

**Professional Development and Training on Assessment for
Learning and Pre-service Teachers' Outcomes in Fundamental
Concepts of Algebra: A Mixed Method Study**

Ayanaw Yigletu Asfaw

**A Thesis Submitted to
The Department of Science and Mathematics Education**

**Presented to the fulfillment of the Requirement
Of Doctor of Philosophy in Mathematics Education**

**Addis Ababa University
Department of Science and Mathematics Education
Addis Ababa, Ethiopia
Jun, 2023**

**Professional Development and Training on Assessment for Learning
and Pre-service Teachers' Outcomes in Fundamental Concepts of
Algebra: A Mixed Method Study**

Ayanaw Yigletu Asfaw

**A Thesis submitted to
The Department of Science and Mathematics Education**

**Presented for the Fulfillment of the Requirement for the
Degree of Doctor of Philosophy in Mathematics Education**

Addis Ababa University

Addis Ababa, Ethiopia

June, 2023

This is to certify that the thesis prepared by Ayanaw Yigletu Asfaw, entitled: *Professional Development and Training on Assessment for Learning and Pre-service Teachers' Outcomes in Fundamental Concepts of Algebra: A Mixed Method Study* and submitted for the fulfillment of the requirements for the Degree of Doctor of Philosophy in Mathematics Education compiles with the regulations of the University and meets the accepted standards with respect to originality and quality.

<u>Prof. Rafael Martínez Planell</u>		<u>07/07/2023</u>
External Examiner	Signature	Date
<u>Prof. Solomon Melesse</u>		<u>07/07/2023</u>
Internal Examiner	Signature	Date
<u> </u>	<u> </u>	<u> </u>
Main Supervisor	Signature	Date
<u>Prof. Mulugeta Atnafu</u>		<u>07/07/2023</u>
Co-Supervisor	Signature	Date
<u> </u>	<u> </u>	<u> </u>
SMED Chair/Coordinator	Signature	Date

ABSACT

Professional Development and Training on Assessment for Learning and Pre-service Teachers' Outcomes in Fundamental Concepts of Algebra: A Mixed Method Study

Ayanaw Yigletu Asfaw

Addis Ababa University, 2023

Due to the extensive culture of summative assessment in teacher education, improving pre-service teachers' achievements and their self-regulated learning skills remains a challenge. The aim of this research was to examine the effect of a job-embedded, context-specific, and content-based comprehensive professional development program for mathematics teacher educators on assessment for learning and training for pre-service primary mathematics teachers on self-assessment and peer assessment on the pre-service primary mathematics teachers' achievement gains, self-regulated learning skills, conceptions of assessment for learning practices, as well as on the implementations of assessment for learning strategies in the teaching and learning of the Fundamental Concepts of Algebra course. In this research, 13 mathematics teacher educators in a teacher education college and 129 pre-service primary mathematics teachers in three different teacher education colleges selected randomly from the ten colleges in Amhara regional state (52 in the intervention group, 38 in comparison group one, and 39 in comparison group two) participated as a sources of data. Three mathematics teacher educators who were assigned to deliver the Fundamental Concepts of Algebra course and their 52 pre-service teachers who took the course participated in the professional development and training, respectively. Thematic analysis of interview data of the conceptions of mathematics teacher educators about assessment for learning revealed that they have an incomplete understanding of assessment for learning. A one-way ANOVA also showed that pre-service primary mathematics teachers in the intervention group outscored pre-service primary mathematics teachers in both comparison groups on their achievement gains in algebra. In addition, a one-way ANOVA showed that pre-service primary mathematics teachers in the intervention group outscored pre-service primary mathematics teachers in both comparison groups on their conceptions of assessment for learning experiences. Furthermore, a paired sample t-test result showed that pre-service primary mathematics teachers' self-regulated learning skills after the intervention were significantly higher than their self-regulated learning skills before the intervention. Besides, the comprehensive professional development and training improved the implementation of assessment for learning in the teaching and learning of the algebra course. The implications of professional development on assessment for learning are discussed, as are recommendations for further study.

Keywords: Professional Development, Assessment for learning, Algebra, Teacher Education

Acknowledgments

I would like to begin by conveying my sincere and profound appreciation to my main supervisor, Dr. Kassa Michael, who has been a constant source of professional and exemplary guidance and continuous encouragement throughout the entire process of my dissertation. His expertise and insightful comments have been invaluable in helping me complete my study. I am also grateful to him for his patience, understanding of my challenges, and assistance during critical times.

My co-supervisor, Professor Mulugeta Atnafu, also deserves my heartfelt gratitude for his professional and exemplary guidance and continuous encouragement throughout the entire process of my dissertation. He has been supportive and helpful in every step of the way.

I am also deeply indebted and thankful to my colleagues Ato Kefialew Shiferaw, Ato Khalid Jemal, Ato Assefa Abate and Ato Ousman Seid, who have been supportive and encouraging in my research. They have provided me with valuable feedback and suggestions that have improved the quality of my work.

I also wish to express my deepest gratitude to the then deans and all of my colleagues at the Dessie College of Teacher Education for their kind support throughout this dissertation. They have created a conducive environment for me to pursue my academic goals. I am also thankful to the Amhara Regional State Education Bureau for giving me this opportunity to further my education. Finally, I would like to acknowledge the contribution of all my friends and colleagues who have helped me directly or indirectly while I conducted this research. They have been a source of motivation and inspiration for me.

Table of Contents

List of Tables	x
List of figures.....	xi
List of Abbreviations and Acronyms	xii
List of publications	xiii
CHAPTER ONE: INTRODUCTION	1
1.1 Background of the Study	1
1.2 Statement of the Problem.....	17
1.3 Research Questions	22
1.4 Objectives of the Study	23
1.5 Significance of the Study	24
1.6 Delimitation of the Study.....	25
1.7 Limitations of the Study.....	25
1.8 Operational Definitions of Key Terms	26
1.9 Organization of the Study	27
Chapter Two: Review of Literature.....	29
2.1 Introduction.....	29
2.2 Theoretical Framework.....	29
2.2.1 Theoretical Framework.....	29
2.3 Assessment for Learning.....	35
2.3.1 Assessment for learning and its historical background.....	35
2.3.2 The Nature, Principles, Strategies and Purposes of Assessment for Learning	39
2.3.3 Conception of Assessment for Learning Experiences	45
2.3.4 Challenges of implementations of assessment for learning	46
2.3.5 Assessment for learning in Global and Ethiopian context.....	48
2.4 Mathematics Education and Assessment for Learning	51

2.4.1 Assessment for learning and the teaching learning of algebra	53
2.4.2 Assessment for Learning and Mathematics Teacher Education	55
2.5 Professional Development and Training on Assessment for learning	58
2.5.1 Qualities of Effective Professional Development and Training in Education .	62
2.5.2 Professional Development and Training on Assessment for Learning.....	64
2.5.3 Teachers' Conceptions of Assessment for learning	65
2.6 Students' Self-Regulated Learning Skills and Assessment for Learning	67
2.6.1 Students' Self-Regulated Learning Skills	69
2.6.2 Ways of Improving Students Self-Regulated Learning Skills	69
2.6.3 Relationship between Assessment for Learning and Self-Regulated Learning Skills	70
2.7 Effects of Assessment for Learning.....	72
2.7.1 Effect of Assessment for Learning on Students' Achievements	72
2.7.2 Effect of Assessment for Learning on Students' Self-Regulated Learning	76
2.7.3 Effect of Teachers' Conceptions of Assessment for Learning	81
2.7.4 Conceptual Framework.....	84
2.8 Summary Review of Literature.....	84
CHAPTER THREE: METHODOLOGY	86
3.1 Introduction.....	86
3.2 Research Approach	86
3.3 Research Paradigm.....	88
3.4 Research Design.....	89
3.5 Variables of the Study	90
3.6 Participants of the Study	91
3.7 Sample Size and Sampling Techniques	92

3.8 The Intervention.....	92
3.8.1 The Comprehensive Professional Development for Mathematics Teacher Educators.....	93
3.8.2 The Training to Pre-service Primary Mathematics Teachers	95
3.8.3 The Improvement of Fundamental Concepts of Algebra Module	95
3.9 Data Collection Instruments	97
3.9.1 Achievement Test	98
3.9.2 Conceptions of Assessment for Learning Practices Questionnaire	99
3.9.3 Self-Regulated Learning Questionnaire.....	100
3.9.4 Classroom Observation Protocol	102
3.9.5 Interviews.....	103
3.10 Data Collection Procedures.....	103
3.11 Data Analyses	104
3.12 Ethical Considerations	105
CHAPTER FOUR: RESULTS AND DISCUSSION.....	106
4.1 Introduction.....	106
4.1 Results.....	106
4.1.1 Mathematics Teacher Educators’ Conception of Assessment for Learning..	106
4.1.2 Mathematics Teacher Educators’ Experiences of the Professional Development on Assessment for Learning.....	125
4.1.3 Effect of the Professional Development on the Implementation of Assessment for Learning	127
4.1.4 Effect of Professional Development in Assessment for learning on Pre-service Primary Mathematics Teachers’ Achievement gain in Algebra	133
4.1.5 Effect of Professional Development on Pre-service Primary Mathematics Teachers’ Gain scores of Conceptions of Assessment for Learning Experience ...	137

4.1.6 Effect of the intervention on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills.....	142
4.2 Discussion.....	144
4.2.1 Mathematics Teacher Educators' Conceptions of Assessment for Learning	144
4.2.2 Mathematics Teacher Educators Experiences of the Professional Development	148
4.2.3 The Effect of the Professional Development on the Practices of Assessment for Learning	151
4.2.4 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Achievement Gains in the Algebra Course.....	154
4.2.5 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Conceptions of Assessment for Learning Experiences.....	158
4.2.6 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills	160
CHAPTER FIVE: SUMMARY, CONCLUSIONS, RECOMMENDATIONS AND IMPLICATIONS	164
5.1 Summary.....	164
5.1.1 Mathematics Teacher Educators' Conceptions of Assessment for Learning	164
5.1.2 Mathematics Teacher Educators' Experiences of the Professional Development on Assessment for Learning.....	165
5.1.3 Effect of the Professional Development on the Implementation of Assessment for Learning Strategies.....	165
5.1.4 Effect of the Intervention on Pre-service Primary Mathematics Teachers' Achievement gains in Fundamental Concepts of Algebra.....	166
5.1.5 Effect of the Intervention on Pre-service Primary Mathematics Teachers' conceptions of Assessment for Learning Experiences.....	167
5.1.6 Effect of the Intervention on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills.....	167

5.2 Conclusion	169
5.3 Recommendation	171
5.4 Implications of the Study	173
References	175
Appendices.....	198

List of Tables

Table 1. The five key strategies of assessment for learning	38
Table 2. Profiles of Mathematics Teacher Educators	91
Table 3. Summary of analysis of Mathematics Teacher Educators' Conceptions of Assessment for Learning.....	123
Table 4. Average observation rating score of the five assessment for learning strategies in the three sections.....	130
Table 5. Average scores of the three sections' implementation of assessment for learning strategies in 6 observations	132
Table 6. Descriptive statistics of achievement gain scores of the three groups.....	135
Table 7. One-Way ANOVA Summary Table of achievement gain scores of the groups	135
Table 8. Multiple Comparisons of Post Hoc tests among the three groups.....	136
Table 9. Descriptive statistics of achievement gain scores of the three groups.....	139
Table 10. One-Way ANOVA Summary Table of gain scores of conceptions of assessment for learning practices of the groups	139
Table 11. Multiple Comparisons of Post Hoc tests among the three groups.....	140
Table 12. Descriptive Statistics of the Pre and Post measures of Self-Regulated Learning Skill Scores	143
Table 13. Paired Sample t-test results.....	143

List of figures

Figure 1. Conceptual framework of the study	84
Figure 2. The research Design	90
Figure 3 Mathematics teacher educators' levels of conceptions of Assessment for Learning	125
Figure 4. Average observation rating score of the five Assessment for Learning strategies in the three sections.....	131
Figure 5. Average scores of the three sections' implementation of assessment for learning strategies in 6 observations	132
Figure 6. Mean achievement gains of Pre-service Primary Mathematics Teachers on Fundamental Concepts of Algebra course	137
Figure 7. Gain scores of conception assessment for learning experiences plot of the three groups.....	141

List of Abbreviations and Acronyms

AfL	Assessment for learning
CTE	College of Teacher Education
MTE	Mathematics Teacher Educator
PA	Peer Assessment
PD	Professional Development
PPMT	Pre-service Primary Mathematics Teacher
SA	Self-Assessment
SRLS	Self-Regulated Learning Skills
SWOT	Strengths, Weaknesses, Opportunities and Treats

List of publications

1. Asfaw, A. Y., Weldeyesus, K. M., & Ayele, M. A. (2022). The effect of assessment for learning on prospective teachers' learning of algebra through a professional development program. *International Electronic Journal of Mathematics Education*, 17(4), em0718. <https://doi.org/10.29333/iejme/12587>
2. Yigletu, A., Michael, K., & Atnafu, M. (2023). The effect of assessment for learning on pre-service mathematics teachers' higher-order thinking skills in algebra. *Journal of Pedagogical Research*, 7(1), 187-202. <https://doi.org/10.33902/JPR.202317679>
3. Yigletu, A., Michael, K., & Atnafu, M. (2023). Professional development on assessment for learning and its effect on pre-service teacher's self-regulated learning. *Cogent Education*, 10(1), <https://doi.org/10.1080/2331186X.2023.2222875>

CHAPTER ONE: INTRODUCTION

1.1 Background of the Study

Every nation that strives for overall sustainable growth, development, and prosperity to secure the welfare of its population needs to enhance its educational standards.

In this regard, several studies worldwide have focused on the role assessment for learning (AfL) plays in enhancing students' achievements and their self-regulated learning skills (SRLS) (Allal, 2016, 2020; Andersson & Palm, 2017; Andrade & Brookhart, 2016; Black & Wiliam, 1998; Boud, 2018; Carless & Boud, 2018; Chemeli, 2019; Hodgen & Wiliam, 2012). To learn effectively, students need to have clear and meaningful goals, expectations, and feedback; teachers need to use various and suitable methods of instruction, with a special focus on the quality and level of feedback for each stage of learning; teachers need to see learning and teaching from the students' perspective; and teachers need to depend on each other (Hattie, 2008).

Since students do not all the time learn what has been taught, teachers need to build strategies for eliciting and assessing information in order to make informed decisions about what their students have learned during instruction (Black & Wiliam, 2018). Assessment means collecting and discussing data from different sources to understand what students have learned, understood, and can do with their knowledge from their educational experiences. The idea that assessment is important for effective education comes from early research on how to personalize learning through Benjamin Bloom's work on how quality feedback affects students' learning in classrooms (Wiliam, 2011). Many activities that teachers and students carry out in the classroom can benefit from assessment.

Classroom activities and oral questions urge students to show their understanding, comprehension, and abilities. Then, after watching and analyzing what the students say and do, conclusions are drawn on how learning may be improved. These assessment processes involve reflection, discourse, and decision-making from students and teachers, and they play a significant part in everyday classroom activities (Assessment Reform Group, 2002). Assessments in the classroom that provide feedback on process and self-regulation have the potential to be very helpful in fostering both achievement and SRLS (Andrade & Heritage, 2018).

Assessment for learning is an important topic of focus within the area of education at all levels and plays a vital role in improving students' learning (Andersson & Palm, 2017; Black & Wiliam, 1998; Sintayehu, 2016; Yiheyis & Getachew, 2014). Assessment for learning (hereafter AfL) is also a means through which students and teachers collect valuable information regarding the learning of students and then apply the information to modify classroom instructions in order to improve students' learning (Swan & Foster, 2016). Moreover, as Black (2015) stated, the importance of AfL lies in its impact on the central aim of education, which is to enhance students' effective, responsible, and independent learning capabilities.

A vital characteristics of AfL is the detection of each and every student's misunderstandings and understandings in the teaching and learning process. As research stretching back decades in mathematics shows and modern work on learning progressions strives to address, students' errors in mathematics typically stem from misconceptions of earlier mathematical experience. Studies on mathematics teaching and learning show that, when students apply what they have learned to new situations, they often make mistakes

by overgeneralizing or extending the rules and formulae in an incorrect way (Burkhardt & Schoenfeld, 2019). Moreover, AfL has the ability to assist students in understanding the specific content of the topic they learn; however, its applications extend beyond the content related to the subject. Assessment for learning can also help achieve other important goals, such as developing the skills that future professionals need to perform their roles in a way that suits the current social situation (López-Pastor & Sicilia-Camacho, 2015).

One of the best ways to enhance both students' learning and teachers' teaching is by implementing AfL strategies appropriately. However, most classrooms lack effective AfL, partly because teachers misunderstand what it is and lack the essential skills to implement it appropriately and effectively (Moss et al., 2009). Furthermore,

Assessment is part of a teaching and learning process where the teacher and students work together to gather and use valuable information about students' learning on an ongoing and regular basis with the clear goal of improving student achievement (Moss et al., 2009).

Understanding the natural and social realities in which we live is made possible by mathematics, a field of study that is incredibly significant and pertinent to nearly all other areas of study. Learning mathematics improves rational decision-making ability, logical reasoning, critical thinking, and problem-solving abilities, all of which are essential in the present world where technology, information, and communication play vital roles (Ferretti et al., 2018). Mathematics not only helps us develop new ways of thinking and applying them to everyday life, but also allows us to critically analyze and evaluate the huge amounts of data generated in today's knowledge society. Moreover, it shapes how we practice citizenship by using logical, reasoned, and justified social decision-making processes. It is nearly impossible to fully access the contemporary world made accessible by information,

communication, and technology without adequate mathematical competency (thinking, logical reasoning, etc.) (Ferretti et al., 2018).

Pre-service teachers should have the necessary general and subject-specific pedagogical knowledge, knowledge of the subject matter they teach, practical and theoretical assessment skills and experiences, and real-life examples of how to teach their students effectively in order to enhance their future students' learning outcomes (DeLuca, Coombs, et al., 2018; Haftu et al., 2018). Despite these arguments, teachers find it challenging to engage in AfL at first. In this regard, mathematics teachers face a special challenge because teaching and learning have traditionally focused on a narrow set of learning objectives, usually involving technical skills taught as routines through observation and repetition; therefore, change is clearly difficult (Burkhardt & Schoenfeld, 2019).

The low scores of students in mathematics tests at different levels of education and the demand for extra training at workplaces have motivated a change in every aspect of mathematics education such as pedagogy, assessment and curriculum (Greenes, 2009). What stands out as an effective classroom assessment for mathematics teachers is to enhance their teaching techniques and strategies in the following three ways: use assessment strategies to identify and designate misconceptions of students; use these identified misconceptions as opportunities for the teaching and learning process; and provide second chances for students to demonstrate their success in learning (Chigonga, 2020).

Algebra is one of the branches of mathematics that many students find challenging. Students' failure rates are frequently high. Because algebra is so critical to a student's future academic life, it's crucial to pay attention to the instructional methods utilized to present algebraic content (Cousins-Cooper et al., 2017). Many authors recognize the challenges in teaching and understanding abstract algebra at the college or university level (algebraic structures theories, especially theories of groups, rings, and fields) (Hausberger, 2018). For a long time, scholars in mathematics education and research have been investigating the algebra problems that students face. Researchers have examined several components of algebraic thinking, which are thought to be crucial in overcoming those obstacles, using a number of theoretical frameworks. Several solutions to these problems have been proposed, and their effectiveness has been studied based on the study findings (Warren et al., 2016).

According to results of a research attempting to identify dominant factors causing difficulty with abstract algebra learning, students' attitudes toward abstract algebra, prior knowledge, and learning materials are three strong factors, while motivation, lecturer performance, and learning technique are three fair elements (Agustyaningrum et al., 2020). Moreover, Agustyaningrum et al. (2020), following interviews with students, discovered that the students' internal factors included their impression of the difficulty of abstract algebra courses, low learning motivation, and insufficient prior knowledge. Meanwhile, external variables include a scarcity of learning resources that might assist students in quickly comprehending material and facilitating self-regulated learning skills (SRLS).

Higher education students' lack of comprehension and understanding of algebraic principles and procedural abilities causes major difficulty in addressing mathematical issues. Assessment for learning is advised to help students learn mathematics (Stewart & Reeder, 2017). It was also shown that AfL can significantly improve achievements of students, although research on how to effectively help instructors apply this approach is lacking (Andersson & Palm, 2018). Even though there are simple and clear basic priorities in assessment practices, there are also radically different and complex assessment profiles across teacher educators in various teacher education institutions (DeLuca, Coombs, et al., 2018). As pre-service teachers learn about assessment, these profiles present them with a variety of views as well as diverse solutions and justifications to challenges of using assessment. While these many viewpoints provide valuable learning opportunities, the problem is to provide pre-service teachers with the time and pedagogical space to integrate, deconstruct, and negotiate these viewpoints together with assessment theories in order to establish their own professional stance (DeLuca, Coombs, et al., 2018).

Present-day strategies to assessment practices in the classroom have shifted away from considering assessment as a sequences of procedures that accurately measure students' learning toward considering it as a social practice characterized by providing continuous information to help students' learning and guidance to the practices of the teacher (Suurtamm et al., 2016). They also argued that assessment practices in the classroom should represent both the mathematics that is necessary to learn and that is seen as valuable. Consider if and how activities measure the complicated nature of mathematics as well as the curriculum or standards being assessed. Assessment design, on both a wide scale and in the classroom, should place a premium on problem solving, modeling, and reasoning.

Preparing beginning well-qualified mathematics teachers for primary schools is vital for any nation that seeks improving quality of education in primary schools. Mathematics teacher education for primary schools has received a lot of attention, in recent years due to its impact and significance on the overall development of children (Alex & Roberts, 2019; Blackley & Howell, 2015), which has piqued our interest in entering the field of mathematics teacher education and training for primary schools (Guerios & Goncalves, 2019). This is very important since well-prepared pre-service teachers have a positively effect to school effectiveness and student achievement (Tasdemir et al., 2020).

Pre-service teachers require opportunities to conduct in-depth investigations of how the classroom's social ecology is generated in order to assist students in monitoring and evaluating their own learning. In addition, an emphasis on discourse is required, as are the types of questions and replies that promote self-regulation in learning. Beyond pre-service education, ongoing professional learning may benefit teachers by engaging them in an in-depth examination of their practice. This analysis should focus on questioning sequences that enable instructors to collect evidence of student learning within a framework of co-regulation (Heritage, 2016). Additionally, teachers need to have adequate time to fully comprehend and understand the mathematical curriculum. To learn how to accomplish elementary school mathematics, why these processes make sense, how to represent these answers using a variety of representations, how the specific feature of material ties to other topics and grades, and at what point children are ready to acquire this content, they need time and intense instructional support (Alex & Roberts, 2019).

Despite the fact that teacher education is regarded to have a big influence on how teachers approach assessment, especially in the fair treatment of students in the assessment process in the classroom (Coombs et al., 2018), AfL principles are only briefly covered in educational study courses and the majority of in-service teachers do not receive enough direct education and training to enable them to become assessment-competent educators (Deluca & Volante, 2016).

In addition, research repeatedly reveals that lecturers do not always teach in ways that research consistently supports as best practices for student learning, despite efforts to educate and train teachers to employ best practices in the classroom. Instead, teachers often overlook the results of educational research in favor of what they believe to be the best methods of instruction. Here, the distinction is crucial. Teachers' beliefs influence both what they do in the classroom and what they view as evidence of learning (Moss et al., 2009).

Pre-service teachers can be significantly shaped by teacher educators' knowledge of teaching, learning, and assessment as well as their responses to pre-service teachers' queries, problems, and prior experiences. These prior understandings of teaching, learning, and assessment may or may not reflect the best methods of contemporary teaching and learning practices (DeLuca, Coombs, et al., 2018). Similar to this, pre-service teachers are more likely to incorporate AfL with ease into their classroom and enhance the learning of their future students when they are exposed to AfL experiences in teacher education institutions in a regular and efficient manner and this helps to create an AfL culture in the schools where they work (Deluca & Volante, 2016).

It is proposed that lecturers, as essential figures in the learning process, use active and innovative learning strategies to encourage students' enthusiasm and good attitudes toward abstract algebra classes. Furthermore, in order to reinforce the students' past knowledge, the lecturer must recall the pre-requisite materials needed to understand the new concepts. Meanwhile, due to a dearth of quality learning materials, lecturers may need to create their own textbooks based on the characteristics of their students and the principles of good teaching materials. The textbooks should allow students to study freely while also assisting in the reinforcement of concepts based on past knowledge that is essential to learning the core subject (Agustyaningrum et al., 2020).

Teachers must have expert knowledge and understanding of their subject matter in order to give students relevant and appropriate feedback as they go through the curriculum levels. They must also be aware of what each student is thinking and knowing in order to design meaningful experiences based on this information (Hattie, 2008). Furthermore, because AfL is a crucial component of good classroom education, Burton et al. (2018) argued that mathematics teachers and mathematics teacher educators must comprehend AfL techniques, be able to use them in mathematics instruction in their classrooms, and spread the word about the tactics to others. In a situation in which teachers have little knowledge of AfL experiences, it's more likely that classroom implementation of AfL will be rigged with flaws and end up deviating from its intended purpose. Furthermore, it is worth noting that effective AfL experience is a function of a teacher's knowledge of the procedure (Amoako et al., 2019).

The employment of the same teaching and assessment strategies by teachers as students has a negative effect on educational systems at non-college and university levels and considerably impedes the development of learning- and improvement-oriented systems of assessment (López-Pastor & Sicilia-Camacho, 2015). It is argued that when teacher candidates are provided with AfL experience in an effective and consistent way, they are presumably inculcating AfL in their classrooms. As a result, AfL integration may enhance future students' learning throughout all of their educational institutions in addition to teacher candidates' learning during the course of their teacher education programs (Deluca & Volante, 2016).

Although the emphasis on AfL seems to correlate extremely well with theories of self-regulated learning and appears to be the main driver of many researchers' interest in AfL, the relationship between the two is disputed (Panadero et al., 2016). Andrade and Heritage (2017) argued that AfL can help students develop SRLS, although they believe that the relationship between AfL and SRLS is primarily theoretical. They also contended that AfL approaches such as offering constructive feedback, both by the teacher and by the students, assist students in taking charge of their own learning. Furthermore, a fully situated, synergistic understanding of SRLS and AfL has significant educational ramifications, including the need for additional pre-service SRLS training, greater support for ongoing teacher PD and reflection opportunities, and an educational system that appropriately values and privileges SRLS and self-assessment as desired and essential outcomes of formal education (Greene, 2020). Students are said to exhibit the self-regulatory traits that seem to be most wanted in students (self-monitoring, self-evaluation, self-assessment, and self-teaching) when they assume the position of a teacher. As a result, the difference is

created by the teachers' and students' apparent teaching and learning (Hattie, 2008). Additionally, in order to do new and difficult tasks in today's fast changing environment, students and pre-service teachers must be able to properly monitor and update their learning (Panadero et al., 2019).

Assessment for learning is believed to have a dramatic effect on students because it engages them in the process of learning how to learn. Students gain more knowledge, become more intelligent, and develop into self-aware learners who can articulate precisely how they arrived at their current location. In other words, kids learn how to manage their own learning and make informed judgments. They are taught to gather evidence of their own learning and to use that evidence to choose from an ever-expanding range of success tactics (Moss et al., 2009).

The relationship between SRLS and AfL has been the focus of research for a while. It is an active area of research that is garnering more attention and in which studies are elucidating critical concerns. Notably, the relationship between students' assessment expertise, which is mostly developed through self- and peer-evaluation and is currently regarded as the development of evaluative judgment, requires more research employing self-regulated learning models (Panadero et al., 2019).

The majority of contemporary self-regulation theorists have resisted defining internal and external control of learning as absolute opposites in favor of a broader, more dynamic view of self-regulation. Not only through hidden cognitive methods, but also through overt behavioral ones like choosing, modifying, or creating advantageous personal environments or asking for social support, students can self-regulate their learning. The sense of self that a learner develops extends beyond the individualized forms of learning and encompasses

self-coordinated collective forms of learning, in which the learner achieves personal goals either through the activities of others—such as family members, teammates, or friends— or through the use of tools or other resources from the physical environment. As a result, behavioral, social, and environmental self-regulatory processes are thought to be mutually dependent on concealed self-regulatory systems (Zimmerman, 2015).

Learners are exhibiting self-regulation of learning when they set goals and then consistently carry out cognitive, affective, and behavioral practices and processes to move toward those goals. To some extent, self-regulated learning is supported by the data acquired through classroom assessments of student learning and accomplishment (Andrade & Brookhart, 2016). Additionally, when students set goals and then control their thoughts, feelings, and actions to attain them, they are engaging in self-regulated learning (Andradel, 2019).

The goal-setting process, for example, is influenced by a teacher's shared success criteria and learning intentions. Feedback from summative and formative assessments has an impact on the progress-monitoring phase. The revision and adjustment phases are influenced by teachers' opportunities for students to use feedback and by the choices they make in response to it (Andrade & Brookhart, 2016). The outcomes of a meta-analysis indicate that self-assessment treatment, which is one component of AfL, benefits students' self-efficacy and SRLS (Panadero et al., 2017). Students' knowledge that their work may be presented to their classmates had an effect on the quality of their work since they reviewed it from a different perspective. This prompted lower-achieving students to participate more completely in assignments, enabling them to take charge of their own learning (Wright et al., 2018).

One of the most significant issues facing teacher education and PD is that teacher education and training (both in-service and pre-service) have not placed a sufficient focus on topic knowledge and contemporary instructional techniques. Teacher education and training programs need to offer pre-service teachers the pedagogical skills necessary to implement active learning and child-centered methods in the classroom (Workneh & Tassew, 2013).

Darling-Hammond and her colleagues examined 35 pieces of research from the previous three decades to identify factors of good professional growth and discovered seven qualities.

According to Darling-Hammond and her colleagues, effective PD is long-lasting, content-driven, involves active learning through adult learning, fosters cooperation often in circumstances where it is ingrained in the workplace, uses models and examples of effective practice, offers coaching and expert support, and provides chances for feedback and reflection (Darling-Hammond et al., 2017). The study of personalized PD is still in its early stages. It is critical to investigate how to build personalized PD that better supports the learning experiences of individual teacher educators, as well as whether this type of PD will improve instructional practice (Zhang et al., 2021).

According to Hamodi et al. (2017), more AfL integration is needed in teacher preparation programs. They discovered recurring trends that might have an impact on regional teacher education programs by looking at chronological, programmatic, and institutional restrictions. The claim is supported by their key premise, which states that teacher candidates are more likely to instill an AfL ethos in their own courses when they are regularly and effectively exposed to AfL. Therefore, when pre-service teachers enter the teaching profession, AfL integration may help not only their learning during initial teacher

education programs but also their respective students' learning throughout educational systems. In order to support mathematics teacher educators' incorporation of AfL techniques in their instruction, it is also crucial to identify barriers that hinder the spirit of AfL and their conceptions and implementation levels in their classrooms, as stated by DeLuca et al. (2019). Furthermore, it was noted that to satisfy the various needs of children and teachers in a range of circumstances, job-embedded, context-specific, and content-based teacher professional development is especially crucial (Darling-Hammond et al., 2017).

Researchers and educators generally concur that teachers' classroom assessment methods, particularly in AfL, lag behind the state of the art in research. This disparity has several root causes, some of which are the traditional nature of school culture and the dearth of adequate professional development in AfL. Additionally, despite the fact that AfL is a crucial element of successful and effective educational institutions, in-service teachers have a limited understanding of how to effectively employ these strategies and practices in their classrooms (Deluca & Volante, 2016). In addition, the environment for teaching and learning both inside schools and at the system level may also hinder the effectiveness of PD. Inadequate funding for professional development, such as suitable curriculum resources, frequently worsens inequity and stymies efforts to reform schools. Important barriers include a dysfunctional school culture and a failure to match policies with a unified set of practices.

Sensitivity to the requirements of educators and students as well as the surroundings in which teaching and learning will take place is also necessary for effective professional development (Darling-Hammond et al., 2017). High-scoring benchmark instances can give "expert models" for teachers to strive to if they are available, and the utilization of an observation technique can help instructors and coaches establish and internalize a shared knowledge of the quality of their work (Lyon et al., 2019). Accordingly, in addition to coaching, teacher peer and self-observation may be another method to promote practice. Time spent learning the protocol could be considered professional development because the rubrics assist to codify the AfL domain, which can help teachers better comprehend the elements of an effective AfL experience (Lyon et al., 2019).

No matter how extensive the research base, it is stated that if concepts are given as general principles that leave it entirely up to the teachers to implement them in classroom practice, teachers will not embrace them. What educators need is a wide range of real-world implementation examples from educators they can relate to and who can inspire confidence in their own abilities. The best way to do things in the real world must be demonstrated for them (Black & Wiliam, 2010). Without taking into account the greater context in which it takes place, assessment cannot be comprehended. According to Black and Wiliam (2018) teachers and schools are constrained, at least in the short term, by cultural traditions, political and public expectations of education, and the norms of the several institutions within which they operate.

In-service and pre-service teacher education students are still taught by teacher educators in Ethiopian colleges of teacher education using a lecture method (Alemu et al., 2019). Geberew and Tigist (2018) also emphasized that there is little information about AfL, assessment as learning, and assessment of learning in the Ethiopian context and that it is unknown how and when these evaluations are used across various stages of the teaching and learning process. To strengthen the broad arguments on the relevance of AfL in teaching, additional research on the impact of professional development on AfL and ways to AfL implementation at different levels and contexts is advised (Burton et al., 2018).

One of the most significant challenges facing teacher education, training and PD in Ethiopia is that training (both pre-service and in-service) has not placed a sufficient focus on topic knowledge and contemporary instructional techniques. This results in the school system being harmed by teacher-centered instruction. Programs for teacher education and training should provide prospective teachers with pedagogical skills necessary to implement active learning and student-centered methods in the classroom (Workneh & Tassew, 2013). In addition, Yiheyis and Getachew (2014) found in their study on the application of AfL at writing classes in one of the twelve teacher education institutes in the Oromia area, that pre-service teachers received more quantitative feedback from their English professors. Additionally, it was discovered that continuous assessment was rarely used for diagnostic purposes and that students had less opportunity to check on their own learning. Furthermore, both self- and peer-assessment processes were underused (Solomon, 2014), and there was a significant preference for summative rather than formative assessment.

In their study of the challenges and opportunities in adopting AfL in three institutions in north Ethiopia, Teklebrhan and Samuel (2015) also found that the implementation of AfL is very poor, with both instructors and students having inadequate comprehension of and unfavorable attitudes toward AfL. Similarly, Sintayehu and Ashenafi (2017) found that instructors' negative attitudes about AfL, gaps in appropriately applying AfL by instructors, and exploiting AfL results for grading purposes were important issues connected to instructors in their studies of challenges in adopting AfL in a south-eastern institution of the country. Additionally, it was determined that obstacles to successfully deploying AfL tactics included students' cheating tendencies, a lack of professional AfL assistance for teachers, and students' reluctance to use AfL techniques.

This study was therefore conducted to design a comprehensive job-embedded, content based, and context-specific PD for MTEs on AfL and training for PPMTs on SA and PA and then investigate their effects on PPMTs' achievement gains, conceptions of AfL practices and SRLS, as well as on the progress of implementation of AfL strategies in the teaching and learning process.

1.2 Statement of the Problem

Since teachers are responsible for interpreting, translating, and implementing educational policies into action, teacher education can be considered crucial in producing quality beginning teachers and improving the quality of education in any nation (Gemechu et al., 2017). There is a claim that many instructors find it challenging to integrate generic principles of quality education into their own teaching and learning practices in the classroom; therefore, discipline-specific examples are valuable to instructors (Quinlan &

Pitt, 2021). Unfortunately, many of the lecturers in teacher education colleges in Ethiopia are subject specialists (who specialize in algebra, analysis, differential equations, optimization, etc.) who are themselves products of a culture that considers subject mastery as a core foundation of quality teaching and learning (Haftu et al., 2018).

Research related to the intervention effects of AfL strategies implementation in teacher education colleges, especially in primary mathematics teacher education programs, is scarce in the Ethiopian context. Sintayehu (2016), in his study of AfL practices in primary schools in one of the towns of the Amhara region, found that the practices of AfL lack consistency and harmony. Besides, he found out about the existence of a discrepancy between actual practices and the perceived purposes of AfL. He also reported a common misconception among primary school teachers: they consider AfL to be continuous testing.

Further investigation into the impact of PD on AfL and strategies for implementing AfL in various settings and levels is advised in order to support the overall justifications for the use of AfL in teaching (Burton et al., 2018). The use of AfL, assessment as learning, and assessment of learning in an Ethiopian setting has also received minimal attention, as noted by Geberew and Tigist (2018), and it is unclear how and when these assessments are employed at various levels of the teaching and learning process. Additionally, while each AfL component might be effective on its own, when combined with other components, the effects could be positive or negative. Therefore, it is crucial that the field understand the effects of an integrated approach to AfL implementation (Lyon et al., 2019).

In secondary, primary, and upper primary schools around the world, a significant amount of study has been conducted on the effect of a PD on AfL for mathematics teachers on students' mathematics achievements and classroom practices of AfL. Andersson and Palm (2017) conducted a quasi-experimental study to examine the effects of implementing combinations of formative assessment strategies in mathematics lessons on year four students' achievements in mathematics after the teachers participated in a PD program that takes six hours of training a week for a total of 144 hours. The PD consisted of lectures on the meaning, importance, and examples of practical activities in formative assessment. The result of the study showed that the achievements of students whose teachers participated in the intervention were significantly higher than those of students whose teachers were in the control group ($p = .036$ and $d = .66$) after controlling for their scores on the pretest.

Additionally, Chemeli (2019) used a pre- and post-test quasi-experimental design with control and treatment groups to investigate the effects of the five core AfL techniques on secondary school students' mathematics achievement in Kenya. His study's findings showed that the treatment group's performance was significantly higher than the performance of the control group's students ($p < 0.05$; $t = 14.82$).

Mathematics teacher educators may have different conceptions and implementation experiences of AfL in providing mathematics courses to PPMTs, and the pre-service teachers' learning of mathematics courses and conceptions of AfL experiences are largely based on what their teachers do. The teacher educators actions also rely on their personal beliefs, conceptions, and experiences to guide their instant decisions in the vastly contextualized setting of their classroom (Black, 2015). In line with this DeLuca, Coombs, et al., (2018) found that teacher educators have complex and vastly different profiles of

assessment, although they have clear common priorities in assessment. Moreover, Geberew and Tigist (2018) strongly suggested that research be conducted to understand what kind of AfL strategies teachers employ when they assess their students across different levels of educational institutions in the Ethiopian context. Besides, it is argued that progress in improving the quality of primary teacher education can be made solely by openly recording related problems and involving individuals taking part in training to become elementary teachers in finding solutions (Alemu et al., 2019).

The study of teachers' conceptions of AfL is an area of study that is expanding globally (Barnes et al., 2017; Fulmer et al., 2015), but more research is needed to understand how pre-service primary math teachers and mathematics teacher educators conceptualize AfL in relation to Ethiopian primary school mathematics teacher preparation. Furthermore, investigations of the effect of a PD on AfL in primary mathematics teacher education will contribute to the global effort to understand the impact of AfL in teacher education institutions. Besides, researchers and educators recommend further research on the professional support mathematics teachers need periodically in implementing AfL (Andersson & Palm, 2018; Deluca & Volante, 2016), to examine the nature of teachers' beliefs about and conceptions of assessment for learning in different cultures and contexts (Barnes et al., 2017; DeLuca et al., 2019; DeLuca, Coombs, et al., 2018), on how AfL is integrated and operationalized across different student learning groups, disciplines, and curricular areas and effective professional learning structures in AfL (Birenbaum et al., 2015), on the impact of implementing PD on AfL (Burton et al., 2018; Pattalitan, 2016), and on how to improve their students' and their own feedback literacy (Carless & Boud, 2018).

Understanding how AfL is being implemented and proposing what can be done to improve existing classroom implementations requires taking the context of the teaching and learning process into account. In agreement with this notion, Black and Wiliam (2018) contended that it is difficult to understand the implementation of assessment without considering the wider context in which that assessment takes place, as schools and teachers are constrained by the political, cultural, and public expectations of education, and recommended further research that is context-specific, job-embedded, and content-based to address the diverse needs of students and teachers. Besides, Deneen et al. (2019) also argued that the way AfL is developed and implemented can vary considerably across different contexts and stakeholder groups and recommended further research on AfL that takes into account the contexts in which it is enacted.

Moreover, there is a need of more systematic studies that investigates and examines the relationship between AfL and SRLS among pre-service teachers in developing countries like Ethiopia. This type of research can give vital clues about the major elements that should be included in a classroom assessment technique geared toward pre-service teachers' SRLS (Allal, 2020). Moreover, despite the widely agreed-upon influences of teacher educators' conceptualizations of, approaches to, practices of, and beliefs about AfL on pre-service teachers' learning and implementations in their future classrooms, few researchers have robustly examined teacher educators' conceptualizations of, approaches to, and practices of AfL (DeLuca, Coombs, et al., 2018).

Additionally, even though there is a growing body of research on the practices of AfL, most of those researches have been done in the context of Western education (Gan et al., 2019). According to some, schools at different levels can enhance their practices of

implementation of AfL strategies in instructional processes by revising and modifying teacher preparation programs, including teacher education institutions in the delivery of PD, and boosting institution-level support for teacher education and training and support that lasts for long period of time (Johnson et al., 2019).

As far as the researcher's knowledge is concerned, little study has been conducted on the effect of context-specific, job-embedded, and content-based comprehensive PD on the implementation of AfL strategies for MTEs and training for PPMTs on how to provide and receive constructive feedback, on SA and PA. This study intends to close this knowledge gap by investigating the conceptions of AfL held by MTEs and the effect of context-specific, job-embedded, and content-based PD for MTEs on AfL; training for PPMTs on SA, PA and receiving and providing quality feedback on PPMTs' SRLS, achievement gains, conceptions of AfL experiences and on the practices of implementation of AfL strategies in the classroom instruction of Fundamental Concepts of Algebra.

1.3 Research Questions

The research questions this study intends to answer are:

1. How do mathematics teacher educators in a CTE in Amhara region conceive the nature, principles, strategies, and purposes of assessment for learning?
2. How is professional development on assessment for learning in Fundamental Concepts of Algebra (Math 221) experienced by mathematics teacher educators?
3. How do mathematics teacher educators and pre-service primary mathematics teachers who participated in PD in Assessment for Learning implement it in the teaching learning of Fundamental Concepts of Algebra (Math 221) course?

4. Is there a significant mean gain difference between the treatment and comparison groups on pre-service primary mathematics teachers' achievement gains in the Fundamental Concepts of Algebra (Math 221)?
5. Is there a significant mean gain difference between the treatment and comparison groups in pre-service primary mathematics teachers' conceptions of assessment for learning experience?
6. Is there a significant difference in pre-service primary mathematics teachers' self-regulated learning skills before and after the intervention?

1.4 Objectives of the Study

The objectives of this study are to examine:

- AfL conceptions of mathematics teacher educators in a teacher education college,
- How the comprehensive PD for MTEs on AfL in the teaching and learning of Fundamental Concepts of Algebra (Math 221) is experienced by them and
- the effect of comprehensive job-embedded, context-specific, and content-based PD on AfL strategies implementation for mathematics teacher educators and their implementation, and training for PPTM on SA and PA in the teaching and learning of Fundamental Concepts of Algebra (Math 221) course, PPMTs' achievement gain in Fundamental Concepts of Algebra (Math 221) course, on their conceptions of AfL experience, and their SRLS.

1.5 Significance of the Study

The majority of educational researchers and educators agree that AfL, if implemented effectively, can improve students' learning of various disciplines at different levels. As a result, implementing AfL strategies in the classroom instruction of Math 221 (Fundamental Concepts of Algebra) in a Teacher Education College was expected to benefit mathematics teacher educators in gaining broader and deeper understandings of the significance of AfL, as well as related challenges of implementing AfL and corresponding solutions for improving pre-service primary mathematics teachers' achievements.

The findings of this study have several implications for improving the quality of mathematics teacher education in CTEs in Ethiopia. First, they provide a glimpse of information about the conceptions of AfL of MTEs in CTEs. This information can help stakeholders, such as college deans, Education Bureau officials, and Ministry of Education experts, to design and implement possible interventions to enhance the incorporation of AfL strategies and the quality of teacher education in CTEs. Second, they provide information to the regional education bureau and the Ministry of Education on the level of practices for implementing AfL strategies in algebra courses in CTEs. This information can help to monitor and evaluate the effectiveness of AfL practices and to identify areas of improvement and support for MTEs and PPMTs. Third, they provide MTEs and PPMTs with real-time experience to implement the five AfL strategies: clarifying learning intentions and success criteria, eliciting evidence of learning, providing feedback, peer assessment, and self-assessment. This experience can help them to develop their skills and confidence in using AfL strategies in their teaching and learning processes. Fourth, they provide information about the effect of a comprehensive, context-specific, job-embedded,

and content-based PD on AfL for MTEs and a training on SA and PA for PPMTs. This information can help to evaluate the impact of the PD and training on the implementation of AfL strategies, on their achievement gains in mathematics courses, on their conceptions of AfL practices in the teaching and learning process, and on their SRLS. Finally, they can be used as a source for further research on assessment for learning in mathematics teacher education.

1.6 Delimitation of the Study

This study was conducted in one of the regional states of Ethiopia, the Amhara Regional State, due to feasibility and time constraints on the researcher. Moreover, in this study, only second-year PPMTs in three different teacher education colleges were involved as sources of data. Furthermore, the study focused on a single course, Fundamental Concepts of Algebra. In terms of instructional approaches, this research delineated the implementation of the five AfL strategies by MTEs who provided fundamental concepts of algebra and by their second-year PPMTs who took the course. Furthermore, this research is limited to achievement gain, conceptions of AfL experiences, implementation of AfL strategies in the classroom instructional process, and SRLS in terms of variables.

1.7 Limitations of the Study

In this study, there are some limitations that were believed to affect the results of the study. One of the limitations of this study is that PPMTs in the intervention group were not asked using interview questions about how the enhanced implementation of AfL influenced their SRLS in the teaching and learning of Fundamental Concepts of Algebra. The second limitation of the study is that interviews were not undertaken to get more data regarding how PPMTs in the intervention group got the training on SA and PA and how they believed

it benefited them. The research may have been much stronger if this had been done. The third limitation of this study is that it used only pre-intervention and post-intervention scores of PPMTs in the treatment group for the SRLS to examine the effect of enhanced implementation of AfL strategies on PPMTs' SRLS. This study could have been strengthened if pre- and post-measures of SRLS of PPMTs in the comparison groups were also taken and compared. Moreover, self-report tools, like questionnaires and surveys, are less capable of capturing key processes at a very fine grain size in terms of real cognitive events or methods utilized by students when studying and learning (Pintrich, 2019).

1.8 Operational Definitions of Key Terms

In this research, the following operational terms are used:

Assessment for Learning (AfL) in this study is defined as a deliberate process by which assessment information is collected by MTEs, PPMTs, and their peers and used to identify gaps and strengths and adjust the classroom instructional process in a lesson.

Conception in this research is inclusive of understandings of, dispositions, attitudes towards, perceptions, and other terms that suggest knowledge and belief about a phenomenon.

Conceptions of assessment for learning experience include awareness and understanding of the implementation of AfL strategies by the students themselves and the teacher in the classroom instruction process of a course (Pat-El et al., 2013).

Achievement gain in this study is a student-level alternative to Hake's (1998) normalized gain measure, called the normalized change score, which is symbolized by c and defined by Marx and Cummings (2007) as:

$$C = \begin{cases} \frac{\text{Post}-\text{Pre}}{100-\text{Pre}} & \text{if post} > \text{Pre} \\ \text{drop} & \text{if Post} = \text{Pre} = 100 \text{ or } 0 \\ 0 & \text{if } 0 < \text{post} = \text{pre} < 100 \\ \frac{\text{Post}-\text{Pre}}{\text{Pre}} & \text{if Post} < \text{Pre} \end{cases}$$

Professional development is a process that involves theoretical discussions and reflection as well as practical trainings that help understand and implement educational strategies to improve the classroom instructional process.

Mathematics teacher educators (MTEs) are lecturers in teacher education colleges who provide mathematics and methods of teaching mathematics courses to PPMTs.

Pre-service primary mathematics teachers (PPMTs) are pre-service mathematics teachers who are attending their three-year diploma program to be mathematics teachers in the primary (grades 1-8) schools.

1.9 Organization of the Study

The first chapter of this study comprises the following: statement of the problem, background of the study, objectives of the study, research questions, significance of the study, limitations of the study, delimitation of the study, and operational definitions.

Chapter two discusses the theoretical framework of the study from the social constructivist theory and sociocultural theory of learning of learning perspectives, the conceptual framework of the study, introduction of AfL, including the principles, strategies, nature,

purposes, and challenges of implementing AfL, mathematics education, and assessment for learning; PD and training on AfL and its challenges; students' SRLS and AfL; research results on the effects of AfL on students' achievements; SRLS, and conception of AfL experiences; and a summary review of the literature.

Chapter three presents the research design, research paradigm, research approach, participants of the study, variables of the study, samples and sampling techniques, the intervention, data collection procedures, data collection instruments, development of instructional material, data analysis, and ethical considerations.

Chapter four presents the qualitative and quantitative results of the research and the qualitative and quantitative discussion of the study.

Finally, chapter five presents the implications, the conclusion, the recommendations, and the summary of the study.

Chapter Two: Review of Literature

2.1 Introduction

This chapter discusses the conceptual and theoretical framework of the study, reviews of literature related to assessment for learning, quality professional development, teachers' conceptions of assessment for learning, the relationship between implementation of AfL strategies in classroom instructions and students' SRLS, and the effects of AfL on students' achievement gains and SRLS.

2.2 Theoretical Framework

2.2.1 Theoretical Framework

Sociocultural Theory of Learning

Learning theories are usually descriptive generalizations about how people learn. They serve as a bridge between educational research and practices and also as a tool to translate and organize the findings of research into suggestions and recommendations for educational practice (Schunk, 2012). Most of the research in mathematics education is guided by learning theories, and every learning theory has its own assumptions about how students learn and how they can be motivated to learn, as well as its own implications for teaching and learning (Simon, 2017). The theoretical foundation of AfL is eclectic, wide-ranging, and usually not explicit. In global education, various learning theory backgrounds lead to differing emphases; for example, the American behaviorist tradition places more emphasis on testing and the teachers' role, while other English-speaking cultures place more emphasis on constructivist and sociocultural thinking to emphasize the role of the student and informal classroom assessment (Baird et al., 2017).

All learning is fundamentally social, requiring the individual's interaction with his or her social surroundings through the use of shared language, tools, norms, and practices (Shepard et al., 2018). Various learning theories influence mathematics teaching and learning at various levels. The abstract nature of algebra, as well as the usage of multiple symbols and the rules for utilizing them effectively, makes teaching and learning difficult. Sociocultural theory may help us understand how knowledge is created collaboratively in communities of practice and, consequently, how meaningful involvement can be damaged in unsupportive learning contexts (Shepard et al., 2018).

The learning theory of Vygotsky is gaining popularity in the area of mathematics education. A student's intellect develops as a result of social engagement with the outside world, speech, social collaboration, and cooperative effort, all of which are significant components of the social world, in accordance with the sociocultural theory of learning (Cottrill, 2003). Additionally, according to sociocultural theory, the objectives of educational classroom assessment should be to (a) discover knowledge and skills that are developing and (b) attempt to foresee what the learner will execute on their own in the future (Scott & Palincsar, 2012).

In order to help teachers create an information literacy community of learners where students interact with one another and help teachers comprehend zone of proximal development (ZPD) so that students can realize their full potential, the sociocultural theory of learning offers good models for building collaborative learning activities that enable students to actively participate in the learning process (L. Wang, 2007).

According to Vygotsky, ZPD is the gap between a student's current degree of growth (working on difficulties) and his or her prospective level of development (working with an adult). This ZPD permits the adult to act as the child's "tool bearer," or conscious controller of the concept, until the infant is able to assimilate external knowledge (Cottrill, 2003). Additionally, a sociocultural theorist could make judgments about an individual's thinking and development based on his or her engagement in culturally organized activities while examining a learning situation by taking into account the larger social structure in which the teaching and learning process occurs (Scott & Palincsar, 2012).

The co-design of assessment, instruction, and curriculum, is made possible by endorsing sociocultural theory as the most successful grand learning theory, allowing for deep learning through time. Fine-grained explanations of learning objectives and knowledge of common related misconception and learning difficulties are also necessary for assessment design, specifically to address the key question "what next?" (Shepard et al., 2018). The sociocultural learning theory also views teachers' professional identities as changing as a result of their interaction in social surroundings. Researchers and scholars contends that sociocultural understandings can similarly be utilized to inform interventional research that involves changing classroom practice, even though the majority of sociocultural studies in mathematics teacher education have focused on investigating teachers' learning (Goos, 2013).

The fact that Vygotsky's theory of learning for PD aims to close the gap between theory and practice gives it a substantial edge over other competing theories. Vygotsky's approach, in contrast to other models that emphasized on practical difficulties, encompasses both theoretical and practical components; it links theory and practice by outlining the intricate

mechanisms of learning processes in actual sociocultural contexts. In order to preserve the effectiveness of teacher education and training, it also underlines the significance of follow-up support systems (Shabani, 2016).

Learning happens through conversation, negotiation, and collaboration, according to a sociocultural approach. Sociocultural theory-informed education pays special attention to practices, norms and the discourse associated with specific communities of discourse and practice. The goal of education is to motivate students to engage in activities, communicate, and use tools in a way that is appropriate for the community to which they are living with (Scott & Palincsar, 2012). Additionally, ZPD describes the gap between what a student can accomplish on their own and what they can accomplish with the assistance of and in collaboration with competent individuals from a sociocultural perspective.

Besides, sociocultural theory of learning, particularly the idea of ZPD, suggests that the goal of assessment (for learning) should be to determine the stage of learning that each student is at and attempt to predict what they can learn independently and what kind of scaffolding they need to proceed with their learning (Scott & Palincsar, 2012). In addition, rather than underlining the acquirement of new knowledge, the theory considers a teacher's learning as a change in engagement in social behaviors that enhance their professional identities.

By focusing on both "who learners are when they join a community" and "who they could become," sociocultural learning theory facilitates the development of treatments that address fairness in educational environments. They enable students to think about what is at risk when they apply new knowledge or skills in a particular setting (Shepard et al.,

2018). According to Goos (2013), sociocultural approaches to inquiries on mathematics teacher education can help to both understand and advance teacher learning. From this perspective, learning is viewed as involvement in social activities that aid teachers in creating their professional characters and have an impact on the growth of pre-service teachers.

The sociocultural theory perspective combines many models of PD into a single cohesive framework with the ability to predict and explain a variety of phenomena of teacher development in various sociocultural circumstances. In comparison to other models, Vygotsky's approach to PD has a significant benefit in that it aims to close the gap between theory and practice. It does this by outlining the complex dynamics of learning processes in actual sociocultural contexts. Furthermore, the learning theory demonstrates the critical need for continual scaffold networks to retain teacher education efficacy (Shabani, 2016). In addition, sociocultural learning theory has been used more successfully in teacher education research to improve our understanding of teachers' learning. According to sociocultural theory of learning, teacher development is not about learning new things or thoughts that come from within the person, but about changing how they take part in social actions that define their professional selves. While assessing teachers' learning has been a major focus of many sociocultural studies on mathematics teacher education, sociocultural views can also be utilized to support interventionist research targeted at enhancing classroom instructions (Goos, 2013).

Constructivist learning theory considers that meaning and knowledge are constructed from interactions between people's experiences, actions, and individual states of mind.

Moreover, advocates of constructivist learning theory encourage mathematics lessons to be more participatory and give increased importance to collaborative and cooperative learning strategies (Stoilescu, 2016). The importance of all facets of interpersonal relationships and social contexts, particularly learner-learner and teacher-learner interactions in learning scenarios, as well as the importance of any activity using signs and symbols in the teaching and learning of mathematics are also heavily emphasized by social constructivist learning theories (Ernest, 2010).

Furthermore, social constructivist learning theories place a strong emphasis on how societal and individual processes interact to co-create knowledge. According to social constructivist feedback research, shared and individual interpretations are created through participant interaction, sense-making, and co-construction of knowledge (Carless & Boud, 2018). Besides, social constructivist theory of learning acknowledges the significance of social processes as well as individual sense-making in the acquisition of mathematical concepts (Ernest, 1994).

Since the study involves a PD and training that require social interaction, identification, and filling of gaps in participants' ZPD, the sociocultural theory of learning and the social constructivist theory of learning guided this study. The theory put forth by Vygotsky is well known in the field of mathematics education. These theories support the premise that social interaction with the outside world, which includes speech, social interaction, and cooperative work, helps students improve their intellect (Cottrill, 2003). Additionally, these learning theories serve as a framework for designing AfL-based learning activities that encourage student participation and assist mathematics teacher educators in understanding the ZPD of their pre-service teachers so that they may support them in

reaching their maximum potential (L. Wang, 2007). Furthermore, according to both of these learning theories, thinking, learning, and relationships among individuals all take place in a world that is shaped by culture and society. These learning theories also support learner-centered methods of instruction and learning, which take into account the cultural and social contexts of knowledge acquisition (L. Wang, 2007).

2.3 Assessment for Learning

It is vital to know the learning intentions, determine whether a student has met those goals, have a good understanding of the student's thinking as they approach the task, and be competent enough in the subject matter to offer meaningful and challenging experiences for their students to be an effective teacher. It requires a skilled teacher who knows various learning strategies to help the student when they seem to struggle, to provide quality feedback in terms of the content being taught to maximize the impact of feedback, and to "step aside" when learning is moving towards the success criteria (Hattie, 2008). Questioning, self- and peer-assessment, constructive formative feedback, and reflection are all components of AfL that aid in eliciting student responses, understanding those responses, and, depending on the data, modifying classroom instruction to enhance student learning. Peer assessment, a part of AfL, empowers students by enabling them to become autonomous, responsible, and reflective learners while also allowing them to learn from one another by evaluating one another's work (Andrade & Brookhart, 2016).

2.3.1 Assessment for learning and its historical background

The prevalent view of education for many years was that educational instructions did not need to be flexible to meet the needs of students as long as it was of a high standard. Either that well-designed education and training would be effective for the majority of students

for whom it was intended or that any failures to learn were the fault of the individual learner (the subject matter was simply too difficult for them, and they should instead explore other, and typically less academic, pathways) were the presumptions made (Wiliam, 2011). Over the past twenty years, AfL has become more and more common in theory and practice. In country after country, educational assessment and evaluation policies and practices have adopted or integrated formative assessment, sometimes known as AfL (Birenbaum et al., 2015). Additionally, during the past 15 years, AfL has developed into a significant area of teacher practice, with worldwide legislative requirements supporting teachers' acceptance of the core ideas behind this pedagogical approach (DeLuca et al., 2019).

The process of acquiring and analyzing data for learners and teachers to use in figuring out where students are in their learning, where they need to go, and the best way to get there is known as assessment for learning. In addition, AfL emphasizes how students learn, is essential to classroom practice, is a crucial professional skill, is sensitive and helpful, generates motivation, promotes comprehension of goals and criteria, aids learners in knowing how to improve, fosters the capacity for SA, and honors all academic success (Assessment Reform Group, 2002). Assessment for learning has been described in a variety of ways, frequently in ways that overlap. Bennett (2011) contends that the phrase "formative assessment" does not refer to a well-defined set of practices and artifacts, but other academics and researchers stress the overlap between different meanings (DeLuca et al., 2019).

Leahy et al. (2005) defined assessment for learning as a set of five broad strategies equally powerful for teachers of all content areas and at all grade levels, described as follows:

1. Clarifying and sharing learning intentions and criteria for success

2. Engineering effective classroom discussions, questions, and learning tasks
3. Providing feedback that moves learners forward
4. Activating students as the owners of their own learning
5. Activating students as instructional resources for one another

Similarly, Black and Wiliam (2009) defined formative assessment as follows:

Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better or better founded than the decisions they would have taken in the absence of the evidence that was elicited.

Likewise, Wiliam and Thompson (2008) suggested that assessment for learning might be conceived as procedures that include the five important practices listed below:

1. Clarifying and sharing learning intentions and criteria for success
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding
3. Providing feedback that moves learners forward
4. Activating students as instructional resources for one another
5. Activating students as the owners of their own learning

Table 1. The five key strategies of assessment for learning

	Where the learner is going	Where the learner is right now	How to get there
Teacher	Clarifying learning intentions and criteria for success	Engineering effective classroom discussions, questions, and learning tasks that elicit evidence of	Providing feedback that moves learners forward
Peer	Understanding learning intentions and criteria for success	Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	Activating students as the owners of their own learning	

Source: *William & Thompson, (2008).*

In a similar way, Moss and Brookhart (2019, p 6) also defined formative assessment as:

Formative assessment is an active and intentional learning process that partners the teacher and the students to continuously and systematically gather evidence of learning with the express goal of improving student achievement.

2.3.2 The Nature, Principles, Strategies and Purposes of Assessment for Learning

The nature of Assessment for Learning

Assessment for learning in education is a vital part of effective classroom instruction, which is essential in supporting students at different levels across the full spectrum of learning mathematical contents (Burkhardt & Schoenfeld, 2019; Chigonga, 2020; Leenknecht et al., 2021). Moreover, AfL links minute-by-minute and daily classroom activities to following three basic enquiries: Where am I going as a learner? Where am I now? and What strategies or technique can help me get there? These three basic questions inform everything a student does, everything a teacher does, and everything teachers and their students do in collaboration. While the three basic questions appear straightforward, they require students and teachers to develop into experienced assessors capable of determining the gap between the learning objective and the students' present level of understanding. Only then can they determine the most effective measures for closing the gap (Moss et al., 2009).

Principles of Assessment for Learning

Through Inside the Black Box, the well-known work of Black and Wiliam, and its follow-up, Assessment for Learning: Beyond the Black Box, the Assessment Reform Group (ARG) has contributed a vital role in conveying research based evidence about the benefits of implementing AfL strategies to the attention of the education and research community. The Principles of AfL were created as part of a continual effort to enhance assessment practices (Assessment Reform Group, 2002). Moreover, the ARG (2002)) identified the

following ten research-based principles of AfL in order to guide classroom implementation of AfL strategies and they are AfL:

1. need to be incorporated in the planning stage of classroom instruction
2. need to emphasis learning of students and how to help them.
3. need to be considered as central part of classroom instruction.
4. need to be considered as an essential skill for teachers for their teaching profession.
5. need to be constructive and sensitive since any assessment has an emotional effect.
6. need to take into consideration of the importance of motivation of the learner.
7. need to support commitment to learning intentions and criteria for success.
8. need to obtain quality and constructive scaffold about how to improve.
9. need to improve students' SA abilities to help them become reflective and independent learners.
10. need to realize the full range of learning of each and every learner.

Strategies of assessment for learning

Sufficient clarity regarding the learning goals to be attained is a vital prerequisite for implementing AfL. Teachers need to determine where they want to take their students and what they need to learn. Teachers can only identify the required information to be collected, the best strategies of getting it, the evidence that can prove learning occurrence, and what they need to focus on during instruction if they have defined learning goals (Ruiz-Primo, 2016). Besides, when learning is appropriately challenging and is the explicit goal, when the student and the teacher seek to determine whether and to what degree the challenging goal is attained, when feedback is given and sought, when there is deliberate practice aimed at attaining the educational goal, and when there are passionate, active, and engaging

collaborators (student, peers, their teacher and so on) participating, visible teaching and learning occur (Hattie, 2008).

As a component of AfL, designing appropriate activities related to learning intentions is crucial for assessing learners, tracking their progress, and gathering appropriate information. Similarly, self-assessment assists students in self-regulating their learning and becoming motivated by evaluating and understanding of their own strengths and weaknesses in the teaching and learning process (Fisseha & Michael, 2016). Furthermore, formative feedback that moves the learner forward, by which students improve their learning approaches by realizing the information from their teacher and peers (Carless & Boud, 2018), plays an important part in enhancing learning of students. As a result, learners need to be trained on how to provide quality feedback to their classmates and receive feedback from their peers as well as teachers (Andrade & Brookhart, 2016) that they can use later when they get evaluated. Improving a reflective character is important since it assists both students and teachers in developing habits of learning that reflect on one's own experiences as well as comments from others.

Explaining the objective of a class activity to students by spending few minutes helps students to move on a learning path and understand where they are going and make connections that would otherwise be overlooked. This type of communication is essential for students to pay attention to what they need to learn from an activity rather than merely completing it. To put it in another way, students will learn more effectively if they fully understand why, what, and how they need to learn, as well as the kinds of evidence that will serve as proof of their success in attaining the intended learning objectives (Ruiz-

Primo, 2016). Moreover, Burkhardt and Schoenfeld (2019) also recommended the following strategies of AfL for mathematics teachers:

- ✓ Provide students with opportunities to think before responding and to reflect on their ideas
- ✓ Provide students with quality and constructive feedback.
- ✓ scaffold students without taking over and modify the classroom instruction with a variety of difficulties
- ✓ Give students freedom to learn independently, but challenge them with tough tactics.
- ✓ Develop and choose effective class activities. (Burkhardt & Schoenfeld, 2019).

Constructive feedback

A process that helps students improve their skills and learning methods by getting feedback from their peers and teacher is called constructive feedback. This process is important for enhancing students' learning outcomes, so students should learn how to receive and give constructive feedback to each other (Andrade & Brookhart, 2016). Giving students constructive, high-quality feedback is a crucial part of implementing assessment for learning. Students should be provided with specific, constructive feedback on how they are doing and what they can do to improve their learning. It is great that teachers are very clear on what they can do to make feedback as specific as possible (Tien et al., 2020).

A student should receive feedback that is specific to the strengths and areas of improvement to his or her work, with advice on how to improve it, and that does not compare him or her to other students (Black & Wiliam, 2010). The feedback students generate from constantly comparing their work to the information they get from the social and material worlds is more important for their learning than planned and occasional comparisons of their work

to feedback from teachers or peers. Without a doubt, it is impossible to properly describe how pupils learn without such continual natural comparisons (D. Nicol, 2021). Likewise, teacher feedback helps students to measure their progress and confirm their own internal goals, criteria, and standard structures. Besides, teachers are much better than peers or students at identifying errors or misconceptions in students' work. As a result, teacher comments can enhance student self-regulation. External input has to be comprehended and integrated by the learner before it can be used to produce positive outcomes in the self-regulation paradigm. There is proof that teachers can affect students' motivation and self-esteem positively or negatively. They can influence both the goals that students choose (learning or performance goals) and their dedication to those goals. Praise for effort and strategic actions, as well as concentrating pupils on learning goals through feedback, results in more accomplishment than praise for ability or intellect (Nicol & Macfarlane-Dick, 2006).

Self-assessment

Self-assessment is the practice of looking at one's own actions and output in order to make adjustments that will enhance learning and performance. Although self-assessment can be summative, the data presented in this review clearly shows that it is most effective in terms of achievement and self-regulated learning when used formatively and in conjunction with training on how to assess oneself. What isn't apparent yet is why and how self-evaluation works (Andradel, 2019). To achieve the unit learning goal(s), teachers and students need to monitor the progress of each lesson within the unit. Each lesson should have a clear learning target and appropriately designed instructional activities that will help students

reach the intended learning target. Teachers should also review and adjust the learning targets and key activities of each lesson as they plan them (Ruiz-Primo, 2016).

In general, students are trustworthy and reliable when evaluating themselves and their peers. However, they may sometimes be too harsh on themselves. The main problem is that students can only effectively evaluate themselves if they have a clear understanding of the learning objectives of the lesson. Unfortunately, many students lack this understanding and seem to view classroom instruction as a series of unrelated activities without any clear purpose (Black & Wiliam, 2010). Students must believe that feedback is beneficial in order to use it. According to the findings, attempts to encourage students to use feedback should concentrate on how teachers offer feedback and how they focus on scaffolding rather than monitoring. While it is important to use feedback to improve, this does not always imply that you are fulfilling the criteria (Kyaruzi et al., 2019).

Purpose of Assessment for learning

The primary goal of implementation of AfL strategies is to enhance classroom instruction by identifying gaps students have, designing appropriate and effective activities, and providing constructive feedback that helps students fill their gaps and move forward. One of the distinctive characteristic of AfL is that teachers use the data they have collected to help students get to where they need to be from where they are. If teachers do not respond thoughtfully to the data gathered, the gap will persist, and learning will not improve. The potential impact of AfL is jeopardized if the cycle is not completed (sharing learning intentions and expectations, collect appropriate data, evaluate and interpret the collected data, and use the data to plan follow-up measures to help students close the gap) (Ruiz-Primo, 2016).

2.3.3 Conception of Assessment for Learning Experiences

The development of conceptions of AfL of teachers grows throughout their experiences in pre-service teacher education, since they experience implicitly and explicitly various conceptions of AfL through courses and in their in-service teaching experiences. Hence, teacher educators and PD programs on AfL should be cognizant of the variability of conceptions of AfL in their endeavor to provide impactful and meaningful support for teachers' practical experience and conceptual understanding related to AfL (Coombs et al., 2018; Deluca & Volante, 2016).

Moreover, as Coombs et al. (2018) emphasized, teachers are neither experiencing effective and up-to-date strategies to implement AfL nor learning about it in their initial teacher education programs, which leads them to depend on their prior assessment experiences, which are predominantly summative. Besides, effective and meaningful implementation of AfL entails a fundamental reconceptualization of the part AfL contributes in the instructional process. Therefore, addressing conceptions of AfL of MTEs has a vital effect for teacher education since classroom practices of AfL are greatly influenced by their conception of assessment.

Whenever possible, experiencing a subject firsthand is the ideal method for learning about it. The employment of AfL enables pre-service teachers to advance their professional abilities (López-Pastor & Sicilia-Camacho, 2015). It is argued that students' perceived implementation of AfL strategies by their teachers is connected with their emotions of autonomy and competence. The more AfL is perceived to be practiced, the greater the sense of autonomy and competence (both in terms of satisfaction and reduced frustration) and the higher the level of autonomous motivation (Leenknecht et al., 2021).

2.3.4 Challenges of implementations of assessment for learning

To use assessment for learning effectively, teachers need to apply a complex and interrelated set of practices in the classroom. The main purpose of AfL is to gather information that can help teachers and students make better decisions about what to do next in instruction, but the results also show that other important factors, such as teachers' planning and preparation, domain knowledge, and ability to involve students in the assessment process, influence how they use that information (Lyon et al., 2019). There are many obstacles to implementing AfL, and there are also many different approaches to overcoming those obstacles. The objective, however, is very necessary: it is to make sure that students acquire the knowledge, abilities, reflective capabilities, and motivations necessary for active engagement in both the present and the future (Allal, 2016).

There is a general agreement among educators and researchers that the classroom assessment practices of teachers are not up to date with the current research findings, especially in AfL. Some of the factors that contribute to this gap are the resistant nature of the school culture and the lack of suitable PD in AfL. Moreover, AfL is becoming a key element of successful and effective educational institutions, but in-service teachers often have a basic and sometimes incomplete knowledge of how to use these practices and strategies effectively in their classrooms during the teaching and learning process (Deluca & Volante, 2016). Moreover, the success of AfL relies heavily on many factors both inside and outside the classroom. These include the types of assessment tasks used, the students' readiness both cognitively and emotionally, the teacher's knowledge of their students and course materials, the characteristics of the course itself, and the physical learning environment (Xu & Brown, 2016).

As with any other reframe of what occurs in schools, the AfL process needs time to grow and develop (Moss et al., 2009). Teachers usually find it very challenging to engage in AfL without proper PD on AfL, and this challenge is particularly severe for mathematics teachers since historically, the teaching and learning of mathematics have focused on a limited set of learning objectives, primarily technical skills taught through demonstration and practice (Burkhardt & Schoenfeld, 2019).

Factors influencing instructors' intentions or implementations for AfL can be divided into two groupings: contextual and personal factors. Instrumental attitude, education and training, and self-efficacy were the most important personal characteristics impacting instructors' intent to undertake AfL. External policy, internal school support, cultural norms, and school atmosphere were all extensively mentioned as contextual factors (Yan et al., 2021). It is recommended that pre-service teacher education and training programs utilize the identified constraints to rigorously analyze their own programs of teacher education and training, although not all constraints will appear in all programs. Using program evaluation approaches such strengths, weaknesses, opportunities and threats (SWOT) analysis, it is possible to pinpoint the strengths, weaknesses, opportunities, and threats to AfL integration. (Deluca & Volante, 2016).

Improving classroom implementation of AfL is not easy, and there is no easy fix that can improve present AfL practice by guaranteeing instant payment. To achieve the significant benefits of implementing AfL in the teaching and learning process that the research evidence suggests, each teacher needs to adapt the lessons and ideas mentioned above to their own classroom practices and to the culture and expectations of their specific school community. This is not a quick process, but a gradual one that requires ongoing PD and

support. This reality does not weaken the message; rather, it shows its honesty because this is how lasting and fundamental changes in teaching and learning have to happen (Black & Wiliam, 2010).

Assessment for learning is a practice, and it will take time to make it part of the classroom's daily rhythm. This means that teachers will vary in how much they adopt formative assessment in their classroom culture and how much all students benefit from it (Moss et al., 2009). All aspects of the education system that hinder the effective implementation of AfL should be examined to see how their undesirable effects can be reduced (Black & Wiliam, 2010). Without a clear understanding of what it involves to apply all of the AfL process elements in the classroom's every day and moment-by-moment work, it is easy for teachers to think that they are already "doing" formative assessment at the highest levels of quality and consistency. That is why Moss et al. (2009) advised conducting a comprehensive and purposeful inquiry into their classroom assessment processes as a starting point and an honest reality check.

2.3.5 Assessment for learning in Global and Ethiopian context

The notion that assessment can both enhance education and measure its benefits has existed for a long period of time, and during the last three decades, evidence concerning assessment's ability to increase learning has gathered. Nonetheless, the systematic use of assessment to promote learning continues to be the exception rather than the rule across the globe (Wiliam, 2018). Numerous educational studies have shown how effective AfL is for improving student learning, which has had an impact on the assessment policies of many nations worldwide (Vlachou, 2015).

Assessment for learning is implemented in educational institutions across the world to varying degrees or levels and the degree of implementation of each of the strategies also varies a lot. A conclusion from a study that examined how AfL was implemented in three big metropolitan public school districts in the United States revealed that master teachers efficiently applied certain parts of AfL, whereas others were used seldom and/or ineffectively (Johnson et al., 2019). Additionally, research that looks at the quality and breadth of AfL implementation among 202 science and mathematics instructors who concluded a two-year, school-based professional development program focusing on AfL revealed that while teachers improved significantly in several areas, certain components of AfL were downplayed, and other trends of implementation quality suggest that additional focused professional development is necessary (Wylie & Lyon, 2015).

In addition, the result of research that evaluated the implementation of AfL in Sweden by Jonsson et al. (2015) showed that the professional development on AfL was successful in influencing teachers' perspectives on classroom instruction and in influencing pedagogical practices of teachers toward AfL. Additionally, research results indicate that AfL practices are predominantly teacher-centered, implying that teachers continue to bear the majority of the assessment burden.

Besides, in a Chinese high school, tests were employed to some extent for the purpose of enhancing student learning via test follow-up procedures (Xiao, 2017). Moreover, teachers in Ghana are exposed to the dynamics of assessment in just one course as part of their teacher training for either a 3-year certificate or a 4-year degree, which is insufficient to provide them with the necessary information for the teaching job. Furthermore, it is argued that teachers in the Metropolis are less likely to receive frequent in-service training in the

field of assessment, particularly AfL, and in this instance, you can only expect them to have some knowledge but not enough for classroom practice (Amoako et al., 2019). Besides, the majority of senior high school mathematics teachers in the Cape Coast Metropolis, in Ghana, had little awareness of AfL procedures, according to Amoako et al. (2019), and it is more likely that teachers' limited knowledge of AfL stems from their education at universities and colleges of education.

In recent years, greater emphasis has been placed on improving AfL implementation in Ethiopia to improve students' learning. While AfL is widely regarded as an effective strategy for enhancing student learning, its potential for learning has been less thoroughly explored in contexts where summative assessment predominates. In their study of the implementation of continuous assessment in eastern Ethiopian universities, Abera et al. (2017) found that the implementation of AfL at these selected universities was substandard, and lecturers were advised to adopt continuous assessment not just for the purpose of assessing and marking students' performance but also for the purpose of improving learning of students.

In their investigation of teachers' professional learning in Ethiopia, Gemedo and Tynjälä revealed that the meaning linked to classroom instruction corresponded to the traditional paradigm of teaching and learning, which depicts learning as passively receiving knowledge and teaching as imparting it. Teachers were positioned at the margins of knowledge development for school improvement in the current professional learning setting, and they became deliverers of pedagogical instructions required by top-down edicts. Furthermore, the results suggested that the currently available professional learning opportunity was unpromising (Gemedo & Tynjälä, 2015). Similarly, in their study of

teacher educators' education and training in Ethiopia, Haftu et al. (2018) identified the teacher dominated nature of classroom instruction and the use of assessment entirely for the purpose of grading as problems in preparing effective initial primary school teachers. Furthermore, in their study of the views of teachers with respect to the main features of assessment practices in the classroom in Ethiopian government secondary schools, Geberew and Tigist (2018) revealed that teachers reported low student motivation, large class sizes, insufficient student background knowledge, limited time for lessons, and inadequate teaching and learning facilities hindered their full use and exploration of different assessment techniques.

Similarly, in their investigation of the implementation of AfL in Ethiopian secondary schools, Figa et al. (2020) revealed that teachers occasionally communicate learning objectives for students, occasionally integrate AfL strategies, and occasionally provide formative feedback, with significant variation in practices. According to the results, the biggest obstacles to implementing AfL were a lack of instructional materials, a lack of laboratory equipment and/or personnel, a lack of teacher competency, large class sizes, and a lack of instructional time.

2.4 Mathematics Education and Assessment for Learning

To successfully use AfL, teachers need to be sensitive to the misconceptions of their students in the content area (e.g., courses in college mathematics) they teach and curiously look out for signs of them in the classroom instructional process. Unfortunately, despite the fact that there are significant misconceptions in various mathematical contents in the

literature, few mathematics teachers are likely to understand the complexities of the misconceptions due to the daily pressure to cover content (Burkhardt & Schoenfeld, 2018).

Mathematics education focuses on how the three parts of an educational unit interact: the teacher, the content, and the students. This means that the quality of education depends not only on the teacher's intellectual and personal abilities but also on how he or she connects with specific students and materials (Chigonga, 2020). Many students have limited knowledge of the idea that making errors followed by corrective feedback improves learning. When predicting the relative efficacy of learning strategies in a hypothetical circumstance, students only significantly chose learning from errors over the study of perfect responses. It does not appear that many students have a strong bias towards believing that creating errors is good for learning; rather, their baseline attitudes tend to be quite indifferent on the matter (Pan et al., 2020).

Assessment for learning has traditionally been viewed as providing teachers with more frequent proof of students' knowledge of standards in order to assist them in making useful instructional decisions and it is intended to enhance students' learning in this manner. The main difference between assessment of learning and AfL is that the former tells teachers how well students have done, while the latter tells students how to improve their own learning. Assessment for Learning is based on the idea that students can also use data to make instructional choices, an idea that has been mostly ignored in our assessment history and past efforts to improve schools (Ferretti et al., 2018).

2.4.1 Assessment for learning and the teaching learning of algebra

Algebra is a key part of students' mathematics education, and many efforts have been made to overcome students' challenges and problems in learning algebra (X. Wang, 2015). Algebra, as one of the hardest subjects, is full of abstract ideas and has developed over many centuries in the history of mathematics (Hodgen et al., 2018). Many students at all levels find it hard to understand basic algebraic concepts and principles, and this is a long-standing issue (Cousins-Cooper et al., 2017; Kanbir et al., 2018). Teaching and learning algebraic concepts are common difficulties for both teachers and students. Teachers have trouble finding and using the best methods to teach algebra, while students have trouble learning algebra topics (Cousins-Cooper et al., 2017; Garzón & Bautista, 2018), and Algebra is hard to teach and learn because of the abstract nature of the subject and the variety of symbols used to teach and learn concepts, methods, and problem-solving solutions (Cousins-Cooper et al., 2017; Ferretti et al., 2018). However, improving students' learning requires using instructional strategies that are based on evidence, and teachers are expected to have a deep knowledge of the content knowledge of algebraic concepts and features of interventions in teaching and learning algebra (Cousins-Cooper et al., 2017; Lee et al., 2020), and helping pre-service mathematics teachers and their educators to better understand difficulties in learning mathematics courses helps inform (Stewart & Reeder, 2017). Therefore, it is important to provide pre-service mathematics teachers with the necessary algebra knowledge through creative instructional approaches.

Many learners at different stages have trouble grasping the basic ideas and rules of algebra; even with a lot of effort, they still face this issue (Cousins-Cooper et al., 2017; Kanbir et al., 2018). Moreover, algebra is seen as an essential part of students' mathematics

education, and a lot of attention has been paid to dealing with students' problems and obstacles in algebra learning (Wang, 2015). The classroom teaching of algebraic concepts is a common difficulty for both students and teachers. Teachers find it hard to choose and apply the best methods to teach algebra, and students find it hard to learn algebra concepts (Cousins-Cooper et al., 2017; Garzón & Bautista, 2018). To help students learn algebra, assessment for learning is one of the suggested instructional strategies (Stewart & Reeder, 2017).

Algebraic concepts are vital areas that students need for success in work and higher education. To improve students' learning needs using evidence based instructional strategies, teachers need to have a deep understanding of algebraic concepts and the features of interventions in algebra teaching in the classroom (Cousins-Cooper et al., 2017; Lee et al., 2020). A poor background in understanding algebraic concepts and procedural skills among college students is a cause of major difficulty in solving mathematical problems. To help students learn algebra, AfL is one of the suggested instructional strategies (Stewart & Reeder, 2017).

Abstract algebra is a challenging course to teach and learn in mathematics education because it involves abstract ideas and many symbols to represent concepts, methods, and problem-solving skills (Cousins-Cooper et al., 2017; Ferretti et al., 2018). It is beneficial for pre-service mathematics teachers and their educators to understand the challenges and difficulties that pre-service teachers face in learning mathematics courses, as this can improve their teaching and learning outcomes (Stewart & Reeder, 2017). Therefore, it is essential to use innovative instructional approaches to help pre-service primary mathematics teachers acquire the necessary algebra knowledge. Three major problems that

were creating difficulties for the students in learning abstract algebra were identified: students' attitudes toward abstract algebra, prior knowledge, and instructional materials. Besides, three other categories that created minor problems; motivation, lecturer performance, and learning style were identified in an investigation conducted by Agustyaningrum et al. (2020).

Errors in mathematics education can be beneficial for learning, but students and teachers often have a negative attitude towards them. Therefore, AfL is important for helping students learn from their mistakes by using them as a chance to improve classroom instruction. Furthermore, teachers should use an analytic approach that focuses on the process of learning from errors, by connecting their feedback to common student mistakes that they find when grading tests or exams and during classroom AfL practices. In addition, teachers should use the results of student assessments, especially mathematics test errors, to support students' learning in areas where they need more help (Kyaruzi et al., 2020).

2.4.2 Assessment for Learning and Mathematics Teacher Education

Developing new ways to invest in mathematics teacher education and training is essential, especially through training courses that involve teachers in collaborative planning and help them to create effective strategies, plans, and methods for teaching mathematics. This requires research on both the content and the pedagogical-didactical learning approaches, especially on classroom assessment, highlighting the importance of AfL and personalized teaching strategies that suit the pace and learning styles of students (Ferretti et al., 2018). Generally, teachers' use of AfL is supported by appropriate education and training. It's worth noting that most of the time, such an impact is achieved by changing teachers' knowledge, abilities, attitudes, or self-efficacy (Yan et al., 2021).

Researchers and educators emphasize the global importance and influence of initial primary school mathematics teacher education on children's growth in recent years (Alex & Roberts, 2019). In addition, researchers have come to the conclusion that it is very difficult to help pre-service teachers become good assessors by teaching theories, principles, and strategies of assessment in an isolated way. Brevik et al. (2017) argued that teaching isolated strategies and principles of assessment for learning to pre-service teachers is limited at best and recommended focusing on assessment for learning as a theoretical principle and as a means to improve students' learning of courses given in teacher education in line with the sociocultural theory of learning.

Deluca and Volante (2016) also emphasized the importance of teacher education modeling a variety of AfL strategies in their coursework to assist pre-service teachers in developing a deeper understanding of the use of various AfL approaches. They also underlined that it is through the experiential method that pre-service teachers can begin to unlearn their misconceptions of AfL and learn the benefits of AfL in improving learning.

A considerable number of researchers call for research that can help better understand how initial teacher education programs prepare pre-service teachers to enact AfL and the roles pre-service teachers play (C. C. Johnson et al., 2019). The way teacher educators think and act about teaching, learning, and assessment influences how pre-service teachers develop their professional knowledge and skills. Teacher educators can help pre-service teachers to rethink their prior beliefs and assumptions about teaching, learning, and assessment, which may be outdated or ineffective (DeLuca, Coombs, et al., 2018). Moreover, when pre-service teachers encounter AfL practices regularly and meaningfully in their teacher education programs, they are more likely to adopt AfL in their own teaching, enhance their

future students' learning outcomes, and foster an AfL culture in their schools (Deluca & Volante, 2016).

Pre-service teacher education contributes a lot to shifting pre-service teachers' orientations towards more impartial approaches to AfL beyond building their AfL knowledge and skills (Coombs et al., 2018), even although, as Deluca and Volante (2016) argued, most teachers lack sufficient education and training in assessment and that AfL concepts are barely covered in educational courses. They also pointed out that the structure of pre-service teacher education programs does not foster the effective use of AfL, resulting in a predominantly summative assessment culture for teaching and learning. Hence, it is suggested that pre-service teacher education programs should adopt a conceptual change model that aligns knowledge and skills with beliefs, values, and attitudes towards AfL (Deneen & Brown, 2016).

Teacher education programs provide potential teachers with several possibilities to participate in dynamic knowledge production and deconstruction processes. As a result, teacher educators' roles should shift from monopolizing and delivering knowledge to assisting potential teachers in learning autonomously and producing knowledge on their own (Gemedä & Tynjälä, 2015). Moreover, how teacher educators respond to teacher candidates' concerns, challenges, and experiences, as well as who we are as teacher educators and our ideas and understandings about schools, teaching, learning, students, and assessment, have enormous ability to shape future teachers and assist their unlearning (DeLuca, Coombs, et al., 2018).

Teacher educators' roles should shift from monopolizing and delivering knowledge to assisting future teachers in learning autonomously and generating knowledge on their own. Teacher education programs should assist instructors in developing the desire to find solutions to tough problems in classroom instruction, the ability to build and update their own practices (Gemedá & Tynjälä, 2015). Furthermore, providing high-quality education requires careful attention to teacher training and PD aspects, such as offering pre-service training and ongoing PD that emphasize content knowledge and pedagogy (Workneh & Tassew, 2013). Also, DeLuca et al. (2019) suggested that to help MTEs use AfL strategies in their teaching, it is essential to first recognize the barriers that prevent the enactment of AfL principles and their understanding and practice of AfL in their classrooms. Moreover, Darling-Hammond et al. (2017), claimed that teacher professional learning that is content-specific, job-embedded, and context-based is vital for addressing the diverse needs of students and their teachers in different settings.

2.5 Professional Development and Training on Assessment for learning

As Johnson et al. (2019) stated, the key aspects of gaining ground in the implementation of AfL in educational institutions are engaging teacher educators in PD on AfL and improving institutional support for institutions across multiple years. Teacher educators need to understand the political and social context of AfL and how these contexts shape assessment-related beliefs and practices if they are interested in changing the future practices of pre-service teachers (Barnes et al., 2014). As Black and Wiliam (2018) argued, it is difficult to understand assessment without considering the wider context within which teaching, learning, and assessment take place. To understand how pre-service primary school mathematics teachers are prepared to be good assessors in teacher education

colleges, it is necessary to conduct research on how AfL strategies are implemented, what AfL means to pre-service primary school mathematics teachers and mathematics teacher educators, and the effect of proper implementation of AfL strategies on their learning of mathematics courses.

In order to successfully integrate AfL strategies into the classroom instruction process, teachers need substantial knowledge and experience in how to implement various AfL strategies. In addition, teachers need supportive materials that developmentally sequence activities and assist them in drawing conclusions about what students learn in relation to the stated objectives. Unfortunately, it is uncertain whether all teachers have the required knowledge about and experience in implementing AfL strategies effectively (Bennett, 2011). Besides, teachers need to understand the difficulties and in order to adjust their instruction to the needs of their students, which are unpredictable and vary across students. Furthermore, in order to reap the substantial benefits of implementing the AfL research evidence promised, each teacher must develop her or his own method of incorporating AfL strategies into classroom activities (Black & Wiliam, 2010).

To equip teachers and teacher educators with the necessary contemporary skills and knowledge about how to support complex and deep learning in their students, PD is a very important strategy. But, as Ruiz-Primo and Brookhart (2017) noted, collaborative learning, administrative support, individualization, providing plenty of time, and taking into consideration the local classroom culture and context are important characteristics of an effective PD on AfL. Moreover, as Deluca and Volante (2016) emphasized, teacher educators need to be provided with proper PD on AfL so as to improve pre-service teachers' learning of various courses. Unfortunately, shifting teachers' or teacher educators'

orientation towards AfL-based teaching and learning is much more challenging than procedurally implementing specific AfL strategies (DeLuca et al., 2019). In addition, assessing participants (e.g., teacher educators, pre-service teachers) before and after a PD is vital in understanding the changes brought to their practices, approaches, attitudes, and teaching and learning practices due to the PD provided (Emery et al., 2019).

Various educational researchers provide lists of characteristics of effective PD in AfL at different levels of education.

According to Allal (2016), teachers need to understand the reasons and principles behind AfL in order to apply it effectively. He suggested that the PD should cover some essential topics, such as: how to define and communicate success criteria; how to give feedback that helps students improve; how to create and use assessment tools, such as criteria, rubrics, and scripts; how to guide students to assess themselves and their peers; and how to ask questions and interact with students in ways that promote learning. Moreover, as Andersson and Palm (2018) stated, the PD may be most effective if it is based on the individual needs of the participants, and information about their needs should be periodically collected and incorporated into the PD. Moreover, the PD should focus on conceptualizing strategies and methodologies rooted within specific content domains (e.g., the fundamental concepts of algebra (Math 221)) to realize maximum benefit from AfL (Bennett, 2011). Besides, providing meaningful, sustained PD that models effective use of AfL in the classroom is essential to promoting mathematics teacher educators' use of AfL (Burton et al., 2018).

In their research on effective teacher PD, Darling-Hammond et al. (2017) identified the following features of teacher PD that promotes teacher learning: it focuses on the content of teaching; it engages teachers in active learning (such as coaching, feedback, and reflection); it fosters collaboration among teachers; it demonstrates effective teaching practices; it provides coaching and expert guidance; it encourages feedback and reflection; and it lasts for a long time. Moreover, although the minimum duration of time required for effective PD has not been identified by research, short-term and one-off workshops of PD cannot be adequate for meaningful professional learning, including AfL. Furthermore, to develop their skills from beginners to experts, teachers need effective professional learning that involves a system of support from peers and specialists and continuous learning opportunities that are embedded in their classroom practice (Darling-Hammond et al., 2017; DeLuca et al., 2019). To maximize its effect on improving teachers' assessment conceptions and practices, AfL PD should be differentiated and targeted based on teachers' teaching experiences and previous assessment education (DeLuca, Valiquette, et al., 2018).

The development of teachers' conceptions of assessment begins first in their experiences as students from elementary to secondary school, then as pre-service teachers in teacher education institutions, and finally in their professional careers as classroom teachers (DeLuca, Coombs, et al., 2018). Consequently, creating a culture of AfL in educational institutions is challenging and requires adequate time to fully integrate AfL into their classroom instructional approaches and a continued commitment to using AfL across different levels and subjects (DeLuca, Chapman-Chin, et al., 2018). There are various challenges at different levels for the effective implementation of a PD for teachers on AfL, and these challenges need differentiated approaches to be resolved. Darling-Hammond et

al. (2017) stated that to facilitate the maximal effectiveness of a PD on AfL, there are four main areas to be taken into account. These are as follows: (1) identifying PD needs; (2) choosing approaches most likely to be effective; (3) implementing approaches with quality and fidelity; and (4) assessing PD outcomes. Besides, a PD designed very carefully may fail to achieve its goals if it is implemented poorly for various reasons (e.g., resource inadequacy, lack of shared vision about what constitutes high-quality instruction, lack of time to implement the newly developed instructional approaches, and lack of appropriate teachers' knowledge) (Darling-Hammond et al., 2017).

There are various challenges in the implementation of AfL, and the ways of resolving those challenges are equally diverse. For the effective implementation of AfL, professional development on AfL to improve classroom practices is required (Allal, 2016; Black, 2015; Black & Wiliam, 2010). Moreover, teachers need a living example of how to implement effective AfL to have the confidence to implement it themselves rather than being oriented to the general principles of implementing AfL (Black & Wiliam, 2010). Besides, effective implementation of AfL requires mathematics teachers to move outside of their comfort zone of customary practices and discuss with others how they can be supported in facing those challenges (Burkhardt & Schoenfeld, 2018).

2.5.1 Qualities of Effective Professional Development and Training in Education

To prepare students for the challenges and opportunities of the twenty-first century, teacher professional learning is becoming more important as a way of helping them acquire advanced skills for further education and careers. These skills include deep knowledge of difficult subjects, critical thinking, complex problem-solving, effective communication and teamwork, and self-direction. To teach these skills, teachers need to use sophisticated

pedagogies. However, research shows that many PD programs are not successful in changing teacher practices and student outcomes (Darling-Hammond et al., 2017). Therefore, professional development that supports teachers in learning and improving these pedagogies is essential.

Most teachers around the world view PD programs that meet their personal development needs as beneficial. Moreover, individual teacher learning experiences and instructional methods improved by cooperation will provide useful insights into the design of customized PD. One new line of inquiry is to look into the use of technology in individual professional development while also expanding collaborative learning among teachers from all over the country and the world (Zhang et al., 2021). Moreover, instructors need to feel comfortable experimenting, evaluating the impact of their innovations, conversing openly, and establishing rules for effective student learning to achieve optimal PD (Wright et al., 2018)

Additionally, context-specific, job-based, and content-based teacher PD is essential for fulfilling the distinct requirements of students (and consequently teachers) in a variety of situations (Darling-Hammond et al., 2017). However, personalized PD research is still in its early stages. It is necessary to examine how to build tailored professional development that better supports individual instructors' learning experiences as well as whether this type of PD would improve instructional practice (Zhang et al., 2021). Moreover, teachers develop as professionals as a result of their long-term and day-to-day work, which is why job-embedded learning opportunities should be at the center of all professional

development activities. Throughout these opportunities, teachers require support from school leaders and peers (Zepeda, 2013).

Darling-Hammond et al. (2017) reviewed 35 studies to identify the features of effective professional development. The studies measured student outcomes while controlling for contextual factors and student attributes, or they used a rigorous experimental or comparison group design. Their analysis revealed seven common elements of effective professional development, which are: 1. focusing on content; 2. applying adult learning theory to involve active learning; 3. fostering collaboration, usually within the scope of a job; 4. using exemplary practice models; 5. offering coaching and expert support; 6. enabling feedback and reflection; and 7. being long-lasting.

2.5.2 Professional Development and Training on Assessment for Learning

Assessment techniques, values, beliefs, and knowledge of teachers are a thriving subject of study that spans the globe. This field is producing a corpus of information that can be interpreted in a number of ways. One promising way of conceptualizing this research is through the micro-, meso-, and macro-level approaches presented below. It has the ability to provide a lens through which to view connections at all levels, both inside and across them (Fulmer et al., 2015). Furthermore, AfL has also been shown to significantly increase students' achievement, but there is a lack of research on the most effective ways to help teachers and students implement this strategy (Andersson & Palm, 2018).

Identifying areas of weakness that cut across numerous teachers is one technique for structuring PD based on observable trends. A second method for finding PD opportunities is to use what we know about establishing AfL experiences to determine tailored next steps for particular teachers (Lyon et al., 2019). Additionally, PD should be built on a well-

defined and openly shared sense of purpose and fundamental beliefs that are regularly examined by the educational community. If teachers have strong beliefs and a clear sense of purpose, they can use their own insights to develop learning experiences for themselves, their students, and their school (Zepeda, 2013).

Despite the fact that AfL is becoming more popular as an educational tool, teachers are finding it challenging to utilize formative assessment procedures in their classrooms. To address this, Kruiper et al. (2021) developed curriculum-based training on AfL and scaffolding. The trained teachers also took part in a post-training interview to discuss how effective the course was. The results suggest that trained teachers exhibited more adaptable behavior than teachers who were not trained. According to the results of the interviews, teachers thought the training was beneficial and that scaffolding theory was helpful. Teachers sought additional time and assistance in putting newly acquired strategies into practice on a regular basis. Similarly, teachers of mathematics who took part in a PD program on AfL improved their AfL classroom practices to the point where it significantly improved students' academic performance, and they believed that the PD program offered them the resources and assistance they required to implement AfL strategies in the teaching and learning process (Andersson & Palm, 2018).

2.5.3 Teachers' Conceptions of Assessment for learning

Teachers need to view AfL as a different activity from summative assessment that is integrated with normal classroom instruction and student participation in their learning to fully benefit from the implementation of AfL, as Johnson et al. (2019) argued. Besides, in order to alter the assessment culture in classrooms, instructors must alter their own ideas of assessment and develop change-oriented intentions. The findings of this study suggest

that in order to increase teachers' self-efficacy in carrying out formative assessment, it was necessary to foster a positive instrumental attitude toward formative assessment. (Yan & Cheng, 2015).

Most teachers are not familiar enough with AfL, nor are they well prepared with the knowledge or skills to use AfL for themselves and their students (Moss & Brookhart, 2019). Many teachers and students lack clear and timely information about their learning progress in the classroom. Teachers struggle to show what their students have learned and how they can improve, because they don't have accurate and up-to-date data. They also find it hard to provide quality and constructive feedback that guides students to the next steps in their learning. Students also face challenges in managing their own learning. They don't know how to set goals, monitor their progress, or evaluate their outcomes. They try to complete tasks without a clear direction, strategy, or criteria for success (Moss et al., 2009).

In recent years, the emphasis on the use of AfL has increased in schools across the United States. While instructors report using formative assessment procedures, their knowledge of what formative assessment truly is in reality is frequently inconsistent (Johnson et al., 2019).

Teachers may have misconceptions about the nature, principles, strategies, and purposes of AfL for different reasons. As Moss et al. (2009) stated, some of the misconceptions of teachers regarding AfL include:

- Assessment for learning is a specialized type of exam or set of tests that teachers learn to utilize to ascertain their pupils' knowledge.
- Assessment for learning is a program that instructors accept and incorporate into their existing practices.
- Assessment for learning includes any method that collects data in order to improve programs or training.

The role of teachers in the implementation of AfL takes many interrelated forms in the classroom, such as aiding classroom discussion, eliciting students' thinking and responses, analyzing students' responses and works, providing quality and timely feedback to students, and using the data obtained from AfL to make informed instructional decisions (Burton et al., 2018).

2.6 Students' Self-Regulated Learning Skills and Assessment for Learning

The study of how students regulate their own learning emerged as a formal research topic in the field of education in the late 20th century, influenced by the broader efforts to understand human self-control in various domains of life. Researchers and educators were inspired by the successful application of self-regulatory processes such as setting goals, rewarding oneself, keeping track of one's progress, and giving oneself guidance in personal areas such as diet and task completion, and they started to explore how students could use

these processes to enhance their academic learning (Zimmerman, 2015). Moreover, there are different theoretical models of self-regulated learning that propose different components and methods for regulating one's learning, but they also have some common underlying assumptions about how learning and regulation work. All the models view learners as active agents who participate in the construction of their own learning. Learners are expected to actively create their own meanings, objectives, and strategies from the information that is available in their external environment as well as from their own mental processes. All the models also suggest that learners have the potential to monitor, control, and regulate some aspects of their own cognitive, motivational, and behavioral processes, as well as some features of their environmental context (Pintrich, 2000)..

Even though it has been demonstrated that promoting SA improves the use of SRLS in many circumstances, it is presently impossible to determine which precise treatments or tools would be most successful in boosting SRLS. A promising way forward is to develop clear criteria for the objectives of SA, which can help students set realistic goals and monitor their progress (both during and after the process). As shown, clear criteria can support all the phases of SRLS, and several suggestions for using SA in the classroom are related to these criteria. For instance, teachers should clarify the criteria that students use to evaluate their work, teach students how to apply the criteria, give students feedback on their SA, help students use the SA information to improve their performance, and provide enough time for revision after the SA (Panadero et al., 2016). It is also claimed that, with the right conditions in place, teacher candidates can graduate from teacher preparation programs with a good understanding of and a mindset for engaging students in activities that foster SRLS (Hill et al., 2017).

2.6.1 Students' Self-Regulated Learning Skills

Self-regulated learning is a process in which learners actively shape their own learning by setting goals and then monitoring, regulating, and controlling how they think, feel, and act in relation to those goals and their learning environment. These self-regulatory actions can influence how individuals interact with their surroundings and how successful they are (Pintrich, 2000). Moreover, self-regulation is shown in practice by the active monitoring and regulation of different learning processes, such as the setting and orientation of learning goals, the strategies used to reach those goals, the management of resources, the effort put in, responses to external feedback, and the products created (Nicol & Macfarlane-Dick, 2006).

2.6.2 Ways of Improving Students Self-Regulated Learning Skills

To self-regulate effectively, learners need to have some goals to compare and judge their performance. In academic settings, goals can be defined by specific objectives, criteria, norms, or other external references (e.g., examples). Feedback tells the student how their current state (of learning and performance) matches these goals and criteria. Self-regulated learners generate their own feedback by monitoring their engagement with learning activities and assignments and checking their progress toward goals. They also actively use external feedback, such as from teachers and peers, to align with their internal goals (Nicol & Macfarlane-Dick, 2006).

One successful strategy for students to build self-regulation is to create opportunities for them to practice controlling areas of their own learning and to reflect on their experience. Students are already evaluating discrepancies between internally defined task goals and the outputs (both internal and external) they generate (Nicol & Macfarlane-Dick, 2006).

Besides, Allal (2020) argued that a major area for future research is to investigate the influence of various AfL strategies on students' SRLS.

2.6.3 Relationship between Assessment for Learning and Self-Regulated Learning Skills

Classroom assessment can be a powerful tool to support students' learning success if it is done with a clear purpose, because it is closely linked to most classroom activities and SRLS strategy instruction works best when it is connected to a realistic learning situation (Andrade & Brookhart, 2016). In addition, studies that showed positive effects of self-regulatory processes such as goal setting, self-reinforcement, self-recording, and self-instruction on personal areas like eating and task completion inspired researchers and educators to explore their use by students in academic learning (Zimmerman, 2015).

Self-regulated learning is about how students take charge of their own learning processes. It is not a fixed ability or skill, but a dynamic process that learners use to enhance their mental talents and develop skills in various domains, such as academics, sports, music, and health. This article will explain what self-regulated learning is and how it emerged as a concept in the intellectual context, how researchers have shifted their focus over time and what they are currently interested in, what methodological challenges are related to the concept, and what future directions for research are (Zimmerman, 2015). Moreover, self-regulated learning involves metacognitive, motivational, and behavioral processes that learners voluntarily initiate to acquire information and skills, such as setting goals, planning, using learning strategies, self-reinforcing, self-recording, and self-instructing. A self-regulated learning perspective changes the way educational research views students' learning abilities and instructional environments from static entities to dynamic processes

that students can actively modify and improve. This approach views learning as an activity that students do for themselves in a proactive way rather than as a passive outcome that happens to them as a result of instructional experiences (Zimmerman, 2015).

One way to assess learning is to involve students and give them feedback. This means that assessment is used to help students learn better. Two important ways to do this are SA and PA. These methods can help students use self-regulated learning strategies more often, but it is hard to know which specific ways or tools work best for this. However, one promising idea is to make clear criteria for self-assessment, so that students can set realistic goals and check their progress (both during and after the learning process). Clear criteria can help students with all the stages of self-regulated learning, and there are some suggestions for how to use self-assessment in the classroom based on them (Panadero et al., 2016).

To make PA more effective, it is important to explain why and how it is done before starting it, let students have a say in the standards, tell them clearly what kind of feedback they should give and receive, and support them with enough guidance and resources. This support is especially helpful when students lack enough knowledge and skills in the subject area (Panadero et al., 2016).

Similarly, Granberg et al. (2021) investigated how one math teacher's use of AfL influenced students' SRLS. They used a questionnaire before and after the eight-month intervention and found a significant difference in students' SRLS compared to two other groups. They also analyzed qualitative data and showed a large improvement in the students' SRLS behavior in the classroom. They examined how the teacher applied AfL and found that it involved different aspects of AfL and provided empirical support for how AfL can enhance students' SRLS and align with SRLS development theories. The study's

findings indicate that AfL had a positive and significant impact on students' SRLS (Granberg et al., 2021).

2.7 Effects of Assessment for Learning

2.7.1 Effect of Assessment for Learning on Students' Achievements

There is compelling evidence that properly implemented AfL can help improve students' learning and their academic self-regulation at different levels or grades (Andrade & Heritage, 2018). Moreover, while AfL benefits all students, it benefits low achiever students more than average achiever students and high achiever students, therefore narrowing the gap of success while improving students total achievements (Black & Wiliam, 1998).

Furthermore, a lot of research has been done on how to help mathematics teachers improve their skills and knowledge of implementation of AfL and how this affects students' math performance in different levels of schooling. Andersson and Palm (2017) did a study where they compared the math scores of year four students whose teachers had received 144 hours of training on formative assessment methods with those whose teachers had not. The training included lectures and examples of how to use formative assessment in math lessons. The study found that the students who had teachers with the training did significantly better on math tests than the students who did not, after taking into account their previous scores ($p = .036$ and $d = .66$).

Using a pre- and post-test quasi-experimental method and an intervention design, Chemeli (2019) examined how the five key assessment for learning techniques proposed by Wiliam and Thompson (2008) affected the mathematics performance of secondary school students

in Kenya. The study involved 534 students and 33 teachers, with 227 students and 15 teachers in the experimental group and 307 students and 18 teachers in the control group. The same mathematics topic was taught to both groups for six weeks, but the experimental group used the five essential AfL techniques while the control group used common methods. The Student Mathematics Achievement Test was used to collect data from pre- and post-tests. An independent samples t-test was used to analyze the data, and the result showed a positive impact on learners' achievement ($p < 0.05$, $t = 14.82$).

Van den Berg et al. (2018) conducted a quasi-experiment to examine how assessment for learning (AfL) practices in primary math education in the Netherlands affected student achievement. The intervention group consisted of 17 teachers who used an AfL approach that involved setting daily and weekly goals, assessing students' progress, and providing immediate feedback to those who needed it. The control group had 17 teachers who changed their usual methods. They used biannual math tests to monitor students' mastery of learning objectives, and they organized weekly pre-teaching sessions for low-performing students. After controlling for student and teacher factors, the posttests showed no significant differences in student outcomes between the two groups. However, there was a positive relationship between the degree of AfL implementation and the performance of fifth-grade students.

In their investigation of the effect of embedded AfL on students' achievement, Yin et al. (2008) found that the embedded AfL in the experimental group's curriculum had no statistically significant effect on students' achievement when compared to students in the control group who used the curriculum without embedded AfL. They argued that the result demonstrated the complexity and significance of performing assessment for learning

properly and that embedding assessments within the curriculum will have no effect on students' learning or motivation unless and until teachers use the assessment data to adapt their instruction.

Achievement Gains and their calculations

Learning gain refers to how students improve their knowledge, skills, work readiness, and personal growth, including their attitudes and values, and how they apply and demonstrate them in specific disciplines and institutions (Evans et al., 2018). Different ways of defining learning gain exist in higher education: as the change in student performance from one stage of their study to another, as a version of the concept of “value added” that is often used to compare school outcomes, or simply as “learning.” Measuring learning gains in higher education is still developing in England, but the UK has been a world leader in creating value-added indicators for school education and has been at the forefront of research and use of value-added models in secondary education (McGrath et al., 2020).

Measuring learning gain is a contentious issue because there is no consensus on what it means, what its purposes are, and who its intended audiences are. Different terms are used to refer to learning gain, which can create confusion or ambiguity. Learning gain definitions often highlight different aspects and combinations of aspects, such as how much progress is made, how much value is added, what specific or general outcomes are achieved (Evans et al., 2018). Moreover, to assess students' learning, objective tests and pre-post comparisons are usually preferred over self-reported measures. Objective tests are more independent of individuals' abilities to accurately judge their own learning improvement since they measure actual learning gains rather than perceived learning gains (Rogaten et

al., 2017). In this study, achievement gain is operationalized as a student-level normalized change score, which is based on Marx and Cummings (2007).

Marx and Cummings (2007) created a student-level alternative to Hake's normalized gain measure called the normalized change score, symbolized by c (Hake, 1998). Instead of computing learning gains at the classroom level with $\langle g \rangle$, Marx and Cummings advocate calculating learning gains at the student level using the following formula:

$$C = \begin{cases} \frac{\text{Post}-\text{Pre}}{100-\text{Pre}} & \text{if post} > \text{Pre} \\ \text{drop} & \text{if Post} = \text{Pre} = 100 \text{ or } 0 \\ 0 & \text{if } 0 < \text{post} = \text{pre} < 100 \\ \frac{\text{Post}-\text{Pre}}{\text{Pre}} & \text{if Post} < \text{Pre} \end{cases}$$

A student-level normalized change score is calculated like a classroom-level normalized gain when students improve their scores from the pre-test to the post-test. However, some students may have special cases: they are excluded if they get 0 or 100 on both tests; they get a 0 if they get the same score on both tests; and they get a negative score adjusted by the maximum points they could have lost if they do worse on the post-test. The normalized change scores of Marx and Cummings are useful because they measure the learning gains of individual students instead of whole classrooms. This means they can compare different interventions within the same classroom and also have more statistical power because of the larger sample size. However, normalized change scores also have a drawback. They do not account for the difficulty of improvement when students get a perfect score or no improvement on the post-test. Their score is 1 or 0 regardless of their pre-score, which could be very low or very high.

2.7.2 Effect of Assessment for Learning on Students' Self-Regulated Learning

Self-regulated learning is the process of learners taking charge of their own learning and transforming their cognitive abilities into academic competencies. This is essential because education's main purpose is to foster lifelong learning skills (Zimmerman, 2000). Assessment for learning and SRLS involve setting goals, evaluating progress towards the goals, and responding to feedback about the discrepancies between students' achievements and the goals (Andrade & Brookhart, 2016). Assessment for learning and self-regulated learning are mutually supportive, as assessment for learning strategies can facilitate students' self- and co-regulation of their learning. Students can self-regulate their learning by setting their own goals, checking their progress, and revising and adapting their goals and progress; in this process, the strategies of AfL are very helpful. Moreover, teachers can support their students' self-regulated learning by helping them plan, monitor, and assess their own learning (Andrade & Brookhart, 2016, 2019; Andrade & Heritage, 2018).

A study on undergraduate students' perception and response to a learning environment where they were expected to take charge of their own learning showed that goals guided them and examples helped them understand what was expected and what good work looked like. Course activities provided evidence of learning and discussions around these activities gave feedback on students' understanding and progress. Evaluating examples improved students' evaluation skills and expertise, while peer review and feedback provided a real context for evaluating and monitoring ongoing work. Each of the AfL tactics helped students self-regulate, but the combined and repeated influence of these strategies on students' thoughts, behaviors, and emotions showed the full impact of AfL as a driving force for self-regulated learning (Hawe & Dixon, 2017). In addition, in their investigation

of the effect of assessment for learning on fifth grade students' self-regulated learning skills, Ozan and Kincal (2018) found that although assessment for learning improved students' self-regulated learning skills, no significant difference between the experimental and control groups was seen.

Self-regulated learners have a potent combination of learning strategies, self-control, and motivation, which allows them to learn more effectively. They are more likely to set learning goals, employ suitable study and thinking tactics, manage their time, seek help when needed, utilize available resources, track their progress, and change up their learning approach when something isn't working. In a nutshell, they've figured out how to learn (Andrade & Heritage, 2018). In other words, teachers and students use AfL effectively to set learning goals (before learning), gather feedback on how well students are meeting those goals (during learning), and use that feedback to adjust teaching strategies and improve learning processes and outcomes (after learning) (Panadero et al., 2018).

According to (Panadero et al., 2018), AfL can enhance students' learning by making them aware of what they want to learn, how they can monitor their progress, and what steps they need to take next. These activities stimulate students' cognitive and motivational abilities, focus their attention on their learning goals, and offer feedback and suggestions to help them reach their desired outcomes. In summary, evaluation for learning can enable students to regulate their own learning.

According to (Zimmerman, 2015), "self-regulated learning" refers to the process of students actively taking charge of their own learning. Self-regulation is not a stable characteristic or ability, but a flexible process that enables learners to use their potential to enhance their skills in various areas, such as academic subjects, sports, music, and health.

Self-regulation can be developed and is not determined by innate factors. All students attempt to regulate their functioning, but some are more effective and consistent than others in doing so.

Furthermore, as highlighted by Andrade and Heritage (2017), the link between evaluation for learning and self-regulated learning is essentially theoretical. In theory, strategies such as providing feedback on progress toward learning objectives would assist students in taking ownership of their learning. They also claimed that teachers can assist students in organizing, monitoring, and evaluating their learning in order to scaffold self-regulated learning. Setting clear learning goals, selecting effective learning tactics, and arranging regular study periods are all part of the planning process. Monitoring entails students paying close attention to whether or not they understand an assignment, reading passage, or lecture, as well as measuring their progress toward their goals using methods such as self-testing. Evaluation comprises determining whether or not their objectives were attained as well as the efficacy of the learning approaches used.

Meusen-Beekman et al.'s (2016) research revealed that employing formative assessment to assist children in gaining self-regulation was helpful for both self-assessment and peer-assessment intervention groups in sixth grade writing tasks. Furthermore, there were no significant differences in self-regulation, motivation, or self-efficacy between the peer-assessment and self-assessment interventions. Chalachew (2018) also discovered that poor use of AfL by university lecturers harmed students' ability to self-regulate their learning in his study on the factors and influences of self-regulated learning.

In addition, although currently AfL is believed to improve student learning, it is not being implemented well in US mathematics instruction, and as Burton et al. (2018) hypothesize, this is because of the inadequacies of its implementation in initial teacher education and the lack of continuous PD for mathematics teachers. Furthermore, because changes in teacher educators' beliefs and attitudes typically take longer than changes in skills and knowledge, planning and implementing PD necessitates a significant budget, time, and attention (Deneen & Brown, 2016).

Similarly, research conducted by Fisseha and Michael (2016) revealed that students who were taught using quality assessment for learning strategies on lessons of a general psychology course reported more self-regulating learning than students who were not taught using quality formative assessment on lessons in their study of the effect of high-quality AfL on self-regulation in university classrooms. As a result, implementing high-quality AfL increases students' engagement and experience with self-regulating learning, with a near-to-high impact size. Zimmerman (2000) also claimed that self-regulation happens in three stages. The first phase is planning, in which goals are set, plans are formed, and methods are chosen. The following phase, performance control, includes processes that take place during the learning process (e.g., self-monitoring and adjusting strategies). Self-reflection, the final stage, comprises behaviors such as self-evaluation, adaptive inferences, and reflection that occur after learning or performing.

Self-regulated learning can be developed through a variety of instructional interventions that assist and lead students in the use of various self-regulated learning methodologies. However, little study has been conducted on the role of evaluation for learning in developing SRLS. Furthermore, it is stated that the link between assessment for learning

and co-regulation of learning merits additional empirical investigation and that research on this relationship should be undertaken in classrooms (Panadero et al., 2018).

Furthermore, research on the influence of assessment for learning implementations on students' self-regulated learning skills before and after intervention is critical, particularly in teacher education institutions. It is also critical to understand how continuing professional development contributes to the assessment of learning processes. As a result, it is critical to investigate the impact of enhanced evaluation for learning strategy implementations provided through continuous PD on students' self-regulated learning skills.

Self-regulation research brings together cognitive, metacognitive, and motivational factors to form a broad picture of how students comprehend and then pursue to attain the intended learning objectives. AfL and SRLS are two pillars that have been connected by the AfL movement. In the past 20 years, researchers have explored how AfL and SRLS are similar in their different types and roles and how they can mutually enhance each other under the right conditions (Panadero et al., 2018). Moreover, various teaching interventions can foster self-regulated learning by helping and directing students to use different learning strategies. However, few studies have looked at the function of assessment in fostering SRLS. Examining that specific relationship is usually restricted to the realm of AfL (Panadero et al., 2018).

Teachers and students use effective AfL to achieve the following steps: First, they set clear and specific learning targets that guide their actions and expectations (this is the preparatory phase goal-setting). Second, they collect and analyze feedback on how well

the students are progressing towards those targets and identify any gaps or misconceptions (this is the performance phase monitoring). Third, they use the feedback to make adjustments to the teaching strategies and methods by teachers and to the learning processes and work products by students (this is the appraisal phase adjustments and revisions) (Panadero et al., 2018).

Furthermore, AfL that provide process and self-regulation feedback have the potential to be extremely beneficial in terms of encouraging both achievement and self-regulation development. When feedback is followed by an opportunity for the teacher to enhance training and/or for students to adapt and improve their work, it is most useful (Andrade & Heritage, 2018). When applied properly, AfL can help students understand what they are trying to learn, how they can monitor their learning progress, and what steps they need to take next. These methods stimulate students' thinking and motivation, focus students on their learning goals, and provide feedback and advice to help them reach their goals. In short, assessment can enable students to regulate their own learning (Panadero et al., 2018).

2.7.3 Effect of Teachers' Conceptions of Assessment for Learning

Hamodi et al. (2017) suggest that initial teacher education courses should use effective AfL strategies to prepare future school teachers for the specific challenges they may face in implementing these assessment systems in schools. However, this requires quality PD for lecturers to enable them to use these strategies effectively. They also stress the importance of developing and strengthening AfL systems in both universities and schools, as well as creating a connection between the faculties where pre-service teachers learn and the schools where they will teach our children. Moreover, they claim that the AfL experiences

that pre-service teachers had as university students were helpful for their professional practice as teachers in schools.

The findings of the research by Amoako et al. (2019), also revealed that there is a link between instructors' knowledge of AfL and the frequency with which they use it in the classroom. This assumes that there is a link between a person's AfL knowledge level and the rate at which they practice AfL in the classroom. In reality, a number of factors play a crucial role in putting what teachers know into practice. Teachers' attitudes and commitment to the teaching profession, as well as the availability of adequate resources, influence teachers' AfL experiences to a considerable extent.

The results of an investigation of the influence of AfL experiences on student learning approaches by Gan et al. (2019) in Chinese higher education showed that students' AfL experiences had a favorable impact on their learning styles. Moreover, all dimensions of students' perceived AfL experience were found to be significantly positively correlated with students' tendency to use a deep approach to learning, implying that AfL experiences contribute to students adopting a meaningful approach to learning in the Chinese and Hong Kong university environments.

It is important for future teachers to learn how to assess their students effectively using current assessment ideas, theories, and methods. They should reflect critically on their own experiences and beliefs about assessment. One way to help them do this is to use AfL in the mathematics courses they take. AfL can show them how to use assessment to improve teaching and learning. Deluca and Volante (2016) argue that if future teachers do not experience this kind of assessment culture, they may develop a negative attitude towards

assessment. They may also rely on outdated or ineffective assessment practices that they are familiar with instead of using a more formative approach to assessment.

One of the key factors for the successful adoption of AfL practices and principles in CTEs is the training of pre-service teachers, as several researchers have highlighted (Andrade & Brookhart, 2019; Brevik et al., 2017; Carless & Boud, 2018). In addition, pre-service teachers should develop feedback literacy skills that enable them to give and receive quality feedback to and from their peers and lecturers. This requires creating a supportive and trusting teaching and learning environment where learners can confidently and openly express their understanding, accept others' opinions constructively, and make sense of the information they receive (Carless & Boud, 2018). Furthermore, pre-service teachers' educators should demonstrate various AfL strategies in their course work to help pre-service teachers gain a deeper understanding of the importance and implementation of these strategies (DeLuca & Volante, 2016). Moreover, pre-service teachers should not only learn about the concept of AfL explicitly but also practice it over time, as AfL is not an innate but a learned approach, according to DeLuca et al., (2018). Finally, using AfL in pre-service teacher education can have positive impacts on the quality of the teaching-learning process and students' learning outcomes, as well as on students' perceptions of its practical value in their future work, as López-Pastor and Sicilia-Camacho, (2015).suggested.

2.7.4 Conceptual Framework

The conceptual framework of the research is briefly presented by the diagram given below.

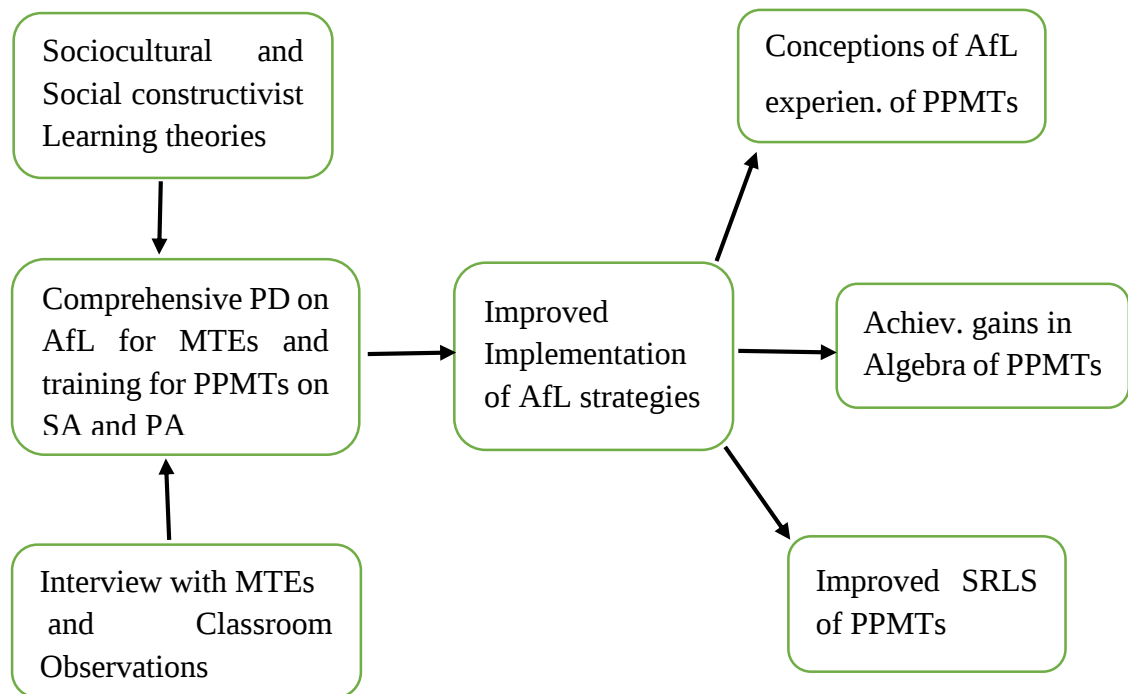


Figure 1. Conceptual framework of the study

2.8 Summary Review of Literature

The studies presented so far show the importance of the implementation of AfL strategies in improving students' learning in different subject areas and at different levels of education worldwide. Moreover, the literature reviewed so far has discussed the challenges and difficulties of implementing AfL strategies in different contexts across different levels, including in teacher education institutions. In addition to the studies presented thus far, which described the current state of AfL implementation in Ethiopia in general and in Ethiopian teacher education institutions in particular, the studies discussed the characteristics of meaningful and effective PD for teachers and teacher educators on

important methodologies, including AfL, and the challenges in implementing effective PDs.

Many researchers who conducted research on the implementation of, the effect of, and the nature of implementation of AfL in different contexts and educational levels recommended future research on different contexts and subjects, especially in teacher education institutions, on how they are implementing AfL, what their conceptions of AfL are, and what the impact of implementation of AfL is on their achievements in different courses and their conceptions (Baird et al., 2017; Barnes et al., 2017; Black & Wiliam, 2010; Burton et al., 2018; Coombs et al., 2018; Johnson et al., 2019; Pattalitan, 2016). Thus, there is a need to design an effective PD on AfL for MTEs and train pre-service mathematics teachers, and to investigate and examine its effects on the proper implementation of AfL strategies, on the achievement gains of pre-service teachers, on pre-service teachers' conceptions of AfL experiences, on pre-service teachers SRLS, and on other theoretically related variables.

CHAPTER THREE: METHODOLOGY

3.1 Introduction

The previous chapter discussed the theoretical and conceptual framework of the research as well as literature related to AfL implementations, challenges, and the current status of AfL implementations in global and local contexts. This chapter discusses the philosophical perspective that oriented the research, the worldview that shaped the research, the research design used in carrying out the study, the selection of participants in the research, the tools used in collecting data, how the tools were validated and their reliability checked, and how the data were analyzed.

3.2 Research Approach

This study contains research questions that demand utilizing both quantitative and qualitative. The qualitative data were gathered in order to investigate the conception of mathematics educators regarding AfL, how professional development is received by mathematics educators, and how the PD impacted the implementation of AfL in an algebra course. Moreover, the quantitative data were collected to gather information about pre-service primary mathematics teachers' conceptions of AfL, SRLS, and achievement gains in algebra. Hence, a mixed-method approach was the appropriate method to conduct this research. Therefore, this study followed a mixed-methods approach.

According to some mixed methods scholars, mixed methods are informed by a single worldview, such as pragmatism, transformational approaches, or critical realism. The dialectical pluralism worldview welcomes the idea that different worldviews may contribute to a mixed-methods inquiry. A third point of view holds that researchers can

choose one of the aforementioned positions for their study, with the choice depending on the type of mixed-methods design used. Another viewpoint is that different scholarly groups may have different worldviews that they develop within their own contexts. To establish a foundation for merging quantitative and qualitative methodologies, the assumptions supporting a mixed-methods study must be recognized and acknowledged, regardless of viewpoint (John & Clark, 2018).

This study used a mixed-methods approach, which allowed the researcher to deal with both qualitative and quantitative data in a single study. As Johnson et al. (2007) argued, mixed-methods research is a way of conducting research that integrates both qualitative and quantitative approaches. It is not just a combination of these two types of research, but a distinct and holistic research paradigm that offers more comprehensive, balanced, and useful insights than either type alone. Mixed methods research is based on the philosophy of pragmatism, which can take different forms depending on the researcher's perspective and values. It also follows a specific logic that guides the design, implementation, and interpretation of the research process. This logic includes the fundamental principle of mixed methods research, which states that the researcher should use both qualitative and quantitative methods in a way that best answers the research question(s). The researcher may also adopt other logical principles from qualitative or quantitative research that are relevant and helpful for producing credible and applicable research findings. Moreover, mixed methods research is aware of and responsive to the social and political contexts and implications of the research, as well as the resources and needs of the researcher and the stakeholders.

3.3 Research Paradigm

The philosophical worldview or research paradigm that guided the research is pragmatism. The paradigm offers this study a philosophical basis that is not dedicated to any one system of philosophy, like in mixed-method research approaches, where researchers are permitted to make both quantitative and qualitative assumptions when conducting their research.

Furthermore, this paradigm gives researchers the freedom and flexibility to choose the most suitable and appropriate research procedures, techniques, and methods that match their objectives and requirements (Creswell & Creswell, 2018). Researchers may use one or more worldviews to inform their study, including constructivism, post-positivism, transformational worldview, and pragmatism. Worldviews are the beliefs and values that researchers bring to their studies (Kuhn, 1962). Each worldview has unique components, and these components are mirrored in various philosophical assumptions on ontology, epistemology, axiology, methodology, and rhetoric (John & Clark, 2018).

The research needs to collect qualitative and quantitative data with multiple instruments. Hence, the pragmatic paradigm will provide the researcher with the opportunity to choose designs, methods, and approaches (Nunez, 2015) that can help achieve the objectives of the study. Furthermore, the paradigm recognizes that research is conducted in social, political, historical, and other contexts, each of which has its own influence on the research findings, and it opens the door to multiple methods and various forms of data collection and analysis for a researcher conducting mixed-method research.

3.4 Research Design

When researchers conduct studies, they use different methods to collect, analyze, interpret, and report the data they obtain. These methods are called research designs and they represent various types of research approaches, each with a specific name and a sequence of steps that researchers follow. Because they establish the logic by which researchers make interpretations at the conclusion of their projects, research designs are helpful in directing the method choices that researchers must make during their studies (John & Clark, 2018).

The researcher wanted to first investigate the conceptions of AfL of MTEs using a semi-structured interview and then design PD for MTEs who provide the Fundamental Concept of Algebra (Math 221) course in one of the CTEs and train PPMTs who are taking the course. Then the researcher wanted to examine the effect on the practices of AfL in the course, on the pre-service teachers' achievement gains in Math 221, on their conceptions of AfL experiences, and on their self-regulated learning skills as well as how MTEs experienced the PD. Based on the research question and objectives of the research, the selected research design for this study was an embedded mixed-method research design that combined quantitative and qualitative data collection and analysis within a single study.

In an embedded design, the researcher aims to gather both numerical and non-numeric data in a single study, either at the same time or one after the other. However, one type of data has a more dominant role than the other type of data. The secondary type of data is collected to enhance or support the primary type of data. For example, in a quantitative experiment that tests the effect of an intervention, the researcher may also collect

qualitative data to explore how the participants in the treatment group perceive and experience the intervention. Alternatively, the researcher may collect qualitative data before or after the experiment to provide additional insights or explanations for the experimental results (Creswell, 2012).

Therefore, the appropriate design for the research is an embedded mixed-method research design since qualitative data were collected before, during, and after the intervention to achieve the research objectives.

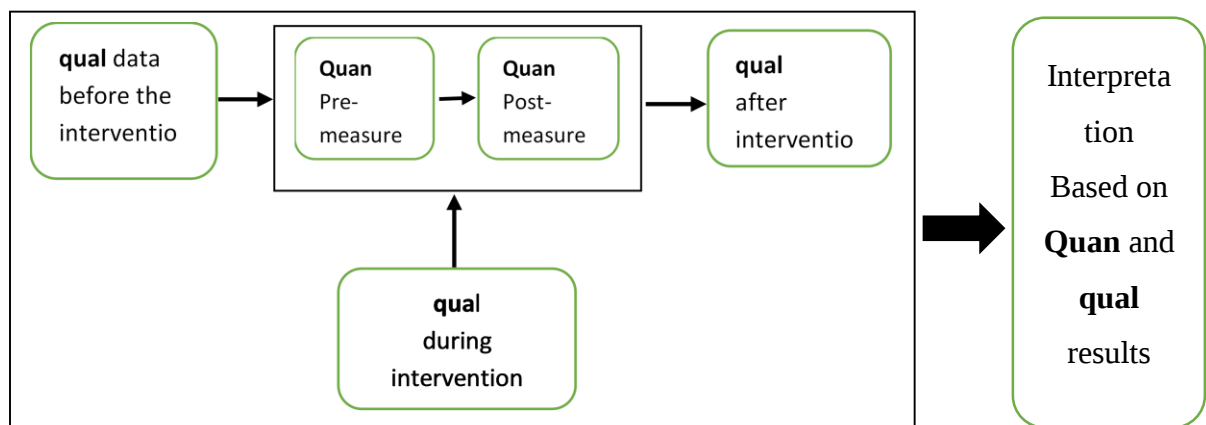


Figure 2. The research Design

3.5 Variables of the Study

This research has one independent variable and four dependent variables. The independent variable in this research is comprehensive PD on AfL for MTEs and training for PPMT on SA and PA. Furthermore, the dependent variables in this study are the PPMT's conceptions of their AfL experience, their achievement gains in the Fundamental Concepts of Algebra course, their SRLS, and the extent to which AfL strategies is implemented in the classroom instruction of the Fundamental Concepts of Algebra course.

3.6 Participants of the Study

The participants of this study were all 129 second-year PPMTs in three selected teacher education colleges in Ethiopia's Amhara regional state in 2021 and all 13 MTEs in one of the three selected teacher education colleges. Among the 129 PPMT, 52 of them were in the intervention group in three different sections; 38 of them were in comparison group one in two different sections; and 39 of them were in comparison group two in two different sections. Whereas the 13 MTEs were all in the treatment group.

Table 2. Profiles of Mathematics Teacher Educators

Code of MTEs	2 nd degree Type	Professional Development about Assessment for Learning
MTE01	M.Sc	Has not Received any kind of PD or training about AfL
MTE02	M.Sc	Has not Received any kind of PD or training about AfL
MTE03	M.Sc	Has not Received any kind of PD or training about AfL
MTE04	M.Ed	Has not Received any kind of PD or training about AfL
MTE05	M.Ed	Has Received some training about AfL
MTE06	M.Sc	Has not Received any kind of PD or training about AfL
MTE07	M.Sc	Has Received some training about AfL
MTE08	M.Sc	Has not Received any kind of PD or training about AfL
MTE09	M.Ed	Has Received some training about AfL
MTE10	M.Ed	Has not Received any kind of PD or training about AfL
MTE11	M.Sc	Has not Received any kind of PD or training about AfL
MTE12	M.Sc	Has Received some training about AfL
MTE13	M.Sc	Has Received some training about AfL

Note. M.Sc = Masters of Science in Mathematics (specialized in Algebra, or in Analysis, or in Differential... and did not conduct educational research for the fulfilment of their degree)

M.Ed = Masters of Education in Mathematics (and conducted an educational research for the fulfilment of their degree.)

3.7 Sample Size and Sampling Techniques

The study took place in the Amhara region of Ethiopia, where there are ten colleges that educate and train teachers for primary schools. The researcher randomly selected three of these colleges by drawing lots, and then randomly assigned one of them to be the treatment group by using the same method of drawing lots. The other two colleges that were not chosen as the treatment group became the comparison groups for the study. The research involved all the second-year PPMTs and their MTEs in the treatment group, as well as two comparison groups. The treatment group had 52 PPMTs in three sections and 13 MTEs, but only three of them taught the Fundamental Concepts of Algebra course to the PPMTs. The comparison group one had 38 PPMTs in two sections, and the comparison group two had 39 PPMTs in two sections.

3.8 The Intervention

The intervention in this research consisted of three parts: 1. job-embedded, context-specific, content-based PD on AfL for MTEs; 2. training for PPMTs on SA, PA, and how to provide and receive quality feedback; and 3. improving the teaching and learning module of the Fundamental Concepts of Algebra course.

3.8.1 The Comprehensive Professional Development for Mathematics Teacher Educators

The comprehensive context-specific, job-embedded, and content-based comprehensive PD on AfL for MTEs consisted of theoretical and practical parts. Theoretically, there were discussions about the strategies, nature, principles, and purposes of AfL among the researcher and the three MTEs. The Three MTEs in the intervention group were first invited to reflect on their conceptions, and then there was a presentation by the researcher and a question-and-answer session. Moreover, prior to the course, MTEs were given the opportunity to implement AfL strategies practically on selected topics of the Fundamental Concepts of Algebra course by practicing them in front of the group (the researcher and the other MTEs) using the improved teaching and learning module that can help with the implementation of AfL in the Math 221 course. In addition, mathematics teacher educators were involved in observing each other's implementation of AfL strategies in the Math 221 course, and there was a discussion and reflection on the implementation of AfL strategies. Moreover, MTEs were given continuous support by the researcher and each other during the normal teaching and learning of the Math 221 course once a week.

The comprehensive PD for the three MTEs was designed taking in to account the information obtained from them through semi-structured interviews about their perceptions of the strategies, principles, nature, and purpose of AfL, as well as the implementation challenges. Furthermore, data collected, prior to the intervention, from observation of the three MTEs' classrooms implementing the five AfL strategies was used as input to design the comprehensive PD.

To make the teaching and learning module of the Fundamental Concepts of Algebra course appropriate for implementing AfL strategies, several enhancements were made. These include specifying the learning objectives for each subtopic of each chapter within the module, providing concise summaries of the key concepts covered in each chapter of the module, and adding additional summary exercises at the end of each chapter to help the pre-service teachers to assess themselves and improve their learning. Moreover, the three MTEs involved in the implementation of the intervention were provided with the rubric of the classroom observation protocol to help them provide quality feedback and scaffolding to each other on how to improve implementations of the five AfL strategies.

In addition, the three MTEs had multiple options for making the learning objectives of the subtopics they were discussing more explicit and clear to their students, as well as for detecting and addressing possible misconceptions and challenges that their students might face. They also benefited from research evidence and their own prior experiences of teaching the course in terms of designing effective oral questions and classroom activities that would engage their students in meaningful learning.

Moreover, they learned various methods for giving constructive and high-quality feedback to their students by recognizing their strengths and areas for improvement, motivating them to assess their own and their peers' work based on the learning goals, and exchanging and receiving feedback that would help them improve their learning. The three MTEs also had the opportunity to observe and evaluate each other's classes by using a classroom observation tool that enabled them to assess how well the five AfL strategies were implemented in a single class session, reflect on their own and each other's practices, and

provide feedback by supporting each other on possible ways to enhance AfL strategy implementation.

3.8.2 The Training to Pre-service Primary Mathematics Teachers

The researcher provided the PPMTs with the training, and their MTEs supported and advised them on enhancing their SA and PA. The training about SA, PA and how to receive and provide quality feedback for PPMT consisted of general discussions of the principles, strategies, and purposes of AfL using materials that were developed by the researcher for this purpose. In addition to gaining practical skills in how to deliver and receive high-quality feedback from MTEs and their fellow students, PPMT also learned how to evaluate their own performance and that of their peers based on the learning objectives of a lesson and criteria of success provided to them by their MTEs.

The intervention group of PPMTs received guidance and examples on the importance of comprehending the definitions, terminologies and theorems used in mathematics, accepting mistakes as a normal and frequent part of the process of teaching and learning mathematics, and using errors as opportunities for improvement at the beginning of the course. Moreover, PPMTs were taught about the significance of responding to oral and textual questions and pose questions that enhance the understanding of mathematical concepts. MTEs were also oriented to help their PPMTs provide and receive quality feedback in the classroom instruction process.

3.8.3 The Improvement of Fundamental Concepts of Algebra Module

The Fundamental Concepts of Algebra course's teaching-learning module was improved by the researcher in collaboration with MTEs to easily implement the five AfL strategies by assisting them in clarifying learning intentions (objectives) to their students. The module

also incorporated guidelines for PPMT on how to use the module appropriately. The guidelines for PPMTs included ways on how to use the teaching and learning module appropriately: the learning objectives, the summary of main ideas, and the summary exercises, the details of which are presented in appendix X.

A. The guidelines on how to use the improved teaching and learning module

The teaching and learning module of Fundamental Concepts of Algebra course is a useful resource for the PPMTs, but they may need some guidance on how to use it most effectively. Therefore, before starting the first chapter of the module, the PPMTs were provided with some helpful guidelines, which is shown in appendix XIII, that explain the objectives, structure, activities and assessment of the module.

B. The learning objectives incorporated at the beginning of each of the sub-topics

The learning objectives of each of the subtopics in each of the six chapters were incorporated to help PPMTs assess themselves easily based on the learning intentions, assess their peers, and regulate their own learning of the Fundamental Concepts of Algebra course, as can be seen from Appendix VIII.

C. Summary of the main ideas which are incorporated at the end of each of the chapters

The main ideas, definitions, and theorems discussed in each of the six chapters are summarized and presented at the end of each of the chapters, as can be seen from Appendix IX. The incorporation of summaries of each of the chapters is aimed at facilitating PPMTs' SA in the intervention.

D. Summary exercises incorporated at the end of the six chapters

In addition to enriching activities at the end of each of the subtopics in each of the chapters, summary exercises were incorporated at the end of each of the chapters, as can be seen from Appendix X, which was aimed at helping PPMTs' SA in the intervention group.

3.9 Data Collection Instruments

To collect data from the participant MTEs and PPMTs, the researcher used an achievement test, two questionnaires, a classroom observation checklist, and semi-structured interviews. The achievement test was used to collect data from the pre-service teachers about their progress in their achievement gains in the Fundamental Concepts of Algebra course. Besides, the test consists of 22 items specially developed for this research. The first questionnaire was used to gather information from pre-service teachers about their conceptions of AfL experiences in the teaching and learning of the algebra course. In addition, the second questionnaire was used to collect data from the pre-service teachers about their self-perceptions of their own SRLS before and after the intervention. Moreover, the classroom observation was conducted to gather information about the progress of the levels of implementation of AfL strategies by MTEs and PPMTs in the teaching and learning of the algebra course every two weeks during the delivery of the course. The first semi-structured interview was conducted to gather data about MTEs' perceptions of the nature, principles, strategies, and purposes of AfL, and the second interview was conducted to collect data from the three MTEs who participated in the comprehensive PD on AfL about their experiences with the PD.

3.9.1 Achievement Test

To examine the effect of implementing enhanced AfL practices on the learning outcomes of PPMTs in the intervention and the two comparison groups, this study designed a specific achievement test that was used as both a pre-test and a post-test. The pre-test was administered to all second-year PPMTs in each group at the beginning of the semester in January 2021, before implementing any AfL strategies. The post-test was given to the same PPMTs in each group at the end of the semester in April 2021, after implementing the AfL strategies for twelve weeks. The difference between the pre-test and post-test scores of the PPMTs in each group was used to measure their achievement gain.

The course on the Fundamental Concepts of Algebra is a part of the curriculum for PPMT students who are in their second year of study and have completed the first semester. The researcher, who conducted the study on the intervention implemented by three MTEs who were also involved in the study, designed an achievement test to measure the learning outcomes of the course. The test items were based on a proportional selection of specific objectives from each chapter of the course content. The test was also reviewed and validated by other MTEs in the treatment group who had expertise and more than 20 years of teaching experience in the college. Three Mathematics educators from Addis Ababa University and the teacher education college where the intervention took place provided feedback on the difficulty levels, face validity, and content validity of the test items, which were taken into account by the researcher in finalizing the test. Before administering the test to the participants of the study, the researcher conducted a pilot test with 35 pre-service primary mathematics teachers who had already taken the course in their third year of study.

Two test items that none of the 35 pre-service teachers answered correctly were replaced with new ones.

To further test the modified achievement exam, another college's 36 third-year pre-service primary mathematics teachers took it as a pilot test. The test's reliability was measured using the Kuder-Richardson 20 (KR-20) test, which is a method of estimating the internal consistency of a test. The KR-20 score was 0.73, which showed that the test had a satisfactory level of internal consistency within the sample. The difficulty level of the achievement test was also calculated, and it showed how easy or hard each of the 22 items was for the test-takers. The difficulty levels ranged from 0.28 to 0.80, with an average of 0.535, which meant that none of the items were too hard or too easy and were therefore in an acceptable range. Moreover, the discrimination indices of all of the 22 items were calculated separately, and they showed how well each item discriminated between high and low achievers. The discrimination indices varied from 0.20 to 0.80, with an average of 0.43, which meant that each item had an acceptable level of discrimination index.

3.9.2 Conceptions of Assessment for Learning Practices Questionnaire

The researchers wanted to find out how PPMTs' views of AfL experiences in the teaching and learning of the Fundamental Concepts of Algebra course changed before and after the intervention among the intervention and the two comparison groups. To do this, they used a questionnaire that had been developed and validated by Pat-El et al. (2013), as can be seen in appendix V. The questionnaire was designed to measure the perceptions of AfL experiences of both students and teachers in classrooms using items that matched each other. The questionnaire had two versions: one for students (SAFL-Q) and one for teachers (TAFL-Q). However, in this research, only the version for students (SAFL-Q) was used.

The SAFL-Q questionnaire had two parts: monitoring and scaffolding. Monitoring was about how the teacher assessed the students' progress, and scaffolding was about how the teacher gave feedback to help the students learn more effectively. The monitoring part had 16 items, and the scaffolding part had 12 items.

The researcher obtained permission to translate the instrument into Amharic and use it for research purposes by properly citing it. The questionnaire was first translated into Amharic by experts, and then it was back translated to English to check the equivalence of the translated and original questionnaires. The questionnaire was pilot tested for reliability on 38 3rd-year PPMTs. The Cronbach's alpha reliability coefficient was found to be 0.92, which showed that the reliability of the instrument is excellent.

3.9.3 Self-Regulated Learning Questionnaire

In this research, PPMTs' perceived SRLS was measured before and after the intervention. The questionnaire used to examine PPMTs' SRLS before and after intervention was a translated version of the learning methods part of Pintrich et al.(1991)'s Motivated Strategies for Learning Questionnaire (MSLQ). The MSLQ is a tool that allows college students to assess their own levels of motivation and the different strategies they employ to learn effectively in a specific college course. The MSLQ consists of several questions that capture the students' goals, beliefs, values, and emotions related to learning, as well as their use of cognitive, metacognitive, and resource management techniques to enhance their academic performance. The MSLQ's fifteen various scales are advised to be used together or independently.

Furthermore, the scales are modular and may be adjusted to match the demands of researchers (Pintrich et al., 1991). The researcher needs no permission to translate and use it for research purposes since it was granted permission as long as the source is cited appropriately and the researcher cited the source properly. The MSLQ appears to be the most reliable and commonly utilized instrument in the SRLS studies (Roth et al., 2016).

The MSLQ's learning strategy section has 50 items, 31 of which are related to students' use of various metacognitive and cognitive methods. The learning techniques section has 19 items pertaining to student resource management. The researcher translated the 50 items into Amharic in consultation with teacher educators, and they were then back translated into English to evaluate if the original and translated meanings matched. Face and content evaluation by language and psychology educators in the treatment group college guaranteed that the original and translated items were congruent.

Before using the questionnaire in the main study, the researcher tested its quality and suitability by administering it to a sample of 38 third-year students who were training to become mathematics teachers. The researcher then calculated the Cronbach's alpha reliability statistic to measure the internal consistency and reliability of the questionnaire items. The result was 0.92, which is a very high value that shows that the questionnaire is a reliable instrument for assessing the self-regulated learning strategies of the pre-service mathematics teachers.

3.9.4 Classroom Observation Protocol

To examine how the intervention group enhanced their use of the five assessment for learning (AfL) strategies in their teaching practices, classroom observations were carried out on a biweekly basis in each of the three sections that participated in the intervention. The teacher educators who were observed received feedback from one another on how they could further improve their application of AfL methods in their classrooms. For the classroom observation of implementations of the five AfL strategies, a classroom observation protocol, developed by Oswalt (2013), as can be seen in Appendix III, was used to record the progress of the level of classroom implementation of AfL strategies in Math 221 courses in the treatment group. The classroom observation tool is a validated and reliable instrument with 23 items rated on a Likert scale from "no evidence of use" (1 point) to "pervasive use" or "highly effective" (5 points). The classroom observations were conducted by a trained mathematics teacher educator in the college where the intervention was implemented and by the researcher once every two weeks to record the improvement of AfL strategies.

The intervention was carried out by three MTEs who had access to the classroom observation protocol and its corresponding rubric. The protocol and the rubric were designed to assess the use of five AfL strategies in a single classroom session. The MTEs were also encouraged to observe each other's classes and use the protocol and the rubric to rate how well they implemented the AfL strategies. They also engaged in reflective discussions with each other and gave feedback and suggestions on how to improve their AfL practices. This was a form of ongoing PD that aimed to enhance their professional capability as MTEs.

3.9.5 Interviews

The researcher, along with teacher educators in the field of measurement and evaluation, designed a semi-structured interview to collect data from the MTEs who participated in the intervention group. The data would help to understand how these MTEs viewed the strategies, nature, principles, and purposes of AfL in their teaching, as can be seen in appendix I. The interview was carried out in person, and the researcher also asked some additional questions to elicit more details or explanations from the MTEs when needed. Similarly, semi-structured interviews were developed by the researcher in collaboration with assessment experts to gather data from MTEs who participated in the comprehensive PD on implementation AfL strategies about how they think they experienced the comprehensive PD, which can be seen in appendix II.

3.10 Data Collection Procedures

Before the intervention, a semi-structured interview with all 13 MTEs in the intervention group was conducted face-to-face to collect data from them about their conception of the nature, principles, strategies, and purposes of AfL. Besides, pre-measures were taken from PPMTs from both of the comparison groups and the intervention group on their conception of the AfL experience, pre-achievements in the Fundamental Concepts of Algebra course, and pre-SRLS. Furthermore, PPMTs were instructed to carefully read each of the questionnaires' items before circling what was true for them, and they were informed that there was no correct or incorrect answer.

During the intervention, data were collected every two weeks from each of the three MTEs using a classroom observation checklist to record the progress of the implementation of AfL strategies in the teaching and learning of the Fundamental Concepts of Algebra course.

Moreover, after the intervention, post-measures using the two questionnaires and the achievement test were collected from PPMTs in the comparison groups and from PPMTs in the treatment group

3.11 Data Analyses

In embedded mixed method research design, the researchers do not combine or integrate the qualitative and quantitative data during the analysis stage. Instead, they keep them separate and analyze them independently. The reason for this separation is that the two datasets usually address different research questions that have different purposes and assumptions (Creswell, 2012). The interview data gathered from MTEs on their conceptions of the nature, principles, strategies, and purposes of AfL was analyzed thematically using pre-defined themes.

The interview data from the MTEs about their experiences with the comprehensive PD was also analyzed thematically. Moreover, the data gathered from classroom observation of the implementation of AfL in the teaching and learning of the algebra course was analyzed thematically. Besides, the data obtained from the achievement test in the comparison and intervention groups were analyzed using ANOVA to examine the difference in their achievement gains. Furthermore, ANOVA was used to analyze data from the comparison and treatment groups' questionnaires about their conceptions of AfL experiences. Moreover, a paired t-test was used to analyze the data obtained from PPMTs in the intervention group before and after the intervention on their SRLS.

3.12 Ethical Considerations

Regarding the tools used in this research, the researcher has received permission from the authors before he uses them. One of the questionnaires does not need to ask permission to be used as long as the source was cited appropriately and the researcher used the questionnaire by citing it appropriately. Regarding the participants of the research, the researcher first got written permission from the Regional Education Bureau, from the teacher education colleges where the participants were found, and from the participant MTEs.

In addition, the researcher used pseudonyms for the colleges and interviewed participants who are MTEs. Moreover, prior to administering the questionnaires and the achievement test to PPMTs in this study, they were informed of its aim, and the researcher verified that each participating PMT had the choice to withdraw participation in the study if they wanted to. In addition to that, the researcher obtained a formal agreement letter from the three MTEs who were involved in the intervention. The letter indicated that they agreed to participate in the research voluntarily, and they were aware that they could withdraw from the research at any point if they wished to do so.

CHAPTER FOUR: RESULTS AND DISCUSSION

4.1 Introduction

The previous chapter discussed the methodology the researcher followed in tackling the research problem. This chapter discusses the results of both the qualitative and quantitative parts of the research. In this research, the main focus was first to examine conceptions of AfL among MTEs and then to investigate how a comprehensive context-specific, job-embedded, and content-based PD on AfL for MTEs and training on SA and PA for PPMT in the teaching and learning of an algebra course affect the practices of AfL in the algebra (Math 221) class, their achievement gain in the course, their conceptions of AfL experiences, and their SRLS. The first three sections describe the qualitative part of the research, and the next three parts describe the quantitative part of the research.

4.1 Results

4.1.1 Mathematics Teacher Educators' Conception of Assessment for Learning

This section presents the findings from semi-structured interviews with MTEs about their conceptions on the strategies, principles, methods, and nature of AfL and their views on the difficulties in implementing AfL in the mathematics courses teaching and learning in CTEs.

4.1.1.1 Mathematics Teacher Educators' Conceptions of the Nature of Assessment for Learning

The responses of MTEs about their conception of the nature or main characteristics of AfL were categorized into five hierarchical levels. The five levels, which range from *complete understanding* of the main characteristics of AfL to *no understanding* of the main characteristics of AfL, are described as follows:

Level 1: *Complete Understanding (CU)*: Responses that included all or almost all of the main components of the validated correct responses, such as: AfL is implemented by the teacher educator, by students as well as by their peers; AfL is implemented day to day and minute by minute while the classroom instruction is carried out; AfL involves interactions among the teacher, students, their peers, and the teaching and learning materials.

Level 2: *Incomplete Understanding (IU)*: Responses that included at least one of the components of the validated correct responses but not all of the components.

Level 3: *Incomplete Understanding with Some Misconception (IUSM)*: Responses that showed some understanding of the concept but also made a statement or statements that demonstrated some misconceptions, such as: AfL is implemented at the end of a lesson, at the end of a chapter, or at the end of a semester; AfL is implemented by the teacher (mainly or entirely); AfL is implemented a fixed number of times (not frequently).

Level 4: *Some Misconception (SM)*: Response that comprised misconceptions like stated in level 3 or other incorrect information about the nature of AfL.

Level 5: *No Understanding (NU)*: When respondents repeated the question; contained unrelated or irrelevant information or vague response or left without responding.

The interview data was analyzed to examine the respondents' comprehension of the nature of AfL. The results showed that approximately 50% of the respondents (6 out of the total 13) possessed a thorough and accurate understanding of what AfL entails and how it works.

These respondents properly identified all or almost all components of the nature of AfL. One of the six mathematics teacher educators' (MTE 05) responses about the nature of AfL is described as follows:

Assessment for learning is implemented or performed primarily by the teacher. Besides, it should be implemented by the student himself. Assessment for learning should be implemented at the beginning, middle, and end of the lesson. Since assessment for learning is a tool for students to bring about change in the classroom instruction process, it is always implemented as long as there is teaching and learning (MTE05, Tuesday, 9 Feb, 2021).

This teacher educator clearly and properly described at which stages of the teaching and learning process, how frequently, and who should implement AfL in the classroom instruction process. Besides, another mathematics teacher educator (MTE 06) described the nature of AfL as follows:

Assessment for learning should be implemented by the teacher and the student himself. I think AfL can be implemented at all three times: before, during, and after a lesson. Before we start the lesson, we can measure the students' knowledge and expand their knowledge based on that, you can use it to fill in the gaps that you see during the daily lesson is being given; we can also give students feedback on what to read and study after the lesson. I think assessment for learning should be implemented frequently. In our current context, there are many topics that need to be addressed and learning objectives can be many, so we need to use assessment for learning frequently (MTE06, Tuesday, 9 Feb, 2021).

The responses of the remaining four teacher educators also revealed that they have sound and complete understanding of the nature of AfL. Moreover, the analysis of interview responses showed that around half of (6 out of 13) the teacher educators have incomplete

understanding of the nature of AfL with some misconceptions. For instance, one of the respondents (MTE 09) described the nature of Assessment for learning as follows:

Assessment for learning must be done by the teacher. He is the teacher who is teaching and AfL should be implemented by the teacher. Assessment for learning should be implemented before starting of the lesson to find out about students' related prior knowledge, while the teaching is being conducted by asking verbal questions and assigning assignments as well as after the lesson is completed. Since students may get bored if we implement assessment for learning very frequently, it should be implemented with some time intervals, from 3 to 6 times in a period (MTE09, Wednesday, 10 Feb, 2021).

The respondent conceived that AfL is implemented by the teacher, although it is implemented by the teacher, the student and his/her peers in the classroom instruction process.

Furthermore, , another participant (MTE 03) also described the nature of Assessment for learning as:

Assessment for learning should be implemented primarily by the teacher. Researchers and the Regional Bureau of Education can assess according to their needs. I don't think it's enough to apply assessment for learning once in a single period. Depending on the reception of the students and the weight of the course, the frequency will vary. Students like tests to be given more frequently; given the limited time available, however, we cannot do so. Since I have no detail

understanding about assessment for learning, I don't have much to say about it (MTE03, Monday, 8 Feb, 2021).

These teacher educators have incomplete understandings of the nature of assessment for learning and also showed to have some misconceptions about it. Moreover, the remaining Mathematics teacher educator (MTE 04) showed that he has some misconceptions about the nature of AfL. He described the nature of AfL as:

I think it would be better if AfL is implemented at the beginning of the class. The assessment will be helpful for learning if the student first assesses himself and then learns the lesson. I believe that the implementation of AfL at the beginning of the lesson will help the learning process (MTE04, Tuesday, 9 Feb, 2021).

In general, most of mathematics teacher educators have been found to have some ideas about the *nature* of assessment for learning even though some of them have some misconceptions.

4.1.1.2 Mathematics Teacher Educators' Conceptions of the Principles of Assessment for Learning

Parallel to the nature of AfL, the responses of mathematics teacher educators about the principles of AfL to guide the teaching and learning practices were categorized into five hierarchical levels. The five levels, which range from complete understanding of the principles of AfL to no understanding of the principles of AfL, are described as follows:

Level 1: *Complete Understanding (CU)*: Responses that included all or almost all of the main components of the validated correct responses about principles of AfL, such as:

it helps with effective planning; it considers the students' learning process; it is essential for classroom practice; it is an important skill for professionals; it influences emotions; it affects how motivated learners are; it fosters engagement with learning goals and assessment criteria; it guides learners on how to improve; it supports self-assessment and acknowledges all achievements.

Level 2: *Incomplete understanding (IU)*: responses that included at least two of the components of the validated correct responses but not all of the components.

Level 3: *Incomplete Understanding with Some Misconception (IUSM)*: Responses that showed some understanding of the concept but also made a statement or statements that demonstrated some misconceptions regarding principles of assessment for learning, such as: It focuses on what students learn from what they are taught; it is not a central component of the classroom instructional process; it has no emotional impact; it is not a key professional skill; and the like.

Level 4: *Some Misconception (SM)*: Response that comprised misconceptions like those stated in Level 3 or other incorrect views regarding the principles of AfL.

Level 5: *No Understanding (NU)*: When respondents repeated the question, contained unrelated or irrelevant information, gave a vague response, or left without responding to the interview question about principles of assessment for learning.

Based on the analysis of the interview data, only one (MTE 11) of the respondents has a complete and sound understanding of the principles of AfL to guide classroom practices. This respondent correctly identified some of the components of the principles of AfL: It is part of effective planning; it focuses on how students learn; it is central to classroom

practice; and it is a key professional skill, as can be inferred from his responses, which are described as:

As long as there is teaching and learning, there is an assessment for learning. If there is teaching-learning, there will be AfL. We must be well prepared to implement AfL. Assessment for learning should be related to the topic of the lesson. We should not just do it to get students results. It should be able to make a difference for the lesson, and it is good to give feedback as soon as possible (MTE11, Thursday, 11 Feb, 2021). .

Likewise, some of the respondents (4 out of 13) have an incomplete understanding of the principles of AfL to guide classroom practices. One of the respondents (MTE 08) stated that it is central to classroom practice and part of effective planning as follows:

In my opinion, AfL is very essential for the classroom instructional process for both teachers and students because, after teaching a topic, we have to assess how the students understand the daily lesson to plan for the next lesson. Assessment for learning and classroom instruction are inseparable, and are implemented side by side MTE08, Wednesday, 10 Feb, 2021).

Another respondent (MTE 05) also described principles of AfL to guide classroom practices as follows:

Assessment for learning is part of teaching and learning and is implemented from the beginning to the end of the lesson. In other words, my daily learning and teaching plan should describe the learning and teaching process (MTE05, Tuesday, 9 Feb, 2021).

In addition, analysis of the interview data revealed that around half (5 out of 13) of the respondents have an incomplete understanding and some misconceptions about the principles of assessment for learning that guide classroom practices. One of the respondents (MTE 06) described it as:

Assessment for learning and classroom instruction are not different; they are one and the same. It gives teachers an in-depth look at what they teach. It helps the teacher to pre-plan a lesson based on the gaps students have and use his knowledge, skills, and methods of teaching to fill those gaps (MTE06, Tuesday, 9 Feb, 2021).

Similarly, another respondent (MTE 12) described the principles of AfL as follows:

Assessment for learning and classroom instruction are interrelated. Assessment for learning is part of classroom learning. Assessment for learning must be implemented to obtain reliable information; it should also be a standardized AfL. Assessment for learning should match the planned training time. It also boosts students' self-confidence (MTE12, Thursday, 11 Feb, 2021).

By the same token, the remaining three respondents clearly stated that they do remember the principles of assessment for learning. However, one of the three respondents (MTE 13) said that "There are no principles of assessment for learning that I remember." (MTE13, Thursday, 11 Feb, 2021)

In general, there is a considerable gap in the conceptions of MTEs about principles of AfL to guide classroom practicing.

4.1.1.3 Mathematics Teacher Educators' Conceptions of the Strategies of Assessment for Learning

According to the strategies of implementation of AfL, the responses of MTEs about the strategies of implementing AfL were categorized into five hierarchical levels. The five levels, which range from complete understanding of strategies for implementing AfL to no understanding of the strategies for implementing assessment for learning, are described as follows:

Level 1: *Complete Understanding (CU)*: Responses that included all or almost all of the main components of the validated correct responses of strategies in implementing AfL, such as: making clear and sharing the learning objectives and criteria for success; designing effective classroom interactions, inquiries, and learning activities; giving feedback that helps learners progress; empowering students to take charge of their own learning; enabling students to support each other's learning.

Level 2: *Incomplete Understanding (IU)*: responses that included at least one of the components of the validated correct responses of strategies in implementing assessment for learning but not all of the components.

Level 3: *Incomplete Understanding with Some Misconceptions (IUSM)*: Responses that showed some understanding of strategies in implementing assessment for learning, but also made a statement or statements that demonstrated some misconceptions about strategies in implementing assessment for learning.

Level 4: *Some Misconception (SM)*: Response that comprised misconceptions or incorrect information about strategies in implementing assessment for learning.

Level 5: *No Understanding (NU)*: When respondents repeated the question, contained unrelated or irrelevant information about strategies for implementing assessment for learning, gave a vague response, or left without responding.

Based on the analysis of the interview data, almost all (12 out of 13) of the respondents have an incomplete understanding of strategies for implementing AfL. The respondents correctly identified only one or two components of the strategies for implementing assessment for learning. As one of the respondents (MTE 05) responds, feedback and questioning are some of the strategies for implementing assessment for learning. He described it as:

We should also prepare assessment tools. We must teach students according to their abilities and gaps. Questioning and feedback in assessment for learning are very important. You will find student gaps when asking questions. You need feedback to fill those gaps. So feedback is also very important. Questioning and feedback are very important things that should not be missed in mathematics (MTE05, Tuesday, 9 Feb, 2021).

Another respondent (MTE 12) also described group work, assignments, and tests as some of the strategies for implementing assessment for learning.

I'm implementing assessment for learning my own way by implementing group work, assignments, and tests. I may have forgotten strategies for implementing assessment for learning. The teacher must make sure the assessment is done on time. The assessment must be accessible to all students. Asking questions and giving feedback are very necessary and important in the classroom instruction of mathematics. If we do not do so, we will not be

able to say that we have implemented assessment for learning (MTE12, Thursday, 11 Feb, 2021).

Moreover, another participant (MTE 08) stated that asking questions and providing feedback are some of the strategies for implementing AfL:

Some of the strategies for implementing AfL are asking questions and interviewing students. Obviously, asking questions and giving feedback in mathematics is very important. If a student responds to a question and corrects it after receiving feedback immediately, he or she will prepare for the next lesson (MTE08, Wednesday, 10 Feb, 2021).

Nevertheless, the remaining participant (MTE 09) has shown no understanding of strategies in implementing assessment for learning, as he stated unrelated ideas when asked about strategies in implementing assessment for learning, which are described below.

If students knew the content of what I taught them, I would say that I had implemented assessment for learning. I understand whether or not I implemented AfL based on the opinions of students (MTE09, Wednesday, 10 Feb, 2021).

4.1.1.4 Mathematics Teacher Educators' Conceptions of the Purposes of Assessment for Learning

Similar to the strategies for implementing AfL, the responses of MTEs about the purpose of AfL were also categorized into five hierarchical levels. The five levels, which range from complete understanding of the nature of AfL to no understanding of the purpose of AfL, are described as follows:

Level 1: *Complete Understanding (CU)*: Responses that included all or almost all of the main components of the validated correct responses, such as: to improve classroom instruction; to diagnose students' difficulties; to promote understanding of learning goals and criteria; to help teachers monitor students' progress towards the expectations; and to help teachers provide timely and constructive feedback to students.

Level 2: *Incomplete understanding (IU)*: responses that included at least one of the components of the validated correct responses about the purposes of assessment for learning in Level 1 but not all of the components.

Level 3: *Incomplete Understanding with Some Misconception (IUSM)*: Responses that showed some understanding of the purpose of AfL, but also made a statement or statements that demonstrated some misconceptions, such as for grading purposes or for placement purposes.

Level 4: *Some Misconception (SM)*: Response that comprised misconceptions like those stated in Level 3 or other incorrect information about the purposes of assessment for learning.

Level 5: *No Understanding (NU)*: When respondents repeated the question; contained unrelated or irrelevant information, gave a vague response, or left without responding.

Based on the analysis of the interview data, some of the respondents (5 out of 13) have a complete and sound understanding of the purpose of AfL. These respondents correctly identified all or almost all components of the purpose of AfL. One of the five respondents (MTE 11) clearly identified that the purpose of AfL is to improve students' learning and to

determine where students are (with respect to their learning intentions), as can be inferred from his response given below.

Assessment for learning is an assessment method we use to understand or determine what students are doing in terms of the academic goals we have set. Assessment should be done to improve student learning. If there is no assessment for learning, we cannot know where students are or whether or not we have achieved the objectives of the lesson. It helps students determine how much they understand the lesson of the day. It also allows teachers to evaluate whether the objectives of the lesson were achieved (MTE11, Thursday, 11 Feb, 2021).

Moreover, another respondent (MTE 07) from these five respondents recognized that the purpose of AfL is to identify students' gaps and improve the teaching and learning process, as can be deduced from his responses given below.

We apply assessment for learning day to day in our classroom to improve the classroom instruction process. The main purpose of AfL is to improve the classroom instruction process. Additional help is needed to identify student gaps and fill them (MTE07, Tuesday, 9 Feb, 2021).

The interview data analysis revealed that more than half (8 out of 13) of the respondents have an incomplete understanding and some misconceptions about the purpose of AfL. One of the respondents (MTE 09) described the purpose of AfL as:

Assessment for learning is a method by which a teacher recognizes and evaluates how well students have understood a topic they have learned. As we assess students on a daily basis, they will have a better understanding of the subject they are

learning. But students may not understand the content of what they have learned in the lesson if the teacher does not assess them. It is important to help students adjust when we find a gap; otherwise, the students may have the wrong idea. Therefore, it is important to assess students' performance and provide appropriate follow-up and support (MTE09, Wednesday, 10 Feb, 2021).

Another respondent (MTE 03) from this group of respondents who has an incomplete understanding of the purpose of AfL and has some misconceptions also describes the purpose of AfL as:

Assessment for learning is one of the best ways to facilitate classroom learning. Assessment for learning is an assessment method used after teaching a certain topic to determine whether or not it reaches the students and whether or not they understand what they are taught. Assessment for learning is necessary to determine whether a student has passed or failed, to decide whether to move on to the next grade level, and for the teacher to self-assess himself or herself (MTE03, Monday, 8 Feb, 2021).

4.1.1.5 Mathematics Teacher Educators' Conceptions of the Challenges in Implementing Assessment for Learning

Mathematics teacher educators were asked to list all possible challenges that hinder the implementation of AfL in mathematics education at the college level. Analysis of the interview data about the perceived challenges of teacher educators in implementing assessment for learning in the classroom instruction of mathematics courses at the college level revealed that more than half of them (8 out of 13) described time constraints as the main challenge to implementing assessment for learning. They argued that if they

implement assessment for learning, they will not be able to cover the course as per the semester schedule. One of the respondents (MTE 07) described the challenges as follows:

The main challenge is time constraints. After the start of the class, time is wasted for a variety of reasons, and as a result, we may not think we will be able to complete our course on time and use the lecture method of teaching without implementing assessment for learning. The gap in students' basic knowledge is another challenge. One is the difficulty of covering the course. Since there is much wastage of time if we apply assessment for learning, we will not be able to cover the course on time (MTE07, Tuesday, 9 Feb, 2021).

Another respondent (MTE 11) from this group of teacher educators also described the challenges of implementing AfL in the classroom instruction of mathematics courses at the college level as follows:

Differences in student achievement make it difficult to implement assessment for learning. It is difficult to determine what each student is doing due to the limited time available for the course. Due to the large number of students in a class, it is difficult to know where each student is and how well each student has understood the topic. The large number of topics covered in a course makes it impossible for us to complete the course on time (MTE09, Thursday, 11 Feb, 2021).

The analysis also showed that around half (6 out of 13) of the respondents described large class sizes as a challenge that negatively influences the implementation of AfL. One of the respondents (MTE 12) described the challenges of implementing AfL as follows:

One challenge is the large number of students in a classroom. The second is a lack of time; students do not complete their assignments on time. Declining students'

self-confidence and trying to cheat with their peers (during exams) is another challenge I have faced when implementing AfL. There is a lack of uniform implementation among teachers in the implementation of AfL (MTE12, Thursday, 11 Feb, 2021).

Moreover, the analysis of the interview data also showed that some of the respondents (5 out of 13) described a lack of PD and a common understanding of AfL as challenges that negatively influence the implementation of AfL. One of the respondents (MTE 08) described the challenges of implementing AfL as follows:

The first challenge is that we teachers do not all have the same conception of AfL. Second, we have an attitudinal problem with implementing assessment for learning. There is a problem of understanding and belief about the benefits of implementing assessment for learning. Third, there are courses that teachers always choose to teach, and teachers do not get assigned to teach the course they prefer, which makes it difficult to implement AfL (MTE08, Wednesday, 10 Feb, 2021).

Besides, the analysis also showed that some of the respondents (3 out of 13) described course overload as a challenge that negatively influences the implementation of AfL. One of the respondents (MTE 11) described the challenges of implementing AfL as follows:

The large number of topics covered in a course makes it impossible for us to complete the course on time. Differences in student achievement make it difficult to implement assessment for learning. It is difficult to determine what each student is doing due to the limited time available for the course. Due to the large number of

students in a class, it is difficult to know where each student is and how well each student has understood the topic (MTE11, Thursday, 11 Feb, 2021).

Moreover, lack of commitment and willingness, a poorly designed course module, declining self-confidence of students, and unhelpful attitudes of teachers and students towards AfL were mentioned as challenges by MTEs in the implementation of AfL in the classroom instruction of mathematics courses at the college level.

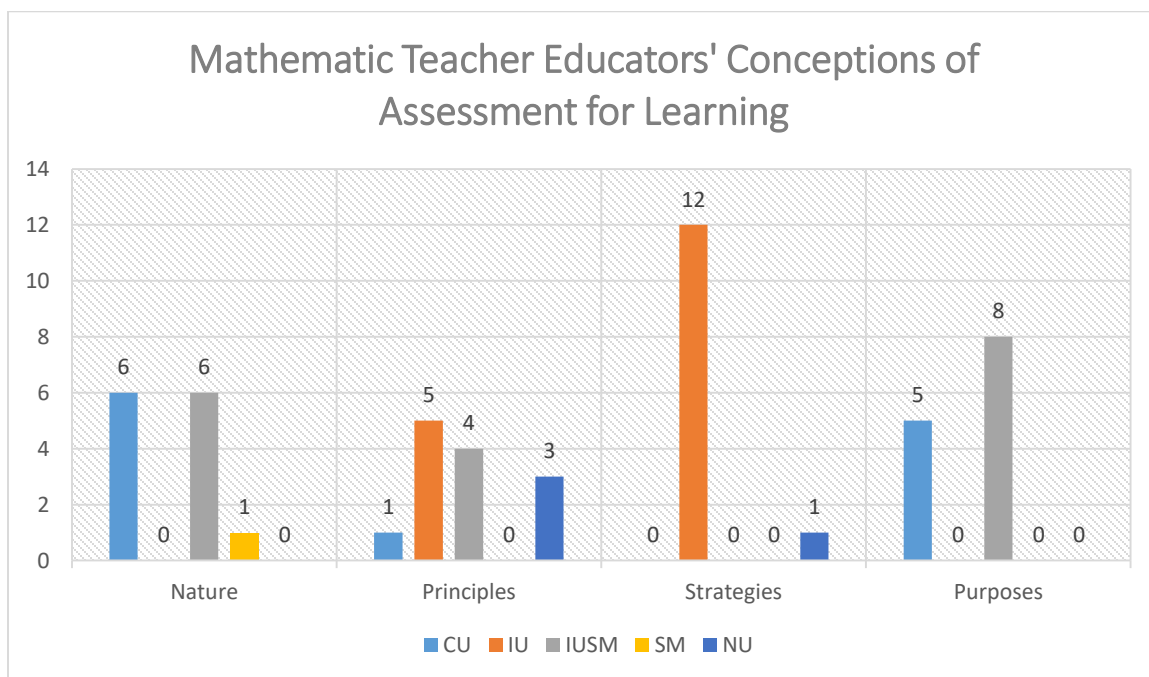
Table 3. Summary of analysis of Mathematics Teacher Educators' Conceptions of Assessment for Learning

	Second degree type	Training Obtained About AfL	Nature of AfL	Principles of AfL	Strategies of AfL	Purposes of AfL
MTE 1	M.Sc	No,	IUSM	NU	IU	IUSM
MTE 2	M.Sc	No,	IUSM	IU	IU	CU
MTE 3	M.Sc	No,	IUSM	NU	IU	IUSM
MTE 4	M.Ed	No,	SM	IUSM	IU	CU
MTE 5	M.Ed	Yes,	CU	IU	IU	IUSM
MTE 6	M.Sc	No,	CU	IUSM	IU	IUSM
MTE 7	M.Sc	Yes,	CU	IU	IU	CU
MTE 8	M.Sc	No,	IUSM	IU	IU	IUSM
MTE 9	M.Ed	Yes,	IUSM	IUSM	NU	IUSM
MTE 10	M.Ed	No,	IUSM	IU	IU	IUSM
MTE 11	M.Sc	No,	CU	CU	IU	CU
MTE 12	M.Sc	Yes,	CU	IUSM	IU	IUSM
MTE 13	M.Sc	Yes,	CU	NU	IU	CU

Note. CU: Complete Understanding (Level 1); IU: Incomplete understanding (Level 2); IUSM: Incomplete understanding with Some Misconception (Level 3); SM: Some Misconception (Level 4); and NU: No Understanding (Level 5)

In general, the conceptions of MTEs about the nature of AfL, the principles that guide classroom implementation of AfL, the strategies for implementing AfL, and the purposes of AfL are below what is expected, and they have varying levels of conceptions of AfL.

Less than half of MTEs have a complete understanding of the nature of AfL and the purposes of AfL (6 and 5 out of 13 respectively), and almost none of the MTEs have a complete understanding of the principles that guide classroom implementations of AfL and the strategies of implementation of AfL (1 and 0 out of 13 respectively). Moreover, nearly half of the MTEs (5 out of 13) have incomplete understandings about the principles that guide classroom implementations of AfL, and nearly all (12 out of 13) have incomplete understandings of strategies for implementing AfL. Furthermore, nearly half (6 out of 13) of MTEs have an incomplete understanding with some misconceptions on the nature of AfL; less than half (4 out of 13) have an incomplete understanding with some misconceptions on the principles that guide classroom implementation of AfL; and more than half (8 out of 13) have an incomplete understanding with some misconceptions on the purposes of AfL.



Note. CU: Complete Understanding (Level 1); IU: Incomplete understanding (Level 2); IUSM: Incomplete understanding with Some Misconception (Level 3); SM: Some Misconception (Level 4); and NU: No Understanding (Level 5)

Figure 3 Mathematics teacher educators' levels of conceptions of Assessment for Learning

4.1.2 Mathematics Teacher Educators' Experiences of the Professional Development on Assessment for Learning

An analysis of the interviews with the three MTEs who were involved in the comprehensive PD on AfL in the classroom instruction of Fundamental Concepts of Algebra course about how they experienced the comprehensive PD revealed that all of them found it very important in the classroom instruction of Fundamental Concepts of Algebra course. One of them (MTE 02) described it as:

The training was important to me and helped me to be aware of how to state learning objectives (to students), how to construct (develop) learning activities, how to assess my students, how to select effective teaching methods, how to provide constructive feedback, how to construct question items that align with stated learning objectives, and how to manage a large class size (MTE02, Monday, 7 June, 2021).

Moreover, all of them stated that the comprehensive PD helped them improve their conceptions of AfL. One of them (MTE 01) described it as:

Yes, the training on assessment for learning improved my conception of implementing AfL. I understand that assessment for learning focuses on effective planning, practicing the plan in the classroom, motivating students learning by

carefully questioning them, and supporting students in their own responsibility for their learning (MTE01, Monday, 7 June, 2021).

Besides, all of them acknowledged that the comprehensive PD helped them improve the implementation of AfL strategies in the classroom instruction of the algebra course. One of them (MTE 03) described it as:

Yes, as we know, the course (Fundamental Concepts of Algebra (Math 221)) is very difficult for our students, but after the professional development and training, there were many reforms starting from the teaching material (course module of Math 221) (MTE03, Monday, 7 June, 2021).

Finally, they were asked what they thought should be done to improve the implementation of AfL in mathematics courses in colleges of teacher education, and they stated that training about AfL should be given to MTEs and that all stakeholders should give due attention to the implementation of AfL in CTEs. One of them (MTE 02) described it as:

Training on assessment for learning should be given in all teacher education colleges. Teacher educators should implement assessment for learning when training trainees in each teacher education college. The Ministry of Education, Regional Education Bureau, and College Deans should give attention to assessment for learning (MTE02, Monday, 7 June, 2021).

In general, the result of the analysis of the interview with the three MTEs who were involved in the comprehensive context-specific, job-embedded, content-based PD revealed that they found the comprehensive PD to be very important for their profession as mathematics teacher educators. Moreover, they believed that the PD was helpful in their

implementation of AfL strategies in the classroom instruction of the algebra course. They believe that the comprehensive PD helped them improve their conception of AfL. Furthermore, the MTEs stated that the comprehensive PD provided them with the essential guidance and support to improve AfL strategies in the classroom instruction of Fundamental Concepts of Algebra.

4.1.3 Effect of the Professional Development on the Implementation of Assessment for Learning

The classroom observation was conducted by a well-experienced mathematics teacher educator who was trained on how to rate classroom implementations of AfL strategies using the rubric specifically developed to rate daily classroom observation by the developer of the classroom observation protocol for AfL and by the researcher. The first classroom observation was conducted before the comprehensive PD on AfL to MTEs and a training to PPMTs on SA and PA were provided.

The next five consecutive classroom observations were made after the delivery of the comprehensive PD to MTEs and training to PPMTs and during the implementation of the intervention. The average ratings of implementation of the components of the five AfL strategies were analyzed, and the analysis of classroom observation ratings of implementation of AfL strategies using the classroom observation protocol revealed that the implementations of the five strategies of AfL by the three MTEs' and their PPMTs' in the classroom of Fundamental Concepts of Algebra (Math 221) improved gradually, as shown in the table in Appendix XII.

The average scores of observation ratings for the five AfL strategies in the first section improved from 1.5 in the first observation to 3.7 in the sixth observation, where 1 represents "no evidence of use" and 5 represents "pervasive use or highly effective." Moreover, the average scores of the observation ratings of the five AfL strategies in the second section improved from 1.6 in the first observation to 4.0 in the sixth observation. Furthermore, the average scores of the observation ratings of the five AfL strategies in the third section improved from 1.5 in the first observation to 4.2 in the sixth observation. Although the initial level of implementation of AfL was approximately the same across the three sections (1.5, 1.6, and 1.5 out of 5 in sections 1, 2, and 3, respectively), there was a slight variation in the scores of the final level of implementation of AfL across the three sections (3.7, 4.0, and 4.2 out of 5 in sections 1, 2, and 3, respectively).

The lowest average rating was observed during the first session of classroom observation before the delivery of the comprehensive PD to MTEs and the training on SA and PA to PPMTs. Moreover, the highest average rating was observed in the last (sixth) observation, where each of the sections implemented all the five AfL strategies better than all previous ratings of observations.

Regarding the implementation of the five AfL strategies, a gradual improvement in the implementation of all five AfL strategies is observed. The implementation of three of the five strategies (*Learning Targets*: Clarifying Learning Intentions and Sharing Criteria for Success; *Self-Assessment*: Activating Students as the Owners of Their Own Learning; and *Peer Assessment*: Activating Students as Instructional Resources for One Another) was very poor at the beginning compared to the remaining two (*Monitoring*: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of

Learning; and *Feedback: Providing Feedback That Moves Learners Forward*) strategies of AfL. Moreover, there is a slight variation in the level of implementation of the five AfL strategies (*monitoring was implemented at 4.13 out of 5; learning targets were implemented at 4.08 out of 5; peer assessment was implemented at 4.00; feedback was implemented at 3.97 out of 5; and self-assessment was implemented at 3.87 out of 5*). Furthermore, the improvement level of implementation of the five AfL strategies also varies to some extent among the strategies, where the highest difference was observed on *peer assessment with a 2.87* difference between the sixth and the first observation and the lowest difference was observed on *feedback with a 1.67* difference between the sixth and the first average observation scores, as can be observed from the following tables and figures below.

Table 4. Average observation rating score of the five assessment for learning strategies in the three sections

		Average observation ratings of implementation of the five AfL strategies of the three MTEs					
		1 st	2 nd	3 rd	4 th	5 th	6 th
	<i>A. Learning Targets: Clarifying Learning Intentions and Sharing Criteria for Success</i>	1.28	2.50	3.5	3.63	3.70	4.08
	<i>B. Monitoring: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of Learning</i>	2.10	2.90	3.63	3.77	3.93	4.13
	<i>C. Feedback: Providing Feedback That Moves Learners Forward</i>	2.3	3.1	3.5	3.83	3.8	3.97
	<i>D. Self-Assessment: Activating Students as the Owners of Their Own Learning</i>	1.07	2.13	2.9	3.40	3.77	3.87
	<i>E. Peer Assessment: Activating Students as Instructional Resources for One Another</i>	1.13	2.17	3.03	3.50	3.77	4.00
	Average	1.58	2.56	3.31	3.63	3.80	4.01

Note. 1= No evidence of use 2= Superficial or ineffective use 3= Minimal use or uncertain effectiveness 4= Frequent Use or Effective and 5= Pervasive Use or Highly Effective

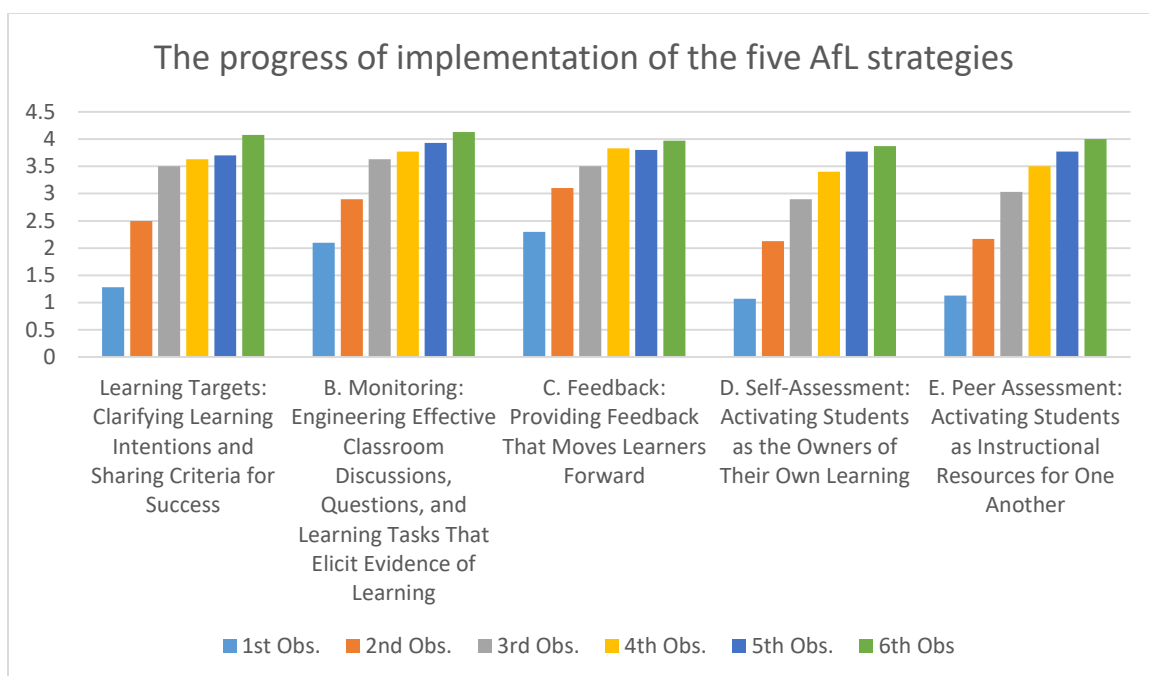


Figure 4. Average observation rating score of the five Assessment for Learning strategies in the three sections

Regarding similarities and differences in the level of improvement of the implementations of AfL strategies in the three sections, although the initial average implementation level of the five AfL strategies in each of the three sections was similar (with a range of 0.08), there was a considerable difference (with a range of 0.49) in the final average level of implementation of the five AfL strategies among the three sections, as can be seen from the table and figure below.

Table 5. Average scores of the three sections' implementation of assessment for learning strategies in 6 observations

	Average scores of implementation of AfL strategies in 6 observations					
	1 st	2 nd	3 rd	4 th	5 th	6 th
Section 01	1.53	2.31	3.01	3.20	3.63	3.69
Section 02	1.61	2.34	3.13	3.49	3.80	4.03
Section 03	1.55	2.82	3.61	3.99	3.82	4.18
Grand Average	1.59	2.33	3.09	3.40	3.74	3.91

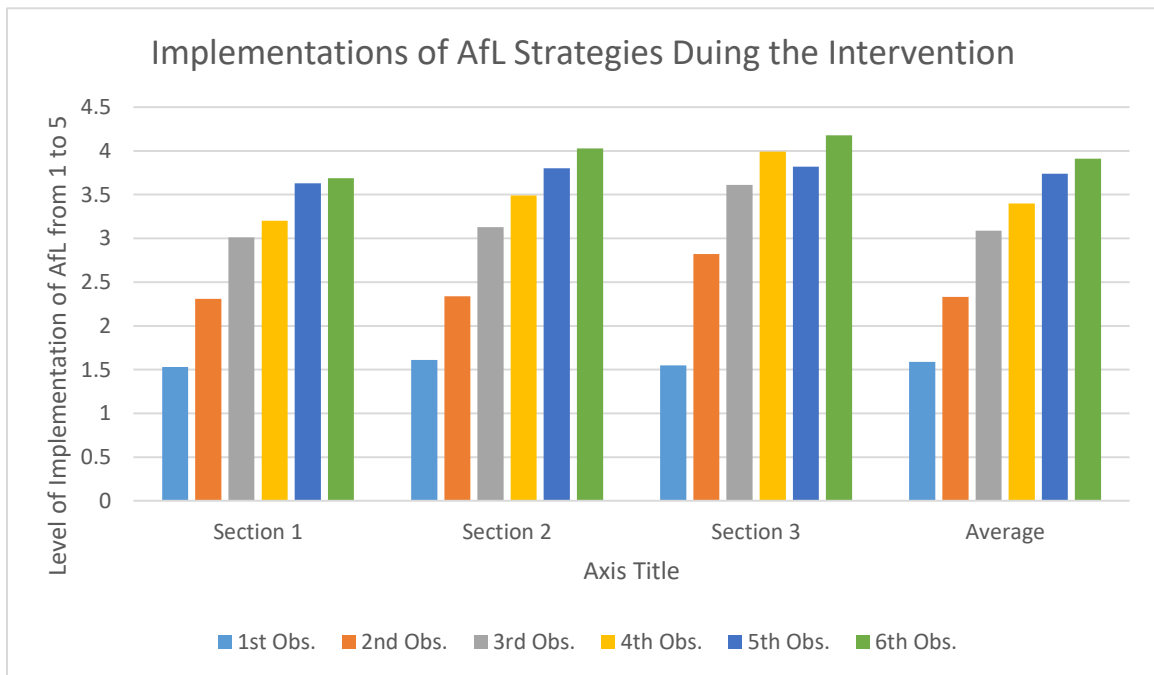


Figure 5. Average scores of the three sections' implementation of assessment for learning strategies in 6 observations

Note. 1= No evidence of use 2= Superficial or ineffective use 3= Minimal use or uncertain effectiveness 4= Frequent Use or Effective and 5= Pervasive Use or Highly Effective

In general, comprehensive PD for MTEs on AfL and training for PPMTs on SA and PA improved classroom implementation of each of the five AfL strategies in the classroom instruction of Fundamental Concepts of Algebra. Moreover, comprehensive PD for MTEs on AfL and training for PPMTs on SA and PA improved classroom implementation of each of the five AfL strategies in each of the three sections. However, although the comprehensive PD for MTEs on AfL and a training for PPMTs on SA and PA improved classroom implementation of AfL (from a grand average score of 1.59 to a grand average score of 3.91 out of 5), the intervention did not bring about pervasive use or highly effective implementation of AfL on each of the three sections in the intervention group and on each of the five strategies of implementation of AfL, and hence there is still room for more improvement in the implementation of AfL strategies.

4.1.4 Effect of Professional Development in Assessment for learning on Pre-service Primary Mathematics Teachers' Achievement gain in Algebra

A one way between groups analysis of variance (ANOVA) was conducted to examine the effect of improved implementation of AfL strategies due to comprehensive context-specific, job-embedded, and content-based PD program on AfL on PPMTs' achievement gains. The normalized achievement gains of each of the PPMTs in the treatment group and in the two comparison groups were calculated based on their pre-test and post-test scores using the student level formula of the normalized change score, formulated by Marx and Cummings (2007), which is symbolized by c and defined as follows:

$$C = \begin{cases} \frac{\text{Post}-\text{Pre}}{100-\text{Pre}} & \text{if post} > \text{Pre} \\ \text{drop} & \text{if Post} = \text{Pre} = 100 \text{ or } 0 \\ 0 & \text{if } 0 < \text{post} = \text{pre} < 100 \\ \frac{\text{Post}-\text{Pre}}{\text{Pre}} & \text{if Post} < \text{Pre} \end{cases}$$

The independent variable was a comprehensive context-specific, job-embedded, and content-based PD training for PPMTs on SA and PA. The dependent variable was the students' individual normalized achievement gains. The assumptions of a one-way ANOVA were not markedly violated. The data gathered from the three groups is independent of one another since each of the three groups is located at least 200 km away from one another. Moreover, the data is approximately normally distributed since the skewness is found to be 0.033, which is between -1 and 1, and the kurtosis is also found to be 0.133, which is also between -1 and 1, hence the data is approximately normally distributed. Furthermore, Levene's test of homogeneity of variance also shows that the variance of the data among the three groups is approximately homogeneous ($p = 0.358 > 0.05$).

The results of a one-way ANOVA between groups show that there is a statistically significant mean achievement gain difference among the three groups ($p = 0.000 < .05$, $F(2, 126) = 63.089$, $\eta^2 = 0.500$) on their normalized achievement gain scores in the Fundamental Concepts of Algebra course, as shown in Table 8, with a very large effect size. The mean achievement gain score ($M = .554$; $SD = .197$) of the treatment group is significantly higher than the mean achievement gain score ($M = .163$; $SD = .199$) of comparison group one and the mean achievement gain score ($M = .213$; $SD = .140$) of comparison group two. Moreover, the test result shows that there is no statistically significant mean achievement gain difference ($p = .461 > .05$) on their mean

achievement gain between comparison group one ($M = .163$; $SD = .199$) and comparison group two ($M = .213$; $SD = .140$) as shown in the Post Hoc tests (Table 9).

Table 6. Descriptive statistics of achievement gain scores of the three groups

Groups	Achievement gain		
	N	M	SD
Treatment group	52	.554	.197
Comparison group one	38	.163	.199
Comparison Group two	39	.213	.140
Total	129	.335	.256

Note. M: mean; SD: Standard Deviation

Table 7. One-Way ANOVA Summary Table of achievement gain scores of the groups

Source	<i>df</i>	SS	MS	F	<i>P</i>	η^2
Achievement gain Scores						
Between Groups	2	4.193	2.096	63.089	.000	0.500
Within Groups	126	4.187	.033			
Total	128	8.379				

Note. SS: Sum of Squares; MS: Mean Square

Table 8. Multiple Comparisons of Post Hoc tests among the three groups

Dependent Variable	(I)Group	(J)Group	Mean Difference (I-J)	Std. Error	Sig.
Normalized Learning Gain	Treatment Group	Comparison group one	.390501	.038903	.000
		Comparison group two	.341067	.038614	.000
	Comparison group one	Comparison group two	-.049434	.041550	.461

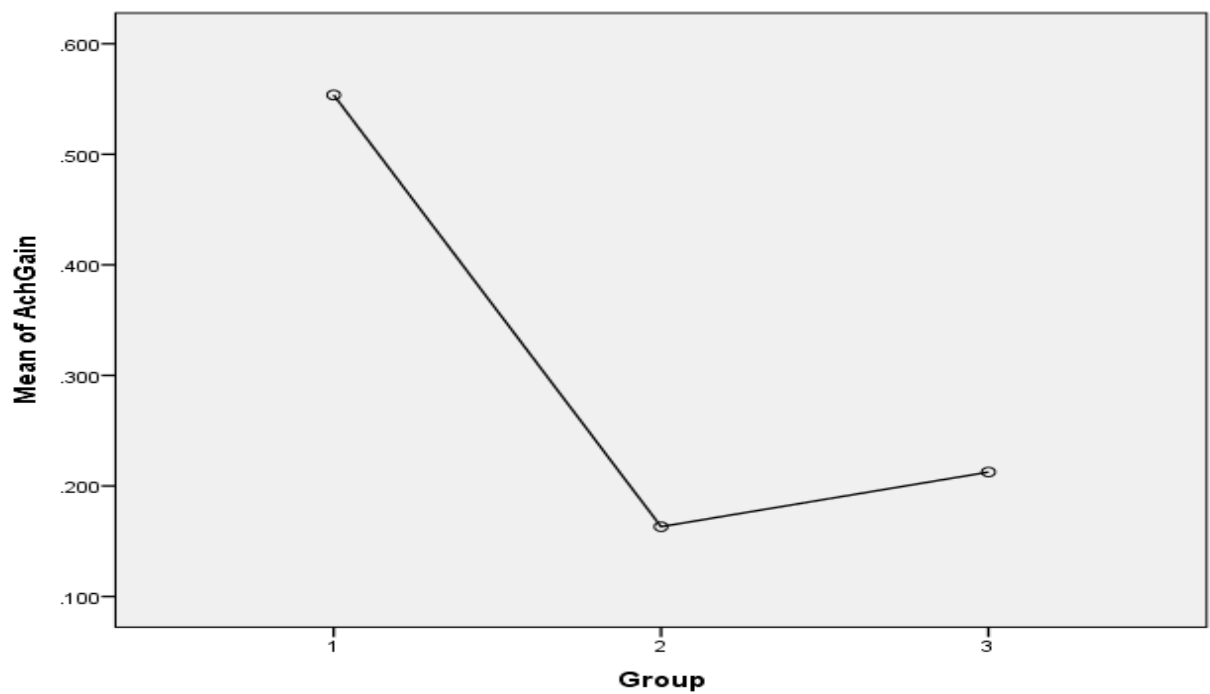


Figure 6. Mean achievement gains of Pre-service Primary Mathematics Teachers on Fundamental Concepts of Algebra course

In general, the intervention, which is comprised of comprehensive context-specific, job-embedded, and content-based PD for MTE and training for PPMTs on SA and PA, brought a statistically significant difference in the mean achievement gain of PPMTs in the intervention group compared to PPMTs in both comparison groups one and two in Fundamental Concepts of Algebra. Moreover, the eta squared ($\eta^2 = 0.50$) value shows that the effect size of the intervention is very large. However, the Post Hoc analysis showed that there was no statistically significant difference in the mean achievement gain of PPMTs in comparison group one and PPMTs in comparison group two in Fundamental Concepts of Algebra.

4.1.5 Effect of Professional Development on Pre-service Primary Mathematics Teachers' Gain scores of Conceptions of Assessment for Learning Experience

To examine the effect of comprehensive PD for MTEs on the implementation of AfL and a training on SA and PA for PPMTs, a questionnaire was administered at the beginning and end of the intervention (during which an intervention was conducted with the treatment group) to PPMTs in the treatment group and to PPMTs in the two comparison groups. The gains scores of conception of AfL experience of each of the PPMTs in the treatment group and in the two comparison groups were calculated based on their pre-test and post-test scores using the student-level formula of the normalized change score, formulated by Marx and Cummings (2007), which is symbolized by c and defined as follows:

$$C = \begin{cases} \frac{\text{Post}-\text{Pre}}{100-\text{Pre}} & \text{if post} > \text{Pre} \\ \text{drop} & \text{if Post} = \text{Pre} = 100 \text{ or } 0 \\ 0 & \text{if } 0 < \text{post} = \text{pre} < 100 \\ \frac{\text{Post}-\text{Pre}}{\text{Pre}} & \text{if Post} < \text{Pre} \end{cases}$$

The assumptions of a one-way ANOVA were checked, and they were not markedly violated. The data gathered from the three groups is independent of one another since each of the three groups is located at least 200 km away from one another. Moreover, the data is approximately normally distributed since the skewness is found to be 0.321, which is between -1 and 1, and the kurtosis is also found to be 0.830, which is also between -1 and 1, hence the data is approximately normally distributed. Furthermore, Levene's test of homogeneity of variance also shows that the variance of the data among the three groups is approximately homogeneous ($p = 0.467 > 0.05$). A one-way between groups ANOVA was conducted to examine the effect of improved implementation of AfL strategies due to a comprehensive, job-embedded, context-specific, and content-based professional development program on AfL in the teaching and learning of Fundamental Concepts of Algebra on PPMTs' conceptions of AfL experiences. The results of a one-way ANOVA between groups showed that there is a statistically significant mean gain difference among the three groups ($p = 0.000 < .05$, $F(2, 126) = 10.249$, $\eta^2 = 0.14$) on their conceptions of AfL experiences, as shown in Table 11, with a large effect size. The mean gain score of conception of AfL experience ($M = .371$; $SD = .297$) of the treatment group was significantly larger than the mean gain score of conception of AfL experience ($M = .063$; $SD = .315$) of comparison group one. But there was no statistically significant mean gain difference on the scores of conception of AfL experience between the treatment group ($M = .371$; $SD = .297$) and comparison group two. Moreover, the Post Hoc tests result

showed that there is a significant mean gain difference on the scores of conception of AfL experience between comparison group one ($M = .163$; $SD = .199$) and comparison group two ($M = .213$; $SD = .140$) as shown in Table 12.

Table 9. Descriptive statistics of achievement gain scores of the three groups

Groups	Gain scores of conceptions of AfL experiences		
	N	M	SD
Treatment group	52	.371	.297
comparison group one	38	.063	.315
comparison Group two	39	.236	.349
Total	129	.239	.341

Note. M: mean; SD: Standard Deviation

Table 10. One-Way ANOVA Summary Table of gain scores of conceptions of assessment for learning practices of the groups

Source	<i>df</i>	SS	MS	F	<i>P</i>	η^2
Conception Gain Scores						
Between Groups	2	2.084	1.042	10.249	.000	0.14
Within Groups	126	12.808	.102			
Total	128	14.891				

Note. SS: Sum of Squares; MS: Mean Square

Table 11. Multiple Comparisons of Post Hoc tests among the three groups

Dependent Variable	(I)Group	(J)Group	Mean Difference (I-J)	Std. Error	Sig.
Conception Gain Scores	Treatment Group	Comparison group one	.308003	.068043	.000*
		Comparison Group two	.135347	.067537	.115
		Comparison group one	-.172656	.072673	.050*
		Comparison Group two			

* The mean difference is significant at 0.05 level

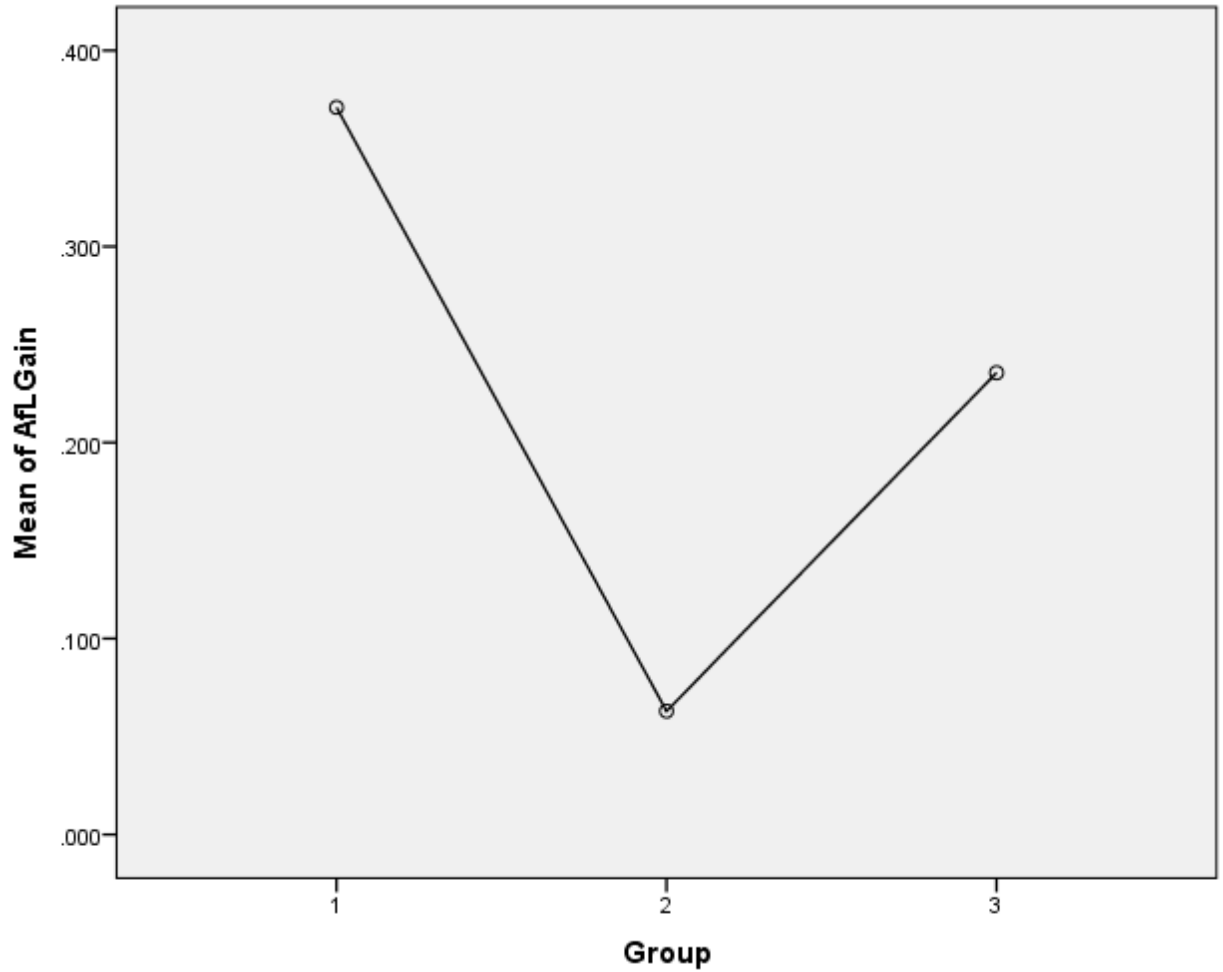


Figure 7. Gain scores of conception assessment for learning experiences plot of the three groups

In general, the analysis of the PPMTs' responses to the students' conceptions of AfL experiences questionnaire showed that there was a statistically significant mean gain score difference between the PPMTs in the intervention group and the PPMTs in comparison group one in their conceptions of AfL experiences in the classroom instruction of Fundamental Concepts of Algebra. Besides, the result of the analysis showed that there was no statistically significant difference in their mean gain score between PPMTs in the intervention group and PPMTs in comparison group 2. Moreover, the analysis showed that

there was a statistically significant mean gain score on the conceptions of AfL experiences between PPMTs in comparison group one and PPMTs in comparison group two.

4.1.6 Effect of the intervention on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills

To examine the effect of the intervention, which consists of a comprehensive context-specific, job-embedded, and content-based PD on AfL for MTEs and a training for PPMTs on SA and PA on the PPMTs' SRLS, a translated version of Pintrich et al. (1991)'s Motivated Strategies for Learning Questionnaire was used. The assumptions of the paired sample t-test were checked, and they were not markedly violated. The two observations are independent of each other. Moreover, the skewness of the data for both the pre- and post-intervention measures of SRLS is between -1 and 1, and the kurtosis is also between -1 and 1, subsequently the data is approximately normally distributed. Furthermore, there were no outliers in the pre- and post-measures of SRLS scores. A paired-samples t-test was used to examine if there was a significant mean difference in SRLS in the pre- and post-scores for PPMTs in the treatment group. The mean scores of measures of SRLS after the intervention ($M = 5.345$, $SD = 0.458$) of the pre-service primary mathematics teacher are significantly higher than their mean pre-intervention scores of measures of self-regulated learning ($M = 4.564$, $SD = 0.807$); $t(51) = 6.338$, $p = .000$, $d = 0.89$ (see tables 13 and 14).

Table 12. Descriptive Statistics of the Pre and Post measures of Self-Regulated Learning Skill Scores

		Mean	N	SD	SEM
Pair	Pre SRLS	4.564	52	.807	.112
	Post SRLS	5.345	52	.458	.063

Note. SD: Standard Deviation; SEM: Standard error mean

Table 13. Paired Sample t-test results

	Paired differences			T	df	P
	MD	SD	SEM			
Pre SRLS – Post SRLS	.782	.890	.123	6.338	51	0.000

Note.MD: Mean difference; SD: Standard Deviation; SEM: Standard error mean

In general, the analysis of the responses of PPMTs to the translated version of Pintrich et al. (1991)'s Motivated Strategies for Learning Questionnaire before and after the intervention showed that there was a statistically significant difference between the scores of SRLS of PPMTs in the intervention group before and after the intervention, with a large effect size.

4.2 Discussion

The main aims of this study were first to investigate MTEs' conceptions of AfL and then to examine the effect of comprehensive context-specific, job-embedded, and content-based PD for MTEs on AfL and training for PPMTs on SA and PA on the implementation of AfL strategies in the classroom instructional process, on PPMTs' achievement gains, on PPMTs' conceptions of AfL experience, and on PPMTs' SRLS by designing an appropriate PD.

4.2.1 Mathematics Teacher Educators' Conceptions of Assessment for Learning

Assessment for learning is a crucial component of the classroom instruction process that actively involves students, their teachers, and their peers and improves the students' overall learning. Besides, teachers' conception of AfL is a growing field of study globally (Barnes et al., 2017; Fulmer et al., 2015), due to its influence on the implementation of AfL in the classroom instruction process in the classroom (Amoako et al., 2019) and hence students' learning.

In addition, teacher educators' practices of AfL are heavily influenced by their conceptions and beliefs about AfL (Barnes et al., 2015, 2017; Fulmer et al., 2015), and investigation of teacher educators' conceptions about AfL is crucial to developing effective PD that can help them change their conceptions and beliefs so that they will be able to implement effective AfL strategies and become a role model for their pre-service teachers. The first question of this research aimed at examining MTEs' conceptions of the nature of AfL in general and the principles that guide classroom implementation of AfL, the strategies of AfL, and the purposes of AfL.

The results showed that MTEs have various levels of perceptions of the AfL and have witnessed misconceptions about the nature of AfL, the principles that guide classroom implementation of AfL, the strategies for implementing AfL, and the purposes of AfL that fall short of expectations. Less than half of MTEs fully comprehend the nature and goals of AfL, and nearly none of the MTEs fully comprehend the principles that guide AfL implementation in the classroom or the methods used to execute AfL in the classroom. Additionally, nearly half of the MTEs are unaware of the principles that govern AfL application in the classroom, and nearly all are unaware of the implementation strategies. Additionally, less than half of MTEs have an incomplete understanding with some misconceptions about the guiding principles for implementing AfL in the classroom; more than half have an incomplete understanding with some misconceptions about the objectives of AfL; and nearly half have an incomplete understanding with some misconceptions about the nature of AfL.

The result of the analysis of interviews with MTEs about their conceptions of AfL is in agreement with expectations, as the implementation of AfL in teacher education colleges in Ethiopia is generally poor and the classroom instruction process is mainly teacher-centered and employs lecture methods. Moreover, the findings of the research that revealed the MTEs' varying levels of conceptions of AfL is in agreement with the findings of DeLuca, Coombs, et al. (2018), who revealed that teacher educators have complex and vastly different profiles of assessment, although they have clear common priorities in assessment, in their investigation of the influence of teacher educators in preparing teachers for assessment in schools.

Furthermore, the result of the interview analysis is in support of the claims of Moss and Brookhart (2019), who stated that the majority of teachers lack the necessary AfL knowledge and expertise to effectively implement AfL strategies for both themselves and their pupils to benefit from them. Besides, the outcomes of the interview analysis are also in agreement with the findings of Teklebrhan and Samuel (2015), who described the implementation of AfL as very poor, with both lecturers and students having inadequate understanding of and unfavorable attitudes about it in their research of obstacles and opportunities in implementing AfL in three universities that are located in north Ethiopia.

The findings of this study are consistent with those of Sintayehu and Ashenafi (2017), who explored the difficulties of applying AfL in a university in the south-east of the country. They found that instructors' lack of positive attitudes toward AfL, their inability to implement AfL properly, and their use of AfL results for grading purposes were major instructor-related challenges. Moreover, the result is in support of the findings of other researchers (Deluca & Volante, 2016; Moss et al., 2009) who claimed that teachers have an inadequate and sometimes superficial understanding of AfL, which was attributed to the idea that most teachers lack sufficient guidance and practice to develop assessment skills, as AfL ideas are barely covered in their teacher education and training courses.(Amoako et al., 2019; Deluca & Volante, 2016).

Furthermore, DeLuca et al. (2019) claimed that it is important to find out what factors hinder MTEs from using effective AfL and how they understand and apply AfL in their classrooms. This can help MTEs to incorporate AfL strategies better in their teaching and learning of mathematics courses. Besides, identifying weaknesses and strengths regarding MTEs' conceptions is crucial to designing effective PD programs that can improve their

understandings and beliefs about AfL and then improve their implementations of AfL to help PPMTs obtain first-hand experiences of the implementation of AfL.

The possible reasons for the inadequate and limited conception of AfL by MTEs may likely stem from their education at college, which did not sufficiently address theoretical and practical aspects of implementation of AfL strategies, and from their not obtaining a quality and effective PD program on AfL in the educational institutions where they are working. Furthermore, it may also likely stem from their primary and secondary education, where the teacher-centered teaching and learning process dominates the classroom culture and no remedial intervention is done to correct their unsupportive assessment experiences, which are dominated by assessment of learning.

The result of the analysis of the interview data also shows that it can be very difficult even for MTEs to fully understand and conceive of the nature of AfL, the principles of AfL that guide classroom implementation of it, the strategies of implementing AfL, and the purposes of AfL. Moreover, the result shows the need for incorporating courses that contain theoretical and practical aspects of AfL in the preparation of MTEs for teacher education colleges. Besides, the result shows the need for quality and effective PD programs for MTEs to help them obtain detailed understandings of the nature, principles, strategies, and purposes of AfL and continuous scaffolds that can help them implement AfL strategies in the teaching and learning of mathematics courses while providing the courses to their pre-service teachers.

Furthermore, since teachers are more comfortable implementing teaching and learning methods that they are familiar with and confident about implementing, MTEs' inadequate conceptions of the nature of AfL, the principles that guide classroom implementation of

AfL, strategies for implementing AfL, and the purposes of AfL will have their own drawbacks in their implementation of quality and effective AfL strategies in the classroom instruction of mathematics courses they deliver to PPMTs. These drawbacks will have a negative consequence on PPMTs' conceptions of AfL experiences, which may also have its own influence on their future implementations of quality and effective AfL strategies in the classroom instruction practices of mathematics to primary students (Hamodi et al., 2017).

4.2.2 Mathematics Teacher Educators Experiences of the Professional Development

The result of the analysis of the semi-structured interviews with the three MTEs about their experiences of the comprehensive PD after the intervention revealed that the MTEs believed the comprehensive PD was very important for them, and they also believed that it helped them broaden their conceptions of AfL. Furthermore, all three of the MTEs acknowledged that the comprehensive PD helped them improve implementations of AfL strategies in the classroom instruction of the Fundamental Concepts of Algebra course, and they recommended a PD on how best to implement AfL to other MTEs and other stakeholders to improve implementation of AfL in teacher education colleges so as to improve pre-service teachers' learning.

The result of the analysis of the semi-structured interviews about how the three MTEs experienced the comprehensive PD is in agreement with the result of research that evaluated the implementation of AfL in Sweden by Jonsson et al. (2015), which showed that the professional development on AfL was successful in influencing teachers' perspectives on teaching and learning and in influencing teachers' pedagogical practices toward implementation of AfL strategies. Moreover, the result of the analysis of the semi-

structured interview supports the research result of Kruiper et al. (2021), who developed a curriculum-based training on AfL and scaffolding and interviewed the trained teachers after the training to discuss how effective the course was. The results suggest that trained teachers exhibited more adaptable behavior than teachers who were not trained, and teachers thought the training was beneficial and that scaffolding theory was helpful.

Similarly, the result of the analysis of post-intervention interviews with the three MTEs was in agreement with the findings of the research conducted by Andersson and Palm (2018), who revealed in their research that teachers of mathematics who took part in a PD program on AfL improved their AfL classroom practices to the point where it significantly improved students' academic performance, and they believed that the PD program offered them the resources and assistance they required to implement AfL strategies in the classroom instruction process. Furthermore, the result of the analysis of the interview is in support of the argument of Johnson et al. (2019), who stated that the key aspects of gaining ground in the implementation of AfL in educational institutions are engaging teacher educators in effective PD on AfL and improving institutional support for educational institutions across various levels.

The quality of the comprehensive PD may have contributed to the positive attitude of the three MTEs toward the PD they engaged in. The comprehensive context-specific, job-embedded, and content-based PD was designed based on literature reviews on the characteristics of a PD which is effective. Darling-Hammond et al. (2017) revealed in their study of effective PD for teachers that the best way to train teachers has seven common features, which are: it focuses on the subject matter; it involves active learning based on how adults learn; it encourages working together, usually in real work situations; it shows

and demonstrates effective practices; it provides guidance and support from experts; it allows for feedback and reflection; and it lasts for a long time.

The designed PD was content-based since it focused on the classroom instruction of the Fundamental Concepts of Algebra course. The PD addressed ways of identifying common students' challenges and misconceptions, both from MTEs' experiences in delivering the course and from research findings in the classroom instruction of algebra. Moreover, the PD involved MTEs in creating and proposing different ways to implement the five AfL strategies in the algebra course. They took part in the planning and execution of strategies through active learning, which allowed them to experience the same kind of learning that they want for their students. Besides, the PD encouraged collaboration by allowing them to exchange ideas with one another on how to better implement the five strategies, sharing the challenges they face in the classroom instruction of the course, and providing constructive feedback and scaffolding based on their classroom observations of their colleagues' implementation of the five AfL strategies.

Furthermore, the PD modeled effective teaching and learning practices by helping them incorporate into their lesson plans the implementation of the five strategies, making observations of their colleagues' classrooms, and providing the opportunity to reflect on one another and scaffold one another. Besides, the PD provided the MTEs with expert support by discussing the nature of AfL, the principles that guide classroom implementation of AfL, the five general strategies of AfL, and the purpose of AfL based on the needs of the three MTEs. In addition, the PD provided MTEs' with feedback and opportunities for reflection. Both the feedback and the reflection assisted MTEs in moving thoughtfully toward expert ideas for the implementation of AfL strategies.

Besides, the PD was conducted throughout the semester for twelve weeks, providing MTEs with adequate time to research, practice, implement, and reflect on ideas that help them make enhancement in their implementation of AfL strategies. In general, the quality of the comprehensive context-specific, job-embedded, and content-based PD may have played a great role in the positive attitude of the three MTEs towards the importance and significance of the PD.

4.2.3 The Effect of the Professional Development on the Practices of Assessment for Learning

The result of the analysis of classroom observation of the implementation of AfL after the comprehensive PD showed that the implementation of AfL strategies improved gradually in each of the three sections during the implementation of the intervention, although there were some differences in the level of improvement among the three sections. Moreover, the implementation of each of the five AfL strategies also improved gradually, as can be seen from Figure 4, although the improvements slightly varied across the five strategies. The comprehensive PD for MTEs on AfL and the training for PPMTs on SA and PA helped to improve the implementation of AfL strategies in the classroom instruction of the Fundamental Concepts of Algebra course. The result of the research is in agreement with the arguments of researchers who claimed that job-embedded, context-specific, and content-based teacher PD is particularly critical for meeting the diverse needs of students and teachers in a variety of contexts (Darling-Hammond et al., 2017).

Furthermore, the effectiveness of the comprehensive PD for MTEs on the implementation of AfL strategies and the training for PPMTs on SA and PA can be attributed to the quality of the design of the intervention that addresses the existing barriers and challenges that are

identified by Sintayehu and Ashenafi (2017), which are the lack of effective professional AfL assistance on the implementation of AfL for teachers and students' unwillingness to employ AfL strategies. Moreover, the result of the study is in agreement with the findings of Andersson and Palm (2018), who found that mathematics teachers who took part in a PD program on AfL improved their AfL classroom practices to the point where it significantly improved students' academic performance, and they believed that the PD program offered them the resources and assistance they required to implement AfL strategies in the classroom instruction process.

Similarly, the result of the study builds on the claim that providing meaningful, sustained PD that models effective use of AfL in the classroom is essential to promote mathematics teacher educators' use of AfL (Burton et al., 2018) and on the argument that it is challenging to offer high-quality education without effectively addressing factors related to teacher training and professional development (PD), such as ensuring top-notch pre-service training and ongoing PD that focuses on subject matter expertise and teaching methods (Workneh & Tassew, 2013). In addition, as Darling-Hammond et al. (2017) argued, the result of the analysis of classroom observation of the implementation of AfL strategies supports the idea that context-specific, job-embedded, and content-based PD for teachers is particularly important for addressing the different needs of students and teachers in various settings.

Besides, the result of the research is in agreement with the claim that it is challenging for teachers to engage in AfL without proper PD on AfL, and this challenge is particularly severe for mathematics teachers since classroom instruction of mathematics have historically addressed a restricted range of learning goals, mostly confined to technical

abilities taught as procedures through observe-and-practice (Burkhardt & Schoenfeld, 2019).

The result of the study also indicates that a PD that is designed based on the needs of participants and informed by concrete evidence of the challenges and gaps participants have in their perceptions of a given innovative strategy and the practical challenges of its implementation can be effective. Moreover, the result of the study shows that the key aspects of gaining ground in the implementation of AfL strategies in educational institutions are engaging teacher educators in comprehensive PD on AfL and improving institutional support for a sustained period of time, as Johnson et al. (2019) argued. Besides, the result of the study shows that the training of PPMTs on SA and PA helps the implementation of AfL strategies by reducing the resistance PPMTs may have when implementing unfamiliar strategies in the classroom instruction process.

Similarly, the improvement of the classroom instruction module assisted MTEs in implementing AfL strategies by providing them with alternative ways of selecting appropriate classroom activities from the module, alternative ways of sharing the learning intentions of each of the sub-topics in the course with their pre-service teachers, and alternative ways of providing quality feedback to their pre-service teachers. The improvement of the teaching and learning module also helped PPMTs assess themselves and their peers based on the learning intentions of the subtopics and on the summaries of the main ideas, definitions, theorems, and postulates included at the end of each of the chapters of the course module.

In general, the result of the analysis of the classroom observation ratings of the implementation of AfL strategies revealed that existing classroom instruction practices can

be enhanced by studying the existing classroom culture, designing effective PD, and providing training to all stakeholders for a sustained period of time in a community of practice where there is an expert scaffold and where participants practice, reflect, and provide constructive feedback and scaffolds to one another.

4.2.4 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Achievement Gains in the Algebra Course

The result of the analysis of PPMTs normalized achievement gains on the Fundamental Concepts of Algebra, which was calculated based on their pre-test and post-test scores using the formula of Marx and Cummings (2007), shows that PPMTs in the treatment group outperformed PPMTs in both comparison groups one and two on their normalized learning mean gains statistically significantly. Moreover, the analysis showed that although there is a difference between the normalized achievement gains of PPMTs in comparison group one and comparison group two, the difference is not statistically significant. The result shows that the comprehensive, context-specific, job-embedded, and content-based PD on AfL for MTEs and the training on SA and PA for PPMTs have a significant positive effect on the PPMTs' achievement gains on the Fundamental Concepts of Algebra.

The comprehensive PD program assisted MTEs in clarifying the learning objectives of the daily lessons when discussing each subtopic with their PPMTs. It also assisted them in designing effective classroom discussions, questions, and learning activities that elicit evidence of PPMTs' learning, identify PPMTs' difficulties and misconceptions during the teaching and learning process of Fundamental Concepts of Algebra, and give constructive feedback to both individual PPMTs and groups, moving them ahead.

Furthermore, the training for PPMTs on SA and PA assisted PPMTs in understanding the learning objectives of each subtopic of each of the chapters in the course, learning from their own and their peers' errors, providing constructive feedback to their peers, and assessing themselves and their peers. By and large, the comprehensive PD assisted MTEs in identifying the gaps, challenges, misconceptions, and strengths of their PPMTs and using these to enhance the classroom instruction process of Fundamental Concepts of Algebra.

The result of the research builds on the research findings of Andersson and Palm (2017), who conducted a quasi-experimental study aimed at examining the effects of implementing combinations of formative assessment strategies in mathematics lessons on year four students' achievements in mathematics in Sweden. The PD for year 4 mathematics teachers was delivered for 144 hours, and the teachers were provided with another 72 hours for reading about formative assessment from literature and planning and reflecting on new formative assessment strategies. The result of their research showed that year 4 students who were taught by mathematics teachers who were involved in the PD outperformed those year 4 students who were taught by mathematics teachers who were not involved in a PD on formative assessment.

The research findings are consistent with Chemeli (2019), who studied how the five main AfL strategies affected the math performance of secondary school students in Kenya using a quasi-experimental design with pre- and post-tests and a control and treatment group. Both groups learned the same topic for six weeks, and the results indicated that the treatment group did better than the comparison group in their achievements and problem-solving skills. Moreover, the large effect size obtained in the findings of this dissertation is in agreement with the finding of Chemeli (2019), who also found a large effect size

between the mathematics scores of the students in the comparison group and the mathematics scores in the treatment group.

This research has a similar outcome to the findings of Kyaruzi et al. (2019) in their study conducted in Tanzania. They claimed that in the educational systems of Sub-Saharan African countries, one of the ways to enhance students' achievement in mathematics is to ensure that teachers provide quality feedback to their students and that students make use of the feedback they receive effectively.

Moreover, this study supports the claims of other researchers (Andrade & Heritage, 2018; Black & Wiliam, 1998) who argued that effective implementations of AfL strategies in the in the classroom instruction process by the teacher and the students have a positive effect on students achievements. The findings of this research contradict the results of some researchers (Van den Berg et al., 2018; Yin et al., 2008), who discovered that there was no statistically significant difference in student achievement between students in the control group who were taught using traditional methods and students in the treatment group who were taught in a teaching and learning environment that implemented AfL strategies. In other words, the use of AfL strategies did not appear to have a significant impact on student achievement according to these researchers.

The study provides empirical evidence that a comprehensive PD program that covers all aspects of the context-specific job-embedded, and content-based on AfL for MTEs who train PPMTs on SA and PA can positively influence how AfL is implemented in the Fundamental Concepts of Algebra course for MTEs and PPMTs. The intervention gave enough support to MTEs and PPMTs to use AfL strategies effectively in teaching and learning the Fundamental Concepts of Algebra in CTE. This had a statistically significant

positive effect on how much PPMTs improved their achievement in the Fundamental Concepts of Algebra course when compared to other groups that did not receive the intervention.

The comprehensive, context-specific, job-embedded, and content-based PD program on AfL provided for MTEs might have improved their conceptions of AfL and improved their confidence to implement AfL strategies in the classroom instruction of the Fundamental Concepts of Algebra course. Moreover, the comprehensive PD may have helped MTEs in easily identifying their PPMTs' misconceptions and difficulties and helping them address those misconceptions and difficulties on time before they create problems in the PPMTs' learning of the Fundamental Concepts of Algebra course.

Furthermore, the training on SA and PA may have helped PPMTs understand the learning intentions of each of the lessons of the course, receive the constructive feedback they get from their MTE and their peers, learn from their errors and the errors of their peers, understand the importance of asking their MTE and their peers questions, and understand the importance of responding to written and oral questions in the classroom instruction of Fundamental Concepts of Algebra. Besides, the improved teaching and learning module of the course might have helped PPMTs understand what they are expected to know and be able to do after each of the subtopics in the course and also to assess themselves based on the learning intentions and the summaries incorporated in each chapter.

4.2.5 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Conceptions of Assessment for Learning Experiences

The analysis of the responses given by the PPMTs about their experiences of AfL in the course of Fundamental Concepts of Algebra reveals that the treatment group had more positive experiences than the comparison group one. This difference can be attributed to the following factors: a PD program on how to implement AfL that was designed for MTEs and that was comprehensive, job-embedded, context-specific, and content-based; a training for PPMTs on how to use SA and PA strategies; and an improvement of the teaching and learning module of the Fundamental Concepts of Algebra course.

However, the analysis showed that there was no statistically significant difference in the conceptions of AfL experiences among PPMTs in the treatment group compared to the conceptions of PPMTs in comparison group two, even though the conceptions of AfL experiences among PPMTs in the treatment group were higher than the conceptions of PPMTs in comparison group one. Moreover, the analysis shows that the conception of AfL experiences of PPMTs in comparison group two is statistically larger than the conception of AfL experiences of PPMTs in comparison group one.

The statistically non-significant mean gain score difference on the conceptions of AfL experiences between the PPMTs in the treatment group and the PPMTs in comparison group two, as well as the statistically significant mean gain score difference between PPMTs in comparison group one and PPMTs in comparison group two may, be because MTEs in comparison group two might have gotten training or PD on AfL implementation. Moreover, as it can be seen from the mean achievement gain in algebra of PPMTs in the three groups in table 7 and figure 6, the mean achievement gain of PPMTs in algebra in

comparison group two is higher than the mean achievement gain of comparison group one, although the difference is not statistically significant, and this may indicate that PPMTs' in comparison group two might have benefited more from this better conception of AfL experiences in the classroom instruction of Fundamental Concepts of Algebra than do PPMTs in comparison group one.

The result of analysis of PPMTs' responses on their conceptions of AfL experiences informs stakeholders in education that pre-service teachers' conceptions of AfL experiences can be improved through a well-designed and effective PD and training to MTEs and pre-service teachers which will have a positive effect for pre-service teachers as Deluca and Volante (2016) argued, who stated that when pre-service teachers are exposed to AfL experiences in teacher education colleges in a consistent and effective manner, they are more likely to integrate AfL with ease in their instruction and improve the learning of their future students, thus contributing to the development of AfL culture in their future classrooms.

Furthermore, the improvement of PPMTs' conceptions of AfL experiences is very important for them as college students and in their future profession, as it is argued that pre-service teachers' higher perceived practice of AfL is connected with more perceived autonomy, competence (both in terms of more competence satisfaction and less frustration), and autonomous motivation (Leenknecht et al., 2021). Besides, the improvement of PPMTs' conceptions of AfL experiences using well-designed PD and training is of utmost importance since the development of teachers' conceptions of assessment begins first in their experiences as students from elementary to secondary

school, then as pre-service teachers in teacher education institutions, and finally in their professional careers as classroom teachers (DeLuca, Coombs, et al., 2018).

In line with the argument made by Gan et al. (2019), the different aspects of students' perceived experience of assessment for learning (AfL) are all positively and significantly related to students' inclination to adopt a deep approach to learning. This means that AfL experiences help students to engage in meaningful and deep learning for themselves as college students, not only in the course that uses AfL, but also in their other courses. Furthermore, this also benefits them in their efforts to become effective beginner teachers who can facilitate deep learning for their future students.

4.2.6 The Effect of Assessment for Learning on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills

The result of the analysis of PPMTs' responses to the SRLS questionnaire before and after the intervention shows that the SRLS of PPMTs in the intervention group improved statistically significantly. The improved implementation of AfL strategies in the teaching and learning of Fundamental Concepts of Algebra was due to the comprehensive, context-specific, job-integrated, and content-based PD for MTEs; training for PPMTs on SA and PA helped PPMTs improve their SRLS.

The large effect size improvement of PPMTs SRLS before and after the intervention indicates that improving AfL implementations in primary mathematics teacher education has a positive effect on preparing initial primary mathematics teachers who can monitor, guide, and evaluate their own learning to become independent learners. This study's findings are consistent with prior research findings that reveal better AfL implementation

has a favorable influence on students' self-regulated learning when compared to comparable groups that were taught the traditional approach (Fisseha & Michael, 2016; Meusen-Beekman et al., 2016). Moreover, the result of the analysis of the PPMTs' responses to the SRLS questionnaire is consistent with the argument of Black (2015), who stated that the importance of AfL lies in its effect on the main aim of education, which is to improve students' effective, responsible, and independent learning capabilities.

The result of the analysis of responses of PPMTs to the SRLS questionnaire is in support of the research result of Granberg et al. (2021), who revealed, in their study of the effect of AfL on students' SRLS, that the implementation of improved AfL strategies by a mathematics teacher who was teaching seventh grade students had a significant positive effect on students' SRLS. Additionally, the result of the study builds on the research findings of Fisseha and Michael (2016), who found that university students who were taught using quality AfL strategies in lessons of a general psychology course reported more SRLS than students who were not taught using quality AfL in lessons in their study of the effect of high-quality AfL on SRLS in university classrooms.

Besides, the result of the study supports the argument of Andrade and Heritage (2017), who argued that AfL can help students develop SRLS abilities, although they also claim that the relationship between AfL and SRLS is primarily theoretical. However, the finding of the study is in contrast to the result of the study that investigated the effect of AfL on fifth-grade students' SRLS, conducted by Ozan and Kıncal (2018), which found that although improved implementation of AfL strategies improved students' SRLS, no significant difference between the experimental and control groups was seen.

When discussing each subtopic with their PPMTs, the comprehensive PD enabled MTEs to easily express the learning objectives of each subtopic to their PPMTs in a variety of ways, and this allowed the PPMTs to realize their own goals when discussing each subtopic. Additionally, the SA and PA assisted PPMTs in monitoring their own learning by evaluating whether they and their peers had met the learning objectives of the subtopics they were learning.

Furthermore, the use of enhanced AfL strategies assisted PPMTs in identifying their own weaknesses and seeking assistance from their peers and their mathematics teachers. The PD that covered various aspects of teaching mathematics also enabled MTEs to design and facilitate effective classroom interactions, such as discussions, questions, and learning activities. These interactions helped them to gather evidence of learning from their students, diagnose their difficulties and misunderstandings, and give them timely and helpful feedback at both individual and group levels. This way, they could support their students to make progress in their learning. Moreover, the training on SA and PA for PPMTs was beneficial for them to comprehend the learning goals of each subtopic in the curriculum, reflect on their own and their colleagues' errors and learn from them, and assess themselves and their peers using appropriate criteria and standards.

By and large, the comprehensive PD helped MTEs pinpoint and use PPMTs' gaps, challenges, and misconceptions to improve teaching and learning of Fundamental Concepts of Algebra. It also encouraged PPMTs to monitor and assess their own learning as well as learn through interactions with the materials, their MTEs, and their classmates. Furthermore, the summaries in the course module allow PMTs to determine whether they understand the major concepts of the topics covered in each chapter.

This research also offered empirical data on the influence of a comprehensive, context-specific, job-embedded, and content-based PD program on AfL for MTEs on AfL and training for PPMTs on SA and PA in the classroom instruction of algebra on the SRLS of PPMTs. Implementation of the five AfL strategies by MTEs and PPMTs has a positive effect on PPMTs' SRLS in the classroom instruction of algebra. The comprehensive PD on AfL that was context-specific, job-embedded, and content-based, as well as follow-up feedback and scaffolding, aided MTEs in successfully implementing AfL in an algebra course, allowing their PPMTs to set their own goals, plan, monitor their own learning, and evaluate their performances in the classroom instruction of algebra.

Moreover, the result of the study gives useful insight into the need to enhance the implementation of AfL strategies in teacher education institutions for MTEs, curriculum developers, and policymakers. According to the findings, teacher educators should receive comprehensive context-specific, job-embedded, and content-based PD on AfL in order to improve the implementation of the five AfL strategies in the preparation of beginning primary mathematics teachers who can learn independently.

CHAPTER FIVE: SUMMARY, CONCLUSIONS, RECOMMENDATIONS AND IMPLICATIONS

The main objectives of this study were to examine MTEs' conceptions of the nature, principles that guide classroom implementation, strategies, and purpose of AfL and then investigate the effect of a comprehensive context-specific, job-embedded, and content-based comprehensive PD on AfL for MTEs and a training to PPMTs on SA and PA on PPMTS' achievement gains, conceptions of AfL experiences, SRLS, and the implementation of AfL strategies in the teaching and learning of Fundamental Concepts of Algebra. This chapter presents a summary of the main findings of this study, the conclusions of the study, and the recommendations and implications of the study.

5.1 Summary

5.1.1 Mathematics Teacher Educators' Conceptions of Assessment for Learning

In general, the MTEs' conception of AfL was not found to be adequate, and a considerable number of MTEs were found to have incomplete understandings about AfL, and some of them also had some kind of misconception about AfL.

- ❖ Regarding the conception nature of AfL, around half of the MTEs were found to have a full understanding of the nature of AfL, and the other half were found to have an incomplete understanding with some kind of misconception.
- ❖ Regarding the principles of AfL that guide classroom implementation of AfL strategies, almost none of the MTEs were found to have a broad conception, most of them were found to have some kind of misconception.

- ❖ Regarding strategies for the implementation of AfL in the classroom instruction process, almost all of the interviewed MTEs were found to have an incomplete understanding without any misconceptions.
- ❖ Regarding the purposes of AfL, below half of the MTEs were found to have a complete understanding, and more than half of the MTEs were found to have an incomplete understanding of the purpose of AfL with some misconceptions.

5.1.2 Mathematics Teacher Educators' Experiences of the Professional Development on Assessment for Learning

- ❖ The result of the analysis of the interviews with the three MTEs who participated in the comprehensive context-specific, job-embedded, content-based PD demonstrated that they valued the comprehensive PD for their profession of mathematics teacher education.
- ❖ Besides, they felt that the PD was beneficial in using AfL principles in the classroom instruction of the algebra course. Additionally, they felt that the thorough PD assisted them in improving their understanding of AfL.
- ❖ Moreover, the MTEs believed that the comprehensive PD provided them with the necessary scaffold that helped them improve implementation of AfL in the classroom instruction of Fundamental Concepts of Algebra.

5.1.3 Effect of the Professional Development on the Implementation of Assessment for Learning Strategies

- ❖ The classroom implementation of each of the five AfL strategies in the classroom instruction of Fundamental Concepts of Algebra was generally enhanced by the comprehensive context-specific, job-embedded, and content-based PD for MTEs on

AfL and training for PPMTs on SA and PA. Moreover, a comprehensive PD for MTEs on AfL and training for PPMTs on SA and PA boosted the use of each of the five AfL strategies in the classroom in each of the three sections.

- ❖ Furthermore, the comprehensive PD for MTEs on AfL and training for PPMTs on SA and PA improved classroom implementation of AfL (from a grand average score of 1.59 to a grand average score of 3.91 out of 5).
- ❖ However, the intervention did not result in widespread use or highly effective implementation of AfL in each of the three sections in the intervention group or for each of the five strategies of implementation of AfL, so there is still room for improvement.

5.1.4 Effect of the Intervention on Pre-service Primary Mathematics Teachers' Achievement gains in Fundamental Concepts of Algebra

- ❖ In general, the intervention, which is comprised of comprehensive context-specific, job-embedded, and content-based PD for MTE and training for PPMTs on SA and PA, brought a statistically significant difference in the normalized mean achievement gain of PPMTs in the intervention group compared to PPMTs in both comparison groups one and two in Fundamental Concepts of Algebra. Moreover, the eta squared value shows that the size of the effect of the intervention is very large.
- ❖ Furthermore, the post hoc analysis showed that there was no statistically significant difference in the normalized mean gain of PPMTs in comparison group one and PPMTs in comparison group two in Fundamental Concepts of Algebra.

5.1.5 Effect of the Intervention on Pre-service Primary Mathematics Teachers' conceptions of Assessment for Learning Experiences

- ❖ The analysis of the PPMTs' responses to the students' conceptions of AfL experiences questionnaire showed that there was a statistically significant normalized mean achievement gain difference between the PPMTs in the intervention group and the PPMTs in comparison group one in their conceptions of AfL experiences in the classroom instruction of Fundamental Concepts of Algebra.
- ❖ Besides, the result of the analysis showed that there was no statistically significant difference in their mean gain score between PPMTs in the intervention group and PPMTs in comparison group two, even though there was a difference between them.
- ❖ Moreover, the analysis showed that there was a statistically significant mean gain score on the conceptions of AfL experiences between PPMTs in comparison group one and PPMTs in comparison group two.

5.1.6 Effect of the Intervention on Pre-service Primary Mathematics Teachers' Self-Regulated Learning Skills

The result of the analysis of the responses of PPMTs to the translated version of Pintrich et al. (1991)'s Motivated Strategies for Learning Questionnaire before and after the intervention showed that the comprehensive context-specific, job-embedded, and content-based PD for MTE and a training for PPMTs on SA and PA statistically improved the SRLS of PPMTs in the intervention group, with a very large effect size.

In general, the conceptions of nature, the principles that guide classroom implementation, and the strategies and purposes of AfL for MTEs were not found to be adequate, and the level of implementation of AfL strategies in classroom instruction was not at a satisfactory

level. Mathematics teacher educators who were involved in the PD found it very important, and they believed that it helped them improve their understanding of AfL and provided them with the necessary guidance and scaffold to improve their implementation of AfL strategies in the classroom instruction of Fundamental Concepts of Algebra. The level of implementation of AfL strategies in the classroom instruction process was also observed to improve gradually, although there was still room for further improvement.

The five AfL strategies that MTEs and PPMTs learned to implement better had a beneficial effect on PPMTs' performance and achievement gains in algebra. By enhancing the teaching and learning module and training pre-service teachers on how to conduct SA and PA, pre-service teachers became more aware of the learning goals and objectives in each subtopic of the course, worked more cooperatively with their classmates, participated more actively in the class discussions, and learned from their own mistakes. MTEs also received PD on AfL that was tailored to their specific context and content area, as well as ongoing feedback and coaching, which helped them implement AfL strategies effectively in algebra courses.

The intervention also changed PPMTs' perceptions of AfL experiences and made them appreciate how AfL can facilitate the teaching and learning processes of the Fundamental Concepts of Algebra course. Furthermore, the intervention enhanced PPMTs' SRLS in the teaching and learning of Fundamental Concepts of Algebra. The intervention gave PPMTs' the necessary guidance and support to keep track of, evaluate, and reflect on their own learning progress and helped them improve their SRLS.

5.2 Conclusion

The objective of this research was to examine conceptions of AfL among MTEs and then, using the information gathered from classroom observation of implementation of AfL strategies, design a comprehensive, context-specific, job-embedded, and content-based PD for MTEs and a training for PPMTs on SA and PA, and then examine the effect on the practices of implementation of AfL in the classroom instruction process of Fundamental Concepts of Algebra, on PPMTs' achievement gains in the course, on their conceptions of AfL experiences, and on their SRLS.

I have argued throughout this dissertation that effective and comprehensive PD for MTEs and training for PPMTs is vital to improving implementation of AfL strategies in the classroom instruction process, to improving students' learning of mathematics courses, especially algebra, to improving pre-service teachers conceptions of AfL experiences, and to improving pre-service mathematics teachers SRLS so as to produce effective beginning primary school mathematics teachers.

I explored qualities of effective PDs for educational practitioners from various researchers and learned that for PDs to be effective, they need to be sustained, designed based on the needs and gaps of the practitioners, provide guidance and multiple examples on how to implement them, and involve all stakeholders. Moreover, I learned that improving or modifying existing teaching and learning materials helps practitioners implement the ideas of the PD with ease.

This research disclosed possible gaps teacher educators can have in their conceptions of AfL and the need to identify the gaps and take necessary measures to help them improve them. Moreover, the research adds to the global argument that comprehensive context-specific, job-embedded, and content-based PD for teacher educators and training for pre-service teachers can help improve implementation of AfL strategies in the classroom instruction process at the college level, and it can also improve pre-service teachers' conceptions of AfL experiences, achievement gains, and SRLS.

It is now very clear that improving the quality of the classroom instruction process in teacher education programs is difficult to attain using a short and general PD that does not consider individual teacher educators needs. Indeed, to improve the education and training of pre-service mathematics teachers, effective and comprehensive PD and training for all stakeholders are vital, in addition to improving the teaching and learning materials.

The study's findings provided useful information to MTEs, curriculum developers, and policymakers on the need to enhance the implementation of AfL strategies in teacher education institutions. Based on the research findings, MTEs should be provided with comprehensive context-specific, job-embedded, and content-based PD in AfL in order to improve implementations of AfL strategies in initial primary mathematics teacher preparation programs.

5.3 Recommendation

Based on the findings of the research, the following recommendations for practice and for further research are made.

Recommendations for Practice

This study has revealed some important aspects that require more focus in order to enhance the use of AfL strategies in mathematics teacher education effectively. These aspects are crucial for improving the learning outcomes of PPMTs and the quality of their preparation in teacher education colleges.

- ✓ The results revealed that MTEs need comprehensive PD and expert support on the implementation of AfL strategies in the classroom instruction process of mathematics courses in teacher education colleges.
- ✓ Pre-service Primary mathematics teachers need to be trained on SA, PA, and the importance of actively participating in asking questions and responding to oral and written questions.
- ✓ The teaching and learning modules of mathematics courses need to be modified so that MTEs and PPMTs will be able to easily implement AfL strategies in the teaching and learning process of mathematics courses.
- ✓ Teacher education colleges could provide continuous professional development and expert support for MTEs to help improve the classroom instruction process culture in their institutions.

- ✓ Mathematics teacher educators, principals of teacher education colleges, the Regional Education Bureau, and policymakers need to collaborate to improve the implementation of AfL in teacher education colleges.

Recommendations for further research

Based on the thorough analysis of the main findings of this study, the following areas for further research are recommended:

- ✓ Action research by MTEs, either individually or in groups, to identify their own gaps and improve the implementation of their own practices of AfL could be conducted.
- ✓ Department heads and teacher education college principals could conduct SWOT analyses of the implementation of AfL strategies in their departments or colleges.
- ✓ Research using comparison groups could be conducted to examine the effect of improved implementations of AfL strategies in the classroom instruction process of other mathematics courses at teacher education colleges on PPMTs' SRLS.
- ✓ Research could be conducted to examine the effect of improved implementation of AfL strategies on the classroom instruction process of mathematics in primary and secondary schools.
- ✓ Research could be conducted to examine the effect of mathematics teachers' conceptions of their own practices of AfL and their actual implementation of AfL strategies in their classrooms.
- ✓ Research could also be conducted to examine the effect of PPMTs' conception of AfL experiences in their studies at the college and secondary school level on their practices of implementation of AfL strategies.

5.4 Implications of the Study

The result of this study highlighted the positive effect of a comprehensive, context-specific, job-embedded, content-based PD for MTEs and a training on SA and PA for PPMTs on the practices of implementation of AfL strategies, on PPMTs' conceptions of AfL experiences, on PPMTs' achievement gains in Fundamental Concepts of Algebra, and on the PPMTs' SRLS. Moreover, the result of the study highlighted the gaps in expectations regarding the status of MTEs' conceptions of nature, principles that guide classroom implementation, and the strategies and purposes of AfL, even though the number of participants was limited and it was not possible to generalize to the whole of the MTEs in every teacher education college in Amhara regional state.

The comprehensive PD has a high quality that matches well with the features of professional development that are known to be effective. The studies used rigorous methods to measure the impact of the PD on student outcomes, such as controlling for variables that could affect the results or comparing different groups of participants. Their analysis revealed seven common elements that make PD effective, and they are: focusing on the content that teachers need to teach; engaging teachers in active learning that respects their prior knowledge and experience; supporting teachers to work together and learn from each other, usually in their own work settings; showing teachers examples of effective practice and how to apply them; providing teachers with coaching and guidance from experts; giving teachers opportunities to get feedback and reflect on their practice; and lasting for a long enough time to allow for meaningful change.

The study serves as a reminder for officials in the Ministry of Education, the Amhara Regional Education Bureau, and teacher education college officials to consider contextual

factors (such as the needs and characteristics of the teachers, the goals of the curriculum, the culture and norm of the learning community ...) when designing training and professional development for mathematics teachers in general and MTEs in particular to attain the required objective of any training and PD. Furthermore, the study serves as a reference for other studies on PD and training on AfL for mathematics teachers, MTEs, and pre-service mathematics teachers aimed at improving the quality of mathematics education.

References

- Abera, G., Kedir, M., & Beyabeyin, M. (2017). The implementations and challenges of continuous assessment in public universities of Eastern Ethiopia. *International Journal of Instruction*, 10(4). <https://doi.org/10.12973/iji.2017.1047a>
- Agustyaningrum, N., Sari, R. N., Abadi, A. M., & Mahmudi, A. (2020). Dominant Factors that Cause Students' Difficulties in Learning Abstract Algebra: A Case Study at a University in Indonesia. *International Journal of Instruction*, 14(1). <https://doi.org/10.29333/IJI.2021.14151A>
- Alemu, M., Kind, V., Basheh, M., Michael, K., Atnafu, M., Kind, P., & Rajab, T. (2019). The knowledge gap between intended and attained curriculum in Ethiopian teacher education: identifying challenges for future development. *Compare*, 51(1), 81–98. <https://doi.org/10.1080/03057925.2019.1593107>
- Alex, J., & Roberts, N. (2019). The Need for Relevant Initial Teacher Education for Primary Mathematics: Evidence From the Primary Teacher Education Project in South Africa. In *27Th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education*, (Vol. 2019, Issue January, pp. 59–72).
- Allal, L. (2016). The Co-Regulation of Student Learning in an Assessment for Learning Culture. In *Assessment for Learning: Meeting the Challenge of Implementation* (pp. 259–273). Springer, Cham. https://doi.org/10.1007/978-3-319-39211-0_15
- Allal, L. (2020). Assessment and the co-regulation of learning in the classroom. *Assessment*

- in *Education: Principles, Policy & Practice*, 27(4), 332–349.
<https://doi.org/10.1080/0969594x.2019.1609411>
- Amoako, I., Asamoah, D., & Bortey, J. (2019). Knowledge of Formative Assessment Practices among Senior High School Mathematics Teachers in Ghana Open Access Knowledge of Formative Assessment Practices among Senior High School Mathematics Teachers in Ghana. *American Journal of Humanities and Social Sciences Research (AJHSSR)*, 3(3), 8–13.
- Andersson, C., & Palm, T. (2017). The impact of formative assessment on student achievement: A study of the effects of changes to classroom practice after a comprehensive professional development programme. *Learning and Instruction*, 49.
<https://doi.org/10.1016/j.learninstruc.2016.12.006>
- Andersson, C., & Palm, T. (2018). Reasons for teachers’ successful development of a formative assessment practice through professional development—a motivation perspective. *Assessment in Education: Principles, Policy and Practice*, 25(6).
<https://doi.org/10.1080/0969594X.2018.1430685>
- Andrade, H., & Brookhart, S. M. (2016). The Role of Classroom Assessment in Supporting Self-Regulated Learning. In D. L. and L. Allal (Ed.), *Assessment for Learning: Meeting the Challenge of Implementation* (pp. 293–309). Springer International Publishing. https://doi.org/10.1007/978-3-319-39211-0_17
- Andrade, H., & Brookhart, S. M. (2019). Classroom assessment as the co-regulation of learning. *Assessment in Education: Principles, Policy & Practice*, 00(00), 1–23.
<https://doi.org/10.1080/0969594X.2019.1571992>

- Andrade, H., & Heritage, M. (2018). Using Formative Assessment to Enhance Learning, Achievement, and Academic Self-Regulation. In *Routledge*. Routledge. <https://doi.org/10.4324/9781315623856>
- Andradel, H. L. (2019). A Critical Review of Research on Student Self-Assessment. In *Frontiers in Education* (Vol. 4). <https://doi.org/10.3389/feduc.2019.00087>
- Assessment Reform Group. (2002). Assessment for Learning: 10 Principles. Research-based principles to guide classroom practice Assessment for Learning. In *Assessment Reform Group*. https://doi.org/10.1007/978-3-319-02600-8_12
- Baird, J. A., Andrich, D., Hopfenbeck, T. N., & Stobart, G. (2017). Assessment and learning: fields apart? *Assessment in Education: Principles, Policy and Practice*, 24(3), 317–350. <https://doi.org/10.1080/0969594X.2017.1319337>
- Barnes, N., Fives, H., & Dacey, C. M. (2015). Teachers' beliefs about assessment. In *International Handbook of Research on Teachers' Beliefs* (pp. 284–300). <https://doi.org/10.4324/9780203108437-25>
- Barnes, N., Fives, H., & Dacey, C. M. (2017). U.S. teachers' conceptions of the purposes of assessment. *Teaching and Teacher Education*, 65. <https://doi.org/10.1016/j.tate.2017.02.017>
- Bennett, R. E. (2011). Formative assessment: A critical review. *Assessment in Education: Principles, Policy and Practice*, 18(1), 5–25. <https://doi.org/10.1080/0969594X.2010.513678>
- Birenbaum, M., DeLuca, C., Earl, L., Heritage, M., Klenowski, V., Looney, A., Smith, K.,

- Timperley, H., Volante, L., & Wyatt-Smith, C. (2015). International trends in the implementation of assessment for learning: Implications for policy and practice. *Policy Futures in Education*, 13(1). <https://doi.org/10.1177/1478210314566733>
- Black, P. (2015). Formative assessment – an optimistic but incomplete vision. *Assessment in Education: Principles, Policy and Practice*, 22(1), 161–177. <https://doi.org/10.1080/0969594X.2014.999643>
- Black, P., & Wiliam, D. (1998). Assessment and classroom learning. In *Assessment in Education: Principles, Policy & Practice* (Vol. 5, Issue 1). <https://doi.org/10.1080/0969595980050102>
- Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability*, 21(1), 5–31. <https://doi.org/10.1007/s11092-008-9068-5>
- Black, P., & Wiliam, D. (2010). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan*, 92(1). <https://doi.org/10.1177/003172171009200119>
- Black, P., & Wiliam, D. (2018). Classroom assessment and pedagogy. *Assessment in Education: Principles, Policy and Practice*, 25(6), 551–575. <https://doi.org/10.1080/0969594X.2018.1441807>
- Blackley, S., & Howell, J. (2015). A STEM narrative: 15 years in the making. *Australian Journal of Teacher Education*, 40(7). <https://doi.org/10.14221/ajte.2015v40n7.8>
- Boud, D. (2018). Assessment could demonstrate learning gains, but what is required for it to do so? *Higher Education Pedagogies*, 3(1).

<https://doi.org/10.1080/23752696.2017.1413671>

Brevik, L. M., Blikstad-Balas, M., & Engelién, K. L. (2017). Integrating assessment for learning in the teacher education programme at the University of Oslo. *Assessment in Education: Principles, Policy and Practice*, 24(2).
<https://doi.org/10.1080/0969594X.2016.1239611>

Burkhardt, H., & Schoenfeld, A. (2018). Assessment in the service of learning: challenges and opportunities or Plus ça Change, Plus c'est la même Chose. *ZDM - Mathematics Education*, 50(4). <https://doi.org/10.1007/s11858-018-0937-1>

Burkhardt, H., & Schoenfeld, A. (2019). Formative Assessment in Mathematics. In *Handbook of Formative Assessment in the Disciplines*.
<https://doi.org/10.4324/9781315166933-3>

Burton, M., Silver, E. A., Mills, V. L., Audric, W., Strutchens, M. E., & Petit, M. (2018). Formative Assessment and Mathematics Teaching: Leveraging Powerful Linkages in the US Context. In D. R. T. B. C. Wright (Ed.), *Classroom Assessment in Mathematics*. Springer. https://doi.org/10.1007/978-3-319-73748-5_13

Carless, D., & Boud, D. (2018). The development of student feedback literacy: enabling uptake of feedback. *Assessment and Evaluation in Higher Education*, 43(8).
<https://doi.org/10.1080/02602938.2018.1463354>

Chalachew, A. A. (2018). Factors and Influences of Self-Regulation Learning among Sport Science Undergraduate Students in Ethiopian Universities. *The International Journal Research Publication's - Research Journal of Social Science and Management (RJSSM)*, 8(7).

- Chemeli, J. (2019). Impact of the Five Key Formative Assessment Strategies on Learner's Achievement in Mathematics Instruction in Secondary Schools: A Case of Nandi County, Kenya. *International Academic Journal of Social Sciences and Education*, 2(1), 212–229.
- Chigonga, B. (2020). Formative Assessment in Mathematics Education in the Twenty-First Century. In *Theorizing STEM Education in the 21st Century*. <https://doi.org/10.5772/intechopen.88996>
- Coombs, A., DeLuca, C., LaPointe-McEwan, D., & Chalas, A. (2018). Changing approaches to classroom assessment: An empirical study across teacher career stages. *Teaching and Teacher Education*, 71. <https://doi.org/10.1016/j.tate.2017.12.010>
- Cottrill, J. (2003). *An Overview of Theories of Learning in Mathematics Education Research*. June, 1–8.
- Cousins-Cooper, K., Staley, K. N., Kim, S., & Luke, N. S. (2017). The effect of the math emporium instructional method on students' performance in college algebra. *European Journal of Science and Mathematics Education*, 5(1). <https://doi.org/10.30935/scimath/9493>
- Creswell, J. W. (2012). Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research. In *Educational Research* (Vol. 148).
- Creswell, J. W., & Creswell, J. D. (2018). *Research Design Qualitative, Quantitative, and Mixed Methods Approaches Fifth Edition* (Fifth Edit, Vol. 148). SAGE Publications, Inc. All.

- Darling-Hammond, L., Hyler, M. E., & Gardner, M. (2017). Effective Teacher Professional Development (research brief). *Effective Teacher Professional Development (Research Brief)*, June.
- DeLuca, C., Chapman-Chin, A. E. A., LaPointe-McEwan, D., & Klinger, D. A. (2018). Student perspectives on assessment for learning. *Curriculum Journal*, 29(1).
<https://doi.org/10.1080/09585176.2017.1401550>
- DeLuca, C., Chapman-Chin, A., & Klinger, D. A. (2019). Toward a Teacher Professional Learning Continuum in Assessment for Learning. *Educational Assessment*, 24(4).
<https://doi.org/10.1080/10627197.2019.1670056>
- DeLuca, C., Coombs, A., & Sherman, A. (2018). Preparing Teachers for Assessment in Schools: The Influence of Teacher Educators. In *In Innovation and Accountability in Teacher Education* (pp. 171–186). Springer. https://doi.org/10.1007/978-981-13-2026-2_11
- DeLuca, C., Valiquette, A., Coombs, A., McEwan, D. L., & Luhanga, U. (2018). Teachers' approaches to classroom assessment: a large-scale survey. *Assessment in Education: Principles, Policy and Practice*, 25(4), 355–375.
<https://doi.org/10.1080/0969594X.2016.1244514>
- DeLuca, C., & Volante, L. (2016). ASSESSMENT FOR LEARNING IN TEACHER EDUCATION PROGRAMS: NAVIGATING THE JUXTAPOSITION OF THEORY AND PRAXIS. *JISTE*, 20(1).
- Deneen, C. C., & Brown, G. T. L. (2016). The impact of conceptions of assessment on assessment literacy in a teacher education program. *Cogent Education*, 3(1).

<https://doi.org/10.1080/2331186X.2016.1225380>

- Deneen, C. C., Fulmer, G. W., Brown, G. T. L., Tan, K., Leong, W. S., & Tay, H. Y. (2019). Value, practice and proficiency: Teachers' complex relationship with assessment for learning. *Teaching and Teacher Education*, 80, 39–47. <https://doi.org/10.1016/j.tate.2018.12.022>
- Emery, N., Maher, J. M., & Ebert-May, D. (2019). Studying Professional Development as Part of the Complex Ecosystem of STEM Higher Education. *Innovative Higher Education*, 44(6). <https://doi.org/10.1007/s10755-019-09475-9>
- Ernest, P. (1994). Social Constructivism and the Psychology of Mathematics Education. In *Constructing Mathematical Knowledge: Epistemology and Mathematics Education* (pp. 62–72). The Faimer Press. <https://doi.org/10.4324/9780203454206>
- Ernest, P. (2010). Reflections on theories of learning. *ZDM - International Journal on Mathematics Education*, 38(1). <https://doi.org/10.1007/BF02655901>
- Evans, C., Howson, C. K., & Forsythe, A. (2018). Making sense of learning gain in higher education. *Higher Education Pedagogies*, 3(1). <https://doi.org/10.1080/23752696.2018.1508360>
- Ferretti, F., Paraskevi, M. C., & Vannini, I. (2018). Formative Assessment for mathematics teaching and learning Teacher Professional Development Research by Videoanalysis Methodologies. *FrancoAngeli s.r.L.*, 3(February 2004), 1–182.
- Figa, J. G., Tarekegne, W. M., & Kebede, M. A. (2020). The Practice of Formative Assessment in Ethiopian Secondary School Curriculum Implementation: The Case of

- West Arsi Zone Secondary Schools. *Educational Assessment*, 25(4).
<https://doi.org/10.1080/10627197.2020.1766958>
- Fisseha, W. M., & Michael, D. J. (2016). The impact of formative assessment on self-regulating learning in university classrooms. *Tuning Journal for Higher Education*, 4(1), 99–118. [https://doi.org/10.18543/tjhe-4\(1\)-2016pp99-118](https://doi.org/10.18543/tjhe-4(1)-2016pp99-118)
- Fulmer, G. W., Lee, I. C. H., & Tan, K. H. K. (2015). Multi-level model of contextual factors and teachers' assessment practices: an integrative review of research. *Assessment in Education: Principles, Policy and Practice*, 22(4), 475–494. <https://doi.org/10.1080/0969594X.2015.1017445>
- Gan, Z., Hoi, C., & Schumacker, R. (2019). Investigating Influence of Assessment for Learning Practice on Student Learning Approaches in Chinese Higher Education. *Measurement*, 17(1), 23–37. <https://doi.org/10.1080/15366367.2018.1479089>
- Garzón, J., & Bautista, J. (2018). Virtual Algebra Tiles: A pedagogical tool to teach and learn algebra through geometry. *Journal of Computer Assisted Learning*, 34(6), 876–883. <https://doi.org/10.1111/jcal.12296>
- Geberew, T., & Tigist, T. (2018). The Nature of Classroom Assessment in Ethiopian Public Secondary Schools: Subject Teachers' Views. *Journal of Education, Society and Behavioural Science*, 26(3), 1–11. <https://doi.org/10.9734/jesbs/2018/43233>
- Gemechu, E., Shishigu, A., Michael, K., Atnafu, M., & Ayalew, Y. (2017). Reforms of teacher education in Ethiopian: a historical analysis. *Research Journal of Educational Sciences*, 5(2), 1–6.

- Gemeda, F. T., & Tynjälä, P. (2015). Professional learning of teachers in Ethiopia: Challenges and implications for reform. *Australian Journal of Teacher Education*, 40(5). <https://doi.org/10.14221/ajte.2015v40n5.1>
- Goos, M. (2013). Sociocultural perspectives in research on and with mathematics teachers: A zone theory approach. *ZDM - International Journal on Mathematics Education*, 45(4). <https://doi.org/10.1007/s11858-012-0477-z>
- Granberg, C., Palm, T., & Palmberg, B. (2021). A case study of a formative assessment practice and the effects on students' self-regulated learning. *Studies in Educational Evaluation*, 68. <https://doi.org/10.1016/j.stueduc.2020.100955>
- Greene, J. A. (2020). Building upon synergies among self-regulated learning and formative assessment research and practice. *Assessment in Education: Principles, Policy and Practice*, 27(4). <https://doi.org/10.1080/0969594X.2020.1802225>
- Greenes, C. (2009). Mathematics Learning and Knowing: A Cognitive Process. *Journal of Education*, 189(3). <https://doi.org/10.1177/002205740918900310>
- Guerios, E., & Goncalves, T. O. (2019). A study on the research on initial training of teachers who teach mathematics in the initial years of schooling. *Educar Em Revista*, 35(78), 27–45.
- Haftu, H., Ahmed, Y., Dawit, A., Meskerem, L., Dawit, T., & Dereje, T. (2018). Revisiting Teacher Educators' Training in Ethiopia: Implications for a New Approach to Curriculum Development. *Bahir Dar Journal of Education*, 17(2), 89–105.
- Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-

- student survey of mechanics test data for introductory physics courses. *American Journal of Physics*, 66(1). <https://doi.org/10.1119/1.18809>
- Hamodi, C., López-Pastor, V. M., & López-Pastor, A. T. (2017). If I experience formative assessment whilst studying at university, will I put it into practice later as a teacher? Formative and shared assessment in Initial Teacher Education (ITE). *European Journal of Teacher Education*, 40(2). <https://doi.org/10.1080/02619768.2017.1281909>
- Hattie, J. (2008). Visible learning: A synthesis of over 800 meta-analyses relating to achievement. In *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. <https://doi.org/10.4324/9780203887332>
- Hausberger, T. (2018). Structuralist Praxeologies as a Research Program on the Teaching and Learning of Abstract Algebra. *International Journal of Research in Undergraduate Mathematics Education*, 4(1), 74–93. <https://doi.org/10.1007/s40753-017-0063-4>
- Hawe, E., & Dixon, H. (2017). Assessment for learning: a catalyst for student self-regulation. *Assessment and Evaluation in Higher Education*, 42(8). <https://doi.org/10.1080/02602938.2016.1236360>
- Heritage, M. (2016). *Assessment for Learning: Co-Regulation in and as Student–Teacher Interaction*. https://doi.org/10.1007/978-3-319-39211-0_19
- Hill, M. F., Ell, F. R., & Eyers, G. (2017). Assessment Capability and Student Self-regulation: The Challenge of Preparing Teachers. *Frontiers in Education*, 2. <https://doi.org/10.3389/feduc.2017.00021>

- Hodgen, J., Oldenburg, R., & Strømskag, H. (2018). Algebraic thinking. *Developing Research in Mathematics Education*, 32–45. <https://doi.org/10.4324/9781315113562-4>
- Hodgen, J., & Wiliam, D. (2012). Mathematics Inside the Black Box: Assessment for Learning in the Mathematics Classroom. *Key Concepts in Teaching Primary Mathematics*.
- John, w. C., & Clark, V. L. P. (2018). Designing and Conducting Mixed Methods Research. In *Organizational Research Methods* (Third Edit). SAGE Publications, Inc.
- Johnson, C. C., Sondergeld, T. A., & Walton, J. B. (2019). A Study of the Implementation of Formative Assessment in Three Large Urban Districts. *American Educational Research Journal*, 20(10), 1–31. <https://doi.org/10.3102/0002831219842347>
- Johnson, R. B., Onwuegbuzie, A. J., & Turner, L. A. (2007). Toward a Definition of Mixed Methods Research. *Journal of Mixed Methods Research*, 1(2). <https://doi.org/10.1177/1558689806298224>
- Jonsson, A., Lundahl, C., & Holmgren, A. (2015). Evaluating a large-scale implementation of Assessment for Learning in Sweden. *Assessment in Education: Principles, Policy and Practice*, 22(1). <https://doi.org/10.1080/0969594X.2014.970612>
- Kanbir, S., Clements, M. A., & Ellerton, N. F. (2018). Identifying a Problem with School Algebra. In *Using Design Research and History to Tackle a Fundamental Problem with School Algebra* (pp. 1–10). Springer, Cham. https://doi.org/10.1007/978-3-319-59204-6_1

- Kruiper, S. M. A., Leenknecht, M. J. M., & Slof, B. (2021). Using scaffolding strategies to improve formative assessment practice in higher education. *Assessment and Evaluation in Higher Education*, 0(0), 1–19.
<https://doi.org/10.1080/02602938.2021.1927981>
- Kuhn, T. (1962). The structure of scientific revolutions. In *Knowledge and Postmodernism in Historical Perspective: Vol. II* (Second Edi, Issue 2).
<https://doi.org/10.5840/philstudies196413082>
- Kyaruzi, F., Strijbos, J. W., & Ufer, S. (2020). Impact of a Short-Term Professional Development Teacher Training on Students' Perceptions and Use of Errors in Mathematics Learning. *Frontiers in Education*, 5.
<https://doi.org/10.3389/feduc.2020.559122>
- Kyaruzi, F., Strijbos, J. W., Ufer, S., & Brown, G. T. L. (2019). Students' formative assessment perceptions, feedback use and mathematics performance in secondary schools in Tanzania. *Assessment in Education: Principles, Policy and Practice*, 26(3).
<https://doi.org/10.1080/0969594X.2019.1593103>
- Leahy, S., Lyon, C., Thompson, M., & Wiliam, D. (2005). Classroom Assessment: Minute by Minute, Day by Day. In *Educational Leadership* (Vol. 63, Issue 3).
- Lee, J., Bryant, D. P., Ok, M. W., & Shin, M. (2020). A Systematic Review of Interventions for Algebraic Concepts and Skills of Secondary Students with Learning Disabilities. *Learning Disabilities Research and Practice*, 35(2).
<https://doi.org/10.1111/ldrp.12217>
- Leenknecht, M., Wijnia, L., Köhlen, M., Fryer, L., Rikers, R., & Loyens, S. (2021).

- Formative assessment as practice: the role of students' motivation. *Assessment and Evaluation in Higher Education*, 46(2).
<https://doi.org/10.1080/02602938.2020.1765228>
- López-Pastor, V., & Sicilia-Camacho, A. (2015). Formative and shared assessment in higher education. Lessons learned and challenges for the future. *Assessment and Evaluation in Higher Education*, 42(1).
<https://doi.org/10.1080/02602938.2015.1083535>
- Lyon, C. J., Nabors Oláh, L., & Caroline Wylie, E. (2019). Working toward integrated practice: Understanding the interaction among formative assessment strategies. *Journal of Educational Research*, 112(3), 301–314.
<https://doi.org/10.1080/00220671.2018.1514359>
- Marx, J. D., & Cummings, K. (2007). Normalized change. *American Journal of Physics*, 75(1). <https://doi.org/10.1119/1.2372468>
- McGrath, C., Guerin, B., Harte, E., Frearson, M., & Manville, C. (2020). Learning gain in higher education. In *Learning gain in higher education*. <https://doi.org/10.7249/rr996>
- Meusen-Beekman, K. D., Joosten-ten Brinke, D., & Boshuizen, H. P. A. (2016). Effects of formative assessments to develop self-regulation among sixth grade students: Results from a randomized controlled intervention. *Studies in Educational Evaluation*, 51.
<https://doi.org/10.1016/j.stueduc.2016.10.008>
- Moss, C. M., & Brookhart, S. M. (2019). Advancing Formative Assessment in Every Classroom: A Guide for Instructional Leaders, 2nd Edition. In ASCD.

- Moss, C. M., Brookhart, S. M., & BROOKHART, C. M. M. S. M. (2009). ADVANCING FORMATIVE ASSESSMENT IN EVERY CLASSROOM A GUIDE FOR INSTRUCTIONAL LEADERS. In ASCD. [https://doi.org/10.1016/0002-9610\(54\)90281-0](https://doi.org/10.1016/0002-9610(54)90281-0)
- Nicol, D. (2021). The power of internal feedback: exploiting natural comparison processes. *Assessment and Evaluation in Higher Education*, 46(5), 756–778. <https://doi.org/10.1080/02602938.2020.1823314>
- Nicol, D. J., & Macfarlane-Dick, D. (2006). Formative assessment and self-regulated learning a model and seven principles of good feedback practice. *Studies in Higher Education*, 31(2), 199–218. <http://dx.doi.org/10.1080/03075070600572090>
- Nunez, I. (2015). Philosophical underlabouring for mathematics education. *Journal of Critical Realism*, 14(2). <https://doi.org/10.1179/1476743015Z.000000000060>
- Oswalt, S. G. (2013). *IDENTIFYING FORMATIVE ASSESSMENT IN CLASSROOM INSTRUCTION: CREATING AN INSTRUMENT TO OBSERVE USE OF FORMATIVE ASSESSMENT IN PRACTICE* (Issue December). BOISE STATE UNIVERSITY.
- Ozan, C., & Kincal, R. Y. (2018). The effects of formative assessment on academic achievement, attitudes toward the lesson, and self-regulation skills. *Kuram ve Uygulamada Egitim Bilimleri*, 18(1). <https://doi.org/10.12738/estp.2018.1.0216>
- Pan, S. C., Sana, F., Samani, J., Cooke, J., & Kim, J. A. (2020). Learning from errors: students' and instructors' practices, attitudes, and beliefs. *Memory*, 28(9), 1105–1122. <https://doi.org/10.1080/09658211.2020.1815790>

- Panadero, E., Andrade, H., & Brookhart, S. (2018). Fusing self-regulated learning and formative assessment: a roadmap of where we are, how we got here, and where we are going. *Australian Educational Researcher*, 45(1), 13–31. <https://doi.org/10.1007/s13384-018-0258-y>
- Panadero, E., Broadbent, J., Boud, D., & Lodge, J. M. (2019). Using formative assessment to influence self- and co-regulated learning: the role of evaluative judgement. In *European Journal of Psychology of Education* (Vol. 34, Issue 3). <https://doi.org/10.1007/s10212-018-0407-8>
- Panadero, E., Jonsson, A., Botella, J., Ernesto, P., Anders, J., & Juan, B. (2017). Effects of self-assessment on self-regulated learning and self-efficacy: Four meta-analyses. *Educational Research Review*, 22, 74–98. <https://doi.org/10.1016/j.edurev.2017.08.004>
- Panadero, E., Jonsson, A., & Strijbos, J.-W. (2016). *Scaffolding Self-Regulated Learning Through Self-Assessment and Peer Assessment: Guidelines for Classroom Implementation*. https://doi.org/10.1007/978-3-319-39211-0_18
- Pat-El, R. J., Tillema, H., Segers, M., & Vedder, P. (2013). Validation of Assessment for Learning Questionnaires for teachers and students. *British Journal of Educational Psychology*, 83(1). <https://doi.org/10.1111/j.2044-8279.2011.02057.x>
- Pattalitan, A. P. (2016). The implications of learning theories to assessment and instructional scaffolding techniques. *American Journal of Educational Research*, 4(9).
- Pintrich, P. R. (2000). The Role of Goal Orientation in Self-Regulated Learning. In

Handbook of Self-Regulation. <https://doi.org/10.1016/b978-012109890-2/50043-3>

Pintrich, P. R. (2019). A Conceptual Framework for Assessing Motivation and Self-Regulated Learning in College Students Author (s): Paul R . Pintrich Source : Educational Psychology Review , Vol . 16 , No . 4 , Measuring Studying and Learning in Higher Education — Conceptual a. *Educational Psychology Review*, 16(4), 385–407.

Pintrich, P. R., Smith, D., Garcia, T., & McKeachie, W. (1991). A manual for the use of the Motivated Strategies for Learning Questionnaire (MSLQ). *Ann Arbor. Michigan*, 48109(August 2016). <https://doi.org/ED338122>

Quinlan, K. M., & Pitt, E. (2021). Towards signature assessment and feedback practices: a taxonomy of discipline-specific elements of assessment for learning. *Assessment in Education: Principles, Policy and Practice*, 28(2). <https://doi.org/10.1080/0969594X.2021.1930447>

Rogaten, J., Rienties, B., & Whitelock, D. (2017). Assessing learning gains. *Communications in Computer and Information Science*, 653. https://doi.org/10.1007/978-3-319-57744-9_11

Roth, A., Ogrin, S., & Schmitz, B. (2016). Assessing self-regulated learning in higher education: a systematic literature review of self-report instruments. In *Educational Assessment, Evaluation and Accountability* (Vol. 28, Issue 3). <https://doi.org/10.1007/s11092-015-9229-2>

Ruiz-Primo, M. A. (2016). *Implementing High Quality Assessment for Learning: Mapping as a Professional Development Tool for Understanding the What to Learn, Why to*

- Learn It, and How to Learn It.* https://doi.org/10.1007/978-3-319-39211-0_13
- Ruiz-Primo, M. A., & Brookhart, S. M. (2017). Using Feedback to Improve Learning. In *Using Feedback to Improve Learning.* <https://doi.org/10.4324/9781315627502>
- Schunk, D. H. (2012). Learning theories: An educational perspective Sixth Edition. In *Pearson* (Vol. 5).
- Scott, S., & Palincsar, A. (2012). Sociocultural Theory. *The Gale Group*, 1–10. <https://doi.org/10.1002/9781405198431.wbeal1084>
- Shabani, K. (2016). Applications of Vygotsky's sociocultural approach for teachers' professional development. In *Cogent Education* (Vol. 3, Issue 1). <https://doi.org/10.1080/2331186X.2016.1252177>
- Shepard, L. A., Penuel, W. R., & Pellegrino, J. W. (2018). Using Learning and Motivation Theories to Coherently Link Formative Assessment, Grading Practices, and Large-Scale Assessment. *Educational Measurement: Issues and Practice*, 37(1). <https://doi.org/10.1111/emip.12189>
- Simon, M. A. (2017). Amidst Multiple Theories of Learning in Mathematics Education
Author (s): Martin A . Simon Source : Journal for Research in Mathematics Education , Vol . 40 , No . 5 (Nov . , 2009), pp . Published by : National Council of Teachers of Mathematics Stable. *Journal for Research in Mathematics Education*, 40(5), 477–490.
- Sintayehu, B. (2016). The Practice of Continuous Assessment in Primary Schools: The CAse of Changi, Ethopia. *Journal of Education and Practice*, 7(31), 24–30.

- Sintayehu, B., & Ashenafi, T. (2017). The Impending Challenges of Continuous Assessment Implementation at Dire Dawa University, Ethiopia. In *International Journal of African and Asian Studies* www.iiste.org ISSN (Vol. 35).
- Solomon, M. M. (2014). The contribution of teachers' peer-and self-assessment for the implementation of active learning strategies: Perceptions of Ethiopia Primary School teachers. *Educational Research and Reviews*, 9(17).
<https://doi.org/10.5897/err2013.1637>
- Stewart, S., & Reeder, S. (2017). Algebra underperformances at college level: What are the consequences? In *And the Rest is Just Algebra*. https://doi.org/10.1007/978-3-319-45053-7_1
- Stoilescu, D. (2016). Aspects of theories, frameworks and paradigms in mathematics education research. *European Journal of Science and Mathematics Education*, 4(2), 140–154. <https://doi.org/10.30935/scimath/9460>
- Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., Silver, E., Ufer, S., & Vos, P. (2016). Assessment in Mathematics Education: Large-Scale Assessment and Classroom Assessment. In *ICME-13 Topical Surveys*. Springer Open. <https://doi.org/10.4324/9780429021015-19>
- Swan, M., & Foster, C. (2016). Formative Assessment Lessons For Concept Development And Problem Solving. *International Congress on Mathematical Education Hamburg, July*, 24–31.
- Tasdemir, M., Iqbal, Z., & Asghar, Z. (2020). A Study of the Significant Factors Affecting Pre-Service Teacher Education in Turkey A Study of the Significant Factors Affecting

- Pre-Service TE in Turkey Background of the Study. *Bulletin of Education and Research*, 42(1).
- Teklebrhan, B., & Samuel, E. (2015). Problems and Prospects of Implementing Continuous Assessment at Adigrat University. In *Journal of Education and Practice* (Vol. 6, Issue 4).
- Tien, N. H., Anh, D. T., Luong, M. Van, Ngoc, N. M., & Thanh, N. (2020). Formative assessment in the teacher education in Vietnam. *Journal of Hunan University Natural Sciences*, 47(8), 1–10. <http://joununs.com/index.php/journal/article/view/429/427>
- Van den berg, M., Bosker, R. J., & Suhre, C. J. M. (2018). Testing the effectiveness of classroom formative assessment in Dutch primary mathematics education. *School Effectiveness and School Improvement*, 29(3). <https://doi.org/10.1080/09243453.2017.1406376>
- Vlachou, M. A. (2015). Does Assessment for Learning Work to Promote Student Learning? The England Paradigm. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 88(3). <https://doi.org/10.1080/00098655.2015.1032194>
- Wang, L. (2007). Sociocultural Learning Theories and Information Literacy Teaching Activities in Higher Education. *Reference and User Services Quarterly*, 47(2), 149–158. <https://doi.org/10.5860/rusq.47n2.149>
- Wang, X. (2015). The Literature Review of Algebra Learning: Focusing on the Contributions to Students' Difficulties. *Creative Education*, 06(02), 144–153. <https://doi.org/10.4236/ce.2015.62013>

- Warren, E., Trigueros, M., & Sonia, U. (2016). RESEARCH ON THE LEARNING AND TEACHING OF ALGEBRA. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education: The Journey Continues* (second, pp. 73–108). SENSE PUBLISHERS. https://doi.org/10.1007/978-94-6300-561-6_9
- William, D. (2011). What is assessment for learning? *Studies in Educational Evaluation*, 37(1), 3–14. <https://doi.org/10.1016/j.stueduc.2011.03.001>
- William, D. (2018). Assessment for learning: meeting the challenge of implementation. *Assessment in Education: Principles, Policy & Practice*, 25(6). <https://doi.org/10.1080/0969594x.2017.1401526>
- William, D., & Thompson, M. (2008). Integrating assessment with learning: what will it take to make it work? *Future of Assessment: Shaping Teaching and Learning*, 1–41.
- Workneh, A., & Tassew, W. W. (2013). Teacher training and development in Ethiopia: improving education quality by developing teacher skills, attitudes and work conditions. In *Young Lives* (Issue October). www.younglives.org.uk
- Wright, D., Clark, J., & Tiplady, L. (2018). *Designing for Formative Assessment: A Toolkit for Teachers*. https://doi.org/10.1007/978-3-319-73748-5_14
- Wylie, E. C., & Lyon, C. J. (2015). The fidelity of formative assessment implementation: issues of breadth and quality. *Assessment in Education: Principles, Policy and Practice*, 22(1), 140–160. <https://doi.org/10.1080/0969594X.2014.990416>
- Xiao, Y. (2017). Formative Assessment in a Test-Dominated Context: How Test Practice

- Can Become More Productive. *Language Assessment Quarterly*, 14(4).
<https://doi.org/10.1080/15434303.2017.1347789>
- Xu, Y., & Brown, G. T. L. (2016). Teacher assessment literacy in practice: A reconceptualization. *Teaching and Teacher Education*, 58.
<https://doi.org/10.1016/j.tate.2016.05.010>
- Yan, Z., & Cheng, E. C. K. (2015). Primary teachers' attitudes, intentions and practices regarding formative assessment. *Teaching and Teacher Education*, 45.
<https://doi.org/10.1016/j.tate.2014.10.002>
- Yan, Z., Li, Z., Panadero, E., Yang, M., Yang, L., & Lao, H. (2021). A systematic review on factors influencing teachers' intentions and implementations regarding formative assessment. *Assessment in Education: Principles, Policy and Practice*, 00(00), 1–33.
<https://doi.org/10.1080/0969594X.2021.1884042>
- Yiheyis, S. B., & Getachew, S. W. (2014). The Implementation of Continuous Assessment in Writing Classes the of Jimma College of Teachers Education. *Ethiopian Journal of Education and Sciences*, 10(1), 109–135.
- Yin, Y., Shavelson, R. J., Ayala, C. C., Ruiz-Primo, M. A., Brandon, P. R., Furtak, E. M., Tomita, M. K., & Young, D. B. (2008). On the impact of formative assessment on student motivation, achievement, and conceptual change. *Applied Measurement in Education*, 21(4). <https://doi.org/10.1080/08957340802347845>
- Zepeda, S. J. (2013). *Professional Development : What Works* (Second Edi). Routledge.
- Zhang, L., Carter, R. A., Zhang, J., Hunt, T. L., Emerling, C. R., Yang, S., & Xu, F. (2021).

Teacher perceptions of effective professional development: insights for design.
Professional Development in Education, 00(00), 1–14.
<https://doi.org/10.1080/19415257.2021.1879236>

Zimmerman, B. J. (2000). Attaining self-regulation: A social cognitive perspective. In
Handbook of Self-Regulation (pp. 13–39).

Zimmerman, B. J. (2015). Self-Regulated Learning: Theories, Measures, and Outcomes.
In *International Encyclopedia of the Social & Behavioral Sciences: Second Edition*.
<https://doi.org/10.1016/B978-0-08-097086-8.26060-1>

Appendices

Appendix I Interview Questions for Mathematics Teacher Educators on their Conception of Assessment for Learning

1. What do you think is assessment for learning?
2. Who do you think should perform assessment for learning?
3. When do you think should assessment for learning be implemented?
4. How often do you think should assessment for learning be implemented?
5. Do you think that students are able to self-assess themselves without getting a training?
6. Do you think that students are able to assess their peers without getting a training?
7. What challenges do you face in implementing assessment for learning in Mathematics courses?
8. What do you think is the Relationship between Assessment for learning and classroom Instruction?
9. What do you think should be done and should not be done during the implementation of assessment for learning?
10. What different strategies do you think can be used in implementing assessment for learning in College of Teacher Education?
11. Can you tell me some of the principles of assessment for learning?
12. What are the benefits of implementations of assessment for learning in your opinion? For students, for lecturers, for CTEs if any.
13. How important do you think Questioning and Feedback are in the teaching learning of Algebra?
14. What do you think are the characteristics of quality questioning and Feedback?
15. What else can you tell me about the nature, principles, strategies and purposes of assessment for learning?
16. Have you ever got a training on Assessment for Learning?

Appendix II Interview questions to Mathematics Teacher Educators on how they experience the professional development

1. How much important do you find the professional development/training/ about assessment for learning?
2. Does it help you improve your understanding/conception/ of assessment for leaning? If yes how?
3. Does it help you improve implementation of assessment for leaning in your algebra course? If yes how?
4. What do you think should be done to improve implementation of assessment for learning in teacher education colleges?

Appendix III Classroom observation protocol of the implementation of the five Assessment for Learning strategies

Classroom Observation Instrument for Assessment for Learning

Teacher's Name: _____ Observation date: _____

Observer: _____ Period of Observation: _____

1= No evidence of use 2 = Superficial or ineffective use 3 = Minimal use or uncertain effectiveness 4 = Frequent Use or Effective and 5 = Pervasive Use or Highly Effective

A. Learning Targets: Clarifying Learning Intentions and Sharing Criteria for Success					
	1	2	3	4	5
1. Does the teacher make certain that students understand the learning intentions for the class session?					
2. Does the teacher make certain that students understand the learning intentions for each activity?					
3. Does the teacher provide examples of high and low quality work?					
4. Does the teacher address potential misunderstandings regarding the criteria for success?					
B. Monitoring: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of Learning					
	1	2	3	4	5
1. Does the teacher make efforts to monitor student learning on an ongoing basis (i.e., minute-to-minute & day-to-day)?					
2. Does the teacher give students a variety of opportunities and methods (e.g., verbal, written, electronic, & visual) to respond to questions?					
3. Does the teacher use effective questioning strategies (e.g., adequate wait time, open-ended questions) to elicit evidence of learning?					
4. Does teacher monitoring seek to elicit evidence from students of both factual/procedural knowledge and of deeper conceptual knowledge?					
5. Does teacher monitoring seek to elicit evidence of whether students can transfer knowledge within and between disciplines/subjects?					
C. Feedback: Providing Feedback That Moves Learners Forward					
	1	2	3	4	5
1. Does the teacher provide meaningful feedback (i.e., information with which a learner can confirm, add to, overwrite, tune, or					

restructure understanding) immediately following formal and/or informal evaluations of student progress?					
2. Does the teacher provide accurate feedback that assists learning?					
3. Does the teacher provide feedback in reference to a criterion based standard, avoiding feedback based in comparison to other students?					
4. Does feedback describe specific areas of needed improvement and suggest alternative strategies for making that improvement?					
5. Does feedback describe specific student strengths and suggest strategies for continued learning in those areas?					
<i>D. Self-Assessment: Activating Students as the Owners of Their Own Learning</i>					
	1	2	3	4	5
1. Does the teacher give students opportunities to use self-regulatory competencies, such as the ability to accurately assess their own knowledge?					
2. Does the teacher make efforts to develop self-monitoring competencies in students (i.e., meta-cognitive skills)?					
3. Are students making decisions related to their own improvement on the basis of ongoing assessment data (i.e., ownership of learning)?					
<i>E. Peer Assessment: Activating Students as Instructional Resources for One Another</i>					
	1	2	3	4	5
1. Does the teacher give students opportunities (e.g., discussions, questions, learning tasks) to engage in peer-monitoring?					
2. Does the teacher utilize the results of peer activities to strengthen ongoing assessment of student learning?					
3. Does the teacher utilize peer activities to help students deepen their understanding of common errors and alternative strategies?					

Appendix IV Achievement test for the calculation of pre-service primary mathematics teachers' individual achievement gain

This is an achievement test for Fundamental concept of Algebra (Math 221) course. You are kindly requested to answer the questions accordingly. The result will be used for research purpose only.

Name: _____ **ID No.:** _____ **College:** _____

I. For each of the questions choose the correct answer from the given alternatives and put the letter of your choice on the space provided.

_____1. Which of the following compound proposition is true if the truth values of p , q and r are *false*, *true* and *false* respectively?

- A) $(\neg p \wedge r) \Leftrightarrow q$ B) $(p \Rightarrow q) \Rightarrow r$ C) $\neg p \Leftrightarrow \neg q$ D) $\neg r \Leftrightarrow q$

_____2. Which of the following pairs of compound propositions are equivalent?

- A) $(\neg p \wedge q); \neg(p \wedge \neg q)$ B) $\neg(p \Rightarrow \neg q); p \wedge q$
C) $p \Leftrightarrow \neg q; \neg p \Leftrightarrow \neg q$ D) $\neg p \Rightarrow q; q \Rightarrow \neg p$

_____3. Which of the following compound propositions is a tautology?

- A) $(p \wedge \neg q) \wedge q$ B) $p \Rightarrow \neg p$ C) $(\neg p \vee q) \vee p$ D) $(p \wedge q) \Leftrightarrow (p \vee q)$

_____4. Which of the following propositions is *false* if the universe is $\mathbb{R}$?

- A) $(\forall x)(x \leq x^3)$ B) $(\exists x)(2x \geq 4x)$ C) $(\exists x)(x^4 < 1)$ D)
 $(\forall x)(1 > -x^2)$

_____5. Which of the following argument is valid?

- A) $p, q \Rightarrow \neg p \vdash q$ B) $p \vee q \vdash p \wedge q$ C) $p \wedge q \vdash p \Rightarrow q$ D) $p \Rightarrow q \vdash p \wedge q$

_____6. If $A \subseteq B$ and $B \subset C$ where A , B and C are sets which of the following relations is *false*?

- A) $C \subseteq A \cup B$ B) $A \cup B \subset C$ C) $A \cap B = A$ D) $A \cup B = B$

_____7. Which of the following relations is true for any sets A, B if $A \subset B$?

- A) $A \triangle B = \emptyset$ B) $B - (B - A) = A$ C) $A \triangle B = A - B$ D) $A \cap (A \triangle B) = A$

_____8. Which of the following relations is an equivalence relation on set $A = \{2,3,4,5\}$?

- A) $R = \{(2,2), (2,3), (3,3), (3,4), (4,4), (5,5)\}$ B) $R = \{(2,2), (2,3), (3,2), (3,3), (4,4), (5,5)\}$ C) $R = \{(2,2), (3,3), (4,4), (4,5), (5,3), (5,5)\}$ D) None

_____9. Which of the following sets **is not** a partition of the set $\{1,2,3,4,5,6,7,8,9,10\}$?

- A) $\{\{1,2,3\}, \{3,4,5,6\}, \{7,8,9,10\}\}$ B) $\{\{1,10\}, \{2,9\}, \{3,8\}, \{4,7\}, \{5,6\}\}$
C) $\{\{1\}, \{2,3,5,7\}, \{4,6,8,9,10\}\}$ D) $\{\{1,3,5,7,9\}, \{2,4,6,8,10\}\}$

_____10. Which of the following total orderings is a well ordering, where $\leq$ is the usual less than or equal to relation?

- A) $(\mathbb{R}, \leq)$ B) $(\mathbb{Q}, \leq)$ C) $(\mathbb{N}, \leq)$ D) $(\mathbb{Z}, \leq)$

_____11. If $f(x) = 3x + 1$ and $g(x) = x^2 - 2x$ then which one of the following is $g \circ f$?

- A) $9x^2 - 1$ B) $x^2 + x + 1$ C) $9x^2 + 3$ D) $9x^2 + 12x + 3$

_____12. Which of the following functions is the inverse of $(x) = x^3 - 4$?

- A) $g(x) = \sqrt[3]{x+4}$ B) $g(x) = \sqrt[3]{x-4}$ C) $g(x) = x^3 + 4$ D) $g(x) = \frac{x+4}{3}$

_____13. Which of the following is the identity element of the group $(\mathbb{Q}, *)$ where $a * b = \frac{3ab}{4}$?

- A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) 1 D) 0

_____14. Which one of the following homomorphisms is not an Isomorphism?

- A) $f: (\mathbb{R}, +) \rightarrow (\mathbb{R}^+, \times)$, defined by $f(x) = 2^x$ B) $f: (\mathbb{Z}, +) \rightarrow (5\mathbb{Z}, +)$ defined by $f(x) = 5x$ C) $f: (\mathbb{R}, +) \rightarrow (\mathbb{R}^+, \times)$, defined by $f(x) = 3^{-x}$ D) $f: (\mathbb{Z}, +) \rightarrow (\mathbb{R}, +)$ defined by $f(x) = x$

_____15. Which of the following sets with the usual multiplication operation, $\bullet$, is a group?

- A) $(\mathbb{R}^-, \bullet)$ B) $(\mathbb{Z}^+, \bullet)$ C) $(\mathbb{Z}, \bullet)$ D) $(\mathbb{Q}^+, \bullet)$

_____16. Which of the following rings is an integral domain?

- A) $(\mathbb{Z}_6, \oplus_6, \odot_6)$ B) $(\mathbb{Z}_7, \oplus_7, \odot_7)$ C) $(\mathbb{Z}_8, \oplus_8, \odot_8)$ D) $(\mathbb{Z}_9, \oplus_9, \odot_9)$

_____17. Which of the following is **not true** for any integers x, y and z ?

- A) $-(x - y) = x + y$ B) $x > y \wedge z < 0 \Rightarrow xz < yz$
C) $x(-y) = -(xy)$ D) $(-x)(-y) = xy$

_____18. Which of the following is the formula for the sum: $3 + 5 + 7 + \dots + (2n + 1), \forall n \in \mathbb{N}$.

- A) $(2n + 1)^2$ B) $2n^2 + 2n$ C) $n^2 + 2n$ D) $n^2 - 2n$

_____19. Which of the following integral domains are Ordered Integral domains?

- A) $(\mathbb{Z}_{17}, \oplus_{17}, \odot_{17})$ B) $(\mathbb{Z}_5, \oplus_5, \odot_5)$ C) $(\mathbb{Z}_{11}, \oplus_{11}, \odot_{11})$ D) $(3\mathbb{Z}, +, \bullet)$

_____20. Which of the following numbers are equal to 1200 in base ten?

- A) $(5322)_{\text{six}}$ B) $(3335)_{\text{seven}}$ C) $(2260)_{\text{eight}}$ D) $(1571)_{\text{nine}}$

_____21. Which of the following is the LCM of 21, 18, 15?

- A) 210 B) 3 C) 5670 D) 630

_____22. If the GCD of a and $b = (a, b) = p$, a prime, what is the values of (a^5, b^3) ?

- A) p B) p^2 C) p^3 D) p^5

Appendix V Conceptions of Assessment for learning practices Questionnaire

Assessment for Learning Questionnaire for Students (SAFL-Q)

This Questionnaire is intended to study conception of assessment for learning of pre-service primary teachers and it does not have any positive or negative effect on your grades in any of the courses you are taking. The information you give will be used for research purpose only and its aim is in improving teacher education programs. There is no single correct answer for any of the statements and circle the scale that best describes your opinion.

Name: _____ ID: _____ Year: 1st 2nd 3rd

Major: _____ Dept.: _____ College: _____

1= I strongly disagree, 2 = disagree, 3 = Undecided, 4 = agree and 5 = Strongly agree

My teacher encourages me to reflect on how I can improve my assignments	1	2	3	4	5
After examining my test results, my teacher discusses the answers I gave to the test with me	1	2	3	4	5
Whilst working on my assignments, my teacher asks me how I think I am doing	1	2	3	4	5
My teachers allow me to think about what I want to learn in school	1	2	3	4	5
My teacher gives me the opportunity to decide on my own learning objectives	1	2	3	4	5
My teachers inquire what went well and what went badly in my work	1	2	3	4	5
My teacher encourages me to reflect on my learning process and to think about how to improve next time	1	2	3	4	5
My teacher stresses my strengths concerning learning	1	2	3	4	5
My teacher identifies my weaknesses concerning learning	1	2	3	4	5
I am encouraged by my teacher to improve my learning process	1	2	3	4	5
My teacher gives me guidance to assist my learning	1	2	3	4	5
My teacher discusses assignments with me to help me understand the subject matter better	1	2	3	4	5
My teacher discusses with me the progress I make	1	2	3	4	5
After each assessment my teacher informs me how to improve the next time	1	2	3	4	5
My teacher discusses with me how to exploit my strengths to improve my assignment	1	2	3	4	5

My teacher and I consider ways to improve my weak points	1	2	3	4	5
When I do not understand a topic, my teacher tries to explain it in a different way	1	2	3	4	5
My teacher provides me with hints to help understand the subject matter	1	2	3	4	5
During class I have an opportunity to show what I have learned	1	2	3	4	5
My teacher asks questions in a way I understand	1	2	3	4	5
My teacher asks questions that help me gain understanding of the subject matter	1	2	3	4	5
My teacher allows for my contribution during the lesson	1	2	3	4	5
I have the opportunity to ask my classmates questions during the lesson	1	2	3	4	5
My teacher makes me aware of the areas I need to work on to improve my results	1	2	3	4	5
There is an opportunity to ask questions during the lesson	1	2	3	4	5
I am aware of the criteria by which my assignment will be evaluated	1	2	3	4	5
When I receive an assignment it is clear to me what I can learn from it	1	2	3	4	5
My assignments allow me to show what I am capable of	1	2	3	4	5

(Amharic version)

የመማር ምዘና ፅንሰ-ሀሳብ መጠይቅ (Assessment for learning Questionnaire)

ይህ መጠይቅ የወደፊት አንደኛ ደረጃ መምህራንን ለመማር ምዘና ትግበራ ፅንሰ-ሀሳብን (Conception of assessment for learning practices) ለማጥናት የታቀደ ሲሆን በምትወስዳቸው/ጃቸው/ በማንኛውም ኮርሶች ውጤትህ/ሽ/ ላይ ምንም አይነት አዎንታዊም ሆነ አሉታዊ ተጽዕኖ የለውም :: የምትሰጠው/ጭው/ መረጃ ለምርምር ዓላማ ብቻ የሚያገለግል ሲሆን ዓላማውም የመምህራን ትምህርት ፕሮግራሞችን ለማሻሻል ነው :: ለተዘረዘሩት እያንዳንዱ መግለጫዎች አንድ ትክክለኛ መልስ ስለሌለ ያንተን/ያንችን/ አስተያየት በተሻለ የሚገልጸው ደረጃ ላይ አክብብ/አክብቢ/:: ፈቃደኝነትክን/ሽን/ ለማርጋገጥ እባክህ/ሽ/ ፈርምልኝ/ሚልኝ/:: ፊርማ: _____ :: ስም _____ የመታወቂያ ቁጥር: _____ ዓመት: 1ኛ 2ኛ 3ኛ ሜጀር: _____ ት/ክፍል: _____ ኮሌጅ _____

ገላጭ ዓረፍተ ነገር	መመዘኛ ስኬል				
የተሰጠኝን ሥራ እንዴት ማሻሻል እንደምችል ጽብረቃ እንዳደርግ አስተማሪዬ ያበረታታኛል	1	2	3	4	5
መምህራ የፈተና ውጤቶቼን ከመረመረ በኋላ ለፈተናው የሰጠኝቸውን መልሶች ከእኔ ጋር ይወያያል	1	2	3	4	5
የተሰጠኝን ስራ እየሠራሁ እያለ አስተማሪዬ እንዴት እየሠራሁት ነው ብዬ እንደማስብ ይጠይቀኛል	1	2	3	4	5
አስተማሪዎቼ መማር ስለምፈልገው ነገር እንዳስብ እድሉን ይሰጡኛል	1	2	3	4	5
አስተማሪዬ የትምህርት ዓላማዎቼ ላይ እንድወስን እድል ይሰጠኛል	1	2	3	4	5
አስተማሪዬ በስራዬ ውስጥ በጥሩ ሁኔታ የተከናወነውን እና በጥሩ ሁኔታ ያልተከናወነውን ይጠይቃል	1	2	3	4	5
አስተማሪዬ በትምህርቴ ሂደት ላይ ጽብረቃ እንዳደርግና እንዴት እንደማሻሻል እንዳስብ ያበረታታኛል	1	2	3	4	5
አስተማሪዬ መማርን በተመለከተ ጠንካራ ጎኖቼን አፅንዖት ይሰጣል	1	2	3	4	5
አስተማሪዬ መማሪያን በተመለከተ ድክመቶቼን ይለያል	1	2	3	4	5
የመማር ሂደቴን ለማሻሻል አስተማሪዬ ያበረታታኛል	1	2	3	4	5
አስተማሪዬ የመማር ሂደቴን እንዳሻሽል አመራር ይሰጠኛል	1	2	3	4	5
አስተማሪዬ የምረጠውን ትምህርት በተሻለ ለመረዳት እንድችል በተሰጠኝ ስራ ዙሪያ ከእኔ ጋር ይወያያል	1	2	3	4	5
አስተማሪዬ በትምህርቴ ላይ ስለማደርገው መሻሻል ከእኔ ጋር ይወያያል	1	2	3	4	5
ከእያንዳንዱ ምዘና በኋላ አስተማሪዬ በሚቀጥለው ጊዜ እንዴት ማሻሻል እንደምችል ያሳውቀኛል	1	2	3	4	5
የተሰጠኝን ስራ ለማሻሻል የእኔን ጥንካሬዎች እንዴት እንደምጠቀም አስተማሪዬ ከእኔ ጋር ይወያያል	1	2	3	4	5
እኔ እና አስተማሪዬ ደካማ ነጥቦቼን ለማሻሻል የተለያዩ መንገዶችን እንመለከታለን	1	2	3	4	5

አንድ ርዕሰ ባለገባች ጊዜ አስተማሪዬ በሌላ መንገድ ለማብራራት ይሞክራል	1	2	3	4	5
አስተማሪዬ የምማረውን ርዕሰ ጉዳይ ለመረዳት የሚያስችለኝ ፍንጭ ይሰጠኛል	1	2	3	4	5
በክፍል ውስጥ በምንማርበት ወቅት የተማርኩትን ለማሳየት እድል አገኛለሁ	1	2	3	4	5
አስተማሪዬ እኔ ልረዳው በምችል መንገድ ጥያቄዎችን ይጠይቃል	1	2	3	4	5
አስተማሪዬ ስለ ርዕሰ ጉዳዩ ግንዛቤ እንዳገኝ የሚረዱኝ ጥያቄዎችን ይጠይቃል	1	2	3	4	5
ክፍል ውስጥ በምንማርበት ጊዜ አስተማሪዬ አስተዋፅዖ እንዳደርግ እድሉን ይሰጠኛል	1	2	3	4	5
በክፍል ውስጥ ስንማር የክፍል ጓደኞቼን ጥያቄ የመጠየቅ እድሉ አለኝ	1	2	3	4	5
ውጤቴን ለማሻሻል የት አካባቢ ላይ መሥራት እንዳለብኝ አስተማሪዬ እንዳውቅ ያደርገኛል	1	2	3	4	5
በክፍል ውስጥ ስንማር ጥያቄዎችን የመጠየቅ እድሉ አለ	1	2	3	4	5
የተሰጠኝ ስራ የሚገመገምበትን መስፈርቶች አውቃቸዋለሁ	1	2	3	4	5
የተሰጡ ስራዎችን ስወስድ ምን መማር እንደምችል ለእኔ ግልፅ ነው	1	2	3	4	5
የተሰጡኝ ስራዎች እኔ ምን ልሰራ እንደምችል ለማሳየት እድሉን ይሰጡኛል	1	2	3	4	5

1 = በጣም አልስማማም ፣ 2 = አልስማማም ፣ 3 = አልወሰንኩም ፣ 4 = እስማማለሁ እና 5 = በጣም እስማማለሁ

Appendix VI Motivated Strategies for Learning Questionnaire Self-Regulated Learning Questionnaire for Pre-service Primary Mathematics Teachers

Your College is participating in a study of college teaching and learning. I would like to ask for your participation in the study. YOUR PARTICIPATION IS VOLUNTARY AND NOT RELATED IN ANY WAY TO YOUR GRADES. All your responses are strictly confidential and only the researcher will see your individual responses. The attached questionnaire asks you about your study habits and learning skills for work in this course. THERE ARE NO RIGHT OR WRONG ANSWERS TO THIS QUESTIONNAIRE. THIS IS NOT A TEST. I want you to respond to the questionnaire by circling the number you choose in the scale as accurately as possible, reflecting your own attitudes and behaviors in this course. Please sign below if you would like to be involved in this study. Thank you for your cooperation. Signature:_____.

Name: _____, ID_____. Gender: Male Female

Class level: 1st year 2nd year 3rd year

1= Not at all true of me 2 3 4 5 6 7= Very true of me

Cognitive and Metacognitive Strategies: Rehearsal							
When I study for this class, I practice saying the material to myself over and over.	1	2	3	4	5	6	7
When studying for this class, I read my class notes and the course readings over and over again.	1	2	3	4	5	6	7
I memorize key words to remind me of important concepts in this class.	1	2	3	4	5	6	7
I make lists of important terms for this course and memorize the lists.	1	2	3	4	5	6	7
Cognitive and Metacognitive Strategies: Elaboration							
When I study for this class, I pull together information from different sources, such as lectures, readings, and discussions.	1	2	3	4	5	6	7
I try to relate ideas in this subject to those in other courses whenever possible.	1	2	3	4	5	6	7

When reading for this class, I try to relate the material to what I already know.	1	2	3	4	5	6	7
When I study for this course, I write brief summaries of the main ideas from the readings and the concepts from the lectures.	1	2	3	4	5	6	7
I try to understand the material in this class by making connections between the readings and the concepts from the lectures.	1	2	3	4	5	6	7
I try to apply ideas from course readings in other class activities such as lecture and discussion.	1	2	3	4	5	6	7

Cognitive and Metacognitive Strategies: Organization	Scale						
When I study the readings for this course, I outline the material to help me organize my thoughts.	1	2	3	4	5	6	7
When I study for this course, I go through the readings and my class notes and try to find the most important ideas.	1	2	3	4	5	6	7
I make simple charts, diagrams, or tables to help me organize course material.	1	2	3	4	5	6	7
When I study for this course, I go over my class notes and make an outline of important concepts.	1	2	3	4	5	6	7
Cognitive and Metacognitive Strategies: Critical Thinking	Scale						
I often find myself questioning things I hear or read in this course to decide if I find them convincing.	1	2	3	4	5	6	7
When a theory, interpretation, or conclusion is presented in class or in the readings, I try to decide if there is good supporting evidence.	1	2	3	4	5	6	7
I treat the course material as a starting point and try to develop my own ideas about it.	1	2	3	4	5	6	7
I try to play around with ideas of my own related to what I am learning in this course.	1	2	3	4	5	6	7
Whenever I read or hear an assertion or conclusion in this class, I think about possible alternatives.	1	2	3	4	5	6	7
Cognitive and Metacognitive Strategies: Metacognitive Self-Regulation	Scale						
During class time I often miss important points because I'm thinking of other things. (REVERSED)	1	2	3	4	5	6	7
When reading for this course, I make up questions to help focus my reading.	1	2	3	4	5	6	7
When I become confused about something I'm reading for this class, I go back and try to figure it out.	1	2	3	4	5	6	7
If course materials are difficult to understand, I change the way I read the material.	1	2	3	4	5	6	7
Before I study new course material thoroughly, I often skim it to see how it is organized.	1	2	3	4	5	6	7
I ask myself questions to make sure I understand the material I have been studying in this class.	1	2	3	4	5	6	7
I try to change the way I study in order to fit the course requirements and instructor's teaching style.	1	2	3	4	5	6	7
I often find that I have been reading for class but don't know what it was all about. (REVERSED)	1	2	3	4	5	6	7
I try to think through a topic and decide what I am supposed to learn from it rather than just reading it over when studying.	1	2	3	4	5	6	7
When studying for this course I try to determine which concepts I don't understand well.	1	2	3	4	5	6	7
When I study for this class, I set goals for myself in order to direct my activities in each study period.	1	2	3	4	5	6	7
If I get confused taking notes in class, I make sure I sort it out afterwards.	1	2	3	4	5	6	7
Resource Management Strategies: Time and Study Environment	Scale						

I usually study in a place where I can concentrate on my course work.	1	2	3	4	5	6	7
I make good use of my study time for this course.	1	2	3	4	5	6	7
I find it hard to stick to a study schedule. (REVERSED)	1	2	3	4	5	6	7
I have a regular place set aside for studying.	1	2	3	4	5	6	7
I make sure I keep up with the weekly readings and assignments for this course.	1	2	3	4	5	6	7
I attend class regularly.	1	2	3	4	5	6	7
I often find that I don't spend very much time on this course because of other activities. (REVERSED)	1	2	3	4	5	6	7
I rarely find time to review my notes or readings before an exam. (REVERSED)	1	2	3	4	5	6	7
Resource Management Strategies: Effort Regulation	Scale						
I often feel so lazy or bored when I study for this class that I quit before I finish what I planned to do. (REVERSED)	1	2	3	4	5	6	7
I work hard to do well in this class even if I don't like what we are doing.	1	2	3	4	5	6	7
When course work is difficult, I give up or only study the easy parts. (REVERSED)	1	2	3	4	5	6	7
Even when course materials are dull and uninteresting, I manage to keep working until I finish.	1	2	3	4	5	6	7
Resource Management: Peer Learning	Scale						
When studying for this course, I often try to explain the material to a classmate or a friend.	1	2	3	4	5	6	7
I try to work with other students from this class to complete the course assignments.	1	2	3	4	5	6	7
When studying for this course, I often set aside time to discuss the course material with a group of students from the class.	1	2	3	4	5	6	7
Resource Management: Help Seeking	Scale						
Even if I have trouble learning the material in this class, I try to do the work on my own, without help from anyone. (REVERSED)	1	2	3	4	5	6	7
I ask the instructor to clarify concepts I don't understand well.	1	2	3	4	5	6	7
When I can't understand the material in this course, I ask another student in this class for help.	1	2	3	4	5	6	7
I try to identify students in this class whom I can ask for help if necessary.	1	2	3	4	5	6	7

Source: Pintrich, P. R., Smith, D. A., Garcia, T., & McKeachie, W. J. (1991). A manual for the use of the Motivated Strategies for Learning Questionnaire (MSLQ) (National Center for Research to Improve Postsecondary Teaching and Learning). Ann Arbor, MI: University of Michigan.

(Amharic Version)

በራስ-ቁጥጥር የሚደረግበት የመማር ችሎታ መጠይቅ

ኮሌጅህ/ሽ/ በኮሌጅ መማር እና ትምህርት ጥናት ላይ እየተሳተፈ ነው ። በጥናቱ ውስጥ ያንተን/ችን/ ተሳትፎ መጠየቅ እፈልጋለሁ ። ተሳትፎህ/ሽ/ በፈቃደኝነት ላይ የተመሰረተ ሲሆን በዚህ ኮርስ ውጤትህ/ሽ/ ላይ ምንም አሉታዊም ሆነ አወንታዊ ተጽዕኖ የለውም። ሁሉንም ምላሾችህን/ሽን/ ከተመራማሪው በስተቀር ማንም ሰው አያያቸውም ፤ ሚስጥራዊነታቸውም የተጠበቀ ነው ። የተያያዘው መጠይቅ በዚህ ኮርስ ውስጥ ስለ ትምህርት ልምዶችህ/ሽ/ እና ስለ የመማር ችሎታህ/ሽ/ ይጠይቃል ። ለነዚህ ገላጭ ዓረፍተነገሮች ትክክለኛ ወይም የተሳሳተ መልስ የለም። በዚህ ኮርስ ውስጥ የራስህን/ሽን/ ትክክለኛ አመለካከቶች እና ባህሪዎች በማንፀባረቅ በመለኪያው የመረጥከውን/ሽውን/ ቁጥር በተቻለ መጠን በትክክል በማክበብ ለጥያቄው መልስ እንድትሰጥ/ጩ/ እፈልጋለሁ ። በዚህ ጥናት ውስጥ ለመሳተፍ ሙሉ ፈቃደኛ መሆንክን/ሽን/ ለመግለጽ እባክህ/ሽ/ ከዚህ በታች ካለ ቦታ ላይ ፈርም/ፈርሚ/። ለትብብርህ/ሽ/ በጣም አመሰግናለሁ። ፈርማ _____ ።

ስም _____ ፣ የመታወቂያ ቁጥር: _____ ፣ ፆታ: ወንድ ☐ ሴት ☐

ዓመት : 1 ኛ ዓመት ☐ 2 ኛ ዓመት ☐ 3 ኛ ዓመት ☐

1 = በጭራሽ ለእኔ እውነት አይደለም 2 = ለእኔ በጣም እውነት አይደለም 3 = ለእኔ እውነት አይደለም
4 = አልወሰንኩም 5 = ለእኔ እውነት ነው 6 = ለእኔ በጣም እውነት ነው 7 = ለእኔ እጅግ በጣም እውነት ነው

ገላጭ ዓረፍተነገር	መለኪያ				
ይሄን ኮርስ ሳጠና የትምህርቱን ይዘቶች ለራሴ እየደጋገምኩ በመናገር እለማመዳለሁ	1	2	3	4	5
ይሄን ኮርስ ሳጠና የክፍል ማስታወሻዎቼን እና የኮርሱን ሞጁል ደጋግሜ አነበባለሁ ።	1	2	3	4	5
በዚህ ክፍል ውስጥ አስፈላጊ ፅንሰ-ሀሳቦችን ለማስታወስ ቁልፍ ቃላትን በቃል እሸምዳለሁ ።	1	2	3	4	5
ለዚህ ኮርስ አስፈላጊ ቃላቶችን ዝርዝር አወጣ እና ዝርዝሮቹን እሸመድዳለሁ	1	2	3	4	5
ይሄን ኮርስ ሳጠና ከተለያዩ ምንጮች ማለትም እንደ የገለጻ ማስታወሻዎች ፣ ንባቦች እና ወይይቶች ያሉ መረጃዎችን በአንድ ላይ አሰባስባለሁ ።	1	2	3	4	5
በሚቻልበት ጊዜ በዚህ ኮርስ ትምህርት ውስጥ ያሉ ሀሳቦችን ከሌሎች ኮርሶች ውስጥ ካሉ ጋር ለማዛመድ እሞክራለሁ ።	1	2	3	4	5
ይህን ኮርስ በማነብበት ጊዜ የማንበበውን ቀድሞውኑ ከማውቀው ጋር ለማዛመድ እሞክራለሁ።	1	2	3	4	5
ይህን ኮርስ ሳጠና ከማነብባቸው ውስጥ የዋና ዋና ሀሳቦችንና ከገለጻዎች ውስጥ ያሉ ፅንሰ-ሀሳቦችን አጠር ያለ ማጠቃለያ እፅፋለሁ ።	1	2	3	4	5
በማነባቸውና በመምህሩ ንግግሮች ውስጥ ካሉ ፅንሰ-ሀሳቦች መካከል ግንኙነቶችን በመፍጠር የኮርሱን ሞጁል ሀሳቦች ለመረዳት እሞክራለሁ ።	1	2	3	4	5
ኮርሱን በማነብበት ወቅት ያገኘኋቸውን ሀሳቦች እንደ ገለጻና እና ወይይት ባሉ ሌሎች የክፍል ተግባራት ውስጥ ለመተግበር እሞክራለሁ ።	1	2	3	4	5
የዚህን ኮርስ ንባቦች ሳጠና ሀሳቦቼን ለማድራጀት የሚረዱኝን ኮርሱን ዋናዋና ሀሳቦች እዘረዝራለሁ	1	2	3	4	5
ይህን ኮርስ ሳጠና ባነበብኳቸውና በክፍል ማስታወሻዎቼ ውስጥ እገባና በጣም አስፈላጊ ሀሳቦችን ለማግኘት እሞክራለሁ ።	1	2	3	4	5
የኮርስ ትምህርቱን ለማድራጀት የሚረዱኝን ቀላል ገበታዎችን ፣ ስዕላዊ መግለጫዎችን ወይም ጠረጴዛዎችን እሠራለሁ ።	1	2	3	4	5
ይህን ኮርስ ሳጠና የክፍል ማስታወሻዎቼን ደጋግሜ አይና አስፈላጊ የሆኑ ፅንሰ ሀሳቦችን እዘረዝራለሁ	1	2	3	4	5
አሳማኝ መሆናቸውን ለመወሰን ብዙውን ጊዜ በዚህ ኮርስ ውስጥ የሰማሁትን ወይም የማነባቸውን ነገሮች እጠይቃለሁ ።	1	2	3	4	5

በዚህ ክፍል ውስጥ አንድ ማረጋገጫ ወይም መደምደሚያ ባነበብኩ ወይም በሰማሁ ቁጥር ፣ ሊሆኑ ስለሚችሉ አማራጮች አስባለሁ ።	1	2	3	4	5
በክፍል ትምህርት ጊዜ ሌሎች ሃሳቦችን ስለማስብ ብዙ ጊዜ አስፈላጊ ነጥቦችን ሳልረዳ እቀራለሁ ።	1	2	3	4	5
ይህን ኮርስ በማነበብበት ጊዜ ንባቤን ለማተኮር የሚረዱ ጥያቄዎችን አወጣለሁ።	1	2	3	4	5
ይህን ኮርስ ሳነበብ ባነበብኩት አንድ ነገር ግራ ሲገባኝ ወደ ኋላ ተመልሼ ለማወቅ እሞክራለሁ ።	1	2	3	4	5
የኮርሱን ሞጁሎችና መጻህፍት ለመረዳት አስቸጋሪ ሲሆኑብኝ ጽሑፉን የማነበበትን መንገድ አለውጣለሁ ።	1	2	3	4	5
አዲስ የኮርስ ሞጁሎችን በጥልቀት ከማጥናቴ በፊት ፣ እንዴት እንደተደራጀ ለማየት ብዙ ጊዜ አለፍ አለፍ እያልኩ አያቸዋለሁ።	1	2	3	4	5
በዚህ ክፍል ውስጥ ያጠናሁትን ሞጁል ወይም መጽሃፍ መረዳቴን ለማረጋገጥ እራሴን ጥያቄዎችን እጠይቃለሁ ።	1	2	3	4	5
ከኮርሱ ባህሪ እና ከአስተማሪው የማስተማር ዘዴ በጋር ለማጣጣም የማጠናበትን መንገድ ለማስተካከል እሞክራለሁ ።	1	2	3	4	5
ብዙ ጊዜ ለኮርሱ ክፍላጊዜ ሳነብ እቆይና ከጨረስኩ በኋላ ምን እያነበብኩ እንደነበረ አላውቅም	1	2	3	4	5
በኮርሱ ውስጥ ሚገኝን አንድ ርዕስ ደጋግሜ ከማነብ ይልቅ ስለርዕሱ በጥልቀት በማስብ ከርዕሱ ምን መማር እንደሚጠበቅብኝ አስቀድሜ ለመወሰን እሞክራለሁ።	1	2	3	4	5
ይህን ኮርስ በማጠናበት ጊዜ በደንብ ያልገቡኝ የትኞቹ ፅንሰ ሀሳቦች እንደሆኑ ለመለየት እሞክራለሁ።	1	2	3	4	5
ይህን ኮርስ በማጠናበት ጊዜ የጥናት እንቅስቃሴዎቼን ለመምራት ለራሴ ግብ እጥላለሁ።	1	2	3	4	5
በክፍል ውስጥ ስማር ማስታወሻዎችን ለመያዝ ግራ ከገባኝ ከወጣሁ በኋላ አንብቤ አስተካክለዋለሁ።	1	2	3	4	5
ብዙውን ጊዜ ትምህርቴን የማጠናው ጥናቴ ላይ ማተኮር የምችልበት ቦታ ላይ ነው	1	2	3	4	5
ለዚህ ኮርስ ያለኝን የጥናት ጊዜዬን በሚገባ እጠቀማለሁ ።	1	2	3	4	5
የጥናት መርሃግብራዊን ሳላስተጓጉል ማጥናት በጣም ይከብደኛል።	1	2	3	4	5
ለጥናት ብቻ የተቀመጠ መደበኛ ቦታ አለኝ።	1	2	3	4	5
ለዚህ ኮርስ ሳምንታዊ ንባቦችን ማንበቤንና የተሰጡ ስራዎችን እየሰራሁ መሆኔን አረጋግጣለሁ ።	1	2	3	4	5
የክፍል ትምህርቴን በመደበኛነት እከታተላለሁ ።	1	2	3	4	5
በሌሎች የተለያዩ ስራዎች ምክንያት ለዚህ ኮርስ ጥናት ብዙ ጊዜ እያጠፋሁ እንዳለሆነ ተረድቻለሁ ።	1	2	3	4	5
ከፊተና በፊት ማስታወሻዎቼን ወይም ንባቤን ለመከለስ ጊዜ አላገኝም።	1	2	3	4	5
ይህን ኮርስ ሳጠና ብዙውን ጊዜ የሰንፍና የመሰልቸት ስሜት ይሰማኝና ለመሰራት ያቀድኩትን ሳልጨርስ አቋርጠዋለሁ።	1	2	3	4	5
የምሰራውን ባልወደውም በክፍል ትምህርት ውስጥ ጥሩ ለመሰራት ጠንከሬ አሰራለሁ።	1	2	3	4	5
በዚህ ኮርስ የምሰራቸው ሥራዎች ክባድ በሚሆኑበት ጊዜ የምሰራውን እተወዋለሁ ወይም ቀለል ያሉ ርዕሶችን ብቻ አጠናለሁ ።	1	2	3	4	5
የኮርሱ ሞጁሎችና መጻህፍት አሰልፎና የማይሰቡ ቢሆኑም እንኳ እስከጨርስ ድረስ መሰራቴን እቀጥላለሁ ።	1	2	3	4	5
ይህን ኮርስ ሳጠና ብዙ ጊዜ የኮርሱን ሃሳቦች ለክፍል ጓደኞቼ ለማብራራት ጥረት አደርጋለሁ።	1	2	3	4	5
በዚህ ኮርስ የሚሰጡትን የሚሰሩ ስራዎች ለማጠናቀቅ ከሌሎች ተማሪዎች ጋር በመሆን አሰራለሁ።	1	2	3	4	5
ይህን ኮርስ ሳጠና ከክፍል ጓደኞቼ ጋር በኮርሱ ዙሪያ በቡድን ለመወያየት ጊዜ አመድባለሁ።	1	2	3	4	5
ምንም እንኳን በዚህ ኮርስ ውስጥ ያሉትን ሃሳቦች ለመረዳት ቢቸግረኝም ፣ ከማንም እርዳታ ሳላገኝ በራሴ ስራውን ለመስራት እሞክራለሁ ።	1	2	3	4	5
በደንብ ያልገቡኝን ፅንሰ-ሀሳቦች አስተማሪዬ እንዲያብራራኝ እጠይቀዋለሁ ።	1	2	3	4	5
በዚህ ኮርስ ውስጥ ያለውን ሃሳብ መረዳት ባልቻልኩ ጊዜ በክፍል ውስጥ ያለ ሌላ ተማሪ እንዲረዳኝ እጠይቃለሁ ።	1	2	3	4	5
በዚህ ክፍል ውስጥ አስፈላጊ በሆነ ጊዜ እንዲያግዘኝ የምጠይቃቸውን ተማሪዎች ለመለየት እሞክራለሁ ።	1	2	3	4	5
አንድ ንድፈ-ሀሳብ ፣ አተረጓጎም ወይም መደምደሚያ በክፍል ውስጥ ወይም በንባቡ ውስጥ ሲቀርብ ፣ ጥሩ ደጋፊ ማስረጃዎች ካሉ ለማወቅ እሞክራለሁ ።	1	2	3	4	5
የኮርሱን ሞጁል እንደ መነሻ አድርጌ በመያዝ በኮርሱ ዙሪያ የራሴን ሀሳቦች ለማዳበር እሞክራለሁ ።	1	2	3	4	5
በዚህ ኮርስ ውስጥ ከምማረው ጋር በሚዛመዱ የራሴ ሀሳቦች ጋር ለመጫወት እሞክራለሁ	1	2	3	4	5

Appendix VII Consent Letter for Mathematics teacher educators

INFORMED CONSENT LETTER

Please read this document carefully before you decide to participate in this study

Title of Research: A Professional Development on assessment for learning in Algebra and its effect on pre-service teachers' Achievements, Self-Regulated Learning skills and conceptions

Researcher Name: Ayanaw Yigletu, PhD candidate in Mathematics Education, College of education and Behavioral studies, Addis Ababa University

Supervisors: 1. Kassa Michael (Asso. Prof.) Department of science and Mathematics Education, College of education and Behavioral studies, Addis Ababa University

2. Prof. Mulugeta Atinafu, Department of science and Mathematics Education, College of education and Behavioral studies, Addis Ababa University

A. Information to Participants

1. **Purpose of the research:** The purpose of the research is to examine the effect of job-embedded, context-specific and content-based Professional Development on Assessment for Learning in Algebra to mathematics teacher educators and a training to pre-service primary mathematics teachers on the implementation of AfL, pre-service primary mathematics teachers' achievements in Fundamental Concepts of Algebra (Math 221) course, on their conceptions of AfL and their self-regulated learning skills.

2. **Research participation:** Participation in this study will involve: Participating in the professional development on assessment for learning, implementing the five strategies of Assessment for Learning in the teaching learning of Fundamental Concepts of Algebra (Math 221) course using the modified course module, being observed when teaching Fundamental Concepts of Algebra (Math 221) course and observing others' classes.

3. **Confidentiality:** The information gathered from this research will be kept confidential. Your identity will be coded, and all data will be kept in a secured, limited access location. Your identity will not be revealed in any publication or presentation of the results of this research.

4. **Risks and benefits:** There is no risk associated with participating in this research. This research is in no way connected to your efficiency. By participating, you are contributing to research to improve the practice of assessment for learning in Algebra to provide quality education.

5. **Right to withdraw:** participation in this study is completely voluntary. You can withdraw at any time with simple oral notice and can ask any question regarding this study.

6. **Payment:** There will be no compensation.

7. **Time Limit:** This study will take 12 weeks for data collection. Specifically, interview, and observation session may take approximately 30 minutes, 50 minutes respectively to complete.

8. **How will results be used:** The results of this research are only for educational purposes. The results will be shared with other researchers and participants through workshops and research symposiums.

B. Consent

I have read and discussed the above information with the researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study. The purpose, nature, risks and benefits of the study has been fully explained to me by Ayanaw Yigletu. I clearly understand what is being asked of me. My participation in this research is voluntary. I may refuse to participate or withdraw from participation at any time without prejudice, on simple oral notice and without having to justify my decisions.

By considering all of the above information, I agree to participate in the study.

Name of the Teacher Educator: _____

Signature: _____

Date: _____

Researcher's Verification of Explanation

I declare that I have carefully explained the purpose and nature of this research to Mathematics Teacher educators who participated in the research and provided answers with the best of my knowledge to all of the questions asked.

Name of the researcher: _____ Sig.: _____ Date: _____

Appendix VIII The learning objectives incorporated at the beginning of each of the sub-topics of each of the six chapters of the teaching learning module.

Chapter one: Elementary Mathematical Logic

General Objectives of the chapter:

- ✓ Understand the rules of logical connectives
- ✓ Understand what an argument is
- ✓ Understand what validity of an argument is.
- ✓ Prove whether a given argument is valid or invalid
- ✓ Prove or disprove a given Mathematical statement

1.1 Propositional Logic

- *Identify whether a given statement is a Proposition/Statement/*

1.2. Propositional Connectives

- *State the rules of the five logical connectives in his/her own words*

1.3. Compound Propositions

- *Determine truth value of a given compound proposition*

1.4. Equivalent Propositions

- *Determine whether or not a two given compound propositions are equivalent*

1.5 Tautology and Contradiction

- *Determine whether a given compound proposition is Tautology or not*
- *Determine whether a given compound proposition is Contradiction or not*

1.6. Predicate Logic

- *Determine truth value of an open proposition at a given value*

1.7 Quantifiers

- *State the rules of Existential and the Universal Quantifiers*
- *Determine truth value of a proposition which contains quantifiers*

1.8. Quantifiers occurring in Combinations

- *State the rules of combinations of the two quantifiers*
- *Determine truth value of a proposition which contains combinations of quantifiers*
- *Translate propositions containing “all” and “some” in to symbolic form*

1.9. Arguments and Validity

- *State in his/her own words what an argument is*
- *Determine whether or not a given argument is valid*

Chapter Two: Theory of Sets/Set Theory/

General Objectives

- ✓ *Understand the concept of set*
- ✓ *Recognize Set Operations*
- ✓ *Know the concept of relation*
- ✓ *Understand the concept of Order relations*
- ✓ *Understand the concept of Function*

2.1 The Concept of a Set

- *Determine whether two given sets are equal*
- *Determine whether a given set is a proper subset of another given set*
- *Solve problems related to set relations*

2.2 Set Operations

- *Solve problems related to set operations*
- *Prove /Disprove mathematical statements related to set operations*
- *Find the intersection of a given family of sets*
- *Find the union of a given family of sets*

2.3 Ordered pairs and Relations

- *Define what relation from set A to set B mean in his/her words*
- *Identify the domain and the range of a given relation*
- *Find the inverse of a given relation*
- *Determine whether or not a given relation is an Equivalence Relation*
- *Determine whether or not a given family of sets is a partition of a given set.*
- *State the definition of Equivalent Class in his/ her own words*
- *State the definition of Quotient Set in his/ her own words*
- *determine the distinct equivalence classes of a given set A based on a given relation R on A*

2.4 Order Relations

- *Determine whether or not a given relation is anti-symmetric*
- *Define Partial Ordering/ Order Relation/ on his/ her own words*
- *Define Partially Ordered Set/Poset/ on his/ her own words*
- *Determine whether or not a given set with a relation is a Poset.*
- *Determine whether or not a partial ordering " $\leq$ " on a given set X is a total ordering*
- *Define lattice on his/ her own words*
- *Determine whether or not a partial ordering " $\leq$ " on a given set X is a lattice*
- *Define complete lattice on his/ her own words*
- *Define well-ordering on his/ her own words*
- *Determine whether or not a total ordering is a well-ordering*

2.5 Functions

- *Define function from set A to set B in his/her own words*
- *Determine whether a given function is one-to-one or not*
- *Determine whether a given function is on-to or not*
- *Find combinations of given functions*
- *Find compositions of given functions*
- *Find the invers function of a given function f.*

Chapter Three: ALGEBRAIC STRUCTURE

General Objectives

- ✓ *Understand binary operation*
- ✓ *Understand algebraic structure*
- ✓ *Understand Morphism*
- ✓ *Understand the concept of Group*
- ✓ *Understand the concept of Ring*

3.1. Binary Operations

- *Define Binary Operation*
- *Solve problems related to binary operation*
- *Find the identity element of a binary operation in a given algebraic structure*

3.2 Algebraic Structure

- *Define an algebraic structure*
- *Determine whether or not a given set with a given operation is an algebraic structure*
- *Find identity element (s) for the binary operation(s) in a given algebraic structure*
- *Find invers element (s) of a given element in a given algebraic structure*
- *Prove given theorems related to algebraic structure*

3.3 Morphisms

- *Define a homomorphism from a given algebraic structure to another algebraic structure*
- *Show whether or not a given homomorphism is monomorphism*
- *Show whether or not a given homomorphism is epimorphism*
- *Determine whether or not a given homomorphism is isomorphism*
- *Show whether or not two given algebraic structures are isomorphic*

3.4 Groups

- *State the definition of Group in his/her own words*
- *Determine whether or not a given set with an operation is a group*

- *Show whether or not a given group is abelian*
- *Show whether or not a given group is a subgroup of another group*

3.5 Rings

- *State the definition of Ring in his/her own words*
- *Determine whether or not a given ring is commutative ring*
- *Determine whether or not a given ring is a ring with unity*
- *Determine whether or not a given ring is an integral Domain*
- *Determine whether or not a given ring is division ring*
- *State the definition of field in his own words*
- *Determine whether or not a given ring is a field*

Chapter Four: THE SYSTEM OF INTEGERS

General Objectives:

- ✓ *Understand properties of Integers*
- ✓ *Understand the concept of Order Axioms*
- ✓ *Understand the concept of Well Ordering Axioms*
- ✓ *Know how to use Mathematical induction in proving Mathematical statements*
- ✓ *Understand the concept of Ordered Integral Domain*

4.1 Properties of Integers

- *Identify properties of integers*
- *Prove theorems related to properties of Integer*

4.2 Order Axioms of the system of Integers

- *State Order axioms of the system of Integers*
- *Prove theorems related to order axioms of integers*

4.3 Well Ordering Axiom (WOA)

- *State Well Ordering Axiom*

- *Prove theorems related to Well Order Axioms of integers*

4.4 Mathematical Induction

- *State principles of Mathematical Induction*
- *Determine whether or not a given statement is true using mathematical induction*

4.5 Ordered Integral Domain

- *Define Ordered integral domain in his/her own words*
- *Determine whether or not a given integral domain is Ordered Integral Domain*
- *Determine whether or not a set S of elements of an ordered integral domain is well-ordered*
- *Prove given Mathematical statements related to Ordered Integral Domain.*

Chapter Five: ELEMENTARY THEORY OF NUMBERS

General Objectives:

- ✓ *Understand the concept of Division and Division Algorithm*
- ✓ *Knows different bases including base ten*
- ✓ *Knows the concept of Greatest Common Factor of positive integers*
- ✓ *Knows the concept of List Common Multiple of positive integers*
- ✓ *Understands the concept of Unique Factorization Theorem*

5.1 Division and the Division Algorithm

- *State the conditions on which an integer a divides an integer b*
- *State the Division Algorithm in his/ her own words*

5.2 Different Bases

- *Convert a number given in base ten to a given different base other than ten*

5.3 The Greatest Common Divisor (GCD)

- *State the definition of Greatest Common Divisor of two or more positive integers*
- *Find the GCD of given positive integers using Euclidean algorithm*
- *Prove given theorems related to Greatest Common Divisor*

5.4 The Least Common Multiple

- *State the definition of Least Common Multiple of two or more positive integers*
- *Prove given theorems related to Least Common Multiple*

5.5 The Unique Factorization Theorem

- *Define prime numbers in his/her own words*
- *State the Fundamental Theorem of Arithmetic in his/her own words*
- *Solve problems related to factorization*

Chapter Six: THE RATIONAL NUMBERS

General Objectives:

- ✓ *Understand how Rational numbers are constructed from integers*
- ✓ *Understand properties of rational numbers*
- ✓ *Understand decimal expansion of rational numbers*

6.1 Construction of the Rational Number from Integers

- *Determine whether or not a given number is a rational number*

6.2 Algebraic Properties of the Rational Numbers

- *List algebraic properties of rational numbers*

6.3 Decimal Expansions of Rational Numbers

- *Find decimal expansion of a given rational number*
- *Convert a given repeating decimal number in to fraction*

Appendix IX Summary of main ideas incorporated at the end of each the chapters in the Module

1. Summary of chapter one (Elementary Mathematical Logic)

Some of the main points discussed in this chapter are the following.

- ✓ A declarative sentence which can be said either “True” or “False” but not both is called Proposition. A Proposition is also called a statement.
- ✓ A word or group of words that joins two or more propositions to form compound proposition are called Propositional Connectives or Logical Connectives. The five commonly used connectives are “but”, “and”, “or”, “if...then...” and “...if and only if...”. The names, symbols and how they are read is summarized in the table below.
- ✓ Given p and q as propositions or statements,

Connective	Symbol	Connective word	Compound proposition
Negation	$\neg$	Not	$\neg p$
Conjunction	$\wedge$	And	$p \wedge q$
Disjunction	$\vee$	Or	$p \vee q$
Implication	$\Rightarrow$	If...then...	$p \Rightarrow q$
Bi implication	$\Leftrightarrow$	...if and only if ...	$p \Leftrightarrow q$

- ✓ Propositions that involve one or more logical connectives are called compound propositions; otherwise they are elementary propositions.
- ✓ In the compound proposition: $p \Rightarrow q$, p is the hypothesis and q is the conclusion.
- ✓ Two compound propositions P and Q are said to be equivalent if they have the same truth value for all possible combinations of truth values for the component elementary propositions occurring in both P and Q .
- ✓ A compound proposition which is always true for all combinations of the truth values of its component elementary propositions is called *tautology*. Whereas, a compound proposition is called a *contradiction* if it is always false regardless of the truth values of its components.
- ✓ An open proposition is a sentence which contains one or more variables and which becomes a proposition when each of its variables is replaced by a

particular individual.

- ✓ A logical deduction (argument form) is an assertion that a given set of statements $p_1, p_2, p_3, \dots, p_n$ called hypotheses or premises yield another statement Q , called the conclusion. Such a logical deduction is denoted by: $p_1, p_2, \dots, p_n \vdash Q$.
- ✓ An argument form $p_1, p_2, \dots, p_n \vdash Q$ is said to be valid if Q is true, whenever all the premise $p_1, p_2, \dots, p_n$ are true; otherwise it is invalid.

2. Summary of chapter two (Theory of Sets/Set Theory/)

- Set A is said to be a subset of a set B if every member of A is also a member of B .
- We say two sets are equal if and only if A is a subset B and B is a subset of A .
- Set A is said to be a proper subset of set B if A is a subset of B and is not equal to B .
- The union of sets A and B , denoted by $A \cup B$, is defined as: $A \cup B = \{x/x \in A \vee x \in B\}$.
- The intersection of sets A and B , $(A \cap B)$, is defined as: $A \cap B = \{x/x \in A \wedge x \in B\}$.
- Two sets A and B are said to be disjoint if they have no common element ($A \cap B = \emptyset$)
- The complement of a set A , denoted by A' , is the set of all elements which are not in A and which are in the universe U . Symbolically: $A' = \{x/x \in U \wedge x \notin A\}$.
- The difference between two sets A and B , denoted by $A - B$, is the set of all elements in A and not in B . Symbolically, by $A - B = \{x/x \in A \wedge x \notin B\}$. Thus, $A' = U - A$.
- The symmetric difference of sets A and B , $A \Delta B$, is defined as $A \Delta B = (A - B) \cup (B - A)$.
- The Cartesian product of sets A and B , denoted by $A \times B$, is the set consisting of all ordered pairs (a, b) , where $a \in A$ and $b \in B$; (i.e. $A \times B = \{(a, b)/a \in A \wedge b \in B\}$).
- A relation is a set whose elements are ordered pairs.
- If A and B are sets, a relation R from A to B is any subset of $A \times B$. That is R is a relation from A to B iff $R \subseteq A \times B$.
- Given a relation R on A , R is said to be:

- i) *Reflexive*: if $\forall x \in A, (x, x) \in R$. That is, $x R x \quad \forall x \in A$.
- ii) *Symmetric*: if $(a, b) \in R$ implies $(b, a) \in R$. That is, $a R b \Rightarrow b R a$.
- iii) *Anti-symmetric*: if $(x, y) \in R \wedge (y, x) \in R \Rightarrow x = y$ (i.e., $(x R y \wedge y R x) \Rightarrow x = y$).
- iv) *Transitive*: if $((a, b) \in R \wedge (b, c) \in R) \Rightarrow (a, c) \in R$. i.e. $a R b \wedge b R c \Rightarrow a R c$.
- A relation R on a set S is said to be an equivalence relation iff R is reflexive, symmetric and transitive.
 - Let $\{B_i\}, i \in I$ be a family of non-empty subsets of A . Then $\{B_i\}, i \in I$ is called a partition of A if the following two conditions are satisfied: i) $A = \bigcup_{i \in I} B_i$ and ii) For any sets B_i, B_j , either $B_i = B_j$ or $B_i \cap B_j = \emptyset$.
 - If R is an equivalence relation on a set A , then the set of all the distinct equivalence classes is called the Quotient Set and is denoted by A / R .
 - A relation R on a set X is called an order relation or partial ordering if R is: reflexive, anti-symmetric and transitive.
 - A partial ordering " $\leq$ " on a set X is called a "total ordering" iff either $x \leq y$ or $y \leq x, \forall x, y \in X$. A set X with a total ordering is called a totally ordered set or a chain.
 - Given a poset $(X, \leq)$: i) An element $a \in X$ is called a first (or smallest) element of X iff $a \leq x \quad \forall x \in X$, i.e, a precedes every element of X . ii) An element $b \in X$ is called a last (or largest) element of X iff $x \leq b \quad \forall x \in X$, i.e., if b - dominates every element belonging to X .
 - Let $(X, \leq)$ be a partially ordered set.
 - i) An element $x_0 \in X$ is called a maximal element if $x_0 \leq x \Rightarrow x \leq x_0$.
 - ii) An element x_0 in X is called a minimal element if $x \leq x_0 \Rightarrow x_0 = x$.
 - Let $(X, \leq)$ be poset, and let S be any subset of X . If the set of all upper bounds of S has a least element, then this element is called the least upper bound of S and is denoted by $\sup S$ or l.u.b. of S . Likewise, if the set of all lower bounds of S has a greatest (last) element, then this element is called the greatest lower bound of S and is denoted by $\inf S$ or g.l.b._of S .
 - A partially ordered set $(A, \leq)$ is said to be a lattice if every pair of elements in A has an infinimum and a supremum.

- A lattice $(A, \leq)$ is said to be a complete lattice if every non-empty subset of A has an infimum and a supremum.
- A total ordering $\leq$ on S is called a well-ordering for S if each non-empty subset A of S has a least element relative to $\leq$.
- A set S with a well-ordering relation $\leq$ is said to be well-ordered.
- A relation whose distinct elements have distinct first coordinates is called a function. Symbolically a relation f is a function if and only if $(x, y) \in f \wedge (x, z) \in f \Rightarrow y = z$.
- Let X and Y be two sets and F be a function. We say F is a function from X into Y if the following conditions are satisfied: i) $F \subseteq X \times Y$ and ii) $\forall x \in X (\exists y \in Y \text{ such that } (x, y) \in F)$
- A function $f: X \rightarrow Y$ is a one-to-one (injective) if $f(x) = f(y) \Leftrightarrow x = y$.
- A function $f: X \rightarrow Y$ is said to be an onto (or surjective) function if it satisfies the following condition: $\forall y \in Y, \exists x \in X$ such that $f(x) = y$ (or if range of $f = Y$).
- If a function $f: X \rightarrow Y$ is both injective and surjective then f is called a bijective function (or 1-1 correspondence).
- Two sets X and Y are equivalent iff there exists a bijective function f from X into Y .
- Any function f from a non-empty set A to $\mathbb{R}$ is called a real valued function.

3. Summary of chapter three (ALGEBRAIC STRUCTURE)

- A binary operation on a set E is a function from $E \times E$ into E .
- A non-empty set with one or two binary operations is called an algebraic structure.
- Given an algebraic structure $(E, *)$, an element $e \in E$ is an identity element of $*$ if $x * e = x = e * x \quad \forall x \in E$.
- Let $(E, *)$ be an algebraic structure, $x \in E$ is invertible (has an inverse) if:
 - i) the identity element for $*$ exists
 - ii) $\exists y \in E$ such that $x * y = e = y * x$. y is called an inverse of x on $*$.
- Let (E, Δ) and (F, ∇) be algebraic structures with one binary operation. Let $f: E \rightarrow F$ be a function from E into F such that $(x \Delta y) = f(x) \nabla f(y) \quad \forall x, y \in E$.
Then, we say f is a homomorphism from (E, Δ) into (F, ∇) .
- Let (E, Δ, ∇) and (F, Δ, ∇) be algebraic structures with two binary operations and let $f: E \rightarrow F$. We say f is a homomorphism from (E, Δ, ∇) to (F, Δ, ∇) if f

- satisfies the following conditions: i) $f(x \Delta y) = f(x) \Delta f(y)$ and ii) $f(x \nabla y) = f(x) \nabla f(y)$.
- Given a homomorphism f from one algebraic structure into another, f is said to be:
 - i) an epimorphism if f is surjective;
 - ii) a monomorphism if f is injective;
 - iii) an isomorphism if f is bijective;
 - Two algebraic structures are said to be isomorphic if there is an isomorphism from one to the other.
 - An algebraic structure with one binary operation (G, Δ) is called a Group if the following properties are satisfied:
 - G1. $a, b \in G \Rightarrow a \Delta b \in G$ (closure)
 - G2: $a \Delta (b \Delta c) = (a \Delta b) \Delta c \quad \forall a, b, c \in G$ (Associativity)
 - G3: $\exists e \in G$ such that $\forall a \in G, a \Delta e = a = e \Delta a$ (Existence of identity)
 - G4: for all $a \in G$, there exists $a' \in G$ such that $a \Delta a' = e = a' \Delta a$
 - A non-empty set H of a group G is a subgroup of G if (H, Δ) is also a group.
 - An algebraic structure with two binary operations (R, Δ, ∇) is called a ring if:
 - i) (R, Δ) is an abelian group.
 - ii) $a, b \in R \Rightarrow a \nabla b \in R$ (closure property of ∇)
 - iii) $a \nabla (b \Delta c) = (a \nabla b) \Delta c, \quad \forall a, b, c \in R$ (Associativity of ∇)
 - iv) (1) $a \nabla (b \Delta c) = (a \nabla b) \Delta (a \nabla c)$
 (2) $(a \Delta b) \nabla c = (a \nabla c) \Delta (b \nabla c), \quad \forall a, b, c \in R$ distributive of ∇ over Δ .
 - A commutative ring R with a unit element which satisfies the condition $a, b \in R \wedge ab = 0 \Rightarrow a = 0 \vee b = 0$ is called an integral Domain.
 - If (R, Δ, ∇) is a ring with unity and if $\forall a \in R \exists a^{-1} \in R$ such that $a \nabla a^{-1} = 1 = a^{-1} \nabla a$ (every element is invertible with respect to ∇) then R is called a division ring.
 - A commutative division ring is called a field.

4. Summary of chapter four (THE SYSTEM OF INTEGERS)

- The System of integers satisfies the following axioms:
 - A₁. Closure property: $x, y \in \mathbb{Z}, \Rightarrow x + y, x \cdot y \in \mathbb{Z}$
 - A₂. Commutative property: $\forall x, y \in \mathbb{Z}$, i) $x + y = y + x$ and ii) $x \cdot y = y \cdot x$
 - A₃. Associativity: $\forall x, y, z \in \mathbb{Z}$, $(x + y) + z = x + (y + z)$ and $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
 - A₄. Existence of Identity Elements:
 - i) Additive identity: $(\exists 0 \in \mathbb{Z})(\forall x \in \mathbb{Z})(x + 0 = x = 0 + x)$

ii) Multiplicative Identity: $(\exists 1 \in \mathbb{Z})(\forall x \in \mathbb{Z})(x \cdot 1 = x = 1 \cdot x)$

iii) $1 \neq 0$

A_5 : Existence of Additive Inverses:

$$(\forall x \in \mathbb{Z})(\exists x \in \mathbb{Z})(x + (-x) = 0 = (-x) + x)$$

A_6 : Distributive property of multiplication over addition

$$\forall x, y, z \in \mathbb{Z}, x \cdot (y + z) = x \cdot y + x \cdot z$$

- Axioms $A_1 - A_6$ establish that $(\mathbb{Z}, +, \cdot)$ is a commutative ring with a unit element.

- There is a set $P \subset \mathbb{Z}$ called the set of positive integers satisfying the following axioms:

$$O_1: x, y \in P \Rightarrow x + y \in P$$

$$O_2: x, y \in P \Rightarrow x \cdot y \in P$$

$$O_3: \text{If } x \in \mathbb{Z}, \text{ exactly one of the following is true: i) } x = 0, \text{ ii) } x \in P, \text{ iii) } -x \in P.$$

- For any $x, y \in \mathbb{Z}$: i) $x > y \Leftrightarrow x - y \in P$ and ii) $x < y \Leftrightarrow y > x$.
- For any $x, y \in \mathbb{Z}$: $x \geq y \Leftrightarrow x > y \vee x = y$.
- If $X \subseteq \mathbb{Z} \wedge X \neq \emptyset$, then $x \in X$ is called a least element of X if $x \leq y, \forall y \in X$.
- Every non-empty set of positive integers has a least element.
- The Integral Domain Axiom: $\forall x, y \in \mathbb{Z}, x \cdot y = 0 \Rightarrow x = 0 \vee y = 0$
- Principle of Mathematical Induction: Suppose that with each positive integer n , there is associated a proposition S_n that is true for all positive integer n if the following hold: (i) S_1 is true and ii) If k is a positive integer such that S_k is true, then S_{k+1} is true.
- An integral domain $(D, \oplus, \odot)$ is said to be an Ordered Integral Domain if $\exists$ subset $D_p \subseteq D$ with the following properties:
 - i) $a, b \in D_p \Rightarrow a \oplus b \in D_p$
 - ii) $a, b \in D_p \Rightarrow a \odot b \in D_p$
 - iii) $\forall a \in D$ exactly one of the following holds: $a = e, a \in D_p, a^* \in D_p$.
- Given an integral domain $(D, \oplus, \odot)$, and $a, b \in D$, then:
 - i) $a > b \Leftrightarrow a \oplus b^* \in D_p$ and ii) $a < b \Leftrightarrow b > a$
- A set S of elements of an ordered integral domain is said to be well-ordered if for each non-empty subset X of $S, \exists x \in X$ such that $x \leq y, \forall y \in X$.

5. Summary of chapter five (ELEMENTARY THEORY OF NUMBERS)

- If $a, d \in \mathbb{Z} \wedge d \neq 0$, we say that d is a divisor (factor) of a if $\exists n \in \mathbb{Z}$ such that $a = nd$.

- If $a, b \in \mathbb{Z} \wedge b > 0$, there exists unique integers q, r such that $a = qb + r, 0 \leq r < b$.
- Let b be a positive integer greater than 1. If a is positive integer, there exists a non-negative integer m such that a can be expressed uniquely in the form: $a = r_m b^m + r_{m-1} b^{m-1} + \dots + r_1 b + r_0$ where $0 < r_m < b$ and $0 \leq r_i < b$, and $i = 0, 1, \dots, m-1$.
- An integer c is a common divisor of integers a and b if $c|b$ and $c|a$.
- If $a \neq 0 \vee b \neq 0$, the greatest among their common divisors is called the greatest common divisor (GCD) of a and b and is denoted by (a, b) .
- Two integers a and b are said to be relatively prime if $(a, b) = 1$.
- Let a and b be non-zero integers. By WOA, there is a smallest positive integer, that is a multiple of both a and b . This integer is called the Least Common Multiple (LCM) of a and b and is denoted by $[a, b]$.
- An integer $p > 1$ is called a prime number (or a prime) if there is no divisor d of p satisfying $1 < d < p$. If an integer $a > 1$ is not a prime, it is called a composite.
- An integer $n > 1$ is either a prime or can be expressed as a product of primes. Besides, the factorization into primes is unique except for the order of the factors.

Appendix X. Summary exercises incorporated at the end of the chapters

1. Summary Exercises of Chapter One

1. Given the compound proposition $p \Rightarrow q$, its converse is $q \Rightarrow p$, its inverse is $\neg p \Rightarrow \neg q$ and its contrapositive is $\neg q \Rightarrow \neg p$. Show that the compound proposition is equivalent to its contrapositive and not equivalent to its converse and its inverse.

2. Find the truth value of the following compound propositions for any proposition p .

- a) $(p \Rightarrow p) \Leftrightarrow (\neg p \wedge p)$ b) $(\neg p \vee p) \Rightarrow (\neg p \Leftrightarrow p)$
c) $\neg(p \Rightarrow p) \Rightarrow p$ d) $(p \vee p) \Rightarrow (p \Leftrightarrow p)$
e) $(p \wedge p) \wedge \neg p$ f) $(p \vee p) \vee \neg p$

3. Check the validity of each of the following arguments.

a) $(p \wedge q) \Rightarrow r, p \Rightarrow \neg r, p \vdash q$

b) $p \Rightarrow \neg q, p, r \Rightarrow q \vdash r$

c) $\neg p \wedge q, (q \vee r) \Rightarrow p \vdash \neg r$

d) If Algebra is a good subject, then it is worth learning. Either the grading system is not fair or Algebra is not worth learning. But the grading system is fair. Therefore, Algebra is not a good subject.

4. Given $P(x, y): xy = 4xy$; $Q(x, y): 3(x + y) \leq 4(x + y)$ and $R(x): (x + 1)^2 > 0$, determine the truth value of the following propositions if the universe is $\mathbb{R}$.

- a) $(\exists x)(\forall y)P(x, y)$ b) $(\forall x)(\exists y)P(x, y)$
c) $(\forall x)(\forall y)Q(x, y)$ d) $(\exists x)(\forall y)Q(x, y)$ e) $(\forall x)(\exists y)Q(x, y)$
f) $(\forall x)R(x)$

5. Given propositions p, q, r , and s with truth values T, F, F, and T respectively, find the truth value of each of the following compound propositions.

- a) $(r \Rightarrow p) \Leftrightarrow (\neg s \wedge q)$ b) $(\neg q \vee p) \Rightarrow (\neg r \Leftrightarrow s)$
c) $p \Rightarrow (s \Rightarrow (r \Rightarrow (\neg q)))$ d) $(p \Rightarrow \neg q) \wedge (\neg s \Leftrightarrow r)$
e) $(\neg p \vee s) \Rightarrow (\neg q \Leftrightarrow r)$ f) $((p \vee q) \vee (r \vee s)) \Rightarrow (q \Rightarrow r)$

6. Given $P(x): x$ is a perfect square, $Q(x): x$ is a multiple of 3 and $U = \mathbb{N}$ find the truth value of each of the following propositions.

a) $P(4) \Rightarrow Q(4)$ b) $\neg P(10) \Rightarrow Q(100)$ c) $\neg Q(38) \wedge P(64)$ d) $P(1) \Leftrightarrow Q(1)$

2. Summary Exercises of Chapter Two

1. Simplify each of the following sets,

a) $(A \cap B') \cup B$ b) $A - (A' - B)$ c) $(A' - B)' \cup (A \cap B)'$

2. Prove each of the following by showing the necessary steps clearly.

a) $(A - B) - A = A \cap B$ b) $A \cap (B - C) = (A \cap B) - (A \cap C)$ c) $A \cup (B - A) = A \cup B$

3. Determine the truth value of each of the following statements for any sets A, B.

a) $(A \cup B) - A = A$ b) $(A \cup B) - (A \cap B)' = A \cap B$ c) $(A - B) \cup B = A$

4. Determine whether or not each of the following relations is reflexive, symmetric, anti-symmetric or transitive.

a) $\{(x, y)/x, y \in \mathbb{N} \wedge x \leq y\}$

b) $\{(x, y)/x, y \in \mathbb{N} \wedge x + y = 2y\}$

c) $\{(x, y)/x, y \in \mathbb{R} \wedge xy = 1\}$

d) $\{(x, y)/x, y \in \mathbb{Z} \wedge |x| = |y|\}$

e) $\{(x, y)/x, y \in \mathbb{Z} \wedge y = 2^x\}$

5. If $f(x) = \sqrt{8 - x}$, $g(x) = \frac{1}{x+2}$, $h(x) = \log_2(x + 5)$ and the universe $U = \mathbb{R}$, then find the domain of each of the following functions.

a) $(f + g)(x)$ b) $(g - h)(x)$ c) $(hxf)(x)$

d) $\frac{f(x)}{g(x)}$ e) $\frac{g(x)}{h(x)}$ f) $\frac{h(x)}{f(x)}$

6. Find the domain and range of each of the following functions. $f: \mathbb{R} \rightarrow \mathbb{R}$.

a) $f(x) = \sqrt{2x - 10}$ b) $g(x) = \frac{1}{x^2}$ c) $h(x) = 10\sin(x)$

7. Determine whether or not each of the following sets with their respective relations is: i) partial ordering, ii) total ordering iii) lattice iv) complete lattice v) well ordering.

a) $(\mathbb{R}, \leq)$ where $\leq$ the usual less than or equal to sign

b) $(\mathbb{Q}, \leq)$ where $\leq$ the usual less than or equal to sign

c) $(\mathbb{Z}, \leq)$ where $\leq$ the usual less than or equal to sign

d) $(\mathbb{N}, \leq)$ where $\leq$ the usual less than or equal to sign

e) $(P(A), \subseteq)$ where $A \neq \emptyset$, $P(A)$ is power set of A and $\subseteq$ is the usual subset relation.

3. Summary Exercises of Chapter Three

1. If a, b, c and d are elements of a ring $(R, +, \cdot)$ then prove that:

a) $(-a) \cdot (b) \cdot (-c) = (a \cdot b \cdot c)$

b) $(-a) \cdot (b) \cdot (c) = -(a \cdot b \cdot c)$

2. Determine whether or not each of the following are fields.

a) $(\{2x/x \in \mathbb{Z}\}, +, \cdot)$

b) $(\{a + b\sqrt{2} / a, b \in \mathbb{Z}\}, +, \cdot)$

c) $(\{a + b\sqrt{3} / a, b \in \mathbb{Q}\}, +, \cdot)$

d) $(\{\bar{0}, \bar{1}, \bar{5}, \bar{7}, \bar{11}\}, \oplus_{12}, \odot_{12})$

3. Given a binary operation Δ on $\mathbb{R}$ defined as $x\Delta y = x + y - 2x^2y^2$:

a) Check whether Δ is commutative, associative and whether it has an identity element

b) Find all the elements of $S = \{x/x \in \mathbb{R} \wedge x \text{ is invertible}\}$

c) Find all the elements of $T = \{x/x \in \mathbb{R} \wedge x \text{ has two inverses}\}$

d) Find the inverses of 1.

4. A non-empty set with an associative binary operation is called a semi group. Determine whether or not each of the following algebraic structures is a semi-group.

a) $(\mathbb{Z}, *)$, where $a * b = a - b$

b) $(\mathbb{Q}, *)$, where $a * b = ab - 1$

c) $(\mathbb{N}, *)$, where $a * b = 2^{ab}$

5. For each of the following, determine whether the subset H under the operation of the given group is a subgroup of G.

a) $G = \{\mathbb{R}, +\}$ H = The set of even integers

b) $G = \{\mathbb{Z}_7, \oplus_7\}$ H = $\{\bar{0}, \bar{2}, \bar{5}\}$

c) $G = \{\mathbb{Z}, +\}$ H = The set of odd integers

d) $G = \{\mathbb{Z}_6, \oplus_6\}$ H = $\{\bar{0}, \bar{2}, \bar{3}\}$

6. Let A and B be a subgroup of a group G. Prove that $A \cap B$ is also a subgroup of G. Is $A \cup B$ necessarily a subgroup of G? Justify your answer with examples.

4. Summary Exercises of Chapter Four

1. Show that $x = -x \Leftrightarrow x = 0$ in $\mathbb{Z}$.

2. Show that $x + 1 > x$, $\forall x \in \mathbb{Z}$.

3. Use the Principle Mathematical Induction to show that:

a) $(\forall x \in P)(\exists y \in P)(y > x)$, i. e., P has no largest element.

b) Let S be an infinite set and n be any positive integer. Then there is a subset $T \subset S$ such that the number of elements in T is n.

4. Using the principle of Mathematical induction, prove that for any positive integer n:

a) $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

b) $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

$$\text{c) } 1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

$$\text{d) } 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{e) } 2 + 4 + 6 + \cdots + 2n = n(n+1)$$

5. Summary Exercises of Chapter Five

1. If n is a composite natural number, show that it has a divisor d such that $1 < d \leq \sqrt{n}$.
2. By using the Euclidean algorithm, find the GCD of each of the following pairs of numbers and express the GCD as a linear combinations of the numbers.
 - a) 1350 and 4500 b) 2450 and 11844 c) 2947 and 3997
3. Prove each of the following.
 - a) If $ac|bc$, then $a|b$.
 - b) If $a|b$ and $c|d$, then $ac|bd$.
 - c) $4 \nmid (n^2 + 2)$ for an integer n .
 - d) If $(a, 4) = 2$ and $(b, 4) = 2$ then $(a + b, 4) = 4$
 - e) If n is odd then $n|(n^2 - 1)$.
 - f) If m and n are odd, then $m^2 + n^2$ is even and $4 \nmid (m^2 + n^2)$.
4. Find all positive integers a, b satisfying the equations $(a, b) = 10$ and $[a, b] = 100$.
5. If $(a, b) = p$, a prime number, what are the possible values of:
 - a) (a^2, b^2) b) (a^2, b^2) c) (a^3, b^4) d) (a^{10}, b^7)

6. Summary Exercises of Chapter Six

1. Prove: Between two rational numbers there is an infinite number of rational numbers.
2. Prove: for each rational number $q > 0$, there is a natural number n such that $\left(\frac{1}{n}\right) < q$.
3. Express each of the following rational numbers as fractions.
 - a) 4.0125 b) 16.003785 c) $0.\bar{9}$ d) $4.38\overline{657}$ e) $23.432\overline{5876}$
4. Express each of the following fractions in decimal form.

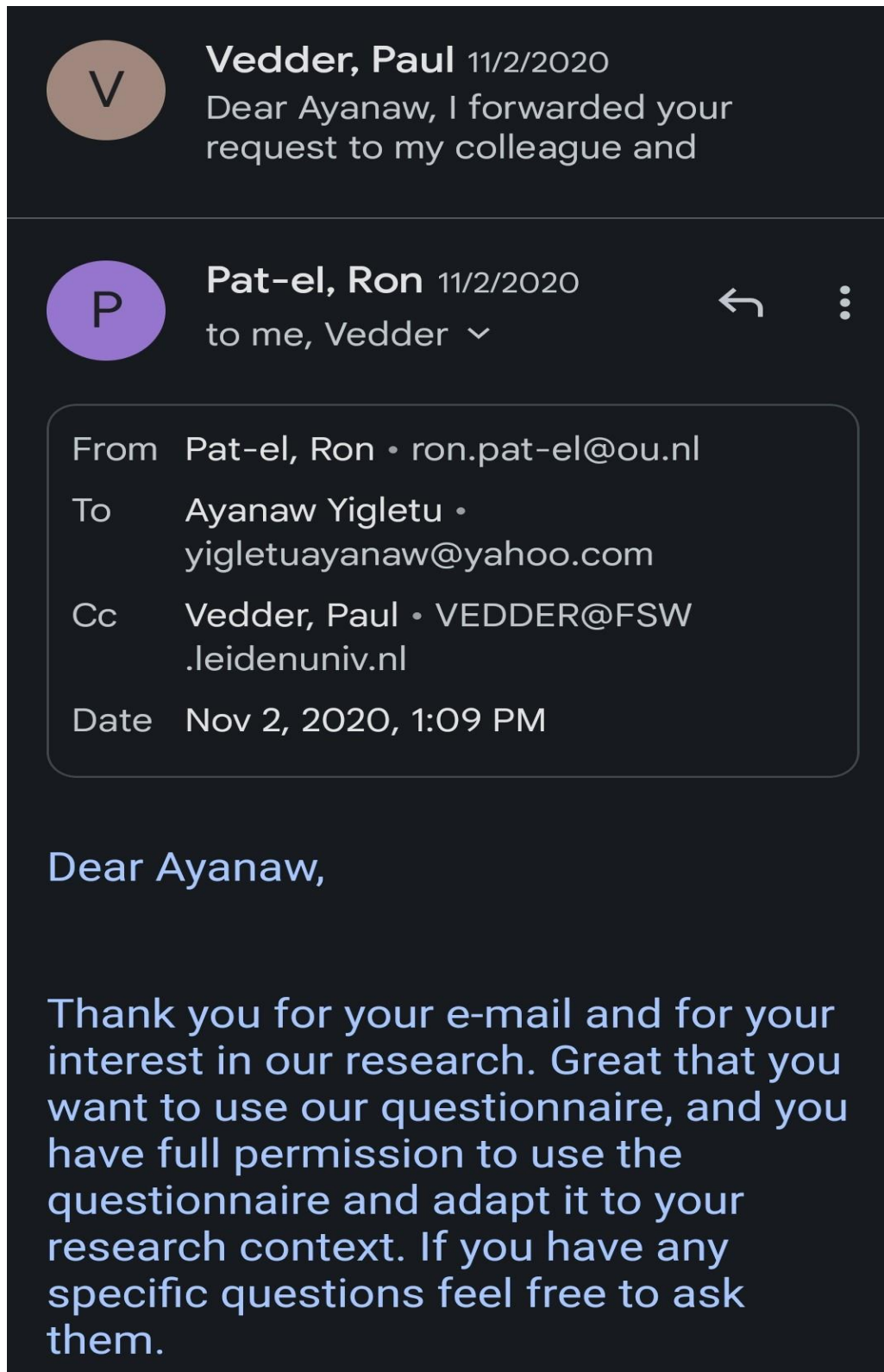
a) $\frac{3}{32}$ b) $\frac{67}{64}$ c) $\frac{29}{11}$ d) $\frac{29}{14}$ e) $\frac{100}{99}$

5. Prove that there are infinite number of rational numbers between any two different rational numbers a and b .

6. Prove that the following real numbers are not rational.

a) $\sqrt{3}$ b) $\sqrt{5}$ c) $\sqrt{7}$ d) $\sqrt{11}$

Appendix XI. Permission to use the questionnaire for students' Conception of Assessment for Learning Experiences



Appendix XII. Average observation ratings of implementation of the five AfL strategies of the three MTEs

		Average observation ratings of implementation of the five AfL strategies of each the three MTEs					
		1 st	2 nd	3 rd	4 th	5 th	6 th
S E C T I O N 0 1	<i>A. Learning Targets: Clarifying Learning Intentions and Sharing Criteria for Success</i>	1.1	1.3	2.3	2.3	2.9	3.1
	<i>B. Monitoring: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of Learning</i>	2.0	2.9	3.7	3.5	3.9	3.8
	<i>C. Feedback: Providing Feedback That Moves Learners Forward</i>	2.2	2.9	3.4	3.7	3.7	3.7
	<i>D. Self-Assessment: Activating Students as the Owners of Their Own Learning</i>	1.2	2.2	2.8	3.0	3.8	3.8
	<i>E. Peer Assessment: Activating Students as Instructional Resources for One Another</i>	1.2	2.3	2.8	3.5	3.8	4.0
	Average	1.5	2.3	3.0	3.2	3.6	3.7
S E C T I O N 0 2	<i>A. Learning Targets: Clarifying Learning Intentions and Sharing Criteria for Success</i>	1.5	2.4	4.0	3.5	3.8	4.1
	<i>B. Monitoring: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of Learning</i>	2.0	2.6	3.1	3.9	4.0	4.1
	<i>C. Feedback: Providing Feedback That Moves Learners Forward</i>	2.4	2.7	3.4	3.9	3.9	3.9
	<i>D. Self-Assessment: Activating Students as the Owners of Their Own Learning</i>	1.0	2	2.7	3.2	3.7	4.0
	<i>E. Peer Assessment: Activating Students as Instructional Resources for One Another</i>	1.2	2.0	2.5	3.0	3.7	4.0
	Average	1.6	2.3	3.1	3.5	3.8	4.0
S E C T I O N	<i>A. Learning Targets: Clarifying Learning Intentions and Sharing Criteria for Success</i>	1.1	2.9	3.3	4.1	3.8	4.3
	<i>B. Monitoring: Engineering Effective Classroom Discussions, Questions, and Learning Tasks That Elicit Evidence of Learning</i>	2.3	3.2	4.1	3.9	3.9	4.5
	<i>C. Feedback: Providing Feedback That Moves Learners Forward</i>	2.3	3.7	3.7	3.9	3.8	4.3

0 3	D. <i>Self-Assessment: Activating Students as the Owners of Their Own Learning</i>	1.0	2.2	3.2	4.0	3.8	3.8
	E. <i>Peer Assessment: Activating Students as Instructional Resources for One Another</i>	1.0	2.2	3.8	4.0	3.8	4.0
	Average	1.5	2.8	3.6	4.0	3.8	4.2

Note. **1**= No evidence of use **2**= Superficial or ineffective use **3**= Minimal use or uncertain effectiveness **4**= Frequent Use or Effective and **5**= Pervasive Use or Highly Effective

Appendix XIII. General Guidelines for Using the Module of the Course

Fundamental Concepts of Algebra (Math 221) is a very important course, and understanding the topics discussed in the course has a positive effect on learning other mathematics courses and teaching elementary school mathematics topics. The pre requisite course for this course is Basic Mathematics I (math 101). Revising this prerequisite course helps a lot in learning Math 221.

This course consists of six chapters: Elementary Mathematical Logic, Set Theory, Algebraic Structure, The System of Integers, Elementary Theory of Numbers, and Rational Numbers.

The following are recommendations you are suggested to practice while you are taking the course to attain the objectives of the course:

- *In each class, there are specific objectives listed below each subtopic that you are expected to attain at the end of the class, and you should read them and understand what you are expected to know or be able to do.*
- *Assess yourself before, during, and after each class to determine whether or not you have attained the intended objectives.*
- *Ask your teacher and your friends questions when you are confused or do not understand some concepts, procedures, steps in solving problems, or other issues.*
- *Respond to your teacher's and your friends' questions appropriately, and do not forget that making mistakes is normal and that students learn a lot from their mistakes in general.*
- *Give constructive feedback based on the objectives set to your friends appropriately when you have the opportunity, and it helps you understand the topics discussed better.*

- *Take all the feedback given to you positively and think about how to use the feedback given to you to improve your understanding of the concepts and your problem-solving skills.*
- *Revise the topics discussed, the problems solved, and the examples given in the class when you go home.*
- *Read the summaries of the chapters at the end of each chapter and compare what you think you know with what is summarized.*
- *Try to solve each of the problems in the exercises and summary questions individually and ask for help from your teacher or your friend(s) if you have difficulties solving them.*
- *Read and solve related problems from additional reference materials from the library, and you can use the internet to read and see how related problems are solved.*

Declaration

I, the understated PhD candidate declare that the dissertation, entitled: Professional Development and Training on Assessment for Learning and Pre-service Teachers' Outcomes in Fundamental Concepts of Algebra: A Mixed Method Study is my own original work and that all the sources that I have used or quoted have been acknowledged.

Phd Candidate: Ayanaw Yigletu Asfaw (GSR/1166/10)



Sign.: _____

Date: 7 July, 2023

This dissertation has been submitted for examination with my approval as a supervisor.

Main Supervisor: Dr Kassa Michael (Associate professor of Mathematics Education)



Sign.: _____

Date: 7 July, 2023

Co-Supervisor: Professor Mulugeta Atinafu (Professor of Mathematics Education)



Sign.: _____

Date: 7 July, 2023