COMPUTERIZED ANALYSIS OF FRAMES WITH NON-PRISMATIC MEMBERS

By

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A Thesis Submitted to the School of Graduate Studies of Addis Ababa University

In Partial Fulfillment of the Requirements For the Degree of Master of Science In Civil Engineering (Structural)

ADDIS ABABA
April 2002

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

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FACULTY OF TECHNOLOGY

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Declaration

The thesis is my original work, has not been presented for a degree in any other university and that all sources of materials used are duly acknowledged.

Name Samson Takele

Signature _______________

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Date of Submission April 30,2002

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DEDICATED
To

My Mother Tabote

And

My Sister Birtukan

ACKNOWLEDGMENT

I praise the almighty GOD for providing me the power and courage to carry out this thesis.

This thesis was carried out under the supervisions of Dr. Shifferaw Taye. I gratefully acknowledge his advice.

My sincere expression goes to my colleagues Dereje Abay, Dawit Hailu and Tezera Firew for their advice and assistance during various stages of this thesis.

I also like to thank Ato Amarebih Refera and Ato Muluneh Admass, for their help during preparation of this thesis.

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A_C	Cross sectional area of section of minimum depth
$\mathbf{A}_C$	Matrix of combined joint loads (in direction of structure axes)
$\mathbf{A}_E$	Matrix of equivalent joint load (in direction of structure axes)
$\mathbf{A}_{FC}$	Matrix of actions corresponding to free joint displacement
$\mathbf{A}_J$	Matrix of loads applied at joints (in direction of structure axes)
$\mathbf{A}_M$	Matrix of member end action (in direction of member axes)
$\mathbf{A}_{MD}$	Matrix of actions at end of member due to joint displacement
$\mathbf{A}_{ML}$	Matrix of actions at ends of restrained members (in direction of member axes)
$\mathbf{A}_{MS}$	Matrix of actions at ends of restrained members (in direction of structure axes)
$\mathbf{A}_R$	Matrix of support reactions (in direction of structure axes)
$\mathbf{A}_{RC}$	Matrix of actions corresponding to restrained joints
$\mathbf{A}_S$	Matrix of member end actions (in direction of structure axes)
A_X	Cross sectional area of any sections distance x from end j or x_I from end k
A_{XL}	Cross sectional area of any sections at end j
A_{XR}	Cross sectional area of any sections at end k
C_X, C_Y	x, y direction cosines of members
C_{jk}	Carry over factor of member jk at end j
C_{kj}	Carry over factor of member jk at end k
d	Ratio of distance from loading point to end j to length of span
d_I	Ratio of distance from loading point to end k to length of span
d_C	Minimum depth of member
$\mathbf{D}_F$	Matrix of free joint displacement (in direction of structure axes)
$\mathbf{D}_J$	Matrix of joint displacement for all joints (in direction of structure axes)
d_L	Depth of member at end j
$\mathbf{D}_M$	Matrix of joint displacement in member axes

d_R	Depth of member at end *k*
$\mathbf{D}_R$	Matrix of joint displacement corresponding to restrained joints
$\mathbf{D}_S$	Matrix of member displacement in structure axes
E	Modules of elasticity (Young's modulus)
E_{LL}	Length of haunch at end *j*
E_{LR}	Length of haunch at end *k*
I_C	Moment of inertia of section of minimum depth
I_X	Moment of inertia of any section distance *x* from end *j* or x_1 from end *k*
I_{XL}	Moment of inertia at end *j*
I_{XR}	Moment of inertia at end *k*
$I_0, I_1, \ldots, I_7$	Integral values
J, K	Joint index
ISN	Structure number
JJ( )	Designation for *j* end of member (joint *j*)
JK( )	Designation for *k* end of member (joint *k*)
JRL	Joint Restraint list
L	Length of member
M_j	Fixed end moment at end *j*
M_k	Fixed end moment at end *k*
NJ	Number of joints
NLJ	Number of loaded joint
NLM	Number of loaded member
NLPM	Number of concentrated load in a member
NLUM	Number of uniformly distributed load in a member
NLS	Number of loading systems
NR	Number of support restraints
NRJ	Number of restrained joint
P	Concentrated load value
R	Rotation matrix
S_{CM}	Stiffness constant of member
$\mathbf{S}_J$	Joint stiffness matrix
$\mathbf{S}_{FF}$	Stiffness matrix for free joint displacements

$\mathbf{S}_M$	Member stiffness matrix for member oriented axes
$\mathbf{S}_{MS}$	Member stiffness matrix for structure oriented axes
$\mathbf{T}$	Rotation transformation matrix
W	Uniform load
W_A	Load intensity at x_A distance from end *j*
W_B	Load intensity at x_B distance from end *j*
x	Distance from variable point to end *j*
x_1	Distance from variable point to end *k*
X( ),Y( )	Coordinate of joint
α	Length of haunch at end *j*
β	Length of haunch at end *k*
γ	Inclination angle of member from structure axes

Introduction

1.1 Objective of Present Study

Beams, columns and other structural components of large-span systems, such as industrial facilities, assembly halls, hangars and bridges, call for the use of large and deep sections in order to provide effective resistance to external loads, their own weight and also to limit the displacements in the system. Efficient and economical response to such demands can be met with the use of non-prismatic members that may have segmented or tapered sections or combinations. The use of non-prismatic members in civil engineering structures usually results in reduced own weight, this in turn alleviate the need to provide undesirably large sections that would have otherwise resulted in heavy and uneconomical structures as well as greater head-room for working space.

The shape of cross-sections at each and every point along the length of each of the constituent members significantly influences the analysis of structural systems that are composed of non-prismatic members, among others. This, in turn, results in more demanding analysis procedure compared to systems composed only of prismatic members. Lack of adequate tools prohibits engineers creativity to propose new structural shapes and designs. As a result, practicing structural engineers tend to avoid the implementation of non-prismatic members in their planning and design undertakings. This thesis explores the concept behind analysis methods and techniques of structures with non-prismatic members with the view of providing design engineers a tool for solving such structural systems to come up with safe, efficient and economical designs.

1.2 Assumptions

The analysis method which is utilized in this thesis is the direct stiffness method .The assumptions which governs the analysis are summarized below:

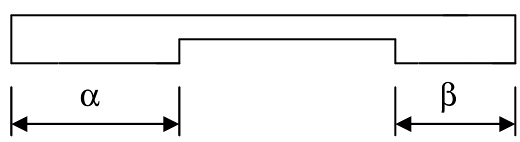
1. Deflections are small.
2. Material is elastic obeying Hooke’s law.
3. Shear deformations are neglected
4. Three degree of freedom are assumed at each joint.

Non-Prismatic Members

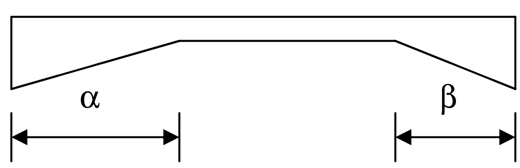
2.1 Member Type

Most common types of Non-prismatic members are shown in the Fig. 2.1.

Type 1 Stepped members



Type 2 Straight haunched members



Type 3 Parabolic haunched members

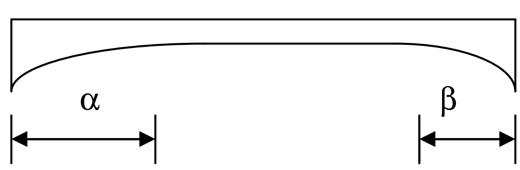
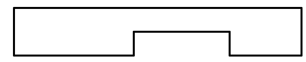
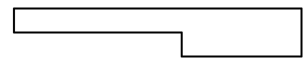
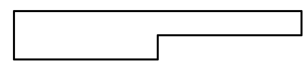
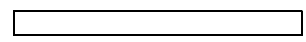
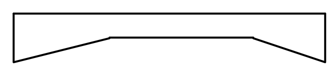
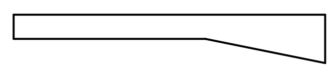
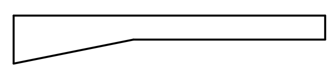
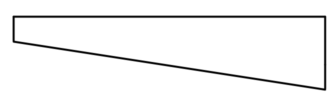
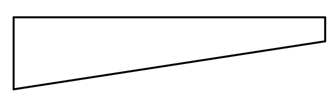
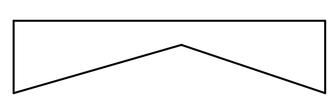
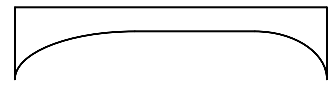
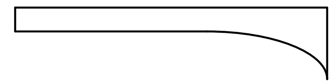
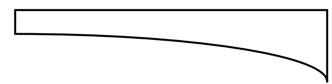
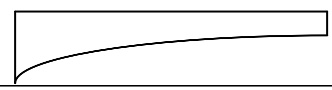
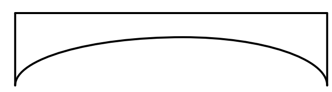


Fig 2.1 Beams with varying rigidity.

Depending on the length of the left haunch α and the right haunch β we may get different types of members which are commonly used and these are shown on Tables 2.1.

Table 2.1 Types of non-prismatic members (For notations refer Fig. 2.1)

Member	α , β	Shape
Type 1	$\alpha \neq 0$. $\beta \neq 0, L-\alpha-\beta>0$	
	$\alpha=0$ $\beta \neq 0$, $\beta<L$	
	$\alpha \neq 0$ $\beta=0$, $\alpha<L$	
	$\alpha=0$ $\beta=0$	
Type 2	$\alpha \neq 0$ $\beta \neq 0$, $L-\alpha-\beta>0$	
	$\alpha =0$ $\beta \neq 0$, $\beta<L$	
	$\alpha \neq 0$ $\beta =0$, $\alpha<L$	
	$\alpha=0$ $\beta=L$	
	$\alpha=L$ $\beta=0$	
	$L- \alpha-\beta=0$ $\alpha \neq 0$, $\beta \neq 0$	

Member	α , β	Shape
	$\alpha \neq 0$ $\beta \neq 0$, $L-\alpha-\beta>0$	
	$\alpha = 0$ $\beta \neq 0$, $\beta<L$	
Type 3	$\alpha \neq 0$ $\beta = 0$, $\alpha<L$	
	$\alpha = 0$ $\beta = L$	
	$\alpha = L$ $\beta = 0$	
	$L- \alpha -\beta=0$ $\alpha \neq 0$, $\beta \neq 0$	

2.2 Derivation of Member Stiffness Matrix for Non-Prismatic Members

Forces and deformations in structures are related to one another by means of stiffness influence coefficients. The matrix containing those coefficients is known as stiffness matrix. A Stiffness Coefficients K_{jk} is a force produced at *j* due to a unit displacement at *k*. hence, the problem of deriving the elements of the stiffness matrix is handled systematically by giving the k^{th} coordinate a unit displacement, holding all other coordinate at zero displacements. The resulting forces at all coordinate due to this unit displacement form the stiffness coefficients of the K^{th} column of the stiffness matrix $\mathbf{S}_{Mi}$. Next we can see how these coefficients can be obtained.

Figure 2.2 shows a frame element, Member *i,* that is fully restrained at both ends, which are denoted as ends *j* and *k*. x_M, y_M, z_M indicates member-oriented axes. The x_M axis is in the direction of the member and is positive in the sense from *j* to *k*.

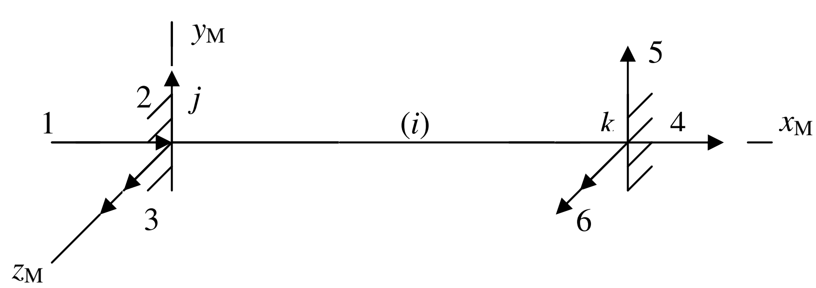


Fig. 2.2 Frame element.

Member stiffness for the restrained member shown in Fig. 2.2 consists of actions exerted on the member by the restraints when unit displacements (translations and rotations) are imposed at each ends of the member.

Complete member stiffness

$$\mathbf{S}_{Mi} = \begin{bmatrix} K_{11} & 0 & 0 & K_{14} & 0 & 0 \\ 0 & K_{22} & K_{23} & 0 & K_{25} & K_{26} \\ 0 & K_{32} & K_{33} & 0 & K_{35} & K_{36} \\ K_{41} & 0 & 0 & K_{44} & 0 & 0 \\ 0 & K_{52} & K_{53} & 0 & K_{55} & K_{56} \\ 0 & K_{62} & K_{63} & 0 & K_{65} & K_{66} \end{bmatrix} \tag{2.1}$$

Values of K_{jk}, j=1,6 and k=1,6 can be obtained as follows

Unit displacements are considered to be induced one at a time while all other end displacements are restrained; also they are assumed to be positive in the x_M, y_M, z_M directions. Thus arrows in Fig. 2.2 indicate the positive senses of the three translations and rotations at each ends of the member. In the figure the single headed arrows denote translations while the double-headed arrows represent rotations.

Case 1: - Unit translation of the j end of the member in the positive x_M direction. Another displacements are zero.

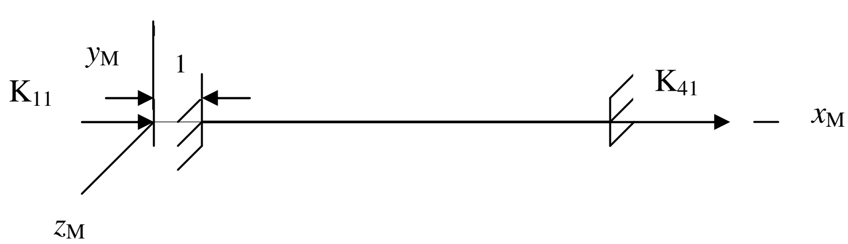


Fig. 2.3 Unit translation at end j.

$$\Delta = \int_0^L \frac{P}{A_X E} dx = p \int_0^L \frac{1}{A_X E} dx$$

$$K_{11} = -K_{41} = \frac{E}{\int_0^L \frac{1}{A_X} dx} \tag{2.2}$$

Case2: -Similarly a unit translation of the *k* end of the member

Fig. 2.4 Unit translation at end *k*.

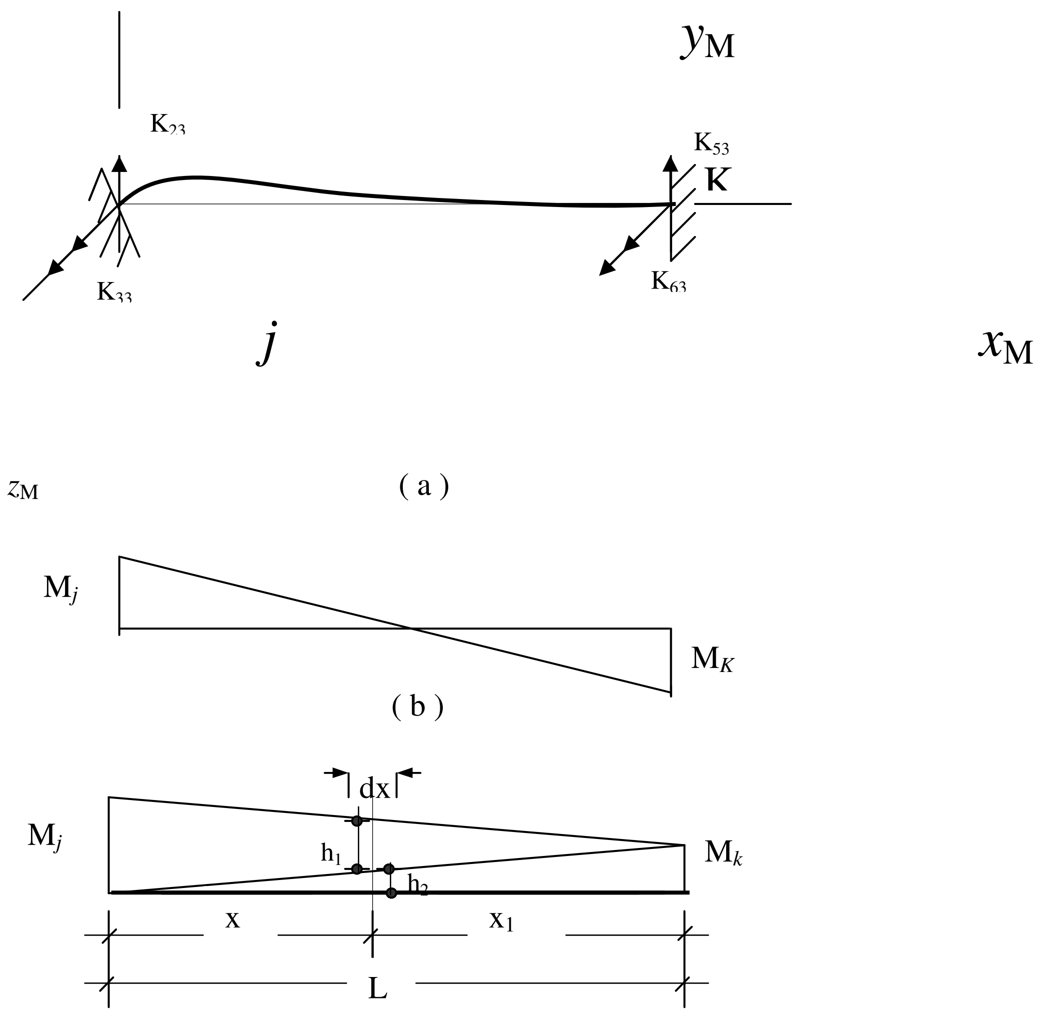
$$K_{44} = -K_{14} = \frac{E}{\int_0^L dx/A_x} \tag{2.3}$$

where A_x is the area of beam cross-section and is a function of *x*.

Rotational stiffness

Rotational stiffness for an end of a member is the end moment required to produce a unit rotation at this end (simple end) while the other end is fixed. For a member with varying moment of inertia we can write rotational stiffness at end *j* as follows.

Case 3: - Unit rotation at the *j* end of the member in the positive z_M direction.



(c)

Fig. 2.5 Unit rotation at the *j* end.

where $h_1 = M_j x_1/L$ and $h_2 = M_k x/L$

In Fig. 2.5b the moment diagram is drawn for beam *jk* and an equivalent form is shown in Fig. 2.5c.According to conjugate beam principle the end reactions of a simply supported beam (conjugate beam) loaded with the moment diagram divided by EI_x are the slope of the tangents to the elastic curve at the support.

Thus, the reaction or slope, at k is for M/EI loading

$\sum M/EI_{x=j} = 0$ (Since the member *j*-*k* is fixed at k the slope must be equal to zero at that point)

$$\frac{1}{L}\int_j^k \frac{M_j x_1/L * x}{EI_x}dx_1 + \frac{1}{L}\int_j^k \frac{M_k x^2/L}{EI_x}dx = 0$$

$$\frac{M_j}{EL^2}\int_0^L \frac{x(L-x)}{I_x}dx + \frac{M_k}{EL^2}\int_0^L \frac{x^2}{I_x}dx = 0 \tag{2.4a}$$

Similarly the reaction or slope at *j* is

$$\sum M/EI_{x=k} = -1 \quad \text{(Since a unit rotation is applied at the j end)}$$

$$\frac{1}{L^2}\int_k^j \frac{M_j x_1^2}{EI_x}dx_1 + \frac{1}{L^2}\int_k^j \frac{M_k x_1(L-x_1)}{EI_x}dx_1 \quad = -1$$

We can use $x_1=L-x$, $dx_1=-dx$

Substituting in to the above equation and simplifying

$$\frac{M_j}{EL^2}\int_0^L \frac{(L-x)^2}{I_x}dx + \frac{M_k}{EL^2}\int_0^L \frac{x(L-x)}{I_x}dx = 1 \tag{2.4b}$$

Using Eq. 2.4a and Eq. 2.4b and after simplification we can obtain the following.

$$K_{33} = M_j = \frac{E\int_0^L \frac{x^2dx}{I_x}}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2} \tag{2.5}$$

$$K_{63} = M_k = \frac{E\int_0^L \frac{(L-x)xdx}{I_x}}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2} \tag{2.6}$$

where I_X is the moment of inertia of the beam cross-section and is a function of *x*.

Case 4: -Like wise when a member is fixed at *j* and free to rotate at end *k*.

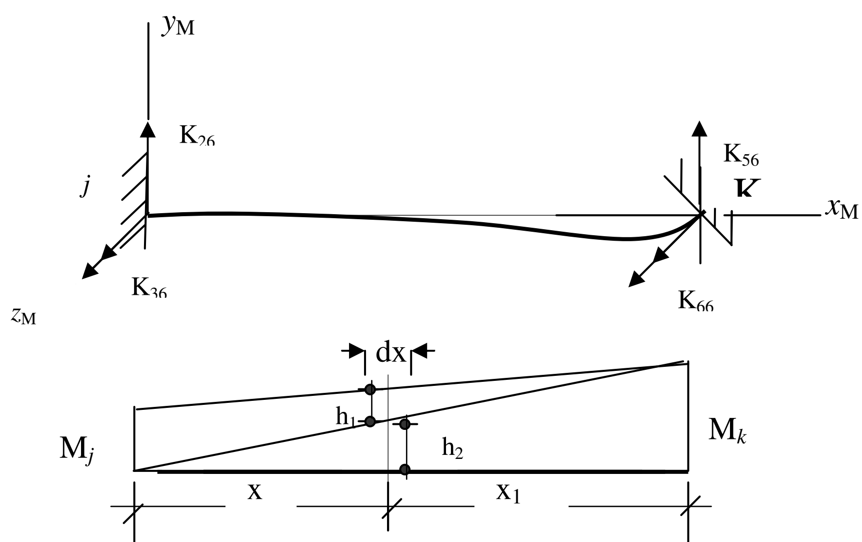


Fig. 2.6 Unit rotation at *k* end.

where $h_1 = M_j x_1/L$ and $h_2 = M_k x/L$

The reaction or slope, at *k* is

$$\sum M_J / EI_{x=j} = -1$$

$$\frac{M_j}{EL^2}\int_0^L \frac{x(L-x)}{I_x}dx + \frac{M_k}{EL^2}\int_0^L \frac{x^2}{I_x}dx = -1 \tag{2.7a}$$

The slope at *j*=0.

$$\frac{M_j}{EL^2}\int_0^L \frac{(L-x)^2}{I_x}dx + \frac{M_k}{EL^2}\int_0^L \frac{x(L-x)}{I_x}dx = 0 \tag{2.7b}$$

Solving Eq. 2.7a and Eq. 2.7b and after simplification we get the following.

$$K_{36} = M_j = \frac{E\int_0^L \frac{(L-x)x\,dx}{I_x}}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{x\,dx}{I_x}\right]^2} \tag{2.8}$$

$$K_{66} = M_k = \frac{E\int_0^L \frac{(L-x)^2dx}{I_x}}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{x\,dx}{I_x}\right]^2} \tag{2.9}$$

The stiffness elements K_{23} and K_{53}; K_{26} and K_{56} can be found from equilibrium equation by using stiffness elements K_{33} and K_{63}; K_{36} and K_{66} respectively.

Therefore

$$K_{23} = \frac{K_{33}+K_{63}}{L} = -K_{53}$$

$$K_{26} = \frac{K_{36}+K_{66}}{L} = -K_{56}$$

Substituting the previous integrals in to the above equations and after simplification we obtain the following

$$K_{23} = \frac{\frac{E}{L}\left[\left(\int_0^L \frac{x^2 dx}{I_x}\right) + \left(\int_0^L \frac{(L-x)x dx}{I_x}\right)\right]}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2 dx}{I_x} - \left[\int_0^L \frac{x dx}{I_x}\right]^2} \tag{2.10}$$

$$K_{26} = \frac{\frac{E}{L}\left[\left(\int_0^L \frac{(L-x)x dx}{I_x}\right) + \left(\int_0^L \frac{(L-x)^2 dx}{I_x}\right)\right]}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2 dx}{I_x} - \left[\int_0^L \frac{x dx}{I_x}\right]^2} \tag{2.11}$$

Carry over factors. It is known that the rotational stiffness for an end of a member is the end moment required to produce a unit rotation at this end (simple end) while the other end is fixed. The carry over factor is the ratio of induced moment at the other end (fixed) to the applied moment at this end. From Fig. 2.5a the induced moment M_K at end *k* is the carry over moment from end *j* and therefore equal to $C_{jk}K_{33}$

Thus we can write

$$C_{jk} = \frac{K_{63}}{K_{33}} = \frac{\int_0^L \frac{(L-x)x}{I_x} dx}{\int_0^L \frac{x^2}{I_x} dx} \tag{2.12}$$

Likewise carry over coefficient from end *k* to *j*

$$C_{kj} = \frac{K_{36}}{K_{66}} = \frac{\int_0^L \frac{(L-x)x}{I_x} dx}{\int_0^L \frac{(L-x)^2}{I_x} dx} \tag{2.13}$$

Case 5: -A unit displacement at end *j*

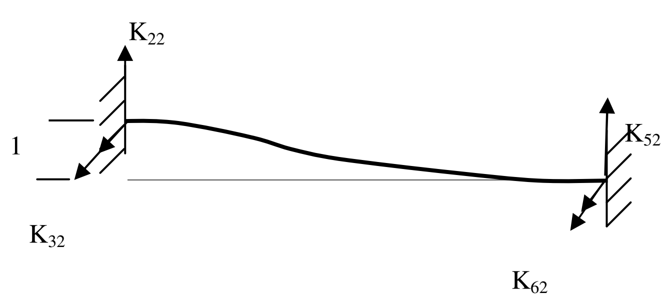


Fig. 2.7 Unit vertical displacement at end *j*.

For a member having varying I_x subjected to a pure relative end translation Δ=1,as shown, the moment resistant at end *j* and *k* can be readily be obtained as.

$$\begin{aligned} M_j &= \left(K_{33} + C_{jk}K_{33}\right)\frac{\Delta}{L} \\ &= K_{33}\left(1 + C_{jk}\right)\frac{1}{L} \end{aligned} \tag{2.14a}$$

$$M_j = K_{32} = \frac{E}{L}\left[\frac{\int_0^L \frac{x^2}{I_x}dx + \int_0^L \frac{(L-x)x}{I_x}dx}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2}\right] \tag{2.14b}$$

For end *k*

$$\begin{aligned} M_k &= \left(K_{66} + C_{kj}K_{66}\right)\frac{\Delta}{L} \\ &= K_{66}\left(1 + C_{kj}\right)\frac{1}{L} \end{aligned} \tag{2.15a}$$

$$M_k = K_{62} = \frac{E}{L}\left[\frac{\int_0^L \frac{(L-x)^2}{I_x}dx + \int_0^L \frac{(L-x)x}{I_x}dx}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2}\right] \tag{2.15b}$$

K_{22} and K_{52} will be obtained using equilibrium equation

$$\begin{aligned} K_{22} &= -K_{52} = \frac{K_{32} + K_{62}}{L} \\ &= \frac{E\left[\int_0^L x^2dx/I_x + 2\int_0^L (L-x)xdx/I_x + \int_0^L (L-x)^2dx/I_x\right]}{L^2\left[\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2\right]} \end{aligned} \tag{2.16}$$

Case 6: -A unit displacement at end *k*

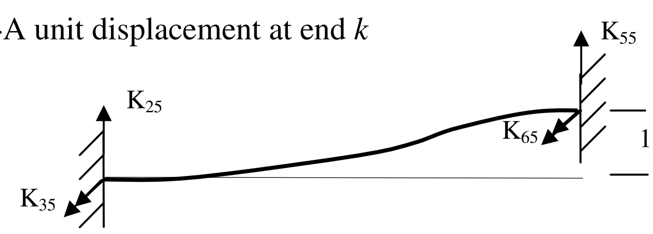


Fig. 2.8 Unit vertical displacement at end *k*.

The resistance at end *j* and *k* can be obtained as

$$M_j = K_{35} = -(K_{33} + C_{jk}K_{33})\frac{\Delta}{L}$$

$$= -\frac{E}{L}\left[\frac{\int_0^L \frac{x^2}{I_x}dx + \int_0^L \frac{(L-x)x}{I_x}dx}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2}\right] = -K_{32} \tag{2.17}$$

$$M_k = K_{65} = -(K_{66} + C_{kj}K_{66})\frac{\Delta}{L}$$

$$= -\frac{E}{L}\left[\frac{\int_0^L \frac{(L-x)^2}{I_x}dx + \int_0^L \frac{(L-x)x}{I_x}dx}{\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2}\right] = -K_{62} \tag{2.18}$$

K_{25} and K_{55} will be obtained by using equilibrium equation

$$K_{25} = -K_{55} = \frac{K_{35} + K_{65}}{L}$$

$$K_{35} = -K_{55} = -\frac{E\left[\int_0^L x^2dx/I_x + 2\int_0^L (L-x)xdx/I_x + \int_0^L (L-x)^2dx/I_x\right]}{L^2\left[\int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2dx}{I_x} - \left[\int_0^L \frac{xdx}{I_x}\right]^2\right]} \tag{2.19}$$

The elements of member stiffness matrix $\mathbf{S}_{Mi}$, which are derived above, are written in matrix form and shown on the Table 2.2.

$$
K = \begin{bmatrix}
\dfrac{\int_0^L dx/I_x \int_0^L x^2\,dx/I_x - \left[\int_0^L x\,dx/I_x\right]^2}{\int_0^L dx/A_x} & 0 & 0 & -\dfrac{\int_0^L dx/I_x \int_0^L x^2\,dx/I_x - \left[\int_0^L x\,dx/I_x\right]^2}{\int_0^L dx/A_x} & 0 & 0 \\
 & \dfrac{1}{L^2}\left[\int_0^L x^2 dx/I_x + 2\int_0^L (L-x)x\,dx/I_x + \int_0^L (L-x)^2 dx/I_x\right] & \dfrac{1}{L}\left[\int_0^L x^2 dx/I_x + \int_0^L (L-x)x\,dx/I_x\right] & 0 & -\dfrac{1}{L^2}\left[\int_0^L x^2 dx/I_x + 2\int_0^L (L-x)x\,dx/I_x + \int_0^L (L-x)^2 dx/I_x\right] & \dfrac{1}{L}\left[\int_0^L (L-x)x\,dx/I_x + \int_0^L (L-x)^2 dx/I_x\right] \\
 & & \int_0^L x^2 dx/I_x & 0 & -\dfrac{1}{L}\left[\int_0^L x^2 dx/I_x + \int_0^L (L-x)x\,dx/I_x\right] & \int_0^L (L-x)x\,dx/I_x \\
 & \text{Symmetric} & & \dfrac{\int_0^L dx/I_x \int_0^L x^2\,dx/I_x - \left[\int_0^L x\,dx/I_x\right]^2}{\int_0^L dx/A_x} & 0 & 0 \\
 & & & & \dfrac{1}{L^2}\left[\int_0^L x^2 dx/I_x + 2\int_0^L (L-x)x\,dx/I_x + \int_0^L (L-x)^2 dx/I_x\right] & -\dfrac{1}{L}\left[\int_0^L (L-x)x\,dx/I_x + \int_0^L (L-x)^2 dx/I_x\right] \\
 & & & & & \int_0^L (L-x)^2 dx/I_x
\end{bmatrix}
$$

Table 2.2 Stiffness matrix for non-prismatic members.

Where

$$\kappa = \frac{E}{\int_0^L dx/I_x \int_0^L x^2 dx/I_x - \left[\int_0^L x dx/I_x\right]^2}$$

2.3 Evaluation of Integrals

Expression given in Table 2.2 is perfectly general and may be used for any shaped members as long as an expression for the moment of inertia I_x and cross- sectional area A_x, can be written as a mathematical equation that is integrable.

The integral values that we need for evaluating the stiffness values are as follows.

$$I_0 = \int_0^L \frac{1}{A_x} dx \tag{2.20a}$$

$$I_1 = \int_0^L \frac{dx}{I_x} \tag{2.20b}$$

$$I_2 = \int_0^L \frac{x dx}{I_x} \tag{2.20c}$$

$$I_3 = \int_0^L \frac{x^2 dx}{I_x} \tag{2.20d}$$

$$I_5 = \int_0^L \frac{(L-x)x dx}{I_x} = L\int_0^L \frac{x dx}{I_x} - \int_0^L \frac{x^2 dx}{I_x} \tag{2.20e}$$

$$I_6 = \int_0^L \frac{(L-x)^2 dx}{I_x} = \int_0^L \frac{L^2 dx}{I_x} - \int_0^L \frac{2Lx dx}{I_x} + \int_0^L \frac{x^2 dx}{I_x} \tag{2.20f}$$

$$I_7 = \int_0^L \frac{dx}{I_x}\int_0^L \frac{x^2 dx}{I_x} - \left[\int_0^L \frac{x dx}{I_x}\right]^2 \tag{2.20g}$$

where I_x and A_x are as a function of x.

After evaluating the above integrals, we can generate the stiffness matrix for any shaped members.

The integrals given in Eq. 2.20 can be evaluated in a systematic way. It can be seen that we need only four basic integrals, i.e. I_0, I_1, I_2 and I_3 and others are a combination of these. Next it is shown how we evaluate these integrals for each type of members.

2.3.1 Stepped Members

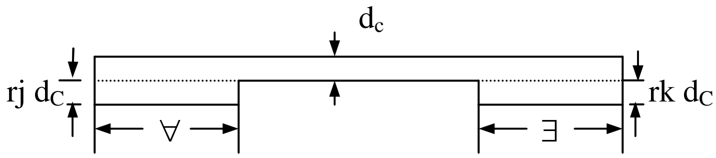


Fig. 2.9 Stepped member.

$$I_c = bd_c^{\,3}/12 \quad , \quad A_c = bd_c$$

$$rj = \frac{(d_L - d_c)}{d_c} \quad , \quad rk = \frac{(d_R - d_c)}{d_c}$$

$$d_x = d_c + r_j d_c$$

$$I_x = \frac{b}{12}\left(d_c + rjd_c\right)^3 = \frac{bd_c^{\,3}}{12}\left(1+rj\right)^3$$

$$I_{xL} = I_C\left(1+rj\right)^3 \quad , \quad A_{xL} = A_C\left(1+rj\right)$$

$$I_{xR} = I_C\left(1+rk\right)^3 \quad , \quad A_{xR} = A_C\left(1+rk\right)$$

Since the member shown in the Fig.2.9 has three parts integration must be made in three steps.

$$I_0 = \int_0^L \frac{1}{A_x}dx = \frac{1}{A_C}\int_0^{\alpha}\frac{dx}{(1+rj)} + \frac{1}{A_C}\int_{\alpha}^{L-\beta}dx + \frac{1}{A_C}\int_{L-\beta}^{L}\frac{dx}{(1+rk)} \qquad (2.21)$$

$$I_1 = \int_0^L \frac{1}{I_x}dx = \frac{1}{I_C}\int_0^{\alpha}\frac{dx}{(1+rj)^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta}dx + \frac{1}{I_C}\int_{L-\beta}^{L}\frac{dx}{(1+rk)^3} \qquad (2.22)$$

$$I_2 = \int_0^L \frac{x}{I_x}dx = \frac{1}{I_C}\int_0^{\alpha}\frac{xdx}{(1+rj)^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta}xdx + \frac{1}{I_C}\int_0^{\beta}\frac{x+L-\beta}{(1+rk)^3}dx \qquad (2.23)$$

$$I_3 = \int_0^L \frac{x^2}{I_x}dx = \frac{1}{I_C}\int_0^{\alpha}\frac{x^2dx}{(1+rj)^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta}x^2dx + \frac{1}{I_C}\int_0^{\beta}\frac{(x+L-\beta)^2}{(1+rk)^3}dx \qquad (2.24)$$

2.3.2 Straight Haunched Members

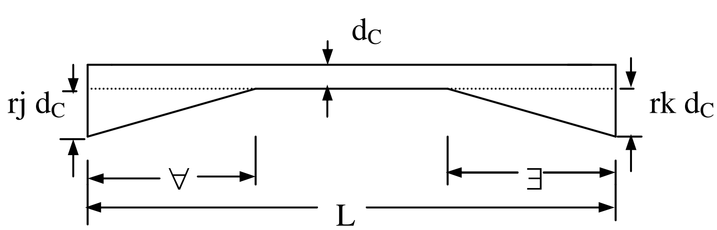


Fig. 2.10 Straight haunched member.

$$I_{xL} = I_C\left[1 + rj(1 - \frac{x}{\alpha})\right]^3$$

$$I_{xR} = I_C\left[1 + rk(1 - \frac{x}{\beta})\right]^3$$

Expression for cross sectional area

$$A_x = A_C(1 + r(1 - x/\alpha))$$

The member has three segments. Therefore integration must be made in three steps, one for each haunch and one for the length having constant depth.

$$I_0 = \int_0^L \frac{1}{A_x}dx = \frac{1}{A_C}\int_0^\alpha dx \Big/ \left[1 + rj(1 - \frac{x}{\alpha})\right] + \frac{1}{A_C}\int_\alpha^{L-\beta} dx + \frac{1}{A_C}\int_{L-\beta}^{L} dx \Big/ \left[1 + rk(\frac{x - L + \beta}{\beta})\right] \quad (2.25a)$$

For the last integral we can change the limits of integral to make more easer

take x-L+∃=∃-u, x=L-u

dx=-du

at x=L-∃, u=∃ and at x=L, u=0

Therefore

$$A_{XR} = A_C(1 + rk(1 - x/\beta)$$

$$I_{XR} = I_C[1 + rk(1 - x/\beta)]^3$$

Using the this expressions and the above integral we get the following

$$I_0 = \int_0^L \frac{1}{A_x}dx = \frac{1}{A_c}\int_0^\alpha dx \Big/ \left[1 + rj(1 - \frac{x}{\alpha})\right] + \frac{1}{A_C}\int_\alpha^{L-\beta} dx + \frac{1}{A_C}\int_0^\beta dx \Big/ \left[1 + rk(1 - \frac{x}{\beta})\right] \quad (2.25b)$$

evaluated integrals are given in Table C1

$$I_1 = \int_0^L \frac{dx}{I_X} = \frac{1}{I_x}\int_0^\alpha \frac{dx}{(1 + rj(1 - x/\alpha))^3} + \frac{1}{I_C}\int_\alpha^{L-\beta} dx + \frac{1}{I_C}\int_0^\beta \frac{dx}{(1 + rk(1 - x/\beta))^3} \quad (2.26)$$

the values for the integrals in Eq. 2.26 may be evaluated by using evaluated integrals 2a and 2b in Table C1.

$$I_2 = \int_0^L \frac{x dx}{I_X} = \frac{1}{I_c}\int_0^\alpha \frac{x dx}{(1 + rj(1 - x/\alpha))^3} + \frac{1}{I_C}\int_\alpha^{L-\beta} x dx + \frac{1}{I_C}\int_0^\beta \frac{(L - x)dx}{(1 + rk(1 - x/\beta))^3}$$

$$I_2 = \int_0^L \frac{x\,dx}{I_X} = \frac{1}{I_C}\int_0^{\alpha}\frac{x\,dx}{(1+rj(1-x/\alpha))^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta} x\,dx + \frac{L}{I_C}\int_0^{\beta}\frac{dx}{(1+rk(1-x/\beta))^3} - \frac{1}{I_C}\int_0^{\beta}\frac{x\,dx}{(1+rk(1-x/\beta))^3} \tag{2.27}$$

The values for the integral in Eq. 2.27 may be obtained by using the evaluated integrals 3a, 3b,and 2a in Table C1.

$$I_3 = \int_0^L \frac{x^2dx}{I_X} = \frac{1}{I_C}\int_0^{\alpha}\frac{x^2dx}{(1+rj(1-x/\alpha))^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta} x^2dx + \frac{1}{I_C}\int_0^{\beta}\frac{(L-x)^2dx}{(1+rk(1-x/\beta))^3}$$

$$\begin{aligned} I_3 = \int_0^L \frac{x^2dx}{I_X} &= \frac{1}{I_C}\int_0^{\alpha}\frac{x^2dx}{(1+rj(1-x/\alpha))^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta} x^2dx + \frac{L^2}{I_C}\int_0^{\beta}\frac{dx}{(1+rk(1-x/\beta))^3} \\ &\quad - \frac{2L}{I_C}\int_0^{\beta}\frac{x\,dx}{(1+rk(1-x/\beta))^3} + \frac{1}{I_C}\int_0^{\beta}\frac{x^2dx}{(1+rk(1-x/\beta))^3} \end{aligned} \tag{2.28}$$

The values for the integral in Eq. 2.28 may be obtained by using evaluated integrals 4a, 4b, 2a and 3a in Table C1.

2.3.3 Parabolic Haunched Members

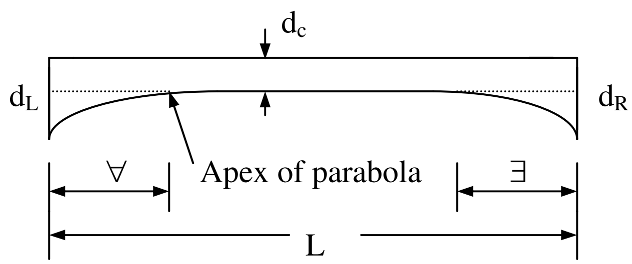


Fig. 2.11 Parabolic haunched member.

Expression for moment of inertia as a function of *x*.

$$I_{xL} = I_C\left[1 + rj(1-\frac{x}{\alpha})^2\right]^3$$

$$I_{xR} = I_C\left[1 + rk(1-\frac{x}{\beta})^2\right]^3$$

I_c is the moment of inertia of the length having constant depth and x is measured from face of support.

Expression for cross sectional area

$$A_x = A_C(1 + r(1 - x/\alpha)^2)$$

For each segments the integration must be made in three steps, one for each haunch and one for the length having constant depth.

$$I_0 = \int_0^L \frac{1}{A_x}dx = \frac{1}{A_c}\int_0^\alpha dx \Big/ \left[1 + rj(1 - \frac{x}{\alpha})^2\right] + \frac{1}{A_C}\int_\alpha^{L-\beta} dx + \frac{1}{A_C}\int_{L-\beta}^{L} dx \Big/ \left[1 + rk(\frac{x - L + \beta}{\beta})^2\right] \quad (2.29a)$$

For the last integral we can change the limits of integral to make more easer

Take x-L+∃=u-∃, x=(L-2∃)+u

dx=du

at x=L-∃, u=∃

and at x=L, u=2∃

Therefore

$$A_{XR} = A_C(1 + rk(1 - x/\beta))$$

$$I_{XR} = I_C[1 + rk(1 - x/\beta)]^3$$

x is measured from right face of member

Using the this expressions and the above integral we get the following

$$I_0 = \int_0^L \frac{1}{A_x}dx = \frac{1}{A_c}\int_0^\alpha dx \Big/ \left[1 + rj(1 - \frac{x}{\alpha})^2\right] + \frac{1}{A_C}\int_\alpha^{L-\beta} dx + \frac{1}{A_C}\int_\beta^{2\beta} dx \Big/ \left[1 + rk(1 - \frac{x}{\beta})^2\right] \quad (2.29b)$$

evaluated integrals are given in Table C2

$$I_1 = \int_0^L \frac{dx}{I_X} = \frac{1}{I_C}\int_0^\alpha \frac{dx}{(1 + rj(1 - x/\alpha)^2)^3} + \frac{1}{I_C}\int_\alpha^{L-\beta} dx + \frac{1}{I_C}\int_\beta^{2\beta} \frac{dx}{(1 + rk(1 - x/\beta)^2)^3} \quad (2.30)$$

The values for the integrals in Eq. 2.30 may be evaluated by using evaluated integrals 2a and 2b in table C2 and 2b in Table C1.

$$I_2 = \int_0^\alpha \frac{xdx}{I_X} = \frac{1}{I_C}\int_0^\alpha \frac{xdx}{(1 + rj(1 - x/\alpha)^2)^3} + \frac{1}{I_C}\int_\alpha^{L-\beta} xdx + \frac{1}{I_C}\int_\beta^{2\beta} \frac{(L - 2\beta + x)dx}{(1 + rk(1 - x/\beta)^2)^3} \quad (2.31)$$

The values for the integral in Eq. 2.31 may be obtained by using the evaluated integrals 3a, 3b,and 2b in Table C2 and 3b in Table C1.

$$I_3 = \int_0^L \frac{x^2dx}{I_X} = \frac{1}{I_C}\int_0^{\alpha} \frac{x^2dx}{(1+rj(1-x/\alpha)^2)^3} + \frac{1}{I_C}\int_{\alpha}^{L-\beta} x^2dx + \frac{1}{I_C}\int_{\beta}^{2\beta} \frac{(L-2\beta+x)^2dx}{(1+rk(1-x/\beta)^2)^3}$$

$$\begin{aligned} I_3 = \int_0^L \frac{x^2dx}{I_X} &= \frac{1}{I_C}\int_0^{\alpha} \frac{x^2dx}{(1+rj(1-x/\alpha)^2)^3} + \frac{1}{I_C}\int_0^{L-\beta} x^2dx + \frac{1}{I_C}\int_{\beta}^{2\beta} \frac{(L-2\beta+x)^2dx}{(1+rk(1-x/\beta)^2)^3} \\ &+ \frac{2}{I_C}\int_{\beta}^{2\beta} \frac{(L-2\beta)xdx}{(1+rk(1-x/\beta)^2)^3} + \frac{1}{I_C}\int_{\beta}^{2\beta} \frac{x^2dx}{(1+rk(1-x/\beta)^2)^3} \end{aligned} \tag{2.32}$$

The values for the integral in Eq. 2.32 may be obtained by using evaluated integrals 4a, 4b, 2b, 3b in Table C2, and 4b in Table C1.

2.4 Element Stiffness Matrix in Global Coordinates

Two distinct types of coordinate systems are used for analysis of structures. i.e. local(element) coordinate defied in the axial and transverse directions for each elements and global(system) coordinates defined in the horizontal and vertical direction for the structure.

The use of element information defined with reference to the local coordinates permitted to work out the stiffness relation in a straightforward way by separating the bending effects from the axial effects.

It is necessary to have all element end displacements corresponding directly to system displacements regardless of the orientation of the element. To do this we need coordinate transformation.

Coordinate transformation is essential step in structural analysis. Physical quantities of interest such as forces and displacements expressed in one coordinate system are required to be transformed to corresponding quantities in another system.

In Fig. 2.12(x_M, y_M) axes (i.e. member axes) are inclined at an angle γ to the (x_S, y_S) axes (i.e. structure axes) .the component of a vector OP (which can be a force or a displacement), can be expressed in either of the system. We can now derive the transformation relationships between the components.

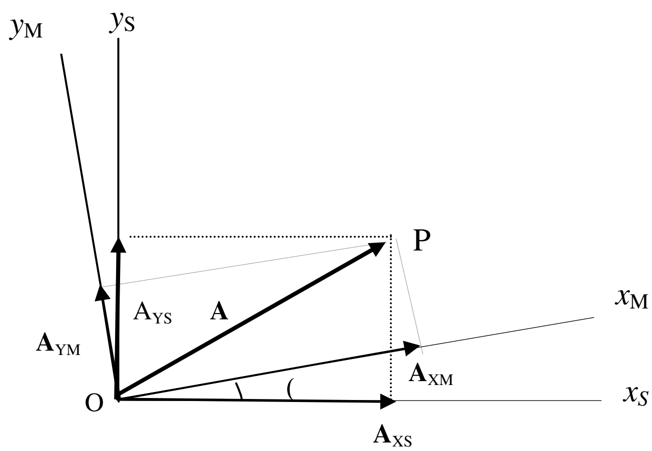


Fig. 2.12 Coordinate transformation.

Component of OP in (x_s, y_s) and (x_M, y_M) coordinate related. From the geometry of Fig. 2.12 we have,

$$A_{XM} = A_{XS} \cos\gamma + A_{YS} \sin\gamma$$

$$A_{YM} = - A_{XS} \cos\gamma + A_{YS} \sin\gamma \tag{2.33a}$$

These, in matrix form, are

$$\begin{Bmatrix} A_{XM} \\ A_{YM} \end{Bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \begin{Bmatrix} A_{XS} \\ A_{YS} \end{Bmatrix} \tag{2.33b}$$

The above may also be stated in concise form by the expression

$$\mathbf{A}_M = \mathbf{R}\,\mathbf{A}_S \tag{2.34}$$

The matrix **R** appearing in Eq. 2.34 is called the rotation matrix. A matrix such as this has the property that its determinant is unity. This result is a very useful (orthogonality) property.

$$\mathbf{R}^{-1} = \mathbf{R}^T$$

Thus solving for $\mathbf{A}_S$ from Eq. 2.34

$$\mathbf{A}_S = \mathbf{R}^{-1}\,\mathbf{A}_M$$

$$\mathbf{A}_S = \mathbf{R}^T\,\mathbf{A}_M \tag{2.35}$$

These relationships can be directly verified from the geometry of Fig. 2.12.Angle γ can be considered as a rotation of a (x_M, y_M, z_M) system about the z axis of an $(x_s, , y_s, z_s)$ system. Thus

z_S and z_M are coincident. If the vector OP has some projection along z_M axis, it is the same as along z_x axis or vice versa. Thus we can write rotation transformation matrix for plane frame member is

$$R = \begin{bmatrix} C_X & C_Y & 0 \\ -C_Y & C_X & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.36}$$

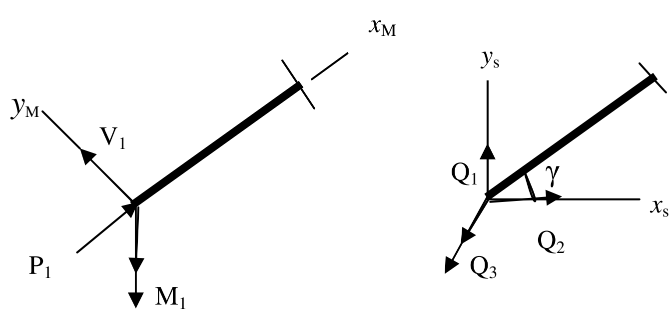
where $C_X = \cos\gamma$ and $C_Y = \sin\gamma$

As an application, consider the transformation required for force vector at node i of a beam element, which is, oriented an arbitrary angle γ to the horizontal as shown in Fig. 2.13a and b. In Fig. 2.13a, the components are along the element axes (x_M,y_M) and in Fig. 2.13b along the global axes (x_S,y_S) by actually resolving the components, we find that

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} P_1 \\ V_1 \\ M_1 \end{Bmatrix} \tag{2.37a}$$

In concise form

$$\mathbf{A}_S = \mathbf{R}^T \mathbf{A}_M \tag{2.37b}$$



(a) Local components (b) global components

Fig. 2.13 Action transformation.

If we apply the above relations to displacement, the components $\mathbf{D}_M$ of displacement D in x_M, y_M, z_M directions may be expressed in terms of the component $\mathbf{D}_S$ in the x_S , y_S , z_S- directions as follows

$$\mathbf{D}_M = \mathbf{R}\, \mathbf{D}_S \tag{2.38}$$

Also, the vector $\mathbf{D}_S$ may be expressed in terms of the vector $\mathbf{D}_M$ by the relationship

$$\mathbf{D}_S = \mathbf{R}^T \mathbf{D}_M \tag{2.39}$$

The action -displacement relationships in the x_M, y_M, z_M directions at the ends of member i may be expressed by the following

$$\mathbf{A}_{Mi} = \mathbf{S}_{Mi}\,\mathbf{D}_{Mi} \qquad (2.40)$$

The member stiffness matrix $\mathbf{S}_{Mi}$ was developed in Sec. 2.3 and presented in Table 2.2.The objective now is to transform this matrix for member axes in to the matrix $\mathbf{S}_{MSi}$ for structure axes.

Writing Eq. 2.40 in partition form

$$\begin{bmatrix} A_{Mj} \\ A_{Mk} \end{bmatrix} = \begin{bmatrix} S_{Mjj} & S_{Mjk} \\ S_{Mkj} & S_{Mkk} \end{bmatrix} \begin{bmatrix} D_{Mj} \\ D_{Mk} \end{bmatrix} \qquad (2.41a)$$

j and k indicates j and k ends of the member. By using the appropriate rotation formulas from Eqs. 2.34 and 2.35 and substituting in to the above

$$\begin{bmatrix} R\ A_{Sj} \\ R\ A_{Sk} \end{bmatrix} = \begin{bmatrix} S_{Mjj} & S_{Mjk} \\ S_{Mkj} & S_{Mkk} \end{bmatrix} \begin{bmatrix} R\ D_{Sj} \\ R\ D_{Sk} \end{bmatrix} \qquad (2.41b)$$

an equivalent form of Eq. 2.41b is

$$\begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} A_{Sj} \\ A_{Sk} \end{bmatrix} = \begin{bmatrix} S_{Mjj} & S_{Mjk} \\ S_{Mkj} & S_{Mkk} \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} D_{Sj} \\ D_{Sk} \end{bmatrix} \qquad (2.41c)$$

to simplify this equation let $\mathbf{T}$ be the rotation transformation matrix for action and displacement at both ends of the member

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & 0 \\ 0 & \mathbf{R} \end{bmatrix} \qquad (2.42)$$

Equation 2.41c rewritten as follows

$$\mathbf{T}\,\mathbf{A}_S = \mathbf{S}_M\,\mathbf{T}\,\mathbf{D}_S \qquad (2.43)$$

$\mathbf{A}_S$ and $\mathbf{D}_S$ consists of the actions and displacements at the ends of the member in the direction of structure axes.

Pre multiplying both sides by $\mathbf{T}^{-1}$ we get

$$\mathbf{A}_S = \mathbf{T}^{-1}\, \mathbf{S}_M\, \mathbf{T}\, \mathbf{D}_S \quad (\text{but } \mathbf{T}^{-1} = \mathbf{T}^T)$$

$$\mathbf{A}_S = \mathbf{T}^T\, \mathbf{S}_M\, \mathbf{T}\, \mathbf{D}_S \tag{2.44}$$

Action displacement equation

$$\mathbf{A}_S = \mathbf{S}_{MS}\, \mathbf{D}_S$$

$\mathbf{S}_{MS}$ is the member stiffness matrix for structure axes can be obtained by

$$\mathbf{S}_{MS} = \mathbf{T}^T\, \mathbf{S}_M\, \mathbf{T} \tag{2.45}$$

Carrying out the matrix multiplications in Eq. 2.45 (using definition of **T** in Eq. 2.42 and the definition of $\mathbf{S}_{Mi}$ in Table 2.2 provides the following for $\mathbf{S}_{MS}$.

Table 2.3 Plane frame member stiffness matrix in structure axes.

$$\begin{bmatrix}
S_{CM1}C_X^2+S_{CM5}C_Y^2 & (S_{CM1}-S_{CM5})\,C_XC_Y & -S_{CM6}C_Y & -S_{CM1}C_X^2-S_{CM5}C_Y^2 & -(S_{CM1}-S_{CM5})\,C_XC_Y & -S_{CM7}C_Y \\
 & S_{CM1}C_Y^2+S_{CM5}C_X^2 & S_{CM6}C_X & -(S_{CM1}-S_{CM5})\,C_XC_Y & -S_{CM1}C_Y^2-S_{CM5}C_X^2 & S_{CM7}C_X \\
 & & S_{CM2} & S_{CM6}C_Y & -S_{CM6}C_X & S_{CM3} \\
 & \text{Sym.} & & S_{CM1}C_X^2+S_{CM5}C_Y^2 & (S_{CM1}-S_{CM5})\,C_XC_Y & S_{CM7}C_Y \\
 & & & & S_{CM1}C_Y^2+S_{CM5}C_X^2 & -S_{CM7}C_X \\
 & & & & & S_{CM4}
\end{bmatrix}$$

Where

$S_{CM1}=E*A_C/(I_0*L)$

$S_{CM2}=E*I_C*I_3/I_7*L$

$S_{CM3}=E*I_C*I_5/I_7*L$

$S_{CM4}=E*I_C*I_6/I_7*L$

$S_{CM5}=E*I_C*(I_3+2I_5+I_6)/(I_7*L^3)$

$S_{CM6}=E*I_C*(I_3+I_5)/(I_7*L^2)$

$S_{CM7}= E*I_C* (I_5+I_6)/(I_7*L^2)$

And I_0, I_1, I_2, I_3, I_5, I_6 and I_7 are defended as Eq. 2.20.

Fixed End Forces

3.1 Derivation of Fixed End Actions

Load on members are taken in to account by calculating the fixed end forces that they produce. Before formulating the load vector we need to find the fixed end action for different non-prismatic member subjected to a general set of interior loads.

3.1.1 Uniformly Distributed Load

The member shown in Fig. 3.1a has varying flexural rigidity and with both ends fixed is subjected to uniformly distributed load. We want to get the fixed end actions.

When a beam is fixed at both ends the slopes of the tangents to the elastic curve at the ends equal zero and, according to conjugate beam principles, they may be expressed as the end reactions of a simply supported beam loaded with M/EI diagram.

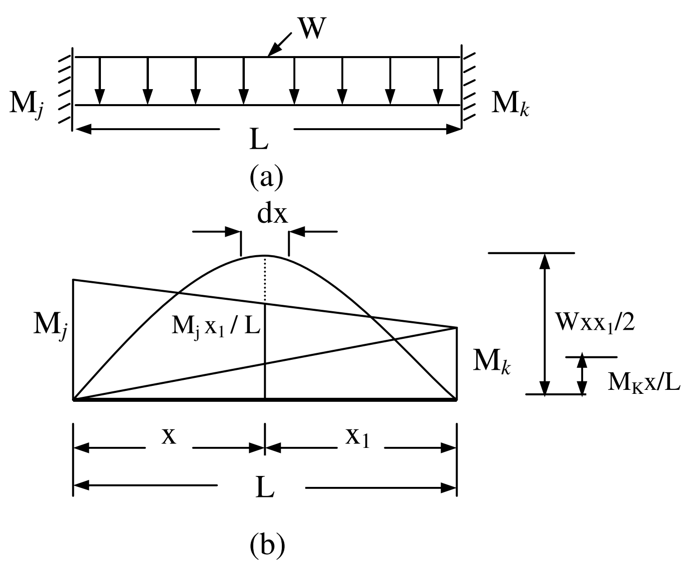


Fig. 3.1 Uniformly distributed load.

Figure 3.1b shows the moment diagram for a fixed end beam with a uniform load of W. The simple beam moment diagram is imposed up on the diagram for fixed end moments, M_j and M_k, and the equation for the reaction or slope at *k* is

$$\sum M/EI_{X=J} = 0$$

$$\int_j^k \frac{M_j}{EI_x}\frac{x_1}{L}xdx + \int_j^k \frac{M_k}{EI_x}\frac{x}{L}xdx + \int_j^k \frac{Wx}{2EI_x}x_1 xdx = 0$$

$$M_j\int_0^L \frac{(L-x)x}{I_x}dx + M_k\int_0^L \frac{x^2}{I_x}dx + \frac{WL}{2}\int_0^L \frac{(L-x)x^2}{I_x}dx = 0 \qquad (3.1)$$

Reaction at *j*

$$\sum M/EI_{X=K} = 0$$

$$\int_k^j \frac{M_j}{EI_x}\frac{x_1}{L}x_1 dx_1 + \int_k^j \frac{M_k}{EI_x}\frac{x}{L}x_1 dx_1 + \int_k^j \frac{Wx}{2EI_x}x_1{}^2 dx_1 = 0$$

substituting x_1=L-x and dx_1=-dx

$$M_j\int_0^L \frac{(L-x)^2}{I_x}dx_1 + M_k\int_0^L \frac{x(L-x)}{I_x}dx + \frac{WL}{2}\int_0^L \frac{(L-x)^2 x}{I_x}dx_1 = 0 \qquad (3.2)$$

The first and the second terms of Eqs. 3.1 and 3.2 are the same as those involved in the derivation of stiffness coefficients.

Solving Eqs. 3.1 and 3.2 simultaneously we get the following

$$M_j = -\frac{WL}{2}(b_2c_1 - b_1c_2)/(a_1b_2 - a_2b_1) \qquad (3.3)$$

$$M_K = -\frac{WL}{2}(a_1c_2 - a_2c_1)/(a_1b_2 - a_2b_1) \qquad (3.4)$$

where

$$a_1 = \int_0^L \frac{(L-x)x}{I_x}dx \quad , \quad b_1 = \int_0^L \frac{x^2}{I_x}dx \quad , \quad c_1 = \int_0^L \frac{(L-x)x^2}{I_x}dx$$

$$a_2 = \int_0^L \frac{(L-x)^2}{I_x}dx \quad , \quad b_2 = \int_0^L \frac{x(L-x)}{I_x}dx \quad , \quad c_2 = \int_0^L \frac{(L-x)^2 x}{I_x}dx$$

Substituting the values of a_1, a_2, b_1, b_2, c_1 and c_2 in to the above equation and simplifying we get the following expressions.

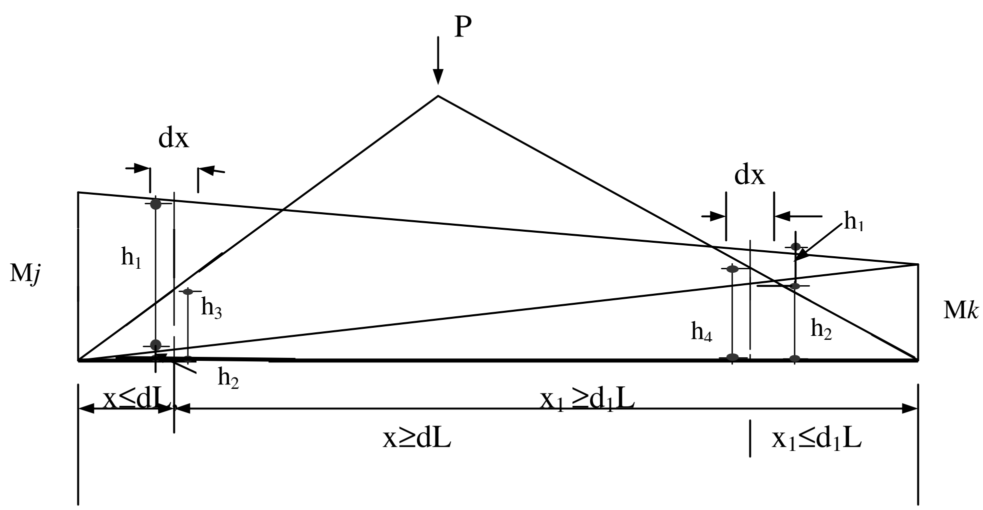
$$M_j = \frac{W}{2}\frac{\left[\int_0^L \frac{x^2}{I_x}dx\right]^2 - \int_0^L \frac{x}{I_x}dx \int_0^L \frac{x^3}{I_x}dx}{\int_0^L \frac{x^2}{I_x}dx \int_0^L \frac{dx}{I_x} - \left[\int_0^L \frac{x}{I_x}dx\right]^2} \tag{3.5}$$

$$M_k = \frac{W}{2}\frac{\left[L^2\int_0^L \frac{x}{I_x}dx\right]^2 + \int_0^L \frac{x^2}{I_x}dx\left[\int_0^L \frac{x^2}{I_x}dx - L\int_0^L \frac{x}{I_x}dx - L^2\int_0^L \frac{dx}{I_x}\right] + \int_0^L \frac{x^3}{I_x}dx\left[L\int_0^L \frac{dx}{I_x} - \int_0^L \frac{x}{I_x}dx\right]}{\int_0^L \frac{x^2}{I_x}dx \int_0^L \frac{dx}{I_x} - \left[\int_0^L \frac{x}{I_x}dx\right]^2} \tag{3.6}$$

All integral terms and their evaluation have been indicated in Sec. 2.3 except the integral $\int_0^L \frac{x^3}{I_x}dx$. For each type of members this can be derived in the same way as before and the evaluated integrals are given in Table C1, C2.

3.1.2 Concentrated Loads

The procedure for determining the fixed end moments for concentrated load is similar to that for uniform load, but the limit of various parts of the integrals will depend up on position of the load, that is, whether it is located with in one of the haunches or in the intervening straight part of the member. Fig. 3.2 shows the moment diagram for a fixed end beam with a concentrated load, P, placed at a distance dL from support *j* and d_1L from support *k*.



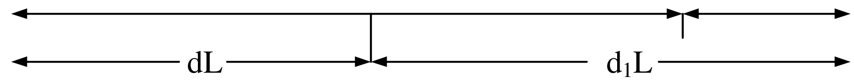


Fig. 3.2 Bending moment for concentrated load.

where $h_1=M_j x_1/L$, $h_2=M_k x/L$, $h_3=Pd_1x$, $h_4=Pdx_1$

The reaction at *k* of M/EI diagram

$$\int_j^k \frac{M_j}{EI_x}\frac{x_1}{L}xdx+\int_j^k \frac{M_k}{EI_x}\frac{x}{L}xdx+\int_j^{dL}\frac{Pd_1x^2}{EI_x}dx+\int_{dL}^k \frac{Pd(L-x)x}{EI_x}dx=0$$

$$M_j\int_0^L \frac{(L-x)x}{I_x}dx+M_k\int_0^L \frac{x^2}{I_x}dx+Pd_1L\int_0^{dL}\frac{x^2}{I_x}dx+PdL\int_{dL}^L \frac{(L-x)x}{I_x}dx=0 \qquad (3.7)$$

The reaction or slope at j of M/EI diagram

$$\int_k^j \frac{M_j}{EI_x}\frac{x_1^{\;2}}{L}dx_1+\int_k^j \frac{M_k}{EI_x}\frac{x}{L}x_1dx_1+\int_k^{d_1L}\frac{Pdx_1^{\;2}}{EI_x}dx_1+\int_{d_1L}^L \frac{Pd_1x_1x}{EI_x}dx_1=0$$

$x_1=L-x$

$$M_j\int_0^L \frac{(L-x)^2}{I_x}dx+M_k\int_0^L \frac{(L-x)x}{I_x}dx+PdL\int_0^{dL}\frac{x_1^{\;2}}{I_x}dx+Pd_1L\int_{dL}^L \frac{(L-x_1)x_1}{I_x}dx_1=0 \qquad (3.8)$$

We have seen before the first two terms of Eq. 3.7 and Eq. 3.8 and their evaluations. The third and fourth terms must be integrated in parts depending up on the location of the load in reference to the haunches.

We can write Eq. 3.7 as

$M_j\, a_1 + M_k\, b_2 + (Pd_1L)\, e1 + (PdL)\, f_1 = 0$

and Eq. 3.8 as

$M_j\, a_2 + M_k\, b_2 + (P\, dL)\, e_1 + (Pd_1L)\, f_2 = 0$

Solving the two equations simultaneously we get the following.

$c_1 = (d_1 * e_1 + d * f_1)$

$c_2 = (d * e_2 + d_1 * f_2)$

Therefore the fixed end moment at the two ends

$$M_j = -PL(b_2c_1 - b_1c_2)/(a_1b_2 - a_2b_1) \qquad (3.9)$$

$$M_K = -PL(a_1c_2 - a_2c_1)/(a_1b_2 - a_2b_1) \qquad (3.10)$$

where a_1, b_1, a_2, b_2 as defined before and

$$e_1 = \int_0^{dL} \frac{x^2}{I_x} dx \quad , \quad f_1 = \int_0^{d_1 L} \frac{(L - x_1)x_1}{I_x} dx_1$$

$$e_2 = \int_0^{d_1 L} \frac{x_1^{\ 2}}{I_x} dx_1 \quad , \quad f_2 = \int_0^{dL} \frac{(L - x)x}{I_x} dx$$

These integrals e1, e2, f_1, and f_2 must be integrated in parts depending up on the location of the load.

3.1.3 Uniform Loads

If we know fixed end moments for concentrated load we can extend it for other types of loads. To use for other types of loads by the use of numerical integration technique consider the following member subjected to loading shown

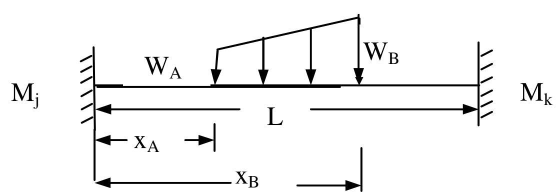
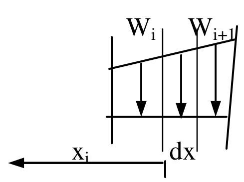


Fig. 3.3 Fixed end member loaded with uniform load.

x_A and x_B are distance of load intensity at point A W_A, and distance of load intensity at point B W_B respectively. First divide the load in to N even numbers



$$dx = \frac{x_B - x_A}{N}, \text{and slope} \quad Sw = \frac{W_B - W_A}{x_B - x_A} \qquad (3.11)$$

$$X_i = X_A + (i-1)*dx \qquad i = 1,2\ldots N+1$$

$$W_{i+1} = W_i + Sw*dx \qquad i = 1,2\ldots N+1$$

$$W_1 = W_A$$

End moments for each load W_i can be obtained by substituting x_i for dL in equation for concentrated load.

Therefore

$$C_{ABi} = P_{AB} * W_i$$

$C_{BAi}=P_{BA}*W_i \quad i=1,N+1$

Using Simpson's rule we get the total value

$$M_j = \frac{dx}{3}\left[C_{AB1} + 4\sum_{j=2}^{N} C_{ABj} + 2\sum_{j=3}^{N-1} C_{ABj} + C_{AB\,N+1}\right] \tag{3.12a}$$

$$M_k = \frac{dx}{3}\left[C_{BA1} + 4\sum_{j=2}^{N} C_{BAj} + 2\sum_{j=3}^{N-1} C_{BAj} + C_{BA\,N+1}\right] \tag{3.12b}$$

After evaluating the fixed end moment, we can calculate the reactions by using equilibrium equations.

After obtaining the fixed end forces using the above derivation we put them in a matrix as follows.

$$A_{MLi} = \begin{bmatrix} (A_{ML})_1 \\ (A_{ML})_2 \\ (A_{ML})_3 \\ (A_{ML})_4 \\ (A_{ML})_5 \\ (A_{ML})_6 \end{bmatrix} \tag{3.13}$$

where, A_{ML1} =force in the x_M direction at the j end

A_{ML2} =force in the y_M direction at the j end

A_{ML3} =moment in the z_M sense at the j end

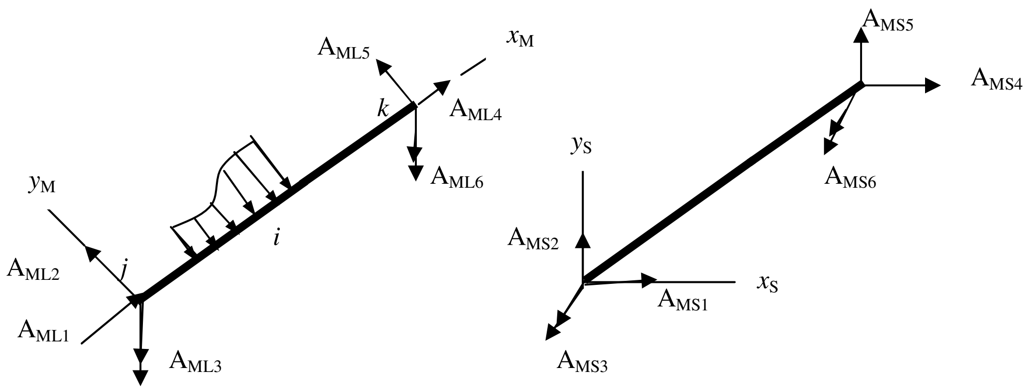
A_{ML4} =force in the x_M direction at the k end

A_{ML5} =force in the y_M direction at the k end

A_{ML6} =moment in the z_M sense at the k end

3.2 Fixed end force in global coordinate

In addition to the transformation of the member stiffness matrix from member axes to structure axes, the rotation of axes concept can also be used for construction of fixed end force $\mathbf{A}_{MSi}$, in structure axes from $\mathbf{A}_{MLi}$



(a) local coordinate (b) global coordinate

Fig. 3.4 Action transformation.

Figure 3.4 (a) shows member *jk* subjected to a general set of interior loads and end force obtained from those loads in member axes.

Figure 3.4 (b) show the equivalent load at *j* and *k*, which receive contributions from member, *i*. Fixed end forces $\mathbf{A}_{\mathrm{MS}i}$, in structure axes, can be obtained using the relation ships

$$\mathbf{A}_{\mathrm{MS}i} = \mathbf{T}^{\mathrm{T}}\,\mathbf{A}_{\mathrm{ML}i} \tag{3.14}$$

Analysis Procedure

4.1General

The basic equation for relating actions and displacement at joints is

$$\mathbf{A}_J = \mathbf{S}_J \, \mathbf{D}_J \tag{4.1}$$

The matrix $\mathbf{S}_J$ can be identified as the joint stiffness matrix relating the action $\mathbf{A}_J$ to the displacement $\mathbf{D}_J$.

It is useful to partition the joint stiffness matrix $\mathbf{S}_J$ in to sub matrices pertaining to the free joint displacement $\mathbf{D}_F$, the support displacements $\mathbf{D}_R$ and their corresponding actions, $\mathbf{A}_F$ and $\mathbf{A}_R$. Thus, the expanded form of Eq. 4.1is

$$\begin{bmatrix} \mathbf{A}_F \\ \mathbf{A}_R \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{FF} & \mathbf{S}_{FR} \\ \mathbf{S}_{RF} & \mathbf{S}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{D}_F \\ \mathbf{D}_R \end{bmatrix} \tag{4.2}$$

At this point it can be seen that the sub matrix $\mathbf{S}_{FF}$ relates the joint actions $\mathbf{A}F$ to the free joint displacement $\mathbf{D}_F$.

Equation 4.2 can be written as two separate matrix equations

$$\mathbf{A}_F = \mathbf{S}_{FF} \, \mathbf{D}_F + \mathbf{S}_{FR} \, \mathbf{D}_R \tag{4.3a}$$

$$\mathbf{A}_R = \mathbf{S}_{RF} \, \mathbf{D}_F + \mathbf{S}_{RR} \, \mathbf{D}_R \tag{4.3b}$$

The first of these equations may be solved for the unknown joint displacement, as follows

$$\mathbf{D}_F = \mathbf{S}_{FF}^{\ -1} \, (\mathbf{A}_F - \mathbf{S}_{FR} \, \mathbf{D}_R) \tag{4.4}$$

Joint displacement due to loads only, becomes

$$\mathbf{D}_F = \mathbf{S}_{FF}^{-1}\,\mathbf{A}_{FC} \tag{4.5}$$

In which $\mathbf{A}_{FC}$ is a vector of combined joint loads (actual and equivalent) corresponding to $\mathbf{D}_F$. Support reactions can be calculated from Eq. 4.3b but if actual or equivalent joint loads applied directly to the joints; they should also be taken in to account. This may be accomplished by adding their negatives to the resultant from Eq. 4.3b as follows

$$\mathbf{A}_R = -\mathbf{A}_{RC} + \mathbf{S}_{RF}\,\mathbf{D}_F + \mathbf{S}_{RR}\,\mathbf{D}_R \tag{4.6}$$

Due to load effect only this equations becomes

$$\mathbf{A}_R = -\mathbf{A}_{RC} + \mathbf{S}_{RF}\,\mathbf{D}_F \tag{4.7}$$

A formula for member end actions caused by joint displacements one member at a time can be obtained by

$$\mathbf{A}_{Mi} = \mathbf{A}_{MLi} + \mathbf{S}_{Mi}\,\mathbf{D}_{Mi} \tag{4.8}$$

All matrices in this expression are referenced to member axes.

4.2 Formulation of Joint Stiffness Matrix

First joint stiffness matrices are formulated in the structure axes for both ends of each members using Table2.3.Then each member contribution is added and the assembled joint stiffness matrix, assuming m members will have the form

$$\mathbf{S}_J = \sum_{i=1}^{M} \mathbf{S}_{MSi} \tag{4.9}$$

$\mathbf{S}_{MSi}$ represents the i^{th} member stiffness matrix with end actions and displacement (for both ends) taken in the directions of structural axes.

Finally the joint stiffness matrix can be rearranged by interchanging rows and columns in such a manner that stiffnesses corresponding to the degrees of freedom are listed first and those corresponding to support restraints are listed second.

The rearranged joint stiffness matrix will have the form

$$\mathbf{S}_J = \begin{bmatrix} \mathbf{S}_{FF} & \mathbf{S}_{FR} \\ \mathbf{S}_{RF} & \mathbf{S}_{RR} \end{bmatrix} \tag{4.10}$$

In this expression the subscripts F and R refers to the free and restrained displacement, respectively.

4.3 Formulation of Load Vector

It is convenient initially to handle the loads at the joints and loads on members separately. The loads applied at joints may be listed in vector $\mathbf{A}_J$ which contains the applied loads corresponding to all possible joint displacements. $\mathbf{A}_J$ are numbered in the same sequence as the joint displacements. The vector $\mathbf{A}_J$ will take the following form

$\mathbf{A}_J = \{(A_J)_1, (A_J)_2, (A_J)_3, \ldots, (A_J)_{3K-2}, (A_J)_{3K-1}, (A_J)_{3K}, \ldots, (A_J)_{3n-2}, (A_J)_{3n-1}, (A_J)_{3n}\}$

The other is load on the member, first fixed end forces are generated using derivation in the previous chapter and then these forces are transformed in to structure axes by the Eq. 3.14. Then the load vector $\mathbf{A}_E$ can be constructed from member contributions, as follows

$$\mathbf{A}_E = -\sum_{i=1}^{M} \mathbf{A}_{MSi} \tag{4.11}$$

$\mathbf{A}_{MSi}$ is a vector of fixed end actions in the direction of structure axes at both ends of member *i*. Using Eq. 3.14, with their sign reversed, represent the incremental portions of $\mathbf{A}_E$ contributed by the i^{th} member.

Thus

$$\begin{aligned}
(\mathbf{A}_E)_{3j-2} &= -\Sigma A_{MS} - C_{Xi} (A_{ML})_{1,i} + C_{Yi} (A_{ML})_{2,i} \\
(\mathbf{A}_E)_{3j-1} &= -\Sigma A_{MS} - C_{Yi} (A_{ML})_{1,i} - C_{Xi} (A_{ML})_{2,i} \\
(\mathbf{A}_E)_{3j} &= -\Sigma A_{MS} - (A_{ML})_{3,i} \\
(\mathbf{A}_E)_{3k-2} &= -\Sigma A_{MS} - C_{Xi} (A_{ML})_{4,i} + C_{Yi} (A_{ML})_{5,i} \\
(\mathbf{A}_E)_{3k-1} &= -\Sigma A_{MS} - C_{Yi} (A_{ML})_{1,i} - C_{Xi} (A_{ML})_{5,i} \\
(\mathbf{A}_E)_{3k} &= -\Sigma A_{MS} - (A_{ML})_{6,i}
\end{aligned} \tag{4.12}$$

The vector $\mathbf{A}_E$ finally will have a form

$\mathbf{A}_E = \{(A_E)_1, (A_E)_2, (A_E)_3, \ldots, (A_E)_{3K-2}, (A_E)_{3K-1}, (A_E)_{3K}, \ldots, (A_E)_{3n-2}, (A_E)_{3n-1}, (A_E)_{3n}\}$

Actual joint load (vector $\mathbf{A}_J$) may be added to equivalent joint loads (vector $\mathbf{A}_E$) to produce the vector of combination joint load $\mathbf{A}_C$ as follows

$$\mathbf{A}_C = \mathbf{A}_J + \mathbf{A}_E \tag{4.13}$$

The next step is rearrangement of combined load vector $\mathbf{A}_C$ in to the form

$$\mathbf{A}_C = \begin{bmatrix} \mathbf{A}_{FC} \\ \mathbf{A}_{RC} \end{bmatrix} \tag{4.14}$$

where $\mathbf{A}_{FC}$ is joint load vector corresponding to free joint displacement

$\mathbf{A}_{RC}$ is joint load vector corresponding to restrained joint displacement

Thus the combined loads corresponding to free joint displacement are separated from those corresponding to support restraint.

4.4 Calculation of results

In the final phase of the analysis, matrices generated in previous steps are used to find unknown displacement $\mathbf{D}_F$, support reaction $\mathbf{A}_R$ and member end-actions $\mathbf{A}_M$. Unknown displacement $\mathbf{D}_F$ are obtained from Eq. 4.5

$$\mathbf{D}_F = \mathbf{S}_{FF}^{-1}\, \mathbf{A}_{FC}$$

Next, the support reactions are found by substituting the matrices $\mathbf{A}_{RC}$, $\mathbf{S}_{RF}$,and $\mathbf{D}_F$ in to Eq4.7

$$\mathbf{A}_R = -\mathbf{A}_{RC} + \mathbf{S}_{RF}\, \mathbf{D}_F$$

Finally member end actions can be calculated by using Eq. 4 .8.We can write Eq. 4.8 as follows

$$\mathbf{A}_{Mi} = \mathbf{A}_{MLi} + \mathbf{S}_{Mi}\, \mathbf{T}_i\, \mathbf{D}_{Ji}$$

Substituting $\mathbf{S}_{Mi}$ from Table 2.2 and $\mathbf{T}_i$ from Eq. 2.40 in to this expression yields the following

$$(\mathbf{A}_M)_{1,j} = (A_{ML})_{1,i} + S_{CM1}\{[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\}$$

$$(\mathbf{A}_M)_{2,j} = (A_{ML})_{2,i} + S_{CM5}\{-[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\}$$

$$(\mathbf{A}_M)_{3,j} = (A_{ML})_{3,i} + S_{CM6}\{-[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\} + S_{CM2}(D_J)_{J3}+S_{CM3}(D_J)_{K3}$$

$$(\mathbf{A}_M)_{4,j} = (A_{ML})_{4,i} - S_{CM1}\{[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\}$$

$$(\mathbf{A}_M)_{5,j} = (A_{ML})_{5,i} - S_{CM5}\{-[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\}$$

$$(\mathbf{A}_M)_{6,j} = (A_{ML})_{6,i} + S_{CM7}\{-[(D_J)_{J1}-(D_J)_{K1}]C_{Xi}+[(D_J)_{J2}-(D_J)_{K2}]C_{Yi}\} + S_{CM3}(D_J)_{J3}+S_{CM4}(D_J)_{K3}$$

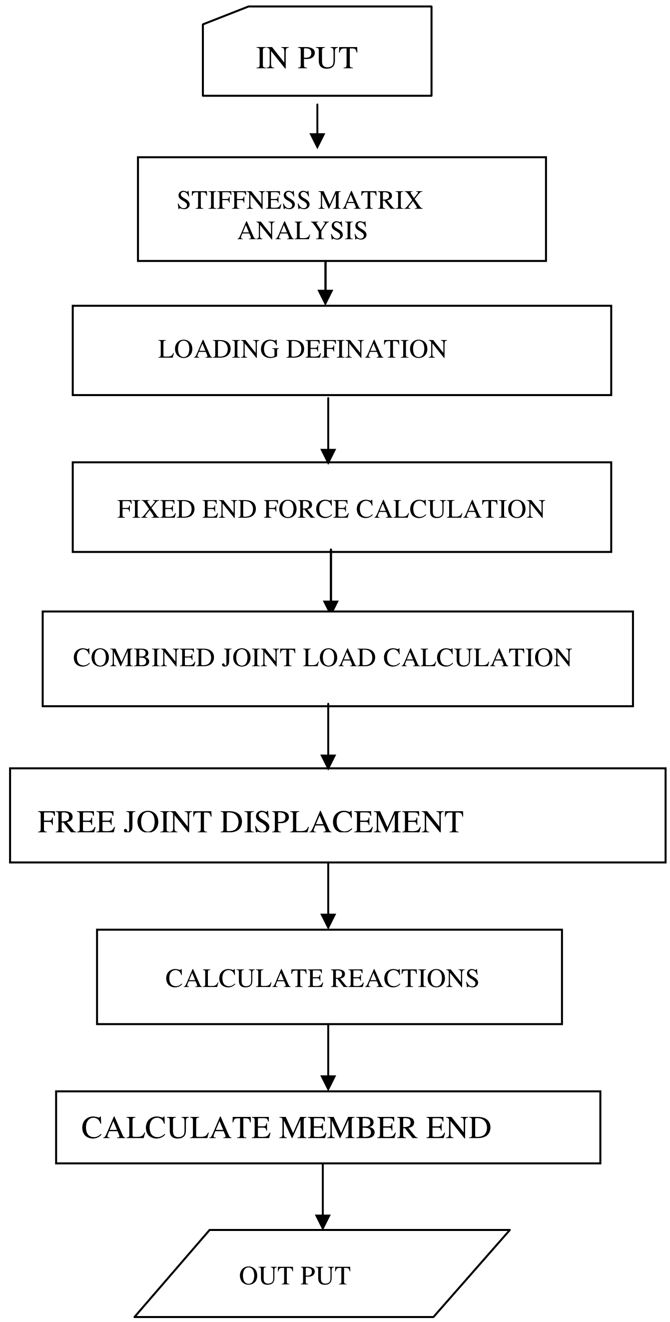
(4.15)

Program Logic

5.1General

In Chapter four it is indicated the analysis procedure for non-prismatic members. A computer program is developed to do this in a systematic way to make it efficient. This program can analyze any two-dimensional frames with non-prismatic members of which the shape is indicated in Table 2.1 for a wide range of external loads (if the input data is properly entered).

Computer language that I have used for coding is Fortran-90 .The flow chart indicated is Fortran-90 oriented charts and is given in appendix A. Next it is shown the programming steps.


IN PUT
STIFFNESS MATRIX ANALYSIS
LOADING DEFINATION
FIXED END FORCE CALCULATION
COMBINED JOINT LOAD CALCULATION
FREE JOINT DISPLACEMENT
CALCULATE REACTIONS
CALCULATE MEMBER END
OUT PUT

5.2 Preparation of input data

The manner in which a program is written depends to some extent up on the form in which the data are supplied. The program is planned to operate on input data provided in the following data group, and generally, in the same order

(i) Program control data

(ii) Structural data

(iii) Load data

Only one set of control and structural data is required for each structure, but there may be any number of sets of load data. The input data required for plane frame with non-prismatic members are given in Table 5.1.

Table 5.1 Preparation of input data for plane frames with non-prismatic members

no	Data	Number of lines	Items on data lines
1	Control data	1	ISN NLS
2	a- structural parameters	1	M NJ NR NRJ E
	b-joint coordinates	NJ	J X (J) Y (J)
	c-member information	M	I JJ (I) JK (I) MT (I) AX(I) ZI(I) dL(I) dC(I) dR(I) ELL(I) ELR(I)
	d-joint restraints list	NRJ	K JRL(3K-2) JRL(3K-1) JRL(3K)
3	a- load parameter	1	NLJ NLM
	b-action at joint	NLJ	K AJ(3K-2) AJ(3K-1) AJ(3K)
	c-load on member	NLM	I NLUM NLPM
		NLUM	X_a W_a X_b W_b
		NLPM	X_c P

(i) **Program control data**

The program control data consists of two parameters that are most useful. These parameters are structure number ISN, and number of loading systems NLS. The number of loading systems NLS indicates the number of sets of load data that accompany a given set of structural data.

(ii) **Structural data**

The structural data group generally consists of the following main subgroups.

(a) *Structural parameters*

Which contains the number of members M, number of joints NJ, the number of support restraints NR, the number of restrained joints NRJ, and the elastic modulus E of the material.

(b) *Joint coordinates*

Each data line for joint coordinates contains a joint number J, the *x* coordinate X (J) of the joint and the *y* coordinate Y (J) of the joint.

(c) *Member information*

On each of the lines containing member information the member number I is listed first, followed by the joint J number JJ (I) and joint K number JK (I) for the two ends of the member. The others are member type MT (I)(1 for stepped, 2 for straight haunched and 3 for parabolic haunched members), cross sectional area of least section AX (I), moment of inertia of least cross section ZI(I), depth of member at left side of the member dL (I), smaller depth of the member dC (I), depth of member at right side of the member dR (I), left haunch length ELL (I) and right haunch length ELR(I).

(d) *Joint restraint list*

Each of the lines in this group (a total of NRJ lines) contains a joint number K and two code numbers, which indicate the conditions of restraint at that joint (0 for active degrees of freedoms and 1 for restrained degrees of freedoms). The term JRL (3K-1) denotes the restraint against translation in x direction, the term JRL (3K-2) denotes the restraint against translation in *y* direction, the term JRL (3K) denotes the restraint against rotation in *z* sense at joint *k*.

(iii) **Load data**

The load data is presented as the last set of data. It contains the following

(a) *Load parameter*

It contains two items. These are the number of loaded joints NLJ and the number of loaded members NLM.

(b) *Action at joint*

Each line of joint load data (a total of NLJ lines) contains a joint number K and the three actions applied at that joint. The terms AJ (3K-2), AJ (3K-1), and AJ (3K) denote the applied forces in the *x* and *y* directions and a moment in the *z* sense.

(c) *Load on member*

Each line load on member data (a total of NLM) contains member number I, number of uniform load on the member NLUM and number of concentrated load on the same member NLPM.

In addition it contains

-Uniform load data (present or not)

Each line (a total of NLUM) contains left load position X_{a}, load intensity at this position Wa , right load position X_b , load intensity at this position W_b .

-Concentrated load data (present or not)

Each line (a total of NLPM) contains load position X_c and concentrated load value P

Numerical Examples

6.1 Example One

Next numerical examples will be solved to demonstrate how to the use the program and the efficiency of the method.

It is required to solve the following concrete frame with the loading indicated

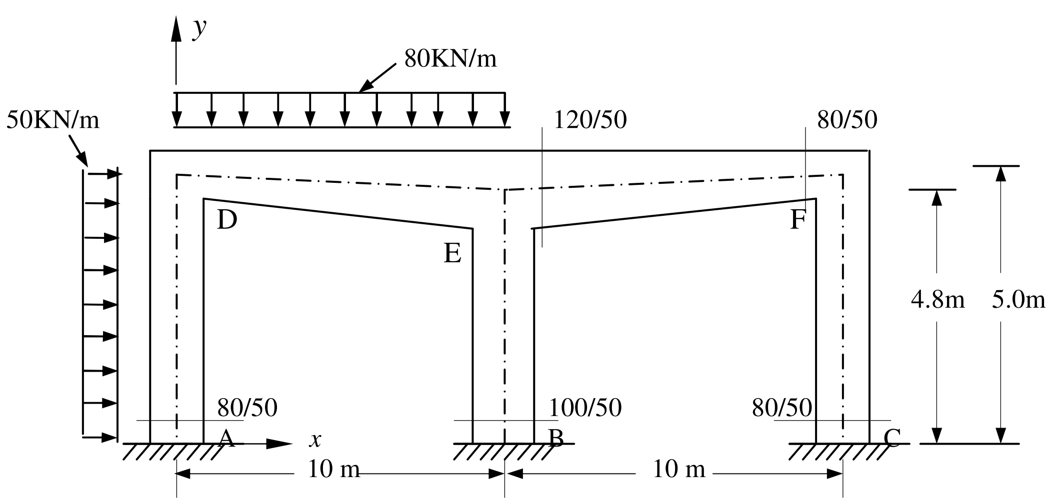


Fig. 6.1 Example 1

Moments at each end that are obtained from three analysis techniques are summarized in the following table.

Table 6.1 Summary of moments for example one

Joint	Member	Manual	SAMT	SAP 2000
A	AD	12.59	13.74	19.20
D	DA	-368.53	-367.92	-366.38
	DE	368.53	367.92	366.38
E	ED	-757.81	-759.07	-718.25
	EB	563.39	565.69	521.89
	EF	194.42	193.38	196.37
B	BE	367.75	370.70	318.90
F	FE	19.73	18.91	15.39
	FC	-19.73	-18.91	-15.39
C	CF	30.73	31.99	31.75

The next diagram shows the bending moment diagram for this frame

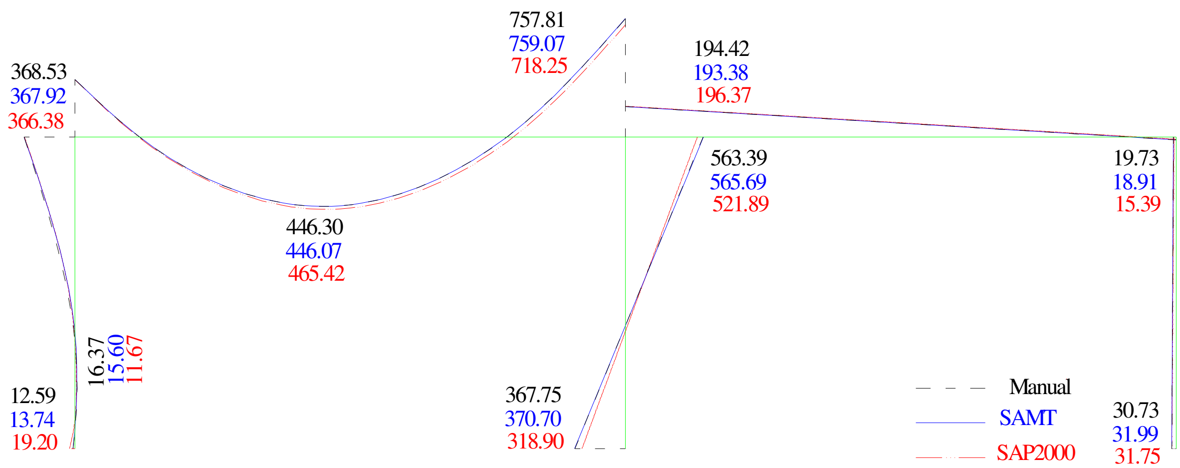


Fig. 6.2 Bending moment diagram for example one.

The required input data for this frame is shown below

```
1 1
5 6 9 3  24821130
1  0.0  0.0
2  0.0  5.0
3 10.0  4.8
4 10.0  0.0
5 20.0  5.0
6 20.0  0.0
1 1 2 1  100000.0  0.02133  0.0   0.0    .80  .80   .80
2 2 3 2  100000.0  0.02133  0.0  10.002  .80  .80  1.20
3 4 3 1  125000.0  0.04167  0.0   0.0   1.00 1.00  1.00
4 3 5 2  100000.0  0.02133 10.002 0.0   1.20  .80   .80
5 6 5 1  100000.0  0.02133  0.0   0.0    .80  .80   .80
1 1 1 1
4 1 1 1
6 1 1 1
0 2
1 1 0
0.0 50.0 5.0 50.0
2 1 0
0.0 80.0  10.00  80.0
```

+---+

ENTER YOUR INPUT FILE NAME fe1

ENTER ECHO OF INPUT DATA FILE NAME fe1in

ENTER YOUR OUTPUT FILE NAME fe1out

```
 *****     **** **    ** ******
******    ***** ***  *** ******
***      ** **  **** ****   **
*****   **  **  *********   **
 ***** ******** ** *** **   **
  *** ******** ** * **   **
****** **   ** **    **   **
*****  **   ** **    **   **
```

(Version 2002)

ECHO OF INPUT DATA FILE NAME: fe1in

YOUR OUTPUT DATA FILE NAME : fe1out

********* A N A L Y S I S   C O M P L E T E **********

```
*****************************************************
*****     E C H O  0 F  I N P U T  D A T A     *****
*****************************************************
```

SAMT 2001 FILE :fe1in KN-m units

DATE : 05:01:2002 TIME : 15:57

STRUCTURE NO. 1 PLANE FRAME

NUMBER OF LOADING SYSTEMS = 1

STRUCTURAL PARAMETERS

M	N	NJ	NR	NRJ	E
5	9	6	9	3	24821130.0

JOINT COORDINATES

JOINT	X	Y
1	0.00	0.00
2	0.00	5.00
3	10.00	4.80
4	10.00	0.00
5	20.00	5.00
6	20.00	0.00

MEMBER INFORMATION

MEMBER	MTP	JNT-1	JNT-2	AX	ZI	LENGTH	AL	BL	hJ	hC	hK
1	1	1	2	100E3	0.021	5.000	0.000	0.000	0.800	0.800	0.800
2	2	2	3	100E3	0.021	10.002	0.000	10.002	0.800	0.800	1.200
3	1	4	3	125E3	0.042	4.800	0.000	0.000	1.000	1.000	1.000
4	2	3	5	100E3	0.021	10.002	10.002	0.000	1.200	0.800	0.800
5	1	6	5	100E3	0.021	5.000	0.000	0.000	0.800	0.800	0.800

JOINT RESTRAINTS

JOINT JR1 JR2 JR3

1 1 1 1

4 1 1 1

6 1 1 1

LOAD CASE 1

NLJ NLM

0 2

MEMBER LOAD

MEMBER	LD NO	DISTANCE-A	VALUE-A	DISTANCE-B	VALUE-B
1	1	0.000	50.000	5.000	50.000
2	1	0.000	80.000	10.000	80.000

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	P-J	V-J	M-J	P-K	V-K	M-K
1	0.000	125.000	104.167	0.000	125.000	-104.167
2	0.000	367.737	516.070	0.000	432.263	-839.566

```
************************************************************

*****        F R A M E  O U T P U T  D A T A        *****

************************************************************


SAMT 2001          FILE :fe1out          KN-m units

DATE : 05:01:2002     TIME : 15:57



     LOAD CASE   1

---------------------------------------------------------


    J O I N T  D I S P L A C E M E N T S

    JOINT      Ux          Uy          Rz

       1   0.00000E+00   0.00000E+00   0.00000E+00

       2   0.65236E-03  -0.72004E-09  -0.81842E-03

       3   0.65236E-03  -0.71792E-09   0.45246E-03

       4   0.00000E+00   0.00000E+00   0.00000E+00

       5   0.65236E-03   0.43443E-10  -0.24034E-03

       6   0.00000E+00   0.00000E+00   0.00000E+00

 ---------------------------------------------------------


     M E M B E R  E N D - A C T I O N S
```

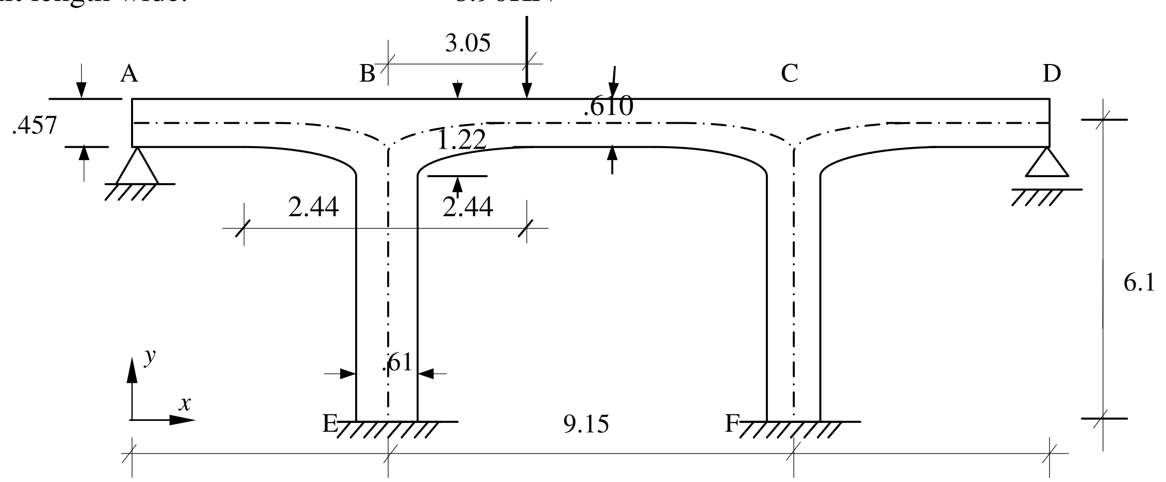
MEMBER	LOC	AXIAL F	SHEAR	MOMENT
1				
	0.00	357.444	54.165	13.738
	5.00	-357.444	195.835	-367.915
2				
	0.00	178.105	360.972	367.915
	10.00	-178.105	439.028	-759.076
3				
	0.00	464.054	195.083	370.703
	4.80	-464.054	-195.083	565.695
4				
	0.00	13.150	21.224	193.381
	10.00	-13.150	-21.224	18.905
5				
	0.00	-21.566	2.618	31.993
	5.00	21.566	-2.618	-18.905

S U P P O R T R E A C T I O N S

JOINT	Fx	Fy	Mz
1	-54.165	357.444	13.738
4	-195.083	464.054	370.703
6	-2.618	-21.566	31.993

6.2 Example Two

It is required to solve the following concrete frame with the loading indicated. Consider a slice of unit length wide.



All dimensions are in m

Fig. 6.3 Example 2

Moments at each end that are obtained from three analysis techniques are summarized in the following table.

Table 6.2 Summary of moments for example two

Joint	Member	Manual	SAMT	SAP 2000
B	BA	-4.19	-4.12	-5.17
	BC	10.74	10.66	11.08
	BE	-6.54	-6.54	-5.91
E	EB	-3.27	-3.26	-2.88
C	CB	- 6.37	-6.43	-6.88
	CF	3.88	3.93	3.65
	CD	2.49	2.49	3.23
F	FC	1.93	1.96	1.78

The next diagram shows the bending moment diagram for frame example two.

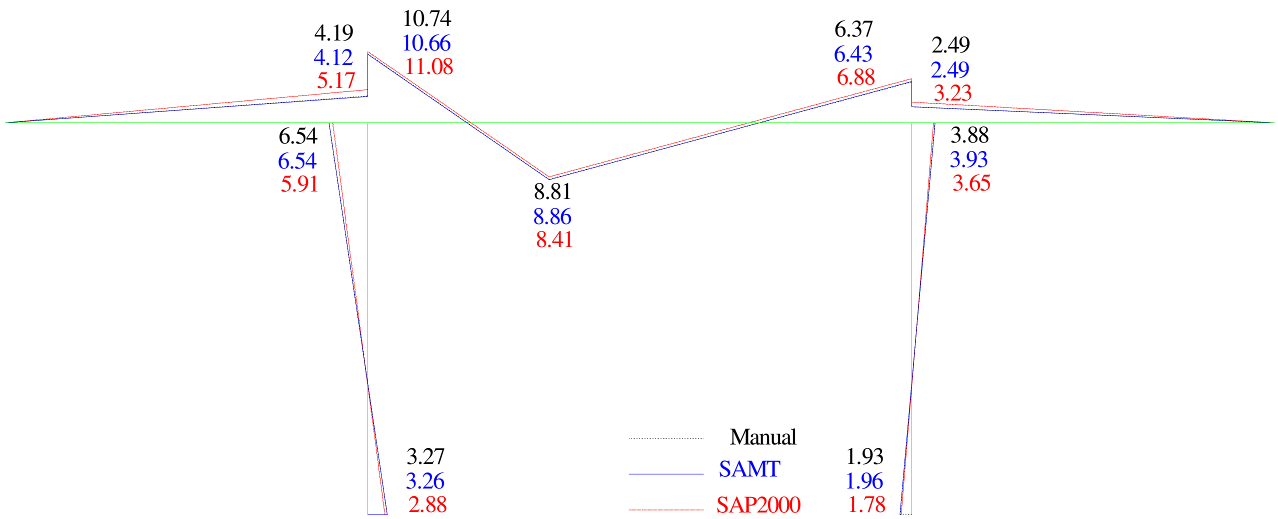


Fig. 6.4 Bending moment diagram for example two.

The required input data for this frame is shown below

```
2 1
5 6 9 4  24821130
1  0.0  6.1
2  6.1  6.1
3  6.1  0.0
4 15.25 6.1
5 15.25 0.0
6 21.35 6.1
1 1 2 3  0.457  0.007954  0.0  2.44 0.457 .457 1.22
2 3 2 1  0.610  0.018915  0.0  0.0  0.61  .61   .61
3 2 4 3   .610  0.018915  2.44 2.44 1.22  .61  1.22
4 5 4 1   .610  0.018915  0.0   .00  .61  .61   .61
5 4 6 3   .457  0.007954  2.44  .0  1.22  .457  .457
1 1 1 0
3 1 1 1
5 1 1 1
6 0 1 0
0 1
3 0 1
3.05 8.90
```

```
        +-----------------------------------------------------+


ENTER YOUR INPUT FILE NAME  fe2

ENTER ECHO OF INPUT DATA FILE NAME  fe2in
```

```
ENTER YOUR OUTPUT FILE NAME  fe2out

                        *****      ****  **      **  ******
                       ******     *****  ***    ***  ******
                       ***       **  **  **** ****     **
                       *****    **   **  *********     **
                        *****  ********  ** *** **     **
                          ***  ********  **  *  **     **
                       ******  **    **  **      **    **
                       *****   **    **  **      **    **

                              ( Version  2002 )
 ECHO OF INPUT  DATA FILE NAME:  fe2in
 YOUR OUTPUT DATA FILE NAME   :  fe2out


   *********  A N A L Y S I S   C O M P L E T E  **********
```

```
**********************************************************

*****     E C H O  O F  I N P U T  D A T A       *****

**********************************************************


 SAMT 2001           FILE :fe2in            KN-m units

 DATE : 05:01:2002     TIME : 15:58

-----------------------------------------------------------

        STRUCTURE NO. 2  PLANE FRAME

        NUMBER OF LOADING SYSTEMS = 1


         STRUCTURAL PARAMETERS

           M   N  NJ  NR  NRJ          E

           5   9   6   9   4    24821130.0

         JOINT COORDINATES
```

JOINT	X	Y
1	0.00	6.10
2	6.10	6.10
3	6.10	0.00
4	15.25	6.10
5	15.25	0.00
6	21.35	6.10

MEMBER INFORMATION

MEMBER	MTP	JNT-1	JNT-2	AX	ZI	LENGTH	AL	BL	hJ	hC	hK
1	3	1	2	0.457	0.008	6.100	0.000	2.440	0.457	0.457	1.220
2	1	3	2	0.610	0.019	6.100	0.000	0.000	0.610	0.610	0.610
3	3	2	4	0.610	0.019	9.150	2.440	2.440	1.220	0.610	1.220
4	1	5	4	0.610	0.019	6.100	0.000	0.000	0.610	0.610	0.610
5	3	4	6	0.457	0.008	6.100	2.440	0.000	1.220	0.457	0.457

JOINT RESTRAINTS

JOINT	JR1	JR2	JR3
1	1	1	0
3	1	1	1
5	1	1	1
6	0	1	0

LOAD CASE 1

NLJ NLM

0 1

MEMBER LOAD

MEMBER	LD NO	DISTANCE-A	VALUE-A	DISTANCE-B	VALUE-B
3	1	3.050	8.900		

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	P-J	V-J	M-J	P-K	V-K	M-K
3	0.000	7.010	15.784	0.000	1.890	-5.932

****

***** F R A M E O U T P U T D A T A *****

****

```
SAMT 2001           FILE :fe2out            KN-m units

DATE : 05:01:2002     TIME : 15:58


LOAD CASE   1

----------------------------------------------------------


   J O I N T  D I S P L A C E M E N T S

   JOINT       Ux            Uy            Rz

      1    0.00000E+00    0.00000E+00    0.17637E-04

      2    0.30408E-06   -0.28487E-05   -0.21323E-04

      3    0.00000E+00    0.00000E+00    0.00000E+00

      4   -0.21282E-06   -0.11737E-05    0.12828E-04

      5    0.00000E+00    0.00000E+00    0.00000E+00

      6   -0.21282E-06    0.00000E+00   -0.10776E-04

-----------------------------------------------------------


   M E M B E R  E N D - A C T I O N S

   MEMBER     LOC      AXIAL F     SHEAR      MOMENT

     1-------------------------------------------------

          0.00      -0.641      -0.675       0.000

          6.10       0.641       0.675      -4.117

     2-------------------------------------------------

          0.00       7.071      -1.607      -3.259
```

6.10 -7.071 1.607 -6.542

3--

0.00 0.966 6.396 10.659

9.15 -0.966 2.504 -6.428

4--

0.00 2.913 0.966 1.959

6.10 -2.913 -0.966 3.933

5--

0.00 0.000 0.409 2.494

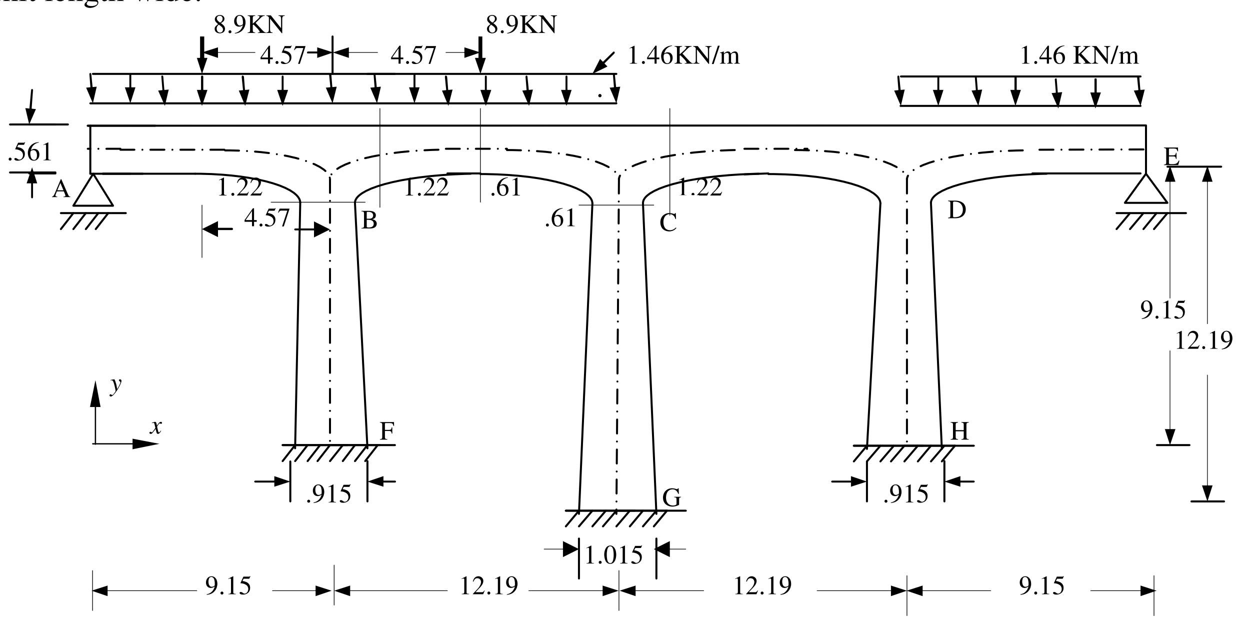
6.10 0.000 -0.409 0.000

S U P P O R T R E A C T I O N S

JOINT	Fx	Fy	Mz
1	-0.641	-0.675	0.000
3	1.607	7.071	-3.259
5	-0.966	2.913	1.959
6	0.000	-0.409	0.000

6.3 Example Three

It is required to solve the following concrete frame with the loading indicated consider a slice of unit length wide.



All dimensions are in m

Fig. 6.5 Example 3

The next table Shows moments that are obtained from three analysis techniques

Table 6.3 Summary of moments for example three

Joint	Member	Manual	SAMT	SAP2000
B	BA	-50.47	-50.03	-53.11
	BC	51. 94	51.55	53.41
	BF	-1.47	-1.51	-.31
F	FB	.47	0.41	0.76
C	CB	-17.51	-17.83	-15.54
	CD	6.89	7.09	6.99
	CG	10.63	10.74	8.54

G	GC	8.86	8.86	6.58
D	DC	-4.45	-4.34	-6.43
	DE	13.45	13.33	14.21
	DH	-9.0	-8.99	-7.78
H	HD	-4.62	-4.61	-4.02

The next diagram shows the bending moment diagram for the three analysis techniques.

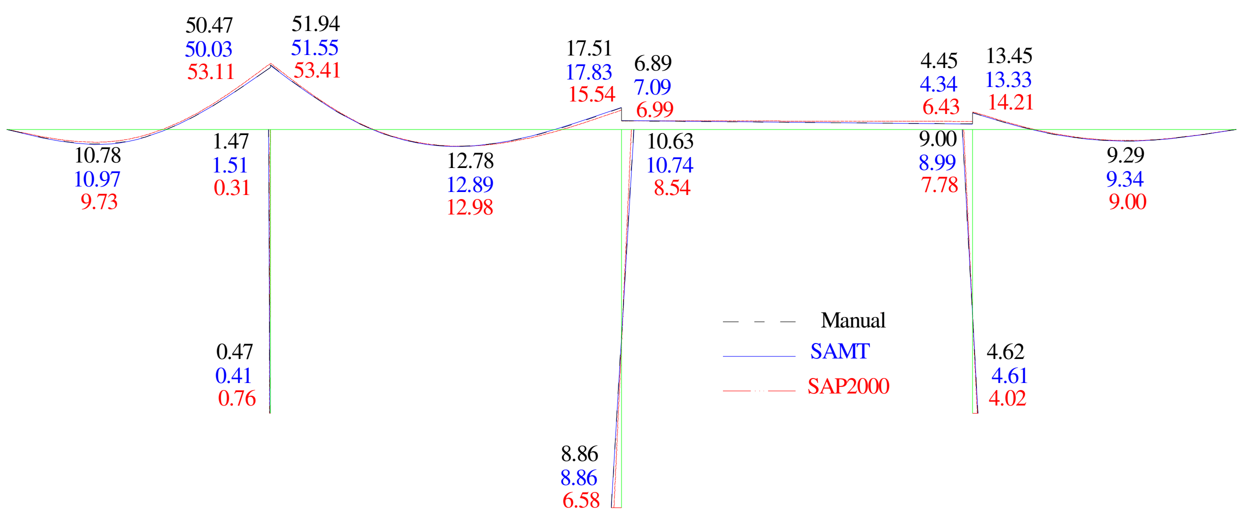


Fig. 6.6 Bending moment diagram for example three.

The required input data for this frame is shown below

```
3 1
7 8 11 5 24821130
1  0.0 9.15
2 9.15 9.15
3 9.15  0.0
4 21.34  9.15
5 21.34 -3.04
6 33.53  9.15
7 33.53  0.0
8 42.68  9.15
1 1 2 3 .561 0.014713  0.0  4.57   .561  .561 1.22
2 3 2 2 .610 0.018915 9.15  0.0    .915  .61   .61
3 2 4 3 .61  0.018915 6.095 6.095 1.22   .61  1.22
4 5 4 2 .61  0.018915 12.19 0.0   1.015  .61   .61
5 4 6 3 .61  0.018915 6.095 6.095 1.22   .61  1.22
6 7 6 2 .61  0.018915 9.15  0.0    .915  .61   .61
7 6 8 3 .561 0.014713 4.57  0.0   1.22  .561  .561
1 0 1 0
3 1 1 1
5 1 1 1
7 1 1 1
8 0 1 0
0 3
1 1 1
0.0 1.46 9.15 1.46
4.57 8.90
3 1 1
0.0 1.46 12.19  1.46
4.57 8.90
7 1 0
0.0 1.46 9.15 1.46
```

```
      +--------------------------------------------------+

ENTER YOUR INPUT FILE NAME  fe3

ENTER ECHO OF INPUT DATA FILE NAME  fe3in

ENTER YOUR OUTPUT FILE NAME  fe3out

                        *****      ****  **     **  ******
                       ******     *****  ***   ***  ******
                       ***       **  **  **** ****    **
                       *****    **   **  *********    **
                        *****  ********  ** *** **    **
                          ***  ********  **  *  **    **
                       ******  **    **  **     **    **
                       *****   **    **  **     **    **

                             ( Version  2002 )
 ECHO OF INPUT  DATA FILE NAME:  fe3in
 YOUR OUTPUT DATA FILE NAME   :  fe3out



    *********  A N A L Y S I S   C O M P L E T E  **********
```

****

*****   E C H O O F I N P U T D A T A   *****

****

SAMT 2001   FILE :fe3in   KN-m units

DATE : 05:01:2002   TIME : 15:59

STRUCTURE NO. 3 PLANE FRAME

NUMBER OF LOADING SYSTEMS = 1

STRUCTURAL PARAMETERS

M	N	NJ	NR	NRJ	E
7	13	8	11	5	24821130.0

JOINT COORDINATES

JOINT	X	Y
1	0.00	9.15
2	9.15	9.15
3	9.15	0.00
4	21.34	9.15
5	21.34	-3.04
6	33.53	9.15

7	33.53	0.00
8	42.68	9.15

MEMBER INFORMATION

MEMBER	MTP	JNT-1	JNT-2	AX	ZI	LENGTH	AL	BL	hJ	hC	hK
1	3	1	2	0.561	0.015	9.150	0.000	4.570	0.561	0.561	1.220
2	2	3	2	0.610	0.019	9.150	9.150	0.000	0.915	0.610	0.610
3	3	2	4	0.610	0.019	12.190	6.095	6.095	1.220	0.610	1.220
4	2	5	4	0.610	0.019	12.190	12.190	0.000	1.015	0.610	0.610
5	3	4	6	0.610	0.019	12.190	6.095	6.095	1.220	0.610	1.220
6	2	7	6	0.610	0.019	9.150	9.150	0.000	0.915	0.610	0.610
7	3	6	8	0.561	0.015	9.150	4.570	0.000	1.220	0.561	0.561

JOINT RESTRAINTS

JOINT	JR1	JR2	JR3
1	0	1	0
3	1	1	1
5	1	1	1
7	1	1	1
8	0	1	0

LOAD CASE 1

NLJ	NLM
0	3

MEMBER LOAD

MEMBER	LD NO	DISTANCE-A	VALUE-A	DISTANCE-B	VALUE-B
1	1	0.000	1.460	9.150	1.460
1	1	4.570	8.900		
3	1	0.000	1.460	12.190	1.460
3	1	4.570	8.900		
7	1	0.000	1.460	9.150	1.460

ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE TO LOADS

MEMBER	P-J	V-J	M-J	P-K	V-K	M-K
1	0.000	8.813	14.125	0.000	13.446	-35.367
3	0.000	15.363	43.992	0.000	11.334	-33.009
7	0.000	7.696	16.723	0.000	5.663	-7.420

****

*****   F R A M E O U T P U T D A T A   *****

****

SAMT 2001 FILE :fe3out KN-m units

DATE : 05:01:2002 TIME : 15:59

LOAD CASE 1

J O I N T D I S P L A C E M E N T S

JOINT	Ux	Uy	Rz
1	0.33793E-04	0.00000E+00	-0.66358E-04
2	0.33793E-04	-0.16574E-04	-0.11594E-04
3	0.00000E+00	0.00000E+00	0.00000E+00
4	0.33716E-04	-0.59865E-05	0.42301E-04
5	0.00000E+00	0.00000E+00	0.00000E+00
6	0.34656E-04	-0.38764E-05	-0.38445E-04
7	0.00000E+00	0.00000E+00	0.00000E+00
8	0.34656E-04	0.00000E+00	0.72328E-04

M E M B E R E N D - A C T I O N S

MEMBER	LOC	AXIAL F	SHEAR	MOMENT
1---				
	0.00	0.000	5.667	0.000
	9.15	0.000	16.592	-50.031
2---				
	0.00	33.820	-0.121	0.406
	9.15	-33.820	0.121	-1.514
3---				
	0.00	0.121	17.228	51.545
	12.19	-0.121	9.470	-17.832
4---				
	0.00	9.696	1.608	8.855

12.19	-9.696	-1.608	10.744

5---

0.00	-1.487	0.226	7.088
12.19	1.487	-0.226	-4.335

6---

0.00	7.910	-1.487	-4.611
9.15	-7.910	1.487	-8.992

7---

0.00	0.000	8.136	13.326
9.15	0.000	5.223	0.000

S U P P O R T R E A C T I O N S

JOINT	Fx	Fy	Mz
1	0.000	5.667	0.000
3	0.121	33.820	0.406
5	-1.608	9.696	8.855
7	1.487	7.910	-4.611
8	0.000	5.223	0.000

In these examples the results from SAMT are compared with those results obtained manually and SAP 2000 and are shown in Table 6.1, 6.2 and 6.3. As we can see, the results obtained using SAMT, and those found using the moment distribution methods or SAP 2000 are very close.

Chapter 7

Conclusions

Economical design of structures may be gained by reducing the member section in the low moment regions. By doing so we get non-prismatic members. In this thesis analysis program has been developed and successfully implemented for non-prismatic members. The program is easy to use and input - out put information are printed in a file. Errors for input data are displayed and also automatic end force generation was implemented for general load case.

This program, not only allows the practicing engineer to obtain a useful tool that meets a particular necessity, but also will serve greatly as a basis for future developments of computerized analysis tools for engineering. The program can also be used for the analysis of flat slab where the design is based on the equivalent frame analysis techniques.

The method used for the preparation of this analysis program may be used for the development of analysis program for members of other shapes.

References

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9. RAO S.S., *The Finite Element Method in Engineering*, Pergamon, Oxford, 1989.
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11. SMITH I.M., *Programming in Fortran 90*,John Wiley, England, 1995
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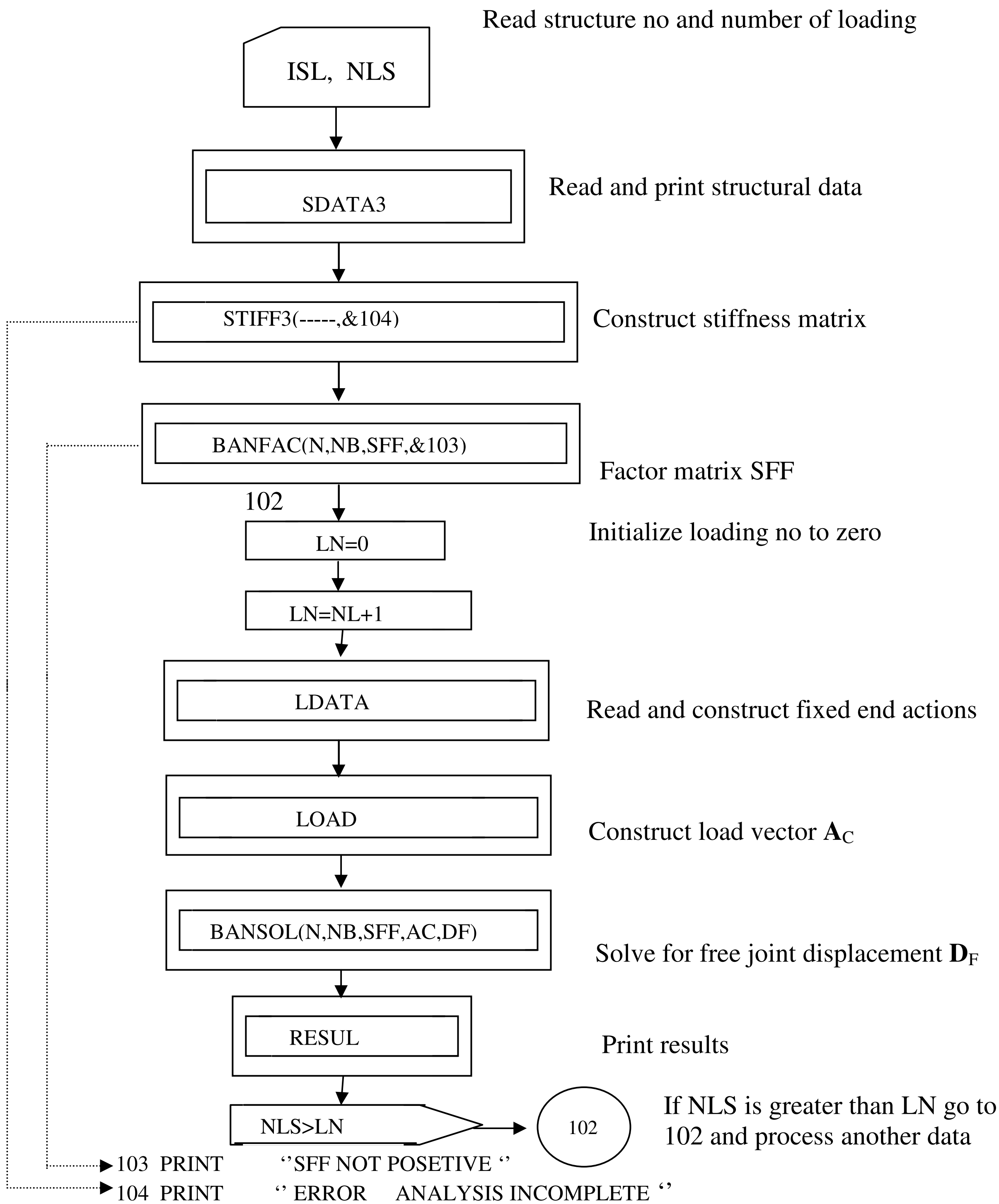
Flow Charts


Read structure no and number of loading
ISL, NLS
SDATA3
Read and print structural data
STIFF3(-----,&104)
Construct stiffness matrix
BANFAC(N,NB,SFF,&103)
Factor matrix SFF
102
LN=0
Initialize loading no to zero
LN=NL+1
LDATA
Read and construct fixed end actions
LOAD
Construct load vector AC
BANSOL(N,NB,SFF,AC,DF)
Solve for free joint displacement DF
RESUL
Print results
NLS>LN
102
If NLS is greater than LN go to 102 and process another data
103 PRINT ''SFF NOT POSETIVE ''
104 PRINT '' ERROR ANALYSIS INCOMPLETE ''

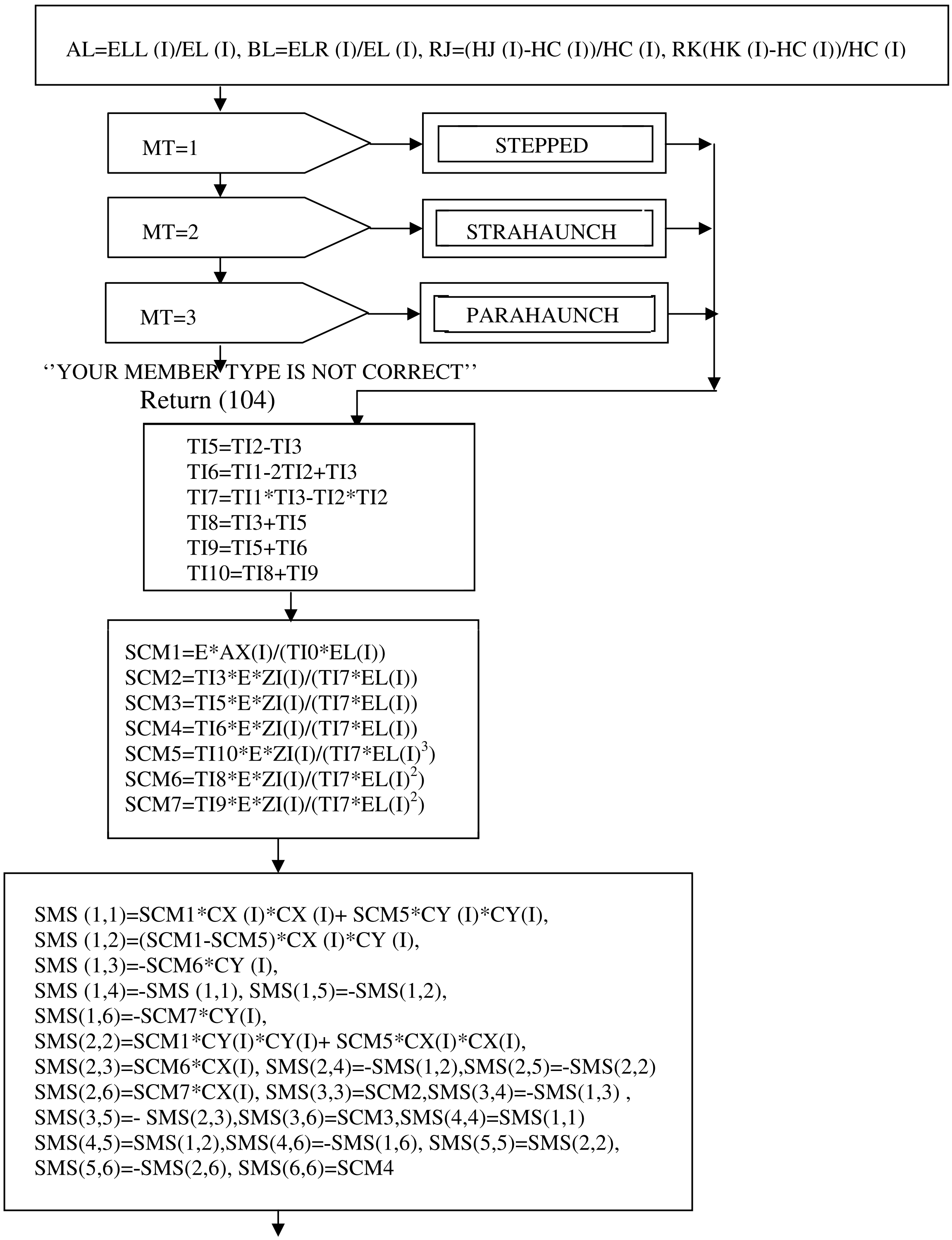
A.1 Subprogram SDATA3

Read and print Structural data

A.2 Subprogram STIFF3

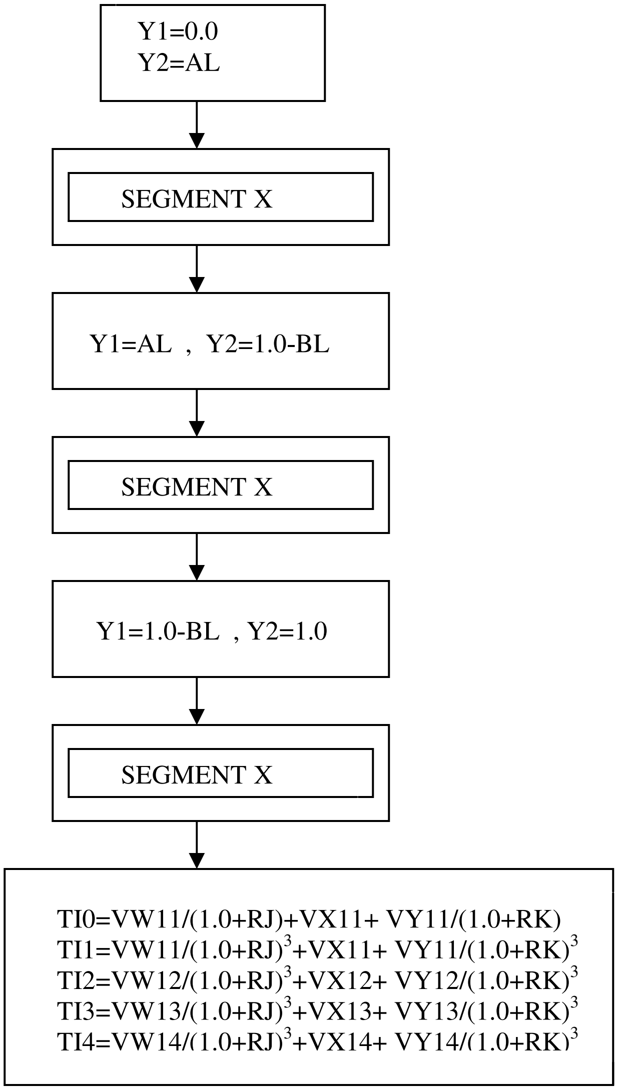
a. <u>Member stiffness</u>

Calculate the values of integral I_0-I_6

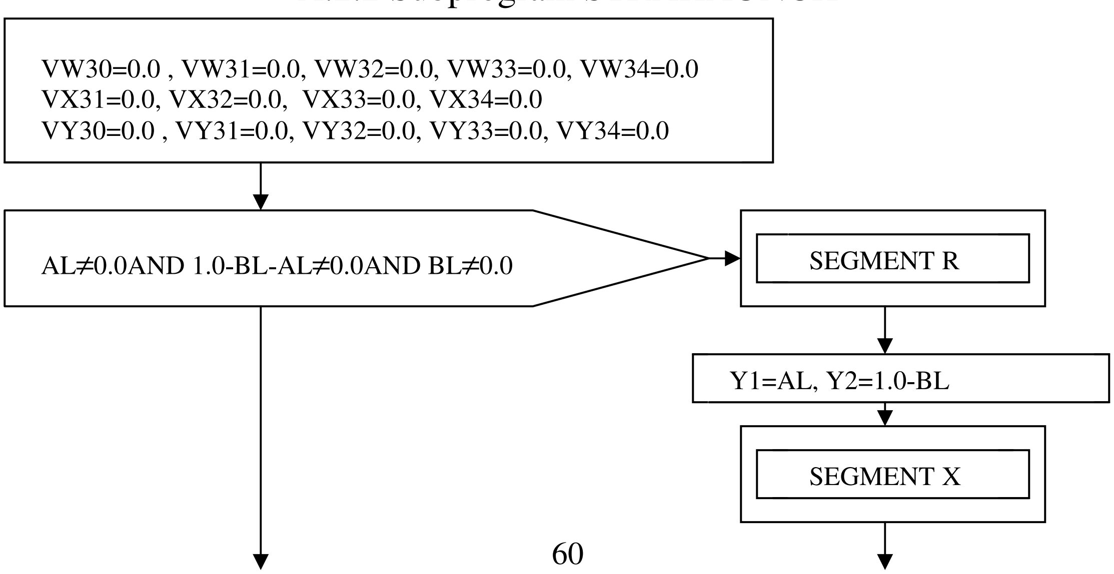


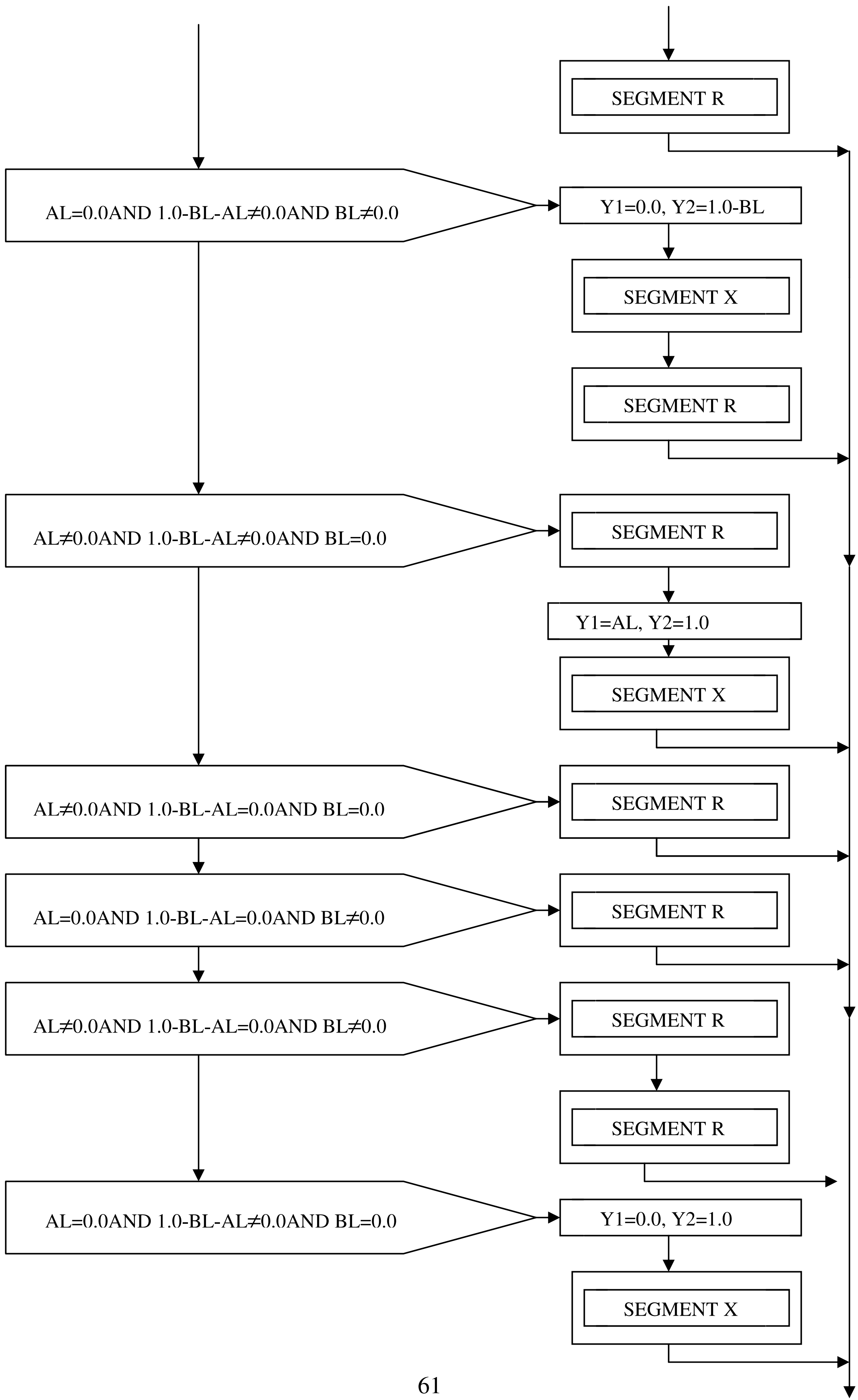
Transfer to joint stiffness matrix

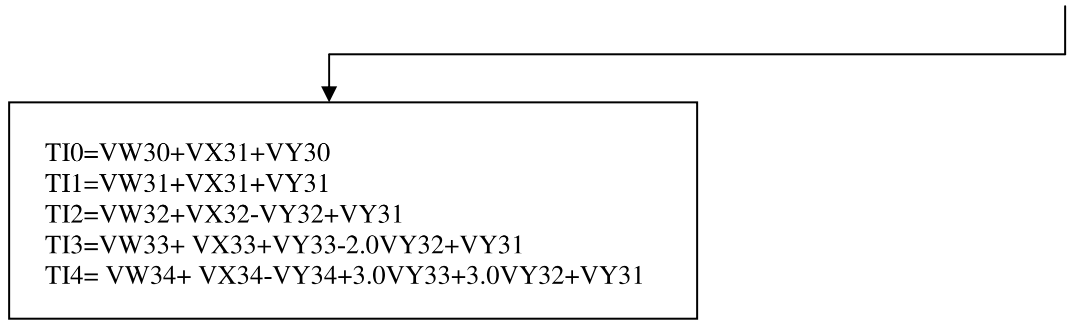
A.2.1 Subprogram STEPPED



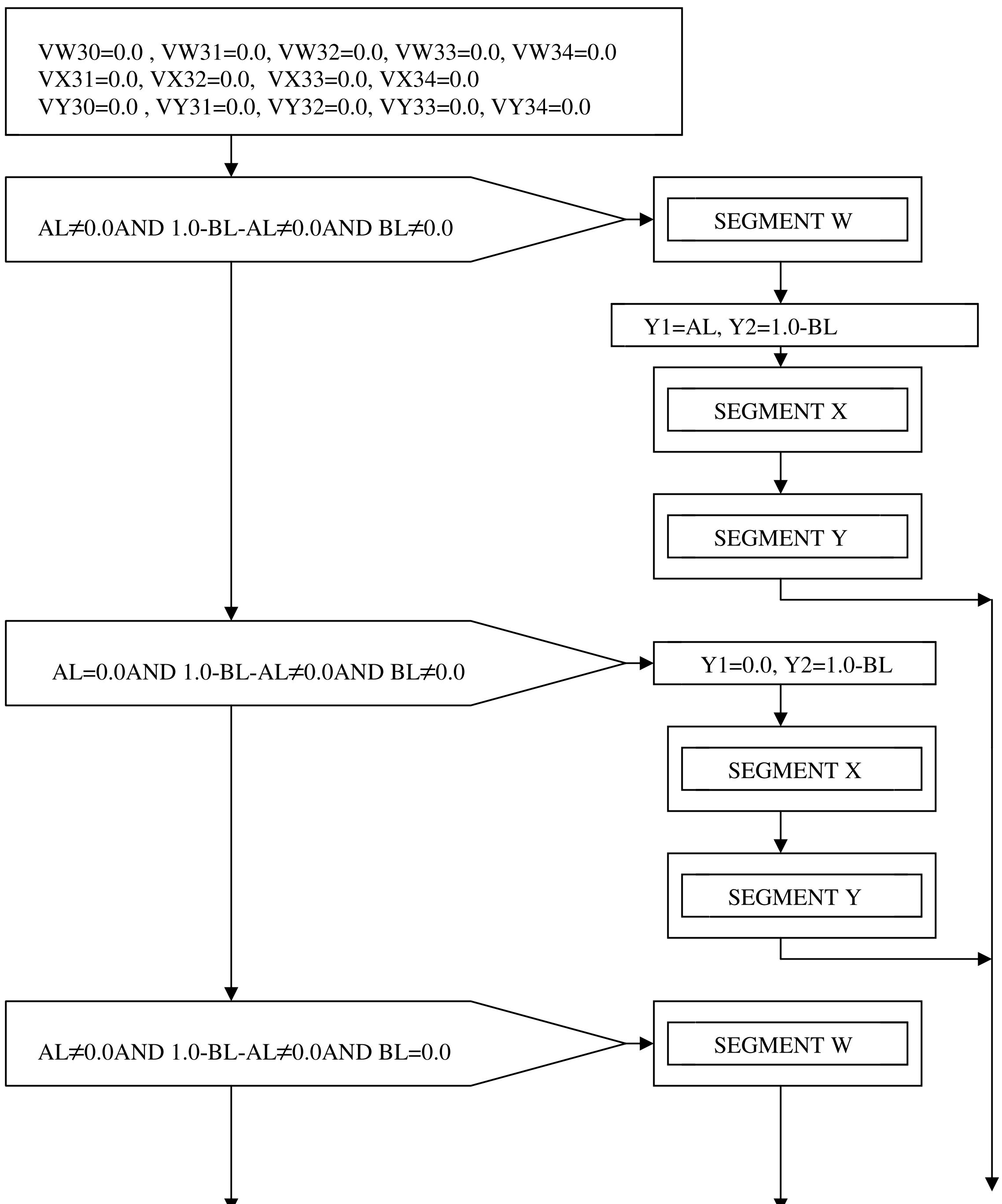
A.2.2 Subprogram STRAHAUNCH

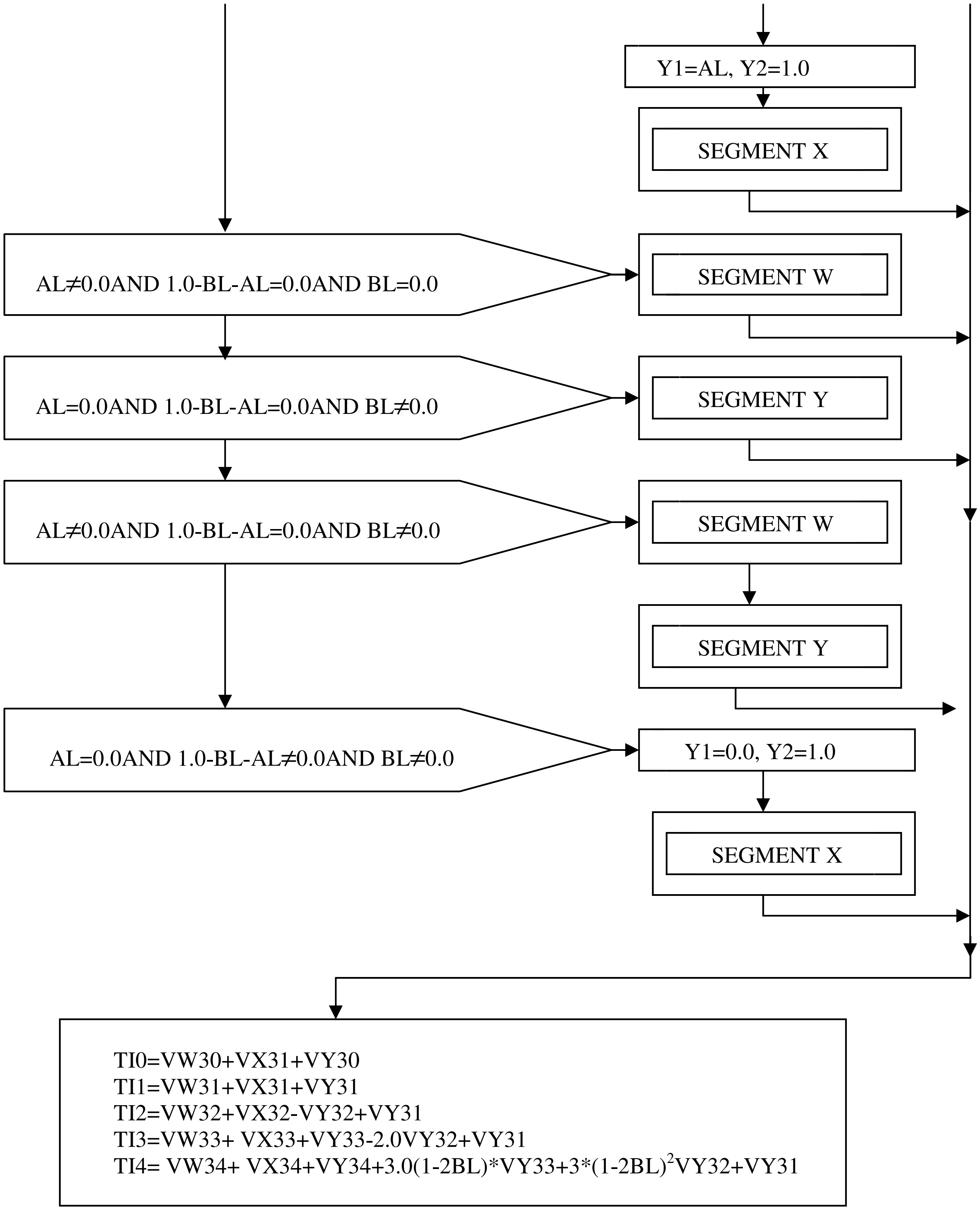



SEGMENT R
AL=0.0AND 1.0-BL-AL≠0.0AND BL≠0.0
Y1=0.0, Y2=1.0-BL
SEGMENT X
SEGMENT R
AL≠0.0AND 1.0-BL-AL≠0.0AND BL=0.0
SEGMENT R
Y1=AL, Y2=1.0
SEGMENT X
AL≠0.0AND 1.0-BL-AL=0.0AND BL=0.0
SEGMENT R
AL=0.0AND 1.0-BL-AL=0.0AND BL≠0.0
SEGMENT R
AL≠0.0AND 1.0-BL-AL=0.0AND BL≠0.0
SEGMENT R
SEGMENT R
AL=0.0AND 1.0-BL-AL≠0.0AND BL=0.0
Y1=0.0, Y2=1.0
SEGMENT X

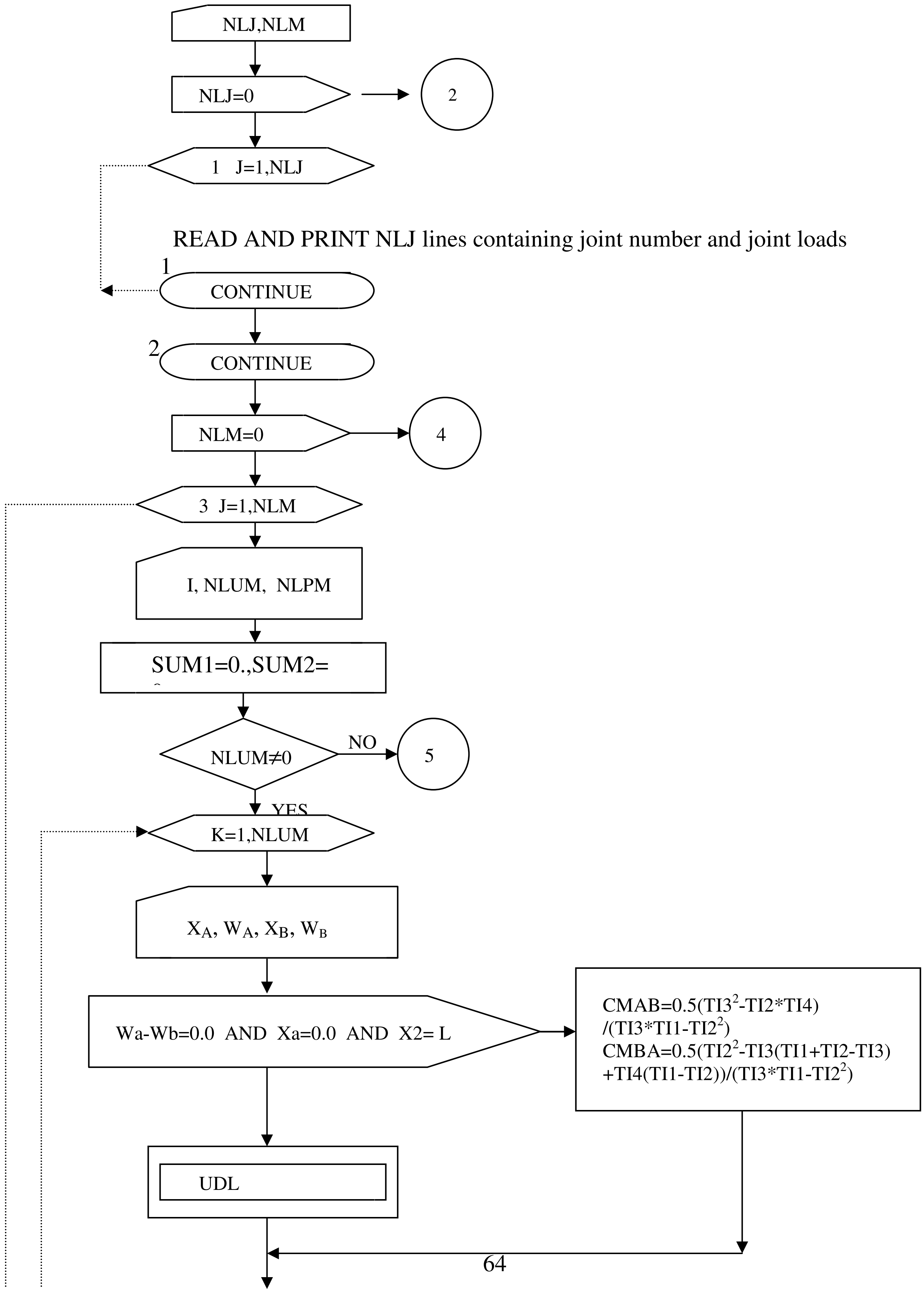


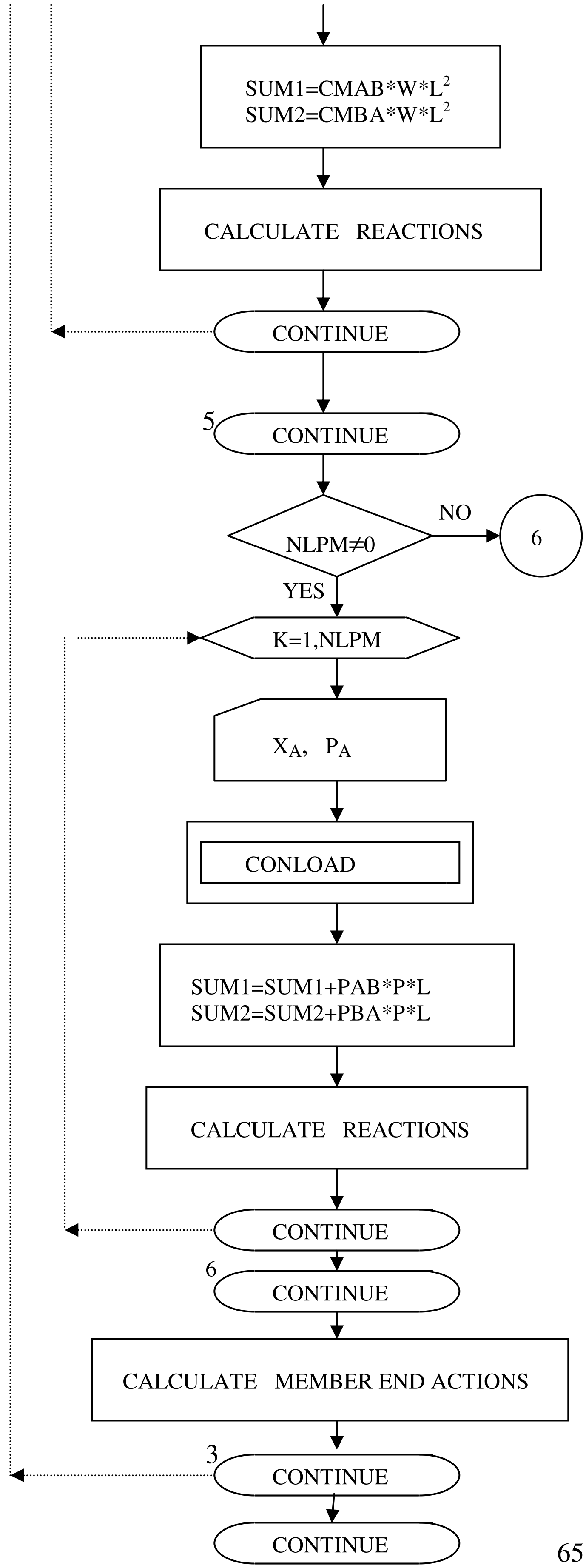
A.2.2 Subprogram PARAHAUNCH




Y1=AL, Y2=1.0
SEGMENT X
AL≠0.0AND 1.0-BL-AL=0.0AND BL=0.0
SEGMENT W
AL=0.0AND 1.0-BL-AL=0.0AND BL≠0.0
SEGMENT Y
AL≠0.0AND 1.0-BL-AL=0.0AND BL≠0.0
SEGMENT W
SEGMENT Y
AL=0.0AND 1.0-BL-AL≠0.0AND BL≠0.0
Y1=0.0, Y2=1.0
SEGMENT X
TI0=VW30+VX31+VY30
TI1=VW31+VX31+VY31
TI2=VW32+VX32-VY32+VY31
TI3=VW33+ VX33+VY33-2.0VY32+VY31
TI4= VW34+ VX34+VY34+3.0(1-2BL)*VY33+3*(1-2BL)^2VY32+VY31

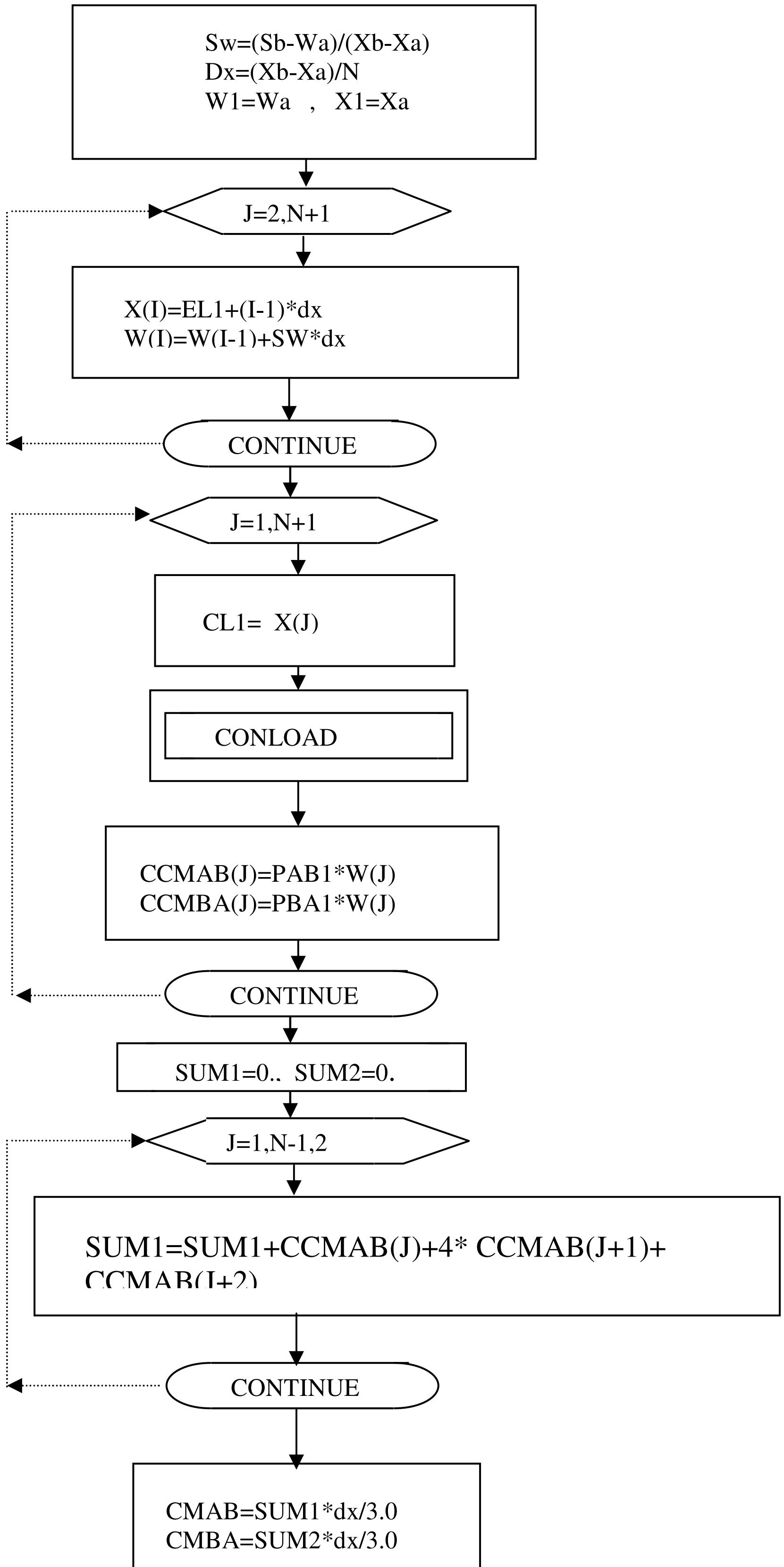
A.3 Subprogram for LDATA3



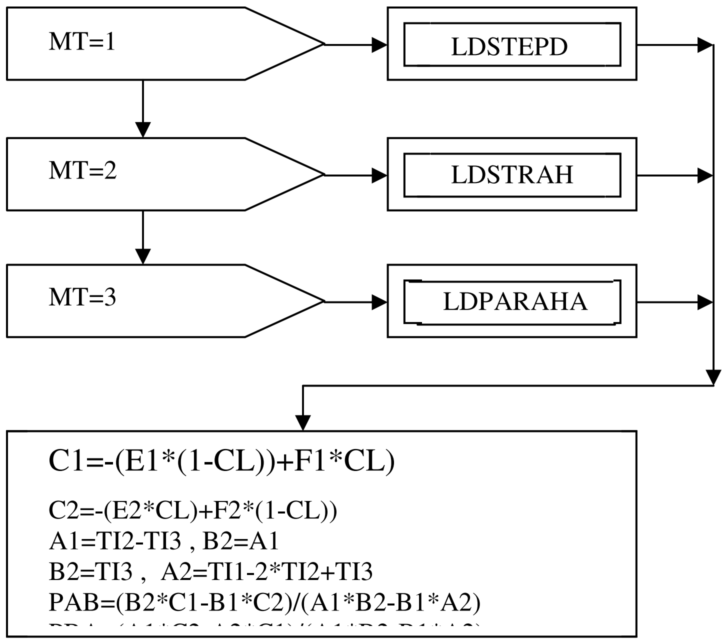

SUM1=CMAB*W*L2
SUM2=CMBA*W*L2
CALCULATE REACTIONS
CONTINUE
5
CONTINUE
NLPM≠0
NO
6
YES
K=1,NLPM
XA, PA
CONLOAD
SUM1=SUM1+PAB*P*L
SUM2=SUM2+PBA*P*L
CALCULATE REACTIONS
CONTINUE
6
CONTINUE
CALCULATE MEMBER END ACTIONS
3
CONTINUE
CONTINUE

4

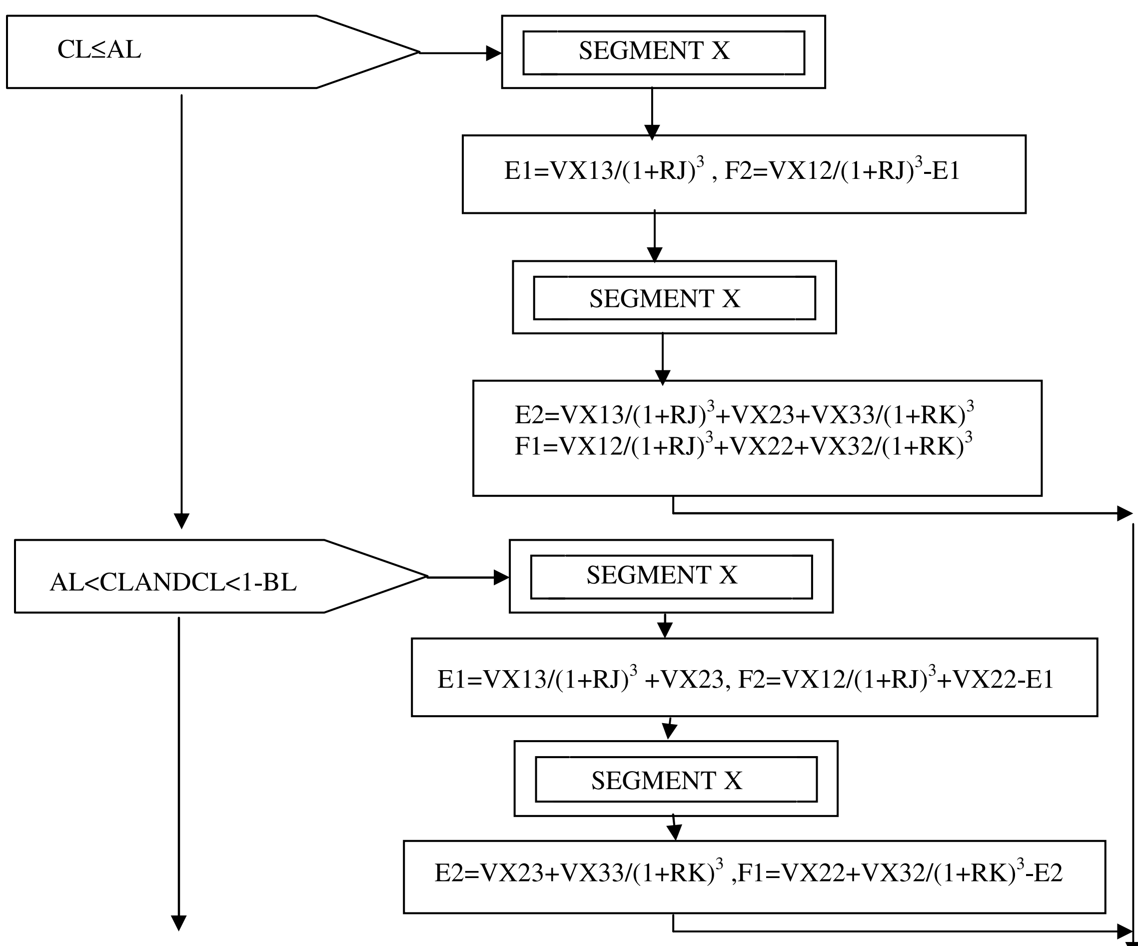
A.3.1 Subprogram for uniform load (UDL)

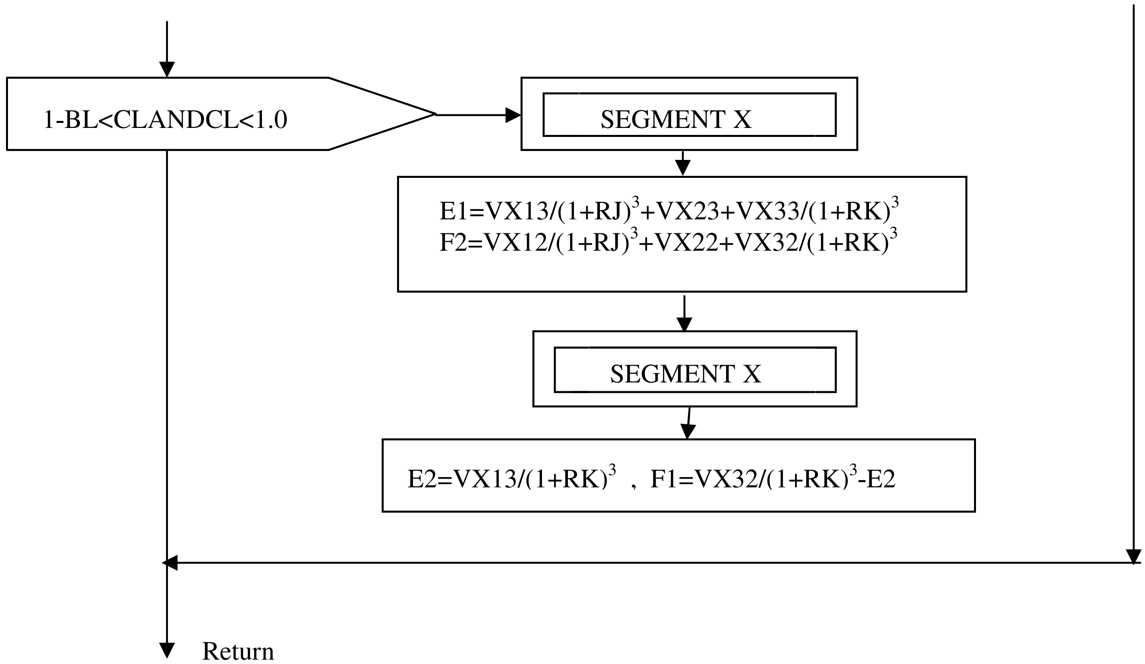


A.3.2 Subprogram for CONCLOAD

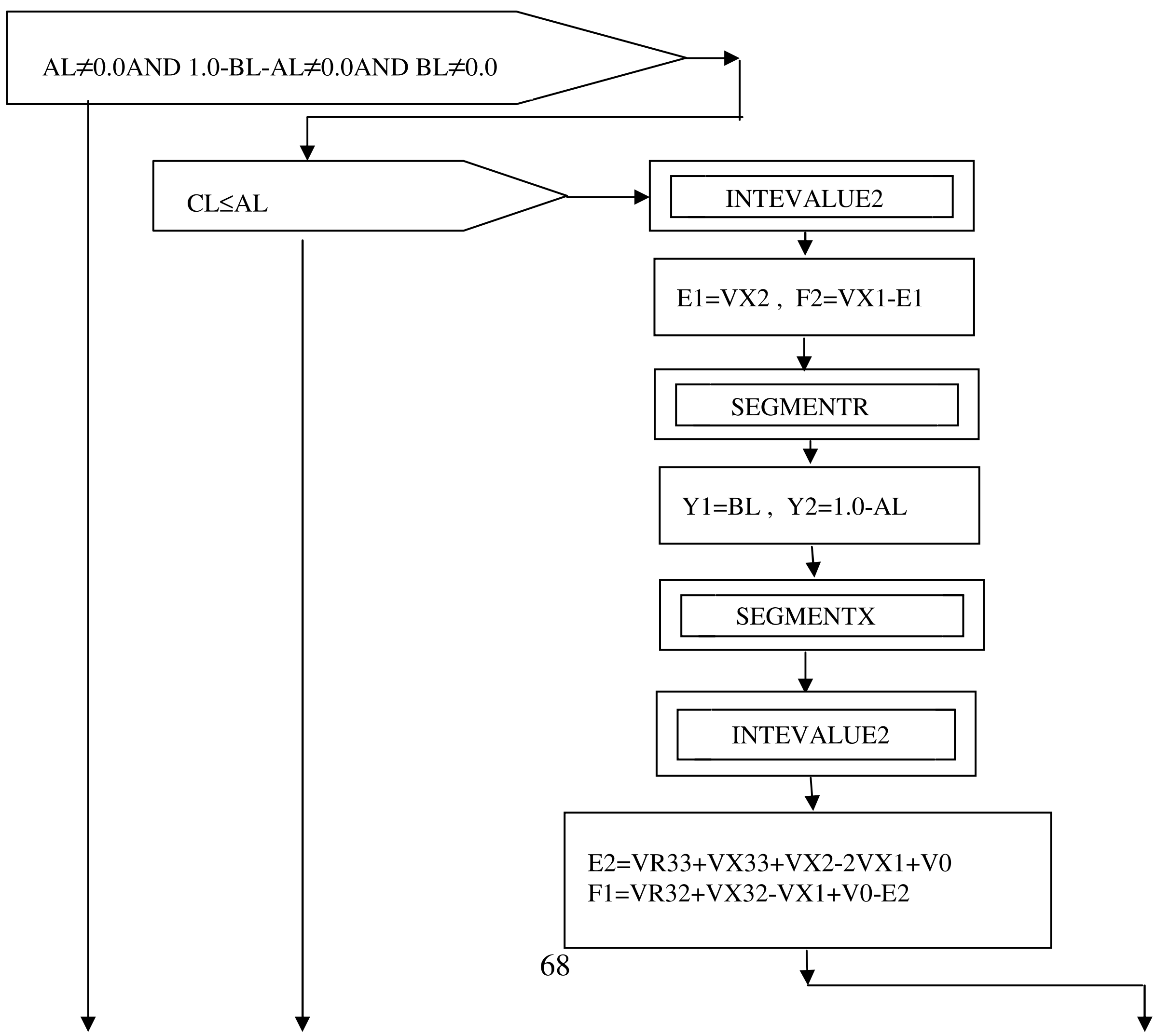


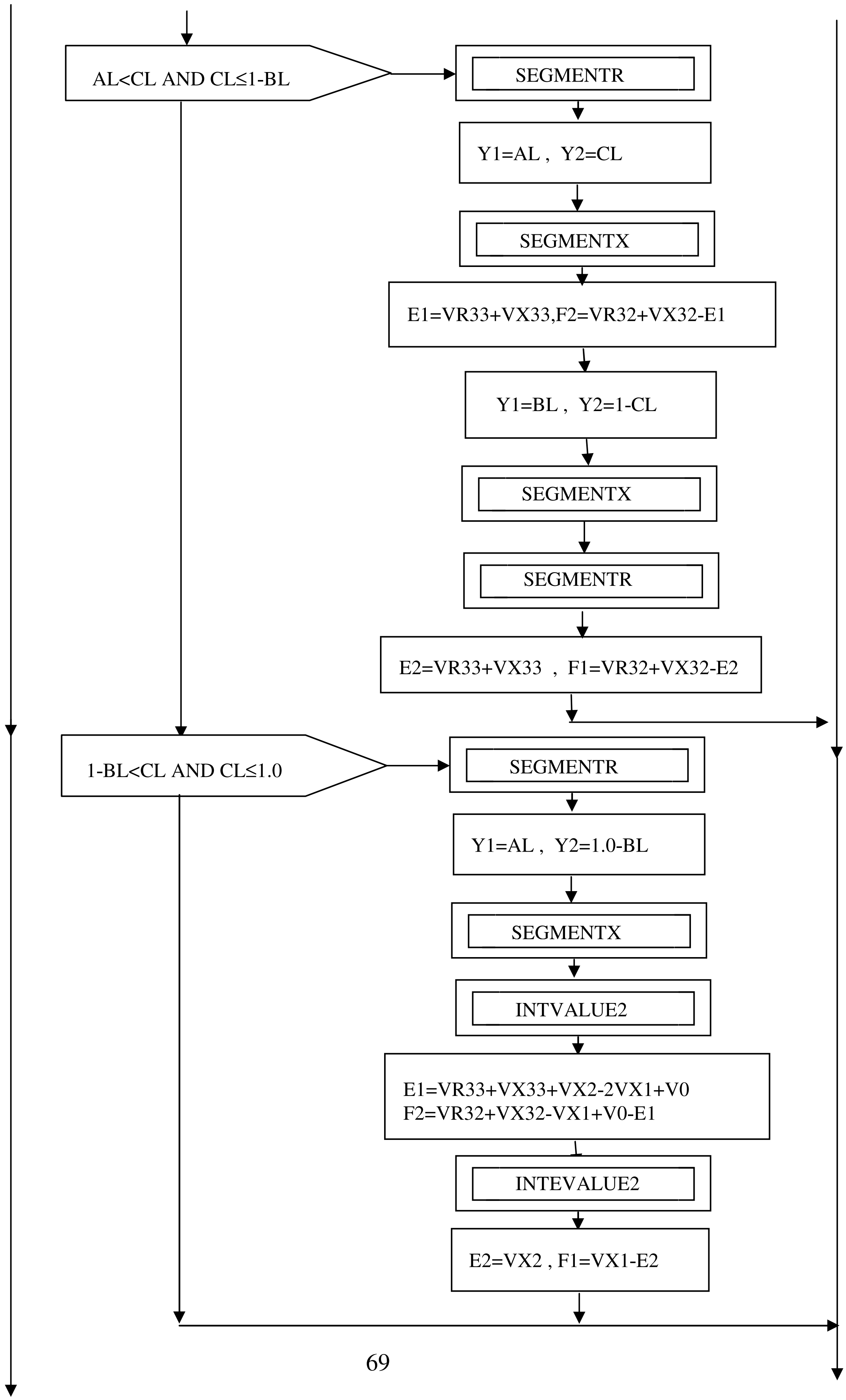
A.3.2.1 Subprogram for LDSTEPD

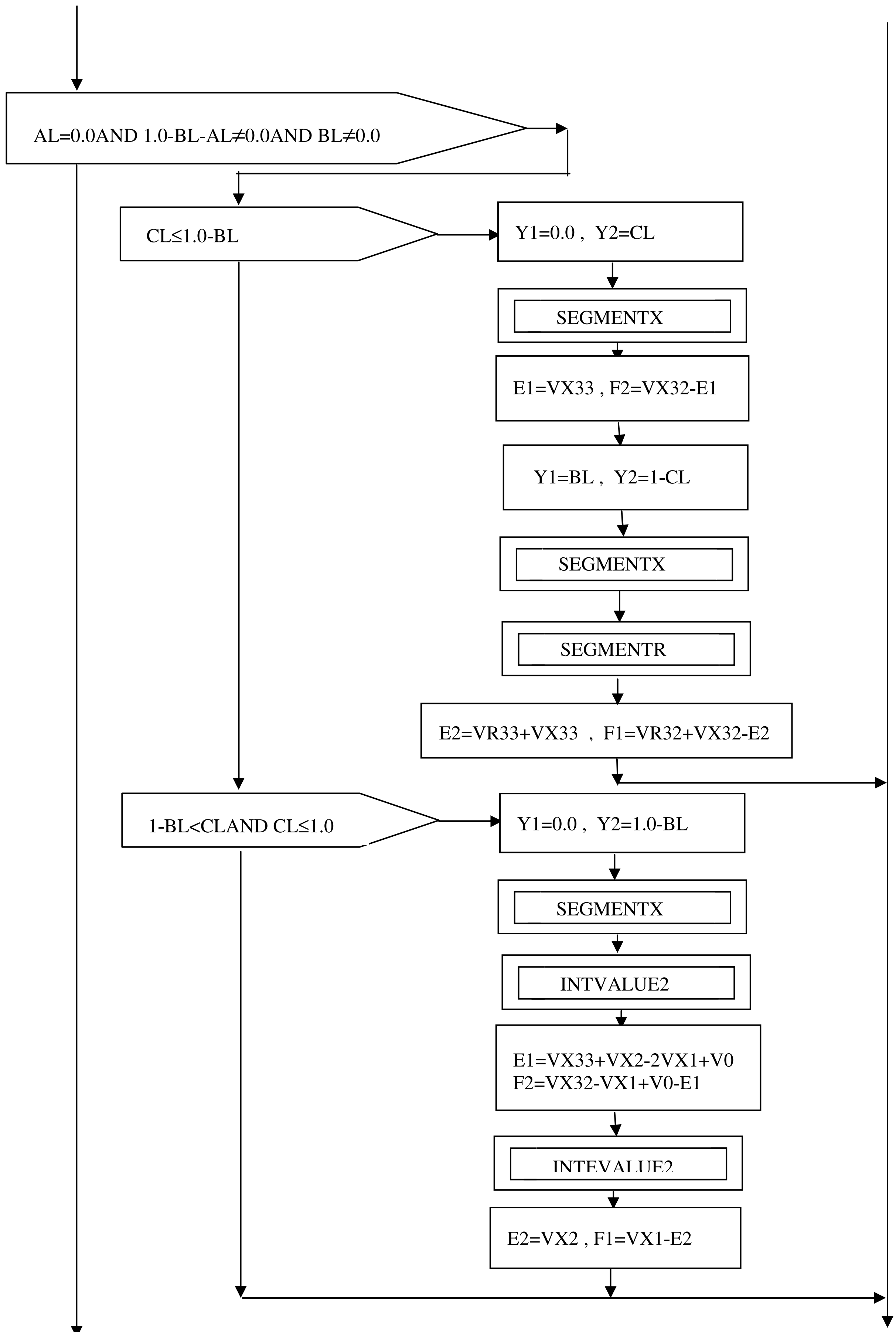


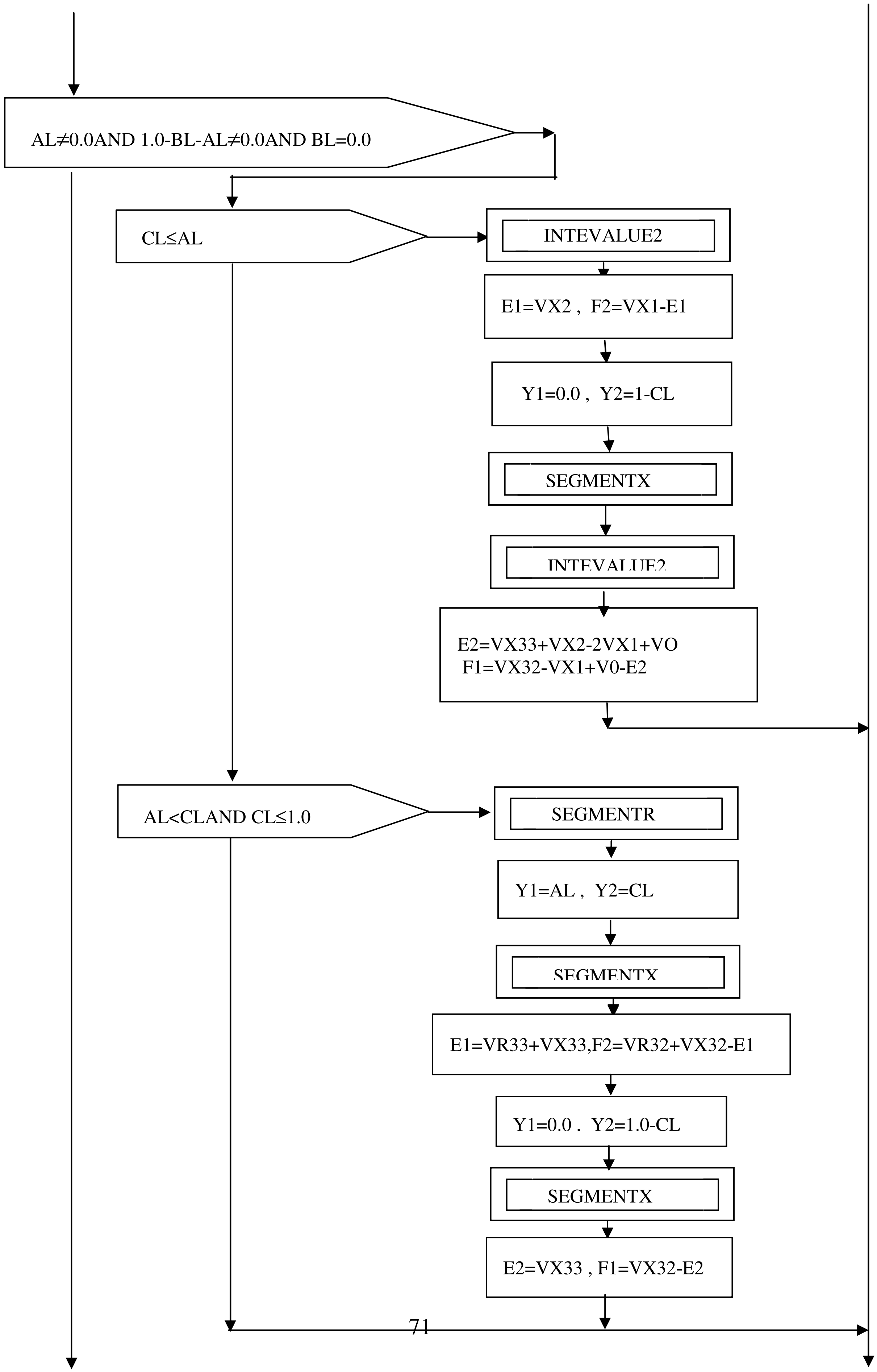


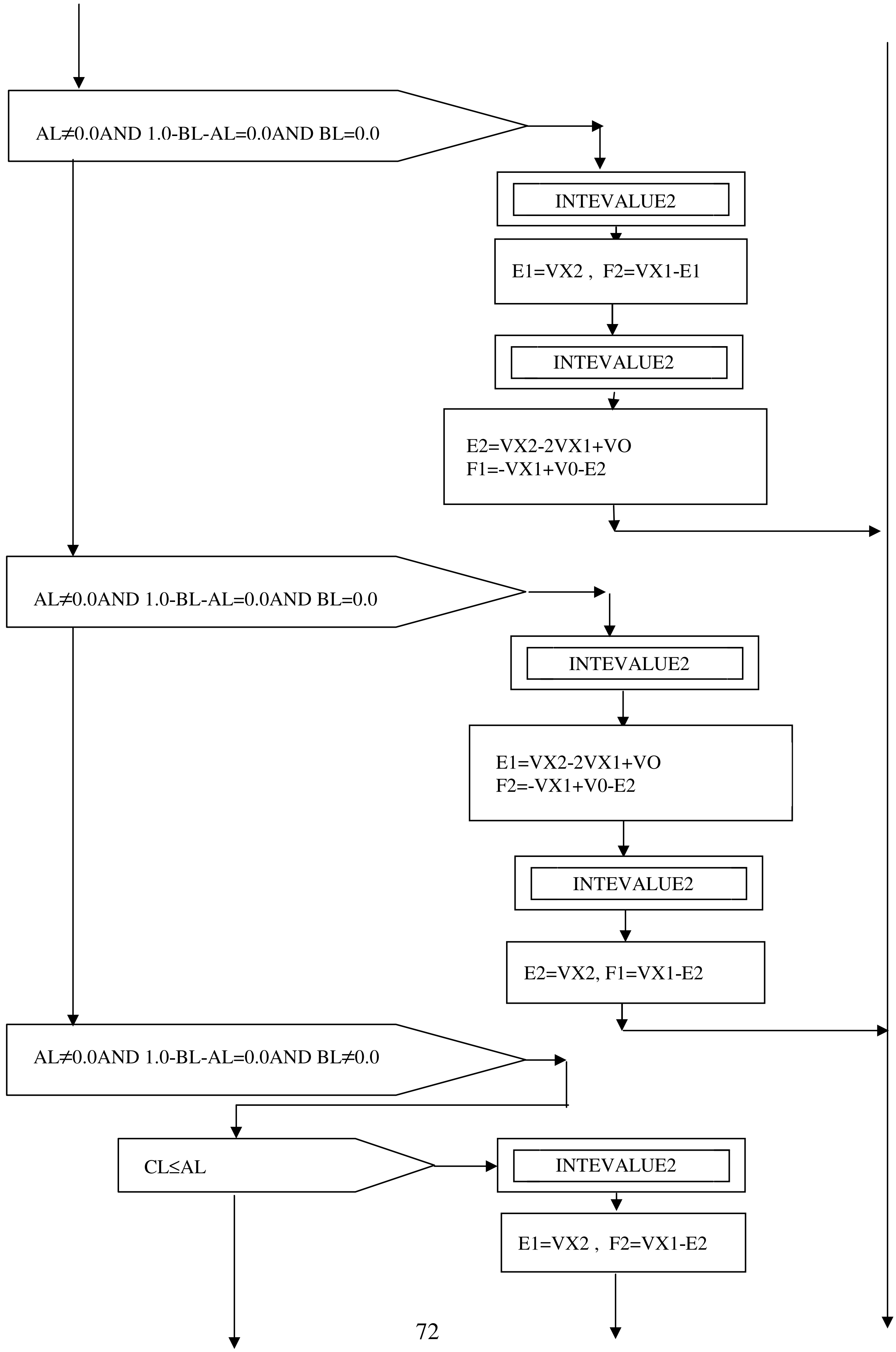
A.3.2.2 Subprogram for LDSTRAHA.

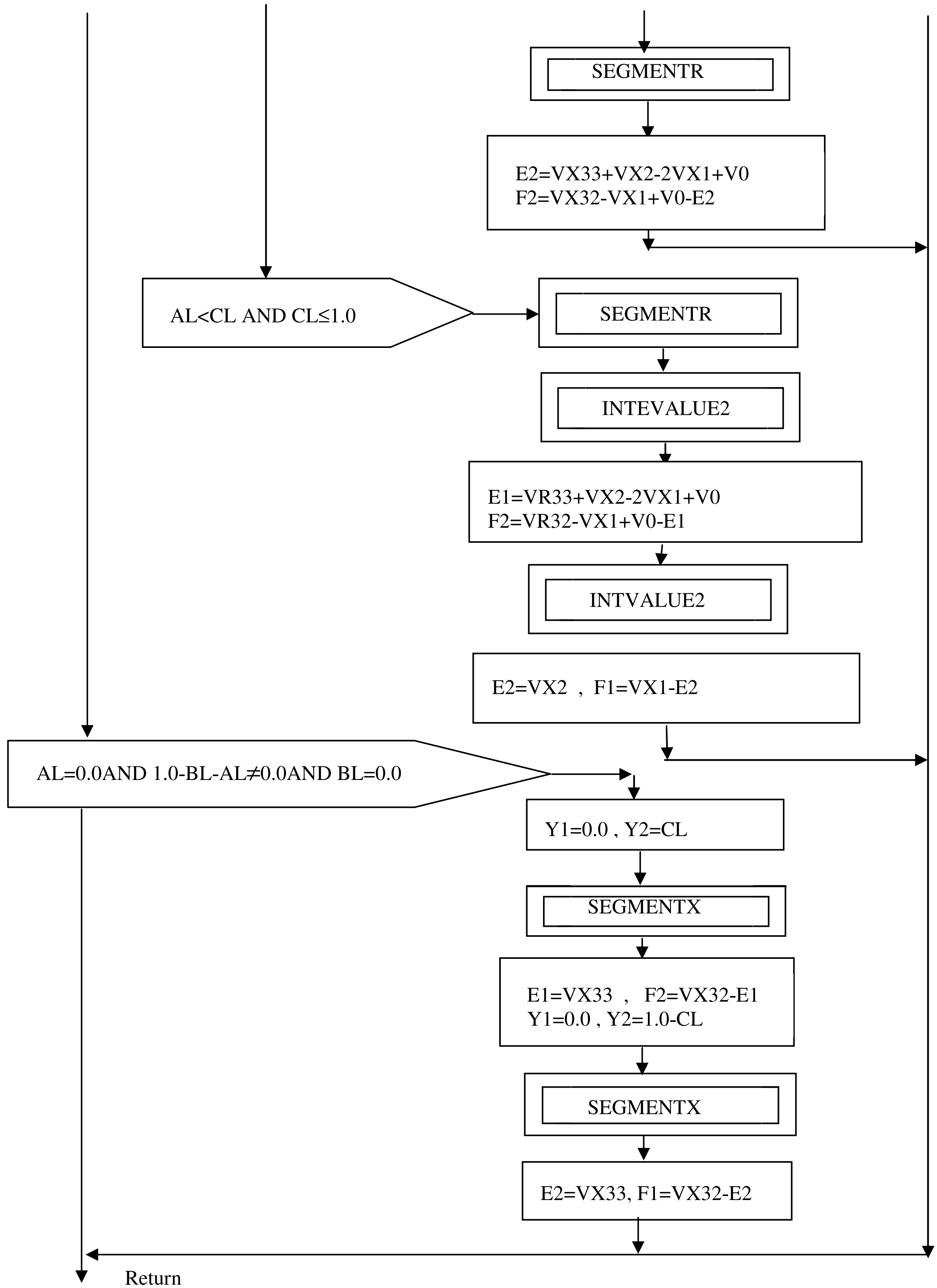



AL<CL AND CL≤1-BL
SEGMENTR
Y1=AL , Y2=CL
SEGMENTX
E1=VR33+VX33,F2=VR32+VX32-E1
Y1=BL , Y2=1-CL
SEGMENTX
SEGMENTR
E2=VR33+VX33 , F1=VR32+VX32-E2
1-BL<CL AND CL≤1.0
SEGMENTR
Y1=AL , Y2=1.0-BL
SEGMENTX
INTVALUE2
E1=VR33+VX33+VX2-2VX1+V0
F2=VR32+VX32-VX1+V0-E1
INTEVALUE2
E2=VX2 , F1=VX1-E2


AL=0.0AND 1.0-BL-AL≠0.0AND BL≠0.0
CL≤1.0-BL
Y1=0.0 , Y2=CL
SEGMENTX
E1=VX33 , F2=VX32-E1
Y1=BL , Y2=1-CL
SEGMENTX
SEGMENTR
E2=VR33+VX33 , F1=VR32+VX32-E2
1-BL<CLAND CL≤1.0
Y1=0.0 , Y2=1.0-BL
SEGMENTX
INTVALUE2
E1=VX33+VX2-2VX1+V0
F2=VX32-VX1+V0-E1
INTEVALUE2
E2=VX2 , F1=VX1-E2


AL≠0.0AND 1.0-BL-AL≠0.0AND BL=0.0
CL≤AL
INTEVALUE2
E1=VX2 , F2=VX1-E1
Y1=0.0 , Y2=1-CL
SEGMENTX
INTEVALUE2
E2=VX33+VX2-2VX1+VO
F1=VX32-VX1+V0-E2
AL<CLAND CL≤1.0
SEGMENTR
Y1=AL , Y2=CL
SEGMENTX
E1=VR33+VX33,F2=VR32+VX32-E1
Y1=0.0 . Y2=1.0-CL
SEGMENTX
E2=VX33 , F1=VX32-E2


AL≠0.0AND 1.0-BL-AL=0.0AND BL=0.0
INTEVALUE2
E1=VX2 , F2=VX1-E1
INTEVALUE2
E2=VX2-2VX1+VO
F1=-VX1+V0-E2
AL≠0.0AND 1.0-BL-AL=0.0AND BL=0.0
INTEVALUE2
E1=VX2-2VX1+VO
F2=-VX1+V0-E2
INTEVALUE2
E2=VX2, F1=VX1-E2
AL≠0.0AND 1.0-BL-AL=0.0AND BL≠0.0
CL≤AL
INTEVALUE2
E1=VX2 , F2=VX1-E2



A.3.2.3 Subprogram for LDPARAHA

Flow chart for this subprogram is similar to flow chart for LDSTRAHA except the definition of the subprograms for evaluating the integrals.

Programs

B.1 Variable Names

AX( )	Cross sectional area of least section
AJ( )	Actions applied at joint
dC	Smaller depth of members
dL	Depth of member at the left side of the member
dR	Depth of member at the right side of the member
E	Modulus of elasticity
ELL( )	Left haunch length
ELR( )	Right haunch length
I	Member index *i*
ISN	Structure number
J, K	Joint indexes
JJ( )	Designation for *j* end of member
JK( )	Designation for *k* end of member
JRL( )	Joint restraint list
M	Number of members
MT( )	Member type
NJ	Number of joints
NLJ	Number of loaded joints
NLM	Number of loaded members
NLPM	Number of concentrated loads in a member
NLS	Number of loaded systems
NLUM	Number of uniform loads in a member
NR	Number of support restraints
NRJ	Number of restrained joints
P	Concentrated load value at X_c distance from end *j*
X( ) ,Y( )	X and Y coordinate of joint

W_a Load intensity at X_a distance from end *j*

W_b Load intensity at X_b distance from end *j*

ZI( ) Moment of inertia of least cross section

B.2 Program

```
PROGRAM MAIN
!    MAIN PROGRAM FOR PLANE FRAMES WITH OR WITHOUT NON-PRISMATIC MEMBERS
!    ------------------------------------------------------------------
 REAL,ALLOCATABLE::SFF(:,:)
 CHARACTER*15,PFI1,PF01,PF02,DATE,TIME,PT1,PT2

REAL::AX(100),ZI(100),EL(100),CX(100),CY(100),DF(300),AML(6,100),AJ(200),AE(20
0),AR(200),AC(200)

REAL::TIL0(100),TIL1(100),TIL2(100),TIL3(100),TIL4(100),ELL(100),ELR(100),HJ(1
00),HC(100),HK(100)
 INTEGER::JJ(100),JK(100),JRL(100),ID(100),MT(100),LML(100)
 PRINT*,'ENTER YOUR INPUT FILE NAME  '
 READ*,PFI1
 PRINT*,'ENTER ECHO OF INPUT DATA FILE NAME  '
 READ*,PF01
 PRINT*,'ENTER YOUR OUTPUT FILE NAME  '
 READ*,PF02
      OPEN(5,FILE=PFI1,STATUS='OLD')
      OPEN(6,FILE=PF01)
      OPEN(7,FILE=PF02)
CALL DATE_AND_TIME(DATE,TIME)
PT2=TIME(1:2)//':'//TIME(3:4)
PT1=DATE(7:8)//':'//DATE(5:6)//':'//DATE(1:4)
WRITE(*,99)
99
FORMAT(25X,5('*'),6X,4('*'),2X,2('*'),5X,2('*'),2X,6('*'),/24X,6('*'),5X,5('*'
),2X,3('*'),3X,3('*'),2X,6('*'),&
&
/24X,3('*'),7X,2('*'),2X,2('*'),2X,4('*'),1X,4('*'),4X,2('*'),/24X,5('*'),4X,2
('*'),3X,2('*'),2X,
9('*'),4X,2('*'),&
/25X,5('*'),2X,8('*'),2X,2('*'),1X,3('*'),1X,2('*'),4X,2('*'),/27X,3('*'),2X,8
('*'),2X,2('*'),2X,
1('*'),2X,2('*'),4X,2('*'),&
/24X,6('*'),2X,2('*'),4X,2('*'),2X,2('*'),5X,2('*'),4X,2('*'),/24X,5('*'),3X,2
('*'),4X,2('*'),2X,
2('*'),5X,2('*'),4X,2('*'),&
& //30X,'( Version  2002 )')
WRITE(6,301)PF01
301 FORMAT(/10X,60('*'),/10X,5('*'),5X,'   E C H O  O F  I N P U T  D A T A
',7X,5('*'),/10X,60('*'),&
&   //,10X,'SAMT 2001             FILE :',A15,'       KN-m units')
WRITE(7,302)PF02
302 FORMAT(/10X,60('*'),/10X,5('*'),5X,'     F R A M E  O U T P U T  D A T A
',6X,5('*'),/10X,60('*'),&
&   //,10X,'SAMT 2001             FILE :',A15,'       KN-m units')
```

```
WRITE(6,201)PT1,PT2
201 FORMAT(10X,'DATE : ',A15,'TIME : ',A15,//)
WRITE(7,202)PT1,PT2
202 FORMAT(10X,'DATE : ',A15,'TIME : ',A15,//)
PRINT*,' ECHO OF INPUT  DATA FILE NAME:  ',PF01
PRINT*,' YOUR OUTPUT DATA FILE NAME   :  ',PF02

101 READ(5, *,END=999)ISN,NLS
    CALL
SDATA3(ISN,NLS,M,MD,NJ,ND,N,NB,JJ,JK,JRL,ID,MT,AX,ZI,EL,CX,CY,E,ELL,ELR,HJ,HC,
HK)
    ALLOCATE(SFF(N,NB))
    CALL
STIFF3(M,MD,NJ,MT,N,NB,JJ,JK,JRL,ID,AX,ZI,CX,CY,EL,E,ELL,ELR,HJ,HC,HK,SFF,TIL0
,TIL1,TIL2,TIL3,TIL4,
    *104)
    CALL BANFAC(N,NB,SFF,*103)
    LN=0
102 LN=LN+1
    CALL
LDATA3(M,ND,LN,MT,EL,ELL,ELR,HJ,HC,HK,TIL1,TIL2,TIL3,TIL4,AJ,AE,AR,AML,NLM,NLJ
,LML,*104)
    CALL LOADS3(M,ND,NLM,LML,ID,JJ,JK,CX,CY,AML,AJ,AE,AC)
    CALL BANSOL(N,NB,SFF,AC,DF)
    CALL
RESUL3(M,NJ,ND,EL,AX,E,ZI,JJ,JK,JRL,ID,CX,CY,DF,AJ,AE,AR,AML,MT,TIL0,TIL1,TIL2
,TIL3)
    IF (NLS>LN) GO TO 102
    GO TO 101
103 WRITE(*,601)
104 WRITE(*,602)
999 STOP
501 FORMAT(3I2)
601 FORMAT(//,10X,'*** ERROR ***',//10X,' SFF NOT POS. DEF. OR UNSTABLE
STRUCTURE')
602 FORMAT(/,'          *****   A N A L Y S I S   I N C O M P L E T E   *****
')
!
CONTAINS
!
! *** READ AND PRINT STRUCTURAL DATA FOR PLANE FRAME ***
!
SUBROUTINE
SDATA3(ISN,NLS,M,MD,NJ,ND,N,NB,JJ,JK,JRL,ID,MT,AX,ZI,EL,CX,CY,E,ELL,ELR,HJ,HC,
HK)
REAL,ALLOCATABLE::X(:),Y(:)
REAL:: AX(:),ZI(:),CX(:),CY(:),EL(:),ELL(:),ELR(:),HJ(:),HC(:),HK(:)
INTEGER::ISN,NLS,JJ(:),JK(:),JRL(:),ID(:),MT(:)
        WRITE(6,505)
!
! A. PROBLEM IDENTIFICATION
      WRITE(6,601)ISN,NLS
! B. STRUCTURAL PARAMETERS
            WRITE(6,602)
            READ(5, *)M,NJ,NR,NRJ,E
            NDJ=3
            ND=NDJ*NJ
```

```
       N=ND-NR
             WRITE(6,603)M,N,NJ,NR,NRJ,E
         ALLOCATE(X(NJ),Y(NJ))
! C. JOINT COORDINATES
             WRITE(6,604)
             DO K=1,NJ
             READ(5, *)J,X(J),Y(J)
             WRITE(6,605)J,X(J),Y(J)
         END DO
! D. MEMBER INFORMATION
       WRITE(6,505)
       WRITE(6,606)
       MD=2*NDJ
       NB=0
       DO J=1,M
       READ(5,*)I,JJ(I),JK(I),MT(I),AX(I),ZI(I),ELL(I),ELR(I),HJ(I),HC(I),HK(I)
       NBI=NDJ*(ABS(JK(I)-JJ(I))+1)
       IF(NBI>NB)NB=NBI
       XCL=X(JK(I))-X(JJ(I))
       YCL=Y(JK(I))-Y(JJ(I))
       EL(I)=SQRT(XCL*XCL+YCL*YCL)
       CX(I)=XCL/EL(I)
       CY(I)=YCL/EL(I)

WRITE(6,607)I,MT(I),JJ(I),JK(I),AX(I),ZI(I),EL(I),ELL(I),ELR(I),HJ(I),HC(I),HK
(I)
       END DO
! E. JOINT RESTRAINTS
         WRITE(6,505)
             WRITE(6,608)
             DO J=1,ND
             JRL(J)=0
         END DO
             DO J=1,NRJ
             READ(5, *)K,JRL(3*K-2),JRL(3*K-1),JRL(3*K)
       WRITE(6,609)K,JRL(3*K-2),JRL(3*K-1),JRL(3*K)
         END DO
! E. DISPLACEMENT INDEXES
             N1=0
             DO J=1,ND
             N1=N1+JRL(J)
             IF (JRL(J)>0) GO TO 4
             ID(J)=J-N1
             GO TO 5
    4        ID(J)=N+N1
    5     END DO
             RETURN
  505  FORMAT(10X,60('-'))
  601        FORMAT(' ', 9X,'STRUCTURE NO.',I2, '  PLANE FRAME',/ &
       10X,'NUMBER OF LOADING SYSTEMS =',I2)
  602        FORMAT(//,10X,' STRUCTURAL PARAMETERS',/,10X,'     M    N   NJ &
        &  NR  NRJ           E')
  603  FORMAT(' ',10X,5I5,F15.1)
  604        FORMAT(' ',10X,'JOINT COORDINATES',/,10X, ' JOINT        X
Y')
  605        FORMAT(13X,I2,2X,2F10.2)
```

```
  606       FORMAT(/,10X,' MEMBER INFORMATION',/,10X,' MEMBER  MTP JNT-1 JNT-2
&
       & AX      ZI   LENGTH   AL     BL     hJ     hC     hK')
  607       FORMAT(' ',10X, I6,I3,3X,2I5,8F7.3)
  608       FORMAT(/,10X,' JOINT RESTRAINTS',/,10X,' JOINT  JR1  JR2  JR3')
  609       FORMAT(' ',10X,4I5)
            END SUBROUTINE SDATA3
!
! *** GENERATE JOINT STIFFNESS MATRIX FOR PLANE FRAME ***
!
      SUBROUTINE
STIFF3(M,MD,NJ,MT,N,NB,JJ,JK,JRL,ID,AX,ZI,CX,CY,EL,E,ELL,ELR,HJ,HC,HK,SFF,TIL0
,TIL1,TIL2,TIL3,TIL4,
       *)
      REAL::
AX(:),ZI(:),CX(:),CY(:),EL(:),TIL0(:),TIL1(:),TIL2(:),TIL3(:),TIL4(:)
      REAL::SMS(6,6),SFF(N,NB),ELL(:),ELR(:),HJ(:),HC(:),HK(:)
      INTEGER::JJ(:),JK(:),JRL(:),ID(:),MT(:),IM(6)
!
! A. MEMBER STIFFNESS
!
      DO J=1,N
      DO K=1,NB
        SFF(J,K)=0.0
      END DO
      END DO
      DO I=1,M
!
!CALCULATE THE VALUES OF INTEGRAL I0-I6
!
      AL=ELL(I)/EL(I);BL=ELR(I)/EL(I)
      RJ=(HJ(I)-HC(I))/HC(I);RK=(HK(I)-HC(I))/HC(I)
      IF(1.0-BL-AL<-0.005)THEN
      WRITE (*,601)
      RETURN 1
      END IF
      IF(RJ<0.0.OR.RK<0.0)THEN
      WRITE (*,602)
      RETURN 1
      END IF
      IF (MT(I)==1) THEN
      CALL STEEPED(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
      ELSE IF(MT(I)==2)THEN
      CALL STRAHAUNCH(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
      ELSE IF(MT(I)==3)THEN
      CALL PARAHAUNCH(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
      ELSE
      WRITE(*,603)
      RETURN 1
      END IF
      TIL0(I)=TI0;TIL1(I)=TI1;TIL2(I)=TI2;TIL3(I)=TI3;TIL4(I)=TI4
      TI5=TI2-TI3
      TI6=TI1-2*TI2+TI3
      TI7=TI1*TI3-TI2*TI2
      TI8=TI3+TI5
      TI9=TI5+TI6
      TI10=TI8+TI9
```

```
          SCM1=E*AX(I)/(TI0*EL(I))
          SCM2=TI3*E*ZI(I)/(TI7*EL(I))
          SCM3=TI5*E*ZI(I)/(TI7*EL(I))
          SCM4=TI6*E*ZI(I)/(TI7*EL(I))
          SCM5=TI10*E*ZI(I)/(TI7*EL(I)**3)
          SCM6=TI8*E*ZI(I)/(TI7*EL(I)**2)
          SCM7=TI9*E*ZI(I)/(TI7*EL(I)**2)
        SMS(1,1)=SCM1*CX(I)*CX(I)+SCM5*CY(I)*CY(I)
        SMS(1,2)=(SCM1-SCM5)*CX(I)*CY(I)
        SMS(1,3)=-SCM6*CY(I)
        SMS(1,4)=-SMS(1,1)
        SMS(1,5)=-SMS(1,2)
        SMS(1,6)=-SCM7*CY(I)
        SMS(2,2)=SCM1*CY(I)*CY(I)+SCM5*CX(I)*CX(I)
        SMS(2,3)=SCM6*CX(I)
        SMS(2,4)=-SMS(1,2)
        SMS(2,5)=-SMS(2,2)
        SMS(2,6)=SCM7*CX(I)
        SMS(3,3)=SCM2
        SMS(3,4)=-SMS(1,3)
        SMS(3,5)=-SMS(2,3)
        SMS(3,6)=SCM3
        SMS(4,4)=SMS(1,1)
        SMS(4,5)=SMS(1,2)
        SMS(4,6)=-SMS(1,6)
        SMS(5,5)=SMS(2,2)
        SMS(5,6)=-SMS(2,6)
        SMS(6,6)=SCM4
!
! B. TRANSFER TO JOINT STIFFNESS MATRIX
!
      IM(1)=3*JJ(I)-2
      IM(2)=3*JJ(I)-1
      IM(3)=3*JJ(I)
      IM(4)=3*JK(I)-2
      IM(5)=3*JK(I)-1
      IM(6)=3*JK(I)
      DO J=1,MD
      I1=IM(J)
      IF(JRL(I1)>0)GOTO 4
      DO K=J,MD
      I2=IM(K)
      IF(JRL(I2)>0)GOTO 5
      IR=ID(I1)
      IC=ID(I2)
      IF(IR<IC)GOTO 2
      ITEM=IR
      IR=IC
      IC=ITEM
  2   IC=IC-IR+1
      SFF(IR,IC)=SFF(IR,IC)+SMS(J,K)
  5   END DO
  4   END DO
      END DO
  601 FORMAT(//25X,'*** ERROR ***',//'          *****  CHECK YOUR DIMENSIONS
:ALFA,BETA,L  *****')
```

```
  602 FORMAT(//25X,'*** ERROR ***',//'          *****  CHECK YOUR DEPTH
:dL ,dC ,dR   *****')
  603 FORMAT(//25X,'*** ERROR ***',//'          *****  YOUR MEMBER TYPE IS NOT
CORRECT  !! *****')
      RETURN
      END SUBROUTINE STIFF3
!
SUBROUTINE STEEPED(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
   REAL::AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4,Y1,Y2
   Y1=0.0;Y2=AL
   CALL SEGMENTX(Y1,Y2,VW11,VW12,VW13,VW14)
   Y1=AL; Y2=1.0-BL
   CALL SEGMENTX(Y1,Y2,VX11,VX12,VX13,VX14)
   Y1=1.0-BL;Y2=1.0
   CALL SEGMENTX(Y1,Y2,VY11,VY12,VY13,VY14)
   TI0=VW11/(1.0+Rj)+VX11+VY11/(1.0+Rk)
   TI1=VW11/(1.0+Rj)**3+VX11+VY11/(1.0+Rk)**3
   TI2=VW12/(1.0+Rj)**3+VX12+VY12/(1.0+Rk)**3
   TI3=VW13/(1.0+Rj)**3+VX13+VY13/(1.0+Rk)**3
   TI4=VW14/(1.0+Rj)**3+VX14+VY14/(1.0+Rk)**3
   END SUBROUTINE STEEPED
     SUBROUTINE STRAHAUNCH(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
     REAL::AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4
     VW30=0.0;VW31=0.0;VW32=0.0;VW33=0.0;VW34=0.0
     VX31=0.0;VX32=0.0;VX33=0.0;VX34=0.0
     VY30=0.0;VY31=0.0;VY32=0.0;VY33=0.0;VY34=0.0
     IF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
          CALL SEGMENTR(rj,AL,VW30,VW31,VW32,VW33,VW34)
          Y1=AL
          Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL SEGMENTR(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
          Y1=0.0
          Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL SEGMENTR(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
          CALL SEGMENTR(rj,AL,VW30,VW31,VW32,VW33,VW34)
          Y1=AL
          Y2=1.0
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL==0.0)THEN
          CALL SEGMENTR(rj,1.0,VW30,VW31,VW32,VW33,VW34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
          CALL SEGMENTR(rk,1.0,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
          CALL SEGMENTR(rj,AL,VW30,VW31,VW32,VW33,VW34)
          CALL SEGMENTR(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
      Y1=0.0; Y2=1.0
      CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
      END IF
      TI0=VW30+VX31+VY30
      TI1=VW31+VX31+VY31
      TI2=VW32+VX32-VY32+VY31
      TI3=VW33+VX33+VY33-2.0*VY32+VY31
```

```
      TI4=VW34+VX34-VY34+3.0*VY33-3.0*VY32+VY31
     END SUBROUTINE STRAHAUNCH
!
  SUBROUTINE PARAHAUNCH(AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4)
  REAL::AL,BL,RJ,RK,TI0,TI1,TI2,TI3,TI4
     VW30=0.0;VW31=0.0;VW32=0.0;VW33=0.0;VW34=0.0
     VX31=0.0;VX32=0.0;VX33=0.0;VX34=0.0
     VY30=0.0;VY31=0.0;VY32=0.0;VY33=0.0;VY34=0.0
     IF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
          CALL SEGMENTW(rj,AL,VW30,VW31,VW32,VW33,VW34)
          Y1=AL
          Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL SEGMENTY(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
          Y1=0.0
          Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL SEGMENTY(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
          CALL SEGMENTW(rj,AL,VW30,VW31,VW32,VW33,VW34)
          Y1=AL
          Y2=1.0
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL==0.0)THEN
          CALL SEGMENTW(rj,1.0,VW30,VW31,VW32,VW33,VW34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
          CALL SEGMENTY(rk,1.0,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
          CALL SEGMENTW(rj,AL,VW30,VW31,VW32,VW33,VW34)
          CALL SEGMENTY(rk,BL,VY30,VY31,VY32,VY33,VY34)
      ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
      Y1=0.0; Y2=1.0
      CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
      END IF
      TI0=VW30+VX31+VY30
      TI1=VW31+VX31+VY31
      TI2=VW32+VX32+VY32+(1.0-2.0*BL)*VY31
      TI3=VW33+VX33+VY33+2.0*(1.0-2.0*BL)*VY32+((1.0-2.0*BL)**2.)*VY31
      TI4=VW34+VX34+VY34+3.0*(1.0-2.0*BL)*VY33+3.0*((1.0-
2.0*BL)**2.)*VY32+((1.0-2.0*BL)**3.)*VY31
      END SUBROUTINE PARAHAUNCH
!
SUBROUTINE SEGMENTW(r,AL,VW30,VW31,VW32,VW33,VW34)
REAL:: r,AL,VW30,VW31,VW32,VW33,VW34
a=ATAN(SQRT(r))
VW30=AL*a/SQRT(r)
VW31=(AL/8.0)*((5.0+3.0*r)/(1.0+r)**2.+3.0*a/SQRT(r))
VW32=(AL**2./(8.0*r))*(r/(r+1.0)+3.0*a*SQRT(r))
VW33=(AL**3./(8.0*(r**2.0)))*(-r+SQRT(r)*(3.0*r+1.0)*a)
VW34=(AL**4./(8.0*(r**2.0)))*(-3.0*r+3.0*SQRT(r)*(r+1.0)*a)
RETURN
END SUBROUTINE SEGMENTW
!
SUBROUTINE SEGMENTY(r,BL,VY30,VY31,VY32,VY33,VY34)
REAL r,BL,VY30,VY31,VY32,VY33,VY34
a=ATAN(SQRT(r))
```

```
VY30=BL*a/SQRT(r)
VY31=(BL/8.0)*((5.0+3.0*r)/(1.0+r)**2.+3.0*a/SQRT(r))
VY32=(BL**2./(8.0*r))*((5.0*r**2.+9.0*r)/(r+1.0)**2+3.0*a*SQRT(r))
VY33=(BL**3./(8.0*(r**2.)))*((7.*r**3.+14.*r**2-
r)/(r+1.)**2.+SQRT(r)*(3.*r+1)*a)
VY34=(BL**4./(8.0*(r**2.)))*((9.0*r**3.+22.0*r**2-
3.0*r)/(r+1.0)**2.+3.0*SQRT(r)*(r+1.0)*a)
RETURN
!
END SUBROUTINE SEGMENTY
   SUBROUTINE SEGMENTR(r,AL,VR30,VR31,VR32,VR33,VR34)
   REAL r,AL,VR30,VR31,VR32,VR33,VR34
   a=LOG(1.0+r)
   VR30=AL*a/r
   VR31=(AL/2.0)*(2.0+r)/(1.0+r)**2
   VR32=AL**2/(2.0*(1.0+r))
   VR33=-((AL/r)**3)*(r*(2.0-r)/2.0-a)
   VR34=((AL/r)**4)*(r*(r**2-3.0*r-6.0)/2.0+3.0*(1.0+r)*a)
   END SUBROUTINE SEGMENTR
!
  SUBROUTINE SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
  REAL:: Y1,Y2,VX31,VX32,VX33,VX34
  VX31=Y2-Y1
  VX32=0.5*(Y2**2.-Y1**2)
  VX33=(Y2**3.-Y1**3.)/3.0
  VX34=(Y2**4.-Y1**4.)/4.0
  RETURN
  END SUBROUTINE SEGMENTX
!
! *** FACTOR JOINT STIFFNESS MATRIX ***
!
      SUBROUTINE BANFAC (N,NB,A,*)
      DIMENSION A(N,NB)
            IF (A(1,1)<=0.0) RETURN 1
            DO J=2,N
            J1=J-1
            J2=J-NB+1
            IF (J2<1) J2=1
            IF (J1==1) GO TO 3
            DO I=2,J1
            I1=I-1
            IF (I1<J2) GO TO 2
            SUM1=A(I,J-I+1)
            DO K=J2,I1
      SUM1=SUM1-A(K,I-K+1)*A(K,J-K+1)
        END DO
            A(I,J-I+1)=SUM1
2           END DO
3           SUM1=A(J,1)
            DO K=J2,J1
            TEMP=A(K,J-K+1)/A(K,1)
            SUM1=SUM1-A(K,J-K+1)*TEMP
            A(K,J-K+1)=TEMP
        END DO
            IF (SUM1<=0.0) RETURN 1
      A(J,1)=SUM1
        END DO
```

```
             RETURN
             END SUBROUTINE BANFAC
!
! *** READ AND PRINT LOAD DATA FOR PLANE FRAME ***
!
      SUBROUTINE
LDATA3(M,ND,LN,MT,EL,ELL,ELR,HJ,HC,HK,TIL1,TIL2,TIL3,TIL4,AJ,AE,AR,AML,NLM,NLJ
,LML,*)
      INTEGER::M,LN

REAL::AJ(:),AE(:),AR(:),AML(:,:),EL(:),TIL1(:),TIL2(:),TIL3(:),TIL4(:),ELL(:),
ELR(:),HJ(:),HC(:),
      HK(:)
      INTEGER::NLM,NLJ,LML(:),MT(:),IDD(M)
! A. LOAD PARAMETERS
        WRITE(6,505)
             WRITE(6,601)LN
        WRITE(7,607)LN
             READ(5, *)NLJ,NLM
             WRITE(6,602)NLJ,NLM
! B. JOINT LOADS
             DO J=1,ND
             AJ(J)=0.0
             AE(J)=0.0
      AR(J)=0.0
        END DO
             IF (NLJ==0) GO TO 2
             WRITE(6,603)
             DO J=1,NLJ
             READ(5, *)K,AJ(3*K-2),AJ(3*K-1),AJ(3*K)
      WRITE(6,604)K,AJ(3*K-2),AJ(3*K-1),AJ(3*K)
        END DO
    2        CONTINUE
! C. MEMBER LOADS
        IF (NLM==0) GO TO 4
      DO J=1,M
             LML(J)=0
        IDD(J)=0
             DO K=1,6
      AML(K,J)=0.0
        END DO
        END DO
             WRITE(6,501)
             DO J=1,NLM
        READ(5,*)I,NLUM,NLPM
        IDD(I)=1
        MTT=MT(I)
        TI1=TIL1(I);TI2=TIL2(I);TI3=TIL3(I);TI4=TIL4(I)
        SUM1=0.0;SUM2=0.0
        SVW1=0.0;SVW2=0.0
        SVP1=0.0;SVP2=0.0
        !UNIFORMLY DIST. LOAD
             IF(NLUM/=0.0)THEN
             DO K=1,NLUM
             READ(5,*)X1,Wa,X2,Wb
             WRITE(6,502)I,K,X1,Wa,X2,Wb
             IF(X1<0.0.OR.X2>EL(I))THEN
```

```
            WRITE(*,608)
            RETURN 1
            END IF
            EL1=X1/EL(I);EL2=X2/EL(I)
            AL=ELL(I)/EL(I);BL=ELR(I)/EL(I)
            RJ=(HJ(I)-HC(I))/HC(I);RK=(HK(I)-HC(I))/HC(I)
            IF (Wa/=0.0) THEN
            Wk=Wa
            ELSE
            Wk=Wb
            END IF
            IF(Wa-Wb==0.0.AND.EL1==0.0.AND.X2-EL(I)==0.0)THEN
            CMAB=0.5*(TI3*TI3-TI2*TI4)/(TI3*TI1-TI2**2.)
            CMBA=0.5*(TI2**2.-TI3*(TI1+TI2-TI3)+TI4*(TI1-TI2))/(TI3*TI1-
TI2**2.)
            ELSE
            CALL UDL(MTT,AL,BL,RJ,RK,Wa,Wb,EL1,EL2,TI1,TI2,TI3,CMAB,CMBA)
            Wk=1.0
            END IF
            SUM1=SUM1+CMAB*Wk*EL(I)*EL(I)
            SUM2=SUM2+CMBA*Wk*EL(I)*EL(I)
IF(Wb-Wa>0.0)THEN
SVW1=SVW1+(3.0*EL(I)-(2.0*X2+X1))*(Wb-Wa)*(X2-X1)/(6.0*EL(I))&
&+(2.0*EL(I)-(X2+X1))*Wa*(X2-X1)/(2.0*EL(I))
SVW2=SVW2+(2.0*X2+X1)*(Wb-Wa)*(X2-X1)/(6.0*EL(I))&
&+(X2+X1)*Wa*(X2-X1)/(2.0*EL(I))
ELSEIF(Wb-Wa<0.0)THEN
SVW1=SVW1+(2.0*X2+X1)*(Wa-Wb)*(X2-X1)/(6.0*EL(I))&
&+(2.0*EL(I)-(X2+X1))*Wb*(X2-X1)/(2.0*EL(I))
SVW2=SVW2+(3.0*EL(I)-(2.0*X2+X1))*(Wa-Wb)*(X2-X1)/(6.0*EL(I))&
&+(X2+X1)*Wb*(X2-X1)/(2.0*EL(I))
ELSE
SVW1=SVW1+(2.0*EL(I)-(X2+X1))*Wa*(X2-X1)/(2.0*EL(I))
SVW2=SVW2+(X2+X1)*Wa*(X2-X1)/(2.0*EL(I))
END IF
           END DO
           END IF
         !CONCENTRATED LOAD
         IF(NLPM/=0.0)THEN
         DO K=1,NLPM
         READ(5,*)X3,P
         WRITE(6,503)I,K,X3,P
         IF(X3<0.0.OR.X3>EL(I))THEN
         WRITE(*,609)
         RETURN 1
         END IF
         CL=X3/EL(I)
         AL=ELL(I)/EL(I);BL=ELR(I)/EL(I)
         RJ=(HJ(I)-HC(I))/HC(I);RK=(HK(I)-HC(I))/HC(I)
         CALL CONCLOAD(MTT,AL,BL,CL,RJ,RK,TI1,TI2,TI3,PAB,PBA)
         SUM1=SUM1+PAB*P*EL(I)
         SUM2=SUM2+PBA*P*EL(I)
         SVP1=SVP1+(EL(I)-X3)*P/EL(I)
         SVP2=SVP2+X3*P/EL(I)
         END DO
         END IF
         AML(1,I)=0.0;AML(2,I)=SVW1+SVP1+(-SUM1+SUM2)/EL(I);AML(3,I)=-SUM1
```

```
         AML(4,I)=0.0;AML(5,I)=SVW2+SVP2+(-SUM2+SUM1)/EL(I);AML(6,I)=SUM2
      LML(I)=1
        END DO
        WRITE(6,505)
        WRITE(6,605)
    4       CONTINUE
        DO I=1,M
        IF (IDD(I)==1)THEN
        WRITE(6,606)I,AML(1,I),AML(2,I),AML(3,I),AML(4,I),AML(5,I),AML(6,I)
        END IF
        END DO
        WRITE(6,505)
        RETURN
  501       FORMAT(/,10X,' MEMBER LOAD',/,10X,'MEMBER  LD NO  DISTANCE-A  &
        &VALUE-A   DISTANCE-B  VALUE-B')
  502       FORMAT(/,11X,2I5,4(F11.3))
  503       FORMAT(/,11x,2I5,2(F11.3))
  505   FORMAT(10X,60('-'))
  601       FORMAT(//,10X,' LOAD CASE  ',I2,/,10X,'   NLJ  NLM')
  602       FORMAT(' ',10X,2I5)
  603       FORMAT(/,10X,' ACTIONS AT JOINTS',/,10X,' JOINT       Px&
     &        Py         M')
  604       FORMAT(' ',10X,I5,3F10.3)
  605       FORMAT(/,10X,' ACTIONS AT ENDS OF RESTRAINED MEMBERS DUE &
     &  TO LOADS',/,                                                 &
     & 10X,'   MEMBER    P-J      V-J      M-J      P-K      V-K&
     &      M-K')
  606       FORMAT(' ',10X,I5,2X,6F9.3)
  607   FORMAT(10X,' LOAD CASE  ',I2)
  608   FORMAT(//25X,'*** ERROR ***',//' *****  CHECK YOUR LOADING  DISTANCE-
A,DISTANCE-B  *****')
  609   FORMAT(//25X,'*** ERROR ***',//'           *****  CHECK YOUR LOADING
DISTANCE-A    *****')
            END SUBROUTINE LDATA3
!UDL
SUBROUTINE UDL(MTT,AL,BL,RJ,RK,Wa,Wb,EL1,EL2,TI1,TI2,TI3,CMAB,CMBA)
PARAMETER (N=50)
REAL::X(N+1),W(N+1),CCMAB(N+1),CCMBA(N+1)
    Sw=(Wb-Wa)/(EL2-EL1)
    dX=(EL2-EL1)/N
    W(1)=Wa
    X(1)=EL1
DO I=2,N+1
X(I)=EL1+(I-1)*dx
W(I)=W(I-1)+Sw*dx
END DO
  DO J=1,N+1
  CL1=X(J)
  CALL CONCLOAD(MTT,AL,BL,CL1,RJ,RK,TI1,TI2,TI3,PAB1,PBA1)
  CCMAB(J)=PAB1*W(J)
  CCMBA(J)=PBA1*W(J)
  END DO
SUM1=0.0;SUM2=0.0
DO J=1,N-1,2
SUM1=SUM1+CCMAB(J)+4.0*CCMAB(J+1)+CCMAB(J+2)
SUM2=SUM2+CCMBA(J)+4.0*CCMBA(J+1)+CCMBA(J+2)
END DO
```

```
CMAB=SUM1*dx/3.0
CMBA=SUM2*dx/3.0
END SUBROUTINE UDL
!CONCENTRATED LOAD
SUBROUTINE CONCLOAD(MTT,AL,BL,CL,RJ,RK,TI1,TI2,TI3,PAB,PBA)
REAL::AL,BL,CL,RJ,RK,TI1,TI2,TI3,PAB,PBA
      IF (MTT==1) THEN
      CALL LDSTEPD(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
      ELSE IF(MTT==2)THEN
      CALL LDSTRAHA(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
      ELSE IF(MTT==3)THEN
      CALL LDPARAHA(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
      END IF
!
 C1=-(E1*(1.0-CL)+F1*CL)
 C2=-(E2*CL+F2*(1.0-CL))
 A1=TI2-TI3;B2=A1
 B1=TI3;A2=TI1-2.0*TI2+TI3
 PAB=(B2*C1-B1*C2)/(A1*B2-B1*A2)
 PBA=(A1*C2-A2*C1)/(A1*B2-B1*A2)
END SUBROUTINE CONCLOAD
!
SUBROUTINE LDSTEPD(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
      IF(CL<=AL)THEN
          CALL SEGMENTX(0.0,CL,VX11,VX12,VX13,VX14)
          E1=VX13/(1.0+Rj)**3;F2=VX12/(1.0+Rj)**3-E1
          CALL SEGMENTX(0.0,BL,VX11,VX12,VX13,VX14)
          CALL SEGMENTX(BL,1.0-AL,VX21,VX22,VX23,VX24)
          CALL SEGMENTX(1.0-AL,1.0-CL,VX31,VX32,VX33,VX34)
          E2=VX13/(1.0+Rj)**3+VX23+VX33/(1.0+Rk)**3
          F1=VX12/(1.0+Rj)**3+VX22+VX32/(1.0+Rk)**3-E2
      ELSEIF(AL<CL.AND.CL<=1.0-BL)THEN
          CALL SEGMENTX(0.0,AL,VX11,VX12,VX13,VX14)
          CALL SEGMENTX(AL,CL,VX21,VX22,VX23,VX24)
          E1=VX13/(1.0+Rj)**3+VX23;F2=VX12/(1.0+Rj)**3+VX22-E1
          CALL SEGMENTX(BL,1.0-CL,VX21,VX22,VX23,VX24)
          CALL SEGMENTX(0.0,BL,VX31,VX32,VX33,VX34)
          E2=VX23+VX33/(1.0+Rk)**3;F1=VX22+VX32/(1.0+Rk)**3-E2
     ELSEIF(1.0-BL<CL.AND.CL<1.0)THEN
          CALL SEGMENTX(0.0,AL,VX11,VX12,VX13,VX14)
          CALL SEGMENTX(AL,1.0-BL,VX21,VX22,VX23,VX24)
          CALL SEGMENTX(1.0-BL,CL,VX31,VX32,VX33,VX34)
          E1=VX13/(1.0+Rj)**3+VX23+VX33/(1.0+Rk)**3
          F2=VX12/(1.0+Rj)**3+VX22+VX32/(1.0+Rk)**3-E1
          CALL SEGMENTX(0.0,1.0-CL,VX31,VX32,VX33,VX34)
          E2=VX33/(1.0+Rk)**3;F1=VX32/(1.0+Rk)**3-E2
     ENDIF
RETURN
END SUBROUTINE LDSTEPD
SUBROUTINE LDSTRAHA(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
IF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
      IF(CL<=AL)THEN
          CALL INTEVALUE2(rj,AL,0.0,CL,VX2,VX1,V0)
          E1=VX2;F2=VX1-E1
          CALL SEGMENTR(rK,BL,VR30,VR31,VR32,VR33,VR34)
          Y1=BL;Y2=1.0-AL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
```

```
          CALL INTEVALUE2(rj,AL,CL,AL,VX2,VX1,V0)
          E2=VR33+VX33+VX2-2.0*VX1+V0
          F1=VR32+VX32-VX1+V0-E2
      ELSEIF(AL<CL.AND.CL<=1.0-BL)THEN
          CALL SEGMENTR(rj,AL,VR30,VR31,VR32,VR33,VR34)
          Y1=AL;Y2=CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E1=VR33+VX33;F2=VR32+VX32-E1
          CALL SEGMENTR(rK,BL,VR30,VR31,VR32,VR33,VR34)
          Y1=BL;Y2=1.0-CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E2=VR33+VX33;F1=VR32+VX32-E2
     ELSEIF(1.0-BL<CL.AND.CL<1.0)THEN
          CALL SEGMENTR(rj,AL,VR30,VR31,VR32,VR33,VR34)
          Y1=AL;Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL INTEVALUE2(rK,BL,1.0-CL,BL,VX2,VX1,V0)
          E1=VR33+VX33+VX2-2.0*VX1+V0
          F2=VR32+VX32-VX1+V0-E1
          CALL INTEVALUE2(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
          E2=VX2;F1=VX1-E2
     ENDIF
ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
     IF(CL<=1.0-BL)THEN
          Y1=0.0;Y2=CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E1=VX33;F2=VX32-E1
          Y1=BL;Y2=1.0-CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL SEGMENTR(rK,BL,VR30,VR31,VR32,VR33,VR34)
          E2=VR33+VX33;F1=VR32+VX32-E2
     ELSEIF(1.0-BL<CL.AND.CL<1.0)THEN
          Y1=0.0;Y2=1.0-BL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL INTEVALUE2(rK,BL,1.0-CL,BL,VX2,VX1,V0)
          E1=VX33+VX2-2.0*VX1+V0
          F2=VX32-VX1+V0-E1
          CALL INTEVALUE2(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
          E2=VX2;F1=VX1-E2
          ENDIF
ELSEIF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
          IF(CL<=AL)THEN
          CALL INTEVALUE2(rj,AL,0.0,CL,VX2,VX1,V0)
          E1=VX2;F2=VX1-E1
          Y1=0.0;Y2=1.0-AL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL INTEVALUE2(rj,AL,CL,AL,VX2,VX1,V0)
          E2=VX33+VX2-2.0*VX1+V0
          F1=VX32-VX1+V0-E2
    ELSEIF(AL<CL.AND.CL<1.0)THEN
          CALL SEGMENTR(rJ,AL,VR30,VR31,VR32,VR33,VR34)
          Y1=AL;Y2=CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E1=VR33+VX33;F2=VR32+VX32-E1
          Y1=0.0;Y2=1.0-CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E2=VX33;F1=VX32-E2
```

```
     ENDIF
ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL==0.0)THEN
          CALL INTEVALUE2(rJ,AL,0.0,CL,VX2,VX1,V0)
          E1=VX2;F2=VX1-E1
          CALL INTEVALUE2(rJ,AL,CL,AL,VX2,VX1,V0)
          E2=VX2-2.0*VX1+V0
          F1=-VX1+V0-E2
ELSEIF (AL==0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
          CALL INTEVALUE2(rK,BL,1.0-CL,BL,VX2,VX1,V0)
          E1=VX2-2.0*VX1+V0
          F2=-VX1+V0-E1
          CALL INTEVALUE2(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
          E2=VX2;F1=VX1-E2
ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
        IF(CL<=AL)THEN
        CALL INTEVALUE2(rJ,AL,0.0,CL,VX2,VX1,V0)
        E1=VX2;F2=VX1-E1
        CALL INTEVALUE2(rJ,AL,CL,AL,VX2,VX1,V0)
        CALL SEGMENTR(rK,BL,VR30,VR31,VR32,VR33,VR34)
        E2=VR33+VX2-2.0*VX1+V0
        F1=VR32-VX1+V0-E2
        ELSEIF(AL<CL.AND.CL<1.0)THEN
        CALL SEGMENTR(rJ,AL,VR30,VR31,VR32,VR33,VR34)
        CALL INTEVALUE2(rK,BL,1.0-CL,BL,VX2,VX1,V0)
        E1=VR33+VX2-2*VX1+V0
        F2=VR32-VX1+V0-E1
        CALL INTEVALUE2(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
        E2=VX2;F1=VX1-E2
        ENDIF
ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
        Y1=0.0; Y2=CL
        CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
        E1=VX33;F2=VX32-E1
        Y1=0.0;Y2=1.0-CL
        CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
        E2=VX33;F1=VX32-E2
END IF
RETURN
END SUBROUTINE LDSTRAHA
!
SUBROUTINE LDPARAHA(AL,BL,RJ,RK,CL,E1,E2,F1,F2)
IF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
      IF(CL<=AL)THEN
          CALL INTEVALUE3(rj,AL,0.0,CL,VX2,VX1,V0)
          E1=VX2;F2=VX1-E1
          CALL SEGMENTW(rK,BL,VW30,VW31,VW32,VW33,VW34)
          Y1=BL;Y2=1.0-AL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          CALL INTEVALUE3(rj,AL,AL,2.0*AL-CL,VX2,VX1,V0)
          E2=VW33+VX33+VX2+2.0*(1.0-2.0*AL)*VX1+((1.0-2.0*AL)**2)*V0
          F1=VW32+VX32+VX1+(1.0-2.0*AL)*V0-E2
      ELSEIF(AL<CL.AND.CL<=1.0-BL)THEN
          CALL SEGMENTW(rj,AL,VW30,VW31,VW32,VW33,VW34)
          Y1=AL;Y2=CL
          CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
          E1=VW33+VX33;F2=VW32+VX32-E1
          CALL SEGMENTW(rK,BL,VW30,VW31,VW32,VW33,VW34)
```

```
            Y1=BL;Y2=1.0-CL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            E2=VW33+VX33;F1=VW32+VX32-E2
       ELSEIF(1.0-BL<CL.AND.CL<1.0)THEN
            CALL SEGMENTW(rj,AL,VW30,VW31,VW32,VW33,VW34)
            Y1=AL;Y2=1.0-BL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            CALL INTEVALUE3(rK,BL,BL,2.*BL-1.0+CL,VX2,VX1,V0)
            E1=VW33+VX33+VX2+2.0*(1.0-2.*BL)*VX1+((1.0-2*BL)**2)*V0
            F2=VW32+VX32+VX1+(1.0-2.*BL)*V0-E1
            CALL INTEVALUE3(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
            E2=VX2;F1=VX1-E2
       ENDIF
  ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL/=0.0)THEN
       IF(CL<=1.0-BL)THEN
            Y1=0.0;Y2=CL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            E1=VX33;F2=VX32-E1
            Y1=BL;Y2=1.0-CL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            CALL SEGMENTW(rK,BL,VW30,VW31,VW32,VW33,VW34)
            E2=VW33+VX33;F1=VW32+VX32-E2
       ELSEIF(1.0-BL<CL.AND.CL<1.0)THEN
            Y1=0.0;Y2=1.0-BL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            CALL INTEVALUE3(rK,BL,BL,2.*BL-1.0+CL,VX2,VX1,V0)
            E1=VX33+VX2+2.0*(1.0-2.0*BL)*VX1+((1.0-2.0*BL)**2)*V0
            F2=VX32+VX1+(1.0-2.0*BL)*V0-E1
            CALL INTEVALUE3(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
            E2=VX2;F1=VX1-E2
            ENDIF
  ELSEIF (AL/=0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
       IF(CL<=AL)THEN
            CALL INTEVALUE3(rj,AL,0.0,CL,VX2,VX1,V0)
            E1=VX2;F2=VX1-E1
            Y1=0.0;Y2=1.0-AL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            CALL INTEVALUE3(rj,AL,AL,2*AL-CL,VX2,VX1,V0)
            E2=VX33+VX2+2.0*(1.0-2*AL)*VX1+((1.0-2.0*AL)**2)*V0
            F1=VX32+VX1+(1.0-2.0*AL)*V0-E2
      ELSEIF(AL<CL.AND.CL<1.0)THEN
            CALL SEGMENTW(rJ,AL,VW30,VW31,VW32,VW33,VW34)
            Y1=AL;Y2=CL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            E1=VW33+VX33;F2=VW32+VX32-E1
            Y1=0.0;Y2=1.0-CL
            CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
            E2=VX33;F1=VX32-E2
       ENDIF
  ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL==0.0)THEN
            CALL INTEVALUE3(rJ,AL,0.0,CL,VX2,VX1,V0)
            E1=VX2;F2=VX1-E1
            CALL INTEVALUE3(rJ,AL,AL,2.0*AL-CL,VX2,VX1,V0)
            E2=VX2+2.0*(1.0-2.0*AL)*VX1+((1.0-2.0*AL)**2)*V0
            F1=VX1+(1.0-2.0*AL)*V0-E2
  ELSEIF (AL==0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
            CALL INTEVALUE3(rK,BL,BL,2.0*BL+CL-1.0,VX2,VX1,V0)
```

```
          E1=VX2+2.0*(1.0-2.0*BL)*VX1+((1.0-2.0*BL)**2)*V0
          F2=VX1+(1.0-2.0*BL)*V0-E1
          CALL INTEVALUE3(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
          E2=VX2;F1=VX1-E2
ELSEIF (AL/=0.0.AND.1.0-BL-AL==0.0.AND.BL/=0.0)THEN
        IF(CL<=AL)THEN
        CALL INTEVALUE3(rJ,AL,0.0,CL,VX2,VX1,V0)
        E1=VX2;F2=VX1-E1
        CALL INTEVALUE3(rJ,AL,AL,2*AL-CL,VX2,VX1,V0)
        CALL SEGMENTW(rK,BL,VW30,VW31,VW32,VW33,VW34)
        E2=VW33+VX2+2.0*(1.0-2*AL)*VX1+((1.0-2.0*AL)**2)*V0
        F1=VW32+VX1+(1.0-2.0*AL)*V0-E2
        ELSEIF(AL<CL.AND.CL<1.0)THEN
        CALL SEGMENTW(rJ,AL,VW30,VW31,VW32,VW33,VW34)
        CALL INTEVALUE3(rK,BL,BL,2.0*BL+CL-1.0,VX2,VX1,V0)
        E1=VW33+VX2+2.0*(1.0-2.0*BL)*VX1+((1.0-2.0*BL)**2)*V0
        F2=VW32+VX1+(1.0-2.0*BL)*V0-E1
        CALL INTEVALUE3(rK,BL,0.0,1.0-CL,VX2,VX1,V0)
        E2=VX2;F1=VX1-E2
        ENDIF
ELSEIF (AL==0.0.AND.1.0-BL-AL/=0.0.AND.BL==0.0)THEN
        Y1=0.0; Y2=CL
        CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
        E1=VX33;F2=VX32-E1
        Y1=0.0;Y2=1.0-CL
        CALL SEGMENTX(Y1,Y2,VX31,VX32,VX33,VX34)
        E2=VX33;F1=VX32-E2
END IF
RETURN
END SUBROUTINE LDPARAHA
!
SUBROUTINE INTEVALUE2(r,AL,X1,X2,VX2,VX1,V0)
REAL Y,X1,X2
Y=1.0-X1/AL
DO I=1,2
dx=(AL/(2.0*r))*(1.0/(1.0+r*Y)**2)
Xdx=((AL/r)**2)*((1.0+r)/(2.0*(1.0+r*Y)**2.)-1.0/(1.0+r*Y))
X2dx=-((AL/r)**3)*(-
(1.0+r)**2/(2.0*(1.0+r*Y)**2)+2.0*(1.0+r)/(1.0+r*Y)+LOG(1.0+r*Y))
IF(I.EQ.2)EXIT
X2dxI=X2dx
XdxI=Xdx
dxI=dx
Y=1.0-X2/AL
END DO
    VX2=X2dx-X2dxI
    VX1=Xdx-XdxI
    V0= dx-dxI
END SUBROUTINE INTEVALUE2
!
SUBROUTINE INTEVALUE3(r,AL,X1,X2,VX2,VX1,V0)
REAL Y,X1,X2
Y=1.0-X1/AL
DO I=1,2
a=ATAN(Y*SQRT(r))
dx=-(AL/8.0)*((2.0*Y+3.0*Y*(1.0+r*Y*Y))/(1.0+r*Y*Y)**2+3.0*a/SQRT(r))
```

```
Xdx=-
(AL**2.0/(8.0*r))*((3.0*r*r*Y**3+5.0*r*Y+2.0)/(1.0+r*Y*Y)**2+3.0*a*SQRT(r))
X2dx=-(AL**3.0/(8.0*(r**2.)))*((((3.0*r**3+r*r)*Y**3)+(5.0*r*r-
r)*Y+4.0*r)/(1.0+r*Y*Y)**2
     +SQRT(r)*(3.0*r+1.0)*a)
IF(I.EQ.2)EXIT
X2dxI=X2dx
XdxI=Xdx
dxI=dx
Y=1.0-X2/AL
END DO
    VX2=X2dx-X2dxI
    VX1=Xdx-XdxI
    V0= dx-dxI
END SUBROUTINE INTEVALUE3
!
! *** CONSTRUCT LOAD VECTOR FOR PLANE FRAME ***
!
      SUBROUTINE LOADS3(M,ND,NLM,LML,ID,JJ,JK,CX,CY,AML,AJ,AE,AC)
      REAL::CX(:),CY(:),AML(:,:),AJ(:),AC(:),AE(:)
      INTEGER::JJ(:),JK(:),ID(:),LML(:),M
!
! A. EQUIVALENT JOINT LOADS
        IF (NLM==0) GO TO 2
        DO I=1,M
        IF (LML(I)==0) GO TO 1
        J1=3*JJ(I)-2
      J2=3*JJ(I)-1
      J3=3*JJ(I)
      K1=3*JK(I)-2
      K2=3*JK(I)-1
      K3=3*JK(I)
      AE(J1)=AE(J1)-CX(I)*AML(1,I)+CY(I)*AML(2,I)
      AE(J2)=AE(J2)-CY(I)*AML(1,I)-CX(I)*AML(2,I)
      AE(J3)=AE(J3)-AML(3,I)
      AE(K1)=AE(K1)-CX(I)*AML(4,I)+CY(I)*AML(5,I)
      AE(K2)=AE(K2)-CY(I)*AML(4,I)-CX(I)*AML(5,I)
      AE(K3)=AE(K3)-AML(6,I)
    1   END DO
    2   CONTINUE
! B. COMBINED JOINT LOADS
        DO J=1,ND
        JR=ID(J)
        AC(JR)=AJ(J)+AE(J)
        END DO
        RETURN
        END SUBROUTINE LOADS3




! *** SOLVE JOINT EQUILIBRIUM EQUATIONS ***
!
      SUBROUTINE BANSOL (N,NB,U,B,X)
      DIMENSION U(N,NB), B(N), X(N)
!
            DO I=1,N
```

```
            J=I-NB+1
            IF (I<=NB) J=1
            SUM1=B(I)
            K1=I-1
            IF (J>K1) GO TO 2
            DO K=J,K1
      SUM1 = SUM1 - U(K,I-K+1)*X(K)
        END DO
    2       X(I) = SUM1
        END DO
            DO I=1,N
      X(I)=X(I)/U(I,1)
        END DO
            DO I1=1,N
            I=N-I1+1
            J=I+NB-1
            IF (J>N) J=N
            SUM1 = X(I)
            K2 = I+1
            IF (K2>J) GO TO 5
            DO K=K2,J
      SUM1 = SUM1 - U(I,K-I+1)*X(K)
        END DO
     5      X(I) = SUM1
        END DO
            RETURN
            END SUBROUTINE BANSOL
!
! *** CALCULATE AND PRINT RESULTS FOR PLANE FRAME ***
!
      SUBROUTINE
RESUL3(M,NJ,ND,EL,AX,E,ZI,JJ,JK,JRL,ID,CX,CY,DF,AJ,AE,AR,AML,MT,TIL0,TIL1,TIL2
,TIL3)
      REAL::DJ(ND),AMD(6),AML(:,:),AM(6),TIL0(:),TIL1(:),TIL2(:),TIL3(:)
      REAL:: DF(:),AX(:),ZI(:),CX(:),CY(:),EL(:),AJ(:),AE(:),AR(:),E
      INTEGER::JJ(:),JK(:),JRL(:),ID(:),MT(:),NJ,M
!
! A. JOINT DISPLACEMENTS
      J=N+1
      DO K=1,ND
      JE=ND-K+1
      IF(JRL(JE)==0)GO TO 1
      DJ(JE)=0.0
      GO TO 2
  1   J=J-1
      DJ(JE)=DF(J)
  2   END DO
        WRITE(7,609)
      WRITE(7,601)
      DO J=1,NJ
      WRITE(7,602)J,DJ(3*J-2),DJ(3*J-1),DJ(3*J)
        END DO
        WRITE(7,609)
! B. MEMBER END ACTIONS
      WRITE(7,603)
      DO I=1,M
      J1=3*JJ(I)-2
```

```
      J2=3*JJ(I)-1
      J3=3*JJ(I)
      K1=3*JK(I)-2
      K2=3*JK(I)-1
      K3=3*JK(I)
      TI0=TIL0(I);TI1=TIL1(I);TI2=TIL2(I);TI3=TIL3(I)
      TI5=TI2-TI3
      TI6=TI1-2*TI2+TI3
      TI7=TI1*TI3-TI2*TI2
      TI8=TI3+TI5
      TI9=TI5+TI6
      TI10=TI8+TI9
          SCM1=E*AX(I)/(TI0*EL(I))
          SCM2=TI3*E*ZI(I)/(TI7*EL(I))
          SCM3=TI5*E*ZI(I)/(TI7*EL(I))
          SCM4=TI6*E*ZI(I)/(TI7*EL(I))
          SCM5=TI10*E*ZI(I)/(TI7*EL(I)**3)
          SCM6=TI8*E*ZI(I)/(TI7*EL(I)**2)
          SCM7=TI9*E*ZI(I)/(TI7*EL(I)**2)
AMD(1)=SCM1*((DJ(J1)-DJ(K1))*CX(I)+(DJ(J2)-DJ(K2))*CY(I))
AMD(2)=SCM5*(-(DJ(J1)-DJ(K1))*CY(I)+(DJ(J2)-
DJ(K2))*CX(I))+SCM6*DJ(J3)+SCM7*DJ(K3)
AMD(3)=SCM6*(-(DJ(J1)-DJ(K1))*CY(I)+(DJ(J2)-
DJ(K2))*CX(I))+SCM2*DJ(J3)+SCM3*DJ(K3)
AMD(4)=-AMD(1)
AMD(5)=-AMD(2)
AMD(6)=SCM7*(-(DJ(J1)-DJ(K1))*CY(I)+(DJ(J2)-
DJ(K2))*CX(I))+SCM3*DJ(J3)+SCM4*DJ(K3)
        DO J=1,MD
        AM(J)=AML(J,I)+AMD(J)
        END DO
        IF(JRL(J1)==1) AR(J1)=AR(J1)+CX(I)*AMD(1)-CY(I)*AMD(2)
      IF(JRL(J2)==1) AR(J2)=AR(J2)+CY(I)*AMD(1)+CX(I)*AMD(2)
      IF(JRL(J3)==1) AR(J3)=AR(J3)+AMD(3)
      IF(JRL(K1)==1) AR(K1)=AR(K1)+CX(I)*AMD(4)-CY(I)*AMD(5)
      IF(JRL(K2)==1) AR(K2)=AR(K2)+CY(I)*AMD(4)+CX(I)*AMD(5)
      IF(JRL(K3)==1) AR(K3)=AR(K3)+AMD(6)
      WRITE(7,604)I,AM(1),AM(2),AM(3),EL(I),AM(4),AM(5),AM(6)
      END DO
      WRITE(7,609)
! C. SUPPORT REACTIONS
      DO J=1,ND
      IF(JRL(J)==0)GO TO 7
      AR(J)=AR(J)-AJ(J)-AE(J)
      7 END DO
      WRITE(7,605)
      DO J=1,NJ
      J1=3*J-2
      J2=3*J-1
      J3=3*J
      N1=JRL(J1)+JRL(J2)+JRL(J3)
      IF(N1==0)GO TO 8
      WRITE(7,606)J,AR(J1),AR(J2),AR(J3)
     8  END DO
        WRITE(*,608)
        WRITE(7,609)
        WRITE(7,607)
```

```
 601 FORMAT(/,10X,'J O I N T  D I S P L A C E M E N T S',/10X,'JOINT
Ux&
     &             Uy              Rz')
  602       FORMAT(' ',10X,I4,3E15.5)
  603       FORMAT(/,10X,' M E M B E R  E N D - A C T I O N S',/,10X,' MEMBER
LOC &
     &      AXIAL F      SHEAR       MOMENT')
  604   FORMAT(10X,I6,50('-'),/23X,'0.00',3F12.3,/22X,F5.2,3F12.3)
  605       FORMAT(/,10X,' S U P P O R T  R E A C T I O N S',/,10X,' JOINT
Fx&
     &             Fy             Mz')
  606       FORMAT(' ',10X,I5,3F15.3)
  607       FORMAT (//,10X,'NOTE:   DIMENTIONS ',//,15X,'LENGTH:  m',/  &
     &  15X,'FORCE:  Kn ',/,15X,'MOMENT:  Kn-m')
  608   FORMAT(//,10X,'*********  A N A L Y S I S   C O M P L E T E
**********')
  609   FORMAT(10X,60('-'))
        RETURN
            END SUBROUTINE RESUL3
        END
```

Evaluated Integrals

Table C1 Evaluated integral for members having straight haunches.

Int. No.	Ix., Ax arbitrary $I_x=I_C(1+r(1-x/al)$ $A_x=A_C(1+r(1-x/al)$		Transformed integral y=1-x/al	Evaluated integrals al =α or al =β
1	$\int_{X_1}^{X_2}\frac{dx}{A_X}$	$\frac{1}{A_C}\int_{X_1}^{X_2}\frac{dx}{(1+r(1-x/al))}$	$-\frac{al}{A_C}\int_{Y_1}^{Y_2}\frac{dy}{1+ry}$	$-\frac{al}{A_C r}\left[Ln(1+ry)\right]_{Y1}^{Y2}$
1a	$\int_{0}^{al}\frac{dx}{A_X}$	$\frac{1}{A_C}\int_{0}^{al}\frac{dx}{(1+r(1-x/al))}$	$-\frac{al}{A_C}\int_{1}^{0}\frac{dy}{1+ry}$	$\frac{al}{A_C r}Ln(1+r)$
1b	$\int_{0}^{L-\beta}\frac{dx}{A_X}$	When Ax=Ac $\frac{1}{A_C}\int_{\alpha}^{L-\beta}dx$		$\frac{1}{A_C}(L-\alpha-\beta)$
2	$\int_{X_1}^{X_2}\frac{dx}{I_X}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{dx}{(1+r(1-x/al))^3}$	$-\frac{al}{I_C}\int_{Y_1}^{Y_2}\frac{dy}{(1+ry)^3}$	$\frac{al}{rI_C}\left[\frac{1}{2(1+ry)^2}\right]_{Y1}^{Y2}$
2a	$\int_{0}^{al}\frac{dx}{I_X}$	$\frac{1}{I_C}\int_{0}^{al}\frac{dx}{(1+r(1-x/al))^3}$	$-\frac{al}{I_C}\int_{1}^{0}\frac{dy}{(1+ry)^3}$	$\frac{al}{2I_C}\left[\frac{2+r}{(1+r)^2}\right]$
2b	$\int_{\alpha}^{L-\beta}\frac{dx}{I_X}$	When Ix=Ic $\frac{1}{I_C}\int_{\alpha}^{L-\beta}dx$		$\frac{1}{I_C}(L-\alpha-\beta)$
3	$\int_{X_1}^{X_2}\frac{xdx}{I_X}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{xdx}{(1+r(1-x/al))^3}$	$\frac{al^2}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)dy}{(1+ry)^3}$	$\frac{al^2}{r^2 I_C}\left[\frac{1+r}{2(1+ry)^2}-\frac{1}{1+ry}\right]_{Y1}^{Y2}$
3a	$\int_{0}^{al}\frac{xdx}{I_X}$	$\frac{1}{I_C}\int_{0}^{al}\frac{xdx}{(1+r(1-x/al))^3}$	$-\frac{al^2}{I_C}\int_{1}^{0}\frac{(y-1)dy}{(1+ry)^3}$	$\frac{al^2}{2I_C}\left[\frac{1}{(1+r)}\right]$
3b	$\int_{\alpha}^{L-\beta}\frac{xdx}{I_x}$	When Ix=Ic $\frac{1}{I_C}\int_{\alpha}^{L-\beta}xdx$		$\frac{1}{2I_C}((L-\beta)^2-\alpha^2)$

4	$\int_{X_1}^{X_2}\frac{x^2dx}{Ix}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{x^2dx}{(1+r(1-x/al))^3}$	$-\frac{al^3}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)^2dy}{(1+ry)^3}$	$-\frac{al^3}{r^3I_C}\left[\begin{matrix}-\frac{(1+r)^2}{2(1+ry)^2}+\frac{2(1-r)}{1+ry}\\+Ln(1+ry)\end{matrix}\right]_{Y1}^{Y2}$
4a	$\int_{0}^{al}\frac{x^2dx}{Ix}$	$\frac{1}{I_C}\int_{0}^{al}\frac{x^2dx}{(1+r(1-x/al))^3}$	$-\frac{al^3}{I_C}\int_{1}^{0}\frac{(y-1)^2dy}{(1+ry)^3}$	$-\frac{al^3}{r^3I_C}\left[\frac{r(2-r)}{2}-Ln(1+r)\right]$
4b	$\int_{\alpha}^{L-\beta}\frac{x^2dx}{Ix}$	When Ix=Ic $\frac{1}{I_C}\int_{\alpha}^{L-\beta}x^2dx$		$\frac{1}{3I_C}((L-\beta)^3-\alpha^3)$
5	$\int_{X_2}^{X_1}\frac{x^3dx}{Ix}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{x^3dx}{(1+r(1-x/al))^3}$	$-\frac{al^4}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)^3dy}{(1+ry)^3}$	$\frac{al^4}{r^4I_C}\left[\begin{matrix}\frac{(1+r)^3}{2(1+ry)^2}-\frac{3(1+r)^2}{(1+ry)}+\\(1+ry)-3(1+r)Ln(1+ry)\end{matrix}\right]$
5a	$\int_{0}^{al}\frac{x^3dx}{Ix}$	$\frac{1}{I_C}\int_{0}^{al}\frac{x^3dx}{(1+r(1-x/al))^3}$	$\frac{al^4}{I_C}\int_{1}^{0}\frac{(y-1)^3dy}{(1+ry)^3}$	$\frac{al^4}{r^4I_C}\left[\begin{matrix}\frac{r(r^2-3r-6)}{2}+\\3(1+r)Ln(1+r)\end{matrix}\right]$
5b	$\int_{\alpha}^{L-\beta}\frac{x^3dx}{Ix}$	When Ix=Ic $\frac{1}{I_C}\int_{\alpha}^{L-\beta}x^3$		$\frac{1}{4I_C}((L-\beta)^4-\alpha^4)$

Table C2 Integral for members having parabolic haunches.

Int. No.	Ix., Ax arbitrary $I_x=I_C(1+r(1-x/al)$ $A_x=A_C(1+r(1-x/al)$		Transformed integral y=1-x/L	Evaluated integrals al =α or al =β
1	$\int_{X_1}^{X_2}\frac{dx}{A_x}$	$\frac{1}{A_C}\int_{X_1}^{X_2}\frac{dx}{(1+r(1-x/al)^2)}$	$-al\int_{Y_1}^{Y_2}\frac{dy}{1+ry^2}$	$-\frac{al}{A_C\sqrt{r}}\left[\tan^{-1}(\sqrt{r}\ y)\right]_{Y1}^{Y2}$
1a	$\int_0^{\alpha}\frac{dx}{A_x}$	$\frac{1}{A_C}\int_0^{\alpha}\frac{dx}{(1+r(1-x/\alpha)^2)}$	$-\frac{\alpha}{A_C}\int_1^0\frac{dy}{1+ry^2}$	$\frac{\alpha}{A_C\sqrt{r}}\tan^{-1}\sqrt{r}$
1b	$\int_{\beta}^{2\beta}\frac{dx}{A_x}$	$\frac{1}{A_C}\int_{\beta}^{2\beta}\frac{dx}{(1+r(1-x/\beta)^2)}$	$-\frac{\beta}{A_C}\int_0^{-1}\frac{dy}{1+ry^2}$	$-\frac{\beta}{A_C\sqrt{r}}\tan^{-1}\sqrt{r}$
2	$\int_{X_1}^{X_2}\frac{dx}{I_x}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{dx}{(1+r(1-x/al)^2)^3}$	$-\frac{al}{I_C}\int_{Y_1}^{Y_2}\frac{dy}{(1+ry^2)^3}$	$-\frac{al}{8I_C}\left[\frac{2y+3y(1+ry^2)}{(1+ry^2)^2}+\frac{3}{\sqrt{r}}\tan^{-1}(\sqrt{r}\ y)\right]_{Y1}^{Y2}$
2a	$\int_0^{\alpha}\frac{dx}{I_x}$	$\frac{1}{I_C}\int_0^{\alpha}\frac{dx}{(1+r(1-x/\alpha)^2)^3}$	$-\frac{\alpha}{I_C}\int_1^0\frac{dy}{(1+ry^2)^3}$	$\frac{\alpha}{8I_C}\left[\frac{5+3r}{(1+r)^2}+\frac{3}{\sqrt{r}}\tan^{-1}\sqrt{r}\right]$
2b	$\int_{\beta}^{2\beta}\frac{dx}{I_x}$	$\frac{1}{I_C}\int_{\beta}^{2\beta}\frac{dx}{(1+r(1-x/\beta)^2)^3}$	$-\frac{\beta}{I_C}\int_0^{-1}\frac{dy}{(1+ry^2)^3}$	$\frac{\beta}{8I_C}\left[\frac{5+3r}{(1+r)^2}+\frac{3}{\sqrt{r}}\tan^{-1}\sqrt{r}\right]$
3	$\int_{X_1}^{X_2}\frac{xdx}{I_x}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{xdx}{(1+r(1-x/al)^2)^3}$	$\frac{al^2}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)dy}{(1+ry^2)^3}$	$-\frac{al^2}{8rI_C}\left[\frac{3r^2y^2+5ry+2)}{(1+ry^2)^2}+3\sqrt{r}\tan^{-1}(\sqrt{r}\ y)\right]_{Y1}^{Y2}$
3a	$\int_0^{\alpha}\frac{xdx}{I_x}$	$\frac{1}{I_C}\int_0^{\alpha}\frac{xdx}{(1+r(1-x/\alpha)^2)^3}$	$-\frac{\alpha^2}{I_C}\int_1^0\frac{(y-1)dy}{(1+ry^2)^3}$	$\frac{\alpha^2}{8rI_C}\left[\frac{r}{(1+r)}+3\sqrt{r}\tan^{-1}\sqrt{r}\right]$
3b	$\int_{\beta}^{2\beta}\frac{xdx}{I_x}$	$\frac{1}{I_C}\int_{\beta}^{2\beta}\frac{xdx}{(1+r(1-x/\alpha)^2)^3}$	$\frac{\beta^2}{I_C}\int_0^{-1}\frac{(y-1)dy}{(1+ry^2)^3}$	$\frac{\beta^2}{8rI_C}\left[\frac{5r^2+9r}{(1+r)^2}+3\sqrt{r}\tan^{-1}\sqrt{r}\right]$

4	$\int_{X_1}^{X_2}\frac{x^2dx}{I_x}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{x^2dx}{(1+r(1-x/al)^2)^3}$	$-\frac{al^3}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)^2dy}{(1+ry^2)^3}$	$\frac{-al^3}{8r^2I_C}\left[\frac{(3r^2+r^2)y^3+(5r^2-r)y+4r}{(1+ry^2)^2}+\sqrt{r}(3r+1)\tan^{-1}(\sqrt{r}\ y)\right]_{Y1}^{Y2}$
4a	$\int_0^{\beta}\frac{x^2dx}{I_x}$	$\frac{1}{I_C}\int_0^{\alpha}\frac{x^2dx}{(1+r(1-x/\alpha)^2)^3}$	$-\frac{\alpha^3}{I_C}\int_1^0\frac{(y-1)^2dy}{(1+ry^2)^3}$	$\frac{\alpha^3}{8r^2I_C}\left[-r+\sqrt{r}(3r+1)\tan^{-1}\sqrt{r}\right]$
4b	$\int_{\beta}^{2\beta}\frac{x^2dx}{I_x}$	$\frac{1}{I_C}\int_{\beta}^{2\beta}\frac{x^2dx}{(1+r(1-x/\beta)^2)^3}$	$-\frac{\beta^3}{I_C}\int_0^{-1}\frac{(y-1)^2dy}{(1+ry^2)^3}$	$\frac{\beta^3}{8r^2I_C}\left[\frac{7r^3+14r^2-r}{(1+r)^2}+\sqrt{r}(3r+1)\tan^{-1}\sqrt{r}\right]$
5	$\int_{X_1}^{X_2}\frac{x^3dx}{I_x}$	$\frac{1}{I_C}\int_{X_1}^{X_2}\frac{x^3dx}{(1+r(1-x/al)^2)^3}$	$-\frac{al^4}{I_C}\int_{Y_1}^{Y_2}\frac{(y-1)^3dy}{(1+ry^2)^3}$	$\frac{-al^4}{8r^2I_C}\left[\frac{(3r^3+3r^2)y^3+4ry^2+(5r^2-3r)y+2(3r+1)}{(1+ry^2)^2}\ 3\sqrt{r}(r+1)\tan^{-1}(\sqrt{r}\ y)\right]_{Y1}^{Y2}$
5a	$\int_0^{\alpha}\frac{x^3dx}{I_x}$	$\frac{1}{I_C}\int_0^{\alpha}\frac{x^3dx}{(1+r(1-x/\alpha)^2)^3}$	$\frac{\alpha^4}{I_C}\int_1^0\frac{(y-1)^3dy}{(1+ry^2)^3}$	$\frac{\alpha^4}{8r^2I_C}\left[-3r+3\sqrt{r}(r+1)\tan^{-1}\sqrt{r}\right]$
5b	$\int_{\alpha}^{2\beta}\frac{x^3dx}{I_x}$	$\frac{1}{I_C}\int_{\beta}^{2\beta}\frac{x^3dx}{(1+r(1-x/\beta)^2)^3}$	$\frac{\beta^4}{I_C}\int_0^{-1}\frac{(y-1)^3dy}{(1+ry^2)^3}$	$\frac{\beta^4}{8r^2I_C}\left[\frac{9r^3+22r^2-3r}{(1+r)^2}+3\sqrt{r}(r+1)\tan^{-1}\sqrt{r}\right]$