Dynamics of Coherently Driven Degenerate Three-Level Atom in Open Cavity

By

Mulugeta Melaku

A THESIS SUBMITTED TO
THE DEPARTMENT OF PHYSICS

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS (QUANTUM OPTICS)

ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared by **Mulugeta Melaku**, entitled "**Dynamics of Coherently Driven Degenerate Three-Level Atom in Open Cavity**" and submitted in partial fulfillment of the requirements for the degree of **Master of Science in Physics (Quantum Optics)** complies with the regulations of the University and meets the accepted standards with respect to originality.

Approved by the Examination Committee:

Advisor: Dr. Fesseha Kassahun Signature:________________Date:__________

Examiner: Dr. Deribe Hirpo Signature:________________Date:__________

Examiner: Dr. Belayneh Mesfin Signature:________________Date:__________

ADDIS ABABA UNIVERSITY

Date: April 14, 2014

Author: **Mulugeta Melaku**

Title: Dynamics of Coherently Driven Degenerate Three-Level Atom in Open Cavity

Department: **Physics**

Degree: **MSc**

Signature of Author

Abstract

In this thesis, we study the quantum properties of the light generated by a coherently driven degenerate three-level atom in an open cavity coupled to a vacuum reservoir. The three-level atom available in the cavity is driven by coherent light from the bottom to the top level. We perform our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Employing the master equation and the quantum Langevin equation we obtain the differential equations of the atomic and cavity-mode operators. Using the steady-state solutions of the resulting equations, we determined the photon statistics, quadrature squeezing and the local mean photon number. It is observed that the mean photon number is greater when $\gamma = 0$ than when $\gamma = 0.1$. We find, like the mean photon number, the variance of the photon number to be greater for $\gamma = 0$ than for $\gamma = 0.1$. Moreover, we have also established that the maximum quadrature squeezing of the light generated by the atom, to be 43.42 % for $\gamma = 0$ and 42.22 % for $\gamma = 0.1$ below the coherent-state level. Thus in the absence of spontaneous emission the mean, the variance and the quadrature squeezing of the cavity light is greater than in the presence of spontaneous emission. Finally, we have found that a large part of the total mean photon number is confined in a relatively small frequency interval.

Acknowledgements

I would like to thank the almighty God, for keeping me alive to accomplish this task. I would like also to express my gratitude to my advisor and instructor Dr Fesseha Kassahun for his tireless supervision, assistance, and contributing ever blooming and indispensable suggestions. His scientific integral view on research, overly enthusiasm and his original work on quantum optics, has made a deep impression on me.

My special acknowledgement goes to my favourite institute Addis Ababa University for sponsoring my MSc studies and my home institute Adwa College of Teacher Education for allowing me to join AAU.

It is also my pleasure to acknowledge Ato Kassa Michael (Asst. Prof.) and Ato Abebe Abadi for their financial and material support. Without their support, I would have not accomplished this task.

My most heart felt thanks goes to my lovely sister Eyerusalem and her husband Ato Girmay Gebreegzabher and my family Ato Desta Seyoum and his wife W/o Kalaeyta Gobeze, Weldegerima Adane (@ACTE), Ato Wehab Nurhussien (Dean ACTE), Ato Awetahegn Assefa (@ACTE), Ato Habtamu Wedaje(AAU), Ato Mola Asmare (PhD candidate @AAU), Ato Anley Gessesse (PhD candidate @AAU), Ato Ataklty Araya (PhD candidate @AAU), Ato Gemechu Fanta (PhD candidate @AAU), Ato Abraham Negash (Asst. Prof.), Ato Ejigu Kebede and all my colleagues for their endless encouragement and providing invaluable sugestions.

Now it is crucial to mention my strong impression and respect to acknowledge the Department of Physics, specially W/o Tsilat Adnew for her endless support.

Finally, I have to express my strong feeling, appreciation, love and respect to my wife Askale Brhane (Furtuna) for her commitment and patience embracing all the responsibilities of our family while I was on study leave.

Dedicated to
The Late PM Meles Zenawi

Contents

Chapter 1

Introduction

A three-level laser is a source of coherent or chaotic light emitted by three-level atoms inside a cavity coupled to a vacuum reservoir [1]. In the quantum theory of a laser, one usually takes into consideration the interaction of the atoms inside the cavity with the vacuum reservoir outside the cavity [1]. Three-level lasers in which the crucial role is played by the coherent superpositions of top and bottom levels of the injected atoms have been studied by several authors [1-6]. These studies have shown that this quantum optical system can generate light in a squeezed state under certain conditions.

The squeezing properties of the cavity modes produced by a nondegenerate three-level laser has been studied [7]. It has been found that the two-mode cavity light exhibits squeezing, if the atoms are initially prepared with more atoms in the bottom than in the upper level, and the degree of squeezing increases with the linear gain coefficient [7-9].

A three-level laser, in which the top and bottom levels of three-level atoms injected into a cavity and coupled by a strong coherent light can also generate light in a squeezed state [8,9].

It appears to be quite difficult to prepare the atoms in a coherent superposition of the top and bottom levels before they are injected into the laser cavity. Moreover, it is certainly hard to find out that the atoms have decayed spontaneously before they are removed from the cavity [10-17].

We seek here to analyze the quantum properties of the light emitted by a degenerate three-level atom available in an open cavity and driven by coherent light. Thus taking into account the interaction of the degenerate three-level atom with resonant coherent light in an open cavity coupled to vacuum reservoir, we first derive the equations of evolution of atomic and cavity-mode operators, applying the quantum Langevin equation and the master equation. Using the steady-state solutions of the resulting equations, we obtain the photon statistics, the quadrature squeezing, and the local mean photon number for the light emitted by the atom. We carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order and considering the interaction of the three-level atom with the vacuum reservoir outside the cavity. To this end, we first determine the operator dynamics for coherently driven degenerate three-level atom in an open cavity.

"If your thinking does not involve light,

then you are like a person in a dark room."

Fesseha Kassahun

Chapter 2

Operator Dynamics

We consider here a degenerate three-level atom available in an open cavity coupled to a vacuum reservoir and driven by coherent light. We denote the top, middle and bottom levels of the atom by $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively.

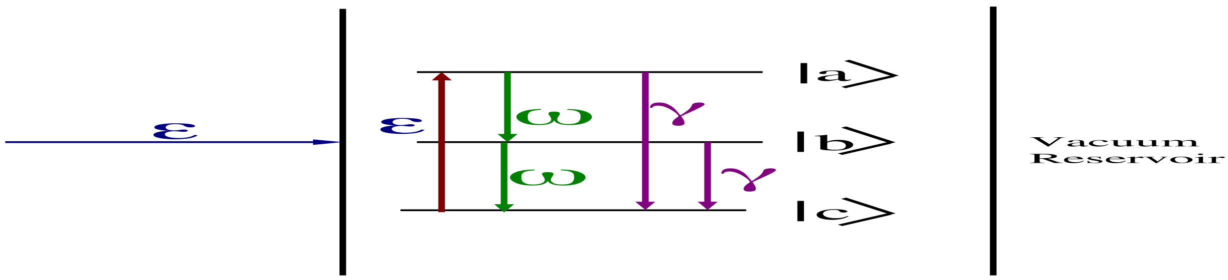

Fig. 2.1 A three-level atom in an open cavity.

In this chapter we seek to determine the operator dynamics for coherently driven degenerate three-level atom in an open cavity, in which the atom is pumped by coherent light coupled to a vacuum reservoir. We first derive the equations of evolution of atomic and cavity-mode operators, applying the master equation and the quantum Langevin equation. Employing the steady-state solutions of the resulting equations, we obtain the expectation value of the atomic operators.

We consider here a degenerate three-level atom available in an open cavity coupled to a vacuum reservoir and driven by coherent light. Thus the interaction of coherent light, represented by a real and constant c-number ε, with the three-level atom can be described at resonance by the Hamiltonian

$$\hat{H}' = \frac{i\Omega}{2}\left(\hat{\sigma}_c^\dagger - \hat{\sigma}_c\right), \tag{2.1}$$

where

$$\Omega = 2g'\varepsilon, \tag{2.2}$$

$$\hat{\sigma}_c = |c\rangle\langle a|, \tag{2.3}$$

with g' being the coupling constant.

Moreover, the interaction of a degenerate three-level atom with the cavity mode can be described at resonance by the Hamiltonian

$$\hat{H}'' = ig(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_b), \tag{2.4}$$

where

$$\hat{\sigma}_a = |b\rangle\langle a|, \tag{2.5}$$

$$\hat{\sigma}_b = |c\rangle\langle b|, \tag{2.6}$$

with g being the coupling constant.

Thus the total Hamiltonian turns out to be

$$\hat{H} = \frac{i\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c) + ig(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_b), \tag{2.7}$$

We recall that the master equation for a three-level atom interacting with a vacuum reservoir has the form [1]

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -i[\hat{H},\hat{\rho}] + \frac{\gamma}{2}(2\hat{\sigma}_c\hat{\rho}\hat{\sigma}_c^\dagger - \hat{\sigma}_c^\dagger\hat{\sigma}_c\hat{\rho} - \hat{\rho}\hat{\sigma}_c^\dagger\hat{\sigma}_c) \\ &+ \frac{\gamma}{2}(2\hat{\sigma}_b\hat{\rho}\hat{\sigma}_b^\dagger - \hat{\sigma}_b^\dagger\hat{\sigma}_b\hat{\rho} - \hat{\rho}\hat{\sigma}_b^\dagger\hat{\sigma}_b),\end{aligned} \tag{2.8}$$

where γ is called the spontaneous emission decay constant.

Now employing Eq. (2.7), the master equation which is the time evolution of the density operator ($\hat{\rho}$) for the three-level atom driven by coherent light, interacting with the cavity mode and vacuum reservoir takes the form

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= \frac{\Omega}{2}(\hat{\sigma}_c^\dagger\hat{\rho} - \hat{\rho}\hat{\sigma}_c^\dagger + \hat{\rho}\hat{\sigma}_c - \hat{\sigma}_c\hat{\rho}) \\ &+ g(\hat{\sigma}_a^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{\sigma}_a^\dagger\hat{a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_a - \hat{a}^\dagger\hat{\sigma}_a\hat{\rho} \\ &+ \hat{\sigma}_b^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{\sigma}_b^\dagger\hat{a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_b - \hat{a}^\dagger\hat{\sigma}_b\hat{\rho}) \\ &+ \frac{\gamma}{2}(2\hat{\sigma}_c\hat{\rho}\hat{\sigma}_c^\dagger - |a\rangle\langle a|\hat{\rho} - \hat{\rho}|a\rangle\langle a|) \\ &+ \frac{\gamma}{2}(2\hat{\sigma}_b\hat{\rho}\hat{\sigma}_b^\dagger - |b\rangle\langle b|\hat{\rho} - \hat{\rho}|b\rangle\langle b|). \end{aligned} \tag{2.9}$$

Using the relation

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right), \tag{2.10}$$

along with the cyclic property of the trace operation, one easily gets

$$\begin{aligned}\frac{d}{dt}\langle\hat{\sigma_a}\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\sigma_a}\hat{\sigma_c}^\dagger - \hat{\rho}\hat{\sigma_c}^\dagger\hat{\sigma_a} + \hat{\rho}\hat{\sigma_c}\hat{\sigma_a} - \hat{\rho}\hat{\sigma_a}\hat{\sigma_c}) \\ &+ g(\hat{\rho}\hat{\sigma_a}\hat{\sigma_a}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_a}^\dagger\hat{a}\hat{\sigma_a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma_a}^2 - \hat{\rho}\hat{\sigma_a}\hat{a}^\dagger\hat{\sigma}_a \\ &+ \hat{\rho}\hat{\sigma_a}\hat{\sigma_b}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_b}^\dagger\hat{a}\hat{\sigma_a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_b\hat{\sigma_a} - \hat{\rho}\hat{\sigma_a}\hat{a}^\dagger\hat{\sigma}_b) \\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_c}^\dagger\hat{\sigma_a}\hat{\sigma_c} - \hat{\rho}\hat{\sigma_a}|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\sigma_a}) \\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_b}^\dagger\hat{\sigma_a}\hat{\sigma_b} - \hat{\rho}\hat{\sigma_a}|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\sigma_a})], \end{aligned} \tag{2.11}$$

so that on account of Eqs. (2.3), (2.5), (2.6), and their adjoints, we have

$$\frac{d}{dt}\langle\hat{\sigma_a}\rangle = -\gamma\langle\hat{\sigma_a}\rangle + \frac{\Omega}{2}\langle\hat{\sigma_b}^\dagger\rangle + g\langle(\hat{\eta}_b - \hat{\eta}_a)\hat{a}\rangle + g\langle\hat{a}^\dagger\hat{\sigma}_c\rangle. \tag{2.12}$$

Where the probability of finding an atom in the top, middle, and bottom levels respectively is expressed as follows

$$\langle\hat{\eta}_a\rangle = \langle|a\rangle\langle a|\rangle, \tag{2.13}$$

$$\langle\hat{\eta}_b\rangle = \langle|b\rangle\langle b|\rangle, \tag{2.14}$$

$$\langle\hat{\eta}_c\rangle = \langle|c\rangle\langle c|\rangle. \tag{2.15}$$

We also notice that

$$\begin{aligned}
\frac{d}{dt}\langle\hat{\sigma_b}\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\sigma_b}\hat{\sigma_c}^\dagger - \hat{\rho}\hat{\sigma_c}^\dagger\hat{\sigma_b} + \hat{\rho}\hat{\sigma_c}\hat{\sigma_b} - \hat{\rho}\hat{\sigma_b}\hat{\sigma_c}) \\
&+ g(\hat{\rho}\hat{\sigma_b}\hat{\sigma_a}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_a}^\dagger\hat{a}\hat{\sigma_b} + \hat{\rho}\hat{a}^\dagger\hat{\sigma_a}\hat{\sigma_b} - \hat{\rho}\hat{\sigma_b}\hat{a}^\dagger\hat{\sigma_a} \\
&+ \hat{\rho}\hat{\sigma_b}\hat{\sigma_b}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_b}^\dagger\hat{a}\hat{\sigma_b} + \hat{\rho}\hat{a}^\dagger\hat{\sigma_b}^2 - \hat{\rho}\hat{\sigma_b}\hat{a}^\dagger\hat{\sigma_b}) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_c}^\dagger\hat{\sigma_b}\hat{\sigma_c} - \hat{\rho}\hat{\sigma_b}|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\sigma_b}) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_b}^\dagger\hat{\sigma_b}^2 - \hat{\rho}\hat{\sigma_b}|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\sigma_b})],
\end{aligned} \tag{2.16}$$

thus on account of Eqs. (2.3), (2.5), (2.6), and their complex conjugates, we get

$$\frac{d}{dt}\langle\hat{\sigma_b}\rangle = -\gamma\langle\hat{\sigma_b}\rangle - \frac{\Omega}{2}\langle\hat{\sigma_a}^\dagger\rangle + g\langle(\hat{\eta}_c - \hat{\eta}_b)\hat{a}\rangle - g\langle\hat{a}^\dagger\hat{\sigma_c}\rangle. \tag{2.17}$$

In addition, we see that

$$\begin{aligned}
\frac{d}{dt}\langle\hat{\sigma_c}\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\sigma}_c\hat{\sigma}_c^\dagger - \hat{\rho}\hat{\sigma}_c^\dagger\hat{\sigma}_c + \hat{\rho}\hat{\sigma}_c^2 - \hat{\rho}\hat{\sigma}_c^2) \\
&+ g(\hat{\rho}\hat{\sigma}_c\hat{\sigma}_a^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_a^\dagger\hat{a}\hat{\sigma}_c + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_a\hat{\sigma}_c - \hat{\rho}\hat{\sigma}_c\hat{a}^\dagger\hat{\sigma}_a \\
&+ \hat{\rho}\hat{\sigma}_c\hat{\sigma}_b^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_b^\dagger\hat{a}\hat{\sigma}_c + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_b\hat{\sigma}_c - \hat{\rho}\hat{\sigma}_c\hat{a}^\dagger\hat{\sigma}_b) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_c}^\dagger\hat{\sigma_c}^2 - \hat{\rho}\hat{\sigma}_c|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\sigma}_c) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_b^\dagger\hat{\sigma}_c\hat{\sigma}_b - \hat{\rho}\hat{\sigma}_c|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\sigma}_c)],
\end{aligned} \tag{2.18}$$

upon introducing here Eqs. (2.3), (2.5), (2.6), and their adjoints into

Eq. (2.18), thus the expression can be written as

$$\frac{d}{dt}\langle\hat{\sigma_c}\rangle = -\gamma\langle\hat{\sigma_c}\rangle + \frac{\Omega}{2}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle) + g\langle(\hat{\sigma_b} - \hat{\sigma_a})\hat{a}\rangle. \tag{2.19}$$

Moreover, we easily get

$$\begin{aligned}
\frac{d}{dt}\langle\hat{\eta}_a\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\eta}_a\hat{\sigma_c}^\dagger - \hat{\rho}\hat{\sigma_c}^\dagger\hat{\eta}_a + \hat{\rho}\hat{\sigma_c}\hat{\eta}_a - \hat{\rho}\hat{\eta}_a\hat{\sigma_c}) \\
&+ g(\hat{\rho}\hat{\eta}_a\hat{\sigma_a}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_a}^\dagger\hat{a}\hat{\eta}_a + \hat{\rho}\hat{a}^\dagger\hat{\sigma_a}\hat{\eta}_a - \hat{\rho}\hat{\eta}_a\hat{a}^\dagger\hat{\sigma_a} \\
&+ \hat{\rho}\hat{\eta}_a\hat{\sigma_b}^\dagger\hat{a} - \hat{\rho}\hat{\sigma_b}^\dagger\hat{a}\hat{\eta}_a + \hat{\rho}\hat{a}^\dagger\hat{\sigma_b}\hat{\eta}_a - \hat{\rho}\hat{\eta}_a\hat{a}^\dagger\hat{\sigma_b}) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma_c}^\dagger\hat{\eta}_a\hat{\sigma_c} - \hat{\rho}\hat{\eta}_a|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\eta}_a) \\
&+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_b^\dagger\hat{\eta}_a\hat{\sigma}_b - \hat{\rho}\hat{\eta}_a|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\eta}_a)],
\end{aligned} \tag{2.20}$$

in view of Eqs. (2.3), (2.5), (2.6), and their complex conjugates, we find

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = -\gamma\langle\hat{\eta}_a\rangle + \frac{\Omega}{2}(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle) + g(\langle\hat{\sigma}_a^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_a\rangle). \tag{2.21}$$

We also notice that

$$\begin{aligned}\frac{d}{dt}\langle\hat{\eta}_b\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\eta}_b\hat{\sigma}_c^\dagger - \hat{\rho}\hat{\sigma}_c^\dagger\hat{\eta}_b + \hat{\rho}\hat{\sigma}_c\hat{\eta}_b - \hat{\rho}\hat{\eta}_b\hat{\sigma}_c)\\ &+ g(\hat{\rho}\hat{\eta}_b\hat{\sigma}_a^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_a^\dagger\hat{a}\hat{\eta}_b + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_a\hat{\eta}_b - \hat{\rho}\hat{\eta}_b\hat{a}^\dagger\hat{\sigma}_a\\ &+ \hat{\rho}\hat{\eta}_b\hat{\sigma}_b^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_b^\dagger\hat{a}\hat{\eta}_b + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_b\hat{\eta}_b - \hat{\rho}\hat{\eta}_b\hat{a}^\dagger\hat{\sigma}_b)\\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_c^\dagger\hat{\eta}_b\hat{\sigma}_c - \hat{\rho}\hat{\eta}_b|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\eta}_b)\\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_b^\dagger\hat{\eta}_b\hat{\sigma}_b - \hat{\rho}\hat{\eta}_b|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\eta}_b)],\end{aligned} \tag{2.22}$$

so that on account of Eqs. (2.3), (2.5), (2.6), and their adjoints, one gets

$$\frac{d}{dt}\langle\hat{\eta}_b\rangle = -\gamma\langle\hat{\eta}_b\rangle - g(\langle\hat{\sigma}_a^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_a\rangle) + g(\langle\hat{\sigma}_b^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_b\rangle). \tag{2.23}$$

Finally, we observe that

$$\begin{aligned}\frac{d}{dt}\langle\hat{\eta}_c\rangle &= Tr[\frac{\Omega}{2}(\hat{\rho}\hat{\eta}_c\hat{\sigma}_c^\dagger - \hat{\rho}\hat{\sigma}_c^\dagger\hat{\eta}_c + \hat{\rho}\hat{\sigma}_c\hat{\eta}_c - \hat{\rho}\hat{\eta}_c\hat{\sigma}_c)\\ &+ g(\hat{\rho}\hat{\eta}_c\hat{\sigma}_a^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_a^\dagger\hat{a}\hat{\eta}_c + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_a\hat{\eta}_c - \hat{\rho}\hat{\eta}_c\hat{a}^\dagger\hat{\sigma}_a\\ &+ \hat{\rho}\hat{\eta}_c\hat{\sigma}_b^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_b^\dagger\hat{a}\hat{\eta}_c + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_b\hat{\eta}_c - \hat{\rho}\hat{\eta}_c\hat{a}^\dagger\hat{\sigma}_b)\\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_c^\dagger\hat{\eta}_c\hat{\sigma}_c - \hat{\rho}\hat{\eta}_c|a\rangle\langle a| - \hat{\rho}|a\rangle\langle a|\hat{\eta}_c)\\ &+ \frac{\gamma}{2}(2\hat{\rho}\hat{\sigma}_b^\dagger\hat{\eta}_c\hat{\sigma}_b - \hat{\rho}\hat{\eta}_c|b\rangle\langle b| - \hat{\rho}|b\rangle\langle b|\hat{\eta}_c)],\end{aligned} \tag{2.24}$$

hence upon putting Eqs. (2.3), (2.5), (2.6), and their complex conjugates into Eq. (2.24), we obtain

$$\frac{d}{dt}\langle\hat{\eta}_c\rangle = -\gamma\langle\hat{\eta}_c\rangle - \frac{\Omega}{2}(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle) - g(\langle\hat{\sigma}_b^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_b\rangle), \tag{2.25}$$

We assume that the cavity is coupled to a vacuum reservoir via a single port mirror. Moreover, we carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators. We can then drop the noise operator and write the quantum Langevin equation for the operator $\hat{a}$ as

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} - i[\hat{a}, \hat{H}], \tag{2.26}$$

where κ is the cavity damping constant. Therefore, with the aid of Eq. (2.7), we easily get

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} - g(\hat{\sigma}_a + \hat{\sigma}_b). \tag{2.27}$$

We observe that Eqs. (2.12), (2.17), (2.19), (2.21), (2.23) and (2.25) are nonlinear differential equations. Hence it is not possible to obtain exact time dependent solutions of these equations. Thus applying the large time approximation scheme, we obtain from Eq. (2.27) the approximately valid relation

$$\hat{a}(t) = -\frac{2g}{\kappa}(\hat{\sigma}_a + \hat{\sigma}_b). \tag{2.28}$$

Now inserting Eq. (2.28) and its adjoint into Eqs. (2.12), (2.17), (2.19), (2.21), (2.23), and (2.25), we obtain

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -(\gamma + \gamma_c)\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^\dagger\rangle, \tag{2.29}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -(\gamma + \frac{\gamma_c}{2})\langle\hat{\sigma}_b\rangle + \gamma_c\langle\hat{\sigma}_a\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a{}^\dagger\rangle, \tag{2.30}$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = -(\gamma + \frac{\gamma_c}{2})\langle\hat{\sigma}_c\rangle + \frac{\Omega}{2}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle), \tag{2.31}$$

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = -(\gamma + \gamma_c)\langle\hat{\eta}_a\rangle + \frac{\Omega}{2}(\langle\hat{\sigma}_c{}^\dagger\rangle + \langle\hat{\sigma}_c\rangle), \tag{2.32}$$

$$\frac{d}{dt}\langle\hat{\eta}_b\rangle = -(\gamma + \gamma_c)\langle\hat{\eta}_b\rangle + \gamma_c\langle\hat{\eta}_a\rangle, \tag{2.33}$$

$$\frac{d}{dt}\langle\hat{\eta}_c\rangle = \gamma(1 - \langle\hat{\eta}_c\rangle) + \gamma_c\langle\hat{\eta}_b\rangle - \frac{\Omega}{2}(\langle\hat{\sigma}_c{}^\dagger\rangle + \langle\hat{\sigma}_c\rangle), \tag{2.34}$$

where

$$\gamma_c = \frac{4g^2}{\kappa}. \tag{2.35}$$

The parameter defined by Eq. (2.35) is called the stimulated emission decay constant. In the absence of spontaneous emission and based on the definition of this decay constant, we infer that the atom in the top or middle-level emits a photon due to its interaction with the cavity light. We certainly identify this process to be stimulated photon emission. The steady-state solutions of Eqs. (2.29), (2.30), (2.31), (2.32), (2.33), and (2.34) are given by

$$\langle\hat{\sigma_a}\rangle = \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma_b}^{\dagger}\rangle, \tag{2.36}$$

$$\langle\hat{\sigma_b}\rangle = \frac{2\gamma_c}{2\gamma+\gamma_c}\langle\hat{\sigma_a}\rangle - \frac{\Omega}{2\gamma+\gamma_c}\langle\hat{\sigma_a}^{\dagger}\rangle, \tag{2.37}$$

$$\langle\hat{\sigma_c}\rangle = \frac{\Omega}{2\gamma+\gamma_c}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle), \tag{2.38}$$

$$\langle\hat{\eta}_a\rangle = \frac{\Omega}{\gamma+\gamma_c}\langle\hat{\sigma_c}\rangle, \tag{2.39}$$

$$\langle\hat{\eta}_b\rangle = \frac{\gamma_c}{\gamma+\gamma_c}\langle\hat{\eta}_a\rangle, \tag{2.40}$$

$$\langle\hat{\eta}_c\rangle = 1 + \frac{\gamma_c}{\gamma}\langle\hat{\eta}_b\rangle - \frac{\Omega}{\gamma}\langle\hat{\sigma}_c\rangle. \tag{2.41}$$

Thus solving Eqs. (2.36) and (2.37) simultaneously, one gets

$$\langle\hat{\sigma_a}\rangle = \langle\hat{\sigma_b}\rangle = 0. \tag{2.42}$$

We recall that [1]

$$\langle\hat{\eta}_a\rangle + \langle\hat{\eta}_b\rangle + \langle\hat{\eta}_c\rangle = 1. \tag{2.43}$$

Now inserting Eq. (2.40) in Eq. (2.43), we find

$$\langle\hat{\eta}_a\rangle\left(\frac{\gamma+2\gamma_c}{\gamma+\gamma_c}\right) + \langle\hat{\eta}_c\rangle = 1, \tag{2.44}$$

upon substituting Eq. (2.38) into Eq. (2.39), we have

$$\langle\hat{\eta}_a\rangle(2\gamma^2+3\gamma\gamma_c+\gamma_c^2+\Omega^2) - \Omega^2\langle\hat{\eta}_c\rangle = 0, \tag{2.45}$$

thus solving Eqs. (2.44) and (2.45) simultaneously, we arrive at

$$\langle\hat{\eta}_a\rangle = \frac{\gamma\Omega^2+\gamma_c\Omega^2}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}, \tag{2.46}$$

$$\langle\hat{\eta}_c\rangle = \frac{\gamma\Omega^2+\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}. \tag{2.47}$$

Furthermore, Putting Eq. (2.46) into Eq.(2.40), we find

$$\langle\hat{\eta}_b\rangle = \frac{\gamma_c\Omega^2}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}. \tag{2.48}$$

In addition, using Eqs. (2.46) and (2.47) in Eq.(2.38), we obtain

$$\langle\hat{\sigma}_c\rangle = \frac{\Omega}{2\gamma+\gamma_c}\left(\frac{2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}\right). \tag{2.49}$$

Chapter 3

Photon Statistics

The photon statistics of the light generated by a three-level atom is described by the mean and variance of the photon number. It would be crucial to classify the photon statistics of a light modes based on the relation between the mean and variance of the photon number. Thus the photon statistics of a light mode for which $(\Delta n)^2 = \bar{n}$ is referred to as Poissonian and the photon statistics of a light mode for which $(\Delta n)^2 > \bar{n}$ is called Supper-Poissonian. Otherwise the photon statistics is said to be Sub-Poissonian [1].

In this chapter we wish to determine the statistical characteristics of the light produced by a degenerate three-level atom, in which the degenerate three-level atom available in an open cavity coupled to a vacuum reservoir is driven by coherent light. We have calculated the steady-state solutions of the expectation value of cavity-mode operators. Upon employing the solutions of these equations, we seek here the mean and the variance of the photon number of the cavity light emitted by a coherently driven degenerate three-level atom. To this end, we first determine the mean photon number.

3.1 The Mean Photon Number

Upon introducing Eq. (2.28) and its adjoint into $\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle$, the mean photon number takes the form

$$\bar{n} = \frac{\gamma_c}{\kappa} \left(\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle \right). \tag{3.1}$$

Now substituting Eqs. (2.46) and (2.48) into Eq. (3.1), we have

$$\bar{n} = \frac{\gamma_c}{\kappa}\left(\frac{\gamma\Omega^2 + 2\gamma_c\Omega^2}{2\gamma\Omega^2 + 3\gamma_c\Omega^2 + 2\gamma^3 + 5\gamma^2\gamma_c + 4\gamma\gamma_c^2 + \gamma_c^3}\right). \tag{3.2}$$

We note for $\gamma = 0$, the mean photon number reduces to

$$\bar{n} = \frac{\gamma_c}{\kappa}\left(\frac{2\Omega^2}{\gamma_c^2 + 3\Omega^2}\right). \tag{3.3}$$

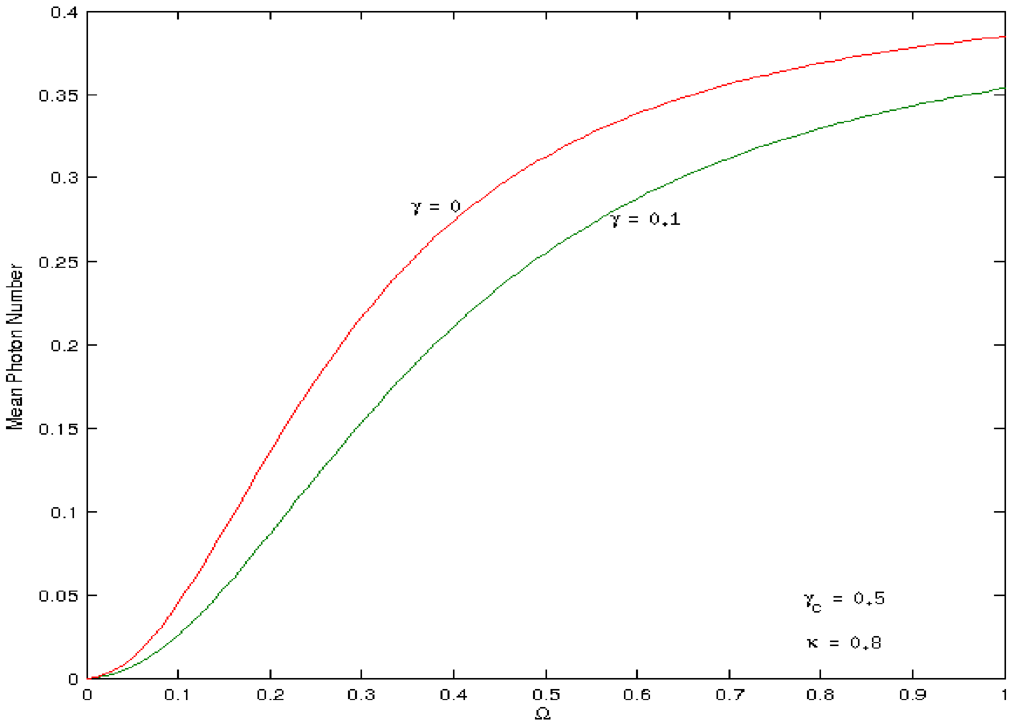

Fig. 3.1 The Mean Photon Number Versus Ω.

From Figure 3.1 we see that the mean photon number for $\gamma = 0$ is greater than for $\gamma = 0.1$.

3.2 The Variance of the Photon Number

Furthermore, the variance of the photon number for the cavity light can be written as

$$(\triangle n)^2 = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2. \tag{3.4}$$

We note from Eq. (3.4) and that $\hat{a}$ is a Gaussian variable with zero mean, one easily obtains [1]

$$(\triangle n)^2 = \langle \hat{a}^\dagger \hat{a}\rangle\langle \hat{a}\hat{a}^\dagger\rangle + \langle \hat{a}^2\rangle\langle \hat{a}^{\dagger 2}\rangle, \tag{3.5}$$

upon applying Eq. (2.28) and its complex conjugate into Eq. (3.5), we find

$$\langle \hat{a}^\dagger \hat{a}\rangle = \frac{\gamma_c}{\kappa}(\langle \hat{\eta}_a\rangle + \langle \hat{\eta}_b\rangle), \tag{3.6}$$

$$\langle \hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}(\langle \hat{\eta}_b\rangle + \langle \hat{\eta}_c\rangle), \tag{3.7}$$

$$\langle \hat{a}^2\rangle\langle \hat{a}^{\dagger 2}\rangle = \left(\frac{\gamma_c}{\kappa}\right)^2 (\langle \hat{\sigma}_c\rangle)^2. \tag{3.8}$$

Now substituting Eqs. (3.6), (3.7), and (3.8) into Eq. (3.5), we obtain

$$(\triangle n)^2 = \bar{n}\frac{\gamma_c}{\kappa}(\langle \hat{\eta}_b\rangle + \langle \hat{\eta}_c\rangle) + \left(\frac{\gamma_c}{\kappa}\right)^2 (\langle \hat{\sigma}_c\rangle)^2. \tag{3.9}$$

On the basis of Eqs. (2.47), (2.48) and (2.49), variance of the photon number turns out to be

$$\begin{aligned}(\triangle n)^2 &= \bar{n}\frac{\gamma_c}{\kappa}\left(\frac{\gamma\Omega^2 + 2\gamma_c\Omega^2 + 2\gamma^3 + 5\gamma^2\gamma_c + 4\gamma\gamma_c^2 + \gamma_c^3}{2\gamma\Omega^2 + 3\gamma_c\Omega^2 + 2\gamma^3 + 5\gamma^2\gamma_c + 4\gamma\gamma_c^2 + \gamma_c^3}\right) + \left(\frac{\gamma_c\Omega}{\kappa(2\gamma + \gamma_c)}\right)^2 \\ &\times \left(\frac{2\gamma^3 + 5\gamma^2\gamma_c + 4\gamma\gamma_c^2 + \gamma_c^3}{2\gamma\Omega^2 + 3\gamma_c\Omega^2 + 2\gamma^3 + 5\gamma^2\gamma_c + 4\gamma\gamma_c^2 + \gamma_c^3}\right)^2. \end{aligned} \tag{3.10}$$

We observe that for $\gamma = 0$, variance of the photon number reduces to

$$\begin{aligned}(\triangle n)^2 &= \left(\frac{\gamma_c}{\kappa}\right)^2 \times \left(\frac{2\Omega^2}{\gamma_c^2 + 3\Omega^2}\right) \times \left(\frac{2\Omega^2 + \gamma_c^2}{\gamma_c^2 + 3\Omega^2}\right) \\ &+ \left(\frac{\Omega}{\kappa}\right)^2 \times \left(\frac{\gamma_c^2}{\gamma_c^2 + 3\Omega^2}\right)^2. \end{aligned} \tag{3.11}$$

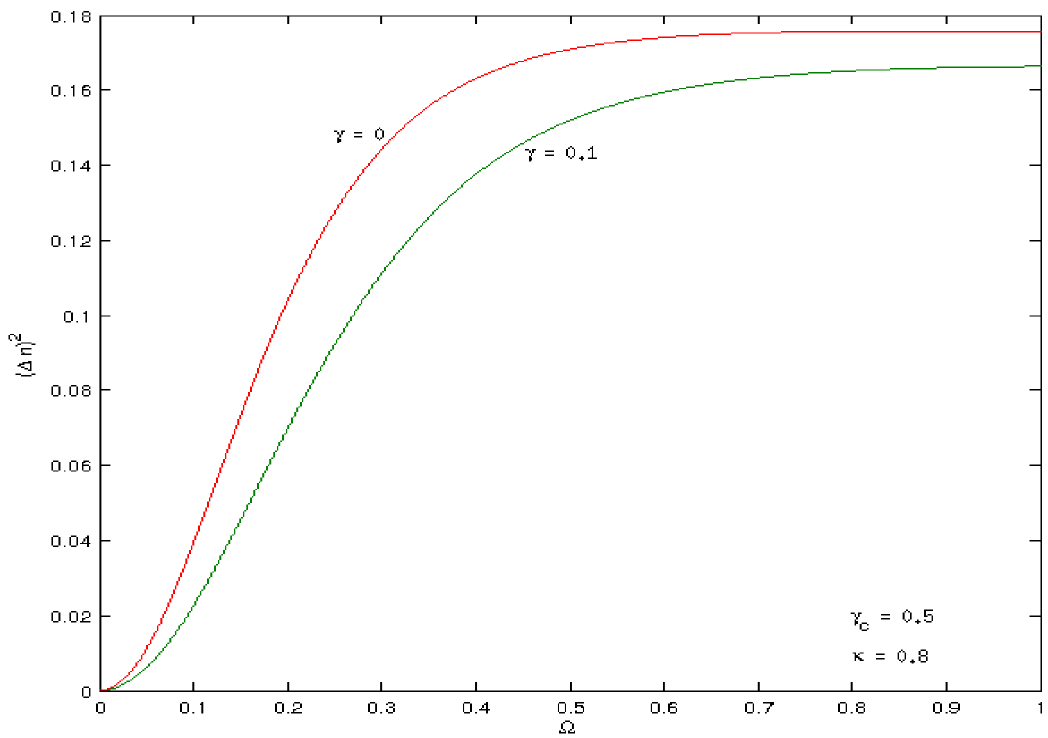

Fig. 3.2 Variance Versus Ω.

From Figure 3.2 we observe that, like the mean photon number, the variance of the photon number is greater for $\gamma = 0$ than for $\gamma = 0.1$.

Chapter 4

Quadrature Squeezing

We wish to calculate the quadrature variance and quadrature squeezing for the plus and minus quadrature operators defined by

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \tag{4.1}$$

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{4.2}$$

Applying the commutation relation for the plus and minus quadrature operators, we obtain

$$[\hat{a}_-, \hat{a}_+] \;\; = \;\; -2i[\hat{a}, \hat{a}^\dagger]. \tag{4.3}$$

It can be easily found that

$$[\hat{a}, \hat{a}^\dagger] \;\; = \;\; \frac{\gamma_c}{\kappa}\left(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle\right). \tag{4.4}$$

Thus, we get

$$[\hat{a}_-, \hat{a}_+] \;\; = \;\; 2i\frac{\gamma_c}{\kappa}\left(\langle\hat{\eta}_a\rangle - \langle\hat{\eta}_c\rangle\right). \tag{4.5}$$

We recall that if $\hat{A}$ and $\hat{B}$ satisfay the commutation relation [1]

$$[\hat{A}, \hat{B}] = i\hat{C}, \tag{4.6}$$

then it can be readily established that

$$\Delta A \Delta B \geq \frac{1}{2}|\langle\hat{C}\rangle|. \tag{4.7}$$

Hence, we readily get

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa} \left| \langle \hat{\eta}_a \rangle - \langle \hat{\eta}_c \rangle \right| . \tag{4.8}$$

The variance of the quadrature operators is expressible as [2]

$$(\triangle a_\pm)^2 = \pm \langle (\hat{a}^\dagger \pm \hat{a})^2 \rangle \mp [\langle \hat{a}^\dagger \rangle \pm \langle \hat{a} \rangle]^2. \tag{4.9}$$

In view of Eq. (2.42), the variance of the quadrature operators takes the form

$$(\triangle a_\pm)^2 = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle \pm \langle \hat{a}^{\dagger 2} \rangle \pm \langle \hat{a}^2 \rangle. \tag{4.10}$$

Substituting Eq. (2.28) and its complex conjugate into the last two terms of Eq. (4.10), we have

$$\langle \hat{a}^2 \rangle = \langle \hat{a}^{\dagger 2} \rangle = \frac{\gamma_c}{\kappa} \langle \hat{\sigma}_c \rangle. \tag{4.11}$$

Now upon introducing Eqs. (3.6), (3.7), and (4.11) into (4.10), we obtain

$$(\triangle a_+)^2 = \frac{\gamma_c}{\kappa} (1 + \langle \hat{\eta}_b \rangle + 2 \langle \hat{\sigma}_c \rangle). \tag{4.12}$$

$$(\triangle a_-)^2 = \frac{\gamma_c}{\kappa} (1 + \langle \hat{\eta}_b \rangle - 2 \langle \hat{\sigma}_c \rangle). \tag{4.13}$$

We see that the cavity light is in a squeezed state and the squeezing occurs in the minus quadrature.

We recall that the light generated by a two-level laser well above threshold is coherent, the quadrature variance of which is given by [2]

$$(\triangle a_\pm)^2_c = \frac{\gamma_c}{\kappa}, \tag{4.14}$$

where $(\triangle a_\pm)^2_c$ is the quadrature variance of coherent light.

We calculate the quadrature squeezing (S) of the cavity light relative to the quadrature variance of the cavity coherent light. We thus define the quadrature squeezing by [2]

$$S = \frac{(\triangle a_\pm)^2_c - (\triangle a_-)^2}{(\triangle a_\pm)^2_c}. \tag{4.15}$$

Hence, employing Eqs. (4.13) and (4.14) into (4.15), the quadrature squeezing turns out to be

$$S = 2\langle\hat{\sigma}_c\rangle - \langle\hat{\eta}_b\rangle, \tag{4.16}$$

Inserting Eqs. (2.48) and (2.49) into Eq. (4.16), we arrive at

$$\begin{aligned} S &= \frac{2\Omega}{2\gamma+\gamma_c}\Big(\frac{2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}\Big) \\ &- \frac{\gamma_c\Omega^2}{2\gamma\Omega^2+3\gamma_c\Omega^2+2\gamma^3+5\gamma^2\gamma_c+4\gamma\gamma_c^2+\gamma_c^3}. \end{aligned} \tag{4.17}$$

There follows

$$S = \frac{2\Omega(\gamma+\gamma_c)^2-\gamma_c\Omega^2}{2\Omega^2(\gamma+\gamma_c)+\gamma_c\Omega^2+(2\gamma+\gamma_c)(\gamma+\gamma_c)^2}. \tag{4.18}$$

When $\gamma = 0$ the above expression reduces to

$$S = \frac{2\gamma_c\Omega-\Omega^2}{3\Omega^2+\gamma_c^2}. \tag{4.19}$$

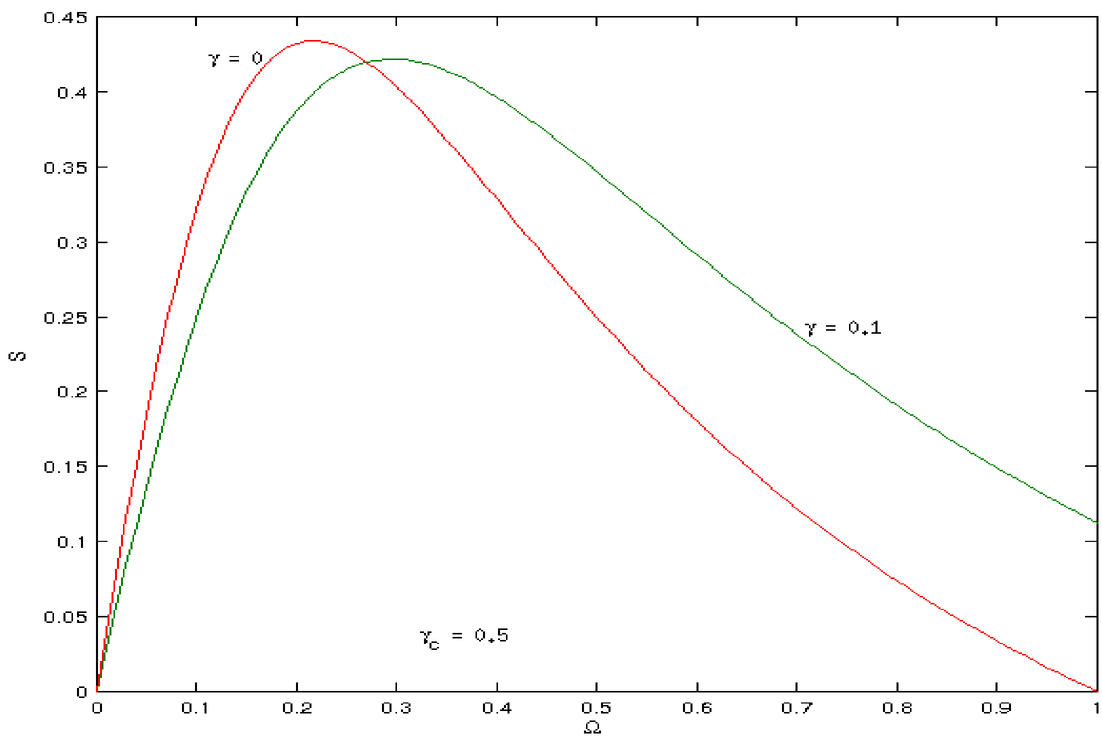

Fig. 4.1 Quadrature Squeezing (S) Versus Ω.

We notice that the plots in Figure 4.1 cross each other at the point $\Omega = 0.27$. We also see that when $\Omega < 0.27$ the quadrature squeezing for $\gamma = 0$ is greater than that for $\gamma = 0.1$ and for $\Omega > 0.27$ the quadrature squeezing when $\gamma = 0$ is less than that for $\gamma = 0.1$. Moreover, we observe that the maximum squeezing of the cavity light is 43.42 % for $\gamma = 0$ and 42.22 % for $\gamma = 0.1$, below the coherent-state level.

Chapter 5

The Local Mean Photon Number

We finally seek to obtain the local mean photon number of the cavity light. To this end, we first determine the power spectrum of the cavity light. The power spectrum of a single-mode light with central frequency ω_0 is given by

$$P(\omega) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty}\langle\hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle_{ss}e^{i(\omega-\omega_0)\tau}d\tau, \tag{5.1}$$

where ss stands for steady-state.

Upon integrating both sides of Eq. (5.1) over ω, we readily get

$$\int_{-\infty}^{\infty}P(\omega)d\omega \;=\; \int_{-\infty}^{\infty}\langle\hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle_{ss}e^{-i\omega_0\tau}d\tau \times \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\tau}d\omega, \tag{5.2}$$

so that using the fact that

$$\delta(\tau) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\tau}d\omega, \tag{5.3}$$

we have

$$\int_{-\infty}^{\infty}P(\omega)d\omega \;=\; \int_{-\infty}^{\infty}\langle\hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle_{ss}e^{-i\omega_0\tau}\delta(\tau)d\tau. \tag{5.4}$$

In view of the relation

$$\int_{-\infty}^{\infty}f(x)\delta(x)dx \;=\; f(x)\big|_{x=0}, \tag{5.5}$$

we obtain

$$\int_{-\infty}^{\infty}P(\omega)d\omega \;=\; \bar{n}, \tag{5.6}$$

with $\bar{n} = \langle\hat{a}^{\dagger}\hat{a}\rangle$ being the steady-state mean photon number. On account of this relation, we observe that $P(\omega)d\omega$ represents the steady-state mean photon number of a light-mode in the interval between ω and $\omega + d\omega$.

We hence realize that the local mean photon number in the interval between $\omega' = -\lambda$ and $\omega' = \lambda$ is expressible as

$$\bar{n}_{\pm\lambda} = \int_{-\lambda}^{\lambda} P(\omega')d\omega', \tag{5.7}$$

in which $\omega' = \omega - \omega_0$.

It proves to be convenient to rewrite Eq. (5.1) as

$$\begin{aligned} P(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{0} \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau \\ &+ \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau, \end{aligned} \tag{5.8}$$

so that replacing t by - τ in the first integral, we find

$$\begin{aligned} P(\omega) &= \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{a}^\dagger(t)\hat{a}(t-\tau)\rangle_{ss} e^{-i(\omega-\omega_0)\tau} d\tau \\ &+ \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau, \end{aligned} \tag{5.9}$$

we observe that one integral is the complex conjugate of the other. Hence the power spectrum can be written as

$$P(\omega) = \frac{1}{\pi} Re \int_{0}^{\infty} \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss} e^{i(\omega-\omega_0)\tau} d\tau, \tag{5.10}$$

in which 'Re' denotes the real part.

Now we proceed to calculate the two time correlation function that appears in Eq. (5.10) for the cavity light. To this end, we realize that the solution of Eq. (2.27) can be written as

$$\begin{aligned} \langle \hat{a}(t+\tau)\rangle &= \langle \hat{a}(t)\rangle e^{-\kappa\tau/2} - g e^{-\kappa\tau/2}\int_0^\tau \langle \hat{\sigma}_a(t+\tau)\rangle e^{\kappa\tau'/2} d\tau' \\ &- g e^{-\kappa\tau/2}\int_0^\tau \langle \hat{\sigma}_b(t+\tau)\rangle e^{\kappa\tau'/2} d\tau'. \end{aligned} \tag{5.11}$$

Applying the large-time approximation scheme, we obtain from Eq. (2.29) the approximately valid relation

$$\langle\hat{\sigma}_a(t)\rangle = \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma}_b^\dagger(t)\rangle. \tag{5.12}$$

We also note that

$$\langle\hat{\sigma}_a^\dagger(t)\rangle = \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma}_b(t)\rangle. \tag{5.13}$$

Employing Eqs. (5.12) and (5.13) in Eq. (2.30), we readily get

$$\frac{d}{dt}\langle\hat{\sigma}_b(t)\rangle = -\frac{\mu}{2}\langle\hat{\sigma}_b(t)\rangle - \frac{\eta}{2}\langle\hat{\sigma}_b^\dagger(t)\rangle, \tag{5.14}$$

where

$$\mu = 2\gamma+\gamma_c+\frac{\Omega^2}{2(\gamma+\gamma_c)}, \tag{5.15}$$

$$\eta = -\frac{\gamma_c\Omega}{\gamma+\gamma_c}. \tag{5.16}$$

We see that the adjoint of Eq. (5.14) is

$$\frac{d}{dt}\langle\hat{\sigma}_b^\dagger(t)\rangle = -\frac{\mu}{2}\langle\hat{\sigma}_b^\dagger(t)\rangle - \frac{\eta}{2}\langle\hat{\sigma}_b(t)\rangle. \tag{5.17}$$

From Eqs. (5.14) and (5.17), one easily gets

$$\frac{d}{dt}\left(\langle\hat{\sigma}_b^\dagger(t)\rangle + \langle\hat{\sigma}_b(t)\rangle\right) = -\frac{(\mu+\eta)}{2}\left(\langle\hat{\sigma}_b^\dagger(t)\rangle + \langle\hat{\sigma}_b(t)\rangle\right), \tag{5.18}$$

$$\frac{d}{dt}\left(\langle\hat{\sigma}_b^\dagger(t)\rangle - \langle\hat{\sigma}_b(t)\rangle\right) = -\frac{(\mu+\eta)}{2}\left(\langle\hat{\sigma}_b^\dagger(t)\rangle - \langle\hat{\sigma}_b(t)\rangle\right). \tag{5.19}$$

The solution of Eqs. (5.18) and (5.19) are given by

$$\left(\langle\hat{\sigma}_b^\dagger(t+\tau)\rangle + \langle\hat{\sigma}_b(t+\tau)\rangle\right) = \left(\langle\hat{\sigma}_b^\dagger(t)\rangle + \langle\hat{\sigma}_b(t)\rangle\right)e^{-(\mu+\eta)\tau/2}, \tag{5.20}$$

$$\left(\langle\hat{\sigma}_b^\dagger(t+\tau)\rangle - \langle\hat{\sigma}_b(t+\tau)\rangle\right) = \left(\langle\hat{\sigma}_b^\dagger(t)\rangle - \langle\hat{\sigma}_b(t)\rangle\right)e^{-(\mu+\eta)\tau/2}. \tag{5.21}$$

Now subtracting Eq. (5.21) from Eq. (5.20), we obtain

$$\langle\hat{\sigma}_b(t+\tau)\rangle = \langle\hat{\sigma}_b(t)\rangle e^{-(\mu+\eta)\tau/2}. \tag{5.22}$$

Taking the complex conjugate of Eq. (5.22), we have

$$\langle\hat{\sigma}_b^\dagger(t+\tau)\rangle \;=\; \langle\hat{\sigma}_b^\dagger(t)\rangle e^{-(\mu+\eta)\tau/2}. \tag{5.23}$$

We note that Eq. (5.12), can be written as

$$\langle\hat{\sigma}_a(t+\tau)\rangle \;=\; \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma}_b^\dagger(t+\tau)\rangle. \tag{5.24}$$

On combining Eqs. (5.23) and (5.24), we easily find

$$\langle\hat{\sigma}_a(t+\tau)\rangle \;=\; \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma}_b^\dagger(t)\rangle e^{-(\mu+\eta)\tau/2}. \tag{5.25}$$

Moreover, from Eq. (5.12), one gets

$$\langle\hat{\sigma}_b^\dagger(t)\rangle \;=\; \frac{2(\gamma+\gamma_c)}{\Omega}\langle\hat{\sigma}_a(t)\rangle. \tag{5.26}$$

Substituting Eq. (5.26) into Eq. (5.25), we see that

$$\langle\hat{\sigma}_a(t+\tau)\rangle \;=\; \langle\hat{\sigma}_a(t)\rangle e^{-(\mu+\eta)\tau/2}. \tag{5.27}$$

Now upon putting Eqs. (5.22) and (5.27) into Eq. (5.11), we arrive at

$$\begin{aligned}\langle\hat{a}(t+\tau)\rangle \;&=\; \langle\hat{a}(t)\rangle e^{-\kappa\tau/2} - g\langle\hat{\sigma}_a(t)\rangle e^{-\kappa\tau/2}\int_0^\tau e^{-[(\mu+\eta)-\kappa]\tau'/2}d\tau' \\ &-\; g\langle\hat{\sigma}_b(t)\rangle e^{-\kappa\tau/2}\int_0^\tau e^{-[(\mu+\eta)-\kappa]\tau'/2}d\tau'.\end{aligned} \tag{5.28}$$

After integration, we obtain

$$\begin{aligned}\langle\hat{a}(t+\tau)\rangle \;&=\; \langle\hat{a}(t)\rangle e^{-\kappa\tau/2} - 2g\langle\hat{\sigma}_a(t)\rangle\left(\frac{e^{-(\mu+\eta)\tau/2}-e^{-\kappa\tau/2}}{\kappa-(\mu+\eta)}\right) \\ &-\; 2g\langle\hat{\sigma}_b(t)\rangle\left(\frac{e^{-(\mu+\eta)\tau/2}-e^{-\kappa\tau/2}}{\kappa-(\mu+\eta)}\right),\end{aligned} \tag{5.29}$$

from which follows

$$\begin{aligned}\langle\hat{a}(t+\tau)\rangle \;&=\; \left[\langle\hat{a}(t)\rangle + \frac{2g}{\kappa-(\mu+\eta)}(\langle\hat{\sigma}_a(t)\rangle\; + \langle\hat{\sigma}_b(t)\rangle)\right]e^{-\kappa\tau/2} \\ &-\; \frac{2g}{\kappa-(\mu+\eta)}(\langle\hat{\sigma}_a(t)\rangle\; + \langle\hat{\sigma}_b(t)\rangle\;)e^{-(\mu+\eta)\tau/2}.\end{aligned} \tag{5.30}$$

The quantum regression theorem states that it is possible under certain conditions to evaluate a two time correlation function employing the explicit form of a one time correlation function, obtained with the aid of some equation of evolution such as the master equation [1].

Now taking into account Eq. (2.28) and application of the quantum regression theorem to Eq. (5.30) leads to

$$\begin{aligned}\langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle &= \frac{\kappa\bar{n}}{\kappa-(\mu+\eta)}e^{-(\mu+\eta)\tau/2} \\ &- \frac{(\mu+\eta)\bar{n}}{\kappa-(\mu+\eta)}e^{-\kappa\tau/2}. \end{aligned} \tag{5.31}$$

Thus on substituting Eq. (5.31) into Eq. (5.10) and carrying out the integration, there follows

$$\begin{aligned}P(\omega) &= \frac{\kappa\bar{n}}{\kappa-(\mu+\eta)}\left[\frac{(\mu+\eta)/2\pi}{(\omega-\omega_0)^2+(\frac{\mu+\eta}{2})^2}\right] \\ &- \frac{(\mu+\eta)\bar{n}}{\kappa-(\mu+\eta)}\left[\frac{\kappa/2\pi}{(\omega-\omega_0)^2+(\frac{\kappa}{2})^2}\right]. \end{aligned} \tag{5.32}$$

Finally, upon introducing Eq. (5.32) into Eq. (5.7) and carrying out the integration, applying the relation

$$\int_{-\lambda}^{\lambda}\frac{dx}{x^2+a^2} = \frac{2}{a}tan^{-1}(\lambda/a), \tag{5.33}$$

we arrive at

$$\bar{n}_{\pm\lambda} = \bar{n}z(\lambda), \tag{5.34}$$

where $z(\lambda)$ is given by

$$\begin{aligned}z(\lambda) &= \frac{2\kappa/\pi}{\kappa-(\mu+\eta)}tan^{-1}(\frac{2\lambda}{\mu+\eta}) \\ &- \frac{2(\mu+\eta)/\pi}{\kappa-(\mu+\eta)}tan^{-1}(\frac{2\lambda}{\kappa}). \end{aligned} \tag{5.35}$$

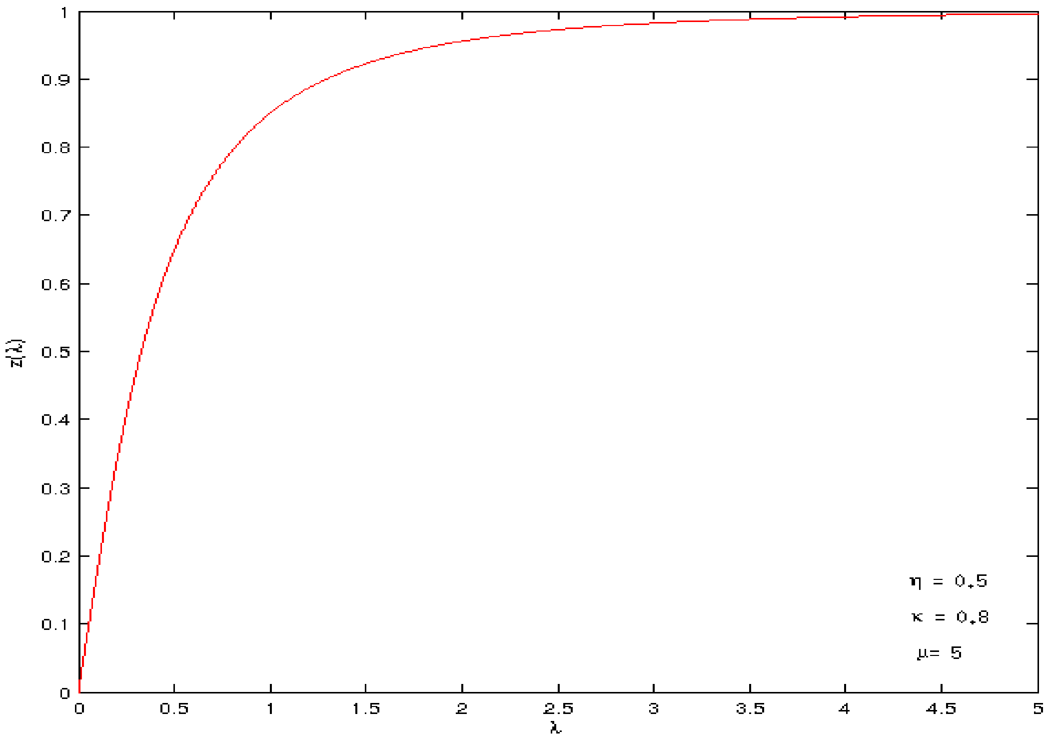

Fig. 5.1 z(λ) Versus λ.

From Eq. (5.35), we easily obtain z(0.5) = 0.65, z(1) = 0.85 and z(2) = 0.96. Then combination of these results with Eq. (5.34) yields, $\bar{n}_{\pm 0.5} = 0.65\bar{n}$, $\bar{n}_{\pm 1} = 0.85\bar{n}$ and $\bar{n}_{\pm 2} = 0.96\bar{n}$. We therefore observe that a large part of the total mean photon number is confined in a relatively small frequency interval.

Chapter 6

Conclusion

We have considered a three-level atom in which the three-level atom available in the open cavity is driven by coherent light from the bottom to the top level. We have carried out our analysis by putting the vacuum noise operators in normal order and applying the large-time approximation scheme. The procedure of normal ordering the noise operators provides the vacuum reservoir to be a noiseless physical entity.

In the absence of spontaneous emission and based on the definition given by Eq. (2.35) we infer that the three-level atom in the top or middle level and inside the cavity emits a photon due to its interaction with the cavity light. We certainly identify this process to be Stimulated Emission.

We have determined that the mean photon number for $\gamma = 0$ is greater than for $\gamma = 0.1$. We have also established that like the mean photon number, the variance of the photon number is greater when $\gamma = 0$ than when $\gamma = 0.1$.

We have obtained that the quadrature squeezing cross each other at the point $\Omega = 0.27$. It is also found that when $\Omega < 0.27$ the quadrature squeezing for $\gamma = 0$ is greater than that of $\gamma = 0.1$ and when $\Omega > 0.27$ the quadrature squeezing for $\gamma = 0$ is less than that of $\gamma = 0.1$. Moreover, we have found that the light generated by the three-level atom is in a squeezed state, with the maximum quadrature squeezing being 43.42 % for $\gamma = 0$ and 42.22 % for $\gamma = 0.1$, thus it is below the coherent-state level.

In the absence of spontaneous emission the mean, the variance and the quadrature squeezing of the cavity light has found greater than in the presence of spontaneous emission.

We have noticed that z(0.5) = 0.65, z(1) = 0.85, z(2) = 0.96. Then combination of these results with Eq. (5.34) yields, $\bar{n}_{\pm 0.5} = 0.65\bar{n}$, $\bar{n}_{\pm 1} = 0.85\bar{n}$ and $\bar{n}_{\pm 2} = 0.96\bar{n}$. Hence we can conclude that the large part of the total mean photon number is confined in a relatively small frequency interval.

References

[1] Fesseha Kassahun, Fundamentals of Quantum Optics (Lulu Press Inc., North Carolina, 2008).

[2] K. Fesseha, arXiv: 1105. 1438v3 [quant-ph] 25 Sep 2012.

[3] K. Fesseha, Opt. Commun. 284, 1357 (2011).

[4] J. Anwar and M.S. Zubairy, Phys. Rev. A49, 481 (1994).

[5] N. Ansari, Phys. Rev. A48, 4686 (1993).

[6] M.O. Scully and M.S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, 1997).

[7] S. Tesfa, Phys. Rev. A74, 04386 (2006).

[8] E. Alebachew and K. Fesseha, OPt. Commun. 265, 314 (2006).

[9] N. Lu, F.X. Zhao and J. Bergou, Phys. Rev. A39, 5189 (1989).

[10] N. Lu, and S.Y. Zhu, Phys. Rev. A41, 2865 (1990).

[11] N. Ansari, Phys. Rev. A46, 1560 (1992).

[12] J. Anwar and M.S. Zubairy, Phys. Rev. A45, 1804 (1992).

[13] N. Ansari, J. Gea-Banacloche, and M.S. Zubairy, Phys. Rev. A41, 5179 (1990).

[14] K. Fesseha, Phys. Rev. A63, 033811 (2001).

[15] K. Fesseha, Opt. Commun. 284 (2011) 1357-1363.

[16] Fesseha Kassahun, The Quantum analysis of light, [CreateSpace Independent Publishing Platform, November7, 2012].

[17] M.O. Scully, K. Wodkiewicz, M.S. Zubairy, J. Bergou, N. Lu, and J. Meyerter Veh, Phys. Rev. Lett. 60, 1832 (1988).

Declaration of Authorship

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

Name: Mulugeta Melaku

Signature of Author

This thesis has been submitted for examination with my approval as University advisor

Name: Dr. Fesseha Kassahun

Signature of Advisor

Place and time of submission:

Physics Department

Addis Ababa University

April 14, 2014