ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

NUMERICAL ANALYSIS ON THE BEHAVIOR OF PRESTRESSED CONCRETE SLEEPER UNDER DIFFERENT SUPPORT CONDITIONS

A Thesis in Civil Engineering under Railway Engineering

By Haftom G/micael

April 1, 2016

Addis Ababa

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science

The undersigned have examined the thesis entitled '**Numerical Analysis on the Behavior of Prestressed Concrete Sleeper under Different Support Conditions**' presented by **HAFTOM G/MICAEL**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

Esayas G/youhannes (Dr.)	______________________	______________________
Advisor	Signature	Date
Mequanent Mulugeta (MSc.)	______________________	______________________
Co-Advisor	Signature	Date
Matias Kabtamu (MSc.)	______________________	______________________
Internal Examiner	Signature	Date
Adil Zekaria (Dr. Ing)	______________________	______________________
External Examiner	Signature	Date
Birhanu Beshah (Dr.)	______________________	______________________
Head, Railway Engineering Center	Signature	Date

UNDERTAKING

I certify that research work titled "Numerical Analysis on the Behavior of Prestressed Concrete Sleeper under Different Support Conditions" is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources it has been properly acknowledged / referred.

Signature with Date:

Haftom Gebremicael

Abstract

Railway sleepers have important roles in the complex railway system. Throughout the international railway community, many methods have been developed to design and analyze the concrete sleepers. Specifically, this paper focuses on how changes in support condition affect the internal behavior of the sleeper. Due to different loading condition, poor maintenance or bad quality of ballast, a random load distribution along the sleeper-ballast interface may occur. A sleeper design, and also the track system design, which do not consider the random load distribution, could influence the performance of the sleeper and even damage the whole railway system. Thus, a numerical analysis for a pre-stressed concrete mono-block railway sleeper is carried out using finite element method. The internal behavior of a single sleeper subjected to a varying sleeper-ballast interaction is studied and, a linear-elastic sleeper analysis model was developed. This model was used to perform a study to determine the sensitivity of the sleeper to changes in ballast reaction along the sleeper. Four different possible scenarios of support conditions for a sleeper with varying contact length are discussed in numerical analysis. Results of vertical displacement and flexural stress of both rail seat and at mid-section of sleeper are presented. According to the simulations conducted in this study, the bending moments due to the applied rail seat load at the sleeper rail seat / center can be exceeded from design bending moments under small variation in distribution of the ballast reaction. Besides very soft support underneath the rail seat and large variation of ballast stiffness along the length of sleeper might cause severe cracking problems especially at the top surface in the middle of the sleeper. The worst support condition is also identified. Moreover, the results of this study were compared to existing design recommendations (Australian Standard AS 1085.14) to find allowable levels of ballast reaction that can occur beneath the sleeper before failure is expected.

Keywords: *Prestressed concrete sleeper, varying support condition, sleeper-ballast interaction, rail seat and center bending stress, and finite element modeling.*

Acknowledgements

First I would like to thank my Advisor Dr. Esayas who gave me the courage to work on the topic of my thesis. His support, motivation and valuable guidance were very important and I thank him for all his time on supervising my thesis.

Appreciation is also extended to my thesis Co-Advisor Mr. Mequanent for his constant guidance, support, supervision and for suggesting valuable comments throughout this thesis work.

I want to thank Mr. Mesfin Kebede as well for his thoughtful guidance, invaluable support, and numerous hours spent helping me to complete this thesis.

Last but not the least, I would like to thank my family for loving and encouraging me.

Most of all I would like to thank the Almighty GOD and his mother the Blessed Saint MARY.

Table of Contents

List of Figures

List of Tables

Appendix

Appendix A: Command used to develop the FE Model

Appendix B: Contour Plots of Stress results according to each cases of Ballast support Pressure Distribution

1. Contour Plots of Stress results for center voided sleeper varying voided portion
2. Contour Plots of Stress results for center void with low stiffness sleeper varying voided portion
3. Contour Plots of Stress results for single hanging sleeper varying voided portion
4. Contour Plots of Stress results for double-side hanging sleeper varying voided portion

Appendix C: Contour Plots of vertical displacement results according to each cases of Ballast support Pressure Distribution

1. Contour Plots of vertical displacement results for center voided sleeper varying voided portion
2. Contour Plots of vertical displacement results for center void with low stiffness sleeper varying voided portion
3. Contour Plots of vertical displacement results for single hanging sleeper varying voided portion
4. Contour Plots of vertical displacement results for double-side hanging sleeper varying voided portion

Notation

A	Cross section of concrete sleeper, in square millimeters
a	Length of pressure distribution (ballast support) beneath each rail seat, in meters
L_v	Unsupported (Voided) length of sleeper
DF	Distribution Factor, in percent
E_c	Young's modulus of a concrete, in gigapascal
E_s	Young's modulus for the rail steel, in gigapascal
f_c	The compressive strength of concrete at 28 days, in megapascals
f_{ctk}	Concrete flexural tensile strength under static load at the age of 28 days, in megapascals
f_{ptk}	Characteristic strength for prestressed steel in megapascals
g	Distance between rail centers, in meters, at the head of the rail
I	Second moment of area for concrete section, in m^4
J	Combined quasistatic and dynamic design load factor
Φ	Impact Factor
L	Length of sleeper
M_{C+}	Maximum positive (design) bending moment at the mid-span (center) of sleeper, in kilonewton meter
M_{C-}	Maximum negative (design) bending moment at the mid-span (center) of sleeper, in kilonewton meter
M_{R+}	Maximum positive (design) moment at the rail seat, in kilonewton meters
M_{R-}	Maximum negative (design) moment at the rail seat, in kilonewton meters
P_{ab}	Maximum ballast pressure, in kilopascals
Q	Static wheel load, in kilonewtons
R	Design rail seat load, in kilonewtons
S	Sleeper spacing, in meters
W	Maximum load per unit length of sleeper, in kilonewton's per meter
ρ_c	Density of concrete in kilogram per cubic meter
ρ_s	Density of tendon in kilogram per cubic meter
ν_c	Poison's ratio of concrete
ν_s	Poison's ratio of steel tendon

Acronyms

AREMA	American Railway Engineering and Maintenance-of-Way Association
AS	Australian Standard
PCS	Prestressed concrete sleeper
NDT	Non-destructive test
FE	Finite element
FEM	Finite element method
FEA	Finite element analysis

1. INTRODUCTION

1.1. Background

Track system comprises many components among which the roles of sleepers are noticeable. Sleepers are rest transversely on the ballast with respect to the longitudinal rail direction. It has been developed for over hundred years, and according to the nature of its purpose, sleeper need to be durable enough to resist the heavy traffic loads. Railway sleepers was first made using timber before evolving to steel, reinforced concrete and to the most common type seen today, prestressed concrete. This evolution is closely related to improve durability and longer service life span.

Several researches of different modes of rail failures show that railway sleeper take significant part as other components like rail, ballast, etc. (Li, 2012). Inspection and maintenance of sleeper is therefore required to insure that the sleeper keeps in good condition to:

- Maintain track gauge and rail inclination;
- Distribute and transmit forces to the ballast bed;
- Provide adequate strength both in the vertical and horizontal direction;
- Insulate electrically one rail from the other; and
- Dampen rail vibration and reduce the influence of sound and impact waves on the environment

Siril Y. (2009) and Li, (2012), reported that a large amount of railway sleepers have been in service for many years in different countries (such as Sweden, Australia, North America etc.) and their inspection is done using non-destructive testing (NDT) methodologies (Visual and Sound), where visual inspection being the basic approach; and also railway engineers do the judgment of the condition of the sleepers largely based on human intuition. However, only basic visual inspection is not sufficient, therefore a reliable and accurate experimental and numerical analysis for sleeper should be carried out under different loading conditions both at the design and operation stage, in order to prevent damage, increase the lifetime and improve the track performance.

1.2. Statement of the Problem

As reviewed by Doyle (1980), one of the main functions of prestressed concrete sleepers is to transfer the vertical loads to the ballast and formation. This vertical loads subject the sleeper to bending moment which is dependent on the pressure distribution exerted by the ballast underneath the sleeper.

The exact contact pressure distribution between the sleeper and the ballast and its variation with time will be of importance in the structural design of sleepers. It is practically impossible to predict the exact distribution for a sleeper in the in-track condition (ORE, 1969). In practice, in order to analyze and calculate the loading and bending occurred in concrete sleepers, various railway regulations consider a simple form of stress distribution beneath the sleeper. However, the contact pressure distribution on the sleeper-ballast interaction is mainly depending on the degree of voids in the ballast beneath the sleeper. The ballast support configurations are often changed by the effects of wheel load, breakage of gravel, or loss of confinement, which therefore creates voids and pockets beneath railway sleeper.

Moreover, because of a poor quality of ballast, a situation happens frequently where sleeper is isolated from ballast or penetrated into a ballast bed. For instance, surface infiltration, and sleeper wear due to repetitive traffic loading may cause ballast fouling and softer deformable ballast structure. Thus, various interactions between sleeper and ballast provide different types of support conditions, which increase the possibility of failure of sleeper.

Therefore, the arrangements of the contact patterns between sleeper and ballast bed can take various shapes. The current design methods do not clearly include the effect of ballast material degradation on the ballast-sleeper pressure distribution pattern (Sadeghi & Youldashkhan, 2005). A sleeper design, and also the track system design, which do not consider the random load distribution, could influence the performance of the sleeper and even damage the whole railway system. Hence, the study on the flexural stress experienced by the sleeper under varying support conditions and how changes in support conditions affect the behavior of sleeper is important in designing the sleeper to withstand vertical loading.

1.3. Scope of Study

The scope of this study is:

a) To review the analysis and design of prestressed monoblock concrete sleepers by referring to Section 4 of Australian Standard (AS 1085.14-2003) and American Railway Engineering and Maintenance-of-Way Association (AREMA 2010, Chapter 30);

b) To perform numerical analysis of sleeper using finite element software, ANSYS, in different kinds of the sleeper-ballast interaction as the controlled variable.

c) The contact support pressure values beneath each sleeper is computed using SAFE V12 software and the maximum limit for the pressure value beneath sleeper is 750 KPa as specified in the Australian Standard.

d) To apply specification of sleeper dimension, material properties and other technical parameters based on the Chinese New Type II sleeper;

e) The preliminary applied rail seat load in the numerical analysis is based on 25 tonne axle load which is the maximum imposed load as regulated by the Ethiopian Railway Corporation.

f) To only consider quasi-static (dynamic ride) wheel loading in the numerical analysis, impact loads due to wheel/rail interaction and/or irregularities are omitted.

1.4. Objective of the study

This thesis focuses on the study of the behavior of prestressed concrete sleeper under varying support conditions. This includes analysis of the sleeper with the ultimate goal of investigating the sensitivity of the sleeper to changes in ballast reaction along the sleeper. The objective is to explore the behaviour of sleeper (such as stress and vertical displacement) compared to existing design recommendations, to find allowable levels of ballast reaction that can occur beneath the sleeper before failure is expected. The specific objectives are stated below:

a) To develop a three-dimensional model of prestressed concrete sleeper using finite element analysis modeling;

b) To investigate the theoretical bending moments experienced by the sleeper under varying support conditions, a linear-elastic sleeper analysis model is developed;

c) To compare the finite element analysis result with the existing design recommendations, to identify which ballast support distribution that affect the sleeper the most;

d) To investigate the influence of different kinds of ballast support on sleeper by judging some remarkable factors such as large bending stress and unacceptable vertical displacement compared to the design recommendations by the Australian Standard;

e) Based on the result of the analyses, recommendations are provided on the maintenance of voids in the ballast to avoid deterioration of the material leading to unsuitable sleeper support conditions.

1.5. Significance of the Study

This study is important to understand how the different kinds of sleeper-ballast interactions affect the behavior of sleeper. This is of paramount importance to the development of health Inspection of a railway in-situ concrete sleeper. Besides, this study has led to a numerical prediction associated with the influences of voids/pockets and contact mechanisms underneath the sleeper. Moreover, the knowledge gained through this research will benefit the maintenance sector to have an awareness of the effect of variations in stiffness and support conditions. This, in turn, can be used to plan and improve track maintenance.

2. LITERETURE REVIEW

2.1. Development of Prestressed Concrete Sleepers

Today the use of concrete sleepers is increasing all over the world as they become an economically competitive alternative to the historical industry standard wooden sleepers, while providing performance which exceeds its competition in terms of durability and capacity. Concrete sleepers can be either reinforced twin block or prestressed monoblock concrete sleepers (Esveld, 2001). At the moment, mono-block PCSs, as the most commonly used types of sleepers (Okonta and Magagula, 2011), are popular and widely used in many countries such as North America, Europe, Asia and Australia. The shape of a typical mono-block sleeper is illustrated in Figure 2.1.1.

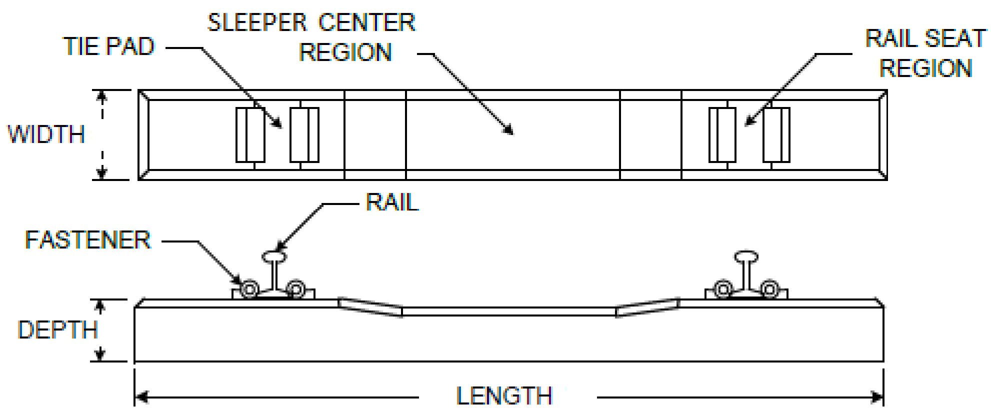


Figure 2.1.1 Monoblock concrete sleeper, Russell H. Lutch (2009)

In the early railways, one reason for starting to use reinforced concrete sleepers was to get a great reduction in the overall cost of track maintenance. The reinforced concrete sleepers (RCSs) were used quite extensively in the 1920s and 1930s in countries such as Italy and India. The use of reinforced concrete sleepers increased the structural stiffness and developed unique problems that are not associated with wooden sleepers, such as flexural cracks which could lead to deterioration of the sleepers (Hwang et al. 2011, J. Taherinezhad, M. Sofi, P. A. Mendis and T. Ngo, 2013).

This fact, together with shortage of good-quality timber during World War II blockades, forced in the development and use of prestressed concrete sleepers (Hwang et al. 2011). These are primarily manufactured using pre-stressed concrete – a technique where internal tension is introduced to the railway sleeper (usually to the high-tensile steel wire skeleton) before it is cast to counteract the external pressure the blocks undergo during service.

Thus, the development of prestressed concrete sleepers solved the early problems associated with the ordinary reinforced concrete sleepers (Lutch R. H., 2009). Since then, the use of prestressed concrete sleepers has increased and become standard in the railway tracks of several countries. PCSs brought many technical and economic advantages to the railway engineering. Some of the advantages of mono-block prestressed concrete sleeper as stated by Esveld (2001), Mundrey (2000), Kaewunruen and Remennikov (2008) and Shan, Li (2012) are:

- Its main advantage is that to keep all cross sections of the sleeper's fully in compression, under either pre-camber or design service loads. Hence, this approach ensures that tension cracks do not occur which can allow access of moisture and corrosion of the embedded steel bar.
- Heavy weight, gives stability to the track;
- Provides good longitudinal and lateral resistance;
- Provides better load distribution to the underlying ballast bed;
- Longer life time and lower maintenance cost, since they are not affected very much by either climate or weather;
- Maintain track gauge in a good manner.

On the other hand some drawbacks of mono-block sleepers according to Esveld (2001), Mundrey (2000), and Shan, Li (2012) are:

- Difficulty in handling, lying and maintenance due to their heavy weight;
- The manufacturing of concrete sleepers, their transport, laying and maintenance requires superior technology, which is not readily available in developing countries. The manufacturing plants generally have a heavy initial cost.
- Risk of damage from impacts (derailment, tamping tines).

2.2. Relationship between Track components and the Sleeper

By considering the static and dynamic loads acting on the track structure, railway track structures is primarily analyzed and designed to avoid excessive loading which may induce damage to the track substructure and superstructure. This include track components such as rails, rail pads, mechanical fasteners and concrete sleepers (superstructure) as well as geotechnical systems such as ballast, sub-ballast and subgrade or formation (substructure). Both superstructure and substructure are mutually important in ensuring the safety and comfort of passengers and quality of the ride. Figure 2.2.1 shows the cross sectional layout of a typical ballasted track (Selig & Waters, Kaewunruen, 2007). This study is focused in the analysis of sleeper; however, it is also important to understand the functions of track components and their load path relationship with the sleeper.

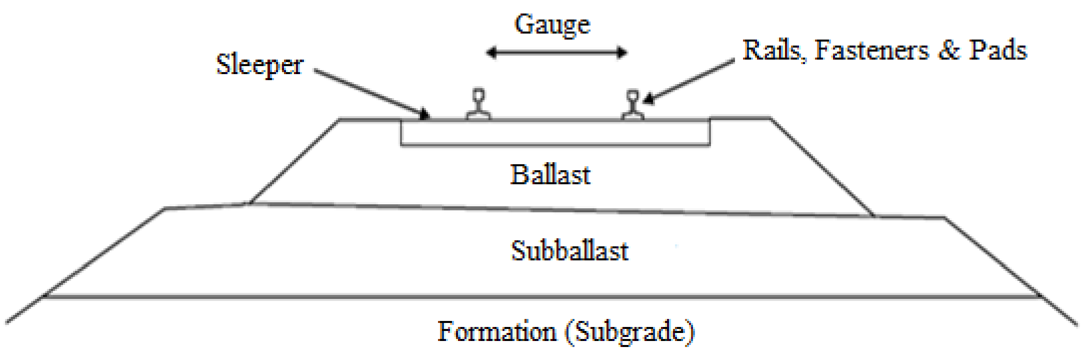


Figure 2.2.1 Cross sectional layout of typical ballasted track (Selig & Waters, Kaewunruen, 2007)

As a longitudinal steel members placed on equally spaced sleepers, rails are the critical component in guiding the rolling stocks. They must have sufficient stiffness to serve as beams that transfer the concentrated wheel/axle loads to the spaced sleeper supports without excessive deflection between supports (Selig & Waters, Kaewunruen, 2007).

Both mechanical fasteners and rail pads are the primary components of the fastening systems. Apart from keeping the rails in position on the sleepers, the mechanical fasteners withstand the three dimensional forces of vertical, lateral and longitudinal as well as the overturning movements of the track. Mechanical fastener also transfer forces caused by wheels, thermal change and natural hazard from rails to the adjacent set of sleepers.

Rail pads which are placed on the rail seat are essential in filtering and transferring the dynamic forces from the rails and mechanical fastener to the sleepers. The dynamic force is predominantly from the traveling rolling stocks and the high damping coefficient of the rail pads considerably reduces the excessive high-frequency force components to the sleepers. The resiliency provided by the rail pads to rail-sleeper interaction has resulted in the alleviation of rail seat cracking and contact attrition (Wan Azilan, 2012).

Underneath the sleepers in providing tensionless elastic support is ballast, a free-draining coarse aggregate layer typically composed of crushed stones, gravel, and crushed gravel which, as a result of internal friction between the grains, can absorb considerable compressive stresses, but not tensile stresses. Depending on the local availability, basalt and granite are usually the preferred material for ballast due to their strength characteristics for load transfer. The bearing strength of the ballast bed in the vertical direction is considerable, but in the lateral direction is clearly reduced. Ballast provides a static and dynamic stability to the sleepers by distributing a uniform load and reduction over the sub ballast and subgrade (Esveld, 2001).

In between the ballast and the underlying subgrade is sub-ballast, commonly composed of broadly graded material that assists in reducing the stress at the bottom of the ballast layer to a tolerable level for the top of the subgrade. The sub-ballast is usually an impervious material that can prevent the inter penetration of the subgrade and ballast, thereby reducing migration of fine material into the ballast which effects drainage and reduce the ballast support pressure below sleepers (S. Kaewunruen and A. M. Remennikov, 2009).

The last support to sustain and distribute the resultant downward dynamic loading along its infinite depth is subgrade or also known as formation. The subgrade facilitates drainage and provides a smooth platform, at an established grade, for the track structure to rest upon.

2.3. Ballast support under sleeper

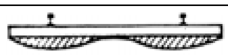
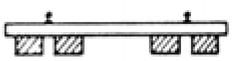
Once wheel loads are applied to the sleeper they must be transferred to the ground through the ballast and sub-ballast materials. Ballast support is crucial to a sleeper’s ability to support load. The bending moments calculated from the applied rail seat loads are a function of the ballast support conditions which varies over the life of the sleeper (Sadeghi, J. & Barati, P., 2010). Poor ballast support results in sleeper cracking and eventually flexural failure (Namura et al. & Lutch, Russell H., 2009).

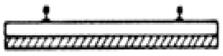
The ballast support is affected by a variety of factors that include loading during train operations, tamping, fouling, and voids. Usually it is very hard to know the real contact situation between sleepers and ballast (Li, 2012). When track is freshly tamped, ballast support is limited to the areas around the rail seat region leaving the center region of the sleeper with little to no support (Figure 4.3.1 a, and b). After the tracks have been in service the contact pressure distribution between the sleeper and the ballast tends towards a uniform pressure distribution, (Sadeghi, J. & Barati, P., 2010). This condition is associated with a gap between the sleeper and the ballast surface below the rail seat (Talbot A.N., Doyle, N.F., 1980); tends to develop the condition of center-binding of sleepers when maintenance is neglected (figure 4.3.1d). Hence, it can be readily seen that the contact pressure distribution between the sleeper and the ballast is a time dependent, i.e. cumulative traffic tonnage, variable.

Lutch R. H., 2009, reported that the sleeper center support (center-binding of sleeper) is as a result of over time cyclic loading applied to the track, causes sleepers to oscillate and deform vertically within the track structure; this deformation produces pumping action which ultimately allows ballast to abrade the bottom of the sleeper and pulverize the ballast beneath the sleeper. However, when maintenance is not carried out regularly, depressions in the pulverized ballast beneath the ends of the sleeper may develop, altering the support condition of the sleeper. The new support condition is a center support, where the sleeper cantilevers from the center over the pulverized ballast depression.

Studies to determine the contact pressure at the ballast-sleeper interface of the track have been undertaken in the past. A.N. Talbot, as chair of American Railway Engineering Association (AREA), (1913-1940), other researchers and standards were focused primarily on the pressure distribution along the length of the sleeper, particularly as an input for sleeper design, and to incorporate into recommended practices. Thus, various hypothetical contact pressure distributions between the sleeper and ballast have been proposed as shown in table 2.3.1 below.

Table 2.3.1 Hypothetical distribution of sleeper bearing pressure (Sadeghi J. M. and Youldashkhan M., 2008)

Distribution of bearing pressure	Developers	Remarks
	ORE (1987), Talbot (1933)	Laboratory Test
	ORE (1987), Talbot (1933), Bartlett (1960), Clark (1957)	Tamped either side of rail

	ORE (1987), Talbot (1933)	Principal bearing over rails
	ORE (1987), Talbot (1933)	Maximum intensity at ends
	Talbot (1933)	Maximum intensity at center
	Talbot (1933)	Center bound
	Talbot (1933)	Flexure of sleeper produces variations in form
	ORE (1987), Talbot (1933), Kerr (2003), Schramm (1961)	Well-tamped sides
	ORE (1987), Talbot (1933)	Stabilized rail seat and sides
	AREMA (2006), Raymond (1977), Talbot (1933)	Uniform pressure

Many railway organizations address the design methodology of railway sleepers based on simple form of support pressure distribution underneath the sleeper. however, the *ORE* (1969) have carried out experiments to determine the contact pressure between the sleeper and the ballast due to the action of known wheel loads passing over the track. Tests were carried out for a range of sleeper types (timber and concrete). Results indicate that the sleeper to ballast contact pressure distribution is very much a function of the ballast packing method used and that it tends to be, "unpredictable and random with a high degree of scatter".

Likewise, recent works show that, because of a poor quality of ballast, a situation happens frequently where sleeper is isolated from ballast or penetrated into a ballast bed (Li, 2012). For instance, Past experience of ballast field performance has shown that the progressive breakdown of ballast materials, such as that caused by traffic load and maintenance tamping, and the intrusion of external materials, such as wagon spillage and infiltration of underlying materials into the ballast, and sleeper wear due to repetitive traffic loading may cause ballast fouling, and softer deformable ballast structure (McHenry, Michael T, 2013). This causes the voids and pockets, or even the poor packing of ballast support underneath railway concrete sleepers (Barke and Chiu, 2005; Kaewunruen and Remennikov, 2007a). A sleeper having voids and pockets underneath is hereafter referred to as ‘voided/hanging sleeper’.

At a hanging/voided sleeper, there is a gap between the sleeper and the ballast. When a gap between the sleeper and the ballast exists, this can be closed by the load of a passing train. The ballast then carries some of the load as soon as the gap is closed. But if the gap is not closed no forces will be carried by the ballast in this section. At hanging sleepers the vertical track stiffness becomes very low. These changes of the vertical track stiffness cause high train/track interaction forces, which will increase the track settlement rate (Berggren, 2009).

According to Calla C., 2003, Voids under sleepers are a common occurrence in the railways and are a major cause of track geometry deterioration and ballast degradation. Voids can be detected by watching vertical movement of the sleeper under traffic or by striking the sleeper with a hammer and subsequently measured by installing instruments like void meters on the track (Figure 2.3.1). Cope and Ellis (2001), stated that voids below sleepers can be easily identified by white powder around the sleepers and less angular ballast on the upper surfaces (See figure 2.14). Voided sleepers occur either as single sleepers supported by adjoining sound sleepers or as a group of loose sleepers again supported by adjoining sound sleepers effectively hanging from the rail. A track with a single voided sleeper if not attended to, soon deteriorates to a group of loose sleepers.

Figure 2.3.1 Showing Pell void meter fixed to bottom flange of the rail (Cope and Ellis, Calla C., 2003)

Figure 2.3.2 Voids below concrete sleeper identified by white powder around the sleeper (Cope and Ellis, Calla C., 2003)

Various interactions between sleeper and ballast provide different types of support conditions, which increase the possibility of fatigue and crack failure of sleeper. Rezaei E., 2010 and Li, 2012, indicate that from previous assessments, five situations of contact boundary conditions are of practical concern in real railway track problems. In particular, these defects (voids or hanging sleepers) can be detected through vibration tests. The stated patterns are center void with no support in the mid-section (figure 4.3.1a), center void with low stiffness in the mid-section (figure 4.3.1b), single hanging (figure 4.3.1c), double hanging (figure 4.3.1d), and double side center hanging. The first four cases are described in chapter four and will be investigated in this thesis.

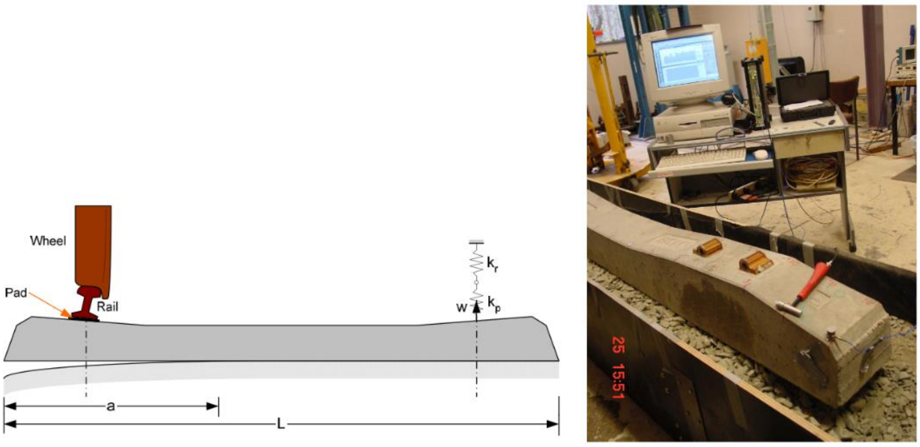


Figure 2.3.3 Schematic model and Experimental setup of a voided railway concrete sleeper (single-hanging) (Kaewunruen S. and Remennikov A. M., 2009)

Kaewunruen S. and Remennikov A. M., (2009), states that the ballast support configurations are often changed by the effects of wheel load, breakage of gravel, or loss of confinement, which therefore creates voids and pockets underneath railway sleeper. Their study focuses on experimental and numerical analysis to better understand free vibration characteristics of voided railway concrete sleepers. A type of void configurations was selected for demonstration in this study, which is called 'the single-hanging sleeper' as illustrated in Figure 2.3.3 above.

Therefore, the contact pressure distribution between the ballast and sleeper is mainly dependent upon the degree of voiding in the ballast under the sleeper. This voiding is caused by traffic loading and due to the gradual change in the structure of the ballast and subgrade. The current design methods do not clearly include the effect of ballast material degradation on the ballast-sleeper pressure distribution pattern (Sadeghi & Youldashkhan, 2005). This means that the sleeper design approach awaits further improvements by the incorporation of long term effects of the loads and in turn, consideration of the ballast degradation. Hence, the study on the internal pattern of sleeper's behavior under varying ballast support pressure is important in designing the sleeper to withstand vertical loading, increase the lifetime and improve track quality.

2.4. Effect of ballast support variations

Different support conditions along a railway track appear due to differences along the line in settlement rates of the ballast, the sub-ballast and the soil in and under the embankment. This is a natural process during the years directly after construction of a new railway and is induced by the weight of the structure. Later in the track life cycle, additional track settlements mostly appear due to the weight of passing trains, but it can also be an effect of ballast fouling (Berggren, 2009).

C. Calla, (2003), states ballast fouling, due to ballast degradation under loads and due to the effect of tamping, can be broadly defined as the filling up of the voids in ballast with particles of size smaller than 6mm. New ballast does not contain more than 2% of the particles smaller than 6mm in size. Over a period of time the proportion of smaller particles increases and beyond a certain limit the smaller particles prevent ballast from providing - support to the sleepers (where sleepers are poorly packed) and drainage of water from the sleepers (that will result in wet spot or slurry spot) (C. Calla, 2003). When the wheel passes over a loose sleeper it causes the sleeper to impact the ballast damaging the soffit of the sleeper and also causing ballast breakage in that area. Also, Poor drainage capacity increases the risk of damage due to frost, and water filled ballast will affect the stability of the track. Uncontrolled water flowing through the track will lead to increased erosion.

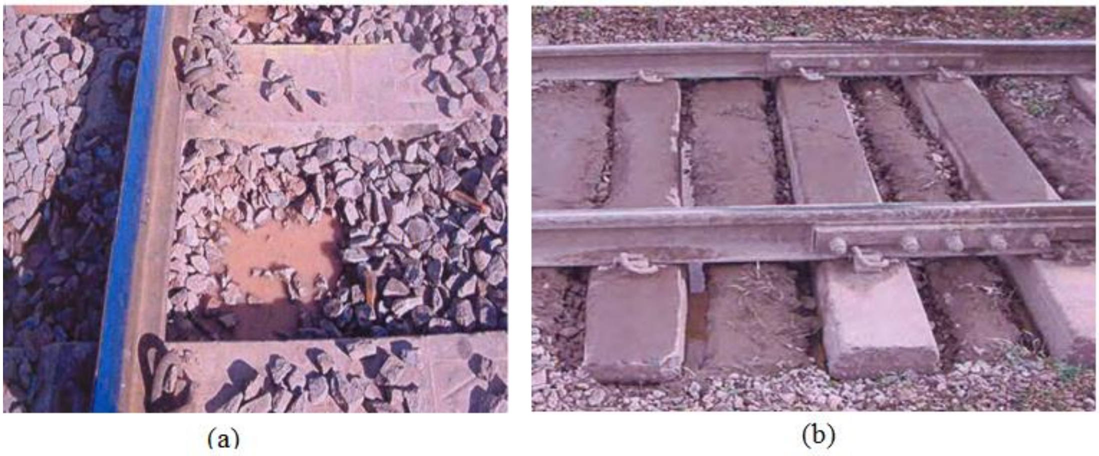


Figure 2.4.1 a) Wet spot with water ponding in the crib, and b) An advanced stage of wet with voided sleepers and fouled ballast (C. Calla, 2003).

Thus, poor drainage and differential settlement of subgrade may lead to some sleepers not being fully supported, or even completely unsupported, resulting in increased deflection and wheel-rail contact force. With growing trends towards heavier axle loads on railway, unsupported sleepers will speed up track deterioration process (Jin Shi, Chan, Burrow, 2012). According to Augustin et al (2002), in reality, a considerable number of sleepers are badly bedded or do not touch the ballast at all.

If one sleeper is totally unsupported (voided or hanging sleeper) the surrounding sleepers need to carry a higher load. This corresponds to a higher quasi static load from passing trains and also an additional dynamic load due to the change in track stiffness caused by the hanging sleeper (Lundqvist and Dahlberg, 2005). This makes a sleeper close to a hanging sleeper likely to crack e.g. due to forces induced by wheel irregularities, such as wheel flats.

Support conditions will differ between sleepers along the track but also along a single sleeper. In research by Li (2012) a static and a dynamic analysis on a single sleeper under varying support conditions is performed. Four cases are considered with fixed length of support and void. The first is a fully supported sleeper; the following two correspond to a central void as described in section 4.3, Figure 4.3.1. Case two with low stiffness and case three with no support in the mid-section. The last case studied corresponds to the double side central void.

The result of the static analysis shows that, of these four cases, the case with double side central void is the worst-case scenario (Li, 2012). The most severe consequence of this support condition is that it induces tensile stresses in parts of the sleeper where the reinforcement is insufficient. Thereby the risk for cracking and tensile failure increases.

The dynamic analysis also singles out the double side-central void case as a worst-case scenario with longer lasting reaction forces, larger vertical displacement and significantly higher stresses (Li, 2012). For the case studied, the tensile stress in the mid-section of the sleeper is estimated to 10 MPa which is a much higher than the tensile capacity of concrete (3 MPa). This means that a sleeper experiencing such a support condition is likely to have damage caused by tensile stresses.

Notable is that Li (2012) did not study the case of a double hanging sleeper (see Figure 4.3.1d). In research by Bolmsvik and Nielsen (2006) this case is studied. They state that a very high bending moment arise in the mid-section of the sleeper when the sleeper is subjected to a support condition corresponding to double hanging.

Likewise, Riessberger, K. (1984), described that the sleeper center support condition, when loaded, large negative moments occur at the sleeper center, resulting in cracking and sleeper failure as the flexural capacity is exceeded. This type of failure is referred to as “center binding”. According to a survey of railroads, concrete sleeper manufacturers, and researchers from around the world, sleeper cracking from center-binding was ranked as the third most critical problem with concrete sleepers (Van Dyk, B. J., 2013).

Hence, it can be concluded that variation in support conditions influence the risk of damage on track and train. The problems with variation in support conditions can be solved by proper maintenance. The fundamental principle of track maintenance is “To maintain a good ‘top’ on a line it must be lifted wherever it is low and the ballast must be packed firmly under the sleepers at the points where it has been lifted” (C. Calla, 2003). So, it is important that such maintenance is performed regularly since damage to the track that has become too severe requires larger and more expensive actions.

3. CURRENT PRACTICES IN THE THEORETICAL ANALYSIS OF SLEEPER DESIGN

3.1. Overview

Nowadays, many organizations (e.g. American Railway Engineering and Maintenance-of Way Association, Australia Standard, European Committee for Standardization, and International Union of Railways) address the design methodology of railway sleepers. Most of them recommend a design methodology for concrete sleepers using permissible stress design concept.

According to the Australian Standard (AS 1085.14—2003), the life cycle of the sleeper is 50 years (Kaewunruen and Remennikov, 2009). The design process relies on the permissible or allowable stress of materials. A load factor is used to increase the static axle load to incorporate dynamic effects. The design load is termed '*combined quasistatic and dynamic load*' which has a specified lower limit of 2.5 times static wheel load. These loads allow for static response of the sleeper due to the mechanism of vertical load transfer between the rail and sleeper as well as the sleeper and ballast interaction.

Past practice has indicated that utilization of this permissible stress design concept is adequate for flexural strength design. AS1085.14 (Standards Australia, 2003) states that, if the design fulfils with AS1085.14, there is no need for consideration to checking stresses other than flexural stresses, because the permissible stress design concept limits the strengths of materials to comparatively low values compared to their true capacity. Under the design loads, the material is kept in the elastic zone so there is no permanent set. In particular, sleepers that comply with AS1085.14 have all cross sections of the sleeper's fully in compression, under either pre-camber or design service loads. This approach ensures that an infinite fatigue life is obtained and no cracking occurs (Kaewunruen and Remennikov, 2009).

3.2. Shape and Dimensions

When designing a sleeper, an evaluation of the loads being transferred and their flow through the track structure is essential. Therefore, forces and pressures of interest include the rail seat load, and the ballast pressure. The required flexural capacity is determined based on the ballast support conditions encountered during the life of a sleeper and the applied rail seat loads. Likewise, it is derived from material properties, tie dimensions and number and type of Prestressing wire used. In this thesis, a Chinese sleeper of type II New is studied, and detailed dimensions are shown below in figure 3.2.1.

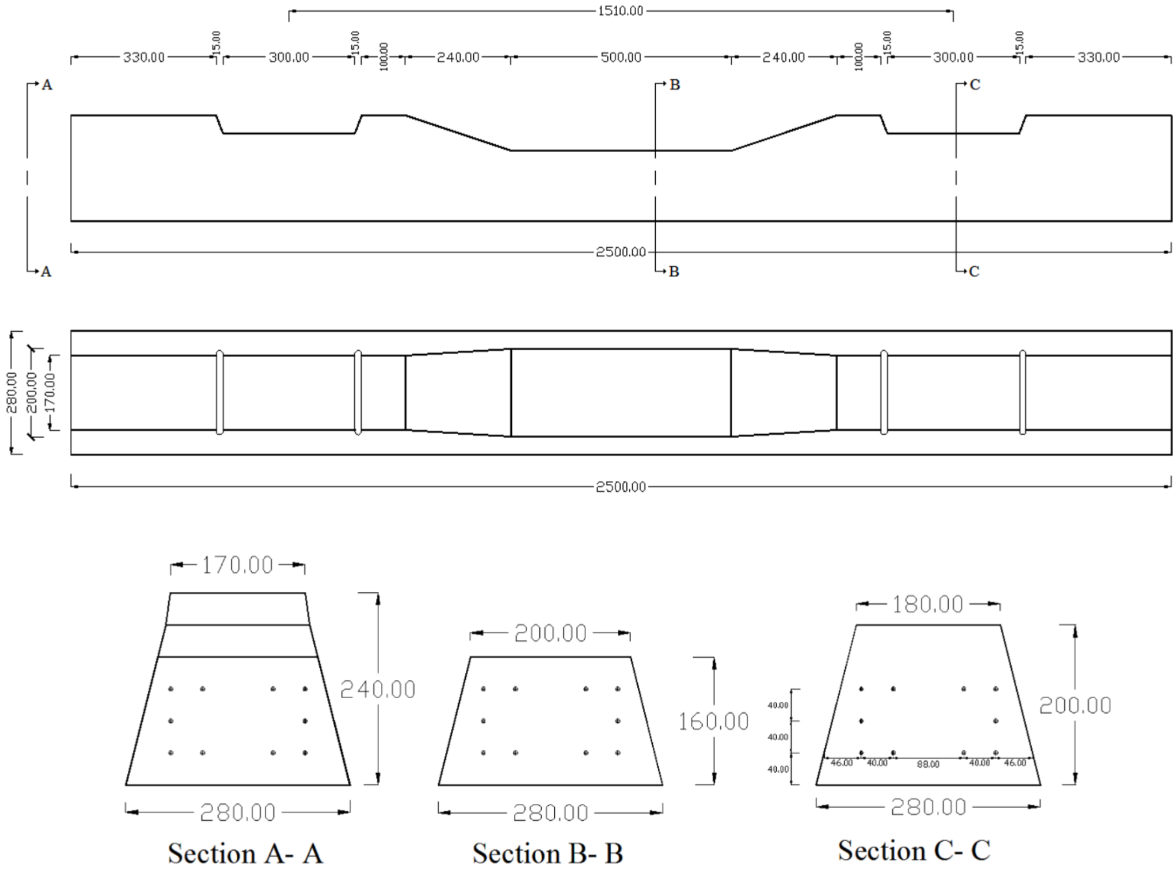


Figure 3.2.1 Sketch of sleeper type II New, all dimensions are in mm

3.3. Material properties

Typical to any prestressed concrete member are early high strength concrete and high tensile strength steel (Russell H. Lutch, 2009).

3.3.1. Concrete

The use of higher strength concrete is necessary in the production of concrete sleepers due to the use of Prestressing. AREMA suggested that the minimum 28-day-design compressive strength of concrete used for concrete sleeper shall be 7,000 psi (48 MPa) as determined by ASTM Method of Test C 39 (AREMA part 4, section 4.2.2). Likewise, the Australian code recommended that the characteristic compressive strength (f'_c) shall be not less than 50 MPa (AS 1085.14-2003, section 2.7.3).

Table 3.3.1.1 Typical properties of concrete used as input data (according to Chinese code for design of Concrete Structures GB50010-2002)

No.	Properties of concrete		Value
1	Density (ρc) in kg/m^3		2400
2	Young's modulus (Es) in MPa		36000
3	Poisson's ratio (ν_s)		0.3
4	Concrete grade		C - 60
5	Characteristic compressive strength (fc) in MPa		60
6	Permissible bending stress	Compressive in MPa	0.4 fc
		Tensile in Mpa	0.04 fc

3.3.2. Pre-stressing steel

Prestressing steel provide the necessary tensile resistance to support the flexural loads imparted by the trains. Prestressing in concrete sleepers consist of either the 7-wire strand or steel wire (AREMA 2010). 5mm wire and 9.3 mm 7-wire strand are the most widely used tendons. Wire tendons shall be stress-relieved indented wires not larger than 8 mm nominal diameter conforming to AS 1310. Similarly, the Chinese code GB50010-2002, recommend stress-relieved steel wires (indented) with nominal diameter of 5mm and 7mm.

Table 3.3.2.1 Typical properties of Prestressing steel used as input data (GB50010-2002)

Item	Properties of wire	Value
1	Density (ρ_s) in kg/m^3	7800
2	Young's modulus (E_s) in GPa	200
3	Poisson's ratio (ν_s)	0.3
5	Number strands (Stress-relieved indented wire)	10ϕ5.0mm
6	Characteristic strength for prestressed steel (f_{ptk}) in MPa	1570

3.3.2.1. Stages of pre-tensioning

Pre-stressing technology has been developed for many years, and it is divided into pre-tension and post-tension. For concrete railway sleeper, pre-tensioning is widely used where tension is applied to tendons before casting of concrete. The stages of pre-tensioning are mainly described below.

The pre-tensioning is carried out in a prestressing bed equipped with end anchorages which take up tension. The high strength reinforcement tendons are pulled between two end anchorages which are fixed upon the prestressing bed before concrete sleeper is cast in the form. Once the concrete has attained a sufficient prestressing strength, the tendons are cut from the end anchorages, and the tensile force is transferred to the concrete through the tendons, because of the bonding between the concrete and the embedded reinforcement (Li, 2012).

The stages of the pre-tensioning procedure are summarized in the following (Li, 2012);

1) Anchoring of tendons against the end abutments
2) Placing jacks
3) Applying tension to the tendons
4) Casting of concrete
5) Cutting of the tendons

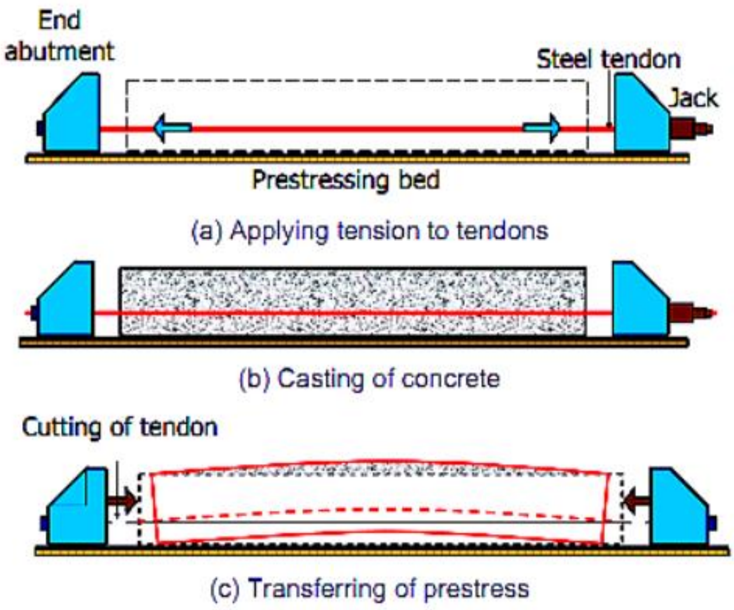


Figure 3.3.2.1.1 Stages of pre-tensioning, Li (2012)

As per the code (GB50010-2002), the value of controlled stress for stretching prestressed reinforcement may not exceed the allowable value of controlled stress for stretching as stipulated in the Table 3.3.2.1.1, and shall be not less than 0.4 f_{ptk}.

Table 3.3.2.1.1 Allowable value of controlled stress for stretching (GB50010-2002, table 6.1.3)

Types of reinforcement	Method of stretching	
	Pre-tensioned	Post-tensioned
Stress relief steel wire, strand	0.75fptk	0.75fptk
Heat treated steel bar	0.7fptk	0.65fptk

Therefore, the stress for stretching of stress relief steel wire is $0.75f_{ptk}$ = 1177.5 Mpa. Then the initial strain of wire can be determined using;

$$E = \frac{\sigma}{\varepsilon};\ \varepsilon = 5.8875\ mm/m$$

3.4. Loading conditions

As a train moves along the track, the load from an axle is distributed amongst several sleepers due to the rigidity of the track (Russell H. Lutch 2009). A single sleeper typically carries between 45 to 55 percent of an axle load directly above it. Factors affecting this load distribution are the sleeper spacing, fastening system, rail stiffness, and ballast and sub-grade conditions with sleeper spacing having the largest effect.

3.4.1. Rail seat load

Concrete sleeper design is based on the rail seat load which is transferred from a single train wheel to the rail seat through the rail, as this load will eventually dictate the applied moment at the critical sections of the sleeper. The first step in determining the magnitude of the rail seat load is considering the maximum load to be found in service. In this study the static axle load is set to 25 tons since it is the highest load allowed on standard tracks. The axle load is then separated between the individual wheels; it must be divided further between the individual sleepers due to the rigid effects of the track structure.

Standards Australia (AS1085.14, 2003), clearly defines the vertical design loads intended for prestressed concrete sleeper design. The quasi-static loads are determined from the static loads and the effects of train speed. The typical values of the quasi-static loads are around 1.4 to 1.6 times the static wheel load. In addition the dynamic loading due to high frequency effects of wheel/rail interaction is required greater than 1.5 times of the static wheel load. Hence the design load is the combination of both quasi-static and dynamic loads. It shall be not less than 2.5 times the static wheel load. At the point of wheel/rail contact over a sleeper, the force will be distributed to the adjacent sleepers in the structural track system due to the rail stiffness, which is considered by introducing the load distribution factor. Standards Australia AS1085.14-2003, Section 4-Design of Sleepers, Part 4.2.1.4, presents the estimation of the axle load distribution for a sleeper under wheel in the track systems varying the sleeper spacing is given in figure 3.4.1.1 below.

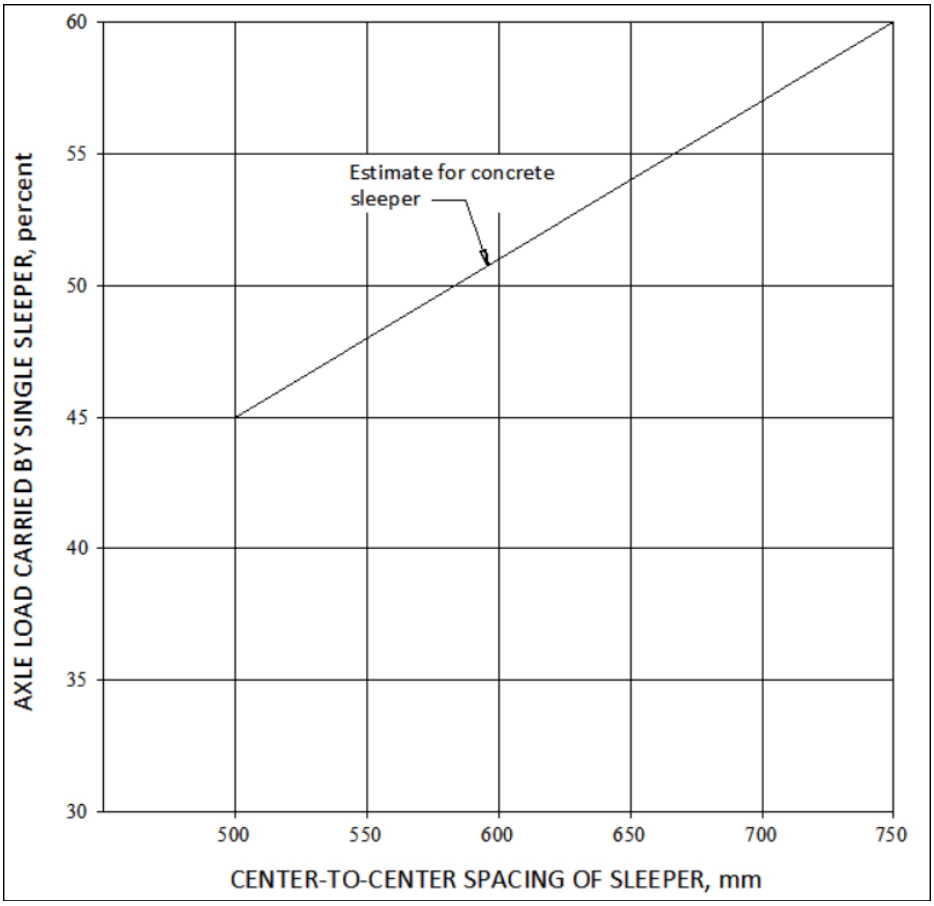


Figure 3.4.1.1 Axle load distribution factor (DF), (AS1085.14, 4.2.1.4)

Based on a typical spacing of 60 cm for concrete sleepers, a single sleeper will carry approximately 51 percent of the axle load. Finally, the value of the rail seat load (R) shall be based on the impact and load distribution factors is calculated as follows:

$$R = \frac{Axle\ Load}{2} * Distribution\ Factor * \left(\frac{Impact\ Factor}{100}\right)$$

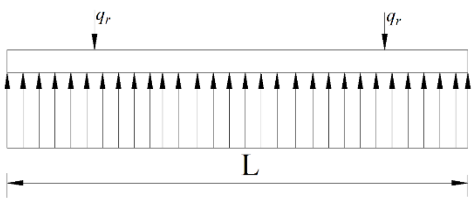
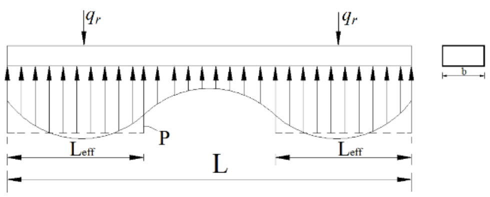
$$R = \frac{250\ KN}{2} * 0.51 * 2.5 = 159.38\ KN$$

3.5. Ballast Pressure

Under sleeper, stresses are the response of the ballast layer against the applied load from the sleeper. It is practically impossible to predict the exact distribution for a sleeper in the in-track condition (ORE, 1969) as explained in chapter 2. Many countries have their own version of design standards or recommended practices in assuming the ballast pressure. As per the AREMA manual (2010), in chapter 30-Ties, Part 1, Article 1.3.6.1 states that sleeper-to-ballast pressure is not uniformly distributed across or along the bottom of a cross sleeper, however an approximate calculation can be made of average pressure at the bottom of the sleeper. It should be noted here that there are differing methods of determining the bearing area of the sleeper for use in ballast pressure calculations.

Article 1.3.6.1 notes the two differing methods presented in the Manual for Railway Engineering for determining *A*. In Chapter 30 – Ties, Part 4, Article 4.1.2.5.1.1, the effective bearing area of the tie appears to be defined as the *entire* footprint of the tie. The AREMA Manual for Railway Engineering, Chapter 16 - Economics of Railway Engineering and Operations, Part 10, Article 10.11.1 defines the bearing area of the tie as *two-thirds* of the tie footprint as shown in Table 3.5.1.

Table 3.5.1 Maximum Ballast-Sleeper Pressure (AREMA, 2010)

Maximum Ballast-Sleeper Pressure	Effective Length of ballast support (L_{eff}) (m)	Ballast-sleeper bearing pressure (P)
	$L_{eff} \cong L$	$P_{ave}, (MPa) = \frac{(2P)\left[1+\frac{IF}{100}\right]\left(\frac{DF}{100}\right)}{A}$ $< 85\ psi$
	$L_{eff} \cong \frac{L}{3}$	$P_a\ , (Kpa) = \frac{3 * q_r}{b * L}$ $< 65\ psi$

Where: P = Wheel load in pounds (KN), IF = Impact factor in percent ((Ø = 2 is assumed), DF= Distribution factor in percent (AREMA, Appendix C, Figure C-4), A = Bearing area of cross sleeper in square inches (millimeters), b = width of sleeper at base; L = length of sleeper, and $q_r = P * DF[1 + IF]$;

According to Standards Australia, Part 4.2.4, the ballast pressure is based on a uniform pressure distribution beneath each rail seat and shall not exceed 750 kPa for high-quality, abrasion-resistant ballast (AS1085.14, 2003).

Table 3.5.2 Maximum ballast-sleeper pressure (AS 1085.14, 2003)

Distance between rail centers (g)	Length of ballast support beneath each rail seat (a) (m)	Maximum ballast bearing pressure (Pa) (kPa)
g > 1.5 m (standard and broad gauge)	$a = L - g$	$P_a = \frac{R}{b * (L - g)}$
1.5 m > g > 1.0 m (narrow gauge)	a = 0.8 (L - g)	$P_a = \frac{R}{0.8b * (L - g)}$

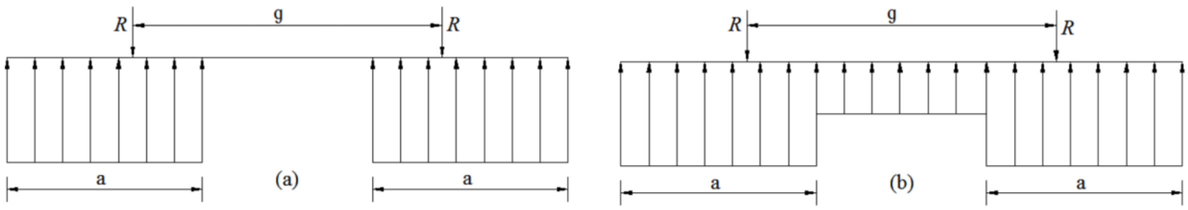


Figure 3.5.1 a) pressure distribution for maximum positive rail seat and center moments, and b) pressure distribution for maximum negative center moments for track gauge of 1600 mm and greater (AS1085.14, 4.2.4)

3.6. Design Moments

The loads applied through the rail, combined with the ballast support reactions at the base of the sleeper, produce flexural stresses in the sleeper (AS1085.14, 2003).

a) Rail seat positive and negative design bending moments (M_{R+} and M_{R-})

The maximum positive bending moment shall be taken to occur at the rail seat producing compressive stress at the top and tensile stress at the underside of the sleeper. The value of this moment, the rail seat positive design bending moment, is based on a uniform ballast support beneath each rail seat as shown in Figure 3.5.1(a). Also, the rail seat negative design bending moment shall be not less than 67 percent of the rail seat positive design bending moment or 14 kNm, whichever is greater.

Table 3.6.1 Maximum positive and negative bending moments at the rail seat (AS1085.14, Part 4.3.2)

Distance between rail centers (g)	Length of ballast support beneath each rail seat (a) (m)	Maximum bending moment at rail seat (kN-m)	Remark
g > 1.5 m (standard and broad gauge)	$a = L - g$	$M_{R+} = \frac{R(L-g)}{8}$	positive
		$M_{R-} = 0.67M_{R+}\ or\ 14$	negative

b) Center positive and negative design bending moments (M_{C+} and M_{C-})

The maximum positive bending moment of the sleeper shall be based on a pressure distribution beneath each rail seat, similar to that shown in Figure 3.5.1(a). And, the value of the maximum center negative bending moment for track gauge of 1435 mm is based on a uniform distribution of ballast pressure on the sleeper soffit.

Table 3.6.2 Maximum positive and negative bending moments at center (AS1085.14, Part 4.3.3)

Distance between rail centers (g)	Length of ballast support beneath each rail seat (a) (m)	Maximum bending moment at center (kN-m)	Remark
g > 1.5 m (standard and broad gauge)	$a = 0.9(L - g)$	$M_{C+} = 0.05R\ (L-g)$	positive
	Full length of sleeper	$M_{C-} = \frac{R(2g-L)}{4}$	negative

4. COMPUTER SIMULATION OF CONCRETE SLEEPER

4.1. Overview

Concrete structural components exist in different forms. Understanding the response of these components during loading is crucial to the development of an overall efficient and safe structure. There are a number of methods for modeling the concrete structures through both analytical and numerical approaches. Finite element analysis (FEA) is a numerical technique widely applied to the concrete structures to study their behavior. It provides a tool that can simulate and predict the responses of reinforced and prestressed concrete members. The use of FEA has increased because of progressing knowledge and capabilities of computer software and hardware. Any attempts for engineering analysis can be done suitably and fast using such versatile FEA packages. It has now become the preferred method to analyze concrete structural components.

A number of commercial FEA codes are available to researchers, along with the advanced modules for complex analysis. In this study, the model is established using Finite Element package – ANSYS12, which is a numerical tool, used to model and simulate the mechanics behaviour and response of sleeper under different possible simple scenarios of support conditions.

4.2. Finite Element model

To create the finite element model in ANSYS, there are multiple tasks that have to be completed for the model to run properly. Models can be created using command prompt line input or the Graphical User Interface (GUI). For this model, the command prompt line input was utilized to create the model (See Appendix A).

4.2.1. Element Types

A variety of element types exists for both steel and concrete materials because the purpose and desired output may vary from model to model. The following section summarizes the elements used in modeling prestressed concrete sleeper. These recommended elements are based on the commercially available software ANSYS.

4.2.1.1. SOLID65 Element

The concrete section is modeled using a three dimensional solid element, SOLID65, which has the material model to predict the failure of brittle elements. This element has eight nodes with three degrees of freedom at each node – translations in the nodal x, y, and z directions. This element is capable of plastic deformation, cracking in three orthogonal directions, and crushing.

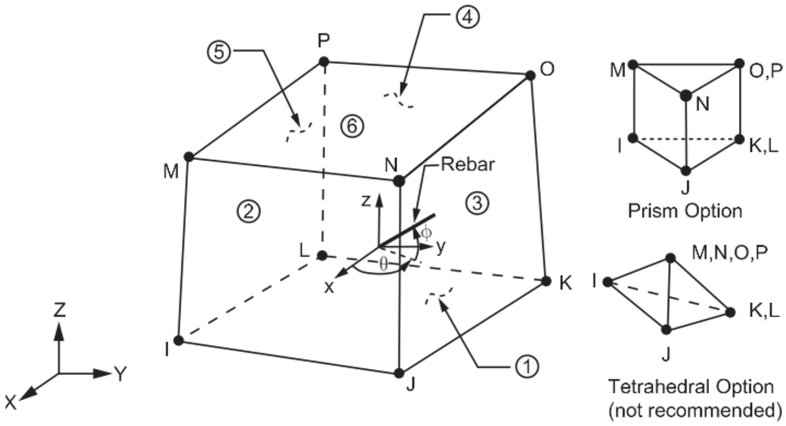


Figure 4.2.1.1.1 Solid65 element geometry (ANSYS element library)

4.2.1.2. LINK8 Element

To simulate the behavior of Prestressing wires, LINK8, were used to withstand the initial strain attributed to Prestressing forces. This is a truss element which is capable of compression and tension with three degrees of freedom at each node: translations in the nodal x, y, and z directions and, also capable of plastic deformation. Each end node is modeled as a pin connection so no bending of the element is considered. LINK8 requires users to input 'real constants' to define reinforcement geometry, material behavior, and prestressing strain.

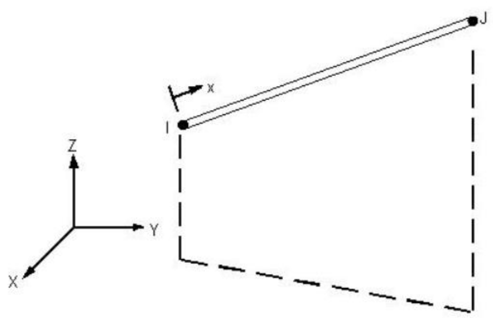
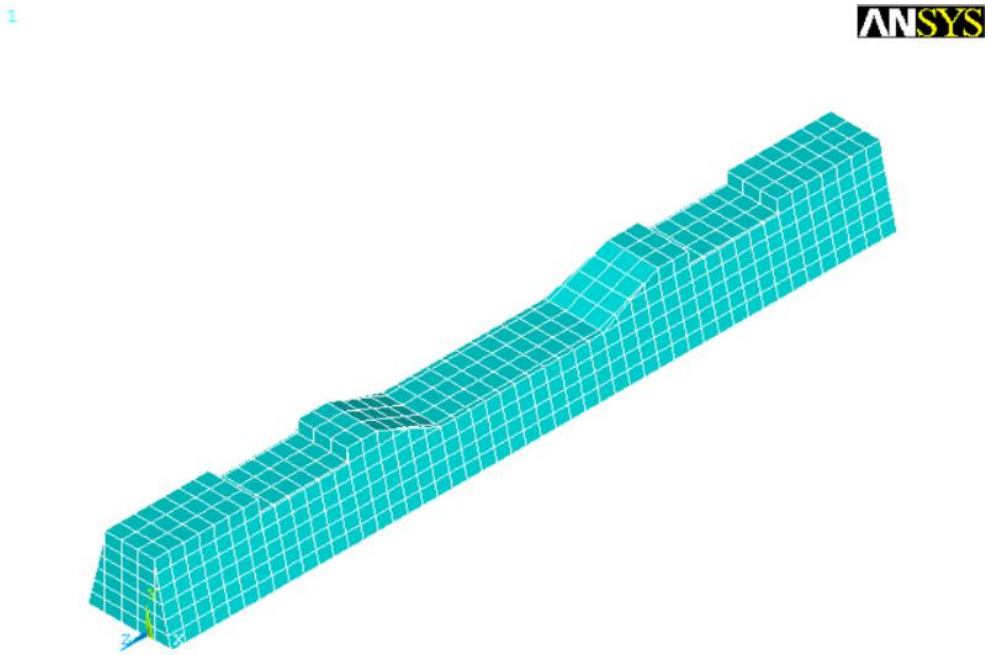
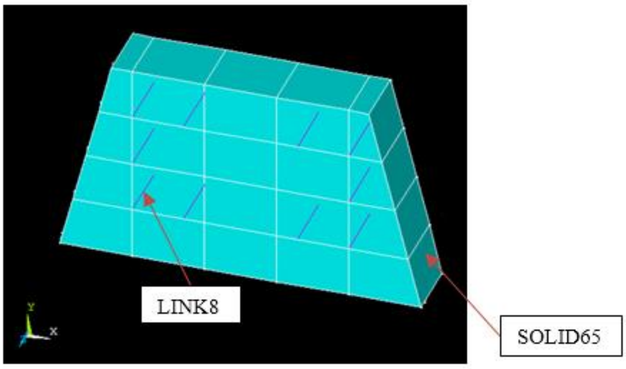


Figure 4.2.1.2.1 LINK8 element geometry (ANSYS element library)

Furthermore, since two independent element types are used for the concrete and steel, the contact between the two elements is defined as a node to node contact. Also, it is assumed that the prestressed reinforcement has perfect bonding to the sleeper. Bond slip between concrete and reinforcement is ignored since global analysis is required (A. Remennikov and S. Kaewunruen 2007, Li, 2012). The material inputs have been given in the previous chapter 3.



a) Three dimensional model of sleeper



b) Connectivity between solid and bar elements

Figure 4.2.1 Finite element mesh of railway prestressed concrete sleeper

4.3. Support conditions

In reality, the sleeper-ballast interaction is neither uniformly distributed nor fixed. Usually the real contact situation between sleeper and ballast is too difficult to be known (Li, 2012). The arrangements of the contact patterns can take various shapes. From previous studies, four different possible situations of contact boundary conditions are of practical concern in real railway track problems (Li 2012, Rezaei 2010, Kaewunruen and Remennikov, 2010). In particular, these defects (voids or hanging sleepers) can be detected through vibration tests (Rezaei, 2010). In this study, the internal pattern of sleeper's behaviour under those typical support conditions is studied and the most severe case will be identified. The specified patterns are shown in Figure 4.3.1.

a. Center void: The void starts from the center of the sleeper and after that the void grows out to the sides symmetrically. This is desirable and is aimed at during construction and maintenance e.g. tamping (Li 2012, Rezaei 2010). In the first case the sleeper is fully supported and then void starts to grow under the sleeper in mid-section. To evaluate the alteration of the behaviour of the sleeper when the voided part increases, the ratio of the central void length to the sleeper length is used as a non-dimensional variable. The relation below presents this value:

$$\alpha_c = \frac{L_c}{L}$$

b. Center void with low stiffness: Over a period of time, because of repeated loads, vibration and crushing of ballast, the ballast will gradually compact, moving away from the areas of greatest concentration (Rezaei, 2010). The sleeper, therefore, settles slightly into the ballast, allowing the center portion of the sleeper to pick up a portion of the load, thereby reducing the amount of load carried by the sleeper ends (Rezaei, 2010). And this will continue until eventually the condition of full support over the entire length of the sleeper is approached. To evaluate the alteration of the behaviour of the sleeper as the end support reduces (from W to W/2), the ratio of the central void-with low stiffness length to the sleeper length is used as a non-dimensional variable.

$$\alpha_{cs} = \frac{L_{cs}}{L}$$

c. **Single hanging:** In this type, the void is assumed to form from one of the ends of the sleeper and it expands incrementally to the other end (Rezaei 2010, Kaewunruen and Remennikov, 2010). Also in this situation the initial case is the fully supported sleeper. As mentioned in the preceding situation, for assessing the variation of the behaviour of the sleeper during this imperfection a non-dimensional parameter is used which is the ratio of the single-side void length to the length of the model. This relation can be written as follows:

$$\alpha_s = \frac{L_s}{L}$$

d. **Double-side hanging:** another contact situation that can occur is double-side hanging sleeper. An unsupported part starts to grow from each end of the sleeper, and the contact remains only in the middle segment of the sleeper (Li 2012, Rezaei 2010). This situation is named the 'double hanging' sleeper. The fourth part of Figure 4.3.1 displays this situation. It is worth highlighting that, in this study, only symmetrical voids at the ends are considered. The Parameter describing in this case is:

$$\alpha_d = \frac{L_d}{L}$$

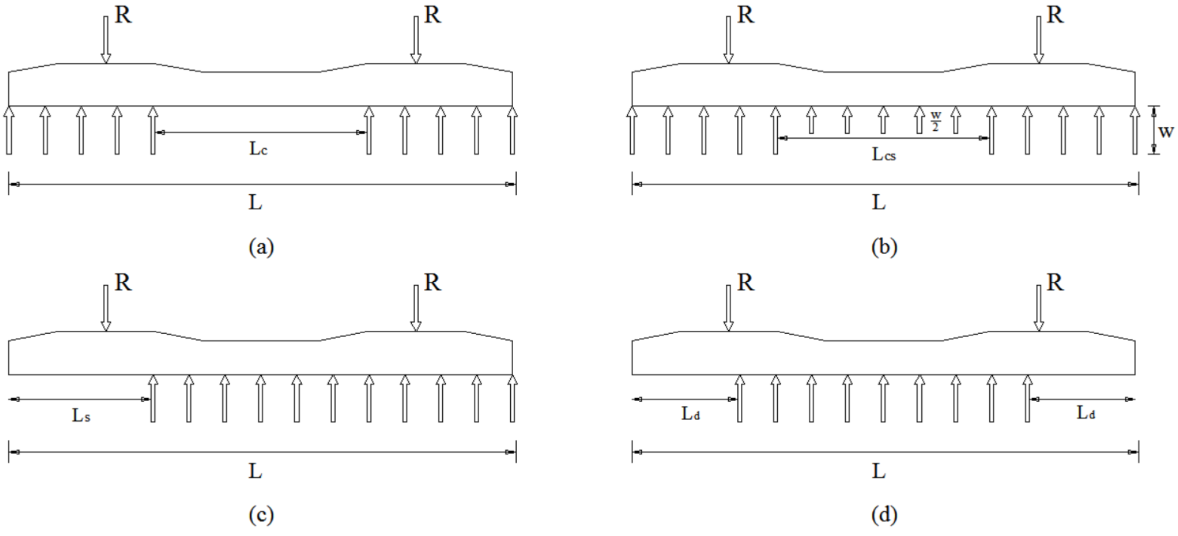


Figure 4.3.1 Sleeper/ballast contact patterns, (a) central void, (b) Center void with low stiffness, (c) single hanging, and (d) double hanging

4.4. Validation of Finite element results

For the validation of the finite element the author used the Australian standard (AS1085.14, 2003) as a reference. The support ballast pressure values are calculated as explained in the previous chapter section 3.5. Also, the rail seat load is used as calculated in section 3.4.1.

4.4.1. Flexural requirements for prestressed concrete sleeper designs according to AS1085.14 standard

Table 4.4.1.1 Design maximum positive and negative bending moments (AS 1085.14)

Length of ballast support beneath each rail seat (a) (m)	Ballast pressure (KPa)	Maximum bending moment (KN-m)	Remark
$a = L - g$	571.43	20.0	rail seat (+ve)
-	-	14.0	rail seat (-ve)
$a = 0.9(L - g)$	634.92	7.20	center (+ve)
Full length of sleeper	457.14	20.0	center (-ve)

4.4.2. Flexural results for prestressed concrete sleeper form finite element analysis

Table 4.4.2.1 Finite element results of maximum positive and negative bending moments

Parts	Area (m^2)	Centroid form bottom (m)	Moment of inertia (m^4)	Section modulus (m^3)		Stress (N/m^2)	
				Z_b	Z_t	+ve	-ve
Rail seat	0.046	0.0928	1.51E-04	1.63E-03	1.41E-03	1.05E+07	6.89E+06
Center	0.0384	0.0756	8.12E-05	1.07E-03	9.61E-04	1.92E+06	1.15E+07

Finite element bending moment results (KN-m)			
Rail seat		Center	
M (+ve)	M (-ve)	M (+ve)	M (-ve)
17.13	9.70	1.84	12.30

4.4.3. Validation of finite element result

Table 4.4.3.1 Comparison of finite element results and standard design bending moments

Location	Rail seat		Center	
	M (+ve)	M (-ve)	M (+ve)	M (-ve)
Finite element bending moment results (KN-m)	17.13	9.70	1.89	12.30
AS1085.14 bending moment results (KN-m)	20.0	14.0	7.20	20.0

Many researchers use Ansys as a finite element tool in different cases to study prestressed concrete sleeper (Kaewunruen and Remennikov 2010, D. Kishore Kumar and K. Sambasivarao 2014, S. Kaewunruen 2007, et.al). The standard (AS1085.14) formulas for the design maximum moment computation are based on real case experimental results. As shown in table 4.4.3 the maximum positive and negative moments obtained from the finite element analysis for both rail seat and center sections of sleeper is less than the moments obtained based on the standard (AS1085.14), hence it is acceptable. Therefore, the obtained values shows that the quality of FE model is good and further analysis using this FE-model can be processed and results are displayed on the next chapter 5.

5. RESULTS AND DISCUSSION

In this study the internal behaviour (bending stress and vertcal displacement) of a fully or partly supported concrete railway sleepers are studied. This is important to know how sensitive the sleeper is to the variation of the length of contact zone between the sleeper and the ballast. The analysis was perfomed for each sleeper as the percentage of sleeper/ballast contact zone would vary from full contact (α =0.00), 90% (α =0.10), 80% (α =0.20), 70% (α =0.30), and 60% (α =0.40), and so on. Thus, evaluation is done how the flexural responses of sleeper are influenced by the boundary conditions (*i.e.* the ballast support). The discussion is based on the stress values at two points i.e. at rail seat and center of sleeper. The value of pressure distribution beneath sleeper is dependent to the changes of length of contact between ballast and sleeper. Hence, the effect of variation of support pressure along the length of sleeper at both rail seat and center section of sleeper will be discussed.

The contact support pressure values beneath each sleeper is computed using SAFE V12 software and the maximum limit for the pressure value beneath sleeper is 750 KPa as specified in the standard (AS1085.14, 2003, Part 4.2.4). The pressure results obtained from SAFE are different at different nodes. For this thesis the maximum pressure values are taken and compared with the maximum pressure recommended by Australian standard.

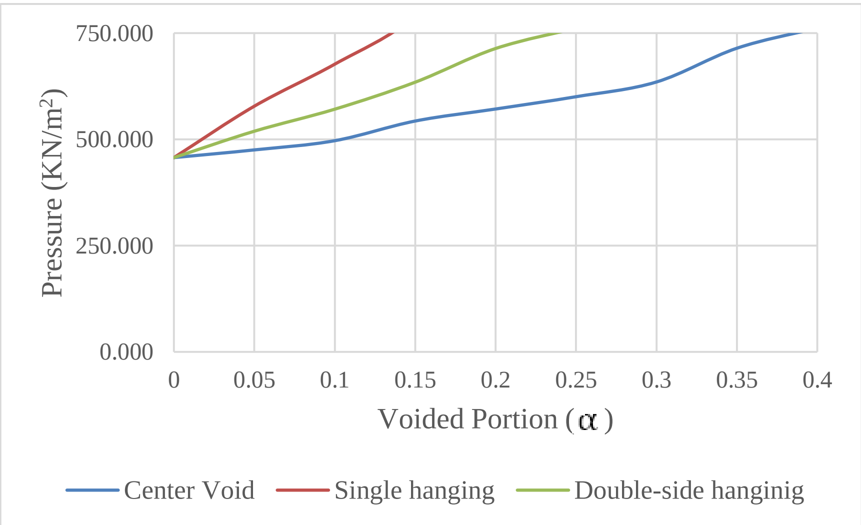


Figure 5.1 Variation of contact pressure with respect to voided portion (SAFE results)

Regarding the mechanical properties of the sleeper, numerical values applied in the calculations presented here are stated in chapter 3. Other parameters, such as geometry, boundary conditions and so on, are given in the modelling section. Results obtained for concrete sleepers for all contact patterns discussed earlier are presented separately in the following sections.

5.1. Computer Simulation Results for different voided support conditions

The results from the simulations are presented in the following sub-chapters one by one.

5.1.1. Center void

Based on Standards Australia, this is where the center portion of sleeper is void (no contact with ballast) and the support is distributed below each rail seat for the length of contact limited to 0.9a and a (where 'a' is the difference between length of sleeper and the distance between rail centers), depending on it is used to calculate the flexural requirements of center and rail seat sections of sleeper. This is desirable and is aimed at during construction and maintenance e.g. tamping. After the track have been in service the contact pressure distribution underneath the sleeper tends towards a uniform pressure distribution, by increasing the length of contact and reducing the capacity of load carried by the ballast at the sleeper ends. The ballast support increment is presented in terms of the variation of percent changes from the initial ballast full contact for different voided portions as shown in Table 5.1.1.1.

Table 5.1.1.1 Variations of support length, voided portion and ballast pressure values along the length of sleeper for center void with no support

Voided portion ($\alpha_c = \frac{L_c}{L}$)	Support length in %	Ballast pressure (KPa)
0.0	Full contact	457.186
0.1	90%	496.947
0.2	80%	571.988
0.3	70%	635.013
0.36	64%	714.402
0.4	60%	762.039 > 750

As mentioned before, the analysis is done based on the limit that the standard maximum ballast pressure beneath sleeper should not exceed 750 KPa. Starting with the full case scenario and then during tamping operation the ballast reaction gets distributed symmetrically below each rail seat leaving the center region with little to no support. For center void it is assumed that there is no contact in the mid-section. The change in voided portion increases until the reduction in contact length results high stress than the standard. However, it is found that the ballast pressure is greater than the standard limit for the voided sleeper occurs at about 62% sleeper/ballast contact. And therefore, this was the reason why the change of voided portion were limited up to Lc/L = 0.40. Thus, at the voided portion ratio of 0.40 the analysis is done with the stress value given by the standard. The results in figure 5.1.1.1 and 5.1.1.2 below shows the bending stress and vertical displacement variation along the length of sleeper.

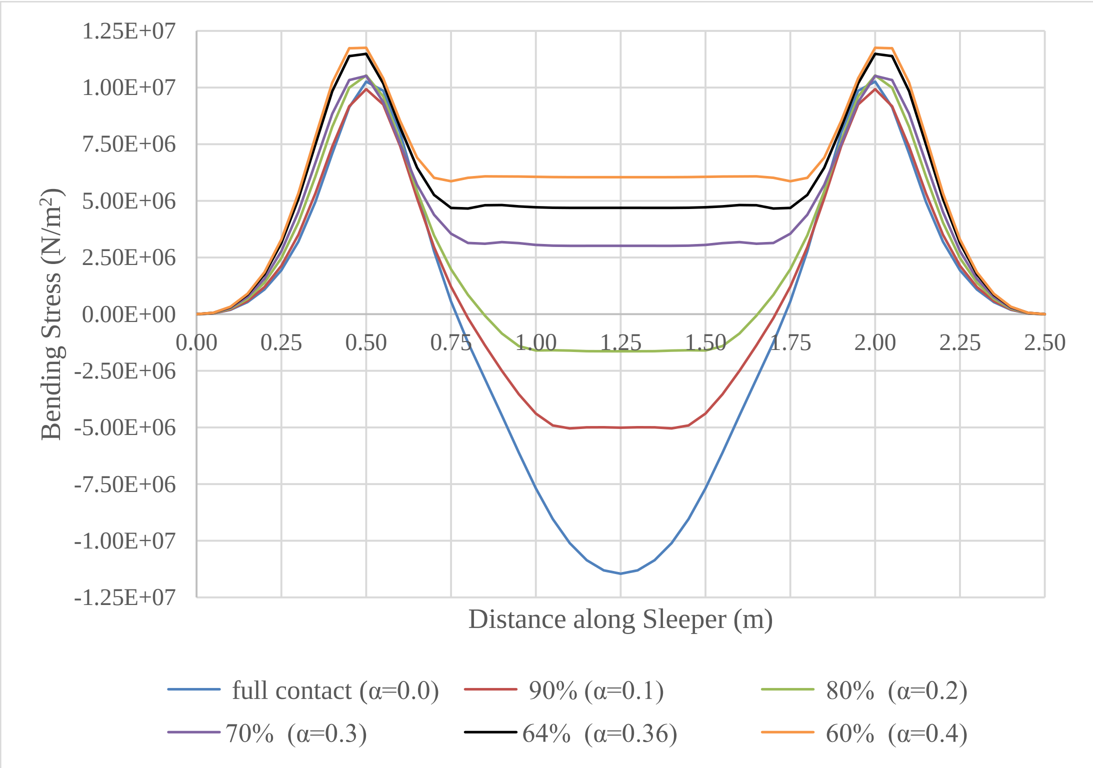


Figure 5.1.1.1 Bending stress along the sleeper

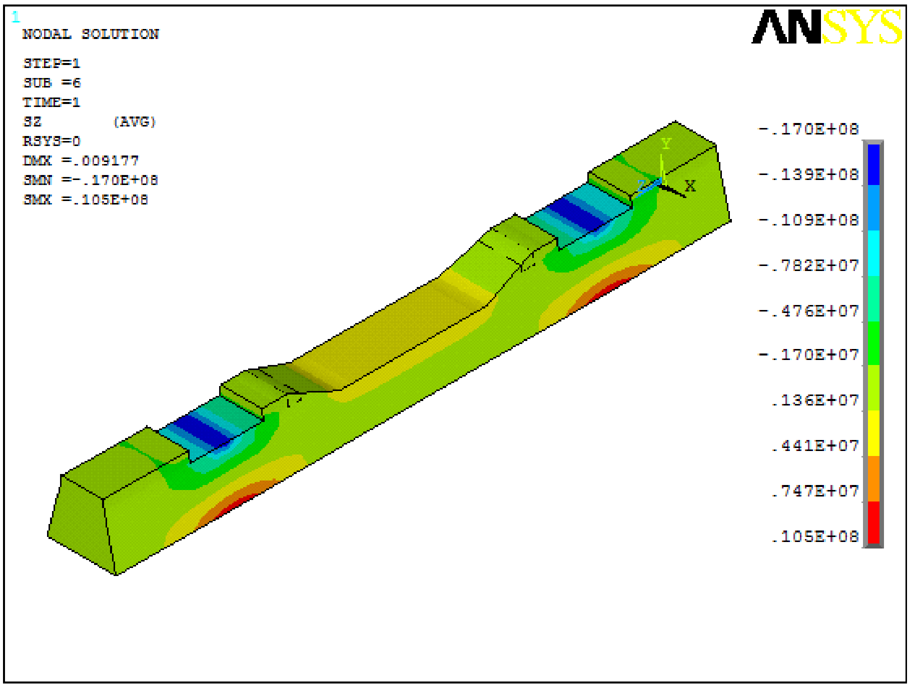


Figure 5.1.1.2 Contour plot of bending stress along the sleeper (for 80% contact)

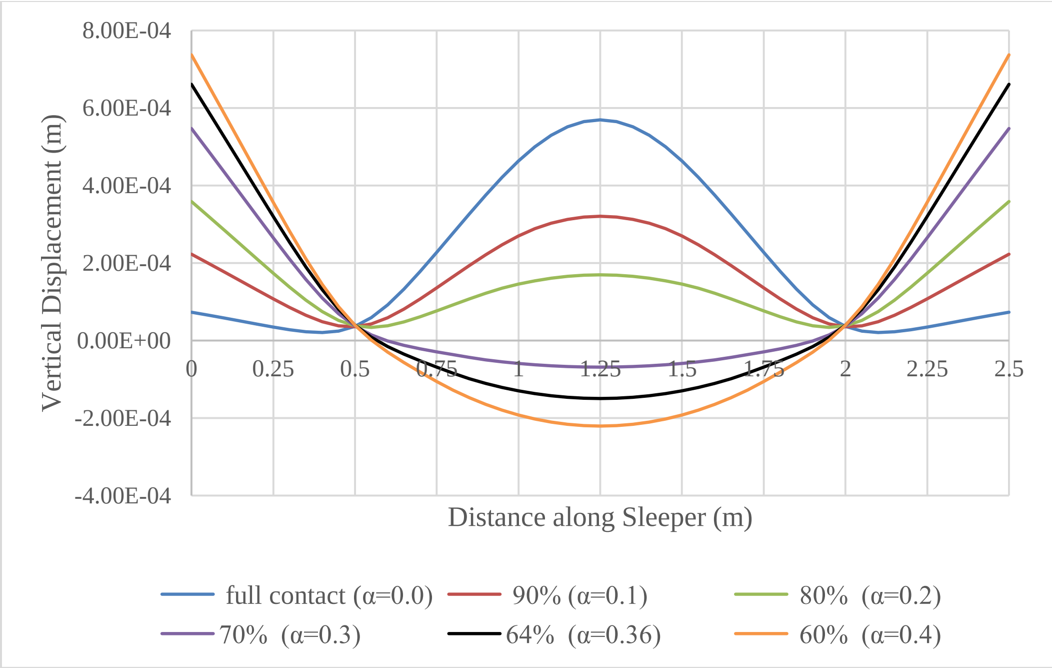


Figure 5.1.1.3 Vertical displacement along the sleeper

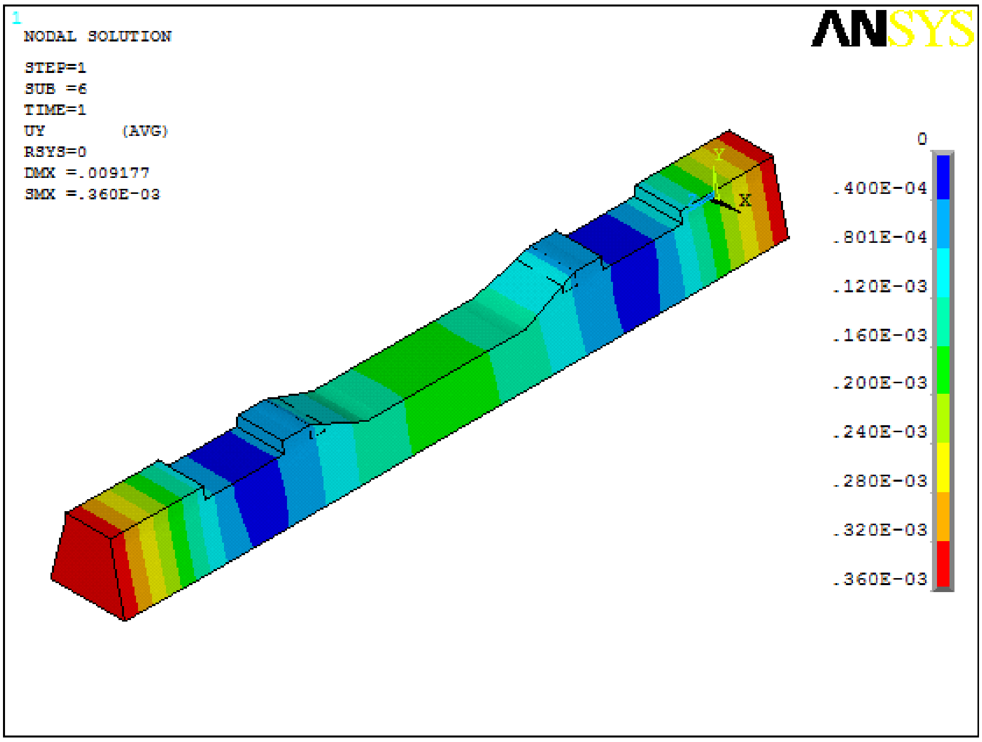


Figure 5.1.1.4 Contour plot of vertical displacement along the sleeper (for 80% contact)

5.1.2. Center void with low stiffness

In this case, it is assumed that the mid-section of sleeper gets little support. This setup is used in different country standards (like the AREMA, AS and CHINA) to compute the maximum negative bending moment at the center. However, the value of ballast reaction in the mid-section is different in these standards, where AREMA specifies a factor of 0.61 times the end support at center, AS considers 50% and the Chinese specifies 75% of end supports. Similarly the effective support length below each rail seat and the contact length of the center portion vary in the standards. In this study, it is assumed that the ballast reaction at voided portion of sleeper is 50% of the end supports as per the AS1085.14, 2003. Hence, to evaluate the alteration of the behavior of the sleeper along its length as the end support (W) reduces and the center portion of the sleeper picks up half (W/2) of the portion of the load carried by the end supports, the ratio of voided length to the sleeper length is used as a non-dimensional variable as defined in section 4.3. Thus, for instance, the ballast reaction at both rail seat and center sections of sleeper for the standard ballast support condition defined in Australia standard is computed as shown below and, the value of ballast pressure beneath the sleeper is computed as shown in the table 5.1.2.1.

$$w = \frac{R}{L-g} = \frac{160\ kN}{2.5-1.51} = 161.62\,\frac{kN}{m}, \qquad And \qquad \frac{w}{2} = 80.81\ kN/m$$

Table 5.1.2.1 Variations of support length, voided portion and ballast pressure values along the length of sleeper for center void with little support

Voided portion ($\alpha_{cs} = \frac{L_{cs}}{L}$)	Support length in %	Ballast pressure (KPa)	
		P	P/2
0.0	Full contact	457.186	228.593
0.1	90%	496.947	248.474
0.2	80%	571.988	285.994
0.3	70%	635.013	317.507
0.36	64%	714.402	357.201
0.4	60%	762.039 > 750	375

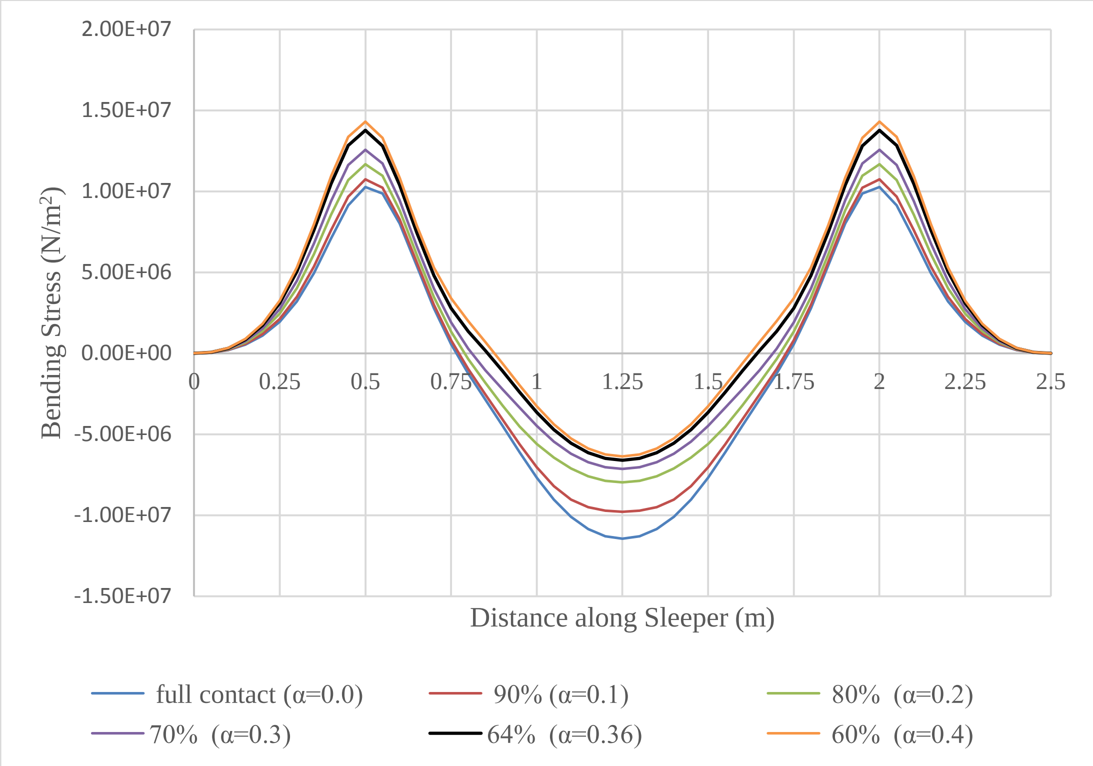


Figure 5.1.2.1 Bending stress along the sleeper

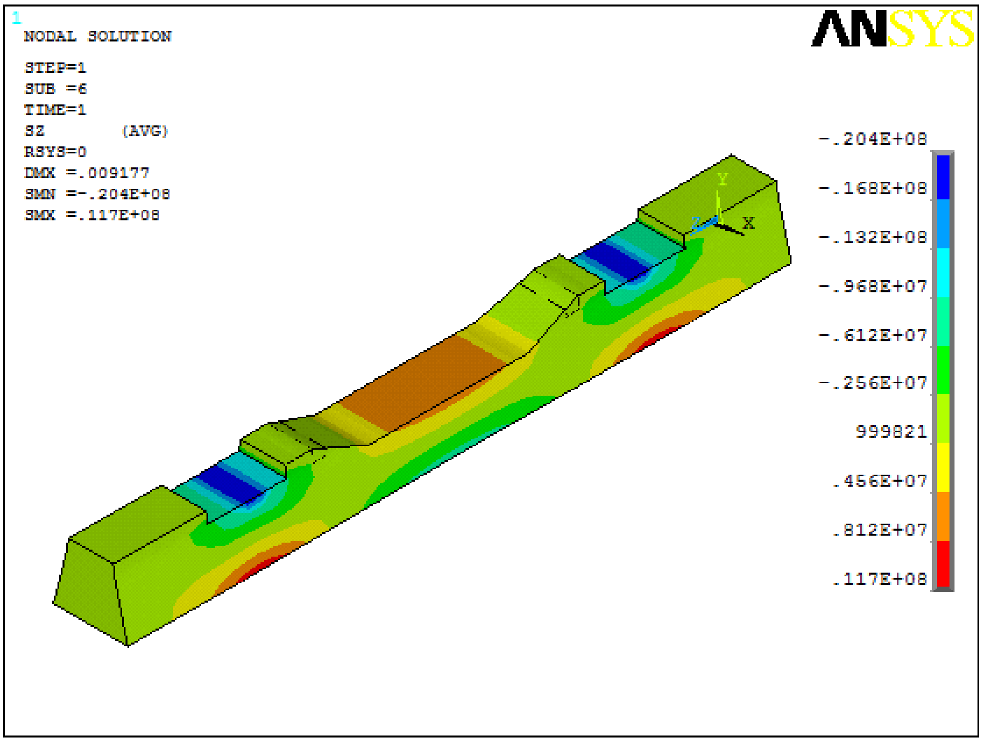


Figure 5.1.2.2 Contour plot of bending stress along the sleeper (for 80% contact)

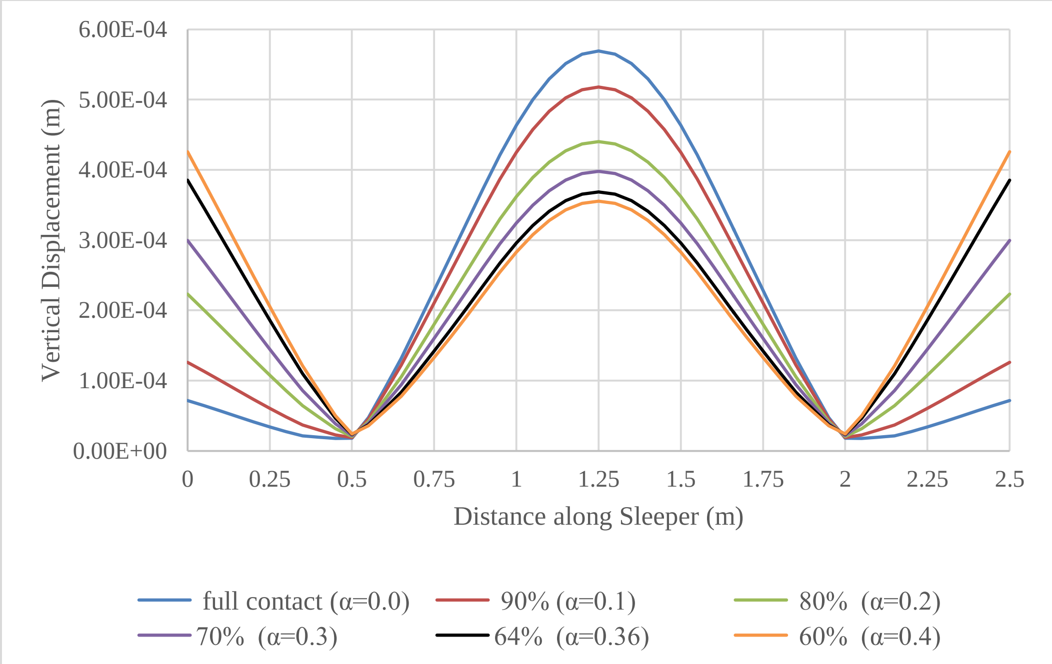


Figure 5.1.2.3 Vertical displacement along the sleeper

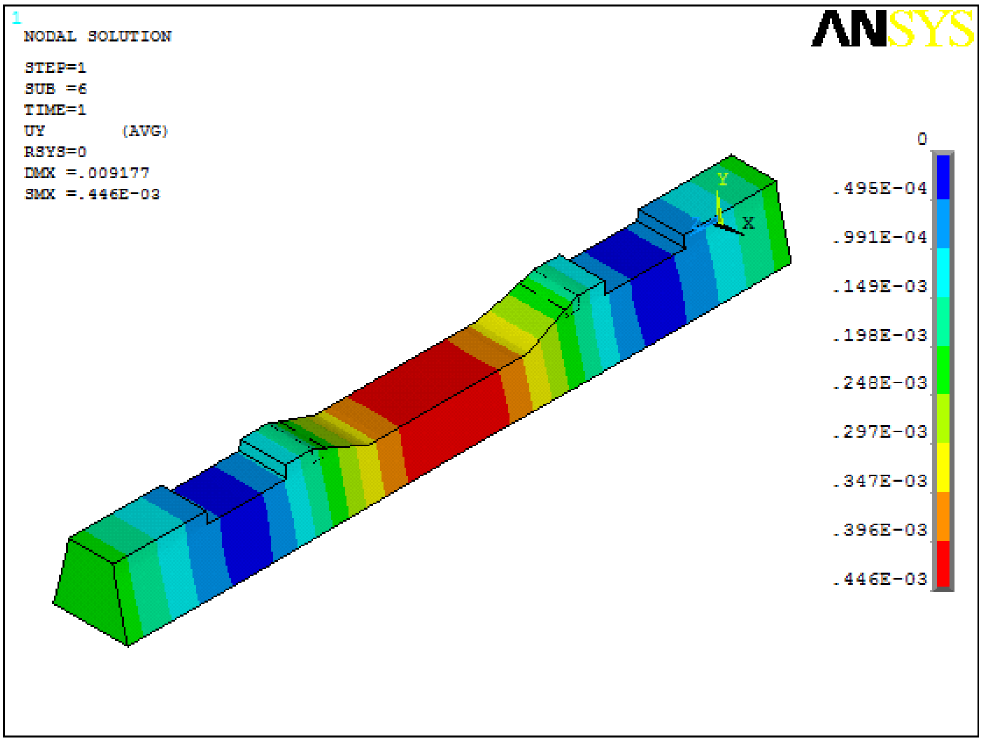


Figure 5.1.2.4 Contour plot of vertical displacement along the sleeper (for 80% contact)

5.1.3. Single hanging

Here, 'single hanging' means that one part at the end of the sleeper is hanging. In the beginning the sleeper is fully supported by the ballast along its full length. Then the contact length between sleeper and ballast diminishes by steps of five percent of the sleeper length until the support pressure value is within the standard limit.

This type of imperfection can be detected through inspection, by watching vertical movement of the sleeper under traffic. Such imperfection affects the sleepers nearby and also causes problems to other track components like loose or missing fittings, missing rail pads, rail weld failure, failure of joints in a jointed track etc., (Cope and Ellis 2001). It would be interesting to study the effect of single hanging sleeper on the response from the track; however this is beyond the scope of this thesis. Thus it is important to keep the rail bearing portion of each sleeper well packed and free from voids.

The ballast reaction in this case as shown in the table 5.1.3.1, is increased as the voided portion increases or with reducing the contact length. Here five cases of voided/ hanging support conditions are investigated. The results of the finite element analysis for bending stress and vertical displacement along sleeper for varying voided portions are shown in the figures below.

Table 5.1.3.1 Variations of support length, voided portion and ballast pressure values along the length of sleeper

Voided portion ($\alpha_s = \frac{L_s}{L}$)	Support length in %	Ballast pressure (KPa)
0.0	Full contact	457.41
0.05	95%	578.45
0.1	90%	677.30
0.15	85%	791.14 > 750
0.2	80%	999.80 > 750

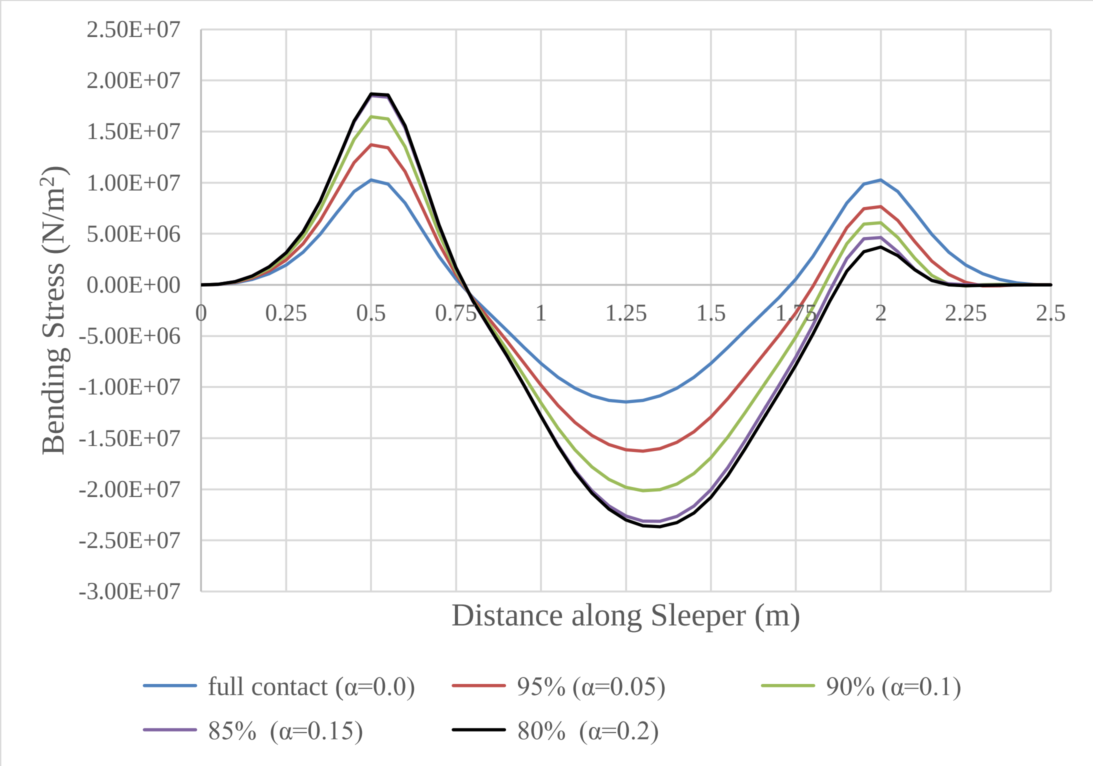


Figure 5.1.3.1 Bending stress along the sleeper

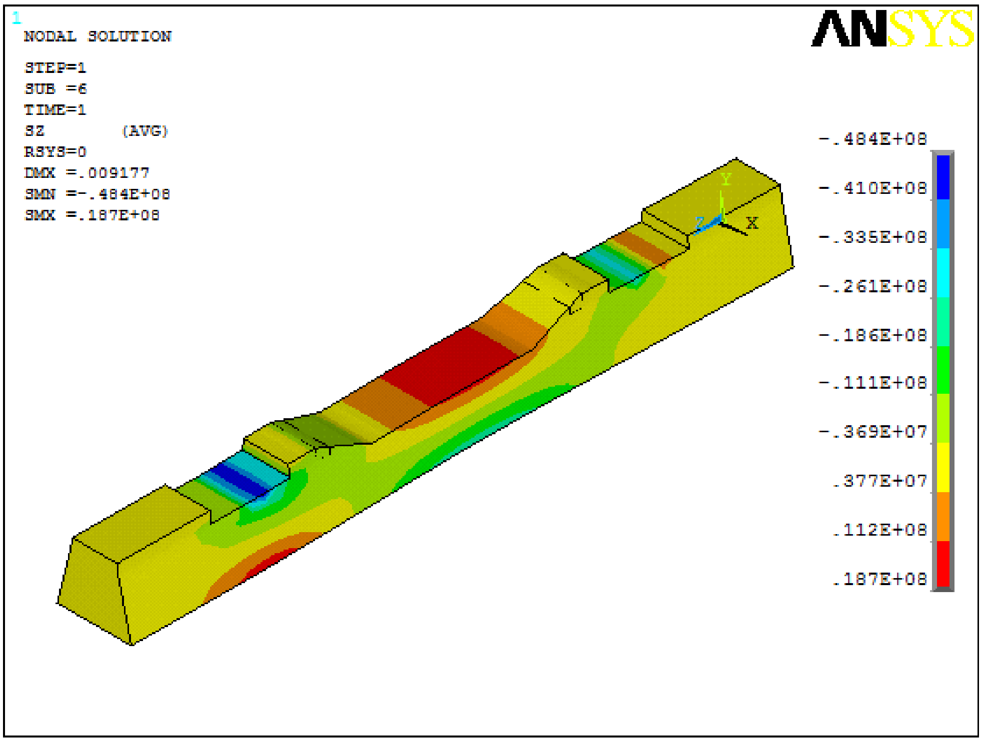


Figure 5.1.3.2 Contour plot of bending stress along the sleeper (for 80% contact)

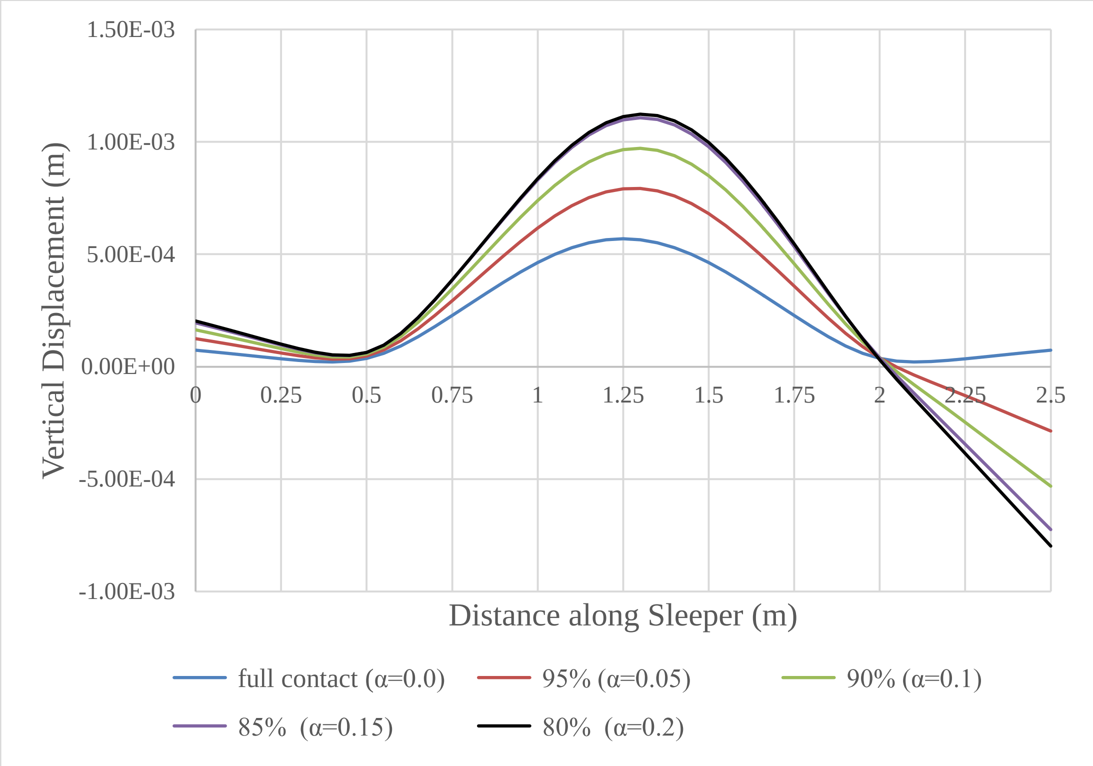


Figure 5.1.3.3 Vertical displacement along the sleeper

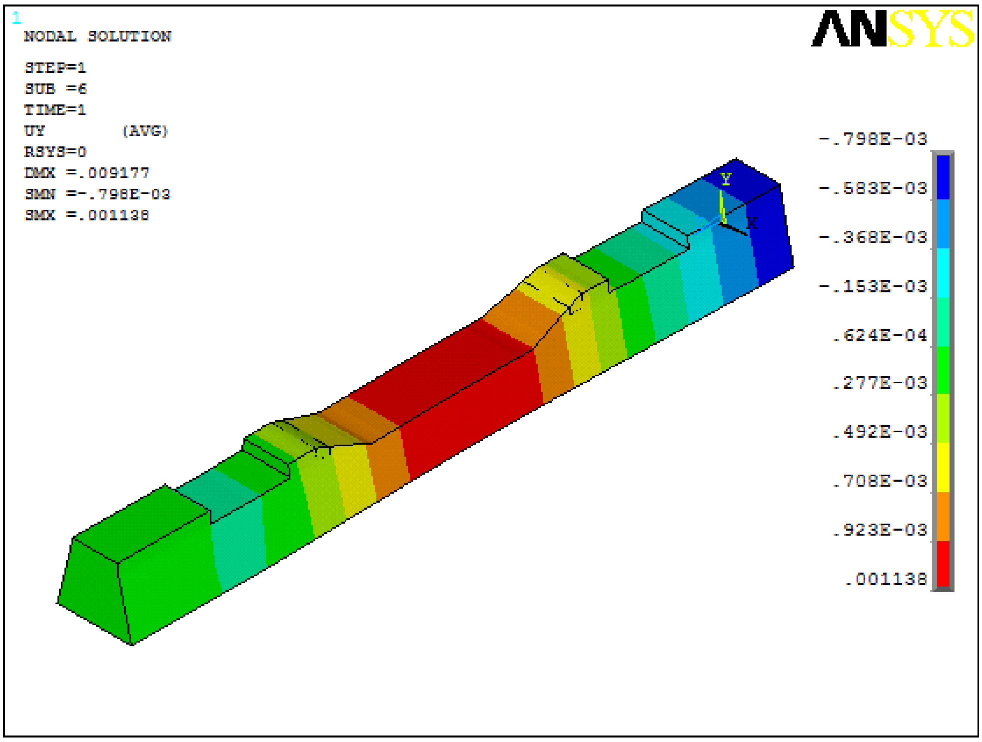


Figure 5.1.3.4 Contour plot of vertical displacement along the sleeper (for 80% contact)

5.1.4. Double-side hanging:

Double hanging is the case where void starts from both ends of the sleeper and increases towards the center. This scenario leads to sleeper center support condition when maintenance is neglected. And when loaded, large negative moments occur at the sleeper center, resulting in cracking and sleeper failure as the flexural capacity is exceeded. This type of failure is known as "center binding" (Lutch R. H., 2009).

In this study, this is a symmetric case, where there is a constant unsupported part at both ends and then the voided part starts to grow and extends through the sleeper. For the given length of support the value of ballast pressure is computed as shown in the table 5.1.4.1. The changes in contact length below sleeper to determine the support pressure goes until the standard ballast pressure limit. Hence, the flexural stresses and vertical displacements for various voided length are shown in the figure shown below.

Table 5.1.4.1 Variations of support length, voided portion and ballast pressure values along the length of sleeper

Voided portion both sides ($\alpha_d = \frac{L_d}{L}$)	Support length in %	Ballast pressure (KPa)
0.0	Full contact	457.41
0.1	80%	519.50
0.2	60%	571.44
0.3	40%	634.93
0.4	20%	714.30
0.5	Free	761.92 > 750

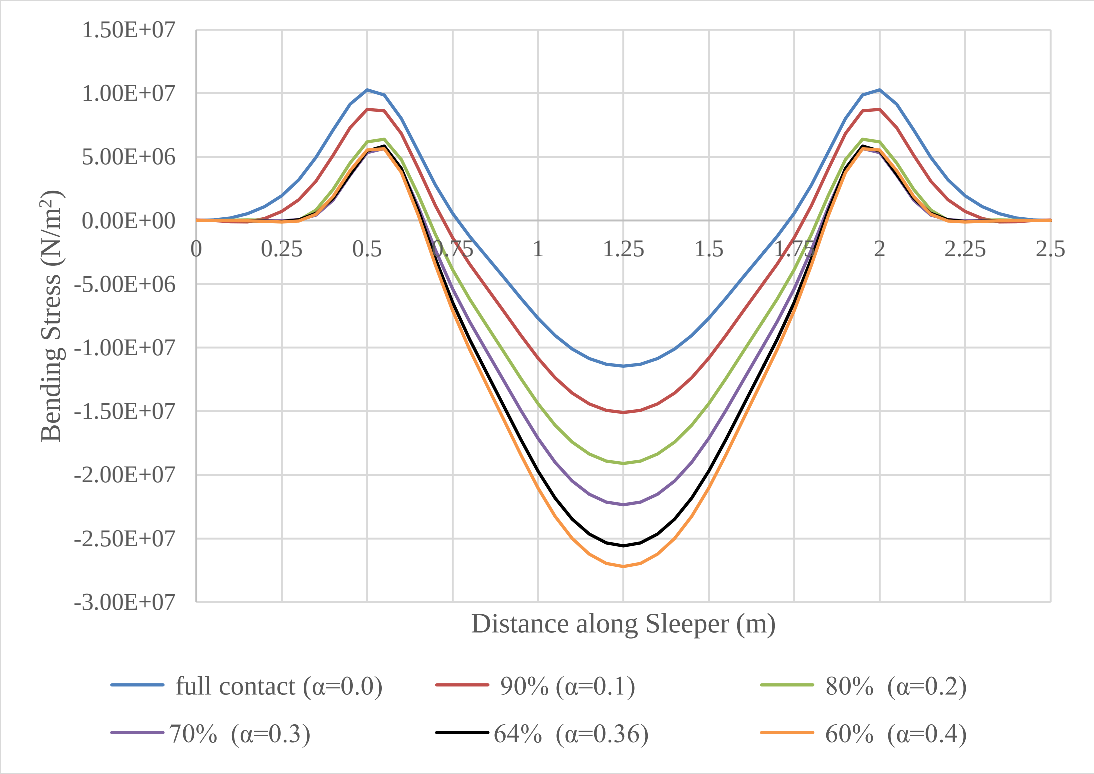


Figure 5.1.4.1 Bending stress along the sleeper

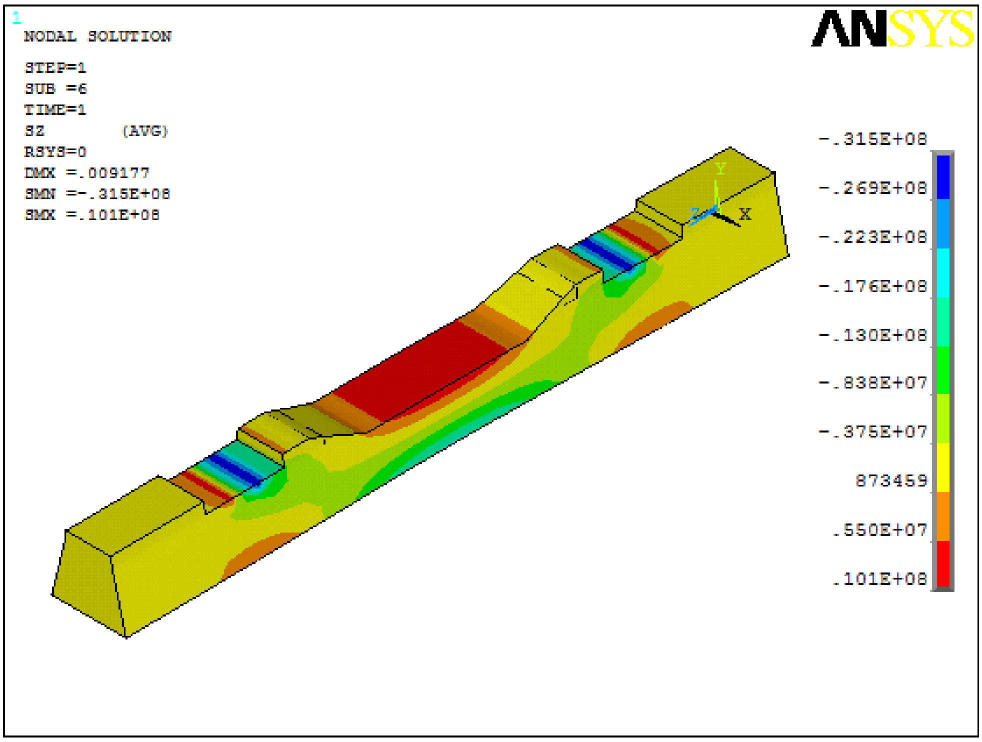


Figure 5.1.4.2 Contour plot of bending stress along the sleeper (for 80% contact)

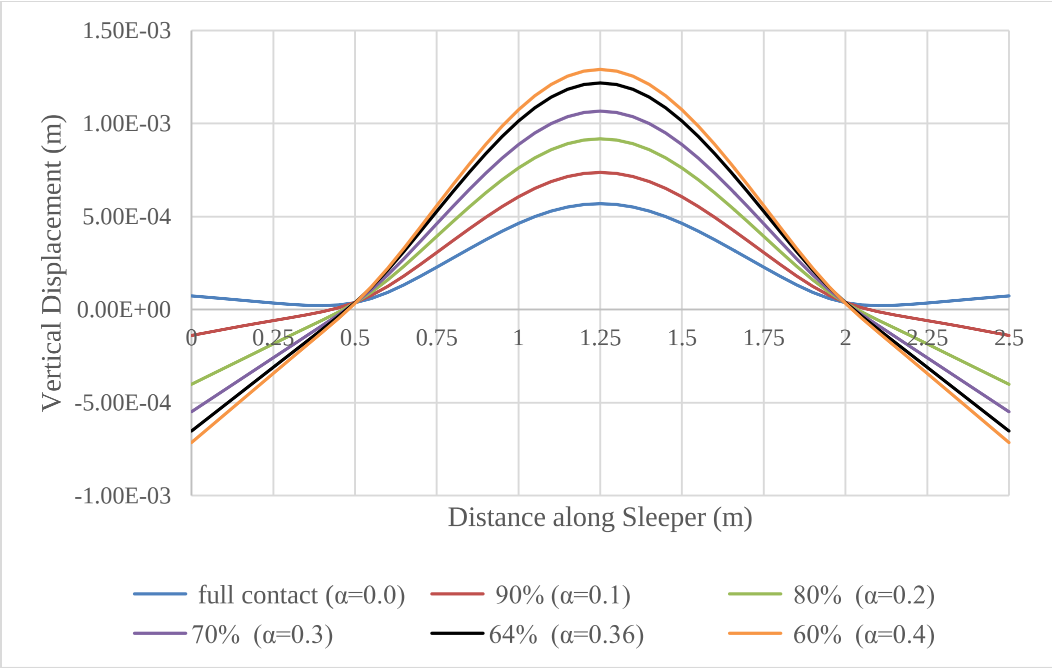


Figure 5.1.4.3 Vertical displacement along the sleeper

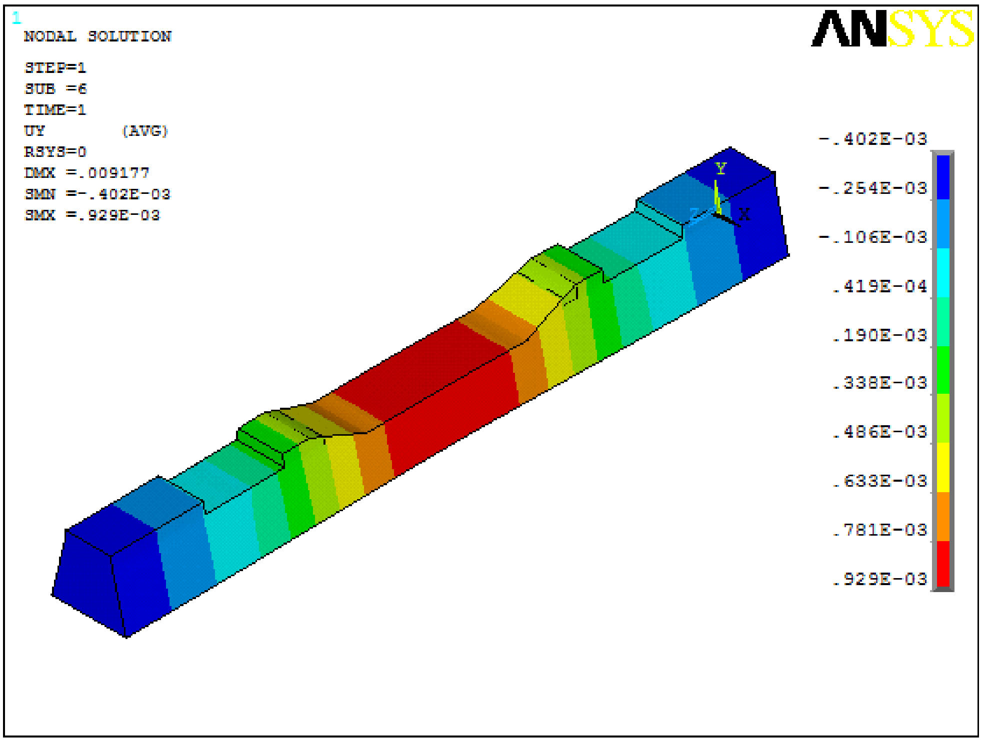


Figure 5.1.4.4 Contour plot of vertical displacement along the sleeper (for 80% contact)

5.2. Discussion

The deformation and stress diagrams of the concrete sleeper due to varying support conditions are explained in the above sections. From the above section, one can see how the variation in the ballast reaction affects the rail seat and center bending moments.

For the center void case, the results of the bending stress and vertical displacement variations for a sleeper with different support pressure values going from central void to full support is displayed in Figure 5.1.1.1. From the figure one can see that the moments have opposite sign in mid-section of sleeper as the unsupported section increases at center. This means that, the sleeper under ballast support where voided portion is less than 0.2, shows the center section is under compression; however, for the voided portions greater than 0.2, the sleeper center section is subjected to tension. Additionally, the bending stress at rail seat section of sleeper increases as the effect of the voided part increases. Hence, both rail seat and center section of sleeper bending stresses are sensitive to the changes in ballast reaction.

Likewise, for the second case with low stiffness in the center, the bending stress variation for a sleeper with different support pressure values are presented in Figure 5.1.2.1. This scenario is used to compute the maximum negative center moment according to AS1085.14 and AREMA recommendations. As shown in the figure, the rail seat moment is increasing with increasing effect of voided portion along sleeper. As the end support pressure increases with reducing length of contact the rail seat section is critical with increasing flexural stress. However, here the center section is under compression, hence looks good. This is due to that the center section picks some load (50% of end support) and is in contact with the sleeper. Also, in this case the lowest contact pressure gives the largest vertical displacement at center. But side displacements are quite large as the support stiffness increases at the ends of sleeper.

For the single hanging case, the voiding starts from one end of the sleeper and increases (bed stiffness is removed by start at one end). Thus, in figure 5.1.3.1 the variation in bending stress along the length of sleeper is shown. As the voided portion increases the center section of sleeper is highly stressed. Also, the stress at the voided part from one end of sleeper is high. This case is critical for both rail seat and center sections of sleeper. The bending moment at a voided portion (Lv/L) of about 0.1 of both rail seat (is 26.77 KNm) and mid-section (-21.28KNm), is greater than the design moments form the standard. This shows that failure may be happen with increasing voided portion. Likewise, the vertical displacement along sleeper is increasing with increasing voided portion as shown in figure 5.3.1.3.

The double hanging case is symmetric as all other cases in that, there is a constant unsupported part at both ends and extends towards the center. The results for the variation in bending stress and vertical displacements are shown in figure 5.1.4.1. From the figure it can be seen that the stress at center is very critical, and increases as the voided part at both ends increases. The flexure at a voided portion of Lv/L=0.2 is, -20.52 KNm, which is greater than the standard design bending moment at mid-section. This support scenario can result flexural cracking with increasing voided portion.

To summarize, it is seen that the rail seat bending moment is always positive and is very sensitive to changes in ballast reaction close to the sleeper ends (i.e. the distance between the rail seat and the end of the sleeper). This is seen in the very high bending stress when the ballast reaction is concentrated at these locations. As discussed earlier, the magnitude of the center bending moment can be very high for either positive or negative bending. The center experiences its maximum positive bending moments when the ballast reaction is concentrated outside of the rail center-to-center spacing. It experiences its maximum negative bending moments when the ballast reaction is concentrated inside of the rail center-to-center spacing. One of the simplest ways to see the sensitivity of ballast reaction is to compare the difference between moments found when the end/center parts of sleeper takes 0% and 100% of the ballast reaction. For the rail seat bending moment, the most sensitive part is found to be the end supports, where the difference between the moment when 100% of the ballast reaction occurs at the end support (19.19 kNm) and when 0% of the ballast reaction occurs in the end (8.20 kNm) is 11.00 kNm. Since the end support is the free end located the greatest distance from the rail seat load, it has the largest moment arm and the greatest effect on bending at the rail seat. Continuing this method of comparison to the center bending moment, the mid-section support has the greatest sensitivity, with a difference in the 0 and 100% reactions of -34.43 kNm. As the reaction moves closer to the sleeper center, the distance between the rail seat load and the centroid of the reaction increases, causing greater magnitudes of negative bending.

6. CONCLUSION AND RECOMMENDATION

6.1. Conclusion

This paper numerically investigates the static effect of different ballast support conditions on the behavior of railway prestressed concrete sleeper. A finite element model of concrete sleeper was developed using ANSYS. The model is validated using the Australian Standard (AS1085.14). The effects of the variation of ballast reaction distribution along the length on the bending and displacement of the railway sleeper were studied. The SAFE V12 software was employed to handle sleeper/ballast contact pressure computation varying contact length.

Analysis of the structural behaviour of prestressed concrete railway sleeper subjected to varying sleeper-ballast interaction clearly shows that the performance and bearing capacity of a sleeper is dependent not only on traffic loading, material characteristics, sleeper geometry, but also upon ballast support condition underneath the sleeper. The gradual change of the ballast distribution takes place with time due to gradual wear and tear of the ballast and it is impossible to forecast the accurate support distribution. Therefore, it is a concern for sleeper designers and manufacturers to investigate the influence of variation of ballast stiffness and distribution.

The numerical study presented shows the sensitivity of both the rail seat and center bending moment as a function of changing support conditions. According to the simulations conducted in this study, the ballast support and ballast support variations along a sleeper, have a high impact on the bending moments and displacements of the sleeper. The bending moments due to the applied rail seat load at the sleeper rail seat/center can be exceeded from design bending moments under small variation in distribution of the ballast reaction. In this thesis, the possible unfavorable ballast support cases are presented. For instance, very soft support underneath the rail seat and large variation of ballast stiffness along the length of sleeper might cause severe cracking problems especially at the top surface in the middle of the sleeper. This demonstrates that frequent tamping to keep the ballast reaction concentrated under the rail seats can prevent very high center negative bending moments that cause cracking. It can, therefore, be concluded that ballast support is an important parameter concerning sleeper durability and improving track performance.

Analyzing the case of center void from a design perspective for the first three plots ($\alpha \leq 0.2$), under rail seat there is positive moment where we place reinforcement in the lower part of sleeper and in the center there is negative moment where we place reinforcement in the upper part of sleeper. But for cases below 80% contact, the moment has opposite sign in the center where the sleeper might have tensile problems because it may not have enough tensile capacity to bear moment.

The bending moment resultants are also affected significantly by the uneven ballast stiffness distribution, particularly in the situation of lacking support (i.e. single hanging and double hanging). In such a case, the nominal bending moment at mid-span could be larger than the standard design moment up to two times. Therefore, understanding bending moment distribution due to varying ballast pressure distribution underneath sleeper will advance the awareness of track engineers for the proper maintenance of the ballast condition.

6.2. Recommendations

Generally, as it is explained in research objective, the numerical simulations are generated so as to investigate the quasi-static bending moment and vertical displacement results of the railway concrete sleeper under varying ballast support reactions against the current design methodology. It is concluded that variation in support conditions influence the risk of failure of sleepers and damage on track. Hence, the following recommendations can be made from the research based on the FEA results:

1. The most severe case is the single hanging which is critical to both rail seat and mid-section bending moments/displacements of sleeper. Therefore, this imperfection which has the worst moment distribution and quite large displacement need to be avoided through regulations, maintenance and safely inspection.

2. The double-side hanging results in large negative center bending moment which might cause severe cracking problems especially at the top surface in the middle of the sleeper. Hence, to prevent center-binding regular maintenance of ballast must be performed to avoid deterioration of the material leading to unsuitable sleeper support conditions.

3. For center void, the allowable contact length of support is 80%, where voided portion is limited to Lv/L=0.2. This contact support gives the sleepers ability to respond well and resist the loads coming from the running vehicle. However, for a voided portion beyond 0.2, the mid-section moment has opposite sign which may cause tensile problem to the sleeper. Thus, it is recommended that this has to be kept through regular maintenance.

4. Finally, the performance and bearing capacity of a sleeper is affected by varying sleeper-ballast interaction, particularly by the voids in the ballast beneath the sleeper. Therefore, it is recommended that voids should be included in the maintenance check list. The track should also be surveyed to look for low spots on track and sections with loose sleepers. The voids beneath sleepers should be fixed by raising sleepers to required level (using a jack) and packing the ballast fills the void below the sleeper. Hence, it is important that such maintenance is performed regularly since damage to the track that has become too severe requires larger and more expensive actions.

6.3. Future research

Future research could be done on studying various ballast stiffness and its distribution based on field measurements so that the sleeper under random ballast stiffness can be analyzed using numerical analysis. Besides, a more accurate result can be obtained to study the sleeper under varying ballast support using experimental approach. In addition it is also a good idea for engineers to design the optimized geometry of sleeper which can function well under varying support condition.

Reference

American Railway engineering and Maintenance-of-Way Association (AREMA), (2010) Manual for Railway Engineering Volume 1, Track, USA.

Standards Australia, (2003). AS1085.14-2003 Railway track material – part 14: Prestressed Concrete Sleepers, Standards Australia

Doyle N. F. (1980), Railway track design: a review of current practice, BHP Melbourne Research Laboratories, Australian Government Publishing Service, Canberra, 1980.

ORE (1965) Stress's in Rails, Question 017, Stresses in the Rails, the Ballast and the Formation Resulting from Traffic Loads, Report D71/RPI/E, Utrecht.

J. Taherinezhad, M. Sofi, P. A. Mendis and T. Ngo (2013), "A Review of Behaviour of Prestressed Concrete Sleepers" Department of Infrastructure Engineering the University of Melbourne, Melbourne, Australia, Special Issue: Electronic Journal of Structural Engineering 13(1) 2013

Hwang, C. L., Chen C. T., Lee L. S., Bui L. A. T., Hou B. S., and Hsieh H. Y.. (2011), "The Material and Mechanical Property of Heavy-Duty Prestressed Concrete Sleeper", Applied Mechanics and Materials 97-98 (September): 408–413.

Esveld, C., "Modern Railway Track", MRT-Productions, the Netherlands, 2001.

Dahlberg T. (2010) Railway Track Stiffness Variations – Consequences and Countermeasures. International Journal of Civil Engineering, vol. 8, no. 1

Siril Y. (2007), Automating condition monitoring of wooden railway sleepers, Engineering IT, October, 2007.

Sadeghi, J., and Youldashkhan, M., "Investigation on the accuracy of current practices in analysis of railway track sleepers", International Journal of Civil Engineering, Vol. 3, No. 1, pp. 9-15, 2005.

Sadeghi J., "Fundamentals of Analysis and Design of Railway Ballasted Track", IUST Publication Survive, Tehran, 2010.

J. Sadeghi and P. Barati, (2010), Evaluation of conventional methods in Analysis and Design of Railway Track System. International Journal of Civil Engineering. Vol. 8, No. 1, March 2010.

J.M. Sadeghi, (2008), Experimental evaluation of the accuracy of current practice in analysis and design of railway track sleepers. Published on the NRC Research Press Web site at cjce.nrc.ca on 19 August 2008.

K. Riessberger. (2012) Increasing Railway Stability with Support element, Civil Engineering and Architecture vol. 55 No. 2.

McHenry, Michael T., "Pressure measurement at the ballast-tie interface of railroad track using Matrix based tactile surface sensors" (2013). *Theses and Dissertations--Civil Engineering.* Paper 15. http://uknowledge.uky.edu/ce_etds/15

Wahid Ferdous, Allan Manalo, (2014), Failures of mainline railway sleepers and suggested remedies - Review of current practice, Centre of Excellence in Engineered Fibre Composites (CEEFC), University of Southern Queensland, Toowoomba 4350, Australia.

D. Kishore Kumar and K. Sambasivarao, Static and Dynamic Analysis of Railway Track Sleeper, International Journal of Engineering Research and General Science Volume 2, Issue 6, October-November, 2014 ISSN 2091-2730.

Li S. (2012) Railway Sleeper Modelling with Deterministic and Non-deterministic Support Conditions. Stockholm: Royal Institute of Technology (Master degree project, Report TSCMT 12-001, Department of Transport Science)

Lutch, Russell H., "Capacity optimization of a prestressed concrete railroad tie", Master's Thesis, Michigan Technological University, 2009.

Anthony J. Wolanski, B.S. (2004), Flexural behavior of reinforced and prestressed concrete beams using finite element analysis, (Master degree project, Marquette University, Milwaukee, Wisconsin, United States)

J. A. Zakeri and R. Abbasi (2012), Field investigation of variation of loading pattern of concrete sleeper due to ballast sandy contamination in sandy desert areas. Journal of Mechanical Science and Technology 26 (12) (2012) 3885~3892.

C. Calla, (2003), two layered ballast system for improved performance of railway track. PhD thesis for the Degree of Doctor of Philosophy in Coventry University, Coventry city, England.

Cope, D.L. Ellis J.B. (2001) British Railway Track vol.4: Plain Line Maintenance, Permanent Way Institution.

Lundqvist A. and Dahlberg T. (2005) Load impact on railway track due to unsupported sleepers. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, vol. 219, F2, pp. 67-77.

Rezaei E. (2010) Vibrations of partly supported concrete railway sleeper. Linköping: University of Linköping (Degree Project, Report LIU-IEI-TEK-A 10/00835-SE, Department Of Management and Engineering)

Rezaei E. and Dahlberg T. (2010) Dynamic behaviour of an in situ partially supported concrete railway sleeper. Proceedings of the Institution of Mechanical Engineer, Part F: Journal of Rail and Rapid Transit, vol. 225, pp. 501-508.

Kaewunruen S, Remennikov AM. Effect of improper ballast packing/tamping on dynamic behaviors of on-track railway concrete sleeper, International Journal of Structural Stability and Dynamics 7(1): (2007) 167-177.

Kaewunruen S, Remennikov AM, (2009), Influence of ballast conditions on flexural responses of railway concrete sleepers, Available at: http://works.bepress.com/sakdirat_kaewunruen/51

Kaewunruen S, Remennikov AM. Investigation of free vibrations of voided concrete sleepers in railway track system, Journal of Rail and Rapid Transit – Proceedings of the Institution of Mechanical Engineers Part F (2007) 495-508.

Kaewunruen, S and Remennikov, AM, Experimental and Numerical Studies of Railway Prestressed Concrete Sleepers under Static and Impact Loads, Civil Computing, 2007, 3, 25-28. Peer-reviewed article, Library: research-pubs@uow.edu.au

Vassilios A. Profillidis. (2006), Railway Management and Engineering, Avebury Technical: England.

Berggren E. (2009) Railway Track Stiffness Dynamic Measurements and Evaluation for Efficient Maintenance. Stockholm: Royal Institute of Technology (PhD thesis, Report TRITA AVE 2009:17, Aeronautical and Vehicle Engineering).

H. Yu, D. Jeong, J. Choros and T. Sussmann, Finite element modeling of prestressed concrete crossties with ballast and subgrade support. Proceedings of the ASME 2011 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference IDETC/CIE 2011 August 2 8-31, 2011, Washington, DC, USA

APPENDICES

Appendix A:

** COMMAND USED TO DEVELOP THE FE MODEL **

```
FINISH
/CLEAR
/PREP7
ET,1,SOLID65
R,1
MP,EX,1,36E9
MP,PRXY,1,0.2
ET,2,LINK8
R,2,1.9635E-05,0.0058875
MP,EX,2,200E9
MP,PRXY,2,0.3
/TRIAD,LBOT
/VIEW,1,1,1,1

N,1,0.0000,0.0000,0.0000
N,2,0.0560,0.0000,0.0000
N,3,0.1120,0.0000,0.0000
N,4,0.1680,0.0000,0.0000
N,5,0.2240,0.0000,0.0000
N,6,0.2800,0.0000,0.0000
N,7,0.0100,0.0400,0.0000
N,8,0.0560,0.0400,0.0000
N,9,0.1120,0.0400,0.0000
N,10,0.1680,0.0400,0.0000
N,11,0.2240,0.0400,0.0000
N,12,0.2700,0.0400,0.0000
N,13,0.0200,0.0800,0.0000
N,14,0.0560,0.0800,0.0000
N,15,0.1120,0.0800,0.0000
N,16,0.1680,0.0800,0.0000
N,17,0.2240,0.0800,0.0000
N,18,0.2600,0.0800,0.0000
N,19,0.0300,0.1200,0.0000
N,20,0.0560,0.1200,0.0000
N,21,0.1120,0.1200,0.0000
N,22,0.1680,0.1200,0.0000
N,23,0.2240,0.1200,0.0000
N,24,0.2500,0.1200,0.0000
N,25,0.0400,0.1600,0.0000
N,26,0.0560,0.1600,0.0000
N,27,0.1120,0.1600,0.0000
N,28,0.1680,0.1600,0.0000
N,29,0.2240,0.1600,0.0000
N,30,0.2400,0.1600,0.0000
N,31,0.0960,0.0400,0.0000
N,32,0.1840,0.0400,0.0000
N,33,0.0960,0.1200,0.0000
N,34,0.1840,0.1200,0.0000
NGEN,50,34,1,34,1,0,0,0.05
E,1,2,8,7,35,36,42,41
E,2,3,9,8,36,37,43,42
E,3,4,10,9,37,38,44,43
E,4,5,11,10,38,39,45,44
E,5,6,12,11,39,40,46,45
E,7,8,14,13,41,42,48,47
E,8,9,15,14,42,43,49,48
E,9,10,16,15,43,44,50,49
E,10,11,17,16,44,45,51,50
E,11,12,18,17,45,46,52,51
E,13,14,20,19,47,48,54,53
E,14,15,21,20,48,49,55,54
E,15,16,22,21,49,50,56,55
E,16,17,23,22,50,51,57,56
```

```
E,17,18,24,23,51,52,58,57
E,19,20,26,25,53,54,60,59
E,20,21,27,26,54,55,61,60
E,21,22,28,27,55,56,62,61
E,22,23,29,28,56,57,63,62
E,23,24,30,29,57,58,64,63
TYPE,2
REAL,2
MAT,2
E,8,42
E,31,65
E,32,66
E,11,45
E,14,48
E,17,51
E,20,54
E,33,67
E,34,68
E,23,57
EGEN,49,34,1,30
N,1701,0.050,0.200,0.000
N,1702,0.056,0.200,0.000
N,1703,0.112,0.200,0.000
N,1704,0.168,0.200,0.000
N,1705,0.224,0.200,0.000
N,1706,0.230,0.200,0.000
N,1707,0.050,0.200,1.600
N,1708,0.056,0.200,1.600
N,1709,0.112,0.200,1.600
N,1710,0.168,0.200,1.600
N,1711,0.224,0.200,1.600
N,1712,0.230,0.200,1.600
NGEN,18,12,1701,1712,1,0,0,0.05
TYPE,1
REAL,1
MAT,1
E,25,1701,1702,26,59,1713,1714,60
E,26,1702,1703,27,60,1714,1715,61
E,27,1703,1704,28,61,1715,1716,62
E,28,1704,1705,29,62,1716,1717,63
E,29,1705,1706,30,63,1717,1718,64
E,59,1713,1714,60,93,1725,1726,94
E,60,1714,1715,61,94,1726,1727,95
E,61,1715,1716,62,95,1727,1728,96
E,62,1716,1717,63,96,1728,1729,97
E,63,1717,1718,64,97,1729,1730,98
E,93,1725,1726,94,127,1737,1738,128
E,94,1726,1727,95,128,1738,1739,129
E,95,1727,1728,96,129,1739,1740,130
E,96,1728,1729,97,130,1740,1741,131
E,97,1729,1730,98,131,1741,1742,132
E,127,1737,1738,128,161,1749,1750,162
E,128,1738,1739,129,162,1750,1751,163
E,129,1739,1740,130,163,1751,1752,164
E,130,1740,1741,131,164,1752,1753,165
E,131,1741,1742,132,165,1753,1754,166
E,161,1749,1750,162,195,1761,1762,196
E,162,1750,1751,163,196,1762,1763,197
E,163,1751,1752,164,197,1763,1764,198
E,164,1752,1753,165,198,1764,1765,199
E,165,1753,1754,166,199,1765,1766,200
E,195,1761,1762,196,229,1773,1774,230
E,196,1762,1763,197,230,1774,1775,231
E,197,1763,1764,198,231,1775,1776,232
E,198,1764,1765,199,232,1776,1777,233
E,199,1765,1766,200,233,1777,1778,234
E,229,1773,1774,230,263,1785,1786,264
E,230,1774,1775,231,264,1786,1787,265
E,231,1775,1776,232,265,1787,1788,266
E,232,1776,1777,233,266,1788,1789,267
E,233,1777,1778,234,267,1789,1790,268
E,263,1785,1786,264,297,1797,1798,298
E,264,1786,1787,265,298,1798,1799,299
E,265,1787,1788,266,299,1799,1800,300
```

E,266,1788,1789,267,300,1800,1801,301
E,267,1789,1790,268,301,1801,1802,302
E,297,1797,1798,298,331,1809,1810,332
E,298,1798,1799,299,332,1810,1811,333
E,299,1799,1800,300,333,1811,1812,334
E,300,1800,1801,301,334,1812,1813,335
E,301,1801,1802,302,335,1813,1814,336
E,331,1809,1810,332,365,1821,1822,366
E,332,1810,1811,333,366,1822,1823,367
E,333,1811,1812,334,367,1823,1824,368
E,334,1812,1813,335,368,1824,1825,369
E,335,1813,1814,336,369,1825,1826,370
E,365,1821,1822,366,399,1833,1834,400
E,366,1822,1823,367,400,1834,1835,401
E,367,1823,1824,368,401,1835,1836,402
E,368,1824,1825,369,402,1836,1837,403
E,369,1825,1826,370,403,1837,1838,404
E,399,1833,1834,400,433,1845,1846,434
E,400,1834,1835,401,434,1846,1847,435
E,401,1835,1836,402,435,1847,1848,436
E,402,1836,1837,403,436,1848,1849,437
E,403,1837,1838,404,437,1849,1850,438
E,433,1845,1846,434,467,1857,1858,468
E,434,1846,1847,435,468,1858,1859,469
E,435,1847,1848,436,469,1859,1860,470
E,436,1848,1849,437,470,1860,1861,471
E,437,1849,1850,438,471,1861,1862,472
E,467,1857,1858,468,501,1869,1870,502
E,468,1858,1859,469,502,1870,1871,503
E,469,1859,1860,470,503,1871,1872,504
E,470,1860,1861,471,504,1872,1873,505
E,471,1861,1862,472,505,1873,1874,506
E,501,1869,1870,502,535,1881,1882,536
E,502,1870,1871,503,536,1882,1883,537
E,503,1871,1872,504,537,1883,1884,538
E,504,1872,1873,505,538,1884,1885,539
E,505,1873,1874,506,539,1885,1886,540
E,535,1881,1882,536,569,1893,1894,570
E,536,1882,1883,537,570,1894,1895,571
E,537,1883,1884,538,571,1895,1896,572
E,538,1884,1885,539,572,1896,1897,573
E,539,1885,1886,540,573,1897,1898,574
E,569,1893,1894,570,603,1905,1906,604
E,570,1894,1895,571,604,1906,1907,605
E,571,1895,1896,572,605,1907,1908,606
E,572,1896,1897,573,606,1908,1909,607
E,573,1897,1898,574,607,1909,1910,608
E,1113,1707,1708,1114,1147,1719,1720,1148
E,1114,1708,1709,1115,1148,1720,1721,1149
E,1115,1709,1710,1116,1149,1721,1722,1150
E,1116,1710,1711,1117,1150,1722,1723,1151
E,1117,1711,1712,1118,1151,1723,1724,1152
E,1147,1719,1720,1148,1181,1731,1732,1182
E,1148,1720,1721,1149,1182,1732,1733,1183
E,1149,1721,1722,1150,1183,1733,1734,1184
E,1150,1722,1723,1151,1184,1734,1735,1185
E,1151,1723,1724,1152,1185,1735,1736,1186
E,1181,1731,1732,1182,1215,1743,1744,1216
E,1182,1732,1733,1183,1216,1744,1745,1217
E,1183,1733,1734,1184,1217,1745,1746,1218
E,1184,1734,1735,1185,1218,1746,1747,1219
E,1185,1735,1736,1186,1219,1747,1748,1220
E,1215,1743,1744,1216,1249,1755,1756,1250
E,1216,1744,1745,1217,1250,1756,1757,1251
E,1217,1745,1746,1218,1251,1757,1758,1252
E,1218,1746,1747,1219,1252,1758,1759,1253
E,1219,1747,1748,1220,1253,1759,1760,1254
E,1249,1755,1756,1250,1283,1767,1768,1284
E,1250,1756,1757,1251,1284,1768,1769,1285
E,1251,1757,1758,1252,1285,1769,1770,1286
E,1252,1758,1759,1253,1286,1770,1771,1287
E,1253,1759,1760,1254,1287,1771,1772,1288
E,1283,1767,1768,1284,1317,1779,1780,1318
E,1284,1768,1769,1285,1318,1780,1781,1319

E,1285,1769,1770,1286,1319,1781,1782,1320
E,1286,1770,1771,1287,1320,1782,1783,1321
E,1287,1771,1772,1288,1321,1783,1784,1322
E,1317,1779,1780,1318,1351,1791,1792,1352
E,1318,1780,1781,1319,1352,1792,1793,1353
E,1319,1781,1782,1320,1353,1793,1794,1354
E,1320,1782,1783,1321,1354,1794,1795,1355
E,1321,1783,1784,1322,1355,1795,1796,1356
E,1351,1791,1792,1352,1385,1803,1804,1386
E,1352,1792,1793,1353,1386,1804,1805,1387
E,1353,1793,1794,1354,1387,1805,1806,1388
E,1354,1794,1795,1355,1388,1806,1807,1389
E,1355,1795,1796,1356,1389,1807,1808,1390
E,1385,1803,1804,1386,1419,1815,1816,1420
E,1386,1804,1805,1387,1420,1816,1817,1421
E,1387,1805,1806,1388,1421,1817,1818,1422
E,1388,1806,1807,1389,1422,1818,1819,1423
E,1389,1807,1808,1390,1423,1819,1820,1424
E,1419,1815,1816,1420,1453,1827,1828,1454
E,1420,1816,1817,1421,1454,1828,1829,1455
E,1421,1817,1818,1422,1455,1829,1830,1456
E,1422,1818,1819,1423,1456,1830,1831,1457
E,1423,1819,1820,1424,1457,1831,1832,1458
E,1453,1827,1828,1454,1487,1839,1840,1488
E,1454,1828,1829,1455,1488,1840,1841,1489
E,1455,1829,1830,1456,1489,1841,1842,1490
E,1456,1830,1831,1457,1490,1842,1843,1491
E,1457,1831,1832,1458,1491,1843,1844,1492
E,1487,1839,1840,1488,1521,1851,1852,1522
E,1488,1840,1841,1489,1522,1852,1853,1523
E,1489,1841,1842,1490,1523,1853,1854,1524
E,1490,1842,1843,1491,1524,1854,1855,1525
E,1491,1843,1844,1492,1525,1855,1856,1526
E,1521,1851,1852,1522,1555,1863,1864,1556
E,1522,1852,1853,1523,1556,1864,1865,1557
E,1523,1853,1854,1524,1557,1865,1866,1558
E,1524,1854,1855,1525,1558,1866,1867,1559
E,1525,1855,1856,1526,1559,1867,1868,1560
E,1555,1863,1864,1556,1589,1875,1876,1590
E,1556,1864,1865,1557,1590,1876,1877,1591
E,1557,1865,1866,1558,1591,1877,1878,1592
E,1558,1866,1867,1559,1592,1878,1879,1593
E,1559,1867,1868,1560,1593,1879,1880,1594
E,1589,1875,1876,1590,1623,1887,1888,1624
E,1590,1876,1877,1591,1624,1888,1889,1625
E,1591,1877,1878,1592,1625,1889,1890,1626
E,1592,1878,1879,1593,1626,1890,1891,1627
E,1593,1879,1880,1594,1627,1891,1892,1628
E,1623,1887,1888,1624,1657,1899,1900,1658
E,1624,1888,1889,1625,1658,1900,1901,1659
E,1625,1889,1890,1626,1659,1901,1902,1660
E,1626,1890,1891,1627,1660,1902,1903,1661
E,1627,1891,1892,1628,1661,1903,1904,1662
E,1657,1899,1900,1658,1691,1911,1912,1692
E,1658,1900,1901,1659,1692,1912,1913,1693
E,1659,1901,1902,1660,1693,1913,1914,1694
E,1660,1902,1903,1661,1694,1914,1915,1695
E,1661,1903,1904,1662,1695,1915,1916,1696
N,1917,0.056,0.240,0.000
N,1918,0.112,0.240,0.000
N,1919,0.168,0.240,0.000
N,1920,0.224,0.240,0.000
N,1921,0.056,0.240,2.100
N,1922,0.112,0.240,2.100
N,1923,0.168,0.240,2.100
N,1924,0.224,0.240,2.100
NGEN,8,8,1917,1924,1,0,0,0.05
TYPE,1
REAL,1
MAT,1
E,1701,1917,1702,1713,1925,1714
E,1702,1917,1918,1703,1714,1925,1926,1715
E,1703,1918,1919,1704,1715,1926,1927,1716
E,1704,1919,1920,1705,1716,1927,1928,1717

E,1705,1920,1706,1717,1928,1718
E,1713,1925,1714,1725,1933,1726
E,1714,1925,1926,1715,1726,1933,1934,1727
E,1715,1926,1927,1716,1727,1934,1935,1728
E,1716,1927,1928,1717,1728,1935,1936,1729
E,1717,1928,1718,1729,1936,1730
E,1725,1933,1726,1737,1941,1738
E,1726,1933,1934,1727,1738,1941,1942,1739
E,1727,1934,1935,1728,1739,1942,1943,1740
E,1728,1935,1936,1729,1740,1943,1944,1741
E,1729,1936,1730,1741,1944,1742
E,1737,1941,1738,1749,1949,1750
E,1738,1941,1942,1739,1750,1949,1950,1751
E,1739,1942,1943,1740,1751,1950,1951,1752
E,1740,1943,1944,1741,1752,1951,1952,1753
E,1741,1944,1742,1753,1952,1754
E,1749,1949,1750,1761,1957,1762
E,1750,1949,1950,1751,1762,1957,1958,1763
E,1751,1950,1951,1752,1763,1958,1959,1764
E,1752,1951,1952,1753,1764,1959,1960,1765
E,1753,1952,1754,1765,1960,1766
E,1761,1957,1762,1773,1965,1774
E,1762,1957,1958,1763,1774,1965,1966,1775
E,1763,1958,1959,1764,1775,1966,1967,1776
E,1764,1959,1960,1765,1776,1967,1968,1777
E,1765,1960,1766,1777,1968,1778
E,1773,1965,1774,1785,1973,1786
E,1774,1965,1966,1775,1786,1973,1974,1787
E,1775,1966,1967,1776,1787,1974,1975,1788
E,1776,1967,1968,1777,1788,1975,1976,1789
E,1777,1968,1778,1789,1976,1790
E,1827,1921,1828,1839,1929,1840
E,1828,1921,1922,1829,1840,1929,1930,1841
E,1829,1922,1923,1830,1841,1930,1931,1842
E,1830,1923,1924,1831,1842,1931,1932,1843
E,1831,1924,1832,1843,1932,1844
E,1839,1929,1840,1851,1937,1852
E,1840,1929,1930,1841,1852,1937,1938,1853
E,1841,1930,1931,1842,1853,1938,1939,1854
E,1842,1931,1932,1843,1854,1939,1940,1855
E,1843,1932,1844,1855,1940,1856
E,1851,1937,1852,1863,1945,1864
E,1852,1937,1938,1853,1864,1945,1946,1865
E,1853,1938,1939,1854,1865,1946,1947,1866
E,1854,1939,1940,1855,1866,1947,1948,1867
E,1855,1940,1856,1867,1948,1868
E,1863,1945,1864,1875,1953,1876
E,1864,1945,1946,1865,1876,1953,1954,1877
E,1865,1946,1947,1866,1877,1954,1955,1878
E,1866,1947,1948,1867,1878,1955,1956,1879
E,1867,1948,1868,1879,1956,1880
E,1875,1953,1876,1887,1961,1888
E,1876,1953,1954,1877,1888,1961,1962,1889
E,1877,1954,1955,1878,1889,1962,1963,1890
E,1878,1955,1956,1879,1890,1963,1964,1891
E,1879,1956,1880,1891,1964,1892
E,1887,1961,1888,1899,1969,1900
E,1888,1961,1962,1889,1900,1969,1970,1901
E,1889,1962,1963,1890,1901,1970,1971,1902
E,1890,1963,1964,1891,1902,1971,1972,1903
E,1891,1964,1892,1903,1972,1904
E,1899,1969,1900,1911,1977,1912
E,1900,1969,1970,1901,1912,1977,1978,1913
E,1901,1970,1971,1902,1913,1978,1979,1914
E,1902,1971,1972,1903,1914,1979,1980,1915
E,1903,1972,1904,1915,1980,1916
N,1981,0.053,0.220,0.650
N,1982,0.056,0.220,0.650
N,1983,0.112,0.220,0.650
N,1984,0.168,0.220,0.650
N,1985,0.224,0.220,0.650
N,1986,0.227,0.220,0.650
N,1987,0.053,0.220,0.700
N,1988,0.056,0.220,0.700

N,1989,0.112,0.220,0.700
N,1990,0.168,0.220,0.700
N,1991,0.224,0.220,0.700
N,1992,0.227,0.220,0.700
N,1993,0.053,0.220,0.750
N,1994,0.056,0.220,0.750
N,1995,0.112,0.220,0.750
N,1996,0.168,0.220,0.750
N,1997,0.224,0.220,0.750
N,1998,0.227,0.220,0.750
N,1999,0.053,0.220,0.800
N,2000,0.056,0.220,0.800
N,2001,0.112,0.220,0.800
N,2002,0.168,0.220,0.800
N,2003,0.224,0.220,0.800
N,2004,0.227,0.220,0.800
TYPE,1
REAL,1
MAT,1
E,1857,1981,1982,1858,1869,1987,1988,1870
E,1858,1982,1983,1859,1870,1988,1989,1871
E,1859,1983,1984,1860,1871,1989,1990,1872
E,1860,1984,1985,1861,1872,1990,1991,1873
E,1861,1985,1986,1862,1873,1991,1992,1874
E,1869,1987,1988,1870,1881,1993,1994,1882
E,1870,1988,1989,1871,1882,1994,1995,1883
E,1871,1989,1990,1872,1883,1995,1996,1884
E,1872,1990,1991,1873,1884,1996,1997,1885
E,1873,1991,1992,1874,1885,1997,1998,1886
E,1881,1993,1994,1882,1893,1999,2000,1894
E,1882,1994,1995,1883,1894,2000,2001,1895
E,1883,1995,1996,1884,1895,2001,2002,1896
E,1884,1996,1997,1885,1896,2002,2003,1897
E,1885,1997,1998,1886,1897,2003,2004,1898
E,1905,1999,1893,1906,2000,1894
E,1906,2000,1894,1907,2001,1895
E,1907,2001,1895,1908,2002,1896
E,1908,2002,1896,1909,2003,1897
E,1909,2003,1897,1910,2004,1898
N,2005,0.056,0.240,0.650
N,2006,0.112,0.240,0.650
N,2007,0.168,0.240,0.650
N,2008,0.224,0.240,0.650
N,2009,0.056,0.240,0.700
N,2010,0.112,0.240,0.700
N,2011,0.168,0.240,0.700
N,2012,0.224,0.240,0.700
N,2013,0.056,0.240,0.750
N,2014,0.112,0.240,0.750
N,2015,0.168,0.240,0.750
N,2016,0.224,0.240,0.750
N,2017,0.045,0.180,0.900
N,2018,0.056,0.180,0.900
N,2019,0.112,0.180,0.900
N,2020,0.168,0.180,0.900
N,2021,0.224,0.180,0.900
N,2022,0.235,0.180,0.900
TYPE,1
REAL,1
MAT,1
E,1981,2005,2006,1983,1987,2009,2010,1989
E,1983,2006,2007,1984,1989,2010,2011,1990
E,1984,2007,2008,1986,1990,2011,2012,1992
E,1987,2009,2010,1989,1993,2013,2014,1995
E,1989,2010,2011,1990,1995,2014,2015,1996
E,1990,2011,2012,1992,1996,2015,2016,1998
E,1999,2013,1993,2001,2014,1995
E,2001,2014,1995,2002,2015,1996
E,2002,2015,1996,2004,2016,1998
E,603,1905,1906,604,637,2017,2018,638
E,604,1906,1907,605,638,2018,2019,639
E,605,1907,1908,606,639,2019,2020,640
E,606,1908,1909,607,640,2020,2021,641
E,607,1909,1910,608,641,2021,2022,642

E,671,2017,637,672,2018,638
E,672,2018,638,673,2019,639
E,673,2019,639,674,2020,640
E,674,2020,640,675,2021,641
E,675,2021,641,676,2022,642
N,2023,0.053,0.220,1.650
N,2024,0.056,0.220,1.650
N,2025,0.112,0.220,1.650
N,2026,0.168,0.220,1.650
N,2027,0.224,0.220,1.650
N,2028,0.227,0.220,1.650
N,2029,0.053,0.220,1.700
N,2030,0.056,0.220,1.700
N,2031,0.112,0.220,1.700
N,2032,0.168,0.220,1.700
N,2033,0.224,0.220,1.700
N,2034,0.227,0.220,1.700
N,2035,0.053,0.220,1.750
N,2036,0.056,0.220,1.750
N,2037,0.112,0.220,1.750
N,2038,0.168,0.220,1.750
N,2039,0.224,0.220,1.750
N,2040,0.227,0.220,1.750
N,2041,0.053,0.220,1.800
N,2042,0.056,0.220,1.800
N,2043,0.112,0.220,1.800
N,2044,0.168,0.220,1.800
N,2045,0.224,0.220,1.800
N,2046,0.227,0.220,1.800
TYPE,1
REAL,1
MAT,1
E,1719,2023,2024,1720,1731,2029,2030,1732
E,1720,2024,2025,1721,1732,2030,2031,1733
E,1721,2025,2026,1722,1733,2031,2032,1734
E,1722,2026,2027,1723,1734,2032,2033,1735
E,1723,2027,2028,1724,1735,2033,2034,1736
E,1731,2029,2030,1732,1743,2035,2036,1744
E,1732,2030,2031,1733,1744,2036,2037,1745
E,1733,2031,2032,1734,1745,2037,2038,1746
E,1734,2032,2033,1735,1746,2038,2039,1747
E,1735,2033,2034,1736,1747,2039,2040,1748
E,1743,2035,2036,1744,1755,2041,2042,1756
E,1744,2036,2037,1745,1756,2042,2043,1757
E,1745,2037,2038,1746,1757,2043,2044,1758
E,1746,2038,2039,1747,1758,2044,2045,1759
E,1747,2039,2040,1748,1759,2045,2046,1760
E,1707,2023,1719,1708,2024,1720
E,1708,2024,1720,1709,2025,1721
E,1709,2025,1721,1710,2026,1722
E,1710,2026,1722,1711,2027,1723
E,1711,2027,1723,1712,2028,1724
N,2047,0.056,0.240,1.700
N,2048,0.112,0.240,1.700
N,2049,0.168,0.240,1.700
N,2050,0.224,0.240,1.700
N,2051,0.056,0.240,1.750
N,2052,0.112,0.240,1.750
N,2053,0.168,0.240,1.750
N,2054,0.224,0.240,1.750
N,2055,0.056,0.240,1.800
N,2056,0.112,0.240,1.800
N,2057,0.168,0.240,1.800
N,2058,0.224,0.240,1.800
N,2059,0.045,0.180,1.550
N,2060,0.056,0.180,1.550
N,2061,0.112,0.180,1.550
N,2062,0.168,0.180,1.550
N,2063,0.224,0.180,1.550
N,2064,0.235,0.180,1.550
TYPE,1
REAL,1
MAT,1
E,2029,2047,2048,2031,2035,2051,2052,2037

E,2031,2048,2049,2032,2037,2052,2053,2038
E,2032,2049,2050,2034,2038,2053,2054,2040
E,2035,2051,2052,2037,2041,2055,2056,2043
E,2037,2052,2053,2038,2043,2056,2057,2044
E,2038,2053,2054,2040,2044,2057,2058,2046
E,2023,2047,2029,2025,2048,2031
E,2025,2048,2031,2026,2049,2032
E,2026,2049,2032,2028,2050,2034
E,1079,2059,1707,1113,1080,2060,1708,1114
E,1080,2060,1708,1114,1081,2061,1709,1115
E,1081,2061,1709,1115,1082,2062,1710,1116
E,1082,2062,1710,1116,1083,2063,1711,1117
E,1083,2063,1711,1117,1084,2064,1712,1118
E,1045,2059,1079,1046,2060,1080
E,1046,2060,1080,1047,2061,1081
E,1047,2061,1081,1048,2062,1082
E,1048,2062,1082,1049,2063,1083
E,1049,2063,1083,1050,2064,1084

Appendix B:

Contour Plots of Stress results according to each cases of Ballast Contact Pressure Distribution

1. Center void

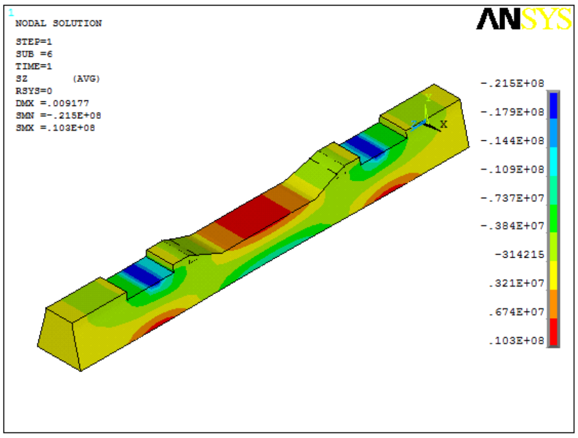


Figure B1-1 Stress at 100% contact

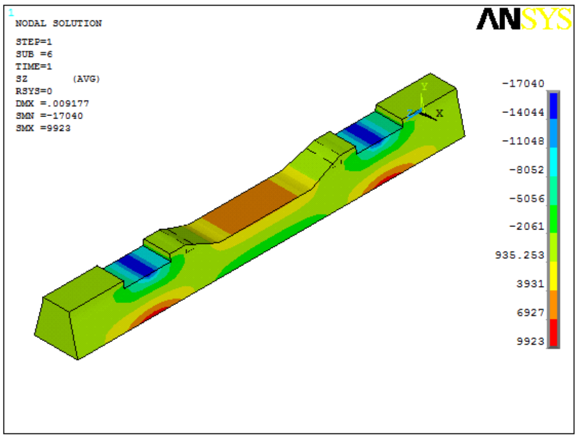


Figure 6.31-2 Stress at 90% contact

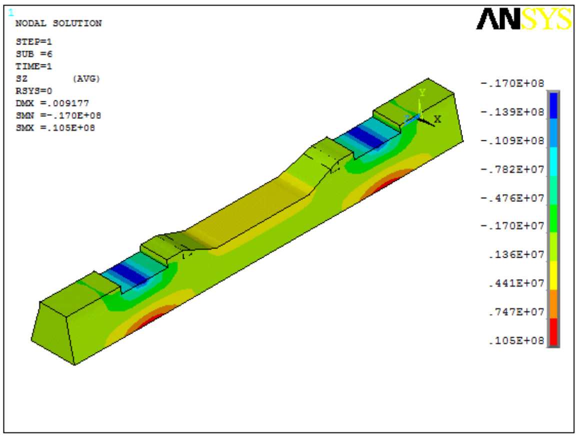


Figure B1-3 Stress at 80% contact

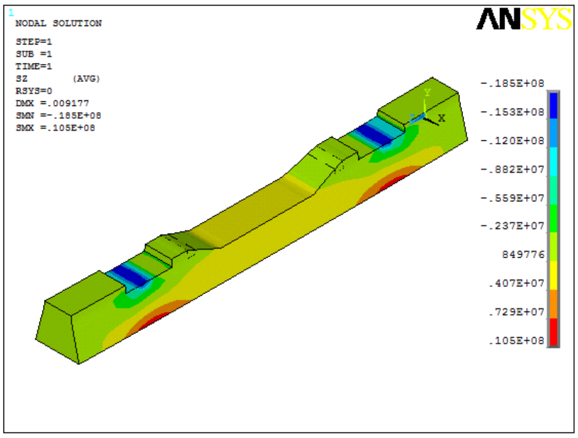


Figure B1-4 Stress at 70% contact

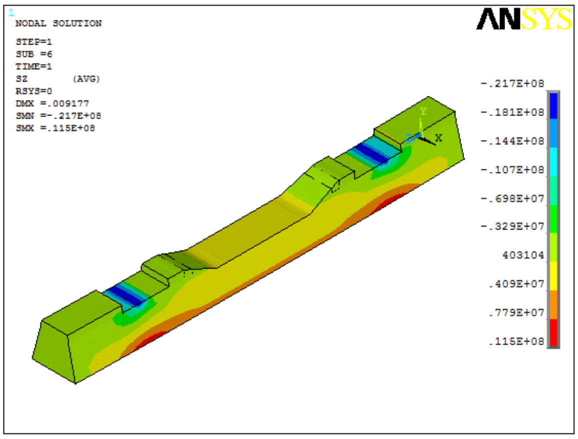


Figure B1-5 Stress at 64% contact

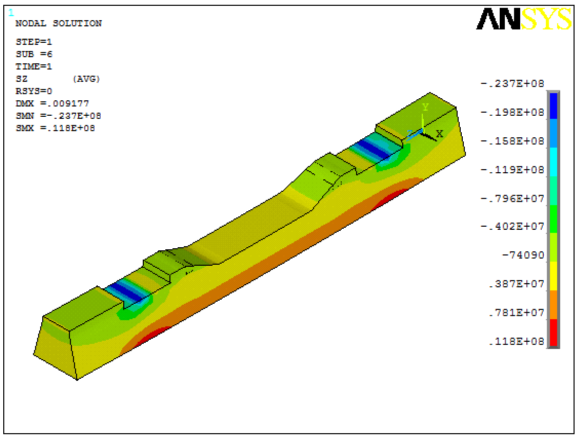


Figure B1-6 Stress at 60% contact

2. Center void with low stiffness

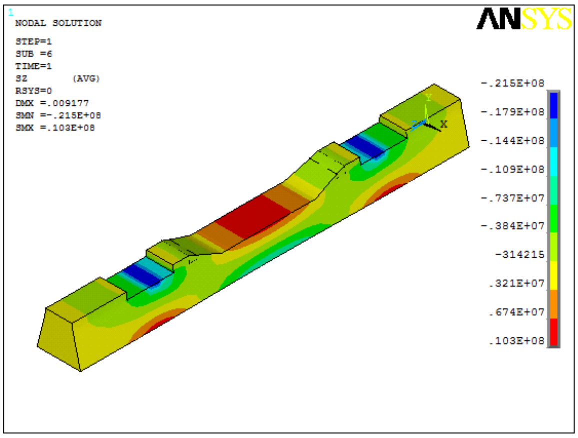


Figure B2- 1: Stress at 100% contact

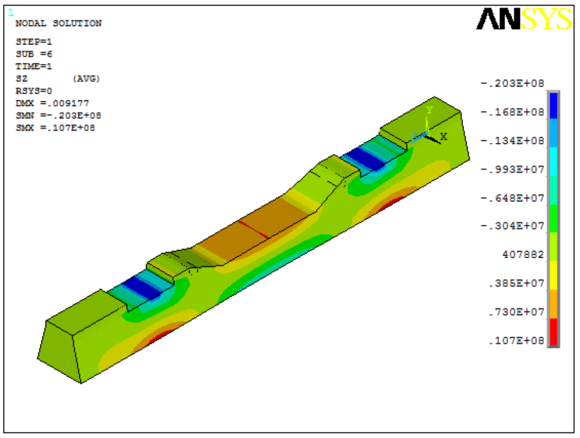


Figure B2- 2: Stress at 90% contact

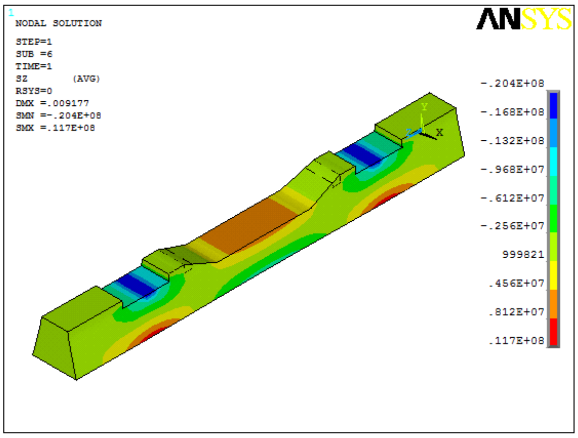


Figure B2- 3: Stress at 80% contact

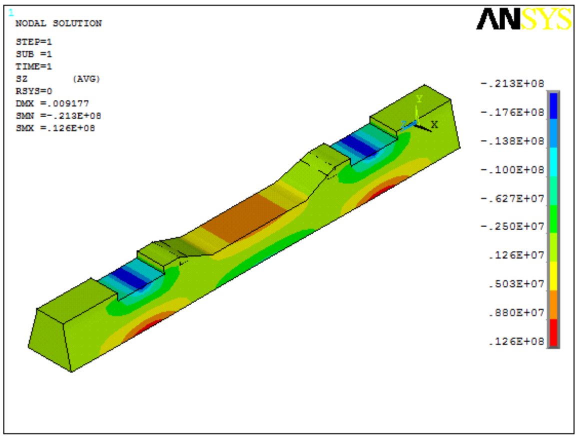


Figure B2- 4: Stress at 70% contact

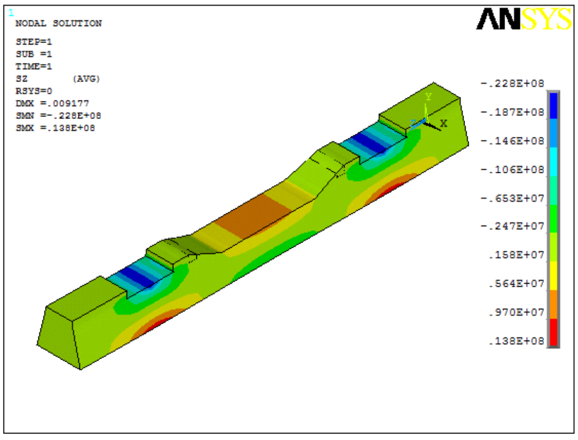


Figure B2- 5: Stress at 64% contact

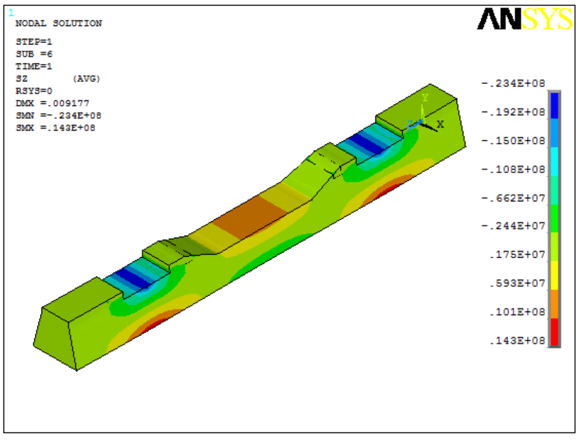


Figure B2- 6: Stress at 60% contact

3. Single hanging

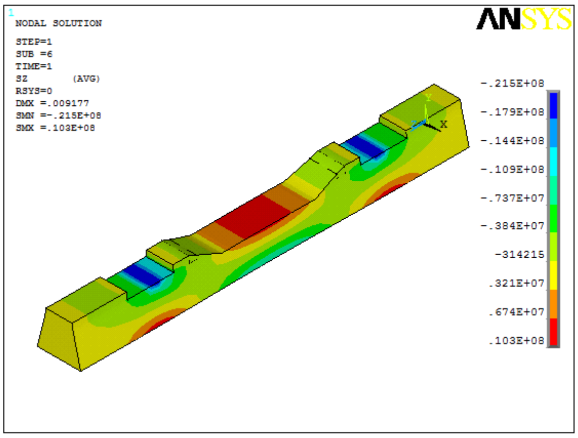


Figure B3- 1: Stress at 100% contact

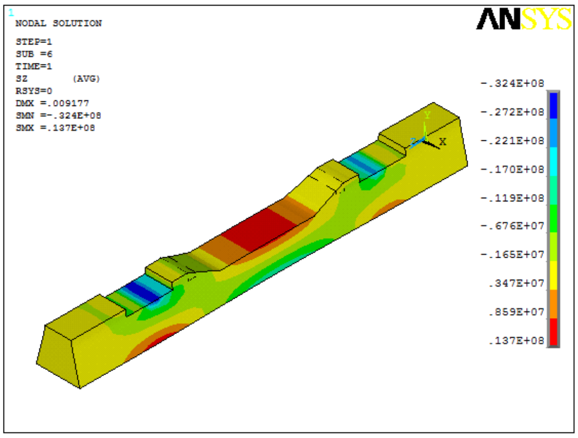


Figure B3- 2: Stress at 95% contact

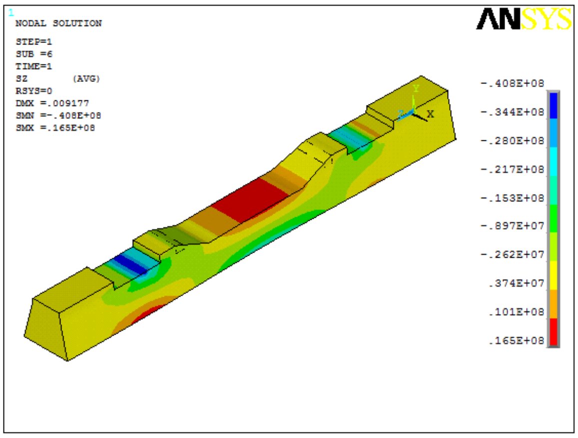


Figure B3- 3: Stress at 90% contact

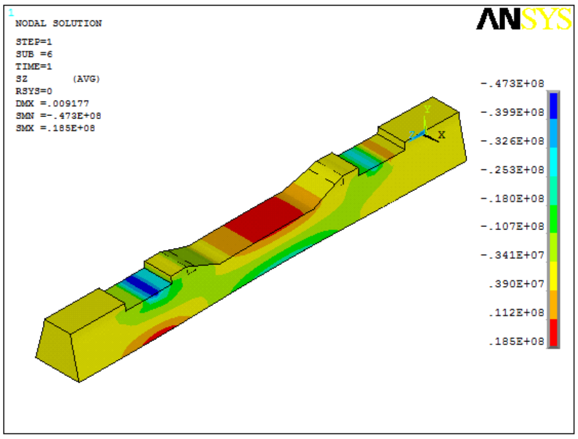


Figure B3- 4: Stress at 85% contact

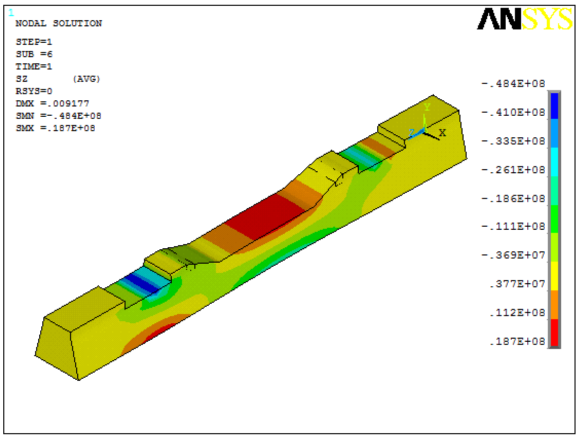


Figure B3- 5: Stress at 80% contact

4. Double-side hanging

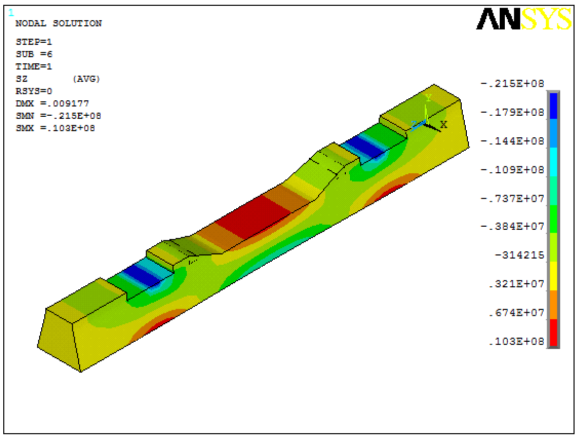


Figure B4- 1: Stress at 100% contact

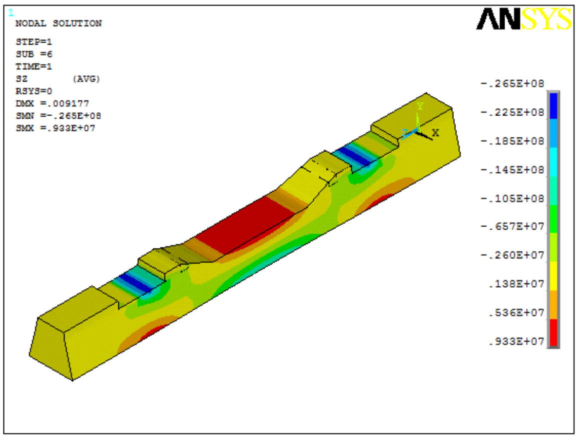


Figure B4- 2: Stress at 90% contact

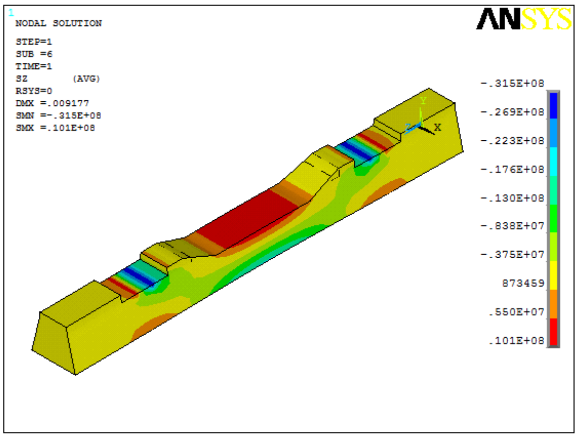


Figure B4- 3: Stress at 80% contact

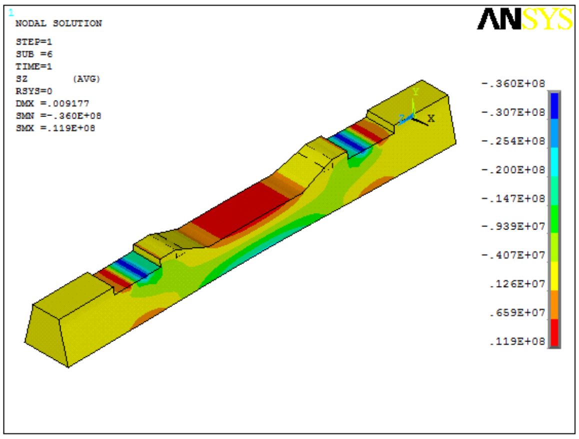


Figure B4- 4: Stress at 70% contact

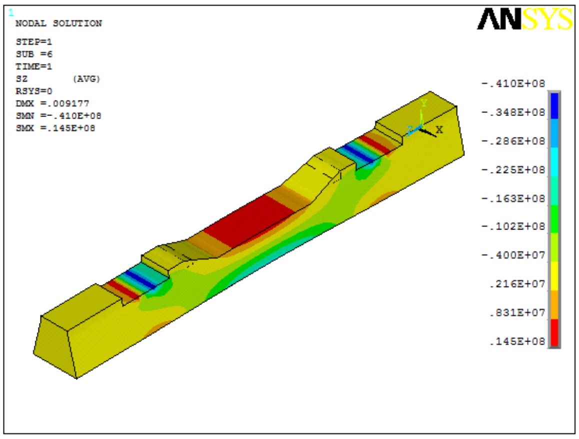


Figure B4- 5: Stress at 64% contact

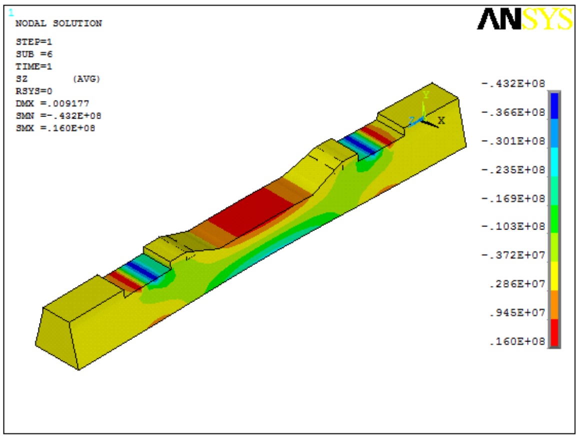


Figure B4- 6: Stress at 60% contact

Appendix C:

Contour Plots of vertical displacement results according to each cases of Ballast support Pressure Distributions

1. Center void

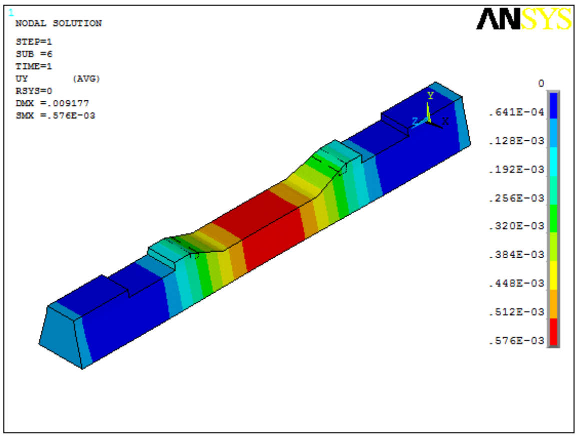


Figure C1- 1: Stress at 100% contact

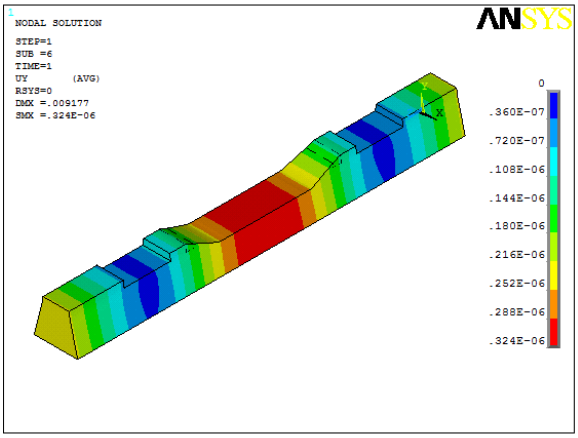


Figure C1- 2: Stress at 90% contact

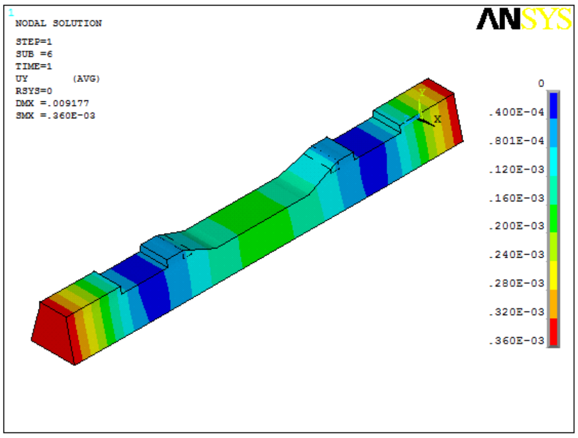


Figure C1- 3: Stress at 80% contact

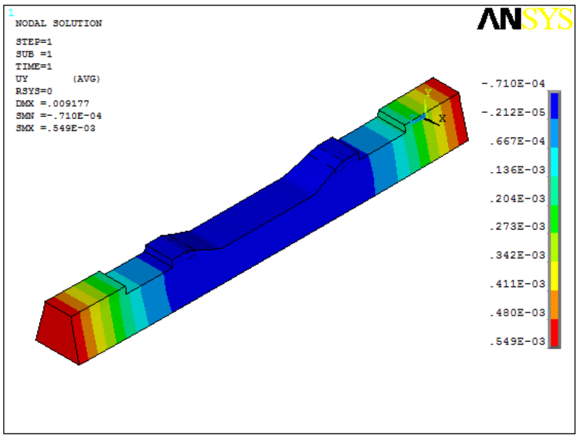


Figure C1- 4: Stress at 70% contact

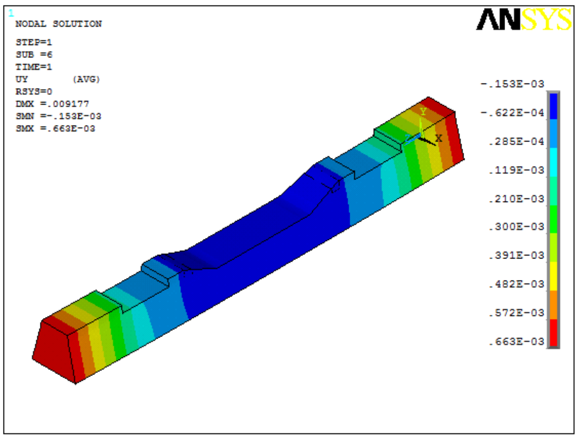


Figure C1- 5: Stress at 64% contact

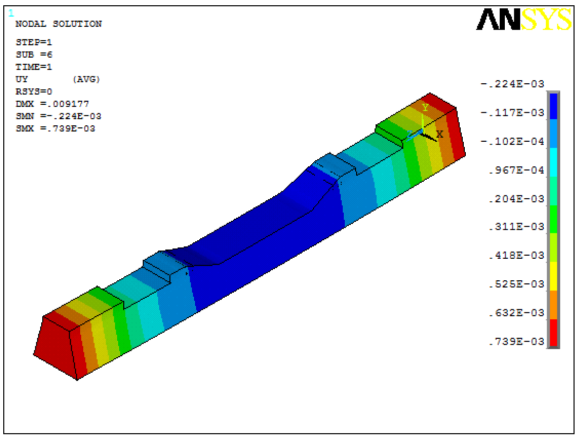


Figure C1- 6: Stress at 60% contact

2. Center void with little stiffness

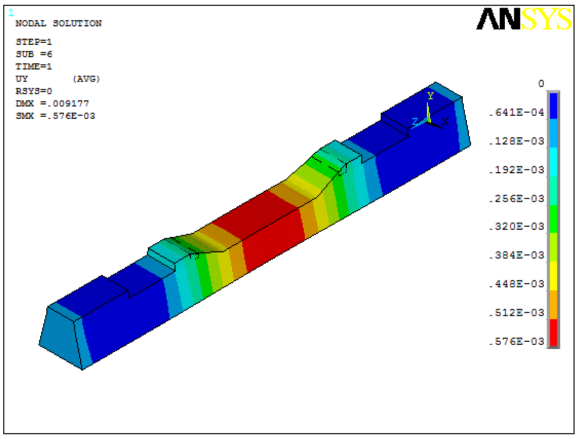


Figure C2- 1: Stress at 100% contact

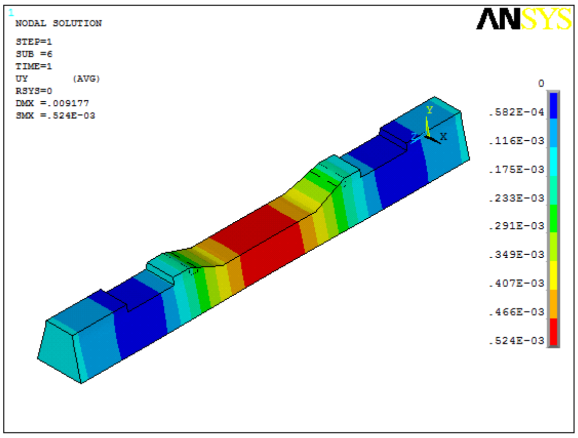


Figure C2- 2: Stress at 90% contact

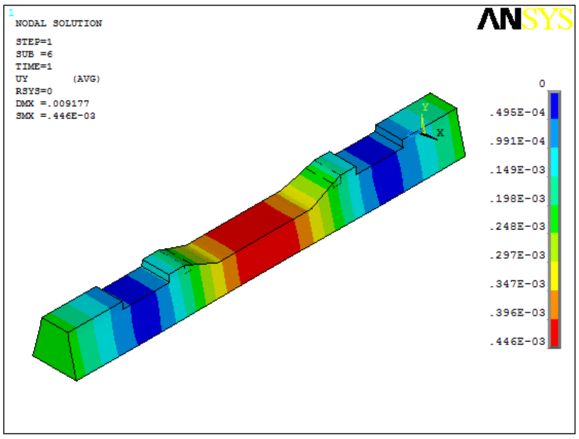


Figure C2- 3: Stress at 80% contact

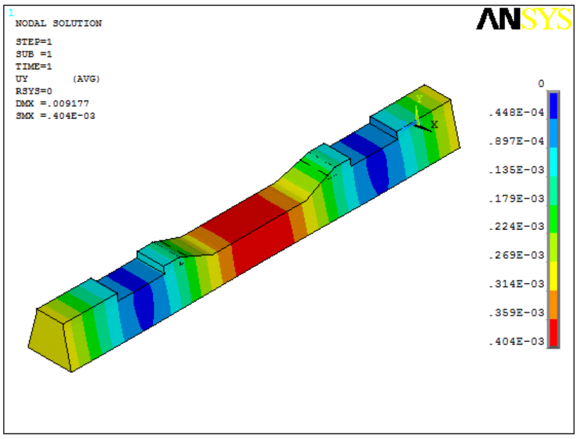


Figure C2- 4: Stress at 70% contact

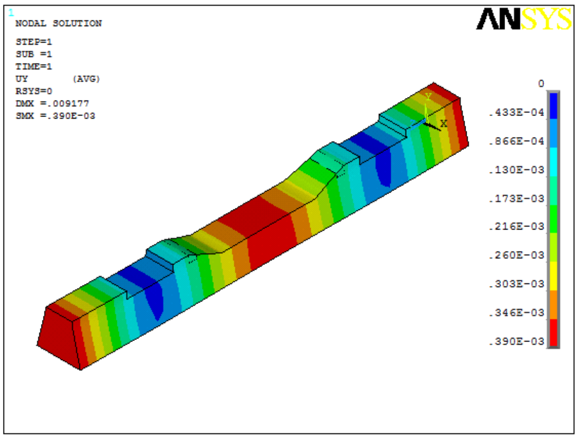


Figure C2- 5: Stress at 64% contact

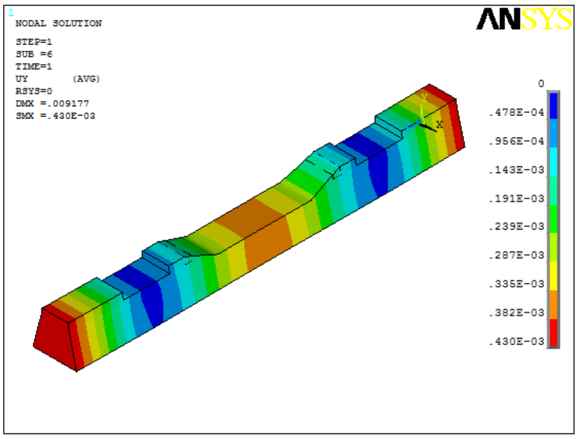


Figure C2- 6: Stress at 60% contact

3. Single hanging

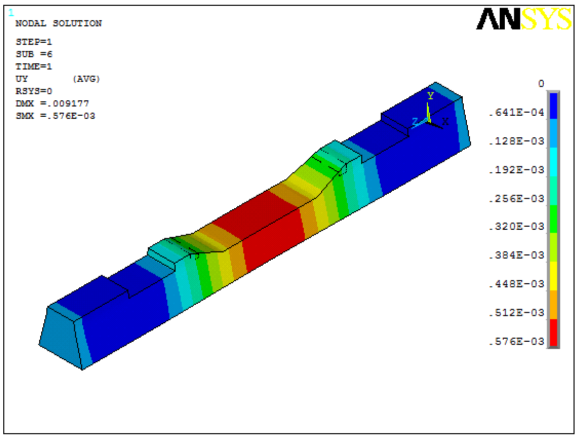


Figure C3- 1: Stress at 100% contact

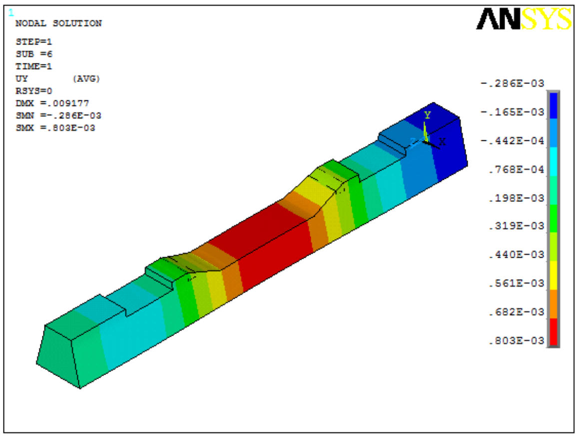


Figure C3- 2: Stress at 95% contact

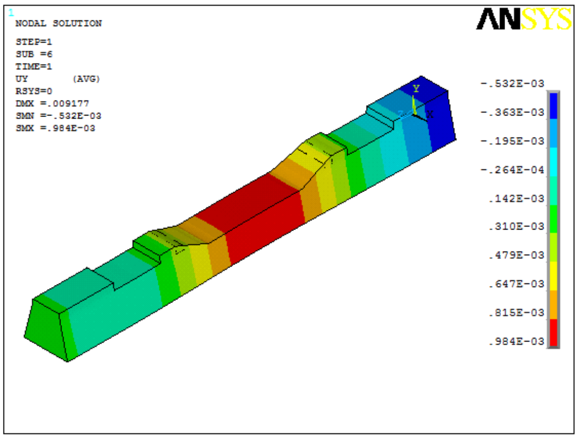


Figure C3- 4: Stress at 90% contact

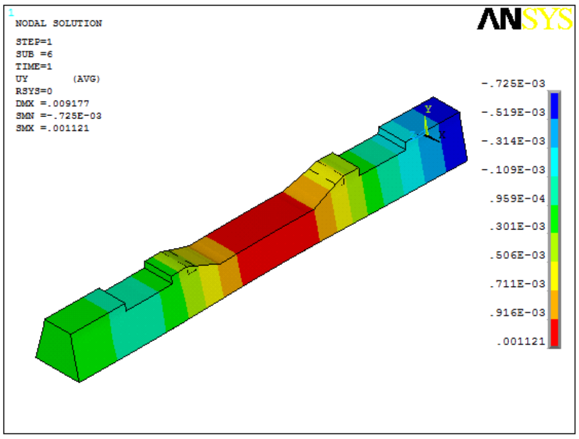


Figure C3- 3: Stress at 85% contact

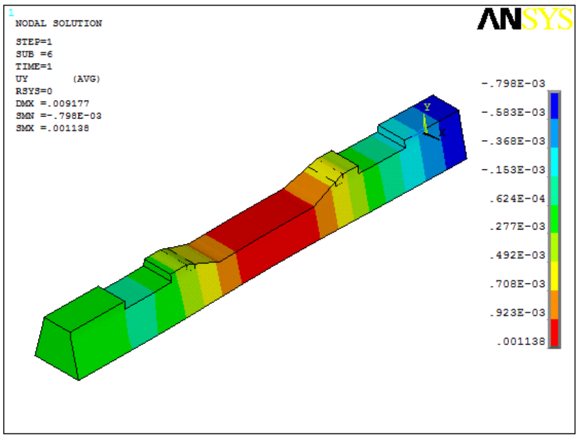


Figure C3- 5: Stress at 80% contact

4. Double-side hanging

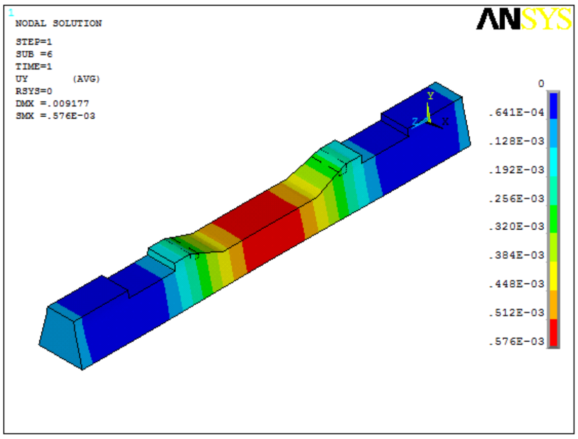


Figure C4- 1: Stress at 100% contact

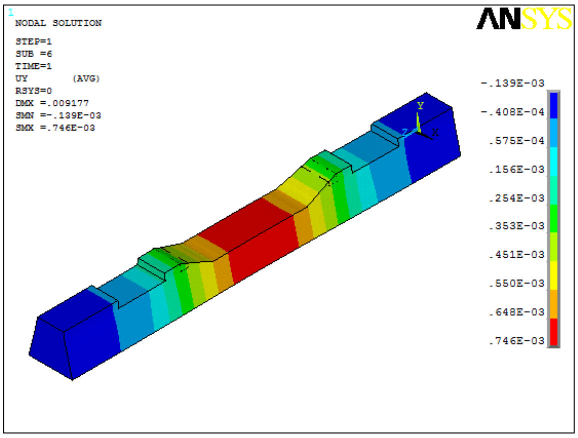


Figure C4- 2: Stress at 90% contact

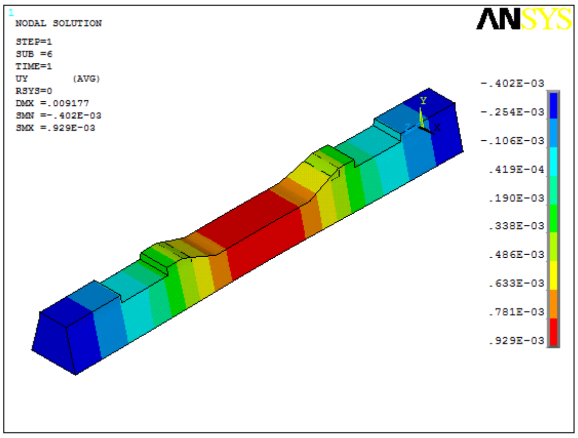


Figure C4- 3: Stress at 80% contact

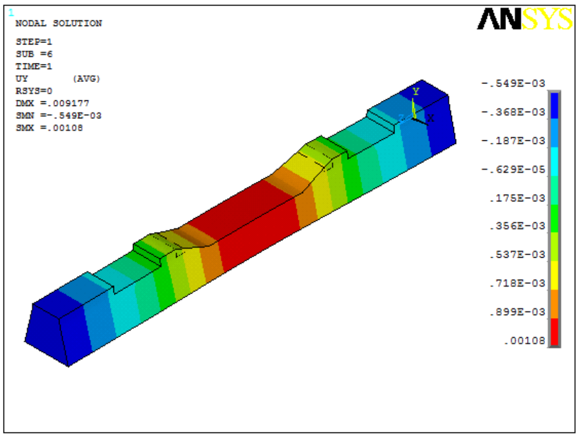


Figure C4- 4: Stress at 70% contact

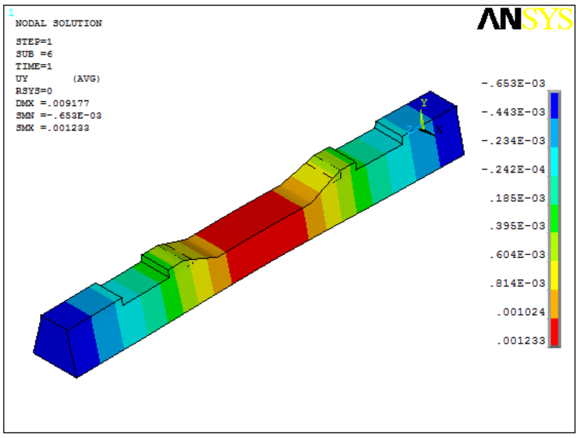


Figure C4- 5: Stress at 64% contact

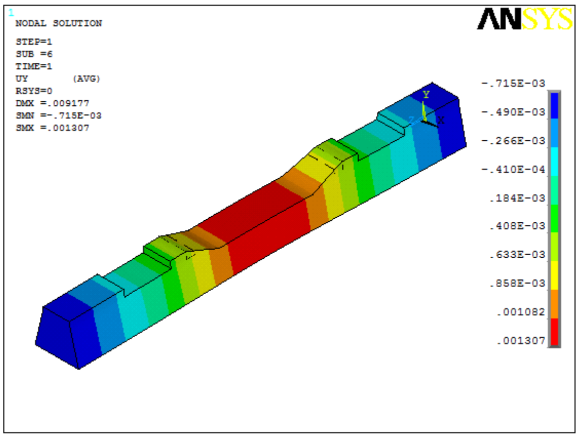


Figure C4- 6: Stress at 60% contact