

A COHERENTLY DRIVEN DEGENARATE THREE-LEVEL ATOM

By

Yosef Terefe

A THESIS SUBMITTED TO
THE DEPARTMENT OF PHYSICS

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE

ADDIS ABABA UNIVERSITY
ADDIS ABABA,ETHIOPIA
JUNE 2013

© Copyright by Yosef Terefe, 2005 E.C.

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared by **Yosef Terefe**, entitled "**A COHERENTLY DRIVEN DEGENARATE THREE-LEVEL ATOM**" and submitted by **Yosef Terefe** in partial fulfillment of the requirements for the degree of **Master of Science**. complies with the regulations of the University and meets the accepted standards with respect to originality.

Signed by the Examining Committee:

Examiner:Dr. Belayneh Mesfin Signature:_____Date:_____
Examiner:Dr. Tadelech Atomsa Signature:_____Date:_____
Advisor: Dr. Fesseha Kassahun Signature:_____Date:_____

Abstract

We analyze the squeezing and statistical properties of a degenerate three-level atom coupled to a coherent light and vacuum reservoir. In the other way, employing the Heseinberg and Quantum Langevin equation associated with the normal ordering noise operator, we study the squeezing properties, power spectrum, variance and also calculate the mean photon number of the cavity mode. It turns out that the generated light exhibits squeezing under certain conditions.

Acknowledgements

I would like to express my strong gratitude and respect to my advisor and instructor, Dr. Fesseha Kassahun, for developing this thesis and for guiding me to use all my efforts on research work besides commenting and supporting me in every area of my work by sharing his very rich experience in the area of Quantum optics.

Table of Contents

Abstract **iii**

Acknowledgements **iv**

Table of Contents **v**

List of Figures **vi**

1 Introduction **1**

2 Dynamics of Atomic and Cavity Mode Operators **3**

3 Photon Statistics **10**

3.1 The mean photon-number 10

3.2 The photon-number variance 12

3.3 Power spectrum 13

4 Quadrature Squeezing **20**

5 Conclusion **25**

Bibliography **26**

List of Figures

2.1 The transition between $|a\rangle - |b\rangle$ and $|b\rangle - |c\rangle$ at frequency ω each are taken to be resonant with the cavity 3

3.1 Plot of steady-state mean photon number versus λ for $\kappa = 0.8$ and different values of $\gamma_c = 0.5$ (blue) and $\gamma_c = 2.4$ (pink). 11
3.2 A plot of Eq. (3.3.34) for $\kappa = 0.8$, $\gamma_c = 0.5$ and $\Omega = 3$ 18

4.1 Plot of the steady-state minus quadrature variance versus λ for $\kappa = 0.8$ and $\gamma_c = 0.5$ 22
4.2 Plot of degree squeezing versus λ. 23

Chapter 1

Introduction

Interaction of three-level atoms with light has attracted a great deal of interest in recent years [1, 2]. Some authors have studied the squeezing and statistical properties of the light produced by three-level atoms in which the crucial role is played by the coupling of the top and bottom levels [1,4,9,10].

The coupling of the top and bottom levels is responsible for the interesting nonclassical feature of the generated light. In general the atomic coherence can be induced in a three-level atom by coupling the levels between which direct transition is dipole forbidden by coherent light or by preparing the atom initially in coherent superposition of these two levels [4,7].

In a three-level atom the top, intermediate, and bottom levels are denoted $|a\rangle, |b\rangle$, and $|c\rangle$ in which direct transition between $|a\rangle$ and $|c\rangle$ is dipole forbidden [4]. When the three-level atom decays form $|a\rangle$ to $|c\rangle$ via level $|b\rangle$, two photons are emitted. If the two photons have the same frequency, then the three-level atom is referred to as a degenerate three-level atom, otherwise it referred to as a nondegenerate three-level atom [7]. It has been established that a three-level atom under certain conditions generates squeezed light [5, 6].

In this thesis we seek to study the squeezing and statistical properties of the light emitted by a degenerate three-level atom in a closed cavity coupled to vacuum reservoir and driven by coherent light. In order to determine the squeezing and statistical properties of the light produced by this quantum optical system, we first derive the equations of evolution of the cavity mode and atomic operators. Then

using the solutions of these equations, we calculate the mean photon number, the photon number variance, and the quadrature variance.

Chapter 2

Dynamics of Atomic and Cavity Mode Operators

In this chapter we consider a degenerate three-level atom interacting with in closed cavity coupled to a vacuum reservoir and the upper and lower level coupled by coherent light. We denote the upper level by $|a\rangle$, the middle level by $|b\rangle$, and the lower level by $|c\rangle$, as shown in Fig. 2.1. The dipole allowed the transition between levels $|a\rangle$ and $|b\rangle$ and between levels $|b\rangle$ and $|c\rangle$ are resonant with the cavity mode. The direct transition between level $|a\rangle$ and $|c\rangle$ is dipole forbidden [3]. The Hamiltonian that describes the interaction between the three-level atom and the coherent light is given by

$$\hat{H}_i = iG(|a\rangle\langle c|\hat{b} - \hat{b}^\dagger|c\rangle\langle a|), \tag{2.0.1}$$

where G is the coupling constant between the coherent light and the three-level atom, and $\hat{b}$ is the annihilation operator for the coherent light. Assuming the the coherent light to be strong, it can be treated classically [4]. In view of this we can

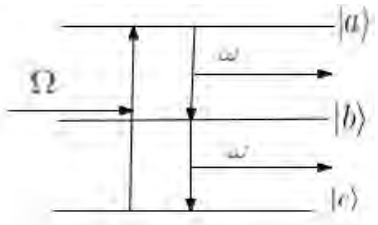


Figure 2.1: The transition between $|a\rangle - |b\rangle$ and $|b\rangle - |c\rangle$ at frequency ω each are taken to be resonant with the cavity

replace the operator $\hat{b}$ by ϵ. Now we rewrite Eq. (2.0.1) as

$$H_i = iG\epsilon(|a\rangle\langle c| - |c\rangle\langle a|), \tag{2.0.2}$$

$$H_i = i\frac{\Omega}{2}(|a\rangle\langle c| - |c\rangle\langle a|), \tag{2.0.3}$$

where

$$\Omega = 2\epsilon G \tag{2.0.4}$$

is the Rabi frequency. When the atom decays from level $|a\rangle$ to $|c\rangle$ via $|b\rangle$, two photons of the same frequency are emitted. The Hamiltonian describing the interaction between the three-level atom and the cavity mode is expressible as

$$H_c = ig[(|a\rangle\langle b| + |b\rangle\langle c|)\hat{a} - \hat{a}^\dagger(|b\rangle\langle a| + |c\rangle\langle b|)], \tag{2.0.5}$$

where g is coupling constant between the three-level atom and the cavity mode. The Hamiltonian that describes the interaction of the three level atom with coherent light and cavity mode is

$$H = i\frac{\Omega}{2}(|a\rangle\langle c| - |c\rangle\langle a|) + ig[(|a\rangle\langle b| + |b\rangle\langle c|)\hat{a} - \hat{a}^\dagger(|b\rangle\langle a| + |c\rangle\langle b|)]. \tag{2.0.6}$$

In order to have more convenient expressions, we introduce the atomic operators

$$\hat{\sigma}_a^\dagger = |a\rangle\langle b|, \tag{2.0.7}$$

$$\hat{\sigma}_a = |b\rangle\langle a|, \tag{2.0.8}$$

$$\hat{\sigma}_b^\dagger = |b\rangle\langle c|, \tag{2.0.9}$$

$$\hat{\sigma}_b = |c\rangle\langle b|, \tag{2.0.10}$$

$$\hat{\sigma}_{ac}^\dagger = |a\rangle\langle c|, \tag{2.0.11}$$

$$\hat{\sigma}_{ac} = |c\rangle\langle a|, \tag{2.0.12}$$

$$\hat{\sigma}_{aa} = |a\rangle\langle a|, \tag{2.0.13}$$

$$\hat{\sigma}_{bb} = |b\rangle\langle b| \tag{2.0.14}$$

and

$$\hat{\sigma}_{cc} = |c\rangle\langle c|. \tag{2.0.15}$$

Note that $\hat{\sigma}_a^\dagger$, $\hat{\sigma}_b^\dagger$, and $\hat{\sigma}_{ac}^\dagger$ are atomic rising operators, while their complex adjoint are atomic lowering operators. On account these operators, Eq. (2.0.6) can be rewritten as

$$H = i\frac{\Omega}{2}(\hat{\sigma}_{ac}^\dagger - \hat{\sigma}_{ac}) + ig[(\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger)\hat{a} - \hat{a}^\dagger(\hat{\sigma}_a + \hat{\sigma}_b)]. \tag{2.0.16}$$

We can find that the time evolution of the atomic operators using the relation

$$\frac{d\langle\hat{A}\rangle}{dt} = -i\langle[\hat{A}, \hat{H}]\rangle. \tag{2.0.17}$$

Employing Eq. (2.0.17), along with Eq. (2.0.16), one readily obtain

$$\frac{d\hat{\sigma}_a}{dt} = \frac{\Omega}{2}\hat{\sigma}_b^\dagger + g(\hat{\sigma}_{bb} - \hat{\sigma}_{aa})\hat{a} + g\hat{a}^\dagger\sigma_{ac}, \tag{2.0.18}$$

$$\frac{d\hat{\sigma}_b}{dt} = -\frac{\Omega}{2}\hat{\sigma}_a^\dagger + g(\hat{\sigma}_{cc} - \hat{\sigma}_{bb})\hat{a} - g\hat{a}^\dagger\sigma_{ac}, \tag{2.0.19}$$

$$\frac{d\hat{\sigma}_{ac}}{dt} = \frac{\Omega}{2}(\hat{\sigma}_{cc} - \hat{\sigma}_{aa}) + g(\hat{\sigma}_b - \hat{\sigma}_a)\hat{a}, \tag{2.0.20}$$

$$\frac{d\hat{\sigma}_{aa}}{dt} = \frac{\Omega}{2}(\hat{\sigma}_{ac}^\dagger + \hat{\sigma}_{ac}) + g(\hat{\sigma}_a^\dagger\hat{a} + \hat{a}^\dagger\hat{\sigma}_a), \tag{2.0.21}$$

$$\frac{d\hat{\sigma}_{bb}}{dt} = -g(\hat{\sigma}_a^\dagger - \hat{\sigma}_b^\dagger)\hat{a} - g\hat{a}^\dagger(\hat{\sigma}_a - \hat{\sigma}_b) \tag{2.0.22}$$

and

$$\frac{d\hat{\sigma}_{cc}}{dt} = \frac{\Omega}{2}(\hat{\sigma}_{ac}^\dagger + \hat{\sigma}_{ac}) - g(\hat{\sigma}_b^\dagger\hat{a} + \hat{a}^\dagger\hat{\sigma}_b). \tag{2.0.23}$$

We assume that the is cavity closed and coupled to vacuum reservoir. In addition, we carry out our calculation by putting the noise operator $\hat{F}$ in normal order. Thus the noise operator will not have any effect on the dynamics of the cavity mode operators. We can then drop the noise operator and rewrite the quantum Langevin equation for the operator $\hat{a}$ as

$$\frac{d\hat{a}}{dt} = -i[\hat{a}, \hat{H}] - \frac{\kappa}{2}\hat{a}, \tag{2.0.24}$$

where κ is cavity decay constant. On account of Eq. (2.0.16), we easily find

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2}\hat{a} - g(\hat{\sigma}_a + \hat{\sigma}_b). \tag{2.0.25}$$

We see that Eqs. (2.0.18-2.0.23) are nonlinear differential equations and it is not possible to find exact time-dependent solutions of these equations. We intend to overcome this problem by applying the large-time approximation. Then using this approximation scheme, we get

$$\hat{a} = -\frac{2g}{\kappa}\left(\hat{\sigma}_a + \hat{\sigma}_b\right). \tag{2.0.26}$$

This expression will be used for operator $\hat{a}$ that appears in a product with an atomic operator. Introducing Eq. (2.0.26) into Eqs (2.0.18 - 2.0.23) and employing the fact that

$$\hat{\sigma}_{aa} + \hat{\sigma}_{bb} + \hat{\sigma}_{cc} = 1, \tag{2.0.27}$$

we readily obtain the first set

$$\frac{d\langle\hat{\sigma}_a\rangle}{dt} = -\gamma_c\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^\dagger\rangle, \tag{2.0.28}$$

$$\frac{d\langle\hat{\sigma}_b\rangle}{dt} = -\frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle + \gamma_c\langle\sigma_a\rangle, \tag{2.0.29}$$

and second set is

$$\frac{d\langle\hat{\sigma}_{ac}\rangle}{dt} = -\frac{\gamma_c}{2}\langle\hat{\sigma}_{ac}\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_{cc} - \hat{\sigma}_{bb}\rangle, \tag{2.0.30}$$

$$\frac{d\langle\hat{\sigma}_{aa}\rangle}{dt} = -\gamma_c\langle\hat{\sigma}_{aa}\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_{ac}^\dagger + \hat{\sigma}_{ac}\rangle, \tag{2.0.31}$$

$$\frac{d\langle\hat{\sigma}_{bb}\rangle}{dt} = \gamma_c(\langle\hat{\sigma}_{aa} - \sigma_{bb}\rangle), \tag{2.0.32}$$

and

$$\frac{d\langle\hat{\sigma}_{cc}\rangle}{dt} = \gamma_c\langle\hat{\sigma}_{bb}\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_{ac}^\dagger + \hat{\sigma}_{ac}\rangle, \tag{2.0.33}$$

where

$$\gamma_c = \frac{4g^2}{\kappa} \tag{2.0.34}$$

is the stimulated emission decay constant. We note that the two set of differential equations are independent. We first deal with the second set to determine the equation of evolution of the density matrix for a coherently driven degenerate three-level atom and coupled to a single-mode vacuum reservoir.We can easily verify that

$$\langle \hat{\sigma}_a \rangle = \rho_{ab}, \tag{2.0.35}$$

$$\langle \hat{\sigma}_b \rangle = \rho_{bc}, \tag{2.0.36}$$

$$\langle \hat{\sigma}_{aa} \rangle = \rho_{aa}, \tag{2.0.37}$$

$$\langle \hat{\sigma}_{bb} \rangle = \rho_{bb}, \tag{2.0.38}$$

$$\langle \hat{\sigma}_{cc} \rangle = \rho_{cc} \tag{2.0.39}$$

and

$$\langle \hat{\sigma}_{ac} \rangle = \rho_{ac}. \tag{2.0.40}$$

Using these equations, we can write time evolution of the density matrix as

$$\frac{d\rho_{ab}}{dt} = -\gamma_c \rho_{ab} + \frac{\Omega}{2}\rho_{cb}, \tag{2.0.41}$$

$$\frac{d\rho_{bc}}{dt} = -\frac{\gamma_c}{2}\rho_{bc} - \frac{\Omega}{2}\rho_{ba} + \gamma_c \rho_{ab}, \tag{2.0.42}$$

$$\frac{d\rho_{aa}}{dt} = -\gamma_c \rho_{aa} + \frac{\Omega}{2}(\rho_{ca} + \rho_{ac}), \tag{2.0.43}$$

$$\frac{d\rho_{bb}}{dt} = \gamma_c(\rho_{aa} - \rho_{bb}), \tag{2.0.44}$$

$$\frac{d\rho_{ac}}{dt} = -\gamma_c \rho_{ac} + \frac{\Omega}{2}(\rho_{cc} - \rho_{aa}) \tag{2.0.45}$$

and

$$\frac{d\rho_{cc}}{dt} = -\gamma_c \rho_{bb} - \frac{\Omega}{2}(\rho_{ca} + \rho_{ac}), \tag{2.0.46}$$

Now we proceed to obtain the steady state solution of $\rho_{aa},\ \rho_{bb},\ \rho_{ac}\ and\ \rho_{cc}$. These solution will be used to study the probability and the squeezing properties

of the emitted light. Subject to this condition the steady state solution of Eq. (2.0.43), (2.0.44), (2.0.45) and (2.0.46) are

$$\rho_{aa} = \hat{\rho}_{bb}, \tag{2.0.47}$$

$$\rho_{aa} = \frac{\Omega}{\gamma_c}(\rho_{ac}), \tag{2.0.48}$$

$$\hat{\rho}_{ac} = \frac{\Omega}{\gamma_c}(\rho_{cc} - \rho_{aa}). \tag{2.0.49}$$

Employing the fact that

$$\rho_{aa} + \rho_{bb} + \rho_{cc} = 1. \tag{2.0.50}$$

And Using Eq. (2.0.47), we easily get

$$\rho_{cc} = 1 - 2\rho_{aa}, \tag{2.0.51}$$

substituting Eq. (2.0.51) into Eq. (2.0.49), we easily get

$$\rho_{ac} = \frac{\Omega}{\gamma_c}(1 - 3\rho_{aa}), \tag{2.0.52}$$

on account of Eq. (2.0.48), we readily obtain

$$\rho_{ac} = \frac{\Omega\gamma_c}{\gamma_c^2 + 3\Omega^2}. \tag{2.0.53}$$

Putting Eq. (2.0.53) into Eq. (2.0.48), we get

$$\rho_{aa} = \frac{\Omega}{\gamma_c}(\frac{\Omega\gamma_c}{\gamma_c^2 + 3\Omega^2}), \tag{2.0.54}$$

then, we can reduce

$$\rho_{aa} = \frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}. \tag{2.0.55}$$

Next substituting Eq. (2.0.55) into (2.0.51), we get

$$\rho_{cc} = 1 - 2\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right), \tag{2.0.56}$$

this can be rewritten as

$$\rho_{cc} = \frac{\gamma_c^2 + \Omega^2}{\gamma_c^2 + 3\Omega^2}. \tag{2.0.57}$$

For a weak coherent light the atom settle close to its lower level($\Omega << \gamma_c,\ \rho_{aa} \backsimeq 0$). For strong coherent light there is nearly equal probability of finding the atom in all levels. For $\Omega >> \gamma$, we see that

$$\rho_{aa} = \rho_{bb} = \rho_{cc} = \frac{1}{3}. \tag{2.0.58}$$

For $\Omega << \gamma$, we readily find

$$\rho_{aa} = \rho_{bb} = 0 \tag{2.0.59}$$

and

$$\rho_{cc} = 1. \tag{2.0.60}$$

By comparing Eq. (2.0.55) and Eq. (2.0.57), we easily see that

$$\rho_{cc} > \rho_{aa}. \tag{2.0.61}$$

Eq. (2.0.61) implies that, at steady state there is a higher probability of finding the atom in the bottom level than the upper or the intermediate level. Moreover, in the absence of the coherent light or $\Omega = 0$ there is no probability of fining the atom either in the upper or intermediate level.

Chapter 3

Photon Statistics

The statistical properties of a light mode are described in terms of the mean and variance of the photon number. In previous chapter we have seen that the expression of the expectation value of atomic and cavity mode operators. Now we seek to study the cavity mode mean photon number, variance of the photon number and power spectrum of coherently driven degenerate three-level atom at steady state.

3.1 The mean photon-number

Now we proceed to determine the mean photon number of the three-level atom at steady-state. Applying the steady-state solution of Eq. (2.0.26), we get

$$\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{4g^2}{\kappa^2} \langle (\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger)(\hat{\sigma}_a + \hat{\sigma}_b) \rangle, \tag{3.1.1}$$

It then follows that

$$\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{4g^2}{\kappa^2} [\langle \sigma_{aa} \rangle + \langle \sigma_{bb} \rangle]. \tag{3.1.2}$$

Substituting Eq. (2.0.37) and Eq. (2.0.38), we easily get

$$\bar{n} = \frac{\gamma_c}{\kappa} (\rho_{aa} + \rho_{bb}). \tag{3.1.3}$$

On account of Eq. (2.0.47), one easily rewrite

$$\bar{n} = \frac{\gamma_c}{\kappa} (2\hat{\rho}_{aa}). \tag{3.1.4}$$

In view of Eq. (2.0.55) the steady-state mean photon number of the cavity light turns out to be

$$\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{\gamma_c}{\kappa} \left[\frac{2\Omega^2}{(\gamma_c^2 + 3\Omega^2)} \right]. \tag{3.1.5}$$

This can be rewritten as

$$\bar{n} = \langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{\gamma_c}{\kappa} \left[\frac{2\lambda^2}{(1 + 3\lambda^2)} \right], \tag{3.1.6}$$

where

$$\lambda = \frac{\Omega}{\gamma_c}. \tag{3.1.7}$$

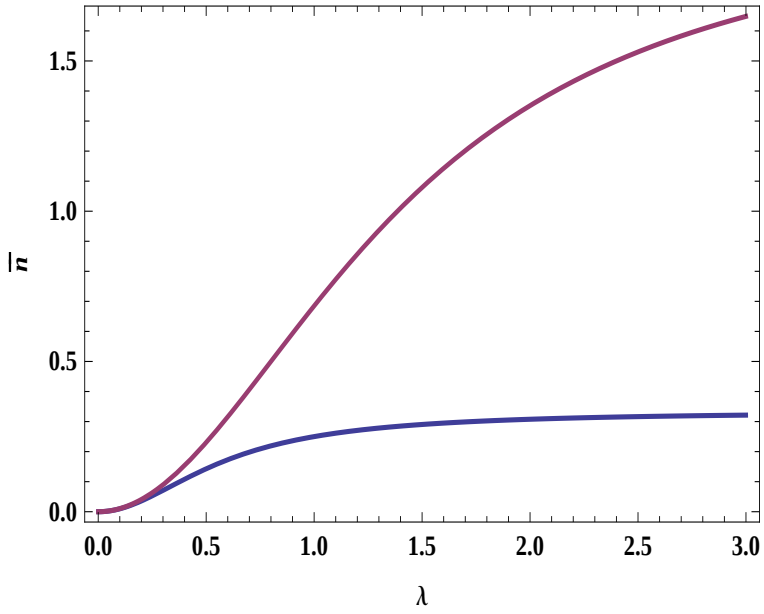


Figure 3.1: Plot of steady-state mean photon number versus λ for $\kappa = 0.8$ and different values of $\gamma_c = 0.5$ (blue) and $\gamma_c = 2.4$ (pink).

For $(\lambda \gg 1)$, Eq. (3.1.6) reduce to

$$\langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{2\gamma_c}{3\kappa}, \tag{3.1.8}$$

which is independent of external field radiation Ω. For weak coupling or $\lambda \ll 1$, We note that

$$\langle \hat{a}^\dagger \hat{a} \rangle_{ss} = \frac{2\Omega^2}{\kappa\gamma_c} \backsimeq 0 \tag{3.1.9}$$

For $\lambda = 1$, we easily get

$$\bar{n} = \frac{\gamma_c}{2\kappa}. \tag{3.1.10}$$

As the figure (3.1) shows that, with increasing the value of λ and γ_c the mean photon number also increases.

3.2 The photon-number variance

In this section we seek to find photon number variance of coherently driven degenerate three-level atoms coupled to vacuum reservoir at steady state. The variance of photon number for the cavity mode is expressible as

$$(\Delta n)^2 = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2. \tag{3.2.1}$$

Now in view of Eq. (2.0.26) and the assumption that the cavity light is initially in a vacuum state,

$$\langle \hat{a} \rangle = 0 \tag{3.2.2}$$

And we observe on the basis of Eqs. (2.0.26) and (3.2.2) that $\hat{a}$ is a Gaussian variable with zero mean. On account of this, we readily get

$$(\Delta n)^2 = \langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{a} \hat{a}^\dagger \rangle + \langle \hat{a}^{\dagger 2} \rangle \langle \hat{a}^2 \rangle. \tag{3.2.3}$$

Its preferable, if we calculate the variable $\langle \hat{a}^\dagger \hat{a} \rangle$, $\langle \hat{a} \hat{a}^\dagger \rangle$, $\langle \hat{a}^{\dagger 2} \rangle$ and $\langle \hat{a}^2 \rangle$ separately. Employing once Eq. (2.0.26), we find

$$\langle \hat{a} \hat{a}^\dagger \rangle = \frac{\gamma_c}{\kappa} (\hat{\sigma}_a + \hat{\sigma}_b)(\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger). \tag{3.2.4}$$

It then follows that

$$\langle \hat{a} \hat{a}^\dagger \rangle = \frac{\gamma_c}{\kappa} [\langle \sigma_{bb} \rangle + \langle \sigma_{cc} \rangle]. \tag{3.2.5}$$

Substituting Eq. (2.0.38) and Eq. (2.0.39), we get

$$\langle \hat{a} \hat{a}^\dagger \rangle = \frac{\gamma_c}{\kappa} (\rho_{bb} + \rho_{cc}). \tag{3.2.6}$$

Similarly with the aid of Eq. (2.0.26), we easily get

$$\langle \hat{a}^2 \rangle = \frac{\gamma_c}{\kappa} \rho_{ac}. \tag{3.2.7}$$

Taking the complex conjugate of Eq.(3.2.7), we find

$$\langle \hat{a}^{\dagger 2} \rangle = \frac{\gamma_c}{\kappa} \rho_{ac} * . \tag{3.2.8}$$

We assume that $\rho_{ac} = \rho_{ac}^*$ and we can see that Eqs.(3.2.7)and (3.2.8) are equal. So that substitute Eqs. (3.1.4), (3.2.6), (3.2.7) and (3.2.8), in to Eq. (3.2.3), we arrive at

$$(\Delta n)^2 = \frac{\gamma_c^2}{\kappa^2} 2\rho_{aa}(\rho_{bb} + \rho_{cc}) + \frac{\gamma_c^2}{\kappa^2}\rho_{ac}^2. \tag{3.2.9}$$

Substitute Eqs. (2.0.51), (2.0.53), (2.0.55) and (2.0.57), into Eq. (3.2.9), we readily get

$$(\Delta n)^2 = \frac{4\gamma_c^2}{\kappa^2}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right)^2 + \frac{3\gamma_c^2}{\kappa^2}\left(\frac{\Omega^2\gamma_c^2}{(\gamma_c^2 + 3\Omega^2)^2}\right). \tag{3.2.10}$$

On account of Eq. (3.1.5), we easily establish that

$$(\Delta n)^2 = \bar{n}^2(1 + \frac{3\gamma_c^2}{4\Omega^2}), \tag{3.2.11}$$

where $\bar{n}$ is steady state mean photon number. We see that for $\Omega \gg \gamma_c$,

$$(\Delta n)^2 = \bar{n}^2. \tag{3.2.12}$$

And for $\Omega = \gamma_c$, we get

$$(\Delta n)^2 = \frac{7}{4}\bar{n}^2, \tag{3.2.13}$$

For $\Omega \gg \gamma_c$ and $\Omega = \gamma_c$,

$$(\Delta n)^2 > \bar{n}. \tag{3.2.14}$$

Eq. (3.2.14) indicate that, the photon statistics of the light mode is supper-Poissonian.

3.3 Power spectrum

In nearly all cases the frequency of a single-mode light is not sharply defined. In general, there is some variation about the central frequency [4]. We wish to obtain the spectrum of the mean photon number, usually known as power spectrum, of light emitted by coherently driven degenerated three-level atoms. We would like to mention that $\hat{a}$ and $\hat{a}^\dagger$ can be cavity mode operators. We define the power spectrum of single mode light with central frequency ω_o by

$$P(\omega) = \frac{1}{\pi} Re \int_0^\infty d\tau e^{i(\omega-\omega_0)\tau} \langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle_{ss}, \tag{3.3.1}$$

$$\int_{-\infty}^{\infty} P(\omega)d\omega = \bar{n}. \tag{3.3.2}$$

In which $\bar{n}$ is the steady-state mean photon number. From this result, we observe that $P(\omega)d\omega$ is the steady-state mean photon number in the interval between ω and $\omega + d\omega$. Now we proceed to determine the two time correlation function that appears Eq. (3.3.1) for the cavity light. To this end, we realize that the solution of Eq. (2.0.25), can also be written as

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\frac{\kappa\tau}{2}} - ge^{-\frac{\kappa\tau}{2}} \int_0^{\tau} e^{\frac{\kappa\tau'}{2}} (\hat{\sigma}_a(t+\tau') + \hat{\sigma}_b(t+\tau')d\tau'. \tag{3.3.3}$$

Now we need to solve $\hat{\sigma}_a(t+\tau')$ and $\hat{\sigma}_b(t+\tau')$ that appears Eq. (3.3.3) from these two deferential equation

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -\gamma_c\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger}\rangle \tag{3.3.4}$$

and

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -\frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger}\rangle + \gamma_c\langle\sigma_a\rangle. \tag{3.3.5}$$

Applying the large time approximation to Eq. (3.3.4), we get

$$\langle\hat{\sigma}_a(t)\rangle = \frac{\Omega}{2\gamma_c}\langle\hat{\sigma}_b^{\dagger}(t)\rangle. \tag{3.3.6}$$

Substituting Eq. (3.3.6) into Eq. (3.3.5), we arrive at

$$\frac{d}{dt}\langle\hat{\sigma}_b(t)\rangle = -\left(\frac{\gamma_c}{2} + \frac{\Omega^2}{4\gamma_c}\right)\langle\hat{\sigma}_b(t)\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger}(t)\rangle \tag{3.3.7}$$

and the complex conjugate of Eq. (3.3.7) is given by

$$\frac{d}{dt}\langle\hat{\sigma}_b^{\dagger}(t)\rangle = -(\frac{\gamma_c}{2} + \frac{\Omega^2}{4\gamma_c})\langle\hat{\sigma}_b^{\dagger}(t)\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b(t)\rangle. \tag{3.3.8}$$

Next by adding and subtract Eq. (3.3.7) and Eq. (3.3.8), we readily get

$$\frac{d}{dt}\langle\hat{\sigma}_b(t) \pm \hat{\sigma}_b^{\dagger}(t)\rangle = -(\frac{\gamma_c}{2} + \frac{\Omega^2}{4\gamma_c} \mp \frac{\Omega}{2})\langle\hat{\sigma}_b(t) \pm \hat{\sigma}_b^{\dagger}(t)\rangle. \tag{3.3.9}$$

The solution of Eq. (3.3.9) is given as

$$\langle\hat{\sigma}_b(t) \pm \hat{\sigma}_b^{\dagger}(t)\rangle = \langle\hat{\sigma}_b(0) \pm \hat{\sigma}_b^{\dagger}(0)\rangle e^{-\eta_{\pm}t}. \tag{3.3.10}$$

We can also rewrite by replacing t $\rightarrow t+\tau$ and $0 \rightarrow t$ as

$$\langle \hat{\sigma}_b(t+\tau) \pm \hat{\sigma}_b^\dagger(t+\tau)\rangle = \langle \hat{\sigma}_b(t) \pm \hat{\sigma}_b^\dagger(t)\rangle e^{-\eta_\pm \tau}. \tag{3.3.11}$$

then, we see that

$$\langle \hat{\sigma}_b(t+\tau)\rangle = \langle \hat{\sigma}_b(t)\rangle(e^{-\eta_+\tau} + e^{-\eta_-\tau}) + \langle \hat{\sigma}_b^\dagger(t)\rangle(e^{-\eta_+\tau} - e^{-\eta_-\tau}) \tag{3.3.12}$$

and

$$\langle \hat{\sigma}_b^\dagger(t+\tau)\rangle = \langle \hat{\sigma}_b^\dagger(t)\rangle(e^{-\eta_+\tau} + e^{-\eta_-\tau}) + \langle \hat{\sigma}_b(t)\rangle(e^{-\eta_+\tau} - e^{-\eta_-\tau}), \tag{3.3.13}$$

where

$$\eta_\pm = \frac{\gamma_c}{2} + \frac{\Omega^2}{4\gamma_c} \mp \frac{\Omega}{2}. \tag{3.3.14}$$

Substituting Eq. (3.3.13) into Eq. (3.3.6), we readily get

$$\langle \hat{\sigma}_a(t+\tau)\rangle = \frac{\Omega}{2\gamma_c}\left[\langle \hat{\sigma}_b^\dagger(t)\rangle(e^{-\eta_+\tau} + e^{-\eta_-\tau}) + \langle \hat{\sigma}_b(t)\rangle(e^{-\eta_+\tau} - e^{-\eta_-\tau})\right]. \tag{3.3.15}$$

Now combination of Eq. (3.3.3) with Eq. (3.3.12) and (3.3.15) yields

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\frac{\kappa\tau}{2}} - ge^{-\frac{\kappa\tau}{2}}\int_0^\tau d\tau' \frac{\Omega}{2\gamma_c}\left[\langle \hat{\sigma}_b^\dagger(t)\rangle(e^{-\eta_+\tau'} + e^{-\eta_-\tau'}) + \langle \hat{\sigma}_b(t)\rangle(e^{-\eta_+\tau'} - e^{-\eta_-\tau'})\right]$$

$$-ge^{-\frac{\kappa\tau}{2}}\int_0^\tau d\tau'[e^{-\frac{\kappa\tau'}{2}}\langle \hat{\sigma}_b(t)\rangle(e^{-\eta_+\tau'} + e^{-\eta_-\tau'}) + \langle \hat{\sigma}_b^\dagger(t)\rangle(e^{-\eta_+\tau'} - e^{-\eta_-\tau'})] \tag{3.3.16}$$

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\frac{\kappa\tau}{2}} - \left[g(\hat{\sigma}_b(t) + \hat{\sigma}_b^\dagger(t))\left(1 + \frac{\Omega}{2\gamma_c}\right)\right]e^{-\kappa\tau}/2\int_0^\tau e^{(\kappa-\eta_+)\tau'}d\tau'$$

$$-\left[g(\hat{\sigma}_b(t) - \hat{\sigma}_b^\dagger(t))\left(1 - \frac{\Omega}{2\gamma_c}\right)\right]e^{\frac{-\kappa}{2}\tau}\int_0^\tau e^{(\frac{\kappa}{2}-\eta_-)\tau'}d\tau'. \tag{3.3.17}$$

carrying out the integration, we easily get

$$\hat{a}(t+\tau) = \hat{a}(t)e^{-\frac{\kappa\tau}{2}} - \left[g(\hat{\sigma}_b(t) + \hat{\sigma}_b^\dagger(t))\left(1 + \frac{\Omega}{2\gamma_c}\right)\right]\left(\frac{2}{\kappa - 2\eta_+}\right)[e^{-\eta_+\tau} - e^{-\frac{\kappa\tau}{2}}]$$

$$- \left[g(\hat{\sigma}_b(t) - \hat{\sigma}_b^\dagger(t)) \left(1 - \frac{\Omega}{2\gamma_c}\right)\right] \left(\frac{2}{\kappa - 2\eta_-}\right) [e^{-\eta_- \tau} - e^{-\frac{\kappa\tau}{2}}]. \tag{3.3.18}$$

Hence using this result, we readily arrive at

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle e^{-\frac{\kappa\tau}{2}}$$

$$- g \left[[\langle \hat{a}^\dagger(t)(\hat{\sigma}_b(t) + \hat{\sigma}_b^\dagger(t))\rangle] \left(1 + \frac{\Omega}{2\gamma_c}\right) \left(\frac{2}{\kappa - 2\eta_+}\right)\right] [e^{-\eta_+ \tau} - e^{-\frac{\kappa\tau}{2}}]$$

$$-g \left[\langle \hat{a}^\dagger(t)(\hat{\sigma}_b(t) - \hat{\sigma}_b^\dagger(t))\rangle \left(1 - \frac{\Omega}{2\gamma_c}\right) \left(\frac{2}{\kappa - 2\eta_-}\right)\right] [e^{-\eta_- \tau} - e^{-\frac{\kappa\tau}{2}}]. \tag{3.3.19}$$

Employing once more the complex conjugate of Eq. (2.0.26) into Eq. (3.3.19), we get

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle e^{-\frac{\kappa\tau}{2}}$$

$$+ \left[[\langle \hat{\sigma}_{bb} + \hat{\sigma}_{ac}\rangle] \left(1 + \frac{\Omega}{2\gamma_c}\right) \left(\frac{\gamma_c}{\kappa - 2\eta_+}\right)\right] [e^{-\eta_+ \tau} - e^{-\frac{\kappa\tau}{2}}]$$

$$+ \left[\langle \hat{\sigma}_{bb} - \hat{\sigma}_{ac}\rangle \left(1 - \frac{\Omega}{2\gamma_c}\right) \left(\frac{\gamma_c}{\kappa - 2\eta_-}\right)\right] [e^{-\eta_- \tau} - e^{-\frac{\kappa\tau}{2}}]. \tag{3.3.20}$$

On account of Eq. (2.0.38) and the complex conjugate of Eq. (2.0.40), we can easily rewrite Eq. (3.3.19) as

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^\dagger(t)\hat{a}(t)\rangle e^{-\frac{\kappa\tau}{2}}$$

$$+ \left[[\rho_{bb} + \rho_{ac}\rangle] \left(1 + \frac{\Omega}{2\gamma_c}\right) \left(\frac{\gamma_c}{\kappa - 2\eta_+}\right)\right] [e^{-\eta_+ \tau} - e^{-\frac{\kappa\tau}{2}}]$$

$$+ \left[[\rho_{bb} - \rho_{ac}] \left(1 - \frac{\Omega}{2\gamma_c}\right) \left(\frac{\gamma_c}{\kappa - 2\eta_-}\right)\right] [e^{-\eta_- \tau} - e^{-\frac{\kappa\tau}{2}}]. \tag{3.3.21}$$

Substituting Eq. (2.0.53) and Eq. (2.0.55) into Eq. (3.3.21), we arrive at

$$\langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle = \langle \hat{a}^{\dagger}(t)\hat{a}(t)\rangle e^{-\frac{\kappa\tau}{2}}$$

$$+\left[\frac{\Omega^2+\Omega\gamma_c}{\gamma_c^2+3\Omega^2}\right]\left(1+\frac{\Omega}{2\gamma_c}\right)\left(\frac{\gamma_c}{\kappa-2\eta_+}\right)[e^{-\eta_+\tau}-e^{-\frac{\kappa\tau}{2}}]$$

$$+\left[\frac{\Omega^2-\Omega\gamma_c}{\gamma_c^2+3\Omega^2}\right]\left(1-\frac{\Omega}{2\gamma_c}\right)\left(\frac{\gamma_c}{\kappa-2\eta_-}\right)[e^{-\eta_-\tau}-e^{-\frac{\kappa\tau}{2}}]. \quad (3.3.22)$$

On account of Eq. (3.1.5). we can rewrite Eq. (3.3.22) as

$$\langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle = \bar{n}e^{-\frac{\kappa\tau}{2}} + \left(\frac{\kappa\bar{n}}{\kappa-2\eta_+}\right)\left[\frac{3}{4}+\frac{\gamma_c}{2\Omega}+\frac{\Omega}{4\gamma_c}\right][e^{-\eta_+\tau}-e^{\frac{-\kappa\tau}{2}}]$$

$$+\left(\frac{\kappa\bar{n}}{\kappa-2\eta_-}\right)\left[\frac{3}{4}-\frac{\gamma_c}{2\Omega}-\frac{\Omega}{4\gamma_c}\right][e^{-\eta_-\tau}-e^{-\frac{\kappa\tau}{2}}], \quad (3.3.23)$$

we can also rewrite in simplified form as

$$\langle \hat{a}^{\dagger}(t)\hat{a}(t+\tau)\rangle = \bar{n}[(1-a-b)e^{\frac{-\kappa\tau}{2}} + ae^{-\eta_+\tau} + be^{-\eta_-\tau}], \quad (3.3.24)$$

where

$$a = \left(\frac{\kappa}{\kappa-2\eta_+}\right)\left[\frac{3}{4}+\frac{\gamma_c}{2\Omega}+\frac{\Omega}{4\gamma_c}\right], \quad (3.3.25)$$

$$b = \left(\frac{\kappa}{\kappa-2\eta_-}\right)\left[\frac{3}{4}-\frac{\gamma_c}{2\Omega}-\frac{\Omega}{4\gamma_c}\right] \quad (3.3.26)$$

and $\bar{n}$ is steady state mean photon number. Substituting Eq. (3.3.24) into Eq. (3.3.1), we easily find

$$P(\omega) = \frac{\bar{n}(1-a-b)}{\pi}Re\int_0^{\infty} d\tau e^{-(\frac{\kappa}{2}-i(\omega-\omega_0))\tau}$$

$$+\frac{a\bar{n}}{\pi}Re\int_0^{\infty} d\tau e^{-(\eta_+-i(\omega-\omega_0))\tau} + \frac{\bar{n}b}{\pi}Re\int_0^{\infty} d\tau e^{-(\eta_--i(\omega-\omega_0))\tau}, \quad (3.3.27)$$

carrying out the integration, we easily get

$$P(\omega) = \bar{n}(1-a-b)\left[\frac{\kappa/2\pi}{(\kappa/2)^2+(\omega-\omega_0)^2}\right]$$

$$+a\bar{n}\left[\frac{\eta_+/\pi}{\eta_+^2+(\omega-\omega_0)^2}\right]+b\bar{n}\left[\frac{\eta_-/\pi}{\eta_-^2+(\omega-\omega_0)^2}\right]. \tag{3.3.28}$$

We realize that the mean photon number in the interval between $\omega' = -\nu$ and $\omega' = \nu$ is expressible as

$$\bar{n}_{\pm\nu} = \int_{-\nu}^{\nu} P(\omega')d\omega' \tag{3.3.29}$$

in which $\omega' = \omega - \omega_o$. Therefore, upon substituting Eq. (3.3.28) into Eq. (3.3.29), we get

$$\bar{n}_{\pm\nu} = \bar{n}(1-a-b)\int_{-\nu}^{\nu}\frac{\kappa/2\pi}{(\kappa/2)^2+\omega'^2}d\omega' + a\bar{n}\int_{-\nu}^{\nu}\frac{\eta_+/\pi}{\eta_+^2+\omega'^2}d\omega'$$

$$+b\bar{n}\int_{-\nu}^{\nu}\frac{\eta_-/\pi}{\eta_-^2+(\omega-\omega_0)^2}d\omega', \tag{3.3.30}$$

and carrying out the integration, applying the relation

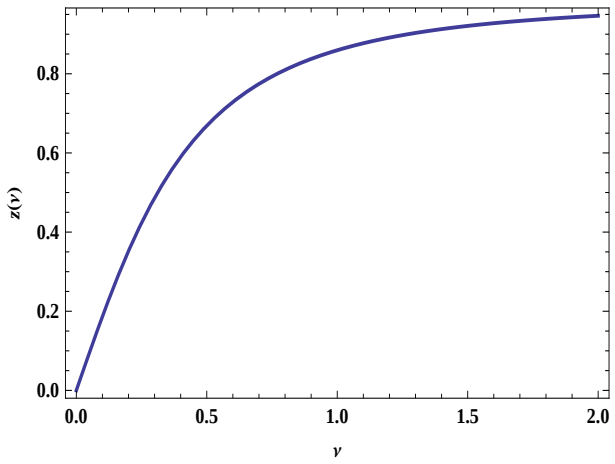


Figure 3.2: A plot of Eq. (3.3.34) for $\kappa = 0.8$, $\gamma_c = 0.5$ and $\Omega = 3$

$$\int_{-\nu}^{\nu}\frac{dx}{x^2+c^2} = \frac{2}{c}tan^{-1}\left(\frac{\nu}{c}\right), \tag{3.3.31}$$

we get

$$\bar{n}_{\pm\nu} = \bar{n}[\frac{2}{\pi}(1-a-b)tan^{-1}\left(\frac{2\nu}{\kappa}\right)+\frac{2a}{\pi}tan^{-1}\left(\frac{2\nu}{\eta_+}\right)+\frac{2b}{\pi}tan^{-1}\left(\frac{2\nu}{\eta_-}\right)], \tag{3.3.32}$$

we can rewrite Eq. (3.3.32) as

$$\bar{n}_{\pm\nu} = \bar{n}z(\nu), \tag{3.3.33}$$

where $z(\nu)$ is given by

$$z(\nu) = \frac{2}{\pi}(1-a-b)tan^{-1}\left(\frac{2\nu}{\kappa}\right) + \frac{2a}{\pi}tan^{-1}\left(\frac{2\nu}{\eta_+}\right) + \frac{2b}{\pi}tan^{-1}\left(\frac{2\nu}{\eta_-}\right). \tag{3.3.34}$$

From the plot in the Fig. 3.2, we easily find z(0.5)=0.66, z(1)=0.86, z(2)=0.96.Then combination of these results with Eq. (3.3.33) yields $\bar{n}_{\pm 0.5} = 0.66\bar{n}$, $\bar{n}_{\pm 1} = 0.86\bar{n}$, $\bar{n}_{\pm 2} = 0.96\bar{n}$.

Chapter 4

Quadrature Squeezing

In a squeezed state the quantum noise in one quadrature is below the coherent state level at the expense of enhanced fluctuations in the conjugate quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation[4]. The squeezing properties of single-mode light are describe by two quadrature operators defined as

$$a_+ = \hat{a}^\dagger + \hat{a}, \tag{4.0.1}$$

$$a_- = i(\hat{a}^\dagger - \hat{a}). \tag{4.0.2}$$

These operators are Hermitian and represent physical quantities called the plus and minus quadratures. We recall that the explicit expression for the cavity mode operator $\hat{a}$ at steady state is given by Eq. (2.0.26). Using this equation,we find

$$[\hat{a}, \hat{a}^\dagger] = \frac{4g^2}{\kappa^2}[(\hat{\sigma}_a + \hat{\sigma}_b), (\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger)], \tag{4.0.3}$$

$$[\hat{a}, \hat{a}^\dagger] = \frac{\gamma_c}{\kappa}(\hat{\sigma}_{cc} - \hat{\sigma}_{aa}). \tag{4.0.4}$$

And we note that

$$[\hat{a}_+, \hat{a}_-] = [(\hat{a}^\dagger + \hat{a}), i(\hat{a}^\dagger - \hat{a})], \tag{4.0.5}$$

$$= i(\hat{a}^{\dagger 2} - \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger - \hat{a}^2 - \hat{a}^{\dagger 2} - \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger + \hat{a}^2), \tag{4.0.6}$$

it then follow that

$$[\hat{a}_+, \hat{a}_-] = 2i[\hat{a}, \hat{a}^\dagger]. \tag{4.0.7}$$

On substituting Eq. (4.0.4) into Eq. (4.0.7), we get

$$[\hat{a}_+, \hat{a}_-] = 2i\frac{\gamma_c}{\kappa}(\hat{\sigma}_{cc} - \hat{\sigma}_{aa}). \tag{4.0.8}$$

If two Hermitian operators satisfy the commutation relation

$$[\hat{A}, \hat{B}] = iC, \tag{4.0.9}$$

then it can be readily established that

$$\Delta A \Delta B \geq \frac{1}{2}|\langle C \rangle|. \tag{4.0.10}$$

According to Eqs. (4.0.8), (4.0.9) and (4.0.10), we readily get

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa}|\langle \hat{\sigma}_{cc} \rangle - \langle \hat{\sigma}_{aa} \rangle|, \tag{4.0.11}$$

Employing $\langle \hat{\sigma}_{cc} \rangle = \rho_{cc}$ and $\langle \hat{\sigma}_{aa} \rangle = \rho_{aa}$ we can rewrite Eq.(4.0.11) as

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa}|\rho_{cc} - \rho_{aa}|, \tag{4.0.12}$$

We now proceed to calculate the variance of the plus and minus quadratures for the cavity mode produced by the degenerate three-level atom. The variance of the plus and minus quadratures is expressible as

$$(\Delta a_\pm)^2 = \langle (\hat{a}_\pm)^2 \rangle - \langle (\hat{a}_\pm) \rangle^2. \tag{4.0.13}$$

Substituting Eqs. (4.0.1) and (4.0.2) into (4.0.13), we easily get

$$(\Delta a_\pm)^2 = \pm \langle [\hat{a}^\dagger \pm \hat{a}]^2 \rangle \mp [\langle \hat{a}^\dagger \rangle \pm \langle \hat{a} \rangle]^2. \tag{4.0.14}$$

On account of Eq. (3.2.2), we have

$$(\Delta a_\pm)^2 = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle \pm \langle \hat{a}^{\dagger 2} \rangle \pm \langle \hat{a}^2 \rangle. \tag{4.0.15}$$

Now employing Eqs. (3.1.4), (3.2.4), (3.2.6) and (3.2.7) in Eq. (4.0.15), we arrive at

$$(\Delta a_+)^2 = \frac{2\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}(\rho_{bb} + \rho_{cc}) + \frac{2\gamma_c}{\kappa}\rho_{ac} \tag{4.0.16}$$

and

$$(\Delta a_-)^2 = \frac{2\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}(\rho_{bb} + \rho_{cc}) - \frac{2\gamma_c}{\kappa}\rho_{ac}. \tag{4.0.17}$$

Substituting Eq. (2.0.53), (2.0.55) and (2.0.57), into Eqs. (4.0.16) and (4.0.17), one can easily get

$$(\Delta a_+)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\Omega^2 + \gamma_c(\gamma_c + 2\Omega)}{\gamma_c^2 + 3\Omega^2}\right] \tag{4.0.18}$$

and

$$(\Delta a_-)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\Omega^2 + \gamma_c(\gamma_c - 2\Omega)}{\gamma_c^2 + 3\Omega^2}\right]. \tag{4.0.19}$$

Where $\lambda = \frac{\Omega}{\gamma_c}$, Eq. (4.0.18) and (4.0.19) can be rewritten

$$(\Delta a_+)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\lambda^2 + 2\lambda + 1)}{1 + 3\lambda^2}\right] \tag{4.0.20}$$

And

$$(\Delta a_-)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\lambda^2 - 2\lambda + 1)}{1 + 3\lambda^2}\right]. \tag{4.0.21}$$

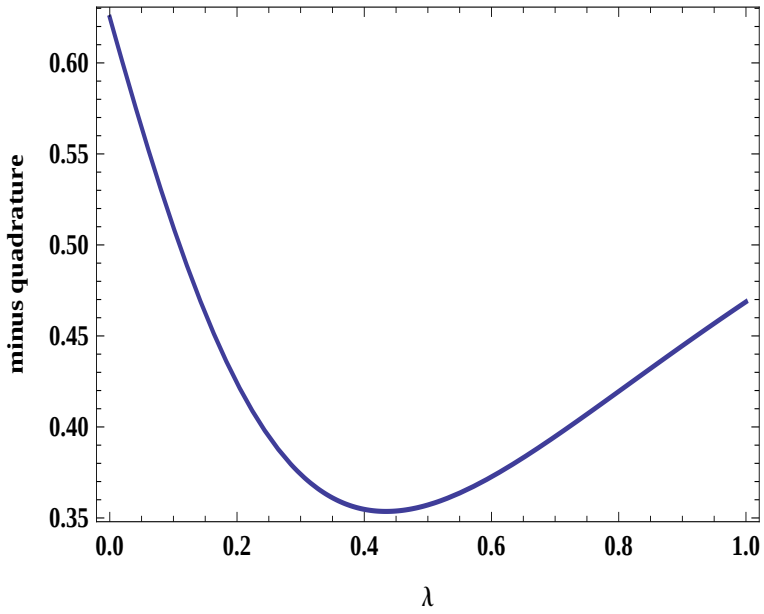


Figure 4.1: Plot of the steady-state minus quadrature variance versus λ for $\kappa = 0.8$ and $\gamma_c = 0.5$

For ($\lambda << 1$), we easily get

$$(\Delta a_-)^2 = (\Delta a_+)^2 = \frac{\gamma_c}{\kappa}. \tag{4.0.22}$$

$$\Delta a_+ \Delta a_- = \frac{\gamma_c}{\kappa} \tag{4.0.23}$$

We define coherent light to be a light mode in which the uncertainties in two quadratures are equal and satisfy minimum uncertainty relation [4]. Hence on the basis of Eqs. (4.0.22) and (4.0.23), we assert that the light emitted by coherently driven degenerate three-level atom is coherent. Figure 4.1, indicated that $(\Delta a_-)^2 < \frac{\gamma_c}{\kappa}$, where $\frac{\gamma_c}{\kappa} = 0.625$ for $\kappa = o.8$ and $\gamma_c = 0.5$, then we see that coherently driven degenerate three level atom is in squeezed state. Squeezing occurs for $\Omega < 2\gamma_c$, with maximum squeezing being stable against small fluctuation of Ω. Now proceed to calculate the degree squeezing of coherently driven degenerate three-level atom. Degree of squeezing expressible as

$$S = \frac{\frac{\gamma_c}{\kappa} - (\Delta a_-)^2}{\frac{\gamma_c}{\kappa}}, \tag{4.0.24}$$

substituting Eq. (4.0.19), we easily get

$$S = \frac{\frac{\gamma_c}{\kappa} - \frac{\gamma_c}{\kappa}\left[\frac{4\Omega^2+\gamma_c(\gamma_c-2\Omega)}{\gamma_c^2+3\Omega^2}\right]}{\frac{\gamma_c}{\kappa}}, \tag{4.0.25}$$

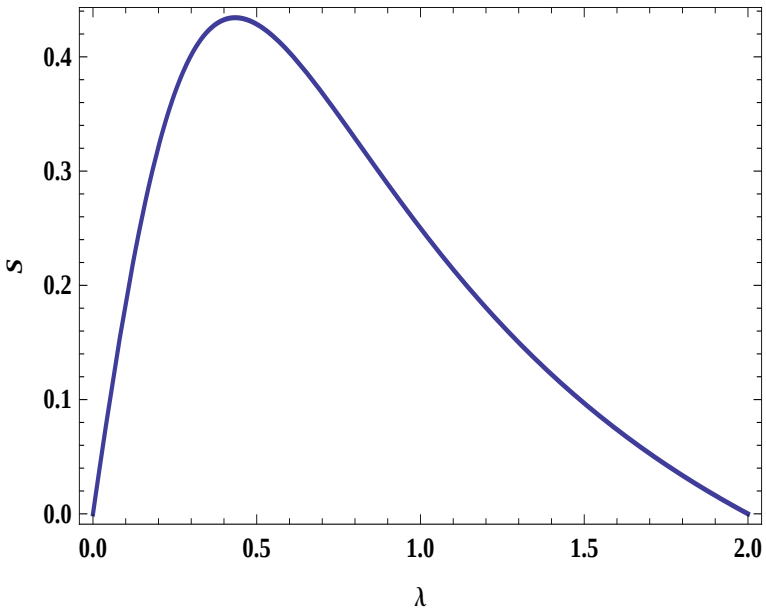


Figure 4.2: Plot of degree squeezing versus λ.

then by rearranging, we arrive at

$$S = \left[\frac{2\Omega\gamma_c - \Omega^2}{\gamma_c^2 + 3\Omega^2}\right], \tag{4.0.26}$$

we can rewrite Eq. (4.0.26), in terms of λ as

$$S = \left[\frac{2\lambda - \lambda^2}{1 + 3\lambda^2}\right]. \tag{4.0.27}$$

As clearly indicated in Fig. 4.2, the degree of squeezing of the radiation increases with the intensity of the biased noise fluctuations in which a higher degree of squeezing is found for relatively smaller values of λ. The coherently driven degenerate three-level atom squeezes 44 % below coherent state. It is believed that the emission initiated by the initially prepared atomic coherent superposition is accountable for the reduction of the fluctuations of the noise in one of the quadrature components below the classical limit [8].

Chapter 5

Conclusion

The squeezing and statistical properties of the light emitted by a degenerate three-level atom in a closed cavity coupled to vacuum reservoir and driven by coherent light is analyzed. For strong coherent light there is equal probability for finding the atom in all levels. We have seen that at steady state $\rho_{cc} > \rho_{aa}$. This implies there is more probability for finding the atom in the bottom level than in the upper or intermediate level. Moreover, in the absence of the coherent light there is no probability for finding the atom either in the upper or intermediate level.

As Fig. 3.1 shows, the mean photon number increases with γ_c and λ. For $\Omega \gg \gamma_c$ and $\Omega = \gamma_c$ the photon number variance is greater than the mean photon number. Hence the photon statistics of the light is supper Poissonian. For $\lambda \ll 1$, we have seen that $(\Delta a_-)^2 = (\Delta a_+)^2 = \frac{\gamma_c}{\kappa}$ and $\Delta a_- \Delta a_+ = \frac{\gamma_c}{\kappa}$. Hence, we assert that the light emitted by the three-level atom is coherent for $\lambda \ll 1$. In addition, we have found that for $\Omega < 2\gamma_c$, the light emitted by the atom is in a squeezed state and the squeezing occurs in minus quadrature. As Fig. 4.2 indicates, the maximum squeezing of the emitted light is about 44% below the coherent level.

Bibliography

[1] Fesseha kassahun, arXivi 1105.1438v3 [quant-ph] 25 Sep 2012.

[2] C. Saaverda, J.C. Retamal, and C. H. Keitel, Phys. Rev.A 55, 3802 (1997).

[3] N. Lu and S.Y. Zhu, Phys Rev A, 41, 2865 (1990).

[4] Fesseha Kassahun, *Fundamentals of Quantum Optics* (Lulu Press Inc., North Carolina, 2010).

[5] M.O. Scully and M.S. Zubairy, Opt Commun, 66, 303 (1988).

[6] N.A. Ansari, Phys Rev A, 48, 4686 (1993).

[7] M.O. Scully, and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, 1997).

[8] S.Tesfa, Phys Rev A, 77, 013815 (2008).

[9] K. Fesseha, Phys. Rev. A, 63, 033811 (2001).

[10] Fesseha Kassahun, Opt. Commun. 284, 1357 (2011).

[11] Gilbert Grynberg, Alian Aspect and Claude, Introduction to Quantum Optics (Cambridge University, 2010).

[12] V.M Akulin, Coherent Dynamics of Complex Quantum system (Paris Spiringer, 2004).

[13] E. Alebachew ,K. Fesseha, Opt. Commun , 265, 314 (2006).

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged

Name: Yosef Terefe

Signature: ———

Place and time of submission: Addis Ababa University, February 2013

This project has been submitted for examination with my approval as University advisor

Name: Dr. Fisseha Kassahun

Signature:———-