ENTRAINING CURRENTS IN HOMOGENEOUS PLASMA

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ABSTRACT

The generation of entraining currents in a homogeneous plasma by means of high frequency electromagnetic waves, with a slowly varying amplitude are considered. The treatment is based on the Vilosov's kinetic equation, from which quasi-stationary additional terms to the distribution function with respect to amplitude are obtained. Using the second order additional term to the distribution function, the densities of the current, energy and energy flux produced by the longitudinal waves are evaluated. In this calculations only the non-resonant particles are taken into account. The condition under which resonant particles can be neglected is also investigated. Results are presented in terms of longitudinal dielectric permitivity and its derivative with respect to frequency. This work is done with a possible application in a steady-state tokamak reactors.

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INTRODUCTION

Plasma may be defined as a large collection of approximately equal number of positively and negatively charged particles. Plasma has a tendency to be electrically neutral on a macroscopic level. Ionized gases are found naturally throughout most of the universe except on the surface of the cold planets, such as the Earth. The study of the property of matter which is highly ionized and where its properties are dominated by the dynamic behavior of the free charges is usually referred as classical plasma. The particles behave classically and the ionization is due to high temperature and low density. Quantum plasma is used to study solids (metals, semiconductors) |1|.

At present, one of the most important branches of plasma research is in the area of controlled thermonuclear reactions. The goal of this research is to generate energy by means of the same reactions that take place in the sun. The fundamental process is that the nuclear fusion of the heavy isotopes of hydrogen into helium, with the subsequent release of energy.

In order for the reaction to take place, the reactants must be heated to a very high temperature ($\sim 10^{8\,0}k$) and must be confined in some convenient device. Tokamaks are installations which are used for this purpose. In these devices, the plasma is heated by toroidal current which is produced inductively. The charged particles are confined by means of the magnetic field produced by the toroidal current and by externally applied magnetic field. The most commonly used method for driving plasma current in these devices is to induce a toroidal electric field in the plasma by means of a time varying magnetic flux. But this method can not operate in a steady-state fashion; therefore alternative methods capable of driving plasma current continuously are desirable. Neutral beams |2|, heavy charged particle beams |3,4| and various waves |5-8,20| have been proposed for maintaining steady-state plasma current by imparting to the electrons the momentum needed to compensate the resistive losses or by making the resistivity asymmetric. Hence, the attractiveness of tokamaks as a theormonuclear reactors would be considerably enhanced if it were possible to provide steady-state toroidal currents in place of the pulsed ohmic heating current.

The main objective of this thesis is, to investigate the generation of entraining currents by using high frequency electromagnetic waves with a slowly varying amplitude. The treatment is based on the Vlasov equation, which is the collisionless Boltzmann equation. The distribution function is determined by successive approximation and using the second order approximation, the densities of the current, energy and energy flux produced in the field of the longitudinal wave will be evaluated. In the evaluation, only the non resonance particles will be taken into account. The condition as when the role of the resonant particles can be neglected will also be investigated.

CHAPTER 1

CONTROLLED THERMONUCLEAR FUSION AND TOKAMAKS

1.1 Thermonuclear Fusion:

The present revival of plasma physics is primarily due to the experimental attempts to produce controlled thermonuclear reactions. The continuous and rapid growth of industry throughout the world raises demands on energy which in the long run cannot be securely satisfied except in one way: by the reliable control of energy that is generated in the interior of the stars, and which in uncontrollable way is also released on Earth in the form of thermonuclear explosions.

Apart from being a virtually inexhaustible source of energy-considering that the reserves of deuterium in the oceans are estimated to suffice for many million of years - thermonuclear generation of energy has several additional advantages:

1. Safe operation of the thermonuclear reactor; Explosion can be virtually ruledout.
2. The extent of radioactive pollution is smaller than with fission type reactors.
3. With controlled thermonuclear reactions taking place in a plasma, thermal energy can be converted directly into electrical energy by magneto hydrodynamic |MHD| generators.

The elementary processes resulting in the release of thermonuclear energy on a macroscopic scale is the fusion (upon their collision) of two light nuclei of deuterium, tritium, etc. into the nucleus of a heavier element. Here are some of the most important reactions:

$$D + T \rightarrow H_e^4 + n + 17.6 \text{ MeV}$$
$$D + D \rightarrow H_e^3 + n + 3.2 \text{ MeV}$$
$$D + D \rightarrow T + P + 4.0 \text{ MeV}$$

The symbols D,T stand here for the nuclei of deuterium and tritium (i.e., hydrogen isotopes of atomic weights 2 and 3) respectively, H_e^3, H_e^4 for nuclei of helium isotopes of atomic weights 3 and 4, and n and P are a neutron and a proton respectively. The number of MeV in each equation indicates the energy released by the fusion of a pair of nuclei.

The process of nuclear fusion, like any elementary process, is defined by its effective cross-section which indicates the probability of the process; on the other hand it defines the size of an inpenetrable target area which one nucleus represents to another moving nucleus. After the impact of the moving nucleus onto this target, both nuclei fuse. In a plasma where the particles move at thermal velocities, the effective cross-section is a function of the relative velocity of the reacting particles or in equilibrium state, functions of the temperature.

Given the effective cross-section for the fusion of two nuclei, for a given density and plasma temperature, the frequency of occurance per unit volume of this reaction can be computed. If we denote the number of reactions in D-T by z , the output released in 1 cm^3 will be N=zx17.6 MeV x se^{-1} |9|. At a given deuterium density, this output rises steeply with the plasma temperature. The radiated output likewise rises with the plasma temperature, though somewhat more slowly. In the energy balance this later output figures as a loss. If the thermonuclear reactor is to supply useful power, the output generated by fusion of nuclei has to exceed this loss. Therefore, a specific temperature limit Tm, called ignition temperature exists for each total particle density n; above this limit the reactor is capable of supplying energy |10,24,27|.

The criteria required for pulsed fusion reactors has been derived by Lawson (1957) by considering the energy balance in a reacting plasma of density n, which was heated instantaneously to a temperature T, and held for some time τ. The criteria is given by the following formula:

$$n\tau > 10^{14}\ cm^{-3}\ sec$$

Thus, inorder to produce useful fusion power it is necessary to heat the plasma to a temperature of 10-20 KeV with a number density of 10^{14}-10^{15} cm^{-3} and confinement time about one second |11|.

Some of the fundamental demands on a thermonuclear reactor are the following:

- Of all the procedures aiming at the utilization of energy released in the fusion of light nuclei, the use of fully ionized high temperature plasma formed by such nuclei seems to hold the best promise.
- The thermonuclear reactor requires a device which permits the plasma to be heated above the ignition temperature of the reaction involved, that is to a temperature at which thermonuclear energy will be released at a rate at least capable of balancing the losses, mainly due to radiation.
- The particle density must be relatively small; otherwise the kinetic pressure p=nkT would exceed the technically sustainable value.
- As the energy loss due to Bremsstrahlung radiation increases with the square of the charge, the penetration into the plasma of the elements that have a higher atomic number must be prevented. Therefore, every thermonuclear reactor needs an ultrahigh vacuum pump which permits the thorough evacuation of the working space before it is field with deuterium or tritium.
- Obviously no material used for the vessel containing the plasma can stand the enormous temperatures that are

required for controlled nuclear fusion. The plasma must therefore be insulated from the vessel in order to prevent it from being contaminated by materials evaporating from the walls. An approach suitable under terrestial condition is the insulation by means of a magnetic field placed inside the vacuum.

- Using the MHD principle the energy released in the thermonuclear reactor can be converted into electric power. Heated by the thermonuclear energy released in the reaction, the plasma expands and cuts across the force lines of magnetic field confining it: an e.m.f. is thus developed in a suitably placed auxiliary windings.

The points listed above refer to the immediate requirements only. There are several additional problems, connected for instance with the expected copious flow of neutrons which cannot be confined in the magnetic vessel, since they carry no electric charge; there are problems regarding the stability of the reactor, etc. At present, research into thermonuclear reaction is mainly concerned with methods of heating and confining the plasma.

1.2 Tokomaks

Since the theme of the thesis is connected with the production of current in tokomaks, let us briefly discuss about these devices. A plasma with a temperature of several million degrees has been sustained for some time in installations known as tokamaks, which are widely regarded as the most promising potential fusion reactors, with a high yield of temperature, number density and confinement time.

The name tokamak (derived from the Russian "toroidal - chamber - magnetic") is applied to axially symmetric toroidal systems in which the plasma is confined by a strong toroidal magnetic field Bt, produced by an external toroidal solenoid, together with a weaker poloidal field B_p , produced mainly by a toroidal current I_p flowing in the plasma itself. The main components of tokamak, together with the toroidal and poloidal magnetic fields are shown in Figures 1 and 2.

As shown in Figure 1, the main part of this device is the vacuum toroidal chamber, with coils to produce toroidal magnetic field B_t. The chamber is filled with hydrogen or its heavy isotopes at a pressure of 10^{-4} - 10^{-5} Torr. and serves as a secondary circuit of the transformer. Current is induced in the plasma by the action of the transformer. This current heats the plasma and produces poloidal magnetic field B_p.

The combination of the two fields, that is, the poloidal and toroidal magnetic fields produce nested toroidal

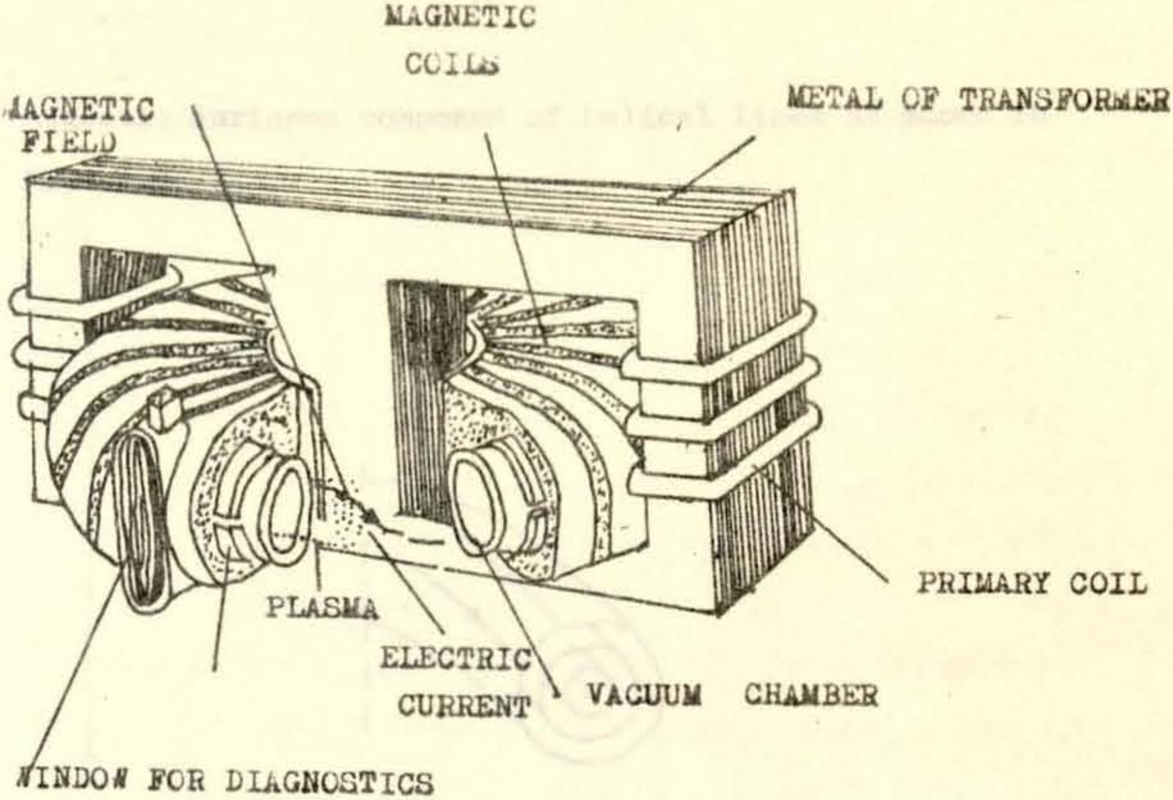

FIG. 1 THE MAIN PARAMETERS OF TOKAMAK

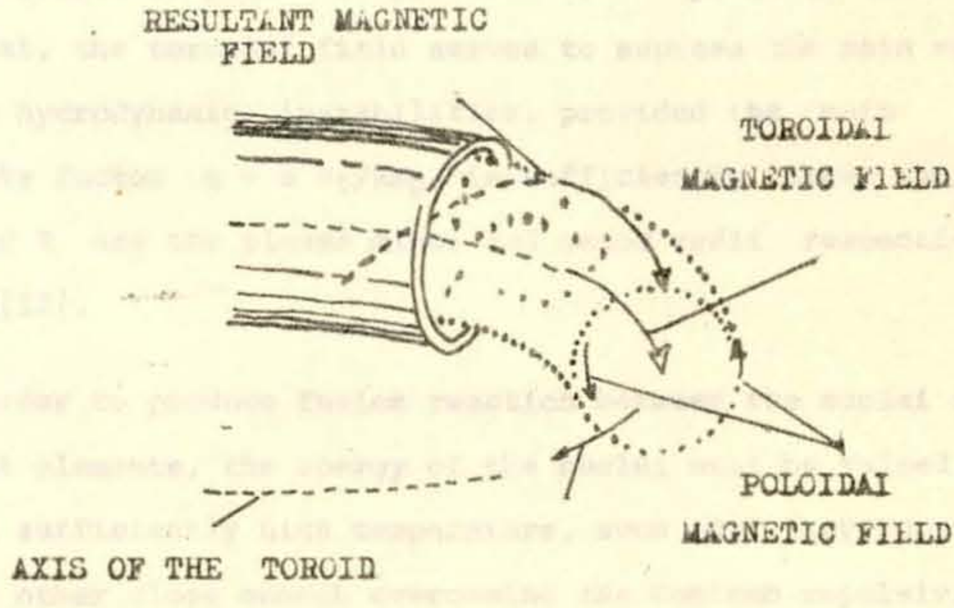

FIG. 2 THE TOROIDAL AND POLOIDAL
MAGNETIC FIELDS

magnetic surfaces composed of helical lines as shown in Figure 3.

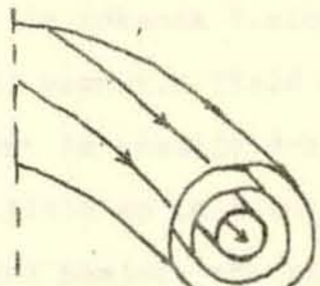

FIG. 3. TOROIDAL MAGNETIC SURFACES IN TOKAMAK.

Equilibrium of plasma is produced by the poloidal field whilst, the toroidal field serves to supress the main magneto hydrodynamic instabilities, provided the main safety factor $q = a\,B_t/RB_p$ is sufficiently large, where a and R are the plasma minor and major radii respectively|12|.

In order to produce fusion reaction between the nuclei of light elements, the energy of the nuclei must be raised to a sufficiently high temperature, such that they approach each other close enough overcoming the Coulomb repulsive barrier and it is necessary to confine the plasma for some time. The toroidal current which is produced inductively is used to heat and confine the plasma.

Eventhough devices such as tokamak are considered as one of the most perspective ways of thermonuclear fusion they have some shortcomings. As has been pointed out earlier, plasma confinement in tokamak fusion devices is maintained in part by poloidal magnetic field sustained by a toroidal current. The current is usually driven by an inductively produced electric field so that the tokamak operates only in a pulsed mode and besides the magnetic field system is one of the most costly item in this device. Therefore, it was necessary to find an alternative method of producing toroidal currents which can be driven continuously so that, the tokamak will operate on a steady-state basis. Different methods of producing these currents have been suggested and will be considered in the next chapter.

CHAPTER 2

PRODUCTION OF STEADY STATE CURRENTS

A number of methods for achieving non-inductive current drive, all employing beams of radiation or particles have been suggested. For the generation of steady state currents with the help of waves, it was suggested to use different types of waves: lower-hybrid, Alfven waves, ion cyclotron, electron cyclotron, electrostatic waves, etc. Before considering the various methods of producing steady-state currents in tokamak, let us briefly discuss the different types of waves and the basic principles for the production of current in a plasma.

2.1 Waves in a Plasma

An ionized gas is capable of a wide variety of oscillatory motions. Considerable theoretical study has been given to three particular types of waves in a plasma: electromagnetic waves, hydromagnetic waves and electrostatic waves. The analysis of these various types of waves is based primarily on Maxwell's equations:

$$\vec{\nabla}.\vec{E} = \rho/\varepsilon \tag{2.1}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{2.2}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{2.3}$$

$$\vec{\nabla}\times\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t} \tag{2.4}$$

If we take the curl of equation (2.3) and differentiate equation (2.4) with respect to time, eliminating $\vec{\nabla}\times\frac{\partial\vec{H}}{\partial t}$, we obtain the basic equation for electromagnetic waves,

$$\nabla^2 \vec{E} - \vec{\nabla} \rho/\varepsilon = \mu \frac{\partial \vec{J}}{\partial t} + \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \quad (2.5)$$

In general, under any given set of circumstances four modes of infinitesimal wave propagation are possible at each frequency, although the phase velocity of some modes may be imaginary. The simplest situation is obtained when there is no external magnetic field. The modes are then of two types, electromagnetic and electrostatic. In electromagnetic waves, the electric field is perpendicular to the direction of propagation. There are two such modes, corresponding to the two directions of polarization. If the frequency is less than a certain critical value, which is called the plasma frequency, which increases with increasing density, the phase velocity becomes imaginary and electromagnetic waves cannot propagate in the absence of magnetic field.

The other two modes in the absence of a magnetic field are electrostatic type, in which the current density $\vec{J}$ and the electric field $\vec{E}$ are parallel to the direction of propagation. In one of these modes the positive ions are essentially unaffected and only the electrons oscillate. These oscillations are called electron or plasma waves. In the other mode, called positive ion waves, the positive ions and electrons, generally move together; the inertia of the ions determines the wave velocity, which is normally less than that of the electron waves. In the absence of a magnetic field, the phase velocity of the electron waves becomes imaginary for frequencies less than the plasma frequency, while the positive ion waves don't propagate above a cut-off frequency equal to $(me/mi)^{\frac{1}{2}}$ times the plasma frequency |13|.

In the presence of a magnetic field, these four modes are profoundly modified, but the number of independent modes remains the same. The term hydromagnetic wave is frequently given to the waves which arise in the presence of a magnetic field at a frequency small compared to the cyclotron frequency of the positive ions. An Alfven wave is a simple type of hydromagnetic waves |13|.

In analysing a dispersion relation, it is frequently helpful to examine the frequencies at which the velocity of the wave is either zero or infinity. The former are called resonances, since these are the frequencies at which a plasma will be in resonance with an applied oscillating transverse electric field. The latter are called cut-offs. At a cut-off, a wave is usually reflected, while at resonance either absorption or reflection may occur depending on the nature of the damping process involved.

When a wave propagates across a magnetic field, the extra ordinary mode, which is partially transverse and partially longitudinal, will have two resonance frequencies at the lower-hybrid and the upper-hybrid, which are defined by:

$$\omega_{h1}{}^2 = \omega_{ce}\,\omega_{ci}$$

$$\omega_{h2}{}^2 = \omega_p{}^2 + \omega_{ce}{}^2 \qquad |13|$$

where:

ω_{h1} is the lower-hybrid frequency

ω_{h2} is the upper-hybrid frequency

ω_p is the plasma frequency

ω_{ci} is the ion cyclotron frequency

and

ω_{ce} is the electron cyclotron frequency

At the upper-hybrid frequency, only the electrons oscillate and the resonant oscillations represent the joint influence of the electrostatic and magnetic forces on gyrating electrons perpendicular to both the magnetic field and the wave front. At the lower-hybrid frequency, electrons and ions oscillate together.

2.2 Generation of Steady-State Currents

Plasma currents driven by traveling waves were first observed by Thonemann, Cowhig and Daveport in 1952 |8,14|. In principle any wave with a net momentum can generate a current via any damping mechanism, caused by the charged particles in the plasma. The process of producing current is based on the transition of momentum of the wave to the electrons and ions in a plasma. This mechanism can be explained using the single particle cold plasma model. In this model only the motion of the electrons in the field of the high frequency electromagnetic wave is considered.

The equation of motion for the electrons is given by:

$$m \frac{d\vec{v}}{dt} = m \frac{d^2\vec{r}}{dt^2} = e\,\vec{E} \qquad (2.6)$$

Taking the electric field of the plane mono-ehromatic wave as:

$$\vec{E}\,(\vec{r},t) = \vec{E}_o \cos\,(\omega t-\vec{k}\cdot\vec{r}) \qquad (2.7)$$

and neglecting the variations of this field on the amplitude of the electron oscillations, by substituting equation (2.7) into equation (2.6) we get:

$$m\ddot{\vec{r}} = e\vec{E}_o \cos\,(\omega t-\vec{k}\cdot\vec{r}) \qquad (2.8)$$

solving for $\dot{\vec{r}}$:

$$\dot{\vec{r}} = \frac{e\vec{E}_o \sin\,(\omega t-\vec{k}\cdot\vec{r})}{m\omega} \qquad (2.9)$$

From equations (2.8) and (2.9) the work done per unit time by the wave is obtained as

$$e\vec{E}\cdot\dot{\vec{r}} = \frac{e^2}{m\omega}|E_o|^2 \;\tfrac{1}{2} \sin 2(\omega t-\vec{k}\cdot\vec{r}) \qquad (2.10)$$

From equation (2.10) , it follows that the average work done by the wave is equal to zero. Therefore, there is no regular interchange of energy between the wave and the electrons. In order to change the average energy of the electrons, it is necessary to have a phase difference between the velocity of the electrons and the field different from $\pi/2$. This can be achieved by using different methods.

a) Scattering by free charges.

If an electromagnetic wave falls on a system of charges, then under its action the charges are set in motion. This motion inturn produces radiation in all directions; there occurs, we say, a scattering of the original wave. The scattering is most conveniently characterized by the ratio of the amount of energy emitted by the scattering system in a given direction per unit time, to the energy flux density of the incident radiation. This ratio clearly has dimensions of area, and is called the effective scattering cross-section. |15,26|.

The occurrence of scattering leads, in particular, to the appearance of a certain force acting on the scattering particle. One can verify this by the following considerations. On the average, in unit time, the wave incident on the particle loses energy $c\langle\omega\rangle\,\sigma$, where $\langle\omega\rangle$ is the average energy density, and σ is the total effective scattering cross-section. Since the momentum of the field is equal to its energy divided by the velocity of light, the incident wave loses momentum equal in magnitude to $\langle\omega\rangle\,\sigma$. The momentum lost by the incident wave is absorbed by the scattering particle. The average force acting on the particle is equal to the average momentum absorbed per unit time, i.e.,

$$\langle\vec{f}\rangle = \sigma\langle\omega\rangle\vec{n} \qquad (2.11)$$

where $\vec{n}$ is a unit vector in the direction of propagation of the incident wave.

The scattering cross-section is given by |16|

$$\sigma = \frac{8\pi}{3} \left(\frac{e^2}{mc^2}\right)$$

b) Collision of Particles:

When electrons collide with the neutral atoms or ions an average force acts on the electrons which is given by |17|

$$<\vec{f}> = \frac{e^2(\vec{k}\cdot\vec{E})\,\nu\,\vec{E}_o}{2\omega m(\omega^2 + \nu^2)} \qquad (2.12)$$

where ν is the collisional frequency. From equation (2.12) , it is seen that the average force is different from zero for longitudianl waves and is directed along the propagation of the wave. Steady-state current is produced if the average force acting on the electrons as given by equation(2.12) is equal to the force of friction which is given by $M\nu\vec{v}$. And when $\nu<\omega$ current does not depend upon the frequency of collision.

c) Interaction of incident wave with resonant particles:

When the velocity of the electrons is close to the phase velocity of the wave, it is necessary to take into account the dependence of the electric field upon the coordinates in integrating the equation of motion of the electrons that is, equation (2.6) . In this case, regular interchange of energy between the electrons and the wave takes place and the current produced by the electrons can be evaluated using the kinetic theory of plasma. The generation of current is connected with the actions of a practically constant electric field which acts on the resonant electrons.

The resonant coupling between the wave and those electrons which are moving with approximately the same velocity as the phase velocity of the wave is known as Landau damping |18| . Particles moving slightly faster than the wave will decrease their average velocity to the phase velocity of the wave through resonant interaction and transfer their extra kinetic energy to the wave. Particles moving slightly slower than the wave will be accelerated and absorb energy from the wave. If the distribution function decreases with increasing velocity damping takes place, since there will be more slow particles to absorb energy from the wave than fast particles to transfer energy to the wave.

d) Non Homogeneouse and Non Stationary Waves:

If the wave changes its amplitude, then, there will be a phase difference between the field and the velocity of the oscillatory particles which is propagational either to the gredient of the amplitude $\frac{\partial A}{\partial r}$ or to the time change of amplitude $\frac{\partial A}{\partial t}$.

.3 Methods Employed for the Generation of Steady-State Currents in Tokamaks

In this section the various methods proposed for the production of continuous toroidal currents in tokamak will be considered. These methods use either waves or beams of particles to produce steady-state currents. For the case of waves, the main idea is, net toroidal momentum is delivered to one of the species of the plasma, usually the electrons, from an external source. The momentum absorption by a particle leads to the production of current. In the other case, many systems for the production of high, energy intense directed beams of plasma electron, ions or neutral atoms are used. These beams are injected into a specially prepared region of plasma confinement where ionization of the neutral particles present can occur.

Fisch |5| has considered the production of continuous toroidal currents by means of radio frequency (r f) waves. These waves which can be injected continuously, have a net toroidal component of momentum, so that when absorbed by electrons, a force is exerted that drives a toroidal current. For practical employment in a reactor, the power required to drive the current must be a small fraction of the fusion out put. Since all types of waves do not incur the same power dissipation, in searching for the most favourable wave, we must consider not only the ease with which the waves can be excited, but also their individual power requirements. There are two types of waves that are attractive in terms of minimizing the power dissipated for a given current generated, namely low and high parallel phase velocity waves. The former region of subthermal phase velocity was suggested by Wort |5| and it is attractive, because in this region waves have a higher momentum content per unit energy. In other words, whereas the momentum in a wave is proportional to its wave number k , the energy carried by the wave is proportional to its freqeuncy ω, so that waves with low ω/k_z have a high parallel momentum content. (k_z is the wave number in the toroidal direction) . When the energy of the wave is absorbed by the electrons, the momentum absorbed is proportionaly higher. Wort envisioned the use of substhermal phase velocity, low compression Alfven waves to achieve current generation.

The alternative approach of using waves with high phase velocities was pointed out by Fisch |18| . Although, these waves have little momentum content, their momentum and energy are absorbed by fast electrons. The electrons that carry the current are relatively collisionless and so retain their momentum longer than the thermal electrons, which carry the current when low phase velocity waves are employed. The relative infrequency of collisions encountered by the current carriers compensate for the dearth of momentum in the driving waves. This approach was originally envisaged employing the lower-hybrid wave with phase velocity parallel to the magnetic field several times the electron thermal velocity. Successful attempts of the production of current by using lower-hybrid waves were reported by Jobs, F, etal |19|.

The production of current by ion cyclotron waves was considered by Tajima |6|. In this method the plasma is irradicated by electromagnetic field in the ion cyclotron range of frequencies. Some of the attractive features of the ion cyclotron heating are:

- the low frequency of this scheme means that high power technology is available for wave generation,
- it would heat the plasma ions directly by either heating the bulk distribution or producing energetic tails which are more active,
- ion cyclotron heating might be an alternative to the expensive neutral beam heating, in terms of good penetration to the centre of the plasma.

In the works of Karney and Fisch |7| the generation of current by means of electron cyclotron wave absorption was considered. These waves are employed to generate toroidal current merely by heating selected electrons, and interestingly without directly injecting substential toroidal momentum to those electrons. The effect of cyclotron heating is to increase primarily the perpendicular component of the velocity of the resonant electrons. The velocity increase lies in this direction because the waves have very little parallel momentum content compared to the energy content, so that when the wave is absorbed by the electrons, the electron energy increases, but by momentum conservation, its parallel momentum barely increases. That the waves themselves have little momentum content to imput to the electrons is a consequence of the super-limineouse parallel velocity. The energy in a wave is proportional to the frequency while its momentum is preportional to the wave number k, and since $\omega/k_{\parallel} > c$ the wave possesses little momentum .

Toroidal currents produced by means of electrostatic waves was considered by Kenematisu Kato |8|. In this work the general formula which gives the current density generated by an electrostatic traveling waves was derived in the frame work of quasi-linear theory. For the case of high frequency electrostatic waves, in which the ion motion can be neglected, it was shown that no net plasma current is produced by the wave particle interaction. Here, the wave momentum is defined as the momentum of the non resonant electrons. When the wave is attenuated or growing, momentum is exchanged between the wave and the resonant electrosn. If the momentum of the electron distribution function, which is the sum of the wave momentum and the momentum of the resonant electrons is initially zero, it must still vanish after the wave has damped out. Not only the resonant electrons but also the nonresonant electrons contribute to the current so that the currents generated by them cancel each other. And next, by considering low frequency electrostatic waves, in which the motions of the ions are taken into account, it is shown that plasma current is generated not by momentum exchange between the electrons themselves but due to the momentum exchange between the electrons and the ions. B ut for the case of electromagnetic wave, which has field momenta as well as particle momenta, as the wave decays, the field momenta is converted to particle momenta therefore, even if the ions are considered to be immobile, the electrons gain momenta so that a plasma current is generated.

Manheimer and Winsor |3| considered the possibility of injecting intense pulsed ion beams into tokamak plasma inorder to heat the plasma and mantain the current. In order to accomplish these objectives, several obstacles must be overcome. First, the ion beams must be produced; second it must be injected into the toroidal chamber, third it must deposite its energy in the plasma. The ion beam is injected into a partially formed plasma and is trapped and the remaining plasma is formed around it. Once the beam is trapped in the torus, the next question is coupling it to the plasma. B y properly choosing the beam and plasma parameters, the energy of the beam can be absorbed by the plasma in a confinment time or less.

The production of steady-state current by high energy α-particles produced by fusion reactor was considered by B harda and Chu |4|. The idea is to use r f power to prohibit the α-particles from slowing down isotropically and to push the α-particles in a preferential direction and thus form an α-particle beam. The α-particle beam will then transfer momentum to the electrons and thus sustain plasma current.

A basically different approach was proposed by Fisch and Boozer |21| in which no net toroidal momentum is injected, but the collisionality of plasma is somehow altered so that, for example electrons moving to the left collide more frequently with ions, than do electrons moving to the right. There would result a net current with ions moving to the left and electrons moving, on the average to the right. The means suggested for achieving this asymmetric resistivity is selective heating of those electrons moving to the right. The electrons being hotter naturally collide less.

In this thesis, the mechanism of generating entraining currents by using high frequency electromagnetic waves with a slowly varying amplitudes is considered which is a basically different approach from the above mentioned works. The current density, energy density and energy flux density which are produced by the passage of such waves will be evaluated using Vlasov's kinetic equation which is considered in the next chapter.

CHAPTER 3

PLASMA KINETIC THEORY: THE BOLTZMANN EQUATION

Kinetic theory attempts to explain the macroscopically observed phenomena of gases by considering the forces of interaction between the molecules of the gases. Free atoms, ions and electrons are considered merely as special types of molecules |22|. If at some instant of time, the position of each molecule of the gas and the forces that acted on each molecule, are known, the subsequent motion of the molecules can be determined in principle by classical mechanics. But it is not possible to get detailed information of the molecules and even if there is, the problem will be too difficult and time consuming to solve.

An alternative approach to the problem is through the methods of statistical mechanics. If we consider a box of gas containing n, molecules at some instant of time, the three components of momentum and the three components of position of each molecule are given by a list of 6n numbers. The entire gas at that instant of time is represented by a single point in the 6n dimensional phase space. If we consider again a box of gas which may be the same box as considered previously, but at some later instant of time, it can be represented by another point in phase space. If the process is continued until all possible states are considered, the total number of points in phase space will be the different number of dynamical states of the particular system. These points are said to form an ensemble of points in phase space.

Statistical mechanics deals with the probability of finding one system of the ensemble, choosen at random, in a particular region of phase space. The assumption of statistical mechanics is that, the macroscopically observed variables of the gas correspond to those dynamical states that are most probable. The fundamental theorem of statistical mechanics is Liouville's theorem which states that, the volume occupied by a given set of points in phase space is constant through out the motion of these points. The solution of Liouville's equation gives the probability of finding a point at a given place in phase space. That is, we would know the probability that each molecule has a specified position and momentum. However, it is often unnecessary to have such a complete description of the gas and it would be sufficient to know only the probability that any single molecule has a given position and momentum. This part of statistical mechanics dealing with one particle distribution forms the framework of kinetic theory. For this case it is necessary to consider only a six dimensional phase space.

The distribution function of molecular velocities is defined as the density of points in phase space. This is a one particle distribution function, because the position and velocity of any given particle is not correlated with the position and velocity of any other particle or particles. If the distribution function of the molecular velocities is known, then the various molecular properties involving the pressure and energy of the gas can be obtained by integrating the quantity of interest times the distribution function over all velocity space.

3.1 The Distribution of Molecular Velocities

Consider a certain volume of gas, where the location of each molecule is represented by its position in a cartesian coordinate system. The differential volume element dr (= $dx_1dx_2dx_3$) is a small, but finite, volume element which is assumed to be large enough to contain a great number of molecules but which is, nevertheless, small compared to the lengths involved in spatial variation of the macroscopic parameters of the gas.

If $N(x_i,t)$ is the number density of molecules in confugaration space, then the number of molecules in dr is Ndr. A molecule is said to be in dr if the molecule is located at a position with its x_1 - component between x_1 and x_1+dx_1, its x_2 - component between x_2 and x_2+dx_2, and its x_3 - component between x_3 and x_3+dx_3. All the molecules in dr have the same value of x_i as far as macroscopic variation of the gas are concerned.

The ndr molecules in dr have a wide range of different velocities. The velocities of the ndr molecules can be represented in a three dimensional velocity space. The density of the ndr points in velocity space is given by $f(v_i, x_i, t)dr$, where f is called the velocity distribution function. If dc(=dv_1 dv_2 dv_3) is a differential volume element in velocity space, then f dr dc is the number of points in the volume element dc. A molecule is said to be in dc if it has a velocity with components between v_1 and v_1+dv_1, v_2 and v_2+dv_2, and v_3 and v_3+dv_3.

Now if f dr dc is the number of molecules in dr whose velocities are in dc, we see that f is the density of points in the six dimensional phase space made up of the three components of position and three components of velocity. Dividing f dr dc by the volume dr we note that f dc is the number of molecules per unit volume with velocity v_i in dc. If we integrate the expression f dr dc over all velocity space, we will obtain the total number of points in the six dimensional phase space. Thus,

$$\begin{aligned} ndr &= dr \int fdc \\ \text{and} \quad N &= \int fdc \end{aligned} \tag{3.1}$$

3.2 Mean Values of Molecular Properties

Let us associate some molecular property $\phi(v_i, x_i, t)$ with each of the ndr moleculars in dr, which is in general a function of velocity v_i , position x_i and time t. At some time t, the values of ϕ for the ndr molecules in dr will depend only on their velocity v_i, because the x_i dependence of ϕ will be the same for all molecules in dr.

If we add up the values of ϕ for the ndr molecules in dr,

$$\phi^{(1)} + \phi^{(2)} + \dots + \phi^{(ndr)} = \Sigma \phi$$

The mean value $< \phi >$ of ϕ for the ndr molecules in dr is defined by the equation

$$\Sigma \phi = < \phi > ndr$$

Some of the ndr values of ϕ making up $\Sigma\phi$ will have essentially the same value: namely, those whose velocities lie in the same velocity element dc. The number of such molecules is f dr dc and each contributes the quantity ϕ to the sum $\Sigma\phi$. Thus, the contribution of all molecules with all velocities is found by integrating over all the velocities. Thus,

$$\Sigma \phi = dr \int \phi f \, dc$$
$$N < \phi > = \int \phi f \, dc \tag{3.2}$$

From equations (3.1) and (3.2) we see that the mean value of any function ϕ is found from

$$< \phi > = \frac{\int \phi f \, dc}{N} = \frac{\int \phi f \, dc}{\int f \, dc} \tag{3.3}$$

Applying the above method we will obtain the mean velocity $< v_i >$ and the peculiar velocity V_i which will be used later.

To find the mean velocity $< v_i >$ let $\phi = v_i$ in equation 3.3, then,

$$< v_i > = \frac{1}{N(x_i, t)} \int v_i \, f(v_i, x_i, t) \, dc \tag{3.4}$$

We note that whereas v_i, x_i and t are independent variables and are thus independent of each other, $< v_i >$ is a function of position x_i and time t. The peculiar velocity V_i is defined as the velocity of a molecule relative to the mean velocity. That is,

$$V_i = v_i - < v_i > \tag{3.5}$$

since $< v_i >$ is a function of x_i and t then, V_i is also a function of x_i and t. From equation 3.5, we see that the mean value of the peculiar velocity is zero since

$$< V_i > = < v_i - < v_i > > = < v_i > - < v_i > = 0$$

3.3 The Boltzmann Equation

The macroscopic properties or the mean values of a gas can be determined from a knowledge of the velocity distribution function f. In order to find the time variation of the macroscopic properties it is necessary to obtain an equation describing the time variation of the velocity distribution function. The equation that describes this time variation is the Boltzmann equation. The Boltzmann equation will be derived in two parts, considering first the form which it takes in the absence of collision, and then deriving the collisional term. The collisional term of the Boltzmann equation takes account of binary collisions. In plasma multiple coulomb collisions must also be taken into account.

3.3.1 The Collisionless Boltzmann Equation

Let R_i be the force per unit mass on a particle in a gas. In most cases the force will not be a function of velocity. However, in case of a plasma a force that is a function of velocity does exist. This is the force on a charged particle moving in a magnetic field given by $q \vec{v} \times \vec{B}$, where q is the charge on the particle. The force per unit mass

R_i is written as the sum of two terms: R_i' which is independent of velocity, and the velocity dependent term $q \vec{v} \times \vec{B}$. That is,

$$R_i = R_i' + q/m \vec{v} \times \vec{B} \tag{3.4}$$

And from Newton's second law we have that $R_i = \dot{v}_i$.

At time t , the number of molecules in volume element $dr^{(t)}$ at x_i with velocities in the velocity element $dc^{(t)}$ at vi is $f(v_i, x_i, t)dc^{(t)} dr^{(t)}$. In the time interval dt, the location of a molecule will change by an amount $\dot{x}_i dt = v_i dt$, while its velocity will change by an amount $\dot{v}_i dt = Ri\ dt$. Thus, at the time $t + dt$, in the absence of any collisions, the number of molecules in the volume element $dr^{(t+dt)}$ at $x_i+v_i\ dt$ with velocities in the velocity element $dc^{(t+dt)}$ at $v_i+R_i\ dt$ is

$f(v_i+R_i\ dt, x_i+v_i\ dt, t+dt).\ dc^{(t+dt)}\ dr^{(t+dt)}$. However, these are exactly the same molecules as considered at time t, so that we can equate the two expressions and obtain:

$$f(v_i,x_i,t)dc^{(t)} dr^{(t)} = f(v_i+R_i dt, x_i+v_i dt, t+dt) \times dr^{(t+dt)}.\ dc^{(t+dt)} \tag{3.5}$$

The relationship between $dr^{(t)}\ dc^{(t)}$ and $dr^{(t+dt)} \times dc^{(t+dt)}$ can be determined in the following way. The changes in the volume element $dc^{(t)}$ and $dr^{(t)}$ are given by the Jacobian transformation

$$dr^{(t+dt)} = J^{(r)} dr^{(t)} = \frac{\partial (x_i+v_i\ dt)}{\partial (x_s)}\ dr^{(t)} \tag{3.6}$$

and

$$dc^{(t+dt)} = J^{(v)}\ dv^{(t)}\ \frac{\partial(v_i + R_i'\ dt + q/m\ \vec{v}\times\vec{B}dt)}{\partial(v_s)}\ dc^{(t)} \qquad (3.7)$$

Since v_i and x_j are independent variables we note that in equation 3.6

$$J^{(r)} = \frac{\partial(x_i + v_i\ dt)}{\partial x_s} = \det \delta\ i\ j = 1$$

where det stands for determinant, so that

$$dr^{(t+dt)} = dr^{(t)} \qquad (3.8$$

Expanding the Jacobian in equation (3.7),

$$\begin{vmatrix} 1 & -q/m\ B_3dt & q/m\ B_2dt \\ q/m\ B_3dt & 1 - q/m\ B_1dt & \\ -q/m\ B_2dt & q/m\ B_1dt & 1 \end{vmatrix} = 1$$

If we neglect terms of order $(dt)^2$, thus, equation 3.7 can be written as

$$dc^{(t+dt)} = dc^{(t)}\ . \qquad (3.9)$$

Using equations (3.8) and (3.9) we can write equation (3.6) as,

$$f(v_i, x_i, t) = f(v_i, + r_i\ dt, x_i + v_i\ dt, t+dt) \qquad (3.10)$$

Expanding the right hand side of equation (3.10) in a Taylor series about dt = 0 we obtain:

$$f(v_i, x_i, t) = f(v_i, x_i, t) + \left| \frac{\partial f}{\partial v_i} R_i dt + \frac{\partial f}{\partial x_i} v_i\ dt + \frac{\partial f}{\partial t} dt \right| +$$

terms of order $(dt)^2$.

Neglecting terms of order $(dt)^2$ and higher terms we have

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + R_i \frac{\partial f}{\partial v_i} = 0 \qquad (3.11)$$

This is the collisionless Boltzman equation expressing the time dependence of the velocity distribution function. The first term, $\frac{\partial f}{\partial t}$, is the local variation of the distribution function. The term $v_i \frac{\partial f}{\partial x_i}$ is the variation of the distribution function resulting from molecules streaming in and out of a given volume element. The term $R_i \frac{\partial f}{\partial v_i}$ is the variation of the distribution function resulting from the external forces acting on the molecules.

The distribution function will always vary as a result of collisions between the molecules of the gas. We can include this variation by writing a collisional term on the right hand side of equation (3.11). Thus,

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + R_i \frac{\partial f}{\partial v_i} = \left(\frac{\partial f}{\partial t}\right)_{coll} \qquad (3.12)$$

The form of this collision term has to be determined. For a gas of low density, where binary collisions only need to be considered, this collision term can be written as an integral known as the collision integral.

3.3.2 The Collision Integral

The term $(\frac{\partial f}{\partial t})_{coll}$ which appears on the Boltzmann equation represents the time rate of change of the distribution function, f due to collisions of the molecules with each other. An explicit expression for this term can be obtained by considering the statistical nature of the binary collisions.

For a given distribution function at a specific time t, there will be a certain number of molecules with velocities in particular regions of the velocity space. That is, there will be a certain number of molecules per unit volume with velocities v_i^A in dc^A. This number is $f(v_i^A)\ dc^A$ which we write as $f^A\ dc^A$. If as a result of collision, the velocity of a molecule changes from v_i^A to $\bar{v}_i^A$, then we will say that the molecules have been knocked out from the velocity element dc^A into the velocity element $d\bar{c}^A$. Let v_i^B and $\bar{v}_i^B$ be the initial and final velocities of a molecule that collided with the above mentioned molecule respectively. That is, to say this molecule was knocked from the velocity element dc^B into the velocity element $d\bar{c}^B$.

Now in order to calculate $(\frac{\partial f}{\partial t})_{coll}$ we need to determine the net gain of molecules with velocities in a certain velocity element due to collisions with molecules of all other velocities, taking as our typical molecules those with velocities v_i^A.

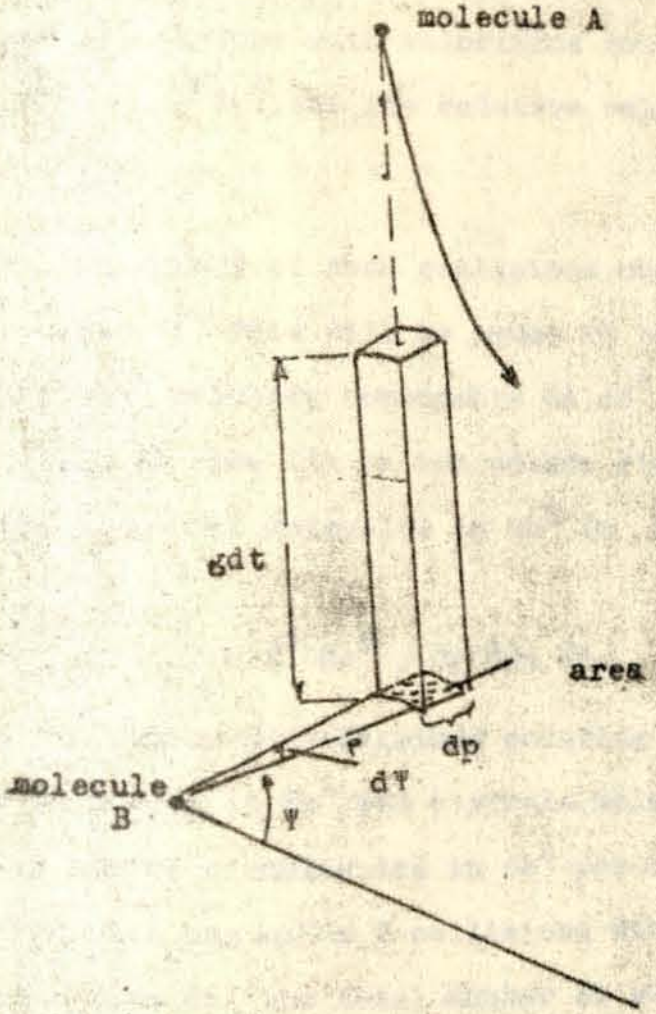

FIG. 4 REPRESENTATION OF COLLISION BETWEEN MOLECULE A AND MOLECULE B

Representation of a collision between molecule A and molecule B.

Consider first the collision in which molecules are knocked out of the velocity element dc^A. The collision occurs between molecule A with velocity in dc^A and molecule B with velocity in dc^B as shown in Figure 4. The number of molecules per unit volume with velocities in dc^B is $f(v_i^B)dc^B$ or $f^B dc^B$ and the relative velocity between the two molecules is g.

Let T be the number of such collisions occuring in the time interval dt. This will be equal to the number of molecules with velocity components in dc^B which at a certain instant of time lie in the volume element p dp dψ g dt. Since the number of molecules in dc^B is $f^B dc^B$, it follows that,

$$T = f^B \, dc^B \, p \, dp \, d\psi g \, dt.$$

This is the number of collisions occuring in the time dt between molecules in dc^B and a single molecule in dc^A. Since the number of molecules in dc^A per unit volume is $f^A \, dc^A$ and each one makes T collisions with molecules in dc^B in the time dt, the total number of such collisions is

$$f^A \, dc^A \, T = f^A \, f^B \, dc^A \, dc^B \, g \, p \, dp \, d\psi \, dt \qquad (3.13)$$

In order to get the total number of molecules that enter into collisions with all molecules of all velocities in the time dt, we must integrate equation 3.13 with respect to dc^B, dp and dψ. Defining this quantity by "out" (molecules knocked out of dc^A), we have from equation 3.13

$$\text{"out"} = dc^A \, dt \int f^A \, f^B \, g \, p \, dp \, d\psi \, dc^B .$$

Now let us consider a collision in which a molecule enters dc^A. For each of the so called direct encounters in which molecules get knocked from dc^A and dc^B in to $d\bar{c}^A$ and $d\bar{c}^B$, there is an inverse encounter in which molecules get knocked from $d\bar{c}^A$ and $d\bar{c}^B$ into dc^A and dc^B. Thus, for an inverse encounter we can consider $\bar{v}_i^A$ to be the pre collision velocity of the molecule which, after collision, has its velocity in dc^A. We then calculate as before the number of collisions $\bar{T}$ between molecules with velocities in $d\bar{c}^A$ and $d\bar{c}^B$, so that the relative speed is $\bar{g}$. The result is

$$\bar{T} = \bar{f}^B \, d\bar{c}^B \, p \, dp \, d\psi \, \bar{g} \, dt .$$

Since each of the $\bar{f}^A \, d\bar{c}^A$ molecules per unit volume in $d\bar{c}^A$ makes $\bar{T}$ collisions with molecules in $d\bar{c}^B$ in the time dt, the total number of this collisions between molecules in $d\bar{c}^A$ and molecules in $d\bar{c}^B$ is

$$f^A d\bar{c}^A \, \bar{T} = \bar{f}^A \, \bar{f}^B \, d\bar{c}^A \, d\bar{c}^B \, \bar{g} \, p \, dp \, d\psi \, dt \tag{3.15}$$

Thus, the total number of molecules that emerge from a collision is found by integrating equation (3.15) . Denoting this quantity by "in" (molecules get knocked into dc^A) we have,

$$\text{"In"} = d\bar{c}^A \, dt \int \bar{f}^A \, f^B \, \bar{g} \, p \, dp \, d\psi \, d\bar{c}^B \tag{3.16}$$

The net gain of molecules is the difference between equation (3.16) and equation (3.14).

Using

$g = \bar{g}$ and $dc^A\, dc^B = d\bar{c}^A\, d\bar{c}^B$ we get:

$$\text{"In"} - \text{"out"} = dc^A\, dt \int (\bar{f}^A \bar{f}^B - f^A f^B)\, g\, p\, dp\, d\psi\, dc^B \qquad (3.17)$$

Now $(\frac{\partial f}{\partial t})_{coll}$ is the rate of change of f^A due to collision. The number of molecules per unit volume with velocities in dc^A at the beggining of dt is $f^A\, dc^A$. The number of molecules per unit volume with velocity in dc^A at the end of dt is $|f^A + (\frac{\partial f^A}{\partial t})_{coll}\, dt|\, dc^A$. Thus, the net gain of molecules in dc^A is,

$$(\frac{\partial f^A}{\partial t})_{coll} = \int (\bar{f}^A \bar{f}^B - f^A f^B)\, g\, p\, dp\, d\ \ dc^B \qquad (3.18)$$

In deriving this equation we have considered v_i^A to be the velocity of our typical molecule. We might as well have choosen any velocity v_i. In this case $f(v_i^A) = f^A$ would be $f(v_i) = f$ and equation (3.18) would be written as

$$(\frac{\partial f}{\partial t})_{coll} = \int (\bar{f}\ \bar{f}^B - f\, f^B)\, g\, p\, dp\, d\psi\, dc^B \qquad (3.19)$$

Using equation (3.19) we can write the Boltzmann equation as

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + R_i \frac{\partial f}{\partial v_i} = \int (\bar{f}\ \bar{f}^B - f\, f^B)\, g\, p\, dp\ \ d\psi\, dc^B \qquad (3.20)$$

3.4 The Equilibrium Solution of the Boltzmann Equation

The equilibrium solution of the Boltzmann equation is used to mean the expression for the velocity distribution function that satisfies the equation when there are no external forces and when interparticle collisions no longer cause any change in the distribution.

Consider a collision between two molecules A and B. If a function ϕ of molecular velocities is associated with each molecule and if the sum of the ϕ's for the two molecules is conserved during a collision, then ϕ is called a summational invariant. Thus, ϕ has a property that,

$$\phi^A + \phi^B = \bar{\phi}^A + \bar{\phi}^B \quad . \tag{3.21}$$

The total energy and the components of linear momentum are conserved during a collision. That is,

$$\tfrac{1}{2} M v_i v_i + \tfrac{1}{2} M^B v_i^B = \tfrac{1}{2} M \bar{v}_i \bar{v}_i + \tfrac{1}{2} M^B \bar{v}_i^B \bar{v}_i^B \tag{3.22}$$

and

$$M v_i + M^B v_i^B = M \bar{v}_i + M^B \bar{v}_i^B \tag{3.23}$$

Thus, $M v_i/2$ and $M v_i$ are summational invariants. A numerical constant is another summational invariant. Equations (3.22) and (3.23) are four equations giving the six quantities $\bar{v}_1$, $\bar{v}_2$, $\bar{v}_1^B$, $\bar{v}_2^B$ and $\bar{v}_3^B$ interms of v_1, v_2, v_3, v_1^B, v_2^B and v_3^B. Thus, solving these four equations in six unknowns leaves two unknowns to be determined. However, there are two geometrical unknowns required to specify the collisions completly - p and ψ in Fig.4. Knowing these the problem is determined by equations (3.23) and (3.24).Thus, any other summational invariant cannot be independent and must be a linear combination of

$$\phi^{(1)}=1, \quad \phi^{(2)}=Mv_i, \quad \phi^{(3)}=\tfrac{1}{2} Mv_j v_j \tag{3.24}$$

We wish to determine the form of the distribution function for the case of a uniform gas ($\frac{\partial f}{\partial x_i} = 0$) on which no external forces are acting. The Boltzmann equation becomes:

$$\frac{\partial f}{\partial t} = \int (\bar{f}\bar{f}^B - ff^B)\, g\, p\, dp\, d\psi\, dc^B \tag{3.25}$$

For equilibrium or steady state solution - that is a solution for which $\frac{\partial f}{\partial t} = 0$, for equation (3.26).This requires that the integral on the right hand side vanish. Clearly this integral will vanish if,

$$\bar{f}\bar{f}^B - ff^B = 0 . \tag{3.26}$$

Equation (3.26) is a necessary and sufficient condition for expression (3.25) to be zero |23|. Taking the natural logarithm of equation (3.26), we obtain,

$$\mathrm{Ln}\bar{f} + \mathrm{Ln}\bar{f}^B = \mathrm{Ln}f + \mathrm{Ln}f^B \tag{3.27}$$

Comparing equations (3.27) and (3.21) we see that Lnf must be a summational invariant. Thus, as we have seen, it must be a linear combination of $\phi^{(1)}$, $\phi_i^{(2)}$ and $\phi^{(3)}$ given by equation (3.22). That is,

$$Lnf = \sum_{n=1}^{3} \alpha^{(n)} \phi^{(n)}$$

$$= \alpha^{(1)} \phi^{(1)} + \alpha_i^{(2)} \phi_i^{(2)} - \alpha^{(3)} \phi^{(3)}$$

$$= \alpha^{(1)} + \alpha_i^{(2)} Mv_i - \alpha^{(3)} \tfrac{1}{2} Mv_j v_j$$

where $\alpha_i^{(2)}$ is a vector having three componets. The sign of the constant $\alpha^{(3)}$ is taken negative for later convenience. Summing over repeated indices gives,

$$Lnf = \alpha^{(1)} + M(\alpha_1^{(2)} v_1 + \alpha_2^{(2)} v_2 + \alpha_3^{(2)} v_3)$$

$$- \tfrac{1}{2} \alpha^{(3)} M(v_1^2 + v_2^2 + v_3^2)$$

$$= Ln\, \alpha^{(0)} - \alpha^{(3)} \tfrac{1}{2} M \left[(v_1 - \frac{\alpha_1^{(2)}}{\alpha^{(3)}})^2 + (v_2 - \frac{\alpha_2^{(2)}}{\alpha^{(3)}})^2 + (v_3 - \frac{\alpha_3^{(2)}}{\alpha^{(3)}})^2 \right]$$

If we write $\frac{\alpha_i^{(2)}}{\alpha^{(3)}} = \beta_i$, we obtain,

$$Lnf = Ln\, \alpha^{(0)} - \alpha^{(3)} \tfrac{1}{2} M |(v_1 - \beta_1)^2 + (v_2 - \beta_2)^2 + (v_3 - \beta_3)^2|$$

or

$$f = \alpha^{(0)} e^{-\alpha^{(3)} (\frac{1}{2}) M |(v_1 - \beta_1)^2 + (v_2 - \beta_2)^2 + (v_3 - \beta_3)^2|} \quad (3.28)$$

The form of f given by equation (3.28) is known as the maxwellian distribution. There are five arbitrary constants, $\alpha^{(0)}$, $\alpha^{(3)}$, β_1, β_2 and β_3 which must be determined.

3.5 The Maxwellian Distribution

The maxwellian distribution of particles given by equation (3.28) is the most important velocity distribution in plasma kinetic theory. It is the distribution of a set of particles in collisional equilibrium with themselves. Under this conditions a temperature may be properly assigned to the gas and the various thermodynamic properties of the gas may be discussed. When this condition is not fulfilled, that is to say, when the external forces are too strong to be masked by collisions, the thermodynamic properties cannot be properly applied to the gas. In such a case it may be a very difficult problem to solve the Boltzmann equation for the distribution function. It is therefore important, in plasma problems to know when the particles posses a maxwellian velocity distribution or when such a distribution may be a good approximation.

In order to determine the five arbitrary constants in equation (3.28), let us make the following change of variables.

$$U_1 = v_1 - \beta_2$$

$$U_2 = v_2 - \beta_2$$

$$U_3 = v_3 - \beta_3$$

Then equation (3.28) becomes

$$f = \alpha^{(0)} e^{-\alpha(3)|U_1{}^2+U_2{}^2+U_3{}^2|} \qquad (3.29)$$

The mean value of any function was given as ,

$$N <\phi> = \int \phi f \, dv_1 \, dv_2 \, dv_3$$

so that,

$$N <\phi> = \int \phi \, \alpha^{(0)} \, e^{-\alpha(3)\frac{1}{2}M|U_1^2+U_2^2+U_3^2|} dU_1 dU_2 dU_3 \qquad (3.30)$$

Let $\phi = U_1$ in equation (3.30), then

$$N <U_1> = \alpha^{(0)} \int_{-\infty}^{\infty} U_1 \, e^{-\alpha(3)\frac{1}{2}M U_1^2} dU_1 \times$$

$$\int_{-\infty}^{\infty} e^{-\alpha(3)\frac{1}{2}MU_2^2} dU_2 \int_{-\infty}^{\infty} e^{-\alpha(3)\frac{1}{2}MU_3^2} dU_3 = 0 \, . \qquad (3.31)$$

Since the fisst integral is the integral from $-\infty$ to ∞ of an odd function U_1, $<U_1> = <v_1-\beta_1> = <v_1> - \beta_1 = 0$ and therefore, $\beta_1 = <v_1>$ is the x_1-component of the mean velocity of the gas. In a similar manner we can show that $\beta_2 = <v_2>$ and $\beta_3 = <v_3>$, so that we can write,

$$U_1 = v_1 - <v_1> = V_1$$

$$U_2 = v_2 - <v_2> = V_2$$

$$U_3 = v_3 - <v_3> = V_3$$

where V_1, V_2 and V_3 are the components of the peculiar velocity V_i. The maxwellian distribution can thus be written as

$$f = \alpha^{(0)} \, e^{-\alpha(3)\frac{1}{2}M\left[V_1^2+V_2^2+V_3^2\right]} \qquad (3.32)$$

by using $N <\phi> = \int \phi \, f dv$ and transforming to spherical coordinates, where $V_i V_i = w^2$ and then by letting $\phi = 1$ and $\phi = MV_i V_i/2$ respectively, we can determine $\alpha^{(0)}$ and $\alpha^{(3)}$. The result is that equaiton (3.32) can be written as,

$$f = N\left(\frac{M}{2\pi kt}\right)^{3/2} e^{-M(w^2/2kt)} \tag{3.33}$$

Thus, we see that for a given number density n, mean velocity $<v_i>$, and temperature T, there is only one possible equilibrium distribution of molecular velocities and that any different distribution will tend to approach this distribution in the absence of external forces.

CHAPTER 4

GENERATION OF ENTRAINING CURRENTS BY A SLOWLY VARYING AMPLITUDE WAVE

4.1 Successive Approximation of the Distribution Function.

The evaluation of the current density, energy density and energy flux density is carried out by means of the kinetic theory. The kinetic theory treatment of plasma is based on the Boltzmann equation and Maxwell's equations. Since the solution of the Boltzmann equation is difficult, some approximation procedures are necessary. For rarefied plasma when collision between particles are negligible the collisional term in the Boltzmann equation can be set equal to zero and we get the Vlasov equation. The velocity distribution function can be determined by successive approximation to any desired degree of accuracy. The successive approximations are given by:

$$f^{(0)} + f^{(1)},\ f^{(0)} + f^{(1)} + f^{(2)},\ \ldots.$$

and

$$f^{(0)},\ f^{(1)},\ f^{(2)} \ldots\ldots f^{(r)}$$

are respectively proportional to n^1, n^0, n^{-1} etc., where n is the number density of the particles |22|. Hence, the later terms in the approximation become relatively more important as the density decreases. When the density of the gas is comparable with that of the atmosphere near the ground, the terms $f^{(r)}$ in f decrease rapidly as r increases and $f^{(0)} + f^{(1)}$ is sufficient good approximation to f for most purposes. In rarer gases, naturally, the later terms are relatively more important, so that some purposes it is worthwhile to determine $f^{(2)}$ |22|.

Consider a homogeneous, isotropic and electrically neutral plasma. There is no constant field which can lead to anisotropic electromagnetic properties and also there is no orderly motion of charged particles which can generate current and bring about non-homogeneouse distribution of particles. Under this conditions, the distribution function does not depend upon the coordinates but depends only upon the absolute value of the impulse of the external wave. For this case, the distribution function can be expanded in power series and our treatment will be limited upto the second order approximation. Thus, we can write the distribution function as:

$$f_{\alpha}(\vec{p},\vec{r},t) = f_{o\alpha}(\vec{p})+f_{1\alpha}(\vec{p},\vec{r},t)+f_{2\alpha}(\vec{p},\vec{r},t) \qquad (4.1)$$

where $f_{o\alpha}(\vec{p})$ is the distribution of α-sort of particles in the unpertubed state of plasma and $f_{1\alpha}(\vec{p},\vec{r},t)$ and $f_{2\alpha}(\vec{p},\vec{r},t)$ are small additional terms to the distribution function.

The Vlasov equation for rarefied plasma is given by

$$\frac{\partial f}{\partial t} + \vec{v}\ \frac{\partial f}{\partial \vec{r}} + e\vec{E}\cdot\frac{\partial f}{\partial \vec{p}} = 0 \qquad (4.2)$$

And for the additional terms f_1 & f_2

$$\frac{\partial f_1}{\partial t} + \vec{V} \cdot \frac{\partial f_1}{\partial \vec{r}} + e\vec{E} \cdot \frac{\partial f_o}{\partial \vec{p}} = 0 \qquad (4,3)$$

$$\frac{\partial f_2}{\partial t} + \vec{V} \cdot \frac{\partial f_2}{\partial \vec{r}} + e\vec{E} \cdot \frac{\partial f_1}{\partial \vec{p}} = 0 \qquad (4,4)$$

When equations (4.3) and (4,4) are integrated in a moment of time t_o, we get

$$f_1(\vec{p},\vec{r},t) = f_1(\vec{p},\vec{r}-\vec{v}(t-t_o), t_o) - e\frac{\partial f_o}{\partial \vec{p}} \int_{t_o}^{t} dt' \quad x$$

$$\vec{E}|\vec{r}-\vec{v}(t-t'), t'| \qquad (4.5)$$

and

$$f_2(\vec{p},\vec{r},t) = f_2(\vec{p},\vec{r}-\vec{v}(t-t_o), t_o) - \int_{t_o}^{t} dt' \quad x$$

$$\vec{E}(\vec{r}-\vec{v}(t-t'),t'| \cdot \frac{\partial f_1}{\partial \vec{p}}|\vec{p},\vec{r}-\vec{v}(t-t'),t'| \qquad (4,6)$$

Now let us consider the electric field of the wave whose amplitude varies slowly with time as

$$\vec{E}(\vec{r}\ t) = \vec{E}_o(1-e^{-\alpha t}) \cos (\omega t-\vec{k}.\vec{r}) \qquad (4.7)$$

where

$\vec{E}_o$ is the amplitude of the wave

$\vec{\omega}$ is the frequency of the wave

$\vec{k}$ is the wave vector and

α is the value characterizing the rate of amplitude growth of teh wave in the plasma.

At a time, $t = 0$ the electric field given by equation (4.7) is equal to zero. And

$$f_1(\vec{p},\vec{r},t) = f_2(\vec{p},\vec{r},t) = 0$$

and when $t \gg 1/\alpha$

$$\vec{E}(\vec{r},t) = \vec{E}_o \cos (\omega t-\vec{k}.\vec{r}).$$

If equation (4.7) is substituted into equations (4.5) and (4.6) we obtain:

$$f_1(\vec{p},\vec{r},t) = -e\frac{\partial f_o}{\partial \vec{p}}\int_o^t dt'\vec{E}_o(1-e^{-\alpha t})\cos|\omega t-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')| \quad (4.8)$$

and

$$f_2(\vec{p},\vec{r},t) = -e\int_o^t dt'\vec{E}_o(1-e^{-\alpha t})\cos|\omega t'-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')|.\frac{\partial f_1}{\partial \vec{p}} \quad (4.9)$$

If the cosine term in equation (4.8) is written in complex form that is, in the form of

$$\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

we get

$$f_1(\vec{p},\vec{r},t) = -\frac{e}{2m} E_{oj} \frac{\partial f_o}{\partial v_j} \int_o^t dt'(1-e^{-\alpha t'}) \times$$

$$|e^{i|\omega t'-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')|} + e^{-i|\omega t'-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')|}|$$

After integration we obtain

$$f_1(\vec{p},\vec{r},t) = -\frac{e}{2m} E_{oj}\frac{\partial f_o}{\partial v_j}\left|\frac{e^{i|\omega t'-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')|}}{i(\omega-\vec{k}.\vec{v})}\right|_o^t$$

$$+\frac{e^{-i|\omega t'-\vec{k}.\vec{r}+\vec{k}.\vec{v}(t-t')|}}{-i(\omega-\vec{k}.\vec{v})}\Big|_o^t - \frac{e^{-\alpha t'+i\omega t'-i\vec{k}.\vec{r}+i\vec{k}.\vec{v}(t-t')}}{-\alpha+i\omega-i\vec{k}.\vec{v}}\Big|_o^t$$

$$+\frac{e^{-\alpha t'-i\omega t'-i\vec{k}.\vec{v}(t-t')+i\vec{k}.\vec{r}}}{-\alpha-i\omega+i\vec{k}.\vec{v}}\Big|_o^t\Big|$$

$$= -\frac{e}{2m}E_{oj}\ \frac{\partial f_o}{\partial v_j}\ \Big|\frac{e^{i\omega t-i\vec{k}.\vec{r}}-e^{-i\vec{k}.\vec{r}+i\vec{k}.\vec{v}t}}{i\,\Omega}$$

$$-\ \frac{e^{-i\omega t+i\vec{k}.\vec{r}}+e^{i\vec{k}.\vec{r}-i\vec{k}.\vec{v}t}}{i\,\Omega} + \frac{e^{-i\vec{k}.\vec{r}+i\vec{k}.\vec{v}t}}{-\alpha+i\omega-i\vec{k}.\vec{v}}$$

$$-\frac{e^{-\alpha t+i\omega t-i\vec{k}.\vec{r}}}{-\alpha+i\Omega}+\frac{e^{-i\vec{k}.\vec{r}-i\vec{k}.\vec{v}t}}{-\alpha+i\Omega}+\frac{e^{-\alpha t-i\omega t+i\vec{k}.\vec{r}}}{\alpha+i\Omega}$$

where $\Omega = \omega - \vec{k}.\vec{v}$

After some rearrangement we arrive at the following expression for $f_1(\vec{p}.\vec{r}.t)$.

$$f_1(\vec{v},\vec{r},t) = -\frac{e}{2m}E_{oj}\frac{\partial f_o}{\partial v_j}\{\,|e^{-\alpha t}\ (\frac{\alpha+i\Omega}{\alpha^2+\Omega^2})\ \frac{-i}{\Omega}|\ x$$

$$e^{i\omega t-i\vec{k}.\vec{r}}\ +\ e^{-\alpha t}\ \frac{(\alpha-i\Omega)}{\alpha^2+\Omega^2}+\frac{i}{\Omega}\quad e^{-i\omega t+i\vec{k}.\vec{r}}$$

$$+ \frac{i\alpha}{\Omega(\alpha-i\Omega)} e^{i\vec{k}.\vec{v}t-i\vec{k}.\vec{r}} - \frac{i\alpha}{\Omega(\alpha+i\Omega)} e^{-i\vec{k}.\vec{v}t+i\vec{k}.\vec{r}} \} \qquad (4.10)$$

The integration of equation (4.9) in a similar manner and using equation (4.10) yields

$$f_2(\vec{v},\vec{r},t) = \frac{1}{i} \left(\frac{e}{2m}\right)^2 E_{oi} E_{oj} \frac{\partial}{\partial v_i} \left(D \frac{\partial f_o}{\partial v_j}\right) \qquad (4.11)$$

which is the additional term to the distribution function, and D is given by

$$D = \frac{1}{\Omega-i\alpha} \left| - \frac{e^{-\alpha t}+1}{\alpha} + \frac{e^{-2\alpha t}-1}{2\alpha} - \frac{e^{+i\Omega t}-1}{i\Omega} - \frac{e^{-\alpha t+i\Omega t}-1}{\alpha-i\Omega} \right|$$

$$- \frac{1}{\Omega+i\alpha} \left| - \frac{e^{-\alpha t}+1}{\alpha} + \frac{e^{-2\alpha t}-1}{2\alpha} + \frac{e^{-i\Omega t}+1}{i\Omega} - \frac{e^{-\alpha t-i\Omega t}-1}{\alpha+i\Omega} \right|$$

$$+ \frac{1}{\Omega} \left| \frac{1}{i\Omega}(e^{-i\Omega t}+ e^{i\Omega t}-2) - \frac{1}{\alpha+i\Omega}(e^{-\alpha t-i\Omega t}-1) + \frac{1}{\alpha-i\Omega}(e^{-\alpha t+i\Omega t}-1) \right.$$

The expression for D can be simplified to yield

$$D = \frac{i}{\Omega^2+\alpha^2} \{ (1-e^{-\alpha t})^2 + 2(-1 + \cos \Omega t - \frac{\alpha}{\Omega} \sin \Omega t)$$

$$- \frac{2(\Omega^2+\alpha^2)}{\Omega^2} (\cos \Omega t-1) + \frac{2\alpha}{\Omega} e^{-\alpha t}\sin \Omega t \} \qquad (4.12)$$

D has the sense of coefficient of diffusion in velocity space |7|.

The expression for D in equation 4.12 can be simplified in two limited cases:

a) when $\alpha < |\Omega|$ or $|\omega/k - v_{\parallel}| > \alpha/k$

$$D \simeq \frac{i}{\Omega^2} \left| (1-e^{-\alpha t})^2 \right| \simeq \frac{i}{\Omega^2} \qquad (4.13)$$

For this case the resonant particles can be neglected.

b) if $\Omega^2 < \alpha^2$ or $|\omega/k - v_{\parallel}| < \alpha/k$

$$D \simeq \frac{i}{\alpha^2} \left| (1-e^{-\alpha t})^2 - 2\frac{\alpha^2}{\Omega^2}(\cos \Omega t - 1) \right| \qquad (4.14)$$

In the interval $|\omega/k - v_{\parallel}| < \omega/k$, the resonant particles must be taken into account.

4.2 Evaluation of the current density, energy density and energy flux density.

In this section, the average velocity, current density, the energy density and the energy flux density are calculated by the second order approximation without taking into account the resonant particles.

$$< \vec{v}(\vec{r}.t) > = \frac{1}{N}\int_{-\infty}^{\infty} \vec{v}\, f_2\, d\vec{v} \qquad (4.15)$$

$$= \frac{1}{iN}\left(\frac{e}{2m}\right)^2 E_{oi}\, E_{oj} \int_{-\infty}^{\infty} \vec{v}\, \frac{\partial}{\partial v_i} \left(D\, \frac{\partial f_o}{\partial v_j}\right) d\vec{v}$$

After substituting for D from equation (4.13)

$$< \vec{v}(\vec{r},t) > = \frac{1}{N}(\frac{e}{2m})^2 E_{oi}E_{oj} \int_{-\infty}^{\infty} \vec{v} \frac{\partial}{\partial v_i} (\frac{1}{\Omega^2} \frac{\partial f_o}{\partial v_j}) d\vec{v}$$

$$= - \frac{1}{N}(\frac{e}{2m})^2 E_{oi}E_{oj} \int_{-\infty}^{\infty} (\frac{1}{\Omega^2} \frac{\partial f_o}{\partial v_j}) d\vec{v}$$

using the definition for the change of longitudnal dielectric permitivity due to the electrons that is,

$$\delta \varepsilon_e = \frac{4\pi e^2}{mk^2} \int_{-\infty}^{\infty} (\vec{k} \frac{1}{\Omega} \frac{\partial f_o}{\partial v}) d\vec{v} \quad |25|$$

the expression for the average velocity can be written in the following form:

$$< \vec{v}(\vec{r},t) > = \frac{\vec{E}_o(\vec{k}.\vec{E}_o)}{16 \pi mn} \frac{\partial}{\partial \omega} \delta \varepsilon_e \tag{4.16}$$

From equation (4.16), it can be seen that the average velocity is different from zero for longitudnal waves and for this case is can be written as:

$$< \vec{v}_{\parallel}(\vec{r},t)> = \frac{k_{\parallel} |\vec{E}_o|^2}{16\pi mn} \frac{\partial}{\partial \omega} \delta \varepsilon_e \tag{4.17}$$

The average current density is given by:

$$< \vec{j}(\vec{r},t) > = eN < v_{\parallel}(\vec{r},t) >$$

and after using eq. (4.17) we obtain,

$$< \vec{j}(\vec{r},t) > = \frac{ek_{\parallel} |\vec{E}_o|^2}{16\pi m} \frac{\partial}{\partial \omega} \delta \varepsilon_e \tag{4.18}$$

This expression is similar with the results obtained in the framework of quasi-linear theory |8|. Also it is necessary to note that equation (4.17) was obtained in the limit as $t \to \infty$ but, for a finite time interval it is necessary to take into account the factor $(1-e^{-\alpha t})^2$ in the expression for coefficient of diffusion which was used for evaluating the average velocity.

The energy density $w(\vec{r},t)$ is evaluated as follows:

$$< w(\vec{r},t) > = \frac{m}{2n} \int_{-\infty}^{\infty} v^2 f_2 \, d\vec{v} \tag{4.19}$$

$$= \frac{m}{2n} \frac{1}{i} \left(\frac{e}{2m}\right)^2 E_{oi} E_{oj} \int_{\infty}^{\infty} v^2 \frac{\partial}{\partial v_i} \left(D \frac{\partial f_o}{\partial v_j}\right) d\vec{v}$$

$$= \frac{1}{N} \left(\frac{e}{2m}\right)^2 E_{oi} E_{oj} \int_{-\infty}^{\infty} \left(\frac{m}{2}\right) \vec{v} \cdot \left(v_i \frac{\partial}{\partial v_i}\right) \left(\frac{1}{\Omega^2} \frac{\partial f_o}{\partial v_j}\right) d\vec{v}$$

integrating this equation by parts and using equation (4.16) we obtain,

$$< w(\vec{r},t) > = \frac{(\vec{E}_o \cdot \vec{k})}{16\pi n k^2} \omega \frac{\partial \delta \varepsilon_e}{\partial \omega} - \frac{(\vec{E}_o \cdot \vec{k})^2}{16\pi n k^2} \delta \varepsilon_e \tag{4.20}$$

Taking into account the longitudnal polarization of the wave equation (4.20) can be written as,

$$< w_{\shortparallel}(\vec{r},t) > = \frac{|\vec{E}_o|^2}{16\pi n} \omega \frac{\partial}{\partial \omega} \delta \varepsilon_e - \frac{|\vec{E}_o|^2}{16\pi n} \delta \varepsilon_e \tag{4.21}$$

This expression is similar with the value obtained for the energy density obtained by Ginzburg |16|.

For the energy flux density $g(\vec{r},t)$ we have,

$$< \vec{g}(\vec{r},t) > = \frac{1}{N}\frac{m}{2}\int_{-\infty}^{\infty} v^2 \, \vec{v} \, f_2 \, d\vec{v} \qquad (4.22)$$

$$= \frac{i}{N}\frac{m}{2i}\left(\frac{e}{2m}\right)^2 E_{oi}E_{oj}\int_{-\infty}^{\infty} v^2\vec{v}\,\frac{\partial}{\partial v_i}\left(\frac{1}{\Omega^2}\frac{\partial f_o}{\partial v_j}\right)d\vec{v}$$

integrating this equation by parts and using eq. (4.16) one gets the following expression for the average energy density

$$< \vec{g}(\vec{r},t) > = \left| \frac{\vec{E}_o(\vec{k}.\vec{E})\omega^2}{32\pi k^2 N} + \frac{\vec{K}(\vec{K}.\vec{E}_o)^2\omega^2}{16\pi k^4 N} \right| \frac{\partial}{\partial\omega}\delta\varepsilon_e$$

$$- \left| \frac{\vec{E}_o(\vec{K}.\vec{E}_o)\omega}{16\pi k^2 N} + \frac{\vec{K}(\vec{K}.\vec{E}_o)\omega}{8\pi k^4 N} \right| \delta\varepsilon_e \qquad (4.24)$$

For longitudnal waves the expression for the energy flux density can be written as,

$$< \vec{g}_{\parallel}(\vec{r},t) > = \frac{3|\vec{E}_o|^2 K_{\parallel}\omega^2}{32\pi k^2 N}\frac{\partial}{\partial\omega}\delta\varepsilon_e - \frac{3K_{\parallel}|\vec{E}_o|^2}{16\pi k^2 N}\delta\varepsilon_e \qquad (4.25)$$

4.3 Evaluation of the Time During which Resonance Particles can be Neglected

The average velocity of the resonant particles can be calculated using the value of the coefficient of diffusion given by equation (4.14).

$$< V_R(\vec{r},t) > = \frac{1}{N}\int_{-\infty}^{\infty} \vec{V}_R \, f_2 \, d\vec{v} \qquad (4.26)$$

$$< \vec{V}_R(\vec{r},t) > = \frac{2}{k}\left(\frac{e}{2m}\right)^2 E_{oi}E_{o\ell}k_j \frac{1}{v_{T_e}^3}\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \frac{1}{k^2} \times$$

$$\int_{-\alpha/k}^{\alpha/k} kv_{\shortparallel} \frac{\cos \theta t-1}{\theta^2} e^{-mv_{\shortparallel}^2/2T} d\theta$$

and using

$$v_{\shortparallel} = v_p^2 , \frac{-m}{T} = - \frac{1}{v_{T_e}^2} , kv_{\shortparallel} = \omega$$

$$\cos \theta t-1 = \cos 2a\, \theta-1 = - 2 \sin^2 a\theta$$

$$< \vec{V}_R(\vec{r},t) > = - \frac{8}{k}\left(\frac{e}{2m}\right)^2 E_{oi}E_{o\ell} k_j \frac{1}{v_{T_e}^3}\left(\frac{1}{2\pi}\right)^{\frac{1}{2}} k_j \frac{\omega}{k^2} \quad X$$

$$\int_{o}^{\alpha/k} \frac{\sin^2 a\theta}{\theta^2} e^{-v_p^2/2v_{T_e}^2} d\theta \qquad (4.27)$$

Denoting the integral $\int_{o}^{\alpha/k} \frac{\sin^2 a\theta}{\theta^2} d\theta$ by I then, we can integrate I by parametric methods as follows:

$$I = \lim_{\substack{b \to a \\ \beta \to o}} \int_{o}^{\alpha/k} e^{-\beta\theta} \frac{\sin a\theta}{\theta} \frac{\sin b\theta}{\theta} d\theta$$

by differentiating with respect to a the above expressin we get,

$$\frac{dI}{da} = \lim_{\substack{b \to a \\ \beta \to o}} \int_{o}^{\alpha/k} e^{-\beta\theta} \left(\frac{\cos a\theta}{\theta} \frac{\sin b\theta}{\theta}\right) \theta d\theta$$

Using the following trigonometric indentities

$$\frac{\sin(a+b)\,\theta - \sin(a-b)\theta}{\theta} = \frac{2 \cos \theta \sin b\,\theta}{\theta}$$

Where $\vec{V}_R$ is the velocity of the resonant particles.

$$< \vec{V}_R(\vec{r},t) > = - \frac{2}{N}\left(\frac{e}{2m}\right)^2 E_{oi}E_{oj} \quad x$$

$$\int_{\infty}^{\infty} \vec{V}_R \frac{\partial}{\partial v_i} \left(\frac{1}{\Omega^2} (\cos \Omega t-1) \frac{\partial f_o}{\partial v_j}\right) d\vec{v}$$

$$= \frac{2}{N} \left(\frac{e}{2m}\right)^2 E_{oi}E_{oj} \int_{\infty}^{\infty} \frac{1}{\Omega^2}(\cos \Omega t-1) \frac{\partial f_o}{\partial v_j} d\vec{v}$$

Substituting the maxwellian distribution for one dimension

$$f_o = N(m/2\pi T) \quad e^{-mv^2/2T}$$

and

$$\frac{\partial f_o}{\partial v_j} = - mv_j/T \quad f_o$$

equation (4.26) can be written as

$$< \vec{V}_R(\vec{r},t)> = -2\left(\frac{e}{2m}\right)^2 E_{oi}E_{o\ell}k_j \left(\frac{m}{T}\right) \left(\frac{m}{2\pi T}\right)^{\frac{1}{2}} \frac{1}{k^2} \quad x$$

$$\int_{-\alpha/k}^{\alpha/k} kv_{\parallel} \frac{\cos \Omega t-1}{(\omega-kv_{\parallel})^2} \quad e^{-mv_{\parallel}^2/2T} dv_{\parallel}$$

by making the following change of variables

$$\omega - kv_{\parallel} = \theta \qquad dv_{\parallel} = - \frac{d\theta}{k}$$

$$v_p = \omega/k \qquad v_{T_e} = \sqrt{T/m} \qquad a = t/2$$

and by considering the region of applicability of equation (4.14) we obtain, Then,

$$\frac{dI}{da} = \lim_{\substack{b \to a \\ \beta \to o}} \frac{1}{2} \left| \int_o^{\alpha/k} e^{-\beta\theta} \frac{\sin(a+b)\theta}{\theta} d\theta - \int_o^{\alpha/k} \frac{\sin(a-b)\theta}{\theta} d\theta e^{-\beta\theta} \right|$$

By letting $\alpha \to \infty$ and using

$$\int_o^{\infty} e^{-qx} \frac{\sin px}{x} dx = \frac{\pi}{2} - \tan^{-1} p/q$$

it can be evaluated to yield:

$$\frac{dI}{da} = \lim_{\substack{b \to a \\ \beta \to o}} \frac{1}{2} \left| \frac{\pi}{2} - \tan^{-1} \frac{(a+b)}{\beta} - \frac{\pi}{2} + \tan^{-1} \frac{(a-b)}{\beta} \right|$$

$$= \lim_{\substack{b \to a \\ \beta \to o}} \frac{1}{2} \left| - \tan^{-1} \frac{(a+b)}{\beta} + \tan^{-1} \frac{(a-b)}{\beta} \right|$$

Using $\int_o^{\infty} \frac{\tan^{-1} x}{a} dx = x \tan^{-1} \frac{x}{a} - \frac{a}{2} \log (a^2 + x^2)$

The above equation can be integrated to yield

$$I = \lim_{\substack{b \to a \\ \beta \to o}} - \left| \frac{a+b}{2} \tan^{-1} \frac{(a+b)}{\beta} - \frac{\beta}{4} \log (\beta^2 + (a+b)^2) \right.$$

$$\left. - \frac{(a-b)}{2} \tan^{-1} \frac{(a-b)}{\beta} + \frac{\beta}{4} \log (\beta^2 + (a-b)^2) \right|$$

$$I = \lim_{\substack{b \to a \\ \beta \to o}} - \left| \frac{a+b}{2} \tan^{-1} \frac{a+b}{\beta} - \frac{(a-b)}{2} \tan^{-1} \frac{a-b}{\beta} \right.$$

$$\left. + \frac{\beta}{4} \log \left(\frac{\beta^2 + (a-b)^2}{\beta^2 + (a+b)^2}\right) + C \right|$$

and after the evaluation of the limit we finally get,

$$I = - \frac{\pi}{2} a \text{ , and since } a = t/2 \text{ , } I = - \frac{\pi t}{4}$$

Substituting this value of I into equation (4.27) we get for the average velocity of the resonant particles the following expression,

$$<\vec{V}_R(\vec{r},t)> = \frac{2}{k^2} \left(\frac{e}{2m}\right)^2 \frac{1}{v_{T_e}^3} \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} |\vec{E}_o|^2 \, e^{-V_p^2/2v_{T_e}^2} \cdot \omega \pi t$$

$$= \frac{1}{2} \sqrt{\frac{\pi}{2}} \, \frac{e^2 \omega t}{m^2 k^2 v_{T_e}^3} \, |\vec{E}_o|^2 \, e^{-V_p^2/2v_{T_e}^2} \qquad (4.8)$$

From equation (4.28) we see that the average velocity of the resonant particles increases with time, whereas the average velocity of the non-resonant particles as given by equation (4.16) is independent of time.

Now, let us estimate the time during which it is possible to neglect the contribution of the resonant particles in evaluating the current density produced by using waves with a slowly varying amplitude. From the condition,

$$< \vec{V}_{N.R}(\vec{r},t) > \gg < \vec{V}_R(\vec{r},t) >$$

where $\vec{V}_{N.R}$ is the velocity of the non resonant particles we get the limitation on time. By using equation (4.16) for the average velocity of the non-resonant particles and equation (4.28) for the resonant particles and after simplification we get,

$$\omega t \ll \frac{k^3 v_{T_e}^3}{\omega_L^2} e^{v_p^2/2v_{T_e}^2} \sqrt{\frac{2}{\pi}} \frac{\partial}{\partial\omega}\delta\varepsilon_e \qquad (4.29)$$

where ω_L is the Langmuir frequency for longitudnal waves,

$$\delta\varepsilon_e = - \omega_L^2/\omega^2 \quad \text{and} \quad \frac{\partial}{\partial\omega}\delta\varepsilon_e = 2\omega_L^2/\omega^2$$

Then equation (4.29) can be written as

$$\omega t \ll 2 \sqrt{\frac{2}{\pi}} (v_{T_e}/v_p)^3 e^{v_p^2/2v_{T_e}^2} \qquad (4.30)$$

From equation (4.30) we note that on the right hand side there is an exponential large term and for a considerable time the current produced is determined by the non-resonance particles. Therefore, during the time given by the inequality of expression (4.30) the effect of the resonant particles can be neglected.

CONCLUSION

In this thesis the mechanism of generation of entraining currents with a slowly varying amplitude high frequency electromagnetic wave was considered. It was shown that non-stationary amplitude leads to the generation of entraining currents. And these currents are produced only if the wave is longitudinal. Using the Vlasov equation the quasi-stationary additional term to the distribution function appearing in the second order approximation was determined. With the help of this term, the current density, energy density and energy flux density in the field of the longitudnal wave were evaluated. The formula for the additional term was investigated as when the resonant and non resonant particles can be neglected. That is when,

$$\alpha < | \omega - KV_{11} |$$

the resonant particles can be neglected and when $\alpha > | \omega - KV_{11}|$ the non-resonant particles can be neglected. It was shown that during the time determined by the inequality

$$\omega t << \sqrt{\frac{2}{\pi}} \quad K^3 \frac{V_{Te}^3}{\omega L_e^2} \quad \frac{\partial}{\partial \omega} \quad \delta \varepsilon_e$$

the group of resonant particles do not play an important role in the production of entraining currents when a slowly varying amplitude high frequency electromagnetic wave is applied to homogeneouse plasma.

The results are presented in terms of the longitudinal dielectric permitivity and its derivatives with respect to frequency. The above method of generating entraining currents may be used in a rarefied plasma when collision between particles are seldom. This method could be used in tokamaks to produce continuous toroidal currents, but it needs further investigation.

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DECLARATION

I hereby declare that the thesis entitled, "Entraining Currents in Homogeneous Plasma", being submitted by me in partial fulfillment for Master of Science Degree in Physics, is my original work, done under the supervision and guidance of Dr. V. Schepilov. Sources of relevant findings and equations taken from books and articles are duly acknowledged in the body of the thesis and the reference.

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The Thesis was submitted to
the Physics Department
Graduate Committee on June,1985

This Thesis has been submitted for examination with my approval as University Advisor.

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