TOWARDS OPTIMIZING TRANSPORT NETWORKS: A LATTICE NUMERICAL MODEL

By

Shewangzaw Nigussie

A THESIS SUBMITTED TO THE
DEPARTMENT OF PHYSICS IN PARTIAL FULFILMENT
OF REQUIREMENT FOR THE DEGREE OF
MASTER OF SCIENCE IN STATISTICAL PHYSICS
AT
ADDIS ABABA UNIVERSITY
ADDIS ABABA
SEPTEMBER 2018

ADDIS ABABA UNIVERSITY
DEPARTMENT OF
PHYSICS

The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled " **Towards optimizing Transport Networks: A Lattice Numerical Model** " by **Shewangzaw Nigussie** in partial fulfillment of the requirements for the degree of **master of science in Statistical physics**.

Dated: September 2018

Supervisor: ______________________
Dr. Tatek Yirgou

examiners: ______________________
Dr.Yitagesu Elfagid

____
Dr. Lemmi Demeyu

ADDIS ABABA UNIVERSITY

Date: **September 2018**

Author: **Shewangzaw Nigussie**

Title: **Towards optimizing Transport Networks: A Lattice Numerical Model**

Department: **Physics**

Degree: **M.Sc.** Convocation: **September** Year: **2018**

Signature of Author

Table of Contents

List of Figures

Abstract

We show the problem of transport network design system in lattice with long-range connections which is limited by the cost constraint.
Our transport network is constructed from two dimensional (d=2) square lattice that can be improved by introducing long-range connections to each other with probability, $p_{ij} \sim \ r_{ij}^{-\alpha}$, where r_{ij} is the Manhattan distance between two sites i and j, and α is a variable exponent. Furthermore, we applied a cost constraint on the total length of the additional links and search optimal transport in the system for $\alpha = 2.4$ which can be applicable for two dimension. This situation remains optimal for lattice size ranging from 60 up to 150.

Acknowledgements

First of all I would like to thank the almighty God for giving me the strength and capacity to learn and score the necessary grades by attending all the courses as well as in doing this thesis at Addis Ababa university. Secondly I am very happy to have the opportunity to express my sincere deepest gratitude to my advisor Dr. Tatek Yirgou for his patience, excellent guidance, kind, support, useful suggestions and advice valuable comments throughout my thesis work. I extend my heartfelt gratitude to all my family especially to my wife Asegedech Getachew for her various supports and encouragement during my study. I would also like to thank my friends who shared their personal experiences in doing this thesis.

Finally I would like to thank the department of physics of all staff members who thought me the necessary courses as well as the FDRE Ministry of Education that gave me the opportunity to learn the master of science in physics.

Chapter 1

Introduction

1.1 Objective Of The Research

1.1.1 General objective

The main objective of the research is to understand the scientific ways of network optimization in transport system.

1.1.2 Specific objectives

- To show the relation between average length and alpha in determining the average shortest path length at a fixed value of alpha.

- To maximize profit as well as to minimize cost and travel time of the transport system.

- To improve and effectively use the transport system with in Ethiopia as well as its global networks in giving the public modern transport system.

1.2 Back Ground Of The Research

Human beings can travel from one place to another place on foot, or else using animals or artificial (man made) transport system. Different types of artificial transport systems give service to the public but their functions and limitations determine the choice of people from each type of artificial transport system in different situations. People travel from place to place for work, trade, or to visit other people, other places, wild animals, buildings, and other tourist sites etc. Nowadays, the artificial transport system can be using cars or buses, trains, aircrafts or airplanes, boats or ships etc. For this case the choice of transport system depends on availability, speed, comfort, cost, profit, revenue and the geographical setting of the land features. Engineers have been improving the artificial transport system from time to time. For mountain areas, hills and highly ups and downs of the high lands where it is difficult to construct roads and long distance travel routes, aircrafts or air planes can be chosen for safely transporting people and precious goods and minerals with short duration of time. But for flat surfaces of the land that has the capacity to construct roads and rail way lines and nearby route connections, cars and trains can be chosen. In transporting enormous amount of goods for import and export purpose in sea water or ocean ships can be chosen. In transporting people and goods in lakes boats can be chosen.

Our focus of attention in this thesis is towards the network optimization in transport system. The financial condition of cost is the fundamental one to limit the number of landing or stopping stations. For example, if a country has a plan with limited budget enough to construct 5 railway stations and rail way lines, only 5 railway stations and railway lines or the routes that connect the stations can be constructed to give long-range connections even though the country needs more than 20. Gradually, the railway stations and railway lines can be increased taking some years step by step but priorities should be given depending on the trade route , work place, import, export, the position of minerals, population, industry sites, distance from the central city, tourist sites, land features, other transport facilities, securities etc. After the cost constraint is satisfied, there is a need to look the time to reach the required place. Optimal navigation with local knowledge and the presence of long-range links in a lattice network without constraints was recently studied by Kleinberg [1]. Much attention has been dedicated to the problem of navigation in complex network geometries $[1-11]$.

Chapter 2

Research model

Any source node s possesses the knowledge of the entire network topology. In this condition the average shortest path length, $< l >$ from source to target becomes the relevant navigation variable to be optimized. If we take for instance, in a passageway network, such as in Manhattan, the travel routes should be planned or changed in such a way that the travel time is minimized for a given limited reconstruction cost. This task is performed by considering the whole structure of the network in terms of its sites and links, namely, by knowing the location of all passageway stations, their connections, and the shortest path between any two stations [12]. Despite the availability of a number of models, we have decided to develop our own research model.

In a two-dimensional regular square lattice with all $N = L^2$ sites present, each site i is connected with its four nearest neighbors. The bonds in the lattice that connect the sites represent the routes of the transport system (see Fig.2.1). In our model, pairs of sites i and j are then randomly chosen to receive long-range connections with probability, $p_{ij} \sim \; r_{ij}^{-\alpha}$, where α is a variable exponent and r_{ij} is the Manhattan distance between the two sites i and j, which is the distance measured as the number of connections to the system ends when their total length (cost), Σr_{ij}, tends to a value Λ. Consider the total length(cost) is proportional to the length of the links in the underlying network which can be expressed by the next equation:

$$\Lambda = AL^2 \tag{2.0.1}$$

, where A is a constant. That is, the budget to improve the system is a fraction of the cost of the current network(without long-range connections)[13,14].

$$r_{ij} = abs(i_1 - i_2) + abs(j_1 - j_2) \tag{2.0.2}$$

where (i_1, j_1) and (i_2, j_2) are the coordinates of the two points i and j respectively. Manhattan distance between two points in a grid is based on strictly horizontal and or vertical path along the grid line. The Manhattan distance is the simple sum of the horizontal and vertical components, where as the diagonal distance is calculated by using Pythagoras theorem[15].

Fig.2.1 shows the connections of a single node or site i. Each node(site) i has four short-range connections to its nearest neighboring nodes or sites (c,d,e,and f). The nodes or sites indicate the stops and the bonds in the lattice indicate the routes of the transport system. A long-range connection may be placed to a randomly chosen site j. Since α controls the average length of the long-range connections, we get for a constant value of Λ, and small values of α, long-range connections, but fewer in number can be added due to the fixed total length limit. We assume that an optimal navigation condition must be reached to adjust the length and the number of connections added to the system.

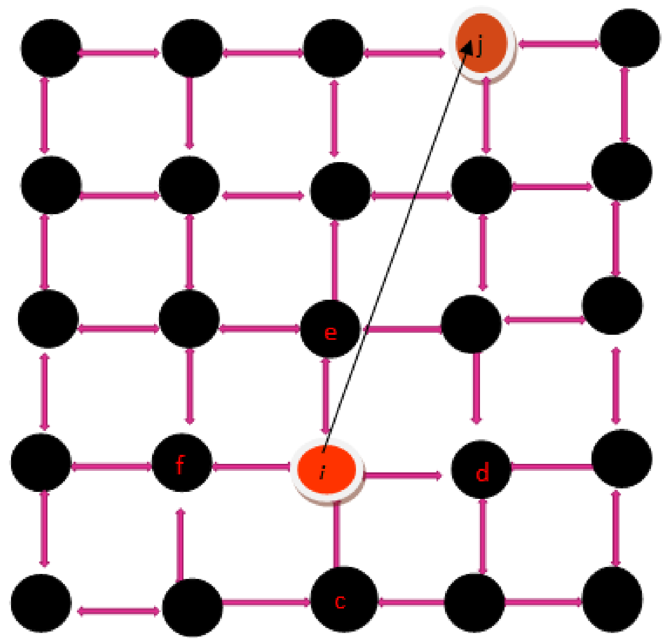

Figure 2.1: Connections of a single node i with its neighboring nodes

The stations can be chosen depending on economy, population, trade route, interest of people, position of minerals, tourist sites and distance between each station. If a unit Manhattan distance between each site is 100 km then the Manhattan distance between site i and j shown in fig.2.1 becomes 400km. The flight may be determined by the number of travelers (cost) and or the time it takes to reach the stations to satisfy the cost constraint and minimizing travel time from a randomly chosen station i to station j which may be the farthest, nearby, moderate connection for transporting direct farthest, nearby, moderate travelers only or else transporting near, moderate, and farthest travelers at a time by stopping at each station in the route. The financial condition of revenue, cost and profit of transport depends on the number, interest and economy of passengers which influences to choose whether to select direct farthest, nearby, moderate or inclusive from each type of travelers at any time. When the farthest travelers are a few in numbers it is better to select inclusive approach, i.e., including nearby, moderate and farthest travelers at a time for a transport system stoping nearby stations so as to minimize cost and increase profit due to the revenue collected from large number of customers attracted by this transport system. It is assumed that the airport network simulation program ($ANSP$) was designed as the first step in the development of an optimization framework able first to investigate model and design complex transport networks in the context of total transport system.

If 9 vehicle stations are randomly placed in a square lattice with side 7 units of Manhattan distance having 49 sites at points (1,2),(1,3),(3,3),(3,4),(2,6),(7,5), (5,2),(5,4),and (6,3), each of the 9 sites are the location of station sites. Therefore, each of the 9 sites are considered to be 0 unit Manhattan distance from the station sites.

The points (1,1),(1,4),(1,6),(2,2),(2,3),(2,4), (2,5),(2,7),(3,2),(3,5),(3,6),(4,2),(4,3),(4,4), (5,1),(5,3),(5,5),(6,2),(6,4),(6,5)(7,3),(7,4),(7,6) are each 1 unit Manhattan distance from the nearest stations.

The points (1,5),(1,7),(2,1),(3,1),(3,7),(4,1),(4,5), (4,6),(5,6),(6,1),(6,6),(7,2),(7,7) are each 2 units Manhattan distance from the nearest stations. The points (4,7),(5,7), (6,7),(7,1) are each three units Manhattan distance from the nearest stations.

The mean Manhattan distance of each site from their nearest station is found by adding the Manhattan distance of each site and divide it by the total number of sites. We can calculate the mean or average Manhattan distance of each site from the nearest station to be 1.245 units Manhattan distance, which is 61 divided by 49. Similarly, on average each station is 3 units Manhattan distance from the central site, since we have 2 stations that are 1 unit Manhattan distance at the points(3,4), and (5,4); a station that is 2 units Manhattan distance at the point (3,3); 2 stations that are 3 units Manhattan distance at the point(5,2), and (6,3); 3 stations that are 4 units Manhattan distance at the points(1,3),(2,6),(7,5); a station that is 5 units Manhattan distance at a point(1,2), which is 27 divided by 9.

We have studied the problem of optimal facilities location which consists of choosing positions for population facility in geographical space such that the mean distance between the member of population and the nearest facility is minimized. So we have also considered the design of optimal networks to connect our facilities together. Given optimally located facilities, we have searched numerically for the network configuration that minimize the sum of maintenance and travel costs. The model tells us the design of transport networks, parcel delivery services and the internat back bone of practical interests. One can also like to find the best way to connect the facilities so as to optimize the performance as a whole.

Consider a situation in which our facilities form the network of stations and the routes of the connection between them form the edges. The efficiency depends on two factors:

1.The smaller the sum of the lengths of all edges, the cheaper the network to construct.

2. The shorter the distance through the network between the stations the faster the network can perform[16].

If we have a cost limitation and only possible to build 5 railway stations and there is a need to put the stations in such a way that the Manhattan distance from each station to the other sites is small as much as possible, we have to put 1 station at the central city (A) and the other 4 stations (A) at 2 Manhattan distance units from the central station (A) as shown in fig.2.2.

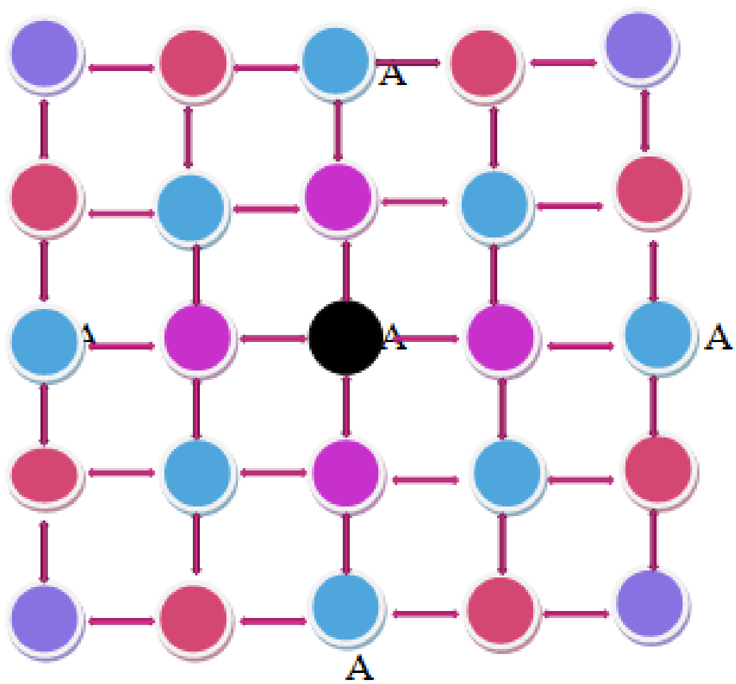

Figure 2.2: 5 railway stations located on 25 lattice sites

In this case 5 of the lattice sites are the location of the stations in the sites, so people living in this sites can directly use train from the stations to travel to the other stations. People living in 12 sites must travel 1 unit Manhattan distance by other means of transport system to use the railway transport system from their vicinity nearby stations. People living in 8 sites must travel 2 units Manhattan distance to use railway transport system from their nearest stations to the other stations. On average people must travel 1.11 units Manhattan distance or 111 km by other means of transport system before they use railway transport system.

Fig.2.2 also shows the first, the second, the third and fourth shell sites that are 1, 2,3,and 4 units of Manhattan distance from the central city station. We have 4 first shell, 8 second shell, 8 third shell, 4 fourth shell, and 1 central station in 25 lattice sites. The central station is black, and the colors of the rest sites are the same for equidistant sites from the central station.

$$k_n = 4n. \tag{2.0.3}$$

Where n =1,2,3, etc. and k_n is the total members of shell depending on lattice size in the first, second, and third shell, etc.

To show the relation between total length and number of links, we generate a single network realization(L=30) for a given value of α and calculate the shortest path length $< l >$ from each node(site) in the network to its central node(site). This calculation is performed as follows:

First we choose the root node (e.g.the central one), then we observe all its nearest stations connected by shortest nearby horizontal and or vertical distances in the connection network that can be classified as shell one nodes, meaning that they are only one step away from the central(root) node. After that, we visit all the neighbors of these nodes that are nearest to shell one and classify them as shell two nodes. Following this procedure for all network nodes, we obtain the l values(time) for each node to be reached from the central(root)node[12].

Large technological systems, such as the internet and the air transport system have never been actively designed in their entirety; instead they evolved gradually, in response to demands. There is a tendency of high demand for the improvement of air transport system due to the growing complexity of air space, via strictly evolutionary changes is problematic. There appears to be a clear need for active and rigorous design methods. Thus evolutionary approaches should be improved and changed with active design methods. The ability to actively design, optimize and control a system assumes the existence of predictive modeling and reasonably well defined functional dependencies between the controllable of the system objective and constraint functions for optimization. Following recent advances in the studies of the effects of network topology structure on dynamic process on transport networks, we have developed a flexible simulation of network topology coupled with flows on the network for use as a platform for computational experiments [17].

We assume the two cities are the cities which can be the starting and the stopping stations. At the first case we assume the transport system starts at station C, which is the central capital city of the country, travels to stations (cities) of D, E and F respectively. The nearest station from the central city is city D, where as the moderate distance from the central city is station E and the farthest distance from the central station is station F.

In the second case we assume farthest connection, the transport system can be arranged for passengers to travel from station C, the central capital city to any site for instance to station F without stopping at stations D and E in between them. In the third case we assume the transport system will have future probabilities to travel from one station to any station in the lattice provided that the profit is maximized and the travel time is minimized.

Ethiopian airline is the largest in revenue and profit from Africa having a star alliance member ship relation with other airlines in the world [18]. Nowadays the American airlines group is the largest by its fleet, size, revenue, profit, Passengers carried and revenue passenger mile from the world[19]. The formula below describes how to calculate costs, net income (profit) and revenue in transport system.

$$C_T = F_c + V_c \tag{2.0.4}$$

where C_T is total cost of transport system. F_c is fixed cost of transport system. V_c is variable cost of transport system [20].

Revenue is collected from service and or sale of technology, machines, vehicles etc in transport system.

Fixed costs include initial construction costs and fixed workers salary.

Variable costs include fuel costs, maintenance costs, bonus etc.

$$Netincome(profit) = Revenue - Totalcost \tag{2.0.5}$$

From this we can understand that increasing revenue increases net income and minimizing total cost also increases net income(profit) of the transport system. One can assume that using some renewable, abundant and cheap energy sources in the design of the transport system will minimize cost and will increase profit.

In reality most cities may already have some urban transport networks. When we build a new network, we should take advantage of the new model over the old or existing model as much as possible. It is possible to build an optimal urban transport network of finite total cost that provides access to all residents from their home to other needed destinations. In building an efficient urban transport network we should focus on the following two conditions. First a reasonable network must be able to provide transportation for every resident to the required destinations. Secondly, the total cost in building the network must be finite. Moreover, after building the network, the total traveling expenses for the entire population such as traveling time and cost on fuels should be as small as possible [21]. Active rigorous design necessitates identification of design variables, as well as objective and constraints whose functional dependence on the independent variables has been established analytically and experimentally. Air transport is a highly complex network. The past several years have seen a surge in the study of both structure of complex networks and the dynamics of flows on complex networks. The effect of network structure on the dynamics is the starting point in our investigation aimed at deriving quantifiable functional relationships between global transport such as through put, delay, capacity and locally controllable structure (connectivity) of transport networks of particular interest is the computational controllability of local variables expressed as an optimization problem. Recent ground breaking work in a variety of domains (Statistical physics, biology, internet router design, and optimal transport networks design) derives models that link static topological network structure such as distribution, average shortest path length, and performance of flows over networks. The existing models operate under a variety of assumptions and predict various effects such as overcrowds and delays[17]. Generally, we are cautiously optimistic that increasing realistic formulations will allow us to progress from abstract networks to models that are appropriate transport networks to a greater degree of confidentiality.

Chapter 3

Simulation Methods

In our simulation method we have used the following procedure.

1. We considered a two dimensional square lattice of sites connected with their nearest neighbors.

2. On a two dimensional square lattice, we assign a site i having a long-range connection to a site j with probability, $p_{ij} \sim r_{ij}^{-\alpha}$. where r_{ij} is the Manhattan distance between sites i and j and α is a variable exponent.

3. The long-range connection stops when the total length reaches a given value $\Lambda = \Sigma r_{ij}$.

4. We introduced a cost constraint on the total length of the long-range connections, the sum of the length of long-range connection is fixed.

$\Lambda = \Sigma r_{ij} = N$, where N is the number of nodes in the underlying lattice.

5. The network model is constructed from a square lattice with L^2 nodes or sites.

6. We have made α to vary starting from 0 up to 4.1 or 5.1.

7. we have generated a random number.

8. By varying the lattice size(L), we observed the average shortest path length as a function of the variable exponent α for different lattice size. After we compile and

ran the code, we drew graphs using grace. We took different lattice sizes by changing the integer parameter lattice size(L) for variety of experiments.

By using probability distributions, variables can have different outcomes occurring. probability describes uncertainty in the occurrence of an event. The simulation is a quantitative technique that calculates results for the random input variables repeatedly using a different set of input values each time.

We extract more quantitative information about this navigation problem by performing extensive simulations for different values of α and different system sizes. In each case, the average shortest path length, $< l >$ is calculated over all realizations, considering all the shortest distance between each pair of nodes for 2XL number of experiments. We also find the average delivery time by dividing the average delivery length for the lattice size, then we drew the average delivery time as a function of alpha and we observed the behavior of the graphs. Further more we drew graphs by taking some points for different values of alpha, average length, and lattice size from the graph of average length versus alpha. Then using this data we drew the graphs of average length, $< l >$ versus lattice size(L), the $log < l >$ versus $log(L)$, the $log < l >$ versus log($logL$). We calculated δs and γs from the ratio of the $log < l >$ and $log(L)$; $log < l >$ and $log(logL)$ respectively , then we observed the behaviors of the graphs of δs versus lattice size(L) and γs versus lattice size (L) respectively. We finally described the behavior of these graphs.

Chapter 4

Results and Discussion

The results presented in Fig. 4.1 clearly indicate the presence of a minimum average length for different system sizes at the same value of the exponent $\alpha = 2.4$, where optimal navigation is achieved at the average shortest paths and cost limitations. There is a constraint in the total length of the long-range connections. As the value of α increases from 0 to 2.4 the value of average length decreases ; whenever the value of α increases from 2.4 to 4 the value of average length increases. The average shortest path length is obtained at $\alpha = 2.4$.

For small lattice size(L) between 35 and 45, our results(data not shown here) agree with Kleinberg's result, that is, the minimum average length or average shortest path length is observed at $\alpha = 2$ for two dimensions[1,2]. The density of long-range connections is defined as N_l/L^2, where N_l is the total number of long-range connections present in the system. The cost limitation imposes that $N_l = \Lambda/ < r >$, and since $\Lambda = AL^2$, we obtain that $\rho = A < r >^{-1}$ [13,14]. where ρ is the expected density and $< r >$ is the average length of the added long-range connections. since $< r >$ is proportional to $\int_{1.4}^{L} r^{1.4-\alpha} dr$ [12], it follows that for $1.4 \leq \alpha < 2.4$, $\rho \sim\ L^{\alpha-2.4}$ and for $\alpha < 1.4$, $< r >$ is limited by the network size leading to $\rho \sim\ L^{-1}$. Since for

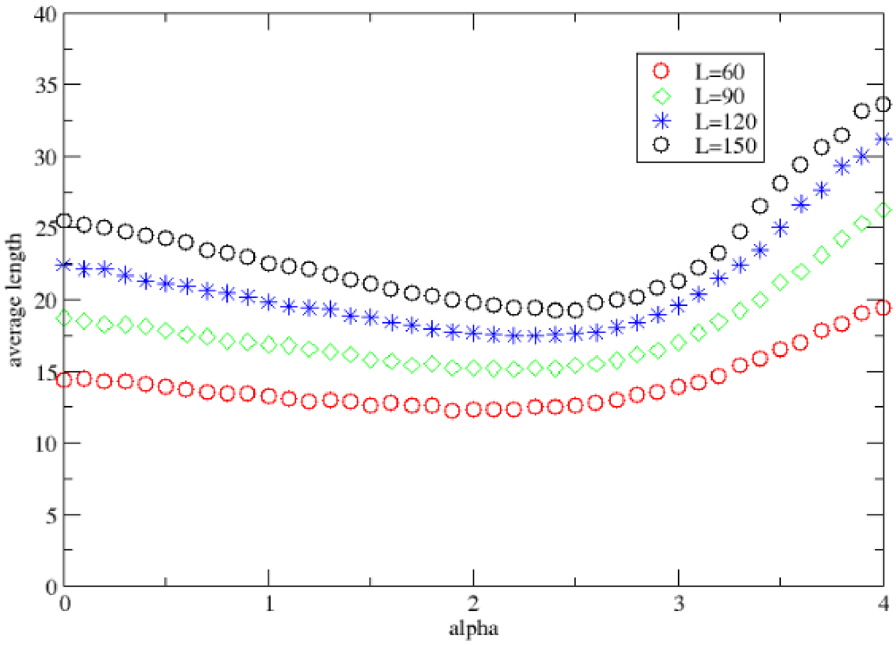

Figure 4.1: Average shortest path length as a function of alpha

the case $1.4 \leq \alpha < 2.4$, $< l >$ decreases with increasing α, the bound $< l >$, average length $> L^{(2.4-\alpha)/d}$ is rigorous and $< l >$ in this range must scale as a power of L. Thus, for all values of $\alpha < 2.4$ the density of the long-range links added, due to the constraint, decreases as a power law with L. Hence, for the values of $1.4 \leq \alpha < 2.4$ the density of the long-range links added, due to a constraint, decreases as a power law with L. As a consequence of this power law decrease in density, $< l >$ must increase as a power of L. For $\alpha > 2.4$ and sufficient networks, $< r >$ is finite and the density becomes independent of the system size, i.e., $\rho \sim \ L^0$. Thus, the effect of the constraint Λ on navigation should become negligible. However the finite value of $< r >$ suggests that long-range links can be neglected and therefore $< l >$ should scale as a power of L. We have clearly observed that the optimal shortest path is obtained at $\alpha = 2.4$.

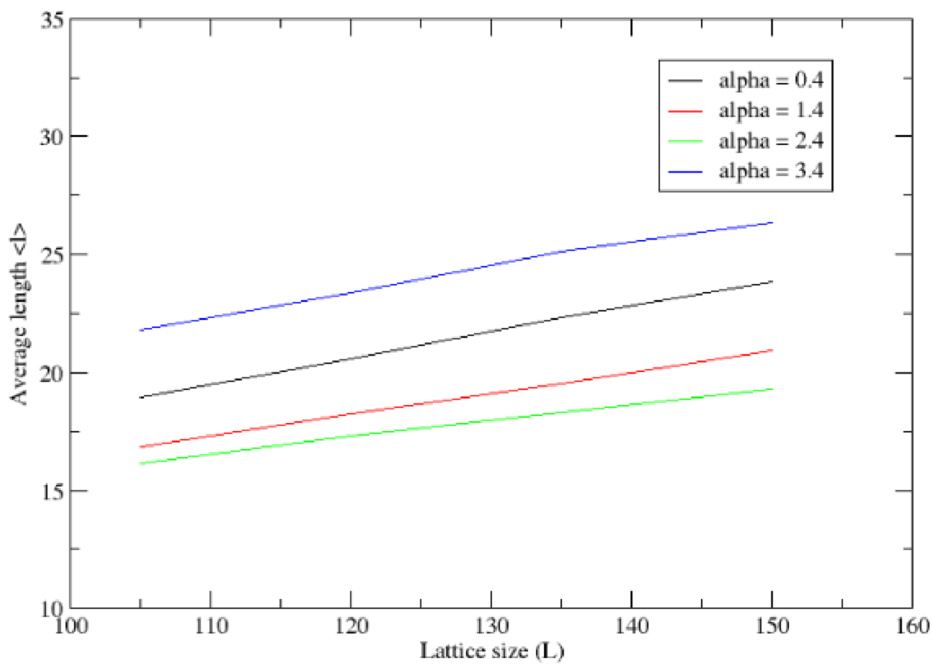

Figure 4.2: average length versus lattice size

As the lattice size increases so does the average length. For the same lattice size the value of average length decreases as alpha increases from 0 to 2.4, but when alpha increases from 2.4 to 3.4 or above the average length value increases. The average shortest path length as a function of lattice size(L) is obtained at $\alpha = 2.4$ as shown in fig.4.2. The way in which average length scales with system size (L), however, seems to follow rather different behaviors, depending on the value of α.

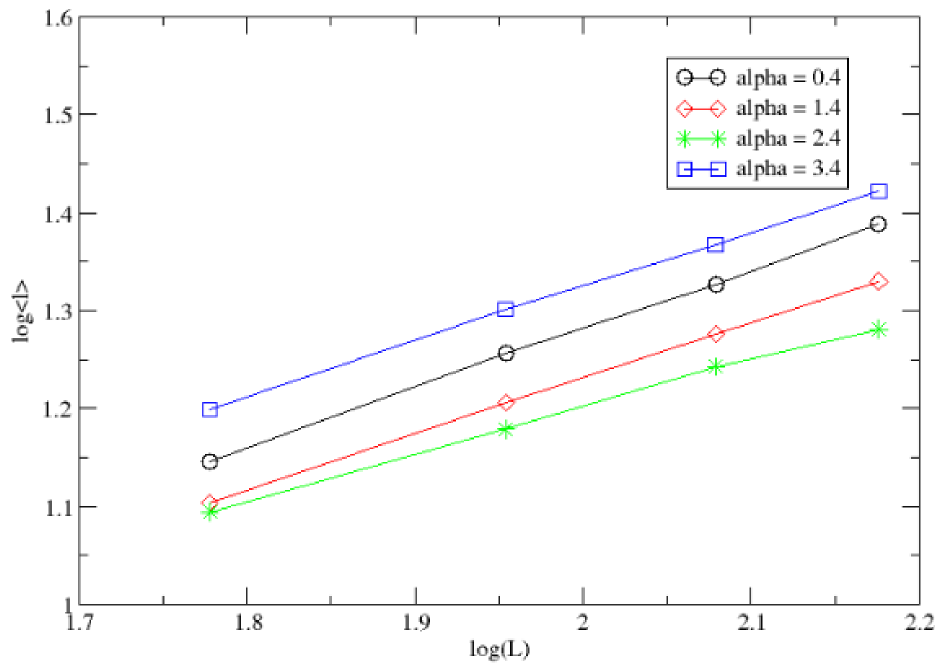

Figure 4.3: $\log < l >$ Versus $\log L$

The minimum value of $\log < l >$ Versus $\log L$ is observed at alpha = 2.4. From fig.4.3, one can easily observe that logarithm of average length is proportional to logarithm of L. This is supported by the plot of successive slopes, δs obtained from logarithm of average length versus logarithm of lattice size(L) which are almost invariant, as shown by the set of graphs in Fig.4.3. In contrast, in the case of $\alpha = 2.4$ the increase with lattice size(L) of average length, $< l >$ appears to be less rapid than the other slopes.

The graph of $\log < l >$ versus $\log(\log L)$ shows similar properties to the other graphs, Which means that $\log < l > \sim \log(\log L)$ as shown in fig.4.4. The minimum

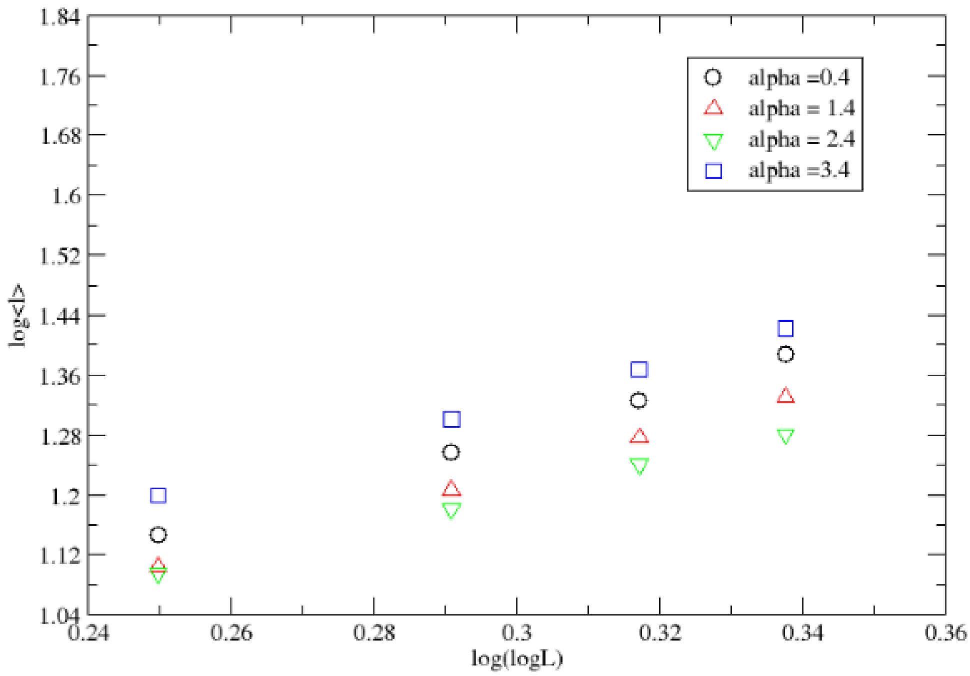

Figure 4.4: $\log < l >$ Versus $\log(\log L)$

value of the slope is observed at $\alpha = 2.4$ showing that the optimal value is reached as shown by the successive slopes γs obtained from the ratio of $\log < l >$ versus $\log(\log L)$, as presented in Fig.4.4. But for large lattice sizes(L) between 512 and 4096, the value of $\alpha = 3$ is the optimal value of average shortest path which can be observed in such a way that the transport will improve even further when the lattice size(L) increases, as suggested by Ref[12].

The values of the graph of δs as a function of lattice size, L generally shows that as the lattice size decreases the value of δs increases, whenever the lattice size increases the value of δs decreases. This is shown by the negative slope except at $\alpha = 0$. We

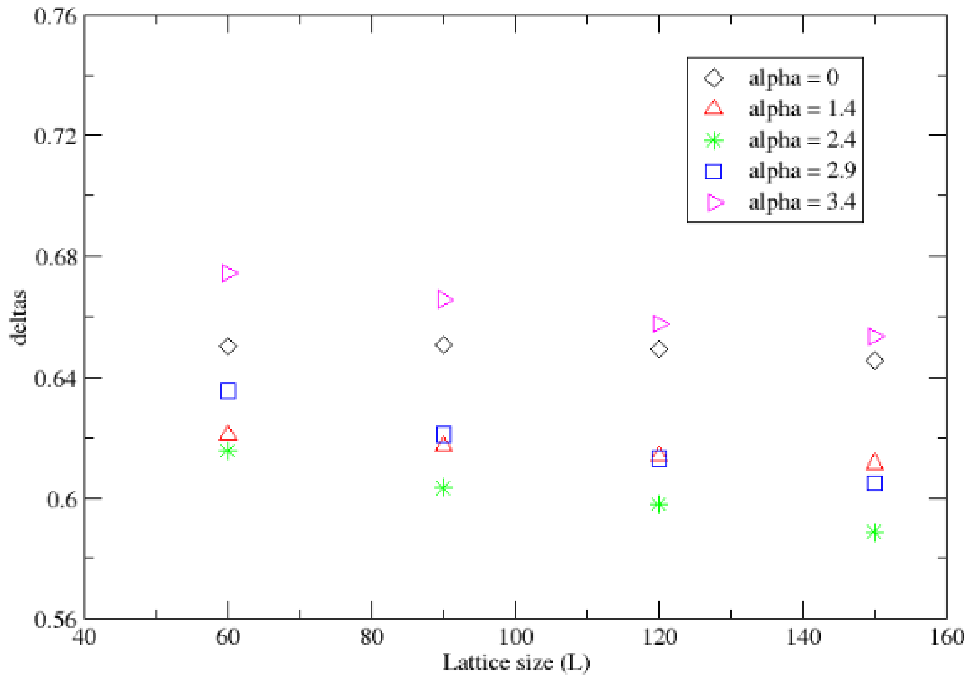

Figure 4.5: deltas versus lattice size

observed that the minimum value of δs at $\alpha = 2.4$ as indicated in fig.4.5. The value of $\delta = 0.65$ is nearly constant at $\alpha = 0$. The value of δs only varies between 0.62 and 0.61, which can be approximated by averaging to 0.615 showing only small deviations at the value of $\alpha = 1.4$. Each of the lattice size yields average length, $< l >= L^{0.65}$ at $\alpha = 0$. We can also approximate $< l >= L^{0.615}$ at $\alpha = 1.4$. This shows that for $\alpha < 2.4$, $< l >\sim\ L^{\delta}$, a power law is observed. Fig.4.6 shows that the value of γs as a function of lattice size(L). The minimum value of of γ is also observed at $\alpha = 2.4$. In the graph of of γs versus α when the lattice size increases the value of γs tends nearly to converge or closer but whenever the value of lattice size decreases the value

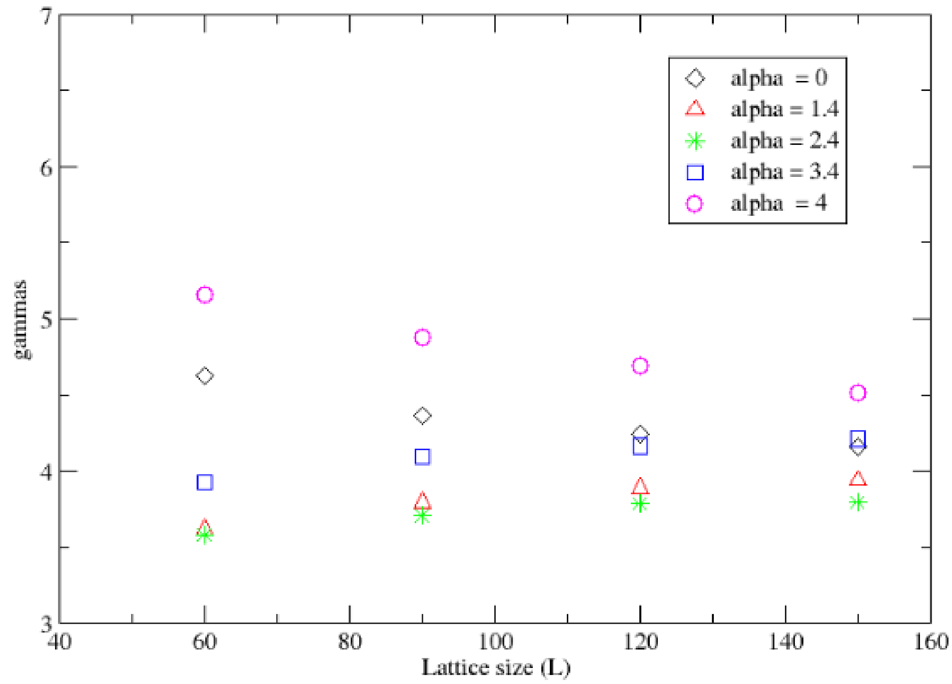

Figure 4.6: gammas versus lattice size

of γs tends to disperse or varies. When the value of α is 1 and 4, the γs vs lattice size slopes are decreasing or negative but as the value of α is 1.4 and 3.4, the γs vs lattice size slopes are increasing or positive slope. The graph of γs as a function of lattice size (L) is nearly constant at $\alpha = 2.4$ showing that $< l >$ increases as a power of the logarithm of L, $< l >$ is proportional to $(\log L)^{\gamma s}$, rather than a power of L, only for $\alpha = 2.4$.

Fig.4.7 shows that the characteristics of average delivery time as a function of α. The graph shows that as the lattice size increases the average delivery time decreases, since it takes longer time to travel by stopping at each station than directly traveling

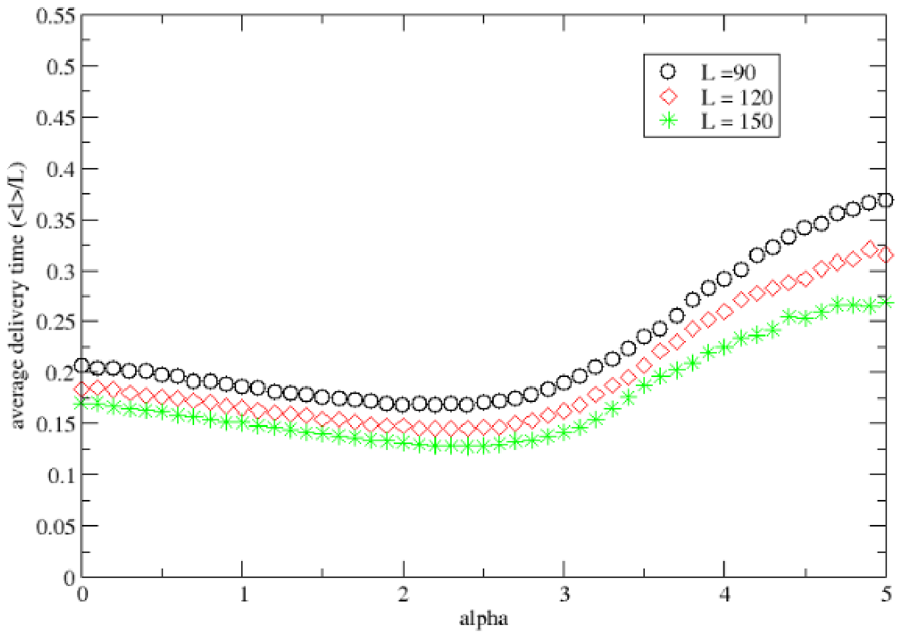

Figure 4.7: average delivery time versus alpha

to farthest station. The average delivery time is obtained by dividing average delivery length to the lattice size. The cost Λ involved to add long-range connections changes the behavior of the density of long-range connections. As a result of that, a minimum is observed at $\alpha = 2.4$. Each data point is a result of 2XL number of experiments and the cost Λ is fixed at L^2. Kleinberg found that $\alpha = 2$ is the optimal value in the navigation with the greedy algorithm [4]. We next ask, what would be the optimal α for the greedy algorithm when cost restriction $\Lambda = AL^2$ is imposed? We find also for the greedy algorithm that the optimal value is obtained at $\alpha = 2.4$. This is shown in Fig.4.7, where we plot the average delivery length $< l >$ that a message travels with only local information of the system geometry. The message is sent from the source node, s to the target node, t through a network generated with the constraint $\Lambda = L^2$. Remarkably, the presence of a minimum is also observed at $\alpha = 2.4$ showing that the type of information (local or global) used by the message holder to pass it through the system during the navigation process becomes unimportant if the network is constructed under length (cost) limitations. However, the two mechanisms display very different and distinct behaviors regarding the scaling with system size. While we observe logarithmic growth for the optimal condition $\alpha = 2.4$, in the case of global information, the time to reach the source, with the greedy algorithm and with cost constraint, appears to increase linearly with system size for all values of α. The linearity of $< l >$ with L is observed in the scaling collapse (Fig.4.7) of the curves of average delivery time versus α.

Chapter 5

Summary and Conclusion

We can maximize profit of transport system by minimizing total cost and increasing revenue. First we have to know the number of passengers in a transport system so as to decide whether to choose inclusive, farthest, moderate or nearby route travelers in order to maximize profit and minimize the travel time. Our results suggest that, regardless of the strategy used by the traveler, based on local or global knowledge of the network structure, the best transportation condition is obtained with an exponent $\alpha = 2.4$. The probability of the journey decays as a power law as distance between stations, $r^{-\alpha}$, where $\alpha = 2.4 \pm 0.2$.

Our results hold for d = 2 and lattice sizes ranging from 60 up to 150. For smaller lattice sizes greater than 35 and less than 45 we obtain the optimum shortest path at $\alpha = 2$ which is similar to Kleinberg's results for 2 dimension. Our results showed that as the lattice size increases the optimal value of alpha also slightly increases. The results recently reported by H.E.Stanley for large lattice size ranging from 512 up to 4096, the shortest path is obtained at $\alpha = 3$ for d = 2 [12]. The result $\alpha = 2.4$ is not the same with the result obtained for unconstrained systems with global and local information, where the optimal conditions are $\alpha = 0$ [6] and $\alpha = d$ [1,2], respectively, where d is the topological dimension of the underlying lattice. While in the unconstrained case the mean length of a link diverges, we find that when cost is considered the mean length is finite. In the case where the traveler has global knowledge of the network, and is able to identify the shortest path for navigation, we obtain a slow growth with size for the transit time at the optimal condition. In the case where the transportation path is decided based on the Manhattan distance to the target, we obtain average delivery time versus α. The average shortest delivery time is obtained at the exponent $\alpha = 2.4$. Finally, our results suggest that the idea of introducing a cost constraint in the navigation problem offers a different theoretical framework to understand the evolving topologies of other important complex network structures in nature, such as subways, trains, or the Internet. Of course, at this point we must emphasize that our approach represents specific model where design principles can be tested to improve the performance of the general network of transport system.

Bibliography

[1]J.M.Kleinberg, Nature (London) 406, 845 (2000); in proceedings of the 32nd ACM symposium on Theory of computing (ACM, New York, 2000),pp. 163-170.

[2] M. R. Roberson and D. ben-Avraham, Phys. Rev. E 74, 017101 (2006).

[3] M.T.Gastner and M.E.J. Newman,Phys.Rev.E 74, 016117 (2006).

[4] S.Carmi etal., Europhys.Lett.102,238702(2009).

[5] C. Caretta Cartozo and P. De Los Rios, Phys. Rev. Lett. 102, 238703 (2009).

[6] K. Komidis et al., Europhys. Lett. 82, 48005 (2008).

[7] C. F. Moukarzel and M. Argollo de Menezes, Phys. Rev. E 65, 056709 (2002).

[8] R. Guimera et al., Phys. Rev. Lett. 89, 248701 (2002).

[9] L. Danon, A. Arenas, and A. Diaz-Guilera, Phys. Rev. E 77, 036103 (2008).

[10] M. C. Santos et al., Phys. Rev. E 77, 041101 (2008).

[11] M. Boguna and D. Krioukov, Phys. Rev. Lett. 102, 058701 (2009).

[12] G.Li,S.D,S.Reis,A.A.Moreira, S.Havlin, H.E. Stanley, and J.S.Andrade,Jr. ,'Towards Design principles for optimal transport system', Phys.Rev.Lett 104,018701 (2010).

[13] Here, we show the case A=1 but we obtained similar results for several values in the range $0 < A < 1$.

[14] The density of long-range connections is defined as N_l/L^2, where N_l is the total

number of long-range connections present in the system. The cost limitation imposes that $N_l = \Lambda / < r >$, and since $\Lambda = AL^2$, we obtain that $\rho = A < r >^{-1}$.

[15] $https://en.wiktionary.org/wiki/Manhattan - distance$.

[16] Optimal design of spatial distribution network by Michael T. Gastner and M.E.J. Newman.

[17] Towards optimal transport network , Natalia Alexandrove,Rex K. Kincaid and Erik P.Vargo.

[18] www.Ethiopian airlines network.com.

[19] www.wikipidia.com/ World's Largest airlines .

[20] Revenue,Cost and Profit - IISS Calamanderi (2016/17)

[21] An application of optimal transport paths to urban transport networks by Qinglan Xia.