INVERSE COMPTON SCATTERING IN CURVED SPACETIME IN THE PRESENCE OF STRONG MAGNETIC FIELD

By

Demisew Teka

A THESIS SUBMITTED TO
THE DEPARTMENT OF PHYSICS

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE

ADDIS ABABA UNIVERSITY
ADDIS ABABA,ETHIOPIA

SEPTEMBER 2014

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared by **Demisew Teka**, entitled "**Inverse Compton Scattering in curved spacetime in the presence of strong magnetic field**" and submitted in partial fulfillment of the requirements for the degree of **Master of Science**. compiles with the regulations of the University and meets the accepted standards with respect to originality.

Signed by the Examining Committee:

Examiner:______________________ Signature:________ Date:______________

Examiner:______________________ Signature:________ Date:______________

Advisor: ______________________ Signature:________ Date:______________

Abstract

We calculate the scattering cross section for inverse compton scattering in curved spacetime in the presence of strong magnetic field. An approximate solution of Dirac equation in a spatially flat RW spacetime has been established. And its solution in Friedman universe is also discussed. The scattering cross section we found contains the expanision parameter of the expanding universe and this makes main difference from the special relativistic results. We investigate the role of the expanission parameter to the scattering cross section of the inverse compton scattering.

Acknowledgements

First of all, I would like to thank the almighty God for letting me to accomplish this study. Secondly, I would like to express my gratitude to Dr.Legesse Wetro, for his advices, continuous support and friendly approach through out this research, specially for the chance he gave me to study this beautifull science.

Addis Ababa University, June, 2014. Demisew T.T.

Table of Contents

Abstract **iii**

Acknowledgements **iv**

Table of Contents **v**

List of Figures **vi**

Introduction **1**

Notations **3**

1 Neutron star and Other Compact objects **4**
- 1.1 An introduction to neutron stars and pulsar 4
- 1.2 other compact objects:white dwarfs and black holes 5
- 1.3 Tests of general relativity: Emission and detection of gravitational waves 6

2 Solution of Dirac Equation in Curved spacetime in the presence of strong magnetic field **8**
- 2.1 Solution of Dirac Equation in Curved spacetime in the presence of strong magnetic field 8
- 2.2 Spinors of the Dirac equation 9
- 2.3 Dirac Equation and it's solution In Robertson-Walker Spacetime . 15
- 2.4 The Dirac field propagator of the electron 19

3 Inverse Compton Scattering in curved spacetime in the presence of strong magnetic field **22**
- 3.1 The S-matrix element 22
- 3.2 The Transition Amplitude and Transition Rate 25
- 3.3 The Scattering Cross Section 27
- 3.4 The Scattering Cross-Section in the electron's rest frame 28

4 CONCLUSION AND DISCUSSION **32**

References **34**

List of Figures

Introduction

The properties of inverse compton scattering in a strong magnetic field has been used in studies of pulsar radiation and have some important influence on theoretical models of pulsars. In astrophysics inverse compton scattering is actually more important than compton scattering. A great number of electrons(or positrons) must be accelerated to Lorentz factor $\gamma = 10^6 - 10^7$ in the gap of a NS according to the Ruderman and Sutherland(1975;RS) model. Meanwhile very strong thermal radiation fields and electromagnetic waves of various frequencies by some other mechanisms exist there too. Thus the scattering of photons by electrons must be analysed carefully in studies of pulsar radiation. In all pulsar theories, very strong magnetic fields($10^{12} - 10^{13}$G) near the polar caps of neutron stars are assumed. If the observed spectral X-ray lies from HerX-I are produced by synchrotron radiation of quantized electrons in a strong magnetic field, this seems to be a direct evidence of the presence of strong magnetic fields. Thus, in the studies scattering of photons by electrons the effect of strong magnetic fields must be taken into account[1].

Compton radiation in a strong magnetic field has higher efficiency than curvature radiation in cetain cases, and it has been the important mechanisms for the energy loss of high energy electrons and for producing high energy photons near the surface of a neutron star. Compton scattering of photons by relativistic electrons is an efficient process to produce high- energy cosmic X- rays and γ- rays[1].

Now, in astrophysical scenario we erfer to the gravitational field produced by the compact object(neutron star or black hole) can also play an important role for the production of high enrgy photons in the polar caps of neutron stars[2,3].

The purpose of this thesis is to calculate the scattering cross section of inverse compton scattering in curved spacetime; in a spatially flat RW spacetime in the presence of stron magnetic field.The first chapter contains a brief introction to neutron stars and the other compact objctcts in relation to NSs and tests of General Theory of Relativity in the study of NSs. In chapter two we discuss the exact solution of Dirac equation in RW space time in the presence of strong magnetic field and we derive the Fynmann propagator of an electron. The next two chapters (chap. 3 and 4)contain the derivation of the differential scattering cross section of inverse compton scattering in the presence strong magnetic field and discussions and conclusions are given.

Notations

Latin indices i,j,k generally run over three spatial coordinate labels, usually, 1,2,3 or x,y,z.

Greek indices $\alpha, \beta, \gamma, \delta$, and so on generally run over the four spacetime inertial labels 0,1,2,3 or t,x,y,z.

Greek indices $\mu, \nu, \kappa, \lambda$, and so on generally run over the four coordinate labels in a general coordinate system.

Repeated indices are summed unless otherwise indicated.

The metric $\eta_{\alpha,\beta}$ in an inertial coordinate system has the signature(+ - - -).

Natural units are used $G = \hbar = c = 1$, except when c.g.s. units are indicated.

Chapter 1

Neutron star and Other Compact objects

1.1 An introduction to neutron stars and pulsar

A star exclusivey composed of neutrons and very dense but without much of unknown is referred to as neutron star(NS). The current knowledge about these compact objects, particularly observational constraints and Theortical modelings, shows that they must involve many aspects of modern physics in order to get a satisfactory description. Some of the aspects that need to be considered, when studing neutron stars are [4]

- a very strong gravitational fields;
- the presence of electromagnetic fields;
- the transition between the neutron (or nuclear) matter and the surface, with the possibility of accretion of matter from the interstellar medium, the formation a crust[4].

A more precise word for NSs could therefore be self-gravitating object at nuclear density, particularly since there are not only neutrons[5]. The historical appreance of the concept of NS has now become some kind of a myth; it is ofen said that, in 1932, the very same day after the discovery of the existance of the neutrons by James Chadwick, Lev Landau discovered(about) the possibility of dense stars made onley of neutrons.

When a star whose mass is above the Chandrasekhar limit, about $n\hbar^{3/2}/M_N^2 G^{3/2}$

reaches the end of its thermonuclear evolution and grows cold. Its internal pressure then fails to support it, and it collapses. One possibility is that the star will simply go on collapsing forever, in which case General Relativity will certainly come into play. Another possibility is that the star will become so heated during its collapse that it will explode, becoming a supernova. It might then blow off enough matter so that its mass drops below the Chandrasekhar limit. It is believed that in this case the highly compressed remnant doesnot find its quietur as a white dwarf, but rather becomes a superdense neutron star[6].

1.2 other compact objects:white dwarfs and black holes

The other types of astrophysical objects sharing some features with NSs are white dwarfs and black holes. All three types of bodies are the so-called compact stars, for which the compactness parameter, Ξ: which measures the ratio between the gravitational potential energy and the mass energy of the star, is becoming to

$$\Xi = v_c^2/c^2 = 2GM/Rc^2$$

unity, meaning that the surface gravity is becoming very strong. All three types of objects also represent the possible final stages for stellar evolution, following the mass M, of the main sequence star(the limitation values for the masses are approximative):

$-M_. < 10M_\odot \rightarrow$ white dwarfs;

$-10M_\odot < M_. < 40M_\odot \rightarrow$ neutron stars;

$-40M_\odot < M_. \rightarrow$ black holes.

White dwarfs have been observed as hot(and therefore white) stars. For instance, one of the firt observed white dwarves, called sirius B, radiates as a black-body at 24000K, and from the flux one can deduce the radius ($\sim 5000km$),the density being therfore $\rho \sim 10^9 kg/m^3$. In these objects,the gravitational force is balanced by the degeneracy pressure of the elctrons, provided that the mass is lower than some critical value, called Chandrasekhar mass M_{ch}:

$$M_{ch} \simeq 1.5 M_{\odot}.$$

If they are not in a binary system, and don't accrete matter, white dwarfs progressively cool down and become fainter and fainter.
More compact than neutron stars, black holes are regions of space-time causally diconnected from asymptotic observers.The boundary of black hole is a pure geometric notion and is called event horizon: it is a closed surface inside which, if a light ray is emitted, it canot reach any outside observer. This is the reason why this object is seen as black. It is called a hole because anything falling inside the horizon is definitely lost. Black holes represent objects where gravity has overcome all other forces and neutron stars seem, from today's theory and observations,the last stable stage where matter is still present and the gravitational field is so intense.

1.3 Tests of general relativity: Emission and detection of gravitational waves

Binary pulsars yield very accurate observations, through the timing of the arrival of pulses. The first binary pulsar to be observed was $PSR1993 + 16$ in 1974 by Hulse and Taylor. The precise timing of this pulsar over a couple of decade allowed for a comparison with general relativistic predictions for the evolution of the binary pulsar parameters and, in particular, the periastron shift and angular momentum loss due to the emission of gravitational waves. The remarcable agreement between observations and predictions was not only a constraining check of the theory of general relativity, but an indirect evidence of the existance of gravitational waves. All these aspects have motivated the award of the Noble Prize to Hulse and Taylor in 1993.
The ground-based gravitational wave detectors, which have been recently built are

aquiring data, and a direct detection of gravitational waves would be very exciting. These detectors are Michelson-type LASER interferometers, which arm-lengths are 3km(for the French-Italian project VIRGO) and 4km(for the US project LIGO) and with almost perfect vacuum. We recall that the gravitational frequency is, within a factor[6] 2, the mechanical frequency of the emitting system. Therefore, many astrophysical sources of such radiation imply NSs[6]. First, supernovae giving birth to neutron srars can produce a fair amount of gravitational waves, although they are not so much efficient because the whole phenomenon remain close to spherical symmetry. Soon after this collapse, proto-neutron stars can undergo a growth of unstable oscillations, inducing deformations and a strong gravitational wave burst. Then, isolated neutron stars can be slightly deformed and thus emit gravitational waves. This might not produce great wave amplitudes, but the signal is very regular(as for radio/X pulsars) and the radiation can be integrated over a long period increase the signal-noise ratio. Finally, binary systems of neutron stars or neutron star/black hole are known to lose angular momentum from the gravitational wave emission. They end up in a violent coalescence, producing a large amount of waves[7].

Chapter 2

Solution of Dirac Equation in Curved spacetime in the presence of strong magnetic field

2.1 Solution of Dirac Equation in Curved spacetime in the presence of strong magnetic field

During the last years a large amount of observational data has been reported showing that our universe is almost isotropic and homogeneous. The study of the structure of the Cosmic Microwave Radiation leads us to conclude that the ratio of the total density to the critical density of the universe Ω_0 is likely to be close to one [8,9,10], favoring a spatially flat Robertson-Walker metric over other topologies.

It is well known that general relativity is a local metrical theory and therefore the cor- responding Einstein field equations do not fix the global topology of spacetime and con- sequently the universe may have compact spatial sections with a nontrivial topology [11,12], then the observational data does not rule out the possibility that our universe possesses a hyperbolic topology [11,13.14,15].

The study of cosmological models with nonstandard topologies is not new and goes back to the works by Zelmanov [16,17], showing that upon different coordinate transformations, spatially closed or flat sections can be transformed into hyperbolic sections and vice versa. The line element associated with an spatially open Friedman universe has the form

$$ds^2 = a(\eta)^2[d\eta^2 - dr^2 - sinh^2(d\theta^2 + sin^2\theta)d\Phi^2]. \tag{2.1.1}$$

Making the coordinate transformation the metric in eqn.(2.1) becomes

$$ds^2 = a^2(\eta)[d\eta^2 - dz^2 - e^{-2z}(dx^2 + dy^2)]. \tag{2.1.2}$$

In order to solve quantum processes in curved spacetime one has to fulfill a preliminary step which constists in having a description of the single-mode solution of the relativistic particles or perturbation in those background field, i.e., exact solution of the relativistic scalar and spinor wave equations. we have different methodes of solving relativistic wave equation in curved space; among them, the methode of separation of varirables is one most widely used.

2.2 Spinors of the Dirac equation

The Dirac equation is a system of coupled partial differential equations which is separable in a very restricted set of metrics. Among the spacetimes where the separability of the Klein-Gordon and Dirac equations has been studied one can mention the St$\ddot{a}$ckel spaces [18], which are those metrics where the Hamilton-Jacobi equation is separable. Nevertheless recently it has been shown that this condition is neither necessary nor sufficient in order to guarantee a complete separability of variables in the Dirac equation . A systematic classification of the gravitational backgrounds where the Dirac equation is separable with the help of the algebraic method is presented[20]. The line element in Eq. (2.2.2) belongs to this family and consequently one can apply the algebraic method of separation. The covariant generalization of the Dirac equation in curved spacetime is[21,22]

$$\bar{\gamma}^\mu(\partial_\mu - \Gamma_\mu - ieA_\mu)\bar{\Psi} + M\bar{\Psi} = 0, \tag{2.2.1}$$

where the curved Dirac matrices $\bar{\gamma}^\mu$ satisfy the anticommutation relation

$$\{\bar{\gamma}^\mu, \bar{\gamma}^\nu\} = 2g^{\mu\nu}, \tag{2.2.2}$$

and the spinor connection Γ_μ are [22]

$$\Gamma_\mu = \frac{1}{4} g_{\lambda\alpha}[(\frac{\partial b^\beta_\nu}{\partial x^\mu})a^\alpha_\beta - \Gamma^\alpha_{\nu\mu}]S^{\lambda\nu}, \tag{2.2.3}$$

where

$$S^{\lambda\nu} = \frac{1}{2}(\bar{\gamma}^\lambda \bar{\gamma}^\nu - \bar{\gamma}^\nu \bar{\gamma}^\lambda), \tag{2.2.4}$$

and $\Gamma^\alpha_{\nu\mu}$ is the affine connection can be computed from the usual

$$\Gamma^\alpha_{\nu\mu} = \frac{1}{2} g^{\delta\alpha}\{\frac{\partial g_{\mu\delta}}{\partial x^\nu} + \frac{\partial g_{\nu\delta}}{\partial x^\mu} + \frac{\partial g_{\mu\nu}}{\partial x^\delta}\} \tag{2.2.5}$$

the matrices b^β_ν, a^α_β establish the connection between the Dirac matrices $\bar{\gamma}^\mu$ on a curved space-time and the flat Dirac matrices γ^μ as follows:

$$\bar{\gamma}_\mu = b^\beta_\nu \gamma_\alpha, \bar{\gamma}^\mu = a^\mu_\beta \gamma^\beta. \tag{2.2.6}$$

from $g_{\mu\nu} = e^a_\mu(x) e^b_\nu(x) \eta_{ab}$ it easy to see that we can also write

$$\eta_{ab} = e^\mu_a e^\nu_b g_{\mu\nu} \tag{2.2.7}$$

since the line element in Eq. (2.1.2) is associated with a diagonal metric, we can workin the tetrad gauge for $\bar{\gamma}^\mu$.

The spinor connections in Eq. (2.2.3) can easily be calculated to give,

$$\Gamma_1 = -\frac{1}{2}\frac{e^{-z}}{a(\eta)}\{a(\eta)\gamma^1\gamma^3 + \frac{da(\eta)}{\eta}\gamma^0\gamma^1\}, \tag{2.2.8}$$

$$\Gamma_2 = -\frac{1}{2}\frac{e^{-z}}{a(\eta)}\{a(\eta)\gamma^2\gamma^3 + \frac{da(\eta)}{\eta}\gamma^2\gamma^0\}, \tag{2.2.9}$$

$$\Gamma_3 = -\frac{1}{2}\frac{da(\eta)}{\eta}\frac{1}{a(\eta)}\gamma^3\gamma^0, \tag{2.2.10}$$

$$\Gamma_0 = 0. \tag{2.2.11}$$

The curved spacetime Dirac matrices in Eq. (2.2.6) can be evaluted in accordance with Eq. (2.2.7). Then substituting the above expressions in Eq. (2.2.1), the

Dirac equation will take the form,

$$\Longrightarrow [\bar{\gamma}^0(\frac{\partial}{\partial\eta} - 0) + \bar{\gamma}^1(\frac{\partial}{\partial x} - \{\frac{1}{2}\frac{e{-}z}{a(\eta)}a(\eta)\gamma^1\gamma^3 - \frac{1}{2}\frac{e{-}z}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^1\gamma^0\} - ieA_1(y))$$
$$+\bar{\gamma}^2(\frac{\partial}{\partial y}-\{\frac{1}{2}\frac{e{-}z}{a(\eta)}a(\eta)\gamma^2\gamma^3-\frac{1}{2}\frac{e{-}z}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0\})+\bar{\gamma}^3(\frac{\partial}{\partial z}-\{\frac{1}{2}\frac{da(\eta)}{d\eta}\frac{1}{a(\eta)}\gamma^3\gamma^0]\bar{\Psi}+M\bar{\Psi}) = 0$$
$$\Rightarrow [\gamma^0(\frac{\partial}{\partial\eta})+\gamma^1 e^z(\frac{\partial}{\partial x}-\{\frac{1}{2}e^{-z}\gamma^1\gamma^3-\frac{1}{2}\frac{e^{-z}}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^1\gamma^0\}-ieA_1(y)+\gamma^2 e^z(\frac{\partial}{\partial y}\{\frac{1}{2}e^{-z}\gamma^2\gamma^3-$$
$$\frac{1}{2}\frac{e^{-z}}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0\}) + \gamma^0(\frac{\partial}{\partial z} + \frac{1}{2}\frac{1}{a(\eta)}\frac{da(\eta)}{d\eta}\gamma^2\gamma^0)]\bar{\Psi} + Ma(\eta)\bar{\psi} = 0.$$

We find that the Dirac equation takes the simple form

$$\{\gamma^0\frac{\partial}{\partial\eta} + \gamma^1 e^z(\frac{\partial}{\partial x} - eA_1(y)) + \gamma^2 e^z\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z} + Ma(\eta)\}\mathit{\Psi} = 0, \tag{2.2.12}$$

where we have introduced the spinor $\bar{\Psi}$

$$\bar{\Psi} = a(\eta)^{\frac{-3}{2}} e^z \mathit{\Psi}, \tag{2.2.13}$$

the factor $a(\eta)^{\frac{-3}{2}}$ was introduced in order to cancel the contribution due to the spinor connections. Regarding Eq. (2.2.12) we should mention that it does exhibits a nonfactorizable structure[23,24]. In order to solve Eq. (2.2.12) we apply the algebraic method of separation of variables[25]. The method consists in rewriting the Dirac equation, Eq. (2.2.12) as a sum of two first order differential operators $\hat{K}_1$,$\hat{K}_2$ satisfying the relation

$$[\hat{K}_1, \hat{K}_2] = 0, \{\hat{K}_1, \hat{K}_2\}\Phi = 0, \tag{2.2.14}$$

where the spinor Φ is related to Ψ via the expression $\gamma^3\gamma^0\Psi = \Phi$ and k is a separation constant. The operators $\hat{K}_1$ and $\hat{K}_2$ read

$$\hat{K}_1(x,y)\Phi = \{\gamma^2\frac{\partial}{\partial y} + \gamma^1(\frac{\partial}{\partial x} - eA_1(y))\}\gamma^3\gamma^0\Phi = ik\Phi, \tag{2.2.15}$$

$$\hat{K}_2(z,\eta)\Phi = e^z\{\gamma^0\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial\eta} + Ma(\eta)\}\gamma^3\gamma^0\Phi = -ik\Phi. \tag{2.2.16}$$

It should be noticed that using the pairwise scheme of separation one has been able to reduce the problem of solving the Dirac equation to finding solutions of the decoupled system of Eqs. (2.2.15) and (2.2.16). A further problem arises when we try to separate variables in Eq. (2.2.16). Here it is not possible to reduce the problem to a set of two commuting first order differential operators. In order to separate variables in Eq. (2.2.16), we re-write it in the following form:

$$(\hat{L_1}\gamma^3\gamma^0 + \hat{L_2})\Phi = 0, \tag{2.2.17}$$

where $\hat{L_1}$,$\hat{L_2}$ are two commuting differential operators given by the expressions

$$\hat{L_1} = \gamma^0\frac{\partial}{\partial\eta} + Ma(\eta), \tag{2.2.18}$$

$$\hat{L_2} = \gamma^0\frac{\partial}{\partial\eta} + ike^z. \tag{2.2.19}$$

In order to separate variables in Eq. (2.2.19) we introduce the auxiliary spinor Y

$$(\hat{L_1}\gamma^3\gamma^0 + \tilde{L_2})Y = \Phi, \tag{2.2.20}$$

where the differential operator $\tilde{L_2}$ is given by the expression

$$\tilde{L_2} = \gamma^0\frac{\partial}{\partial z} - ike^z \tag{2.2.21}$$

substituting Eq. (2.2.21) into Eq. (2.2.19) we obtain that Y satisfies the following equation

$$\{\hat{M_1}, \hat{M_2}\} = 0 \tag{2.2.22}$$

with $[\hat{M_1}, \hat{M_2}] = 0$, and

$$(\hat{M_1} + \tilde{\lambda})Y = (-\frac{\partial^2}{\partial z^2} - i\gamma^0 ke^{2z} + \tilde{\lambda})Y = 0 \tag{2.2.23}$$

$$(\hat{M_2} - \tilde{\lambda})Y = (-\frac{\partial^2}{\partial\eta^2} - i\gamma^0 M\frac{da(\eta)}{d\eta} + M^2a^2(\eta) - \tilde{\lambda})Y = 0 \tag{2.2.24}$$

where $\tilde{\lambda}$ is separation constant. Introducing the new variable $u = 2ke^z$, we have that Eq. (2.2.24) can be written as

$$(\frac{\partial^2}{\partial u^2} + \frac{i}{2u}\gamma^0 - \frac{1}{4} + (\frac{1}{4} - \lambda)\frac{1}{u^2})S = 0, \tag{2.2.25}$$

where

$$u^{\frac{-1}{2}}S = y. \tag{2.2.26}$$

Choosing the following representation of the Dirac matrices,

$$\gamma^0 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix}. \tag{2.2.27}$$

We readily obtain that the spinor Φ has the following structure

$$[\sigma_1 \frac{\partial}{\partial y} - i\sigma_2(k_x - A_1(y))]\Phi_1 = ik\Phi_2, \tag{2.2.28}$$

$$[-\sigma_1 \frac{\partial}{\partial y} + i\sigma_2(k_x - A_1(y))]\Phi_2 = ik\Phi_1, \tag{2.2.29}$$

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \Phi = \begin{pmatrix} \phi(y) \\ F\sigma^3\phi(y) \end{pmatrix} exp(ik_x x) \tag{2.2.30}$$

where

$$\phi(y) = \begin{pmatrix} A(y) \\ B(y) \end{pmatrix} \tag{2.2.31}$$

Using the representation (2.2.27) we obtain that the solution of Eq. (2.2.25) can be written in terms of Whittaker functions

$$S_{1,2} = D_1 W_{-1/2,\sqrt{\lambda}(u)} + D_2 M_{-1/2,\sqrt{\lambda}(u)}, S_{3,4} = D_3 W_{1/2,\sqrt{\lambda}(u)} + D_4 M_{1/2,\sqrt{\lambda}(u)}, \tag{2.2.32}$$

where D_1 , D_2 , D_3 , D_4 do not depend on the variable u. Looking at (2.2.28) and (2.2.29) we have that, for regular solutions at u = 0, the spinor Y has the following structure:

$$Y = \begin{pmatrix} a(y)c_1(\eta)u^{-1/2}M_{+\frac{1}{2},\sqrt{\lambda}(u)} \\ b(y)c_1(\eta)u^{-1/2}M_{+\frac{1}{2},\sqrt{\lambda}(u)} \\ c(y)c_2(\eta)u^{-1/2}M_{-\frac{1}{2},\sqrt{\lambda}(u)} \\ d(y)c_2(\eta)u^{-1/2}M_{-\frac{1}{2},\sqrt{\lambda}(u)} \end{pmatrix} exp(ik_x x) \tag{2.2.33}$$

Substituting (2.2.33) into Eq. (2.2.20) and noticing that Eq. (2.2.24) is equivalent to the following system of equations

$$(\frac{\partial}{\partial \eta} - iMa(\eta)c_1(\eta)) = \sqrt{\lambda}c_2(\eta), \tag{2.2.34}$$

$$(\frac{\partial}{\partial \eta} + iMa(\eta)c_2(\eta)) = \sqrt{\lambda}c_1(\eta), \tag{2.2.35}$$

we obtain that the spinor Φ has the following structure

$$\Phi = \begin{pmatrix} A(\nu)c_1(\eta)u^{-z/2}M_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ B(\nu)c_1(\eta)u^{-z/2}M_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ iA(\nu)c_2(\eta)u^{-z/2}M_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ -iB(\nu)c_2(\eta)u^{-z/2}M_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \end{pmatrix} exp(ik_x x), \tag{2.2.36}$$

where $A(\nu)$ and $B(\nu)$ satisfy the system coupled system of equations

$$(\frac{d}{dy} - (k_x - A_1(y)))B = ikA, \tag{2.2.37}$$

$$(\frac{d}{dy} + (k_x - A_1(y)))A = ikB, \tag{2.2.38}$$

where

$$\nu = \frac{A_1 y - k_x}{\sqrt{A_1}}. \tag{2.2.39}$$

The corresponding solution of Eq. (2.2.36) in terms of the Whittaker functions $W_{k,\mu}(z)$ has the form

$$\Phi = \begin{pmatrix} i\sqrt{\tilde{\lambda}}A(\nu)c_1(\eta)u^{-z/2}W_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ -i\sqrt{\tilde{\lambda}}B(\nu)c_1(\eta)u^{-z/2}W_{-\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ A(\nu)c_2(\eta)u^{-z/2}W_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \\ B(\nu)c_2(\eta)u^{-z/2}W_{\frac{1}{2},\sqrt{\tilde{\lambda}}(2ke^z)} \end{pmatrix} exp^{i(k_x x + k_y y + k_z z)} \tag{2.2.40}$$

Let us look for solutions of the system (2.2.37) and (2.2.38) when the electromagnetic potential has the simple functional dependence $A_1(y) = A_1 y$. In this case one can obtain exact solutions for $A(\nu)$ and $B(\nu)$ in terms of hypergeometric functions. After making the change of variable (2.2.39) and using the recurrence relations

$$(b-1)M(a, b-1, z) = (b-1)M(a, b, z) + z\frac{dM(a, b, z)}{dz}, \tag{2.2.41}$$

$$\frac{1}{a}\frac{dM(a, b, z)}{dz} + M(a, b, z) = M(a+1, b, z), \tag{2.2.42}$$

$$\frac{dU(a, b, z)}{dz} - U(a, b, z) = -U(a, b+1, z), \tag{2.2.43}$$

we find that the general solution of the system of equations (2.2.37) and (2.2.38) reads

$$A = \sqrt{\frac{2A_1}{ik}} e^{-1/2}\nu^2 (c_1 M(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{1}{2}, \nu^2)) + c_2 U(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{1}{2}, \nu^2)), \quad (2.2.44)$$

$$B = e^{-\frac{1}{2}\nu^2} \nu (c_1 M(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{3}{2}, \nu^2)) + c_2 U(-\frac{k^2}{4A_1} + \frac{1}{2}, \frac{3}{2}, \nu^2)). \quad (2.2.45)$$

2.3 Dirac Equation and it's solution In Robertson-Walker Spacetime

Consider the following spatially flat isotropically changing RW metrics given by[8,13]

$$ds^2 = dt^2 - a^2(t) dx^i dx_j, \quad (2.3.1)$$

where $a(t)$ is the scale factor of the expanding Universe. we suppose that the cosmological scale factor has an arbitrary time dependence that asymptotically approaches constant values at early and late values of the cosmic time to this cosmic time t is the proper time of a set of clocks on a geometric worldlines that remains at constant values of spatial coordinates (x,y,z) we take[26]

$$a(t) \sim \begin{cases} a_1, t \to -\infty \\ a_2, t \to +\infty \end{cases}, (2.3.2)$$

interms of the conformal time parameter given by

$$\eta = \int_t \frac{dt}{a(t)}, \quad (2.3.3)$$

the line element (2.3.1) to be

$$ds^2 = a^2(\eta)(d\eta^2 - dx^i dx_i). \quad (2.3.4)$$

For a particle of charge eQ, the Dirac equation in presence of a magnetic field is given by[27,28]

$$(i\gamma^\mu \partial_\mu - ma)\psi = 0;$$

$$(i\gamma^0 \vec{\alpha}.(\vec{\nabla} - ieQ\vec{A}) - \beta ma)\psi = i\gamma^0 \partial_0 \psi, \quad (2.3.5)$$

where $\vec{\alpha}$, γ^0 and β are the Dirac matrices,a is expantion and $\vec{A}$ is the vector potential. In our convention, e is the positive unit of charge, taken as usual to be equal to the proton charge. For stationary states. we can write,

$$\psi = e^{iE_c t}\begin{pmatrix} \Phi \\ \chi \end{pmatrix}, \tag{2.3.6}$$

where ϕ and χ are 2-component objects. We use the Pauli-Dirac representation of the Dirac matrices, in which

$$\alpha = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.3.7}$$

where each block represents a 2 x 2 matrix, and $\vec{\sigma}$ are the Pauli matrices. With this notation, we can write Eq. (2.3.5) as

$$(E_c - m)\phi = \frac{1}{a}\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})\chi, \tag{2.3.8}$$

$$(E_c + m)\chi = \frac{1}{a}\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})\Phi. \tag{2.3.9}$$

Eliminating χ, we obtain

$$(E_c^2 - m^2)^2 a^2 \phi = [\vec{\sigma}.(-i\vec{\nabla} - eQ\vec{A})]^2\Phi. \tag{2.3.10}$$

We will work with a constant magnetic field $\vec{B}$. Without loss of generality, it can be taken along the z-direction. The vector potential can be chosen in many equivalent ways. We take the external magnetic field $A_\mu = (0, A(x))$ is a c number and the assymetry gauge is taken here

$$A(x) = (0, -By, 0). \tag{2.3.11}$$

With this choice, Eq. (2.3.10) reduces to the form

$$(E_c^2 a^2 - m^2 a^2)^2 \phi = [-\vec{\nabla}^2 + (eQB)^2 y^2 - eQB(2iy\frac{\partial}{\partial x} + \sigma_z)]\phi. \tag{2.3.12}$$

Noticing that the co-ordinates x and z do not appear in the equation except through the deriva tives, we can write the solutions as

$$\phi = e^{i\vec{P}.\vec{X}_y} f(y), \tag{2.3.13}$$

where $f(y)$ is a 2-component matrix which depends only on the y-coordinate, and possibly some momentum components, as we will see shortly. We have also introduced the notation $\vec{X}$ for the spatial co-ordinates (in order to distinguish it from x, which is one of the components of $\vec{X}$), and $\vec{X}_y$ for the vector X with its y-component set equal to zero. In other words, $\vec{p}.\vec{X}_y = p_x x + p_z z$, where p_x and p_z denote the eigenvalues of momentum in the x and z directions. There will be two independent solutions for $f(y)$, which can be taken, without any loss of generality, to be the eigenstates of σ_z with eigenvalues $s = \pm 1$. This means that we choose the two independent solutions in the form

$$f_+(y) = \begin{pmatrix} F_+(y) \\ 0 \end{pmatrix}, f_-(y) = \begin{pmatrix} 0 \\ F_-(y) \end{pmatrix}. \tag{2.3.14}$$

Since $\sigma_z F_s = sF_s$, the differential equations satisfied by F_s is

$$\frac{d^2F_s}{dy^2} - (eQB_y + p_z)^2 F_s + (E_c^2 a^2 - m^2 a^2 - p_z^2 + eQBs)F_s = 0, \tag{2.3.15}$$

which is obtained from Eq. (2.3.12). The solution is obtained by using the dimensionless variable

$$\xi = \sqrt{eQB}(y + \frac{p_x}{eQB}), \tag{2.3.16}$$

which transforms Eq. (2.3.15) to the form

$$\frac{d^2F_s(\xi)}{d\xi^2} - \xi^2 F_s(\xi) + a_s f_s(\xi) = 0, \tag{2.3.17}$$

where

$$a_s = \frac{(E_c^2 a^2 - m^2 a^2 - p_z^2 + eQBs)}{eQB}. \tag{2.3.18}$$

This is a special form of Hermites equation, and the solutions exist provided $a_s = 2\nu + 1$ for $\nu = 0, 1, 2,$ [29]. This provides the energy eigenvalues

$$E_c^2 = \frac{1}{a^2}(m^2 a^2 + p_z^2 - eBQs + eB(2\nu + 1)), \tag{2.3.19}$$

and the solutions for F_s are

$$F_s(\xi) = (\frac{\sqrt{eQB}}{2^\nu \nu! \pi^{1/2}})^{1/2} e^{-\xi^2/2} H_\nu(\xi) = I_\nu(\xi), \tag{2.3.20}$$

where H_ν are Hermite polynomials of order ν, and N_ν are normalizations which we take to be

$$N_\nu = (\frac{\sqrt{e|Q|B}}{2^\nu \nu! \pi^{1/2}})^{1/2}. \tag{2.3.21}$$

We stress that the choice of normalization can be arbitrarily made, as will be clarified later. With our choice, the functions I_ν satisfy the completeness relation

$$\sum_\nu I_\nu(\xi) I_\nu(\xi_*) = \sqrt{e|Q|B}\delta(\xi - \xi_*) = \delta(y - y_*), \tag{2.3.22}$$

where ξ_* is obtained by replacing y by y_* in Eq. (2.3.16). So far, Q was arbitrary. We now specialize to the case of electrons, for which $Q = 1$. The solutions are then conveniently classified by the energy eigenvalues

$$E_{cn}^2 = \frac{1}{a^2}(m^2 a^2 + p_z^2 + 2neB), \tag{2.3.23}$$

which is the relativistic form of Landau energy levels. The solutions are two fold degenerate in general: for $s = 1$, $\nu = n - 1$ and for $s = -1$, $\nu = n$. In the case of $n = 0$, only the second solution is available since ν cannot be negative. The solutions can have positive or negative energies. We will denote the positive square root of the right side by E_{cn} . Representing the solution corresponding to this n^{th} Landau level by a superscript n, we can then write for the positive energy solutions,

$$f_+^n = \begin{pmatrix} f_+(\xi) \\ 0 \end{pmatrix} = \begin{pmatrix} I_\nu(\xi) \\ 0 \end{pmatrix} = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \end{pmatrix},$$

$$f_-^n = \begin{pmatrix} 0 \\ f_-(\xi) \end{pmatrix} = \begin{pmatrix} 0 \\ I_\nu(\xi) \end{pmatrix} = \begin{pmatrix} 0 \\ I_n(\xi) \end{pmatrix}. \tag{2.3.24}$$

For $n = 0$, the solution f_+ does not exist. We will consistently incorporate this fact by defining

$$I_{-1}(y) = 0, \tag{2.3.25}$$

in addition to the definition of In in Eq. (2.3.20) for non-negative integers n. The solutions in Eq. (2.3.24) determine the upper components of the spinors through Eq. (2.3.13). The lower components, denoted by χ earlier, can be solved using

Eq. (2.3.9), and finally the positive energy solutions of the Dirac equation can be written as

$$e^{-ip.X_y} U_s(y, n, \vec{p_y}), \tag{2.3.26}$$

where X^ν denotes the space-time coordinate. And U_s are given by

$$U_+(y, n, \vec{p_y}) = \begin{pmatrix} I_{n-1}(\xi) \\ 0 \\ \frac{p_z}{a(E_{cn}+m)} I_{n-1}(\xi) \\ -\frac{\sqrt{2neB}}{a(E_{cn}+m)} I_n(\xi) \end{pmatrix}, U_-(y, n, \vec{p_y}) = \begin{pmatrix} 0 \\ I_n(\xi) \\ -\frac{\sqrt{2neB}}{a(E_{cn}+m)} I_{n-1}(\xi) \\ -\frac{p_z}{a(E_{cn}+m)} I_n(\xi) \end{pmatrix}. \tag{2.3.27}$$

A similar procedure can be adopted for negative energy spinors which have energy eigenvalues $E = E_{cn}$. In this case, it is easier to start with the two lower components first and then find the upper components from Eq. (2.3.8). The solutions are

$$e^{-ip.X_y} V_s(y, n, \vec{p_y}), \tag{2.3.28}$$

Where

$$V_+(y, n, \vec{p_y}) = \begin{pmatrix} \frac{p_z}{a(E_{cn}+m)} I_{n-1}(\tilde{\xi}) \\ \frac{\sqrt{2neB}}{a(E_{cn}+m)} I_n(\tilde{\xi}) \\ I_{n-1}(\tilde{\xi}) \\ 0 \end{pmatrix}, V_-(y, n, \vec{p_y}) = \begin{pmatrix} \frac{\sqrt{2neB}}{a(E_{cn}+m)} I_{n-1}(\tilde{\xi}) \\ -\frac{p_z}{a(E_{cn}+m)} I_n(\tilde{\xi}) \\ 0 \\ I_n(\tilde{\xi}) \end{pmatrix}, \tag{2.3.29}$$

where $\tilde{\xi}$ is obtained from ξ by changing the sign of the p_x-term.

2.4 The Dirac field propagator of the electron

It is obvious that the spinors $u_{s,n}(y, p), v_{s,n}(y, p)$,$s = 1, 2$ form a complete and orthogonalized basis in the four-dimensional vector space. Therefore the dirac field operator can be expanded as(for the rest of the calculation we used the gauge $A(x) = (0, Bx_1, 0)$

$$\psi(x) = \sum_{n=0}^{\infty} \sum_{s=1}^{2} \int \frac{dp_2 dp_3}{2\pi} [c_{s,n}(p, t) u_{s,n}(y, p) + d^+_{s,n}(p, t) v_{s,n}(y, p)] exp(ip_1 x_1 + ip_3 x_3), \tag{2.4.1}$$

$$\psi^+(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\int \frac{dp_2 dp_3}{2\pi}[c^+_{s,n}(p,t)u^+_{s,n}(y,p)+d_{s,n}(p,t)v^+_{s,n}(y,p)]exp(-ip_1x_1-ip_3x_3). \tag{2.4.2}$$

From the commutation relation

$$|\psi_\alpha(x,t),\psi^+_\beta(y,t)| = \delta_{\alpha\beta}\delta^3(x-y), \tag{2.4.3}$$

the commutation relations between the annihilation and destruction operators can be derived as

$$\{c_{s,n}(p_1,t), c^+_{r,m}(p_2,t)\} = \delta_{sr}\delta_{nm}\delta^3(p_1-p_2), \tag{2.4.4}$$

$$\{d_{s,n}(p_1,t), d^+_{r,m}(p_2,t)\} = \delta_{sr}\delta_{nm}\delta^3(p_1-p_2). \tag{2.4.5}$$

Therefore the Feynman propagator of an electron is defined by

$$S_F(x,y) = -i\langle 0|T\psi(x)\bar{\psi}(y)|0\rangle. \tag{2.4.6}$$

Now substituting Eq. (2.4.1) and (2.4.2) into Eq. (2.4.6)gives

$$S_F(x,y) = -i\langle 0|\sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L^2}\sum_{p_2,p_3}[c_{s,n}(p,t)u_{s,n}(x_1,p)c^+_{s,n}(p,t)u^+_{s,n}(y_1,p)]$$
$$\times exp(ip_2(x_2-y_2)+ip_3(x_3-y_3))\gamma^0|0\rangle. \tag{2.4.7}$$

Then employing the commutation and anticommutation relations and summation over s gives after some algebra

$$S_F(x,y) = -\frac{i}{L^2}\sum_{n=0}^{\infty}\sum_{p_2,p_3}[u_{1,n}(x_1,p)u^+_{1,p}(y_1,p)+u_{2,p}(x_1,p)u^+_{2,p}(y_1,p)$$
$$\times exp(ip_2(x_2-y_2)+ip_3(x_3-y_3))\gamma^0], \tag{2.4.8}$$

where operators are all the interaction operators. This can further be simlified to give

$$S_F(x,y) = \frac{1}{L^2}\sum_{p_2,p_3}\int\frac{d\omega}{2\pi}\sum_{n=0}^{\infty}\frac{S_n(x_1,y_1,p)}{(\omega+i\epsilon)(E_n+m)}$$
$$\times exp(-i\omega(t_x-t_y)+ip_2(x_2-y_2)+ip_3(x_3-y_3)), \tag{2.4.9}$$

Where

$$S_n(x_1, y_1, p) = \begin{pmatrix} (E_n+m)I_{n-1}I_{n-1} & 0 & -p_3 I_{n-1}I_{n-1} & i\sqrt{2neB}I_{n-1}I_n \\ 0 & (E_n+m)I_nI_n & -i\sqrt{2neB}I_nI_{n-1} & p_3I_nI_n \\ p_3I_nI_n & -i\sqrt{2neB}I_{n-1}I_n & -(E_n-m)I_{n-1}I_{n-1} & 0 \\ i\sqrt{2neB}I_nI_{n-1} & -p_3I_nI_n & 0 & -(E_n-m)I_nI_n \end{pmatrix} \tag{2.4.10}$$

in which I_nI_m is the abbrevation of $I_n(x_1, k_2)I_m(y_1, k_2)$ and E_n stands for E_{cn} given by Eq. (2.3.23).

In the initial Minkowski spacetime the metric is that of Eq. (3.2.1) with $a(t) = a_1$. The coordinates can be rescaled with $x'^i = a_1 x^i$, so that we have the usual Minkowski metric, and x'^i is the physical or measured distance. The appropriate physical momentum is then taken $k'^i = k^i/a_1$, and the physical energy of the particle is $|\vec{k'}| = k/a_1$. The physical volume is $a_1^3 V$, and in the final Minkowski metric the physical quantities will have the same form but in this case a_2 replaces a_1.

Chapter 3

Inverse Compton Scattering in curved spacetime in the presence of strong magnetic field

In many scattering problems in curved space time most authors use the plane wave solution for a free particle not affected by gravitational fields [30,31]. These authors introduce the effect of gravitational field as a perturbation in the expansion of the S-matrix. In our case we have the quantized fields in a strong magnetic field and the curvature effects are neglected from the S-matrix i.e. a spatially flat RW space time behaves like a Minkowskian space time in the asymptotic regions. Hence, we can use the vacuum state of the Minkowskain space time for the creation of particles. We have not included the Bogolibov transformation for the creation and annihilation operators of the in and out modes.

3.1 The S-matrix element

Charged particles residing on excited Landau levels have a much short life-time τ due to the cyclotron radiation, $\tau \leq 10^{-15}$. Hence, it is worth to assume that an electron either before or after the scattering should reside on the ground Landau level.The ground Landau level is not degenerate due to $u_{1,0}(x_1, p) = 0$, therefore the initial and final states of an electron-photon scattering system can be represented by

$$|i, t_i\rangle = c_0^+(a_i, p_i, t_i) a_{\lambda_i}^+(k_i, t_i)|0\rangle, \tag{3.1.1}$$

$$|f, t_f\rangle = c_0^+(a_f, p_f, t_f) a_{\lambda_f}^+(k_f, t_f)|0\rangle. \tag{3.1.2}$$

In the above two expressions c_0^+ denotes $c_{2,0}^+$, $p_{i(f)}$ and $a_{i(f)}$ means that the incident (outgoing) electron has a momentum $p_{i(f)}$ in the direction of the magnetic field and the center of the Landau orbit is $-\lambda^2 a_{i(f)}$. We stress here that the operators in these two expressions are Heisenberg ones, but not the free ones(interactions have been considerd).The scattering matrix can be generally expressed as

$$S_{fi} = \lim_{t_i \to -\infty, t_f \to \infty} \langle 0|T[c_0(a_f, p_f, t_f) a_{\lambda_f}(k_f, t_f) c_0^+(a_i, p_i, t_i) a_{\lambda_i}^+(k_i, t_i)]|0\rangle. \tag{3.1.3}$$

Introducing the incoming interaction picture and making pertubation expanisions, this becomes under Born approximation[2]

$$\begin{aligned} S_{fi} = \lim_{t_i \to -\infty, t_f \to \infty} e^2 \int a_1^3 d^4x_1 a_2^3 d^4x_2 \langle 0|T c_0(a_f, p_f, t_f)\bar{\psi}(x_1)|0\rangle \gamma_\mu \langle 0|T\psi(x_1)\bar{\psi}(x_2)|0\rangle \gamma_\nu \\ *[\langle 0|T a_{\lambda_f}(k_f, t_f) A_\mu(x_1)|0\rangle \langle 0|T a_{\lambda_i}^+(k_i, t_i) A_\nu(x_2)|0\rangle \\ +\langle 0|T a_{\lambda_f}(k_f, t_f) A_\nu(x_2)|0\rangle \langle 0|T a_{\lambda_i}^+(k_i, t_i) A_\mu(x_1)|0\rangle] \\ *\langle 0|T\psi(x_2) c_0^+(a_i, p_i, t_i)|0\rangle, \end{aligned} \tag{3.1.4}$$

where a_1 and a_2 are expanision constants of the remote past and the remote future of the cosmic time.

It is convenient to impose into the normilization constant periodic boundary conditions in a box having sides of coordinate length L and coordinate volume $V = L^3$, for the cancellation of the volume factor V. So now, introducing all the operators into Eq. (3.1.4) and manupilating all the scalar products by imposing the commutation relation between the annihilation and creation operators(after the same mathematical trick as in [32]) the scattering matrix reads

$$S_{fi} = \frac{(2\pi)^2}{VL^2} \frac{e^2 a_1^3 a_2^3}{sqrt(4\omega_i \omega_f)} [(\frac{(E_i + m)(E_f + m)}{4E_i E_f})^{1/2} exp[-\frac{\lambda^2}{4}(\omega_i^2 \sin^2\theta_i + \omega_f^2 \sin^2\theta_f)]$$

$$\times exp[-i\lambda^2 a_i(k_{ix}-k_{fx})+i\frac{\lambda^2}{2}(k_{ix}k_{iy})+k_{fx}k_{fy}]X$$

$$\times\delta(E_i+\omega_i-E_f+\omega_f)\delta(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f)\delta(a_i+k_{iy}-a_f-k_{fy}), \quad (3.1.5)$$

where θ_i and θ_f are incoming and outgoing angles of the electron and X is

$$X=\sum_{n=0}^{\infty}\frac{1}{n!}[(\frac{\lambda^2 k_i^- k_f^+}{2})^n exp(i\lambda^2 k_{iy}k_{fx})(\frac{X_1}{(E_i+\omega_i)^2-E_{i,n+1}^2}+\frac{X_2}{(E_i+\omega_i)^2-E_{f,n}^2})$$

$$+(\frac{\lambda^2 k_i^- k_f^+}{2})^n exp(i\lambda^2 k_{ix}k_{fy})(\frac{X_i'}{(E_i-\omega_f)^2-E_{f,n+1}^2}+\frac{X_2'}{(E_i-\omega_f)^2-E_{f,n}^2})], \quad (3.1.6)$$

in which $k_i^\pm=k_{ix}\pm ik_{iy}$, $k_f^\pm=k_{fx}\pm ik_{fy}$,

$$E_{i,n}^2=\frac{1}{a_1^2}(m^2+(p_i+\omega_i\cos\theta_i)^2+2neB), \quad (3.1.7)$$

$$E_{f,n}^2=\frac{1}{a_2^2}(m^2+(p_i-\omega_f\cos\theta_f)^2+2neB), \quad (3.1.8)$$

and X_i, X_i', $i=1,2$ are given by

$$X_1=[\frac{(E_i+\omega_i+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)}+(E_i+\omega_i-m)]e_i^-e_f^+$$

$$+(\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m})$$

$$\times[k_f^+e_{fz}e_i^-+k_i^-e_{iz}e_f^+-(p_i+\cos\theta_i)e_f^+e_i^-], \quad (3.1.9)$$

$$X_2=[\frac{(E_i+\omega_i+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)}+(\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m})$$

$$\times(p_i+\omega_i\cos\theta_i)+(E_i+\omega_i-m)]e_{fz}e_{iz}, \quad (3.1.10)$$

$$X_1'=(\frac{(E_i-\omega_f+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_f+m)(E_i+m)}+(E_i-\omega_f-m))e_i^+e_f^-$$

$$-(\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f}+m)$$

$$\times[k_i^+e_{iz}e_f^-+k_f^-e_{fz}e_i^++(p_i-\omega_f\cos\theta_f)e_i^+e_f^-], \quad (3.1.11)$$

$$X_2'=[\frac{(E_i-\omega_f+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_f+m)(E_i+m)}+(E_i-\omega_f-m)$$

$$+(\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m})(p_i-\omega_f\cos\theta_f)]e_{iz}e_{fz}, \qquad (3.1.12)$$

where e_i and e_f are polarization of the incident and scattered photons and

$$e_i^{\pm}=e_{ix}\pm ie_{iy}, e_f^{\pm}=e_{fx}\pm ie_{fy}.$$

3.2 The Transition Amplitude and Transition Rate

The diffeferential cross section is proportional to $|S_{fi}|^2$; the transition amplitude, and thus squaring Eq. (3.1.5) and neglecting unimportant phase factors the transition amplitude becomes;

$$|S_{fi}|^2=\frac{(2\pi)^2}{L^4V^2}\frac{e^4a_1^6a_2^6}{4\omega_i\omega_f}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)]$$
$$\times exp[-i\lambda^2a_i(k_{ix}-k_{fx})-i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]|X|^2$$
$$\times\delta^2(E_i+\omega_i-E_f-\omega_f)\delta^2(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f)$$
$$\times\delta^2(a_i-k_{iy}-a-f-k_{fy}). \qquad (3.2.1)$$

In the expressions X_1, X_2, X_1' and X_2' employing the conservation of momentum, i.e. $p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f=p_f$, we get the following,

$$X_1=\{[A-B(p_i+\omega_i\cos\theta_i)]e_f^+e_i^-+B(k_f^+e_{fz}e_i^-+k_i^-e_{iz}e_f^+)\}(E_f+m)^{-1}(E_i+m)^{-1}, \qquad (3.2.2)$$

$$X_2=\{[A+B(p_i+\omega_i\cos\theta_i)]e_{fz}e_{iz}\}(E_f+m)^{-1}(E_i+m)^{-1}, \qquad (3.2.3)$$

$$X_1'=\{[A'-B(p_i-\omega_f\cos\theta_f)e_f^-e_i^+-B(k_f^-e_{fz}e_i^++k_i^+e_{iz}e_f^-)\}(E_f+m)^{-1}(E_i+m)^{-1}, and \qquad (3.2.4)$$

$$X_2'=\{[A'+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}\}(E_f+m)^{-1}(E_i+m)^{-1}. \qquad (3.2.5)$$

In the above two expressions the coefficients A, A', B are defined by

$$A=(E_i+\omega_i+m)p_ip_f+(E_i+\omega_i-m)(E_i+m)(E_f+m), \qquad (3.2.6)$$

$$A' = (E_i - \omega_f + m)p_i p_f + (E_i - \omega_f - m)(E_i + m)(E_f + m), \tag{3.2.7}$$

and

$$B = p_i(E_f + m) + p_f(E_i + m) \tag{3.2.8}$$

respectively.

Substituting these results back into the expression for X, one can get after some algebra;

$$X = \sum_{n=0}^{\infty} \frac{1}{n!}[(\frac{\lambda^2\omega_i\omega_f \sin\theta_i \sin\theta_f}{2} exp(-i(\phi_i - \phi_f)))^n exp(i\lambda^2\omega_i\omega_f \sin\theta_i \sin\theta_f \cos\phi_f \sin\phi_i)$$

$$\times(\frac{[A - B(p_i + \omega_i\cos\theta_i)]e_f^+ e_i^- + B(k_f^+ e_{fz} e_i^- + k_i^- e_{iz} e_f^+)(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i + \omega_f)^2 - E_{i,n+1}^2}$$

$$+\frac{[A + B(p_i + \omega_i\cos\theta_i)]e_{fz}e_{iz}(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i + \omega_f)^2 - E_{i,n}^2})$$

$$+(\frac{\lambda^2\omega_i\omega_f \sin\theta_i \sin\theta_f}{2} e^{i(\phi_i - \phi_f)})^n exp(i\lambda^2\omega_i\omega_f \sin\theta_i \sin\theta_f \sin\phi_f \cos\phi_i)$$

$$\times(\frac{[A' - B(p_i - \omega_f\cos\theta_f)e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^-)(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i - \omega_f)^2 - E_{i,n+1}^2}$$

$$+[\frac{[A' + B(p_i - \omega_f\cos\theta_f)]e_{fz}e_{iz}(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i - \omega_f)^2 - E_{f,n}^2})]. \tag{3.2.9}$$

Now defining $Y = (Y_1 + Y_2) = X(E_f + m)(E_i + m)$ i.e. $X = \frac{Y}{(E_f+m)(E_i+m)}$ implying,

$$|X|^2 = \frac{|Y|^2}{(E_f + m)^2(E_i + m)^2}. \tag{3.2.10}$$

And substituting $|X|^2$ interms of $|Y|^2$ into $|S_{fi}|^2$ gives;

$$|S_{fi}|^2 = \frac{(2\pi)^2}{L^4V^2}\frac{e^4 a_1^6 a_2^6}{4\omega_i\omega_f}[\frac{(E_i + m)(E_f + m)}{4E_iE_f}]exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)]$$

$$\times exp[-i\lambda^2 a_i(k_{ix} - k_{fx}) - i\lambda^2(k_{ix}k_{iy} + k_{fx}k_{fy})]\frac{|Y|^2}{(E_f + m)^2(E_i + m)^2}$$

$$\times\delta^2(E_i + \omega_i - E_f - \omega_f)\delta^2(p_i + k_i\cos\theta_i - p_f - k_f\cos\theta_f)$$

$$\times\delta^2(a_i - k_{iy} - a - f - k_{fy}). \tag{3.2.11}$$

3.3 The Scattering Cross Section

In deriving the scattering cross-section, we take T and V finite, to begin with. In this case, the transition probability per unit time

$$w = |S_{fi}|^2/T \tag{3.3.1}$$

involves the factor of the type $[\delta_{\gamma\nu}(\sum p'_f - \sum p_i)]^2$. For large values of T and V we can then take

$$\delta_{TV}(\sum p'_f - \sum p_i) = (2\pi)^4\delta^(4)(\sum p'_f - \sum p_i), \tag{3.3.2}$$

and

$$[\delta_{TV}(\sum p'_f - \sum p_i)]^2 = TV(2\pi)^4\delta^(4)(\sum p'_f - \sum p_i), \tag{3.3.3}$$

with errors which tends to zero as $T \to \infty$ and $V \to \infty$. To find the transition probability in the intervals $(p'_f, p'_f + dp'_f)$, $f = 1, ..., N$, we must multiply w by the number of these states which is

$$\Pi_f V \frac{d^3 p'_f}{(2\pi)^3}. \tag{3.3.4}$$

We take in Eq. (3.3.4) the final states of the scattered photon and electron to be $\frac{Vd^3\omega_f}{(2\pi)^3} = \frac{V\omega_f^2 d\Omega_f}{(2\pi)^3}$ and $\frac{Vd^3p_f}{(2\pi)^3}$ respectively.

The differential cross-section is the transition rate into this group of final states for one scattering center and unit incident flux. With our choice of normilization for the stares, the volume V which we are concidering contains v_{rel}/V where v_{rel} is the relative velocity of the colliding particles. Combining these results with the equation for w, we obtain the required exppresion for the differential cross-section

$$d\sigma = w\frac{V}{v_{rel}}\frac{V\omega_f^2 d\Omega_f}{(2\pi)^3}\frac{Vd^3p_f}{(2\pi)^3}, \tag{3.3.5}$$

where $v_{rel} = \frac{(E_i - p_i \cos\theta_i)}{E_i}$. Therefore, substituting the expression for w in the above equation and using the usual rule$(L^3 = V)$,

$$|\delta(E_i + \omega_i - E_f - \omega_f)|^2 = \frac{T}{2\pi}\delta(E_i + \omega_i - E_f - \omega_f), \tag{3.3.6}$$

and

$$|\delta(p_i + k_i\cos\theta_i - p_f - k_f\cos\theta_f)|^2 = \frac{L}{2\pi}\delta(p_i + k_i\cos\theta_i - p_f - k_f\cos\theta_f), \quad (3.3.7)$$

the differential cross-section becomes

$$d\sigma = \frac{(2\pi)^6 m^2 r_0^2 a_1^6 a_2^6}{V^3 4\omega_i\omega_f}(\frac{1}{E_f})exp(\frac{-\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f))$$
$$\times exp(-2i\lambda^2 a_i(k_{ix} - k_{fx}) - i\lambda^2(k_{ix}k_{iy} + k_{fx}k_{fy}))$$
$$\frac{|Y|^2}{(E_i+m)(E_f+m)}\frac{1}{(E_i - p_i\cos\theta_i)}\frac{V^3\omega_f^2 d\Omega_f}{(2\pi)^3}\frac{d^3p_f}{(2\pi)^3}\delta^2(a_i - k_{iy} - a_f - k_{fy}), \quad (3.3.8)$$

where $|Y|^2$ is the exppresion given in Eq. (3.2.10) and $r_0^2 = e^4/16\pi^2 m^2$ is the classical electron radius.

Integrating Eq. (3.3.8) over p_f' by letting

$\vec{k_i} = \omega_i(\sin\theta_i\cos\phi_i, \sin\theta_i\sin\phi_i, \cos\theta_i)$, and $\vec{k_f} = \omega_f(\sin\theta_f\cos\phi_f, \sin\theta_f\sin\phi_f, \cos\theta_f)$ and therefore,

$$(\frac{\lambda^2}{2}k_i^- k_f^+)^n = [\frac{\lambda^2}{2}\omega_i\omega_f\sin\theta_i\sin\theta_f e^{-i(\phi_i-\phi_f)}]^n \quad (3.3.9)$$

and

$$(\frac{\lambda^2}{2}k_i^+ k_f^-)^n = [\frac{\lambda^2}{2}\omega_i\omega_f\sin\theta_i\sin\theta_f e^{i(\phi_i+\phi_f)}]^n, \quad (3.3.10)$$

where $k_i^- k_f^+ = (k_{ix} - ik_{iy})(k_{fx} + ik_{fy})$, the differential cross-section will becomes to be

$$\frac{d\sigma}{d\Omega_f} = \frac{r_0^2}{4}\frac{\omega_f}{\omega_i}\frac{m^2 a_1^6 a_2^6}{(E_i+m)(E_f+m)}\frac{exp[\frac{-\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)]|Y|^2}{(E_i - p_i\cos\theta_i)(E_f - (p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)\cos\theta_f)}. \quad (3.3.11)$$

3.4 The Scattering Cross-Section in the electron's rest frame

To simplify calculations, we choose the coordinate system which enables the momentum $\vec{k_i}$ of the incident photon lying within its x-z plane, i.e. $\phi_i = 0$. denoting

$\phi_f = \phi$, we then obtain

$$\vec{k_i} = \omega_i(\sin\theta_i, 0, \sin\theta_i), \vec{k_f} = \omega_f(\sin\theta_f \cos\phi, \sin\theta_f \sin\phi, \cos\phi) \quad (3.4.1)$$

Accordingly, polarization of the incident and scatterd photons can be taken as

$$e_i^{(1)} = (-\cos\theta_i, 0, \sin\theta_i), e_i^{(2)} = (0, -1, 0), \quad (3.4.2)$$

$$e_f^{(1)} = (-\cos\theta_f \cos\phi, -\cos\theta_f \sin\phi, \sin\theta_f), e_f^{(2)} = (\sin\phi, -\cos\phi, 0). \quad (3.4.3)$$

Averaging over the polarization of incident photons and summing over those of scatterd photons, the differential cross section will be;

$$\frac{d\sigma}{d\Omega_f} = \frac{1}{2} \sum_{e_i^{(1)}, e_f^{(1)}, e_i^{(2)}, e_f^{(2)}} \frac{r_0^2}{4} \frac{\omega_f}{\omega_i} \frac{m^2 a_1^6 a_2^6}{(E_i + m)(E_f + m)}$$
$$\times \frac{exp[\frac{-\lambda^2}{2}(\omega_i^2 \sin^2\theta_i + \omega_f^2 \sin^2\theta_f)]|Y|^2}{(E_i - p_i\cos\theta_i)(E_f - (p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)\cos\theta_f)}. \quad (3.4.4)$$

Now we define reduced quantities as

$$\Delta_i = \frac{\omega_i}{m}, \Delta_f = \frac{\omega_f}{m}, \Delta_0 = \frac{\omega_0}{m}, \quad (3.4.5)$$

where $\omega_0 = eB/m$ is the cyclotron frequency. We denote the reduced Doppler frequencies by

$$\Delta_{ir} = \gamma_i(1 - \beta_i\cos\theta_i)\Delta_i, \Delta_{fr} = \gamma_f(1 - \beta_f\cos\theta_f)\Delta_f. \quad (3.4.6)$$

We can now write Eq. (3.4.4) interms of Eqs. (3.4.5) and (3.4.6), by keeping the effect of Eq. (2.3.2)

$$\frac{d\sigma}{d\Omega_f} = \sum_{e_i^{(1)}, e_f^{(1)}, e_i^{(2)} e_f^{(2)}} \frac{|Y|^2}{m^2} \frac{r_0^2 a_1^6 a_2^6}{8} \frac{\Delta_f}{\Delta_{ir}(1+\gamma_i)(1+\gamma_i+\Delta_i-\Delta_f)}$$
$$\times \frac{e^{\frac{-B_c}{2B}(\Delta_f^2 \sin^2\theta_f + \Delta_i^2 \sin^2\theta_i)}}{[\gamma_i(1 - \beta_i\cos\theta_f) + \Delta_i(1 - \cos\theta_i\cos\theta_f) - \Delta_f \sin^2\theta_f]}, \quad (3.4.7)$$

in which $B_c = \frac{m^2}{e} \approx 4.414 \times 10^9 T$ is called the critical magnetic field and where $Y = Y_1 + Y_2$ and is given by:

$$\frac{1}{m^2}(|Y(1i \to 1f)|^2 + |Y(1i \to 2f)|^2 + |Y(2i \to 1f)|^2 + |Y(2i \to 2f)|^2. \quad (3.4.8)$$

Here $\lambda_i \to \lambda_f$,$\lambda_{i(f)} \to 1_{i(f)}, 2_{i(f)}$ represents the scattering from the polarization $\lambda_i \to \lambda_f$. Defining $\xi = \frac{B_c}{2B}\Delta_i\Delta_f \sin\theta_i \sin\theta_f$ and $\eta = 2\xi \sin\phi'$ and substituting the polarization of the incident and scatterd photons and integrating over ϕ, the most simlified expession for $\sum |Y|^2/2\pi$ interms of the reduced quantities defined in Eq. (3.4.5) and Eq. (3.4.6) will become for $n = 0$;

$$Y_r = \frac{|Y|^2}{2\pi m^2} = C_1^2 S_{i,1}^2 + C_2 S_{f,1}^2 + [(A_+ S_{i,0} - A'_- S_{f,0})^2 + 2(1 - J_0(\xi))A_+ A'_+ S_{i,0} S_{f,0}]$$

$$(\sin\theta_i \sin\theta_f)^2 + 2[(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)(A'_- \cos\theta_f + B_1)$$

$$-D_1 D_2 - A_- A'_-] J_2(-\xi) S_{i,1} S_{f,1} - 2[D_1 A'_+ S_{i,1} S_{f,0} + D_2 A_+ S_{i,0} S_{f,1}] \sin\theta_i \sin\theta_f J_1(-\xi), \quad (3.4.9)$$

where;

$$S_{i,n+1} = \frac{1}{(2(\Delta_{ir} - n\Delta_0) + \Delta_i^2 \sin^2\theta_i)}, \quad (3.4.10)$$

$$S_{f,n+1} = \frac{1}{(2(\Delta_{ir} + n\Delta_0) - \Delta_f^2 \sin^2\theta_f)}, \quad (3.4.11)$$

$$D_1 = (A_- \cos\theta_f - B_1)\cos\theta_i - B_2 \cos\theta_f, \quad (3.4.12)$$

$$D_2 = (A'_- \cos\theta_f + B_1)\cos\theta_i + B_2 \cos\theta_f, \quad (3.4.13)$$

$$C_1 = [D_1^2 + (A_- \cos\theta_i - B_2)^2 + (A_- \cos\theta_f - B_1)^2 + A_-^2], \quad (3.4.14)$$

$$C_2 = [D_1^2 + (A_- \cos\theta_i)^2 + (A'_- \cos\theta_f + B_1)^2 + {A'_-}^2], \quad (3.4.15)$$

$$A_\pm = a(\beta_i\gamma_i + \Delta_i \cos\theta_i), A'_\pm = a'(\beta_f\gamma_f - \Delta_f \cos\theta_f), \quad (3.4.16)$$

$$B_1 = B\Delta_f \sin^2\theta_i, \quad (3.4.17)$$

$$a = \beta_i\gamma_i(1+\gamma_f-\Delta_f)(\beta_i\gamma_i+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)+(\gamma_i-1+\Delta_i)(1+\gamma_i)(1+\gamma_i+\Delta_i-\Delta_f), \quad (3.4.18)$$

$$a' = \beta_i\gamma_i(1+\gamma_i-\Delta_f)(\beta_i\gamma_i+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)+(\gamma_i-1+\Delta_f)(1+\gamma_i)(1+\gamma_i+\Delta_i-\Delta_f), \tag{3.4.19}$$

$$B = \beta_i\gamma_i(1+\gamma_i+\Delta_i-\Delta_f)(\beta_i\gamma_i+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma_i), \tag{3.4.20}$$

and $J_n(\xi)$ are the Bessel functions[29].

Therefore, the total scattering cross section will be;

$$\sigma = \frac{\pi r_0^2 a_1^6 a_2^6}{4} \frac{\Delta_f e^{\frac{-B_c}{2B}} \Delta_f \sin^2\theta_f}{\Delta_{ir}(1+\gamma_i)(1+\gamma_i+\Delta_i-\Delta_f)} \frac{Y_r}{[\gamma_i(1-\beta_f\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]} \tag{3.4.21}$$

where Y_r is given in Eq. (3.4.9).

Chapter 4

CONCLUSION AND DISCUSSION

The effect of the expanision parameter constants on the scattering cross section can be seen from Eq. (3.4.21). In this expression the expanision parameter constants affects for example, all the reduced quantities because of the dependence of the physically measured quantities like, the energy, momentum, and distance.

If we naively set $a_1 = a_2 = a$, the cofficient of the scattering cross section in Eq. (3.4.21) is proportional to a^{12}. In the electron-rest frame of reference $p_i = 0$, Eq. (3.4.21) is reduced to Herold's expression, with the presence of the expanision parameter constants as a coefficient.

In fact these expanision constants also affect the expression for Y_r in Eq. (3.4.9) by the rest of the physically measured quantities like in the reduced quantities defined earlier. Therefore, the main feature of our result can be deduced for a particullar solution of Einstein's equation for a spatially flat isotropic Universe. The expanision parameter constants in the remote past and future of the cosmic time can therefore be deduced from the solution of Einstein's equation for our Universe. Our result can also predict the cosmological evolution of the Universe. This can be understood from the conservation of momentum in the process. Conservation of momentum in the z direction imply;

$$p_f + k_f \cos\theta_f = p_i + k_i \cos\theta_i.$$

Now if we take $\theta_i = 0$, in the electron-rest frame of reference the above equation interms of the physically measured quantities can be written as;

$$\frac{a_2}{a_1} \leq \frac{k_f}{k_i}(1 + \frac{p_f}{k_f}),$$

which is resonable and in accordance to mathematical and physical principles.
This expression is derived from conservation of momentum to give an approximate expression for possible limit in which our Universe will expand in the future.
In our study as a beginning researcher we have used the most approximate solution of Einstein's equation for a spatially flat isotropic Universe[26].
Finally a more general study can be conducted using the exact solution of Einstein's equation. And this will be beyond our scope for now and we left this for future study.

References

[1] Xia, X.Y., Qiao, G.J, and Hou, Y.A.: Astron. Astrophys. 152, 93 (1985)

[2] Xu, H. H., Xu, C. H.: phys. Rev., D52, 6116 (1995)

[3] Anderson, J.O., Haugset, T.: phys. Rev., D51, 3073 (1995)

[4] Sandulescu, N.: Eur.Phys.J.special topics.156,265(2008)

[5] Glendenning, N.K.:Compact stars: Nuclear physics, particle physics, and general relativity.Springer, New York. USA(1979)

[6] Weinberg, S.: Gravitation and cosmology. Jhon Willey and Sons, Inc, New York (1972)

[7] Novak, J.: Eur. Phys. J. Special Topics. 156, 151 (2008)

[8] de Bernardis, P. et al. Proc. CAPP2000 (Verbier, 17-28 July 2000) (Preprint astro-ph/0011469)

[9] Perlmutter, S. et al: Astrophys J. 517, 565 (1999)

[10] Perlmutter, S., Turner, M. S. and White, M.: Phys. Rev. Lett. 83, 670 (1999)

[11] Levin, J., Barrow, J., Bunn, E. F. and Silk, J.: Phys. Rev. Lett. 79, 974 (1997)

[12] Sokolov, D. D. and Starobinskii, A. A.: Sov. Astron. 19, 629 (1976)

[13] Gomero, G. I., Rebou as, M. J., and Tavakol, R.: Class. Quantum Grav. 18, 4461 (2001)

[14] Gomero, G. I., Rebou as, M. J. and Texeira, A. F.: Int. J. Mod. Phys. D. 9 678 (2000)

[15] Gomero, G. I., Rebou as, M. J. and Tavakol, R.: Class. Quantum Grav. 18 L145 (2001)

[16] Zelmanov, A. L., Dokl. A.N.: URSS 124, 1030 (1959)

[17] Vladimirov Yu. S.: Sistemy otscheta v teorii gravitatsii (Energoizdat, Moscow,

1982)

[18] Stäckel, P.: Math. Ann. 42, 537 (1893)

[19] Varaksin, L. O., and Shirokov, I. V. Izvestya.: Vuzov, Fizika 1, 31 (1996)

[20] Andrushkevich, I. E., and Shishkin, G. V.: Theor. Math. Phys. 70, 204 (1987)

[21] Birrel, N. D. and Davies, P. C. W.: Quantum Fields in Curved. Cambridge University Press, Cambridge (1982)

[22] Brill, D. and Wheeler, J. A. Rev. Mod. Phys. 29, 465 (1957)

[23] Villalba, V. M.: Mod. Phys. Lett A. 8, 2351 (1993)

[24] Fels, M. and Kamran , N.: Proc. R. Soc. London A 425, 229 (1990)

[25] Villalba, V. M.: Mod. Phys. Lett A. 8, 2351 (1993)

[26] Parker, L.E., and Toms, D.J.: Quantum Field Theory in curved space-time.cambrigde University press, New York(2009)

[27] Moradi, S.: Int. J. Theor. phys. 48: 969 (2009)

[28] Sarkar, N.G., Biswas, S., Shaw, A., Pranama -5. phys. 53, 4 (1999)

[29] Arfken, G.B., Weber, H.J. : Mathematical Methods for Scientists. 5th ed., Elsevier Science academic press, USA (2001)

[30] Coriano, C., Rose, L.D., Gabrielli, E. and Trentadue, L.: phys. Rev. JHEP 03, 136 (2014)

[31] Pagandeh, F., Mehrafarin, M., and Jakook, M.V.: sci. china. ser. G. phys. Mech. Astron. Vol.52, 212 (2009)

[32] Mokonnen, G.: MSc Thesis. AAU(2007)

DECLARATION

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

Name: Demisew Teka

Signature:................

This thesis has been submitted for examination with my approval as university advisor.

Name: Dr.Legesse Wetro

Signature:....................

Place and time of submission:

Depatement of Physics

Addis Ababa University

September 2014