Primary Pre-service Mathematics Teachers' Geometry Learning with Technology-Integrated Guided Inquiry-Based Learning Approach

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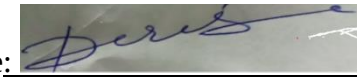
Signature: 

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ABSTRACT

Primary Pre-service Mathematics Teachers' Geometric Learning with Technology-Integrated Guided Inquiry-Based Learning Approach

Dereje Eshetu
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The purpose of this study was to examine the effect of the technology-integrated guided inquiry-based learning (TGIBL) approach on pre-service primary mathematics teachers' (PSMTs') learning of plane geometry. In the study, a mixed-methods approach with a non-equivalent quasi-experimental design was employed. A total of 116 second-year PSMTs, divided into two experimental and one comparison groups, participated. A random sampling technique was used to select the sample of two intact classes for intervention and comparison groups, which were then assigned as experimental groups 1, 2, and the comparison group. In the experimental group 1, the TGIBL approach was used; in the experimental group 2, the GIBL approach was used; and in the comparison group, the traditional teacher-centered instructional method was used for ten weeks. The Geometry Attitude Scale Questionnaire, Geometry Conceptual Understanding Test, Geometry Procedural Knowledge Test, and Geometry Problem Proof Test were administered to all groups as pretests and posttests to collect quantitative data while Semi-structured interviews and classroom observation were used for qualitative data. The quantitative data were analyzed using descriptive and inferential statistics such as mean, standard deviation, MANOVA, ANOVA, ANCOVA, paired sample t-test and regression analysi whereas thematic analysis used for qualitative data. The analysis revealed that PSMTs taught by the TGIBL approach showed significant improvements in geometry learning outcomes when compared to PSMTs in traditional teacher-centered methods and the GIBL approach. Moreover, PSMTs in the GIBL approach showed a statistically significant difference from the traditional teacher-centered instruction method in geometry learning outcomes. In regression analysis, reasoning and attitude have significant influence on conceptual and procedural knowledge. The qualitative data also revealed that PSMTs who are taught using the TGIBL approach have attained a positive attitude towards geometry when compared to PSMTs who are taught using the GIBL approach or the traditional teacher-centered method. However, there were no significant differences between the GIBL approach and traditional teacher-centered methods in attitude towards geometry. Based on the findings, mathematics teachers and colleges of teacher education must use the TGIBL approach, and pre-service mathematics teacher education courses need to be incorporated with digital technologies and pedagogies to improve PSMTs' learning of geometric concepts.

Key words*:* Guided Inquiry-Based Learning, Conceptual Understanding, Procedural Knowledge, Reasoning Skills, Attitude, Technology

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LIST OF ACRONYMS AND ABBREVIATIONS

ANCOVA- Analysis of Covariance

ANOVA- Analysis of Variance

ATT- Attitude

CU- Geometry Conceptual Understanding

EFA- Exploratory Factor Analysis

ENLA- Ethiopian National Learning Assessment

ESDP V – Education Sector Development Program V

GASQ- Geometry Attitude Scale Questionnaire

GIBL- Guided Inquiry-Based Learning

GPKT- Geometry Procedural Knowledge Test

GPPT- Geometry Problem Proof Test

MANOVA- Multivariate Analysis

MoE- Ministry of Education

MoFED- Ministry of Finance and Economic Development

PK- Procedural knowledge

PSMT- Pre-service Primary Mathematics Teacher

RS- Reasoning Skill

TGIBL- Technology integrated Guided Inquiry –Based Learning

TRAD- Traditional Teacher-centered

TTGDT-Two-tiered Geometric Diagnostic Test

CHAPTER ONE: INTRODUCTION

Mathematics education, when compared to other fields of study, is the basis for all sciences and technological knowledge, as it's applicable in various areas of study. However, learners' mathematics learning outcomes have declined all over the world, including in Ethiopia (Pohjolainen, Nykänen, Venho, & Kangas, 2018; Preez, 2018; Kiwanuka, Damme, Noortgate, Anumendem, & Namusisi, 2015; Simamora, Simamora, Sinaga, & Calderon, 2017; MoE, 2015; Plenty & Heubeck, 2013).

Geometry, in particular, is a mathematical content identified as a major area where learners fail to develop an adequate understanding of the concept, reasoning abilities and problem-solving skills (MoE, 2017; Mamali, 2015; Adolphus, 2011). Furthermore, studies have also demonstrated that teachers' knowledge of mathematics, beliefs about teaching mathematics, and classroom instructional practices all make a significant impact on preservice mathematics teachers' (PSMTs') learning and performance (MoE, 2015; Teshome, 2012; Hill, Rowan, & Ball, 2005; Xue & Meisels, 2004).

Accordingly, in order to tackle the poor learning outcomes of geometry among primary PSMTs, instructional methods that support learners in developing a deep understanding of the concepts and rationales behind these concepts as well as encourage them to construct their own meaningful learning were recommended (MoE, 2020; Carpenter & Lehrer, 1999; Darling-Hammond, 1997). In addition, research also suggests that the integration of technology with instructional methods in classroom practice can support the learning of mathematics and geometry (Eshetu et al., 2022; Charles-Ogan, 2015). In

line with this, in Ethiopia, the mathematics curriculum and teaching mathematics in colleges of Teacher Education (CTEs) have been revised (MoE, 2015).

This particular study investigated the conceptual understanding, procedural knowledge, reasoning skills, and attitude towards learning the plane geometry of mathematics students who are studying to become PSMTs. To conduct such an investigation, the PSMTs participated in a series of problem-solving and guided inquiry-oriented learning activities that were augmented with suitable digital technology on the plane geometry course (Math-111) delivered in CTEs. Constructivism and Skemp's theory served as the study's guiding theoretical framework.

For this purpose, the study employed a mixed-method research approach with a concurrent embedded design in which qualitative is embedded within quantitative approach. Within this framework, a non-equivalent quasi-experimental method with a pretest-posttest comparison group was employed. In this chapter, the background of the study, the research problem and the significance of the research study, the delimitation, and then the outline of the thesis are presented.

1.1 Background of the Study

Currently, the world is experiencing infinite changes in scientific and technological developments. As such, mathematics education is regarded as essential for a country's scientific and technological advancement because it serves as the foundation for other fields of knowledge (MoE, 2010; MoFED, 2014; Hindeya et al., 2017). Mathematics education accomplishment is also used to compare the socioeconomic status of nations (MoFED, 2014). According to Wikipedia, mathematics is the discipline that underpins

science and technology, without which a country would never achieve economic independence and prosperity.

Furthermore, the mathematical ideas and skills underlying geometric reasoning assist learners in understanding high-level mathematical concepts (Jacobsen & Lehrer, 2000) and help in supporting a conceptual understanding of other mathematical concepts (Igiri, Odiong, & Ubi, 2018; Daher & Jaber, 2010; French, 2004). This shows that learners' mathematical competencies are associated with their understanding of geometry concepts. According to Gutierrez, Leder, and Boero (2016), the attainment of geometric problem-solving skills assists PSMTs in improving their reasoning skills and logical thinking abilities and helps them visualize abstract mathematical concepts.

In addition, geometry concepts are used to formulate and perpetuate scientific ideas and theories to specify and establish the relationship of laws and concepts in science subjects such as chemistry, physics, biology, calculus, etc. (Lozano-Rbledo, 2019). As well, geometry is a key to the study of other contents of mathematics (Adelabu, 2018). According to Choudhury and Das (2012) geometry is a combining theme to the entire mathematics curriculum and as such is a rich foundation of visualization for arithmetical, algebraic, and statistical concepts.

Even though mathematics and geometry are keys for technological and scientific developments (Schoenfeld, 2002), research shows low achievements are continuing among learners all over the world (Pohjolainan et al., 2018; Preez, 2018; MoE, 2017; Simamora et al., 2017; Kiwanuka et al., 2015; Herholdt & Sapire, 2014). For instance, survey studies by PISA and TIMSS that provide general evidence about the status of

school science and mathematics education in the world in terms of test scores and issues related to curricula and instruction, students' attitudes, resources for teaching, teacher preparation and professional development, classroom methods and activities, and technology used in instruction, etc., have shown that the quality, achievements, and interest of students toward mathematics and science education are low in developing countries (ICSU, 2011).

Furthermore, Mji and Makgato (2006) also confirmed that learners from developed countries outperform those from developing countries. Similarly, the reports from the International Mathematics Union (IMU) also indicated that "*most African nations have poor primary and secondary mathematics education, which lowers the number of outstanding students who could choose to study mathematics in higher education*" (IMU, 2020). It is quite evident that poor quality, low achievement, and negative attitudes toward mathematics education hinder the ability to solve different day-to-day problems.

The situation in Ethiopia is not different from what is mentioned above. For instance, the report from the Ethiopia National Learning Assessment (ENLA) conducted at different times depicts underperformance, and the declines in mathematics and geometry achievement among students are far below the national standard. Different factors, such as the nature of the subject (Adolphus, 2011), the instructional method (Gradwohl & Eichler, 2018), the attitudes (Huang, 2011; Eshetu, Atnafu, & Woldemichael, 2023), poor visualization abilities and geometric language (Idris, 2006), etc., are some that cause poor achievements among PSMTs in learning geometry.

In fact, plane geometry is a mathematical content knowledge in mathematics education that deals with the concepts of plane figures such as triangles, quadrilaterals, circles and related concepts. However, its dual nature, i.e., abstractness as well as concreteness, makes it difficult for learners to understand (Fujita & Jones, 2002). For instance, geometry topics like perimeter and area (Ma, 1999), triangle congruence and similarity (Alatorre, Flores, & Mendiola, 2012; Dündar & Gündüz, 2017), quadrilaterals, areas, and volumes of geometric figures (Fujita & Jones, 2006; Jones, Mooney & Harries, 2002; Ding, Fujita & Jones, 2006), geometric transformation (Kambilombilo & Sakala, 2015), etc. are some of challenging concepts.

On the other hand, a plethora of research revealed that PSMTs' conceptual understanding of geometric concepts influenced by the instructional approaches used and implemented in the classroom (i.e., pedagogical approach) (Darling-Hammond, 1997; Fosnot, 2005). However, Bekele (2012) and MoE (2015) indicated that higher education in Ethiopia practices a teacher-centered, traditional approach. This approach is less important for attaining conceptual knowledge (Fosnot, 2005). In traditional approach, unidirectional knowledge transformation and PSMTs are passive participants, expected to recall facts and concepts, take notes, and master basic facts and information for assessment purposes (Khalafi, 2018; Kitti, 2014). In this case, the focus is given to formula-based rote problem-solving rather than recognizing the concepts behind it (Antwi, Hanson, Savelsbergh, & Eijkelhof, 2011).

Accordingly, a traditional teacher-centered approach might not be an effective method to prepare PSMTs for the new information age since this approach is insignificant for meaningful learning such as critical thinking, conceptual understanding of mathematical

concepts, and cooperative learning (Cronhjort, Filipsson, & Maria, 2013; Freeman et al., 2014; MoE, 2020), as well as not instigating motivation, confidence, attitude, and enjoyment among PSMTs (Eshetu et al., 2023; Weimer, 2002). This indicates the need for alternative and innovative approaches for teaching and learning geometry in CTEs to promote meaningful learning, problem-solving skills, and critical thinking among PSMTs.

To this effect, learner-centered instructional models based on the principle of constructivism (Baeten, Dochy, & Struyven, 2012) such as cooperative learning (Soyibo & Evans, 2002), anchored instruction (Bransford, Sherwood, Hasselbring, Kinzer, & Williams, 1990), inquiry-based learning (IBL), problem-based learning (PBL), and experiential learning (Kirschner, Sweller, & Clark, 2006; Laursen & Rasmussen, 2019), have been developed. Of these approaches, guided inquiry-based learning (GIBL) is an alternative to the traditional teacher-centered method of teaching (Hayward et al., 2016; Laursen & Rasmussen, 2019; Mackenzie, 2016).

GIBL is a learner-centered pedagogy that focuses on PSMTs engagement in sequenced and scaffolded learning rather than the instructor transmitting content (Laursen & Rasmussen, 2019; Lazonder, 2014), and where PSMTs are active learners who learn by doing (Reiser & Dempsey, 2012). In this instructional approach meaningful task such as cases, projects, and research are used to situate learning (Avsec & Kocijancic, 2016). In this case, PSMTs are encouraged to ask questions and solve real-world problems that enable them to identify problems, offer and examine solutions, and communicate their thoughts (Laursen & Rasmussen, 2019). Through these practices, PSMTs develop conceptual understanding in addition to procedural knowledge (Lewis & Estis, 2020) and

skills such as critical thinking, reasoning, problem-solving, and mathematical thinking (Hassi & Laursen, 2015; MoE, 2013).

Unlike the traditional teacher-centered approach, in GIBL approach the teacher acts as a guide or facilitator by posing questions, problems, and scenarios (Artigue, 2012; Bankov, 2013; Clark, Kirschner, & Sweller, 2012) than as a source of knowledge. In the process, PSMTs are encouraged to inquire and attempt geometrical questions, think mathematically and produce solutions collaboratively, devise procedures to explore solutions, and justify the conclusions depending on their prior knowledge (Maxwell, Lambeth, & Cox, 2015; Dorier & Maass, 2020). GIBL approach helps PSMTs develop processing skills, deep understanding of the mathematical concepts, and retain geometric knowledge longer (Celikten et al., 2012; Larsen et al., 2015; Simsek & Kabapinar, 2010). Similarly, the curriculum framework for PSMTs (MoE, 2013) also stresses that PSMTs should be taught through inquiry to acquire mathematical and geometric knowledge.

However, research indicates that while the GIBL approach is effective in science education, it is still not a widely used in mathematics classrooms and even inappropriately implemented (Kim, Hannafin, & Bryan, 2007; Gardner, 2012; MoE, 2015; Caswell & Labrie, 2017). The amount of classroom time needed to develop activities, experience, and insufficient teacher background, insufficient incorporation of abstract concepts with inquiry, and PSMTs' constraints in the learning context (Kim et al., 2007; Hwang, Chiu, & Cheng, 2014; MoE, 2015; Sebsibe & Feza, 2019) are some of the problems to implement GIBL approach in geometric classrooms. Concurrently, there has been a proliferation of interest in and research on the use of technology in mathematics education (Heid & Blume, 2008, MoE, 2012a; 2015). Accordingly,

integrating educational technology is recommended as an alternative approach to overcome these concerns of implementation of GIBL approach through a technology-integrated guided inquiry-based learning (TGIBL) approach (Kim et al., 2007; MoE, 2012a).

In the TGIBL approach, classrooms become more dynamic, encourage exploratory learning (to explain, explore, and model geometrical concepts and their relationships), and impact teachers' conceptions regarding mathematics teaching (Dikovic, 2013; Dockendorff & Solar, 2018; Tatar & Zangin, 2016). The integration of technology into geometric learning activities can promote inquiry, communication, and conceptualization (Moreno-Armella & Hegedus, 2020; MoE, 2012a; Baki, 2015; Kasa, 2015) if appropriate technology is implemented in the classroom.

Furthermore, MoE (2015) also stated that teaching and learning in CTEs should adopt learner-centered teaching methods supported by technology. According to the NCTM (2000), technology influences and enhances PSMTs' geometry learning. The use of technology, such as dynamic geometry software, allows learners and instructors to construct figurative, functional, and relational samples, resulting in higher-level thinking, better problem-solving skills, understanding concepts, and reasoning abilities about geometric concepts (Battista, 2002; Bokosmaty, Mavilidi, & Paas, 2017). In addition, dynamic geometry software supports for the manipulation of geometric figures by moving, dragging, and animating for immediate feedback.

Geometric Sketchpad (GSP) is one a dynamic geometry software used to visualize abstract geometrical structures (Arbain & Shukor, 2015; Pilli & Aksu, 2013; Gurbuz &

Birgin, 2012; Arini & Dewi, 2019). In this respect, GSP supports and offer opportunities to develop geometric ideas to enhance PSMTs' geometric understanding and enables geometrical investigations. Thus, the GSP design can be used in GIBL settings effectively (Aventi, 2014; Yeo & Webel, 2022). When technology used, teachers can expand curriculum objectives, make use of more problem-solving examples and utilize an inquiry-based approach to learning (Jurdak, 2004; Donevska-Todorova & Trgalova, 2017).

As aforementioned, GIBL and TGIBL have received extensive attention from educators in order to improve the quality of education (MoE, 2012a; 2013; Artigue & Blomhøj, 2013; Huang, Liao, Huang & Chen, 2014; Dorier & Maass, 2020). In developed countries, for instance, several countries prioritize a technology-supported inquiry learning approach (Hakverdi-Can & Sonmez, 2012). According to Sinclair (2020), employing existing technology with guided inquiry can change the content and methods of teaching mathematics and geometry in paticular.

The use of technology and visualization in mathematics has been researched (Hoyles, 2008). Even though technology blended with appropriate pedagogy to support the learning of mathematics in education, the results of a recent review of empirical research about the use of technology in mathematics education stress that technology itself (Viberg & Mavroudi, 2018; Bray & Tangney, 2017; MoE, 2015; 2020). This indicates that the use of technology-integrated guided inquiry is still a research area to understand the complexity of technological innovation and change to support improved teaching and learning of geometry in different contexts (Howard et al., 2019).

1.2 Context of the Study

Ethiopia, as a developing country, strives to build competent and scientifically literate, critical-thinking, creative, and culturally innovative students in science, technology, engineering, and mathematics (STEM) in order to address the nation's all-inclusive development in the 21st century. In doing so, quality education plays an important role. To maintain quality education, however, teacher education has a greater impact than other factors (Darling-Hammond, 2000; MoE, 2015; MoFED, 2015). Thus, the quality of school education is a direct product of the quality of teacher education, and vice versa.

Teacher training in Ethiopia is a recent phenomenon that began in 1944 (MoE, 2004). Since then, however, the teacher education program has been passed through a sequence of different policy reforms and practices. However, such reform activities, according to Tirusew (2006), were liable for: a poor approach to content presentation; a poor medium of instruction; poor integration and coordination in the structure of education (primary, junior, and secondary education); a mismatch between the expansion of education and the poor competence of teachers (Bishaw & Lasser, 2012).

Although quality education is believed to be the foundation for development and that quality education cannot be realized without competent, motivated, and capable teachers, CTE is viewed as the least attractive form of higher education. According to Teklehymanot (2000), teacher education in Ethiopia does not attract academically vibrant and professionally passionate students towards the teaching stream. Generally, teacher education programs attract students who are unable to pursue careers in medicine, engineering, or other more attractive fields in east Africa (Ottevanger, van den Akker, & de Feiter, 2007).

In addition, according to Amare (2009) and Lemlem (2010), the quality of teacher education remained poor in terms of content knowledge and pedagogical knowledge, disregarding the context of Ethiopian schools. Furthermore, trainees' low motivation and dedication to the profession, combined with their low competence, had a significant impact on the production of competent primary school teachers.

Consequently, *the Curriculum Framework for Primary Pre-service Teacher Education* was developed (MoE, 2013a) to envisage Ethiopian teachers with all the necessary skills and knowledge depending on the current demands of teacher education. This new mathematics curriculum was designed to incorporate key standards that emphasize a constructivist view of teaching and learning mathematics. This new mathematics curriculum aimed to address the limitations of the past mathematics curriculum, such as content knowledge gaps, delivery of content knowledge, and contextual aspects (MoE, 2013). In this situation, PSMTs study modules that apply modern pedagogies for teaching mathematics, are in line with the objectives and content of the present mathematics curriculum, and promote digital technology integration.

As part of their CTE studies, the PSMTs take a number of mathematics subject courses such as geometry, calculus, and algebra with a broader scope than what they learned in high school. Geometry is taught in pre-service teacher education courses to assist PSMTs in developing an in-depth conceptual understanding of the content, solving real-life problems, and visualizing, analyzing, and illustrating real-life situations in mathematical terms (MoE, 2013a). In such a way, the curriculum for CTEs was revised in accordance with the primary curriculum's emphasis on competency-based learning, active learning,

and learner-centeredness. However, the adoption of new teaching practices is inadequate, as stated in ESDP V (MoE, 2015:20):

> "*…the practices of primary education and teacher education within CTEs remain weak in the content knowledge of teacher trainees. This implies that the implementations of active learning methods are not properly and sufficiently employed as well as stated teachers' subject matter competence is also inadequate*".

This weakness in CTE graduates suggests that PSMTs are poorly prepared. The above-mentioned problem is one of the parts considered in this study.

In the same manner, the document also (MoFED, 2015; MoE, 2015:58) explains that to increase the training of education and to assert the quality of education, one of the indicators involved in the document is ICT/technology integration in the teaching and learning process in CTEs.

> "*ICT will be fully integrated in teacher's training courses and supported with practice so that teachers better equipped to use technology and to teach and assist their students with technology to enhance their thinking and to further their conceptual understanding*."

To support the technology-driven mathematics curriculum, PSMTs are expected to be equipped with digital technology knowledge and skills to enhance their students' conceptual understanding of mathematics and problem-solving competence. However, the emphasis continued to be on mastery rather than integrating technology into instructional strategies in CTEs (MoE, 2015).

The development of PSMTs' conceptual understanding and problem-solving skills, and in turn the influence this might have upon PSMTs' subsequent teaching of mathematics concepts in primary school, has been the stimulus to conceptualize this research study. It

is within this context that this study was conducted, aimed at improving PSMTs' geometry learning using a technology-integrated guided inquiry strategy.

1.3 Statement of the Problem

Despite the importance of geometry learning, poor performance has been persistent among learners at all levels. One of the causes is the absence of conceptual understanding that helps PSMTs learn concepts (Handhika, Cari, Soeparmi, & Sunarno, 2016). According to Pavethira and Leong (2017), Salifu (2020), and Adolphus (2011), learning geometry is challenging, and PSMTs fail to attain conceptual understanding, procedural knowledge, reasoning skills, problem-solving skills and many PSMTs simply memorize proofs or rules (Masilo, 2018).

There have been lots of studies on conceptual and procedural learning to determine whether PSMTs learn conceptual or procedural knowledge first (Haapasalo & Kadijevich, 2000). According to Rittle-Johnson and Siegler (1998), there is no fixed order between conceptual and procedural knowledge. Procedural knowledge and conceptual understanding are assumed to be distinct yet related (Areaya & Sidelil, 2012). Some literature also states that teaching conceptual knowledge first leads to the acquisition of procedural knowledge, but the converse is not true (Schneider & Stern, 2005). However, researchers have discovered that emphasizing flexibility as the source of knowledge or conceptual knowledge that can be taught first is more effective in teaching mathematics (Awang & Zakaria, 2012; Mahir, 2009).

PSMTs' mathematics teaching and learning is also related to their attitude (Farooq & Shah, 2008; Atnafu, 2012). In addition, PSMTs' negative attitudes toward mathematics

are because of their weak mathematics backgrounds and their experiences with mathematics, and this also affects mathematics teacher educators' practices (Tsao, 2014). Moreover, researchers revealed a strong connection between attitudes towards mathematics and understanding of mathematical concepts (Akdemir, 2006; Grootenboer & Marshman, 2016; Hourigan & Leavy, 2019; Atnafu, 2012).

In Ethiopia, a significant number of PSMTs fail to develop adequate understanding of geometry concepts, problem-solving skills, and overall achievement (MoE, 2017; 2020). The ENLA survey showed that the performance of students in geometry and other domains of mathematics are inadequate (MoE, 2020). The fact that the mean geometry score for grade 10 is 33.59% and the mean score for grade 12 is 37.63% indicates that both are below the minimum standard (MoE, 2017). In addition, it has been demonstrated that during the past three years, academic achievement among pupils has decreased in the area where the research site was located in an inconsistent pattern (Kene et al., 2021).

In the same manner, the needs assessment study conducted by Ahmed, Getahun, Debele, Tiruneh, Gebremeskel, and Wondema (2017) showed pre-service teachers have low performance in mathematics and other disciplines. Furthermore, Tadesse (2014) and Michael (2015) showed that PSMTs' geometric competency was inadequate, and Brodie (2007) and Wilburne and Long (2010) also found that PSMTs have limited conceptual understanding of geometric contents. Yurniwati and Soleh (2019) also identified prospective teachers with poor conceptual understanding of geometric concepts.

Several factors can be mentioned for this problem, but teaching methods, absence or inappropriate use of technology, and problems related to curriculum design can be a few.

Among these, instruction should be given priority as it has a direct and significant impact affects PSMTs' attitudes and learning of geometric concepts, researchers stated that poor learning of geometric concepts is associated with teachers' failure to use effective instructional approaches and the attitudes of the PSMTs towards the subject matter (Adunola, 2011; Ahmed & Abimbola, 2011; Hong, Choi, Hwang, & Runnalls, 2017; MoE, 2020).

The GIBL approach is an alternative approach that helps PSMTs in learning geometry as it entails activities, promotes mathematical thinking, and provides an effective teaching and learning environments (Artigue & Blomhøj, 2013; Huang, Liao, Huang & Chen, 2014; MoE, 2015). However, the implementation of the GIBL approach in the mathematics classroom is challenging due to time limits, abstract geometric concepts, and attitudes of learners as well as instructors' attitude lead to meager implementations (Cheung, 2007). In addition, integration of technology into the GIBL approach used to overcome challenges such as the abstractness of geometrical concepts (Ocal, 2017; Akcay, 2017; Othman, Tarmuji, & Hilmi, 2017; Zulnaidi & Oktavika, 2018).

The teaching and learning process through technology supported approach have shown significant impact on PSMTs' learning of geometric contents (Battista, 2007; Laborde et al., 2006; Kakana, 2017; Kotu & Weldeyesus, 2022) and mathematical practices (Moreno-Armella & Hegedus, 2020; Uygun, 2019; Uygun & Akyuz, 2019; Ocal & Simsek, 2017). According to studies by Yanik and Porter (2009), and Furner & Marinas (2013), integrating technology into math education and geometry can make it more engaging. The technology gives PSMTs more time to investigate and create more activities to do more investigations.

However, the design of learning environments enhanced by technology is a question that requires attention and a through examination of mathematical practices in these teaching and learning environments. Lack of expertise and knowledge of how to utilize technology, or attitudes with utilizing it, is another typical challenge for instructors to integrate it into their courses. On the other hand, there are insufficient pre-service teacher education programs that prepare teachers to use technology-integrated inquiry learning and yet transformed into practice at CTEs (Gerard et al., 2011; MoE, 2015; 2018; 2020).

Furthermore, in spite of considerable research has been conducted about the general use of technology in teaching and learning of geometric concepts, there is still little research about the use of GSP in developing countries such as Ethiopia in the context of CTEs (Akyuz, 2014; MoE, 2015; 2020). As such, this indicated that there is a need to investigate the understanding of concepts and practices on plane geometry concepts through GSP supported GIBL on PSMTs' in context of CTE.

Therefore, this study was aimed to investigate the effects of the GIBL approach supported by technology on PSMTs' learning of plane geometric concepts based on **5E** (**E**ngage, **E**xplore, **E**xplain, Extend, **E**valuate) lesson model. Consequently, this study employed the guided inquiry approach with GSP. The findings are expected to inform preservice mathematics teacher education practices by exemplifying effective learning environments and methodologies.

1.4 Objectives of the Study

In the twenty-first century, learning geometry remains important. Teaching geometry, therefore, can help PSMTs be successful in their mathematical understanding and

competencies. Thus, the primary goal of the study is to investigate the impact of technology-integrated guided inquiry-based learning on PSMTs' geometry learning and attitude toward geometry, as well as how the interventions facilitated the learning process in experimental groups.

The study involved three groups. Experimental group 1 (EG1), Experimental group 2 (EG2), and Comparison group (CG). These groups were taught using a technology-integrated guided inquiry approach, a guided inquiry approach, and traditional teacher-centered approaches, respectively. In addition, the 5E lesson plan was used for lesson delivery in both experimental groups. Per the general objective of the current study, the following specific objectives are formulated:

Specific Objectives of the Study were:

1. To investigate the effect of technology-integrated guided inquiry-based learning approach on PSMTs' learning of plane geometry concepts.

2. To examine the impacts of technology-integrated guided inquiry-based learning approach on attitude towards plane geometry of PSMTs.

3. To explain how attitude and reasoning skills affect PSMTs' conceptual and procedural knowledge of plane geometry concepts.

4. Examine how the implementation of a technology-integrated guided inquiry-based learning approach during intervention classes improves the learning plane geometry.

1.5 Research Questions

To achieve the aforementioned purpose of the study, the following basic research questions have been set:

1. Is there a significant mean difference between experimental and comparison groups?
   1.1 On the conceptual understanding of plane geometry concepts?
   1.2 On the procedural knowledge of plane geometry concepts?
   1.3 On the reasoning skills of plane geometry concepts?
2. Is there a significant mean difference in attitude towards geometry between the experimental and comparison groups?
3. What is the effect of attitude and reasoning skills on the conceptual understanding and procedural knowledge in learning of plane geometry concepts?
4. How technology-integrated guided inquiry-based learning approach affected the learning of plane geometry during intervention?

1.6 Significance of the Study

The curriculum framework for primary pre-service teachers has been advocating active learning and/or student-centered approaches (MoE, 2013; 2015). To this effect, different innovative pedagogical approaches in geometry instruction are needed to promote meaningful teaching of geometry and increase geometry performance among PSMTs (Eshetu et al., 2009; Jones, Fujita, & Ding, 2006; MoE, 2009; NCTM, 2000). Hence, studying the knowledge of pre-service teachers is significant since teachers' knowledge influences the mathematics achievement of their students (Baumert *et al*., 2010; Hill,

Rowan, & Ball, 2005). Ng, Shi, and Ting (2020) found that primary teachers who had better mathematical knowledge preferred to use new, innovative, and non-traditional instructional methods, whereas teachers who had lower mathematical knowledge were likely to use traditional instructional methods for teaching.

To do so, the type of instructional method used to promote meaningful learning is important. As a result, one of the instructional approaches based on the constructivist approach, the guided inquiry-based learning approach supplemented by technology, which facilitates PSMTs' learning by visualizing processes related to plane geometry, was designed and implemented with the 5E learning cycle model.

As a result, the findings of this study may be of great interest to all concerned bodies of pre-service primary teacher education and other stakeholders in the educational sectors. Therefore, the findings of this study may be significant:

- In preparing primary teachers, professional development, and the teaching profession, the government, curriculum developers, and in particular the Ministry of Education, regional bureaus, colleges of teacher education, and teacher educators can get information for further improvement of the quality of primary teachers' training practices and develop new ways of training strategies by using digital technologies. To give some ideas to curriculum developers about how to design effective instruction to increase PSMTs' learning in plane geometry concepts.
- The findings of this study may have value for mathematics teacher educators who are engaging in the preparation of teachers in CTEs.

- May help to monitor the professional qualities of teachers by helping to ensure that only suitably qualified candidates for mathematics teacher education programs are enrolled; candidates who would subsequently be better prepared to take up teaching positions in primary schools.
- To address the effects of mathematics instruction supported by technology-integrated guided inquiry on PSMT performance in geometry.

To summarize, it is expected that this study may add to the literature on the effect of a technology-integrated guided inquiry-based learning approach in geometry learning and mathematics education as a whole, as well as how to use technology in accordance with the inquiry approach. Moreover, the findings of this study are believed to offer useful information for mathematics teachers in planning instructional activities within a technology-integrated guided inquiry-based instruction environment.

1.7 Delimitation of the Study

This study was conducted on the plane geometry (Math-111) course, which is delivered as a major course to pre-service mathematics teachers (12+3 level) in CTEs at the beginning of their second year on their first semester and who will be upcoming teachers to teach upper primary grades after their graduation. The module includes concepts such as point, line, and plane; triangle; triangle congruency and similarity; quadrilaterals (their properties and area); and circle and its measures (prepared by the Oromia Education Bureau).

Since the study was designed to investigate the impact of a technology-integrated guided inquiry-based learning approach on PSMTs' learning, it is wise to look for an area where

there is access to technology and feasibility for the intervention. As a result, CTEs in Oromia Regional State were selected to be studied. The CTEs are selected because of their ease of accessibility, availability of technology (internet, Wi-Fi, computer laboratories, power), availability of the mathematics department, and time of CTE establishment using purposive sampling. Finally, the colleges were assigned to experimental and comparison groups at random. GSP, PowerPoint, and downloaded videos from YouTube were among the technological tools that were implemented in this study. For an ethical purpose, the colleges were named CTE1, CTE2 and CTE3. Accordingly, CTE1 assigned for EG1 & EG2, CTE2 is for CG, and CTE3 as the pilot study. Further, the study was delimited to the variables conceptual understanding, procedural knowledge, reasoning skills, and attitude, so other variables that might affect the study are beyond the scope of the study.

1.8 Operational Definitions of Terms

In the following, the operational definitions of key terms used in the study are given below.

Geometry Learning: in this study geometry learning refers to PSMTs' conceptual understanding, procedural knowledge, and reasoning skills of plane geometry concepts.

Conceptual understanding: ability to comprehend geometric concepts, definitions, rules, theorems, propositions, and reason out relations among concepts of plane geometry which is measure through two-tiered geometry diagonostic test (TTGDT).

Procedural knowledge: ability to formulate, represent and solve geometric concepts, application of algorithms, techniques and procedures to solve geometric problems which was measured through geometry procedural knowledge test (GPKT).

Reasoning skills: the ability to proof geometric statements with its justifications, using facts, theorems, proved statements logically either inductively or deductively which measured through geometry problem proof test (GPPT).

Attitude: a positive and negative feeling about learning plane geometry in TGIBL approach measured through geometry attitude scale questionnaire (GASQ).

Technology: refers to digital technologies that were used during this study, such as computers, projectors, and laptops; YouTube videos and GSP software incorporated during the classroom; and PowerPoints used to introduce the lesson.

Guided inquiry based learning: a method in which learning and teaching are facilitated in collaboration between teachers and small groups and independently as well by asking questions, making connections, reflecting on processes, and discussing their ideas and conclusions using hands-on activities. Think-Pair-Share is mainly used during lesson delivery.

Technology integrated guided inquiry based learning: an approach used for enhancing PSMTs plane geometry learning by integrating available interactive technology (GSP, Videos) into guided inquiry approach to support PSMTs and mathematics teacher educators during geometry lesson delivery to make learning and teaching more interactive.

5E learning model: A five-staged instructional model for teaching and learning based on **E**ngage, **E**xplore, **E**xplain, **E**xtend/**E**laborate, and **E**valuate

Traditional teacher-centered instruction: the convetional method of teaching practiced in CTEs suh as lecture, demonstration and sometimes includes group discussion.

1.9 Structure of the Dissertation

There are five chapters in this thesis. In first chapter, the background and statement of the problem of the research has been illustrated. In chapter two, theoretical framework, literatures on geometry teaching and learning were reviewed, the concepts of technology integrated guided inquiry approach and its empirical results and research gap were discussed. In addition, conceptual framework of the study also illustrated at the end of the chapter. Chapter three presented the research paradigm, research design and data collection methods and procedures including the instruments designed, sample selection, and the procedure data analysis. In chapter four, the analyses of data obtained through the interventions were analyzed. Finally, in chapter five, the discussion of the results, the conclusion and recommendations of the study were discussed and described.

CHAPTER TWO: REVIEW OF RELATED LITERATURE

2.1 Introduction

The educational process is centered on the teaching and learning processes. According to research, there is a direct link between teaching quality and students' achievement (Darling-Hammond & Youngs, 2002). In a teaching and learning process, effective teaching methods are essential to involving learners in the teaching and learning process (Ding & Sherman, 2006).

The philosophical foundation and underlying concept of the teacher-centered instructional strategy are positivism and behaviorism. In this case, learning is viewed as a system of behavioral responses to physical stimuli that is driven by reinforcement, practice, and external motivation.

Because of the limitations of the traditional teacher-centered teaching method, a new paradigm called learner-centered teaching evolved. Interpretivism and constructivism are the philosophical underpinnings and guiding principles of the learner-centered approach to education. It is therefore important to investigate the learning theory of constructivism that supports critical thinking and can help learners gain a deeper understanding using a variety of teaching approaches.

This chapter reviews the existing theoretical and empirical studies related to the study. In the first part, constructivism is explained as the theoretical background of the study. In the second part, inquiry-based learning is described in detail, and related studies that investigate the effect of inquiry-based learning on geometry learning and attitudes among pre-service mathematics teachers are examined.

2.2 Constructivism Learning Theory

The theories of the teaching and learning process in the twenty-first century demand that learners actively participate in the construction of knowledge. This is because learning is a complex process that requires learners' active engagement in activities, curiosity, motivation, and self-regulation to take place. In this century, mathematics education supports the concept and objective of mathematics literacy, although the methods used to achieve this goal vary. As a result of the complex nature of mathematics learning, devising an appropriate way of delivering approaches and pedagogy is significant for understanding most of the concepts in mathematics. Thus, various ideas that provide guidance on how mathematics should be taught in this regard were devised (Guffin, 2008).

For many years in mathematics education, behaviorist learning theory and a teacher-centered method were prevalent in the teaching and learning process. In this case, the most common teaching approach was the lecture method, which didn't support learners in understanding mathematical concepts beyond memorizing mathematical facts, formulas, and rules. A behaviorist teaching approach in mathematics education emphasizes rote learning and memorizing formulae, one technique for solving problems, and adherence to routines and drills. In a behaviorist approach, the teaching and learning process concentrates on covering wide subject areas, leaving learners with little time to look beyond the facts and solve problems, reducing independent and autonomous learning. Little emphasis is placed on learners' attitudes and devotion to program goals.

However, according to Santrock (2001), meaningful learning occurs when learners actively construct knowledge from prior understanding. Thus, this issue has stimulated

researchers to look for new learning theories and techniques to address the problem. In opposition to behaviorism, the pioneering learning theory in the first half of the twentieth century, constructivism arose as a learning theory. In this theory, learners' prior experiences in constructing knowledge are also emphasized (Ausubel, 1968; Bischoff & Anderson, 2001). According to Goodwin and Webb (2014), constructivism transforms the learner's position from that of a passive receiver of knowledge to an active participant in the learning process. The learners apply their prior knowledge and real-world experience, learn to generate and test hypotheses, and finally evaluate their findings (Goodwin & Webb, 2014).

In recent literature, radical or cognitive constructivism and social constructivism are among the most prominent and espoused variants of constructivism (Derry, 1996; Phillips, 1995; von Glasersfeld, 1984).

2.2.1 Cognitive Constructivism

Knowledge, according to cognitive constructivism, is something that learners actively construct through a process based on their current cognitive structures rather than something that people passively receive (Bruner, 1966; Piaget, 1972). Cognitive psychologists, such as Jean Piaget and Lev Vygotsky, have long advocated the constructivist theory of learning. Piaget said (in Wilson, Cooney, & Stinson, 2005) that learners learn better when they can articulate knowledge through inquiry and experimentation instead of passively acquiring facts.

According to Piaget's theory of cognitive development, humans must generate their own knowledge rather than receive it in order to comprehend and use it (Piaget, 1972). He

views knowledge as more actively constructed by the student than passively received from the environment. Piaget emphasizes cognitive processes over environmental influences on learning and believes that speech and language are simply manifestations of what has already been learned (Gredler, 1997). In Piaget's theory of cognitive development, learners grasp concepts at an increasing level of abstraction depending on their level of maturity through four distinct stages.

The ***Sensorimotor stage*** (0 - 2 years old) a pre-linguistic stage characterized by kinesthetic understandings and experiential organizations (Swan, 2005). At this stage, children begin to discover their surroundings, first through their own senses and physical activity, and then through language.

The ***Pre-operational stage*** (2 - 7 years old) is characterized by egocentricity (viewing the world from one dimension of view) and the organization of knowledge relative to oneself (Swan, 2005). At this stage, children are able to use symbols (language), but their thinking is unsystematic and illogical. According to Piaget, this stage includes "symbolic function," in which children begin to distinguish pictures or symbols for many objects in their environment, and "intuitive thought," in which children ask a variety of questions about everything in their environment. The ***Concrete Operational stage*** (7 - 11 years old) in which knowledge is organized into logical categories but still linked to concrete experiences (Swan, 2005). In this stage, the children begin to replace "intuitive thought" with their own logical reasoning.

The ***Formal operational stage*** (11 – adulthood) in which knowledge is abstracted from experience and formal reasoning can occur (Swan, 2005). At this stage, students are able

to deal logically with multifaceted situations, hypothetical or deductive reasoning, inductive reasoning (moving from specific facts to formulate general principles and conclusions), reflective abstractions, and the ability to reason in a symbolic or abstract manner. In this case, learners start using higher-order thinking or abstract ideas to solve problems. Therefore, this stage of development includes higher education students, but learners in the same class may not always have the same cognitive abilities. As a result, determining learners' cognitive levels is critical since it can inform the instructional process.

Table 2.1 Piaget's stages of cognitive development

Phases	**Range of Age (Years)**	**Explanation**
Sensorimotor	below 2	Use common sense, exercise, and later language for environment exploration.
Preoperational	2 – 7	Developing one's own language yet unable to comprehend the views of others.
Concrete	7 – 11	Using one's own logical reasoning to take the place of intuition.
Formal operational	> 11	Use higher order thinking to solve problems.

Piaget (1972) also distinguished three types of knowledge and how students learn, i.e., external knowledge (social knowledge, physical knowledge), and internal knowledge (logico-mathematical knowledge). Social knowledge can only be gained through interaction with others (Lutz & Huitt, 2004), while physical knowledge (empirical knowledge) is acquired through practical interaction with the environment or discovered through exploration. Exploring concepts with concrete material can help students gain physical knowledge. Physical knowledge forms the basis for logical-mathematical thinking (Piaget, 1972). When students form abstract associations between objects in their minds, they develop more sophisticated knowledge, logical-mathematical

knowledge. It is knowledge of an abstract reasoning method that can be used beyond the physical interaction with a concrete stimulus (Lutz & Huitt, 2004).

According to Piaget, learners construct their own knowledge (schema) through a continuous self-construction process that incorporates three essential concepts: assimilation, accommodation, and equilibration, which are accepted as the kernel of constructivism theory. In the assimilation process, new knowledge is incorporated into previous schemas for further knowledge construction. On the other hand, accommodation occurs when new knowledge conflicts with pre-existing schemas; a person adjusts their knowledge to a new concept. In this case, the learning of new knowledge occurs when new knowledge disturbs the pre-existing organization of knowledge (Savin-Baden & Wilkie, 2004). Equilibration occurs whenever learners progress from one stage to another.

Consequently, student-centered and discovery-oriented learning processes are considered learning methods by cognitive or radical constructivists. Piaget's cognitive constructivism theory emphasizes the importance of individual students' learning demands and the pace of knowledge construction. Belbase (2011) claims that group discussion, engagement, and idea exchange do not imply that knowledge is socially constructed. Social interaction facilitates knowledge construction in the individual's mind. As a result, lecturers should take this into consideration and plan mathematics teaching and learning around the individual PSMT's cognitive processes to help with the equilibration, assimilation, and accommodation processes in the classroom. In addition, concrete materials are also considered for the interaction of students with abstract concepts.

2.2.2 Social Constructivism

After Piaget's cognitive constructivism theories, social constructivism (realism) was established by Lev Vygotsky. In contrast to cognitive constructivism, social constructivism emphasizes social and cultural dimensions of learning. In social constructivism, knowledge is structurally and internally created by learners in response to interactions with their environment. According to social constructivist theory, cognitive structures must be explained as the results of social interaction because language and culture are the frameworks through which humans' experience, communicate, and understand reality (Vygotsky, 1962). The Vygotsky's philosophy are generally intricate in social constructivism and language development, such as cognitive negotiation, the zone of proximal development (ZPD), social interaction, culture, and inner speech (Vygotsky, 1962).

Within a social constructivist instructional context, learners are primarily given opportunities to cooperate with their peers for the purpose of discussing, producing, and sharing knowledge (Roessingh & Chambers, 2011). As a result, a student can learn from a more knowledgeable peer or adult through mutual understanding and negotiation. Vygotsky also proposed the concept of ZPD, which states that learners can do something they couldn't do on their own by collaborating with more capable peers or adults through scaffolding (Bruner, 1966; Wood, 1998).

According to social constructivism, cognitive development begins at a social level and then moves to an individual level, as a function of ZPD (Vygotsky, 1962). Thus, instructors who are facilitators in social constructivism provide initial assistance to students, and this assistance gradually fades out as learners begin to learn independently

of themselves (Amineh & Asl, 2015). To accomplish this, learners will need to collaborate with one another, with a more capable peer, or with a teacher or an adult to complete the task. In this case, cooperative learning is an integral part of creating a deeper understanding. Cooperative learning is part of creating a social constructivist classroom. In this manner, classroom teachers plan activities in which students collaborate to build knowledge while embracing diversity among learners.

According to Ernest (1994), the social constructivist theory recognizes both social processes and individual senses as playing a vital and essential role in mathematics learning. As a result, social constructivism has influenced instruction and curriculum design within the educational contexts of teacher education (Jones & Brader-Araje, 2002). Therefore, social constructivism is a significant approach to teaching mathematical concepts at the college level to pre-service mathematics teachers since the pedagogical approach is designed towards an active learner-centered, knowledge-centered, and community-centered environment. As such, pre-service mathematics teachers are acculturated into their learning community and encouraged to construct their own understandings and then validate, through social negotiation, these new perspectives (Ertmer & Newby, 2013; MoE, 2013).

To sum up, even though diverse views exist about constructivism, the common foundation is that the acquisition of knowledge is described as a meaning-making process in which knowledge is actively constructed by individuals or social communities. For instance, radical or cognitive constructivism stresses individuals' meaning-making in the knowledge-building process, while social constructivists focus on the social and collaborative processes of knowledge construction.

Constructivism is a theory of knowing and learning that aims to explain what knowledge is and how it is acquired in contrast to the behaviorist tradition in learning (Duit, 1996). In constructivism, learning or meaning-making is a process in which learners construct their own understandings as a result of interactions between prior knowledge, beliefs, and ideas, as well as new knowledge and prior knowledge (Richardson, 2003). In this case, learning is seen as a dynamic and social process in which learners actively generate new meaning from their experiences and past knowledge. On the other hand, behaviorism is based on a "banking" paradigm in which the lecturer provides students with deposits of information that is assumed to be actual knowledge and the students receive it passively (Kim, 2005).

To this effect, the knowledge gained in the behaviorist approach is not adequately integrated with prior knowledge and depends on rote memorization. According to Snyder and Snyder (2008), these instructional methods do not promote conceptual understanding. In contrast to lecturer-centered methods, constructivist approaches can produce greater internalization and deeper understanding. Consequently, the constructivist method discourages rote memorization and instead promotes the creation of meaningful knowledge and understanding (Richardson, 2003).

As suggested in different literatures, constructivism approaches are more effective than traditional instructional approaches at different levels of education in mathematics, science, and social science (Kim, 2005; Karaduman & Gultekin, 2007; Pfeiffer, 2017; Beker & Maunsaiyat, 2004).

On the other hand, although constructivism has offered mathematics teachers important approaches to understanding learning, reconstructing mathematics teaching on the basis of a constructivist view of learning has a significant problem. It doesn't tell us how to teach mathematics; in other words, it doesn't specify a method (Simon, 1995).

To sum up, cognitive constructivism regards knowledge schemes as being constructed by learners and filtered through previous experiences. Moreover, interaction with the environment was emphasized so that learners could assimilate complementary components of the external world into their existing schema (Piaget, 1972). On the other hand, social constructivism relies on the social context of learning. According to Vygotsky (1962), cultural history, language, and social context have a significant impact on the learning process of learners. In addition, ZPD is the main concept in that the learning process is scaffolded by more experienced individuals (Vygotsky, 1962).

Despite their differences, cognitive constructivism and social constructivism share similarities in at least three areas. First, students build information that is "relevant and meaningful" for them based on prior knowledge (Powell & Kalina, 2009:241). Second, they emphasize inquiry learning, in which challenging scenarios are given and learners are asked to resolve them based on prior evidence (Jaworski, 2006). Thirdly, lecturers and professors are viewed as guides who assist students in creating their own knowledge (Powell & Kalina, 2009). Using these two theories as a foundation, the literature proposes a number of strategies for improving teaching and learning, such as:

- Challenging learners with questions to find out whether learners have struggle with the concept and serving them to alter their misconception(s);

- Students working on projects while receiving assistance from their teachers; and
- Learners completing projects and assignments with instructor assistance; and
- Encouraging learners to discuss their within their context and how the topic relates to their context (Powell & Kalina, 2009).

According to the study's setting, constructivism has a significant impact on the New Ethiopian General Curriculum Framework (MoE, 2020). The phrasing of its vision, values, essential competencies, objectives, and principles all use constructivist terminology. For instance, students are encouraged to embrace inquiry and technology since they are believed to be at the core of teaching and learning in terms of principles and values (MoE, 2020), which is a component of constructivism. Critical thinking and problem solving are key constructivist competencies (e.g., developing understanding and constructing knowledge).

However, constructivism is not a panacea that will solve all of our educational issues. Gordon (2009) highlights that, when used appropriately and judiciously, the constructivist approach to education may provide impressive outcomes. According to Gordon, if it is misused or misunderstood, it can lead to unproductive learning, and learners may still exhibit low performance. Although constructivism has provided mathematics teachers with useful ways to understand learning and learners, the task of reconstructing mathematics pedagogy on the basis of a constructivist view of learning is a considerable challenge because of the abstractness of the concepts in mathematics. According to Eison (2010), the use of technology can be one of the strategies that transform the learning process, and this way a teacher can create excitement and enhance learning.

2.2.3 Constructivism and Educational Technology

In order to generate meaningful learning experiences, 21st century learners involve "self-learning opportunities, interactive settings, numerous sorts of feedback, and project preference that makes use of numerous resources" (Barnes, Marateo, & Ferris, 2007:2). Additionally, incorporating technology into the classroom will support the growth of a positive learning approach (Sahin, 2003). As a result, technology may be utilized to help students access more rich learning environments, improve communication, and have collaborative discussions for problem-solving both inside and outside of the classroom. According to Gagliardi (2007), technology integration into the teaching and learning process will have the following effects:

- Technology supports learners to represent their knowledge, ideas, understanding and beliefs.
- Technology can be help for search of new information, connecting learners with learners' prior knowledge, ideas, and understandings.

Thus, the integration of technology into classroom based on constructivism is beneficial for learners to improve their learning and skills.

2.3 The Skemps' Mathematical Learning Theory

The major thoughts in connection with mathematical learning include Skemp's theories of learning. Because this study examines PSMTs' learning of a mathematical subject, plane geometry, it explores why mathematical learning is crucial. According to Skemp's theory, understanding is the incorporation of something into an existing proper schema. In this

case, "schema" is considered a general psychological term for a mental construction that assimilates existing knowledge and serves as a means for acquiring new knowledge.

In Skemp's theory (1976:2), two categories of understanding and learning were discussed: instrumental (knowing "*rules without reasons*") and relational (*knowing both what to do and why).* Instrumental mathematical learning involves learning "fixed plans" that enable learners to solve mathematical problems using a specific starting point, whereas in relational mathematical learning, learners construct a conceptual construction (schema) that helps them create several plans that can be used for solving mathematical problems (Skemp, 1979). Furthermore, logical understanding is a third type of understanding defined as the ability to show that what has been stated follows logically from the premises given along with appropriately chosen items from what is accepted as recognized mathematical knowledge (axioms and theorems) through a chain of inference (Skemp, 1979:47).

Regardless of the difference in meaning between them, there is no clear boundary between the two, and occasionally they are not independent (Rittle-Johnson & Schneider, 2014; Mahir, 2009). Procedural knowledge denotes knowing the distinct symbols of a mathematical system and the rules or procedures for solving mathematical problems, while conceptual knowledge is a connected web of knowledge where the linking relationships are as prominent as the discrete piece of information. According to Rittle-Johnson and Schneider (2014), conceptual knowledge is knowledge of concepts in which an abstract or generic idea is generalized from particular instances, while procedural knowledge is the ability to execute an action sequence (procedure) or step to solve problems. As a result, learners may be unable to solve problems or may produce solutions

but fail to understand what they are doing if conceptual and procedural knowledge are not connected (Hiebert & Lefevre, 1986). As a result, there are four different theoretical stances on the link between procedural and conceptual knowledge.

Table 2.2 The relationship between procedural and conceptual knowledge

Approach	Description
Concept-first	Concept taught first the procedure can be derived
Procedure-first	Mathematical procedure taught first the conceptual knowledge abstracted
Inactivation	Both should be learned independently
Iterative	Concept is a source of procedure and vice versa & inseparable

The iterative approach, however, is the most accepted approach in mathematical disciplines (Rittle-Johnson & Schneider, 2014). The initial knowledge might be conceptual or procedural, depending on one's previous understanding of the problems; what matters is that having one type of knowledge fosters the growth of the other (Rittle-Johnson & Schneider, 2014). To this effect, conceptual knowledge helps in selecting the best methods for solving problems, but procedural knowledge acquired through experience may aid in conceptual understanding. As a result, research has shown that teaching mathematics at all levels can be successfully accomplished via an iterative connection of knowledge (Awang & Zakaria, 2012).

Nevertheless, most research literature states that teachers and learners have been emphasizing more procedure-first instruction, almost ignoring concept-based instruction. This can be one of the most difficult concepts for students to understand. This research study is an intervention study that heavily borrows and utilizes technology intended to improve geometry learning.

In line with constructivism learning theory and Skemp's theory, different learner-centered approaches that allow PSMTs to actively engage and construct knowledge individually as well as cooperatively among each other should be designed to enrich the learning process. Thus, these theoretical frameworks will contribute to the reform of the pre-service mathematics classroom.

2.4 Constructivist Teaching Strategies

The traditional teacher-centered teaching approach is characterized by a one-way flow of knowledge from teachers to students, with students passively listening and following teachers' instructions (Kitti, 2014). According to Bekele (2012) and MoE (2015), the majority of the higher institutions, including CTEs, in Ethiopia practice the traditional lecture method. In traditional method of teaching more focus is given to formula based problem-solving rather than acknowledging the concepts behind (Antwi, Hanson, Savelsbergh & Eijkelhof, 2011). Furthermore, learners in class instruction tends to be dominated by the traditional teacher-centered find boring and useless for real-life, ultimately leading to low success rates (Givvin & Thompson, 2010; Sebsibe & Feza, 2019; MoE, 2020).

Davis (2008) stated that the traditional way of teaching, which relies on rote memorization and focused on teaching isolated facts, may allow students to pass a test but will leave them without understanding and unable to use and apply the knowledge in different situations. This approach fails in developing the conceptual understanding, critical thinking, and reasoning skills of mathematics and 21st century skills in general (Cronhjort, Filipsson, & Maria, 2013). Thus, in order to achieve mathematics learning in PSMTs, an appropriate instructional approach must be chosen in a constructivism setting.

Currently, the pedagogical approaches that are recommended by constructivism learning theory in mathematics education are cooperative learning, inquiry learning, problem-based learning, and discovery learning methods (Jaworski, 2006; Bissell & Lemons, 2006; Kirschner, Sweller & Clark, 2006; Ubaidah & Aminudin, 2019). An inquiry-based learning approach is one of the applications of this theory, which advocates that learning should be active and constructed by the learner (Kuhlthau, 2001). Inquiry-based instruction fits with a constructivist perspective in that it poses activities, fosters mathematical thinking, and provides advantages for understanding mathematics conceptually (Cobb, Wood, & Yackel, 1990; Jaworski, 2006).

The following sub-section of the chapter explores the literature regarding such constructivist-oriented teaching pedagogies; inquiry-based learning (IBL) is the focus of the current study. Therefore, in the following section, IBL and its levels, the 5E learning cycle model, and studies conducted so far will be discussed.

2.4.1 Inquiry Based Learning Approach

The theoretical bases for an inquiry-based learning (IBL) approach are Vygotsky (social constructivism), Dewey (experiential learning), and Piaget (cognitive constructivism). In IBL instructional process, knowledge is created through the collaboration between individuals and groups, using language for the advancement of knowledge (Vygotsky, 1962), as well as through practice and reflection on that experience (Piaget, 1972). IBL is a student-centered pedagogy that focuses on student engagement in sequenced and scaffolded learning rather than the instructor transmitting content (Laursen & Rasmussen, 2019; Lazonder, 2014), and where students are active learners who learn by doing (Reiser & Dempsey, 2012). PSMTs solve real-world problems that allow them to recognize

problems, offer and examine solutions, and communicate their thoughts (Laursen & Rasmussen, 2019).

IBL was implemented as a teaching and learning model in classrooms by using problem-based, discovery-based, and project-based learning. In this case, the IBL instructional method supports the construction of new knowledge based on existing knowledge, in parallel with individual interest and experience in social interaction within the real world. In this way, in the IBL instructional approach, learners are encouraged to ask questions, look for solutions, and build a connection to prior knowledge using problem solving rather than memorizing facts or rules as that traditional instructional method.

IBL also increases PSMTs' interest and motivation, aids in problem-solving, fosters the development of critical thinking, and motivates them to participate actively in their education (Angel & Marcus, 2017). It also inspires learners to participate in innovations (Padaste et al., 2015). It motivates learners to learn, think critically, analyze problems, and be engaged, cooperative, and involved in learning activities, which leads to higher-level learning outcomes (Spronken-Smith, 2012). In this respect, the IBL instructional method is effective when PSMTs engage in learning activities that make-sense for them (Love, Hodge, Corritore, & Ernst, 2015). During the process, teachers must encourage learners via questioning to realize mathematical concepts to overcome misconceptions. Furthermore, Artigue and Baptist (2012) and Douglass and Horstman (2011), PSMTs must think and ask questions, explore, discover, and prove mathematical concepts in order to gain a deeper understanding of mathematical contents, while teachers facilitate and provide feedback to clear misconceptions.

Similarly, Karagiorgi and Symeou (2005) stated that, PSMTs in IBL can practice mathematical concepts with hands-on activities to formulate mathematical questions, identify and collect appropriate evidence, present results mathematically, analyze and interpret results, formulate mathematical conclusions, and finally evaluate and conclude the results. In addition, Saunders-Stewart et al. (2012) also added that the implementation of IBL instructional method enhances PSMTs' achievement, knowledge application, thinking and problem-solving skills, and attitudes towards learning, and also promotes their self-confidence and motivation toward learning independently.

Additionally, IBL enables PSMTs to develop their own senses and sense-making abilities. According to Lewis and Estis (2020), PSMTs gain both procedural and deep conceptual comprehension through these encounters. Effective IBL promotes participation in tasks and offers possibilities for peer collaboration through group projects or presentations in class (Hassi & Laursen, 2015; Johnson et al., 2016; Kogan & Laursen, 2014; LaForce et al., 2016; Magee & Flessner, 2012). IBL is not controlled by what the instructor transmits, in contrast to conventional lecture-based learning. Instead, it enables PSMTs to seek, cooperate, analyze their responses, and comprehend the mental processes of others (Darling-Hammond, 2008; Sawyer, 2006).

IBL helps PSMTs develop processing skills, develop a deep understanding of mathematical concepts, and retain information longer (Johnson et al., 2020), whereas traditional transmission-based (lecture) learning involves PSMTs as passive recipients of information and the instructor as the main source of knowledge (Sawyer, 2006). The traditional conventional way of teaching is concentrated on teaching of procedural

knowledge, whereas IBL facilitates a deep understanding that occurs when PSMTs are active participants and involved in shaping their learning experiences.

According to Von Secker (2002), IBL has a significant positive effect on PSMTs' conceptual understanding and effectively improves PSMTs' reasoning ability. However, some PSMTs cannot learn the concept that becomes the target of inquiry. Therefore, appropriate guidance is required. Appropriate guidance can enable PSMTs at low levels to follow the ongoing activities and avoid the PSMTs at high levels from dominating those activities. In doing so, PSMTs will develop their mathematical concepts by following certain steps under the supervision and guidance of a teacher.

Finally, IBL offers PSMTs the chance to work as scientists or mathematicians (Dorier & Maab, 2020). In this sense, learners learn to inquire by asking questions and exploring phenomena, as well as evaluating and communicating their findings. The lowest common denominator for starting all inquiry-based pedagogies is a "big question" which is a problem or project (Albion, 2015). To progressively develop mathematical concepts and structures, Maaß and Artigue (2013) suggest that PSMTs should be invited to generalize their findings and methods.

2.4.2 Levels of Inquiry-Based Learning Approach

The literature states that the IBL approach can be divided into four levels based on the amount of teacher direction and the freedom provided to learners throughout course activities (Zion & Sadeh, 2007). According to Llewellyn (2013) classification, the levels are confirmation (level 1), structured (level 2), guided (level 3), or open (level 4). Separating inquiry into its component ideas, guided and unguided, enables us to discern

these varied levels of student-centeredness in inquiry (Guisti, 2008). In this classification, as the level increases, teachers' engagement declines.

Table 2.3 Characteristics of four levels of inquiry

	Confirmatory (Level1)	Structured (Level 2)	Guided (Level 3)	Open/self-directed (Level 4)
Posing question	Teacher	Teacher	Teacher	Student
Planning the procedure	Teacher	Teacher	Student	Student
Formulating the result	Teacher	Student	Student	Student

The research results show that the implementation of different levels of inquiry has an effect on the development of different abilities. The confirmation and structured inquiry levels are the least likely to enhance the learners' learning outcome sent of different abilities. The confirmation and structured inquiry levels are the least likely to enhance the learners' learning outcomes. Learners at this level are not engaged in a deep learning process; rather, simple facts and concepts are learned, which enhance only low-order thinking (Bell, Smetana, & Binns, 2005; Martin-Hansen, 2002). In addition, studies have shown that the verification inquiry has the least impact on learners learning's (Blanchard et al., 2010). In contrast, guided or open levels are important to enhance higher-order thinking and learners' abilities (Bell et al., 2010).

When open inquiry was compared with other levels of inquiry, learners in open inquiry achieved more cognitive skills, procedural skills, and critical and scientific thinking than those in guided and structured inquiry (Zion & Mendelovici, 2012; Sadah & Zion, 2009). However, not all students can benefit from open inquiry because it can lead to misunderstandings and frustration when problems cannot be solved. Moreover, the

implementation of open inquiry leads to lower cognitive learning results than the implementation of guided inquiry.

According to Lazonder and Harmsen (2016), guidance is crucial for inquiry-based learning to be effective during the teaching of science and mathematics education. Therefore, the question in the guided inquiry approach is structured to encourage learners to think critically and analytically while the teacher assists their learning. Thus, it is important for the classroom teacher to facilitate, mentor, and supervise the process to achieve maximum learning outcomes in the inquiry approach.

Although research indicates that both open and guided inquiry-based learning techniques are important, the effectiveness of these approaches in the teaching and learning process is controversial (Sadeh & Zion, 2009). However, research has shown that inquiry-based learning, when supported with appropriate scaffoldings, can assist PSMTs in understanding abstract mathematical concepts (Kirschner et al., 2006). A guided inquiry method is therefore more effective in preventing time waste, lowering cognitive load, and soothing learners' frustrations (Trautman, Makinster, & Avery, 2004).

2.4.3 Guided Inquiry-Based Learning Approach

Guided inquiry-based learning (GIBL) applies to a range of structured investigative activities where the instructor provides the problem and encourages students to work out the procedure to resolve it (Guisti, 2008). It is an activity-oriented approach to learning where learners are closely monitored by their teachers to find a solution to instructional problems through investigation. According to Martin-Hansen (2002), the GIBL approach is a carefully planned, closely supervised, ongoing assessed and targeted intervention that

gradually leads students towards independent learning. In this case, PSMTs are given activities through which they develop knowledge and understanding of concepts, ideas, and course material by engaging them in hands-on, mind-on tasks while the teacher functions as the facilitator. It requires teachers to actively engage students in developing a conceptual understanding of mathematical concepts (Chapman, 2011).

Furthermore, Clark (2012) stated that in the GIBL approach, learners engage in class discussion in which every individual participates in activities. In support of this notion, Capaldi (2015:283) agrees that GIBL is "a pedagogical method that encourages students to conjecture, discover, solve, explore, collaborate, and communicate." According to Case et al. (2015), further aspects of GIBL in mathematics include the acquisition of mathematical activities and language through cooperative participation in mathematical discussions and reasoning while engaged in the process of solving challenging problems.

Therefore, GIBL is student-focused and problem or question-based, and the use of communication, cooperation, and reflection skills is essential to the instructional process. GIBL offers the opportunity to generate and create new perspectives on mathematical problems and content by giving students questions in the context of the curriculum, starting from their previous experience, and experimenting with multiple solutions (Chapman, 2011). This shows that the GIBL approach in the mathematics classroom is based on the construction of mathematical rules, concepts, and principles by PSMTs beyond the direct presentation of mathematical concepts, rules, and processes by the teacher.

Teaching and learning through inductive methods are a key concept in GIBL and refers to constructivist teaching and learning methods such as problem-based learning. Pedaste et al. (2015) argued that student-student and teacher-student communication can assist learners in sharing outcomes and ideas in order to obtain feedback. In GIBL, PSMTs are asked questions such as "If this happens, what should we do?" to encourage them to inquiry and explore. Utilizing such questions in the teaching process may improve PSMTs' ability to recognize and adopt mathematical concepts and rules. Because the learning steps in the GIBL approach require the participation of learners, motivation and interest can have a positive influence on the implementation of the learning steps in the inquiry model (Pohan, 2019).

However, despite different research revealing that learners benefited from the GIBL approach for mathematics learning, implementing the approach in a classroom is challenging, and the practice has been underutilized and given little emphasis by mathematics teachers (Gardner, 2012; Rooney, 2012). These challenges are more pronounced in developing countries, where basic learning technologies and other educational infrastructures are grave problems.

Teachers sometimes refrain from using the GIBL technique in mathematics classrooms for reasons such as the insufficiency of pupils, the lack of time for students to investigate and study due to test preparation pressures, and the lack of autonomous working abilities among students (Engeln, Euler, & Maass, 2013). Additionally, Handal (2003) discovered that many teachers choose not to utilize GIBL because they think that since math is made up of rules and principles, learning arithmetic relies more on memory than on investigation. Moreover, inadequate scaffolding and a lack of visualization tools are

among the challenges. Furthermore, the inquiry approach is mainly implemented in science subjects rather than mathematics. More recently, various education reforms have put exploration, sense-making, and empirical understanding of concepts at the center of mathematical instruction.

As stated by Sanni (2007), learning geometry is an investigation rather than an instructive process. As pointed out by van Hiele (1986), geometry is full of visual concepts such as planar concepts and relations among them, such as congruency, that provide opportunities to construct relationships. This feature is best embodied through the dragging operation of dynamic geometry tools, but it can also be visualized with traditional tools. This implies that the integration of technologies has the potential to increase the effectiveness of an inquiry-based learning approach.

To remain within the set of timeframes of school lessons, visualization technologies increasingly aid inquiry studying which will permit high-level, differentiated studying without being time-consuming. Thus, digital tools offer an effective way to decrease time consumption and increase the adoption of inquiry strategies in regular lessons (Sotiriou, Bybee, & Bogner, 2017).

2.5 Technology-Integrated Guided Inquiry-Based Learning Approach

Technology has the potential to expand and extend learning, affecting every aspect of a mathematics classroom and education as a whole. Due to this fact, educators and educational researchers have been working to integrate technology into classroom activities (Ruggiero & Mong, 2015). The technological resources that impact educational activities include such things as computers, portable devices (smartphones and tablets),

social media platforms, software programs, e-learning, web-based technologies, etc. As a result, teachers can integrate any available technology that fits into the content used as a source of data, supportive material, reference, etc. (Harrell & Bynum, 2018). Technology can also alter teachers' educational activities in effective ways to include all types of learners and measure their understanding in a variety of ways (Abdullah & Ward 2016).

Technological practice not only improves learners' learning and understanding, but it also fosters their enthusiasm for studying, which is essential for learning, collaborative learning, and the development of problem-solving skills (Blau et al., 2020). Furthermore, technology-enhanced learning environments are used to "scaffold" learners in sense-making, activity management, problem solving, the articulation of ideas, and learning reflection in the inquiry approach (Quintana et al., 2018). Thus, technology can scaffold PSMTs' inquiry by sustaining their motivation, providing supports or tools in the inquiry steps, and reducing cognitive load (Bell et al., 2010; Clark et al., 2003; Hmelo-Silver et al., 2007; Jermann, Soller, & Lesgold, 2004; van Joolingen et al., 2007).

During the inquiry process, technology can scaffold learners in formulating questions, designing and planning investigations, collecting and analyzing data, building and testing, producing and manipulating objects, and developing deep explanations (Linn & Eylon, 2006; Slotta & Linn, 2009). As a result, the integration of technology such as dynamic software into mathematics education assists PSMTs in learning mathematics topics through visual and multiple representations (Hollebrands, 2003). In this case, teachers can provide innovative ways to promote PSMTs' learning and foster the acquisition of mathematical knowledge and skills by incorporating educational tools into their normal teaching practice.

In general, the technological environment provides new learning opportunities with the potential to engage PSMTs of various mathematical abilities and levels of understanding in mathematical tasks and activities. According to Liu and Bera (2005), technology can increase PSMTs' mathematics competency when appropriately integrated into the teaching and learning process and decrease the cognitive load of mathematics learning through various software applications. Multiple technologies, in particular, are important for improving teachers' instruction and PSMTs' active engagement and learning opportunities when teaching mathematical content. Therefore, technology-based learning environments in mathematics enable PSMTs to operate at higher cognitive levels, facilitate constructive learning, and improve inquiry and conceptual change by providing chances for active learning (Chee, Horani & Daniel, 2005; Ayub, Mokhtar, Luan & Tarmizi, 2010).

For this goal, a variety of virtual educational environments and programs (e.g., simulations and modeling tools, GeoGebra, the Computer Algebra System, web resources, spreadsheets, databases, and Geometric Sketchpad, etc.) are available for mathematics and geometry instruction. Hence, there is potential to use these technologies to enhance the quality of PSMTs' mathematics and geometry learning for further improvement of quality education. As a result, these technologies have the potential to increase the quality of PSMTs' mathematical learning and, in turn, improve the quality of education (MoE, 2012a).

Technology such as dynamic geometry software establishes a setting where the PSMTs will perform some construction by drawing points, lines, curves, and polygons, then manipulate a geometric construction to make interpretations about the result and

conjecture their observations. Geometer's Sketchpad, one of the pieces of dynamic geometry software, provides PSMTs with ways to visualize mathematical structures and abstract concepts and helps in building PSMTs' understanding of mathematical concepts (Arini & Dewi, 2019).

To this effect, educational scholars believe that incorporating technology into PSMTs' mathematics and geometry learning activities is considered a determinant factor (Moreno-Armella & Hegedus, 2020). To motivate teachers to use technology in the classroom, more professional development is needed, according to a study, since otherwise they are frequently reluctant to do so (Philipsen, Tondeur, Pareja Roblin, Vanslambrouck, & Zhu, 2019). According to a study by Garrison, Anderson, and Archer (1999), employing technology in conjunction with content and pedagogical understanding has been more effective in training teachers to use technology in the classroom.

In relation to this, digital technologies have developed into significant teaching aids for mathematical content. Teaching mathematics in technologically advanced environments is more efficient than utilizing conventional paper and pencil teaching techniques. The use of technological software by teachers in the mathematics classroom can establish a foundation for students' learning achievement (Arbain & Shukor, 2015; Pilli & Aksu, 2013) and also assist in minimizing mathematical misconceptions (Birgin, Gurbuz, & Birgin, 2012). Thus, mathematical software facilitates access to the many representations of mathematical concepts and has the potential to improve mathematics education. Hence, integrating digital tools into teaching and learning improves favorable attitudes

toward mathematics, particularly geometry, among PSMTs and helps them learn (Han, Luká, Sekerák, & Veda, 2011).

In addition, Andraphanova (2015) stated that technology has the ability to change learners' attitudes towards difficult geometry concepts and the traditional process of memorizing concepts into clearer concepts that are well understood. In this case, the use of technology has facilitated exploration and visualization (Bulut, Akçakn, Kaya, & Akçakn, 2016), generalizations, and proofs (Oner, 2008). In conclusion, Malabar and Pountney (2002), Amarin and Ghishan (2013), and others confirm that technology can support the constructivist approach to learning if used properly and effectively. Bretscher (2010) stated that technology can be potentially important to teach geometry since the static nature of the environment is transformed into a dynamic interface where students can visualize and manipulate lines and angles.

With regard to geometry, there are numerous claims about the benefits of using digital technologies to improve PSMTs' understanding of geometric concepts (Akcay, 2017; Zulnaidi & Oktavika, 2018). With the help of technology, we can show or use multiple representations of mathematical concepts in the instructional phase to see and explore different constructions and views of an object. In this way, we may create technology-based learning experiences that are both conceptual and technical. The same goes for procedural activities like developing intuition, identifying patterns, investigating, proving, and testing. According to Laborde, Kynigos, and Strasser (2006), this technical construct is suitable for teaching geometry, increasing understanding of geometrical ideas as well as resolving conceptual challenges (Ocal, 2017).

One such technology is Geometric Sketchpad (GSP), which is dynamic geometry software that helps to create hot links for mathematical objects to show their both symbolic and visual representations simultaneously. The GSP software integrates interactive geometry software elements with spreadsheets, a computer algebra system, and other functions. Both instructors and students can use its user-friendly interface. PSMT may handle the geometric objects in this scenario and track changes across various representations of the items (Battista, 2002; Laborde, Kynigos, & Strasser, 2006). Hence, using GSP, users may see, record, modify, and anticipate geometrical objects, ideas, and theorems (Forsythe, 2007; Hill & Hannafin, 2001). Also, it promotes numerous representations and experiential learning, where users construct and engage with mathematical objects (Dikovic, 2009).

Moreover, GSP offers a solution to improving mathematics education from middle school through college by making algebraic and geometric concepts easier to understand. The GSP can be used by users (primarily educators, instructors, and students) to explain, investigate, and model mathematical topics and their links (Dolonen & Ludvigsen, 2012). Tusiime, Maniraho, and Uwurukundo (2021) assert that GSP may be used to simulate real-world situations, aid in problem-solving, provide visualizations and interactive examples, boost learner motivation and cognitive growth, and improve learner-centered pedagogy. It may also be utilized to further connect the ideas of geometry (Samur & Akyuz, 2016) and trigonometry (Zengin, Furkan, & Kutluca, 2012).

However, despite the fact that employing IT resources and computer software may improve PSMTs' learning results, they are still uncommon in schools and institutions worldwide (French, 2004). By offloading and dispersing cognitive learning processes,

technology transforms abstract concepts into concrete representations, which can aid students' reasoning and learning processes (Sedig & Liang, 2006). However, researchers contend that only well-designed tools can successfully engage students and support their learning of mathematics (Liang & Sedig, 2010). As a result, developing high-quality tools for mathematics requires careful design, and it is important to prioritize integrating technology to enhance the learning process in terms of depth and scope.

2.5.1 Geometer's Sketchpad as a Technology for Teaching Geometry

GSP is one of several dynamic geometry software programs used as a technology for teaching and learning mathematics and geometry (Chew & Lim, 2011a). According to Battista (2002), GSP preserves the basic properties of a geometric construction, which enables the learner to explore the generality of the consequences of the construction. The dynamic capability of GSP, inquiry-based tasks, as well as student-student and researcher-student interactions, deepened PSMTs' conception of two-dimensional shapes (Driskell, 2004).

Currently, GSP is not only used for creating, exploring, and analyzing in the field of geometry but also in algebra, calculus, pre-calculus, trigonometry, and other areas of mathematics. It is interactive software that can help PSMTs understand geometry, starting from points, lines, and angles to more complex understandings such as arches, turns, and transformations.

Like any new software to be used in education, GSP has its own tools, buttons, and menus, which must be taught to the students via individual tutorials or full classroom presentations and follow-ups. Again, the students try to find new ideas or prove old ones

with their model. We are then able, for example, to prove that the sum of the angles of a triangle is 180 degrees by using the animation features and the formula features. Teachers can use GSP to create learning environments where the students are able to help teach themselves by answering the "*What if*...?" questions themselves with the help of GSP by using the animation and formula tools and repeatedly testing their hypotheses to develop a deeper understanding of geometrical concepts. As a result, learners are kept engaged as they ask questions. This enables teachers to address learners' misconceptions. The following figure shows the window of GSP used for this study.

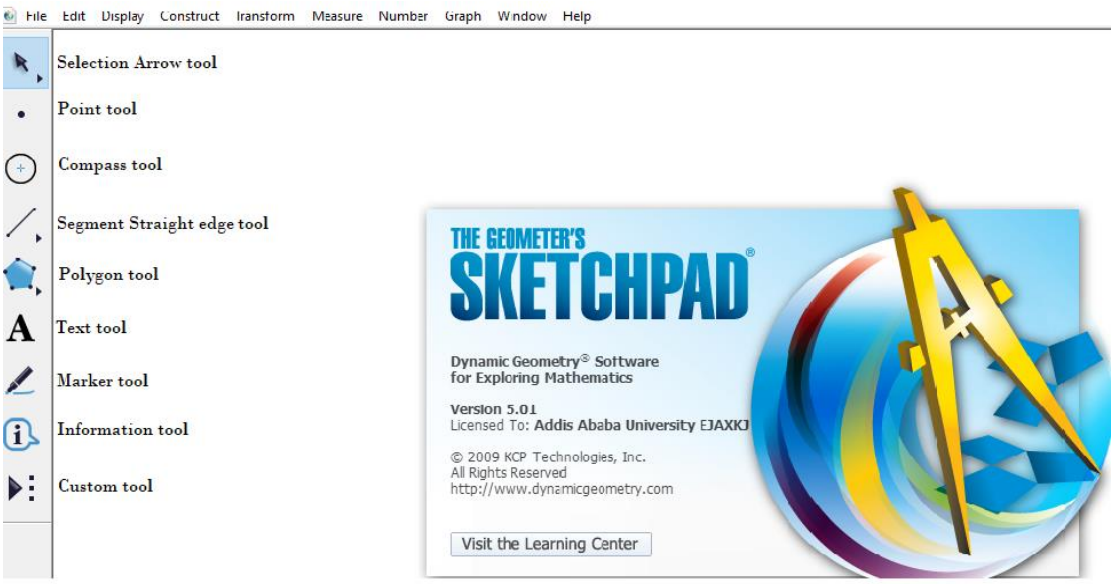


Figure 2.1 Window of Geometer's Sketchpad Version 5.01. Licensed to: AAU EJAXKJ

GSP's menu bar consists of "File, Edit, Display, Construct, Transform, Measure, Number, Graph, Window, and Help" listed across the top. The Display menu commands allow one to control the appearance of objects in a sketch and the tools to work with them. The Construct menu runs commands for doing many key geometric constructions. The Transform menu commands apply geometric transformations to figures in sketches,

allowing one to create translations, rotations, dilations, reflections, tessellations, scale models, kaleidoscopes, fractals, etc. The Measure menu commands allow measuring the numerical properties of selected objects. The Graph menu allows creating and manipulating coordinate systems, creating parameters and functions, find the derivative of a function, plot points and functions on the coordinate axes, and tabulate measured values.

In the teaching and learning procedure of geometry, GSP can be used as a scientific inquiry tool to observe patterns, discover rules, formulate hypotheses, and make conjectures as students are construct and manipulate various geometric objects on a computer (Hoyles & Noss, 2003; Idris, 2009; Ruthvan, 2008). For instance, the GSP software displays that as the triangle's size and form change as a result of dragging one of the triangle's vertices, the sum of the remaining angles stays constant. The following hypothesis might be made by the PSMTs when they transform the triangle: despite variations in the angles' measurements, a triangle's sum of interior angles is always 180 degrees.

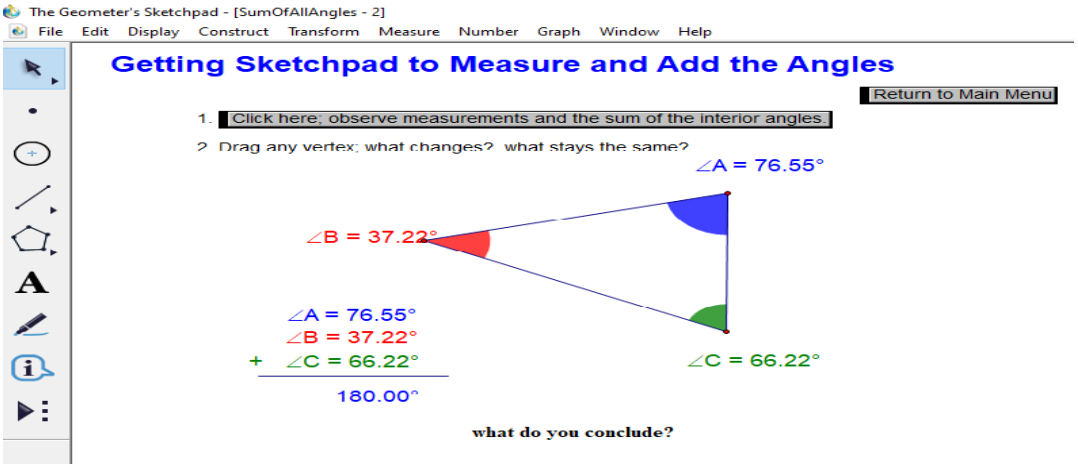


Figure 2.2 The sum interior angles of a triangle

The GSP software can be used to facilitate making and testing conjectures, which are crucial elements of mathematical discovery in formulating proof hypotheses (Battista & Clements, 1995). Users of the GSP can come up with definitions, posit hypotheses, solve considerable problems, preserve their generalizations, and provide innovative proofs as a result of their use.

According to De Villiers (2004), university and high school students are not given the opportunity to experience the process of discovering or creating new results during their teaching and learning process. A study by Govender and De Villiers (2004) used a similar approach, but with the GSP, where a geometric configuration could be continuously dragged into different shapes to check for invariance. PSMTs were provided with worksheets with sketches satisfying given conditions and had to find the conclusion based on the conditions.

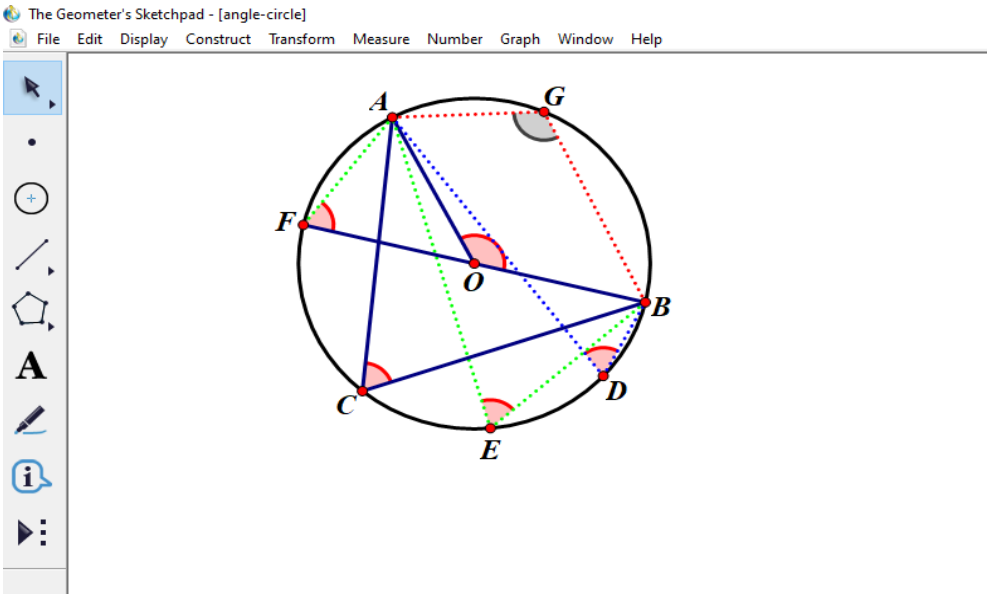


Figure 2.3 The relationship of angles at center and circumference & cyclic quadrilateral

In this application, learners are dragged to see the different angles, or many examples of the angle at the circumference, and a non-example of the angle at the circumference is double the angle at the center. In this case, PSMTs explored the size of the angle at the circumference by dragging its vertex, C. Vertices E, D, F, and G represent the different positions of vertex C. PSMTs can be given similar activities such as dragging vertex C to E, D, and F, which are all in the same segment, but they will notice that this is not the case at G, which is in a different segment than $A\hat{O}B$.

In addition, the property of the opposite angles of the cyclic quadrilateral can also be explored under different conditions and with non-examples of cyclic quadrilaterals. That is, in this case, AOBG. Furthermore, PSMTs were given different GSP activities to explore all the theorems and corollaries of circle geometry prescribed in the module. This activity was therefore consistent with the inquiry by not starting with proofs of theorems or definitions but starting with exploration.

2.5.2 The Application of Technology-Integrated Guided Inquiry in Geometry

One of the potential uses of technology in mathematics education is the use of dynamic geometry software to improve the teaching and learning of geometry. As aforementioned above, the integration of technology into teaching and learning is important, but how it is implemented into classroom activities has an impact on its purpose and whether it becomes an effective teaching tool. Therefore, the process of interaction between teachers, learners, and technology plays a vital role in the formation of a positive learning environment and cognitive development (Manca & Ranieri, 2013).

The difficulty of establishing pedagogical changes appears to be one of the limiting elements in the development of technologically improved approaches to education (Karp & Fletcher, 2014). Teachers must utilize suitable pedagogical techniques to effectively use technology in learning processes, but an emphasis on strengthening teachers' digital technology abilities while ignoring associated pedagogical consequences appears unlikely to be sufficient (Karp & Fletcher, 2014). Using the framework of technological pedagogical content knowledge (TPCK), Angeli, Valanides, and Christodoulou (2016) indicate that a wide variety of knowledge is necessary to use technology effectively. Understanding the representation of concepts and pedagogy while utilizing technology, as well as awareness of how technology might solve learners' conceptual challenges, are all part of this foundational knowledge.

One of the potential uses of technology in mathematics education is the use of dynamic geometry software to improve the teaching and learning of geometry. As aforementioned above, the integration of technology into teaching and learning is important, but how it is implemented into classroom activities has an impact on its purpose and whether it becomes an effective teaching tool. Therefore, the process of interaction between teachers, learners, and technology plays a vital role in the formation of a positive learning environment and cognitive development (Manca & Ranieri, 2013).

In the same manner, technology can use instructional videos such as presentation software, mathematical software, calculator emulators, and simulation applications, all of which can represent content. YouTube videos are applicable for PSMTs when learning difficulties are experienced in abstract science topics (Astriyani & Fajriani, 2020). Hence, mathematics is an abstract science and one of the compulsory subjects at school (Isnaini

& Azhar, 2021). Moreover, mathematics teachers may use audio-visual media to develop learning independence and deal with mathematics problems. These videos allow cheap learning, enhancing knowledge and skill development (Copper & Semich, 2019). Its use increases the PSMT's interest in mathematics subjects (Buzzetto-More, 2014), encourages self-directed learning and enhances conceptual understanding (Lee et al., 2017).

Furthermore, video tutorials help improve PSMTs' motivation to learn by enhancing their knowledge and understanding of mathematical lessons and achievements (Lalian, 2019). Sharma (2018) reported that students who were consistently watching instructional videos in mathematics performed better in the assessments than those who were receiving traditional instruction. Similarly, learners claimed that watching video tutorials improved their understanding of mathematics concepts (Sharma, 2018).

Instructional videos tend to be short and are designed to teach a specific skill (Pai, 2014), but they are neither dynamic software nor interactive. Videos are short (3-4 minutes) and to the point, designed to focus on one or two main ideas. They are designed with the intent to hold viewers attention for the duration of the video, and a single one can provide knowledge about several topics. Integrating instructional videos into the classroom is not a new concept but one that is increasingly popular (Shipper, 2013).

2.5.2.1 The 5E Instructional Lessen Plan

In mathematics classrooms, inquiry-primarily based questions also can stand up in summary a structure which includes numbers, shapes, and algebraic structures (Schäfer, 2019). The 5E guided inquiry model is an instructional framework that develops the

learning process through an inquiry-based approach using a learning cycle called the 5Es (Bybee, 2009). The 5E guided-inquiry model stands for *engage, explore, explain, elaborate,* and *evaluate* (Bybee, 2009). This is the most practical model in the constructivist pedagogical approach, as it enables learners to analyze and synthesize new information in the constructivist classroom.

In the 5Es Model, students are expected to work collaboratively to solve problems and plan investigations. This is supported by Yilmaz's (2010) finding that when collaborative learning is directed toward inquiry, students succeed in making their own discoveries. The model promotes PSMTs' ability to think critically and creatively and enhances their understanding of concepts in mathematics (Alshehri, 2016; Dahlan, Herman, & Oktaviyanthi, 2018; Schäfer, 2019).

Engagement: In this phase, PSMTs are presented with a problem, and thereby, learners are motivated to participate in the learning activity. The aim here is to encourage PSMTs to produce different ideas and ask questions. In this manner, the PSMTs' prior knowledge of a concept is elicited to make the connection between prior knowledge and new topics.

Thus, before beginning a new topic in geometry, teachers should determine what kinds of prior knowledge their students have and provide the related information, such as shapes or real materials, which can allow PSMTs to observe or manipulate and discover or understand some knowledge by doing so. This phase is the most important step, since if the material is not presented correctly; PSMTs may not be able to establish the essential connections to fully engage in the remaining phases.

Exploration: This is the most active phase of the cycle. It is the discovery stage, which is student-centered; in which PSMTs organize what they have found by developing hypotheses. PSMTs explore geometric knowledge through concrete materials and hands-on activities. In this case, teachers should deliberately prepare sequential activities for inquiry and guide PSMTs to discover the rules, formula, or principles of each geometry topic through manipulations or activities to make abstract generalizations.

In this phase, the teacher acts as a facilitator and should encourage group discussions cooperatively by posing guiding questions while PSMTs test the geometrical concepts by investigating new information (Cepni, Sahin, & Ipek, 2010).

Explanation: In this phase, PSMTs should be encouraged to express and share their thoughts or points of view about the geometric topic and discuss it with the teacher and their peers to clarify their geometric knowledge. At this point, the teacher can employ a variety of teaching strategies and methods, including technology such as videos, software, and books. In this phase, PSMTs are asked to explain events, and subsequently, formal geometrical language is presented.

Elaboration: in this phase, PSMTs are encouraged to further extend their understanding of the geometrical concepts they have learned in the previous three phases. Worksheets, model preparation, and activities such as sketching and problem solving are some of the learning methods employed. The purpose of this phase is to move PSMTs from the physical experience to the logical-mathematical operation of thinking to generalize about geometrical concepts. They must give reasons for the steps they have taken and explain not just what they have done but why they have done it.

Evaluation: it is the final phase but does not always occur at the end of the cycle (NCTM, 2000). The evaluation can involve open-ended questions and demonstrations, and it commonly asks probing questions that lead to the next inquiry. By doing this, learners can assess their understanding and growth in relation to the concept they are learning by using the 5E lesson inquiry model. Both the teacher and the student received feedback from the evaluation (NCTM, 2000). The following diagram shows the process followed during the 5E learning cycle of lessen plan.

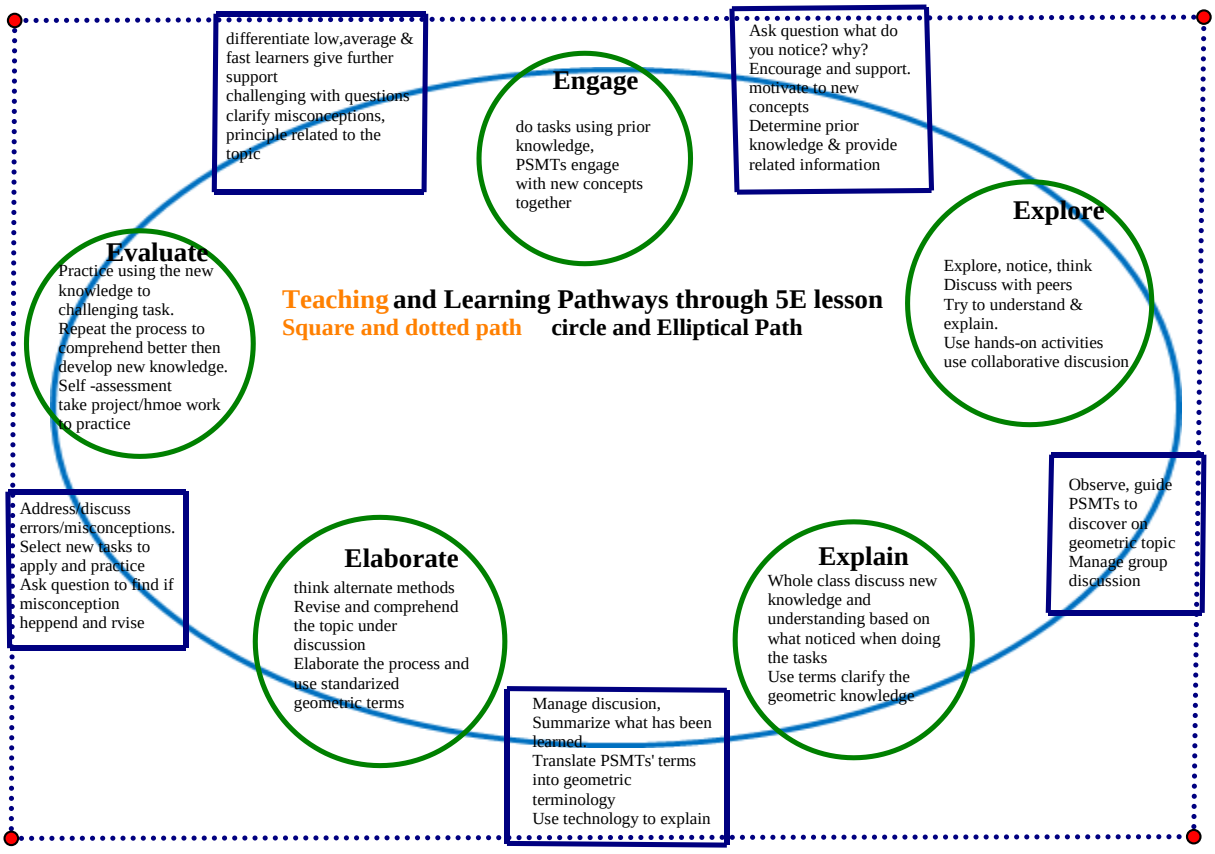


Figure 2.4 Activities of teaching and learning in 5E lesson plan

The figure illustrates a typical student-centered 5E lesson plan. The ellipse shows the learning pathway or sequence of student activities, and these activities are described in

the circles. The rectangle shows the teacher's role in the lesson, and the interactions with the PSMTs are described in the square boxes.

The relevance of the 5E inquiry paradigm to mathematics instruction has been researched over the past ten years, and the findings are encouraging. Tuna and Kacar (2013) examined the effect of using the 5E inquiry model for teaching trigonometry and found students' achievement increased following the application of 5E model. Similarly, in Omotayo and Adeleke's (2017) study, students' overall performance and interest in mathematics advanced due to utilizing the 5E inquiry model. A recent study's findings by Nopasari, Ikhsan, and Johar (2020) also showed that teaching mathematics using the 5E approach can improve students' understanding of mathematical concepts. Furthermore, other studies by Runisah, Herman, and Dahlan (2016) and Walia (2012) indicate the positive effect of the 5E model on students' creative thinking skills in mathematics education.

According to the literature, the 5E lesson inquiry model has been shown to improve learners' academic performance in mathematics, and studies have suggested that it be included in pre-service teacher education courses (Okafor, 2017; Tuna & Kacar, 2013). For instance, the 5E lesson plan model improved learners' mathematical critical thinking skills (Runisah, Herman, & Dahlan, 2017), retention (Alshehri, 2016), and conceptual understanding of mathematics topics such as trigonometry (Tuna & Kacar, 2013), geometry (Palut, 2009), and quadrilaterals (Wafirah, Waluya, & Suyitno, 2016). In summary, the 5E inquiry model developed for science education can also be effectively

applied to teaching mathematics through inquiry. Thus, the 5E lesson plan has been used in this research for the teaching and learning of geometry in CTEs.

2.6 Learning of Geometry

Geometry is a branch of mathematics that has many applications in engineering, economics, science, and technology, as well as other mathematical topics like calculus and algebra. An understanding of geometry permits the modeling of many real-world situations and the solution of many problems for which other mathematical concepts are insufficient (Al-Shehri, Al-Zoubi, & Rahman, 2011; Fyfe, McNeil, & Borjas, 2015; Gagatsis, Sriraman, Elia, & Modestou, 2006). Similarly, Jones (2002) indicated that geometry is an important field of mathematics to teach for solving problems in other mathematical content and real-world problems. Generally, Jones listed the following purposes for teaching and learning geometry:

- To encourage spatial awareness, visualization, and geometrical intuition.
- To give students practice with geometrical concepts in both two and three dimensions;
- To gain knowledge and understanding of geometrical properties, theorems, and its applications;
- To encourage the development and application of conjectures, logical reasoning, and proofs;
- To gain the ability to use theorems and the attributes of geometric forms in problem-solving;
- To promote the use of geometric knowledge in modeling and problem-solving for real-world scenarios;
- To increase the understanding of geometry in society and its modern applications;

- To enhance practical integrated technology (IT) skills in scenarios using geometry; and
- To encourage a positive attitude of mathematics.

There are many different types of geometric concepts in mathematics. These are hyperbolic geometry, projective geometry, Euclidean geometry, non-Euclidean geometry, analytic geometry, plane geometry, and vector geometry. In the current school mathematics curriculum of Ethiopia, most geometry topics covered in elementary and secondary school as well as college teacher education fall under Euclidean geometry.

However, PSMTs encounter difficulties when learning geometry (French, 2004; Lo, Cox, & Mingus, 2006; Fujita & Jones, 2002; MoE, 2017; Michael, 2015). Furthermore, as researchers Dündar and Gündüz (2017) discovered, PSMTs had difficulty mitigating various geometric and mathematical concepts in general. However, these difficulties are attributed to the currently-applied lecture method, which is a teaching method that focuses on facts, concepts, and procedures as knowledge to be memorized (MoE, 2012a; 2015; 2017). Further, Luneta (2015) attributes the errors and difficulties experienced by learners to the conventional way of teaching. According to Gourgey (2001), demonstration and rote teaching approach, which is common in traditional mathematics classrooms, encourages memorization rather than understanding.

Meaningful mathematical learning has occurred if cognitive relationships between units of knowledge are created or recognized (Hiebert & Lefevre, 1986). Meanwhile, rote learning, however, is defined as learning where cognitive relationships between units of knowledge are not established. According to Anderson and Krathwohl (2001), meaningful learning takes place when learners make an effort to link new ideas and

schemas to pre-existing ideas and propositions in their cognitive framework. Learning that is meaningful involves the formation of new knowledge as learners work to interpret their past experiences, which occurs progressively through an active interaction process.

A focus on meaningful learning is consistent with the view of learning as knowledge construction, in which learners seek to make sense of their experiences. Interactions between students and students-teachers are required for effective learning. Additionally, for learning to be successful, learners must take responsibility for their own learning through self-reflection and self-evaluation. In these terms, various constructivist pedagogical strategies that will support the learning of abstract mathematical concepts were devised. However, there is no one-size-fits-all approach to learning; different models, including group interaction and technology integration, all contribute to effective learning among students (Vallori, 2014).

In a constructivist learning approach, students actively process their thoughts, paying attention to related information as it comes in and cognitively structuring that knowledge into a coherent picture (Mayer, 1999). The constructivist learning approach, such as discovery learning, inquiry learning, and cooperative learning, encourages learners to engage in critical thinking, problem solving, and decision-making. Nowadays, with the availability of advanced technological tools, the integration of digital technologies into mathematics education has become common (Alacaci & McDonald, 2012; Brahier, 2020). This focuses the learning contexts on the learners and allows them to be more autonomous in their education process. Teaching in a technology-integrated context has been proven to increase and deepen PSMTs' learning by supporting instructional goals. Educators, teachers, and students can use technology to explain, explore, and model

mathematical concepts and their relationships (Hohenwarter & Jones, 2007). It offers both teachers and students effective teaching and learning settings and visual representations.

Furthermore, Martinovic and Manizade (2014) stated that technology-based geometric activities have promoted pre-service teachers' mathematical reasoning via carefully structured activities and engaged technological skills that enabled them to develop and evaluate geometric conjectures. In addition, technology might enhance PSMTs' learning outcomes if utilized properly. According to Christensson and Sjöström (2014), technologies can increase educational output by enhancing PSMTs' motivation and engagement as well as the acquisition of mathematical concepts. Therefore, finding alternate teaching and learning techniques that use technology is critically important.

2.6.1 Studies on Understanding and Learning of Geometry

One area of teacher education that still requires improvement is geometry instruction. French (2004) asserts that teaching and learning geometry is more challenging than teaching algebra because solving problems in geometry relies more on intuition and less on procedural methods. For instance, since teachers cannot draw an infinite line on the chalkboard to demonstrate some geometric concepts, it is logical that PSMTs would mistake lines for segments.

A gap in learners' prerequisite understanding of geometrical concepts causes learners to not grasp further concepts. Since mathematical and geometrical concepts are interrelated and linked to one another and build on one another (NTCM, 2000). As mentioned previously, solid understanding requires making connections or establishing relationships

between existing knowledge and new knowledge (Hiebert & Carpenter, 1992). At this point, it is obvious that learners' previously learned concepts and skills and the relations among them play a crucial role in making sense of mathematics.

Thus, teachers must assess the PSMTs' prior knowledge in order to prevent misconceptions. As a result, concept maps, prediction-observation-explanation, discussions about concepts and occurrences, discussions about ideas and associations, and diagnostic tests are some strategies that may be used to identify PSMTs' misconceptions (Vosniadou, 2020). In addition, teachers must use effective teaching techniques to minimize misconceptions and foster conceptual development (Kurniawan et al., 2020).

Mathematical and geometric misconceptions occur when mathematical rules and concepts are misused. According to Morapeli (2017), mathematical misconceptions happen when: (1) concepts are taught to PSMTs in the form of concept names and definitions; (2) PSMTs create their own conceptual images using cognitive structures connected to the concepts that the teacher taught them; (3) new knowledge cannot be anchored to the existing knowledge or concept image; and (4) concept images are poorly constructed as a result of poor conceptual understanding.

Geometry is one of the most challenging areas of mathematics to teach and learn (Adolphus, 2011). Many PSMTs fail to develop an adequate understanding of geometrical concepts, reasoning, and problem-solving skills (Khoo & Clements, 2001; Kasa, 2015). In addition, the results of some research (Jones, Mooney, & Harries, 2002; Mooney, Fletcher, & Jones, 2003) showed British primary school teachers performed

poorly on basic-level questions on geometric knowledge and generally lacked confidence for teaching geometry.

The abstract nature of mathematical and geometric concepts, instructional approaches, tools used, and language complexity are the problems identified that make geometry content challenging. The abstractness of mathematics concepts and structures is the main difficulty for learners' conceptual and procedural understanding of mathematical concepts. Many concepts in geometry require learners to visually perceive concepts and identify their properties by comparing them with their previous experiences involving similar objects. Thus, visual activities are recommended to help learners understand geometric concepts. In addition, geometry topics are undervalued in the curriculum in many countries (French, 2004).

2.6.2 Conceptual and Procedural Understanding in Geometry

Several learning and instruction theories assign knowledge a central role in cognitive development and learning. A large body of research and theory in cognitive science favors the idea that a learner's capacity for representing and connecting pieces of knowledge is a key factor in developing deep understanding (Kilpatrick Swafford & Findell, 2001). Thus, learners are encouraged to make connections between mathematical ideas, facts, procedures, and relationships in order to gain a deeper understanding of the principle of mathematics. Schneider and Stern (2006) discovered that at least two types of information, conceptual knowledge and procedural knowledge, have an impact on one's understanding and behavior.

In perspective of mathematics education, conceptual and procedural knowledge are considered crucial for a learner to understand mathematics meaningfully and become

competent mathematically (Hiebert & Carpenter, 1992; Rittle-Johnson & Alibali, 1999). Thus, in order to address mathematical problems and contents, PSMTs must have both conceptual and procedural knowledge (Sangguroa & Surifa, 2019).

However, mathematical concepts or fact is also fully understood if it is connected to pre-existing conceptual networks. Conceptual knowledge is the understanding of geometric concepts and theoretical principles, while procedural knowledge is the knowledge of algorithms, rules, and step-by-step procedures used during the process of solving geometric problems (Haapasalo & Kadijevich, 2000). According to Star (2005), "conceptual knowledge" includes both what is understood and how concepts can be known. It is a set of mathematical concepts with an understanding of "why" and "when" to use mathematical algorithms. According to Slesnick (1982), learners use their conceptual understanding of mathematical concepts when they recognize and apply rules, apply definitions and facts, and compare and contrast related concepts of mathematics and geometry.

On the other hand, Kilpatrick et al. (2001) express conceptual understanding as the comprehension of mathematical concepts, operations, and relations. Thus, learners with conceptual understanding know more than isolated facts and methods. Because facts and methods learned with understanding are connected, easier to remember and use, and can be reconstructed. Therefore, conceptual understanding is an integrated and practical knowledge of mathematical concepts (Kilpatrick et al., 2001). Conceptual understanding helps students avoid errors of particular magnitude. In addition, a conceptual knowledge network develops as new knowledge is linked to prior knowledge (Hiebert & Wearne,

1986). Procedural knowledge specifies not only about what is known (knowledge of procedures) but also the way that procedures (algorithms) can be used.

There are several studies that demonstrated the ability of many learners to handle procedural difficulties, but they were unable to grasp the fundamental concepts (Sangguroa & Surifa, 2019), since conceptual knowledge needs a sophisticated thinking process to generate meaningful learning. Knowing the explanation behind a concept or a method is not vital to procedural knowledge; what matters is that you know how to apply it. However, understanding geometrical ideas is crucial for conceptual knowledge (Rittle-Johnson & Schneider, 2015). In this aspect, the majority of students have a lack of conceptual knowledge. This idea is supported by the fact that many PSMTs can solve problems that involve procedures easily but are challenged by questions that involve concepts (Cracolice et al., 2008). This indicates the observation that PSMTs can recognize and apply formulas and procedures even if they lack understanding of the key concepts. Nonetheless, knowing mathematics conceptually is frequently inextricably linked to knowing mathematics procedurally.

According to Areaya and Sidelil (2012), conceptual and procedural knowledge are assumed to be distinct but related. However, no fixed order whether learners acquire conceptual or procedural knowledge first when learning mathematical concepts (Rittle-Johnson & Alibali, 1999; Schneider, Rittle-Johnson, & Star, 2011; Baroody, 2003). Some research highlights that mathematical procedures and skills precede mathematical concepts (Baroody, 2003). In this case, PSMTs learn procedures and then grasp concepts through repeated practice (imitating, using trial-and-error), abstraction, and reflection (Rittle-Johnson et al., 2001; Schneider & Stern, 2010).

Several studies on mathematics education, on the other hand, place a premium on conceptual knowledge across multiple mathematical domains. Through applying conceptual knowledge, procedures are generated and used to solve mathematical problems (Rittle-Johnson et al., 2001). However, investigations have shown that these orders were not always exact (Yurniwati, 2018). To this effect, mathematical procedure linked with conceptual knowledge enhances the use of procedures effectively; similarly, connecting concepts with procedure also supports the advancement of conceptual knowledge.

Nevertheless, most literature states that learning procedural knowledge first does not promise the appropriate acquisition of conceptual knowledge. This situation would have been taken as one of the reasons why learning advanced mathematics has become challenging. Although knowing the conceptual underpinnings of procedures is not necessary for the proper execution of such processes, it is considered that doing so will increase knowledge (Baroody, 2003; Star, 2005). However, obtaining conceptual knowledge is necessary for acquiring procedural knowledge. In other words, obtaining conceptual knowledge almost always guarantees the acquisition of procedural knowledge (Kaya, Ministry, & Keşan, 2012). Thus, researchers share the opinion that a concept-based approach has to be stimulated more in the teaching and learning of mathematical contents.

On the other hand, there is no clear distinction between conceptual and procedural knowledge, which adds to the difficulty of those kinds of knowledge. As a result, not all knowledge is best categorized as conceptual or procedural. Some knowledge appears to be neither, while others appear to be a little of both. In this case, however, both types of

knowledge developed when a) learners had both types of knowledge partially, b) having greater knowledge of one type was correlated with greater knowledge of the other, and c) advances in one type of knowledge could lead to improvements in the other type of knowledge (Rittle-Johnson et al., 2001).

Accordingly, in the iterative model in which the relationship between the two types of knowledge is assumed to be bi-directional and a causal approach to teaching mathematical concepts is emphasized (Rittle-Johnson et al., 2001; Haaapasalo & Kadijevich, 2000), In this case, the order of instruction and the source of instruction are both flexible. Hence, conceptual knowledge is the source of procedural knowledge and vice versa, and the two are inseparable entities that are mutually enhanced in learning mathematical concepts depending on learners' prior experiences. For instance, Awang and Zakaria (2012), Mahir (2009), and Zulnaidi and Zakaria (2012) have concluded that the iterative approach of mathematics lessons is significant from the elementary to higher levels of education. The iterative approach put forth by Rittle-Johnson et al. (2001), which contends that one sort of knowledge does not often evolve before the other, has my backing. One type of knowledge might be better developed at a particular point in time, but this does not imply that the other type is completely overlooked.

The content of geometry is appropriate both for the development of lower-level mathematical thinking (i.e., the recognition of shapes) and for higher-order thinking (i.e., the procedural, conceptual, and reasoning skills). In this approach, procedural problems are viewed as being less complicated than conceptual ones. A procedural problem is one in which all the information required to solve it is present, the techniques for solving it are well known, and the objective is clearly stated (Surif et al., 2014). When it comes to

assessing whether PSMTs have understood particular mathematical concepts, open-ended questions are particularly helpful. PSMTs will generally have to use their understanding of the key ideas that go along with the mathematical issue to answer it. According to some academics, problem solving is essential for procedural, conceptual, and open-ended issues (Randles & Overton, 2015).

Conceptual knowledge is defined as the ability to demonstrate, interpret, and relate the veracity of mathematical concepts correctly to a variety of problem-solving situations (Star & Stylianides, 2013). PSMTs use their conceptual knowledge to identify what and when to use definitions, rules, and procedures, and to distinguish associated concepts and evaluate results (Schneider & Stern, 2005), while procedural knowledge can be measured by checking the procedure of solving problems. According to Star and Stylianides (2013), a concept map, an interview about concepts, an open-ended test, and diagnostic tests can be used to evaluate conceptual understanding. The traditional multiple-choice question (MCQ) item evaluates simple subject knowledge (Bayrak, 2013). Open-ended questions, on the other hand, are preferred because they reveal not only the students' final answer but also how they arrived at it.

Open-ended questions, on the other hand, are preferred because they reveal not only the students' final answer but also how they arrived at it. Furthermore, Bayrak (2013) observes that normal MCQ items tend to overestimate PSMTs’ knowledge as they may not be able to adequately explain their choices. They should therefore be made to justify their choices in the MCQ items so that more information can be obtained on the PSMTs’ knowledge. This is aimed at promoting higher-order thinking and reasoning skills. Tamir

(1989) and Kanli (2014) report the use of justifications when answering MCQ items to be a sensitive and effective way of assessing meaningful learning among PSMTs and address the extent of the limitations of traditional MCQ items. Two-tier diagnostic tests consist of at least two sections. The first section aims to diagnose how the individual interprets discipline knowledge. In the second section, students are asked to state the reason(s) for their answer in the first section (Kanli, 2015).

Two-tier diagnostic tests consist of at least two sections. The first section aims to diagnose how the individual interprets content knowledge. In the second section, PSMTs are asked to state the reason(s) for their answer in the first section (Kanli, 2015). Each multiple-choice item's first tier consists of content questions with two to four options, one of which is the correct answer, and the others are distractors. The second tier of each item includes a list of four possible explanations for the first part's answer. The reasons comprise the selected correct answer together with the identified students' conceptions and/or misconceptions. The reasons are from the PSMTs' responses given to each open-ended question or information gathered from the literature or via interview (Adadan & Savasci, 2012). Cheong, Treagust, Kyeleve, and Hiew (2010) state that with the use of an instrument before a course on a subject, a teacher will be able to target misconceptions in an attempt to change students' misunderstandings and help them consider expertly acceptable conceptions. If teachers use the diagnostic tests for summative assessment, they will see the impact of their instructional strategies in a positive or negative way, which will serve as feedback for later instructional planning.

2.6.3 Reasoning and Proof in Geometry

Reasoning skills are an important component of education, especially as a means for developing concepts for understanding mathematical ideas (NCTM, 2000). It is the ability to think coherently and logically and draw inferences or conclusions from facts known, assumed, or accepted to be true. According to Umay (2003), reasoning is a skill or ability exhibited when thinking at an advanced level or during problem-solving processes, and it is high-order mathematical thinking.

The concept of mathematical reasoning has been widely researched in mathematics education (Sumpter, 2013). Mathematical reasoning is, according to English (2013), reasoning with structures that emerge from concrete experiences as we interact with the environment and transform such experiences into models of abstract thought. The main value of mathematical reasoning lies in explaining why a particular statement is true (Carpenter, Franke, & Levi, 2003). From these views, it can be argued that mathematical reasoning is the logical communication of the developed mathematical constructs or thoughts constructed through learning experiences.

Mathematical reasoning is also a procedure executed to obtain a conclusion based on mathematical evidence and related facts and foundations that have been supposed to be true (Hasanah et al., 2019; Kadioglu et al., 2020). Morsanyi, Prado, and Richard (2018) define mathematics as a multifaceted system of complex relationships that involve and invoke reasoning. This indicates that mathematical reasoning has a cognitive structure in terms of how learners attain the ability to reason. PSMTs' overall ability to reason about mathematical ideas and make justifications for why a mathematical concept makes sense or why a procedure should be used is a powerful and necessary part of learning

mathematics. Thus, mathematical reasoning is the ability to think reasonably and logically and draw inferences or conclusions from known or implicit mathematical facts, axioms, or pre-defined concepts.

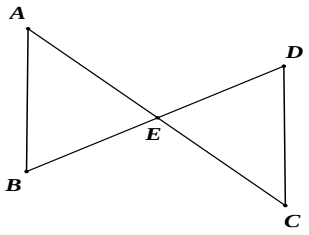
The PSMTs' ability for conjecture, justification, and reasoning is correlated with their regular mathematical activity. According to NCTM (2000), reasoning is important for learning mathematics because it encourages the creation and discovery of concepts, the interpretation of outcomes, and the application of hypotheses. Because of this, mathematical reasoning is a crucial component of PSMTs' mathematical experience. PSMTs also need to collaborate with one another in order to develop and test their conjectures, as well as to communicate and understand various interpretations. Toole (2001) has emphasized that there is a definite relationship between reasoning ability and mathematical accomplishment, with those with higher reasoning abilities also having superior problem-solving and communication skills. Because of this, PSMTs who struggle with thinking will learn to view mathematics as a set of strict rules, formulae, and diagrams.

Several researchers argue that a PSMT's performance on geometric tasks is the best indicator of a PSMT's ability to reason mathematically. Geometric reasoning requires mathematical reasoning. In other words, when good mathematical reasoning develops, so does geometric reasoning. Stefanowicz (2014) claims that a proof is a complex chain of logical arguments that explains why a certain assertion is true through the use of statements that relate to one another.

The skill of formulating, illustrating, and justifying solutions to geometrical issues is known as geometry reasoning (Kilpatrick et al., 2001). Geometric reasoning is the study of form and space using formal conceptual frameworks. PSMTs' deficiency of understanding and their weak ability to reason and prove is evident in the field of geometry, where they encounter difficulties in applying theories linking geometric concepts with different definitions and relationships, inferring new relationships, and drawing new conclusions (Adolphus, 2011). To describe spatial connections inside and across geometric forms, these systems make use of concepts like angle measure, length measure, congruence, and parallelism (Battista, 2007). Geometric reasoning may also be viewed as a dialectical interaction between a figure and the geometrical concepts it expresses. In this case, geometric reasoning is considered the ability to give logical explanations, arguments, verifications, or proofs to arrive at a conclusive solution to geometrical problems.

Although proofs are decisive to the teaching and learning of geometry, PSMTs often face difficulties with geometric proof (Battista, 2007). Moreover, Kunimune (2000) also added that PSMTs have difficulties for understanding proof in geometry. Moreover, Stevens (2012) warns that learners may struggle with the complex thinking processes needed to build proofs. Two-column, paragraph, and flowchart proofs are the three most popular geometric proof techniques used to measure geometric reasoning abilities. Each of them offers a unique strategy for presenting evidence and information in a way that makes geometric ideas easy to understand.

Table 2.4 The two-column proof method



Given: E is the midpoint of AC and BD
Proof: $\Delta ABC \equiv \Delta CDE$

Statement	Reasons
1. E is the midpoint of AC and BD	1. Given
2. $AE \equiv CE, BD \equiv DB$	2. Midpoint of a segment divides into two congruent segments.
3. $< BEA \equiv < CED$	3. Vertical angles are congruent.
4. $\Delta ABC \equiv \Delta CDE$	4. SAS theorem. If two sides and included angle of a triangle are congruent to the corresponding two sides and included angle of a triangle, then the triangles are congruent.

To provide a further justification for each action, the second column employs deductive reasoning. These explanations are either theorems, definitions, or postulates (assumptions derived from mathematical reasoning) or rules demonstrated through formulas (assumptions derived from mathematical reasoning). It is typical practice in school geometry classrooms to employ two-column proofs since they provide a straightforward means of illustrating each step. PSMTs cannot thus succeed in a proof-intensive geometry course without the required reasoning abilities.

According to Greenstein (2011), "geometric reasoning" is the application of geometric properties and relationships in the problem-solving process. In this case, the conceptual understanding of figures and their properties and relationships to geometric concepts is significant in geometric reasoning. Such properties can be provided or determined by inspecting the diagram or by logical deduction. Thus, geometric reasoning can help learners organize their knowledge in ways that enhance their conceptual development in geometry.

Similarly, Evans (2007) indicated that the development of geometric reasoning enables PSMTs to work with geometric proof. Brown et al. (2004) illustrate that geometric reasoning is important in developing learners' knowledge and understanding of and use of geometrical properties. Learners can use geometric properties to communicate why a particular proof is valid. Duval (2017) also viewed reasoning as a process that provides depth to the existing knowledge of geometric understanding. Moreover, according to Lithner (2000), learners' reasoning skills are closely related to the depth of their conceptual knowledge and corresponding associative skills. Thus, reasoning involves unproven and meaningless reasoning processes resulting from insufficient conceptual and procedural knowledge of geometric concepts.

However, approaches to teaching geometry have been shown to highly affect learners' reasoning abilities (Nassar, 2010). Thus, teaching mathematics and geometry must focus on encouraging learners to express their reasoning behind the concepts (Brandt & Tatsis, 2009). Traditional teaching methods that Gunhan (2014) views as characterized by teachers giving definitions and making no use of concrete materials or practical ways to explain mathematical concepts hinder learners' geometric learning outcomes. For effective geometry instruction, a closer look at the geometry learning process may be necessary. Three theoretical frameworks may give insights into understanding the geometry learning of PSMTs. Duval (1998) argues that geometric thinking combines three cognitive processes, which may be visualization, construction, and reasoning. In order for PSMTs to draw inferences about geometric concepts, they must be able to recognize geometric figures in various dimensions; hence, visualization is essential for geometry learning. Visualization and reasoning are the essential mental skills required for

mathematics, and these cognitive processes are interconnected, promoting students' success in geometry (Duval, 1998).

The proof forms properties and relations within and among figures is an essential part of the study of geometry which begins from some known fact and proceeds a step-by-step where each step is deduced from the result of the previous, until the unknown fact is justified. Each of the sequential statements contains a proposition supported by valid reasons (Serra, 1997). De Villiers (2004) viewed a proof as a formal written argument of the complete thinking procedures that are used to reach a valid conclusion; the steps of these procedures are supported by theorems, postulates, or definitions verifying the validity of each step and explaining why these steps are achievable. The inclusion of the study of formal proof as part of the study of geometry provides a means for the development of deductive reasoning skills (Mudaly & de Villiers, 2004).

In geometric proof problems, PSMTs find it difficult to make the verbal-symbolic or graphic-visual connection (Alcock & Weber, 2010; Zazkis, Weber, & Mejía-Ramos, 2016). Therefore, it can be argued that how teachers teach influences how learners learn. Researchers (Jones, Fujita, & Ding, 2006; Jones & Herbst, 2012) suggest that to promote geometrical reasoning, teachers are supposed to use various instructional techniques and strategies.

Furthermore, visualization and reasoning skills can be improved through instruction methods (Jones, 2000). According to NCTM (2000) and Smith (2010), the implementation of digital technology into the mathematics classroom helps promote reasoning skills and geometric understanding. The use of manipulatives and technology

in teaching geometry and mathematics is also suggested (Dorier, Gutiérrez, & Strässer, 2003; Sriraman & English, 2005; Aksoy, Bayazit, & Soybaş, 2010) as useful in facilitating PSMTs' progression to higher levels of geometric thinking and favoring the advance of deductive reasoning.

In technology-integrated guided inquiry, teachers help PSMTs use their own language for constructing knowledge by encouraging active listening and questioning. While exploring and investigating a problem, teachers guide students to talk through their thinking, which supports the development of PSMTs' reasoning, mathematical thinking and problem solving, and math communication skills. Teachers can ask questions that provoke thinking and provide their own ideas for possible answers that stimulate further questions. For example, a teacher can show students visual area problems and guide them to identify the patterns and relationships between the circumference and diameter of spherical shapes to foster geometric reasoning.

2.6.4 Attitudes toward Learning Plane Geometry

While learning geometry and mathematics is mostly a cognitive process, the affective domain of PSMTs (such as attitudes, emotions, and beliefs) is equally important. Thus, to understand and account for the variation in PSMTs' performance in mathematics, the attitudes of PSMTs must be taken into account (Mohamed & Waheed, 2011; Nicolaidou & Philippou, 2003). Di Martino and Zan (2010) and Recbe (2018) all state that attitude is the learned predisposition to react positively or unfavorably to a certain item, concept, or person. Moreover, attitude refers to how one feels, thinks, or has been trained to see a person or thing (Khan & Ali, 2012). In general, PSMTS' attitude toward geometry is

reflected in their comprehensive evaluation of the subject. However, no exact meaning for "attitude" can be found.

Several researchers believe that the attitude of PSMTs is an important factor in determining whether learners perform well in mathematics (Mohamed & Waheed, 2011; Mata, Monteiro, & Peixoto, 2012; Ngussa & Mbuti, 2017; Hannula, 2002). PSMTs' attitudes toward geometry are formed through their various experiences with geometry (Davadas & Lay, 2017). As accumulated experiences with a certain object or subject influence PSMTs' psychological state, they develop a positive or negative attitude toward such experiences. Moreover, PSMTs tend to behave and think in a certain way that matches their attitude.

For instance, PSMTs with a positive attitude toward geometry tend to enjoy the subject, understand its value or usefulness, and have confidence in it; thus, they are likely to put effort into and prioritize the study of geometry (Kiwanuka, Van Damme, Van den Noortgate, & Reynolds, 2020; Mullis, Martin, Foy, Kelly, & Fishbein, 2020), which could lead to high performance. However, PSMTs with a negative attitude toward mathematics tend to hate geometry, think of it as a useless subject, feel scared to engage in it, and tend to avoid mathematics-related activities (Guo et al., 2015).

Attitudes were also found to be shaped by the learning environments one experiences (Graham & Fennel, 2001), and teachers who understand their PSMTs' attitudes are better able to create learning environments conducive to positive attitudes and better achievement. This concern with PSMTs' attitudes towards mathematics has led educators to develop instructional techniques and create a positive learning environment, leading to

greater motivation to learn and more positive attitudes towards the subject (Graham & Fennel, 2001).

The NCTM (2000) reports that PSMTs' understandings of geometry, their confidence in themselves as learners of geometry, and their motivations to learn geometry are all shaped by the teaching methods used in the classroom. The teachers' teaching method significantly influences PSMTs' attitudes (Mensah et al., 2013). Teachers can do many things to facilitate classroom learning to alleviate PSMTs' engagement level and confidence in learning mathematics (Attard, 2012; Kele & Sharma, 2014). By adopting meaningful activities that are integrated into actual circumstances, this may be accomplished (Kacerja, 2012).

Researchers such as Di Martino and Zan (2010) have pointed out that attitude are still a confusing construct. Similarly, Recber, Isiksal and Koç (2018) highlighted the inconsistency of research findings in mathematics regarding the relationship between attitude toward mathematics and achievement. While some researchers discovered a statistically significant relationship between math attitude and math achievement (Recber, et al., 2018), others discovered a statistically significant but weak relationship.

Researchers have suggested the presence of numerous dimensions of a multidimensional attitude toward geometry. For instance, Syyeda (2016) asserts that attitudes are multidimensional and considers affect, cognition, and behavior as three factors of attitude. Emotions, visions, and beliefs all contribute to the attitude factor. Davadas and Lay (2017) suggested motivation, enjoyment, self-confidence, and value as components of an attitude toward mathematics. Likewise, Di Martino and Zan (2011) suggested

emotional disposition, vision of mathematics, and perceived competence as components of attitude. Similarly, in the Trends in International Mathematics and Science Study (TIMSS), researchers examined students' attitudes toward mathematics through three components (Mullis et al., 2020): enjoyment of mathematics, value of mathematics, and confidence in mathematics.

In summary, while different researchers have used different terms, they have commonly measured attitude toward mathematics using four components: self-confidence in geometry, value in geometry, enjoyment in geometry, and motivation in geometry (Davadas & Lay, 2017; Tapia & Marsh, 2004).

2.6.4.1 Self-Confidence in Learning of Plane Geometry

The belief in oneself and one's abilities is referred to as "self-confidence." According to Kanza (2016) and Perkins (2018), it is a learner's belief in successfully completing a task. It is PSMTs' beliefs and capability to learn and achieve good results in geometry (Adelson & McCoach, 2011; Lee & Kung, 2018). Tapia and Marsh (2004) also looked at PSMTs' self-confidence and self-concept of their performance in geometry learning.

The results of Hannula, Maijala, and Pehkonen (2004) indicate that self-confidence is the main factor that impacts PSMTs' learning of geometry, which in turn affects their performance in geometry. According to Van der Bergh (2013), PSMTs who have high levels of self-confidence are more likely to overcome their fear of failing by having faith in their ability to learn geometry, while PSMTs with low self-confidence do not believe in themselves, hence they avoid taking geometry tasks (Adelson & McCoach, 2011).

Thus, it is a significant factor in geometry since it makes PSMTs sure of themselves when solving non-routine problems and learning new concepts.

On the other hand, self-confidence also affects PSMTs' enjoyment and interest in geometry, thereby getting them more involved in geometry tasks. Akbari and Sahibzada (2020) reported that self-confidence had a significant effect on the learning process among learners, including the sharing of opinions, developing relations with their peers as well as teachers, seeking goals, controlling anxiety, and developing interest in lessons to learn more effectively. Raniere et al. (2020) reported that the teaching strategies used, and teachers' behavior had a significant effect on PSMT's self-confidence in learning. The authors suggest that PSMTs' self-confidence might be improved by creating a classroom environment rich with strong ties to everyday mathematical applications where PSMTs can learn to transfer their classroom learning to the real world. Thus, constructivist teaching strategies like inquiry-based instruction and group work fostered PSMTs' problem-solving skills to a greater extent than traditional teaching strategies such as direct instruction (Ziegler & Yan, 2001).

2.6.4.2 Enjoyment in Learning of Plane Geometry

PSMTs enjoy doing and learning geometry, which is another component of attitude (Kupari & Nissinen, 2013). It is a sub-domain used to measure the degree to which PSMTs enjoy working with geometry and geometry classes in general (Tapia & Marsh, 2004). The enjoyment of PSMTs in geometry learning is an important factor because it might affect how ready PSMTs are to engage in geometric activities that will help them get better at the subject. According to Stipek (2002), the enjoyment of PSMTs in geometry improves the learners' engagement.

When PSMTs did not enjoy geometry class, they became nervous, frustrated, and, as a result, had a limited understanding of geometric concepts and avoided learning geometry (Wadlington & Wadlington, 2008). When PSMTs were engaged in entertaining, interesting, and relevant problem-solving activities, they would find more enjoyment in their learning as it became more purposeful (Altieri, 2009). In addition, Curtis (2006) found that instructional strategies like group activities, classroom discussions, and problem-based learning help improve mathematical confidence and increase students' enjoyment of college algebra and their general attitude towards mathematics.

2.6.4.3 Usefulness/Value of Learning Plane Geometry

The value or usefulness of geometry measures PSMTs' beliefs in the usefulness, relevance, and worth of mathematics and geometry in their lives now and in the future (Tapia & Marsh, 2004). According to Adelson and McCoach (2011), perceived usefulness is determined by how PSMTs view geometry's role in daily life and its potential future use. Syyeda (2016) emphasized that PSMTs will be encouraged to study, practice, and learn mathematics if they recognize its relevance in everyday life. It is also a positive predictor of achievement in the subject matter. Kadijevich (2006) posits that attitudes toward geometry are frequently saturated by either the usefulness of mathematics or by self-confidence in mathematics. Value gives someone the will and determination to work in mathematics and geometry because PSMTs are more likely to participate in mathematical assignments they value (Seah, 2013).

Awareness of usefulness can boost self-confidence, which leads to learning and improved performance. PSMTs engage in activities if they feel motivated, either extrinsically or intrinsically (Ryan & Deci, 2000). According to Ryan and Deci (2000), intrinsic

motivation is generated by the interest or pleasure of an activity in itself, and extrinsic motivation is related to the value of the usefulness of the activity. Thus, the usefulness or value of mathematical knowledge in general and geometric concepts is achieved when using hands-on activities and we see geometry as relevant to real-life activities (Hernandez-Martinez & Vos, 2018). Hands-on activities, on the other hand, that aid in the applications of geometric content in reality can increase the value of mathematics' usefulness beyond standard applications (Niss, Blum & Galbraith, 2007).

According to Seah (2013), the values/usefulness learners have for learning geometry will improve their conceptual understanding, thus maximizing their learning. Therefore; different teaching methods must be employed to change the learners' beliefs about learning geometry.

2.6.4.4 Motivation in Learning of Plane Geometry

Motivation is another factor that is used to measure PSMTs' interest in learning geometry and the desire to pursue further studies in geometry (Tapia & Marsh, 2004) and has an essential role in mathematics education (Hannula, 2006; Walter & Hart, 2009). Motivation is a method used to inspire PSMTs' interests towards the subject, and this contributed to the observed trend of positive attitudes toward mathematics (Singh, Granville, & Dika, 2002). Numerous studies have shown that mathematics and geometry performance is strongly related to PSMTs' motivation towards geometry learning (Kim, Park, & Cozart, 2014; Mullis et al., 2012).

Motivation is often thought of as either intrinsic or extrinsic, and it establishes a continuum of concepts rather than discrete categories. Intrinsic motivation shows a

preference for inherent satisfactions over some separate outcome (Atnafu, 2012); while extrinsic motivation, on the other hand, is carried out in order to obtain something from others. Thus, intrinsic and extrinsic incentives may both exist at the same time, which makes learning much easier.

According to Rowel and Hong (2013), PSMTs who were motivated valued and enjoyed their learning. Similarly, intrinsically motivated PSMTs are often on task, monitor their own progress, and engage in challenging activities. Thus, motivation and achievement are linked (Akomolafe, Agunmakin, & Fassoto, 2013; Rowell & Hong, 2013).

The research observed a variety of factors that contributed to PSMTs' motivation to participate in mathematics. The factors that influenced favorable dispositions toward learning mathematics included classroom environment and teacher-student interaction (Mueller, Yankelewitz, & Maher, 2011). Therefore, well-designed content, activities, and materials are needed in geometry education to increase PSMTs' motivation (Cakir et al., 2018).

Technology-based approaches are making mathematics instruction increasingly engaging in the real world (Kuhn & Dempsey, 2011), which allows PSMTs to practice skills in meaningful contexts (Berry & Ritz, 2004). Technology not only enhanced students' communication and collaboration but also improved the quality of instructional activities (Suh, 2011).

Regarding attitude assessment, the Attitude towards Mathematics Inventory (ATMI) measures a student's attitude using six factors: confidence, anxiety, value, enjoyment,

motivation, and parent/teacher expectations. It contains 49 items. The items were constructed using a 5-point Likert scale, with 1 = strongly disagree to 5 = strongly agree.

This initial 49-item instrument was reduced to 40 items after deleting these nine items, whose item-to-total correlation was less than 0.50. In addition, two of the original factors (confidence and anxiety) were combined, and the parent/teacher expectations factor was eliminated. Furthermore, Tapia and Marsh (2005) revalidated the ATMI with 40 items to measure attitude at the college level with students enrolled in mathematics classes and determined reliable alpha values of 0.96, 0.93, 0.88, and 0.87 for self-confidence, value, enjoyment, and motivation, respectively. Hence, in order to assess PSMTs' attitudes towards learning geometry in different instructional methods, the current study adapted the ATMI. (For more information, see Chapter 3). After the expert review and pilot analysis, exploratory factor analysis (EFA) was conducted (using communalities >.05, verimax rotation, loading factor > 0.40, and Eigen value > 1), and the original 40-item scale was reduced to comprise 29 items to measure PSMTs' attitude towards geometry learning.

2.6.5 Effect of Attitude and Reasoning skills on Conceptual Understanding and Procedural Knowledge

Attitudes are crucial for optimal learning. Kilpatrick (1992) also concurs that attitudes are crucial to the learning process. The career and educational paths of PSMTs may depend on how confident they are in their capacity to complete mathematical tasks (Hackett & Betz, 1995). Many studies have been carried out in many nations to determine the variables that affect pupils' success in mathematics (Bramlett & Herron, 2009; Mohamed & Waheed, 2011). Researchers came to the conclusion that PSMTs' attitudes toward

mathematics are one of the significant elements that have both direct and indirect relationships with PSMTs' academic performance (Bramlett & Herron, 2009). PSMTs with a high self-concept level showed considerably greater mathematics learning outcome than PSMTs with a moderate or low self-concept level (Akiba et al., 2008). Additionally, according to Ismail and Anwang's (2009) research, PSMTs that have a favorable attitude toward mathematics do better academically in both science and mathematics. Similar to confidence, motivation has a significant impact on PSMTs' learning outcomes in mathematics (Mayer, 1998; McDonald & Hannafin, 2003).

The analytical thinking necessary for issue resolution comes from mathematical reasoning (OECD, 2021a). Reasoning ability is a fundamental concept in mathematics and is critical to mathematical learning (Kollosche, 2021). When learning mathematics, mathematical reasoning is crucial for problem-solving and communicating concepts (Ayal et al., 2016). Furthermore, a thorough conceptual understanding of mathematics may also be developed by pushing PSMTs to think critically and reasonably about mathematics. PSMTs' understanding of mathematics will grow deeper if there are more possibilities for them to explore, apply, verify, and express the subject. According to Basra and Fauzi (2017), reasoning abilities help PSMTs understand mathematical ideas and find solutions to issues in daily life. Therefore, for PSMTs to develop completely, they must have the capacity to reason or think things through.

Additionally, one of the PISA assessment indicators that are examined is mathematical reasoning (Hidayah et al., 2021; Pratiwi, 2019). According to earlier studies, mathematical reasoning skills are crucial for assisting PSMTs in understanding, resolving

problems, drawing conclusions, proving statements, and solving problems in mathematics (Sumartini, 2015; Adamura & Susanti, 2018; Lemmer et al., 2020). Meanwhile, previous study stated that mathematical reasoning is the basis of mathematics in drawing conclusions from understanding the concepts, ideas, and their relationships to solve mathematical problems (Gravemeijer et al., 2017; Wahyuni et al., 2019). Geometric reasoning is essential for learners to process mathematical proof, which is defined as a chain of deductions derived from axioms and previously established propositions. Additionally, Brown, Jones, Taylor, and Hirst (2004) assert that learners can better understand geometric problems by generating and assessing logical arguments regarding figures and their attributes. It can help learners organize their mathematical knowledge and enhance their conceptual development in geometry (Battista, 2007; Budi, 2011; NCTM, 2002).

Conceptual knowledge is a must to have an explicit or implicit understanding of the principles, central concepts, their interrelations, and further generalizations (Rittle-Johnson & Alibali, 1999; Schneider & Stern, 2005; Hiebert, 1986), whereas procedural knowledge is the sequences of activities valuable to solve the problems (Rittle-Johnson & Alibali, 1999).The conceptual and procedural understanding are complementary to every other, and each of them need to co-exist inan iterative and interactive way to make the learning thorough, interesting, and purposeful (Rittle-Johnson & Alibali, 1999). On the other hand, the main emphasis of mathematics in schools is on procedural knowledge (procedural knowledge) or known as procedural expertise (procedural fluency) (Balka & Miles, 2019). Procedural know-how is constructed on the idea of conceptual understanding. Procedural knowledge involves understanding the rules and routines of

mathematics while conceptual knowledge involves an understanding of mathematical relationships (Forrester & Chinnappan, 2010).

2.7 Empirical Review

In this section, the empirical evidence from a few reviewed studies on PSMTs' attitudes and geometry learning, the guided inquiry approach and PSMTs' learning of geometry, the technology-integrated guided inquiry approach and PSMTs' learning of geometry, and the association between the learning approach and PSMTs' learning will be discussed.

2.7.1 Technology, Attitude and Learning of Plane Geometry

A PSMT's attitude toward geometry is defined as his or her self-reported enjoyment, interest, and level of self-confidence in learning geometry (Pilli, 2008). It was revealed that attitude towards geometry affects PSMTs' learning and achievement in geometry.

In the literature, attitudes have been examined in relation to different topics, including achievement, technology use, and school life. These studies' findings are inconsistent. According to several research studies, achievement and attitude are positively correlated (Boyraz, 2008; Gün, 2011; Mehdiyev, 2009; Myers, 2009). Other studies, like those by Olsen (2020) and Slavik (2015), found no significant correlation between them.

Numerous studies demonstrate that the use of technology in the mathematics classroom increases students' motivation and attitudes, which in turn increases their performance (Davies & West, 2014). PSMTs' achievement is positively impacted by an improved attitude and motivation. The experiences, examples, and resources accessible in the mathematics classroom are also extended by technology. Academic achievement

outcomes are influenced by these improved teaching tools and students' improved attitudes and motivation in the classroom (Clements & Kamau, 2018).

Gün (2011) examined the seventh-grade students' attitudes toward mathematics in terms of cognitive, affective, and behavioral components. The participants of the study were selected from 19 public elementary schools. According to the results, PSMTs' attitudes toward mathematics significantly explained their confidence in learning mathematics and their beliefs about the usefulness and importance of mathematics. These results indicated that seventh grade students' attitudes towards mathematics can be interpreted in terms of cognitive, affective, and behavioral components based on their confidence and beliefs.

Genç (2010) investigated the effect of using GeoGebra on 5th grade students' achievement, retention, and attitude. Mixed design, including experimental design and qualitative methods, was used. This model combined the results of qualitative and quantitative research. 70 students participated in the study; the experimental group (n = 35) and control group (n = 35) were randomly assigned. During the 5-week periods, the experimental group received computer-assisted instruction, while the control group received the lessons alongside regular classroom activities. Polygons and Quadrilaterals Test and Mathematics attitude scale were used to collect data. Results of the study illustrated that using DGS was more effective in improving students' success on polygons and quadrilaterals compared to regular classroom activities. Moreover, it was concluded that the experimental group showed more positive attitudes and more enthusiasm towards mathematics. Qualitative results showed positive opinions towards the use of GeoGebra, such as easily understandable procedures, applicable practices, and free accessibility.

Saunders-Stewart et al. (2012) also emphasize that inquiry-based instruction enhances students' achievement, knowledge application, thinking and problem-solving skills, and attitudes towards learning. This instruction method does not only affect achievement and attitude but also fosters PSMTs' motivation towards learning independently and their self-confidence (Aulls & Shore, 2008). Malone (2008) also emphasizes the benefits of the inquiry-based model, postulating that it enhances self-efficacy, confidence, and independence of the learners.

Technology can also be used by teachers to improve their own practice, which in turn affects student motivation. Research studies have shown that the use of technology by students during their learning not only keeps them more engaged in their work but also helps them stay motivated to keep learning and to be better students. For example, Bester and Brand (2013) tested the role that technology plays in motivation and concentration in the teaching of lessons in math, English, and geography. Based on survey responses from a control learning group of students who did not use technology and a second group who did, researchers concluded that there was a significant increase in motivation and engagement levels in the group using technology. Additionally, researchers saw increased achievement scores in the technology group, which they attributed to higher engagement.

On the other hand, Zengin (2011) examined the effects of the dynamic mathematics software GeoGebra on students' attitudes toward and achievements in trigonometric functions. The number of participants in the study was 51 high school students, of whom 25 were in the experimental group and 26 were in the comparison group. The experimental group received computer-assisted instruction, and the comparison group

received the usual traditional approach. The result of the study showed no meaningful difference between the attitudes of the experimental and control groups (Zangin, 2011). As a result, the researcher concluded that technology did not have an effect on the students' attitudes towards mathematics.

In a similar manner, the study conducted by Boyraz (2008) examined the effects of dynamic geometry based on computer instruction on 7th grade students' attitudes towards geometry, technology, and spatial abilities and examined the students' ideas related to the effects of computer-based instruction, which lasted only for two weeks. 57 seventh-grade students from a private school participated in the study. Data were gathered using interviews, a spatial ability test, attitude scales for mathematics and technology, and a geometry attitude scale. The findings of the study revealed that dynamic geometry-based instruction did not have a significant effect on students' attitudes toward geometry (Boyraz, 2008).

Although it is difficult to demonstrate a direct causal link between attitude and performance, the link is more obvious in certain groups (e.g., gender, socioeconomic, or ethnic ones). A multitude of circumstances can readily change attitudes since they are not fixed constructions. One's attitude toward mathematics can change, especially if technology is used. However, more research is still needed to determine how technological advancements impact people's attitudes and views about mathematics (McLeod, 1992). Investigating how the use of technology in the classroom affects students' attitudes toward mathematics is crucial. Studying these trends in developing countries is especially important because access to technology is still viewed as a sign of modernity in these nations (Ursini & Sanchez, 2008).

2.7.2 Guided Inquiry Approach and Learning of Plane Geometry

The guided inquiry approach is widely used in science lessons (Ajaja & Eravwoke, 2012; Dorji, Panjaburee, & Srisawasdi, 2015; Supasorn & Promarak, 2015). Due to its effectiveness in learners' learning processes, it has also been used in mathematics classrooms (Tuna & Kacar, 2013; Walia, 2013; Okafor, 2017).

In addition to these studies conducted on students, some researchers studied with pre-service teachers in Turkey to investigate the influence of inquiry-based instruction. For instance, Göksu (2011) studied with pre-service science teachers to investigate their misconceptions about force and motion and to overcome these misconceptions through inquiry-based laboratory instruction and to examine the relationships between achievements, misconceptions, and epistemological beliefs. The findings of this pre- and post-test semi-experimental study revealed that inquiry-based instruction has a meaningful effect on preservice science teachers' achievement, their overcoming misconceptions about force and motion, and their epistemological beliefs.

Tuna and Kacar (2013) used experimental research to investigate the effectiveness of the guided inquiry based on the 5E lesson plan in teaching trigonometry. There were 25 students in the experimental group and 24 students in the control group, chosen randomly. The students in the experimental group learned trigonometry in a course taught by the researcher using the 5E Inquiry Model, while students in the control group learned the same trigonometry content from a mathematics teacher, where the activities in the official mathematics curriculum were used (teacher-centered method of teaching). The trigonometry course was taught for eight weeks in both groups. The data collected from the pre-test and post-test of both groups revealed that students in the experimental group

have higher academic achievement than those in the control group. The difference between these groups was statistically significant and was in favor of the experimental group.

In order to determine the impact of the 5E guided inquiry approach on students' achievement and retention in learning geometry, Okafor (2017) carried out a quasi-experimental research study. Two schools were chosen for this study, in which the experimental group received the 5E inquiry approach whereas the comparison group received the traditional teacher-centered approach. The 5E Inquiry Model was successful in increasing secondary students' achievement and retention in studying geometry, according to the study's findings (Okafor, 2017). The researchers suggested that the 5E inquiry approach be used to teach mathematics in secondary schools and that it be included in the curricula of PSMTs' courses.

2.7.3 Technology-Integrated Guided Inquiry and Learning of Plane Geometry

Nowadays, technology integration into the mathematics classroom has become a common denominator for all-inclusive economic development (Alacaci & McDonald, 2012). Because of this, educational scholars believe it is imperative to include technology in learners' mathematics learning activities (Moreno-Armella & Hegedus, 2020). The research by Arbain and Shukor (2015), Mereku and Mereku (2015), and Clark-Wilson and Hoyles (2017) suggests that infusing technology into the mathematical classroom is useful for achievement and has the potential to clear mathematical misconceptions. Furthermore, technology use encourages PSMTs' engagement, motivation, attitudes, high-level thinking, and logical problem-solving opportunities. According to Voogt and

McKenney (2017), technology can motivate and engage learners while making learning exciting and improving their attitudes (Perry, Catapano, & Ramon, 2016; Getenet, 2017).

For instance, several studies have been done on the impact of technology integration on the teaching and learning of geometry classes at various levels of education (Agyei & Benning, 2015; Pittalis, Christou, & Pitta-Pantazi, 2012).

In a similar study, Almeqdadi (2000) investigated students' understanding of some geometrical concepts by using DGS. The statistical results showed a significant difference between the means of the students' scores. The experimental group had a higher mean score than the control group regarding the posttest scores. The use of the GSP had a positive effect on PSMTs' understanding of the geometrical concepts. Because technology helps create more student-centered instruction, supports cooperative learning, and enhances teacher-student interaction (Almeqdadi, 2000). The mathematics teacher participants also mentioned that this teaching approach should apply to other mathematics courses as it builds an engaging environment with dynamic learning elements that provide visualization and prospects for learning mathematics. These were created through practice and exercises, enabling thorough understanding and exposition of skills and paving the way for conceptual learning instead of rote learning (Tatar & Zengin, 2016).

Myers (2009) led a study to probe the influence of GSP on the success of grade 10 geometry learners. He sampled an experimental group where GSP was used in teaching geometry and a control group where it was not used. The MANCOVA test results showed a significant difference between the two groups at Λ (5.78) =.773, p = .001.

Saha, Ayub, and Tarmizi (2010) have conducted a non-equivalent post-test only control group quasi-experimental design study in which a total of 53 secondary mathematics learners from Kuala Lumpur were divided into two groups. The data were analyzed by an independent t-test. The experimental group was trained in coordinate geometry by means of GeoGebra, while the comparison group used a common traditional teaching method. The findings of their study showed that the experimental group taught with GeoGebra achieved better than the comparison group taught with a traditional teaching method. This implies that the approach used in facilitating learning is very important, and as a result, teachers should look carefully at their methods of teaching.

In his study, Ljajko (2013) separated learners into two groups: the control group and the experimental group. The control group learns ellipses without the use of technology, while the experimental group is taught ellipses using technology (GeoGebra). The results of the tests in this study revealed that learners in the experimental group had better problem-solving skills in the concepts of ellipse as compared to learners in the control group. This implies that the use of technology ensures that learners gain skills that make them better learners. The incorporation of technology has helped learners understand complex mathematics concepts by increasing their deductive reasoning processes, allowing them to assimilate many complex ideas about the subject (De Moura Fonseca & De Oliveira Lino Franchi, 2016). In conclusion, De Moura Fonseca and Lino Franchi (2016) write that technology equips learners with different visual representations of the same mathematical concepts and those learners are interacting with a high level of inductive analysis, helping them to attain multiple related perspectives of the same concepts.

In Dogan and Içel's (2011) study, technology was used to teach about triangles. In this study, the participants were 8th grade students separated into an experimental and a comparison group. The experimental group was given activities that were constructed using GeoGebra so as to make the subject dynamic, visual, and concrete. After the intervention, students in the experimental group attained more conceptual knowledge about a triangle than the comparison group. Learners from the experimental group wished that they could learn more mathematical concepts using technology (Dogan, 2011). The implication is that the use of technology improved both their motivation and their understanding of mathematical concepts. Similarly, Xistouri and Pitta-Pantazi (2013) also used a 5E lesson plan based on an inquiry approach and found that learners understood geometric reflection concepts and properties. They added that technology offered a rich context for designing a variety of activities with different cognitive requirements and objectives.

The study conducted by Nurjanah, Dahlan, and Wibison (2017) aimed to compare the effects of hands-on activity media, computer-based media, and a traditional teacher-centered approach on the conceptual understanding of mathematics and reasoning ability. The study used three groups: two experimental and one comparison. A quasi-experimental, nonequivalent pretest–posttest comparison group design was used. The research data were obtained using tests of conceptual understanding and mathematical reasoning, and an ANOVA and Scheffe test were used to analyze the data. The ability of groups of students who use computer-based media to understand mathematical concepts differs significantly from that of those who use hands-on activity media and a traditional approach (Nurjanah et al., 2021).The use of both hands-on activity and computer-based

media contributes to the achievement of student reasoning and conceptual understanding, especially on the topic of plane geometry being studied (Nurjanah et al., 2021). In a similar manner, the study by Aronin and Floyd (2013) also confirmed that technology integration has been shown to improve learning outcomes.

In a separate quasi-experimental study, Abdullah and Zakaria (2013) chose 94 students from secondary schools together with two instructors to assess the effectiveness of van Hieles' phase-based learning on students' geometric understanding. The comparison group (traditional group) made up half of the group, and the experimental group made up the other half (taught through the use of GSP). The study's goal was to assess the role played by the van Hiele phases in the GSP-based understanding of geometry. The VGT was administered to both groups. The results showed that the experimental group had higher levels of geometric understanding than the comparison group (Abdullah & Zakaria, 2013). Learners can also use geometric proofs and appreciate geometry as a whole (Muyeghu, 2008). With this insight, learners may use axioms without proving them and, therefore, become efficient and flexible in geometric reasoning.

Moreover, for exceptional children as well, integrating technology has been shown to increase learning outcomes (Aronin & Floyd, 2013). According to the authors of the article, the use of iPad technology increased the self-confidence and independence of preschoolers with learning disabilities. Likewise, the adoption of instructional iPad apps helped students with limited motor abilities engage more fully in class activities (Aronin & Floyd, 2013). Moreover, sophisticated assistive technology helped visually impaired learners with advanced mathematics (DePountis, Pogrund, Griffin-Shirley, & Lan, 2015). In this study, a mixed qualitative and quantitative study used 82 participants with visually

impaired students. Technologies such as a personal computer, an Electronic Refreshable Braille Notetaker (ERBN), and a talking scientific calculator are used as tools for instruction. DePountis et al. (2015) discovered that visually impaired students can learn algebraic expressions and advanced mathematics content such as algebra I and algebra II.

Al-Shehri, Al-Zoubi, and Rahman (2011); Tamer (2013); and Pandiscio and Knight (2011) also found a significant and positive impact of a technology-based learning approach on the academic achievement of PSMTs. Similarly, a study conducted by Al-Ghassani et al. (2015) indicated that blended learning improved students' understanding of the course concepts (Calculus I).

In further quasi-experimental research conducted by Kandil (2016), the effect of inquiry-based instruction enriched with origami activities on 7th grade students' achievement, attitude toward geometry, and self-efficacy in geometry was investigated. The study used a random sampling method to assign students to an experimental and comparison group, in which the experimental group was taught through an inquiry-based approach and the comparison group with a traditional approach. The data were analyzed using ANCOVA tests. The findings of the study showed an inquiry-based approach enriched with origami activities has a significant effect on students' achievement, attitude towards geometry, and self-efficacy in geometry when compared to a traditional approach (Kandil, 2016; Deringöl, 2020; Kandil & Işıksal-Bostan, 2019).

In addition, Gemechu, Michael, and Atnafu (2013) conducted research using the Geometer's Sketchpad based on a quasi-experimental design. The findings showed a

positive effect on students' attitudes, confidence, and engagement in geometry. Generally, the learning and teaching of geometry in teacher education are affected by different factors. According to the NCTM (2000), classroom instruction has an impact on PSMTs' understandings of geometry, self-confidence in their ability to master the subject, and motivation to do so. Teaching methods that actively involve learners would likely lead to a more positive attitude and meaningful learning in mathematics and geometry than teaching methods where learners remain passive in the classroom. The ability to manipulate learning, develop team and group work, promote discussions and questioning, engage thinking and reasoning, reflect, and solve problems can be more engaging with technology. Technology can also support teachers in assessing the instructional experience (Albarracin & Shavitt, 2018).

2.8 Current issues of Mathematics Education in Ethiopia and Research Gap

Mathematics is critical to the advancement of science and technology. Due to this fact, achievement in mathematics is essential for the economic advancement of nations, particularly in developing countries (Vista, 2016; Bosman & Schulze, 2018). However, learners' poor performance in mathematics continued to be a global concern. For instance, the assessment findings of the Southern Africa Consortium for Monitoring Educational Quality [SACMEQ] (2011) showed learners' engagement and performance in mathematics are declining in many African countries.

Similarly, there is evidence that learners in Ethiopia's elementary and secondary schools perform insufficiently in mathematics. According to the Ethiopian National Learning Assessment (NLA), which is administered every four years, learners score lower in mathematics than they do in other courses. For instance, the data from the NLA, which

was collected from different grade level confirmed the mean score of students' performance in mathematics is below the standard mean of 50% (MoE, 2017).

In general, students achieved very low scores in all mathematical content domains, but particularly in algebra, geometry, trigonometry, and statistics (MoE, 2017), and the mean scores on these content domains ranged from 28.65% to 33.59%. In addition to their poor performance in mathematics, students generally score lowest on concept understanding and low in higher-order thinking skills such as problem solving and reasoning skills (MoE, 2017). The percentages of students in Grade 10 who scored 50% (the standard that the ministry set as a pass mark and designated proficient) or more in mathematics were only 14.7% (NLA, 2010). In addition to achievement, attitude toward mathematics is one of the affective variables related to learning mathematics and the learning environment (Reyes, 1984).

The findings showed that students were not meeting the mathematical performance standards at both the elementary and secondary levels. These dismal findings indicated that there is a necessity for improvement in both the teaching and learning of mathematics. The instructional strategies employed so far (conventional methods) in the Ethiopian context have not improved students' achievement and attitude toward the subject to a considerable extent (Eshetu et al., 2009). Therefore, in order to enhance science and mathematics education in Ethiopia, new pedagogical techniques that include technology are required.

The significance of technology integration in teaching and learning mathematics is widely acknowledged by scholars because the constructivist framework places more

focus on context or the learning environment than on instruction. According to Ndlovu, Wessels, and De Villiers (2011), there is an increasing global push to incorporate technology into mathematics instruction and learning. Thus, MoE (2015:58) stated that ICT should be incorporated into teacher training programs and supported by practice in order to improve teachers' abilities to utilize technology, as well as their ability to educate and support students using technology. In addition, teacher educators must possess the ability to incorporate modern technologies as necessary for education, according to a different policy document called Content and Pedagogical Standards for Mathematics Teachers (MoE, 2012a). The document also requires mathematics teachers to use technology for a variety of courses, including Euclidean plane geometry, coordinate geometry, transformational geometry, calculus, etc., employing manipulative and suitable technology to represent mathematical concepts and solve problems. Michael (2015) also contends that integration of technology, pedagogy, and content is necessary to raise educational quality.

However, teachers continue to use technology for low-level academic tasks without supporting students' active learning (Hare, 2007; Gebremariam, 2004). Although considerable research has been conducted regarding the general use of technology in the teaching and learning of mathematics, there is still little research about the use of technology in developing countries like Ethiopia (Bansilal, 2015).

In GIBL, students are given a chance to work like scientists or mathematicians (Dorier & Maaß, 2020). It is a student-centered instructional approach that enables students to freely explore, raise questions, and make attempts to justify their answers (Artigue & Blomhøj,

2013). According to a number of studies, teaching mathematics based on the GIBL approach could enhance students' mathematical understanding. Its applicability to mathematics education has improved mathematical understanding with promising results (Tuna & Kacar, 2013; Omotayo & Adeleke, 2017; Nopasari, Ikhsan, & Johar, 2020). However, inquiry is still not a widespread concept in classrooms, incorporates insufficient abstract concepts, and is often inappropriately used (Gencturk & Turkmen, 2007).

To the best of the researcher's knowledge, there has been no qualitative or quantitative research on this crucial aspect of mathematics instruction in Ethiopian CTE contexts, despite the fact that GIBL approaches are frequently used in teaching and learning and are recognized for their importance in science and mathematics education. The majority of higher education institutions in Ethiopia employ traditional lecture methods that are ineffective for deep learning (MoE, 2015; Bekele, 2012).

Many scholars feel that technology can help teachers overcome the difficulties of representing abstract concepts in mathematics (Mousoulides, 2011; Ocal, 2017). Because of this, educational scholars believe it is imperative to include technology in students' mathematics learning activities (Moreno-Armella & Hegedus, 2020). However, sufficient empirical research exploring how a technology-integrated guided inquiry-based learning strategy could improve PSMTs' learning outcomes is lacking (Philipsen, Tondeur, Pareja Roblin, Vanslambrouck, & Zhu, 2019). Thus, it needs to undertake more research to inform better practices specific to the field of mathematics teaching (Borba et al., 2016). Consequently, the current study's goal is to investigate the effects of a technology-

integrated guided inquiry-based approach on PSMTs' attitudes toward geometry, procedural and conceptual knowledge, and reasoning skill in geometry in college teacher education, as well as hypothetically fill a gap in the literature.

2.9 Conceptual Framework

A conceptual framework is a textual or visual description of the key concepts under study. It is a road map for achieving the goal of the investigation. Thus, the conceptual framework demonstrates the investigation's hypothetical stances as well as the relationship between independent and dependent variables in the study. In this section, the research's conceptual framework is described. It is based on an analysis of the literature on conceptual knowledge, procedural knowledge, reasoning abilities, attitude, guided inquiry-based learning, and the theoretical underpinnings of the study. The main factors influencing conceptual knowledge, procedural knowledge, reasoning skills, and attitude toward geometry and mathematics are outlined in the conceptual framework.

The problem situation that was identified and discussed in Chapter 1 served as the impetus for the current investigation's conceptual framework, which was developed primarily through a review of the literature that contained theories and research findings relevant to the study's goals and objectives. The highlighted mathematical issue and the paucity of research on the particular subject under investigation, namely, plane geometry, drove the necessity for the current study.

In this study, constructivism and Skemp’s theory are used to create a theoretical framework for the exploration of the research topics. It specifically explored how the IBL approach relates to the teaching and learning of mathematics in Ethiopian CTE classrooms. In this study, the experience under investigation is the learning of geometry

through a technology-integrated guided inquiry-based and 5E model using digital technology. The 5Es model has therefore evolved into a method for teaching mathematics and is crucial to PSMTs' learning. The lesson plan for structuring the GIBL method in was the 5E model. The figure below shows the conceptual framework of the study.

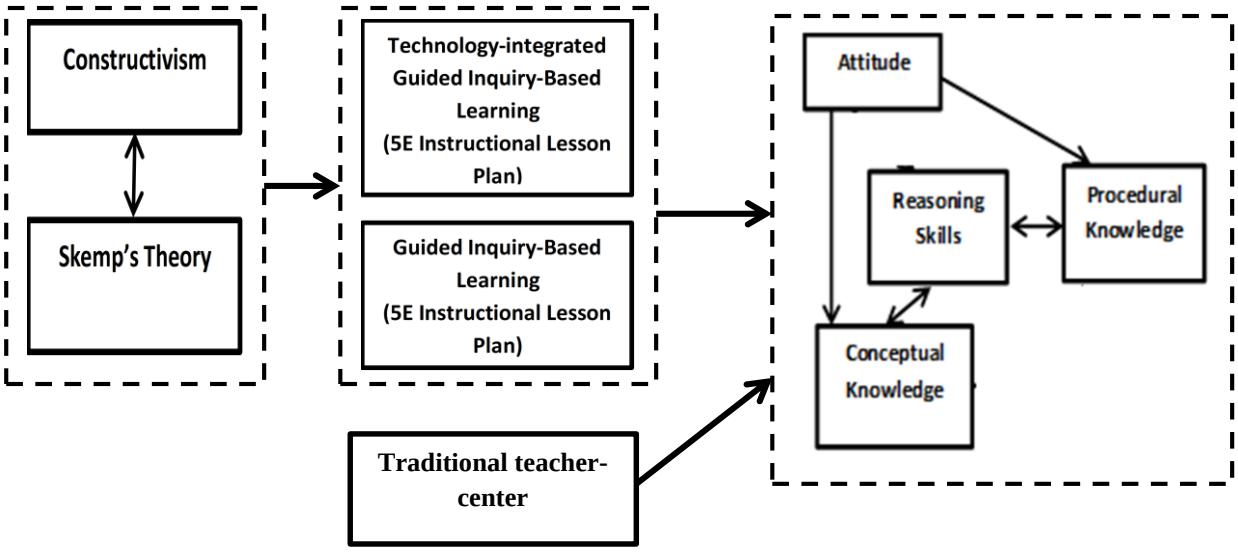


Figure 2.5 Conceptual Framework for the study

CHAPTER THREE: RESEARCH METHODOLOGY

Introduction

This chapter is devoted to elucidating the research methodology of the study. Here are the research paradigm, research method, research design, population, and sampling technique. Also, the development and validation of learning materials, the development and validation of instruments for data collection, variables, and instruments used to collect data, procedures, treatments, data analysis methods, and ethical issues are presented in depth.

3.1 Research Paradigm

A researcher must first recognize the underlying philosophical paradigm that underpins the inquiry before devising a complete plan for the research activities. According to Creswell (2014), a "paradigm" is a philosophical outlook on the nature of the universe and views that a researcher conveys through a study. A paradigm affects how knowledge is researched, perceived, and acquired (Mackenzie & Knipe, 2006). A paradigm is a way of describing a worldview that is based on philosophical presuppositions that serve as the foundation for ontology (beliefs about social reality), epistemology (how the researcher knows and what can be known), axiology (ethics in research), and methodology (the means by which the knower comes to know) (Creswell, 2014).

Research paradigms were broadly categorized by Mertens (2015) under four groups: post-positivism, interpretivism, transformative, and pragmatism. A post-positivist paradigm encourages the use of quantitative techniques because it sees the world as a collection of visible and quantifiable facts (Glesne & Peshkin, 1992).

An interpretivism paradigm prefers qualitative methods such as case studies because they provide the detailed information that the constructivist requires to understand contexts (Thanh & Thanh, 2015). A transformative paradigm, which commonly supports mixed methodology, is based on the belief that knowledge is socially and traditionally situated through interactive links between the researcher and participants (Lee, 2012). Similarly, a pragmatism supports a mixed method as the researcher uses different approaches and is founded on the idea that research is determined by what the researcher thinks is appropriate for that specific study. Post-positivism, interpretivism, and pragmatism are the three most commonly used paradigms in education and the social sciences. Based upon these paradigms, the approach of research is determined (Anney, 2014; Guba & Lincoln, 1994; Mertens, 2015).

Pragmatism is considered to be the most appropriate paradigm for mixed-method research. It contains a set of knowledge and review assumptions that undermine the mixed-methods approach. Some might think the term "pragmatic" denotes a deficiency of principles fundamental to a course of action. According to Feilzer (2010), pragmatism may serve as a guide for grounded inductive or deductive research as well as top-down deductive research design. Pragmatists are putting their attention on the research questions and planning data gathering and analysis in a way that offers insights into the issue without relying on alternative perspectives (Creswell, 2014; Morgan, 2007). Therefore, the most suitable strategy for responding to the research questions stated earlier in the study is a pragmatic one.

Although the main approach is quantitative, which focused on the results of the paper and pencil test, the qualitative component is evidenced by the results of learners' interviews

as well as lesson observations. In this methodology, a researcher investigates a certain quantitative finding and justifies it using qualitative information (Creswell & Plano Clark, 2007). A flexible research viewpoint, according to pragmatism, enables the researcher to employ a variety of techniques. It lessens prejudice in this instance (Guba & Lincoln, 1994; Clark, 1998).

In the pragmatic inquiry practice, the researcher does not take a strong position on whether reality and knowledge are objective or subjective. Thus, the researcher can incorporate constructivist and post-positivist features and their approaches to combine quantitative and qualitative components. The strength to address a single study issue in several ways is one of the advantages of this research tradition. Some argue that it is not really possible to be so flexible in one's view of reality and truth, and that pragmatism is often a disguised form of post-positivism (Denzin, 2010).

The intent for this philosophical position is the need for multiple perspectives, to validate the quantitative measures with qualitative experiences, to explain quantitative results, and to better contextualize the intervention for the purpose of exploring its effectiveness in enhancing learning outcomes and attitudes toward geometry. In the current research study, the researcher used a mixed-methods approach based on pragmatism and its philosophical assumptions. This allows the researcher to use both quantitative and qualitative approaches in responding the research question(s).

3.2 Research Method

For this study, a mixed-method approach has been used. In educational research, there are three common research approaches: quantitative, qualitative, and mixed methods (the

eclectic-mixed pragmatic mode) that dominate in the educational setting (Reeves & Hedburg, 2003). In a quantitative approach, numerical data are gathered and typically evaluated using inferential and descriptive statistical techniques (Leedy & Omrod, 2001); in contrast, qualitative research is focused on qualitative phenomena, that is, phenomena linked to or involving quality. Results from a qualitative research technique are either non-quantitative or not subjected to rigorous quantitative examination (Kothari, 2004).

On the other hand, mixed-methods research combines both quantitative and qualitative research methods in different ways, with each approach adding something to the understanding of the phenomenon. Mixed-method research combines both qualitative and quantitative methodologies of data collection and analysis. In this case, Ponce and Pagan-Maldonado (2014) agree that in a mixed-methods study, quantitative and qualitative approaches are combined and integrated at different points of the research process.

Thus, this study employed a mixed-methods approach to investigate the effectiveness of a technology-integrated guided inquiry-based learning approach through both quantitative and qualitative data collection (Creswell, 2012). The rationale for choosing a mixed-methods design was to provide the researcher with greater scope and understanding of the research questions posed in this study (Johnson & Onwuegbuzie, 2004; Almalki, 2016). A mixed-methods approach provides the researcher with the opportunity to compensate for the inherent weaknesses that are apparent in a purely quantitative or qualitative approach (Almalki, 2016). Quantitative methods test a hypothesis to produce a conclusion that empirically describes the relationship among the tested variables; however, qualitative methods recognize the difficulty of accurately measuring social

construct variables (e.g., learning) and the importance of an insider's perspective to fully understand such complex situations (Almalki, 2016).

In investigating the PSMTs' learning of plane geometry using TGIBL, both quantitative (i.e., post-positivism) then qualitative (i.e., interpretive) methods were used. It is also argued that the application of only one research method is insufficient to obtain practical results (Hevner, March, Park, & Ram, 2004). Quantitative approaches are useful for comparing PSMTs' results from different treatment groups by using statistical inferences to determine whether there is any significant difference between the performance and knowledge of PSMTs at these respective levels, and a qualitative method was used because it relies on understanding a phenomenon from the PSMTs' perspective in the setting it happens (Stake, 2011).

For instance, one of the purposes of this study was to investigate PSMTs' conceptual understanding of geometry. In this case, a quantitative approach is used to express statistical results that are insufficient to explain the situation and behavior behind them. Because of this, it is useful to utilize a qualitative method in addition to a quantitative method to assess PSMTs' conceptual understanding in order to gather specific data on their in-depth understanding. In this study, therefore, the results from the qualitative data aid in complementing the conclusions from the quantitative data. Therefore, the qualitative data obtained through semi-structured interviews had to be added to the quantitative information from the two-tier multiple choice tests in order to have strong evidence for a conclusion through triangulation. In addition, through classroom observation and interviews with PSMTs, qualitative information regarding the actual classroom practice and their opinions of the intervention is collected.

It is these principles that underlie the choice of the mixed method, which combines both quantitative and qualitative approaches to understand more thoroughly than any single approach (Cohen, Manion, & Morrison, 2007; Creswell, 2014). To do so, mixed-method research uses data collection techniques that can be done sequentially or concurrently (Creswell, 2014).

3.3 Research Design

A research design is the set of procedures for collecting, analyzing, interpreting, and reporting data in research studies (Creswell & Plano Clark, 2007). As a result, it is the method through which the researcher attempted to comprehend phenomena in their context. When compared to using a single study approach, the mixed-methods approach might give a more comprehensive picture of educational issues and make it easier to investigate this complicated social phenomenon (Mertens, 2015). As a result, the mixed-methods approach is crucial for looking at topics in social science and education.

Specifically, this current study employed a concurrent embedded mixed method design to compare, corroborate, and cross-validate its findings (Creswell, 2014). The design includes first producing quantitative data to test and explain the theoretical framework. Then it is followed with qualitative data that complement, refine, and enrich the quantitative results. More importantly, the concurrent embedded mixed methods design provides interpretation in the qualitative phase if unexpected findings emerge from the initial quantitative phase. This design ensures the trustworthiness, credibility, and reliability of the research findings, as well as producing more complete knowledge to inform theory and practice.

In sequential phases, two research methods are used separately, in which the researcher begins with one method and then incorporates the other at a later point (Ponce & Pagan-Maldonado, 2014; Creswell & Plano Clark, 2007). Quantitative research in this situation looks for broad cause-and-effect relationships, makes quantitative predictions, and generalizes research findings, whereas qualitative research aims to deeply comprehend individual perceptions, provides detailed explanations of individual meanings, and vividly interprets how each participant constructs their meanings and why. The embedded mixed research design (Figure 3.1) as adapted from Creswell and Plano Clark (2007) is the research design used in this study.

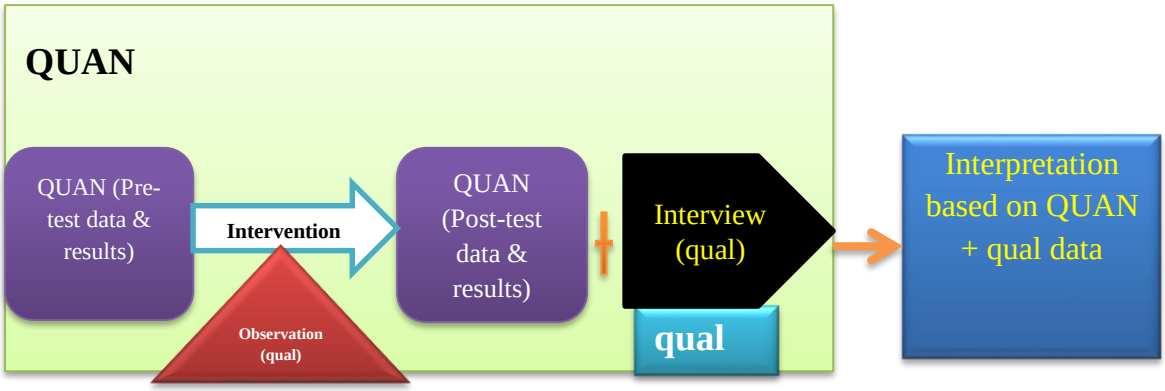


Figure 3.1 Research approach (adapted from Creswell & Plano Clark, 2007)

Both quantitative and qualitative forms of data were gathered concurrently (before, during, and after the intervention) (Ary et al., 2010; Creswell, 2014). Thus, the concurrent embedded mixed methods design in the current study could provide a more complete portrait of conceptual understanding, in relation to how teachers and PSMTs use technology-integrated guided inquiry-based learning to develop conceptual understanding, procedural knowledge, reasoning skills, and an attitude towards geometry.

In educational research, particularly in mathematics education, it is challenging to conduct a true experiment because all variables cannot be controlled completely, and randomization is also impossible. As a result, in educational research, a quasi-experimental has emerged as an alternative to true experimental research. Therefore, a quasi-experimental procedure employed because complete randomization is not possible. In place of randomly constructed samples, intact classes were used. Among various types of quasi-experimental method, the researcher selected "pretest-posttest non-equivalent group design" because this design is often used in classroom experiments when experimental and control groups were such naturally assembled group's intact classes. This is due to the fact that courses at CTEs exist as whole groups, and it is unethical to regroup and reorganize them for research reasons as it would cause some sort of disturbance. In this method the participants are not randomly assigned (Shadish, Cook & Cambell, 2002).

In this method, one comparison and two experimental groups with pretests and posttests were employed. The study used a comparison group that was similar to the treatment groups in order to obtain information about the effects of the treatment (Mackey & Gass, 2005). A quasi-experimental method was appropriate for this study because of the inability to control for all variables in a middle CTE setting. Although the research could not control all variables, by using a comparison group, the researcher expected to find that the main effects of the uncontrollable variables affected both the experimental groups and the comparison group equivalently (Mackey & Gass, 2005).

PSMTs in group one were subjected to technology-integrated guided inquiry-based learning (TGIBL); PSMTs in group two were exposed to guided inquiry-based learning

(GIBL); and PSMTs in group three were exposed to a traditional instructional approach. However, this type of design is exposed to the threat of internal validity because pre-existing disparities between the groups may impact the study's results. Therefore, in order to reduce the impact of discrepancies between the two experimental and comparison groups, different measures were taken. Prior to the interventions, pre-tests were administered to all groups to compare their performance on the analyzed learning outcomes (Creswell, 2014). This study thus used a covariate analysis to overcome this problem.

Similarly, to minimize the teachers' effect, two initial group teacher educators with equivalent academic qualifications (a master's degree) and experience of teaching in CTEs were assigned to the comparison and experimental groups. They used the same course outline and modules (Math 111). At last, the researcher adapted similar questions for pre-test and post-test to reduce the effect of test differences, but the time range was set with the purpose of decreasing the remembered influence of PSMTs on test topics. In any quasi-experimental approach, absolute control is not possible; therefore, the control group in this study is labeled as the comparison group.

Table 3.1 Research method Layout

Group	Pre-test	Intervention	Post-test
Experimental Group1 (EG1)	X1	TGIBL	X2
Experimental Group1 (EG2)	X1	GIBL	X2
Comparison Group (CG)	X1	TRAD	X2

TGIBL: Technology integrated Guided inquiry-based learning, **GIBL:** Guided inquiry-based learning, **TRAD**: Traditional teacher-centered, **X1**: pre-test, **X2**: Post-test

Furthermore, qualitative data were collected to complement the quantitative data and gain an in-depth examination of the intervention's implementation during the teaching and

learning process (Creswell, 2014). Besides these tests and questionnaires, semi-structured interviews and observations were conducted with twelve PSMTs (nine of them from two experimental groups) and three from comparison groups. The study was conducted over two months. There were two periods in the week, each lasting two hours.

3.4 Research Site, Population and Sampling Procedure

The study was conducted in Western Zones Oromiya Regional State, Ethiopia, in public CTEs with PSMTs who were taking the course plane geometry (Math-111) at the department of mathematics education. The colleges are located in Oromiya Regional State of two zones named Qellam Wallaga Zone and Horro Gudduro Wallaga Zone. Msabila and Nalaila (2013) state that there are numerous motivating factors that could affect the researcher's decision regarding the study site, including the nature and occurrence of the problem, the research time frame, data accessibility, interest in and instructions, resource availability, performance in a particular field, and the study's goals and objectives. The reason for the choice of those two CTEs is that both are established at the same time frame, the avail of mathematics department, data accessibility, avail of computer lab etc.

The population of the study was second-year PSMTs enrolled for the 2021 academic year. A total of 116 PSMTs participated in the study. This is because teacher education is an important sector of an educational system for several reasons. The production of high-quality professionals in different fields and the quality of education in the country in general depend on the quality of teacher education. At this level of education, therefore, preservice teachers must be prepared with the required skills and knowledge to produce qualified learners in their future careers. Moreover, the researcher has exposure to the

problems that exist in CTE mathematics teaching because of his appointment as a teacher of mathematics at a college of teacher education.

As the study employs a quasi-experimental method, it entails three groups taking the same course, and as a result, three CTEs were purposefully selected. Those CTEs were selected based on their hypothetical contexts (e.g., biographies, geographical settings, and background factors), computer facilities (computer labs, internet access), and teachers' qualifications and experiences in teacher education.

According to Cohen et al. (2007), the researcher can choose groups that are as similar as possible, even while differences between groups in a non-equivalent quasi-experimental study design naturally arise. The purpose of having three CTEs is to prevent information contamination during data collection and intervention. Then, the two CTEs, CTE1 and CTE2 were assigned into the experimental and control groups using a simple random sampling method. Accordingly, CTE1 is for EG1 and EG2, CTE2 for CG while CTE3 for the pilot group. One mathematics teacher educator from each of the two CTEs was involved. First, relatively well qualified and experienced mathematics teachers and educators in teaching mathematics at CTEs were assigned; they have taught for over 12 years and have a master's degree.

The setting in experimental CTE1 was such that the PSMTs in groups EG1 and EG2 of intact groups were in opposite shifts. As of the CTE schedule, to cover the uncovered courses due to the COVID-19 pandemic, the college used two shifts. Every week, the shifts were switched. When PSMTs in TGIBL left for practicum for a week, the GIBL group stayed in CTE for intervention. Similarly, when the GIBL group left for practicum,

the TGIBL group stayed in CTE for interventions. Furthermore, the intervention material was locked away in the computer laboratory and only used for interventions. As a result, the two intact groups did not meet in the so-called CTE1.

In order to have the most representative samples, the PSMTs for interviews were chosen by taking into account their post-geometry attitude score, geometry conceptual test score, and procedural knowledge test score. Participants were four PSMTs from the EG2 and five PSMTs from the EG1. After the interviewees' selection, the researcher conducted individual interviews with each one of them while going through the interview questions sequentially. Participants' replies were recorded and processed to determine the common responses.

The study was conducted on plane geometry (Math 111) at the course level. This course is offered for year II preservice mathematics teachers. The topics covered in this module are chapter 1 (point, line, and plane); chapter 2 (triangles); chapter 3 (quadrilaterals and polygons); and chapter 4 (circles). The reason for the selection of plane geometry is that, as already mentioned in the background, the concepts of geometry are reported as being difficult concepts for students at all levels of education to learn and hence need suitable instructional approaches to enhance PSMTs' learning. Learners' difficulties in learning mathematics are mainly affected by the teaching approach implemented and misconceptions in geometry.

3.5 Variables of the study

Variables, according to Leech, Barrett, and Morgan (2005), are features of the participants or environment for a certain study that have various values. It is the

conditions or features that the researcher can manipulate, regulate, and/or notice (Best & Kahn, 2006). The variables are classified as dependent, independent (active or attributable), mediator, and extraneous. The dependent variables are the conditions that can be changed as the independent variables are removed or changed in an effort to determine their relationship to observed phenomena (Best & Kahn, 2006). The present study has seven main variables, i.e., three independent and four dependent variables.

3.5.1 Independent Variables

An independent variable is the one that is altered to see how it impacts on a dependent variable, called an outcome variable. It can be changed for researchers to evaluate how it affects the dependent variable. The instructional methods are the independent variables in this study. As a consequence, intervention groups and time points comprised the study's independent variables. The intervention groups have three levels. The independent variables TGIBL and GIBL were applied to the two experimental groups while keeping other conditions the same for the comparison group. Time points have two levels as well (pre-test and post-test).

3.5.2 Dependent Variables

The variables that the independent variable affects are known as dependent variables. These variables are expected to change as a result of the experimental treatments for the independent variable(s). The dependent variables in the study were PSMTs' conceptual understanding, procedural knowledge, and reasoning skills scores from the geometry test, as well as their attitudes towards geometry as measured by the geometry attitude scale

questionnaire (GASQ) with subscales (self-confidence, value, enjoyment, and motivation).

3.5.3 Extraneous Variables

Due to the direct or indirect impact of extraneous variables, the experimental research's findings were open to questions (Best & Khan, 2006). To ensure the validity of the research's findings, therefore, the researcher should control the extraneous variable. In this research, selection of CTEs, instructor, subject matter, teaching materials, experimental time, and availability of the computers, internet access, electricity, maturation and test were considered as some of the extraneous variables.

In this research, two CTEs selected were inaugurated at the same time and has approximately similar academic staff in quality and quantity. The same test papers were conducted for all groups before the experimentation. The researcher has given training on the materials for the intervention for mathematics teacher educator in CTE so that, the teacher to manage the intervention and the tools and materials were managed as well to minimize the extraneous variable. Similarly, the researcher had provided equal time to all groups and the same test papers were conducted for all groups after the experimentation.

3.6 Instruments for Collecting Data

In order to address the research questions, multiple data collection instruments were employed. The data gathering tools were plane geometry learning (conceptual understanding, procedural knowledge, and reasoning skills) questions that were adapted and partly prepared by the researcher, a geometry attitude scale questionnaire (GASQ) adapted from literature, semi-structured interviews, and classroom observation. The

features and the process of developing these instruments are described in detail in the following section.

3.6.1 Geometry Conceptual Understanding Test

The Two-Tiered Geometry Diagnostic Test (TTGDT) was used to measure PSMTs' conceptual understanding of plane geometry concepts. TTGDT is a two-tiered multiple-choice exam in which the first level contains questions concerning the subject being assessed, and the second level contains the justifications or reasons for each response to the question at the first tier (Treagust, 2006). The format of the second dimension can be a multiple choice or a multiple choice with one optional open-ended (Bayrak, 2013). One benefit of the two-tier multiple choice tests, according to Treagust (2006), is that they can examine students' cognitive ability at a higher level (higher order thinking skills).

Thus, the Two-Tiered Geometry Diagnostic Test (TTGDT) was used for this research to measure PSMTs' conceptual understanding of geometry concepts. Of the 30 TTGDT items employed in this research, 20 items were adapted from France's (2019) unpublished PhD thesis dissertation with permission. The rest items were developed by the researcher. In the next section, the preparation procedure of TTGDT will be discussed.

Developmental Pattern of TTGDT

The development of the tools that can be used to diagnose for conceptual understanding about plane geometry concept was done following a model by Treagust (2006). In this model, there are three stages and ten phases to be followed. The three stages are: defining the content area, identification of students' conceptions and instrument development. The

scheme development process of the TTGDT is based on three stages (see Figure) using the procedure proposed by Treagust (1995) is shown below.

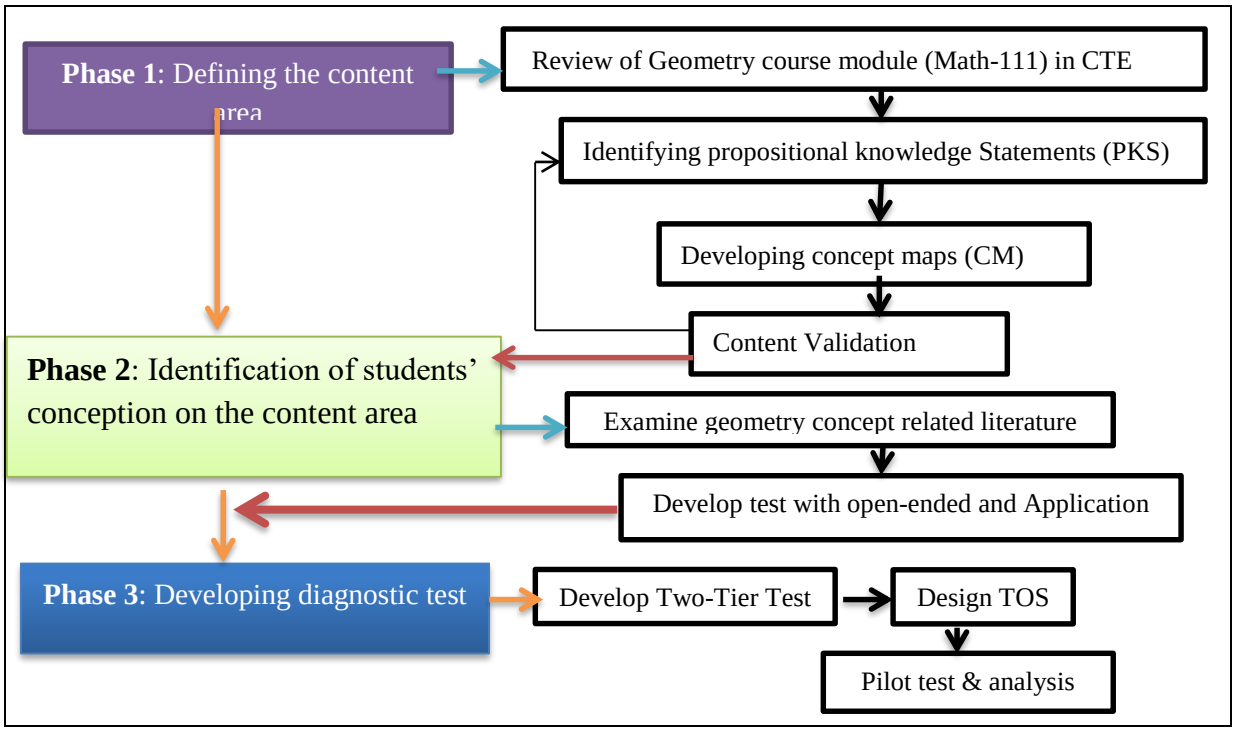


Figure 3.2 The flow chart of TTGDT instrument development

(a) Stage 1: Defining Content Area of the Study

This stage involved the researcher defining the content limits, which were further defined in terms of concept maps and propositional knowledge statements representing the knowledge required to understand the geometry knowledge as specified in the teacher education plane geometry curriculum. The concept map's objective was to illustrate the essential concepts that made up the content of the under-researched geometrical notions and how these concepts were connected to one another in a hierarchical and integrated way.

The propositional knowledge statements represented the relationship between the key concepts. The content, propositional knowledge statements and concept maps were based on the geometry content tested, including plane shapes and their properties; similarities and differences among the plane shapes; perimeter and area of plane shapes; and identification of 2D shapes and angles (MoE, 2013). After identification of propositional knowledge statements, the second step in phase I is the development of concept map. However, it is difficult to put all concepts in to one concept map. Hence, the researcher developed three concept maps.

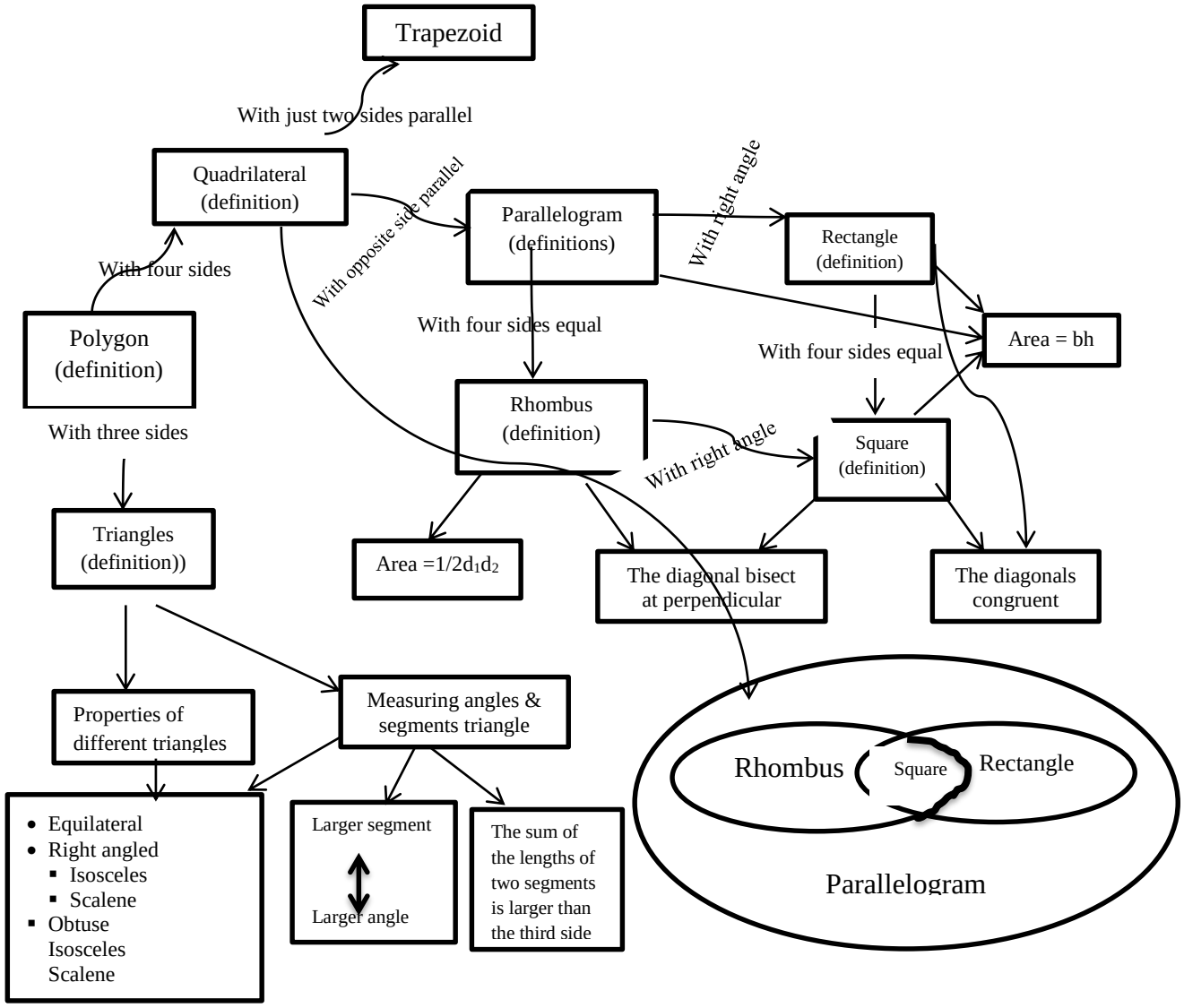


Figure 3.3 The concept map of polygon

The following figure shows the concept map for the circles its concepts. Circle is also one of the concepts in plane geometry lesson in the curriculum of PSMTs.

1. there is one and only one circle passing through three given non-collinear points	2. Equal chords of circle subtend equal angles at the center	3. The perpendicular from the center of a circle to a chord bisects the chord
4. Equal chords of a circle are equidistant from the center	5. The angle subtended by an arc at the center double the angle subtended by it at any poi part of the circle.	6. Angles in the same segment of a circle are equal.

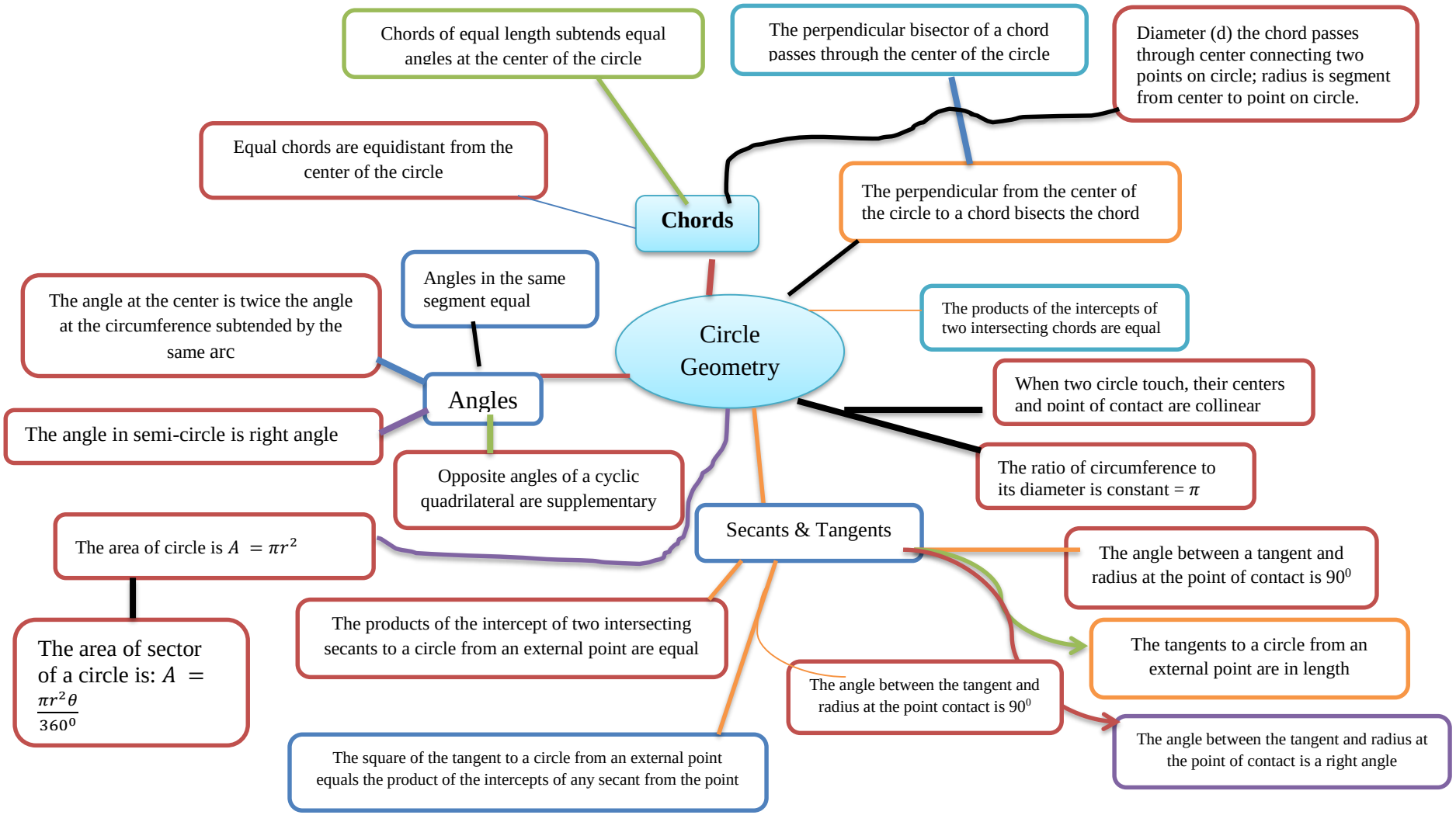


Figure 3.4 Concept map for Circle

Stage 2: Identifications of the conceptions of Geometry concepts/open-ended test

At this stage, the information about PSMTs' conceptual knowledge about geometry concepts was collected through an open-ended test. In this case, the researcher initially prepared a type of two-tier geometry diagnostic test (TTGDT) consisting of open-ended questions to elicit PSMTs' conceptual understanding from their written responses. In preparing the items, the objectives of the topics in the CTE geometry module were used as a source.

The test items were multiple-choice questions with open-ended responses that required PSMTs to supply the answers as well as provide advance reasons for their answers. The developed test had 30 items. After face and content validation and evaluation for the language level of the items and the scope of the items by experts from mathematics education, the test was administered to 30 PSMTs from CTE selected at random. The purpose of the test was to find PSMTs' correct and incorrect conceptions or misconceptions. The misconceptions identified from PSMTs' responses to each multiple-choice open-ended question and literatures were used to develop the instrument (the concept, generalization, and reasoning parts). The source of item options in the two-tier multiple-choice test or primary instrument was the most often identified answers and justifications for the answers (correct and incorrect) provided by PSMTs on each item of the open-ended questions.

Stage 3: Two Tier Instrument Development

Based on the data obtained, the researcher developed the Two-Tier Geometry Diagnostic Test (TTGDT). The developed test item consisted of a two-tier multiple-choice question

for each test. The first tier consists of a standard multiple-choice content question with four answers (one correct, three distractors) based on PSMTs' assumptions, propositional knowledge statements, and a concept map developed. The second-tier question consisted of a multiple-choice set of reasons PSMTs had given that are associated with the answer they gave to the first tier, in addition to the mathematically accepted reasons, and one blank space to write reasons that are not included in the alternative's distracters.

Furthermore, the table of specifications was prepared to check for the fair distribution of propositional knowledge statements recognized from the plane geometry content. The following table shows the specifications.

Table 3.2 Table of specification

Content	*Sub-topic of the content area*	*No of item*	*Propositional Knowledge*
Plane Geometry (Math-111)	Triangle	12	1,3,5,11,12,19,26, 28,31
	Quadrilaterals	8	2,4,7,8,9,17,18,20,21,24,25
	Circles	10	10,14,22,27

Finally, after the development of the item, the instrument was refined. The items were then reviewed, and each item was revised again based on comments from professionals in mathematics education and the PSMTs' responses. The final version of the TTGDT for assessing the plane geometry concept consisted of 30 items, which were retained and prepared for a pilot study. At this stage, the implementation of the pilot study starts.

In scoring the two-tiered tests, there are different ways to analyze the two-tier multiple-choice items. For example, Chandrasegaran, Treagust, and Mocerino (2007) and Cengiz (2009) considered the two-tiered test item a correct answer if both the content (first tier) and reason (second tier) were correctly answered. The following table shows the scoring methods of PSMTs' answers on TTGDT.

Table 3.3 The scoring criteria of TTGDT and its explanations

S. No	Tiers	Explanation	Score	Remark
1	**T – T** (True – True)	Two correct answers to first and second tiers	1	Understanding
2	**T – F** (True – False)	One correct answer to first tier	0	Misconception
3	**F – T** (False – True)	One correct answer to second tier	0	Guess
4	**F – F** (False – False)	Two incorrect answers to first and second tiers	0	Not understanding
5	**F – F** (False – False)	Answering more than one choice/no answer at all	0	Not understanding

PSMTs' answer to each item was scored correct when both the correct choice (answer) and reasons are correct with full credit (1). The instrument consisted of 30 two-tier multiple-choice objective test items.

3.6.2 Geometry Procedural Knowledge Test

The geometry procedural knowledge tests (GPKT) were developed by the researcher. The development of these tests was based on the objectives of the module, questions in the Math-111 module, and those questions prepared for the exit-exam (COC) of PSMTs. GPKT was used in this study and comprises 15 items each developed by the researcher to measure PSMTs' geometric procedural knowledge before and after the intervention. Furthermore, a table of specifications based on the revised Bloom's taxonomy was used in the preparation of the GPKT items. The GPKT is used to measure procedures, geometric rules, formulas, and algorithms used to solve geometric problems.

Then, the tests were pilot tested with a group of year III PSMTs who had completed the course. While administering the pilot test, teacher educators were requested to record any questions or clarifications raised by PSMTs and the adequacy of the stipulated 30 minutes for completing the test. Based on the teachers' educator observations and suggestions and the PSMTs' verbal feedback and answers on the tests, further modification of some questions was made. Thus, out of 15 workout items developed, three of them were removed, and a total of 12 items were used (see the appendix). The final 12 items were administered to all three groups to assess PSMTs' procedural knowledge of plane geometry before and after the intervention.

The GPKT scoring scheme was adapted from various sources. Based on the following guidelines, a maximum of 4 points and a minimum of 0 points were assigned. The rubric in Table 3.4 was used to assess the learners' procedural knowledge, while the rubric in Table 4 was used to assess their conceptual knowledge.

Table 3.4 Scoring rubric for geometry test: Procedural Knowledge Test

Scores	Indicators of Procedural knowledge
4	Representing using appropriate symbols, drawing, formula and procedures efficiently Executing all the steps correctly with appropriate reasons. Work showed clearly with logical reasoning.
3	Not precise in using geometry symbols, formula and procedures Executing more than half, but not all, of the step on solving the problem correctly. Calculations are completely correct with partially correct reasoning.
2	Executing correctly at most half of the relevant steps on solving the problem. The problem was correctly solved but then redirected to incorrect procedures. The calculations are mostly correct but contain minor errors or incomplete reasoning.
1	Presenting only one correct step on solving the problem. Applying wrong procedures to solve the problem/or error in fundamental reasoning. Calculation contains major error.
0	No answer given Incorrect answer given. A limited of work shown. Calculations are completely incorrect, incoherent or irrelevant.

The rubrics were developed by modifying the rubrics suggested by Khashan (2014) and Poh, Lai, Hoe, and Muthoosamy (2015) to suit the current study. Tularam and Hulsman (2014) used two separate rubric schemes for evaluating the procedural knowledge and conceptual knowledge of first-year university mathematics students in algebra (e.g., linear and quadratic functions, limits). Khashan (2014), on the other hand, recommended a general marking rubric that could be used to evaluate and score primary school teachers' procedural knowledge and conceptual understanding of rational numbers.

3.6.3 Geometry Reasoning Skill Test

The geometry problem proof test (GPPT) was used to assess PSMTs' reasoning skills on plane geometry concepts. These test items were adapted from different literature depending on the objectives of the module (Math-111) used for teaching the plane geometry course in CTE for primary pre-service mathematics teachers.

Following a review of various tools for assessing geometry reasoning skills, a contextualized instrument, the GPPT, was adapted from Tobin and Capie (1981) and used as a pre- and post-test. After a review and feedback from peers and mathematics teacher educators, the original eight task items were revised, and finally the revised five task items were used to collect data. The scoring scheme of the test was based on the guidelines provided by McCrone and Martin (2004). Thus, the rubrics with a 5-point rating scale used on written answers to measure geometric concepts of reasoning were developed and checked by members of two panels of mathematics teacher educators. For each question's answer points, the rubrics were assigned as follows:

Table 3.5 The rubric scoring criteria for GPPT item

Score	Explanation
0	Incorrect answer and invalid evidence/reasoning.
1	Correct answer and invalid evidence/reasoning or incorrect answer and slightly valid reasoning.
2	Correct answer and vague/ambiguous reasoning.
3	Correct answer and slightly valid reasoning.
4	Correct answer and valid reasoning.

3.6.4 Geometry Attitude Scale

The tools such as Fennema-Sherman Mathematics Attitudes Scales (FSMAS) and the Mathematics Attitudes Scales (MAS) other tools that have been used to measure learners' attitudes towards mathematics. However, Attitudes Toward Mathematics Inventory (ATMI) is one of the latest instruments, with a more distinct and cohesive factor structure than FSMAS, which includes attitudes towards mathematics, perceptions about mathematics as a male-dominant learning area, and perceptions about parental and teacher support. The ATMI is one of the newest tools to assess students' attitudes toward mathematics (Chamberlin, 2010). It was made to offer a cost-effective substitute for the current mathematics attitude measures (Tapia & Marsh, 2004).

Indeed, existing instruments designed to measure learners' attitudes toward mathematics are either old, or provide data on validity and reliability only with western samples. The ATMI originally consisted of 40 items with four sub-scales, such as self-concept, value, motivation, and enjoyment. However, after feedback from experts and a pilot test analysis, eight items were excluded because of their low inter-item correlation with the overall item. It is a 5-point Likert scale that assesses positive and negative dimensions of attitude toward geometry. Students select among the five alternatives, namely: strongly agree = 5, agree = 4, neutral = 3, disagree = 2, and strongly disagree = 1.

The wording of the adapted item has been changed to fit the issues of this study. For instance, the item "*I am always under a terrible strain in a mathematics class*" was improved to "*I am always under a terrible strain in a geometry class.*" The negative items were scored in reverse order. (See appendix).

3.6.5 Interview

The qualitative data was embedded into the quantitative data for credibility purposes or to increase the integrity of the quantitative findings. The validity and consistency of the findings of the study are unquestionably improved by the collection of data from different sources (Cohen & Manion, 1989). In addition to quantitative data collection, qualitative data collected through semi-structured interviews was used to assess PSMTs' understanding of plane geometry concepts and the benefit they gained from the intervention. Thus, a semi-structured interview was used to examine PSMTs' experiences, attitudes, and favorites during their intervention. Further, semi-structured interviews were conducted after the intervention and post-test in order to elaborate further, enhance, illustrate, and clarify the results of the pre- and post-tests. It is also the most commonly used data collection method in qualitative research (Yin, 2003).

The semi-structured interviews were most beneficial because probing was possible to understand PSMTs' experiences when working with technology integrated guided inquiry. Interviewees were interviewed at their respective CTEs after the post-test. Therefore, for the purpose of this research, three PSMTs from each group (a total of nine PSMTs) were taken based on their score on conceptual understanding: i.e., one PSMT whose performance was good, one whose performance was average, and one whose

performance was low using simple random sampling. An interview with each PSMT continued for 30 to 50 minutes and was audio recorded and transcribed.

3.6.6 Observation

In order to learn more about the phenomenon being studied, observation might be used in specific contexts for individual behaviors (Driscoll, 2011). In this instance, observations in the classroom were made to evaluate how well the intervention was being implemented and to assess the teaching and learning process in general. For this purpose, the observation data were manually collected weekly for eight weeks. For evaluation of proper implementation of the intervention, a researcher prepared an observation check list based on a TGIBL-based 5E instructional model and a GIBL-based 5E instructional lesson plan. An observation checklist consisting of 20 items with a Yes (1), Moderate (.5), and No (0) scale was prepared by the researcher and used during observation to evaluate the extent of the implementation (see Appendix).

During the observations, researcher also verified that the technology being used was the GSP. In addition, researcher noted how these PSMTs interacted with and used the technology and how they interacted with both the course instructor and their classmates. These observations were also used to verify the fidelity of the GSP activities and how these GSP activities were used by the instructor to promote students' sense-making and conceptual understandings of the concepts.

3.7 Validity of Instrument

Validity refers to making sure that the research tool measures what it is intended to measure. Validity is the extent to which a test, questionnaire, or other measurement tool

captures the anticipated outcome (Tavakol, Dennick, & Tavakol, 2011). The validity of the instruments was tested in various ways i.e., validated through feedback from experts in research and mathematics education. In addition, validity also determines the degree of the accuracy of study findings (Polit & Beck, 2004).

The two most frequently used aspects of validity are content and face validity. A content validity measures how well it adequately covers the subject being studied (Kothari, 2004). The instrument must demonstrate that it fairly and completely covers the domain or items that it professes to cover in order to demonstrate this form of validity (Cohen et al., 2007). Additionally, content validity determines whether the questions that are used in the tests are relevant for what they are intended to test and the adequacy of the content covered in the instruments (Kimberlin & Winterstein, 2008) while face validity is determined by judging on the appearance of the instrument (Cook & Beckman, 2006).

Accordingly, validity can be determined by using a panel of experts who shall judge how well the measuring instrument meets the standards (Kothari, 2004). Thus, experts from mathematics education conducted both the content and the face validity without using statistical tests (Kimberlin & Winterstein, 2008). As a result, the content and face validity of the instruments were evaluated by professionals from curriculum and instruction, psychology, and mathematics education.

To this effect, experts in mathematics education were given for the evaluations of data gathering tools, such as plane geometry procedural knowledge, geometry reasoning and conceptual understanding. To ensure the content validity, a PhD candidate in mathematics education, supervisors and two geometry mathematics teacher educators

from CTEs reviewed the test items to check the consistency between what is in the module (objectives, concepts) and items, adequacy, clarity and representation for measuring geometric knowledge and understanding and mistakes in the answer key. Moreover, the table of specification was employed to ensure content validity. Finally, amendments were made by considering the feedbacks and comments obtained from these panels of experts.

In a similar way, the Geometry Attitude Scale Questionnaire (GASQ) was translated into Afan Oromo by Afan Oromo language experts and given to experts in the Afan Oromo language for further reconciliation. The attitude survey was revised in light of the feedback from Afan Oromo language specialists. Without having access to the original material, the Afan Oromo translation was likewise translated back into English. An expert reviewed the back-translations for inconsistencies, compared them to the original text to find them, and discussed any necessary revisions with the translator who completed the reconciliation. This translation process was adapted in many ways, but the fundamental steps are as follows: forward translation, back translation review and discussion, and finalization (Tyupa, 2011). The review procedure, in which a reviewer compares the back-translation with the original and looks for any errors or issue areas, is, in my opinion, the most crucial step in this process.

Finally, after completing the validation of materials, a pilot study was conducted to examine for instrument reliability and item analysis. In the following section, the procedures of pilot testing, item analysis and reliability checking will be discussed.

3.8 Piloting the Materials for Intervention and Instruments

Before the commencement of this research, the instructional materials and tools for data collection were piloted. The piloted CTE undertaken to donate 4 hours over the course of two weeks to help with the pilot research. The main purpose of the pilot study was for further evaluation of the applicability of module prepared the level of difficulty and discrimination of test items and reliability of instrument. The researcher conducted piloting after agreed to pilot with third year PSMTs who were completed the Math-111 course. In addition, in the harmonized curriculum of CTE, PSMTs took introduction to computer and fundamentals of data. This was a good opportunity for the pilot intervention, as it resolves problems of basic computer knowledge. Due to the time limitation, the pilot study could only examine the feasibility of instructional materials and achievement assessments.

First, PSMTs and mathematics teacher educators received an orientation on the TGIBL approach based on the 5E lesson plan. The researcher conducted the orientation for an hour in the CTE computer lab. Second, a two-hour implementation of the GIBL approach using a 5E lesson plan took place in a normal classroom. Throughout the pilot, data were gathered through classroom observations and conversation with participants (teacher's educator and some selected PSMTs).

In the piloting process, the method of implementation of the model was observed and revised based on the feedback from participants. For instance, feedback was gathered regarding whether the subject needed more supplementation for visualization through manipulatives and YouTube videos in the lesson activities utilized during the elaboration phase. A mathematics teacher educator suggested disconnecting the internet during

instructional phases in case the accessibility of the internet unfocused participants' attention.

On the same manner, the final versions of data collection instruments such as geometry procedural knowledge test (GPKT), geometry conceptual understanding test (TTGDT), geometry reasoning skill test (GPPT) and geometry attitude scale questionnaire (GASQ) were piloted to check for their reliability. The instruments were administered to 80 third year primary PSMTs those who were completed the course content. In the following sections the item analysis and reliability of the test will be discussed.

3.9 Reliability of the Instruments

According to Leech et al. (2005), reliability refers to the degree to which many items, measures, or evaluations are consistent with one another and the degree to which each measure is free of measurement error. Reliability is the consistency of scores or the extent to which participants and/or rate scores are free from error.

There are five forms of reliability estimates such as: (1) stability (measured by giving the same instrument twice/test-retest reliability); (2) equivalence (correlation of two forms of the same test); (3) equivalence and stability estimate (two forms given at different times); (4) internal consistency (correlation of items measuring the same trait); and (5) agreement (existence of some type of co-efficient agreement, expressed as either a correlation or as percentage of agreement/inter-rater reliability).

In this study, different reliability estimates are used. The most widely used method for estimating internal consistency reliability are Cronbach's coefficient alpha for scales and Kuder-Richardson 20 (KR20) coefficient alpha for testes (Ary, et al., 2010). KR20 and

Cronbach's alpha are the most frequently reported estimates for internal consistency, which is a measure based on correlations between different test items on the same test. KR20 can only be applied if the test items are scored dichotomously. Cronbach's alpha coefficient is recommended as measure for internal consistency when the test items in a study have more than two possible scores (Wells & Wollack, 2003; Gay, Mills & Airasian, 2006).

Thus, for this study, the researcher used Cronbach's coefficient alpha for scales (GASQ), KR20 test for conceptual understanding and inter-rater reliability (ICC) for GPKT score and GRS score as checking mechanism of reliability of instruments after pilot testing. During the calculation of TTGDT scores, correct responses for both tiers (first and second tier) were coded as 1 and incorrect responses in any one of the two tiers were coded as 0, hence, the maximum score was 30 and the minimum was 0. To determine the reliability of the item TTGDT, final version was pilot tested on 36 PSMTs in CTE that wasn't included in the sample study (CTE1). The reliability coefficient of the TTGDT test item was computed to be KR20 = 0.87. This reliability coefficient showed a high level of internal consistency and hence the instrument was considered reliable for the study (Anikweze, 2013). For instance, Andale (2017) concurred that a test must have a reliability coefficient KR20 of 0.5 and above to be acceptable and reasonably reliable.

Furthermore, the test items were subjected to psychometric analysis. This was an attempt to determine the quality of the test in terms of how difficult and discriminating the test items might be using the classical approach. These were calculated by computing the difficulty and discrimination indices for each of the 30 items of the TTGDT.

Moreover, a psychometric analysis of the test items was performed. Thus, difficulty (P) and discrimination (D) indices computed to evaluate the quality of the test items. The difficulty and discrimination indices for each of the 30 items of the TTGDT computed and the value are D = .49 and P = .33. This value is in accepted range (Metha & Mokhasi, 2014). (See **Appendix A1**)

To analyze the rubrics obtained from GPKT and GRST for measuring PK and RS in plane geometry, the inter-rater reliability (IRR) for the work-out or open-ended items are used (Liao, Hunt, & Chen, 2010). IRR is considered to be acceptable if the intera-class coefficient (ICC) is greater than 0.75. IRR is considered to be very good if the ICC is greater than 0.9 (Portney & Watkins, 2008). ICC is an analysis technique which includes more than two raters used to assess different scoring sets, and in cases where there is missing data, it has three basic models: the one-way random effects model, the two-way random effects model, and the two-way mixed model. The two-way mixed model, each item is scored by all raters in the population concerned. In essence, ICC is used for data at equal intervals and ratio scale level; it is also used for data at ranking level (Gisev et al., 2013). For calculating the ICC, SPSS software was utilized.

Cohen's kappa and intra-class correlation (ICC) are two methods for calculating IRR. Cohen’s kappa is utilized for nominal or categorical data while ICC used for continuous data (Iacobucci, Grayson, & Rust, 2001). IRR differs in the way it quantifies the relationship between two raters over the evaluation of multiple subjects. IRR is usually reported in terms of linear correlation or ANOVA analysis (Tinsley & Weiss, 1975). A correlation coefficient, or ICC, measures how consistently judges rate a group of items

(Field, 2013). The IRR of the researcher's scores for both GPKT and GRST were compared to the scores of his colleague's scores.

The calculated average measure of ICC of the pilot test was .879 and .827 for GPKT and GRST respectively are acceptable values (Graham, Milanowski, & Miller, 2012). The p-values for both tests were $p < .001$ and this indicates that there were no significant differences between the scores of the raters. If the ICC is greater than 0.75, the reliability is considered to be acceptable (Morgan, Leech, Gloeckner, & Barret, 2013). (See **Appendix A2 & A3**)

Geometry Attitude Scale Questionnaire (GASQ) is an instrument adapted from Tapia (2004). For instance, "*Mathematics does not scare me all*" changed to "*Geometry does not scare me at all*." when used as a GASQ. The instrument contains 40 positive and negative items under four subscales: self-confidence (15 items), value (10 items), enjoyment (10 items) and motivation (5 items), was employed for pilot study in CTEs. The original English questionnaire was translated into the Afan Oromo language, the medium of instruction in CTEs by lecturer from CTE who specializes in teaching the language. After the pilot study, exploratory factor analysis (EFA) was conducted. EFA is primarily used to provide a concise summary of data for easy interpretation and comprehension of the patterns and correlations between the measured variables.

Researchers might narrow their emphasis on fewer elements that explain the structure of their studies by regrouping the observed variables into useful categories. However, EFA uses flexible statistical techniques, and the accuracy of the results is strongly correlated with the researcher's ability to make good selections (Tabachnick & Fidell, 2007). In

order to employ the proper processes in EFA, the researcher must make deliberate and well-informed judgments. In fact, there is not another statistical method in the literature that is designed as powerfully as EFA and causes many debates on its correct use (Goretzko, Pham, & Bühner, 2021).

Despite the fact that sample size is a key issue in EFA, the literature offers a variety of suggestions and a number of general guidelines (Tabachnick & Fidell, 2007). Several academics believe that a sample size of 300 would be enough. Some researchers, however, believe that smaller sample sizes are likewise sufficient. For instance, Sürücü, Sesen, and Maslakçı (2021) considered 200 samples and Beavers (2013) assumed 150 samples to be sufficient. Also, Mundfrom, Shaw, and Ke (2005) claimed that even samples smaller than 100 are sufficient for factor analysis when the population is high enough and the factors are represented by many items. Moreover, Winter, Dodou and Wieringa (2009), expanded on these views, arguing that a sample size of 50 or less may be sufficient in behavioral studies.

In general, the larger the sample size, especially in relation to the number of variables, the more reliable the resulting factors. Insofar as the communalities of the items with the other items are large, or at least somewhat high and changeable, sample size is less important for component analysis. Sample sizes can really be rather modest when communalities are strong (> .60) and each component is described by a number of items (Henson & Roberts, 2006). Others, such as Guadagnoli and Velicer (1988), stated that solutions with correlation coefficients greater than 0.80 necessitate smaller sample sizes, while Sapnas and Zeller (2002) argued that even 50 cases may be adequate.

In order to gain more information about the GASQ, researcher explored it from a variety of perspectives about the adapted questionnaire (Tapia & Marsh, 2004; Chamberlin & Powers, 2013). EFA analysis was used in the creation, improvement, and assessment of tests, scales, and measurements (Williams et al. 2010). Before computing the factor analysis, the overall indicators of correlation factor using Bartlett's Test of Sphericity and Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy must be checked. Both of these methods test whether sufficiently large relationships exist within the data set of interest to perform EFA. The researcher can acquire more about how to organize survey items through sampling adequacy. It may be easier to understand the constructions being studied if things are grouped into a collection of interpretable components. The EFA correlation matrix's measures of sampling adequacy assess how strongly one item is associated with other items (Laura, Burton, Stephanie, & Mazerolle, 2011).

The KMO test indicates whether or not each component predicts enough items. The KMO value should be greater than 0.70 and not less than 0.50. According to Netemeyer et al (2003) stated that a KMO correlation above 0.70 is considered adequate for analyzing the EFA. The result of Bartlett's test for Sphericity is a chi-square that must be significant. The Bartlett test should be significant (i.e., a significance value of less than .05); this means that the variables are correlated highly enough to provide a reasonable basis for factor analysis (Tabachnick & Fidell, 2007). The table below shows such results.

Table 3.6 KMO and Bartlett's Test

Kaiser-Meyer-Olkin (**KMO**) Measure of Sampling Adequacy		.884
Bartlett's Test of Sphericity	*Approx. Chi-Square*	3441.484
	df	406
	Sig	.000

From the table above, the KMO test showed the value .884 which is greater than the threshold value .70 and the Bartlett's test of Sphericity was significant (χ^2=3441.464; *df* = 406; $p < .001$). Therefore, this indicates that there are enough items to predict each factor and the variables are correlated highly enough to provide a reasonable basis for factor analysis (Leech, Barrett & Morgan, 1994).

Factor extraction was then calculated. There are several techniques for factor extraction in this situation. Principal axis factoring (PAF), image factoring, maximum likelihood (ML), alpha factoring, unweight least squares, generalized least squares, and canonical correlation analysis are a few of the most popular (Tabachnich & Fidell, 2007; Thompso, 2004). These techniques each show distinct advantages and disadvantages. However, the most often utilized methods in research are principal component analysis (PCA) and principal axis factoring (PAF) (Tabachnick & Fidell, 2007; Henson & Roberts, 2006). The decision whether to use PCA or PAF is fiercely debated among analysts (Henson & Roberts 2006), although the practical differences between the two are often insignificant. PCA is suggested for use when no prior theoretical basis or model exists (Gorsuch 1983). According to Costello and Osborne (2005), factor analysis is preferable to PCA, which is only a data reduction approach. If a researcher has initially developed an instrument with several items and is interested in reducing the number of items, then the PCA is useful (Netemeyer et al. 2003).

The researcher must choose how many constructs to keep for rotation after the extraction step. In comparison to earlier phases, factor retention is more crucial. There are several criteria available to help with these choices, but they don't necessarily provide the same or even comparable outcomes (Zwick & Velicer, 1986; Thompson & Daniel, 1996). The Scree test (Cattell, 1966), Kaiser's criterion (eigenvalue > 1 rule) (Kaiser, 1960), the cumulative percent of extracted variance, and parallel analysis (Horn, 1965) are the techniques for keeping factors. According to Kaiser's (1960) technique, only constructions with eigenvalues larger than one should be kept for interpretation. Because of its theoretical foundation and usability, this method may be the best recognized and most often applied in practice (Fabrigar et al. 1999). Furthermore, in order to produce a more interpretable and simplified solution, a rotation that maximizes high item loadings and minimizes low item loadings must be computed.

In addition, through loading values, factor analysis establishes the number of factors within a collection of variables. Frequently, variables corresponding to a factor are kept, while matching to some factors or no factors are eliminated (Tabachnick & Fidell, 2007). There are different ideas about the cut-off for factor loadings. According to Yong and Pearce (2013), the researcher often uses his expert decision to determine the cut-off point of factor loadings & the number of factors. Thus, the pilot study's estimated cutoff point for factor loadings was 0.40.

Table 3.7 Factor Loadings for Rotated Factors

Extracted items	Factor Loading				Communality
	1	2	3	4	
SC3: My mind goes blank and unable to think clearly when working with geometry.	.84				.72
SC10: Geometry does not scare me at all.	.82				.67
SC4: Studying geometry makes me feel nervous.	.78				.65
SC9: I have selected geometry as my area of emphasis.	77				.65
SC14: I am always confused in my Geometry class.	.76				.61
SC2: When I hear the word geometry, I have a feeling of dislike.	.75				.55
VA19: Geometry is important in everyday life.	.73				.55
SC7: I cannot recall many geometry concepts that were hard for me to understand.	.73				.59
VA22: I think of many ways that I use geometry outside of school.	.68				.56
SC6: I am always under terrible strain in geometry class.	.62				.41
VA18: Geometry helps develop the mind and teaches a person to think.		.83			.68
VA21: Geometry courses would be very helpful no matter what grade level I taught.		.81			.72
VA16: Geometry is a very worthwhile and necessary subject.		.80			.68
VA20: Geometry is one of the most important subjects for people to study.		.77			.63
SC15: I believe I am good at solving geometry problems.		.77			.66
VA17: I want to develop my geometrical skills.		.71			.56
VA24: I believe studying geometry helps me with problem solving in other areas.		.69			.55
SC5: Geometry makes me feel uncomfortable.		.68			.48
VA25: A strong geometry background could help me in my professional life.		.60			.50
VA23: I think studying advanced geometry is useful.		.55			.44
EJ30: I would prefer to do an assignment in geometry than to write an essay.			.75		.83
EJ29: Geometry is enjoyable and stimulating to me.			.73		.71
EJ27: Geometry is very interesting and I have usually enjoyed it.			.71		.72
EJ34: Geometry is dull and boring.			.46		.68
MT38: I find learning the geometry is motivating.				.82	.63
MT37: I have taken geometry classes even though it was not required.				.78	.56
MT36: It wouldn‘t bother me at all to take more geometry course.				.62	.56
MT40: I liked and motivated geometry class of my college.				.49	.47
Eigenvalues	6.05	5.86	2.63	2.61	
% of variance	20.86	20.21	9.08	8.99	

Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization. Loadings < .40 were omitted.

As seen from the table 3.7 above, these initial communalities represent the relation between the variable and all other variables (i.e., the squared multiple correlation between the item and all other items) before rotation. The initial communalities are all positive and above 0.30, which is good. Also, the eigenvalues of the four factors are larger than 1.0, which is a typical requirement for a factor to be effective. The Eigenvalues of the factors 1, 2, 3 and 4 were respectively, 6.05, 5.86, 2.63 and 2.61. The percentage of variance explained by each of the factors is 20.86, 20.21, 9.08 and 8.99. The total percentage of variance explained was 59.14.

The researcher by making thorough examination further reduced the variables based on four factors. These factor loadings found questions with common themes that load into the same factors (sub-scales). These four factors primarily influenced PSMTs' attitude. They are self-confidence (SC), enjoyment of mathematics lessons (EJ), Motivation (MT) and value (VA) towards learning geometry. The findings are similar to those of Awang, Ilias, Che Hussain, and Mokhtar (2013). After EFA, twenty-eight variables with their corresponding factor loadings with SC (10 items), VA (10 items), MT (4 items) and EJ (4 items) were extracted. Furthermore, the reliability of each sub-scale was checked using Cronbach's alpha.

Table 3.8 Reliability Coefficients of the Variables

Variables	**No of items**	**Cronbach alpha**
Self-confidence	10	.921
Value	10	.897
Motivation	4	.892
Enjoyment	4	.739
Attitude	28	.913

From the table above, all the Cronbach alpha value are above the acceptable value of .70 (Nuunnally, 1978). During the reliability check, 2 items were also deleted due to low inter-item correlation and finally a total of 27 items were used for final study. Therefore, the test items were considered as reliable and valid instrument to be conducted in the current study. The following table shows the summary of the reliability of the tools used for the current study.

Table 3.9 Cronbach alpha, KR20 and ICC reliability coefficient value of the test items

		Item Index		**Reliability coefficient**		
Tests type	No	**P**	**D**	**Cr. Alpha**	**KR20**	**Remark**
TTGDT	30	.49	.33		.87	Reliable
GPKT	15			.88		Reliable
GPPT	12			.83		Reliable
GASQ	**28**			**.913**		**Reliable**
Self-concept			10	.921		Reliable
Value			10	.897		Reliable
Enjoyment			4	.739		Reliable
Motivation			4	.892		Reliable

Subsequently the development and necessary amendments were made on instruments, the final version of the tests and questionnaires were administered as pretest and posttest to determine PSMTs procedural knowledge, conceptual understanding, reasoning skill and attitude before and after the interventions.

3.10 Procedures of Data collection

Two CTEs, two mathematics teacher educators, and three sections were first chosen and randomly assigned to treatment and comparison groups. Two treatment groups and one comparison group were used in this study. From two sections, one of the sections in the

selected CTEs was assigned to the TGIBL approach and the second section assigned to the GIBL approach randomly by lottery method. These were considered experimental groups in one of the CTE. The third section of the other CTE, which was considered as a comparison group, was assigned to traditional teacher-centered approach.

Before the implementation of the intervention, the mathematics teacher educator who was assigned as a research assistant had given training about the purpose and details of the intervention process. The training on the TGIBL and GIBL instructional material (the 5E lesson plan) was given to the mathematics teacher educator who was assigned to the treatment groups. At the training, the researcher provided brief information about the goal of the study, methods for implementing the intervention, and tasks to be completed throughout the intervention. The details of the implementation process, including the application of the activities at each phase of the 5E learning cycle, were discussed and clarified. The training also consisted of how the 5E lesson can be prepared with practical examples.

For each lesson, detailed lesson plans for 5E learning cycle model instruction together with appropriate activities and technology integrated were prepared by the researcher and before each lesson; they were shared with and explained to classroom teacher. The researcher coached and supervised the regular class teachers of the experimental groups using a mock teaching exercise, lesson plan prepared by the researcher while the teacher in control group was taught normally using the lecture method. The researcher kept monitoring the teacher of experimental group to ensure that he did not deviate from the procedure of instruction given to them.

In the same manner, side by side, PSMTs of the experimental group were trained for the usage of basic tools of GSP for one week (four class hours) in a computer laboratory. They were trained the usage of the essential tools of the software and how to make the basic constructions in GSP using these tools, such as constructing a point, line, circle, regular polygon, rotating an object around a point, or reflecting an object. One week before the pretest administration, the training session was completed.

About a week before the interventions underway, the geometry conceptual understanding test (TTGDT), geometry procedural knowledge test (GPKT), geometry problem proof test (GPPT), and Geometry Attitude Scale towards plane geometry learning (GASQ) were administered to PSMTs in all three groups as pre-tests in their respective college classrooms by their classroom teachers.

PSMTs were informed about the purpose of the tests and then asked to complete the questions on their own. The geometry conceptual understanding test (TTGDT) and Geometry Procedural Knowledge (GPKT) were applied in order to determine PSMTs' level of conceptual understanding and procedural knowledge of geometry concepts respectively. Similarly, geometry proof problem test (GPPT) was applied to see the reasoning skills PSMTs on geometry problems. In the same manner, the purpose of using the Geometry Attitude Scale toward geometry learning (GASQ) as a pre-test was to measure PSMTs' attitude toward plane geometry as a school subject. After the interventions, the same instruments were also administered to all three groups as a post-test in order to examine the effect of treatment.

Before the starting of the interventions, PSMTs in each group were pre-tested, experimental groups were heterogeneously divided into groups with five and four members each, depending on their academic ability levels as obtained from their pretest results with the help research assistant teacher. The PSMTs' in the comparison group were not divided into groups. All groups of experimental and comparison were instructed for a time span of eight weeks (32 class hours in total for each group) with different teaching methods. There is four hours of mathematics lessons in each week, and each lesson hour lasted 1:40 minutes in all groups.

During the intervention, normal class schedule was used. The following is an example of how the intervention was implemented in one of the experimental groups which was assigned with TGIBL using the topic circle plane geometry module (Math-111) unit. All groups were used the same geometry module Math-111.

In the traditional teacher-centered/comparison group conventional method implemented; teacher mainly used lecturing method to teach plane geometry concepts. The comparison group was taught based on teacher explanations, questioning, discussion, and theorems and activities suggested in the module. The teacher explained, defined, and described each concept as it is on the module. PSMTs listened to the teacher, took notes, followed the teacher from the textbook throughout the lessons. After finishing his explanation, he directed questions to whole class to lead class discussions on the key concepts. Majority of the class time was used for explanation and discussion of the questions directed by the teacher. The remaining time were used for worksheet practices that required written responses or reading assignments from module. The lessons generally ended with discussing the answers to worksheet questions with a teacher directed strategy. Teacher

gave homework, assignments from the module, which can be reviewed in the coming classroom session. He also applied quizzes from time to time to assess students' learning process. Providing clear and detailed information to PSMTs was the main idea behind the teacher-centered traditional instruction.

The lessons in EG were conducted by using the lesson plans, activity sheets and worksheets which were developed by considering the objectives of the CTEs geometry module suggested by Ministry of Education. For the experimental groups, TGIBL and GIBL approach has been used based on 5E lesson plan, which is a student-centered rather than teacher-centered approach. In this approach the teacher, act as a facilitator and provided appropriate guidance.

The worksheets were designed in such a manner that the instructor could lead the PSMTs as they explored plane geometry ideas in a dynamic geometry setting. Besides, PSMTs conjectured and explored geometric concepts and ideas using GSP software. PSMTs constructed, dragged and resized the objects which were displayed on the screen dynamically. In this learning environment PSMTs created their own understanding about geometry. In addition, the PSMTs were imagining, communicating, exploring and expressing their ideas. While the PSMTs were dealing with the activities, the teacher gave feedback on the PSMTs' errors and guided them about their questions.

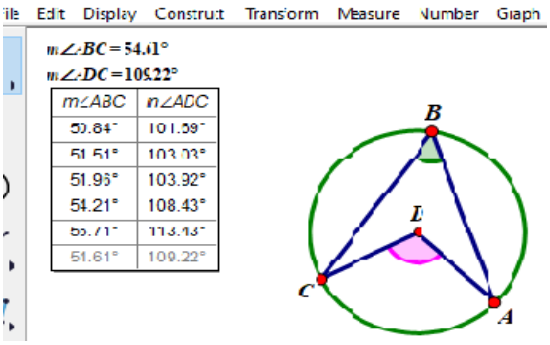
3.10.1 Treatment in the EG

The EG1 group was taught by the research assistant teacher during the treatment period. He also taught the intact experimental group in the mainstream class using a guided inquiry approach. Lessons were held in the classroom in single-period sessions and in the

computer-lab allocated for double-period sessions. In the lab, PSMTs conjectured and explored geometry topics by using GSP software, and PSMTs were given worksheets in the classroom sessions to ensure as much consistency as possible in the teaching of the unit. In each computer lab class, the learning activities prepared by the researcher were discussed by groups interactively with each other.

The following is an example of how the intervention was implemented in one of the experimental groups that was assigned TGIBL using the topic circle in the plane geometry unit. A sample of 5E lesson plan on circle activities are shown below.

Table 3.10 Sample lesson plan for EG1 (Central Angle Theorem)

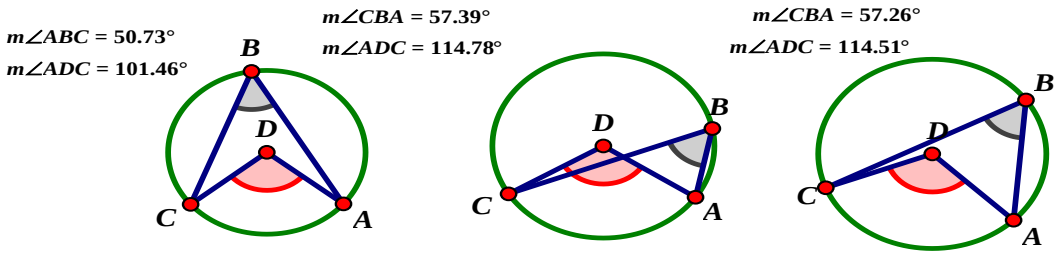
<table>
<tr><td colspan="3">Prior Knowledge:
• The measure of arcs,
• Central angles in a circle,
• Solving linear equations.</td></tr>
<tr><td colspan="3">Exclusions: PSMTs will not cover
➢ Cyclic quadrilateral,
➢ Solving problems, the measure of all inscribed angles subtended by equal arc are congruent.</td></tr>
<tr><td colspan="3">Objectives: At the end of the lesson PSMTs will:
➢ Determine the relationship between the inscribed angle and central angle of a circle.</td></tr>
<tr><td>Materials</td><td colspan="2">GSP worksheet (investigating inscribed and central angles), Computer</td></tr>
<tr><td></td><td>Teacher activities</td><td>Students Activities</td></tr>
<tr><td>Engagement</td><td>Initiate the question</td><td></td></tr>
<tr><td>Exploration</td><td>• Encourage and/or focus PSMTs' exploration.
• Guide the activities
• Probing questions</td><td>• In groups of four to five PSMTs will explore different activities, use GSP to construct central and its inscribed angle, and then explain their reasoning for their results.
• PSMTs should derive formula for the relationship between the inscribed and its central angles.
• PSMTs explore the following GSP activity themselves supported by their teacher and peers.
 

PSMTs work on the provided GSP worksheet to grasp the geometric concept(s) in an explorative manner and start to formulate observations and tentative explanations.</td></tr>
</table>

Explanation	Teacher will probe PSMTs for responses about their conclusions. (How? why? what if…?)	• After completing the exploration in groups, the PSMTs will return to a whole-group setting and present their groups' answers. Other PSMTs are responsible questioning the groups while comparing their work with the groups. • PSMTs share their experiences and start formulating and justifying their findings and explanations in groups.
Elaboration	Teacher will clear up any misconceptions and allow PSMTs to ask questions.	• PSMTs will present their findings to the class. • Discuss their finding, conjectures. • $\alpha = 2\theta$, where β is central angle, θ is the inscribe angle. • apply the concepts in other topics
Evaluation	PSMTs work evaluated Feedback for further investigations Assess learning progress	Solve more problems Answer open-ended questions; and evidence of development and change in thinking/behavior.

The following Figure shows how the GSP software used to show the relationship between the central angle and its inscribed angle. The research assistant asked PSMTs to make different similarities of the central angle theorem and estimate the angles in the diagrams that they have drawn on their own.

Table 3.11 Sample activities for EG1 group



What's the relation between the angle at the center $\angle ADC$ and the inscribed angle at the circumference$\angle ABC$?
You can use the cursor to move point A, and hence to alter angle$\angle ADC$. You can also move point C around the circumference.
If you move point C, then angle $\angle ADC$ and the size of the circle both change. Notice what happens to angle $\angle ABC$ as you make these changes.

What do you notice the relation between $mADC$ (central angle) and $m\widehat{AC}$?

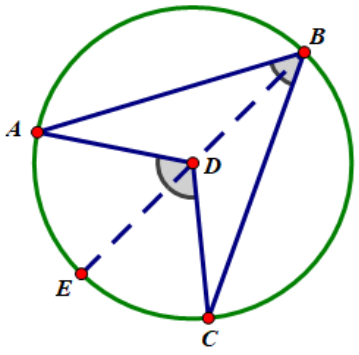
What do you conclude about the relationship between central angle, its inscribed angle and arcs of central angle?

The activity is about the angles subtended by an arc of a circle at its center. In this activity, PSMTs have been investigating the central angles of a circle and comparing the measurement of the arc to the measurement of the angle. In this activity, the measurement of the arc that is bounded by an inscribed circle also investigated.

After reviewing prior knowledge about concepts of circle geometry, the teacher asked PSMTs to discuss on the activity to explore the relationship between the measure of the central and its inscribed angle using GSP. A GSP applet are created by each groups of PSMTs, and inquired to discover what happens to the measures of $m(A\hat{B}C)$, central angle $m(A\hat{D}C)$, and arc $m(\widehat{AC})$ as:

- Point C moves around circle centered at point D?
- What if point A moves around a circle D?
- What do you conclude about the relationship between angle B and D?

And then the teacher initiated a whole-class to explore and explain or discuss on their exploration. In the class discussion, the teacher asked PSMTs to elaborate and discuss to clarify why angle the measure of angle B is half of angle D.



draw line EB that passes through points B and D.
triangle ABD and CBD are an isosceles triangle. **why?**
∠DBA ≅ ∠BAD and ∠DCB≅ ∠CBD. **why?**
∠DBA + ∠BAD = ∠ADE. **why?**
2(∠DBA) = ∠ADE. **why?**
similarly,
∠CBD + ∠DCB = ∠CDE. **why?**
2(∠CBD) = ∠CDE. **why?**

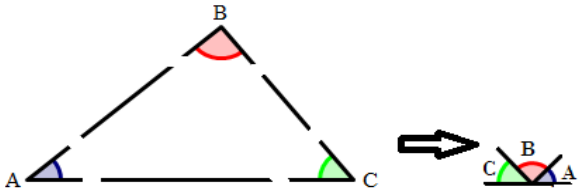
∠ADC = ∠ADE + ∠CDE. **why?**
∠ADC = 2(∠CBD) + 2 (∠CDE). **why?**
∠ADC = 2 (∠CBD + ∠CDE)
∠ADC = 2 (∠ABC). **why?**
∴ <ABC = $\frac{1}{2}$(∠ADC)

Finally, based on the unique activities employing GSP, the teacher gave a formal proof to the entire class. Although the relationship between inscribed angle and central angle

theorem was verified and physically tested, the teacher and PSMTs jointly developed an abstract mathematical proof of the theorem.

During the interventions period, the EG1 group's research assistant also taught the EG2 group. In EG1 and EG2, identical worksheets were used, and similar lesson plans were used. All lessons were delivered in the mainstream classroom, where only laptop and projector were used to show the instruction of how to conduct experiment with hands-on manipulatives and no use of GSP. Similar to EG1, PSMTs in EG2 were given paper and pencil activities and structures to review all they had learned in class.

Table 3.12 Sample 5E lesson plan for EG2 (activities on triangles)

<table>
<tr><td colspan="3">Prior Knowledge:
• the concept of an exterior angle,
• the angle measure of straight line,
• Parallel lines and traversals and corresponding angles.</td></tr>
<tr><td colspan="3">Exclusions: PSMTs will not cover
➢ Cyclic quadrilateral,
➢ Solving problems using the fact that, the measure of all inscribed angles subtended by the same arc are equal.</td></tr>
<tr><td colspan="3">Objectives: At the end of the lesson PSMTs will:
➢ prove that the sum of the measures of the interior angles of a triangle is equal to 180∘,
➢ comprehend that the exterior angle of a triangle is the supplementary angle to the adjacent interior angle,
➢ prove that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles,
➢ Find the measure of missing angles using the property of an exterior angle of a triangle being equal to the sum of the measures of the opposite interior angles.</td></tr>
<tr><td>Materials</td><td colspan="2">worksheet (the angle sum and properties of angle in triangles), manipulatives,</td></tr>
<tr><td></td><td>Teacher activities</td><td>Students Activities</td></tr>
<tr><td>Engagement</td><td>Initiate the question</td><td>Think-pair-Share</td></tr>
<tr><td>Exploration</td><td>• Encourage and/or focus PSMTs' exploration.
• Guide the activities
• Probing questions</td><td>• In groups of four to five PSMTs will be given triangles of different sizes and types.
• PSMTs explore the following activity themselves supported by their teacher and peers.
•
 

PSMTs will be instructed to tear off each angle (corner) of the triangle and fit the vertices together to see the measure of</td></tr>
</table>

		angle formed.
Explanation	Teacher will probe PSMTs for responses about their conclusions. (How? why? what if…?)	• After completing the exploration in groups, the PSMTs will return to a whole-group setting and present their groups' answers. Other PSMTs are responsible questioning the groups while comparing their work with the groups. • PSMTs share their experiences and start formulating and justifying their findings and explanations in groups.
Elaboration	Teacher will clear up any misconceptions and allow PSMTs to ask questions.	• PSMTs will present their findings to the class. • Discuss their finding, conjectures. • $A + B + C = ?$, • Relations between sum of angles in triangle and a straight angle • PSMTs deducing that the size of triangles does not affect the sum of their interior angles.
Evaluation	PSMTs work evaluated Feedback for further investigations Assess learning progress	Solve more problems Pupils will be given various types of triangles (scalene, isosceles and equilateral) with one unknown angle. Students will be asked to determine the size of the unknown angle in each triangle.

3.10.2 Treatment in the CG

The traditional teaching approach was used to teach plane geometry topics to PSMTs in the CG. The traditional instruction environment was based on the objective of the module (MoE, 2015). The homework assignments were also given from this textbook. PSMTs were given the same worksheets as the other classes. In this class, generally, the teacher acted as an information presenter and filled knowledge to the PSMTs. PSMTs were given definitions by the classroom teacher, and the teacher solved some examples on the blackboard by writing and drawing. The lessons were continued by solving questions on worksheets. The PSMTs in this group were passive receivers, meaning they were just responsible for listening to the teacher, taking notes, and solving the questions the teacher asked at their own pace.

In the ongoing of the intervention, the researcher discussed with research assistant teacher educator how to implement the lesson plan and carry out the activities in the intervention. After completions of the interventions, posttests (TTGDT, GPKT, GPPT,

and GASQ) were administered to all PSMTs in the three groups. Furthermore, in the process of the intervention, classroom observation was conducted to collect data; an interview was also conducted following the intervention after posttest data was taken. The following table summarizes the intervention and data collection matrix. The table 3.13 displays the methods for collection of both types of data and the timeline for acquisition of the data.

Table 3.13 Summary of intervention procedure and data collection Matrix

Group	Week1			Week2-to-week9	Week10	Week11
	Pretest	Training	Trainees	Intervention	Posttest	
EG1	Yes	Yes (TGIBL)	Teacher & PSMTs	Yes (Obs.)	Yes	interview
EG2	Yes	Yes (GIBL)	Teacher	Yes (Obs.)	Yes	interview
CG	Yes	No	No	No	Yes	interview

3.11 Method of Data Analysis

All of the data that the researcher collected using the various instruments was coded and assessed. Themes from interviews and classroom observations were identified and evaluated for the qualitative data, while descriptive and inferential statistics were used for the quantitative data analysis. The analyses of the quantitative data were given much weight and priority and were conducted using the Statistical Package for Social Sciences (SPSS) version 20.0 program.

To answer the research questions, the researcher used a MANOVA to compare the intervention group and the comparison group on several different, but related, All of the data that the researcher collected using the various instruments was coded and assessed.

Themes from interviews and classroom observations were identified and evaluated for the qualitative data, while descriptive and inferential statistics were used for the quantitative data analysis. The analyses of the quantitative data were given much weight and priority and were conducted using the Statistical Package for Social Sciences (SPSS) version 20.0 program.

A one-way MANOVA was conducted in order to compare the mean scores of the two experimental groups and the comparison group on conceptual knowledge, procedural knowledge, and reasoning skills in geometry as a whole to reveal whether these differences are significant or not. One-way ANOVA follow-up analyses were done to examine the impact of the teaching method on each dependent variable because the MANOVA findings only revealed significant differences between groups on the collective dependent variables.

The paired sample t-test is used when the two means to be compared are obtained from the same population (i.e., repeated measurement). When comparing two variables in individuals in a population, this method of analysis is used to draw some sort of conclusion about the observed correlation (Taylor, 2020; Field, 2005). Finally, multiple linear regressions were used to determine the strength of the relationship between the PSMTs' attitudinal constructs and their conceptual and procedural knowledge as dependent variables.

Two possible approaches are suggested for performing further analysis of an ANOVA: post hoc tests and planned contrasts. The Benforoni and Schaffe tests were applied for the post-hoc analysis. Before the analyses were made, all the required test assumptions were

verified. The descriptive statistics based on pre-test and post-test scores were reported as well. In this way, univariate and multivariate normality, homogeneity of regression slopes, homogeneity of variances, and variance-covariance homogeneity assumptions were analyzed (Field, 2013). The Skewness-kurtosis coefficients were calculated for each intervention group and variable in terms of univariate normality. To determine extreme values in terms of multivariate normality, Mahalanobis distance values for each dependent variable were calculated. Regression slopes, homogeneity of variances, and matrices of variance-covariance have been defined as homogenous for extreme dependent variables. The hypotheses were tested at a significance level of .05, a value that is commonly used in mathematics education and educational research.

For the qualitative data analysis, thematic analysis was used to identify themes within the data. The qualitative data collected through semi-structured interviews was transcribed and categorized according to themes. On the other hand, themes were not predetermined but rather emerged from the data. Hence, they were data-driven themes. I started out by listening to each individual audio recording before I started constructing themes from the data. I listened to the audio recording to familiarize myself with the content and fix any transcription errors. To acquaint myself with the participant's experience after that, I read the transcript (Creswell, 2009). Following that, each transcript was manually coded, paying special attention to each line of text. These codes were made based on the participant's use of certain words or phrases (Gillham, 2010). In a Word document, I kept track of the several codes that each transcript produced and the folder it created (Lichtman, 2013). Before proceeding to the next round of coding, I went through each group of codes and analyzed them, looking for emerging patterns and similarities.

Following the coding of all the data, the codes were clustered depending on how similar they were, and themes were found for each grouping (Braun, Clarke, & Terry, 2014). Finally, the themes were used in the qualitative data analysis of the study to interpret the findings based on how they related to the research questions.

In addition, Ponce and Pagan-Maldonado (2015) assert that the analysis of mixed data entails arranging and combining quantitative and qualitative data in order to accomplish the following goals in relation to the research topic: (1) triangulation of data; (2) complementing data; and (3) deepening the analysis. This research first provides the analysis of quantitative data, then the analysis of qualitative data, and finally a combination, or mixed-methods analysis. The qualitative data in a mixed-methods data study supports and complements the quantitative data. According to Mathison (1988), the best research practice necessitates the researcher to use triangulation to improve the validity of the findings, in which inferences founded on quantitative data are reinforced by qualitative data and vice versa. Braun et al. (2014) added triangulation as the process of using different data collection methods to cross-validate data and findings. In this case, the experimental and comparison groups' performances were disclosed by the quantitative pre- and post-tests, and the reasons for these performances were revealed by observations and interviews, thus ensuring triangulation.

3.12 Ethical Considerations

In order to endorse research ethics, researchers must obtain consent to gather information from individuals and locations. According to Creswell and Plano Clark (2007), individuals include those in charge of research locations, those who supply the data (teachers, students, etc.), and institutional review boards located on campuses.

In light of this, ethical clearance was requested from the Department of Science and Mathematics Education, AAU for the permission to collect data from CTEs. Approval was sought from the academic vice dean of the participating CTEs to do research at the respective colleges. Prior to data collection or engaging with participants, a clear explanation of the research purpose and procedures was given to the teachers and PSMTs as participants. Each participant was requested to sign a consent form (assent form for students), and the following factors were clarified with participants: Participants had a right to withdraw or refuse to participate if they so wished. PSMTs were not asked to write their name; rather, their ID code was used. Anonymity and confidentiality were respected and assured. Pseudonyms used by interviewees would be considered when publishing the findings, and the findings might be published for academic purposes where applicable. Afan Oromo, the language of instruction in mathematics, was used as the main language in CTEs. The researcher translated into the mother tongue where necessary to ensure that every participant understood the expectations and their role in the study.

CHAPTER FOUR: DATA ANALYSIS AND INTERPRETATION

4.1 Introduction

The main purpose of this study was to investigate the effects of a technology-integrated guided inquiry-based learning (TGIBL) approach based on the 5E instructional model on PSMTs' learning outcomes and attitudes towards geometry. The findings on the effect of the TGIBL approach on PSMTs' learning outcomes and attitudes towards learning geometry are presented in the following subsection of this chapter. Following the main goal of this study, which was to determine how the TGIBL method affected the teaching and learning of plane geometry in connection to learning outcomes and attitude, the following questions served as the basis for the investigation:

1. Is there a significant mean difference between experimental and comparison groups of PSMTs:
    1.1 On conceptual understanding of plane geometry?
    1.2 On procedural knowledge of plane geometry?
    1.3 On reasoning skills of plane geometry?
2. Is there a significant mean difference among PSMTs' attitudes towards learning plane geometry between the experimental and comparison groups?
3. What is the effect of attitude and reasoning skills on conceptual understanding and procedural knowledge test scores in learning plane geometry?
4. How technology-integrated guided inquiry-based learning approach does enhance PSMTs the learning of plane geometry during the intervention?

4.2 Quantitative Data Analysis

In quantitative data analysis, quantifiable and verifiable data are systematically collected and evaluated. It encompasses a statistical appliance for assessing or analyzing quantitative data (Cohen et al., 2007). The primary goal of quantitative analysis is to quantify hypothetical data scenarios. As a result, prior to starting the investigation, the researcher must establish the degree of measurement associated with the quantitative data. Thus, the variables assessed, and the tools used for this study are listed in the following table 4.1. Four continuous dependent variables (DV), one nominal independent variable (IV) (treatment with three levels: TGIBL, GIBL, CG), and three different groups of participants involved in this study could be taken as factors for utilizing the aforementioned statistical tools (Kandemir & Demibag-Kaskin, 2019).

Table 4.1 The variable types and its measurement

Variables	Type	Tools for Measuring	Nature of data
Conceptual understanding	DV	TTDGT	Continuous
Procedural knowledge	DV	GPKT	Continuous
Reasoning skill	DV	GPPT	Continuous
Attitude	DV	GASQ	Continuous
Group	IV		Categorical

DV: Dependent Variable, IV: independent Variable

4.2.1 Assumptions in Statistical Testing

Every statistical test, whether it is parametric or non-parametric, starts with a number of assumptions about the data that it will be applied to. The parametric tests typically include rigid assumptions regarding the underlying population distribution. Parametric statistics (e.g., t-test, ANOVA, etc.) are grounded on a specific distribution, such as a normal distribution. In general, parametric tests are regarded as being more powerful than

their non-parametric counterparts in detecting the differences existing among the groups, but this statement is tenable only if four parametric assumptions are met: normality of distribution, homogeneity of variance, interval data, and independence (Field, 2005). However, non-parametric (e.g., chi-square, Mann-Whitney, etc.) tests do not assume such assumptions and are assumption-free statistics (Rachon, Gondan, & Kieser, 2012). As a result of their unmet assumptions, the researchers may come to the wrong conclusions regarding the data, and the inferences formed will be erroneous. Still, certain parametric techniques are "robust" against certain hypotheses while preserving the validity of the results (Field, 2013).

Of the aforementioned assumptions, normality is the most predominant assumption about distributions. As part of exploratory data analysis, normality should be verified before conducting any inferential statistics. If any of the independent samples' normality is violated, parametric testing shouldn't be used (Rietveld & van Hout, 2015). For independent samples, tests of normality should be run separately for each sample (Orcan, 2020). Normality could be tested in a variety of ways, some of which are the Kolmogorov-Smirnov (KS) test and Shapiro-Wilk (SW) test, skewness and kurtosis, as well as the graphing of the Q-Q plot and Box's plot. For instance, both KS and SW assume the data is normal if the statistical value is insignificant. According to Field (2009), the KS test is not useful and is less accurate in practice. Another way to check the normality of the distribution of the data is through the use of skewness and kurtosis. Although the use of skewness and kurtosis values is common in practice for checking the normality of a distribution, there is no consensus about the values that indicate normality.

In this research study, the skewness and kurtosis values of an absolute value of 2 are used to define a normal distribution (George & Mallery, 2003).

Moreover, before applying non-parametric tests, the researcher may wish to consider altering the data to correct the non-normal distributions if the normality assumption is violated. This would enable us to deal with outliers as opposed to discarding them. In the following subsection, the details of the statistical assumptions that are employed in this study will be inspected and addressed rigorously.

4.2.1.1 Sample and Independence of observations

The independent of observations is one of the assumptions for quantitative analysis. That is, the results for one subject are not affected by another. Non-independent observations introduce bias, causing our statistical test to generate an excessive number of false positives (Type I error). As a result, the researcher's goal in this study is to examine the mean differences of three independent groups picked at random from two distinct CTEs. Each PSMT in the study were answered the test items on their own independent of each other. Therefore, the assumption of independence of observation was not violated.

4.2.1.2 Normality Test for Dependent Variables

Many statistical tests require a determination of the data's normality since parametric testing assumes normal data as a fundamental tenet. The normality of a given dataset can be assessed graphically or numerically. Normality is one of the essential assumptions for drawing reliable inferences about the underlying population of data (Field, 2009).

Normal distribution analysis for study variables was done in this study among the TIGBL group, GIBL group, and Traditional Method (TRAD) group. The skewness and kurtosis

measurements were used to check whether the data for each of the groups were normally distributed (George & Mallery, 2003). The following table 4.2 shows the skewness and kurtosis of the dependent variables of the pretest and posttest results.

Table 4.2 Normality Analysis for the PSMTs' on pretest results

Variables	Group	N	*M*	*SD*	*Skewness*	*Kurtosis*
Pre-CU	EG1	48	17.58	5.58	-.364	-.394
	EG2	38	18.00	4.18	.142	-.813
	CG	30	17.77	4.89	-1.066	1.876
Pre-PK	EG1	48	15.33	5.62	.738	-.799
	EG2	38	15.21	4.48	.596	-.810
	CG	30	16.03	5.50	.346	-1.481
Pre-RS	EG1	48	10.46	4.42	-.169	-.969
	EG2	38	10.53	3.56	-.182	-.865
	CG	30	11.70	3.96	-.233	-.863

The research factors for the conceptual understanding test; the procedural knowledge test, and the reasoning skill test across the three groups are shown in Table 4.2 above. The skewness and kurtosis values of all factors range between -2 and +2. Thus, according to George and Mallery (2003), all the factors are normally distributed. In the same manner, all the posttest factors are also normally distributed. The normality distribution of the posttest values is shown in table 4.3 below.

Table 4.3 Normality distribution analysis for PSMTs' Posttest results

Variables	Group	N	*M*	*SD*	*Skewness*	*Kurtosis*
Post-CU	EG1	48	27.38	6.70	-.364	-.394
	EG2	38	23.79	4.63	.142	-.813
	CG	30	21.13	6.89	-1.066	1.876
Post-PK	EG1	48	26.91	2.74	.738	-.799
	EG2	38	23.34	2.69	.596	-.810
	CG	30	21.02	2.87	.346	-1.481
Post-RS	EG1	48	16.98	2.25	-.169	-.969
	EG2	38	14.45	2.66	-.182	-.865
	CG	30	12.87	3.22	-.233	-.863

The skewness and kurtosis values of all factors range between -2 and +2. Thus, according to George and Mallery (2003), all the posttest factors are normally distributed.

4.2.1.3 Checkup for Outliers in the Dependent Variables

Finding an outlier in the data set is a researcher's first step. A data point that deviates from the norm in the dataset is referred to as an "outlier" (Dattalo, 2013). This kind of data might arise from inaccurate entry or the use of an extreme score during the data collection. Box plots were used to seek out outliers with regard to univariate normality as part of the first data screening (Dattalo, 2013). During the analysis, no univariate outliers among the intervention groups were found. The Mahalanobis distance was calculated to look for outliers in multivariate normality. The crucial chi square value must be understood in order to detect an outlier. The multivariate normality is the normality distribution of all the values of the dependent variables together. Thus, according to Pallant (2013), calculating Mahalanobis distance is adequately sufficient to validate multivariate normality and outliers. In order for multivariate normality to be assumed, the maximum Mahalanobis distance should be smaller than the critical value.

The critical Mahalanobis value for three dependent variables in the table of critical values is 16.27. The Mahalanobis distance for the post-test (GCUT, GPKT, and GRST) data in this study was 14.48, which is less than the critical value of 16.27 and less than the Cook's distance of 0.1. In addition, a small Mahalanobis distance rejects the existence of outliers as well. The following Table 4.4 shows the maximum Mahalanobis and Cooks distances of posttest data.

Table 4.4 Mahalanobis and Cook's distances for post test data

Variable		*Min*	*Max*	*Mean*	*SD*	*N*
	Maha. Distance	.08	14.48	2.97	2.21	116
Post test	Cooks Distance	.00	.11	.01	.02	116

Dependent variable: posttest group.

From Table 4.4, it can be seen that the maximum Mahalanobis distance is smaller than the critical value of 16.27, and in addition, the Cooks distance is less than 0.1. Thus, these indicate that multivariate normality is tenable as well as that there are no outliers in the samples under study.

4.2.1.4. Homogeneity of Variance for Dependent Variables

The parametric test presupposes homogeneity of variances to verify if the samples are from populations with comparable variation in a certain criterion. The equality of variances assumption assures that deviations from averages in subgroups are homogeneous, whereas the normality assumption guarantees that the data distribution is symmetric. The residuals, which account for the variations in sample data, are the outcome of deviations from normality.

There are many statistical tests that may be run to determine if the variance is homogeneous. Cochran's, Levene's, Hartley's Fmax, Barlett's, and Box's M tests are a few

of them (Dattalo, 2013). Since it was shown that several of these tests were too sensitive to non-normality, they are no longer often used. The most commonly used tests for variance homogeneity in univariate analysis (for multivariate analysis) are Levene's test and Box's M test. The F-test is employed in Levene's test to evaluate the null hypothesis that group variance is equal. If the p-value is less than .05., the assumption has been violated. In this case, another test, such as the non-parametric equivalent of the analysis, is most likely used, or appropriate post-hoc tests, such as the Games Howell test, are chosen for unequal variance (Morgan et al., 2004).

Similarly, Box's M test is a multivariate statistical test that checks the equality of multiple variance-covariance matrices. In multivariate and mixed models of variance, the Box's M test is commonly employed to check the homogeneity of the variance-covariance assumption. If the homogeneity of variance-covariance assumption is met, Wilks' lambda (λ) is used in multivariate analysis of variance to determine if there are statistically significant differences between the group means on a variety of variables. In contrast, Pillai's Trace, a more reliable multivariate test statistic, may be used to assess the multivariate results when the homogeneity of variance-covariance is violated. The following table 4.5 shows the results of Levene's test of homogeneity of variances of PSMTs' conceptual, procedural, and reasoning skills across three groups.

Table 4.5 Levene's Test on the PSMT's pre- and post-test scores of CU, PK, and RS

DV	*Levene's Statistic*	*df1*	*df2*	*p*
Pre-CU	1.938	2	113	.15
Pre-PK	1.364	2	113	.26
Pre-RS	2.387	2	113	.10
Post-CU	1.70	2	113	.19
Post-PK	.22	2	113	.80
Post-RS	2.68	2	113	.07

In Table 4.5 above, the Levene's Test results for the pre-conceptual understanding test (p =.15), the pre-procedural knowledge test (p =.26), the pre-reasoning skill test (p =.10), the post-conceptual understanding test (p =.19), the post-procedural knowledge (p =.80), and the post-reasoning skill test (p =.07) indicate (p >.05) that there was no significant difference across all groups. This shows that the assumption of homogeneity of variance was tenable. This indicates that the variance is similar across the group for the given pretest and posttest dependent variables. In the same manner, the homogeneity of variance-covariance matrices for multivariate was checked through Box's M test of equality.

4.2.1.5 Homogeneity of Variance-Covariance Matrices for Dependent Variables

The variance-covariance matrices are equal across all the cells produced by between-subjects' effects in the multivariate analysis, and the vectors of the dependent variables are assumed to follow a multivariate normal distribution. The Box's M test is used to determine if the variance-covariance matrices for MANOVA were homogeneous. Box's M test correlates all dependent variables with one another and determines whether the correlation matrices are similar. Because the Box's M test is very sensitive, unlike most other tests, the level of significance is usually .001 (Hahs-Vaughn, 2016). If the statistic is not significant at p =.001, the variance-covariance matrices must be equal. The following table 4.6 shows the homogeneity of variance-covariance matrices.

Table 4.6 Box's M Test of equality of covariance Matrices of dependent variable

	Box's M Test of Equality of Variance-Covariance Matrices [a]				
Variable	*Box's M*	*F*	*df1*	*df2*	*p*
Posttest	17.19	1.38	12	45506.02	.17

a. Design = intercept + group

According to Table 4.7, the assumption of equality for variance-covariance is tenable because the p-value is not significant. This statistic test showed that the variance–covariance matrices are the same in all three groups. Therefore, if the matrices are equal (and therefore the assumption of homogeneity is met) this statistic should be non-significant. For the linear combination of the three dependent variables of conceptual, procedural, and reasoning skill test scores, the Box's M value (17.19) is related to p =.17, which is non-significant (p >.001) for MANOVA. Hence, according to Pallant (2020), the homogeneity of the variance-covariance is not violated.

4.2.1.6 Multicollinearity or Singularity among Dependent Variables

Correlations between the dependent variables are used to determine whether or not there is multicollinearity. Any correlation higher than 0.80 indicates multicollinearity exists, even if all of the dependent variables should be connected in some way (Pallant, 2020). The Pearson moment correlations can be used to determine the linearity assumption. In this study, the Pearson product correlations have been used to determine the association between the matrices. According to Pallant (2020), MANOVA works best when the dependent variables are moderately associated. The following table shows the correlation between dependent variables.

Table 4.7 Pearson product correlations of dependent variables

Variable	*Post-GCUT*	*Post-GPUT*	*Post-GRST*
Post-GCUT	1		
Post-GPUT	.333**	1	
Post-GRST	.521**	.385**	1

***. Correlation is significant at the level .01 level (2-tailed).*

From Table 4.7 above, it can be seen that the correlation between dependent variables is not higher than the threshold of 0.8 ($r < 0.8$). Hence, linearity is not violated (Pallant, 2005).

4.2.2 Quantitative Data Analysis of Group Differences on Pre-test Scores

To investigate the effects of interventions on PSMTs' conceptual understanding, procedural knowledge, and reasoning skills, the outcomes of the pre-test and post-test were analyzed based on the aforementioned research questions. The first subsection of the analysis was done on pre-test results to identify the difference between the three groups (EG1, EG2, and CG) at baseline to see if the pretest could be a covariate for the posttest, and finally the quantitative analysis was done post-intervention to see how the interventions affected them.

The quasi-experimental research design necessitates the establishment of requirements for instructional interventions that employ technology-integrated guided inquiry-based learning in the course's lessons. Before the interventions, the pretests were administered to check whether the baseline data were equivalent or not. To confirm the actual treatment effects, to avoid bias in treatment effects, and to ensure the existence of statistical independence between treatment (posttest) and covariate (pretest), a mean difference test between variables at the pretest stage must be computed. Therefore, the intent of the pretest is to determine whether there is a difference between the three groups on geometry outcomes and attitude before the commencement of the interventions.

Prior to investigating the effectiveness of the interventions in the experimental groups and the comparison group, a one-way ANOVA test was conducted. The primary

hypothesis of the study was that the intervention groups would be comparable. Thus, according to Wiersma and Juurs (2005), the researcher must look at the homogeneity of the intervention groups before applying the instruction. Therefore, the scores obtained from the pre-tests of conceptual understanding, procedural knowledge, and reasoning skills were analyzed using a one-way ANOVA, which compares the means of the three groups as shown in tables 4.8 and 4.9 below.

Table 4.8 Descriptive Statistics of Pre-test scores on CU, RS and PK

Variable	Group	*N*	*Mean*	*SD*
Pre-CU	EG1	48	17.58	5.58
	EG2	38	18.00	4.18
	CG	30	17.77	4.89
Pre-RS	EG1	48	10.46	4.42
	EG2	38	10.53	3.56
	CG	30	11.70	3.96
Pre-PK	EG1	48	15.33	5.62
	EG2	38	15.21	4.48
	CG	30	16.03	5.50

From Table 4.8 above, the mean score of EG2 (M = 18, SD = 4.18) is somewhat higher than the mean score of EG1 and CG on pre-conceptual understanding, the mean score of CG (M = 11.70, SD = 3.96) is higher than both experimental groups on pre-reasoning skills, and the mean score of CG (M = 16.03, SD = 5.50) is higher than both experimental groups. However, to check for the significance of the difference, a one-way ANOVA was conducted. Table 4.9 below shows the ANOVA test on the pre-test score between groups.

Table 4.9 A one-way ANOVA summary table on CU, RS and PK before interventions

DV	*Sources*	*SS*	*df*	*MS*	*F*	*P*
Pre-CU	Between Groups	3.682	2	1.841	.074	.93
	Within Groups	2807.033	113	24.841		
	Total	2810.716	115			
Pre-RS	Between Groups	32.749	2	16.375	1.043	.36
	Within Groups	2807.033	113	24.841		
	Total	2810.716	115			
Pre-PK	Between Groups	12.973	2	6.487	.236	.79
	Within Groups	3105.949	113	27.486		
	Total	3118.922	115			

From Table 4.9 above, the ANOVA results revealed that there was no statistically significant mean difference between the EG1 (M = 17.58, SD = 5.58), EG2 (M = 18, SD = 4.18), and the CG (M = 17.77, SD = 4.69) at (F (2, 113) =.074, p = .93); in terms of geometry conceptual understanding, according to the groups' pre-GCUT. There was no statistically significant mean difference between the EG1 (M = 10.46, SD = 4.42), the EG2 (M = 10.53, SD = 3.56), and the CG (M = 11.70, SD = 3.96) at (F (2, 113) = 1.043, p =.36), and the EG1 (M = 15.33, SD = 5.62), the EG2 (M = 15.21, SD = 4.48), and the CG (M = 16.03, SD = 5.50) at (F (2, 113) = .236, p =.79) in terms of conceptual understanding, procedural knowledge, and reasoning skills This means that, prior to the intervention procedure, there were no significant differences between the groups in terms of the dependent variables of the study (Pallant, 2011). This result shows that the three groups were approximately equivalent on conceptual, procedural, and reasoning skills before the interventions. Therefore, it can be deduced that the difference observed on the posttest score is attributable to the implementation of the interventions.

4.2.3 Quantitative Data Analysis of Groups Difference on Posttest Scores

A one way MANOVA test was employed to examine the differences among the groups of PSMTs in terms of dependent variables conceptual understanding, procedural knowledge and reasoning skill (Pallant, 2011). MANOVA is a statistical technique that explores the relationship between several independent variables (usually referred to as treatments) and two or more dependent variables (Nimon, 2012). Furthermore, univariate analyses (ANOVA) may lead to a greatly inflated overall Type I error rate, therefore, using this approach of fragmented univariate tests results in an overall error rate which is entirely too uncertain. The use of MANOVA includes a condition that mainatains the overall rate error rate at the .05 or whatever $\alpha -$ level is pre-selected (Harris, 2001).

Although it is not always the case, these numerous dependent variables are usually composed of several measurements of the same item (Aron & Aron, 1999). Oftentimes, these multiple dependent variables consist of different measures of essentially the same thing (Aron & Aron, 1999), but this need not always the case. At the minimum, the DVs should have some degree of linearity and share a common conceptual meaning (Stevens, 2012). The fact that a MANOVA contains more dependent variables than a univariate ANOVA is clearly advantageous. To avoid inflating the Type-I error rate in the follow-up ANOVAs and post-hoc comparisons, a MANOVA was first run on the means. However, the dependent variables must, at the very least, be linear and have a combined conceptual meaning (Stevens, 2012).

Prior to conducting the MANOVA, a Pearson correlation between the dependent variables was conducted to evaluate the MANOVA assumption that the dependent variables would be moderately associated with each other ($0.2 < r < 0.8$) (Meyers et al.,

2006). The dependent variables of conceptual, procedural, and reasoning skill test scores showed a significant pattern of correlations, indicating that a MANOVA is acceptable to conduct (see Table 4.8).

Secondly, the multivariate normality distribution was examined; the means of each dependent variable in each cell, as well as all linear combinations of dependent variables, were found to be roughly normal. It is probabily most important that both ANOVA and MANOVA are robust to moderate violations of normality, provided the violation is created by skewness and not by outliers (Tabachnick & Fidell, 1996). The assumption of equal covariance matrices is a necessary condition for multivariate normality. Specifically, possible violations of this assumption may be assessed by interpreting the results of Box's test. On the other hand, if homogeneity of variance-covariance is violated, a more robust multivariate test statistic, pillai's trace, can be selected when interpreting the multivatiate results.

Since homogeneity of variance-covariance is attest assumption for MANOVA and has implications in how to interpret the multivatiate tests, the results of box's test should be evaluated. Typically, if Box's test is significant at $p < .001$ and group sample size are extremely unequal, then robustness cannot be assumed due to unequal variances among groups (Tabachnick & Fidell, 1996). In such a case, a more robust MANOVA test statistic, Pillai's Trace, is utilized. If equal variances are assumed, Wilks' Lambda is commonly used as the MANOVA test statistic. Since, the M-value of the Box's Test was 17.19, with a p-value of 0.17, which is non-significant then the covariance matrices between the groups was assumed to be equal for the purposes of the MANOVA (see Tables 4.11 and 4.12).

Accordingly, a MANOVA test was conducted to explore whether there was a statistically significant difference between the posttest scores of the experimental groups and the comparison group in terms of the combined score of the CU, PK, and RS test scores after the intervention. The following section presents the descriptive statistics of post-CU, post-PK, and post-RS test scores in relation to groups of PSMTs.

Table 4.10 Means and standard deviations comparing the three groups after interventions

	Groups								
	EG1			*EG2*			*CG*		
Variables	*N*	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*
Post-CU	48	27.38	6.70	38	23.79	4.63	30	21.13	6.89
Post-PK	48	26.91	2.74	38	23.34	2.69	30	21.02	2.87
Post-RS	48	16.98	2.25	38	14.45	2.66	30	12.87	3.22

As it can be seen from the table above (Table 4.10), the mean scores of the EG1, EG2, and CG on the post-CU, post-PK, and post-RS tests were different. The mean score for EG1 (27.38) is higher than the other groups in post-CU. In post-PK, the mean score for this group (26.91) is also higher than the other groups. Likewise, the mean score for EG2 (23.34) is slightly higher than the CG (21.02) groups in post-RS. The mean score for the CG is lower than the others in all three variables.

In general, the result of the descriptive statistics revealed that there was a mean score difference between groups in relation to the posttest scores of the CU, PK, and RS test scores. Then, a MANOVA was performed to assess whether there was statistically significant post-test mean differences between the three groups under study. To assess if there were statistically significant post-test mean score differences between the three groups, a one-way MANOVA was conducted. The following table shows MANOVA tests on the dependent variables.

Table 4.11 Multivariate Analysis of Variance Results: multivariate test

	Wilk's Lambda	*f*	*Df*	*Error df*	*p*	η^2	*O power*
Groups	.459	17.62[b]	6	220	.000	.32	1.00

a. Design: Intercept + Group
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level.
d. Computed using alpha = .02

Thus, the MANOVA (Table 4.11) results revealed a statistically significant difference in post-test means between the three groups: Wilk's $\Lambda = .459$, F (6, 220) = 17.62[b], p < .001, multivariate ($\eta^2 = .32$), observed power = 1.00. The eta-square value is much larger than the typical value (Cohen, 1988). This eta squared (η2) value indicated that 32% of multivariate variance of posttest mean scores was associated with the interventions. In addition, the observed power was found to be 1.0, showing that there was a 100% chance that the results could have come out of significant. Hence, the difference found between the groups stemmed from the treatment effect and this difference had a practical value.

When multivariate significance is found, the univariate ANOVA results can indicate the degree to which groups differ for each DV. Thus, follow-up ANOVAs were conducted to test for between-subjects' effects. Each ANOVA was tested using the Bonferroni method. According to Tabachnick, Fidell, and Ulman (2007), a Bonferroni type adjustment should be applied in this situation to guarantee a smaller Type-I error on multiple comparisons. As a result, to make the adjustment, the previously used alpha of .05 is divided by the total number of dependent variables. Therefore, 0.05 divided by 3 (the number of dependent variables) yields 0.02, which is an adjusted alpha for the multiple comparisons. In the following table, we show the ANOVA test between-subjects effect results.

Table 4.12 ANOVA Tests of Between-Subjects Effects

Independent Variable	*Dependent Variable*	*Type III Sum of Squares*	*Df*	*F*	*p*	η^2
Groups	Post-GCUT	755.11	2	9.98	.000	.15
	Post-GPKT	683.49	2	44.82	.000	.44
	Post-GRST	799.84	2	23.69	.000	.29

Using the Bonferroni method, each ANOVA was tested at a .02 alpha level. The results of the Between-Subjects Effects tests (Table 4.12) showed that there was a significant mean difference between groups in Post-GCUT (F (2,113) = 9.98, p < .001), partial eta squared (η^2) = .15, observed power = 1.00; post-GPKT (F(2, 113) = 44.82, p < .001), partial eta-squared (η^2) =.44 observed power =1.00 and post-GRST (F(2, 113) =23.69, p < .001), partial eta-squared (η^2) = .29. The effect size was large for all three ANOVAs. The strength of the relation between the intervention and dependent variables was large in all cases (Cohen, 1988). The partial eta squared (η^2) values are .15, .44 and .29 respectively for post-GCUT, post-GPKT and post-GRST indicating that 15%, 44% and 29% of variance of the dependent variables are associated with the intervention. This deduces that the difference between the groups in dependent variables was attributed to the intervention.

Even though the results of the Tests for Between-Subject Effects indicated that there was a statistically significant difference in mean scores between treatment groups for all dependent variables, they did not indicate which group was different from the other. Thus, in order to determine which group differs from the others, post hoc multiple comparisons were performed at .02 adjusted alpha levels. A more conservative alpha level should be applied using the Bonferroni adjustment. Post hoc results can then

indicate which groups are significantly different for the DV if univariate significance is found for particular DV (Pallant, 2010). The following table shows the result of the post-hoc multiple comparisons.

Table 4.13 Bonferroni Post-hoc multiple comparison

Dependent Variables	*(I) group*	*(J) group*	*Mean Difference (I - J)*	*SE*	p^b
Post-GCUT	EG1	EG2	3.59*	1.34	.03
		CG	6.24*	1.43	.00
	EG2	CG	2.66	1.50	.24
Post-GPUT	EG1	EG2	3.56*	.60	.00
		CG	5.89*	.64	.00
	EG2	CG	2.33*	.67	.00
Post-GRST	EG1	EG2	2.53*	.58	.00
		CG	4.11*	.62	.00
	EG2	CG	1.58*	.65	.05

*. The mean difference is significant at the .05 level.

[b]. Adjustment for multiple comparisons: Bonferroni.

As it can be revealed from Table 4.13 above, post hoc multiple comparison result revealed that there was statistically significant mean difference between each pair of the group ($p < .001$) except between EG2 and CG ($p = .24$) in CU mean scores. PSMTs in EG1 performed better than EG2 and CG with mean gain of 3.59 and 6.24 respectively while no significant mean difference between EG2 and CG groups.

In the same way, post hoc multiple comparison result revealed that there was statistically significant mean difference between each pair of the groups ($p = .000$) in PK mean scores. PSMTs in EG1 performed better than EG2 and CG with mean gain of 3.56 and 5.89 respectively. Similarly, students in EG2 performed better than CG with mean gain of 2.33 in PK mean scores. The TGIBL helped PSMTs to score significantly higher mean scores in both post-CU and post-PK indicating its effectiveness over the other methods in enhancing procedural knowledge and conceptual understanding followed by GIBL.

In terms of post-RS, the post hoc multiple comparison result revealed a statistically significant mean difference between pairs of groups ($p < .001$), with the exception of EG2 and CG ($p = .05$). PSMT in EG1 has a higher mean score than EG2 and CG, with mean gains of 2.53 and 4.11, respectively. However, though there was a difference in mean score between pairs of groups (EG1 and CG) in post-RS, it was not statistically significant. This implies that GIBL has no significant effect on PSMTs' geometry reasoning, but the TGIBL approach is effective. The TGIBL approach was significantly better than the other instructional approaches in improving PSMTs' conceptual understanding, procedural knowledge, and reasoning skills of concepts in geometry, followed by the GIBL approach. Therefore, PSMTs that used the TGIBL approach showed higher conceptual, procedural, and reasoning skills in geometry when compared to PSMTs who learned using both the GIBL approach and the traditional teacher-centered method. Similarly, PSMTs who used the GIBL approach showed a higher learning outcome when compared to PSMTs who learned using the traditional teacher-centered method.

The responses of PSMTs to all conceptual understanding scores were analyzed using a MANOVA test. Moreover, some items were selected and analyzed to confirm further the significant differences between the experimental and comparison groups. Therefore, items 5, 6, 7, 8, 9, 12, and 18 from GCUT were presented (See **Appendix**).

For instance, items 5 and 12 were prepared to assess PSMTs' conceptual understanding of concepts in angles and triangles. When item 5 was analyzed, 65.71%, 54.84%, and 56.67% from EG1, EG2, and CG respectively answered it. PSMT responses for items 5 and 12 were analyzed and are shown in the table below (Table 4.14).

Table 4.14 Percentage of PSMTs' response to item no. 5 and 12

	EG1	EG2	CG
5. Of the following which can represent a degree measure of scalene triangle?			
A. 80^0, 40^0, 50^0	10	19.03	0
B. 40^0, 110^0, 30^0 *****	**65.71**	**54.84**	**56.67**
C. 30^0, 120^0, 30^0	24.29	10	40
D. 60^0, 60^0, 60^0	0	16.13	3.33
Sababa/Reason for my choice:			
A. All the three sides are equal in length and all interior angles are also equal.	4	16.13	31
B. Two sides are not equal in length and two angles are equal.	10	29.03	12.33
C. All sides are different in length and all angles are different. *****	**65.71**	**54.84**	**56.67**
D. Only one angle is a right angle with different sides	20.29	0	5
E. Other reason:_____	0	0	0
12. How many diagonal and sides respectively does a triangle have?			
A. 0, 3*****	**82.86**	**58.06**	**26.67**
B. 3, 3	0	22.59	0
C. 3, 0	5.71	19.35	10.00
D. 0, 0	11.43	0	63.33
Sababa/Reason for my choice:			
A. A triangle is three sided with three median diagonals.	5.71	16.13	63.33
B. A triangle has three sides and three slant edge of diagonals.	11.43	25.81	16.67
C. A triangle has three sides with no diagonals. *	**82.86**	**58.06**	**26.67**
D. Other reason:_____	0	0	0

KEY: ***** sign are the responses that are the correct answers and correct reasoning

In Item 5, PSMTs had to choose one of four scalene triangles with specified angle measurements. A relatively higher percentage (65.71%) of the PSMTs from EG1 followed by those from CG (56.67%) understood the concept of angle and type of triangle, while 43.33% of PSMTs from CG misunderstood scalene triangles as isosceles triangles. The high percentage (43.33%) of PSMTs from CG demonstrates that, they lacked a conceptual understanding of isosceles triangles as well as familiarity with the term "scalene." However, no significant misunderstandings from the TGIBL group were found.

Similarly, Item 12 was also taken as an example from the TTGDT test item to show the analysis of the conceptual understanding of the triangle and its diagonals. Thus, only

82.86%, 58.06%, and 26.67% of EG1, EG2, and CG, respectively, understood the concept of the diagonal, while 17.13%, 41.48%, and 73.33% had difficulty understanding the concepts of diagonals and the number of sides of a triangle, respectively. A quite higher percentage of PSMTs in EG1 understood the concept (82.86%), followed by EG2 (58.06%). On the other hand, a higher percentage of EG2 (41.48%) and CG (73.33%) misunderstood the concepts. Moreover, a high number of PSMTs from EG1 and CG considered the median of a triangle to be the diagonal of a triangle.

On the other hand, items 8 are, for instance, used to assess the PSMT's sub-concepts of quadrilaterals such as rectangles, squares, rhombuses, parallelograms, and trapezium. This item was prepared to measure PSMTs' knowledge of ordering rectangles, squares, parallelograms, and rhombuses in terms of their properties.

Table 4.15 Percentage of PSMTs' response to item no. 8

Item 8	**EG1**	**EG2**	**CG**
8. Write one correct statement relating any two of the following quadrilaterals: rhombus, parallelogram, rectangle and square. For example; square is a rhombus.			
A. A rectangle is a square.	1.97	10.83	71
B. A square is a rectangle.*	**88.03**	**64.51**	**6.67**
C. A parallelogram is a rhombus.	3.71	13.66	0
D. Parallelogram is a square.	6.29	11	22.33
Sababa/Reason for my choice:			
A. All have four sides and opposite sides are parallel.	15.6	60.92	6.1
B. Opposite sides are equal with equal diagonals.	7	3.6	14.10
C. Their diagonals are intersects at right angle.	17.4	0	13.13
D. Opposite sides are equal and parallel and each angle is a right angle.*	**60**	**35.48**	**66.67**
E. Other reason:_____	0	0	0

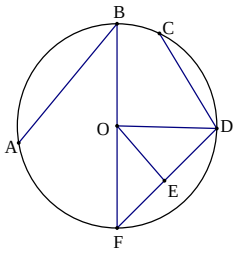
From Table 4.15 above, 60% of PSMTs from EG1 answered both tiers of the test item, followed by 35.48% of PSMTs from EG2. This indicates that highly comparable PSMTs from EG1 answered the item. On the other hand, from Table 4.15 above, 28.03%, 29.03%, and 6.67% of PSMTs from EG1, EG2, and CG respectively answered first-tier questions only, while 5.71%, 3.23%, and 60% of PSMTs from EG1, EG2, and CG

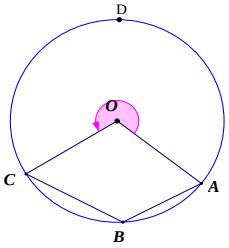
respectively answered second-tier questions only. From this item, a higher percentage of PSMTs in EG1 (60%) understood the concept, whereas relatively higher percentages of PSMTs in EG2 (64.52%) and CG (93.33%) have difficulties with the ordering of the properties, that is, misconceptions about that a square, a rectangle, and a rhombus can be a parallelogram. This suggests that many CG participants have only a cursory understanding of geometry beyond recognizing it as a discrete shape. This supports the assertion that most PSMTs in CG were unable to understand the class inclusion properties of quadrilaterals.

For the concepts related to circle geometry, items 10 and 14 were used to examine PSMTs' conceptual understanding of the concepts about the chords and the relationship between the central angle and its inscribed circle angle. The items from the properties of circles and circle geometry were analyzed as examples in the following table below.

Table 4.16 Percentage of PSMTs' response to item no. 10 and 14

Item s 10 and 14	EG1	EG2	CG
10. In following figure, which represents the chords of the circle with center O?			
A. $\overline{AB}$ only	5.71	0	6.67
B. $\overline{AB}$, $\overline{CD}$ and $\overline{DF}$ only	8.58	12.9	26.66
C. $\overline{OE}$ only	0	0	0
D. $\overline{AB}$, $\overline{CD}$, $\overline{BF}$ and $\overline{FD}$ only*	85.71	87.10	66.67
Sababa/Reason for my choice:			
A. A chord is a line segment that passes through the center of a circle.	5.71	0	6.66
B. A chord is any line segment joining two points on the circumference of a circle. *	85.71	87.10	66.67
C. A chord is the line segment from the center of a circle to any point on the circle.	8.58	12.9	26.67
D. Other reason:_____	0	0	0
14. In following circle with center O, if $m(C\hat{O}A) = 110^0$ then what is $m(A\hat{B}C) =$ ___			
A. 55^0	14.29	25.81	36.67
B. 125^0*	62.85	38.71	6.67
C. 70^0	22.86	16.13	56.66
D. 110^0			
Sababa/Reason for my choice:			
A. Because ABCO is a cyclic quadrilateral and $m(A\hat{B}C)$ is it's supplementary.	8.57	26.38	53.53
B. The $m(A\hat{B}C)$ is one-half of the measure of central angle $A\hat{O}C$.	14.29	25.81	32.67
C. The measure of an angle $m(A\hat{B}C)$ is one-half the degree measure of arc $\widehat{ADC}$. *	62.85	38.71	6.67
D. Vertically opposite angle are equal.	14.29	9.1	3.01
E. Other reason _____	0	0	4.12





KEY: * sign are the responses that are the correct answers and correct reasoning

When we look at an item 10 from the table 4.16 above, the result showed that relatively high percentage of PSMTs 85.71%, 87.10%, and 66.67% of EG1, EG2, and CG, respectively, understood the concepts of chords of circles while only 14.29% from EG1, 13.9% from EG2 and 33.33% from CG misunderstood about the concepts of the chords of circle. According to the findings, PSMTs fromCG (26.67%) did not consider diameter to be a chord and instead considered radius to be a circle's chord. In this item, PSMTs from EG2 have performed better than both PSMTs from EG1 and CG.

Similarly, when item 14 was considered, a high percentage of PSMTs from CG (93.33%) and EG2 (61.29%) misunderstood the relation between the central angle and its inscribed angle. Those PSMTs claimed that and gave the reason as vertically opposite angles. PSMTs, on the other hand, consider a cyclic quadrilateral and assume that the opposite angle of a cyclic quadrilateral is equal, and compare the inscribed angle with angle ABC; in general, those PSMTs did not calculate the inscribed angle with respect to the reflex angle AOC. PSMTs also compared $A\hat{B}C$ with $A\hat{O}C$ visually. However, a higher proportion of PSMTs from EG1 (54.29%) understood the relationship between the central angle and its inscribed angle than PSMTs from EG2 (38.71%) and CG (6.67%). These item analyses also strengthen the result of the ANOVA that the mean score of PSMTs in TGIBL was significantly larger than the other groups. This implies that a technology-integrated guided inquiry approach afforded them an opportunity to understand geometry concepts and abstract mathematical concepts better than other groups of PSMTs. That is, PSMTs in technology-integrated classrooms were able to grasp basic conceptual relationships after the intervention.

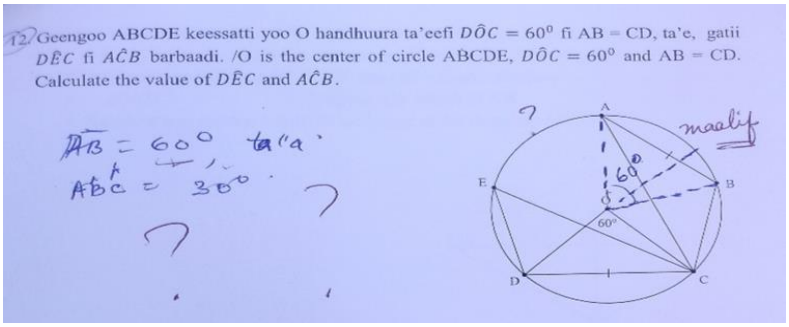
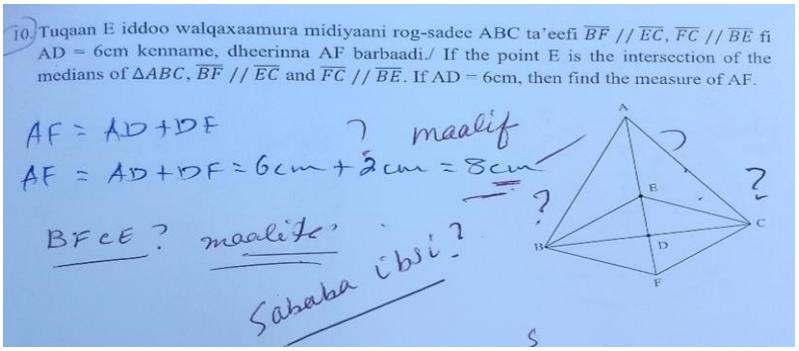
	This problem was solved without indicating the reason and procedure followed.
	The answer for the problem is correct but the procedure and reason is not indicated.

Figure 4.1 PSMT from CG responses to some items from GPKT in the posttest

In this case, the PSMT has calculated the final answer only, without the steps to solve the items. This PSMT assumed that the segment $\overline{AB}$ = 60^o and drew a triangle AOB to compare angles with the chord visually to determine the measure of an angle for item 12. In the same manner, this PSMT from CG could not show the reason how AF = 8cm, and nor provide sufficient reasons to conclude his answer for item 10.

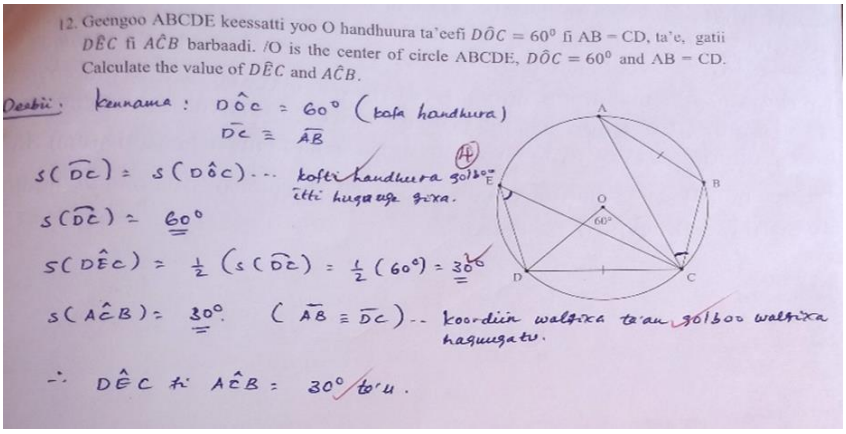
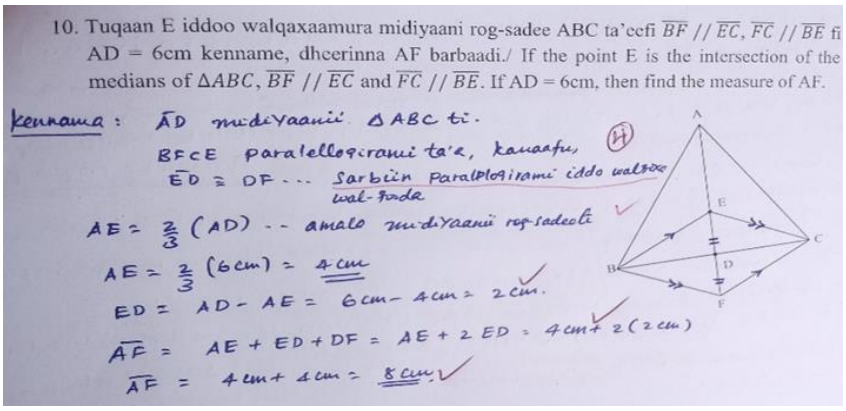
	In this item, PSMT showed all steps necessary to arrive at the solution with its reasons
	In this item, PSMT from EG1 showed all steps necessary to arrive at the solution with its reasons

Figure 4.2 PSMT from EG1 response to some item from GPKT

PSMT from EG1 get the correct answer with the correct procedure and completely correct reasons for each procedure that s/he followed. The PSMT even demonstrated all of the steps he took to reach the conclusion that he reasoned "equal chords subtended an equal arc on the circle" for item 12. Similarly, PSMT responded to item 10 with precise procedures and reasons. In this item, the PSMT understands "the division ratio of the median of a triangle and the diagonals of a parallelogram bisect each other."

In the post-test, the PSMTs from EG1 demonstrated greater learning gains on procedural geometry knowledge than the PSMTs from CG. They also began to use plane geometry arguments to justify arguments with necessary and sufficient reasons to reach their

conclusions. The PSMT's responses to GPKT items showed that he gained procedural knowledge of plane geometry and was able to apply this knowledge after being taught using designed activities with a technology-integrated guided inquiry-based approach.

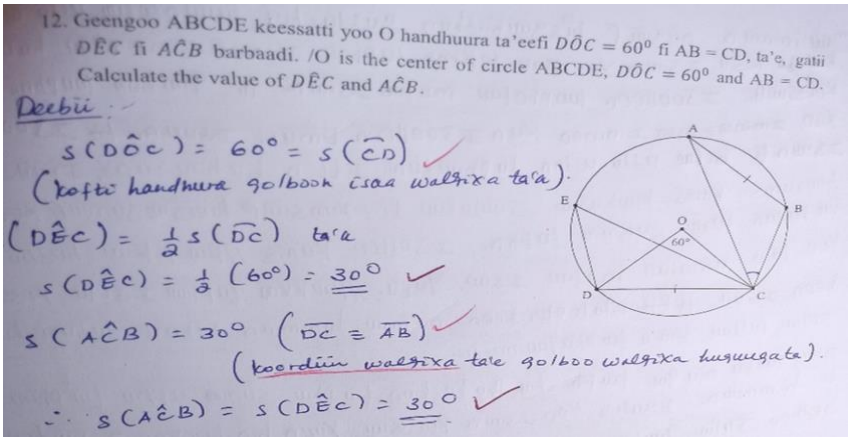
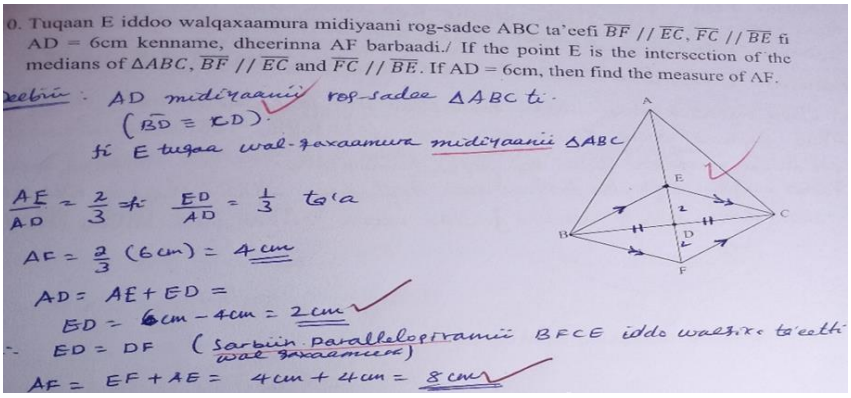
	In this item, PSMT showed all steps necessary to arrive at the solution with its reasons.
	In this item, PSMT showed all steps necessary to arrive at the solution with its reasons.

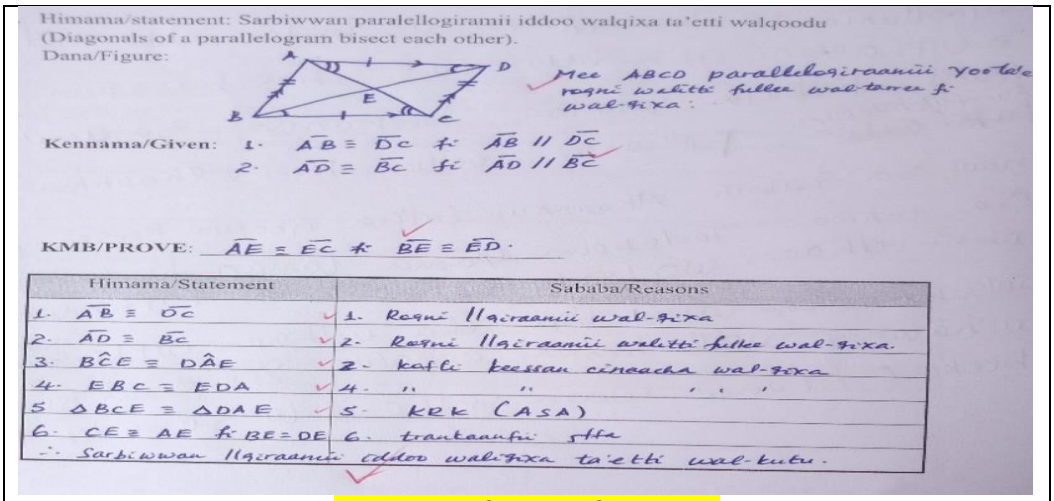
Figure 4. 3 PSMT from EG2 response to for some items from GPKT

Similarly, PSMT from EG2 also gets the correct answer with the correct procedure and the correct reasons for each procedure he followed. The PSMT showed all the necessary procedures to concluded that, $D\hat{E}C = A\hat{C}B = 30^0$, and gave why "equal chords are subtended by equal arcs." Similarly, PSMT answered item 10 with precise procedures and reasons.

The aforementioned figures also revealed that there was an enhancement of knowledge and understanding in plane geometry among the experimental group compared to the comparison group after the interventions. PSMTs exposed to a technology-integrated guided inquiry approach performed generally better than both the guided inquiry-based learning approach and the traditional teacher-centered approach in each of the tasks for geometry reasoning skills and geometry procedural knowledge. This also backs up the MANOVA results obtained.

In addition to quantitative analysis on geometry reasoning skills, the current study also used some selected items to investigate how well PSMTs performed on the proof test items administered at the end of the interventions. For the sake of this research, a "proof" is defined as an argument that follows a logical chain of reasoning, relies on established axioms, definitions, postulates, theorems, etc., and establishes the truth of the antecedent while demonstrating the truth or falsehood of the consequent of a statement. The method of evaluating reasoning frequently entails a sequence of situations in which PSMTs are tracked step-by-step as they respond to each one.

In Task 1, the PSMTs were asked to draw and label the figure illustrating the statement "diagonals of a parallelogram bisect each other" and then use the figure to explain and support their claims with their respective reasons. PSMTs are tested on their ability to recognize and state the hypothesis, claim parts of a geometric statement, and visually represent the given information with an appropriate sketch in this task. Finally, the task was proved using a two-column format: the first column states the statement, and in the respective second column, the reasons for every statement in the first column are explained.



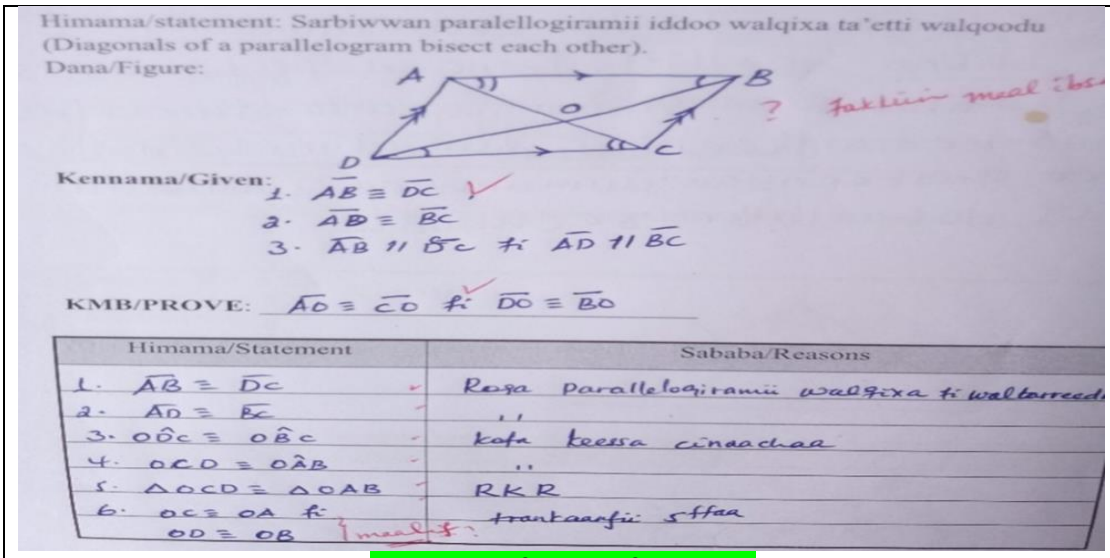
Response of PSMT from EG1

Statement	**Reason**
1.$\overline{AB} \equiv \overline{DC}$……..	1. The opposite sides of parallelogram are congruent.
2.$\overline{AD} \equiv \overline{BC}$……..	2. Opposite sides of parallelogram are congruent.
3. $< OCD \equiv < OAB$……	3. Alternate angles are congruent.
4. $\Delta OCD \equiv \Delta OAB$……..	4. SAS congruency
5. OC ≡ OA and $OD \equiv \overline{OB}$ ……..	5. From SAS congruency step.

Therefore, the diagonals of a parallelogram bisect each other.

Figure 4.4 The response of PSMT from EG1 to task-1 of GPPT after being intervention

The PSMTs in EG1 showed all the steps followed to conclude that the diagonals of a parallelogram bisect each other. The PSMT used reasons to justify his arguments and provided the correct reasons statement in the proof of the proposition. Thus, this PSMT was able to reason out and understand the reasonable deductions between steps and was able to generalize and specify the concepts, procedures, properties, and propositions. Similarly, PSMTs from EG2 also checked how well they performed on the item.



Response of PSMT from EG2

Statement	**Reason**
1. $\overline{AB} \equiv \overline{DC}$…….	1. the opposite sides of parallelogram are congruent
2. $\overline{AD} \equiv \overline{BC}$… …….	2. Opposite sides of parallelogram are congruent.
3. $< OCD \equiv < OAB$……..	3. Alternate angles are congruent.
4. $\Delta OCD \equiv \Delta OAB$……..	4. SAS congruency
5. $\overline{OC} \equiv \overline{OA}$ and $\overline{OD} \equiv \overline{OB}$	5. From step-4

Figure 4.5 PSMT in EG2 response to task-1 of GPPT after intervention

The PSMT in EG2 answered the questions by utilizing reasons to justify his arguments for every statement. He was able to follow simple deductions and understand the reasons, but s/he did not deduce the final step. This shows PSMT was unable to build a relationship between condition and conclusion and partially generalize and specify existing geometrical concepts, procedures, properties, and propositions.

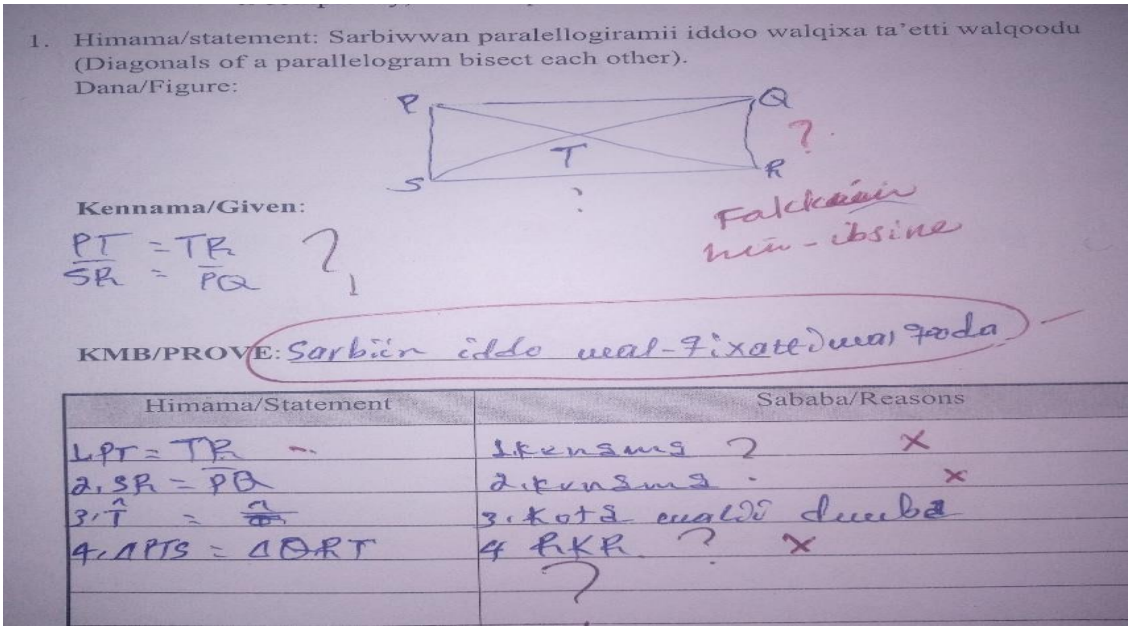
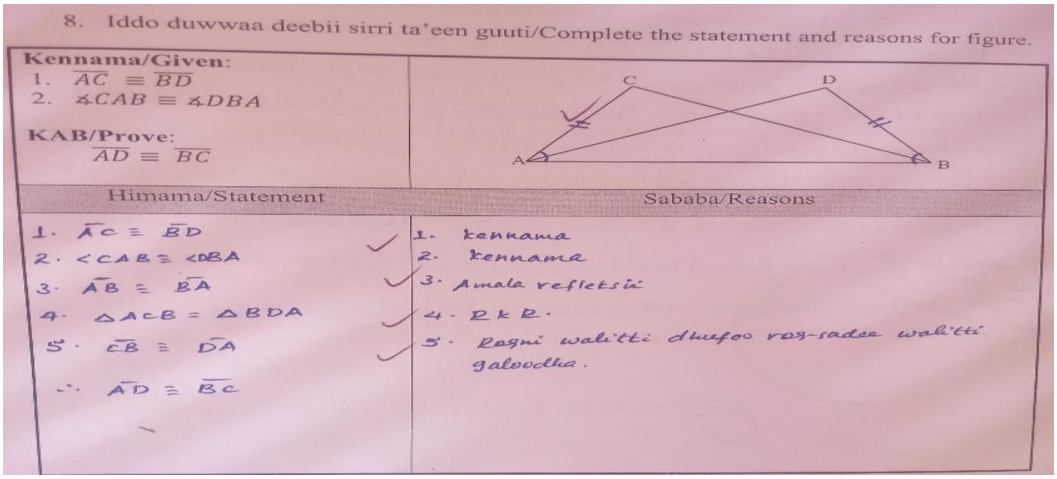


Figure 4.6 The response of PSMT from CG to task-1 in the post-test

PSMT in CG did not answer the tasks correctly and did not give sufficient reasons why the diagonals of a parallelogram bisect each other. Furthermore, PSMTs in CG incorrectly draw a parallelogram. For instance, he drew a rectangle instead of a parallelogram or a diagonal connecting the midpoints of the quadrilateral's opposing sides and did not give all the reasons. These PSMTs were not able to recognize the two parallel lines from the parallelogram and apply the alternate interior angles theorem to be congruent. Another common error pattern observed was a lack of ability to recognize and apply reasonable inferences. Furthermore, the key abilities examined in the other task item are the PSMTs' ability to recognize, apply, and link geometric postulates with reasonable inferences.


8. Iddo duwwaa deebii sirri ta'een guuti/Complete the statement and reasons for figure.

Kennama/Given:
1. $\overline{AC} \equiv \overline{BD}$
2. $\measuredangle CAB \equiv \measuredangle DBA$

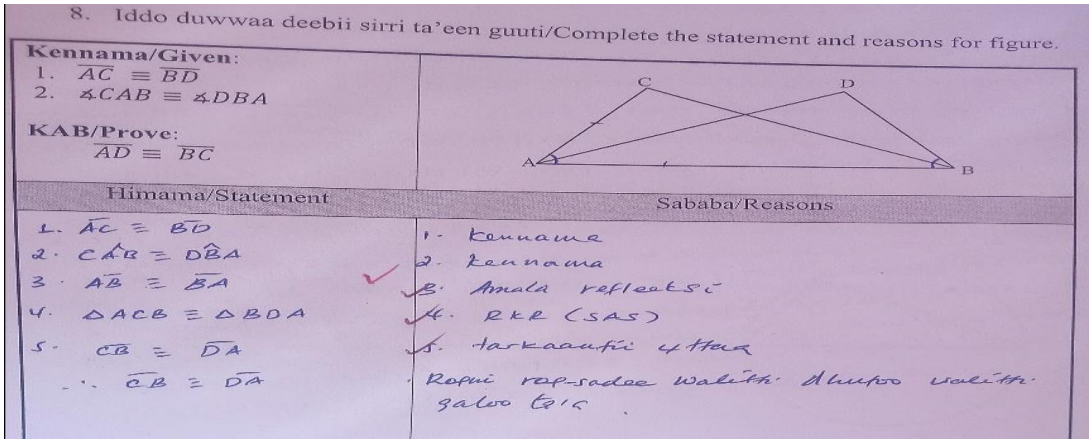
KAB/Prove:
$\overline{AD} \equiv \overline{BC}$

Himama/Statement	Sababa/Reasons
1. $\overline{AC} \equiv \overline{BD}$	1. kennama
2. $\angle CAB \equiv \angle DBA$	2. kennama
3. $\overline{AB} \equiv \overline{BA}$	3. Amala refleksii
4. $\triangle ACB \equiv \triangle BDA$	4. R k R.
5. $\overline{CB} \equiv \overline{DA}$	5. Bagni walitti dhufoo ros-sadee walitti galoodha.
$\therefore \overline{AD} \equiv \overline{BC}$	

Statement	**Reason**
1. $\overline{AC} \equiv \overline{BD}$ ……	1. Given
2. $C\hat{A}B \equiv D\hat{B}A$ …….	2. Given
3. $\overline{AB} \equiv \overline{BA}$ ……	3. Properties of reflexive
4. $\Delta ACB \equiv \Delta BDA$ …	4. SAS
5. $\overline{CB} \equiv \overline{DA}$ ……	5. corresponding sides of congruent figure are congruent

Figure 4.7 PSMT from EG1 response to item 8 from GPPT

In figure 4.7 above, PSMT's response from EG1 showed that he gained knowledge of the theorems and corollaries of triangle congruency and that he was able to apply this knowledge after being taught by the designed activities and technology-integrated guided inquiry approach. This indicates that PSMT developed reasoning skills in geometry learning. In the post-test, he began to apply reasons to justify his arguments for all assumptions made during the proof of triangle congruency.


8. Iddo duwwaa deebii sirri ta'een guuti/Complete the statement and reasons for figure.

Kennama/Given:
1. $\overline{AC} \equiv \overline{BD}$
2. $\measuredangle CAB \equiv \measuredangle DBA$

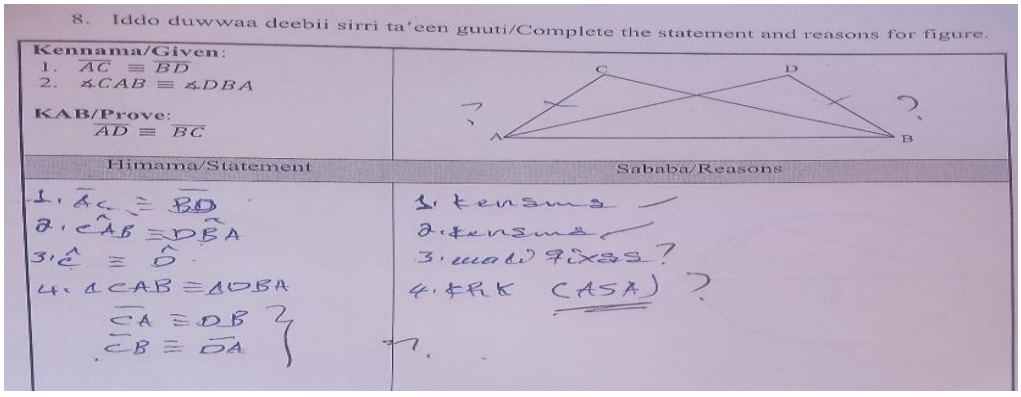
KAB/Prove:
$\overline{AD} \equiv \overline{BC}$

Himama/Statement	Sababa/Reasons
1. $\overline{AC} \equiv \overline{BD}$	1. Kennama
2. $C\hat{A}B \equiv D\hat{B}A$	2. Kennama
3. $\overline{AB} \equiv \overline{BA}$	3. Amala reflexsi
4. $\Delta ACB \equiv \Delta BDA$	4. RKR (SAS)
5. $\overline{CB} \equiv \overline{DA}$	5. tarkaanfii 4ffaa
$\therefore \overline{CB} \equiv \overline{DA}$	Ropui rop-sadee walii'n dhufee walii'n galee ta'e

Statement	**Reason**
1. $\overline{AC} \equiv \overline{BD}$	1. Given
2. $C\hat{A}B \equiv D\hat{B}A$	2. Given
3. $\overline{AB} \equiv \overline{BA}$	3. Properties of reflexive
4. $\Delta ACB \equiv \Delta BDA$	4. SAS
5. $\overline{CB} \equiv \overline{DA}$	5. corresponding sides of congruent figure are congruent

Figure 4.8 The response of PSMT from EG2 to item from GPPT

In a similar manner, PSMT from EG2 also proved task-8 of a triangle problem in geometry requiring two or more lines of reasoning. Following the post-test PSMT, there were reasons to justify arguments and also state sufficient reasons to conclude his answer in a guided inquiry approach.


8. Iddo duwwaa deebii sirri ta'een guuti/Complete the statement and reasons for figure.

Kennama/Given:
1. $\overline{AC} \equiv \overline{BD}$
2. $\measuredangle CAB \equiv \measuredangle DBA$

KAB/Prove:
$\overline{AD} \equiv \overline{BC}$

Himama/Statement	Sababa/Reasons
1. $\overline{AC} \equiv \overline{BD}$	1. kennama
2. $C\hat{A}B \equiv D\hat{B}A$	2. kennama
3. $\hat{C} \equiv \hat{D}$	3. walii fixaa?
4. $\measuredangle CAB \equiv \measuredangle DBA$	4. RKR (ASA)
$\overline{CA} \equiv \overline{DB}$	
$\overline{CB} \equiv \overline{DA}$	

Statement	**Reason**
1. $\overline{AC} \equiv \overline{BD}$	1. Given
2. $C\hat{A}B \equiv D\hat{B}A$	2. Given
3. $\hat{C} \equiv \hat{D}$.....	3. equal from the figure
4. $\Delta ACB \equiv \Delta BDA$	4. ASA
$\overline{CB} \equiv \overline{DA}$	----------

Figure 4.9 The response of PSMT rom CG to item of GPPT

PSMT from the CG group attempted to answer task 8. The PSMTs listed only wrong statements with wrong reasons. For instance, he estimated the figure by its appearance and compared it visually to be congruent, not based on reasonable conclusions. This suggests that PSMT in the traditional teaching approach lacks a thorough understanding of geometric reasoning and theorem concepts and properties. In addition, PSMTs ignored some conditions that applied to proving geometry concepts.

This further demonstrated that a technology-integrated approach assisted PSMTs in engaging in stimulating and meaningful exploratory activities to build and test hypotheses and gave them a better understanding of the formal reasoning and proving process. Additionally, the technology assisted PSMTs in understanding the value of formal deductive reasoning. PSMTs exposed to a technology integrated guided inquiry approach performed generally better than both the guided inquiry-based learning approach and the traditional teacher-centered approach in each of the tasks for the geometry reasoning skills test items. This trend would suggest that PSMTs exposed to a technology-integrated guided inquiry approach may have developed skills for success in proofreading in addition to the reasoning skills investigated. This also supports the results of the ANOVA obtained in the above table.

4.2.4 Quantitative Data Analysis of Group Differences in Pre-Post-Test Mean Gain

To investigate the mean gain between the pretest and posttest of each group, it is appropriate to conduct a paired sample t-test related to each group with dependent variables. The dependent variables for each group are normally distributed, and hence the assumption of the paired t-test was not violated (see Tables 4.2 and 4.3).

Table 4.17 Paired sample t-test on the mean gain of PSMTs in TGIBL groups

Observation	Group	Paired Differences					t	df	p
					95% CI				
		M	SD	SE	Lower	Upper			
Post-CU-Pre-CU	EG1	9.792	6.591	.951	7.8776	11.7056	10.29	47	.000
	EG2	5.789	4.503	.731	4.3093	7.26964	7.93	37	.000
	CG	1.333	4.257	.778	-.45615	2.7228	1.46	29	.156
Post-PK –Pre-PK	EG1	11.573	6.338	.9148	9.7326	13.4133	12.65	47	.000
	EG2	8.132	4.655	.7552	6.6014	9.6617	10.77	37	.000
	CG	2.733	4.514	.8241	1.0478	4.4188	3.32	29	.002
Post-RS – Pre-RS	EG1	6.5208	3.196	.4613	5.5929	7.4488	14.14	47	.000
	EG2	3.9211	2.972	.4821	2.9443	4.8978	8.13	37	.000
	CG	.0667	1.659	.3029	-.55301	.68634	.22	29	.827

As shown in table 4.17 above, a paired-samples t-test revealed that PSMTs' conceptual test score improved significantly from pre-intervention (pre-test) to post-intervention (post-test) (t (47) = 10.29, p < .001, d = 1.485) for the TGIBL group, and the effect size was very large (Cohen, 1988), but not for the comparison group (t (29) = 1.46, p =.156). The PSMTs in GIBL also made a significant improvement on the conceptual test score (t (37) = 7.93, p < .001, d = 1.285), and the effect size is very large (Cohen, 1988).

In terms of procedural knowledge, the paired sample t-test result confirmed that the procedural test score of PSMTs in the TGIBL group (t (47) = 12.65, p < .001, d = 1.826) was statistically significant with a typical effect size. Similarly, PSMTs in the comparison group showed significant improvement in procedural knowledge (t (29) = 3.32, p =.002, d =.605), showing a medium effect size (Cohen, 1988). Similarly, PSMTs in the GIBL group improved their procedural knowledge significantly (t (37) = 10.77, p < .001, d = 1.747), indicating a large effect size (Cohen, 1988).

Regarding the variable reasoning skill, a paired sample t-test result showed non-significant improvement by PSMTs in the comparison group (t (29) =.22, p =.827). However, there is statistically significant improvement of reasoning skills for PSMTs in the TGIBL and GIBL groups (t (47) = 14.14, p < .001, d = 2.04) and (t (37) = 8.13, p < .001, d = 1.319, respectively) which shows a large effect size.

4.2.5 Group Differences in Attitude towards Geometry

According to research, a cognitively oriented teaching and learning process might have negative effects on student achievement (Lu, Geng, Chen, & Zhang, 2011). Affective variables in learning must therefore be taken into consideration. For the purpose of this study, the attitude of PSMTs has been identified and considered a dependent variable. Thus, the attitude has been defined by four dimensions: a) confidence in learning geometry, which refers to PSMTs' perspectives of their geometry capacity and performance; b) usefulness of geometry, referring to PSMTs' viewpoints regarding the practicality of geometry; c) motivation for learning geometry, which refers to the extent of PSMTs' active efforts to explore geometry; d) enjoyment of learning geometry, which refers to the degree to which PSMTs' enjoy working with geometry and taking geometry classes.

4.2.5.1 Analysis of Group Difference on Attitude (its dimension) Pretest Score

The objective of this subsection is to examine the effects of independent factors on PSMTs' attitudes about geometry, and the findings are presented in descriptive and inferential statistics. Prior to conducting any statistical tests, the normality of the assumptions on the pretest and posttest scores on attitude (its dimensions) must be checked to determine whether the data is parametric or not. Due to their simplicity,

skewness and kurtosis are used to check for normality in the data obtained from pretest and posttest scores. To determine if the assumption of normality is acceptable or not, a combination of visual examination, skewness assessment, and kurtosis assessment might be utilized. If we believe the data to be significantly out of normality, we may either alter the data, such as by calculating logarithms, or use a nonparametric approach such that the normality assumption is not necessary (Pallant & Manual, 2010).

In addition, the variances within the subgroups must be comparable, or there must be homogeneity of variance, for parametric tests to be valid when comparing differences between two or more groups. This assumption is especially important when groups of unequal size are being compared (Tabachnick, Fidell, & Ullman, 2007). The following table shows the descriptive statistics of attitude (its dimensions).

Table 4.18 Descriptive statistics of attitude (its dimensions) in three groups

		Pre-ATT		*Pre-SC*		*Pre-VA*		*Pre-EJ*		*Pre-MT*	
Group	N	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
EG1	48	2.98	.38	2.87	.49	3.14	.46	3.03	.56	2.91	.63
EG2	38	3.06	.32	2.95	.54	3.08	.51	2.91	.53	3.10	.51
CG	30	2.74	.29	2.51	.49	2.77	.61	2.91	.41	2.78	.49
		Post-ATT		*Post-SC*		*Post-VA*		*Post-EJ*		*Post-MT*	
Group	N	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
EG1	48	3.69	.28	3.82	.55	3.65	.58	3.65	.48	3.66	.47
EG2	38	3.46	.29	3.50	.63	3.48	.55	3.36	.58	3.49	.37
CG	30	3.25	.30	3.11	.45	3.29	.55	3.32	.54	3.28	.37

From table 4.18 above, we see that the means of attitude (its dimensions) are different at pre-test and post-test among the groups.

As normal data is a fundamental presumption of parametric testing, it is necessary to evaluate the normality of the data before performing various statistical tests, as was previously mentioned. Graphical and numerical (including statistical tests) are the two main methods used for assessing normality. Statistical tests have the advantage of making an objective judgment of normality but have the disadvantage of sometimes being too sensitive at low sample sizes or overly sensitive at large sample sizes. The normality of the continuous data may be tested in a number of different ways. The Shapiro-Wilk test, Kolmogorov-Smirnov test, skewness, kurtosis, histogram, box plot, P-P plot, Q-Q plot, and mean with SD are among the techniques that are most widely used. For the sake of simplicity, the skewness and kurtosis of the normal distribution are used in this particular study.

Table 4.19 The Skewness & kurtosis of pre-test & posttest score of Attitude (dimensions)

			Pre-test score		*Posttest score*	
DV	*Group*	*N*	*Skewness*	*Kurtosis*	*Skewness*	*Kurtosis*
ATT	EG1	48	.797	-.455	.437	.026
	EG2	38	-.303	-.902	.441	-.143
	CG	30	.014	-.666	-.046	-.599
SC	EG1	48	-.104	-.243	.149	-.639
	EG2	38	-.026	-1.340	.248	-.103
	CG	30	-.495	-.600	1.173	1.536
VA	EG1	48	.546	.009	.026	.026
	EG2	38	.434	.156	-.298	-.428
	CG	30	-.026	-.056	-.488	1.045
EJ	EG1	48	.263	-.489	.058	-506
	EG2	38	-.010	-.515	.828	1.018
	CG	30	1.006	1.658	-.263	-.540
MT	EG1	48	.500	-.263	.055	-.322
	EG2	38	-.186	-.870	.948	1.110
	CG	30	.490	-.264	-.763	1.075

From table 4.19 above, it can be seen that the data are normally distributed (George & Mallary, 2010) as the values of both skewness and kurtosis in the pre- and post-tests of attitude (its dimensions) are between -2 and 2. In addition, the homogeneity of variance must be checked for further analysis of the data. The assumption of homogeneity of variance is the second statistical assumption that needs to be tested when comparing three or more independent groups on a continuous outcome with ANOVA. Levene's Test for Equality of Variances is used to determine whether a variance is homogeneous. To satisfy the variance of the homogeneity assumption, the p-value for Levene's test should be non-significant. If Levene's test is significant, the assumption of variance in homogeneity is violated (Pallant & Manual, 2010). If the result had been significant (i.e., the assumption had not been met), then an adjusted F-test such as the Welch statistic or the Brown-Forsythe statistic can be used but normality and homogeneity violated, a non-parametric test such as Kruskal-Wallis can be used.

Table 4.20 Levene's Test results for pretest and posttest score of attitude (its dimension)

	Levene's F-Test for Equality of Variances			
DV	*df1*	*df2*	*F*	*P*
Pre-ATT	2	113	1.32	.27
Pre-SC	2	113	1.03	.36
Pre-VA	2	113	.91	.41
Pre-EJ	2	113	2.60	.08
Pre-MT	2	113	2.11	.13
Post-ATT	2	113	.09	.92
Post-SC	2	113	1.16	.32
Post-VA	2	113	.34	.71
Post-EJ	2	113	.16	.86
Post-MT	2	113	1.83	.17

The ANOVA output shown in table 4.20, along with Levene's test, checks the assumption that the variances of the three groups are equal (i.e., not significantly different). Thus, the assumption of homogeneity of variance is tenable (i.e., not violated) for this sample. Therefore, the ANOVA test can be used to determine if there are differences between the groups on the dependent variables after the treatment has been completed.

4.2.5.2 The Analysis of Group Differences on the Posttest Score of Attitude

The researcher used a one-way ANOVA to evaluate the PSMTs' mean post-test scores on the dependent variables to see if treatment had an influence on their attitude towards learning geometry. This will be done after the assumptions are all not violated (see table 4.19, 4.20). However, an ANOVA test must be conducted on pre-test scores to examine the initial difference (if any) among the three groups on attitude towards learning geometry. The following table shows the analysis of ANOVA tests on the pre-test score of attitudes before the commencement of the interventions. As it can be seen from the table 4.30 below the ANOVA tests for Pre-ATT, Pre-SC and Pre-VA are F (2,113) = 7.36, $p < .001$, F (2,113) = 7.08, $p < .001$ and F (2,113) = .01 respectively shows statistically significant differences before the interventions. This indicates PSMTs' attitude, value and self-confidence in learning geometry before are at different level. Thus, pre-ATT, pre-VA and pre-SC are used a covariate for respective posttest scores. On the other hand, the ANOVA test on pre-MT and pre-EJ are F (2,113) = .77, p = .46 and F (2,113) = 2.89, p = .06 respectively shows no statistically significant difference. This indicates PSMTs were at an equal level in motivation and enjoyment before the commencement of the interventions (see table 4.21).

Table 4.21 A one-way ANOVA test on attitude (its dimensions) before the interventions

DV	*Sources*	*SS*	*df*	*MS*	*f*	*p*
Pre-ATT	Between Groups	1.73	2	.87	7.36	.000
	Within Groups	13.32	113	.12		
	Total	15.05	115			
Pre-SC	Between Groups	3.70	2	1.85	7.08	.000
	Within Groups	29.49	113	.26		
	Total	33.18	115			
Pre-VA	Between Groups	2.72	2	1.36	5.11	.01
	Within Groups	30.03	113	.27		
	Total	32.74	115			
Pre-EJ	Between Groups	.41	2	.21	.77	.46
	Within Groups	29.94	113	.27		
	Total	30.35	115			
Pre-MT	Between Groups	1.81	2	.91	2.89	.06
	Within Groups	35.41	113	.31		
	Total	37.22	115			

Thus, it can be concluded that the ANOVA test is employed to evaluate the posttest score differences between the groups of PSMTs in enjoyment and motivation (Pallant & Manual, 2010).

Table 4.22 One-Way ANOVA Summary table comparing groups on Post-MT & Post-EJ

DV	*Sources*	*SS*	*df*	*MS*	*f*	*p*	η^2
Post-EJ	Between Groups	2.602	2	1.30	4.67	.01	.08
	Within Groups	31.449	113	.28			
	Total	34.051	115				
Post-MT	Between Groups	2.659	2	1.33	7.75	.000	.12
	Within Groups	19.396	113	.17			
	Total	22.055	115				

As it can be seen from Table 4.22 above, the result of the ANOVA shows that the difference between the groups is statistically significant at (F (2,113) = 4.67, p = .01, η^2= .08) and (F (2,113) = 7.75, p < .001, η^2 = .12) for enjoyment and motivation,

respectively. The eta squared (η2) values for enjoyment and motivation are .08 and .12 respectively, indicating that interventions explained 8% and 12% variance of dependent variables. The eta squared values showed moderate effect size (Cohen, 1988). However, there is no evidence that any two groups differ statistically in their motivation and enjoyment towards learning geometry. Thus, the Scheffé post hoc pairwise comparison tests were used to do so.

Table 4.23 Scheffé post hoc multiple pairwise comparison on Post-EJ and Post-MT

DV	*(I) group*	*(J) group*	*Mean Difference (I - J)*	*SE*	*p*
Post-EJ	EG1	EG2	.29*	.11	.04
		CG	.32*	.12	.03
	EG2	CG	.04	.13	.96
Post-MT	EG1	EG2	.17	.09	.17
		CG	.38*	.09	.00
	EG2	CG	.21	.10	.12

*. The mean difference is significant at the 0.05 level.

From table 4.23 above, Scheffe's post-hoc analyses revealed that the EG1 group's enjoyment of learning geometry (M = 3.65, SD =.48) was significantly higher than that of the EG2 group (*M* = 3.36, *SD* =.58, *p* =.04) and the CG group (*M* = 3.32, *SD* =.54, *p* =.03). Similarly, PSMTs in the EG1 group, motivation to learn geometry was significantly higher (*M* = 3.66, *SD* =.47) than in the CG group (*M* = 3.28, *SD* =.37, *p* < .001). Meanwhile, there was no significant mean difference between PSMTs in EG2 and the CG on enjoyment and motivation (*p* >.05) as well as between EG1 and EG2 on motivation towards learning geometry (*p* >.05). Hence, it is evident from the study that technology-integrated guided inquiry-based learning supported PSMTs to be more enjoyed and motivated in learning geometry than PSMTs groups in other two approaches. Furthermore, the graph below shows an enhancement in motivation and enjoyment in learning geometry from the TRAD group to GIBL group, and from the GIBL group to the

TGIBL group. The following figure also indicates that the difference is visible between PSMTs in the intervention and comparison groups.

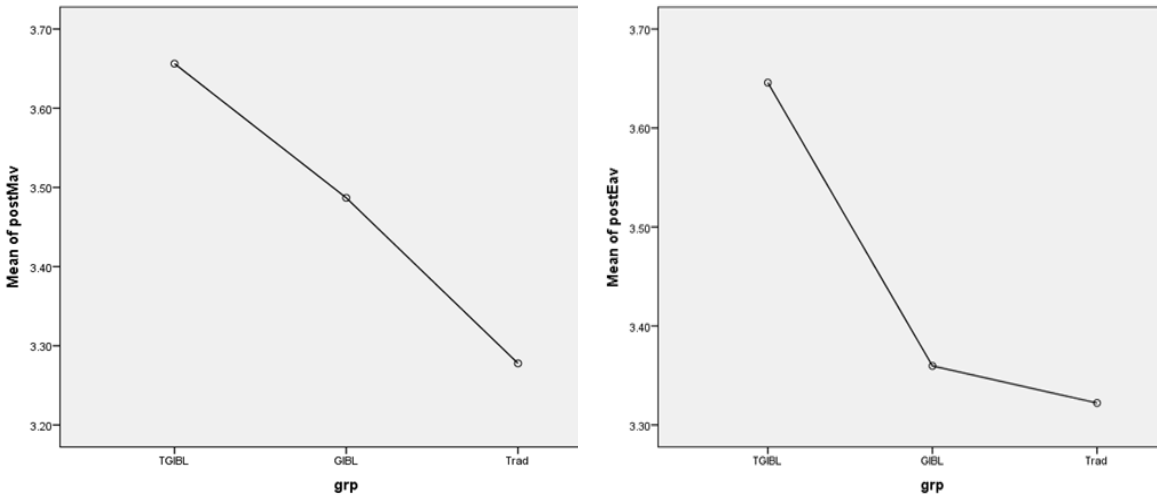


Figure 4.10 The Effect of Intervention on PSMTs' Motivation & Enjoyment among TGIBL, GIBL and CG Groups

On the other hand, since ANOVA tests on PSMTs' pre-attitude, pre-self-confidence, and pre-value showed statistically significant differences, then an ANCOVA test should be employed to analyze the impact of interventions on the dependent variables. Thus, ANCOVA is used to assess the effectiveness of interventions by taking into account the pretest scores of attitudes, self-concept, and values as covariate factors. ANCOVA was used because it eliminates the effect of the covariate factor and reduces error variance (Morgan et al., 2004). As a result, the ANCOVA's additional assumption of interaction between independent variables and covariates must be verified. In this case, pre-ATT, pre-SC, and pre-VA are used as covariates to check the assumptions of homogeneity of the regression slope. The following table shows the test of assumption.

Table 4.24 Homogeneity of Regression Slopes considering the dependent post-test score

Source	*Type III SS*	*df*	*MS*	*f*	*p*
Group	.01	2	.00	.03	.97
Pre-ATT	.07	1	.07	.85	.36
Group* Pre-ATT	.06	2	.03	.33	.72
Corrected Error	115				
Group	1.95	2	.98	3.34	.04
Pre-SC	1.99	1	1.99	6.82	.01
Group*Pre-SC	.87	2	.44	1.49	.23
Corrected Error	115				
Group	1.54	2	.77	2.50	.09
Pre-VA	.02	1	.02	.07	.79
Group*Pre-VA	1.59	2	.79	2.58	.08
Corrected Error	115				

As shown in table 4.24 above, there is no custom interaction between group and pre-test scores ($p > .05$). Therefore, the assumption of the homogeneity of regression has been verified. As a result, the assumption of ANCOVA is not violated (Pallant & Manual, 2010). Thus, ANCOVA tests whether the independent variable still influences the dependent variable after the removal of the influences of the covariates. The ANCOVA analysis on the following variable after intervention is shown in the table below.

Table 4.25 Means and Variability for Post Attitudes (its Subscale) Score Using Pretest Score as Covariate Variable

			Unadjusted		*Adjusted*	
Variable	*Groups*	*N*	*M*	*SD*	*M*	*SE*
Post-ATT	EG1	48	3.69	.29	3.69	.04
	EG2	38	3.46	.29	3.44	.05
	CG	30	3.25	.30	3.27	.06
Post-SC	EG1	48	3.82	.55	3.80	.08
	EG2	38	3.50	.63	3.46	.09
	CG	30	3.11	.45	3.18	.10
Post-VA	EG1	48	3.65	.58	3.62	.08
	EG2	38	3.48	.55	3.50	.09
	CG	30	3.29	.55	3.31	.11

As it can be seen from Table 4.25 above, the results indicated that after statistically controlling the effect of the covariates (pretest score), the mean score of EG1 is higher than both EG2 and CG on the attitude posttest. The analysis of covariance presented in Table 4.26 below further computed the significant difference between the experimental and comparison groups in the attitude (subscale) posttest score.

Table 4.26 ANCOVA result for attitude (subscale) posttest score

Variable	*Source*	*df*	*MS*	*f*	*p*	η^2
Post-ATT	Pre-ATT	1	.15	1.76	.19	.02
	Groups	2	1.62	19.45	.00	.26
	Error	112	.08			
Post-SC	Pre-SC	1	1.66	5.62	.02	.05
	Groups	2	3.48	11.80	.00	.17
	Error	112				
Post-VA	Pre-VA	1	.01	.04	.84	.00
	Groups	2	1.15	3.63	.03	.06
	Error	112				

The ANCOVA test indicated a significant mean difference between groups in attitude (F (2, 112), $p < .001$, η^2=.26); self-confidence (F (2, 112) = 5.62, $p < .001$, η^2 =.17); and value score (F (2, 112) = 3.63, p =.03, η^2=.06). The η^2=.26 shows a large effect size, η^2 =.17 shows a large effect, and η^2 =.06 shows a medium effect for attitude, self-confidence, and value respectively. Thus, it can be noticed that the interventions made 26%, 17%, and 6% variation in attitude, self-confidence, and value respectively, in learning geometry without the covariates' effects.

Although the overall result shows a significant difference in attitudes between the experimental and comparison groups following the intervention, it does not specify which group differs from the other. The post hoc Bonferroni test was used to further investigate each pair of group differences in attitude, as shown in Table 4.27 below.

Table 4.27 Bonferroni post hoc multiple comparison of groups on attitude (subscale)

Variable	*(I) group*	*(J) group*	*Mean Difference (I – J)*	p^b
Post-ATT	EG1	EG2	.24*	.00
		CG	.42*	.00
	EG2	CG	.17	.07
Post-SC	EG1	EG2	.34*	.01
		CG	.62*	.00
	EG2	CG	.28	.15
Post-VA	EG1	EG2	.17	.51
		CG	.37*	.03
	EG2	CG	.19	.50

[b]. Adjustment for multiple comparisons: Bonferroni.

As it was presented in Table 4.27 above, the post hoc Bonferroni test shows a significant mean difference in attitude between EG1 (M = 3.69, SD =.29) and CG (M = 3.25, SD

=.30, $p < .001$) and also between EG1 ($M = 3.69$, SD =.29) and EG2 ($M = 3.46$, SD =.29, $p < .001$), while there is no significant mean difference between EG2 and CG. Furthermore, the graph below shows an improvement in attitude toward learning geometry from the comparison group to the GIBL group, and from the GIBL group to the TGIBL group.

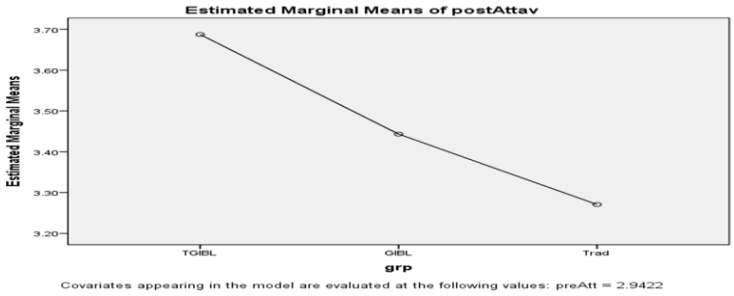


Figure 4.11 The Effect of Intervention on PSMTs' post-attitude among TGIBL, GIBL and CG Groups

In the same way, the post hoc test shows a significant mean difference in self-confidence between EG1 ($M = 3.82$, $SD = .55$) and CG ($M = 3.11$, $SD = .45$, $p < .001$) and between EG1 and EG2 ($M = 3.82$, $SD = .55$, $p = .01$) meanwhile no significance mean difference between EG2 and CG ($p>.05$). Furthermore, the graph below shows the improvement in self-confidence of learning geometry from the TRAD group to GIBL group, and from the GIBL group to the TGIBL group.

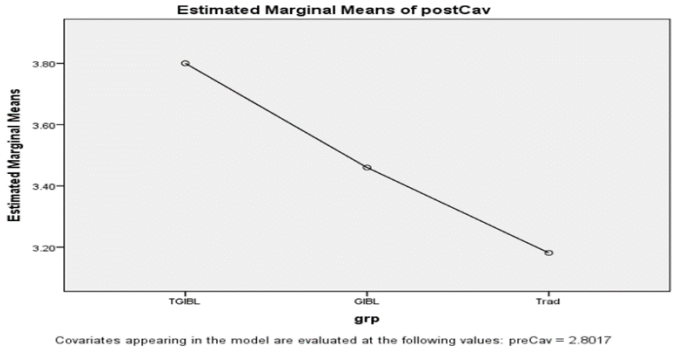


Figure 4.12 The Effect of Intervention on PSMTs' post-Self-Confidence among TGIBL, GIBL and CG Groups

Similarly, the post hoc test shows a significant mean difference in value of learning geometry among EG1 and CG (p = .03). Furthermore, the graph below shows an improvement in value/usefulness of learning geometry from the TRAD group to GIBL group, and from the GIBL group to the TGIBL group.

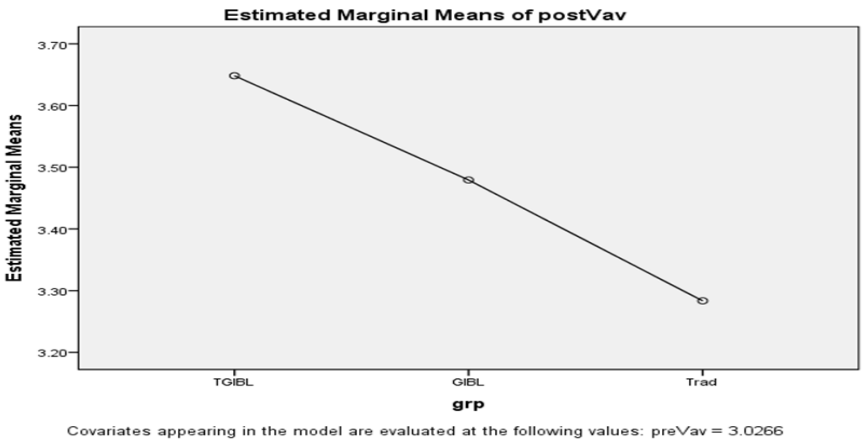


Figure 4.13 The Effect of Intervention on PSMTs' post-value of learning geometry among TGIBL, GIBL and CG Groups.

4.2.5.3 Analysis of Group Differences on mean gain of pre-posttest Score of Attitude

For this purpose, a paired-samples t-test was performed to determine if the attitude scores of PMTs had changed from the pre-intervention (pre-test) to the post-intervention (post-test). A paired t-test statistical test was conducted on the pre- and post-test results of 48 PSMTs of the EG1, 38 PSMTs of the EG2, and 30 PSMTs of the CG. Prior to using a paired t-test, the normality of the data was checked (see Table 4.28). The following table shows the paired sample t-test of PSMTs' attitudinal change between pretest and posttest.

Table 4.28 A paired sample t-test results of mean gain in attitude scores

Group		Paired Differences					t	df	p
					95% CI				
		M	SD	SE	Lower	Upper			
EG1	Post-Pre	.75583	.42147	.06083	.63345	.87822	12.42	47	.000
EG2	Post-Pre	.37202	.39528	.06412	.24209	.50194	5.802	37	.000
CG	Post-Pre	.18311	.66523	.12145	-.06529	.43151	1.508	29	.142

There was a statistically significant difference between the TGIBL group's pretest and posttest attitudes, as shown in table 4.28 above (t (47) = 12.42, p < .001, d = 1.79). The d effect size is large (Cohen, 1988). Similarly, in the GIBL group, there was a statistically significant difference between pretest and posttest attitude scores (t (37) = 5.8, p < .001, d =.94). The size of the effect is also large. However, there was no statistically significant difference between the pretest and posttest attitude of scores in the traditional group (t (29) = 1.51, p =.142).

In conclusion, the attitude towards learning the plane geometry of PSMTs in TGIBL and GIBL changed positively between the pre-test and post-test. This indicated that the technology-based learning approach and guided inquiry approach have changed their attitude towards learning geometry. On the other hand, the traditional teacher-centered approach did not change the attitudes of PSMTs in the comparison group towards learning geometry.

4.2.6 Effect of Attitude and Reasoning Skills on Conceptual Understanding and Procedural Knowledge

According to Pallant (2013), multiple linear regressions (MLR) are used to examine the association between a continuous dependent variable and two or more continuous

independent variables. It also enables us to determine the relative contribution of each predictor to the overall variance explained as well as the overall fit of the model (variance explained). MLR analysis is a statistical tool used to predict a dependent variable from multiple independent variables (Harlow, 2005; Stevens, 2009). This analysis method was used to estimate the relationship between independent and dependent variables and to predict the effect of these variables (Cohen et al., 2007: Mertler & Vannatta, 2001).

However, it is critical to recognize that MLR is inherently a correlation technique and cannot explain the causalities that may underlie the relationship between the variables being observed (Steven, 2009). Nevertheless, before conducting the analysis, the researcher must check the necessary assumptions for MLR (Tabacknick & Fidell, 2013). The assumptions of MLR include the assumptions of linearity, normality, independence, and homoscedasticity, which will be discussed separately in the following sections.

Assumption #1: The relationship between the independent variables and the dependent variable is linear. To check the multicollinearity assumption, a Pearson correlation analysis was carried out to identify any relationship between them.

Table 4.29 Pearson Correlation Coefficient between Variables (n =116)

Variables	RS	CU	PK	ATT
RS	1			
CU	.521**	1		
PK	.385**	.333*	1	
ATT	.218*	.211*	.363**	1

*$p < 0.05$, **$p < 0.01$.
RS: Reasoning Skill, CU: Conceptual Understanding, PK: Procedural Knowledge, & ATT: Attitude

The Pearson correlation analysis revealed that significant relationships existed between geometry reasoning skills and each of the variables of conceptual understanding,

procedural knowledge, and the geometry attitude score. Although all of these values were statistically significant and none were greater than 0.80, there is no evidence of multicollinearity (Alpar, 2003; Leech et al., 1994).

Assumption #2: The value of the residuals should be independent. The residual is the discrepancy between the dependent variable's actual value and its expected value. A Durbin-Watson statistic was calculated to assess the assumption that the values of the residuals are independent, which suggested that this assumption is not violated as the obtained value is close to 2. The Durbin-Watson value must be between 1 and 3 (Pituch & Stevens, 2015).

Table 4.30 Models of Durbin-Watson for CU

Model	R	R Square	Adj. R Square	SE	Durbin-Watson
1	.530[a]	.281	.268	5.65741	1.794

a. Predictors: (Constant), ATT, RS
b. Dependent Variable: CU

Table 4.30 shows that the Durbin-Watson d = 1.794, which is between the two critical values. As a result of the Durbin-Watson statistic's assumption that the residuals are uncorrelated, the independence assumption is met for this analysis.

Table 4.31 Models of Durbin-Watson for PK

Model	R	R Square	Adj. R Square	SE	Durbin-Watson
1	.479[a]	.230	.216	3.24543	1.481

a. Predictors: (Constant), ATT, RS
b. Dependent Variable: PK

The Durbin-Watson d = 1.481, as shown in table 4.31, which is between the two critical values. Thus, the assumption of the Durbin-Watson statistic indicating that the residuals are uncorrelated; therefore, the independence assumption is met for this analysis.

Another assumption that must be verified is the relationship between variance inflation factors (VIF) and tolerance values. If the tolerance value is less than (1- R2) and the VIF value is greater than 10, there is a multicollinearity problem (Tabachnick & Fidell, 2013). The VIF and Tolerance values of the predictors are shown in the table 4.32 below.

Table 4.32 Models of VIF and Tolerance

Model		Unstandardized Coefficients		Standardized Coefficients			Collinearity Statistics	
		B	SE	Beta	*t*	*Sig*	Tolerance	VIF
1	(Constant)	1.789	5.647		.317	.752		
	RS	1.049	.172	.498	6.097	.000	.952	1.050
	ATT	1.992	1.596	.102	1.248	.215	.952	1.050

RS: Reasoning Skill, ATT: Attitude, DV: CU

As it can be shown on table 4.32 above, it can be said that there is no multicollinearity problem in the multiple regression analysis process, since the smallest calculated tolerance value for the data pattern is .952 and the highest VIF is 1.050.

Table 4.33 Models of VIF and Tolerance

Model		Unstandardized Coefficients		Standardized Coefficients			Collinearity Statistics	
		B	SE	Beta	*t*	*Sig*	Tolerance	VIF
1	Constant	7.484	3.239		2.310	.023		
	RS	.375	.099	.321	3.795	.000	.952	1.050
	ATT	3.166	.916	.293	3.457	.001	.952	1.050

RS: Reasoning Skill, ATT: Attitude, DV: PK

There is no multicollinearity problem in the multiple regression analysis process, as shown in Table 4.33 above, because the smallest calculated tolerance value for the data pattern is .952 and the highest VIF is 1.050.

Assumption #3: The values of the residuals are normally distributed. Showing for normality is an important early step when conducting MLR, as assuming residuals are normally distributed is an assumption (Stevens, 2009; Tabachnick & Fidell, 2006). The normalized residuals' normal probability plots (P-P plots), which may be visually inspected, can also be used to evaluate the assumption of normality. To evaluate the notion that the residual values follow a normal distribution, a P-P plot was made. A random distribution of plots around a 45-degree line in a P-P plot represents the normal distribution. Figure 4.14 portrays an example of a normal P-P plot of the standardized residuals (Tabachnick & Fidell, 2006). The dots in the normal P-P are close to the line. The plot did not show the violation of this assumption.

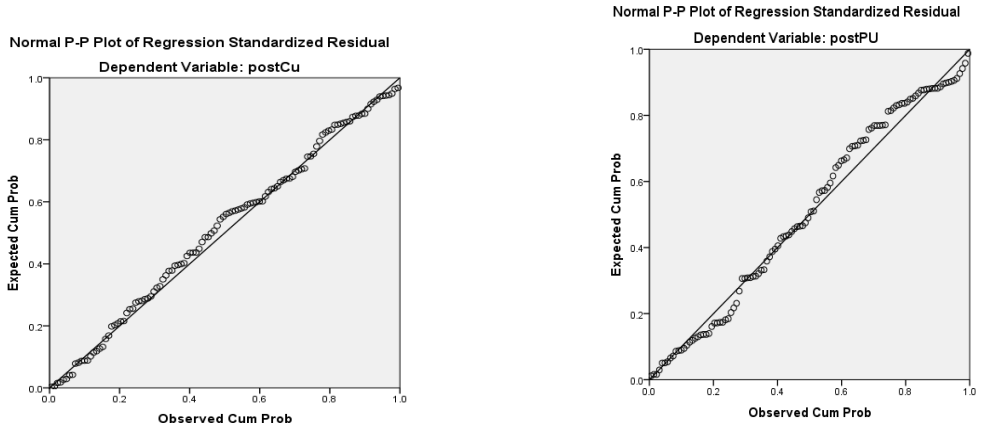


Figure 4.14 **P-P** plots of residuals

Assumption #4: Linearity (the relationship between the independent and dependent variables is linear). A simple way to check this is by producing scatterplots of the relationship between each independent variable and dependent variable.

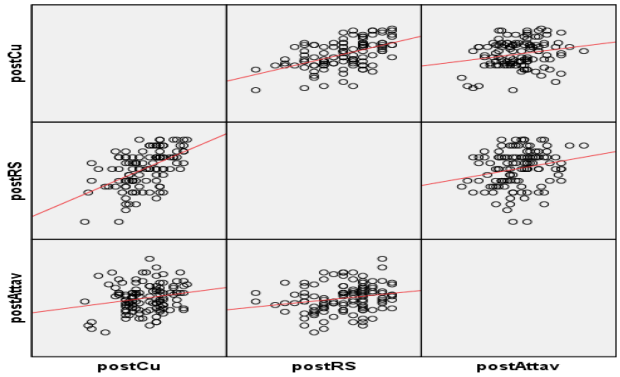


Figure 4.15 The linear relationship between IVs and DV

Assumption #5: The variance of the residuals is constant (homoscedasticity). The assumption of homoscedasticity indicates that the variance of errors is equal and constant across all levels of the variables (Osborne & Waters, 2002; Stevens, 2009). Homoscedasticity and the presumption of normality are connected because the connection between the variables is homoscedastic when the presumption of normality is fulfilled (Tabachnick & Fidell, 2006). This test shows that the variation in the residuals (or amount of error in the model) is similar at each point of the model. This plots the standardized values our model would predict against the standardized residuals obtained. If the graph looks like a funnel, then it is likely that this assumption has been violated. This plot did not indicate a violation of this assumption.

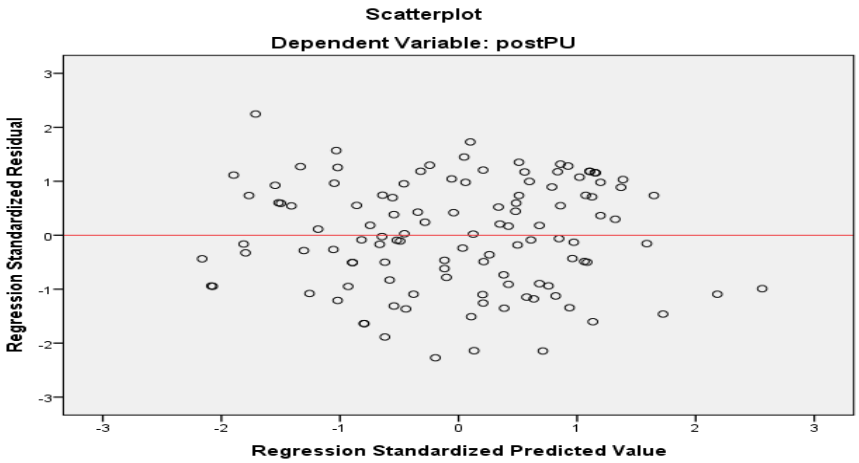


Figure 4.16 Variance of residuals

As shown in figure 4.16 above, the homoscedasticity assumption has been met, and the residuals will present as being randomly scattered around the horizontal line depicting xi = 0. Figure 4.16 portrays an example of a residual plot demonstrating a relative equal clustering of residuals along the horizontal line in a rectangular shape; therefore, the homoscedasticity assumption seems to have been met.

Normality, linearity, homoscedasticity, residual independence, and sample size assumptions have all been verified. According to a scatterplot of standardized predicted values against standardized residuals, the data met the conditions of homogeneity of variance and linearity, and the residuals were almost normally distributed. Therefore, all the necessary assumptions for MLR were not violated. Hence, after proving all the assumptions, the analysis of MLR has been conducted as shown in the following model. In the analysis, it was hypothesized that conceptual understanding and procedural knowledge would be predicted from the reasoning skills and attitude towards learning geometry. The model description, ANOVA results, and coefficients of MLRs for conceptual and procedural test scores are shown in tables 4.34 and 4.35. The following table shows the model of multiple regressions.

Table 4.34 Model summary of Regression Analysis on Conceptual understanding of plane geometry as predicted from attitude and reasoning skills

Multiple R = .530					R^2 = .281
ANOVA Table					
	SS	*df*	*MS*	*F*	*Sig*
Regression	1413.429	2	706.714	22.08	.000
Residual	3616.709	113	32.006		
Total	5030.138	115			
Variables in the Equation					
Variables	*R*	*B*	*SE*	*β*	*t*	*sig*
Constant		1.789	5.647		.317	.752
RS	.521	1.049	.172	.498	6.097	.000
ATT	.211	1.992	1.596	.102	1.248	.215

Predictors :(Constant), reasoning skill, attitude
Dependent Variable: conceptual understanding

As shown on table 4.34 above, the geometry conceptual understanding test score was regressed from the predictor variables of reasoning skills and attitude towards learning plane geometry. The independent variables significantly predict conceptual knowledge test scores, F (2, 113) = 22.08, $p < .001$, which indicates that the two factors under study have a significant impact on conceptual knowledge test score. Furthermore, the R^2 = .281 value indicates that the model accounts for 28.1% of the variance in conceptual understanding score.

When the individual predictors were further inspected, it was revealed that reasoning skill (β =.498, t = 6.097, $p < .001$) was positively significant predictor in model, whereas attitude (β = .102, t = 1.248, p = .215) was not a significant predictor that influenced PSMTs' conceptual understanding of plane geometry. Furthermore, the beta weight suggested the geometric reasoning ability predicts conceptual understanding more than attitude towards learning geometry. According to the findings, the regression model for

the prediction of conceptual understanding in geometry concepts is given by: CU = 1.789+ 1.049*RS +1.992*ATT.

Furthermore, the percentage of the effect or contribution of each component—geometry reasoning skill and attitude towards geometry (RS and ATT)—on the conceptual understanding variances is 25.96% (high effect) for reasoning skills and 2.15% (null effect) for attitude.

Table 4.35 Model summary of Regression Analysis on Procedural knowledge of plane geometry as predicted from attitude and reasoning skills

Multiple R = .479					R^2 = .230
ANOVA Table					
	SS	*df*	*MS*	*F*	*Sig*
Regression	354.902	2	177.451	16.847	.000
Residual	1190.211	113	10.533		
Total	1545.112	115			
Variables in the Equation					
Variables	*R*	*B*	*SE*	*β*	*t*	*Sig*
Constant		7.484	3.239		2.310	.023
RS	.385	.375	.099	.321	3.795	.000
ATT	.363	3.166	.916	.293	3.457	.001

Predictors :(Constant), reasoning skill, attitude
Dependent Variable: procedural knowledge

As shown on Table 4.35 above, the geometry procedural test score was regressed from the predictor variables of reasoning skills and attitude towards plane geometry. F (2, 113) = 16.847, $p < .001$, indicating that the independent variables have a significant impact on the procedural knowledge test score. Furthermore, with an R^2 = .230, the model accounts for 23% of the variance in procedural knowledge.

When the individual predictors were further inspected, and it was revealed that reasoning skills (β=.321, t = 3.795, p < .001) and attitude (β = .293, t = 3.457, p = .001) were significant predictor that influenced PSMTs' plane geometry procedural knowledge score. Moreover, the beta weight indicated that geometric reasoning ability contributed more in predicting procedural knowledge than attitude towards learning geometry. According to the findings, the regression model for the prediction of procedural knowledge in geometry concepts is: PK = 7.484 + .375*RS + 3.166*Att.

Furthermore, the percentage of the effect or contribution of each component on procedural knowledge variances is 12.36% (a medium effect) for reasoning skills and 10.64% (a small effect) for attitude.

4.3 Results of Qualitative Data

To answer the study's final research question, a descriptive case study was commenced. A case study is can be taken as an applicable research approach for a qualitative study (Roberts, Dowell, & Nie, 2019). A constructivist paradigm is based on a qualitative method such as a case study, as it provides in-depth evidence that is important for the constructivist to understand contexts (Thanh & Thanh, 2015). The constructivist paradigm was used to recognize the PSMTs' realities and describe geometry learning and attitudes in this study.

In qualitative research, the comprehensive and in-depth insights about subjects and realities are exolained than quantifying results. According to Thomas (2015), a case study emphasis on a specific issue in-depth from different perspectives. In addition, case could can give a thick description depending on the information obtained from observations,

interviews, and videos. Yin (2017) argues that an interview is an essential tool for collecting case study information. Interviews can be classified as structured, unstructured or semi-structured (Bryman, 2008).

For the purpose of this study, because of its flexible nature semi-structured interview was used with ten PSMTs who were purposively selected from experimental group. By selecting cases, and individuals, documents, and artificats within the case, purposeful sampling allows to focus in depth on a phenomenon. It allows to explore information-rich cases from which you can learn a great deal about issues of centeral importance to the research (Patton, 2002). All PSMTs choosen for semi-structured interviews decided to partake in the interviews. Semi-structured interviews are often used in qualitative analysis and the interviewer does not do the research to test a specific hypothesis (David & Sutton, 2004). The semi-structured interviews were most favorable because examining was possible to understand PSMTs' experiences when working with GSP integrated in guided inquiry approach. Semi-structured interview may also help to comprends PSMTs' perspective on the effectiveness of technology (GSP) in teaching and learning of geometry in the context of higher education. For interviews, the interview protocols, or a list of questions and prompts used to interview PSMTs (**See Appendix**).

An audio recorder and field notes were used to document the qualitative data; thus, themes were used for analysis. Thematic analysis is a method of qualitative data analysis that is often employed because it is conceptually flexible, allowing it to be applied within various frameworks to address research issues. According to Braun, Clarke, and Terry (2014), thematic analysis uses groupings of coding that focus on discovering patterns of meaning throughout a dataset. Gabriel (2013) discusses different strategies for

undertaking theme analysis, i.e., deductive and inductive: an existing concept directs the development of codes and themes in a deductive way, while data content directs the development of codes and themes in an inductive way. Coding was therefore employed as an analytical technique.

In the analysis of data, I first listened to each individual audio recording before starting to create themes. In this step, I examined and re-examined the data, as well as read and re-read the transcript, to become familiar with the experiences of the various participants. According to Patton (2002:446), “coding was to read the data I collected over and over and over, the more I interacted with tha data, the more patterns and categories began to ‘jump out’ at me”. Thus, familiaring with the data is the first step for coding to create themes (Creswell, 2009).

Each transcript was then manually coded, carefully reading each line of text. I identified the number of codes generated from each transcript of a Word document. Before beginning the next cycle of coding, I examined each set of codes to seek out expanding patterns and similarities before analyzing the data. This process involved applying and improving codes that highlight significant data properties and seem pertinent to addressing the study issues. The selected codes were then improved over time when data from multiple data sources was coded.

I then employed rigorous coding; at this stage, I looked over the coded data to find "themes," or recurring patterns of meaning. I searched for the existing codes and put them in groups according to their commonality. Following that, a preliminary title was given after establishing a category for it. I used manual examination of their frequencies and

collecting relevant evidence for each topic to ascertain the reasonableness of each identified theme.

I looked at the categories and supporting codes for a more in-depth analysis. The primary categories were identified using axial coding, which was then linked to subcategories. On the codes, each participant's pseudonym was recorded. This provided participants with a visual aid for studying and evaluating the codes they created. For a profound analysis, I looked at the categories and auxiliary codes. Axial coding was used to determine the main categories, which were then connected to subcategories. Each participant's pseudonym was recorded on the codes. Participants now have a visual aid for learning and assessing the codes they produced.

During this process, I also reviewed and checked categories to determine whether the identified themes were substantial in regards to responding to the research question. To this effect, the themes were refined, which sometimes involved being fragmented, united, or discarded. The final themes and sub-themes were structured for the analysis and writing, which were thoroughly interwoven to form the analysis for the findings report.

After that, four principal themes and eight subthemes emerged from the data analysis. The four principal themes that emerged were (a) the effectiveness of a technology-integrated guided inquiry approach, (b) opinions or attitude, (c) challenges with the strategies and (d) recommandations. The subthemes that emerged from the data analysis were cooperative and group work, connection and concept construction, multiple representations, visualization, formulating relationships, motivation and eagerness, perception, the PSMTs' backgrounds, instant feedback, and endorsement. The emerging

data is discussed in detail in the following subsection, along with typical quotes that reflect its proper meaning.

4.3.1 Effectiveness of Technology-integrated Guided Inquiry approach

Connection and Concept Construction

This is the subtheme from the transcription of PSMT's interviews. During their previous classroom experiences, none of the interviewees' mathematics teachers used technology [GSP] to teach geometry and mathematics. The interview results revealed that every interviewee concurred that using technology-based learning geometry has improved their understanding of concepts in the subject. In this regard, the following excerpt was taken from the transcripts of some PSMTs:

> *A23: Yes, I found that using a computer to learn was more intuitive than using a pencil and paper to complete geometric concepts. Because I get that the concepts are connected when I see them on a computer screen.*

> *A5: It was really a mind-catching experience to view geometric concepts by animating figures with technology [GSP]. Not only was it exciting, but I now have a much broader and more in-depth understanding; for instance, in circle geometry, such as the relation between the central angle and its inscribed angle, cyclic quadrilaterals, the equality of two angles subtended by the same chord, etc.*

> *A35: It would be so boring if we studied these topics on the blackboard. We wouldn't be that active, and we wouldn't understand the topics that well. I think I learned very quickly and easily because I was very active in the learning process, I constructed objects and dragged them myself. Computers increased our interaction in the classroom.*

> *B16: It would be quite challenging for us to grasp the concepts if we learned these concepts as normal through rote memorization. For example, using hands-on activities improved our exploration of theorems to test conjectures such as the sum of interior angles of a triangle being 180^0; the sum of opposite angles of a cyclic quadrilateral being supplementary, and so on.*

This demonstrates how technology [GSP] integration would improve PSMTs' understanding and learning of geometry's from EG1 said that their learning was meaningful, since they learned the concepts themselves rather than memorizing definitions and concepts.

Visualization and Multiple Representations

This subtheme also appeared in the transcripts of the interview. The incorporation of technology [GSP] made it easier to visualize abstract ideas in geometry. However, PSMTs from EG2 suggested that they would learn more effectively if computers were employed alongside hands-on activities. Thus, by incorporating educational tools into teaching practices, educators can provide innovative opportunities for supporting PSMTs' learning. In this regard, technology helped them to visualize abstract concepts using multiple representations. Thus, they stated as follows:

> *A35: What appears to be a complicated and abstract concept is actually simple. My understanding of geometry concepts had improved thanks to the technology [GSP]. We have access to get immediate feedback, and that helped me understand the course content.*
>
> *A5: Technology [GSP]... is such a powerful tool in mathematics (geometry) because, you know, it provides the visual that we need to process in order to understand and apply the concept.*
>
> *B3: The use of hands-on manipulatives in learning geometrical concepts simplifies the concept. I think it enhanced our learning process.*
>
> *B25: Hands-on activities added to my understanding of the concepts of the course. But I think it would have been much better if we had used other tools, such as computers.*

The PSMTs' responses confirmed that technology assisted them in visualizing plane geometry concepts, making it easier for them to understand concepts. In general, it [technology] assisted them to better understand geometric concepts by visualizing abstract geometric concepts, conceptualizing their own ideas, and analyzing geometrical concepts better. Furthermore, technology [GSP] enables seeing and understanding by manipulating geometric objects.

Cooperative and Group Work

This subtheme was inspired by an interview and an observation of a classroom in which PSMTs emphasized the importance of technology for cooperative and group work. The integration of technology into their instruction clearly promoted PSMT interaction in class. They believed that they had more discussion when they used their technology [GSP] to solve problems. During my observation, I also noticed that the interaction among them [PSMTs] was to share ideas and communicate with their teacher to receive further support. They discuss and share ideas, then compare all the answers in a group.

> *A3: The method we used earlier was inadequate to grasp everything about the course. However, the learning approach we used for this semester was important and helpful, since it created a meaningful way to learn and provided sufficient opportunities to collaborate within the groups.*
>
> *A5: ... yes. We also benefited greatly from group discussions while participating in computer activities. We worked in collaboration with each other under the guidance of our teacher.*

Most of the participants preferred to learn in a group with collaboration among their peers. Furthermore, I saw the teacher in the classroom brainstorming and challenging PSMTs during their activities by connecting their prior knowledge to meaning-making. PSMTs were discussed in groups and engaged in cooperative learning. In addition, the learning strategies of engagement and exploration include experiential learning,

explaining ideas, investigating and discovering using GSP and manipulatives, writing to communicate ideas, and presenting to share ideas.

Problem-Solving and Content Knowledge

This is another subtheme that is primarily seen in the transcription of the interview and observations. PSMTs learned geometric concepts and problem-solving. As one PSMT stated, "I learned how to conjecture, how to solve geometric problems, and obtained an understanding of ideas in Euclidean geometry." As a matter of fact, the technological environment enables the PSMTs to move mathematical objects, giving them the chance to empirically validate or test their findings.

> *…...technology supported classroom activities were realistic because in the past it was basically just theory; you just had to know this comes after this and that, and you really didn't know why it was like that. But now, I could link concepts together, and by linking them, I understood it better when we were working on the computer.*

4.3.2 Opinions

Motivation and Enjoyments of PSMTs

The PSMTs in the experimental groups seemed to be making significant improvements in showing interest in and becoming more and more involved in each activity of the training, as shown in both the interview and the observation. It was discovered because the interviewees were so fascinated by technologically based mathematical activities. The following excerpt, which was taken from PSMTs' transcripts, reflects their perceptions in this respect.

> *A10: I enjoyed my geometry classes this semester since we employed a computer to learn. It was very interesting. Learning geometry using a computer is remarkable.*

A23: At first, studying geometry on the computer seemed a little strange, but I actually enjoyed it. Studying geometry just with the computer instead of paper and pencil was very exciting and engaging. It was, in my opinion, the most interesting subject we have studied so far.

A35: I liked computer-based geometry instruction, and I found that I learned the topic well. I wish computers could always be used for studying mathematics.

Attitudes towards learning geometry

The overall positive attitudes toward geometry were revealed in the PSMTs' interviews. According to interviewees, the use of technology in their geometry classroom enables them to reach their potential for achievement in geometry and has developed their self-confidence. This shows that geometry learning via technology-integrated inquiry has improved their attitudes.

A35: *Although I don't mind learning in a traditional classroom setting, the computer-based training was much better for our training. The first time we had lessons there, I was quite pleased and eager to go to class.*

A5: *Even though I don't particularly appreciate geometry, I thoroughly enjoyed these sessions. Although I could solve problems in the past, I never found them to be very interesting or easy. Now, I believe geometry is not all that challenging.*

A10: *I have to admit that I was very nervous about using computers to learn geometry because I am not much of a math person. But I have found that I have been doing so much better than I had expected. I noticed that there are some topics that I like in geometry.*

A23: *I realized that there are mathematics topics that I can enjoy. Mathematics was just numbers for me before, and it was really boring for me .Now I think that geometry is fun.*

A12: *I've always had to work so hard to succeed in mathematics, but this time I didn't. Learning more about the subject was interesting. I discovered that I can succeed in geometry and mathematics.*

B22: *When we got hands-on lessons, I was more eager to engage since I knew it would be enjoyable. I believe I truly understood the topic we covered.*

A9: *Normally we work on math topics on the board, and it's so boring. That's why we enjoyed learning subjects with computers so much, because computers are new and interesting.*

B29: *Most of the time, we simply write down what the instructor writes on the board. In contrast, in these sessions, I was so engaged and active instead of just taking notes.*

B17: *Usually, finishing up college by taking notes is an ache. I am also quite bored. However, I had a good time in my last classes.*

None of the interviewees had a negative perspective towards a technology-integrated [GSP] approach.

4.3.3 Potential Issues in a Technology-Integrated Inquiry Approach

Background skills in Computer usage

This is the most considerable subtheme of difficulties portrayed within the transcripts of the PSMTs' interview. The biggest challenge was the lack of computer skills during the implementation of both interventions. One of the interviewees said:

> *... Since I have no prior knowledge of computer skills, I was slightly exasperated. The syntax of GSP confused me a bit. Nonetheless, it helped me in terms of circle geometry, and we could actually see that the angle at the center is twice the circumference angle.*

Mathematics Teacher

Mathematics teachers inspired PSMT participants in this study to learn mathematics and even pursue careers as math teachers. From my observations, their teacher in experimental groups didn't act as an information disseminator as usual. However, the teacher's position did not change significantly in CG.

A23: *The principles were previously explained by the teacher while writing on the board. However, in this approach [TGIBL], the teacher didn't explain the concepts directly; instead, he simply guided and assisted us.*

B16: *In our previous lessons, the teacher would explain the topics. The instructor didn't provide as much direct explanation of the topics in these lessons as in previous ones. He asked us more questions and provided hints to help us find the answers.*

4.3.4 Final Ideas

Renovating of classroom with technology

Technology offers new ways of working and offers learners new ways of interacting with information and gaining knowledge, understanding, and skills. The use of technology-based geometric activities was favored and valued by both instructors and PSMTs. PSMTs from the GIBL and TGIBL groups agreed that interventions had helped them to facilitate meaningful learning and learn difficult geometric concepts more readily, resulting in greater and more meaningful student outcomes. The following was taken directly from the TGIBL teacher's interview transcripts, which acknowledged the significance of this issue as an enabler.

> *...since technology touches almost every aspect of life, to link abstract mathematical concepts with concrete life experiences... especially the implementation of a student-centered, especially technology-integrated inquiry approach, is very motivating. It was good. He also recommended that to verify geometric properties and theorems, technology-based activities should be integrated into the mathematics classroom in order to enhance discovery.*

In conclusion, according to the PSMTs' interviews, the technology integration improved the instruction because teachers could show what they explained while they taught. PSMTs believed that technology would help them visualize abstract mathematical concepts. They also thought the way teachers presented the instructional materials was effective and helped them visualize the concepts. Thus, PMSTs had alternative ways of approaching, analyzing, and understanding geometrical concepts when they used technology to study mathematics. Another reason why technology made learning

geometry more effective was because it allowed PSMTs to approach a geometric problem in multiple ways, which is by watching, discussing, and manipulating objects.

4.4 Summary of the Findings

4.4.1 Summary of the Findings of Descriptive Statistics

Descriptive statistics, such as sample size, mean, standard deviation, skewness, and kurtosis, provided information on the sample under study. According to the results, the mean of PSMTs in the EG1 > EG2 > CG, which shows PSMTs taught using a technology-integrated guided inquiry-based learning approach had greater scores than PSMTs in the guided inquiry approach and comparison group, which were taught using traditional teacher-centered instruction.

4.4.2 An overview of findings from inferential statistics

- The purpose of the current study was to investigate the effect of technology-integrated guided inquiry-based instruction on PSMTs' geometry learning outcome and attitude towards geometry. In this sense, ANOVA, MANOVA, ANCOVA, and MLR were conducted to examine the impact of this instructional method on geometry learning outcomes. Assumptions of independence of observation, normality and outliers, reliability of covariates, multicollinearity, linearity, homogeneity of regression, and homogeneity of variance were all checked, and the assumptions were not violated for each dependent variable. Thus, statistical analyses of quantitative data were summarized in relation to the research question guiding the study.
- A MANOVA result showed there was a significant mean difference between groups on the collective dependent variables of the PSMTs' posttest scores on

conceptual understanding, procedural knowledge, and reasoning skill test on geometry learning.

- The ANOVA and ANCOVA results showed there was a significant mean difference between PSMTs' group posttest scores on the geometry conceptual understanding test, the procedural knowledge test, the reasoning skill test, and the GASQ (its dimensions) test when PSMTs' pretest scores on the geometry attitude scale, value, and self-confidence are controlled. Finally, the effect sizes calculated on the post-test scores of Post-CU, Post-PK, Post-RS, and Post-ATT ranged from moderate to large for independent variables, thus indicating practical significance. Furthermore, a paired sample t-test showed that there was a significant change from pretest to posttest with a very large effect size for PSMTs in EG1 and EG2 in learning plane geometry.
- A MLR demonstrated conceptual understanding; procedural geometry knowledge was predicted by geometry reasoning skills, but attitude toward geometry was not statistically predicted as conceptual geometry understanding.
- There was no significant mean difference between the post-ATT scores of the EG2 and the CG. On the other hand, there was a significant mean difference between the EG1 and the CG, and between the EG1 and the EG2.
- There was no significant mean difference between the Post-CU, Post-PK, and Post-RS scores of the EG2 and the CG, while there was a significant mean difference between the EG1 and the CG and between the EG2 and the EG1.

4.4.3 An overview on the Findings from Qualitative Data

PSMTs stated that technology created a learning environment that supported their development and helped them to explore mathematics in a far more meaningful way. They stated that their learning was meaningful for them since they discovered the topics themselves through group work and cooperation instead of memorizing properties and definitions that the teacher said. Using a technology-supported approach gave PSMTs a visual way to explore and understand geometry concepts. PSMTs specified that technology provided a visual representation that improved their understanding of geometry concepts.

PSMTs also stated that learning geometry using the TGIBL approach was exciting and interesting. The use of technology affected PSMTs' attitudes towards geometry positively, as most of the PSMTs stated that they were more willing to participate in the class and offer answers to questions. According to PSMTs in EG1 and EG2, the instructor's role included being both a mentor and a learning partner.

Chapter Five: Discussion, Conclusion and Recommendation

5.1 Summary and Discussion of the Findings

In the previous chapter, the researcher presented the data analysis results and interpretation of both quantitative and qualitative data. In this section, the findings from the data analysis results on conceptual understanding, procedural knowledge, reasoning skills, and attitude are presented, triangulated, and discussed depending on the research questions and findings from different related literature.

As indicated in the preceding chapter, the purpose of this study was to investigate the effects of the TGIBL approach on year-2 PSMTs' learning of plane geometry. The learning of plane geometry means conceptual understanding, procedural knowledge, reasoning skills, and attitudes towards geometry, which were measured through geometry conceptual understanding test, geometry procedural knowledge test, geometry reasoning skill test, and geometry attitude scale questionnaire. For this purpose, a quasi-experimental research design was employed. To that end, two CTEs were purposefully chosen, and the intact classrooms were randomly assigned as experimental and comparison groups into three groups. The three groups were assigned to TIGBL (EG1), GIBL (EG2), and TRAD (CG), which have 48, 38, and 30 participants, respectively.

Prior to the implementation of the interventions, pre-tests were administered to all three groups. Thus, the descriptive statistics of the pretest result reveal that the mean score of the groups seems somewhat different (see Table 4.8). An ANOVA test was further conducted to analyze the results of the pre-tests in order to determine if the differences between groups existed before the interventions (see Table 4.9). The result of the ANOVA showed that there was no significant difference between the groups in terms of

Pre-CU, Pre-PKT and Pre-RS test scores. So, the three groups were assumed to be similar before the interventions. Therefore, the final differences observed are attributed to the effectiveness of the interventions and not the PSMTs' prior differences. Nevertheless, a prior significant difference was found between the three groups with respect to their pre-attitude, pre-self-confidence, and pre-value/usefulness of geometry (see Table 4.30). Therefore, pre-attitude, pre-self-confidence, and pre-value/usefulness scores of PSMTs were used as a covariate.

The interventions were carried out for 10 weeks, using four credit hours per week. The mathematics teacher educator who implemented the interventions in EG1 and EG2 was trained about materials that were going to be applied, activities that were going to be used, and lesson plans for the instructions that were going to be implemented for the intervention. After the implementation of the interventions, Post-CU, Post-PK, Post-RS, and Post-GASQ were administered to collect quantitative data. In addition, semi-structured interviews and classroom observation were conducted to collect qualitative data to triangulate with the findings of the quantitative data. In order to analyze quantitative data, MANOVA, ANOVA, ANCOVA, t-test and MLR were used, while theme was applied to analyze qualitative data. Following a thorough evaluation of all statistical assumptions, the post-test scores were analyzed with each dependent variable in relation to its independent factors and finally triangulated with the theme results.

5.1.1 The Effects of interventions on PSMTs' Learning Plane Geometry

The study's first research question was anticipated to look into whether there were any significant mean differences in the conceptual understanding, procedural knowledge, and reasoning skills of PSMTs across the groups after the interventions. A one-way analysis

of variance was used to determine how interventions affected the conceptual, procedural, and reasoning skills of PSMTs. The study's participants were divided into three groups: those who received the traditional teacher-centered method (CG), EG2 those who received a guided inquiry-based learning approach (GIBL), and EG1 those who received a technology-integrated guided inquiry-based learning approach (TGIBL).

According to the results of the conceptual, procedural, and reasoning skill post-tests, the post-test mean scores for the three groups were different. The mean conceptual test scores for the experimental groups (EG1 and EG2) were M = 27.38, SD = 6.70, and M = 23.79, SD = 4.63, respectively, whereas the mean for the comparison group was M = 21.13, SD = 6.89, showing PSMTs in EG1 and EG2 scored a higher mean score than CG. Similarly, the mean procedural posttest scores of PSMTs in EG1, EG2, and CG (M = 26.91, SD = 2.74; M = 23.34, SD = 2.69; and M = 21.02, SD = 2.87) show that PSMTs in CG scored a lower mean score as compared to EG1 and EG2. Also, the mean reasoning posttest scores of PSMTs in EG1 and EG2 (M = 16.98, SD = 2.25 and M = 14.45, SD = 2.66) respectively have achieved higher reasoning mean scores than PSMTs in CG (M = 12.87, SD = 3.22) (see Tables 4.10). As it can be seen from the descriptive statistics, all groups have different mean scores in conceptual understanding, procedural knowledge and reasoning skills of geometric concepts. When placed in descending order, the mean scores for EG1 > EG2 > CG.

A one-way MANOVA was used to evaluate the effect of the intervention on three dependent variables: conceptual understanding, procedural knowledge, and reasoning skills. Using Wilk's Lambda λ= .459, F (6, 222) = 17.62, p < .001, multivariate$\eta^2 = .32$, there was a significant difference on the three dependent variables among PSMTs in the

study. The multivariate partial eta square shows that the group factor was responsible for about 32% of the multivariate variation of the dependent variables (See Table 4.11). For further analysis, follow-up univariate ANOVAs were conducted and revealed significant effects of interventions on conceptual, procedural knowledge and reasoning skills (Table 4.12).

The finding of the ANOVA test showed the interventions had a significant effect on conceptual test scores (F (2, 113) = 9.98, $p < .001$,$\eta^2 = .15$), procedural test scores (F (2, 113) = 44.82, $p < .001$, $\eta^2 = .44$) and reasoning test scores (F (2, 113) = 23.69, $p < .001$, $\eta^2 = .29$) (see Tables 4.12, 4.18, and 4.21). Thus, the partial eta indicated that the effect size was large in all cases (Cohen, 1988). The variances of 15%, 44%, and 29% of posttest scores on conceptual, procedural, and reasoning, respectively, among the group were associated with the effectiveness of the interventions.

Additionally, a post hoc comparison Bonferroni test was computed to assess pairwise differences between group means. The Bonferroni post-hoc analysis specified a significant mean difference among the three groups: the TGIBL group and the CG group, the TGIBL group and the GIBL group. However, PSMTs who were treated with GIBL did not significantly differ from the CG group (see Tables 4.13). Hence, PSMTs treated with TGIBL and GIBL methods showed better performance on conceptual, procedural, and reasoning test scores than those who thought using a traditional teacher-centered method.

The analysis of PSMTs' post-test score items of CU, PK, and RS also showed that PSMTs in EG1 who used the TGIBL approach did well on each item when compared to

PSMTs in EG2 who were treated with the GIBL approach and PSMTs who were taught with the traditional teacher-centered instructional method. When we categorize the percentage of PSMTs' responses based on conceptual understanding, the percentage of PSMTs' responses from the TGIBL approach is higher than those from both the GIBL approach and traditional approaches. For instance, if we see conceptual test item 5, item 12, item 8, item 10, and item 14, the percentage of PSMTs from TGIBL who answered correctly are 65.17%, 82.86%, 60%, 85.71%, and 54.29%, respectively, and only 43.33%, 73.33%, 80%, 33.33%, and 93.33% percent of PSMTs from CG did not respond correctly to the given items, respectively (see details from Tables 4.14, 4.15, and 4.16). Furthermore, on conceptual geometry test items, a higher percentage of PSMTs from the GIBL approach were scored than PSMTs from traditional teacher-centered methods. On item 6, for example, PSMTs were asked to determine their understanding of the anticipated response for item "The relation between the central angle and its inscribed angle." The percentage of PSMTs with 54.29% in EG1 suggested a complete understanding, but with 6.67% in the comparison group, it suggested an unsatisfactory understanding.

Similarly, when items from the procedural knowledge test were considered, a high number of PSMTs from TGIBL and GIBL groups had no difficulties in solving geometric procedural problems. The PSMT participants had solved the test items by showing all the necessary steps and procedures (see Figures 4.1, 4.2, and 4.3). Ferrer (2016) stated that writing whole steps without skipping the steps was just attributed to the PSMTs' in-built mathematical knowledge and skills from basic mathematics. Likewise, when we categorize the percentage of PSMTs' responses to reasoning test items, we find

that PSMTs from the TGIBL group had no difficulty in the proving and reasoning of geometric questions, followed by PSMTs from the GIBL group. The item analysis of these groups showed that, for all questions asked, PSMTs from the TGIBL group answered the questions with sufficient reasons behind their explanation, but PSMTs from the other groups did not give correct explanations with reasons behind. The PSMTs in the TGIBL group showed sufficient reasons to justify and conclude their answers for each reasoning test item.

The findings from the interviews also indicate that using the TGIBL approach affected their learning of geometric concepts positively. The theme analysis of qualitative data from EG1 and EG2 interviewees indicated computer-assisted activities helped them to visualize concepts and analyze geometric problems, which assisted in understanding abstract geometric concepts. In addition, PSMTs from the EG1 stated that their learning was very meaningful since technology aided in discovering, exploring, and analyzing geometric concepts in multiple ways. They also stated that the use of computers allowed them to have independent practice and that they actively participated in the learning process. Hence, PSMTs in the TGIBL group achieved higher, understood the problems better, and reasoned about them better than the other groups. The result from both quantitative and qualitative data analysis revealed that PSMTs taught using the TGIBL approach outperformed the other groups.

To determine if there was a significant difference between the means of the pretest and posttest score measures for each group, a paired sample t-test was also performed. The conceptual, procedural, and reasoning test scores for groups of PSMTs in TGIBL and GIBL were significantly higher in the post-test than they were in the pre-test (Table

4.17), indicating that the intervention program was successful in enhancing the PSMTs' procedural knowledge, reasoning skills, and understanding of plane geometry concepts. The degree of the difference between the pre- and post-test mean scores was also determined by calculating the effect size (Cohen's d). The effect size values are high (d > 0.8), implying relatively large mean gain between pretest and posttest for each group as well. Nevertheless, when the comparison group was taken into account, there were no significant variations between the pretest and post-test results for any of the measures.

The results of this study's findings support several researchers' suggestions on the benefits of employing technology to enhance learning (Drijvers, 2015; Arbain & Shukor, 2015; Pilli & Aksu, 2013). In accordance with this, technology like computer-assisted instruction offers a new dimension to teaching abstract mathematical concepts and can minimize misconceptions (Awodeyi, Akpan, & Udo, 2014; Gurbuz & Bairgin, 2012; Daher, Baya'a, & Anabousy, 2018). Sari and Bakr-Güven (2013), for instance, studied pre-service teachers' learning of modern physics using inquiry-based instruction supported by technology (simulation, videos, and animation). Their findings revealed that pre-service teachers using technology-supported inquiry-based instruction outperformed those taught with a traditional approach in terms of achievement and conceptual understanding. With the use of technology, learners' reasoning and learning processes may be facilitated, and abstract concepts can be represented in concrete ways.

The impact of 5E guided inquiry supplemented by computer animation on geometry learning is being investigated by Bulbul (2010). The experimental group used 5E guided inquiry assisted by computer animation, whereas the control group received teacher-centered computer representation. The results revealed that students in the experimental

group outperformed students in the comparison group on tests of conceptual knowledge. In the end, the researcher came to the conclusion that using technology-supported classes in a student-centered manner rather than a teacher-centered approach increases their effectiveness. In a student-centered approach, Sabnem (2008) also looked at the effects of technology-based hands-on activities. Random assignments were made to two treatment groups and one control group. Students who employed technology-based hands-on activities achieved higher post-test conceptual understanding scores than students who received technology-based activities with a teacher-centered and traditional approach. The authors came to the conclusion that a technology-based approach is useful only in a student-centered approach with teacher-guided learning activities.

Similarly, Donevska-Todorova (2015) conducted a design-based study to investigate the impact of technology-based classrooms on students' conceptual understanding of vector dot products. The study argued that using technology allows pupils to learn this particular subject in more depth through a variety of representations. Moreover, Martinovic and Manizade (2014) found that technology-based geometric exercises improved the reasoning skills of pre-service teachers. Positive findings included the fact that these pre-service teachers were able to explore designed activities and use technology to construct and assess geometric conjectures. According to the study, technology has a vital role to play in promoting professional integrity among aspiring teachers. Moreover, Ljajko and Ibro (2013) found that mathematical concepts and procedures learned using technology are long-lasting and better incorporated into learners' cognitive structures.

Many studies on how different technologies affect learning have found that, when combined with effective pedagogical methods, technology improves learning and

learning outcomes (Shanwal, 2017; Pfeiffer, 2017). GIBL and technology can also improve PSMTs' learning outcomes (Schmidt & Williamson-Kefu, 2020; Khasawneh, Hodge-Zickerman, York, Smith, & Mayall, 2023; Bedada & Machaba, 2022; Dem, 2020; Ocal, 2017; Wassie & Zergaw, 2019). The TGIBL methodology is more successful than other methods at improving PSMTs' knowledge of geometry ideas, according to the results of this study, which generally supported other research findings. The findings indicate that a guided inquiry approach is essential for enhancing PSMT's learning, but that using technology to support the guided inquiry is even more successful in enhancing PSMT's learning. The finding confirmed that using a guided inquiry approach is important in enhancing PSMT's learning, but supporting the guided inquiry with technology is even better for increasing PSMT's learning.

Nevertheless, the present findings are not concurred with those of Alotaibi (2013) and Cracraft (2015), whose research found no statistically significant difference in the achievement between students, taught using dynamic geometry software and those taught using the conventional technique. In addition, researchers assert that only well-designed materials can successfully include PSMTs and help their learning of mathematics and geometry (Liang & Sedig, 2010).

5.1.2 The Effects of Interventions on PSMTs' Attitudes

The research asserts that effective teaching approaches are essential for developing attitudes and emotional responses, or a mindset toward mathematics, that persist over time. In order to alter PSMTs' attitudes toward comprehending geometry and mathematics in general, guided inquiry-based learning activities that integrate technology into the learning process should be created. In order to examine the impact of the

interventions while adjusting for the covariate of pre-test attitude, the ANOVA and ANCOVA tests were used.

The ANOVA result revealed that the three groups differed significantly in terms of enjoyment and motivation at (F (2,113) = 4.67, p=.01, η^2= .08) and (F (2,113) = 7.75, p < .001, η^2 = .12) respectively. The eta squared indicates that 8% and 12% of the variance is associated with interventions, respectively, for enjoyment and motivation. This eta squared showed moderate effect size (Cohen, 1988) (Table 4.31). The post-hoc analysis showed PSMTs in the TGIBL group were more motivated and enjoyed learning geometry lessons when compared to the other two groups (Table 4.32).

Similarly, the ANCOVA result exposed that the three groups are significantly different in terms of attitude, self-confidence, and value towards learning geometry (F (2, 112), p < .001, η^2=.26); (F (2, 112) = 5.62, p < .001, η^2 =.17); and (F (2, 112) = 3.63, p =.03, η^2=.06). The η2 =.26 shows a large effect size, η2 =.17 shows a large effect, and η2 =.06 shows a medium effect for attitude, self-confidence, and value respectively (Table 4.35). The Bonferroni post-hoc analysis indicated that PSMTs who were treated with the TGIBL approach had a more progressive attitude and self-confidence to learn geometry than PSMTs who were taught using the GIBL and traditional methods of teaching (Table 4.35).

Furthermore, the paired sample t-test was conducted to test if the means of the pretest and the posttest of attitudes towards learning plane geometry differed significantly for each group. The paired samples test results revealed that the paired differences between the attitude towards geometry posttest and pretest are.76 and.37 for PSMTs in TGIBL and

GIBL, respectively, revealing statistically significant differences. The effect size values are high ($d > 0.8$), implying a relatively large mean gain between pretest and posttest for each group as well (see Table 4.28). This showed that the technology-based learning approach and guided inquiry approach changed PSMTs' attitudes towards learning geometry. However, there is no difference between the two means of the pretest attitude and the posttest attitude for PSMTs in the traditional teacher-centered approach.

In the same manner, the results from the interview also strengthened the results from the quantitative analysis. Findings from the interviews also indicated that a technology-based approach had positive effects on PSMTs' attitudes and motivations towards learning geometry. Some of the PSMTs from the TGIBL group stated that their learning was very meaningful and enjoyable since they discovered the topics by themselves instead of memorizing properties and definitions of geometric concepts. They also stated that technology-augmented geometric activities helped them practice with peers and stay actively engaged in the learning process, and their interest improved as a result of visualizing geometric concepts through technology-based learning and connecting geometrical ideas and activities. Mathematics teachers inspired PSMT participants in this study to learn mathematics and even pursue careers as math teachers.

The findings of the study were further corroborated by the study's analysis of classroom observations. It was observed that PSMTs in the experimental group seemed very eager, motivated, and enjoyed attending activities, discussing questions, communicating, and sharing their conclusions. PSMTs were excited and enjoyed when technology was integrated into geometry class, and as a result, attitudes toward geometry have been

affected positively. The attitude that PSMTs have toward mathematical topics affects their ability, learning and determines their engagement, interest, or self-confidence.

According to the literature, the teaching methods employed in the classroom play a significant role in changing the attitudes of the learners (Williams & Williams, 2011; Sani & Salahudeen, 2016). Active teaching methods, as opposed to passive teaching methods, are more likely to affect positive attitudes and meaningful learning (Shihusa & Keraro, 2009). In this regard, according to Sakar (2010), guided inquiry-based learning is an effective teaching strategy that accounts for PSMTs' positive attitudes toward geometry and mathematics. In addition, Moore (2017) asserts that technology can provide a cognitive process and engage learners to increase their interest in and enjoyment of geometry and mathematics.

Several research studies have shown that using technology-based activities in the classroom enhanced PSMTs' interest in geometry in particular as well as mathematics in general (Celen, 2020; Murni, Sariyasa, & Ardana, 2017; Tamam & Dasari, 2020; Yimer & Feza, 2019). Furthermore, Adegoke (2016), Almeqdadi (2018), Komar (2019), Sudihartinih and Purniati (2018), and Samur and Akyuz (2016) discovered that PSMTs' attitudes changed after being taught using technology-based activities, which is consistent with the current findings. According to Mousoulides (2011) and Ocal (2017), technology has also become one of the most effective methods for teaching mathematical concepts, as it has the potential to overcome the conceptual difficulties faced by students.

Moreover, learners' self-confidence towards geometry were positively changed after technology-based instructional methods (Hidayati & Kurniati, 2018; Şeker & Erdoğan,

2017; Zetriuslita, Nofriyandi & Istikomah, 2020). On the other hand, the current finding of the study is inconsistent with Slavik's (2015) study, which found that the effect of spreadsheet-based learning is insignificant in changing college students' attitudes towards mathematics.

Similarly, Pulat (2009) discovered that learners who were taught using the 5E guided inquiry approach demonstrated a significant change in attitude when compared to learners who were taught using the traditional approach. It was claimed that this was because the participants were actively involved in the learning process guided by the instructions provided in the activities. The research findings of the current study also supported the findings from different literature mentioned above because PSMTs taught with a TGIBL approach, actively engage learners, are more motivated, and gain a more positive attitude than PSMTs taught with a traditional method of instruction. This is because technology integrated guided inquiry is the inquiry method that helps PSMTs actively engage in learning. Similarly, PSMTs taught using the TGIBL approach were found to be significantly more motivated than those taught with the GIBL approach or conventional instruction.

Contrary to expectations, the use of the GIBL approach did not have a significantly greater effect on PSMTs' motivation, enjoyment, and usefulness of geometry than conventional instruction. This is not in parallel with the findings of research conducted by Masilo (2018), Kandil (2016), and Pulat (2009), which revealed that the use of guided inquiry approaches has a positive effect on PSMTs' attitudes towards learning mathematics. As a result, further study is needed in this area.

5.1.3 Conceptual and Procedural Knowledge of Plane Geometry as a Function of attitude and reasoning skills

A MLR analysis was computed to determine whether conceptual and procedural knowledge in geometry accounted for a significant portion of the variation in scores of the reasoning skill test and geometry attitude scale scores.

The independent variables significantly predict the conceptual knowledge test scores, F (2, 113) = 22.08, $p < .001$, which indicated that the two factors under study have a significant impact on the conceptual knowledge test scores. Besides, the finding revealed that 28.1% of the conceptual knowledge test scores can be explained by the predictors (R^2 = .281). The individual predictors were examined further and indicated that reasoning skills (β = .498, t = 6.097, $p < .001$) were positively significant predictors in the model, while attitude score (β = .102, t = 1.248, p = .215) was not a statistical predictor that influenced PSMTs' conceptual knowledge test scores. The affective domain was not one of the two predictor factors in this study that predicted conceptual knowledge test scores (see Table 4.34).

Similarly, a significant regression equation was found (F (2,113) = 16.847, $p < .001$), which indicated that the two factors under study have a significant effect on the procedural knowledge test score. Furthermore, the predictor's model explained 23% of the variance in the procedural knowledge test score. Taking individual predictor contributions into account, the results revealed that reasoning skills (β = .321, t = 3.795, $p < .001$) and attitude (β = .293, t = 3.457, p = .001) were positively significant predictors of procedural knowledge test score (see Table 4.35).

According to Yankelewitz et al. (2010), reasoning is essential in the construction and justification of mathematical arguments. In addition, Ball and Bass (2003) stated that mathematical understanding is insufficient due to a lack of reasoning skills. The ability to reason is therefore a significant factor in PSMT's learning of mathematics and geometry.

Numerous studies on reasoning ability discovered that PSMTs needed to be able to back their mathematical solutions with arguments as well (Francisco & Maher, 2005; Maher, 2005; Yackel & Hanna, 2003). Since mathematics and mathematical reasoning are inextricably linked, reasoning skills demand PSMTs' complete learning. In order to solve mathematical problems, one can use logical reasoning, conceptual understanding, mathematical reasoning, and thinking abilities (Faradillah, Hadi, & Tsurayya, 2018).

The studies noted above support the importance of conceptual understanding for improving mathematical achievement. Erickson and Lanning (2014) discovered that transferability of knowledge occurs when there is conceptual understanding.In the same way, Zulnaidi and Zakaria (2010) found a relationship between conceptual knowledge and mathematical achievement and reasoning ability.

In this regard, the finding lends support to the literature's contention that there is a significant relationship between conceptual understanding, reasoning skills, and achievement (Istikomah & Nor-Sakinah, 2013), implying that conceptual understanding was one of the indicators influencing reasoning ability among learners.Accordingly, Marlina and Nurhidayah (2010) also stated that a higher student reasoning ability demonstrated a higher conceptual understanding.

Similarly, research has established that procedural skill is related to both concurrent and future mathematics achievement (LeFevre et al., 2010; Mazzocco & Thompson, 2005). According to models of mathematical cognition (Geary, 2004), degrees of conceptual understanding and procedural competence depend on domain-general skills, including but not limited to reasoning memory, which in turn depend on levels of these competencies.

In addition to significant factors, the current investigation also yielded non-significant results. Attitude towards geometry was found to have no impact on the reasoning abilities scores of PSMTs. Despite some inconsistencies that contradicted the current study, attitude was found to be positively and significantly associated with, as well as predict, learners' learning outcomes (Hayat et al., 2020; Nabizadeh et al., 2019).

The same result was found by Rittle-Johnson and Star (2007), who found a significant contribution of conceptual knowledge on procedural knowledge. This depicts how having previous knowledge of mathematical concepts would greatly help PSMTs enhance their procedural knowledge. Furthermore, according to Hiebert et al. (2003), in order to construct the concepts and definitions in relation to a specific concept, procedures and skills should be studied in conjunction with other techniques and representations rather than in isolation. PSMTs should organize mathematical knowledge into a conceptual schema that provides the basis for understanding, reasoning, and problem-solving.

This result is caused by the fact that technology can provide easy steps for students to acquire concepts and procedures for solving a problem. This research supports the claim made by Rittle-Johnson and Koedinger (2005) that the procedures or activities used to solve an issue may help kids learn. For instance, conceptual knowledge assists in

choosing suitable procedures during problem-solving and practicing several procedures (procedural knowledge) may help achieve a better understanding of the concept (Rittle-Johnson & Schneider, 2014). This piece of information is a very important indicator for educators in their efforts to enhance PSMTs' conceptual and procedural knowledge, particularly for the mathematics subject.

5.2. Conclusion

Mathematics is now, in the 21st century, the common standard for evaluating a nation's level of educational and social development. The nation's social, economic, and technological advancement is accelerated in large part by mathematics. A mathematically skilled person and societies at large will have many possibilities and alternatives for determining their destiny in this rapidly changing environment.

Nowadays, geometry is used extensively in many mathematical applications. Geometric concepts and methods are used to address issues in science, engineering, calculus, algebra, and other disciplines. It is a subfield of mathematics that has applications in many other domains and disciplines. A geometric concept teaches students the fundamentals of visualization, critical thinking, conjecturing, logical argument, problem-solving, proof, and deductive reasoning. In addition, learning geometry helps to develop the skills of applying geometrical concepts through modeling and problem-solving in real-world contexts, and the mathematical ideas and skills underlying geometric reasoning assist learners in understanding high-level mathematical concepts and provide support for the development of their conceptual understanding.

Results from national learning assessments, however, showed that learners' performance in mathematics and geometry was inadequate. According to the literature, the classroom's teaching strategy is one of the factors contributing to low achievement. Therefore, there is a need for the development and implementation of teaching strategies that have a positive impact on the learners' learning outcomes. Furthermore, the integration of technology into a mathematics classroom can promote more meaningful and result oriented learning. As a result, the impacts of a technology-integrated guided inquiry approach on PSMTs' conceptual and procedural knowledge, reasoning skills, and attitude towards geometry were investigated in this study.

From the findings of this study, the TGIBL approach is found to be more effective in enhancing PSMTs' learning than the other instructional approaches. This implies that using a technology integrated GIBL approach improved PSMTs' ability to learn geometry concepts more than the other methods tested in the study. The most evident outcome of this research is that adopting technology-based mathematics classrooms resulted in PSMTs' learning of geometry. Furthermore, it has been discovered that when a teacher employed technology-based learning in the mathematics classroom, PSMTs' collaboration, discussion, making sense of ideas, and sharing of knowledge improved, leading to an effective learning environment that supports the understanding of plane geometry concepts. In addition, the GIBL approach was also found to be more effective than the traditional teacher-centered instructional method in enhancing PSMTs' learning of plane geometry concepts.

Moreover, a technology-integrated guided inquiry approach had opened up windows for PSMTs to learn mathematics and geometry in new ways in order to increase their

learning geometry as well as their attitude. Furthermore, the integration of technology into guided inquiry provides opportunities for the visualization of abstract concepts of geometry, increases engagement and participation in activities, and makes the learning more exciting, enjoyable, and self-explanatory.

As a result, using the TGIBL approach can significantly improve PSMTs' conceptual understanding, procedural knowledge, geometric concept reasoning skills, and positive attitude toward geometry compared to the other instructional approach, provided that additional research is conducted. Finally, the researcher found that if mathematics teacher educators and PSMTs implement a technology-integrated guided inquiry approach in the teaching and learning of mathematics and geometry, they are better able to improve their learning.

5.3 Recommendations

The Ethiopian Ministry of Education intends to implement the technology settings outlined in the strategy for the growth of the educational sector (MoE, 2015; 2018). However, the plan was limited to providing digital resources with the curriculum rather than integrating them into the subject matter. The strategies that help generate resources in order to strengthen the effective practices of teacher education programs and methodologies need to be designed and implemented.

PSMTs should be well-prepared and equipped with mathematical and geometric knowledge because poor preparation of pre-service mathematics graduates, in turn, contributes to another generation of mediocre teachers repeating the cycle. Based on the

findings of this study, the following recommendations are made for teachers, curriculum designers, and teacher educators:

Curriculum Designers

Curriculum designers who design mathematics and geometry curriculum or modules should pay attention to the integration of technology into the instructional approach and the mathematical contents. Thus, curriculum developers should offer examples of how to use the guided inquiry approach and technology activities in mathematics lessons effectively.

The Ministry of Education

The Ministry of Education (e.g., Division of Curriculum Supervisory, Teacher Professional Provision Division) should scrutinize CTEs to initiate professional development prospects for mathematics teacher educators and how to integrate technology in mathematics classrooms in order to promote pre-service mathematics teachers' learning outcomes. Mathematics teacher educators should receive regular training and professional development on how to use a technology-integrated guided inquiry approach in teaching and learning mathematics.

Teacher Education Organizers and CTE Proprietors

In order to effectively use technology with pedagogy and mathematical content, teachers and educators need to receive training. Additionally, CTEs should regularly provide seminars, symposiums, workshops, and training sessions for teacher educators on how to use technology in the teaching and learning of mathematics to provide a platform to

explore digital software (e.g., GSP, GeoGebra, etc.) that can be used in the mathematics classroom. Moreover, teacher education supervisors should regularly make inventories and support teacher educators to give adequate constructive feedback to improve the learning and teaching processes of teacher education.

PSMT education colleges need to inculcate technology-integrated instruction rigorously and adequately with the reformed mathematics curriculum. Preservice teacher training programs should also involve a course to inform prospective teachers about the benefits of technology-based instruction, and pre-service mathematics education courses need to incorporate digital technologies and pedagogies.

Teachers Educator

Teacher educators must be technologically literate in order to support their PSMTs' understanding of mathematical and geometric concepts and, ultimately, to foster positive attitudes learning mathematics and plane geometry. They must also understand how to use technology and select the appropriate technological software in order for teacher educators to have a synthesis of all knowledge (technology, pedagogy, and content knowledge).

This, in turn, aided them in implementing well-designed technology-integrated inquiry-based instruction in the mathematics classroom to improve the effectiveness of the teaching and learning processes. In order to instill inquiry, classroom discussion, self-regulation, and student-centered learning among PSMTs, a mathematics teacher educator must use digital technology in diverse ways. This will help them determine how, why, and when to use strategies and resources at all levels of knowledge and understanding.

Similar to this, in order to implement constructivism in their future classrooms, PSMTs must get training in technology-integrated inquiry-based educational settings.

5.4 Limitations of the Study

The researcher is conscious of a number of limitations. The duration of data collection was the main study restriction. The participants were actively involved and fully engaged in college co-curricular activities after their studies in order to make up for the courses that had fallen behind due to the COVID-19 outbreak. The researcher carefully scheduled a period between classes and extracurricular activities to conduct the interventions and interviews with the assistance of the mathematics teacher educator.

Intact classrooms, which could jeopardize internal validity, were the only ones allowed in the research. As complete environmental control could not be maintained, great internal validity could not be asserted. Qualitative methodologies were applied in a unique way to allay validity worries. Both techniques were employed to collect sufficient proof to back up the validity. Another problem of this study is the bias in the qualitative data analysis. There is a chance that the data interpretation may have been impacted by the researcher's personal experiences with learning and teaching mathematics, as well as by the ideas he has, even if the researcher remained sensitive and followed the procedure of research ethics. Lastly, a potential drawback is that PSMTs may not have benefited completely from the strategy due to their lack of technological background knowledge.

5.5. Implications for Future Research

The purpose of the current study is to determine how a guided inquiry-based learning strategy that integrates technology affects the conceptual understanding, procedural

knowledge, reasoning abilities, and attitudes toward geometry of PSMTs. Several recommendations for more research may be made based on the study's findings.

Only PSMTs chosen from two CTEs in Oromiya Regional State participated, and the study only focused on plane geometry; as a result, more research is needed to assess the effects of a technology-integrated guided inquiry-based learning approach on PSMTs' learning outcomes in various courses (such as calculus, algebra, physics, etc.) in various CTE and secondary school contexts. To analyze all cognitive information, including metacognitive knowledge, future study on TGBL is also required.

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Appendices

Appendix-A: Pilot Test Analysis

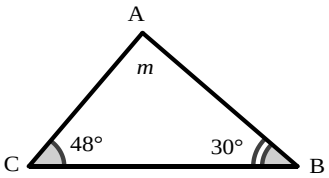
A1: A Two-Tiered Geometry Diagnostic test (TTGDT)

ID No: _____ Section: _____
College: _____.
Dear participant,

This test is prepared for the purpose of partial fulfillment of my PhD thesis work. The test was prepared in a two-tiered format. The first tier is the main question, and the second tier is the reason for the main question. Please try all the test items as much as possible. Notice that a full mark is obtained only when both tiers are answered correctly.

Thank you in advance for your cooperation!

_____1.Find the value of the marked angle "*m*" in the figure below



A. 18^0
B. 12^0
C. 102^0
D. 78^0

_____ Reason for your answer

A. The sum of interior angle of a triangle is 90^0.
B. The sum of interior angles a triangle is equal 180^0.
C. The sum of interior angles equal 360^0 in a triangle.
D. The sum of interior angles equal 180^0 in a straight angle

_____2.A plane shaped quadrilateral that is both rectangle and rhombus is called…

A. Square B. Trapezium C. Parallelogram D. Equaliteral

_____ Reason for your answer

A. It is closed shape which has three or more straight lines.
B. All its pairs of opposite sides are parallel.
C. It has all sides equal; both pairs of opposite sides are equal and parallel.
D. It has four sides and four angles.

_____3.A triangle with sides 3cm, 5cm and 6cm represents

A. Right Angled triangle C. Isosceles triangle
B. Obtuse angled triangle D. scalene triangle

_____ Reason for your answer

A. A triangle with all the three sides or line segments of different lengths.
B. a triangle with all sides equal in length and its angle is equal.
C. A triangle with all the three angles of different values.
D. A triangle with all the angles less than 90^0.

_____4.A square has a perimeter of 20cm. What is the length of one side of the square?

A. 2cm B. 4cm C. 5cm D. 20cm

_____ **Reason for your answer**

A. A square has all its four sides equal. Its perimeter is the sum of all the sides.
B. A square has all its four sides equal. Its perimeter is equal to the sides.
C. A square is a quadrilateral with one pair of sides equal. Its perimeter is total sum of the sides.
D. A square is a quadrilateral with pair of opposite sides equal. Its perimeter is the product of its sides.

_____5.Of the following which can represent a degree measure of scalene triangle?

A. 80^0, 40^0, 50^0 C. 30^0, 120^0, 30^0
B. 40^0, 110^0, 30^0 D. 60^0, 60^0, 60^0

_____**Reason for your answer**

A. All the three sides are equal in length and all interior angles are also equal.
B. Two sides are not equal in length and two angles are equal.
C. All sides are different in length and all angles are different.
D. Only one angle is a right angle with different sides.

_____6.Which one pair of angles is complementary?

A. 120^0 & 60^0 B. 120^0 & 30^0 C. 30^0 & 60^0 D. 20^0 & 60^0

_____**Reason for your answer**

A. A pair of angles is complementary if their product is two right angles.
B. A pair of angles is complementary if their division is a right angle.
C. A pair of angle is complementary if their sum is two right angles.
D. A pair of angles is complementary if their sum is a right angle.

_____7.If one pair of opposite sides of a quadrilateral is parallel, then the quadrilateral is a parallelogram.

A. True B. False

_____**Reason for your Answer**

A. Because it is rectangle
B. Because it is rhombus
C. Because it is square
D. Because it is a trapezium

_____8.Write one correct statement relating any two of the following quadrilaterals: rhombus, parallelogram, rectangle and square. For example; square is a rhombus.

A. A rectangle is a square.
B. A square is a rectangle.
C. A parallelogram is a rhombus.
D. Parallelogram is a square.

_____**Reason for your answer**

A. All have four sides and opposite sides are parallel.
B. Opposite sides are equal with equal diagonals.
C. Their diagonals are intersects at right angle.
D. Opposite sides are equal and parallel and each angle is at right angle.

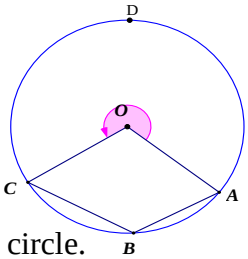
_____9.Which statement makes the difference between a square and a rectangle?

A. A square has all sides equal.
B. A square has all angles 90^0.
C. A square has its diagonals bisect each other.
D. The two pair of opposite side is parallel.

_____ **Reason for your answer**

A. A rectangle has only two parts equal.
B. A rectangle is not equal to square.
C. A rectangle and square have different lengths.
D. A rectangle has only opposite sides equal.

_____10.In following figure, which represents the chords of the circle with center O?



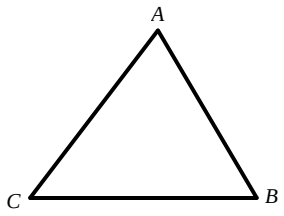
A. $\overline{AB}$ only.
B. $\overline{AB}$, $\overline{CD}$ and $\overline{DF}$ only.
C. $\overline{OE}$ only.
D. $\overline{AB}$, $\overline{CD}$, $\overline{BF}$ and $\overline{FD}$ only.

_____**Reason for your answer**

A. A chord is a line segment that passes through the center of a circle.
B. A chord is any line segment joining two points on the circumference of a circle.
C. A chord is the line segment from the center of a circle to any point on the circle.

_____11.In a given figure, $AB > BC$.

If we assume that $m\angle A = m\angle C$, it follows that $AB = BC$. This contradicts the given statement that $AB > BC$. What conclusion can be drawn from this contradiction?



. A. $m\angle A = m\angle B$ C. $m\angle A = m\angle C$
B. $m\angle A \neq m\angle B$ D. $m\angle A \neq m\angle C$

_____ **Reason for your answer**

A. The triangle is an isosceles.
B. The measure angle opposite to largest angle largest.
C. The base angles are different.
D. The angle opposite to side AB is obtuse.

_____12.How many diagonal and sides respectively does a triangle have?

A. 0, 3 B. 3, 3 C. 3, 0 D. 0, 0

_____**Reason for your answer**

A. A triangle is three sided with three median diagonals.
B. A triangle has three sides and three slant edge of diagonals.
C. A triangle has three sides with no diagonals.

_____13.A quadrilateral which has perpendicular diagonals is only a rhombus.

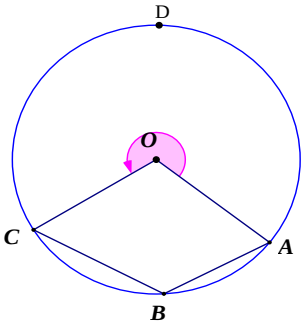
A. True B. False

_____**Reason for your answer**

A. Rectangle has perpendicular diagonals.
B. Trapezium has perpendicular diagonals.
C. A kite has perpendicular diagonals.
D. A parallelogram has perpendicular diagonals.

_____14.In following circle with center O, if $m(\widehat{COA}) = 110^0$ then what is $m(\widehat{ABC}) =$

A. 55^0
B. 125^0
C. 70^0
D. 110^0



_____**Reason for your answer**

A. Because ABCO is a cyclic quadrilateral and $m(\widehat{ABC})$ is it's supplementary.
B. The $m(\widehat{ABC})$ is one-half of the measure of central angle $\widehat{AOC}$.
C. The measure of an angle $m(\widehat{ABC})$ is one-half the degree measure of arc $\widehat{ADC}$.
D. Vertically opposite angle are equal.

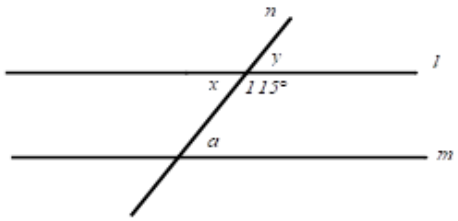
____15.What is the length of a square whose area is $169m^2$.

A. 84.5m B. 42.3m C. 12m D. 13m

_____**Reason for your answer**

A. Its length equals to division of area by four sides.
B. Its length equal to division of area by two sides.
C. Its length is square root of its area.
D. The length of a square is a measure of its area.

____16.The value of the marked angle *y* in the figure below is…



A. 25^0 B. 65^0 C. 90^0 D. 115^0

_____**Reason for your answer**

A. Angle *x* and *y* are vertically opposite angles. Vertically opposite angles are equal in value.
B. Angle *x* and *y* are alternate angles. Alternate angles are equal in value.
C. Angle *x* and *y* are alternate angles. Sum of alternate angles are equal.
D. Angle *x* and *y* are corresponding angles. Corresponding angles are equal

_____17.What is the main difference between a parallelogram and a rhombus?

A. Opposite sides of parallelogram are parallel.
B. Opposite angles of a parallelogram are equal.
C. The diagonal of parallelogram bisect each other.
D. Diagonals of a rhombus bisect at right angle.

_____**Reason for your answer**

A. Consecutive angles in a parallelogram are supplementary.
B. A parallelogram has its diagonal only bisect each other and not at right angle.
C. A parallelogram has only the opposite sides equal.
D. A rhombus has all its sides equal.

_____18.The top of a rectangular dining table measures 110cm by 75cm. What is its perimeter?

A. 185cm B. 370cm C.825cm D.350cm

_____Reason for your answer

A. Opposite sides of a rectangle are equal and parallel; its perimeter is the product of its length and breadth.
B. The perimeter of the rectangular table is the sum of the given measures.
C. Opposite sides of rectangle are equal and parallel. Its perimeter is the sum of its four sides.
D. The perimeter of the table is the sum of its lengths.

_____19.**Theorem**: ***A triangle has at most one obtuse angle.***

One student is proving the theorem above by contradiction. She began by assuming that in ΔABC, $\angle A$ and $\angle B$ are both obtuse. Which theorem will Eduardo use to reach a contradiction?

A. If two angles of a triangle are equal, the sides opposite the angles are equal.
B. If two supplementary angles are equal, the angles each measure 90°.
C. The largest angle in a triangle is opposite the longest side.
D. The sum of the measures of the angles of a triangle is 180°.

State Reason for your answer

_____

_____

_____20.What is the area of a rectangle measuring 20cm by 8cm?

A. $28cm^2$ B. $56cm^2$ C.$160cm^2$ D. $200cm^2$

_____**Reason for your answer**

A. The area of a rectangle is the sum of its lengths and width.
B. The area of a rectangle is a product of its length and width.
C. The area of a rectangle is amount of surface covered by it.
D. The area of a rectangle is a product of the lengths.

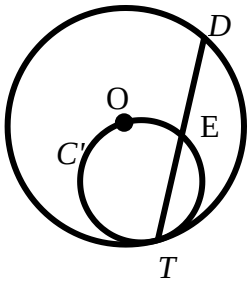
_____21.Calculate the perimeter of a square of length 12cm:

A. 144cm B. 24cm C.48cm
D.68cm

_____Reason for your answer

A. A square has all sides equal. Its perimeter is the product of the sides.
B. A square has all sides equal. Its perimeter is square of its sides.
C. A square has all sided equal. Its perimeter is the sum of its lengths.
D. A square has all sides equal. Its perimeter is sum of two lengths.

_____22.Let C be a circle with center O and C' be another circle with common tangent at T with C and passing through O (see figure below). If $\overline{TD}$ is a chord of the circle C intersecting C' at E, then which of the following is t



A. ΔTEO is a right angle.
B. $\Delta TOE \equiv \Delta DOE$
C. E bisects TD
D. ΔTEO has one-half area as ΔOED

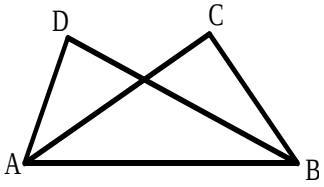
State Reason for your answer

_____

_____

_____23.In the figure $m(\angle ACB) < m(\angle ADB) < 2m(\angle ACB)$, the which of the following is true about point D, with respect to the circle through points A, B and C?

A. D is the center of the circle.
B. D is an exterior point.
C. D is on the circle.
D. D is an interior point.



State Reason for your answer

_____

_____

_____24.The following information is known about the quadrilateral:

- $\overline{BC}$ is parallel to $\overline{AD}$
- $\overline{AB}$ is not congruent to $\overline{CD}$
- $\angle CDA$ is a right angle.

Which must be true of the quadrilateral ABCD?

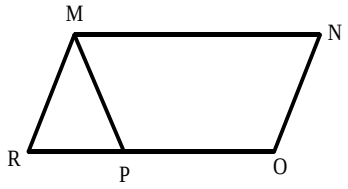
A. ABCD is a rhombus
B. ABCD is a rectangle
C. ABCD is a trapezium
D. ABCD is a parallelogram.

Reason for your answer

_____

_____

_____25.Figure MNOP is an isosceles trapezium, and MNOR is a parallelogram. If $m(\angle MPR) = 62^0$, what is $m(\angle RMP)$?



A. 56^0
B. 62^0
C. 118^0
D. 136^0

_____**Reason for your answer**

A. Triangle ΔRMP formed is an isosceles.
B. MP is the diagonals for trapezium
C. $m(\angle R) = m(\angle O)$

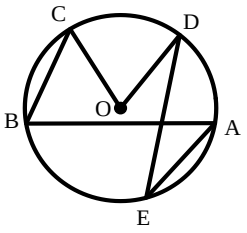
_____26.If a, b, c are the measures of the sides of a triangle, which of the following is not true?

A. $a - b - c > 0$
B. $a + b - c > 0$
C. $a + c - b > 0$
D. $b + c - a > 0$

_____**Reason for your answer**

A. The sum of the two sides of a triangle is less than its third side.
B. The difference of two sides of a triangle is greater than its third side
C. The sum of the two sides of a triangle is greater than its third side.

_____27.In the figure, o is the center of the circle, $m(\angle ABC) = 60^0$ and $m(\angle AED) = 30^0$. Then what is the value of $m(\angle DOC)$?



A. 60^0
B. 45^0
C. 30^0
D. 50^0

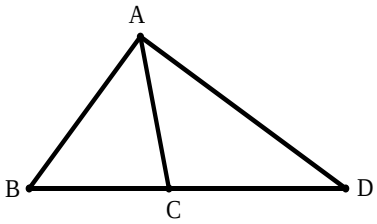
State Reason for your answer

_____

_____

_____28.If in the figure $m(\angle ABC) = m(\angle BAC)$, then which of the following is true?

A. $\mathrm{AD} > \mathrm{BD}$
B. $\mathrm{AD} = \mathrm{BD}$
C. $\mathrm{AD} < \mathrm{BD}$
D. $\mathrm{AD} > \mathrm{AC} + \mathrm{DC}$



State Reason for your answer

_____

_____

_____29.Which one is correct about a trapezium?

A. A Trapezium has both pairs of opposite sides parallel.
B. A Trapezium is a rectangle with all angles equal.
C. A Trapezium has exactly one pair of opposite sides parallel.
D. A Trapezium has all the sides equal.

_____**Reason for your answer**

A. It is a quadrilateral or a polygon with four sides.
B. It is a quadrilateral bounded by three sides.
C. It is a plane shape bounded by two straight sides.
D. It is a quadrilateral whose two pairs of adjacent sides are equal.

_____30.Find the perimeter of a ΔABC with $AB = 10cm$, $BC = 12cm$ and $AC = 13cm$ respectively

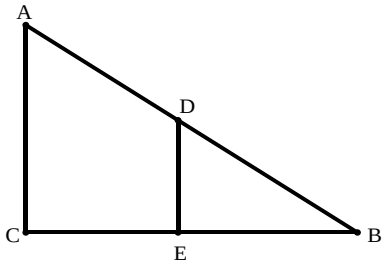
A. 38cm B. 25cm C. 33cm D. 35cm

_____**Reason for your answer**

A. Perimeter is the length round the triangle, which is the sum of its three sides.
B. Perimeter is the distance round the surface of triangle, which is the sum of its surface.
C. Perimeter is the distance round the triangle, which is the product of its three sides.
D. Perimeter is the distance between sides of triangle, which is equal to the area of triangle.

_____31.Which of the following facts would be sufficient to prove that triangles ABC and DBE are similar?

A. $\overline{CE}$ and $\overline{BE}$ are congruent.
B. $\angle ACE$ is a right angle.
C. $\overline{AC}$ and $\overline{DE}$ are parallel.
D. $\angle A$ and $\angle B$ are congruent.



_____**Reason for your answer**

A. If two parallel line cut by transversal line the corresponding angles are congruent.

B. Two triangles are similar if corresponding angle are similar.

C. Two triangle are similar if two corresponding forms ratio.

D. Two triangles are similar if two corresponding sides are similar.

_____32.In ΔPQR, $PQ = 8$, $QR = 12$, and $RP = 13$. Which statement about the angles of must be true?

A. $m\angle Q > m\angle P > m\angle R$ C. $m\angle Q > m\angle R > m\angle P$

B. $m\angle R > m\angle P > m\angle Q$ D. $m\angle P > m\angle R > m\angle Q$

_____**Reason for your answer**

A. The angle opposite to largest side is obtuse.

B. The angle opposite to smallest side is acute.

C. The angle opposite to the largest side is longest angle.

D. The interior angles are all acute.

____33.A student cut a quadrilateral from a piece of cardboard with the diagonals having the following characteristics.

- Congruent
- Perpendicular
- bisect each other

Which type of quadrilateral must have cut out?

A. Parallelogram B. Rectangle C. Rhombus D. Square

State Reason for your answer

_____

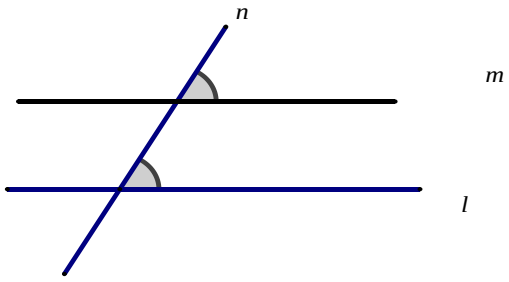
_____

_____

____34.In figure below, the line *m* is parallel to line *l* and *n* is a transversal line. Then the two marked angles are equal

A. True

B. False



____**Reason for your answer**

A. If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.

B. If two lines are cut by a transversal and the interior angles on the same side are supplementary, the lines are parallel.
C. If two lines are perpendicular to the same line they are parallel.
D. If two lines are cut by a transversal and the corresponding angles are congruent, they are parallel.

A2: Geometry Procedural Knowledge Test (GPKT)

LWE / ID No:

_____Daree/Section_____

Saala/Sex: _____

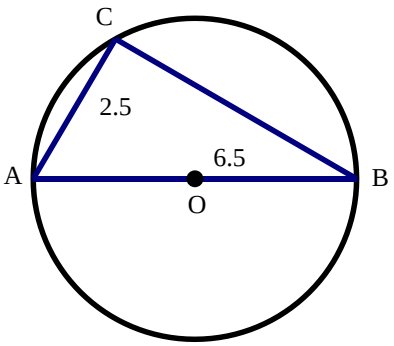
Ajaja/Instruction: ***Gaaffiwwan armaan gadii hojiin agarsiisi. Hojii kee hundaa ibsi***.

Please read each question carefully and answer it to the best of your ability. Show each necessary step and explain each of your steps.

1. Danaa armaan gadii keessatti, yoo AB diyaameterii geenoo O keessa darbeefi AB = 6.5, AC = 2.5 ta'e, gatii kannen armaan gadii barbaadi?

   In the figure below, ΔABC is inscribed in a circle O. AB is the diameter and AB = 6.5, AC = 2.5, find the following.

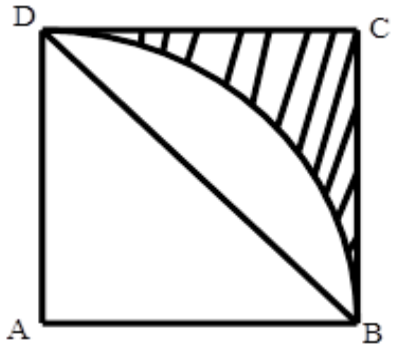
   i. $m(BC)$
   ii. a(ΔABC)



2. Danaa armaan gadii kessatti yoo ABCD iskuwerii dheerina rogaa 2cm fi A handhura geengoo B fi D keessa darbuu ta'e, gatii armaan gadii barbaadi./

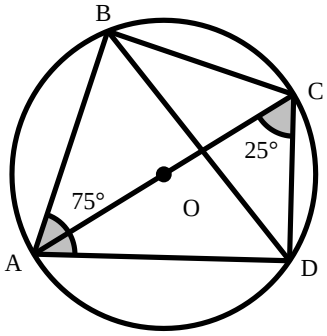
In the figure below, ABCD is a square of side 2cm. If point A is the center of a circle passing through B and D, then find the following.

i. Dheerina golboo AB
   Measure of arc $m(\widehat{AB})$
ii. Balinna seektarii ABD
   Area of sector ABD
iii. Balinna dibamaa
   Area of shaded region



3. Tuqaalee A, B, C fi D tuqaa geengoo O fi AC diyameterii ta'e fi <BAD =750, <ACD $=25^0$ kenname safara <BDC barbaadi.

A, B, C and D are points on a circle; center O such that AC is a diameter of the circle. If ∠BAD = 75° and ∠ACD = 25°. Find the measure of ∠BDC.



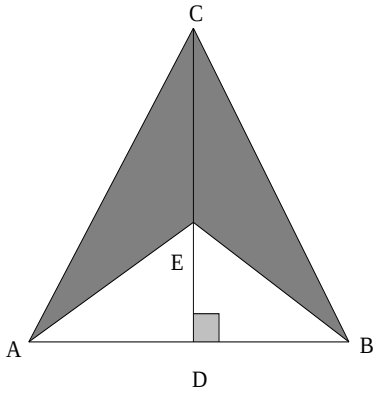
4. Yoo sarbiwwan rombasii 10cm fi 24cm ta'e, kannen arman gadii barbaadi.

If the diagonals of a rhombus are 10cm and 24cm, then find the following.

i. Dheerina roga rombasii/The length of the sides of rhombus
ii. Nannawwaa rombasii/The perimeter of the rhombus
iii. Balinna rombasii barbaadi/The area of the rhombus.

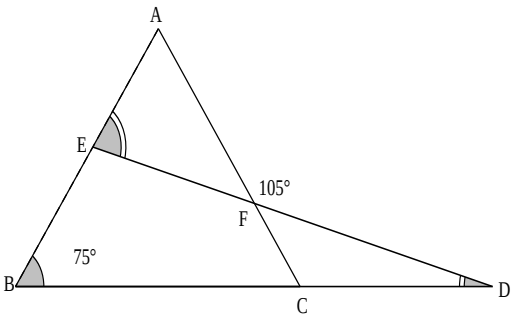
5. Yoo AB = 10cm fi CE =2cm ta'e, balinni AEBC meeqa ta'a?

Find the area of shaded region for the following figure.



6. Danaa armaan gaditti kennamef gatii $\angle BAC + \angle BDE$ barbaadi?

In the figure below find the sum of angles $\angle BAC + \angle BDE$?

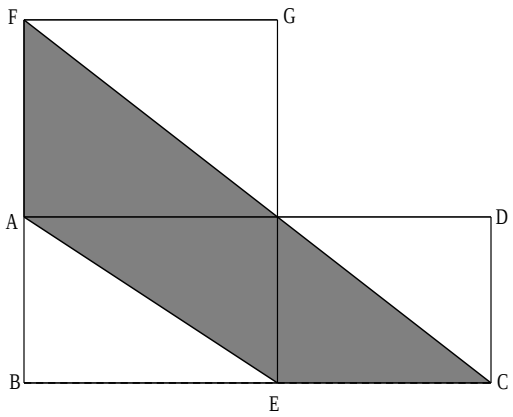


7. Yoo hojjawwan parallelogiramii tokkoo 2cm fi 4cm ta'eefi nannawaan parallelogiramii kanaa 24cm ta'e, dheerina roga isaa barbaadi.

If the length of the two altitude of a parallelogram are 2cm and 4cm and its perimeter is 24cm, then find the length of the sides of the parallelogram?

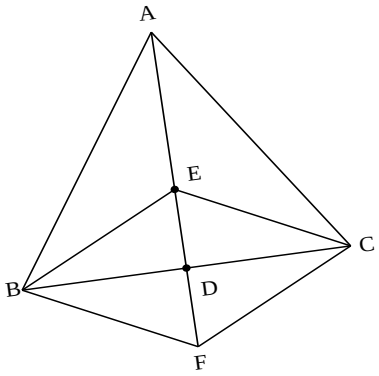
8. Rektangilii ABCD kessatti AB =3cm fi BC =6cm fi BE = 4cm ta'a. Yoo balinni rektangilii FBEG fi ABCD walqixa ta'e balinna dibamee barbaadi.

ABCD is a rectangle with AB = 3cm and BC =6cm and E is a point on BC with BE = 4cm. if rectangle FBEG and ABCD have equal areas, then the area of shaded region FAEC is:



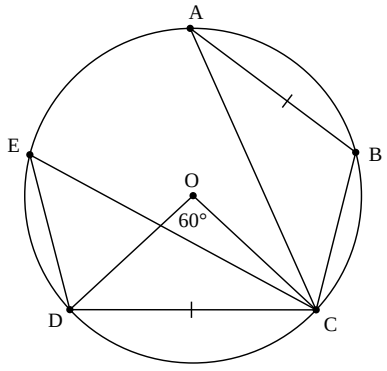
9. Yoo Tuqaan E iddoo walqaxaamura midiyaani rog-sadee ABC ta'eefi $\overline{BF} \,//\, \overline{EC}$, $\overline{FC} \,//\, \overline{BE}$ fi AD = 6cm kenname, dheerinna AF barbaadi./

If the point E is the intersection of the medians of ΔABC, $\overline{BF} \,//\, \overline{EC}$ and $\overline{FC} \,//\, \overline{BE}$. If AD = 6cm, then find the measure of AF.



10. Geengoo ABCDE keessatti yoo O handhuura ta'eefi $D\hat{O}C = 60^0$ fi AB = CD, ta'e, gatii $D\hat{E}C$ fi $A\hat{C}B$ barbaadi.

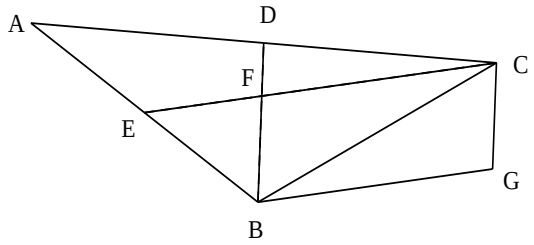
If O is the center of circle ABCDE, $D\hat{O}C = 60^0$ and AB = CD. Calculate the value of $D\hat{E}C$ and $A\hat{C}B$.



11. Rgosadee ΔABC keessatti D fi E tuqaa walakkessoo roga AC fi AB duraaduban ta'efi F tuqaa walqaxaamura midiyanii CE fi BD ta'a. Yoo $BG \parallel FC$ fi $CG \parallel BF$ kenname kannen armaan gadii mirkanessi.

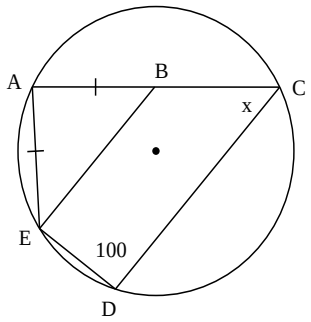
In ΔABC, D is the midpoint of AC, E is a midpoint of AB, F is the intersection of CE and BD, $BG \parallel FC$ and $CG \parallel BF$, then show the following.

i. EF: EC = 1:3 ii. ED:BC = 1:2

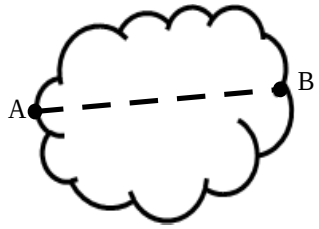


12. Danaa ACDE keessatti EB || DC fi AB = AE ta'e, gatii x barbaadi.

In the figure, ACDE is a circle with EB || DC and AB = AE. Find the value of *x*.



13. As shown in figure, A and B were located at the end of a pond. A surveyor wanted to measure the distance between A & B with the rope, but the rope was not long enough.She came up with an idea: first took a point C on the ground that could reach A,and B at the same time, linked AC and extend to D, made DC = AC, linked BC and extend to E, made EC = BC, linked DE and measured its length, the length of DE was the distance between A and B. why? According to the above description, draw your figure and explain why?



● C

A3: Geometry Problem Proof Test

LWE/ID No: _____ Daree/Section: _____

College: _____

Ajaja/Instruction: Battalle kun qaama qorannooti. Qorronnichi jijjiirama teknoolojiin akkaata baru-barsiisu barnoota gosa herreega "Ji'oometrii" irratti fidu ilaala. Gaaffileewwan armaan gadii akkaata kennama isaattiin ilaalihojjedhu.

This test is given to you as part of a research project. This study examines how the use of technology integrated approach has changed the way geometry is taught and learned. Please try your best on this test and take it seriously.

Galatoomi/Thank You!

1.Himamoota kennaman kannen mirkaneessi. i. Himamoota kanneen bifa "yoo….ta'e" ta'een barressi. ii. Fakkii kaasi akkasumas moggasi. iii. Mirkaana isa agarsisi.

Suppose you wished to prove each of these statements. Using your figure, write what is given and what is to be proved and write the proof.

1. Himama: **Sarbiiwwan parallelogiramii iddo wiqixa ta'eetti wal-qooda**.

Statement: **Diagonals of a parallelogram bisect each other.**

(i) bifa yoota'en barresi /write in conditional statement:

_____

(ii) Fakkii isaa kaasi, moggaasi/ Draw its' supposed figure and label it:

(iii) Kennama/Given: _____ KAB/To Prove: _____

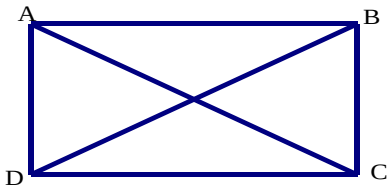
Himama/Statement	Sababa/Reason

2. Yoo sararri handhuura geengoo keesa qaxxaamurufi koordiiti parpeendikularii ta'e, koordicha iddoo wiqixa ta'etti qooda. /A line drawn from the center of a circle perpendicular to a chord bisects the chord.

(i) fakkii kaasi/draw figure and label

(ii) KMB/ To prove: _____.

3. Sarbiiwwan rektaangilii walittigaloo ta'uu mirkaneessi/Prove that the diagonals of a rectangle are congruent.

Kennama/Given: ABCD rektangiliidha. **KMB/PROVE**: $\overline{AC} \equiv \overline{BD}$	
Himama/Statement	Sababa/Reasons
1. $\overline{AD} \equiv \overline{BC}$	1.
2. $\overline{DC} \equiv \overline{CD}$	2.
3. $< ADC \equiv < BCD$	3. kofa sirii rektangilii walitti galoodha
4. Δ_____ ≡ Δ_____	4.
5. $\overline{AC} \equiv \overline{BD}$	5.

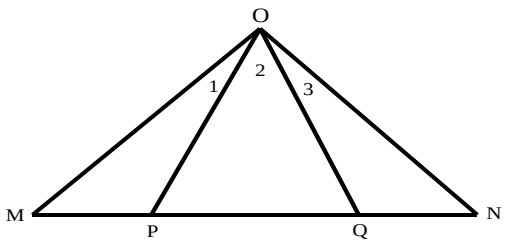
4. Bakka duwwaa guuti /Write statements and reasons to complete this proof.

Kennama: $\overline{OM} \equiv \overline{ON}$

$\hat{1} \equiv \hat{3}$

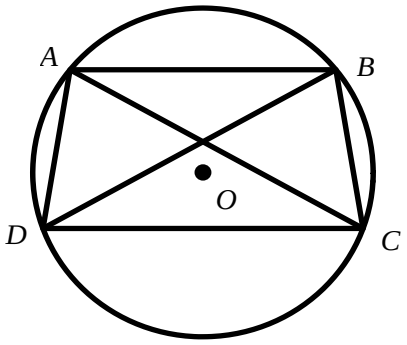
KMB/PROVE:

ΔOPQ Ayisoosilasii ta'a/ an isosceles.

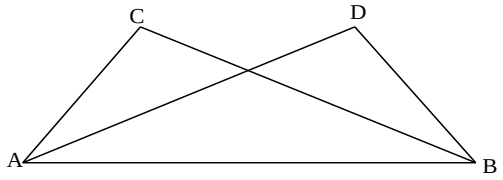


Himama/Statement	Sababa/Reasons
1. $\overline{OM} \equiv \overline{ON}$	1. Kennama
2. $\hat{1} \equiv \hat{3}$	2. Kennama
3. _____	3. Kofti hundee rog-sadee ayisosilasii walitti galoodha.
4. $\Delta OMP \equiv \Delta ONQ$	4. _____
5. $\overline{OP} \equiv \overline{OQ}$	5. _____
6. ΔOPQ Ayisoosilasii ta'a	6. _____

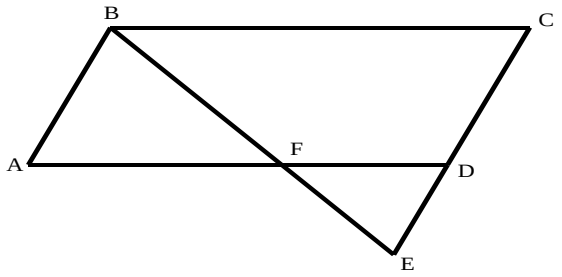
5. Fakii armaan gadii kessatti yoo ABCD', rog-arfee geengoo ta'efi $\overline{AB} \parallel \overline{DC}$ akkasumas $\overline{AC}$ fi $\overline{BD}$ sarbii ta'an, (i) $\Delta ACD \cong \Delta BDC$ fi (ii) ABCD saayikilikii ta'uu mirkannesi. In the diagram below, quadrilateral *ABCD* is inscribed in circle *O*, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn. Prove that (i) $\Delta ACD \cong \Delta BDC$ and (ii) ABCD is cyclic.



6. Iddo duwwaa deebii sirri ta'een guuti/Complete the statement and reasons for figure.

Kennama/Given: 1. $\overline{AC} \equiv \overline{BD}$ 2. $\measuredangle CAB \equiv \measuredangle DBA$ **KAB/Prove**: $\overline{AD} \equiv \overline{BC}$	
Himama/Statement	Sababa/Reasons

7. Himama fi sababa isaa mirkanessi/Write a Statement and Reason proof.

Kennama/Given: 1. ABCD Parallelogiramidha. 2. $\overline{BE}$ fi $\overline{AD}$ tuqaa F irratti walqaxaamuru. 3. Tuqaa D sarara $\overline{CE}$ irra jira. **KAB/Prove**: $\Delta ABF \sim \Delta DEF$	
Himama/Statement	Sababa/Reasons
1._____ 2. _____ 3. _____ 4. _____ 5. _____	1._____ 2._____ 3._____ 4._____ 5._____

6._____	6._____

A4: Psychometric and Reliability Analysis of a TTGDT of the Pilot Test

Items	Difficulty index (P)	Discrimination Index (D)	Remark
Q1	.61	.24	SELECTED
Q2	.67	.30	SELECTED
Q3	.69	.24	SELECTED
Q4	.49	.24	SELECTED
Q5	.18	.04	REJECTED
Q6	.36	.36	SELECTED
Q7	.15	-.06	REJECTED
Q8	.42	.24	SELECTED
Q9	.55	.36	SELECTED
Q10	.42	.24	SELECTED
Q11	.39	.30	SELECTED
Q12	.42	.24	SELECTED
Q13	.39	.24	SELECTED
Q14	.45	.30	SELECTED
Q15	.30	.24	SELECTED
Q16	.48	.24	SELECTED
Q17	.67	.30	SELECTED
Q18	.52	.24	SELECTED
Q19	.69	.30	SELECTED
Q20	.55	.54	SELECTED
Q21	.42	.36	SELECTED
Q22	.48	.24	SELECTED
Q23	.17	.36	REJECTED
Q24	.61	.42	SELECTED
Q25	.39	.48	SELECTED
Q26	.30	.36	SELECTED
Q27	.39	.36	SELECTED
Q28	.39	.36	SELECTED
Q29	.55	.48	SELECTED
Q30	.55	.55	SELECTED
Q31	.55	.36	SELECTED
Q32	.42	.55	SELECTED
Q33	.67	.24	SELECTED
Q34	.79	.12	REJECTED

AVERAG (P) = **.49** AVERAGE (D) =**.33** KR20 = **.87**

Criteria for Difficulty Index (P) & Discrimination Index (D)

Interpretation of Difficulty index (P)				***Interpretation of Discrimination Index (D)***			
Range	No	Description	Remark	Range	No	Description	Remark
< .20	3	Very Difficult	Removed	>.40	6	Very Good Item	Retained
.20 - .4	8	Difficult	Retained	.30 - .39	12	Good Item	Retained
.40 - .60	15	Average	Retained	.20 - .29	13	Fair Item	Retained
.60 - .80	8	Easy	Retained	< .20	3	Poor Item	Discarded
> .81	0	Very Easy	Removed				

A5: The Reliability Analysis of GPKT score of Pilot Test

	Rater-One	Rater-Two
S1	20	28
S2	23	23
S3	16	16
S4	21	21
S5	19	22
S6	16	19
S7	17	17
S8	22	26
S9	23	23
S10	19	19
S11	18	18
S12	21	21
S13	20	20
S14	23	23
S15	24	24
S16	27	30
S17	18	21
S18	22	22
S19	17	21
S20	19	19
S21	21	25
S22	28	28
S23	22	21
S24	18	18
S25	8	8
S26	19	23
S27	22	19
S28	13	8
S29	19	9
S30	12	13

Reliability Statistics

Cronbach's Alpha	No of Items
.879	2

Intraclass Correlation Coefficient (ICC)

	Intraclass Correlation[b]	95% Confidence Interval		F Test with True Value 0			
		Lower Bound	Upper Bound	Value	df1	df2	Sig
Single Measures	.784[a]	.597	.891	8.278	29	29	.000
Average Measures	.879[c]	.748	.942	8.278	29	29	.000

Two-way mixed effects model where people effects are random and measures effects are fixed.

a. The estimator is the same, whether the interaction effect is present or not.

b. Type A intraclass correlation coefficients using an absolute agreement definition.

c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

A6: The Reliability Analysis of GPPT score of Pilot Test

	Rater-One	Rater-Two
S1	15	18
S2	22	22
S3	18	20
S4	21	19
S5	19	19
S6	13	20
S7	22	19
S8	23	23
S9	26	27
S10	18	18
S11	19	19
S12	12	17
S13	20	16
S14	18	20
S15	16	19
S16	15	15
S17	18	17
S18	21	19
S19	24	24
S20	20	21
S21	20	20
S22	12	15
S23	17	14
S24	23	23
S25	21	19
S26	14	14
S27	15	19
S28	19	22
S29	21	26
S30	22	22

Reliability Statistics

Cronbach's Alpha	N of Items
.834	2

Intraclass Correlation Coefficient (ICC)

	Intraclass Correlation[b]	95% Confidence Interval		F Test with True Value 0			
		Lower Bound	Upper Bound	Value	df1	df2	Sig
Single Measures	.705[a]	.470	.847	6.008	29	29	.000
Average Measures	.827[c]	.640	.917	6.008	29	29	.000

Two-way mixed effects model where people effects are random and measures effects are fixed.

a. The estimator is the same, whether the interaction effect is present or not.

b. Type A intraclass correlation coefficients using an absolute agreement definition.

c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

A7: GASQ score Analysis (Pilot)

Geometry Attitudes Scale Questionnaire (GASQ) for Pre-Service Mathematics Teachers towards Plane Geometry (Pilot)

Kabajamoo Hirmaattota/Dear participants,

Bar-gaaffiin kun qorannoo Dirgii 3ffaa Kaadhimatoota Barsiisota Herrega irratti mata duree "***Pre-service Mathematics Teachers' Geometry Learning with Technology integrated Guided inquiry approach***"' jedhurratti qorannoo gaggeessuuf ragaalee funaanuuf kan wixineeffamedha. Kanaafuu, dhamaatii fi amanamummaan bargaaffiilee asii gaditti dhihaatan deebisi. Safaartu tokko qofa filadhu.

The closed-ended questionnaire is prepared for collecting data on "Pre-service Mathematics Teachers' Geometry Learning with Technology-Integrated Guided Inquiry-Based Approach" for my PhD thesis work. So, please fill out the questionnaire honestly. Choose only one scale for each statement.

Hub: Odeeffannoon kennitu icitiin kan qabamuufi dhimma qorannoo kanaa qofaaf oola.

Hirmaannaa keetif Galatoomi! Thank you!

Qajeelfama/Instruction:

1) Gaaffiilee mara tartiiba isaaniin deebisi; gaaffii kamiyyuu bira hin darbiin.
2) Gaaffiilee filannoo lakkoofsa kenname itti mari.

Dhimmoonni ilaalcha (Attitude) barnoota Ji'ometrii akkata waliigalaatti barnoota Herregaa irratti qabdu akkaata sadarkaa 1-5 tti tarreeffamaniiru.

Isaanis:

1= **Guutummaatti itti walii hingalu**,	1= Strongly Disagree (SD)
2= **Itti waliihingalu**,	2= Disagree (D)
3 = **Hinmurteessu**,	3= Undecided (UD)
4 = **Ittin walii gala**, fi	4= Agree (A)
5 = **Sirriittan waliigala**.	5= Strongly Agree (SA)

Haluma Kanaan, sadarkaa waliigaltee keeti lakkoofsa gabatee keessatti kenname jalatti maruun akka agarsiistan kabajaadhan isin gaafadha.

LWE/ID NO_____ Daree/Section_____

College: _____

	Statement	Himama	Guutummaatti itti walii hingalu (SD)	Itti walii hingalu (D)	Hinmurteessu (UD)	Ittin walii gala (A)	Sirriittan waliigala (SA)
1SC	Geometry is one of my dreaded subjects.	Ji'oometriin barumsa sodaadhudha.	1	2	3	4	5
2SC	When I hear the word Geometry, I have a feeling of dislike	Yeroon Jecha Ji'oometri iedhu dhagahuu miira jibbuutu natti dhagahama.	1	2	3	4	5
3SC	My mind goes blank and unable to think clearly when working with geometry.	Ji'oometriin sammun kiyya dhuwwaa taasiisa.	1	2	3	4	5
4SC	Studying Geometry makes me feel nervous.	Ji'oometrii barachuutti nan dhiphadha.	1	2	3	4	5
5SC	Geometry makes me feel uncomfortable.	Ji'oomeetrii barachuun natti hin tolu.	1	2	3	4	5
6SC	I am always under terrible strain in Geometry class.	Ji'oometrii barachuutti baayyen itti rakkadha.	1	2	3	4	5
7SC	I cannot recall many geometry concepts that were hard for me to understand.	Barnoota koorsi ji'oometrii keessatti gaaffiwwan (concept) natti ulfaate hin-jiru.	1	2	3	4	5
8SC	I am always confused in my Geometry class	Ji'oometri barachun na jeeqa.	1	2	3	4	5
9SC	I have selected geometry as my area of emphasis	Barumsi ji'oometriitti baayyeen xiyyeefadha.	1	2	3	4	5
10SC	Geometry does not scare me at all	Ji'oomeetrii barachuu hinsodadhu.	1	2	3	4	5
11SC	I have a lot of sense-confidence when it comes to Geometry.	Ji'oometrii yeroon baradhu ofitti amanamumma natti dhagahama.	1	2	3	4	5
12SC	I believe I am good at solving Geometry problems.	Pirobleemota Ji'oomeetrii furuun natti hin ulfaatu.	1	2	3	4	5
13SC	Geometry concept is usually easy for me to understand	Yaad-rimeen Ji'oometri hubachuun natti salphata.	1	2	3	4	5
14SC	I have selected geometry as my area of emphasis	Barumsa ji'oometriitti baayyeen xiyyeefadha.	1	2	3	4	5
15SC	I believe I am good at solving Geometry problems.	Piroobleemota Ji'oometri furuun baayyee natti salphata.	1	2	3	4	5
16VA	Geometry is a very worthwhile and necessary subject	Barnootni Ji'oometrii baayyee barbaachisoodha.	1	2	3	4	5
17VA	I want to develop my geometrical skills.	Dandeetti barnoota Ji'oometrii gabbifachuun fedha.	1	2	3	4	5
18VA	Geometry helps develop the mind and teaches a person to think.	Ji'oomeetriin sammuu namaa gabbisuu fi akka sababeessan nama deegara.	1	2	3	4	5
19VA	Geometry is important in everyday life.	Barnoonni Ji'oometrii hojii guyya guyyaa keessatti nama gargaara	1	2	3	4	5
20VA	Geometry is one of the most important subjects for people to study	Barnootni Ji'oometrii barnoota nama fayyaduu waan ta'eef baratamuu qaba.	1	2	3	4	5

	Statement	Himama	Guutummaatti itti walii hingalu (SD)	Itti walii hingalu (D)	Hinmurteessu (UD)	Ittin walii gala (A)	Sirriittan waliigala (SA)
21VA	Geometry courses would be very helpful no matter what grade level I taught.	Barnoonni Ji'oometrii Sadarkaa barnoota kamittuu baayyee nama fayyada.	1	2	3	4	5
22VA	I think of many ways that I use geometry outside of school	Ji'oomeetrii mana barmusaa alatti fayyadamuu nana yaala.	1	2	3	4	5
23VA	I think studying advanced geometry is useful	Ji'oometrii karaa waara te'een barachuun faayidaa qaba.	1	2	3	4	5
24VA	I believe studying geometry helps me with problem solving in other areas.	Ji'oometrii barachuu kiyyaan piroobileemiiwwan koorsii biro furuu keessatti nan fayyadama.	1	2	3	4	5
25VA	A strong geometry background could help me in my professional life	Dandeetti buu'ura Ji'oometrii qabaachuun fuladura hojii kiyyaaf baayyee nafayyada.	1	2	3	4	5
26EJ	I get a great deal of satisfaction out of solving geometry problem	Piroobilemii ji'oometrii furuu kiyyatti baayyen gamadda.	1	2	3	4	5
27EJ	I have usually enjoyed studying Geometry in school	Yeroon Ji'oometrii baradhu bayyeen gamadda.	1	2	3	4	5
28EJ	I like to solve new problems in Geometry.	Piroobilemii Ji'oometri furuu baayyeen jaaladha.	1	2	3	4	5
29EJ	Geometry is enjoyable and stimulating to me	Barumsi Ji'oometrii jallatamaa fi hawwataadha.	1	2	3	4	5
30EJ	I would prefer to do an assignment in geometry than to write an essay	Gaaffiwwan herregaa biro irra kan Ji'oometrii hojjechun filadha.	1	2	3	4	5
31EJ	I have always enjoyed studying geometry in college.	Koorsii Kollejji keessa ji'oometri barachu kiyyatti nan gammada.	1	2	3	4	5
32EJ	Geometry is a very interesting subject	Ji'oomeetrin koorsii itti gamaddu keessaa tokkodha.	1	2	3	4	5
33EJ	Geometry is dull and boring	Ji'oomeetriin barumsan jibuudha.	1	2	3	4	5
34EJ	Geometry is very interesting and I have usually enjoyed it	Barumsi ji'oometrii kan na hawwatuufi itti gammadudha.	1	2	3	4	5
35EJ	I am comfortable answering questions in geometry class	Ji'oometrii barachuutti baayyee itti gammada.	1	2	3	4	5
36MT	It wouldn't bother me at all to take more geometry course	Koorsii ji'oometrii dabalatan yoo fudhe itti dhibadhu.	1	2	3	4	5
37MT	I have taken geometry classes even though it was not required	Barumsi Ji'oometri barbaachisaafi barachuun fedha.	1	2	3	4	5
38MT	I find learning the geometry is motivating.	Ji'oometrii barachuun baayye na hawwata .	1	2	3	4	5
39MT	I am avoiding taking geometry classes in college.	Ji'oometrii barachuun nan hawwatu.	1	2	3	4	5
40MT	I liked and motivated geometry class of my college	Ji'oometriin Kollejjiitti baradhe baayyee miira kiya nakakaase.	1	2	3	4	5

A8: Exploratory factor Analysis for GASQ on pre-test

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.884
Bartlett's Test of Sphericity	Approx. Chi-Square	3441.464
	Df	406
	Sig.	.000

Rotated Component Matrix[a]

	Component			
	1	2	3	4
SC3	.837			
SC10	.818			
SC4	.776			
SC9	.771			
SC14	.758			
SC2	.745			
VA19	.728			
SC7	.728			
VA22	.678			
SC6	.620			
VA18		.834		
VA21		.808		
VA16		.803		
VA20		.770		
SC15		.768		
VA17		.713		
VA24		.691		
SC5		.679		
VA25		.604		
VA23		.553		
EJ30			.751	
EJ29			.732	
EJ27			.711	
EJ34			.463	
EJ33			.438	
MT38				.817
MT37				.775
MT36				.616
MT40				.486

Extraction Method: Principal Component Analysis.

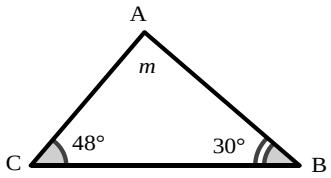
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 5 iterations.

Appendix-B: Test for Pretest and Posttest

B1: Geometry Conceptual Understanding Test

_____1.Find the value of the marked angle "*m*" in the figure below



A. 18^0
B. 12^0
C. 102^0
D. 78^0

_____ **Reason for your answer**

A. The sum of interior angle of a triangle is 90^0.
B. The sum of interior angles a triangle is equal 180^0.
C. The sum of interior angles equal 360^0 in a triangle.
D. The sum of interior angles equal 180^0 in a straight angle

_____2.A plane shaped quadrilateral that is both rectangle and rhombus is called…

A. Square B. Trapezium C. Parallelogram D. Equilateral

_____ **Reason for your answer**

A. It is closed shape which has three or more straight lines.
B. All its pairs of opposite sides are parallel.
C. It has all sides equal; both pairs of opposite sides are equal and parallel.
D. It has four sides and four angles.

_____3.A triangle with sides 3cm, 5cm and 6cm represents

A. Right Angled triangle C. Isosceles triangle
B. Obtuse angled triangle D. scalene triangle

_____ **Reason for your answer**

A. A triangle with all the three sides or line segments of different lengths.
B. a triangle with all sides equal in length and its angle is equal.
C. A triangle with all the three angles of different values.
D. A triangle with all the angles less than 90^0.

_____4.A square has a perimeter of 20cm. What is the length of one side of the square?

B. 2cm B. 4cm C. 5cm D. 20cm

_____ **Reason for your answer**

A. A square has all its four sides equal. Its perimeter is the sum of all the sides.
B. A square has all its four sides equal. Its perimeter is equal to the sides.
C. A square is a quadrilateral with one pair of sides equal. Its perimeter is total sum of the sides.
D. A square is a quadrilateral with pair of opposite sides equal. Its perimeter is the product of its sides.

_____5.Which one pair of angles is complementary?

A. 120^0 & 60^0 B. 120^0 & 30^0 C. 30^0 & 60^0 D. 20^0 & 60^0

_____**Reason for your answer**

A. A pair of angles is complementary if their product is two right angles.

B. A pair of angles is complementary if their division is a right angle.

C. A pair of angle is complementary if their sum is two right angles.

D. A pair of angles is complementary if their sum is a right angle.

_____6.Write one correct statement relating any two of the following quadrilaterals: rhombus, parallelogram, rectangle and square. For example; square is a rhombus.

A. A rectangle is a square.
B. A square is a rectangle.
C. A parallelogram is a rhombus.
D. Parallelogram is a square.

_____**Reason for your answer**

A. All have four sides and opposite sides are parallel.
B. Opposite sides are equal with equal diagonals.
C. Their diagonals are intersects at right angle.
D. Opposite sides are equal and parallel and each angle is at right angle.

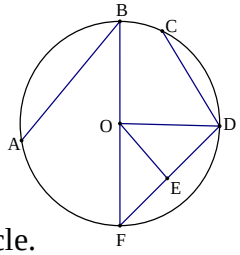
_____7.Which statement makes the difference between a square and a rectangle?

A. A square has all sides equal.
B. A square has all angles 90^0.
C. A square has its diagonals bisect each other.
D. The two pair of opposite side is parallel.

_____ **Reason for your answer**

A. A rectangle has only two parts equal.
B. A rectangle is not equal to square.
C. A rectangle and square have different lengths.
D. A rectangle has only opposite sides equal.

_____8.In following figure, which represents the chords of the circle with center O?



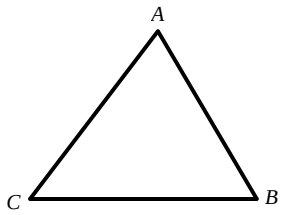
A. $\overline{AB}$ only.
B. $\overline{AB}$, $\overline{CD}$ and $\overline{DF}$ only.
C. $\overline{OE}$ only.
D. $\overline{AB}$, $\overline{CD}$, $\overline{BF}$ and $\overline{FD}$ only.

_____**Reason for your answer**

A. A chord is a line segment that passes through the center of a circle.
B. A chord is any line segment joining two points on the circumference of a circle.
C. A chord is the line segment from the center of a circle to any point on the circle.

_____9.In a given figure, $AB > BC$.

If we assume that $m\angle A = m\angle C$, it follows that $AB = BC$. This contradicts the given statement that $AB > BC$. What conclusion can be drawn from this contradiction?



. A. $m\angle A = m\angle B$ C. $m\angle A = m\angle C$

B. $m\angle A \neq m\angle B$ D. $m\angle A \neq m\angle C$

_____ **Reason for your answer**

A. The triangle is an isosceles.
B. The measure angle opposite to largest angle largest.
C. The base angles are different.
D. The angle opposite to side AB is obtuse.

_____10.How many diagonal and sides respectively does a triangle have?

A. 0, 3 B. 3, 3 C. 3, 0 D. 0, 0

_____**Reason for your answer**

A. A triangle is three sided with three median diagonals.
B. A triangle has three sides and three slant edge of diagonals.
C. A triangle has three sides with no diagonals.

_____11.A quadrilateral which has perpendicular diagonals is only a rhombus.

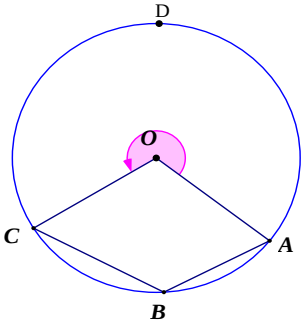
A. True B. False

_____**Reason for your answer**

A. Rectangle has perpendicular diagonals.
B. Trapezium has perpendicular diagonals.
C. A kite has perpendicular diagonals.
D. A parallelogram has perpendicular diagonals.

_____12.In following circle with center O, if $m(\widehat{COA}) = 110^0$ then what is $m(\widehat{ABC}) =$ ___

A. 55^0
B. 125^0
C. 70^0
D. 110^0



_____**Reason for your answer**

A. Because ABCO is a cyclic quadrilateral and $m(\widehat{ABC})$ is it's supplementary.
B. The $m(\widehat{ABC})$ is one-half of the measure of central angle $\widehat{AOC}$.
C. The measure of an angle $m(\widehat{ABC})$ is one-half the degree measure of arc $\overset{\frown}{ADC}$.
D. Vertically opposite angle are equal.

____13.What is the length of a square whose area is $169m^2$.

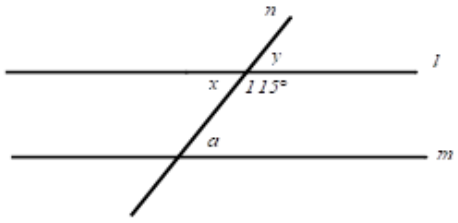
A. 84.5m B. 42.3m C. 12m D. 13m

_____Reason for your answer

A. Its length equals to division of area by four sides.
B. Its length equal to division of area by two sides.
C. Its length is square root of its area.
D. The length of a square is a measure of its area.

____14.The value of the marked angle *y* in the figure below

A. 25^0 B. 65^0 C. 90^0 D. 115^0



_____Reason for your answer

A. Angle *x* and *y* are vertically opposite angles. Vertically opposite angles are equal in value.
B. Angle *x* and *y* are alternate angles. Alternate angles are equal in value.
C. Angle *x* and *y* are alternate angles. Sum of alternate angles are equal.
D. Angle *x* and *y* are corresponding angles. Corresponding angles are equal

_____15.What is the main difference between a parallelogram and a rhombus?

A. Opposite sides of parallelogram are parallel.
B. Opposite angles of a parallelogram are equal.
C. The diagonal of parallelogram bisect each other.
D. Diagonals of a rhombus bisect at right angle.

_____Reason for your answer

A. Consecutive angles in a parallelogram are supplementary.
B. A parallelogram has its diagonal only bisect each other and not at right angle.
C. A parallelogram has only the opposite sides equal.
D. A rhombus has all its sides equal.

_____16.The top of a rectangular dining table measures 110cm by 75cm. What is its perimeter?

A. 185cm B. 370cm C.825cm D.350cm

_____Reason for your answer

A. Opposite sides of a rectangle are equal and parallel; its perimeter is the product of its length and breadth.
B. The perimeter of the rectangular table is the sum of the given measures.
C. Opposite sides of rectangle are equal and parallel. Its perimeter is the sum of its four sides.
D. The perimeter of the table is the sum of its lengths.

_____17.**Theorem**: ***A triangle has at most one obtuse angle.***

One student is proving the theorem above by contradiction. She began by assuming that in ΔABC, $\angle A$ and $\angle B$ are both obtuse. Which theorem will Eduardo use to reach a contradiction?

A. If two angles of a triangle are equal, the sides opposite the angles are equal.
B. If two supplementary angles are equal, the angles each measure 90°.
C. The largest angle in a triangle is opposite the longest side.
D. The sum of the measures of the angles of a triangle is 180°.

State Reason for your answer

_____

___

_____

___

_____18.What is the area of a rectangle measuring 20cm by 8cm?

A. $28cm^2$ B. $56cm^2$ C.$160cm^2$ D. $200cm^2$

_____**Reason for your answer**

A. The area of a rectangle is the sum of its lengths and width.
B. The area of a rectangle is a product of its length and width.
C. The area of a rectangle is amount of surface covered by it.
D. The area of a rectangle is a product of the lengths.

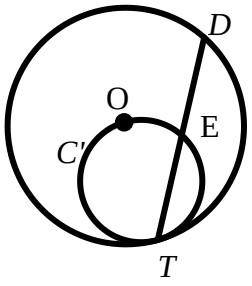
_____19.Calculate the perimeter of a square of length 12cm:

A. 144cm B. 24cm C.48cm D.68cm

_____Reason for your answer

A. A square has all sides equal. Its perimeter is the product of the sides.
B. A square has all sides equal. Its perimeter is square of its sides.
C. A square has all sided equal. Its perimeter is the sum of its lengths.
D. A square has all sides equal. Its perimeter is sum of two lengths.

_____20.Let C be a circle with center O and C' be another circle with common tangent at T with C and passing through O (see figure below). If $\overline{TD}$ is a chord of the circle C intersecting C' at E, then which of the following is t



A. ΔTEO is a right angle.
B. $\Delta TOE \equiv \Delta DOE$
C. E bisects TD
D. ΔTEO has one-half area as ΔOED

State Reason for your answer

_____

_____

_____21.The following information is known about the quadrilateral:

- $\overline{BC}$ is parallel to $\overline{AD}$
- $\overline{AB}$ is not congruent to $\overline{CD}$
- $\angle CDA$ is a right angle.

Which must be true of the quadrilateral ABCD?

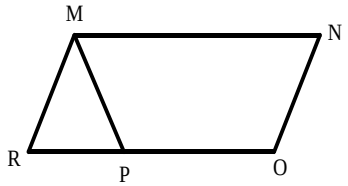
A. ABCD is a rhombus
B. ABCD is a rectangle
C. ABCD is a trapezium
D. ABCD is a parallelogram.

Reason for your answer

_____

_____

_____22.Figure MNOP is an isosceles trapezium, and MNOR is a parallelogram. If $m(\angle MPR) = 62^0$, what is $m(\angle RMP)$?



A. 56^0
B. 62^0
C. 118^0
D. 136^0

_____**Reason for your answer**

A. Triangle ΔRMP formed is an isosceles.
B. MP is the diagonals for trapezium
C. $m(\angle R) = m(\angle O)$

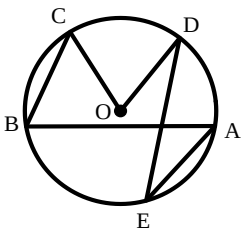
_____23.If a, b, c are the measures of the sides of a triangle, which of the following is not true?

A. $a - b - c > 0$
B. $a + b - c > 0$
C. $a + c - b > 0$
D. $b + c - a > 0$

_____**Reason for your answer**

A. The sum of the two sides of a triangle is less than its third side.
B. The difference of two sides of a triangle is greater than its third side
C. The sum of the two sides of a triangle is greater than its third side.

_____24.In the figure, o is the center of the circle, $m(\angle ABC) = 60^0$ and $m(\angle AED) = 30^0$. Then what is the value of $m(\angle DOC)$?



A. 60^0
B. 45^0
C. 30^0
D. 50^0

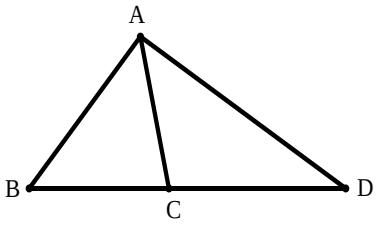
State Reason for your answer

_____

_____

_____

_____25.If in the figure $m(\angle ABC) = m(\angle BAC)$, then which of the following is true?



A. $AD > BD$
B. $AD = BD$
C. $AD < BD$
D. $AD > AC + DC$

State Reason for your answer

_____

_____

_____26.Which one is correct about a trapezium?

A. A Trapezium has both pairs of opposite sides parallel.
B. A Trapezium is a rectangle with all angles equal.
C. A Trapezium has exactly one pair of opposite sides parallel.
D. A Trapezium has all the sides equal.

_____**Reason for your answer**

A. It is a quadrilateral or a polygon with four sides.
B. It is a quadrilateral bounded by three sides.
C. It is a plane shape bounded by two straight sides.
D. It is a quadrilateral whose two pairs of adjacent sides are equal.

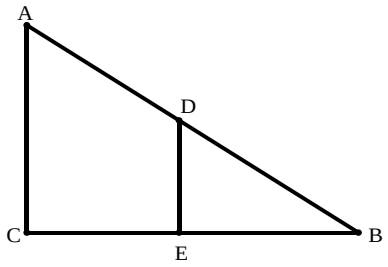
_____27.Find the perimeter of a ΔABC with $AB = 10cm$, $BC = 12cm$ and $AC = 13cm$ respectively

A. 38cm B. 25cm C. 33cm D. 35cm

_____**Reason for your answer**

A. Perimeter is the length round the triangle, which is the sum of its three sides.
B. Perimeter is the distance round the surface of triangle, which is the sum of its surface.
C. Perimeter is the distance round the triangle, which is the product of its three sides.
D. Perimeter is the distance between sides of triangle, which is equal to the area of triangle.

_____28.Which of the following facts would be sufficient to prove that triangles ABC and DBE are similar?



A. $\overline{CE}$ and $\overline{BE}$ are congruent.
B. $\angle ACE$ is a right angle.
C. $\overline{AC}$ and $\overline{DE}$ are parallel.
D. $\angle A$ and $\angle B$ are congruent.

_____Reason for your answer

A. If two parallel line cut by transversal line the corresponding angles are congruent.
B. Two triangles are similar if corresponding angle are similar.
C. Two triangle are similar if two corresponding forms ratio.
D. Two triangles are similar if two corresponding sides are similar.

_____29.In ΔPQR, $PQ = 8$, $QR = 12$, and $RP = 13$. Which statement about the angles of must be true?

A. $m\angle Q > m\angle P > m\angle R$
B. $m\angle R > m\angle P > m\angle Q$
C. $m\angle Q > m\angle R > m\angle P$
D. $m\angle P > m\angle R > m\angle Q$

_____Reason for your answer

A. The angle opposite to largest side is obtuse.
B. The angle opposite to smallest side is acute.
C. The angle opposite to the largest side is longest angle.
D. The interior angles are all acute.

____30.A student cut a quadrilateral from a piece of cardboard with the diagonals having the following characteristics.

- Congruent
- Perpendicular
- bisect each other

Which type of quadrilateral must have cut out?

B. Parallelogram B. Rectangle C. Rhombus D. Square

State Reason for your answer

_____

_____

B2: Geometry Procedural knowledge Test

LWE / ID No: _____Daree/Section_____

Saala/Sex: _____

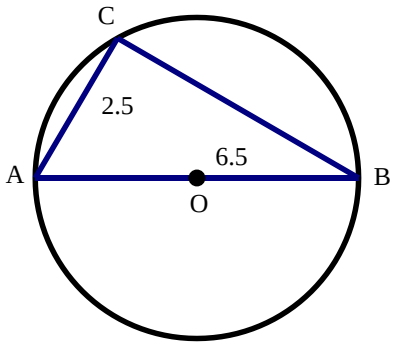
Ajaja/Instruction: ***Gaaffiwwan armaan gadii hojiin agarsiisi. Hojii kee hundaa ibsi***.

Please read each question carefully and answer it to the best of your ability. Show each necessary step and explain each of your steps.

1. Danaa armaan gadii keessatti, yoo AB diyaameterii geenoo O keessa darbeefi AB = 6.5, AC = 2.5 ta'e, gatii kannen armaan gadii barbaadi?

   In the figure below, ΔABC is inscribed in a circle O. AB is the diameter and AB = 6.5, AC = 2.5, find the following.

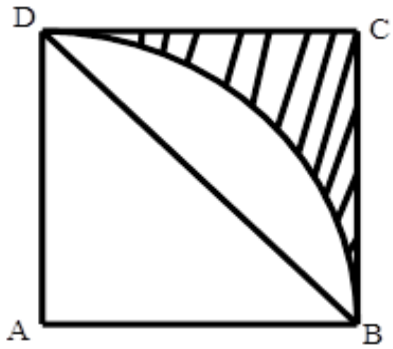
   iii. $m(BC)$
   iv. a(ΔABC)



2. Danaa armaan gadii kessatti yoo ABCD iskuwerii dheerina rogaa 2cm fi A handhura geengoo B fi D keessa darbuu ta'e, gatii armaan gadii barbaadi./

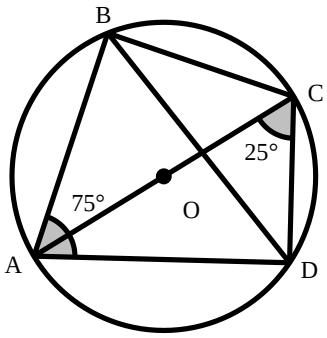
In the figure below, ABCD is a square of side 2cm. If point A is the center of a circle passing through B and D, then find the following.

iv. Dheerina golboo AB
Measure of arc $m(\widehat{AB})$
v. Balinna seektarii ABD
Area of sector ABD
vi. Balinna dibamaa
Area of shaded region



3. Tuqaalee A, B, C fi D tuqaa geengoo O fi AC diyameterii ta'e fi <BAD =750, <ACD $=25^0$ kenname safara <BDC barbaadi.

A, B, C and D are points on a circle; center O such that AC is a diameter of the circle. If ∠BAD = 75° and ∠ACD = 25°. Find the measure of ∠BDC.



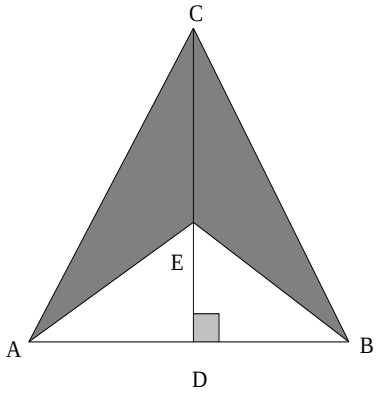
4. Yoo sarbiwwan rombasii 10cm fi 24cm ta'e, kannen arman gadii barbaadi.

If the diagonals of a rhombus are 10cm and 24cm, then find the following.

iv. Dheerina roga rombasii/The length of the sides of rhombus
v. Nannawwaa rombasii/The perimeter of the rhombus
vi. Balinna rombasii barbaadi/The area of the rhombus.

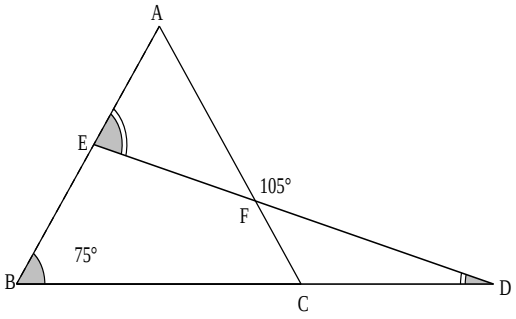
5. Yoo AB = 10cm fi CE =2cm ta'e, balinni AEBC meeqa ta'a?

Find the area of shaded region for the following figure.



6. Danaa armaan gaditti kennamef gatii $\angle BAC + \angle BDE$ barbaadi?

In the figure below find the sum of angles $\angle BAC + \angle BDE$?

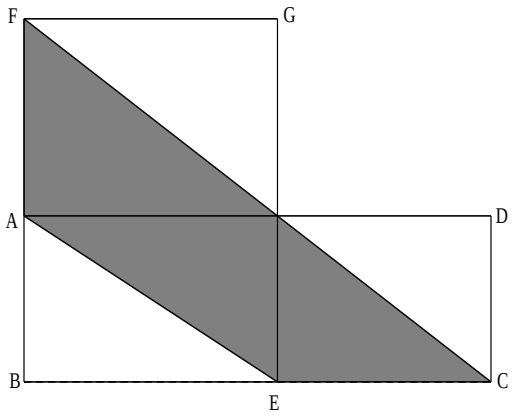


7. Yoo hojjawwan parallelogiramii tokkoo 2cm fi 4cm ta'eefi nannawaan parallelogiramii kanaa 24cm ta'e, dheerina roga isaa barbaadi.

If the length of the two altitude of a parallelogram are 2cm and 4cm and its perimeter is 24cm, then find the length of the sides of the parallelogram?

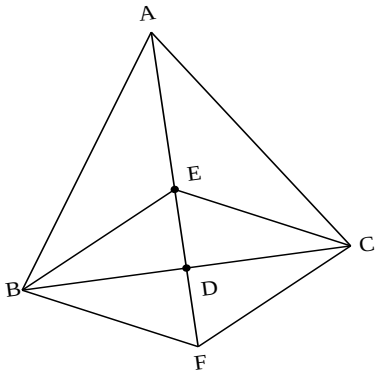
8. Rektangilii ABCD kessatti AB =3cm fi BC =6cm fi BE = 4cm ta'a. Yoo balinni rektangilii FBEG fi ABCD walqixa ta'e balinna dibamee barbaadi.

ABCD is a rectangle with AB = 3cm and BC =6cm and E is a point on BC with BE = 4cm. if rectangle FBEG and ABCD have equal areas, then the area of shaded region FAEC is:



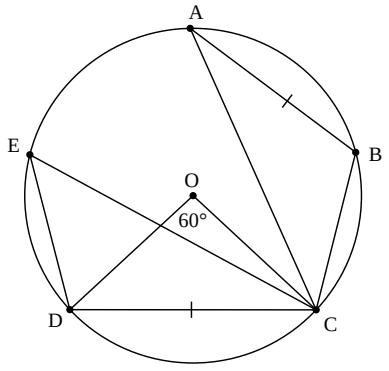
9. Yoo Tuqaan E iddoo walqaxaamura midiyaani rog-sadee ABC ta'eefi $\overline{BF} \,//\, \overline{EC}$, $\overline{FC} \,//\, \overline{BE}$ fi AD = 6cm kenname, dheerinna AF barbaadi./

If the point E is the intersection of the medians of ΔABC, $\overline{BF} \,//\, \overline{EC}$ and $\overline{FC} \,//\, \overline{BE}$. If AD = 6cm, then find the measure of AF.



10. Geengoo ABCDE keessatti yoo O handhuura ta'eefi $D\hat{O}C = 60^0$ fi AB = CD, ta'e, gatii $D\hat{E}C$ fi $A\hat{C}B$ barbaadi.

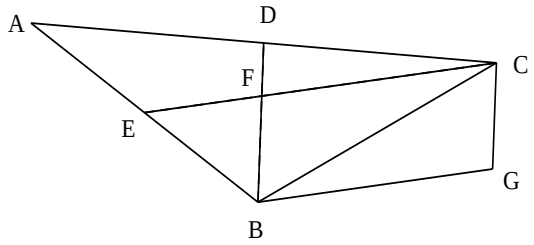
If O is the center of circle ABCDE, $D\hat{O}C = 60^0$ and AB = CD. Calculate the value of $D\hat{E}C$ and $A\hat{C}B$.



11. Rgosadee ΔABC keessatti D fi E tuqaa walakkessoo roga AC fi AB duraaduban ta'efi F tuqaa walqaxaamura midiyanii CE fi BD ta'a. Yoo $BG \parallel FC$ fi $CG \parallel BF$ kenname kannen armaan gadii mirkanessi.

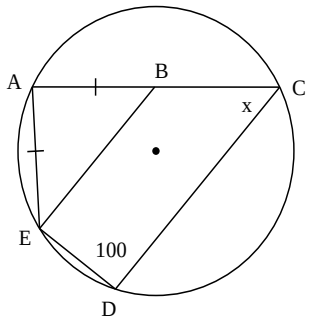
In ΔABC, D is the midpoint of AC, E is a midpoint of AB, F is the intersection of CE and BD, $BG \parallel FC$ and $CG \parallel BF$, then show the following.

ii. EF: EC = 1:3 ii. ED:BC = 1:2

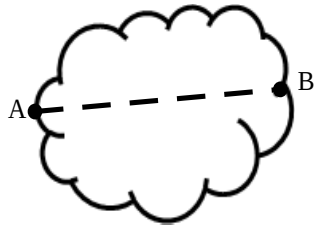


12. Danaa ACDE keessatti EB || DC fi AB = AE ta'e, gatii x barbaadi.

In the figure, ACDE is a circle with EB || DC and AB = AE. Find the value of *x*.



13. As shown in figure, A and B were located at the end of a pond. A surveyor wanted to measure the distance between A & B with the rope, but the rope was not long enough.She came up with an idea: first took a point C on the ground that could reach A,and B at the same time, linked AC and extend to D, made DC = AC, linked BC and extend to E, made EC = BC, linked DE and measured its length, the length of DE was the distance between A and B. why? According to the above description, draw your figure and explain why?



● C

B3: Geometry Problem Proof Test

LWE/ID No: _____ Daree/Section: _____

College: _____

Ajaja/Instruction: Battalle kun qaama qorannooti. Qorronnichi jijjiirama teknoolojiin akkaata baru-barsiisu barnoota gosa herreega "Ji'oometrii" irratti fidu ilaala. Gaaffileewwan armaan gadii akkaata kennama isaattiin ilaalihojjedhu.

This test is given to you as part of a research project. This study examines how the use of technology integrated approach has changed the way geometry is taught and learned. Please try your best on this test and take it seriously.

Galatoomi/Thank You!

1.Himamoota kennaman kannen mirkaneessi. i. Himamoota kanneen bifa "yoo….ta'e" ta'een barressi. ii. Fakkii kaasi akkasumas moggasi. iii. Mirkaana isa agarsisi.

Suppose you wished to prove each of these statements. Using your figure, write what is given and what is to be proved and write the proof.

1. Himama: **Sarbiiwwan parallelogiramii iddo wiqixa ta'eetti wal-qooda**.

Statement: **Diagonals of a parallelogram bisect each other.**

(i) bifa yoota'en barresi /write in conditional statement:

_____

(ii) Fakkii isaa kaasi, moggaasi/ Draw its' supposed figure and label it:

(iii) Kennama/Given: _____

KAB/To Prove: _____

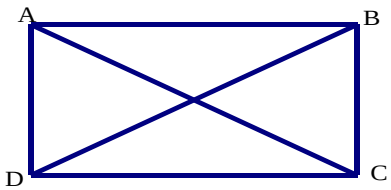
Himama/Statement	Sababa/Reason

2. Yoo sararri handhuura geengoo keesa qaxxaamurufi koordiiti parpeendikularii ta'e, koordicha iddoo wiqixa ta'etti qooda. /A line drawn from the center of a circle perpendicular to a chord bisects the chord.

(i) fakkii kaasi/draw figure and label

(ii) KMB/ To prove: _____.

3. Sarbiiwwan rektaangilii walittigaloo ta'uu mirkaneessi/Prove that the diagonals of a rectangle are congruent.

Kennama/Given: ABCD rektangiliidha. **KMB/PROVE**: $\overline{AC} \equiv \overline{BD}$	
Himama/Statement	Sababa/Reasons
1. $\overline{AD} \equiv \overline{BC}$	1.
2. $\overline{DC} \equiv \overline{CD}$	2.
3. $< ADC \equiv < BCD$	3. kofa sirii rektangilii walitti galoodha
4. Δ_____ ≡ Δ_____	4.
5. $\overline{AC} \equiv \overline{BD}$	5.

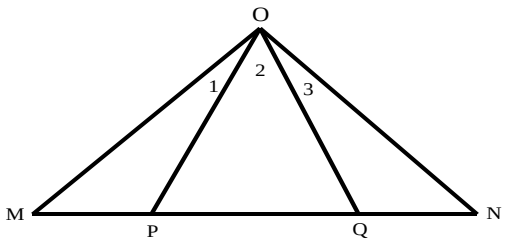
4. Bakka duwwaa guuti /Write statements and reasons to complete this proof.

Kennama: $\overline{OM} \equiv \overline{ON}$

$\hat{1} \equiv \hat{3}$

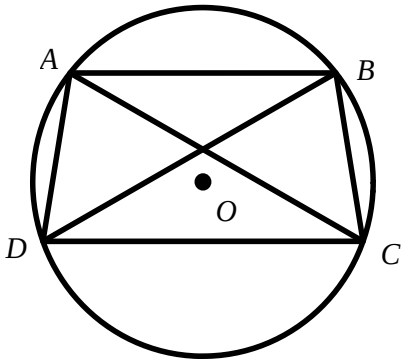
KMB/PROVE:

ΔOPQ Ayisoosilasii ta'a/ an isosceles.

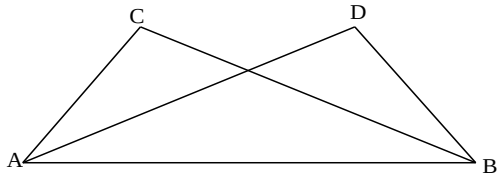


Himama/Statement	Sababa/Reasons
1. $\overline{OM} \equiv \overline{ON}$	1. Kennama
2. $\hat{1} \equiv \hat{3}$	2. Kennama
3. _____	3. Kofti hundee rog-sadee ayisosilasii walitti galoodha.
4. $\Delta OMP \equiv \Delta ONQ$	4. _____
5. $\overline{OP} \equiv \overline{OQ}$	5. _____
6. ΔOPQ Ayisoosilasii ta'a	6. _____

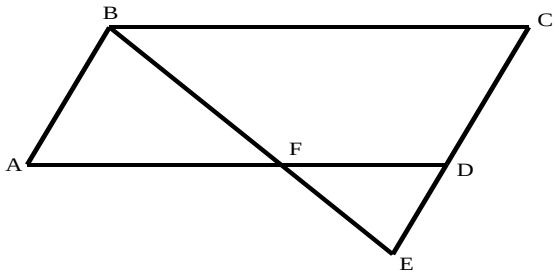
5. Fakii armaan gadii kessatti yoo ABCD', rog-arfee geengoo ta'efi $\overline{AB} \parallel \overline{DC}$ akkasumas $\overline{AC}$ fi $\overline{BD}$ sarbii ta'an, (i) $\Delta ACD \cong \Delta BDC$ fi (ii) ABCD saayikilikii ta'uu mirkannesi. In the diagram below, quadrilateral *ABCD* is inscribed in circle *O*, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn. Prove that (i) $\Delta ACD \cong \Delta BDC$ and (ii) ABCD is cyclic.



6. Iddo duwwaa deebii sirri ta'een guuti/Complete the statement and reasons for figure.

Kennama/Given: 3. $\overline{AC} \equiv \overline{BD}$ 4. $\measuredangle CAB \equiv \measuredangle DBA$ **KAB/Prove**: $\overline{AD} \equiv \overline{BC}$	
Himama/Statement	Sababa/Reasons

7. Himama fi sababa isaa mirkanessi/Write a Statement and Reason proof.

Kennama/Given: 1. ABCD Parallelogiramidha. 2. $\overline{BE}$ fi $\overline{AD}$ tuqaa F irratti walqaxaamuru. 3. Tuqaa D sarara $\overline{CE}$ irra jira. **KAB/Prove**: $\Delta ABF \sim \Delta DEF$	
Himama/Statement	Sababa/Reasons
1._____ 2. _____ 3. _____ 4. _____ 5. _____ 6._____	1._____ 2._____ 3._____ 4._____ 5._____ 6._____

B4: Geometry Attitude Scale Questionnaire

Geometry Attitudes Scale Questionnaire (GASQ) (Pre-posttest)

Kabajamoo Hirmaattota/Dear participants,

Bar-gaaffiin kun qorannoo Dirgii 3ffaa Kaadhimatoota Barsiisota Herrega irratti mata duree "***Pre-service Mathematics Teachers' Geometry Learning with Technology integrated Guided inquiry approach***"' jedhurratti qorannoo gaggeessuuf ragaalee funaanuuf kan wixineeffamedha. Kanaafuu, dhamaatii fi amanamummaan bargaaffiilee asii gaditti dhihaatan deebisi. Safaartu tokko qofa filadhu.

The closed-ended questionnaire is prepared for collecting data on "Pre-service Mathematics Teachers' Geometry Learning with Technology-Integrated Guided Inquiry-Based Approach" for my PhD thesis work. So, please fill out the questionnaire honestly. Choose only one scale for each statement.

Hub: Odeeffannoon kennitu icitiin kan qabamuufi dhimma qorannoo kanaa qofaaf oola.

Hirmaannaa keetif Galatoomi! Thank you!

Qajeelfama/Instruction:

1) Gaaffiilee mara tartiiba isaaniin deebisi; gaaffii kamiyyuu bira hin darbiin.
2) Gaaffiilee filannoo lakkoofsa kenname itti mari.

Dhimmoonni ilaalcha (Attitude) barnoota Ji'ometrii akkata waliigalaatti barnoota Herregaa irratti qabdu akkaata sadarkaa 1-5 tti tarreeffamaniiru.

Isaanis:

1= **Guutummaatti itti walii hingalu**,	1= Strongly Disagree (SD)
2= **Itti waliihingalu**,	2= Disagree (D)
3 = **Hinmurteessu**,	3= Undecided (UD)
4 = **Ittin walii gala**, fi	4= Agree (A)
5 = **Sirriittan waliigala**.	5= Strongly Agree (SA)

Haluma Kanaan, sadarkaa waliigaltee keeti lakkoofsa gabatee keessatti kenname jalatti maruun akka agarsiistan kabajaadhan isin gaafadha.

LWE/ID NO_____ Daree/Section_____

College: _____

	Statement	Himama	Guutummaatti itti walii hingalu (SD)	Itti walii hingalu (D)	Himmurteessu (UD)	Ittin walii gala (A)	Sirriittan waliigala (SA)
	Self-Confidence	**Ofitti Amanuu**					
1SC	My mind goes blank and unable to think clearly when working with geometry.	Ji'oometriin sammun kiyya dhuwwaa taasiisa.	1	2	3	4	5
2SC	Geometry does not scare me at all	Ji'oomeetrii barachuu hinsodadhu.	1	2	3	4	5
3SC	Studying Geometry makes me feel nervous.	Ji'oometrii barachuutti nan dhiphadha.	1	2	3	4	5
4SC	I have selected geometry as my area of emphasis	Barumsi ji'oometriitti baayyeen xiyyeefadha.	1	2	3	4	5
5SC	I have selected geometry as my area of emphasis	Barumsa ji'oometriitti baayyeen xiyyeefadha.	1	2	3	4	5
6SC	When I hear the word Geometry, I have a feeling of dislike	Yeroon Jecha Ji'oometri iedhu dhagahuu miira jibbuutu natti dhagahama.	1	2	3	4	5
7VA	Geometry is important in everyday life.	Barnoonni Ji'oometrii hojii guyya guyyaa keessatti nama gargaara	1	2	3	4	5
8SC	I cannot recall many geometry concepts that were hard for me to understand.	Barnoota koorsi ji'oometrii keessatti gaaffiwwan (concept) natti ulfaate hin-jiru.	1	2	3	4	5
9VA	I think of many ways that I use geometry outside of school	Ji'oomeetrii mana barmusaa alatti fayyadamuu nana yaala.	1	2	3	4	5
10SC	I am always under terrible strain in Geometry class.	Ji'oometrii barachuutti baayyen itti rakkadha.	1	2	3	4	5
	Value	**Faayidaa**					
11VA	Geometry helps develop the mind and teaches a person to think.	Ji'oomeetriin sammuu namaa gabbisuu fi akka sababeessan nama deegara.	1	2	3	4	5
12VA	Geometry courses would be very helpful no matter what grade level I taught.	Barnoonni Ji'oometrii Sadarkaa barnoota kamittuu baayyee nama fayyada.	1	2	3	4	5
13VA	Geometry is a very worthwhile and necessary subject	Barnootni Ji'oometrii baayyee barbaachisoodha.	1	2	3	4	5
14VA	Geometry is one of the most important subjects for people to study	Barnootni Ji'oometrii barnoota nama fayyaduu waan ta'eef baratamuu qaba.	1	2	3	4	5
15SC	I believe I am good at solving Geometry problems.	Piroobleemota Ji'oometri furuun baayyee natti salphata.	1	2	3	4	5
16VA	I want to develop my geometrical skills.	Dandeetti barnoota Ji'oometrii gabbifachuun fedha.	1	2	3	4	5
17VA	I believe studying geometry helps me with problem solving in other areas.	Ji'oometrii barachuu kiyyaan piroobileemiiwwan koorsii biro furuu keessatti nan fayyadama.	1	2	3	4	5
18SC	Geometry makes me feel uncomfortable.	Ji'oomeetrii barachuun natti hin tolu.	1	2	3	4	5
19VA	A strong geometry background could help me in my professional life	Dandeetti buu'ura Ji'oometrii qabaachuun fuladura hojii kiyyaaf	1	2	3	4	5

		baayyee nafayyada.					
	Statement	Himama	Guutummaatti itti walii hingalu (SD)	Itti walii hingalu (D)	Himurteessu (ID)	Ittin walii gala	Sirriittan waliigala (SA)
20VA	I think studying advanced geometry is useful	Ji'oometrii karaa waara te'een barachuun faayidaa qaba.	1	2	3	4	5
	Enjoyment	**Gammaduu**					
21EJ	I would prefer to do an assignment in geometry than to write an essay	Gaaffiwwan herregaa biro irra kan Ji'oometrii hojjechun filadha.	1	2	3	4	5
22EJ	Geometry is enjoyable and stimulating to me	Barumsi Ji'oometrii jallatamaa fi hawwataadha.	1	2	3	4	5
34EJ	Geometry is very interesting and I have usually enjoyed it	Barumsi ji'oometrii kan na hawwatuufi itti gammadudha.	1	2	3	4	5
35EJ	Geometry is dull and boring	Ji'oomeetriin barumsan jibuudha.	1	2	3	4	5
	Motivation	**Kaka'umsa**					
26MT	I find learning the geometry is motivating.	Ji'oometrii barachuun baayye na hawwata .	1	2	3	4	5
27MT	I have taken geometry classes even though it was not required	Barumsi Ji'oometri barbaachisaafi barachuun fedha.	1	2	3	4	5
28MT	It wouldn't bother me at all to take more geometry course	Koorsii ji'oometrii dabalatan yoo fudhe itti dhibadhu.	1	2	3	4	5
29MT	I liked and motivated geometry class of my college	Ji'oometriin Kollejjiitti baradhe baayyee miira kiya nakakaase.	1	2	3	4	5

Adapted from ATMI (Tapia & Marsh, 2004)

B5: Observation Schedule Checklist

Name: _____ Date: _____
Class Observed: _____ Time: _____
Observer: _____ Observation No: _____

This is classroom observation checklist. Please put thick (√) to yes if the procedures are found and (**x**) if the procedures are not found during classroom observation.

Note: 1 = Not observed 2 = inadequate 3 = adequate

1. New learning was connected to prior learning. • The teacher poses questions that elicit learner thinking about information and concepts in the content areas as well as learner application of critical thinking skills (such as inference making, comparing, and contrasting).	1	2	3
2. The Geometric content presented was accurate • The teacher accurately and effectively communicates concepts, processes and knowledge, and how to guide learners to accurate conceptual understanding.			
3. Teacher used precise and accurate mathematical language. • The teacher accurately and effectively communicates concepts, processes and knowledge in the discipline, and how to guide learners to accurate conceptual understanding. • Defined unfamiliar terms, concepts, and principles.			
5. PSMTs used precise and accurate mathematical language to explain their thinking. • PSMTs questions and responses demonstrated progress toward the learning goals of the class session. • The PSMTs seemed like having a high curiosity and always asked something in learning process.			
5. Teacher uses questioning strategies. • The teacher adapts instruction and uses modified materials, resources, tools, and technology to address learners need. • The teacher incorporates technology in a variety of innovative ways in planning. • The teacher models questioning skills related to content discipline (e.g., generating hypotheses, taking multiple perspectives, using metacognitive processes) and engages learners in activities that develop these skills.			
6. Teacher provides wait time.			

• The teacher gives PSMTs opportunities to participate by sharing their ideas. • The teacher also provides PSMTs with feedback.			
7. Instructional Structure: Guided practice and discussion • Pair or group work • Independent practice • Cooperative learning • Hands-on activities/technology-based			
8. PSMTs engage in the following problem-solving behaviors (select all that applied): • Collaborate with others • Use varied/appropriate strategies • Construct and discover ideas • Make multiple attempts			
9. PSMTs utilizes various representations to demonstrate their thinking • Concrete materials (manipulatives) • Digital technologies			
10. The PSMTs were active to share their ideas during the discussion time.			
11. PSMTs feel motivated being taught.			
12. PSMTs enjoyed doing activities in Math's class.			
13. PSMTs feel excited when they communicate in classroom.			
14. PSMTs pay attention when the teacher is explaining the lesson.			

B6: Semi-structured Interview Protocol

The purpose of this interview is to understand more about your opinions and perceptions about the teaching strategy implemented during this semester. For each question, there is no right or wrong response. But you must be as truthful as you can in your response by expressing your emotions. I appreciate your willingly contributing to this research.

1. How would you describe yourself as a mathematics student? (i.e., any struggles or challenges, comfort with subject matter, etc.)
    a) What are some things that motivate you to learn geometry? What aspect of geometry do you like/find enjoyable?
    b) In what ways has the geometry you've learned this semester been useful to you? Please give an example.
2. Compare the use of technology as a teaching and learning tool vs. the traditional way of teaching of last year or before.
3. Do you think that the technology [GSP] approach helped you with your geometry learning? Give reasons.
4. Do you feel you are better able to understand geometry as a result of using hands-on instructional strategies (manipulatives, cooperative groups and technology)? Did GSP help you to have a better understanding of certain geometry concepts? What concepts and what can you say how did you experience it?
5. Would you be more interested in learning geometry if hands-on instructional strategies (manipulatives, cooperative groups, and technology) were used as an instructional tool? Did you enjoy working with GSP? Why?
6. What did you find most challenging about a TGIBL & GIBL approach? Was there anything that you did not like whilst working with GSP?
7. So, what changes you suggest to create better learning environment?

Questions for teachers

8. What strategy/teaching method do you apply in teaching Euclidean Geometry?
9. What challenges do you encounter in teaching Euclidean Geometry?
10. How can you rate your teaching and your students' performance in Euclidean Geometry last year and this year (poor, average, good)?
11. What is the main cause of such performance?
12. What improvements would you like to make for better facilitation and learning in Euclidean Geometry?
13. How was it doing Mathematics on a computer this year? Explain. (Depending on participant's responses)
14. Do you think more training is needed before we use GSP as learning tool in the computer lab? Explain.

Appendix C: Module used for Plane Geometry

Department	Mathematics
Course Title	Plane Geometry
Module code	Math111
Pre-requisite	None
Status of the course	Compulsory
Credit hour	3
Contact hours	4
Target Group	Second Year Pre-service Mathematics Teachers
Requirements	Required for al Math's, Chemistry and Physics major diploma students.

Course Descriptions: In this course basic concepts such as points; lines; planes; triangles; congruency and similarity of polygons; area and perimeter of polygons; Pythagoras' theorem; basic constructions, circles; arcs, chords, tangent lines, secant lines, segments and sector of a Circle; central angles and their measures; area of segment and of a sector; radian measure of an angle will be treated.

Course Objectives: At the end of the course each students should be able to

- Define basic concepts such as points; lines and planes.
- Use axioms, postulates and theorems to prove congruencies and similarity of triangles.
- Use formulas to find area and perimeters of polygons.
- Apply Pythagorean Theorem.
- Know basic constructions.
- Identify the relation between circles, lines and angles.
- Find the area of circles, sectors, and segments of a circle.

Period	Course Content
Week one	**Unit I Undefined Terms, Definitions And Axioms** 1.1 **The undefined terms**: - Point, Line, Plane 1.2 **Definition**: Line segment, half-lines, half-plane, angle, Exterior and interior of an angle, adjacent and vertical angles. 1.3 Axioms 1.3.1 Axioms on connection 1.3.2 Axioms of order 1.3.3 Axioms on congruency 1.3.4 Axioms on parallel lines 1.3.5 Axioms on continuity of a line 1.3.6 Axiom of completeness
Week three	**Unit II Measurement and Units of Measurement** 2.1 Definition

	2.2 Units of measurement of length 2.3 Units of measurement of angles 2.4 Some postulates about angles 2.5 Units of measurement for area of plane figure
Week four and five	**Unit III Triangles** 3.1 Definition of triangles 3.1.1 Types of triangles 3.1.2 Equilateral, Isosceles, Scalene and a right angled triangles 3.2 Definition of altitudes and medians of a triangle 3.3 Congruence of triangles: 3.3.1 Definition of congruent triangles 3.3.2 Congruence theorems for triangles, namely: ASA, SSS, AAS & RHS 3.3.3 Examples of application of the congruence theorems 3.4 Theorems on concurrence of: 3.4.1 Perpendicular side bisectors of a triangles 3.4.2 Bisectors of angles of triangles 3.4.3 Altitudes and medians of a triangle 3.5 Similarity of triangles: 3.5.1 Some properties of parallelogram 3.5.2 Theorems on proportional divisions of sides of a triangle and a parallel projection 3.5.3 Definition of similar triangles 3.5.4 Examples and applications of similar triangles 3.5.5 Pythagoras theorem
Week six and seven	**Unit IV Polygons** 4.1 Definition of a polygon, interior and exterior angles of a polygon, perimeter of a polygon, convex and concave polygons, diagonals of a polygon, equiangular, equilateral, regular, congruent and similar polygons. 4.2 Congruence theorem on perpendicular side bisectors of a regular polygon 4.3 Quadrilaterals: trapezium, parallelogram, rectangle, rhombus. 4.4 Area of polygons: 4.4.1 Area postulates 4.4.2 Area formulae for: 4.4.2.1 Rectangles, triangles, Parallelograms 4.4.2.2 Trapeziums and regular polygons 4.4.2.3 Area of similar polygons
Week one and two	**Unit V Basic Constructions** 5.1 Copying an angle and a line segment 5.2 Bisection a given angle and a line segment 5.3 Dropping a perpendicular line to a given line through a given point not on the line 5.4 Construction of the common polygons and regular polygons 5.5 Construction of special angles 30^0,60^0, 45^0, 90^0 5.6 Construction of a triangle that has an area which is equal to the area of a given quadrilateral.
	Unit VI Circles 6.1 Definition of chords, arcs, secant and tangent lines, sectors and

Week eight and nine	segments of a circle, central and inscribed angles and arc length. 6.2 Circumference and area of circle sector and segment. 6.3 Relationships between arc measures and angles formed by lines Intersecting inside or outside circles. 6.4 Radian measure of angles 6.5 Definition and dependence of the radian measure on the radius of the circle 6.6 Transformation of degree measure to radian measure and vice versa.
References	**1. Smart: Introductory Geometry** **2. Moise: Elementary Geometry from advanced standpoint** **3. Ministry of Education: Geometry C3**

Appendix D: 5E lesson plan Model for TGIBL and GIBL

Lesson Plan for grade level: PSMTs		**Length of the lesson:** 2hr
Subject area course: Plane Geometry**.**		**Date:**
Materials: LCD projectors, Marker, laptop and internet		
Lesson Objectives:		
Differentiation strategies to meet diverse learners need:		
	Teacher's Activities	**PSMTs' Activities**
Engagement		
Exploration		
Explanation		
Elaboration		
Evaluation		

Appendix-E: Sample Tasks for Experimental Groups

Below are examples of activities that were considered:

- Put PSMTs into small groups of four or five and asked them to collaborate on completing activities via GSP; an interactive questions that provides a diagram of different geometry concepts with blank labels and requires PSMTs to drag and drop the proper labels in place from an answer key.
- PSMTs will compare the different geometric theorem properties and make conclusions regarding the differences they find.
- Each group will be asked to infer what might happen when different geometric properties are considered. Each group will compile evidence to make their conjecture.

Lesson One Subject: Math111

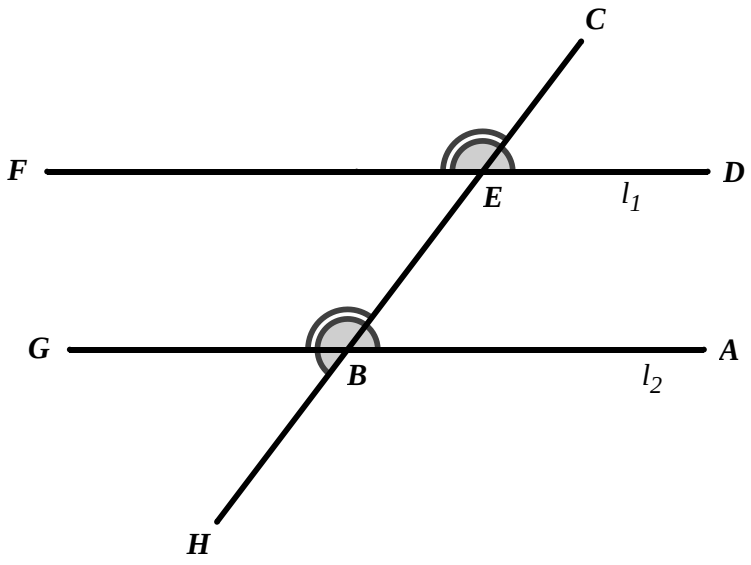
Topic: Topic: Angles between lines

Lesson Objectives: By the end of the lesson students should be able to identify and conjecture the relationship between angles of:

- Straight line angles
- Vertically Opposite angle
- Corresponding angles
- Alternate interior angles

In the following diagram lines l_1 and l_2 are parallel to each other. Segment HC is a transversal of these parallel lines. Using the measurements provided make as many conjectures as you can about the angles formed by transversals and parallel lines.

***m∠ABC* = 52.44°**
***m∠DEC* = 52.44°**
***m∠CEF* = 127.56°**
***m∠CBG* = 127.56°**
***m∠GBH* = 52.44°**



$m(\angle DEC)$	$m(\angle CEF)$	$m(\angle DEB)$	$m(\angle FEB)$	$m(\angle GBE)$	$m(\angle ABE)$	$m(\angle GBH)$	$m(\angle ABH)$

Conjecture(s): If two parallel lines are cut by a transversal line then:

Chapter 2: TRIANGLES

Various properties of a triangle will be examined. GSP allows students to construct visual representations of triangles; this helps the students solve the problems in each lesson. Each lesson provides opportunity to use the various commands in GSP used in the previous chapter and also introduces new commands specific to this chapter.

1. The sum of the interior angle is 180 degree.
2. The sum of any two sides of a triangle is greater than the third side of a triangle.
3. The sum of two adjacent interior angles is the same as the exterior angle.

$m\angle ABC = 59.28°$

$m\angle CAB = 49.70°$

$m\angle BCA = 71.02°$

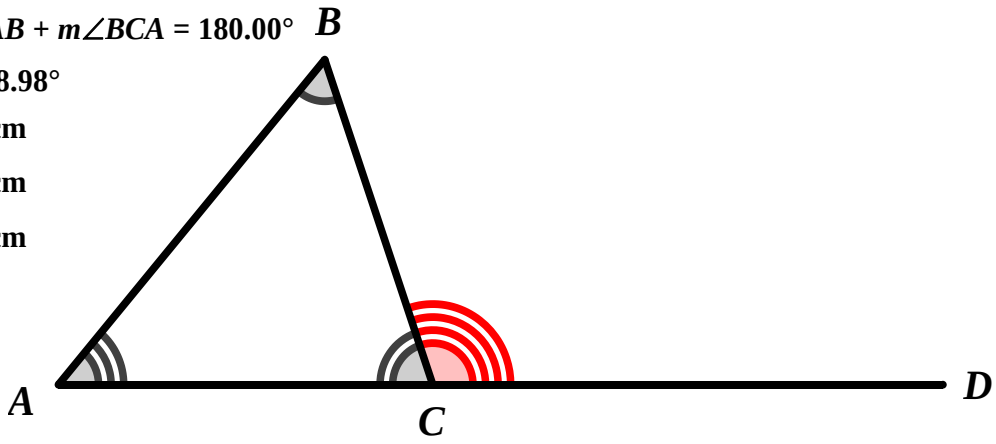
$m\angle ABC + m\angle CAB + m\angle BCA = 180.00°$

$m\angle DCB = 108.98°$

$m\ \overline{BC} = 3.50$ cm

$m\ \overline{AB} = 4.34$ cm

$m\ \overline{CA} = 3.94$ cm



$m\ \overline{BC} + m\ \overline{AB} = 7.83$ cm

$m\ \overline{BC} + m\ \overline{CA} = 7.44$ cm

$m\ \overline{AB} + m\ \overline{CA} = 8.28$ cm

$m(\angle ABC)$	$m(\angle CAB)$	$m(\angle BCA)$	$m(\angle ABC) + m(\angle CAB) + m(\angle BCA)$	$m(\angle DCB)$	$m\overline{BC}$	$m\overline{AB}$	$m\overline{CA}$	$m\overline{BC} + m\overline{AB}$	$m\overline{BC} + m\overline{CA}$	$m\overline{AB} + m\overline{CA}$

Conjecture(S):

- The sum of the interior angle of any triangle
- The sum of length of two sides of a triangle is
- The angle of any triangle opposite to the largest side is
- The angle of any triangle opposite to the smallest side is

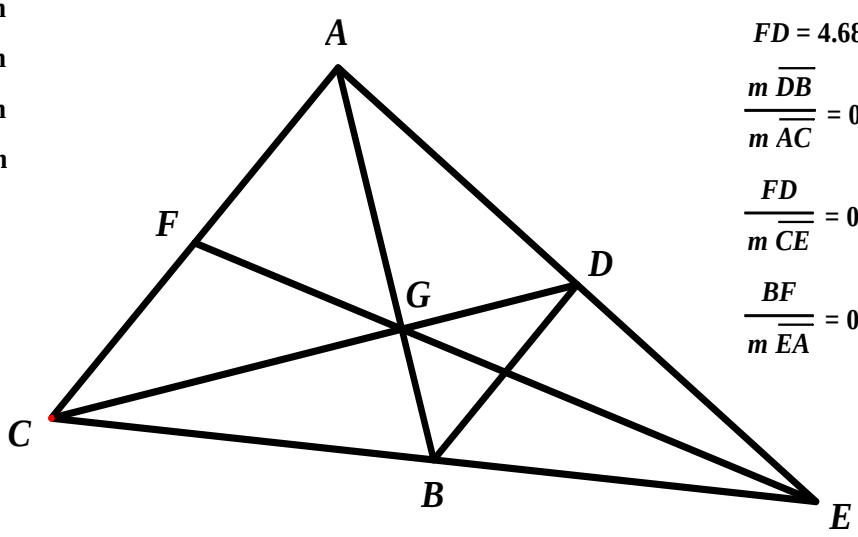
The Median of the side of a triangle is the line through a vertex and the midpoint of the opposite side.

Question:

Where is the Centroid located (inside or outside the triangle, or on an edge) when a triangle is moved and transformed?

a. When the triangle is an ACUTE triangle (no matter how moved)
b. When the triangle is an OBTUSE triangle (no matter how moved)
c. When the triangle is a RIGHT triangle (no matter how moved)
d. When the triangle is a EQUILATERAL triangle (no matter how moved)

$m\overline{AC}$ = 5.23 cm
$m\overline{EA}$ = 7.55 cm
$m\overline{CE}$ = 9.36 cm
$m\overline{FE}$ = 8.09 cm
$m\overline{AB}$ = 4.51 cm
$m\overline{DC}$ = 6.57 cm
EG = 5.40 cm
AG = 3.00 cm
CG = 4.38 cm
$\frac{EG}{m\overline{FE}}$ = 0.67
$\frac{AG}{m\overline{AB}}$ = 0.67
$\frac{CG}{m\overline{DC}}$ = 0.67



$m\overline{DB}$ = 2.61 cm
BF = 3.78 cm
FD = 4.68 cm
$\frac{m\overline{DB}}{m\overline{AC}}$ = 0.50
$\frac{FD}{m\overline{CE}}$ = 0.50
$\frac{BF}{m\overline{EA}}$ = 0.50

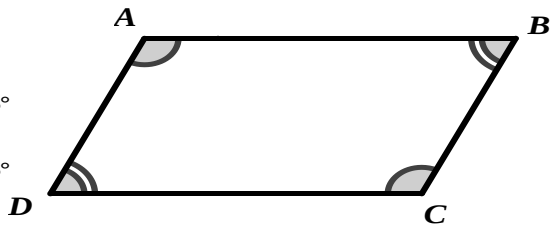
Conjecture(s): the ratio of the length of median from vertex to the point of intersection to the its total length of the median

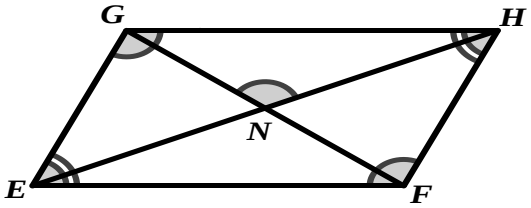
The ratio of length of the length of joining two midpoints of sides of a triangle:

Properties of Quadrilateral
The PSMTs will observe the properties, conjecture and complete the table

Parallelogram

$m\overline{AB}$ = 4.63 cm
$m\overline{DC}$ = 4.63 cm
$m\overline{AD}$ = 2.75 cm
$m\overline{BC}$ = 2.75 cm
$m\angle DAB$ = 115.25°
$m\angle ABC$ = 64.75°
$m\angle BCD$ = 115.25°
$m\angle CDA$ = 64.75°

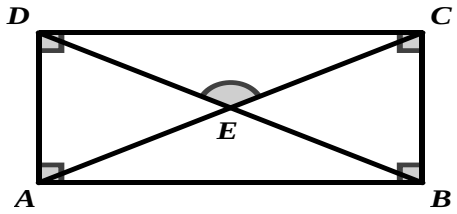




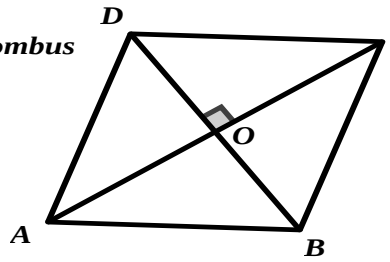
$m\overline{FG}$ = 4.26 cm
$m\overline{EH}$ = 6.31 cm
$m\angle GHE$ = 23.20°
$m\angle FEH$ = 23.20°
$m\angle HFG$ = 79.52°
$m\angle FGE$ = 79.52°
FN = 2.13 cm
NG = 2.13 cm
HN = 3.16 cm
EN = 3.16 cm
$m\angle HNG$ = 121.07°

Rectangle

$m\overline{DB}$ = 5.67 cm
$m\overline{CA}$ = 5.67 cm
ED = 2.83 cm
BE = 2.83 cm
AE = 2.83 cm
CE = 2.83 cm
$m\angle CED$ = 127.35°



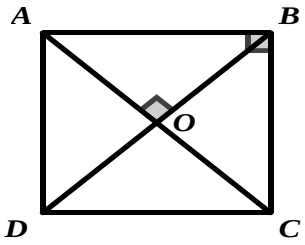
Rhombus



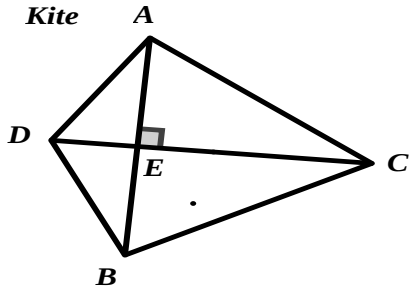
$m\overline{AB}$ = 3.34 cm
$m\overline{CB}$ = 3.34 cm
$m\overline{DC}$ = 3.34 cm
$m\overline{DA}$ = 3.34 cm
$m\overline{CA}$ = 5.36 cm
$m\overline{DB}$ = 3.98 cm
$m\angle COD$ = 90.00°

Square

$m\overline{AC}$ = 4.27 cm
BD = 4.27 cm
CO = 2.13 cm
AO = 2.13 cm
DO = 2.13 cm
BO = 2.13 cm



Kite



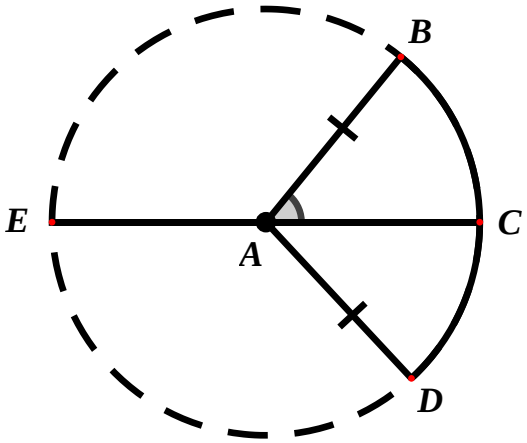
$m\overline{BD}$ = 2.16 cm
$m\overline{DA}$ = 2.16 cm
$m\overline{CA}$ = 3.61 cm
$m\overline{CB}$ = 3.61 cm

The PSMTs are expected to respond to the following conjectures using GSP and animating the each figure.
Is a square a rectangle?
Is a rhombus a parallelogram?
Is square a rhombus?

Properties	Parallelogram	Rectangle	Rhombus	Square	Kite	Trapezium
Only one pair of sides are parallel						
Opposite sides are parallel						
Opposite sides are congruent						
All sides are congruent						
Two pair of adjacent sides are congruent						
Opposite angles are congruent						
All angles are congruent						
Only on diagonal bisect the other one						
Diagonals bisect each other						
Diagonals are congruent						
Diagonals are perpendicular						
Only one diagonal is bisecting opposite angles						
Both diagonals bisect the angles						

Exploring circle Geometry Activities using by GSP

Terminology in Circles



$\overline{AO}$, $\overline{OB}$ *are radius/ Raadiyeesidha*

$\overline{CD}$ *is diameter/Diyaameetriidha.*

$\overset{\frown}{BED}$ *major arc/ Golboo guddaa*

$\overset{\frown}{BCD}$ *is minor arc/ Golboo Xiqqaa*

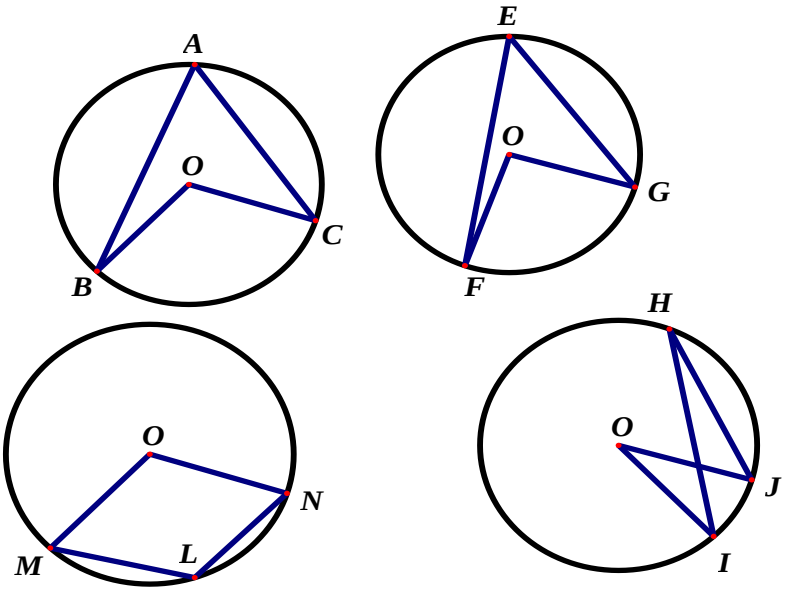
$\angle BAC$ *central angle/Kofa handhuuraa*

Learning objective: To investigate

- Activity-1: The angle at the center of a circle is twice the angle at the circumference of the circle from the same arc.
- Activity-2: The angle formed in a semicircle is always a right angle.
- Activity-3: Angles from the same arc in the same segment are congruent.

$m\angle CAB$ = 58.08°
$m\angle COB$ = 116.16°

$m\angle COB$	$m\angle CAB$
93.60°	46.80°
99.70°	49.85°
102.71°	51.36°
104.19°	52.09°
104.38°	52.19°
116.16°	58.08°

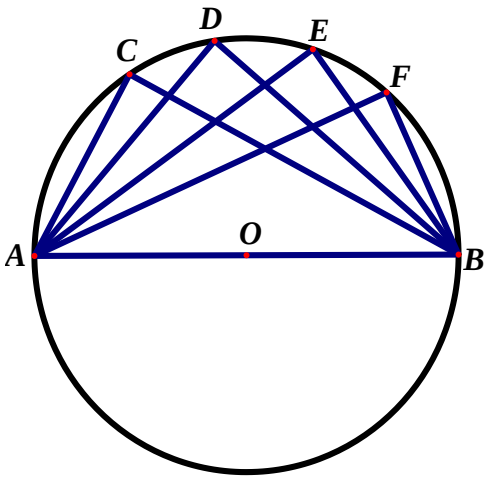


Conjecture(s): what is the relationship between the central angle and its inscribed angle?

Activity-2: Is angles inscribed in Semi-circle is a right angle?

$m\angle ACB = 90.00°$

$m\angle ADB = 90.00°$



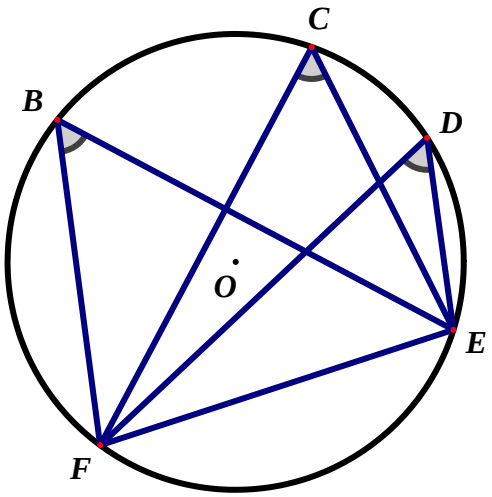
Conjecture(s): If a triangle is inscribed in a semi-circle then….

Activity-3: is angles from the same arc in the same segment are congruent?

$m\angle EBF = 54.57°$

$m\angle ECF = 54.57°$

$m\angle FDE = 54.57°$

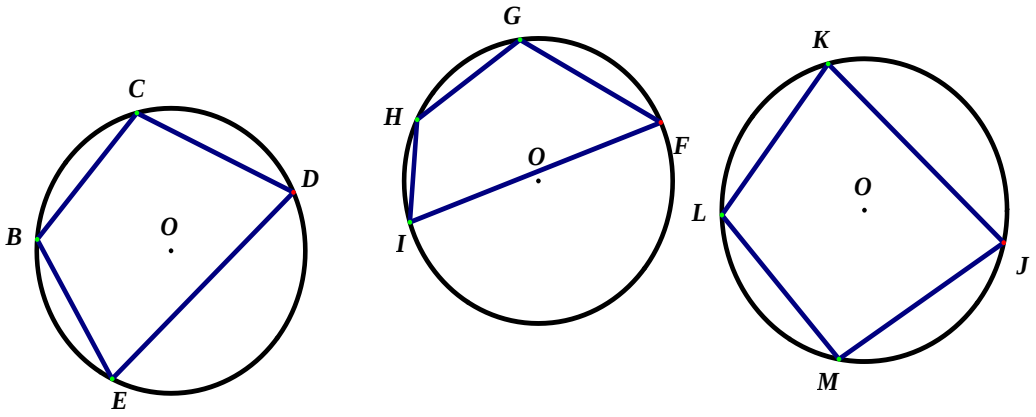


Conjecture(s):

Activity-4: Cyclic Quadrilateral

In the figure below a quadrilateral is inscribed in a circle. Make is many conjectures as you can about inscribed quadrilaterals.

$m\angle CBE = 110.37°$
$m\angle DCB = 104.41°$
$m\angle CDE = 69.63°$
$m\angle DEB = 75.59°$



Conjecture(s): If a quadrilateral is inscribed in a circle then…….

Appendix-F: Some sample activities for groups of PSMTs in GIBL approach

Lesson one

Course: Plane Geometry (math-111)

Class: PSMTs Time: 2hrs

Topic: Angles between lines.

Lesson objective: PSMTs should be able to identify and calculate the measure of angles:

i) Vertically opposite angles

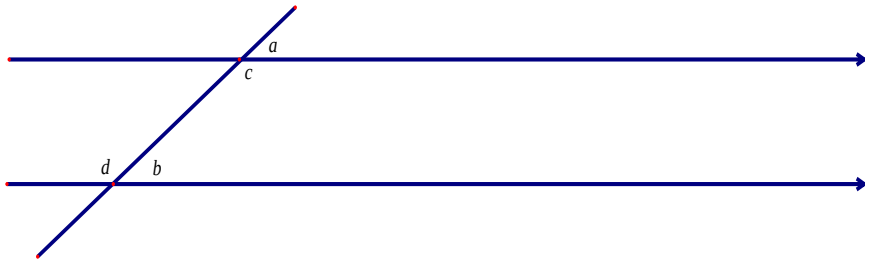
ii) Corresponding angles

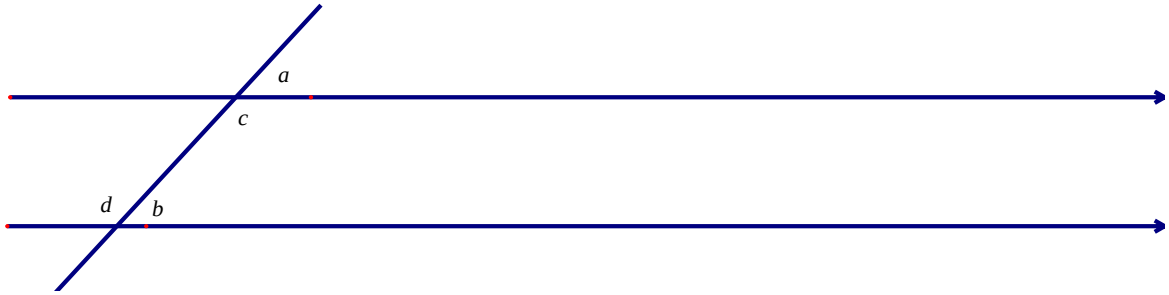
iii) Alternate angles

Teaching material: Ruler, Protractor, paper cutter

Instructional Technique: Encourage group activities, allow students interactions through independent work, and encourage asking of questions.

Students are to draw two parallel lines with a transversal as in the following figures.





Hence, PSMTs measure angle *a* and *b* which are corresponding similarly *c* and *d* which are alternate. Then, PSMTs are to discuss within groups and conjecture the following in their own way

The PSMTs draw the conclusion that the following are true if two parallel lines are crossed by a transversal line:

a. The alternate angles are equal.

b. The corresponding angles are equal.

c. The interior angles on the same side of the transversal are supplementary.

The PSMTs also observed that there were pairs of alternate and corresponding angles from lines that are not parallel and the pairs of interior angles too were not supplementary.

Lesson-8: Area of circle

Course: Math-111

Topic: Area of circle

Learning Aids: Paper, paper cutter, Compass, Ruler, Plaster, different color markers

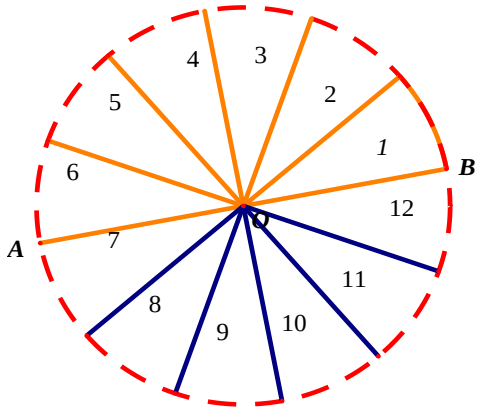
Draw different circle with different radius in each groups. Then divide the circle with 30 degree sector into 12 equal sectors.

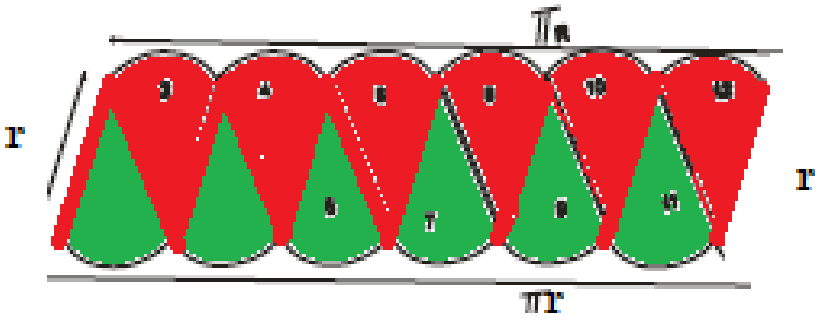
BA = 6.92 cm

Circumference $\odot OB$ = 21.74 cm

$$\frac{\text{Circumference } \odot OB}{BA} = 3.14$$

BA	Circumference $\odot OB$	$\frac{\text{Circumference } \odot OB}{BA}$
7.06 cm	22.17 cm	3.14
7.09 cm	22.28 cm	3.14
7.10 cm	22.29 cm	3.14
6.84 cm	21.49 cm	3.14
7.44 cm	23.37 cm	3.14
7.98 cm	25.07 cm	3.14
6.92 cm	21.74 cm	3.14





Conjecture: area = $\pi r(r) = \pi r^2$

Lesson: Five

Topic: The properties and relationships of the properties of quadrilateral

Teaching tools: set square, paper cutter, square charts

Sketch all quadrilaterals on the square chart (trapezium, parallelogram, rhombus, kite, rectangle, squares).

Describe each of the following as:

i. A quadrilateral is a plane shape with four sides.
ii. A trapezium is a quadrilateral which has one pair of opposite side parallel.
iii. A parallelogram is a quadrilateral which has two pairs of parallel sides.
iv. A rhombus is a quadrilateral with four sides equal in length.
v. A kite is quadrilateral which has one diagonal as a line of symmetry.
vi. A rectangle is a quadrilateral in which each angle is a right angle
vii. A square is a rectangle with all four sides equal in length.

PSMTs are to discern the following relations:

i. All squares are rectangles but not all rectangles are squares.
ii. Squares, rectangles and rhombus are special parallelogram but there are many parallelograms that are not square, rectangles etc.
iii. All rhombuses are kites
iv. All squares are kites, not rhombus.
v. But all kites are not rhombuses

Evaluation: Write down 2 shapes which are examples of (i) Trapezium (ii) Parallelogram.

Appendix-G: The consent form for PSMTs

Please complete the consent form and return back to the researcher

I _____, understand the context of the research study titled: *Primary Pre-service Mathematics teachers' geometry learning with technology integrated guided inquiry based learning approach*. I am aware that optionally I can allow audio recording of my participation during interview sessions. I am aware that anonymity and confidentiality will be adhered to in this study. I am informed that I may withdraw my agreement to participate at any time without penalty by advising the researcher. I agree on my free will to participate in the research study.

Signature: .....

Date: .....

Appendix-H: TEACHER CONSENT MEMO

Teacher's participation consent

I _____understand the context of the research study titled:

Primary Pre-service Mathematics teachers' learning geometry with technology integrated guided inquiry based learning approach. I am aware that optionally I can allow audio recording during participation. I am aware that anonymity and confidentiality will be adhered to in this study. I am informed that I may withdraw my agreement to partake at any time without punishment by advising the researcher. I agree on my free will to participate in the research study.

Signature: .....

Date: .....