THE EXTENT TO WHICH TEACHERS USE HEURISTIC PROBLEM SOLVING APPROACH IN TEACHING GRADE SEVEN MATHEMATICS

A THESIS SUBMITTED TO THE SCHOOL
OF GRADUATE STUDIES OF
ADDIS ABABA UNIVERSITY

IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF MASTER OF ARTS IN CURRICULUM AND
INSTRUCTION

BY

METASEBIA DEMISSIE

JUNE 1999

ADDIS ABABA

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

THE EXTENT TO WHICH TEACHERS USE HEURISTIC PROBLEM SOLVING APPROACH IN TEACHING GRADE SEVEN MATHEMATICS

BY

METASEBIA DEMISSIE

APPROVED BY THE BOARD OF EXAMINERS

Marew Zewdie	
CHAIRMAN, DEPARTMENT GRADUATE COMMITTEE	SIGNATURE
Nardos Abebe	
ADVISOR	SIGNATURE
Thomas Tilson	
EXAMINER, EXTERNAL	SIGNATURE
Azeb Desta	
EXAMINER, INTERNAL	SIGNATURE

THE EXTENT TO WHICH TEACHERS USE HEURISTIC PROBLEM SOLVING APPROACH IN TEACHING GRADE SEVEN MATHEMATICS

A THESIS SUBMITTED TO THE SCHOOL
OF GRADUATE STUDIES OF
ADDIS ABABA UNIVERSITY

IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE DEGREE
OF MASTER OF ARTS IN CURRICULUM AND
INSTRUCTION

BY
METASEBIA DEMISSIE

JUNE 1999
ADDIS ABABA

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

THE EXTENT TO WHICH TEACHERS USE HEURISTIC PROBLEM SOLVING APPROACH IN TEACHING GRADE SEVEN MATHEMATICS

BY

METASEBIA DEMISSIE

APPROVED BY THE BOARD OF EXAMINERS

Marew Zewdie	
CHAIRMAN, DEPARTMENT GRADUATE COMMITTEE	SIGNATURE
Nardos Abebe	
ADVISOR	SIGNATURE
Thomas Tilson	
EXAMINER, EXTERNAL	SIGNATURE
Azeb Desta	
EXAMINER, INTERNAL	SIGNATURE

ACKNOWLEDGEMENTS

I am greatly indebted to my advisor, Ato Nardos Abebe, for his generous help and effective guidance throughout the study. I would also like to express my deep gratitude to Dr. Azeb Desta for her encouraging remarks and assistance in giving valuable suggestions and comments since the identification of the problem till the end of this work.

I am very grateful to Dr. Demissu Gemeda, Dr. Yismaw Alemu, and Ato Mihiretu K/Tsadik for their encouragement and valuable comments which helped me a great deal in effecting the aim of the study.

I am particularly indebted to my friends Ato Germew Semu, Ato Seme Negere, Ato Nigussie T/Mariam, Ato Tsgaye Kassa, and Ato Asrat Assefa for the great concern and help they rendered during my stay in the graduate program.

I also wish to thank W/o Yeshi G/Wold who typed the thesis neatly.

Finally, I would like to extend my heart felt and deep gratitude to my wife, Rediat Tesfaye, who had been a source of constant encouragement, advice, and material support without which the completion of this program could have been very difficult.

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF APPENDICES

ABSTRACT

The purpose of the study was to investigate the extent to which problem solving is taught in grade seven mathematics classes using heuristic problem solving approach in Addis Ababa City Administrative Region. The sources of data were grade seven mathematics textbook and sample grade seven mathematics teachers. Three data collection instruments were used: content analysis, observation, and questionnaire. Accordingly, the content of the textbook was analyzed, samples of eleven teachers were observed, and 35 teachers (including the eleven teachers observed) were made to fill the questionnaire.

The result of the study revealed that problem solving was found not incorporated in most of the topics (contents) of the textbook. Heuristic problem-solving approach is applied only 25%, which is very low. Because of this the textbook is said to be following the traditional approach. Regarding the teachers' use of the heuristic approach while teaching problem solving to the students, two results were obtained. The result of the data gathered through questionnaire administration showed that the teachers applied 71.75% heuristic approach whereas the data obtained through classroom observation showed the teachers applied 34.5% heuristic approach. However, it was also found from both instruments that the teachers gave more emphasis to teaching the computational work or performing the mathematics needed than teaching the solution process. This is also considered as the teachers used the traditional approach while teaching problem solving. An investigation was also made regarding the appropriate use of effective teaching strategies while teaching problem solving. The use of thought-provoking questions was chosen for investigation from among other effective teaching strategies. Accordingly, the data gathered through classroom observation revealed that the teachers used less number of thought - provoking questions (35.35%) than memory and factual questions. The majority of these thought-provoking questions (51.32%) were asked for the

purpose of facilitating the computational skills of the students. The result of the data obtained through questionnaire showed that the teachers ask thoughtful questions (in line with the heuristic elements indicated) rated as "sometimes". Therefore, since the teachers were not found to use the thought questions properly and in sufficient number for the purpose of teaching most of the solution processes of a problem, it is said that the teachers did not use this teaching strategy appropriately.

Thus, curriculum developers and other pertinent bodies should see to it that the textbook be revised and incorporate non-routine and multi-step problems that provide contexts for problem solving to occur in every content of the textbook. Together with this, when problems that are presented for illustrative examples are solved, heuristic problem solving approach must be used regularly. In addition, the teacher's guide should provide the solution of every problem that is presented in the textbook by applying the heuristic approach. Teachers should get the necessary knowledge and skills of using heuristic problem solving approach with the appropriate teaching strategies. This can be made possible through a course in problem solving to be offered in either pre-service or in-service training program.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

The New Education and Training Policy (E.T.P) of Ethiopia gives emphasis for the strengthening of the individual's and society's problem-solving capacity at all levels starting from basic education. As stated in the E.T.P (1994:7), One of the objectives of education is to "develop the physical and mental potential and the problem-solving capacity of individuals by expanding education, and in particular, by providing basic education for all". In addition to the stated objectives it is also mentioned that "the preparation of curriculum will be based on the stated objectives of education, ensuring that the relevant standard and the expected profile of students are achieved." (Ibid:12)

From the above statements it can be realised that the curriculum to be developed will ensure the contents of the various subjects to incorporate the necessary knowledge and skills that can help develop the problem solving ability of students. The emphasis given for the development of problem solving skill in the education policy of the country is timely and proper. As Tuma and Reif (1980:ix) noted, since problem solving plays a central role in all the sciences and in most other fields "there is an increasing need to teach improved problem-solving skills to students who must be adequately prepared to cope with a world characterised by growing complexity, rapid change, and vastly expanding knowledge". Tuma and Reif further noted that both intellectual and technological development suggest that the present time is favourable for examining the current knowledge about problem solving and its relevance to practical education. Good & Brophy (1990:260) also asserted that students, in addition to being able to read with comprehension and study efficiently, need to learn to solve problems efficiently-not only in mathematics, but also in any other subject area. Gage and Berliner (1991:304), in support of the idea above, witnessed that in recent years many researches were conducted on how people learn to solve problems. These studies have led to a rapid increase and spread of programs for teaching problem solving, thinking and reasoning-which are often known as the higher mental processes.

From the above ideas given by different scholars it can be realised that problem solving is an essential component of education that has received universal importance. So, as it is mentioned earlier, the focus given for problem solving by the new education policy of the country is proper and timely.

It is generally accepted that to improve education, a well-developed methodology of problem-solving skill is highly needed that would be useful for students in all disciplines and professions (Cyert in Tuma and Reif, 1980:5). To this effect, as noted by Husen and Postlethwaite (1985:4054), the curriculum should be organized in such a way that instruction be aimed at improving problem-solving abilities. So, with this understanding, as indicated by Good and Brophy (1990:264-265), students should be provided not only with frequent opportunities to solve problems but also with instruction in problem-solving processes.

Mathematics is one of the disciplines that incorporates problem solving as its essential component. For instance, Pimm and Love (1991:264) confirmed that "Problems are at the heart of mathematics, and that solving problems-and learning how to solve problems-is an essential component of learning Mathematics." Musser and Burger (1988:5) also asserted that problem solving is the main goal of teaching mathematics as it is of social sciences. Further, according to Morris (1989:79), because of the global importance problem solving has, the National Council for Teachers of Mathematics, which is the main group in Mathematics education in the United

States of America, recommended in its 1980 Agenda for Action that "Problem solving be the focus of school Mathematics in the 1980s."

Based on the above mentioned reasons, that is, because of the belief and the growing need for the development of problem solving skills both at international and national level it was considered to be very essential to study and uncover the present status of the teaching of problem solving skills in our school system.

1.2 Statement of the Problem

A problem exists when a person needs to achieve some goal but does not immediately know how to achieve it (Good and Brophy, 1990:260). The problem at hand may not be very complex to require great skill and effort for its solution; or it may be so simple that solving it is almost automatic. But, in any case, as Callahan and Clark (1988:241) noted, the problem-solving activity is one that requires thought and a search for a solution. It is an activity to which one can not find the answer by the process of simple recall.

The importance of problem solving in Mathematics and the complex nature it has, as indicated by Schoenfeld (1985:5), necessitated the development of heuristics (a general procedural guideline that helps for successful problem solving) whose investigation was pioneered by George Polya. In 1945 George Polya developed the four-step process which serves as a tool to help promote successful problem-solving skills. Several mathematicians like Cruikshank and Sheffield (1988); Billstein, et al., (1981); Musser and Burger (1988) accepting this popular four-step process used it for teaching how to solve problems in their textbooks and recommended to teachers to apply it whenever they teach problem solving.

Since the study focused on the teaching and learning process of grade seven mathematics, it was found that the syllabus which was prepared by

Institute for Curriculum Development and Research (ICDR) and the teacher's guide prepared by Region 14 Education Bureau both emphasize the use of pre-planned problem-solving approach when teaching problem solving to students. Especially in the teacher's guide it was recommended that the teachers follow certain steps which serve as heuristic problem solving approach whenever they teach problem solving to their students. Many scholars, such as, Krulik and Rudnick (1987) believed that these heuristics are very helpful instruments for solving different problems at different times if used regularly with appropriate teaching strategies.

So, the study aimed to investigate to what extent problem solving is taught using heuristic problem solving approach in grade seven mathematics classes. Therefore, the study tried to provide answers for the following research questions.

1. Is problem solving incorporated in every topic (content) of grade seven-mathematics textbook?
2. Is heuristic problem solving approach applied properly in the text book?
3. Do grade seven mathematics teachers apply heuristic problem solving approach properly when they teach problem solving to students?
4. Do the teachers use effective teaching strategies appropriately when they teach problem solving to students?

1.3 Significance of the Study

It is believed that learning should not be limited to knowledge acquisition involving facts and rules which are classified as lower levels of behavioural complexity. Rather, most of the time students should be engaged in learning concepts and abstractions through inquiry and problem solving which require higher form of thinking process. This helps students to cope with the rapidly changing world and technological advancement. To be a successful problem solver one needs to be equipped with the necessary knowledge and skill of problem solving.

Because problem solving plays a central role in Mathematics as in the other fields of study, trying to reveal to what extent problem solving is taught properly using the apprpriate heuristic problem solving approach is believed to be helpful (by attracting the attention of curriculum developers, teachers, and other scholars in the field) to contribute towards further investigation so that the necessary measures would be taken for more effective teaching of problem solving.

1.4 Delimitation of the Study

In order to make the study manageable the study is delimited to one grade level, subject, and region. i.e. grade seven mathematics in Addis Ababa city. It is also delimited to considering only word problems since it is assumed that the model selected is more appropriate for checking such problems than other types of problems at this grade level. Therefore, the result of the study should not be generalized to other grade levels, subjects, problem types, and regions.

1.5 Limitation of the Study

The following points are found to be the main limitations of the study.

1. Due to time and financial constraints it was not possible to use larger sample size for classroom observation to get more reliable data. Therefore, the attempt to use adequate sample size, i.e., above one third of grade seven mathematics teachers, necessitated to administer questionnaire (with different rating scale from the rating scale used for observation). As a result, the employment of the questionnaire masked to certain extent the true picture of the data gathered.

2. Since the study is confined to investigating only aspects of problem-solving, generalizing the results of the study to other aspects of the textbook and teachers activity is curtailed.

1.6 Definition of Terms

1. **Word Problem**: A problem so stated in words (rather than in symbols) that the operations necessary for solving the problem must be determined (Good, 1973).

2. **Teaching Strategy:** A plan of attack to bring about the desired learning (Clark and Starr, 1986).

1.7 Organization of the Study

The content of the study has been organized into five chapters. Chapter one introduces the background of the study and also contains the statement of the problem, significance of the study, delimitation of the study, limitation of the study and definition of terms. Chapter two presents review of relevant selected literature and research findings that are related to the problems under investigation. Chapter three and four deal with the method and procedures of the study and the report of the result and interpretation of the findings respectively. Chapter five summarizes the work of the investigation and presents conclusions drawn from the results of the investigation. Recommendations are also provided based on the findings of the study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

This chapter presents review of the relevant literature and research findings that are considered to be related to the research problems under investigation. Accordingly, first, the concept of problem solving and heuristics, second, significance of teaching problem solving via heuristics, and finally, the process of teaching and learning problem solving in mathematics are presented.

2.1 <u>The Concept of Problem Solving and Heuristics</u>

2.1.1 Problem Solving

Before discussing problem solving it is felt appropriate to have a clear understanding of what a problem is. The term "problem" has been given different meanings by different scholars. For some of them, a problem is of two types which differ in degree of structure. For others, it is only one type with a clear and precise meaning. For instance, according to Fredericksen (1984) and Simon (1979) cited in Good and Brophy (1990:260), there are two types of problems-well-structured and ill-structured. Well-structured problems are problems that constitute clearly defined goals and all of the information needed to solve the problems. This type of problems are solved by using appropriate algorithms which are fixed rules or procedures that guarantee correct answers if followed precisely. The rules for whole-number addition or subtraction are a good example of this. On the other hand ill-structured problems are the type of problems that are very difficult to define and solve. Problems of this type may not be clear for a person to find out the information needed to solve them. Such problems are attacked using certain procedural guidelines designed for processing information and solving such problems.

But these procedural guidelines do not guarantee solutions the way algorithms do.

Other scholars like Musser and Burger (1988); Houston (1986); Krulik and Rudnick (1987); Schoenfeld (1985) viewed a problem as one type only. For instance, Schoenfeld (1985:74) believed that if one has ready access to a solution for a mathematical task, that task is not considered as a problem but as an exercise. For him a problem is a difficult question that requires thought and mental exercise. According to Musser and Burger (1988:5), to solve a problem one has to pause, think, reflect, and may also take a new step never taken before to arrive at a solution. On the other hand, if a routine procedure is applied to arrive at the answer it will simply be considered as an exercise. Billstein (1981) and Schoenfeld (1985) stressed that for a given situation to be problem, it should be considered in a relative sense depending upon the individual who confronts the situation. A problem for one person may simply be an exercise for another. For example, solving $15 \times 3 = \square$ is an exercise for high school students, but it is a problem for most first or second grade students.

From these explanations, it can be said that a given situation is a problem to an individual if the situation is new and unfamiliar to the individual and also, if he / she does not have readily available means to find a solution for it. Therefore, it seems convincing to consider a problem to be of one type with such a situation and take the other situation, which can be solved by previously learned rules or procedures, as an exercise.

Houston (1986:19) referred to a problem as a situation in which an individual seeks to reach some goal but is blocked from achieving it by some obstacle or obstacles. In a more comprehensive way Krulik and Rudnick (1987:3) defined a problem as "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which the individual sees no apparent or obvious means or path to obtain the solution." From these definitions it can be inferred that a given situation is not a problem for a person if he/she can find its solution by apparently known means or by previously learned rules or procedures. These scholars agree that if a situation is presented to reinforce a previously learned skill or algorithm it is an exercise not a problem. It is on the basis of this understanding about a problem that most scholars seem to agree upon the assertion that problem solving does not take place if a solution strategy is immediately apparent. This was confirmed by De Vault (1981:40), stating that "problem solving occurs when an individual seeks to answer a question for which that individual has no readily available strategy for determining the answer." In fact, the final answer is not considered to be a solution. As noted by Krulik and Rudnick (1987), the solution is the process by which the answer is obtained. They insisted that "problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation." (P.4). The individual must synthesise what he/she has learned to solve the problem at hand. This differs from what has been said about the specific rules or procedures, which are directly applicable to a given situation usually, for drill and practice. Therefore, it can be concluded that problem solving is a process in which an individual who confronts the problem is required to apply his previously acquired knowledge in order to fulfil the demands of the problem situation.

2.1.2 Heuristics

According to Good and Brophy (1990:261), heuristics are defined as "general rules of thumb and procedural guidelines for processing information and solving problems." They include: identifying what information is given and what is needed, reasoning by analogy from other familiar problems, working backwards from possible solutions, and so on. Heuristics do not guarantee solutions the way algorithms do. However, they can be applied to wider range of problems and allow individuals to discover solutions. Krulik and

Rudnick (1987:21) further described that heuristics are a set of suggestions and questions that an individual must follow in order to resolve an ambiguous situation.

Therefore, from the above explanations it can be said that heuristics are procedural guidelines which provide the direction needed for an individual to understand and find a solution for a given problem. They are considered as rules of thumb, because, the suggestions and questions which serve as procedural guidelines are developed from practice or past experience.

There are several sets of heuristics for problem solving developed by mathematicians. But most of them seem to be similar. This will be discussed in detail in section three of this chapter. Many scholars, for instance, Musser and Burger (1988); Gioradano (1992); Schoenfeld (1985); Wheeler (1984) reported that George Polya (1945) is the first mathematician and researcher who pioneered the development of a four-step approach to problem solving process. The four steps are:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan
4. Looking back

In brief, the first step, understanding the problem, is a stage where the problem solver analyses the problem so that he/she can identify the unknown, the data (or what is given), and the condition to satisfy the unknown. The second, devising a plan, is a stage where the individual identifies a strategy or strategies, which enable him to find the solution. These strategies, mostly known as heuristic strategies, according to Schoenfeld (1985:23), are "rules of thumb for successful problem solving." They help an individual to understand a problem better or to make progress toward its solution. There are many heuristic strategies that can be applied for solving different problems. Among these the most common ones include: looking for a pattern, setting up an equation, drawing a diagram or a model, making a systematic list or table, looking for a related easier problem, and so on (Musser and Burger, 1988; Higgins, 1993; Wheeler, 1984). In most cases what researchers used to measure the test of learning to solve problems is the ability to apply or use a number of strategies that have been learned in new or a variety of situations (Ornstein, 1995:193). The third step is simply a stage where the individual carries out the plan or does the computational work based on the strategy selected until the final answer is secured. The last stage is where the problem solver checks the correctness of the answer and the reasonableness of the argument.

According to Krulik and Rudnick (1987:21), there is no single set of heuristics for problem solving. There are several workable models developed by different people. They suggested that whether a student follows the one designed by Polya or others is not important. What is more important is that "students learn some set of carefully developed heuristics, and that they develop the habit of applying these heuristics in all problem-solving situations." This suggestion also expresses the firm stand of the writer.

2.2 Significance of Teaching Problem Solving Via Heuristics

According to Ausubel and Robinson (1969), cited in Good and Brophy (1990:126), if students are trying to memorise new information without relating it to their existing knowledge it is considered that they are engaging in rote learning. But, if they try to relate the new information to what they already know and thereby make sense of it, it is said that they are engaging in meaningful learning. They further noted that "meaningful learning is retained longer than rote learning." As Ornstein (1995:20-21) also stated, new information can be made more meaningful to students if it is related to knowledge they already have acquired. So teachers are advised to assist their students in making meaningful connections between prior knowledge and new information instead of teaching them facts, rules and procedures only and let them practise in an isolated manner. Ornstein believed that these connecting processes are more important than the isolated acquisition of particular content or learning basic facts.

In support of the idea given above, scholars such as Clark and Starr (1986); Callahan and Clark (1988) asserted that if students are to function successfully in subsequent grades and also in the world outside of classroom they should be involved in learning contents that require them higher-level of thinking process, i.e., contents that require synthesis, analysis, and decision making. Also as indicated by Thompson and Rathmell (1988:17), problem solving is believed to be "a process which furnishes a context for the meaningful learning of concepts and skills and fosters the development of students' higher-level thinking process."

As discussed in the preceding section, therefore, if a student is confronted by a problem he/she cannot obtain the answer by relying on rote mechanical procedures. Solving the problem calls for relating or rearranging learned concepts or procedures with new ideas generated by the problem. Thus, a student's understanding of the concepts and the transfer of this understanding to new and different situation is decisive in problem solving.

It is indicated that problem solving plays a central role in mathematics and in most other fields (Tuma and Reif, 1980; Husein and Postlethwaite, 1985; and Pimm, et al, 1991). Because of this Tuma and Reif (1980:ix) witnessed the growing need "to teach improved problem-solving skills to students who must be adequately prepared to cope with a world characterised by growing complexity, rapid change, and vastly expanding knowledge." Supporting this idea, Copple, et al. (1984:42) also suggested that the reason for teaching problem solving is primarily because of its adaptiveness. According to them, the individual who grows exercising the ability to solve problems successfully is able to transfer this ability to real-life problems by employing the techniques and strategies to find solutions. As Giordano (1992:89), citing Suydam (1980), also noted,

> research studies have indicated that successful problem solvers visualize spatial relationships, estimate, attend to patterns, distinguish relevant from irrelevant details, generalise and exhibit personal characteristics such as poise, confidence, and versatility.

Therefore, it can be concluded that successful problem solving practices can be extended beyond mathematics and beyond the classroom.

As also stated by Schoenfeld (1987:6), because of the universal importance problem solving has, the National Council of Supervision of Mathematics, in the United States of America, asserted that "learning to solve problems is the principal reason for studying mathematics." Three years later the National Council of Teachers of Mathematics (NCTM), in USA, which is the main professional group in mathematics education in its (1980) Agenda for Action, made its Recommendation No 1 that "problem solving must be the focus of school mathematics in the 1980's." In addition to this, more recently as indicated by scholars such as Brown (1990); Campbell and Bamberger (1990), in March 1989, the NCTM published Curriculum and Evaluation Standards for School Mathematics. This document contains a set of standards for mathematics curricula K-12 which made problem solving the cornerstone of mathematics curriculum and instruction.

Despite the centrality of problem solving in mathematics, as reported by Hussen and Postlethwaite (1985:4054), its role in the school mathematics curriculum was not clearly established. According to them, some mathematics educators view the task of teachers to be providing students with the knowledge and skills needed for subsequent problem-solving activity and see this task alone as sufficient. Whereas others believe that the curriculum should be organised around the process of solving problems and instruction aimed at improving problem solving ability.

On the other hand, according to Higgins (1993) and Schoenfeld (1985), a number of studies have revealed that the teaching of problem solving via heuristics does not positively affect one's problem solving ability. For instance, Begle (1979) after summarising 75 empirical studies reported that the result was not as expected to show a positive correlation with students' performance. Also as indicated by Schoenfeld (1985:71), Wilson (1967) and Smith (1973) reported that "general heuristics did not, as hypothesised, transfer well to new situations." But others, as cited by Higgins (1993:6), such as Bruner (1963); Lester (1985); Kantowski (1974); Webb (1975); Schoenfeld (1985); and Mayes (1980) have confirmed that explicitly teaching problem solving skills positively affects the problem-solving ability of an individual.

In Schoenfeld's (1987:18-19) view, the reason for the results of different studies about the effect of teaching problem solving via heuristics to be equivocal is that even though Polya's characterisation of problem solving strategies (since he believed most researchers used George Polya's (1945) approach) were accurate descriptions, they were not prescriptive details which characterise a procedure precisely so that they can serve as a guide for implementing the strategies. Schoenfeld believed that the description merely characterises a procedure in sufficient detail for it to be recognised but did not provide enough information to enable the student to use them. For the strategies to be applicable, it was necessary to create prescriptive versions of the strategies at the right level of detail. For example, "under the following conditions, you should try the following things, in the following ways." (p.19). Krulik and Rudnick (1987:21) also agreed in its being prescriptive in approach to actually apply the strategies rather than simply describing them for the purpose of understanding only.

By making the necessary adjustments, i.e., by presenting the strategies with sufficient detail, Schoenfeld (1985) reported the results of two studies which were conducted using college students as subjects. The first one was a study on a small-scale laboratory experiment to explore students' mastery of heuristic strategies after explicit heuristic training in problem solving. Two comparisons of pre-test-to-post-test gains indicated that the experimental group significantly out performed the control group, which showed that more problem-solving practice alone without explicit training is not enough for better performance.

The other study was about paper-and-pencil measures of problem solving processes which included three matched pre-tests and post-tests that focussed on what the student did while trying to solve problems rather that on the final solution he/she produced. The measures provided clear evidence that students can master heuristic strategies and use them for closely related, somewhat related, and completely unrelated problems. He concluded that "this indicates substantial degree of heuristic transfer, which as one would expect, tails off as the problems become less and less familiar." (Ibid:241)

More recently, Higgins (1993) conducted a research on middle-school students to investigate the effects of teaching directly problem-solving skills on the students' attitudes, beliefs, and abilities in problem solving and mathematics. In the study, two 6^{th} –grade teachers and four 7^{th}-grade teachers as well as their students participated. Half of the teachers at each grade level received training in teaching problem solving skills in mathematics and used the materials with their students over one-year period (he referred to these students as the heuristic students). The other half of the teachers did not receive training on the use of the teaching materials which enabled them to apply the instructional approach as the others.

The study used both quantitative and qualitative components where the quantitative component consisted of the results from a student questionnaire and problem solving inventory and the qualitative component consisted of student interviews.

The result of the quantitative component of the study revealed that:

1. There were little, if any, differences in problem-solving ability between the heuristic and non-heuristic students.

2. The heuristic students believed that real mathematics problems could be solved by common sense and were less dependent on the teacher and the textbook for finding out about incorrect answers to mathematics problems.

3. The heuristic students placed less emphasis on the role of memorisation.

4. The heuristic students expected their teachers to ask thoughtful questions and did not answer questions when they did not know the answer.

The result of the qualitative component of the study also revealed that:

1. The heuristic students preferred problems that made them "think" and believed that mathematics was useful, regardless of ability.

2. Whereas, many of the non-heuristic students equated how fast they could solve a problem with how much they understood a problem, the heuristic students claimed that they could tell they understood a problem when they could solve the problem in different ways and explain the answer to someone else.

3. The heuristic students had greater task persistence in solving problems than the non- heuristic students.

Higgins, based on the findings, concluded that teaching problem solving skills by making the use of many different heuristic strategies explicit on a regular basis may positively affect students' beliefs and attitudes in a way encouraged by the NCTM's Curriculum and Evaluation Standards. But, regarding problem-solving ability he commented that to draw conclusions through the use of instrument alone was not possible. He observed that the differences he found could have been the result of the students' ability levels or certain teachers' behaviours. In addition, he continued, some of the problem-solving strategies seemed to be used intuitively by all students regardless of training how and when to use them (for example, "guess and check", "look for a pattern").

Other researchers like Newell and Simon (1972), cited in Giordano (1992:90), commented that "the impact of heuristic strategies is difficult to evaluate not only because such strategies are resistant to experimental isolation but also because the strategies are by nature tenuous and variable."

From the foregoing discussions it can be concluded that students should not be limited to learning acquisition of knowledge which involves memorisation of facts, rules and procedures if they are to function successfully in subsequent grades and in real-life situations. Rather, they should have to be engaged in learning that can develop high-level thinking skills. This can be achieved by giving students frequent opportunities to synthesize and analyse content and to find relationships between knowledge already acquired and the new information. Since problem solving calls for rearranging, relating learned concepts or procedures with new ideas generated by the problem at hand it requires high-level thought process. Because of this it is believed to be highly important to teach problem solving regularly at every grade level. The teaching of problem solving is found to be more effective using heuristic problem solving approach. Although there are equivocal results reported by different researchers, there seems a consensus reached among many scholars in the field that teaching problem solving via heuristics is helpful in promoting the development of students' problem-solving abilities and building their attitudes and beliefs positively towards problem solving in particular and mathematics in general. It is with this understanding that this study aimed at investigating the use of heuristic problem solving approach in teaching mathematics in grade seven of Addis Ababa Region.

2.3 The Process of Teaching and Learning Problem Solving in Mathematics

The preceding sections have discussed the concept of problem solving and heuristics, and also, the significant role problem solving has in the development of students' abilities so that they can deal effectively with problems in real-life situations.

This section discusses the effective way of teaching and learning problem solving in order to attain the intended objective-the development of problem solving skill. Therefore, first, the traditional approach in brief; second how problem solving is taught by heuristics; third, what is required from the curriculum in order to promote the development of problem solving skill, and finally, effective teaching strategies used while teaching problem solving in classroom are presented.

2.3.1 Traditional Approach

It is believed that problems are at the heart of mathematics and, learning how to solve the problems is, an essential component of learning mathematics. The term problem solving is used for both an activity (actually solving problems) and for a set of approaches to solving problems that can be taught (Pimm and Love, 1991:264). In the traditional approach, problem solving is not considered as a process which can be taught by a set of approaches. Rather, according to Husen and Postlethwaite (1985:4054), the task of problem solving is considered only as providing students with the knowledge and skills needed for getting the answer for subsequent problems.

As krulik and Rudnick (1987:4) noted, educators assume that "expertise in problem solving develops incidentally as one solves many problems."

The problems most commonly used in mathematics are "word problems" which are sometimes termed as " verbal problems" or "story problems". They are used mostly for practising the application of mathematical procedures that have been learned before. A word problem in this sense, describes a situation where some information is given and other information is to be found by applying one or more mathematical procedures (Husen and Postlethwaite, 1985:4054).

It was also stated in the document of National Council of Teachers of Mathematics (NCTM, 1989:76) that traditional word problems most of the time do not offer opportunities for true problem solving since they only provide contexts for using particular formulas or algorithms.

Similarly, according to Krulik and Rudnick (1987:4), most textbooks of mathematics contain sections labelled "word problems" or "verbal problems" among which few are considered to be real problems. In many cases, the teacher presents a model solution in class and the student is required merely to apply this model to a series of similar "problems" in order to solve them. Those which require higher-order thinking skills are few in number. In such cases the student is learning algorithms-techniques that guarantee success if mechanical errors are avoided. However, if a student sees these "word problems" for the first time they could be problems to him.

Furthermore, in the traditional approach, problem solving, that is, for those problems which are put in non algorithmic fashion is taught only for getting the final answer without teaching the process. As Polya (1990:3) noted, the techniques or the strategies used by teachers for solving problems are seldom articulated which make it very difficult for the students to use the strategies in different problem situations. In this approach the problem solving skill is thought to be developed by engaging students frequently in solving different problems. This was confirmed by Husen and Postlethwaite (1985:4054) as put as follows:

> ...mathematics instruction is influenced by the view that the pupil learns to solve problems chiefly through practice and that the teacher's role is simply to provide an ample number of problems and opportunities to solve them.

To sum up, in the traditional approach, most textbook word problems are presented for using particular formulas or algorithms. Those which provide contexts for true problem solving are few in number. In this approach problem solving is not considered as a process, rather it is considered as an activity where the finding of a final answer is regarded as sufficient. This task is performed mechanically without using any systemically developed set of approaches. It is also believed that problem solving ability is developed by giving individuals frequent opportunities to solve different problems.

2.3.2 Teaching Problem Solving by Heuristic Problem Solving Approach.

As discussed in the first section of this chapter problem solving is a process. This process, as noted by Krulik and Rudnick (1987:21), starts when the individual initially encounters with the problem and ends when the obtained answer is reviewed based on the information given. It is believed that the process of problem solving is a teachable skill (Callahan and Clark, 1988; Good and Brophy, 1990; Krulik and Rudrick, 1987). The development of this problem solving skill could be promoted by teaching the process of solving the problems using heuristics which are general rules of thumb and procedural guidelines that help the problem solvers to approach, understand, and obtain answers to problems that confront them (Good and Brophy, 1990; Krulik and Rudnick, 1987). It has been also found that the frequent and explicit use of heuristic strategies in teaching problem solving, not only increases the ability of solving different problems, but also, helps to develop positive attitude toward mathematics in general and problem solving in particular (Higgins, 1993; Schoenfled, 1985). So as Krulik and Rudnick (1987:21) also stressed, "children must learn this process if they are to deal successfully with the problems they will meet in schools and else where."

As indicated in the first section of this chapter George Polya (1945) is believed to be a pioneer in developing a four-step process for solving mathematical problems. The four steps include: understanding the problem, devising a plan, carrying out the plan, and looking back. This four-step problem solving approach which includes questions and suggestions under each step that help the problem solver comprehend, analyze, synthesise, formulate a strategy and review the solution process, has become very popular in mathematics education.

It is also true that different mathematicians have developed other models which they claim to be more workable than Polya's. For instance, Krulik and Rudnick (1987:22) suggested the following model as more workable. 1) Read 2) Explore 3) Select a strategy 4) Solve 5) Review and extend (see Appendix-G for full detail). Others like Bransford and Stein (1985),cited in Good and Brophy (1990:262), suggested: 1-Identify the problem 2-Define it 3-Explore possible strategies 4-Act on the strategies 5-Look at the effects of your efforts. There are still others like John Dewey (1910) (with his classical model) and Rechard Cyert (1980) who suggested different models which are applicable for all subjects. For the purpose of this study it is preferred to focus on the models forwarded by Polya and Krulik and Rudnick to other models not only because they are popular, but also, because they are found to be more helpful to meet the objectives of the study. In fact the two models are quite similar in the main, differing only in terminology or degree of elaboration. This is shared by Schoenfeld (1985) and Krulik and Rudnick themselves, who suggested that Polya's model is descriptive in nature and should be made prescriptive, that is, it should be made to serve the actual purpose by pinpointing when and how to use them. Therefore, it is decided to modify

Polya's model a little, in order to make it more applicable to fulfil the purpose of the study, by including some questions and suggestions in the steps taken mainly from Krulik and Rudnik, and also, from other scholars (see Appendix - H).

The development of problem solving skill, as discussed earlier, is not secured by providing only frequent opportunities to solving problems for students but also, by giving instruction in problem solving processes using heuristics. According to Krulik and Rudnick (1987:21), students should "develop the habit of applying these heuristics in all problem-solving situations." They insisted that since applying heuristics is a difficult skill in itself, teachers must show students how and when to use each of the elements of the heuristics.

As indicated by Polya (1990:2-3), the questions and suggestions that serve as procedural guidelines are grouped in four phases by keeping their order in which they are most likely to occur. They are generally applicable for all sorts of problems without restriction to any subject matter. But the problems, must be "problems to find" not "problems to prove", which require different questions and suggestions. These questions and suggestions are considered to be simple and obvious that proceed from common sense and come naturally to any person who is seriously concerned with solving the problem at hand. But in Polya's view, the person (the problem solver) in most cases does not express them in clear words. And, he claimed that the list of questions and suggestions grouped in the four steps expressed them.

In order to have a clear idea, following are descriptions of how each of the four phases of the process is applied when solving problems.

Step 1. **Understanding The Problem**

This phase of the problem solving process is fundamental. Students may overlook important ideas and implications that could give hints to them or they may not even understand the problem at all let alone trying to solve it. Therefore, the teacher should make sure that the student understands the verbal statements of the problem first. Key words must be identified and their meanings given. The principal parts of the problem, i.e., the unknown, the data, and the condition must be identified. This is to mean, the problem solver must identify what relevant information is given, what is required to be found, and determine whether there is sufficient condition to satisfy the unknown by forming relationship between the data and the unknown (Polya, 1990; Musser and Burger, 1988; Krulik and Rudnick, 1987; Wheeler, 1984). In order to refer to model example that illustrates how the four-step process is applied see Appendix B).

Step 2. **Devising a Plan**

According to Houston (1986:349), this phase is assumed by many mathematicians to be the most critical phase in problem solving. It is a phase where the problem is to be reduced to a set of directly solvable tasks. As Polya (1990:12) noted, the idea of the solution is conceived in this phase. To succeed in devising the plan for attacking the problem one needs to have formerly acquired knowledge, concentration upon the purpose, and good luck.

In this phase, the problem solver selects strategies often called heuristic strategies (Higgins, 1993:6), which are considered to be artful means to an end (Musser and Burger, 1988:6). They include: look for a pattern, guess and check, set up an equation, draw a diagram or a model, make a table or a systematic list, look for a related easier problem, and use a formula (Higgins,1993; NCTM, 1989; Musser and Burger, 1988; Wheeler, 1984). There are also other strategies that can be applicable to varied problems to any grade level. (Some of them are listed in appendix I). These are selected for the purpose of the study assuming that they are most commonly used in middle grades. Higgins (1993:13) noted that "As teachers solve problems and model the problem-solving process, they should discuss why they used particular skills and strategies to solve the problems." This helps students to use the strategies for some other related problems. As indicated in the second section of this chapter, most research studies were conducted to test their effect on students' ability and attitude towards problem solving and mathematics, after explicit training in when and how to use them.

Step 3. Carrying Out the Plan

Once the problem has been understood and a strategy has been selected the next stage is to perform the computational work necessary to arrive at the final answer. Polya (1990:13) stressed here that checking the correctness of each step while performing the computational work is highly important in order to find the correct answer.

Step 4. Looking back

This is the last phase of the problem solving process where the problem solver is required to see if his/her solution makes sense. But in most cases, individuals skip this phase once the final answer is found (Polya, 1990; Wheeler,1984). But, Krulik and Rudnick (1987:28) insisted that the solution is not the final answer. Rather, "the solution is the process by which the answer is obtained." According to them, the answer is meerly the final outcome of the solution. Most of the time individuals miss this very important and instructive phase of the process. Polya (1990:4-5) expressed the importance of the phase as follows:

> By looking back at the completed solution, by reconsidering and re-examining the result and the path that led to it, they [the students] could consolidate their knowledge and develop their ability to solve problems. A good teacher should

> understand and impress on his students the view that no problem whatever is completely exhausted. There remains always something to do, with sufficient study and penetration, we could improve any solution, and, in any case, we can always improve our understanding of the solution.

So, with this understanding, it is found to be very essential that teachers whenever they teach problem solving they should make their students check the result of the problem and the argument used, derive the result differently, and use the method and the result for some other problem (Polya, 1990; Wheeter, 1984; Krulik and Rudnick, 1987).

So far, it has been discussed that problem solving is a process and a teachable skill. The skill can be developed by using a set of heuristics involving questions and suggestions which serve as procedural guidelines grouped in a four-phase process, i.e., understanding the problem, selecting workable strategies, performing the mathematics needed according to the strategies selected, and finally reviewing the solution process. Teachers should regularly assist their students so that they can develop the habit of using this set of heuristics to solve the problems that confront them.

2.3.3 Curricular Requirements for Teaching Problem Solving

It was indicated earlier that in the traditional approach, since problem solving is not considered as the focus of school mathematics, most text book problems are presented for the purpose of practising the use of formulas and algorithms that have been learned before. It is also common to label "word problems" by giving a separate section in which model solutions are presented beforehand so that students can apply these models for the rest of the problems.

Some scholars, for instance, Musser and Burger (1988); Billstein, et al. (1981); Cruikshank and Sheffield (1988), even if they claimed to believe that problem solving is a major goal of learning mathematics and apply the four-step problem solving approach in the text books they prepared, they used separate sections usually at the beginning of each chapter for teaching problem solving. Good and Brophy (1990:263) also witnessed that some books give separate topic labelling "brain teasers" for practising heuristic problem solving approach. But, according to the Curriculum and Evaluation Standards for School Mathematics which was developed by the National Council of Teachers of Mathematics (NCTM) in the United States of America, problem solving should not take an isolated topic. The document, NCTM (1989:23) stated that:

> Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.

The Document further indicated that problem solving should be incorporated in the topics and ideas across the entire curriculum. For example, an instructional activity might involve problem solving and use geometry, probability, statistics, rational numbers, and so on (Ibid:75). In support of this idea, Krulik and Rudnick (1987:6) also suggested that problem solving should be taught using various mathematical ideas at all times.

The above mentioned document presented standards that serve for grades k-12. The grades are categorised in three level, i.e., k-4, 5-8, and 9-12. There are thirteen standards for each k-4 and 5-8 level and fourteen for 9-12 level among which standard 1, for grades k-12, is "Mathematics as problem solving."

Since the study is concerned with Grade 7 mathematics teaching, the level 5-8 is taken as reference. Therefore, problem-solving standard for 5-8 as stated in NCTM (1989:75) is indicated below.

> In grades 5-8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can:
>
> - use problem-solving approaches to investigate and understand mathematical content;
> - formulate problems from situations within and outside mathematics;
> - develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non routine problems;
> - verify and interpret results with respect to the original problem situation;
> - generalise solutions and strategies to new problem situations;
> - acquire confidence in using mathematics meaningfully.

From the above standard it can be understood that except the second and the last points the rest of them are similar to that of the elements found in the set of heuristics discussed in the preceding sections, i.e., understand the problem, select and apply variety of strategies and review or look back the solution process in order to check or verify results and use the results or arguments to some other problems.

So it can be said that when the curricular materials such as curriculum guides, syllabii; teachers guides, textbooks, and other workbooks are organised; it should be made certain that problem solving is emphasised and integrated in all mathematical activities. The textbooks prepared for each grade level should incorporate non-routine and multistep problems in all the topics for the purpose of teaching the process of solving in addition to other routine problems used for practising particular formulas and algorithms. (see Appendix J for examples that illustrate routine, one-step problem and multi-step problem).

2.3.4 Effective Teaching Strategies Used While Teaching Problem Solving.

In the preceding section, it has been discussed that problem solving should pervade in all mathematics program instead of being treated as an isolated topic in the curriculum. The contents of textbooks should be selected and organised to promote the development of problem-solving abilities by giving opportunities for teaching the process of problem solving. However, this alone does not guarantee the proper attainment of the intended objective-the development of problem solving skills. Teachers should also be able to use appropriate and effective methods and strategies in the classrooms in order to assist students in the utilisation of heuristics in their development of problem solving skills.

As noted by Krulik and Rudnick (1987:21), simply providing students with a set of heuristics to follow has little value. Since applying heuristics is a difficult skill in itself teachers must constantly show their students how and when to use each of the heuristics when solving the problems. According to Polya (1990:2-5), while teaching problem solving infront of the class the teacher has two purposes. That is: "First, to help the student to solve the problem at hand. Second, to develop the student's ability so that he may solve future problems by himself." These purposes can effectively be achieved by asking the questions and the suggested steps (which were listed in the four-step process) to the students as often as he can do so naturally. For instance the teacher can ask: What is the unknown? What are the data? What is the condition? In addition, the teacher should dramatise his ideas and also put to himself the same questions which he uses when helping the students. This helps the students to eventually discover the right use of these questions and suggested steps so that they can apply them independently. So, the teacher who wishes to develop his students' ability to solve problems must give plenty of opportunity for imitation and practice.

Ornstein (1995:194) also said that the teacher should reveal or model his own thought process, i.e., what he is thinking and how he is tackling a problem while solving problems. He should also ask his students to reveal their thinking during problem-solving tasks. Good and Brophy (1987:181) too noted that when teachers solve problems in front of the class they should regularly think out loud so that students can see them model the thought processes. They believed that to think out loud is the basic method of modelling. In other words, teachers should "verbalise each step in their thinking processes from beginning to end." This helps students to see the way the problem is approached and arrived at an answer logically following a chain of reasoning. This was supported by Higgins (1993:13) who conducted a research recently on sixth and seventh grade students (which was described in the second section of this chapter). Higgins recommended that teachers should make explicit and model how they decided on the strategy they used and how they solved the problems.

According to Krulik and Rudnick (1987:58-65) teachers should guide their students through a solution by using open ended questions such as "How many...?" "Find all...". In addition, throughout the problem-solving process, the questions posed by the teacher should cause the students to reflect back on the problem solution. They should ask many "What if...?" questions. For example, what if the conditions of the problem were changed to...? What if we use some other strategy or strategies? What other related problem does it remind us? Further more, in the Curriculum and Evaluation Standards (NCTM, 1989:23,77) it was stated that in order to promote a problem solving approach teachers should use thought-provoking questions and frequently allow group and classroom discussion so that the students can explore and examine a variety of strategies, verify results, interpret solutions and questions whether a solution makes sense.

It is true that discussions are considered to be helpful in fostering the ability to think critically (Gage and Berliner, 1992:420), but they are more effective if the class size is small or the class is divided into smaller groups and the seating arrangement is made in such a way that students can sit face to face in order to exchange views and ideas more purposefully (Cruickshank, et al, 1995:176). Regarding the use of questions, as noted by Perrott (1982:47-48), questions can be classified according to the thinking process required as lower-order questions and higher-order questions. Lower-order questions are memory and factual questions. They require to remember specific facts or information which were previously taught or which are general knowledge. On the other hand, higher-order questions are considered to be thought questions that require thoughtful responses by changing the form of information given in order to compare, contrast, explain, summarise, analyse, synthesise or evaluate. Perrott explained that even though both types of questions have their part to play in teaching and learning, "a heavy reliance on lower-order questioning encourages rote learning and does little to develop higher-order thinking processes." (p.55). Nevertheless, Perrott, Good and Brophy (1987) and Clark and Starr (1986) believed that factual questions often are needed to ascertain if students have basic information before posing thought-provoking questions.

Therefore, since problem solving requires a higher-order thinking process, it is appropriate to use more thought questions than memory and factual questions. In general, it can be concluded that the effective way of teaching problem solving is promoted by modelling or making explicit the thought processes needed for each step. Together with this, teachers should use thought-provoking questions and allow (if there are favourable conditions) group and classroom discussions.

CHAPTER THREE

METHOD AND PROCEDURES OF THE STUDY

This chapter presents the research method employed, the source of data, the sampling procedures used, the instruments of data collection selected, and the procedures of data collection followed.

3.1 Research Method

The research method applied to examine the extent to which heuristic problem-solving approach is used in teaching grade seven mathematics is descriptive survey.

3.2 Source of Data

Grade seven mathematics textbook, which was prepared by Addis Ababa City Education Bureau and selected grade seven mathematics teachers of Addis Ababa city were the sources of data. The main reason for the grade to be chosen is, because it is a grade where the medium of instruction is changed from Amharic to English, with the assumption that the grade needs special attention in order to make the students able to learn properly using the language.

3.3 Sampling Procedure

The study involved the government schools only assuming that the other non-government schools may not provide the true reflection of the teaching-learning activities in the region's school system.

CHAPTER THREE

METHOD AND PROCEDURES OF THE STUDY

This chapter presents the research method employed, the source of data, the sampling procedures used, the instruments of data collection selected, and the procedures of data collection followed.

3.1 Research Method

The research method applied to examine the extent to which heuristic problem-solving approach is used in teaching grade seven mathematics is descriptive survey.

3.2 Source of Data

Grade seven mathematics textbook, which was prepared by Addis Ababa City Education Bureau and selected grade seven mathematics teachers of Addis Ababa city were the sources of data. The main reason for the grade to be chosen is, because it is a grade where the medium of instruction is changed from Amharic to English, with the assumption that the grade needs special attention in order to make the students able to learn properly using the language.

3.3 Sampling Procedure

The study involved the government schools only assuming that the other non-government schools may not provide the true reflection of the teaching-learning activities in the region's school system.

There are six zones, 54 Junior Secondary Schools, and 90 grade seven mathematics teachers (See Appendix A) in Addis Ababa City. Since the geographic distribution of the Junior Secondary Schools is widely scattered, it was preferred to select two zones, namely, Zone.3 and Zone.4 in order to make the study manageable within the given limit of time and financial support. The criteria for selecting the two zones is the relative abundance of the schools and proximity of their location to each other in the two zones as compared with that of the other zones. In order to get proper information about the application of heuristic problem solving approach by the teachers two instruments (observation and questionnaire) were used. Classroom observation was used, as suggested by Best and Kahn (1989:174), to gather more reliable data. But, since it was found that it is very costly to observe a sufficient number of sample teachers, questionnaire was employed, in addition to classroom observation, to use adequate sample size and also to get elaborate information. The selection of sample teachers was made as follows.

First, the list of the schools and the number of respective grade seven mathematics teachers, which were obtained from Zone 3 and Zone 4 Education Departments, were identified and coded. From among the coded teachers 35 teachers were randomly selected to fill out the questionnaire. This number is above one third of the total number of grade seven mathematics teachers in the city. Second, twelve teachers were selected randomly from among the 35 teachers for classroom observation. The 35 teachers were found to be distributed in 21 schools (Six in Zone.3 and 15 in Zone.4) and the twelve teachers in eight schools (three in Zone.3 and five in Zone.4)

3.4 Instruments of Data Collection

The instruments used to investigate the extent to which problem solving is taught using heuristic problem-solving approach are content analysis , observation, and questionnaire.

3.5 Procedures of Data Collection

Before discussing the data collection procedures it is felt appropriate to explain how the data collecting instruments were developed. Therefore, how the checklists for content analysis and classroom observation and the questionnaire items were developed are presented as follows.

In order to meet the objectives of the study the data needed were:

i. Problems that provide situations for problem solving to occur.

ii. Elements of heuristic problem solving approach applied when solving problems that are found in grade seven mathematics text book.

iii. Elements of heuristic problem solving approach used by grade seven mathematics teachers while teaching problem solving to students.

iv. Appropriate teaching strategies practiced by the teachers while teaching problem solving to students.

To investigate to what extent heuristic problem-solving approach is exercised in grade seven mathematics textbook and is employed by the mathematics teachers of grade seven, George Polya's four-phase problem-solving approach was chosen as a model. The model was selected due to:

i. Its wide acceptance in the field of mathematics (Musser and Burger, 1988; schoen feld, 1985). For instance, with the approval of the National Council of Teachers of Mathematics, which is the main professional group concerned with mathematics teaching and learning in U.S.A., textbooks produced by scholars such as Billstein, et al., (1981); Musser and Burger (1988); wheeler (1984) apply the model as an instrument that helps for solving mathematical problems.

ii. The presence of some elements that fit into the model in the textbooks (grades 5 - 7) prepared by Addis Ababa City Education Bureau.

The model is modified a little to serve the purpose of the study through a pilot-study and on the basis of relevant literature. Based on the model three different data gathering instruments were developed as indicated below.

The first one was designed to check the presence or absence of the elements of heuristic approach in the process of solving word problems in grade seven mathematics textbook. The draft checklist which consisted of four categories and ten sub-categories was first tried on some problems from different textbooks by checking the steps taken towards their solutions. From this try-out it was found that using the third category from Polya's four-step process, i.e., "carrying out the plan", and three other sub-categories were very difficult to measure reliably the solution processes of the problems. Therefore, upon consultation with experts in Addis Ababa University a checklist with three categories and six sub-categories was considered as adequate for the final use. Finally, to establish reliability the solution processes of six word problems were checked together with a graduate student from mathematics department and coder agreement percentage, i.e., percentage of the number of sub-categories on which the coders agree to the total number of sub-categories marked, was computed and 94.44% agreement was secured (see Appendix D for the finalized checklist).

The second instrument was designed to check the proper use of heuristic approach while teaching problem solving in grade seven mathematics classroom. The checklist has four categories and ten sub-categories . For each sub-category a 3- point rating scale was used i.e, proper, moderately proper, and not at all proper with values 2,1,0 respectively so that the observer can check the degree of occurrences according to the suggestions given for rating. The observation checklist was tried with three teachers while solving two word problems each in classrooms. After having made some modifications in the rating items it was discussed with the same experts in Addis Ababa University mentioned above. In addition, in order to establish reliability of the checklist using (viewing) the video cassette records of the above three teachers selected for the try-out, a graduate student from mathematics department and the writer checked together the solution processes of the six problems solved. Finally, since 91.67% agreement was secured the checklist was made ready for the final use (see Appendix. B for the finalized copy).

The third data gathering instrument was a 30-item questionnaire involving both structured and unstructured items. The items constructed were composed of mainly from the items of observation schedule No.1 designed for checking the proper use of heuristic approach. There are also some items that are concerned with asking questions. The questionnaire was designed to get appropriate information about how often the grade seven mathematics teachers apply heuristic approach and ask thought questions while solving word problems to students. A 5 - point rating scale was used: Always, Frequently, Sometimes, Rarely, and Never with values 4,3,2,1,0 respectively. The questionnaire was tried using five grade seven mathematics teachers that are different from the sample teachers. On the basis of information obtained from these teachers selected for the try-out, the questionnaire was revised and finalized (see Appendix. E for the finalized sample copy).

For the purpose of investigating whether a teacher applies appropriate teaching strategies or not while teaching problem solving using heuristic approach, it was intended to use Flanders' Classroom Interaction Analysis Category (FIAC). But, trying it in a pilot-study the writer found the instrument to be less effective to meet the particular purpose of the study. Because of this, a more suitable instrument was developed and employed. As it is discussed in the last section of chapter two, the effective way of teaching problem solving via heuristics is promoted using teaching strategies such as: modeling, asking questions and allowing group and classroom discussion. However, the instrument developed was only for checking the type and purpose of questions and responses made by both the teacher and the students while teaching and learning problem solving in the class. This was done because of the following reasons.

As it is also indicated in the last section of chapter two, modeling, in this context, occurs when teachers verbalize the thinking process or think out loud when they solve problems to students. This nearly coincides with the proper use of heuristics, i.e., explicitly showing the thought process needed in each step. Since an instrument (see Appendix. B) was already developed for investigating the proper use of heuristics it was found unnecessary to design another one.

Regarding the use of group or classroom discussion, as scholars such as Cruickshank, et.al., (1995) suggested, it should preferably be exercised in classrooms with smaller class size and where the seating arrangement is made face-to-face. Because of this, it was found highly important to assess some of the classrooms of Junior Secondary Schools. Therefore, before developing an instrument that measures the proper employment of discussion in classrooms, the writer observed classrooms of six schools found in three zones (two from each zone), namely, Zone.1,Zone.4, and Zone 5. As a result, it was found that:

i. The seating arrangements were traditional, i.e., they were arranged in rows and columns facing to one direction, particularly, to the chalk board.

ii. The average number of students observed was 63.

Therefore, because of the above reasons, the writer decided that trying to measure the proper employment of discussion as a means of facilitating the effective way of teaching problem solving would be unfair. Hence, the writer developed one observation checklist only which was intended to check the type and purpose of questions and responses made by both the teacher and students while teaching and learning problem solving.

The checklist has two parts. The first part was designed to check the type of questions and responses that are classified as memory / factual and thought-provoking. It has two main categories: teacher talk and student talk. Each category is further sub-devided into initiation and response. In order to establish reliability of the checklist the writer and a

graduate student from mathematics department checked together the number of memory and factual questions and thought questions that were asked while solving two problems each by the above mentioned three teachers selected for try-out. Using Pearson's product-moment coefficient of correlation formula reliability coefficient was computed for each type of questions. As a result, reliability coefficient of 0.86 for memory and 0.91 for thought questions were secured. The second part was designed to identify from among the thought questions and responses the purposes of the question and responses and classify them according to which of the categories of the four-step problem-solving process they belong (See Appendix C).

3.5.1. Content Analysis

The following two steps were taken to analyze the content of the textbook.

Step 1. To investigate whether problem solving is incorporated in every mathematical content of the textbook, first, all " word problems" that are presented for illustrative purposes and exercises were identified. Second from among the " word problems" those "problems" which provide "problem" situations for direct application of a formula or a procedure that are already learned were identified from those problems which provide problem situations that require other arithmetic operation or solution strategy different from the formula or procedure already learned, i.e., non-routine, multi-step problems. The number of occurrences of the two types of " problems" were then presented in accordance with the units and sub-units they belong. Finally, simple frequency counts and percentages were calculated and reported with their interpretations.

<u>Step.2.</u> To investigate to what extent heuristic problem-solving approach is exercised in the textbook, those word problems which provide situations that require other arithmetic operation and solution strategy (different from immediate application of a formula or a procedure) that are presented as illustrative examples were considered. Thus for each problem the steps taken towards the solution were checked using the checklist prepared for the purpose. Finally, simple frequency counts and percentages were calculated and reported.

3.5.2. Classroom Observation

The purpose of the classroom observation was to check to what extent grade seven mathematics teachers:

i. apply heuristic problem-solving approach while teaching problem solving to students.

ii. use appropriate teaching strategies while teaching problem solving to students.

As indicated earlier in this section, twelve teachers were randomly selected to be observed from the two zones which are distributed in eight schools. To fulfill the purposes mentioned above, two types of observation schedules were designed and video recording was used with the help of a camera man employed for the purpose. This enabled to perform the checking more reliably by viewing repeatedly using replay mechanism. Prior arrangement was made with the teachers using their annual and weekly lesson plans two-three weeks earlier before the actual observation schedule. The schedule was confirmed again a week before the observation day. In order to make the observation program agreeable with that of the lesson plans of the respective teachers and based on the information assessed from the teachers, it was decided to observe a teacher while solving four different word problems in two different sections (two word problems in each section) of the same shift. This makes the total number of word problems to be 48. The four problems were chosen by the teachers themselves from among the word problems already identified (that are found in the textbook) by the writer.

For the purpose of the observation, the following new arrangements were made with the help of the teachers concerned and their respective school directors. Twenty minutes were taken from the period allotted for other subject immediately preceding the period intended for the actual observation in order to familiarize the teacher and the students with the video camera. One period has 40 minutes. Therefore, one observation session was arranged to last for almost 60 minutes. That is, the first 20 minutes was allotted for the purpose of familiarizing with the camera (without actually recording) while the teacher teaches other lessons different from the two word problems selected for observation. The process of solving the two word problems was accomplished within the rest of the 40 minutes. One teacher was observed for two sessions either in the morning shift or in the afternoon shift. (The shift is applied only for the students).

3.5.3 Questionnaire Administration.

The questionnaire was administered to 35 teachers (including the observed teachers) which are distributed in 21 schools found in the two zones in the presence of the writer. Each observed teacher was made to fill the questionnaire after the two observation sessions were over in order to get elaborate information about the use of heuristic problem solving approach.

CHAPTER FOUR

PRESENTATION AND INTERPRETATION OF DATA

This chapter deals with the presentation and interpretation of the data collected through three types of instruments: content analysis, observation and questionnaire. Accordingly, first, the result of the textbook analysis, second, the result of the data collected through classroom observation, and finally, the data collected through the questionnaire are presented.

4.1 Data Obtained Through Content Analysis

The analysis of the textbook was performed in two stages. The first stage was concerned with investigating whether or not problem solving is incorporated in every content of the textbook. The second stage was for investigating the proper use of heuristic problem solving approach. The findings of the two stages of analysis are presented as follows.

4.1.1 The Presence of Problem Solving in the Contents

The mathematics textbook of grade seven is divided into five units and twenty sub-units. The topics of the units are:

Unit 1.	Ratio, Proportions, and Percentage
Unit 2.	Rational Numbers
Unit 3.	Equations and Inequalities in One Variable
Unit 4.	Plane Geometry
Unit 5.	Measurement of Area and Volume

In the units and sub-units different mathematical facts, concepts, operations, and procedures are introduced and explained. These are mostly followed by different types of "problems" which are meant for illustrative examples and exercises stated under the topics labeled "Example" and "Exercise" respectively. At the end of each unit there is also a topic labeled

"Miscellaneous Exercises" . These examples and exercises include "problems" to find, construct, prove, or derive a formula depending on the topics of the contents. The "problems" are in the form of "word problems" or otherwise. As indicated in chapter two section 2.3 the study is concerned with the type of "problems" to find only since the other types of "problems" require different approaches. From among "problems" to find the study considered those "problems" which are stated in words rather than in symbols known by "word problems" or "verbal problems", where part of the information is given in the "problem" and the other part is to be found by the problem solver himself by applying one or more mathematical procedures. There are also other types of "problems", which the study did not consider, where some of the information is given in the stated "problem" itself and the other information to be found with the help of a given graph, table, diagram, coordinate plane, and the like.

Table 1 and 2 present the distribution of the "word problems" (which were considered in the study) and the other types of "problems" stated as examples and exercises respectively that are found in every unit of the textbook.

It should be noted that in all the tables found in this chapter percentages are presented rounded to two decimal places.

Table 1. Distribution of "Problems" Presented as Illustrative Examples.

Unit	"Word Problems"	Other Types of "Problems"	Total	%
1	24	24	48	33.33
2	-	47	47	32.64
3	4	27	31	21.53
4	-	6	6	4.17
5	9	3	12	8.33
Total	37	107	144	100
%	25.69	74.31	100	

Table 2. Distribution of "Problems" Presented as Exercises

Unit	"Word Problems"	Other Types of "Problems"	Total	%
1	83	45	128	30.33
2	5	101	106	25.12
3	22	48	70	16.59
4	-	80	80	18.96
5	25	13	38	9.0
Total	135	287	422	100
%	31,99	68.01	100	

Table 1 shows that the textbook consists of 144 "problems" presented to serve as illustrative examples. Of the 144 examples 37 examples are "word problems" which is 25.69% of the total. Unit 2 and Unit 4 which are concerned with "Rational Numbers" and "Plane Geometry" do not present "word problems" as illustrative example.

Table 2. shows that the total number of "problems" presented as exercises is 422. Of these "problems" 135 "problems" (31.99%) are "word problems". Unit 4, which is about "Plane Geometry", does not provide "word problems" as exercise. The maximum number of "problems", which is 30.33% of the total number of "problems" presented in the textbook, is presented in Unit 1 as exercises. The minimum number of "problems" (9%) is presented in Unit 5 which is about Measurement of Area and Volume.

In order to investigate to what extent problem solving is incorporated in the contents of the textbook, the "word problems" are further classified as indicated below.

i. "Word problems" which provide "problem" situation for direct application of a formula or a procedure that have been already learned

Table 2. Distribution of "Problems" Presented as Exercises

Unit	"Word Problems"	Other Types of "Problems"	Total	%
1	83	45	128	30.33
2	5	101	106	25.12
3	22	48	70	16.59
4	-	80	80	18.96
5	25	13	38	9.0
Total	135	287	422	100
%	31,99	68.01	100	

Table 1 shows that the textbook consists of 144 "problems" presented to serve as illustrative examples. Of the 144 examples 37 examples are "word problems" which is 25.69% of the total. Unit 2 and Unit 4 which are concerned with "Rational Numbers" and "Plane Geometry" do not present "word problems" as illustrative example.

Table 2. shows that the total number of "problems" presented as exercises is 422. Of these "problems" 135 "problems" (31.99%) are "word problems". Unit 4, which is about "Plane Geometry", does not provide "word problems" as exercise. The maximum number of "problems", which is 30.33% of the total number of "problems" presented in the textbook, is presented in Unit 1 as exercises. The minimum number of "problems" (9%) is presented in Unit 5 which is about Measurement of Area and Volume.

In order to investigate to what extent problem solving is incorporated in the contents of the textbook, the "word problems" are further classified as indicated below.

i. "Word problems" which provide "problem" situation for direct application of a formula or a procedure that have been already learned

ii. Word problems which provide problem situation that require other arithmetic operation or solution strategy different from the formula or procedure already learned.

Table 3 and 4 present the distribution of these two types of "word problems" stated as illustrative examples and exercises respectively.

Table 3. Types Of "Word Problems" Presented As Examples.

Unit	"Problems" for Direct Application of a Formula or a Procedure	Problems That Require Additional Solution Strategy	Total	%
1	20	4	24	64.86
2	-	-	-	-
3	-	4	4	10.81
4	-	-	-	-
5	9	-	9	24.32
Total	29	8	37	100
%	78.38	21.62	100	

Table 3 shows the distribution of two different types of "word problem". As it can be seen, from among the total of 37 "word problems" 29 "word problems" (78.38%) are found to be "problems" which provide situations for direct application of a formula or a procedure. The rest of the problems which are found in Unit 1 and Unit 3 are word problems that require a solution strategy or strategies other than or in addition to the application of a formula. Such type of problems constitute 21.62% of the total. The writer believes that it is with this type of problems (as discussed in the first section of chapter – two) that true problem solving occurs since the solution strategy is not readily available. Therefore, with this understanding, it was found appropriate to check the presence or absence of heuristic approach by considering only these eight word problems.

Table 4. <u>Types Of "Word Problems" Presented As Exercises</u>

Unit	"Problems" for Direct Application of a Formula or a Procedure	Problems That Require Additional Solution Strategy	Total	%
1	61	22	83	61.48
2	4	1	5	3.70
3	3	19	22	16.30
4	-	-	-	-
5	15	10	25	18.52
Total	83	52	135	100
%	61.48	38.52	100	

Table 4 shows that there are 135 "word problems" that are presented as exercises in the textbook. Of these "problems" 38.52% constitute word problems that require additional arithmetic operation or solution strategy. The rest of the "word problems" are meant for direct application of a formula or a procedure only. It was from among these word problems (which constitute 38.52% of the total "problem") that the problems for classroom observation were selected with the belief that they can provide a situation for the problem solving to occur. As it can be seen from Table 4, Unit 4, which is about "Plane Geometry", does not have any type of word problem. Unit 2, which is about "Rational numbers", has a total of 5 "word problems" (3.70%) of which, only 1 word problem requires additional solution strategy. Of the 52 word problems of this type the maximum number (42.31%) is found in Unit 1 which is about "Ratio, Proportions and Percentages".

The other point to consider about the organization of the textbook is about the separate section given for teaching problem solving. In Unit 3, which deals with "Equations and Inequalities in One Variable", there is a section labeled "Solving Word Problems". In this section, model solution processes of four word problems were presented for illustrations. However, even if they are presented for teaching how to solve word problems to students, the solution processes of the four problems were not found performed consistently using the elements of heuristic approach (Refer to Table – 5 to get adequate information).

To sum up, in the textbook there are 138 "problems" presented as illustrative examples. Of these "problems" eight problems, which is 5.8% of the total, are word problems which provide problem situation for true problem solving. These problems are found in two units only --- four problems in Unit 1 and the other four problems in Unit 3. The textbook has also 422 "problems" which are presented as exercises. Of these "problems" 52 problems (12.32%) are word problems which provide problem situations for true problem solving. There is not any such type of word problem seen in Unit 4, Plane Geometry. In Unit 2, which is about Rational Number, there is only one word problem found where problem solving can occur. In the textbook there is a separate section labeled "Solving Word Problems" where model solutions are given for four problems inorder to teach problem solving. This implies that the rest of the contents of the textbook are not intended for incorporating the teaching of problem solving. In general, since problem solving is not integrated in most of the mathematical activities of the text book, it can be said that the textbook follows traditional approach. As it is discussed in chapter two section 2.3.3, problem solving should not be treated as an isolated topic. It rather should permeate in the entire mathematical activities. In other words, textbooks should incorporate non-routine, and multi-step problems in all mathematical topics for the purpose of teaching the process of solving problems.

4.1.2 *The Use of Heuristic Problem Solving Approach*

As mentioned earlier in this section, the checking of the proper application of heuristic problem solving approach was performed by considering eight word problems that are found in Unit 1 and Unit 3 presented as illustrative examples. These word problems are the only problems that are found in the textbook for true problem solving to be exercised. This is because, the solution strategies for these problems are not provided for the problem solver. As it is discussed in chapter two section 2.1.1, if a solution strategy, a formula or a procedure is readily available for a given situation, then it will be considered as an exercise and not as a problem. Table 5 reveals the result of the extent to which heuristic problem solving approach is applied in the textbook.

Table 5. Distribution Of The Presence Or Absence Of Elements Of Heuristic Problem Solving Approach.

Category	Subcategory	Unit -1				Unit - 3				Total			
		Yes		No		Yes		No		Yes		No	
		f	%	f	%	f	%	f	%	f	%	f	%
1. Understanding The Problem	1.1 The data is identified	-	-	4	100	3	75	1	25	3	375	5	62.5
	1.2 The unknown is identified	-	-	4	100	3	75	1	25	3	37.5	5	62.5
	1.3 The condition is Identified	1	25	3	75	1	25	3	75	2	25	6	75
2. Devising a plan	2.1 The Strategy is indicated	-	-	4	100	1	25	3	75	1	12.5	7	87.5
3. Looking Back	3.1 The result is checked	-	-	4	100	3	75	1	25	3	37.5	5	62.5
	3.2 the result is discussed	-	-	4	100	-	-	4	100	-		8	100
TOTAL		1	4.17	23	95.83	11	45.83	13	54.17	12	25	36	75

As table 5 shows, under the first category, i.e., "understanding the problem", the data and the unknown are identified three times each which is 37.5% of the total. These sub-categories are not treated in any one of the problems of Unit 1. The condition is identified twice (once in each unit) which is 25% of the total. The Second category, "devising a plan", is treated only once, i.e., the strategy used to solve the problem is indicated clearly for one problem only which is 12.5% of the total problems. This problem is found in Unit 3 . In the third category, "looking back", the results of three problems were checked which is 37.5% of the total problems. These problems are found in Unit 3. The other sub-category, which is about discussing the result and the method, is not treated at all in any one of the problems.

From the above discussion it can be understood that heuristic approach is not applied consistently in the solution processes of the problems. The data, the unknown, and the condition (which are known by the principal parts of the problem) are not identified for most of the problems. But, these principal parts help the students to understand the problem situation properly (in order to find a solution strategy) if they are identified and stated clearly and separately. The solution strategy is indicated clearly for one problem only. However, if the strategies are indicated in all problems students will be able to apply them if they are confronted with other similar or related problems. This enables them to develop their problem solving skills as intended. The third phase, "looking back", is a phase where the students consolidate the knowledge they acquired in the problem-solving process. In this phase, the results should be verified, alternative strategies should be pinpointed, the methods and the arguments should be discussed in relation to other similar or related problems. But, in the textbook these are not fulfilled.

In general, according to the evaluation mechanism devised, it can be concluded that 25%heuristic problem solving approach is applied in the `mathematics textbook of grade seven. But this is very far from adequate use of the approach.

4.2 Data Obtained Through Classroom Observation

In this section the data obtained through classroom observation (with the help of video tape recorder) is presented. Accordingly, in sub-sections 4.2.1 and 4.2.2, the answers to the research questions which ask about the proper use of heuristic problem solving approach and effective teaching strategies by the teachers observed are given respectively.

Even though it was intended to observe twelve teachers that are found in eight schools, while solving four word problems each, it was only possible to observe eleven teachers found in seven schools. This was because of unforeseen interruptions that had happened twice in the school's program. Therefore, eleven teachers were observed while solving four word problems each which makes the total number of problems to be 44. The teaching load of the teachers observed was found to be either 20 periods or 25 periods per week. That is, eight teachers, which is approximately 73%, have 20 periods and the rest three teachers have 25 periods. The number of students in a classroom ranges from 42 to 88 with an average number of approximately 72. The other characteristics of the observed teachers are indicated in tables 6-12.

Table 6. Distribution of Observed Teachers by School and Zone.

Zone	School	Sex		Total
		M	F	
3	Misrak Ber	-	2	2
	Wendirad	2	-	2
	Berhaneh Zare	1	-	1
4	Misrak Goh	1	1	2
	Tinsae Berhan	1	-	1
	Urael	1	-	1
	Minilik	2	-	2
	Total	8	3	11
	%	72.73	27.27	100

Table 6 shows that of the eleven teachers observed three were female and the other eight were male. Five teachers are found in Zone 3 and six teachers in Zone 4.

In the following tables (7-12) the symbols A, B, C, D, E, F, G are used to substitute for the names of the schools:1. Misrak Goh 2. Tinsae Berhan 3. Urael 4. Wendirad 5. Misrak Ber 6. Berhanh Zare 7. Minilik respectively.

Table 7. Distribution of Teachers by Age.

School	Year					
	below 36	36-40	41-45	46-50	51-55	Total
A	-	-	1	1	-	2
B	-	-	-	1	-	1
C	-	-	-	1	-	1
D	-	-	-	1	1	2
E	-	2	-	-	-	2
F	-	1	-	-	-	1
G	-	-	-	2	-	2
Total		3	1	6	1	11
%		27.27	9.09	54.55	9.09	100

Table 7 shows that the majority (54.55%) of the teachers are in the age range of 46-50 years. It can be also seen that there is not any teacher with age below 36 years.

Table 8. Teachers' Years of Service in Teaching Profession.

School	Year				Total
	Below 16	16-20	21-25	26-30	
A	-	-	1	1	2
B	-	-	-	1	1
C	-	-	-	1	1
D	-	1	-	1	2
E	-	1	1	-	2
F	-	-	1	-	1
G	-	-	1	1	2
Total		2	4	5	11
%		18.18	36.36	45.45	100

In table 8 the number of service years in teaching profession that the teachers have is presented. As it is shown there are only two teachers who make 18.18% of the total with number of years of teaching service between 16-20 years. The rest are above 20 years of teaching service.

Table 9. Teachers' Years of Service in Teaching Mathematics in Any Grade

School	1-5	6-10	11-15	16-20	21-25	26-30	Total
A	-	-	-	-	1	1	2
B	1	-	-	-	-	-	1
C	-	-	-	1	-	-	1
D	-	1	-	-	-	1	2
E	-	1	-	1	-	-	2
F	-	-	-	1	-	-	1
G	-	-	-	-	1	1	2
Total	1	2	-	3	2	3	11
%	9.09	18.18	-	27.27	18.18	27.27	100

As table 9. shows the number of teachers that taught mathematics for the years in the range of 16-20, 21-25 and 26-30 are 3, 2, 3 respectively. The number of these teachers add up to eight which is 72.73 of the total number of teachers observed. By referring back to the data on table – 8 it can safely be concluded that these teachers have spent most of their teaching experience in teaching mathematics.

Table 10. Teachers' Years of Service in Teaching Grade Seven Mathematics

School	Year						
	1-5	6-10	11-15	16-20	21-25	26-30	Total
A	1	-	1	-	-	-	2
B	1	-	-	-	-	-	1
C	-	1	-	-	-	-	1
D	1	-	1	-	-	-	2
E	1	-	1	-	-	-	2
F	1	-	-	-	-	-	1
G	-	1	-	-	1	-	2
Total	5	2	3	-	1	-	11
%	45.45	18.18	27.27	-	9.09	-	100

Regarding teaching mathematics in grade seven, as shown in table – 10, there is only one teacher who has taught for 21-25 years. There are five teachers (45.45%) who have least number of service, that is, between 1-5 years.

Table 11A. Teachers' Level of Education

School	Level			
	12+1	12+2	12+3	Total
A	1	1	-	2
B	-	1	-	1
C	-	1	-	1
D	2	-	-	2
E	-	2	-	2
F	-	-	1	1
G	2	-	-	2
Total	5	5	1	11
%	45.45	45.45	9.09	100

Table 11B. Teachers' Level of Education by Major and Minor Subjects.

Level	Subject			
	Major Maths	Minor Maths	Major or Minor other	Total
12+1	4	-	1	5
12+2	1	-	4	5
12+3	-	-	1	1
Total	5	-	6	11
%	45.45	-	54.55	100

Table – 11A shows the level of education of the observed teachers. As it can be seen there are equal number of teachers with level of education 12+1 and 12+2 where each level constitutes 45.45% of the total number of teachers. There is only one teacher with the level of education 12+3.

When table – 11B is observed, which indicates the type of subjects that the teachers majored or minored, there are only five teachers (45.45%) with mathematics as their major subject. The rest are with major or minor subject other than mathematics. Of the five teachers majoring in mathematics only one teacher is with level of education 12+2. According to the document of Standard for Kindergarten and Primary Schools Set by Ministry of Education (MOE, 1995), the teachers who are eligible to teach in the upper primary level (5-8) are those teachers with level of education 12+2. Therefore, from among the teachers observed 90.91% of the teachers are not eligible to teach mathematics in grade seven. This includes also the teachers majoring in subjects different from mathematics.

4.2.1 Result of The Proper Use of Heuristic Problem Solving Approach

As indicated in chapter three the data gathered are of two types, i.e., the proper use of heuristic approach by the teachers and the type and purpose of questions and answers that were asked and responded by both the teacher and the students while the teacher is solving word problems in classrooms. Since each of the two types is checked by two different observation schedules their results are presented in separate tables.

In the tables that follow, i.e. Table 12 , Table 13 and Table 15 the categories and sub-categories prepared for checking heuristic approach are represented by the numbers that correspond to the items as indicated below. The observed teachers are also represented by the symbols T_1, T_2, T_3, ...T_{11} in the tables 13 and 14 unless stated otherwise.

<u>Category 1.</u> Understanding the problem

1.1 gives meaning for key words

1.2 restates the problem in easier words

1.3 identifies the data (what is given)

1.4 identifies the unknown (what is needed)

1.5 identifies the condition to determine the unknown (i.e. looks for sufficient condition by finding relationships between the known and the unknown)

<u>Category 2</u> – Devising a plan

2.1 Indicates the use of one or more strategies as a means for finding the solution to the problem

<u>Category 3</u> – Carrying out the plan

3.1 Solves the problem by checking or proving the correctness of each step

<u>Category 4</u> – Looking back

4.1 checks the result

4.2 derives the result differently

4.3 discusses the argument and the result.

Table 12 presents the frequencies – occurrences of the proper use of heuristic elements while solving the problems according to the rating scale devised: proper, moderately proper, and not at all proper.

Table 12. Frequency of the Proper Use of Heuristic Approach in The Problems Solved by the Teachers.

Rating Scale									
Category	Sub Category	Proper		Moderately Proper		Not at All Proper		Total	
		f	%	f	%	f	%	f	%
1	1.1	14	33.33	21	50	7	16.67	42*	100
	1.2	-	-	22	50	22	50	44	100
	1.3	16	36.36	23	52.27	5	11.36	44	100
	1.4	2	4.55	33	75	9	20.45	44	100
	1.5	5	11.36	32	72.73	7	15.91	44	100
2	2.1	-	-		-	44	100	44	100
3	3.1	33	75	4	9.09	7	15.91	44	100
4	4.1	6	13.64	15	34.09	23	52.27	44	100
	4.2	-	-	-	-	44	100	44	100
	4.3	-	-	-	-	44	100	44	100
	TOTAL	76	17.35	150	34.25	212	48.40	438	100

* Checking for two problems was left because of vagueness.

Table 12 shows that of all the categories, category 3, "carrying out the plan", which deals with performing the calculation needed was best applied by the teachers. That is, 75% of the problems were solved properly by checking the correctness of each step till the final answer was obtained. On the other hand, Category-2, "devising a plan," which is about indicating the use of a strategy, and from Category-4, "looking back," two sub-categories,, 4.2 and 4.3 which are about deriving the result differently and discussing the argument and the result respectively were not treated at all. Especially, when the teachers devise a plan or select strategies in order to attack the problem, none of them was found to indicate them explicitly and express their purposes. The other one to consider is sub-category 1.2, which is about restating the problem in easier words. In this sub-category it can be seen that no student was given a chance to restate the problem in his own words. Moreover, 50% of the problems were restated by the teachers (moderately proper) and the rest of the problems were not restated at all.

As to sub-category 4.1, which is about checking the result obtained, it is found that more than half of the results of the problems (52.27%) were not checked at all. The rest of the problems were checked, but those that were checked by considering (revising) the condition that determined the relationship between the known and the unknown explicitly are 13.64%.

Table 13, presents the distribution of the data in terms of the values given for the rating scale, i.e., Proper – 2 , Moderately proper – 1 , and Not al all proper 0. The symbols T_1, T_2, T_3,.... T_{11} represent the individual teachers observed. The numbers under each symbol, corresponding to the heuristic items, represent the average value given to the observed teacher while solving the four problems.

Table 13. Distribution of The Use of Heuristic Approach by the Teachers in Terms of Values Given for The Rating Scale

Category	Sub category	Observed Teachers											Total	$\bar{X}$	%
		T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}			
1	1.1	1	1.5	1	1.25	1	0	1	1.75	2	1.25	0.5	12.25	1.11	55.5
	1.2	0.5	0.5	1	1	0	0	0.5	0.25	1	0.25	0.5	5.50	0.5	25.0
	1.3	1.75	1.5	0.75	0.5	1	1.75	1.75	1.25	1.25	1	1.25	13.75	1.25	62.5
	1.4	0.5	1	0	0.5	1	1	1	1	0.75	1.5	1	9.25	0.84	42
	1.5	1	1.25	0.25	0.25	1	1	1	1.25	1.25	1.25	1	10.50	0.95	47.5
2	2.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3.1	2	2	1.25	0	2	0.25	2	2	2	2	2	17.50	1.59	79.5
4	4.1	1.25	1.5	0.25	0	0	0	0.5	1	1.25	0.5	0.5	6.75	0.61	30.5
	4.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Total	8	9.25	4.50	3.50	6	4	7.75	8.50	9.50	7.75	6.75	75.50	6.86	
	$\bar{X}$	0.8	0.93	0.45	0.35	0.6	0.4	0.78	0.85	0.95	0.78	0.68	7.55	0.69	34.5

From table 13, it can be seen that there is not any teacher who used the heuristic approach properly or who was nearer to use it properly. There are only two teachers observed (T_2 and T_9) who are at least very near to moderately proper use of the approach with mean values 0.93 and 0.95 respectively. The teacher (T_9) with the highest mean score is also found to be consistent in giving the meanings of key words (subcategory 1.1) with mean value 2. Three teachers. T_3, T_4, and T_6 with mean values 0.45, 0.35, and 0.4 respectively are found to be nearer to not proper use of the approach. This number makes 27.27% of the total number of the teachers observed.

Regarding the data, the unknown, and the conditions (mostly known as the principal parts of a problem) which are sub-categories 1-3, 1-4, and 1-5 respectively, no teacher was observed consistently identifying them in an explicit manner. Most of the teachers were found to state these principal parts clearly and separately for some problems and not clearly and separately for others. the mean value indicates 1.25, 0.84, and 0.95 for identifying the data, the unknown, and the condition respectively which are considered to be nearer to moderately proper.

In general, by considering the overall mean value, which is 0.69, it can be said that the teachers observed use heuristic problem solving approach by far below moderately proper. In other words, 34.5% heuristic problems solving approach was exercised by the observed teachers.

4.2.2. <u>Result of the Type and Purpose of Questions and Responses</u>

The second type of data gathered through classroom observation is the type and purpose of questions and responses made by the teachers and the students. This was done as follows.

First, the type of questions and responses, i.e., memory/factual questions and thought questions were identified and tallied using the checklist prepared for the purpose. Second, the thought questions and responses were further classified according to the categories of the four-step heuristic approach they belong in order to determine for which purpose they were asked and responded.

The result of the above mentioned type and purpose of the questions and responses are presented in Table 14 and `15 respectively.

In the tables, the items of the questions and responses are represented by the symbols that correspond to them as indicated below.

1. Teacher talk

A. Initiation

A_1 - teacher asks memory and factual questions

A_2 – teacher asks thought questions

B. Response

B_1 – teacher gives response to memory and factual questions asked by students

B_2 – teacher gives response to thought questions asked by students

II. Student talk

C – Initiation

C_1 – student asks memory and factual questions

C_2 – student asks thought questions

D - Response

D_1 – student gives response to memory and factual questions

D_2 – student gives response to thought questions

Table 14. Frequency of the Type of Questions and Responses by the Teachers and Students.

Category	Subcategory		Observed Teachers											Total	%
			T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}		
I	A	A_1	10	10	13	12	16	10	14	14	16	13	11	139	64.65
		A_2	4	5	8	8	9	6	9	8	7	5	7	76	35.35
	Total		14	15	21	20	25	16	23	22	23	18	18	215	100
	B	B_1	-	-	-	-	-	-	-	-	-	-	-	-	-
		B_2	1	1	-	-	-	-	-	-	-	-	-	2	100
	Total		1	1	-	-	-	-	-	-	-	-	-	2	100
II	C	C_1	-	-	-	-	-	-	-	-	-	-	-	-	-
		C_2	1	1	-	-	-	-	-	-	-	-	-	2	100
		Total	1	1	-	-	-	-	-	-	-	-	-	2	100
	D	D_1	10	10	13	12	16	10	14	14	16	13	11	139	68.14
		D_2	4	4	8	7	8	5	9	6	4	4	6	65	31.86
		Total	14	14	21	19	24	15	23	20	20	17	17	204	100

As it can be seen from table 14 the total number of questions asked by the teachers while solving four word problems each (44 word problems altogether) is 215. This makes the average number of questions asked per word problems to be approximately five. Of the total questions asked 76 questions, which is 35.35%, were found to be thought-provoking questions The rest 139 questions were memory and factual questions. The average number of thought questions asked per word problem was approximately two. This shows that the teachers were more inclined to ask memory and factual questions than thought questions. According to 'Perrott (1982:53), asking more factual questions than thought questions encourages rote learning and has little effect in developing higher-order thinking ability.

As to the responses made by the students, the table shows that all memory and factual questions asked by the teachers have got responses from the students. However, from among the thought questions asked, eleven questions, which is 14.47% did not get responses from the students.

Regarding the questions asked by the students the table shows that only two thought questions were asked throughout. These questions have got also

Table 14. <u>Frequency of the Type of Questions and Responses by the Teachers and Students.</u>

Category	Subcategory		Observed Teachers											Total	%
			T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}		
I	A	A_1	10	10	13	12	16	10	14	14	16	13	11	139	64.65
		A_2	4	5	8	8	9	6	9	8	7	5	7	76	35.35
	Total		14	15	21	20	25	16	23	22	23	18	18	215	100
	B	B_1	-	-	-	-	-	-	-	-	-	-	-	-	-
		B_2	1	1	-	-	-	-	-	-	-	-	-	2	100
	Total		1	1	-	-	-	-	-	-	-	-	-	2	100
II	C	C_1	-	-	-	-	-	-	-	-	-	-	-	-	-
		C_2	1	1	-	-	-	-	-	-	-	-	-	2	100
		Total	1	1	-	-	-	-	-	-	-	-	-	2	100
	D	D_1	10	10	13	12	16	10	14	14	16	13	11	139	68.14
		D_2	4	4	8	7	8	5	9	6	4	4	6	65	31.86
		Total	14	14	21	19	24	15	23	20	20	17	17	204	100

As it can be seen from table 14 the total number of questions asked by the teachers while solving four word problems each (44 word problems altogether) is 215. This makes the average number of questions asked per word problems to be approximately five. Of the total questions asked 76 questions, which is 35.35%, were found to be thought-provoking questions The rest 139 questions were memory and factual questions. The average number of thought questions asked per word problem was approximately two. This shows that the teachers were more inclined to ask memory and factual questions than thought questions. According to 'Perrott (1982:53), asking more factual questions than thought questions encourages rote learning and has little effect in developing higher-order thinking ability.

As to the responses made by the students, the table shows that all memory and factual questions asked by the teachers have got responses from the students. However, from among the thought questions asked, eleven questions, which is 14.47% did not get responses from the students.

Regarding the questions asked by the students the table shows that only two thought questions were asked throughout. These questions have got also responses from the part of the teachers. From this it can be said that the students almost did not show any initiation to ask questions which is quite the opposite of the responsive act they showed when they were provoked or initiated by their teachers.

Table 15. Frequency Distribution of Thought Questions and Responses According to The Four-Step Problem Solving Approach.

Questions and Responses	Categories of Heuristic Approach																			
	1						2		3		4								Total	
	1.3		1.4		1.5		2.1		3.1		4.1		4.2		4.3					
	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%		
A_2	17	22.37	12	15.79	-	-	-	-	39	51.32	8	10.53	-	-	-	-	76	100		
B2	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	2	100		
C2	-	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	2	100		
D2	14	21.54	8	12.31					39	60	4	6.15	-	-	-	-	65	100		

Table 15 presents the frequency distribution of the thought questions and responses made by the teachers and the students according to the four-step heuristic problem solving approach: Understanding the problem, devising a plan, carrying out the plan and looking back.

Table 15. reveals that no thought question was asked about 1.5 identifying the condition, 2.1- indicating a strategy, 4.2 deriving the result differently and 4.3 discussing the argument and the result. As it is indicated in table 12., which is concerned with the proper use of heuristic approach, except identifying the condition the rest of these sub-categories also were not treated at all. As to the condition of the problem, even if the presence of sufficient conditions (by relating the data with the unknown) to solve the problems were determined for some of the problems, they were done by the teachers themselves without allowing the students to participate.

Regarding the other principal parts of a problem, 1.3-the data and 1.4 the unknown, 17 questions (22.37%) were asked for the purpose of identifying what is given (the data) and 12 questions (15.79%) for the purpose of identifying what is required (the unknown). In Category 4, "looking back", eight questions (10.53%) were asked for the purpose of 4.1-checking the result. Concerning

Category-3, more than half of the total questions asked (51.32%) were asked while performing the mathematics needed according to the strategy devised. Most of the questions asked in this phase of the problem solving process were about how to solve the equation already set up or how to perform the calculation needed (using different arithmetic operations) to get the final answer.

In order to get some idea about the appropriate use of the thought questions by the teachers observed, one can consider, for instance, the frequency of the questions asked for the purpose of identifying the data and compare it with the total number of problems performed. Although only one thought question can not be made to correspond with one word problem (since more than one question could be asked for the purpose of one sub-category, say, the data) a teacher is expected to ask, as discussed in chapter two section 2.3.4, at least one question about each of most of the sub-categories indicated in the four-step process. That is, the teacher should ask what the data, the unknown, and the condition is (understanding the problem), what strategy is needed (devising a plan); how to proceed in performing the mathematics needed (carrying out the plan); how the result is checked and questions that ask to apply other strategy or provoke to discuss the argument and the result (looking back). Therefore, with this understanding, if we consider the number of questions asked for identifying the data, 17 questions were asked out of the expected 44 questions which is 38.64%. This is the highest proportion that could be about the data since there is a possibility for asking more than one question per problem concerning the data. By the same way 12 questions (27.27%) about the unknown, 39 questions (88.64%) about carrying out the plan, and eight questions (18.18%) about checking the result were asked. Because the writer feels that it is very awkward to tell the appropriate use of thought questions by calculating the average, it is preferred to leave the individual data as it appears and suggest that: the teachers did not use the thought questions in sufficient number and for appropriate purposes. Relatively more questions were asked for the purpose of performing the calculation. This implies that the teachers are more interested in the computational work than the other solution processes which are believed to be more helpful for the development of problem solving skills. This will be discussed more in section 4.3. Regarding the questions asked by the students, both questions were found to be concerned with performing the calculation needed. Perhaps this could have happened because of the interest they have more in getting the final answer than the process as their teachers have.

4.3 Data Obtained Through Questionnaire

In this section the data obtained through questionnaire is presented. The questionnaire was constructed with the intention of getting elaborate information about the two type of data which were gathered through the observation schedules No. 1 & 2 . They were data about the proper application of heuristic approach and the purpose of questions asked by the teachers while solving problems in classrooms (see Appendices B, C, and E)

The questionnaire was administered to 35 teachers distributed in 21 schools that are found in Zone 3 and Zone 4 (see Appendix A). All copies of the questionnaire were filled and returned. According to the data gathered from the respondents, it was found that six respondents were female, which is 17.14% of the total, and the rest 29 respondents were male. The average teaching load of the respondents is found to be 22.86. This indicates that almost half of the respondents have teaching load of 20 periods and the other half 25 periods per week. The number of students in a classroom they teach ranges from 35-88 with the average number of approximately 68. The other characteristics of the respondents are indi cated in tables 16-21.

Table 16. Distribution of Respondents by Age

Sex	Year						Total	%
	Below 30	30-35	36-40	41-45	46-50	51-55		
M		2	1	4	20	2	29	82.86
F	-	-	2	4	-	-	6	17.14
Total	-	2	3	8	20	2	35	100
%	-	5.71	8.57	22.86	57.14	5.71	100	

Table 16. shows that the majority (57.14%) of the respondents are in the age range of 46-50 years. 5.71% of the teachers are in the age range of 30-35 and the other 5.71% in the age range of 51-55. It can be seen that there is not any one teacher below 30 years of age.

Table 17. Respondents' Years of Service in the Teaching Profession.

Sex	Year							Total	%
	1-5	6-10	11-15	16-20	21-25	26-30	Above 30		
M	-	1	2	1	6	17	2	29	82.86
F	-	-	-	1	4	1	-	6	17.14
Total	-	1	2	2	10	18	2	35	100
%	-	2.86	5.71	5.71	28.57	51.43	5.71	100	

In table 17. it can be seen that most of the teachers have teaching service more than 20 years. The majority of them (51.43%) have served for 26-30 years. There is only one teacher (2.86%) who has years of service in teaching for 6-10 years. There is not any teacher who has years of teaching service below 6 years. This shows that most of the respondents are rich in experience.

Table 18. Respondents' Years of Service in Teaching Mathematics In Any Grade.

Sex	Year							Total	%
	1-5	6-10	11-15	16-20	21-25	26-30	Above 30		
M	1	7	6	5	7	3	-	29	82.86
F	1	2	1	2	-	-	-	6	17.14
Total	2	9	7	7	7	3	-	35	100
%	5.71	25.71	20	20	20	8.57	-	100	

Table 18. shows that there are only two teachers (5.71%) who taught mathematics in different grades for 1-5 years. On the other hand there are three teachers (8.57%) who taught mathematics for 26-30 years. These teachers seem to be spending all of their teaching service years in teaching

mathematics only. The majority of the teachers (68.57%) have taught mathematics for 11-30 years.

Table 19. Respondents' Years of Service in Teaching Grade Seven Mathematics.

Sex	Year							Total	%
	1-5	6-10	11-15	16-20	21-25	26-30	Above 30		
M	12	11	4	1	1	-	-	29	82.86
F	5	-	1	-	-	-	-	6	17.14
Total	17	11	5	1	1	-	-	35	100
%	48.57	31.43	14.29	2.86	2.86	-	-	100	

As shown in table 19 almost one half of the teachers (48.57%) have least number of service years in teaching mathematics in grade seven. That is, between 1-5 years.

Table 20 Respondents' Level of Education.

Sex	Level				Total	%
	TTI	12+1	12+2	12+3		
M	1	12	13	3	29	82.86
F	-	2	4	-	6	17.14
Total	1	14	17	3	35	100
%	2.86	40	48.57	8.57	100	

Table 21. Respondents' Level of Education by Major and Minor Subjects.

	Maths Major		Maths Minor		Majoring Other Subject		No Major Subject		Total	
Level	No	%	No	%	NO	%	No	%	NO	%
TTI	-	-	-	-	-	-	1	2.86	1	2.86
12+1	12	34.29	-	-	2	5.71	-	-	14	40
12+2	9	25.71	-	-	8	22.86	-	-	17	48.57
12+3	-	-	-	-	3	8.57	-	-	3	8.57
Total	21	60	-	-	13	37.14	1	2.86	35	100

Table 20 shows that there is only one teacher (2.86%) with a certificate, i.e., from Teacher Training Institute. There are 3 teachers (8.57%) with level of education, 12+3. Almost one half of the teachers (48.87%) are with level of education, 12+2 and 40% with 12+1. However, when table 21 is observed, which indicates about the type of subjects that the teachers majored or minored in, there are 12 teachers (34.29%) with level of education, 12+1 and majoring in mathematics and 9 teachers (25.71%) with level of education, 12+2 and majoring in mathematics. The rest of the teachers (40%) were not found to be either majoring or minoring in mathematics. From this it can be concluded that those teachers who are eligible to teach, according to the standard set by MOE, mathematics in grade seven are nine in number which is 25.71% of the total number of teachers. These teachers are with level of education, 12+2 and majoring in mathematics.

Other than the items which ask about the bio-data of the respondents, there are two items (item No.27 and 29) which ask them to indicate if they have participated in any training program which is concerned with the teaching of the development of problem-solving skills and if they refer to the teacher's guide and apply according to the suggestions given for teaching respectively. For item No.27 the respondents indicated that they have not participated in any training program which is concerned with the teaching of the development of problem-solving skills. As to item No.29 all of them responded "Yes". That is, they apply according to the suggestions given for teaching in the teacher's guide which was prepared in 1997 by Region-14 Education Bureau. In the teacher's guide, it was stated that the teachers should be able to see the students learn: 1) analyzing the text (which is similar to understanding the problem), 2) working out the method of solution (devising a plan), and 3) performing the solution with checking (carrying out the plan and looking back) (p.2). It was also mentioned that these steps should be used continuously (p.109). From this it can be said that the teacher's guide recommended the application of a kind of heuristic problem solving approach, even though it is not in sufficient detail.

Sub-section4.3.1 refers to the result of the data gathered on the proper application of heuristic problem-solving approach while teaching problem-solving in classroom.

4.3.1 Data on the Application of Heuristic Approach

In the questionnaire the items that refer to the heuristic approach are classified by item numbers 9,10,11,12,13,14,17,18,21,22,23 and 25. These items are made to correspond to the sub-categories indicated in observation schedule No. 1 with the exception of sub-categories 1.2 and 2.1 (see Appendices B and E) . Each of these sub-categories was expressed using two items (item No. 10 and 11 and item No.17 and 18 respectively) in the questionnaire in order to get appropriate information. These item numbers are also made to serve as codes of the items stated in the questionnaire (for tables 22, 23, 24 and 25) unless indicated otherwise.

Table 22, 23, 24 and 25 refer to the data about the four categories of heuristic approach. Namely, understanding the problem, devising a plan, carrying out the plan, and looking back respectively. In the tables, the frequency distribution of the responses made by the respondents for each item, according to the rating scale and their respective values (always -4, frequently --3, sometimes – 2, rarely – 1 and never – 0), is presented. In the tables the sum total is indicated by multiplying each number of frequency by their respective values. That is, Total = sum of Vxf. The mean value is also indicated by dividing the sum of the product of the number of frequencies and their respective values by the total number of respondents. That is,

$$\bar{X} = \frac{\Sigma V \times f}{N}, \text{ where } \bar{X} = \text{mean},$$

Σ = Sum of, v = value, f= frequency, and N = number of respondents.

Table 22. Frequency Distribution of The Responses Given to Understanding the Problem.

Item No.	Rating Scale with Values										Total ΣVxf	$\bar{X} = \frac{\Sigma V \times f}{N}$
	Always -4		Frequently-3		Sometimes-2		Rarely -1		Never-0			
	f	%	f	%	f	%	f	%	f	%		
9	21	60	7	20	4	11.43	3	8.57	-	-	116	3.31
10	17	48.57	9	25.71	6	17.14	3	8.57	-	-	110	3.14
11	9	25.71	5	14.29	15	4286	3	8.57	3	8.57	84	2.40
12	19	54.29	12	34.29	3	8.57	1	2.86	-	-	119	3.40
13	24	68.57	5	14.29	4	11.43	2	5.71	-	-	121	3.46
14	17	48.57	7	20	8	22.86	3	8.57	-	-	108	3.09
Total	107	50.95	45	21.43	40	19.05	15	7.14	3	1.43	658	3.13

Table 22. shows that item no. 9, which is about giving the meaning of difficult words, 60% of the respondents give the meaning of difficult words always. Whereas the mean value (3.31) is closer to frequently giving the meanings. This item was made to substitute for sub-category 1.1. in the observation schedule No 1 which is about giving the meaning of key terms. The change was made because of the confusion that arose in the try-out process. Item No 10 refers to restating the problem in easier words. The responses given for this item indicated that 48.57% of the respondents always restate the problem in easier words. Whereas 17.14% of the respondents sometimes and 8.57% of the respondents rarely restate the problem in easier words. Concerning item No. 11 which is about asking students to restate the problem in their own words, most of the respondents indicated that they either sometimes (42.86%) or rarely (8.57%) or never (8.57%) ask students to restate the problem. The respondents who responded either "sometimes " or "rarely" or "never" (for item nos. 9-11) were asked to give their reasons. Accordingly, two respondents gave their reasons about item Nos. 9 and 10 by saying that they are not supposed to give the meaning of difficult words and/or restate the problems in easier words. Eventhough the number of respondents who gave this reason is only two, most of mathematics and science teachers, as reported by Herber (1970) and Devine (1989), regard the task of teaching reading (reading with comprehension) the sole responsibility of language teachers.

However, because of this wrong belief, many students are blocked right in the beginning from following or actively participating in the process of solving problems, let alone trying to solve the problem independently. Therefore, teachers are expected to make sure first, whether the students have understood the problem before they proceed to the next step.

Regarding item No. 11, thirteen respondents alleged that the students are unable to express their ideas in English. These teachers seem to believe that because the medium of instruction is English they expected their students to restate the problem in English always. But, as long as English is a foreign language for the students, and having in mind that the language is made the medium of instruction just starting from the particular grade they are in, the writer believes that the students should be allowed at least sometimes to restate the problem in Amharic. This is because, since the main objective of the lesson is to enable students to solve problems, their learning should not have to be blocked by the problem of language. Therefore, it seems preferable to encourage the students to express their ideas using both languages side by side until they are ready to express their ideas in English only.

Regarding item No.12, which is about identifying the data and item No 13, which is about identifying the unknown, the majority of the respondents (54.29% and 68.57% respectively) indicated that they identify them explicitly always. The mean values for identifying the data is 3.40 and for identifying the unknown 3.46 which are very near to each other. Concerning item No.14 also, which refers to identifying the condition, a good number of respondents (48.57%) indicated that they always identify the condition of the problem by changing the structure. The mean value of this item is 3.09. Those respondents who filled "sometimes" or "rarely" for these principal parts of a problem (the data, the unknown, and the condition) did not give their reasons for not identifying them explicitly always or frequently. However, as Polya (1990: 6-7) noted, the teacher must always explicitly identify what relevant information is given, what is required to find and show whether there is sufficient condition to satisfy the unknown by forming relationship between the data and the unknown.

In general, although the mean value of the first category (understanding the problem) is 3.13, which indicates almost frequent application of the heuristic element, it can not be taken as sufficient since this phase of the problem solving process is considered to be fundamental. This is because, unless a student is assisted to understand the problem thoroughly from the beginning, it cannot be said that the student is engaged in meaningful learning for the next successive phases of the problem solving process.

Table 23. Frequency Distribution of The Responses Given to Devising a Plan

Item No	Rating Scale with Values										Total	$\bar{X} = \frac{\Sigma V \times f}{N}$
	Always -4		Frequently-3		Sometimes-2		Rarely-1		Never-0		Total	
	f	%	f	%	f	%	f	%	f	%	Σv x f	
17	22	62.86	9	25.71	3	8.57	1	2.86	-	-	122	3.49
18	3	8.57	6	17.14	7	20	18	51.43	1	2.86	62	1.77
Total	25	35.71	15	21.43	10	14.29	19	27.14	1	1.43	184	2.63

Table 23. shows that item No 17 which asks about the selection of strategies, 62.86% of the respondents indicated that when they select strategies they always tell their purposes clearly to the students. However, this response totally contradicts with that of the data obtained through observation. According to this data (see table 12), it was found that no teacher applied the strategies by indicating explicitly and expressing their purpose. But, for item No. 18, which asks about indicating the usefulness of the strategies selected, the majority of the respondents (51.43%) replied that they rarely indicate the usefulness of the strategies for other related or similar problems. As it is discussed in chapter two section 2.3.2 this phase is the most critical phase in the process of problem solving. Since the main objective of teaching problem solving is to enable students develop and apply strategies to solve problems (NCTM; 1989: 24), they should get the help of their teachers in order to gain the experience on how to select strategies, discuss why particular strategies are applied so that they can be independent problem solvers. Also, the development of problem solving skill will not be possible unless the students are engaged in the process of exploring strategies and are made to be aware of the usefulness of the strategies selected to other related problems. According to the data obtained

through classroom observation, no teacher was found to perform adequately this critical phase of problem solving. This data was somehow supported by the responses given particularly for item No. 18 of the questionnaire. The respondents replied that they either sometimes (20%) or rarely (51.43%) or never (2.86%) (which add up to 74.29%) indicate the usefulness of strategies selected for other related problems. No respondent was found to give reason for this.

Table 24. Frequency Distribution of the Responses Given to Carrying out The Plan.

Item No	Rating Scale with Values										Total $\sum v \times f$	$\bar{X} = \frac{\sum V \times f}{N}$
	Always -4		Frequently-3		Sometimes-2		Rarely-1		Never-0			
	f	%	f	%	f	%	f	%	f	%		
21	24	68.57	7	20	4	11.43	-	-	-	-	125	3.57

Item No. 21, in table-24, is an item which asks whether the teacher checks the correctness of the steps or not while doing the computational work. As it can be seen, the response for this item has got the highest value – 3.57. There are only 11.43% of the respondents who sometimes check the steps. The rest responded that they either always (68.57%) or frequently (20%) check the correctness of the steps while performing the calculation needed. The data obtained through classroom observation also revealed (according to the rating scale devised) that for most of the problems (75%) the teachers performed the computational work by checking the correctness of each step till the final answer was obtained (see table 12.). This seems to be encouraging even though there are some who overlook the importance of convincing the students by checking the correctness of the steps using formal rules or intuitive insight. If the students are not convinced or learned this way they will rely on their teacher to get the final answer always. Therefore, to make them independent learners they should see that the final answer is obtained by logical reasoning.

through classroom observation, no teacher was found to perform adequately this critical phase of problem solving. This data was somehow supported by the responses given particularly for item No. 18 of the questionnaire. The respondents replied that they either sometimes (20%) or rarely (51.43%) or never (2.86%) (which add up to 74.29%) indicate the usefulness of strategies selected for other related problems. No respondent was found to give reason for this.

Table 24. Frequency Distribution of the Responses Given to Carrying out The Plan.

Item No	Rating Scale with Values										Total	$\bar{X} = \frac{\Sigma V \times f}{N}$
	Always -4		Frequently-3		Sometimes-2		Rarely-1		Never-0			
	f	%	f	%	f	%	f	%	f	%	$\Sigma v \times f$	
21	24	68.57	7	20	4	11.43	-	-	-	-	125	3.57

Item No. 21, in table-24, is an item which asks whether the teacher checks the correctness of the steps or not while doing the computational work. As it can be seen, the response for this item has got the highest value – 3.57. There are only 11.43% of the respondents who sometimes check the steps. The rest responded that they either always (68.57%) or frequently (20%) check the correctness of the steps while performing the calculation needed. The data obtained through classroom observation also revealed (according to the rating scale devised) that for most of the problems (75%) the teachers performed the computational work by checking the correctness of each step till the final answer was obtained (see table 12.). This seems to be encouraging even though there are some who overlook the importance of convincing the students by checking the correctness of the steps using formal rules or intuitive insight. If the students are not convinced or learned this way they will rely on their teacher to get the final answer always. Therefore, to make them independent learners they should see that the final answer is obtained by logical reasoning.

Table 25 . Frequency Distribution of The Responses Given to Looking Back.

Item No	Rating Scale with Values										Total	$\bar{X} = \frac{\Sigma V \times f}{N}$
	Always -4		Frequently-3		Sometimes-2		Rarely -1		Never-0			
	f	%	f	%	f	%	f	%	f	%	Σv x f	
22	20	57.14	11	31.43	4	11.43	-	-	-	-	121	3.46
23	7	20	8	22.86	5	14.29	12	34.29	3	8.57	74	2.11
25	2	5.71	4	11.43	5	14.29	13	37.14	11	31.43	43	1.23
Total	29	27.62	23	21.90	14	13.33	25	23.81	14	13,33	238	2.27

Table – 25, shows that for item No. 22, which refers to checking the results, 57.14% of the respondents indicated that they always check the result in the original problem. But, according to the data gathered through observation (refer to table 12) the result of only 13.64% of the problems was found to be checked properly in the original problem, where as, 52.27% of the problems were not checked at all.

Concerning item No 23 which refers to deriving the result differently, 20% of the respondents indicated that they always derive the result using different strategies. There are only 8.57 % of the respondents who never use different strategies. The mean value of this item is 2.11, which is nearer to "sometimes". As to item No 25, which is about discussing the method and the result, 31.43% of the teachers responded that they never discussed the method. The mean value of this item is 1.23 which is nearer to "rarely". According to the data obtained through classroom observation (refer to table-12), no teacher was found to derive the result differently or discuss the method and the result.

As it can be understood from the reports given above, there are lots of discrepancies between the data obtained through the questionnaire and classroom observation. The writer has also found that some of the items were filled by the observed teachers as if they are performed always, whereas, they were not performed at all during the observation sessions. The reason for these discrepancies to occur may be due to:

1. vagueness of the items which may not be understood by the respondents the way they are understood by the writer,

2. the nature of the items that directly ask the efficiency of the respondents, which as a result, leads the respondents to cover their weakness and safeguard their job security, and

3. negligence in filling the questionnaire, i.e. respond to the items without reading them properly and patiently.

Regarding the responses given to item Nos. 22,23,and 25; the respondents were made to give their reasons if their responses were not "always" or "frequently". Accordingly, four respondents gave their reasons for the three items that there is no sufficient time for doing them "always" or "frequently". This shows that the teachers are more interested to cover the content, or in this context, to get the final answer than the solution process. This was also reported by Kroulik and Rudnick (1987:58) that too often teachers in the interest of time tend to turn away from a problem after an answer has been found in order to move on the next problem. However, because of this students will miss the chance of consolidating the knowledge they acquire in the solution process. In this phase of the solution process students can be made to develop their ability to solve problems by giving them chance to reconsider or reexamine the result and the argument that led to the solution, explore alternative strategies, and find relationship with the solution process of prior problems.

In general, according to the rating scale devised, when the overall mean is computed it gives 2.87. From this it can be concluded that as the respondents indicated they apply heuristic problem solving approach almost frequently. In other words, 71.75% heuristic problem solving approach is exercised by the respondents.

4.3.2 Data On Asking Questions

In the questionnaire the items that refer to the purpose of questions are item Nos. 15, 19, 24, and 25 (see Appendix E). These item numbers also serve as codes of the items stated in the questionnaire unless indicated otherwise. Items that require responses about carrying-out the plan and checking the result were cancelled from the questionnaire after the try-out process because of the ambiguity and confusion they created. Table-26 refers to the data that indicate how often the teachers ask questions about the heuristic elements.

Table 26. Frequency Distribution of The Questions Asked According to the Heuristic Elements.

Item No		Rating Scale with Values										Total	$\bar{X} = \frac{\Sigma V \times f}{N}$
		Always -4		Frequently-3		Sometimes-2		Rarely-1		Never-0		Total	
		f	%	f	%	f	%	f	%	f	%	Σv x f	
15	A	18	51.43	11	31.43	4	11.43	2	5.71	-	-	115	3.29
	B	13	37.14	13	37.14	7	20	2	5.71	-	-	107	3.06
	C	6	17.14	7	20	14	40	3	8.57	5	14.29	76	2.17
19		4	11.43	8	22.86	15	4286	6	17.14	2	5.71	76	2.17
24		4	11.43	3	8.57	7	20	12	34.29	9	25.71	51	1.46
25		2	5.71	4	11.43	5	14.29	13	37.14	11	31.43	43	1.23
Total		47	22.38	46	21.90	52	24.76	38	18.10	27	12.86	468	2.23

In table 26, item No. 15 A-C deals with questions asked about the principal parts of a problem, i.e., 15A-the data, 15B-theunknown, and 15C- the condition. The table shows that 51.43% of the respondents ask questions about the data The mean value of this item is 3.29. And also, 37.14% of the respondents ask questions always and the other 37.14% of them ask frequently questions about the unknown. The mean value of this item is 3.06. The mean values of both items indicate that the respondents almost frequently ask questions about the data and the unknown. According to the data gathered through observation, it was found that questions were asked for 38.64% of the problems solved about the data and 27.27% of the problems about the unknown (see page 60). Regarding the condition, item No. 15C, as the table shows the mean value is 2.17 which implies that the respondents sometimes ask questions about the presence of sufficient condition in order to solve the problem under consideration. But, in the classrooms observed no teacher was found asking questions about the condition of a problem. In any of the cases, the teachers were not found to ask questions about the principal parts of a problem as often as it should have been. As Polya (1990:1-2) noted teachers should make sure whether their students have understood the problem or not by asking the same type of questions always for every problem under consideration. The questions are: What are the data? What is required? What is the condition? Find the relationship between what is given and what is required, and the like. In fact, the words may vary from problem to problem and be asked in many different ways. However, as the result of the data gathered through both instruments (observation and questionnaire) revealed, there are many instances where the students get answers to which they donot know what the problem is about.

Item No 19, which is about asking students to select a strategy, has got a response with mean value 2.17 This implies that the respondents sometimes ask students or give them chance to suggest a strategy. This was not supported by the data gathered through observation. All the teachers observed were not found giving chance to students for indicating a strategy. The respondents did not give any reason for not asking questions.

Item No. 24 and 25 refer to the last phase of problem solving-"looking back". Item No 24 is about asking students to suggest other alternative strategies and derive the result obtained differently. Regarding this item, most of the respondents indicated either rarely (34.29%) or never (25.71%) ask students to suggest alternative strategies. Concerning item No. 25 also, which is about asking "what if ...?" questions in order to discuss the argument and the result, most of the respondents indicated that they either "rarely" (37.14%) or "never" (31.43%) give chance to students to discuss the argument and the result by asking questions like: what if the data is changed to ...? what if the unknown is changed to ...? what if the condition is changed to...? Both of these items (item No 24 and 25) were not found treated by any one of the observed teachers. As table 26. reveals the overall mean value is 2.23 which implies that the respondents sometimes ask thoughtful questions for the purpose of the heuristic elements indicated. The only reason given by the respondents is the reason which is mentioned in section 4, 3.1, i.e., shortage of time to cover the content.

In general, whatever the case may be, if the teachers rush to get the final answer and after getting the answer turn immediately to the next problem and repeat the same procedure again and again, the endeavour to develop the problem solving skill of their students will be in vain. To develop the problem solving skills of the students teachers should take time and allow them to explore strategies, verify results, interpret solutions, examine alternative strategies, and reflect their views back to the solution process.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

The main objective of the study was to investigate the extent to which heuristic problem solving approach is used by grade seven mathematics teachers while teaching problem solving to students. In due course of the study it was intended to get answers for the following research questions.

1. Is problem solving incorporated in every mathematical activities of grade seven mathematics textbook?
2. Is heuristic problem solving approach applied properly in the textbook?
3. Do grade seven mathematics teachers apply heuristic problem solving approach while teaching problem solving to students?
4. Do the teachers use effective teaching strategies appropriately while teaching problem solving to students?

In order to get the appropriate data three data gathering instruments were used: content analysis, observation and questionnaire. The sources of data were grade seven mathematics textbook and selected grade seven teachers of mathematics. That is, eleven teachers for observation and 35 teachers (including the eleven teachers) for questionnaire administration. Accordingly, the following results were obtained in accordance with the research questions.

5.1.1 In the textbook, problem solving was not found incorporated in most of the mathematical topics. It was found that:

a) of 138 "problems" presented as illustrative examples eight problems (5.8%) and

b) of 422 "problems" presented for exercises 52 problems (12.32%) were found to be problems which provide situations for problem solving to occur.

These problems are not also distributed in every topic of the textbook. All the problems which are meant for illustrative examples are found in Unit-1 (50%) and Unit-3 (50%) only. Of the 52 problems presented as exercises most of the problems are found in Unit-1 (42-31%) and Unit-3 (36.54%) with the total of 78.85%. In addition to this, there is a separate section labeled "Solving Word Problems" which is found in unit three, where model examples are given for teaching how to solve word problems.

5.1.2 In the textbook, by considering those word problems that are presented for illustrative examples and which provide situation for problem solving to occur, the steps taken towards their solution were analyzed using an evaluation checklist composed of the elements of the heuristic problem-solving approach. As a result, it was found that 25% heuristic approach is exercised in the textbook.

5.1.3 Regarding the use of heuristic approach by the teachers while teaching problem solving in classrooms, the data gathered through classroom observation revealed that, according to the rating scale devised, the mean value was found to be 0.69, which is to mean, 34.5% heuristic problem solving approach was exercised by the observed teachers. According to the data gathered through questionnaire administration the mean value was found to be 2.87. That is, the teachers responded that they exercise the proper use of heuristic approach almost frequently. This implies that 71.75% heuristic problem solving approach is exercised by the respondents.

5.1.4 As to the proper use of appropriate teaching strategy while teaching problem solving, the data gathered through classroom observation showed that the observed teachers used more memory and factual questions than thought questions. Of the total questions asked 35.35% was found to be thought questions. Moreover, the greater number of these thought questions (51.32%) were asked for the purpose of facilitating the computational skills of the students in order to get the final answer. On the other hand, no thought question was asked for the purpose of most of the other phases of the problem solving process, i.e., identifying the condition, indicating a solution strategy, deriving the result of a problem differently, and discussing the result and the method. The result of the data obtained through questionnaire showed that the respondents sometimes (with mean value 2.23) ask thoughtful questions in line with the heuristic elements indicated while teaching problem solving. Two items (carrying out the plan and checking the result) were not included for the purpose of asking questions in the questionnaire because of ambiguity.

5.2 Conclusion

Problem solving is considered to be a process which provides a context for meaningful learning of concepts and skills. It requires higher-level thought process since it calls for relating and rearranging the past knowledge already acquired with the new information available in order to make sense of it . It is believed that if the problem-solving skill of individuals is developed they will successfully deal with the world characterized by rapid change and technological advancement. It is also believed that the development of problem-solving skill will be effective if individuals are taught problem solving with the regular use of heuristic problem-solving approach and appropriate teaching strategies.

As it is indicated in chapter one the New Education and Training Policy (E.T.P, 1994) stated that the development of individual's and society's problem-solving capacity should be strengthened by giving the necessary education for all. In line with this, the grade seven mathematics syllabus which was prepared by the Institute for Curriculum Development and Research (ICDR) and the teacher's guide which was prepared by Region 14 Education Bureau, both indicated that problem solving should be taught to students using premeditated approach. In the teachers guide it was further mentioned about the regular use of certain steps (which are almost similar with that of Polya's four-step problem-solving approach) by the teachers while teaching problem solving to students.

Based on the aforementioned information, the study tried to uncover the extent to which problem solving is taught in grade seven mathematics classroom using heuristic problem-solving approach. Accordingly, it was found that:

5.2.1 In the grade seven textbook problem solving is not found to be integrated in most of the topics. The textbook should have incorporated problems that are non-routine and multi-step, i.e., problems that provide situations for problem solving to occur in every mathematical content. Instead, a separate section is given for teaching how to solve problems. The steps taken towards the solution for the problems solved in the textbook do not also show the consistent application of the elements of the set of heuristic problem-solving approach. It was found that 25% heuristic approach is applied in the textbook which is considered to be very low. Because of these reasons it can be concluded that the textbook follows what is known as traditional approach.

5.2.2 Regarding the proper use of heuristic approach in classrooms, although there are lots of discrepancies realized between the result of the data gathered through the questionnaire and that of classroom observation, there is an indication of the teachers being more interested in getting the final answer than teaching the solution process while solving problems. As the results from the two data gathering instruments revealed, the highest score recorded was for the item which asks about the proper use of the computational work or performing the mathematics needed by checking each step. This helps, of course, the students to realize that the final answer is obtained by logical reasoning and not by chance or some kind of miracle. On the other hand, the respective items which ask about: 1) giving the chance to students to restate the problems in their own words, 2) the use of the solution strategies by indicating their usefulness to other similar or related problems, 3) deriving the result differently, and 4) discussing the argument and the result; got very low scores. However, without making sure whether the students have understood the problem situation or not, merely showing how to perform the calculation needed perfectly can be considered as giving the answer to which the students do not know what the problem is about. Even if it is assumed that this condition is fulfilled (understanding the problem), unless the students are always told explicitly the use of one or more strategies for the problem at hand and for other related problems, their ability to solve problems independently will remain under question. Moreover, if the students are not allowed to explore alternative solution strategies, reexamine the result and the argument, and find relationship with the solution process of prior problems, the knowledge they acquired in the solution process can not be consolidated. As a result, the development of their problem-solving skill will not be actualized.

Therefore, from the foregoing discussion it can be said that the teachers do not seem to have the proper knowledge of how problem-solving skill of an individual can be developed, even though they responded that they teach according to the suggestions given in the teacher's guide. This is because of the absence of either pre-service or in-service training for the teachers. Worse than this, it was found that 40% of the sample teachers teach mathematics without majoring or minoring in the subject. This makes the teaching and learning of problem solving in mathematics classes more difficult. In general, since it was found that the teachers are more inclined to giving emphasis to the computational work (in order to get a correct answer) than teaching the other solution processes, it can be concluded that the teachers are following the traditional approach.

5.2.3 As to the use of appropriate teaching strategy, since problem solving requires higher-level thought process it is expected that teachers ask more thought questions than memory and factual questions in order to get thoughtful responses. As noted by Good and Brophy (1987) and Perrott (1982), memory and factual questions are most of the time needed to make sure if students have basic information before posing thought-provoking questions. But, giving more emphasis to asking memory and factual questions is believed to be encouraging rote learning which has little effect on developing higher-level thinking process.

The result of the data obtained through observation showed that the teachers ask more factual and memory questions than thought questions. In addition to this, according to the result obtained through both the questionnaire and observation, the teachers were not found to use the thought questions properly and in sufficient number for the purpose of teaching the greater part of the solution process. It can be said that the teachers' use of appropriate teaching strategy was very low. This seems also it is because the teachers lack proper understanding of the importance of asking thought-provoking questions while teaching problem solving.

5.3 Recommendations

Based on the findings presented in this study, the following recommendations are provided so that the necessary adjustments will be made by the curriculum developers, teachers, and other concerned scholars and officials in the field.

5.3.1 To make problem solving an essential component of mathematics curriculum:

5.3.1.1 Non-routine and multi-step problems that provide contexts for problem solving to occur should be incorporated in every topic (content) of the textbook side by side with the other routine problems presented for practicing formulas and procedures. For example, a mathematical activity might involve problem solving and use rational numbers, geometry, probability, statistics, proportions, and so on.

5.3.1.2 The textbook should use heuristic problem-solving approach regularly while solving problems that are presented for illustrative examples so that students can develop the habit of applying it.

5.3.1.3 The teacher's guide should provide the solution of every problem that are presented in the textbook by applying the heuristic approach in order to assist teachers with some technical matters. In addition to this, the teacher's guide should present guidelines for the teachers to follow by emphasizing the importance of the regular use of the approach.

5.3.2 Teachers should get the necessary knowledge and skills of using heuristic problem-solving approach with the appropriate teaching strategies that enable them help to develop the problem-solving skills of their students. This needs to be made possible through a course in problem solving to be offered in either pre-service or in-service training program.

BIBLIOGRAPHY

Best, J.W. and Kahn, J.V. (1989). Research in Education(6^{th} ed). New Jersey: Prentice-Hall, Inc.

Billstein, R. et.al., (1981). A Problem Solving Approach to Mathematics for Elementary Teachers. Menlopark: The Benjamin/ Commings Publishing Company.

Borich, G.D. (1988). Effective Teaching Methods. New York Macmillan Publishing Company.

Brown, Sve. "Integrating Manipulatives and Computers in Problem Solving Experiences." Arithmetic Teacher. Vol. 38. (Oct. 1990): p.8.

Callahan, J.F. and Clark, L.H. (1988). Teaching in the Middle and Secondary Schools: Planning for competence (3^{rd}.ed.). New York: Macmillan Publishing Company.

Campbell, P.F. and Bamberger, H.J. "The Vision of Problem Solving in the Standards." Arithmetic Teacher. Vol. 37. (May 1990): pp. 14-15.

Clark, L.H. and Starr, I.S. (1986). Secondary and Middle School Teaching Methods (5^{th} ed.) New York: Macmillan Publishing Company.

Copple, C. et.al., (1984). Educating the Young Thinker: Classroom Strategies for Cognitive Growth. Hillsdale: Lawerence Elbaum Associates, Inc.

Cruickshank, D.R., et.al., (1995). The Act of Teaching. New York: McGraw-Hill, Inc.

Cyert, R.M. (1980). Problem Solving and Educational Policy. In Tuma, D.T. and Reif, F. (ed.), Problem Solving and Education: Issues in Teaching and Research. New York: Lawerence E. Associates, Publishers.

De Vault M.V. "Doing Mathematics Is Problem Solving." Arithmetic Teacher. Vol.28. (April 1981): pp. 40-43.

Devine, T.G. (1989). Teaching Reading in the Elementary School: From Theory to Practice. Boston: Allyn and Bacon, Inc.

Gage, N.L. and Berliner, D.C. (1992). Educational Psychology (5th ed.). Boston: Houghton Mifflin Company.

Giordano, G. "Heuristic Strategies: An Aid for Solving Verbal Mathematical Problem." Journal of Emotional and Behavioral Disorders. Vol. 1. (Jan. 1993): pp 1-72.

Good, C.V. (ed.) (1973). Dictionary of Education (3rd. ed.). New York: Mac Graw-Hill, Inc.

Good, T.L. and Brophy, J.E. (1990). Educational Psychoiogy: A Realistic Approach (4th ed.). New York: Longman.

________________________ (1987). Looking in Classrooms (5th ed.). New York: Harper and Row Publishers, Inc.

Herber, H.L. (1970). Teaching Reading in Content Areas. New Jersey: Prentice-Hall, Inc.

Higgins, K.M. "An Investigation of the Effects on Students' Attitudes, Beliefs, and Abilities in Problem Solving and Mathematics After One Year of a Systematic Approach to the Learning of Problem Solving." Paper Presented at the Annual Meeting of the American Educational Research Association. Atlanta, GA, April 12-16, 1993.

Houston, J.P. (1986). Fundamentals of Learning and Memory (3rd ed.). New York: Harcourt Brace Jovanovich, Inc.

Hussen, T. and Postlethwaite, T.N. (editors). (1985). The International Encyclopedia of Education. (Vol.7). Oxford: Pergamon Press.

Institute for Curriculum Development and Research. Syllabus: Grade 7. Addis Ababa: Jan., 1996.

Krulik, S. and Rudnick, J.A. (1987). Problem Solving: A Hand Book for Teachers (2nd.ed). Boston: Allyn and Bacon, Inc.

Ministry of Education. Education and Training Policy. Addis Ababa. April, 1994.

Morris, R. (ed.) (1989). Studies in Mathematics Education. (Vol.7). UNESCO.

Musser, G.L. and Burger, W.F. (1988). Mathematics for Elementary Teachers: A Contemporary Approach. New York: Macmillan Publishing Company.

National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School. Reston Virginia: NCTM.

Ornstein, A.C. (1995). Strategies for Effective Teaching (2nd ed.). Madison: Wm.C. Brown Communications, Inc.

Perrott, Elizabeth. (1982). Effective Teaching: A Practical Guide to Improving Your Teaching. London: Longman, Inc.

Pimm, D and Love, E. (ed.) 1991). Teaching and Learning School Mathematics. London: Hodder and Stoughton.

Polya, G. (1990). How To Solve It (2nd.ed.). London: Pengvin Books.

Region 14 Education Bureau. Mathematics: Grade 7 Student Text. Addis Ababa: Sept., 1990 E.C.

________________________. Mathematics: Grade 7 Teacher's Guide. Addis Ababa: Nov., 1997.

Schoenfeld, A.H. (1985). Mathematical Problem Solving. San Diego: Academic Press, Inc.

______________________, (Editor). (1987). Cognitive Science and Mathematics Education. Hillsdale: Lawerence Erlbaum Associates.

Servas, W and Varga, T. (1971). Teaching School Mathematics: A Unesco Source Book. UNESCO.

Thompson, C.S. and Rathmell, E.C. "NCTM's Standards for School. Mathematics, k-12". Arithmetic Teacher. Vol.35. (May 1988) p.17.

Tuma, D.T. and Reif, F.(Editors). (1980). Problem Solving and Education: Issues in Teaching and Research. New York: Kawerence Erlbaum Associates, Inc.

Wheeler, R.E. (1984). Modern Mathematics: An Elementary Approach (6th ed.). Monterey: Brooks / Cole Publishing Company.

የአፀደ ሕፃናት ፕሮግራምና የመጀመሪያ ደረጃ ትምህርት ስታንዳርድ፡፡ ትምህርት ሚኒስቴር፡፡ (ያልታተመ)፡፡ ሚያዝያ 1987፡፡

Appendix - A

List of Junior Secondary Schools with the number of 7th grade sections and the number of Mathematics teachers (1998-1999).

Zone 1

No	Name of the School	Number of Sections	Number of Teachers
1.	Edget Besira	8	2
2.	Balcha Aba Nefso	14	3
3.	Tesfa Kokeb	10	2
4.	Umer Semetir	9	2
5.	Yekatit 23	18	4
	Total	**59**	**13**

Zone - 2

1.	Keranio Medhanealem	5	1
2.	Mekdela	6	2
3.	Mekanissa AKababi	4	1
4.	Boslios	6	2
5.	Selam Ber	7	2
6.	Biruh Tesfa	4	1
7.	Repi	4	1
8	Edget Behibret	10	2
9	Woriha Yekatit	5	1
10.	Lake Adgeh	6	2
11.	Tebay Maremia	1	1
12.	Addis Ababa Wehni	2	1
	Total	**60**	**17**

No	Name of the School	Number of Sections	Number of Teachers
		Zone - 3	
1.	Misrak Ber**	12	3
2	Birhane Zare**	4	1
3.	Bole Gerji*	6	2
4.	Temenja Yaje	4	1
5.	Sibiste Negasi*	8	2
6.	Megabit - 28*	4	1
7.	Wendirad**	10	3
8.	Kara Ilo	3	1
9.	Alpha Liyou (A school of students with hearing impairment)		
	Total	**51**	**14**
		Zone - 4	
1.	Ethiopia Tikdem*	9	2
2.	Urael**	10	2
3.	Entoto Amba*	8	2
4.	kuskuam Taitu Bitul*	6	2
5.	Dil Betigil	4	1
6.	Tsehai Chora*	6	2
7.	Miazia - 23*	8	2
8.	Hibret Fire*	7	2
9.	Biherawi Betemengist	3	1
10.	Minilik II**	9	2
11.	Kebena*	3	1
12.	Misrak Goh**	8	2
13.	Kokebe Tsibah*	10	2
14.	Kelemwork*	10	2
15.	Tinsae Berhan**	4	1

** Schools where the observed teachers and who filled the questionnaire are found.

* Schools where the teachers who filled the questionnaire are found

No	Name of the School	Number of Sections	Number of Teachers
16.	Berhan Guzo*	5	1
17.	Mekane Hiwot	7	2
18.	Arbegnoch*	7	2
	Total	**124**	**31**

Zone - 5

1.	Medhanealem	7	2
2.	Kechene Debraselam	5	1
3.	Belay Zeleke	5	1
4.	Meskerem -2	4	1
5.	Kolfe	10	2
6.	Fitawrari Habte Giorgis	9	2
Total		**40**	**9**

Zone - 6

1.	Fitawrari Abayneh	5	1
2.	Akaki Mengist	9	2
3.	Kaliti	5	1
4.	Akaki Cherkacherk	6	2
Total		**25**	**6**
Grand Total		**359**	**90**

Total Number of Schools is 54.

Appendix - B

Classroom Observation Schedule No.1

School ______________________________

Grade __________ Section __________________

Name of teacher observed __________________

Class size ______________________________

Observer's name __________________________

Date of Observation ______________________

Observation session 1st, 2nd (circle one)

This observation schedule is designed to check the proper use of heuristic problem solving approach while teaching problem solving in grade 7 mathematics classroom. The schedule has four categories and ten sub-categories as indicated below.

Category 1- Understanding the problem

1.1 gives meaning for key words

1.2 restates the problem in easier words

1.3 identifies the data (what is given)

1.4 identifies the unknown (what is needed)

1.5 identifies the condition to determine the unknown (i.e., looks for sufficient condition by finding relationships between the known and the unknown)

Category 2 - Divising a plan

2.1 Indicates the use of one or more strategies as a means for finding a solution to the problem.

Category 3 - Carrying out the plan

3.1 Solves the problem by checking or proving the correctness of each step.

Category 4 - Looking back

4.1 checks the result

4.2 derives the result differently

4.3 discusses the argument and the result.

Direction for the Use of the Schedule

Checking using the above check-list will be made possible only when the lesson involves solving word problems. For each sub-category a 3- point rating scale will be used: Proper, Moderately Proper, Not At All Proper with values 2,1,0 respectively so that the observer can check the degree of occurrences according to the suggestions given for rating.

Note

If occurrences are found to be vague or if activities that correspond to a particular category can not be performed, because of the nature of the problem or any other known reason, that particular category will be left unrated.

Suggestions for Checking and Illustrative Examples

The following suggestions are intended to serve as guidelines for giving values according to the rating scale devised. For certain categories examples are given to minimize confusion.

Category 1 - Understanding the problem.

1.1 gives meaning for key words.

Category 3 - Carrying out the plan

3.1 Solves the problem by checking or proving the correctness of each step.

Category 4 - Looking back

4.1 checks the result

4.2 derives the result differently

4.3 discusses the argument and the result.

Direction for the Use of the Schedule

Checking using the above check-list will be made possible only when the lesson involves solving word problems. For each sub-category a 3- point rating scale will be used: Proper, Moderately Proper, Not At All Proper with values 2,1,0 respectively so that the observer can check the degree of occurrences according to the suggestions given for rating.

Note

If occurrences are found to be vague or if activities that correspond to a particular category can not be performed, because of the nature of the problem or any other known reason, that particular category will be left unrated.

Suggestions for Checking and Illustrative Examples

The following suggestions are intended to serve as guidelines for giving values according to the rating scale devised. For certain categories examples are given to minimize confusion.

Category 1 - Understanding the problem.

1.1 gives meaning for key words.

Example - 1

Kebede weighs 50 kilograms. His sister Sara weighs 40 kilograms Getachew weighs 30 kilograms more than Sara. What is the average weight of all the three persons?

Here the key words are "more than" and "average"

a) If all the key words are identified and their meanings are given-2

b) if the meanings of only some of the key words are given -1

c) if no key word is identified and given any meaning - 0

1.2 restates the problem in easier words

a) if the problem is restated in students' own words - 2

b) if the problem is restated but not in students' own words - 1

c) if the problem is not restated in one of the above forms - 0

1.3 identifies the data

a) if the data or what is given is identified explicitly and stated clearly -2

b) if it is identified but not stated clearly and separately -1

c) if the problem is solved without identifying clearly and explicitly the data from the unknown - 0

1.4 identifies the unknown

a) if the unknown or what is needed is identified explicitly and stated clearly -2

b) if it is identified but not stated clearly and separately - 1

c) if the problem is solved without identifying clearly and explicitly the unknown from the data - 0

1.5 identifies the condition

a) if the presence of sufficient conditions to satisfy the unknown is determined by restating the problem (changing the structure) to find relationships between the data and the unknown explicitly - 2

b) if the condition is determined without restating the problem explicitly - 1

c) if the problem is solved without identifying the condition to satisfy the unknown - 0

Example - 2

Problem

In a class there are 72 students. If the number of boys is twice the number of girls how many boys and how many girls are there in the class?

Solution

Understanding the problem

the key word here is "twice"

The data - Total number of students is 72

The number of boys is two times the number of girls.

The unknown - to find the number of boys and the number of girls.

The Condition

If we add the number of girls and the number of boys we can get the total number of students.

i.e., number of girls + number of boys = number of students (the condition is sufficient to determine the unknown).

Devise a plan

1. Use a variable - strategy

   let x be the number of girls then the number of boys is equal to 2x

2. Set up an equation - strategy

   $x + 2x = 72$

   $3x = 72$

Carry out the Plan

Solving the equation.

$3x = 72$

$\frac{1}{3} \times 3x = 72 \times \frac{1}{3}$

$x = 24$

The number of girls is 24.

The number of boys is $2 \times 24 = 48$

Looking back

- Check the result

  We have said that if we add the number of boys and the number of girls we get the total number of students.

  $24 + 48 = 72$ is correct

- derive the result differently

  What if we let x to be the number of boys

  Then the number of girls will be $\frac{1}{2}x$

  $x + \frac{1}{2}x = 72$

  ⋮

- discuss the method and the result

Ask what if ... questions

Examples

- What if the No. of girls equals to the No. of boys.
- What if the No. of girls is three times the No. of boys etc...

Category 2. Devising a Plan

2.1 Uses one or more strategies

Note

A strategy is an artful means to an end (Musser & Burger, 1987). Strategies include: look for a pattern, make a chart or a table, guess and test, form an equation, use a model, use a formula, use an algorithm, draw a diagram, etc.

a) If one or more strategies are indicated explicitly by relating their application with other familiar or similar problems solved before, that are already known to the students, or expressing their usefulness in finding the solution for the type of such problem at hand - 2

b) if the strategies are suggested explicitly and applied without trying to indicate their usefullness to other similar problems - 1.

c) if the strategies are applied without indicating explicitly and expressing their purpose - 0

Example - 3

The perimeter of a rectangle is 120cm. the length of the rectangle is 10cm greater than its width. Find the length and the width of the rectangle.

Devise a pan -- use strategies

1) draw a model (diagram)
2) label the unknown using a variable
3) use a formula
4) form an equation

Category - 3 Carrying out the plan

3.1 Solves the problem by checking or proving the correctness of each step.

a) While applying the strategy selected if the problem is solved by checking or proving the correctness of each step using intuitive insight or formal rules explicitly - 2

b) Solves as above but by checking or proving some of the steps only - 1

c) Solves without checking or proving the steps - 0

Category - 4 Looking back

4.1 Checks the result

a) checks the correctness of the result by considering (revising) the condition that determined the relationship between the known and the unknown explicitly - 2

b) checks the result without explicitly considering the relationship between the known and the unknown - 1

c) does not check the result - 0

Example. Refer to example 2

4.2 Derives the result differently

a) derives the result in some other way(s) and compares them with the previous one explicitly -2

b) derives the result differently but does not compare them with the previous one - 1

c) does not attempt to derive the result differently - 0

Refer to example 2 and also Ex. 4 below.

Example - 4

A farmer has some pigs and some chicken. He finds that together they have 70 heads and 200 legs. How many pigs and how many chicken does he have?

The above problem can be solved algebraically using two equations in two variables or by a series of successive approximations. Or still, we can use the idea of a one-to-one correspondence - All chicken stand on one leg, all pigs stand on hind legs. Thus the farmer will see 70 heads and 100 legs. The extra 30 legs must belong to the pigs, since the chicken have one leg per head. Thus there are 30 pigs and 40 chicken.

4.3 Discusses the argument and the result.

- a) if the method and the solution are interpreted or considered from various sides, or if relationships are examined with some other related problems or formerly acquired knowledge and at least one example is performed -2
- b) if the above is fulfilled without performing an example -1
- c) does not discuss the argument or the result at all - 0

Appendix C

Classroom Observation Schedule No.2.

School ____

Grade __________________ Section ___________________

Name of teacher observed _____________________________

Class size ____

Observer's name ___________________________________

Date of observation _________________________________

Observation session 1st, 2nd (Circle one).

This observation schedule has two parts. Part I serves for checking the type of questions and responses made by both the teacher and students. Part II serves for classifying the thought questions asked by the teachers according to the four step problem solving approach.

PART - I

This part of the observation schedule is designed to check the type of questions and responses made by both the teacher and students in teaching and learning the process of problem solving in grade 7 mathematics classrooms. The schedule has two main categories - teacher talk and student talk. Each category is further subdivided into initiation and response as indicated below.

I. **Teacher Talk**

A. Initiation

A1- teacher asks memory and factual questions

A2 - teacher asks thought questions

B. Response

B1 - teacher gives responses to memory & factual questions asked by students.

B2 - teacher gives responses to thought questions asked by students.

II. Student talk

C. Initiation

C_1 - student asks memory and factual questions

C_2 - student asks thought questions.

D. Response

D_1 - student gives response to memory & factual questions

D_2 - student gives response to thought questions.

Direction for the Use of the Schedule

Checking will be made possible when the lesson is solving word problems only. The observer puts a mark in the appropriate place when utterance is heard belonging to the particular category.

Putting marks according to the occurrences against the respective categories begins when the process of solving a word problem starts, and ends when the process is over. By the same way the process of solving successive word problems will be treated separately.

Note

Memory and factual questions are considered to be questions that require to remember specific facts or information which were previously taught or which are general knowledge. The questions focus on memory and recall of information that do not test understanding or problem-solving skills.

Example.

"what is the definition of ....?"

"what is the formula for ...?

"What is the sum, difference, product or quotient of numbers?"

Thought questions (thought provoking) are considered to be questions that require thoughtful responses by changing the form of information given in order to compare, contrast, explain, summarize, analyze, synthesize or evaluate. They are questions that make students to search and discover the answer or give ideas generated by creative thinking.

Example

Questions such as : "What would happen if ....?"
"How do you solve ...?"
"How do you construct ...?"
"What other solution strategy do you suggest?" etc...

PART II

This checklist is designed to classify the thought questions according to the four-step heuristic problem-solving approach indicated below.

1. Understanding the problem.
   - 1.1 identifying the data
   - 1.2 identifying the unknown
   - 1.3 identifying the condition.

2. Division a plan
   - 2.1 indicating a solution strategy.

3. Carrying out the plan.
    3.1 performing the computational work in order to get the final answer.

4. Looking back.
    4.1 Checking the result.
    4.2 Deriving the result differently.
    4.3 Discussing the method and the result.

Direction for Use

The observer first identifies the thought question. Then, he puts a mark against the respective category and subcategory it belongs so as to classify it.

Appendix - D

Textbook Evaluation Checklist

This checklist is designed to check the application of heuristic approach in the process of solving word problems in grade 7 mathematics textbook. The checklist has 3 categories and 6 sub-categories as indicated below.

Category 1. Understanding the problem

1.1 the data is identified and stated clearly & separately.

1.2 the unknown is identified and stated clearly & separately.

1.3 Sufficient condition to determine the unknown is examined by restating the problem (changing the structure) and forming relationships between the known & the unknown.

Category 2. Devising a plan.

2.1 the use of one or more strategies is indicated clearly as a means for solving the problem.

Category 3. Looking back.

3.1 the result is checked by considering the condition to determine the unknown.

3.2 the method and the result are interpreted or considered from various sides and examined its relationship with formerly acquired knowledge.

Note

The checklist serves only for those word problems already solved for illustrative purposes that are found in every unit of the textbook. A mark will be put in the appropriate category if there is an element of the set of heuristic approach which is applied in the process of finding the solution to the problem under consideration.

Appendix - E

Addis Ababa University
School of Graduate Studies
Faculty of Education
Department of Curriculum & Instruction

Questionnaire to be filled by selected Grade 7 Mathematics teachers of Addis Ababa Region.

Objective: The purpose of this questionnaire is to gather appropriate information as to how Grade 7 mathematics teachers teach problem solving to students. Since the reliability of the information depends on the objectivity of your responses you are kindly requested to be as frank and as honest as possible.

Direction:

1. Writing name is not necessary.
2. Please mark a (✓) in the appropriate box that corresponds to your choice (always, frequently, sometimes, rarely, never) or write the information needed in brief whenever necessary.
3. Please read the additional information provided on a separate sheet of paper attached that may help you understand better some of the items of the questionnaire.
4. If you may not understand an item please leave the item unchecked.

Note:

Throughout the questionnaire "problem-solving" refers to the process of solving "word problems" only.

1. Sex _____ 2. Age ____ 3. Level of Education ____ Major __ Minor.
4. Years of service in teaching ________
5. Years of service teaching mathematics in any grade ______
6. Years of service teaching mathematics in Grade 7 _______
7. Total Teaching load per week at present _________
8. Average number of students in classes you are teaching at present _____

When you teach problem solving to your students in the class:

	Always	Frequently	Sometimes	Rarely	Never
9. Do you give the meanings of difficult words?					
10. Do you restate the problem in easier words (in students words)?					
11. Do you ask students to restate the problem in their own words?					
12. Do you identify the data (what is given) explicitly and write it separately and clearly on the blackboard?					
13. Do you identify the unknown (what is needed) explicitly and write it separately and clearly?					
14. Do you identify the condition? i.e., do you show clearly whether there is sufficient condition to satisfy the unknown by restating the problem (changing the structure)? See example on the separate sheet of paper.					
15. Do you ask students (give them chance) to identify					
a) the data?					
b) the unknown?					
c) the condition?					

16. If your answer for one or more of the questions stated 9-15 is either "sometimes" or "rarely" or "never" please give your reasons for not doing them "always" or "frequently" at the back of the paper in short. (You can use Amharic to respond if you prefer).

Before trying to solve the problem at hand you select strategy or strategies-which are the means in which you convert or reduce the word problem to solvable computational stage. Example: setting up an equation, drawing a diagram, looking for a pattern, using a formula, making a chart or a table, and so on.

	Always	Frequently	Sometimes	Rarely	Never
17 When you select the strategies do you indicate their purposes clearly to the students?					
18. Do you indicate their use-fullness for other related or similar problems?					
19. Do you ask students (give them chance) to select strategies by themselves.					

20. Please, give your reasons at the back if your answer is not "always" or "frequently" for one or more of the questions 17-19.

	Always	Frequently	Sometimes	Rarely	Never
21. When you do the computational work, according to the strategy selected, do you take time and check the correctness of each step in order to convince the students?					
22. After you have found the solution to the problem do you check the result in the original problem?					
23. Do you derive the result differently using other alternate strategy or strategies?					
24. Do you ask students (give them chance) to suggest other strategies and derive the result differently?					

	Always	Frequently	Sometimes	Rarely	Never
25. Do you discuss the method and the result by asking students one or more of the following questions? What if the data is changed to ...? What if the unknown is changed to ...? What if the condition is changed to...?					

26. Please give your reasons at the back if your answer is not "always" or "frequently" for one or more of the questions 21-25.

27. Have you participated in any training program (regular program, summer program, workshop, seminar or other) which is concerned with the teaching of the development of problem-solving skills? Yes ___ No ___

    If "yes", please specify the type of program you participated in, when and where you participated.

    ____

    ____

28. Have you participated in any one of the training programs above for the purpose of acquainting yourself with the new mathematics content, teaching method or evaluation mechanism? Yes ___ No ___

    If "Yes", please specify the type of program, when and where you participated.

    ____

    ____

29. Do you refer to the teacher's guide and apply according to the suggestions given for teaching? Yes ______ No ____

30. If "No", please give your reasons ______________________________

    ____

Problem

In a class there are 72 students. If the number of boys is twice the number of girls how many boys and how many girls are there in the class?

Solution

Understanding the problem

the key word here is "twice"

The data - Total number of students is 72

The number of boys is two times the number of girls.

The unknown - To find the number of boys and the number of girls.

The condition:

If we add the number of girls and the number of boys we can get the total number of students.

i.e., number of girls + number of boys = number of students (the condition is sufficient to determine the unknown).

Devise a plan

1. Use a variable - strategy.

   Let x be the number of girls then the number of boys is equal to 2x.

2. Set up an equation - strategy

$$x + 2x = 72$$

$$3x = 72$$

Carry out the plan

Solve the equation.

$$3x = 72$$

$$\frac{1}{3} \times 3x = 72 \times \frac{1}{3}$$

$$x = 24$$

The number of girls is 24.

The number of boys is 2 x 24 = 48.

Looking back.

- Check the result.

  We have said that if we add the number of boys and the number of girls we get the total number of students.

  24 + 48 = 72 is correct

- Derive the result differently

  What if we let x to be the number of boys?

  The number of girls will be 1/2x

- Discuss the method and the result

  Ask what if ... questions.

Example - What if the no. of girls equals to the no. of boys?

- What if the no. of girls is three times the no. of boys? etc.,

Appendix F

George Polya's Four-step Heuristic Problem-solving Approach

HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.
You have to understand the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? O r is it insufficient? Or redundant? Or contradictory?
Draw a figure. Introduce suitable notation.
Separate the various parts of the condition. Can you write them down?

DIVISING A PLAN

Second.
Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.

Have you seen it before? Or have you seen the same problem in slightly different form?
Do you know a related problem? Do you know a theorem that could be useful?
Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.
Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
Could you restate the problem? Could you restate it still differently? Go back to definitions.

CARRYING OUT THE PLAN

Third.
Carry out you plan.

Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.
Examine the solution obtained.

Can you check the result? Can you check the argument?
Can you derive the result differently? Can you see it at a glance?
Can you use the result, or the method, for some other problem?

source: Polya (1990)

Appendix G

Krulik and Rudnick's Set of Heuristics for Solving Problems.

1. Read
    a) Note the key words
    b) Describe the problem setting and visualize the action.
    c) Restate the problem in your own words.
    d) What is being asked for?
    e) What information is given?

2. Explore
    a) Organize the information.
    b) Is there enough information.
    c) Is there too much information.
    d) Draw a diagram or construct a model.
    e) Make a chart or a table.

3. Select a strategy.
    a) Pattern recognition.
    b) Working backwards.
    c) Guess and test.
    d) Simulation or experimentation.
    e) Reduction/solve a simpler problem.
    f) Organized listing / exhaustive listing.
    g) Logical deduction
    h) Divide and conquer.

4. Solve
    a) Carry through your strategy.
    b) Use computational skills.
    c) Use geometric skills.

d) Use Algebraic skills.

f) Use elementary logic.

5. Review and extend.

   a) Verify your answer

   b) Look for interesting variations on the original problem.

   c) Ask "What if ..." questions.

   d) Discuss the solution.

Source: Krulik and Rudnick (1987)

Appendix H

A Modified Version of Polya's Model.

1. Understanding the problem.
    - 1.1 gives meaning for key words.
    - 1.2 restates the problem in easier words.
    - 1.3 identifies the data (what is given).
    - 1.4 identifies the unknown (what is needed).
    - 1.5 identifies the condition to determine the unknown (i.e., looks for sufficient condition by finding relationships between the known and the unknown).

2. Devising a plan
    - 2.1 indicates the use of one or more strategies as a means for finding the solution to the problem.

3. Carrying out the plan.
    - 3.1 Solving the problem by checking or proving the correctness of each step.

4. Looking back
    - 4.1 Checks the result
    - 4.2 derives the result differently
    - 4.3 discusses the argument and the result.

Appendix I

List of Some Strategies That are Applicable for Solving Problems

1. Guess and test.
2. Use a variable.
3. Look for a pattern.
4. Make a list.
5. Solve a simpler problem.
6. Draw a picture.
7. Draw a diagram.
8. Use direct reasoning.
9. Use indirect reasoning.
10. Use properties of numbers.
11. Solve an equivalent problem.
12. Work backward.
13. Use cases.
14. Look for a formula.
15. Form an equation.
16. Do a simulation.
17. Use a model.
18. Use dimensional analysis.
19. Identify sub-goals.
20. Use coordinates.
21. Use symmetry.

Source: Musser and Burger (1988)

Appendix - J

Examples of One-step and Multi-step Problem.

Example:

Bekele bought a soft drink for birr 1.50 and a piece of bread for birr 0.30. How much did he spend all together?

This is an example of a one-step problem. The student merely adds birr 1.50 and birr 0.30 to obtain the answer birr 1.80. However, this can be made into a two-step problem as follows:

Example:

Bekele bought a soft drink for birr 1.50 and a piece of bread for birr 0.30. How much change did he receive from a ten birr note.

Just as before, the student must find out first how much Bekele spent all together. Then he must subtract this sum from birr ten in order to find the amount of the change Bekele receives.

The following problem represent a three-step problem.

Example

Bekele had a ten birr note. After buying a soft drink for birr 1.50 and a piece of bread for birr 0.30, he gave the change he received for two of his sisters. How much is the share of one of his sisters.

DECLARATION

I, the undersigned, hereby declare that this thesis is my original work done under the guidance of Ato Nardos Abebe. All sources of materials used for the thesis have been duly acknowledged.

Name: Metasebia Demissie

Signature: ____________

Place : Addis Ababa

Date: June, 1999.

This thesis has been submitted for examination with my approval as university advisor.

Nardos Abebe (Ato)

____