ADDIS ABABA INSTITUTE OF TECHNOLOGY

GRADUATE SCHOOL

TRAFFIC GENERATED POWER

A Thesis Submitted to the School of Graduate Studies of Addis Ababa Institute of Technology in Partial Fulfillment of the Requirements for the Degree of Master in Structural Engineering

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by

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Train of ideas may flock to one's mind, abundantly, every minute. At times, developing and making these ideas relevant may not be an easy task. It could even be startlingly and disappointingly prohibitive. The fate of the ideas, in such occasions, lies directly on the noble contributions of all those individuals who are taking part in developing the ideas. Special gratitude is due to my advisor Dr. Shifferaw Taye for his technical assistance during the preparation of this thesis, not to mention his encouragement when things seem to be prohibitive. I would also like to thank co-advisor Dr-Ing Tamirat Tesfaye (department of mechanical engineering) for his invaluable technical advices during design of the mechanical system which is a backbone for the thesis. I am indebted to those 'invisible hands' that had been tireless to make my environment flawless during the preparation of this thesis, my father Tesfaye Melaku and my mother Kelemework Tadesse, I thank you.

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ABSTRACT

The objective of this thesis has been to investigate the possibility of tapping energy from a moving vehicle that would have been released and wasted to the ground otherwise. This has been done by designing a mechanical system to be installed across a road way such that vehicles running over it would not be disrupted. Then analysis model has been developed to study the interaction between the vehicles and the mechanical system. Emphasis has been made on observing the performances of the interacting bodies, estimating the amount of power that can be trapped, illustrating the technical feasibility of power generation, and finally recommending alternative areas that should be considered to increase the efficiency of the system.

The thesis starts by taking a vehicle representative (in terms of dynamic properties) of the traffic flow of the roadway under consideration. The vehicle is modeled as a quarter-car having only one mass-spring-damper components with a single tire attached. This quarter-car model is allowed to roll over a flat spring-supported mechanical system that is linked to a rotor below the surface of the original ground, establishing more like a shock-absorber system. Then dynamic analysis is performed to see the response of the vehicle and that of shock-absorber (supporting mechanical system). Analysis is also done for a sine-curved supporting system instead of the flat support.

Both analyses results have demonstrated that the rotor (main component of the shock-absorber) will be accelerated to a satisfactory rotational speed causing very little disturbance on the vehicle. This illustrates the possibility trapping energy of magnitude more than 250 joule per single vehicle moving over a specially designed mechanical equipment to be installed across a roadway. Furthermore, qualitative description of this power transmitting mechanical system confirms the technical possibility of the traffic generated power system.

CHAPTER ONE: INTRODUCTION

1.1 GENERAL

The energy consumption of countries in general and per capita demand of individuals in particular is increasing at an increasing rate. Number of reasons can be given for this alarming rate. The urbanization of states for the developing countries on one hand and yet an increase in standard of living in the case of developed countries, coupled with an increase in population could justify the recorded increment in power demand of the globe. World population doubles every thirty-five years. Furthermore, there is a much faster increase in world energy consumption to double every ten to twelve years due to a natural desire to improve living standards (1).

To satisfy this ever increasing demand, therefore, various sources of energy are being utilized despite their harm to the environment and inconvenience to implement. Petroleum, for example, is a ‘necessary’ energy source throughout the world, at least until this century and yet its utilization dangerously affecting the atmosphere, not to mention the insecurity in the natural reserve of this resource. Hydropower and wind power could be taken relatively as environmental friendly sources of energy and convenient for utilization, although they need suitable geographic or hydrologic resource which might be scarce in many countries.

This thesis introduces the concept of generating energy from vehicular traffic to be referred as traffic generated power (TGP) system, an alternative source of energy, which is proposed to address the aforementioned drawbacks of existing sources of energy. The thesis presents detail analysis to estimate the expected amount of energy from the TGP system and demonstrates the mechanism of absorbing the estimated energy.

1.2 BACKGROUND

Naturally, human beings make their first journey starting from “home” to “destination” using their four limbs as a means of locomotion during the early days of their life. The instance could be taken as their first encounter to friction and understanding of the principles of motion. Imagine a baby crawling on a spring mattress. The three basic steps the little baby has to learn to reach its “destination” are pushing down the springs of the mattress, grab firmly the fabrics of the mattress with its four limbs and then strain its muscles to push itself forward. It is worthy to have a closer look at the three steps and discuss in a bit detail since they represent all sorts of friction-based motions and are basic for the thesis.

Firstly, the baby has to push down and compress the springs to get enough support from the mattress that would, otherwise, accelerate and sink the baby down. The spring compression prolongs until the downward forces on the limbs of the baby (gravitational weight distributed over the limbs) are counter balanced by the upward forces of the compressed springs. In effect, the baby will get sufficient support and he will be in a position to move forward. Note that it does some work while compressing the spring. The next important step for crawling is to hold firmly the fabrics of the mattress. This safeguards slipping. Finally, the baby strains its muscles and exerts a tractive effort (force) on the mattress in opposite direction to its motion in order to propel itself forward. All these steps show application and interpretation of Coulomb’s laws that are basic to all friction-based motions, like the motion of a rubber-tired vehicle on asphalt road (2).

Though the qualitative explanation seems crude and rudimentary, one fundamental statement can be withdrawn to shade light on the concepts underlying this thesis. An object moving or intending to move horizontally on an elastic support (half-space) by friction mechanism and subjected to downward gravitational force has to do some work on the supporting medium in order to enhance its movement. A car moving over an asphalt road, or man walking on a ground both do some work on the respective support systems. The next section briefs the basic principles as to how energy tapping will be possible.

1.3 UNDERLYING PRINCIPLE

How much energy is propagated to the ground when a vehicle moves on a gravel road? How can we sense it and absorb it? When vehicles move on an uneven ground, it is a common experience to see the ground shaking, nearby building units like windows and doors quake. This is one way to sense the propagated energy due to a moving vehicle. To mention other, asphalt road sections after their long service time exhibit cracks along and across the roadway. Without neglecting the environmental effects and other possible causes, this is predominantly contributed by the repetitive loading due to vehicles moving over the road. Therefore, this could be taken as another effect of the propagated energy. These two common occurrences, in addition to confirming the propagation of energy to the supporting ground, suggest the possibility of absorbing energy from moving vehicular loads. The absorption of this energy and conversion and usage as an alternative source of energy (power), called traffic generated energy (power) TGE/P, could be realized if innovative technological system is devised.

1.4 SCOPE

TGP system encompasses a wide variety of disciplines ranging from traffic engineering, to tribology (in order to closely study the behavior of friction between the tire and energy tapping equipment), from theory of vibrations to vehicular dynamics. In addition, depending on the form of energy it is intended to convert the TGP, other areas of study may emerge; for example, if the energy from the traffic is intended to be converted to purely mechanical system, then mechanical engineering design know-how will be important. In other occasion, a need may arise to use the trapped energy in the form of electrical energy, and hence the knowledge of electromechanical systems will be crucial. Thus, detail analyses have to be done in all the respective areas before starting to implement the system. Strictly speaking, discussion of this thesis is limited to traffic analysis, dynamical analysis and mechanical system design among the wide spectrum of analyses required. This is not only because complete analysis is cumbersome, but doing so needs a specialty in all those areas. The procedural chart below (figure 1-1) shows a summarized description of the overall TGP system design procedure.

Design of the overall system starts by selecting a road section based on a preliminary data about the number of traffic flow, the axle load of the traffic and suitability of the road section for TGP system installation. Selecting a road with high traffic flow with heavy axle load is preferable due to its higher power generation potential.

As shown in the chart, traffic flow analysis is the next important step while designing TGP system. The analysis provides adjusted traffic flow parameters: traffic count and axle load quantities that are of paramount importance for dynamical analysis and overall estimation of power (expected from the TGP system). In addition to this, traffic flow analysis helps the determination of the installation period, i.e. the time at which installing TGP system on the selected road will be feasible (if at present, it is found out to be unfeasible).

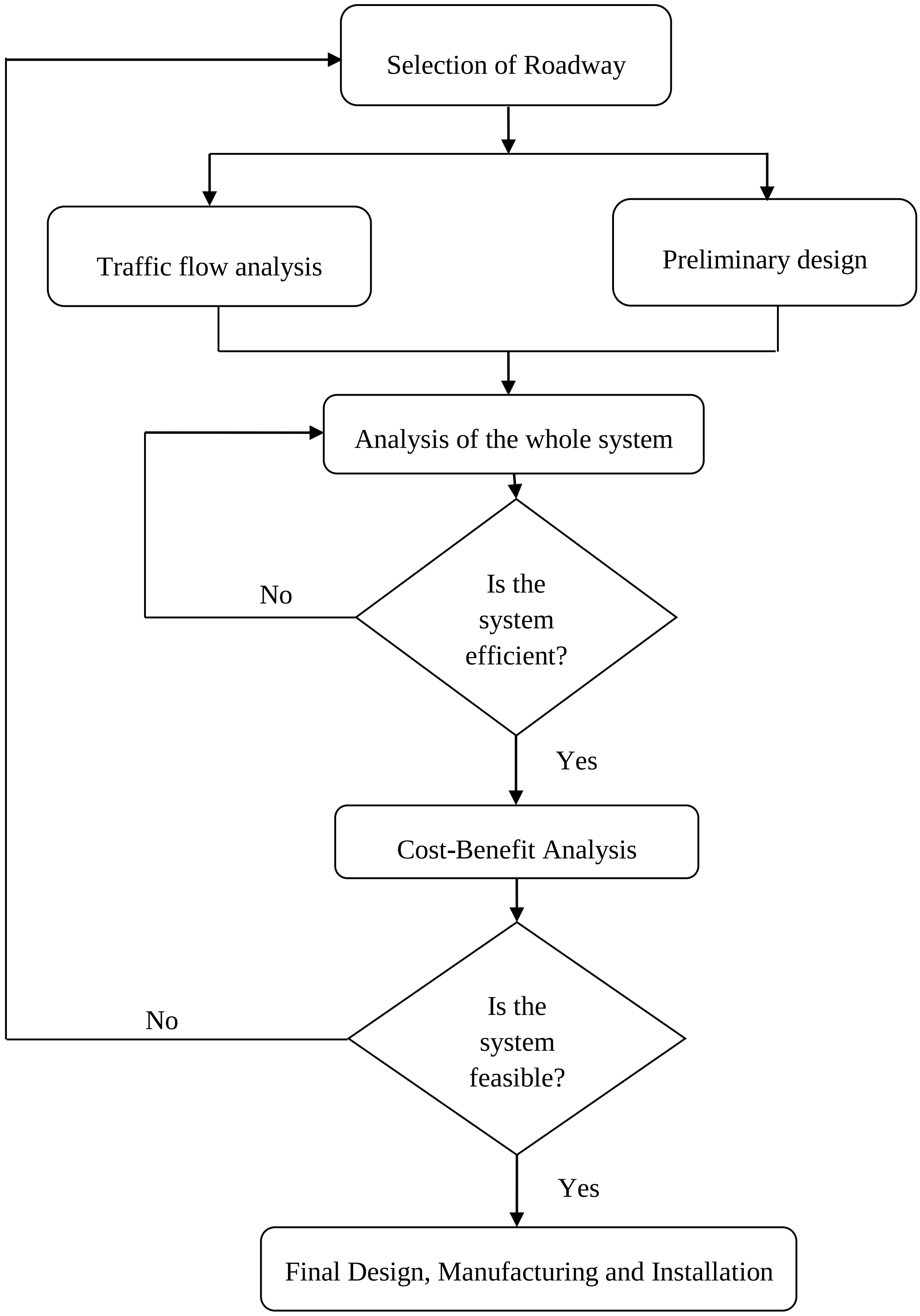


Fig. 1-1 Schematic representation of the TGP system design procedure

The other step is the system design process, which is a process of designing the actual physical elements and their mechanical of operation that make up TGP system. It is during this process that innovative ideas will be highly relevant. A mechanical engineer may suggest purely mechanical system to trap the energy from moving vehicles; a material engineer may think of a special pavement material that can absorb the strain energy due to the imposed load as vehicles run over the material. This shows that various design outputs can be achieved from the different innovative ideas involved. In this thesis, an electro-mechanical system is preferred and designed.

Outputs from the traffic flow analysis design will make analysis of the overall TGP system possible. Dynamic analysis of the vehicle-mechanical system, estimation of trappable energy from TGP system, and performance evaluation are some of the outputs from the overall system analysis procedure. The outputs will provide enough ground to evaluate the efficiency of the proposed mechanical system.

How can the proposed system be recommended for manufacturing and installation once it is proved efficient? It may not be a simple task decide but all the advantage of installing the system on the selected road have to be weighted with the cost of installing the system (cost-benefit analysis) in order to come up with a justified decision. However, for a system analyzed to be inefficient has to be optimized or redesigned and checked for its performance. This iterative procedure prolongs until an efficient but not feasible. This happens if an expensive investment is needed for the implementation of the system than it can be benefited from the investment. The only option, in such cases, is to search for another site (design road) or wait for the implementation until the traffic flow grows large enough.

CHAPTER TWO: LITERATURE REVIEW

2.1 GENERAL

As stated in the introduction Chapter, traffic generated power (TGP) system covers different disciplines: analysis of traffic flow, analysis of vehicles, the design of mechanical power transmission systems, and tribology (study of friction) are the main areas to be dealt with. It appears that all these disciplines have no relation with each other. However, they must be considered as parts of an emerging subject than unrelated segments from various disciplines. Thus, literatures reviewed in this thesis seem to have a wide gap in their content. But going through the whole chapters will explain the relevance of the literatures to TGP system analysis.

Literatures surveyed in sections 2.2 and 2.3 focus on traffic analysis. Traffic analysis involves the study of traffic loading and traffic volume that a given road section is expected to carry within a certain period of time. As it will be shown in the subsequent chapters, both components of the analysis (traffic loading and traffic volume) are essential variables that directly affect the magnitude of energy that could be obtained from TGP system. This necessitates the detail analyses of traffic flow. Fortunately, already established procedures for designing road pavements may be adopted since they provide startup guidelines, though not complete. In order to fill this gap between established procedures and TGP system requirements, therefore, customization is crucial and will be made in the next chapter.

Here, it should be emphasized that it is never a coincidence to use the same analysis procedures for pavement design and TGP system design with slight modification for two apparently different purposes. Instead, the energy that contributes for the deterioration of paved roads is, after all, what TGP system tries to restore to some other form of energy instead of deteriorating the pavement and the supporting system.

Apart from traffic, vehicular dynamic analysis is another area about which literatures are assessed. Various literatures suggest various approaches in order to analyze a vehicle under different circumstances. This disparity in approach may arise depending on the level of accuracy anticipated form the analysis output. In any case, literatures serve as a foundation to the theoretical analysis of vehicles in relation to extraction of energy from vehicular traffic.

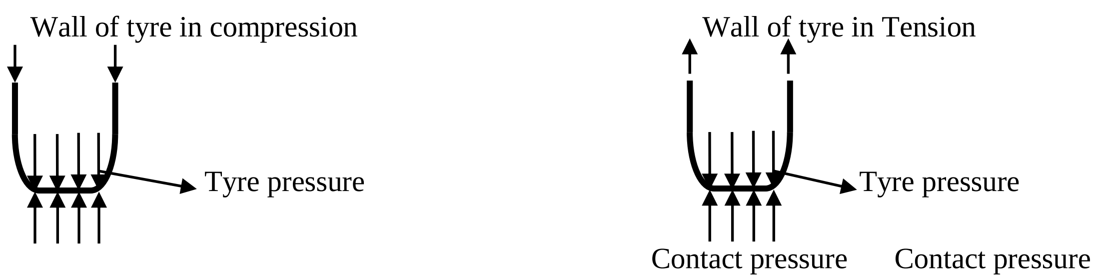
In general, the reviewed literatures will give a good background by presenting approved procedures and important guidelines that are necessary to obtain the important parameters in TGP system.

2.2 TRAFFIC LOADING

2.2.1 Contact Area:

As a first step in traffic loading analysis, it is necessary to know the contact area between tire and pavement to estimate the pressure imparted on the paved ground by the wheels of the vehicle. In addition to its help in estimating the contact pressure of the wheels on the ground during pavement design, calculation of the contact area is helpful in sizing of the submergible hump during the design of traffic generated power system that is to be discussed in the next Chapter.

Referring to [3], the axle load can be assumed to be uniformly distributed over the contact area. The size of this contact area depends on the contact pressure.



(a) Low pressure tyre

(b) High pressure tyre

Fig. 2-1 Relationship between contact and tyre pressure [3]

As indicated in Fig. 2-1(a), the contact pressure is greater that the tire pressure for low-pressure tires, because that wall of tires is in compression on the sum of vertical forces due to wall and tire pressure must be equal to the force due to contact pressure. Whereas, the contact pressure is smaller than the tire as in Fig. 2-1(b), this is because the wall of the tires is in tension. In pavement design, however, the contact pressure is generally assumed to be equal to the tire pressure.

(a) Actual contact area

(b) Equivalent area

Fig. 2-2 Dimensions of tire contact area [3]

Fig 2-2(a) shows the approximate shape of contact area for each tire, which is composed of a rectangle and two semi-circles that is approximated, further, by an equivalent rectangular area as in Fig 2-2(b). By assuming length L and width 0.6L, the area of contact Ac:

$$A_c = \pi\,(0.3L)^2 + (0.4L)(0.6L) = 0.5227L^2$$

$$L = \sqrt{(A_c/0.5227)}$$

in which A_c = contact area, which can be obtained by dividing the load on each tyre by the tyre pressure.

2.2.2 Static Load Analysis

a) Stresses and deflections of elastic foundation

The simplest way to characterize the behavior of a flexible pavement under wheel loads is to consider it as a homogenous half-space. A half-space has an infinitely large area and an infinite depth with a top plane on which the loads are applied. The original Boussinesq theory was based on a concentrated load applied on an elastic half-space. The theory assumes a weightless mass which is elastic and isotropic with no initial displacement. The stresses, strains, and deflections due to a concentrated load can be integrated to obtain those due to a circular loaded area.

Fig. 2-3 shows a homogenous half-space subjected to a circular load with radius ‘a’ and a uniform pressure ‘q’.

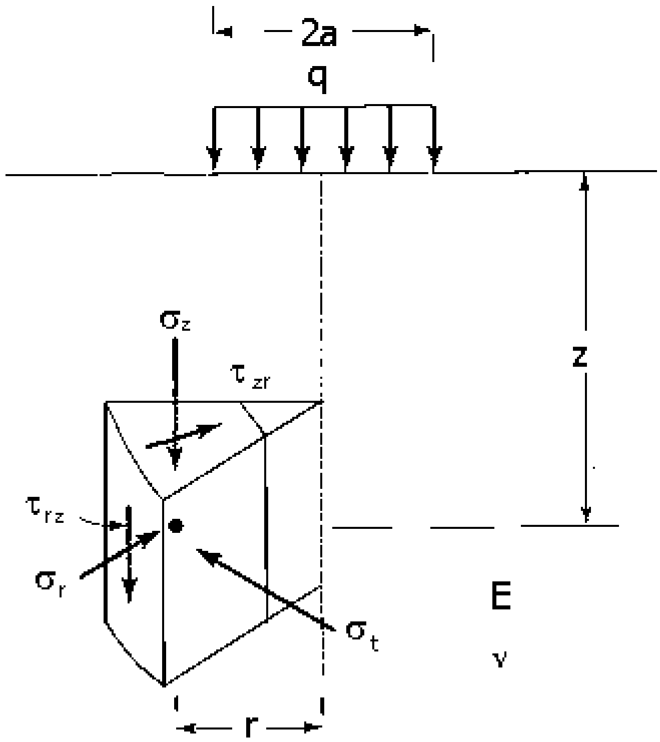


Fig. 2-3 Components of stresses under Axisymmetric loading [3]

The half-space has an elastic modulus ‘E’ and a Poisson ratio ‘v’. A small cylindrical element with center at a distance ‘z’ below the surface and ‘r’ from the axis of symmetry is shown. Due to axisymmetry, there are only three normal stresses, σ_z, σ_r and σ_θ, and one shear stress, τ_{rz}, which is equal to τ_{zr}. These stresses are functions of q, r/a, and z/a.

b) Circular area loading

It is discussed in (3) that the load applied from tyre to pavement is similar to a flexible plate with a radius ‘a’ and a uniform pressure ‘q’. In fact, the shape of this plate could have different shapes depending on the tyre pressure as presented in the previous section. The Stresses beneath the center of the plate are the axis of symmetry, on which the most critical stress, strain and deflection occur, where $\tau_{rz} = 0$, and $\sigma_r = \sigma_\theta$, so σ_z and σ_r are the principal stresses. These stresses and deflection can be determined by:

On the axis (r=0)

Stresses:

$$\sigma_z = q\left[1 - \left\{\frac{1}{1 + (^a/_z)^2}\right\}^{\frac{3}{2}}\right]$$

Equation 2-1 (a)

$$\sigma_r = \sigma_t = \frac{q}{2}\left[(1 + 2v) - \frac{2(1 + v)z}{\sqrt{(a^2 + z^2)}} + \frac{z^3}{(a^2 + z^2)^{\frac{3}{2}}}\right]$$

Equation 2-1 (b)

Deflections:

$$w = \frac{(1+2v)qa}{E}\left\{\frac{2}{\sqrt{(a^2+z^2)}} + \frac{(1-2v)}{a}\left[\sqrt{(a^2 + z^2)} - z\right]\right\}$$

Equation 2-2 (a)

When ν=0.5 it can be simplified to:

$$w = \frac{3qa^2}{2E\sqrt{(a^2+z^2)}}$$

Equation 2-2(b)

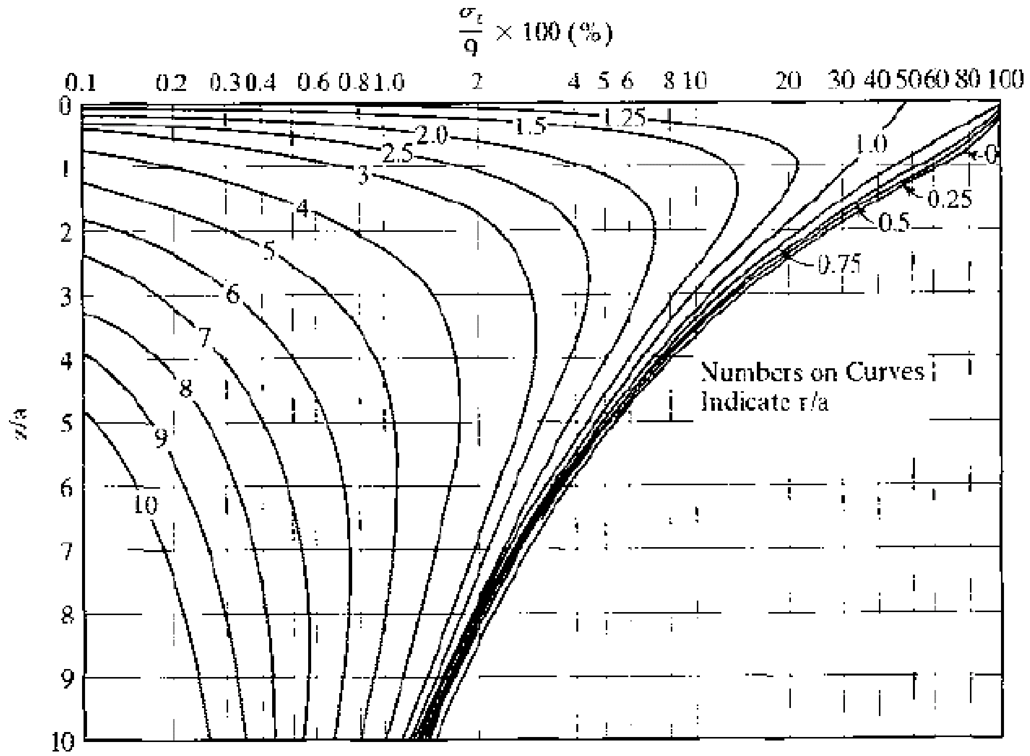
On the surface of the half-space, z=0:

$$w_0 = \frac{2(1-v^2)qa}{E}$$

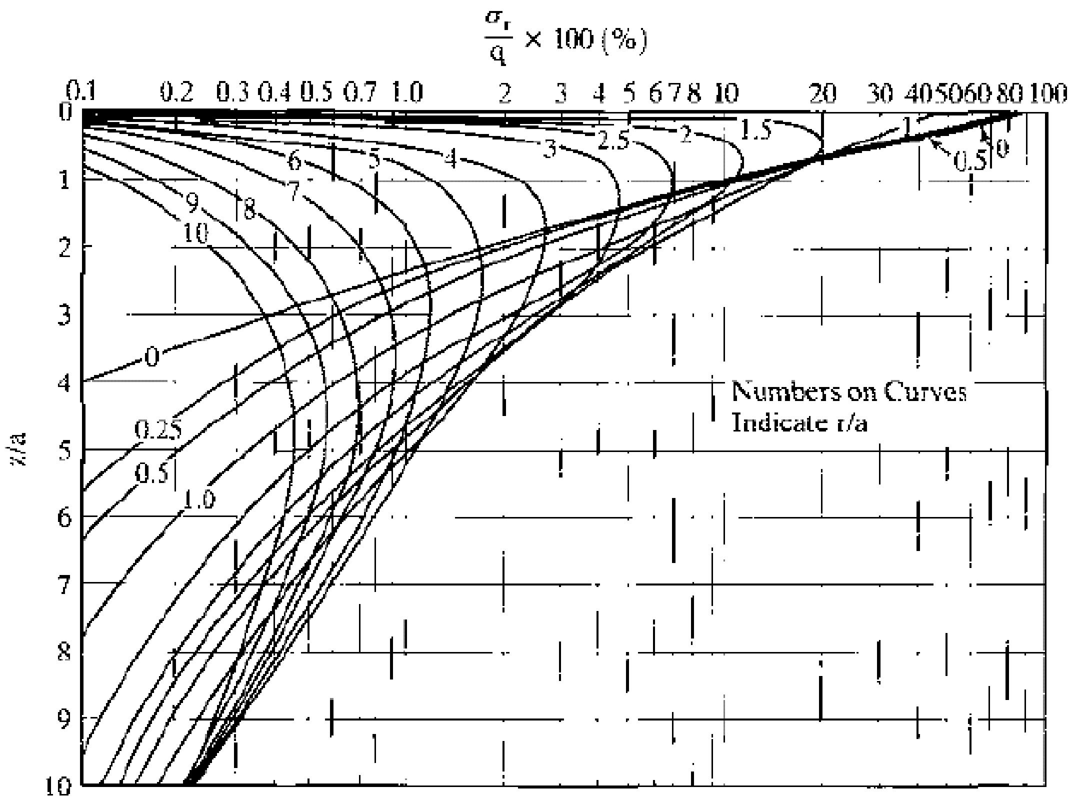
Equation 2-2 (c)

c) Solutions by charts

Charts could be generated for determining vertical stress, σ_z, radial stress, σ_r, tangential stress, σ_θ, and vertical deflection, w, as shown in Fig 2.4 through 2.5. The load is applied over a circular area with radius ‘a’ and an intensity ‘q’. Because Poisson ratio has a relatively small effect on stresses and deflections, it would be sufficient to take one value for the Poisson ratio, ν, equal to 0.5 so only one set of charts is needed instead of one for each Poisson ratio.

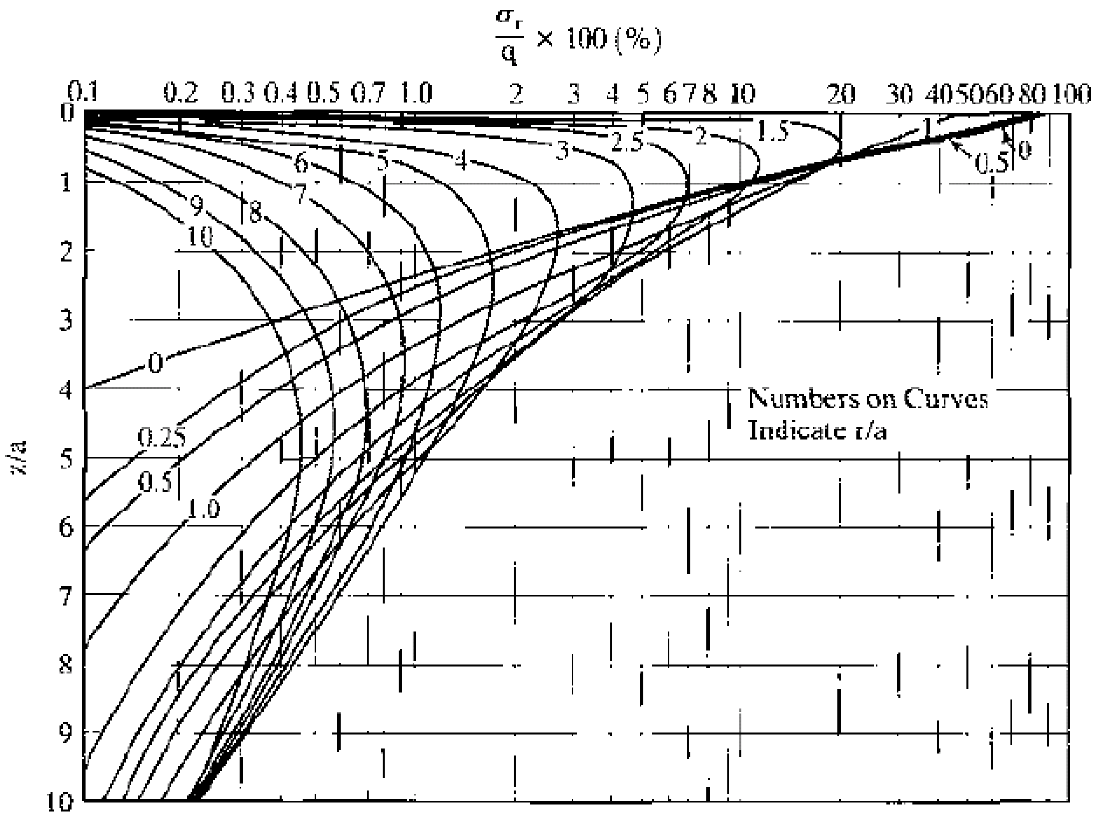


(a)

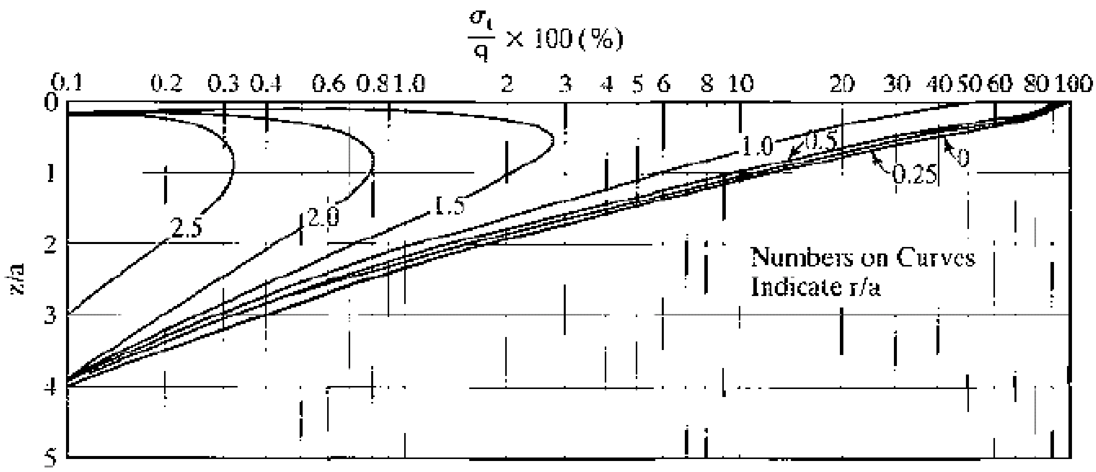


(b)

Fig. 2-4 Graphical solutions of the elastic half-space subjected to a uniform circular load [3]



(c)



(d)

Fig. 2-5 Graphical solutions of the elastic half-space subjected to a uniform circular load [3] (continued)

2.2.3 Analysis of moving loads

Method-1: Axel load of vehicles at a given point which passes to the ground through the contact area estimated above can be analyzed using a simplified method. In this method, it is assumed that the intensity of load varies with time according to a haversine function as shown in Fig. 2-6. With t=0 at the peak, the load function is expressed as:

$$L(t) = q \sin^2(\frac{\pi}{2} + \frac{\pi t}{d})$$

Equation 2-3

in which d is the duration of load from a given point, or $t = \pm d/2$, the load above the point is zero, or $L(t) = 0$. When the load is directly above the given point, or t=0, the load intensity is q.

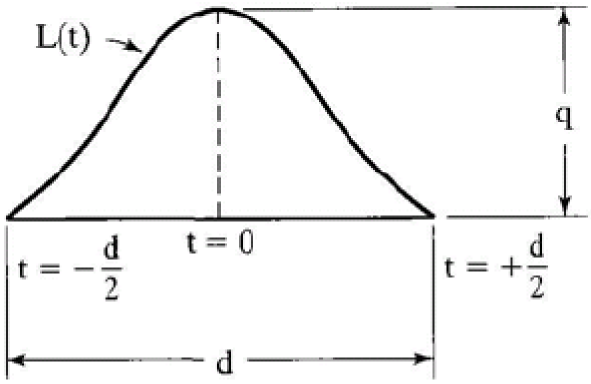


Fig. 2-6 Moving load as a function of time [3]

The duration of load depends on the vehicle speed s and the tire contact radius a. A reasonable assumption is that the load has practically no effect when it is at a distance of 6a from the point, or d=12a/s.

Method-2: Another method showed in (3) is that the vertical stress pulse can be approximated by a haversine or a triangular function, as shown in Fig. 2-7 below. The type and duration of loading used in the method should simulate that actually occurring in the field. It is therefore reasonable to assume the stress pulse to be a haversine or a triangular function, the duration of which depends on the vehicle speed and the depth of the point below the pavement surface.

$$\sigma_V = \sigma_{max} \sin(\frac{\pi}{2} + \frac{\pi t}{d})$$

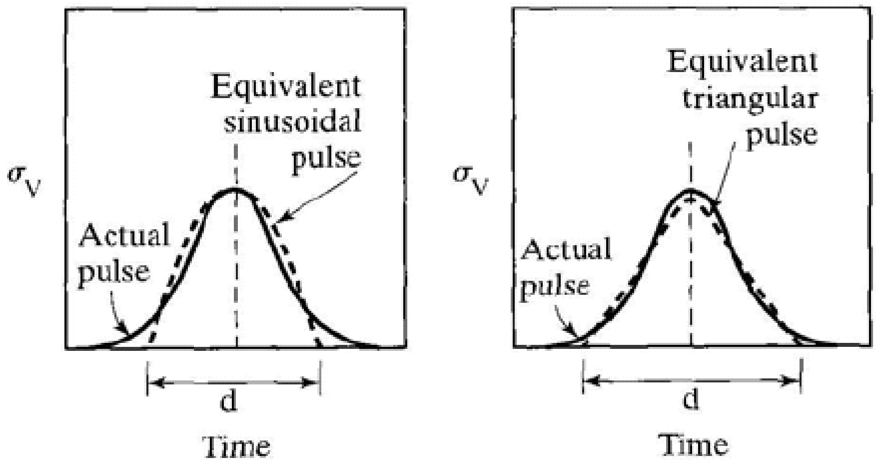


Fig. 2-7 Equivalent haversine & triangular pulse [3]

After considering the inertial and viscous effects based on the vertical stress pulses measured in many field experiments, the stress pulse time can be related to the vehicle speed and depth, as shown in Fig. 2-8**Error! Reference source not found.**. Because of these effects, the loading time is not inversely proportional to the vehicle speed.

It is possible to derive the loading time for a bituminous layer as a function of vehicle speed and layer thickness. The loading time is based on the average pulse time for stresses in the vertical and horizontal directions at various depths in the bituminous layer.

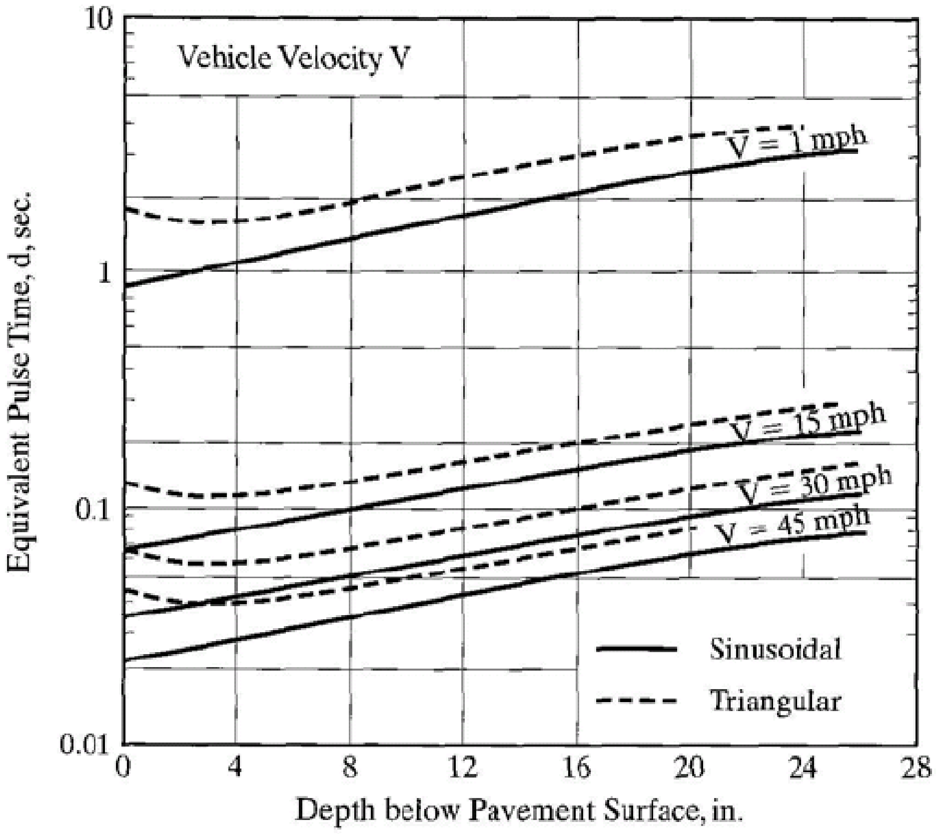


Fig. 2-8 Vertical stress pulse time under haversine or triangular loading (1in. = 25.4mm, 1mph = 1.6km/h) [3]

2.3 TRAFFIC FLOW

In this section, the general guideline in counting and classifying traffic flow is described. In addition, actual data and work methods for Addis Ababa city is illustrated.

2.3.1 Traffic Counts

Traffic counts [4] shall be carried out in a manner that results in a grouping of vehicle types into the categories given in Table 2-1. Inclusion of other vehicle categories or a further sub-division of the categories may be desirable for other purposes. However, manipulation of data shall not obscure the prescribed division into the four heavy vehicle categories.

Table 2-1 Vehicle Classification System [4]

Category	Cars	Light	Medium	Heavy	Articulated
Axles	2	2	3	4	>4
Tires	4	6	10	14	>14
Length	<3m	3m-7.5m	3m-7.5m	>7.5m	>7.5m
GVW	<3.5T	3.5T-12T	>12T	>12T	>12T
Includes	Cars Utility Minibus 4WD	Bus 1Axle Truck	2Rear Axle Truck	4 Axle Truck	

In the absence of detailed classification of traffic counts completed in accordance with the Traffic and Axle Load Study Manual, Table 2-2, Table provides typical values for each road class in Addis Ababa for the year 2002.

Table 2-2 Typical Values for Addis Ababa Traffic [4]

Road Classification	AADT (two way)	Typical percentage				
		Car	Light	Medium	Heavy	Articulated
Freeway	-	-	-	-	-	-
Arterial	10,000	80%	17%	2%	0%	1%
Sub-Arterial	9,000	89%	9%	1%	0%	1%
Collector	4,000	91%	8%	1%	0%	0%
Local	1,500	97%	3%	0%	0%	0%

2.3.2 Axel Loads and Equivalency Factors

a) Axel load

It is determined from weighbridge measurement usually undertaken for the purpose of enforcing axle load limits.

b) An equivalent factor

i. For pavement design

It is the damage per pass to a pavement by the axel in question relative to the damage per pass of a standard axel load, usually the 80kN single-axel load. According to [4], the equivalency factor is expressed as:

$$Equivalency\ Factor\ = \left[\frac{Axle\ Load}{ESA}\right]^{4.5};$$

Equation 2-4

Where ESA= Equivalent Single Axel Load (8,160kg)

In the absence of exact axle load survey results or studies for the specific roadway, generic values may be used modifying results presented for the purpose of pavement design from studies completed in Addis Ababa for roads in Addis Ababa. These are shown in Table 2-3. Default values should be used based on the lower and upper values listed in the table but with caution.

Table 2-3 Typical Equivalency Factors for Addis Ababa Traffic [4]

Vehicle Class	Typical	Lower	Upper
Car	1.07	0.00	1.39
Light	2.17	1.89	2.34
Medium	2.48	2.17	2.69
Heavy	2.59	2.36	2.77
Articulated	3.00	2.76	3.19

In order to determine the initial daily traffic loading from the survey data, the following procedure should be followed:

- Determine the daily traffic flow for each class of vehicle from the results of this traffic survey and other recent traffic count information that is available in (4)
- Adjust the flows for two way traffic, lane factors and seasonal adjustment.
- Determine the mean equivalence factor for each class of vehicle from the results of the axle load survey or default values from Table 2-3.
- Determine the initial daily traffic loading by summing the products of the daily flow in each class by the mean equivalence factor for each respective class:

$$N=N_L EF_L+N_M EF_M+N_H EF_H+N_A EF_A$$

where N=initial daily Equivalent Standard Axles

N_L=daily number of light vehicles in the current year

N_M=daily number of medium vehicles in the current year

N_H=daily number of heavy vehicles in the current year

N_A=daily number of articulated vehicles in the current year

EF_L=equivalency factor for the average light vehicle

EF_M=equivalency factor for the average medium vehicles

EF_H=equivalency factor for the average heavy vehicles

EF_A=equivalency factor for the average articulated vehicles

i. For pavement design

Equivalency factor for TGP system design purpose is similar to, but not identical, in appearance and meaning with the equivalency factor used for the purpose of pavement design which is given above. This adjusted equivalency factor is the relative GVW of other vehicle categories with respect to GVW of Car vehicle category and is expressed by:

$$Equivalency\ Factor\ = \left[\frac{Axle\ Load(other\ categories)}{Axel\ Load\ (Cars\ category)}\right]\ ;$$

Equation 2-5

where Axel load (cars category) = 3,500kg

Note that we are not interested in the damaging effect of vehicles, instead, simply the relative value of the vehicle's axel load to a standard axel load. Hence, modification is made by assuming and substituting the equivalent standard axel (ESA) with value of axel load for Cars category having an axel load of 3,500kg and neglecting the exponent part.

It is possible to modify the typical factors for Addis Ababa traffic (i.e. Table 2-3) if the actual traffic survey data is obtained. In addition, to determine the initial daily traffic loading form the survey data, the procedure mentioned for pavement design case should be followed with no adjustment. Nevertheless, the initial daily equivalent standard axel (N) calculation must be modified in such a way to include the daily number of cars vehicle category. This category will be neglected in the pavement design stage for its significance in determining the amount of N which, in turn, is due to fact that the ratio of car axel load to equivalent single axel (ESA) load is small and the ratio being raised to 4.5 will make the resulting figure irrelevantly small.

$$N= N_CEF_C+N_LEF_L+N_MEF_M+N_HEF_H+N_AEF_A$$

where N=initial daily Equivalent Standard Axles

N_C=daily number of cars vehicles in the current year

N_L=daily number of light vehicles in the current year

N_M=daily number of medium vehicles in the current year

N_H=daily number of heavy vehicles in the current year

N_A=daily number of articulated vehicles in the current year

EF_L=equivalency factor for the average light vehicle

EF_M=equivalency factor for the average medium vehicles

EF_H=equivalency factor for the average heavy vehicles

EF_A=equivalency factor for the average articulated vehicles

2.3.3 Rate

a) Design period Design Period and Traffic Growth

i) For pavement design

Determining an appropriate design period is the first step towards pavement design. Many factors may influence this decision, including budget constraints. However, the designer should follow certain guidelines in choosing an appropriate design period, taking into account the conditions governing the project. Some of the points to consider include:

- Functional importance of the road
- Traffic volume
- Location and terrain of the project
- Financial constraints
- Difficulty in forecasting traffic

It generally appears economical to construct roads with longer design periods, especially for important roads and for roads with high traffic volume. Where rehabilitation would cause major inconvenience to road users, a longer period may be recommended. For roads in difficult locations and terrain where regular maintenance proves to be costly and time consuming because of poor access and non-availability of nearby construction material sources, a longer design period is also appropriate. Problems in traffic forecasting may also influence the design. When accurate traffic estimates cannot be made, it may be advisable to reduce the design period to avoid costly overdesign [6].

ii) For TGP system design

Some roads may entertain low volume of traffic having lighter GVW. In such circumstances, it would be repulsive to consider that road as a potential TGP source. The fact that traffic volume increases with time, for many reasons, might make the road considerable without even mentioning the probable increment of GVW associated with the increase in traffic volume. The installation (design) period is simply , therefore, the length of time expressed in years it has to be waited until the traffic flow grows well enough to justify the economic feasibility for the installation of TGP system on that specific road. The traffic flow is assumed to grow at a constant rate and this rate is defined as follows.

b) Growth rate

A given road segment may not be suitable for the installation of energy absorbing equipment due to its low capacity of traffic, both in number and vehicular weight. Naturally, however, this traffic capacity increases with time making the road segment to be suitable for energy storage purpose. Therefore, it necessitates estimation of the traffic growth rate so that decisions whether to install the energy absorbing system would be made evidently and easily.

The forecasting of traffic growth shall include separate estimates for the 5 vehicle categories. It is necessary to assess future traffic in respect of the following types:

- Normal traffic that would use the route regardless of the condition of the road
- Diverted traffic that moves from an alternative route due to the improvement of the road, but at otherwise unchanged origin and destination
- Generated traffic
- Additional traffic occurring due to the improvement of the road

There is considerable uncertainty and risk of making large errors in estimations of traffic growth since a number of individually uncertain factors are brought together in the analysis. Where little information is available, historical data, origin-destination surveys and records for Addis Ababa roads, for example, from Addis Ababa City Roads Authority (AACRA) design manual are among the sources of information for assessment of traffic growth. The decision maker may have to resort to the use of growth figures for GDP in the estimation of movement of goods.

Based on road traffic survey information, it is reasonable, in most circumstances, to assume that traffic volumes will increase geometrically either for the entire design period or up to a stage where “road capacity” is reached (in which case traffic volumes are assumed to remain constant for the remainder of the design period.

Table 2-4 Cumulative Growth Factors (GF) [5]

Design Period (years)	Growth Rate (%pa)					
	0	2	4	6	8	10
5	5	5.2	5.4	5.6	5.9	6.1
10	10	10.9	12.0	13.2	14.5	15.9
20	20	24.3	29.8	36.8	45.8	57.3
30	30	40.6	56.1	79.1	113.3	164.5
40	40	60.4	95.0	154.8	259.1	442.8

For geometric traffic growth throughout the design period, total traffic over the design period is determined by multiplying the total traffic in the first year by the appropriate Cumulative Growth Factor from Table 2-4 or calculated exactly using the following equation:

$$GF = \frac{(1 + 0.01i)[(1 + 0.01i)^y - 1]}{0.01i}$$

Equation 2-6

where i =Growth Rate (%pa)

y=Intended Life Time of the Road (years)

GF=Growth Factor

For instance, in the absence of specific traffic survey and study results of Addis Ababa city roads, typical values for growth rates can be adopted with caution based on the economic growth zones shown in (4) and the zone/road classification matrix presented in Table 2-5. These values should be used with the intended service life of the road in Eq. 2-6 to determine the appropriate approximate Growth Factor.

The sensitivity of the design to these default values should be checked if these values are adopted for the final design parameters. Otherwise, they should serve purely as a guide and check for more detailed survey and investigation results will be mandatory.

Table 2-5 Growth Rate Matrix (% pa) [5]

Road Classification	Economic Growth Zone					
	1	2	3	4	5	6
Arterial	7.0	11.2	7.0	11.1	7.2	9.3
Sub-Arterial	6.5	9.3	6.5	9.2	6.6	8.2
Collector	6.0	7.4	6.0	7.4	6.1	6.8
Local	5.5	5.5	5.5	5.5	5.5	5.5

2.3.4 Calculation of Design Traffic

At the end of the day, from the traffic analysis it is the number of vehicles having a standardized axel load passing a given point, i.e. design traffic, is what is required. This value is then calculated as the design number of standard axles passing a point within a second (N_t):

$$N_t \text{ (No. of traffic/sec)} = N \text{ x } GF/\ (24\text{x}3600)$$

Equation 2-7

where GF is the cumulative growth factor from Table 2-4 or Eq. 2-6 and

N is the initial daily traffic loading in accordance with the Traffic and Axle Load Study Manual (4)

2.4 MODELING VEHICLES

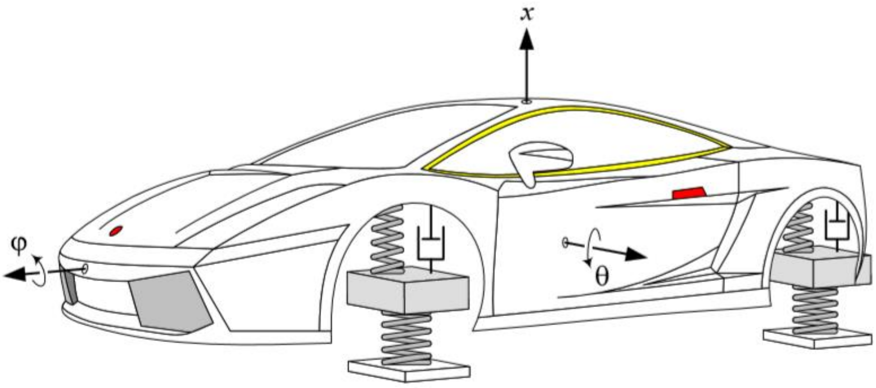
Vehicles are the basic units of the traffic flow system. These vehicles may vary in different aspects such as weight, dimensions of their parts, etc. Despite their dissimilarity, however, it is possible to represent their main body parts by equivalent mechanical elements. Such simplification of the actual vehicle is a crucial step if one needs to study dynamic behavior and performance of moving vehicles.

When dealing with TGP system, one of the primary steps to be carried out is dynamic analysis for which modeling of a representative is the first step. A good model or representation of this vehicle will better simulate the actual condition. Eventually, the dynamic analysis performed using this model will result a fair estimation of power expected form TGP system. Considering this, literatures are reviewed as to how vehicles are modeled.

2.4.1 Simplified Model

Vehicle is used to refer any moving object having a certain weight and moves by traction force between the tires of the vehicle and the ground. The vehicle in this thesis is simplified as a two dimensional object having only one wheel which is allowed to move in both horizontal and vertical directions. It is further simpli9fied and modeled, for dynamic analysis purpose, as a mass (being equal to the entire vehicular mass) to which a spring-damper system is attached.

According to some literatures (4), vehicles are modeled to include a mass, spring-dampers to represent the suspension-shock absorber system at each tire location and another spring to represent the stiffness of the tires themselves, see Fig. 2-9 (a).



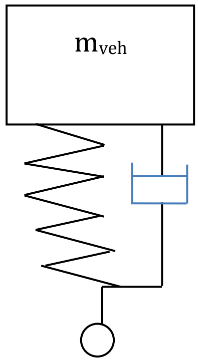


Fig. 2-9 A full car vibrating model of a vehicle (a)two-springs [4] (b)one-spring [6]

Vehicles can be modeled in a more simplified way. Some other literatures (6) neglect the stiffness of the tire and simply consider one spring to signify the effect of suspension system, see Fig. 2-9(b).

2.4.2 Vehicle Moving Over a Bump

The primary aim of this section is to illustrate the procedure needed in order to mathematically model a vehicle moving over a bumpy ground. This specific area is selected because it is fundamental for dynamic analysis of traffic generated power system (Chapter 3). AS shown in Fig 2-10, a moving vehicle is subjected to a bump of half-sine curve. Our interest is to write the equation of motion for the vehicle in the vertical direction along its way over the bump. The figure depicts that the vehicle is represented by a mass m, a spring having a stiffness k, and a damper with damping c.

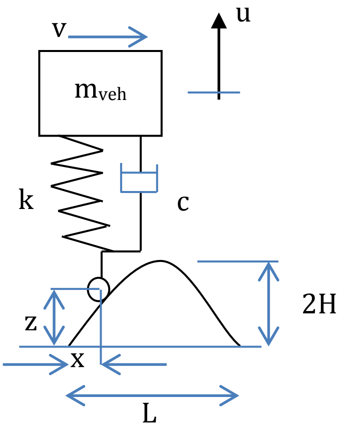


Fig. 2-10 A vehicle model moving with speed v on a bumpy road

Where: k, c = stiffness and damping of the vehicle

V = horizontal vehicular speed

U = vertical displacement

H = amplitude of the sine curve (hump)

L = wave length of sine curve

Z = vertical position of the tires as it covers a horizontal distance x on the bump

x = horizontal position of the tire

The equation of motion is given as:

$$m\ddot{u} + \dot{u}c + uk = kz + c\dot{z}$$

But $z = Hsin\left(\frac{\pi x}{L}\right) = Hsin(\frac{\pi vt}{L})$

$$\dot{z} = \frac{\pi Hv}{L}\cos\left(\frac{\pi vt}{L}\right)$$

$$m\ddot{u} + \dot{u}c + uk = k\left\{Hsin\left(\frac{\pi vt}{L}\right)\right\} + c\left\{\frac{\pi Hv}{L}\cos\left(\frac{\pi vt}{L}\right)\right\}$$

Equation 2-8

The solution of the above equation is adequate to evaluate and graph the response of the vehicle. The derivation and solution of a similar system can be found in [6].

2.5 SUMMARY

In this chapter, literatures have been surveyed regarding the procedures to calculate the amount of traffic flow, the gross vehicular weight (GVW) of the traffic and other relevant values from pavement design perspective. Wherever necessary, slight modifications are performed to suggest and accommodate cases typical to traffic in TGP system.

In addition, it is tried to review literatures on how to model a moving vehicle since a vehicle is the basic unit of traffic flow system and hence, a fundamental element for TGP system analysis. As shown above, different literatures suggest different approaches in order to model a vehicle. However, this opts for the simpler model form the tow modeling techniques where the stiffness of the tire is neglected.

In general, the surveyed materials can give a good background and serve as a basis for understanding TGP system.

CHAPTER THREE: DYNAMICA ANALYSIS

3.1 GENERAL

Given the amount of traffic flow and gross vehicular weight (GVW) of the traffic carried by a given paved road using the procedures discussed in Chapter 2. Assume a representative vehicle moving over this road encounters a special mechanical system installed at the ground level and provides a submergible support to the tyres. If someone is interested to observe the behavior of the vehicle as it runs over the 'obstacle', s/he has to perform a detail dynamic analysis of the overall system. Then, it will be possible to determine not only the dynamic behavior of the vehicle but also the response of the submergible mechanical system characterized by its mass-spring-damper parameters. The detail of this Chapter is to formulate and interpret the dynamic system described.

It should be emphasized that the selection and discussion of this kind of problem emanates not from the mere interest to see the behavior of a moving vehicle as it runs over a submergible support. Instead, the problem and its solution have a far reaching implication and application. Basically, the above mentioned problem arises in an effort to study the amount of dynamic stress that is caused by traffic load on the ground and investigate the possibility of absorbing the imposed stress. This is done by proposing a mechanical system mentioned which is linked to appropriate electro-mechanical system and then modeling and analyzing the dynamic system. As will be shown in this chapter, the result of the analysis helps estimate the amount of possible energy that could be stored from the traffic load.

Dynamic analysis of vehicle-submergible support (shock absorber) system is quite decisive in a way that it enables the determination of dynamic load which is going to be transferred from the tyres of a moving vehicle to inertial mass below the surface of the ground. This inertial mass is linked to the tyre of the vehicle by means of a linking rod whose toping is flat or humped and installed to reach the road level. The magnitude of the dynamic load transferred is dependent on various parameters like: the hump width-velocity ratio (L/V), the mass of the vehicle (m_1), the mass of the hump-inertial mass combination (m_2), stiffness and damping of the vehicle's spring-dashpot system (K_1 & C_1), stiffness and damping of recoiling spring-dashpot system attached to the hump (K_2 & C_2). It is quite interesting to investigate the response of the two masses (vehicle and submergible support).

Before getting into the detail analysis of the dynamic system, it is imperative to layout the general scheme and important components of the system and terminologies used in the analysis. Hence, sections 3.2 and section 3.3 attempt to give clear picture of the dynamic system and useful assumptions to properly analyze the system. The next section (section 3.4) is the core issues of the Chapter that extensively explains the problem statement and equation derivation. The Chapter is summarized by pointing out limitations of the derived equations.

3.2 **KEY FEATURES AND TERMINOLOGIES**

Basically, the traffic generated power system is proposed to consist of the following main components to ensure the transmission of vehicular load and generate power: moving vehicle, submergible-support system (shock-absorber) and generator. The submergible-support (shock-absorber) is composed of topping (crown), link rod, recoiling spring-dashpot, and equivalent mass of rotor as shown in Fig. **3-3**.

3.2.1 Spring-Dashpot System

The system is installed to be fixed to the ground at its bottom end and to the topping (crown) at its upper end. This helps the crown and linking rod recoil back to their original position after the passage of traffic load so that they would be ready to transmit the vertical load due to the succeeding vehicle.

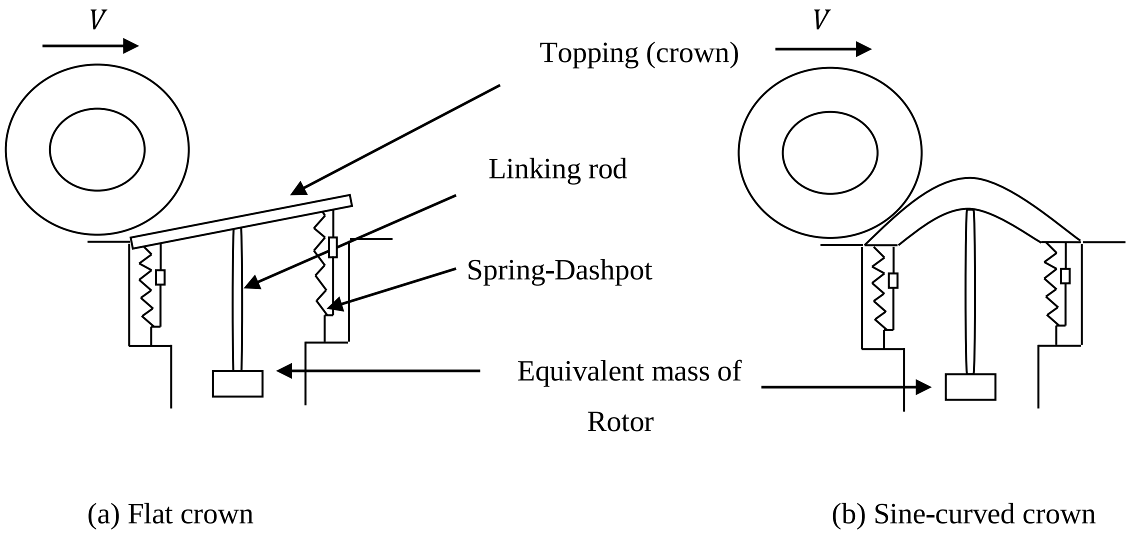


Fig. 3-1 Submergible hump

3.2.2 Submergible-Support (Shock-Absorber)

This is composed of simple mechanical devices (topping/crown, linking rod, recoiling spring-dashpot, and equivalent mass of rotor) serially linked with each other to transmit the load imposed at its topping from a ramming vehicle. The whole purpose of the submergible mechanical system is to substitute the natural ground support in a way that does not disturb the comfort of the traveler and absorb the energy which would have been taken up by the ground. Had there been a depression or a rigid hump on the ground instead of a submergible support, the moving vehicle would have experienced a terrible shock as a result of the sudden interrupted/deformed ground. However, the introduction of the submergible support behaves much more like a shock absorber. As a result, the name shock absorber is used interchangeably with submergible support.

3.2.3 Rotating System (Rotor):

This is a mechanical part linked to the linking rod by a one-way pawl and ratchet mechanism or rack-pinion mechanism. It is characterized by its rotational inertia, I_{Rot}, which can be converted to equivalent mass (also shown in Fig 3-2) for the purpose of simplicity during analysis of the dynamic system of shock absorber. This equivalent mass is calculated by equating the energy required to spin the rotating system having inertia, I_{Rot}, with the energy required to linearly accelerate an equivalent mass, m_2'.

$$E_{Rot}=E_{Lin}$$

$$=> \quad \left(\frac{I\omega^2}{2}\right)_{Rot} = \left(\frac{mv^2}{2}\right)_{Lin}$$

$$=> \quad m = \frac{I\omega^2}{v^2} \qquad \text{but} \quad v = \omega r$$

$$=> \quad m = \frac{I}{r^2} = m_2$$

$$=> \quad m_2 = \frac{I_{Rot}}{r^2}$$

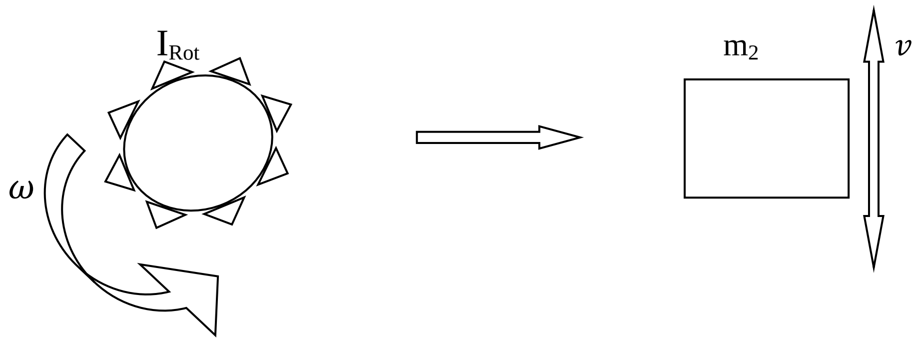


Fig. 3-2 Converting rotational inertia to an equivalent, linearly moving mass

This is employed in the analysis section. Then, the rotating system by virtue of its rotation generates electricity when coupled with a generator or elevates water if it is coupled with pump.

3.3 ASSUMPTIONS

I. The wheels of the car do not slip as they pass over the energy-stealing artificial hump.

II. The spring which supports the artificial hump recoils back to its original position before another axel (vehicle) comes to compress it.

III. The roadway is straight and leveled.

The first assumption is logical as long as the wheel gets enough support from the shock-absorber and grips crest (topping) firmly which will guarantee the wheel from slipping.

The second assumption will hold true if the natural period of the system is made shorter than the duration it takes for two successive axels to cross a point. So, the stiffness of the spring (K_2) must be selected taking due consideration.

- Shortest duration between two consecutive axles passing a point = the time gap (t_{gap}) for the front and rear axels of the shortest vehicle moving with the design speed to cross a point.

i.e. $\mathrm{t_{gap}} = \frac{L}{V}$;

where L = spacing between the front and rear axles of the shortest vehicle using the road

V = the design speed

$$\omega = \sqrt{\frac{\mathrm{k_2}}{\mathrm{m'_2}}}$$

$$\Rightarrow \mathrm{k_2} = \mathrm{m'_2}\,\omega^2 = \mathrm{m'_2}\,(\frac{2\pi}{\mathrm{T}})^2$$

$$\text{But} T = 4 * t_{gap} = \frac{4L}{v}$$

$$\Rightarrow k_2 = m'_2 \left(\frac{2\pi}{\frac{4L}{v}}\right)^2 = m'_2 \left(\frac{\pi v}{2L}\right)^2 = \frac{m'_2 (\pi v)^2}{4L^2}$$

Therefore, the stiffness of the second spring should be made stiffer than $k_2 = \frac{m'_2 (\pi v)^2}{4L^2}$ in order for the submergible hump to get back to its original position before another axel load comes to compress it. Without assuming a leveled and straight road, i.e. the third assumption, the modeling task will be more complicated due to the necessity of considering the consequences of sloping and curving of the roadway on the normal force exerted by the vehicle.

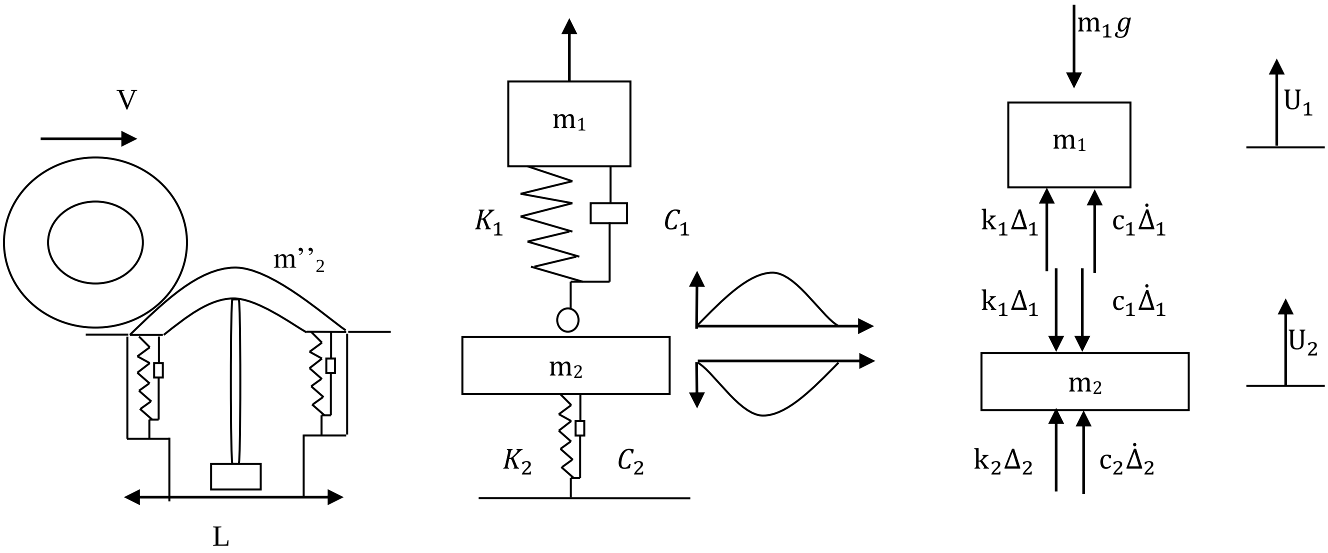
3.4 DERIVATION OF EQUATION OF MOTION

The dynamic system is composed of two masses, the vehicular mass and the rotating system (rotor) whose rotational inertia is converted to an equivalent mass. They are modeled as lumped masses with two degrees of freedom system. The lumped masses are coupled with a spring and damper system to model the actual displacement resistant and motion resistant forces, respectively. To better represent the actual dynamic condition of the system, the possibility of decoupling which might happen if the tyre of the vehicle departs from the submergible support should be considered.

After modeling the dynamic system with mechanical objects, equations of motion are derived as follows. First, we decompose the forces in this dynamic system into a part that is associated with the elongation, Δ_1, of the spring between masses m_1 (the vehicle) and m_2 (the equivalent inertial mass) and elongation, Δ_2, of the second spring between m_2 and the ground in addition to gravity force. All components are shown in the free body diagram, Fig. **3-3**(b). The gravity on the second mass is disregarded intentionally because it represents a static effect on this mass thus gravity is considered to act only on the first mass.

The elongation of the first spring, Δ_1, results due to two reasons. The first one is due to relative vertical positions of the two masses. An upward displacement, U_1, of mass m_1 elongates the spring, k_1, and dashpot, c_1, by that amount, thereby inducing 'tensile' forces, k_1U_1 and $c_1\acute{U}_1$, that push mass m_1 downward while thrusting m_2 upwards (had the spring been attached with mass m_2). Similarly, an upward displacement U_2 of mass m_2 shortens the spring, k_1 , and dashpot, c_1, by that amount, thereby inducing compressive forces k_1U_2 and $c_1\acute{U}_2$, that push mass m_1 upward. The second cause for the elongation, Δ_1, of the first spring is the shape of the top part of the shock-absorber, crown. As the vehicle runs over it, moving with a certain horizontal speed, the spring will be compressed by the amount $Z = H_o \sin\frac{\pi vt}{L}$ (assuming that the crown has a shape of a half sine wave function) inducing forces k_1Z and $c_1\dot{Z}$ on masses m_1 and m_2. The forces on m_1 is in the upward direction while on m_2 in the downward direction. Notice that this elongation is independent of the vertical positions of the two masses, U_1 & U_2. Instead, the elongation depends on the shape of the hump and the velocity of the car provided that the wheel is in touch (coupled) with the crown.

This coupling is secured only when the force in spring-1 is compressive or zero. Otherwise, it would mean the tyre is glued with the topping of submergible support for the spring force to be in tension mode, which is an unrealistic condition. Instead, under certain conditions, a real vehicle dynamics shows that after the vehicle and the shock absorber (having a sine-curved crown, for example) exchanged some momentum, the vehicle departs from the ground and momentarily jumps up while the equivalent mass (m_2) is accelerated downwards. It indicates that the two masses stop interacting once the tyre of the vehicle detaches from the supporting "ground". At this moment, the suspension spring force changes from compression to tension. Therefore, it is possible to conclude that the equations to be derived hereinafter will be valid only if the spring force is compressive as it will be stated later.



(a) (b) (c)

(a) Overall dynamic system (b) Representation by mechanical systems (c) Free body diagram

Fig. 3-3 Representation of the vehicle-submergible support dynamic system

3.4.1 Equations of motion

Mass-1

From Fig. **3-3**(c), the equation of motion for m_1 can be written as follows.

$$m_1\ddot{u}_1 = c_1\dot{\Delta}_1 + k_1\Delta_1 - m_1 g$$

But $\Delta_1 = (u_2 + H_o \sin\frac{\pi v t}{L}) - u_1$; $\dot{\Delta}_1 = (\dot{u}_2 + \frac{\pi H_o v}{L}\cos\frac{\pi v t}{L}) - \dot{u}_1$

$$\Rightarrow \quad m_1\ddot{u}_1 = c_1\left\{(\dot{u}_2 + \frac{\pi H_o v}{L}\cos\frac{\pi v t}{L}) - \dot{u}_1\right\} + k_1\left\{(u_2 + H_o \sin\frac{\pi v t}{L}) - u_1\right\} - m_1 g = F_{12}(t)$$

$$\ddot{u}_1 + 2\xi_1\omega_{n_1}\dot{u}_1 + \omega_{n1}^2 u_1 = (\frac{2\xi_1\omega_{n_1}\pi H_o v}{L}\cos\frac{\pi v t}{L} + \omega_{n1}^2 H_o \sin\frac{\pi v t}{L}) + (2\xi_1\omega_{n_1}\dot{u}_2 + \omega_{n1}^2 u_2) - g$$

Where $\xi_1 = c_1/(2m_1\omega_{n_1})$, $\omega_{n1}^2 = k_1/m_1$,

Mass-2

$$m_2\ddot{u}_2 = (\mathrm{c}_2\dot{\Delta}_2 + \mathrm{k}_2\Delta_2) - (\mathrm{c}_1\dot{\Delta}_1 + \mathrm{k}_1\Delta_1)$$

But $\Delta_2 = -u_2$; $\dot{\Delta}_2 = -\dot{u}_2$; Δ_1 & $\dot{\Delta}_1$ are as defined above

$$\Rightarrow \quad m_2\ddot{u}_2 = \mathrm{c}_2(-\dot{u}_2) + \mathrm{k}_2(-u_2) - \mathrm{c}_1\left\{\left(\dot{u}_2 + \frac{\pi H_o v}{L}\cos\frac{\pi vt}{L}\right) - \dot{u}_1\right\} - \mathrm{k}_1\left\{\left(u_2 + H_o\sin\frac{\pi vt}{L}\right) - u_1\right\} = F_{21}(t)$$

$$m_2\ddot{u}_2 + \dot{u}_2(\mathrm{c}_2 + \mathrm{c}_1) + u_2(\mathrm{k}_2 + \mathrm{k}_1) = \left\{\left(-\mathrm{c}_1\frac{\pi H_o v}{L}\cos\frac{\pi vt}{L}\right) + \mathrm{c}_1\dot{u}_1\right\} + \left\{\left(-\mathrm{k}_1 H_o\sin\frac{\pi vt}{L}\right) + \mathrm{k}_1 u_1\right\}$$

$$\ddot{u}_2 + \left(2\xi_2\omega_{n2} + 2\xi_{12}\omega_{n12}\right)\dot{u}_2 + (\omega_{n2}^2 + \omega_{n12}^2)u_2 = \left(2\xi_{12}\omega_{n12}\right)\left\{\left(-\frac{\pi H_o v}{L}\cos\frac{\pi vt}{L}\right) + \dot{u}_1\right\} + (\omega_{n12}^2)\left\{\left(-H_o\sin\frac{\pi vt}{L}\right) + u_1\right\}$$

Where $\xi_2 = \mathrm{c}_2/(2\mathrm{m}_2\omega_{\mathrm{n}_2})$, $\xi_{12} = \mathrm{c}_1/(2\mathrm{m}_2\omega_{\mathrm{n}_{12}})$, $\omega_{\mathrm{n}2}^2 = \mathrm{k}_2/\mathrm{m}_2$, $\omega_{\mathrm{n}12}^2 = \mathrm{k}_1/\mathrm{m}_2$

The equations to be solved:

$$\ddot{\mathrm{u}}_1 + 2\xi_1\omega_{\mathrm{n}1}\dot{\mathrm{u}}_1 + \omega_{\mathrm{n}1}^2\mathrm{u}_1 = \left(\frac{2\xi_1\omega_{\mathrm{n}_1}\pi\mathrm{H_o v}}{\mathrm{L}}\cos\frac{\pi\mathrm{vt}}{\mathrm{L}} + \omega_{\mathrm{n}1}^2\mathrm{H_o}\sin\frac{\pi\mathrm{vt}}{\mathrm{L}}\right) + \left(2\xi_1\omega_{\mathrm{n}_1}\dot{\mathrm{u}}_2 + \omega_{\mathrm{n}1}^2\mathrm{u}_2\right) - \mathrm{g}$$

$$\ddot{\mathrm{u}}_2 + \left(2\xi_2\omega_{\mathrm{n}2} + 2\xi_{12}\omega_{\mathrm{n}12}\right)\dot{\mathrm{u}}_2 + (\omega_{\mathrm{n}2}^2 + \omega_{\mathrm{n}12}^2)\mathrm{u}_2$$
$$= -\left(\frac{2\xi_{12}\omega_{n12}\pi H_o v}{L}\cos\frac{\pi vt}{L} + \omega_{n12}^2 H_o\sin\frac{\pi vt}{L}\right)$$
$$+ \left(2\xi_{12}\omega_{n12}\dot{u}_1 + \omega_{n12}^2 u_1\right)$$

Eq. 3-1

The solution of the above equations can be found using numerical method, specifically, central difference method. This method is based on a finite difference approximation of the time derivatives of displacement (i.e., velocity and acceleration). Taking constant time steps, $\Delta t_i = \Delta t$, the central difference expressions for velocity and acceleration at time *i* are:

$$\dot{u}_{1(i)} = \frac{u_{1(i+1)} - u_{1(i-1)}}{2\Delta t} \qquad \ddot{u}_{1(i)} = \frac{u_{1(i+1)} - 2u_{1(i)} + u_{1(i-1)}}{(\Delta t)^2}$$

$$\dot{u}_{2(i)} = \frac{u_{2(i+1)} - u_{2(i-1)}}{2\Delta t} \qquad \ddot{u}_{2(i)} = \frac{u_{2(i+1)} - 2u_{2(i)} + u_{2(i-1)}}{(\Delta t)^2}$$

Equation 3-2

Substituting the approximate central difference expressions (Equation 3-2) for the velocity and acceleration terms and rearranging the equations yields:

$$u_{1(i+1)} = \left\{\frac{(\Delta t)^2}{1+\Delta t\xi_1\omega_{n1}}\right\}\left(\frac{2\xi_1\omega_{n1}\pi H_o v}{L} cos\frac{\pi vt}{L} + \omega_{n1}^2 H_o\, sin\frac{\pi vt}{L} - g\right) + \left\{\frac{\xi_1\omega_{n1}\Delta t-1}{(1+\Delta t\xi_1\omega_{n1})}\right\}u_{1(i-1)} + \left\{\frac{2-\omega_{n1}^2(\Delta t)^2}{(1+\Delta t\xi_1\omega_{n_1})}\right\}u_{1(i)} + \left\{-\frac{\xi_1\omega_{n_1}\Delta t}{1+\Delta t\xi_1\omega_{n_1}}\right\}u_{2(i-1)} + \left\{\frac{\omega_{n1}^2(\Delta t)^2}{1+\Delta t\xi_1\omega_{n_1}}\right\}u_{2(i)} + \left\{\frac{\xi_1\omega_{n_1}\Delta t}{1+\Delta t\xi_1\omega_{n_1}}\right\}u_{2(i+1)}$$

$$u_{2(i+1)} = -\frac{(\Delta t)^2}{(1+\Delta t\xi_2\omega_{n2})}\left(\frac{2\xi_{12}\omega_{n12}\pi H_o v}{L}\cos\frac{\pi vt}{L} + \omega_{n12}^2 H_o \sin\frac{\pi vt}{L}\right) + \left\{\frac{\xi_2\omega_{n2}\Delta t-1}{(1+\Delta t\xi_2\omega_{n2})}\right\}u_{2(i-1)} + \left\{\frac{2-\omega_{n2}^2(\Delta t)^2}{(1+\Delta t\xi_2\omega_{n2})}\right\}u_{2(i)} + \left\{-\frac{\xi_{12}\Delta t}{(1+\Delta t\xi_2\omega_{n2})}\right\}u_{1(i-1)} + \left\{\frac{\omega_{n12}^2(\Delta t)^2}{(1+\Delta t\xi_2\omega_{n2})}\right\}u_{1(i)} + \left\{\frac{\xi_{12}\omega_{n2}\Delta t}{(1+\Delta t\xi_2\omega_{n2})}\right\}u_{1(i+1)}$$

In these equations $u_{1(i)}$, $u_{1(i-1)}$, $u_{2(i)}$ & $u_{2(i-1)}$ are assumed known from implementation of the procedures for the preceding time steps.

The equations can further be made handy by introducing coefficients:

$$u_{1(i+1)} = A + Bu_{1(i-1)} + Cu_{1(i)} + Du_{2(i-1)} + Eu_{2(i)} + Fu_{2(i+1)}$$
$$u_{2(i+1)} = A' + B'u_{2(i-1)} + C'u_{2(i)} + D'u_{1(i-1)} + E'u_{1(i)} + F'u_{1(i+1)}$$

Equation 3-3

where

$$A = \left\{\frac{(\Delta t)^2}{1+\Delta t\xi_1\omega_{n1}}\right\}\left(\frac{2\xi_1\omega_{n1}\pi H_o v}{L}\cos\frac{\pi vt}{L} + \omega_{n1}^2 H_o \sin\frac{\pi vt}{L} - g\right)$$

$$A' = \left\{\frac{-(\Delta t)^2}{(1+\Delta t\xi_2\omega_{n2})}\right\}\left(\frac{2\xi_{12}\omega_{n12}\pi H_o v}{L}\cos\frac{\pi vt}{L} + \omega_{n12}^2 H_o \sin\frac{\pi vt}{L}\right)$$

$$B = \left\{\frac{\Delta t\xi_1\omega_{n1}-1}{(1+\Delta t\xi_1\omega_{n1})}\right\} \qquad B' = \left\{\frac{\Delta t\xi_2\omega_{n2}-1}{(1+\Delta t\xi_2\omega_{n2})}\right\}$$

$$C = \left\{\frac{2-(\Delta t)^2\omega_{n1}^2}{(1+\Delta t\xi_1\omega_{n1})}\right\} \qquad C' = \left\{\frac{2-(\Delta t)^2\omega_{n2}^2}{(1+\Delta t\xi_2\omega_{n2})}\right\}$$

$$D = \left\{-\frac{\Delta t\xi_1\omega_{n_1}}{1+\Delta t\xi_1\omega_{n_1}}\right\} \qquad D' = \left\{-\frac{\Delta t\xi_{12}}{(1+\Delta t\xi_2\omega_{n2})}\right\}$$

$$E = \left\{\frac{(\Delta t)^2\omega_{n1}^2}{1+\Delta t\xi_1\omega_{n1}}\right\} \qquad E' = \left\{\frac{(\Delta t)^2\omega_{n12}^2}{(1+\Delta t\xi_2\omega_{n2})}\right\}$$

$$F = \left\{\frac{\Delta t\xi_1\omega_{n1}}{1+\Delta t\xi_1\omega_{n1}}\right\} \qquad F' = \left\{\frac{\Delta t\xi_{12}\omega_{n2}}{(1+\Delta t\xi_2\omega_{n2})}\right\}$$

To start the iteration process using the central difference method, initial conditions are supposed to be calculated beforehand or should be known quantities. Thus $u_{1(0)}$ and $u_{1(-1)}$ are required to determine $u_{1(1)}$; the specified initial displacement u_0 is known. To determine $u_{1(-1)}$, we specialize (Equation 3-2) for $i = 0$ to obtain:

$$\dot{u}_{1(0)} = \frac{u_{1(1)} - u_{1(-1)}}{2\Delta t} \qquad \ddot{u}_{1(0)} = \frac{u_{1(1)} - 2u_{1(0)} + u_{1(-1)}}{(\Delta t)^2}$$

$$\dot{u}_{2(0)} = \frac{u_{2(1)} - u_{2(-1)}}{2\Delta t} \qquad \ddot{u}_{2(i)} = \frac{u_{2(1)} - 2u_{2(0)} + u_{2(-1)}}{(\Delta t)^2}$$

Solving for $u_{1(1)}$& $u_{2(1)}$ from the velocity equations ($\dot{u}_{1(0)}$ & $\dot{u}_{1(0)}$) and substituting in the acceleration equations ($\ddot{u}_{1(0)}$ & $\ddot{u}_{1(0)}$) gives

$$u_{1(-1)} = u_{1(0)} - (\Delta t)\dot{u}_{1(0)} + \frac{(\Delta t)^2}{2}\ddot{u}_{1(0)} \qquad u_{2(-1)} = u_{2(0)} - (\Delta t)\dot{u}_{2(0)} + \frac{(\Delta t)^2}{2}\ddot{u}_{2(0)}$$

The initial displacements $u_{1(0)}$ & $u_{2(0)}$ and initial velocities ($\dot{u}_{1(0)}$ & $\dot{u}_{1(0)}$) are given, and the equation of motion at $(t = t_0)$,

$$\ddot{u}_{1(0)} + 2\xi_1 \omega_{n1}\dot{u}_{1(0)} + \omega_{n1}^2 u_{1(0)} =$$
$$\left(\frac{2\xi_1 \omega_{n_1} \pi H_o v}{L}\cos\frac{\pi v t_0}{L} + \omega_{n1}^2 H_o \sin\frac{\pi v t_0}{L}\right) + (2\xi_1 \omega_{n_1}\dot{u}_{2(0)} + \omega_{n1}^2 u_{2(0)}) - g$$

$$\ddot{u}_{2(0)} + \left(2\xi_2 \omega_{n2} + 2\xi_{12}\omega_{n12}\right)\dot{u}_{2(0)} + (\omega_{n2}^2 + \omega_{n12}^2)u_{2(0)} = -\left(\frac{2\xi_{12}\omega_{n12}\pi H_o v}{L}\cos\frac{\pi v t_0}{L} +\right.$$
$$\left.\omega_{n12}^2 H_o \sin\frac{\pi v t_0}{L}\right) + \left(2\xi_{12}\omega_{n12}\dot{u}_{1(0)} + \omega_{n12}^2 u_{1(0)}\right)$$

provides the acceleration at time $t_0 = 0$:

$$\ddot{u}_{1(0)} = \left(\frac{2\xi_1 \omega_{n_1} \pi H_o v}{L}\right) + \left(2\xi_1 \omega_{n_1}\dot{u}_{2(0)} + \omega_{n1}^2 u_{2(0)}\right) - (2\xi_1 \omega_{n_1}\dot{u}_{1(0)} + \omega_{n1}^2 u_{1(0)}) - g$$
$$\ddot{u}_{2(0)} = -\left(\frac{2\xi_{12}\omega_{n12}\pi H_o v}{L}\right) + \left(2\xi_{12}\omega_{n_{12}}\dot{u}_{1(0)} + \omega_{n12}^2 u_{1(0)}\right) - (2\xi_2 \omega_{n2}\dot{u}_{2(0)} + \omega_{n2}^2 u_{2(0)})$$

Equation 3-4

Hence, in the above set of equations (Equation 3-4) only two items in each of the two equations ($u_{1(i+1)}$ and $u_{2(i+1)}$) are unknown. They can be solved easily by substituting the first equation in the second one. i.e.

$$u_{1(i+1)} = \alpha\{\beta + \gamma u_{1(i-1)} + \delta u_{1(i)} + \varepsilon u_{2(i-1)} + \varphi u_{2(i)}\}$$
$$u_{2(i+1)} = \alpha\{\beta' + \gamma' u_{2(i-1)} + \delta' u_{2(i)} + \varepsilon' u_{1(i-1)} + \varphi' u_{1(i)}\}$$

Equation 3-5

where

$\alpha = \left[\frac{1}{1-FF'}\right]$

$\beta = (A + A'F)$ $\qquad$ $\beta' = (A' + AF')$

$\gamma = (B + D'F)$ $\qquad$ $\gamma' = (B' + DF')$

$\delta = (C + E'F)$ $\qquad$ $\delta' = (C' + EF')$

$\varepsilon = (D + B'F)$ $\qquad$ $\varepsilon' = (D' + BF')$

$\varphi = (E + C'F)$ $\qquad$ $\varphi' = (E' + CF')$

The time step is chosen considering the stability criteria: $\frac{\Delta t}{T_n} < \frac{1}{\pi}$, where T_n is the natural period of the overall system as it is used by Chopra (5).

3.4.2 Limitations of the equations

The validation of the equations lies on the following presumptions:

a) The interaction between the vehicle and shock-absorber stays until the tire of the vehicle completes its journey over the topping of the shock-absorber. The journey will be completed after passing over the full length (L) of the crown (topping). This takes time $t \leq L/V$. The inequality is to assert the possibility of early departure of the tyre of the vehicle from the crown that may happen depending on the shape of the crown. Such a case will be discussed in the numerical experience section in the next Chapter.

b) The spring force between masses m1 and m2 should be compressive. This is an important checkpoint to guarantee the coupling of the two masses which are coupled in one direction only (unidirectional coupling) at the point of contact between the tire and submergible support. Therefore, unless the transmitted force is compressive, the tire and the supporting surface will not be in touch, hence jeopardizes the assumption of a coupled system. This can be verified in either of the two ways:

i) Decomposing and analyzing the forces acting on mass m_1 (gravity and spring forces) and then observe if the spring force is compressive, i.e. $F_{spring-1}(t) \geq 0$. Or

ii) Determine the point at which the downward force exceeds the weight of m_1 ($F_1(t) \geq m_1 g$). As exceeding the weight implies the existence of a downward pull, then it can be concluded that the cause for the increament is the tensile force of the spring.

c) Mass m_2 should not be displaced above the ground level, i.e. performs only quarter of its cycle. This assumption is necessary to avoid resonance of m_2 by damping down m_2 before it encounters another excitation force, i.e. $U_2 \leq 0$.

CHAPTER FOUR: NUMERICAL EXPERIENCE

4.1 GENERAL

The solution of the already established finite difference equations (Eq. (3.4)) describes the behavior (response) of the two masses under different circumstances. The anticipated responses are displacement, velocity and acceleration of the vehicle and the equivalent inertial mass of the rotating system to be installed below the ground.

It is found out that the responses are sensitive to speed of vehicle (L/V), mass of vehicle (m_1) and equivalent inertial mass of the rotating system (m_2), stiffnesses (K_1 & K_2) and dampings (C_1 & C_2) of attached spring-dashpot system and shape of hump, as it is expected intuitively. In dealing with the responses, first certain values are assumed for each of these key parameters and the responses of the masses are calculated. Next, some of these parameters of our interest are allowed to vary while holding the rest parameters to investigate the effect of varying those parameters on the response of the masses. For this purpose, the following parameters are assumed to have a startup value indicated.

V (m/s) = 13.89 (50km/hr)	K_1 (N/m) = 15,000.0	m_1 (kg) = 1,200.0
L (m) = 0.30	K_2 (N/m) = 10,577.0	m_2 (kg) = 50.0
H_o (m) = 0. 0	C_1 (N-sec/m) = 2,000.0	g (m/s^2) = 9.81
	C_2 (N-sec/m) = 1,058.0	

Here, the assigned values must be considered as reference values (not as representative values) based on which actual values may be compared with. Expect for the known constant, g, and the value k_2 whose minimum value is set as $k_2 \geq m_2\left(\frac{\pi V}{2Lcar}\right)^2$ =10,577.0N/m according to the assumption made in section[1] 3.3, all the remaining values are arbitrary constants which can be changed when ever dependable data is obtained.

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[1]Spacing between the front and rear axles (Lcar) of the shortest vehicle is assumed to be 1.8m

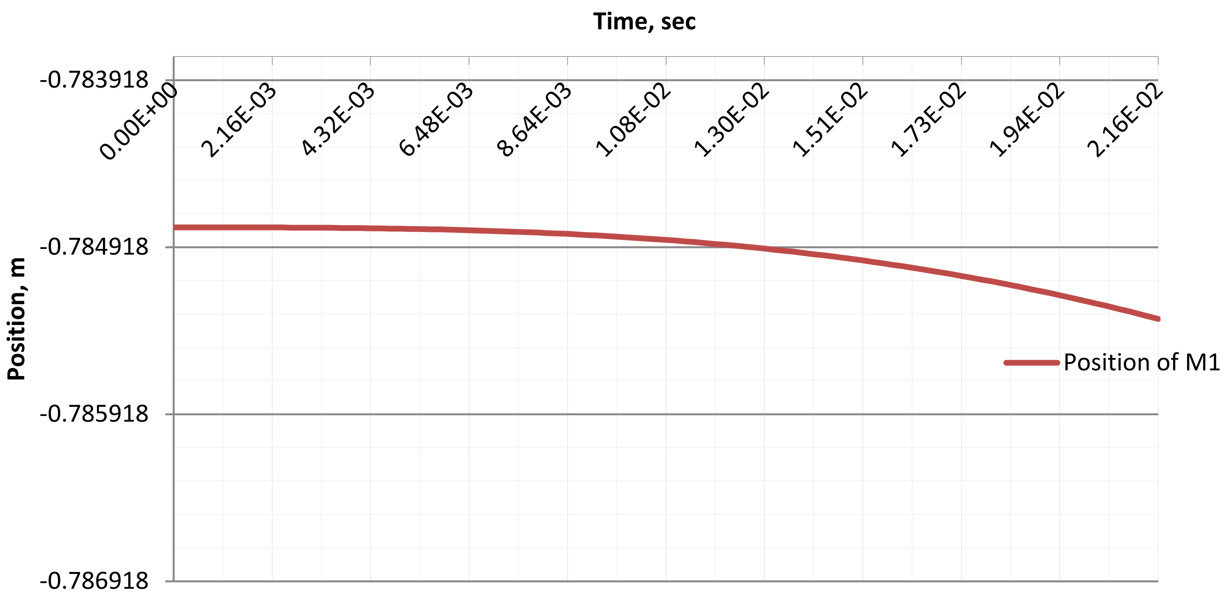
4.2 FLAT SHOCK ABSORBER

According to

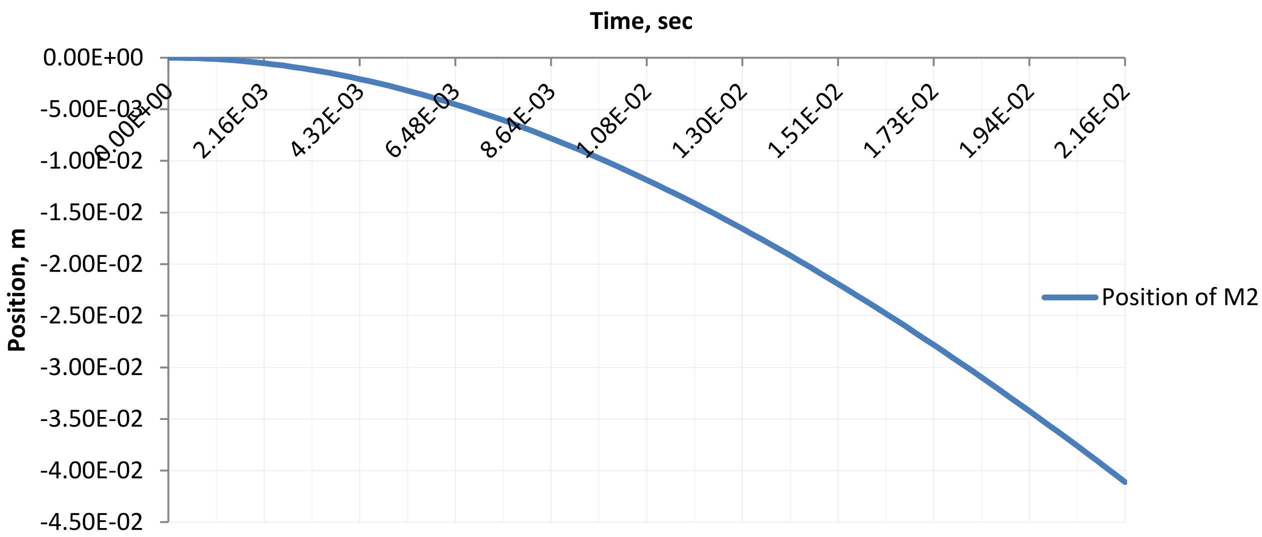
Fig. 4- (a) and (b), the displacement response of two masses m_1 & m_2, show that they will interact for 21.6 milliseconds. In other words, the vehicle completes its journey over the submergible support within 21.6milli-seconds, the tire being kept in touch with the surface of the submergible support. Note that during the interaction between the two masses, mass of the vehicle (m_1) is displaced alone while the submergible mass (m_2) is displaced together with the tyre which implies the fact that the vehicle body is separated from the tyre by suspension spring and, hence, allows separate motion.

Originally, mass m_1 is displaced 78.48cm downwards due to initial compression of the attached spring (suspension spring) as a result of gravitational force (W_1/K_1). This can be noticed from (a). The figure also shows that m_2 is displaced (about 40mm downward) more than m_1 (0.6mm downward) at the end of the journey. This indicates that there is little disturbance on mass m_1 (i.e the vehicle) as it rolls over the supporting mass. Also in terms of velocity and acceleration the disturbance can be verified to be slight.

A careful observation of the vertical acceleration and vertical force diagrams Fig. 4-3 and Fig. 4-4 show that the vehicle initially possesses zero net acceleration and zero net force as it leaves the pavement and just starts interacting with the submergible mass. However, as it gets back to the original pavement (at t=21.6milli-seconds), it will have a vertical acceleration of 5.8m/s^2 in the direction of gravity while its velocity reaches a magnitude of 0.04m/s downwards which is too small to be sensed by the traveler. Meanwhile, mass m_2 is accelerated downwards to a higher velocity 3.28m/s. In fact, the magnitude of its acceleration decreases when the magnitude of acceleration of mass m_1 increases (see Fig. 4-4 (d)). This happens due to the fact that the spring-dashpot force ($k_2\Delta_2 + c_2\dot{\Delta}_2$) prevails over the spring-dashpot force ($k_1\Delta_1 + c_1\dot{\Delta}_1$) which inturn results due to the sustained compression,Δ_2, of the spring that supports the submergible mass and relaxation/extension, Δ_1, of suspension spring as time proceeds.



(a)



(b)

Fig. 4.1 Displacement responses of vehicle-shock absorber system when the interaction surface is flat

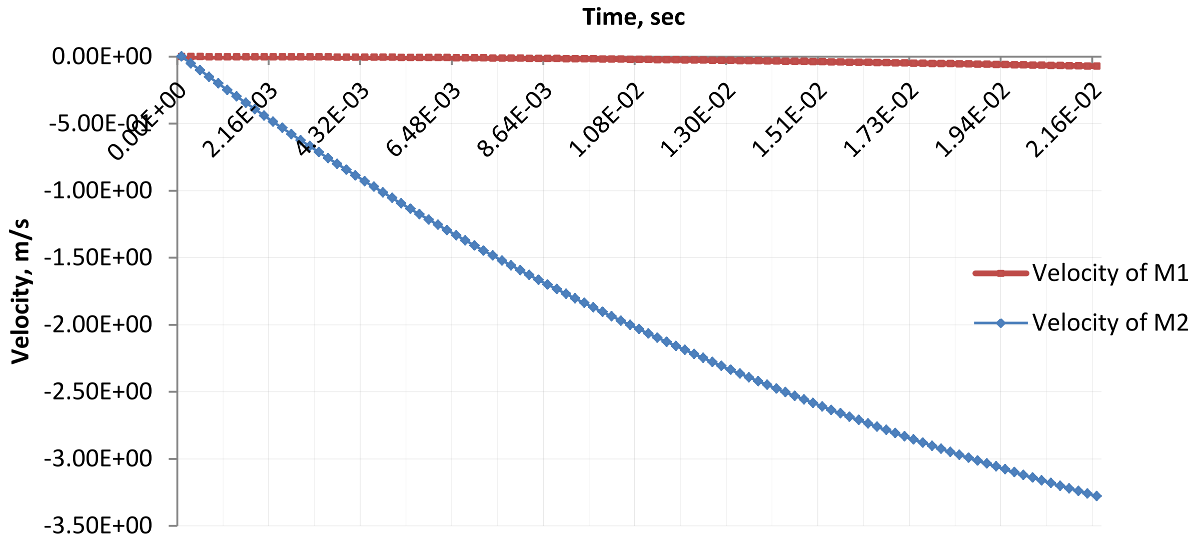


Fig. 4-2 Velocity Responses of vehicle shock absorber system when the interaction surface is flat

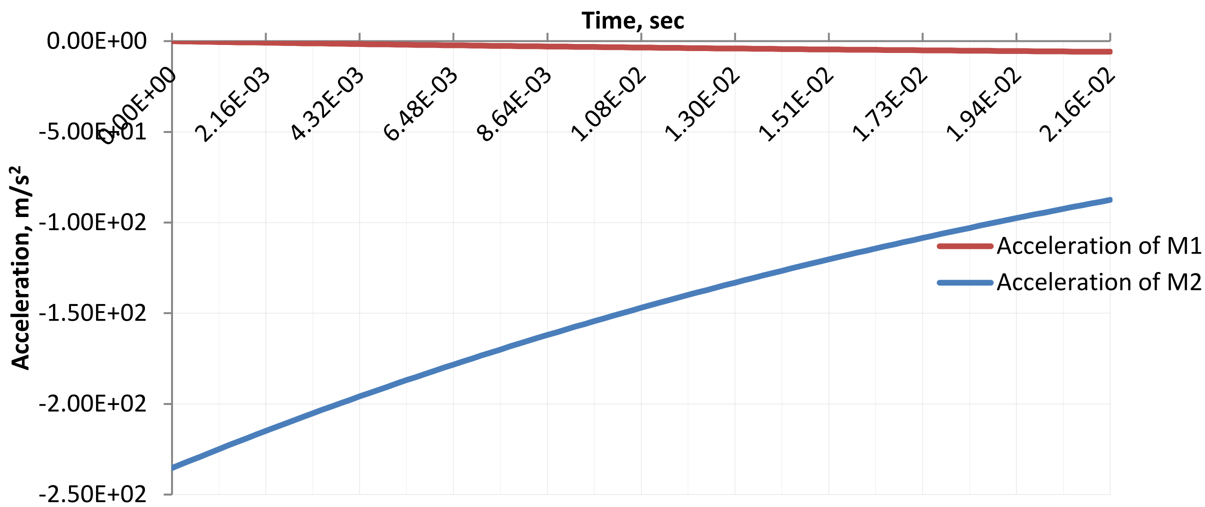


Fig. 4-3 Acceleration Responses of vehicle-shock absorber system when the interaction surface is flat

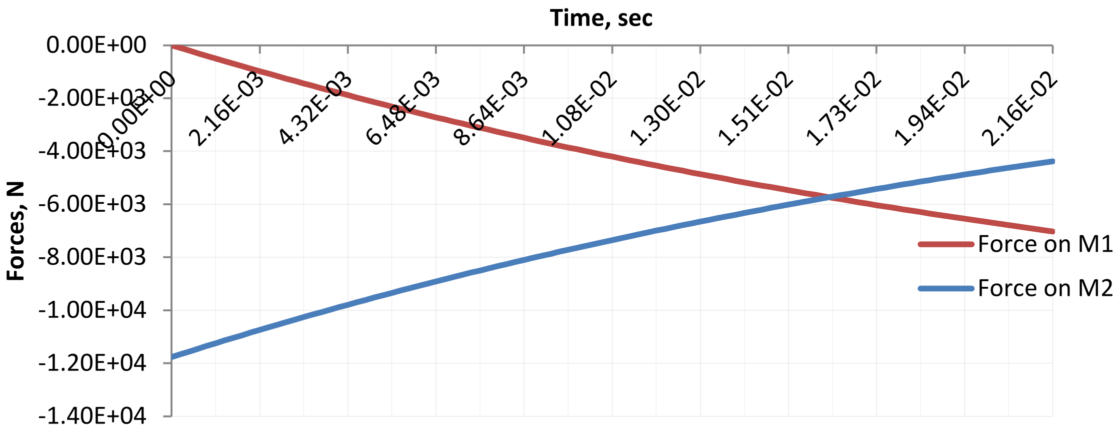


Fig. 4-4 Interaction between vehicle-shock absorber system when the interaction surface is flat

The analysis of the dynamic system explaining the response of a vehicle moving over a flat, submergible support which is linked to inertial mass can be generalized as in the figure below. Actually, the chart illustrates only the velocity response of the equivalent mass (m_2). However, it provides important information regarding the momentum transferred to m_2 from vehicles having different masses (800kg, 1000kg and 1200kg) moving at different velocities (up to 145km/h).

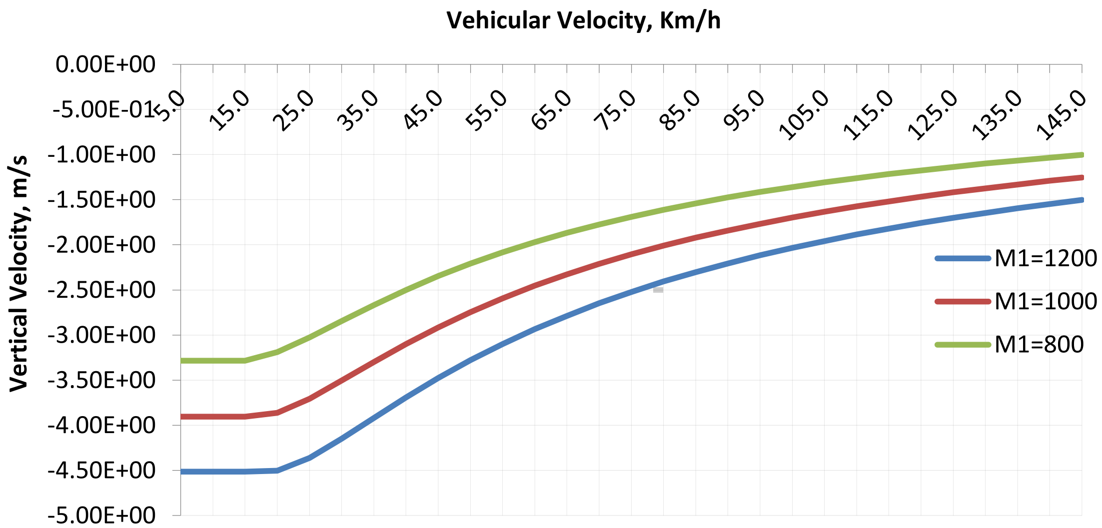


Fig. 4-5 Velocity of mass-2 when vehicles of different mass with different velocity pass over a flat shock absorber

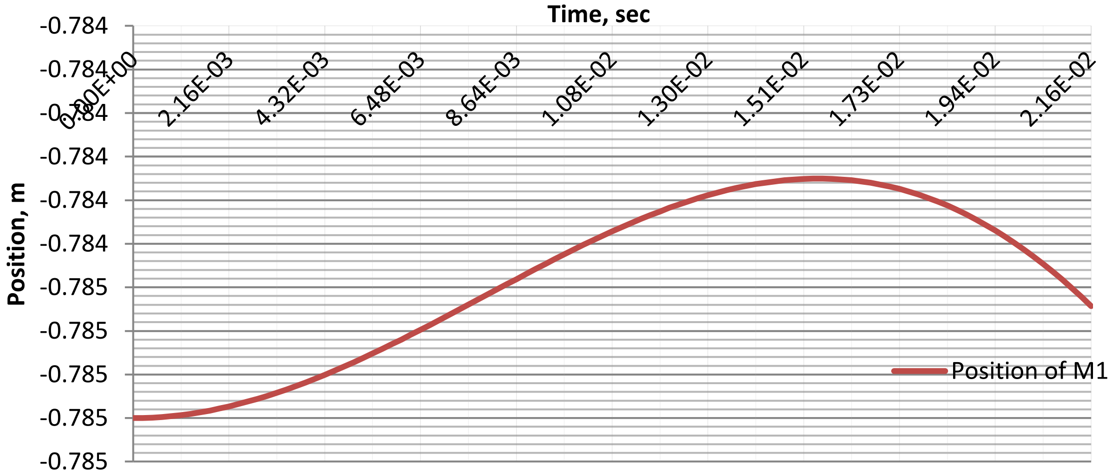
According to the chart, the slower the velocity of the vehicle, the higher velocity m_2 attains. It is clear that slow moving vehicle spends more time on the 30cm width submergible support. This longer duration gives enough time for the suspension spring to release the energy it stores in the form of spring potential energy. The effect of this force is manifested in terms of the higher velocity attained by m_2 after the end of the longer interaction time. Slowing down the speed, however, does not increase indefinitely the velocity of m_2. Below a certain horizontal speed of the vehicle, the attained speed of m_2 reaches to its peak value and tends to decrease afterwards.

Another important but obvious thing that can be concluded from the design chart is the effect of vehicular weight on the response of m_2. As can be noted, the attained velocity of m_2 increases with increasing the mass of the vehicle. Here, the frequency of the vehicle is kept constant. In other words, the stiffness of the spring is proportionally increased to produce the chart which looks like Fig. 4-5.

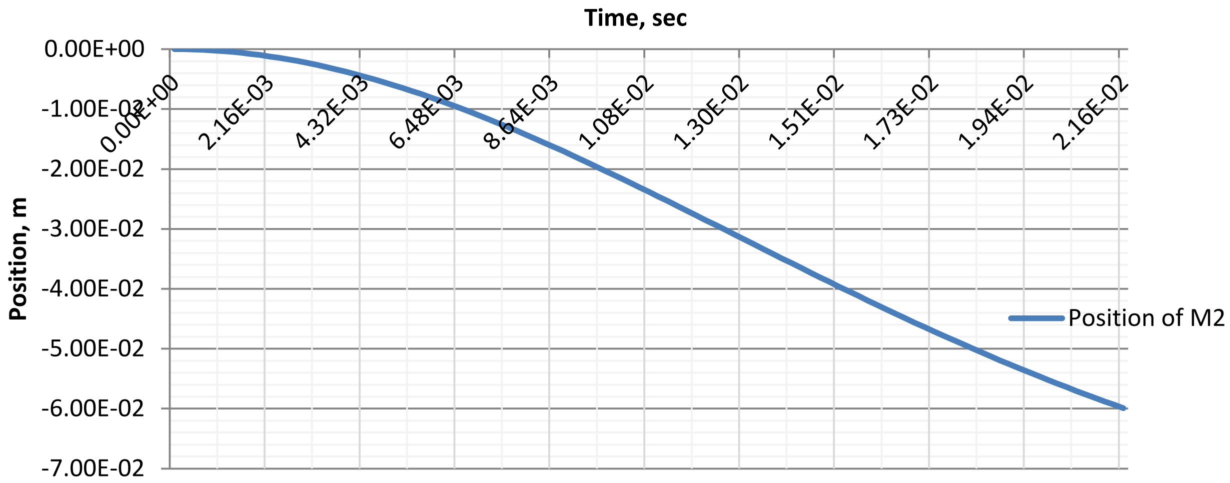
4.3 SINE-CURVED SHOCK ABSORBER

What happens to a moving vehicle characterized by the above basic parameters when subjected to a hump having a half-sine curve with amplitude H_o and wavelength 2L? Common sense tells that a moving vehicle will be raised from the ground as it rolls over a bumped ground, as can be observed at speed-breaker spots, for instance. The dynamic system concerned in this section, i.e. the interaction between a moving vehicle and sine-curved shock absorber, is very much similar to the speed-breaker phenomenon except the non-submergibility of the speed-breaker. Unlike the case of speed-breaker, the sine-curved artificial hump is linked to an inertial mass and allowed to sink down. In addition, the hump is attached to spring-damper system to support it and recoil it back to its original position after passage of the vehicle.

Shown below in Fig. 4-6 up to Fig. 4-9 are the responses of the vehicle and shock absorber when the interaction surface is sine-curved with amplitude 5cm. The rest of the parameters defining the overall dynamic system are considered to be the same as in the previous section (i.e. with a flat shock absorber system). Fig. 4-6 (a) shows the displacement responses of m1 and m2. As can be observed from the figure, m1 (vehicle) has an initial vertical deflection of 78.4cm as it enters to the sine-curved and submergible supporting system. The vehicle starts rising up from its original elevation to a height of 0.6mm after moving for about 15.6 milliseconds on the curved and submergible ground support. But then it descends down to reduce its raised elevation by half (0.29mm) just before reaching the original ground (t=21.6ms). On the other hand, m2 is continuously displaced downwards to a depth of 6cm at t=21.6ms.



(a)



(b)

Fig.4-6 Displacement responses of vehicle shock absorber system when the interaction surface is sine-curved

The velocity and acceleration graphs, Fig. 4-7 and Fig. 4-8, suggest that the accelerations of both masses change their magnitude and direction during the interaction period. In effect, m2 attains a maximum downward speed of only 3.7m/s at t=1.27ms which decreases afterwards to 2.6m/s while m1 attains a maximum upward speed of 0.06m/s at t=8.0ms and decreases to zero to reverse its direction, finally reaching a downward speed of about 0.1m/s.

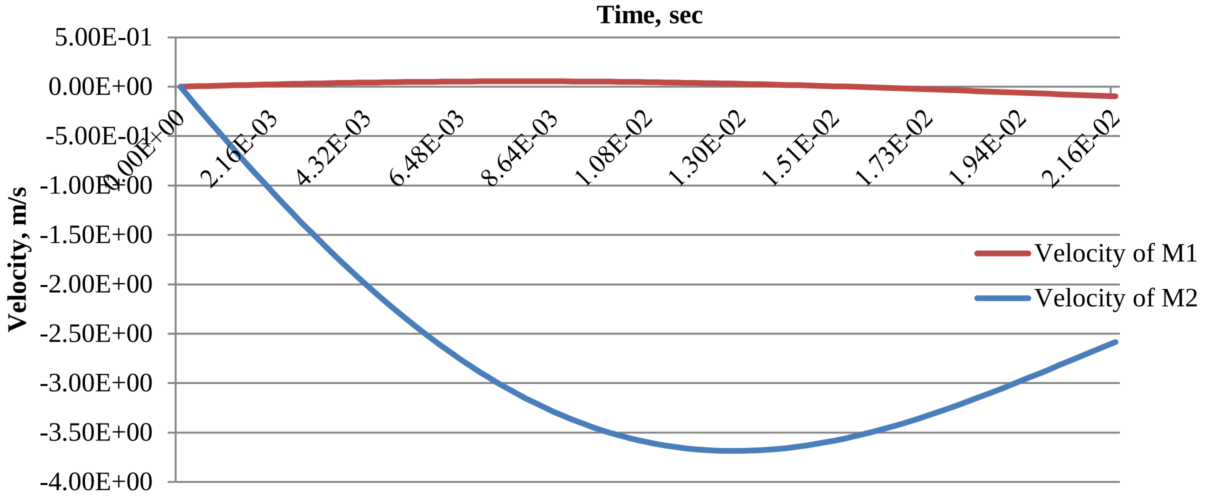


Fig 4-7 Velocity responses of vehicle shock absorber system when the interaction surface is sine curved.

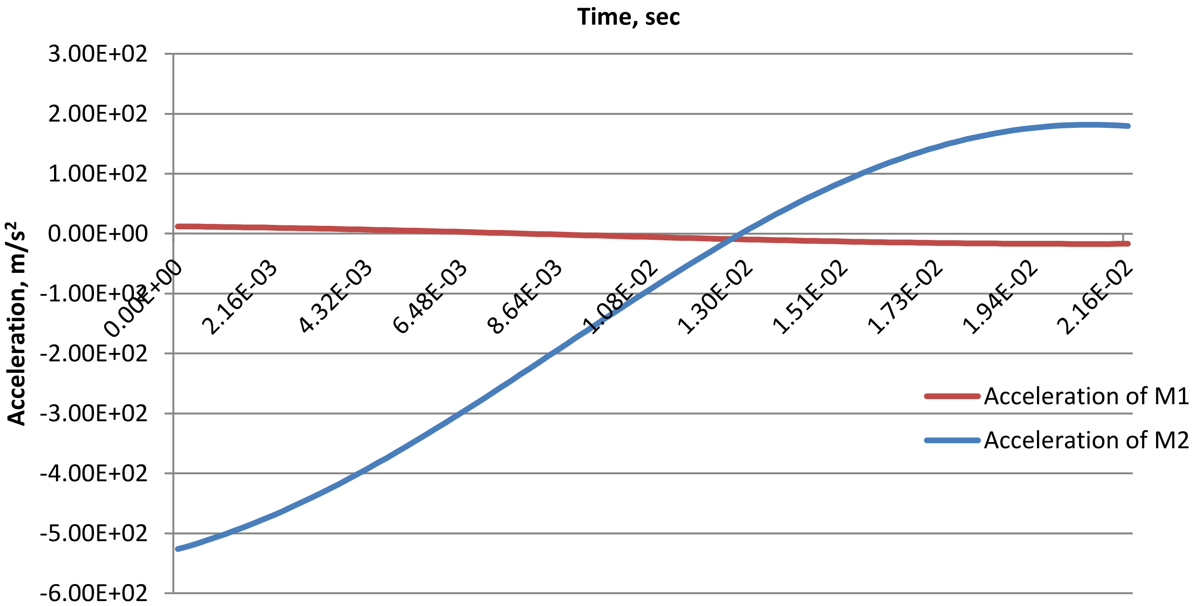


Fig 4-8 Acceleration responses of vehicle shock absorber system when the interaction surface is sine-curved

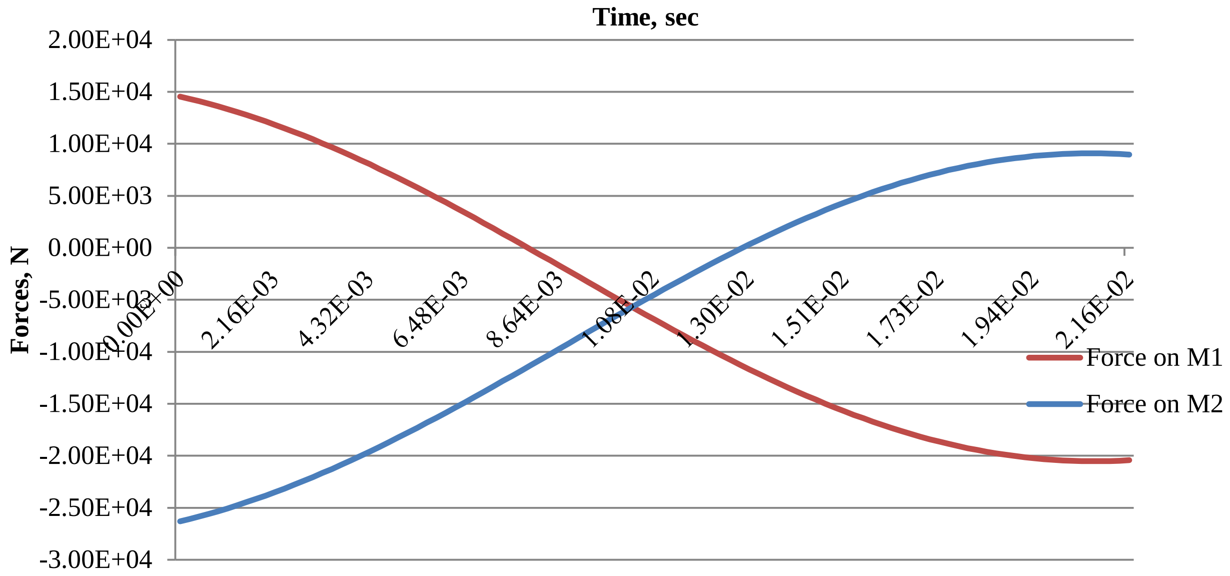


Fig 4-9 Interaction force between vehicle shock absorber systems when the interaction surface is sine-curved

Fig 4-9 assists to visualize the overall dynamic system. But before that, it is noteworthy understanding the difference between spring force and net force acting on the masses. Spring force is a force imposed on a mass which is attached with a spring. If, for example, a spring is attached with two masses (like the suspension spring), then equal but opposite force is imposed on the two masses. On the other hand, net force is the resultant of all the forces acting on a mass. Fig 4-9 shows the net forces acting on m_1 and m_2. The net force on m_1 is the resultant of the spring force and the gravitational force acting on it. Meanwhile, the net force on m_2 is the resultant of the two spring forces acting on it. After having a clear understanding of the net force and spring force, it is possible to interpret force diagram.

According to the fig 4-9, m_1 is subjected to huge upward net force (much greater than the gravitational force) as it starts moving over the humped-shock absorber. The upward force is a resultant of the gravitational weight and the spring force (major force). This huge spring force is caused by the sine-curved hump which forces to compress the suspension spring upwards by virtue of its shape. Referring to Newton's 3rd law, this upward spring force has reaction force which pushes m_2 downward.

Here, it is good to recall that m_2 is submergible, being able to move vertically. Though the magnitude of the spring force imparted on the two masses is equal, the resulting acceleration is significantly different. It is because the force accelerates lighter mass (m_2) more than heavier mass (m_1). Consequently, the separation between the two masses, m_1 and m_2, increases rapidly, thereby, relaxing the suspension spring. Thus, the force magnitude of the spring (pushing m_1 upward and pressing m_2 downward) decreases. At t=8.64ms, the force on m_1 (but not on m_2) becomes zero. Instead of being zero, the net force on m_2 has a magnitude of about 11,800N downward. It suggests that at this moment the suspension spring, after relaxing continuously, reaches a compression level to just support the weight of m_1 as it used to be when the vehicle was resting on the natural pavement ground (prior to the entrance of the submergible support zone). After this moment, the support force provided to m_1 by the relaxing suspension spring vanishes as the extension of the spring becomes zero. The zero extension moment is the moment when m_1 is subjected to only gravitational force (free fall), as can be noted from the force diagram at t=12.7ms. After this instant, the equation of motion becomes invalid because the two masses will stop interacting and start moving independently. In fact, this invalidation criterion was stipulated during the derivation of the equations of motion in this chapter.

The dynamic analysis of vehicle-shock absorber (having a sine-curved crown) system can be generalized to see the effect of mass and velocity of vehicle on the momentum transferred to shock absorber. Of course, the generalization is made taking into consideration all the invalidation criteria mentioned earlier. Fig 4-10 explains that vehicles affect a submergible hump more when the vehicles have slower speed and larger mass. However, at higher vehicular speed, the sensitivity of the hump to vehicular speed and vehicular weight vanishes. The submergible hump attains an asymptotic value of 2m/s at higher speed of the vehicle.

It should be noted that when generating the chart in fig.4-10, the frequency of the vehicle is kept constant allowing variation of the vehicular mass only. This helps to identify the effect of varying the mass of the vehicle alone, without including the effect of the frequency variation.

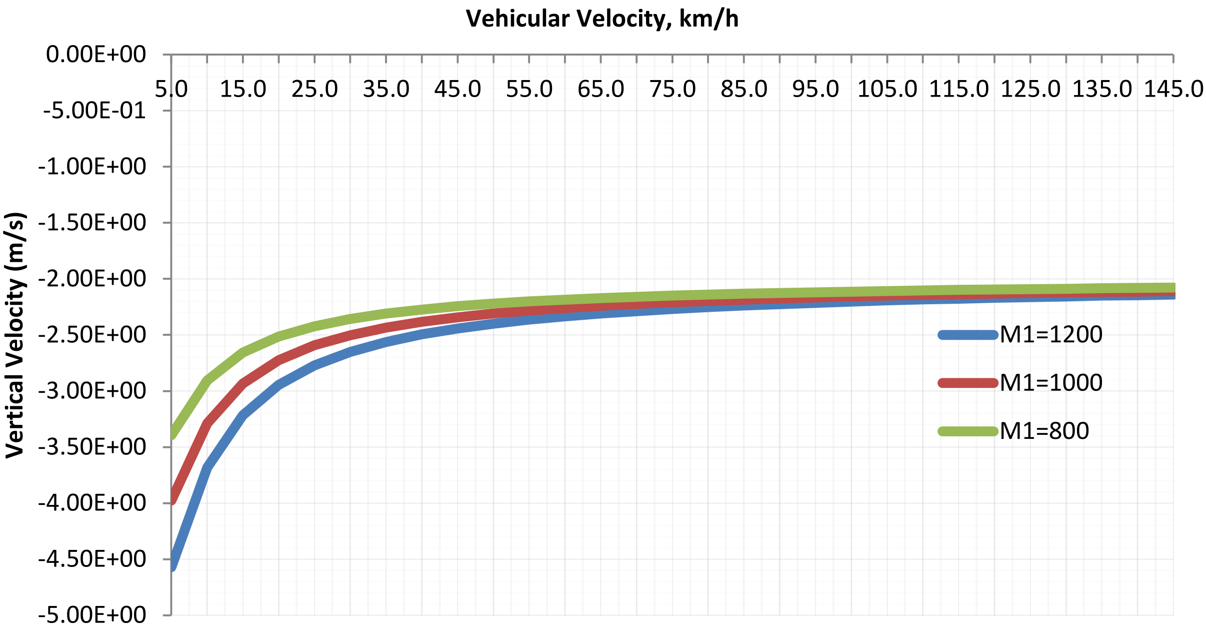


Fig. 4-10 Velocity response of mass-2 when vehicles of different mass with different velocity pass over a sine-curved shock absorber

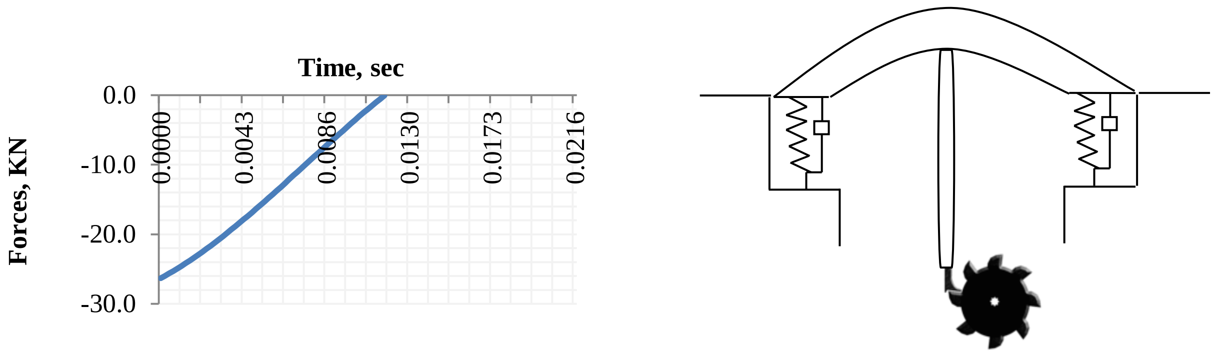
In general for a car moving with a certain horizontal speed, it is possible that the suspension spring pushes down and accelerates a linked, submergible mass even if the topping of this mass is not bulged up. As shown in section 4-2 , the flat supporting system submerges m_1 (vehicle) a magnitude of 0.6mm from its original position as it completes its journey over the shock-absorber. However, changing the shape of the topping (crown) entirely changes the path, velocity and acceleration of both masses. When the crest is changed to a sine-curved shape, section 4-3, the vehicle is forced to jump up, its tire losing contact from the crown before even moving halfway over the crown. The responses of m_2 (submergible equivalent mass) in the two cases also varies significantly.

To our satisfaction, the shape of the crown largely affects the responses of m_1 and m_2. This invites a search for another suitable shape of crown which provides maximum comfort to the traveller as the same time transfers maximum momentum (energy) to m_2.

The attained (generated) velocity of m2 is what is sought from sections 4-2 and 4-3 in order to estimate the amount of power that could be generated from vehicular traffic. The detail discussion as to how energy is tapped will be made next.

4.4 ENERGY AND POWER

Where does the energy come from? What is the mechanism of trapping it? Work is said to be done when a force displaces an object to a certain distance. The downward traffic load F exerted on the topping of the shock absorber will be transmitted through the linking rod and applied directly to the teeth of a ratchet wheel of radius r. Thus, a torque $T = F.r$ will be developed. Shown below is the applied force F from the tire of a vehicle having a mass 1,200kg as it passes over the sinusoidal hump at a speed of 50km/hr. In the graph, adapted from the previous chapter, the negative value indicates the downward direction of application of the force. Further below, a shock absorber system which receives and transmits the imposed impulse load is illustrated.



(a) (b)

Fig. 4-11 (a) Dynamic load imposed by a vehicle on the sine-curved shock absorber
(b) Complete shock absorber system

The input torque T applied at the teeth of the ratchet will give rotational displacement θ to the shaft to which ratchet wheel, flywheel, output torque and other rotating units are attached. Therefore it can be said that work (W) is done by the tire load on the ratchet wheel. The magnitude of W is given by:

$$W = T\theta$$

Without considering the cause of motion, i.e. the input torque T, another expression for work done can be developed. This is by calculating change in energy of the shaft (and all the attached masses) or, in other words, calculating the energy difference before and after the application of load. The energy of the rotating objects is a function of their moment of inertia and angular velocity. So the change in energy (ΔE) which is the work done due to the application of a torque is:

$$\Delta E = \frac{1}{2} I_{Rot}(\omega_2^2 - \omega_1^2) = W = T\theta$$

$$I_{Rot} = I_S + I_{RW} + I_{FW} + I_{OT}$$

Eq. (4-1)

Where I_{Rot} = total moment of inertia of the rotor shaft and all attached masses

I_S = moment of inertia of the shaft and other minor attachements

I_{RW} = moment of inertia of ratchet wheel

I_{FW} = moment of inertia of flywheel

I_{OT} = moment of inertia of output torque

ω_1= angular frequency before input torque is applied

ω_2= angular frequency after input torque is applied

Detail analysis was performed regarding the responses (displacement, velocity, etc.) of the equivalent mass m_2 in the previous chapter. in sections 4-2 and 4-3. Responses of m_2 from this analysis can be converted readily to rotational responses (angular displacement, angular velocity, etc.) of the rotor using the above equation.

$$m_2 = \frac{I_{Rot}\omega^2}{v^2}$$

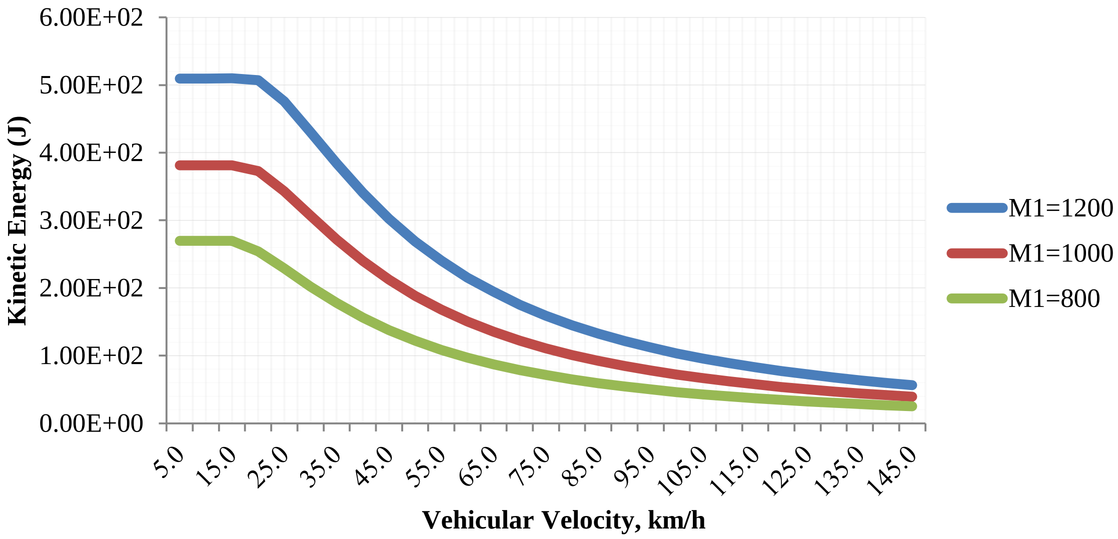
$$=> \qquad \omega = v\sqrt{\frac{m_2}{I_{Rot}}}$$

Eq. (4-2)

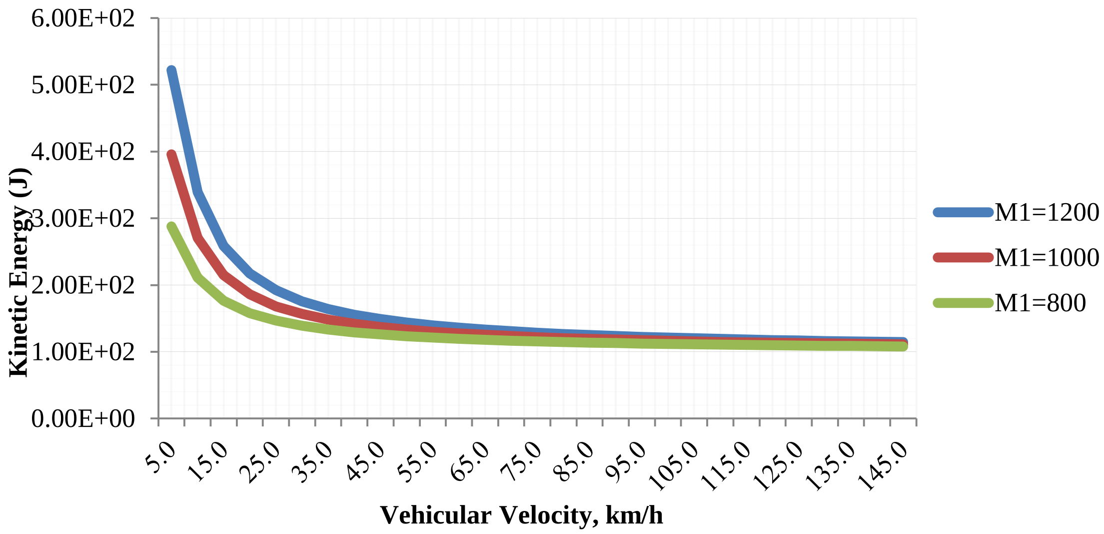
The angular speed of the rotor, for example, can be determined by first assuming the value for I_{Rot} and then inserting the already known values of the m_2 and its linear velocity V at every instant of time. Furthermore, integration and differentiation of the angular velocity will give angular displacement and angular acceleration, respectively. Now, it should be noted that the amount of kinetic energy m_2 gains ($= \frac{1}{2} m_2 v^2$) has the same value with the energy stored in the rotating system ($= \frac{1}{2} I_{Rot}\omega^2$) whose angular velocity ω is just calculated using Eq. (4-2).

4.4.1 Energy

As it can be testified from the equation 4.1, the kinetic energy of m_2 is equal to the energy stored in the rotor shaft and all the attached masses. This statement implies that we are given a freedom to utilize the velocity of m_2 (from the dynamic analysis section, see Fig 4.5 and Fig 4.10) to calculate the kinetic energy of m_2 such that the obtained kinetic energy can be taken as the estimation of the absorbable energy from TGP system. Therefore, the amount of energy which could be extracted from a vehicle moving over a flat and sine curved shock absorber is summarized in the following charts.



(a)



(b)

Fig. 4-12 Kinetic energy of mass-2 when vehicles of different mass with different velocity pass over (a) flat crown shock absorber (b) sine-curved crown shock absorber

The charts illustrate that around 0.25kJ of energy could be stored from 1200Kg vehicle moving up to 25Km/hr over a flat crowned shock absorber. However, for the same vehicle moving over sine-curved crown shock absorber, the same or even greater amount of energy could be stored if the vehicle moves at a speed slower than 55km/hr. for such a vehicle and shock absorber system, a peak amount 0.365J of energy can be gained. More generally, the energy from moving vehicles can be calculated using the following formula:

$$W = \int_{t=0}^{t=\tau} F(t)du$$

Eq. (4-3)

In this formula, F(t) is tire load exerted on the shock absorber and du is an infinitesimal displacement of the shock absorber due to the application of the tire load, F(t). The numerical solution for the system of second order non-homogeneous differential equation (Eq. (3.1)) in chapter 3 which explains the dynamic system is quit helpful to get the integration result within the interaction interval: $0 \leq t \leq \tau$. To quantify the amount of work done by the tire load completely, the efficiency of the overall system η, has to be determined. Unfortunately, in this thesis estimating the efficiency is not attempted since it needs a thorough discussion of mechanical engineering concept which is beyond the scope of the thesis.

4.4.2 Power

Power is the rate of work done. The term work and its rate of application widely vary as the sources of power vary innumerably. The term work for traffic generated power system, as the name implies, comes from vehicular traffic load. Amount of this work is nothing but the one estimated above meanwhile the rate at which this work is done depends on the rate of traffic flow on the selected roadway. This implies that the higher the number of standard axles passing a point on a given highway within a second (N_t) the more power it is expected from the installed TGP system. When expressed mathematically, it gives:

$$P = N_t W \quad (a)$$

Inserting the values for W from Eq. 4-1:

$$P = T\theta \, N_t \quad (b)$$

Or $$P = \frac{1}{2} I_{Rot}(\omega_2^2 - \omega_1^2) * N_t \quad (c)$$

Eq. (4-4)

Eq. 4-4 (b) & (c) are valid if the energy absorbing mechanism involves a rotor. Eq. 4-4 (b) requires the input torque $\mathbf{T}$ and angular displacement θ of the rotor in order to estimate the amount of power. The value of both quantities can be readily determined from dynamic analysis and little manipulation of the analysis result, as explained above. On the other hand, Eq. 4-4 (c) gives estimation of power based on the angular velocities of the rotor before and after the application of the torque ($\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_1}$), provided that moment of inertia of the rotor $\mathbf{I_{Rot}}$ is known beforehand.

CHAPTER FIVE: CONCLUSION AND COMMENDATION

5.1 CONCLUSION

- As vehicles move over the topping (crown) of a specially designed mechanical system, called shock absorber, they do some work on the system unleashing the energy they stored in their suspension spring.
- Mathematically, the amount of energy that can be trapped from the passage of a single vehicle over a single shock-absorber system can be calculated by:

$$W = \int_{t=0}^{t=\tau} F(t)du \text{ (t)}$$

Where:

W = expected energy (J)

η =overall system efficiency (%)

F (t) = Dynamic axle/tire load (kN) as a function of time

u (t) = vertical displacement of axle (mm), also can be given in terms of time

τ = duration the vehicular tire stays in contact with crown of the shock absorber (sec)

- At least 250J of energy could be stored from vehicles moving up to 25km/h over a flat crowned shock absorber.
- For vehicles moving over sine-curved crown shock absorber, up to 365J of energy could be stored if the vehicle moves at a speed slower than 15km/h.
- More than 250W power can be generated depending on type of traffic.
- It is Illustrated that the generation and storage of power from vehicular traffic is technically possible.
- Flywheels solve the problem of discontinuity of loading which is another characteristic feature of TGP system.
- Shape of crown highly determines the amount of energy that can be trapped from the system.

5.2 RECOMMENDATION

- Search for supporting material satisfying the dynamic properties of the shock absorber which can absorb the strain energy imparted on it.
- Estimate the overall efficiency of the system.
- Validate the obtained theoretical results with experiment.
- Similar procedure can be followed to include the case of energy loss due to friction.
- Study the response of vehicle and the amount of trapped energy when the shape, of the crown, stiffness of the vehicle and that of the support are changed to other values.
- Performance evaluation of vehicles (such as fuel consumption) subjected to the shock-absorber system
- It should be emphasized that the proposed mechanical system is one out of the many other possible engineering solutions that could be employed to enhance the absorption of energy from moving vehicles. This is no way means that the proposed methods are futile. Rather, they are basic mechanisms that may guide scientists and engineers to understand the problem clearly and reach to other innovative solutions. Therefore, it is recommended that that looking for other possible innovative designs could increase the overall efficiency and suitability of TGP system.
- Investigating the possible occurrence of resonance and suggesting solutions.

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DECLARATION

I, the undersigned, declare that this thesis is my original work, and that all the sources of material used for the thesis have been duly acknowledged.

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