ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY

POST GRADUATE PROGRAM

STUDY ON THE BEHAVIOR OF REINFORCED CONCRETE CYLINDRICAL SHELL STRUCTURES UNDER VARYING PARAMETER

A thesis submitted to the school of Graduate Studies in Partial fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering (Structures)

BY

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DECLARATION

I, the undersigned, declare that this thesis is my work and all sources of materials used for the thesis have been duly acknowledged.

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ABSTRACT

Architects and civil engineers are continuously looking for new and efficient methods to cover large spaces using the least materials and minimum number of columns. Thin concrete cylindrical shells can cover the roofs of various buildings efficiently and aesthetically. Common theories adopted for analysis of cylindrical shells interpret the design behavior in different ways. Nowadays it is easy to study shells to a desired level as tedious hand calculations are minimized due to the appearance of computer. As a result their behavior, specially their structural behavior, can be studied and designed by varying various parameters of the shell.

The parameters, which are studied in this paper, are named as length, radius and thickness of the shell. Many researchers use computer knowledge to study shell behavior under variable parameters. This research was done using commercially available software MATLAB, Excel and manuals. Appropriate shell geometry, loading condition, material as well as other necessary parameters were assumed and selected according to manuals, codes and standards to achieve accuracy. The analysis was performed for stiffened and unstiffened single and multiple spans of reinforced concrete cylindrical shells with varying shell thickness, length, and radius keeping other things constant. Three models and seven trials with variable parameters for both single and multiple spans have been analyzed under dead and live load to obtain variable force, moment, stress and displacements.

Results obtained from this research indicated that shells are varied in their behavior as one or two parameters changed as shown from Table (8-16) and Figure (5-7). Forces, moments, stresses and displacements were increased under increasing length and radius of shell. However, stresses and displacements were decreased and forces and moments were increased with increasing thickness of the shell. This study also adhere the reduction of internal effects when stiffeners were introduced and increasing depth edge beam were reduced the forces, moments, stresses and displacements. Finally it was also plotted a graph along longitudinal and circumferential directions of the shell for the purpose of reinforcement detailing.

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1. INTRODUCTION

1.1. BACK GROUND

Reinforced concrete shells can be defined as curved shape slabs whose thickness is very small compared to other dimensions. The curved structures resisted more applied forces than flat plate with less deformation and stresses. Also shell structure are much efficient than other structure having the same span and dimensions because these shapes have a high strength to weight ratio. There are various type of shells depend upon their size, shape, type of load, material used etc. Due to this large variation, many practical difficulties were occurring. To solve these difficulties many researchers have introduced theories for design of shell.

Generally shells have wide range of applications in engineering practice. Indeed, a list of familiar examples will be useful which enabling us to pick out some structural features in a qualitative way in order to provide an introduction to the main body of theoretical work. It is instructive to assemble a list of applications from a historical point of view, and to take as a connecting theme for a sequence of brief sketches the way in which the introduction of the thin shell as a structural form has made an important contribution to the development of several different branches of engineering.

A cylindrical shell presents one of the most generally acceptable and optimal structural form of shells employed in engineering. It is successful in combining simplicity, compactness, and almost an ideal technological effectiveness. The latter enables one to develop items by usual rolling-up a given sheet skelps. The common type of shell used in field is cylindrical reinforced concrete shell to cover large space. Generally long shells and short shells are the two common form of cylindrical shells. Normally long shells are used for roof factories and short shells for aircraft hangers.

Therefore, pattern cutting of a material and the subsequent process of manufacturing of a given structural form requires a minimum of technological effort. On the other hand, unlike the other shell forms say, noncircular shells of revolution, the governing differential equations of the moment theory of a circular cylindrical shell of constant thickness represent a system of equations with constant coefficients, which gives us the possibility to analyzing their solutions in the general form.

1.2. STATEMENT OF THE PROBLEM

Although shell structures are economically, aesthetically and structurally efficient, they are not practically available in Ethiopia. Besides, their analysis were limited to what codes, researchers and manuals postulated. Nowadays, the appearance of computer will minimize the tedious hand calculations. Current study presents the structural behavior of open circular reinforced concrete cylindrical shell structures for different geometric parameters of shells using MATLAB computer application. This will help designers, especially for Ethiopians, to analysis these structures with and without stiffeners as well as single and multiple spans with in short period of time and enable them how the internal effects behaves elsewhere in the shell sections.

1.3. SCOPE AND LIMITATION

Under the variation of shell parameters one may predict their structural behavior manually and using software. This research was done using commercially available software MATLAB (2009a), Excel and manuals. It was performed by selecting and assuming appropriate shell geometry, loading condition, material as well as other necessary parameters from manuals, codes and standards. The analysis was performed for single and multiple spans of open circular cylindrical reinforced concrete shells with varying shell thickness, length and radius keeping the other dimensions, materials and support conditions constant. Three models with variable parameters for both single and multiple spans were analyzed under dead, and live. The research is not complete analysis of all components of these structures as it was behavioral study and comparison.

1.4. OBJECTIVE

1.4.1. General objective

The main objective of this research is to study the structural behavior of reinforced concrete cylindrical shell structures under the variation of shell length, radius and thickness geometrical parameters.

1.4.2. Specific objective

It is intended to study the Stress, deflection, force and moment effects of circular reinforced concrete cylindrical shells by varying shell length, radius and thickness geometrical parameters.

In addition, it is aimed to do detail analytical investigations on how the internal effects would be affected with the provision stiffeners. Also aimed to study how the dimensions of the edge beam fluctuates the shell effects either in compression or tension.

Furthermore, this study tries to develop a graph that show the variation of internal effects at any section of the shell.

2. LITERATURE REVIEW

2.1. INTRODUCTION

Shell construction began in the 1920s. The shell emerged as a major long-span concrete structure after World War II. Thin parabolic shell vaults stiffened with ribs have been built with spans up to about 90 m. More complex forms of concrete shells have been constructed, including hyperbolic paraboloid or saddle shapes and intersecting parabolic vaults less than 1.25cm thick.

The reinforced concrete shells can be defined as curved shape slabs whose thickness is verysmall compared to their other dimensions. The curved structures resisted more applied forces than flat plate with less deformation and stresses. Also shell structure are much efficient than other structure having the same span and dimensions because these shapes have a high strength to weight ratio. There are a different type of shell depend upon their size, shape, type of load, material used etc. Due to this large variation, many practical difficulties were occurring. To solve these difficulties many researchers introduced their theory for design of shell.

The behavior of shell structures is, in various aspects, different from that of so-called "framed structures". This feature originates mainly from the geometrical features of shells which make the intern al force system in shells differ from those in other types of structural forms. The internal force distribution in shells is, in general three dimensional, i.e. Spatial. Moreover, shell structures carry the applied forces mostly by the so-called membrane forces, whereas other structural forms carry the applied loads by bending mechanisms. These unique features of shells are also reflected in their design as well as in their method of construction. [6]

Shell structures support applied external forces efficiently by virtue of their geometrical forms. Shells, having their spatial curvature, are much stronger and stiffer than other structural forms. For this reason shells are sometimes referred to as form resistant structures. The strength to weight ratio of a shell structure is usually much smaller than that of other structural systems having the same span and overall dimensions. [6]

2.2. CLASSIFICATION OF SHELLS

Shell structures can be classified into number of categories based on several factors. According to Gaussian curvature shell structures can be classified as Synclastic (positive Gaussian curvature), developable (zero Gaussian curvature) and anticlastic (negative Gaussian curvature) surfaces. They can be also classified based on surface of geometric behavior; whether the surface is ruled or not. Surfaces that can be obtained entirely by straight lines are termed as ruled surfaces and outside these categories are classified as shells with no ruled surface. In addition to the above classifications shells can also classified according to manner of generation; as shells of revolution, shells of translation and cylindrical shells. Shells of revolution are obtained when a plane curve is rotated about an axis of symmetry. Shells of translation on the other hand are generated when one plane of curve moves parallel to itself along another curve. Whereas cylindrical shells are singly curved shell in which the generatrix or directrix is a straight line.

2.3. CYLINDRICAL SHELL

As mentioned above, a cylindrical shell surface is generated by translating a straight line generator over a plane curve directrix. An arc of a circle, semi ellipse, parabola, cycloid, and catenary are some of the most common directrices to form cylindrical surface. Shells for which the plane curve is arc of a circle are called circular cylindrical shells. All other which have their generating plane curves other than circular arc are collectively referred to as non-circular cylindrical shells. Open cylindrical shells are mostly used for the construction of roof for industrial building, sport and exhibition halls, garages and hangars etc...

According to *J.N.Bandyopadhyay* the cylindrical shell is bounded by two longitudinal edges parallel to the axis of the cylinder and two curved transverse edges which are perpendicular to the axis of the cylinder. Along the longitudinal edges, there may or may not edge beams and along the transverse edges, the shell is supported by traverses. The shell is considered supported on the traverses which may be a solid, diaphragm, tied arch or rigid frame. The traverses are assumed to be rigid in their own planes, but flexible out of their planes. Thus they are capable of resisting loads in their own plane, but cannot resist loads in direction perpendicular to them.

2.3.1. Classification of cylindrical shell

Cylindrical shells can be classified as long, short, and sometimes intermediate shells. The object of this classification, principally, is to help the designer to determine whether a shell of particular dimension is susceptible for an approximate method of analysis. There are three classification methods.

2.3.1.1. Classification based on radius and longitudinal length

A cylindrical shell are usually described as either long or short depending upon the ratio of the length (L) and radius (R). Thus:

Long shells: $R/L \leq 0.2$

Short shells: $R/L > 0.2$ without edge beam (2.0)

The demarcation will be raised to $R/L < 0.33$ with edge beam in the above case.

2.3.1.2. Classification based on the magnitude of edge disturbance

Based on the extent to which the disturbances arising from the springing edges penetrate into the body of the shell itself. Based on this consideration, once again the radius and the longitudinal length of the shell are indicative parameter.

Long shells: $R/L < 0.63$

Short shells: $R/L > 0.63$ (2.1)

2.3.1.3. Classification based on geometric dimension and thickness

This method is based on the numerical value of the parameter $\phi = B/(L^2Rt)^{0.25}$

Long shells: $\Phi < 3$;

Intermediate shells: $3 < \Phi < 5$;

Short shells: $\Phi < 5$; (2.2)

2.4. LOAD CARRYING BEHAVIOR OF SHELLS

In a shell structure subjected to applied external loading, temperature changes, support settlements, and deformation constraints, some internal stresses may develop. These internal stresses are shown on the shell element of Figure (2- 3) below. As we see, the general state of stress in a shell element consists of membrane normal and shear stresses lying in the shell surface, as well as the transverse shear stresses. In thin shells, the component of stress normal to the shell surface, compared with other components of the intern al stresses, is very small and is neglected in the classical shell theories.

The mechanical properties of a shell element describe its resistance to deformation in terms of separable stretching and bending effects. Loads which are applied to the shell are carried in general by a combination of bending and stretching actions, which vary over the surface. One of the leading difficulties in the theory of shell structures is to find a relatively simple way of describing the interaction between the two effects. This aspect of the theory has been troublesome from the beginning.

The *membrane forces,* as the name implies, are the resultant internal forces which lie “inside" the mid-surface of the shell. The membrane force field causes the stretching or contraction of the shell, as a membrane, without producing any bending and /or local curvature changes. The membrane force field consists of two membrane normal resultant forces and a membrane shear force. The second group of internal forces are called the *bending forces,* since they cause bending and twisting of the shell cross-sections. The bending force field consists of bending moments, twisting couples, and transverse shear forces.

Shell structures carry the applied extern al forces mostly by the mechanism of membrane action. In some regions of the shell a bending force field may develop to satisfy specific equilibrium or deformation requirements. The range of influence of the bending field is local and is confined to the vicinity of loading and geometrical discontinuities and/or the deformation incompatibilities. The rest of the shell is virtually free from bending actions and can be analyzed and designed as a membrane. Depending on the nature of the applied forces, this membrane shell may be in tension or compression or partly both. The extent of the domain of influence of bending depends on the particular shell geometry and its edge and loading conditions.

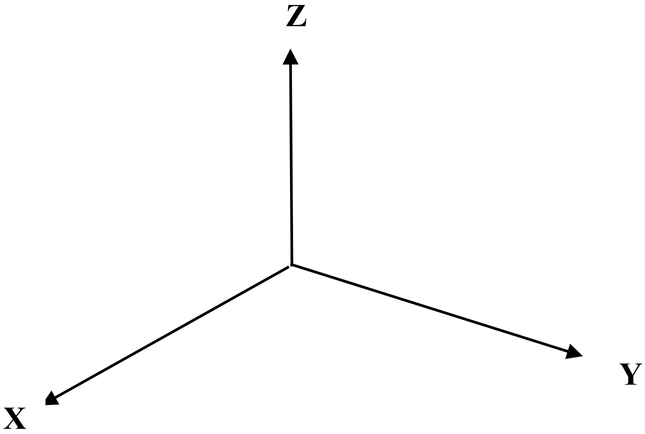


Figure.1. Coordinate system (positive orientation)

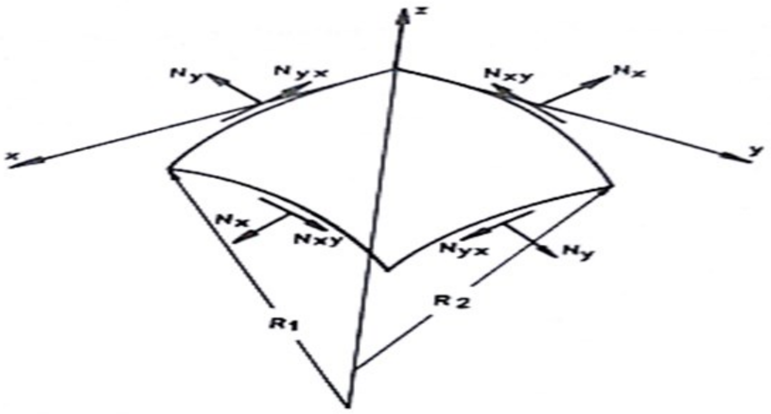


Figure.2. Membrane stress resultants

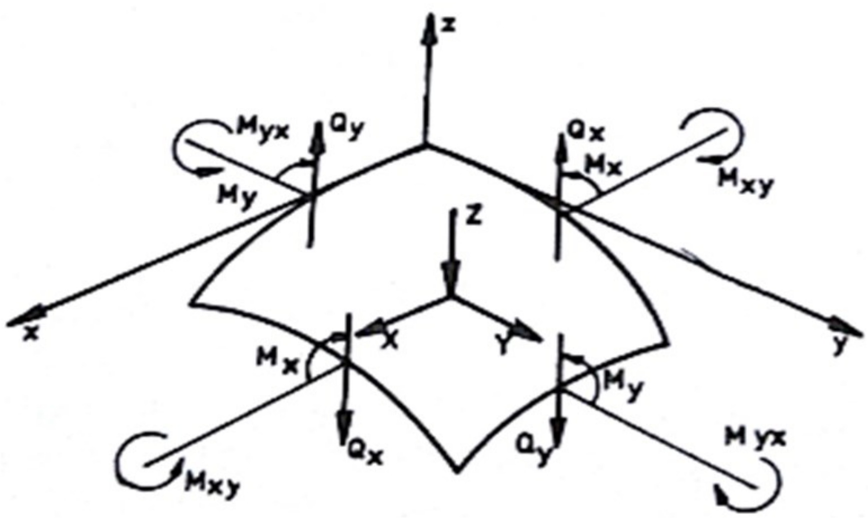


Figure.3. Bending, twisting, shear and load resultants

The load-carrying mechanism, i.e., the internal forces at any point of a shell, consist of ten component internal force resultants (**Nx, Ny, Nxy, Nyx, Mx, My, Mxy, Myx, Qx, Qy).** These components, can be separated into two groups, entitled membrane and bending internal force field, as follows: Membrane field: Nx, Ny, Nxy, Nyx and Bending field: Mx, My, Mxy, Myx, Qx, Qy .

In this terminology, Mx and My stand for bending moments while Mxy and Myx represent the twisting couples. Qx and Qy represent the out-of-plane shear forces. For a material body in spatial equilibrium there are six governing equilibrium equations. Since there are more than six force resultants, we conclude that a shell is, in general, an internally *statically indeterminate* structure.

In general the forces transmitted across any cut in a structure have a resultant which may be expressed as a force passing through an arbitrary point, together with a couple. It is convenient to regard the resultant force on each face as acting through the center of the face; then to resolve in the orthogonal x, y and z-directions, and finally to express the components as stress resultants having the units of force/length. Thus the force transmitted across an x-face (i.e. a face perpendicular to the x-axis) is expressed as three Stress resultants Nx, Nxy and Qx. The other two are 'shear' stress resultants, acting tangentially, and we use the symbol Nxy for the component in the y direction (thus Nx and Nxy may be called 'in-plane' or 'tangential' stress resultants) but symbol Q_x for the out-of-plane or normal or transverse shear-stress resultant.

The symbol Qx is really an abbreviation of Q_{xz} but the double subscript is unnecessary on account of the use of the symbol Q as distinct from N. The sign convention is that used for stress resultants on a three-dimensional element: on the positive x-face, i.e. the one which overlooks the positive x-direction, the positive sense of Q is in the positive z direction. Conversely, on the negative x-face, the positive sense of Qx is in the negative direction of z. The same convention is used for Nxy and Nyx: on the positive x-face, the positive sense of Nxy is in the positives y-direction, etc.

2.5. REQUIRED RELATIONSHIPS

The work of *C.R.Calladine* in the analysis of cylindrical shell structures is similar as others in driving the required relationship formula. However, the method of assumption he proposed is different from others. Accordingly, he has first assumed mode of displacements and then determine stress resultants and then the corresponding load distribution in which he divide the applied load for the contribution of membrane and bending analysis separately. In addition he has been worked for specific problems for single and multiple cylindrical shells upon supported edge and fee edges. For a better understanding please see the reference to the work published by [2].

The work of *M.Farshad, J.N.Bandyopadhyay and K.Chadrashekhara*, in the analysis of cylindrical shell structures are similar except *J.N.Bandyopadhyay* assumed a little different coordinate orientations. Therefore, the following work has been derived from these authors work, particularly the work of K.Chadrashekhara is fully used.

Nilesh .S.Lende and Rajshekhar.S.Talikoti has been investigate the variation of stresses, forces and moments in their journal as they published in international journal of research in engineering and technology. They were used finite element software SAP2000 for their study under radius and thickness variation one at a time for multiple cylindrical shells.

Srinivasan Chandrasekaran, S.K.Gupta and Fedreico Carannante have been proposed, design curves for fixed support reinforced concrete cylindrical shells under uniformly distributed load which vary sinusoidal and remain constant once at a time under varying parameters, in their journal as they published in international journal of research in engineering and technology.

Present work is an investigation of reinforced concrete cylindrical open barrel shells under different parameters namely radius, thickness and length of the shell using MATLAB as one or two variables are varied alternatively for both single and multiple stiffened and unstiffened cases.

2.5.1. Membrane analysis of cylindrical shells

Our immediate objective is to find a set of relations between the stress resultants acting on the element and the distortions which they produce. Since the material of which the element is made is linear-elastic we may call these relations 'generalized Hooke's law'. We are concerned only with small distortions.

2.5.1.1. Force equilibrium relations of membrane cylindrical shell

By applying equilibrium equations for the determinate shell structures we can determine the membrane stress resultants namely normal stress along X axis (**N_x**) normal stress along circumferential direction(**N_θ**) and shear stress(**$N_{\theta x}$** or **$N_{x\theta}$**).

$$(\partial Nx)/\partial x + (\partial N\theta x)/R\partial\theta + X = 0, \qquad \textit{along X direction.}$$

$$(\partial N\theta)/\partial\theta + (\partial N\theta x)/\partial x + Y = 0, \qquad \textit{along } \theta \textit{ direction} \tag{2.3}$$

$$N\theta/R + Z = 0, \qquad \textit{along Z direction.}$$

Solving these differential equations finally give us the following membrane stress resultants.

$$N_\theta = -ZR \tag{2.4}$$

$$N_{\theta x} = -\int \frac{1}{R}\left(\frac{\partial N_\theta}{\partial\theta}\right)dx - \int Y dx + F_1(\theta)\,, \tag{2.5}$$

$$N_x = -\int \frac{1}{R}\left(\frac{\partial N_{\theta x}}{\partial\theta}\right)dx - \int X dx + F_2(\theta)\,, \tag{2.6}$$

Where,

'X' is generalized load component along X axis, 'Y' is generalized load component along Y axis, 'Z' is generalized load component along Z axis and, $F_1(\theta)$ and $F_2(\theta)$ are constant integrations and are determined from boundary conditions.

The above equations are for any shape of curved directrix for generalized loads X, Y and Z. However; in many practical cases these load components vary only along **θ** and remains constant along **X**. thus they are functions of **θ** only.

2.5.1.2. Strain displacement relations

$$\varepsilon_{xo} = \frac{\partial u}{\partial x} \tag{2.7}$$

$$\varepsilon_{\theta o} = \frac{\partial v}{R\partial\theta} + \frac{w}{R} \tag{2.8}$$

$$\gamma_{x\theta o} = \frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x} \tag{2.9}$$

The general equations for the displacements can now be obtained in terms of the stress resultants.

$$\frac{\partial u}{\partial x} = \frac{1}{Eh}(N_x - vN_\theta)$$

$$\frac{\partial v}{R\partial\theta} + \frac{w}{R} = \frac{1}{Eh}(N_\theta - vN_x) \tag{2.10}$$

$$\frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x} = \frac{N_{\theta x}}{Gh}$$

The equation of displacement can now be obtained from simple integration of the above equations

$$u = \frac{1}{Eh}\int(N_x - vN_\theta)dx + F_3(\theta) \tag{2.11}$$

$$v = \frac{1}{Gh}\int(N_\theta)dx - \frac{1}{R}\int\left(\frac{\partial u}{\partial\theta}\right)dx + F_4(\theta) \tag{2.12}$$

$$w = R\left[\frac{1}{Eh}(N_\theta - vN_x) - \frac{\partial v}{R\partial\theta}\right] \tag{2.13}$$

$F_3(\theta)$ and $F_4(\theta)$ are constant integrations and are determined from boundary conditions.

2.5.2. Bending analysis of cylindrical shells

2.5.2.1. Introduction

The membrane theory gives a fair idea of the forces for certain dimensions of cylindrical shells only and even in this case, the membrane theory would be valid only for portion of shells far away from the edges. The membrane analysis would give forces along the longitudinal edges, which cannot be normally attained in practice by providing a suitable support. Any corrective force or displacement applied along these edges, so as to satisfy the actual support condition, would lead to bending of the shell. As a result, most of the cylindrical shell roof structures have to be analyzed considering bending, to get a more realistic picture of force distribution in the shell.

The traverses are normally intended to take large forces in their own plane, but can seldom resist a force normal to this plane. This type of support is often referred to as simple support and a shell structure, which has only two traverses, is said to be simply supported. If the shell is supported on three or more traverses, then it is referred as a continuous shell. The traverse could be a solid wall, an arch rib or a bowstring girder. Often, several circular-cylindrical shells are placed side by side or one after another, connecting them along the longitudinal edge. Such shells is called a multiple cylindrical shell

The governing differential equations for bending analysis circular-cylindrical shell given by Flügge and also revised in Chandrashekhara are also presented here.

2.5.2.2. Equilibrium equations

The middle surface of an element of circular-cylindrical shell is described by (x, θ) coordinate system, where $\boldsymbol{x}$ is measured in the direction of the generatrix and $\boldsymbol{\theta}$ is measured in the direction of the directrix.

The above forces and moments have to satisfy the six static equilibrium equations; three concerning the forces and the other three the bending moments.

$$\begin{aligned}
&\Rightarrow \frac{\partial N_x}{\partial x} + \frac{\partial N_{\theta x}}{R\partial \theta} + X = 0, \quad \text{equilibrium for forces in X direction} \\
&\Rightarrow \frac{\partial N_\theta}{R\partial \theta} + \frac{\partial N_{\theta x}}{\partial x} - \frac{Q_\theta}{R} + Y = 0, \quad \text{equilibrium for forces in Y direction} \\
&\Rightarrow \frac{N_\theta}{R} + \frac{\partial Q_\theta}{R\partial \theta} + \frac{\partial Q_x}{\partial x} + Z = 0, \quad \text{equilibrium for forces in radial direction}
\end{aligned} \tag{2.17}$$

$$\Rightarrow \frac{\partial M_{\theta x}}{R\partial\theta} + \frac{\partial M_x}{\partial x} - Q_x = 0, \quad \textit{moment equilibriumabout X direction}$$
$$\Rightarrow \frac{\partial M_{x\theta}}{\partial x} + \frac{\partial M_\theta}{R\partial\theta} - Q_\theta = 0, \quad \textit{moment equilibrium about Y direction} \tag{2.18}$$
$$\Rightarrow \frac{M_{\theta x}}{R} + (N_{x\theta} - N_{\theta x}) = 0, \quad \textit{moment equilibrium about Z direction}$$

The above four equilibrium equations have eight unknown stress resultants and the problem is statically indeterminate internally. Hence, additional equations are necessary to complete the analysis from deformation of the shell.

2.5.2.3. Strain displacement relationship

The strain-displacement relationship can be obtained purely from geometrical consideration by comparing the deformed shape of the shell element with the original. If the u, v and w are the components of displacement at the mid-plane in X, θ and Z directions. The strain-displacement relation valid at the mid-plane of the shell are presented from Equation (2.7-2.9). However, in the case of the shell subjected to bending, the displacements will vary over the thickness of the shell. Hence, the displacements at any arbitrary points at a distance z from the middle surface can be given by:

$$\varepsilon_x^a = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{2.19}$$

$$\varepsilon_\theta^a = \frac{\partial v}{R\partial\theta} - \frac{1}{R}\left(\frac{z}{R+z}\right)\frac{\partial^2 w}{\partial\theta^2} + \frac{w}{R+z} \tag{2.20}$$

$$\gamma_{x\theta}^a = \left(\frac{1}{R+z}\right)\frac{\partial u}{\partial\theta} + \left(\frac{R+z}{R}\right)\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial\theta\partial x}\left(\frac{z}{R+z} + \frac{z}{R}\right) \tag{2.21}$$

2.5.2.4. Stress-strain relationship

If σ_x and σ_θ are the normal stresses and $\tau_{x\theta}$ the shear stress in the shell, then from Hooke's law, the stress-strain relationship can be written as:

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\theta) \tag{2.22}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_x) \tag{2.23}$$

$$\tau_{x\theta} = \frac{E}{2(1+\nu)}(\gamma_{x\theta}) \tag{2.24}$$

The stress resultant expressions can be determined based on these stress on the shell in the following manner.

2.5.2.5. Force-displacement relationship

The force-displacement relations are obtained by substituting the stress-diplacements relations in the above stress resultant equations and integrate with respect to z and with some assumptions and simplifications as follows.

$$\boldsymbol{N_x} = K\left[\frac{\partial u}{\partial x} + \upsilon\left(\frac{\partial v}{R\partial\theta} + \frac{w}{R}\right)\right] - \frac{D}{R}\left(\frac{\partial^2 w}{\partial x^2}\right), \tag{2.25}$$

$$\boldsymbol{N_\theta} = K\left[\left(\frac{\partial v}{R\partial\theta} + \frac{w}{R}\right) + \upsilon\frac{\partial u}{\partial x}\right] + \frac{D}{R^3}\left(\frac{\partial^2 w}{\partial\theta^2} + w\right) \tag{2.26}$$

$$\boldsymbol{N_{x\theta}} = \left(\frac{1-\upsilon}{2}\right)\left[K\left[\frac{\partial v}{\partial x} + \left(\frac{\partial u}{R\partial\theta}\right)\right] - \frac{D}{R}\left(\frac{\partial^2 w}{\partial\theta\partial x}\right) + \left(\frac{D}{R^2}\frac{\partial v}{\partial x}\right)\right], \tag{2.27}$$

$$\boldsymbol{N_{\theta x}} = \left(\frac{1-\upsilon}{2}\right)\left[K\left[\frac{\partial v}{\partial x} + \left(\frac{\partial u}{R\partial\theta}\right)\right] + \frac{D}{R}\left(\frac{\partial^2 w}{\partial\theta\partial x}\right) + \left(\frac{D}{R^2}\frac{\partial u}{\partial\theta}\right)\right], \tag{2.28}$$

$$\boldsymbol{M_x} = D\left[\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{R^2\partial\theta^2}\right] - \frac{D}{R}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{R\partial\theta}\right), \tag{2.29}$$

$$\boldsymbol{M_\theta} = D\left[\frac{\partial^2 w}{R^2\partial\theta^2} + \upsilon\frac{\partial^2 w}{\partial x^2}\right] + \frac{Dw}{R^2} \tag{2.30}$$

$$\boldsymbol{M_{x\theta}} = \frac{D(1-\upsilon)}{R}\left[\frac{\partial^2 w}{\partial\theta\partial x} - \frac{\partial v}{\partial x}\right] \tag{2.31}$$

$$\boldsymbol{M_{\theta x}} = \frac{D(1-\upsilon)}{R}\left[\frac{\partial^2 w}{\partial\theta\partial x} - \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{R\partial\theta}\right)\right] \tag{2.32}$$

$$\boldsymbol{Q_x} = D\left[\frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 w}{R^2\partial\theta^2\partial x}\right] \tag{2.33}$$

$$\boldsymbol{Q_\theta} = D\left[\frac{\partial^3 w}{R^3\partial\theta^3} - \frac{\partial^3 w}{\partial\theta\partial x^2}\right] \tag{2.34}$$

As we noticed above in the force displacement relations $\mathbf{N_{x\theta}} \neq \mathbf{N_{\theta x}}$ *and* $\mathbf{M_{x\theta}} \neq \mathbf{M_{\theta x}}$ are not equal although $\tau_{x\theta} = \tau_{\theta x}$. However, if we take the primary terms only, this equality will be satisfied.

2.6. BOUNDARY CONDITION

To solve a particular problem one must investigate the general question of boundary conditions', for it is clear that these will play an important part in the solution of the differential equations, just as they do in the analysis of beams.

One of the advantages of using models to study the behavior of shell structures is their adaptability to various boundary conditions. The internal work of the system corresponding to a virtual displacement can be formulated in terms of strains. Since the geometric and the force boundary conditions can be expressed by strain-displacement relations the internal work of the deformable nodes on or near the boundary is easily obtained by the use of appropriate strains. Whenever the strain-displacement relations at the deformable nodes on or near the boundary include displacements of points outside the model new strain-displacement relations must be formulated. These expressions are obtained according to the restraints at the edge and include only displacements of points within the model.

Appling boundary condition in shell structures, especially to those concerned models will in able us to obtain additional equations in order to solve the indeterminate structures. The following are some of the boundary conditions for cylindrical shells. [4].

2.6.1. Single span cylindrical shell

The boundary condition along the longitudinal edges ($\theta = \pm\theta_o$) of the shell depends on the methods of supporting the edges.

Edge free: Since the edge is free, the forces and moments along the edges will be zero.

Edge support on unyielding wall: Wall restricts vertical and horizontal displacements but allows rotation.

Edge simply supported: these are characterized by the following expressions:

At $\theta = \pm\theta_o, \; N_\theta = M_\theta = \varepsilon_x = w = 0$ (2.35)

Edge clamped: Complete restriction for the displacements and rotations.

Edge supported on a beam: edge of the shell is monolithically connected to the beam and the forces acting on the edge of the shell gets transmitted onto the beam. As a result the beam displaces longitudinal (u_b), lateral (v_b), vertical (w_b) as well as twisting effect(β_b), and the longitudinal (u_s), lateral (v_s), vertical(w_s) as well as twisting effects(β_s) of the shell component will be equal two the beam respectively. Hence;

$$u_s = u_b\ , \qquad \beta_s = \beta_b, \qquad \Delta w_s = w_b, \quad \Delta H_s = v_b \tag{2.36}$$

$where, \Delta H_s = v_s \cos\theta + w_s \sin\theta\ , \qquad \Delta w_s = w_s \cos\theta - v_s \sin\theta$

However; if the beam is torsionally stiff and cannot resist lateral thrust the conditions are simplified as follows:

At $\theta = \pm\theta_o$)RH $= \mathrm{N}_\theta \cos\theta - Q_\theta{}' \sin\theta = \beta\mathrm{s} = 0,\ u_s = u_b\ , \Delta w_s = w_b,$ (2.37)

2.6.2. Multiple span cylindrical shell

Multiple span of cylindrical shells may appear with and without edge beams. Multiple span without edge beams could be either free end and/or with intermediate beam. At the two free ends from the center line of each shell, the circumferential normal stress and bending stress, the normal shear, the resultant transverse shear stresses are zeros. However, at the junction of the two shells from their centerline, longitudinal, lateral, vertical as well as twisting effects for the two shells are equal. Similarly the circumferential normal stress and bending stress, the normal shear, and the resultant transverse shear stresses of the two shells are equal. Modified equation for inner shell is given by;

$$\beta_s = \Delta H_s = \Delta F = \frac{\partial N_{x\theta}}{\partial \theta} = 0, where\ \Delta F = \mathrm{N}_\theta \sin\theta + Q_\theta{}' \cos\theta \tag{2.38}$$

Multiple span with edge beams are characterized by the presence of edge beams everywhere in the shell junctions and free ends. At the two ends from the center line of each shell, longitudinal, lateral, vertical as well as twisting effects of the shell and the beam are equal. However, at the junction of the two shells from their centerline, longitudinal, lateral, vertical as well as twisting effects of each shell and the beam at junction are equal. The modified equation for inner shell is given by;

$$u_s = u_b\ , \qquad \beta_s = 0, \qquad \Delta w_s = w_b, \quad \Delta H_s = 0 \tag{2.39}$$

3. MATERIALS AND METHODS

3.1. GEOMETRICAL DIMENSION SELECTION

The geometry of the open barrel cylindrical shells for this thesis were selected according the work of Michele Melaragno, an introduction to shell structures (the art and science of vaulting) Table 12.3 page 263 for shells with edge beams. However, some of the geometrical dimension models were interpolated upon these dimensions provided.

Table 1
Geometrical dimension selection

Orginal		Model 1		Model 2		Model 3	
φ	R/L	φ	R/L	φ	R/L	φ	R/L
2.75	2.74	2.69	2.61	2.69	2.51	2.83	2.74
2.83	2.64	2.79	2.57	2.78	2.46	2.92	2.64
2.84	2.60	2.80	2.54	2.80	2.44	2.92	2.60
2.95	2.63	2.91	2.57	2.91	2.49	2.95	2.63
2.92	2.55	2.89	2.50	2.88	2.43	2.85	2.55
2.93	2.57	2.90	2.52	2.90	2.45	2.87	2.57
3.00	2.76	2.89	2.53	2.97	2.64	2.94	2.76

$\phi = B/(L^2Rt)^{0.25}$

$\Phi < 3$; Long shell

$3 < \Phi < 5$; Intermediate shell

$\Phi < 5$; Short shell

$R/L < 0.33$ with edge beams

As indicated in Table 1 the selected dimensions in Table 2 are classified as long shells. However, for single span without edge beams the dimensions does not bring satisfactory result for the given concrete materials. Therefore, some body may refer to Table 3 in order to have safe results for single spans with free edges.

Table 2
Shell dimension and model

Orginal dimension									Model 1		Model 2		Model 3	
	Length	Rise	Radius	Thickness	Longitudinal edge		Transverse arched		Length	Rise	Radius	Rise	Thickness	Rise
Trial	(m)	(m)	(m)	(m)	Width(m)	Depth(m)	Width(m)	Depth(m)	(m)	(m)	(m)	(m)	(m)	(m)
1	30.50	3.00	11.13	0.09	0.25	0.81	0.61	0.30	29	3.00	12.13	2.68	0.08	3.00
2	35.00	3.30	13.25	0.09	0.25	0.81	0.61	0.30	34	3.30	14.25	3.00	0.08	3.30
3	40.00	3.70	15.36	0.10	0.30	1.22	0.61	0.30	39	3.70	16.36	3.41	0.09	3.70
4	45.70	4.30	17.39	0.10	0.25	1.22	0.81	0.51	44.7	4.30	18.39	4.00	0.10	4.30
5	50.00	4.50	19.61	0.11	0.30	1.22	0.81	0.51	49	4.50	20.61	4.22	0.12	4.50
6	55.00	5.00	21.41	0.12	0.36	1.63	1.07	0.81	54	5.00	22.41	4.72	0.13	5.00
7	61.00	6.10	22.11	0.13	0.36	1.63	1.07	0.81	56	6.10	23.11	5.75	0.14	6.10

As a general guideline, a thickness to chord width ratio of range of (1/300 to 1/200) could be considered asound choice for concrete shell thickness. The selected thicknesses are in the given range.

The rise of the shell is also almost larger than (1/10) of the chord width for all models. As well as for single cylindrical shell it should be in the interval of (1/16 to 1/12) of the chord width but it should be not less than (1/15) of the chord width. The rise should include the height of the edge beam.

The central angles are also almost in the range of (60 to 80). In Table 2 the width of the selected dimensions is half of the longitudinal length of the shell.

The radius and central angle of circular-cylindrical shell can be determined by simple mathematical calculations to the given geometry of the shell.

By applying the following formula for radius and central angle the above table were obtained.

$$\sin \emptyset = \left(\frac{B/2}{R}\right), \qquad \text{central angle determination} \tag{3.1}$$

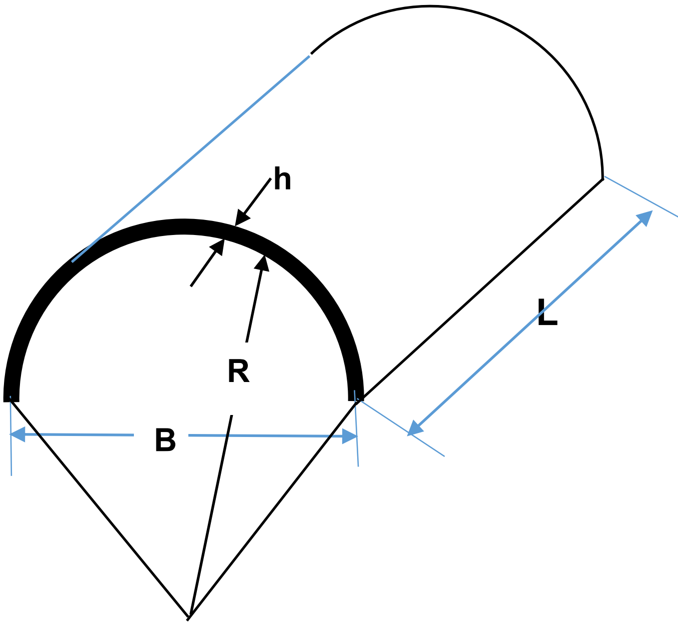
$$R^2 = x^2 + (B/2)^2 , \qquad \text{radius of shell determination} \tag{3.2}$$

Table 3

Dimensions for single span without edge beams.

L	B	R	φ	h
20.00	10.00	11.00	27.04	0.08
25.00	12.50	10.00	38.68	0.08
28.00	14.00	12.00	35.69	0.10
32.00	16.00	14.00	34.85	0.13
36.00	18.00	15.00	36.87	0.14
40.00	20.00	12.00	33.75	0.12
45.00	15.00	13.00	35.23	0.13

Table 3 are used determination forces, moments, stresses and displacements of single span without edge beams. However, the dimensions indicated in Table 2 were also used for single span without edge beam for comparison purpose only.



where

'H' is height from crown to edge

'X' is distance obtained by subtracting height from radius

'R' is radius of shell

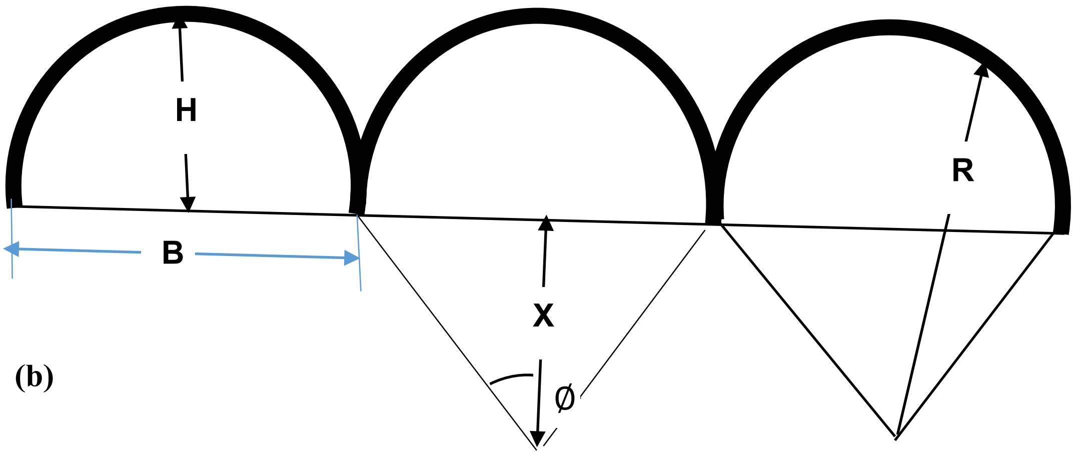
'B' is chord length (horizontal projection of the arc length)

$'\emptyset'$ *is half central angle*

'L' is Length of shell

'h' is thickness of the shell

(a)



(b)

Figure.4. proposed single (a) and multiple (b) cylindrical shells

3.2. LOAD DETERMINATION

Most of circular-cylindrical shell roof structures are subjected to two types of loads. These loads are dead load acting per unit are of the shell and snow (live load) which is assumed uniform over the horizontal projection of the shell. Sometimes wind load may applied depending on the central angle.

Both types of loads have components along longitudinal, circumferential as well as radial directions. However, the component along the longitudinal direction is negligible and hence it is assumed zero. For long shells the following loads are recommended. [8]

Table 4

Recommended weights and strength of materials

Materials	Concrete	27.6 mPa
	Steel	276 mPa
Dead load	Self-weight	24 kN/m^3
	Roofing	0.25 kPa
	Miscellaneous	0.25 kPa
Live load	1.5 kPa	

Dead load is obtained by adding its components indicated in Table 3.

Dead load =self-weight +roofing +miscellaneous

= (24 KN/m^3 *thickness) + (0.25 KPa) + (0.25 KPa)

= (24 KN/m^3 *thickness) + (0.5 KPa)

The value of (24 KN/m^3 *thickness) is dependent on the value of thickness of the shell and is tabulated in Table 4 for different models.

The dead and live loads can be combined according to Ethiopian Building Code of Standard 1995. Although the combination of loads for shell structures is not straight forward as framed structures, one may combined by changing the live load to its equivalent form. The equivalent form of live load is the projected areal distribution of the shell. Therefore, the load combination will follow the usual combination of structures.

Factored load =1.3*(dead load) +1.6*(live load) (1.3)

3.3. LOAD COMPONENTS

For dead and live loads, there are different components along X, θ and Z directions since the distribution of loads are different. In addition to this the component load along X direction is almost negligible and as a result the load is distributed to wards θ and Z directions. Hence:

Components of dead load (say, dead load=W_g in KN/m^2 and acts down ward)

Along θ direction (W_θ) is given by

$$W_\theta = W_g \sin\theta \tag{3.4a}$$

Along Z direction (W_z) is given by

$$W_z = W_g \cos\theta \tag{3.4b}$$

Components of live load (say, live load=W_p in KN/m^2 and acts down ward)

Along θ direction (W_θ) is given by

$$W_\theta = W_p \sin\theta \cos\theta \tag{3.5a}$$

Along θ direction (W_z) is given by

$$W_z = W_p \cos^2\theta \tag{3.5b}$$

3.4. ANALYSIS PROCEDURE

The analysis was performed for all models upon the following conditions using MATLAB. The name MATLAB stands for MATrix LABoratory. This was written originally to provide easy access to matrix software developed by the LINPACK (linear system) and EISPACK (Eigen system package) projects. [7]

It has been used this software as it writes automatically expressions for the equilibrium equations of shells and minimize tedious hand calculations. It also explore the exact solutions which require long iterations.

The analysis was performed for open circular reinforced concrete cylindrical shell structures for single span without edge beam, single span with edge beams, multiple span without edge beams and multiple span with edge beams at all ends. It is important to notice that the arch ribs are always appear unlike edge beams.

Therefore, this study addresses to analyze reinforced concrete circular cylindrical structures using MATLAB. It was done by optimized the differential and integration equations of shell structures. The First task was to get the solutions of displacement functions for shell structures. Following to this the strains, stresses, forces and moments functions were determined from these displacement functions.

Solutions of these displacements can be obtained from finite difference method, finite element method and exact solutions. This research determined displacement functions using the exact solutions of single trigonometry series. Since most of cylindrical shell roof are analyzed using single trigonometry series, it was also addressed here these solution methods. Although it was used these solution methods, this work only used the idea for constant determination. In addition the constants were determined using MATLAB.

The general expression for displacement determination is given by;

$\omega = \omega_o + \omega_1$, where (3.6)

ω = total displacement

ω_o = homogeneous solution

ω_1 = particular solution

$$\omega_o = \sum_{m=1,3}^{\infty}\big[(A_1 \cos b\theta + A_2 sinb\theta)e^{a\theta} + (A_3 \cos b_1\theta + A_4 sinb_1\theta)e^{a_1\theta} + (A_5 \cos b\theta + A_6 sinb\theta)e^{-a\theta} + (A_7 \cos b_1\theta + A_8 sinb_1\theta)e^{-a_1\theta}\big] \cos \alpha_m x \quad (3.6a)$$

$$\omega_1 = \sum_{m=1,3}^{\infty}[CE_m \cos \alpha_m x \cos \theta] \quad (3.6b)$$

Generally for any force or displacement it can be written as

$$F = \sum_{m=1,3}^{\infty} MF\Big[\big((A_1\alpha_1 - A_2\alpha_2) \cos b\theta - (A_1\alpha_2 + A_2\alpha_1) sinb\theta\big)e^{a\theta} + \big((A_3\alpha_3 - A_4\alpha_4) \cos b_1\theta - (A_3\alpha_4 + A_4\alpha_3) sinb_1\theta\big)e^{a_1\theta} \pm \big((A_5\alpha_1 - A_6\alpha_2) \cos b\theta + (A_5\alpha_2 + A_6\alpha_1) sinb\theta\big)e^{-a\theta} \pm \big((A_7\alpha_3 - A_8\alpha_4) \cos b_1\theta + (A_7\alpha_4 + A_8\alpha_3) sinb_1\theta\big)e^{-a_1\theta}\Big] \quad (3.7)$$

Where, F denotes the force or displacement, MF denotes the multiplying factor where as $\alpha_1, \alpha_2, \alpha_3$ *and* α_4 are coefficients, and all are defined in Table 5 and Table 6. It has been simply arranged these tables for this thesis purpose. In these equations the only unknowns are the constants label **A_1** to **A_8** which are determined from the boundary conditions of the shell edge at $\theta = \pm\theta_o$. It can be also observed that the constants may have equal values if the shell is subjected to symmetric load and has symmetric boundary conditions along $\theta = \pm\theta_o$, hence; $A_1 = A_5, A_2 = A_6, A_3 = A_7, A_4 = A_8$. And the plus and minus sign indicates plus for even and minus for odd functions.

$$\lambda = (\sqrt[8]{3(1-\nu^2)})(\sqrt{R\alpha_m})(\sqrt[4]{\frac{R}{h}}) \;, \qquad j = \sqrt{\frac{\sqrt{(1+(1+\varepsilon)^2)}+(1+\varepsilon)}{2}} \qquad (3.8a)$$

$$\varepsilon = \frac{1}{\sqrt[4]{3(1-\nu^2)}}(\sqrt{\frac{h}{R}})(R\alpha_m) \;, \qquad k = \sqrt{\frac{\sqrt{(1+(1+\varepsilon)^2)}-(1+\varepsilon)}{2}} \qquad (3.8b)$$

$$j_1 = \sqrt{\frac{\sqrt{(1+(1-\varepsilon)^2)}-(1-\varepsilon)}{2}} \;, \qquad k_1 = \sqrt{\frac{\sqrt{(1+(1-\varepsilon)^2)}+(1-\varepsilon)}{2}} \qquad (3.8c)$$

$$a = \lambda j,\; a_1 = \lambda j_1,\; b = \lambda k,\; b_1 = \lambda k_1 \qquad (3.9)$$

$$C = -\frac{R^4}{D}\left[\frac{(R\alpha_m)^4+(4+\nu)R^2(\alpha_m)^2+2}{(1+R^2(\alpha_m)^2)^4+R^6(1-\nu^2)(\alpha_m)^4\left(\frac{12}{h^2}\right)}\right] \qquad (3.10)$$

$$\alpha_m = {}^{m\pi}/_{L} \;, \qquad E_m = (-1)^{(m-1)/2} * \frac{4W}{m\pi} \qquad (3.11)$$

W = combined load

(The combination of total dead load and equivalent live load combined according to EBCS 1995).

m = postitive integer varied from one to infinity.

R = radius of the shell, h = thickness of the shell, D = flexural rigidity,

ν = poisson ratio, L = length of the shell

Table 5

Shell forces and displacements from complementary function (even)

F	MF	α_1	α_2	α_3	α_4
N_x	$\frac{2DR\alpha_m{}^4}{\varepsilon^3}\cos\alpha_m x$	-1	1+ε	1	1-ε
N_θ	$-\frac{2DR\alpha_m{}^4}{\varepsilon^2}\cos\alpha_m x$	0	1	0	-1
M_x	$-D\alpha_m{}^2\cos\alpha_m x$	$\left[1-\frac{\nu\lambda^2(1+\varepsilon)}{(\alpha_m R)^2}\right]$	$-\frac{\nu\lambda^2}{(\alpha_m R)^2}$	$\left[1-\frac{\nu\lambda^2(\varepsilon-1)}{(\alpha_m R)^2}\right]$	$-\frac{\nu\lambda^2}{(\alpha_m R)^2}$
M_θ	$\frac{D}{R^2}\cos\alpha_m x$	$(\lambda^2(1+\varepsilon)$ $-\nu(\alpha_m R)^2)$	(λ^2)	$(\lambda^2(\varepsilon-1)$ $-\nu(\alpha_m R)^2)$	(λ^2)
Q_x	$-\frac{D\alpha_m{}^3}{\varepsilon}\sin\alpha_m x$	1	1	-1	1
u	$\frac{h^2R\alpha_m{}^3}{6(1-\nu^2)\varepsilon^3}\sin\alpha_m x$	-1	$(1+\varepsilon(1$ $+\nu))$	1	$(1-\varepsilon(1$ $+\nu))$
w	$\cos\alpha_m x$	1	0	1	0

Table 6

Shell forces and displacements from complementary function (odd)

F	MF	α_1	α_2	α_3	α_4
$N_{x\theta}$	$\frac{2DR\alpha_m{}^4}{(\sqrt{\varepsilon})^5}\sin\alpha_m x$	-k	j	k_1	j_1
$M_{x\theta}$	$-\frac{D\alpha_m}{R}\sin\alpha_m x$	a	b	a_1	b_1
Q_θ	$\frac{D\alpha_m{}^3}{(\sqrt{\varepsilon})^3}\cos\alpha_m x$	$j-k$	$j+k$	$-(j_1+k_1)$	(j_1-k_1)
Q_θ’	$\frac{D\alpha_m{}^3}{(\sqrt{\varepsilon})^3}\cos\alpha_m x$	$j(1-\varepsilon(1$ $-\nu))-k$	$k(1-\varepsilon(1$ $-\nu))+j$	$-j_1(1+\varepsilon(1$ $-\nu))-k_1$	$-k_1(1$ $+\varepsilon(1-\nu))$ $+j_1$
v	$-\frac{h^2R\alpha_m{}^3}{6(1-\nu^2)(\sqrt{\varepsilon})^7}*$ $(\cos\alpha_m x)$	$k(1-\varepsilon(1$ $+\nu))+j$	$j(1-\varepsilon(1$ $+\nu))-k$	$k_1(1+\varepsilon(1$ $+\nu))-j_1$	$-j_1(1$ $+\varepsilon(1+\nu))$ $-k_1$
β*	$\cos\alpha_m x$	$\frac{-a+MF\alpha_1}{R}$	$\frac{-b+MF\alpha_2}{R}$	$\frac{-a_1+MF\alpha_3}{R}$	$\frac{-b_1+MF\alpha_4}{R}$

*The coefficients for β are from v coefficients

Where

$g_1 = (-1)^{(m-1)/2} * \frac{4W}{m\pi} cos\, \alpha_m x\, cos\, \theta$, $g_3 = (-1)^{(m-1)/2} * \frac{4W}{m\pi} sin\, \alpha_m x\, cos\, \theta$

$g_2 = (-1)^{(m-1)/2} * \frac{4W}{m\pi} sin\, \alpha_m x\, sin\, \theta$, $g_4 = (-1)^{(m-1)/2} * \frac{4W}{m\pi} cos\, \alpha_m x\, sin\, \theta$

$$H_3 = \frac{CD}{R^4}((\alpha_m R)^2 + 1)^2$$

Displacement functions of edge beams interns of shell forces are given by:

$$w_b = \frac{1}{EI_y} \sum_{m=1,3}^{\infty} \frac{1}{\alpha_m{}^4} \left[Q_\theta{}' \cos\theta_o + N_\theta \sin\theta - \frac{\alpha_m d}{2} N_{x\theta} - g' \right] \cos\alpha_m x \qquad (3.12)$$

$$u_b = -\sum_{m=1,3}^{\infty} \frac{1}{E(\alpha_m{}^2)} \left[\frac{N_{x\theta}}{bd} - \left\{ \frac{6}{\alpha_m b d^2} \left(Q_\theta{}' \cos\theta_o + N_\theta \sin\theta - \frac{\alpha_m d}{2} N_{x\theta} - g' \right) \right\} \right] \sin\alpha_m x \qquad (3.13)$$

Table 7

Shell forces and displacements from particular function

F	N_x	N_θ	$N_{x\theta}$	M_x	M_θ	$M_{x\theta}$
PP	$-\frac{1}{R(\alpha_m)^2}(H_3 + 2)g_1$	$-R(H_3 + 1)g_1$	$-\frac{1}{\alpha_m}(H_3 + 2)g_2$	$-D(\alpha_m{}^2 + \frac{v}{R^2})Cg_1$	$-D(v\alpha_m{}^2 + \frac{1}{R^2})Cg_1$	$D(1 - v)\left(\frac{\alpha_m}{R}\right) * Cg_2$
F	Q_x	Q_θ	w	u	v	β
PP	$D(\alpha_m{}^3 + \frac{\alpha_m{}^2}{R})Cg_3$	$D(\frac{1}{R^2} + \frac{\alpha_m{}^2}{R})Cg_4$	Cg_1	$\frac{1}{Eh\alpha_m}\left[-\frac{1}{R(\alpha_m)^2} * (H_3 + 2) + vR(H_3 + 1)\right] g_3$	$\frac{1}{Eh\alpha_m{}^2}\left[v(H_3 + 3) + (H_3 + 2)(2 + \frac{1}{(\alpha_m R)^2})\right] g_4$	$\frac{1}{R}(C + V^*)g_4$

V* =particular integral of V without g terms

The detail and other unknown constants can be obtained from the book written by [4] from page 150 to 170 or in the MATLAB code written in Appendix B.

Where, d and b are depth and width of edge beams but W is now for edge beams.

$E_m = g'$,

w_b = vertical displacement of edge beam,

u_b = longitudinal displacement of edge beam,

I_y = moment of inertia for edge beam,

E = modulus of elasticity of edge beam

$$Q_\theta{}' = Q_\theta + \frac{\partial M_{x\theta}}{\partial x} \qquad (3.14)$$

Therefore; the program was developed using the aforementioned relationships and applying the condition presented at section of literature review under boundary condition.

The whole procedure done in this research was:

- Determination of constants $\mathbf{A_1}$ to $\mathbf{A_8}$ using MATLAB for all cases
- Determination of displacement function using MATLAB in symbolic expressions
- Determination of strains, stresses, forces and moments in symbolic expressions
- Symbolic substitutions for all symbolic expressions
- Obtained an output of these effects
- Plotting these functions in MATLAB all conditions together for each functions obtained.
- Discussion on the maximum Results of all effects

4. RESULT

So far, the researcher has shown estimates of the structural effects of reinforced concrete cylindrical shell structures under the action of dead and live load by varying the shell parameters; length, radius and thickness, and with various assumptions using MATLAB code in order to compare results with original dimension of structural effects and plotting their effect along longitudinal and circumferential directions of the shell. The calculated results were presented using tables as shown from Table (9-16) which indicated negative and maximum values of forces, moments, stresses and displacements of shell structures for all models.

Although the research was worked for seven trials it has discussed only the first trials tabulated on Table 8.Inaddition it is possible to determine any values of the structural effects at any point on the structure using MATLAB code written at Appendix B and samples for stresses as shown in Figure (9-11) which are important for reinforcement detailing.

Therefore, Table 8 indicates percentage comparison of original values of the first trial with values when the length, radius, and thickness, are increased by 15 meter, 6.71 meter and 0.03 meter respectively. The table shows percentage increment or decrement of the original value when length, radius, thickness are changed respectively. Positive value indicates increasing by the indicated amount from the original whereas negative value indicates decreasing by the indicated amount from the original value.

In addition it could be shown that in Table (17) and Figure (8); the shell forces, moments, stresses and displacements will reduce with increase in depth of the edge beam. It is also interesting to note that the assumed dimensions of the edge beam result in either compression and/or tension effects on the shell.

Forces, moments, stresses and displacements along longitudinal and circumferential directions are also plotted in Figure (A1-A5) in the Appendix A.

Under the application of external loadings for reinforced concrete structures, the internal effects may be experienced tension and compression, bending and twisting effects. As a result maximum positive and negative results were presented in Table (9-16).

Furthermore, the stress resultants plotted at closer intervals of **θ** shown in Figure (9-11) can be useful for detailing of reinforcement layout in reinforced concrete shells.

Table 8

Comparison of structural effects under varying length, thickness and radius

	a	b	c		a	b	c		a	b	c		a	b	c
	Longitudinal force				Moment along X. (Mx)				Longitudinal stress				Longitudinal displacement		
SF	42%	18%	0%		13%	18%	25%		39%	18%	-34%		62%	18%	-34%
SB	52%	38%	33%		49%	-13%	46%		58%	32%	22%		69%	38%	15%
MF	-12%	37%	17%		43%	-16%	5%		0%	33%	-16%		26%	36%	-13%
MB	27%	19%	26%		59%	11%	28%		34%	13%	-5%		51%	18%	0%
	Circumferential force				Moment along ɵ. (Mɵ)				Circumferential stress				Circumferential displacement		
SF	-35%	37%	18%		-16%	10%	19%		-17%	12%	-41%		71%	3%	-53%
SB	-22%	32%	20%		46%	-30%	37%		38%	-14%	-11%		75%	8%	-5%
MF	-12%	33%	14%		37%	2%	15%		40%	10%	-41%		39%	32%	-21%
MB	-8%	31%	15%		51%	-11%	21%		50%	-11%	-38%		48%	23%	-8%
	Tangential force				Torsional moment (Mxɵ)				Shear stress				Vertical displacement		
SF	23%	30%	12%		20%	7%	18%		22%	15%	-36%		63%	18%	-77%
SB	23%	23%	14%		44%	-46%	39%		29%	12%	-6%		73%	9%	-20%
MF	0%	32%	13%		40%	1%	6%		5%	30%	-18%		42%	48%	-33%
MB	3%	26%	16%		58%	-13%	31%		14%	23%	-13%		45%	45%	-14%
	Vertical shear force (Qx)				Vertical shear force(Qx)				Vertical shear force (Qθ)				Vertical shear force (Qθ)		
SF	-76%	8%	18%	MF	7%	1%	15%	SF	-23%	12%	20%	MF	2%	7%	15%
SB	25%	-36%	41%	MB	28%	-15%	20%	SB	-1%	37%	28%	MB	21%	-5%	19%

SF=Single span with free edge, SB=Single span with edge beam, MF=multiple span without edge beam, MB=multiple span with edge beam, a=ΔL=15m, b=ΔR=6.71m, c=ΔT=0.03m, ΔL= increase in length, ΔR= increase in radius, ΔT= increase in thickness

Table 9

Negative and positive maximum forces and moments of cylindrical shell for single span with free edge

Without edge beam single span																
	Maximum negative value								Maximum positive value							
`	Nx (kN/m)				Mx (kNm/m)				Nx (kN/m)				Mx (kNm/m)			
Trial	Mo	M1	M2	M3	Mo	M1	M2	M 3	Mo	M1	M2	M3	Mo	M1	M2	M 3
1	406.00	937.00	630.00	723.00	3.69	2.70	4.52	7.07	1934.0	2571.0	2732.0	3469.0	4.45	7.35	6.22	7.68
2	579.00	735.00	654.00	723.00	5.58	6.41	7.40	7.07	2796.0	2966.0	3111.0	3469.0	6.21	8.56	8.52	7.68
3	698.00	698.00	698.00	698.00	8.17	8.17	8.17	8.17	3337.0	3337.0	3337.0	3337.0	9.56	9.56	9.56	9.56
4	936.00	853.00	838.00	831.00	12.26	9.62	8.21	9.12	4134.0	3980.0	3592.0	3346.0	15.53	10.73	10.36	12.59
5	975.00	1087.00	838.00	910.00	15.95	9.46	8.21	9.26	4720.0	4673.0	3592.0	3382.0	18.85	11.14	10.36	13.69
6	1683.00	2102.00	2157.00	2251.00	7.90	7.51	0.00	0.00	4870.0	5402.0	4708.0	4915.0	6.18	7.58	7.89	7.37
7	1880.00	1880.00	2379.00	2379.00	10.68	10.68	0.00	0.00	5656.0	5656.0	5139.0	5139.0	8.01	8.01	9.07	9.07
Trial	Nθ (kN/m)				Mɵ (kNm/m)				Nθ (kN/m)				Mɵ (kNm/m)			
1	87.00	165.00	68.00	124.00	19.19	24.92	20.38	34.93	0.00	0.00	0.00	0.00	0.35	0.42	0.43	0.51
2	103.00	156.00	109.00	124.00	27.60	36.49	36.10	34.93	0.00	0.00	0.00	0.00	0.44	0.61	0.65	0.51
3	147.00	147.00	147.00	147.00	42.36	42.36	42.36	42.36	0.00	0.00	0.00	0.00	0.71	0.71	0.71	0.71
4	204.00	130.00	181.00	177.00	66.82	46.58	45.45	52.09	0.00	0.00	0.00	0.00	1.16	0.74	0.72	0.96
5	200.00	113.00	181.00	186.00	81.47	43.83	45.45	55.09	0.00	0.00	0.00	0.00	1.48	0.57	0.72	1.04
6	137.00	162.00	155.00	161.00	48.94	52.90	26.53	25.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	157.00	157.00	169.00	169.00	64.72	64.72	26.54	26.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Trial	Nxθ (kN/m)				Mxɵ in (kNm/m)				Nxθ (kN/m)				Mxɵ in (kNm/m)			
1	197.00	332.00	237.00	307.00	6.61	8.26	9.26	12.20	197.00	332.00	237.00	307.00	6.61	8.26	9.26	12.20
2	254.00	328.00	281.00	307.00	9.71	12.18	13.10	12.20	254.00	328.00	281.00	307.00	9.71	12.18	13.10	12.20
3	334.00	334.00	334.00	334.00	14.48	14.48	14.48	14.48	334.00	334.00	334.00	334.00	14.48	14.48	14.48	14.48
4	443.00	342.00	386.00	376.00	22.58	16.88	15.14	17.59	443.00	342.00	386.00	376.00	22.58	16.88	15.14	17.59
5	468.00	377.00	386.00	391.00	28.35	17.76	15.14	18.58	468.00	377.00	386.00	391.00	28.35	17.76	15.14	18.58
6	425.00	496.00	463.00	481.00	11.28	12.10	6.24	5.90	425.00	496.00	463.00	481.00	11.28	12.10	6.24	5.90
7	486.00	486.00	506.00	506.00	14.98	14.98	6.31	6.31	486.00	486.00	506.00	506.00	14.98	14.98	6.31	6.31
Trial	Qx (kN/m)				Qθ (kN/m)				Qx (kN/m)				Qθ (kN/m)			
1	3.05	3.68	2.37	3.99	6.95	9.34	5.87	9.22	3.05	3.68	2.37	3.99	6.95	9.34	5.87	9.22
2	3.53	4.57	4.14	3.99	8.09	10.76	9.41	9.22	3.53	4.57	4.14	3.99	8.09	10.76	9.41	9.22
3	4.81	4.81	4.81	4.81	11.03	11.03	11.03	11.03	4.81	4.81	4.81	4.81	11.03	11.03	11.03	11.03
4	6.58	4.68	5.10	5.82	15.28	10.77	11.99	13.61	6.58	4.68	5.10	5.82	15.28	10.77	11.99	13.61
5	7.20	3.94	5.10	6.12	16.34	9.57	11.99	14.41	7.20	3.94	5.10	6.12	16.34	9.57	11.99	14.41
6	3.78	4.02	2.49	2.38	11.95	13.05	9.58	9.15	3.78	4.02	2.49	2.38	11.95	13.05	9.58	9.15
7	4.46	4.46	2.38	2.38	13.98	13.98	9.63	9.63	4.46	4.46	2.38	2.38	13.98	13.98	9.63	9.63

Table 10

Negative and positive maximum stresses and displacements of cylindrical shell for single span with free edge

Without edge beam single span																
	Maximum negative value								Maximum positive value							
	Sx(kPa)				Ux (m)				Sx(kPa)				Ux (m)			
Trial	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3
1	7567	10994	6882	11653	0.01	0.01	0.01	0.02	28340	30120	31052	50564	0.01	0.01	0.01	0.02
2	9405	11181	8076	11653	0.01	0.01	0.01	0.02	40773	34793	36228	50564	0.01	0.01	0.01	0.02
3	10362	10362	10362	10362	0.01	0.01	0.01	0.01	39101	39101	39101	39101	0.01	0.01	0.01	0.01
4	11193	10346	13122	9615	0.01	0.02	0.01	0.01	37315	46232	42131	30208	0.01	0.02	0.01	0.01
5	9908	12209	13122	9338	0.02	0.02	0.01	0.01	39483	53408	42131	28348	0.02	0.02	0.01	0.01
6	24548	25521	21783	25183	0.03	0.03	0.02	0.02	58693	58567	51816	60067	0.03	0.03	0.02	0.02
7	25212	25212	24712	24712	0.03	0.03	0.02	0.02	61362	61362	56835	56835	0.03	0.03	0.02	0.02
Trial	Sɵ (kPa)				Vɵ (m)				Sɵ (kPa)				Vɵ (m)			
1	19074	16602	12880	34301	0.04	0.03	0.07	0.11	16906	13304	11571	31193	0.04	0.03	0.07	0.11
2	27154	23449	22751	34301	0.08	0.06	0.08	0.11	24588	20336	20569	31193	0.08	0.06	0.08	0.11
3	26884	26884	26884	26884	0.08	0.08	0.08	0.08	23951	23951	23951	23951	0.08	0.08	0.08	0.08
4	25293	29249	29081	19853	0.08	0.12	0.08	0.05	22154	26643	25461	17132	0.08	0.12	0.08	0.05
5	26372	27395	29081	18197	0.10	0.17	0.08	0.05	23508	25198	25461	15533	0.10	0.17	0.08	0.05
6	37775	33362	17466	20489	0.21	0.19	0.10	0.12	34720	30114	14375	16914	0.21	0.19	0.10	0.12
7	40395	40395	17617	17617	0.25	0.25	0.11	0.11	37263	37263	14229	14229	0.25	0.25	0.11	0.11
Trial	Sxɵ(kPa)				Wz m)				Sxɵ(kPa)				Wz (m)			
1	8436	7962	7835	15026	0.17	0.15	0.24	0.58	8436	7962	7835	15026	0.01	0.00	0.03	0.06
2	12059	10312	10514	15026	0.37	0.26	0.33	0.58	12059	10312	10514	15026	0.04	0.00	0.04	0.06
3	11692	11692	11692	11692	0.37	0.37	0.37	0.37	11692	11692	11692	11692	0.02	0.02	0.02	0.02
4	11129	13406	12597	8906	0.36	0.54	0.40	0.22	11129	13406	12597	8906	0.01	0.06	0.00	0.00
5	11700	14255	12597	8268	0.44	0.71	0.40	0.19	11700	14255	12597	8268	0.03	0.10	0.00	0.00
6	12619	11714	7953	9221	0.67	0.60	0.30	0.39	12619	11714	7953	9221	0.00	0.00	0.00	0.00
7	13393	13393	8399	8399	0.80	0.80	0.35	0.35	13393	13393	8399	8399	0.00	0.00	0.00	0.00

Table 11

Negative and positive maximum forces and moments of cylindrical shell for single span with edge beam

With edge beam single span																
	Maximum negative value								Maximum positive value							
`	Nx (kN/m)				Mx (kNm/m)				Nx (kN/m)				Mx (kNm/m)			
Trial	Mo	M1	M2	M3	Mo	M1	M2	M 3	Mo	M1	M2	M3	Mo	M1	M2	M 3
1	443.92	404.11	444.49	440.49	3.02	2.75	3.08	2.31	333.58	293.71	344.38	240.67	3.02	2.75	3.08	2.31
2	579.89	547.80	578.68	577.07	4.10	3.86	4.11	3.08	497.59	461.98	508.71	372.20	4.10	3.86	4.11	3.08
3	551.43	525.05	552.63	536.06	3.15	2.99	3.19	2.47	291.01	276.61	306.35	212.49	3.15	2.99	3.19	2.47
4	720.58	689.33	721.95	738.91	3.62	3.50	3.73	4.62	437.17	411.39	443.98	563.39	3.62	3.50	3.73	4.62
5	885.06	848.82	883.85	896.95	6.42	6.21	6.56	7.80	704.16	670.49	717.48	859.05	6.42	6.21	6.56	7.80
6	790.93	762.53	793.35	817.91	3.50	3.37	3.55	4.36	306.22	296.95	320.11	390.75	3.50	3.37	3.55	4.36
7	1005.00	849.80	1012.40	1038.50	5.27	4.57	5.44	6.49	514.06	413.91	515.08	634.37	5.27	4.57	5.44	6.49
Trial	Nθ (kN/m)				Mɵ (kNm/m)				Nθ (kN/m)				Mɵ (kNm/m)			
1	97.04	99.45	109.46	88.78	10.59	9.98	11.31	8.31	0.00	0.00	0.00	0.00	10.31	9.48	10.66	8.15
2	115.89	117.73	128.47	105.84	14.20	13.84	15.14	11.10	0.00	0.00	0.00	0.00	13.86	13.28	14.35	10.96
3	136.38	137.63	148.02	126.91	12.23	11.75	12.67	9.84	0.00	0.00	0.00	0.00	11.21	10.60	11.30	8.98
4	142.32	143.90	153.75	153.52	13.69	13.49	14.57	17.29	0.00	0.00	0.00	0.00	13.17	12.71	13.55	16.53
5	185.22	186.95	198.17	198.67	24.07	23.56	25.13	29.01	0.00	0.00	0.00	0.00	22.87	22.08	23.35	27.45
6	182.96	184.30	194.21	194.89	14.84	14.56	15.49	18.01	0.00	0.00	0.00	0.00	13.16	12.66	13.33	16.08
7	193.98	201.90	206.38	206.48	19.89	18.95	21.34	24.32	0.00	0.00	0.00	0.00	19.53	16.86	20.18	23.68
Trial	Nxθ (kN/m)				Mxɵ in (kNm/m)				Nxθ (kN/m)				Mxɵ in (kNm/m)			
1	235.61	230.20	251.84	231.59	2.31	2.12	2.39	1.81	235.61	230.20	251.84	231.59	2.31	2.12	2.39	1.81
2	288.05	283.88	304.32	279.89	3.10	2.97	3.21	2.43	288.05	283.88	304.32	279.89	3.10	2.97	3.21	2.43
3	322.40	316.90	334.66	312.09	2.48	2.34	2.50	1.97	322.40	316.90	334.66	312.09	2.48	2.34	2.50	1.97
4	363.89	358.50	376.34	374.12	2.86	2.77	2.96	3.63	363.89	358.50	376.34	374.12	2.86	2.77	2.96	3.63
5	448.30	443.35	465.24	472.20	5.05	4.89	5.17	6.10	448.30	443.35	465.24	472.20	5.05	4.89	5.17	6.10
6	451.27	444.87	463.37	468.09	2.82	2.73	2.87	3.49	451.27	444.87	463.37	468.09	2.82	2.73	2.87	3.49
7	514.10	483.59	529.08	529.84	4.14	3.66	4.32	5.10	514.10	483.59	529.08	529.84	4.14	3.66	4.32	5.10
Trial	Qx (kN/m)				Qθ (kN/m)				Qx (kN/m)				Qθ (kN/m)			
1	1.16	1.12	1.20	0.91	4.76	4.61	4.97	3.80	1.16	1.12	1.20	0.91	4.76	4.61	4.97	3.80
2	1.37	1.34	1.40	1.07	5.68	5.60	5.92	4.53	1.37	1.34	1.40	1.07	5.68	5.60	5.92	4.53
3	0.98	0.96	1.01	0.79	8.82	9.10	9.74	8.82	0.98	0.96	1.01	0.79	8.82	9.10	9.74	8.82
4	0.98	0.97	1.02	1.23	8.81	9.08	9.67	8.70	0.98	0.97	1.02	1.23	8.81	9.08	9.67	8.70
5	1.56	1.54	1.61	1.88	9.80	10.07	10.61	9.59	1.56	1.54	1.61	1.88	9.80	10.07	10.61	9.59
6	0.87	0.86	0.90	1.05	14.39	14.65	15.34	14.69	0.87	0.86	0.90	1.05	14.39	14.65	15.34	14.69
7	1.08	1.08	1.13	1.32	13.61	15.12	14.73	13.76	1.08	1.08	1.13	1.32	13.61	15.12	14.73	13.76

Table 12

Negative and positive maximum stresses and displacements of cylindrical shell for single span with edge beam

	With edge beam single span															
	Maximum negative value								Maximum positive value							
	Sx(kPa)				Ux (m)				Sx(kPa)				Ux (m)			
Trial	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3
1	6008	5540	6118	6567	0.002	0.002	0.002	0.002	5944	5246	6043	5054	0.002	0.002	0.002	0.002
2	8567	7989	8699	8577	0.003	0.003	0.003	0.003	8567	7989	8699	7449	0.003	0.003	0.003	0.003
3	6694	6413	6784	7160	0.003	0.003	0.003	0.003	3891	3609	3925	2975	0.003	0.003	0.003	0.003
4	9533	9187	9673	8914	0.005	0.004	0.005	0.004	6375	5938	6346	7601	0.005	0.004	0.005	0.004
5	9758	9416	9848	9936	0.005	0.005	0.005	0.005	8857	8394	8924	9936	0.005	0.005	0.005	0.005
6	9534	9246	9657	9001	0.005	0.005	0.005	0.005	3462	3477	3772	3684	0.005	0.005	0.005	0.005
7	10750	9364	10951	10258	0.007	0.005	0.007	0.007	5399	3908	5279	6483	0.007	0.005	0.007	0.007
Trial	Sɵ (kPa)				Vɵ (m)				Sɵ (kPa)				Vɵ (m)			
1	8919	8497	9592	8897	0.023	0.019	0.023	0.024	7401	6610	7447	7181	0.023	0.019	0.023	0.024
2	11807	11556	12641	11725	0.037	0.033	0.037	0.038	10230	9466	10093	9715	0.037	0.033	0.037	0.038
3	8703	8427	9083	8700	0.029	0.026	0.029	0.029	6192	5829	6220	6076	0.029	0.026	0.029	0.029
4	11720	11588	12502	11911	0.053	0.049	0.052	0.052	9082	8749	9340	9291	0.053	0.049	0.052	0.052
5	13620	13384	14264	13741	0.063	0.058	0.063	0.062	10665	10283	10878	10802	0.063	0.058	0.063	0.062
6	10732	10580	11234	10704	0.052	0.049	0.052	0.052	7140	6894	7350	7255	0.052	0.049	0.052	0.052
7	11628	11230	12460	11855	0.076	0.055	0.075	0.075	8905	7623	9201	9126	0.076	0.055	0.075	0.075
Trial	Sxɵ(kPa)				Wz m)				Sxɵ(kPa)				Wz (m)			
1	3592	3458	3837	3672	0.076	0.064	0.079	0.082	3592	3458	3837	3672	0.018	0.013	0.016	0.023
2	4562	4473	4838	4647	0.133	0.121	0.138	0.144	4562	4473	4838	4647	0.034	0.029	0.032	0.042
3	3672	3577	3822	3729	0.093	0.084	0.094	0.095	3672	3577	3822	3729	0.022	0.017	0.018	0.026
4	4791	4677	4936	4724	0.187	0.172	0.192	0.181	4791	4677	4936	4724	0.057	0.051	0.056	0.050
5	5307	5226	5520	5346	0.229	0.213	0.234	0.220	5307	5226	5520	5346	0.061	0.054	0.058	0.053
6	4659	4588	4773	4467	0.168	0.157	0.171	0.164	4659	4588	4773	4467	0.054	0.048	0.051	0.048
7	5094	4634	5220	5066	0.247	0.178	0.253	0.245	5094	4634	5220	5066	0.085	0.052	0.085	0.077

Table 13

Negative and positive maximum forces and moments of cylindrical shell for multiple span with free edge

	Without edge beammultiple span															
	Maximum negative value								Maximum positive value							
`	Nx (kN/m)				Mx (kNm/m)				Nx (kN/m)				Mx (kNm/m)			
Trial	Mo	M1	M2	M3	Mo	M1	M2	M 3	Mo	M1	M2	M3	Mo	M1	M2	M 3
1	367.71	381.75	421.33	349.35	1.74	1.60	1.81	1.65	0.00	0.00	0.00	0.00	1.07	1.04	1.16	0.96
2	446.19	448.84	491.45	426.34	2.35	2.24	2.42	2.21	0.00	0.00	0.00	0.00	1.38	1.35	1.47	1.24
3	549.35	551.89	594.45	525.94	3.26	3.13	3.35	3.08	0.00	0.00	0.00	0.00	1.91	1.87	2.01	1.73
4	603.23	604.71	645.75	628.66	3.96	3.85	4.10	4.22	0.00	0.00	0.00	0.00	2.12	2.09	2.23	2.34
5	751.00	752.92	797.76	781.65	5.40	5.25	5.53	5.68	0.00	0.00	0.00	0.00	3.03	2.99	3.16	3.30
6	795.74	796.71	840.00	827.23	6.09	5.95	6.26	6.47	0.00	0.00	0.00	0.00	3.21	3.17	3.34	3.51
7	851.75	858.66	899.70	883.18	7.44	6.76	7.72	7.92	0.00	0.00	0.00	0.00	3.82	3.66	4.02	4.17
Trial	(Nθ in kN/m)				. (Mɵ in kNm/m)				(Nθ in kN/m)				. (Mɵ in kNm/m)			
1	92.96	94.29	103.46	87.47	10.44	9.78	11.00	9.75	13.39	14.61	16.70	12.53	4.47	4.23	4.74	4.12
2	111.26	112.17	121.71	104.64	13.92	13.42	14.53	12.98	16.80	17.61	20.05	15.71	5.89	5.71	6.18	5.42
3	136.45	137.38	147.41	128.79	19.32	18.69	20.02	18.09	20.97	21.81	24.38	19.69	8.16	7.93	8.50	7.55
4	144.51	145.55	155.07	153.29	22.99	22.44	23.92	24.66	21.64	22.48	24.91	23.11	9.43	9.26	9.88	10.25
5	183.32	184.33	194.83	193.06	31.73	30.92	32.66	33.63	28.75	29.63	32.30	30.41	13.23	12.95	13.68	14.16
6	188.05	189.15	199.13	198.89	35.26	34.55	36.39	37.70	28.86	29.74	32.27	30.73	14.36	14.14	14.92	15.54
7	199.97	205.94	211.87	211.12	42.81	39.35	44.59	45.79	27.57	32.34	31.31	29.34	17.20	16.21	18.07	18.64
Trial	Nxθ (kN/m)				Mxɵ in (kNm/m)				Nxθ (kN/m)				Mxɵ in (kNm/m)			
1	250.22	251.13	275.56	238.34	0.55	0.51	0.57	0.51	250.22	251.13	275.56	238.34	0.55	0.51	0.57	0.51
2	302.03	302.30	327.03	288.09	0.73	0.70	0.76	0.68	302.03	302.30	327.03	288.09	0.73	0.70	0.76	0.68
3	370.71	370.87	396.91	354.36	1.01	0.98	1.05	0.95	370.71	370.87	396.91	354.36	1.01	0.98	1.05	0.95
4	406.21	405.76	430.74	424.10	1.20	1.18	1.25	1.30	406.21	405.76	430.74	424.10	1.20	1.18	1.25	1.30
5	503.94	503.70	531.12	525.18	1.66	1.62	1.71	1.76	503.94	503.70	531.12	525.18	1.66	1.62	1.71	1.76
6	532.92	532.11	558.48	554.95	1.84	1.80	1.90	1.97	532.92	532.11	558.48	554.95	1.84	1.80	1.90	1.97
7	578.30	574.48	605.39	600.60	2.23	2.06	2.32	2.40	578.30	574.48	605.39	600.60	2.23	2.06	2.32	2.40
Trial	Qx (kN/m)				Qθ (kN/m)				Qx (kN/m)				Qθ (kN/m)			
1	1.06	1.05	1.12	0.99	12.59	12.53	13.49	11.78	1.06	1.05	1.12	0.99	12.59	12.53	13.49	11.78
2	1.24	1.23	1.29	1.16	14.77	14.72	15.59	13.80	1.24	1.23	1.29	1.16	14.77	14.72	15.59	13.80
3	1.50	1.49	1.56	1.41	17.98	17.91	18.82	16.88	1.50	1.49	1.56	1.41	17.98	17.91	18.82	16.88
4	1.57	1.57	1.63	1.68	18.85	18.84	19.74	20.13	1.57	1.57	1.63	1.68	18.85	18.84	19.74	20.13
5	1.98	1.97	2.03	2.09	23.77	23.69	24.63	25.14	1.98	1.97	2.03	2.09	23.77	23.69	24.63	25.14
6	2.00	2.00	2.06	2.14	24.17	24.15	25.07	25.73	2.00	2.00	2.06	2.14	24.17	24.15	25.07	25.73
7	2.19	2.19	2.28	2.34	26.26	26.45	27.49	27.94	2.19	2.19	2.28	2.34	26.26	26.45	27.49	27.94

Table 14

Negative and positive maximum stresses and displacements of cylindrical shell for multiple span with free edge

	Without edge beammultiple span																
	Maximum negative value								Maximum positive value								
	Sx(kPa)				Ux (m)				Sx(kPa)				Ux (m)				
Trial	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3	
1	4506	4655	5149	4796	0.002	0.002	0.002	0.002	1291	1189	1337	1546	0.002	0.002	0.002	0.002	
2	5416	5523	6061	6026	0.002	0.002	0.002	0.002	1739	1663	1794	2075	0.002	0.002	0.002	0.002	
3	6023	6096	6605	6609	0.003	0.003	0.003	0.003	1956	1880	2008	2284	0.003	0.003	0.003	0.003	
4	7936	7837	8331	7193	0.004	0.004	0.004	0.004	2934	2854	3036	2530	0.004	0.004	0.004	0.004	
5	7721	7644	8071	7179	0.004	0.004	0.005	0.004	2680	2601	2742	2368	0.004	0.004	0.005	0.004	
6	9603	9492	9972	8789	0.005	0.005	0.006	0.005	3654	3572	3757	3210	0.005	0.005	0.006	0.005	
7	9612	9132	9996	8867	0.006	0.005	0.006	0.006	3687	3351	3829	3300	0.006	0.005	0.006	0.006	
Trial	Sə (kPa)				Və (m)				Sə (kPa)				Və (m)				
1	7878	7408	8334	9294	0.008	0.008	0.009	0.009	7878	7408	8334	9294	0.008	0.008	0.009	0.009	
2	10501	10135	10986	12365	0.012	0.011	0.012	0.013	10501	10135	10986	12365	0.012	0.011	0.012	0.013	
3	11799	11432	12257	13617	0.015	0.014	0.016	0.016	11799	11432	12257	13617	0.015	0.014	0.016	0.016	
4	17273	16875	17993	15028	0.022	0.021	0.023	0.020	17273	16875	17993	15028	0.022	0.021	0.023	0.020	
5	15994	15603	16487	14266	0.023	0.023	0.025	0.022	15994	15603	16487	14266	0.023	0.023	0.025	0.022	
6	21442	21028	22156	18972	0.032	0.031	0.033	0.029	21442	21028	22156	18972	0.032	0.031	0.033	0.029	
7	21477	19808	22394	19325	0.035	0.031	0.037	0.033	21477	19808	22394	19325	0.035	0.031	0.037	0.033	
Trial	Sxə(kPa)				Wz m)				Sxə(kPa)				Wz (m)				
1	2937	2933	3218	3163	0.017	0.016	0.019	0.020	2937	2933	3218	3163	0.000	0.000	0.000	0.000	
2	3560	3553	3838	3841	0.027	0.025	0.030	0.032	3560	3553	3838	3841	0.000	0.000	0.000	0.003	
3	3935	3926	4197	4200	0.034	0.033	0.038	0.040	3935	3926	4197	4200	0.000	0.000	0.000	0.004	
4	4842	4830	5126	4531	0.060	0.057	0.064	0.051	4842	4830	5126	4531	0.014	0.011	0.011	0.007	
5	4887	4875	5137	4649	0.060	0.058	0.064	0.053	4887	4875	5137	4649	0.006	0.003	0.003	0.001	
6	5731	5716	5999	5406	0.092	0.089	0.098	0.080	5731	5716	5999	5406	0.025	0.021	0.022	0.015	
7	5655	5598	5922	5372	0.100	0.086	0.106	0.087	5655	5598	5922	5372	0.034	0.016	0.032	0.024	

Table 15

Negative and positive maximum forces and moments of cylindrical shell for multiple span with edge beam

	With edge beam multiple span															
	Maximum negative value								Maximum positive value							
`	Nx (kN/m)				Mx (kNm/m)				Nx (kN/m)				Mx (kNm/m)			
Trial	Mo	M1	M2	M3	Mo	M1	M2	M 3	Mo	M1	M2	M3	Mo	M1	M2	M 3
1	877.21	840.14	951.18	777.12	2.24	1.96	2.19	2.11	877.2	840.1	951.2	777.1	1.77	1.68	1.88	1.52
2	1157.60	1132.10	1240.80	1033.80	3.08	2.87	3.04	2.88	1157.6	1132.1	1240.8	1033.8	2.31	2.23	2.41	1.99
3	1045.40	1012.90	1097.40	923.20	3.74	3.50	3.70	3.49	1045.4	1012.9	1097.4	923.2	2.69	2.60	2.78	2.34
4	1353.90	1321.40	1417.30	1506.90	4.90	4.68	4.93	5.29	1353.9	1321.4	1417.3	1506.9	3.15	3.06	3.25	3.60
5	1810.50	1776.00	1890.70	1980.50	6.71	6.40	6.69	7.09	1810.5	1776.0	1890.7	1980.5	4.67	4.55	4.80	5.24
6	1304.10	1268.50	1344.40	1464.90	6.67	6.42	6.72	7.21	1304.1	1268.5	1344.4	1464.9	4.07	3.97	4.18	4.62
7	1716.50	1536.80	1776.80	1889.20	8.94	7.53	9.07	9.66	1716.5	1536.8	1776.8	1889.2	5.38	4.82	5.55	6.05
Trial	(Nθ in kN/m)				. (Mɵ in kNm/m)				(Nθ in kN/m)				. (Mɵ in kNm/m)			
1	100.44	101.31	111.27	93.87	14.28	12.90	14.47	13.04	19.17	19.43	21.88	17.02	6.79	6.19	6.90	6.06
2	120.85	121.50	131.77	112.89	19.32	18.27	19.57	17.64	24.17	24.46	27.10	21.55	9.07	8.61	9.21	8.11
3	143.52	144.02	154.46	134.66	23.22	22.03	23.44	21.24	18.66	18.37	20.24	16.06	10.70	10.19	10.82	9.61
4	153.82	154.57	164.65	164.19	29.42	28.25	29.89	32.23	22.35	22.40	24.52	25.52	13.26	12.77	13.51	14.78
5	196.45	197.12	208.32	207.90	41.25	39.66	41.66	44.42	31.79	31.85	34.37	35.16	18.97	18.29	19.19	20.73
6	194.93	195.63	205.90	207.27	39.64	38.30	40.20	43.36	17.37	17.04	18.59	20.70	17.46	16.91	17.75	19.40
7	210.44	214.72	222.58	223.33	52.67	45.30	53.79	57.45	22.38	22.08	24.64	25.78	23.26	20.26	23.84	25.77
Trial	Nxθ (kN/m)				Mxɵ in (kNm/m)				Nxθ (kN/m)				Mxɵ in (kNm/m)			
1	231.44	230.75	253.81	215.73	0.78	0.71	0.78	0.71	231.44	230.75	253.81	215.73	0.78	0.71	0.78	0.71
2	281.27	281.12	304.44	260.13	1.05	0.98	1.05	0.97	281.27	281.12	304.44	260.13	1.05	0.98	1.05	0.97
3	318.43	316.96	338.07	302.18	1.25	1.18	1.25	1.15	318.43	316.96	338.07	302.18	1.25	1.18	1.25	1.15
4	351.25	349.81	370.51	369.92	1.62	1.55	1.63	1.77	351.25	349.81	370.51	369.92	1.62	1.55	1.63	1.77
5	442.67	441.82	466.56	472.23	2.25	2.15	2.26	2.41	442.67	441.82	466.56	472.23	2.25	2.15	2.26	2.41
6	440.18	437.76	458.52	462.11	2.15	2.08	2.17	2.36	440.18	437.76	458.52	462.11	2.15	2.08	2.17	2.36
7	488.58	478.22	509.26	510.55	2.90	2.46	2.95	3.17	488.58	478.22	509.26	510.55	2.90	2.46	2.95	3.17
Trial	Qx (kN/m)				Qθ (kN/m)				Qx (kN/m)				Qθ (kN/m)			
1	1.44	1.36	1.45	1.32	16.33	15.77	16.91	14.94	1.44	1.36	1.45	1.32	16.33	15.77	16.91	14.94
2	1.70	1.65	1.72	1.56	19.34	18.97	19.91	17.71	1.70	1.65	1.72	1.56	19.34	18.97	19.91	17.71
3	1.79	1.74	1.81	1.65	20.58	20.16	21.05	18.89	1.79	1.74	1.81	1.65	20.58	20.16	21.05	18.89
4	2.00	1.96	2.03	2.19	22.71	22.40	23.33	24.77	2.00	1.96	2.03	2.19	22.71	22.40	23.33	24.77
5	2.55	2.50	2.57	2.74	29.15	28.73	29.73	31.32	2.55	2.50	2.57	2.74	29.15	28.73	29.73	31.32
6	2.24	2.20	2.27	2.45	25.85	25.52	26.42	28.12	2.24	2.20	2.27	2.45	25.85	25.52	26.42	28.12
7	2.69	2.51	2.74	2.93	30.37	28.93	31.29	32.92	2.69	2.51	2.74	2.93	30.37	28.93	31.29	32.92

Table 16

Negative and positive maximum stresses and displacements of cylindrical shell for multiple span with edge beam

	With edge beam multiple span															
	Maximum negative value								Maximum positive value							
	Sx(kPa)				Ux (m)				Sx(kPa)				Ux (m)			
Trial	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3	Mo	M1	M2	M 3
1	11409	10785	12189	11691	0.004	0.003	0.004	0.004	11409	10785	12189	11691	0.004	0.003	0.004	0.004
2	15145	14702	16038	15625	0.006	0.005	0.006	0.006	15145	14702	16038	15625	0.006	0.005	0.006	0.006
3	12696	12231	13193	12842	0.005	0.005	0.006	0.005	12696	12231	13193	12842	0.005	0.005	0.006	0.005
4	18675	18147	19399	18241	0.009	0.008	0.009	0.009	18675	18147	19399	18241	0.009	0.008	0.009	0.009
5	19786	19317	20504	19460	0.011	0.010	0.011	0.011	19786	19317	20504	19460	0.011	0.010	0.011	0.011
6	17044	16537	17477	16893	0.009	0.009	0.010	0.009	17044	16537	17477	16893	0.009	0.009	0.010	0.009
7	20037	17705	20652	19770	0.012	0.010	0.013	0.012	20037	17705	20652	19770	0.012	0.010	0.013	0.012
Trial	Sə (kPa)				Və (m)				Sə (kPa)				Və (m)			
1	10790	9773	10960	12439	0.019	0.018	0.021	0.020	10790	9772.6	10960	12439	0.019	0.018	0.021	0.020
2	14579	13802	14795	16807	0.029	0.028	0.031	0.030	14579	13802	14795	16807	0.029	0.028	0.031	0.030
3	14118	13403	14266	15911	0.030	0.029	0.032	0.031	14118	13403	14266	15911	0.030	0.029	0.032	0.031
4	22037	21173	22415	19592	0.048	0.046	0.051	0.046	22037	21173	22415	19592	0.048	0.046	0.051	0.046
5	20744	19955	20971	18799	0.055	0.053	0.057	0.053	20744	19955	20971	18799	0.055	0.053	0.057	0.053
6	23959	23152	24305	21688	0.058	0.055	0.060	0.056	23959	23152	24305	21688	0.058	0.055	0.060	0.056
7	26320	22665	26898	24153	0.071	0.059	0.074	0.068	26320	22665	26898	24153	0.071	0.059	0.074	0.068
Trial	Sxə(kPa)				Wz m)				Sxə(kPa)				Wz (m)			
1	3105	3053	3340	3324	0.034	0.032	0.040	0.038	3105.3	3053.2	3339.7	3324	0.000	0.000	0.000	0.000
2	3779	3739	4015	4058	0.056	0.053	0.063	0.062	3778.5	3739.2	4014.8	4058	0.000	0.000	0.000	0.000
3	3876	3827	4070	4105	0.060	0.057	0.066	0.066	3875.6	3827.2	4069.9	4105	0.000	0.000	0.000	0.000
4	4918	4862	5136	4626	0.107	0.103	0.115	0.097	4917.5	4861.6	5136.2	4626	0.000	0.000	0.000	0.000
5	4975	4928	5174	4751	0.116	0.112	0.125	0.108	4975.2	4927.5	5174.1	4751	0.000	0.000	0.000	0.000
6	5461	5401	5650	5188	0.139	0.133	0.146	0.126	5461	5400.8	5649.6	5188	0.000	0.000	0.000	0.000
7	5635	5372	5835	5380	0.164	0.136	0.173	0.150	5634.5	5372	5835.2	5380	0.010	0.000	0.002	0.000

Graphical presentation of effects of Varying Parameters

a) For Single Span Without Edge Beam

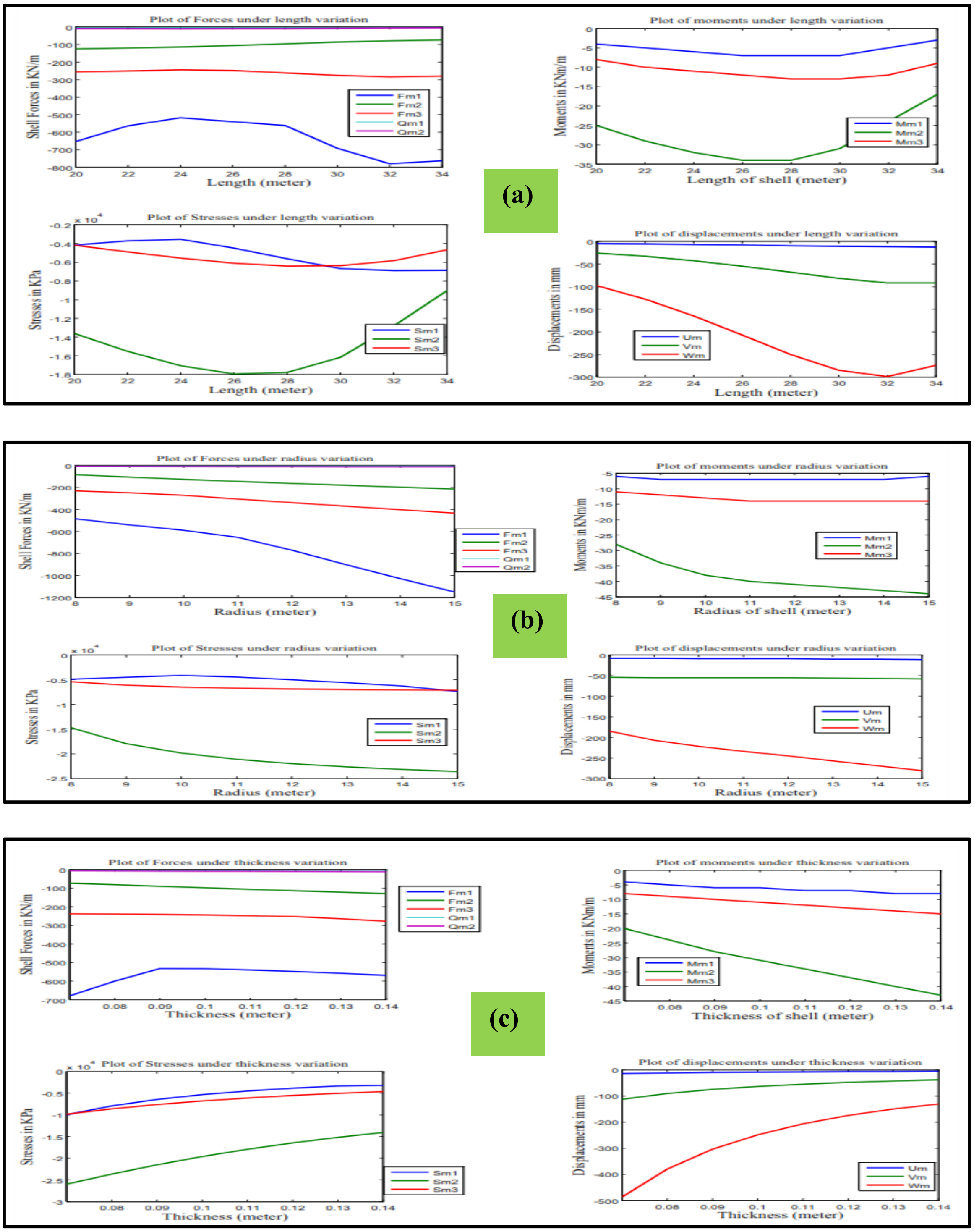


Figure.5. Internal effect variation; (a) length variable, (b) radius variable and (c) thickness variable for single span without edge beams

b) For Single Span With Edge Beam

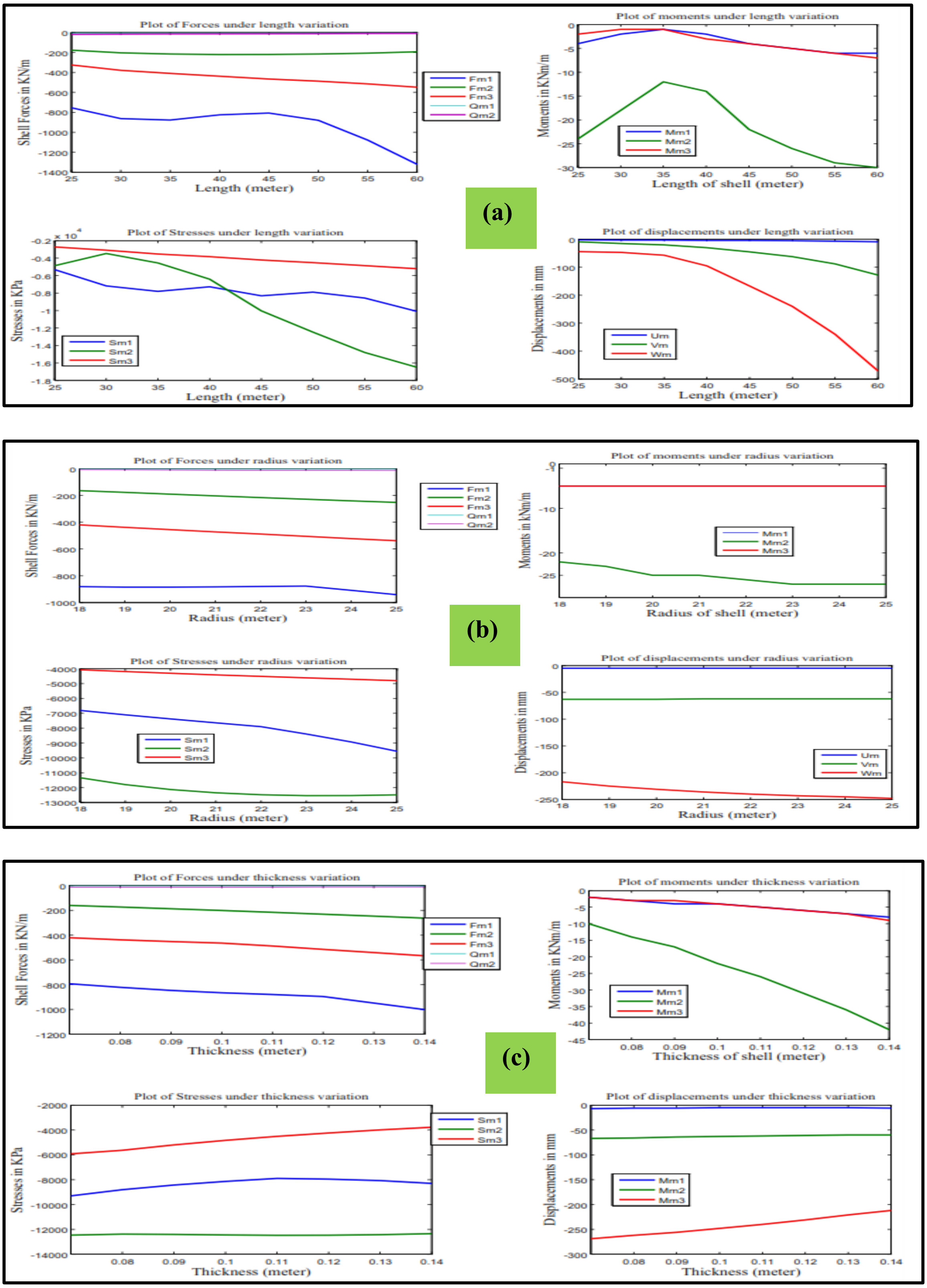


Figure.6. Internal effects variation; (a) length variable, (b) radius variable and (c) thickness variable for single span with edge beams

c) For Multiple Span With Edge Beam

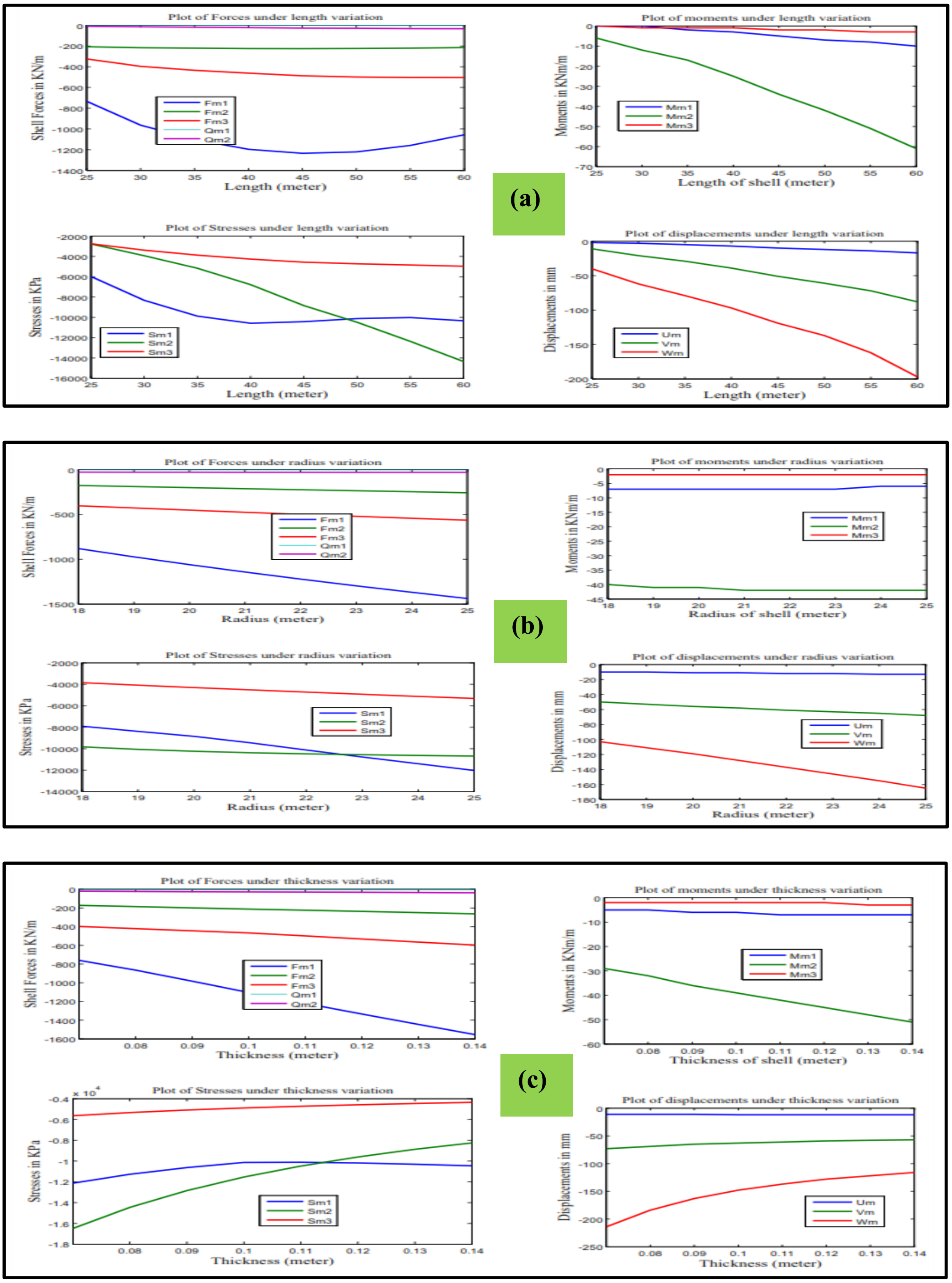


Figure.7. Internal effects variation; (a) length variable, (b) radius variable and (c) thickness variable for multiple span with edge beams

The Effect of Depth of Edge Beam

The results shown in Table (17) are the maximum value of forces, moments, stresses and displacements of constant value of shell geometric parameters but varying of depth of edge beam. In the table absolute maximum values are indicated, which does not represent either tension or compression effect, combined effect. However, the plotted graph shown in Figure (8) indicates how the compression part, which comprises the major part of the shell, would be affected as depth of the edge beam will increase.

Table 17

Effect of depth of edge beam

	Nx	Nθ	Nxθ	Mx	Mθ	Mxθ	Qx	Qθ	Sx	Sθ	Sxθ	U	V	W
0.8	1572	250	549	14.2	45.4	9.6	3.4	13.0	18930	20477	7512	0.001	0.009	0.037
0.1	1298	244	531	12.0	38.8	8.3	2.9	11.5	15545	18207	6849	0.001	0.008	0.032
0.2	1080	238	515	10.1	34.1	7.1	2.4	10.2	12738	16178	6265	0.001	0.007	0.027
0.3	911	234	503	8.4	29.8	6.1	2.1	10.5	10429	14379	5743	0.001	0.006	0.023
0.4	844	230	491	7.1	26.1	5.2	1.8	12.0	8542	12796	5281	0.000	0.005	0.020
0.5	817	226	479	6.1	22.9	4.4	1.6	13.4	8121	11407	4873	0.000	0.005	0.017
0.6	793	223	467	5.3	20.0	3.7	1.4	14.6	7926	10194	4515	0.000	0.004	0.014
0.7	782	221	456	4.5	17.5	3.1	1.2	15.7	7772	9136	4200	0.000	0.004	0.012

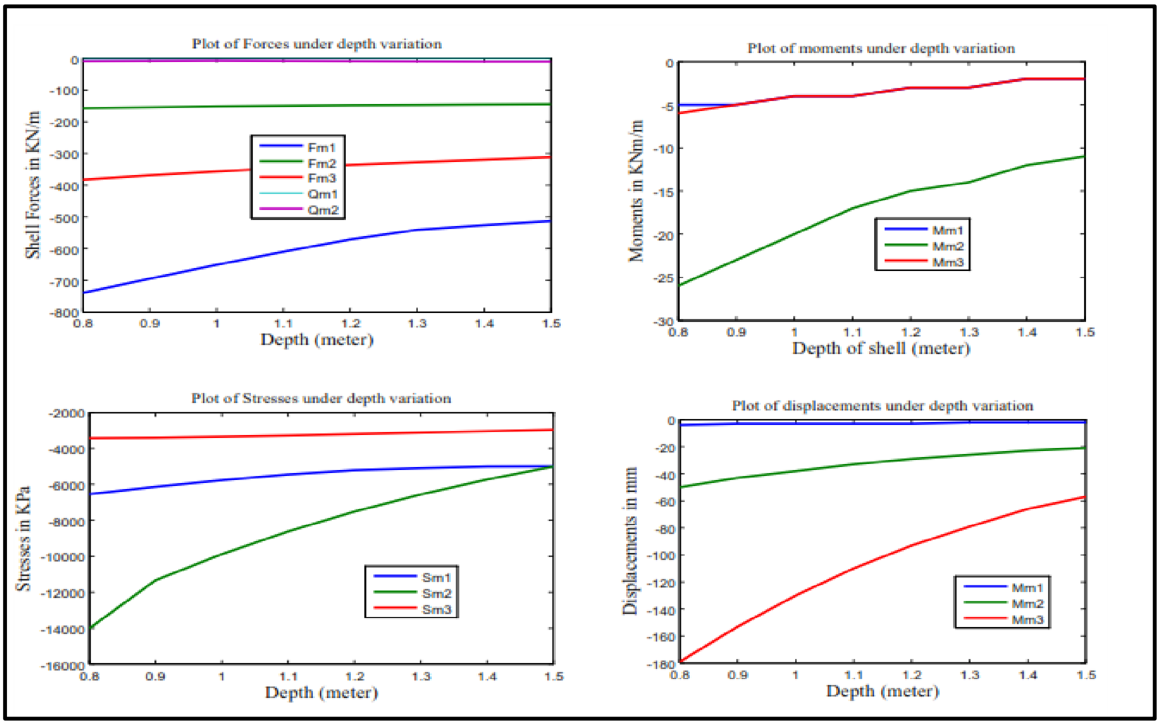


Figure.8. Effect of depth of edge beam

Variation of Shell Stresses at Closed Interval of θ

This parametric study also provides an important contribution to designers that they can get values of stresses at any section of the shell and let them to distinguish the compression and tension zones of the shell from the analyzed values. The graph shown in Figure (9-11) are the stresses for single and multiple spans of shells with and without stiffeners plotted together.

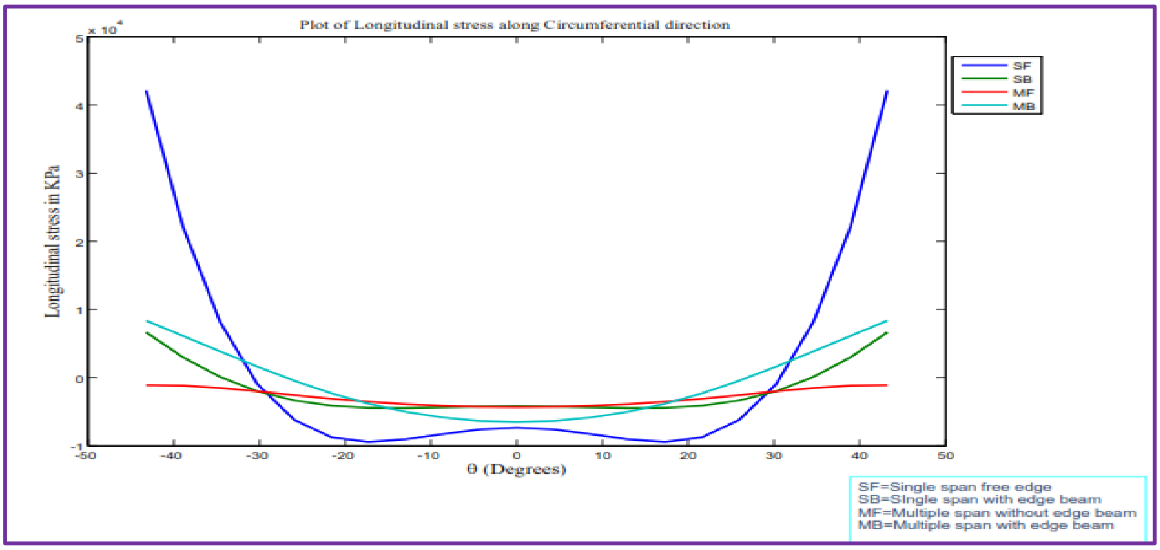


Figure.9. Longitudinal stress at closed interval of θ

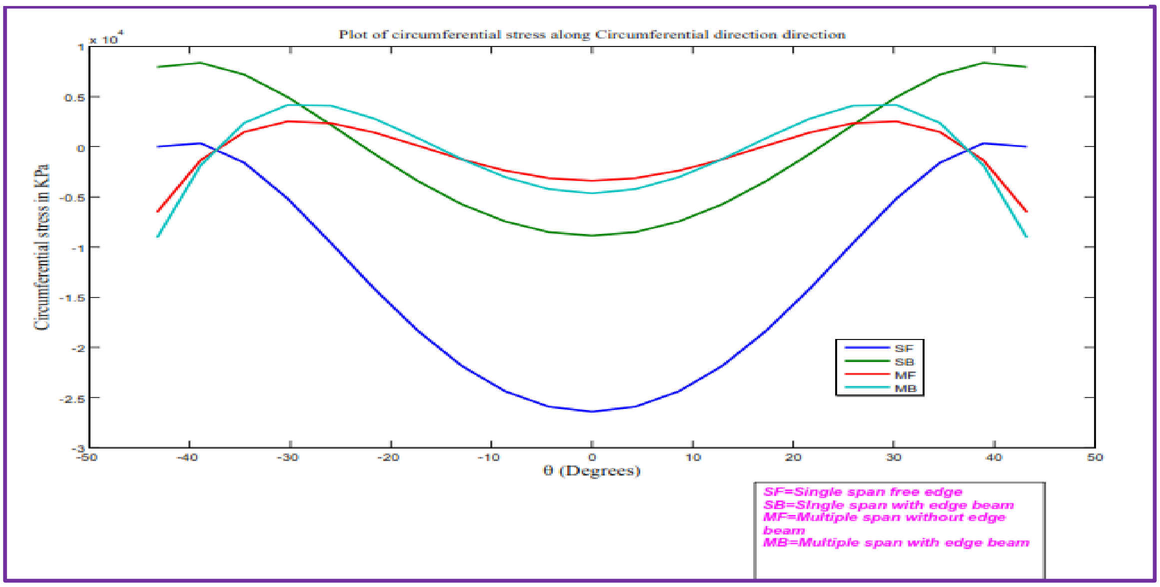


Figure.10. Circumferential stress at closed interval of θ

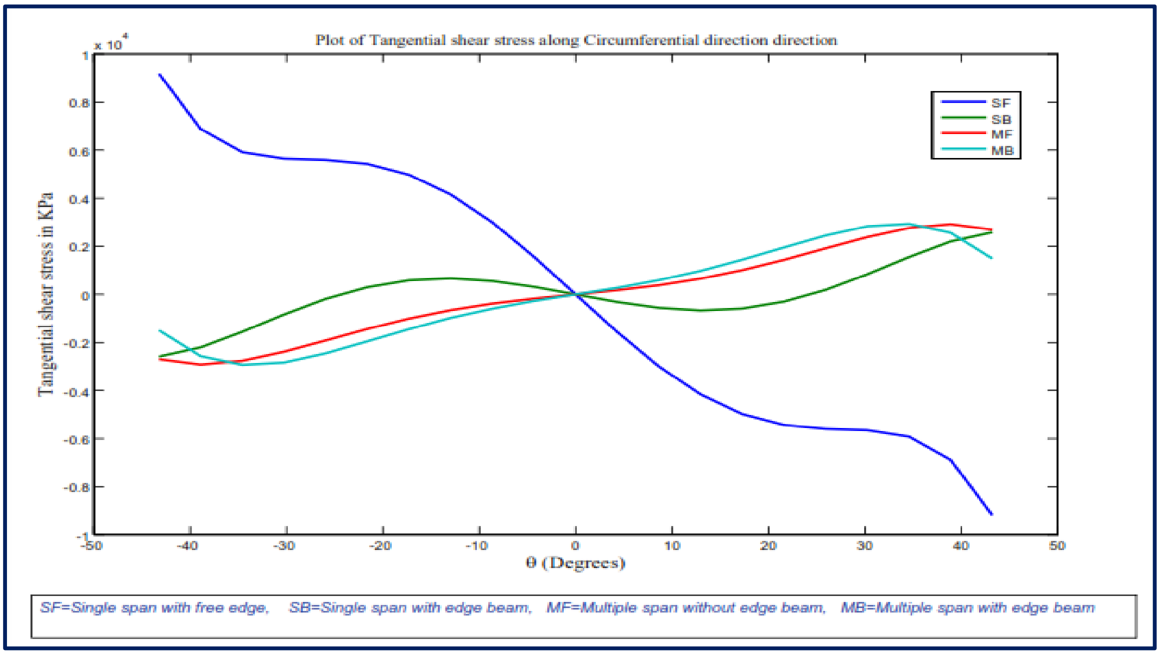


Figure.11. Tangential stress at closed interval of θ

5. DISCUSSION

From the results obtained in this paper, the following important aspects can be highlighted:

The study also use graphical presentation to show internal effects fluctuation as one or two of the geometrical parameters changed. Accordingly one may predict how the internal effects behaves at valley, crown or at any desired sections of the shell from graphical presentations of Figure (A1-A5) in Appendix A. In addition, graphical comparison of internal effects was observed when parameters varied as it was plotted all at one graph. The provision of stiffeners like edge beams will reduce the internal effects as shown in Table (17) and Figure (8).

5.1. RESULTANT FORCES

Although the degree of increment is different for all parameters, all values of forces are increased from their original values as indicated in Table 8 with increased values of parameters for absolute maximum values. The graphs plotted in Figure (5-7) indicates how the shell forces for major constitute component which is the compression effects are plotted against the variable parameters. As shown in the table and graphs the shell forces increases in magnitude when length, radius and thickness parameters increased. However, for single spans without edge beams, when length is increased the magnitude of the forces increases and finally decreased and changed to tension effects.

The shell forces will decrease as the edge beams are introduced as it can be noticed from Table (9-10) and Table (11-12) and decrease in magnitude as depth of edge beam is increased as shown in Table (17) and Figure (8).

5.2. RESULTANT MOMENT

As indicated in Table 8 the magnitude of resultant moment increases with increasing length, radius and thickness parameters for absolute maximum values. In addition the graphs plotted in Figure (5-7) indicates how the shell resultant moments are affected when the length, radius and thickness are increased. As the graph shows the negative maximum moments are increased when length, radius and thickness of the shell are increased.

The shell moments will decrease as the edge beams are introduced and decrease in magnitude as depth of edge beam is increased.

5.3. DISPLACEMENTS

As shown in Table 8 and Figure (5-7) the shell displacements increases in magnitude when length, radius parameters increased and decrease when thickness is increased.

The shell displacements will decrease as the edge beams are introduced and decrease in magnitude as depth of edge beam is increased.

5.4. RESULTANT STRESS

Most of the value of stresses are increased with an increasing length and radius of the shell. However; the stresses are decreased in magnitude as shell thickness is increased as shown in Table 8 and in Figure (5-7).

The shell stresses will decrease as the edge beams are introduced and decrease in magnitude as depth of edge beam is increased.

In designing of shell structures, the variation of stresses at any section of the shell is necessary in order to give actual response. Therefore, Figure (9-11) shows how the stresses behave at any section of the shell and thereby important to provide the compression and tension reinforcements.

The magnitude of stresses in single span without edge beams with the dimensions provided in Table 2 are large enough that cause failure of the structures. Even though with small dimensions as Table 3 there are tension effects around the longitudinal edges. Therefore, we need to provide either edge beams or additional reinforcements around the edges.

6. CONCLUSION AND RECOMMENDATION

6.1. CONCLUSION

In this research, open circular reinforced concrete cylindrical shell structures (barrel shells) were presented using commercially available software, MATLAB computer application. Forces, moments, stresses and displacements effect were investigated.by changing the parameters of length, thickness and radius of the shell keeping other things constant.

From the results obtained, it has been concluded that the structural behavior of shells is changed when one or two parameters were changed.

An important point which can be noted as a result of this work is that the provision of stiffeners like edge beams will reduce the internal effects. Increasing depth of edge beam decreases the magnitude of internal effects. It is also interesting to note that the assumed dimensions of the edge beam result in either compression and/or tension effects on the shell.

The other interesting thing observed was, it may give a clue for designers and other interesting professionals how the internal effects behaves at valley, crown or at any desired sections of the shell from graphical presentations of Figure (A1-A5) in the Appendix A. Moreover, they can obtained internal effects by changing parameters of the shell at any desired point on the shell from the code written at Appendix B. Furthermore, the stress resultants plotted at closer intervals of **θ** can be useful for detailing of reinforcement layout in reinforced concrete shells as shown in Figure (9-11).

6.2. RECOMMENDATIONS

The researcher recommend that further researches should be investigated concerning for nonlinear analysis including all load types acting on shells.

It could be also recommended that the application of MATLAB to other form shells and other civil works.

Besides of this both manual and computer applications should be also undertaken.

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Appendix A

Graph of forces, moments, stresses and displacements along circumferential and longitudinal direction.

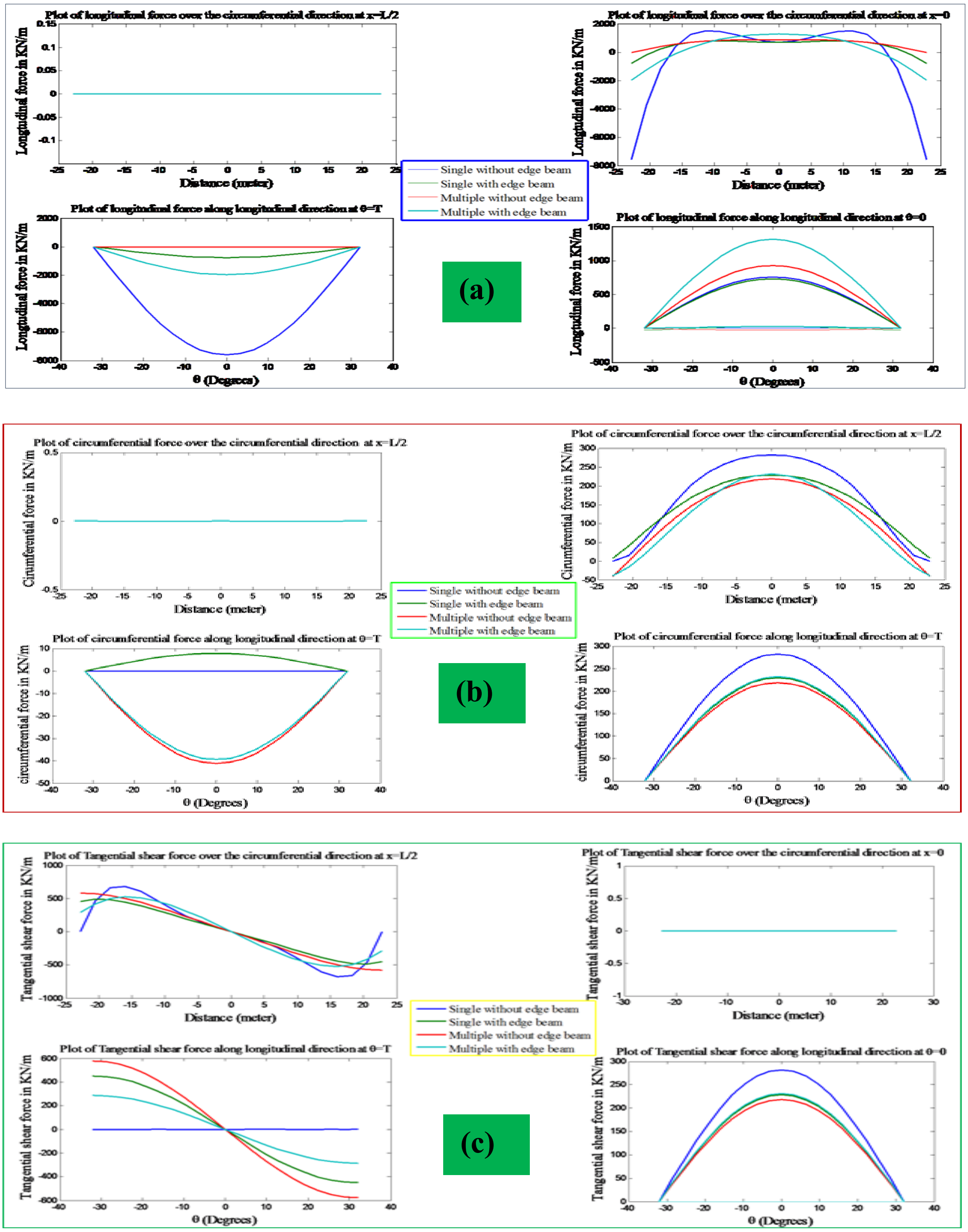

Figure. A1. (a) Longitudinal and (b) circumferential forces, (c) tangential shear force; Where T is half central angle

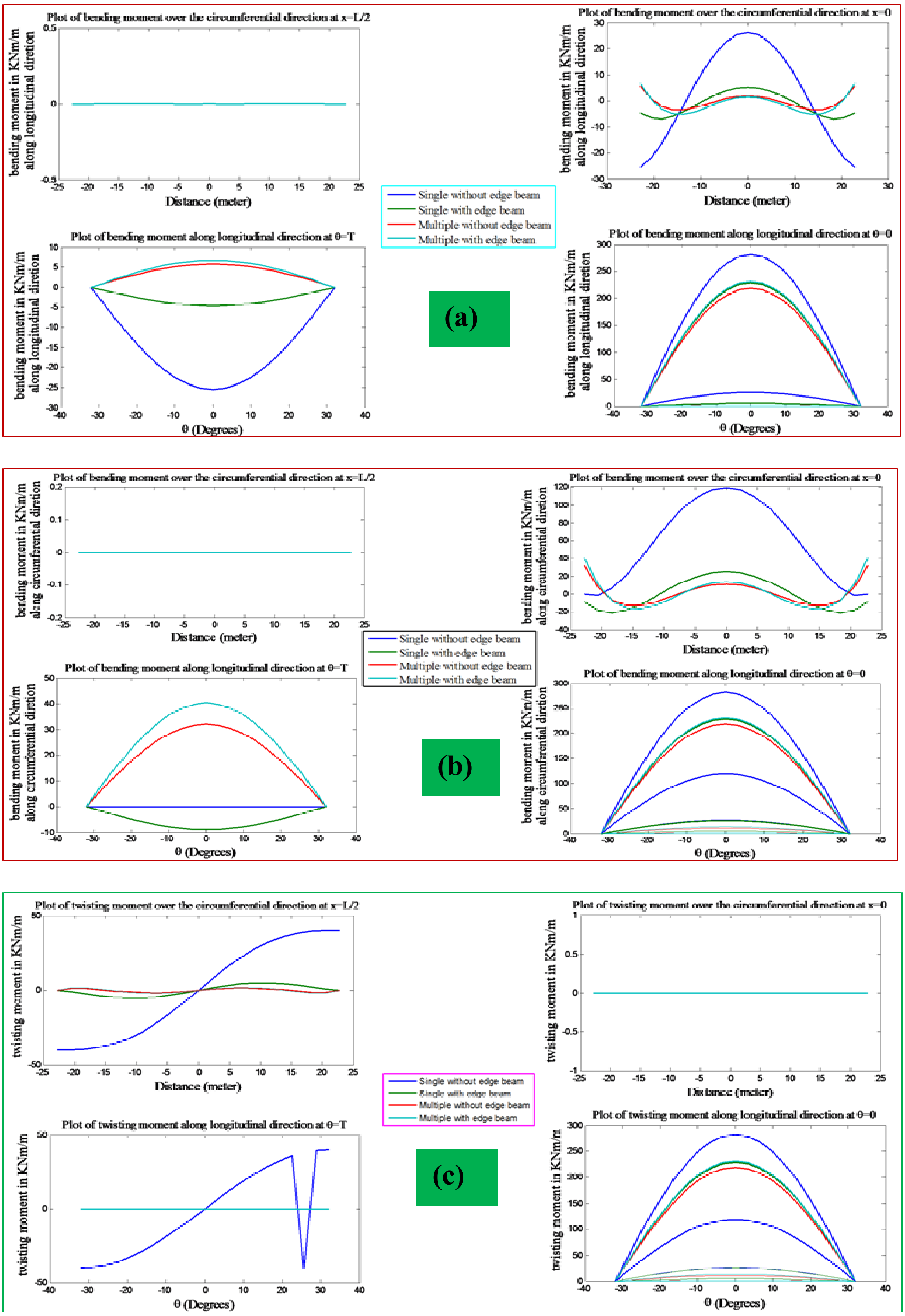

Figure. A2. (a) Bending moment along longitudinal, (b) bending moment along circumferential and (c) twisting moment

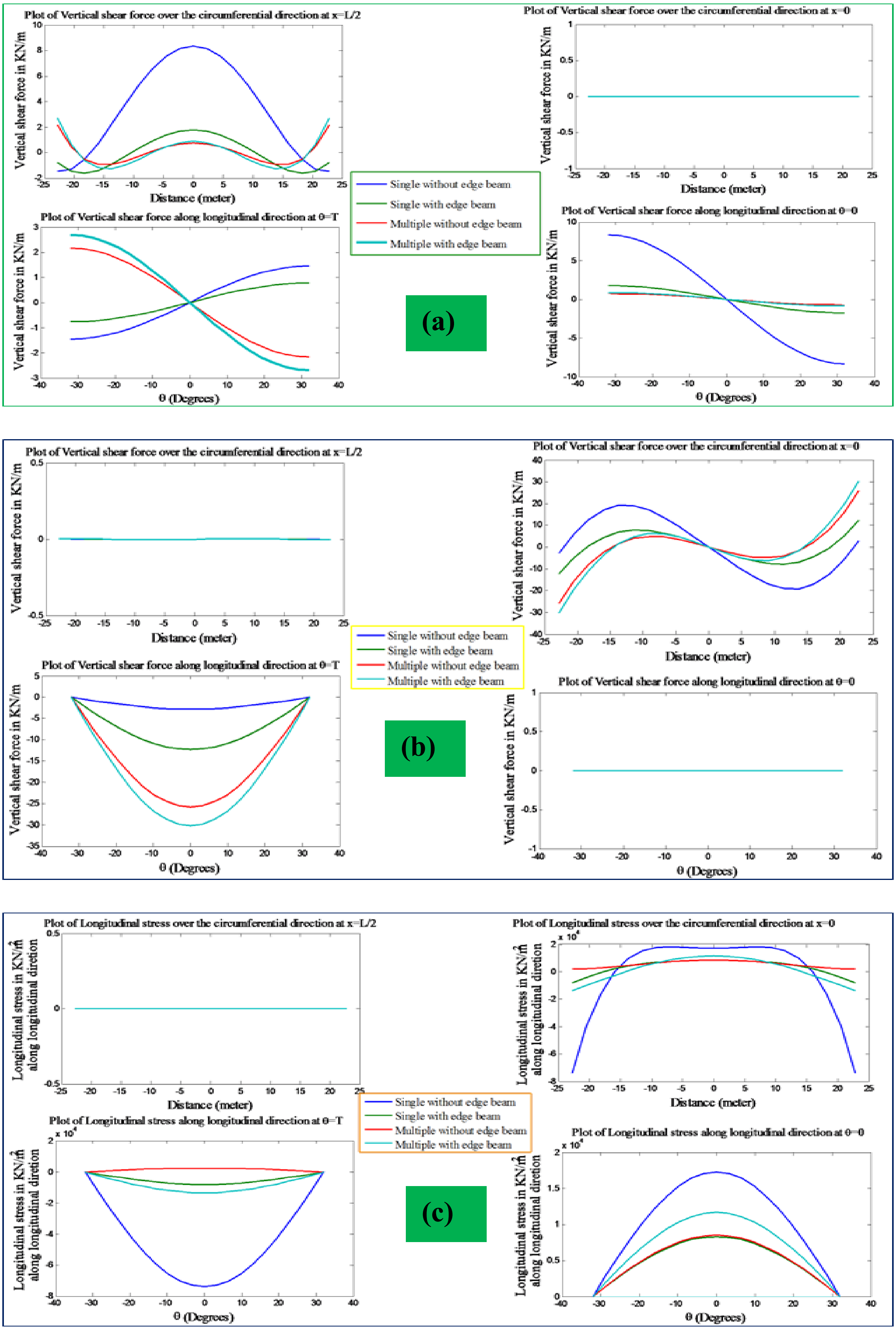

Figure. A3. Vertical shear force towards (a) longitudinal and (b) circumferential direction, (c) longitudinal shear stress

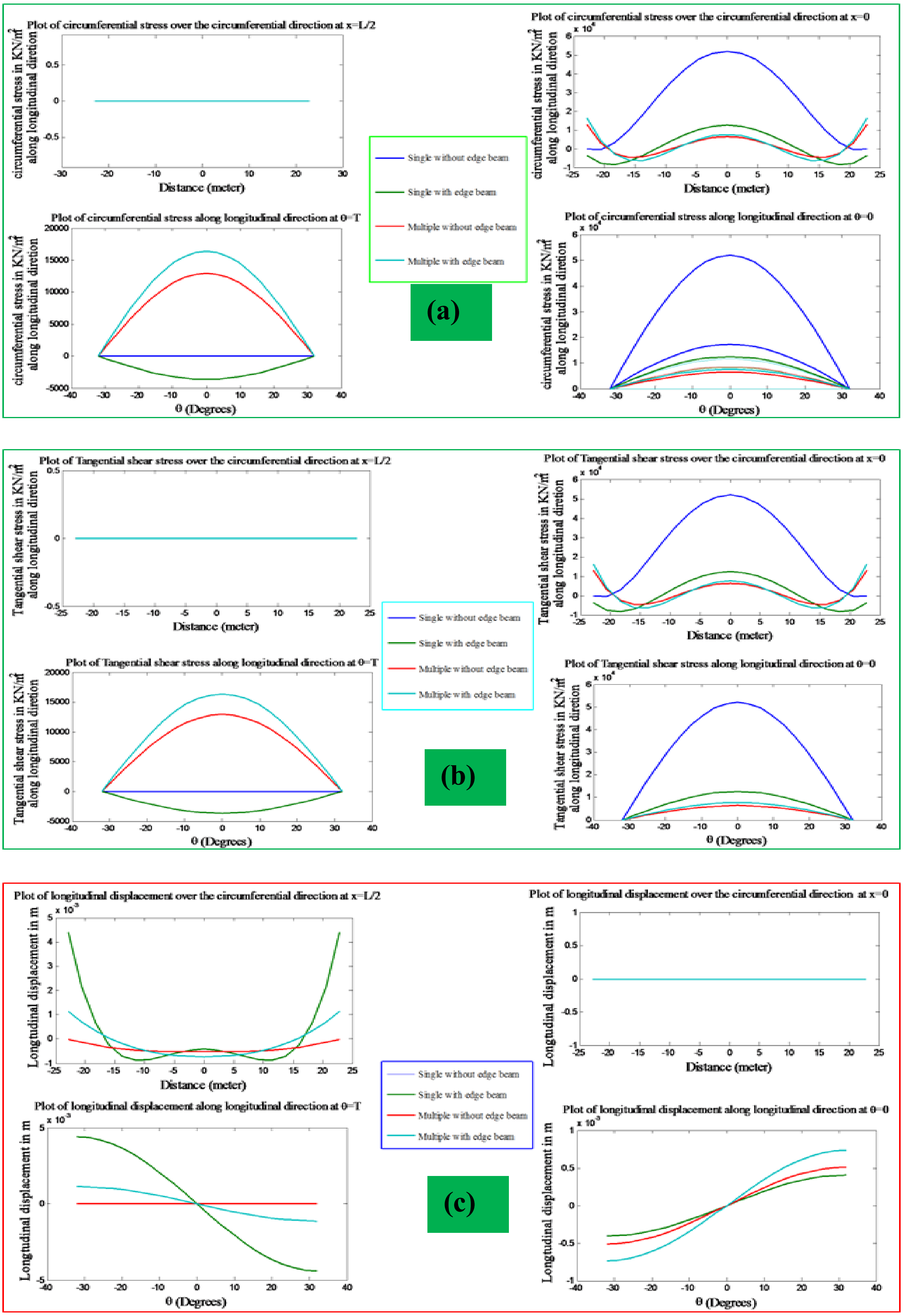

Figure. A4. (a) Circumferential stress, (b) tangential stress and (c) longitudinal displacement

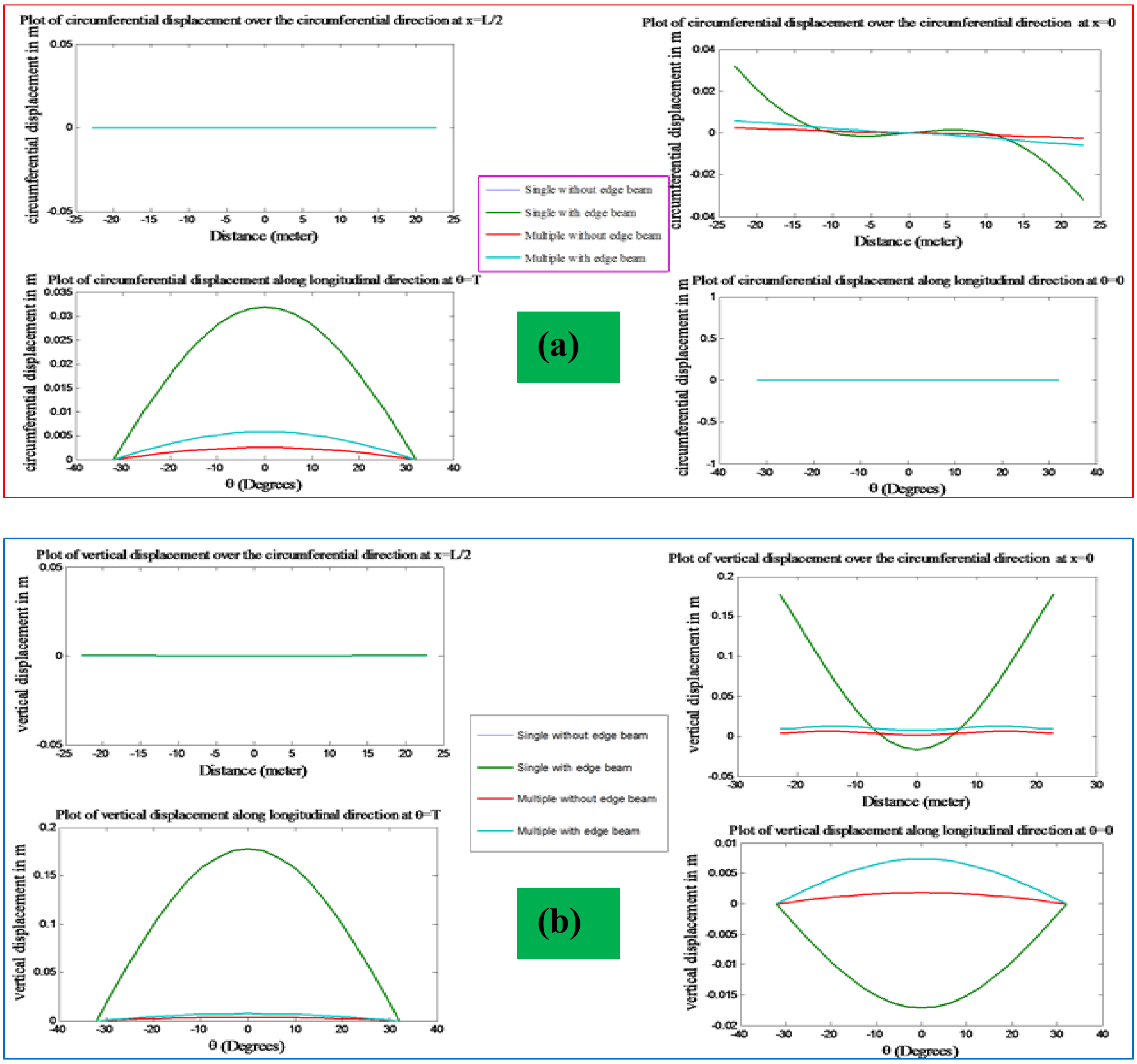

Figure. A5. (a) Circumferential displacement and (b) Vertical displacement

Appendix B

MATLAB CODE

The code written will analyze open circular reinforced concrete cylindrical shell structures to any dimension for both stiffened and unstiffened conditions of single and multiple spans.

NB: For the purpose of avoiding repetitions, being everything is similar except the constant determination criteria, it has been written all together. Therefore, if one tries to analyze it use constant determination for each respective cases and conditions. If it is intended to analyze simple span without edge beams, use constants for simple span without edge beams only and do same for others.

```
%ANALYSIS OF REINFORCED CONCRETE
%CYLINDERICAL SHELL ROOF STRUCTURES
%ADDIS ABABA UNIVERSITY
%SCHOOL OF CIVIL  AND ENVIRONMENTAL ENGINEERING
%STRUCTURE STREAM
%NAME: ABRHA EYASSU BEYENE
%ID: GSR/0663/06
%SUBMITED TO: AAIT Post raduate program
format short
syms x
bw=0.30;%wedith of beam in meter
db=1.22;%depth of beam in meter
Fc=27.6; %compressive strength of concrete in MPa
Ee=4700*sqrt(Fc); %in MPa
E=Ee*1000;% Ee in Kpa
Iy=bw*db^3/12;% moment of inertia along y axis for the edge beam
%determine poisson's ratio from the given weight and compressive strength
v=0.18; %poisson ratio
G=E/(2*(1+v));                      %shear modulus
for    h=0.09;                      %thickness of shell in meter
       K=E*h/(1-v^2);               %extentional rigiditry
       D=(E*h^3)/(12*(1-v^2));      %flexural rigiditry
       L=30.5;                      %longitudinal length
       R=11.13;                     %Radius of the shell
       tt=asind(L/(4*R));           %semi central angle in degree
       t=asin(L/(4*R));             %semi central angle in radiun
      Wg=24;                        %unit weight of concrete in KN/m^3
       gs=Wg*h;               %uniform self weigth over the area of shell in KN/m^2
       gr=0.250;                    % load from roofing in KN/m^2
       gm=0.250;                    % load from miscellaneous in KN/m^2
       gt=gs+gr+gm;                 % total dead load
       pl=1.5;     % uniform live load distributed over the projected area of shell in KN/m^2
       pd=pl*(sind(tt)/t);          %equivalent live load distributed over the shell area
       g=(1.3*gt+1.6*pd);           %combined load in KN/m^2 according to EBCS
%t=theta,l=lambda,e=epslon,v=mu,s=alpha
%w=w1+w2 solution of differential equations
      m=1;
         for i=m
```

```
            B=i*pi/L;
             l=((3*(1-v^2))^(1/8)*sqrt(B*R)*(R/h)^(1/4));
            e=(1/((3*(1-v^2))^(1/4)))*(sqrt(h/R))*(B*R);
            mc=cos(B*x);      %Multiples of M1,M2,M3,M4,M7,M10,M11,M12,M13.g1,g4
            ms=sin(B*x);                  %Multiples of M5,M6,M8,M9,g3,g2
            M1=((2*D*R*(B)^4)/((e)^3));
            M2=((-2*D*R*(B)^4)/((e)^2));
            M3=((-D*(B)^2));
            M4=(D/((R)^2));
            M5=(-D*((B)^3)/e);
            M6=(R*h^2*(B)^3)/((6*(1-v^2)*e^3));
            M7=1;
            M8=(2*D*R*(B)^4)/((sqrt(e))^5);
            M9=(-D*(1-v)/R)*(B);
            M10=(D*(B)^3)/((sqrt(e))^3);
    %M11=M10+diff(M9,x)
            M12=-(R*h^2*(B)^3)/((6*(1-v^2)*((sqrt(e))^7)));
            M13=1;
            g11=(-1)^((i-1)/2)*(4*g/(i*pi));
            g1=g11*cos(t);
            g2=g11*sin(t);
            g3=g11*cos(t);
            g4=g11*sin(t);
C=-(R^4/D)*(((B*R)^4+(4+v)*(B*R)^2+2)/((1+(B*R)^2)^4+R^6*(1-v^2)*(12*(B)^4/h^2)));
           H3=(C*D/R^4)*((B*R)^2+1)^2;
           N1=(-1/((B)^2*R))*(H3+2)*g1;
           N2=-R*(H3+1)*g1;
           N3=-D*((B)^2+v/R^2)*C*g1;
           N4=-D*(v*(B)^2+1/R^2)*C*g1;
           N5=D*((B)^3+(B)^2/R)*C*g3;
           N6=(1/(E*h*(B))*((-1/((B)^2*R)*(H3+2)+v*R*(H3+1))))*g3;
           N7=C*g1;
           N8=(-1/((B))*(H3+2))*g2;
           N9=D*(1-v)*((B)/R)*C*g2;
           N10=D*((B)^2/R+1/R^3)*C*g4;
           n11=B*N9;
           N11=(N10+n11);
           N12=(1/(E*h*(B)^2)*(v*(H3+3)+(H3+2)*(2+1/((B)^2*R^2))))*g4;
           N13=(1/R)*(C+(1/(E*h*(B)^2)*(v*(H3+3)+(H3+2)*(2+1/((B)^2*R^2)))))*g4;
    %N11=diff(N9,x)+N10
           p1=((B)^2);
           p2=((B)^3);
           p3=((B)^4);
           J=sqrt(((sqrt(1+(1+e)^2)+(1+e))/2));
           J1=sqrt(((sqrt(1+(1-e)^2)-(1-e))/2));
            k=sqrt(((sqrt(1+(1+e)^2)-(1+e))/2));
            k1=sqrt(((sqrt(1+(1-e)^2)+(1-e))/2));
            a=l*J;a1=l*J1;b=l*k;b1=l*k1;
            s1=[-1 1+e 1 1-e];
            s2=[0 1 0 -1];
s3=[(1-(v*l^2*(1+e)/(((B)^2)*R^2))) -((v*l^2/((B)^2)*R^2)) ((1-(v*l^2*(e-1)/((B)^2)*R^2))) -
((v*l^2/((B)^2)*R^2))];
            s4=[(l^2*(1+e)-v*((B)^2)*R^2) l^2 (l^2*(e-1)-v*((B)^2)*R^2) l^2];
            s5=[1 1 -1 1];
            s6=[-1 (1+e*(1+v)) 1 (1-e*(1+v))];
            s7=[1 0 1 0];
            s8=[-k J k1 -J1];
            s9=[a b a1 b1];
            s10=[(J-k) (J+k) -(J1+k1) (J1-k1)];
```

```
        s11=[(J*(1-e*(1-v))-k) (J+k*(1-e*(1-v))) (-k-J1*(1+e*(1-v))) (J1-k1*(1+e*(1-v)))];
        s12=[(J+k*(1-e*(1+v))) (k-J*(1-e*(1+v))) (-J1+k1*(1+e*(1+v))) (-k1-J1*(1+e*(1+v))
s13=[(-a/R+(M12*s12(1))/R) (-b/R+(M12*s12(2))/R) (-a1/R+(M12*s12(3))/R) (-
b1/R+(M12*s12(4))/R)];
format short e
        cc=cosh(a*t)*cos(b*t);   % asignments
        ss=sinh(a*t)*sin(b*t);
        sc=sinh(a*t)*cos(b*t);
        cs=cosh(a*t)*sin(b*t);
        ca=cosh(a1*t)*cos(b1*t);
        sa=sinh(a1*t)*sin(b1*t);
        sb=sinh(a1*t)*cos(b1*t);
        cb=cosh(a1*t)*sin(b1*t);
        r1=(M10*s10(1)+B*M9*s9(1));
        r2=(M10*s10(2)+B*M9*s9(2));
        r3=(M10*s10(3)+B*M9*s9(3));
        r4=(M10*s10(4)+B*M9*s9(4));
        K1=M2*s2(1);
        K2=M2*s2(2);
        K3=M2*s2(3);
        K4=M2*s2(4);
        R1=M8*s8(1);
        R2=M8*s8(2);
        R3=M8*s8(3);
        R4=M8*s8(4);
        Ha=M13*s13(1);
        Hb=M13*s13(2);
        Hc=M13*s13(3);
        Hd=M13*s13(4);
        Z1=M6*s6(1);
        Z3=M6*s6(3);
        Z2=M6*s6(2);
        Z4=M6*s6(4);
% constant determination
% for single span without edge beam
        L1=M4*s4(1);
        L2=M4*s4(2);
        L3=M4*s4(3);
        L4=M4*s4(4);
d1=[K1*cc-K2*ss -K2*cc-K1*ss K3*ca-K4*sa -K4*ca-K3*sa];
d2=[R1*sc-R2*cs -R2*sc-R1*cs R3*sb-R4*cb -R4*sb-R3*cb];
d3=[L1*cc-L2*ss -L2*cc-L1*ss L3*ca-L4*sa -L4*ca-L3*sa];
d4=[r1*sc-r2*cs -r2*sc-r1*cs r3*sb-r4*cb -r4*sb-r3*cb];
d=[d1;d2;d3;d4];
        NM2=-N2;
        NM4=-N4;
        NM8=-N8;
        NM11=-N11;
T1= [NM2 NM8 NM4 NM11]';
A=d\T1;

% constant determination for single span with edge beam

        Iy=bw*db^3/12;
        Z5=(6/(E*bw*db^2*p2))*M2*s2(1)*sin(t);
        Z6=(6/(E*bw*db^2*p2))*M2*s2(2)*sin(t);
        Z7=(6/(E*bw*db^2*p2))*M2*s2(3)*sin(t);
        Z8=(6/(E*bw*db^2*p2))*M2*s2(4)*sin(t);
```

```
            Y1=(4/(E*bw*db*p1))*M8*s8(1);
            Y2=(4/(E*bw*db*p1))*M8*s8(2);
            Y3=(4/(E*bw*db*p1))*M8*s8(3);
            Y4=(4/(E*bw*db*p1))*M8*s8(4);
            Y5=(6/(E*bw*db^2*p2));
            K5=M7*s7(1)*cos(t);
            K6=M7*s7(2)*cos(t);
            K7=M7*s7(3)*cos(t);
            K8=M7*s7(4)*cos(t);
            E1=(1/(E*Iy*p3))*M2*s2(1)*sin(t);
            E2=(1/(E*Iy*p3))*M2*s2(2)*sin(t);
            E3=(1/(E*Iy*p3))*M2*s2(3)*sin(t);
            E4=(1/(E*Iy*p3))*M2*s2(4)*sin(t);
            E5=(db/(2*E*Iy*p2))*M8*s8(1);
            E6=(db/(2*E*Iy*p2))*M8*s8(2);
            E7=(db/(2*E*Iy*p2))*M8*s8(3);
            E8=(db/(2*E*Iy*p2))*M8*s8(4);
            H1=(1/(E*Iy*p3))*cos(t);
            H2=M12*s12(1)*sin(t);
            H3=M12*s12(2)*sin(t);
            H4=M12*s12(3)*sin(t);
            H5=M12*s12(4)*sin(t);
            H6=1/(E*bw*db*p1);
            H7=1/(E*bw*db^2*p2);
            H8=1/(E*Iy*p3);
            H9=1/(2*E*Iy*p2);
% constant determination for single span with free edge
d11=[(Z1-Z5)*cc+(Y1-Y5*r1*cos(t))*sc+(-Z2+Z6)*ss+(-Y2+Y5*r2*cos(t))*cs ...
   (-Z2+Z6)*cc+(-Y2+Y5*r2*cos(t))*sc+(-Z1+Z5)*ss+(-Y1+Y5*r1*cos(t))*cs ...
   (Z3-Z7)*ca+(Y3-Y5*r3*cos(t))*sb+(-Z4+Z8)*sa+(-Y4+Y5*r4*cos(t))*cb ...
   (-Z4+Z8)*ca+(-Y4+Y5*r4*cos(t))*sb+(-Z3+Z7)*sa+(-Y3+Y5*r3*cos(t))*cb];
d12=[K1*cc*cos(t)-r1*sc*sin(t)-K2*cos(t)*ss+r2*sin(t)*cs ...
   -K2*cc*cos(t)+r2*sc*sin(t)-K1*cos(t)*ss+r1*sin(t)*cs ...
    K3*ca*cos(t)-r3*sb*sin(t)-K4*cos(t)*sa+r4*sin(t)*cb ...
   -K4*ca*cos(t)+r4*sb*sin(t)-K3*cos(t)*sa+r3*sin(t)*cb];
d13=[(K5-E1)*cc+(E5-H1*r1-H2)*sc+(-K6+E2)*ss+(-E6+H1*r2+H3)*cs ...
   (-K6+E2)*cc+(-E6+H1*r2+H3)*sc+(-K5+E1)*ss+(-E5+H1*r1+H2)*cs ...
   (K7-E3)*ca+(E7-H1*r3-H4)*sb+(-K8+E4)*sa+(-E8+H1*r4+H5)*cb ...
   (-K8+E4)*ca+(-E8+H1*r4+H5)*sb+(-K7+E3)*sa+(-E7+H1*r3+H4)*(cb)];
d14=[Ha*sc-Hb*cs -Hb*sc-Ha*cs Hc*sb-Hd*cb -Hd*sb-Hc*cb];
d1=[d11;d12;d13;d14];
           MN1=-N6-4*N8*H6+6*N11*cos(t)*H7+6*N2*sin(t)*H7-6*g11*H7;
           MN2=N11*sin(t)-N2*cos(t);
           MN4=-N13;
           MN3=-N7*cos(t)+N12*sin(t)+N11*cos(t)*H8+N2*sin(t)*H8-N8*db*H9-g11*H8;
T=[MN1 MN2 MN3 MN4]';
 A=d1\T;

% constant determination for multiple span without edge beam
           K5=M7*s7(1)*sin(t);
           K6=M7*s7(2)*sin(t);
           K7=M7*s7(3)*sin(t);
           K8=M7*s7(4)*sin(t);
           H2=M12*s12(1)*cos(t);
           H3=M12*s12(2)*cos(t);
           H4=M12*s12(3)*cos(t);
           H5=M12*s12(4)*cos(t);
d21=[R1*(a*cc-b*ss)-R2*(b*cc+a*ss) R2*(b*ss-a*cc)-R1*(b*cc+a*ss) ...
    R3*(a1*ca-b1*sa)-R4*(b1*ca+a1*sa) R4*(b1*sa-a1*ca)-R3*(b1*ca+a1*sa)];
```

```
d22=[K5*cc+H2*sc-K6*ss-H3*cs -K6*cc-H3*sc-K5*ss-H2*cs ...
     K7*ca+H4*sb-K8*sa-H5*cb -K8*ca-H5*sb-K7*sa-H4*cb];
d23=[K1*cc*sin(t)+r1*sc*cos(t)-K2*sin(t)*ss-r2*cos(t)*cs ...
    -K2*cc*sin(t)-r2*sc*cos(t)-K1*sin(t)*ss-r1*cos(t)*cs ...
     K3*ca*sin(t)+r3*sb*cos(t)-K4*sin(t)*sa-r4*cos(t)*cb ...
    -K4*ca*sin(t)-r4*sb*cos(t)-K3*sin(t)*sa-r3*cos(t)*cb];
d24=[Ha*sc-Hb*cs -Hb*sc-Ha*cs Hc*sb-Hd*cb -Hd*sb-Hc*cb];
d2=[d21;d22;d23;d24];
              MM1=-(N8*cos(t)/sin(asin(L/(4*R))));
              MM2=-N12*cos(t)-N7*sin(t);
              MM3=-N11*cos(t)-N2*sin(t);
              MM4=-N13;
       T2=[MM1 MM2 MM3 MM4]';
A=d2\T2;

% constant determination formultiple span with edge beam
               Iy=(bw/2)*db^3/12;
               Z5=(6/(E*(bw/2)*db^2*p2))*M2*s2(1)*sin(t);
               Z6=(6/(E*(bw/2)*db^2*p2))*M2*s2(2)*sin(t);
               Z7=(6/(E*(bw/2)*db^2*p2))*M2*s2(3)*sin(t);
               Z8=(6/(E*(bw/2)*db^2*p2))*M2*s2(4)*sin(t);
               Y1=(4/(E*(bw/2)*db*p1))*M8*s8(1);
               Y2=(4/(E*(bw/2)*db*p1))*M8*s8(2);
               Y3=(4/(E*(bw/2)*db*p1))*M8*s8(3);
               Y4=(4/(E*(bw/2)*db*p1))*M8*s8(4);
               Y5=6/(E*(bw/2)*db^2*p2);
                K5=M7*s7(1)*cos(t);
                K6=M7*s7(2)*cos(t);
                K7=M7*s7(3)*cos(t);
                K8=M7*s7(4)*cos(t);
                E1=(1/(E*Iy*p3))*M2*s2(1)*sin(t);
                E2=(1/(E*Iy*p3))*M2*s2(2)*sin(t);
                E3=(1/(E*Iy*p3))*M2*s2(3)*sin(t);
                E4=(1/(E*Iy*p3))*M2*s2(4)*sin(t);
                E5=(db/(2*E*Iy*p2))*M8*s8(1);
                E6=(db/(2*E*Iy*p2))*M8*s8(2);
                E7=(db/(2*E*Iy*p2))*M8*s8(3);
                E8=(db/(2*E*Iy*p2))*M8*s8(4);
                H1=(1/(E*Iy*p3))*cos(t);
                H2=M12*s12(1)*sin(t);
                H3=M12*s12(2)*sin(t);
                H4=M12*s12(3)*sin(t);
                H5=M12*s12(4)*sin(t);
                H6=1/(E*(bw/2)*db*p1);
                H7=1/(E*(bw/2)*db^2*p2);
                H8=1/(E*Iy*p3);
                B5=db/(2*E*Iy*p2);
                B6=2*Y5*cos(t);
                B7=H1*tan(t);

d31=[(Z1-Z5)*cc+(Y1-Y5*r1*cos(t))*sc+(-Z2+Z6)*ss+(-Y2+Y5*r2*cos(t))*cs ...
    (-Z2+Z6)*cc+(-Y2+Y5*r2*cos(t))*sc+(-Z1+Z5)*ss+(-Y1+Y5*r1*cos(t))*cs ...
    (Z3-Z7)*ca+(Y3-Y5*r3*cos(t))*sb+(-Z4+Z8)*sa+(-Y4+Y5*r4*cos(t))*cb ...
    (-Z4+Z8)*ca+(-Y4+Y5*r4*cos(t))*sb+(-Z3+Z7)*sa+(-Y3+Y5*r3*cos(t))*cb];
d32=[K5*cc*tan(t)+H2*sc*cot(t)-K6*tan(t)*ss-H3*cot(t)*cs ...
    -K6*cc*tan(t)-H3*sc*cot(t)-K5*tan(t)*ss-H2*cot(t)*cs ...
     K7*ca*tan(t)+H4*sb*cot(t)-K8*tan(t)*sa-H5*cot(t)*cb ...
    -K8*ca*tan(t)-H5*sb*cot(t)-K7*tan(t)*sa-H4*cot(t)*cb];
```

```
d33=[(K5-E1)*cc+(E5-H1*r1-H2)*sc+(-K6+E2)*ss+(-E6+H1*r2+H3)*cs ...
   (-K6+E2)*cc+(-E6+H1*r2+H3)*sc+(-K5+E1)*ss+(-E5+H1*r1+H2)*cs ...
   (K7-E3)*ca+(E7-H1*r3-H4)*sb+(-K8+E4)*sa+(-E8+H1*r4+H5)*cb ...
   (-K8+E4)*ca+(-E8+H1*r4+H5)*sb+(-K7+E3)*sa+(-E7+H1*r3+H4)*(cb)];
d34=[Ha*sc-Hb*cs -Hb*sc-Ha*cs Hc*sb-Hd*cb -Hd*sb-Hc*cb];
d3=[d31;d32;d33;d34];
               MN1=-N6-4*N8*H6+6*N11*cos(t)*H7+6*N2*sin(t)*H7-6*g11*H7;
               MN2=-N12*cos(t)-N7*sin(t);
               MN4=-N13;
               MN3=-N7*cos(t)+N12*sin(t)+N11*cos(t)*H8+N2*sin(t)*H8-N8*B5-g11*H8;
    f=[MN1 MN2 MN3 MN4]';
 A=d3\f;

               x1=M7*mc;
               x2=M6*ms;
               x3=M12*mc;
               x4=N7*mc;
               x5=N6*ms;
               x6=N12*mc;
% changing to symblic expression
         syms t
          t1=cosh(a*t)*cos(b*t);
          t2=sinh(a*t)*cos(b*t);
          t3=sinh(a*t)*sin(b*t);
          t4=cosh(a*t)*sin(b*t);
          t5=cosh(a1*t)*cos(b1*t);
          t6=sinh(a1*t)*cos(b1*t);
          t7=sinh(a1*t)*sin(b1*t);
          t8=cosh(a1*t)*sin(b1*t);
         y1=(A(1)*s7(1)-A(2)*s7(2))*t1;
         y2=(A(1)*s6(1)-A(2)*s6(2))*t1;
         y3=(A(1)*s12(1)-A(2)*s12(2))*t2;
         y4=(A(1)*s7(2)+A(2)*s7(1))*t3;
         y5=(A(1)*s6(2)+A(2)*s6(1))*t3;
         y6=(A(1)*s12(2)+A(2)*s12(1))*t4;
         y7=(A(3)*s7(3)-A(4)*s7(4))*t5;
         y8=(A(3)*s6(3)-A(4)*s6(4))*t5;
         y9=(A(3)*s12(3)-A(4)*s12(4))*t6;
         y10=(A(3)*s7(4)+A(4)*s7(3))*t7;
         y11=(A(3)*s6(4)+A(4)*s6(3))*t7;
         y12=(A(3)*s12(4)+A(4)*s12(3))*t8;
         W1h=(x1*((y1-y4+(y7-y10))));        % homogeneous solution of vertical
          U1h=(x2*((y2-y5+(y8-y11))));       % homogeneous solution of longitudinal
          V1h=(x3*((y3-y6+(y9-y12))));       % homogeneous solution of circumferential
          W1p=x4*cos(t)/cos(asin(L/(4*R)));    % particular solution of vertical
          U1p=x5*cos(t)/cos(asin(L/(4*R)));    % particular solution of longitudinal
          V1p=x6*sin(t)/sin(asin(L/(4*R)));    % particular solution of circumferential
          W1=W1h+W1p;U1=U1h+U1p;V1=V1h+V1p;
       end
 syms t z
W=W1;
U=U1;
V=V1;

%resultant forces and moments

Nx=K*(diff(U,x)+v*(1/R*diff(V,t)+W/R));
Nt=K*(v*diff(U,x)+(1/R*diff(V,t)+W/R));
```

```
Nxt=K*((1-v)/2)*(diff(V,x)+(1/R*diff(U,t)));
Mx=D*(diff(W,x,2)+v/R^2*(diff(W,t,2)));
Mt=D*(v*diff(W,x,2)+1/R^2*(diff(W,t,2)));
Mxt=D*((1-v)/R*(diff(diff(W,t),x)));
Q1=D*((diff(W,x,3)+1/R^2*(diff(diff(W,t,2),x))));
Q2=D*((1/R^3*diff(W,t,3)+1/R*(diff(diff(W,t),x,2))));
Q12=(Q2+diff(Mxt,x));
Sxp=(Nx/h) +12*(Mx*z)/h^3;                    %longitudinal stress for +ve
Sxm=(Nx/h)-12*(Mx*z)/h^3;                     %longitudinal stress for -ve
Stp=(Nt/h) +12*(Mt*z)/h^3;                    % circumferential stress for +ve
Stm=(Nt/h)-12*(Mt*z)/h^3;                     % circumferential stress for -ve
Sxtp=(Nxt/h) +12*(Mxt*z)/h^3;                 %tangential shear stress for +ve
Sxtm=(Nxt/h)-12*(Mxt*z)/h^3;                  % tangential shear stress for -ve res

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Symbolic substitution and numerical results

z=h/2;tt=asin(L/(4*R));
t=-tt:tt/5:tt;
Fx=subs(Nx)';        % longitudinal force as a function of longitudinal axes
Ft=subs(Nt)';        % circumferential force as a function of longitudinal axes
Fxt=subs(Nxt)';
MX=subs(Mx)';
MT=subs(Mt)';
MXT=subs(Mxt)';
QX=subs(Q1)';
QT=subs(Q2)';
Sx=subs(Sxp)';
Sx1=subs(Sxm)';
St=subs(Stp)';
St1=subs(Stm)';
Sxt=subs(Sxtp)';
Sxt1=subs(Sxtm)';                     %flexural rigidity
Ux=(subs(U))';                          %longitudinal displacement
Vt=(subs(V))';                          %circumferential displacement
Wz=(subs(W))';                          %vertical displacement
x=-L/2:L/10:L/2;
%Determination of internal effects at any desired location on the shell
Fm1=(subs(Fx));
Fm2=(subs(Ft));
Mm1=(subs(MX));
Mm2=(subs(MT));
Qm1=(subs(QX));
Sm1=(subs(Sx));
Sm2=(subs(St));
Sm3=(subs(Sx1));
Sm4=(subs(St1));
Um=(subs(Ux));
Wm=(subs(Wz)); % displacement in mili meter
F1=subs(Fxt,x(1));
F2=subs(Fxt,x(2));
F3=subs(Fxt,x(3));
F4=subs(Fxt,x(4));
F5=subs(Fxt,x(5));
F6=subs(Fxt,x(6));
F7=subs(Fxt,x(7));
F8=subs(Fxt,x(8));
F9=subs(Fxt,x(9));
```

```
F10=subs(Fxt,x(10));
F11=subs(Fxt,x(11));
Fm3=([F1 F2 F3 F4 F5 F6 F7 F8 F9 F10 F11]);
M1=subs(MXT,x(1));
M2=subs(MXT,x(2));
M3=subs(MXT,x(3));
M4=subs(MXT,x(4));
M5=subs(MXT,x(5));
M6=subs(MXT,x(6));
M7=subs(MXT,x(7));
M8=subs(MXT,x(8));
M9=subs(MXT,x(9));
M10=subs(MXT,x(10));
M11=subs(MXT,x(11));
Mm3=round([M1 M2 M3 M4 M5 M6 M7 M8 M9 M10 M11]);
Q1=subs(QT,x(1));
Q2=subs(QT,x(2));
Q3=subs(QT,x(3));
Q4=subs(QT,x(4));
Q5=subs(QT,x(5));
Q6=subs(QT,x(6));
Q7=subs(QT,x(7));
Q8=subs(QT,x(8));
Q9=subs(QT,x(9));
Q10=subs(QT,x(10));
Q11=subs(QT,x(11));
Qm2=([Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q10 Q11]);
S1=subs(Sxt,x(1));
S2=subs(Sxt,x(2));
S3=subs(Sxt,x(3));
S4=subs(Sxt,x(4));
S5=subs(Sxt,x(5));
S6=subs(Sxt,x(6));
S7=subs(Sxt,x(7));
S8=subs(Sxt,x(8));
S9=subs(Sxt,x(9));
S10=subs(Sxt,x(10));
S11=subs(Sxt,x(11));
Sm5=round([S1 S2 S3 S4 S5 S6 S7 S8 S9 S10 S11]);
S11=subs(Sxt1,x(1));
S21=subs(Sxt1,x(2));
S31=subs(Sxt1,x(3));
S41=subs(Sxt1,x(4));
S51=subs(Sxt1,x(5));
S61=subs(Sxt1,x(6));
S71=subs(Sxt1,x(7));
S81=subs(Sxt1,x(8));
S91=subs(Sxt1,x(9));
S101=subs(Sxt1,x(10));
S111=subs(Sxt1,x(11));
Sm6=round([S11 S21 S31 S41 S51 S61 S71 S81 S91 S101 S111]);
V1=subs(Vt,x(1));
V2=subs(Vt,x(2));
V3=subs(Vt,x(3));
V4=subs(Vt,x(4));
V5=subs(Vt,x(5));
V6=subs(Vt,x(6));
V7=subs(Vt,x(7));
V8=subs(Vt,x(8));
```

```
V9=subs(Vt,x(9));
V10=subs(Vt,x(10));
V11=subs(Vt,x(11));
Vm=([V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11]);
Fr1=[min(min(Fm1)) min(min(Fm2)) min(min(Fm3)) min(min(Qm1)) min(min(Qm2))];
Mr1=[min(min(Mm1)) min(min(Mm2)) min(min(Mm3))];
Dr1=[min(min(Um)) min(min(Vm)) min(min(Wm))];
Sr1=[max(min(min(Sm1)),min(min(Sm3))) max(min(min(Sm2)),min(min(Sm4)))
max(min(min(Sm5)),min(min(Sm6)))];
fr1=[max(max(Fm1)) max(max(Fm2)) max(min(Fm3)) max(max(Qm1)) max(max(Qm2))];
mr1=[max(max(Mm1)) max(max(Mm2)) max(max(Mm3))];
dr1=[max(max(Um)) max(max(Vm)) max(max(Wm))];
sr1=[max(max(max(Sm1)),max(max(Sm3))) max(max(max(Sm2)),max(max(Sm4)))
max(max(max(Sm5)),max(max(Sm6)))];
end
FF=[Fr1 Mr1 Sr1 Dr1];   % insert next row matrices for any variable parameter
FR=[fr1 mr1 sr1 dr1];    % insert next row matrices for any variable parameter

% graphical presentation
% this shows for eigth value of length for maximum negative values
% repeat FF=[Fr1 Mr1 Sr1 Dr1]; for next values of length and change to
% subscript 1 to 2-8
% this will plot all internal effects with variable length only change to
% other parameters when they are required to be changed
L=25:5:61;
hold off;
subplot(2,2,1),plot(L,FF(1:8,1:5)),legend('Fm1','Fm2','Fm3','Qm1','Qm2');
xlabel ('Length (meter)','FontName','Times','FontSize',14)
ylabel ('Shell Forces in KN/m','FontName','Times','FontSize', 14)
title ('Plot of Forces under length variation','FontName','Times','FontSize',12)
subplot(2,2,2),plot(L,FF(1:8,6:8)),legend('Mm1','Mm2','Mm3');
xlabel('Length of shell (meter)','FontName','Times','FontSize',14)
ylabel('Moments in KNm/m','FontName','Times','FontSize',14)
title('Plot of moments under length variation','FontName','Times','FontSize',12)
subplot(2,2,3),plot(L,FF(1:8,9:11)),legend('Sm1','Sm2','Sm3');
xlabel ('Length (meter)','FontName','Times','FontSize',14)
ylabel('Stresses in KPa','FontName','Times','FontSize',14)
title ('Plot of Stresses under length variation','FontName','Times','FontSize',12)
subplot(2,2,4),plot(L,FF(1:8,12:14)),legend('Um','Vm','Wm');
xlabel ('Length (meter)','FontName','Times','FontSize',14)
ylabel('Displacements in mm','FontName','Times','FontSize',14)
title ('Plot of displacements under length variation','FontName','Times','FontSize',12)
hold on
```