



ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

Assessment of Shear Strength of Reinforced Concrete Beam-Column Joints

A thesis submitted to the school of School of Civil and Environmental Engineering in Partial
Fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering

By

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Advisor Dr. Abrham Gebre

(December 2017)



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ABSTRACT

Even though beam-column joint is one of the most critical regions in reinforced concrete frames, a little attention is paid to the design of beam-column joints in Ethiopian practice. In this study shear strength of beam-column joints RC building frames designed to old EBCS is investigated. Shear strength of selected beam- column joints of a G+5 building has been assessed. The building is analyzed for seismic actions and the shear strength of the beam-column joint is estimated using both empirical expressions and analytically by using VecTor2 finite element computer program. Joints in the fourth story are selected to be assessed under monotonic loading for a constant column axial load. The study showed that there is a significant variation between result from analysis in VecTor2 and provisions in EBCSEN1998-1 (2013). From the investigation on the interior joints, the interior joint satisfies both ductility and shear strength requirement after column jacketing. The maximum shear stress occurred at a story drift angle of 1.01% before column jacketing and it occurred at a story drift angle of 4.63% after column jacketing. This study is not enough in exposing the problems regarding design and detailing of beam-column joints in Ethiopian practice. Laboratory investigation is highly recommended due to its accuracy compared to software simulations.

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SYMBOLS

A_g	Gross cross sectional area of a column
A_{jh}	Effective joint area
A_{s1}	Area of the beam tensile reinforcement
A_{s2}	Area of the beam compressive reinforcement
A_{sjh}	Total area of horizontal shear reinforcement
A_{sjv}	Total area of vertical shear reinforcement
A_{sw}	Cross-sectional area of the shear reinforcement
C_c'	Compressive force in the concrete
C_s'	Compressive force in top longitudinal reinforcing bars in beam
C_c	Compressive force in the concrete in bottom fiber
C_s	Compressive force in bottom longitudinal reinforcing bars in beam
E_s	Modulus of elasticity of steel
L	Length
M_{Rb}	Moment resistances of the beams
M_{Rc}	Moment resistances of columns
N	Column axial load
P_{tot}	Total gravity load at and above the story considered in the seismic design
S	Spacing of the stirrups
T	Tensile forces in top reinforcing bars in beam passing through the connection
T'	Tensile forces in bottom reinforcing bars in beam
T_I	Fundamental period of the building in the horizontal direction of interest
T_C	Corner period at the upper limit of the constant acceleration region of the elastic spectrum
V_c	Column shear force
V_{ch}	Contribution of strut mechanism on joint shear strength
V_j	Joint shear force

V_j	Joint shear demand
V_{jh}	Horizontal joint shear strength
V_{sh}	Contribution of truss mechanism on joint shear strength
V_{tot}	Total seismic story shear
$V_{Rd,c}$	Design shear resistance of the member without shear reinforcement
$V_{Rd,s}$	Design value of shear force sustained by the yielding of shear reinforcement
b_b	Width of beam cross section
b_c	Width of a column parallel to the width of b_w a beam framing into the column
b_c	Width of column cross section
b_j	Effective width of beam-column joint
b_o	Minimum dimension of the concrete core
b_w	Width of a web of a beam
c	Depth of the flexural compression zone of the elastic column
d	Effective depth of RC section
d_{bw}	Diameter of the hoops
d_{bL}	Minimum diameter of the longitudinal bars
dr	Design inter story drift
d_{sb}	Average diameter of beam tensile reinforcement
f_{cd}	Design compressive strength of concrete
f_{ck}	Cylinder compressive strength of concrete
f_n	Cyclic frequency
f_{cta}	Design axial tensile strength of concrete
f_y	Yield strength of reinforcement
f_{yd}	Design yield strength of reinforcement
f_{jhy}	Yield strength of horizontal shear reinforcement
f_{jvy}	Yield strength of and vertical shear reinforcement
f_{ywd}	Design yield strength of the shear reinforcement
h	Inter story height

h	Overall depth of a cross-section
h_b	Height of beam cross section
h_c	Height of column cross section
k_w	Factor reflecting the prevailing failure mode in structural systems with walls
n_b	Maximum number of the top and the bottom beam bars
q	Behavior factor
q_0	The basic value of the behavior factor
s	Spacing of the hoops
z	Inner lever arm of the bending moment in the element under consideration
Δ	Deflection
σ_c	Concrete normal stress
ε_c	Compressive strain in the concrete
ε_{cm}	Compressive strain in the concrete at the peak stress
ε_{cu}	Ultimate compressive strain in the concrete
ε_s	Tensile strain in the steel
ν_d	Normalized axial force in the column above the joint
γ_I	Importance factor
θ	The inter story drift sensitivity coefficient
λ_0	Overstrength factor
ν_1	Strength reduction factor for concrete cracked in shear
α_{cw}	Coefficient taking account of the state of the stress in the compression chord
ω_n	Circular frequency
χ_b	Beam bar index

ABBREVIATIONS

ACI	American Concrete Institute
ASCE	American Society of Civil Engineers
DCH	Ductility Class High
DCL	Ductility Class Low
DCM	Ductility Class Medium
DSFM	Disturbed Stress Field Model
EBCS	Ethiopian Building Code Standard
ELF	Equivalent Lateral Force
EN	European Standard
FEM	Finite Element Method
MCFT	Modified Compression Field Theory
NA	Neutral Axis
NLFEA	Nonlinear Finite Element Analysis
RC	Reinforced Concrete

1. INTRODUCTION

1.1. Background

A beam-column joint is a portion of the column within the depth of a beam that frames into the column. Beam-column joint should be designed and detailed properly as it is an important component of a reinforced moment resisting frame especially when the frame is subjected to seismic forces. As soon as lateral loads i.e.; seismic forces, comes into picture it will become a critical problem. An attempt to tackle this problem has to be made in our design practice. In normal design practice for gravity loads, the design check for joints is not critical and hence not warranted. But, the failure of reinforced concrete frames during many earthquakes has demonstrated heavy distress due to shear in the joints that culminated in the collapse of the structure. In Ethiopia large building failures due to earthquake is not witnessed currently in big cities. But this shall not hold engineers back from designing beam-column joints for seismic forces as there have been some earth quake occurrences in some towns.

In reinforced concrete moment resisting frame structures, the functional requirement of a joint, which is the zone of intersection of beams and columns, is to enable the adjoining members to develop and sustain their ultimate capacity. The demand on this finite size element is always severe and more complex due to the possible two-way actions in three-dimensional frame structures. However, the codes consider one direction of loading at a time and arrive at the design parameters for the joint.

Inadequate performance of some structures could be attributed to shear failure of beam-column joints. So it becomes apparent to assess the shear resistance of beam-column joints in multistory reinforced concrete frames subjected to a certain amount of ground motion. To utilize the energy dissipating capacity of structural members, the joint connecting the beams and columns must function without brittle failure taking place and without excessive loss of stiffness.

As a consequence of seismic moments in columns of opposite signs immediately above and below the joint, and similar beam moment reversal across the joint, the joint region is subjected to horizontal and vertical shear forces whose magnitude is typically many times higher than in the adjacent beams and columns. If not designed for, joint shear failure can result. The reversal in moment across the joint also means that the beam reinforcement is required to be in compression on one side of the joint and at tensile yield on the other side of the joint. (Paulay and Priestley 1992) The basic requirement of design is that the joint must be stronger than the adjoining hinging members, usually the beams or columns.

Shear deformation of beam-column joints have a significant contribution to reduction of lateral stiffness of reinforced concrete structures. Most of the available analysis programs assume rigid joints regardless of reinforcement details in the joint region. Post-earthquake inspections of damaged reinforced concrete buildings as well as laboratory tests of non-seismic RC beam-column joints were demonstrated that non-seismically detailed beam-column joints suffered serious damage and significantly affect the global structural performance and cause to structural collapse (Jalil Shafaei 2014). In RC moment-resisting frames, assessment of shear strength of beam-column joints detailed according to old codes needs to be carried out. The assessment will be reasonably useful as the shear the behavior of beam-column joints affects overall earthquake resistance of the system.

Ethiopian practice towards design and construction of reinforced concrete beam-column joints may lead to weak beam-column joints. The joints may be efficient in supporting gravity loads. But there is no guarantee that seismic forces due to large magnitude of earthquake could not happen. It may be experienced that many buildings in the past years were detailed even without transverse reinforcement in the beam-column joint region. Due to these reasons assessing the shear strength of beam- column joints designed to the old EBCS is important.

EBCS-8, 1995 gives detailing provisions within beam-column joints to enhance the strength of the system. Of course it needs some research to answer how many buildings are designed and detailed according to the provisions in the old code. Whatever it is assessment of the shear strength of beam-column joints of frames designed to old EBCS is essential to ensure safety of existing RC buildings.

1.2. Statement of the Problem

Even though beam-column joint is one of the most critical regions in reinforced concrete frames, a little attention is paid to the design of beam-column joints in Ethiopian practice. Old EBCS has provisions for detailing of reinforced concrete beam-column joints though it is not exactly known how many buildings were designed accordingly. As knowledge of lateral loads especially earth quake loads increased the new EBCS EN has wider provisions regarding earth quakes and structures including joint mechanisms. So shear forces coming to RC beam-column joints analysed according to old EBCS and new EBCS EN become different.

Beam-column joint shear failure causes reduction in stiffness of structural members leading to the overall collapse. In many cases frame beam-column joints are detailed only for gravity loads. Their shear strength when lateral load occurs is not estimated most of the time. It is therefore helpful to assess shear strength of reinforced concrete beam-column joints of RC frames designed to the old EBCS.

1.3. Objective of the Study

General Objective

To assess shear strength of reinforced concrete beam-column joints of RC frames designed to the old EBCS.

Specific Objectives

The specific objectives of this study are to:

- Review seismic analysis of RC building frame designed to old EBCS.
- Promote seismic evaluation of reinforced concrete beam-column joints of existing buildings to new EBCS EN.
- Analytically estimate horizontal shear strength of the beam-column joint in the x-direction
- Compare the results against results from EBCS EN code.

1.4. Scope of the Study

The scope of present research work is limited to following structural considerations:

- I. Material property data and analysis of the building structure is based on the new EBCS EN codes.
- II. Fixity is assumed in all the column ends. Soil structure interaction is neglected.
- III. Only selected beam-column joints in the fourth story are considered in the present study.
- IV. The joints with concentric beams are investigated; the axes of the column and beams are coincident.
- V. The building under investigation is Monolithic RC structure
- VI. Contribution of floor slabs to joint shear strength is not considered.
- VII. Estimation of only horizontal joint shear strength is carried out in this paper.

1.5. Thesis Organization

This thesis consists of six chapters. The study focuses on horizontal shear strength of beam-column joints of an existing RC building designed to old EBCS. Accordingly, inputs that are crucial in estimating shear strength of RC beam-column joints are discussed in each chapter of the study.

The general background is presented in Chapter one. Various literatures regarding shear strength of beam-column joints and design of beam-column joints in old EBCS, i.e, EBCS 8,1995 are reviewed in Chapter two. Moreover shear strength models are discussed under this chapter. The materials and methods used to carry out this study are discussed under Chapter three. Chapter four discusses about the seismic analysis in general and the importance of seismic analysis in this study. The main objective of the study is presented in Chapter five of this paper. In this chapter, shear strength of the selected joints are estimated based on provisions in building codes and the shear strength of the joints are tried to be estimated using VecTor2. In last chapter, Chapter six conclusions and recommendations are presented.

2. LITERATURE REVIEW

Designers have to carefully consider the shear strength and the ductility performance of beam–column connections under lateral loads. Various literatures are reviewed in this chapter concerning the shear strength modeling of RC beam-column joints. Several existing shear strength models are categorized with respect to the type of underlying basic concept. Various researchers tried to develop shear strength models based on experimental test results.

2.1. Types of Beam-Column Joints in RC Frames

The report by Joint ACI-ASCE Committee 352, i.e., Recommendations for Design of Beam-Column Connections in Monolithic Reinforced Concrete Structures, classifies beam-column joints into interior, exterior and corner joints based on their geometrical configuration.

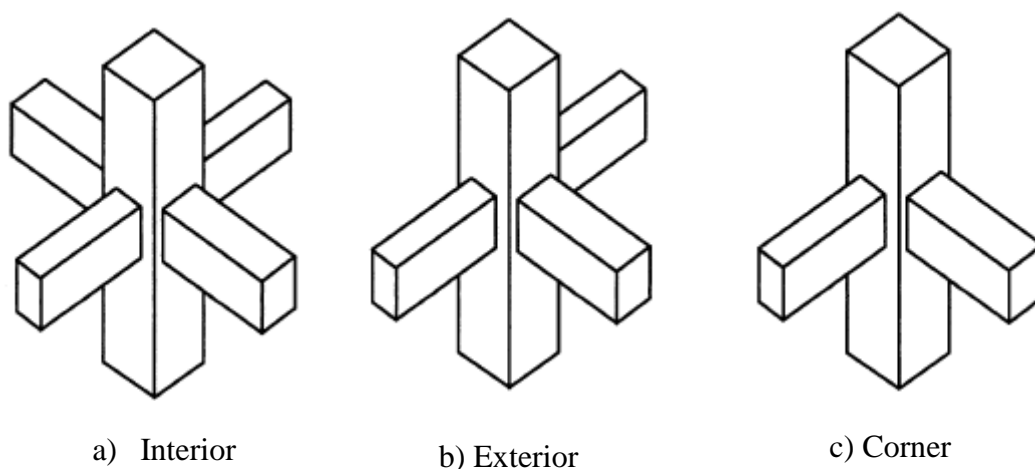


Figure 2.1: Classification of Joints Based on Geometrical Configuration (ACI352R-02 2002)

Paulay and Priestley (1992) classified beam-column joints as **elastic** and **inelastic joints** based on structural response. RC joints remain elastic when inelastic deformations do not occur or cannot occur in the beams and columns adjacent to a joint. Smaller amounts of joint shear reinforcement are generally sufficient under such circumstances. Joints in which inelastic strains along the reinforcing bars of the beams penetrate in to the joint are classified as inelastic joints. They require larger amounts of joint shear reinforcement.

Another classification of beam-column joints in the Joint ACI-ASCE Committee 352 is based on loading conditions. Accordingly beam-column joints are classified as **Type-1** and **Type-2** joints. Type-1 joints are joints designed and detailed for gravity loads. Type-2 joints are connections that have members that are required to dissipate energy through reversals of deformation into the inelastic range. These joints designed and detailed for seismic loads.

2.2. Design of RC Beam-Column Joints Using EBCS 8, 1995

According to the previous code, the area included between the edges of a beam and a column framing into each other shall be appropriately designed, taking into account that

- a. A damaged joint reduces the amount of energy that can be dissipated by the framing elements and
- b. The repair of damaged joints poses more severe difficulties than those experienced in the repair of structural elements. (EBCS-8 1995)

It is apparent to present the provisions of EBCS 8, 1995 regarding RC beam-column joints to make successive discussions clear.

Horizontal Joint Shear Demand

The determination of horizontal shear force acting around the core of a joint should take into account the most adverse conditions under seismic loading.

EBCS-8, 1995 gives the following simplified expressions for the shear force acting on the concrete core of the joints as:

- (a) For interior beam-column joints:

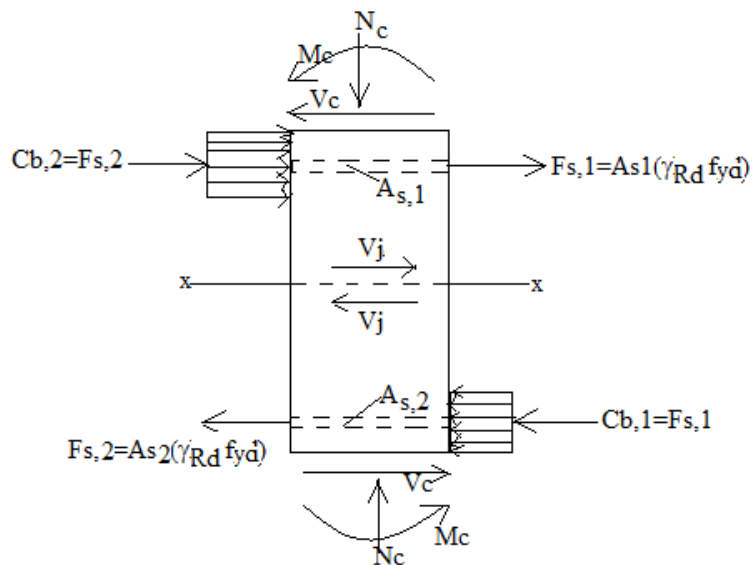
$$V_j = \gamma_{Rd} \left(\frac{2}{3} \right) (A_{s1} + A_{s2}) f_{yd} - V_c \quad 2.1$$

- (b) For exterior beam-column joints:

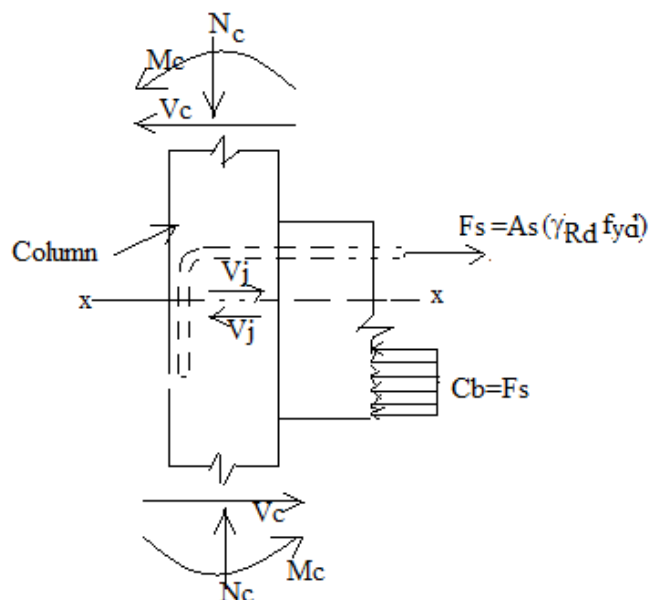
$$V_j = \gamma_{Rd} \left(\frac{2}{3} \right) A_{s1} f_{yd} - V_c \quad 2.2$$

Where;

- A_{s1} and A_{s2} ; beam tension and compression area of reinforcement respectively
- γ_{Rd} ; 1.25 for joints of DC''H'' and 1.15 for joints of DC''M''
- The reduction factor 2/3 accounts for the part of inclined bond forces flowing sideways out of the core of the joint
- V_c ; column shear and should be taken as obtained from the analysis for the combination considered.



(a) Interior beam-column joints



(b) Exterior beam-column joints

Figure 2.2 : Horizontal Shear Forces Acting on Beam-Column Joints

Horizontal Joint Shear Resistance

In absence of a more precise model, the requirement of the above statement may be satisfied by means of the subsequent rules:

(a) For interior beam-column joints:

$$V_j \leq 5.0 f_{ctd} b_j h_c \quad 2.3$$

(b) For exterior beam-column joints:

$$V_j \leq 3.5 f_{ctd} b_j h_c \quad 2.4$$

Where;

V_j ; joint shear demand

f_{ctd} ; design axial tensile strength of concrete

b_j ; effective width of beam-column joint

h_c ; width of column in the direction of a beam framing into the column

Effective width of beam-column joints is taken as the width of the narrower member plus a distance included between lines on a slope of 1:2. The effective joint width b_j may be taken as:

(a) If $b_c > b_w$

$$b_j = \min\{b_c; (b_w + 0.5h_c)\}$$

(b) If $b_c < b_w$

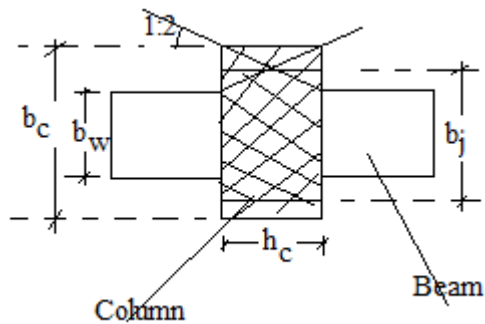
$$b_j = \min\{b_w; (b_c + 0.5h_c)\}$$

Where ;

b_w ; width of a web of a beam

b_c ; width of a column parallel to the width of b_w a beam framing into the column

a) $b_c > b_w$



b) $b_c < b_w$

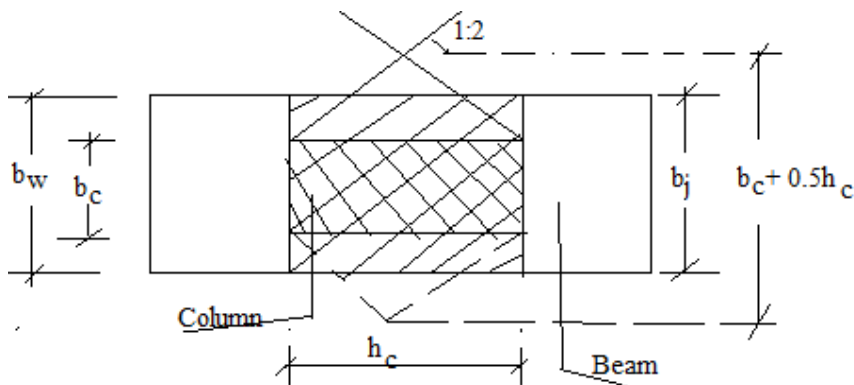


Figure 2.3 : Effective joint width from plan view of beam-column joints

2.3. Joint Response to Lateral Loading

The response of beam-column joints is a significant factor that affects the overall behavior of RC framed structures subjected to lateral loads. This is affected by various parameters in the joint. Some of the parameters affecting joint response to lateral loading are:

i. Concrete Strength

Measured concrete compressive strength, observed maximum joint shear strength as per codes and studies and nominal joint shear stress have influences on shear strengths of RC

beam-column joints. An increase in nominal joint shear strength normalized by the square root of concrete compressive strength of concrete, $\tau_{nom}/\sqrt{f_{ck}}$, results in an increase in the probability of joint shear failure prior to beam yielding. (Mitra 2007)

It is shown in the paper by M. T. Tran (2016) that concrete compressive strength has higher influence on joint shear strength. The joint shear strength was considered as a function of the square root of the concrete compressive strength is highly influenced by concrete compressive strength than the four remaining factors studied in the paper. (M. T. Tran 2016)

ii. Axial Load in the Column

Studies by Beckingsale (1980) and Birss (1978) indicate that column axial load in the column enhance joint shear resistance. The influence of column axial load is usually expressed in terms of column axial load ratio. Expected benefit of axial compression lies in the probability that the bond environment for the beam bars should be improved in joints with heavier axial loads. However, Mitra (2007) pointed out that it has also been concluded that column axial load affects the deformation but not the joint strength.

iii. Reinforcement in the Joint

Both beam top and bottom flexural reinforcements and column flexural reinforcements have major influences on shear resistance of RC beam-column joints. The quantity and strength of the beam flexural reinforcement provides the input shear forces for joint analysis and design. On the other hand the composition of the beam reinforcement may also have some influence on the resistance of the joint to the applied shear.

Horizontal and vertical joint shear reinforcements highly improve shear resistance of joints. Up to a limit it has been found that the greater the amount of transverse shear reinforcement in the joint, the greater the shear strength. Researchers have found, however, that the increase in shear strength was not proportional to the shear reinforcement used. (Beckingsale 1980)

Dowel action has also been identified as one of the shear resisting mechanisms. It may be a major source of shear resistance when all the other sources are exhausted, i.e. when yield of the joint reinforcement and deterioration of the diagonal concrete strut have taken place. (Birss 1978)

iv. Geometric Parameters

Beam and column sizes, effective joint width and eccentricity are the major geometrical parameters affecting joint strength. Both beam and column sizes usually expressed in terms of, joint aspect ratio, defined as the ratio of the height of the beam section, h_b , to the height of the column section, h_c , has influence on joint shear strength. Codes make recommendations for the effective joint width to be used in assessing nominal joint stresses. Problems will arise in joints in which the beam and column center lines do not intersect. Additional stresses arise due to torsional moments because of eccentricity.

v. Bond Stress

The problem of bond deterioration in a beam-column joint undergoing severe reversed loading has been noted by researchers. Referring specifically to interior RC beam-column joints, two failure modes are typically considered. These two modes are joint shear failure and anchorage (or bond) failure. (Brooke and Ingham 2013) In exterior joints a better anchorage for beam reinforcement is possible. So, additional joint strength is obtained from this anchorage. While, in internal joints anchorage from beam bars is hardly obtained.

Bond stress can be influenced by various factors. Paulay & Priestley (1992) pointed out that confinement, bar diameter, compression strength of concrete, clear distance between bars and bar deformation are parameters that influence bond response. It is generally recognized that the maximum bond stress that can be developed between steel reinforcement and concrete in a beam-column joint core is related to the tensile strength of concrete, and hence proportional to the compressive strength of concrete.

vi. Plastic Hinges in Beams

The work by Fenwick and Megget (1993) showed that reinforced beam-column joints suffered stiffness degradation and increasing shear deformation in plastic hinge zones. (Megget, Elongation and Load Deflection Characteristics of Reinforced Concrete Members Containing Plastic Hinges 1993) It is generally desired plastic hinge to form in beams in lateral load resisting systems. The plastic hinges can form at some distance away from the column faces by efficiently reinforcing the beams. This relocation of the plastic hinges will

result in the sections adjacent to the column faces remaining essentially elastic. This condition prevents penetration of yield strain into the joint core. (Beckingsale 1980)

2.4. Shear Strength Models in RC Beam-Column Joints

According to Paulay & Priestley (1992) shear strength of beam-column joints in reinforced concrete frames is based on diagonal concrete strut and truss mechanisms. Internal forces transmitted from adjacent members to the joint result in joint shear forces in both the horizontal and vertical directions (Paulay and Priestley 1992) (T. Paulay 1988) (O.Jirsa 1982). These shear forces lead to diagonal compression and tension stresses in the joint core.

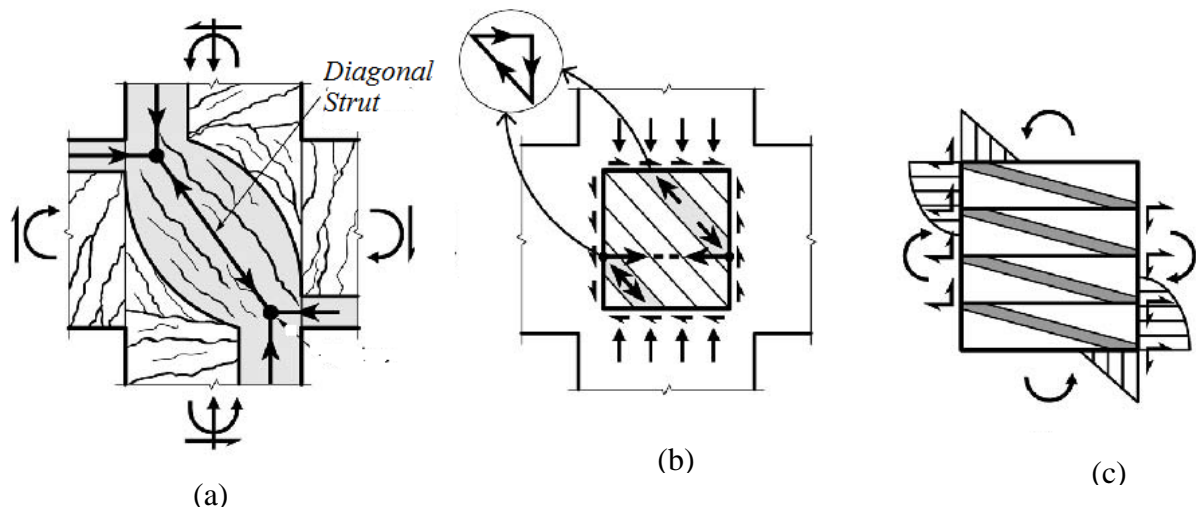


Figure 2.4: (a) Mechanisms of shear at an interior joint (b) Sub-strut mechanism. (c) Truss mechanism. (E. Englekirk 2003)

Basic mechanisms of shear transfer are shown for an interior joint in Figure 2.4 shown above. Internal forces generated in the concrete will combine to develop a diagonal strut. As the principal diagonal strut reaches its strength limit and bond stresses deteriorate, the strain in the concrete that creates the strut increases and load is shed to a truss mechanism which is similar to the sub-strut mechanism described in Figure 2.4(a) and a broader compression field is created (Figure 2.4b). Ultimately this compression field will also break down and diagonal cracking will become severe. Now, a (pure) truss mechanism forms (Figure 2.4c) and the role of the transverse reinforcement becomes dominant.

Forces transmitted to the joint core from beam and column bars by means of bond develop a truss mechanism. Using model shown above the total shear strength of an interior beam column joint can be estimated. The joint shear strength can be derived from the superposition of the two mechanisms, i.e., concrete strut mechanism and truss mechanism. Contributions of each strut mechanism and truss mechanism are estimated separately and superposed. Therefore, from Paulay & Priestley (1992) the horizontal joint shear strength, V_{jh} , is usually proposed to be composed of two components:

$$V_{jh} = V_{ch} + V_{sh} \quad 2.5$$

where, V_{jh} ; horizontal joint shear strength

V_{ch} ; contribution of strut mechanism on joint shear strength

V_{sh} ; contribution of truss mechanism on joint shear strength

In the above equation (2.1) contribution of strut mechanism is related to the concrete strength and contribution of truss mechanism relates to the horizontal and vertical joint reinforcement.

Shiohara (2001) developed a new model for joint shear failure from different observed experimental results. Shear deformation of the beam-column connection increases mainly due to diagonal expansion, despite remaining shear resisting capacity of the connection is reserved. On the contrary, story shear gradually decreases (Shiohara 2001). This new model rules out existing behavior model for beam-column connections, such as the truss mechanism or the strut mechanism stating that they are not useful to account for these facts accurately. This new behavior model for shear failure of the interior beam-column connection is illustrated in Figure. 2.5. In this model, shear deformation in the connection is not uniformly distributed and is due to the rotation of the four triangular concrete segments and the crack opening.

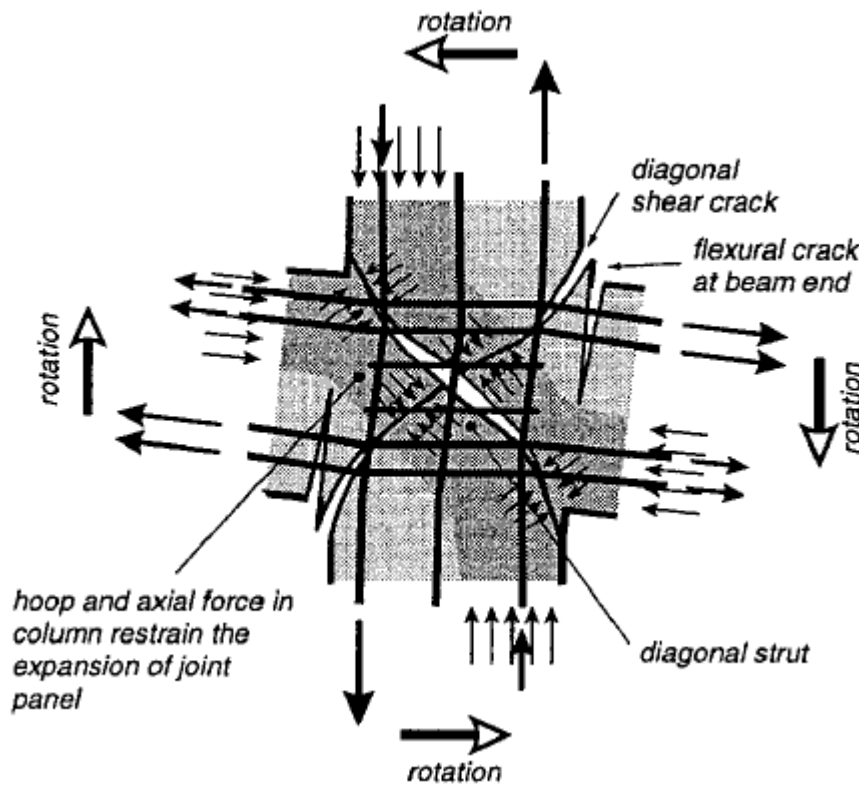


Figure 2.5: Behavior Model for Joint Shear Failure and Force Flow in Beam-Column Connection (Shiohara 2001)

Tran et al.(2014) proposed a new empirical model to estimate the joint shear strength of both exterior and interior beam-column connections. In the model, including parameters that have the most influence on joint shear strength, additional parameters to other models are considered (Tran, Hadi and Pham 2014). New parameter is introduced to consider the bond condition and the possibility of beam bars transferring joint shear force into the columns. In a work by Tran et al.(2014), to calibrate the model, a large database of 98 RC exterior and 73 RC interior beam-column connections displaying joint failure mode was compiled from the literature.

Tran et al.(2014) focused on the bond between beam bars and the concrete at the joint core rather than total cross-sectional area and yield strength of beam reinforcement. Therefore, the new model is proposed in which the bond condition of beam bars is considered via the new parameter, beam bar index. The number of beam bars and their diameters are thus examined instead of their total cross sectional area and yield strength. The contribution of joint shear

reinforcement is also included. In the new shear strength model by Tran et al.(2014), the horizontal shear strength of RC beam-column joints is given by:

$$V_{jh} = \left(\gamma_1 + \frac{N}{b_c h_c f_{ck}} + 1.2 \chi_b \right) A_{jh} (f_{ck})^{0.5} + \gamma_2 (A_{sjh} f_{jhy} + A_{sjv} f_{jvy}) \quad 2.6$$

$$\chi_b = \frac{n_b d_{sb} h_c}{b_b h_b} \leq 0.4 \quad 2.7$$

A_{jh} ; effective joint area which is equal to $b_j h_c$

N ; column axial load

b_c ; width of column cross section

h_c ; height of column cross section

A_{sjh} ; total area of horizontal shear reinforcement

A_{sjv} ; total area of vertical shear reinforcement

f_{jhy} ; yield strength of horizontal shear reinforcement

f_{jvy} ; yield strength of and vertical shear reinforcement

χ_b ; beam bar index

b_b ; width of beam cross section

h_b ; height of beam cross section

d_{sb} ; average diameter of beam tensile reinforcement

n_b ; maximum number of the top and the bottom beam bars

$\gamma_1 = 0.81$ and $\gamma_2 = 0.14$ for interior joints

$\gamma_1 = 0.34$ and $\gamma_2 = 0.22$ for interior joints

A report by joint ASCE – ACI Committee 352 on recommendations for design of beam-column connections in monolithic reinforced concrete structures it is stated that all connections should be designed for the most critical combination that results from the interaction of the multidirectional forces that the members transmit to the joint, including axial load, bending, torsion, and shear. (ACI352R-02 2002) It also states that the connection should resist all forces that may be transferred by adjacent members, using those

combinations that produce the most severe force distribution at the joint, including the effect of any member eccentricity.

ACI352R-02 (2002) ignore the contribution of contribution of truss mechanism, thus the joint shear strength is expressed as a function of concrete compressive strength and the joint geometry. According to ACI352R-02 (2002) the horizontal shear strength of RC beam-column joint is given as:

$$V_{jh} = 0.083\gamma\sqrt{f_{ck}}b_jh_c \quad 2.8$$

Where, b_j ; the effective joint width.

h_c ; depth of the column in the direction of joint shear being considered.

f_{ck} ; cylinder compressive strength of concrete.

Table 2.1: Values of γ for Beam-to-Column Connections (ACI352R-02 2002)

Joint type	Interior	Exterior	Corner
Joints with a continuous column	20	15	12
Joints with a discontinuous column (Joints at roof level)	15	12	8

EBCSEN1998-1 (2013) gives the following expression for estimation of joint shear strength in interior beam-column joints.

$$V_{jh} = \eta f_{cd} \sqrt{1 - \frac{v_d}{\eta}} b_j h_c \quad 2.9$$

$$\eta = 0.6(1 - f_{ck} / 250) \quad 2.10$$

Where,

f_{cd} ; design compressive strength of concrete

v_d ; normalized axial force in the column above the joint

For exterior beam-column joints joint shear strength is taken as 80% of the value given by Equation 2.9.

Zhou (2009) analyzed collecting experimental data of RC joints under cyclic loading collected from the literature. Altoontash (2003) developed a model to represent the response of reinforced-concrete beam-column joints under reversed-cyclic loading using general one-

dimensional hysteretic load-deformation model. Based on this Lowes (2007) developed and evaluated a model for use in simulating the response of reinforced concrete interior beam–column joints using an extensive experimental data set. (Altoontash 2003) (Lowes 2007) (Zhou 2009)

Various joint problems occur in existing buildings designed by old codes. Scholars tried to estimate the shear strength of reinforced concrete beam-column joints of existing buildings both experimentally and analytically. The works by Dinh (2017), C. E. Chalioris *et al.* (2007) and Ciro Del Vecchio *et al.* (2014) showed the performance of RC joints in old buildings. Those joints had various structural deficiencies. Some of the various structural deficiencies observed in those joints;

- a) Inadequate confining effects in the potential plastic regions
- b) Insufficient amount, if any, of transverse reinforcement in the joint regions
- c) Insufficient amount of column longitudinal reinforcement, when considering seismic lateral forces
- d) Inadequate anchorage detailing, for both longitudinal and transverse reinforcement
- e) Lapped splices of column reinforcement just above the floor level
- f) Lower quality of materials (concrete and steel) when compared to current practice

The experimental work to simulate shear strength of RC beam-column joints was performed by preparing prototype structure specimens which are detailed according to the old building codes. (Dinh 2017) (C. E. Chalioris 2007) (Ciro Del Vecchio 2014) Finite Element Modeling is also an important tool in simulating shear strength of beam-column joints. Compared with experimental studies on reinforced concrete joints subjected to cyclic loading under progressive collapse, predictions using Finite Element Modeling are much more economic, convenient and efficient. Non-linear finite element analysis computer programs can efficiently predict the shear strengths of RC beam-column joints. (C. Del Vecchio 2015) (Zhangcheng Pan 2017) (G. Sagbas 2011) (Sagbas 2007)

The structural system consists of frames. All beams have size of 200mmx450mm for 1st – 5th floor beams and 200mmx400mm for ground and top-tie beams. Column size varies in different stories of the building. The floor slab is solid and 150mm thick. Isolated footings represent the foundation system of the building.

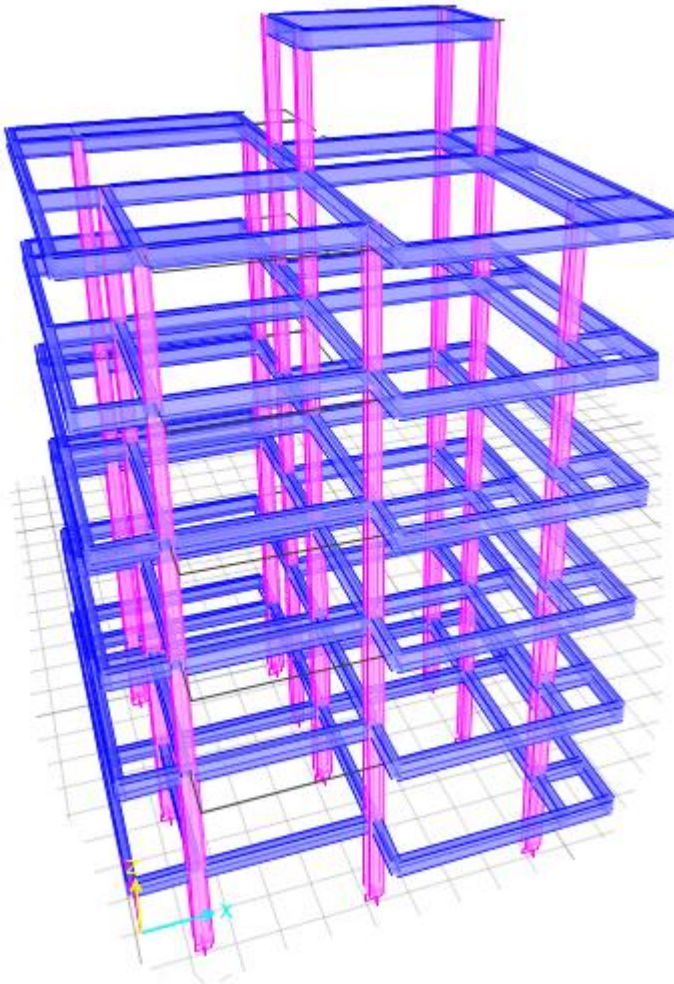


Figure 3.2: Three Dimensional Structural Model of the G+5 Building

The materials used for the construction of the building are specified. Concrete C25 is used for all structural elements. Steel S400 is used for reinforcing bars. Additional properties of these materials used for successive analysis are obtained in the new EBCS EN code.

The structure may be subjected to various actions. Seismic actions, permanent loads and live loads are the major actions are considered in the analysis of the structure in this paper. Wind load is not considered as seismic action governs for Addis Ababa city. The seismic action is

represented by the **elastic response spectrum**, which depends on reference peak ground acceleration, soil type, importance factor, behavior factor, etc. The permanent loads are represented by the self-weight of the structure and additional permanent load.

3.2. Methodology

The primary methods and procedures followed in this paper are literature review and data collection. Data about the multistory Guest House building in Addis Ababa described above is collected from a design office. The building is analyzed for seismic actions using ETABS software. The outputs from ETABS are used to estimate shear demand of beam-column joints. The shear strength of selected beam-column joints is estimated using existing shear strength models in the literature and building codes. The shear strength of the beam-column joint is also estimated using VecTor2 finite element computer program.

4. SEISMIC ANALYSIS OF RC BUILDINGS

4.1. General

Ground motions from earthquakes present unique challenges to the design of structures. The forces that a structure must resist in an earthquake result directly from the distortions caused by motion of the ground that supports it. So, it is important to ensure that large magnitude forces of short duration produced by earthquake must be resisted by a structure without causing collapse and preferably without significant damage to the structural members.

There are many historically recorded earthquake occurrences in Ethiopia. The damages recorded regarding structures may not be severe in the past when there were no high-rise buildings and other huge infrastructures. Recently, expansion in the planning and construction of major building structures is taking place in the capital of Ethiopia, Addis Ababa. The capital is located close to the western edge of the Ethiopian Rift Valley.

Despite only moderate seismicity Addis Ababa is facing seismic vulnerability due to weak infrastructure and lack of preparedness. The presence of high-rise buildings and sky-scrapers in the city and lack of proper seismic design for buildings in the past draws a great attention towards seismic analysis and design of structures.

The damage earthquakes cause to structures is huge. The damaging effect of earthquake ranges from formation of minor cracks to total collapse of structures. Earthquake damage on structures can be attributed to various factors. Damage as a result of problem soils, damage as a result of structural problems and secondary cause of structural damage are the major factors causing damages on structures.

Liquefaction, landslides and weak clay are damage causing factors as a result of problem soils. Foundation failure, foundation connections, soft story failure, torsional moments, shear failure, flexural failure, connection problems and problem structures (e.g. unreinforced masonry structures, tilt-up buildings) are among the major factors causing structural damage as a result of structural problems. Surface faulting and damage caused by nearby structures and lifelines are secondary cause of structural damage. These effects are considered as secondary because they are not always present during an earthquake.

The concern of structural engineers is mainly avoiding the damages caused by structural problems. This can be achieved by performing seismic analysis and design of each structural element. Most of the time design of beam-column joints is overlooked by many engineers. Contrary to this, the most catastrophic type of structural damage is the failure of joints. RC structures must be carefully designed to allow the shear transfer at joints. Sufficient concrete and reinforcement must be provided to resist the large tension and compression forces in moment-resisting joints during earthquakes. Joints of moment-resisting RC frames should be stronger than the elements that join them.

4.2. Seismic Analysis

The general philosophy of earthquake resistant design for buildings is based on the principles that they should:

1. Resist minor earth quakes without damage.
2. Resist moderate earthquakes without structural damage but accepting the probability of no-structural damage.
3. Resist average earthquakes with the probability of structural as well as non-structural damage, but without collapse. (Bryan and Alex 1991)

Seismic analysis of the building under investigation has been helpful even though it can be possible to estimate shear strengths of the beam-column joints of the building without carrying out seismic analysis. Seismic responses are tried to be obtained after carrying out seismic analysis of structure. Seismic analysis will be helpful in obtaining column shear, column axial load, and story drift and shear forces and bending moments around the joints which are important in estimating the shear strength of beam-column joints. Due to these reasons carrying out seismic analysis becomes apparent. The analysis can be performed on the basis of external action, the behavior of structure or structural materials, and the type of structural model selected. The two major issues in seismic analysis are structural modeling and method of analysis.

4.2.1. Structural Modeling

The estimation of seismic demands requires the development of a mathematical model of the building. The model should incorporate all components that influence the mass, stiffness, and strength of the building. Most simple, convenient, realistic and widely used approaches must be used in relevant common assumptions used in structural modeling. Geometric idealizations, loading conditions and soil-structure interactions are issues that need careful considerations in structural modeling.

Soil–structure interaction must be carefully evaluated before a decision is made to include or exclude the soil–foundation system in the final model. It is also important to carefully examine how non-structural components are deliberately and properly separated from structure. The model should also properly account for gravity loads that comprise dead loads and other permanent fixtures and live loads.

A structural model also includes specification of the expected behavior of all of the elements used to develop the building model. A linear elastic analysis requires only the estimation of the effective stiffness of each element, whereas a nonlinear analysis demands a more concerted effort to establish the expected local behavior of every element in the overall structural model.

4.2.2. Methods of Analysis

The primary objective of structural analysis is to determine forces and deformations both at the global and at the local level when the structure is subjected to seismic loads that characterize the hazard at the building site. The estimation of demands can be accomplished using a variety of available methods. Analysis methods can be classified based on the type of external action and behavior of structure. Both EC8 and EBCSEN1998-1, 2013 specify two broad methods of analysis for estimation of seismic design forces. These methods are:

- A. Linear-elastic method
 - a. Lateral force method of analysis (Equivalent Lateral Force method)
 - b. Modal response spectrum analysis

- B. Non-linear method
 - a. Non-linear static (pushover) analysis
 - b. Non-linear time history analysis

Equivalent Lateral Force Method

In the equivalent lateral method, the inertial forces are specified as static forces using empirical formulas. The procedure does not require dynamic analysis, however, it account for the dynamics of building in an approximate manner. This linear static method is the simplest one-it requires less computational efforts and is based on formulate given in the code of practice. ELF uses simple estimate of fundamental period of the structure and the anticipated maximum ground acceleration, or velocity, together with other relevant factors, to determine maximum base shear. Horizontal loading equivalent to this shear is then distributed in some prescribed manner throughout the height of the building to allow a static analysis of the structure.

In EC8 and EBCSEN1998-1, 2013 equivalent lateral method is applied to buildings which satisfy the following two conditions:

- a) They have fundamental periods of vibration, T_1 in the two main directions which are smaller than the following values

$$T_1 \leq \begin{cases} 4.T_c \\ 2.0s \end{cases} \quad 4.1$$

Where T_c is given in Tables 4.1

- b) they meet the criteria for regularity in elevation given in EBCSEN1998-1, 2013

Table 4.1 : Values of T_C from EBCSEN1998-1, 2013

Ground Type	T_C (s)	
	Type 1 elastic response spectra	Type 2 elastic response spectra
A	0.25	0.40
B	0.25	0.50
C	0.25	0.60
D	0.30	0.80
E	0.25	0.50

Modal Response Spectrum Analysis Method

Modal Response spectrum analysis method is the linear dynamic analysis method. In this method the peak response of structure during an earthquake is obtained under linear range. It is used when criteria for using ELF is not satisfied and thus more accurate for structural design applications than ELF.

The Modal Response spectrum analysis utilizes the response spectra to give the structural designer a set of possible lateral forces developed in structure and deformations a real structure would experience under earthquake loads thus facilitates in earthquake-resistant design of structures. The response spectra are plots of values of response quantities as a function of the natural vibration period, T_n of the system, or a related parameter such as circular frequency, ω_n or cyclic frequency, f_n . (Chopra 1995) Modal Analysis is performed in Modal Response spectrum analysis for a MDOF (multi degree of freedom) system with 'n' degree of freedom. Then the required response quantity of interest (displacement, shear force, bending moment etc.) of the structure can be obtained in each mode of vibration. The final maximum response shall be obtained by commonly used modal combinations rules for obtaining the peak response quantity of interest.

Non-Linear Static (Pushover) Analysis Method

Non-linear static (pushover) analysis is carried out under conditions of constant gravity loads and monotonically increasing horizontal loads. It is the method of seismic analysis in which

behavior of the structure is characterized by capacity curve that represents the relation between the base shear force and the displacement of the roof.

Non-Linear Time History Analysis Method

Non-linear time history analysis method is a step-by-step analysis of the non-linear dynamic response of a structure to a specified loading that may vary with time. It is more time consuming and accurate than other methods of analysis.

The seismic analysis of the case study building is carried out by using ETABS2015. The method of analysis used in for the analysis of the building is modal response spectrum method.

Seismic Parameters

Ground type C

Seismic zone Addis Ababa (Zone 2)

Importance factor (γ_I) =1 Importance class II

Behavior factor (q)

$$q = q_o k_w \geq 1.5$$

where,

q_o ; the basic value of the behavior factor, dependent on the type of the structural system and on its regularity in elevation

k_w ; the factor reflecting the prevailing failure mode in structural systems with walls and is 1.0 for frame systems

For Frame system, q_o in DCM is $3.0\alpha_u/\alpha_1$. For multistory, multi-bay frames α_u/α_1 is 1.3.

For buildings which are not regular in elevation, the value of q_o should be reduced by 20% .

4.3. Analysis Output

ETABS2015 gives various analysis outputs after the analysis complete. However all the analysis outputs are not needed in this study. The analysis output presented here are gravity load and seismic responses for frames on axis-2 and-3 because they are frame of interest as the beam-column joints to be investigated are found in this frames. Shear forces and bending moments are presented in the following figures for both gravity and seismic loads. The story drifts and inter story drift sensitivity coefficient are also presented in Tables 4.2 and 4.3.

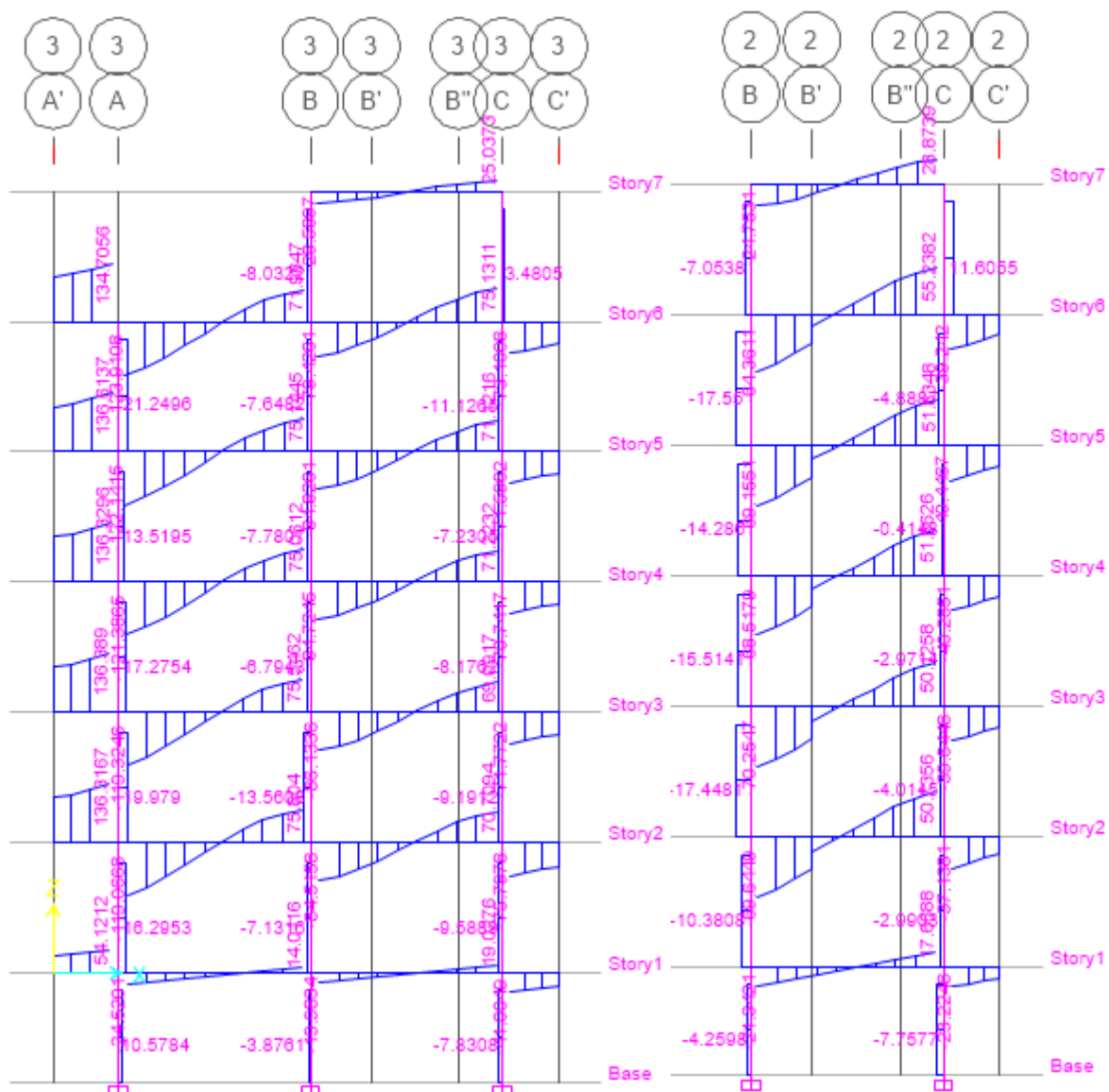


Figure 4.1: Shear Force Diagrams from Gravity Loads for frame on Axis-3 and -2

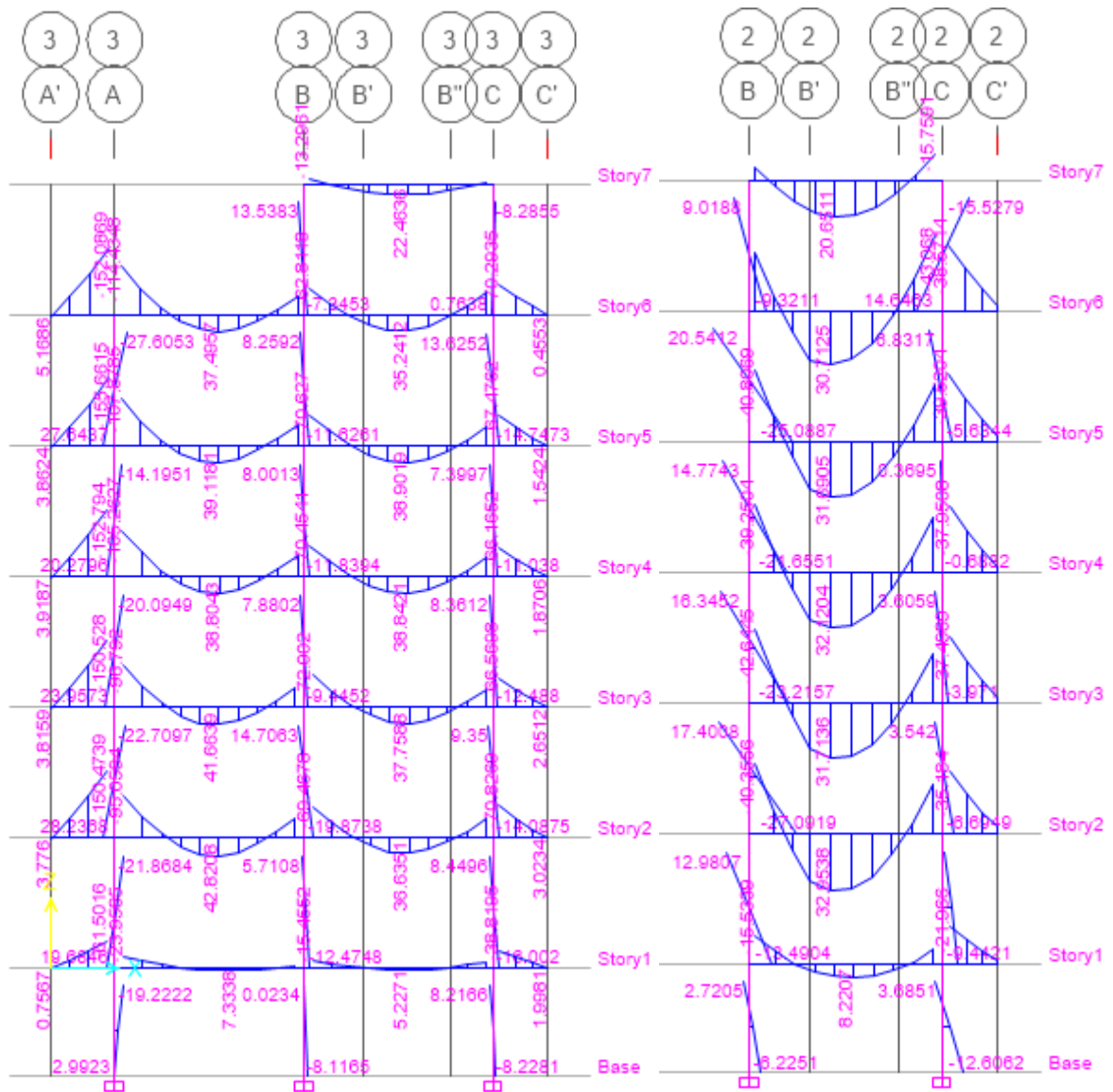


Figure 4.2: Bending Moment Diagrams from Gravity Loads for frame on Axis-3 and -2

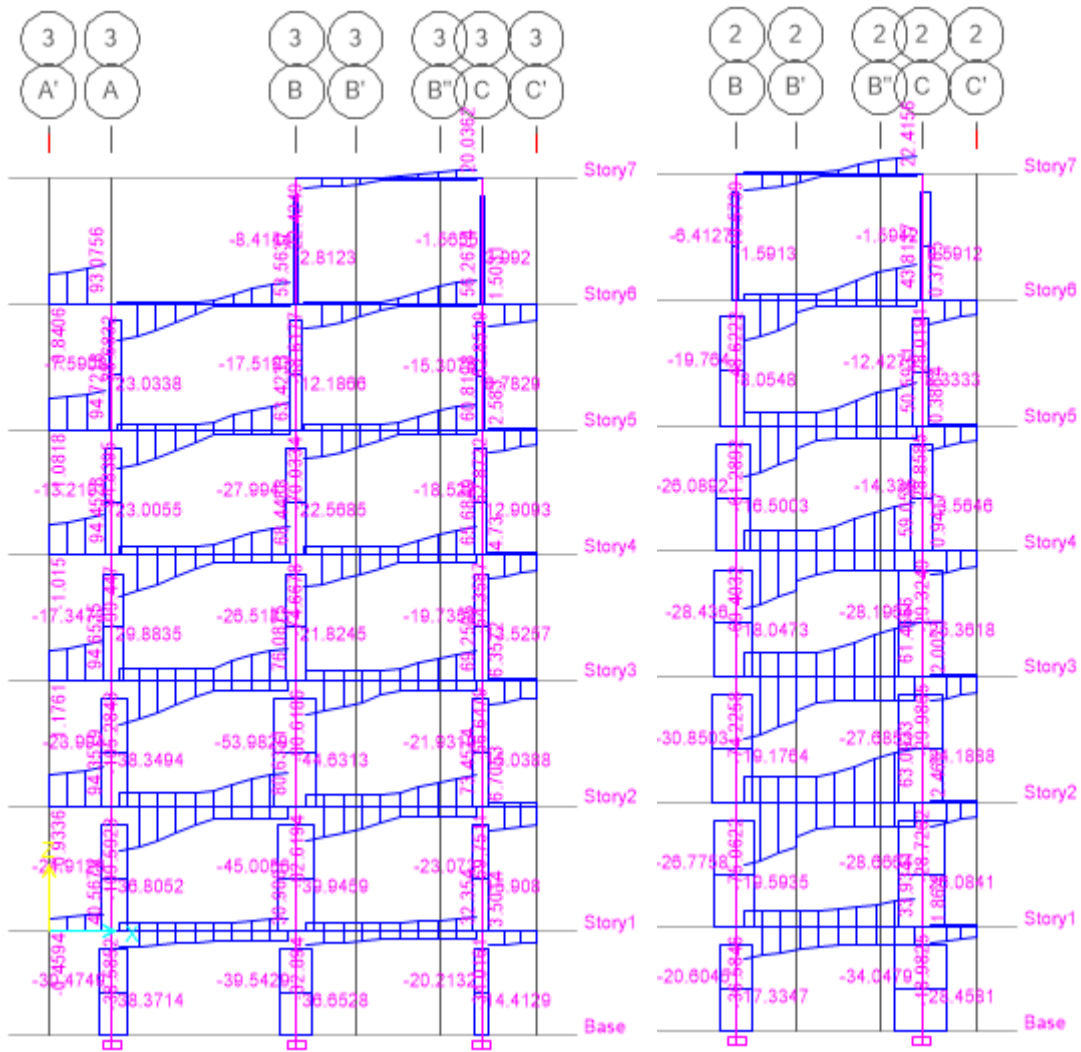


Figure 4.3: Shear Force Envelopes for frame on Axis-3 and -2

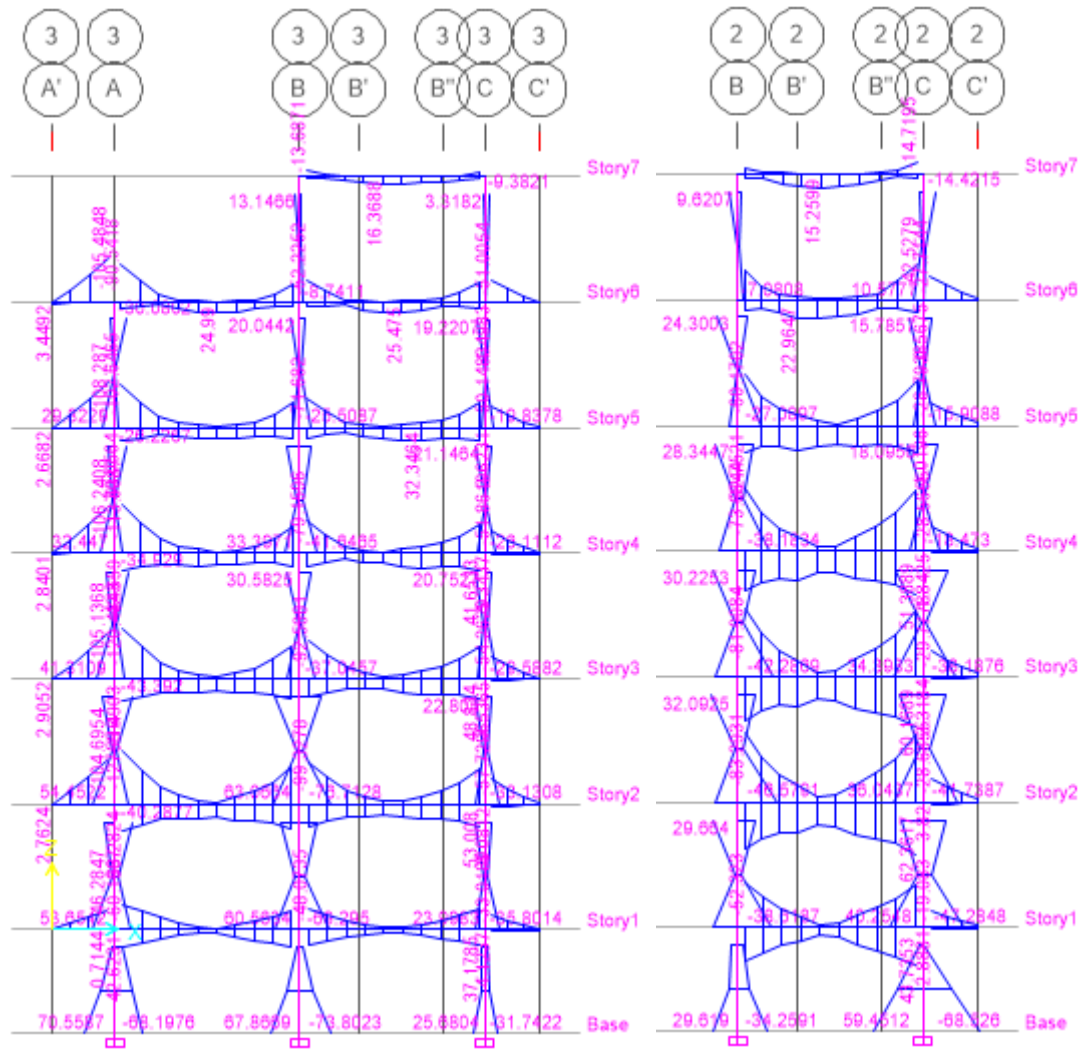


Figure 4.4: Bending Moment Envelopes for Frame on Axis-3 and -2

Story Drift

Table 4.2: Story Drift

Story Level	Story Drift (%)
7 (Roof Level)	0.0396
6 (Fifth Floor)	0.1090
5 (Fourth Floor)	0.1733
4 (Third Floor)	0.1936
3 (Second Floor)	0.2033
2 (First Floor)	0.2110
1 (Ground Floor)	0.1055
0 (Base)	0

Inter story Drift Sensitivity Coefficient

The inter story drift sensitivity coefficient is given as the following expression according to EBCSEN1998-1 (2013). Second order effects are not taken to account if the condition in the expression is fulfilled. It is also a measure of stability of the frames and used to classify columns in a given story as sway column or non-sway column.

$$\theta = \frac{P_{tot} \cdot d_r}{V_{tot} \cdot h} \leq 0.1 \quad 4.2$$

where

θ ; the inter story drift sensitivity coefficient

P_{tot} ; the total gravity load at and above the story considered in the seismic design situation

d_r ; the design inter story drift

V_{tot} ; the total seismic story shear and

h ; the inter story height

Table 4.3: Inter story Drift Sensitivity Coefficient

Story Level	P_{tot} (kN)	d_r (mm)	V_{tot} (KN)	h (m)	$\theta = \frac{P_{tot}d_r}{V_{tot}h}$	Classification
7	161.0296	4.998	6.403	3.0	0.0033	Non-Sway
6	1913.1652	10.771	74.524	3.0	0.0093	Non-Sway
5	3683.1737	17.328	135.609	3.0	0.0157	Non-Sway
4	5467.904	19.853	184.321	3.0	0.0191	Non-Sway
3	7263.578	20.742	220.361	3.0	0.0223	Non-Sway
2	9057.5614	21.018	242.554	3.0	0.0262	Non-Sway
1	9748.3393	10.802	245.334	2.5	0.0167	Non-Sway

5. ASSESSMENT OF THE SHEAR STRENGTH OF RC BEAM-COLUMN JOINT

In RC frames loads and moments are transferred from beams to columns through beam-column joints. The behavior of RC beam column joints is different under gravity and lateral loads. Bending moments in beams due to gravity loading normally have the same sign at opposite faces of their joint with a vertical member. Both bending moments are hogging moments causing tensile stresses to form at top fibers of the beams and compressive stresses at the bottom fibers of the beams. So, under gravity loads transmits the tensions and compressions at the ends of the beams and columns directly through the joint. By contrast, beam bending moments due to seismic loading have opposite sign at opposite faces of the joint. Therefore, seismic shear forces are very high in the joint itself. When the frames sway to the right bending moment at the joint in the left beam is hogging causing top fibers of the beam under tension. But the bending moment at the joint in the right beam is sagging causing top fibers of the beam under compression. When the frames sway to the left bending moment at the joint in the left beam is sagging causing top fibers of the beam under compression. But the bending moment at the joint in the right beam is hogging causing top fibers of the beam under tension. Therefore, joints in a laterally loaded frame require diagonal tensile and compressive forces within the joint. Cracks develop perpendicular to the tension diagonal in the joint and at the faces of the joint where the beams frame into the joint. (Fardis 2009) (Wight and MacGregor 2012)

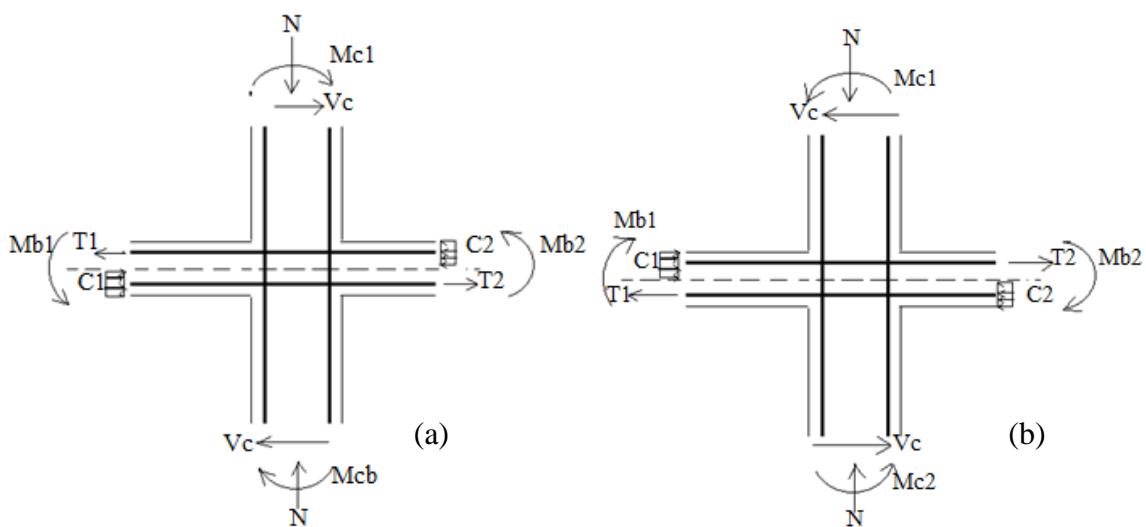


Figure 5.1: Free Body Diagram of Beam-Column Joints (a) Sway to the Right (b) Sway to the Left

Joint behavior is influenced mainly by factors discussed in chapter two of this paper. The factors are mainly from the joint behavior itself and the behavior of adjacent members. Shear input from beam, bond deterioration of beam bars and lack of confinement for joint core concrete lead to joint failure. In beam-column joints it is important to avoid brittle joint failure. This can be ensured by controlling the penetration of beam plastic hinges into the joint core. In other words it is preferable to make joints in the elastic range throughout the inelastic response of the structure. This is affected by behavior of adjacent members. Plastic formation in columns or penetration of plastic hinges in beam-column joints is undesirable in RC frames resisting lateral loads. This is ensured by capacity design rule, i.e., ensuring the requirement of strong column-weak beam. In another way Fardis (2009) pointed out that the weaker of the beam and the column framing into the joint is to deliver shear. (Fardis 2009)

In EBCS EN1998-1 the requirement of strong column-weak beam is ensured by the following expression:

$$\sum M_{Rc} \geq 1.3 \sum M_{Rb} \quad 5.1$$

Where,

$\sum M_{Rc}$; the sum of moment resistances of columns framing into the joint. Column moment resistance corresponding to column axial forces produced by the seismic design situation should be used in expression

$\sum M_{Rb}$; the sum of moment resistances of the beams framing in to the joint.

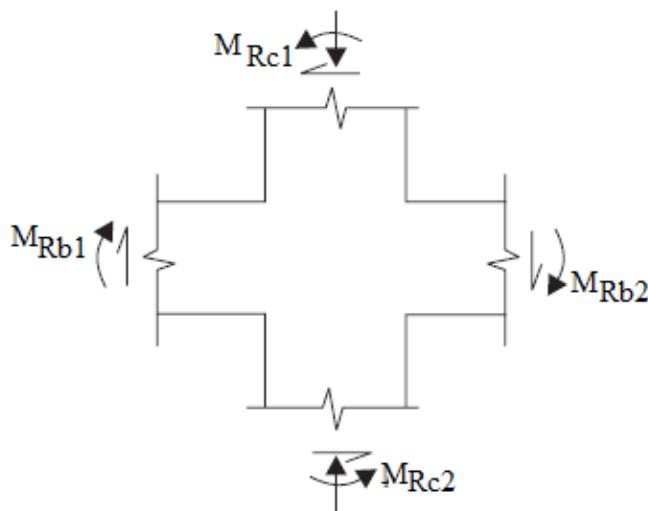


Figure 5.2: Strong Column-Weak Beam Concept in Interior Beam Column Joint.

Both column and beam moment resistances should be calculated in the direction under consideration when assessing shear strength of beam-column joints.

5.1. Shear Strength Modeling

5.1.1. Joint Shear Demand

Many researchers follow similar procedures to calculate the horizontal joint shear demand in RC beam-column joints. Shear forces in the joint are induced from over strengths of adjacent members. From equilibrium criteria in RC beam-column joints, joint shear demand is larger than that of the adjacent members. The method of estimation of joint shear demand adopted in this paper is the method used by many researchers and incorporated in building codes. When estimating shear demands at joints it is always recommended to calculate moment resistance of beams and columns. If the sum of moment resistances of the beams framing into a joint is less than that of the columns the shear input in the joint is governed by the beams. (Fardis 2009)

Interior joints

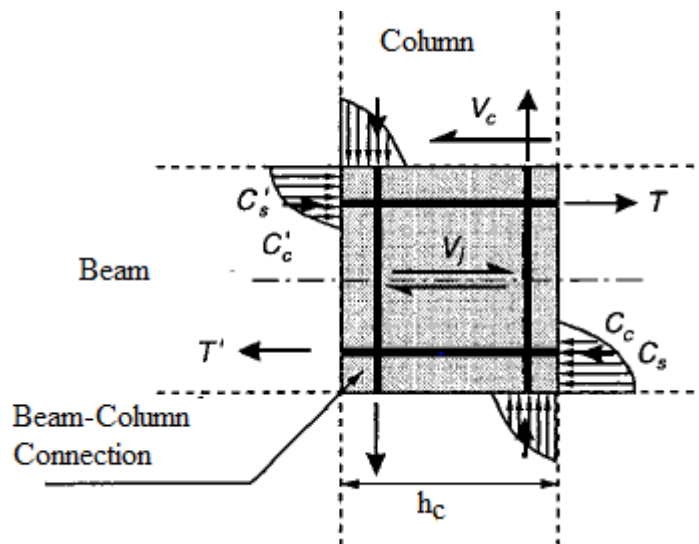


Figure 5.3: Horizontal Joint Shear in Interior RC Beam-Column Joints

The following expression from, Paulay and Priestley (1992), is widely used in the estimation of horizontal joint shear strength of interior beam-column joints.

$$V_j = C_c' + C_s' + T - V_c = C_c + C_s + T - V_c \quad 5.2$$

Approximation, $T' = C_c' + C_s'$, may be made and joint shear can be written as

$$V_j = T + T' - V_c \quad 5.3$$

V_j ; joint shear force

C_c' ; compressive force in the concrete

C_s' ; compressive force in top longitudinal reinforcing bars in beam passing through the connection

C_c ; compressive force in the concrete in bottom fiber

C_s ; compressive force in bottom longitudinal reinforcing bars in beam passing through the connection

T ; tensile forces in top reinforcing bars in beam passing through the connection;

T' ; tensile forces in bottom reinforcing bars in beam passing through the connection;

V_c ; column shear force

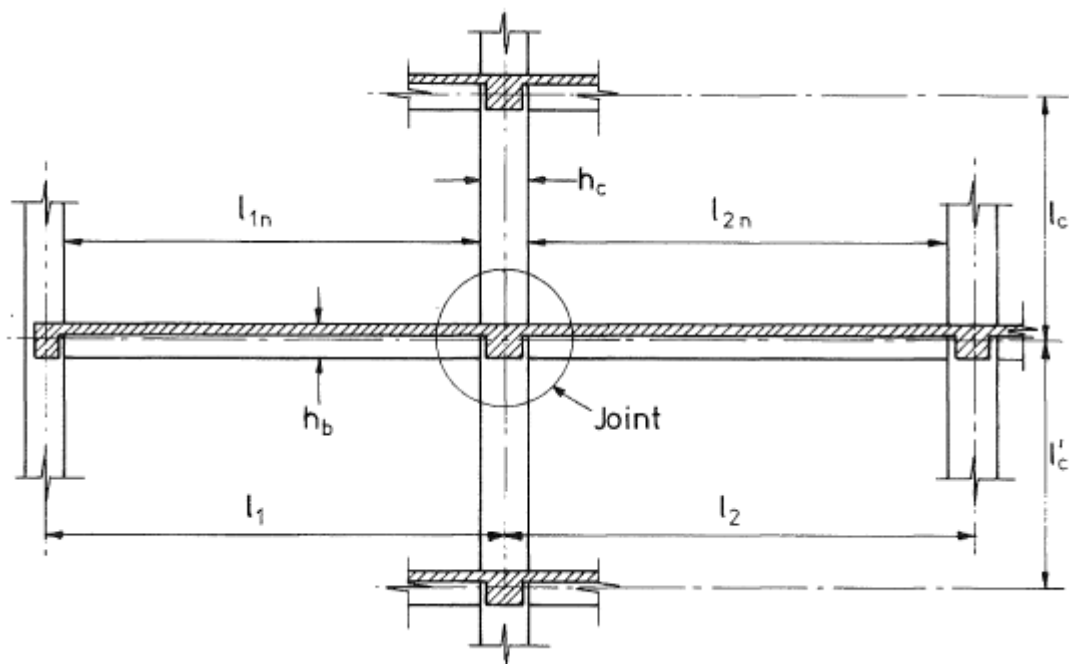


Figure 5.4: Interior Beam-Column Assemblage (Paulay and Priestley 1992)

According to Paulay and Priestley (1992), V_c is average of column shears above and below the joint obtained from beam flexural strength at column faces.

$$V_c = \frac{2 * \left(\frac{l_1}{l_{1n}} M_{Rb1} + \frac{l_2}{l_{2n}} M_{Rb2} \right)}{(l_c + l_c')} \quad 5.4$$

M_{Rb1} and M_{Rb2} are moment resistances of left and right beams of the joint.

The expression given in the old EBCS code, EBCS-8, 1995 for estimating horizontal joint shear demand in interior joints is,

$$V_j = \gamma_{Rd} \left(\frac{2}{3} \right) (A_{s1} + A_{s2}) f_{yd} - V_c \quad 5.5$$

Eurocode8 and EBCS EN1998-1 give the following expression for estimation of horizontal joint shear demand in interior joints.

$$V_j = \gamma_{Rd} (A_{s1} + A_{s2}) f_{yd} - V_c \quad 5.6$$

A_{s1} ; area of the beam tensile reinforcement

A_{s2} ; area of the beam compressive reinforcement

γ_{Rd} ; a factor to account for over-strength due to steel strain-hardening and should be not less than 1.2.

V_c ; column shear and should be taken as obtained from the analysis for the combination considered according to EBCS 8,1995, EC8 and EBCS EN1998-1.

Exterior joints

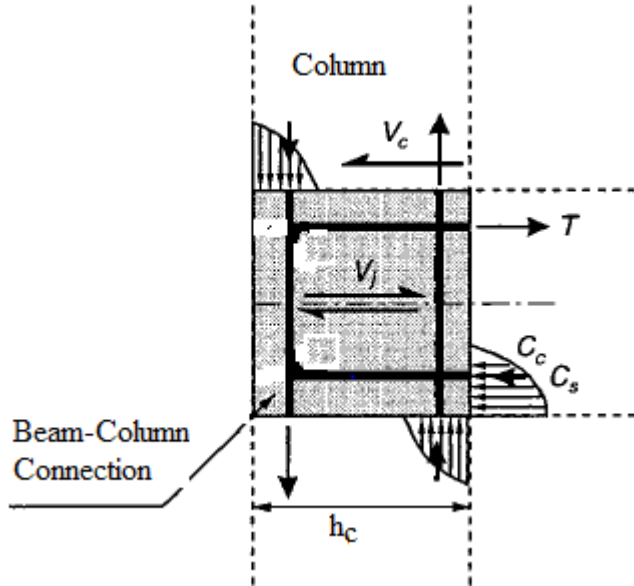


Figure 5.5: Horizontal Joint Shear Exterior RC Beam-Column Joints

The following expression from, Paulay and Priestley (1992), is widely used in the estimation of horizontal joint shear strength of exterior beam-column joints.

$$V_j = T - V_c = C_c + C_s - V_c \quad 5.7$$

Approximation, $= C_c + C_s$, may be made and joint shear can be written as

$$V_j = T - V_c \quad 5.8$$

The expression given in the old EBCS code, EBCS-8, 1995 for estimating horizontal joint shear demand in exterior joints is,

$$V_j = \gamma_{Rd} \left(\frac{2}{3} \right) A_{s1} f_{yd} - V_c \quad 5.9$$

Eurocode8 and EBCS EN1998-1 give the following expression for estimation of horizontal joint shear demand in interior joints.

$$V_j = \gamma_{Rd} A_{s1} f_{yd} - V_c \quad 5.10$$

5.1.2. Joint Shear Strength

Existing joint shear strength models from literatures and codes are considered. The joint shear strength models were discussed in literature review. Codes give empirical expressions limiting the shear capacity of joints.

According to Paulay and Priestley (1992) the horizontal shear resistance of RC beam column joints is given by an expression:

$$V_{jh} = V_{ch} + V_{sh} \quad 5.11$$

In internal joints the contribution of the strut mechanism to horizontal joint shear strength is given by

$$V_{ch} = (1.55 \frac{c}{h_c} + \frac{A_{s2}}{A_{s1}} - 0.55)T - V_c \quad 5.12$$

T ; the maximum tension force applied to the top beam reinforcement and is

$$T = \lambda_0 f_{yd} A_{s1} \quad 5.13$$

λ_0 ; overstrength factor due to strength enhancement of the constituent materials.

c ; depth of the flexural compression zone of the elastic column and can be approximated by

$$c = (0.25 + 0.85 \frac{N}{f_{ck} A_g}) h_c \quad 5.14$$

N ; minimum compression force acting on the column

A_g ; gross cross sectional area of a column

The contribution of truss mechanism to horizontal joint shear strength in internal joints is given by

$$V_{sh} = (1.15 - 1.3 \frac{N}{f_{ck} A_g}) T \quad 5.15$$

In external joints the contribution of the strut mechanism to horizontal joint shear strength is given by

$$V_{ch} = (1 - \frac{A_{s2}}{\lambda_0 A_{s1}} + \frac{1.4c A_{s2}}{\lambda_0 h_c \frac{A_{s1}}{A_{s2}}}) T - V_c \quad 5.16$$

The contribution of truss mechanism to horizontal joint shear strength in external joints is given by

$$V_{sh} = \frac{A_{s2}}{\lambda_0 A_{s1}} (0.7 - \frac{N}{f_{ck} A_g}) T \quad 5.17$$

The shear stress associated with joint shear demand is given by the following expression.

$$v_j = \frac{V_j}{A_j} \quad 5.18$$

$$A_j = b_j h_j \quad 5.19$$

Where; A_j ; effective joint area

b_j ; effective joint width obtained according to Figure 2.3

h_j ; the same as h_c in Figure 2.3

5.2. Numerical (Finite Element) Analysis for Joint Shear

The Finite Element Method (FEM) is a numerical analysis for obtaining approximate solutions to a wide variety of engineering problems. In this paper VecTor2 is used for conducting the finite element analysis of the RC Beam-Column joint.

VecTor2 is one of VecTor suite of programs developed at the University of Toronto. It is a finite element computer program for nonlinear analysis of two-dimensional reinforced concrete structures. The VecTor suite of programs are nonlinear finite element programs which are used analyze different types of structures are applicable to various types of materials including reinforced concrete, structural steel, masonry, and wood. The theoretical basis of VecTor programs is the Modified Compression Field Theory (MCFT) and the Disturbed Stress Field Model (DSFM). (Sagbas 2007) (F. J. Vecchio 2000) (F. Vecchio 2001) (P. Collins 1987)

Formworks-Plus is a universal pre-processor for the entire VecTor software suite. User's manual provides the main steps to be followed in Formworks-Plus to create an analysis job of a reinforced concrete structure to be analyzed using VecTor2. These steps can be summarized in the following list:

1. Starting-up
2. Material Definition
3. Structure Definition
4. Job Definition
5. Load Definition
6. Saving the Input Files and Running VecTor2

As advanced nonlinear finite element analysis (NLFEA) software, the VecTor programs generate a considerable volume of data as a result of their analyses. After the analysis is completed, the user can use the post-processor, Augustus, for a graphical display of the results.

5.2.1. Element Types

Selection of proper element types is an important criterion in Finite Element Analysis. For beam-column joints the concrete portion was modeled by using an element available in the element library of VecTor2, namely four-noded rectangular element. The reinforcement was modeled by using Truss element. The details of both the elements used in the analysis are explained below briefly.

Four-Noded Rectangular Element

Four-noded rectangular element is used for 2-D modeling of solids with or without reinforcing bars (rebar). The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions.

Truss Element

Truss element is a two-noded element and useful in a variety of engineering applications. The element is used to model reinforcement bars in reinforced concrete structures. The element is a uniaxial tension-compression element with two degrees of freedom at each node: translations in the nodal x and y directions.



Figure 5.6 : Rectangular and Truss Element Types Used in VecTor2

5.2.2. Material Models

Material Model for Concrete

Material plays an important role in modeling of reinforced concrete structures using finite element softwares. Correct values of material properties have to be given as input in softwares as much as possible to obtain good results. The challenging task in modeling the Beam -Column joints is the development of the behavior of concrete. Concrete is purely non-linear material and it has different behavior in compression and tension.

In compression, the stress-strain curve of concrete is linearly elastic up to about 40% of the maximum compressive strength. (EBCSEN1992-1-1:2013 2013) Above this point, the stress increases gradually up to the maximum compressive strength, and then descends into a softening region, and eventually crushing failure occurs at an ultimate strain, ϵ_{cu} . In tension, the stress-strain curve for concrete is approximately linearly elastic up to the maximum tensile strength. After this point, the concrete cracks and the strength decreases gradually to zero. Various concrete models are available in Formworks-Plus to choose from. Therefore in analyzing beam-column joints using VecTor2 material model for concrete shall be properly chosen.

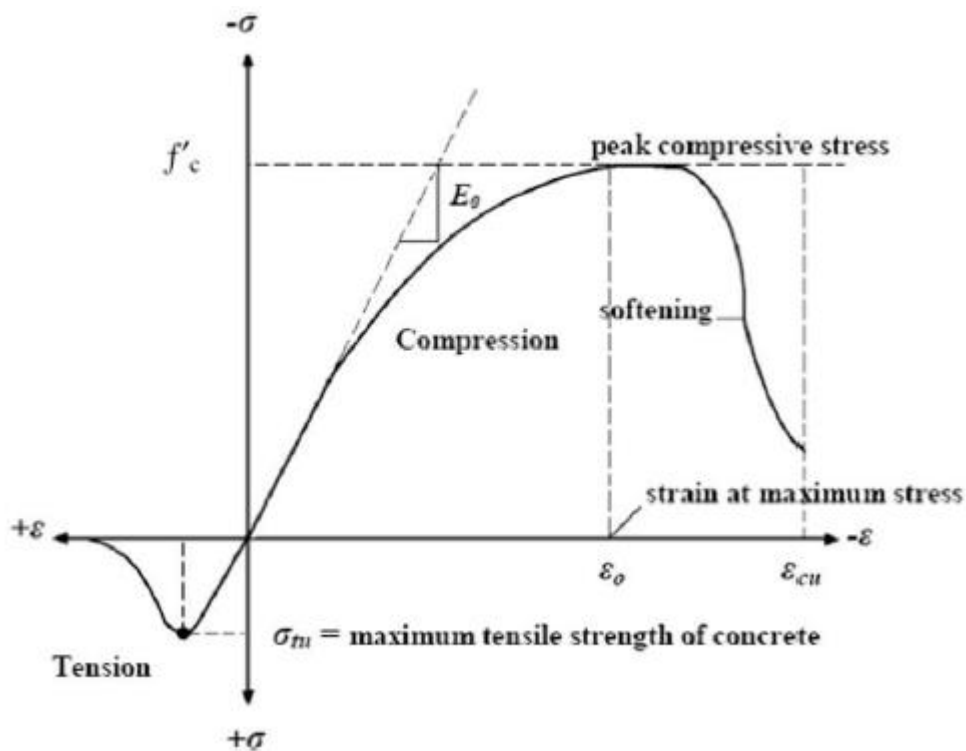


Figure 5.7: Uniaxial Compressive and Tensile Stress–Strain Curve for Concrete
(R.A.Hawileh 2009)

Material Model for Steel Reinforcement bars

Properties like Modulus of elasticity, yield strength and ultimate strength for the steel reinforcement used as an input. Reinforcement materials are only to be used only for truss elements in Formworks-Plus.

5.2.3. Loading and Boundary Conditions

Loading conditions differ in different literatures in the assessment of shear strength of beam-column joints in experimental simulations. The load can be applied at top column or at the tip end of a beam. The load can be applied in the form of displacements and can be monotonic or cyclic. There is no clear rule which loading condition is better to estimate the strength of beam-column joints. The generally accepted modeling and boundary condition of beam-column joints are based on the assumption that the points of infection of the prototype building subjected to lateral forces are at mid-span and mid-story height. Due to this specimens are modeled based on prototype structure with a half scale.

Lateral load is applied at the upper column end in addition to a constant axial load. Displacement boundary conditions are needed to constrain the model to get a unique solution. The lower column end is provided with hinged boundary conditions. Both the beam ends were provided with roller boundary conditions. The upper column end is allowed to move horizontally to simulate story drift. (Hamahara, et al. 2007) (Dinh 2017) (Filippou and Issa 1988) (R.A.Hawileh 2009)

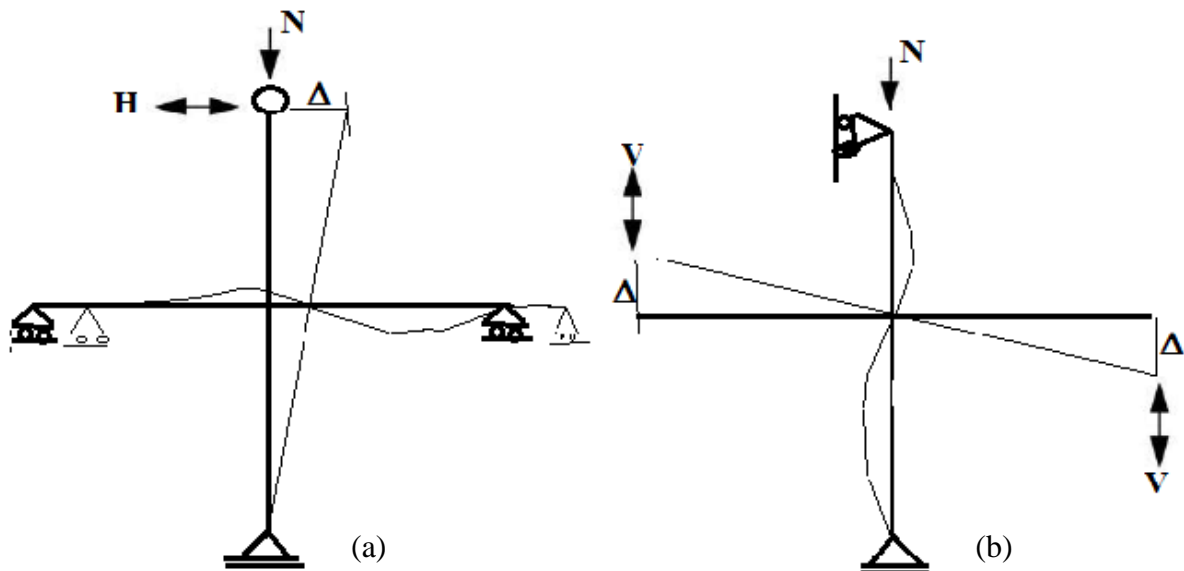


Figure 5.8: Loading and Boundary Conditions

Another loading and boundary condition scenario is when the beam end is subjected to a lateral load. In this case the lower column end is provided with hinged boundary conditions. The upper column end is not allowed to move horizontally. A constant axial load may be

applied at the upper column end. A cyclic or monotonic load is applied at the beam end in the form of displacements. Story drift can be calculated as story drift angle as shown in Figure 5.7(b): (El-Salakawy 2015) (C. E. Chalioris 2007) (R.A.Hawileh 2009) (Tomohiko Kamimura 2000)

$$\text{Story drift angle} = 2\Delta/L \quad 5.20$$

Δ ; the deflection shown in Figure 5.8(b)

L ; the sum of lengths of right and left beams

Which of the loading and boundary conditions are more appropriate to choose is the question that may be raised. In this paper the problem is addressed based on the strengths of the joining members. (Fardis 2009) The weaker of beam and column governs shear input to joint core. The shear input in the joint is governed by the beam if the sum of moment resistances of the beams framing into a joint is less than that of the columns. In another word it is appropriate to apply loads at the beam end in this case. The shear input in the joint is governed by the column if the sum of moment resistances of the columns framing into a joint is less than that of the beams.

5.2.4. Finite Element Modeling of Beam Column Joints

Modeling is one of the important features in Finite Element Analysis. Improper modeling of the structures leads to the unexpected errors in the solution. Hence, proper care should be taken for modeling the structures. A good idealization of the geometry, loading and boundary conditions reduces the running time of the solution considerably. After the application of loads, stresses and strains are calculated at integration points of these small elements.

Finite Element modeling of beam-column joints in Formworks-Plus consist of proper selection of element types, material properties and models, loading and boundary conditions and finally saving all input files and running the VecTor program.

5.2.5. Verification

The verification example carried out to verify the reliability of VecTor2 is the interior beam-column joint tested by Noguchi and Kashiwazaki (1992). The interior joint was one of the many RC interior joints tested and named as Specimen OKJ-2. (Kashiwazaki 1992), (Lee 2014) The beam-column joint is modeled in VecTor2 using the available models in the program. For the specimen OKJ-2, monotonic loading is applied on the ends of two beams with constant column axial load of 756kN.

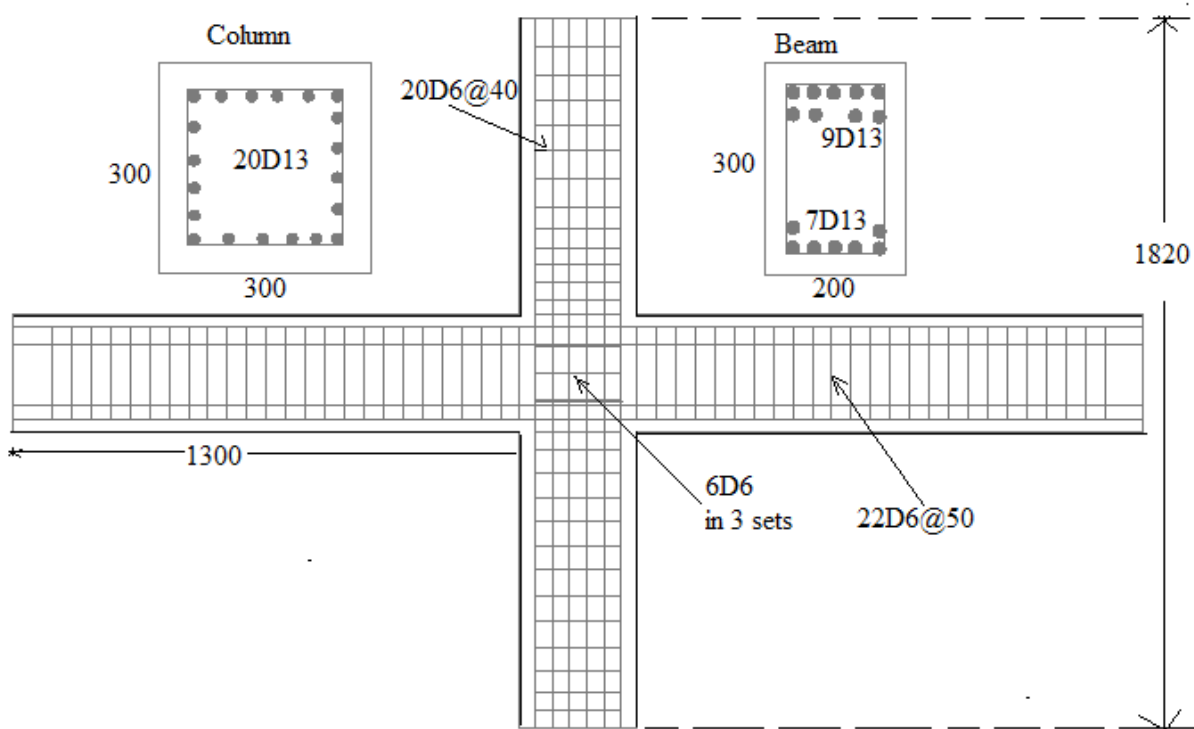


Figure 5.9: Beam-Column Joint Tested by Noguchi and Kashiwazaki (1992).

Concrete compressive strength, f_c' : 70.0MPa

Steel bar yield strength, f_y : 955Mpa for D6 bars and 718MPa for D13 bars

Steel bar ultimate strength, f_u : 1140MPa for D6 bars and 767MPa for D13 bars

The modeling of the interior beam-column joint tested by Noguchi and Kashiwazaki (1992) in VecTor2 is shown in Figure 5.10 below. A constant column axial load of 756kN is applied at column top end and a monotonic load is applied at beam ends. The bottom column end is provided with hinged boundary condition and the top column end is provided with roller

boundary condition. The load-displacement response of the joint shown in Figure 5.11 plotted using story drift (mm) versus story shear force (kN).

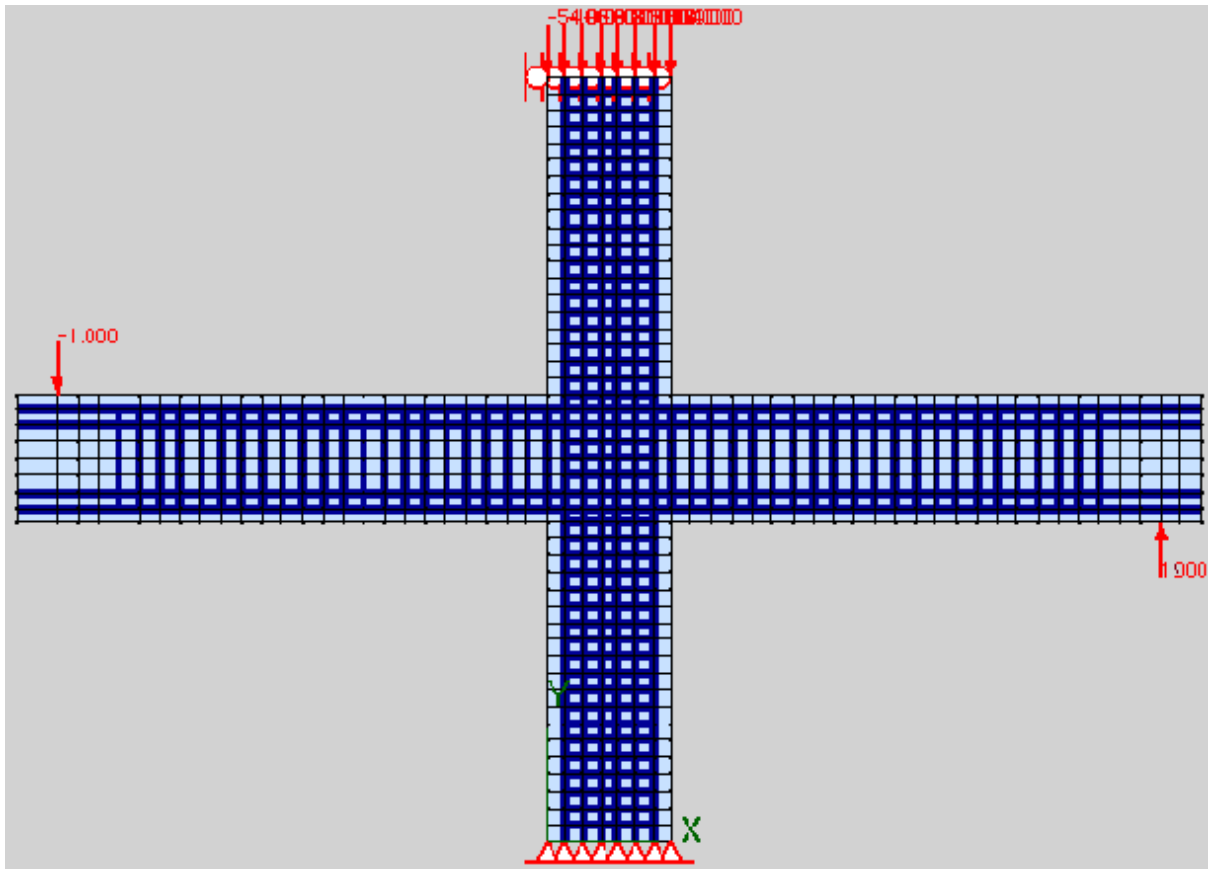


Figure 5.10: VecTor2 Model for the Beam-Column Joint of Verification Example

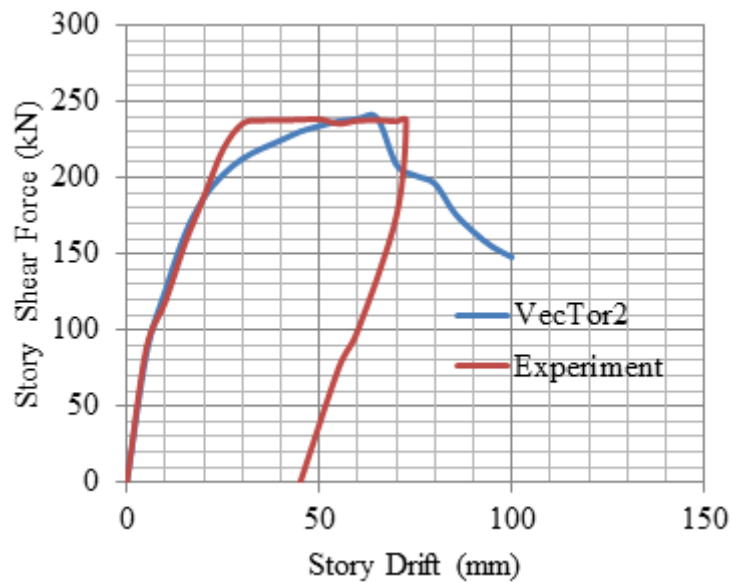


Figure 5.11: Load-Displacement Response of a Beam-Column Joint tested by Noguchi and Kashiwazaki (1992).

5.3. Shear Strength of Beam-Column Joint of the Case Study Building

The building is designed according to old EBCS. The beam column joints in this building are not properly designed and detailed for lateral loads. It is important here to evaluate the detailing of the beam column joint according to both old and new codes.

Various provisions are given in EBCS 8, 1995 regarding RC beam column joints. For joints of DC”L”, the code states that the horizontal confinement reinforcement in beam column joints shall be equal to that provided along the critical regions of the column.

The provision in the code for joints of DC”M” states that within the beam column joints, horizontal hoops shall be provided satisfying the following conditions:

- (a) The diameter, d_{bw} of the hoops is not less than 6mm.
- (b) The spacing, s of the hoops is not greater than:

$$s = \min(h_c/2; 150mm) \quad 5.21$$

EBCSEN1998-1 (2013) also gives clear provisions for detailing of beam column joints in DC”M”. It states that the horizontal confinement reinforcement in joints of primary seismic beams with columns should fulfill the following criteria:

- a) Hoops and cross-ties, of at least 6mm in diameter,
- b) The spacing, s , of the hoops (in millimeters) does not exceed:

$$s = \min\{b_o/2; 175; 8d_{bL}\}$$

where,

b_o ; the minimum dimension of the concrete core (to the centerline of the hoops) in millimeters

d_{bL} ; the minimum diameter of the longitudinal bars (in millimeters)

Again both codes provide that at least one intermediate (between column corner bars) vertical bar shall be provided at each side of a joint of primary seismic beams and columns. Based on the provisions of both codes it can be arrived at the conclusion that the joints of the building under investigation is not properly designed and detailed for lateral loads, especially, seismic load. The provisions given by the codes are important such that a minimum ductility is ensured and local buckling of longitudinal bars is prevented.

Joint strength should be more than that in the adjacent members. In this study three types joints are selected to be investigated. These joints are the interior joint in the 4th story, the exterior joint in the 4th story and the corner joint in the 4th story. The assessment of these joints is discussed briefly in the subsequent sections.

5.3.1. The Interior Joint

Joint Horizontal Shear Demand

To estimate the joint shear demand in the interior beam-column joint selected for investigation in this paper, estimation of moment resistance of beams, tension force in the top and bottom bars of beam longitudinal reinforcement, compression force in the concrete of beam and column shear is necessary. The detail of the joint is described in figures of Appendix A. The estimations are based on sway-to-the right. Accordingly beam sagging and hogging moment capacities are calculated. In the case of sway-to-the right top fibers of the left beam are under tension and top fibers of the right beam are under compression. This causes hogging moment in the left beam and sagging moment in the right beam. So, the moment capacity of the right beam, MRb1, is the sagging moment capacity and the moment capacity of the left beam, MRb2, is the hogging moment capacity.

Another important issue in beam-column joints is to check the requirement of strong column-weak beam by taking the flexural strength ratio of column and beams adjacent to the joint with respect to the direction of lateral load under consideration. This is carried out by using equations in Appendix B.

Using equations in appendix B the sagging and hogging moment capacities are;

$$M_{Rb2} = 106.70kNm$$

$$M_{Rb1} = 105.8kNm$$

Again using the same equation in appendix B, the moment capacity of the column above and below the joint core corresponding to a certain minimum column axial load is calculated. Here, moment capacity of columns is calculated with zero axial loads. However, this approach is conservative and in actual practice it desirable to take moment capacity of columns corresponding to actual minimum axial load in the column obtained from different load combinations. Putting this fact under consideration the moment capacity of columns of this internal joint are found to be:

$$M_{Rc1} = M_{Rc2} = 52.044kNm$$

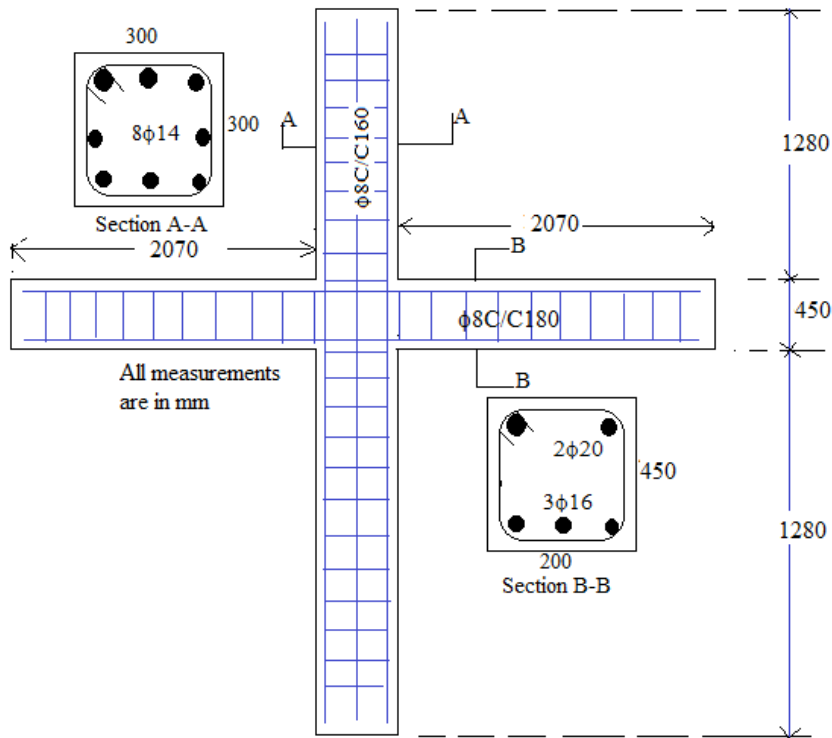


Figure 5.12 : Interior Joint under Investigation

Check for the requirement of strong column-weak beam

$$\frac{\sum M_{Rc}}{\sum M_{Rb}} \geq 1.3 \Rightarrow \frac{2 * 52.044}{106.7 + 105.8} = 0.245$$

The requirement of strong column-weak beam is not satisfied as per EBCS EN1998. The column is weaker than the joining beams. To satisfy ductility condition according to EBCS EN1998, plastic hinges are preferred to be formed in beams. However from the check for the requirement of strong column-weak beam shown above columns are found weaker. This makes ductility of the interior beam-column joint anticipated to originate from columns. Then plastic hinges form in columns and column failure occurs before beam failure under lateral loads, especially under seismic loads. This situation leads to global failure of the structure as failure of column is more critical than failure of beams.

Table 5.1 : Horizontal Joint Shear Demand for the Interior Joint

	Horizontal Joint Shear Demand	
	V_j (kN)	v_j (MPa)
Paulay and Priestley (1992)	465.44	5.17
EBCS-8 (1995)	318.68	3.54
EBCSEN1998-1 (2013)	490.02	5.44

Horizontal joint shear demand calculated according to EBCS-8 (1995), EBCSEN1998-1 (2013) and Paulay and Priestley (1992) is presented in the Table 5.1 above. Horizontal joint shear demand calculation according to Paulay and Priestley (1992) is presented here because of its widely acceptance among many researchers. Horizontal joint shear demand calculation according to ACI352R-02 (2002) is not included in the table since it directly adopts that of Paulay and Priestley (1992). Horizontal joint shear stress is calculated by dividing horizontal joint shear force by effective joint area as shown in Equation 5.18.

Joint Horizontal Shear Strength

The Horizontal Joint Shear Strength for the Interior Joint is presented in Table 5.2 shown below. The results shown are compared against the respective method of horizontal joint shear demand and remarked as ‘OK!’ and ‘NOT OK!’.

Table 5.2 : Horizontal Joint Shear Strength for the Interior Joint

	Horizontal Joint Shear Strength		Remark
	V_{jh} (kN)	v_{jh} (MPa)	
Paulay and Priestley (1992)	227.41	2.53	NOT OK!
EBCS-8 (1995)	450	5.00	OK!
EBCSEN1998-1 (2013)	284.74	3.16	NOT OK!
ACI352R-02 (2002)	556.76	6.19	OK!

Identification of a structural member which governs shear input into beam-column joints is important. Accordingly, the column in the interior joint is weaker than beams. Therefore a monotonic lateral load is applied in the form of displacements at upper column end when modeling in VecTor2. A constant axial load of 416.09KN obtained from seismic analysis is also applied upper column end. The lower column end is provided with hinged boundary conditions and both the beam ends were provided with roller boundary conditions. The upper column end is allowed to move horizontally. This is shown in the following Figure 5.13.

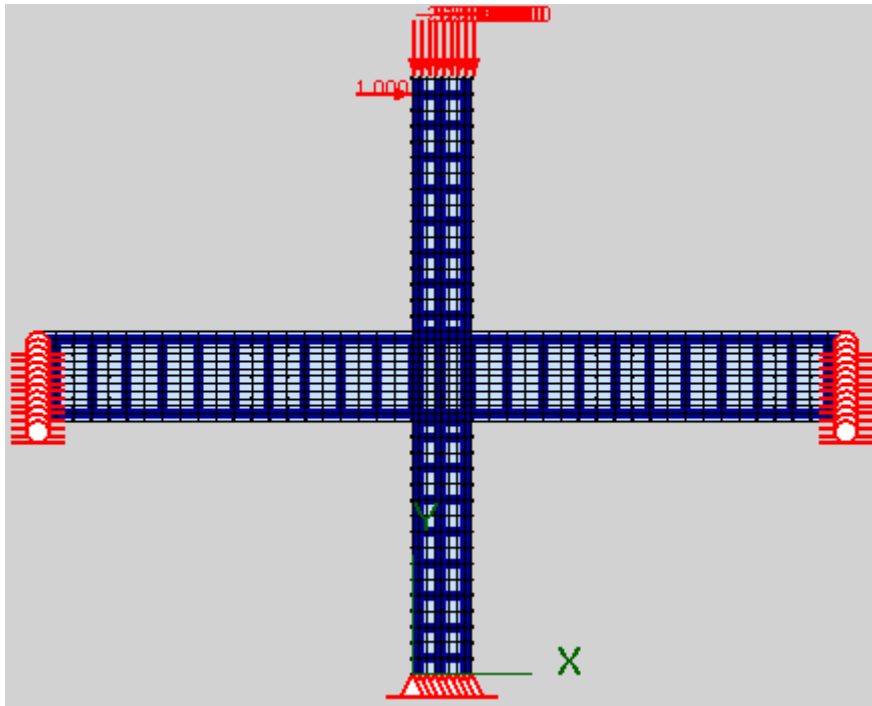


Figure 5.13 : Interior Beam-Column Joint Model in VecTor2

The default models were chosen for the parameters used in modeling both concrete and reinforcement. The behavioral models that were used for the concrete are given in Table 5.3.

Table 5.3: Material Behavior Models for Concrete

Material Property	Model
Concrete Compression Pre-Peak Response	Hognestad Parabola
Concrete Compression Post-Peak Response	Modified Park-Kent
Concrete Compression Softening	Vecchio 1992-A (e1/e2-Form)
Concrete Tension Stiffening	Modified Bentz 2003
Concrete Tension Softening	Linear
Concrete Confined Strength	Kupfer/Richard Model
Concrete Dilation	Variable Kupfer
Concrete Cracking Criterion	Mohr-Coulomb (Stress)
Concrete Crack Slip Check	Vecchio-Collins 1986
Concrete Crack Width Check	Agg/2.5 Max Crack Width

Including the geometric data, yield strength, ultimate strength and modulus of elasticity the material models shown in Table 5.4. below were used for reinforcement material in modeling of beam-column joints in VecTor2.

Table 5.4: Material Behavior Models for Reinforcement

Material Property	Model
Reinforcement Dowel Action	Tassios Model (Crack Slip)
Reinforcement Buckling	Akkaya 2012 (Modified Dhakal-Maeka)

The shear strength obtained according to EBCSEN1998-1 (2013) is 3.16MPa. It is found to be 5.47MPa from VecTor2 as shown in Figure 5.14.

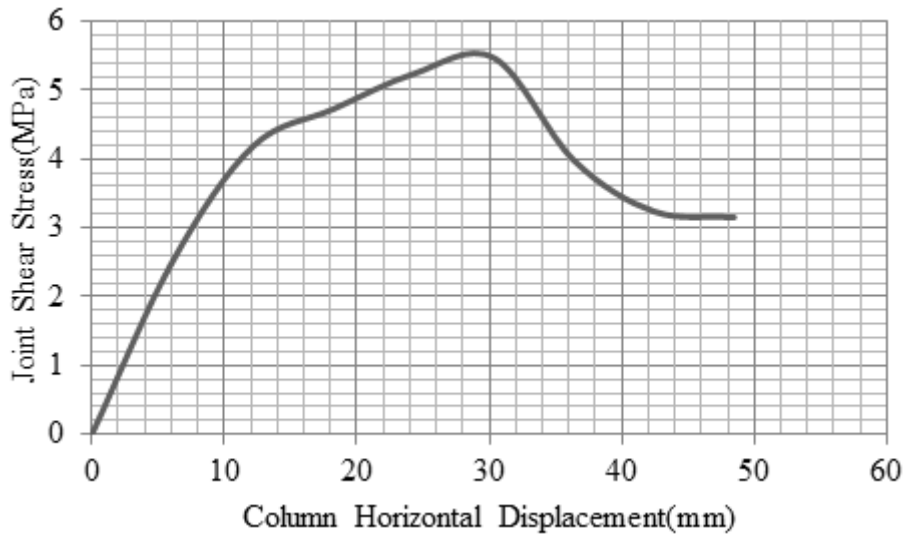


Figure 5.14: Shear Strength of Joint Core for the Interior Joint from VecTor2

5.3.2. Enhancement of the Shear Strength of the Interior Joint

The interior joint investigated does not satisfy both capacity design rule and shear strength requirement of beam-column joints provided in EBCSEN1998-1 (2013). The column of the joint is retrofitted to investigate the joint behavior regarding both ductility and shear strength in the following discussion.

5.3.2.1. Retrofitting of the Interior Column

The existing interior column investigated in this study is enough in resisting all axial, shear and moment responses from the seismic analysis. However it doesn't meet the capacity design rule specified in EBCSEN1998-1 (2013). Both the shear (79KN) and flexural (52.04kNm) capacity of the column are less than the shear (97.5KN for both beams) and flexural (108.6kNm for right beam and 112.6kNm for left beam) capacities of the joining beams. The moment capacity of the column shall be increased to enhance the strength, stiffness, ductility, stability, and integrity of the building. The shear strength of the joint estimated using EBCSEN1998-1 (2013) also shows the joint does not satisfy the shear strength requirement of EBCSEN1998-1 (2013). To handle these two problems the retrofitting of the column in the interior joint becomes important. In this study retrofitting is carried out to strengthen the existing columns by concrete jacketing. Concrete jacketing of columns increases in shear capacity of columns and improves the column flexural strength.

5.3.2.2. Design of the Column Jacketing

1. Obtain the design axial force and bending moments

The design axial force (N) is found to be 428kN for the seismic analysis. The design bending moment (M) shall be calculated putting the flexural capacities of joining beams under consideration to meet capacity design rule. This obtained from Equation 5.1 and the value is taken 143.78kNm for both axes of the column cross-section.

2. Estimate column size and section details using design axial force and bending moments obtained above.

A trial section of (450X450) mm is taken.

$$\nu = \frac{N}{f_{cd}bh} = \frac{428*1000N}{11.33N/mm^2 * 450mm * 450mm} = 0.2$$

$$\mu_b = \mu_h = \frac{M}{f_{cd}bh^2} = \frac{143.78*10^6 Nmm}{11.33N/mm^2 * 450mm * (450mm)^2} = 0.14$$

Using the biaxial chart for design of reinforced columns;

$$\omega = 0.2 \rightarrow A_{s,tot} = \omega \frac{A_c f_{cd}}{f_{yd}} = 0.2 * \frac{450mm * 450mm * 11.33N/mm^2}{347.826N/mm^2} = 1319.24mm^2$$

Minimum area of reinforcing steel shall be checked and it is;

$$A_{s,tot\min} = 0.8\% * A_c = 0.008 * 450mm * 450mm = 1620mm^2$$

Therefore, the total area of steel reinforcement required is 1620mm².

3. Jacketing details of the column cross section

Amount of concrete and steel to be provided in the jacket is obtained by deducting the existing column size and amount of reinforcement from the total size and reinforcement amount obtained above.

$$b = (450 - 300) / 2 = 75mm \quad h = (450 - 300) / 2 = 75mm$$

The jacket thickness 75mm would be since minimum jacket thickness may be taken as 75-100mm.

The reinforcement area in the jacket is;

$$A_s' = 1620\text{mm}^2 - (8 * 14^2 \pi / 4)\text{mm}^2 = 389.12\text{mm}^2$$

To account for losses the total area required in the jacket is taken as;

$$A_s = (4/3)A_s' = (4/3) * 389.12 = 518.83\text{mm}^2$$

Assuming 14mm Ø bars, the number of bars (n) can be obtained as;

$$n = A_s * 4 / (\pi * 14^2) = 518.83 * 4 / (\pi * 14^2) = 4 \text{ bars}$$

4. Details of the jacketed cross-section

The jacketed cross-section is now 450mm by 450mm and provided with 4 Ø14 longitudinal reinforcing bars. Minimum transverse reinforcement is enough and so the reinforcement provided for the existing column would be sufficient. Therefore, provide Ø8 with a spacing of 160mm.

The jacketed column cross section now has a shear capacity of 140.35KN and flexural capacity of 145.34kNm.

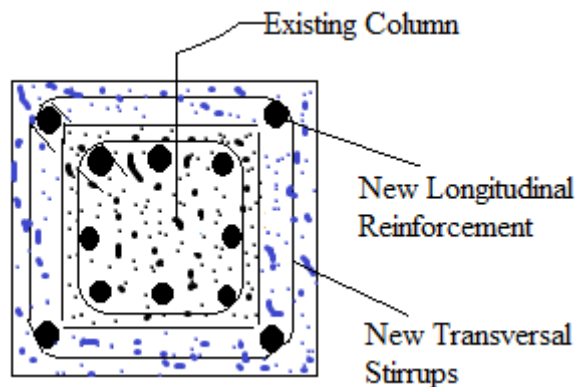


Figure 5.15 : Jacketing Detail of the Column

5.3.2.3. Horizontal Shear Strength of the Interior Joint after Column Jacketing

The flexural capacity ratio of the joint is greater than 1.3 after column retrofitting. So, the capacity design rule is satisfied according to EBCSEN1998-1 (2013). The horizontal shear demand (V_j) is 614.20kN and the horizontal joint shear strength (V_{jh}) is 973.96kN. The joint shear strength requirement is satisfied after column retrofitting.

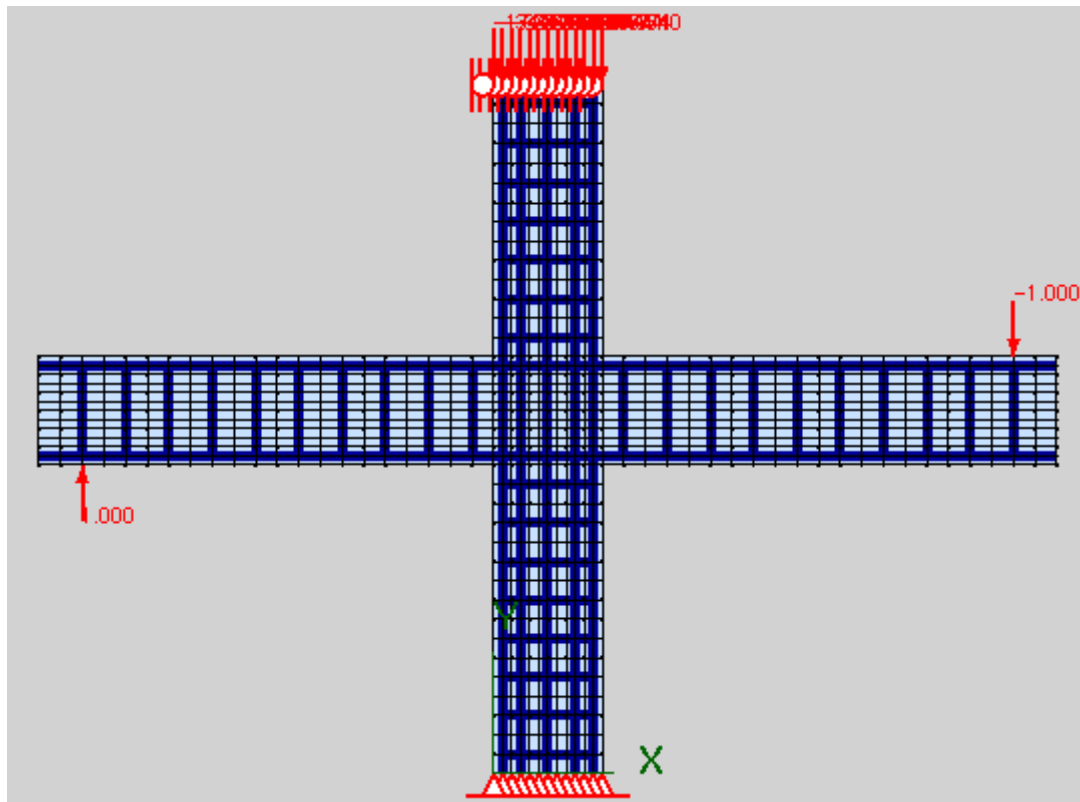


Figure 5.16 : Interior Joint Model in VecTor2 after Column Jacketing

A significant difference is observed in the interior joint after column jacketing. The joint does not satisfy both ductility and shear strength requirement before column jacketing and it does after column jacketing. The joint shear strength based on EBCSEN1998-1 (2013) has improved from 3.16MPa to 5.093MPa. The maximum shear stress occurred at a story drift angle of 1.01% before column jacketing and it occurred at a story drift angle of 4.63% after column jacketing.

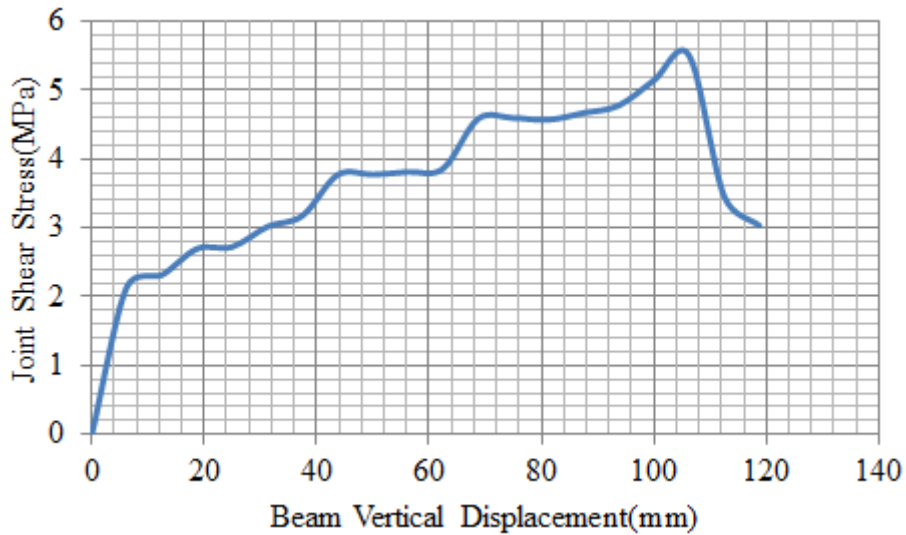


Figure 5.17 : Shear Strength of Joint Core for the Interior Joint from VecTor2 after Column Jacketing

5.3.3. The Exterior Joint

Joint Horizontal Shear Demand

The shear transfer mechanism of exterior joints is similar to that of interior joints. So, the same procedure is followed in calculating shear force demands in the joints.

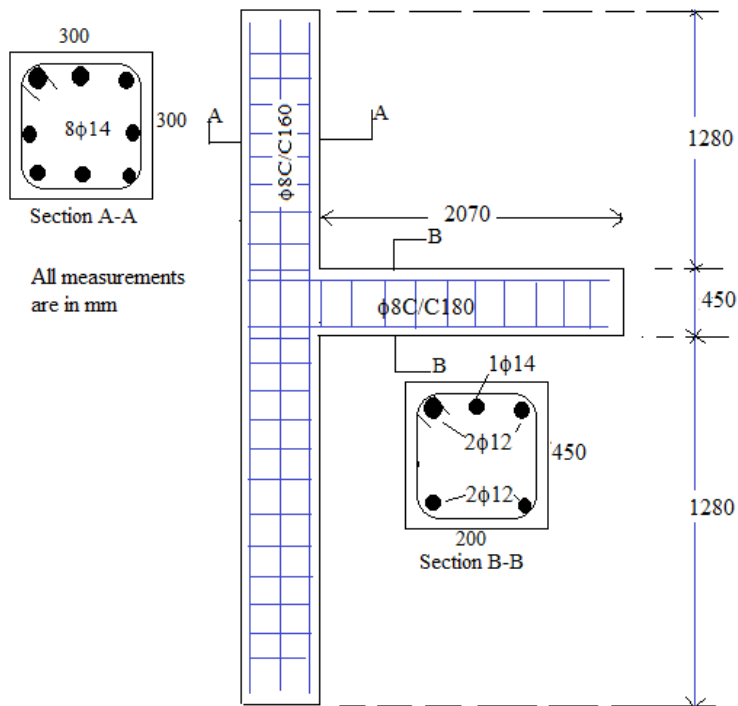


Figure 5.18 : Exterior Joint under Investigation

$$M_{Rb} = 82.8kNm \quad M_{Rc1} = M_{Rc2} = 52.044kNm$$

$$\frac{\sum M_{Rc}}{\sum M_{Rb}} \geq 1.3 \Rightarrow \frac{2 * 52.044}{82.8} = 1.26$$

The requirement of strong column-weak beam is not satisfied as per EBCS EN1998. However, the column is stronger than the joining beams.

Table 5.5 : Horizontal Joint Shear Demand for the Exterior Joint

	Horizontal Joint Shear Demand	
	V _j (kN)	v _j (MPa)
Paulay and Priestley (1992)	137.675	1.53
EBCS-8 (1995)	87.246	0.97
EBCSEN1998-1 (2013)	140.134	1.56

Joint Horizontal Shear Strength

Table 5.6 : Horizontal Joint Shear Strength for the Exterior Joint

	Horizontal Joint Shear Strength		Remark
	V _{jh} (kN)	v _{jh} (MPa)	
Paulay and Priestley (1992)	105.08	1.17	NOT OK!
EBCS-8 (1995)	315.00	3.50	OK!
EBCSEN1998-1 (2013)	269.00	2.99	OK!
ACI352R-02 (2002)	417.58	4.64	OK!

Beam in this joint is a structural member which governs shear input into the joints. Therefore downward vertical monotonic load monotonic load is applied at the beam end in the form of displacements when modeling in VecTor2. Lower column end is provided with hinged boundary conditions. The upper column end is not allowed to move horizontally. A constant axial load of 361.77KN is applied at the upper column end.

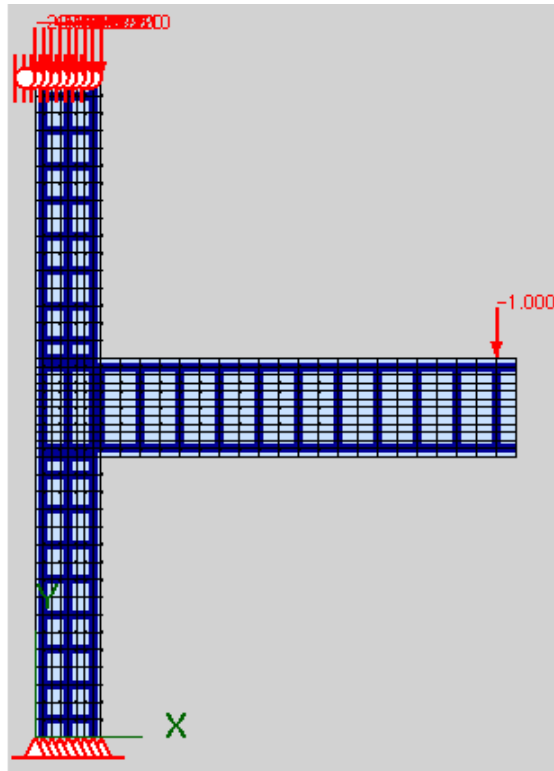


Figure 5.19 : Exterior Beam-Column Joint Model in VecTor2

The shear strength obtained according to EBCSEN1998-1 (2013) is 2.99MPa. It is found to be 1.225MPa from VecTor2 as shown in Figure 5.20.

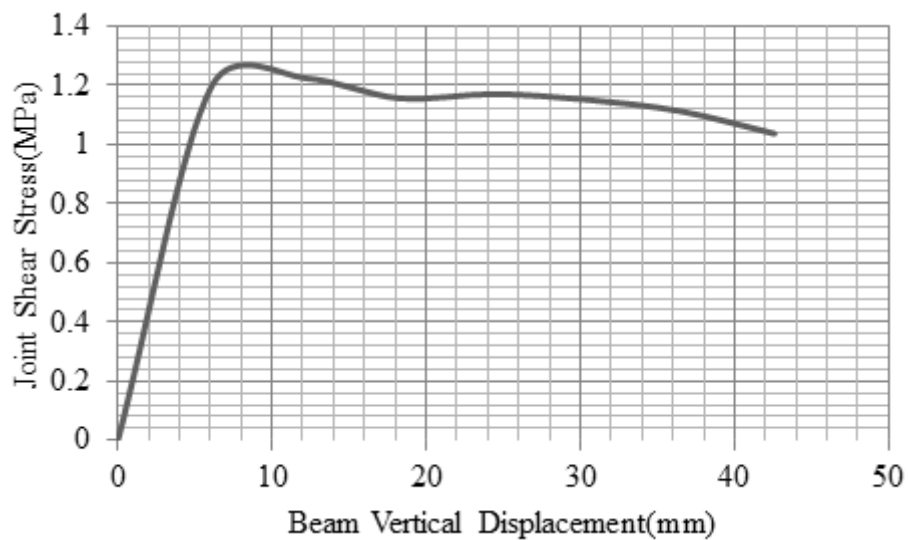


Figure 5.20: Shear Strength of Joint Core for the Exterior Joint from VecTor2

5.3.4. The Corner Joint

The shear transfer mechanism of corner joints is similar to that of interior and exterior joints, especially to exterior joints as corner joint is one type exterior joints. So, the same procedure is followed in calculating shear force demands in the joints.

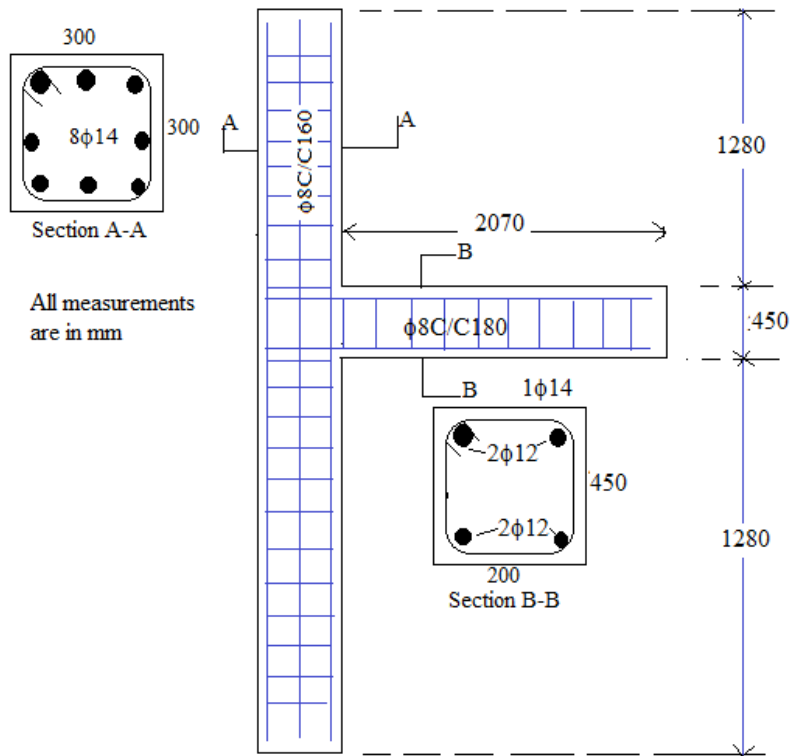


Figure 5.21 : Corner Joint under Investigation

$$M_{Rb} = 44.2 \text{ KNm} \quad M_{Rc1} = M_{Rc2} = 52.044 \text{ KNm}$$

$$\frac{\sum M_{Rc}}{\sum M_{Rb}} \geq 1.3 \Rightarrow \frac{2 * 52.044}{44.2} = 2.355$$

The requirement of strong column-weak beam is satisfied as per EBCS EN1998. The column is stronger than the joining beam.

Table 5.7 : Horizontal Joint Shear Demand for the Corner Joint

	Horizontal Joint Shear Demand	
	V_j (kN)	v_j (MPa)
Paulay and Priestley (1992)	83.62	0.93
EBCS-8 (1995)	48.21	0.54
EBCSEN1998-1 (2013)	79.68	0.885

Joint Horizontal Shear Strength

Table 5.8 : Horizontal Joint Shear Strength for the Corner Joint

	Horizontal Joint Shear Strength		Remark
	V_{jh} (kN)	v_{jh} (MPa)	
Paulay and Priestley (1992)	45.81	0.51	NOT OK!
EBCS-8 (1995)	315.00	3.50	OK!
EBCSEN1998-1 (2013)	333.22	3.70	OK!
ACI352R-02 (2002)	334.07	3.71	OK!

Beam in this joint is a structural member which governs shear input into the joints. Therefore, downward vertical monotonic load shall be applied at beam tip when modeling in VecTor2. The loading and boundary conditions applied are similar to exterior joints discussed above. The shear strength obtained according to EBCSEN1998-1 (2013) is 3.70MPa. It is found to be 1.235MPa from VecTor2 as shown in Figure 5.22.

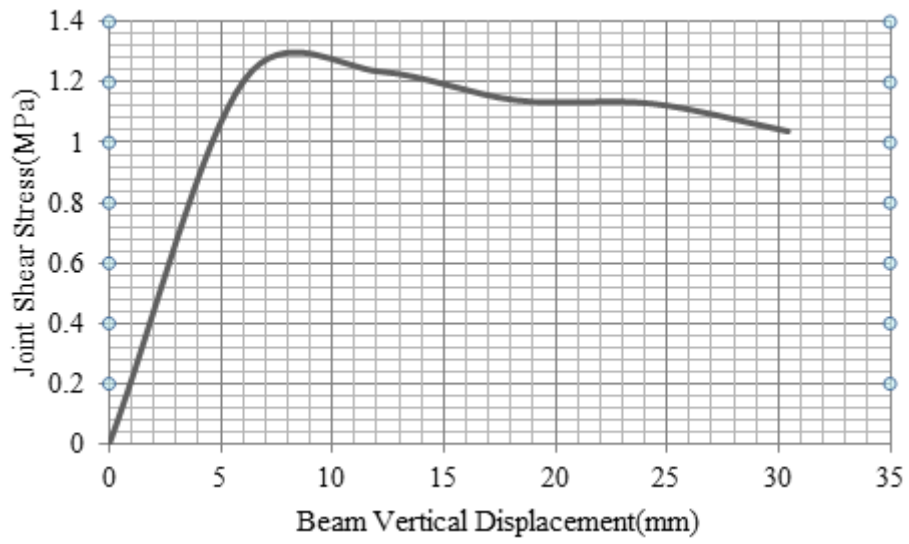


Figure 5.22: Shear Strength of Joint Core for the Corner Joint from VecTor2

6. CONCLUSIONS AND RECOMMENDATIONS

6.1. Conclusions

The building investigated in this project is designed to old EBCS codes. The joints investigated are not properly designed and detailed for seismic loads. The interior and exterior beam-column joints do not satisfy the strong column-weak beam requirement. However, the joints pass the shear strength requirements of the old EBCS code. All joints investigated in this paper fail to pass the strength requirement expressions given by Paulay and Priestley (1992). However, only the interior joint fails to pass the shear strength requirement of EBCSEN1998-1 (2013). All the joints have fulfilled the shear strength requirement of ACI352R-02 (2002). This is may be due to the fact that ACI code gives higher joint shear strength values for concrete strengths less than 40MPa. (K.Jain 2006)

The study shows that there is a significant variation between result from analysis in VecTor2 and provisions in EBCSEN1998-1 (2013). The investigation on the interior joint revealed that column jacketing improves both ductility and shear strength of the joint and maximum shear stress occurred at larger story drift angle for the joint with jacketed column.

6.2. Recommendations

This study is not enough in exposing the problems regarding design and detailing of beam-column joints in Ethiopian practice. The shear strength of beam-column joints of existing buildings are not only investigated by the methods discussed in this study. The author put forward the following recommendations based on the results:

- I. Modeling a number of joints in a number of buildings is important to reach at a good finding regarding shear strength of beam-column joints in Ethiopian practice.
- II. Only horizontal shear strength for earthquake in the x-direction is checked in this paper. However, checking the shear strength of joints for earthquake in y-direction is important.
- III. Laboratory investigation is highly recommended due to its accuracy compared to software simulations.
- IV. Modeling of beam-column joints in other finite element softwares is also possible and shall be worked on.
- V. Since the failure of joints is catastrophic, proper design and detailing of RC beam column joints is highly recommended.

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APPENDICES

A. Beam and Column Section Detail of Beam-Column Joints Investigated in this Paper

A.1. Interior Joint

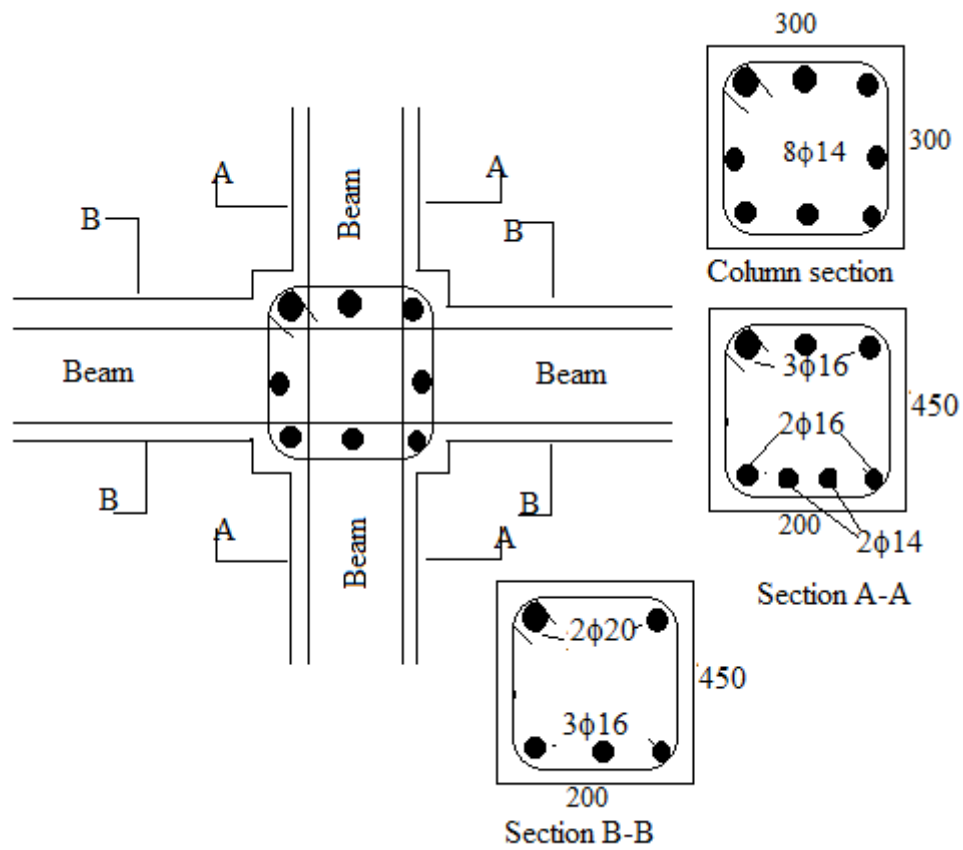


Figure A.1 : The Interior Joint in Plan

A.2. Exterior Joint

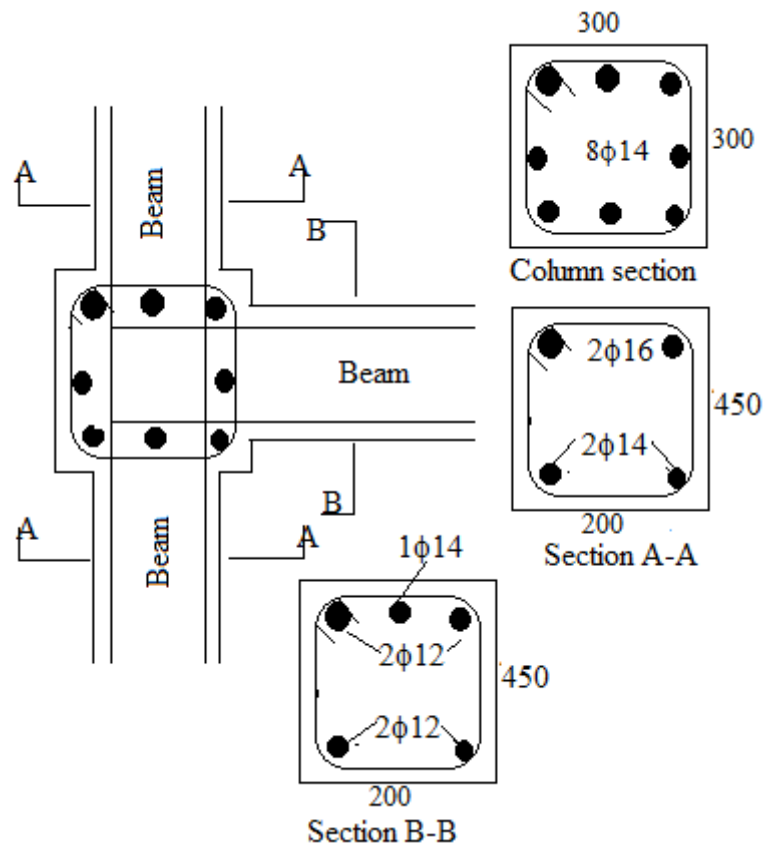


Figure A.2 : The Exterior Joint in Plan

A.3. Corner Joint

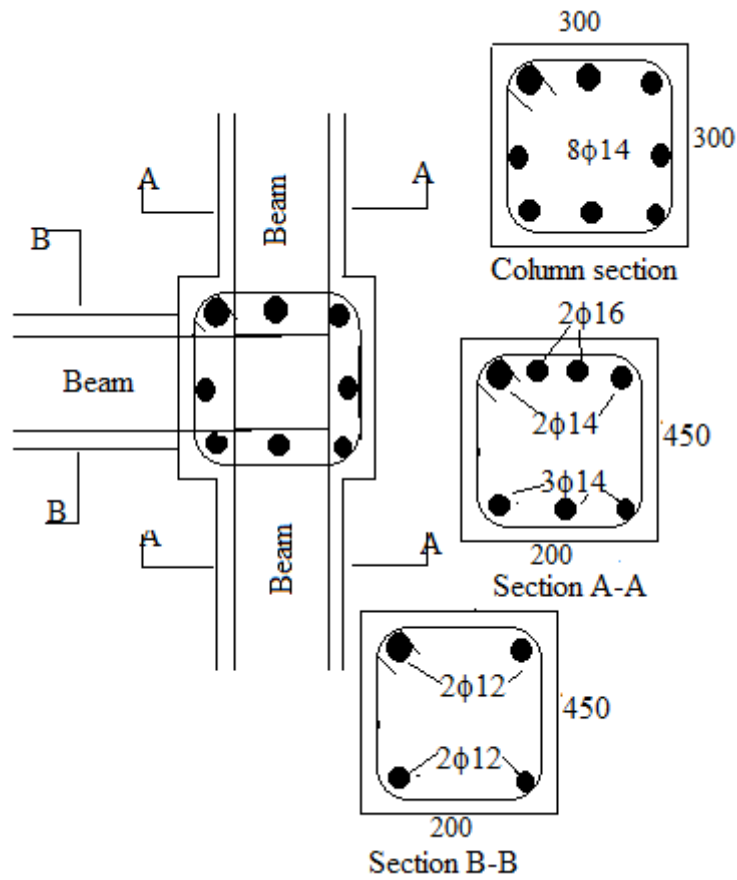


Figure A.3 : The Corner Joint in Plan

B. Moment Capacity of Rectangular Reinforced Concrete Cross-Section

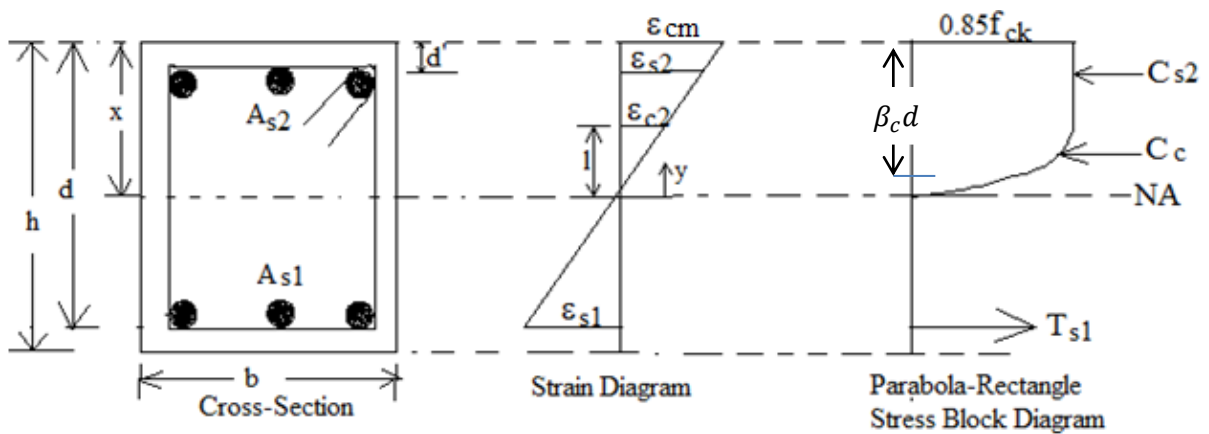


Figure B.1 : Strain and Stress Distribution over a Reinforced Rectangular Concrete Section

$$C_{s2} = \varepsilon_{s2} E_s A_{s2} \quad \text{B.1}$$

If compression bar has yielded it is given as;

$$C_{s2} = A_{s2} f_{yd} \quad \text{B.2}$$

Assuming the tension bar has yielded we have;

$$T_{s1} = \varepsilon_{s1} E_s A_{s1} \quad \text{B.3}$$

If tension bar has yielded it is given as;

$$T_{s1} = A_{s1} f_{yd} \quad \text{B.4}$$

Normally the tension bar is assumed to be yielded in estimation of flexural capacities of reinforced concrete sections if;

$$\varepsilon_{s1} \geq \varepsilon_{yd} = f_{yd} / E_s \quad \text{B.5}$$

$$C_c = 0.85 f_{ck} b(x-l) + \int_0^l \sigma_c b d_y \quad \text{B.6}$$

$$\sigma_c = 0.85 f_{ck} \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c2}} \right)^n \right] \quad \text{for } 0 \leq \varepsilon_c \leq \varepsilon_{c2} \quad \text{B.7}$$

$$\sigma_c = 0.85 f_{ck} \quad \text{for } \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu} \quad \text{B.8}$$

The exponent, n and ε_{c2} are obtained from EBCS EN1992-1-1 (2013).

For $f_{ck} \leq 50 \text{MPa}$;

$$n = 2$$

$$\varepsilon_{c2} = 2\text{‰}$$

N.A is within the section and we have two cases;

$$\text{I) } \varepsilon_{cm} \geq 2\text{‰}$$

$$\text{II) } \varepsilon_{cm} \leq 2\text{‰}$$

Case I

From similarity of triangles

$$\begin{aligned}\varepsilon_c &= \frac{y}{x} \varepsilon_{cm} \\ \varepsilon_{s2} &= \varepsilon_{cm} \frac{x-d'}{x} \\ l &= \frac{2x}{\varepsilon_{cm}}\end{aligned}$$

Therefore;

$$C_c = 0.85 f_{ck} b(x-l) + 0.85 f_{ck} b \int_0^l \left[1 - \left(1 - \frac{\varepsilon_c}{2} \right)^2 \right] dy \quad \text{B.9}$$

Integrating and Substituting ε_c by $(y/x)*\varepsilon_{cm}$ and l by $(2x)/\varepsilon_{cm}$ in the above expression we arrive at

$$\begin{aligned}C_c &= \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} * 0.85 f_{ck} bx \\ x &= k_x d \\ C_c &= \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} k_x * 0.85 f_{ck} bd \rightarrow \alpha_c = \frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} k_x \\ C_c &= \alpha_c * 0.85 f_{ck} bd\end{aligned} \quad \text{B.10}$$

The neutral axis depth, x , is obtained from equilibrium of forces,

$$T_{s1} = C_c + C_{s2} \quad \text{B.11}$$

If there is a compressive axial force (N)

$$N = C_c + C_{s2} - T_{s1} \quad \text{B.12}$$

Taking the distance of C_c from the top fiber as $\beta_c * d$ the following expression is obtained for β_c ;

$$\beta_c = \frac{\varepsilon_{cm} (3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm} (3\varepsilon_{cm} - 2)} k_x \quad \text{B.13}$$

The moment resistance is obtained finally after knowing the value of x as

$$M_{Rd} = C_c(x - \beta_c d) + C_{s2}(x - d') + T_{s1}(d - x) \quad \text{B.14}$$

Alternatively the moment resistance can be given by an expression;

$$M_{Rd} = 0.85 f_{ck} b \int_0^l \left[1 - \left(1 - \frac{\varepsilon_c}{2} \right)^2 \right] y d_y + 0.85 f_{ck} b (x - l) \left(\frac{x + l}{2} \right) + C_s (x - d') + T_s (d - x) \quad \text{B.15}$$

Case II

$$C_c = 0.85 f_{ck} b \int_0^x \left[1 - \left(1 - \frac{\varepsilon_c}{2} \right)^2 \right] dy \quad \text{B.16}$$

$$\varepsilon_c = \frac{y}{x} \varepsilon_{cm}$$

$$\varepsilon_{s2} = \varepsilon_{cm} \frac{x - d'}{x}$$

Integrating and Substituting ε_c by $(y/x) * \varepsilon_{cm}$ in the above expression we arrive at

$$C_c = \frac{\varepsilon_{cm} (6 - \varepsilon_{cm})}{12} * 0.85 f_{ck} b x$$

$$x = k_x d \quad \text{B.17}$$

$$C_c = \frac{\varepsilon_{cm} (6 - \varepsilon_{cm})}{12} k_x * 0.85 f_{ck} b d \rightarrow \alpha_c = \frac{\varepsilon_{cm} (6 - \varepsilon_{cm})}{12} k_x$$

$$C_c = \alpha_c * 0.85 f_{ck} b d$$

The neutral axis depth, x , is obtained from equilibrium of forces,

$$T_{s1} = C_c + C_{s2} \quad \text{B.18}$$

If there is a compressive axial force (N)

$$N = C_c + C_{s2} - T_{s1} \quad \text{B.19}$$

Taking the distance of C_c from the top fiber as β_c*d the following expression is obtained for β_c ;

$$\beta_c = \frac{8 - \varepsilon_{cm}}{4(6 - \varepsilon_{cm})} k_x \quad \text{B.20}$$

The moment resistance is obtained finally after knowing the value of x as

$$M_{Rd} = C_c(x - \beta_c d) + C_{s2}(x - d') + T_{s1}(d - x) \quad \text{B.21}$$

Alternatively the moment resistance can be given by an expression ;

$$M_{Rd} = 0.85 f_{ck} b \int_0^l \left[1 - \left(1 - \frac{\varepsilon_c}{2} \right)^2 \right] y d_y + C_{s2}(x - d') + T_s(d - x) \quad \text{B.22}$$

C. Shear Capacity of RC Sections

The shear resistance of a member with shear reinforcement is equal to

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \quad \text{C.1}$$

$$V_{Rd,c} = \left[C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} + k_1 \sigma_{cp} \right] b_w d \quad \text{C.2}$$

With a minimum of

$$V_{Rd,c} = (v_{\min} + k_1 \sigma_{cp}) b_w d$$

Where ;

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \quad d \text{ is in mm}$$

$$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.002$$

b_w is the smallest width of the cross-section in the tensile area in mm

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} < 0.2 f_{cd} \text{ (MPa)}$$

N_{Ed} is axial load (N)

A_c is area of concrete cross-section (mm²)

$V_{Rd,c}$ is in N

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \quad \gamma_c \text{ is partial factor for concrete material}$$

$$k_1 = 0.15$$

$$V_{Rd,s} = \min \left\{ \begin{array}{l} \frac{A_{sw}}{S} z f_{ywd} \cot \theta \\ \alpha_{cw} b_w z v_1 f_{cd} / (\cot \theta + \tan \theta) \end{array} \right. \quad \text{C.3}$$

A_{sw} ; cross-sectional area of the shear reinforcement

S ; spacing of the stirrups

f_{ywd} ; design yield strength of the shear reinforcement

v_1 ; strength reduction factor for concrete cracked in shear

α_{cw} ; coefficient taking account of the state of the stress in the compression chord

z ; inner lever arm corresponding to the bending moment in the element under consideration.