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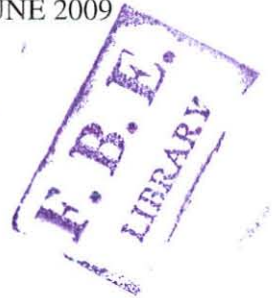
MODELING THE DYNAMIC MACROECONOMIC THEORIES OF GEBREHIOWT

BAYKEDAGN

A thesis submitted to the School of Graduate Studies of Addis Ababa University in partial fulfillment of the requirements for the Degree Masters of Science in Economic Policy Analysis in the Department of Economics

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“Modeling the Dynamic Macroeconomic Theories
of Gebrehiwot Baykedagn.”

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Abstract

A mathematical model is specified for the dynamic macroeconomics theories of Gebrehiwot Baykedagn on conflict, unequal exchange, and public spending on human capital formation. Based on the theory, through time total output increases with higher labour force participation resulted from public spending on human capital formation, political stability and trade equivalence and it declines with land degradation resulted from political instability (conflict) and unequal exchange in the international trade through time. Hence, to show how the total output reach at the steady state, we have derived the equilibrium path of the model under three different scenarios both graphically and mathematically. Based on the specified model, in the scenario of which the rate of growth in total output due to labour force participation through time is equal to the rate of decrease in total output due to land degradation, the economy will be at the steady state, otherwise in the downward or upward trend of the equilibrium path. Furthermore, in order to see how the overall economy moves, a counter factual dynamic simulation is experimented and found to be consistent with the model under the scenario of a shock to public spending on human capital formation and a shock to political stability.

Chapter one

1.1 Introduction

Before king Menilik had taken power, the contribution and influences of Ethiopian intellectuals, who went abroad for studies, in their country's socio economic problem were almost insignificant (see Alem, 1973 and Baharu, 2002). This is because, firstly, the intention of their trip to Europe was not directed to the economic, social and political problems of the country but rather it was under the religious expansion program of the European Catholics and Protestants churches. Consequently, up on their return to Ethiopia, rather than addressing the then problems of the country, they either preach the religion dogmas of protestant and catholic churches or served as a translator to the emperor and European visitors. Secondly, they were seen as culturally alienated and dubious in their deeds and behaviour. Thirdly, they were so few in number that they could not have power to influence the rulers and bring change in the country's socio economic activities (see Alem, 1973 and Fekade, 1988).

However , in the last decade of nineteenth and early twenty century, whatever the background and nature of their formation, whether they were educated abroad or at home, acquired formal qualification or just had spell of foreign exposure, the intellectuals were ready to serve their country as a diligent functionaries (see Baharu,2002). Moreover, they were aware of their country's backwardness and became committed to the introduction of reforms as the sole guarantee of their country's independence existent (see Baharu, 2002).

Among the intellectuals, who put forward solutions for the then problems, *Naggadras* Gebrehiwt Baykedagn was the leading individuals in terms of bringing change in the administrative system of king Halie-Silasse and in the degree of organization and flows of

ideas (see Baharu, 1995). His books of *Atse Menilik Na Ethiopia* and *Mengest Na Yezbe Astedader* have addressed almost all economic and social problems and issues of the early twenty century Ethiopia. Particularly his work of *Mengest Na Yezbe Astedader* has various issues, which have been viewed from different angles and makes him a pioneer in different field of studies in Ethiopia. For instance, studies on Ethiopian history categorized him as a reformist intellectuals of the early twentieth century in the country (see Baharu,1991,1995 and 2002); economics study of Ethiopia made him a pioneer development and political economist of the country (see Abdulhakim, 1997;Alemayehu, 2002 and 2008; Ayele, 1981;Sosina, 1999 and Tenkir, 1995) and environmental studies of Ethiopia viewed him as the first Ethiopian thinker on environment and development (see Emmanuel,2004). Moreover, Alemayehu (2002; 2008) and Sosina, (1999) specify a linear static mathematical model for his macroeconomic thoughts.

This being the case, however, so far in economics there is Gebrehiowt's ideas which have not overlooked (see Alemayehu, 2002) but never been done. For instance, his discussion on conflict, unequal exchange, public investment on human capital formation and infrastructural facilities even if reviewed by different scholars and individuals(see Abdulhakim,1997; Alemayehu,2002; Ayele,1981; Bahru,2002; Emmanuel,2004 sosina,1999 and tenkir,1995) still no one has formulated it in dynamic terms although the theories being very dynamic. Hence, this paper will fill this gap by incorporating his dynamic theories on human capital formation, politics (conflict) and environmental science (land degradation in relation to unequal exchange and conflict) to the macroeconomic model of Gebrehiowt Baykedagn.

1.2 Objective of the Study

The major objective of this paper is to build a mathematical model to the dynamic macroeconomic theories of Gebrehiwt Baykedagn discussed on his book of *Mengest Na Yezbe Astedader*. Whereas, the specific objectives of this paper include:

- Delineating the socio economic condition of Ethiopia in the early twenty century as well as reviewing the major economic theories of Gebrehiwt Baykedagn discussed on his book of *Mengest Na Yezbe Astedader*.
- Summarizing his dynamic theories on conflict, unequal exchange, taxation and public spending on human capital formation and infrastructural facilities.
- Driving the equilibrium path of the model both mathematically and graphically under three different scenarios.
- Simulate dynamically the specified model of Gebrehiwt Baykedagn in the scenario of a shock in public spending on human capital formation and conflict/political stability.

1.3 Scope and Significant of the Study

Gebrehiwt's book of *Mengest Na Yezbe Astedader* have discussed dynamic economic theories ranging from domestic agricultural production at the house hold level to the banking and monetary sector of the economy as well as international trade and the foreign direct investment on manufacturing sector. However, in this paper I will model only his dynamic theories on conflict, unequal exchange and public spending on human capital formation since they still have relevance to the current realm of developing countries (Ethiopian) economy.

Moreover, the model links agricultural production, domestic private investment, trade balance, and government tax revenue through public spending on human capital formation and conflict to the major macroeconomic variables of developing countries. Hence, building a macroeconomic model which links the aforementioned variables by maintaining the balance of payment (trade balance) and budget deficit (increasing tax revenue) at tolerable level in the condition of political instability (conflict) will have significance for the developing countries policy makers. Moreover, the paper will be a source of literature review for additional studies on Ethiopian intellectuals' development in general and Gebrehiwot economic theories in particular.

1.4 Organization of the paper

The rest of the paper organized as follows. In chapter two, in order to give the background and the context under which Gebrehiwot gave his analysis, the socio economic conditions of the early twenty century, the major economic theories of Gebrehiwot Baykedagn and his biography is reviewed. Gebrehiwot's dynamic theories on conflict, public spending on human and physical capital formation, unequal exchange and taxation are described in the third chapter. In chapter four, Gebrehiwot's dynamic mathematical model, graphical and mathematical representation of the equilibrium path under three different scenarios is presented. In the fifth chapter, a dynamic counter factual simulation is experimented in the scenario of a shock to public spending on human capital formation and in the scenario of a shock to political stability. In the sixth chapter, concluding remark is given.

Chapter Two

2. Reviews of Literature

2.1 The Socio Economic Condition in Ethiopia Between 1889-1920's¹

The last decade of nineteenth and the early twenty century marks the beginning of modern state in Ethiopia (see Baharu, 2001 and Pankhurst, 1967). The major successes that the country achieved by that time include; establishments of centralized government, changes in relation between social classes; emergence of new means of communication and transportation; improvement of trade and commerce of the country domestically and internationally; emergence of organized banking and monetary revolution; and materialization of modern education as well as modern health care systems (see Pankhurst et al: 1967).

The successes and changes outlined above, the majority of which in one way or another affected the vast bulk of the population, were supplemented by other development which thought at first confined to a minority of people, often at the capital or other towns (see Pankhurst et al: 1976). Such change, however significant, left vast areas in which traditional life continued more or less as in previous centuries (see Pankhurst et al: 1976).

For instance, politically, in spite of the largely achieved reunification, power by no means easily centralized. In the same way, despite the ending of the *Zemene-Mesafint*, which was characterized by major civil wars of the country, the underlying tensions remained as in the previous state (see Baharu, 2001 and Pankhurst, 1976). The country's social structure likewise

¹ The period of 1889-1920's marks the beginning of modernization and secular thought in Ethiopia.

remained unaltered throughout the period. The ownership of slaves was still widespread, only a fraction of the total slave population managed to emancipate (see Baharu, 2002).

Economically, notwithstanding with the considerable efforts to improve transport, there were a few thousand miles of roads .In the land, most journeys even on the main trade routes, being possible only by mule, while the capital's only efficient mode of transport with the outside world was by means of the railway. The expansion of trade, though particularly noticeable after the foundation of Addis Ababa, left the greater part of the population in a purely subsistence economy. The advent of money was slow to have much influence in the country as a whole since "primitive money" being still used at the end of the first decade of the twenty century, and paper money scarcely known by the public at large. The impact of banking was similarly restricted to a handful of towns and it was under the monopoly position of foreigners (see Baharu, 2001 and Pankhurst, 1967).

Agricultural sector, in which the vast majority of the populations were engaged, underwent little appreciable development. Despite the increased utilization of imported iron for the manufacture of tools, in the first two decades of twenty century, the plough and other agricultural implements remained essentially unchanged. There was no change in agricultural techniques or crop cycles except temporarily (see Pankhurst et al, 1976). Although the establishment of model farms and foreign plantation led to significant expansion of certain cash crops, largely for export, it happened without materially affecting the way of life of the average peasant (see Baharu, 2001 and Pankhurst, 1967).

Although modern school opened in the capital of the country, it was only the royal family members and government officials' family who were allowed to attend to school. Epidemic

disease (like small pox and malaria) continues to kill the vast population of the country in spite of the emergence of new health care systems (see Baharu, 2001). A further unresolved weakness inherited from the past decades were, the country's ethnic, cultural and linguistic heterogeneity, which in the absence of any strong and widespread feeling of nationalism could only surmounted by the most charismatic of the emperors (see Pankhurst et al, 1976).

Having aware the problems mentioned above, the early twenty century intellectuals began to take the burden of addressing the problems of Ethiopia's modernization. For each problem that the country had already faced, the then intellectuals were suggesting their thoughts using local news letter and writing letter to the kings (see Alem, 1973 and Baharu, 2002). For instance, *Bejerond* Tekele-Hawryat Tekele-Maryam (1884-1977), Melaku Beyene, Mikael Tessema (1900-1963) and Tedla Haile were tried to address the problem of administrative efficiency. *Naggadras* Afewerq Gebre-Yesus (1868-1947), Tekele-Hawryat Tekele-Maryam, and Takele Welde-Hawaryat tried to address the problem of social injustice and slavery (see Bahru, 2002). *Hakim* Werqene Eshete (1864-1952) and *Blatta* Deressa Amente (1887-1952) concentrated on the problem of modern education and knowledge dissemination (see Baharu, 2002 and Alemayehu, 2008).

In addition to the aforesaid problems raised by the above intellectuals to be addressed, *Naggadras* Gebrehiowt Baykedagn went far beyond by signifying the critical structural problems of the country. He raised the problem of engaging in unequal exchange, the recurrence of conflict (civil war), the presence of multiplicity of custom posts (implying excessive tax burden on workers), the monopoly of banking sector by the foreigners, and the unbalanced growth of infrastructure development, and modern education and health care

systems (see Abdulhakim, 1997; Alemayehu, 2002 and 2008; Ayele, 1981 and Baharu, 2002 Gebrehiwot, 1924 and Tenkir, 1995).

Before reviewing his ideas on the aforementioned areas, describing his biography would be logical as it may give the reason of his peculiarity from the rest of twenty century intellectuals.

2.2 The Biography of Gebrehiwot Baykedagn

Gebrehiwot Baykedagn was born in 1886 in the village of May Masha in the district of Adwa, Tigray region of northern Ethiopia (see Alemayehu,2003; Ayele, 1981 and Baharu,2002).The early 1890s were a period of exceptional turbulence in Tigray where the political disintegration and psychological void created by the death of Emperor Yohannes, the ravages of one of the longest and most devastating famines of the country had ever known, and the depredation that attended Emperor Menilik's campaign of 1890 to assert his new imperial authority ,all combined to produce great instability (Alemayehu,2002 and Baharu, 2002).It was in this circumstances that he fled with some of his companion to Eritrea at the age of seven and joined the Swedish mission school at Menkullu, on the main land off Massawa (see Abdulhakim,1997; Alemayehu, 2002; 2008 Baharu,2002; Sosina, 1999 and Tenkir, 1995).

According to Baharu (2002), a trip to port of Massawa that Gebrehiwot subsequently made with his friends was to change the course of his life. Along with them, he got permission from the captain of a German ship docked there to go abroad and look around. He was curious to know how the ship is working and lost in the process. On arrival at the destination, the captain entrusted him to the rich Austrian family, which adopted him (see Alemayehu, 2002; 2008; Bahru,2002; Ayele, 1981 and Baharu, 2002).Subsequently, he learned the German language and went on to study medicine at Berlin University (see Alemayehu, 2002;2008; Ayele, 1981;

Baharu, 2002 and Sosina 1999). Having completed his studies in Germany, he returned to Ethiopia, learned the Amharic language and reportedly made the private secretary and interpreter to the emperor Menilik (see Alemayehu, 2002; 2008 and Baharu, 2002). In November 1909, he chose to exile himself in the British colony of Sudan, apparently having difficulties with the empress Taytu (Abdulhakim, 1997; Alemayehu, 2002; Alemayehu, 2008; Ayele, 1981 and Baharu, 2002). Having returned from Sudan he wrote his first book entitled, *Atse Menilik na Ethiopia* in Asmara, which literally can be translated as Emperor Menilik and Ethiopia (see Ayele, 1981 and Baharu, 2002). He returned from Sudan being critically ill. He recovered after being hospitalized at Massawa and subsequently wrote his second treatise on *Mengest Na Yezbe Astedader* means government and public administration or more appropriately it can be translated as the political economy of development (see Abdulhakim, 1997; Alemayehu, 2002; Alemayehu, 2008 and Baharu, 2002).

2.3 The Major Economic Ideas of Gebrehiwot Baykedagn

The major economic theories, which will be reviewed in this section from his book of *Mengest Na Yezbe Astedader* contain the theory of value, international trade, taxation and money and banking.

2.3.1 Theory of Value

Unlike most of the classical economist whose theory of value is ambiguous and incomplete (see Bhatia, 1989)², Gebrehiwot's theory of value evidently laid its base on the labour theory

² H L Bhatia (1989) have mentioned as Adam Smith's theory of value covers more than one strand's and at the same time without a complete synthesis (pp-86). He also mentioned as David Ricardo's theory of value is incomplete since it is not clear as to how the labour equivalent of different capital inputs is estimated (pp-118).

of value only (see Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995). The labour theory of value, according to Baharu (2002), is Gebrehiowt's linchpin idea. He also elaborated and illustrated the theory with the then Ethiopian existing situation. The value of a commodity, Gebrehiowt explained, is determined by the amount of labour time expended to produce it (see Abdulhakim, 1997; Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Gebrehiowt, 1924; Sosina, 1999 and Tenkir, 1995). Concomitantly, the value of a commodity declines with the refinement of the means of producing it (see Alemayehu, 2002; Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995). This brings him to another important consideration: the centrality of knowledge and education, which are better understood in the sense of technology (Alemayehu, 2002; Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995). According to Baharu (2002) and Gebrehiowt (1924) himself, it is the edge that the European manufacturer has in knowledge that reduces the Ethiopian peasant to paying with many days of labour for European glasses and bottles which are produced within minutes.

As Baharu (2002) pointed out, Gebrehiowt lends his labour theory of value a clear social and political content. Writing about the impact of war, he underscored, the national wealth can be expected to grow only when the consumers do not outnumber the labour force since feeding an unproductive person is a waste of resources (see Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995). Moreover, for Gebrehiowt merchants represent the non-labour force class. According to him when there are too many merchants in a country, it does the public a lot of harm since more merchants means more soldiers; more robbers (see Abdulhakim, 1997; Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995).



In more logical framework, Alemayehu (2002) modeled Gebrehiowt's labour theory of value³. According to him, like that of the Marxist labour theory of value, the value of a commodity for Gebrehiowt is determined by the amount of labour power embodied in it. On account of the assumption of perfect mobility of resources, any variation in the value of a commodity from the amount of labour embodied in it will be adjusted quickly. The labour embodied in a commodity, however, is inversely related to the physical investment in the production of those goods (see equation (2) in the footnote). A far more interesting notion introduced by Gebrehiowt's model is that a commodity can be considered as wealth and hence, has value when only people know how to use it effectively (see Alemayehu, 2002; Gebrehiowt, 1924 and Tenkir 1995).

2.3.2 International Trade

In his analysis of international trade, Gebrehiowt gave more attention to the tragedy of trade between developing countries (Ethiopia) and developed countries (Europe), arguing that trade always work for the developed countries (see Alemayehu, 2002; Baharu, 2002; Emmanuel, 2004; Gebrehiowt, 1924 and Tenkir, 1995). To explain this concept, he used the labour theory of value noting that the labour endowment of the developing country is less whenever it is equated with skilled labour since it has to be plenty in terms of quantity for equality to be established (see Alemayehu, 2002; Emmanuel, 2004).

³ The value of agricultural product for Gebrehiowt, according to Alemayehu (2002), is given by the following set of equations;

$$V_a = P_a Q_a = f(L^+) \text{ -----(1) ;}$$

$L = f(I)$ -----(2), this equation in Gebrehiowt's model is important as it link the agricultural sector with the non-agricultural and government sector.

$$P_1 I = f(HK^+) \text{ -----(3)}$$

where $V_a = P_a Q_a$ = Value of agricultural production; P_a = Price of agricultural product and L = The amount of labour embodied in a commodity. P_1 = price of investment (goods) and HK = Human capital.

Using this theory, he also showed the implication of unequal exchange with different level of skills, namely between people with skill (developed countries) and people without skill (developing countries), whereby the former plunder the latter. (see Alemayehu, 2002; Emanuel, 2004; Gebrehiwt, 1924; Tenkir, 1995). Moreover, as long as developing countries export agricultural products, which embodied unskilled labour due to its low level of development while importing processed commodity from Europe, which constitute skilled labour, Gebrehiwt explained, unequal exchange will persist (see Alemayehu, 2002). Gebrehiwt analysis of unequal exchange, Alemayehu (2002) noted, prefigures the works of Prebisch (1962), Singer (1950), Emmanuel (1972) and Amin (1974).

Gebrehiwt also noted that with unequal exchange, terms of trade deterioration, trade deficit and foreign debt for the developing countries will follow (see Emmanuel, 2004; Gebrehiwt, 1924 and Tenkir, 1995). While explaining how terms of trade deterioration follows unequal exchange, as Alemayehu (2002) aptly noted, Gebrehiwt said even if trade between two countries is in balance, the developing country will lose since it sells primary commodities while buying manufactured goods⁴. This is because, the cost of transportation for the developed country has a tendency to decrease while for the developing one it has a tendency to rise since the commodity needs to be shipped for processing and coming back as a finished good⁵.

⁴ Alemayehu (2002), modeled Gebrehiwt's international trade theory as $X_D = C_D + I_D = M_U$ where X, C, I and M stands for export, cost of production, profit and import respectively where as D and U represents developed and undeveloped countries.

⁵ According to Alemayehu (2002), Gebrehiwt in his analysis of terms of trade deterioration, prefigured the Prebisch-Singer hypothesis of terms of trade. The theoretical underpinning in his case, however, is the cost of production and degree of processing.

Having analyzed the 1912 import and export data of Ethiopia with Europe (see Gebrehiwot, 1924 and Tenkir, 1995), Gebrehiwot also showed how developing countries encounter the problem of trade deficit and foreign debt when they trade with developed countries (see Emmanuel, 2004; Gebrehiwot, 1924 and Tenkir, 1995). Furthermore, he also revealed the danger international trade poses for developing countries, owing to the uncompetitiveness of their domestic infant industry and consequently, vulnerability, entailing loss of little skills they have, unemployment and further dependency (see Ayele, 1981; Baharu, 2002; Emmanuel, 2004; Gebrehiwot, 1924 and Tenkir, 1995).

2.2.3 Tax Policy

Gebrehiwot also emphasized that a rational system of tax policy is the best guarantee for the maintenance of country's development (see Ayele, 1981; Baharu, 2002; Gebrehiwot, 1924 and Tenkir, 1995). He viewed taxation from different angles: domestic versus foreign; and tax on consumer goods versus on investment goods (see Alemayehu, 2002; Gebrehiwot, 1924 and Tenkir, 1995). Regarding to domestic taxation, he argued that imposition of tax on internal trade, particularly on domestic produce discourages domestic production, subsequently lower government revenue (see Alemayehu, 2002; Ayele, 1981; Emmanuel, 2004; Gebrehiwot, 1924 and Tenkir, 1995). This is because domestic taxation would give domestic traders and producers an additional strain on that of the one braved seeking one another (see Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Gebrehiwot, 1924 and Tenkir, 1995). Hence, it encourages subsistence farming in rural areas since peasants' after tax income may not be worth his additional work effort (see Alemayehu, 2002). According to Alemayehu (2002), such an adverse effect of domestic taxation is aggravated by Gebrehiwot's model assumption that processed/manufactured goods are not absolutely required in the rural areas. Consequently,

Peasants' choice for leisure instead of work limits the urban manufacturing activity, which implies lower government revenue⁶.

The other category of tax identified by Gebrehiwot is related to tax on foreign trade activity (tariff) (see Gebrehiwot, 1924 and Tenkir, 1995). This category comprises those tax levied on luxury items and those on investment goods (see Alemayehu, 2002). Gebrehiwot advocate a zero tax on imported machineries and equipments provided that they are essentials (see Alemayehu, 2002; Ayele, 1981; Baharu; Gebrehiwot, 1924 and Tenkir, 1995). But at the same time he recommended that textiles, luxuries and other imports, which compete with domestically produced goods, should be taxed highly (see Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Emmanuel, 2004; Gebrehiwot, 1924 and Tenkir, 1995). This is because, doing so for Gebrehiwot will make importers to sell the imported machineries and equipments at cheap price to the producers and peasants (see Emmanuel, 2004; Gebrehiwot, 1924; Tenkir, 1995). Moreover, it discourages imports of finished goods since the imported goods become dearer, subsequently stimulating production domestically (see Gebrehiwot, 1924 and Tenkir, 1995).

Gebrehiwot's idea of protective tax policy on foreign trade clearly distinguishes him from the classical economists' idea, particularly that of David Ricardo, who unconditionally upheld free trade (see Ayele, 1981 and Emmanuel, 2004). Alemayehu (2002) also mentioned

⁶Concerning tax in Gebrehiwot model, Alemayehu(2002), specify that

$$T_d = t_d QP, \text{-----(1)}$$

$$QP = f(t_d) \text{-----(2)}$$

$$T = f(I^+) \text{-----(3)}$$

Where P is a weighted average price of agricultural and non agricultural product. Q stands for output, t_d is the tax rate, T_d is domestic tax revenue and I is investment. Hence Eq (1) implies that domestic tax revenue is a function of domestic tax rate and output. Eq (2) showed that output is negatively related to tax rate. Eq (3) depicts Tax revenue is positively related to investment.

Gebrehiowt's tax policy on foreign trade prefigures Kalecki's (1976) famous work on financing development and that of 1950s and 1960s import substitution policy advocates.

2.2.4 Money and Banking

Another interesting point that Gebrehiowt raised in his treatise of *Mengest an Yezbe Astedader* was the idea of banking and money in developing country (Ethiopia) (see Gebrehiowt, 1924 and Tenkir, 1995). Although Gebrehiowt acknowledge the role of banks in creation of money, he pointed out, in a country where people lacks knowledge and are not engage in a various field of production activities the drawbacks of banks surpass their merit (see Abdulhakim, 1997; Alemayehu,2002; Ayele,1981, Baharu,2002;Gebrehiowt,1924 and Tenkir,1995). He argued, the fact that a developing country does not have educated labour force implies the country is incapable of utilizing the mobilized resource placed in the banks (see Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Gebrehiowt, 1924 and Tenkir, 1995). Hence, the accumulated capital in the bank could benefit only foreigners (European) (see Alemayehu, 2002) for they know how to use the resource, and even can afford to borrow at the exorbitant rate (see Alemayehu, 2002; Gebrehiowt, 1924 and Tenkir, 1995).He substantiated his argument using data from Ethiopia and the activity of the British bank in Ethiopia at the turn of the century (see Alemayehu, 2002; Baharu, 2002; Gebrehiowt; 1924 and Tenkir, 1995)⁷.

For Gebrehiowt, it is absolutely wrong to put an institution like bank, which is directly associated with the day to day activity of the people, under the monopolistic control of foreigners (see Abdulhakim, 1997; Ayele, 1981; Baharu, 2002; Gebrehiowt, 1924 and Tenkir,

⁷ Of the 1,220,850 thalers (unit of measurement for the currency) borrowed from the bank of Abyssinia (the British bank in Ethiopia) in the year 1913, only about 50,000(i.e. less than 5 percent) were borrowed by Ethiopians (see Alemayehu, 2002; Baharu, 2002; Gebrehiowt, 2002 and Tenkir, 1995).

1995). Moreover, he argued, the absence of competition in the banking sector is bad not only for public and government but also for the bank itself (see Gebrehiwt, 1924; Sosina, 1999 and Tenkir, 1995). This is because its monopoly position will allow the bank to charge higher interest rate on loan, which consequently lowers the number of its customers and profit (see Gebrehiwt, 1924; Sosina, 1999 and Tenkir, 1995). Thus, for banking sector to be effective in facilitating the economic activity of developing countries, he set two preconditions: educating the people and abandoning particularly the monopolist position of foreigners (developed countries' investor) from the sector (see Gebrehiwt, 1924 and Tenkir, 1995).

Another subject addressed by Gebrehiwt was the theory of money. He pointed out the underlying need for currency is the ultimate inadequacy of a barter system (see Baharu, 2002). After tracing the historical evolution of the means of exchange, from cattle (in ancient Greece) and slaves (in Britain) to 'amole' or salts bars (in Ethiopia), he showed how the world has finally arrived at non-perishable basis of exchange through the use silver and gold (see Ayele, 1981; Baharu, 2002; Gebrehiwt, 1924 and Tenkir, 1995). As per Baharu (2002), Gebrehiwt went further into a discussion of the relative merits of the two precious metals, gold and silver. Having realized that silver currency fluctuates with the value of metal as opposed to the relative stability of gold based currency, Gebrehiwt recommended the adoption of the gold standard currency for Ethiopia (see Alemayehu, 2002; Ayele, 1981; Baharu, 2002; Gebrehiwt, 1924; Sosina, 1999 and Tenkir, 1995).

In his discussion of money and banking, Gebrehiwt also underscored interest rate as an important monetary policy instruments (see Alemayehu, 2002; Gebrehiwt, 1924 and Tenkir, 1995). For him the rate of interest is an indicator of country's level of development, that is to say the higher the rate of interest, the lower the level of development of a country (see

Alemayehu, 2002; Ayele, 1981; Gebrehiwot, 1924 and Tenkir, 1995). This is because, according to Alemayehu (2002) and Gebrehiwot (1924) himself, development entails expansion of both physical infrastructure and financial capacity, which increase the velocity of money and economic activity through facilitating exchange. This in turn discourages hoarding and hence increases the amount of money supply in the economy, which subsequently lower the rate of interest. Additionally, he noted, monetary policy through inflation tax, can help to raise government revenue (see Alemayehu, 2002). However, he was against such kind of tax since it is not transparent and could lead to unfair distribution of income by raising the relative value of physical asset to that of financial holdings (see Alemayehu, 2002; Gebrehiwot, 1924; Sosina, 1999 and Tenkir, 1995).

Another important monetary issue that Gebrehiwot discussed is demand for money (see Alemayehu, 2002; Gebrehiwot, 1924 and Tenkir, 1995). He underscored the importance of monitoring one of the components of demand for money (currency in circulation) and the need to satisfy that demand (see Alemayehu, 2002 and Sosina, 1999). If such policy is not pursued, he argued, foreigners will grasp this opportunity to exploit the domestic market by supplying the required demand and thus extracting surplus in the process as the currency was a commodity money (see Alemayehu, 2002). Finally, in order to have efficient monetary system, Gebrehiwot recommended, the government of Ethiopia (which represent developing country) need to study the demand for currency outside the bank; restrict the responsibility of issuing currency to the state and expand infrastructure to ensure efficient communication (see Alemayehu, 2002; Ayele, 1981; Gebrehiwot, 1924 and Tenkir, 1995).

In sum, Gebrehiwot's work of *Mengest Na Yezbe Astedader* has various issues, which have been viewed from different angles by different field of studies. Although much has been done

on his economic, politics (see Abdulhakim, 1997; Alemayehu, 2002 and 2008; Ayele, 1981; Sosina, 1999 and Tenkir, 1995) and environment (see Emmanuel, 2004) thought, still no one has formulated them in dynamic term while they are very dynamic. Hence, in the next chapter, we will outline the major dynamic theory of Gebrehiowt Baykedagn, which will be modeled in the subsequent chapter.

Chapter Three

3.1 Dynamic Macroeconomic Theories of Gebrehiwot Baykedagn

3.1.1 Internal Dynamics

A. The Dynamics of Conflict.

War, according to Gebrehiwot, is the principal internal culprit responsible for the state of underdevelopment as compared to any other natural obstacles for many developing countries (Gebrehiwot, 1924, p:31). Explaining the source of war, he acknowledged the existence of reckless but powerful people in a given society who have their own chief. Furthermore, he noted, these people sustain themselves by exploiting peasants in the form of taxation rather than working (p: 31). However, in spite of high tax burden that levied on them, he said, the peasants expand their land and wealth by descending from the edge of the mountain to the marshy areas along the river basins (p:31). Through time, they reach near the territory of land cultivated and grazed by another chief. As a result, the two neighbouring chief become jealous each other, and subsequently engaged in war (p:32)⁸.

After the war, the productivity of the defeated peasants will decline. Clarifying the reason, he said;

የተሸነፈው ቆሮ ገባሮቹ ያንን ቡብዙ ድካም ያቀኑትን ሜዳና የወንዝ ዳር ትተው ለመመከት ይመቻቸዋልና ዱሮ ወደ ነበሩበት ስፍራ ወደ ተራራው ጥግ

⁸ The reference mentioned in this chapter is taken from Gebrehiwot's Book of Mengest Na Yezbe Astedader; hence we would mention the page number only where ever appropriate.

ይሸሻሉ።አነሆም ያ በብዙ ድክም የቀና ሜዳ ተመልሶ እንደጥንቱ ደን ይሆናል።በድክም ብዛትም ያደረቁት የወንዙ ዳር ተመልሶ ምድረበዳ ይሆናል።

The peasants of the defeated chief will abandon the plain and river basin, which they had developed and retreat to their earlier settlements on the edges of the mountains as these are easier to defend. The land, which had developed with a lot of effort, will revert to bush and reclaimed marshland along the basin of the river and it will become a desert (p:32).

Nevertheless, after some time, Gebrehiowt said there would be a peace agreement between the two chiefs and restart their job. Although the defeated group starts to work from scratch, it would not be effective easily. For him this is because,

በጦርነቱ ውስጥ ብዙ ሰዎች ሞተውባቸው ስላነሱ ጉልበት አጥራቸዋልና መሬቱን በቶሎ እንደጥንቱ መመለስ ይቸግራቸዋል።

The defeated peasant lack labour force due to the losses sustained in battle, they will find it difficult to restore the land into its former productivity (p:32).

This implies that the war, which takes place at any time, has a negative impact on the productive labour force participation of the economy for the current and subsequent years. Low labour force participation, Gebrehiowt added, will result in a decline in the productivity agricultural output and the tax revenue accruing to the government; hence, the king will no more be strong (p:33). As a result of this,

ባገሩ ውስጥም ሽፍታ ቢነሳ ሰራዊቱ በፍጥነት የሚሄድበት የተበጀ መንገድና የምድር ባቡር ስለሌለው ሽፍታ ያለበትንና የተነሳበትን አገር በፍጥነት ሊረዳው አይችልም።ንጉሱ በገዛ እጁ በሾማቸው መከንንት ሙሉ ስልጣን የለውምና ምንም

እነርሱ ባንድ ንጉስ ውስጥ ቢያድሩም ቢገዙም ርስ በርሳቸው መዋጋትን አይተውም። ይልቁንም ጉልበት ሲሰማቸው ንጉሱን አጥፍተው ራሳቸው ለመንገስ ያስባሉ። በእንደዚ ያለ ምክንያት ጦርነት ይነሳል።.....

....rebellion cannot be put down due to lack of road and railways. Though the local leaders are under the leadership of one king, they will not refrain from fighting each other. This is because the king will not have absolute authority over the officers due to lack of wealth. When they feel that they are sufficiently powerful, they even attempt to get rid of the king and enthrone themselves instead. Consequently, the war will continues.... (pp: 33, 34)

Hence, for him, low labour force participation by itself is a constraint to avoid political instability since the tax revenue accrues to the government and hence public spending on infrastructure and human capital formation.

With a special emphasis on the history of Ethiopia, he underscored the dynamics of conflict (political instability) and land productivity stating that;

የኢትዮጵያ ሕዝብ ሀብቱና ዕውቀቱ ሊያድግ ሲገባም ጠቅላይ ግድብ አዲስ ሀክትና ጦርነት ይነሳል። ስለዚህም የኢትዮጵያ ሕዝብ አቅንቶት የነበረውን ደህና መሬት እየተወ ሀክትና ጦርነት በተነሳ ቁጥር ለመመከት እንዲመቸው ሲል ዱሮ ወደነበረበት ወደ መጥፎው መሬት ይመለስ ነበር።

Each time the Ethiopian people made a head start to accomplish in terms of knowledge and wealth, the country ran into fresh conflict and war. Hence, in order to defend themselves, the Ethiopian people used to abandon the fertile lands they claimed and return to inferior lands they left a while back (pp:40-41).

In sum, for Gebrehiwot, war at any time affects the labour force participations negatively, which has a negative consequence on the productivity of the land to be cultivated and agricultural output of the economy for the current and subsequent years.

B. The Dynamics of Public Spending on Human Capital Formation and Infrastructure

Gebrehiwot also discussed the constraints that underdeveloped countries are facing due to the absence of well-developed infrastructure, education and health services. For him, public spending on human capital formation and investment on infrastructure is positively related to private investment in non agricultural sector and negatively related to the cost of production and hence the price level since it reduces the time taken to produce a particular product and it largely shuns the profit margin goes to the merchants (pp-49). Justifying this fact, he said,

ስለዚህ ዕውቀት ሲበዛ ለነገር የሚያስፈልገው ማናቸውንም ነገር ለማግኘት መስናክሎ ያንሳል። የማናቸውም ዋጋ ሁሉ ዝቅ ይላል። ዕውቀት እያነሰ ሲሄድ ግን ለነገር የሚያስፈልገው ዕቃ ሁሉ ይወደዳል። እንደዚህም ባንድ አገር ውስጥ የተሰራው የጋሪው መንገድና የውሃ ድልድይ ሲበላሽ በቶሎ ያልታደሰ እንደሆነ ባለበርኖሱ ጤፍ ሲፈልግ ባለጤፍም በርኖስ ሲፈልግ እንደፈለጉት በጋሪ ሊመጣላቸው አይችልም።.....በዚህ የተነሳ ጤፍ ለባለበርኖሱ በርኖሱም ለባለጤፍ ይወደድበታል። ስለዚህም ባንድ አገር ውስጥ ዕውቀት ሲበዛ የነጋዴው ጥቅም ያንሳል የሠራተኛው ጥቅም ግን እንዲያድግ ግልጽ ነው።

When there is more knowledge, the obstacles to acquire the necessities of life will be less. The price of all goods will be lower. If the cart trails and the bridges over the rivers are not repaired soon, the carts cannot deliver the goods...hence; the price of *teff* will be high for the wool dealer and vice versa. Therefore, in the country where

there is an abundance of knowledge, the margin going to the merchants decreases (pp:49-50).

Moreover, he also noted, as the government invest more on roads and infrastructure, education and health services, investment and hence total output of the economy will increase (p:60). The increase in total output of the economy will lead to an increase in the tax revenue accruing to the government. The government could in turn use this tax revenue to finance its investment on education, railways and health stations, which will further boost up the country's output (p:60).

Concluding his discussion on the dynamics of public investment on human and physical capital formation, Gebrehiwot said,

ስለዚህ ባንኩ ውስጥ ያሉትን ሠራተኞች ቢያሰለጥንና ቢያቃርብ ቢያተጋቸውም ወይም ለጤናቸውና በደህና አካላቸው እንዲኖሩለት ቢጥር ገንዘብም ቢያወጣ ዋናውን ሳያጠፋ በየቀኑና በየወሩ በያመቱ ብዙ ወለድ በሚሰጥ በታመነ ባንክ ገንዘብን አደራ እንዳስቀመጠ ያህል ነው። ስለዚህ ባንኩ ሀብት የበለጠ ሀብት ይገኛል ብንል የተከከለ ቃል ነው።

Therefore, if the government strives to invest money, which aims to train and bring people together, motivates and provides medical care to improve their living standard: it is equivalent to depositing a capital asset in a secure bank earning daily, monthly and yearly. Thus, it is accurate if we say that more wealth is earned from the initial ones (pp: 60- 61).

In sum, government investment on human capital formation and infrastructural facility, for Gebrehiwt, lowers cost of production, which encourages private investment and hence total output that subsequently increases the tax revenue.

C. The Dynamics of Tax Rate

As part of an additional means of relaxing the constraints to economic growth of a country, Gebrehiwt raised the advantage of avoiding excessive taxation on domestic investors⁹. This is because the lower the tax rate, the lower the cost of production for workers, which encourages peasants, artisans, tailors and craftsmen to invest more and hence improves the tax revenue accruing to the government. Conversely, for him, higher taxation results in lower tax revenue. This is because

.....አራሹና ሸማኔው ሊገዙ የሚፈልጉት ነገር ጥቂት ስለሆነ የሚሰሩትም ሥራ ያንሳል። አራሹ እኸል ሰብስቤ ከሆዴ የተረፈውን ለሌላ ነገር ልለውጠው ብል ድካሜ ከበዛ እኸለ ቢቆለል ለኔ ምን ይሆናል ይላል። ለምግብ የሚበቃውን ያኸል አርሶ ከዘራ በዋላ ሥራ ፈቶ ይቀመጣል። ሸማኔውም እንዲ ያስባል.....። ሠራተኞቹም ሥራ ሲፈቱ ባገሩ ውስጥ የሚሠራው ሥራ ያንሳል መንግሥትም ብዙ ግብር ሊያገኝ አይችልም።

The farmers would say that if what I produce as a surplus from my consumption needs cannot be exchanged for other necessary goods, what will be the benefit of having a heap of grain. Hence, he only sows seeds adequate to his needs and remain

⁹ Investors in Gebrehiwt's model include artisans, tailors and individuals engaged in non agricultural sector of the economy.

idlewhen the workers remain unemployed, the total output will decline and hence the tax revenue accruing to the government.

In sum, Gebrehiwt recommended the optimal tax rate on domestic producers to be at a level of which it does not discourage domestic investment in non agricultural sector for this sector supplies the needs of peasants as well as it absorbs their produce. Moreover, the reduction in investment in non agricultural sector results in lower tax revenue. His analysis on tax rate and tax revenue prefigured the work of famous American economic advisor of Reagan administration Arthur Laffer after whom the curve commonly called Laffer curve (see Collins, 2000).

3.1.2 External Dynamics

A. The Dynamics of Unequal Exchange

Another problem addressed by Gebrehiwt, is the problem developing countries encountering when they partake in international trade with the developed ones. Explaining this concept with the labour theory of value, he said when people with no/less skills exchanged their produce with those who are more skilled, they greatly lose from such trade (p: 67). In order to substantiate this concept empirically, he used the 1912 export and import price of leather from Ethiopia to Europe and vice versa, which subsequently resulted in a trade deficit of the country (p: 68). Moreover, he also emphasized, trade deficit can only be filled at the cost of degrading the country's natural resource more specifically the soil, which subsequently results low level of production (p:69).

While explaining the intertwined pitfall of unequal exchange in relation to agricultural output and labour force participation, he said

ስለዚህም ያገራችን መሬት የዝሆን ጥርስ ዝባድ ቆዳ ቡና እህል በሬ ላም ፈረስ አህያ በቅሎ እየሆነ በያመቱ ወደ ውጭ አገር ቢሄድ አሁን ለጊዜው ለኛ ባይሰማን የኢትዮጵያ ሕዝብ በዝቶ መሬት እየጠበበ ሲሄድ ጉዳቱን ያገኘዋል። ሕዝቡ በሥራ ሠልጥኖ ሥራውንም ሁሉ እንደመጡ ተከፋፍሎ የሚያስፈልገውን ነገር ሁሉ ባገሩ ለማበጀት ቢችል ግን ፍጥ ባገሩ ይቀርላት ነበር። ስለዚህም ዕውቀት ያነሰው ሕዝብ ዕውቀታቸው ከፍ ከለ ሕዝቦች ጋር ሲገበያይ መዝናኛ ትርፍ ድክም ይሰጣቸዋል ። ሁለተኛም መሬቱ ወደነሱ እየተጋዘ ይሄዳል። እንደዚህ ያለ ዕውቀት ያነሰውም ሕዝብ ሲበዛ መሬቱ የሚሰጠው ፍሬ አይበቃውም ። ስለዚህም በጊዜ ብዛት ሠርቼ እበላለው ሲል መሬቱ ወደ ሄደበት አገር ተከትሎ ዕውቀት ወዳላቸው ሕዝቦች አገር ይሰደዳል።

Our land... flees every year in the guise of ivory, civet, hide, coffee, cereals, oxen, cows, horses, donkeys and mules; although it is not being felt as such at the moment, it is bound to impact painfully when the population multiplies and land becomes scarce. Had the people been entitled to advance in terms of skills and an adequate division of labour, producing at home all that they consume, the manure of the country would have stayed at home. Nevertheless, unfortunately, when people lacking in skills indulge in trade with people imbued with it, first the result will be loss since they cede extra labour and then their land follows suit. Moreover, when the number of people lacking knowledge increases in number, the fruit of the land fails to satiate their needs. Hence, in order to survive such people will migrate to the country where their land went (Ibid, pp-70).

To sum up his idea on trade, he noted that whenever developing countries (people with low level of skill and technology) trade with the developed one (people with high level of skill and technology), firstly, they face trade deficit since the gap between the import and export price

of developing countries is vast. Consequently, the productivity of land diminishes since the loss from trade would not allow maintaining the productivity the degraded environment, which reduces agricultural output. Low agricultural output in turn brings a reduction in the total labour supply since it would be difficult for the peasant to stay in the place of where the produce of the land is minimal.

Summarizing his dynamic theories; with political instability (conflict), the labour force participation in agricultural sector declines, hence the land hitherto cultivated will be left barren, which finally leads to a decrease in agricultural output of the economy through time. Likewise, with higher spending on human capital formation and infrastructural facilities, and lower taxation, domestic investment in non agricultural sector will be encouraged with time and hence an increased in total output of the economy. Moreover, unequal exchange, which is an exchange between unskilled labours embodied output (most commonly agricultural products) and skilled labour embodied output (which is manufactured products) results in trade deficit. Trade deficit in turn would restrain the productivity of the degraded natural resources (particularly land) not to be maintained through time, and hence agricultural output of the economy declines.

Chapter Four

4.1 Model Specification

Although we disaggregated Gebrehiowt's dynamic economic theory into internal and external dynamics in the previous chapter for the sake of simplicity, his theory on internal dynamics is interlinked with the external ones. For instance, according to him, when government increases its investment on education and health, the number of skilled labour force participation will increase and hence, the terms of trade and trade balance improves, which allows the country to be competitive in the international market. The improvement in the balance of trade helps the country to maintain the productivity of the degraded land, which subsequently increases agricultural output of the country. Similarly, with political instability (conflict) skilled labour force participation declines and hence the ability of that particular country to produce manufactured (internationally competitive) output declines with time and hence terms of trade and trade balance deteriorates. Together with the direct impact of low labour force participation in agricultural sector due to war/conflict, terms of trade deterioration will exacerbate the problem of land degradation and resulted lower agricultural output.

Thus, in modeling Gebrehiowt's theories it is inevitable to untie the external dynamics with the internal one. However, given the relationship between the major variables of each sector of the model, we can view the theory from the internal and external sectors model stand. The internal sector of the model contains domestic production of agricultural sector and non agricultural sector as well as the government sector. The external sector holds the relationship between trade balance and terms of trade.

4.1.1 Internal Sector

Domestically, total output produced in the economy at any time t is given by the sum of agricultural output and non agricultural output of the same period, as given below;

$$Y_t = Y_{a,t} + Y_{na,t} \dots\dots\dots (4.1)$$

Where Y_t is total output (income in the economy), $Y_{a,t}$ agricultural output in the economy at time t , $Y_{na,t}$ is non agricultural output in the economy at time t .

A. Agricultural Output

Agricultural output at any time t is a function of the same period total labour force engaged in the agricultural sector and the total productive land in use for the same period. In linear form it is given by;

$$Y_{a,t} = \beta_1 + \beta_2 L_{a,t} + \beta_3 PL_t \dots\dots\dots (4.2)$$

Where, $\beta_1 > 0$ is level of output, which is not affected by labour and productive land, $0 < \beta_2 < 1$ is marginal product of agricultural labour. $0 < \beta_3 < 1$ is the marginal product of productive land¹⁰. $Y_{a,t}$ is agricultural output in period t . $L_{a,t}$ is total number of labour force in agricultural sector. PL_t is the total productive land for agricultural production.

¹⁰ The whole model in this paper is specified in the assumption of where there is linear relationship between the major variables. Based on this assumption there is constant return to scale to labour and land. (I.e. $\beta_2 + \beta_3 = 1$).

A.1. Labour Force in Agricultural Sector.

Labour force participations in the sector divided in to skilled and unskilled, which is given by;

$$L_{a,t} = L_{sa,t} + L_{usa,t} \dots\dots\dots (4.3)$$

Where $L_{sa,t}$ is total number of skilled labour force in agricultural sector, $L_{usa,t}$ total number of unskilled labour force in agricultural sector.

Moreover, skilled labour force at any time t is a function of the current and previous period's conflict, and investment made by the government on human capital formation, which is given by;

$$L_{sa,t} = \delta_1 + \delta_2 W_t + \delta_3 W_{t-1} + \delta_4 PHC_t + \delta_5 PHC_{t-1} \dots\dots\dots (4.4)$$

Where, W is war/conflict and PHC is public spending on human capital formation. $\delta_2, \delta_3 < 0$ are the rate of skilled labour force destruction as a result of conflict, $\delta_4, \delta_5 > 0$ are the rate of increase in skilled labour formation (the proportion of unskilled labour force becomes skilled as a result of spending on human capital formation). $\delta_1 > 0$ is total number of skilled labour force in agricultural sector, which is not affected by conflict and spending on human capital formation.

However, unskilled labour in agricultural sector, for Gebrehiwt, determined by the current and previous period conflict/war only as specified below;

$$L_{usa,t} = \alpha_1 + \alpha_2 W_t + \alpha_3 W_{t-1} \dots\dots\dots (4.5)$$

Where, $\alpha_1 > 0$ is the total number of labour force, which is not affected by war/conflict. $\alpha_2, \alpha_3 < 0$ are the rate of unskilled labour force destruction as a result of conflict/war.

A2. Productive Land in Agricultural Sector

Productive land at any time t is a function of the return from the period t and $t-1$ trade balance and the period t number of labour force applied on it. Hence, in linear form, it is given by;

$$PL_t = \rho_1 TB_t + \rho_2 TB_{t-1} + \rho_3 L_{at} \dots \dots \dots (4.6)$$

Where, PL_t is the productive land at time t , TB_t and TB_{t-1} is balance of trade at time t and $t-1$. L_{at} is labour force in the agricultural sector of period t . $\rho_1, \rho_2 > 0$ are the rate at which the productivity of the degraded land maintained as a result of trade surplus due to equivalent exchange in the international trade. $\rho_3 > 0$ is the rate of increase in the productivity of land as a result of increase in labour force due conflict/war.

B. Non Agricultural Output

Output in non agricultural sector is a function of investment in the sector as given by;

$$Y_{n,t} = \mu_1 + \mu_2 I_{na,t} + \mu_3 I_{na,t-1} \dots \dots \dots (4.7)$$

Where, $Y_{n,t}$ is non agricultural output at time t , $I_{na,t}$ is investment in non agricultural sector at time t , μ_1 is the total non agricultural output produced with zero investment in the

sector. $\mu_{2,t} > 0$ are the rate at which output increases as a result of investment in non agricultural sector of period t and t-1.

B.1. Investment in Non Agricultural Sector.

Investment in non agricultural sector at time t is a function of the same period skilled labour force, total output /income, tax rate, and period t-1 public spending on physical capital formation. In linear form it is given by;

$$I_{na,t} = \varphi_1 + \varphi_2 Y_t + \varphi_3 L_{sna,t} + \varphi_4 PIF_{a,t-1} + \varphi_5 T_t \dots\dots\dots (4.8)$$

Where, Y_t is total output (income) in period t, $L_{sna,t}$ skilled labour force in non agricultural sector in period t, $PIF_{a,t-1}$ is public spending on physical capital formation in period t-1, T_t is tax rate at time t. $\varphi_2 > 0$ is the rate of increase in investment in non agricultural sector as a result of increase in total output, $\varphi_3 > 0$ is the rate of increase in investment in non agricultural sector as a result of the availability of skilled labour force, $\varphi_4 > 0$ is the rate of increase in investment as a result of improvement in the availability of infrastructural facilities. $\varphi_5 > 0$ for $T \leq t$ and $\varphi_5 < 0$ for $T \geq t$, is the rate at which investment increases or decrease as a result of tax rate. Where, t is the optimal tax rate beyond which investment is discouraged since the cost would be greater than the benefits.

B.2. Labour Force in Non Agricultural Sector

Likewise, labour force participation in non agricultural sector is also divided into skilled and unskilled labour force.

$$L_{na,t} = L_{sna,t} + L_{usna,t} \dots\dots\dots(4.9)$$

Skilled labour force in non agricultural sector is determined by the current and previous period government spending on human capital formation and conflict/war. Linearly, it is specified as,

$$L_{sna,t} = \gamma_1 + \gamma_2PHC_t + \gamma_3PHC_{t-1} + \gamma_4W_t + \gamma_5W_{t-1} \dots\dots\dots(4.9)$$

Where, $\gamma_1 > 0$ is the total number of skilled labour force, which is not affected by public spending on human capital formation. $\gamma_2, \gamma_3 > 0$ are the rates of increase in skill labour force in non agricultural sector due to public spending on human capital formation. $\gamma_4, \gamma_5 < 0$ are the rate of decrease in skilled labour force in the sector as a result of conflict/war.

Nevertheless, unskilled labour force in non agricultural sector is determined by the current and previous period conflict/war only as specified below;

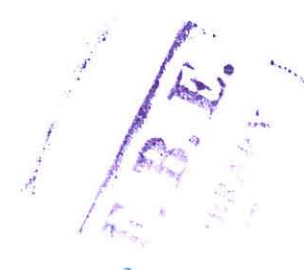
$$L_{usna,t} = \pi_1 + \pi_2W_t + \pi_3W_{t-1} \dots\dots\dots (4.10)$$

Where, $\pi_1 > 0$ is the total number of labour force, which is not affected by war/conflict. $\pi_2, \pi_3 < 0$ is the rate of unskilled labour force destruction as a result of conflict/war.

C. Government Sector

For Gebrehiwot, spending on human and physical capital formation is financed by the tax revenue obtained from international trade and domestic production. Hence, following Alemayehu, (2008), the fiscal stance of Gebrehiwot's model is given by ;

$$PHC_t + PFI_t = T_{d,t} + T_{f,t} = TR \dots\dots\dots(4.11)$$



$T_{d,t}$ is total tax revenue from domestic production, $T_{f,t}$ is total tax revenue from foreign trade activity and TR is total tax revenue. Tax revenue is also a function of total output at time t. hence, in linear form it is specified as;

$$T_{d,t} = \tau_1 Y_t \dots\dots\dots (4.12)$$

Where, $\tau_1 > 0$ is the rate at which domestic tax revenue increase with income. Similarly, tax revenue from foreign trade at any time t is a linear function of necessary ($M_{n,t}$) and luxury ($M_{l,t}$) import item, which is given by;

$$T_{f,t} = \tau_2 M_{n,t} + \tau_3 M_{l,t} \dots\dots\dots (4.13)$$

Where, $\tau_3 > \tau_2 > 0$ are the tariff rates on luxury and necessary import items respectively.

4.1.2 External Sector

The external sector of Gebrehiowt model consists of trade balance (net export) and terms of trade. For him, trade balance at any time t is determined by terms of trade of the same period. In linear form it is given by;

$$TB_t = \theta_1 TOT_t \dots\dots\dots (4.14)$$

TOT_t is terms of trade at time t, $\theta_1 > 0$ is the rate of improvement in the balance of trade as a result of improvement in the terms of trade.

Moreover, terms of trade for a given country in any period t is determined by the ratio of skilled to unskilled labour force, which determines the external stance of the country.

$$TOT_t = \omega \frac{L_{s,t}}{L_{us,t}} \dots\dots\dots (4.15)$$

Where, $\omega > 0$ is the rate of improvement in terms of trade as a result of the increase in the ratio of skilled to unskilled labour force, which will make the country's output competitive in the international market.

4.2 The Dynamics of Gebrehiwt Model

In Gebrehiwt model, the dynamics of the state variables, which includes total output, agricultural output, and non agricultural output, labour force in both sectors(agricultural and non agricultural), productive land and trade balance, comes from the change in the control variables ; war, public spending on human and physical capital formation , tax rate and terms of trade (unequal exchange). For instance, the movement of labour force in both agricultural and non agricultural sector from period t to period t+1 is depends upon the period t conflict and spending on human capital formation. Likewise, the transition of total productive land from period t to period t+1 is depends upon the period t conflict and unequal exchange. Moreover, the shift of investment in non agricultural sector from period t to period t+1 depends up on availability of infrastructural facility, income, public spending on human capital formation and the level of tax rate at period t. The cumulative impact of the above state variables (i.e. labour force, productive land and investment in non agricultural sector) determines the movements of total output from time t to time t+1.

In order to assess how output moves through time, in the next subsection we will analyze the transition of labour force in agricultural sector, productive land and investment in non agricultural sector from period t to period t+1.

4.2.1 Transitional Dynamics of the Labour force

As we have already discussed, labour force in agricultural and non agricultural sector is a function of conflict and public spending on human capital formation. Disaggregating the total labour force into skilled and unskilled labour as we did earlier, skilled labour force at time t+1 will be the sum of those skilled labour forces that survived from the time t conflict/war and the proportion of the survived unskilled labour force that become skilled as result of public spending on human capital formation. Mathematically it is given by,

$$L_{s,t+1} = (1 - \varphi)L_{s,t} + \gamma(1 - \sigma)L_{us,t} \dots \dots \dots (4.16)$$

Where, $\varphi \in (0,1)$ is the proportion of skilled labour force destructed as a result of war/conflict and $1 - \varphi$ is the proportion of skilled labour force survived from war/conflict of time t. Likewise, $\sigma \in (0,1)$ is the proportion of unskilled labour force destructed as a result conflict and $1 - \sigma$ is the proportion of unskilled labour force survived from war/conflict.¹¹ Moreover, $\gamma \in (0,1)$ is proportion of the survived unskilled labour force at time t that becomes skilled at time t+1 as a result of spending on human capital formation.

Similarly, unskilled labour force at time t+1 is the proportion of the unskilled labour forces that survived form conflict but remain unskilled at time t. This relationship is given by;

$$L_{us,t+1} = (1 - \sigma)L_{us,t} \dots \dots \dots (4.17).$$

¹¹ In the model, it is assumed that the proportion of unskilled labour force destruction in war/conflict is greater than the proportion of skilled labour force destruction. i.e. $\sigma > \varphi$

The total labour force at period t+1 is given by the sum of skilled and unskilled labour force in the sector of the same period, which is given by;

$$L_{t+1} = (1-\varphi)L_{s,t} + \gamma(1-\sigma)L_{us,t} + (1-\sigma)L_{us,t}$$

$$L_{t+1} = (1-\varphi)L_{s,t} + (1-\sigma)(\gamma L_{us,t} + L_{us,t}) \dots\dots\dots(4.18)$$

4.2.2 Transitional Dynamics of Productive Land

Productive land, as we have observed, is a function of the return from international trade and the number of labour force applied on it. The total productive land area at any time t+1 is the difference between the productive land at time t, and the land which degraded as a result of unequal exchange at time t and the land hitherto cultivated but left barren at period t as a result of lack of labour force due to conflict/war.¹² Hence, mathematically it is given by;

$$PL_{t+1} = PL_t - \rho PL_t - \delta PL_t$$

$$PL_{t+1} = (1 - \rho - \delta) PL \dots\dots\dots(4.19)$$

Where, $\rho \in (0,1)$ is the proportion of the productive land degraded as result of unequal exchange and $\delta \in (0,1)$ is the proportion of the productive land left barren as a result of lack of human power.

¹² Maintenance of the degraded land as a result of trade surplus and cultivation of the barren land as a result of higher labour force participation assumed not to increase the total cultivable productive land beyond its maximum since it is impossible to maintain the productivity of the uncultivated land as well as it is impossible to cultivate the degraded land.

4.2.3 Transitional Dynamics of Investment

The transition of investment from time t to time $t+1$ is determined by the availability of infrastructure, taxation and skilled labour. Thus, investment at time $t+1$ is the difference between the sum of the previous period investments in the sector and the proportion of new additional investment ($\lambda \in (0,1)$) resulted from higher income, lower tax rate, improved infrastructural facility and labour quality, and the proportion of investment which shut down their business due to the deterioration of the quality of infrastructure ($\theta \in (0,1)$), high level of taxation ($\omega \in (0,1)$), the loss in skilled labour force at time t ($\vartheta \in (0,1)$). Hence, mathematically it is given by;

$$I_{na,t+1} = (I_{na,t} + \lambda I_{na,t+1}) - (\theta I_{na,t} + \omega I_{na,t} + \vartheta I_{na,t}) \dots\dots\dots(4.20)$$

Rearranging the above equation, the state of transition for investment from time t to time $t+1$ is given by;

$$(1 - \lambda)I_{na,t+1} = (1 - \theta - \omega - \vartheta)I_{na,t}.$$

$$I_{na,t+1} = \frac{(1 - \theta - \omega - \vartheta)}{(1 - \lambda)} I_{na,t} \dots\dots\dots(4.21)$$

As we have already observed, the change in the above state variables comes into effect due to the dynamics of conflict, human and physical capital formation, unequal exchange and taxation. In the next sub section, we will discuss the dynamics of each control variables outlined above except taxation and physical capital formation since their link to the overall model is minimal.

4.2.4 Dynamics of Conflict

Given the model stated above, an incidence of war at time t leads to a reduction of the skilled and unskilled labour force participation in both agricultural and non agricultural sector for the same period. The reduction of labour forces in agricultural sector leave the land hitherto cultivated barren for the subsequent period. Consequently, agricultural output will decline. Similarly, due to the effect of war on skilled and unskilled labour force, the terms of trade deteriorates and hence reduce total productive land and agricultural output. Moreover, the reduction of labour force in non agricultural sector at time t will lead to a decrease in investment in non agricultural sector and output in the sector. Given, the transitional dynamics of labour force, the reduction in the number of labour force in both sectors in period t will result lower labour force participation in period $t+1$ and hence total output in the economy of period $t+1$.

4.2.5. Dynamics of Unequal Exchange

The deterioration of terms of trade as a result of unequal exchange at period t results a loss (deficit) in the international trade. Since the export product of the smaller country is assumed to be agricultural item in Gebrehiwt model, the loss in trade will not allow the productivity of the used up land to be maintained as it is intensively cultivated. Degradation of the land further leads to a reduction of agricultural output for the same period. Moreover, given the transitional dynamics of the productive land, environmental degradation at time t results in lower productivity of the land at time $t+1$, which will bring lower output at time $t+1$.

.2.6 Dynamics of Human Capital Formation

In Gebrehiwot model, an increase in public spending on human capital formation results in an increase in the number of skilled human power on both agricultural and non agricultural sector. The increase in labour force in agricultural sector leads to an increase in the amount of the total cultivable productive land. Together with the direct effect of high labour force participation, the increase in the amount of productive land will increase output in agricultural sector. Moreover, the increase in skilled labour force participation in non agricultural sector also results in an increase in investment in the sector, which will increase non agricultural output. Likewise, increasing spending on human capital formation has an impact of increasing the competitiveness of the country's product in the international market since the quality of labour embodied on internationally tradable item and hence terms of trade improves . Improvement in terms of trade further brings positive trade balance, which will help the country to maintain the productivity of the used up land. Furthermore, conditioned up on the transitional dynamics of investment, the increase investment in the sector at time t leads to an increase in the time $t+1$ investment and hence, non agricultural output of time $t+1$.

4.3 Equilibrium Path of Gebrehiwot Model

As we have observed, in Gebrehiwot model, the impact of conflict on labour force does not only affect productive land and agricultural output of the economy but also the external sector stance of the economy. For instance, with the incidence of conflict, the number of skilled labour forces participation declines, which will make the product of the country uncompetitive in the international market and hence, deterioration of terms of trade and trade balance. Similarly, an increase of public spending on human capital formation will allow the country's

product to be competitive in the international market and improve its terms of trade and trade balance. Terms of trade deterioration on the other hand results in environmental (land) degradation and hence reduces agricultural output. The decline in agricultural output further affects investment in non agricultural sector through its impact on total output.

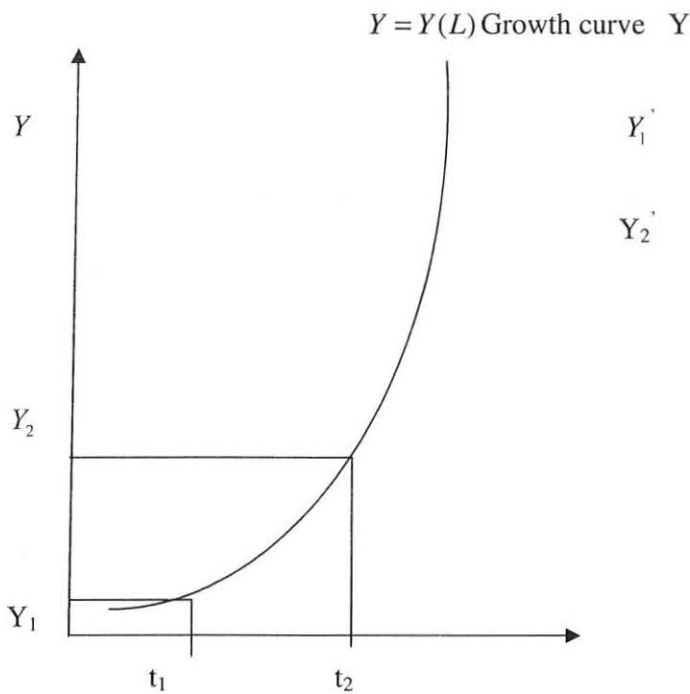
Hence, in his model, total output decreases with conflict/political instability and unequal exchange, which results in land degradation. Conversely, total output of the economy grows with public spending on human capital formation, political stability *and trade equivalence*, which determines the total labour forces participation. Taking total output, which is the sum of agricultural and non agricultural output, labour force and productivity of land(land degradation) as the state variables, and conflict (political stability), trade equivalence and spending on human capital formation as a control variable, we will show the equilibrium path of developing country's (Ethiopian) economy in Gebrehiwot model graphically.

The four quadrant graph depicted below shows the relationship between output (Y), land degradation (LD), labour force (both skilled and unskilled) (LF), spending on human capital formation (PHC) and political stability and trade equivalence (TE). Quadrant I shows the relationship between land degradation and total output, quadrant II shows the relationship between stability and land degradation as well as trade equivalence and land degradation, quadrant III shows the relationship between spending on human capital formation and total labour force (skill and unskilled), stability and labour force as well as trade equivalence and labour force, and quadrant IV depicts the relationship between labour force and total output.

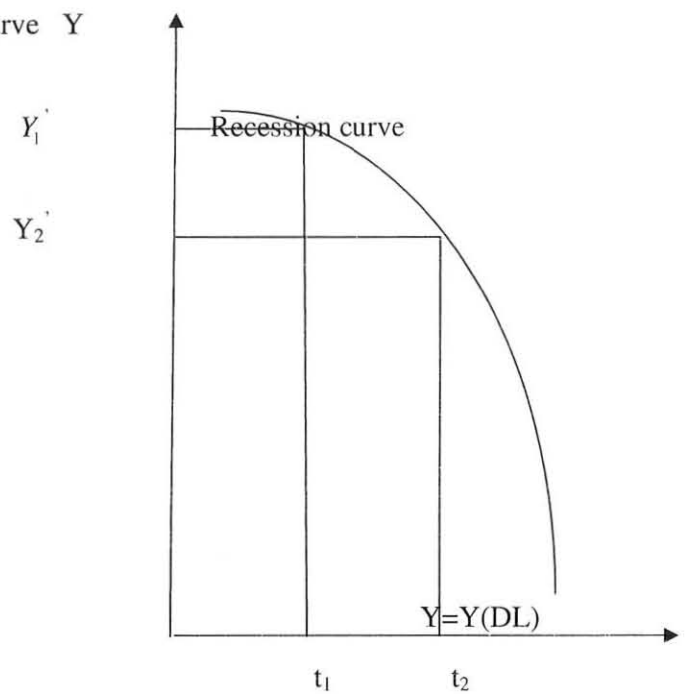
From the graph below, deterioration in the condition of political stability and equivalence of trade between trading partners' at any time t from point OC_2 to OC_1 worsens the level of land

degradation from OX_2 to OX_1 since the destruction of labour force with political instability and trade deficit from unequal exchange is enormous. The movement in the level of land degradation from OX_2 to OX_1 results the fall in output from OY_2 to OY_1 of the same period. In contrast, an increase in public spending on human capital formation and improvement in the condition of political stability and trade equivalence from OC_1 to OC_2 at time t results in an improvement in the number of total labour force (skilled and unskilled) from OX'_1 to OX'_2 and hence total output from OY_1 to OY'_1 .

Hence, from quadrant IV, we can conclude that total output grows with labour force participation through time resulted from spending on human capital formation, political stability and trade equivalence. Likewise, from quadrant I, we can see as it declines with land degradation resulted from unequal exchange and low level of labour force participation due to conflict. Hence, putting total output in the vertical axis and time in the horizontal axis, the dynamics of growth and recession curve of Gebrehiwot model can be depicted graphically as follow,



Growth Curve Figure (4.2)



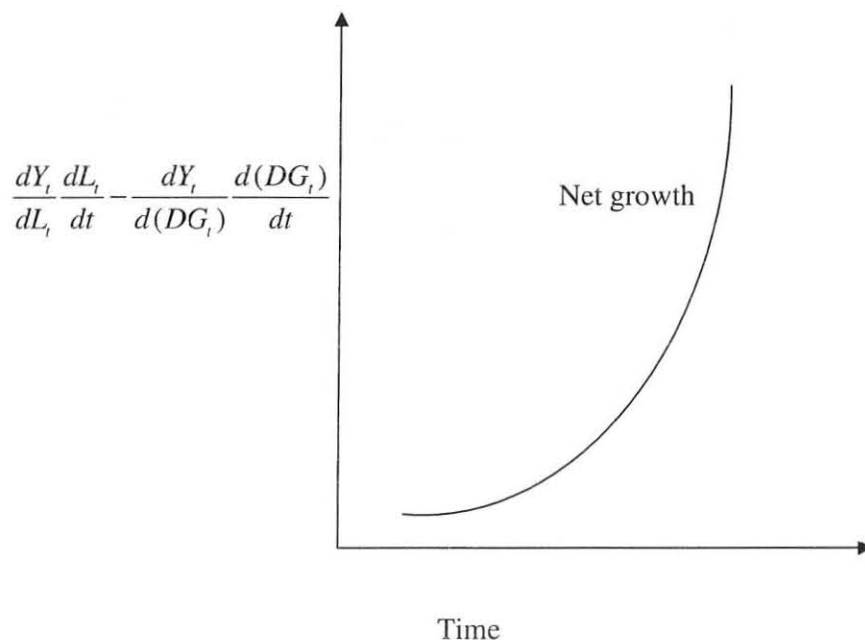
Recession Curve Figure (4.3)

Depending on the net effect of growth and recession, the economy grows or decline through time. Graphically, if the slope of growth curve is greater than the slope of recession curve

(i.e. $\frac{Y_2 - Y_1}{t_2 - t_1} > \frac{Y'_2 - Y'_1}{t_2 - t_1} \Rightarrow \frac{dY}{dt} > \frac{dY'}{dt}$), the net growth curve will have a positively up ward slope

as depicted below. In this is case, the economy is an upward trend of the equilibrium path.

Figure (4.4). Positively upward Equilibrium Path of net growth curve

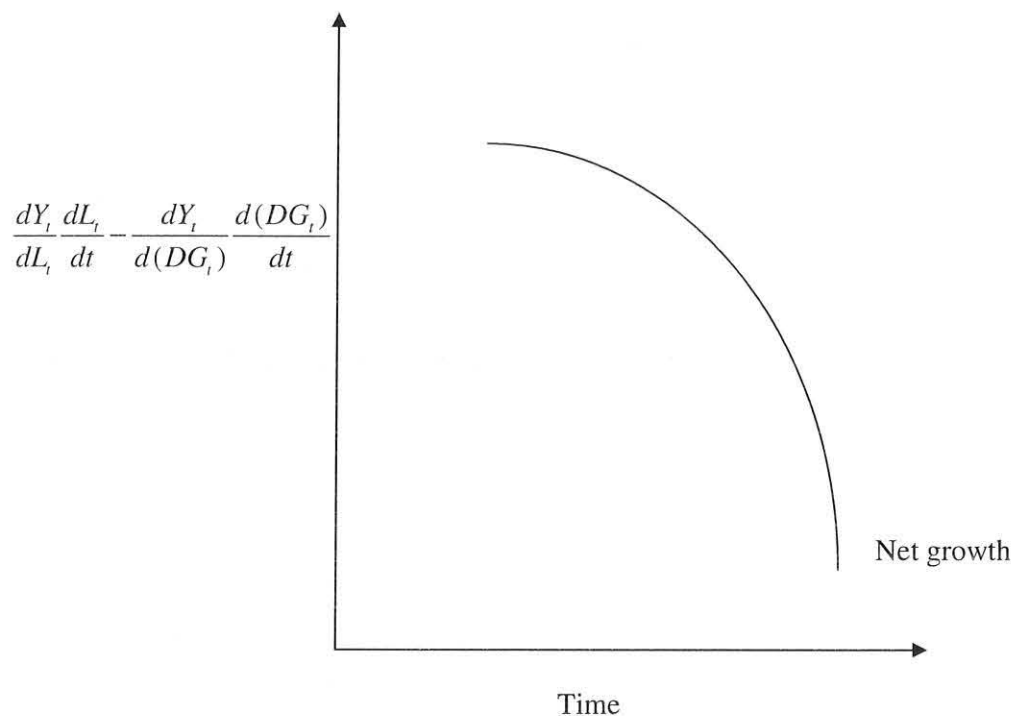


Conversely, if the slope of the growth curve is less than the slope of recession curve

(i.e. $\frac{Y_2 - Y_1}{t_2 - t_1} < \frac{Y'_2 - Y'_1}{t_2 - t_1} \Rightarrow \frac{dY}{dt} < \frac{dY'}{dt}$), the net growth will have a negatively downward curve as

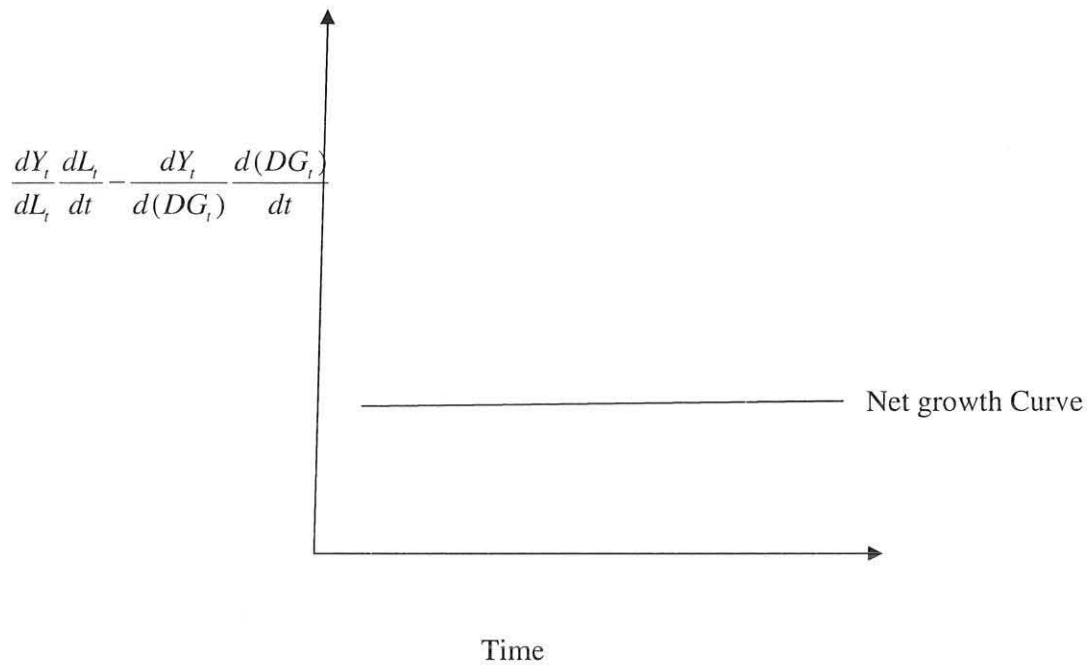
depicted below. This is the case, when the economy is in down ward trend of the equilibrium path.

Figure (4.5). Negatively downward Equilibrium Path of net growth curve



If the slope of the growth curve is equal to the slope of recession curve (i.e. $\frac{Y_2 - Y_1}{t_2 - t_1} = \frac{Y'_2 - Y'_1}{t_2 - t_1} \Rightarrow \frac{dY}{dt} = \frac{dY'}{dt}$), the net growth will have horizontal line curve, in which case the economy is at the steady state through time.

Figure (4.6) Steady state net Growth Curve



- Mathematically, the growth in output through time is given by $\frac{dY_t}{dt}$ is the same as the decline
- (recession) of output through time given by $\frac{dY_t}{dt}$. Based on the theory stated above, output grows with labour force participation, hence using the chain rule, the growth of output can be rewritten as $\frac{dY_t}{dt} = \frac{dY_t}{dL_t} \frac{dL_t}{dt}$. Likewise, total output also falls with land degradation, hence it can be rewritten as $\frac{dY_t}{dt} = \frac{dY_t}{d(DG_t)} \frac{d(DG_t)}{dt}$.

Hence, the equilibrium path of the model is depend up on either the slope of the growth equation is greater or less or equal to the slope of the recession curve given by

$$\frac{dY_t}{dL_t} \frac{dL_t}{dt} >, = \text{ or } < \frac{dY_t}{d(DG_t)} \frac{d(DG_t)}{dt} \dots\dots\dots(4.22)$$

From equation (4.1), (4.2), (4.7) and (4.8) we have $Y_t = Y_{at} + Y_{nat}$, $Y_{at} = f(L_t, PL_t)$, $Y_{nat} = f(I_{nt})$, and $I_{n,t} = f(Y_t, PIF_t, L_{sna,t}, T_t)$ respectively. Hence, the left hand side of equation of (4.22) will take the following form as given by;

$$\frac{dY}{dL} \frac{dL}{dt} = \left[\frac{df(L_{at}, PL_t)}{dL_t} + \frac{dY_{na}}{dI_{na,t}} \frac{df(Y, L_{nas,t}, PIF_t, T_t)}{dL} \right] \frac{dL_t}{dt} \dots\dots\dots (4.23)$$

Moreover, total labour force which is given by the sum of labour force in agricultural and in non agricultural is given by the model;

$$L_t = L_{a,t} + L_{na,t} \dots\dots\dots (4.24)$$

Hence, agricultural labour force at any time t is given by;

$$L_{a,t} = L_t - L_{na,t} \dots\dots\dots (4.25)$$

Substituting equation (4.9) into equation (4.24) and solve for skilled labour force in non agricultural sector gives us;

$$L_{sna,t} = L_t - L_{a,t} - L_{sna,t} \dots\dots\dots (4.26)$$

Substituting equation (4.25) and (4.26) into equation (4.23), the slope of growth curve through time will be given by:

$$\frac{dY}{dL} \frac{dL}{dt} = [\beta_2 + \mu_2 \phi_2] \frac{dL}{dt} \dots\dots\dots (4.27)$$

where $\beta_2 = \frac{df(L_t - L_{na,t}, PL_t)}{dL_t}$, derived from equation (4.2),

$$\mu_2 = \frac{dY_{n,t}}{dI_{n,t}}, \text{ derived from equation (4.7) and}$$

$$\varphi_3 = \frac{df(Y_t, (L_t - L_{a,t} - L_{usna,t}), PIF_t, T_t)}{dL_t}, \text{ derived from equation (4.8).}$$

The marginal product of labour force participation in the economy, which is the sum of marginal product of labour in agricultural sector (β_2) and non agricultural sector ($\mu_2\varphi_3$) is given by;

$$\frac{dY}{dL} = [\beta_2 + \mu_2\varphi_3] \dots\dots\dots (4.28)$$

Moreover, assuming the rate of skilled and unskilled labour force destruction equal i.e. $\sigma = \varphi$ in equation (4.18), the growth in labour force can be derived as follow

$$L_{t+1} = (1 - \varphi)L_{s,t} + (1 - \varphi)(\gamma L_{us,t} + L_{us,t})$$

$$\Rightarrow L_{t+1} = (1 - \varphi)L_t + \gamma L_{us,t} \dots\dots\dots(4.28A)$$

Since $L_t = L_{s,t} + L_{us,t}$

Moreover, $L_t = L_{s,t} + L_{us,t} \Rightarrow L_t = (1 - \phi_t)L_t + \phi_t L_t$, Where, $\phi_t \in (0,1)$,

$$\text{Hence, } L_{us,t} = \phi_t L_t \dots\dots\dots(4.28B)$$

Substituting equation (4.28B) into equation (4.28A), the growth in labour force from time t to time t+1 gives us

$$\Rightarrow \frac{L_{t+1} - L_t}{L_t} = \frac{dL_t}{dt} = \phi_t \gamma - \varphi \dots \dots \dots (4.29)$$

Substituting equation (4.29) into (4.27), the growth in output with respect to labour force participation in Gebrehiowt model is given by:

$$\frac{dY}{dL} \frac{dL}{dt} = (\phi_t \gamma - \varphi) [\beta_2 + \mu_2 \varphi_3] \dots \dots \dots (4.30)$$

Equation (4.30) is the rate of growth in output due to an increase in the labour force participation. Where, $(\phi_t \gamma - \varphi)$ captures the rate of growth in labour forces participation through time as a result of spending on human capital formation and political stability.

Similarly the right hand side of equation (4.22) can be rewritten as

$$\frac{dY_t}{dDG_t} \frac{dDG_t}{dt} = \left[\frac{dY_{a,t}(L_{a,t}, PL_t)}{dDG_t} + \frac{dY_{na,t}}{dDG_t} \right] \frac{dDG_t}{dt} \dots \dots \dots (4.31)$$

Using the chain rule, the change in non agricultural output with respect to land degradation can be derived as:

$$\frac{dY_{na,t}}{dDG_t} = \frac{dY_{na,t}}{dI_{na,t}} \frac{dI_{na,t}}{dY_t} \frac{dY_t}{dDG_t} \dots \dots \dots (4.32)$$

From equation (4.2), since we have assumed $PL_t = -DG_t$

$$\frac{dY_{a,t}(L_{a,t}, -DG_t)}{d(DG_t)} = \frac{dY_{a,t}(L_{a,t}, -DG_t)}{d(DG_t)} = -\beta_3 \dots \dots \dots (4.33)$$

$$\text{, from equation (4.7), } \frac{dY_{na,t}}{dI_{na,t}} = \mu_2 \dots \dots \dots (4.34)$$

and from equation (4.8) $\frac{dI_{na,t}}{dY_t} = \varphi_2$ (4.35)

Substituting equation (4.33), (4, 34) and (4.35) into equation (4.31), the slope of the recession curve can be rewritten as ;

$$\frac{dY_t}{d(DG_t)} \frac{d(DG_t)}{dt} = (-\beta_3 + \mu_2 \varphi_2) \frac{dY_t}{d(DG)} \frac{d(DG_t)}{dt}, \text{ ignoring } \frac{d(DG_t)}{dt} \text{ for a moment,}$$

$$\frac{d(Y_t)}{d(DG_t)} = -\beta_3 + \mu_2 \varphi_2 \frac{d(Y_t)}{d(DG_t)} \Rightarrow \frac{d(Y_t)}{d(DG_t)} (1 - \mu_2 \varphi_2) = -\beta_3,$$

$$\Rightarrow \frac{d(Y_t)}{d(DG_t)} = \frac{-\beta_3}{1 - \mu_2 \varphi_2}, \text{ (4.36)}$$

Equation (4.36) is the marginal deterioration of total output due to land degradation.

$$\Rightarrow \frac{dY}{dDG} \frac{dDG}{dt} = \frac{-\beta_3}{1 - \mu_2 \varphi_2} \frac{d(DG_t)}{dt} \text{ (4.37)}$$

Since the theory states the productivity of the land always declines with unequal exchange

and land degradation , the rate of degradation ($\frac{dDG_t}{dt}$) is equal to the declining in the

productivity of the land ($\frac{dPL_t}{dt}$) . Hence, from equation (4.9) the rate of land degradation can

be derived as;

$$\frac{PL_{t+1} - PL_t}{PL_t} = \frac{dPL_t}{dt} = \frac{d(DG_t)}{dt} = -\rho - \delta \text{ (4.38)}$$

Substituting equation (4.38) into (4.37), the slope of the recession curve as result of land degradation can be rewritten as;

$$\Rightarrow \frac{dY}{dDG} \frac{dDG}{dt} = \frac{-\beta_3}{1-\mu_2\phi_2}(-\rho-\delta) \Rightarrow \frac{dY}{dDG} \frac{dDG}{dt} = \frac{\beta_3}{1-\mu_2\phi_2}(\rho+\delta) \dots\dots\dots(4.39)$$

Where, $\rho+\delta$ captures the rate of land degradation due to unequal exchange and conflict.

Since the marginal productivity of investment in non agricultural sector increase at increasing rate at the beginning (when there is low investement) i.e $\mu_2\phi_2 > 1$, equation (4.39) represents the rate of fall in output due to land degradation through time.

The net growth in output according to Gebrehiwot is depending up on the difference between the rate of growth in output due to labour force participation and the absolute value of the rate of recession due to land degradation .i.e.

$$\frac{dY_t}{dL_t} \frac{dL_t}{dt} - \frac{dY_t}{dDG_t} \frac{dDG_t}{dt} = ((\gamma\phi_t - \varphi)(\beta_2 + \varphi_3\mu_2) - |\frac{\beta_3}{1-\mu_2\phi_2}(\rho+\delta)|) >, <= 0 \dots\dots\dots(4.40)$$

Based on equation (4.40) we have three scenarios in the equilibrium path of Gebrehiwot dynamic macroeconomic model.

Scenario one; $((\gamma\phi_t - \varphi)(\beta_2 + \varphi_3\mu_2) - |\frac{\beta_3}{1-\mu_2\phi_2}(\rho+\delta)|) = 0$, is the case when the economy is at equilibrium or at steady state since the increase in total output as a result of increasing labour force participation due to spending on human capital formation, trade equivalence and political stability equally offset the negative impact of land degradation to total output due to political instability and unequal exchange.

Scenario two; $((\gamma\phi_t - \varphi)(\beta_2 + \varphi_3\mu_2) - \frac{\beta_3}{1 - \mu_2\varphi_2}(\rho + \delta)) > 0$ is the case when the economy is growing and there is an upward trend in the equilibrium path since the positive impact of labour force participation to the total output due to spending on human capital formation, trade equivalence and political stability more than offset the negative impact of land degradation to the total output due to conflict (political instability) and unequal exchange.

Scenario three; $((\gamma\phi_t - \varphi)(\beta_2 + \varphi_3\mu_2) - \frac{\beta_3}{1 - \mu_2\varphi_2}(\rho + \delta)) < 0$, is the case when the forces of land degradation as a result of political instability and unequal exchange less than offset the positive impact of labour force participation due to spending on human capital formation, political stability and trade equivalence and hence downward trend of the equilibrium path. This scenario, according to Gebrehiwot, represents most developing countries (especially Ethiopia's economy) case where the recurrence of conflict (political instability) and unequal exchange has been determining the overall economic activity.

In sum, Gebrehiwot's model has fifteen state variables, which is determined with in the model and five control variables, which determines the movement of the over all model¹³. Moreover, it has eight transitional dynamic and five identity equations. Using the Ethiopian data on the model variables for the years covering 1981-2006, in the next chapter we have simulated the model dynamically in scenario of a shock to public spending on human capital formation and political stability.

¹³ All equations of the model along with the estimated and calibrated coefficients are presented in the appendix.

Chapter five

5.1 The Data

The data employed for simulation purpose in this study is obtained from World Bank development indicators of 2008 and the ministry of finance and economic development of Ethiopia (MOFED). Specifically, data for agricultural output, non agricultural output, total output, tax rate, domestic tax revenue, public spending on human capital formation and public spending on infrastructure are obtained from MOFED. Whereas data for trade balance, investment, terms of trade, total land area under cereal production, which is a proxy for total productive land, military spending, which is a proxy for war, skilled and unskilled labour force participation and labour force participation in agricultural and non agricultural sector are found from the World Bank development indicator of 2008.

Data for tax rate, terms of trade, skilled and unskilled as well as agricultural and non agricultural labour force participation are not available directly from the aforementioned sources. Hence, in order to find tax rate we have taken the ratio of real tax revenue to real GDP for each year as a proxy variable. As for skilled and unskilled labour force, we multiplied the total labour force of each year by the respective year average literacy rate. Furthermore, in calculating agricultural and non agricultural labour force, we have multiplied each year's labour force by the percentage of labour force engaged in agricultural and non-agricultural sector. Moreover, in solving the terms of trade we followed two procedures, first we have taken the ratio of volume of export at current price to volume of export at constant price and volume of import at current price to volume of import at constant price so as to find the unit price of export and import respectively. Having obtained the unit price of export and import,

we have taken their respective ratio to find the terms of trade. The value of public spending on human capital formation is the sum of public spending on health and education. Whereas, the value of public spending on infrastructure is given by the sum of public investment on road construction and transport and communication.

5.2 Dynamic Simulation of the Model

In Gebrehiowt's model, since specifications run from highly abstract and theoretical equation to very handful and easy identities, the parameters of the model is chosen in a fairly eclectic way. The ideal method would be a full system estimation of the entire model, which would respect all the relevant cross restrictions. Nevertheless, full system estimation is unfortunately not realistic for several reasons. For instance, the amount of parameters is reasonably large and the model frequency is annual, so that the degree of freedom would be short. On the other hand, we cannot fully rely entirely on an eclectic calibration approach that borrows from the literature since the current literature on macroeconomic modeling hardly represents Gebrehiowt's model. Hence, in practice we end up with a mixed strategy which we can compactly state as "estimate when possible, then calibrate to generate reasonable behaviour as a system".

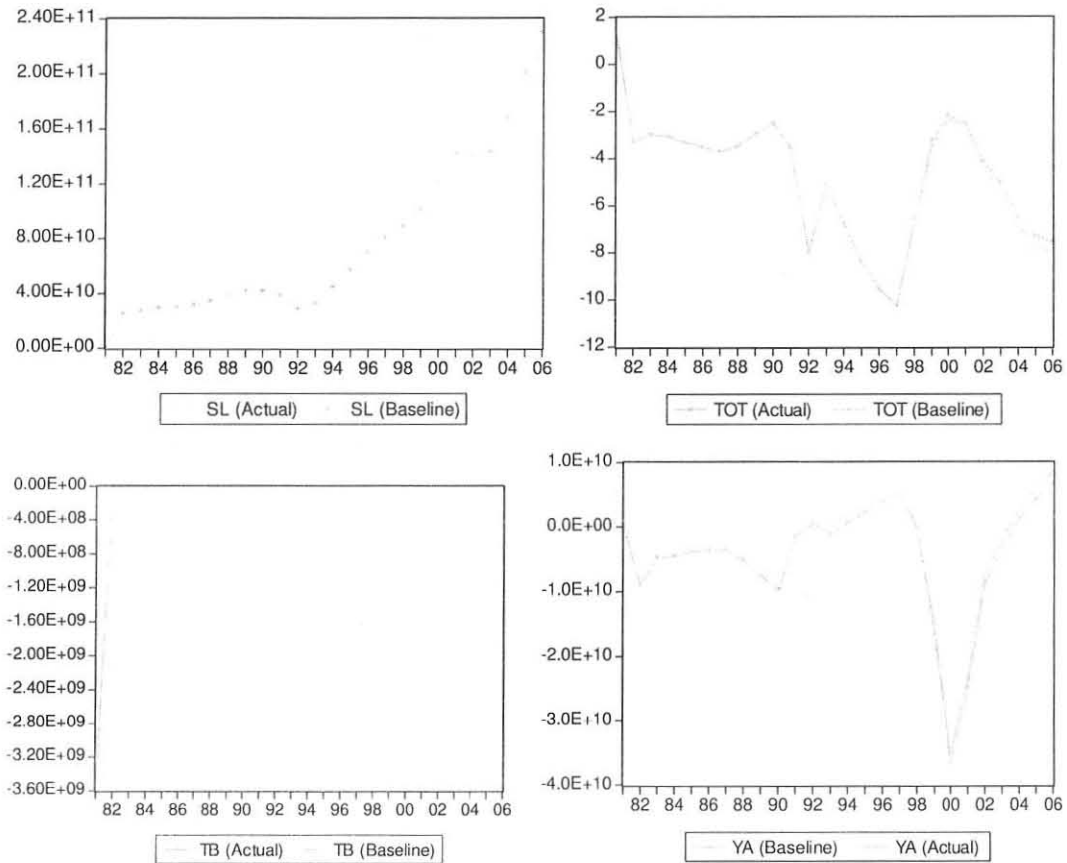
Using the dynamic deterministic solution and Gauss- Seidle iteration algorithm¹⁴, we have solved the whole model for the year covering 1981 to 2006. Moreover, in order to use the model for policy analysis, a counter factual simulation is conducted dynamically in the scenario of a shock to public spending on human capital formation and a shock to political instability.

¹⁴ The description about Gauss-Seidel iteration algorithm is presented in appendix C.

5.2.1 A Shock to Public Spending on Human Capital Formation

For Gebrehiwot, increasing spending on human capital formation has a crucial role in boosting skill labour force participation, domestic production (both agricultural and non agricultural), government tax revenue as well as in moderating the loss from international trade. To see the impact, a permanent 5% increase in spending on human capital formation is introduced for the years covering 1998 to 2006. It results an increase in the skilled labour force participation by 37 % in 1998, 6.2% in 2002 and 5.7% in 2006. The increase in the number of skilled labour force participation brings an improvement in the terms of trade and trade balance by 37%, 6.2% and 5.7% for the year 1998, 2002 and 2006 respectively. The improvement in the terms of trade and trade balance increases the productivity of the land by 10.5%, 15.5% and 29.5% for the respective years of 1998, 2002 and 2006. This further brings an increase in the total agricultural output of the economy by 10.5%, 15.5% and 29.5% for the same periods. The increase in agricultural output results an improvement of tax revenue by 0.06% in 1998, by 0.063% in 2002 and 0.065% in 2006. Graphically, the impact of a 5% increase in spending on human capital formation on the aforementioned variables is depicted below

Figure 5.1 The impact of a 5% increase in public spending on human capital formation from 1998-2006¹⁵



5.2.2. A Shock to Political Stability

For Gebrehiwot, with the incidence of conflict, the labour force participation (both skilled and unskilled) in agricultural and non-agricultural sector declines, the decrease in labour force participation leave the land hitherto cultivated barren and results a fall in agricultural

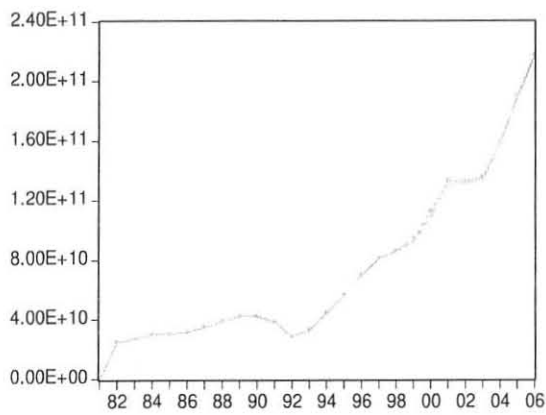
¹⁵ Where, SL= Skilled labour force, TOT= Terms of trade, TB= Trade Balance, YA = Agricultural output and TR=Tax revenue

production. On the other hand, it also affects the competitiveness of the country's product in the international market since its impact on skilled and unskilled labour force determines the terms of trade and balance of trade.

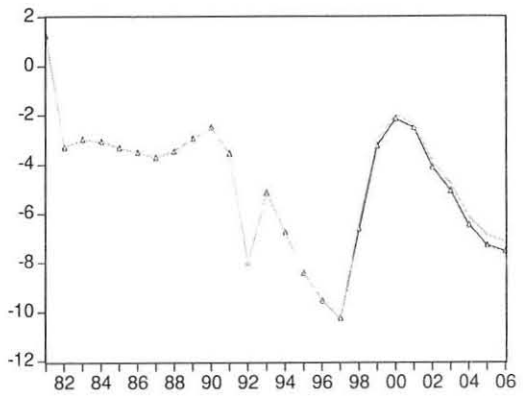
Thus, to examine its impact, a permanent 5% increase in military spending is introduced for the years covering 1998 to 2006. It is found that skilled labour force participation in agricultural sector decreased by 0.8% in 1998, 1.5% in 2002 and 0.9% in 2006. The reduction in skilled labour force participation brings a deterioration of terms of trade by 3.9%, 5.9% and 5.5% for the respective years of 1998, 2002 and 2006. Deterioration in the terms of trade further worsens trade deficit by 3.91%, 5.93% and 0.06% for the same periods. Together with the direct impact of the decline in labour force participation in agricultural sector, deficit in the balance of trade reduces the productivity of the land by 16.9%, 20% and 24.6% for the year 1998, 2002 and 2006. Consequently, agricultural production for the same period decreases by 16.3%, 20.39 % and 24.5% respectively. The fall in agricultural production along with a 0.05%, 0.09% and 0.14% decrease in non agricultural production results a 0.013%, 0.05%, 0.07% reduction in tax revenue for the year 1998, 2002 and 2006. Graphically, the impact of an incidence of conflict, which results a 5% increase in military spending on the aforementioned variables is given below.

Figure 5.2 The impact of 5% increase in military spending due to the incidence of conflict.¹⁶

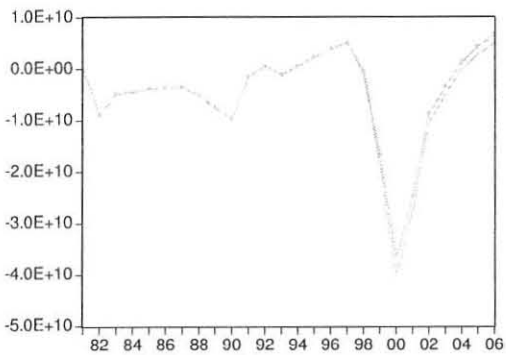
¹⁶ Where, SL=skilled labour force, TOT= terms of trade, YA=Agricultural output, PL= productive land, TB=trade balance, TR=Tax revenue and YNA= non agricultural output.



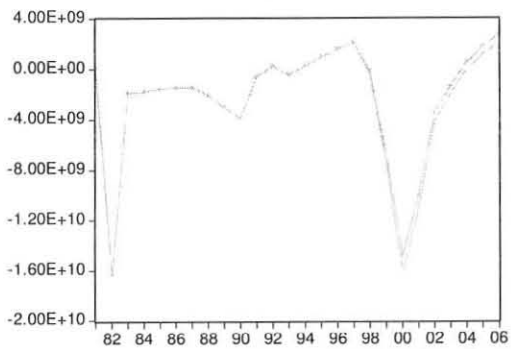
— SL (Baseline) — SL (Actual)



—△— TOT (Actual) — TOT (Baseline)



— YA (Baseline) — YA (Actual)



— PL (Baseline) — PL (Actual)

Chapter Six

6.1 Conclusion

In our study, an attempt is made in order to build a mathematical model for Gebrehiowt's dynamic theories. Specifically, we have modeled his dynamic theories emerged from conflict, unequal exchange, public spending on human capital formation. The dynamic theories state the number of labour force participation declines with conflict (political instability). This further result, the land hitherto cultivated to be left barren and hence a fall in agricultural production. Likewise, terms of trade and trade balance deteriorate with unequal exchange in the international trade, which do not allow the productivity of the used up land (natural resource) to be maintained. This further reduces agricultural production for the subsequent periods. Moreover, an increase in the skilled labour force participation resulted from spending on human capital formation brings terms of trade improvement, which reduces the loss from international trade. The improvement in balance of trade together with an increase in the skilled labour force participation increases the productivity the barren and degraded land, which subsequently increases the total output of the economy.

Moreover, in driving the equilibrium path of the model, it is found that the overall economy grows with labour force participation and declines with land degradation. The net effect of labour force participation and land degradation is depend up on the rate of growth in output with respect to labour force participation and the rate of decrease in output with respect to land degradation. If the rate of growth of output with respect to labour force greater than the rate of decrease in output with respect to land degradation, the economy will be in the upward trend of the equilibrium path. In the scenario of which, the rate of growth of output with respect to

labour force participation is less than the rate of decreases of output with respect to land degradation, the economy will be in the downward trend of the equilibrium path. When the rate of increase in output with respect to labour force participation is equal to the rate of decrease in output with respect to land degradation, the over all economy will be at the steady state.

Given the estimated and calibrated parameters, a dynamic counter factual simulation is also conducted under the scenario of a shock to political stability (conflict) and a shock to spending on human capital formation. The simulated result on both scenarios is found to be consistent with the theory.

Bibliography

Abdulhakim, Abubeker. (1997). "Development of Economic Ideas in Ethiopia", Bachelor's thesis, Department of Economics, Addis Ababa University

Alemayehu, Geda (2002). "The Gebrehiwot model: A Pioneer African Development Macroeconomists", Ethiopian Journal of Development Research. Vol. 24, 1. Addis Ababa: Addis Ababa University Printing Press.

Alemayehu, Geda, (2008). "Ethiopian Macroeconomic Modeling in Historical Perspectives Bringing Gebrehiwot and His Contemporaries in to the Ethiopian Macroeconomic Realm", Journal of East African Studies. Vol.10, 1. Michigan State University Press.

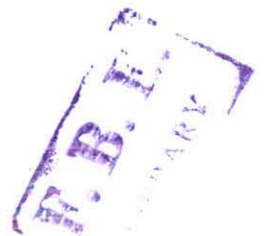
Amin, Samir, (1974) "Accumulation on a world scale: A critique of the theory of Underdevelopment" Vol I and II, London Monthly review Period.

Ayele, Zewege, (1980). "Naggadras Gebrehiwot Baykedagn's Political-Economy Ideas: A Review of His Book Government and Public Administration". Meskrem Magazine, no. 3 and 5

Alem, Eshete (1973). "Foreign Educated Ethiopian between 1800-1889", Journal of Education, Volume 6. pp115-148, Addis Ababa University press.

Baharu, Zewde (1991). "The Concept of Japanization in the intellectual History of Modern Ethiopia", Proceeding of The Fifth Seminar of the Department of History Ethiopian Studies, Addis Ababa University Press

Baharu, Zewde (1995), "The Intellectual and State in Twentieth Century Ethiopia". Paper Presented at the 12th International Conference of Ethiopian Studies, Addis Ababa University



Baharu, Zewde (2002), "Pioneers of Change in Ethiopia: The Reformist Intellectuals Of the Early Twentieth Century". Oxford: James Currey.

Bahtia, H.L (1989), "History of Economic Thought", University of Delhi, Vikas Publishing House Private Ltd.

Collins M. Robert (2000), "The Politics of Economic growth in Post war America", Oxford printing press.

Emmanuel, A., (1972). "Unequal Exchange: A study of Imperialism of Trade", London, Monthly Review Press.

Emmanuel, Malifu (2004). "Gebrehiwt Baykedagn Pioneer Thinker On Environment and Development", A Masters Thesis on an environmental science, Department of Environmental Science, Addis Ababa University.

Fekade, Azeze (1988) "The Role and Position of foreign Educated interpreters in Ethiopia (1800-1889)", Journal of Ethiopian Studies, Addis Ababa .Volume 11, number 1 pp17-27.

Gebrehiwt, Baykedagn (1912). "Atse Menilik Na Ethiopia", Asmara, Berhan Yeun.

Gebrehiwt, Baykedagn (1924). "Mengest Na Yezbe Astedader". Addis Ababa: Teferi Mekonnen Printing Press.

Pankhurst, Richard (1976), "A Brief Note on the Economic History of Ethiopia", Department of history, Halie-Silasse I university.

Prebisch, Raul, (1962), "The economic Development of Latin America and Its Principal Problems" Economic Bulletin for Latin America, 7,1: 1-22.

Singer, H.W., (1950) US Foreign Investment in Underdeveloped Areas: The distribution of Gains between Investing and Borrowing Countries, *The American Economic Review: Papers and Proceeding*, 40, 2: 473-485.

Sosina, Bezu. (1999). "Behind the Heavy Curtain: Formalization of Gebrehiwt Baykedagn Model". A Bachelor's thesis on Department of Economics, Addis Ababa University.

Tenkir, Bongor, (1995). "The State and Economy in Early 20th Century Ethiopia by Gebrehiwt Baykedagn". Trans.. London: Karnack House.

Appendix

Appendix A

A.1. Equations of Transitional Dynamics

$$Y_{t+1} = Y_{a,t+1} + Y_{na,t+1} \dots\dots\dots(A1)$$

$$Y_{na,t+1} = \mu_1 + \mu_2 I_{na,t+1} + \mu_3 I_{na,t} \dots\dots\dots(A2)$$

$$Y_{a,t+1} = \beta_1 + \beta_2 L_{a,t+1} + \beta_3 PL_{t+1} \dots\dots\dots(A3)$$

$$L_{s,t+1} = (1 - \varphi) L_{s,t} + \gamma(1 - \sigma) L_{us,t} \dots\dots\dots(A4)$$

$$L_{us,t+1} = (1 - \gamma)(1 - \sigma) L_{us,t} \dots\dots\dots(A5)$$

$$L_{t+1} = (1 - \varphi) L_{s,t} + (1 - \sigma) L_{us,t} \dots\dots\dots(A6)$$

$$PL_{t+1} = (1 - \rho - \delta) PL_t \dots\dots\dots(A8)$$

$$I_{na,t+1} = \frac{(1 - \theta - \omega - \vartheta)}{(1 - \theta^*)} I_{na,t} \dots\dots\dots(A9)$$

A.2. Identity Equations

$$L_{a,t} = L_{sa,t} + L_{usa,t} \dots\dots\dots(A10)$$

$$L_{na,t} = L_{sna,t} + L_{usna,t} \dots\dots\dots(A11)$$

$$L_{s,t} = L_{sa,t} + L_{sna,t} \dots\dots\dots(A12)$$

$$L_{usa,t} = L_{usa,t} - L_{usna,t} \dots\dots\dots(A13)$$

$$L_t = L_{a,t} + L_{sna,t} \dots\dots\dots(A14)$$

$$Y_t = Y_{a,t} + Y_{na,t} \dots\dots\dots(A15)$$

A.3.The Equilibrium relationship of the model

$$Y_{n,t} = \mu_1 + \mu_2 I_{na,t} + \mu_3 I_{na,t-1} \dots\dots\dots(A16)$$

$$Y_{a,t} = \beta_1 + \beta_2 L_{a,t} + \beta_3 PL_t \dots\dots\dots(A17)$$

$$I_{na,t} = \varphi_1 + \varphi_2 Y_t + \varphi_3 L_{sna,t} + \varphi_4 PIF_{a,t-1} + \varphi_5 T_t \dots\dots\dots(A18)$$

$$PL_t = \rho_1 TB_t + \rho_2 TB_{t-1} + \rho_3 L_{at} \dots\dots\dots(4.19)$$

$$L_{sna,t} = \delta_1 + \delta_2 W_t + \delta_3 W_{t-1} + \delta_4 PHC_t + \delta_5 PHC_{t-1} \dots\dots\dots(A20)$$

$$L_{usa,t} = \alpha_1 + \alpha_2 W_t + \alpha_3 W_{t-1} \dots\dots\dots(A21)$$

$$L_{sna,t} = \gamma_1 + \gamma_2 PHC_t + \gamma_3 PHC_{t-1} + \gamma_4 W_t + \gamma_5 W_{t-1} \dots\dots\dots(A22)$$

$$L_{usna,t} = \pi_1 + \pi_2 W_t + \pi_3 W_{t-1} \dots\dots\dots(A23)$$

$$T_{d,t} = \tau_1 Y_t \dots\dots\dots(A24)$$

$$TOT_t = \omega \frac{L_{s,t}}{L_{us,t}} \dots\dots\dots (A25)$$

$$TB_t = \theta_1 TOT_t \dots\dots\dots (A26)$$

Appendix B. The Estimated and Calibrated Parameters used for Model Simulation

The parameters employed for simulation purpose in this paper, which is found in eclectic way of estimation and calibration is given below

$$\beta_1 = 15687, \beta_2 = 0.63, \beta_3 = 0.37,$$

$$\delta_1 = 3289, \delta_2 = -0.054, \delta_3 = -0.014, \delta_4 = 0.15, \delta_5 = 0.01$$

$$\alpha_1 = 4366, \alpha_2 = -0.035, \alpha_3 = -0.046,$$

$$\rho_1 = 6.7, \rho_2 = 4.3, \rho_3 = 0.3,$$

$$\mu_1 = 34400, \mu_2 = 0.75, \mu_3 = 0.67$$

$$\varphi_1 = 1089.83, \varphi_2 = 1.28084, \varphi_3 = 0.41103, \varphi_4 = 0.0216, \varphi_5 = -0.2732$$

$$\gamma_1 = 1625, \gamma_2 = 0.73, \gamma_3 = 0.46, \gamma_4 = -0.023, \gamma_5 = -0.014$$

$$\pi_1 = 2357, \pi_2 = -0.031, \pi_3 = -0.028$$

$$\theta_1 = 0.57, \tau_1 = 0.35, \omega = 0.83$$

Appendix C Description of Gauss Seidle Iteration Algorithm

Given a square system of n linear equations with unknown \mathbf{x} :

$$A\mathbf{x} = \mathbf{b}$$

$$\text{Where, } A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \text{ and } b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Then A can be decomposed into a lower triangular component L_* , and a strictly upper triangular component U :

$$A = L_* + U \text{ where } L_* = \begin{pmatrix} a_{11} & 0 \cdots & 0 \\ a_{21} & a_{22} \cdots & 0 \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{pmatrix} \text{ and}$$
$$U = \begin{pmatrix} 0 & a_{21} \cdots & a_{n1} \\ 0 & 0 \cdots & a_{n2} \\ 0 & 0 \cdots & 0 \end{pmatrix}$$

The system of linear equations may be rewritten as:

$$L_*\mathbf{x} = \mathbf{b} - U\mathbf{x}$$

The Gauss–Seidel method is an iterative technique that solves the left hand side of this expression for \mathbf{x} , using previous value for \mathbf{x} on the right hand side. Analytically, this may be written as:

$$\mathbf{x}^{(k+1)} = L_*^{-1}(\mathbf{b} - U\mathbf{x}^{(k)}).$$

However, by taking advantage of the triangular form of L_* , the elements of $\mathbf{x}^{(k+1)}$ can be computed sequentially using forward substitution:

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j>i} a_{ij} x_j^{(k)} - \sum_{j<i} a_{ij} x_j^{(k+1)} \right), \quad i = 1, 2, \dots, n.$$

The procedure is generally continued until convergence achieved to some desired tolerable level.

Declaration

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all source of materials used for the thesis have been duly acknowledged.

The examiners' comments have been dully incorporated.

Declared by:

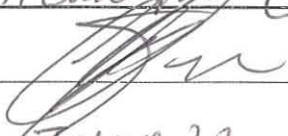
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