions through pedagogical strategies (group circulation) and the genetic decomposition of the APOS theory in order not to suffer to appear from the misconception.

5.3 Recommendation

The researcher recommends that in the teaching learning process, teachers should use methodologies to facilitate conceptual change of the students. The teaching learning process of mathematics needs serious conceptual change strategies. The mathematics subject needs more strategies or methods in order to understand about it. Even to have a conceptual change or an internal development on mathematics, students should be guided to the correct teaching learning methodologies. Nowadays if we see for example the preparatory students of Ethiopia, in particular in mathematics subject students are not getting the concept of the given topic or sub topic. Why students afraid to learn mathematics? Why don't they understand it?

The reason for the former is that they do not understand mathematics. But for the latter the reason is that they are not learning through conceptual change strategies or other methods. For example if we see the response of the students in the first eleven questions of the analysis in chapter 4 there are students who did not even guess any solution. In addition there are also students whose mental construction (conception) is limited in a certain stages of APOS theory. The researcher found that the teachers of these students did not use the way that students would follow in order to make mental constructions. In the same chapter, in the analysis of after group circulation, the researcher used conceptual change strategies in the teaching learning process and found that there is a change (transformation) in mental structure conception. In other words, it is clear that for the success of teaching learning process students should have conceptual change of their mental construction. But in the researchers' investigation of this research, the researcher found that there are students who are learning mathematics in different schools without having the expected conceptual change.

From the perspective of the concept of polynomial and rational equations and functions the researcher can conclude that there are a number of students who are suffering from getting the concept of what they are learning. Therefore for this lack of conceptual structure (mental development) there should be a solution. One of the solutions is using the conceptual change strategies through the stages of APOS theory.

REFERENCE

Asiala, M., Brown,A., Devries, D., Dubinsky,E., Mathews,D., and Thomas,K (1996). A framework for research and curriculum development in undergraduate mathematics education.

Valsiner, J. (1987). Culture and development of children's action. Chichester:Wiley.

LipingMA, (1992). Teachers' Understanding of mathematics in elementary school.

Vygotsky, L.s (1962). Thought and Language. Cambridge, MA: MIT Press.

Johnson,D., Johnson,R., and Smith K. (1991a). Cooperative Learning: Increasing College faculty instructional productivity.

Piaget, Jean. (1972). The principles of genetic epistemology.

Clark, J. , Kraut, G., Mathews, D., and Wimbish, J. (2003). The fundamental Theorem of Statistics: Classing student understanding of basic statistical concept. Retrieved October,20,2007.

Breidenbach, D.,Dubinsiky, E,Hqwks, J., Nicholas, D. (1992). Development of the process conception of Function. Educational studies in Mathematics, 23, 247-285.

Dubinsky, E. and Mc Donald, M.A: 2001, APOS: A constructivist theory of learning in undergraduate mathematics education research.

David son, N.(1990). Cooperative learning in mathematics: A hand book of teachers.

Davidson,N.and kroll,D.L. (1991). An overview of research on cooperative learning related to mathematics. Journal for research in mathematics Education, 22(5), 362-365.

AKKOC,H., and Tall D. (2005). A mismatch between curriculum design and student learning:

The case of the function concept.

Sfard, Anna. (1992). Operational origins of mathematical objects and the quandary of reification the case of Function: Aspects of epistemology and pedagogy (pp.59-84).

Stard, Anna. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as different sides of the same coin. Educational studies in mathematics, 22, 1-36.

Piaget,Jean.(1972).The principle of genetic epistemology. London: Routledge and keganpaul.

Dienes, zoltan p. (1960). Building up mathematics London: Hutchinson Educational.

Davis, Robert B. (1984). Learning mathematics: The cognitive science approach to mathematics education. Norwood, NJ: Ablex.

Cotrill,Jim,Dubinsky (1996). Understanding the limit concept: Beginning with a coordinated process schema. Journal of mathematical Behavior, 15, 167-192,

D.Vidakovic (1993). Cooperative learning: differences between group and individual processes of construction of the concept of inverse function. Unpublished doctoral dissertation, Purdue University, 1993.

David Tall, Michael Thomas, Gary Davis, Eddie Gray and Adrian Simpson (What is the object of Encapsulation of a process?), Jornal of mathematical behavior,18(2),223-241,Available on line in 2000.

ED Dubinsky,Kirk Weller, Michael A. MC Donald and Anne Brown.(Some historical issues and paradoxes regarding the concept of infinity: An Apos Based analysis: Part 1, Educational Studies in mathematics (2005) 58: 335-359.

Williams, Steven (1990). The understanding of limit: Three perspectives. Proceedings of the Four tenth Conference for the Psychology of mathematics Education (pp. 101-108).

Dubinsky,Ed.(1991). Reflective abstraction in advanced mathematical thinking. In Davi O. Tall (Ed.), Advanced mathematical thinking(pp. 95-123).

Dubinsky,E(2010,January). The Apos theory of learning mathematics: Pedagogical applications and results. Paper presented at the Eighteenth Annual Meeting of the Southern African Association for Research in mathematics, Science and Technology Education.Durban, South Africa.

APPENDIX A: TESTS

Tests to analyze Students' Understanding of the concept of Polynomial and Rational Equations and Functions

1. *When do we say that an expression which is written in the form of* $\frac{p(x)}{Q(x)}$ *is rational? Why?*

____

Because ____

2. *Is the function f(x) =* $\sqrt{9-x2}$ *a rational function? Why?*

____

Because ____

3. *Is the function* $f(x) = x^2+1 / x^2-1$ *a polynomial function? Why?*

____

Because ____

4. *To decompose a rational expression* $\frac{p(x)}{Q(x)}$*, what should be the degree of p(x) in relation to the degree of Q(x)? Why?*

____

Because ____

5. *How many zeros does* $x^4-1=0$ *have? Why?*

____

Because ____

6. Can a polynomial function have more zeros than its degree? Why?

____

Because ____

7. What is polynomial function?

____

8. What is rational function? Why?

____

Because ____

9. For the polynomial function, suppose the degree of the dividend is n and the degree of the divisor is m. If n>m, then what will be the degree of the quotient, q?

____

10. Is the graph of the function $f(x) = |x|$*, a polynomial function? Why?*

____

Because ____

11. Do the functions $y = x^2 + 2$ *and* $y = -x^2 - 2$ *have intersection point (a point in common)?Why?*

____

Because ____

APPENDIX B: TESTS OF THE GROUP CIRCULATION

1. *For the polynomial function, suppose the degree of the dividend is n and the degree of the divisor is m. If $n>m$, then what will be the degree of the quotient, q?*

Because ____

2. *Do the functions $y = x^2 + 2$ and $y = -x^2 -2$ have intersection point (a point in common)?Why?*

____

Because ____

3. *What values of a, b and c in the function $f(x) = ax^2 + bx + c$ make the equation to be a quadratic function? Why?*

____

Because ____

APPENDIX C: OBSERVATION CHECKLIST

1. Students understanding in the classroom

v.good ☐ good ☐

Needs Improvement ☐ Other______________________________

2.Usage of conceptual change strategies

v.good ☐ good ☐

Needs Improvement ☐ Other______________________________

3.Students' participation

v.good ☐ good ☐

Needs Improvement ☐ Other______________________________

4.Assessing students' understanding

v.good ☐ good ☐

Needs Improvement ☐ Other______________________________

5.Teachers' Subject knowledge

v.good ☐ good ☐

Needs Improvement ☐ Other______________________________

APPENDIX D: ADDITIONAL GIVEN ACTIVITIES AND SOME OF STUDENTS' RESPONSE

1. *What is the degree of any constant polynomial function? Why?*

Table 1: question 1 analysis of student response (N=20)

No.	**Respondents' response**	**Number of respondents**	**Percent**	**APOS level**
1	1	7	35%	Level 2
2	Totally no response (no answer)	4	20%	—
3	0, because $2x^0 = 2(1) = 2$ Constant function, any variable or number the power of zero is one.	9	45%	Level 1

2. ***What is the domain of*** $\frac{1}{x^2 - 1}$ **? why ?**

Table 2: question 2 analyses of students response (N=20)

No.	**Respondents' response**	**Number of respondents**	**Percent**	**APOS level**
1.	All real number without 0 because any number divided by zero is undefined.	2	10%	level 1
2.	All real number because it is defined.	1	5%	level 1
3.	All real number without 1 and -1 because the denominator will be zero making the expression zero.	10	50%	level 3
4.	Totally no response (no answer)	7	35%	__

3. What values a, b and c in the function $f(x) = ax^2 + bx + c$ make the equation to be:

3.1 A constant function? Why?

Table 3: Question 3.1 analysis of student response (N= 20)

No.	Respondents' Response	Number of Respondents	Percent	APOS level
1	Totally no response (no answer)	6	30%	__
2	a and b are 0	4	20%	level 2
3	If a = b= 0, then the function will be constant function because when the variables are multiplied by zero, we are left with constant c	10	5%	Level 4

3.2 A linear function? Why?

Table 4: Question 3.2 analysis of student response (N = 20)

No.	**Respondents' Response**	**Number of respondents**	**Percent**	**APOS level**
1	a=0	2	10%	level 2
2	Totally no response (no answer)	7	35%	—
3	If a and c are zero and if b is 1	11	55%	Level 1

4. *What is a rational equation? Why?*

____

Because ____

5.When is the division of polynomial function exact? Why?

____

Because ____

6.What is the domain of a rational function $f(x) = \frac{1}{x}$*? Why? Draw its Graph.*

____

Because ____

7. Is the following a rational equation? Why? What is its domain? Why?

$$\frac{x}{x+4} - \frac{4}{x-4} = \frac{x2+16}{x2-16}$$

Declaration

I the undersigned hereby declare that this thesis entitled "Conceptual change Strategies on students' Mathematics Achievement and its Challenge" is my original work done under supervision of Dr.Kassa Michael. I also confirm that this thesis has not been presented to any other university for the award of degree. All sources used for the thesis work have been dully acknowledged.

Name: Esmael Mohamed

Signature:_____________________________

Date:_________________________________

Approval

This is to certify that the thesis entitled "Conceptual change Strategies on students' Mathematics Achievement and its Challenge" is the original work of Esmael Mohamed done under my supervision as a university advisor.

Name: Dr.Kassa Michael

Signature:_____________________________

Date:_________________________________

August,2015

Addis Ababa University