ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING

ANALYSIS AND DESIGN OF SIMPLY SUPPORTED PRE STRESSED PRECAST-CAST IN-SITU CONCRETE COMPOSITE RAILWAY BRIDGES: COMPARISON OF BALLASTED AND SLAB TRACK ALTERNATIVES

By

Ashenafi Oycha Oyda

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A thesis submitted to the School of Graduate Studies of the Addis Ababa University in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering

(Railway civil Engineering)

By

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March 12, 2015

DECLARATION

I hereby declare that this thesis is my original work performed under the supervision of my thesis advisor Dr. Asnake Adamu and has not been presented as a thesis for a degree in any other university. All sources of materials used for this thesis have also been duly acknowledged.

Ashenafi Oycha Oyda
(Student)

Date

This is certifying that the above declaration made by the candidate is correct to the best of my knowledge.

Dr. Asnake Adamu
(Advisor)

Date

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ABSTRACT

Bridge is one of the most important components of railway line for development of transportation. It is known that Addis Ababa light rail transit project construction started two years ago. Also the construction of national railway project started. In these projects, the construction of bridges need highly skilled man power and highest construction cost.

The railway bridge commonly contains two types of tracks which are ballasted and slab track. There is a difference in construction cost of bridge between ballasted track case and slab track case.

Thus, this study has attempted to analyze, design and compare both ballasted and slab track type laying on the surface of bridge. This study mainly focuses on the superstructure of bridge supporting track rails attached to the bridge to support trains.

The intention of this thesis is therefore to compare the construction cost of railway bridges with ballasted and slab track cases and to select the economical track type laying on the surface of railway bridge.

Finally, from the overall analysis result of this thesis, the study conclude that slab track is economical than ballasted track for span length greater than 15m. On the other hand, ballasted track seems economical than slab track for span length less than 15m.

TERMINOLOGY

Anchorage	A device generally used to enable the tendon to impart and maintain pre-stress in concrete.
Ballast	Gravel or coarse stone used to form the bed of a railway track or the substratum of a road.
Bars	A tendon can be made up of a single steel bar. The diameter of a bar is much larger than that of a wire.
Cable	A group of tendons form a pre-stressing cable.
Curb	A rim, especially of joined stones or concrete, along a bridge or roadway, forming an edge for a sidewalk.
Dead load	A type of load that is relatively constant over time, including the weight of the structure itself and also known as permanent loads.
Jacking force	The temporary force applied to tendons by a jacking device to produce the tension in pre-stressing tendons.
Jacking stress	The maximum stress that occurs during the stressing of a precast concrete tendon.
Jacking plate	A steel bearing plate used during jacking operations to transmit the load of the jack to the pile.
Live load	A temporary load for short duration or a moving load. Bridge live loads are produced by vehicles traveling over the deck of the bridge.
Mild steel	Steel with less than 0.15% carbon.

Post-tensioning A method of pre-stressing concrete by tensioning the tendons against hardened concrete. In this method, the pre-stress is imparted to concrete by bearing.

Pre-stressed concrete Is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from the external loads are counteracted to a desired degree.

Pre-stressing wire A single unit made of steel.

Pre-tensioning A method of pre-stressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the concrete is introduced by bond between steel& concrete.

Rail A bar or pair of parallel bars of rolled steel making the railway along which railroad cars or other vehicles can roll**.**

Sleeper A track tie usually made of concrete or wood that is laid across a railroad bed to support the rails.

Strands Two, three or seven wires are wound to form a pre-stressing strand.

Tendons A group of strands or wires are wound to form a pre-stressing tendon.

SYMBOLS

A_c	Area of concrete.
A_{cc}	Area of composite concrete.
A_{pc}	Area of precast concrete.
A_s	Area of tendon.
e	Eccentricity of pre-stressing force.
E_s	Modulus of elasticity of steel.
E_c	Modulus of elasticity of concrete.
f_{br}	Permissible range of stresses at the bottom of the cross section.
f_{cd}	The compressive design strength of concrete.
f_{ck}	The characteristic cylinder compressive strength of concrete.
f_{cp}	Compressive pre-stress at the centroid of a section.
f_{ct}	Allowable compressive Stresses in concrete at transfer.
f_{ctd}	The tensile design strength of concrete.
f_{ctk}	Tensile strength of concrete.
f_{cu}	The value of the cube strength of concrete.
f_{cw}	Allowable compressive Stresses in concrete under working.
f_d	The design strength for a given material.
f_k	The characteristic strength for all materials has the notation.
f_{lw}	Stresses after all losses of pre stress and the full working load acts on the lower fiber.
f_{pe}	Effective stress in tendons after loss.
f_{pi}	Stress in tendon at transfer.

f_{pk}	Characteristics strength of tendon.
f_{tr}	Permissible range of stresses at the top of the cross section.
f_{tt}	Allowable tensile stress in concrete at transfer.
f_{tw}	Allowable tensile stress in concrete under working.
f_y	Characteristic strength of reinforcing steel.
f_{yd}	The tensile or compressive design strength of reinforcing steel.
$\bar{I}_c$	Second moment of Inertia for composite structure.
I_p	Second moment of Inertia for precast.
M_d	The bending moment at the cross-section where shear capacity is calculated.
M_{g1}	Bending moment caused by the self –weight.
M_{g2}	Bending moment caused by the superimposed load.
M_o	Bending moment caused by the eccentricity and pre-stressing force.
M_{q2}	Bending moment caused by the live load.
η	Reduction factor for loss of pre-stress.
P	Pre-stressing force at transfer on the cross section
P_e	Pre-stressing force under working condition (after all losses) on cross section.
V_c	Flexural Shear force.
V_{cw}	Web shear force
V_d	Design shear force
V_{Rd}	Limiting value of shear force against diagonal compression.
W_{eq}	Equivalent uniform load.
Z	Section modulus.

Z_{bc}	Section modulus, at bottom fiber for composite structure.
Z_{bp}	Section modulus, at bottom fiber for precast structure.
Z_{tc}	Section modulus, at top fiber for composite structure.
Z_{tp}	Section modulus, at top fiber for precast structure.
Δ_{LL+I}	Deflection due to live load and impact load.
Δ_g	Deflection due to self-weight load.
Δ_p	Deflection due to pre-stressing force.
Δ_s	Deflection due to superimposed load.
$\bar{y}_c$	Centroid of composite structure.

ABBREVIATIONS

AASHTO	American Association of State Highway and Transportation Officials.
AREMA	American Railway Engineering Maintenance of Way Association.
DL	Dead load.
EBCS	Ethiopian building code standard.
KMPH	Kilo meter per hour.
LL	Live load.
LRT	Light rail transit.
PSC	Pre-stressed concrete.
RC	Reinforced concrete.
RCC	Rolled compacted concrete.
SDL	Superimposed dead load.
ULS	Ultimate limit state.
ACI	American concrete institute.

LIST OF TABLES

LIST OF FIGURES

TABLE OF CONTENTS

CHAPTER ONE

1 Introduction

1.1 Background

The development of infrastructure especially railway lines has a great share to the economic development of a nation. But the developing countries are not well developed in railway transportation. Therefore railway transportation is intended to solve such problem in our country.

To implement this type of transportation, Ethiopian Railways Corporation was started the construction of railway lines in both Addis Ababa light rail transit and national railway project which are eight corridors from Addis Ababa to investment areas throughout in the country.

But the topography of our country is difficult to construct highway as well as railway especially northern part of our country. These ups and downs of landscape are a great challenge when selecting routes and constructing railway lines. In addition to this, railway lines that pass through these topographies need a number of bridges.

In Addis Ababa also a railway lines need a number of bridge to make overpass and reduce the congestion or traffic jam. Now in Addis Ababa, the construction of railway lines is under progress and these lines have different forms. That is at grade level, under pass and elevated. In the case of elevated, the railway line needs bridge and therefore bridge is mandatory for construction of the railway line.

The construction cost of railway line too high as compared with highway lines. Out of the whole railway project, the construction cost of railway bridge takes a very large portion of the annual budget of the project.

A bridge is a structure providing passage over an obstacle. The obstacle may be a river, valley, road or railway. The passage may be for highway or railway traffic, pedestrian, canal or pipeline. Bridges provide essential links in highways and railways at obstacles. Therefore bridge construction becomes mandatory for development of railway and hence for the development of transportation. Since, bridge is the key element in a transportation system; balance must be achieved between handling future traffic volume and loads and the cost of heavier and wider bridge structure.

The intension of this thesis is to minimize significant construction cost of railway bridge since construction and maintenance of bridge is highly expensive and it needs highly skilled man power. In addition to this, for a wide gorge bridge the construction period also long.

Thus, this study is intended to address these problems and seeks one alternative solution by preparing analysis and design of simply supported pre stressed precast-cast in-situ concrete composite railway bridges, with due regard to comparison of ballasted and slab track alternatives.

Finally, from the overall analysis result of this thesis, the study conclude that slab track is economical than ballasted track for span length greater than 15m. On the other hand, ballasted track seems economical than slab track for span length less than 15m.

1.2 Objectives of the Study

1.2.1 Objectives

The main objective of this thesis is to compare analysis result of ballasted and slab railway bridges and economic analysis of both cases.

The specific objectives of the thesis are to compare slab and ballasted railway bridge in terms of construction cost, to select economical track type for the railway bridge and to recommend better track type for railway bridges in Ethiopia.

1.2.2 Scope of the Study

The scope of the study has been limited to the preparation of analysis and design of simply supported pre-stressed precast-cast in-situ concrete composite railway bridges; with due regard to comparison of ballasted and slab track alternatives. Also this study considered only single track, not covered dynamic analysis of bridge as well as track but the dynamic effect considered through impact factor, study based on only superstructure, the bridge is assumed to have a linear behavior, pre-stressed concrete is assumed to have an elastic behavior and to be un-cracked and lateral forces, lateral accelerations or displacements are not considered, some load effects, such as snow, water pressure, and wind and centrifugal force are not considered.

The study area conducted in view of the light rail transit at Addis Ababa. The total length of 34.25 km (north-south line 16.9 km and east-west line 17.35 km) two lines (i.e. north-south and east-west lines) use common track of about 2.7km and standard gauge (1.435 meters).

1.3 What is in this Study

The study of this thesis mainly focuses on pre-stressed post tensioned railway bridge girder, advantage of ballasted track over slab track and vice versa, and the advantage of post tension over pre tension. The reference material used for this thesis are AREMA, and books reference for pre-stress, railway bridge, ballasted track, and slab track.

The study basically focused on selection of economical track type laying on railway bridge and developing analysis and design of simply supported pre stressed precast-cast in-situ concrete composite railway bridges. Since the study based on comparative analysis of railway bridge, the study has been took counter examples of both ballasted and slab railway bridge.

The study took the same values of design parameters, span of the bridge and depth of girder for both cases. In addition to this, the depth of ballast and slab are predetermined in literature review. Here the study took the same parameters and compare initial pre-stress force for both cases. Finally, the study used software, Auto CAD to draw the different figures in this text.

To attain the objective of the study which is based on economical point of view, the results of analysis of both cases have been compared and contrasted. Finally, from the above result the study has concluded the whole idea of the study and recommended the better track type for practice in the railway project.

This thesis contains five chapters. The first chapter deals with introductory part of the thesis, second chapter deals with literature review of the thesis, third chapter deals with analysis and design of pre-stressed bridges, fourth chapter deals with design examples and discussion , fifth chapter deals with the summary, conclusion and recommendation of the study.

CHAPTER TWO

2 Literature Review

A number of studies dealing with pre stressed concrete, railway bridges, ballasted or slab tracks are important for this study. An important literature review has been discussed below.

2.1 Pre-Stressed Concrete for Railway Bridges

The classic everyday example of pre-stressing is this: a row of books can be lifted by squeezing the ends together:

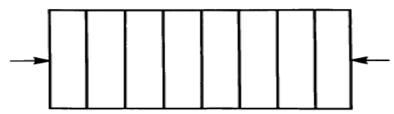


Figure 2-1 Pre-stressed row of books

The structural explanation is that the row of books has zero tensile capacity. Therefore the ‘beam’ of books cannot even carry its self-weight. To overcome this we provide an external initial stress (the pre-stress) which compresses the books together. Now they can only separate if the tensile stress induced by the self-weight of the books is greater than the compressive pre-stress introduced.

Concrete is very strong in compression but weak in tension. In an ordinary concrete beam the tensile stress at the bottom are taken by standard steel reinforcement: But we still get cracking, which is due to both bending and shear:

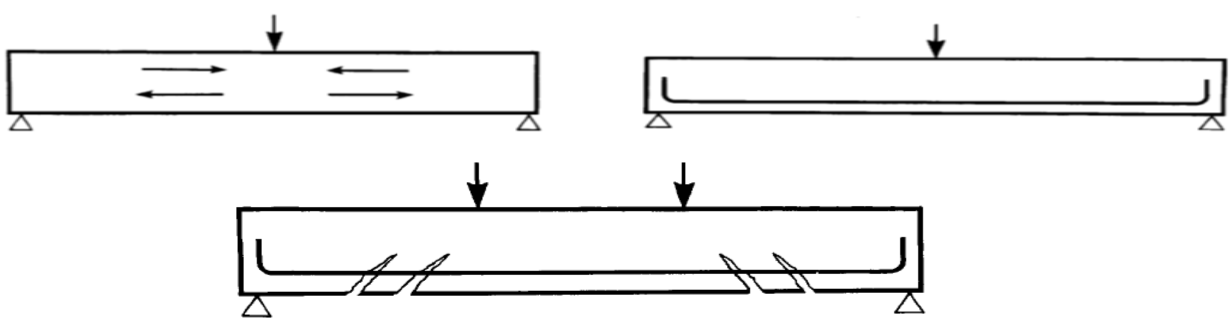


Figure 2-2 Reinforced concrete

In pre-stressed concrete, because the pre-stressing keeps the concrete in compression, no cracking occurs. This is often preferable where durability is a concern [10].

2.1.1 Definition

Pre-stress is defined as a method of applying pre-compression to control the stresses resulting due to external loads below the neutral axis of the beam tension developed due to external load which is more than the permissible limits of the plain concrete. The pre-compression applied (may be axial or eccentric) will induce the compressive stress below the neutral axis or as a whole of the beam resulting either no tension or compression [28].

Pre-stressed concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from the external loads are counteracted to a desired degree [28].

2.1.2 Methods of Pre-Stressing

There are two methods of pre-stressing:

- Pre-tensioning: Apply pre-stress to steel strands before casting concrete; and
- Post-tensioning: Apply pre-stress to steel tendons after casting concrete.

1. Pre-Tensioning

In the pre-tensioning systems, the tendons are first tensioned between rigid anchor-blocks cast on the ground or in a column or unit-mould type's pre tensioning bed, prior to the casting of concrete in the mould. The tendons comprising individual wires or strands are stretched with constant eccentricity or a variable eccentricity with tendon anchorage at one end and jacks at the other. With the forms in place, the concrete is cast around the stressed tendon. The system is shown in Figure below [28].

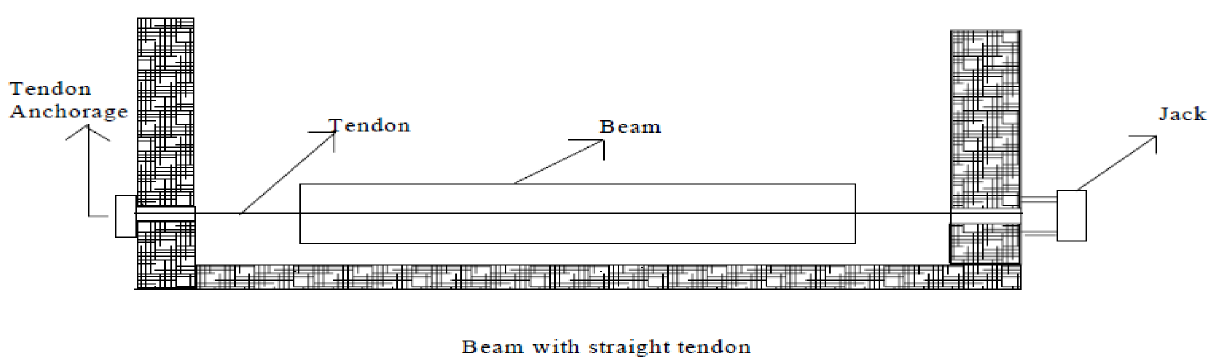


Figure 2-3 Pre- tensioning of tendon

Pre-tensioning is the most common form for precast sections. In Stage 1, the wires or strands are stressed; in Stage 2, the concrete is cast around the stressed wires/strands; and in Stage 3, the pre-stressed in transferred from the external anchorages to the concrete, once it has sufficient strength [10].

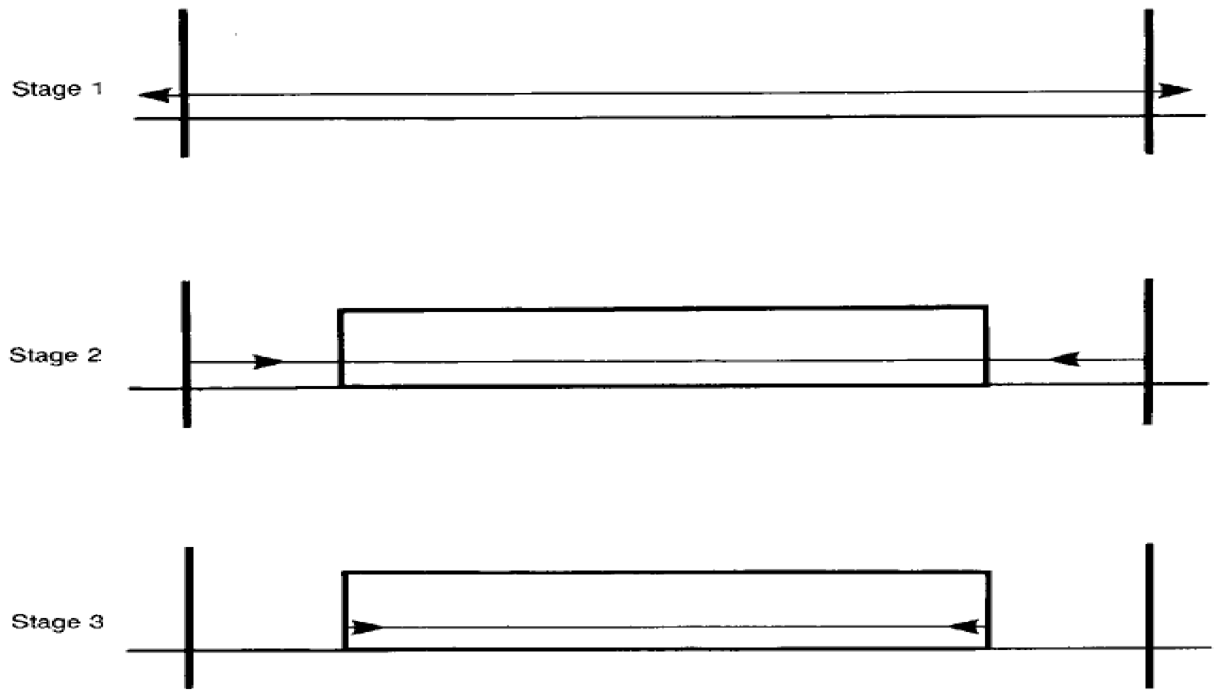


Figure 2-4 Stages of pre-tensioning

In pre-tensioned members, the strand is directly bonded to the concrete cast around it. Therefore, at the ends of the member, there is a transmission length where the strand force is transferred to the concrete through the bond [10].

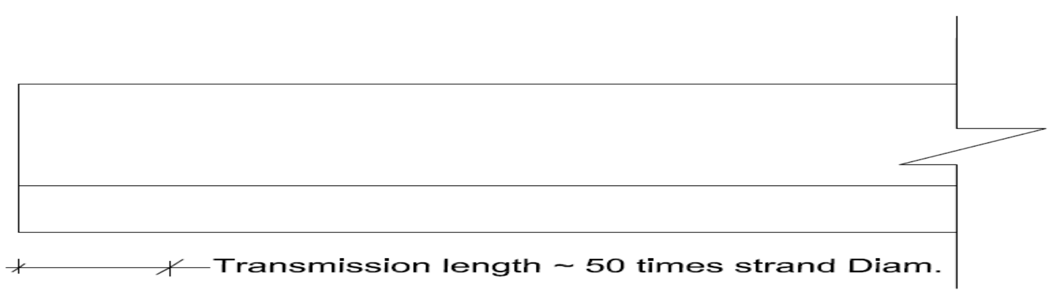


Figure 2-5 Transmission length

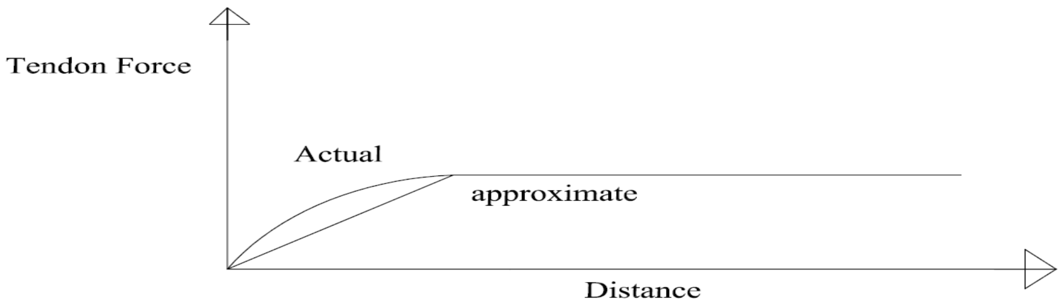


Figure 2-6 Transmission of tendon force

In pre tensioned concrete components, the pre-stressing steel is tensioned before the concrete is placed, as illustrated in figure below. The pre-stressing tendons are typically un-sheathed so that the concrete bonds to the tendons [23].

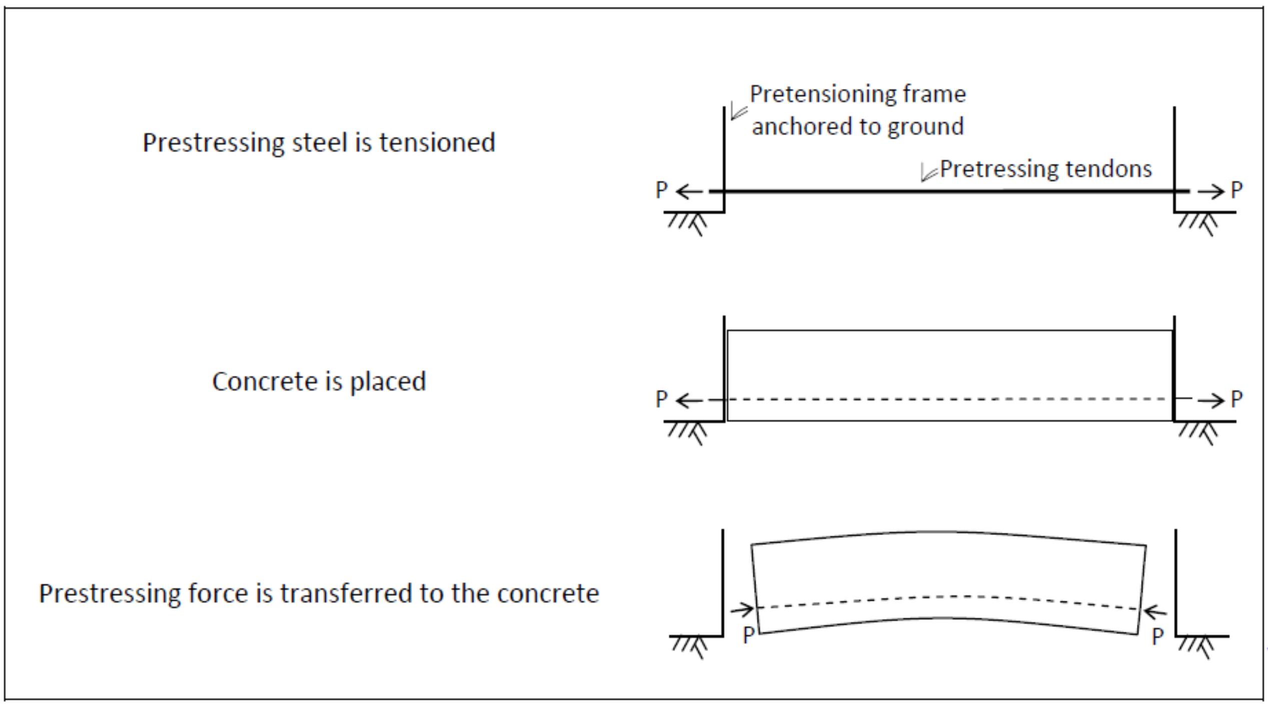


Figure 2-7 Construction sequence for pre-tensioned concrete beam

2. Post-Tensioned

In this method, the concrete has already set but has ducts cast into it. The strands or tendons are fed through the ducts (Stage 1) then tensioned (Stage 2) and then anchored to the concrete (Stage3) [10].

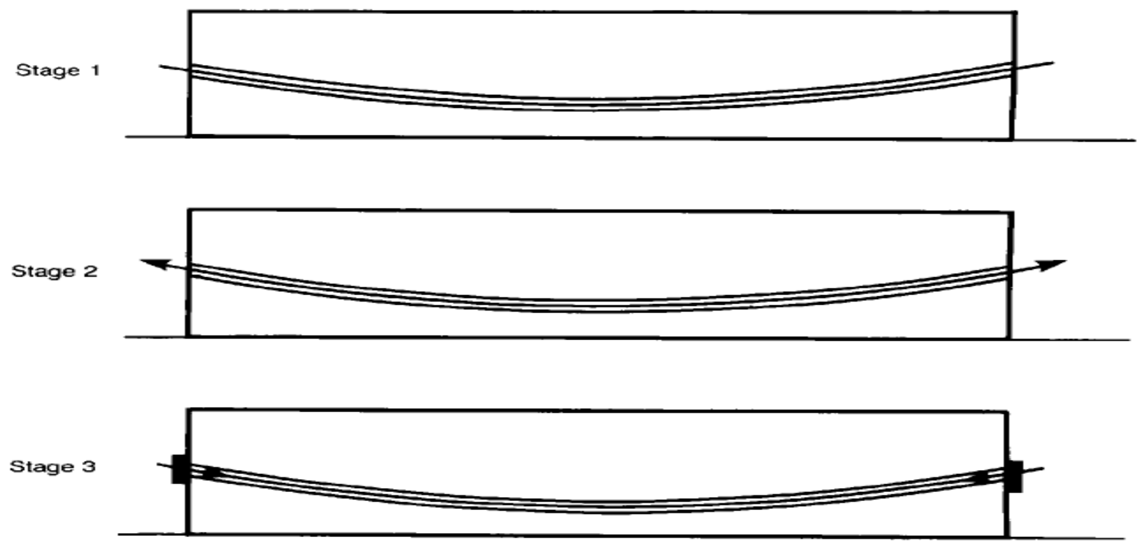


Figure 2-8 Stages of post tensioning

In post-tensioned concrete, the concrete is pre-stressing steel is tensioned after the concrete is placed, as illustrated in figure below. The pre-stressing tendons are sheathed or placed in ducts so that the concrete and tendons are unbounded. Ducts can be grouted after post-tensioning to protect the tendons from moisture and corrosion [23].

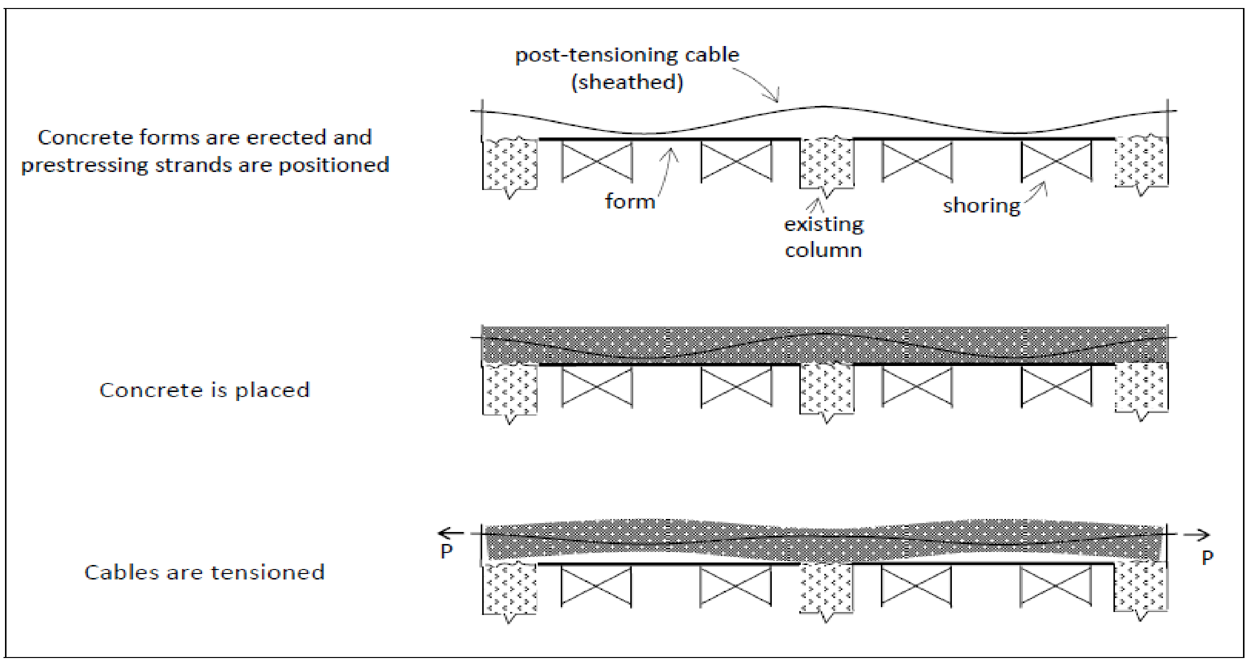


Figure 2-9 Post tensioned cable profile

In post-tensioning the concrete unit are first cast by incorporating ducts or grooves to house the tendons. When the concrete attains sufficient strength, the high-tensile wires are tensioned by means of jack bearing on the end of the face of the member and anchored by wedge or nuts. The forces are transmitted to the concrete by means of end anchorage and, when the cable is curved, through the radial pressure between the cable and the duct. The space between the tendons and the duct is generally grouted after the tensioning operation [28].

Most of the commercially patented pre-stressing systems are based on the following principle of anchoring the tendons:

1. Wedge action producing a frictional grip on the wire.
2. Direct bearing from the rivet or bolt heads formed at the end of the wire.
3. Looping the wire around the concrete.

To summarize that, pre-stressing systems are two types. Pre-tensioning and post-tensioning. The comparisons between them have been discussed in the table below [28].

Table 2-1 Comparative study: pretension Vs post-tensioned member

Pretension member	**Post-tensioned member**
1. In pre-tensioned pre-stress concrete, steel is tensioned prior to that of concrete. It is released once the concrete is placed and hardened. The stresses are transferred all along the wire by means of **bond**.	1. Concreting is done first then wires are tensioned and anchored at ends. The stress transfer is by **end bearing** not by bond.
2. Suitable for short span and precast products like sleepers, electric poles on mass production.	2. Suitable for long span bridges
3. In pre-tensioning the cables are basically straight and horizontal. Placing them in curved or inclined position is difficult. However the wires can be kept with eccentrically. Since cables cannot be aligned similar to B.M.D. structural advantages are less compare to that of post-tensioned.	3. The post tensioning cables can be aligned in any manner to suit the B.M.D due to external load system. Therefore it is more economical particularly for long span bridges. The curved or inclined cables can have vertical component at ends. These components will reduce the design shear force. Hence post-tensioned beams are superior to pre-tensioned beams both from flexural and shear resistances point.
4. Pre-stress losses are more compare to that of post-tensioned concrete.	4. Losses are less compare to pre-tensioned concrete

Advantage of Pre-Stressed Concrete [28]

- The use of high strength concrete and steel in pre-stressed members results in lighter and slender members than is possible with RC members.
- In fully pre-stressed members the member is free from tensile stresses under working loads, thus whole of the section is effective.
- In pre-stressed members, dead loads may be counter-balanced by eccentric pre-stressing.
- Pre-stressed concrete member possess better resistance to shear forces due to effect of compressive stresses presence or eccentric cable profile.
- Use of high strength concrete and freedom from cracks, contribute to improve durability under aggressive environmental conditions.
- Long span structures are possible so that saving in weight is significant & thus it will be economic.
- Pre-stressed members are tested before use.
- Pre-stressed concrete structure deflects appreciably before ultimate failure, thus giving ample warning before collapse.

Disadvantages of Pre-Stressed Concrete [28]

- The availability of experienced builders is scanty.
- Initial equipment cost is very high.
- Availability of experienced engineers is scanty.
- Pre-stressed sections are brittle
- Pre-stressed concrete sections are less fire resistant.

2.1.3 Materials of Pre Stressing

1. Concrete

The main factors for concrete used in PSC are:

- Ordinary Portland cement-based concrete is used but strength usually greater than 50 N/mm^2;
- A high early strength is required to enable quicker application of pre-stress;
- A larger elastic modulus is needed to reduce the shortening of the member;
- A mix that reduces creep of the concrete to minimize losses of pre-stress [28].

2. **Steel**

The steel used for pre-stressing has nominal yield strength of between 1550 to 1800N/mm^2. The different forms the steel may take are:

- **Wires:** individually drawn wires of 7 mm diameter;
- **Strands:** a collection of wires (usually 7) wound together and thus having a diameter that is different to its area;
- **Tendon**: A collection of strands encased in a duct – only used in post tensioning;
- **Bar:** a specially formed bar of high strength steel of greater than 20 mm diameter.

Figure 2-10 Pre-stressing steel (wires, strands, tendons and bars)

3. **A Typical Tendon Anchorage**

The anchorages to post-tensioned members must distribute a large load to the concrete, and must resist bursting forces as a result. A lot of ordinary reinforcement is often necessary [10].

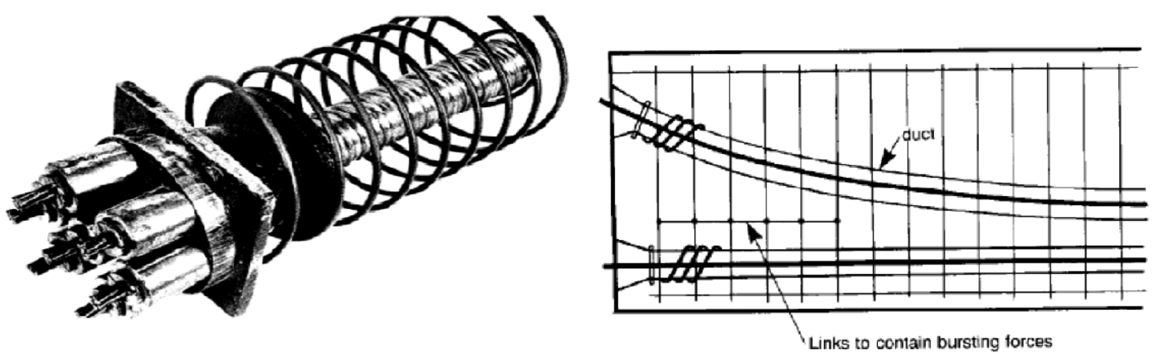


Figure 2-11 a) Tendon anchorage b) Reinforcement of end of a post-tensioned member

4. **Post-Tensioning Ducts**

Post-tensioning ducts for longitudinal post-tensioning tendons in precast spliced I-girders shall be made of rigid galvanized spiral ferrous metal to maintain standard girder concrete cover requirements. The radius of curvature of tendon ducts shall not be less than 20 feet except in anchorage areas where 12 feet may be permitted [29].

2.1.4 What is Composite Precast Pre-Stressed - in Situ Concrete

Precast Concrete

Pre-casting involves the casting of concrete away from the site of final position. It can also be produced near the site. The member of precast element produced either in a permanent plant or temporary arrangement and eventually erected at the final position. Pre-casting permits better control in mass production. It is also economical [14].

Precast concrete is a construction product produced by casting concrete in a reusable form which is then cured in a controlled environment, transported to the construction site and lifted into place. The precast girder supports the cast in place part of the slab and standard precast girders may be grouped together or spread apart to accommodate heavy or light loads, respectively. This precast is post tensioned or pre tensioned is known as precast pre-stressed concrete.

Cast in place

Concrete cast in situ is poured into site-specific forms and cured on site. Cast in place concrete require more formwork and false work per unit of product but save the cost of transportation and erection, and it is better used for large and heavy members. The cast in place top slab ties the structure together and gives a uniform continuous surface.

Therefore, the combination of precast pre-stressed concrete and concrete cast in situ is called composite precast pre-stressed - in situ concrete. As shown in following figure below; by using composite construction, it is possible to save much form work and false work as compared to fully cast in place construction.

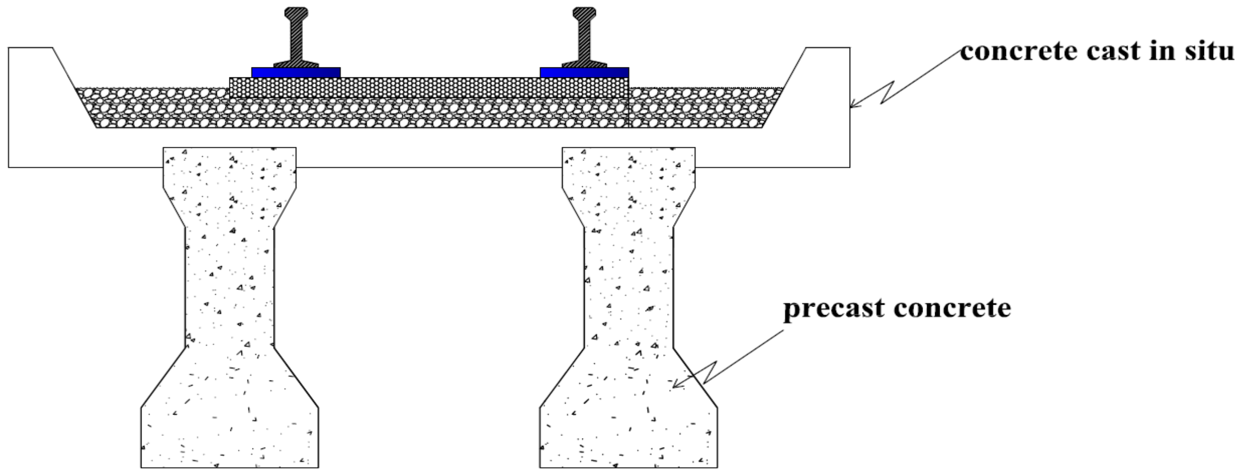


Figure 2-12 Composite precast pre-stressed - in situ concrete

2.1.5 Differences of Pre-Stressed Concrete over Reinforced Concrete:

1. In pre-stress concrete member steel plays active role. The stress in steel prevails whether external load is there or not. But in R.C.C., steel plays a passive role. The stress in steel in R.C.C members depends upon the external loads (*i.e.* no external load, no stress in steel).
2. In pre-stress concrete the stresses in steel is almost constant where as in R.C.C the stress in steel is variable with the lever arm.
3. Pre-stress concrete has more shear resistance, whereas shear resistance of R.C.C is less.
4. In pre-stress concrete members, deflections are less because the eccentric pre-stressing force will induce couple which will cause upward deflections, where as in R.C.C, deflections are more.
5. In pre-stress concrete fatigue resistance is more compare to R.C.C. because in R.C.C. stress in steel is external load dependent where as in P.S.C member it is load independent.
6. Pre-stress concrete is more durable as high grade of concrete is used which are denser in nature. R.C.C. is less durable.
7. In pre-stress concrete, dimension is less because external stresses are counterbalance by the internal stress induced by pre-stress. Therefore reactions on column & footing are less as a whole the quantity of concrete is reduced by 30% and steel reduced by about 60 to 70%. R.C.C. is uneconomical for long span because in R.C.C. dimension of sections are large requiring more concrete & steel. Moreover as self-weight increases more reactions acted on columns & footings, which requires higher sizes [28].

2.2 Rail Track on Bridges

Traditionally, railway tracks are ballasted. However, efficient and safe solutions that do not use ballasts have been developed in the past 40 years, called slab tracks [25].

2.2.1 Ballast Track

Ballast is gravel or coarse stone used to form the bed of a railway track or the substratum of a road. Ballast has many functions. The most important functions are to retain track position, reduce the sleeper bearing pressure for the underlying materials, store fouling materials, provide drainage for water falling onto the track, and rearrange during maintenance to restore track geometry. Thus, ballast materials are required to be hard, durable, and angular, free from dust and dirt, and have relatively large voids. Since ballast is a type of granular material, behavior of such a material is well documented in granular materials literature. Past experience of ballast field performance has shown that the progressive breakdown of ballast materials, such as that caused by traffic load and maintenance tamping, and the intrusion of external materials, such as wagon spillage and infiltration of underlying materials into the ballast results in major track deterioration. The response of fouled ballast is highly dependent on the types of fouling materials, the quantity of fouling materials and water content [28].

The ballasted track has relatively low construction costs comparing to slab track, it has high elasticity and high maintainability at relatively low cost. Another important advantage of the ballasted track is the high noise absorption [7]. The 150 years' experience with ballasted tracks makes the engineers more confident to deal with ballasted track problems and avoid the higher risk of dealing with problems concerning a relatively new design [30].

Many recent advances in ballasted track construction make them even more competitive against slab track. Few examples of these advances are the following [30].

- Introduction of reinforced concrete or steel sleepers.
- Optimization of the rail geometry improves the wheel-rail contact condition offering a better load bearing capacity.
- Optimized rail pads and fastening providing better attenuation of the wheel- rail contact forces and of noise.

- Stone blowing and tamping machines provide more accurate corrections and faster maintenance.
- Better track resilience and resistance to settlements through the use of under sleeper mats as well as geo grids and geo synthetics between the different layers.
- The use of geotechnical retro-fitting techniques (e.g. lime cement columns), improving the bearing capacity of the sub grade, minimizing the possibility of settlements.

The final decision for the construction is taken considering both advantages and disadvantages of each system. The disadvantages which may lead the decision-makers to a slab track construction are the following [7].

- The ballasted track does not provide good lateral or longitudinal resistance with result to have “floating” track effects.
- Limited lateral resistance of the ballasted track, resulting to lower speeds in curves.
- The deterioration of the ballast creates particles which are damaging the rail and the wheels.
- The wear of the ballast plus the intrusion of the fine particles from the sub grade, contaminate the structure making it impermeable.
- Ballasted track is heavier and higher structure demanding stronger structures and larger foundations in case of viaducts and bridges.
- The track elements and the way they are put together during construction will highly influence the rate of the track deterioration.
- In bridges and tunnels where continuous ballast bed is the case, extra elasticity must be supplied by application of ballast mats, rail fastenings with increased elasticity. Nonetheless the usual maintenance must be provided at regular basis.

2.2.2 Slab Track on Bridge

The bridge deck supports the slab track on a bridge. Structural elements that are part of the slab track are subject to mainly compression forces. All components of the slab track have to be designed in such a way that the vertical and horizontal forces are transmitted and resisted in a safe way [25].

Generally speaking the advantages of the slab track design are the significantly reduced maintenance need in combination with its higher serviceability life, as well as its higher structural track stability. According to bibliography [24], the reasons that may lead to the construction of a slab track system instead a ballasted one are the following:

- Lower maintenance need during its life cycle. No need for tamping, ballast cleaning and track lining results to a reduced cost approximately 20-30% for repairs comparing to that in ballasted track.
- Lower traffic hindrance or interruption costs.
- Higher life cycle, around 50-60 years compared to ballasted track (30-40 years) and possibility of almost full replacement at the end of the service life.
- More cost effective line positioning (as narrower curves at high super elevation and super elevation deficiency can be applied)
- No ballast or solid particles are whirled up on slab track.
- Higher safety against lateral forces and accommodation of higher axle loads.
- The eddy current brake can be applied without problems any time. (This is an advantage against ballasted track only in certain places such as signals or at station entrances. It cannot be seen as an advantage on plain line track.)
- Emergency vehicles and fire brigade vehicles can drive on the slab track in tunnels easily.
- Cost of vegetation control is either excluded or very reduced.
- Near maximum availability of the line and barely causes disturbances to the residents for maintenance during night shifts.
- Optimum design for high speed trains since it does not experience any problems such as drag forces at ballast.

- The slab track can compensate any excess in super elevation and in cant deficiency with freight trains or passenger trains without fears for dislocation of the track.
- Possible corrections up to 26 mm in vertical and up to 5 mm in horizontal position can be applied to counterbalance minor displacements.
- Reduced height and weight of the structure.
- The lack of suitable aggregates for a ballasted track in a certain area can also lead to a slab track design.
- Slab track maybe also more suitable in cases where the noise emissions and the vibration nuisance do not cause problems and are acceptable.
- In places where the release of dust from the ballast bed must be prevented for environmental reasons the slab track is a good solution.
- Excellent riding comfort at high speed.
- Better load distribution, hence reduced dynamic load of subsoil.
- Excellent load distribution, thereby reducing the pressure on unconfined soil layers and the sub grade.
- Lower wear of vehicle running gear through good retention of track geometry.
- The higher braking forces enable for shorter braking distances.
- Slab tracks allow for steeper route gradients.
- The rail can be laid in lower temperatures since buckling is less of a concern comparing to ballasted track.
- Lower construction costs in case track and rolling stock are adjusted to one another.

Slab track systems have various disadvantages too. In general the higher investment costs combined with the longer manufacture and installation time needed for its construction as well as the limited options in adjustments after construction and the higher air-borne vibration emissions, are few of the main reasons slab track is not the dominant track type used. The disadvantages of slab track are stated below according to the available bibliography [24]:

- The danger to have wrongly selected this design because of its lower maintenance cost due to the influence of the operational organization which is fully responsible for the railways maintenance after the construction. In many railways a part or the whole construction costs of a track are usually covered by governmental financial subsidies for sums invested in infrastructure. The total cost must be assessed and carefully examined in a fair way in order to select the most suitable track design.

- The deterioration of the track geometry in case the operational strength of the concrete slab track has been reached, can occur very suddenly and unforeseeably. Thus the operational strength of the slab track might be compared to the occurrence of rail fracture.
- Small adaptability to large displacements in the embankment. Large displacements in track can be compensated only by significant amounts of work.
- Slab track has an estimated life cycle of 50-60 years. Of course this is valid only if the presupposition that the expected acceptable settlements will occur. In case of a derailment or any other unforeseeable events which could cause greater damage than the expected one (damage in sensitive fastening elements) can result to long term and expensive track closures. Unfortunately due to the short age of slab track there is not enough information on the actual performance during its life time in order to assess and examine this issue with high validity.
- Slab track by its rigid structure it is ensured that its life time will be at least 50-60 years. The nature of the slab track does not allow for easy adjustments and repairs after its construction. That means that its quality during the construction must be checked and reassured carefully because any defect on its quality would either remain for the entire life cycle either high costly measures should be taken in order to eliminate it.
- Not many possibilities to apply any innovation (improvement) or future updates after construction.
- Slab track cannot be built in soft clays, earthquake areas or embankments on soft peat layers.
- Slab track requires homogeneous sub layers which are capable to carry the imposed loads with minor or no settlements. This means that in many cases and especially in earth structures special attention should be given in the foundation preparations. The high costs which are associated with the above mentioned fact is the main reason for the limited use of the slab track.
- Higher noise emissions. To handle the increased noise, extra treatment is needed which result to higher construction costs.

- Very expensive repair concepts and long term closures due to the curing and hardening procedures of the concrete.
- The frost protective layer in earth structures must be applied in any case and it is much thicker comparing to the ballasted one. This is a prerequisite in order to reassure a lengthy life cycle.
- The cost of the reconstruction (after it has reached the end of its life cycle) of the slab track is not considered. One or two standardized types of slab track seem to be optimal solutions.
- Transitions between ballasted track and slab track require special attention.
- In many cases new mechanisms needed for production and repair.

2.2.3 Comparison Between Ballasted and Slab Track

The slab track can ensure very good geometrical stability of the track comparing to the ballasted track, on the other side it produces higher noise emissions. The extra protective measures need to be taken, increase the costs of the slab track construction significantly. In tunnels though, the slab track has been proved in many cases to be a more economical efficient solution due to its structure (lower height and no need for special preparation of the subsoil). Slab track is more beneficial in more favorable locations of the lines which can be adapted to the terrain and to existing structures by applying higher super elevation and super elevation deficiency values. One more advantage of the slab track is the easier and more economic vegetation control comparing to conventional track. In general the ballasted track is considered as a better solution in earth structures due to the lower costs of the construction [24]. On the other side, slab track maybe is more expensive to construct but the lower demand for track maintenance during the years and its high serviceability life time, suggest that in a long term perspective is economically more efficient [24].

The comparison of ballasted track and slab track as follows [1].

Table 2-2 Comparisons of ballasted track and ballast-less track:

S.N.	DESCRIPTION	BALLASTED TRACK	SLAB TRACK
1	Maintenance Input	Frequent maintenance for geometry.	No frequent maintenance for geometry.
2	Cost comparison	Relatively low construction costs but higher life cycle cost.	Relatively high construction cost but lower life cycle cost.
3	Elasticity	High elasticity due to ballast.	Elasticity is achieved through use of rubber pads and other artificial materials.
4	Riding Comfort	Good riding comfort at speeds up to 250 – 280 KMPH.	Excellent riding comfort even at speeds greater than 250 KMPH.
5	Life expectation	Poor Life expectation	Good life expectation.
6	Stability	Over time, the track tends to "float", in both longitudinal and lateral directions, as a result of non-linear, irreversible behavior of the materials.	No such problem.
7	Lateral resistance	Limited non compensated Lateral acceleration in curves, due to the limited lateral resistance offered by the ballast.	High lateral resistance to the track which allows future increase in speeds in combination with tilting coach technology.
8	Noise	Relatively High noise.	Relatively low noise and vibration nuisance.

9	Churning up of Ballast	Ballast can be churned up at high speeds, causing serious damage to rails and wheels.	No such damage to rails and wheels.
10	Permeability	Reduced permeability due to contamination, grinding-down of the ballast and transfer of fine particles from the sub grade.	High impermeability
11	Construction cost of Bridges/Tunnels/etc.	Ballast is relatively heavy, leading to an increase in the costs of building bridges and viaducts if they are to carry a continuous ballasted track.	Less cost of construction of bridges and viaducts due to lower dead weight of the slab track.
12	Construction Depth	Depth of Ballasted track is relatively high, and this has direct consequences for tunnel diameters and for access points.	Reduced height
13	Availability of Material.	Limited availability of suitable ballast material.	No problem of material.
14	Accessibility to road vehicles.	Track is not accessible to road vehicles.	The track can be accessible to road vehicles
15	Dust pollution requirement.	Release of dust from the ballast into the environment thus causing environmental pollution.	Less environment pollution.
16	Suitability on the basis of requirement of maintenance input.	Less suitable due to limited availability of traffic block in the present scenario of high traffic growth.	Highly suitable in the present and future scenario due to very less maintenance

2.3 Structural Initial Dimension of Slab for Rail Tracks

2.3.1 Thickness of the Deck

Appropriate thickness need to be provided on the basis of deflection requirement. The minimum thickness of reinforced concrete depends on the support condition and the support spacing. This criterion is defined for concrete flexural members so that they are provided with adequate stiffness to resist excessive deflection. The deflection of simple or continuous slabs shall not exceed L/800 of the span length [14]. The thickness of the composite slab can be

$$t = \frac{1.2\,(S+3.05)}{30} \quad \text{for simply supported slab} \qquad (2.1)$$

$$t = \frac{(S+3.05)}{30} \quad \text{for continuous slab} \qquad (2.2)$$

Where, t=thickness of deck in meter.

S=effective span in meter.

2.3.2 Determination of Ballast Depth in the Bridge

Depth of ballast in bridge is limited and numerically specified as follows;

For a railway bridge, the ballast height should be at least 0.6m. The ballast has a weight density of $20kN/m^3$ [25].

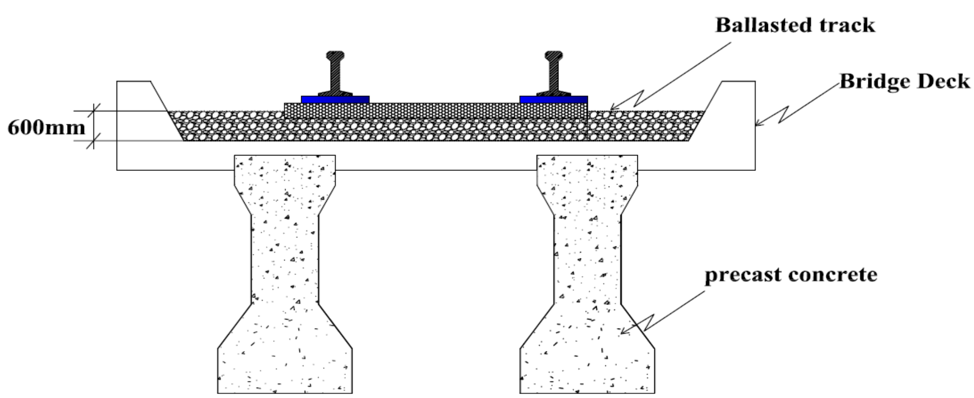


Figure 2-13 Depth of ballast in bridge

2.3.3 Determination of Slab Depth in the Bridge

The minimum depth of slab track is 200mm as shown in the figure below [24].

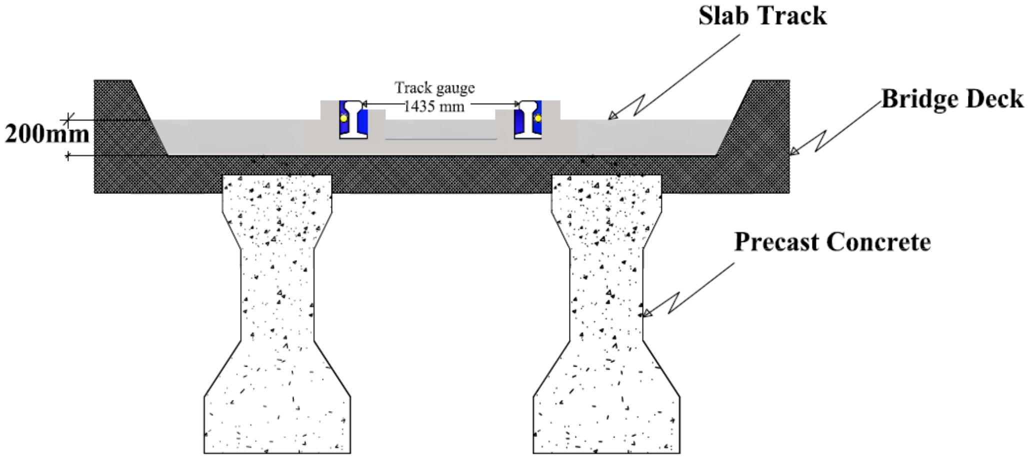


Figure 2-14 Depth of slab track on bridge

2.4 Design Standards and Methods

2.4.1 Design Standards

In order to form a consistent basis for design, design standards loading are applied to the design model of structure. Now a day's in Ethiopia, the railway project constructed by china companies and used their design standard. However, china design standards are not available. For the availability as well as many country adoptions, the study used design standards: American Railway Engineering and Maintenance of Way Association (AREMA) and American Association of State Highway and Transportation Officials (AASHTO).

2.4.2 Loadings on Railway Bridges

Bridges must be designed to carry the specified dead loads, live loads and impact, as well as, loads from continuous welded rail, loads from ballast, loads from curbs and load from sleeper. The forces and stresses from each of these specified loads should be a separate part of the design calculations. Also, because rail cars have changed in size and weight over the years and frequently are run in unit consists; the designer should be alert to live loadings that may be more severe than those used in some specifications.

The following loads and forces shall be considered in the design of railway concrete structures supporting tracks [4].

1) Dead load.
2) Live load.
3) Centrifugal force.
4) Lateral force due to wind load and nosing of locomotives.
5) Longitudinal force.
6) Impact load.
7) Earth quack load

Dead Load

Dead loads are those that are constant in magnitude and fixed in location throughout the lifetime of the structure. Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material.

1. The dead load shall consist of the estimated weight of the structural member, plus that of the track, ballast, fill, and other portions of the structure supported thereby.
2. The unit weight of materials comprising the dead load, except in special cases involving unusual conditions or materials, shall be assumed as follows [4].
   - Track rails, inside guardrails and fastenings – 200 lb per linear foot of track. (3kN/m)
   - Ballast, including track ties – 120 lb per cubic foot. (1900 kg/m^3)
   - Reinforced concrete – 150 lb per cubic foot. (2400 kg/m^3)
   - Earth filling materials – 120 lb per cubic foot. (1900 kg/m^3)
   - Waterproofing and protective covering – estimated weight.

Live Load

Live loads consist chiefly of occupancy loads in buildings and traffic loads on bridges. They may be either fully or partially in place or not present at all, and may also change in location. Railway bridges have a higher live-to-dead load ratio because the mass of the railway loading is generally large, relative to that of the bridge [4].

The recommended live load for each track of main line structure is Cooper E-80 loading with axle loads and axle spacing as shown in figure below. On branch lines and in other locations where the loading is limited to the use of light equipment, or cars only, the live load may be reduced, as directed by the engineer.

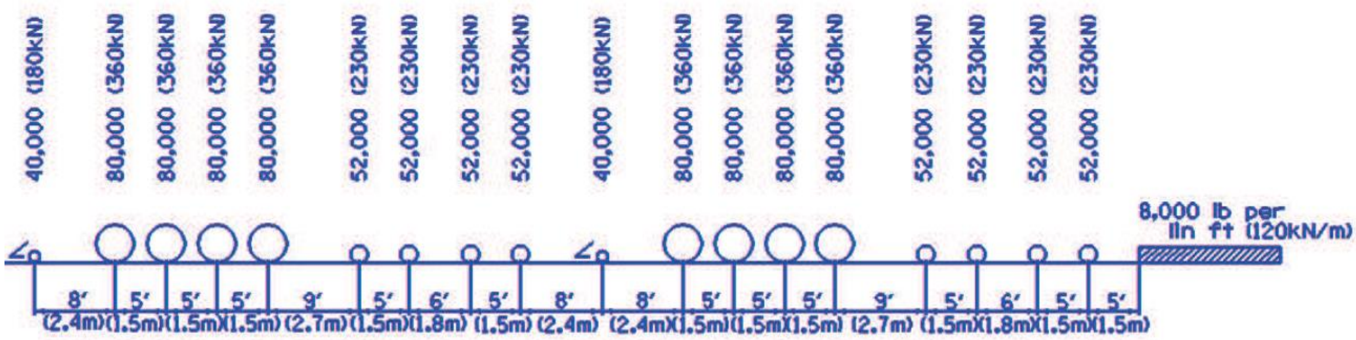


Figure 2-15 Cooper E- 80 Live load

To determine maximum moments, and shears force for cooper E-80 live load, refer table 2-3 and tabulated values are used for only simple spans [4].

Table 2-3 Maximum moments, and shears of LL for one rail (one - half track load)

span length (Ft)	maximum moment (Ft-Kips)		maximum moment Quarter point (Ft-Kips)		maximum shears (Kips)						Maximum pier reaction (Kips) (2)	
					At End		At Quarter Point		At center			
	E-80	Alt.	E-80	Alt.	E-80	Alt.	E-80	Alt.	E-80	Alt.	E-80	Alt.
5	50.00	62.50	37.50	46.88	40.00	50.00			20.00	25.00	40.00	50.00
6	60.00	75.00	45.00	56.25	46.67	58.33	30.00	37.50	20.00	25.00	53.33	58.33
7	70.00	87.50	55.00	68.75	51.43	64.29	31.43	39.29	20.00	25.00	62.86	71.43
8	80.00	100.00	70.00	87.50	55.00	68.75	35.00	43.75	20.00	25.00	70.00	81.25
9	93.89	117.36	85.00	106.25	57.58	72.22	37.78	47.23	20.00	25.00	75.76	88.89
10	112.50	140.63	100.00	125.00	60.00	75.00	40.00	50.00	20.00	25.00	80.00	95.00
11	131.36	164.20	115.00	143.75	65.45	77.27	41.82	52.28	21.82	27.28	87.28	100.00
12	160.00	188.02	130.00	162.50	70.00	83.33	43.33	54.17	23.33	29.17	93.33	108.33
13	190.00	212.83	145.00	181.25	73.84	88.46	44.61	55.76	24.61	30.76	98.46	115.39
14	220.00	250.30	165.00	200.00	77.14	92.86	47.14	57.14	25.71	32.14	104.29	121.43
16	280.00	325.27	210.00	250.00	85.00	100.00	52.50	62.50	27.50	34.38	113.74	131.25
18	340.00	400.24	255.00	318.79	93.33	111.11	56.67	68.05	28.89	36.11	121.33	138.89
20	412.50	475.00	300.00	362.50	100.00	120.00	60.00	72.50	28.70	37.50	131.10	145.00
24	570.42	668.75	420.00	500.00	110.83	133.33	70.00	83.33	31.75	41.67	147.92	154.17
28	730.98	866.07	555.00	650.00	120.86	142.86	77.14	92.86	34.29	46.43	164.58	
32	910.85	1064.06	692.50	800.00	131.44	150.00	83.12	100.00	37.50	50.00	181.94	
36	1097.30	1262.50	851.50	950.00	141.12	155.56	88.90	105.56	41.10	55.56	199.06	
40	1311.30	1461.25	1010.50	1100.00	150.80	160.00	93.55	110.00	44.00	60.00	215.90	
45	1601.20	1710.00	1233.60	1287.48	163.38	164.44	100.27	114.45	45.90	64.45	237.25	
50	1901.80	1959.00	1473.00	1481.05	174.40		106.94	118.42	49.73	68.00	257.52	
55	2233.10		1732.30		185.31		113.58	120.91	52.74	70.91	280.67	
60	2597.80		2010.00		196.00		120.21	123.33	55.69	73.33	306.42	
70	3415.00		2608.20		221.04		131.89		61.45	77.14	354.08	
80	4318.90		3298.00		248.40		143.41		67.41	80.00	397.70	
90	5339.10		4158.00		274.46		157.47		73.48	82.22	437.15	
100	6446.30		5060.50		300.00		173.12		78.72	84.00	474.24	
120	9225.40		7098.00		347.35		202.19		88.92		544.14	
140	12406.00		9400.00		392.59		230.23		101.64		614.91	

In calculating the maximum live loads on a structural member due to simultaneous loading on two or more tracks, the following proportions of the specified live load shall be used:

- For two tracks – full live load,
- For three tracks – full live load on two tracks and one-half on the other track,
- For four tracks – full live load on two tracks, one-half on one track, and one-fourth on the remaining track [4].

Centrifugal Force

a. On curves, the centrifugal force in percentage of the live load is:

   $0.00117S^2D$ (2.3)

   Where:

   S = Speed in miles per hour

   D = Degree of curve

   (Because of the limited duration of the loads, centrifugal force need not be considered in the design of stringers.)

b. The effect of centrifugal force may be reduced by the compensating effect of the actual amount of super elevation provided [4].

Impact Forces

- Impact forces, applied at the top of rail, shall be added to the axle loads specified. For rolling equipment without hammer blow (diesels, electric locomotives, tenders alone, etc.), the impact shall be equal to the following percentages of the live load:

  For $L \leq 4$ meters $I = 60$ (2.4)

  For 4 meters $< L \leq$ 39 meters $I = \frac{125}{\sqrt{L}}$ (2.5)

  For $L \geq 39$ meters $I = 20$ (2.6)

  Where L is the span length in (meters)

- For continuous structures, the impact value calculated for the shortest span shall be used throughout.
- Impact may be omitted in the design for massive substructure elements which are not rigidly connected to the superstructure.
- For steam locomotives with hammer blow, the impact shall be increased by 20% [4].

Longitudinal Force

The longitudinal force for E-80 loading shall be taken as the larger of:

- Force due to braking, as prescribed by the following equation, acting 8 feet (2450mm) above top of

  Longitudinal braking force (kips) = 45+1.2L (2.7)

  (Longitudinal braking force (kN) = 200+17.5L) (2.8)

  Where L is the length in feet (meters) of the portion of the bridge under consideration

- Force due to traction, as prescribed by the following equation, acting 3 feet (900mm) above top of rail.

  Longitudinal traction force (kips) = 25 $\sqrt{L}$ (2.9)

  (Longitudinal traction force (kN) = 200$\sqrt{L}$) (2.10)

  Where L is the length in feet (meters) of the portion of the bridge under consideration

Wind Load

A wind load of 300lb per linear foot (4.4kN/m) on the train shall be applied 8 feet (2450mm) above the top of rail in a horizontal direction perpendicular to the centerline of the track [4].

Earthquake Loading

Earth quake loading is a product of natural loading which depend on geometric location of the bridge. Seismic force primarily affects bridge substructure components. The earth quake forces can be described as a function of the acceleration coefficient, Soil type, the fundamental period. The effect of earthquake in Ethiopia is grouped in to four different zones. Bridge in seismic zone 1-3 need not be analyzed for seismic loads, regardless of their importance and geometry. Seismic analysis is not required for single-span bridge, regardless of the seismic zone (10) [14].Since the study area is Addis Ababa (zone-2), and no need of design of seismic load in this zone.

Load Path

The path of the load from the wheels through the rail and into the tie is either directly to the supporting beams, or through a ballast bed to a deck and hence into the supporting beams or girders and then load to the abutments of the bridge [4].

Combination of Loads

The loads which are considered in the thesis are dead, superimposed, and live and impact loads. These loads combination is as follows:

Loads combination = 1.25 DL + 1.75 LL [4] (2.11)

Loads combination = 1.4(D+5/3(L+I)) [18] (2.12)

2.4.3 Material Properties of Pre-Stressed and Non-Pre-Stressed Concrete and Steel

Concrete

Concrete is stone like material obtained artificially by hardening of the mixture of cement, inert-aggregate materials (fine & course) and water in predetermined proportions. When these ingredients are mixed, they form a plastic mass which can be poured in suitable moulds (forms) and set-on standing into hard solid mass, as a result of exothermic chemical reaction between cement and water. To produce a workable mix, more water is used over and above that needed for this chemical reaction (water-cement ratio required for complete chemical reaction is about 0.25). The reaction between cement and water is relatively slow and requires time and favorable temperature for its completion [21].

Compressive Strength of Concrete: -A wide range of strength properties can be obtained for concrete by appropriate adjustment of the proportions of the constituent materials, using different degree of the compaction and the conditions of temperature and moisture under which it is placed and cured. Water cement ratio is the main factor affecting the strength of concrete, as shown in figure below.

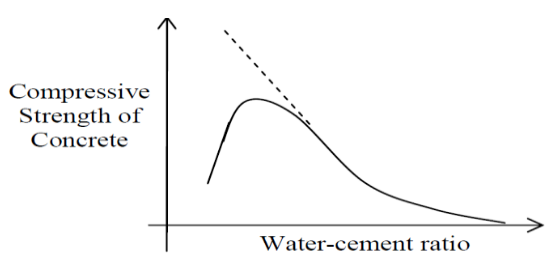


Figure 2-16 Relationship between compressive strength of concrete and water-cement ratio

Standard test specimens of 150mm cube are taken at the age of 28days to determine the compressive strength of concrete. At age of 7days, concrete may attain approximately about 2/3 of the full compressive strength of concrete. In some national standard (example ACI code), cylinder specimens of 150mm diameter by 300mm high are taken. Although the load is applied uni-axially, the friction between the loading plate and the contact faces of the test specimen has more effect on cube strength than the cylinder strength. Because of this, the cube strength gives more strength than the true compressive strength of concrete, whereas, cylinder strength gives reasonably the true compressive strength. On average, cube strength is taken as 1.25 times cylinder strength. If large size aggregates are used, a cube mold with side 200mm may be used to determine compressive strength of concrete. And strength of concrete is converted to 150mm cube compressive strength by factor of 1.05.

The strength of concrete for design purposes will be based on compressive tests made on cubes at an age of 28 days unless there is satisfactory evidence that a particular testing regime is capable of predicting the 28-day strength at an earlier age. These 28-day characteristic strengths determine the grade of the concrete and it is important to select the correct grade appropriate for use. The concrete has to provide the durability for the environmental conditions as well as adequate strength for the loading requirements [21].

Table 2-4 Grades of concrete

Class	Permissible Grades of concrete							
I	C5	C15	C20	C25	C30	C40	C50	C60
II	C5	C15	C20					

In accordance with Ethiopian Standards, compressive strength of concrete is determined from tests on 150mm cubes at the age of 28 days. Cylindrical or cubical specimens of other sizes may also be used with conversion factors determined from a comprehensive series of tests. In the absence of such tests, the conversion factors given in Table below may be applied to obtain the equivalent characteristic strength on the basis of 150mm cubes [21].

Table 2-5 Conversion factors for strength

Size and type of Test Specimen	Conversion Factor
Cube (200mm)	1.05
Cylinder (150mm diameter 300mm height)	1.25

The characteristic cylinder compressive strength f_{ck} are given for different grades of concrete in Table below

Table 2-6 Grades of concrete and characteristic cylinder compressive strength f_{ck}

Grades of concrete	C 15	C 20	C 25	C 30	C 40	C 50	C 60
f_{ck}	12	16	20	24	32	40	48

Characteristic Tensile Strength of Concrete: the characteristic tensile strength of concrete can be determined statistically by the same equation given above using test results obtained from split-cylinder test or from beam-test. It can also be determined using empirical relation obtained from a number of tests in terms of characteristic compressive strength of concrete given by codes. According to EBCS-2, characteristic tensile strength of concrete is obtained using

$$f_{ctk} = 0.21(0.8f_{cu})^{2/3} \quad (f_{cu} \;\&\; f_{ctk} \text{ in MPa}) \qquad (2.13)$$

Reinforced concrete has a weight density usually set, for design purposes; to 25 kN/m^3 (unreinforced concrete has a weight density of 24 kN/m^3) [25].

Design Strength of Material in Limit State: The design strength for a given material and limit state is given by:

Design Strength for Concrete [11].

a) In compression $f_{cd} = \frac{0.68 f_{cu}}{\gamma_c}$ or $f_{cd} = \frac{0.85 f_{ck}}{\gamma_c}$ (2.14)

b) In tension $f_{ctd} = \frac{f_{ctk}}{\gamma_c}$ (2.15)

Design Strength for Steel

In tension and compression $f_{yd} = \frac{f_{yk}}{\gamma_s}$ (2.16)

Idealized Stress-Strain Diagrams: For s design purpose, most codes adopt idealized stress-strain diagrams in predicting the ultimate strength of sections in plastic-theory. In EBCS-2, a parabola-rectangle stress-strain diagram is given for concrete in compression as shown in figure below.

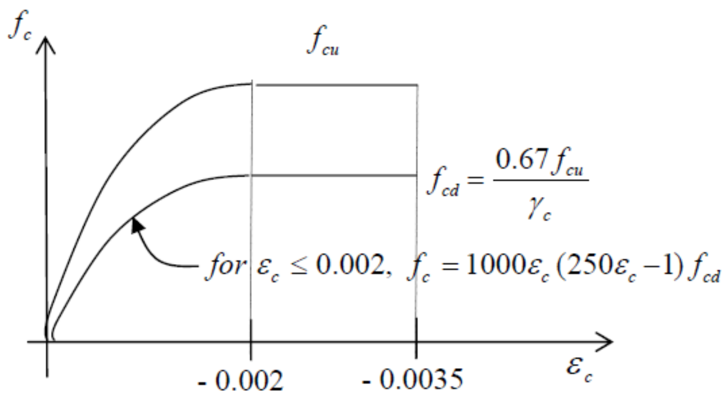


Figure 2-17 Idealized stress- strain diagram

Modulus of Elasticity of Concrete: According to EBCS-2/95, mean value of the secant modulus, E_c is given as shown in table below.

Table 2-7 Modulus of elasticity of concrete

f_{cu}(MPa)	C-15	C-20	C-25	C-30	C-40	C-50	C-60
E_c (GPa	26	27	29	32	35	37	39

Reinforcing Steel

Steel reinforcements are available in the form of round bars and welded wire fabric. The most commonly used bars have projected ribs on the surface of bar. Such bars are called deformed bars. The ribs of deformed bar improve the bond between steel and the surrounding concrete in RC members by providing mechanical keys. A wide range of reinforcing bars is available with nominal diameter ranging 6mm to 35mm. Most bars except 6mm diameter are deformed one. Some of the common bar size with their application in concrete works are given in table below.

Table 2-8 Reinforcing steel

	For stirrups		For slabs		For beams & columns						
Diam. (mm)	Ø6	Ø8	Ø10	Ø12	Ø14	Ø16	Ø18	Ø20	Ø22	Ø 25	Ø 28
Area (cm^2)	0.28	0.50	0.785	1.13	1.54	2.01	2.52	3.14	3.80	4.90	6.20
Weight (kg/m)	0.222	0.395	0.617	0.888	1.21	1.57	2.00	2.47	3.00	3.90	4.80
per. (cm)	1.88	2.51	3.14	3.77	4.40	5.02	5.65	6.28	6.90	7.85	8.79

Strength of Reinforcing Steel: - Reinforcing steel is capable of resisting both tension and compression. Compared with concrete, it is a high strength material. For instance, the strength of ordinary reinforcing steel is about 10 & 100 times, the compressive & tensile strength of common structural concrete. Typical stress-strain curves for mild-steel and high-yield (cold-worked) steel are shown in figures below.

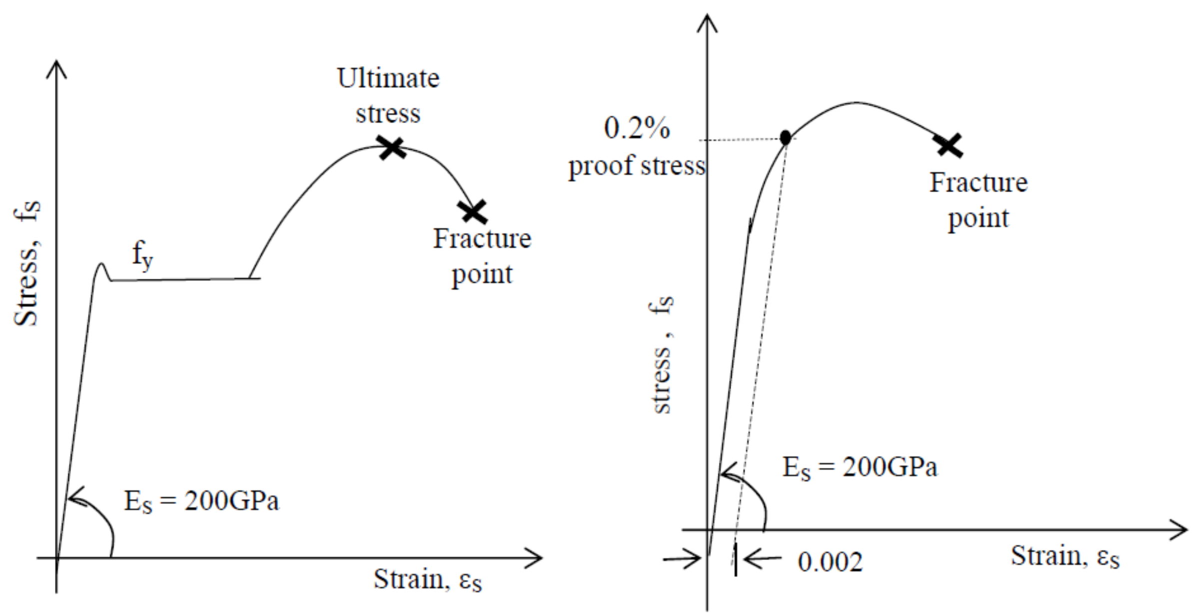


Figure 2-18 a) Stress-strain curve for mild steel b) Stress-strain curve for high-yield steel
(S- 250MPa, S- 300MPa) (S- 420MPa, S- 460MPa, S- 500MPa)

The strength of mild steel is taken as yield point or yield stress of steel whereas for high-yield steel is based on specified proof stress of steel. 0.2% proof stress is specified in most codes to determine strength of high yield steel. A 0.2% offset is drawn parallel to the linear part of the stress-strain curve to determine 0.2% proof stress.

The shape of the stress-strain curve is similar for all steel, and differs only in the value of strength of steel, the modulus of elasticity, E_s being for all practical purposes constant. E_s is taken as 200GPa. For a design of RC members, reinforcing steel up to grade of 550MPa can be used. If steel with grade beyond 550MPa is used for RC member, the sections are under utilizing the reinforcement. This is because the width of concrete crack is wide if the steel is fully stressed.

EBCS-2, also idealized the stress-strain diagram for steel with ultimate strain of 0.01 as shown in figure below. It is a portion of stress-strain diagram of steel. The maximum strain of steel, ε_S, max = 0.01 permitted by code assumed to limit width of concrete crack in tension zone to acceptable limit.

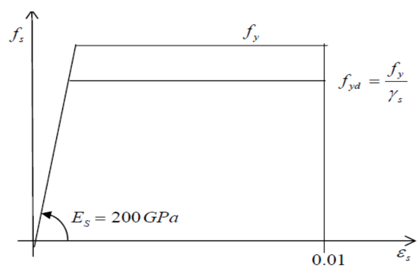


Figure 2-19 Idealized the stress-strain diagram for steel with ultimate strain of 0.01

Reinforced Concrete

It is known that plain concrete is quite strong in compression, weak in tension. On the other hand, steel is a high cost material which able to resist both tension & compression. The two materials (plain concrete & reinforcing steel) are best be utilized in logical combination if steel bars are embedded in the plain concrete in tension zone close to the surface. In this case, plain concrete is made to resist the compressive stresses and reinforcing steel resists the tensile stresses. Both plain concrete & reinforcing steel bar together assumed to act as one composite unit and it is termed as reinforced concrete (RC). The tensile stresses developed in the section are transferred to reinforcing steel by the bond between the interfaces of the two materials [21].

In all RC members, strength design is made on the assumption that concrete does not resist any tensile stresses. All the tensile stresses are assumed to be resisted by the reinforcing steel imbedded in tension zone. Sometimes if necessary, reinforcing steel is provided in compression zone to assist the concrete resisting compression in addition to reducing creep deformation.

Reinforcing steel & concrete may work readily in combinations due to the following reasons.

1. Bond between the bars & the surrounding concrete prevents slip of the bars relative to the concrete. Adequate concrete cover for steel bar and embedment length of bar are required to transfer stress between steel and concrete without slipping.
2. Proper concrete mixes provide adequate impermeability of concrete against bar corrosion.
3. Sufficiently similar rates of thermal expansion (0.00001/^{0}C to 0.000013/^{0}C for concrete and 0.000012/^{0}C for steel) introduce negligible stresses between steel and concrete under temperature changes [21].

Advantages of Reinforced Concrete:

1. It is monolithic. This gives it more rigidity.
2. It is durable. It does not deteriorate with time.
3. While it is plastic, it can be moldable into any desired shape.
4. It is fire, weather and corrosion resistant.
5. By proper proportioning of mix, concrete can be made water-tight.
6. It maintenance cost is practically nil.

Disadvantages of Reinforced Concrete:

1. It is difficult to demolish in case of repair of modification.
2. It is too difficult to inspect after the concrete has been poured [21].

2.4.4 Design Criteria: Strengths, Deflections and Shears

2.4.4.1 Material Properties

To satisfy the design objective of a structural member, the design strength for ULS of both concrete and steel are required. The instantaneous and time dependent properties of concrete and steel at typical in-service stress level are also required [6].

a) Concrete

High strength concrete are used, concrete grade larger than C-30 class I works high compressive strength at a reasonably early age, & comparatively higher tensile strength as compared with ordinary RC member, low shrinkage, minimum creep characteristics and high young's modulus are necessary [6].

Concrete grade larger than

- C-40, for pre tensioned,
- C-30, for post tensioned members

The stress permitted in concrete at the stage of transfer and service loads are defined in terms of the corresponding compressive strength of the concrete at each stage.

Allowable stresses in concrete [6].

At transfer

$f_{ct} \leq 0.6\ f_{ck(t)}$ in compression due to bending. (2.17)

$f_{tt} \leq 0.21\ f_{ck(t)}^{2/3}$ in tension due to bending. (2.18)

Under working loads

$f_{cw} \leq 0.50\ f_{ck}$ in compression due to bending. (2.19)

$f_{tw} \leq 0.00$ in tension for fully pre-stressing. (2.20)

b) Pre-Stressing Steel

Three types of high strength steel termed as tendons are used with $f_{pu} \geq 1000$ MPa.

For design purpose the following idea stress – strain curve is adopted.

In design the stress in pre stressing steel at ULS limited to $0.9f_{pk}/\Upsilon_s$ and strain $\in_u \leq 0.01$.

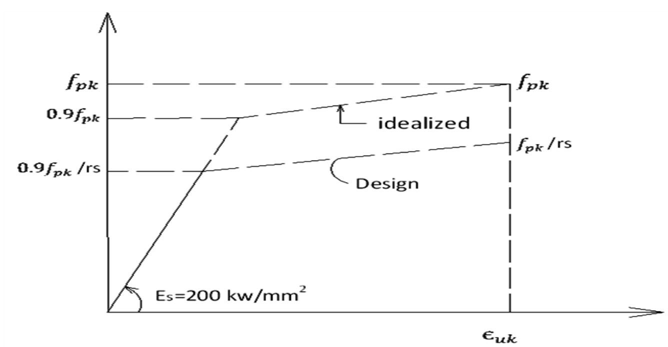


Figure 2-20 Stress-strain curve for typical high strength steel

Immediately after pre stress transfer

$f_{pi} \leq 0.75f_{pk}$ (2.21)

Stress in tendons after all losses (under working)

$f_{pe} \leq \eta*(0.75f_{pk}) = 0.6\ f_{pk}$ (2.22)

Where, η- is loss factor which is 0.78 or 0.82 for pre tension and post tension respectively.

2.4.4.2 Pre-Stress Losses

Lump Sum Estimate of Pre Stress Losses

For average steel and concrete properties, cured under average air condition, the following values may be taken as representative of average losses [6].

Table 2-9 Pre-stress losses

Loss due to	% loss	
	Pre-tensioning	Post-tensioning
Elastic shortening and bending of concrete	4	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Steel relaxation	5	6
Total loss	22	18

These values are based on the assumption that over tensioning has been applied to overcome friction and anchorage losses.

An average of 22% loss in case of pretension and 18% in post-tension may be assumed with condition of over tensioning has been applied to overcome friction and anchorage losses.

2.4.4.3 Deflection

Deflections can take place: upwards due to pre stressing (-) & down wards due to transverse loads (+). The deflection can be calculated using the ordinary deflection formulas because there are no cracks

Deflections due to loads: ordinary formula can be applied depending on

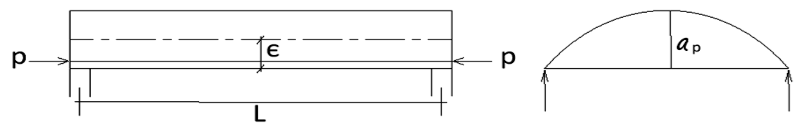
- The intensity of loading
- Restraint condition and span

Deflection due to Self-Weight or uniformly distributed Superimposed Load

$$\Delta = \frac{5 \times W \times L^4}{384 \times E_c \times I} \tag{2.23}$$

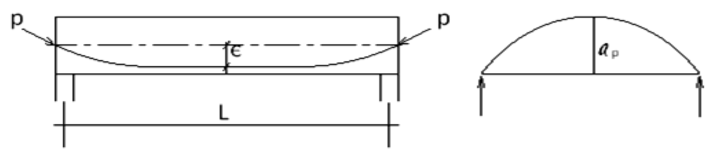
Deflection due to Pre Stressing Force [6]

a) **Straight tendons**



$$\Delta_p = \frac{-P \times e \times L^2}{8EL} \qquad (2.24)$$

b) **Parabolic Tendons (Central Anchors)**



$$\Delta = \frac{-5 \times P \times e \times L^2}{48 \times E_c \times I} \qquad (2.25)$$

2.4.4.4 Shear

Two major modes of shear cracks may be observed namely web shear and flexure shear cracks

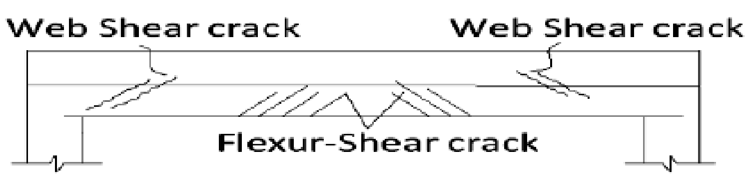


Figure 2-21 Web and flexural shear crack

Web Shear Crack: - it is governed by limiting value of the principal tensile stress as developed in concrete. It is more likely to occur in highly pre stressed members with thin webs.

Flexure Shear Cracks: - primarily initiated by flexural cracks and are developed when the combined shear and flexural tensile stresses produce a principal tensile stress exceeding the design tensile strength of concrete. When the shear force exceeds the limiting value against diagonal compression the section must be changed.

When $V_c > V_d$, only minimum shear reinforcement shall be provided which may be expressed as

$$S_{max} = \frac{a_v}{\rho_w \times b} = \frac{a_v \times f_{yk}}{0.4 \times b} \quad (2.26)$$

The shear force sustained by stirrups may be computed for

$$V_s = \frac{a_v \times d \times f_{yd}}{S} \quad (2.27)$$

2.4.4.5 Designs for Bearings

In the case of post-tensioned members, the pre stress is transferred to concrete by means of external anchorage, the bearing pressure developed behind the anchorages suitably controlled to prevent crushing failure of the end block zone. To such effect, limit bearing plates, both at initial pre stress and after losses are provided [6].

At initial pre-stress

$$f_b = 0.8 \times f_{ci} \times \sqrt{\left(\frac{A_{br}}{A_{pum}} - 0.2\right)} \leq 1.25 \times f_{ci} \quad (2.28)$$

After losses

$$f_b = 0.6 \times f_{ci} \times \sqrt{\frac{A_{br}}{A_{pum}}} \leq f_{ci} \quad (2.29)$$

Where,

f_{ci} is cube strength of concrete at the stage of stress transfer

A_{pun} = Punching area and is generally contact area of the anchoring device

A_{br} = Bearing area and is the maximum area of that portion of the member, which is geometrically similar and concentric to the effective punching area.

2.5 Advantage of Simple Span over Continuous Span Bridge

Many railroads prefer simple span bridges to continuous structures, finding them easier install and maintain. Since they are structurally determinate, simple span are better able to handle problems such as support, settlement, and thermal effects than some continuous bridges. Precast concrete elements are particularly suited to simple span construction. Additional reasons many railroads prefer simple supported bridges to continuous span bridges include the following [18].

- If repair or replacement of superstructure elements is necessary, less interruption to train traffic is incurred with a simple span bridge than with a continuous span bridge.
- Installation of simple spans can be accomplished more quickly than continuous spans.
- If a bridge experiences substructure problems such as settlement, a continuous span bridge may require immediate and more extensive work, thereby resulting in greater interruptions to train traffic.
- Simple span bridges have a proven history of performing well [18].
-

2.6 Disadvantage of Simple Span over Continuous Span Bridge

In simply supported bridges the bending moment anywhere in the span is considerably greater than that in case of continuous supported span. Such increment of bending moment ultimately results in the less economic section for the bridge.

In continuous bridges the stresses are reduced due to negative moments developed at pier or supports. Thus continuous span bridges have considerable saving compared to simply supported bridge construction. However; this study considered simple supported bridge due to the above advantages.

CHAPTER THREE

3 Analysis and Design of Pre-Stressed Concrete Bridge

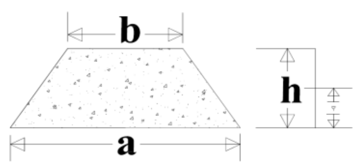
3.1 Analysis of Bridge with Ballasted Track System

Assumptions

Concrete is homogeneous elastic material and within the range of working stresses both materials concrete & steel behave elastically

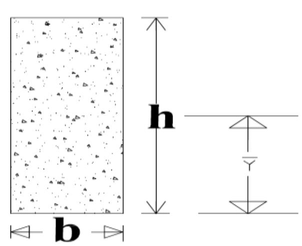
- Plane section before bending remain plane
- Sign convention :- in the foregoing computation
- (+) is for compression
- (-) is for tensile stress
- An average of 22% loss in case of pretension and 18% in post-tension may be assumed with condition of over tensioning has been applied to overcome friction and anchorage losses.

3.1.1 Section Properties of Pre-Stressed Precast Girder



Centroid of trapezium (form top of trapezium), Y= $\frac{h(2a+b)}{3(a+b)}$ (3.1)

Second moment of inertia at center of trapezium is $\bar{I} = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$ (3.2)



Centroid of rectangle, $\bar{y} = \frac{h}{2}$ (3.3)

Second moment of inertia at center of rectangle is $\bar{I} = \frac{b\times h^3}{12}$ (3.4)

Section modulus, Z is two types which is

$$Z_t = \frac{\bar{I}}{h-\bar{y}} \text{ and} \quad (3.5)$$

$$Z_b = \frac{\bar{I}}{\bar{y}} \quad (3.6)$$

Where, $\bar{y}$ is measured from base or bottom of rectangle.

3.1.2 Section Properties of Pre-Stressed Precast - in Situ Composite Deck Girder

Centroid of composite structure is $\bar{y} = \frac{\sum A_i \times \bar{y}_i}{\sum A}$ (3.7)

Where,

- A_i - Area of components of composite structure
- $\bar{y}_i$ - Centroid of components of composite structure
- A- Area of composite structure

When a composite area is composed of a large number of parts, it is convenient to tabulate the results for each of parts in terms of its area A, its centroidal moment of inertia I, the distance d from its centroidal axis to the axis about which the moment of inertia of the entire section is being computed, and the product $A \times d^2$. For any one of the parts the moment of inertia about the desired axis by the transfer of axis theorem is $\bar{I} + A \times d^2$. Thus, for the entire section the desired moment of inertia becomes $I = \sum \bar{I} + \sum A \times d^2$, and the tabulation include

Part	Area, A	d_x	d_y	$A \times d_x^2$	$A \times d_y^2$	$\bar{I}_x$	$\bar{I}_y$
Sums	$\sum A$			$\sum A \times d_x^2$	$\sum A \times d_y^2$	$\sum \bar{I}_x$	$\sum \bar{I}_y$

From the sums of the four columns, then, the moment of inertia for the composite area about the x- and y- axes become

$$I_x = \sum \bar{I}_x + \sum A \times d_x^2, \quad (3.8)$$

$$I_y = \sum \bar{I}_y + \sum A \times d_y^2 \quad (3.9)$$

3.1.3 Analysis Procedure for Different Stages of Loading

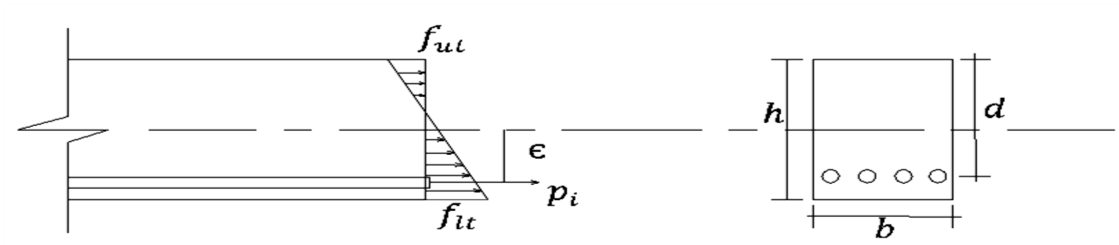
1. **For precast**

Bending stress analysis of a PC member has two cases and calculation may be made:

a) At transfer

- Self-weight &
- Pre-stress force

Stresses Immediately after Pre Stressing



$f_{ut} = P_i/A_{pc} - P_i \times e/Z_{tp} + M(g_p)/Z_{tp} \geq - f_{tt}$ (3.10)

$f_{lt} = P_i/A_{pc} + P_i \times e/Z_{bp} - M(g_p)/Z_{bp} \leq f_{ct}$ (3.11)

P_i = pre stressing force immediately after pre stressing

A_{pc} = b×h, area of concrete, Z_{tp} = section modulus top,

Z_{bp} = section modules bottom

M_{gp} = bending moment caused by the own weight of precast.

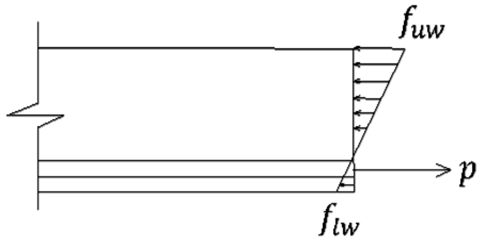
f_{ct} = permissible compressive stresses in concrete at the moment of pre stressing.

f_{tt} = permissible tensile stresses in concrete at the moment of pre stressing.

b) Under Working (at Service)

- Self-weight
- Cast-in situ deck &
- Pre-stressing force

Stresses after all Losses of Pre Stress



$f_{lw} = P_e/A_{pc} + P_e \times e/Z_{bp} - M(g_{p+}g_{in\text{-}situ}) / Z_{bp} \geq -f_{tw}$ (3.12)

$f_{uw} = P_e/A_{pc} - P_e \times e/Z_{tp} + M(g_{p+}g_{in\text{-}situ}) / Z_{tp} \leq f_{cw}$ (3.13)

Where, P_e - is the effective pre stressing force after all losses.

$M_{(g_p+g_{in})}$ = Bending moment caused by maximum total load of precast and cast-in situ.

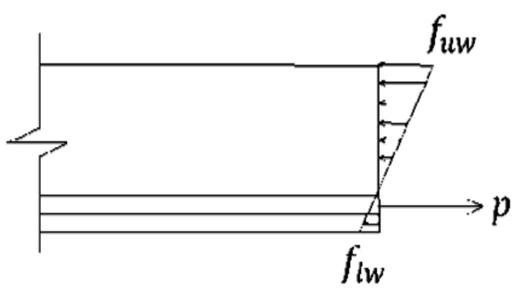
2. **For Precast-Cast-in Situ Composite Structure**

In this case, only one stage has been discussed, because pre-stress force already transferred in the stage of precast.

Under Working (at Service)

- Self-weight (precast plus cast-in situ deck)
- Superimposed dead load
- Live load &
- Pre-stressing force

Stresses after all Losses of Pre Stress and the full Working Load acts on the Member



$f_{lw} = P_e/A_{pc} + P_e \times e/Z_{bp} - M(g_1+g_2+q) / Z_{bc} \geq -f_{tw}$ (3.14)

But $-f_{tw}$ is zero for fully pre-stressed

$f_{uw} = P_e/A_{pc} - P_e \times e/Z_{tp} + M(g_1+g_2+q) / Z_{tc} \leq f_{cw}$ (3.15)

Where, P_e - is the effective pre stressing force after all losses.

$M_{(g1+\ g2+q)}$ = bending moment caused by maximum total load (self-weight, superimposed load and live load).

Ultimate Limit State of a Cross Section

Consider the PC member when stressed nearly to failure.

$N_P = A_P \times f_{pyd}$, (3.16)

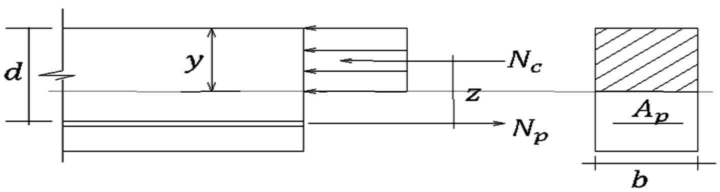
Where, f_{pyd} is the design strength of tendon

$f_{pyd} = 0.9 \times f_P$ (3.17)

$f_P = f_{pk}/\Upsilon_s$, (3.18)

Where, Υ_s is partial safety factor (1.15)

A_P = area of tendons' cross section



$\sum F_h = o$

$\rightarrow \quad N_p = N_c = y \times b \times f_{cd}$ (3.19)

$\rightarrow \quad y = \frac{N_p}{b \times f_{cd}}, \quad Z = d - y/2$ (3.20)

$\sum M_o = o$

$\rightarrow M_u = N_c \times Z = N_p \times Z$ (3.21)

Design moment must be less than ultimate moment ($M_d \leq M_u$)

3.2 Analysis of Bridge with Slab Track System

The procedure for analysis of bridge with slab track system is similar with ballasted track system.

3.2.1 Section Properties of Pre-Stressed Precast Girder

As discussed above, section properties of pre-stressed precast girder in ballasted & slab case are the same. Therefore the reader can refer the sub topic 3.1.1 in this thesis.

3.2.2 Section Properties of Pre-Stressed Precast-Cast in Situ Composite Deck Girder

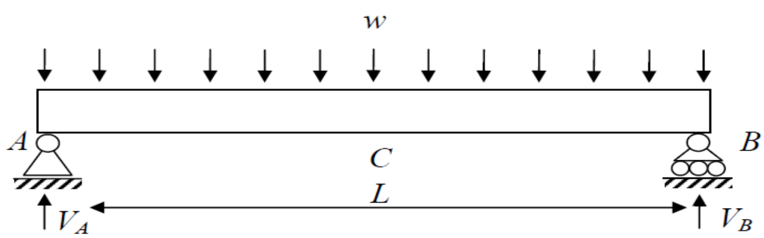
Also section properties of pre-stressed precast – cast in situ composite deck girder in ballasted & slab case are the same. Therefore the reader can refer the sub topic 3.1.2 in this thesis.

3.2.3 Analysis Procedure for Different Stages of Loading

The study used same procedure for this case as discussed above and refer sub topic 3.1.3 in this thesis.

Basic principle of pre-stressed concrete is discussed below.

Consider the basic case of a simply-supported beam subjected to a uniformly distributed load of w kN/m:



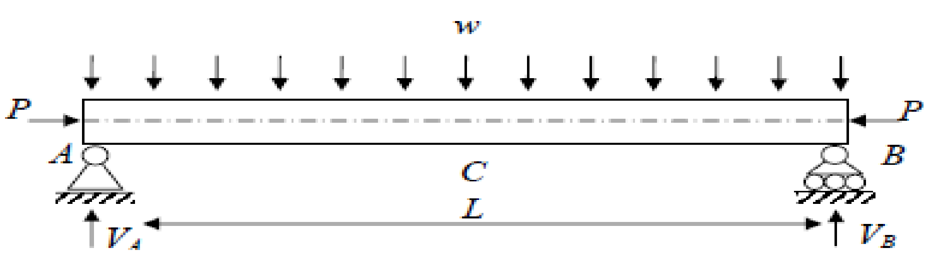
In this case, we have the maximum mid-span moment as:

$$M = \frac{WL^2}{8} \tag{3.22}$$

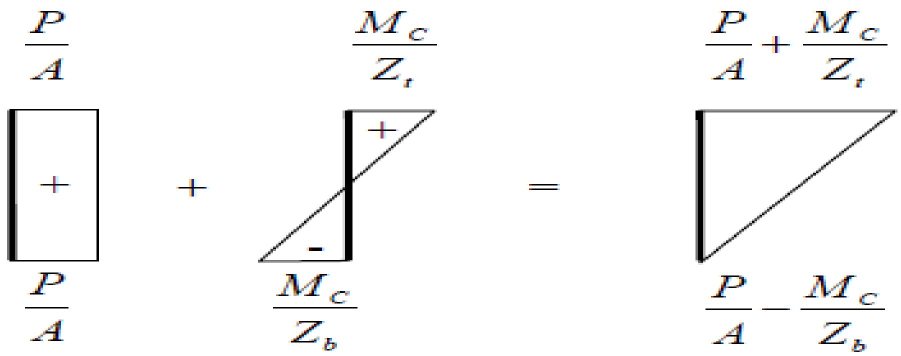
Also, if we assume a rectangular section as shown, we have the following section properties:

<u>Case one:</u>

We consider the same beam, but with centroidal axial pre-stress (eccentricity equals zero) as shown below:



Now we have two separate sources of stress:



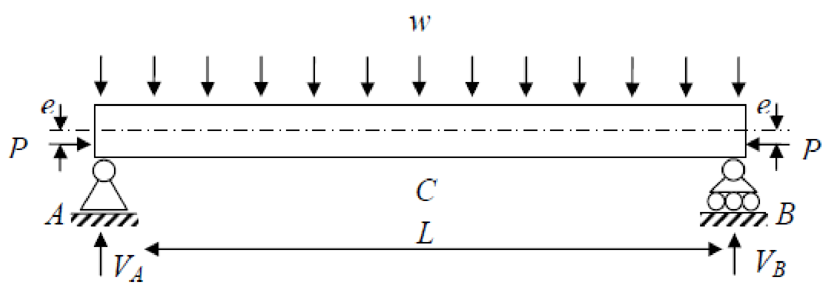
For failure to occur, the moment caused by the load must induce a tensile stress greater than $\frac{P}{A}$. Hence, just prior to failure, we have:

$$\frac{M_c}{Z_b}=\frac{P}{A}=\frac{WL^2}{8Z_b} \tag{3.23}$$

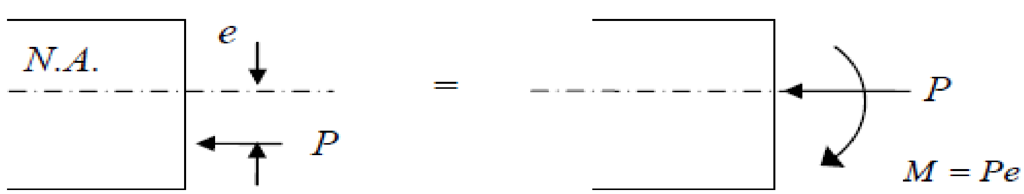
$$W_I=\frac{8P\times Z_b}{A\times L^2} \tag{3.24}$$

<u>Case two:</u>

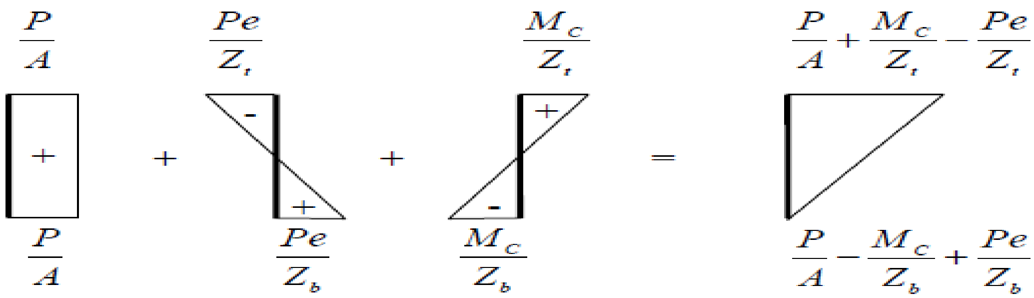
In this case we place the pre-stress force at an eccentricity:



Using an equilibrium set of forces as shown, we now have three stresses acting on the section:



Thus the stresses are:



Hence, for failure we now have:

$$\frac{M_c}{Z_b} = \left(\frac{P}{A} + \frac{Pe}{Z_b}\right) \tag{3.25}$$

But $M_c = \frac{W_{II} \times L^2}{8}$

$$W_{II} = \frac{8 \times Z_b}{L^2}\left(\frac{P}{A} + \frac{Pe}{Z_b}\right) \tag{3.26}$$

If, for example, we take $e = \frac{d}{6}$, then:

$$W_{II} = \frac{8 \times Z_b}{L^2}\left(\frac{P}{A} + \frac{\frac{Pd}{6}}{\frac{bd^2}{6}}\right) = \frac{16 \times Z_b}{L^2} \times \frac{P}{A} = 2 \times W_I \tag{3.27}$$

So the introduction of a small eccentricity has doubled the allowable service load.

Deflection

Deflections can take place: upwards due to pre stressing (-) & down wards due to transverse loads (+). The deflection can be calculated using the ordinary deflection formulas shown in the equation 2.23 and 2.25.

Deflections due to loads: ordinary formula can be applied depending on

: The intensity of loading

: Restraint condition and span

3.2.4 Design of Section in Ballasted Track System

The minimum depth of ballast on the top of bridge deck is 600mm. In the analysis, the study considered the weight of ballast is including sleeper weight. As shown in the figure below, the cross section of ballasted railway bridges include deck, precast girder, ballast, sleeper, fastening, rail and etc. The design parameters like compressive strength of concrete, tensile strength of tendon, span length, depth of precast girder, and eccentricity of tendon are the same for both ballasted and slab railway bridge.

3.2.5 Design of Section in Slab Track System

The minimum depth of slab on the top of bridge deck is 200mm. In the analysis, the study considered the weight of slab is including sleeper weight. As shown in the figure below, the cross section of slab railway bridges include deck, precast girder, slab on the surface of bridge deck, sleeper, fastening, rail and etc.

3.3 Other Design Consideration

3.3.1 Design Consideration of Ballasted Track System for Shear

Two major modes of shear cracks may be observed namely web shear and flexure shear cracks.

Web Shear Crack: - it is governed by limiting value of the principal tensile stress as developed in concrete. It is more likely to occur in highly pre stressed members with thin webs [6].

$$V_{cw} = b_w \times (I/Q) \times \sqrt{f^2{}_{ctd} + f_{cp} \times f_{ctd}} \tag{3.28}$$

However, I/Q range between 0.67h, (for rectangle) to 0.85h (for flanged section)
Thus, it is normally recommended to use

$$V_{cw} = 0.67 \times b_w \times h \times \sqrt{f^2{}_{ctd} + f_{cp} \times f_{ctd}} \tag{3.29}$$

Where

f_{cp} is compressive pre stress at the centroid of a section

b_w is the breath of web, h is gross depth.

Flexure Shear Cracks: - primarily initiated by flexural cracks and are developed when the combined shear and flexural tensile stresses produce a principal tensile stress exceeding the design tensile strength of concrete [6].

The section capacity in this case on the basis of experiments is given by

$$V_c = \beta_1 \times 0.05 \times b \times d \times f_{ctd} \quad (3.30)$$

Where, $\beta_1 = 1 + \frac{M_o}{M_d} \leq 2.0$

M_d is the bending moment at the cross section where shear capacity is calculated.
Mo = P(Z/A_c + e) , Z is section modulus where flexure shear likely to occur , P is pre-stressing force at the cross section however, when the shear force exceed the limiting value against diagonal compression the section must be changed.
When $V_c > V_d$, only minimum shear reinforcement shall be provided which may be expressed in the equation 2.26 and 2.27.

3.3.2 Design of Slab Track System in Shear

The study took the same procedure for both ballasted and slab track system in shear. Therefore the reader can refer sub topic 3.4.1 of this thesis.

3.3.3 Bearings

Designs for Bearings

In the case of post-tensioned members, the pre stress is transferred to concrete by means of external anchorage, the bearing pressure developed behind the anchorages suitably controlled to prevent crushing failure of the end block zone. To such effect, limit bearing plates, both at initial pre stress and after losses are calculated by the equation 2.28 and2.29.

CHAPTER FOUR

4 Design Examples and Discussion

As we know that, now a day's a railway project is the big project next to grand renaissance dam in our country. This great project should be studied to make the project economical. From this project, railway bridge shares great percent and that is one needs to study on railway bridge.

Therefore, this chapter is made to covers analysis of railway bridge as follows. Analysis is made to make comparative study of bridges design on ballasted and slab track and compare construction costs.

The study has been taken counter examples of both ballasted and slab railway bridge. In this example the design parameter, span and cross sections of the bridge are the same for both cases.

Example 1: 15m Span Bridge with Ballasted Track System

A simple span precast post tensioned pre-stressed concrete girder bridge is to be designed for a railway bridge with cast in situ deck slab, cross section of which shown in figure below. The span length is 15m and cast in place concrete deck slab is of 300 mm thick, while the ballast is of 600 mm in thickness. The bridge is to serve for the Cooper E-80 rail load (live load) configuration and the following design data may be used.

Material Properties

Concrete C-50

Mild steel S-500

Pre-stressing tendons f_{pk} = 1800MPa

Unit weight of plain concrete =24kN/m^3

Unit weight of reinforced concrete RC =25kN/m^3

Unit weight of steel= 78.5kN/m^3

Unit weight of sand, gravel and ballast =19kN/m^3

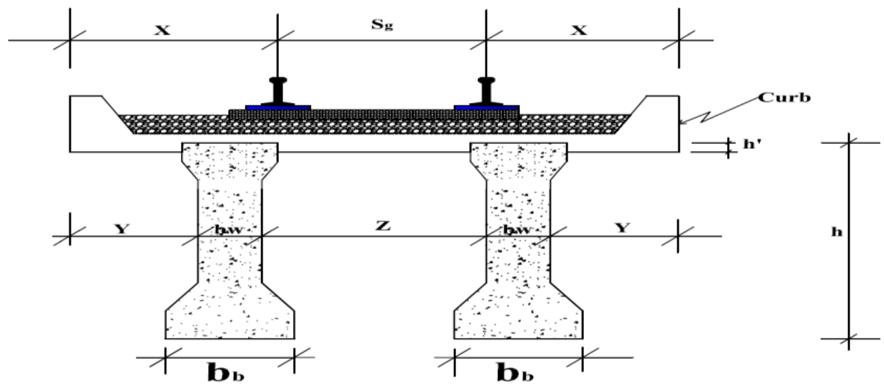


Figure 4-1 Cross section of ballasted railway bridge

Table 4-1Dimensions of bridge

Span (mm)	Rail gauge dist.S_g (mm)	Girder clear spacing Z (mm)	Live load configuration	precast web, b_w (mm)	Y(mm)	h'(mm)	X(mm)
15000	1435	1500	E-80	570	1680.00	20	2282.5

Design parameters as follows:

Table 4-2 Design parameters as follows

C-50 (Mpa)	S-500 (Mpa)	f_{pk} (Mpa)	Υ_c (Mpa)	Υ_{Rc} (Mpa)	Υ_{steel} (Mpa)	$\Upsilon_{ballast}$ (Mpa)
50	500	1800	24	25	78.5	19

- ✓ f_{ck}=50/1.25=0.8×50 = 40Mpa
- ✓ $f_{pi} = 0.75 \times f_{pk} = 0.75\times1800= 1350$Mpa
- ✓ $f_{pe} = 0.82\times1350$Mpa = 1107Mpa since pre-stress is post-tension type
- ✓ $f_{cd} = \frac{0.85\times f_{ck}}{1.5} = \frac{0.85\times 40}{1.5} = 22.667$Mpa
- ✓ $f_{ctd} = f_{ctk}/\Upsilon_c$, but $f_{ctk} = 0.21\times f_{ck}^{\frac{2}{3}}$,
- ✓ From above, f_{ck} = 40 MPa so, $f_{ctk} = 0.21\times 40^{\frac{2}{3}} = 2.46$ MPa

  Therefore, $f_{ctd} = f_{ctk}/\Upsilon_c = 2.46/ 1.5 = 1.64$Mpa
- ✓ For S-500(Mpa), f_{yk} = 500Mpa.

  $f_{yd} = f_{yk} / \Upsilon_s = 500 / 1.15 = 434.78$ Mpa.

Now find permissible stress in different stages:

i. Permissible stresses immediately at transfer

$f_{ct} = 0.6 \times f_{ck} = 0.6\times40 = 24$MPa

$f_{tt} = 0.21\times f_{ck}^{\frac{2}{3}} = 0.21\times 40^{\frac{2}{3}} = 2.46$ MPa

ii. Permissible stresses under working condition (after all losses)

$f_{cw} = 0.5 \times f_{ck} = 0.5 \times 40 = 20$MPa

$f_{tw} = 0.75\times 0.21 \times f_{ck}^{\frac{2}{3}} = 0.75 \times 0.21 \times 40^{\frac{2}{3}} = 1.84$ MPa

iii. Permissible range of stresses at the top of the cross section

$$f_{tr} = (f_{cw} + \eta \times f_{tt}) \tag{4.1}$$

Where, η-is pre-stress loss factor between 0.8-0.85 for post tensioned case and this study took average of pre-stress loss factor, 0.82.

$f_{tr} = 20+0.82 \times 2.46 = 22$Mpa

iv. Permissible range of stresses at the bottom of the cross section

$$f_{br} = \eta \times f_{ct} + f_{tw} \tag{4.2}$$

$f_{br} = 0.82 \times 24 + 1.84 = 21.5$ Mpa

Checking of Section Modulus in Different Stage

$$z_t > (M_q+(1-\eta) \times M_{g1})/f_{tr} \quad \text{or} \quad z_t > (M_q+M_{g2}+(1-\eta) \times M_{g1})/f_{tr} \tag{4.3}$$

$$z_b > (M_q+(1-\eta) \times M_{g1})/f_{br} \quad \text{or} \quad z_b > (M_q+M_{g2}+(1-\eta) \times M_{g1})/f_{br} \tag{4.4}$$

To check section modulus, first we should determine the value of the following variables.

1. **Find maximum moment and shear force due to live load**

From the table 2-3, the maximum moment due to live load (Cooper E-80) for span length 15m is 2,514.10kN-m and moment due to impact load (20% of live load). Therefore moment due to impact load equals 20% of 2,514.10kN and equals 502.82kN.The maximum moment due to live load (M_q) equals 2,514.10 plus 502.82kN and equals 3,016.92kN.

Similarly, from the above table 2-3, the maximum shear force due to live load for span length 15m is 767.99kN and impact load from live load is 20% of live load. Therefore impact load equals 20% of 767.99kN and equals 153.60kN. The maximum shear force due to live load equals 767.99kN plus 153.60kN and equals 921.59kN.

2. **Find maximum moment and shear force due to self -weight.**

i. **precast**

From the appendix A, area of precast is A = 1,380,825 mm^2

Self -weight = $\Upsilon_c \times A_P$ = 25kN/m^3 ×1.38 m^2 = 34.52kN/m

Since our bridge is simple span, so

a. Maximum moment (M_{g1}) = $\frac{w\times L^2}{8} = \frac{34.52\times 15^2}{8}$ = 970.89kN-m

b. Maximum shear force = $\frac{w\times L}{2} = \frac{34.52\times 15}{2}$ = 258.90kN

ii. Composite structure

From the appendix A, area of composite structure is A =2,265,625mm^2

Self-weight = $\Upsilon_c \times A_c$ = 25 kN/m^3 × 2.27m^2 = 56.64kN/m

Since our bridge is simple span, so

a. Maximum moment (M_{g1}) = $\frac{w\times L^2}{8} = \frac{56.64\times 15^2}{8}$ =1,593kN-m

b. Maximum shear force = $\frac{w\times L}{2} = \frac{56.64\times 15}{2}$ = 424.80kN.

3. **Find maximum moment and shear force due to superimposed load.**

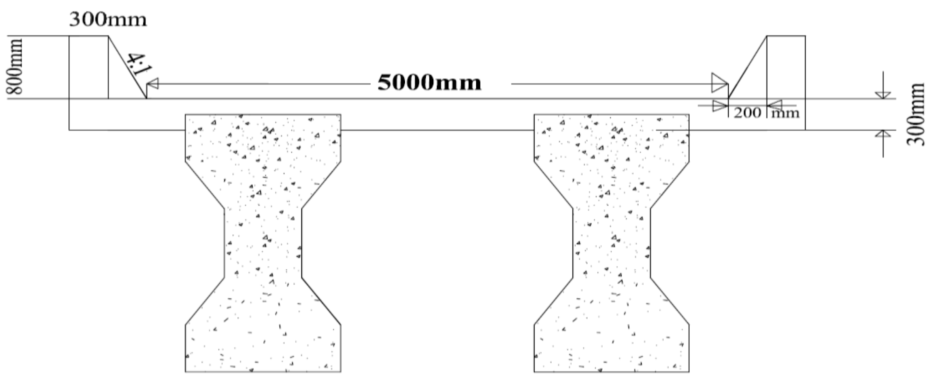


Figure 4-2 Girder with superimposed load

i. **Ballast**

Unit weight of sand, gravel and ballast equals 19kN/m^3 from [4].

A= 1/2(5+5.3) ×0.6 =3.09m^2

Ballast distributed load equals $3.09m^2$ times $19kN/m^3$ and equals 58.71kN/m but ballast weight is shared to two girders. Therefore, for one precast girder ballast weight is half of total ballast weight. i.e. ballast weight equals half of 58.71 and equals 29.355kN/m

ii. Rail

Rail self-weight is assumed 1.5kN/m [4].

iii. Curb

From the above figure, area of curb is A= 1/2(0.3+0.5) ×0.8 = $0.32m^2$

Self- weight = $\Upsilon_c \times A_{curb}$ = 25 kN/m^3 × 0.32 m^2 = 8kN/m

From above (i, ii, and iii), total superimposed load is 29.355+1.5+8 = 38.855kN/m

Since our bridge is simple span, so

a) Maximum moment (M_{g2}) = $\frac{w \times L^2}{8} = \frac{38.855 \times 15^2}{8}$ = 1,092.80kN-m

b) Maximum shear force = $\frac{w \times L}{2} = \frac{38.855 \times 15}{2}$ = 291.41kN.

A. Check section modulus at transfer

i. $[M_q + (1-\eta) \times M_{g1}] / f_{tr}$ = [3,016.92 + (1- 0.82)) × 970.89] ×1,000,000 / 22

= 1.45×10^8 mm^3

1.45×10^8 $mm^3 \leq 1.06 \times 10^9$ mm^3 ----------------------Ok!!

ii. $[M_q + (1-\eta) \times M_{g1}] / f_{br}$ = [3,016.92 + (1- 0.82) × 970.89] × 1.000,000 / 21.5

= 1.48×10^8 mm^3

=1.48×10^8 $mm^3 \leq 1.30 \times 10^9$ mm^3 ----------------------Ok!!

B. Check section modulus at working

i. $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{tr}$ = [3,016.92 + 1,092.80 + (1-0.82) × 1,593.02]×1,000,000/22

= 2.00×10^8 mm^3

= 2.00×10^8 $mm^3 \leq 1.38 \times 10^9$ mm^3 ---------------Ok!!

ii. $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{br}$ = [3,016.92 + 1,092.80 + (1-0.82) × 1,593.02]×1,000,000/21.5

= 2.04×10^8 mm^3

= 2.04×10^8 $mm^3 \leq 9.09 \times 10^8$ mm^3------------- Ok!!

Load combination

Load factor group I loading =1.4(DL +5/3 (LL +I))

Table 4-3 Load combination

Self Wt (g)	Dead Load (SDL)	Live +impact load (LL +I)	Total Service Load	Total Factored Load	Maximum BM	Maximum SF
56.64	38.86	107.27	202.76	383.99	10,799.62	2,879.90

Determination of initial pre - stressing force to be applied

Minimum pre - stressing force to be applied on concrete may be expressed as equation (4.5) and e = ȳ-concrete cover-1/2×Ø where, Ø – is diameter of tendon which may be assumed 100mm.

e =853- 50 - 0.5×100 = 753mm

$f_{lw} = P_e/A_{cp} + P_e \times e/Z_{bp} - M(g_1+g_2+q)/Z_{bc} \geq -f_{tw}$ but $-f_{tw}$ is zero for fully pre-stressed, then

$P_e/A_{cp} + P_e \times e/Z_{bp} = M(g_1+g_2+q)/Z_{bc}$

$P_e = M(q+g_2+g_1)/Z_{bc} \times (A_{cp} \times Z_{bp} / (Z_{bp} + A_{cp} \times e))$

$$P_e = \frac{M_{(q+g_2+g_1)} \times A_{cp} \times Z_{bp}}{Z_{bc}(Z_{bp} + A_{cp} \times e)} \quad (4.5)$$

$$P_e = \frac{(3016.92 + 1{,}092.80 + 1{,}593.02) \times 1000000 \times 1{,}380{,}825 \times 1{,}297{,}027{,}973.54}{909{,}617{,}191.20 \times (1{,}297{,}027{,}973.54 + 1{,}380{,}825 \times 753)}$$

P_e = 4,804.02kN

p = P_e/0.82 = 4,804.02/ 0.82 = 5,858.56kN.

Number of high strength steel wires as follows:

Area of tendon $= \frac{P_e}{f_{Pe}} = \frac{4{,}804.02\text{kN}}{1107\text{Mpa}}$ = 4,339.67mm^2

Assume wire diameter is 7mm, therefore; area of one wire is as follows:

Area of one wire = $\frac{\pi \times d^2}{4} = \frac{\pi \times 7^2}{4} = 38.47\text{mm}^2$

Number of high strength steel wires required (n) = $\frac{\text{Area of tendon}}{\text{Area of one wire}} = \frac{4{,}339.67\ \text{mm}^2}{38.47\text{mm}^2}$ = 113

n = 113 wires are necessary to pre-stress the concrete.

Deflection

I. Deflection due to pre-stress

Since pre-stressing method used in this study is post tensioning, assuming eccentricity profile to be parabolic and calculated by using equation 2.25 but E_c is unknown and determined by equation 4.6.

$E_c = w_c \times 1.5 \times 0.043 \times (f'_c)^{1/2}$ [4]. (4.6)

w_c = Unit weight of concrete in kg/m^3 is $2500 kg/m^3$

Again, f'_c = Characteristics compressive strength of concrete

$f_{ck}=50/1.25=0.8\times50 = 40$Mpa, therefore;

$E_c = 2500 \times 1.5 \times 0.043 \times 40^{0.5} \times 10^{-3}$GPa

E_c= 33.99 GPa and from appendix A, I_p = 1,106,811,332,931mm^4, therefore;

$$\Delta_p = \frac{-5\times 5{,}858{,}558.61 \times 753 \times 15000^2}{48\times 33.99 \times 10^3 \times\ 1{,}106{,}811{,}332{,}931}$$

Δ_p = - 2.70mm ………………..(Upwards)

II. Deflection due to beam self-weight at transfer:

By using equation 2.23, the deflection due to self-weight at transfer is calculated as follows:

$$\Delta_g = \frac{5\times 34.52 \times 15^4 \times 10^{12}}{384\times 33.99 \times 10^3 \times 1{,}106{,}811{,}332{,}931}$$

Δ_g = 0.60mm ………….. (Downwards)

III. Deflection due to uniformly distributed superimposed dead load:

By using equation 2.23, the deflection due to superimposed dead load is calculated as follows:

$$\Delta_s = \frac{5\times 38.855 \times 15^4 \times 10^{12}}{384\times 33.99 \times 1000 \times 1{,}195{,}060{,}251{,}655.53}$$

Δ_s =0.63mm ……………. (Downwards)

IV. Deflection due to live load:

By using equation 2.23, the deflection due to live load is calculated as follows:

Before calculating for deflection, the value of W_{eq} is calculated from the live load moment.

From the above, we have live load moment M_{LL+I} = 3,016.92kN-m

Now the value of distributed live load is, $W_{eq.} = \frac{3{,}016.92\times 8}{15^2}$

$W_{eq.}$ = 107.27kN/m, therefore;

$$\Delta_{LL+I} = \frac{5 \times 107.27 \times 15000^4}{384 \times 33.99 \times 1000 \times 1,195,060,251,655.53}$$

Δ_{LL+I} = 1.74mm ……………. (Downwards)

But, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4].

Therefore, L/640 = 15000/640 = 23.44mm, where L-is in mm.

Δ_{max} = 1.74mm < 23.44mm …………………………………………..OK!!

V. Total immediately deflection after pre stressing - parabolic tendons

Total immediate deflection is the sum of deflection due to initial pre-stressing force and deflection due to self-weight at transfer

$\Delta_i = \Delta_p + \Delta_g$ = - 2.70mm + 0.60mm = -2.10mm…….. (Upwards)

VI. Deflection due to self-weight under working:

By using equation 2.23, the deflection due to self-weight under working is calculated as follows:

$$\Delta_g = \frac{5 \times 56.64 \times 15^4 \times 10^{12}}{384 \times 33.99 \times 10^3 \times 1,195,060,251,655.53}$$

Δ_g = 0.92mm ………….. (Downwards)

VII. Long time deflection:

Long time deflection is the result of creep and shrinkage of concrete and relaxation of stresses in steel. Therefore, the total long time deflection Δ_l may be obtained from

$$\Delta_l = \Delta_c \times (1+\emptyset) - \Delta_p \times [(1- \Delta P/P) + \emptyset \times (1- \Delta P/P)] \text{ [6]}. \qquad (4.7)$$

Where

Δ_c = Initial deflection due to transverse loads

Δ_l = Long time deflection

Δ_p = Initial deflection due to pre stress force

$\emptyset$ = Coefficient which account creep, shrinkage etc. ($\emptyset$~2.0)

$\Delta P = P-P_e$ = loss of pre stressing force due to relaxation, shrinkage & creep.

But,

$$\Delta_c = \frac{5 \times W \times L^4}{384 \times E_c \times I_c}$$

Where; W- is the sum of live load, composite self-weight and superimposed dead load.

W = 107.27kN/m + 56.64kN/m + 38.84kN/m =202.76kN/m

$$\Delta_c = \frac{5\times202.76\times15^4 \times 10^{12}}{384\times33.99 \times10^3 \times1,195,060,251,655.53} = 3.29\text{mm.}$$

$\Delta P = P - \eta \times P = P\text{-}P_e$,

Where; η-is pre-stress loss factor = 0.82 for post tensioned case.

$\Delta P = P\text{-}P_e$ = 5,858.56 - 0.82×5,858.56= 0.18×5,858.56 =1054.54kN

$\Delta_l = \Delta_c \times (1+\emptyset) - \Delta_p \times [(1- \Delta P/P) + \emptyset \times (1- \Delta P/P)]$

Δ_l = 3.29× (1+2) – 2.70× [(1- 1054.54/5,858.56) + 2 × (1- 1054.54/5,858.56)]

Δ_l = 3.29× (3) – 2.70× [0.82+ 1.64] =3.11mm………. (Downwards)

Check web shear and design for flexural shear

Web shear and flexural shear crack are two different cracks as shown in the figure 2.21 and both are governed by limiting value of the principal tensile stress expressed in equation 3.29 and 3.30.

I. **Web shear:**

In equation 3.29, f_{cp} is unknown. To get the value of f_{cp}, use the following formula.

$f_{cp} = P_e/A_{pc}$, but $P_e = A_s \times f_{pe}$, therefore; $f_{cp} =(A_s \times f_{pe})/A_{pc}$ or $f_{cp} = P_e/A_{pc}$, hear already p_e calculated above and its equals P_e = 4,804,018.06N, and from appendix A, A_{pc} = 1,380,825mm^2, f_{cp} = 4,804,018.06N/1,380,825mm^2 f_{cp} = 3.48Mpa

From above, b_w = 570mm, h = 2,180.00mm, f_{ctd} = 1.64MPa & V_d = 2,879.90kN.

Therefore; $V_{cw} = (0.82\times 570\times2,180 \times \sqrt{1.64^2 + 3.48 \times 1.64}) \times (10^{-3})$ = 2,949.29kN and

V_{cw} = 2,949.29 > V_d = 2,879.90kN……………………………………………Ok!

II. **Flexural shear:**

The section capacity (V_c), on the basis of experiments is given by

$V_c = \beta_1 \times0.5\times b\times d\times f_{ctd}$ where, β_1 = 1+ Mo / M_d and now we can calculate the value of

$M_o = P (Z/A_c + e)$. Already, we have the values for P, Z_{bp}, A_c and e.

P = 5,858,558.61, e = 753mm, A_{cp} = 1,380,825mm^2, Z_{bp} = 1,297,027,973.54 mm^3

$M_o = P (Z_{bp}/A_{cp} + e)$ = 5,858,558.61N × (1,297,027,973.54/1,380,825+ 753) × (10^{-6})

M_o = 9,916.54kN-m but M_d =10,799.62kN-m & then,

β_1 = 1 +9,916.54/10,799.62 = 1.92 ≤ 2.00 …………….……………………………Ok!

Therefore; we can take β_1 = 1.92 and now calculate for $V_c = \beta 1 \times 0.5 \times b \times d \times f_{ctd}$

Where, b = 570mm, d = h_c – cover- half of diameter of tendon = 2180 – 50-0.5×100= 2,080mm & f_{ctd} = 1.64 MPa and V_c = (1.92 ×0.5×570×2080×1.64) × (10^{-3}) = 1,861.99kN < V_d = 2,879.90kN……………Not ok.

When $V_c < V_d$, Therefore; shear force sustained by stirrups may be computed for $V_s = a_v \times d \times f_{yd} / s$

Assume that spacing of stirrup is S =150mm , $V_s = V_d - V_c$ =2,879.90 - 1,861.99 =1017.91kN and a_v = (Vs × S / d × f_{yd}) = 1017.91×1000 × 150 / (2080× 434.78) = 169mm². Assume that stirrup diameter is Ø8. And area of one stirrup is $A = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4}$ = 50.27mm²

Number of stirrup (n) = 169/50.27 = 4

III. Limiting value against diagonal compression:

When shear force exceed the limiting value against diagonal compression, then the section must be changed. Let us see the following.

$V_{Rd} = 0.25 \times f_{cd} \times b_w \times d$ = 0.25×22.667×570×2080 × (10^{-3}) = 6,718.40kN > 2,879.90kN……Ok!

Therefore, the section is adequate for design load

Example 2: 15m Span Bridge with Slab Track System

A simple span precast post tensioned pre-stressed concrete girder bridge is to be designed for a railway bridge with cast in situ deck slab, cross section of which shown in figure below. The span length is 15m and cast in place concrete deck slab is of 300 mm thick, while the slab track is 200 mm in thickness. The bridge is to serve for the Cooper E-80 rail load (live load) configuration.

Checking of Section Modulus in Different Stage

To check section modulus, first we should determine the value of the following variables.

1. **Find maximum moment and shear force due to live load**

From example 1, the result is as follows:

- The maximum moment due to live load (M_q) = 3,016.92kN-m
- The maximum shear force due to live load = 921.59kN.

2. **Find maximum moment and shear force due to self -weight.**

i. precast

From example 1, the result is as follows:

- The maximum moment due to self-weight of pre-cast ((M_{g1})) = 970.89kN-m
- The maximum shear force due to self-weight of pre-cast = 258.90kN

ii. Composite structure

- The maximum moment due to self-weight of composite structure ((M_{g1})) = 1,593kN-m
- The maximum shear force due to self-weight of composite structure = 424.80kN.

3. **Find maximum moment and shear force due to superimposed load.**

i. Slab instead of ballast

- Unit weight of RC concrete is 25kN/m^3 &

$A= 1/2(5+5.1) \times 0.2= 1.01m^2$

Slab distributed load is 1.01m^2 times 25kN/m^3 and equals 25.25kN/m but slab weight is shared to two girders. Therefore, for one precast girder, slab weight is half of total slab weight. i.e. Slab weight = ½ × (25.25) = 12.625 kN/m

ii. Rail

Rail self -weight is assumed 1.5kN/m [4].

iii. Curb

From the above figure, area of curb is A= ½ × (0.3+0.5) × 0.8 = 0.32m^2

Self-weight = $\Upsilon_c \times A_{curb}$ = 25kN/m^3 × 0.32 m^2 =8kN/m

From above (i, ii, and iii), total superimposed load is 12.625+1.5+8=22.125kN/m

Since our bridge is simple span, so

a. Maximum moment (M_{g2}) = $\frac{w\times L^2}{8} = \frac{22.125\times 15^2}{8}$ = 622.27kN-m

b. Maximum shear force = $\frac{w\times L}{2} = \frac{22.125 \times 15}{2}$ = 165.94kN.

A. Check section modulus at transfer

a) $[M_q + (1-\eta) \times M_{g1}] / f_{tr}$ = [3,016.92 + (1- 0.82)) × 970.89] ×1,000,000 / 22

= 1.45 × 10^8 mm^3

1.45 × 10^8 mm^3 ≤ 1.06 ×10^9 mm^3 ----------------------Ok!!

b) $[M_q + (1-\eta) \times M_{g1}] / f_{br}$ = [3,016.92 + (1- 0.82) × 970.89] × 1.000,000 / 21.5

= 1.48 ×10^8 mm^3

=1.48 × 10^8 mm^3 ≤ 1.30 ×10^9 mm^3 ----------------------Ok!!

B. Check section modulus at working

a) $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{tr}$ = [3,016.92 + 622.27+ (1-0.82) × 1,593.02]×1,000,000/22

= 1.78 ×10^8 mm^3

= 1.78 ×10^8 mm^3 ≤ 1.38 × 10^9 mm^3 ---------------Ok!!

b) $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{br}$ = [3,016.92 + 622.27 + (1-0.82) × 1,593.02]×1,000,000/21.5

= 1.82 × 10^8 mm^3

= 1.82 × 10^8 mm^3 ≤ 9.09 × 10^8 mm^3------------- Ok!!

Load combination

Load factor group I loading =1.4× (DL +5/3× (LL +I))

Table 4-4 Load combination

Self Wt (g)	Dead Load (SDL)	Live +impact load (LL +I)	Total Service Load	Total Factored Load	Maximum BM	Maximum SF
56.64	22.13	107.27	186	361	10,140.88	2,704.23

Determination of initial pre - stressing force to be applied

Minimum pre - stressing force to be applied on concrete may be expressed as equation (4.5) and

$P_e = \frac{(3016.92+622.27 \ + 1{,}593.02)\times 1000000\times 1{,}380{,}825\times 1{,}297{,}027{,}973.54}{909{,}617{,}191.20 \times (1{,}297{,}027{,}973.54+1{,}380{,}825\times 753)} = 4{,}407.64\text{kN}$

p = P_e/0.82 = 4,407.64/ 0.82 = 5,375.17kN.

Number of high strength steel wires as follows:

Area of tendon $= \frac{P_e}{f_{Pe}} = \frac{4{,}407.64\text{kN}}{1{,}107\text{Mpa}}$ = 3,981.61mm^2

Assume wire diameter is 7mm, therefore; area of one wire is as follows:

Area of one wire = $\frac{\pi\times d^2}{4} = \frac{\pi\times 7^2}{4} = $ 38.47mm^2

Number of high strength steel wires required (n) = $\frac{\text{Area of tendon}}{\text{Area of one wire}} = \frac{3{,}981.61\ \text{mm}^2}{38.47\text{mm}^2}$ = 104

n = 104 wires are necessary to pre-stress the concrete.

Deflection

I. Deflection due to pre-stress

Since pre-stressing method used in this study is post tensioning, assuming eccentricity profile to be parabolic and calculated by using equation 2.25.

$$\Delta_p = \frac{-5\times\ 5{,}375{,}170.37\ \times 753\times 15000^2}{48\times 33.99\times 10^3\times\ 1{,}106{,}811{,}332{,}931}$$

Δ_p = - 2.5mm ………………..(Upwards)

II. Deflection due to beam self-weight at transfer:

By using equation 2.23, the deflection due to self-weight at transfer is calculated as follows:

$$\Delta_g = \frac{5\times 34.52\times 15^4\times 10^{12}}{384\times 33.99\times 10^3\times 1{,}106{,}811{,}332{,}931}$$

Δ_g = 0.60mm ………….. (Downwards)

III. Deflection due to uniformly distributed superimposed load:

By using equation 2.23, the deflection due to superimposed dead load is calculated as follows:

$$\Delta_s = \frac{5\times 22.13\times 15^4\times 10^{12}}{384\times 33.99\times 1000\times 1{,}195{,}060{,}251{,}655.53}$$

Δ_s = 0.36mm ……………. (Downwards)

IV. Deflection due to live load:

By using equation 2.23, the deflection due to live load is calculated as follows:

$$\Delta_{LL+I} = \frac{5 \times W_{eq} \times L^4}{384 \times E_c \times I_c} = \frac{5 \times 107.27 \times 15000^4}{384 \times 33.99 \times 1000 \times 1{,}195{,}060{,}251{,}655.53}$$

Δ_{LL+I} = 1.74mm ……………. (Downwards)

But, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4].

Therefore, L/640 = 15000/640 = 23.44mm, where L-is in mm.

Δ_{max} = 1.74mm < 23.44mm …………………………………………..OK!!

V. Total immediately deflection after pre stressing - parabolic tendons

Total immediate deflection is the sum of deflection due to initial pre-stressing force and deflection due to self-weight at transfer

$\Delta_i = \Delta_p + \Delta_g$ = - 2.5mm + 0.60mm = -1.90mm…….. (Upwards)

VI. Deflection due to self-weight under working:

By using equation 2.23, the deflection due to self-weight under working is calculated as follows:

$$\Delta_g = \frac{5 \times 56.64 \times 15^4 \times 10^{12}}{384 \times 33.99 \times 10^3 \times 1{,}195{,}060{,}251{,}655.53}$$ = 0.92mm ………….. (Downwards)

VII. Long time deflection:

By using equation 4.7, the deflection due to long time is calculated as follows:

First calculate for W = 107.27kN/m + 56.64kN/m + 22.13kN/m =186.04kN/m,

$$\Delta_c = \frac{5 \times 186.04 \times 15^4 \times 10^{12}}{384 \times 33.99 \times 10^3 \times 1{,}195{,}060{,}251{,}655.53}$$ = 3.02mm, and then $\Delta P = P - P_e$

=5,375.17 - 0.82×5,375.17 = 0.18×5,375.17 =967.53kN

Therefore; Δ_l = 3.02× (1+2) – 2.50× [(1- 967.53/5,375.17) + 2 × (1- 967.53/5,375.17)]

= 3.02× (3) – 2.50× [0.82+ 1.64] = 2.91mm……. (Downwards)

Check web shear and design for flexural shear

Web shear and flexural shear crack are two different cracks as shown in the figure 2.21and both are governed by limiting value of the principal tensile stress expressed in equation 3.29 and 3.30.

I. Web shear:

In equation 3.29, f_{cp} is unknown. To get the value of f_{cp}, use the following formula.

f_{cp} = P_e/A_{pc} , but P_e= A_s × f_{pe} , therefore; f_{cp} =(A_s × f_{pe})/A_{pc} or f_{cp} = P_e/A_{pc}, hear already p_e calculated above and its equals P_e =4,407,639.71N, and from appendix B, A_{pc} = 1,380,825mm^2, f_{cp} = 4,407,639.71N /1,380,825mm^2 f_{cp} = 3.19Mpa

From above, b_w = 570mm, h = 2,180.00mm, f_{ctd} = 1.64MPa & V_d = 2,704.23kN

Therefore; V_{cw} = (0.82× 570×2,180 × $\sqrt{1.64^2 + 3.19 \times 1.64}$) × (10^{-3}) = 2,865.36kN and

V_{cw} = 2,865.36> V_d =2,704.23kN ……………………………………..……………………Ok!

Flexural shear:

The section capacity (V_c), on the basis of experiments is given by

$V_c = \beta_1 \times 0.5 \times b \times d \times f_{ctd}$, where, β_1 = 1+ Mo / M_d and now we can calculate the value of

M_o = P (Z/A_c + e). Already, we have the values for P, Z_{bp}, A_c and e.

P = 5,375,170.37, e = 753mm, A_{cp} = 1,380,825mm^2, Z_{bp} = 1,297,027,973.54 mm^3

M_o = P (Z_{bp}/A_{cp} + e) = 5,375,170.37 × (1,297,027,973.54/1,380,825+ 753) × (10^{-6})

M_o =9,098.33kN-m but M_d =10,140.88kN-m & then,

β_1 = 1+ Mo / M_d = 1 +9,098.33/10,140.88 = 1.90≤ 2.00 ……………………………………Ok!

Therefore; we can take β_1 = 1.90 and now calculate for $V_c = \beta1 \times 0.5 \times b \times d \times f_{ctd}$

Where, b = 570mm, d = h_c - cover - half of diameter of tendon = 2180 – 50-0.5×100= 2,080mm & f_{ctd} = 1.64 MPa and V_c = (1.90×0.5×570×2080×1.64) × (10^{-3}) = 1,841.57kN < V_d =2,704kN…………………………………….. Not ok.

When V_c < V_d, Therefore; shear force sustained by stirrups may be computed for $V_s = a_v \times d \times f_{yd} / s$

Assume that spacing of stirrup is S =150mm , V_s = V_d -V_c =2,704kN -1,841.57kN=862.67 kN and a_v = (Vs × S / d × f_{yd}) = 862.67 ×1000 × 150 / (2080× 434.78) = 143.09mm^2 Assume that stirrup diameter is Ø8. And area of one stirrup is $A = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4}$ = 50.27mm^2

Number of stirrup (n) = 143.09/50.27 = 3

IV. Limiting value against diagonal compression:

When shear force exceed the limiting value against diagonal compression, then the section must be changed. Let us see the following.

V_{Rd} = 0.25 × f_{cd} × b_w ×d = 0.25×22.667×570×2080 × (10^{-3}) = 6,718.40kN > 2,704kN ……Ok!

Therefore, the section is adequate for design load

Example 3: 40m Span Bridge with Ballasted Track System

A simple span precast post tensioned pre-stressed concrete girder bridge is to be designed for a railway bridge with cast in situ deck slab, cross section of which shown in figure below. The span length is 40m and cast in place concrete deck slab is of 300 mm thick, while the ballast is of 600 mm in thickness. The bridge is to serve for the Cooper E-80 rail load (live load) configuration.

Checking of Section Modulus in Different Stage

To check section modulus, first we should determine the value of the following variables.

1. **Find maximum moment and shear force due to live load**

From the table 2-3, the maximum moment due to live load E-80 for span length 40m is 14929.33kN-m and moment due to impact load (20% of live load). Therefore moment due to impact load equals 20% of 14929.33 kN and equals 2985.87kN. The maximum moment due to live load (M_q) equals 14929.33 plus 2985.87kN and equals 17915.20kN.

Similarly, from the above table 2-3, the maximum shear force due to live load for span length 40m is 1658kN and impact load from live load is 20% of live load. Therefore impact load equals 20% of 1658kN and equals 331.60kN. The maximum shear force due to live load equals 1658kN plus 331.60kN and equals 1989.60kN.

2. **Find maximum moment and shear force due to self-weight**

   i. **For precast**

   From the above table, area of precast is A_{cp} = 1,851,300 mm^2

   Self-weight = $\Upsilon_c \times A_P$ = 25kN/m^{3} * 1.85m^2 = 46.25kN/m

   Since our bridge is simple span and span length is 40m,

   a) Maximum moment (M_{g1}) = $\frac{w \times L^2}{8} = \frac{46.25\times 40^2}{8}$ = 9,250kN-m

   b) Maximum shear force = $\frac{w \times L}{2} = \frac{46.25\times 40}{2}$ = 925kN.

   ii. **For Composite structure**

   From the above table, area of composite structure is A_{cc} = 2,733,700mm^2

   Self-weight = $\Upsilon_c \times A_c$ = 25 kN/m^3 × 2.73m^2 = 68.34kN/m

   Since our bridge is simple span and span length is 40m,

a. Maximum moment (M_{g1}) = $\frac{w\times L^2}{8} = \frac{68.34\times 40^2}{8}$ = 13,668.50kN-m

b. Maximum shear force = $\frac{w\times L}{2} = \frac{68.34\times 40}{2}$ = 1,366.85kN.

3. Find maximum moment and shear force due to superimposed load.

From the example 1, total superimposed load is 38.855kN/m

Since our bridge is simple span and span length is 40m,

a. Maximum moment (M_{g2}) = $\frac{w\times L^2}{8} = \frac{38.855\times 40^2}{8}$ = 7771kN-m

b. Maximum shear force = $\frac{w\times L}{2} = \frac{38.855\times 40}{2}$ = 777kN.

A. Check section modulus at transfer

a) $[M_q + (1-\eta)\times M_{g1}] / f_{tr}$ = [1,7915.20 + (1- 0.82) × 9,250] ×1,000,000 /22

$= 8.90\times 10^8\ mm^3$

$= 8.90\times 10^8\ mm^3 \leq 1.64\times 10^9\ mm^3$-------------------- ok!!

b) $[M_q + (1-\eta)\times M_{g1}] / f_{br}$ = [1,7915.20+ (1- 0.82) × 9,250] × 1.000,000 / 21.5

$= 9.11\times 10^8\ mm^3$

$= 9.11\times 10^8\ mm^3 \leq 2.01\times 10^9\ mm^3$-------------------- Ok!!

B. Check section modulus at working

a) $[M_q + M_{g2} + (1-\eta)\times M_{g1}]/f_{tr}$ = [17915.20 + 7771+ (1-0.82) × 13,650] ×1,000,000 /22

$= 1.28\times 10^9\ mm^3$

$= 1.28\times 10^9$ mm3 $\leq 1.77\times 10^9 mm^3$--------------------------- ok!!

b) $[M_q + M_{g2} + (1-\eta)\times M_{g1}]/f_{br}$ = [17915.20 + 7771 + (1- 0.82) ×13,650] ×1.000,000 / 21.5

$= 1.31\times 10^9\ mm^3$

$= 1.31\times 10^9\ mm^3 \leq 1.32\times 10^9 mm^3$------------------------ ok!!

Load combination

Load factor group I loading =1.4(DL +5/3 (LL +I))

Table 4-5 Load combination

Self Wt (g)	Dead Load (SDL)	Live +impact load (LL +I)	Total Service Load	Total Factored Load	Maximum Design BM	Maximum Design SF
68.34	38.86	89.58	196.77	359.09	71817.43	7181.74

Determination of initial pre - stressing force to be applied

Minimum pre - stressing force to be applied on concrete may be expressed as equation (4.5) and

$P_e = \frac{(17915.20+7771+13,668.5)\times 1000000\times 1,851,300 \times 2,013,522942.45}{1,319,126,548.95\times (2,013,522,942.45+1,851,300 \times 888)} = 30,404.91\text{kN}$

$p = P_e/0.82 = 30,404.91/0.82 = 37,079.15\text{kN}$.

Number of high strength steel wires as follows:

Area of tendon $= \frac{P_e}{f_{Pe}} = \frac{30,404,905.85\ \text{N}}{1107\text{Mpa}} = 27,466.04\text{mm}^2$

Assume wire diameter is 7mm, therefore; area of one wire is as follows:

Area of one wire $= \frac{\pi\times d^2}{4} = \frac{\pi\times 7^2}{4} = 38.47\text{mm}^2$

Number of high strength steel wires required (n) $= \frac{\text{Area of tendon}}{\text{Area of one wire}} = \frac{27,466.04\ \text{mm}^2}{38.47\text{mm}^2} = 714.05$

n = 715 wires are necessary to pre-stress the concrete.

Deflection

I. Deflection due to pre-stress

Since pre-stressing method used in this study is post tensioning, assuming eccentricity profile to be parabolic and calculated by using equation 2.25.

$\Delta_p = \frac{-5\times 37,079,153.48\times 888\times 40000^2}{48\times 33.99\times 10^3 \times\ 1,989,527,364,025} = -81.15\text{mm}$ ………………..(Upwards)

II. Deflection due to beam self-weight at transfer:

By using equation 2.23, the deflection due to self-weight at transfer is calculated as follows:

$\Delta_g = \frac{5\times 46.28 \times 40^4 \times 10^{12}}{384\times 33.99 \times 10^3 \times 1,989,527,364,025} = 22.81$ mm ………….. (Downwards)

III. Deflection due to uniformly distributed superimposed load:

By using equation 2.23, the deflection due to superimposed dead load is calculated as follows:

$\Delta_s = \frac{5\times 38.855\times 40^4 \times 10^{12}}{384\times 33.99\times 1000\times 1,875,977,587,106}$

$\Delta_s = 20.31$ mm ……………. (Downwards)

IV. **Deflection due to live load:**

By using equation 2.23, the deflection due to live load is calculated as follows:

$$\Delta_{LL+I} = \frac{5\times W_{eq}\times L^4}{384\times E_c\times I_c} = \frac{5\times 89.58\times 40000^4}{384\times 33.99\times 1000\times 1{,}875{,}977{,}587{,}106}$$

$\Delta_{LL+I} =$ 46.82 mm ……………. (Downwards)

But, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4].

Therefore, L/640 = 40000/640 = 62.50mm, where L-is in mm.

Δ_{max} = 46.82 mm < 62.50 mm …………………………………………..OK!!

V. **Total immediately deflection after pre stressing - parabolic tendons**

Total immediate deflection is the sum of deflection due to initial pre-stressing force and deflection due to self-weight at transfer

$\Delta_i = \Delta_p + \Delta_g$ = - 81.15mm + 22.81 mm = -58.34 …….. (Upwards)

VI. **Deflection due to self-weight under working:**

By using equation 2.23, the deflection due to self-weight under working is calculated as follows:

$$\Delta_g = \frac{5\times 68.34\times 40^4\times 10^{12}}{384\times 33.99\times 10^3\times 1{,}875{,}977{,}587{,}106}$$ = 35.72 mm ………….. (Downwards)

VII. **Long time deflection:**

By using equation 4.7, the deflection due to long time is calculated as follows:

First calculate for W = 68.34kN/m + 38.855 kN/m + 89.58 kN/m =196.78kN/m,

$$\Delta_c = \frac{5\times 196.78\times 40^4\times 10^{12}}{384\times 33.99\times 10^3\times 1{,}875{,}977{,}587{,}106}$$ = 103mm, and then $\Delta P = P - P_e$

=37,079 – 0.82×37,079 = 0.18×37,079 = 6,675N

Therefore; Δ_l = 103× (1+2) - 81.15× [(1- 6675/37,079) + 2 × (1- 6675/37,079)]

= 103× (3) - 81.15× [0.82+ 1.64] =109.30mm………. (Downwards)

Check web shear and design for flexural shear

Web shear and flexural shear crack are two different cracks as shown in the figure 2.21 and both are governed by limiting value of the principal tensile stress expressed in equation 3.29 and 3.30

I. **Web shear:**

In equation 3.29, f_{cp} is unknown. To get the value of f_{cp}, use the following formula.

f_{cp} = P_e/A_{pc} , but P_e= A_s × f_{pe} , therefore; f_{cp} =(A_s × f_{pe})/A_{pc} or f_{cp} = P_e/A_{pc}, hear already p_e calculated above and its equals P_e = 30,404,905.85N & from appendix C, A_{pc} = 1,851,300mm^2, f_{cp} = 30,404,905.85/ 1,851,300 = 16.42Mpa

From above, b_w = 660mm, h = 2,480 mm, f_{ctd} = 1.64MPa & V_d = 7,181.74kN

Therefore; V_{cw} = (0.82× 660 × 2,480 × $\sqrt{1.64^2 + 16.42 \times 1.64}$) × (10^{-3}) = 7,303.42kN and V_{cw} = 7,303.42 > V_d = 7,181.74kN……………….……………………Ok!

II. Flexural shear:

The section capacity (V_c), on the basis of experiments is given by

$V_c = \beta_1 \times 0.5 \times b \times d \times f_{ctd}$, where, β_1 = 1+ Mo / M_d and now we can calculate the value of

M_o = P (Z/A_c + e). Already, we have the values for P, Z_{bp}, A_c and e.

P =37,079,153.48 N, e = 888 mm, A_{cp} = 1,851,300mm^2, Z_{bp} = 2,013,522,942.45mm^3

M_o = P (Z_{bp}/A_{cp} + e) =37,079,153.48 N × (2,013,522,942.45/1,851,300 + 888) × (10^{-6})

M_o =73,258kN-m but M_d =71,817.43kN-m & then,

β_1 = 1+ Mo / M_d = 1 + 73,258/71,817.43= 2.02 ≥ 2.00 ……………………………Not Ok!

Therefore; we can take β_1 = 2 and now calculate for $V_c = \beta 1 \times 0.5 \times b \times d \times f_{ctd}$

Where, b = 660mm, d = h_c – cover- half of diameter of tendon = 2480 – 50-0.5×100= 2,380mm & f_{ctd} = 1.64 MPa and V_c = (2 ×0.5×660×2380×1.64) × (10^{-3}) = 2,597.90kN < V_d = 7,181.74 kN…………….…………………Not ok.

When $V_c < V_d$, Therefore; shear force sustained by stirrups may be computed for $V_s = a_v \times d \times f_{yd}/s$

Assume that spacing of stirrup is S =150mm , and $V_s = V_d - V_c$ = 7,181.74-2,597.90 =4,583.85kN and a_v = (Vs × S / d × f_{yd}) = 4,583.85 ×1000 × 150 / (2380× 434.78) = 665mm^2. Assume that stirrup diameter is Ø8. And area of one stirrup is $A = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4}$ = 50.27mm^2

Number of stirrup (n) = 665/50.27 = 14

III. Limiting value against diagonal compression:

When shear force exceed the limiting value against diagonal compression, then the section must be changed. Let us see the following.

V_{Rd} = 0.25 × f_{cd} × b_w ×d = 0.25×22.667×660×2380 × (10^{-3}) = 8,901.20kN > 7,181.74kN……Ok!

Therefore, the section is adequate for design load

Example 4: 40m Span Bridge with Slab Track System

A simple span precast post tensioned pre-stressed concrete girder bridge is to be designed for a railway bridge with cast in situ deck slab, cross section of which shown in figure below. The span length is 40m and cast in place concrete deck slab is of 300 mm thick, while the slab track is 200 mm in thickness. The bridge is to serve for the Cooper E-80 rail load (live load) configuration.

Check section modulus in different stage

To check section modulus, first we should determine the value of the following variables.

1. **Find maximum moment and shear force due to live load**

From example 3, the result is as follows:

- The maximum moment due to live load (M_q) = 17,915.20kN.
- The maximum shear force due to live load = 1989.60kN.

2. **Find maximum moment and shear force due to self-weight.**

   i. **For precast**

From example 3, the result is as follows:

- The maximum moment due to self-weight of pre-cast ((M_{g1})) = 9,250kN-m
- The maximum shear force due to self-weight of pre-cast = 925kN.

   iii. **For composite structure**

From example 3, the result is as follows:

- The maximum moment due to self-weight of composite structure (M_{g1}) = 13,668.50kN-m
- The maximum shear force due to self-weight of composite structure = 1,366.85kN.

3. **Find maximum moment and shear force due to superimposed load.**

From above the example 2, total superimposed load is 12.625+1.5+8=22.125kN/m

Since our bridge is simple span, so

a. Maximum moment (M_{g2}) $= \frac{w \times L^2}{8} = \frac{22.125 \times 40^2}{8}$ = 4425kN-m

b. Maximum shear force $= \frac{w \times L}{2} = \frac{22.125 \times 40}{2}$ = 442.50kN.

A. Check section modulus at transfer

a) = [1,7915.20 + (1- 0.82) × 9,250] ×1,000,000/22

$= 8.90 \times 10^8 \text{ mm}^3$

$=8.90 \times 10^8 \text{ mm}^3 \leq 1.64 \times 10^9 \text{ mm}^3$-------------------- ok!!

b) $[M_q + (1-\eta) \times M_{g1}] / f_{br}$ = [1,7915.20+ (1- 0.82) × 9,250] × 1.000,000 / 21.5

$= 9.11 \times 10^8 \text{ mm}^3$

$= 9.11 \times 10^8 \text{ mm}^3 \leq 2.01 \times 10^9 \text{ mm}^3$-------------------- Ok!!

B. Check section modulus at working

a) $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{tr}$ = [17915.20 + 4,425+ (1-0.82) × 13,650] ×1,000,000 /22

$= 1.13 \times 10^9 \text{ mm}^3$

$= 1.13 \times 10^9$ mm3 $\leq 1.77 \times 10^9 \text{mm}^3$--------------------------- ok!!

b) $[M_q + M_{g2} + (1-\eta) \times M_{g1}]/f_{br}$ = [17915.20 + 4,425 + (1- 0.82) ×13,650] ×1.000,000 / 21.5

$= 1.15 \times 10^9 \text{ mm}^3$

$= 1.15 \times 10^9 \text{ mm}^3 \leq 1.32 \times 10^9 \text{mm}^3$------------------------ ok!!

Load combination

Load factor group I loading =1.4× (DL +5/3× (LL +I))

Table 4-6 Load combination

Self Wt (g)	Dead Load (SDL)	Live +impact load (LL +I)	Total Service Load	Total Factored Load	Maximum BM	Maximum SF
68.34	22.13	89.58	180	336	67,133.03	6,713.30

Determination of initial pre - stressing force to be applied

Minimum pre - stressing force to be applied on concrete may be expressed as equation (4.5) and

$P_e = \frac{(17915.20+4{,}425+13{,}668.5)\times 1000000 \times 1{,}851{,}300 \times 2{,}013{,}522942.45}{1{,}319{,}126{,}548.95 \times (2{,}013{,}522{,}942.45+1{,}851{,}300 \times 888)} = 27{,}820\text{kN}$

p = P_e/0.82 = 27,819,832/ 0.82 =33,926,624.39 N = 33,926.62kN.

Number of high strength steel wires as follows:

Area of tendon $= \frac{P_e}{f_{Pe}} = \frac{27819832 \text{ N}}{1107\text{Mpa}} = 25{,}130.83\text{mm}^2$

Assume wire diameter is 7mm, therefore; area of one wire is as follows:

Area of one wire = $\frac{\pi \times d^2}{4} = \frac{\pi \times 7^2}{4} = 38.47\text{mm}^2$

Number of high strength steel wires required (n) = $\frac{\text{Area of tendon}}{\text{Area of one wire}} = \frac{25{,}130.83\ \text{mm}^2}{38.47\text{mm}^2}$ = 653.34

n = 654 wires are necessary to pre-stress the concrete.

Deflection

I. Deflection due to pre-stress

Since pre-stressing method used in this study is post tensioning, assuming eccentricity profile to be parabolic and calculated by using equation 2.25.

$$\Delta_p = \frac{-5 \times 33{,}926{,}624.39 \times 888 \times 40000^2}{48 \times 33.99 \times 10^3 \times 1{,}989{,}527{,}364{,}025}$$

Δ_p = - 74.20mm ………………..(Upwards)

II. Deflection due to beam self-weight at transfer:

By using equation 2.23, the deflection due to self-weight at transfer is calculated as follows:

$$\Delta_g = \frac{5 \times 46.28 \times 40^4 \times 10^{12}}{384 \times 33.99 \times 10^3 \times 1{,}989{,}527{,}364{,}025}$$

Δ_g = 22.81 mm ………….. (Downwards)

III. Deflection due to uniformly distributed superimposed dead load:

By using equation 2.23, the deflection due to superimposed dead load is calculated as follows:

$$\Delta_s = \frac{5 \times 22.13 \times 40^4 \times 10^{12}}{384 \times 33.99 \times 1000 \times 1{,}875{,}977{,}587{,}106}$$

Δ_s = 11.56 mm ……………. (Downwards)

IV. Deflection due to live load:

By using equation 2.23, the deflection due to live load is calculated as follows:

$$\Delta_{LL+I} = \frac{5 \times W_{eq} \times L^4}{384 \times E_c \times I_c} = \frac{5 \times 89.58 \times 40000^4}{384 \times 33.99 \times 1000 \times 1{,}875{,}977{,}587{,}106}$$

Δ_{LL+I} = 46.82 mm ……………. (Downwards)

But, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4].

Therefore, L/640 = 40000/640 = 62.50mm, where L-is in mm.

Δ_{max} = 46.82 mm < 62.50 mm ……………………………………………..OK!!

V. Total immediately deflection after pre stressing - parabolic tendons

Total immediate deflection is the sum of deflection due to initial pre-stressing force and deflection due to self-weight at transfer

$\Delta_i = \Delta_p + \Delta_g$ = - 74.20mm + 22.81 mm = -51.39 ……………….. (Upwards)

VI. Deflection due to self-weight under working:

By using equation 2.23, the deflection due to self-weight under working is calculated as follows:

$$\Delta_g = \frac{5\times 68.34 \times 40^4 \times 10^{12}}{384\times 33.99 \times 10^3 \times 1{,}875{,}977{,}587{,}106}$$

Δ_g = 35.72 mm ………….. (Downwards)

VII. Long time deflection:

By using equation 4.7, the deflection due to long time is calculated as follows:

First calculate for W = 68.34kN/m + 22.13kN/m + 89.58kN/m =180.05kN/m,

$$\Delta_c = \frac{5\times 196.78 \times 40^4 \times 10^{12}}{384\times 33.99 \times 10^3 \times 1{,}875{,}977{,}587{,}106}$$ = 94.24mm, and then $\Delta P = P - P_e$

= 33,927 – 0.82 × 33,927= 0.18 × 33,927= 6,107 N

Therefore; Δ_l = 94.24× (1+2) - 74.20× [(1- 6,107/33,927) + 2 × (1-6,107/33,927)]

= 94.24× (3) – 74.25× [0.82+ 1.64] =100.07mm……. (Downwards)

Check web shear and design for flexural shear

Web shear and flexural shear crack are two different cracks as shown in the figure 2.21 and both are governed by limiting value of the principal tensile stress expressed in equation 3.29 and 3.30

II. Web shear:

In equation 3.29, f_{cp} is unknown. To get the value of f_{cp}, use the following formula.

$f_{cp} = P_e/A_{pc}$, but $P_e = A_s \times f_{pe}$, therefore; $f_{cp} = (A_s \times f_{pe})/A_{pc}$ or $f_{cp} = P_e/A_{pc}$, hear already p_e calculated above and its equals P_e = 27,819,832N and from appendix D, A_{pc} = 1,851,300mm^2,

f_{cp} = 27,819,832/ 1,851,300 = 15.03MPa

From above, b_w = 660mm, h = 2,480 mm, f_{ctd} = 1.64MPa & V_d =6,713.30kN

Therefore; $V_{cw} = (0.82\times 660 \times 2{,}480 \times \sqrt{1.64^2 + 15.03 \times 1.64}) \times (10^{-3})$ = 7,011.18kN and V_{cw} = 7,011.18 > V_d = 6,713.30kN………………………………Ok!

III. Flexural shear:

The section capacity (V_c), on the basis of experiments is given by

$V_c = \beta_1 \times 0.5 \times b \times d \times f_{ctd}$, where, $\beta_1 = 1 + Mo / M_d$ and now we can calculate the value of $M_o = P (Z/A_c + e)$. Already, we have the values for P, Z_{bp}, A_c and e.

P = 33,926,624.39N, e = 888 mm, A_{cp} = 1,851,300mm^2, Z_{bp} = 2,013,522,942.45mm^3

$M_o = P (Z_{bp}/A_{cp} + e)$ = 33,926,624.39 N × (2,013,522,942.45/1,851,300 + 888) × (10^{-6})

M_o =67,029kN-m but M_d =67,133.03kN-m & then,

$\beta_1 = 1 + Mo / M_d$ = 1 + 67,029/67,133.03= 1.998

$\beta_1 = 1.998 \leq 2.00$ ……………………………………Ok!

Therefore; we can take β_1 = 1.998 and calculate for $V_c = \beta 1 \times 0.5 \times b \times d \times f_{ctd}$

Where, b = 660mm, d = h_c - cover - half of diameter of tendon = 2480 - 50 - 0.5×100 = 2,380mm & f_{ctd} = 1.64 MPa and V_c = (1.998 ×0.5×660×2380×1.64) × (10^{-3}) = 2,570.12kN < V_d = 6,713.30kN…………………………………Not ok.

When $V_c < V_d$, Therefore; shear force sustained by stirrups may be computed for $V_s = a_v \times d \times f_{yd} / s$

Assume that spacing of stirrup is S =150mm , and $V_s = V_d - V_c$= 6,713.30 - 2,570.12 = 4,143.19kN and $a_v = (V_s \times S / d \times f_{yd})$ = 4,143.19 ×1000 × 150 / (2380× 434.78) = 600.59mm^2. Assume that stirrup diameter is Ø8. And area of one stirrup is $A = \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4}$ = 50.27mm^2

Number of stirrup (n) = 600.59/50.27 = 12

IV. Limiting value against diagonal compression:

When shear force exceed the limiting value against diagonal compression, then the section must be changed. Let us see the following.

$V_{Rd} = 0.25 \times f_{cd} \times b_w \times d = 0.25 \times 22.667 \times 660 \times 2380 \times (10^{-3})$ = 8,901.20kN > 6,713.30kN……Ok!

Therefore, the section is adequate for design load

4.1 Discussion of the Results

The previous chapter target is only analysis and design of superstructure in both ballasted and slab cases. Now in this section, discussion based on the result from the different loading of both ballasted and slab track cases are addressed. The analysis considered two different span lengths with corresponding depth of girder. The first analysis considered 15m span length and the second analysis considered 40m span length and the effect of span difference also discussed below.

The study considered the same result of loading of live load in simply supported bridges for both ballasted and slab track as shown in the table 2-3, the maximum moment for span length 15m due to live load (Cooper E-80) is 2,514.10kN-m and the maximum moment due to impact load (impact load is 20% of live load). Therefore, the maximum moment for span length 15m due to impact load is 502.82kN. The maximum moment for span length 40m due to live load (Cooper E-80) is14, 929.33kN-m and the maximum moment for span length 40m due to impact load is 2985.87kN. Similarly, the maximum shear force for span length 15m due to live load is 767.99kN and the maximum shear force for span length 15m due to impact load is 153.60kN. The maximum shear force for span length 40m due to live load is 1,658kN and the maximum shear force for span length 40m due to impact load is 331.60kN.

Here the study considered the same depth of precast girder and deck of bridge for this study and this assumption leads to conclude the self-weight of bridge is the same for both ballasted and slab bridge. Since our bridge is simple supported, so the maximum moment for span length 15m due to self-weight is 1,593kN-m and its shear force is 424.80kN.The maximum moment for span length 40m due to self-weight is 13,668.50kN-m and its shear force is 1,366.85kN.

The basic difference between both cases is superimposed dead load. In ballasted case, the depth of ballast on bridge surface already mentioned above is 600mm and its cross sectional area is 3.09m^2. Therefore its weight is 58.71kN/m. But ballast weight is shared to two girders. Therefore, for one precast girder, ballast weight is half of total ballast weight (i.e. ballast weight is ½ times 58.71kN/m is 29.355kN/m). Besides, rail self-weight assumed 1.5kN/m from [4]. In addition to this curb of bridge weight calculated and which is 8kN/m. Therefore total superimposed load is 38.855kN/m. So the maximum bending moment and shear force for span length 15m due to superimposed dead load is 1,092.80kN-m and 291.41kN respectively. The maximum bending moment and shear force for span length 40m due to superimposed dead load is 7,771kN-m and 777kN respectively.

For slab case, the depth of slab instead of ballast on bridge surface already mentioned above and which is 200mm and its cross sectional area is 1.01m^2. Therefore, slab weight is 25.25kN/m but slab weight is shared to two girders. Therefore, the share of one precast girder is half of total slab weight. i.e. half of 25.25kN/m, is equals 12.625kN/m. Besides, rail self-weight is assumed 1.5kN/m [4]. In addition to this curb of bridge weight calculated and which is 8kN/m. Therefore total superimposed load is 22.125kN/m. So the maximum bending moment and shear force for span length 15m due to superimposed dead load is 622.27kN-m and 165.94kN respectively. The maximum bending moment and shear force for span length 40m due to superimposed dead load is 4,425kN-m and 442.50kN respectively.

To address the objective of this thesis, the result of initial pre - stressing force in both cases is necessary. Here the initial pre - stressing force introduced in the ballasted case for 15m span length is 5,858.56kN and the initial pre - stressing force introduced in the slab case for 15m span length is 5,375.17kN. From this result, the slab case saved 483.39kN. Again, the initial pre - stressing force introduced in the ballasted case for 40m span length is 37,079.15kN and the initial pre - stressing force introduced in the slab case for span length 40m is 33,926.62kN. From this result, the slab case saved 3,152.53kN.

As the study discussed above, in the ballasted case for span length 15m, the deflection immediately after pre-stressing is 2.70mm upwards, deflection due to beam self-weight at transfer is 0.60mm downwards, and deflection due to superimposed dead load is 0.63mm downwards, deflection due to self-weight under working is 0.92mm downwards, total immediately deflection(combination of deflection due to pre-stress and deflection due to beam self-weight at transfer) is 2.10mm upwards, long term deflection is 3.11mm downwards and deflection due to live load 1.74mm downwards but, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4] and therefore, L/640 is 15000/640 which is 23.44mm, where L-is in mm. Since deflection due to live load is 1.74mm which is less than 23.44mm and from this result the study conclude that the cross section is adequate for this live load (Cooper E-80).

As the study discussed above, in the ballasted case for span length 40m, the deflection immediately after pre-stressing is 81.15mm upwards, deflection due to beam self-weight at transfer is 22.81mm downwards, and deflection due to superimposed dead load is 20.31mm downwards, deflection due to self-weight under working is 35.72 mm downwards, total immediately deflection(combination of deflection due to pre-stress and deflection due to beam self-weight at transfer) is 58.34mm upwards, long term deflection is 109.30mm downwards and deflection due to live load 46.82mm downwards but, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4] and therefore, L/640 is 40000/640 which is 62.50mm, where L-is in mm. Since deflection due to live load is 46.82mm which is less than 62.50 mm and from this result the study conclude that the cross section is adequate for this live load (Cooper E-80).

In the slab case for 15m span length, deflection immediately after pre-stressing is 2.5mm upwards, deflection due to beam self -weight at transfer is 0.60mm downwards, and deflection due to superimposed dead load is 0.36mm downwards, deflection due to self-weight under working is 0.92mm downwards, total immediately deflection (combination of deflection due to pre-stress and deflection due to beam self-weight at transfer) is 1.90mm upwards, long term deflection is 2.91mm downwards and deflection due to live load 1.74mm downwards but, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4] and therefore, L/640 = 15000/640 which is 23.44mm, where L-is in mm. Since deflection due to live load is 1.74mm which is less than 23.44mm and from this result the study conclude that the cross section is adequate for this live load (Cooper E-80).

In the slab case for 40m span length, deflection immediately after pre-stressing is 74.2mm upwards, deflection due to beam self -weight at transfer is 22.81mm downwards, and deflection due to superimposed dead load is 11.56mm downwards, deflection due to self-weight under working is 35.72 mm downwards, total immediately deflection (combination of deflection due to pre-stress and deflection due to beam self-weight at transfer) is 51.39mm upwards, long term deflection is 100.07mm downwards and deflection due to live load 46.82 mm downwards but, the maximum allowable deflection due to live load plus impact shall not exceed L/640 [4] and therefore, L/640 = 40000/640 which is 62.50mm, where L-is in mm. Since deflection due to live load is 46.82mm which is less than 62.50 mm and from this result the study conclude that the cross section is adequate for this live load (Cooper E-80).

The web shear of ballasted case for 15m span length is 2,949.29kN which is greater than design shear force which is 2,879.90kN and this satisfies pre condition of web shear. But flexural shear for this case is 1,861.99kN which is less than design shear force which is 2,879.90kN and this is not satisfy pre condition of flexural shear. To satisfy this condition, the study needs extra stirrups and number of stirrup from above 4 Ø8 @ 150mm has been used.

The web shear of ballasted case for 40m span length is 7,303.42kN which is greater than design shear force which is 7,181.74kN and this satisfies pre condition of web shear. But flexural shear for this case is 2,597.90kN which is less than design shear force which is 7,181.74kN and this is not satisfy pre condition of flexural shear. To satisfy this condition, the study needs extra stirrups and number of stirrup from above 14 Ø8 @ 150mm has been used.

For 15m span length of slab case, the web shear is 2,865.36kN which is greater than design shear force which is 2,704.23kN and this satisfies pre condition of web shear. But flexural shear for this case is 1,841.57kN which is less than design shear force which is 2,704.23kN and this is not satisfy pre condition of flexural shear. To satisfy this condition, the study needs extra stirrups and number of stirrup from above 3 Ø8 @ 150mm has been used.

For 40m span length of slab case, the web shear is 7,011.18kN which is greater than design shear force which is 6,713.30kN and this satisfies pre condition of web shear. But flexural shear for this case is 2,570.12kN which is less than design shear force which is 6,713.30kN and this is not satisfy pre condition of flexural shear. To satisfy this condition, the study needs extra stirrups and number of stirrup from above 12 Ø8 @ 150mm has been used.

When shear force exceed the limiting value against diagonal compression, then the section must be changed. From the analysis result, limiting value against diagonal compression for 15m span length of ballasted case is 6,718.40kN which is greater than that of design shear force which is 2,879.90kN. Therefore the section provided above is safe. Again, in slab case, the result of limiting value against diagonal compression is 6,718.40kN which is greater than that of design shear force which is 2,704kN. Therefore the section provided above is safe also for this case.

When shear force exceed the limiting value against diagonal compression, then the section must be changed. From the analysis result, limiting value against diagonal compression for 40m span length of ballasted case is 8,901.20kN which is greater than that of design shear force which is 7,181.74kN. Therefore the section provided above is safe. Again, in slab case, the result of limiting value against diagonal compression is 8,901.20kN which is greater than that of design shear force which is 6,713.30kN. Therefore the section provided above is safe also for this case.

4.2 Summary of Discussion

As discussed before, the overall idea of this thesis is to compare analysis results of ballasted and slab railway bridges, economic analysis result of both cases, and to recommend better track type for rail way bridge in Ethiopia.

The study discussed in review literature about pre-stressing of tendon, types of pre-stress, advantage of post tensioning over pre tensioning, advantage of slab track over ballasted track on bridge and vice versa, and finally the study took counter examples with 15 and 40 meter san length of slab and ballasted railway bridge.

This thesis took the same values of design parameters, span of the bridge and depth of girder for both slab and ballasted cases and compared initial pre-stress force for both cases. The analysis addressed two different span length (15m and 40m) to compare construction cost of ballasted & slab track and selected preferable track type for Addis Ababa light rail transit.

According to analysis result, the following table shows us comparative study of initial pre-stressing force of tendon in both slab and ballasted track in different span lengths.

Table 4-7 Pre-stressing force versus span length in both ballasted and slab railway bridge

Description	Span Length			
	15m		40m	
	Ballasted track	Slab track	Ballasted track	Slab track
Initial pre-stressing force of tendon (kN)	5,858.56	5,375.17	37,079.15	33,926.62
Difference due to span change (kN)	483.39		3,152.53	
Percentage saved by slab track (%)	1.30		9.29	

The above table results presented in the following graph and show us the difference of four points in both slab and ballasted cases. Let us consider the first point (15, 5,375.17), second point (15, 5,858.56), third point (40, 33,926.62) and fourth point (40, 37,079.15). In the first and second point; there is insignificant difference in value, which means the first point saved 1.30% from second point. But in third and fourth point; there is significant difference in value which means the third point saved 9.29% from fourth point.

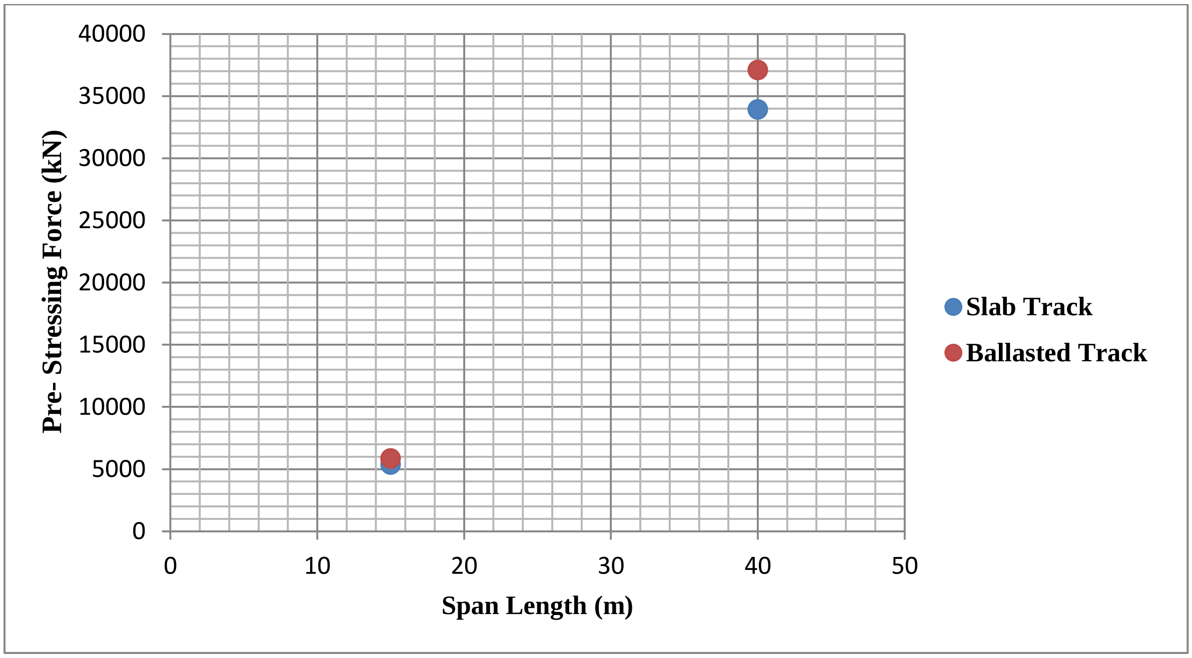


Figure 4-3 Pre-stressing force versus span length in both slab and ballasted railway bridge

The graphical presentation shows us comparative analysis result in three parts. They are below 15m span length, between 15m and 40m span length, and above 40m span length. Above 40m span length, slab track more economical than ballasted track, again in between 15m and 40m span length, slab track economical than ballasted track but below 15m span length, ballasted track seems economical than slab track.

CHAPTER FIVE

5 Conclusions and Recommendation

5.1 Conclusion

In Addis Ababa Light Rail Transit (LRT), a number of overpasses (railway bridges) with different span length in different area are considered to study construction cost of both slab and ballasted cases.

The purpose of this study is to select economical track type for railway bridges. Based on the thesis objective, the analysis considered 15 and 40meter span length railway bridge for slab and ballasted cases.

Hear numerical value saved by slab track in 15m span length is 1.30% and in 40m span length is 9.29%. Therefore the study conclude that slab track is economical than ballasted track for span length greater than 15m. On the other hand, ballasted track seems economical than slab track for span length less than 15m.

Moreover; the study considered pre-stressed post tensioned concrete and this consideration leads to use long span bridge due to the fact that pre-stressed post tensioned concrete suitable for long span bridges than pre-stressed pre-tensioned concrete and hence the study concluded from this consideration and analysis result, slab track is economical for long span than ballasted track.

Finally, from over all analysis result, discussion and conclusion, the study selected slab track type than ballasted track and which is preferable & should be adopted in Ethiopian Railway Projects (Addis Ababa Light Rail Transit).

5.2 Recommendation

The slab track can ensure very good geometrical stability of the track comparing to the ballasted track. This stability of geometry is necessary for railway bridge because of maintenance of bridge takes a lot of time and this may leads to interruption of transportation. Therefore the study recommended slab track than ballasted track for railway bridge.

Ballasted track cannot be accessible to road vehicles and slab track can be accessible to road vehicles especially at level crossing. Therefore, around level crossing, the study recommended slab track.

Release of dust from the ballast into the environment is causing environmental pollution but slab track has less environment pollution as compared with ballasted track. Therefore, for Addis Ababa Light Rail Transit project, the study recommended slab track than that of ballasted track.

The final result of this study show us the economical track type for long span bridge is slab track and therefore the study recommended that, for long span railway bridge, slab track type should be adopted.

Besides the objective of the study, this thesis used for national railway project to select economical track type for simply supported railway bridges. Therefore, the one who interested in this study area can improve this thesis by considering the limitation of this thesis.

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7 Appendix

Appendix-A

Arithmetic calculation of area of pre-cast, area of composite structure, centroid of precast, centroid of composite structure, second moment of inertia of precast, second moment of inertia of composite structure, section modulus of precast and section modulus of composite structure in ballasted case (15m span) as follows:

First, find the relevant dimensions of the pre-cast concrete girder

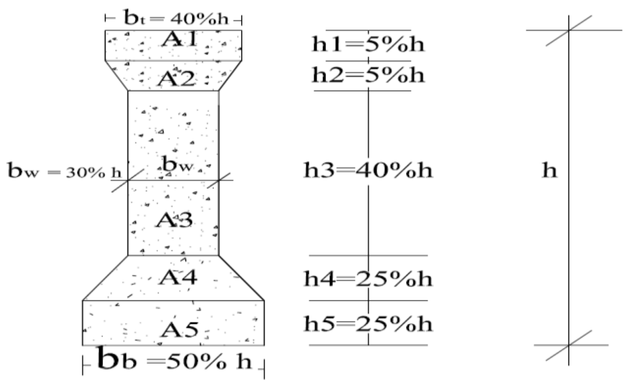

Figure 7-1 Cross section of precast

After so many trial the study selected pre-cast depth d = 1,900mm

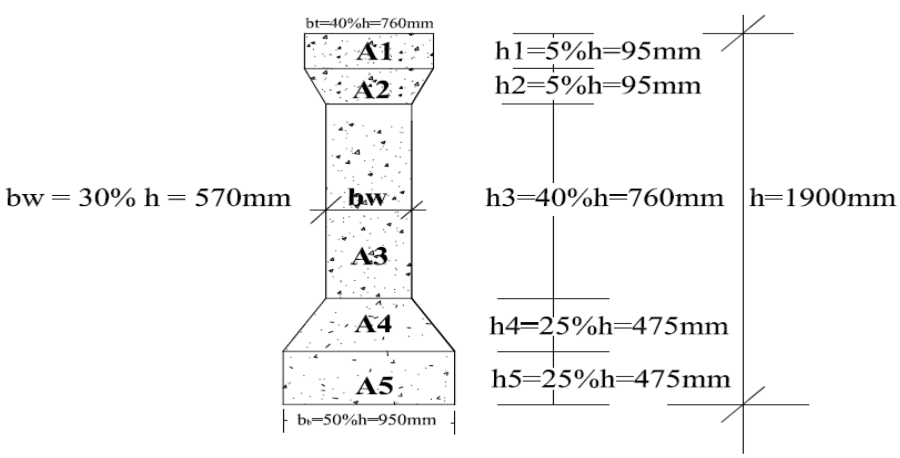

Figure 7-2 Relevant dimensions of the precast concrete girder

Section properties of precast

Precast and composite structure section properties discussed below. The following figures show us, neutral axis, second moment of inertia and total area of precast structure. All dimensions are expressed in terms of height of precast.

Table 7-1 First trial dimension of precast

h	b_t	b_b	h_1	h_2	h_3	h_4	h_5	b_w
h	0.40h	0.50h	0.05h	0.05h	0.40h	0.25h	0.25h	0.30h
1900	760	950	95	95	760	475	475	570

N.B: all dimensions are in mm.

Determination of the centroid of the precast girder

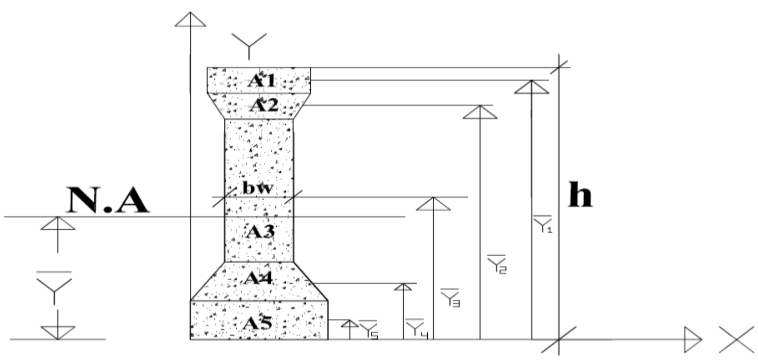

Figure 7-3 Centroid of precast measured from x-axis

$A_p \times \bar{y}_p = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5$

$$\bar{y}_p = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5}{A_P}$$

Table 7-2 Determination of centroid of precast

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i $(mm)^2$	$\bar{Y}_i$ (mm)	$A_i * \bar{y}_i$ $(mm)^3$
A1	95	760	760	72,200	1,853	133,750,500
A2	95	760	570	63,175	1,760	111,172,958
A3	760	570	570	433,200	1,330	576,156,000
A4	475	570	950	361,000	693	250,067,708
A5	475	950	950	451,250	238	107,171,875
∑=				1,380,825	∑	1,178,319,042
$\bar{y}$				853	mm	

N.B: Area of trapezium and rectangle solved by the following formulae.

$A_T = \frac{1}{2} \times (a + b) \times h$ and $A_R = b \times h$

Determination of the second moment of inertia of the precast girder

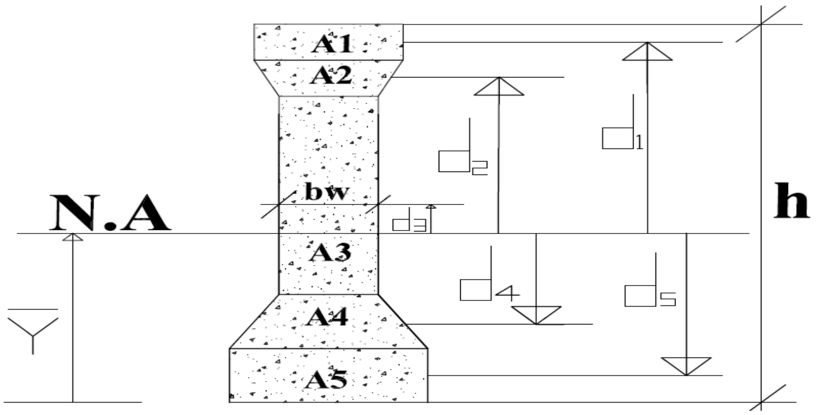

Figure 7-4 Second moment of inertia of precast

Table 7-3 Determination of second moment of inertia for precast

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Īi $(mm)^4$	Area A_i $(mm)^2$	d_i (mm)	d_i^2 $(mm)^2$	$A_i * d_i^2$ $(mm)^4$
A1	95	760	760	54,300,417	72,200	999	998,312	72,078,145,140
A2	95	760	570	47,189,648	63,175	906	821,593	51,904,138,238
A3	760	570	570	20,851,360,000	433,200	1,330	1,768,900	766,287,480,000
A4	475	570	950	6,646,144,748	361,000	161	25,804	9,315,204,362
A5	475	950	950	8,484,440,104	451,250	616	379,264	171,142,930,274
∑				36,083,434,917	1,380,825	∑		1,070,727,898,014
$\bar{I}_P$		1,106,811,332,931		mm^4				

$\bar{I}_p = (\bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4 + \bar{I}_5) + A_1 \times D_1^2 + A_2 \times D_2^2 + A_3 \times D_3^2 + A_4 \times D_4^2 + A_5 \times D_5^2$

$\bar{I}_p = \sum \bar{I}_i + \sum A_i \times d_i^2$, i= 1, 2, 3 ……

$\bar{I}_p = 1{,}106{,}811{,}332{,}931\ mm^4$

Section modulus of precast structure

$$Z_{bp} = \frac{\bar{I}p}{\bar{y}} = \frac{1{,}106{,}811{,}332{,}931 mm^4}{853mm} = 1.30 \times 10^9\ mm^3$$

$$Z_{tp} = \frac{\bar{I}p}{(h-\bar{y})} = \frac{1{,}106{,}811{,}332{,}931 mm^4}{(1900-853)\ mm} = 1.06 \times 10^9\ mm^3$$

The following figures show us, precast and cast in situ composite structure neutral axis, second moment of inertia and total area of composite structure.

Determination of the centroid of the composite girder

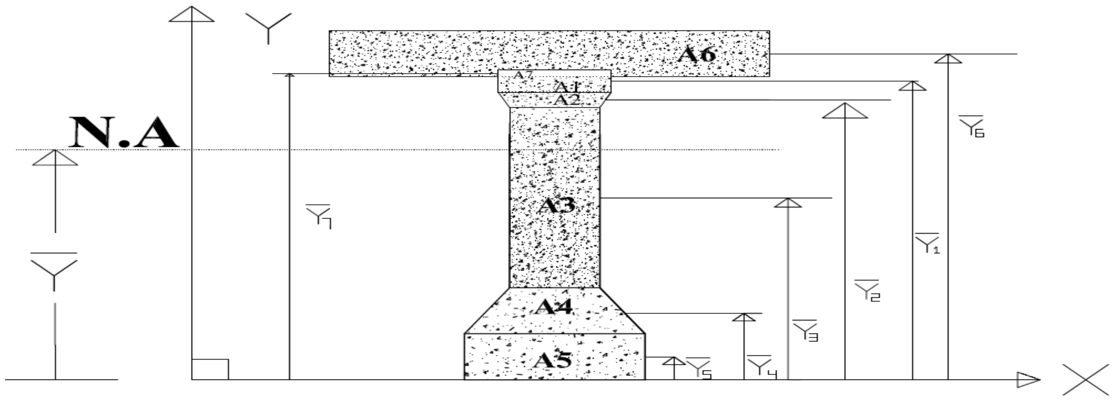

Figure 7-5 Centroid of precast and cast in situ composite structure

$A_c \times \bar{y}_c = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7$

$\bar{y}_c = \dfrac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7}{A_c}$

Table 7-4 Determination of the centroid of the composite girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ $(mm)^3$	$A_i * \bar{y}_i$ $(mm)^3$
A1	95	760	760	72,200	1,853	133,750,500
A2	95	760	570	63,175	1,760	111,172,958
A3	760	570	570	433,200	1,330	576,156,000
A4	475	570	950	361,000	693	250,067,708
A5	475	950	950	451,250	238	107,171,875
A6	300	3,000	3,000	900,000	2,030	1,827,000,000
A7	20	760	760	15,200	1,890	28,728,000
∑=				2,265,625	∑	2,976,591,042
$\bar{y}$				1,314	mm	

Determination of the second moment of inertia of the composite structure

The determination of second moment of inertia of the composite structure is shown in the table below.

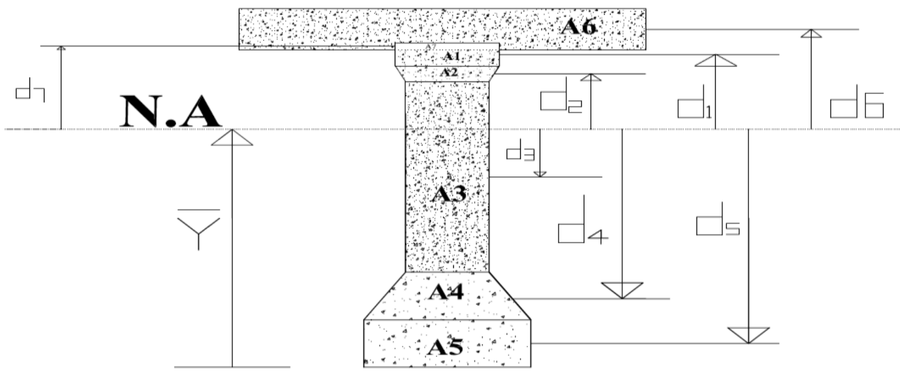

Figure 7-6 Second moment of inertia of composite structure

Table 7-5 Determination of the second moment of inertia of the composite girder

Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}_i$ $(mm)^4$	Area A_i (mm^2)	d_i (mm)	d_i^2 $(mm)^2$	$A_i * d_i^2$ $(mm)^4$
A1	95	760	760	54,300,417	72,200	539	290,192	20,951,829,738
A2	95	760	570	47,189,648	63,175	446	198,877	12,564,050,403
A3	760	570	570	20,851,360,000	433,200	(16)	262	113,609,003
A4	475	570	950	6,646,144,748	361,000	621	385,762	139,260,060,451
A5	475	950	950	8,484,440,104	451,250	1,076	1,158,434	522,743,325,500
A6	300	3,000	3,000	6,750,000,000	900,000	716	512,934	461,640,846,335
A7	20	760	760	506,667	15,200	576	332,000	5,046,398,024
∑ ==				42,832,928,250	2,265,625	∑		1,152,227,323,405
Īc		**1,195,060,251,655**		**mm⁴**				

Section modulus of composite structure

$$Z_{bc} = \frac{\bar{I}c}{\bar{y}_c} = \frac{1{,}195{,}060{,}251{,}655 \ \text{mm}^4}{1314\text{mm}} = 9.09 \times 10^8 \ \text{mm}^3$$

$$Z_{tc} = \frac{\bar{I}c}{(h-\bar{y}_c)} = \frac{1{,}195{,}060{,}251{,}655 \ \text{mm}^4}{(2180-1314)\text{mm}} = 1.38 \times 10^9 \ \text{mm}^3$$

Appendix-B

Arithmetic calculation of area of pre-cast, area of composite structure, centroid of precast, centroid of composite structure, second moment of inertia of precast, second moment of inertia of composite structure, section modulus of precast and section modulus of composite structure in slab case (15m span) as follows:

As discussed before in chapter 2, the depth of precast is the same for both ballasted track and slab track and which is h = 1,900mm.

Section properties of precast

Table 7-6 Dimension of precast

h	b_t	b_b	h_1	h_2	h_3	h_4	h_5	b_w (mm)
100%h	0.40h	0.50h	0.05h	0.05h	0.40h	0.25h	0.25h	0.30h
1900	760	950	95	95	760	475	475	570

N.B: all dimensions are in mm.

Determination of the centroid of the precast girder

$A_p \times \bar{y}_p = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5$

$$\bar{y}_p = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5}{A_P}$$

Determination of the centroid of the precast girder

Table 7-7 Determination of the centroid of the precast girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i * \bar{y}_i$ $(mm)^2$
A1	95	760	760	72,200	1,853	133,750,500
A2	95	760	570	63,175	1,760	111,172,958
A3	760	570	570	433,200	1,330	576,156,000
A4	475	570	950	361,000	693	250,067,708
A5	475	950	950	451,250	238	107,171,875
$\sum$=				1,380,825	$\sum$	1,178,319,042
$\bar{y}$				853	mm	

N.B: Area of trapezium and rectangle solved by the following formulae.

$A_T = \frac{1}{2} \times (a + b) \times h$ *and* $A_R = b \times h$

Determination of the second moment of inertia of the precast girder

Table 7-8 Determination of the second moment of inertia of the precast girder

Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}_i$ $(mm)^4$	Area A_i $(mm)^2$	d_i (mm)	d_i^2 $(mm)^2$	$A_i * d_i^2$ $(mm)^4$
A1	95	760	760	54,300,417	72,200	999	998,312	72,078,145,140
A2	95	760	570	47,189,648	63,175	906	821,593	51,904,138,238
A3	760	570	570	20,851,360,000	433,200	1,330	1,768,900	766,287,480,000
A4	475	570	950	6,646,144,748	361,000	161	25,804	9,315,204,362
A5	475	950	950	8,484,440,104	451,250	616	379,264	171,142,930,274
∑				36,083,434,917	1,380,825	∑		1,070,727,898,014
$\bar{I}_P$		1,106,811,332,931		mm^4				

Section modulus of precast structure

$$Z_{bp} = \frac{\bar{I}p}{\bar{y}} = \frac{1{,}106{,}811{,}332{,}931 mm^4}{853 mm} = 1.30 \times 10^9 \ mm^3$$

$$Z_{tp} = \frac{\bar{I}p}{(h - \bar{y})} = \frac{1{,}106{,}811{,}332{,}931 mm^4}{(1900 - 853)\ mm} = 1.06 \times 10^9 \ mm^3$$

Determination of the centroid of the composite girder

$$A_c \times \bar{y}_c = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7$$

$$\bar{y}_c = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7}{A_c}$$

Table 7-9 Determination of the centroid of the composite girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ $(mm)^3$	$A_i * \bar{y}_i$ $(mm)^3$
A1	95	760	760	72,200	1,853	133,750,500
A2	95	760	570	63,175	1,760	111,172,958
A3	760	570	570	433,200	1,330	576,156,000
A4	475	570	950	361,000	693	250,067,708
A5	475	950	950	451,250	238	107,171,875
A6	300	3,000	3,000	900,000	2,030	1,827,000,000
A7	20	760	760	15,200	1,890	28,728,000
∑=				2,265,625	∑	2,976,591,042
ȳ				1,314	mm	

Determination of the second moment of inertia of the composite structure

The determination of second moment of inertia of the composite structure is shown in the table below

Table 7-10 Determination of the second moment of inertia of the composite girder

Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}_i$ $(mm)^4$	Area A_i (mm^2)	d_i (mm)	d_i^2 $(mm)^2$	$A_i * d_i^2$ $(mm)^4$
A1	95	760	760	54,300,417	72,200	539	290,192	20,951,829,738
A2	95	760	570	47,189,648	63,175	446	198,877	12,564,050,403
A3	760	570	570	20,851,360,000	433,200	(16)	262	113,609,003
A4	475	570	950	6,646,144,748	361,000	621	385,762	139,260,060,451
A5	475	950	950	8,484,440,104	451,250	1,076	1,158,434	522,743,325,500
A6	300	3,000	3,000	6,750,000,000	900,000	716	512,934	461,640,846,335
A7	20	760	760	506,667	15,200	576	332,000	5,046,398,024
∑ ==				42,832,928,250	2,265,625	∑		1,152,227,323,405
Īc		**1,195,060,251,655**		**mm4**				

Section modulus of composite structure

$$Z_{bc} = \frac{\bar{I}c}{\bar{y}_c} = \frac{1{,}195{,}060{,}251{,}655\ \text{mm}^4}{1314\text{mm}} = 9.09 \times 10^8\ \text{mm}^3$$

$$Z_{tc} = \frac{\bar{I}c}{(h-\bar{y}_c)} = \frac{1{,}195{,}060{,}251{,}655\ \text{mm}^4}{(2180-1314)\text{mm}} = 1.38\times 10^9\ \text{mm}^3$$

Appendix-C

Arithmetic calculation of area of pre-cast, area of composite structure, centroid of precast, centroid of composite structure, second moment of inertia of precast, second moment of inertia of composite structure, section modulus of precast and section modulus of composite structure in ballasted case (40m span) as follows:

After trial, took height of precast girder, H = 2.2m for 40m span.

Table 7-11 Second trial dimensions of precast

h	b_t	b_b	h_1	h_2	h_3	h_4	h_5	b_w
h	0.40h	0.50h	0.05h	0.05h	0.40h	0.25h	0.25h	0.30h
2200	880	1100	110	110	880	550	550	660

N.B: all dimensions are in mm.

Determination of the centroid of the precast girder

$A_p \times \bar{y}_p = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5$

$$\bar{y}_p = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5}{A_P}$$

Table 7-12 Second trial, determination of the centroid of precast girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i \times \bar{y}_i$ (mm^3)
A1	110	880	880	96,800	2,145	207,636,000
A2	110	880	660	84,700	2,038	172,586,333
A3	880	660	660	580,800	1,540	894,432,000
A4	550	660	1,100	484,000	802	388,208,333
A5	550	1,100	1,100	605,000	275	166,375,000
			∑=	1,851,300	∑	1,829,237,667
			$\bar{y}$	988mm		

Determination of the second moment of inertia of the precast girder

Table 7-13 Determination of the second moment of inertia of the precast girder

Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}_i$ $(mm)^4$	Area A_i (mm^2)	d_i (mm)	d_i^2 $(mm)^2$	$A_i \times di^2$ $(mm)^4$
A1	110	880	880	97,606,667	96,800	1,157	1,338,457	129,562,679,599
A2	110	880	660	84,824,841	84,700	1,050	1,101,526	93,299,282,594
A3	880	660	660	37,480,960,000	580,800	1,540	2,371,600	1,377,425,280,000
A4	550	660	1,100	11,946,649,306	484,000	186	34,596	16,744,365,935
A5	550	1,100	1,100	15,251,041,667	605,000	713	508,487	307,634,673,417
			∑	64,861,082,480	1,851,300		∑	1,924,666,281,544
$\bar{I}_P$		1,989,527,364,025	mm⁴					

A_{cp} = 1,851,300 mm^2

Section modulus of precast structure

$$Z_{bp} = \frac{\bar{I}p}{\bar{y}} = \frac{1{,}989{,}527{,}364{,}025mm4}{988mm} = 2{,}013{,}691{,}664mm^3 = 2.01 \times 10^9\ mm^3$$

$$Z_{tp} = \frac{\bar{I}p}{(h-\bar{y})} = \frac{1{,}989{,}527{,}364{,}025\ mm^4}{(2200-988)\ mm} = 1{,}641{,}524{,}228mm^3 = 1.64 \times 10^9\ mm^3$$

Determination of the centroid of the composite girder

$$A_c \times \bar{y}_c = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7$$

$$\bar{y}_c = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7}{A_c}$$

Table 7-14 Determination of the centroid of the composite girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i \times \bar{y}_i$ (mm^3)
A1	110	880	880	96800	2145.00	207636000
A2	110	880	660	84700	2037.62	172586333.3
A3	880	660	660	580800	1540.00	894432000
A4	550	660	1100	484000	802.08	388208333.3
A5	550	1100	1100	605000	275.00	166375000
A6	300	3000	3000	900000	2330.00	2097000000
A7	20	880	880	17600	2190.00	38544000
∑=				2,733,700	∑	3887693667
			ȳ	1422.14 mm		

Determination of the second moment of inertia of the composite structure

The determination of second moment of inertia of the composite structure is shown in the table below.

Table 7-15 Determination of the second moment of inertia of the composite girder

Determination of the second moment of inertia of the composite girder								
Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}i$	Area Ai (mm^2)	di (mm)	di^2 (mm)	Ai × di^2 $(mm)^4$
A1	110	880	880	97,606,667	96,800	723	522,532	50,581,107,939
A2	110	880	660	84,824,841	84,700	615	378,819	32,085,983,208
A3	880	660	660	37,480,960,000	580,800	118	13,892	8,068,404,372
A4	550	660	1,100	11,946,649,306	484,000	620	384,466	186,081,315,919
A5	550	1,100	1,100	15,251,041,667	605,000	1,147	1,315,921	796,132,451,990
A6	300	3,000	3,000	6,750,000,000	900,000	908	824,217	741,795,049,239
A7	20	880	880	586,667	17,600	768	589,615	10,377,221,376
∑ =				71,610,495,813	2,733,700	∑		1,804,367,091,292
$\bar{I}_c$	1,875,977,587,106			mm^4				

Section modulus of composite structure

$$Z_{bc} = \frac{\bar{I}c}{\bar{y}_c} = \frac{1{,}875{,}977{,}587{,}106 mm^4}{1422 mm} = 1{,}319{,}252{,}874 mm^3 = 1.32 \times 10^9 mm^3$$

$$Z_{tc} = \frac{\bar{I}c}{(h - \bar{y}_c)} = \frac{1{,}875{,}977{,}587{,}106 \ mm^4}{(2480 - 1422) \ mm} = 1{,}773{,}135{,}716 mm^3 = 1.77 \times 10^9 mm^3$$

Appendix-D

Arithmetic calculation of area of pre-cast, area of composite structure, centroid of precast, centroid of composite structure, second moment of inertia of precast, second moment of inertia of composite structure, section modulus of precast and section modulus of composite structure in slab case (40m span) as follows:

As discussed before in chapter 2, the depth of precast is the same for both ballasted track and slab track and which is h = 2,200mm.

Section properties of precast

Table 7-16 Dimension of precast

h	b_t	b_b	h_1	h_2	h_3	h_4	h_5	b_w (mm)
100%h	0.40h	0.50h	0.05h	0.05h	0.40h	0.25h	0.25h	0.30h
2200	880	1100	110	110	880	550	550	660

N.B: all dimensions are in mm.

Determination of the centroid of the precast girder

$A_p \times \bar{y}_p = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5$

$$\bar{y}_p = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5}{A_P}$$

Determination of the centroid of the precast girder

Table 7-17 Determination of the centroid of the precast girder

Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i \times \bar{y}_i$ (mm^2)
A1	110	880	880	96,800	2,145	207,636,000
A2	110	880	660	84,700	2,038	172,586,333
A3	880	660	660	580,800	1,540	894,432,000
A4	550	660	1,100	484,000	802	388,208,333
A5	550	1,100	1,100	605,000	275	166,375,000
∑=				1,851,300	∑	1,829,237,667
$\bar{y}$				988	mm	

N.B: Area of trapezium and rectangle solved by the following formulae.

$A_T = \frac{1}{2} \times (a + b) \times h$ *and* $A_R = b \times h$

Determination of the second moment of inertia of the precast girder

Table 7-18 Determination of the second moment of inertia of the precast girder

Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}_i$ $(mm)^4$	Area A_i (mm^2)	d_i (mm)	d_i^2 $(mm)^2$	$A_i \times di^2$ $(mm)^4$
A1	110	880	880	97,606,667	96,800	1,157	1,338,457	129,562,679,599
A2	110	880	660	84,824,841	84,700	1,050	1,101,526	93,299,282,594
A3	880	660	660	37,480,960,000	580,800	1,540	2,371,600	1,377,425,280,000
A4	550	660	1,100	11,946,649,306	484,000	186	34,596	16,744,365,935
A5	550	1,100	1,100	15,251,041,667	605,000	713	508,487	307,634,673,417
∑				64,861,082,480	1,851,300	∑		1,924,666,281,544
$\bar{I}_P$		1,989,527,364,025		mm^4				

A_{cp} = 1,851,300mm^2

Section modulus of precast structure

$$Z_{bp} = \frac{\bar{I}p}{\bar{y}} = \frac{1{,}989{,}527{,}364{,}025mm^4}{988mm} = 2.01 \times 10^9\ mm^3$$

$$Z_{tp} = \frac{\bar{I}p}{(h-\bar{y})} = \frac{1{,}989{,}527{,}364{,}025\ mm^4}{(2200-988)\ mm} = 1.64 \times 10^9\ mm^3$$

Determination of the centroid of the composite girder

$A_c \times \bar{y}_c = A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7$

$$\bar{y}_c = \frac{A_1 \times \bar{y}_1 + A_2 \times \bar{y}_2 + A_3 \times \bar{y}_3 + A_4 \times \bar{y}_4 + A_5 \times \bar{y}_5 + A_6 \times \bar{y}_6 - A_7 \times \bar{y}_7}{A_c}$$

Table 7-19 Determination of the centroid of the composite girder

Determination of the centroid of the composite girder						
Area	Height (mm)	base 1 (mm)	base 2 (mm)	Area A_i (mm^2)	$\bar{y}_i$ (mm)	$A_i \times \bar{y}_i$ (mm^3)
A1	110	880	880	96800	2145.00	207636000
A2	110	880	660	84700	2037.62	172586333.3
A3	880	660	660	580800	1540.00	894432000
A4	550	660	1100	484000	802.08	388208333.3
A5	550	1100	1100	605000	275.00	166375000
A6	300	3000	3000	900000	2330.00	2097000000
A7	20	880	880	17600	2190.00	38544000
∑=				2,733,700	∑	3887693667
ȳ				1422.14	mm	

Determination of the second moment of inertia of the composite structure

The determination of second moment of inertia of the composite structure is shown in the table below.

Table 7-20 Determination of the second moment of inertia of the composite structure

Determination of the second moment of inertia of the composite girder								
Area	height (mm)	base 1 (mm)	base 2 (mm)	$\bar{I}i$	Area Ai (mm^2)	di (mm)	di^2 $(mm)^2$	$Ai \times di^2$ $(mm)^4$
A1	110	880	880	97,606,667	96,800	723	522,532	50,581,107,939
A2	110	880	660	84,824,841	84,700	615	378,819	32,085,983,208
A3	880	660	660	37,480,960,000	580,800	118	13,892	8,068,404,372
A4	550	660	1,100	11,946,649,306	484,000	620	384,466	186,081,315,919
A5	550	1,100	1,100	15,251,041,667	605,000	1,147	1,315,921	796,132,451,990
A6	300	3,000	3,000	6,750,000,000	900,000	908	824,217	741,795,049,239
A7	20	880	880	586,667	17,600	768	589,615	10,377,221,376
$\sum$ =				71,610,495,813	2,733,700	$\sum$		1,804,367,091,292
$\bar{I}_c$		1,875,977,587,106		mm^4				

Section modulus of composite structure

$$Z_{bc} = \frac{\bar{I}c}{\bar{y}_c} = \frac{1{,}875{,}977{,}587{,}106 mm^4}{1422.14 mm} = 1.32 \times 10^9 \ mm^3$$

$$Z_{tc} = \frac{\bar{I}c}{(h-\bar{y}_c)} = \frac{1{,}875{,}977{,}587{,}106 \ mm^4}{(2480-1422.14) \ mm} = 1.77 \times 10^9 \ mm^3$$