ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIROMENTAL ENGINEERING

ASSESSING CODE BASED SHEAR PREDICTION VALUES OF REINFORCED CONCRETE BEAMS USING MODIFIED COMPRESSION FIELD THEORY

A thesis submitted to the School of Graduate Studies of Addis Ababa University in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering (Structural Engineering)

By

Surafel Tsegaye

December, 2016

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DECLARATION

I, the undersigned, declare that research work titled "Assessing Code based Shear Prediction Values of Reinforced Concrete Beams using Modified Compression Field Theory" is my original work and has not been presented for a degree in any other university and all sources of material used for this thesis have been properly acknowledged.

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ABSTRACT

Though different researches have been conducted so far on shear strength of reinforced concrete beam, it is yet not definite to predict it in accurate manner. As a result, different design codes use different empirical formulae based on truss analogy.

The truss analogy which was developed independently in 1899 and 1902 by Swiss engineer Ritter and by a German engineer Mörsch respectively. This theory is still deficient in representing the actual response of reinforced concrete element that is subjected to shear.

In recent years researchers at the University of Toronto came up with a more rational theory called "The Modified Compression Field Theory (MCFT)"; that can explain the response of a reinforced concrete elements subjected to shear. However, the complexity of this theory necessitate to convert it is procedure in to a computer program.

Therefore this research transact code algorithm of MCFT and evaluated both American Concrete Institute (ACI) 318-11 (2011) and EuroCode 2 (2004) design codes procedures.

Analysis and design of three different beams that are exposed to different types of loading is conducted using the above two codes. Results have been compared with those obtained from MCFT and have shown a safe results. Also a comparison between this two codes shows, EuroCode 2 gives more economical shear reinforcement spacing than ACI, for beams that require minimum shear reinforcement.

Key words: shear; strength; reinforced concrete; computer program; design code.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF NOTATION

a	= maximum aggregate size
A_c	= concrete area
A_s	= area of steel reinforcement
A_{s1}	= area of tensile reinforcement in beam section
A_v	= area of shear reinforcement
$A_{v,min}$	= minimum shear reinforcement area
b	= width of the beam
b_w	= width of the beam
C	= compressive force
D	= the vertical components of the diagonal compression force
d	= effective depth
E_c	= modulus of elastic of concrete (initial tangent stiffness)
E_s	= modulus of elasticity of reinforcement
ϵ_1	= principal tensile strain in concrete (positive quantity)
ϵ_2	= principal compressive strain in concrete (negative quantity)
ϵ_c'	= strain in concrete cylinder strength at peak stress (negative quantity)
ϵ_{cr}	= strain in concrete at cracking
ϵ_{cx}	= strain in concrete in x-direction
ϵ_{cy}	= strain in concrete in y-direction
ϵ_{sx}	= strain in reinforcement steel in x-direction
ϵ_{sy}	= strain in reinforcement steel in y-direction
ϵ_b	= bottom fiber strain in beam section
ϵ_{bot}	= bottom fiber strain in beam section
ϵ_t	= top fiber strain in beam section
ϵ_{top}	= top fiber strain in beam section
ϵ_x	= strain in x-direction
ϵ_y	= strain in y-direction
ϵ_{yx}	= yield strain in x-reinforcement
ϵ_{yy}	= yield strain in y-reinforcement
f_c'	= maximum compressive stress observed in cylinder test (negative quantity)

f_{c1}	= principal tensile stress in concrete
f_{c2}	= principal compressive stress in concrete (negative quantity)
f_c	= compressive stress
f_t	= tensile stress
f_1	= principal stress in orthogonal plane
f_2	= principal stress in orthogonal plane
f_{ci}	= compressive stress on crack surface (positive quantity)
f_{cr}	= stress in concrete at cracking
f_{cx}	= stress in concrete in x-direction
f_{cy}	= stress in concrete in y-direction
f_{sx}	= average stress in x-reinforcement
f_{sy}	= average stress in y- reinforcement
f_{sxcr}	= stress in x- reinforcement at crack location
f_{sycr}	= stress in y- reinforcement at crack location
f_x	= stress applied to element in x-direction
f_y	= stress applied to element in y-direction
f_{yx}	= yield stress in x- reinforcement
f_{yy}	= yield stress in y- reinforcement
f_{yt}	= yield stress in y- reinforcement
h	= depth of a concrete layer
H	= overall depth of beam cross-section
I	= the second moment of area
M	= bending moment
N	= axial Force
Q	= the first moment of area about the neutral axis
s	= spacing of shear reinforcement measured along the longitudinal axis of the structural member.
S_{max}	= maximum spacing of shear reinforcement
S_θ	= spacing of cracks inclined at θ
s_{mx}	= average spacing of cracks perpendicular to x-reinforcement
s_{my}	= average spacing of cracks perpendicular to y-reinforcement
T	= tensile force

v_{ci} = shear stress on crack surface

v_{cimax} = maximum shear stress a crack of given width can resist

v_{cx} = shear stress on x- face of concrete

v_{cy} = shear stress on y- face of concrete

v_{cxy} = shear stress on concrete relative to x, y axes

v_{sx} = shear stress on x- reinforcement

v_{sy} = shear stress on y- reinforcement

v_u = maximum shear stress element can resist

V_{Ed} = design shear force

$V_{Rd,c}$ = design shear resistance of the member without shear reinforcement

$V_{Rd,s}$ = design value of the shear force which can be sustained by the yielding shear reinforcement.

V_u = factored design shear force

V_n = nominal shear strength of a beam section

V_c = nominal shear strength of concrete

V_s = nominal shear strength of shear reinforcement

$V_{s,max}$ = maximum shear a beam section can carry

v_{xy} = shear stress on element relative to x, y axes

V = shear force

w = crack width

y = distance from an element to the neutral axes of the beam

$\bar{y}$ = distance from the top fiber to the centroid of the beam

τ = shear stress

θ = angle between concrete compression strut (principal strains) and the beam longitudinal axis

θ_c = angle of inclination of principal stresses in concrete to x-axis

ρ_{sx} = reinforcement ratio for reinforcing steel in x–direction

ρ_{sy} = reinforcement ratio for reinforcing steel in y–direction

$\emptyset$ = strength reduction factor of ACI

λ = modification factor for lightweight concrete

1. INTRODUCTION

1.1. Background

Current design procedures for reinforced concrete beam in shear are largely based on the truss analogy developed by Ritter and Mörsch nearly a century ago. In discussing the angle of inclination of the concrete struts θ, Mörsch concluded that it was mathematically impossible to determine the slope, but 45^0 was a conservative assumption, became known as the truss equation for shear. It has formed the basis of many of the design procedures for shear used since. [1]

Accurate prediction of Shear failure of reinforced concrete which commonly named to be diagonal tension failure is difficult. Despite many decades of experimental researches and the use of highly sophisticated analytical tools, this accurate prediction is not yet fully understood. Furthermore, if a beam is designed without proper shear reinforcement and subsequently vulnerable to failure due to overloading, shear collapse is likely to occur swiftly, with no advance warning of distress, which is appalling. [2]

The manner in which shear failure can occur varies widely with the dimensions, geometry, loading, and properties of the members. For this reason, we cannot define a unique design method for shear. [3]

Since 1971, there have been intensive researches efforts aimed at improving design methods for shear. The researches disproved that the angle of inclination of the concrete compression is 45^0, rather equations based on a variable truss angle provides a more realistic basis for shear design. In addition, tests on reinforced concrete panels subjected to pure shear make better the understanding of the stress-strain characteristics of diagonally cracked concrete. This stress-strain relationship made it possible to develop an analytical model, called the Modified Compression Filed Theory (MCFT) which has become the tool in accurately predicting the response of reinforced concrete subjected to shear. [4]

This method provide consistent and reliable prediction of the ultimate shear resistance of reinforced concrete beams, though, it is complex and time consuming in use; because of that it’s not completely adapted by any design codes.

1.2. Statement of Problem

The shear prediction formulas that are used by different country code vary significantly to one another, because of the way their formulas are developed is based on continuous researches which deliver valuable experimental data. Because of this variety in the design codes, a beam that's exposed to the same conditions of geometry, loading and material can give you different shear strength for different design codes.

This variation of shear strength as recognized above shows that there exists understandable disparity in those equations used for the prediction of shear strength; and to make sure whether the codes we usually use are safe and economical, we need to evaluate them using another mechanisms, and for sure, one of the best theories in the shear strength prediction is "The Modified Compression Filed Theory".

1.3. Objective

An experimental and analytical research conducted at the University of Toronto led to a more rational theory called "The Modified Compression Field Theory". This theory can explain the response of reinforced concrete elements subjected to shear, but it's too complex to use it for design purposes. Thus, my thesis will formulate an algorithm based on this theory (MCFT) to accurately calculate the desired outputs and finally compare these results with those that is obtained using the current design code procedures.

1.4. Significance of the Research Study

It is known that due to our county codes revision, the peak ground acceleration value and zonal demarcation in regard to earth quick calculations has changed significantly. As a result, for the sake of safety, it is mandatory to setup a system of assessing the capacities of existing structures.

One of the method to check their performance is by evaluating the shear strength using one of the design codes, which for sure does not show the actual response of the reinforced concrete structure, especially beams when it is subjected to shear. However, this can be resolved by the use of Modified Compression Theory, which helps in evaluating "to what extent do the exciting codes ensures level of safety of structures?'' and finally we can use MCFT to do the essential performance assessment of structures.

1.5. Limit of the Research Study

The program 3RCB is written only for a shallow depth rectangular cross-section beams with reinforcement placed in a single row. Because of the time limitation to do this thesis. It is actually possible to be doing general program for all sections and loading types, but this program is sufficient for this thesis as it meets the objective of the thesis. If someone needs the program to work on other sections, it can be modified in the code at "User interface for Section properties", and the Appendix can give a clue how to cope it.

1.6. Organization of the Paper

This thesis is organized in six chapters. In the first chapter, background and justification of this thesis work and the objectives to be achieved are discussed. In chapter two, a review of literatures relevant to this thesis work, which have been investigated by different researchers, are given. Chapter three is about algorithm development for Modified Compression Filed Theory. Chapter four analysis of three different beam, in chapter five, results of the analysis are summarized and discussions are made based on the outputs of the analysis. Finally, chapter six gives conclusion and recommendation achieved from this thesis work and propose future work in this field of study.

2. LITERATURE REVIEW

2.1. Shear stresses in homogenous rectangular beams [5]

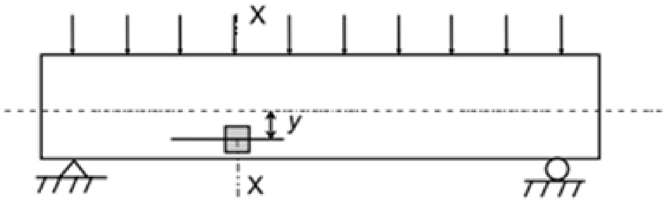
In order to gain an insight into the causes of flexural shear failure in reinforced concrete, the stress distribution in a homogeneous elastic beam of rectangular section is reviewed here. In such a beam, loaded as shown in Figure 2.1(a), any transverse section (marked 'XX'), in general, is subjected to a bending moment *M* and a transverse shear force *V*.

From basic mechanics of materials , it is known that the flexural (normal) stress f_x and the shear stress τ at any point in the section, located at a distance *y* from the neutral axis, are given by:

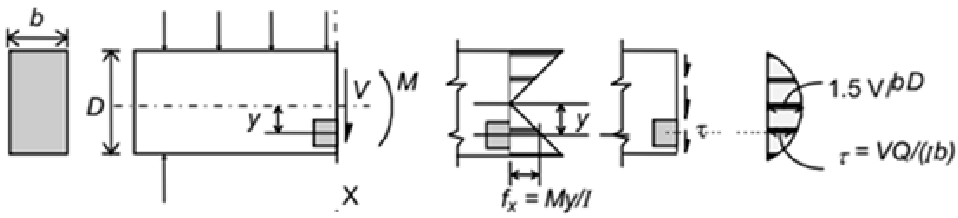
$$f_x = \frac{M.y}{I} \qquad \tau = \frac{VQ}{Ib} \qquad \dots\dots\dots\dots\dots\dots \; 2.1$$

Where *I* is the second moment of area of the section about the neutral axis, *Q* the first moment of area about the neutral axis(NA) of the portion of the section above the layer at distance *y* from the NA, and *b* is the width of the beam at the layer at which τ is calculated. The distributions of f_x and τ are depicted in Figure 2.1(b). It may be noted that the variation of shear stress is parabolic, with a maximum value at the neutral axis and zero values at the top and bottom of the section.

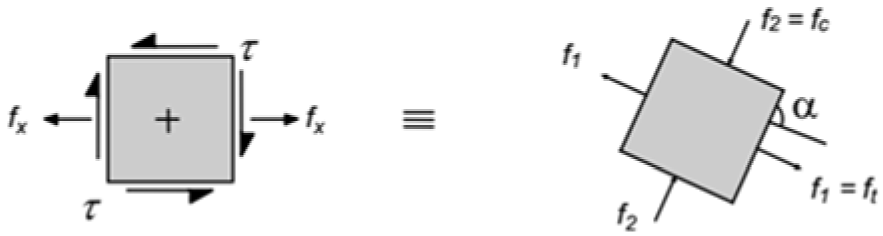
Considering an element at a distance *y* from the NA [Figure 2.1(c)], and neglecting any possible vertical normal stress f_y caused by the surface loads, the combined flexural and shear stresses can be resolved into equivalent principal stresses f_1 and f_2 acting on orthogonal planes, inclined at an angle α to the beam axis as shown in Figure 2.1.



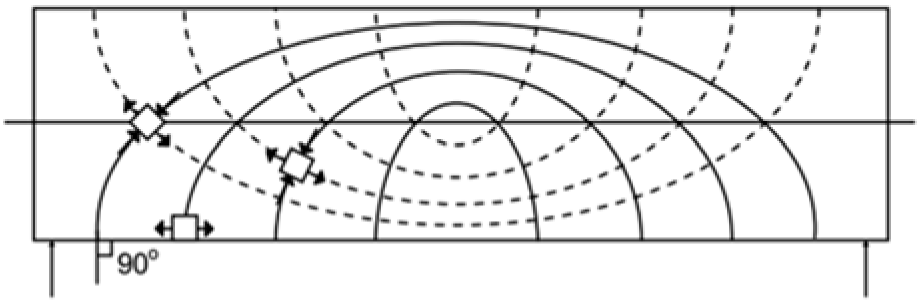
(a) Loading



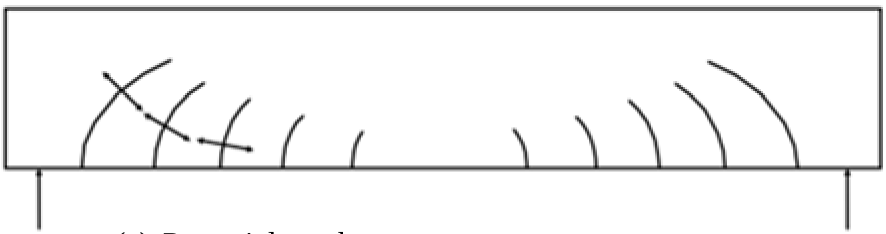
(b) Flexural and Shear



(c) Principal



(d) Principal stresses



(e) Potential crack pattern

Figure.2.1 Stress distribution in homogeneous beam of rectangular section

$$f_{1,2} = \frac{1}{2} f_x \pm \sqrt{(\frac{1}{2} f_x)^2 + \tau^2} \qquad \text{… … … … … … 2.2}$$

$$tan\, 2\alpha = \frac{2\tau}{f_x} \qquad \text{… … … … … … 2.3}$$

In general, the stress f_1 is tensile (say = f_t) and f_2 is compressive (say = f_c). The relative magnitudes of f_t and f_c and their directions depend on the relative values of f_x and τ [Eq. 2.2, 2.3]. In particular, at the top and bottom fibers where shear stress τ is zero, it follows from Eq. 2.3 that $\alpha = 0$, indicating that one of the principal stresses is in a direction parallel to the surface, and the other perpendicular to it, the latter being zero in the present case. Thus, along the top face, the nonzero stress parallel to the beam axis is f_c, and along the bottom face, it is f_t.

On the other hand, a condition of 'pure shear' occurs for elements located at the neutral axis (where τ is maximum and $f_x = 0$), where by $f_t = f_c = \tau_{max}$ and $\alpha = 45^o$. The stress pattern is indicated in Figure 2.1(d), which depicts the principal stress trajectories in the beam.

In a material like concrete which is weak in tension, tensile cracks would develop in a direction that is perpendicular to that of the principal tensile stress. Thus the compressive stress trajectories [firm lines in Figure 2.1(d)] indicate potential crack patterns (depending on the magnitude of the tensile stress), as shown in Figure 2.1(e). It should be noted, however, that once a crack develops, the stress distributions depicted here are no longer valid in that region, as the effective section gets altered and the above equations are no longer valid.

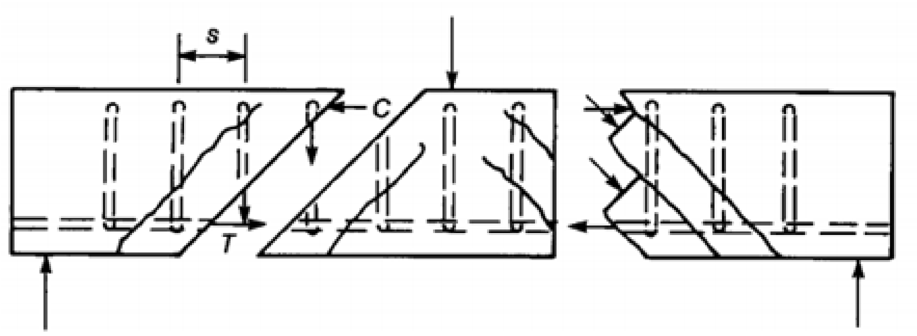
2.2. Truss Model of the behavior of slender beam failing in shear [3]

The behavior of beams failing in shear must be expressed in terms of a mechanical-mathematical model before designers can make a use of this knowledge in design. The best model for beams with web reinforcement is the truss model.

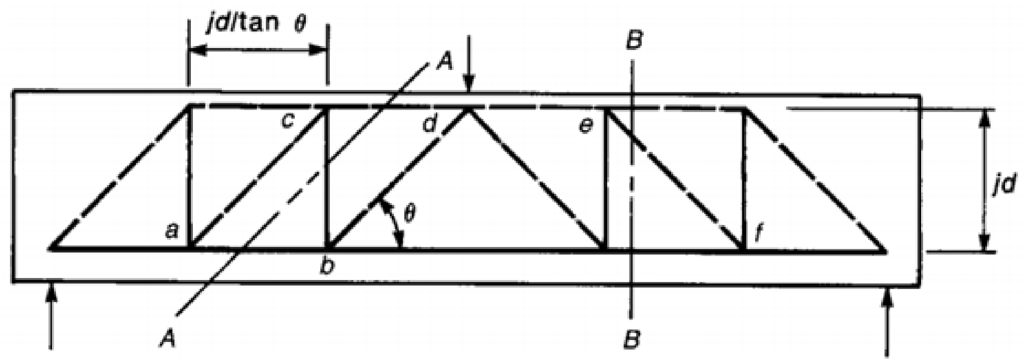
In 1899 and 1902, respectively, the Swiss engineer Ritter and the German engineer Mörsch, independently, publish papers proposing the truss analogy for the design of reinforced concrete beams for shear. These proposing provide an excellent conceptual model to show the forces that exist in a cracked concrete beam.

As shown in the Figure 2.2a, a beam with inclined cracks develops compressive and tensile forces, *C* and *T*, in its top and bottom "flanges," vertical tension in the stirrups, and inclined compressive forces in the concrete "diagonals" between the inclined cracks. This highly indeterminate system of forces can be replaced by an analogous truss.

Several assumptions and simplifications are needed to derive the analogous truss. In Figure 2.2b, the truss has been formed by lumping all of the stirrups cut by section A-A into one vertical member b-c and all the diagonal concrete members cut by section B-B into one diagonal member e-f. This diagonal member is stressed in compression to resist the shear on section B-B. The compression chord among the top of the truss is actually a force in the concrete but is shown as a truss member. The compression members in the truss are shown with dashed lines to imply that they are really forces in the concrete, not separate truss members. The tensile members are shown with solid lines.



(a) Internal forces in a cracked beam



(b) Pin-jointed truss.

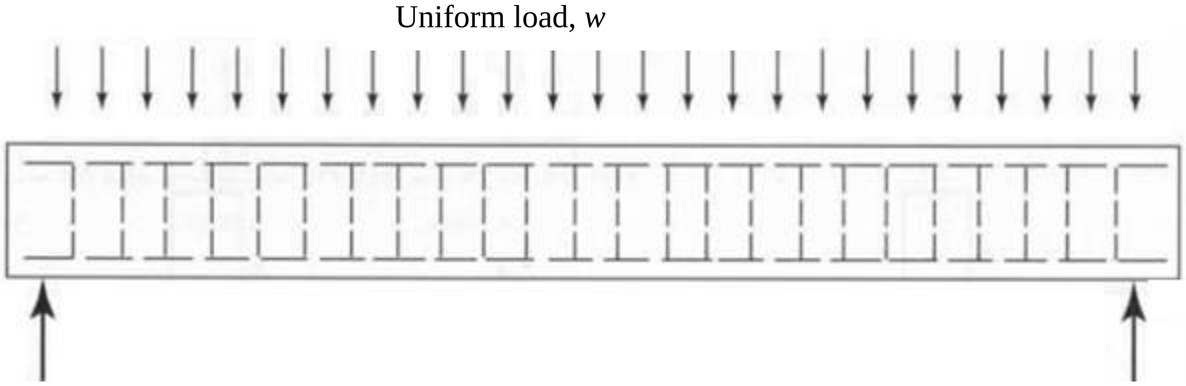
Figure 2.2 Truss analogy

In design, the ideal distribution of stirrups would corresponds to all stirrups reaching yield by the time the failure load is reached. It will be assumed, therefore, that all the stirrups have yielded and that each transmits a force $A_v f_{yt}$ across the crack, where A_v is the area of the stirrup legs and f_{yt} is the yield strength of the transverse reinforcement. When this is done, the truss becomes statically determinates. The truss in Figure 2.2b is referred to as the plastic-truss model, because we are depending on plasticity in the stirrups to make it statically determinate. The beam will be proportioned so that the stirrups yield before the concrete crushes, so that it will not depend on plastic action in the concrete.

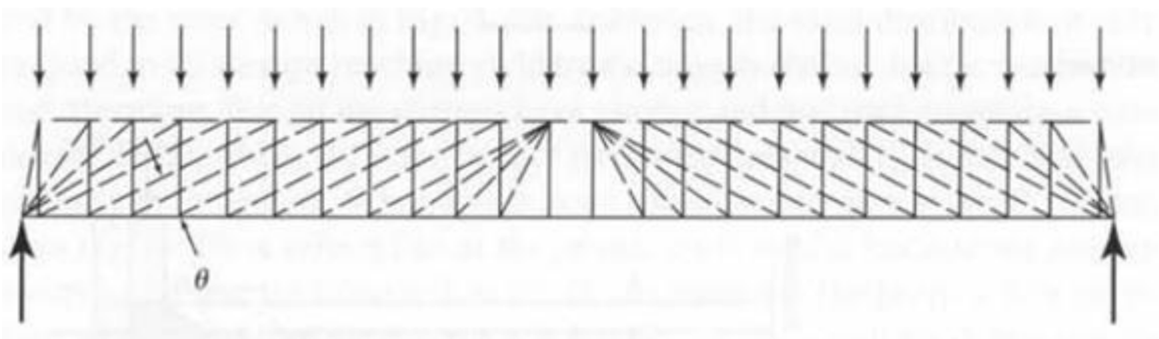
2.2.1. Simplified truss analogy

A statically determinate truss analogy can be derived via the method suggested by Peter Marti [6] and [7]. Figure 2.3a and b shows a uniformly loaded beam with stirrups and a truss model incorporating all the stirrups and representing the uniform load as a series of concentrated loads at the panel points. The truss in Figure 2.2b is statically indeterminate, but it can be solved if it is assumed that the forces in each stirrups cause that stirrup to just reach yield. For design, it is easier to represent the truss as shown in Figure 2.3c, where the tension forces in each vertical member represents the force in all the stirrups within a length $jd\ cot\ \theta$. The uniform load has been idealized as a concentrated loads of $w(jd\ cot\ \theta)$ acting at the panel points. The truss in Figure 2.3c is statically determinate.

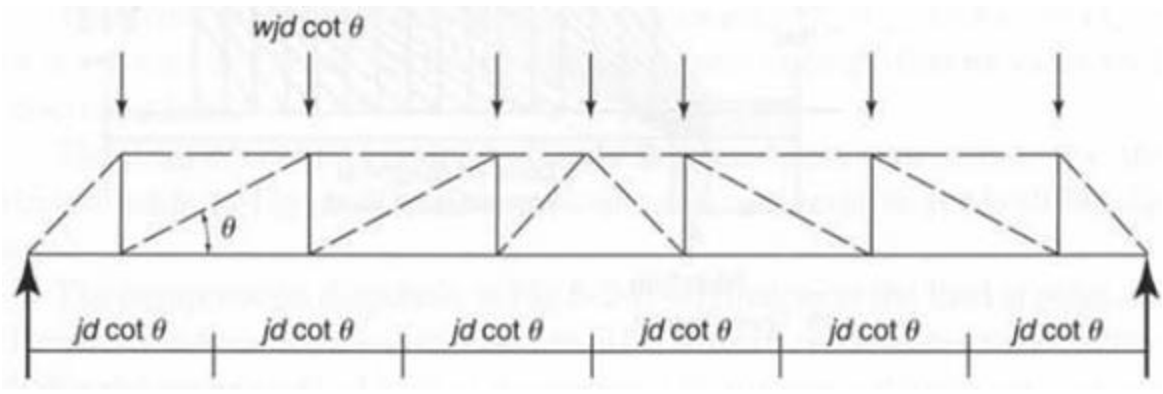
Uniform load, *w*



(a) Beam and reinforcement



(b) Truss model



(c) Statically determinate truss

Figure 2.3 Truss Model for design

2.2.2. Internal Forces in the Plastic-Truss Model

If we consider the free-body diagram cut by section A-A parallel to the diagonals in the compression field region in Figure 2.4a, the entire vertical components of the shear force is resisted by tension forces in the stirrups crossing this section. The horizontal projection of section A-A is $jd\cot\theta$, and the number of stirrups it cuts is $jd\cot\theta/s$. The forces in one stirrup is $A_v f_{yt}$, which can be calculated from

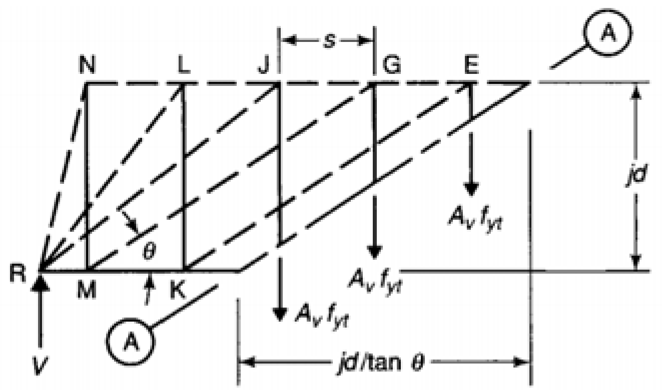
$$A_v f_{yt} = \frac{V \times s}{jd\,cot\,\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.4$$

The free body shown in Figure 2.4b is cut by a vertical section between G and J in Figure 2.4b. Here, the vertical force, acting on the section is resisted by the vertical components of the diagonal compression force *D* (Figure 2.4c). The width of the diagonal is $jd\cos\theta$, as shown in Figure 2.4b, and expressing *D* as $V/\sin\theta$, the average compressive stress in the diagonals is

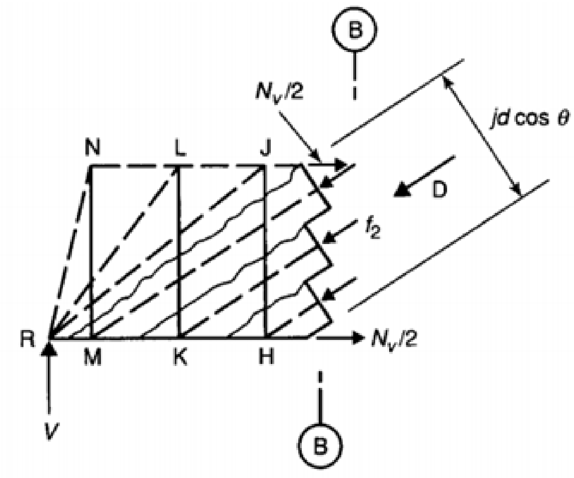
$$f_2 = \frac{V}{b_w jd\cos\theta\sin\theta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.5$$

With the use of trigonometric identities, this equation becomes

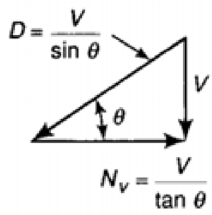
$$f_2 = \frac{V}{b_w jd}(tan\,\theta + \frac{1}{tan\,\theta}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.6$$



(a) Calculation of forces in stirrups



(b) Calculation of stress in compression diagonals.



(c) Replacement of V with internal forces of D and N.

Figure 2.4 Forces in stirrups and compression diagonals.

Where b_w is the thickness of the web. If the web is very thin, this stress may be cause the web to crush, as shown in Fig 4.5.

The shear V on section B-B can be replaced by the diagonal compression force

$$D = \frac{V}{\sin\theta} \qquad \text{……………… 2.7}$$

And an axial tension force

$$N_v = \frac{V}{\tan\theta} \qquad \text{……………… 2.8}$$

As shown in Fig 2.4c

If it's assumed that the shear stress is constant over the height of the beam, the resultant of D and N_v act at mid height. As a result, a tension of $N_v/2$ acts in each of the top and bottom chords. This reduces the force in the compression chord and increases the forces in the tension chord.

When a reinforced concrete beam with stirrups is loaded to failure, inclined cracks initially develop at an angle of 35^0 to 45^0 with the horizontal. With further loading. The angle of the compression stress may cross some of the cracks. For this to occur, aggregates interlock must exist.

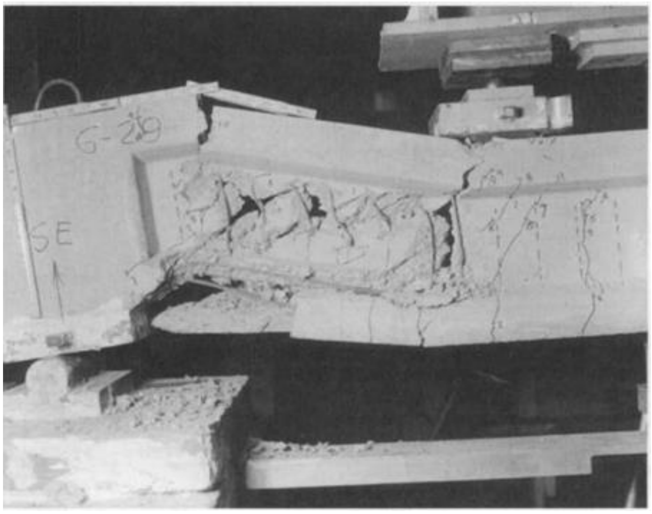


Figure 2.5 Web crushing failure

In design, the values of θ should be in the range $25° \leq \theta \leq 65°$. The choice of a small value of θ reduces the number of stirrups required, but increases the compression stress in the web and increases N_v. The opposite is true for larger angles.

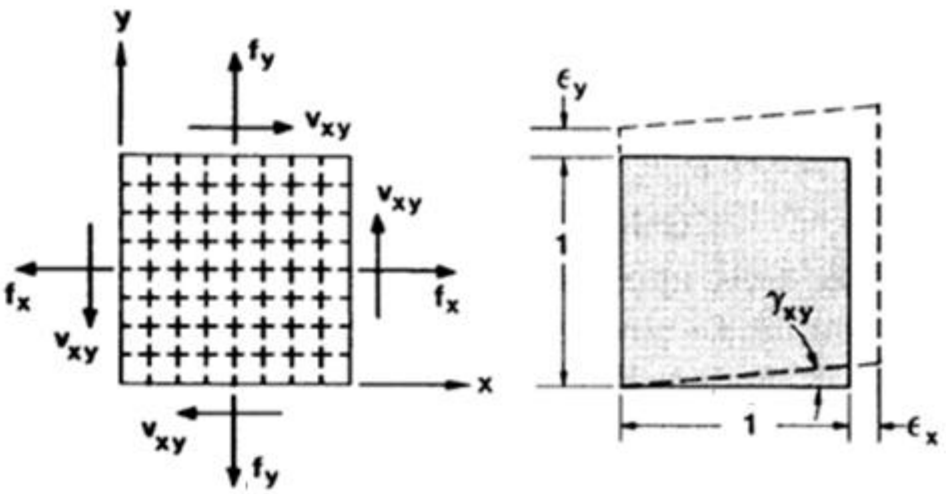
In the analysis of a given beam, the angle θ is determined by the number of stirrups needed to equilibrate the applied loads and reactions. The angle should be within the limited it's given, except in compression-fan regions where the angle θ varies. In the design of beam, the crack angles is free choice that leads to values of other unknowns.

2.3. The Modified Compression Field Theory [8]

In this model, cracked concrete is treated as a new material with its own stress-strain relationships are formulated in terms of average stresses and average strains. Consideration is also given to local stress conditions at crack location.

The modified compression-field theory presented here has been developed from the compression-field theory for reinforced concrete in torsion and shear. In both models, the cracked concrete is treated as a new material with its own stress-strain characteristic. Equilibrium, compatibility, and stress-strain relationships are formulated in terms of average stresses and average strains. While the original compression-field theory ignored tension in the cracked concrete, this model takes into account tensile stresses in concrete between the cracks, and employs experimentally verified average stress-average strain relationships for cracked concrete.

The membrane element shown in Figure 2.6 represent a portion of reinforced concrete structures. It is taken to be of uniform thickness and relatively small size, and contains an orthogonal grid of reinforcement (x) and transverse (y) axes chosen to coincide with the reinforcement directions. Loads acting on the element's edge planes are assumed to consist of the uniform axial stresses f_x and f_y and uniform shear stress v_{xy}. Deformation of the element is assumed to occur such that the edges remain straight and parallel. The deformed shape is defined by the two normal strains ϵ_x and ϵ_y and the shear strains, γ_{xy}.



Loading Deformation

Figure 2.6 Membrane elements

The problem at hand is to determine how the three in-plane stresses $f_x, f_y,$ and v_{xy} are related to three in-plane strains $\epsilon_x, \epsilon_y,$ and τ_{xy}. In solving this problem, the following additional assumptions will be made:

1. For each strains state there exists only one corresponding stress state; situations in which the influence of loading history is significant will not be treated.
2. Stresses and strains can be considered in terms of average values when taking over areas or distances larger enough to include several cracks.
3. The concrete and the reinforcing bars are perfectly bonding together at the boundaries of the element (i.e., no overall slip).
4. The longitudinal and transverse reinforcing bars are uniformly distributed over the elements.

Tensile stresses and tensile strains will be treated as positive quantities while compressive stresses and strains will be taken as negative.

2.3.1. Compatibility Conditions

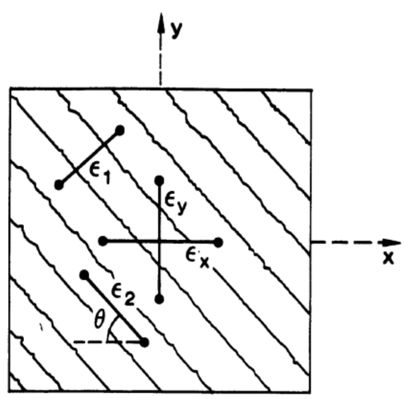
Having assumed that the reinforcement is anchored to the concrete, compatibility requires that any deformation experienced by the concrete must be matched by an identical deformation of the reinforcement. Any change in concrete strain will be accompanied by equal change in steel strain.

Non-prestressed reinforcement has the same initial strain as the surrounding concrete. Hence

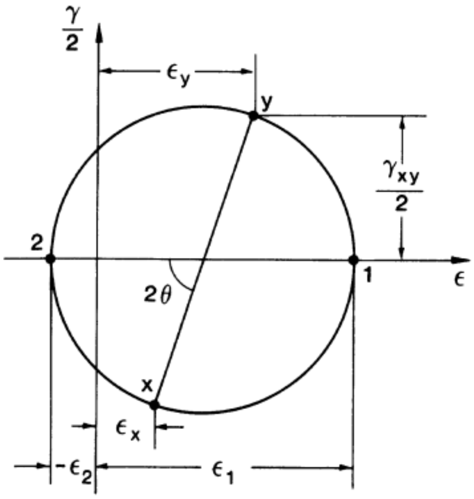
$$\epsilon_{sx} = \epsilon_{cx} = \epsilon_x \qquad \dots\dots\dots\dots\dots\dots \; 2.9$$

And

$$\epsilon_{sy} = \epsilon_{cy} = \epsilon_y \qquad \dots\dots\dots\dots\dots\dots \; 2.10$$



(a) Average Strains in Cracked Element



(b) Mohr's Circle for Average Strains

Figure 2.7 Compatibility conditions for cracked element

Where ϵ_1 is the principal tensile strain and ϵ_2 is the principal compressive strain.

If the three strains components $\epsilon_x, \epsilon_y,$ and τ_{xy} are known, then the strain in any other direction can be found from geometry. The Mohr's circle of strain shown in Figure 2.7b elegantly summarizes the transformations involved. Useful relationships which can be derived from its geometry include

$$\gamma_{xy} = \frac{2(\epsilon_x - \epsilon_2)}{\tan\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.11$$

$$\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.12$$

$$tan^2\,\theta = \frac{\epsilon_x - \epsilon_2}{\epsilon_y - \epsilon_2} = \frac{\epsilon_1 - \epsilon_y}{\epsilon_1 - \epsilon_x} = \frac{\epsilon_1 - \epsilon_y}{\epsilon_y - \epsilon_2} = \frac{\epsilon_x - \epsilon_2}{\epsilon_1 - \epsilon_x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.13$$

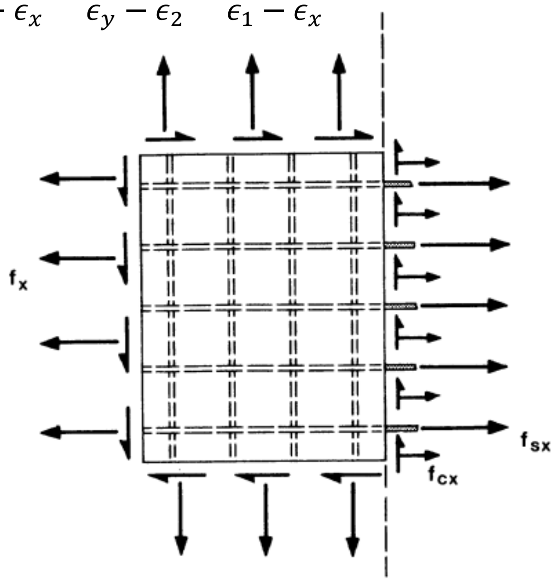
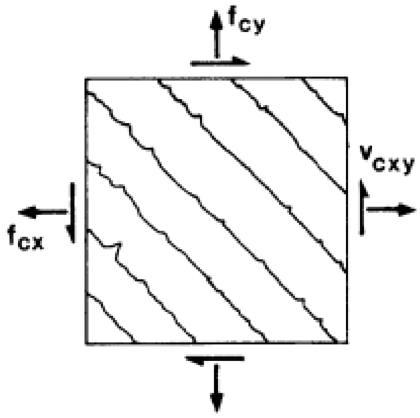
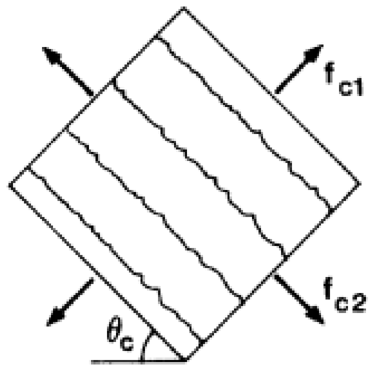


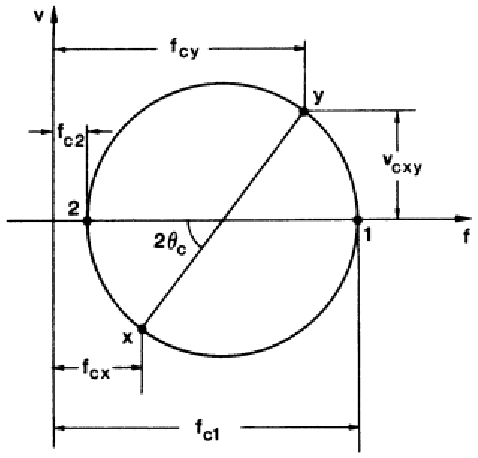
Figure 2.8 Free-body diagram of element



(a) Average Concrete Stresses



(b) Principal Stresses in Concrete



(c) Mohr's Circle for Average Concrete Stresses

Figure 2.9 Stresses in cracked conrete

2.3.2. Equilibrium Conditions

The forces applied to the reinforced concrete element are resisted by stresses in the concrete and stresses in the reinforcement. For the free-body diagram shown in Figure 2.8, the requirement that the forces sum to zero in x-direction can be written as

$$\int_A f_x \, dA = \int_{A_c} f_{cx} dA_c + \int_{A_s} f_{sx} dA_s \qquad \text{………………… 2.14}$$

Ignoring the small reduction in concrete cross-sectional area due to the presence of reinforcing bars, Eq. 2.14 becomes

$$f_x = f_{cx} + \rho_{sx} \cdot f_{sx} \qquad \text{………………… 2.15}$$

In a similar fashion, the flowing equilibrium conditions can be derived.

$$f_y = f_{cy} + \rho_{sy} . f_{sy} \qquad \dots\dots\dots\dots\dots\dots \; 2.16$$

$$v_{xy} = v_{cx} + \rho_{sx} . v_{sx} \qquad \dots\dots\dots\dots\dots\dots \; 2.17$$

And

$$v_{xy} = v_{cy} + \rho_{sy} . v_{sy} \qquad \dots\dots\dots\dots\dots\dots \; 2.18$$

Assuming that

$$v_{cx} = v_{cy} = v_{cxy} \qquad \dots\dots\dots\dots\dots\dots \; 2.19$$

The stress condition in the concrete are fully defined if f_{cx}, f_{cy} and v_{cxy} are known.

The Mohr's circle for the concrete stresses shown in Figure 2.9c yields the following useful relationships.

$$f_{cx} = f_{c1} - \frac{v_{cxy}}{\tan\theta_c} \qquad \dots\dots\dots\dots\dots\dots \; 2.20$$

$$f_{cy} = f_{c1} - v_{cxy} . \tan\theta_c \qquad \dots\dots\dots\dots\dots\dots \; 2.21$$

And

$$f_{c2} = f_{c1} - v_{cxy} . (\tan\theta_c \;\; + \frac{1}{\tan\theta_c}) \qquad \dots\dots\dots\dots\dots\dots \; 2.22$$

2.3.3. Stress-Strain Relationships

Constitutive relationships are required to link average stresses to average strains for both reinforcements and the concrete. These average stress-average strain relations may differ significantly from standard material tests. Furthermore, the average stress-average strain relationships for the reinforcement and concrete will not be completely independent of each other, although this will be assumed to maintain the simplicity of the model.

The axial stress in the reinforcement will be assumed to depend on only one strain parameter, the axial strain in the reinforcement. It will be assumed further that the average shear stress on the plan normal to the reinforcement resisted by the reinforcement is zero. In relating stress-strain relationships shown in Figure 2.10 will be adopted. Thus

$$f_{sx} = E_s \, \epsilon_x \leq f_{yx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.23$$

$$f_{sy} = E_s \, \epsilon_y \leq f_{yy} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.24$$

$$v_{sx} = v_{sy} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.25$$

In regard to the concrete, it will be assumed that the principal stress axes and principal strain axes coincide

$$\theta_c = \theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.26$$

To complete the model, relationships between the principal compressive stress and principal compressive strain and between the principal tensile stress and the principal tensile strain are required.

2.3.4. Experimental Program

To obtain the necessary information, reinforced concrete elements were subjected to simple well-defined loading conditions. While the majority of the tests were conducted in monotonic pure shear, some elements where subjected to uniaxial compression, combined biaxial compression and shear, combined biaxial tension and shear, reversed cyclic shear, and changing load rations. In addition to loading conditions, the prime variables included percentage of transverse reinforcement, percentage of longitudinal reinforcement, and concrete strength.

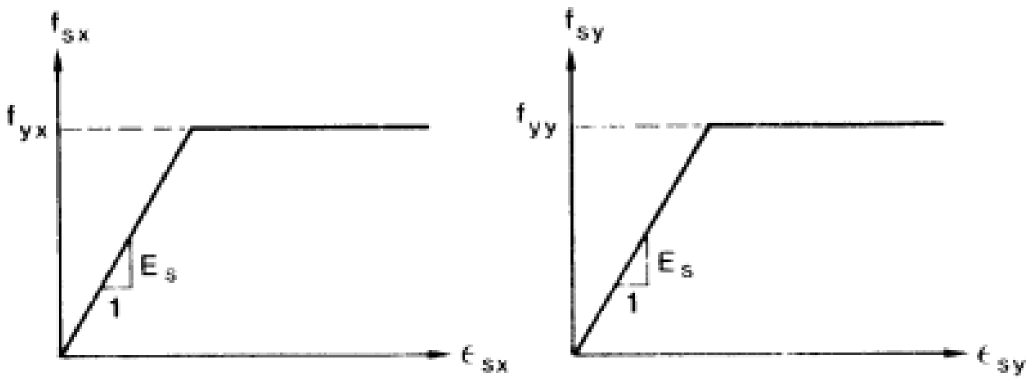


Figure 2.10 Stress-strain relationships for reinforcements

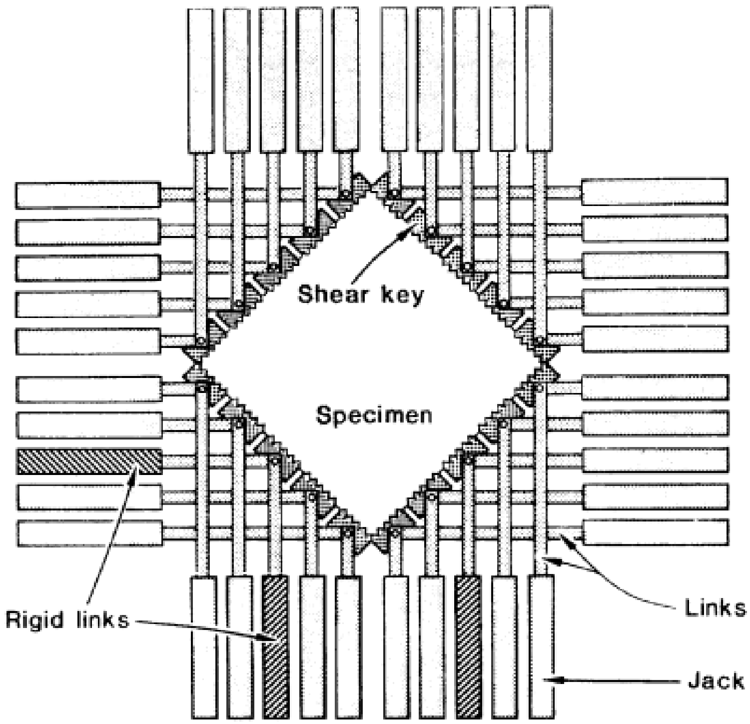


Figure 2.11 Jack-and-link assembly used to apply shear and normal stresses

Figure 2.12 Membrane Element Tester

2.3.5. Average Stress-Average Strain Response of Concrete

The direction of principal strains in the concrete deviated somewhat from the direction of principal stress in the concrete (see Figure 2.13). However, it remains a reasonable simplification to assume that the principal strain axes and the principal stress axes for the concrete coincide.

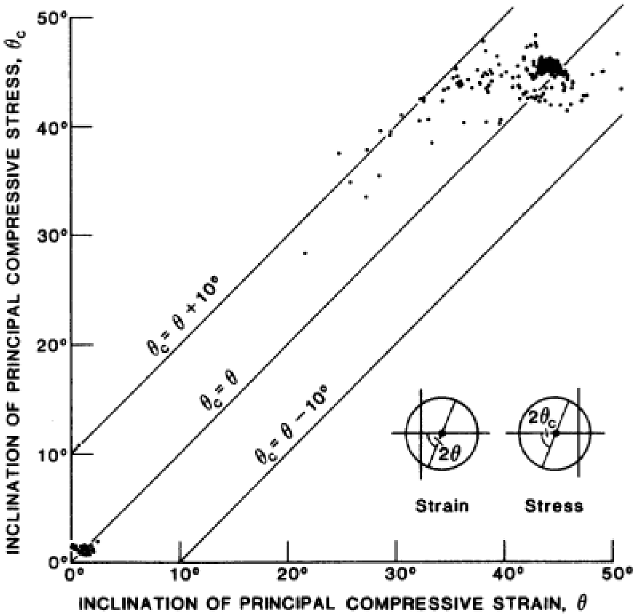


Figure 2.13 Comparison of principal compressive stress direction with principal compressive strain direction

The principal compressive stress in the concrete f_{c2} was found to be a function not only of the principal compressive strain ϵ_2 but also of the co-existing principal tensile strains ϵ_1. Thus, cracked concrete subjected to high tensile strain in the direction normal to the compression is softer and weaker than concrete in standard cylinder test (Figure 2.14). The relationships suggested is

$$f_{c2} = f_{c2max}\left[2\left(\frac{\epsilon_2}{\epsilon_c'}\right) - \left(\frac{\epsilon_2}{\epsilon_c'}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.27$$

Where

$$\frac{f_{c2max}}{f_c'} = \frac{1}{0.8 - 0.34\,\epsilon_1/\epsilon_c'} \leq 1.0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 2.28$$

Note that as ϵ_c' is a negative quantity (usually -0.002), increasing ϵ_1 will reduced f_{c2max}/f_c'.

The relationship between the average principal tensile stress in the concrete and average principal tensile strain is nearly linear prior to cracking and then shows decreasing values of f_{c1} with increasing values of ϵ_1 (see Figure 2.14). The relationship suggested prior to cracking (i.e., $\epsilon_1 \leq \epsilon_{cr}$) is

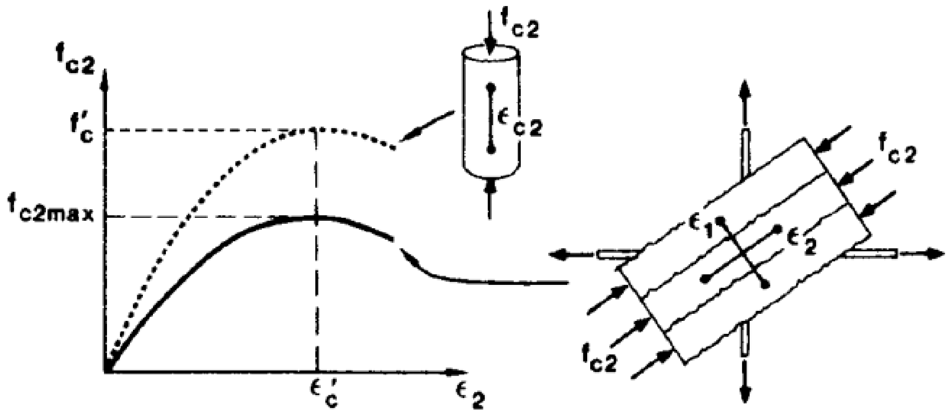
$$f_{c1} = E_c \epsilon_1 \qquad \dots\dots\dots\dots\dots\dots 2.29$$

Where E_c is the modulus of elasticity of concrete which can be taken as $2f_c'/\epsilon_c'$. The relationship suggested after cracking.

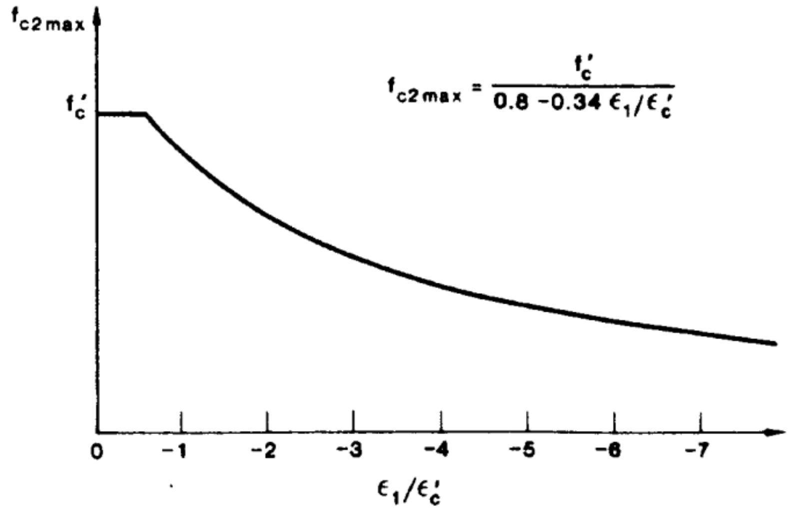
$$f_{c1} = \frac{f_{cr}}{1 + \sqrt{200\epsilon_1}} \qquad \dots\dots\dots\dots\dots\dots 2.30$$

2.3.6. Transmitting Loads across Cracks

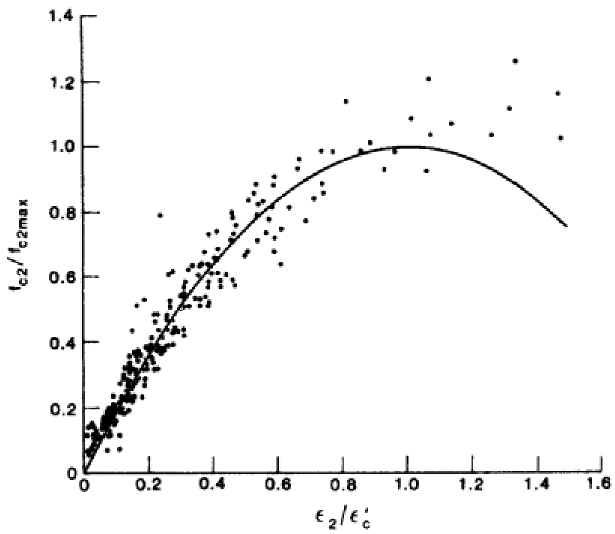
The stress and strain formulation described deal with average values and do not give information regarding local variations. At a crack, the tensile stresses in the reinforcement will be higher than average, while midway between cracks they will be lower than average. The concrete tensile stresses, on the other hand, will be zero at a crack and higher than average midway between cracks. These local variation are important because the ultimate strength of a biaxial stresses element may be governed by the reinforcement's ability to transmit tension across the cracks.



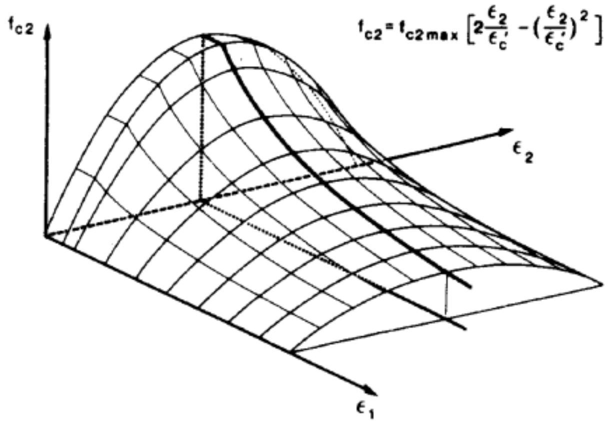
(a) Stress-Strain Relation for Cracked Concrete in Compression



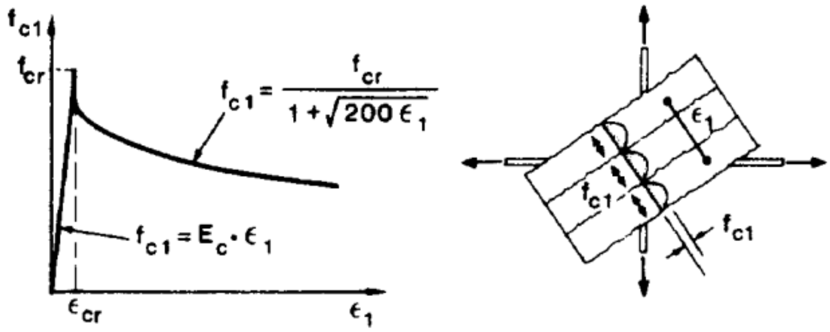
(b) Proposed Relationship for Maximum Compressive Stress



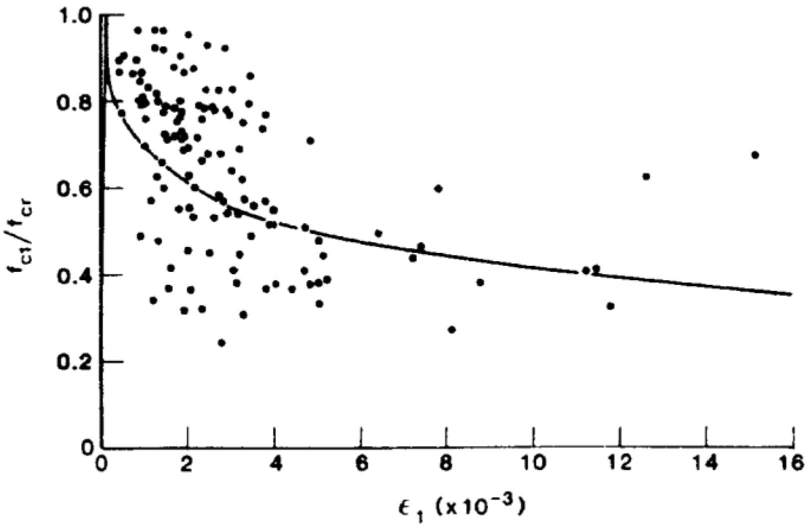
(c) Correlation of Test Data for Cracked Concrete in Compression



(d) Three-Dimensional Representation of Compressive Stress-Strain Relationship

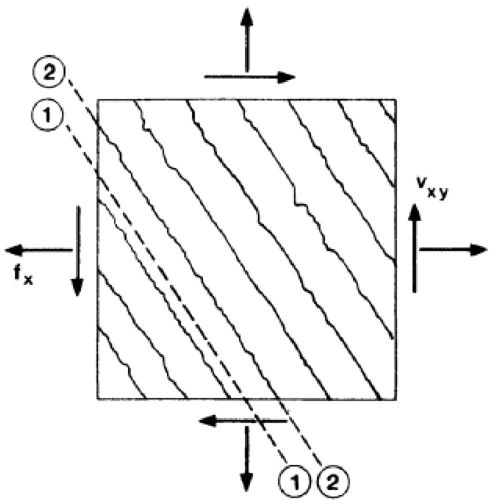


(e) Average Stress-Strain Relationship for Cracked Concrete in Tension

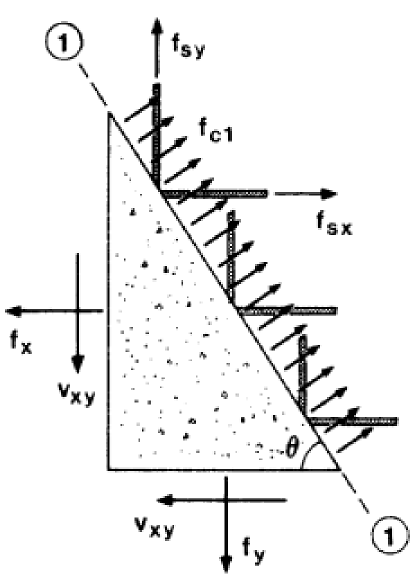


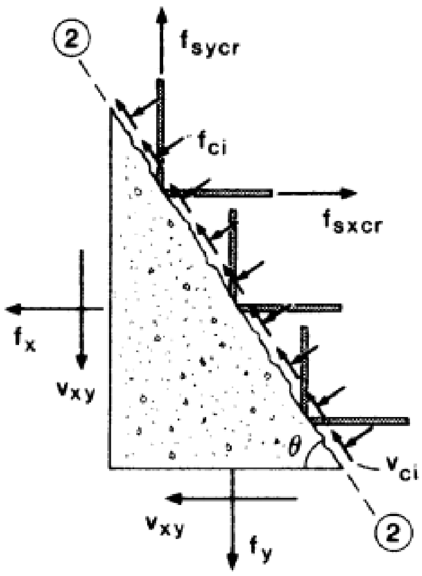
(f) Correlation of Test Data for Cracked Concrete in Tension

Figure 2.14 Stress-Strain Relationships for Cracked Concrete



(a) Stresses Applied to cracked Element





(b) Calculated Average Stresses (c) Local Stresses at a Crack

Figure 2.15 Comparison of local stresses at a crack with calculated average stresses

Figure 2.15 compares the calculated average stresses with the actual local stresses that occur at a crack. The critical crack direction is assumed normal to the principal tensile strain direction. While the calculated average shear stress on plane 1 is zero. (In terms of average stresses it is a principal plan), there may be local shear stresses on plane 2. These shear stresses v_{ci}, may be accompanied by small local compressive stresses f_{ci}, across the crack.

As the applied external stresses f_x, f_y and v_{xy} are fixed, the two sets of stresses shown in Figure 2.15 must be statically equivalent. Assuming a unit area for both plane 1 and plane 2, the requirement that the two sets of stresses produce the same forces in the x-direction is

$$\rho_{sx} f_{sx} \sin\theta + f_{c1} \sin\theta = \rho_{sx} f_{sxcr} \sin\theta - f_{ci} \sin\theta - v_{ci} \cos\theta \qquad \text{2.31}$$

This equation can be rearranged as

$$\rho_{sx}(f_{sxcr} - f_{sx}) = f_{c1} + f_{ci} + \frac{v_{ci}}{\tan\theta} \qquad \text{2.32}$$

The requirement that the two sets of stresses on plane 1 produced the same force in the y-direction is

$$\rho_{sy} f_{sy} \cos\theta + f_{c1} \cos\theta = \rho_{sy} f_{sycr} \cos\theta - f_{ci} \cos\theta - v_{ci} \sin\theta \qquad \text{2.33}$$

This equation can be rearranged as

$$\rho_{sy}\left(f_{sycr} - f_{sy}\right) = f_{c1} + f_{ci} - v_{ci} \tan\theta \qquad \text{2.34}$$

Equilibrium Eq. 2.32 and Eq. 2.34 can be satisfied with no shear stress on the crack and no compressive stresses on the crack only if

$$\rho_{sy}\left(f_{sycr} - f_{sy}\right) = \rho_{sx}(f_{sxr} - f_{sx}) = f_{c1} \qquad \text{2.35}$$

However, the stress in the reinforcement at a crack cannot exceed the yield strength, that is

$$f_{sxcr} \leq f_{yx} \qquad \text{2.36}$$

$$f_{sycr} \leq f_{yy} \qquad \text{2.37}$$

Hence, if the calculated average stress in either reinforcement is high, it may not be possible to satisfy Eq. 2.35. In this case, equilibrium will require shear stresses on the crack.

For the vast majority of concretes, cracking will occur along the interface between the cement paste and aggregate particles. The resulting rough cracks can transfer shear by aggregate interlock (see Figure 2.16).

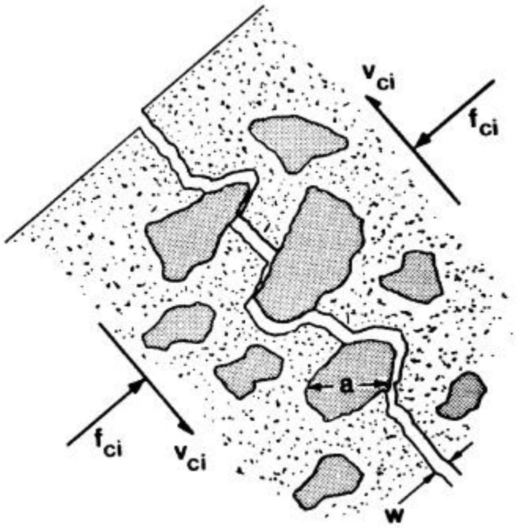


Figure 2.16 Transmitting shear stresses across cracked by aggregate interlock

The relation between the shear across the crack v_{ci}, the crack width *w*, and the required compressive stress on the crack f_{ci} have been experimentally studied by a number of investigator, including Walraven. Based on Walraven's work, the following relationship was derived. (see Figure 2.17)

$$v_{ci} = 0.18 v_{cimax} + 1.64 f_{ci} - 0.82 \frac{f_{ci}^2}{v_{cimax}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.38$$

Where

$$v_{cimax} = \frac{\sqrt{-f_c'}}{0.31 + 24\, w/(a + 16)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.39$$

And where a is the maximum aggregate size in millimeters and the stresses are in Mpa.

The crack width *w* to be used in Eq. 2.39 should be the average crack width over the crack surface. It can be taken as the product of the principal tensile strain and the crack spacing s_θ; that's

$$w = \epsilon_1 s_\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.40$$

Where

$$s_\theta = \frac{1}{\frac{\sin\theta}{s_{mx}} + \frac{\cos\theta}{s_{my}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.41$$

And where s_{mx} and s_{my} are the indicators of the crack control characteristics of the x-reinforcement and the y-reinforcement, respectively.

The average crack spacing in the longitudinal and transverse direction can be estimated as follows. [9]

$$s_{mx} = 2 \times \left(C_x + \frac{S_x}{10}\right) + 0.25 \times K_1 \times \frac{d_{bx}}{\rho_x} \quad \text{… … … … … …} \quad 2.42$$

$$s_{my} = 2 \times \left(C_y + \frac{S}{10}\right) + 0.25 \times K_1 \times \frac{d_{by}}{\rho_y} \quad \text{… … … … … …} \quad 2.43$$

Where :- d_{bx} and d_{by} are the diameters of the reinforcing bars in the x and y direction.

$K_1 =$ 0.4 for deformed bars and 0.8 for plain bars.

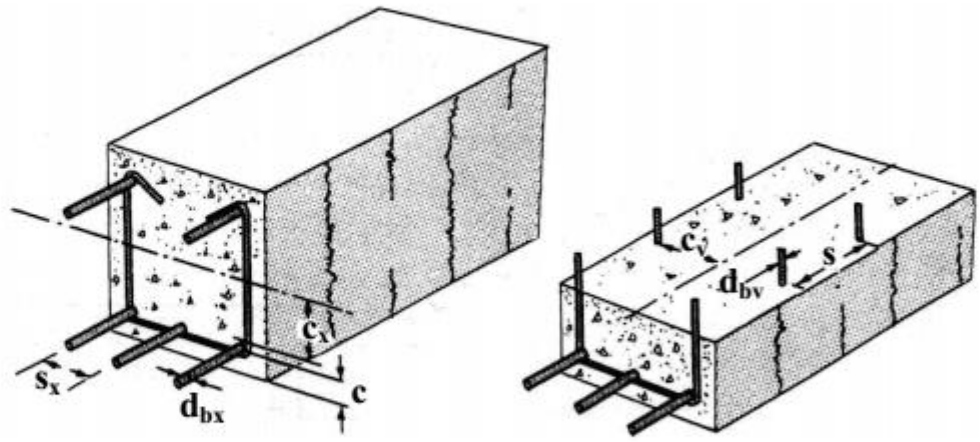


Figure 2.17 Parameters influencing crack spacing

Thus, in checking stress condition at the crack surface, a combination of the shear and compressive stresses v_{ci} and f_{ci} must be determined to satisfy Eq. 2.31 through 2.37. If, because of steel yielding at the crack, a solution is not possible, then the calculated average principal tensile stress f_{ci} must be reduced until a solution is possible.

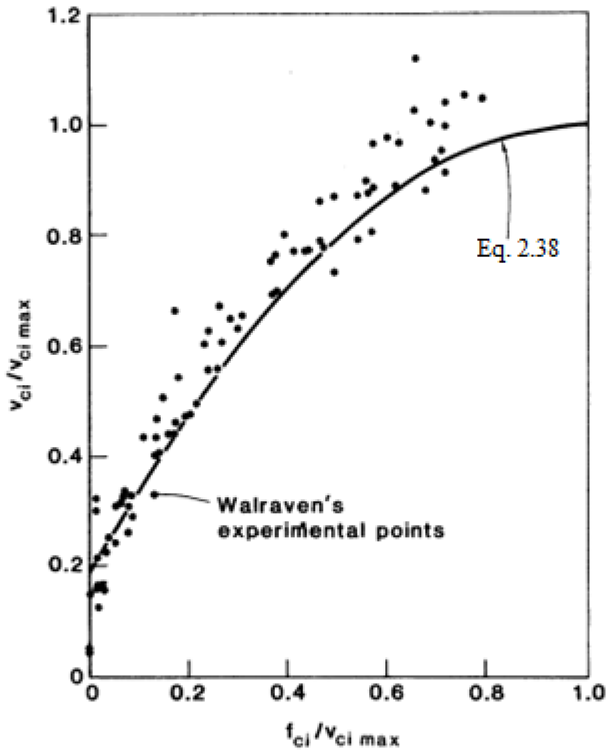


Figure 2.18 Relationship between shear transmitted across crack and compressive stress on crack

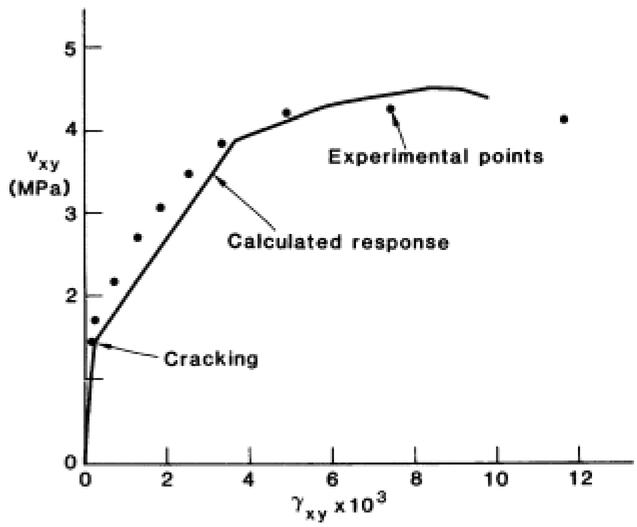


Figure 2.19 Comparison of calculated and observed response

2.4. Sectional Analysis

The modified compression filed theory can be used for the analysis of reinforced concrete beams by considering the beam section is composed of a series of concrete layers and longitudinal steel elements. As it is shown in Figure 2.20, each layer is defined by its width b, depth h, the amount of transverse reinforcement ρ_y and its position from the top fiber y_c. The longitudinal steel elements are defined by their cross sectional area A_s, yield strength f_{yx}, and its position from the top fiber y_s. Common properties for the entire beam cross section are the concrete cylinder strength f_c', concrete strain at peak stress ϵ_c', yield strength of transverse reinforcement f_{yy}, and Young's modulus for steel E_s. This layered model can be used for beams of unusual cross sectional areas and reinforcement details.

The concrete layers and longitudinal steel elements are analyzed individually, although conditions of compatibility and equilibrium must be satisfied for the section as a whole. The only section compatibility requirement used is plain section remain plane. Thus, the longitudinal strain in each of the concrete layers and reinforcing bar elements will be fixed by defining the top and bottom fiber strains. [1]

$$\epsilon_{xi} = \epsilon_t + \frac{(\epsilon_b + \epsilon_t)}{H} . y_i \qquad \text{… … … … … …} \quad 2.44$$

Where:-

ϵ_{xi} - Longitudinal strain at the point of interest.

ϵ_t –Top fiber strain.

ϵ_b –Bottom fiber strain.

y_i –location of the point from the top fiber.

H – Overall depth of the beam.

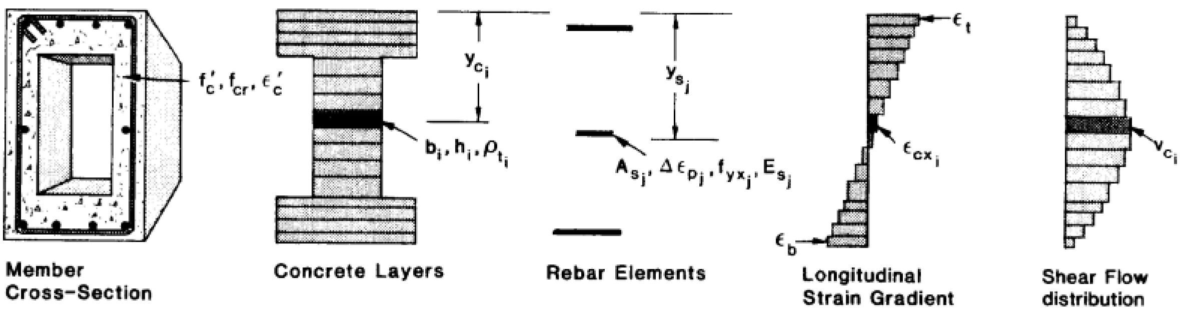


Figure 2.20 Beam section for layered model

Both equilibrium and compatibility requirement should be satisfied for sectional analysis. The equilibrium requirement include balancing of applied shear, moment and axial forces with their cross sectional forces. Condition of compatibility and equilibrium are determined using modified compression field theory and it's assumed that a uniform stress condition will exist in each concrete layer and longitudinal rebar elements.

The analytical procedure requires that estimate be made of the longitudinal strain distribution and shear stress distribution across the section, then each concrete layers and reinforced bar elements can be analyzed separately. The stress in the longitudinal reinforced bar elements be determined from the longitudinal strains by using a bilinear stress-strain relationship. The longitudinal stress in the concrete layers are determined by using the formulation of modified compression field theory; in the process we can find the concrete longitudinal compressive stress f_{cx} for each layer.

For the given longitudinal strain and shear flow distribution, longitudinal stresses are determined for each concrete layers and reinforced bar elements. The resultant of these stresses must balance the applied section forces. For a beam section discretized into m concrete layers and n longitudinal reinforcing bar elements, the elemental stresses must satisfy the following conditions. [1]

$$N = \sum_{i=1}^{m} f_{cxi} b_i h_i + \sum_{j=1}^{n} f_{sxj} A_{sj} \qquad \dots\dots\dots\dots\dots\dots \; 2.45$$

$$M = \sum_{i=1}^{m} f_{cxi} b_i h_i (y_{ci} - \bar{y}) + \sum_{j=1}^{n} f_{sxj} A_{sj} (y_{sj} - \bar{y}) \qquad \dots\dots\dots\dots\dots\dots \; 2.46$$

$$V = \sum_{i=1}^{m} V_{ci} b_i h_i \qquad \dots\dots\dots\dots\dots\dots \; 2.47$$

Where:-

N- Axial load, M- Moment, V- Shear at the centroid of the section.

$\bar{y}$ – Distance from the top to the centroid of the section.

If the above conditions are not satisfied, it becomes necessary to readjust the assumed longitudinal strain gradient and repeat the analysis until equilibrium is achieved.

2.5. Code based procedure for Shear Design of Reinforced Concrete Beam

In this thesis only two design codes are covered, the Building Code Requirements for Structural Concrete (American Concrete institute (ACI) 318-11) and Eurocode 2: Design of concrete structures EN 1992-1-1 (2004).

2.5.1. ACI General Procedure for Shear Design [10]

Design of cross section subjected to shear shall be based on

$$V_u \leq \emptyset V_n \qquad \dots\dots\dots\dots\dots\dots \quad 2.48$$

Where:-

$\emptyset$ is strength-reduction factor for shear and torsion and equal to 0.75 (ACI Code section 9.3.2.3).

V_u is the factored shear force at the considered and V_n is nominal shear strength computed by

$$V_n = V_c + V_s \qquad \dots\dots\dots\dots\dots\dots \quad 2.49$$

Where, V_c is nominal shear strength provided by concrete and V_s is nominal shear strength provided by shear reinforcement.

For members subjected to shear and flexure only,

$$V_c = \frac{\lambda\sqrt{f_c'}b_w d}{6} \qquad \dots\dots\dots\dots\dots\dots \quad 2.50$$

λ is used to account for the use of lightweight concrete, unless specifically noted otherwise, a modification factor λ appears as a multiplier of $\sqrt{f_c'}$ in all applicable equations. For normal weight concrete $\lambda = 1$. (ACI Code section 8.6.1).

According to ACI Code Section 11.4.6.1; A minimum area of shear reinforcement, $A_{v,min}$, shall be provided in all reinforced concrete flexural members where $V_u \geq 0.5\emptyset V_c$

The minimum web reinforcement shall be (ACI Code Section 11.4.6.3)

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} \qquad \dots\dots\dots\dots\dots\dots \quad 2.51$$

But not less than

$$A_{v,min} = \frac{b_w s}{3 f_{yt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ 2.52$$

To check if the section is large enough (ACI Code Section 11.4.7.9) gives the maximum shear in the stirrups as

$$V_{s,max} = \frac{2}{3}\left(\sqrt{f_c'} b_w d\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \ 2.53$$

To find the maximum stirrup spacing (ACI Code Section 11.4.5.1 and 11.4.5.3)

Spacing of shear reinforcement placed perpendicular to axis member shall not exceed $d/2$, or 600mm. If $V_s > 4\sqrt{f_c'} b_w d$, then the maximum spacing will be $d/4$, or 300mm whichever is less.

To compute the stirrup spacing required to resist the shear force (ACI Code Standard Section 11.4.7.2)

$$V_s = \frac{A_v f_{yt} d}{s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ 2.54$$

2.5.2. Eurocode General Procedure for Shear Design [11]

The shear resistance of a member with vertical shear reinforcement is equal to (Eurocode 2 Section 6.2.1)

$$V_{Rd} = V_{Rd,s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ 2.55$$

In regions of the member where $V_{ED} \leq V_{Rd,c}$ no calculated shear reinforcement is necessary.

In regions where $V_{ED} > V_{Rd,c}$ sufficient shear reinforcement should be provided in order that $V_{ED} \leq V_{Rd}$.

Where:-

V_{Ed} is the design shear force in the section considered resulting from external loading.

$V_{Rd,c}$ is the design shear resistance of the member without shear reinforcement.

$V_{Rd,s}$ is the design value of the shear force which can be sustained by the yielding shear reinforcement.

Members not requiring design shear reinforcement (Eurocode 2 Section 6.2.2) The design value for the shear resistance $V_{Rd,c}$ is given by:

$$V_{Rd,c} = [C_{Rd,c}k(100\rho_1 f_{ck})^{1/3} + k_1\sigma_{cp}]b_w d \qquad \dots\dots\dots\dots\dots\dots\ 2.56$$

With a minimum of

$$V_{Rd,c} = (v_{min} + k_1\sigma_{cp})b_w d \qquad \dots\dots\dots\dots\dots\dots\ 2.57$$

Where:

f_{ck} is in Mpa.

$k = 1 + \sqrt{200/d}\ \leq 2.0$, with d in mm.

$\rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$

A_{s1} is the area of the tensile reinforcement, which extends $\geq (l_{bd} + d)$ beyond the section considered (see Figure 2.21).

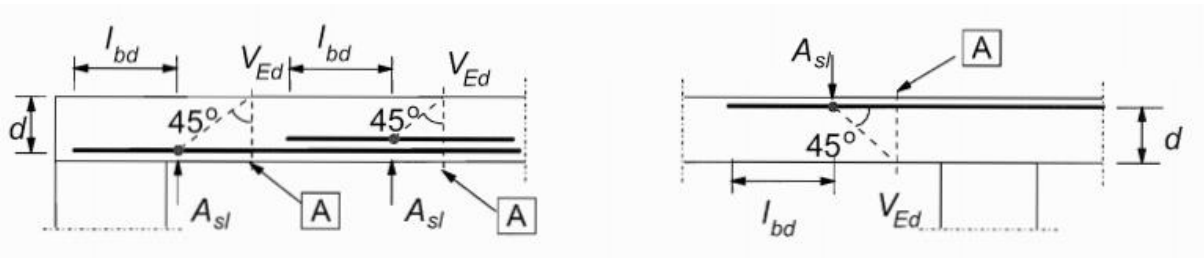
b_w is the smallest width of the cross-section in the tensile area (mm)

$\sigma_{cp} = N_{Ed}/A_c < 0.2f_{cd}$ (Mpa)

N_{Ed} is the axial force in the cross-section due to loading or prestressing (N)

A_c is the area of concrete cross section (mm2)

The values of $C_{Rd,c}, v_{min}$ and k_1 for use in a country may found in its National Annex. The recommended values for $C_{Rd,c} = 0.18/\gamma_c$, that for $v_{min} = 0.035k^{3/2}f_{ck}^{1/2}$ and that for $k_1 = 0.15$



A - Section considered

Figure 2.21 Definition of A_{s1}

Members requiring design shear reinforcement (Eurocode 2 Section 6.2.3)

The design of members with shear reinforcement is based on a truss model (Figure 2.22).

In Figure 2.22 the following notations are shown:

α is the angle between shear reinforcement and the beam axis perpendicular to the shear force (measured positive as shown in Figure 2.22)

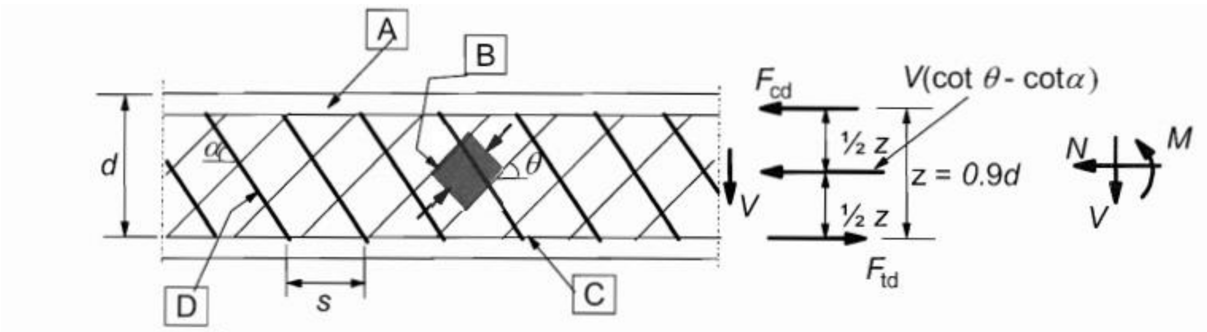
θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force

F_{td} is the design value of the tensile force in the longitudinal reinforcement

F_{ed} is the design value of the concrete compression force in the direction of the longitudinal member axis.

b_w is the minimum width between tension and compression chords

z is the inner lever arm, for a member with constant depth, corresponding to the bending moment in the element under consideration. In the shear analysis of reinforced concrete without axial force, the approximate value $z = 0.9d$ may normally be used.



A - Compression chord, B - struts, C - tensile chord, D - shear reinforcement

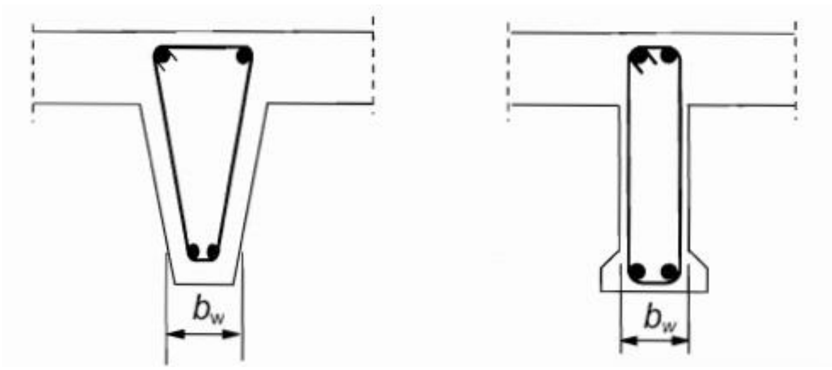


Figure 2.22 Truss model and notation for shear reinforcement members

Determine Maximum Design Shear Force

To prevent crushing of the concrete compression struts, the design shear force V_{ED} is limited by the maximum sustainable design shear force, $V_{Rd,max}$. If the design shear force exceeds this limit, a failure condition occurs. The maximum sustainable shear force is defined as:

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} / (\cot\theta + \tan\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.58$$

The coefficient α_{cw} takes account of the state of stress in the compression chord and is taken equal to one, which is recommended for non-prestressed structures. The strength reduction factor for concrete cracked in shear, v_1 is defined as:

$$v_1 = 0.6(1 - \frac{f_{ck}}{250}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.59$$

For most load combination θ can be taken as 45^0, otherwise take a value between 21.8^0 and 45^0.

Required shear Reinforcement

For members with vertical reinforcement, if V_{Ed} is greater than $V_{Rd,c}$ and less than $V_{Rd,max}$, the required shear reinforcement in the form of stirrups or ties per unit spacing, A_{sw}/s, is calculated as:

$$V_{Rd,s} = \frac{A_{sw}}{S} z f_{yd} \cot\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.60$$

The minimum shear reinforcement is given by:

$$\rho_{w,min} = 0.08\sqrt{f_{ck}}/f_{yk} \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.61$$

$$\frac{A_{sw,min}}{S} = \frac{0.08\sqrt{f_{ck}} b_w}{f_{yk}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.62$$

The maximum spacing between bars should not exceeded

$$S_{max} = 0.75d \leq 600mm \quad \ldots\ldots\ldots\ldots\ldots\ldots \; 2.63$$

3. ALGORITHM DEVELOPMENT

The procedure that's is followed by the modified compression filed theory to determine the response of reinforced concrete element as its shown below its an iterative procedure too tedious and hard to solve it by hand, but by converting this procedure in to an algorithm the solution can be found easily.

3.1. Solution technique for determining response Reinforced concrete beam.

Step 1. Define Section properties.

f_c' - Concrete cylinder strength.

ϵ_c' - Concrete strain at peak stress.

E_c – Young's modulus of concrete.

f_{yy} – Yield strength of transverse steel.

E_s = 200,000Mpa

H – Overall depth of the beam.

m –Number of concrete layers.

n – Number of longitudinal steel components.

If the initial tangent stiffness of the concrete is known, or a stress-strain curve from a cylinder test is available an estimate of the strain at peak stress may be made. If neither are available, then the following method is suggested. [1], [12], [13] or [14]

$$E_c = 3320\sqrt{-f_c'} + 6900 \text{ Mpa}$$

$$\epsilon_c' = \frac{-2f_c'}{E_c} \times \frac{n_c}{n_c - 1}$$

$$n_c = 0.8 + \frac{-f_c'}{17}$$

Where: - n_c- Curve fit parameter

The number of concrete layer can be taken greater than or equal to ten for fair response results , if a large number of concrete layer is taken, then it will required more computation time, so it will make the program to run slower. An estimation must be made to avoid errors and still not too much large to make the program run faster.

Step 2. Define properties of each concrete layer.

b_i - Width of the layer.

h - Height of the layer.

ρ_y - Amount of transverse steel, as a ratio of the concrete area.

y_{ci} - Distance from the centroid of the layer to the top fiber.

Step 3. Define properties of each longitudinal steel components.

A_{sj} – Cross sectional area.

y_{yxj} - Yield strength of longitudinal bars.

y_{sj} - Distance from the centroid of the steel components to the top fiber.

Steel is assumes to have an ultimate strength of 50% higher than the yield strength and 10% strain at the peak stress.

Step 4. Specify Section loads.

V – Shear force.

M – Moment.

To find the maximum shear Strength of the section we can start the shear force value from zero and increasing the value up to failure. In this thesis it's assumed the beam is subjected to shear force and bending moment only.

$V = V_i + \Delta V$

N – Axial force.

$N = 0$

Step 5. Estimate shear flow distribution (q).

There are three different methods that can be used to estimate the shear flow distribution according to Vecchio and Collins[1] ,dual section analysis, constant shear flow and parabolic shear strain.

The dual section analysis uses two section that are s distance apart approximately $H/6$. Both section are analyzed for the same shear stressed distribution, satisfying section equilibrium in each case. The assumed shear stresses are then checked by examining the static equilibrium of each layer. If the shear stresses assumed initially are not correspond to the one calculated then revised the assumed shear flow and repeat the analysis. Even if the dual section analysis can represent the actual response it will take too much computational time and as reported by Bentz[13] it show some instability and cause the algorithm not to converge.

The parabolic shear strain assumption gives a good result for axially loaded members, but for members subjected to moments, it gives as more diverted result when we compare to dual section analysis as shown in the Figure 3.1.

A constant shear flow analysis gives a more precise results for members subjected to shear and moments compare to dual section analysis as shown in the Figure 3.1 and it can produce a stabile algorithm than dual section analysis.

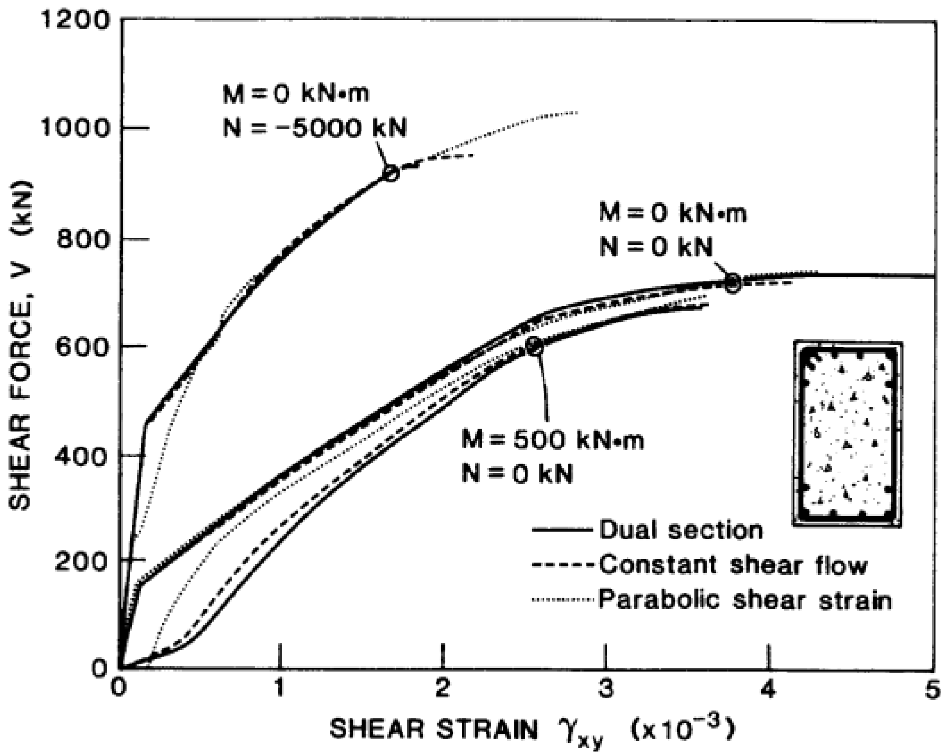


Figure 3.1 Shear load-deformation response of a beam using dual section, constant shear flow and parabolic shear strain methods. [1]

In this thesis I use a constant shear flow analysis because of the beam section here are subjected to shear and as it's shown in the above figure it will give us a very close result as dual section analysis and it will give us a more stable algorithm.

$$q = \frac{V}{H}$$

$$V_{xy} = \frac{q}{b_i}$$

Where:-

q - Shear flow.

V_{xy} - Shear stress

Step 6. Estimate longitudinal strain gradient. Define top and bottom fiber strains, ϵ_{top} and ϵ_{bot} respectively.

Using longitudinal strain gradient as a longitudinal strain controlling variable we can define the longitudinal strain at the centroid of the section (ϵ_{sx}) using geometrical relationships of the longitudinal strain distribution diagram as it is shown in Figure 3.2. Choose the value of ϵ_{sx} to start the calculation start with zero.

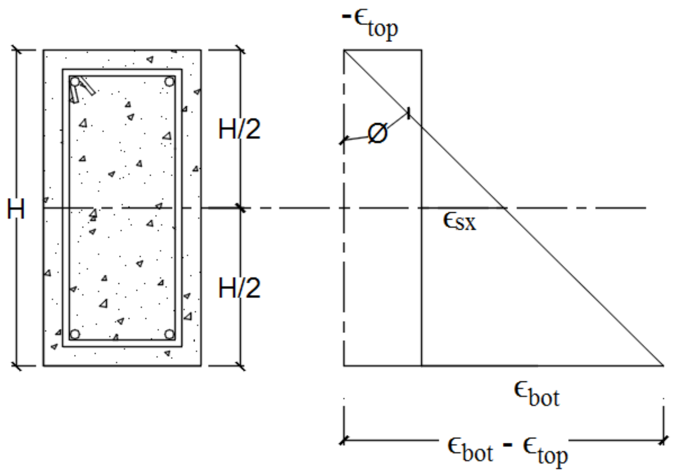


Figure 3.2 Longitudinal strain distribution

$$\tan \emptyset = \frac{\epsilon_{sx} - \epsilon_{top}}{H/2} = \frac{\epsilon_{bot} - \epsilon_{top}}{H}$$

$$\epsilon_{top} = \epsilon_{sx} - \frac{H}{2} \tan \emptyset$$

$$\epsilon_{bot} = \epsilon_{top} + H \tan \emptyset$$

Step 7. Determine longitudinal steel stresses.

$$f_{sxj} = \epsilon_{sxj} E_s \leq f_{yxj}$$

Step 8. Determine each concrete layer stress-strain conditions using modified compression field theory (MCFT).

Step 8.1. Given.

Longitudinal strain for a layer at a depth of y_i from the top fiber, ϵ_{xi}.

$$\epsilon_{xi} = \epsilon_{top} + \frac{(\epsilon_{bot} - \epsilon_{top})}{H} . y_i$$

Normal shear stress, V_{xy}.

$$V_{xy} = \frac{V}{H . b_i}$$

Step 8.2. Estimate concrete principal tensile strain, ϵ_1.

Give an initial values to start the iteration.

Step 8.3. Concrete principal tensile stress.

Calculate average crack width w using Eq. (2.40).

Calculate average tension in the concrete f_{c1} using Eq. (2.29) and (2.30), subject to the condition that

$$f_{c1} \leq f_{c1,max}$$

$$f_{c1,max} = v_{cimax}(0.18 + 0.3k^2)\tan\theta + \rho_{sy}(f_{yy} - f_{sy})$$

Where $k = 1.64 - 1/\tan\theta$, but $k \geq 0$; and where v_{cimax} is given by Eq. (2.39).

Step 8.4. Determine principal compressive stress direction θ.

(i) Concrete transverse compressive stress.

$$f_{cy} = -\rho_y f_{sy}$$

$$f_{cy} = f_{c1} - V_{xy}\tan\theta$$

(ii) Transverse tensile strain

$$\epsilon_y = \epsilon_1 - (\epsilon_1 - \epsilon_x)\tan^2\theta$$

(iii) Transverse steel tensile stress.

$f_{sy} = \epsilon_y E_s$, where $f_{sy} \leq f_{yy}$

We can find the angle positively using the above relationships.

$$f_{cy} = -\rho_y f_{sy} = f_{c1} - V_{xy}\tan\theta$$

$$-\rho_y \epsilon_y E_s = f_{c1} - V_{xy}\tan\theta$$

$$-\rho_y E_s(\epsilon_1 - (\epsilon_1 - \epsilon_x)\tan^2\theta) = f_{c1} - V_{xy}\tan\theta$$

$$-\rho_y E_s \epsilon_1 + \rho_y E_s(\epsilon_1 - \epsilon_x)\tan^2\theta - f_{c1} + V_{xy}\tan\theta = 0$$

$$\big(E_s(\epsilon_1 - \epsilon_x)\big)\tan^2\theta + \left(\frac{V_{xy}}{\rho_y}\right)\tan\theta - \left(E_s\epsilon_1 + \frac{f_{c1}}{\rho_y}\right) = 0$$

$$\tan\theta = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Where: - $A = \big(E_s(\epsilon_1 - \epsilon_x)\big)$

$$B = \left(\frac{V_{xy}}{\rho_y}\right)$$

$$C = -\left(E_s\epsilon_1 + \frac{f_{c1}}{\rho_y}\right)$$

Step 8.5. Determine remaining strain quantities.

(i) Transverse tensile strain.

$$\epsilon_y = \epsilon_1 - (\epsilon_1 - \epsilon_x)\tan^2\theta$$

(ii) Principal compressive strain.

$$\epsilon_2 = \epsilon_x - (\epsilon_1 - \epsilon_x)\tan^2\theta$$

(iii) Normal shear strain.

$$\gamma_{xy} = 2(\epsilon_1 - \epsilon_x)\tan\theta$$

Step 8.6. Determine remaining stress conditions.

(i) Concrete longitudinal compressive stress.

$$f_{cx} = f_{c1} - V_{xy}/\tan\theta$$

(ii) Transverse steel tensile stress.

$$f_{sy} = \epsilon_y E_s$$

$$f_{sy} \le f_{yy}$$

(iii) Principal compressive stress calculated from the given shear stress.

$$f'_{c2} = f_{c1} - V_{xy}\left[\tan\theta + \frac{1}{\tan\theta}\right]$$

(iv) Concrete principal compressive stress.

$$f_{c2} = f_{c2max}\left[2\left(\frac{\epsilon_2}{\epsilon'_c}\right) - \left(\frac{\epsilon_2}{\epsilon'_c}\right)^2\right]$$

$$\beta = 0.8 - 0.34\ \epsilon_1/\epsilon'_c$$

$$f_{c2max} = \frac{f'_c}{\beta} \le 1.0$$

Check that $f_{c2}/f_{c2max} \le 1.0$. If it's greater than 1.0, then solution is not possible; choose θ closer to 45^0 or increase ϵ_1.

Step 8.7. Check equilibrium.

Is $f_{c2} = f'_{c2}$?

If yes, go to Step 8.8

If no, go to Step 8.2

Step 8.8. Calculate the stresses on crack v_{ci} and f_{ci}.

$\Delta f_{c1} = f_{c1} - \rho_y (f_{yy} - f_{sy})$

If $\Delta f_{c1} \leq 0$, then $v_{ci} = 0$ and $f_{ci} = 0$. Go to Step 8.9.

If $\Delta f_{c1} > 0$, then $C = \frac{\Delta f_{c1}}{\tan\theta} - 0.18 v_{cimax}$

If $C \leq 0$, then $f_{ci} = 0$ and $v_{ci} = \Delta f_{c1} / \tan\theta$

Otherwise

$A = 0.82 / v_{cimax}$ and $B = \frac{1}{\tan\theta} - 1.64$

$f_{c1} = (-B - \sqrt{B^2 - 4AC}) / 2A$

$v_{ci} = (f_{c1} + \Delta f_{c1}) / \tan\theta$

Step 8.9. Calculate transverse reinforcement stresses at crack f_{sycr}.

$f_{sycr} = f_{sy} + (f_{c1} + f_{ci} - v_{ci} \tan\theta) / \rho_y$

Step 8.10. If at failure:

i. f_{c1} is limited by the condition in Step 8.3, then slipping on the crack govern the failure.
ii. f_{c2} is limited by f_{c2max}, then crushing or shear failure of the concrete governs.
iii. If $f_{sycr} \geq f_{yy}$, then the reinforcement is not capable of transmitting the loads across the crack.

Step 9. Compute resulting sectional loads.

$$N' = \sum_{i=1}^{m} f_{cxi} b_i h_i + \sum_{j=1}^{n} f_{sxj} A_{sj}$$

$$M' = \sum_{i=1}^{m} f_{cxi} b_i h_i (y_{ci} - \bar{y}) + \sum_{j=1}^{n} f_{sxj} A_{sj} (y_{sj} - \bar{y})$$

Step 10. Check sectional equilibrium

Is $N' = 0$ and $M' = M$

If yes, go to Step 11

If no, go to Step 6; to iterate curvature values until the moment and axial load equilibrium is reached.

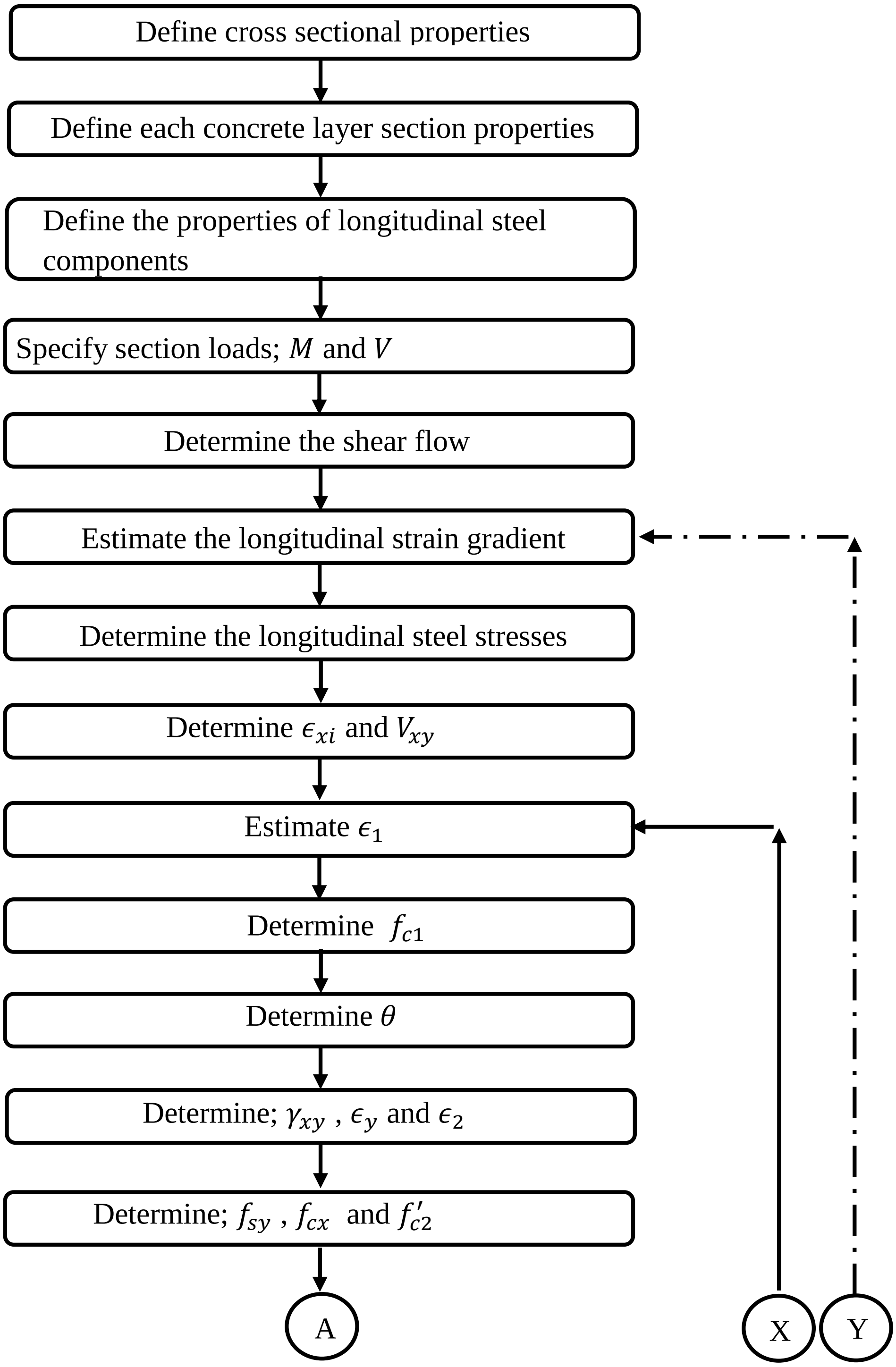
Step 11. Extract required information regarding behavioral response to given loading condition.

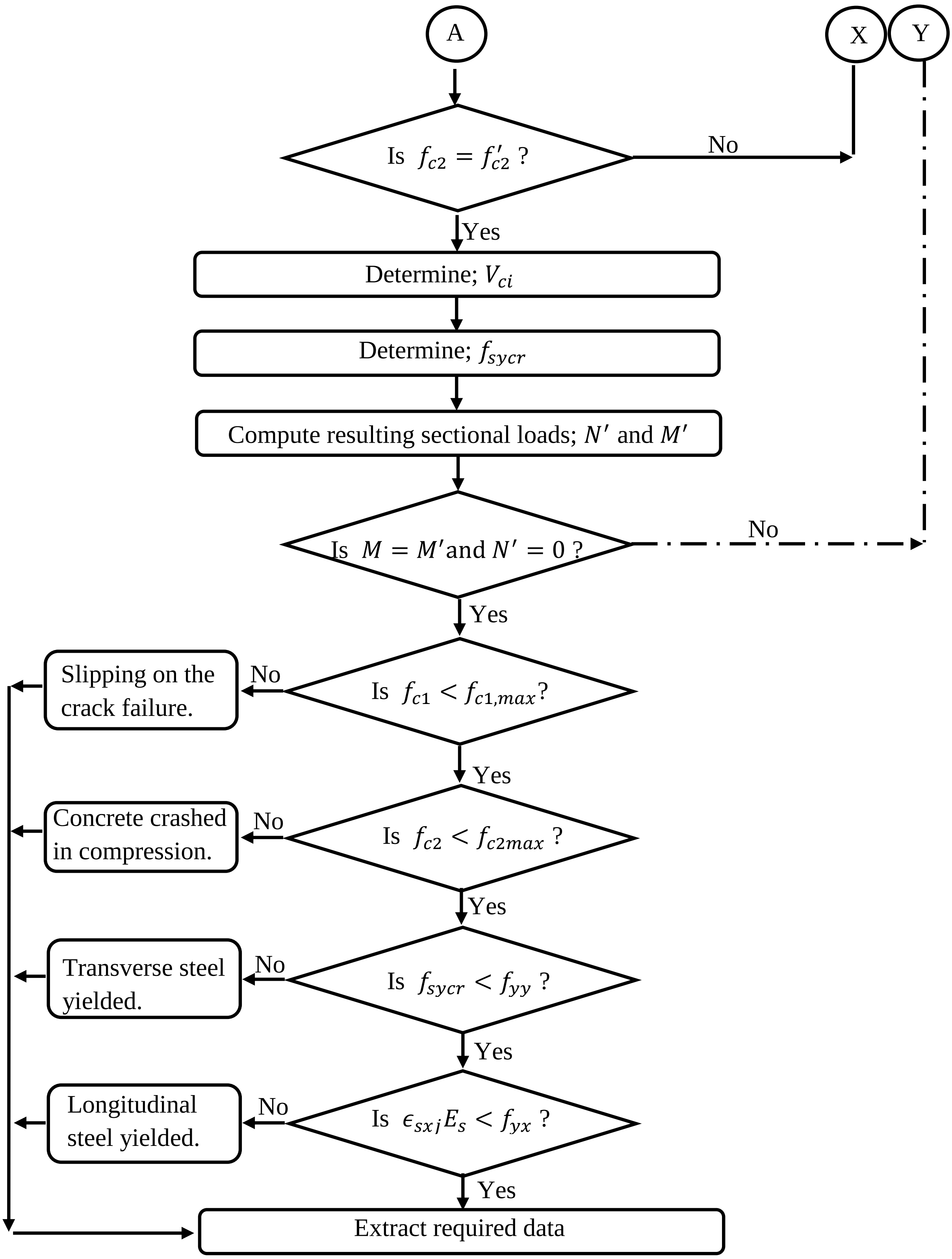
Repeat step 4 through 10 for an increment of shear force and the corresponding bending moment value up to failure.

$V = V_i + \Delta V$

3.2. Flow Chart for the response of rectangular reinforced concrete beam.

The solution procedure that is described earlier is summarized here and a computer program source code can be found in the Appendix.



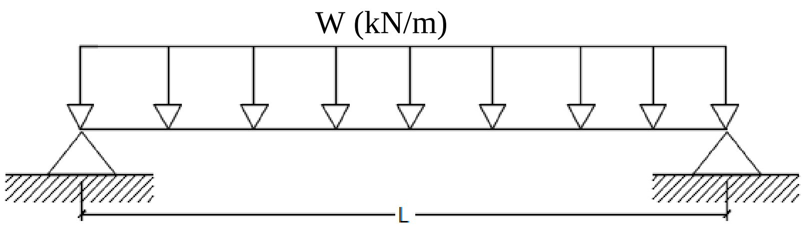


Note: - To show the continuity of the flow chart, I used solid line, broken line and letter A, X and Y.

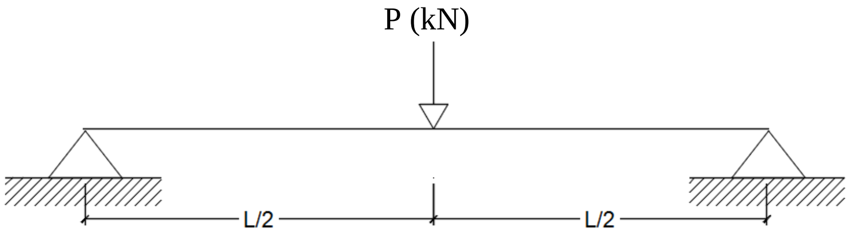
4. ANALYSIS

We consider two simply supported and one cantilever beam with different loading conditions. An estimation is made for the beam design load and flexural reinforcement to avoid the difference in load factors and flexural reinforcement between different codes.

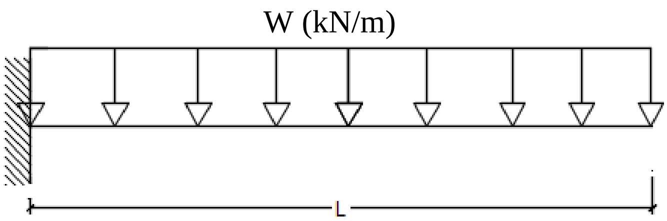
Here is the detail of the three beams:



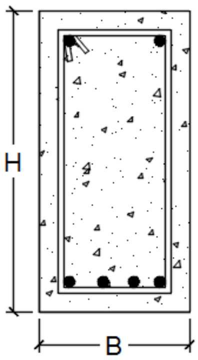
Beam 1
W = 30 kN/m
L = 6m
Maximum Shear
Vmax = 90kN



Beam 2
P = 80 kN
L = 4m
Maximum Shear
Vmax = 40kN



Beam 3
W = 100 kN/m
L = 1.7m
Maximum Shear
Vmax = 170kN



Beam 1	Beam 2	Beam 3
H = 400mm	H = 300mm	H = 350mm
B = 250mm	B = 200mm	B = 250mm
Reinforcement	Reinforcement	Reinforcement
Top 2Ø10	Top 2Ø10	Top 4Ø24
Bottom 4Ø24	Bottom 3Ø20	Bottom 2Ø10
Material	Material	Material
C20/25 and S300	C20/25 and S300	C20/25 and S300

Figure 4.1 Beam Detail for analysis

To see the difference between the ACI and EuroCode procedures and to compare their output with MCFT results, the analysis will follows the outline below:

I. We design the beams using ACI procedure.
II. Design the beams using EuroCode procedures.
III. Calculate the shear strength using MCFT.

4.1. Design for Shear using ACI procedure.

Design of beam 1

Figure 4.1 shows the cross section of a rectangular simply supported beam. This beam is subjected to a design load of 30kN/m and column size of (40cmX40cm).

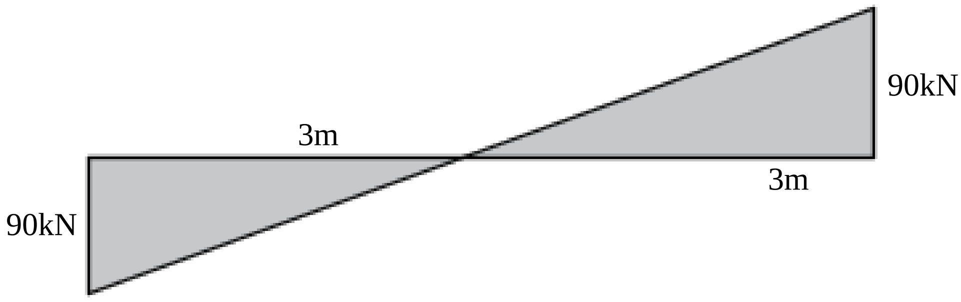


Figure 4.2 Shear force diagram of beam 1

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø8 for shear reinforcement

$$d = 400 - 25 - 8 - 12 = 355mm = 0.355m$$

$$V_u = 90 * \left(\frac{3 - 0.2 - 0.355}{3}\right) = 73.35\text{kN}$$

$$V_n = \frac{V_u}{Ø} = \frac{73.35}{1} = 73.35\text{kN}$$

Shear reinforcement are not required if $V_n \leq V_c/2$

$$V_c = \frac{\lambda\sqrt{f_c'}b_w d}{6} = \frac{1 \times \sqrt{20} \times 250mm \times 355mm}{6} = 66150.34\text{N} = 66.15\text{kN}$$

$V_n = 73.35 > \frac{V_c}{2} = 33.08$kN, shear reinforcement are required.

Check if the section is large enough?

$$V_{s,max} = \frac{2}{3}\left(\sqrt{f_c'}b_w d\right) = \frac{2}{3}\left(\sqrt{20} * 250 * 355\right) = 264601.38\text{N} = 264.6\text{kN}$$

$$\left(\frac{V_u}{Ø}\right)_{max} = V_c + V_{s,max} = 66.15 + 264.6 = 330.75\text{kN}$$

$V_u/Ø = 73.35\text{kN} < 330.75\text{kN}$, the section is large enough.

Using Ø8 double-leg stirrups.

$$A_v = 2(\pi \times 4^2) = 100.53mm^2$$

We anchored the stirrups by a standard 135° hook around a longitudinal bars.

Find the maximum stirrup spacing.

$$S_{max} \leq \begin{cases} \begin{cases} 0.5d \\ 600 \end{cases}, \text{if } V_s \leq (1/3)\sqrt{f_c'}b_w d \\ \begin{cases} 0.25d \\ 300 \end{cases}, \text{if } V_s > (1/3)\sqrt{f_c'}b_w d \end{cases}$$

$$\left(\frac{1}{3}\right)\sqrt{f_c'}b_w d = \left(\frac{1}{3}\right)\sqrt{20} \times 250 \times 355 = 132300.69N = 132.3\text{kN}$$

$$V_s = \frac{V_u}{Ø} - V_c = 73.35 - 66.15 = 7.2\text{kN} < 132.3\text{kN}$$

$$S_{max} \leq \begin{cases} 0.5d = 0.5 * 355 = 177.5 \\ 600 \end{cases} = 170mm$$

Maximum spacing based on minimum shear reinforcement

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w S}{f_{yt}} \geq \frac{1}{3}\frac{b_w S}{f_{yt}}$$

$$\frac{1}{16}\sqrt{20} = 0.28 < \frac{1}{3}$$

$$S_{max} = \frac{3A_v f_{yt}}{b_w} = \frac{3 \times 100.53 \times 300}{250} = 361.91mm$$

Therefore, the maximum spacing based on the beam depth governs.

$$S_{max} = 170mm$$

Compute the stirrup spacing required to resist the shear force.

$$S = \frac{A_v f_{yt} d}{\frac{V_u}{Ø} - V_c} = \frac{100.53 \times 300 \times 355}{(73.35 - 66.15) \times 1000} = 1487mm$$

$$S = 170mm$$

The total shear strength of the beam for the calculated spacing will be:

$$V_c + \frac{A_v f_{yt} d}{S} = 66.15 + \frac{100.53 \times 300 \times 355}{170 \times 1000} = 129.13\text{kN}$$

Design of beam 2

Figure 4.1 shows the cross section of a rectangular simply supported beam. This beam is subjected to a design load of 80kN.

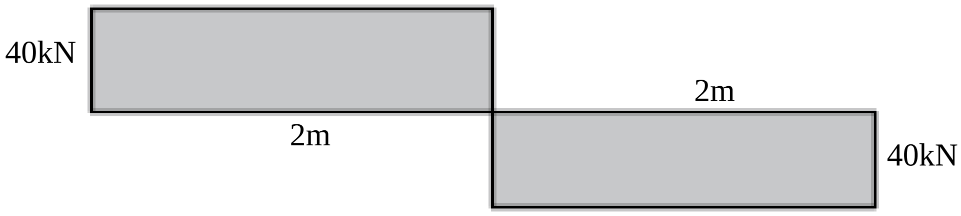


Figure 4.3 Shear force diagram of beam 2

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø6 for shear reinforcement

$$d = 300 - 25 - 6 - 10 = 259mm = 0.259m$$

$$V_u = 40\text{kN}$$

$$V_n = \frac{V_u}{\text{Ø}} = \frac{40}{1} = 40\text{kN}$$

Shear reinforcement are not required if $V_n \leq \frac{V_c}{2}$

$$V_c = \frac{\lambda\sqrt{f_c'}b_w d}{6} = \frac{1 \times \sqrt{20} \times 200mm \times 259mm}{6} = 38609.44N = 38.61\text{kN}$$

$V_n = 40 > V_c/2 = 19.31\text{kN}$, shear reinforcement are required.

Check if the section is large enough?

$$V_{s,max} = \frac{2}{3}\left(\sqrt{f_c'}b_w d\right) = \frac{2}{3}\left(\sqrt{20} * 200 * 259\right) = 154437.76\text{N} = 154.44\text{kN}$$

$$\left(\frac{V_u}{\text{Ø}}\right)_{max} = V_c + V_{s,max} = 38.61 + 154.44 = 193.05\text{kN}$$

$$V_u/\text{Ø} = 40\text{kN} < 193.05\text{kN the section is large enough.}$$

Using Ø6 double-leg stirrups.

$$A_v = 2(\pi \times 3^2) = 56.55mm^2$$

We anchored the stirrups by a standard 135° hook around a longitudinal bars.

Find the maximum stirrup spacing.

$$\left(\frac{1}{3}\right)\sqrt{f_c'}b_w d = \left(\frac{1}{3}\right)\sqrt{20} \times 200 \times 259 = 77218.88\text{N} = 77.22\text{kN}$$

$$V_s = \frac{V_u}{\text{Ø}} - V_c = 40 - 38.61 = 1.39\text{kN} < 77.22\text{kN}$$

$$S_{max} \leq \begin{cases} 0.5d = 0.5 * 259 = 129.5 \\ 600 \end{cases} = 129.5mm$$

Maximum spacing based on minimum shear reinforcement

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w S}{f_{yt}} \geq \frac{1}{3}\frac{b_w S}{f_{yt}}$$

$$\frac{1}{16}\sqrt{20} = 0.28 < \frac{1}{3}$$

$$S_{max} = \frac{3A_v f_{yt}}{b_w} = \frac{3 \times 56.55 \times 300}{200} = 254.475mm$$

Therefore, the maximum spacing based on the beam depth governs.

$$S_{max} = 120mm$$

Compute the stirrup spacing required to resist the shear force.

$$S = \frac{A_v f_{yt} d}{\frac{V_u}{Ø} - V_c} = \frac{56.55 \times 300 \times 259}{(40 - 38.61) \times 1000} = 3161.104mm$$

$$S = 120mm$$

The total shear strength of the beam for the calculated spacing will be:

$$V_c + \frac{A_v f_{yt} d}{S} = 38.61 + \frac{56.55 \times 300 \times 259}{120 \times 1000} = 75.23\text{kN}$$

Design of beam 3

Figure 4.1 shows the cross section of a rectangular cantilever beam. This beam is subjected to a design load of 100kN/m and column size of (40cmX40cm).

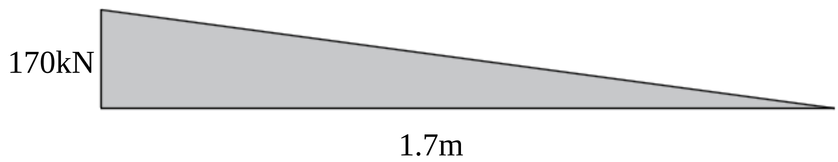


Figure 4.4 Shear force diagram of beam 3

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø8 for shear reinforcement

$$d = 350 - 25 - 8 - 12 = 305mm = 0.305m$$

$$V_u = 170 * \left(\frac{1.7 - 0.2 - 0.305}{1.7}\right) = 119.5\text{kN}$$

$$V_n = \frac{V_u}{Ø} = \frac{119.3}{1} = 119.5\text{kN}$$

Shear reinforcement are not required if $V_n \leq \frac{V_c}{2}$

$$V_c = \frac{\lambda\sqrt{f_c'} b_w d}{6} = \frac{1 \times \sqrt{20} \times 250mm \times 305mm}{6} = 56833.39\text{N} = 56.83\text{kN}$$

$V_n = 119.5 > V_c/2 = 28.42$kN, shear reinforcement are required.

Check if the section is large enough?

$$V_{s,max} = \frac{2}{3}\left(\sqrt{f_c'}b_w d\right) = \frac{2}{3}\left(\sqrt{20} * 250 * 305\right) = 227333.58\text{N} = 227.33\text{kN}$$

$$\left(\frac{V_u}{Ø}\right)_{max} = V_c + V_{s,max} = 56.83 + 227.33 = 284.16\text{kN}$$

$$V_u/Ø = 119.5\text{kN} < 284.16\text{kN the section is large enough.}$$

Using Ø8 double-leg stirrups.

$$A_v = 2(\pi \times 4^2) = 100.53mm^2$$

We anchored the stirrups by a standard 135° hook around a longitudinal bars.

Find the maximum stirrup spacing.

$$S_{max} \leq \begin{cases} \begin{cases} 0.5d \\ 600 \end{cases}, \text{if } V_s \leq (1/3)\sqrt{f_c'}b_w d \\ \begin{cases} 0.25d \\ 300 \end{cases}, \text{if } V_s > (1/3)\sqrt{f_c'}b_w d \end{cases}$$

$$\left(\frac{1}{3}\right)\sqrt{f_c'}b_w d = \left(\frac{1}{3}\right)\sqrt{20} \times 250 \times 305 = 113666.79\text{N} = 113.67\text{kN}$$

$$V_s = \frac{V_u}{Ø} - V_c = 119.5 - 56.83 = 62.67\text{kN} < 113.67\text{kN}$$

$$S_{max} \leq \begin{cases} 0.5d = 0.5 * 305 = 152.5 \\ 600 \end{cases} = 152.5mm$$

Maximum spacing based on minimum shear reinforcement

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w S}{f_{yt}} \geq \frac{1}{3}\frac{b_w S}{f_{yt}}$$

$$\frac{1}{16}\sqrt{20} = 0.28 < \frac{1}{3}$$

$$S_{max} = \frac{3A_v f_{yt}}{b_w} = \frac{3 \times 100.53 \times 300}{250} = 361.91mm$$

$$S_{max} = 152.5mm$$

Compute the stirrup spacing required to resist the shear force.

$$S = \frac{A_v f_{yt} d}{\frac{V_u}{Ø} - V_c} = \frac{100.53 \times 300 \times 305}{(119.5 - 56.83) \times 1000} = 146.78mm$$

$$S = 140mm$$

The total shear strength of the beam for the calculated spacing will be:

$$V_c + \frac{A_v f_{yt} d}{S} = 56.83 + \frac{100.53 \times 300 \times 305}{140 \times 1000} = 122.53\text{kN}$$

4.2. Design for Shear using EuroCode procedure.

Design of beam 1

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø8 for shear reinforcement

$$d = 400 - 25 - 8 - 12 = 355mm = 0.355m$$

$$V_{ED} = 90 * \left(\frac{3 - 0.2 - 0.355}{3}\right) = 73.35\text{kN}$$

Shear reinforcement are not required if $V_{ED} \leq V_{Rd,c}$

$k = 1 + \sqrt{200/d} = 1 + \sqrt{200/355} = 1.751 < 2.0$ OK

$A_{s1} = 4 \times \pi \times 12^2 = 1809.56mm^2$

$\rho_1 = \frac{A_{s1}}{b_w d} = \frac{1809.56}{250 \times 355} = 0.021 > 0.02$ Not OK take 0.02

$\sigma_{cp} = N_{Ed}/A_c = 0/(250 \times 400) = 0$ (Mpa)

$\gamma_c = 1$

$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1} = 0.18$

$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} = 0.035 \times 1.751^{3/2} \times 20^{1/2} = 0.3627$

$k_1 = 0.15$

$V_{Rd,c} = \left[C_{Rd,c}k(100\rho_1 f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right]b_w d \geq (v_{min} + k_1\sigma_{cp})b_w d$

$= \left[0.18 \times 1.751(100 \times 0.02 \times 20)^{\frac{1}{3}} + 0.15 \times 0\right]250 \times 355 = 95663.66\text{N}$

$\geq 0.3627 \times 250 \times 355 = 32189.625\text{N}$ OK

$V_{ED} = 73.35\text{kN} > V_{Rd,c} = 95.66\text{kN}$, shear reinforcement are required.

Check if the section is large enough?

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{ck}/1.5(\cot\theta + \tan\theta)$$

$\alpha_{cw} = 1$

$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.6(1 - 20/250) = 0.552$

$z = 0.9d = 0.9 \times 355 = 319.5mm$

$$V_{Rd,max} = 1 \times b_w \times 0.9d \times \frac{0.6\left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)} = \frac{0.54 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)}$$

$21.8° \leq \theta \leq 45°$

For $\theta = 21.8°$

$$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 21.8° + \tan 21.8°} = 0.186$$

For $\theta = 45°$

$$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 45° + \tan 45°} = 0.27$$

$$V_{Rd,\max(21.8°)} = 0.186 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck} = 0.186 \times 250 \times 355 \times \left(1 - \frac{20}{250}\right) 20 = 303738\text{N}$$

$V_{ED} = 73.35KN < V_{Rd,\max(21.8°)} = 303.74\text{kN}$ OK

$$V_{Rd,\max(45°)} = 0.27 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck} = 0.27 \times 250 \times 355 \times \left(1 - \frac{20}{250}\right) 20 = 440910\text{N}$$

$V_{ED} = 73.35\text{kN} < V_{Rd,\max(45°)} = 440.09\text{kN}$ OK

Determine angle θ

$$V_{ED} = V_{Rd,max} = \frac{0.54 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)}$$

$$\frac{1}{(\cot\theta + \tan\theta)} = \sin\theta\cos\theta = 0.5\sin 2\theta$$

$$V_{ED} = V_{Rd,max} = (0.5\sin 2\theta) \times 0.54 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$V_{ED} = V_{Rd,max} = 0.27 \times \sin 2\theta \times b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$\sin 2\theta = V_{ED}/0.27 \times b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$\theta = 0.5\sin^{-1}\left\{V_{ED}/0.27 \times b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}\right\}$$

Or

$$\theta = 0.5\sin^{-1}\{V_{ED}/V_{Rd,\max(45°)}\} = 0.5\sin^{-1}\left(\frac{73.35}{440.09}\right) = 4.80° < 21.8°$$

Therefore take $\theta = 21.8°$ and $V_{Rd,max} = V_{Rd,\max(21.8°)} = 303.74\text{kN}$

The maximum spacing between bars should not exceeded

$$S_{max} = 0.75d = 0.75 \times 355 = 266.25mm \leq 600mm$$

$S_{max} = 266.25mm$

Compute the stirrup spacing required to resist the shear force.

$$\frac{A_{sw}}{S} = \frac{V_{Ed}}{0.9 d f_{yk} \cot\theta} = \frac{73.35 \times 1000}{0.9 \times 355 \times 300 \times \cot 21.8°} = 0.306$$

The minimum shear reinforcement is given by:

$A_{sw} = 2 \times \pi \times 4^2 = 100.53mm^2$, for a double-leg stirrups.

$$\frac{A_{sw,min}}{S} = \frac{0.08\sqrt{f_{ck}}b_w}{f_{yk}} = \frac{0.08 \times 250 \times \sqrt{20}}{300} = 0.298$$

$$\frac{A_{sw}}{S} = 0.306 > \frac{A_{sw,min}}{S} = 0.298 \quad \text{OK}$$

$$S = \frac{A_{sw}}{0.306} = \frac{100.53}{0.306} = 328.53mm > S_{max} = 266.25mm$$

Therefore take $S = 260mm$ (use Ø8 c/c 260mm).

The total shear strength of the beam will be:

$$V_{Rd} = V_{Rd,s} = \frac{A_{sw}}{S} z f_{yd} \cot\theta = \frac{A_{sw}}{S} \times \left(\frac{0.9}{1}\right) d \times f_{yk} \cot\theta$$

$$= \left(\frac{100.53}{260}\right) \times \left(\frac{0.9}{1}\right) \times 355 \times 300 \times \cot 21.8° \times 10^{-3} = 92.66\text{kN}$$

<u>Design of beam 2</u>

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø6 for shear reinforcement

$$d = 300 - 25 - 6 - 10 = 259mm = 0.259m$$

$$V_{ED} = 40\text{kN}$$

Shear reinforcement are not required if $V_{ED} \leq V_{Rd,c}$

$k = 1 + \sqrt{200/d} = 1 + \sqrt{200/259} = 1.88 < 2.0$ OK

$A_{s1} = 3 \times \pi \times 10^2 = 942.48mm^2$

$\rho_1 = \frac{A_{s1}}{b_w d} = \frac{942.48}{200 \times 259} = 0.0182 < 0.02$ OK

$\sigma_{cp} = N_{Ed}/A_c = 0/(200 \times 300) = 0$ (Mpa)

$\gamma_c = 1$

$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1} = 0.18$

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035 \times 1.88^{3/2} \times 20^{1/2} = 0.404$

$k_1 = 0.15$

$$V_{Rd,c} = \left[C_{Rd,c} k (100\rho_1 f_{ck})^{\frac{1}{3}} + k_1 \sigma_{cp}\right] b_w d \geq (v_{min} + k_1\sigma_{cp}) b_w d$$

$$= \left[0.18 \times 1.88 (100 \times 0.0182 \times 20)^{\frac{1}{3}} + 0.15 \times 0\right] 200 \times 259 = 58093.46\text{N}$$

$$\geq 0.41 \times 200 \times 259 = 21238\text{N} \quad \text{OK}$$

$V_{ED} = 40\text{kN} < V_{Rd,c} = 58.09\text{kN}$, shear reinforcement are not required.

Check if the section is large enough?

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{ck} / 1.5(\cot\theta + \tan\theta)$$

$\alpha_{cw} = 1$

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.6(1 - 20/250) = 0.552$$

$z = 0.9d = 0.9 \times 259 = 233.1mm$

$$V_{Rd,max} = 1 \times b_w \times 0.9d \times \frac{0.6\left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)} = \frac{0.54 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)}$$

$21.8° \leq \theta \leq 45°$

For $\theta = 21.8°$

$$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 21.8° + \tan 21.8°} = 0.186$$

For $\theta = 45°$

$$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 45° + \tan 45°} = 0.27$$

$$V_{Rd,\max(21.8°)} = 0.186 b_w d \left(1 - \frac{f_{ck}}{250}\right) f_{ck} = 0.186 \times 200 \times 259 \times \left(1 - \frac{20}{250}\right) 20$$
$$= 177280.32\text{N}$$

$V_{ED} = 30\text{kN} < V_{Rd,\max(21.8°)} = 177.28\text{kN}$ OK

The maximum spacing between bars should not exceeded

$$S_{max} = 0.75d = 0.75 \times 259 = 194.25mm \leq 600mm$$

$S_{max} = 194.25mm$

The minimum shear reinforcement is given by:

$A_{sw} = 2 \times \pi \times 3^2 = 56.55mm^2$, for a double-leg stirrups.

$$\frac{A_{sw,min}}{S} = \frac{0.08\sqrt{f_{ck}}b_w}{f_{yk}} = \frac{0.08 \times 200 \times \sqrt{20}}{300} = 0.239$$

$S = \frac{A_{sw}}{0.239} = \frac{56.55}{0.239} = 236.61mm > S_{max} = 194.25mm$

Therefore take $S = 190mm$ (use Ø6 c/c 190mm).

The total shear strength of the beam will be:

$$V_{Rd} = V_{Rd,s} = \frac{A_{sw}}{S} z f_{yd} \cot\theta = \frac{A_{sw}}{S} \times \left(\frac{0.9}{1}\right) d \times f_{yk} \cot\theta$$

$$= \left(\frac{56.55}{190}\right) \times \left(\frac{0.9}{1}\right) \times 259 \times 300 \times \cot 21.8° \times 10^{-3} = 52.04\text{kN}$$

<u>Design of beam 3</u>

The critical shear section is at *d* distance from the support

Using 25mm concrete cover and Ø8 for shear reinforcement

$$d = 350 - 25 - 8 - 12 = 305mm = 0.305m$$

$$V_{ED} = 170 * \left(\frac{1.7 - 0.2 - 0.305}{1.7}\right) = 119.5\text{kN}$$

Shear reinforcement are not required if $V_{ED} \le V_{Rd,c}$

$k = 1 + \sqrt{200/d} = 1 + \sqrt{200/305} = 1.81 < 2.0$ OK

$A_{s1} = 4 \times \pi \times 12^2 = 1809.56mm^2$

$\rho_1 = \frac{A_{s1}}{b_w d} = \frac{1809.56}{250 \times 305} = 0.0237 > 0.02$ Not OK take 0.02

$\sigma_{cp} = N_{Ed}/A_c = 0/(250 \times 350) = 0$ (Mpa)

$\gamma_c = 1$

$C_{Rd,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1} = 0.18$

$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} = 0.035 \times 1.81^{3/2} \times 20^{1/2} = 0.381$

$k_1 = 0.15$

$V_{Rd,c} = \left[C_{Rd,c}k(100\rho_1 f_{ck})^{\frac{1}{3}} + k_1\sigma_{cp}\right] b_w d \ge (v_{min} + k_1\sigma_{cp}) b_w d$

$= \left[0.18 \times 1.81(100 \times 0.02 \times 20)^{\frac{1}{3}} + 0.15 \times 0\right] 250 \times 305 = 84959.3\text{N}$

$\ge 0.381 \times 250 \times 305 = 29051.25\text{N}$ OK

$V_{ED} = 119.5\text{kN} > V_{Rd,c} = 84.96\text{kN}$, shear reinforcement are required.

Check if the section is large enough?

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{ck}/1.5(\cot\theta + \tan\theta)$$

$\alpha_{cw} = 1$

$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.6(1 - 20/250) = 0.552$

$z = 0.9d = 0.9 \times 305 = 274.5mm$

$V_{Rd,max} = 1 \times b_w \times 0.9d \times \frac{0.6\left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)} = \frac{0.54 b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)}$

$21.8° \leq \theta \leq 45°$

For $\theta = 21.8°$

$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 21.8° + \tan 21.8°} = 0.186$

For $\theta = 45°$

$\frac{0.54}{(\cot\theta + \tan\theta)} = \frac{0.54}{\cot 45° + \tan 45°} = 0.27$

$V_{Rd,\max(21.8°)} = 0.186 b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck} = 0.186 \times 250 \times 305 \times \left(1 - \frac{20}{250}\right) 20 = 260958\text{N}$

$V_{ED} = 119.5\text{kN} < V_{Rd,\max(21.8°)} = 260.96\text{kN}$ OK

$V_{Rd,\max(45°)} = 0.27 b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck} = 0.27 \times 250 \times 305 \times \left(1 - \frac{20}{250}\right) 20 = 378810\text{N}$

$V_{ED} = 119.5\text{kN} < V_{Rd,\max(45°)} = 378.81\text{kN}$ OK

Determine angle θ

$$V_{ED} = V_{Rd,max} = \frac{0.54 b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}}{(\cot\theta + \tan\theta)}$$

$$\frac{1}{(\cot\theta + \tan\theta)} = \sin\theta\cos\theta = 0.5\sin 2\theta$$

$$V_{ED} = V_{Rd,max} = (0.5\sin 2\theta) \times 0.54 b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$V_{ED} = V_{Rd,max} = 0.27 \times \sin 2\theta \times b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$\sin 2\theta = V_{ED}/0.27 \times b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}$$

$$\theta = 0.5\sin^{-1}\left\{V_{ED}/0.27 \times b_w d\left(1 - \frac{f_{ck}}{250}\right) f_{ck}\right\}$$

Or

$$\theta = 0.5\sin^{-1}\{V_{ED}/V_{Rd,\max(45°)}\} = 0.5\sin^{-1}\left(\frac{119.5}{378.81}\right) = 9.2° < 21.8°$$

Therefore take $\theta = 21.8°$ and $V_{Rd,max} = V_{Rd,\max(21.8°)} = 260.96\text{kN}$

The maximum spacing between bars should not exceeded

$$S_{max} = 0.75d = 0.75 \times 305 = 228.75mm \leq 600mm$$

$S_{max} = 228.75mm$

Compute the stirrup spacing required to resist the shear force.

$$\frac{A_{sw}}{S} = \frac{V_{Ed}}{0.9df_{yk}\cot\theta} = \frac{119.5 \times 1000}{0.9 \times 305 \times 300 \times \cot 21.8°} = 0.58$$

The minimum shear reinforcement is given by:

$A_{sw} = 2 \times \pi \times 4^2 = 100.53mm^2$, for a double-leg stirrups.

$$\frac{A_{sw,min}}{S} = \frac{0.08\sqrt{f_{ck}}b_w}{f_{yk}} = \frac{0.08 \times 250 \times \sqrt{20}}{300} = 0.298$$

$$\frac{A_{sw}}{S} = 0.58 > \frac{A_{sw,min}}{S} = 0.298 \quad \text{OK}$$

$$S = \frac{A_{sw}}{0.58} = \frac{100.53}{0.58} = 173.21mm > S_{max} = 228.75mm$$

Therefore take $S = 170mm$ (use Ø8 c/c 170mm).

The total shear strength of the beam will be:

$$V_{Rd} = V_{Rd,s} = \frac{A_{sw}}{S}zf_{yd}\cot\theta = \frac{A_{sw}}{S} \times \left(\frac{0.9}{1}\right)d \times f_{yk}\cot\theta$$

$$= \left(\frac{100.53}{170}\right) \times \left(\frac{0.9}{1}\right) \times 305 \times 300 \times \cot 21.8° \times 10^{-3} = 121.75\text{kN}$$

4.3. Calculate the shear strength using MCFT.

Using a program called 3RCB that is develop in this thesis for response of reinforced concrete beam subjected shear, we calculated the maximum shear strength of the three beams. The source code of this program can be found in the Appendix.

	Beam 1	Beam 2	Beam 3
Width (W)	250mm	200mm	250mm
Depth (H)	400mm	300mm	350mm
Flexural Reinforcement			
Top	2Ø10	2Ø10	4Ø24
Bottom	4Ø24	3Ø20	2Ø10
Shear Reinforcement			
ACI design	Ø8 c/c 170	Ø6 c/c 120	Ø8 c/c 140
EuroCode design	Ø8 c/c 260	Ø6 c/c 190	Ø8 c/c 170

Table 1 ACI and EuroCode design summary.

4.3.1. Sample calculation for Beam 2 using 3RCB (Only Shear) program

3RCB program uses three input windows forms and one main windows form, to communicate with users.

The first form is called "Material" as it is shown in Figure 4.5, this form will allow the user to input the material strength of concrete and reinforcement bars.

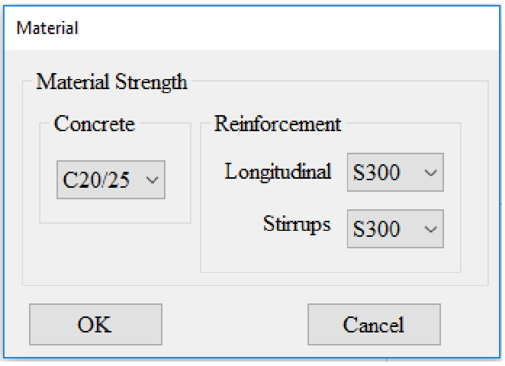


Figure 4.5 User input for material

The second form is called “Section Properties” as it is shown in Figure 4.6, this form will allow the user to input beam dimensions and reinforcement details. Using values from Table 1, we can fill the “Section Properties” form as it is shown in Figure 4.6.

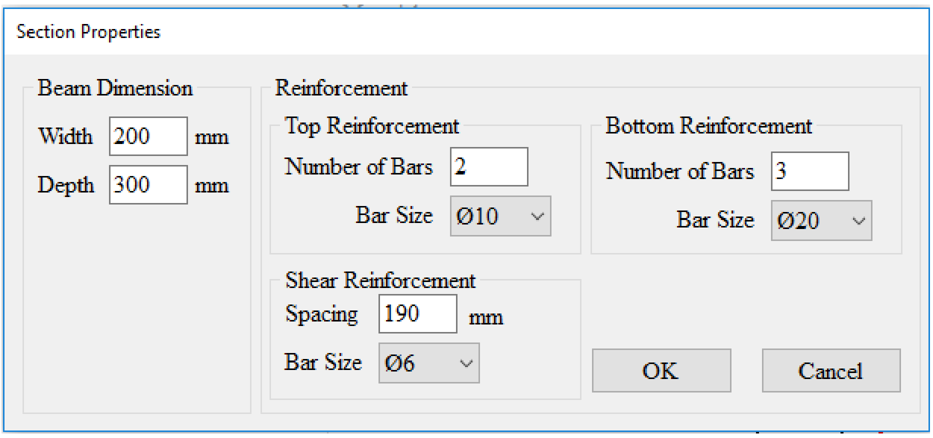


Figure 4.6 User input for Section detail of the beam

The third form is called “Output” as it is shown in Figure 4.7, this for will allow the user to select the desired output, from the section analysis.

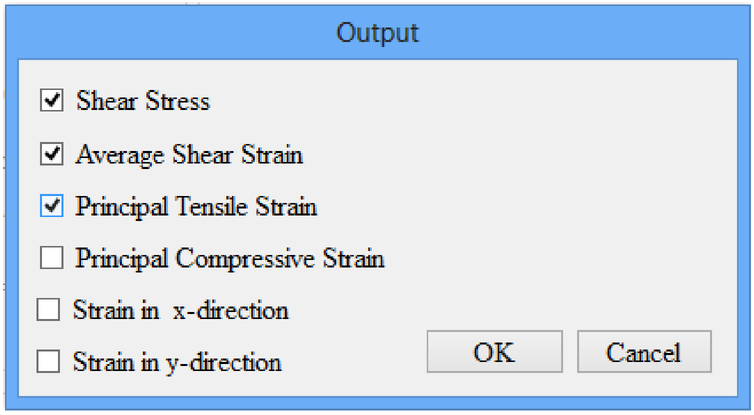


Figure 4.7 Output from the analysis

The main windows is called “3RCB (Only Shear)”, this window allow the user to review the input from “Material” and “Section Properties” forms as its shown in Figure 4.8.

The program works in a way, for the given shear force it can give you the section response, that you are selected and the output values can be found in a folder where you put 3RCB application. As you can see in Figure 4.8 for the design shear force of 40kN the beam can carry it without any failure, but by increasing the shear force value we find the maximum shear strength of this beam, which is 88.9kN.

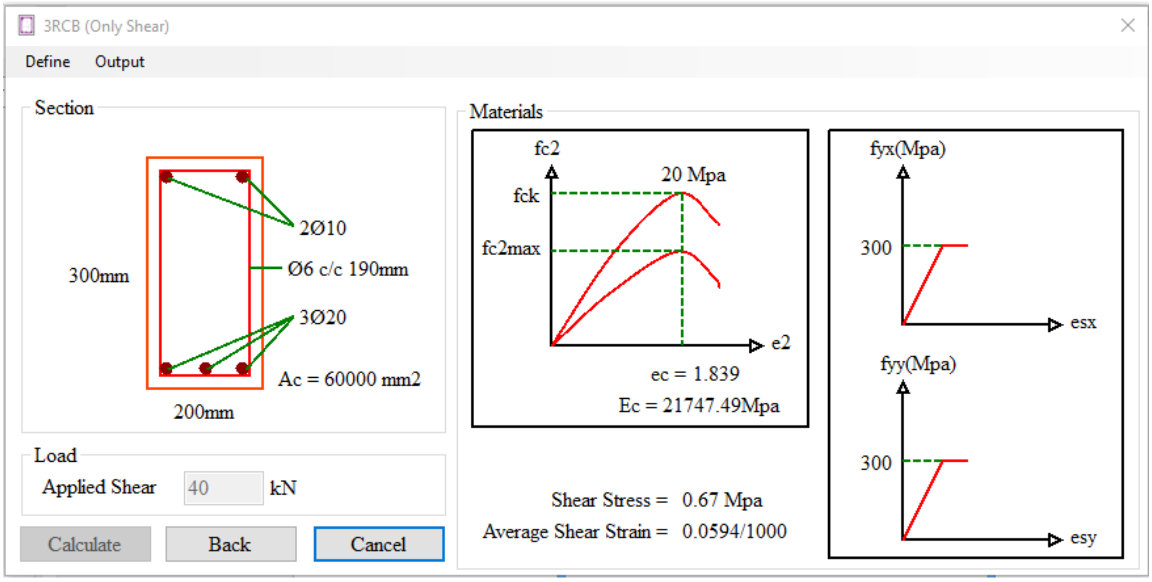


Figure 4.8 Main Window of the program

4.3.2. Sample calculation for Beam 3 using 3RCB (Cantilever) program

To include the effect of moment, we modify the "Section Properties" form, as it is shown in Figure 4.9. This form will allow the user to input the dimension of beam, column and reinforcement bars.

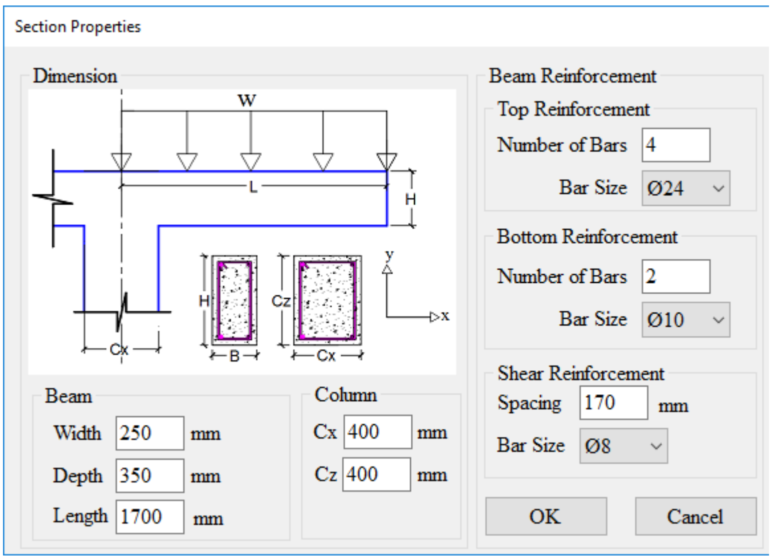


Figure 4.9 User input for Section detail for the cantilever beam

The main window of the program is modified as it is shown in Figure 4.10, this window can show the value of moment corresponding to the design shear force. This program works only for cantilever beam, which is subjected to a distributed load.

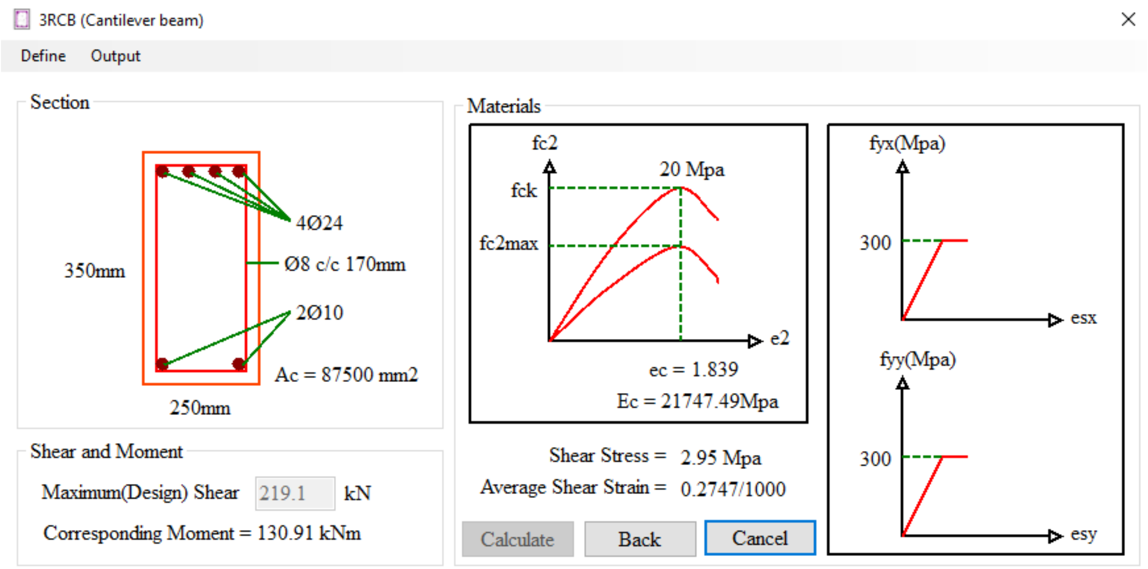


Figure 4.10 Main Window for the cantilever beam program

Method	Beam 1		Beam 2		Beam 3	
	Spacing	Shear Resistance (kN)	Spacing	Shear Resistance (kN)	Spacing	Shear Resistance (kN)
ACI Design	Ø8 c/c 170	129.13	Ø6 c/c 120	75.23	Ø8 c/c 140	122.53
EuroCode Design	Ø8 c/c 260	92.66	Ø6 c/c 190	52.04	Ø8 c/c 170	121.75
MCFT	Ø8 c/c 170	140.0	Ø6 c/c 120	180.9	Ø8 c/c 140	231.1
	Ø8 c/c 260	137.0	Ø6 c/c 190	88.9	Ø8 c/c 170	219.1

Table 2 Summary of shear strength.

5. DISCUSSION

5.1. Compare the result using MCFT

As it is shown in Table 2, this codes gives us different shear strength values, for the same design shear force and cross section.

For beam 1, the design shear is large enough to make both of the codes to provide a shear reinforcement, even if they gives as different reinforcement spacing. The ACI design procedure gives us nearly similar results to that of MCFT rather than EuroCode, as it is shown in Figure 5.1 and 5.2.

The shear strength equation in Eurocode consider all the shear force should be carried by the shear reinforcement alone, in a way reduced the direct contribution of concrete section. As the result the EuroCode will procedure a lesser shear strength than ACI for beam 1 as shown in Table 2.

Beam 2 resulted minimum shear reinforcement in both codes and the design procedure of EuroCode follows 0.75 times the effective depth as maximum shear reinforcement spacing, whereas ACI consider 0.5 times the effective depth as the maximum spacing between shear reinforcements. This difference can cause over estimation in shear strength and which is why EuroCode gives us a closer result to that of MCFT for beam 2, as shown in Figure 5.1 and 5.2.

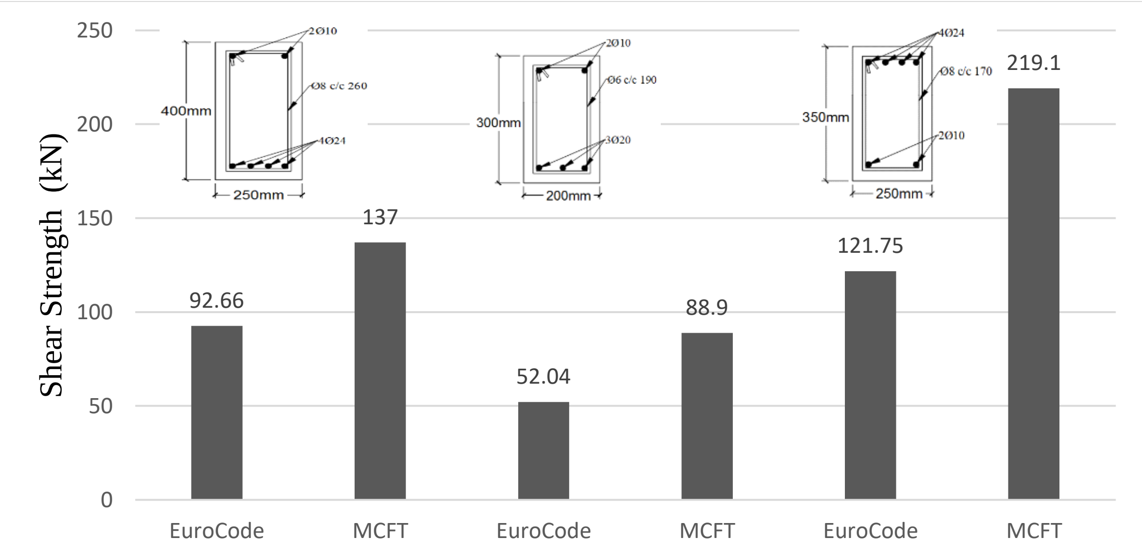


Figure 5.1 EuroCode and MCFT shear strength for beam 1, 2 and 3

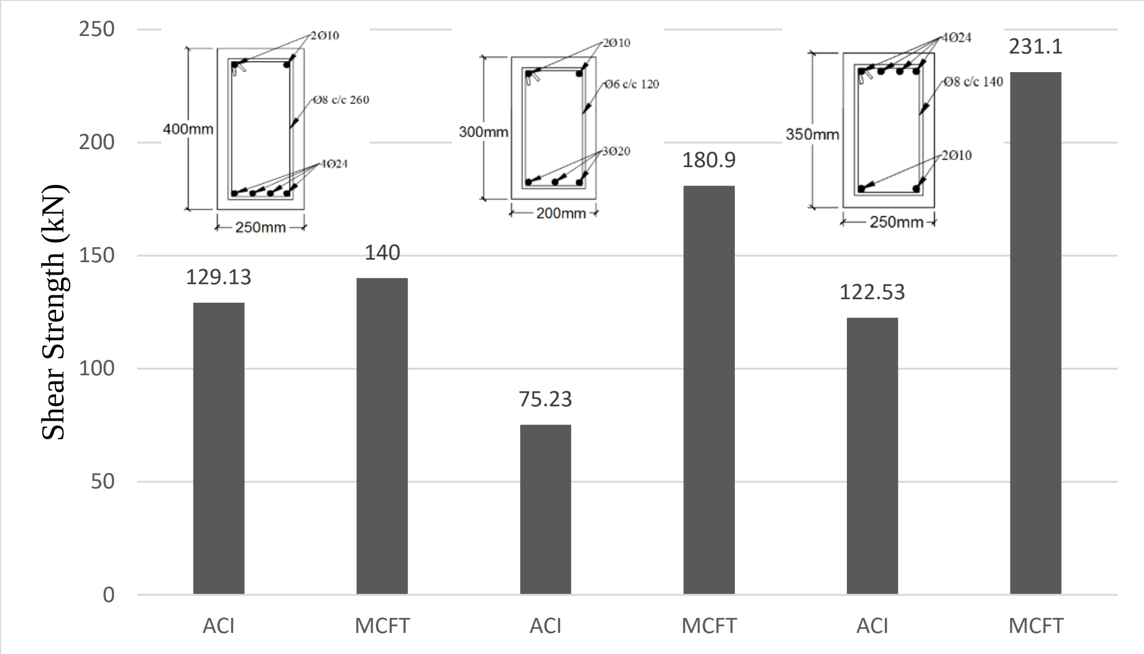


Figure 5.2 ACI and MCFT shear strength for beam 1, 2 and 3

On the other hand ACI gives a more close result for larger concrete section when we compare it to MCFT, as shown in Figure 5.2. This is because of ACI consider the direct contribution of concrete and reinforcement steel individually.

For practical purpose we consider the effect of bending moment on shear strength in beam 3 and as its shown in Figure 5.1 and 5.2; both of the codes gives us nearly similar shear strength, but EuroCode gives us a more economical spacing than ACI as shown in Table 2.

6. CONCLUSION AND RECOMMENDATION

6.1. Conclusion

The shear strength calculation in MCFT include the contribution of the concrete section, the aggregate interlock, the shear reinforcements, and the longitudinal reinforcements. Because of that MCFT gives us higher shear strength value than both the ACI and EuroCode producers.

For beams that require minimum shear reinforcement, ACI procedure provides narrowly spaced shear reinforcements ($S_{max} = 0.5d$). Thus when we consider this spacing the MCFT gives higher shear strength value than that of ACI.

EuroCode procedure is more concentrated on the shear strength of the reinforcement by discarding the direct contribution of the concrete section, especially the beam width does not directly participate in the shear design equations. This will cause some under estimation in shear strength, when we compare EuroCode to MCFT.

In general, both EuroCode and ACI procedures gives us a safe shear reinforcement spacing, but for beams that require a minimum shear reinforcement, EuroCode gives more economical shear reinforcement spacing than ACI.

6.2. Recommendation

The following points are recommend for future work on Modified Compression Filed Theory as the extensions of this research.

- To incorporate Modified Compression Filed Theory in our design code, by simplifying the procedure or by developing design charts and tables based on this theory.
- Develop a program that can calculate the response of reinforced concrete beam for different cross-sections other than rectangular sections, like T, I, and Inverted L section.
- Develop a program that can be used for the response of reinforced concrete column that's subjected to axial, bending moment and shear force.
- Develop a program that can calculate the response of reinforced concrete slab that is subjected to bending moment and shear force.

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APPENDIX

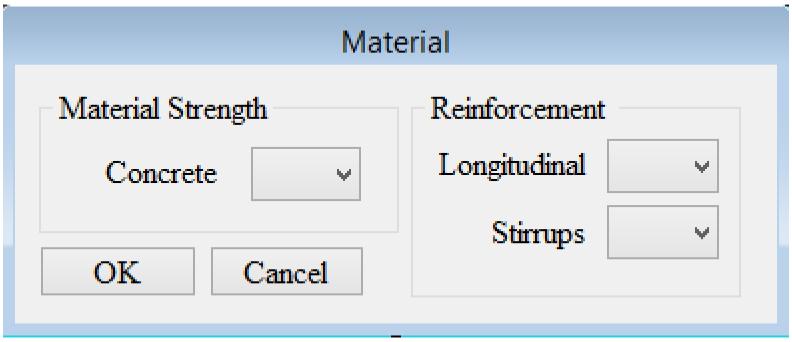
Program Source Code

For 3RCB (only Shear) Program

The procedure is written in Microsoft Visual Studio, it’s an object oriented programing. For comment the line will start with “ ' ”. The program is approximately 629 lines (without the code that makes the interface).

We have material, Cross section and output forms for user input.

User interface for material




```
' ****************** Variable Declaration ***********************************
    Public fck As Double 'concrete cylinder strength
    Public eo As Double 'concrete strain at peak stress
    Public fyy As Double 'yield strength of transverse steel
    Public Es As Double 'Young's modulus of steel
    Public Ec As Double 'Young's modulus of concrete
    Public fcr As Double 'Cracking stress
    Public ecr As Double 'Cracking strain
    Public fyx As Double 'yield strength of longitudinal rebar
' ****************** Code *********** ************************************
   Private Sub cmdOK_Click(sender As Object, e As EventArgs) Handles cmdOK.Click
       If COBConcrete.SelectedItem = Nothing Then
           MsgBox("Please Select Concrete strength", MsgBoxStyle.Critical)
           Me.Show()
           COBConcrete.Focus()
           Exit Sub
       ElseIf COBSteelL.SelectedItem = Nothing Then
           MsgBox("Please Select Reinforcement yield strength", MsgBoxStyle.Critical)
           Me.Show()
           COBSteelL.Focus()
           Exit Sub
       ElseIf COBSteelT.SelectedItem = Nothing Then
           MsgBox("Please Select Reinforcement yield strength", MsgBoxStyle.Critical)
           Me.Show()
```

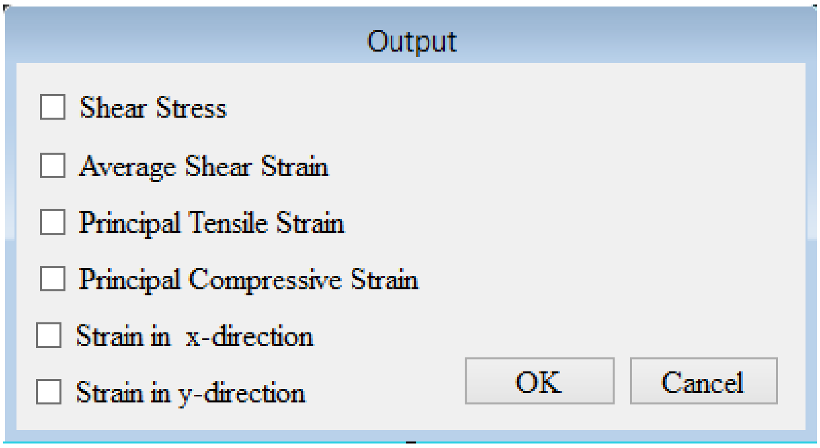
```
    COBSteelT.Focus()
    Exit Sub
  Else
    If COBConcrete.SelectedItem = "C12/15" Then
      fck = -15 / (1.25) 'Mpa
    ElseIf COBConcrete.SelectedItem = "C16/20" Then
      fck = -20 / (1.25) 'Mpa
    ElseIf COBConcrete.SelectedItem = "C20/25" Then
      fck = -25 / (1.25) 'Mpa
    ElseIf COBConcrete.SelectedItem = "C25/30" Then
      fck = -30 / (1.25) 'Mpa
    ElseIf COBConcrete.SelectedItem = "C30/37" Then
      fck = -37 / (1.25)  'Mpa
    End If
    If COBSteelL.SelectedItem = "S260" Then
      fyx = 260  'Mpa
    ElseIf COBSteelL.SelectedItem = "S300" Then
      fyx = 300  'Mpa
    ElseIf COBSteelL.SelectedItem = "S400" Then
      fyx = 400 'Mpa
    End If
    If COBSteelT.SelectedItem = "S260" Then
      fyy = 260  'Mpa
    ElseIf COBSteelT.SelectedItem = "S300" Then
      fyy = 300  'Mpa
    ElseIf COBSteelT.SelectedItem = "S400" Then
      fyy = 400 'Mpa
    End If
  End If
     '**************************************************
  Ec = 3320 * Math.Sqrt(-fck) + 6900 'Mpa
  eo = (2 * fck) / Ec
  Es = 200000 'Mpa
  fcr = 0.33 * Math.Sqrt(-fck)
  ecr = fcr / Ec
  frmmain.lblfck.Text = -fck & " Mpa"
  frmmain.lblEc.Text = "Ec = " & Math.Round(Ec, 2) & "Mpa"
  frmmain.lbleo.Text = Math.Round(-eo * 1000, 3)
  frmmain.lblfykL.Text = fyx
  frmmain.lblfykT.Text = fyy
  frmmain.GRBmaterial.Visible = True
  frmmain.SectionToolStripMenuItem.Enabled = True
  frmmain.Enabled = True
  frmmain.BringToFront()
```

```
'Material Drawing
Dim bit As Bitmap = New Bitmap(frmmain.pic1.Width, frmmain.pic1.Height)
Dim g As Graphics = Graphics.FromImage(bit)
Dim myPen As Pen = New Pen(Color.Black, 2)
Dim penRed As Pen = New Pen(Color.Red, 2)
Dim penGreen As Pen = New Pen(Color.Green, 2)
'to draw an axis for concrete stress
Dim Cpoint1 As New Point(64, 40)
Dim Cpoint2 As New Point(68, 40)
Dim Cpoint3 As New Point(64, 32)
Dim Cpoint4 As New Point(60, 40)
Dim Cpoint5 As New Point(64, 40)
Dim Cpoint6 As New Point(64, 168)
Dim Cpoint7 As New Point(216, 168)
Dim Cpoint8 As New Point(216, 172)
Dim Cpoint9 As New Point(224, 168)
Dim Cpoint10 As New Point(216, 164)
Dim Cpoint11 As New Point(216, 168)
Dim ClinePoints As Point() = {Cpoint1, Cpoint2, Cpoint3, Cpoint4, Cpoint5,
Cpoint6, Cpoint7, Cpoint8, Cpoint9, Cpoint10, Cpoint11}
g.DrawLines(myPen, ClinePoints)
'to draw a stress curve maximum
Dim C1point1 As New Point(64, 168)
Dim C1point2 As New Point(112, 96)
Dim C1point3 As New Point(160, 52)
Dim C1point4 As New Point(188, 72)
Dim C1point5 As New Point(192, 76)
Dim C1linePoints As Point() = {C1point1, C1point2, C1point3, C1point4,
C1point5}
g.DrawCurve(penRed, C1linePoints)
Dim DashPen As Pen
DashPen = New Pen(Brushes.Green, 2)
Dim DashpenRed = New Pen(Color.Red, 2)
DashPen.DashStyle = Drawing2D.DashStyle.Dash
g.DrawLine(DashPen, 164, 52, 164, 168) 'vertical for ec
g.DrawLine(DashPen, 164, 52, 64, 52) 'horzontal for fck
'to draw a stress for fc2max
Dim C2point1 As New Point(64, 168)
Dim C2point2 As New Point(108, 128)
Dim C2point3 As New Point(160, 96)
Dim C2point4 As New Point(188, 116)
Dim C2point5 As New Point(192, 124)
Dim C2linePoints As Point() = {C2point1, C2point2, C2point3, C2point4,
C2point5}
```

```
        g.DrawCurve(DashpenRed, C2linePoints)
        g.DrawLine(DashPen, 164, 96, 64, 96) 'horzontal for fc2max
        'to make a border
        Dim C3point1 As New Point(4, 4)
        Dim C3point2 As New Point(260, 4)
        Dim C3point3 As New Point(260, 230)
        Dim C3point4 As New Point(4, 230)
        Dim C3point5 As New Point(4, 4)
        Dim C3linePoints As Point() = {C3point1, C3point2, C3point3, C3point4,
C3point5}
        g.DrawLines(myPen, C3linePoints)
        frmmain.pic1.Image = bit
        'to draw an axis for Steel stress fyx
        Dim bit1 As Bitmap = New Bitmap(frmmain.pic2.Width, frmmain.pic2.Height)
        Dim g1 As Graphics = Graphics.FromImage(bit1)
        'to make a border
        Dim S1point1 As New Point(4, 4)
        Dim S1point2 As New Point(228, 4)
        Dim S1point3 As New Point(228, 330)
        Dim S1point4 As New Point(4, 330)
        Dim S1point5 As New Point(4, 4)
        Dim S1linePoints As Point() = {S1point1, S1point2, S1point3, S1point4, S1point5}
        g1.DrawLines(myPen, S1linePoints)
        'to draw an axis
        Dim Sxpoint1 As New Point(60, 40)
        Dim Sxpoint2 As New Point(64, 40)
        Dim Sxpoint3 As New Point(60, 32)
        Dim Sxpoint4 As New Point(56, 40)
        Dim Sxpoint5 As New Point(60, 40)
        Dim Sxpoint6 As New Point(60, 152)
        Dim Sxpoint7 As New Point(172, 152)
        Dim Sxpoint8 As New Point(172, 156)
        Dim Sxpoint9 As New Point(180, 152)
        Dim Sxpoint10 As New Point(172, 148)
        Dim Sxpoint11 As New Point(172, 152)
        Dim SxlinePoints As Point() = {Sxpoint1, Sxpoint2, Sxpoint3, Sxpoint4, Sxpoint5,
Sxpoint6, Sxpoint7, Sxpoint8, Sxpoint9, Sxpoint10, Sxpoint11}
        g1.DrawLines(myPen, SxlinePoints)
        Dim SSxpoint1 As New Point(60, 152)
        Dim SSxpoint2 As New Point(90, 92)
        Dim SSxpoint3 As New Point(110, 92)
        Dim SsxlinePoints As Point() = {SSxpoint1, SSxpoint2, SSxpoint3}
        g1.DrawLines(penRed, SsxlinePoints)
        g1.DrawLine(DashPen, 60, 92, 90, 92) 'horzontal for fyx
```

```
        'to draw an axis for steel stress fyy
        Dim Sypoint1 As New Point(60, 204)
        Dim Sypoint2 As New Point(64, 204)
        Dim Sypoint3 As New Point(60, 196)
        Dim Sypoint4 As New Point(56, 204)
        Dim Sypoint5 As New Point(60, 204)
        Dim Sypoint6 As New Point(60, 316)
        Dim Sypoint7 As New Point(172, 316)
        Dim Sypoint8 As New Point(172, 320)
        Dim Sypoint9 As New Point(180, 316)
        Dim Sypoint10 As New Point(172, 312)
        Dim Sypoint11 As New Point(172, 316)
        Dim SylinePoints As Point() = {Sypoint1, Sypoint2, Sypoint3, Sypoint4, Sypoint5,
Sypoint6, Sypoint7, Sypoint8, Sypoint9, Sypoint10, Sypoint11}
        g1.DrawLines(myPen, SylinePoints)
        Dim SSypoint1 As New Point(60, 316)
        Dim SSypoint2 As New Point(90, 256)
        Dim SSypoint3 As New Point(110, 256)
        Dim SSylinePoints As Point() = {SSypoint1, SSypoint2, SSypoint3}
        g1.DrawLines(penRed, SSylinePoints)
        g1.DrawLine(DashPen, 60, 256, 90, 256) 'horzontal for fyy
        frmmain.pic2.Image = bit1
        '------------------------------------------------
        Me.Hide()
    End Sub
'*********************** *********** *********************************
```

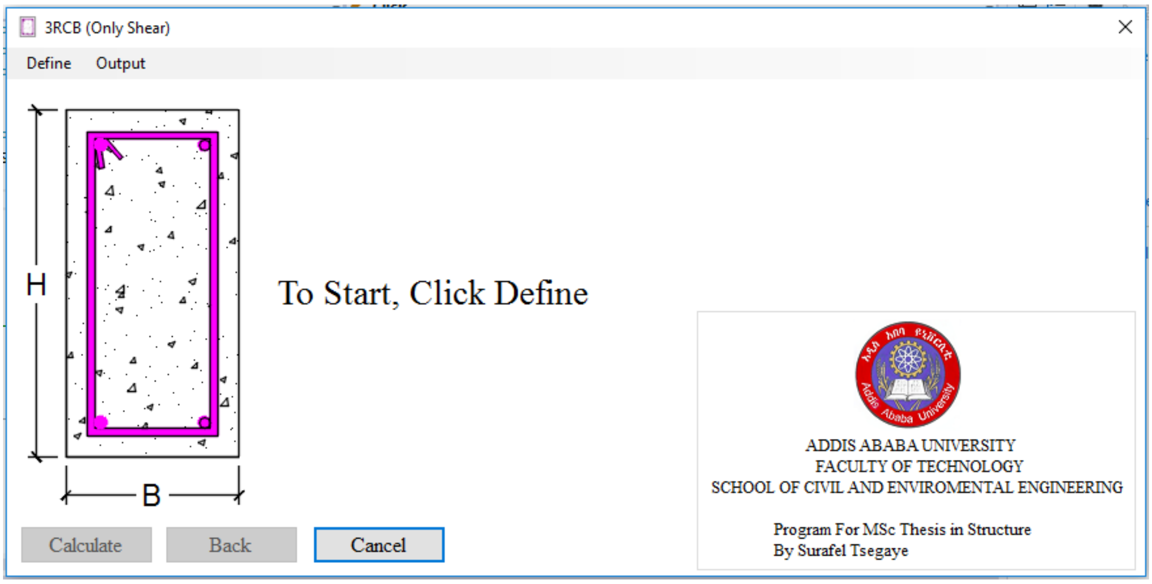
User interface for output




```
' ****************** Code ***************************************
Private Sub cmdCancel_Click(sender As Object, e As EventArgs) Handles
cmdCancel.Click
        Me.Close()
        frmmain.Enabled = True
        frmmain.BringToFront()
    End Sub
    Private Sub cmdOK_Click(sender As Object, e As EventArgs) Handles cmdOK.Click
        On Error Resume Next
        If CHBVxy.Checked Then
            FileOpen(1, "Vxy.Txt", OpenMode.Output, OpenAccess.Default,
OpenShare.Default)
        End If
        If CHBGamaxy.Checked Then
            FileOpen(2, "Gamaxy.Txt", OpenMode.Output, OpenAccess.Default,_
            OpenShare.Default)
        End If
        If CHBe1.Checked Then
            FileOpen(3, "e1.Txt", OpenMode.Output, OpenAccess.Default,
OpenShare.Default)
        End If
        If CHBe2.Checked Then
            FileOpen(4, "e2.Txt", OpenMode.Output, OpenAccess.Default,
OpenShare.Default)
        End If
        If CHBex.Checked Then
            FileOpen(5, "ex.Txt", OpenMode.Output, OpenAccess.Default,
OpenShare.Default)
        End If
        If CHBey.Checked Then
            FileOpen(6, "ey.Txt", OpenMode.Output, OpenAccess.Default,
OpenShare.Default)
```

```
        End If
        frmmain.Enabled = True
        frmmain.BringToFront()
        Me.Hide()
    End Sub
```

User interface for main window




```
' ***************** Variable Declaration **********************************
    Public V As Double
    Public M As Double
    Public Vxy As Double 'Shear flow
    Public etop As Double 'Top fiber strain
    Public ebot As Double 'Bottom fiber strain
    Public esT As Double 'longitudinal strain of rebar top
    Public esB As Double 'longitudinal strain of rebar Bottom
    Public fslT As Double 'longitudinal steel stresses top
    Public fslB As Double 'longitudinal steel stresses bottom
    Public ex(0 To 19) As Double
    Public fcx(0 To 19) As Double
    Public e2(0 To 19) As Double
    Public e1 As Double
    Public ey(0 To 19) As Double
    Dim fc2(0 To 19) As Double
    Dim fc2n(0 To 19) As Double
    Public fc2max(0 To 19) As Double
    Public Gamaxy(0 To 19) As Double
    Dim Beta(0 To 19) As Double
    Dim ep(0 To 19) As Double
    Dim fc1(0 To 19) As Double
    Dim fcy(0 To 19) As Double
```

```
    Dim fsy(0 To 19) As Double
    Dim GamaM(0 To 19) As Double
    Dim K2(0 To 19) As Double
    Dim w(0 To 19) As Double
    Dim Teta As Double
    Dim Teta1 As Double
    Dim Teta2 As Double
    Dim Nrs As Double 'Normal resultant section load.
    Dim Mrs As Double 'resultant sectional moment
    Dim fcxsum As Double 'Resultant of concrete forces
    Dim X As Double
    Dim Vci(0 To 19) As Double 'shear stress on cracked surfaces
    Dim fsxcr As Double 'stress in x-reinforcement at crack location
    Dim fsycr(0 To 19) As Double 'stress in x-reinforcement at crack location
    Dim fci(0 To 19) As Double 'compressive Stress on crack surafce
    Dim Vcimax(0 To 19) As Double
    Dim Dfc1 As Double
    Dim G As Double
' ****************** Code *********** *************************************
Private Sub cmdCancel_Click(sender As Object, e As EventArgs) Handles
cmdCancel.Click
        Application.Exit()
    End Sub
    Private Sub MaterialToolStripMenuItem_Click(sender As Object, e As EventArgs)
Handles MaterialToolStripMenuItem.Click
        frmMaterial.Show()
        Me.Enabled = False
        frmMaterial.BringToFront()
    End Sub
    Private Sub SectionToolStripMenuItem_Click(sender As Object, e As EventArgs)
Handles SectionToolStripMenuItem.Click
        frmSectionProperties.Show()
        Me.Enabled = False
        frmSectionProperties.BringToFront()
    End Sub
    Private Sub SelectOutputsToolStripMenuItem_Click(sender As Object, e As
EventArgs) Handles SelectOutputsToolStripMenuItem.Click
        frmOutput.Show()
        Me.Enabled = False
        frmOutput.BringToFront()
    End Sub
    Private Sub frmmain_Load(sender As Object, e As EventArgs) Handles MyBase.Load
        GRBSection.Visible = False
        GRBmaterial.Visible = False
```

```
        GRBLoad.Visible = False
        SectionToolStripMenuItem.Enabled = False
        SelectOutputsToolStripMenuItem.Enabled = False
        cmdCalculate.Enabled = False
        cmdBack.Enabled = False
    End Sub
        Private Sub cmdCalculate_Click(sender As Object, e As EventArgs) Handles
cmdCalculate.Click
        Dim N As Double
        V = Val(txtShear.Text)
        Vxy = (V * 1000) / frmSectionProperties.SecArea
        If V <= 400 Then
            N = 10000
        ElseIf V < 600 Then
            N = 200
        ElseIf V < 1000 Then
            N = 10
        Else
            N = 5
        End If
        X = frmSectionProperties.H / 2
lineB:  etop = frmMaterial.eo / N
lineA:  ebot = -etop * ((frmSectionProperties.H - X) / X)
        esT = etop * ((X - frmSectionProperties.ysT) / X)
        esB = -etop * ((frmSectionProperties.ysB - X) / X)
        If Math.Abs(esT * frmMaterial.Es) <= frmMaterial.fyx Then
            fslT = esT * frmMaterial.Es 'Mpa
        ElseIf Math.Abs(esT * frmMaterial.Es) > frmMaterial.fyx Then
            fslT = frmMaterial.fyx
        End If
        If Math.Abs(esB * frmMaterial.Es) <= frmMaterial.fyx Then
            fslB = esB * frmMaterial.Es 'Mpa
        ElseIf Math.Abs(esB * frmMaterial.Es) > frmMaterial.fyx Then
            fslB = frmMaterial.fyx
        End If

'*************************************************************************
        Dim j As Integer
        e1 = 0
        For j = 0 To 19 Step 1
            'longitudinal Strain ex
            ex(j) = etop * ((X - frmSectionProperties.yc(j)) / X)
line1:      'Concrete prinicipal tensile stress, fc1
            If 0 <= e1 <= frmMaterial.ecr Then
```

```
        fc1(j) = e1 * frmMaterial.Ec
    ElseIf e1 > frmMaterial.ecr Then
        fc1(j) = frmMaterial.fcr * (1 / (1 + (Math.Sqrt(e1 / 0.005))))
    ElseIf e1 < 0 Then
        fc1(j) = frmMaterial.fck * ((2 * (e1 / frmMaterial.eo)) - ((e1 / frmMaterial.eo)
^ 2))
    End If
    'Calculate angle of inclination of principal compressive strain "Tan(Teta)"
    Dim A As Double
    Dim B As Double
    Dim C As Double
    A = (e1 - ex(j) * frmMaterial.Es)
    B = (Vxy / frmSectionProperties.Rowy)
    C = -(fc1(j) / frmSectionProperties.Rowy) - (e1 * frmMaterial.Es)
    Teta1 = (-B + Math.Sqrt(B ^ 2 - (4 * A * C))) / (2 * A)
    Teta2 = (-B - Math.Sqrt(B ^ 2 - (4 * A * C))) / (2 * A)
    If (180 * Math.Atan(Teta1) / Math.PI) >= 0 Then
        Teta = Teta1
    ElseIf (180 * Math.Atan(Teta2) / Math.PI) >= 0 Then
        Teta = Teta2
    Else
        MsgBox("unabile to find Teta Reivew the code ", MsgBoxStyle.Exclamation)
    End If
    'Transverse tensile strain, ey
    ey(j) = e1 - ((e1 - ex(j)) * Teta ^ 2)
    'Principal compressive strain, e2
    e2(j) = ex(j) - ((e1 - ex(j)) * Teta ^ 2)
    'Normal shear strain
    Gamaxy(j) = 2 * (e1 - ex(j)) * Teta
    'Concrete longitudinal compressive stress, fcx
    fcx(j) = fc1(j) - Vxy / Teta
    'Transverse steel tensile stress, fsy
    If Math.Abs(ey(j) * frmMaterial.Es) >= frmMaterial.fyy Then
        fsy(j) = frmMaterial.fyy
    Else
        fsy(j) = ey(j) * frmMaterial.Es
    End If
    'Concrete principal compressive stress,fc2 form Mohr's circle
    fc2(j) = fc1(j) - Vxy * (Teta + 1 / Teta)
    'Concrete principal compressive stress,fc2n from stress strain relation
    Beta(j) = 0.8 - (0.34 * (e1 / frmMaterial.eo))
    fc2max(j) = frmMaterial.fck / Beta(j)
    If -fc2max(j) > -frmMaterial.fck Then
        fc2max(j) = frmMaterial.fck
```

```
        End If
        fc2n(j) = fc2max(j) * Math.Abs((2 * (e2(j) / frmMaterial.eo)) - ((e2(j) /
frmMaterial.eo) ^ 2))
        'Check equilibrium
        If Math.Round((fc2(j) - fc2n(j)), 1) > 0 Then
          e1 = e1 + 0.0000001
          GoTo line1
        ElseIf Math.Round((fc2(j) - fc2n(j)), 1) < 0 Then
          e1 = e1 + 0.0000001
          GoTo line1
        End If
        'Average crack width w
        Dim Steta(0 To 19) As Double
        Steta(j) = 1 / ((Math.Sin(Math.Atan(Teta)) / frmSectionProperties.Smx) +
(Math.Cos(Math.Atan(Teta)) / frmSectionProperties.Smy))
        w(j) = e1 * Steta(j)
        'Calculate stresses on crack
        Vcimax(j) = Math.Sqrt(-frmMaterial.fck) / (0.31 + (24 * w(j) /
(frmSectionProperties.a + 16)))
        Dfc1 = (fc1(j) - (frmSectionProperties.Rowy * (frmMaterial.fyy - fsy(j))))
        If Math.Round(Dfc1, 4) <= 0 Then
          Vci(j) = 0
          fci(j) = 0
        Else
          Dim D As Double
          Dim F As Double
          G = (Dfc1 / Teta) - (0.18 * Vcimax(j))
          If Math.Round(F, 4) <= 0 Then
            fci(j) = 0
            Vci(j) = 0.18 * Vcimax(j)
          Else
            D = 0.82 / Vcimax(j)
            F = (1 / Teta) - 1.64
            fci(j) = (-F - Math.Sqrt(F ^ 2 - (4 * D * G))) / (2 * D)
            Vci(j) = (fc1(j) + Dfc1) / Teta
          End If
        End If
        'Calculate reinforcement stress at crack
        If e1 > frmMaterial.ecr Then
          fsycr(j) = fsy(j) + ((fc1(j) + fci(j) - (Vci(j) * Teta)) /
frmSectionProperties.Rowy)
        Else
          fsycr(j) = fsy(j)
        End If
```

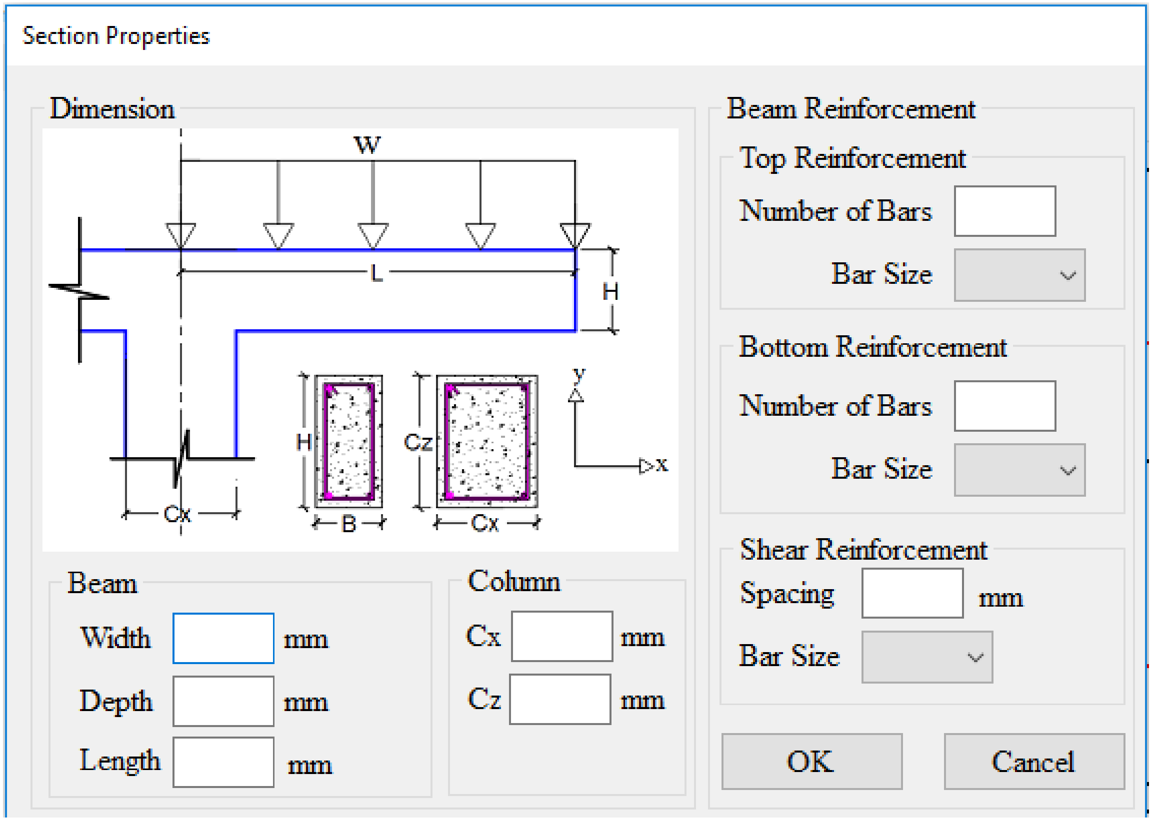
```
        Next
'**************************************************************************
*

  'compute resulting section loads.
        fcxsum = fcx(0) + fcx(1) + fcx(2) + fcx(3) + fcx(4) + fcx(5) + fcx(6) + fcx(7) +
fcx(8) + fcx(9) _
            + fcx(10) + fcx(11) + fcx(12) + fcx(13) + fcx(14) + fcx(15) + fcx(16) + fcx(17) +
fcx(18) + fcx(19)
        Nrs = (fcxsum * frmSectionProperties.b * frmSectionProperties.hi) + (fslT *
frmSectionProperties.AsT) + (fslB * frmSectionProperties.AsB)
        Mrs = ((fcx(0) * frmSectionProperties.yc(0) + fcx(1) * frmSectionProperties.yc(1) _
                + fcx(2) * frmSectionProperties.yc(2) + fcx(3) * frmSectionProperties.yc(3) _
                + fcx(4) * frmSectionProperties.yc(4) + fcx(5) * frmSectionProperties.yc(5) _
                + fcx(6) * frmSectionProperties.yc(6) + fcx(7) * frmSectionProperties.yc(7) _
                + fcx(8) * frmSectionProperties.yc(8) + fcx(9) * frmSectionProperties.yc(9) _
                + fcx(10) * frmSectionProperties.yc(10) + fcx(11) *
frmSectionProperties.yc(11) _
                + fcx(12) * frmSectionProperties.yc(12) + fcx(13) *
frmSectionProperties.yc(13) _
                + fcx(14) * frmSectionProperties.yc(14) + fcx(15) *
frmSectionProperties.yc(15) _
                + fcx(16) * frmSectionProperties.yc(16) + fcx(17) *
frmSectionProperties.yc(17) _
                + fcx(18) * frmSectionProperties.yc(18) + fcx(19) *
frmSectionProperties.yc(19)) _
            * frmSectionProperties.b * frmSectionProperties.hi) + (fslT *
frmSectionProperties.AsT * frmSectionProperties.ysT) + (fslB *
frmSectionProperties.AsB * frmSectionProperties.ysB)
'**************************************************************************
        If Math.Round(Nrs, 1) < -10 Then
            X = X - 0.1
            GoTo lineA
        ElseIf Math.Round(Nrs, 1) > 10 Then
            X = X + 0.1
            GoTo lineA
        End If
        If Math.Round(Mrs * 10 ^ -6, 1) > 0.1 Then
            N = N + 100
            GoTo lineB
        End If
        For j = 0 To 19 Step 1
            If Math.Round(-fc2(j), 2) >= Math.Round(-fc2max(j), 2) Then
```

```
            MsgBox("Concrete is crashed in Compression", MsgBoxStyle.Critical)
            GoTo line7
        ElseIf Math.Round(Math.Abs(esT * frmMaterial.Es), 2) >= frmMaterial.fyx
Then
            MsgBox("Bottom bar yield", MsgBoxStyle.Critical)
            GoTo line7
        ElseIf Math.Round(Math.Abs(esB * frmMaterial.Es), 2) >= frmMaterial.fyx
Then
            MsgBox("Top bar yield", MsgBoxStyle.Critical)
            GoTo line7
        ElseIf Math.Round(Math.Abs(fsycr(j))) >= frmMaterial.fyy Then
            MsgBox("The Shear reinforcement is not capable of transmitting the loads
across the crack.", MsgBoxStyle.Critical)
            GoTo line7
        End If
        Dim k1 As Double
        Dim k As Double
        k1 = 1.6 - (1 / Teta)
        If k1 >= 0 Then
            k = k1
        Else
            k = 0
        End If
        Dim fc1n(0 To 19) As Double
        fc1n(j) = Vcimax(j) * (0.18 + (0.3 * k ^ 2)) * Teta + frmSectionProperties.Rowy
* (frmMaterial.fyy - fsy(j))
        If Math.Abs(fc1(j)) > fc1n(j) Then
            MsgBox("The beam is subjected to Shear stress on crack behold its Strength .",
MsgBoxStyle.Critical)
            GoTo line7
        End If
    Next
    Timer1.Interval = 1000
    Timer1.Start()
    Dim Sec As Integer
    Sec = Sec + 1
    If Sec > 10000 Then
        MessageBox.Show("Sorry the program is Unable to converge within short sec;
Change the Shear value and try", "3RCB", _
              MessageBoxButtons.OK, MessageBoxIcon.Error)
        GoTo line7
    End If
    For j = 0 To 19 Step 1
        If frmOutput.CHBVxy.CheckState = CheckState.Checked Then
```

```
            WriteLine(1, Math.Round(Vxy, 3))
          End If
          If frmOutput.CHBGamaxy.CheckState = CheckState.Checked Then
            WriteLine(2, Math.Round(Gamaxy(j) * 100000, 4))
          End If
          If frmOutput.CHBe1.CheckState = CheckState.Checked Then
            WriteLine(3, Math.Round(e1 * 1000, 4))
          End If
          If frmOutput.CHBe2.CheckState = CheckState.Checked Then
            WriteLine(4, Math.Round(e2(j) * 1000, 4))
          End If
          If frmOutput.CHBex.CheckState = CheckState.Checked Then
            WriteLine(5, Math.Round(ex(j) * 1000, 4))
          End If
          If frmOutput.CHBey.CheckState = CheckState.Checked Then
            WriteLine(6, Math.Round(ey(j) * 1000, 4))
          End If
        Next
        MessageBox.Show("Response calculation completed increase the value of shear to
find Maximum Shear Strength .", "3RCB", MessageBoxButtons.OK,
MessageBoxIcon.Information)
line7:
        cmdCalculate.Enabled = False
        txtShear.Enabled = False
        cmdBack.Enabled = True
    End Sub
    Private Sub cmdBack_Click(sender As Object, e As EventArgs) Handles
cmdBack.Click
        txtShear.Enabled = True
        txtShear.Clear()
        txtShear.Focus()
        cmdCalculate.Enabled = True
        cmdBack.Enabled = False
    End Sub
    Private Sub frmmain_FormClosing(sender As Object, e As FormClosingEventArgs)
Handles MyBase.FormClosing
        If MessageBox.Show("Do you want to exit?", "3RCB", _
          MessageBoxButtons.YesNo, MessageBoxIcon.Question) _
          = DialogResult.Yes Then
          e.Cancel = False
        Else
          e.Cancel = True
        End If
    End Sub
```

Modification for 3RCB (Cantilever Beam) Program




```
Public Class frmSectionProperties
    Public H As Double 'overall depth of the beam
    Public b As Double 'Width of the beam
    Public d As Double 'effective depth
    Public Colx As Double 'column size in the x direction
    Public Colz As Double 'column size in the z direction
    Public BeamL As Double 'Beam Length
    Public BarsizeLongT As Double 'bar size longitudinal top
    Public BarsizeLongB As Double 'bar size longitudinal bottom
    Public BarNoLongT As Integer  'bar no longitudinal top
    Public BarNoLongB As Integer  'bar no longitudinal top
    Public Sp As Double 'Spacing b/n stirrups
    Public ASR As Double 'Area of Shear reinforcement
    Public BarsizeShear As Double 'Reinforcement Bar size of shear reinforcement
    Public hi As Double 'hight of a strip
    Public AsT As Double 'Area of steel Top
    Public AsB As Double 'Area of steel Bottom
    Public ysB As Double 'The position of Bottom Reinforcement from top of the beam
    Public ysT As Double 'The position of Top Reinforcement from top of the beam
    Public Rowy As Double 'transverse steel ratio
    Public ConCover As Double 'concrete cover
    Public SecArea As Double
```

```
    Public yc(0 To 19) As Double 'distance from the top of the beam to the centroid of the
strip
    Public i As Integer
    Public a As Double 'Maximum aggregate size use 10mm
    Dim w As Double 'clear space for bar placement
    Public SpBbT As Double 'Spacing b/n bars at top
    Public SPBbB As Double 'Spacing b/n bars at bottom
    Public Smx As Double
    Public Smy As Double
    Public Cx As Double
    Public Cy As Double
    Public Sx As Double
    Dim K1 As Double
    Public Rowx As Double 'steel ratio

    Private Sub cmdCancel_Click(sender As Object, e As EventArgs) Handles
cmdCancel.Click
        Me.Close()
        frmmain.Enabled = True
        frmmain.BringToFront()
    End Sub

    Private Sub cmdOK_Click(sender As Object, e As EventArgs) Handles cmdOK.Click
        "**********Input Data From the user ***********************************
        If txtDepth.Text = Nothing Or txtWidth.Text = Nothing Or txtLength.Text =
Nothing _
            Or txtColumnCx.Text = Nothing Or txtColumnCz.Text = Nothing Then
            MsgBox("Please fill all beam dimension", MsgBoxStyle.Exclamation)
            Me.Show()
            txtWidth.Focus()
            Exit Sub
        ElseIf txtNoBarT.Text = Nothing Or txtNoBarB.Text = Nothing Or txtSpacing.Text
= Nothing Then
            MsgBox("Please fill all reinforcement values ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtNoBarT.Focus()
            Exit Sub
        ElseIf Not IsNumeric(txtDepth.Text) Or Not IsNumeric(txtWidth.Text) Or Not
IsNumeric(txtLength.Text) _
            Or Not IsNumeric(txtColumnCx.Text) Or Not IsNumeric(txtColumnCz.Text)
Then
            MsgBox("Use Numbers only", MsgBoxStyle.Exclamation)
            Me.Show()
            txtWidth.Focus()
```

```
            Exit Sub
        ElseIf Not IsNumeric(txtNoBarT.Text) Or Not IsNumeric(txtNoBarB.Text) Or Not
IsNumeric(txtSpacing.Text) Then
            MsgBox("Use Numbers only", MsgBoxStyle.Exclamation)
            Me.Show()
            txtNoBarT.Focus()
            Exit Sub
        ElseIf Val(txtDepth.Text <= 0) Or Val(txtWidth.Text <= 0) Or Val(txtLength.Text
<= 0) _
            Or Val(txtColumnCx.Text <= 0) Or Val(txtColumnCz.Text <= 0) Then
            MsgBox("Please Put a positive number ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtWidth.Focus()
            Exit Sub
        ElseIf Val(txtNoBarT.Text <= 0) Or Val(txtNoBarB.Text <= 0) Or
Val(txtSpacing.Text <= 0) Then
            MsgBox("Please Put a positive integer number", MsgBoxStyle.Exclamation)
            Me.Show()
            txtNoBarB.Focus()
            Exit Sub
        ElseIf Val(txtNoBarT.Text < 2) Or Val(txtNoBarB.Text < 2) Then
            MsgBox("At least you need two bars at top and bottom",
MsgBoxStyle.Exclamation)
            Me.Show()
            txtNoBarB.Focus()
            Exit Sub
        ElseIf COBBarSizeB.SelectedItem = Nothing Or COBBarSizeT.SelectedItem =
Nothing _
                Or COBBarSizeShear.SelectedItem = Nothing Then
            MsgBox("Please Select all Reinforcement bar sizes ", MsgBoxStyle.Critical)
            Me.Show()
            COBBarSizeB.Focus()
            Exit Sub
        ElseIf Val(txtDepth.Text) >= 900 Then
            MsgBox("Please Put a smaller depth than 900mm ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtDepth.Focus()
            Exit Sub
        ElseIf Val(txtWidth.Text) < 102 Then
            MsgBox("Please Put a large width than 101mm ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtWidth.Focus()
            Exit Sub
        ElseIf Val(txtColumnCx.Text) < 150 Or Val(txtColumnCz.Text) < 150 Then
```

```
            MsgBox("Please Put a large width >= 150mm ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtColumnCx.Focus()
            Exit Sub
        ElseIf Val(txtWidth.Text) >= Val(txtDepth.Text) Then
            MsgBox("Please Put a smaller width than its Depth ",
MsgBoxStyle.Exclamation)
            Me.Show()
            txtWidth.Focus()
            Exit Sub
        ElseIf Val(txtSpacing.Text) < 100 Then
            MsgBox("Please Put a larger strirrup spacing ", MsgBoxStyle.Exclamation)
            Me.Show()
            txtSpacing.Focus()
            Exit Sub
        Else
            H = Val(txtDepth.Text)
            b = Val(txtWidth.Text)
            BeamL = Val(txtLength.Text)
            Colx = Val(txtColumnCx.Text)
            Colz = Val(txtColumnCz.Text)
            BarNoLongT = Val(txtNoBarT.Text)
            BarNoLongB = Val(txtNoBarB.Text)
            Sp = Val(txtSpacing.Text)
            '-----------------------------------------------Reinforcement bar Top
            If COBBarSizeT.SelectedItem = "Ø6" Then
                BarsizeLongT = 6
            ElseIf COBBarSizeT.SelectedItem = "Ø8" Then
                BarsizeLongT = 8
            ElseIf COBBarSizeT.SelectedItem = "Ø10" Then
                BarsizeLongT = 10
            ElseIf COBBarSizeT.SelectedItem = "Ø12" Then
                BarsizeLongT = 12
            ElseIf COBBarSizeT.SelectedItem = "Ø14" Then
                BarsizeLongT = 14
            ElseIf COBBarSizeT.SelectedItem = "Ø16" Then
                BarsizeLongT = 16
            ElseIf COBBarSizeT.SelectedItem = "Ø20" Then
                BarsizeLongT = 20
            ElseIf COBBarSizeT.SelectedItem = "Ø24" Then
                BarsizeLongT = 24
            ElseIf COBBarSizeT.SelectedItem = "Ø30" Then
                BarsizeLongT = 30
            ElseIf COBBarSizeT.SelectedItem = "Ø32" Then
```

```
    BarsizeLongT = 32
ElseIf COBBarSizeT.SelectedItem = "Ø40" Then
    BarsizeLongT = 40
End If
'----------------------------------------------Reinforcement bar Bottom
If COBBarSizeB.SelectedItem = "Ø6" Then
    BarsizeLongB = 6
ElseIf COBBarSizeB.SelectedItem = "Ø8" Then
    BarsizeLongB = 8
ElseIf COBBarSizeB.SelectedItem = "Ø10" Then
    BarsizeLongB = 10
ElseIf COBBarSizeB.SelectedItem = "Ø12" Then
    BarsizeLongB = 12
ElseIf COBBarSizeB.SelectedItem = "Ø14" Then
    BarsizeLongB = 14
ElseIf COBBarSizeB.SelectedItem = "Ø16" Then
    BarsizeLongB = 16
ElseIf COBBarSizeB.SelectedItem = "Ø20" Then
    BarsizeLongB = 20
ElseIf COBBarSizeB.SelectedItem = "Ø24" Then
    BarsizeLongB = 24
ElseIf COBBarSizeB.SelectedItem = "Ø30" Then
    BarsizeLongB = 30
ElseIf COBBarSizeB.SelectedItem = "Ø32" Then
    BarsizeLongB = 32
ElseIf COBBarSizeB.SelectedItem = "Ø40" Then
    BarsizeLongB = 40
End If
'----------------------------------------------Reinforcement bar Shear
If COBBarSizeShear.SelectedItem = "Ø6" Then
    BarsizeShear = 6
ElseIf COBBarSizeShear.SelectedItem = "Ø8" Then
    BarsizeShear = 8
ElseIf COBBarSizeShear.SelectedItem = "Ø10" Then
    BarsizeShear = 10
ElseIf COBBarSizeShear.SelectedItem = "Ø12" Then
    BarsizeShear = 12
ElseIf COBBarSizeShear.SelectedItem = "Ø14" Then
    BarsizeShear = 14
ElseIf COBBarSizeShear.SelectedItem = "Ø16" Then
    BarsizeShear = 16
ElseIf COBBarSizeShear.SelectedItem = "Ø20" Then
    BarsizeShear = 20
ElseIf COBBarSizeShear.SelectedItem = "Ø24" Then
```

```
        BarsizeShear = 24
    End If
End If
'*********************************************************
ConCover = 25 'mm
a = 10 'mm
w = b - (2 * ConCover) - (2 * BarsizeShear)
'Check the beam length
d = (H - ConCover - BarsizeShear - (BarsizeLongT / 2))
If Val(txtLength.Text) < d Then
    MsgBox("Please Put a large beam Length >=" & d & "mm" _
        , MsgBoxStyle.Exclamation)
    Me.Show()
    txtLength.Focus()
    Exit Sub
End If
'*********************************************************
If BarsizeLongT >= 20 And BarsizeLongT >= (a + 5) Then
    SpBbT = BarsizeLongT
ElseIf BarsizeLongT <= 20 And 20 >= (a + 5) Then
    SpBbT = 20
ElseIf BarsizeLongT <= (a + 5) And 20 <= (a + 5) Then
    SpBbT = (a + 5)
End If
If BarsizeLongB >= 20 And BarsizeLongB >= (a + 5) Then
    SPBbB = BarsizeLongB
ElseIf BarsizeLongB <= 20 And 20 >= (a + 5) Then
    SPBbB = 20
ElseIf BarsizeLongB <= (a + 5) And 20 <= (a + 5) Then
    SPBbB = (a + 5)
End If
Dim NoBarTop As Double 'calulated number of bar
Dim NobarBot As Double 'calulated number of bar
NoBarTop = (w + SpBbT) / (BarsizeLongT + SpBbT)
If NoBarTop < BarNoLongT Then
    MsgBox("Please put less number of top bars", MsgBoxStyle.Critical)
    Me.Show()
    txtNoBarT.Focus()
    Exit Sub
Else
    SpBbT = (w - (BarNoLongT * BarsizeLongT)) / (BarNoLongT - 1)
End If
NobarBot = (w + SPBbB) / (BarsizeLongB + SPBbB)
If NobarBot < BarNoLongB Then
```

```
    MsgBox("Please put less number of bottom  bars", MsgBoxStyle.Critical)
    Me.Show()
    txtNoBarB.Focus()
    Exit Sub
Else
    SPBbB = (w - (BarNoLongB * BarsizeLongB)) / (BarNoLongB - 1)
End If

Dim Spacingbar As Double
If SPBbB >= SpBbT Then
    Spacingbar = SPBbB
Else
    Spacingbar = SpBbT
End If
Cx = (H / 2) - ConCover - BarsizeShear - BarsizeLongB
Cy = (b / 2) - ConCover - BarsizeShear
K1 = 0.4
ASR = 2 * (Math.PI * (BarsizeShear / 2) ^ 2) 'mm2
Rowy = (ASR) / (b * Sp)
AsT = BarNoLongT * (Math.PI * (BarsizeLongT / 2) ^ 2)
AsB = BarNoLongB * (Math.PI * (BarsizeLongB / 2) ^ 2)
ysT = ConCover + BarsizeShear + (BarsizeLongT / 2)
ysB = H - ConCover - BarsizeShear - (BarsizeLongB / 2)
SecArea = H * b
Rowx = (AsT + AsB) / SecArea
Sx = Spacingbar
Dim barSize As Double
If BarsizeLongB > BarsizeLongT Then
    barSize = BarsizeLongB
Else
    barSize = BarsizeLongT
End If
Smx = 2 * (Cx + (Sx / 10)) + (0.25 * K1 * (barSize / Rowx))
Smy = 2 * (Cy + (Sp / 10)) + (0.25 * K1 * (barSize / Rowy))
hi = H / 20
For Me.i = 0 To 19
    yc(i) = i * hi + (hi / 2)
Next
frmmain.lblConArea.Text = "Ac = " & Math.Round(SecArea, 2) & " mm2"
frmmain.lblWidth.Text = b & "mm"
frmmain.lblDepth.Text = H & "mm"
frmmain.lblToprein.Text = BarNoLongT & "Ø" & BarsizeLongT
frmmain.lblBotrein.Text = BarNoLongB & "Ø" & BarsizeLongB
frmmain.lblShear.Text = "Ø" & BarsizeShear & " c/c " & Sp & "mm"
```

```
frmmain.GRBSection.Visible = True
frmmain.cmdCalculate.Enabled = True
frmmain.GRBLoad.Visible = True
frmmain.Enabled = True
frmmain.BringToFront()
frmmain.OutputToolStripMenuItem1.Enabled = True
'To draw a Reinforcement Detail
Dim bit As Bitmap = New Bitmap(frmmain.pic3.Width, frmmain.pic3.Height)
Dim g As Graphics = Graphics.FromImage(bit)
Dim myPen As Pen = New Pen(Color.Black, 2)
Dim penRedCir As Pen = New Pen(Color.DarkRed, 2)
Dim penRedlin As Pen = New Pen(Color.Red, 2)
Dim pengreen As Pen = New Pen(Color.Green, 2)
Dim penorange As Pen = New Pen(Color.OrangeRed, 2)
'To Draw a rectangle
Dim myRectangle As New Rectangle
Dim myRectangle1 As New Rectangle
myRectangle.X = 88
myRectangle.Y = 24
myRectangle.Width = 88
myRectangle.Height = 176
myRectangle1.X = 88 + 10
myRectangle1.Y = 24 + 10
myRectangle1.Width = 88 - 2 * 10
myRectangle1.Height = 176 - 2 * 10
g.DrawRectangle(penorange, myRectangle)
g.DrawRectangle(penRedlin, myRectangle1)
'To Draw a Circle
g.DrawEllipse(penRedCir, 156, 180, 8, 8)
Dim myBrush As Brush = New SolidBrush(Color.DarkRed)
g.FillEllipse(myBrush, 156, 180, 8, 8)
g.DrawEllipse(penRedCir, 156, 34, 8, 8)
g.FillEllipse(myBrush, 156, 34, 8, 8)
Dim NoBarB As Integer
Dim SpacB As Integer
Dim NoBarT As Integer
Dim SpacT As Integer
NoBarB = BarNoLongB
SpacB = Math.Round(((68 - (8 * NoBarB)) / (NoBarB - 1)), 0)
NoBarT = BarNoLongT
SpacT = Math.Round(((68 - (8 * NoBarT)) / (NoBarT - 1)), 0)
Dim Xo As Integer
Dim Yo As Integer
Dim Lx As Integer
```

```
        Dim Ly As Integer
        Xo = 98
        Yo = 180
        Lx = 8
        Ly = 8
        Dim intcount As Integer
        For intcount = 0 To NoBarB - 2 Step 1
            g.DrawEllipse(penRedCir, Xo + intcount * (Lx + SpacB), Yo, Lx, Ly)
            g.FillEllipse(myBrush, Xo + intcount * (Lx + SpacB), Yo, Lx, Ly)
        Next
        For intcount = 0 To NoBarT - 2 Step 1
            g.DrawEllipse(penRedCir, Xo + intcount * (Lx + SpacT), Yo - 146, Lx, Ly)
            g.FillEllipse(myBrush, Xo + intcount * (Lx + SpacT), Yo - 146, Lx, Ly)
        Next
        For intcount = 0 To NoBarB - 1 Step 1
            g.DrawLine(pengreen, Xo + 5 + intcount * (Lx + SpacB), Yo + 5, 200, 144)
        Next
        For intcount = 0 To NoBarT - 1 Step 1
            g.DrawLine(pengreen, Xo + 5 + intcount * (Lx + SpacT), Yo - 146 + 5, 200, 76)
        Next
        'Line for stirrups
        g.DrawLine(pengreen, 166, 108, 166 + 40, 108)
        frmmain.pic3.Image = bit
        '-------------------------------------------------------
        Me.Hide()
    End Sub
End Class
```

'There is only small modification in the main windows code; it's the iteration of calculated and applied moment, as well the calculation of the design moment based on the design shear fore.

```
'*********************************************************************
M = V * (frmSectionProperties.BeamL - ((frmSectionProperties.Colx / 2) +
frmSectionProperties.d)) / 2000

        If Math.Round(Nrs, 1) < -10 Then
            X = X - 0.1
            GoTo lineA
        ElseIf Math.Round(Nrs, 1) > 10 Then
            X = X + 0.1
            GoTo lineA
        End If
        If Math.Round(Math.Abs(M - (Mrs * 10 ^ -6)), 1) > 0.1 Then
            N = N - 10
            GoTo lineB
        End If

'*********************************************************************
```