THE QUANTIZED HALL EFFECT

By

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a project work entitled "THE QUANTIZED HALL EFFECT." by Lemma Kurie in partial fulfillment of the requirements for the degree of Master of Science.

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ABSTRACT

The study of the Hall Effect in the two-dimensional electron Systems (2DES) showed new quantum effects. The Experimental or theoretical works show that at high magnetic fields and low temperatures, instead of increasing linearly with increasing magnetic field as in CHE, the Hall resistivity exhibits a series of step up increasing plateaus. In these same intervals of magnetic fields, the longitudinal resistivity vanishes. For each plateau, the Hall resistivity is given by the Plank's constant h divided by the square of the electron charge e multiplied by an integer i , which represents the number of completely filled Landau levels. In the extreme (high magnetic fields and low temperatures) case, the Lowest Landau Levels (LLL) is partially filled it results in FQHE where $i = \textit{the fraction}$ number. Bits of magnetic field can get attached to each electron, creating other objects. Such particles have properties very different from those of the electrons. They sometimes seem to be unaware to huge magnetic fields and move in straight lines, although any bare electron would orbit on a very tight circle. All of these strange phenomena occur in two dimensional electron systems at low temperatures exposed to a high magnetic field only electrons and a magnetic field. We see a brief, high light on classical Hall Effect and overview on description of the integer and fractional quantized Hall effects, and the basic conditions associated with their occurrence. The way in which these conditions are satisfied, the integer quantized Hall Effect can be understood in terms of non interacting electrons, using flux quantization. The fractional quantized Hall Effect depends fundamentally on electron-electron interactions. Which lead to peculiar highly correlated elementary quasi particle excitations. We shall overview and briefly discuss such phenomena.

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CHAPTER ONE

1 INTRODUCTION

In Chapter chapters (one and two) we will review the property of charged particles, (in our case free electrons in a conductor), under the influence of electric and magnetic fields.

Further, in chapter three we will develop understandings of the formation of Two-Dimensional electron systems (2DES) and how to confine them within a solid to the interface between a semiconductor and insulator or to the interface between two different semiconductors. To conduct electricity i.e. for access of free electrons, in semiconductors require addition of some impurities, known as doping. We then go to chapter four to review that has been observed in theoretical [1, 4,45] and experimental[2,3 ,47]cases of the behavior of the two-dimensional electron Gas (2DEG), while decreasing temperatures and increasing magnetic fields Quantum cases. Then after we would see the observations, verified about the behavior of the 2DEG in extreme cases (at low temperatures and high magnetic fields) that has indicated new and interesting quantum phenomena.

1.1 GENERAL REVIEW OF HALL EFFECT

It is well known that one “key” to gain insight about the internal (electronic) properties. Of the conducting materials is achieved by the transport measurements. When an electric current passes through a metal strip there is a voltage drop along the strip, which can be measured. The Hall Effect refers to the potential which an electric current is flowing, created by a magnetic field applied perpendicular to the current. . The magnetic field B normal to a gold leaf exerts a Lorentz force on a current I flowing longitudinally along the leaf. That force separates charges and builds up a transverse voltage now known as" Hall voltage" between the conductor's lateral edges. In 1879, E. H. Hall discovered that by applying a perpendicular magnetic field to a gold leaf mounted on a glass plate (perpendicular to the direction of a current flow through the metal) a voltage V_H so called the Hall voltage is produced across the metal strip, due to the accumulation of charge. The appearance of the Hall voltage V_H is known as the classic Hall Effect (CHE) [1]. The occurrence of the classic Hall Effect is due to the deflection of the charge carriers by the Lorentz force to the edge of the sample. The equilibrium is achieved when the Lorentz force $\vec{F}_L = q\vec{v} \times \vec{B}$ is balanced by the electrostatic force $\vec{F} = qV_H / w$ from the build up of charges at the edge. Where, (q is the charge of the carrier, v is its drift velocity, $\vec{B}$ is the perpendicular magnetic field, and $\vec{E}$ is the electric field,). The origin of Hall Effect is Classical Electrodynamics. Nowadays the Hall Effect is thoroughly investigated and well understood in metals and semiconductors. By measuring the Hall voltage across the element, one can determine the strength of the magnetic field. Hall voltage (V_H) is the result of the interaction of the magnetic field and the current. Thus we can write the Hall voltage as a function of the input current, $V_H \propto I \times B$. (See fig 1.1).Note that the direction of the current I in the diagram is that of conventional current, so that the motion of electrons is in the opposite direction. This can be expressed as the Hall voltage,

$$V_H = \frac{IB}{ned} \qquad 1.1$$

Where, d is the depth of the plate, e is the electron charge, and n is the bulk density of the carrier electrons. Thus from Eqn 1.1 the Hall resistance is apparent .

Thus in a normal conductor the resistance R_H depends linearly on the strength of the magnetic field B .[i.e. $R_H \propto B$] (See Fig 1.2). In 1980 Klaus von Klitzing discovered that, for the case of two dimensional electrons systems, the dependence is very different.

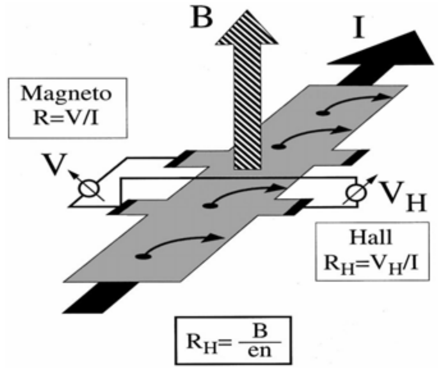

Fig 1.1: Geometry for measurement of the magneto resistance R and Hall Resistance R_H as a function of the current I and magnetic field B. V Represents the longitudinal voltage, which is dropping along the current path, and V_H the Hall voltage, which is dropping perpendicular to the current path. The electron density per cm^2 is denoted as n_s and the electron charge $,e$. The black dots represent electrons that are forced toward one side of the bar following the Lorentz force from the magnetic field. Edwin Hall's 1878 experiment was the first demonstration of the Hall effect. A magnetic field B normal to a gold leaf exerts a Lorentz foece on a current I flowing longitudinally along the leaf. That force separates charges and builds up a transverse "Hall voltage" between the conductors's lateral edges. [3]

The higher the field, the bigger the push, the bigger R_H. But, the lower the density of electrons, the higher R_H. To generate the same current, fewer electrons need to travel faster. Faster electrons experience a stronger Lorentz Force and create a bigger V_H and, hence, a bigger R_H. In its final form, we get $R_H = B / n_s e$ where n_s is the electron density per (unit area) in the sample, which is equal to the electron density n per (unit volume) times the thickness of the specimen, and where e is the elementary charge of an electron. Fig 1.2 indicates the graph that Hall drawn. The vertical axis is proportional to the Hall voltage.

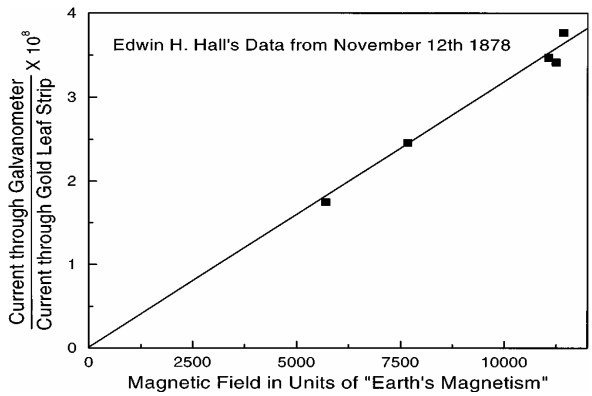

Fig.1.2: Edwin Hall's data of 1879 as plotted from a table in his publication. The vertical axis is proportional to the Hall voltage V_H of (fig 1.3) and the Horizontal axis is proportional to the magnetic field of fig 13. A linear relationship between V_H and B, hence between R_H and B is apparent. (Source: [3])

Due to its independence from all intrinsic and extrinsic parameters, the Hall Effect has become a standard tool for the determination of the density of electrons in conductor, and in semiconductors. Therefore, Classically, we would expect the Hall conductivity to vary linearly with $1/B$. From Ohm's Law, the voltage, V, along the current I path, which when divided by the current , represented the electrical resistance R of the material.

The transverse voltage, V_{across} which is across the current path, which was expected to be zero since the current ran perpendicular to it. This was Hall's Observation until he applied a magnetic field B normal to the metal sheet. It gave rise to a nonzero voltage V_H across the current path. From his experiments, Hall deduced that V_H was proportional to the current I and proportional to the magnetic field B. Hence, V_H / I is an electrical resistance R_H yielded $R_H \propto B$.

1.2 EVOLUTION OF RESISTANCE CONCEPTS

Electrical characterization of materials evolved in three levels of understanding. In the early 1800s, the resistance R and conductance G were treated as measurable physical quantities obtainable from two-terminal $I-V$ measurements (i.e., current - voltage). Later, it became obvious that the resistance alone was not comprehensive enough since different sample shapes gave different resistance values. This led to the understanding (second level) that an intrinsic material property like resistivity (or conductivity) is required that it is not influenced by the particular geometry of the sample. For the first time, this allowed scientists to quantify the current-carrying capability of the material and carry out meaningful comparisons between different samples. By the early 1900s, that resistivity was not a fundamental material parameter, since different materials can have the same value. Also, a material might exhibit different values of resistivity, depending upon how it was synthesized. This is especially true for semiconductors, where resistivity alone could not explain all observations. Theories of electrical conduction were constructed with varying degrees of success, but until the advent of quantum mechanics, no generally acceptable solution to the problem of electrical transport was developed. This led to the definitions of carrier density n and mobility μ_{el} (third level of understanding) which are capable of dealing with even the most complex electrical measurements today. Through this chapter, we have qualitatively reviled the Hall Effect .Also we have identified that the Hall resistance has linear relation with the normal magnetic field ($R_H \propto B$). Thus, the conductivity is inverse relation with the field i.e. $\sigma_H \propto \frac{1}{B}$. Above all, we observed that the origin of Hall Effect is the classical electrodynamics.

CHAPTER TWO

2 TRANSPORT PROPERTIES OF THE CLASSICAL TWO DIMENSIONAL ELECTRON GAS IN MAGNETIC AND ELECTRIC FIELDS

In this chapter, we will see some quantitative approach of the classical Hall Effect, then we merge to the magneto and the Hall resisitivities. Finally, we see the property of electron gas in the low limit magnetic field.

The theory of classical Electrodynamics reveals that when a moving electron gas (or conduction) electron enters in the region where magnetic field exists, the particle will experience the electromagnetic force. If the magnetic field is in such a way that in a direction normal to the plane on which the electron is moving, then, the electron gas will follow a circular/or cyclotron path on this plane. Then we can observe the behavior that the electron reveals such as in (its orbital velocity/cyclotron frequency and magnetic length), when we varies such quantities as magnetic and electric fields, temperature and/ or when we place another particle near by.

2.1 THE CLASSICAL MOTION OF ELECTRON GAS SUBJECTED TO ELECTRIC AND MAGNETIC FIELDS

Magneto transport measurements are used to determine some properties of a material (the electron density n_s and electron mobility μ_{el}) subjected to crossed electric and magnetic fields ($E \times B$). An electric field exerts a force on a charged particle and induces a drift velocity of the particle in the direction of the field lines. The magnetic field exerts a force, only on moving charged particles. Its direction is perpendicular to both the particle's direction of motion and the direction of the magnetic field (viewed in Fig 2.1). Thus, the particle experiences curved paths with the radius of this circle, inversely proportional to the strength of the magnetic field i.e. ($r = m^* v / eB$). This means that higher B values lead to smaller orbits for the charge carriers, in the particle trajectory. The classical motion of an electron of charge –e confined in a two-dimensional plane (x, y), and subject to a constant field $B\hat{z}$ perpendicular to this plane. The Newtonian equations of motion due to the Lorentz force are given by:

$$\begin{pmatrix} \dfrac{d^2x}{dt^2} \\ \dfrac{d^2y}{dt^2} \end{pmatrix} = \frac{eB}{m} \begin{pmatrix} -\dfrac{dy}{dt} \\ \dfrac{dx}{dt} \end{pmatrix} \qquad 2.1$$

m is the mass of the particle. In complex notations $z = x + iy$, with ω_c defined as: $\omega_c = eB/m$ The above eqn Rewrites:

$$d^2z/dt^2 = i\omega_c dz/dt \qquad 2.2$$

The solution is given by;

$$z(t) = z_0 + re^{i\omega t} \qquad 2.3$$

The trajectory is a circle of radius $|r|$ run at a constant angular velocity. The frequency ω_c is independent of the initial conditions and fixed by the magnetic field, the charge and mass of the particle.

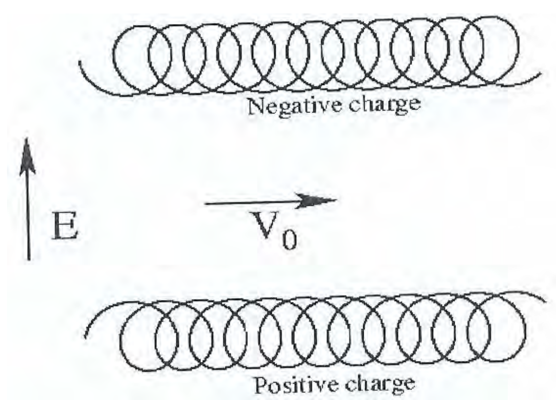

Fig 2.1: Illustration of cyclotron motion for a classical charged particle in the presence of uniform perpendicular magnetic and electric fields. The magnetic field is perpendicular to the plane of the figure, pointing upwards. The electric field E lies in the plane as shown.

The drift velocity is also represented. We have drowned trajectories for both possible signs of the particle electric charge.

The occurrence of crossed electric and magnetic fields ($\vec{E} \times \vec{B}$) induces a cycloidal path of the charged particle (see fig 2.1). The magnetic field causes the electron to move on orbits around the circle center, known as the guiding center. But the applied electric field makes the guiding center to move in a straight line, perpendicular to both the electric field and the magnetic field. It is called cyclotron frequency. The average position $z_0 = x_0 + iy_0$, of the particle over the time is arbitrary, and is called the guiding center. The radius $|r|$ of the trajectory is proportional to the speed of the particle times its mass. In a Fermi liquid, the speed of the electrons times their mass is frozen and equal to the Fermi momentum. The measurement of the cyclotron radius can thus be used to determine the Fermi momentum. Let us add to the magnetic field an electric field $E\hat{y}$ in the y direction. The equation of motion now becomes,

$$d^2 z / dt^2 = i\omega_c dz / dt - ie\vec{E} / m \qquad 2.4$$

This equation can be put into the form ($d^2 z / dt^2 = i\omega dz / dt$) if we use the variable $z' = z - Et / B$. It results from the fact that the electric field can be eliminated through a Galilean transformation to the frame moving at the speed E / B in the x- direction perpendicular to E with respect to the Laboratory frame As a result, in presence of an electric field E, the Guiding center move perpendicularly to the electric field at a speed:

$$v_0 = \frac{E \times B}{B^2} \qquad 2.5$$

2.2 THE CLASSICAL HALL EFFECT OF A TWO-DIMENSIONAL ELECTRON GAS

In this section, we review the classical description of electric magneto transport of conduction electrons. This is important in order to introduce the concepts necessary to determine properties of the system. Let us see the transport properties of the Electron gas in a uniform magnetic field. Classically, an electron in a static uniform magnetic field B will move in a circle due to the Lorentz force, $\vec{F}_L$, with the angular frequency ($\omega = 2\pi f$) is defined above to be

$$\omega_c = eB/mc \qquad 2.6$$

where e is the elementary charge and m^* is the effective mass of the electron. The electron experiences a force $-e\vec{E}$, due to the transverse electric field $\vec{E}$, and a force of the magnetic field, $-e\vec{v}\times\vec{B}$ due to $\vec{B}$,

$$\vec{F}_L = -e(\vec{E}+\vec{v}\times\vec{B}) = m^*\left(\frac{d}{dt}+\frac{1}{\tau}\right)\vec{v} \qquad 2.7$$

Let us assume that the particles move in a plane and that the magnetic field is orthogonal to this plane. We may write the relation between the current density $\vec{j}$ and the electric field $\vec{E}$ in the medium.

Where $\tau = m^*\sigma/(e^2 n_s)$ is the momentum relaxation time.

In a steady state. The drift velocity is

$$\vec{v}_d = -\frac{e\tau}{m^*}\cdot\left(\vec{E}+\vec{v}_d\times\vec{B}\right) \qquad 2.8$$

In the components form become:

$$v_x = -\frac{e\tau}{m[1+(\omega_c\tau)^2]}\left(E_x - \omega_c\tau E_y\right) \qquad 2.9$$

$$v_y = -\frac{e\tau}{m^*\left(1+(\omega_c\tau)^2\right)}\left(E_y + \omega_c\tau E_x\right) \qquad 2.10$$

The electric current density in a constant E-field with the help of the component eqns comparing with Ohms Law $\vec{j} = \hat{\sigma}\vec{E}$, we can determine the two-dimensional DC conductivity tensor

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix} \qquad 2.11$$

The longitudinal component will be

$$\sigma_l = \sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1+\left(\omega_c\tau\right)^2} \qquad 2.12$$

In addition, the Hall component becomes

$$\sigma_H = -\sigma_{xy} = \sigma_{yx} = \sigma \cdot \frac{\omega_c \tau}{1 + (\omega_c \tau)^2} \qquad 2.13$$

Where $\sigma_0 = n_s e^2 \tau / m^*$, is the DC conductivity for $B = 0$, $\sigma_l = \sigma_{xx} = \sigma_{yy} = \sigma_0$ and $\sigma_H = -\sigma_{xy} = \sigma_{yx} = 0$, the resistivity tensor $\hat{\rho}$, defined as the inverse the conductivity tensor $\hat{\sigma}$ and the resistivity tensor be written as:

$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \frac{1}{\sigma} = \frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2} \cdot \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} = \rho_0 \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \qquad 2.14$$

Where $\rho_0 = \frac{1}{\sigma_0} = \frac{m^*}{n_s e^2 \tau}$ is the DC resistivity and the longitudinal component is given, where. $\rho_{xx} \to 0$ Then $\sigma_{xx} \to 0$ whenever $\rho_{xy} \neq 0$.

$$\rho_l = \rho_{xx} = \rho_{yy} = \rho_0 = \frac{m^*}{n_s e^2 \tau} = \frac{1}{e \mu_{el} n_s} \qquad 2.15$$

$$\rho_0 = \frac{1}{\sigma_0} = \frac{m^*}{n_s e^2 \tau} = \frac{1}{e \mu_{el} n_s} \qquad 2.16$$

In addition, the Hall component becomes

$$\rho_H = -\rho_{xy} = \rho_{yx} = \rho_0 \omega_c \tau = \frac{B}{e \cdot n_s} \qquad 2.17$$

In addition, the mobility $\mu_{el} = e\tau / m^*$ is found. Thus, the longitudinal resistivity is independent of B while, in most bulk metals and semiconductors, the Hall resistance is linear in the magnetic field. Hence the transverse (Hall resistivity) is linearly proportional to the strength of the magnetic field B (see chap 1 fig1.2).The linear dependence of the $\rho_{xy}(B)$ is known as the classical Hall effect (CHE). By using the last Eqn (2.16) we can determine the electron density n_s and by inserting the obtained value of the electron density in eqn (2.14) the electron mobility μ_{el} is determined. Consider, A special sample geometry of QH device that may be used (corbino geometry)' which allows a direct measurements of the longitudinal conductivity σ_{xx} . Further Corbino samples were used in the first magneto transports [2].By applying a voltage between the inner (source-S) and outer (drain-D) contacts of the Corbino disc, radial current flows in between the contacts (See Fig 2.2). If a uniform and constant magnetic field is applied perpendicularly to the Corbino disc, a circular current will flow inside the disc. If the applied source-drain voltage V_{SD} exceeds a critical value the electrons leave the circular trajectory around the S contact, and a radial current between S and D contacts establishes. This corresponds to the breakdown of QHE in QH devices with Corbino geometry.

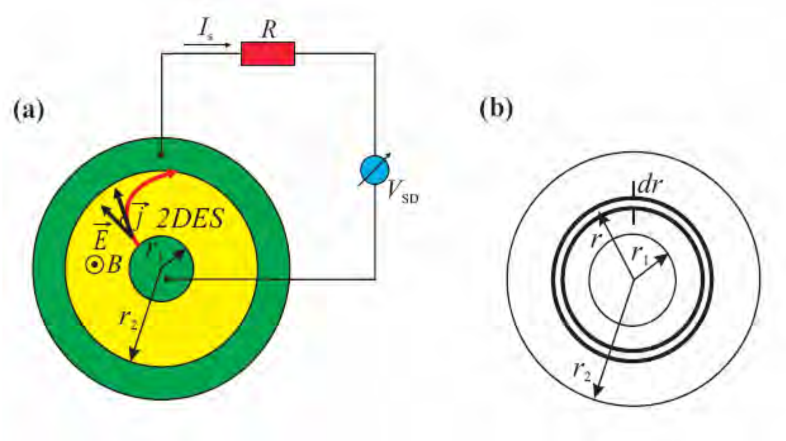

Figure 2.2: (a) A schematic view of the magneto transport measurement setup on a QH device with Corbino geometry (green - the contacts where the source-drain voltage is applied, red – resistor where the current I_S is measured, yellow - 2DES. (b) A scheme used to calculate the resistance of the length element $L = 2\pi r$ placed at distance r from the center.

Corbino geometry (r_1 and r_2 are the inner and outer radii) placed in perpendicular magnetic field (see fig 2.1[a]) is used to measure Hall Resistance. By applying a constant voltage between the source (S) and the drain (D) contacts .a constant electric field E is maintained The electrons drift velocity in direction $E \times B$. During the magneto transport measurements, the electrons move in cycloid paths, and they reach after certain time the outer (drain) contact, this time is shorter if the magnetic field is weaker; but at high magnetic field, electrons complete many cycloidal orbits before they reach the outer contact. Then current I_S is established between the source and drain contacts. It can be measured through a serial resistor with the sample, as a function of the magnetic field B.We need to calculate resistance of the QH device with Carbino geometry, consider a length $L = 2\pi r$ Of width dr ,at distance r from center of the device with carbine geometry for a three-dimensional (3D) system, conductor, the resistance is proportional to its electrical resistivity ρ and the length l of the conductor but inversely proportional to the area A of the cross-section, $R = L \cdot \rho / A$ However, in the case of a two dimensional system, the area A is replaced by the appropriate length element. $L = 2\pi \cdot r$ The length l is replaced by the width .Thus, the resistance can be calculated as

$$dR = \rho_{xx} \cdot \frac{dr}{L} = \rho_{xx} \cdot \frac{dr}{2\pi \cdot r} \Rightarrow R_{XX} = \frac{\rho_{xx}}{2\pi} \cdot \int \frac{dr}{r} = \frac{\rho_{xx}}{2\pi} \ln\left(\frac{r_2}{r_1}\right), \qquad 2.18$$

The resistance is given by $R_{xx} = V_{SD} / I_S$ where V_{SD} , I_S are source-drain voltage and the current respectively. Now we can determine the longitudinal electrical resistivity

$$\rho_{xx} = \frac{V_{SD}}{I_S} \cdot \frac{2\pi}{\ln\left(\frac{r_2}{r_1}\right)} \qquad 2.19$$

For a carbine for disc the longitudinal conductivity is measured,

$$\vec{j} = \sigma \cdot \vec{E} \Leftrightarrow \begin{pmatrix} j_a \\ j_r \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \cdot \begin{pmatrix} E_a \\ E_r \end{pmatrix} \qquad 2.20$$

The azimutal component is zero $E_a = 0$ and symmetry relations

$\sigma_{xx} = \sigma_{yy}$ And $\sigma_{xy} = -\sigma_{yx}$, Thus eqn (2.15) becomes

$$\begin{pmatrix} j_a \\ j_r \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ E_r \end{pmatrix} = \begin{pmatrix} \sigma_{xy} \cdot E_r \\ \sigma_{xx} \ E_r \end{pmatrix} \qquad 2.21$$

Since the radial current density j_r is

$$\vec{j}_r = \sigma_{xx} \cdot E_r = \frac{I_S}{2\pi \cdot r} \qquad 2.22$$

And the longitudinal conductivity σ_{xx} is calculated as:

$$\sigma_{xx} = \frac{I_S}{2\pi} \cdot \frac{1}{V_{SD}} \cdot \int_{r_1}^{r_2} \frac{dr}{r} = \frac{I_S}{V_{SD}} \cdot \frac{\ln\left(r_2 / r_1\right)}{2\pi}, \qquad 2.23$$

Where I_S is the measured sample current through the resistor, $V_{SD} = \int E_r dr$ is the applied Source-drain voltage and r_1, r_2 are the inner and the outer radii of contacts of the QH device with Corbino geometry. Whereas the longitudinal conductivity σ_{xx} is determined directly from the measured current I_S and applied source-drain voltage V_{SD}.The Hall conductivity σ_{xy} is not directly measured. However, it can be determined from the longitudinal ρ_{xx} and Hall ρ_{xy} resistivities measured on samples. Thus the Hall conductivity is determined:

$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad 2.24$$

Thus in the classical point of view we have analyzed that the Hall resistivity is linear with the magnetic field. Also we can also solve for the electron mobility thus using classical Hall Effect we can analyze the properties of the electron gas when it is in magnetic field that is not very strong.

CHAPTER THREE

3.0 TWO DIMENSIONAL ELECTRON GAS FORMATIONS

3.1 THE CAUSE FOR QUANTIZED HALL EFFECT.

The discovery of the Quantized Hall Effect (QHE) was the result of systematic measurements of the two-dimensional electron Gas on silicon field effect transistors-the most important device in microelectronics[l]. Such devices are important for applications. The pioneering work by Fowler, Fang, Howard and Stiles has shown that new quantum phenomena become visible if the electrons of a conductor are confined within a typical length of 10 nm. Their discoveries opened the field of two-dimensional electron Gas systems. It has been demonstrated that this field is important for the description of electrical properties of microelectronic devices. A two-dimensional electron gas is necessary for the observation of the Quantized Hall Effect, and the formation and properties of such a system will be discussed in this chapter. The confined electrons within a two-dimensional layer, have another quantization - the Landau quantization of the electron motion in a strong magnetic field - is essential for the interpretation of Quantized Hall Effect.

3.2 THE TWO DIMENSIONAL ELECTRON GAS

Electrons can be confined to the surface of liquid He or to the surface of some insulator. They can be kept there by an electric field, which pushes them against a highly impenetrable barrier. One method to create Two-Dimensional electron systems (2DES) is to confine them within a solid to the interface between a semiconductor and insulator or to the interface between two different semiconductors [2]. Silicon MOSFET (metal oxide semiconductor field-effect transistor), in which the 2DES is confined to the interface between silicon and silicon oxide; [see fig (3.1(a)]. Electrons reside at the silicon side of the interface; pushed against the highly importable, insulating silicon oxide glass by an electric field from a metal electrode (called the gate) it makes an ideal transistor. In a MOSFET, electrons can move along the (x-y) plane of interface but are bound to it in the perpendicular (z-axis) direction; due to Quantum Mechanics, the electrons cannot move in this direction at all. The electric field from the electrode pushes the carriers strongly against the glass and the electrons become captured in this direction that only a set of discrete states are Quantum Mechanically allowed in this dimension (see figure 3.1[c]). At low temperature, much lower than the energetic spacing between these orbits, and at sufficiently low density, all electrons reside in the lowest of these states. [3]

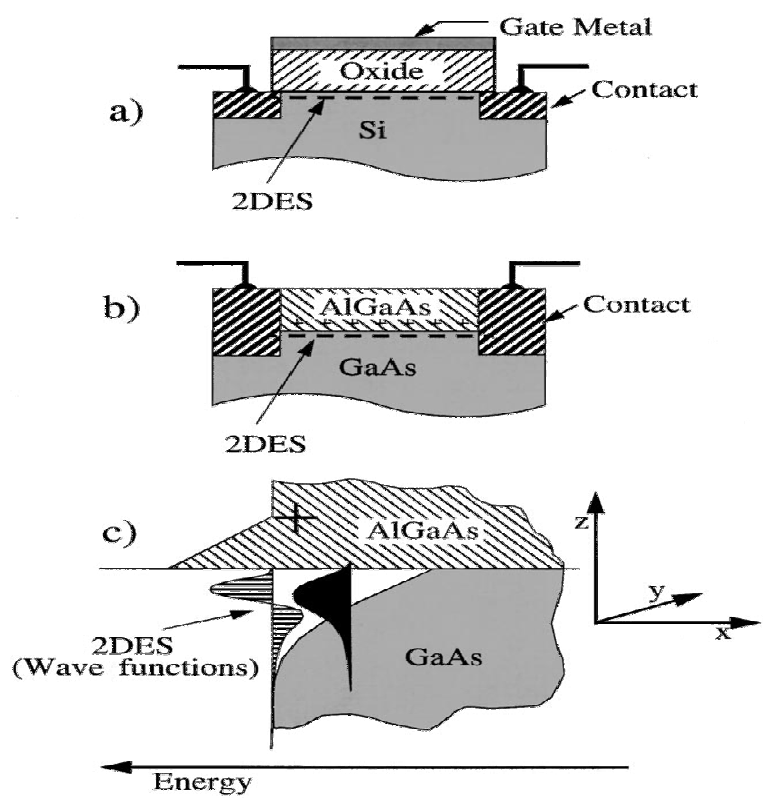

Fig 3.1: Schematic drawings of a silicon metal-oxide semiconductor field-effect transistor (MOSFET). (a) The two-dimensional electron system (2DES) resides at the interface between silicon and silicon oxide. Electrons are held against the oxide by the electric field from the gate metal; (b) Schematic drawings of a modulation-doped gallium arsenide / aluminum gallium arsenide (GaAs / AlGaAs) heterojunction. The 2DES resides at the interface between GaAs and AlGaAs. Electrons are held against the AlGaAs by the electric field from the charged silicon do pants (+) in the AlGaAs; (c) Energetic condition in the modulation-doped structure (very similar to the condition in the MOSFET). Energy increases to the left. Electrons are trapped in the Triangular

Shaped Quantum Well at the interface. They assume discrete energy states in the z direction (black and horizontally striped). At low temperatures and low electron concentration only the lowest (black) electron state is occupied. The electrons are totally confined in the z direction but can move in the x-y plane[3].

Electrons, bound to the interface between two different semiconductors, should make for an even "better" 2DES than in silicon MOSFET. Modulation-doped gallium arsenide/ aluminum- gallium arsenide (GaAs / AlGaAs) hetrostructures have provided such a superior system for research and for applications. For the MOSFET system the electrons are provided by the Si semiconductor while for the GaAs/AlGaAs hetrostructure, electrons are coming from the positively charged donors Si

3.2.1 MODULATION DOPING

Semiconductor materials are grown on top of each other. A first layer of GaAs is grown as substrate, followed by a layer of un doped AlGaAs on top of it as space layer. The samples during the experiments are (at liquid Helium temperature 4K and below), the undoped semiconductors are insulating and therefore the doping of the AlGaAS layer with Si atoms is compulsory. Hence we see the formation of a 2DES at the interface between GaAs undoped,but (p -type due to the growth conditions) and AlGaAs (n -type due to the Si atoms implanted).When the GaAs layer comes in contact with the AlGaAs the electrons flow across the GaAs/AlGaAs interface.

The Coulomb attraction by the donors left behind on the other side. It pulls the electrons towards the interface, creating a two-dimensional electron gas (2DEG) inside a triangular Quantum well. Because the electrons are specially separated from their donors, impurity scattering is reduced and the electron mobility is enhanced. An undoped spacer region is left adjacent to the interface, which further decreases the scattering. In [Fig 3.1(c)] we see that at the GaAs/AlGaAs interface, at the conduction band discontinuity, a nearly triangular potential well is formed. The conduction electrons are confined by this potential to a layer of a few nanometers

In its most common implementation, the 2DES in an MBE- grown GaAs/AlGaAs sandwich resides at the GaAs side of a single interference with AlGaAs; see Fig 3..1(b).

A several μm thick GaAs layer is grown onto a $1/2-mm$ thickGaAs substrate. The substrate provides a Pattern for the arriving atoms as well as mechanical support for silicon impurities are introduced into the AlGaAs material at a distance of about $0.1\mu m$ from the interface. Each silicon impurity has one more outer-shell electron than the gallium atom, which it replaces in the solid. It easily loses this additional electron, which wanders around the solid as a conduction electron. Seeking the energetically lowest state (see both figs 3..2), the electron projects over the energetic cliff and falls "down" into the GaAs material, only $0.1\mu m$ away(Fig3..2). In the highly pure GaAs layer such conduction electrons can move practically unimpeded by their parent silicon impurities, which remain in the AlGaAs layer, on the other side of the barrier (Fig 3..2). The attraction from all those positively charged (loss of one electron) stationary silicon ions pull the mobile electrons against the AlGaAs barrier of the interface [see Figs 3.1 (c) and]. Transitors from such modulation-doped material represent today's lowest-noise, highest-frequency transistors and are extensively used in mobile technology.

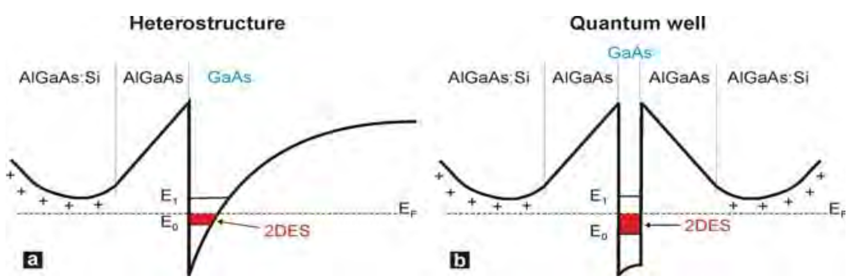

Figure 3.2:The conduction band of an AlGaAs/GaAs. (a) and quantum well (b) are schematically shown. A 2DES forms in the GaAs layer with quantized energies in z-direction. At low temperature ($K_B T << E_{sub\,band}$) only the lowest sub band is populated. The electrons, supplied from a Si-donorlayer,fill the 2DES up to E

3.2.2 CALCULATION OF ENERGY SUB BANDS AND DENSITY OF STATES (DOS)

One easy way to obtain a 2DES is to use two semiconductor materials with different energy gaps. Thus, by enclosing a layer of one semiconductor with a smaller gap between two layers of another semiconductor with a larger gap, a quantum well is established. The band discontinuity leads to a potential well inside the layer with the smaller energy gap. This potential well is characterized by a quantization of the electron motion perpendicular to the semiconductor interface.

The fundamental properties of the QHE are a consequence of the fact that the energy spectrum of the electronic system used for experiments is a discrete energy spectrum. Normally, the energy E of mobile electrons in a semiconductor is quasicontinuous and can be compared with the kinetic energy of free electrons with wave vector k but with an effective mass m*

$$E = \frac{\hbar^2}{2m}\left(k_{x^2} + k_{y^2} + k_{z^2}\right) \qquad 3.1$$

If the energy for the motion in one direction (usually z -direction) is fixed, one obtains a quasi-two-dimensional electron gas (2DEG), and a strong magnetic field perpendicular to the two-dimensional plane will lead-as a fully quantized energy spectrum which is necessary for the observation of the QHE.

At low temperature ($T < 4K$) and small carrier densities for the 2DEG (Fermi energy E_F relative to the lowest electric sub bands E_0 small compared with the sub band separation $E_1 - E_0$), only the lowest electric sub band is occupied with electrons (electric quantum limit), which leads to a strictly two dimensional electron gas with an energy spectrum

$$E = E_0 + \frac{\hbar^2 k_{x,y}^2}{2m} \qquad 3.2$$

Here $k_{x,y}$ is a wave vector within the two-dimensional plane (x, y). The electrons are trapped in the potential and a 2DES forms in the GaAs layer since they are only free to move in the x-y-plane while their energies are quantized in the z-direction (perpendicular to the interface). The energy spectrum is given by: If the width of this potential well is small compared to the de Broglie wavelength of the electrons, the energy of the carriers are grouped in so-called electric subbands E_i corresponding to quantized levels for the motion in z-direction,(See Fig 3.2) the direction normal to the surface. In lowest approximation, the electronic subbands can be estimated by calculating the energy eigenvalues of an electron in a triangular potential with an infinite barrier at the surface

($z=0$) and a constant electric field $\vec{E}$ for $z \geq 0$, which keeps the electrons close to the surface. The electrons are confined at the interface by the electrostatic potential, if the potential $\Phi(z)$ is approximated by a triangular potential

$$E = E_z^i + \hbar^2 k_x^2 / 2\mathrm{m} + \hbar^2 k_y{}^2 / 2\mathrm{m} \qquad 3.3$$

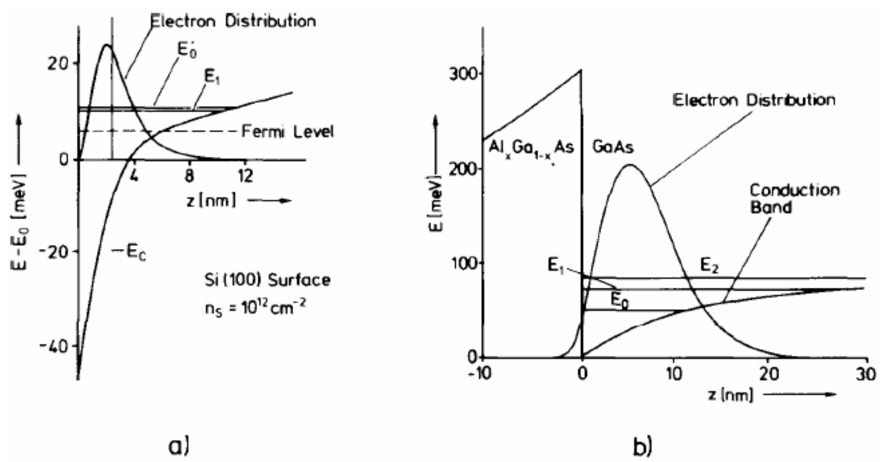

Fig 3.3: Calculations of electronic subbands and the electron distribution within the surface channel of a silicon MOSFET (a) and a GaAs-$Al_xGa_{1-x}As$ hetrostructure(b) [source: von K. Klitzing Lecture]

$$\Phi(z) = -z \cdot \vec{E} \qquad 3.4$$

Where $\vec{E}$ is electric field normal to the interface, due to the positive charges [36], which causes a drop in the electron potential toward the interface, then the energy of the electrons, is quantized in electric sub bands E_z^i corresponding to the quantized i levels for the motion in the z-direction. Experimentally, the separation between electric sub bands, which is of the order of 10meV, can be measured by analyzing the resonance absorption of electromagnetic waves with a polarization of the electric field perpendicular to the interface [5]. By assuming an electron in an (asymmetric) triangular potential with an infinite barrier $V(z) = \infty$ at the interface $z = 0$ (See Fig 3.3) and a linear potential $V(z) = eF_z$) at $z > 0$, the electric sub bands could be determined by solving the time-independent Schrödinger equation

$$\left[-\frac{\hbar^2\partial^2}{2\mathrm{m}\partial z^2}+V(z)\right]\Psi(z)=E\Psi(z) \qquad 3.5$$

The energy sub bands are the eigenvalues determined from Eqn.(3.4)

$$E_i^z = a_i \cdot \left[\frac{(e\hbar F)^2}{2\mathrm{m}}\right]^{\frac{1}{3}}, \qquad 3.6$$

Where $i = 0,1,...$ and a_i are zeros of Airy function Ai(z)

Then the eigen functions are given by

$$\Psi_i(z) = \frac{1}{N_i}\cdot Ai(\frac{eFz - E_i^z}{E_0^z}) \qquad 3.7$$

Where N_i ,the normalized constants, for

$$z_0 = \left[\frac{\hbar^2}{(2\mathrm{m}eF)}\right]^{\frac{1}{3}} \qquad 3.8$$

,and for

$$E_0^z = e\ E\ Z_0 \qquad 3.9$$

are the units of length and energy, respectively. It follows that by increasing i the energy spacing of two adjacent levels decreases. This is because the triangular well broadens by increasing the energy[37] The occupation of more than one electric sub band means that electrons have different position in the z -direction (perpendicular to the plane where the 2DES exists), we recall the time-independent Schrödinger equation ;

$$\left[-\frac{\hbar^2\nabla^2}{2\mathrm{m}}+V(\vec{r})\right]\Psi(\vec{r})=E\Psi(\vec{r}) \qquad 3.10$$

Where $V(r)$ is the electric potential energy and the position r = (x; y; z). in our Case, the potential energy depends only on z and the electrons are free to move in the x-y Plane. now we can write the above eqn:

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(z)\right]\Psi(x,y,z)=E\Psi(x,y,z) \qquad 3.11$$

In the 2DES the electrons are quasi-free to move in x-y plane but quantized in the z-direction and the potential energy depends only on z in turn; that the wave functions in the x-y plane; assumed to be plane waves.

$$\Psi(x,y,z) = \exp i(k_x x + k_y y)\cdot \Psi_i(z) \qquad 3.12$$

where k_x and k_y are the wave vectors in the plane of 2DES, $\Psi_i(z)$ are the eigenfunctions defined in (3.7. By inserting eqn.(3.11) into eq.(3.10) and after some algebra steps we obtain (z-dependent) time-independent Schrödinger equation .The initial three-dimensional time-independent Schrödinger eqn (3.10), from solving of eqn. (3.12) for the motion of electrons in x-y plane, gives the eigenvalues

$$E_i(k_x,k_y) = E_i^z + \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right) \qquad 3.13$$

Where $i = 0,1,2,..., E_i$ are the total energy of the 2D sub bands, k_y and k_y are the wave vectors in the plane of 2DES and $E_i^{\,z}$ are the electrical sub band levels. In eqn.(3.13), for $\vec{K} = 0$ ($\vec{K} = (k_x, k_y, 0)$) , total energy E_i is given by the electrical sub band levels $E_i^{\,z}$. Thus, E_i (for $\vec{k} = 0$) $= E_i^z$.

Since the magnetotransport measurements are performed at low temperatures ($T < 4K$) and the 2DES has usually thin electron density, the electrons occupy only the lowest electric subband E_0^z.This situation leads to the electric quantum limit, for $E_1^z - E_0^z - E_F >> k_B T$. The motion of electrons in the $\vec{K}$-space (in the $x-y$ plane) is described by the kinetic energy

$$E_k = \frac{\hbar^2}{2m}k^2 = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right) \qquad 3.14$$

The electrons in the lowest electric sub band may be considered as points inside a circle, the so-called the Fermi circle in the $\vec{k}$-space. We consider the 2DES having area $A = L_x L_y$ where L_x and L_y are the lengths of the 2DES in the $x-$ and $y-$directions, each electron occupies an $\Delta k_x \Delta k_y = 4\pi^2\left(L_x L_y\right)$ area element in the $\vec{k}$-space[32].The magnitude of the wave vector at the Fermi circle is the Fermi wave vector k_F and the energy at the edge of the Fermi circle is the Fermi energy .

$$E_F = \frac{\hbar^2}{2\mathrm{m}} k_F^2 \quad . \qquad 3.14$$

This is free electron energy. it occupies an area element $\Delta k_x \cdot \Delta k_y$ in the $\vec{k}$ -space and the area of the Fermi circle A_F is defined by πk_F^2, we can determine the number of electrons N_{el} existing in the Fermi circle , $N_{el} = \frac{A_F}{\Delta k_x \Delta k_y}$

We can calculate the electron density in 2DES by dividing the number of electrons N_{el} by the area A of the 2DES, $n_s = \frac{N_{el}}{A}$. By counting both allowed spin states. Thus,

$$k_F = \sqrt{2\pi \cdot n_s} \qquad 3.15$$

and then we have,

$$E_F = \frac{\pi\hbar^2}{m} \cdot n_S \qquad 3.16$$

As indicated in eqn.(.3.16), the Fermi energy is a measure of the electron density($E_F \propto n_S$). This is very important: Since the electron density does not vary with temperature. Another very important characteristic of the 2DES, is the density of states $D(E)$, which describes the distribution of energy in the system.

$$D(E) = \frac{\partial n_s}{\partial E} = \frac{m}{\pi\hbar^2} \qquad 3.17$$

The density of states (DOS) is independent of the energy E for the 2DES. it is a step function of $\frac{m}{\pi\hbar^2}$ step height and $E_{i+1}^z - E_i^z$ step width. Thus, each electrical sub band contributes with the same amount of electrons to the total DOS.

3.3 EVALUATION OF ELECTRON MOBILITY FOR 2DEG

By applying the electrical field, the electrons drift according to the Newton's law we assume that, due to the presence of impurities and dislocations, the electron are scattered. The effect of scattering on the 2DES can be modeled by introducing a frictional term in the Newton's equation of motion for the average velocity.

$$\vec{F} = -e\vec{E} = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\vec{v} \tag{3.20}$$

The velocity approaches steady state at large times. $\frac{d\vec{v}}{dt} = 0$ .Thus, it becomes

$$v_x = \frac{-e\tau}{m} \cdot E_x \tag{3.21}$$

$$v_y = -\frac{e\tau}{m} E_y \tag{3.22}$$

The electric current density in a constant electric field is

$$\vec{j} = -e \cdot n_s \cdot \vec{v} \tag{3.23}$$

The QHE are a consequence of the fact that the energy spectrum of the electronic system used for the experiments is a discrete energy spectrum.

Normally, the energy E of mobile electrons in a semiconductor is quasicontinuous and can be compared with the kinetic energy of free electrons with wave vector k but with an effective mass m* . The current density vector in its $x - y$ components:

With the help of Ohm's law, $\vec{j} = \sigma \cdot \vec{E}$ we can define the Drude electrical conductivity as:

$$\sigma = \frac{e^2 n_s \tau}{m} = e\mu_{el} n_s \tag{3.24}$$

With n_s, the electron density and $\mu_{el} = e\tau / m$ the electron mobility.

3.4 THE PROPERTIES OF 2DEG IN MAGNETIC FIELD

There has been much interest in the effects of a perpendicularly applied magnetic field on the 2D electron gas. Because of the confinement, there is no dispersion in the direction of the magnetic field so that a magnetic field applied perpendicular to the plane of the electron gas creates highly degenerate electron energy levels. There has been much interest in the effects of a perpendicularly applied magnetic field on the 2D electron gas. Because of the confinement, there is no dispersion in the direction of the magnetic field so that a magnetic field applied perpendicular to the plane of the electron gas creates highly degenerate electron energy levels. In a single particle picture and with no disorder in the 2d electron gas, the magnetic field essentially places electrons in identical harmonic oscillator potential wells of number equal to the number of magnetic flux quanta passing through the 2D system. The quantum energy levels, equally spaced in energy, in these harmonic oscillator potential wells are known as "Landau levels" (LLs) Depending on the magnetic field strength, these levels can have enormous degeneracy. In the single particle picture with no scattering of electrons, the addition of a magnetic field perpendicular to the plane of the 2D electron gas produces a series of Dirac delta functions in the Density of states(DOS) spectrum (See Fig 3.4 a) . Of course, the DOS spectrum does not involve true delta functions: instead, one expects that these levels should contain some broadening. The magnetic field produces Landau Quantization of motion parallel to the interface. The density of states D(E) consists of broadened δ function; minimal overlap is achieved if the magnetic field is sufficiently high.(See Fig 3.4b)

In the presence of a perpendicular magnetic field, the energy levels of a two-dimensional electron collapse, as a result of Landau Quantization of its cyclotron orbits, into discrete Landau levels separated by the cyclotron energy Quantum. Scattering broadens the Landau levels and gives rise to two dimensional (2D) magneto-transport described by the Ando-Uemura Theory.

In a single particle picture and with no disorder in the two dimensional (2D) electron gases, the magnetic field essentially places electrons in identical harmonic oscillator potential wells of number equal to the number of magnetic flux quanta passing through the 2D system. The quantum energy levels, equally spaced in energy, in these harmonic oscillator potential wells are known as "Landau levels". Depending on the magnetic field strength, these levels can have enormous degeneracy. In the single particle picture with no scattering of electrons, the addition of a magnetic field perpendicular to the plane of the 2D electron gas produces a series of Dirac delta functions in the DOS spectrum (See Fig 3.4).

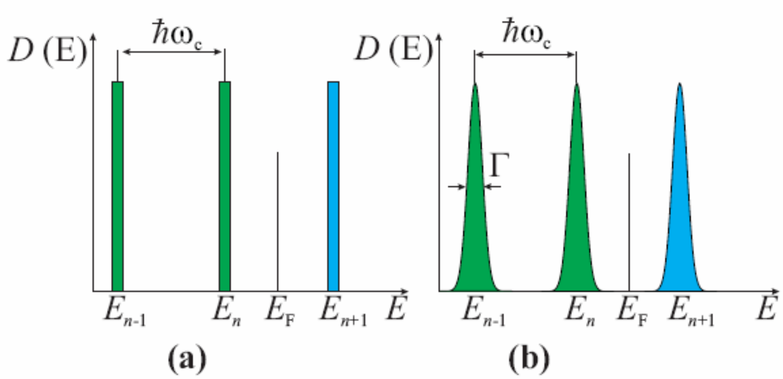

Figure 3.4: A schematic view of (a) DOS of the 2DES with δ - Functions Landau levels for the ideal sample (no scattering). (b) DOS with the broad Landau Levels for the real sample (with scattering). Green - denotes the filled Landau level and blue - Denotes the empty Landau level.

In order to describe the 2DEG properties in a magnetic field B, we need the time-independent Schrödinger equation.

$$\left[\frac{\hbar^2}{2\mathrm{m}}\left(-i\nabla+\frac{e}{\hbar}\vec{A}\right)^2+g\mu_B\vec{B}\vec{S}+V(z)\right]\Psi(\vec{r})=E\Psi(\vec{r}) \qquad 3.25$$

Where we assumed the vector potential in the Landau gauge, $\vec{A}=(0,Bx,0),g$ is the effective gyro magnetic factor, $\mu_B=9.274\times10^{-24}\,JT^{-1}$ is the Bohr magneton and $\vec{S}$ is the spin vector. As a result of the orbital quantization, the energy levels of the 2DEG can be written in the form

$$E_n=\left(n+\frac{1}{2}\right)\cdot\hbar\omega_c+S\cdot g\mu_B\cdot B \qquad 3.26$$

Where $n=0,1,2,\ldots$ is the principal quantum number, $\omega_c=eB/m$ is the cyclotron angular frequency as discussed before, the Zeeman energy can be defined as

$$\Delta E_S=g\,\mu_B B \qquad 3.27$$

Depends linearly on the magnetic field B . At small values of B the spin splitting energy is small and the DOS contains both spin states. By increasing B the spin splitting also increases and therefore the spin states start to separate.

When we solve equation(3.25)it gives:

$$\Psi_{k_y,n}(x,y) \propto H_n\left(\frac{x + k_y l_B{}^2}{l_B}\right)\exp\left[-\frac{\left(x + k_y l_B{}^2\right)^2}{2l_B{}^2}\right]\exp(ik_y y) \qquad 3.28$$

Where H_n are the Hermite polynomials, $l_B = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length. As mentioned the DOS in magnetic fields undergoes the transition from the step-like spectrum into discrete spectrum(series of δ-function).

Bohr –Sommer field model, the electrons can be considered as points inside a circle for each value of n and corresponding to LL. Consider two adjacent LL'sand therefore two adjacent circles (corresponding to $n = i$ and $n = i + 1$).The area between two adjacent circles in k -space is given by

$$A_{i,i+1} = \pi\Delta\left(k^2\right) = \frac{2\pi m^*\omega_c}{\hbar} = \frac{2\pi e\vec{B}}{\hbar} \qquad 3.29$$

Where we solve for $\Delta k^2 = 2m^*\Delta E/\hbar^2$, and $\Delta E = \hbar\omega_c$

is free electron energy and $\omega_c = eB/m^*$ is the cyclotron angular frequency

An electron occupies an area element $\Delta k_x \cdot \Delta k_y$ in the k-space.

The number of electrons N_L existing in one circle LL is

$$N_L = \frac{A_{i,i+1}}{\Delta k_x \cdot \Delta k_y} = \frac{L_x L_y eB}{2\pi\hbar} = \frac{L_x L_y eB}{h} \qquad 3.30$$

We can calculate the electron density in each Landau level (LL), by dividing the number of electrons N_L per Landau level by the area $A = L_x L_y$ of the 2DES.

$$n_L = \frac{N_L}{A} = \frac{eB}{h} = \frac{m\omega_c}{2\pi\hbar} = \frac{1}{2}\cdot\frac{m}{\pi\hbar^2}\cdot\hbar\omega_c \qquad 3.31$$

When we calculate the number v of filling factor the electron density in the 2DES to the electron density in each (LL). Thus each spin-splitted Landau level. It contains half of the number of states as the 2DES (Without magnetic field) has, multiplied by $\hbar\omega_c$, the energy difference between two adjacent LL's. .It is the number of electrons per cell

$$\nu = \frac{n_s}{n_{LL}} = \frac{n_s h}{eB} = 2\pi \cdot l_B^2 \cdot n_s \qquad 3.32$$

Due to the Pauli principle, a cell can be occupied by one electron only per energy level. Therefore, each time the filling facror reaches an integer, an energy level gets filled and the next electron must be added to the next energy level. By keeping constant the electron density, and by changing the magnetic field; the filling factor is varied, equation (2.32) shows an inverse proportionality between filling factor and magnetic field. In this chapter we have analyzed the two dimensional Electron Gas, their formation, the property that the 2DEG gas will display within magnetic field $B \neq 0$ and with $B = 0$ Thus in this chapter we have seen, the, DOS of 2DES in magnetic field behaves as a series of delta-functions. This is true only for an ideal sample, where electrons are not scattered by impurities, defects or/and phonons. But, in a real sample, due to the impurities or defects, the electrons are scattered the energy of scattered electrons fluctuates and leads to a broadening of the *D(E)* . Quantum mechanically, there exists only a discrete set of allowed orbits at a discrete set of energies. These so-called Landau Levels represent an equally spaced ladder of states having energies $E_i = (i-1/2)heB/(2\pi m), where, (i = 1,2,3,...)$ This is, Proportional to the magnetic field B. Here m is the mass and h is the Plank's. Electrons can only reside at these energies, but not in the large energy gaps in between. The existence of the gaps is crucial for the occurrence of the IQHE. In 2DES, the number of electrons fitting into each Landau level is exactly quantized. It reflects the number n of orbits that can be packed per Landau level into each cm^2 of the specimen. This turns out to be $n_\phi = eB/h$. This capacity per Landau level is called its degeneracy, apart from natural constants, depends only on the magnetic field B. None of the materials parameters enters in any way. It is therefore a universal measure, independent of the material employed.

CHAPTER FOUR

4 THE QUANTIZED HALL EFFECT

For more than two and a half decades, experiments on two dimensional electron systems in strong magnetic fields, at low temperatures have given rise to a series of surprising phenomena, known collectively as the Quantum Hall Effects.

One usually plots the resistivities along the direction of the current (ρ_{xx}) and the direction perpendicular to it (ρ_{xy})as a function of the field B. Very schematically, for certain range of the field (ρ_{xx}) is nearly equal to zero, and for other ranges it develops a bump. On the average ρ_{xy} grows linearly with the field, but in the regions where ρ_{xx} is equal to zero, ρ_{xy} presents a flat plateau which is a fraction times ${h}/{e^2}$ to an extraordinary accuracy. This is the Quantized Hall Effect, which has led two Nobel prizes, one in 1985 to Von Klitzing for the discovery of the integer Hall Effect, and the other in 1992 to Laughlin, Stormer and Tsui for the fractional Hall Effect.

The basic experimental observation is best recast using the conductivities σ_{xx} and σ_{xy} which gives components of the inverse of resistivity tensor that is $\left[\sigma_{xx} = \frac{\rho_{xx}}{\sqrt{\rho_{xx}^2 + \rho_{xy}^2}}, \sigma_{xy} = \frac{\rho_{xy}}{\sqrt{\rho_{xx}^2 + \rho_{xy}^2}}\right]$. The Quantized Hall Regime corresponds to a nearly vanishing dissipation $\sigma_{xx} \to 0$, accompanied by the Quantization of the Hall conductivity $\sigma_{xy} = v\frac{e^2}{h}$ In the integer Hall Effect case υ is an integer with precision of about 10^{-10}. In the fractional case, υ is a fraction, which reveals the bizarre properties of many electron physics. The fractions are universal and independent of the type of semiconductor material, purity of the sample and so forth. The effect occurs when the electrons are at a particular density in the fraction υ as if the electrons locked their separation at particular values. Changing the electron density by a small amount does not destroy the effect but changing it by a large amount does, this is the origin of the plateaus.

In this minute chapter, we shall see the basic ideas, methods and tools that has been used in different outers to understand these phenomena. Finally, we give some hints of Quasi Particles resulted from the attachment of magnetic flux to electron in vortex which reduces the Coulomb electron-electron interactions, to explain the occurrence of fractional Hall Effect

4.1 MAGNETO TRANSPORT IN THE QUANTUM REGIME

The linear dependence of the Hall resistivity ρ_{xy} to the strength of the magnetic field B applies only at low magnetic fields where the number of filled Landau Lebels(LL's) is larger. However, by increasing the magnetic field and reducing the number of filled LL's, the resistivity shows a different behavior. Figure 4.1 shows a typical magneto transport curve taken for a QH device with Hall bar geometry (Fig 2.2). For the specified sample, at low magnetic fields, up to 0.4T, the 2DES behaves classically (See Fig 4.1). The Hall resistivity shows a linear increase with the magnetic field ($\rho_{xy} \propto B$) (eqn 2.14) and the longitudinal resitivity ρ_{xx} almost remains constant with slight decrease with the magnetic field (See eqn 2.12).

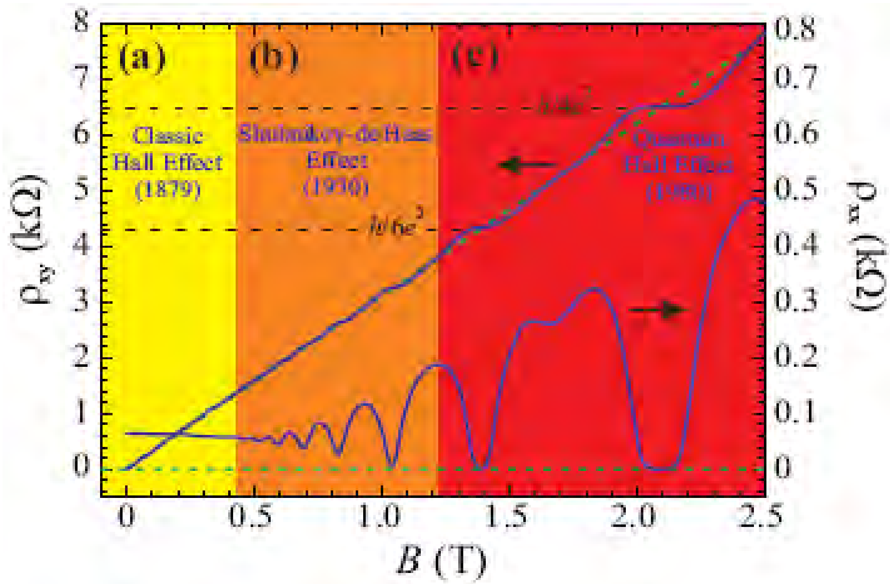

Figure 4.1: A typical magneto transport curve taken at a QH device with Hall bar geometry curve taken at a QH device with Hall bar geometry ($n_s \approx 2\times10^{11} cm^{-2}$). This curve shows three regimes of the magneto transport at the 2DES with increasing B: (a) Classical Hall Effect, (b) Shunpike-de Haas Effect (c) Quantum Hall Effect and (c) Quantum Hall Effect (QHE)[36]

At low fields ($B < 0.4T$), ρ_{xy} increases linearly with B while ρ_{xx} remains constant, as expected from the classical magneto transport theory of a 2DES. However at higher fields, a series of plateaus in the Hall resistivity,accompanied by a vanishing longitudinal resistivity becomes the most striking features in the curve. That is by increasing the strength of the magnetic field B further, the 2DES leaves the CHE regime and enters a new regime called the Shubnikov-de Haas regime. In this regime, the Hall Resistivity starts to deviate from the linear behavior, and the longitudinal resistivity oscillates strongly with B The amplitude of these oscillations(of the longitudinal resistivity) increases with B. At high magnetic field, for the 2DES, shows very remarkable behavior, the minima of the longitudinal resistivity oscillations tend toward zero. Around the B values where the longitudinal resistivity ρ_{xx} ,tends to zero, the Hall resistivity ρ_{xy}, becomes independent of B and exhibits plateaus, which is called Quantum Hall regime.

4.1.1 DE HAAS (SDH)EFFECT

Fig 4.2. Shows that oscillating chemical potential at zero and finite temperatures as a function of the magnetic field B. By increasing the strength of the magnetic field B, the LL's increase their energies and the chemical potential μ_{ch} starts to oscillate since it has to follow the occupation of the Landau levels(LL's), until at integer filling factor $v = i$ (at $B_{v=i} = hn_s / ev$, the LL gets depopulated and the chemical potential jumps to the next lower (LL). With the spin-splitting included, at even filling factors $v = 2\text{i}, (i - \text{int}eger)$, the magnitude of the jump of the chemical potential is $\hbar\omega - g\mu_B B$;at odd filling factor, $v = 2\text{i} + 1, (i - \text{int}eger)$ the magnitude of the jump of the chemical potential is $-g\mu_B B$ .At finite temperature T the oscillations of the chemical potential die out at low magnetic fields, even if collision broadening of the LL's is not considered. The oscillatory behavior of μ_{el} is reflected in the oscillatory behavior of the longitudinal resistivity.The oscillations of longitudinal resistivity are known as the Shubnikove-de Haas oscillations Minima of the SdH oscillations occur at magnetic field $B_{v=i} = hn_s / ev$

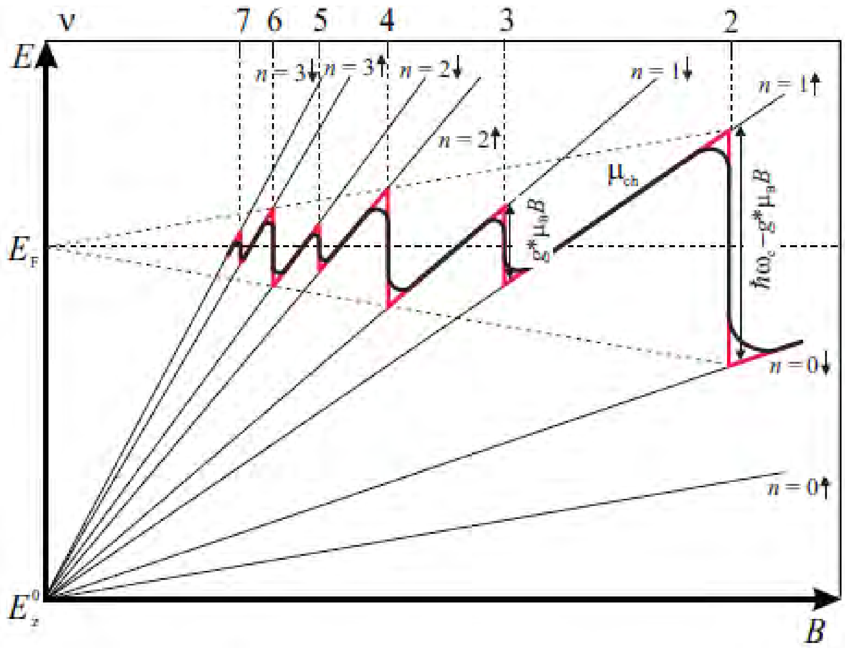

figure 4.2: The Landau fan with Landau energies E_n for $n = 0,1,2,3$ for both the spin-up and the spin –down states and the oscillating chemical potential μ_{el} as a function of magnetic field B .(black line of μ_{el} at $T = 0$,red line of μ_{el} at $T \neq 0$ [36]).

This is because for integer filling factor, the highest LL is completely filled and therefore the chemical potential is in between the highest occupied and lowest unoccupied LL's. From the periodicity of SdH oscillations, from magnetotrnasport measurement, the electron density n_s can be determined by plotting the longitudinal conductivity or resistivity as a function of inverse magnetic field with the help of equation (3.32).

$$B_\upsilon = \frac{h \cdot n_s}{e \cdot v} \Leftrightarrow \frac{1}{B} = \frac{e \cdot v}{h \cdot n_s} \Rightarrow \Delta\left(\frac{1}{B}\right) = \frac{2e}{hn_s}, \qquad 4.1$$

4.1.2 THE INTEGER QUANTUM HALL EFFECT (IQHE)

4.1.2.1 GALILEAN INVARIANT SYSTEMS

For a two dimensional Galilean invariant system, in the absence of impurities or boundaries a full Quantum mechanical treatment yields the same resistivity tensor as for the pure classical system, namely $\rho_{xx} = 0$ and $\rho_{xy} = B/(ne)$, or equivalently, $\sigma_{xx} = 0$

and $\sigma_{xy} = ne/B$.To check this, Let us consider the Hamiltonian, for a system of N interacting electrons in the presence of uniform time independent perpendicular magnetic $(\vec{B})$ and Electric fields $(\vec{E})$:

$$H = \frac{1}{2m}\sum_{j=1}^{N}((\vec{p}_j + e\vec{A}(\vec{r}_j))^2 - eV(\vec{r}_j)) + \frac{1}{2}\sum_{i\neq j}U(r_i - r_j) \qquad 4.2$$

Where, $\vec{B} = \vec{\nabla}\times\vec{A} = \nabla V$, and U is the pair interaction potential. This classical case let us now see the transformation on the N-particle wave function $\Psi(r_1, r_2, ..., r_N, t)$:

$$\Psi(r_1, r_2, ..., r_N, t) = \exp\left(\frac{i}{\hbar}\sum_{j=1}^{N}\theta(r_j, t)\right)\Psi'(r_1', r_2', ..., r_N', t) \qquad 4.3$$

Where r_j denotes the position of particle j in the Laboratory frame and r_j' its position in the moving frame, with constant v_0 given by Eqn. 2.5.

Therefore we have the relation: $r_j' = r_j - v_0 t$. It is possible to choose the phase $\theta(r,t)$ does depend on the choice of gauge. For instance, in the radial gauge $\vec{A} = \frac{1}{2}\vec{B}\times\vec{r}$, we have

$$\theta(r,t) = m\vec{v}_0\vec{r} - \frac{1}{2}\left(mv_0{}^2 + e\vec{E}\vec{r}\right)t \qquad 4.4$$

The phase- factor does alter the classical compos ion rule for currents, and we get;

$$\left\langle\Psi\middle|\vec{J}(\vec{r})\middle|\Psi\right\rangle = -nev_0 + \left\langle\Psi'\middle|\vec{J}(r')\middle|\Psi'\right\rangle \qquad 4.5$$

Now since there is no driving electric field in the moving frame, $\left\langle\Psi'\middle|J(r')\middle|\Psi'\right\rangle = 0$ so $\left\langle\Psi\middle|J(r)\middle|\Psi\right\rangle = -nev_0$ which is exactly the classical result. The natural way to measure, the electronic density for a two-dimensional Quantum system in a magnetic field is the filling factor $v = \frac{nh}{eB}$. Therefore, we end up with

$$\sigma_{xy} = v\frac{e^2}{h} \qquad 4.6$$

Although this expression does involve Plank’s constant, it is important to note that it is identical to the classical prediction for a uniform fluid of electrons of area density n once . This prediction for a Galilean invariant system coincides with the experimental result for the Hall conductivity when the filling factor v is an integer. The existence of Quantized plateaus of the form $\sigma_H = n\frac{e^2}{h}$, with n integer clearly indicates the breakdown of Galilean invariance in real samples. Since there is always a random electrostatic potential induced by the impurities, which are quantized to generate charge carriers at the interface between two semi-conductors. Despite this random potential (without which there would be no observable Hall Quantization), the measured plateau values are universal with a very high accuracy.

4.1.2.2 DISCRETE ENERGY SPECTRUM OF A 2DEG IN STRONG MAGNETIC FIELDS

Magnetic field B_z, normal to the interface causes the electrons in the two-dimensional layer to move in cyclotron orbits parallel to the surface. As a result orbital quantization, the energy levels of 2DEG can be written in the form:

$$E_n = E_0 + (n+1/2)\hbar\omega_c + s \cdot g \cdot \mu_B \cdot B \qquad 4.7$$

$n = 0,1,2,\ldots$

With cyclotron energy $\hbar\omega_c = \hbar eB/m$, the spin quantum numbers $s = \pm\frac{1}{2}$ the Lannde factor g and the Bohr magneton μ_B .

The wave function of a 2DEG in a strong magnetic field can be put in the form where the y-coordinate y_0, the center of the cyclotron orbit [5]

$$\psi = e^{ikx}\Phi_n\left(y - y_0\right) \qquad 4.8$$

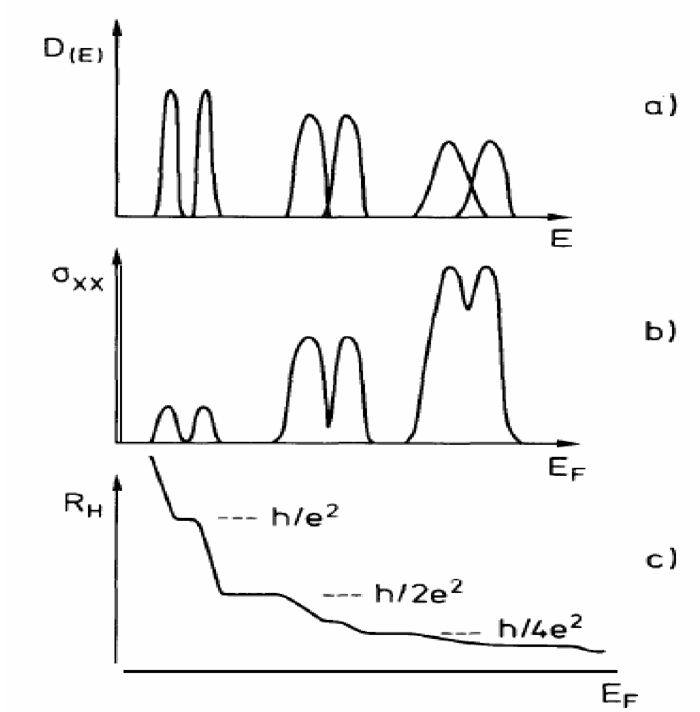

Fig 4.3: Sketch for energy dependence of the density of states(a), conductivity σ_{xx} (b), Hall Resistivity ρ_H (c) at a fixed magnetic field.

Where Φ_n is the solution of the harmonic oscillator equation

$$\frac{1}{2\mathrm{m}}\left[p_{y^2} + (eB)^2 y^2\right]\Phi_n = (n+1/2)\hbar\omega_c\Phi_n \qquad 4.9$$

And y_0 is related to k by

$$y_0 = \hbar k / e\vec{B} \qquad 4.10$$

The degeneracy factor for each Landau level, given by the number of center of coordinates y_0. For given device with dimension $L_x \cdot L_y$,

$$\Delta y_0 = \frac{\hbar}{eB}\Delta k = \frac{\hbar}{eB}\frac{2\pi}{L_x} = \frac{h}{eBL_x} \qquad 4.11$$

Degeneracy factor, $N_0 = L_y / \Delta y_0$ also, $N_0 = L_x L_y eB / h$ is the number of flux quanta within the sample. The degeneracy factor per unit area:

$$N = \frac{N_0}{L_x L_y} = \frac{eB}{h} \qquad 4.12$$

Degeneracy factor for each Landau level is independent of the parameter of the conductor. The commutator for the center coordinates of cyclotron orbit $[x_0, y_0] = i\hbar / eB$ equivalent, that each state occupies in real space area $A = h / eB$, to the area of a flux quantum. The expression for Hall voltage V_H of a 2DEG with surface charge density n_s is

$$V_H = \frac{B}{n_s \cdot e} \cdot I \qquad 4.13$$

where I , the current through the sample, Hall Resistance $R_H = V_H / I$, if i energy levels are fully occupied $(n_s = iN)$, leads to the expression for the quantized Hall resistance

$$R_H = \frac{B}{iNe} = \frac{h}{ie^2} \qquad 4.14$$

$i = 1,2,3$. Quantized Hall resistance is expected if the carrier density n_s and the magnetic field B are adjusted in such a way that the filling factor i of energy levels:

$$i = \frac{n_s}{eB / h} \qquad 4.15$$

is integer. Under this condition the conductivity σ_{xx} (current flow in the direction of the electric field) becomes zero, since electrons are moving like free particles exclusively perpendicular to the electric field and no diffusion (originating from scattering) in the direction of the electric field is possible. Born approximation [9], the discrete energy spectrum broadens as shown in Fig 4.4 (a). This theory predicts that the conductivity σ_{xx} is mainly proportional to the square of the density of states at the Fermi energy E_F, which leads to a vanishing conductivity σ_{xx} in the Quantum Hall regime and quantized plateaus in the Hall Resistance R_H (Fig 4.4 c).Hall Effect on (one-electron picture) of two-dimensional system in a strong magnetic field leads to the correct value for the Quantized Hall resistance (Eqn. 4.9) at integer filling factors of the Landau levels. The value $\frac{h}{ie^2}$ of Quantized Hall resistance is predicted if the conductivity σ_{xx} is zero. In experiment σ_{xx} becomes immeasurably small at high magnetic fields and low temperatures. The fact that the value of the quantized Hall resistance seems exactly correct for $\sigma_{xx} = 0$ has led to the conclusion that microscopic details of the device is not necessary for calculation of the quantized value.

4.1.2.3 THE FLUX QUANTIZATION

As a result Laughlin [10] tried to deduce the result from gauge invariances. See fig 4.4; a ribbon of two-dimensional system is bent into a loop and pierced everywhere by a magnetic field B normal its surface. A voltage drop V_H is applied between the two edges of the ring. Under the condition of vanishing conductivity $\sigma_{xx} = 0$,energy is conserved and Faraday's law of induction in a form, the current I in the loop is the adiabatic derivative of the total energy E of the system with respect to the magnetic flux ϕ threading the loop

$$I = \frac{\partial E}{\partial \phi} \qquad 4.16$$

If the flux is varied by a flux Quantum $\Delta\phi = hc/e$ (Fig 4.4), the wave function enclosing the flux must change with phase factor 2π with a translation of a state, wave vector k into $k + \frac{2\pi}{L}$ where L is circumference of the ring. The total change in energy corresponds to a transport of states from one edge the other, to transfer one state per Landau level from the left side of the sample to the right, i.e. to transfer one electron across the sample per occupied Landau level. If the potential difference between the two sides is V_H we have

$$\Delta E = i \cdot e \cdot V_H \qquad 4.17$$

The integer i corresponds to the number of filled Landau levels

From Eq.(4.12) the relation between the Hall current and the Hall voltage can be solved

$$I = i \cdot e \cdot V_H \,/\, \Delta\varphi = i \cdot e \cdot V_H \cdot \frac{e}{hc} \qquad 4.18$$

This leads to quantum Hall Resistance:

$$R_H = \frac{h}{ie^2} \quad 4.19$$

Imagine a sequence of flux values to solve the problem

$$H_\phi \left| \Psi_\phi \right\rangle = E_\phi \left| \Psi_\phi \right\rangle \quad 4..20$$

For each one, so that the many-body ground state $\left| \Psi_\phi \right\rangle$ have energy value . By Hellman-Feynman theorem we have

$$\left\langle \Psi_\phi \right| \frac{\partial H_\phi}{\partial \phi} \left| \Psi_\phi \right\rangle = \frac{\partial}{\partial \phi} \left\langle \Psi_\phi \right| H_\phi \left| \Psi_\phi \right\rangle = \frac{\partial E_\phi}{\partial \phi} \quad 4.21$$

The total current at any given value of ϕ is just adiabatic derivative of the total energy with respect to ϕ. For slowly changing ϕ creates an electromotive force around the loop, which does work on the system if current is flowing. The effect is just Faraday's law of induction. The adiabatic derivative may replaced by a differential:

$$I = \frac{c\Delta E}{\Delta \phi} \quad 4.22$$

Where the denominator is the flux quantum hc/e.The energy can have increased only through repopulation of the original states. The orbitals in the presence of nonzero flux are

$$\psi_{k,n}(x,y) = \frac{1}{\sqrt{2^n n! \sqrt{\pi} L_x}} e^{ikx} e^{(y+y_0-k-\alpha)^2/2} \left(\frac{\partial}{\partial y} \right)^n e^{-(y+y_0-k-\alpha)^2} \quad 4.23$$

Where $\alpha = el\phi_0 / \hbar c L_x$. As ϕ advanced from 0 to $\Delta\phi$ they simply slide over like a shift register, the net result being to transfer one state per Landau level from the left side of the sample to the right, i.e. to transfer one electron across the sample per occupied Landau level. If the potential difference between the two sides is V, we have

$$I = c\frac{neV}{\Delta \phi} = n\frac{e^2}{h} \quad 4.24$$

In this picture the main reason for the Hall quantization is the flux quantization h/e and the quantization of charge into elementary charges e. In analogy, the fractional quantum Hall Effect, is interpreted on the basis of elementary excitations of Quasi-particles with a charge $e^* = \frac{e}{3}, \frac{e}{5}, \frac{e}{5} etc$.

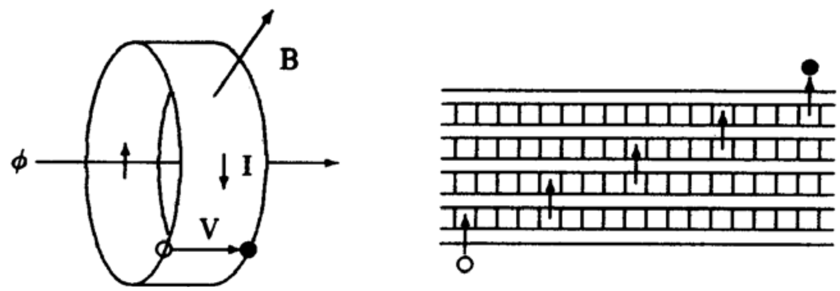

Fig 4.4: Model of a two-dimensional metallic loop, for the derivation of quantum Hall resistance. As magnetic flux $\Delta\phi = hc/e$ is adiabatically forced through the loop, one electron per Landau level is transferred from one edge to the other. In the absence of disorder the transformation is accomplished by a mechanical shift of the 1-body wave function that evolves each wave function in to its neighbor. When small amounts of disorder are present the number of electrons transferred must be exactly the same, but the mechanism of transfer is not, for the wave functions are violently distorted by even the smallest perturbation.[4]

If the state of matter in question does map to a non-interacting prototype then a flux-winding experiment either dissipates or pumps an integral number of electrons across the sample. When the latter occurs, the Hall conductance is accurately quantized. It measures the charge of particle being pumped, in this case the charge of electron.

The simple theory predicts that the ratio between the carrier density and the magnetic field has to be adjusted with very high precision in order to get exactly integer filling factor Eq 4.10 and therefore quantized values for the Hall resistance , fortunately, the Hall quantization is observed not only at special magnetic field values but also but in a wide range , so that an accurate fixing of magnetic field or carrier density for high precision measurements of the quantized resistance value is not necessary. The Hall plateau can be explained if localized states in the tails of the Landau levels are assumed. Theoretical investigations have shown that a mobility edge exists in the tails of Landau levels separating extended states from localized states The Mobility edges are located close to the center of Landau level for long -range potential fluctuations. [17]. (See Fig 4.6)

4.1.2.4 TRIVIALMODEL HAMILTONIAN FORMULATION

Consider that the situation in (Fig 4.5) of a translational invariant strip of charge density ρ in a normal magnetic field B. Flowing current along the strip is the same as Lorentz force by speed v , which gives current $j = v\rho$ an electric field $E = vB/c$, a Hall conductance :

$$\sigma_{xy} = \frac{j}{E} = \frac{\rho \cdot c}{B} \ , \tag{4.25}$$

In a real field-effect transistor or hetrostructure ρ is fixed by doping and the gate voltage and is not accurately quantized. Often it is the variable against which the Hall plateaus are plotted. Thus this result is inconsistent with all Quantum Hall experiments. Sample imperfection is implicated in the formation of plateaus because it is the only agent in the problem, other than sample ends, capable of destroying translational invariance.

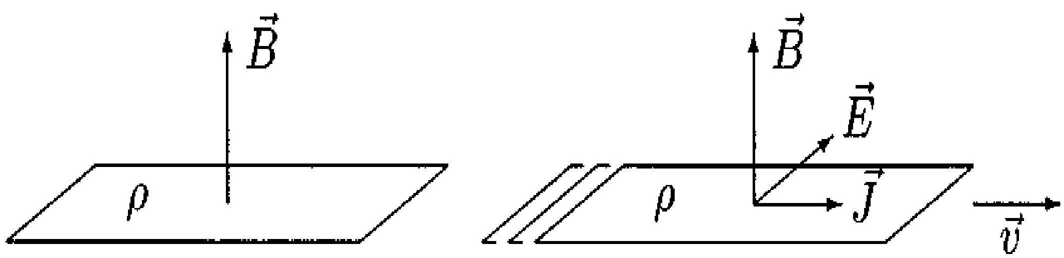

Fig 4.5: When a disordered 2D electron gas is placed in magnetic field it cane Loretz boosted to generate a current and a transverse electric field. The ratio of these equation gives, which is inconsistent with the quantum Hall experiment This shows that disorder, is essential for the platea formation. $\sigma_{xy} = J/E = \rho \cdot c/B$

When a non-interacting metal is subjected to a sufficiently large random potential. All the eigenstate of 1-electron Hamiltonian below certain energy have finite spatial extent, so occupy with electrons. Real metals, in which the electrons interact, have similar metal-insulator transition. The trivial model The Hamiltonian

$$H = \sum_{j}^{N} \left\{ \frac{1}{2m} \left[\frac{\hbar}{i} \vec{\nabla}_j - \frac{e}{c} A(\vec{r}_j) \right]^2 + Eey_j \right\}$$

4.26

where the vector potential; it is convenient to choose the Landau guage

$$A(\vec{r}) = \vec{B}y\hat{x} \quad , \qquad 4.25$$

the classical cyclotron orbital radius.

$$l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar c}{eB}}\,(\omega_c = \frac{eB}{mc})\,, \qquad 427$$

One finds the wave functions

$$\psi_{k,n}(x,y) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}\,L_x}} e^{ikx} e^{\left(y+y_0-k\right)^2/2} (\frac{\partial}{\partial y})^n e^{-\left(y+y_0-k\right)^2} \qquad 428$$

$y_0 = eEl/\hbar\omega_c$, the energy of which are

$$E_{k,n} = (n+1/2)\hbar\omega_c + \hbar ck\left(\frac{E}{B}\right) - \frac{mc^2}{2}\left(\frac{E}{B}\right)^2 \qquad 4.29$$

If the electric field is small, there is a large gap between Landau levels n and n+1, and the chemical potential is adjusted to lie in this gap, then the number of electrons in the sample is $N = nL_xL_y/(2\pi \cdot l_{B^2})$, the charge density is $\rho = ne/(2\pi \cdot l_B^2)$ and Eqn. (1.23) becomes $\sigma_{xy} = \rho c/B = ne^2/h$.On this result we note that ,

$$\frac{1}{m}\iint \psi^*_{k,n}\left(\frac{\hbar}{i}\frac{\partial}{\partial x} - \frac{e}{c}A_x\right)\psi_{k,n}(x,y)dxdy = c\frac{\vec{E}}{\vec{B}} \qquad 4.30$$

Thus the current carried by each orbital is e times classical drift velocity $c\vec{E}/\vec{B}$. Adding this up and dividing by the sample area, we find that

$$J = \frac{ne^2}{\hbar}E \qquad 4.31$$

We can imagine turning on a small impurity potential in the interior of the ribbon. If the potential is very small that the gaps between Ladau Level is clean, no change in the out come of this Landau level into the disordered region on the left and pulls one on the right. Thus any one electron Hamiltonian that can be adiabatically evolved in to Landau levels without having states cross the Fermi Level has an exactly quantized Hall conductance. I-electron density of states of the Hamiltonian

$$H = \sum_{j}^{N} \left\{ \frac{1}{2m} \left[\frac{\hbar}{i} \vec{\nabla}_j - \frac{e}{c} \vec{A}(\vec{r}_j) \right]^2 + V_{random}(\vec{r}_j) \right\} \tag{4.32}$$

For various strengths of V_{random}. When it is small, V_{random} its effect is to break the degeneracy of the Landau level. The states in the tails of the distribution that results are localized.

For real Quantum Hall Effect. A field-effect transistor is a capacitor. It stores charge in response to the application of a gate voltage by the rule $Q = CV_g$, where C is determined by the oxide thickness, sample area A sweep of gate voltage is a sweep of Q not of chemical potential. The chemical potential simply adjust itself to whatever is required to fix the charge at Q.If the Fermi Level lies in a region of localized states then the chemical potential can move about freely, populating or depopulating the localized states at will with no effect what ever on the Hall conductance. Another nicely accounted by localization is lack of parallel resistance. In the limit that the electrons do not interact the parallel conductance σ_{xx}, the electric dipole translations from states just below Fermi level to just above. If the states are localized, σ_{xx} must be zero. Localization causes insulation. However, the resistivity and conductivity tensors are inverses of each other, so that if $\sigma_{xy} = ne^2 / h$ we also have

$$\begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}^{-1} = \frac{h}{ne^2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{4.18}$$

Note that experiments that tune the magnetic field strength instead of V_g are not fundamentally different, as they simply vary the length scale against which the charge density is measured. It is believed that localized states are responsible for the observed stabilization of the Hall resistance at certain quantized values. The explanation of the QHE is based on disorder, induced by the random potential from impurities or defects in real samples. This approach assumes that the disorder causes localization, where the localized states are situated energetically in the tails of the broadened LL's (see Fig. 4.6). In these localized states, the electrons are localized in the vicinity of an impurity or defect and cannot carry any net current (do not contribute to the electron transport in the 2DES). On the other hand, the center of the LL's contains so-called extended (delocalized) states which carry the current and thus contribute to the electron transport. In the delocalized state, the electrons can flow over a large area of the crystal. Therefore, the increase of the magnetic field would correspond to a sweep of the chemical potential through regions of alternatively localized and delocalized states. Ando [57].

4.1.2.5THETHEORETICAL ANALYSIS COMPARED WITH THE EXPERIMENTAL RESULT

The theory for the Hall conductivity is much more complicated, and In the lowest approximation one expects that the Hall conductivity σ_{xy} deviates from the classical curve (where n_s is the total number of electrons in the two-dimensional system per unit area). (Fig 4.7) shows the observed data for the parallel conductivity (σ_{xx}) with respect to the gate voltage at different values of fixed magnetic fields $B=0$, $B=10T$,and $B=18T$ .At $B=0$ (σ_{xx}) is approximately linear with the gate voltage. How ever with the existence of fixed magnetic field, (σ_{xx}),oscillates with amplitude which will be greater for the larger

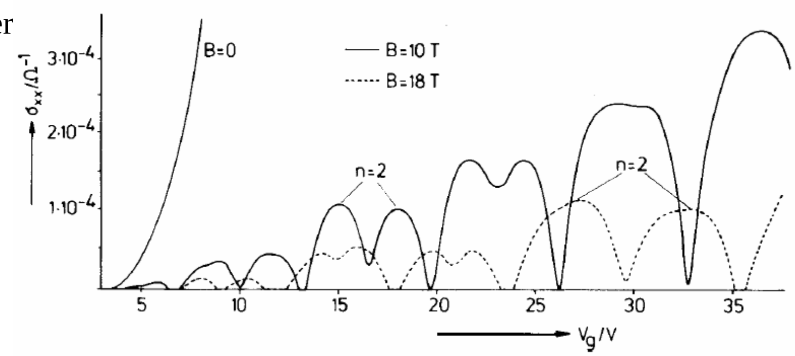

Fig 4.7: conductivity σ_{xx} of silicon MOSFET at different magnetic fields B as a function of the Gate voltage Where as the conductivity is σ_{xx} can be measured directly by using a corbino disc geometry(chap 2: fig 2.4) for the sample, the Hall conductivity is not directly accessible in an experiment but can be calculated from the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} measured on samples with Hall geometry shown in chapter 2 fig (2.2). [1]

$$\sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \;, \; \sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad 4.19$$

The classical curve $\sigma_{xy}{}^0 = -\frac{n_S c}{B}$, the plateau value $\sigma_{xy} =$ constant. (Observable in the gate voltage region where σ_{xx} becomes zero) should change with the width of the plateau. Wider plateau should give smaller values for σ_{xy}. The values of the Hall resistance in the plateau region are not influenced by the plateau width.

The first high-precision measurement of plateau value in $R_H(V_g)$ showed that these resistance values are quantized in integer parts of h/e^2 = 25812.8Ω with experimental uncertainty 3ppm [KLAUS von KLITZING 's Original Discovery]

The Hall resistance $R_H = \rho_{xy}$ increases step like with plateaus in the magnetic field region where the longitudinal resistance ρ_{xx} vanishes. The width of the ρ_{xx}-peaks in the limit of zero temperature can be used for a determination of the amount of extended states and the analysis[29] shows that only few percent of the states of a Landau level are not localized. The fraction of extended states within one Landau level decreases with increasing magnetic field but the number of extended states within each level remains approximately constant since the degeneracy of each Landau level increases proportionally to the magnetic field. Due to the high accuracy of measuring the Hall resistivity, the QHE is also used to give a highly accurate value of the fine-structure constant $\alpha = e^2 / \hbar c = 1137.035999$. The most striking difference between the IQHE and the CHE is that, the Hall resistance, in IQHE, develops a series of plateaus, in the same intervals of magnetic fields, where the Hall resistance exhibits plateaus, the longitudinal resistance vanishes. The vanishing longitudinal resistance is a sign that for those magnetic fields the current flows from one end to the other of the 2DES without dissipating any energy in the interior of the sample. For an integer i, which represents the number of completely filled Landau, levels

($R_{xy} = h/ie^2 = 25812.807\Omega / i = R_{K-90} / i$ Where $R_{K-90} = h/e^2$ is the von Klitzing's constant) The value of R_H at the position of the plateaus of the steps is quantized. $R_H = h/ie^2$, where i is an integer and h is Planck's constant ($R_H \approx 25.812...k\Omega$ *for* $i = 1$).In 1990 , h/e^2, the quantum of resistance as measured via the integral Quantum Hall Effect (IQHE), became the world's new resistance standard.

4.2 THE FRACTIONAL QUANTIZED HALL EFFECT

The fractional Quantum Hall Effect (FQHE) is observed in high-mobility samples, GaAs-$Al_xGa_{1-x}As$ hetrostructures in the extreme Quantum limit in the regime where the electron-electron interaction dominates. It manifests the many body interaction physics of the 2DEG in the intense B field. We can observe into the so-called extreme quantum limit, where the lowest Landau level is only partially occupied with electrons. We can investigate this regime for signs of the so-called Wigner solid, an electron crystal in two dimensions. In 1983,R.B Laughlin presented a theory with a variation ground state and excited-state wave functions, which describes the condensation of a 2DES into a new state of matter. This state of matter is described by an incompressible Quantum liquid where the excitations over its ground state carry a fractional electric charge.

4.2.1 THE ORIGIN OF FQHE

The largely linear relationship between Hall resistivity ρ_H and magnetic field B is evident. The IQHE can be understood solely on the basis of the quantized motion of individual 2D electrons in the presence of a magnetic field and random fluctuations of the interface potential, which creates localized states. The existence of all fellow electrons enters only in the simplest of ways—as a filler of empty states of the Landau levels. The electrostatic interaction (so-called Coulomb interaction) between the like-charged carriers is irrelevant to the understanding of the IQHE. It is therefore called a single-particle effect. Deviations at low field indicate the emergence of the IQHE. Knowledge of the electron density, as well as the values of the resistance steps [$R_H = h/ie^2$, $i = 1,2,3,...$], clearly identify these features as the IQHE. With the last ($i = 1$) step occurring at the critical magnetic field $B_{critical} \approx 5T$ for all fields beyond this point the electrons had to reside in the lowest Landau level, filling it to only a fraction v of its capacity. As the sample was cooled to 1.5 K (See Fig 4.8 and 4.9), the IQHE features firmed up, developing the familiar flat plateau. The IQHE, arising from exact filling of Landau levels, could not have been at work, since above ($B_{critical} \approx 5T$) the lowest level was only partially occupied. Furthermore, the Hall resistance in the vicinity of this change in slope far exceeded the largest possible of IQHE resistances of $R_H = h/e^2 \approx 25k\Omega$

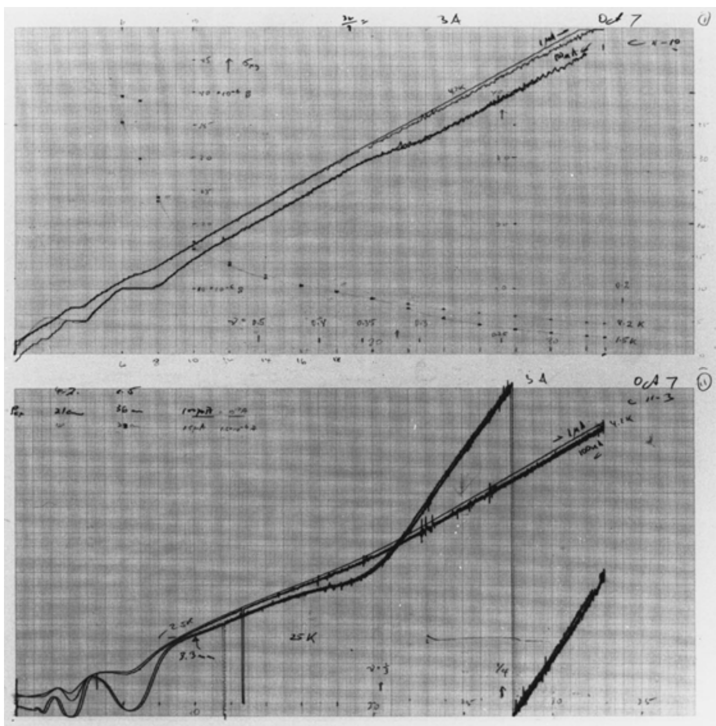

Fig. 4.8: Photograph of a GaAs/AlGaAs sample. The size is about $6 \times 1.5mm$. Data of October 7, 1981, on the specimen # 6-19-81(3) on millimeter paper. The top panel shows the Hall resistance R_H at temperatures 4.2 K and 1.5 K verses magnetic field B. The bottom panel shows the magneto resistance R vs. B at similar temperatures. 1 T is equivalent to ;1.5 cm. Features at ;3 cm and ;7 cm are due to the IQHE. Weaker features at ;21 cm are due to the FQHE.(Source[3]).

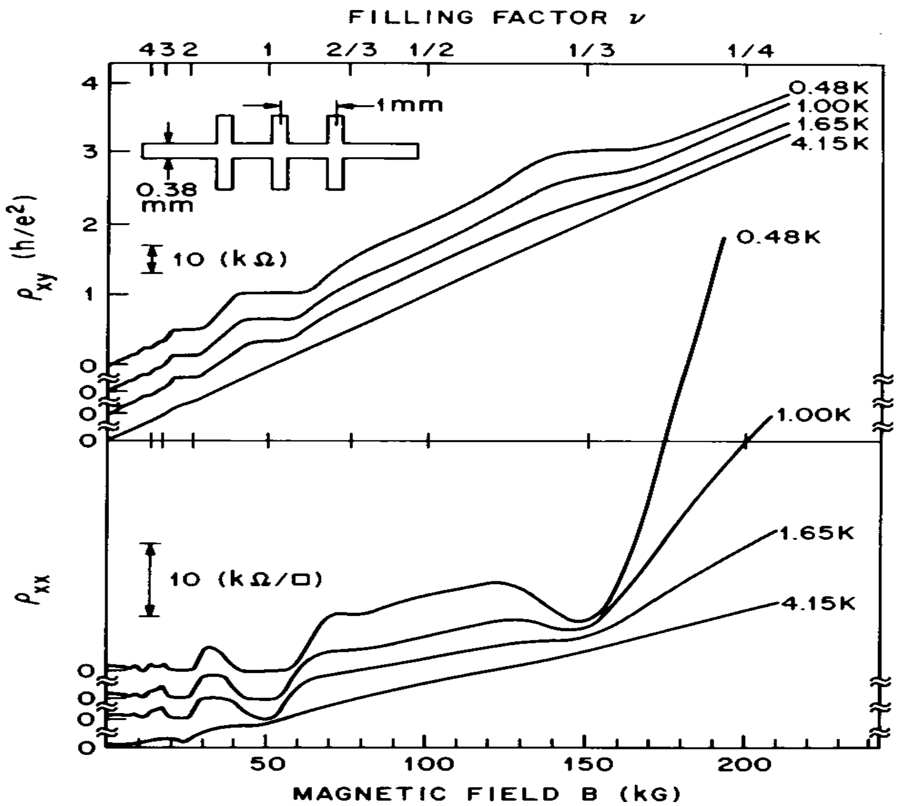

Fig.4.9: First publication on the FQHE. Hall resistance data (here ρ_{xy}) and magnetoresistance data (here ρ_{xx}) are from the same specimen as in Fig. 4.11. The filling factor v of the Landau level is indicated on the top. The features at $v = 1,2,3,...$ are due to the IQHE. The features at $v = 1/3$ are due to the FQHE.Sample dimensions and sample temperatures are indicated [3].

In the IQHE, the quantum numbers are integers identified with the number of completely filled Landau Levels, the energy gap is the Landau gap of a cyclotron energy Quantum the existence of an energy gap ,so crucial for the exact quantization in the IQHE is also essential for the occurrence of the FQHE. However, all magnetic-field-induced energy gaps have been exhausted by the IQHE and have emerged as integral quantization of the Hall resistance to $R_H = h/ie^2, i = 1,2,3,...$ other energy gaps, at fractional filling of a Landau level, must be of a different origin. The origin of the FQHE is interaction between electrons. It is therefore termed a many-particle effect or an electron correlation effect, since the charged electrons are avoiding each other by correlating their relative motion in a multifaceted manner. In the IQHE, electrons have no freedom to avoid one another. Occurring at exact integral Landau-level filling, electrons are already ‘‘close packed’’ with no option for further avoidance. At fractional filling this is different. There is much ‘‘space’’ in a Landau level. Electrons have the freedom to avoid each other in the energetically most advantageous fashion. The electron solid, in which electrons reside at fixed positions of maximum mutual distance, would have represented a static pattern that minimizes electron interaction.

Confined to the lowest Landau Level. Under those exceptional conditions, thus at fractional filling v of this level, the Hall resistance is found to be quantized to $\rho_{xy} = \frac{h}{ie^2}$, where i is a simple rational fraction. Parallel, resistivity ρ_{xx} drops towards zero. So far this effect has been observed close to $v = 1/3, 2/3, 4/3, 5/3, 2/5, 3/5, 4/5,$ and $2/7$ with Quantum numbers $i = v$ quantized, to better than 1 part 10^4

4.2.2 PECULIAR PROPERTIES

The Fermi energy in the bulk is in the middle of a partially filled Landau level, the energy gap in this case must come entirely from electron-electron interactions. The ground states for fractional quantized Hall systems states are strongly correlated many-body states with very peculiar properties.

4.2.3 ELECTRONS AND FLUX QUANTA

In a classical model, 2D electrons behave like charged billiard balls on a table [Fig. 4. 110(a)]. They are distinguishable. Quantum mechanically, electrons are smeared out over the table. They are inherently indistinguishable... They behave like an undistinguished liquid [Fig. 4.13(b)]. But the motions of the electrons are correlated., shown in Fig. 4.10 (c) in a classical representation. They also do this in the quantum-mechanical liquid Fig. 4.10 (b). It affects the probability of finding one electron here having detected another electron there. We realize that a magnetic field B, creating tiny whirlpools, so-called vortices, in this lake of charge one for each flux quantum $\phi_o = h/e$ of the magnetic field [Fig. 4.10 (d)]. Inside the vortex, electronic charge is displaced, dropping to zero in the center and recovering to the average surrounding charge density at the edge of the vortex. The extent of a vortex is roughly the size of the area, which contains one quantum of magnetic flux ($area \times B = \phi_0$). Of course, just as the electrons are spread out uniformly over the plane, so are the vortices. As required by quantum mechanics, [Fig. 4.10 (e)]. However, the picture of electrons and vortices provides an intuitive way of looking at electron - electron correlation in the presence of a magnetic field. Electrons and vortices are opposite objects, one representing a package of charge, the other the absence of charge.

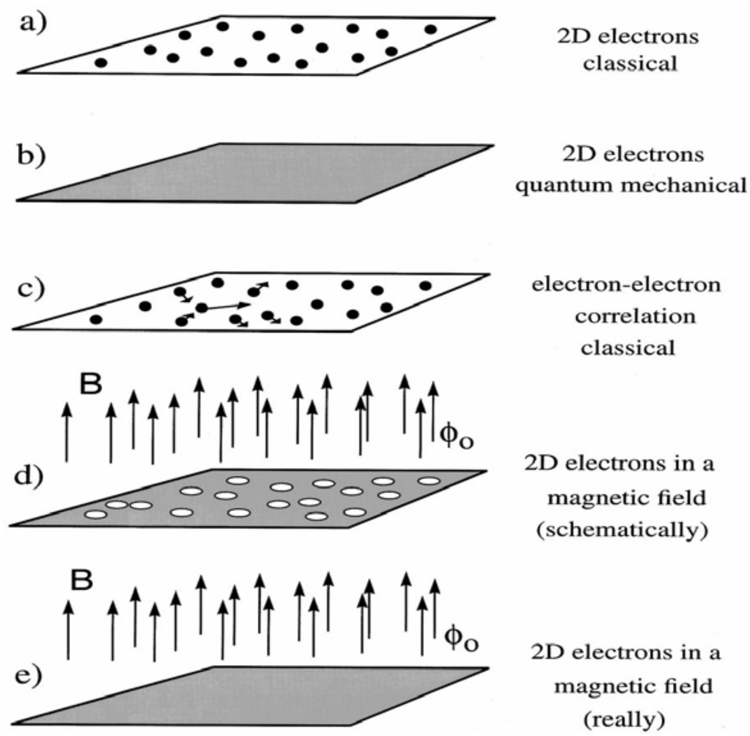

Fig 4.10: Schematic drawing of 2DES in various approximations. Black dots represent electrons. White holes represent Vortices. Arrows represent magnetic flux quanta ϕ_0 of the magnetic field B. [3]

Each electron always needs to be surrounded by one vortex. It is system's way of satisfying the Pauli Exclusion Principle for electrons, which, in this situation, requires that no two electrons can be in the same position. At complete filling of the lowest Landau level, where the number of electrons equals the number of flux quanta, the arrangement the Pauli principle —one vortex per electron, (see Fig. 4.11). This is the condition of the $i=1$ IQHE. It can easily be extended to more Landau levels $i = 2,3,4,...$. It is another way of expressing that the existence of other electrons enters the IQHE only in the simplest of ways—as a filler of empty states.

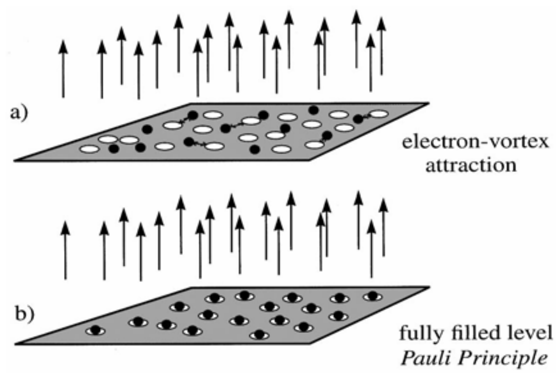

FIG. 4.11: Schematic drawing of electron vortex attraction of a 2DES in the presence of a magnetic field: (a) In the fully filled Landau level $\nu=1$, there are as many vortices as there are electrons, and the Pauli Exclusion Principle forces the vortices onto the electrons (b). (The spin of the electron is neglected throughout.) [3]

At magnetic fields higher than $\nu=1$, the IQHE, the stronger magnetic field provides more flux quanta and hence there are more vortices than there are electrons. The electron system can considerably reduce its electrostatic Coulomb energy by placing more vortices onto each electron [Fig4.12 (b)]. More vortices on an electron generate a bigger surrounding whirlpool, pushing further away all fellow electrons, thereby reducing the repulsive energy. The so-established relative motion of electrons is driven by the reduction in Coulomb energy. This is the central principle underlying electron-electron correlation in 2DES in a magnetic field.

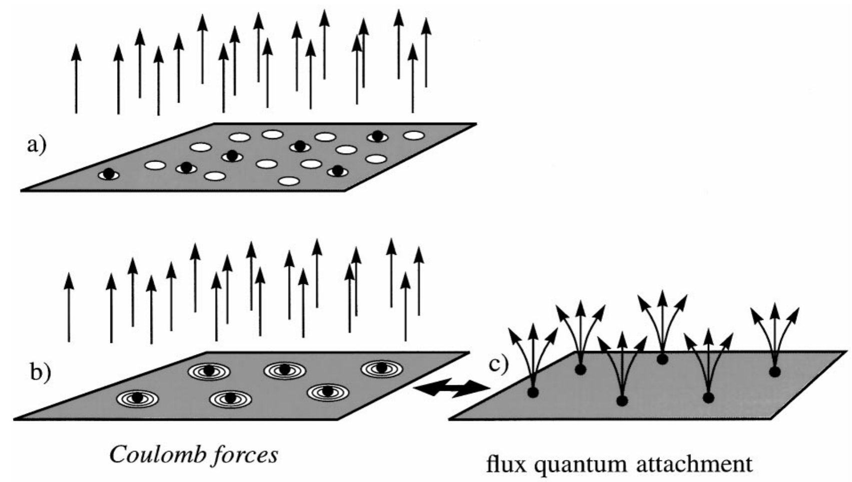

Fig. 4.12: Schematic drawing of electron vortex attraction at fractional Landau-level filling $v=1/3$. Now there are three times as many vortices as there are electrons. (a) The Pauli principle is satisfied by placing one vortex onto each electron. (b) Placing three vortices onto each electron reduces electron- electron (Coulomb) repulsion. Vortex attachment can be viewed as the attachment of magnetic flux quanta to the electrons, transforming them to composite particles (c).

4.2.3.1 COMPOSITE PARTICLES

A weakly interacting gas of a new type of particles, called "composite fermions" have the same quantum number as electrons, but are distinct from the electrons in their property, that they experience a much reduced electromagnetic field. [Fig. 4.12 (b), (c)]. Electrons plus flux quanta can be viewed as new entities, which have come to be called composite particles; CPs. composite particles are almost void of mutual interactions. All of the external magnetic field has been incorporated into the particles via flux quantum attachment to the electrons. They inhabit an apparently field-free 2D plane.

4.2.3.2 COMPOSITE PARTICLE STATISTICS

Electrons are fermions. As one slowly moves two electrons in a 2D electron system around each other and exchanges them, the wave function undergoes the sign reversal expected from fermions. It is different for CPs (Fig. 4.13). The attached flux quanta need to be taken into account, and their presence changes the particles' statistics. As one slowly moves two CPs around each other and exchanges them, the electrons by themselves reverse the sign of the wave function, but each attached flux quantum creates an extra ''twist,'' multiplying it by an extra -1. As a result, CPs can be either fermions or bosons, depending on the number of attached flux quanta. An electron plus an even number of flux quanta becomes a composite fermions (CF), since the wave function is multiplied by -1 an odd number of times, i.e., by -1. An electron plus an odd number of flux quanta becomes a composite boson (CB), since the wave function is multiplied by -1 an even number of times, i.e., by $+1$. This so-called transmutation of the particle statistics through flux quantum attachment is deeply rooted in the two-dimensionality of the system. It represents a deep connection between space and particle statistics.

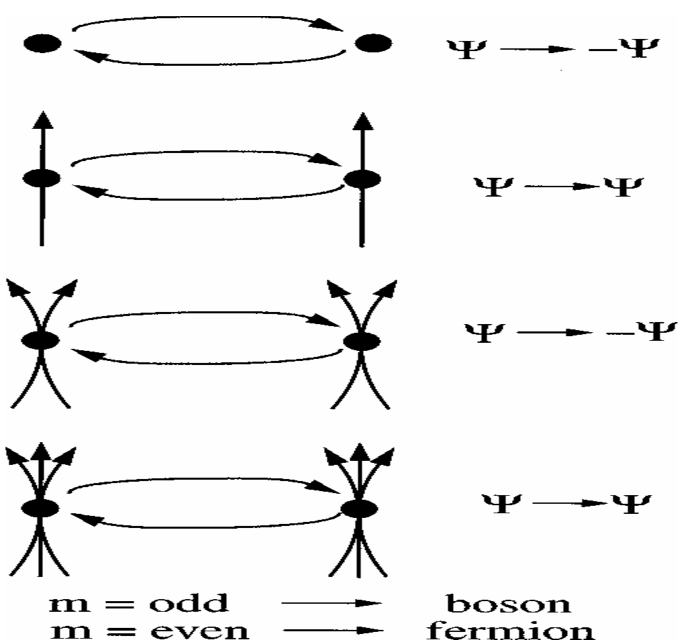

Fig 4.13: Statistics of electrons and composite particles. Exchange of two particles affects the wave function Ψ, which describes the quantum-mechanical behavior of the system. For electrons, Ψ is multiplied by -1, identifying the particles as fermions. With the attachment of an odd number of flux quanta, Ψ remains unchanged under exchange (multiplication by $+1$), identifying these particles as bosons. Attachment of an even number of flux quanta returns the particles to fermions. Here m is the number of flux quanta.

4.2.3.3 THE STATE OF $v = 5/2$

Electrons with two attached flux quanta are fermions. They fill up sequentially the lowest energy states and are the starting point for multiple sequences of FQHE states. However, they themselves cannot be FQHE states. Yet the 5/2 state is exactly that. It has all the characteristics of a FQHE state, including energy gap and quantized Hall resistance; in spite of its even denominator, classification .The $v = 5/2$ state resides in a higher Landau level $5/2 = 2 + 1/2$. Discovered more than a decade ago, its true origin remains mysterious.

4.2.3.4 $1/3$ FRACTIONAL QUANTUM HALL STATE

At $\boldsymbol{1/3}$ filling of the lowest Landau level ($v = \boldsymbol{1/3}$), the magnetic field contains three times as many flux quanta per unit area as there are electrons in the 2D system. Therefore the electron liquid contains three times as many vortices as there are carriers. To minimize electron-electron interaction, each electron accepts three vortices, which keeps fellow electrons optimally at bay. This is equivalent to the attachment of three magnetic flux quanta to each electron, which cause to be CPs [Fig. 4.12(c)]. Since all the external magnetic field has been incorporated into the particles, they live in an apparently magnetic-field-free region. Consisting of an electron plus an odd number of flux quanta, resulting in composite bosons (CBs). Being bosons and living in apparently zero magnetic field, these CBs Bose condense into a new ground state with an energy gap, this is the sought-after energy gap required for quantization of the Hall resistance and vanishing resistance to arise. It has been measured by various experimental techniques. As the magnetic field deviates from exactly $v = \boldsymbol{1/3}$ filling to higher fields, more vortices are being created (Fig 4.12). They are not attached to any electrons, since this would disturb the symmetry of the condensed state. The amount of charge shortfall in any of these vortices amounts to exactly 1/3 of an electronic charge. These quasi holes (whirlpool in the electron lake) are effectively positive charges as compared to the negatively charged electrons. Quasi particles can move freely through the 2D plane and transport electrical current. They are the famous 1/3 charged particles of the FQHE that have been observed by various experimental means, most recently by measurement of the amount of electrical noise that they generate. Plateau formation in the FQHE arises, in analogy to plateau formation in the IQHE from potential fluctuations and the resulting localization of carriers. In the case of the FQHE the carriers are not electrons, but, instead, the bizarre fractionally charged quasiparticles. The FQHE at $v = 1/5, 1/7, etc.$, with quasi particles of charge e/5, e/7, etc., can be accounted for in total analogy to the 1/3 FQHE by attaching 5, 7, etc. flux quanta to each electron. In fact, even states at $v = 2/3, 4/5, 6/7, etc.$ and $v = 1+1/3, 1+1/5, etc.$ can be covered by this procedure, for example, the $v = 2/3$ state as a full Landau level with $1/3$ missing electrons. In this way all fractions at Landau-level filling factor

4.2.3.5 THE STATE AT $v = 1/2$

At first sight, the $v = 1/2$ state should be similar to the 1/3 state, yet it turns out to be very different. At half filling of the lowest Landau level, the magnetic field contains two times as many flux quanta per unit area and hence creates two times as many vortices, which keep the others at bay (Fig. 4.14). However, the attachment of an even number of magnetic flux quanta to each electron makes these objects composite fermions (CFs) and not composite bosons. As at $1/3$ so also at $v = 1/2$, the entire external magnetic field has been incorporated into the particles and they live in at apparently zero magnetic field.

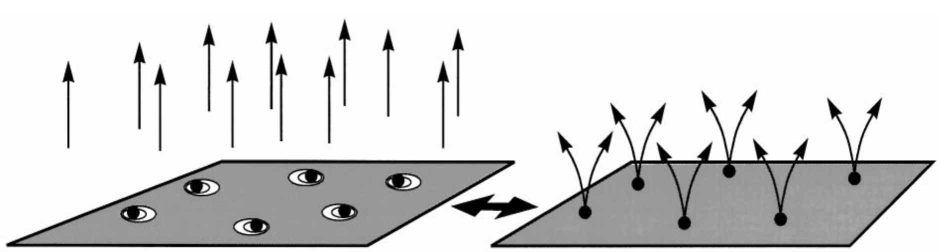

FIG.4.14: Schematic representation of the state at Landau-level filling factor $v = 1/2$. Two vortices are bound to each electron, equivalent to the attachment of two flux quanta. The slight pay off the second vortex is meant to represent the formation of tiny in-plane dipoles [3].

However, being fermions they are prevented from condensing into the lowest energy state. Instead, they fill up successively the sequence of lowest-lying energy states, until a maximum is reached and all CFs have been accommodated. The process is equivalent to the filling of states by electrons at $B = 0$.

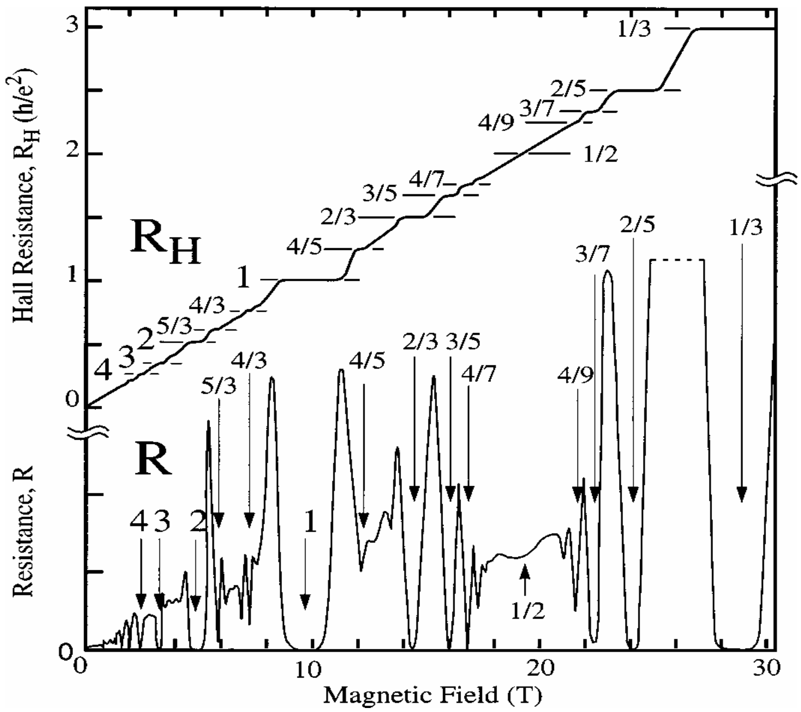

Fig. 4.15: The FQHE as it appears today in ultrahigh-mobility modulation-doped GaAs/AlGaAs 2DESs. Many fractions are visible. The most prominent sequence, $v = p/(2p \pm 1)$, converges toward $v = 1/2$.

In spite of the huge external magnetic field at half filling of the Landau level, *CFs* are moving in a similar fashion to electrons moving in zero fields. This has been directly observed in experiment. Flux quantum attachment has transformed these earlier electrons and they are propagating along straight trajectories in a high magnetic field, whereas normal electrons would orbit on very tight circles. The mass of a *CFs*, usually considered to be a property of the particle, is unrelated to the mass of the underlying electron. Instead, the mass depends on the magnetic field and only on the magnetic field. In fact, it is a mass of purely many-particle origin, arising solely from interactions, rather than being a property of any individual particle. It is another one of these mysterious implications of $e-e$ interactions in high magnetic fields. The absence of condensation and the lack of an energy gap prevent the $v=1/2$ state from showing a quantized Hall resistance. Instead the Hall line is featureless, just as it is for electrons around $B=0$ (see Fig. 4.15). The difference between $v=1/3$ and $v=1/2$ is striking. One is a Bose-condensed many-particle state showing a quantized Hall Effect and giving rise to fractionally charged particles. The other is a Fermi sea.

4.2.3.6 THE OTHER FQHE STATES

Following Laughlin's gedanken experiment and accepting quantization of the Hall resistance to measure the charge of the particle, a plateau three times as high as the last IQHE plateau meant the appearance of a charge $q=\phi_0/(3h/e^2)=e/3$. Bob Laughlin had the correct theoretical insight and invented an elegant wave function which described the quantum-mechanical behavior of all those electrons in the $1/3$ FQHE as well as all other $1/q$ FQHE states Bose condensation of CBs consisting of electrons and an odd number of flux quanta rationalizes the appearance of the FQHE at the primary fractions around Landau-level filling factor $v=i\pm 1/q$ with quantized Hall resistances $R_H = h/ve^2$ and deep minima in the corresponding magneto resistance R. However, a multitude of other FQHE states has been discovered over the years. Figure 4.19 shows one of the best of today's experimental traces on a specimen with a multimillion cm2/V sec mobility. The composite fermions model offers an extraordinarily clear picture. The sequence of prominent fractions $2/5, 3/7, 4/9, 5/11, \ldots$,

(i.e., $v = p/(2p\pm 1), p=2,3,4\ldots$) around $v=1/2$. At half filling the electron system has been transformed into CFs consisting of electrons which carry two magnetic flux quanta. All of the external magnetic field has been incorporated into the particles and they reside in an apparently field-free 2D plane. Since they are fermions, the system of CFs at $v=1/2$ resembles a system of electrons of the same density at $B=0$. For electrons their motion becomes quantized into electron-Landau orbits. They fill up their electron-Landau levels, encounter the energy gaps, and exhibit the well-known IQHE. CFs around $v=1/2$ follow the same route. As the magnetic field deviates from exactly $v=1/2$, the motion of CFs becomes quantized into CF-Landau orbits. They fill up their CF-Landau levels, encounter CF energy gaps, and exhibit an IQHE. However, this is not an IQHE of electrons, but an IQHE of CFs. This IQHE of CFs arises exactly at $v = p/(2p \pm 1)$, which are the positions of the FQHE features.

In all above of this chapter we have reviewed Some points in Quantum Magneto transport, in that The integer Quantized Hall Effect is characterized in:

1) For the ratio of ϕ_0 to e, one can regard ρ_H as being a very precise measure of the electron charge when expressed as $e = \phi_0 /(i\rho_H)$.From this purview, Klaus von Klitzing's experiment has provided a highly accurate electrometer to determine the charge of the current carrying filling factor $v =$ Number of occupied LL's :

$$v = \frac{n\Phi_0}{B} \text{ , } (n = 2\mathrm{D}\ \textit{electron density})$$

2) The Physics of the IQHE lies in the formation of Quantized Landau Levels for non-interacting electrons, a gap occurs only at $v = \text{int}\,eger$

3) The precision of quantization does not depend on the shape and size of the specimen, or on the particular care taken to define its contact regions. In a quirk of nature, the existence and precision of the IQHE plateaus requires the existence of imperfections in the sample.

4) The Fermi energy in the bulk is in the middle of a partially filled Landau level, the energy gap in this case must come entirely from electron-electron interactions. The ground states for fractional quantized Hall systems states are strongly correlated many-body states. Each electron always needs to be surrounded by one vortex. It is system's way of satisfying the Pauli Exclusion Principle for electrons, which, in this situation, requires that no two electrons can be in the same position

5 CONCLUSION

In this project I have reviewed few points of some findings that have been discovered and synthesized by some distinguished Scholars and Scientists. Thus in my project work, title "The Quantized Hall Effect", I have confined these points in four chapters that I conclude as follow.

In most bulk metals and semiconductors, the Classical Hall resistivity is linear in the magnetic field $\rho_{xy}(B)$. Due to its independence from all intrinsic and extrinsic parameters, the classical Hall Effect became a standard tool for the determination of the density of electrons in conductor, and in semiconductor. In order to observe the QHE, a 2DEG at low temperatures (below $T < 4K$) and high magnetic fields are required. A two-dimensional electron Gas can be formed at the interface of a semiconductor. Like silicon or gallium arsenide where the surface is usually in contact with a material, which acts as an insulator, SiO2 for silicon field effect transistors and (GaAs/AlGaAs hetrostructure). Due to the trapped electrons the approximately triangular potential well at the discontinuity of the conduction bands is formed. In the 2DES, the electrons move quasi-free in two spatial dimensions. Their energy Quantized in the third spatial dimension. Applying a perpendicular magnetic field to the 2DES leads to the Landau quantization of energies. As a result, Quantum Hall effects are observed : the SdH effect, the IQHE (the LL's are full filled at filling factors equal to $i = 1,2,3, etc.$). In case of the FQHE, the Landau level is partially filled at filling factors equal to rational fractions $i = 1/3, 1/5, 1/7, 2/3, 4/5, 6/7$. In a QHE, the Hall resistivity is observed to be quantized at; $\rho_{xy} = h/ie^2$.Since electrons are moving as free particles entirely perpendicular to the electric field, the conductivity σ_{xx} (current flow in the direction of the electric field) becomes zero, thus no dissipation of energy.

The Integral Quantum Hall resistivity where , it is written in the form $\rho_H = \alpha^{-1}\mu_0 c/2i$, $i = 1$, μ_0 is the permeability of free space, exactly equal to $4\pi \times 10^{-7}\, Hm^{-1}$, and c is speed of light in vacuum which is equal to $299792458 ms^{-1}$ with current uncertainty $0.004 ppm$, is exact quantized essentially derived the fine structure constant, $\alpha \approx \frac{1}{137}$. Thus in its application, the Quantum Hall Effect can be used for the determination of the fundamental constant h/e or for the realization of a voltage standard. In analogy, the QHE can be used for the determination of h/e^2 or as a resistance standard.

The IQHE can be understood solely on the basis of the quantized motion of individual 2D electrons in the presence of a magnetic field and random fluctuations of the interface potential, which creates localized states. In the integral or fractional Quantum Hall Effects, the essential thing is the accuracy of Quantization. In a single particle picture in the 2d electron gas, the magnetic field essentially places electrons in identical harmonic oscillator potential wells of number equal to the number of magnetic flux quanta passing through the 2D system. The quantum energy levels, equally spaced in energy, these harmonic oscillator potential wells are known as “Landau levels”.

Bits of magnetic field can get attached to each electron, creating yet other objects such objects known as composite particles. They have properties very different from those of the electrons. They sometimes seem to be insensible to huge magnetic fields and move in straight lines, although any bare electron would orbit on a very tight circle. Their mass is unrelated to the mass of the original electron but arises solely from interactions with their neighbors. The origin of the FQHE is interaction between electrons. It is therefore termed, a many-particle effect or an electron correlation effect, since the electrons are avoiding each other being in touched in vortices, and correlating their relative motion in an obscure manner. Two-dimensional electron systems, in extreme low temperatures and in extreme high magnetic fields, reveal FQHE, totally new many-particle physics. Confined to a plane and exposed to a magnetic field, such electrons display an enormously diverse spectrum of fascinating new properties: totally unexpected new electron states with fractional quantum numbers, the attachment of magnetic flux to electrons, termed as quasi particles. **Future:** All what I did in this project paper is just, collecting the findings that have already been done on the Quantized Hall effect. I put those things comparatively the CHE, IQHE, and the FQHE.However the Real obscurity of Quantized Hall Effect even untouched in this work. Particularly the secrecy of FQHE remained. It is left for future inquiry.

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