Addis Ababa University

Addis Ababa Institute of Technology

School of Mechanical and Industrial Engineering

Analysis of Train Energy Consumption Reduction by Passing Low Passenger Flow Stations in Off-peak Hour for Addis Ababa LRT
Case Study on the Line of E-W Addis Ababa LRT

A Thesis Submitted to the School of Graduate Studies of Addis Ababa University in Partial Fulfillment of the Degree of Masters of Science in Railway Mechanical Engineering

By

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Addis Ababa, Ethiopia

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Declaration

I, the undersigned, declare that this thesis is my original work and has not been presented for any degree in any university and all the sources of materials used for the thesis have been fully acknowledged.

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This thesis work has been submitted for examination with my approval as a university advisor.

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Abstract

In this thesis in order to make the analysis of train energy consumption reduction, the E-W line of AA LRT has been taken. The length of the route is about 16.76 km, and there are 22 stations. This gives an average of about 798 m between each two stations. The passenger flow of E-W line is forecasted as 734.4 thousands persons/day based on the passenger transport survey. There are 41 trains in four directions and it will enable the light rail transit to provide transportation service to 15,000 people per hour in one direction.

The objective of the research is to analyze energy consumption reduction of AA LRT E-W line for one complete cycle trip. The problem has been formulated by reducing number of stations in off-peak hour of train operation considering low passenger flow of stations. There has been selection of six low passenger flow stations which are EW5, EW7, EW8, EW14, EW18 and EW20. They are selected based on passenger flow of stations and spacing between stations.

The model of the train motion has been formulated by considering the important acting force components, such as tractive effort and train resistances. The method of analyzing train energy consumption model was based on power and time, and the software which is used to create the simulation of the train energy consumption has been analyzed using MATLAB SIMULINK loop. It has been observed that energy consumption of train has increased significantly with increasing number of train stops at stations. The results have shown that there is a very large difference in the energy consumption in the case of number of train stops at stations. From the analysis the train, by passing six low passenger flow stations, consumed only about 79.42% of energy consumption of the train stopping at all stations for one complete cycle trip per train. That is, reducing the number of train stops at stations gives a large reduction of energy consumption.

Keywords: Predict low passenger flow stations, model of train energy consumption, energy consumption reduction, Matlab Simulink simulation of energy consumption

Table of Contents

List of Tables

List of Figures

Nomenclature

X_i	variable traveling distance of train (m)
t_i	variable traveling time (s)
t	elapsed time (s)
V_i	variable train speed (m/s) in acceleration motion
V_c	constant train speed (m/s) in constant speed motion
a	train acceleration (m/s^2)
X_a	acceleration distance (m)
X_b	braking distance (m)
m	sum of vehicle and passengers (kg)
m_{tot}	total mass (kg)
η	motor efficiency
I	current (A)
V	applied voltage (V)
R	resistance (Ω)
P_{in}	input power per motor (W)
P_{out}	output mechanical power per motor (W)
$P_{in,tot}$	total input motors power (W)
$P_{out,total}$	total output mechanical motors power
P_{tot}	total power (W)
TE	Tractive effort (N)
T	axle torque (Nm)

N	axle revolution speed (rpm)
ω	angular speed (rad/s)
F_t	tractive force (N)
F_a	aerodynamic resistance force (N)
F_r	rolling resistance force (N)
F_g	gradient resistance force (N)
F_c	curve resistance force
F_A	acceleration resistance (N)
F_b	braking force (N)
F_{tot}	total sum of resistance forces (N)
ρ	air density (kg/m^3)
C_a	coefficient of aerodynamic resistance
A_{fr}	frontal area of train (m^2)
C_r	coefficient of rolling resistance
g	acceleration of gravity (m/s^2)
α	angle of the gradient
r_g	gradient coefficient (%)
R_c	radius of curvature (m)
K	constant value of coefficient of curve resistance (%)
r_c	coefficient of curve resistance (%)
F_{adh}	adhesion force
μ	adhesion coefficient

f_m	mass factor
X_L	distance of level track (m)
X_U	distance of uphill track (m)
X_D	distance of downhill track (m)
E-W	East-West
EW1, EW2….EW22	station names
LRT	light rail transit
AA	Addis Ababa
S	stop at station
P	pass the station
E_{tot}	Train total energy consumption (J)
E_i	variable energy consumption with distance (J)
E_{ai}	acceleration motion energy consumption (J)
E_{ci}	constant speed motion energy consumption (J)
E_{La}	level acceleration energy consumption (J)
E_{Lc}	level constant speed energy consumption (J)
E_{Ua}	uphill acceleration energy consumption (J)
E_{Uc}	uphill constant speed energy consumption (J)
E_{Da}	downhill acceleration energy consumption (J)
E_{Dc}	downhill constant speed energy consumption (J)

Chapter One

Introduction

1.1 Background

The term light rail transit has been used to describe electric rail systems since the 1970s, with no formal definition until 1989, when the transportation research board (TRB) developed a standard definition [1]. Hamilton uses a modified definition based on that of the TRB which defines LRT as: A lightweight metropolitan electric railway system characterized by its ability to operate single cars or short trains along exclusive right-of-way at street level. These vehicles are usually powered by overhead electrical wires, and offer a frequent, fast, reliable, comfortable and high quality service that is environmentally sustainable [2].

LRT is often identified by its right-of-way and vehicle weight and size. When compared with a regional railway or metro, the system is lighter in terms of actual system weight. However, when compared with modern low-floor trains, LRT is heavier because the vehicles are usually wider or there are two to three vehicles coupled together. The terms ‘heavy’ or ‘light’ do not solely refer to weight, but also to the flexibility of a system to deal with different types of right-of-way and to the ability to be integrated into a variety or urban streetscapes [3].

Almost all of today’s companies have concerned about reduction of energy consumption. Reduction of energy consumption might refer to the energy consumed during production operations, supply chain-transportation and/or the energy consumed during functioning of the products after sales [4].

Concerned about rising energy costs, rail transit operators have implemented energy conservation strategies to maintain sustainability of rail operations. To improve overall operation efficiency, various methods to reduce energy consumption of passenger railway service were made, such as energy efficient locomotives, peak demand control of power supply, and reduction of train resistance between wheels and tracks, and regenerative braking system, which are very expensive [5].

The problem of reducing energy consumption in railway systems has received lot of attention in recent years because of its known impact on economy and environment [4]. The goal of this research is to find low passenger flow stations in off-peak hour and then the train drivers are made pass these stations within a given time schedule to reduce the usage of electrical energy. This task can be accomplished by providing train drivers with satisfying timetables in off-peak hour.

1.2 Addis Ababa Light Rail Transit

Addis Ababa is the capital of Ethiopia and it is growing exponentially every year. As the city grows so does the need for mass transportation needs. The Addis Ababa Light Rail Transit will have a total of 41 trains each with a capacity to carry 286 passengers. This will enable the light rail transit to provide transportation service to 15,000 people per hour in one direction and 60,000 in all four directions. Addis Ababa Light Rail Transit Project is a semi-closed urban rail transit system in the composed of the E-W and N-S lines. For the project, modern tram car (DC750V power supply type) is used as the passenger train; DC750V diversified power supply system is adopted for the power supply system; the vehicles are 70% low-floor articulated 6-axle modern trams, consisting of three modules, bidirectional driving with maximum operating speed of 70km/h; two tramcars are able to operate with double heading [6].

The planned main line of the project has a total length of about 75km. The scope of works designed only include E-W and N-S in phase I works, covering the main line in total length of about 31.025 km (including a common rail section of about 2.662km long). The starting point of the main track of E-W line is located near to the south gate of Tor Hailoch Hospital and the terminal point located at Ayat. The starting point of the main track of N-S line is located at the east of St. George Church and the terminal point near to Kality Loop. For E-W line, Ayat Depoted is set near to the terminal point; for N-S line, Kality Depoted is set near to the terminal point [15].

In this research the case study is taken on the line of E-W Addis Ababa light rail transit. A total of 22 stations are designed on the E-W line among which there are 5 common rail stations with N-S. The maximum and minimum intervals between stations are 1.26km and 0.435km respectively. The average Station interval is 0.798km [15].

1.3 Train driving modes

A driving pattern can describe a train operation either as a function of the elapsed time or the number of driven distance. Each of these is described with a set of equations that describe the operation in any phase. In general train motion behaviors are separated in the regimes of acceleration, constant speed, coasting (rolling), braking and stop time as [8].

Acceleration: The train's speed is increasing. This can be due to start from a station, signal or in connection with a change in speed limits. Some limiting cases would be constant acceleration, constant traction force, or constant locomotive power. During the acceleration, the traction force required is usually significantly larger than the aerodynamic, rolling or possibly gradient resistance.

Constant speed operation: Since the speed is considered constant, the required traction force is equal to the sum of the driving resistances (aerodynamic, rolling and gradient). In model developed here, a minimum acceleration is defined, under which it is assumed that the acceleration is zero.

Coasting: Both rolling and braking are portions of the operation where the speed decreases. In the case of rolling, the braking force and the traction force are both equal to 0. That is, it is alone the driving resistances that determine the speed of the train.

Braking: In this condition the brakes are active. That is, the traction force is 0, while both the braking and driving resistances slow the train down. Roughly there are two different types of braking; regenerative braking and mechanical braking. During regenerative braking returning energy to the grid is translated into negative energy consumption. During mechanical braking the train is neither saving nor consuming energy.

The different modes of the train movement based on speed are shown below in figure 1.1:

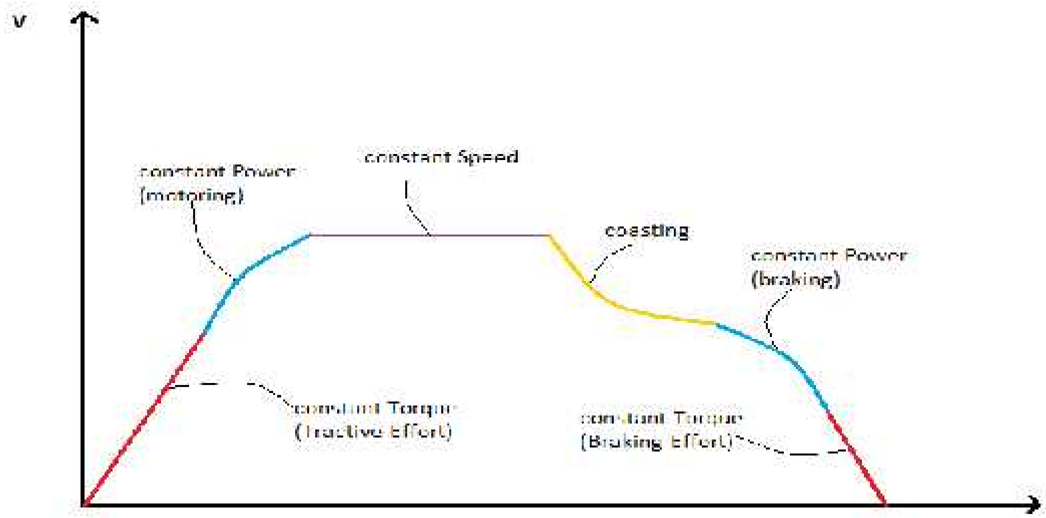


Figure 1.1: Train modes based on power [4]

1.4 Train acting forces

There are mainly three types of forces acting on train movement. These are train resistance , tractive effort and braking force. Trains resistance is defined in terms of force required to encounter resistance arising due to vehicle, track, grade, curve, acceleration, air at different time and place etc. The train Resistance is divided into internal and external resistance. The internal resistance is internal to the train and prevailing track geometry over the entire train run. Internal resistance plays different role during start and running and subdivided so. External resistances are situational in nature and those which are not fixed and depend on varying terrain (gradient and curve) [28].

The force which a locomotive can exert when pulling a train is called its tractive effort, and depends on various factors. For electric locomotives, which obtain their power by drawing current from an external supply, the most important are weight and speed [29]. The term tractive effort is often qualified as starting tractive effort, continuous tractive effort and maximum tractive effort. These terms apply to different operating conditions, but are related by common mechanical factors: input torque to the driving wheels, the wheel diameter, coefficient of friction (μ) between the driving wheels and supporting surface, and the weight applied to the driving wheels (m). The product of μ and m is the factor of adhesion, which determines the maximum torque that can be applied before the onset of wheel spin or wheel slip [28]. The general curve of resistance and tractive effort are shown in figure 1.2 below.

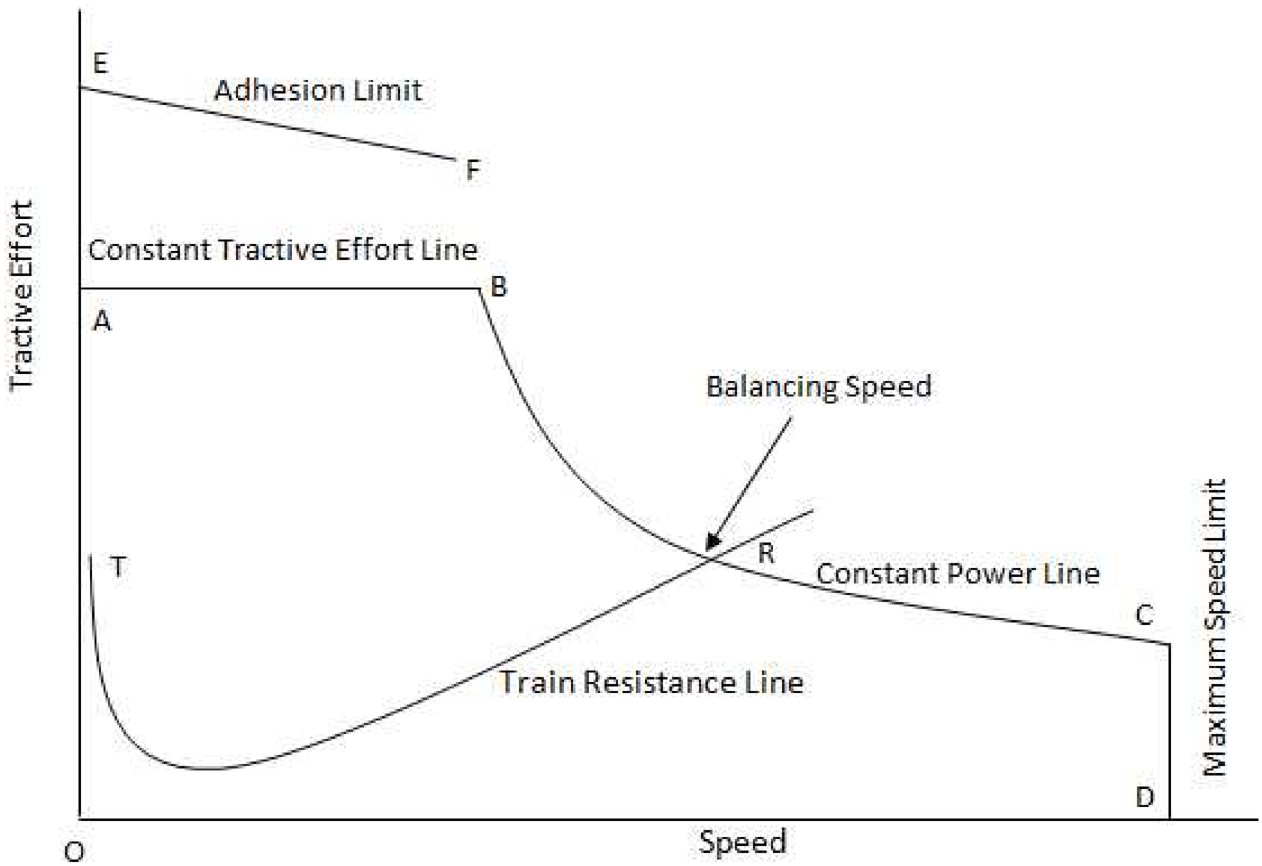


Figure 1.2: Tractive effort and resistance as a function of speed [28]

AB is constant tractive effort line. Point B is the maximum power point. Both train speed and traction motor power increase simultinesly up to point B and the power becomes constant after point B. OA is decided based on the continuous rating of Traction Motor and adhesion limits. This curve is lifted upward by overloading traction motor utilizing thermal capacity of the traction motor.

BC This is a constant power line in which tractive effort drops with increasing speed in inverse proportion. For passenger locos, need for tractive effort is less as compared to speed therefore tractive effort characteristics at lower end is at reduced power.

CD Maximum limit of speed is imposed on the mechanical design of the traction motor and mechanical transmission.

TR This is train resistance curve. Point of intersection determines the speed at which tractive effort equals train resistance. Acceleration reserve exists up to the point of intersection. The curve TR moves up and down while running on up and down gradient.

EF This line gives the limit of adhesive weight and tractive effort has to be within this limit.

1.5 Principle of power transmission for electric trains

The evolution of electric motors for railways and the development of electrification from the middle of the 20the century meant that this kind of motor was suitable for railways. Nowadays, practically all commercial locomotives are powered by electric motors. The major drawback of electrical traction is the high cost of the infrastructure required to carry the electrical energy to the point of usage. This requires constructing long electrical supply lines called *"catenary"*, (Figure 1.3). In addition, the locomotives need devices that enable the motor to be connected to the catenary: the most common being "pantographs" or the so-called "floaters". In its favour, electrical traction can be said to be clean, respectful of the environment and efficient, as an optimum regulation of the motors can be achieved [7].

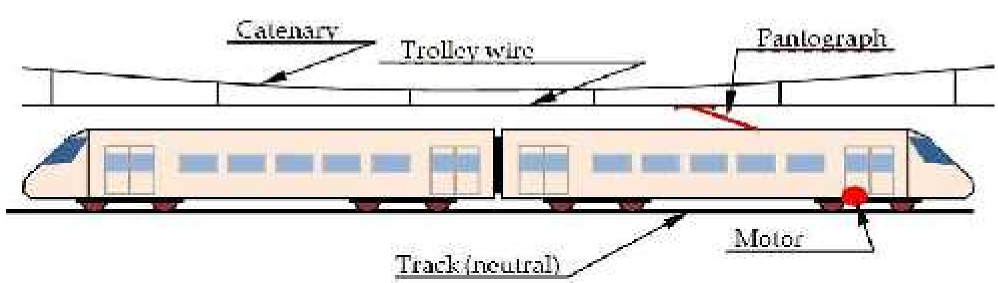


Figure 1.3: General outline of electric railway traction [7]

The locomotive receives electricity for the electric traction motors directly from the electricity lines along the track. Electrical equipment on the locomotive adapts the electricity to that type used by the traction motors as shown in figure 1.4 and 1.5.

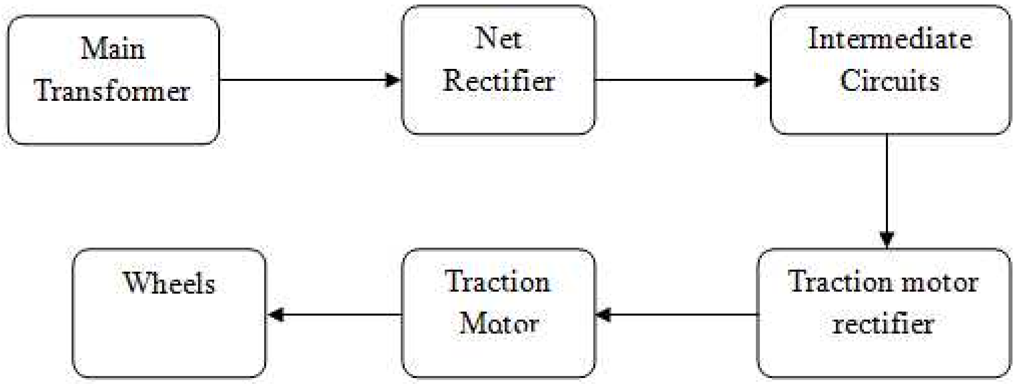


Figure 1.4: Principle of transmission for electric trains [8]

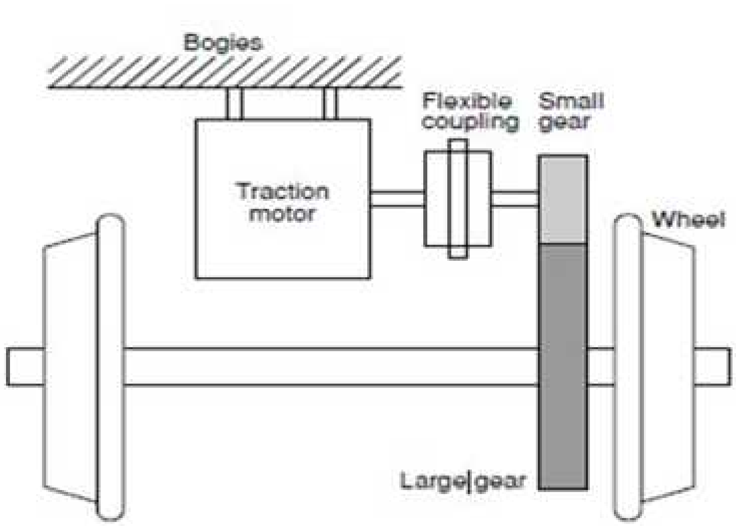


Figure 1.5: Transmission of force (power) from the power source to the wheel sets [9]

1.6 Statement of the problem

In train operation the main problem is the wastage of energy. The motion of the train between two consecutive stations is constrained by various parameters such as the alignment of the track, the status of the signals, the separation of the stations, the nature of the train and various requirements of the passengers. The numbers of passengers is variable in case of time and place. There are usually specified traveling time requirements by passengers. At the same time, the operating company needs to conserve the operating energy. These two requirements of the operating company and of the passengers are usually antagonistic to one another that it is usually difficult to satisfy both of them at the same time. But there is made balancing between theses antagonistic to reduce energy consumption and to satisfy the passengers.

The problem to be solved in this research is to reduce the energy consumption of Addis Ababa LRT E-W line by passing low predicted passenger flow stations in off-peak hour and how can this get further improved to reduce energy consumption in order to be able to be used on work by the ERC.

Due to this, there are different questions to be answered.
The current research questions that can be answered:

- How can we study and select low predicted passenger flow station?
- What are the forces that affect passenger train movement?
- What are the methods that can be followed for analyzing train acting forces in this research?
- How can the energy consumption be affected by train resistance?
- How can we formulate dynamic model of train operation?
- How can the train energy consumption be affected by number of stations?
- What is the method to analyze energy consumption of the train?
- How can we simulate energy consumption of the train operation?
- How can we show graphically the result of the simulation?
- How can we compare the energy consumption reduction in percentage?

Many other questions can be diverted to solve the problem in the case of train energy consumption.

1.7 Objective of the research

1.7.1 Major Objective

The major objective of this research is to analyze the train energy consumption reduction of Addis Ababa LRT E-W line by passing the predicted low passenger flow stations in off-peak hour by taking the two cases the train stops at all stations at the time of peak hour and the train passes low passenger flow stations at the time of off-peak hour.

1.7.2 Specific Objectives

- Study passenger load at each stations of line EW of AA LRT and predict low passenger flow stations.
- Determine the important forces acting on the train motion and modeling the train resistances.
- Modeling the motion and energy consumption of the train.
- Simulate energy consumption of the train using MATLAB simulink software for different number of train stops at stations.
- Make a comparison of the energy consumption in percentage between the train stops at all stations and the train passes the selecting low passenger flow stations.

1.8 Scope and Limitation of the Research

The scope of this study is to calculate energy consumption for train stops at all intermediate station and passes selecting stations in off-peak hour from Ayat to Torhailoch of the east-west line of Addis Ababa LRT of the stations and show the energy consumption reduction. There are 22 train stations from which the cases are taken as: the train stops at all station and the train passes low passenger flow stations in off-peak hour to reduce energy consumption of train movement. The train energy consumption is analyzed in both direction of the train movement from Ayat to Torhailoch and Torhailoch to Ayat for one complete cycle trip. In this thesis the train energy consumption is analyzed the energy used only by traction motors. The train resistances can take a number of factors to increase energy consumption. However, in this research, only main resistances are analyzed in detail due to time restrictions.

1.9 Research Methodology

This research proposes the methods to study the energy consumption reduction of Addis Ababa LRT passenger train.

The method includes the following task to achieve the objectives of the research:

- The research method used in this research is purely quantitative.
- Literature Reviewing: This includes reading related books, research paper, articles and software tools etc.
- Using conventional method based on power and time method to analyze the train energy consumption.
- Using conventional method to model train resistances for analyzing energy consumption.
- Using MATLAB simulink software to simulate the train energy consumption.

After deciding the method, the next step is collecting data to analyze the energy consumption of train. In quantitative research the form of data collected is quantitative data. In this research, the data are divided into primary data and secondary data.

1. **Primary Data**

The primary data is data which are collected in the field based on the existing and reality condition.

1 **Secondary Data**

The secondary data is data which is collected to support the primary data and as comparison to other studies. The secondary data in this research are obtained from the institution or organizations associated with the research object.

For the purpose of this research, and in order to achieve the objectives listed in the sub topic of specific objective of the research the data will be collected through secondary data collection method.

1.10 Significance of the research

Addis Ababa LRT, as servicing industry, is highly concerned about environmental issues in general and more specifically in energy-saving systems in trains. The whole research has a view to result in an advisory system for organization that will lead to reduce energy consumption.

The primary significance of the research is to study the train energy consumption reduction for power saving and cost minimization in Addis Ababa LRT.

This research has the specific significant of reducing shortage of power, reducing wear of wheel and brake pads, reducing traveling time duration, reducing to use limited resources etc.

This work is also believed to contribute to the understanding of the different parameters which are important for determining energy consumption of trains. Energy demand of trains in railway networks are of a growing concern for operators, railway administrators, time table designers and train suppliers. This is due to present and future energy cost and demands of the infrastructure, and also due to the aspect of use of limited resources and pollution, which is associated with the production of the energy needed for train operation.

1.11 Organization of the research

This research is organized into five chapters.

Chapter one introduced about background of LRT and energy consumption, the situation of Addis Ababa LRT, theory of train motion and discussed the requirements for the determination of the energy consumption to reduce costs of operation.

Chapter two summarizes the efforts of previous studies related to the current study.

Chapter three presents the model and analysis of train energy consumption that can be used to compute various train parameters.

Chapter four discusses the results of simulation.

Chapter five concludes the study, gives recommendation and suggests for future work.

Chapter Two

Literature review

2.1 Introduction

Various researches have been conducted in the previous to come up the reduction of train energy consumption using different techniques such as optimal train movement, regenerative braking system, reduction of train resistance between wheels and tracks, energy efficient locomotives, optimal train front shape etc. Some of these techniques have disadvantages in case of expensive to apply on work. The main advantage of the current research there is no any added cost to apply on work. This chapter reviews some of the most important previous works related to the current work.

2.2 Previous Related Worked Papers

The train energy consumption reduction was studied by analyzing the influence of number of train stops at stations. Simulations were performed for express trains, rush hour trains without some suburban stops, and trains with stops at all stations. The result of energy consumption from simulation, Operation without stop results in a reduction of energy consumption of about 50-56% relative to stopping at all stations. By stopping at a selected number of stations, energy consumption was reduced by about 20-36% relative to stopping at all stations as [8]. An operation with many station stops and thus a lot of accelerations and decelerations makes it possible to regenerate more energy. This does not mean however that energy consumption is reduced by having many station stops; the net usage is still higher than compared with an operation with few stops as [12]. The main energy consumption of train operation is during acceleration motion. To show the effect of accelerations, there are considered a number of stops at stations, N_{stops}, along the route, where the train stops, and afterwards accelerates to a maximum speed V_{max}. The velocity is considered to be constant over the entire trip at the average speed, V_{ave}. Assume also, that the steady state load is applied throughout the entire trip. This will compensate for the assumption that the load is estimated at the average speed and not the maximum speed. There was concluded that the acceleration energy consumption plays a dominating role in the many number of stops as [11]. There are different parameters that affect energy consumption of a railway system. Traction energy is used for moving train sets on the line, and its consumption depends on many parameters. From these parameters, line geometry which includes gradients, number of passenger stations and their locations, curves, speed restrictions etc., has big effect on energy consumption of train as [13].

There are different methods of calculating energy consumption of electric train as presented below in different papers.

In paper [18] four main stages of train movement including acceleration, balancing, coasting, and deceleration are proposed. There was showed that the total energy consumed in train operations is the product of force and displacement. There is also proposed a statistical method for estimating energy. However, this approach is an actual measurement and thus, contributes less functions in energy saving. The estimation of energy consumption is usually based on train movement module. In order to obtain the parameters for calculating energy consumption at every time step, a train dynamic model is required. The energy consumed by traction motors is utilized to produce sufficient tractive effort for train movements. It is the majority of total energy consumption of electric trains. In order to study energy saving problems, an accurate energy estimation model is necessary [17]. Estimation of energy consumption can be categorized mainly into electric-power approach and kinematics approach as. The electric-power method calculates electric energy that is directly imported into the train. The kinematics method estimate energy consumption via kinetic energy and efficiency factor. In the proposed model, the kinematics based approach is selected to estimate energy consumption. The reason is that the input data of electric-power based model implies the concept of motor efficiency [17].

The energy consumption of a train can be estimated by conventional methods using the equations of vehicle dynamics. In these methods, the rolling resistance, aerodynamic resistance and gradient resistance can be calculated to give the load required to pull a train at a given speed. The method is based upon the idea of integrating the steady state load and the acceleration load over a given route. The energy consumption for a trip can be obtained by integrating the sum of train resistance forces in addition to the energy required for the acceleration of the train. [11].

There are two main ways to calculate the energy consumption. The first one is based on distance while the second one is based on time as [4]. In the paper [16] a mathematical model of the train has been developed, that allows calculating energy consumption of train. The energy consumption of the trains running in both directions has been calculated for the train movement during traction, speed holding, regenerative braking and blended braking.

The motion of a train can be modeled and represented by the various force components and the motion quantities that act on it at a particular time and location. The force components that act on the train include weight of the train, Tractive Effort (TE), rolling resistance, air resistance, gradient resistance, curvature resistance, brake effort and adhesion. Figure 2.1 shows a simplified diagram of the forces acting on a three car train [30].

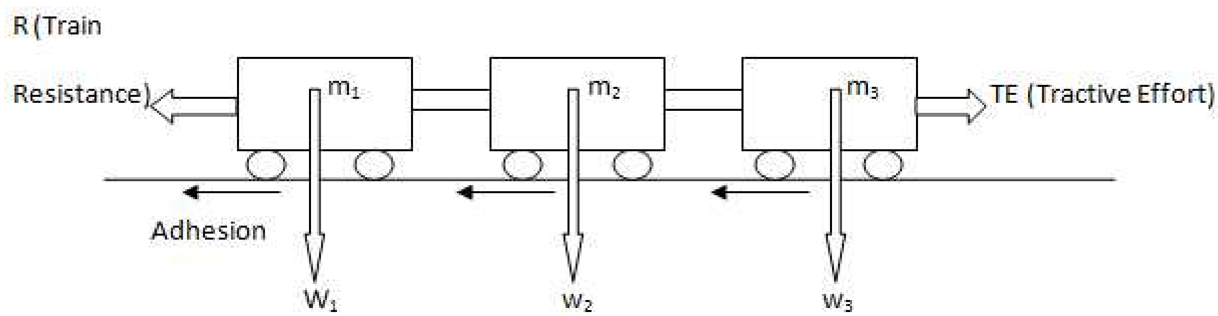
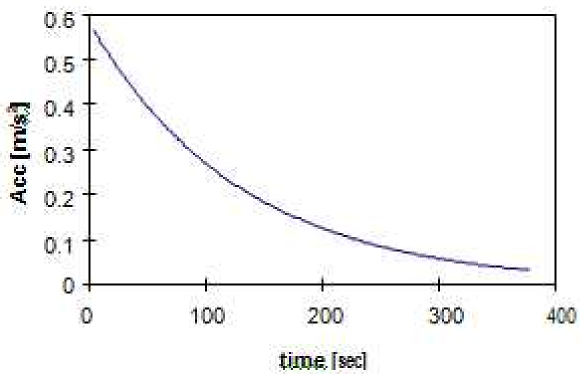


Figure 2.1: Simplified force diagram of a three car train [30]

In order to determine train energy consumption for a variety of situations, or in cases where it is necessary to make estimates, a simulation model is developed to describe the dynamic of the train motion under acceleration as [11]. The model that calculates the acceleration from the formula 2.1 as:

$$a = \frac{\frac{P}{m}\eta}{V} - \sum \frac{F}{m} \quad (2.1)$$

Where P/m is the motor power divided by the mass of the train and F/m is the previously calculated train resistance in Newton per kg and η is the transmition efficiency. The acceleration is a function of the train speed because the train resistance varies with the train speed. The curve of acceleration and speed obtained from simulation model are shown in figure 2.2 and 2.3.



Figur 2.2: Sketch of the train acceleration curve from the simulation model [11]

The velocity is calculated from the equation 2.1 as

$$dv = a(v).dt \tag{2.2}$$

The new velocity is calculated as

$$V_{t+dt} = V_t + dv \tag{2.3}$$

The movement of the train under the total acceleration period distance is formulated as follow:

$$ds = V.dt \tag{2.4}$$

$$S_{t+dt} = S_t + ds \tag{2.5}$$

The deceleration is regarded as a constant negative acceleration.

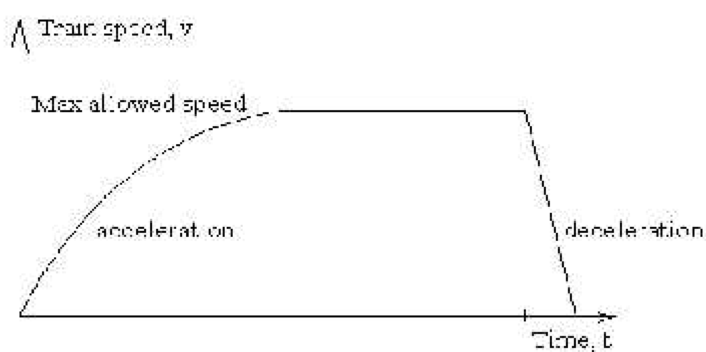


Figure 2.3: Sketch of the train speed curve from the simulation model [11]

Train control for most transit operations represents a cycle of different motion regimes, including acceleration, cruising, coasting, and braking. Figure 2.4 shows the most commonly used train control types. Control I is used when the distance to travel is shorter. Control II operation drives shorter travel time but consumes more energy, compared to those in controls III and IV. Control III operation is commonly used for reducing energy consumption. By using control IV operation, the consumed energy can be further reduced, despite the longest travel time [21].

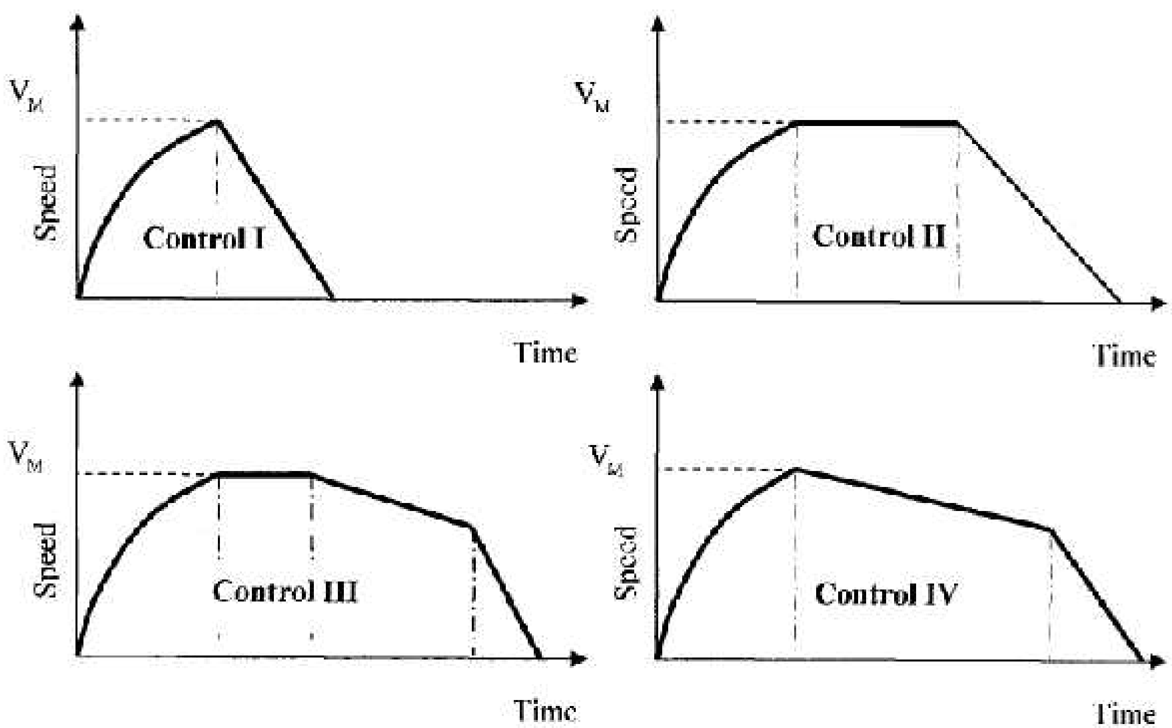


Figure 2.4: Four cases of inter-station train control regimes [21].

For different stations spacing, there can be created different regimes of speed profile based on the distance of between the stations as shown in figure 2.4 above. In this research for all inter-stations control II operation is applied since the AA LRT maximum train operation speed is low which is about 19.44 m/s and the traveling time is very short to reach on this maximum operation speed.

Moving a train along a route involves many force components, including TE, resistance, braking force and train weight. While the TE provides a necessary force to move a train, resistance is the force that opposes the movement and speed of the train. To accelerate or decelerate a train, the

TE must be transferred between wheels and the running surface of the rail through a friction force, called adhesion [21]. TE is the mechanical force available at the rim of the driving wheels of the train. It is usually related to the speed of the train as shown in equation below [36].

$$TE = P*\eta/v \tag{2.6}$$

Equation 2.6 shows that the relationship between the maximum TE and the speed of train is an inverse relation. P is the maximum power developed by the traction motors and is usually a constant value. The transmission efficiency (η) corresponds to the nature of the mechanical force transmission system. Generally the total traction power (P_{tot}) is the sum of the individual maximum power values developed by each of the traction motors [37].

Note In practical situations, not all amount of induced tractive effort can be transmitted to the train, mainly due to the following three conditions [39]:

1) There will be a power efficiency factor η during the process of effort transmission within train's own mechanical structure.
2) In order to prevent a train's motor from overheating, the maximum power is limited to a certain value;
3) Tractive effort should always be lower than an adhesion value, otherwise wheels will just do useless spinning.

Chapter Three

Modeling and Analysis of Train Energy Consumption

3.1 Introduction

For a railway to operate efficiently and safely, its locomotives should be powerful enough to accelerate their trains rapidly to the maximum allowed line speed, and the braking systems must be able to bring a train reliably to a standstill at a station, even on an adverse gradient. There are many different and complex considerations that must be included in a realistic model of railway operation. Here, just some of the main important issues are identified and examined, in order to show how mathematical analysis can be used to provide an indication of expected results [29].

During the course of the train movement, the train dynamics such as the changes of train position, velocity, and acceleration with respect to the change of time are governed by Newton's Second Law of Motion, which is the second-order ordinary differential equation (ODE) of distance with respect to time. The forces acting on a moving train may include tractive effort, train running resistance, alignment resistance, and braking force. Tractive effort provides the propulsion to overcome resistances and to accelerate the train. Running resistance is the force opposing the movement of the train. Alignment resistance is composed of grade resistance and curve resistance. Both are due to the geometry of railway alignment. Braking force is used to decelerate the train and to bring it to full stop. For coasting operation, the tractive effort and braking force of the train are cut off. It is affected only by running and alignment resistance [26].

The overall goal of this section is to predict low passenger flow stations of line E-W and analyze energy consumption of train for Addis Ababa LRT E-W line train operation using matlab simulink in peak hour and off-peak hour time. This can be done from different points of view at different number of stops of train at stations.

3.2 Predicted passenger flow of AA LRT E-W line stations

Based on the passenger transport survey the passenger flow of E-W route in Addis Ababa LRT is forecasted as 734.4 thousands persons/day. Line E-W goes along the important E-W transportation corridor in Addis Ababa. This line goes through city center, and is one of the most important contacting lines between city center and city western parts and western suburbs. The eastern and central parts of this line go through highly populated areas, the busiest commercial areas and Ayat residential area with large traffic flow; the western part of the line goes through western city and western suburbs (10).

It is forecasted that the maximum unidirectional section passenger flow of Line E-W in peak hours will take place in the road section from Lideta Light to La Gare shared with Line N-S. The section from Lideta Light to La Gare of line E-W locates in Addis Ababa center with high population. This section is the convergence of E-W route and N-S route which have the largest passenger flow. As the population density decreases along the line and the road section is not shared by two lines, passenger flow decreases also. This passenger flow distribution conforms to railway transportation characteristics in cities (10).

The predicted number of passengers flow on/off trains per hour of each station time are indicated in table 3.1 and 3.2 for the long term by taking the number of passenger flow stations in the peak hour of morning from paper [15]. The peak hour passenger flow of each station is determined by multiply with peak hour factor. Based on this passenger flow of each station, there is selection of the relative low passenger flow of stations to make the train passes these low passenger stations in off-peak hour. There is consideration of the spacing between stations to pass only one station from two sequence low number of passenger's stations except very low passenger flow sequence stations. To select low number of passengers, there is taken limitation by finding the total average number of passengers flow for both direction of W-E and E-W. It is considered that the limitation becomes less than the average number of passengers getting on/off train per hour for each station. So the average number of passengers getting on/off train per hour for W-E and E-W line is 1467 and 1478 respectively.

Table 3.1: Predicted Passenger Flow of line W-E direction stations (15)

S/N	Station Name	Station location	W-E line direction		
			Number of Passengers Getting On Trains per hour	Number of Passengers Getting Off Trains per hour	Total Number of Passengers Getting On/off Trains per hour
1	EW1	Ayat2	0	1622	1622
2	EW2	Ayat1	6	1530	1536
3	EW3	CMC Intersection	23	1204	1227
4	EW4	Kotebe Mikael Church	67	1182	1249
5	EW5	Municipal Service Institute	114	929	1043
6	EW6	S-ealite Mihret Church	141	679	820
7	EW7	Ethiopia Communication	116	430	546
8	EW8	Ethiopia Geological Survey	148	377	525
9	EW9	Megenagna	608	947	1555
10	EW10	Lem Hotel	1895	1459	3354
11	EW11	Togo Intersection (Traffic Police HQ)	1605	1499	3104
12	EW12	Holiday Hotel	675	840	1515
13	EW13	Kidus Urael	560	951	1511

		Church			
14	EW14	Yordanos Hotel	543	711	1254
15	EW15	St.Estifanos Church	667	670	1337
16	EW16	Addis Ababa Stadium	896	576	1472
17	EW17	Gambia Intersection (Legahar)	1097	373	1470
18	EW18	Ethiopia Road Department	952	145	1097
19	EW19	Mexico Square	1064	74	1138
20	EW20	Lideta Mariam Church	1111	38	1149
21	EW21	Coca Cola Intersection	1464	16	1480
22	EW22	Torhailoch Hospital	2250	0	2250

Table 3.2: Predicted Passenger Flow of line E-W direction stations (15)

S/N	Station Name	Station location	E-W line direction		Total Number of Passengers Getting On/off Trains per hour
			Number of Passengers Getting On Trains per hour	Number of Passengers Getting Off Trains per hour	
1	EW1	Ayat2	1635	0	1635
2	EW2	Ayat1	1511	6	1517
3	EW3	CMC Intersection	1210	23	1233
4	EW4	Kotebe Mikael Church	1193	68	1261
5	EW5	Municipal Service Institute	936	115	1051
6	EW6	S-ealite Mihret Church	686	142	828
7	EW7	Ethiopia Communication	435	117	552
8	EW8	Ethiopia Geological Survey	380	149	529
9	EW9	Megenagna	756	612	1368
10	EW10	Lem Hotel	1409	1909	3318
11	EW11	Togo	1513	1617	3130

		Intersection (Traffic Police HQ)			
12	EW12	Holiday Hotel	845	679	1524
13	EW13	Kidus Urael Church	958	564	1522
14	EW14	Yordanos Hotel	716	547	1263
15	EW15	St.Estifanos Church	675	680	1355
16	EW16	Addis Ababa Stadium	580	904	1484
17	EW17	Gambia Intersection (Legahar)	376	1105	1481
18	EW18	Ethiopia Road Department	146	963	1109
19	EW19	Mexico Square	76	1072	1148
20	EW20	Lideta Mariam Church	38	1122	1160
21	EW21	Coca Cola Intersection	16	1742	1758
22	EW22	Torhailoch Hospital	0	2284	2284

The low number of passengers flow of stations is the same for both directions of W-E and E-W. Therefore the selection of stations to pass by train can be the same for both route to make uniform the schedule of train operation. The overall selection of stations to stop or pass the stations by train during off-peak hour is shown in table 3.3.

Table 3.3: Selecting stations for the train stopping (S) and passing (P)

S/N	Station Name	Selecting stations for stopping and passing			
		Stopping at all stations for both line of **E-W** and **W-E** during peak hour	Stopping and passing stations for line of **E-W** during off-peak hour	Stopping and passing stations for line of **W-E** during off-peak hour	Recommendation
1	EW1	S	S	S	The train always stops here since the station is the place of starting and ending of a train
2	EW2	S	S	S	high passenger flow
3	EW3	S	S	S	Low passenger flow but better pass EW7 and EW8 by stopping here
4	EW4	S	S	S	Low passenger flow but better pass EW7 and EW8 by stopping here
5	EW5	S	P	p	Low passenger flow
6	EW6	S	S	S	Low passenger follow but balance the distance
7	EW7	S	P	P	Low passenger flow
8	EW8	S	P	P	Low passenger flow
9	EW9	S	S	S	Low passenger flow but balance the distance
10	EW10	S	S	S	High passenger flow
11	EW11	S	S	S	High passenger flow
12	EW12	S	S	S	High passenger flow
13	EW13	S	S	S	High passenger flow

14	EW14	S	P	P	Low passenger flow
15	EW15	S	S	S	Low passenger flow but balance the distance
16	EW16	S	S	S	High passenger flow
17	EW17	S	S	S	High passenger flow
18	EW18	S	P	P	Low passenger flow
19	EW19	S	S	S	Low passenger flow but balance the distance
20	EW20	S	P	P	Low passenger flow
21	EW21	S	S	S	High passenger flow
22	EW22	S	S	S	The train always stops here since the station is the place of starting and ending of the train

The above table 3.3 shows the selected stations for train stopping or passing stations during operation based on the data of tables 3.1 and 3.2. The symbol S and P indicate stop and pass respectively. At the time of peak hour, the train is made stop at all stations but at the time of off-peak hour, the train is made pass selected low passenger flow stations. There is considering spacing between stations to pass only one station from two low number of passenger's sequence stations. But two very low passenger flow sequence stations like EW7 and EW8 are made pass by train to compensate other low passenger stations such as EW3 and EW4. The sum of number of passengers of stations EW7 and EW8 is less than Stations EW3 and EW4.

In general, the stations that are determined to pass by train are indicated in figure 3.1 and 3.2 by red color for both directions of line W-E and E-W.

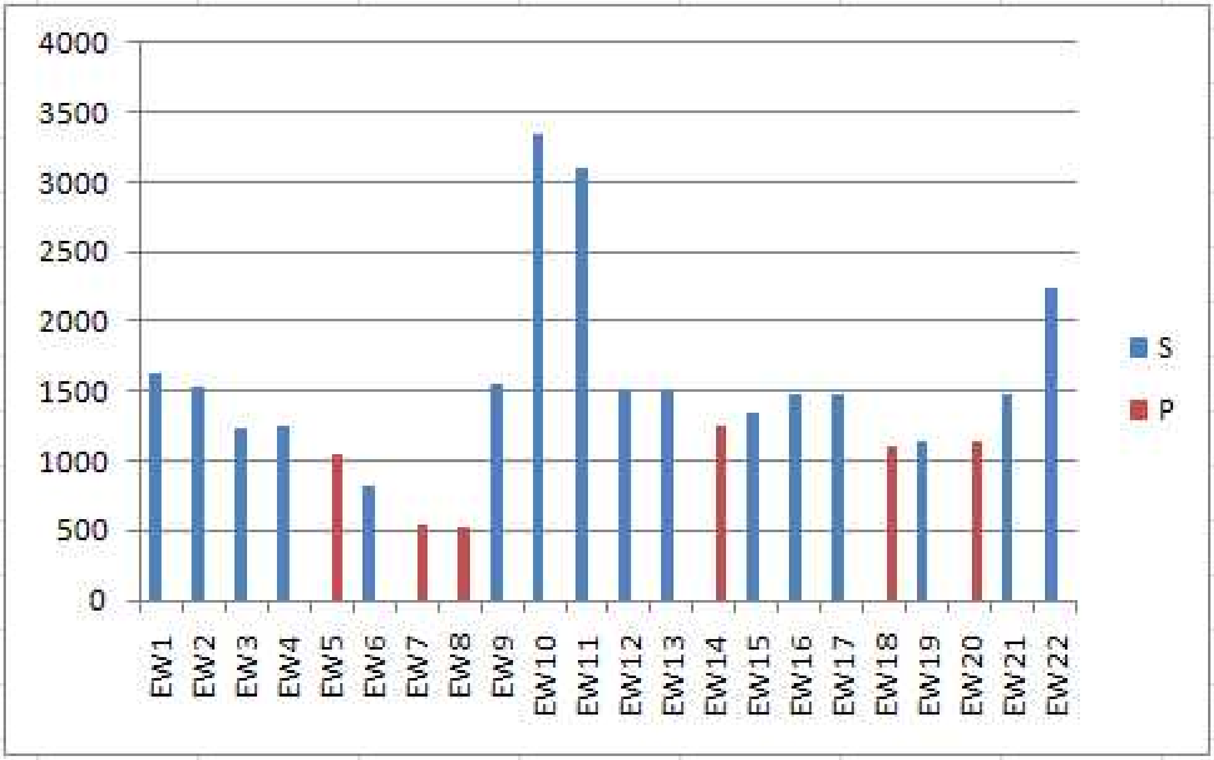


Figure 3.1: Chart of selecting stations to pass (P) or stop (S) for W-E direction stations

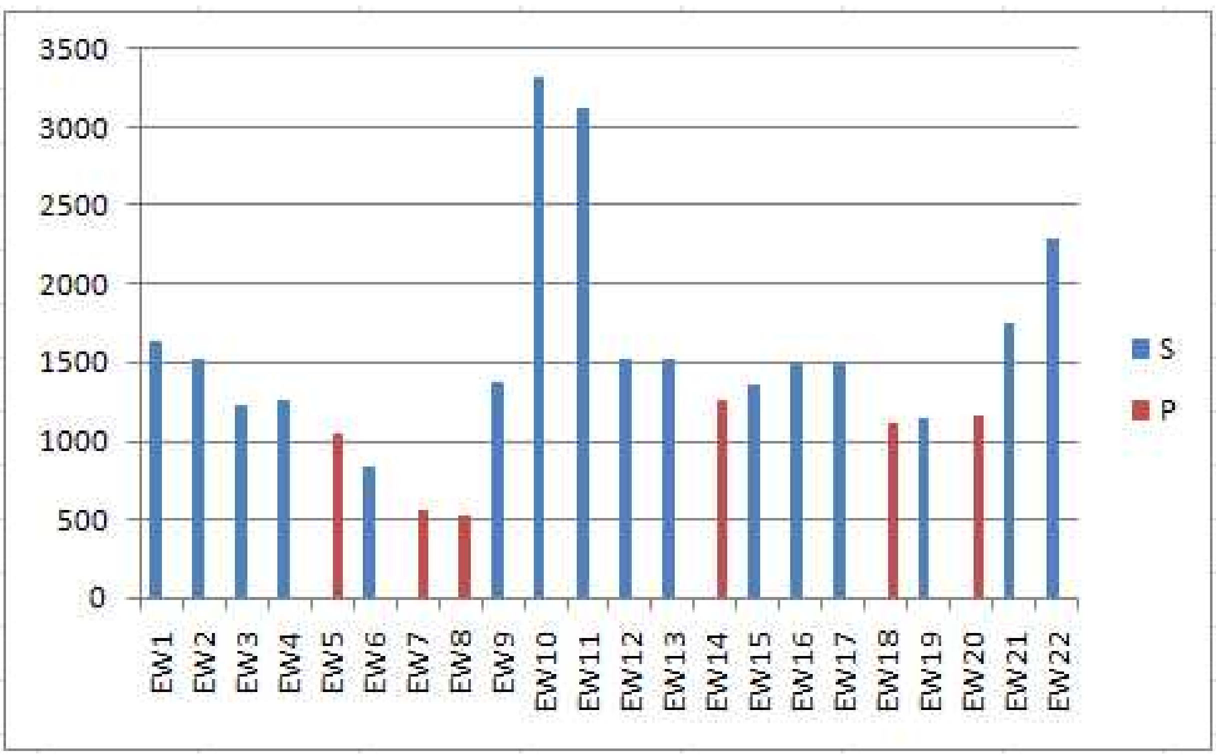


Figure 3.2: Chart of selecting stations to pass (P) or stop (S) for E-W direction stations

3.3 Simulation

Simulation is defined as the construction of a mathematical model to reproduce the characteristics of a phenomenon, system, or process, often using a computer, in order to solve problems. In this research the software which is used to simulate the model is Matlab simulink.

Simulink provides an interactive, graphical environment for modeling, simulating, and analyzing of dynamic systems. It enables rapid construction of virtual prototypes to explore design concepts at any level of detail with minimal effort. For modeling, Simulink provides a graphical user interface (GUI) for building models as block diagrams. It includes a comprehensive library of pre- defined blocks to be used to construct graphical models of systems using drag-and-drop mouse operations. It supports linear and nonlinear systems, modeled in continuous-time, sampled time, or hybrid of the two. The interactive nature of Simulink encourages you to try things out, you can change parameters “on the fly” and immediately see what happens, for “what if” exploration [40].

There are six steps to get the required results from the simulation [41]

1. **Defining the System**

The first step in modeling is defining the dynamic system of the train. There are a large system that can be broken into parts, there should be modeled each subcomponent on its own. These are train resistance forces such as acceleration, aerodynamic, rolling, gradient, curve resistance etc.

2. Identifying System Components

The second step in the modeling process is to identify the system components.

There are three types of system components:

• **Parameter**: System values that remain constant unless they are changed, such as rolling coefficient, aerodynamic coefficient, air density, gravitational acceleration etc.

• **State**: Variables in the system that change over time, such as train speed, acceleration, aerodynamic force, tractive force, energy consumption etc.

• **Signals**: Input and output values that change dynamically during a simulation. The input value such as speed and acceleration are changed, and then the output value of energy consumption is changed.

3. **Modeling the System with Equations**

The third step in modeling a system is to formulate the mathematical equations that describe the system. For each subsystem, use the list of system components that they are identified to describe the system mathematically.

The model may include:

- Algebraic equations
- Logical equations
- Differential equations, for continuous systems
- Difference equations, for discrete systems

These equations are used to create the block diagram in Simulink at different track profile of

- level position
- uphill position
- downhill position
- horizontal curvature position

4. Building the Simulink Block Diagram

After the mathematical equations are defined that describe each Subsystem, there can be constructed a block diagram of model in Simulink.

5. Running the Simulation

After the Simulink block diagram is constructed, simulate the model and analyze the results. Simulink allows us to interactively define system inputs, simulate the model and observe changes in behavior.

6. Validating the Simulation Results

After the results are obtained, it must be checked by comparing with expectation results.

3.4 Passenger Train Movement Model

For determining train energy consumption, it is important to know the driving modes and the forces acting on train's motion.

During the train trip from its start station to end station, by alternating powering, coasting, and braking modes, the train has four operation regimes, including acceleration, deceleration, constant speed, and stop. The movement of a train along a route is governed by the equation of motion and constrained by speed limits. The motion of a train along a route is influenced by many forces, including tractive effort, train resistance, braking forces, and equivalent mass of train [20].

The tractive effort and braking force can be controlled by train driver or ATC (automatic train control) computer. Therefore, there are three operating modes in driving a train, as listed in Table 3.4 below.

Train operating mode		Tractive effort condition	
		$F_t = 0$	$F_t > 0$
Braking Force condition	$F_b = 0$	Coasting mode	Powering mode
	$F_b > 0$	Braking mode	______

Table 3.4: Train driving Modes [20]

The operation of a train along a line must obey the equation of motion and the speed constraints. As a result, station to station movement may consist of four operation regimes, such as stop, acceleration, constant speed, and deceleration. The conditions of the net force acting on the train and its velocity for each regime are summarized in Table 3.5 below.

Operation Regimes	Net force	Velocity
Stop	$F_t - F_{tot} - F_b = 0$	$V = 0$
Acceleration	$F_t - F_{tot} > 0$	$0 \leq V \leq V_{max}$
Constant Speed	$F_t - F_{tot} = 0$	$V = V_{max}$
Deceleration	$F_t - F_{tot} - F_b < 0$	$0 \leq V \leq V_{max}$

Table 3.5: Force and Velocity Conditions for Four Operation Regimes [20]

As shown in table 3.5 a locomotive's tractive force is required to overcome the total resistance (F_{tot}) to motion of both locomotive and train. When the tractive force is greater than the total resistance, then the train will accelerate in accordance with Newton's law of motion. If the tractive force is equal to the total resistance, then the train will travel at constant speed. If the tractive force is less than the sum of total resistance and braking force, then the train will decelerate or slow down to stop.

It is known that the formula that relates the sum of forces with the acceleration is Newton's second law as shown in figure 3.3:

$$\sum F = m_{tot} * a \tag{3.1}$$

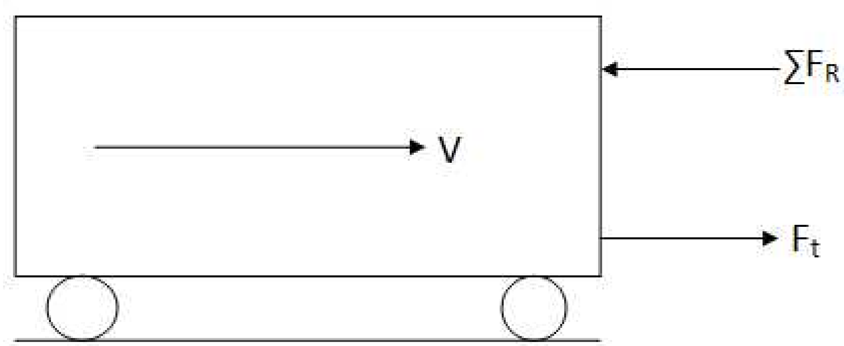


Figure 3.3: Longitudinal train dynamics [7]

$\Sigma \mathbf{F_R}$ is the sum of resistance force

In this case, **a** is the acceleration that the train is experiencing when all the different forces F are applied to it. Thus, the connection between the train resistances and motion is given from Newton's 2nd law as follow (20).

$$f_m * m * a = F_t - F_r - F_a - F_c \pm F_g \quad (3.2)$$

Where,

m: sum of vehicle and passengers mass

f_m: mass factor

a: train acceleration

F_t: the locomotive's traction force at the wheels

F_r: the rolling resistance

F_a: the aerodynamic resistance

F_g: the gradient resistance

F_c: the curve resistance

The plus or minus of gradient resistance shows that for train moves down taken as plus and when the train moves uphill taken as minus.

The ability to calculate the energy needed is the traction force needed at the wheels which will result in the train overcoming the resistances and reach the desired acceleration and speed.

$m*a$ is normally called the acceleration force F_A, or acceleration resistance. So the equation (3.2) becomes:

$$F_A = F_t – F_r - F_a - F_c \pm F_g \quad (3.3)$$

By rearranging the total train resistance from equation 3.3

$$F_t = F_{tot} = F_r + F_a \pm F_g + F_c + F_A \quad (3.4)$$

In the case where the tractive force is greater than the sum of the total resistance, then the train can accelerate, or climb a grade.

In the case of speed holding motion of train, the equation motion is expressed as

$$F_t - F_r - F_a - F_c \pm F_g = 0 \quad (3.5)$$

During this condition since acceleration is zero, acceleration resistance becomes zero.

By rearranging the total train resistance from equation 3.5

$$F_t = F_{tot} = F_r + F_a + F_c \pm F_g \tag{3.6}$$

Here, model of train deceleration motion is not considered since no power consumption in the deceleration motion since tractive effort becomes zero**.**

3.5 Model of train resistance

The train resistance equations of aerodynamic, rolling, track gradient and track curvature which are used in the analysis of train energy consumption presented as follow.

3.5.1 Aerodynamic resistance

The air in front of and around a vehicle in motion causes resistance to the movement of the vehicle, and the force required to overcome this resistance is known as air resistance. The magnitude of this force depends on the square of the velocity at which the vehicle is traveling and the cross-sectional area of the vehicle. The formula presented by [24] for the calculation of aerodynamic resistance is:

$$F_a = 0.5 * \rho * C_a * A_{fr} * V^2 \tag{3.7}$$

Where

F_a: total aerodynamic resistance (N)

V: train's speed (m/s)

ρ: air density (kg/m^3)

C_a: aerodynamic coefficient

A_{fr}: the frontal area (m^2)

Note take variable speed V_i for energy consumption and acceleration calculation during acceleration motion. The value of aerodynamic coefficient depends on the train type and it is taken from technical specification of table 3.6.

3.5.2 Rolling resistance

Rolling resistance is the sum of the mechanical forces, exclusive of windage and braking, acting to impede the forward motion of a train traveling at constant speed on level track under operating conditions. Two factors normally determine the rolling resistance of a vehicle. The first is the weight, which is not a train parameter, but an operational variable. The second is the rolling resistance coefficient, C_r. This is a parameter for the type of train/wagon involved in the calculation. The formula as presented in [24] for the calculation of total rolling resistance is:

$$F_r = C_r * m * g \tag{3.8}$$

Where,

F_r: total rolling resistance (N)

C_r: rolling resistance coefficient

m: sum of vehicle and passenger mass (kg)

g: acceleration of gravity (9.82 m/s2)

The weight of the total train is given; g is a constant and $C_{r\ coefficient}$ depends on train type and it is taken from technical specification table 3.6.

3.5.3 Gradient resistance

In a rail vehicle rolling along a straight level track, the force component perpendicular to the direction of gravity is zero. However, when the plane of the track is inclined (when the train runs uphill or downhill), a force component F_g develops parallel to the plane of the track (Figure 3.4). In the case of an uphill gradient this component is an additional resistance to vehicle motion and in the case of a downhill gradient this component is an additional force to make move the train (26).

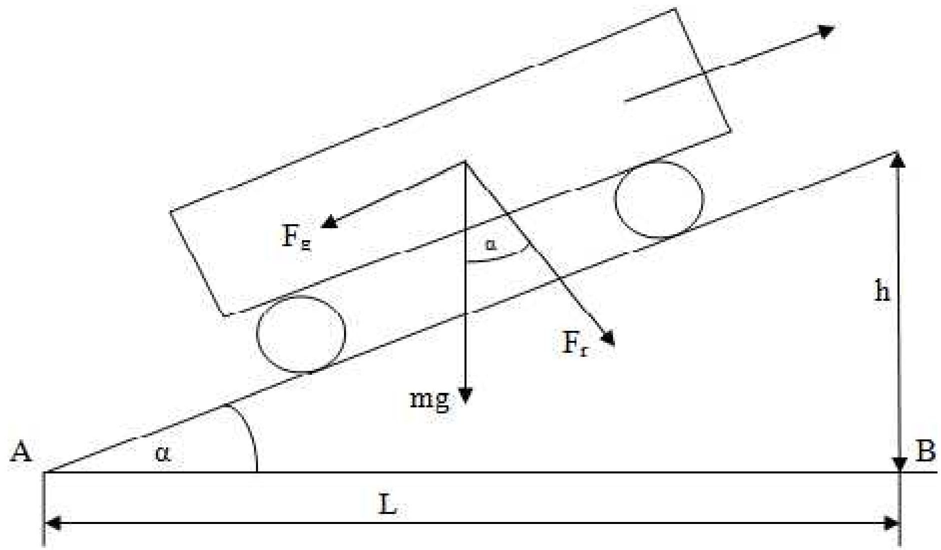


Figure 3.4: Illustrate gradient resistance (26)

The formula as presented in [26] for the calculation of gradient resistance is:

$$F_g = m*g*\sin\alpha = m_{tot}*g*\frac{r_g}{1000} \quad (3.9)$$

Where,

F_g: total gradient resistance (N)

m: sum of vehicle and passengers mass (kg)

g: acceleration of gravity (9.81 m/s2)

α: angle of the gradient

r_g: gradient (‰) which is taken from case study data table 3.7

3.5.4 Curve resistance

When the train travels on a curve section of its travel way, external forces act on the vehicle. Certain components of these forces tend to retard the forward motion of the vehicle. The sum of these components is the curve resistance. The formula as presented in [26] for the calculation of curve resistance is:

$$F_c = r_c mg/1000 \quad (3.10)$$

Where,

$r_c = K/Rc$

F_c = curve resistance (N)

r_c = coefficient of curve resistance (‰)

R_c = radius of horizontal curvature

For a specific rail system, the above factors are considered to be known data except radius, which varies with alignment geometry. Hence, r_c can be expressed as a constant K multiplied by the reciprocal of the radius R_c (m). The typical value of K in equation above ranges from 500 to 800, depending on rail systems. The value of K is 700 commonly used for passenger train [26]. Therefore the value of K is 700 used in this research.

3.6 Model of Energy consumption

The total train energy consumption can be calculated in several ways as presented in the literature review section. There are two main ways to calculate the energy consumption. The first one is based on distance while the second one is based on power and time. The model that was preferred for this thesis for analyzing energy consumption using simulink is the one based on power and time as shown in equation 3.12. The reason, this model is best for matlab simulink simulation. For analytical analysis, energy consumption calculation is based on knowledge of train resistance and distance as shown in Equation 3.11 [4].

The energy consumption of train is studied only the energy used by the traction motors. Energy generation during regenerative braking was calculated in some papers because of train speed is high. But in this research the train speed is low which is about maximum 70 km/h. Whether the regenerative braking is available or no, it has no significant value in reduction of energy consumption. So the regenerative braking analysis is not considered in this research.

a. Energy consumption calculation based on knowledge of train resistance and distance

By integrating the instantaneous force over the traveled distance:

$$E = \int_{x1}^{x2} F_{tot}\, dx = F_{tot} \Delta X \qquad \text{...(11)}$$

Where F_{tot} is the sum of the train resistances

b. Energy consumption calculation based on power and time

The total power P_{tot} needed can be calculated as

$$P_{tot} = F_{tot} * V_i \quad (3.12)$$

Where,

P_{tot}: total power (W)

V_i: train speed (m/s)

F_{tot}: sum of the train resistances (N)

Based on that the energy consumption is obtained by integrating the total power needed for the total trip time. This gives

$$E = \int_{t_{i-1}}^{t_i} P_{tot}\, dt \quad (3.13)$$

Where,

E: total energy consumption (J)

P_{tot}: total power (W)

t: elapsed time (s)

If P_{tot} is constant, the energy consumption is:

$$E = P_{tot} * t \quad (3.14)$$

Where $\mathbf{F_{tot}}$ is the sum of the train resistance as discussed above in equation 3.4 is generalized as follow:

For acceleration motion:

$$F_{tot} = F_A + F_r + F_a + F_c \pm F_g \quad (3.15)$$

For constant speed motion:

$$F_{tot} = F_r + F_a + F_c \pm F_g \quad (3.16)$$

The total train resistance force of different track profile is formulated as follow:

For acceleration and constant speed motion in the level motion of train:

$$F_{tot} = F_A + F_r + F_a + F_c \tag{3.17}$$

$$F_{tot} = F_r + F_a + F_c \tag{3.18}$$

For acceleration and constant speed motion in the uphill motion of train:

$$F_{tot} = F_A + F_r + F_a + F_c + F_g \tag{3.19}$$

$$F_{tot} = F_r + F_a + F_c + F_g \tag{3.20}$$

For acceleration and constant speed motion in the downhill motion of train:

$$F_{tot} = F_A + F_r + F_a + F_c - F_g \tag{3.21}$$

$$F_{tot} = F_r + F_a + F_c - F_g \tag{3.22}$$

In this research it is assumed that energy consumption or energy regeneration during deceleration are not present, that is, there is no considering regenerative braking.

3.7 Model of train motion for simulation

The following variables are used in the model formulation by discretizing the system as shown in the figure 3.5.

t_i represents train traveling time

X_i represents the location of the train

V_i represents the speed of the train

a_i train acceleration

E_i represents the energy consumed by the train

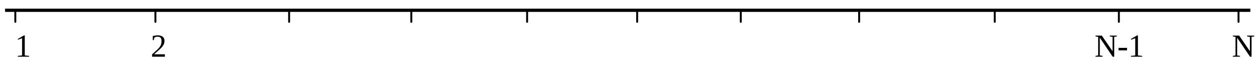


Figure 3.5: Discretization of the system into N elements.

Assuming that there are N discrete values in the figure (3.5) and the following results are obtained from simulation:

$$t_i \in \{t_1, t_2, t_3, \ldots t_N\}$$

$$X_i \in \{X_1, X_2, X_3, \ldots X_N\}$$

$$V_i \in \{V_1, V_2, V_3, \ldots V_N\}$$

$$a_i \in \{a_1, a_2, a_3, \ldots a_N\}$$

$$E_i \in \{E_1, E_2, E_3, \ldots E_N\}$$

Where i = 1, 2, 3,……………N

3.7.1 Model of acceleration motion

Using train motion equations from equation 3.2

$$f_m * m * a_i = F_t(v) - F_r - F_a(v) \pm F_g - F_c \quad (3.23)$$

Express train acceleration interim of other parameters as presented in the literature review (equation 2.2)

$$a_i = \frac{F_t(v) - F_r - F_a(v) - F_c \pm F_g}{f_r * m} \quad (3.24)$$

Where,

$$F_t(v) = \frac{\eta * P_{in,tot}(v)}{V_i} \quad (3.25)$$

$F_t(v)$ is tractive force

$P_{in,tot}$ is total input motors power in watts (W)

η is motor efficiency

V_i is train speed

The input one motor power is produced by applying the voltage and entering the current into the motors as:

$$P_{in} = I * V \tag{3.26}$$

The current can be expressed by dividing voltage by resistance as:

$$I = \frac{V}{R} \tag{3.27}$$

Then the input motor power:

$$P_{in} = I^2 * R \tag{3.28}$$

Where,

I is current in amperes (A) V is applied voltage in voltage (V)

R is resistance in ohm (Ω)

The total output mechanical power of the motors to turn the axles is obtained from input motors power by multiply with motor efficiency.

$$P_{out,tot} = P_{in,tot} * \eta \tag{3.29}$$

Then, equation 3.24 is obtained from the following driving equations:

$$P_{in,tot}(v) = \frac{P_{out,tot}(v)}{\eta} \tag{3.30}$$

$$P_{out,tot} = T * \omega \tag{3.31}$$

$$P_{in}(v) = \frac{T * N * 2\pi}{60 * \eta} = \frac{V_i * F_t(v)}{\eta} \tag{3.32}$$

Where,

$$\omega = \frac{N * 2\pi}{60} \tag{3.33}$$

$$T = \frac{r * F_t}{G} \tag{3.34}$$

$$N = \frac{60 * V_i * G}{2\pi * r} \quad (3.35)$$

$P_{out,tot}$ is output mechanical power of the motor (W)

T is the torque in Nm ω is angular speed

N is the motor speed in rpm r is the wheel radius in m

G is the gear ratio

Express the train acceleration interim of other parameters for level motion:

$$a_i = \frac{P_{in,tot}(v) * \frac{\eta}{V_i} - F_r - F_a(v)}{f_r * m} \quad (3.36)$$

Integrate the train acceleration to obtain the traveling speed:

$$V_i = \int_{t_{i-1}}^{t_i} a_i \, dt = a_i * (t_i - t_{i-1}) + V_{i-1} \quad (3.37)$$

Integrate the train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} a_i t \, dt = \frac{a_i}{2} * (t_i - t_{i-1})^2 + X_{i-1} \quad (3.38)$$

Express acceleration interim of other parameters for uphill motion:

$$f_m * m * a_i = F_t(v) - F_r - F_a(v) - F_g \quad (3.39)$$

$$a_i = \frac{P_{in,tot}(v) * \frac{\eta}{V_i} - F_r - F_a(v) - F_g}{f_m * m} \quad (3.40)$$

Integrate train acceleration to obtain the traveling speed:

$$V_i = \int_{t_{i-1}}^{t_i} a_i \, dt = a_i * (t_i - t_{i-1}) + V_{i-1} \quad (3.41)$$

Integrate train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} a_i t \, dt = \frac{a_i}{2} * (t_i - t_{i-1})^2 + X_{i-1} \quad (3.42)$$

Express acceleration (a) interim of other parameters for downhill motion:

$$f_m * m * a_i = F_t(v) - F_r - F_a(v) + F_g \tag{3.43}$$

$$a_i = \frac{P_{in,tot}(v) * \frac{\eta}{V_i} - F_r - F_a(v) + F_g}{f_r * m} \tag{3.44}$$

Integrate train acceleration to obtain the rated traveling speed:

$$V_i = \int_{t_{i-1}}^{t_i} a_i \, dt = a_i * (t_i - t_{i-1}) + V_{i-1} \tag{3.45}$$

Integrate the train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} a_i t \, dt = \frac{a_i}{2} * (t_i - t_{i-1})^2 + X_{i-1} \tag{3.46}$$

Energy consumption of train acceleration motion based on power and time

$$E_{ai} = \int_{t_{i-1}}^{t_i} F_{tot} V_i dt \tag{3.47}$$

3.7.2 Model of constant speed motion of train

$$F_t(v) - F_r - F_a(v) \pm F_g - F_c = 0 \tag{3.48}$$

Express maximum train speed interim of other parameters for level motion:

$$F_a(v) = F_t(v) - F_r \tag{3.49}$$

$$0.5 * \rho * A_{fr} * C_a * (V_c)^2 = P_{in,tot}(v) * \frac{\eta}{V_c} - F_r \tag{3.50}$$

$$V_c = \sqrt{\frac{P_{in,tot}(v) * \eta / V_c - F_r}{0.5 * \rho * A_{fr} * C_a}} \tag{3.51}$$

Integrate maximum train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} V_c \, dt = V_c * (t_i - t_{i-1}) + X_{i-1} \tag{3.52}$$

Express maximum train speed interim of other parameters for uphill:

$$F_a(v) = F_t(v) - F_r - F_g \tag{3.53}$$

$$0.5*\rho*A_{fr}*C_a*(V_c)^2 = P_{in,tot}(v) * \frac{\eta}{V_c} - F_r - F_g \tag{3.54}$$

$$V_c = \sqrt{\frac{P_{in,tot}(v)*\frac{\eta}{V_c} - F_r - F_g}{0.5*\rho*A_{fr}*C_a}} \tag{3.55}$$

Integrate maximum train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} V_c\, dt = V_c * (t_i - t_{i-1}) + X_{i-1} \tag{3.56}$$

Express maximum train speed interim of other parameters for downhill:

$$F_a(v) = F_t(v) - F_r + F_g \tag{3.57}$$

$$0.5*\rho*A_{fr}*C_a*(V_c)^2 = P_{in,tot}(v) * \frac{\eta}{V_c} - F_r - F_g \tag{3.58}$$

$$V_c = \sqrt{\frac{P_{in,tot}(v)*\frac{\eta}{V_c} - F_r + F_g}{0.5*\rho*A_{fr}*C_a}} \tag{3.59}$$

Integrate maximum train speed to obtain the traveling distance:

$$X_i = \int_{t_{i-1}}^{t_i} V_c\, dt = V_c * (t_i - t_{i-1}) + X_{i-1} \tag{3.60}$$

Energy consumption of train constant speed motion based on power and time

$$E_{ci} = \int_{t_{i-1}}^{t_i} F_{tot}\, V_c dt \tag{3.61}$$

The general formula to calculate train energy consumption of train analytically based on distance is presented as follow.

The acceleration and braking distance is expressed as:

$$V_{max}{}^2 = V_0{}^2 \pm 2aX \tag{3.62}$$

Where,

V_{max} is maximum operation speed in m/s

V_o is initial speed in m/s ($V_0 = 0$)

X is acceleration or braking distance in m

a is acceleration or deceleration m/s^2

(a is **positive** for acceleration motion and a is **negative** for deceleration motion)

Therefore, for acceleration motion the acceleration distance is:

$$X_a = \frac{V_{max}{}^2}{2*a} \tag{3.63}$$

And, for deceleration motion the braking distance is:

$$X_b = \frac{V_{max}{}^2}{2*a} \tag{3.64}$$

The energy consumption of train analysis equation based on distance:

$$E_i = F_{tot} X_i \tag{3.65}$$

3.8 Technical Specifications and track profile data of AA LRT E-W line

The data were collected from Addis Ababa LRT project office of Ayat area. They are comprised of the AA LRT rolling stock and track profile.

Table 3.6: Technical specifications of Addis Ababa LRT rolling stock (6)

S/N	Description of the Specification	Quantity
1	Number of seats	65
2	Number of rated passenger capacity (seated + standing)	65 + 189 = 254
3	Number of Overload Passenger capacity (seated + standing)	65 + 252 = 317
4	Empty train vehicle weight	44000 kg
5	Average weight of passengers	60kg
6	Rated passenger capacity	15240 kg
7	Motor efficiency (η)	87%
8	Input power per electric motor (There are four motor for one vehicle)	130 KW
9	Gravitational acceleration	9.81 m/s^2
10	Aerodynamic resistance coefficient (C_a)	1.8
11	Rolling resistance coefficient (C_r)	0.002
12	Mass factor (f_m)	1.05
13	Maximum operation speed	70 km/h
14	Train acceleration for 70km/h	≥ 0.5 m/s^2
15	Service brake deceleration	1.1m/s^2
16	Air density	1.2 kg/m^3
17	Frontal area of the vehicle (normal) (width X height)	2.65*3.70 = 9.805 m^2

Table 3.7: Case study track profile and stations spacing of AA LRT E-W line (15)

S/N	Station Rout	Stations Spacing (m)	Track Profile
1	EW1-EW2	1260	$0 \le X_L \le 170$ $170 \le X_D \le 419.995$ (38.5‰) $419.995 \le X_L \le 929.995$ $929.995 \le X_U \le 1153.995$ (50‰) $1153.995 \le X_L \le 1260$
2	EW2-EW3	1092	$0 \le X_L \le 1092$
3	EW3-EW4	863	$0 \le X_L \le 435.078$ $435.078 \le X_D \le 711.078$ (37.5‰) $711.078 \le X_L \le 863$
4	EW4-EW5	860	$0 \le X_L \le 860$
5	EW5-EW6	724.821	$0 \le X_L \le 724.821$
6	EW6-EW7	970	$0 \le X_L \le 128.912$ $128.912 \le X_D \le 348.52$ (53.6‰) $348.52 \le X_L \le 970$
7	EW7-EW8	1081.761	$0 \le X_L \le 70.52$ $70.52 \le X_U \le 421.017$ (29‰) $421.017 \le X_L \le 1081.761$
8	EW8-EW9	805	$0 \le X_L \le 509.156$ $509.156 \le X_D \le 714.156$ (43.5‰) $714.156 \le X_L \le 805$
9	EW9-EW10	802	$0 \le X_L \le 426.156$ $426.156 \le X_D \le 581.156$ (35‰) $581.156 \le X_L \le 802$

10	EW10-EW11	746	$0 \le X_L \le 746$
11	EW11-EW12	771	$0 \le X_L \le 164.087$ $164.087 \le X_U \le 489.087$ (55‰) $489.087 \le X_L \le 771$
12	EW12-EW13	950	$0 \le X_L \le 633.974$ $633.974 \le X_D \le 829.974$ (43.5‰) $829.974 \le X_L \le 950$
13	EW13-EW14	675	$0 \le X_L \le 675$
14	EW14-EW15	583	$0 \le X_L \le 95.326$ $95.326 \le X_U \le 449.33$ (47.5‰) $449.33 \le X_L \le 583$
15	EW15-EW16	650	$0 \le X_L \le 650$
16	EW16-EW17	435	$0 \le X_L \le 435$
17	EW17-EW18	570	$0 \le X_L \le 570$
18	EW18-EW19	688	$0 \le X_L \le 688$
19	EW19-EW20	735	$0 \le X_L \le 61.313$ $61.313 \le X_D \le 226.313$ (46‰) $226.313 \le X_L \le 386.313$ $386.313 \le X_U \le 664.313$ (55‰) $664.313 \le X_L \le 735$
20	EW20-EW21	732	$0 \le X_L \le 229.313$ $229.313 \le X_D \le 666.462$ (55‰) $666.462 \le X_L \le 732$
21	EW21-EW22	769	$0 \le X_L \le 342.462$ $342.462 \le X_U \le 484.462$ (55‰) $484.462 \le X_L \le 769$

3.9 Analysis of energy consumption

The train movement energy consumption are analyzed for the train motion of acceleration and constant speed by using different data from technical specification and case study of track alignment table 3.6 and 3.7 by calculating the forces such as acceleration, aerodynamic, rolling and gradient. The operating data parameters are entered into the model of the system to obtain the required results.

The energy consumption of train in the route of station EW1-EW2 is calculated analytically as follow. In this route there is level, downhill and uphill track profile. The spacing between these stations is 1260 m. The track profile in the direction of E-W is 170 m level, 250 m downhill, 510 m level, 224 m uphill and 106 m level.

For analytical analysis, the maximum operation speed, preferable acceleration and deceleration of train are 19.44 m/s, 0.8 m/s^2 and 1.1 m/s^2 respectively. To calculate energy consumption of train between these stations, the acceleration and braking distances are calculated as follow.

Acceleration distance:

$$X_a = V_{max}^2/(2*a) = (19.44)^2/(2*0.8) = 236.20 \text{ m}$$

Braking distance:

$$X_b = V_{max}^2/(2*a) = (19.44)^2/(2*1.1) = 171.78 \text{ m}$$

At level distance of 170 m the speed of train is:

$$X_a = V_{max}^2/(2*a) = 170 = V_{170}^2/(2*0.8) , V_{170} = 16.49 \text{ m/s}$$

For the analysis of acceleration resistance the equation is taken from train acceleration motion equation 3.2. In this research, the mass of (vehicle + passenger weight) is taken as 44000 + 15240 = 59240 kg which is rated passenger capacity. The value of mass factor is 1.05 which is constant taken from paper [42].

The acceleration resistance force is:

$$F_A = f_m*m*a = 1.05*59240*0.8 = 49761.60 \text{ N}$$

For the analysis of aerodynamic resistance the equation is taken from equation 3.7. The value of train speed is taken from 0-19.44 m/s throughout the train trip for summation of resistance force in energy consumption during acceleration motion. It is variable with time. For constant speed motion it is taken the maximum speed of 19.44 m/s. But for analytical analysis, the value of train speed is taken as average which is about 8.25 m/s for level distance of 170 m and 17.97 m/s for remaining acceleration distance of 66.20 m during acceleration motion.

For acceleration motion, the aerodynamic force in distance of 170 m is:

$$F_a = 0.5*\rho*C_a*A_{fr}*V_{tr}^2 = 0.5*1.2*1.8*9.805*(8.25)^2 = 720.74 \text{ N}$$

For acceleration motion, the aerodynamic force in distance of 66.20 m is:

$$F_a = 0.5*\rho*C_a*A_{fr}*V_{tr}^2 = 0.5*1.2*1.8*9.805*(17.97)^2 = 3419.54 \text{ N}$$

For constant speed motion, the aerodynamic force is:

$$F_a = 0.5*\rho*C_a*A_{fr}*V_{tr}^2 = 0.5*1.2*1.8*9.805*(19.44)^2 = 4001.88 \text{ N}$$

For the analysis of rolling resistance the equation is taken from equation 3.8. The value of rolling resistance force is the same throughout the trip of train. Since it doesn't depend on the train speed.

Therefore in this research the rolling force is:

$$F_r = C_r*m*g = 0.002*59240*9.81 = 1162.29 \text{ N}$$

For the analysis of gradient resistance the equation is taken from equation 3.9. The value of gravitational force depends on the gradient r_g (‰). In this research there is different value of gradient r_g which is equal to or less than 55‰ as shown in table 3.7. Therefore the value of r_g is variable. In the route of EW1-EW2 there are 38.5‰ for downhill and 50‰ for uphill.

For downhill gradient r_g = 38.5‰ the gravitational force is:

$$F_g = m*g*r_g/1000 = 59240*9.81*38.5/1000 = 22374.06 \text{ N}$$

For uphill gradient r_g = 50‰ the gravitational force is:

$$F_g = m*g*r_g/1000 = 59240*9.81*50/1000 = 29057.22\ N$$

For level acceleration motion, the energy consumption:

$$E_1 = (F_A + F_a + F_r)X_1 = (49761.60 + 720.74 + 1162.29)*170$$

$$= 8779587\ J$$

For downhill acceleration motion, the energy consumption:

$$E_2 = (F_A + F_a + F_r - F_g)*X_2 = (49761.60 + 3419.54 + 1162.29 - 22374.06)*(236.20-170)$$

$$= 2116372\ J$$

For downhill constant speed motion, the energy consumption:

$$E_3 = (F_a + F_r - F_g)*X_3 = (4001.88 + 1162.29 - 22374.06)*(250-(236.20-170))$$

$$= -3163178\ J$$

For level constant speed motion, the energy consumption:

$$E_4 = (F_a + F_r)*X_4 = (4001.88 + 1162.29)*510$$

$$= 2633727\ J$$

For uphill constant speed motion, the energy consumption:

$$E_5 = (F_a + F_r + F_g)*X_5 = (4001.88 + 1162.29 + 29057.22)*(224-(171.78 - 106))$$

$$= 5414508\ J$$

Therefore, the total energy consumption of train movement in the route of EW1-EW2:

$$E_{1\text{-}2} = E_1 + E_2 + E_3 + E_4 + E_5$$

$$E_{1\text{-}2} = 8779587 + 2116372 + -3163178 + 2633727 + 5414508$$

$$= 15781016\ J$$

The energy consumption of train movement in the route of EW1-EW2 using matlab simulink:

$$E_{1\text{-}2} = E_a + E_c = 11097272 + 4841201 = 15938473 \text{ J}$$

The error difference of energy consumption calculated above for the route of EW1-EW2 in percentage is about 0.99%. So the difference is very small and the analysis result which is done using simulink throughout this research has been worked in similar way.

The analysis of energy consumption of train movement using analytical and simulink is approximately similar. But using matlab simulink simulation is better to obtain the nearest energy consumption of train. Since during acceleration motion the speed of train is taken as average and the acceleration is taken one preferable acceleration.

3.10 Building of Simulink Loop for analysis of energy consumption

Before building the loop diagram of simulink, there is constructed flow chart in figure 3.6 to use it as guide during building of simulink diagram. The simulink loop diagram which is used to analyze the energy consumption shown below in three different simulink loop diagrams. Figure 3.7 shows combined simulink loop diagram of energy consumption analysis; figure 3.8 shows acceleration motion simulink loop diagram energy consumption analysis; figure 3.9 shows constant speed motion simulink loop diagram energy analysis.

The total energy consumption which is indicated in flow chart below, it is the total energy consumption of each interval stations such as EW1-EW2, EW2-EW3, EW3-EW4 etc. As an example in the route of EW1-EW2, first calculate the acceleration energy consumption for level distance of 170 m and the remaining acceleration distance energy consumption is calculated in downhill of 3.85%; second calculate the speed constant energy consumption in the downhill of 3.85%; third calculate the speed constant energy consumption in level distance of 510 m and finally find the speed constant energy consumption in the uphill of 5%. After analyzing the energy consumption of train movement, the total energy consumption of each interval route of stations is shown in table 4.1 and 4.2.

* By entering the necessary operating inputs into simulink loop diagram simulate the system and observe the results.

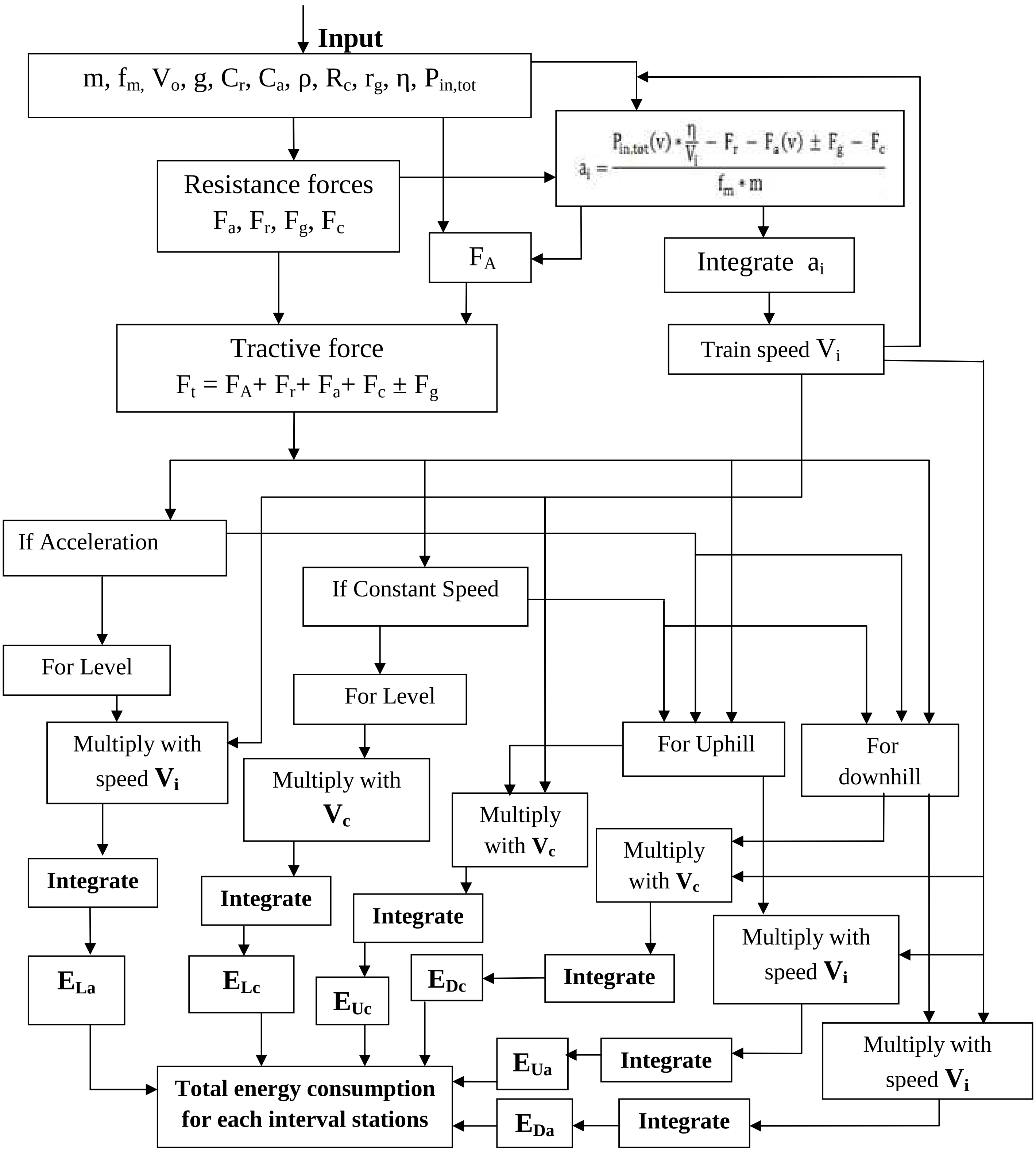


Figure 3.6: Flow chart of train energy consumption analysis

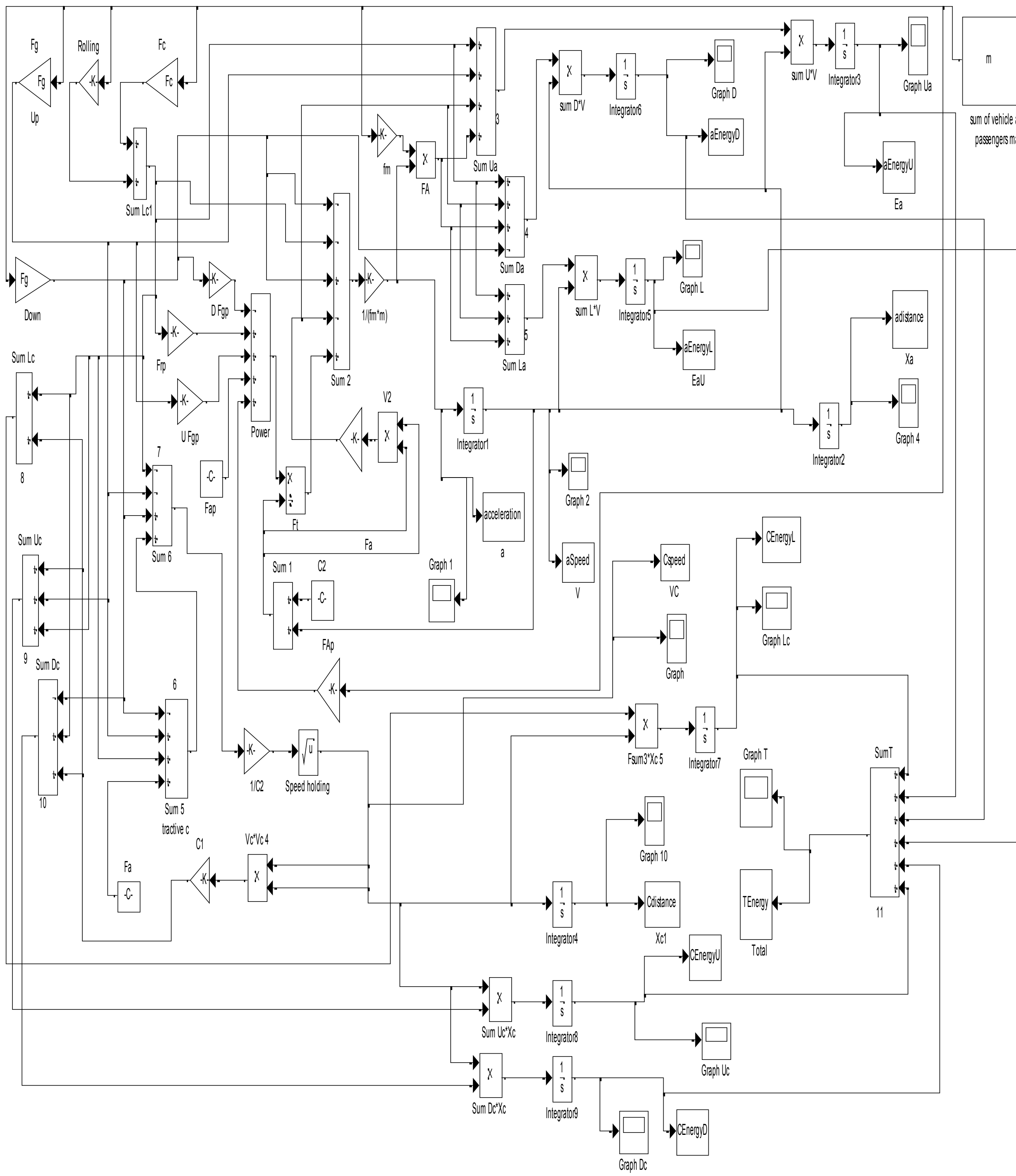


Figure 3.7**:** Combined simulink loop diagram of train energy consumption analysis

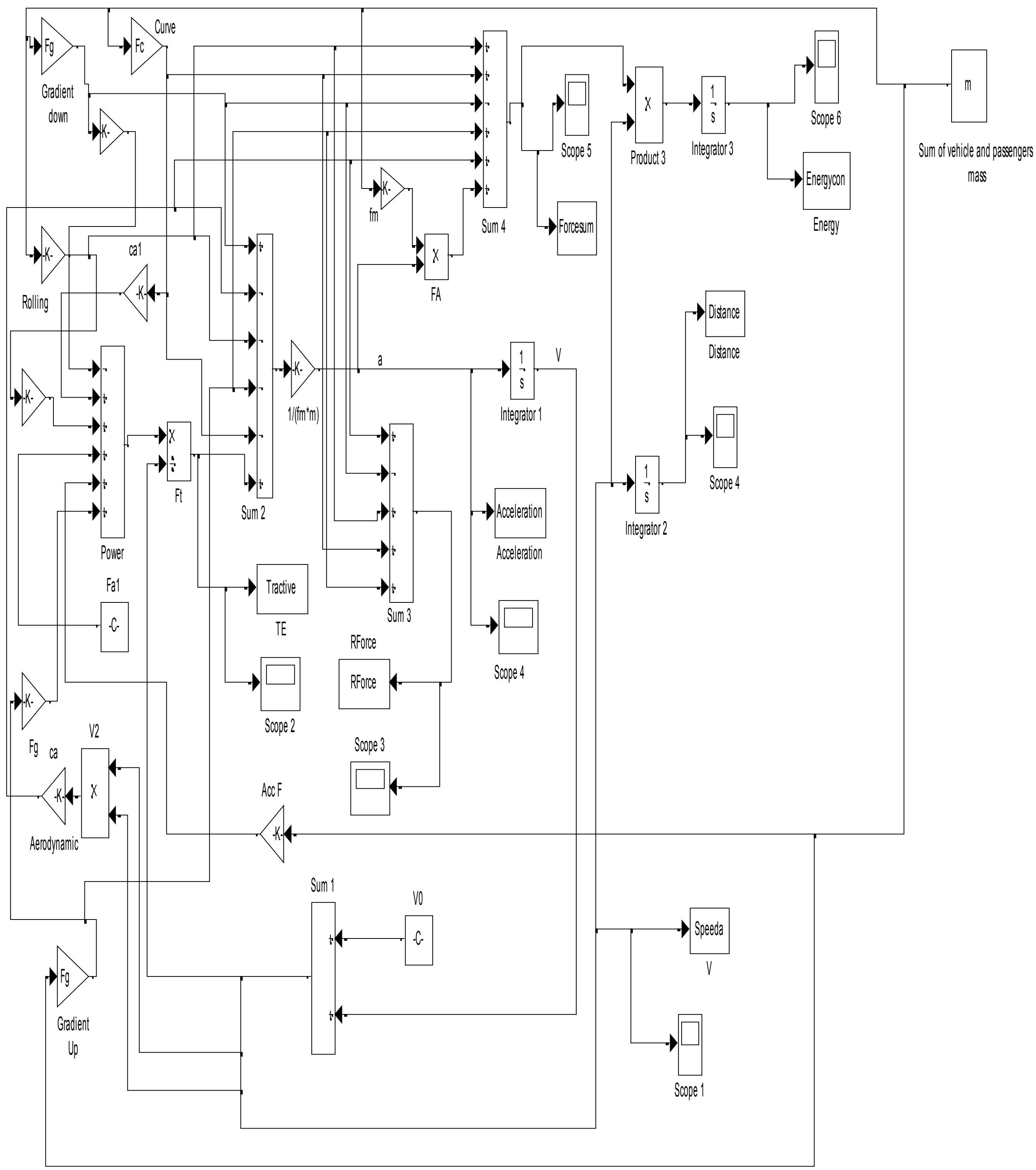


Figure 3.8: Simulink loop diagram of acceleration train energy consumption analysis

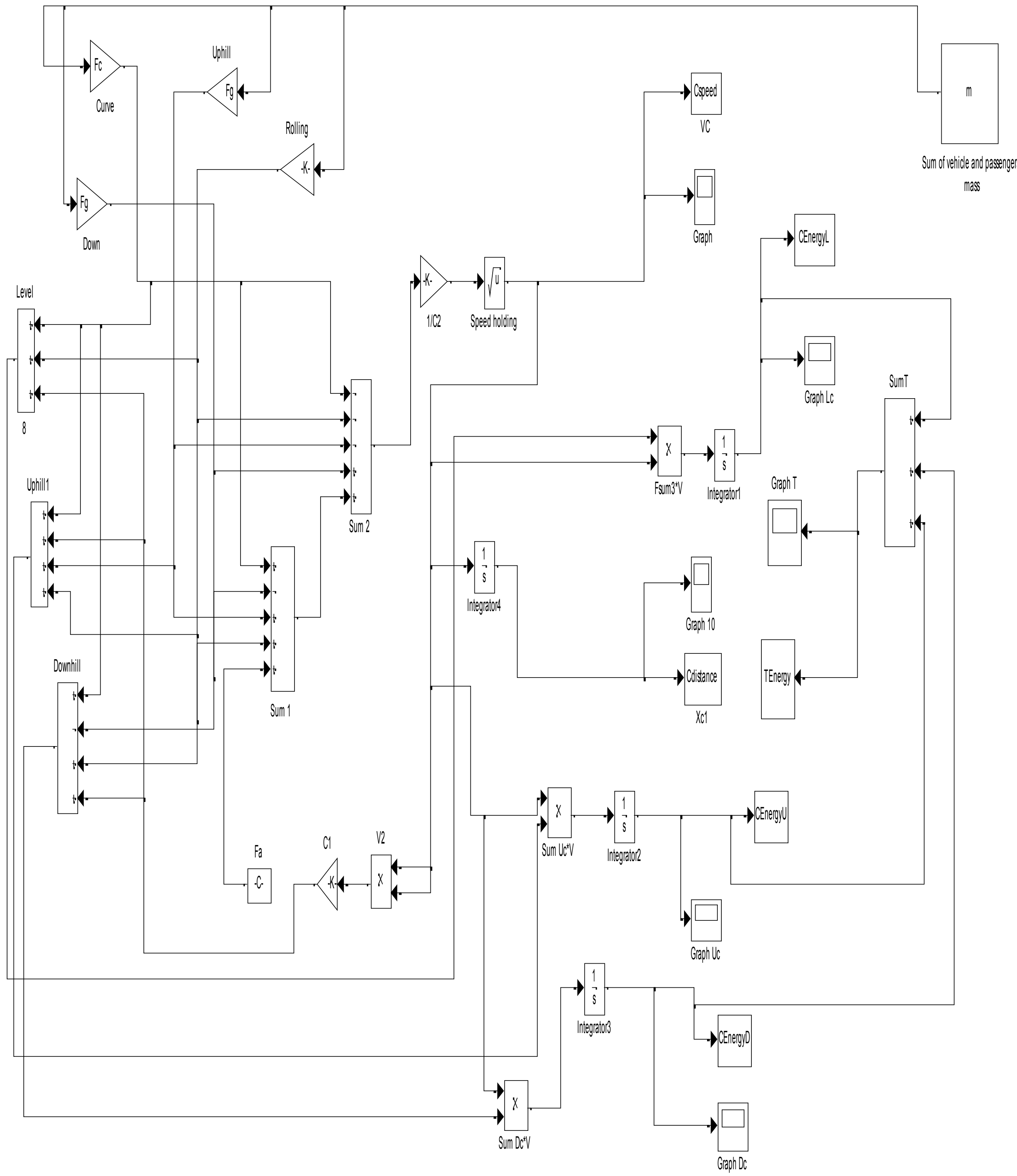


Figure 3.9: Simulink loop diagram of constant speed motion of train energy consumption analysis

Chapter Four

Results and Discussion

4.1 Results

In this section the results has been obtained from the matlab simulink loop simulations in chapter three regarding on train energy consumption during train acceleration and maximum speed operation. The solution has been analyzed for different cases to get the total energy consumption of the train stops at all stations and passes the six selected low passenger flow stations in non-peak hour for one complete trip.

The graph of acceleration curve, speed profile, tractive effort, running resistance and acceleration energy consumption curve have been obtained from simulation as shown below.

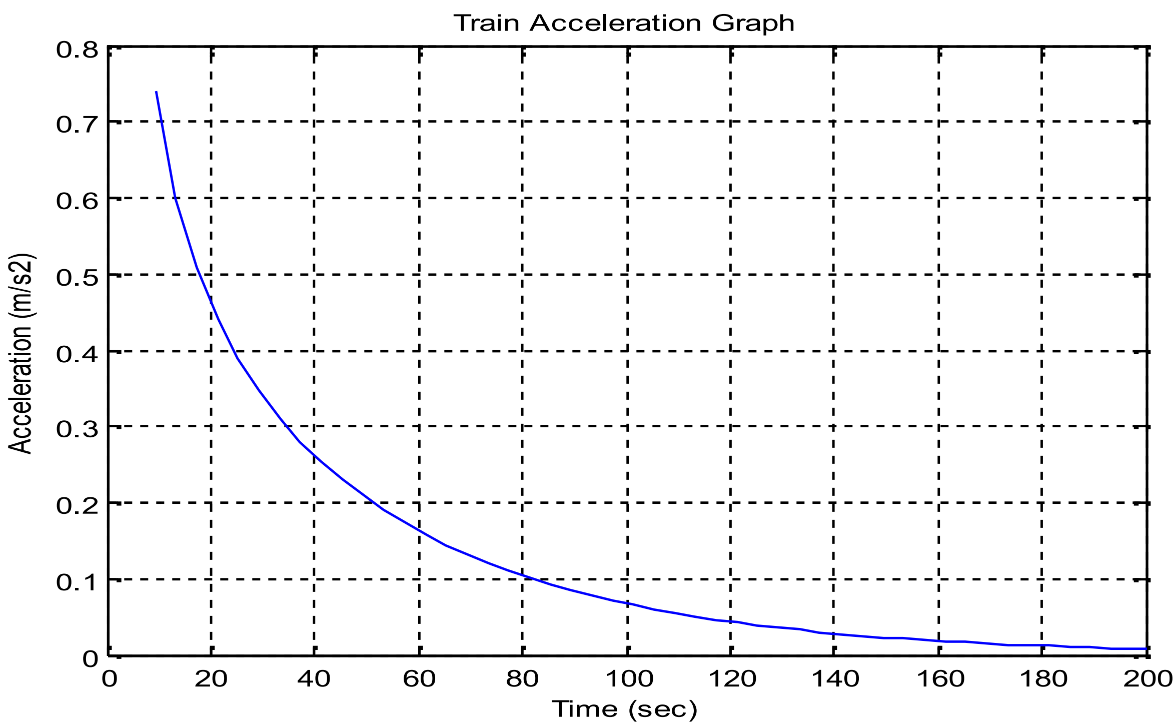


Figure 4.1: Train acceleration curve

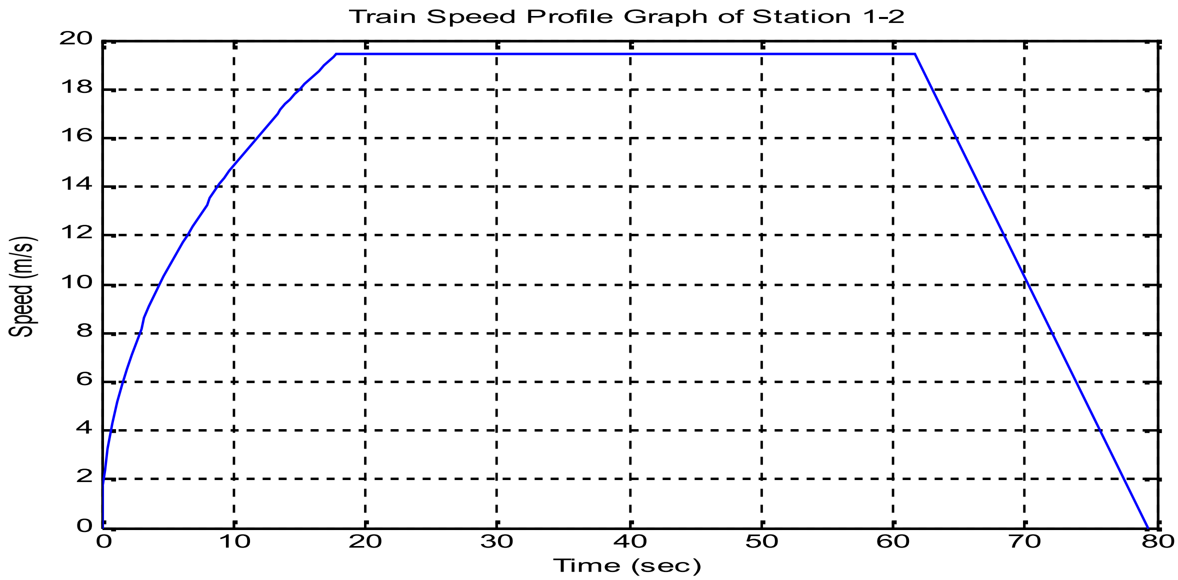


Figure 4.2: Train speed profile of highest distance stations spacing

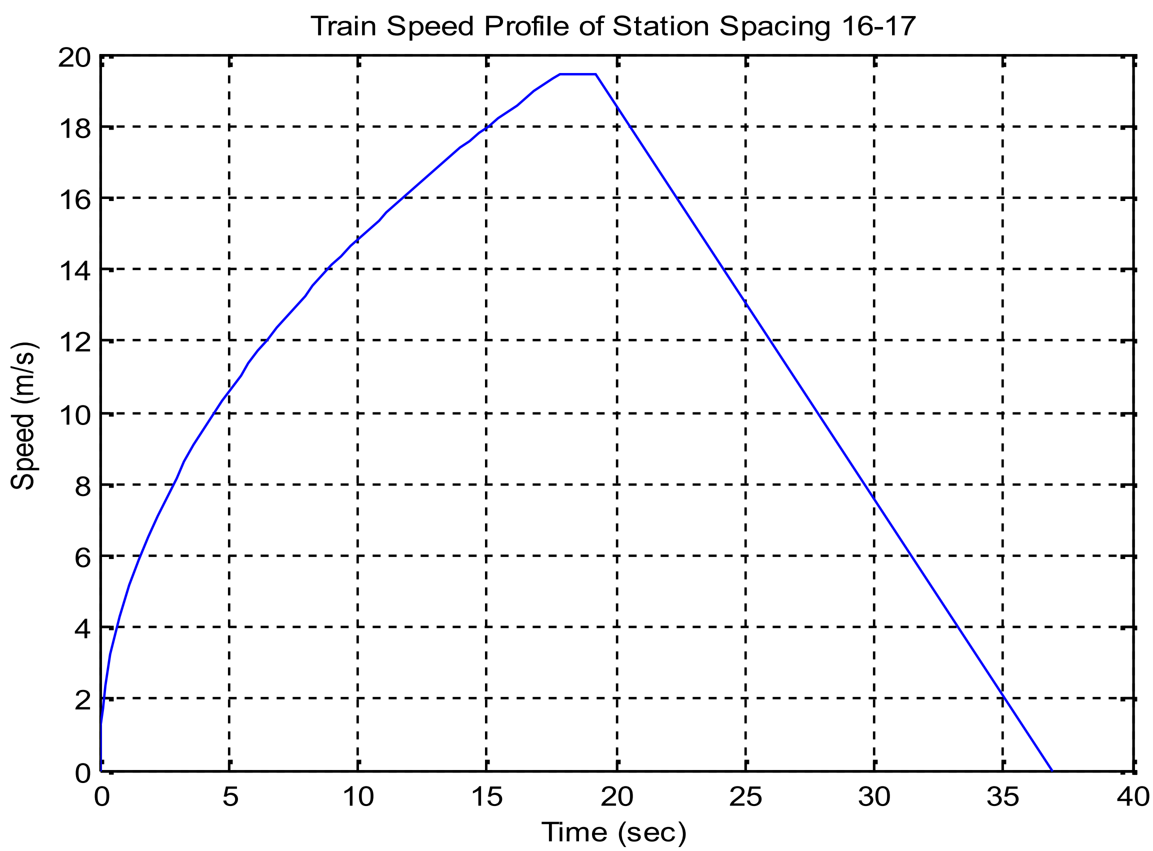


Figure 4.3: Train speed profile of lowest distance stations spacing

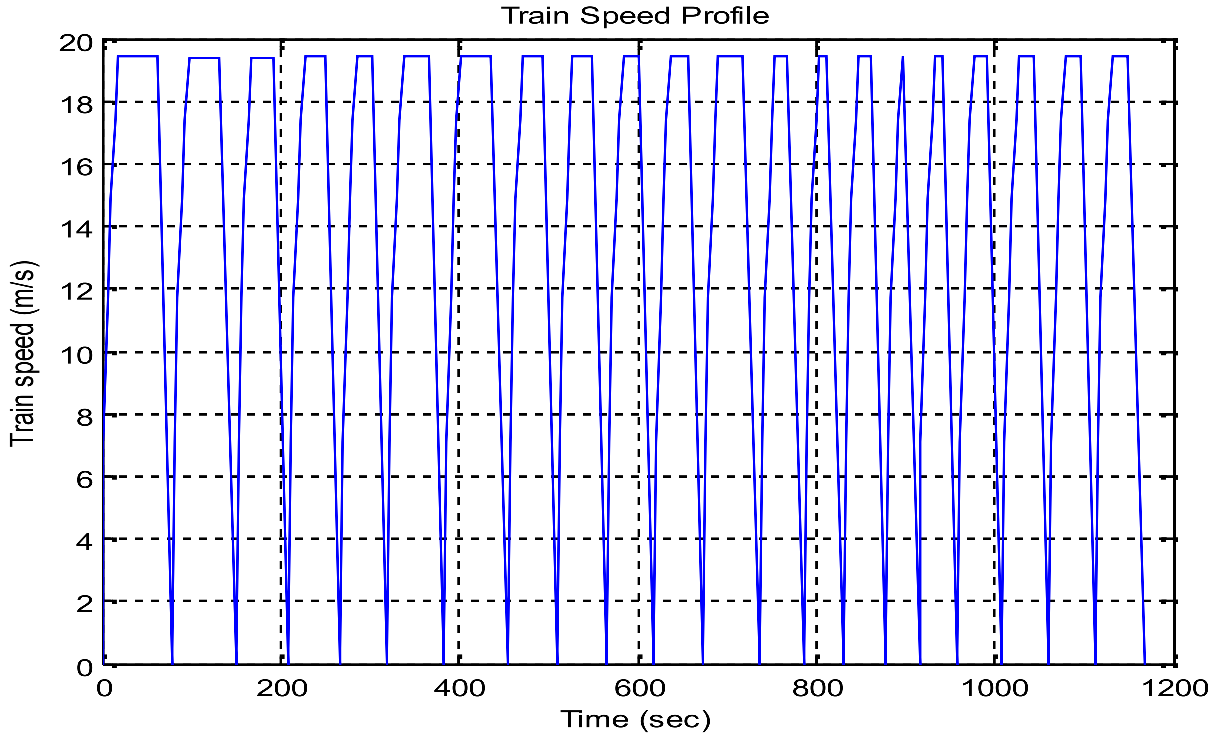


Figure 4.4: Combined train speed profile

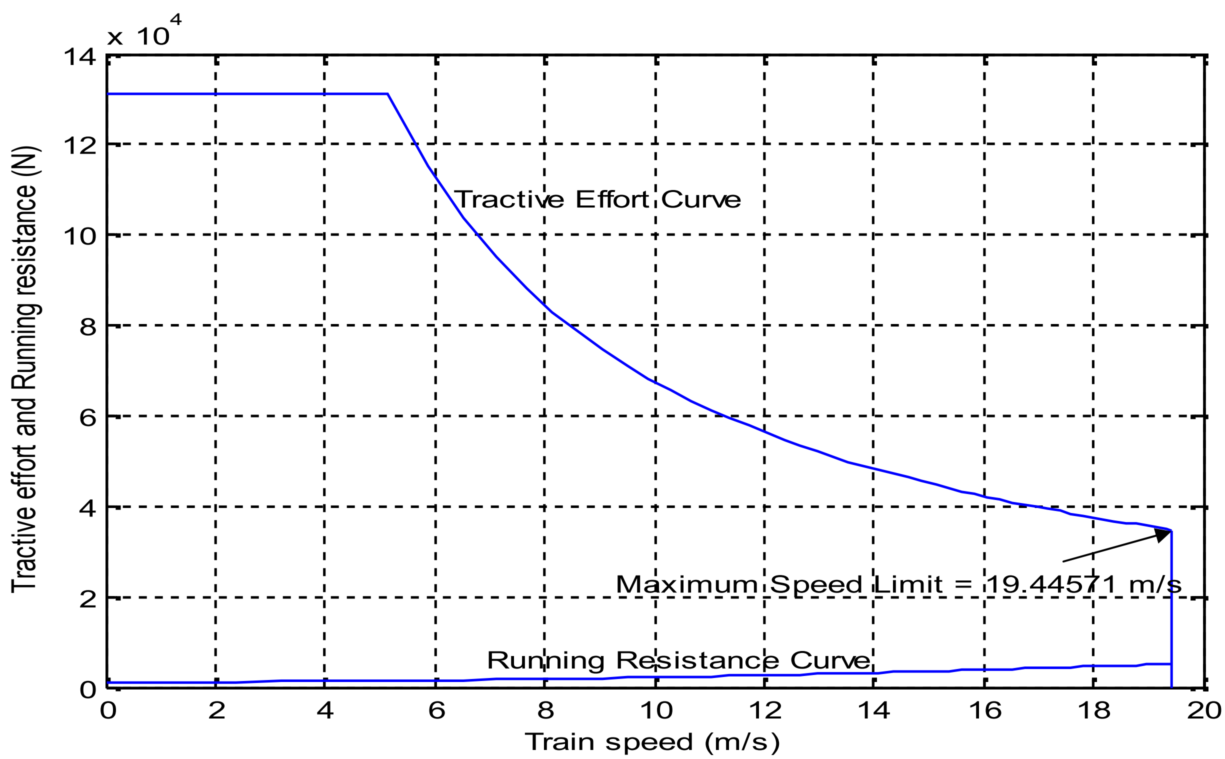


Figure 4.5: Tractive effort and running resistance curves

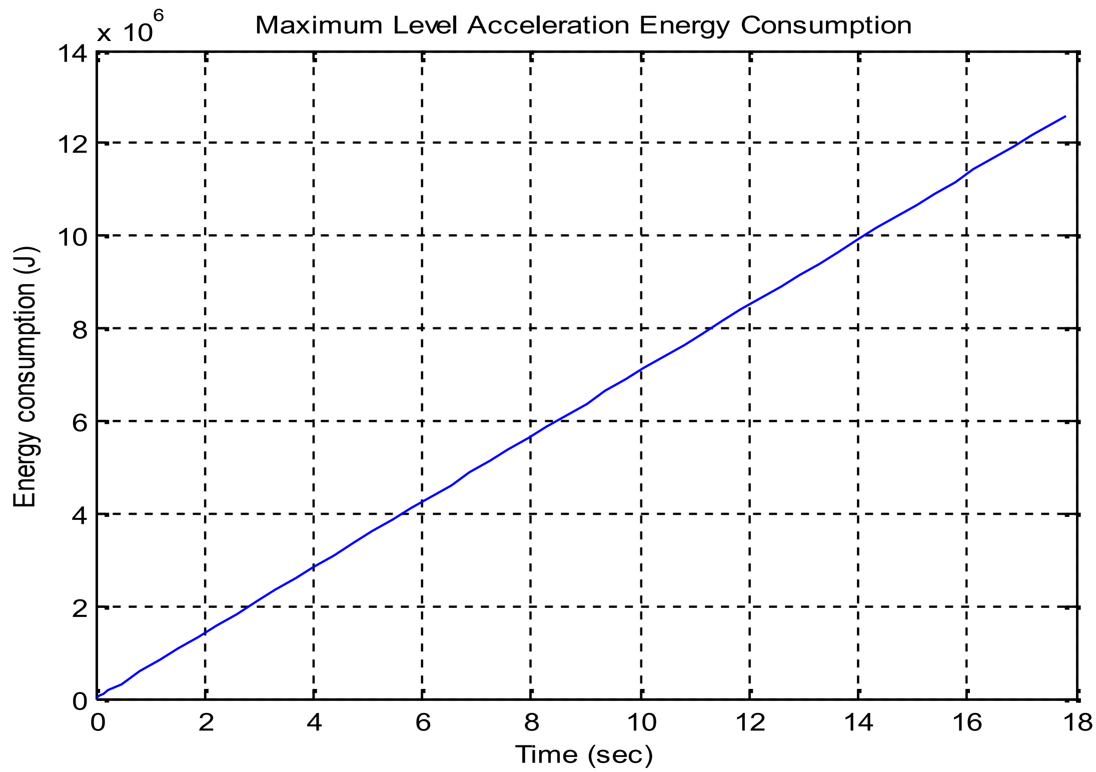


Figure 4.6: Train acceleration motion energy consumption versus time

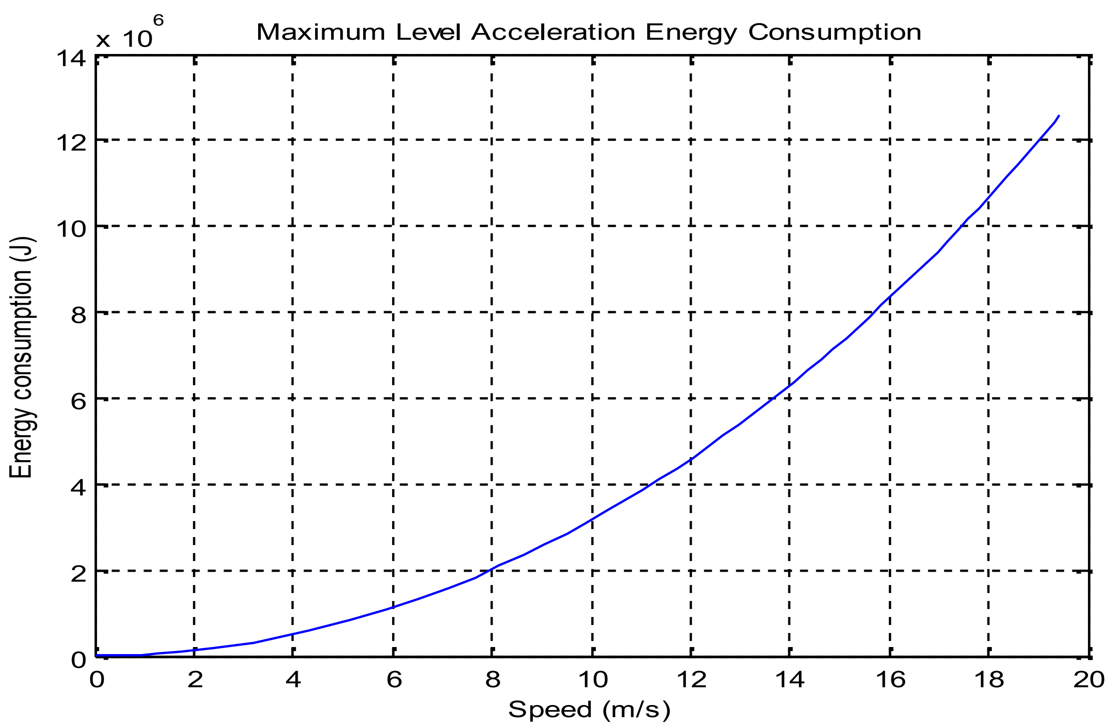


Figure 4.7: Train acceleration motion energy consumption versus speed

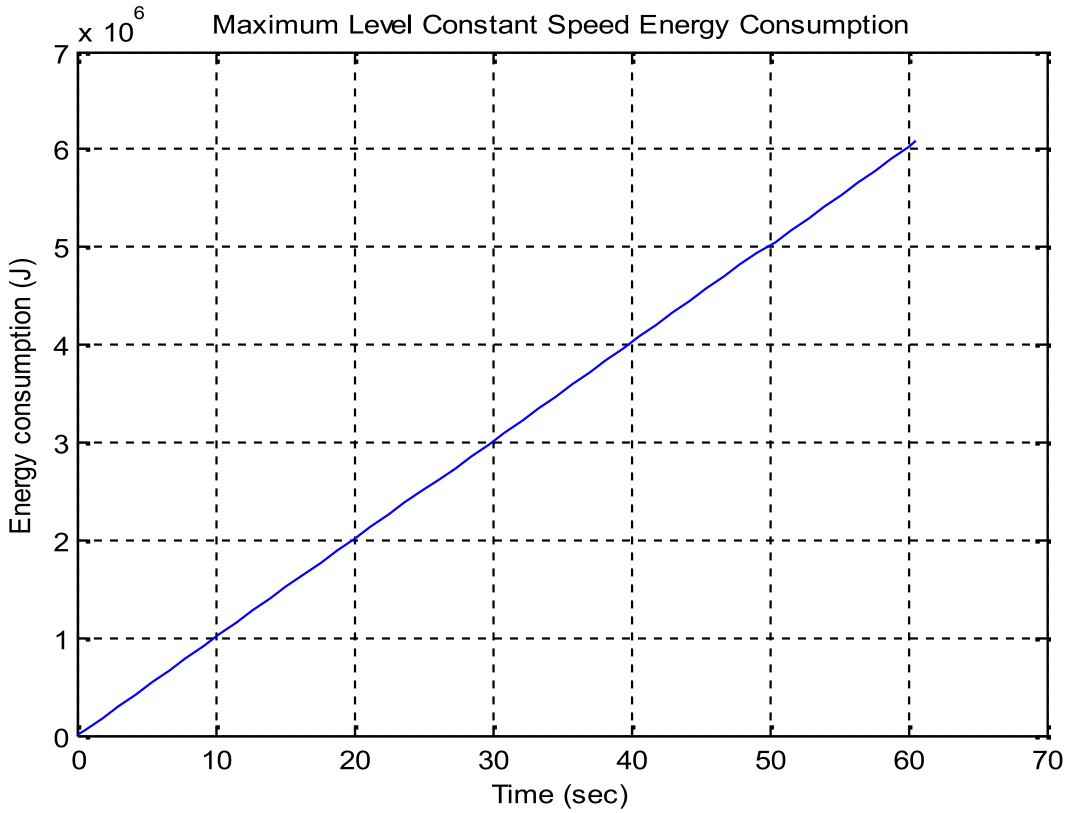


Figure 4.8: Train constant speed motion energy consumption versus time

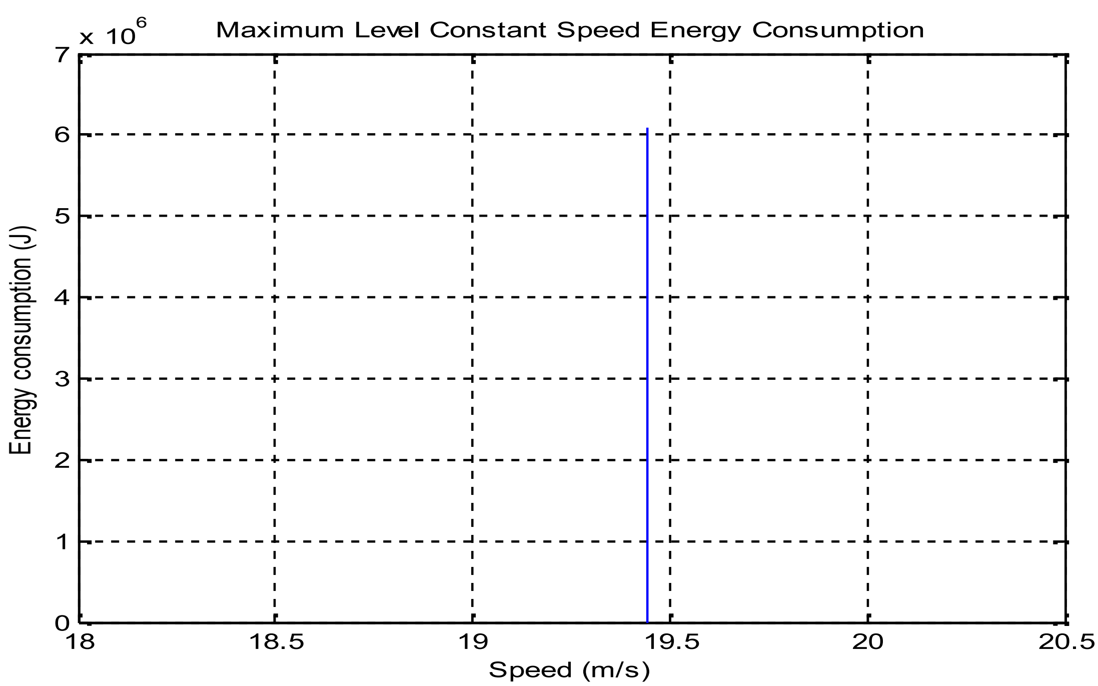


Figure 4.9: Train constant speed motion energy consumption versus speed

The graphs which are obtained from simulation, they have similar profile with theory and literature papers curves. The train acceleration and speed curves are discussed in literature review on picture 2.2, 2.3 and 2.4. The simulation acceleration and speed curves results have been obtained the same with these curves. The tractive effort curve is discussed in introduction section on picture 1.2. The simulation result of tractive effort curve is also obtained similar with theory curve. The curve of train energy consumption can be validated based on the model of equations. When the speed of the train increases from zero to maximum operation speed, the curve of train energy consumption increases. For constant speed motion, the curve of energy consumption also increases with time.

The energy consumption results of train operation in the line of E-W Addis Ababa LRT have been indicated in table 4.1 and 4.2 below from simulation for two cases of one complete trip.

Table 4.1: Energy consumption results for train stopping at all stations

S/N	Station Rout	For **E-W** line		For **W-E** line (Reverse direction)		Total (J)
		Acceleration motion (J)	Speed Constant motion (J)	Acceleration motion (J)	Speed constant motion (J)	
1	EW1-EW2	11097272	4841201	9378456	7171003	**32487932**
2	EW2-EW3	12606604	3543853	12606604	3543853	**32300914**
3	EW3-EW4	12606604	-3218127	14531470	6588663	**30508610**
4	EW4-EW5	12606604	2345349	12606604	2345349	**29903906**
5	EW5-EW6	12606604	1647019	12606604	1647019	**28507246**
6	EW6-EW7	8983983	-668560	12606604	8427026	**29349053**
7	EW7-EW8	15861707	6643733	12606604	-704240	**34407804**
8	EW8-EW9	12606604	-1073465	16780049	3628564	**31941752**
9	EW9-EW10	12606604	-1106978	12848553	4934004	**29282183**
10	EW10-EW11	12606604	1756429	12606604	1756429	**28726066**

11	EW11-EW12	14984591	10030943	12606604	-8251818	**29370320**
12	EW12-EW13	12606604	-829994	15720497	4883223	**32380330**
13	EW13-EW14	12606604	1389645	12606604	1389645	**27992498**
14	EW14-EW15	16970559	5800337	9589392	-3971584	**28388704**
15	EW15-EW16	12606604	1260496	12606604	1260496	**27734200**
16	EW16-EW17	12606604	149813	12606604	149813	**25512834**
17	EW17-EW18	12606604	847219	12606604	847219	**26907646**
18	EW18-Ew19	12606604	1456803	12606604	1456803	**28126814**
19	EW19-EW20	7054998	7357031	6418648	-532567	**20298110**
20	EW20-EW21	12447905	-8735790	19000425	10282117	**32994657**
21	EW21-EW22	12606604	6413972	12606604	-2663482	**28963698**
Total energy consumption for one trip		**263893471**	**39850929**	**268153342**	**44187535**	**616085277**
		303744400		**312340877**		

Table 4.2: Energy consumption results for train passing stations

S/N	Station Rout	For **E-W** line		For **W-E** line (Reverse direction)		Total (J)
		Acceleration motion (J)	Speed Constant motion (J)	Acceleration motion (J)	Speed constant motion (J)	
1	EW1-EW2	11097272	4841201	9378456	7171003	**32487932**
2	EW2-EW3	12606604	3543853	12606604	3543853	**32300914**
3	EW3-EW4	12606604	-3218127	14531470	6588663	**30508610**
4	**EW4-EW6**	12606604	6089750	12606604	6089750	**37392708**
5	**EW6-EW9**	8983983	11822872	16780049	13823283	**51410187**
6	EW9-EW10	12606604	-1106978	12848553	4934004	**29282183**
7	EW10-EW11	12606604	1756429	12606604	1756429	**28726066**
8	EW11-EW12	14984591	10030943	12606604	-8251818	**29370320**
9	EW12-EW13	12606604	-829994	15720497	4883223	**32380330**
10	**EW13-EW15**	12606604	13124361	9589392	-2610087	**32710270**
11	EW15-EW16	12606604	1260496	12606604	1260496	**27734200**
12	EW16-EW17	12606604	149813	12606604	149813	**25512834**
13						

	EW17-EW19	12606604	4401404	12606604	4401404	**34016016**
14	**EW19-EW21**	7054998	3787059	19000425	6636985	**36479467**
15	EW21-EW22	12606604	6413972	12606604	-2663482	**28963698**
Total energy consumption for one trip		**180793488**	**62067054**	**198701674**	**47713519**	**489275735**
		242860542		**246415193**		

AS shown in table 4.1 and 4.2 above the total energy consumption of train movement for one complete cycle trip per train, by stopping at all stations it is 616085277 J and by passing the six stations it is about 489275735 J. The reduction of energy consumption for one complete cycle trip per train in percentage is about 20.58%. There has been separately observed that about 20.04% of energy consumption is reduced in the direction of E-W line, and about 21.11% of energy consumption is reduced in the direction of W-E line by passing the six stations. The energy consumption of train in the direction of E-W and W-E line is different in case of track profile.

4.2 Discussion

From simulink loop simulation, it has been obtained the results of different parameters in acceleration and constant speed motion as shown in appendix table 1 and 2. The appendix table results show the maximum traveling distance of train for level movement during acceleration and speed holding motion. But some track profile for acceleration movement is not fully level. Some part of its distance would be uphill and downhill gradient. Let us take the route E-W line between stations 1 and 2. The train accelerates until 170 m with level track profile. But after 170 m there is downhill gradient. The driver should decrease the power to move the train with necessary speed in the downhill motion. During downhill motion energy consumption is decreased in the case of gravitational force in the direction of train movement. But in the case of uphill motion the driver should increase the power and the energy consumption is highly increased to overcome gravitational force against the train motion. In the simulation process to obtain the energy consumption of train in downhill motion after 170 m until maximum distance of acceleration, we can take the speed 17.59327 m/s from appendix table 1 and insert in the initial speed of simulink tool with gradient value of 38.5‰. The traveling time is obtained by decreasing 14.34889 seconds from maximum traveling time of acceleration. For constant speed and other cases, the method is the same.

Some part of track route, except acceleration and deceleration distance, has no show changing in energy consumption when the train passes the stations. Let us take a train starts to move from station 4 to stop at station 6 without stopping at station 5. Acceleration energy consumption is calculated from station 4 and speed holding energy consumption is calculated from sum of unchanged distance of speed holding, and deceleration and acceleration distance of station 5. Because the train didn't stop at station 5 and the train travels the distance of decelerating and acceleration distance of station 5 at maximum operating constant speed.

From simulink simulation result, for stopping at all stations, the total energy consumption of E-W and W-E line is 303744400 J and 312340877 J respectively. The maximum acceleration energy consumption of train is 16970559 J at station 14 and the minimum energy consumption is 7054998 J at station 19 in the direction of E-W line. For direction of W-E line the maximum acceleration energy consumption is 19000425 J at station 21 and the minimum energy consumption is 6418648 J at station 20. The total energy consumption of motion in the direction of E-W is less than W-E line direction in the case of track profile difference.

For passing selected 6 stations the total calculated energy consumption of E-W and W-E line is 242860542 J and 246415193 J respectively. The maximum acceleration energy consumption of train is 14984591 J at station 11 and the minimum energy consumption is 7054998 J at station 19 in the direction of E-W line. For direction of W-E line the maximum acceleration energy consumption is 19000425 J at station 21 and the minimum energy consumption is 9378456 J at station 2. The total calculated energy consumption in the direction of E-W is less than W-E line direction in the case of track profile difference.

Generally the total calculated energy consumption of both direction, for stopping at all stations, is 616085277 J and the total calculated energy consumption of both direction, for passing selected 6 stations, is 489275735 J. As shown in the result, the energy consumption can be saved by 20.58% of energy consumption of stopping at all stations by passing selected 6 stations in non-peak hour for one complete trip.

Chapter Five

Conclusion, Recommendation and Future Works

5.1 Conclusion

The objective of this research is to reduce energy consumption of trains operation on the AA LRT network by passing low passenger flow stations in off-peak hours. The dynamics of train motion was represented by a set of kinematic equations by including the important forces that act on the train movement. The train resistances such as acceleration, aerodynamic, rolling, track gradient and horizontal track curvature were considered in the model.

Simulink loop was constructed to calculate acceleration, speed, and distance and train energy consumption for every type of riding mode in each two station space. Energy consumption reduction is considered to solve in this research.

Various settings were made on the built in MATLAB simulink loop to ensure results convergence and accuracy. After the loop was created by feeding necessary inputs and simulate it, the results were outcome for the operation of trains on every section from Ayat station to Torhailoch station of Addis Ababa LRT. The results have shown large difference in reducing the energy consumption of the train by passing six low passenger flow stations compared to stop at all stations.

From simulink simulation result, by passing six stations, it has been observed that about 20.04% of energy consumption is reduced in the direction of E-W line, and about 21.11% of energy consumption is reduced in the direction of W-E line.

The total energy consumption was 616085277 J by stopping at all stations and 489275735 J by passing six low passenger flow stations for one complete cycle trip. It has been observed that much amount of energy consumption was reduced in passing selected stations compared to using all stations. The main cause of increasing energy consumption was the acceleration mode. In this research, the total energy consumption reduction of one complete cycle trip per train has been obtained **20.58%** by passing six low passenger flow stations.

Generally, reducing the number of train stops at stations gives a large reduction in the energy consumption of the train operation. In this thesis energy consumption of train, in the combination of trains stop at all stations during peak hour and the trains pass the selected low passenger flow stations during off-peak hour, has be reduced highly for one complete cycle route of E-W.

5.2 Recommendation

From the simulink simulation result of train energy consumption analysis in the both direction of East-west line of Addis Ababa LRT is shown in table 4.1 and 4.2. The total energy consumption reduction of the train movement from Ayat to Torhailoch by passing six stations in off-peak hour is about 20.58% for one complete cycle trip. This is very important in the power saving and cost reduction of the organization. Therefore it must be recommended to the Organization of ERC to pass low passenger flow stations in off-peak hour to reduce power consumption.

5.3 Future Works

Future research areas related to the current research are listed below:

- Making scheduling for this research by passing low passenger flow stations of train operation in off-peak hour during operation in future.
- Making scheduling of train operation that the train passes the selected stations and make the next coming train stops at stations of the previous train passed them.
- Study the cost reduction of Organization per month by passing low passenger flow in off-peak during operation in the future.
- Making optimization to improve the balance between energy consumption and passenger station flow stations to pass stations in off-peak hour.
- Analysis of energy consumption reduction by reducing the number of train in off-peak hour.
- Analysis of energy consumption reduction by making limited the length of a train trip.

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Appendix table 1: Acceleration motion results for maximum level track

Time (s)	Acceleration (m/s^2)	Train speed (m/s)	Traveling distance (m)	Energy Consumption (J)
0	1.14E+61	1.00E-60	0	0
1.76E-65	623387.6	1.83E-05	3.22E-70	1.25E-59
3.62E-65	623387.6	1.83E-05	6.63E-70	2.57E-59
1.29E-64	623387.6	1.83E-05	2.37E-69	9.18E-59
5.95E-64	623387.6	1.83E-05	1.09E-68	4.22E-58
2.92E-63	623387.6	1.83E-05	5.35E-68	2.07E-57
1.46E-62	623387.6	1.83E-05	2.67E-67	1.03E-56
7.27E-62	623387.6	1.83E-05	1.33E-66	5.16E-56
3.64E-61	623387.6	1.83E-05	6.66E-66	2.58E-55
1.82E-60	623387.6	1.83E-05	3.33E-65	1.29E-54
9.09E-60	623387.6	1.83E-05	1.66E-64	6.45E-54
4.54E-59	623387.6	1.83E-05	8.32E-64	3.23E-53
2.27E-58	623387.6	1.83E-05	4.16E-63	1.61E-52
1.14E-57	623387.6	1.83E-05	2.08E-62	8.07E-52
5.68E-57	623387.6	1.83E-05	1.04E-61	4.03E-51
2.84E-56	623387.6	1.83E-05	5.20E-61	2.02E-50
1.42E-55	623387.6	1.83E-05	2.60E-60	1.01E-49
7.10E-55	623387.6	1.83E-05	1.30E-59	5.04E-49
3.55E-54	623387.6	1.83E-05	6.50E-59	2.52E-48
1.78E-53	623387.6	1.83E-05	3.25E-58	1.26E-47
8.88E-53	623387.6	1.83E-05	1.63E-57	6.30E-47
4.44E-52	623387.6	1.83E-05	8.13E-57	3.15E-46
2.22E-51	623387.6	1.83E-05	4.06E-56	1.58E-45
1.11E-50	623387.6	1.83E-05	2.03E-55	7.88E-45
5.55E-50	623387.6	1.83E-05	1.02E-54	3.94E-44
2.77E-49	623387.6	1.83E-05	5.08E-54	1.97E-43
1.39E-48	623387.6	1.83E-05	2.54E-53	9.85E-43
6.93E-48	623387.6	1.83E-05	1.27E-52	4.92E-42
3.47E-47	623387.6	1.83E-05	6.35E-52	2.46E-41
1.73E-46	623387.6	1.83E-05	3.17E-51	1.23E-40
8.67E-46	623387.6	1.83E-05	1.59E-50	6.16E-40
4.33E-45	623387.6	1.83E-05	7.94E-50	3.08E-39
2.17E-44	623387.6	1.83E-05	3.97E-49	1.54E-38
1.08E-43	623387.6	1.83E-05	1.98E-48	7.69E-38

5.42E-43	623387.6	1.83E-05	9.92E-48	3.85E-37
2.71E-42	623387.6	1.83E-05	4.96E-47	1.92E-36
1.35E-41	623387.6	1.83E-05	2.48E-46	9.62E-36
6.77E-41	623387.6	1.83E-05	1.24E-45	4.81E-35
3.39E-40	623387.6	1.83E-05	6.20E-45	2.40E-34
1.69E-39	623387.6	1.83E-05	3.10E-44	1.20E-33
8.46E-39	623387.6	1.83E-05	1.55E-43	6.01E-33
4.23E-38	623387.6	1.83E-05	7.75E-43	3.01E-32
2.12E-37	623387.6	1.83E-05	3.88E-42	1.50E-31
1.06E-36	623387.6	1.83E-05	1.94E-41	7.51E-31
5.29E-36	623387.6	1.83E-05	9.69E-41	3.76E-30
2.65E-35	623387.6	1.83E-05	4.84E-40	1.88E-29
1.32E-34	623387.6	1.83E-05	2.42E-39	9.39E-29
6.61E-34	623387.6	1.83E-05	1.21E-38	4.70E-28
3.31E-33	623387.6	1.83E-05	6.06E-38	2.35E-27
1.65E-32	623387.6	1.83E-05	3.03E-37	1.17E-26
8.27E-32	623387.6	1.83E-05	1.51E-36	5.87E-26
4.13E-31	623387.6	1.83E-05	7.57E-36	2.94E-25
2.07E-30	623387.6	1.83E-05	3.78E-35	1.47E-24
1.03E-29	623387.6	1.83E-05	1.89E-34	7.34E-24
5.17E-29	623387.6	1.83E-05	9.46E-34	3.67E-23
2.58E-28	623387.6	1.83E-05	4.73E-33	1.83E-22
1.29E-27	623387.6	1.83E-05	2.37E-32	9.17E-22
6.46E-27	623387.6	1.83E-05	1.18E-31	4.59E-21
3.23E-26	623387.6	1.83E-05	5.91E-31	2.29E-20
1.61E-25	623387.6	1.83E-05	2.96E-30	1.15E-19
8.07E-25	623387.6	1.83E-05	1.48E-29	5.73E-19
4.04E-24	623387.6	1.83E-05	7.39E-29	2.87E-18
2.02E-23	623387.6	1.83E-05	3.70E-28	1.43E-17
1.01E-22	623387.6	1.83E-05	1.85E-27	7.17E-17
5.04E-22	623387.6	1.83E-05	9.24E-27	3.58E-16
2.52E-21	623387.6	1.83E-05	4.62E-26	1.79E-15
1.26E-20	623387.6	1.83E-05	2.31E-25	8.96E-15
6.31E-20	623387.6	1.83E-05	1.16E-24	4.48E-14
3.15E-19	623387.5	1.83E-05	5.78E-24	2.24E-13
1.58E-18	623387.5	1.83E-05	2.89E-23	1.12E-12
7.88E-18	623387.4	1.83E-05	1.44E-22	5.60E-12
3.94E-17	623386.7	1.83E-05	7.22E-22	2.80E-11
1.97E-16	623383.4	1.83E-05	3.61E-21	1.40E-10
9.85E-16	623366.6	1.83E-05	1.80E-20	7.00E-10
4.93E-15	623283	1.83E-05	9.02E-20	3.50E-09

2.46E-14	622865.5	1.83E-05	4.51E-19	1.75E-08
1.23E-13	620790.6	1.84E-05	2.26E-18	8.75E-08
6.16E-13	610718	1.87E-05	1.14E-17	4.37E-07
3.08E-12	566808.5	2.01E-05	5.93E-17	2.19E-06
1.54E-11	435603.5	2.62E-05	3.46E-16	1.09E-05
7.70E-11	249754.5	4.57E-05	2.62E-15	5.47E-05
3.13E-10	132268.8	8.63E-05	1.87E-14	0.000222
8.40E-10	81816.89	0.00014	7.94E-14	0.000597
2.64E-09	46433.82	0.000246	4.35E-13	0.001875
7.20E-09	28158.62	0.000405	1.95E-12	0.005112
2.06E-08	16640.45	0.000686	9.45E-12	0.014656
8.78E-08	8108.492	0.001408	8.29E-11	0.062376
1.59E-07	6011.925	0.001899	2.01E-10	0.112889
3.72E-07	3924.81	0.002909	7.20E-10	0.263971
8.57E-07	2582.958	0.00442	2.52E-09	0.608581
1.97E-06	1701.877	0.006709	8.82E-09	1.400931
4.54E-06	1121.503	0.010181	3.08E-08	3.225138
1.05E-05	739.0576	0.015449	1.08E-07	7.425653
2.41E-05	487.0291	0.023443	3.76E-07	17.0982
5.54E-05	320.9434	0.035574	1.31E-06	39.37142
0.000128	211.4935	0.053982	4.59E-06	90.66105
0.000294	139.3665	0.081915	1.61E-05	208.7699
0.000677	91.83502	0.124304	5.61E-05	480.7552
0.001559	60.51196	0.188627	0.000196	1107.114
0.003259	41.84398	0.27274	0.000593	2314.645
0.006202	30.32938	0.376219	0.001556	4404.388
0.011416	22.35055	0.510405	0.003885	8107.367
0.021131	16.42408	0.694358	0.009783	15007.04
0.039077	12.07394	0.944115	0.024599	27751.9
0.072275	8.87442	1.283723	0.061866	51327.2
0.133667	6.521901	1.7453	0.155567	94922.84
0.247192	4.791937	2.372513	0.391121	175535.9
0.457094	3.519564	3.224436	0.983094	324574.1
0.813294	2.633765	4.297784	2.331983	577463.5
1.169494	2.192588	5.15026	4.019307	830323.5
1.525694	1.916406	5.87882	5.986493	1083156
1.881894	1.722599	6.525128	8.19768	1335960
2.238094	1.576856	7.111646	10.62793	1588737
2.594294	1.46203	7.652124	13.25857	1841486
2.950494	1.368475	8.155708	16.07493	2094208
3.306694	1.290291	8.628843	19.06509	2346901

3.662894	1.223645	9.076276	22.21907	2599565
4.019094	1.165917	9.501623	25.52841	2852199
4.375294	1.115255	9.907713	28.98574	3104804
4.731494	1.070302	10.29681	32.58464	3357378
5.087694	1.030041	10.67075	36.31939	3609922
5.443894	0.993692	11.03107	40.18487	3862435
5.800094	0.960646	11.37905	44.17646	4114916
6.156294	0.930417	11.71577	48.28997	4367364
6.512494	0.902615	12.04217	52.52155	4619780
6.868694	0.876923	12.35904	56.86768	4872163
7.224894	0.853077	12.66711	61.32509	5124513
7.581094	0.830859	12.96697	65.89075	5376829
7.937294	0.810083	13.25918	70.56185	5629110
8.293494	0.790594	13.54423	75.33574	5881357
8.649694	0.772259	13.82254	80.20996	6133569
9.005894	0.754963	14.09451	85.18217	6385745
9.362094	0.738606	14.36048	90.25017	6637885
9.718294	0.723102	14.62079	95.4119	6889990
10.07449	0.708375	14.87571	100.6654	7142058
10.43069	0.694359	15.12552	106.0088	7394089
10.78689	0.680994	15.37045	111.4402	7646083
11.14309	0.66823	15.61073	116.9581	7898039
11.49929	0.656019	15.84657	122.5608	8149958
11.85549	0.644319	16.07814	128.2467	8401838
12.21169	0.633094	16.30563	134.0144	8653681
12.56789	0.622311	16.52921	139.8624	8905485
12.92409	0.611938	16.74902	145.7893	9157250
13.28029	0.601948	16.9652	151.7939	9408975
13.63649	0.592317	17.17789	157.8749	9660662
13.99269	0.583022	17.38721	164.0311	9912309
14.34889	0.574042	17.59327	170.2612	10163916
14.70509	0.565358	17.79619	176.5641	10415483
15.06129	0.556954	17.99606	182.9388	10667010
15.41749	0.548812	18.19299	189.3842	10918496
15.77369	0.540919	18.38707	195.8992	11169941
16.12989	0.53326	18.57837	202.4828	11421346
16.48609	0.525824	18.76699	209.1341	11672710
16.84229	0.518599	18.95299	215.8521	11924033
17.19849	0.511574	19.13646	222.6359	12175314
17.55469	0.50474	19.31746	229.4846	12426554
17.81	0.499953	19.44571	234.4329	12606604

Appendix table 2: Constant speed motion results for maximum level track

Traveling time (s)	Speed (m/s)	Distance (m)	Energy consumption (J)
0	19.4444	0	0
2.00E-09	19.4444	3.89E-08	0.0002
1.20E-08	19.4444	2.33E-07	0.00121
6.20E-08	19.4444	1.21E-06	0.00623
3.12E-07	19.4444	6.07E-06	0.03135
1.56E-06	19.4444	3.04E-05	0.15694
7.81E-06	19.4444	0.00015	0.78491
3.91E-05	19.4444	0.00076	3.92477
0.0002	19.4444	0.0038	19.6241
0.00098	19.4444	0.01899	98.1205
0.00488	19.4444	0.09497	490.603
0.02442	19.4444	0.47484	2453.01
0.1221	19.4444	2.3742	12265.1
0.61051	19.4444	11.871	61325.4
1.82301	19.4444	35.4474	183120
3.03551	19.4444	59.0237	304915
4.24801	19.4444	82.6001	426710
5.46052	19.4444	106.176	548505
6.67302	19.4444	129.753	670300
7.88552	19.4444	153.329	792095
9.09802	19.4444	176.906	913890
10.3105	19.4444	200.482	1035685
11.523	19.4444	224.058	1157480
12.7355	19.4444	247.635	1279275
13.948	19.4444	271.211	1401070
15.1605	19.4444	294.787	1522865
16.373	19.4444	318.364	1644660
17.5855	19.4444	341.94	1766455
18.798	19.4444	365.516	1888250
20.0105	19.4444	389.093	2010045
21.223	19.4444	412.669	2131840
22.4355	19.4444	436.246	2253636
23.648	19.4444	459.822	2375431
24.8605	19.4444	483.398	2497226

26.073	19.4444	506.975	2619021
27.2855	19.4444	530.551	2740816
28.498	19.4444	554.127	2862611
29.7105	19.4444	577.704	2984406
30.9231	19.4444	601.28	3106201
32.1356	19.4444	624.857	3227996
33.3481	19.4444	648.433	3349791
34.5606	19.4444	672.009	3471586
35.7731	19.4444	695.586	3593381
36.9856	19.4444	719.162	3715176
38.1981	19.4444	742.738	3836971
39.4106	19.4444	766.315	3958766
40.6231	19.4444	789.891	4080561
41.8356	19.4444	813.467	4202356
43.0481	19.4444	837.044	4324151
44.2606	19.4444	860.62	4445946
45.4731	19.4444	884.197	4567741
46.6856	19.4444	907.773	4689536
47.8981	19.4444	931.349	4811331
49.1106	19.4444	954.926	4933126
50.3231	19.4444	978.502	5054921
51.5356	19.4444	1002.08	5176716
52.7481	19.4444	1025.65	5298511
53.9606	19.4444	1049.23	5420306
55.1731	19.4444	1072.81	5542101
56.3856	19.4444	1096.38	5663896
57.5981	19.4444	1119.96	5785691
58.8106	19.4444	1143.54	5907486
60.0231	19.4444	1167.11	6029281
60.6251	19.4444	1178.82	6089750