## ADDIS ABABA INSTITUTE OF TECHNOLOGY

# Validation of EGM - 08 using GPS and LEVELING in Addis Ababa

**A THESIS**

**SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES OF ADDIS ABABA UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTERS OF SCIENCE IN GEODESY AND GEOMATICS**

**Submitted by**

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## DECLARATION

I, the undersigned, declare that this thesis work is my original work carried out under the supervision of Ass. Prof. Tulu Besha Bedada. It has not been presented for a degree in any other universities and all sources of materials used for the thesis work have been properly acknowledged.

Name Signature Date

Place: Addis Ababa

This thesis has been submitted for examination with my approval as a university advisor.

Name Signature Date

**Addis Ababa University**

**Addis Ababa Institute of Technology**

**School of Civil and Environmental Engineering**

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# List of Abbreviations

| | |
|---|---|
| ArcGP | Arctic Gravity Project |
| BD | Block-Diagonal |
| $C^e$ | Ellipsoidal harmonic coefficients |
| CHAMP | Challenging Mini satellite Payload |
| DNSC | Danish National Space Center |
| DOT | Dynamic Ocean Topography |
| DTM | Digital Topographic Model 2006.0 |
| EGM | Earth Gravitational Model |
| EGM96 | Earth Gravitational Model 96 |
| EGM08 | Earth Gravitational Model 2008 |
| EMA | Ethiopian Mapping Agency |
| GOCE | Gravity filed and steady state Ocean Circulation Explorer |
| GNSS | Global Navigation Satellite System |
| GPS | Global Positioning System |
| GRACE | Gravity Recovery And Climate Experiment |
| IAG | International Association of Geodesy |
| IGFS | International Gravity Field Service |
| LSC | Least Squares Collocation |

| | |
|---|---|
| MSL | Mean Sea Level |
| MSS | Mean Sea Surface |
| NGA | National Geospatial-Intelligence Agency |
| NOAA | National Oceanic and Atmospheric Administration |
| NQ | Numerical Quadrature |
| PGM | Preliminary Gravitational Models |
| RTM | Residual Terrain Model |
| PPS | Precise Positioning Service |
| SA | Selective Availability |
| SIO | Scripps Institution Oceanography |
| SLR | Satellite Laser Ranging |
| SPS | Standard Positioning Service |
| SSH | Sea Surface Heights |
| SST | Satellite-to-Satellite Tracking |
| SRTM | Shuttle Radar Topographic Mission |
| Sx | Complete error covariance matrix |

# Abstract

The Global Positioning System (GPS) observations which is a component of the Global Navigation Satellite System (GNSS) is being referenced to the World Geodetic System 1984 (WGS84) ellipsoid with geographic coordinates defined in terms of latitude, longitude and ellipsoidal heights ($\varphi$,$\lambda$,$h$). This prevalent situation makes it difficult to apply standard forward transformation equation for direct conversion of ellipsoidal heights (h) to a practical orthometric height (H) within Ethiopian local geodetic reference network. In order to overcome such a challenge, many researchers resort to various methods of determining the geoidal undulations for a local and national geodetic network and improving the recent New Earth Gravitational Model accuracies and its performances. This present study therefore seeks to validate the accuracy of the geoid heights derived from the Earth Gravitational Model 2008 (EGM08) in Addis Ababa. The estimated geoid heights obtained by the EGM08 model were compared with 9 geometrical geoid heights derived from co-located GPS and orthometric heights obtained by running first order geodetic leveling with a tie to the existing Blue Nile Bench Mark(1957- 1961). The methods applied include estimating the geoidal heights using the EGM08 model and a geometric method. The statistics of the differences between derived geoid heights by the geometric approach and corresponding geoid heights obtained from the Earth Gravity Model (EGM08) suggests an precedented accuracy (~2.27cm) of the geoid heights computed from EGM08.

# CHAPTER ONE

## 1. Introduction

### 1.1 Background of the study

Height is a vertical displacement of a point above a specified surface of constant potential; distance is measured along the direction of gravity between the desired field point and the equipotential surface (Meyer et al., 2005). It is the third component of a spatial point positional coordinates defined in the form of geopotential height or in any form of heights derived from it, Traditionally, heights are measured as elevation differences (vertical distances) between points of interest and the Mean Sea Level (MSL) using either direct or indirect measurement techniques (El-Hassan and Ali, 2011). The elevation differences thus measured could be mathematically expressed as; $N = h - H$

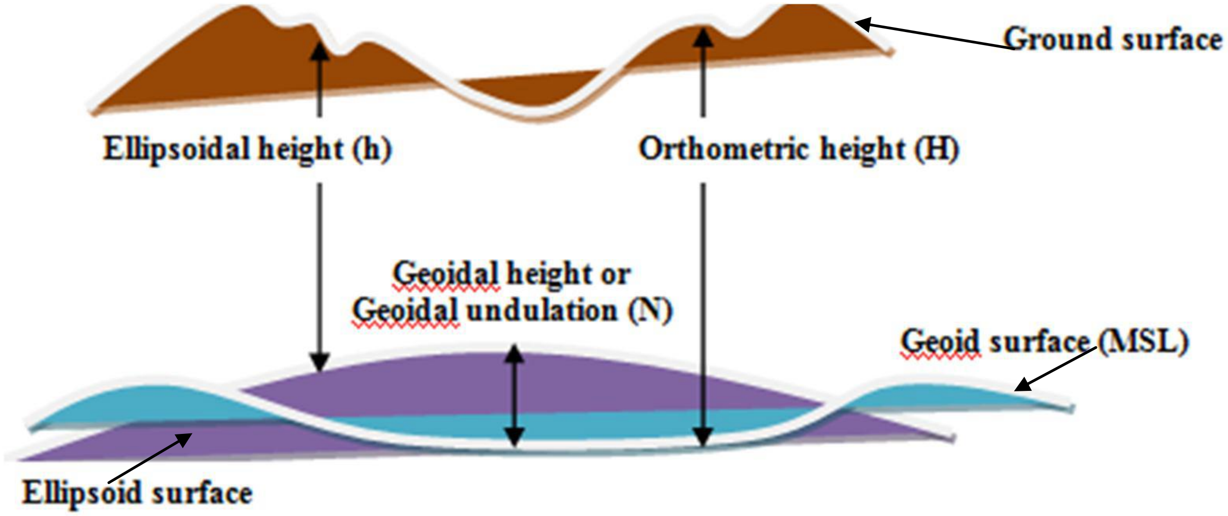


Figure 1.1. Graphical representation of height(elevation) of a field point with reference to the MSL.

However, MSL does not have equal elevation at all points for several reasons, mainly variation in local mass redistribution caused by climate change (e.g. Melting of Greenland, Antarctica, Alaska), atmospheric pressure, tides, winds, and spatial variability of temperature and salinity of open seas and ocean water. Therefore, it has become necessary to determine physical heights with a tie to the geoid. For example, orthometric height (H) can be derived from geopotential number in the form:

$$H = \frac{c}{\bar{g}}$$

Where: c is the geopotential number and it represents the difference in potential between the constant value at the geoid ($\omega_o$) and the potential at the point *p* on the surface ($\omega_p$) :

$$c = \omega_o - \omega_p$$

$\bar{g}$ is measured gravity value at observation points.

Orthometric heights are used to uniquely trace level surfaces on the ground and very useful for most civil engineering applications that require to model energetic flow of materials.

However, one of the most interesting and challenging tasks in the field of geodetic surveying is the accurate determination of orthometric heights, specifically by combining the gravimetric geoid height with ellipsoidal height acquired from processing of GPS data.. This poses a challenge for high order engineering works such as engineering surveys or 3D coordinate transformation and mapping (Featherstone *et al*., 2001; Fotopoulos, 2003).

This has therefore drawn the attention of many researchers in the area of orthometric height determination. the conversion of an ellipsoidal height /h/ to a physical meaning height (e.g ortometric height) require the determination of the geoid undulation /N/ of the area (Shen and Han, 2013; Dumrongchai *et al*., 2012).

Great achievement in the fields of geodesy and geophysics were achieved in determining physical heights. without necessarily carrying out the tedious and time-consuming operation of geometric or trigonometric leveling (Hirt, *et al*., 2011, Gruber *et al*., 2011).

An accurate geoid model is essential for determining orthometric heights using the GNSS technology, which is being accepted globally for geodetic purposes (Fotopoulus *et al*., 1999).

Now a days, the earth gravitational models are used to determine the geoid undulations an area to provide orthometric height via ellipsoidal height for GPS measurements. The original technique that was used to compute the geoid undulation was the Stokes' integral (Heiskanen and Moritz, 1967). Presently, the 2008 Earth Gravitational Model /EGM08/ has been the most widely used for computing geoid or geopotential height, relatively with high accuracy (Bedada, 2010).

The EGM08 is good enough for determining a vertical datum that best approximate the Mean Sea Level (MSL) (Yilmaz *et al*., 2010; Abeho *et al*., 2014). The EGM08 derived geoid heights can reach the accuracy of regional or local geoid models after modeling the differences between the GPS/leveling geoid heights and EGM08 derived geoid heights at identified control points (Dawod, 2008; Dawod *et al*., 2010; Soycan, 2014).

The EGM08 was preceded by EGM96 which had a lower degree of accuracy (Pavlis *et al*., 2008). In this modern era, EGM08 is capable of obtaining a sufficiently accurate model of the gravity field over the surface of the earth (Kotsakis *et al*., 2009; Kotsakis and Sideris, 1999). The aim of this study is to assess the accuracy of the EGM08 derived geoid heights against geometric geoid model computed from co-located measurements of GPS and leveling data, in the region of Addis Ababa city.

### 1.2 Statement of the Problem

Accurate estimation of orthometric heights is one of the current research areas for geodesists. Orthometric heights are computed from geopotential heights, and this process requires approximation of gravity variation along the plumb-line from the surface to the geoid. Normal heights on the other hand are less laborious to compute and do not require knowledge of actual gravity at measurement points.

The choice of a height system is a matter of preference and it may be related to cost of data production. With respect to the geometrically simpler shape of a perfect ellipsoid, the Earth's topographic surface is irregular and complex. The task of mapping a level surface on the surface of the earth is challenging.

Traditionally, a level surface is determined by spirit leveling to control and design the engineering works. However, leveling is expensive, time consuming and prone to accumulation of systematic and an un modeled random errors. Also, it needs a long way leveling to connect a particular field point to a known mean sea level tide gauge height datum station.

Traditional leveling height cannot be predicted for any isolated field point independently, without connecting to the local network of leveled heights. Although, thus classical approach of height definition cannot define a globally absolute height system as mean sea level changes due to earth's mass redistribution. Hence, leveling heights must be combined with gravity to determine the geopotential height difference between two points. Due to a development of GNSS, Now a days we can obtain accurate geometrical heights with physical meaning without necessarily carrying out the tedious and time-consuming procedures of geometric or trigonometric leveling (Hirt, *et al*., 2011, Gruber *et al*., 2011).

Although ellipsoidal heights can be useful in some applications, they are not applicable in engineering projects; where heights referenced to an equipotential surface (geoid) are required. So, there is a need for a new approach which computes height of the desired filed point by referring to a global reference surface called geoid using the Earth Gravity Model (EGM2008) and GPS technology. This research will investigate how the gravimetric based orthometric height can best simulate to GPS leveling data.

## 1.3 Objective of the study

### 1.3.1 General Objective

The main aim of this study is validating the accuracy of the Earth Gravitational Model 2008 derived geoid height in Addis Ababa using in-situ co-located measurement of Global Positioning System (GPS) measurement and classical leveling data.

### 1.3.2 Specific objectives

- To determine geoid heights and its residual based on EGM2008 global gravity model , GPS and leveled orthometric heights
- Validate the accuracy of orthometric heights determined from EGM08 via GPS ellispidal height against in-situ leveled height.

## 1.4 Scope of the study

This research was conducted in Addis Ababa in parts of Arada, Yeka, and Gulele sub cities. The study area has different topography. Because of the very rugged nature of the earth's surface, the task of determining height of the field point is very difficult and tedious work for geodesists. Precise determination of physical height is necessary for different application especially in engineering fields. Scientists create numerous methodologies in order to determine heights of a point on the surface of the earth. i.e. from classical spirit leveling to the modern satellite based GNSS technology .The choice of the height determination depends on physical definition and required, There for, this study is concerned to validate the Earth Gravity Model of 2008 for its capability in providing accurate Orthometric height.

## 1.5 Significance of the study

The significance of the study could be to give information about precise determination of heights on the surface of the earth, It gives information about different geoid models and their significance for accurate height determination. The current study could be used as an input for further assessment of geopotential height across Ethiopia.

### 1.6 Limitation of the study

This research attempted with all possible efforts in acquiring the required inputs in the form of primary and secondary data collection, interpretation and analysis. However, the study has encountered certain limitations . Some of the limitations was  financial and shortage of time.

### 1.7 Thesis Structure

This thesis consists of six chapters and it is summarized as follows: Chapter two aims to present theory and fundamentals of height system , earth gravity model 2008 and Global positioning system/GPS. Chapter three describes the study area , research design and methodology. It is focus on data acquisition processing and presentation. Chapter four presents  result and discussions of the research. Chapter five aims to present the key research conclusions and recommendations.

# CHAPTER TWO

## 2. Literature review

### 2.1 Height systems

To determine heights of earth points, an origin surface with zero height must be described and the vertical distance of points from the surface must be determined. For heights, a variety of surfaces can be taken as reference. Of these, the most important surface is the geoid.

Reference surfaces and vertical distances of points from the surfaces have different physical and geometric meanings. Geodetic heights also have different physical and geometric meanings. Thus, scholars introduces several height systems amongst all some basic height systems were examined as follows.

#### 2.1.1 Geopotential heights

Geopotential height or geopotential number is a surface point's potential difference to the geoid potential. Put differently, it is the difference between *A* surface point's $W_A$ potential with *O* geoid point's $W_O$ potential.

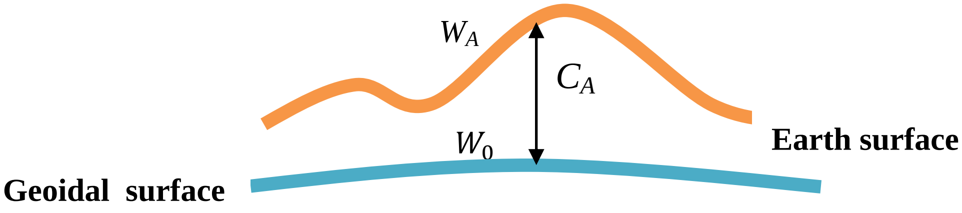


Figure 2.1 Graphical representation of potential differences /$C_A$/ between surface point and geoid point.

The difference can be obtained by the integral from

$$C_A = W_O - W_A = -\int_0^A dW = \int_0^A g\,dn$$

Where,

$C_A$ is the geopotential number and is path-independent;
$W_0$ is the geoid potential;
$W_A$ is a point's potential;
$dW$ is the potential difference in differential meaning;
$dn$ is the height difference in differential meaning;
$g$ is gravity.

$\boldsymbol{C}$ is equal in the same equipotential surface Although it has no distance dimension, it is the natural criterion for heights. The geopotential number is measured in kgal•m.

#### 2.1.1.1 Computation of geopotential heights

The geopotential height difference between two points

$$C_B - C_A = \Delta C_{AB} = W_A - W_B = \int_{\boldsymbol{A}}^{\boldsymbol{B}} \boldsymbol{gdn} \cong \sum_A^B gdn$$

is the geopotential height difference between points $A$ and $B$; $g$ is the measured actual gravity along the route; d$n$ is the height difference in the meaning of the differential.

### 2.1.2 Dynamic heights

Dynamic heights are the difference in height between two points measured normal to gravity*. Dynamic heights have been obtained The geopotential number is divided by the $\gamma_0$constant where $\gamma_0$ is the normal gravity value at 45° latitude.

The value at $\gamma_{45^0}$ = 9.806199203 $ms^{-2}$ = 980.6294 gal for international ellipsoid.

Dynamic height will be given by a formula

$$\boldsymbol{H^{din}} = \boldsymbol{C}/\boldsymbol{\gamma_0}$$

#### 2.1.2.1 Computation of dynamic height

In practice, height differences ($\Delta n_{AB}$) measured by leveling are converted into dynamic heights by adding a correction magnitude. This correction is called a dynamic correction ( $DC_{AB}$ ). It is calculated as follows:

$$\Delta \mathbf{H}_{\mathbf{AB}}^{\mathbf{din}} = \Delta n_{AB} + DC_{AB}$$

### 2.1.3 Orthometric height

Orthometric height is the height of a point above the geoid (equipotential surface of the Earth), and the orthometric height difference would be the difference between the orthometric heights of two points. Orthometric heights are the distance of a surface point along the plumb line to the geoid, which is taken as the reference surface, is named the orthometric height. It is also a natural "heights above sea level", that is, heights above the geoid. Therefore, they have an unequalled geometrical and physical significance.

It can be obtained as follows:

$$H = C / g$$

For the computation of actual average gravity $g$ along the plumb line, actual gravity is required between the geoid and earth surface. However, gravity inside the earth cannot be measured so a hypothesis regarding the mass distribution must be formed and $g$ is computed on this basis. The orthometric height cannot be determined without a hypothesis.

### 2.1.3.1 Computation of orthometric height

Height differences ( $\Delta n_{AB}$ ) measured by leveling are converted into orthometric heights by adding a correction magnitude. This correction is called an orthometric correction ( $OC_{AB}$ ).

It is calculated as follows:

$$OC_{AB} = \sum_{A}^{B} \frac{g - \gamma_0}{\gamma_0} \delta n + \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_A^* - \frac{\bar{g}_B - \gamma_0}{\gamma_0} H_B^*$$

$$\bar{g} = \frac{1}{2}(g - g_0)$$

Where, $g$ is the actual gravity measured along the route , $\bar{g}_A \bar{g}_B$ $g$ are the actual average gravity. Actual average gravity values can be computed either by using this equation $\bar{g} = g + 0.042\ 4H^*$ , where $g$ is the gravity measured on point $A$: $g_0$ is the gravity measured on point $A_0$ on the geoid corresponding to point $A$. Here, $g$ is accepted to be linearly changing along the plumb line. Consequently, orthometric height difference is given by

$$\Delta H_{AB}^* = \Delta n_{AB} + OC_{AB}$$

In correction computations, $\gamma_0$is taken as a constant gravity value (usually normal gravity $45^0$ $\gamma$ for the geographic latitude $\phi = 45^0$.

### 2.1.4 Normal height

The ellipsoid is the reference surface best resembling the geoid, which is the earth's basic shape. An ellipsoid's gravity field is called a *normal gravity field*. The Earth's actual gravity field is slightly different from the normal gravity field. The difference is called the *disturbing potential*. We are going to assume that the actual gravity field is equal to the normal gravity field.

In other words, we assume

$$W = U, g = \gamma, T = 0.$$

Hence, orthometric heights which correspond to this approximation are named normal heights and are shown by $H^N$. They can be obtained as follows:

$$H^N = \frac{C}{\bar{Y}}$$

Where,

W is the actual gravity potential;

U is the normal gravity potential;

T is the disturbing potential;

$\bar{Y}$ is the normal average gravity along a normal plumb line.

A surface point *P* has $W_P$ actual and $U_P$ normal gravity potential and these potentials are not equal ($W_P \neq U_P$). But on the plumb line that crosses point *P*, there is a *Q* point. In this *Q* point, the normal potential is equal to the actual potential ( $W_P = U_Q$ ). The surface for which $W_P=U_Q$ holds for every point is called a “Telluroid”. The normal $H^N$of a point *P* is equivalent to the height of the corresponding telluroid point *Q* above the ellipsoid.

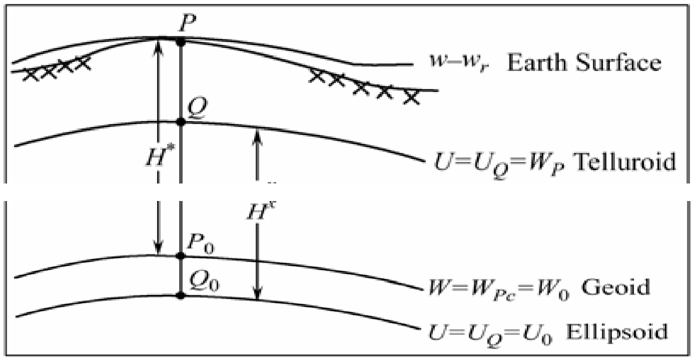


Figure 2.2 Graph shows a point where the normal potential is equal to the actual potential

#### 2.1.4.1 Computation of normal height

Height differences ( $\Delta n_{AB}$ ) measured by leveling are converted into normal heights by adding a correction magnitude. This correction is called a normal correction ( $NC_{AB}$ ). The normal correction can be obtained by changing *g* with $\bar{Y}$□and $H^*$ with *H* is calculated as follows;

$$NC_{AB} = \sum\nolimits_{A}^{B} \frac{g - \gamma_0}{\gamma_0} \delta n + \frac{\bar{Y}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{Y}_B - \gamma_0}{\gamma_0} H_B$$

Where: *g* is the actual gravity measured along the route; $\gamma_0$ is the normal gravity $\gamma_0 = \gamma_0^{45^0}$; $\bar{Y}$ is the normal average gravity. At points *A* and *B* normal average gravity values are obtained from normal gravity values γ at points *A* and *B.*

### 2.2 Earth Gravitational Model

Accurate knowledge of the gravitational potential of the Earth, on a global scale and at very high resolution, is a fundamental prerequisite for various geodetic, geophysical and oceanographic investigations and applications. Over the past 50 or so years, continuing improvements and refinements to the basic gravitational modeling theory have been paralleled by the availability of more accurate and complete data and by dramatic improvements in the computational resources available for numerical modeling studies. These advances have brought the state-of-the-art from the early spherical harmonic models of degree 8 [Zhongolovich, 1952], to the present solution that extends to degree 2190. Rapp [1998] provided a brief review of the major developments in global gravitational field modeling over the 20th century.

There are numerous uses for these high degree potential coefficient models [Tscherning, 1983]. In recent years, two types of applications have played a major role in emphasizing the need for high resolution, accurate global gravitational models.

1. In areas of GPS positioning and gravimetrically determined geoid heights offer the possibility of determining orthometric heights and height differences without the need for leveling [Schwarz et al., 1987]. A global high degree model may be used here, either as a reference to support the development of more detailed regional geoids, or to provide the geoid heights on its own.
2. In oceanic areas , the need to determine the absolute Dynamic Ocean Topography (DOT) and its slopes, from altimeter-derived Sea Surface Heights (SSH) and a global gravitational model, puts very stringent accuracy and resolution requirements on global high degree models [Ganachaud et al., 1997]. Furthermore, a unique, accurate, global high degree gravitational model may be used to provide the reference surface for the realization of a global vertical datum [Rapp and Balasubramania, 1992].

### 2.2.1 Evolution of EGM 2008

The first decade of the new millennium has been called “The Decade of Geopotentials” and has seen the launch of three dedicated gravity field mapping missions: CHAMP [Reigber et al., 1996] launched in July 2000, GRACE [GRACE, 1998] launched in March 2002, and GOCE [European Space Agency, 1999] launched in March 2009. Considering these advances, and in particular the expected availability of very accurate long wavelength gravitational models from GRACE, the National Geospatial-Intelligence Agency (NGA) decided to embark on the development of a new Earth Gravitational Model (EGM) to serve as:

(a) a replacement of EGM96 [Lemoine et al., 1998], and,

(b) a candidate (pre-launch) reference model for the analysis of data to be acquired from GOCE.

It was decided early on that the new EGM would be developed by combining the best available GRACE-derived satellite-only model, with the most comprehensive compilation of a global 5 arc-minute equiangular grid of area-mean free-air gravity anomalies that NGA could furnish.

In this fashion, the highly accurate long wavelength information provided by the GRACE data would be complemented with the short wavelength information contained within the 5 arc-minute gravity anomaly data. The accuracy goal for the new EGM was set to _15 cm global Root Mean Square (RMS) geoid undulation commission error. The analytical and numerical work required to ensure technical readiness for the development of the new EGM began in earnest around 2000. The status and progress of the project was demonstrated with the development of Preliminary Gravitational Models (PGM) that were presented in 2004 [Pavlis et al., 2005], 2006 [Pavlis et al.,2006a], and 2007 [Pavlis et al., 2007a].

Following the example of EGM96, PGM2007A [Pavlis et al., 2007a] was also provided for evaluation to an independent Special Working Group, functioning under the auspices of the International Association of Geodesy (IAG) and the International Gravity Field Service (IGFS). Based in part on the feedback received from this Working Group, The development of a new Earth Gravitational Model (EGM 2008) to degree 2160 is progressing with the availability of improved versions of a complete and accurate 5′×5′ worldwide global gravity anomaly database and GRACE-derived satellite solutions . That takes advantage of all the latest data and modeling for both land and marine areas worldwide. The development of the final model, designated EGM2008, was completed in late March 2008, and EGM2008 was presented and released to the scientific community on April 17, 2008 [Pavlis et al., 2008].

**The Development of  EGM 96:**The development of EGM96 involved the analysis of various types of satellite tracking data, acquired over many years, from 40 satellites. It is the predecessor of the EGM 2008 model, has a composite solution in which different estimation techniques were used to compute different spectral bands of the model (Lemoine et al. [1998] for details). The lower degree portion of EGM96 (up to degree 70), was estimated from the combination of the satellite-only model EGM96S (complete to degree and order 70), with surface gravity data (excluding altimetry-derived values) and satellite altimetry in the form of “direct” tracking. In this mode, satellite altimeter data are treated as ranges from the spacecraft to the ocean surface whose upper endpoint senses through the orbit dynamics attenuated gravitational signals, both static and time-varying, while their lower endpoint senses the combined effects of geoid undulation, DOT as well as tides and other time-varying effects, without any attenuation. In this manner, altimeter data contribute to the estimation of the satellite's orbit, as well as the estimation of the DOT and of the potential coefficients. Fully occupied normal matrices were formed and combined with appropriate relative weights, in order to estimate this “comprehensive" low degree portion of the EGM96 model. This analysis involved the simultaneous estimation of several parameter sets besides the gravitational potential coefficients and the spherical harmonic coefficients representing the DOT, which were its main products. Beyond degree 70, and up to degree 359, the fully occupied normal matrix associated with EGM96S was combined with a Block-Diagonal (BD) approximation of the normal equations resulting from the analysis of a complete global 30 arc-minute equiangular grid of gravity anomalies, which used altimetry-derived values over most oceanic areas. Over areas without adequate gravity anomaly data, the 30 arc-minute grid used in EGM96 was filled with composite “fill-in values, computed from the low degree part of EGM96S, augmented with coefficients of the topographic-isostatic potential (Lemoine et al. 1998,). In the specific approximation (BD3) used in EGM96, each block corresponds to all the unknown coefficients of the same order, and the rest of the matrix is all zeroes. Finally, the EGM96 coefficients of degree 360 were estimated from this 30 arc-minute grid using the Numerical Quadrature (NQ technique. (Pavlis, 1998b).

### 2.2.2 Design and Rationale of EGM 2008

EGM2008 is a spherical harmonic model of the Earth’s gravitational potential, developed by a least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arc-minute equiangular grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, their spectral content was supplemented with gravitational information implied by the topography.

EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/ Leveling values are on the order of _5 to _10 cm. EGM2008 vertical deflections over USA and Australia are within _1.1 to _1.3 arc-seconds of independent astrogeodetic values.

These results indicate that EGM2008 performs comparably with contemporary detailed regional geoid models. EGM2008 performs equally well with other GRACE-based gravitational models in orbit computations. Over EGM96, EGM2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy, depending on gravitational quantity and geographic area. EGM2008 represents a milestone and a new paradigm in global gravity field modeling, by demonstrating for the first time ever, that given accurate and detailed gravimetric data, a single global model may satisfy the requirements of a very wide range of applications.

a degree and order 2160 model with a geoid accuracy of 15 centimeters RMS with the considerations EGM 96, it became clear that a very high degree (2159) combination solution could now be developed, not as a composite solution anymore, but rather using a single least squares adjustment estimation technique.

Data's used in the analysis of EGM 2008 was the ITG-GRACE03S Model, Digital Topographic Model DTM 2006.0, gravity anomalies derived from satellite altimetry, gravity anomalies estimated from terrestrial data, fill-in gravity anomalies using RTM forward modeling and 5 Arc-Minute global merged gravity anomaly file.

Evaluation of EGM2008 has been done using orbit fit tests, comparisons with GPS/Leveling data, comparisons with astrogeodetic deflections of the vertical, comparisons with TOPEX altimeter data and dynamic ocean topography comparisons.

### 2.3 Global Positioning System /GPS

It all started with Sputnik. What seemed at the time like a major defeat in the Cold War, turned out to be the catalyst for one of the most important technologies of the 20th century, and maybe the 21st. It was October 4th, 1957. Scientists at MIT noticed that the frequency of the radio signals transmitted by the small Russian satellite increased as it approached and decreased as it moved away. This was caused by the Doppler Effect, the same thing that makes the timbre of a car horn change as the car rushes by. The Russians launched the Sputnik satellite in 1957, surprising the world. This gave the scientists a grand idea. Satellites could be tracked from the ground by measuring the frequency of the radio signals they emitted, and conversely, the locations of receivers on the ground could be tracked by their distance from the satellites. That, in a nutshell, is the conceptual foundation of modern GPS.

As a result GPS is a satellite navigation system that provides positioning and clock time to the terrestrial user. Currently, the system consists of a total of 31 GPS satellites orbiting the earth twice a day. While the satellites make up the space segment, the system also includes a control segment that monitors and maintains the satellites, as well as the user segment ( http://tycho.usno.navy.mil/gps.html for more information.)

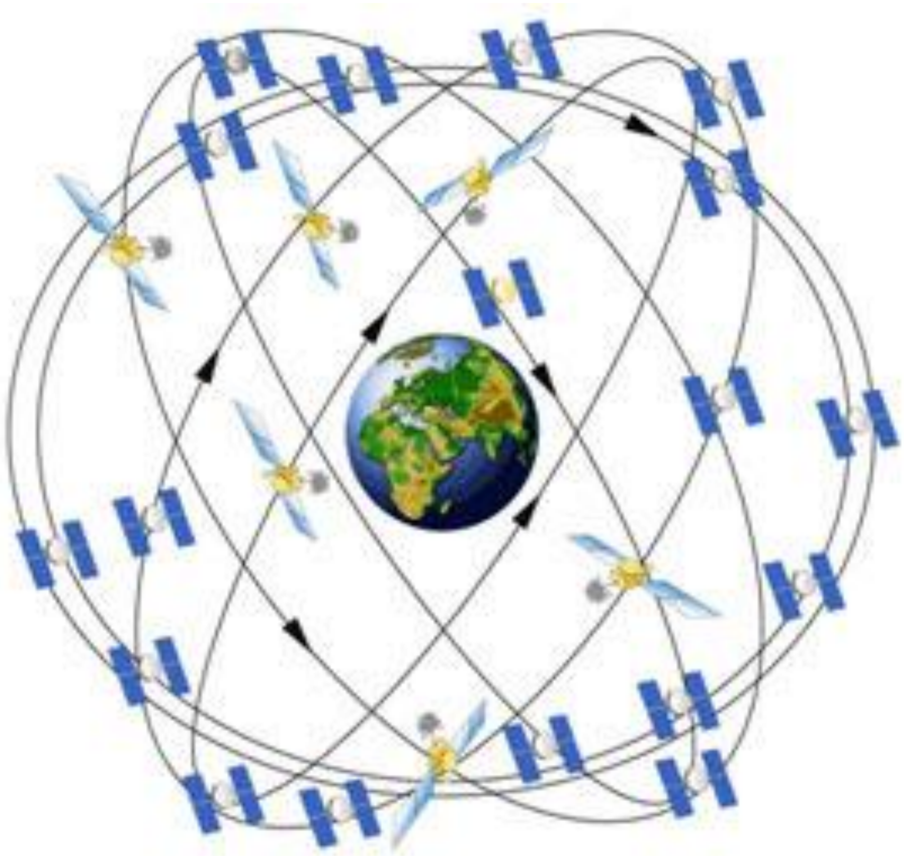


Figure 2.3. GPS satellites with their orbit

As GPS system is a composition of space, control and user segment. The space segments consists of twenty four satellites, of which four satellites present in each six orbital plane carrying atomic clock and inclined at 550 with respect to earth's equatorial plan. They are distributed in such a way that from any point on the earth surface four or more satellites are seen above the local horizon. The satellites that make the space segment are orbiting the earth at about 20,000 km altitude above the earth surface or at orbital radius of 26,600 km. They travel at a speed of 11,000 kilometer in an hour. They are moving constantly and making two complete orbits in less than twenty four hours (Keith, 2002).

GPS satellites are powered by solar energy. They have backup batteries onboard to keep them running in the event of a solar eclipse and also has got small rocket boosters on each satellite so as to keep them flying in the correct path. The control segment is consists of a number of ground-based monitoring stations, which continually gather information from the satellite. These data are sent to Master Control Station in Colorado Springs which analyzes the constellation and projects the satellite ephemerides and clock behavior forward for the next few hours. This information is then uploaded to the satellite for transition to user. The user segment consists of all geodetic standard GPS receivers that are receiving signals transmitted from the satellites and enabling us to determine position, velocity and time at the desired field points on or above the earth’s surface. GPS satellite transmits its position and other navigational information via the L-band radio signal, L1 (1575.43MHz) and L2 (1227.60MHz). The L- band carrier signal ismodulated with data carrying information such as the satellite status, the satellite clock error and the ephemeris (Hofmann-Wellenhof et al., 1997). The L1 carrier signal is modulated with a precision code (p-code) known as Precise Positioning Service (PPS) code and a Coarse Acquisition Code (C/A-code), known as Standard Positioning Service (SPS) code. On the other hand, the L2 carrier signal is modulated with only the precise positioning service (Remondi, 1985).

GPS receiver takes information and use trilateration to calculate the user’s exact location (Witte & Wilson, 2004). The datum used for GPS refers to the World Geodetic System (WGS84) ellipsoid (NIMA). Essentially, the GPS receiver compares the time which a signal is transmitted by a satellite with the time it is received; this elapsed time interval is scaled to be a distance by multiplying with the speed of light and tells how far the GPS receiver is away from the satellite. GPS receiver must receive signal at least from four satellites to calculate latitude, longitude and ellipsoidal height of the field point with centimeter and even millimeter accuracy.

However, GPS signal is influenced by many factors such as satellite orbit error,satellite clock error, receiver clock error, antenna phase center variation, tropospheric delay, ionspheric delay, multipath, electromagnetic interface and signal attenuation. In practice, the errors associated with GPS signals are modeled and removed from carrier phase and range observation. The error correction applied in calculating position using pseudo range and carrier phase’s measurements

### 2.3.3 GPS Error and Biases

GPS pseudorange and carrier-phase measurements are both affected by several types of random errors and biases (systematic errors). These errors may be classified as those originating at the satellites, those originating at the receiver, and those that are due to signal propagation (atmospheric refraction) (Ahmed El-Rabbany, 2002).

The errors originating at the satellites include ephemeris, or orbital, errors, satellite clock errors, and the effect of selective availability. The latter was intentionally implemented by the U.S. DoD to degrade the autonomous GPS accuracy for security reasons. Some of the errors are:-

- **Atmosphere**: Ionospheric and Tropospheric refraction can delay the signal and cause ranging errors
- **Multipath**: Reflecting or bouncing signals not traveling directly to the antenna can cause ranging errors, e.g.buildings, tree trucks, canyons.
- **Satellite Geometry** (Dilution of Precision or DOP): Bad satellite geometry can result in weak positional solutions. These DOPs can be separated into Vertical, Horizontal, Positional (3D) and Geometric (with time).
- **Selective Availability:** The US government's ability to degrade positional accuracy by "dithering" or slightly altering the satellite clocks and by changing the broadcast ephemeris to report a slightly different satellite position.(Switched off on May 1, 2000, but can be reinstated at any time.)
- **Anti Spoofing:** To prevent hostile outside sources from degrading the P-Code, the (Y) Code replaces the P Code, creating an encryption that can only be demodulated by special hardware.

# CHAPTER THREE

## 3. Study area, Data source and research method

### 3.1 Back ground of the Study Area

Addis Ababa is located in the central highlands of Ethiopia. Geographically, it is located at 9º 38’ 0’’N latitude and 38º 42’ 0’’E longitude , with the lowest elevation of 2020m above Mean Sea Level; in the southern periphery, and the highest over 3000m above MSL, in the north part of the city.

For administrative purpose,Addis Ababa city is divided in to 10 sub-cities and 120 woredas. This study is carried out in three sub- cities i.e , Arada, Gulele and Yeka. The accuracy of the geoid model is validated along the baseline: Legehar - Hilton - 4 kilo - Shiromeda Road. Geodetic leveling was conducted to along the main asphalt road running from the Ethiopian Geospatial Information Institute (formerly named Ethiopian Mapping Agency) to 4 kilo ( Addis Ababa University, Science Faculty Campus), 5 kilo(Addis Ababa Institute of Technology), 6 kilo (Addis Ababa University Main Campus), Addis Ababa University Faculty of Bussines and Economicsto the Entoto TEVT.

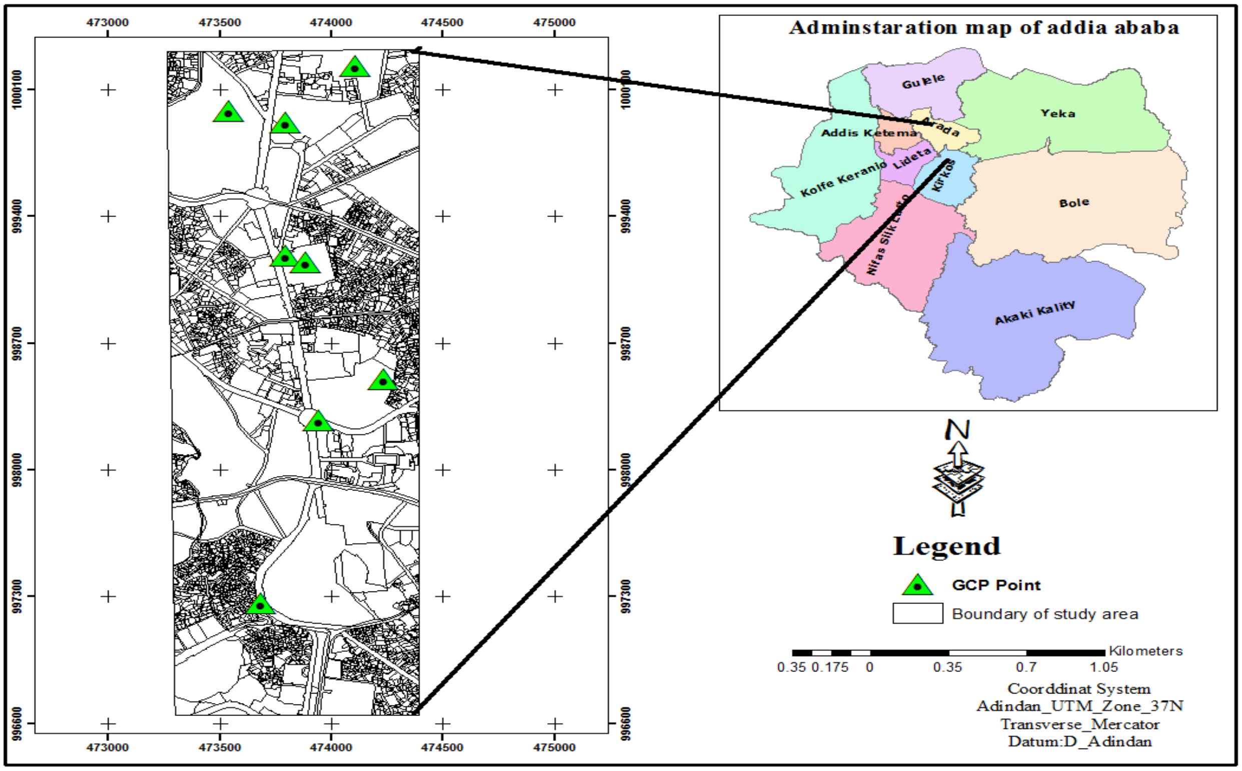


Figure 3.1: Location map of the study area

## 3.2 Back ground of the Ethiopian geodetic leveling height network

The history of the Ethiopian geodetic leveling height network dates back to around 1950's. The Blue Nile river basin geodetic control project of 1957 – 1961 was accomplished under the authority of agreements between the imperial Ethiopian government and the government of the united states of America specifically the third operational agreement for the extension of the program for the study of the water resources of Ethiopia for the multipurpose Blue Nile river basin investigation'' dated June 26,1956. The technical requirements of the program include aerial photography, the preparation of photo mosaic maps and the establishment of precise geodetic control with a view to using the data in the preparation of topographic maps. The geodetic networks were established with the first order methods and procedures by a combined organization of Ethiopian and united states personnel, financed by a joint imperial Ethiopian government and united states government cooperative service.

The Ethiopian Geodetic control project area covers the Ethiopian watershed of the Blue Nile River basin as determined from existing maps, extended to one half degree latitude and longitude lines to assure inclusion of all tributaries.

The general route of horizontal and vertical surveys throughout the project area were specified as well as the accuracy requirements. The specifications required that the surveys be connected to Sudanese triangulation and leveling in order that the 30th meridian African datum and mean sea level elevation could be provided in the river basin. The project was Also included the coverage of areas required to make the connection to Sudanese control and to existing triangulation in Ethiopia.

The triangulation extended from Kashmul el- Gibra in Kassala province of Sudan, eastward across the Ethiopia – Sudan border in to western Eritrea, thence southward along the border to the vicinity of metema in Ethiopia and thence into the blue nile river basin project area. The leveling survey extended from Gedaraf Sudan, southeastward across the Ethiopia – Sudan border and in to the river basin at metemma Ethiopia. From Metemma the leveling survey continued to Azozo, Addis Zemen, Dessie, Addis Ababa, Lechempti, Asosa, Gubba, Metekel, Bahar dara with a loop tie-back in to Addis Zemen. Spur lines were extended from Azozo to lake Tana gorgora (a second connection to the lake surface was made at Bahrdar) from addis ababa to Ducam and from Combolsia to Asseb on the red sea coast of Ethiopia where a connection was made to mean sea level of the red sea. (Ethiopian geodetic survey, 1957- 1961).

Field work methods and procedures as prescribed by the manual of the First order leveling were followed throughout the survey. All position computations were made using the Clarke Spheroid of 1880. All lines were double run although the specifications permitted single run through rough terrain. The type of bench mark established on the survey was adopted to provide marks more suitable to local conditions. Brass disks and Monel metal rivet were used in cities and towns and chiseled burrs on rock out crops or large boulders were used in remote areas(Ethiopian geodetic survey, 1957-61).

### 3.3 Data acquisition and materials

#### 3.1.1 Leveling Data

Geodetic spirit leveling is used to measure elevation differences between ground points. In this method a horizontal line of sight is established by using a sensitive level bubble in a level vial. The instrument is leveled and the line of sight of the instrument describes a horizontal plane. The difference in elevation between a known elevation and the height of instrument is determined. Next, the difference in elevation from the height of instrument to an unknown point is derived by measuring the vertical distance with precise or semi-precise level and leveling rods.

For the conventional spirit leveling, a SOKKIA455701 B20 010369 and LEICA LS 15 Digital automatic level were used with 2 metric(centimeter graduations) Philadelphia leveling rods. Level has 32x Magnification Rapid, Accurate, and Stable Automatic Compensation, Ultra-Short 20cm Focusing, All-Weather Dependability, Clamp less, Endless Fine Horizontal Adjustment, Sub-millimeter accuracy when used in conjunction with invar staff and OM-5 Plate Micrometer.

The spirit leveling was performed in 5 loops with two surveying crew EMA crew and the research crew. The first loop run from a known control point at the gate of EMA denoted as EMA 1 to a Known point AAU1 in front of the IGS station found in Addis Ababa university 4 kilo campus on July 16/2018 G.C and back turning on the same points. The second crew run the remaining double 4 loops from a known point AAU1 to ET01 i.e,

(BM - 5K to 5K - 01 , 5K - 02 , 6K - 01), (6K - 01 to FB - 01) ,( FB - 01 to ET - 01 ) from august 8 to 11/2018 G.C . Turning points were set unevenly. All readings were recorded in a standard survey field book. EMA 1, was used as the orthometric vertical control point for the level loop. EMA 1 was established by the Ethiopian Mapping Agency of the Blue Nile Network in 1957. It is also established baselines between BMP 1 and BMP 3.

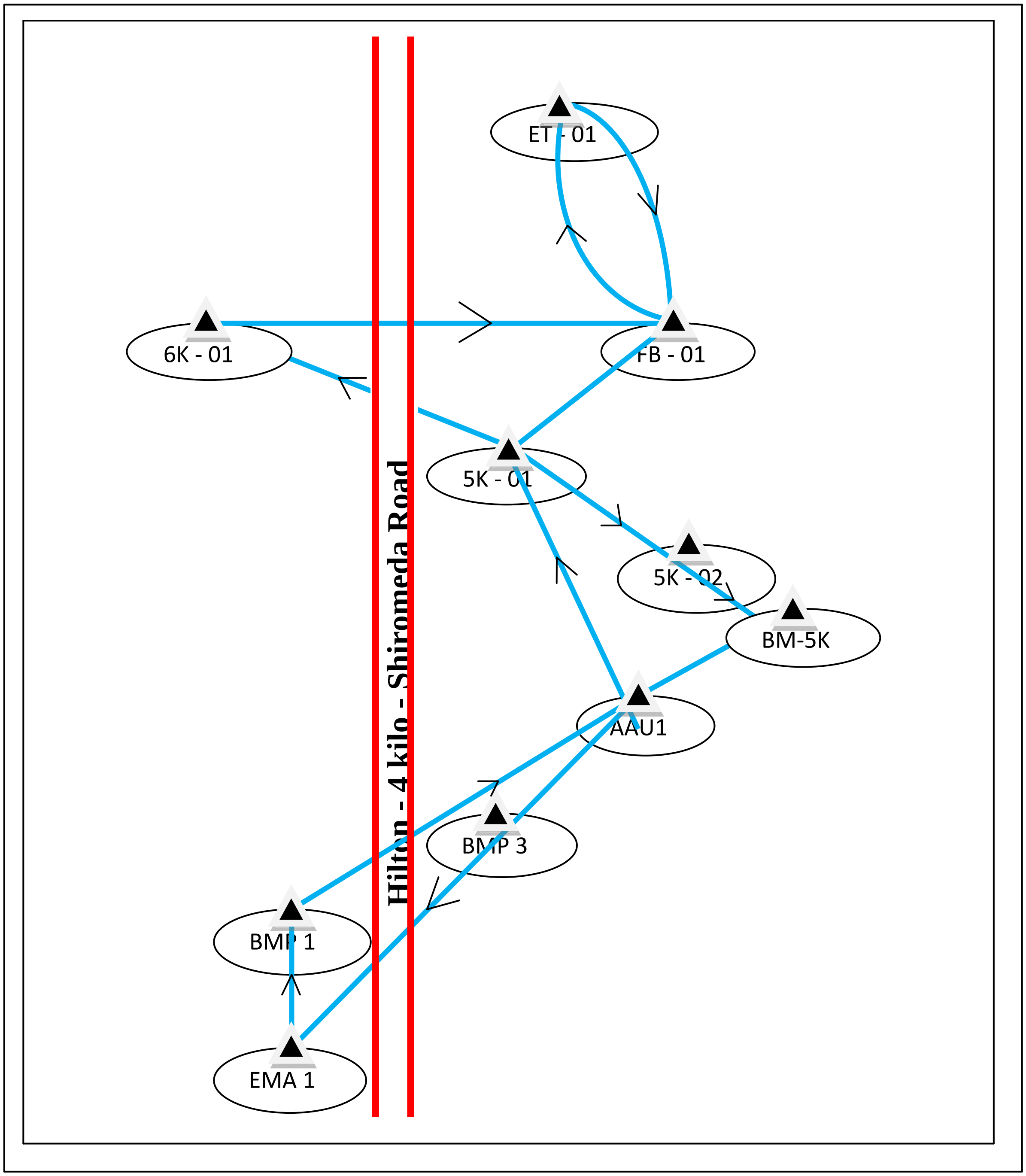


Figure 3.2 Shows the leveling loop

**Ground Control Points, or GCPs**: are marked points on the ground that have a known geographic location. Control points ensure that the latitude and longitude of any point on your map corresponds accurately with actual coordinates. This is especially important in situations where precision mapping and true global accuracy are needed.

Table 3.1 Description of GCP's

| **No** | Name of GCP | Description | LOCATION | Elevation(m) | Remark |
|---|---|---|---|---|---|
| **1** | EMA1 | Bronze plate tagged on a Concrete culvert | At the main gate of EMA | 2385.25 | Known Established by EMA |
| **2** | BMP1 | Pin like Iron plate tagged on road side walkway | Around the curb of grand palace to Sheraton Addis road | 2406.13 | Newly established |
| **3** | BMP3 | Pin like Iron plate tagged on road side walkway | Behind of 4 kilo roundabouts | 2433.06 | Newly established |
| **4** | AAU1 | Concrete monument with tag of bronze plate at the top | Inside 4 kilo campus In front of the IGS station | 2443.586 | Newly established |
| **5** | 5K01 | Concrete monument | Inside 5kilo campus at the curb of upper gate | 2470.245 | Newly established |
| **6** | 5K02 | Concrete monument | Inside 5 kilo campus green area found In front of the library | 2467.746 | Newly established |
| **7** | 6K01 | Concrete monument | In front of Lidet Hall at AAU 6 kilo campus | 2494.655 | Newly established |
| **8** | FB01 | Concrete monument | Around stadium of FBE campus | 2498.726 | Newly established |
| **9** | ET01 | Concrete monument | In front of metal workshop in Entoto TVET | 2498.988 | Newly established |
| **10** | BM- 5K | Iron pin tagged at the bottom beam of a building |  |  |  |

### 3.1.2 Global Positioning System (GPS) Data

Geodetic coordinates (latitude, longitude, ellipsoidal heights) are determined at every leveling bench mark using static differential GPS measurements. GPS data is processed using differential approach with a tie to International GNSS service (IGS). This technique uses both the L1 and L2 carrier frequencies broadcast by the GPS satellites in order to measure baselines and determine positions to the centimeter (cm) level. The length of observation time at every bench mark is 5 hour at a data logging rate of 5 seconds. Loss of lock, when moving between stations, can also occur with no effect on the results since each baseline is processed independently of each other.

For the DGPS survey, LEICAVIVA GS-14 receivers were used. For the data processing, a LENOVO core i7 computer with Gamit/GLOBK software was used.

The DGPS survey was performed in three session from July 28 to 30/2018 G.C. Three GPS receiver was set up over 2 EMA known points( EMA1and AAU1) and other 7 newly established unknown points (BMP 1, BMP 3,5K - 01 , 5K - 02 , 6K - 01, FB - 01 , ET - 01 ) collected data in a static mode. Each station was observed for 5 hours;

- During the 1st session Colored by Magenta EMA1,BMP1 and BMP3 were observed.
- AAU1,5K01 and 5K02 stations were measured during the 2$^{nd}$ session.
- In the 3rd session Colored by Blue 6K01,FB01 and ET01 were acquired.

The GPS data was processed using Gamit/GLOBK post processing software. Baselines were processed between (EMA1, BMP1 and BMP3), (AAU1, 5K - 01 and 5K - 02), (6K - 01 , FB - 01 to ET - 01 ).

The observed baselines are shown in Figure 4.2, Latitude, Longitude and Ellipsoidal height, were computed for each control point . The GPS coordinate point values were then used as input in the geoid modeling software to determine the geoid heights for each point. These geoid heights were then subtracted from the GPS ellipsoidal heights to obtain orthometric e height. Ellipsoidal heights (h), orthometric heights from spirit leveling ($H_{\text{level}}$) and the orthometric height derived from the geoid modeling ($H_{\text{model}}$) are the basic parameters used for evaluation of the geopotential model. The differences between the gravimetric geoid model (derived from EGM08) and the geometric geoid (calculated from leveled heights and ellipsoidal heights) were calculated at all bench marks (i.e. EMA1 , BMP 1, BMP 3, AAU1,5K - 01 , 5K - 02 , 6K - 01, FB - 01 , ET - 01). Besides, the residuals between leveled heights ( $\Delta H_{\text{level}}$ ) and orthometric height ( $\Delta H_{\text{model}}$) were determined.

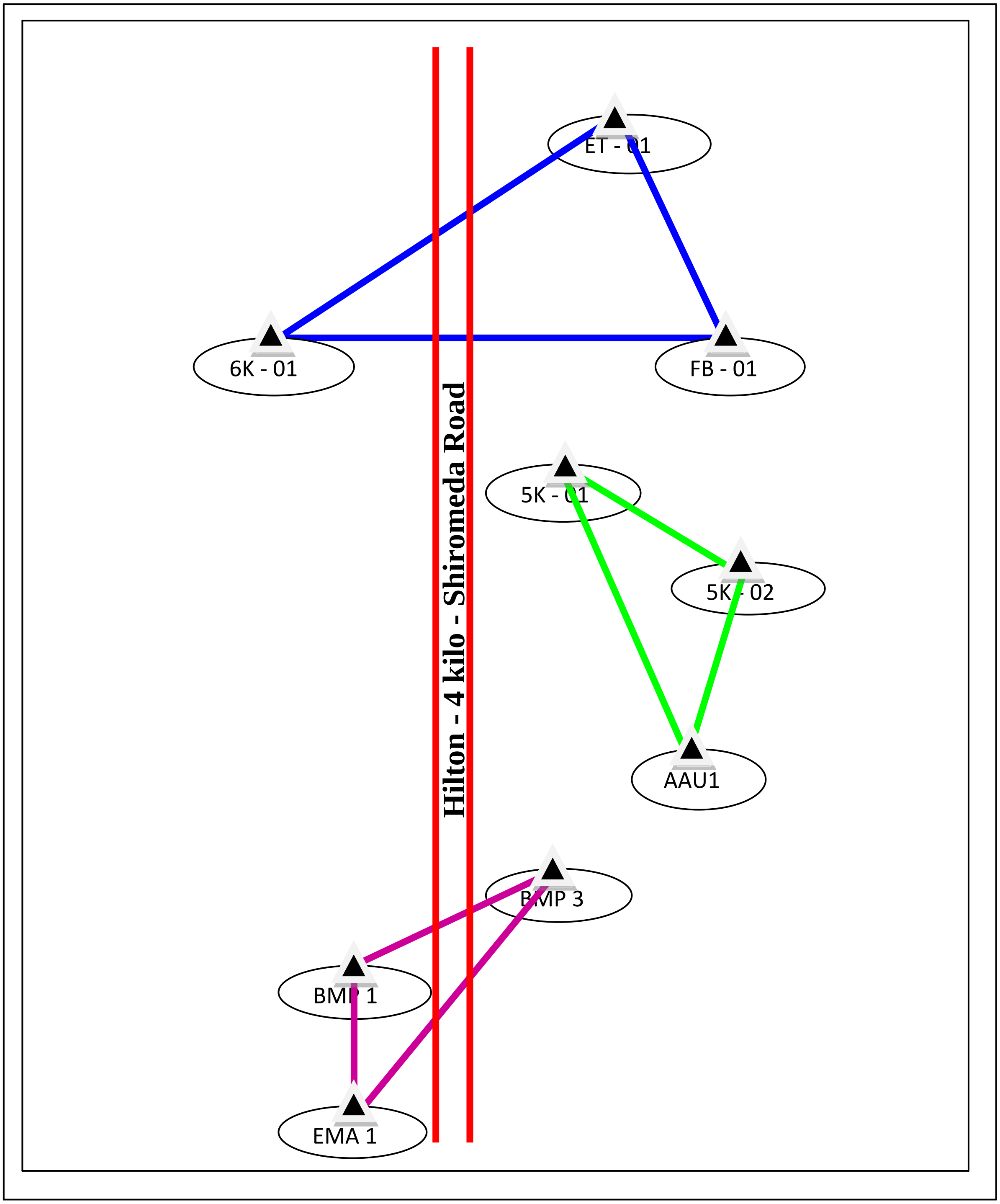


Figure 3.3 Shows GPS base line

**IGS Networks**

In order to improve positional accuracy of the GPS data a total of 8 IGS stations were used for a reference propose i.e Adis from Ethiopia, BHR4 from Bahrain, Drag , Mal2, Mbar, NKLG, Ramo and Zamb. The stations were chosen based on proximity to the local network stations and stability based on the velocity solutions as presented in ITRF14.

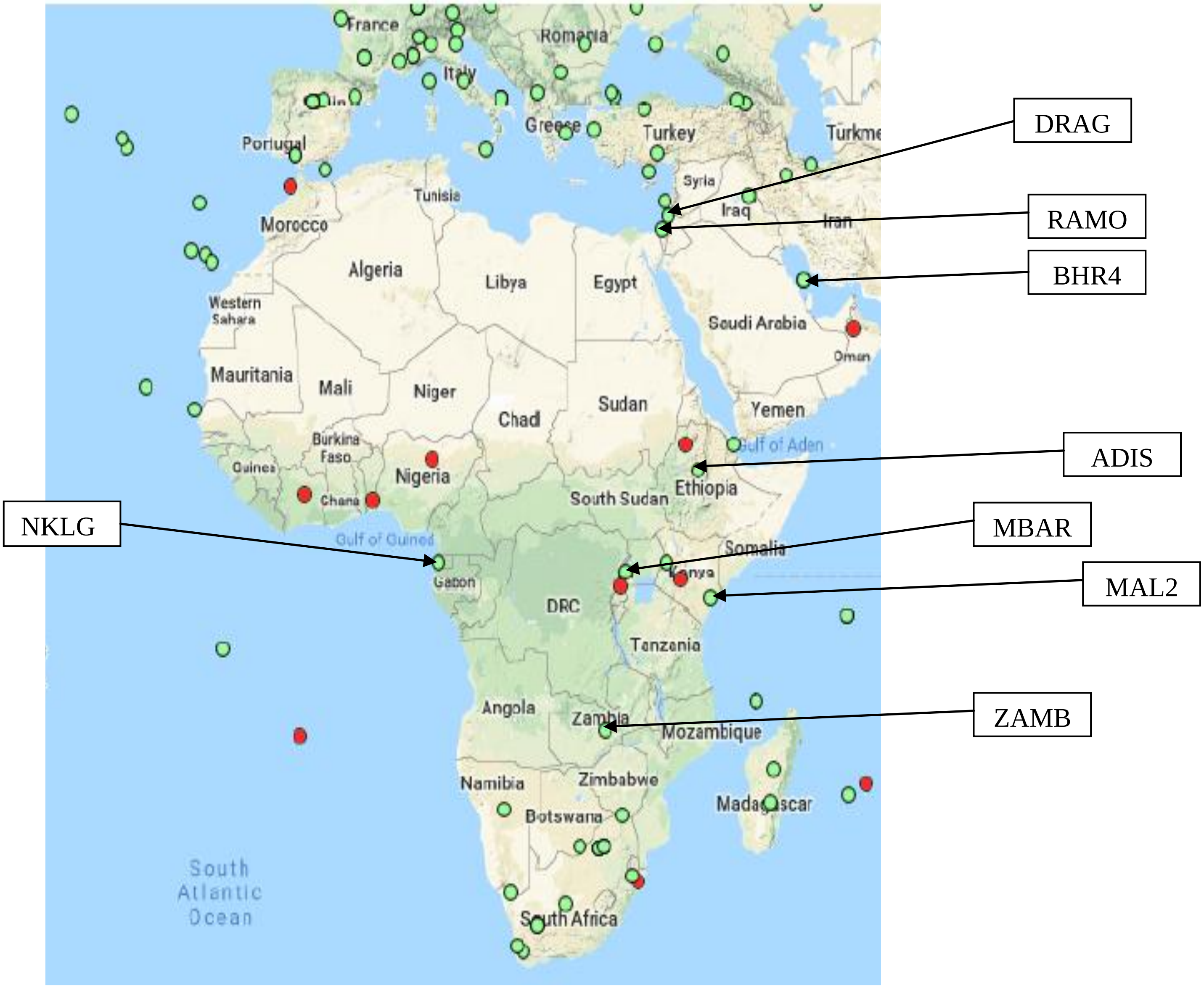


Figure 3.4 show Map of IGS station used in this study

### 3.1.3 Earth Gravitational Model / EGM/ Data

The GPS derived heights are based on the WGS84 ellipsoid (ellipsoidal heights), but the heights determined from EGM (orthometric height) are tied to the geoid. Ellipsoidal height is a very useful geometrical parameter that exploits the advantage of leveled heights for evaluating the accuracy of the gravimetric geoid model.

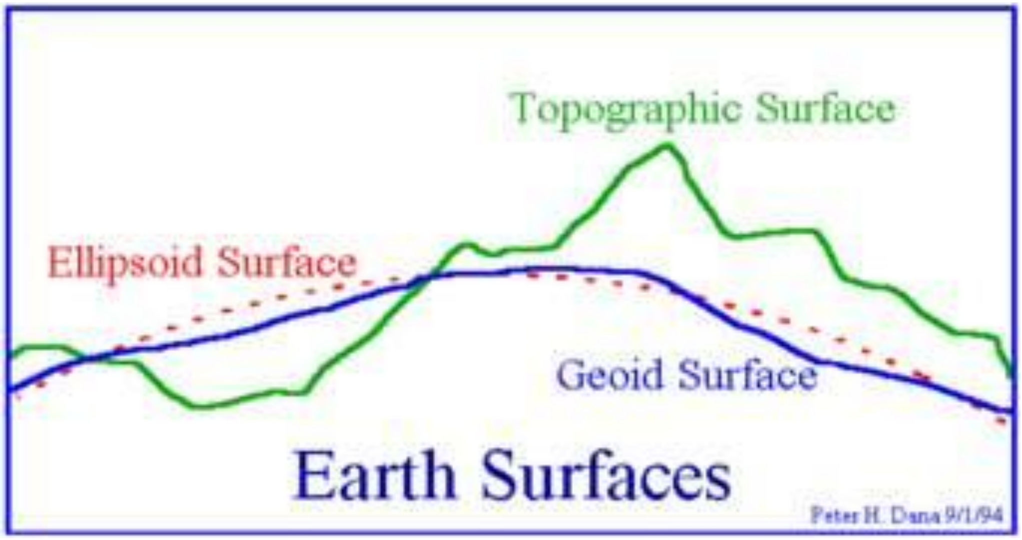


Figure 3.5 Shows Geoid and Ellipsoid relationship.

Different EGMs are used for computing the geoid. These include EGM96, EGM08, etc. In this study, we used in-situ ellipsoidal heights and leveled heights in order to evaluate the accuracy of the EGM08 derived geoid model.

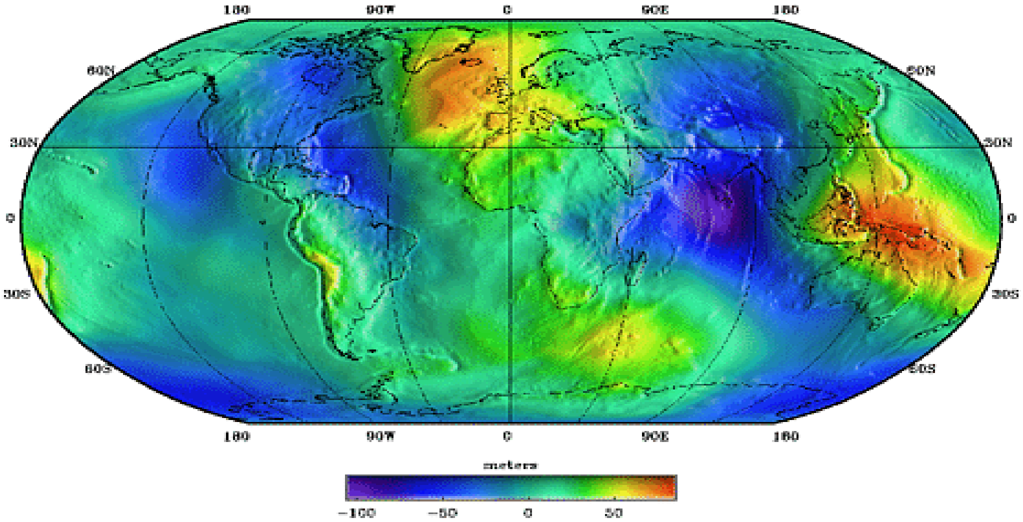


Figure 3.6 Earth Gravity Model/EGM 08

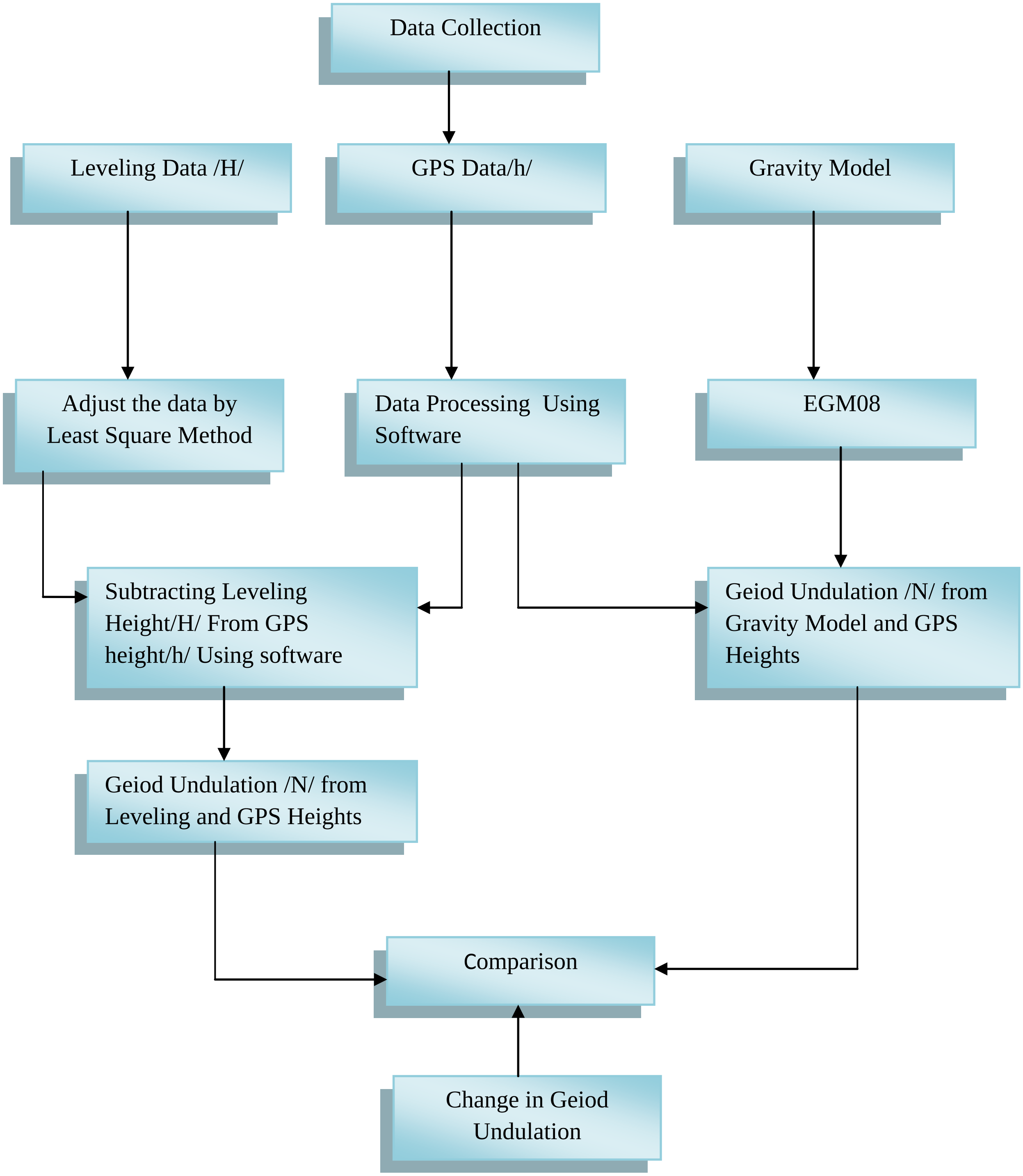


Figure 3.7 Data processing flow chart showing the general methodology

CHAPTER FOUR

## 4. Results and Discussions

Precise determination of orthometric heights based on geodetic leveling is expensive, time consuming and tedious. The Global Positioning System (GPS) has emerged as a successful technology in providing precise position of points on the surface of the earth with reference to the 1984 world geodetic system reference ellipsoid, with an achievable accuracy at millimeter level. most practical application in various geosciences disciplines necessitate the tasks of transforming ellipsoidal heights into orthometric heights. this involves computing the geoid from EGM. The EGM 2008, a global geoid model is widely employed for providing sub meter accuracy.

This study used EGM08 complete to degree and order 2160 spherical harmonic coefficient to determine gravimetric geoid and orthometric height via ellipsoid height and evaluate them against an independent level heights ( orthometric height) and geometric geoid heights estimated at the selected 9 control points.

Using the EGM 2008 geoid model, the geoidal undulation (geoidal separation-N) for the corresponding points were obtained.

On the other hand, leveling data were collected and adjusted at every selected Bench Marks. The processed data from leveling provided the leveling height of these points. The final adjusted leveled heights of all the 9 control points are provided in Table 5.1

Besides, co-located measurement of GPS data were collected at all leveling Bench Marks. GPS data were collected at 9 leveling benchmarks for three consecutive days using Leica Viva GS 14 GPS receivers. Ellipsoidal heights are determined from raw GPS data at a millimeter accuracy level using GAMIT/GLOBK software package. The geodetic coordinates obtained from the processed GPS data are presented in the form of easting, northing and ellipsoidal height.

The orthometric height of these points are computed using the ellipsoid height/h/ and the geoidal undulation/N/. These gravity-based orthometric heights are compared with the orthometric heights obtained from leveling/H/. The difference between these two height systems was used to compute the Root Mean Square Error (RMSE). Hence, the RMSE will help us to assessing the accuracy of EGM 2008 derived geoid model that converts ellipsoidal height to orthometric height.

### 4.1 Leveling survey

This research conducts **t**he most accurate method of Deferential leveling by which instrument locked in position, readings are made on two calibrated staffs held in an upright position ahead and behind the instrument was operated in order to measure orthometric elevations of the 9 control points that run from EMA1 to Entoto TVET. The difference between readings is the difference in elevation between the consecutive points. The measured data from leveling provided the leveling height of these points. A total length of 41,438.82 m leveling survey was done with a Forward and backward reading in 4 closed loops. The leveled heights are combined with ellipsoidal heights to determine geometric geoid heights. For more information, the site descriptions of the 9 control points and each level readings are presented in previous table 4.1 and Appendix A respectively.

The first loop running from EMA1 to AAU1 has two known points with an elevation of 2385.25m at EMA1 and 2443.586m at AAU1 of Addis Ababa University 4 kilo campus. The remaining bench marks such as BMP1,EMA2,and EMA3 points are newly established control points. A total length of 4390.72 m leveling survey was done with a closed loop. The elevation of EMA 1 with a tie to AAU1 benchmark has involved the task of surveying 4390.72 m leveling distance with a leveling increments of 58.34 m. The leveled heights of EMA 1,BMP1.EMA2,EMA3 and AAU1 benchmarks are determined to be 2385.30 m, 2406.13m, 2424.39m, 2433.06m and 2443.59 m above mean see level with a closure error of 0.5mm.

The 2nd loop was run from AAU1 to BM 5K it was done single run because the level was start and end with known points of AAU1 and BM 5K. With this level run there is newly established control points namely 5k 01 and 5k 02. These task involved 1995.5 m leveling distance with a leveling increments of 23.93 m. The leveled heights of AAU1, 5k 01, 5k 02 and BM 5K benchmarks are determined to be 2443.586 m, 2470.245m, 2467.746 and 2467.5125 m above mean see level respectively with a total closure error of 0.0925 mm.

The 3rd, 4th and 5th loop was done from BM 5k to 6k 01 , 6K 01 to FB 01, and FB 01 to ET 01 in a double run of forward and backward leveling. The task involved 2156.8 m, 1414.3 m 1935.7 m total leveling distance in every closed loop with a leveling increments of 27.199m, 4.069 m, and -0.279 m. The leveled heights of 6k 01 , FB 01, and ET 01 benchmark are estimated to be 2443.586 m, 2498.78 m, and 2498.509 m above mean see level with a closure error of -0.3409 m,0.0055m and -0.0172 respectively.

| | **From** | **TO** | **∑Back sight** | **∑ foresight** | **Dist. Go** | **Dist. Back** | **RL(m)** | **Remark** |
|---|---|---|---|---|---|---|---|---|
| **1st Loop** | EMA1 | BMP1 | 20.8857 | -20.8366 | 514.86 | 513.63 | 2385.3 | 2385.25(known) |
| | BMP1 | EMA2 | 18.2562 | -18.259 | 846.25 | 846.15 | 2406.13 | |
| | EMA2 | EMA3 | 8.6686 | -8.6677 | 260.75 | 260.68 | 2424.39 | |
| | EMA3 | AAU1 | 10.5276 | -10.5286 | 572.38 | 576.02 | 2433.06 | |
| | AAU1 | | | | | | 2443.59 | 2443.586(known) |
| | ***Total*** | | ***58.3381*** | ***-58.2919*** | ***2194.24*** | ***2196.48*** | | |
| **2nd Loop** | AAU1 | 5k01 | 45.032 | 18.373 | 6.322 | 6.042 | 2443.586 | 2443.586(known) |
| | 5k01 | 5k02 | 1.339 | 1.699 | 307 | 142 | 2470.245 | |
| | 5k02 | BM-5k | 1.033 | 1.276 | 188 | 206 | 2467.746 | |
| | BM-5k | | | | | | 2467.5037 | 2467.42(known) |
| | ***Total*** | | ***48.249*** | ***24.331*** | ***7088*** | ***6861*** | | |
| **3rd Loop** | BM-5k | 6k 01 | **36.619** | 9.328 | 5.328 | 5.358 | 2467.42(known | 2467.42(known) |
| | 6ko1 | Bm5k | 14.8748 | 42.5087 | 4.876 | 6.006 | 2494.711 | |
| | BM-5k | | | | | | 2467.0791 | |
| | ***Total*** | | ***51.4938*** | ***51.8367*** | ***10.204*** | ***11.364*** | | |
| **4th Loop** | 6k01 | FB 01 | 20.8909 | 16.8223 | 3.568 | 3.409 | 2494.711 | |
| | FB 01 | 6K 01 | 16.6964 | 20.7595 | 3.619 | 3.547 | 2498.7796 | |
| | ***Total*** | | ***37.5873*** | ***37.5818*** | ***7.187*** | ***6.956*** | | |
| **5th Loop** | FB 01 | ET 01 | 26.4566 | 26.7277 | 4.935 | 4.667 | 2498.7796 | |
| | ET 01 | FB 01 | 27.522 | 27.2681 | 4.659 | 5.096 | 2498.5085 | |
| | ***Total*** | | ***53.9786*** | ***53.9958*** | ***9.594*** | ***9.763*** | | |

**Table 4.1 Summary of surveying results of 9 control points**

## 4.2 GPS Data Processing

GPS measurement was made at all 9 leveling bench marks using LEICA VIVA GS 14 GNSS receivers. At each bench mark, GPS observations was made for the duration of five hours. Co located GPS measurements at all bench marks was carried out in three days from July 28, 2018 to July 30, 2018.

After the field observations, the observation data was downloaded from LEICA VIVA GS 14 GNSS in raw data format. The Leika Geo office rinex converter program was used to convert the raw data to RINEX (Receiver Independent Exchange) format .The Rinex data for each of the benchmark station was processed using GAMIT/GLOBK software to compute the geodetic latitude, longitude and ellipsoidal heights in the ITRF 2014 reference frame using IGS orbit product precise ephemeris.

**GAMIT** ("**G**NSS **A**t **MIT**") is collection of programs to process phase data to estimate three-dimensional relative positions of ground stations and satellite orbits, atmospheric zenith delays, and Earth orientation parameters. As the software is composed of distinct programs which perform the functions of preparing the data for processing, generating reference orbits for the satellites , computing residual observations and partial derivatives from a geometrical model, detecting outliers or breaks in the data and performing a least-squares analysis.

**Table 4.2:** Computed Coordinates of the 9 control points with their ellipsoidal heights in meter

| Station | Longitude(N) | Latitude(E) | Ellipsoidal Height (U) in m |
|---|---|---|---|
| AAU1 | 9.0350022706 | 38.7661556069 | 2437.53594 |
| EMA1 | 9.0201275087 | 38.7631697525 | 2381.92561 |
| EMA2 | 9.0235709470 | 38.7614239603 | 2396.90899 |
| EMA3 | 9.0327244799 | 38.7637606911 | 2423.23256 |
| ET01 | 9.0504953403 | 38.7652641905 | 2492.55483 |
| FB01 | 9.0478527564 | 38.7627510480 | 2492.77368 |
| 5K01 | 9.0410153698 | 38.7624008679 | 2464.20479 |
| 5K02 | 9.0406353942 | 38.7631512669 | 2461.68860 |
| 6K01 | 9.0481252062 | 38.7600491229 | 2488.69491 |

Using GAMIT, the coordinates for each of the benchmark on the respective time were computed to millimeter accuracy in the North, East and Up (NEU) components with reference to WGS84 reference ellipsoid. All coordinates were computed with a tie to eight IGS network stations called ADDIS, BHR4, DRAG, NKLG, MAL2, MBAR, RAMO, and ZAMB. The stations were chosen based on proximity to the local network stations and stability based on the velocity solutions as presented in ITRF14.

GPS data processing can be effectively done with shell scripts, where a sequence of batch files setup a driver module for modeling, editing and estimation. Data editing is performed automatically so that the solution residuals can be displayed or plotted to identify the outliers. The residual values (*r*) shows the degree of accuracy of coordinates of the stations.

Table 4.3 Coordinates and residual values of control points residual are in meter

| **Station** | **North** | **East** | **Up** | ***r* in (N)** | ***r* in (E)** | ***r* in (U)** |
|---|---|---|---|---|---|---|
| EMA1 | 9.0201275087 | 38.7631697525 | 2381.92561 | - | - | - |
| EMA2 | 9.0235709470 | 38.7614239603 | 2396.90899 | - | - | - |
| EMA3 | 9.0327244799 | 38.7637606911 | 2423.23256 | - | - | - |
| AAU1 | 9.0350022706 | 38.7661556069 | 2437.53594 | 0.0053 | 0.0070 | 0.0266 |
| ET01 | 9.0504953403 | 38.7652641905 | 2492.55483 | 0.0062 | 0.0075 | 0.0299 |
| FB01 | 9.0478527564 | 38.7627510480 | 2492.77368 | 0.0061 | 0.0069 | 0.0291 |
| 5K01 | 9.0410153698 | 38.7624008679 | 2464.20479 | 0.0067 | 0.0161 | 0.0342 |
| 5K02 | 9.0406353942 | 38.7631512669 | 2461.68860 | 0.0072 | 0.0112 | 0.0413 |
| 6K01 | 9.0481252062 | 38.7600491229 | 2488.69491 | 0.0066 | 0.0073 | 0.0299 |

Table 4.4 Coordinates and residual values of IGS networks residual are in meter

| **Station** | **North** | **East** | **Up** | ***r* in (N)** | ***r* in (E)** | ***r* in (U)** |
|---|---|---|---|---|---|---|
| ADDIS | 9.0351372042 | 38.7663058953 | 2439.12631 | 0.0010 | 0.0014 | 0.0047 |
| BHR4 | 26.2091449877 | 50.6081497811 | -13.89786 | 0.0024 | 0.0018 | 0.0066 |
| DRAG | 31.5932031565 | 35.3920736145 | 31.85042 | 0.0023 | 0.0010 | 0.0061 |
| MAL2 | -2.9960532748 | 40.1941459484 | -20.92625 | 0.0010 | 0.0012 | 0.0038 |
| MBAR | 0.6014671599 | 30.7378788162 | 1337.54024 | 0.0009 | 0.0008 | 0.0036 |
| NKLG | 0.3539088565 | 9.6721276776 | 31.49407 | 0.0010 | 0.0020 | 0.0045 |
| RAMO | 30.5976078883 | 34.7631427167 | 886.84048 | 0.0022 | 0.0010 | 0.0058 |
| ZAMB | -15.4255385856 | 28.3110146687 | 1324.91456 | 0.0018 | 0.0009 | 0.0046 |

As we can see from the above two tables 4.3 and 4.4;  From Tables 4.3 and 4.4, it is observed that the residual values are relatively higher for the local GPS leveling benchmarks due to short session of observations (~6hrs)  As a result of this, it was not possible to calculate residual values for all the stations, see Table 4.3.

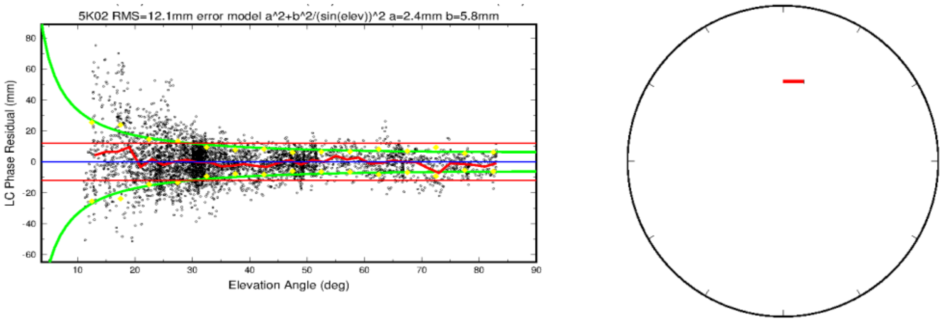
The residual values *r* of  the IGS stations in the north and east coordinates are below  0.0024 m. The accuracy of the east and north coordinates of the local stations has a median value of 0.0107m. The RMS error estimate of the vector is a good indicator - usually not more than 15 millimeters . This justifies that the majority of  horizontal position coordinates are within the acceptable range (< 15mm).

In principle, the error budget in the vertical coordinate is expected to be higher compared to errors in the horizontal coordinates. In this study, the vertical (up) coordinates of the local GPS stations have been determined with a maximum error of 0.0413m.

The dominant errors in GPS observations are not random rather it correlates with the error sources affecting the GPS signal propagation. In respective of the quality of the data; sky plot is one of the tools to check the nature of the GPS data. The geometry of satellites constellation  can change rapidly and frequently. This causes different satellites to be used in the position computation. Satellite constellation has a large effect on the quality of the data collected, as different satellite constellations cause different bias in the data.

Constantly changing constellations result in data that is inconsistent and has poor relative accuracy. The sky plot helps us to identify multipath (repeatable noise) and level of atmospheric noise called water vapor (time varying noise) in the observation. The plot of phase versus elevation angle can be used to identify multipath, water vapor, antenna phase center shift providing a unique opportunity to know the stability of the GPS station.

For example 5k 01and 5k 02 has relatively large rms value 0.034m and 0.041m sky plot and residual vs. elevation plot ( see figure 4.1)



(a)

(b)

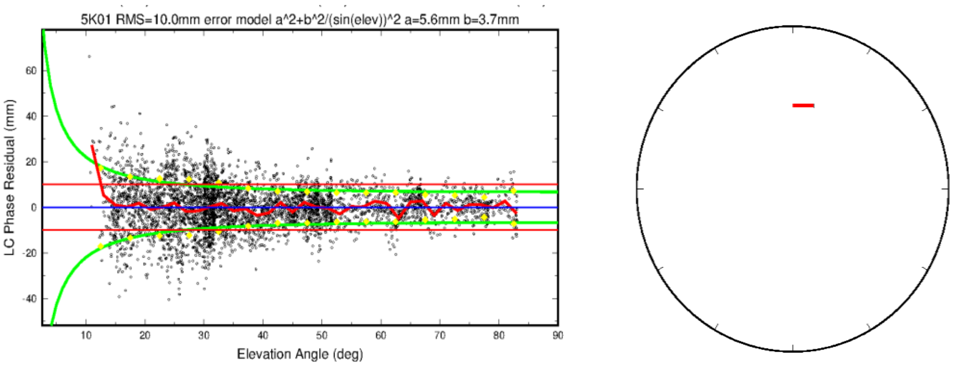


Figure 4.1 Multipath and Phase residual vs. Elevation angle of 5k01(a) and 5k02(b) benchmark

In sky plot red elliptical curve is satellite track, yellow and green elliptical curves are residuals, and red bar is scale (10mm). As it can be seen from the sky plot , Figure 4.1 there is small residual in the same place at different times corresponding to small multipath effect during satellite observation (between -120 to -150 degree). Similarly, there is small residual appearing in a given place only at one time because of water vapor.

The plot phase residual versus elevation angle gave normal pattern showing the stability of station, except instability for 70 to 80 degree of elevation angle. Red lines are smoothing of individual values and show no strong systematic error. Green lines show the elevation dependent noise model and used to re-weight the data. In general, even if station 5k 01 and 5k 02 has relatively large *r* compared to the other stations, there is no large significant multipath and atmospheric effect on the sky plot. The phase versus elevation angle plots also showed that physical performance of this station is slightly good and stable.

### 4.3 EGM2008-based geiod heights

Co-located measurements of ellipsoidal height and precise geodetic leveled height provide a unique opportunity to validate the accuracy of the gravimetric geoid model i.e EGM 08. Now, GPS technology provides ellipsoidal height of a particular field point accurate to millimeter level. This enables us to determine orthometric height for any isolated field point independently from gravity and GPS without any need to connect to a local network of leveled heights. In essence, the difference between ellipsoidal height and gravimetric geoid height gives an orthometric height.

$$\mathbf{H\ (ortho) = h - N_{gravity\ model}}$$

Traditionally, orthometric heights are determined using a precise geodetic leveling network with a tie to a local mean sea level through one or more tide gauge stations. Now, the gravity and GPS satellite missions have revolutionized the way geopotential heights (orthometric and normal heights) are determined directly from the global gravity model (e.g. EGM2008) and GPS coordinates (α, λ, h).

### 4.3.1 Computation of Geoid heights/N/ from GPS derived and leveled heights

The geoid heights at the 9 points were computed based on ellipsoidal heights and leveled heights i e given by , $N^{GPS/Lev} = h - H$ ,where $N^{GPS/lev}$ is the geoid height determined by GPS/leveling and provides the data set upon which the evaluation tests are performed. Table 4.5 shows the computed geoid heights from ellipsoidal heights and leveled height.

The GPS derived heights are ellipsoidal heights (h) with respect to the reference ellipsoid while the orthometric heights (H) are levels with respect to the geoid (MSL). The separation of the geoid and the reference ellipsoid is known as geoid undulation/separation (N). N is positive when reference ellipsoid is above geoid & negative when it is below.

Table 4.5 Computed geoid height /N/ of 9 G.C.P ( from ellipsoidal/h/ and leveled/H/ heights )

| **Station** | **Longitude(N)** | **Latitude(E)** | **Ellipsoidal Height (h) in m** | **Leveled Heights(H) in m** | **$N^{GPS/Lev} = h - H$** |
|---|---|---|---|---|---|
| EMA1 | 9.0201275087 | 38.7631697525 | 2381.92561 | 2385.25 | -3.32439 |
| EMA2 | 9.0235709470 | 38.7614239603 | 2396.90899 | 2406.13 | -9.22101 |
| EMA3 | 9.0327244799 | 38.7637606911 | 2423.23256 | 2433.06 | -9.82744 |
| AAU1 | 9.0350022706 | 38.7661556069 | 2437.53594 | 2443.586 | -6.05006 |
| 5K01 | 9.0410153698 | 38.7624008679 | 2464.20479 | 2470.245 | -6.04021 |
| 5K02 | 9.0406353942 | 38.7631512669 | 2461.68860 | 2467.746 | -6.0574 |
| 6K01 | 9.0481252062 | 38.7600491229 | 2488.69491 | 2494.711 | -6.01609 |
| FB01 | 9.0478527564 | 38.7627510480 | 2492.77368 | 2498.7796 | -6.00592 |
| ET01 | 9.0504953403 | 38.7652641905 | 2492.55483 | 2498.5085 | -5.95367 |

There are some differences in the results between leveling and GPS observations because datum reference. GPS provide ellipsoid heights with respect to WGS84 , while the leveling heights are based on a level surface called the geoid (M.S.L).

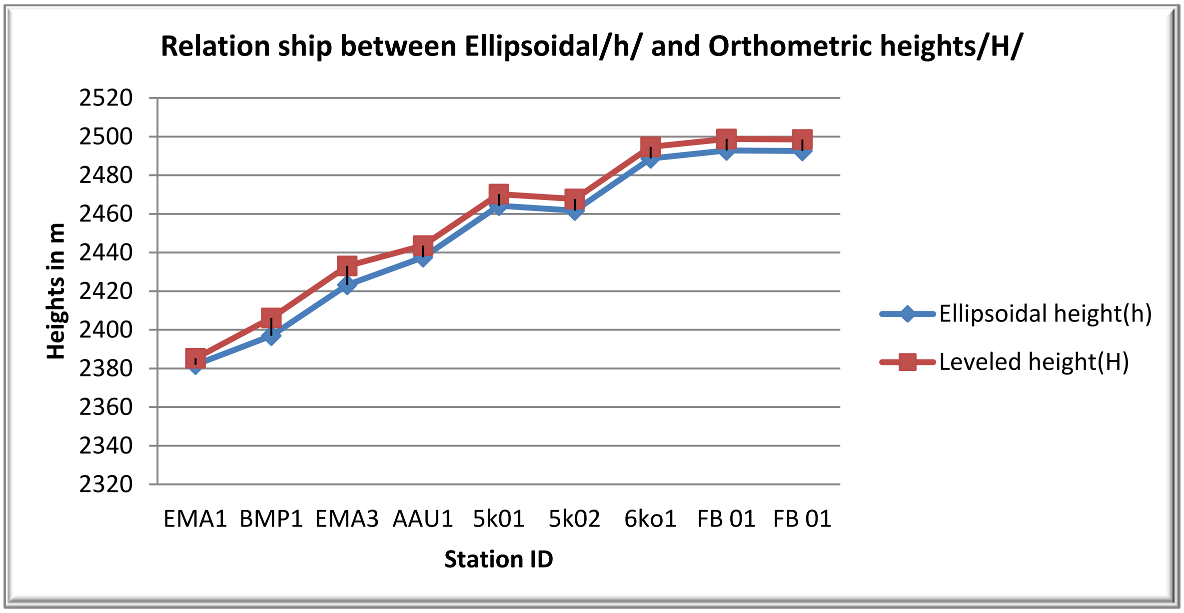


Figure 4.2 The relationship between Ellipsoidal height and Orthometric height

### 4.3.2 Computation of Geoid heights/N/ from ellipsoidal heights/h/ and EGM 08

The Geoid Model EGM2008 is adopted to calculate the Geoid Undulation (N) for the same Control Points in this study, using online geoid calculations of the geoidEval utility, see the link :*http://geographiclib.sourceforge.net/cgibin/GeoidEval?input=33+45&option=Submit*

```
                         Example
Preview for output of AAU1 from geoidEval online processing
Geoid heights
lat lon = 9.03500 38.76616 (09°02'06"N 038°45'58"E)
    geoid heights (m)
        EGM2008 = -7.0594
        EGM96   = -7.4388
        EGM84   = -7.0189
```

Table 4.6 Computed geoid height /N/ of 9 control points from ellipsoidal/h/ and EGM08

| **Station** | **Longitude(N)** | **Latitude(E)** | **Ellipsoidal Height (h) in m** | **$N^{GPS/EGM08}$** |
|---|---|---|---|---|
| EMA1 | 9.0201275087 | 38.7631697525 | 2381.92561 | -7.1494 |
| EMA2 | 9.0235709470 | 38.7614239603 | 2396.90899 | -7.1235 |
| EMA3 | 9.0327244799 | 38.7637606911 | 2423.23256 | -7.0690 |
| AAU1 | 9.0350022706 | 38.7661556069 | 2437.53594 | -7.0594 |
| 5K01 | 9.0410153698 | 38.7624008679 | 2464.20479 | -7.0130 |
| 5K02 | 9.0406353942 | 38.7631512669 | 2461.68860 | -7.0169 |
| 6K01 | 9.0481252062 | 38.7600491229 | 2488.69491 | -6.9629 |
| FB01 | 9.0478527564 | 38.7627510480 | 2492.77368 | -6.9696 |
| ET01 | 9.0504953403 | 38.7652641905 | 2492.55483 | -6.9570 |

Figures- 4.3 and 4.4 below show statistical figures that declare the relationship between points distributed along the survey line against the Geoid undulation. It can be noticed that, the EGM 2008 is a global geopotential that can serve as a vertical datum for defining national or worldwide height system.

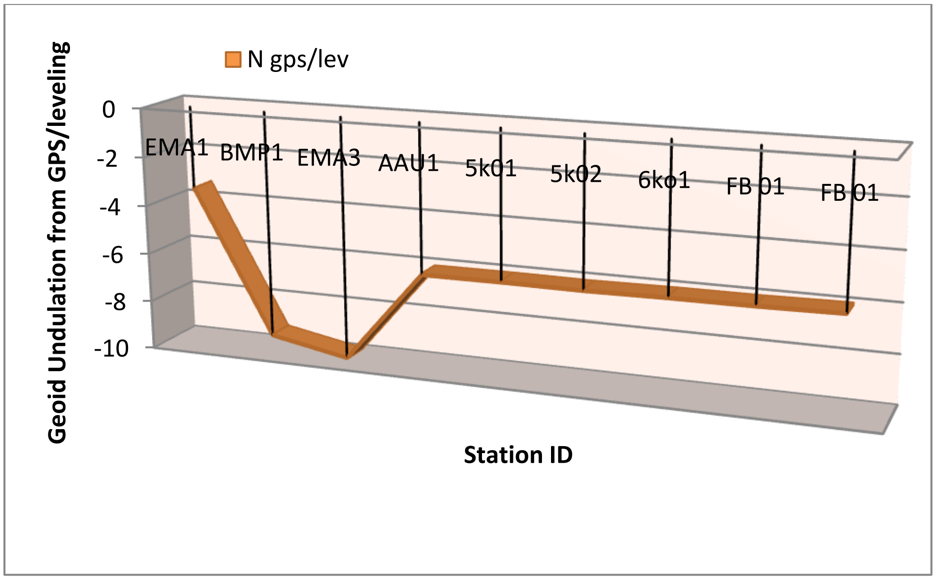


Figure 4.3 Geoid Undulations/N/ computed from GPS/leveling data

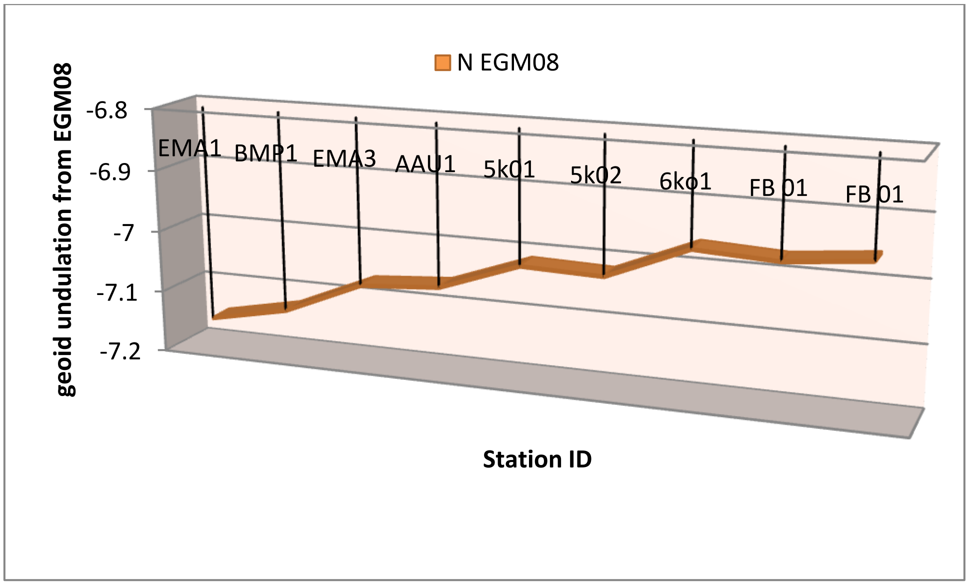


Figure 4.4 EGM08 derived Geoid Undulations

*Note:* Presents the difference between orthometric height obtained from EGM08 (hGPS-N) with leveled heights (in Tabular form)

### 4.4 Comparison of EGM 08 derived height with GPS/Leveling heights

Validation of EGM08 is based on the comparison with the external data like GPS/leveling observations. The objective of this research is to validate the quality of EGM08 model. Various researches have been conducted at different countries to validate the accuracy of EGM08 by using GPS/leveling. For example, Bedada (2010) compared leveling heights with the EGM08 derived Geopotential heights in Ethiopia and obtained an improved standard deviation of 3.9 cm accuracy.

Comparison of the geoid undulations /N/ derived from GPS/leveling and EGM08 model alone was used for validation of this study.

Table .7 The difference in geoid undulation/Δ in N/ derived from GPS/Leveling and EGM08

| **Station** | **$N^{GPS/Lev} = h - H$** | **$N^{GPS/EGM08}$** | **Δ in<br>$N^{GPS/Lev} - N^{GPS/EGM08}$** |
|---|---|---|---|
| EMA1 | -3.32439 | -7.1494 | 3.82501 |
| EMA2 | -9.22101 | -7.1235 | -2.09751 |
| EMA3 | -9.82744 | -7.0690 | -2.75844 |
| AAU1 | -6.05006 | -7.0594 | 1.00934 |
| 5K01 | -6.04021 | -7.0130 | 0.97279 |
| 5K02 | -6.0574 | -7.0169 | 0.9595 |
| 6K01 | -6.01609 | -6.9629 | 0.94681 |
| FB01 | -6.00592 | -6.9696 | 0.96368 |
| ET01 | -5.95367 | -6.9570 | 1.00333 |

The Maximum and minimum values for the Geoid Undulation are -6.957m and -7.1494 m respectively with average -7.0356 meter by using EGM 2008. maximum and the minimum values of the Geoid Undulation are -3.32439 m, and -9.82744m with average -6.5 m respectively. Finally, the determined Geoid Undulation values using the model EGM-2008 gives; ΔN value between the two different methods is a maximum 3.82501 m, a minimum -2.75844 m, the average value 0.5361 m, and a standard deviation SD = ±2.268m.

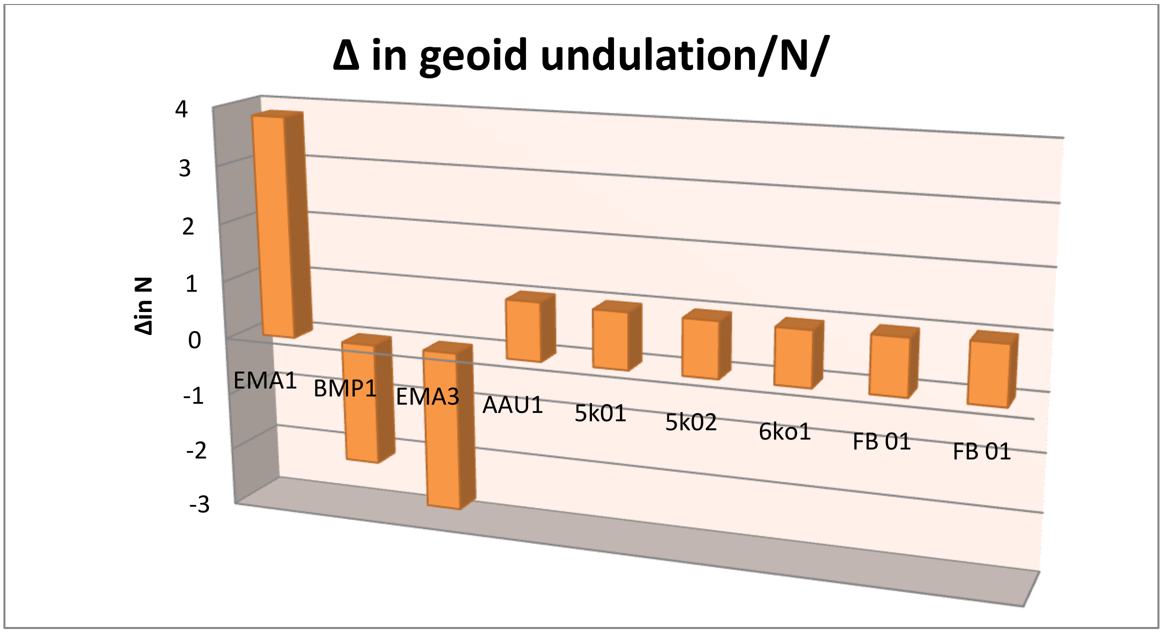


Figure 4.5 Mismatch between geometrical and gravimetric geoid undulations will attain/ $N^{GPS/Lev}$ - $N^{EGM0}$ /

.

## CHAPTER FIVE

## 5.Conclusions and Recommendation

### 5.1 Conclusion

In this study the average standard deviation of the EGM2008 computed geoid heights compared with those computed from the combination of leveled and GPS-based geodetic heights at 9 stations reached the level of *2.268 c*m. The observation errors in leveling relatively higher closure error was noticed on known bench marks of EMA1, AAU1 and BM 5k than the newly established control points may due to lack of update in the GCP's establishments since 1950's.

Since the positional accuracy of GPS derived height of the GCP's depends on duration of measurement time and the base line length Between points. The residual of local GPS leveling benchmarks has relatively higher value than the IGS stations.

As error in the ellipsoidal height is directly transferable to the Geoidal height the accuracy of determination of the Geoid undulation /N/ depend on number and distribution GCPs, time duration of the observed data and the methods used to determine the geoid undulation (N).As a result the GPS position coordinates have to be determined with a millimeter accuracy in order to get geoid heights accurate to few centimeter.

### 5.2 Recommendation

The following recommendations are derived from the findings of the results:

I recommend that an independent check is done to verify the leveled heights assigned to the fundamental benchmarks i.e referenced to the mean sea level at the port of Alexandria

EGM2008 should be modified locally with detailed topographic data; this could add more detail to gravity information.

Further assessment has to be done conducting long distance leveling co-located with very accurate GPS data.

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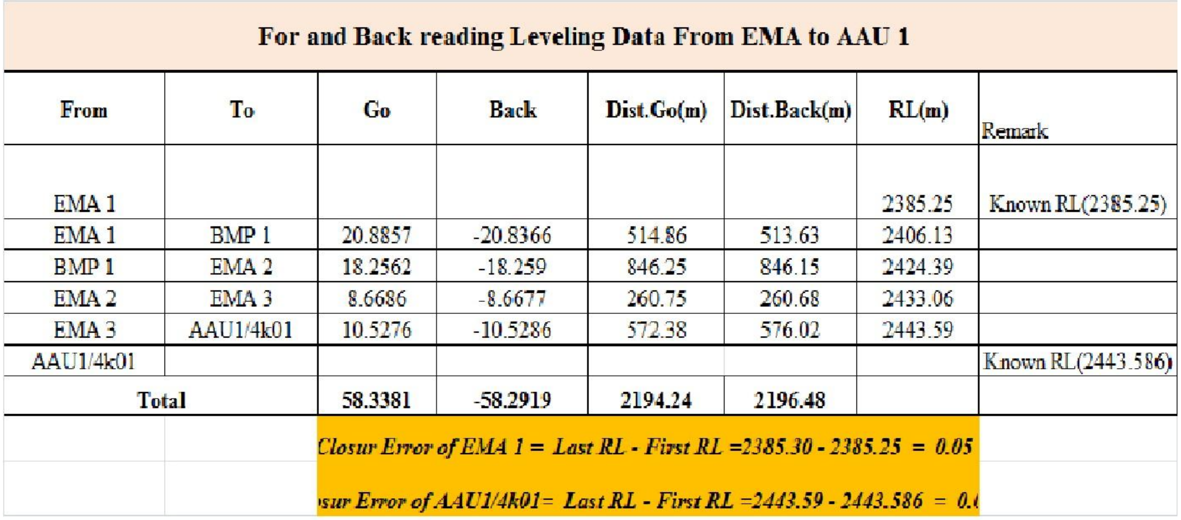
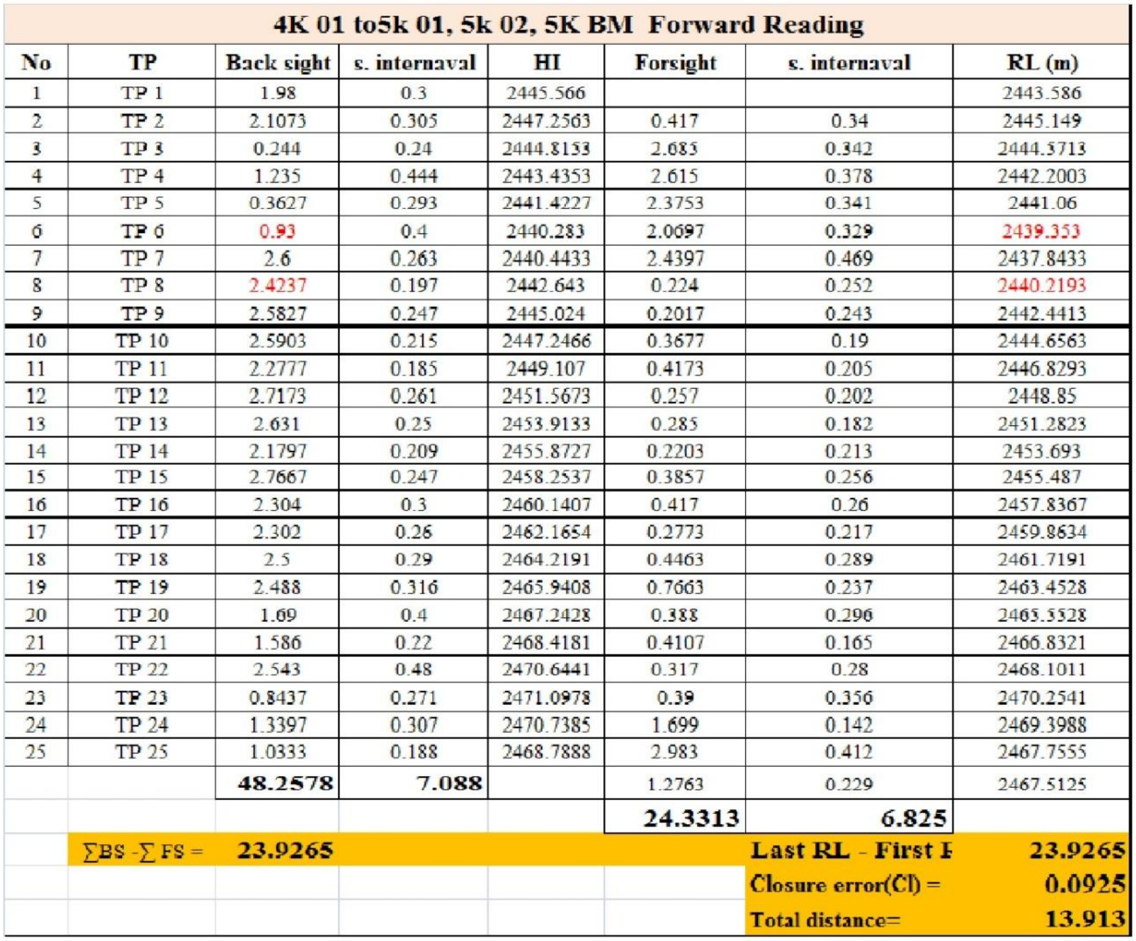
# Appendix A

## Leveling Data



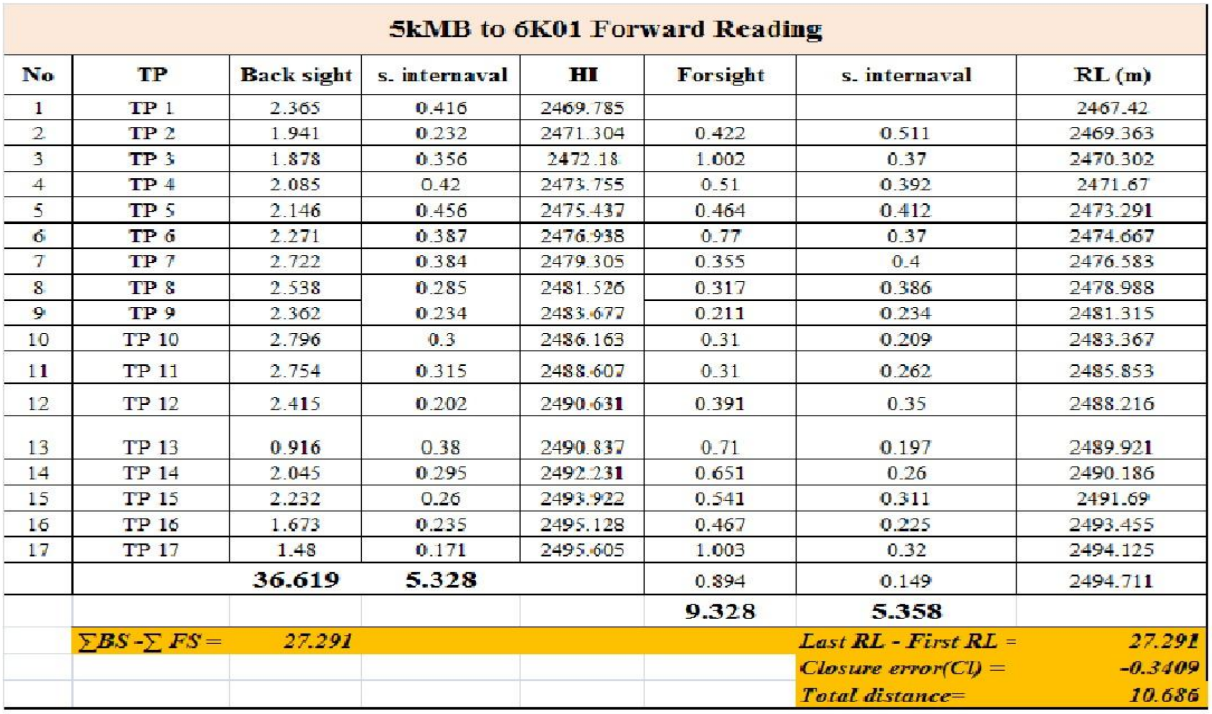
**For and Back reading Leveling Data From EMA to AAU 1**

| From | To | Go | Back | Dist.Go(m) | Dist.Back(m) | RL(m) | Remark |
|---|---|---|---|---|---|---|---|
| EMA 1 | | | | | | 2385.25 | Known RL(2385.25) |
| EMA 1 | BMP 1 | 20.8857 | -20.8366 | 514.86 | 513.63 | 2406.13 | |
| BMP 1 | EMA 2 | 18.2562 | -18.259 | 846.25 | 846.15 | 2424.39 | |
| EMA 2 | EMA 3 | 8.6686 | -8.6677 | 260.75 | 260.68 | 2433.06 | |
| EMA 3 | AAU1/4k01 | 10.5276 | -10.5286 | 572.38 | 576.02 | 2443.59 | |
| AAU1/4k01 | | | | | | | Known RL(2443.586) |
| **Total** | | **58.3381** | **-58.2919** | **2194.24** | **2196.48** | | |
| | | ***Closur Error of EMA 1 = Last RL - First RL =2385.30 - 2385.25 = 0.05*** | | | | | |
| | | ***sur Error of AAU1/4k01= Last RL - First RL =2443.59 - 2443.586 = 0.*** | | | | | |



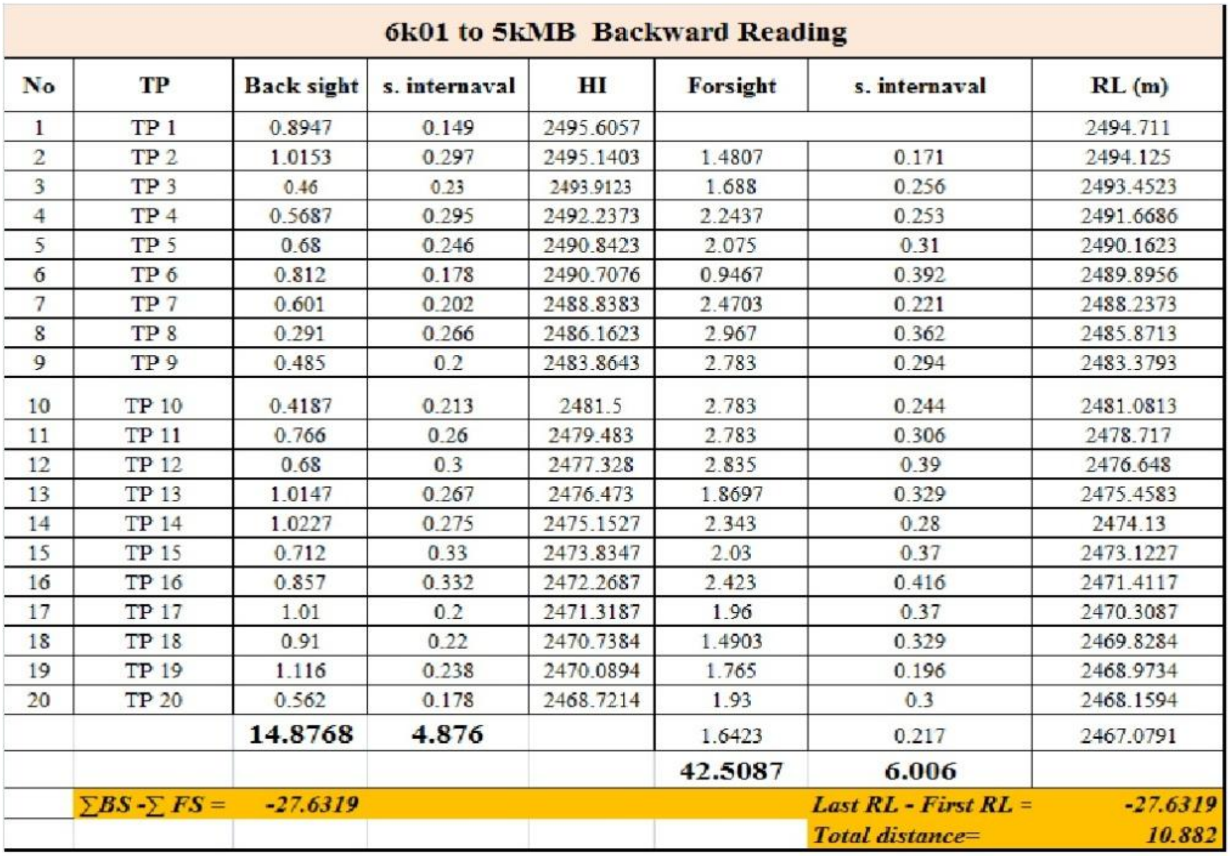
**4K 01 to5k 01, 5k 02, 5K BM Forward Reading**

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 1.98 | 0.3 | 2445.566 | | | 2443.586 |
| 2 | TP 2 | 2.1073 | 0.305 | 2447.2563 | 0.417 | 0.34 | 2445.149 |
| 3 | TP 3 | 0.244 | 0.24 | 2444.8133 | 2.683 | 0.342 | 2444.5713 |
| 4 | TP 4 | 1.235 | 0.444 | 2443.4353 | 2.615 | 0.378 | 2442.2003 |
| 5 | TP 5 | 0.3627 | 0.293 | 2441.4227 | 2.3753 | 0.341 | 2441.06 |
| 6 | TP 6 | 0.93 | 0.4 | 2440.283 | 2.0697 | 0.329 | 2439.353 |
| 7 | TP 7 | 2.6 | 0.263 | 2440.4433 | 2.4397 | 0.469 | 2437.8433 |
| 8 | TP 8 | 2.4237 | 0.197 | 2442.643 | 0.224 | 0.252 | 2440.2193 |
| 9 | TP 9 | 2.5827 | 0.247 | 2445.024 | 0.2017 | 0.243 | 2442.4413 |
| 10 | TP 10 | 2.5903 | 0.215 | 2447.2466 | 0.3677 | 0.19 | 2444.6563 |
| 11 | TP 11 | 2.2777 | 0.185 | 2449.107 | 0.4173 | 0.205 | 2446.8293 |
| 12 | TP 12 | 2.7173 | 0.261 | 2451.5673 | 0.257 | 0.202 | 2448.85 |
| 13 | TP 13 | 2.631 | 0.25 | 2453.9133 | 0.285 | 0.182 | 2451.2823 |
| 14 | TP 14 | 2.1797 | 0.209 | 2455.8727 | 0.2203 | 0.213 | 2453.693 |
| 15 | TP 15 | 2.7667 | 0.247 | 2458.2537 | 0.3857 | 0.256 | 2455.487 |
| 16 | TP 16 | 2.304 | 0.3 | 2460.1407 | 0.417 | 0.26 | 2457.8367 |
| 17 | TP 17 | 2.302 | 0.26 | 2462.1654 | 0.2773 | 0.217 | 2459.8634 |
| 18 | TP 18 | 2.5 | 0.29 | 2464.2191 | 0.4463 | 0.289 | 2461.7191 |
| 19 | TP 19 | 2.488 | 0.316 | 2465.9408 | 0.7663 | 0.237 | 2463.4528 |
| 20 | TP 20 | 1.69 | 0.4 | 2467.2428 | 0.388 | 0.296 | 2465.5528 |
| 21 | TP 21 | 1.586 | 0.22 | 2468.4181 | 0.4107 | 0.165 | 2466.8321 |
| 22 | TP 22 | 2.543 | 0.48 | 2470.6441 | 0.317 | 0.28 | 2468.1011 |
| 23 | TP 23 | 0.8437 | 0.271 | 2471.0978 | 0.39 | 0.356 | 2470.2541 |
| 24 | TP 24 | 1.3397 | 0.307 | 2470.7385 | 1.699 | 0.142 | 2469.3988 |
| 25 | TP 25 | 1.0333 | 0.188 | 2468.7888 | 2.983 | 0.412 | 2467.7555 |
| | | **48.2578** | **7.088** | | 1.2763 | 0.229 | 2467.5125 |
| | | | | | **24.3313** | **6.825** | |
| | ∑BS -∑ FS = | **23.9265** | | | | **Last RL - First F** | **23.9265** |
| | | | | | | **Closure error(Cl) =** | **0.0925** |
| | | | | | | **Total distance=** | **13.913** |



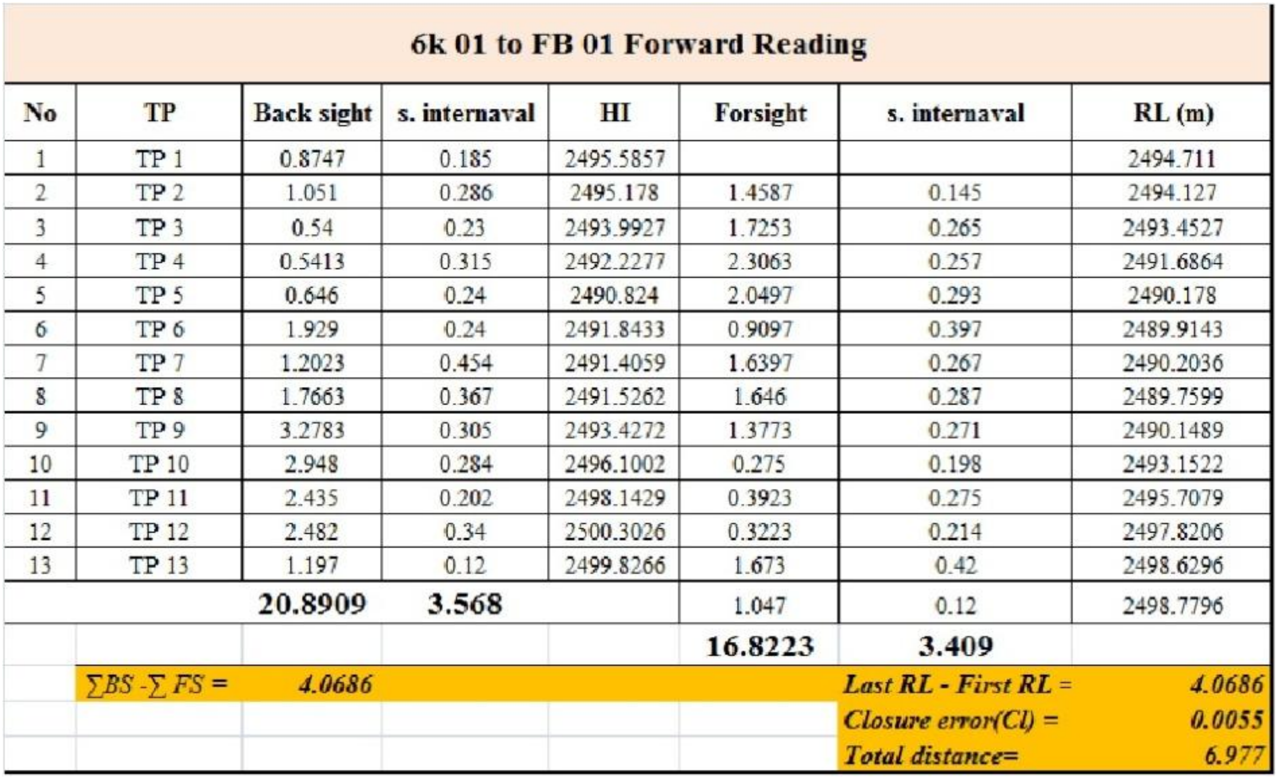
**5kMB to 6K01 Forward Reading**

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 2.365 | 0.416 | 2469.785 | | | 2467.42 |
| 2 | TP 2 | 1.941 | 0.232 | 2471.304 | 0.422 | 0.511 | 2469.363 |
| 3 | TP 3 | 1.878 | 0.356 | 2472.18 | 1.002 | 0.37 | 2470.302 |
| 4 | TP 4 | 2.085 | 0.42 | 2473.755 | 0.51 | 0.392 | 2471.67 |
| 5 | TP 5 | 2.146 | 0.456 | 2475.437 | 0.464 | 0.412 | 2473.291 |
| 6 | TP 6 | 2.271 | 0.387 | 2476.938 | 0.77 | 0.37 | 2474.667 |
| 7 | TP 7 | 2.722 | 0.384 | 2479.305 | 0.355 | 0.4 | 2476.583 |
| 8 | TP 8 | 2.538 | 0.285 | 2481.526 | 0.317 | 0.386 | 2478.988 |
| 9 | TP 9 | 2.362 | 0.234 | 2483.677 | 0.211 | 0.234 | 2481.315 |
| 10 | TP 10 | 2.796 | 0.3 | 2486.163 | 0.31 | 0.209 | 2483.367 |
| 11 | TP 11 | 2.754 | 0.315 | 2488.607 | 0.31 | 0.262 | 2485.853 |
| 12 | TP 12 | 2.415 | 0.202 | 2490.631 | 0.391 | 0.35 | 2488.216 |
| 13 | TP 13 | 0.916 | 0.38 | 2490.837 | 0.71 | 0.197 | 2489.921 |
| 14 | TP 14 | 2.045 | 0.295 | 2492.231 | 0.651 | 0.26 | 2490.186 |
| 15 | TP 15 | 2.232 | 0.26 | 2493.922 | 0.541 | 0.311 | 2491.69 |
| 16 | TP 16 | 1.673 | 0.235 | 2495.128 | 0.467 | 0.225 | 2493.455 |
| 17 | TP 17 | 1.48 | 0.171 | 2495.605 | 1.003 | 0.32 | 2494.125 |
| | | **36.619** | **5.328** | | 0.894 | 0.149 | 2494.711 |
| | | | | | **9.328** | **5.358** | |
| | *∑BS -∑ FS =* | *27.291* | | | | *Last RL - First RL =* | *27.291* |
| | | | | | | *Closure error(Cl) =* | *-0.3409* |
| | | | | | | *Total distance=* | *10.686* |



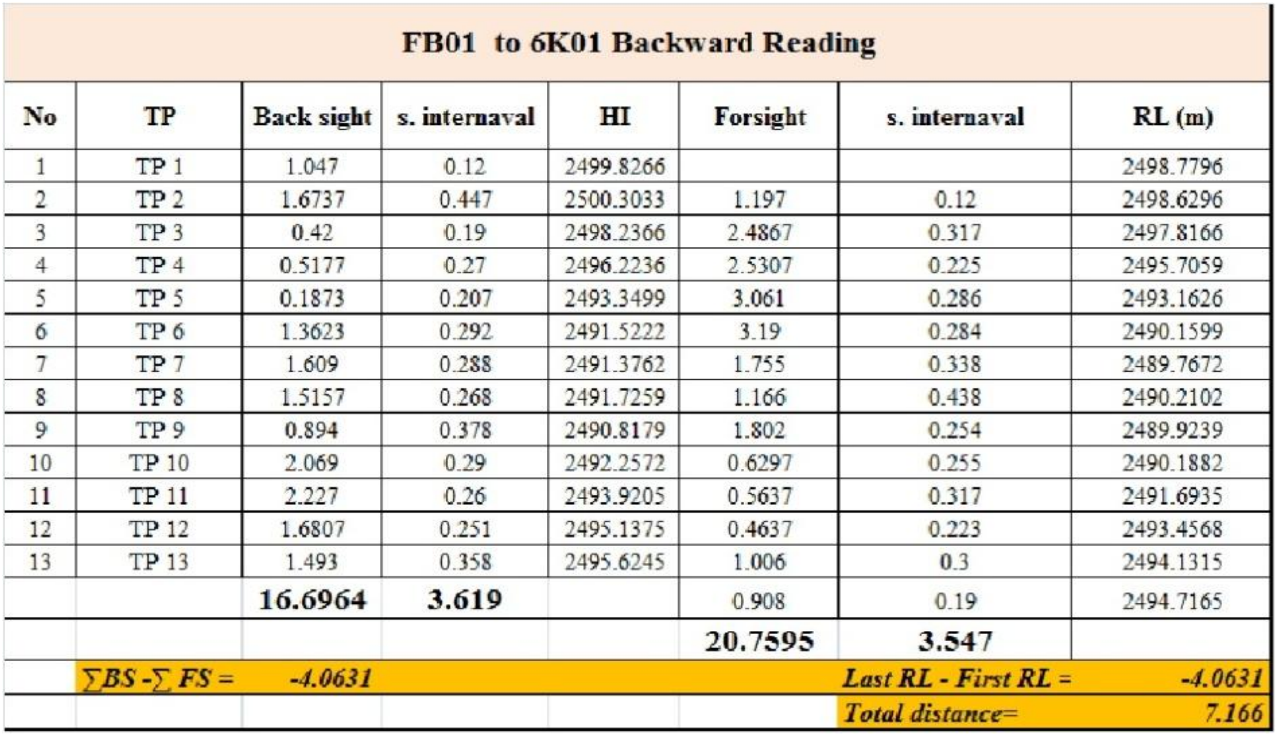
**6k01 to 5kMB Backward Reading**

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 0.8947 | 0.149 | 2495.6057 | | | 2494.711 |
| 2 | TP 2 | 1.0153 | 0.297 | 2495.1403 | 1.4807 | 0.171 | 2494.125 |
| 3 | TP 3 | 0.46 | 0.23 | 2493.9123 | 1.688 | 0.256 | 2493.4523 |
| 4 | TP 4 | 0.5687 | 0.295 | 2492.2373 | 2.2437 | 0.253 | 2491.6686 |
| 5 | TP 5 | 0.68 | 0.246 | 2490.8423 | 2.075 | 0.31 | 2490.1623 |
| 6 | TP 6 | 0.812 | 0.178 | 2490.7076 | 0.9467 | 0.392 | 2489.8956 |
| 7 | TP 7 | 0.601 | 0.202 | 2488.8383 | 2.4703 | 0.221 | 2488.2373 |
| 8 | TP 8 | 0.291 | 0.266 | 2486.1623 | 2.967 | 0.362 | 2485.8713 |
| 9 | TP 9 | 0.485 | 0.2 | 2483.8643 | 2.783 | 0.294 | 2483.3793 |
| 10 | TP 10 | 0.4187 | 0.213 | 2481.5 | 2.783 | 0.244 | 2481.0813 |
| 11 | TP 11 | 0.766 | 0.26 | 2479.483 | 2.783 | 0.306 | 2478.717 |
| 12 | TP 12 | 0.68 | 0.3 | 2477.328 | 2.835 | 0.39 | 2476.648 |
| 13 | TP 13 | 1.0147 | 0.267 | 2476.473 | 1.8697 | 0.329 | 2475.4583 |
| 14 | TP 14 | 1.0227 | 0.275 | 2475.1527 | 2.343 | 0.28 | 2474.13 |
| 15 | TP 15 | 0.712 | 0.33 | 2473.8347 | 2.03 | 0.37 | 2473.1227 |
| 16 | TP 16 | 0.857 | 0.332 | 2472.2687 | 2.423 | 0.416 | 2471.4117 |
| 17 | TP 17 | 1.01 | 0.2 | 2471.3187 | 1.96 | 0.37 | 2470.3087 |
| 18 | TP 18 | 0.91 | 0.22 | 2470.7384 | 1.4903 | 0.329 | 2469.8284 |
| 19 | TP 19 | 1.116 | 0.238 | 2470.0894 | 1.765 | 0.196 | 2468.9734 |
| 20 | TP 20 | 0.562 | 0.178 | 2468.7214 | 1.93 | 0.3 | 2468.1594 |
| | | **14.8768** | **4.876** | | 1.6423 | 0.217 | 2467.0791 |
| | | | | | **42.5087** | **6.006** | |
| | *∑BS -∑ FS =* | *-27.6319* | | | | *Last RL - First RL =* | *-27.6319* |
| | | | | | | *Total distance=* | *10.882* |



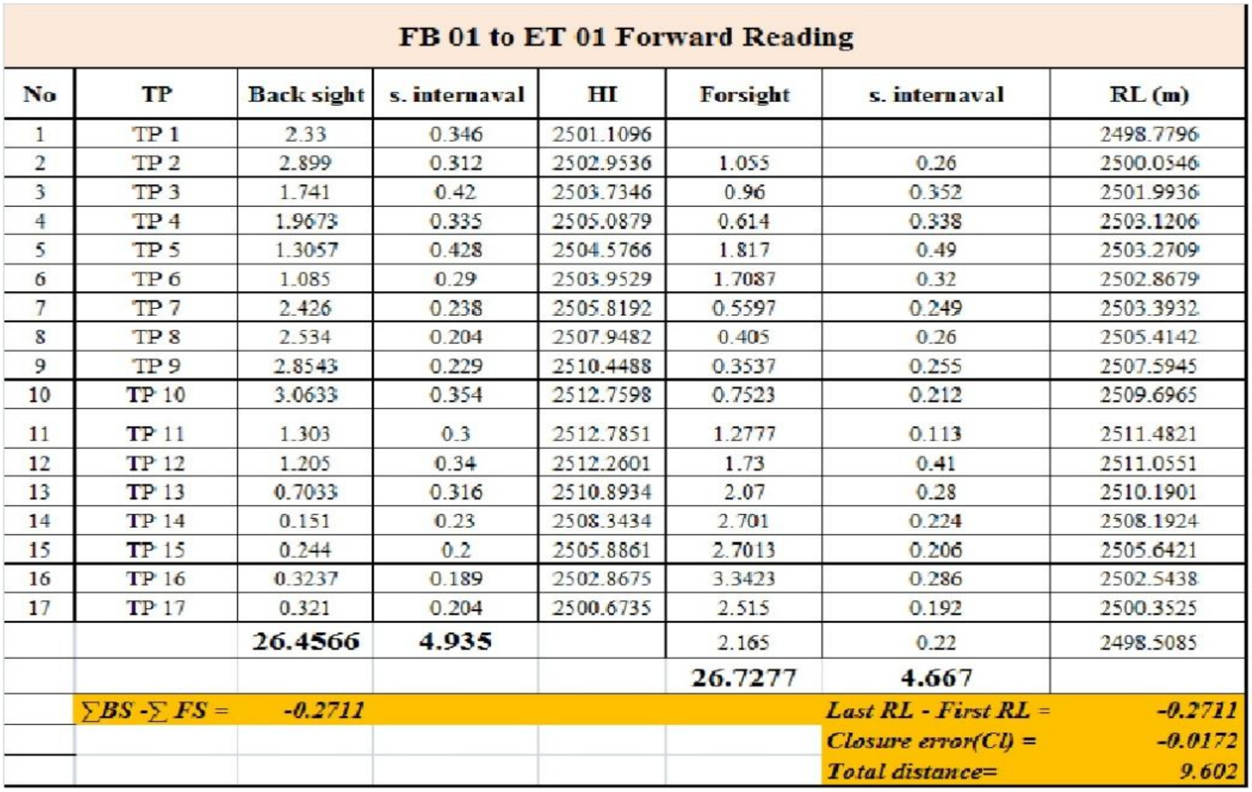
### 6k 01 to FB 01 Forward Reading

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 0.8747 | 0.185 | 2495.5857 | | | 2494.711 |
| 2 | TP 2 | 1.051 | 0.286 | 2495.178 | 1.4587 | 0.145 | 2494.127 |
| 3 | TP 3 | 0.54 | 0.23 | 2493.9927 | 1.7253 | 0.265 | 2493.4527 |
| 4 | TP 4 | 0.5413 | 0.315 | 2492.2277 | 2.3063 | 0.257 | 2491.6864 |
| 5 | TP 5 | 0.646 | 0.24 | 2490.824 | 2.0497 | 0.293 | 2490.178 |
| 6 | TP 6 | 1.929 | 0.24 | 2491.8433 | 0.9097 | 0.397 | 2489.9143 |
| 7 | TP 7 | 1.2023 | 0.454 | 2491.4059 | 1.6397 | 0.267 | 2490.2036 |
| 8 | TP 8 | 1.7663 | 0.367 | 2491.5262 | 1.646 | 0.287 | 2489.7599 |
| 9 | TP 9 | 3.2783 | 0.305 | 2493.4272 | 1.3773 | 0.271 | 2490.1489 |
| 10 | TP 10 | 2.948 | 0.284 | 2496.1002 | 0.275 | 0.198 | 2493.1522 |
| 11 | TP 11 | 2.435 | 0.202 | 2498.1429 | 0.3923 | 0.275 | 2495.7079 |
| 12 | TP 12 | 2.482 | 0.34 | 2500.3026 | 0.3223 | 0.214 | 2497.8206 |
| 13 | TP 13 | 1.197 | 0.12 | 2499.8266 | 1.673 | 0.42 | 2498.6296 |
| | | **20.8909** | **3.568** | | 1.047 | 0.12 | 2498.7796 |
| | | | | | **16.8223** | **3.409** | |
| | *∑BS -∑ FS =* | ***4.0686*** | | | | ***Last RL - First RL =*** | ***4.0686*** |
| | | | | | | ***Closure error(Cl) =*** | ***0.0055*** |
| | | | | | | ***Total distance=*** | ***6.977*** |



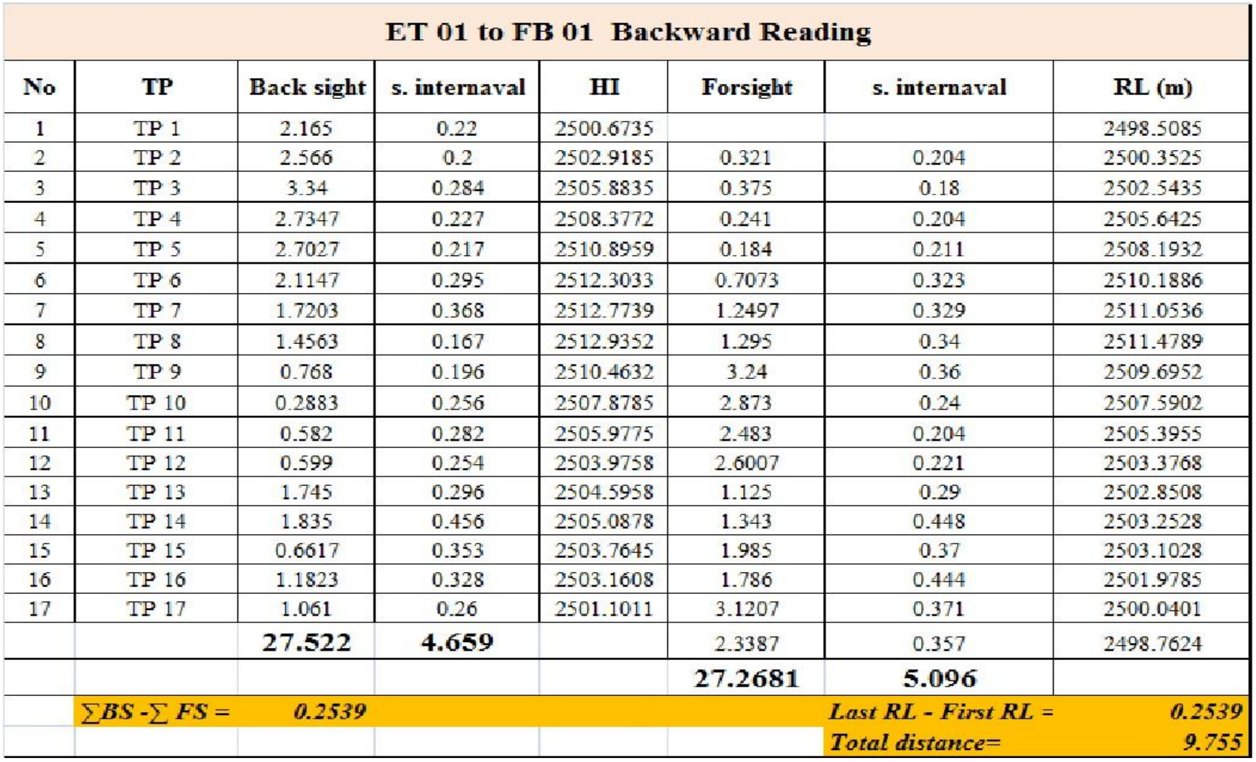
### FB01 to 6K01 Backward Reading

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 1.047 | 0.12 | 2499.8266 | | | 2498.7796 |
| 2 | TP 2 | 1.6737 | 0.447 | 2500.3033 | 1.197 | 0.12 | 2498.6296 |
| 3 | TP 3 | 0.42 | 0.19 | 2498.2366 | 2.4867 | 0.317 | 2497.8166 |
| 4 | TP 4 | 0.5177 | 0.27 | 2496.2236 | 2.5307 | 0.225 | 2495.7059 |
| 5 | TP 5 | 0.1873 | 0.207 | 2493.3499 | 3.061 | 0.286 | 2493.1626 |
| 6 | TP 6 | 1.3623 | 0.292 | 2491.5222 | 3.19 | 0.284 | 2490.1599 |
| 7 | TP 7 | 1.609 | 0.288 | 2491.3762 | 1.755 | 0.338 | 2489.7672 |
| 8 | TP 8 | 1.5157 | 0.268 | 2491.7259 | 1.166 | 0.438 | 2490.2102 |
| 9 | TP 9 | 0.894 | 0.378 | 2490.8179 | 1.802 | 0.254 | 2489.9239 |
| 10 | TP 10 | 2.069 | 0.29 | 2492.2572 | 0.6297 | 0.255 | 2490.1882 |
| 11 | TP 11 | 2.227 | 0.26 | 2493.9205 | 0.5637 | 0.317 | 2491.6935 |
| 12 | TP 12 | 1.6807 | 0.251 | 2495.1375 | 0.4637 | 0.223 | 2493.4568 |
| 13 | TP 13 | 1.493 | 0.358 | 2495.6245 | 1.006 | 0.3 | 2494.1315 |
| | | **16.6964** | **3.619** | | 0.908 | 0.19 | 2494.7165 |
| | | | | | **20.7595** | **3.547** | |
| | ***∑BS -∑ FS =*** | ***-4.0631*** | | | | ***Last RL - First RL =*** | ***-4.0631*** |
| | | | | | | ***Total distance=*** | ***7.166*** |



## FB 01 to ET 01 Forward Reading

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 2.33 | 0.346 | 2501.1096 | | | 2498.7796 |
| 2 | TP 2 | 2.899 | 0.312 | 2502.9536 | 1.055 | 0.26 | 2500.0546 |
| 3 | TP 3 | 1.741 | 0.42 | 2503.7346 | 0.96 | 0.352 | 2501.9936 |
| 4 | TP 4 | 1.9673 | 0.335 | 2505.0879 | 0.614 | 0.338 | 2503.1206 |
| 5 | TP 5 | 1.3057 | 0.428 | 2504.5766 | 1.817 | 0.49 | 2503.2709 |
| 6 | TP 6 | 1.085 | 0.29 | 2503.9529 | 1.7087 | 0.32 | 2502.8679 |
| 7 | TP 7 | 2.426 | 0.238 | 2505.8192 | 0.5597 | 0.249 | 2503.3932 |
| 8 | TP 8 | 2.534 | 0.204 | 2507.9482 | 0.405 | 0.26 | 2505.4142 |
| 9 | TP 9 | 2.8543 | 0.229 | 2510.4488 | 0.3537 | 0.255 | 2507.5945 |
| 10 | TP 10 | 3.0633 | 0.354 | 2512.7598 | 0.7523 | 0.212 | 2509.6965 |
| 11 | TP 11 | 1.303 | 0.3 | 2512.7851 | 1.2777 | 0.113 | 2511.4821 |
| 12 | TP 12 | 1.205 | 0.34 | 2512.2601 | 1.73 | 0.41 | 2511.0551 |
| 13 | TP 13 | 0.7033 | 0.316 | 2510.8934 | 2.07 | 0.28 | 2510.1901 |
| 14 | TP 14 | 0.151 | 0.23 | 2508.3434 | 2.701 | 0.224 | 2508.1924 |
| 15 | TP 15 | 0.244 | 0.2 | 2505.8861 | 2.7013 | 0.206 | 2505.6421 |
| 16 | TP 16 | 0.3237 | 0.189 | 2502.8675 | 3.3423 | 0.286 | 2502.5438 |
| 17 | TP 17 | 0.321 | 0.204 | 2500.6735 | 2.515 | 0.192 | 2500.3525 |
| | | **26.4566** | **4.935** | | 2.165 | 0.22 | 2498.5085 |
| | | | | | **26.7277** | **4.667** | |
| | ***∑BS -∑ FS =*** | ***-0.2711*** | | | | ***Last RL - First RL =*** | ***-0.2711*** |
| | | | | | | ***Closure error(Cl) =*** | ***-0.0172*** |
| | | | | | | ***Total distance=*** | ***9.602*** |



## ET 01 to FB 01 Backward Reading

| No | TP | Back sight | s. internaval | HI | Forsight | s. internaval | RL (m) |
|---|---|---|---|---|---|---|---|
| 1 | TP 1 | 2.165 | 0.22 | 2500.6735 | | | 2498.5085 |
| 2 | TP 2 | 2.566 | 0.2 | 2502.9185 | 0.321 | 0.204 | 2500.3525 |
| 3 | TP 3 | 3.34 | 0.284 | 2505.8835 | 0.375 | 0.18 | 2502.5435 |
| 4 | TP 4 | 2.7347 | 0.227 | 2508.3772 | 0.241 | 0.204 | 2505.6425 |
| 5 | TP 5 | 2.7027 | 0.217 | 2510.8959 | 0.184 | 0.211 | 2508.1932 |
| 6 | TP 6 | 2.1147 | 0.295 | 2512.3033 | 0.7073 | 0.323 | 2510.1886 |
| 7 | TP 7 | 1.7203 | 0.368 | 2512.7739 | 1.2497 | 0.329 | 2511.0536 |
| 8 | TP 8 | 1.4563 | 0.167 | 2512.9352 | 1.295 | 0.34 | 2511.4789 |
| 9 | TP 9 | 0.768 | 0.196 | 2510.4632 | 3.24 | 0.36 | 2509.6952 |
| 10 | TP 10 | 0.2883 | 0.256 | 2507.8785 | 2.873 | 0.24 | 2507.5902 |
| 11 | TP 11 | 0.582 | 0.282 | 2505.9775 | 2.483 | 0.204 | 2505.3955 |
| 12 | TP 12 | 0.599 | 0.254 | 2503.9758 | 2.6007 | 0.221 | 2503.3768 |
| 13 | TP 13 | 1.745 | 0.296 | 2504.5958 | 1.125 | 0.29 | 2502.8508 |
| 14 | TP 14 | 1.835 | 0.456 | 2505.0878 | 1.343 | 0.448 | 2503.2528 |
| 15 | TP 15 | 0.6617 | 0.353 | 2503.7645 | 1.985 | 0.37 | 2503.1028 |
| 16 | TP 16 | 1.1823 | 0.328 | 2503.1608 | 1.786 | 0.444 | 2501.9785 |
| 17 | TP 17 | 1.061 | 0.26 | 2501.1011 | 3.1207 | 0.371 | 2500.0401 |
| | | **27.522** | **4.659** | | 2.3387 | 0.357 | 2498.7624 |
| | | | | | **27.2681** | **5.096** | |
| | ***∑BS -∑ FS =*** | ***0.2539*** | | | | ***Last RL - First RL =*** | ***0.2539*** |
| | | | | | | ***Total distance=*** | ***9.755*** |

## Appendices B

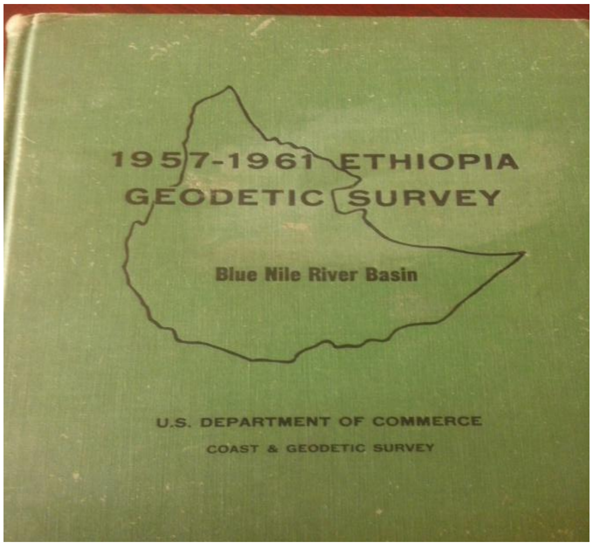
### Instrument used in the data collection process

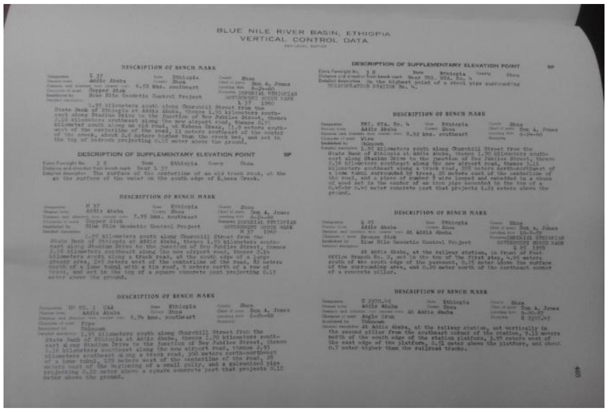
SOKIA Sprit level Instrument used in the data collection

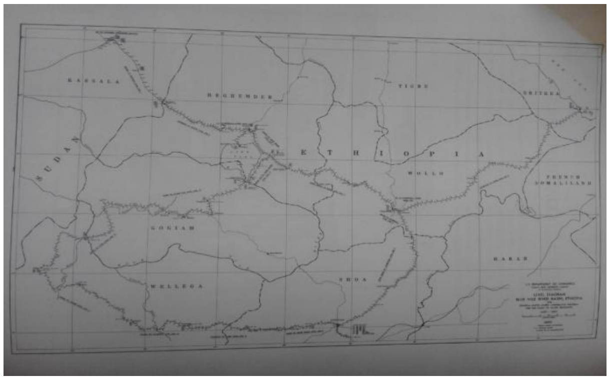
Leika Viva GS 14 DGPS instrument was used during data collection

## Appendices C

## The green book used for reference about Ethiopian geodetic leveling network




BLUE NILE RIVER BASIN, ETHIOPIA
VERTICAL CONTROL DATA



## Appendices D :

**Output of the 9 control points from geoidEval online processing Geoid heights**

link :*http://geographiclib.sourceforge.net/cgibin/GeoidEval?input=33+45&option=Submit*

1. AAU1

lat lon = 9.03500 38.76616 (09°02'06"N 038°45'58"E)

geoid heights (m)

EGM2008 = -7.0594

EGM96 = -7.4388

EGM84 = -7.0189

2.EMA 1

lat lon = 9.02013 38.76317 (09°01'12"N 038°45'47"E)

geoid heights (m)

EGM2008 = -7.1494

EGM96 = -7.5014

EGM84 = -7.0594

3.EMA 2

lat lon = 9.02357 38.76142 (09°01'25"N 038°45'41"E)

geoid heights (m)

EGM2008 = -7.1235

EGM96 = -7.4820

EGM84 = -7.0490

4. EMA 3

lat lon = 9.03272 38.76376 (09°01'58"N 038°45'50"E)

geoid heights (m)

EGM2008 = -7.0690

EGM96 = -7.4446

EGM84 = -7.0243

5. ET 01

lat lon = 9.05050 38.76526 (09°03'02"N 038°45'55"E)

geoid heights (m)

EGM2008 = -6.9570

EGM96 = -7.3658

EGM84 = -6.9751

6. FB 01

lat lon = 9.04785 38.76275 (09°02'52"N 038°45'46"E)

geoid heights (m)

EGM2008 = -6.9696

EGM96 = -7.3732

EGM84 = -6.9815

7. 5K 01

lat lon = 9.04102 38.76240 (09°02'28"N 038°45'45"E)

geoid heights (m)

EGM2008 = -7.0130

EGM96 = -7.4038

EGM84 = -7.0005

8. 5K02

lat lon = 9.04064 38.76315 (09°02'26"N 038°45'47"E)

geoid heights (m)

EGM2008 = -7.0169

EGM96 = -7.4070

EGM84 = -7.0018

9. 6K01

lat lon = 9.04813 38.76005 (09°02'53"N 038°45'36"E)

geoid heights (m)

EGM2008 = -6.9629

EGM96 = -7.3669

EGM84 = -6.9796