



**MODELING TRAFFIC FLOW PROBLEM:  
COMPARISON OF MULTILANE  
ROUNDBOUT VERSUS TRAFFIC LIGHT  
CONTROL**

By  
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# Abstract

The vehicular traffic is controlled by a self-organized scheme in which traffic lights are absent at traffic junctions. This controlling method incorporates a yield-at-entry strategy for the vehicles approaching to the circulating traffic flow in the roundabout. Vehicular dynamics are simulated within the framework of the probabilistic cellular automata and the throughputs experienced at each individual street are evaluated for specified time intervals to determine the performance of the roundabout. We used Multi-stream Minimum Acceptable Space (MMAS) Cellular Automata (CA) model for the description of vehicular traffic at a roundabout. In this thesis inconsistency of driver behavior and interactions in cross traffic at entrances of roundabouts are simulated by incorporation of four different categories of driver behavior (i.e., conservative, moderate, urgent and radical). Our results give the critical throughput in which the intersection should be controlled in a self-organized manner is approximately 4500vph. This proves that below certain congestion, the roundabout efficiency is higher than fixed-time signalized junction point. In general, average throughputs for two-lane roundabout are lower than the signalized intersection, even for a fixed-time signalization.

# Chapter 1

## Introduction

Undoubtedly, traffic management is nowadays considered as one of the basic ingredients of modern societies and large sums are invested by governments in order to increase its efficiency. The rapidly growing volume of vehicular traffic flow, limitations on expanding the construction of new infrastructure, hazardous environmental impact due to the emission of pollutants, together with unfavorable delays suffered in congested traffic jams, are among the basic features which necessitate the search for new control as well as optimization schemes, for vehicular traffic flow. Inevitably, this task would not be significantly fulfilled unless a comprehensive survey of vehicular dynamics, within a mathematical framework, is developed. This has motivated researchers to carry out extensive numerical, as well as analytical research, in the discipline of traffic flow theory [2].

A safe and efficient transportation system is a key component of state of quality of life. It has direct attitude on mobility and the accessibility of people and goods to employment, shopping, and other opportunities, economic development, and the patterns of growth in the nation [1]. However the application of vehicle traffic controlling method at some intersections enhance the safety and efficiency of these intersections

and the transportation purposes they serve.

Broadly speaking, the traffic flow theory can be categorized into two parts: high way traffic and city traffic, and now there is vast literature in both domains. In this thesis we focus our attention on two particular aspects of city traffic, the so-called roundabout and traffic light controlling.

The vehicular traffic is controlled by traffic light and a self-organized scheme in which traffic lights are absent; this controlling method incorporates a yield-at-entry strategy for the approaching vehicles to the circulating traffic flow in the roundabout.

The traffic light, also known as traffic signal or stop light, is a signaling device positioned at a road intersection which is used to control vehicles and passengers so that traffic can flow smoothly and both drivers and passengers are safe.

Roundabouts permit a continual flow of traffic to flow through the intersection, whereas a signalized intersection requires traffic to stop completely in one direction [31].

Nowadays, it is common to see traffic congestion at junctions in Addis Ababa at peak hours in the morning and afternoon. Traffic polices have to intervene in the situation to regulate the traffic flow by over-riding the traffic control devices at roundabouts. Otherwise, it would be impossible to have normal traffic flow, especially at junction points. This problem will continue and it may worsen in the future due to the rapid growth of population and vehicle numbers in Addis Ababa.

The study of traffic problems proposes to answer several questions: where to install traffic controlling mechanisms or traffic controls; for instance: duration of traffic lights; where to construct entrances, exits, and overpasses. The principal aim is to discover traffic phenomena in order to eventually take decisions which may alleviate

congestion, maximize flow of traffic, and eliminate accidents [27].

Although traffic signal lights are relatively commonplace, they are critical for ensuring the safety of the driving public. A traffic signal light is used to control vehicles and passengers so that traffic can flow smoothly and both drivers and passengers are safe. Traffic signal lights have been around for years and are used to efficiently control traffic through intersection [28].

The traffic at intersection points in most of urban areas has been controlled by signalized devices. Modern roundabouts have quite recently come into play as alternatives to signalized intersections, a roundabout is a form of intersection design and control, which accommodates traffic flow in one direction around a central island operating with yield control at the entry points, and giving priority to vehicles within the roundabout (circulation flow). Several characteristics such as safety, deflection and turning movements and construction costs distinguish a modern roundabout from the more general form of a traffic circle.

The era of modern roundabouts began in the United Kingdom in the 1950's with the construction of the first "yield-at-entry" roundabouts. In 1966, a nationwide yield-at-entry rule launched the modern roundabout revolution in UK. Yield-at-entry is the most important operational element of a modern roundabout, but it is not the only one. Deflections of the vehicle path and entry flare are also important characteristics that distinguish the modern roundabout from the non-conforming traffic circle, which does not have these characteristics. The primary characteristics of the modern roundabout reduce many of the safety hazards of traditional intersections and nonconforming traffic circles [9].

Gap-acceptance models have received much attention in modeling traffic flow at the entrances of roundabouts and intersections [12]. Treating the entrances of roundabouts similarly to those of intersections obviously does not reveal the operational characteristics of roundabouts.

**Cellular automata (CA)** models provide an efficient alternative way to model traffic flow for highway and urban networks [11]. CA models have been widely used to simulate traffic flow with a single direction of movement, such as those found on highways, one-way streets. Some research has been conducted on bi-directional traffic flow [14, 10]. However, limited research has been conducted on cross traffic in particular, unsignalized cross traffic flow, which is one of the most important features in urban traffic networks.

A more realistic CA model was first proposed by Ruskin and Wang [8], which simulates the cross traffic at unsignalized intersection reflects the give-way rule appropriately, and describes the details of vehicle movements and interactions from different entry roads. The model was based on the Minimum Acceptable sPace (MAP) method proposed by Wang and Ruskin. MAP can be designed to describe both heterogeneous and inconsistent driver behavior, as well as random interaction between individual vehicles in cross traffic flow, independent of headway distribution considerations. As such, the MAP method can be applied to most features of traffic flow and has been used for the single-lane roundabout and intersection [8, 13].

In this thesis we consider a Multi-stream Minimum Acceptable Space (MMAS) CA model based on the MAP method, to simulate the cross traffic at unsignalized two-lane roundabouts. Performance measurements for roundabouts include **throughput** (the maximum number of vehicles that can navigate a roundabout) and **capacity** (the

maximum number of vehicles that can pass through a roundabout from a given road) as well as queue lengths, waiting time, passage time and so on. All the performance measures mentioned can be readily obtained by Multi-stream Minimum Acceptable Space (MMAS) CA models, as above.

Modeling multi-stream traffic flow is a challenging task. Particularly, the heterogeneous nature of human behavior, the random interactions between drivers, the highly nonlinear dynamics and the large dimensions of the system under investigation combine to create considerable complexity.

The physical configuration of a modern roundabout with a deflected entry and yield-at-entry, forces a driver to reduce speed during his approach at entry and movement within the roundabout. This is contrary to an intersection where many drivers are encouraged by a green or yellow light to accelerate to get across the intersection quickly in order to "beat the red light". It has always been a subject of argument whether to control an intersection under a signalized or non-signalized scheme via roundabouts.



Figure 1.1: Aerial shoot of Gerji Imperial Roundabout(Source: Google Earth)

In Ethiopia, there are various road sections under constructions and almost all the intersection points are constructed using roundabouts only, but their effects in terms of safety and operational efficiency are unclear because there is no statistical data that compare traffic light verses roundabout without traffic light.

Better traffic control is the only way to solve problems caused by an increasing number of vehicles in our cities. It is still more and more complicated and expensive to magnify the existing traffic infrastructure. Roundabouts in Ethiopia are increasingly adopted by the transportation community as a new traffic control concept in terms of enhanced vehicular safety, traffic operation, emissions, capacity and aesthetics

The basic question which arises is under what circumstances should one control an intersection by signalized traffic lights? And under what circumstance the self organized control scheme becomes inefficient? To address these fundamental questions, we try to explore and analyze some basic characteristics of a typical roundabout, such as throughputs, which suffice to give an integrated picture of roundabout performance for various traffic situations in order to find a quantitative understanding, and at the same time we demonstrate special variation of traffic density along the road. In what follows we try to illustrate these fascinating aspects through computer simulations.

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The existing literature is vast and characterized by various contributions taking in to account modeling aspects, qualitative analysis of the existing models and simulations

related to applications

Previous studies by Tewodros G. [30] have been carried out to evaluate the capacity of roundabouts in Addis Ababa. The capacity analysis is done based on empirical gap acceptance method, the relation between a roundabout performance measure and capacity is often expressed in terms of degree of saturation (Demand volume-capacity ratio) [30]. The result indicates number of entry lane, number of circulatory lane, high traffic flow and pedestrian volume are the major causes of their over saturation.

## 1.1 Statement of the problem

There have been major advances in developing algorithms for tracking individual traffic controlling mechanisms but less emphasis has been given on characterizing the comparisons between traffic light and roundabout without traffic light i.e., the study lacks to address the subject of argument whether to control an intersection under a signalized or non-signalized scheme via roundabouts. But this problem has not been researched yet and this thesis tries to compare the efficiency of the two junction controlling mechanisms.

The basic question which arises is under what circumstances should one control an intersection by signalized traffic lights? And under what circumstance the self organized control scheme becomes inefficient? To address these fundamental questions, we try to explore and analyze some basic characteristics of a typical roundabout, such as throughputs, which suffice to give an integrated picture of roundabout performance for various traffic situations in order to find a quantitative understanding, and at the same time we demonstrate special variation of traffic density along the road. In what follows we try to illustrate these fascinating aspects through computer simulations.

## 1.2 Limitation of the study

The availability of the operational data, which is evidently impacted by the intersection, will directly affect the quality of the model. The necessary peak hour traffic data was collected on two locations of roundabout and traffic light. Traffic composition for the study area was collected at the Imperial roundabout and at Olympia road for Traffic light which is located in Addis Ababa, Ethiopia. These locations were chosen based on the principle that these places can be considered as possible representative of the target population of roundabout and traffic light in terms of flow, geometry and size.

In the data gathering process, there were three limitations. Firstly the collected data does not include vehicles from the third lane in both cases because they were not directly administered through traffic light and the give way rule of the roundabout as well. Secondly, there was continuous interference of traffic police on both places. And finally financial shortage limited the data gathering process from being supported by professional camera man and advanced video recorder.

This thesis is organized as follows. In chapter 2, the limitations of gap-acceptance models have been discussed. In Chapter 3 the description of the problem, lane-allocation patterns and the MMAS method are introduced. In Chapter 4, the modeling of vehicles traffic flow using conservation laws is described by a first-order nonlinear PDE. In Chapter 5 results are given and some findings are described. In Chapter 6 comparison of traffic light versus roundabout is discussed. Finally conclusion and Recommendation are given.

# Chapter 2

## Literature Review

Fundamentally, two distinct theories or methodologies can be used to assess the capacity of the roundabouts and traffic lights. These theories are the analytical or gap acceptance based methods and the empirical methods

A common deficiency of all previous models that study cross traffic flow is the assumption that drivers are consistent and homogeneous [12]. In reality, both inconsistent and heterogeneous behavior is endemic. This realization provides a principal motivation for much of the work described here.

Empirical capacity models have some advantages, one of which is clearly that there is no need to describe or to understand the driver's behavior, as the data is from the real world, which has already taken many factors that influence capacity into account (such as the driver's behavior). There are also some drawbacks, e.g. the significant amount of data that has to be collected to ensure reliability of results. Entry data has to be collected at saturation (or at capacity) level, and this method is sometimes inflexible under unfamiliar circumstances, for example, when the value is far out of the range of regressed data.

Based on Kimber's study in 1980 the first empirical approach is a linear approximation used to determine the entry capacity of roundabout [13].

Kimber's capacity formula is

$$Q_e = F - f_c Q_c \quad (2.1)$$

where  $Q_e$  =Entering Capacity (vph);

$Q_c$  =Circulating Flow (vph); and

$F, f_c$  =Parameters defined by roundabout geometry.

Kimber used a number of parameters to describe the geometry, which are the entry width, the inscribed diameter, the effective length, the approach road half width, the entry radius and the angle of entry.

Kimber also found that the angle of entry and the radius as their effect was small, he decided to modify the above equation by including a correction factor

$$Q_e = k(F - f_c Q_c) \quad (2.2)$$

where  $k = 1.151 - 0.00347\phi - 0.978/r$ ;

$r$  =The entry radius (m); and

$\phi$  =The angle of entry (degree).

For Kimber's values of  $k$ , it can be generally expected within 0.9 to 1.1.

The gap-acceptance model was developed originally for "priority rule" intersection (i.e. without traffic lights) and was based on Tanner's capacity model in 1962[13].

The basic assumption of this model is that the driver will enter the intersection when there is a safe opportunity or "gap" in the traffic. **Gaps** are measured in time and

are equal to **headway** (=distance/ speed). The Tanner's model was developed to analyze the delays at an intersection of two streams, in which the major stream had priority as shown in Fig.1.1. If the headway( $\Delta$ ) between the two major stream vehicles is greater than the **critical gap** ( $T$ ), the minor stream vehicle then enters the intersection. If the available gap is large enough, then more than one minor stream vehicle can enter the intersection. The vehicles can follow at intervals of  $T_0$  seconds (follow-up time). The values of  $T=3-5.2$  seconds,  $T_0=2-3$  seconds, and  $\Delta=1$  or 2 seconds were recommended [13].

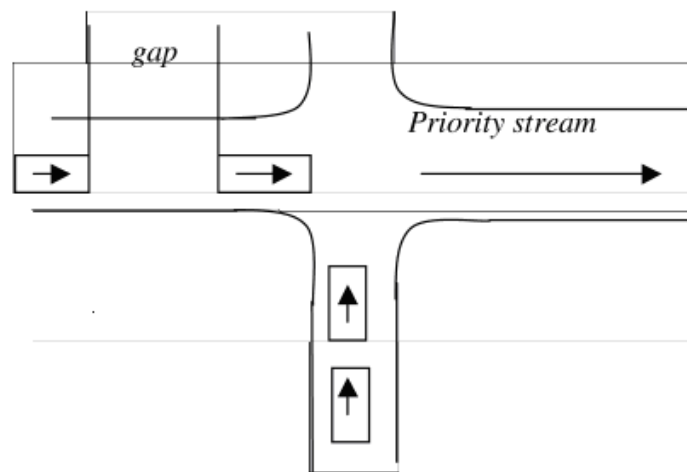


Figure 2.1: Illustration of two-stream intersection

An advantage of this method is that the gap acceptance technique offers a logical basis for the evaluation of capacity. Secondly, it is easy to appreciate in the model the meaning of the parameter used and to make adjustments for unusual conditions. Moreover, gap acceptance theory conceptually relates traffic interactions at roundabouts with the availability of gap in the traffic streams [13]. Gap-acceptance models

have long been used in modeling cross traffic at the entrance of roundabouts and intersections and has primarily been used for single lane roundabouts or intersections. Gap-acceptance models assume that a driver enters the intersection when a safe opportunity or gap occurs in the traffic. Gaps are measured in time and correspond to headway (defined as distance divided by speed). Critical gap and follow-up time are the two key parameters (where critical gap is defined as the minimum time interval required for one minor-stream vehicle to enter the intersection).

Gap-acceptance models are, however, unrealistic in assuming consistent driver behavior [17, 21]. A consistent driver would be expected to behave in the same way in all similar situations, while in a homogenous population, all drivers have the same critical gap and are expected to behave uniformly [33]. More realistically, the driver's type may differ and the critical gap for a particular driver should be represented by a stochastic distribution [7], and this should be reflected in the simulation.

In gap-acceptance models, estimation of the critical gap has attracted much attention with use of a mean critical gap also proposed. Maximum likelihood estimation of the mean critical gap has gained wide acceptance [22], but the basic assumption remains the same, namely that all drivers are consistent. Investigation of the factors affecting critical gap and follow-up time concluded that drivers use a shorter critical gap at higher flow and delay conditions [35]. However, a critical value obtained for any given situation is unlikely to be generally applicable.

Further, at roundabouts in an urban network, adjacent intersections with traffic lights will have grouped the vehicles into a queue (or queues) during the red signal phases and platoons will thus be present (e.g., a filtering effect). The filtering of traffic flow by traffic signals or human intervention has a significant impact on capacity

and performance [18]. In particular, the gap-acceptance model can be applied only when no platoon is present [32]. Otherwise, no minor-stream vehicle can enter the intersection or roundabout, as the mean headway within a platoon is supposed to be less than the critical gap. If traffic signal cycles are known and coordinated, the platoon pattern may be predictable. If it is not predictable, traditional gap-acceptance cannot readily be applied and moreover, does not specifically allow for modeling directional flow [35].

Priority sharing is another phenomenon that gap-acceptance models fail to take into account. This occurs at the entrance to a roundabout where circulating vehicles may deliberately give way to entering vehicles [24]. This appears to lead both to reduction in the critical gap and in the average follow-on time for entering vehicles, which causes inaccuracy in the capacity model based on the gap-acceptance criteria. Troutbeck and Kako [25] have tried to overcome this by adding an additional factor "C" in the capacity formula to justify the priority sharing effects. This  $C$  value ranges from 0 to 1 and depends on the headway distribution. Even if this modification can improve the accuracy of previous gap-acceptance models, it obviously provides little help in analyzing the cross traffic operation unless priority sharing can be directly related to the headway distribution.

Although road networks are crucial to economic sustainability, over-burdening is common particularly in urban areas. Modern traffic management depends heavily on the efficiency of mechanisms, such as the controlled intersection and multi-lane roundabout. Vehicle throughput at any such configuration is intuitively influenced by rules governing maneuverability and by the driver's observance, as well as by traffic density. Various computational approaches have been explored to obtain insight on flows and

causes of congestion, to predict capacity and to assess transportation strategies [6]. Many continuum traffic flow models can be described by a system of hyperbolic PDEs. The first of these models was by Lighthill and Whitham [25] which relies on the assumption that there exists an equilibrium speed-density relationship  $v = v(\rho)$  based on the mass conservation, i.e., traffic conservation, and is described by a first-order nonlinear PDE. The model proposes that their dynamics is described by the following PDE:

$$\rho_t + f(\rho)_x = 0 \tag{2.3}$$

The Minimum Acceptable sPace (MAP) method uses analogous, but more flexible methodology to that of gap-acceptance (e.g., spatial and temporal details of vehicle interactions can be described using MAP). This not only facilitates understanding of the interaction between drivers, but can also be applied to situations for which headway distributions are insufficient to describe traffic flow (e.g., traffic flow filtered by traffic signals). Researchers used a multi-CA ring to extend previous work on the single-lane roundabout to the two-lane case [13].

Note that the notion of gap-acceptance has been used in modeling two-lane traffic, with overtaking allowed if the gap on the other lane is large enough [3]. However, these models again are for single directional multi-lane traffic flow, which is different from traditional gap-acceptance models and that developed in this thesis.

# Chapter 3

## Traffic flow model for a two-lane roundabout

### 3.1 Major geometric features of a modern roundabout

A roundabout is a type of circular intersection on the road. It is useful to define, identify and clearly understand the geometric features or elements of a roundabout. But not all circular intersections are classified as roundabouts for operational analysis and design purposes according to the capacity study of roundabouts in the UK, geometrical elements of roundabouts played an important part in the efficiency of roundabout operational performance. Good geometrical design will improve not only capacity but also safety, which is a major concern for road design [4]. Basic elements for design consideration of roundabout are:

**Central Island:** is the raised area in the center of a roundabout around which traffic circulates.

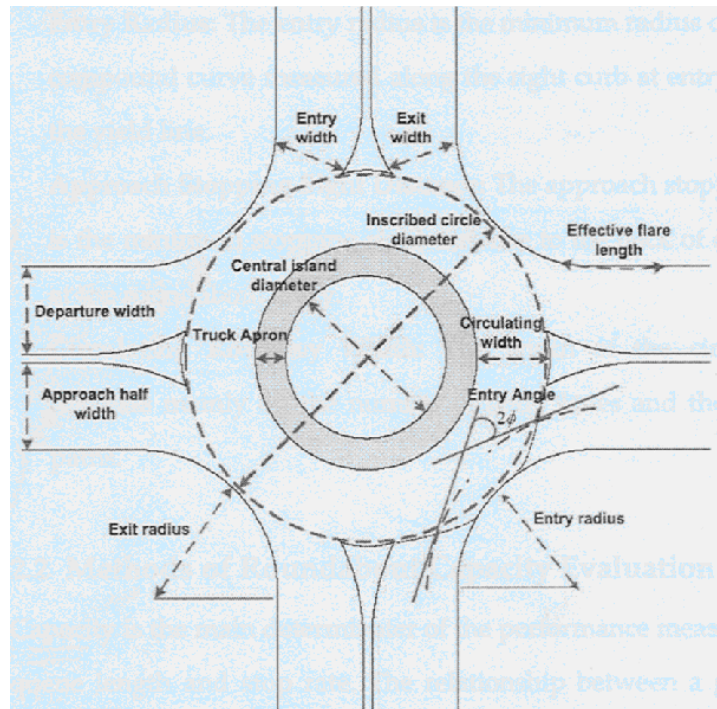


Figure 3.1: Major Geometric features of modern roundabout

**Truck Apron:** is a traversable portion of the raised center island to accommodate the wheel path of oversized vehicles.

**Splitting Island:** is placed within the leg of a roundabout to separate entering and exiting traffic and provide vehicle deflection prior to entering the roundabout.

**Approach Width:** refers to the width of the entering lanes before flaring or any other influence from the roundabout.

**Exit Width:** is the perpendicular distance from the right curb line of the exit to the intersection of the left edge line and the inscribed circle.

**Departure width:** refers to the width of the lanes departing from the roundabout at a point where the width is no longer influenced by the roundabout.

**Effective flare length:** a flare may be used to increase the entry width and capacity of a roundabout by providing additional lanes at the entry. The effective flare length is equal to the distance from the entry width to a point where the approach width is equal to half the sum of the entry width and the approach traveled way width prior to influence from the roundabout.

**Entry Radius:** is the minimum radius of curvature for the compound curve measured along the right curb at entry beginning before the yield line.

**Circulating roadway width:** depends mainly on the number of entry lanes and the radius of vehicle paths.

## 3.2 Description of the problem

Let us first discuss the basic driving principles applied to roundabouts. In its most general form, a roundabout connects four incoming, as well as four outgoing flow directions. In principle, each incoming vehicle approaching the roundabout can exit from each of the four out-going directions via making appropriate turning maneuvers around the central island of the roundabout. Fig.3.2 illustrates the situation.

Under the offside-priority rule, the vehicles waiting at a roundabout entrance needs to give way the vehicles on the roundabout. Drivers need to determine how much space on the roundabout is sufficient for them to drive to the required position and to gain enough speed so that their car does not obstruct an oncoming vehicles. Determination of the opportunity to drive onto the roundabout is a complicated decision-making process.

In this thesis we only investigate a simplified version in which all the streets, including the circulating lane around the central island, are assumed to be double-lane. Let us

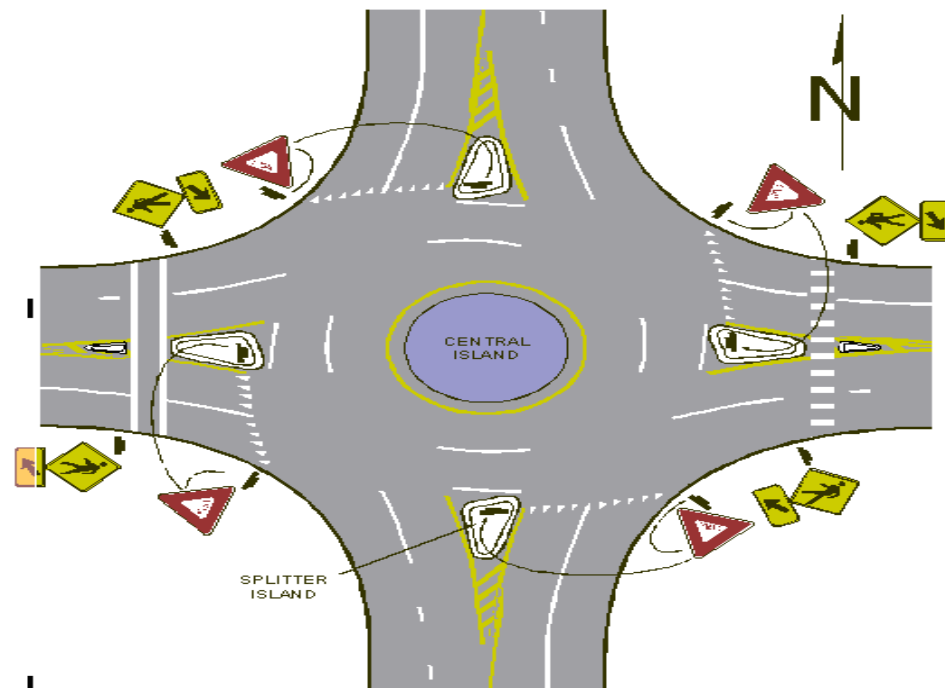


Figure 3.2: Roundabout Intersections with Two-Lane Approaches

explain the entrance regulations in some details.

Each approaching vehicles from the left lane to the entry points of the roundabout should decelerate and simultaneously look at the left-ward quadrant of the roundabout. If there is any vehicle in inner lane quadrant of the roundabout, then the approaching vehicle should come to a complete stop until it has enough space to enter the roundabout without interrupting any on coming vehicle; otherwise, it has to slow down and stop.

This is possible due to stochastic fluctuation in the space gap of the flowing direction. Once such an appropriate space gap has been found, the stopped car is allowed to enter the roundabout. This procedure is continuously applied to all approaching vehicles. Now returning to those vehicles which are moving around the interior island of the roundabout. Once a vehicle is permitted to enter the roundabout, it continues

moving until it reaches to its aimed exit direction. Depending on the predetermined out-going direction, each interior vehicle moves a portion of the way around the central island. These turning movements are classified as: right-turn, straight ahead, left-turn and U-turn as shown in the Fig. 3.3 and 3.4. For those who tend to make a U-turn, the whole circumference should be traveled.

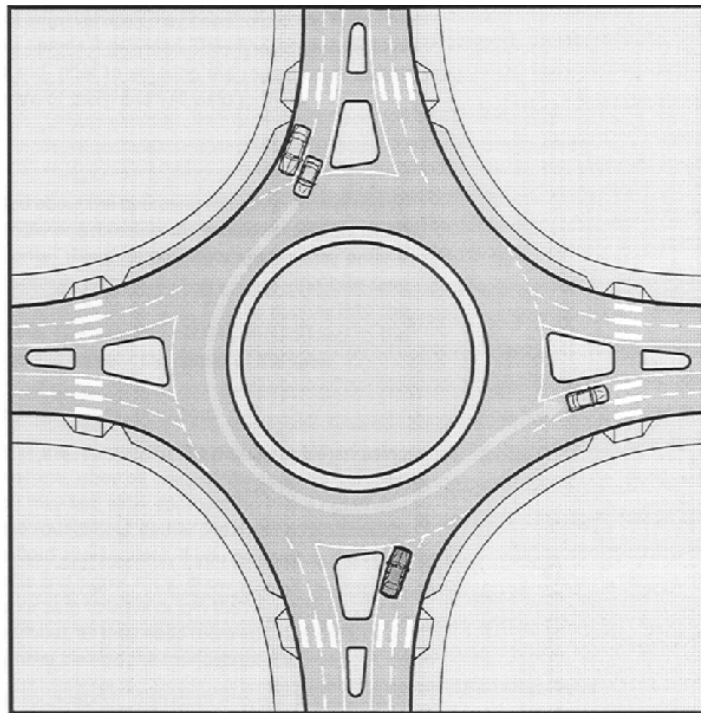


Figure 3.3: Turning left at a roundabout

One of the benefits of roundabout installation is the improvement in overall safety performance, that is roundabout may improve the safety of intersections by eliminating or altering conflict types, by reducing speed differentials and intersections and by forcing drivers to decrease speeds as they proceed into and through the intersection. The frequency of crashes at an intersection is related to the number of conflict points at an intersection, as well as the magnitude of conflicting flows at each conflicting

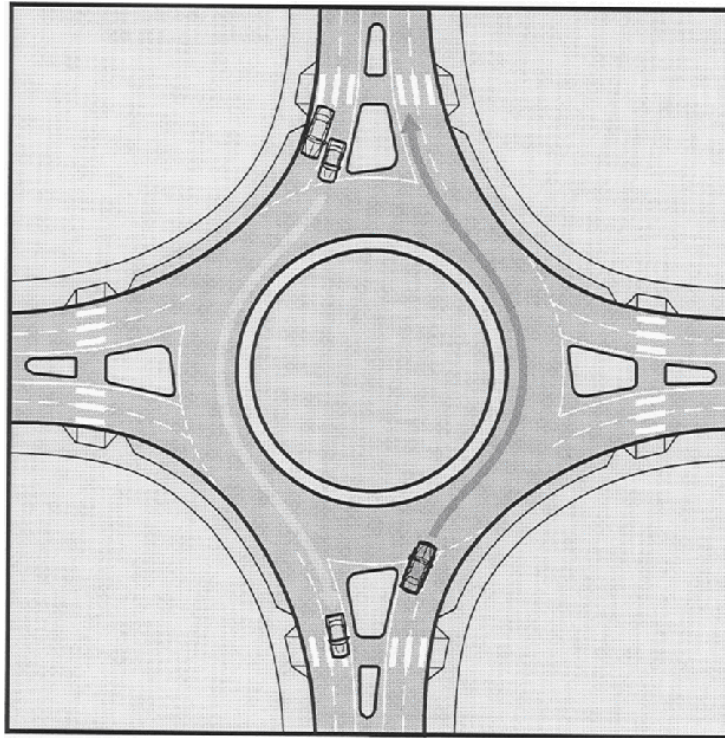


Figure 3.4: Driving straight through a roundabout

points. A conflict point is a location where the paths of two vehicles merge, or cross each other. The types of conflicts present in two-lane roundabouts occur when drivers use the incorrect lane or make an improper turn.

The most severe crashes at signalized intersections occur when there is a violation of the traffic control device designed to separate conflicts by time e.g. a right angle collision due to running a red light, and vehicle-pedestrian collisions.

### 3.3 Cellular automata (CA)

A cellular automaton (CA) is a discrete model studied in microstructure modeling. It consists of a regular grid of cells, each in one of a finite number of states, such as

”On” and ”Off”. The grid can be in any finite number of dimensions. For each cell, a set of cells called its neighborhood (usually including the cell itself) is defined relative to the specified cell. An initial state (time  $t = 0$ ) is selected by assigning a state for each cell and a new generation is created (advancing  $t$  by 1), according to some fixed rule (generally, a mathematical function) that determines the new state of each cell in terms of the current state of the cell and the states of the cells in its neighborhood. In this chapter, we develop a multi-state CA rings in order to characterize vehicle destinations. The state in each cell has three physical meanings. If  $C = 0$ , it means that there is no vehicle in this cell. If larger than null  $C > 0$ , then there is a vehicle in this cell and the actual value indicates how many cells the car needs to traverse to arrive at the destination exit.

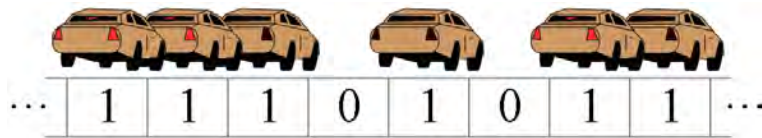


Figure 3.5: Each cell corresponding to a vehicle and each vehicle moves forward only if it has open space in front of it.

The number of cells in the ring is determined by the real dimension of the roundabout. If real dimensions of roundabout are known, then the number of cells in each ring will be known, although the requirement for the program is obviously to be flexible enough to allow size to be varied.

Vehicles on single and double-lane roundabouts are assumed to observe the same give-way rule (the priority rule), as follows: entering vehicles must give way to the vehicles already on a roundabout. In addition, for double-lane roundabouts, vehicles

on the outer lanes must give way to vehicles leaving from the inner lanes.

Priority-sharing or give-way rules can also be replaced by fixed waiting periods determined by traffic light cycles or human interventions. This model is for single directional multi lane traffic flow, which uses multi-CA ring to multi-lane cases.

Vehicles on entry roads move onto the corresponding lanes of the roundabout, while navigation through a roundabout is subject to the following processes:

1. Vehicle arrival at start of entrance road (e.g., 100 cells away from the roundabout).
2. Predetermined destination (pre allocation to a lane of the entry road).
3. Lane allocation (on entry road).
4. Vehicle movement along entrance roads.
5. Position delay: vehicles on the right lane of an entrance road may be halted for position delay time (PDT), if view is impeded by the adjacent vehicle in order to adjust position/check opportunity to enter roundabout.
6. Entry (interaction between drivers at the entrance and vehicles on the roundabout).
7. Navigation on the roundabout.
8. Exit.

It is more realistic to assume that the destination is predetermined and remains unchanged for all vehicles throughout the roundabout maneuver. Therefore, all the

vehicles in our model will randomly be assigned a destination according to turning rates when entering the system. Their destinations will remain unchanged during their maneuvers. Assignment to lanes are random on vehicle arrival to the entry road, and assuming a constant speed of one cell moved in one time step for a given driver's destination.

Factors that influence the driver's decision include the driver skills, the weather, the car performance, motivation of travel etc, and may vary for each individual driver. However, the important common factor is the space available on the roundabout (gaps required to enter the roundabout).

In this model, we use the space available on a roundabout as the only parameter to describe the driver's behavior. Optimum condition for a vehicle to move onto the roundabout is the space available on the roundabout without interrupting any oncoming vehicle.

## 3.4 Algorithms

### 3.4.1 Update Rule for the Roads

The update algorithm can be expressed in terms of the sequential cells. If there is a vacant cell in front of the cell occupied by a vehicle, the vehicle will move forward one cell in the current time step. If the cell in front is occupied too, the vehicle will stop there in that time step.

The state ( $C_n^{t+1}$ ) of the  $n^{th}$  cell in the next time step is decided by the states of its own cell  $C_n^t$ , the state of the cell in front and the cell behind in this time step, where

$t$  and  $n$  express time and position sequence respectively. The states can only be 0 or 1, where 0 means the cell is vacant and 1 that a car occupies the cell.

The update rule can be expressed therefore as follows:

$$\text{If } C_n^t = 1 \text{ and } C_{n+1}^t = 0, \text{ then } C_n^{t+1} = 0 \text{ and } C_{n+1}^{t+1} = 1$$

$$\text{If } C_n^t = 0 \text{ and } C_{n+1}^t = 0, \text{ then } C_{n+1}^{t+1} = 0$$

$$\text{If } C_n^t = 1 \text{ and } C_{n+1}^t = 1, \text{ then } C_n^{t+1} = 1$$

### 3.4.2 Predetermined Exit on the roundabout

Drivers clearly have their own destination in mind, so they make decisions which exit is appropriate before entering. Characterizing a given exit for each vehicle before entry is more realistic. This approach is used to characterize each car by randomly giving them a different number. These numbers are equal to the number of cells that the car needs to pass to arrive at the destination exit. For instance, for a roundabout which has four entrance road the CA model has a total of  $(a+b+c+d)$  cells, where  $a$ ,  $b$ ,  $c$  and  $d$  are the cells between entrance road (arms) 1 and 2, 2 and 3, 3 and 4, 4 and 1 respectively. The vehicle that comes from arm 1 are signed  $a$ ,  $(a+b)$ ,  $(a+b+c)$  and  $(a+b+c+d)$  randomly as shown in Table 1. The four numbers represent the number of cells to pass prior exiting at respective points. If the distribution of vehicles from arm 1 exits at the four different exits then they are known as  $m_1\%$ ,  $n_1\%$ ,  $o_1\%$  and  $p_1\%$  respectively, we then randomly assign  $m_1\%$  of the vehicles with  $a$ ,  $n_1\%$  with  $(a+b)$ ,  $o_1\%$  with  $(a+b+c)$  and  $p_1\%$  with  $(a+b+c+d)$  as shown in Table 1.

$$\text{Also } m_i\% + n_i\% + o_i\% + p_i\% = 1$$

$$i = 1, 2, 3, 4.$$

Table 1: The numbers assigned for the vehicles

	Arm1	Arm2	Arm3	Arm4
	$\cdot m_1 \% a$	$\cdot m_2 \% b$	$\cdot m_3 \% c$	$\cdot m_4 \% d$
	$\cdot n_1 \% (a + b)$	$\cdot n_2 \% (b + c)$	$\cdot n_3 \% (c + d)$	$\cdot n_4 \% (d + a)$
	$\cdot o_1 \% (a + b + c)$	$\cdot o_2 \% (b + c + d)$	$\cdot o_3 \% (c + d + a)$	$\cdot o_4 \% (d + a + b)$
	$\cdot p_1 \% (a + b + c + d)$	$\cdot p_2 \% (a + b + c + d)$	$\cdot p_3 \% (a + b + c + d)$	$\cdot p_4 \% (a + b + c + d)$

### 3.4.3 Up-date rule on the Roundabout

The update rule for the roundabout is as follows. If the state in cell  $n$  at time step  $t$  is larger than one ( $C_n^t > 1$ ), then it shows that there is a vehicle in cell  $n$ , the state ( $C_{n+1}^t$ ) in cell  $(n+1)$  in front must be checked to see if it is vacant. If it is vacant ( $C_{n+1}^t = 0$ ), then the number decreases by one when it moves forward into the cell  $(n+1)$  in front ( $C_{n+1}^{t+1} = C_n^t - 1$ ) and cell  $n$  will become zero ( $C_n^{t+1} = 0$ ) in time step  $(t+1)$  as shown below. If the state in cell  $(n+1)$  is not zero ( $C_{n+1}^t \geq 1$ ) the number in cell  $n$  ( $C_n^{t+1}$ ) will be kept unchanged ( $C_n^{t+1} = C_n^t$ ) as shown below. As the car moves, its number will finally become equal to one ( $C_n^t = 1$ ) indicating that the car will leave the roundabout in the next time step if the exit is free, and there will be no car in this cell in the next time step ( $C_n^{t+1} = 0$ ). If the exit is not free, the car will remain there.

Using the same notation as above, the update rule can be simply expressed as follows:

If  $C_n^t > 1$  and  $C_{n+1}^t = 0$ , then  $C_n^{t+1} = 0$  and  $C_{n+1}^{t+1} = C_n^{t+1} - 1$ ;

If  $C_n^t > 1$  and  $C_{n+1}^t \geq 1$ , then  $C_n^{t+1} = C_n^t$ ;

If  $C_{n-1}^t = 0$ ,  $C_n^t = 0$ , then  $C_n^{t+1} = 0$ ;

If  $C_n^t = 1$ , then  $C_n^{t+1} = 0$ , if it is able to exit; and

If it can not,  $C_n^{t+1} = C_n^t$

The update rule on the roundabout is shown in Fig 3.6.

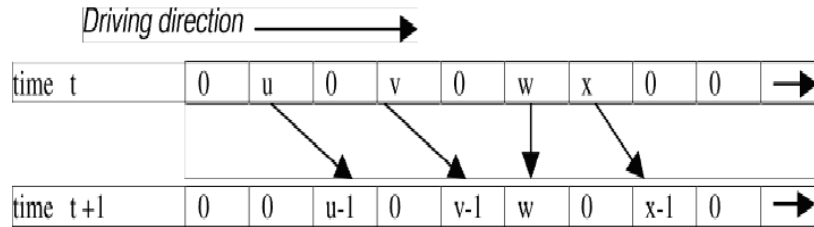


Figure 3.6: The update rule on the roundabout

There are several advantages of using this notation instead of just 0 and 1. Firstly the number does not only mean that the cell is occupied or vacant, but also indicates where its occupant will go. The update rule is the same for any cell on the roundabout as well. When a car drives out of the roundabout, the number automatically becomes zero.

Secondly if we want to visualize the car on the roundabout in the future, its turning indicators may also be visualized, we can simply define it as : if number  $C_n^t \geq (a + b + c + d)/2$ , its left turning indicator is on; if  $C_n^t \leq (a + b + c + d)/4$  the right indicator is on.

Thirdly, this method makes multiple entrance/exit programming possible and it can be applied to simulation of traffic flow where the origins and destinations are known, by assigning the vehicle with a number corresponding to the steps needed to arrive at its destination exit.

### 3.4.4 Lane allocation

In the lane allocation process, a vehicle chooses which lane it will use to approach a roundabout. The lane allocation process at a two-lane roundabout is similar to that

of the major roads of a two-lane Two-Way-Stop-Controlled (TWSC) intersection [27]. However, criteria for lane allocation differ slightly. We detect two possible patterns in the real world:

- Left-turning (LT) vehicles use left lane only. Straight-through (ST) and RT Vehicles use right lane only, i.e., only LT vehicles use the inner lane of a roundabout (Pattern A).

- LT vehicles use left lane only. RT vehicles use right lane only, and ST vehicles using both lanes (Pattern B).

In the first scenario, the vehicles traveling on the outer lane of the roundabout are RT vehicles only. The advantage of this system is that entry vehicles need to check the space on the inner lane of the roundabout only, since there is no oncoming vehicle from the outer lane. The maneuver is a merging process between the vehicles in the circulating flow on the inner lane of the roundabout, and vehicles on the left lanes of entry roads and no cross interaction occur. Unfortunately, in reality ST vehicles will take the left lane if the right lane of the entrance road is saturated.

The second scenario is used to give greater flexibility to the ST vehicles. Selection by the driver of a lane might be based on e.g., perception of waiting time or similarly. As for ST vehicles they can use the outer lane, some passing interaction occurs when vehicles on the inner lane of the roundabout are exit.

### 3.4.5 Minimum Acceptable sPace (MAP)

We briefly outline the MAP method [8, 13] before describing Multi-stream Minimum Acceptable Space (MMAS) models. In the MAP method, the driver's behavior is categorized into four groups: radical, urgent, moderate and conservative. We use a single-lane roundabout to illustrate the MAP method in Fig 3.7. If a driver accepts a 3-cell space (between circulating vehicles) as the MAP enters the roundabout, behavior is designated as moderate

Radical behavior requires one cell space. The driver takes any space on the intersection without consideration of safety. Consequently, this vehicle may delay the oncoming vehicle on the roundabout for two time steps and may generate gridlock [13]. A 2-cell space corresponds to urgent behavior, which may be the result of such things like misjudgment, over confidence in vehicle acceleration, bad driving habits, urgency on traveling or the phenomenon of priority sharing. The effect is to delay an oncoming vehicle on the roundabout. Conservative behavior corresponds to  $MAP \geq 4$  cells.

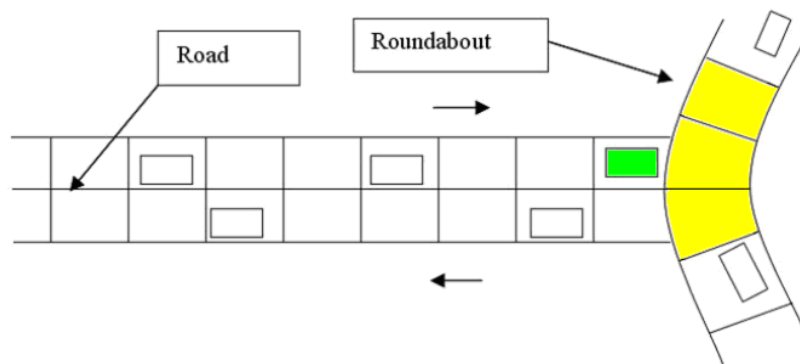


Figure 3.7: A road and its entrance to a roundabout.

MAP and MMAS are combinations of both spatial and temporal conditions. For example, MAP is determined by the number of cells needed to represent the driver's behavior of the various types, when entering a single-lane roundabout. In the model the vehicle can move forward one cell in each time step on a roundabout. Therefore, the number of cells of MAP corresponds to the headway between the two circulating vehicles on a roundabout. On the other hand, the space for the vehicle to enter a roundabout must have clearance of specific cells for entry, otherwise the entering vehicle must wait (see Fig. 3.7 for MAP and Fig. 3.8 for MAAS) to join different roundabout lanes. Therefore, MAP and MMAS describe both temporal and spatial behavior of drivers in cross traffic.

### 3.4.6 Interaction at entrances of double-lane roundabouts

In [13], a multi-state CA ring is developed in order to characterize vehicle destinations. In other words, the states of a vehicle on a roundabout depends on the distance (the number of cells) to its destination. In this thesis, multi state two- CA rings are used to model two-lane roundabouts (two cellular automata rings with the same center but different diameters). All rings have the same number of cells, and vehicles can move ahead one cell in each time step when navigating the roundabout. In other words, we assume that vehicles in all lanes traverse the same number of radians in the same period of time. This is permitted by the assumption of an adjustment on the speed of the vehicles on a roundabout with different radius. The shorter the radius, the lower the speed at which that vehicle moves. The state of a vehicle thus depends on the distance (the number of cells) to its destination and which lane it is in.

In this thesis, we use vehicles in the left lane of entrances to two-lane roundabout to

show the interactions which occur. In order to simplify the representation, the shape of the arc of the roundabout with entry road can be stretched to resemble Fig. 3.8, which looks like a T-intersection. The paths of vehicles in the entry road are shown in Fig. 3.8(d), while the paths of vehicles exiting from the roundabout are shown in Fig.3.8 (c).

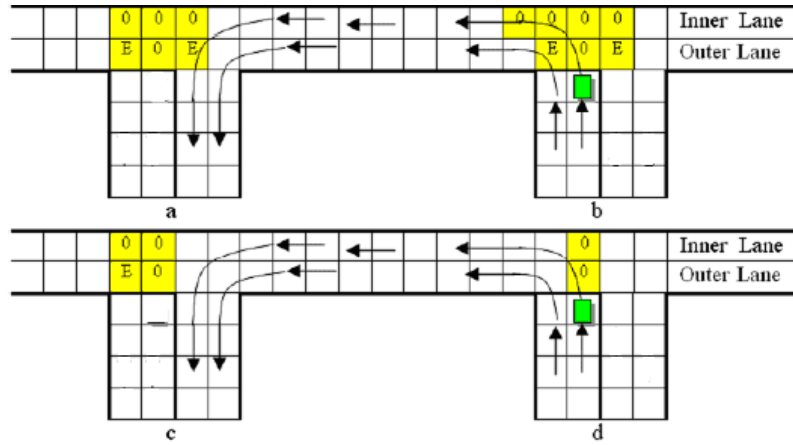


Figure 3.8: Vehicle on the left lane of the two-lane entrance road with behavior of (a) moderate, (b) conservative, (c) urgent and (d) radical

When the vehicle in the left lane of the entrance road needs to change lane from the outer lane to the inner lane on the roundabout, it crosses the two cells diagonally. Likewise, this is true for the vehicles coming out from the inner lane to the outer lane. In other words, when a vehicle changes lane on the roundabout, it also moves one cell ahead at the same time.

Similar to the MAP method, we use figures to explain our MMAS model and the conditions that are required by vehicles from entrance roads. The Driver's behavior is categorized into four groups: conservative, moderate, urgent and radical, with associated probabilities [15].

For a two-lane roundabout, the required conditions for the target vehicle (shaded) on the left lane of the entry road in this time step are indicated by the spaces required (marked cells) in Fig. 3.8(a) and 3.8(b), based on two different types of driver's behavior.

The requirement for each cell is indicated by "0" or "E", where "0" means the cell must be vacant and "E" means the cell is either vacant or occupied by a non-circulating vehicle. A non-circulating vehicle is either just entering the roundabout from an entrance road or going to leave the roundabout in the next time step. All the spaces required are indicated cell by cell (with the same notation "0" or "E"). Theoretically, the right lanes of entrance roads and the outer lanes of roundabouts are designed for right-turning vehicles only. Thus, the right-turning vehicles do not need to check conditions before entering. However, in practice, checking is clearly necessary for safety reasons and is built into the models.

Further suggestions are that driver's heterogeneous behavior is partially determined by their types and individual characteristics, such as sex, age and driving experience, among others [29], not by their locations in different lanes. Some investigations indicate that age is an important factor in determining not only the driver's reaction time, but also behavior [29, 34], our view is that all possibilities strongly reflect the individual driver.

### 3.4.7 Interaction on roundabouts

Immediately after entering a roundabout, vehicles from the left lane of the entrance roads move from the outer lane into the inner lane. Thus they are assumed to move along the inner lane until they arrive at the destination (exit road). In other words,

they do not change lanes except for entering and exiting. This assumption is supported by the fact that unnecessary lane-changing on roundabouts is not common [15]. Nevertheless, exit of the vehicle on the inner lane may be blocked by vehicles driving on the outer lane. The Give-Way Rate (GWR) is a probability assigned to this random result of the driver's interaction. The probability ranges from 0 (no drivers give way) to 1 (all drivers give way).

### 3.5 Data Requirements

The analysis described above requires the specification of traffic volume for each approach to a roundabout, including the throughput for each directional movement. In general intersection volume counts are made at the intersection stop bar, with an observer noting the number of vehicles that pass the point over a specified time period. For the roundabout, the data of interest for each approach are the throughput, delay and Queue length each measure provides a unique perspective on the quality or performance of service at which a roundabout will perform under a given set of traffic and geometric condition.

The relationship between the standard origin-to-destination turning movements at an intersection and circulating and entry flows at roundabout is important.

**Throughput (flow)** is simply the sum of the throughput, left and right turn movement on an approach.

**Circulating flow** is the sum of the vehicles from different movements passing in front of the adjacent upstream splitter island. These flows can simply be measured in the field. Right turns are included in approach volumes and required capacity, but are not included in the circulating volumes downstream because they exit before the

next entrance.

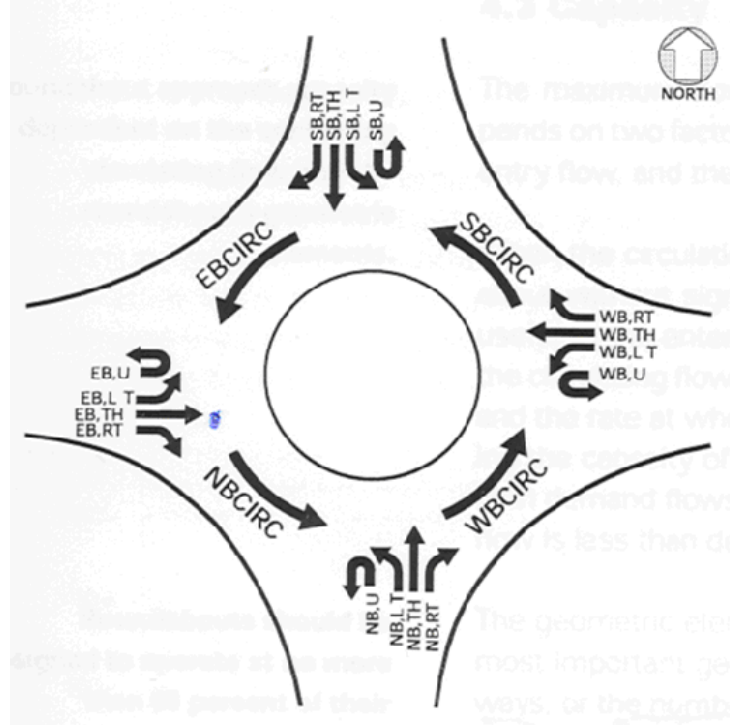


Figure 3.9: Traffic flow parameters

The following equations can be applied to determine conflicting (circulating) flow rates. As shown graphically in Fig 3.9.

$$V_{EB,circ} = V_{WB,LT} + V_{SB,LT} + V_{SB,TH} + V_{NB,U-turn} + V_{WB,U-turn} + V_{SB,U-turn} \quad (3.1)$$

$$V_{WB,circ} = V_{EB,LT} + V_{NB,LT} + V_{NB,TH} + V_{SB,U-turn} + V_{EB,U-turn} + V_{NB,U-turn} \quad (3.2)$$

$$V_{NB,circ} = V_{EB,LT} + V_{EB,LT} + V_{SB,TH} + V_{WB,U-turn} + V_{SB,U-turn} + V_{EB,U-turn} \quad (3.3)$$

$$V_{SB,circ} = V_{WB,LT} + V_{WB,TH} + V_{NB,LT} + V_{EB,U-turn} + V_{NB,U-turn} + V_{WB,U-turn} \quad (3.4)$$

For existing roundabouts, when approaching, right-turn, circulating, and exit flows are counted, directional turning movements can be computed as shown in the following

equation. Eqn (3.5) shows the throughput movement flow rate for the east bound approach as a function of the entry flow rate for that approach, the exit flow rate for the opposing approach, the right turn flow rate for the subject approach, the right turn flow rate for the approach on the right and the circulating flow rate for the approach on the right. Other throughput movement flow rates can be estimated using a similar relationship.

$$V_{EB,TH} = V_{EB,entry} + V_{WB,exit} - V_{EB,RT} - V_{NB,RT} - V_{NB,circ} \quad (3.5)$$

The left turn flow rate for an approach is a function of the entry flow rate, the through flow rate and the right turn flow rate for that same approach, as shown in Eqn (3.6). Again, other movement flows are estimated using similar equations.

$$V_{EB,LT} = V_{EB,entry} - V_{EB,TH} - V_{EB,RT} \quad (3.6)$$

While the above method is mathematically correct, it is somewhat sensitive to errors and inconsistency in the input data so we count all the locations of the roundabout made simultaneously. At the same time the sum of the entering and exiting volumes were checked and adjustments were made to ensure that the same amount of traffic enters and leaves the roundabout.

The geometrical elements of the roundabout affect the rate of entry flow. The most important element is the width of the entry and circulating road ways, or the number of lanes at the entry and on the roundabout. Roundabout approach capacity which is the maximum flow rate that can be accommodated at roundabout entry is dependent on the conflicting circulating flow and the roundabout geometric elements. The

capacity forecast is based on the simplified British regression relationship as shown in Appendix A.

**Delay** is a standard parameter used to measure the performance of intersections. It is the primary measure of effectiveness for both signalized and unsignalized intersection, with level of service determined from the delay estimate.

**Control delay** is the time that a driver spends queuing and then waiting for an acceptable gap in the circulating flow while at the front of the queue. The formula for computing this delay, is given in Eqn (3.7) and shows control delay as a function of capacity and entering flow.

$$d = \frac{3600}{C_{m,x}} + 900T \left[ \frac{V_x}{C_{m,x}} - 1 + \sqrt{\left(\frac{V_x}{C_{m,x}} - 1\right)^2 + \frac{\left(\frac{3600}{C_{m,x}}\right)\left(\frac{V_x}{C_{m,x}}\right)}{450T}} \right] \quad (3.7)$$

where  $d$  = average control delay, sec/veh;

$V_x$  = flow rate for movement  $x$ , veh/h;

$C_{m,x}$  = capacity of movement  $x$ , veh/h; and

$T$  = analysis time period,  $h$  ( $T = 0.25$  for a 15-minute period).

**Queue length** is useful for comparing roundabout performance with other intersection forms that use intersection delay as an input. The average queue length ( $L$  vehicles) can be calculated by using the following equation:

$$L = \frac{Vd}{3600} \quad (3.8)$$

where  $V$  = entry flow, veh/h; and

$d$  = average delay, sec/h.

## 3.6 Implementation

Let us now specify the physical value of our time and space units. Ignoring the possibility of existence of long vehicles such as buses, trucks etc, the length of each cell is taken to be 6 meters which is determined by the length of the vehicles and the spaces that drivers leave between them in a waiting queue. We want to emphasize the roundabout size i.e., the circumference of the central island which is approximately 69.08 m and the width of the lane is 3.7m and the number of cells for the inner lane is 11 and for the outer lane the number of cell is 13. We have also set the streets lengths as  $L$  cells.

The algorithm in Section (3.4) can be implemented as follows, it handles data input which include:

- The number of entering road in our case 4;
- The length of the selected entering road (which is the number of cell the road contains);
- The number of cells between entrances which is determined by the real dimension of the diameter to the central island; and
- The length of time for the simulation.

Furthermore, the means of arrival and turning rates of each road and the driver behavior probabilities are adjustable.

A program is developed to evaluate the capacity of the roundabout that contains two classes, a road class and a roundabout class. Both classes contain the following functions: driving-in functions which assign lanes to the vehicles at the entrance, update function for the vehicles on the road which is given above as "Update Rule

for the Roads” i.e. each vehicle moves forward only if it has open space in front of it for both the right and the left lane of the entrance road, when vehicles reach at the intersection they have given the number which does not only mean that the cell is occupied or vacant, but also indicate where its occupant will go and at the same time we can calculate the following:

**Throughput** equal to the maximum number of vehicles that can navigate a roundabout

**Density**( $\rho$ ) which is the number of vehicles on a road/roundabout divided by the total number of cells of the road/roundabout:

$$\rho = \frac{Q}{N} \quad (3.9)$$

where  $Q$  = Number of vehicles; and

$N$  = Total number of cells.

**Queue length** of each road is the number of vehicles waiting to enter on the roundabout.

The information output functions give us all the information about the operation of the roundabout. This includes information such as:

- The number of cars entering the roundabout;
- The numbers of vehicle remained on the road and on the roundabout;
- The number of vehicles that have passed through roundabout (throughput); and
- The density and queue length of each road.

## Chapter 4

# The modeling of vehicles traffic flow using conservation laws

When one thinks of modeling automobile traffic, it is natural to reason out the phenomena from personal experience and to visualize the car and the driver as a coupled system. The driver responding to the surrounding vehicles and operating the car to make it become a part of the flow of freeway and city traffic. Thus the traffic is not just a mechanical process but one in which human decisions are involved.

Here we refer to the papers [19, 27] for the description of the mathematical model of traffic flow on road networks. Traffic operations on roadways can be improved by field research and field experiments of real-life traffic flow. Traffic flow models designed to characterize the behavior of the complex traffic flow system have become an essential tool in traffic flow analysis and experiments.

The treatments of these problems are based on the fundamental traffic variables: velocity, density and flow. The nonlinear partial differential equation is the consequence of conservation of cars and experimental relationships between car velocity and traffic density.

For the description of the mathematical model of traffic flow on road networks, there is a close relationship between the three fundamental traffic variables: velocity, density and flow. It is quite realistic to think the flow  $q$  the number of cars per time unit-as a function of the only density  $\rho$ . More precisely the flow will be expressed as

$$\text{Flow} = (\text{density}) \times (\text{mean velocity})$$

that means

$$q = \rho * v \tag{4.1}$$

Now imagine a road segment with cars spread along it. The traffic density on this road associated with a given position  $x$  and time  $t$ , is the average number of vehicles per unit length of road at the position and time specified. Clearly to measure a density we need a stretch of road with enough cars on it to allow a reasonable statistical average. At the same time, we want to talk about the spatial variation of traffic density along the road, so the length over which we take the average should not be too long either, or else we will be getting to the natural scale of variation of the density. Thus  $\rho(x, t)$  is the average number of cars per unit length at the position  $x$  and time  $t$ .

If all vehicles have length  $L$  (or else  $L$  is a good average length) and the spacing (or average spacing) between the cars is  $d$ , then each vehicle takes up  $L + d$  units of road, so that approximately  $\frac{1}{L+d}$  vehicles will be present per unit length of road. Thus the constant density of the traffic in this case is  $\rho = \frac{1}{L+d}$ .

The other thing we need to think about is the common usage of the term traffic flow. We mean by this the number of cars per unit time which cross a given point on the road. We have seen the line across a road that counts passing vehicles. This is being used to determine the traffic flow.

## 4.1 Conservation of the number of cars

Consider the traffic flow of cars on road segment where overtaking is not possible. Let us fix a certain segment  $(A, B)$  and we are assuming that no cars are created or destroyed in the interval, then the changes in the number of cars result from crossing at  $x = B$  only. We deduce the cars entered at point  $A$  at a certain time will exit through point  $B$ . If more cars flow into the segment  $AB$  than flow out of it, then the number of vehicles within the segment will increase, and similarly if more flow out than in, it will decrease. We can express this mathematically in terms of the flux at  $A$  and  $B$ . Namely, the rate of change of the number of vehicles in the segment, with respect to time, should equal the difference in flow rate or flux. If  $N_{AB}(t)$  is this number of vehicles, then

$$\frac{dN_{AB}(t)}{dt} = -q(B, t) + q(A, t) \quad (4.2)$$

On the other hand we know that  $N_{AB}$  can be computed from the density by integration:

$$N_{AB}(t) = \int_A^B \rho(x, t) dx, \quad (4.3)$$

Thus we can rewrite our relation as:

$$\frac{d}{dt} \int_A^B \rho(x, t) dx = -q(B, t) + q(A, t) \quad (4.4)$$

We say that this last result is a global conservation law for the vehicles on the road. Note that the signs on the right are consistent. If  $q(B, t) > q(A, t)$  more cars flow out than in, so  $N_{AB}$  will decrease in time. From above we have that

$$\frac{d}{dt} \int_A^B \rho(x, t) dx = \int_A^B \frac{\partial \rho}{\partial t} dx \quad (4.5)$$

Note the use of partials on the right, since  $\rho$  also depends upon  $x$ .

To get the local relation we use the fundamental theorem of calculus:

$$\int_A^B \frac{\partial q(x, t)}{\partial x} = q(B, t) - q(A, t) \quad (4.6)$$

Note that we have used a partial derivative with respect to  $x$  here, since  $q$  depends on both  $x$  and  $t$ . Using the above three, we have

$$\int_A^B \left[ \frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} \right] dx = 0 \quad (4.7)$$

Now this relation holds over an interval  $AB$ . We now use the following result (which we do not prove here): Let  $f(x)$  be continuous on some closed interval  $[\alpha, \beta]$  and assume that

$$\int_A^B f dx = 0 \quad (4.8)$$

For any interval  $AB \in (\alpha, \beta)$ . Then  $f = 0$  for  $\alpha < x < \beta$ .

Using this result, we see that Eqn (4.7) implies that

$$\left[ \frac{\partial \rho(x, t)}{\partial t} + \frac{\partial q(x, t)}{\partial x} \right] = 0 \quad (4.9)$$

Using Eqn (4.1) we can write Eqn (4.9) as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \quad (4.10)$$

## 4.2 Velocity as a function of density

The equation we now have for vehicle conservation,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \quad (4.11)$$

is one relation involving two unknowns. Conventionally, we would need another relation to close the system in two unknowns. A major assumption that is often made is that velocity may be reasonably assumed to be a function of density alone.

That is, we can assume  $v = v(\rho)$  and our equation becomes a relation in  $\rho$  and its derivatives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v(\rho))}{\partial x} = 0 \quad (4.12)$$

Such an equation is called a partial differential equation (*PDE*) of first order. It is a *PDE* because of the two variables involved,  $x$ ,  $t$ , and the partial differentiations with respect to these variables. It is a first-order equation because only first partials are involved. To see this set  $F(\rho) = \rho v(\rho)$ , then equation (4.12) can be written

$$\frac{\partial \rho}{\partial t} + F'(\rho) \frac{\partial \rho}{\partial x} = 0 \quad (4.13)$$

This is a partial differential equation and it has to be supplemented by the initial condition

$$\rho(x, 0) = \rho_0(x), \quad x \in R$$

We will focus later on how to solve this equation once  $v(\rho)$  is given. For the moment let us see whether or not this assumption is justified, and then what the function  $v(\rho)$  should be.

To begin, consider a section of the road with only a single vehicle on it. Under these conditions, the density will be very low (*veh/km*) and the driver will be able to travel at a speed close to the design speed of the road. This speed is referred to as the free flow speed because vehicle speed is not inhibited by the presence of other vehicles. As more and more vehicles begin to use a section of the road the traffic density will increase and the average operating speed of vehicles will decline from the free-flow value as drivers slow to allow for the maneuvers of other vehicles. Eventually the road section will become so congested. As the density increases (meaning there are more and more cars per unit length), the velocity of cars diminishes. Thus we make the hypothesis that the velocity of a car at any point of the road is a regular strictly decreasing function of the density:

$$v = v(\rho) \tag{4.14}$$

If there are no other cars on the highway (corresponding to very low traffic densities), then the car would travel at the maximum speed  $v_{max}$ ,

$$v(0) = v_{max} \tag{4.15}$$

$v_{max}$  is sometimes referred to as the "mean free speed" corresponding to the velocity cars that would travel if they were free from interference from other cars. At a certain density cars stop before they touch each other. This maximum density  $\rho_{max}$  usually corresponds to what is called bumper-to-bumper traffic:

$$v(\rho_{max}) = 0 \tag{4.16}$$

Mathematically such a relationship can be expressed as

$$v = v_{max} \left(1 - \frac{\rho}{\rho_{max}}\right) \quad (4.17)$$

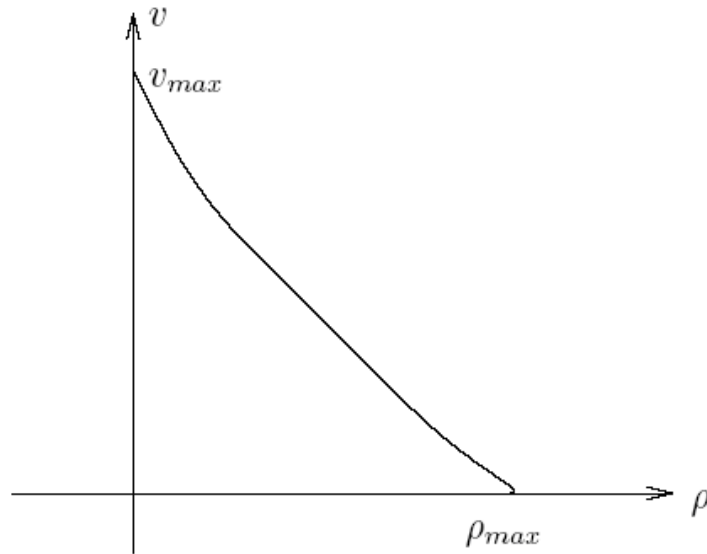


Figure 4.1: A fluid dynamic model for traffic flow on road network

The advantage of using a linear representation of the speed-density relationship is that it provides a basic insight into the relationships among traffic flow, speed and density interactions.

### 4.3 Implementation

Here we want to demonstrate special variation of traffic density along the road, and for that we have to solve the above pde equation and find the relation between speed and density, in order to see the relation between the two variables from the collected data

we use an estimated regression function (the technique of predicting the value of one variable called the dependent variable from measurements of the other called the independent variable) as shown in the Appendix B.

As we have stated above the partial differential equation has to be supplemented by the initial condition for that we use our data to see how the density varies as a function of time as shown in Fig.4.2

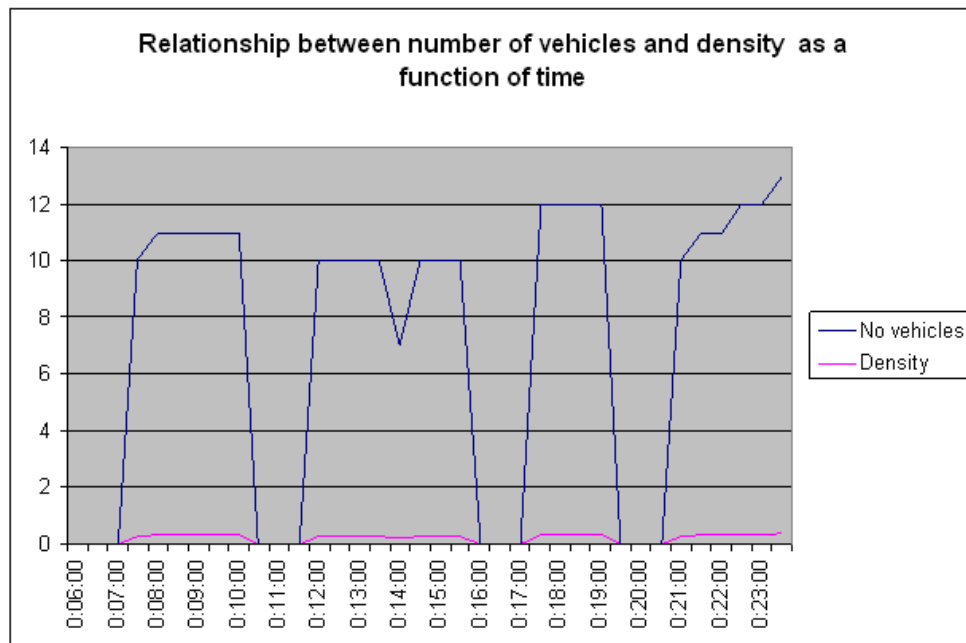


Figure 4.2: Relationship between number of vehicles and density as a function of time

We observe that the density has a periodic behavior with time, so we can approximate the initial density using periodic function  $f(x) = 0.2\sin(\pi x/20)$  and we clip the results to maintain densities between 0 and 1.

# Chapter 5

## Analysis and Discussions

### 5.1 Analysis results for The density distribution

For all simulations, the range of time is on the  $x$ -axis, which goes from 0 to 50 units. The  $y$ -axis is the length along the road, which ranges from 0 to 20.

The first result shown in Fig.5.1 is rather simple. Here the initial traffic distribution is given by  $f(x) = 0.2 \sin(\pi x/20)$ . The algorithm produces the following. Note how the peak traffic density propagates backwards along the road as time moves forward. The peak traffic density also diminishes slightly as the traffic spreads itself out along the length of the road.

The second result depicted in Fig.5.2 has the same initial distribution, but includes an inflow function  $g_1(t) = 0.05 \sin(\pi t/50)$  at  $x = 10$ . Notice the sudden increase in density at the center, where the initial distribution and inflow function achieve their maxima.

The third result shown in Fig.5.3 uses the same initial distribution and inflow function and also has an outflow function at  $x = 7$ , which is defined by  $g_2(t) = -g_1(t)$ . This

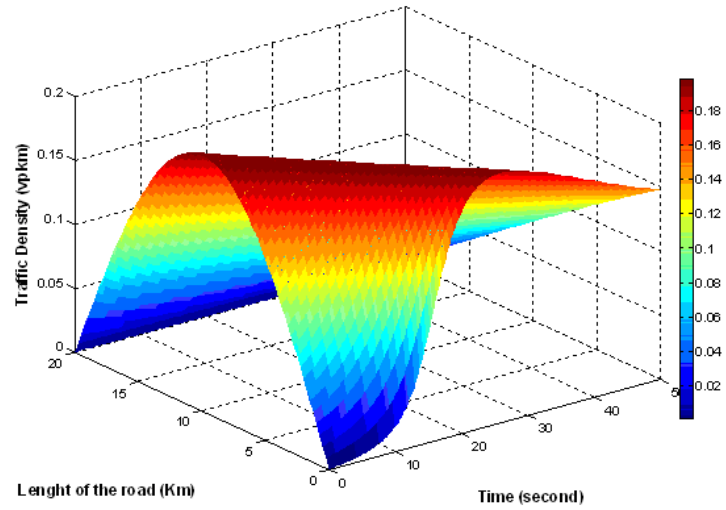


Figure 5.1: Evolution of car density for the initial distribution function  $f(x) = 0.2 \sin(\pi x/20)$ .

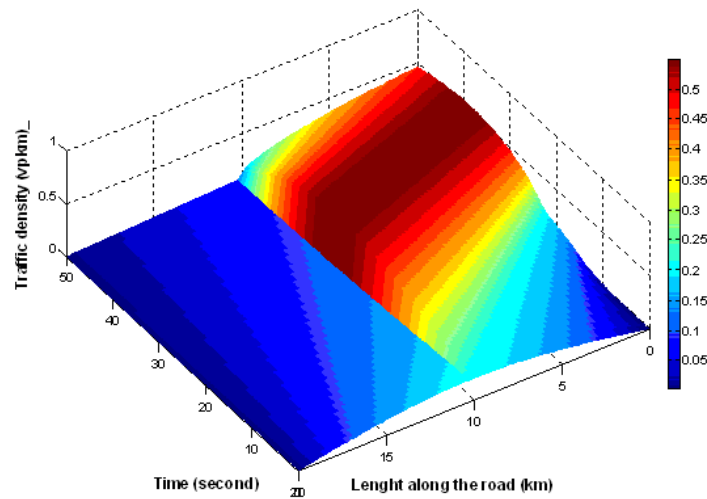


Figure 5.2: Evolution of car density for the initial distribution function  $f(x) = 0.2 \sin(\pi x/20)$  and inflow function  $g_1(t) = 0.05 \sin(\pi t/50)$  at  $x = 10$ .

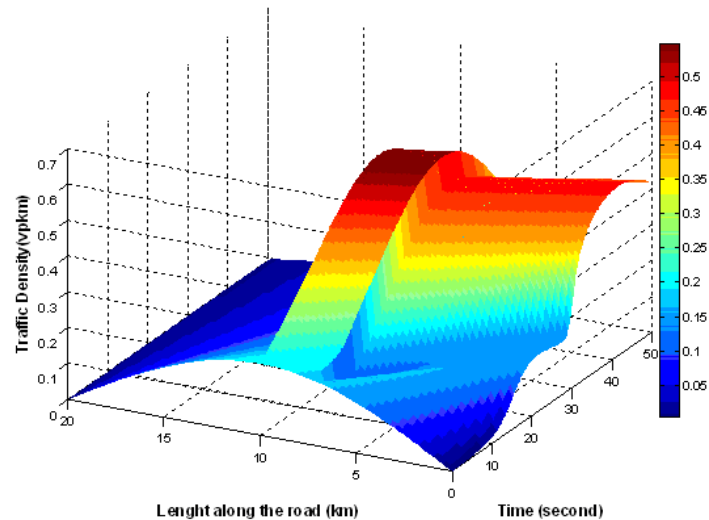


Figure 5.3: Evolution of car density for the initial distribution function  $f(x) = 0.2 \sin(\pi x/20)$  inflow at  $x = 10$  and outflow at  $x = 7$ .

could simulate an offramp and onramp on a road way and the density is reduced as cars exit and then increases as new cars enter.

## 5.2 Analysis results for two-lane roundabout

The focus here is based on results from two-lane roundabouts model using Multi-stream Minimum Acceptable Space (MMAS) Cellular Automata (CA) models to investigate traffic flow at two-lane roundabouts and to study a roundabout performance.

The method allow to assess the operational performance of the given roundabout and we use only Pattern A (only allows RT vehicles to use the right lane of the entrance road).

The model estimate Performance measures to obtain the broadest possible evaluation of the performance of a given roundabout by using the following relations

1. Throughput versus Arrival rates.
2. Throughput versus Turning rates.
3. Position Delay Time (PDT).
4. The Driver's behavior versus Throughput.
5. Road performance (i.e., queue lengths).

Results from (1) and (4) are presented explicitly here below. Others are discussed only briefly.

### 5.2.1 Relationship between Throughput and Arrival Rates

Table 2: Throughput of roundabout when the driver behavior at four special situations

		Arrival Rates				
Driver Behavior	0.2	0.25	0.30	0.35	0.40	0.45
P <sub>co</sub> =1	2000	2088	3001	3056	3245	3305
P <sub>mo</sub> =1	2808	3235	3416	3516	3818	3904
P <sub>ur</sub> =1	2877	3198	3215	3356	3589	3658
P <sub>ra</sub> =1	65	72	96	99	98	95

Throughput of the two-lane roundabout continues to increase with arrival rate as shown in fig.5.4 below. Therefore, when arrival rates increase, throughput also increases.

For single-lane roundabouts since vehicles arriving from any road needs to wait for the minimum acceptance space to enter the roundabout throughput and arrival rates are

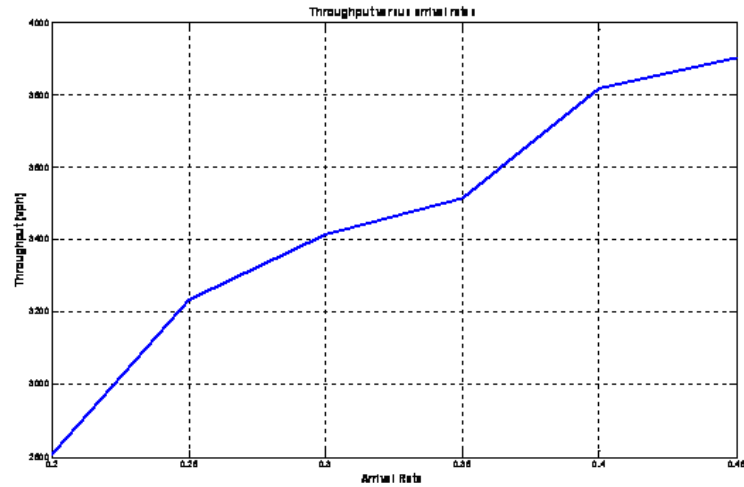


Figure 5.4: Throughput versus arrival rates

closely related [13]. However, for two-lane roundabouts, throughput also depends on turning rates; in particular, it is dependent on right-turning rates on the roundabout as right-turning vehicles on the outer and right lanes of the roundabout and the entrance road respectively are theoretically free flow. For larger proportion of right-turning vehicles, the throughput will be higher. Thus, the relation above focuses on the relationship between throughput, straight-ahead and left-turning vehicles.

### 5.2.2 Relationship between Throughput and the Driver's behavior

The impact of the driver's behavior on throughput in Pattern A can be shown as follows, we assume that the sum of probabilities of conservative ( $P_{co}$ ), moderate ( $P_{mo}$ ), urgent ( $P_{ur}$ ) and radical ( $P_{ra}$ ) behavior is equal to 1. In this case, and for simplicity, the outcomes for all drivers of one type are considered.

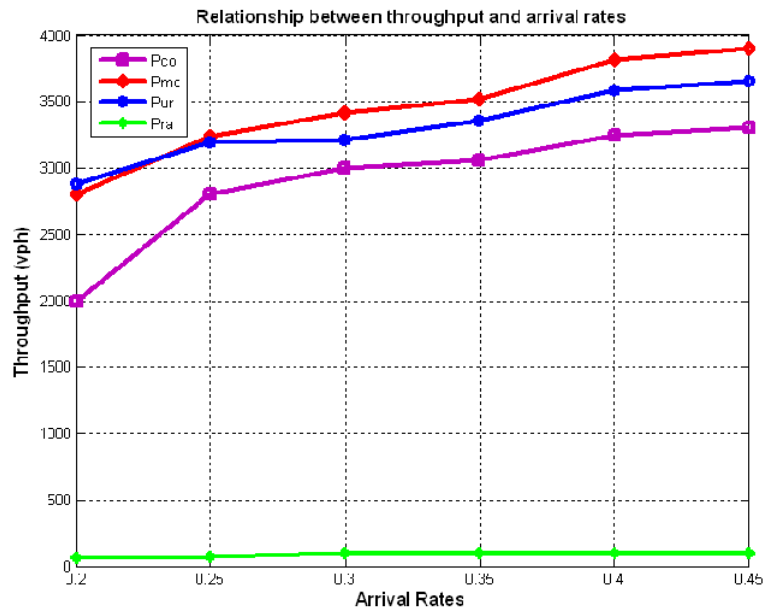


Figure 5.5: Throughput versus Driver's behavior

Thus, as for single-lane roundabouts in [13], collective conservative behavior decreases throughput. In contrast, collective radical behavior can cause congestion on the roundabout and decrease in throughput compared to collective moderate behavior. Certainly, a distribution of driver behavior is more appropriate, but our result reproduce for example the phenomenon of congestion on a two-lane roundabout, due to the fact that too many drivers may not observe the give-way rules.

There is some difference between moderate and urgent behavior with respect to throughput. Compared to moderate, urgent behavior has better performance with higher throughput when all arrival rates are  $< 0.25$ . In particular, when  $0.45 >$  all arrival rates  $> 0.25$ , saturation occurs for moderate behavior.

Now comparing the throughput of the roundabout with the simulated one the gap between the simulated and the real line is close at the initial stage and as the time

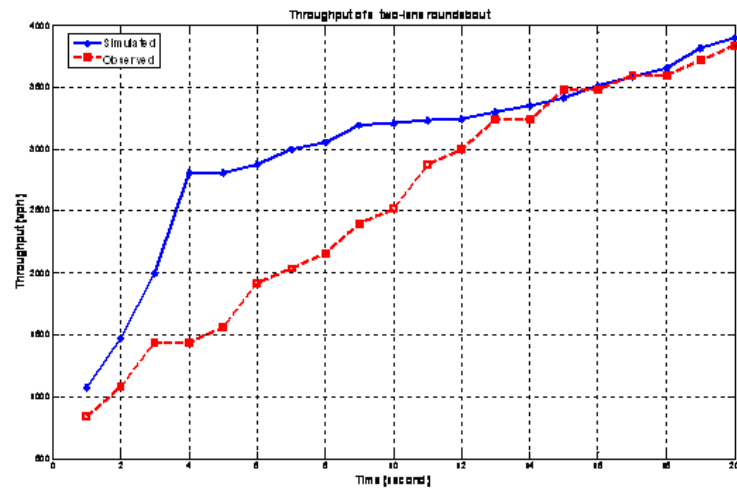


Figure 5.6: Throughput versus Driver's behavior

goes (increases) indefinitely it eventually become equal although the gap is wider at the middle.

# Chapter 6

## Comparison to other Controlling Schemes

### 6.1 Data Collection

Vehicular traffic operation is critical to the identification and evaluation of alternative treatment for improved access to the transportation facility, for the development of recommendation regarding to their treatment it is important to collect empirical data. As much as possible, the traffic data collected should indicate the existing peak hour (the morning, noon and evening sessions) traffic conditions. Even if the Addis Ababa road Authority Traffic Engineering Department has established a computerized database system for traffic data, the data collected does not relate to peak hour traffic and it does not show the turning movements on the junctions. Only the Average Annual Daily Traffic (AADT) along some of Addis Ababa's major and minor roads was stored in the database. Essential operational data for the evaluation and to test the capability of a selected model are the traffic movements on the legs, traffic throughput and the number of vehicles on a fixed time interval, the headway (the time measure from head to head of each of successive vehicles) and the queue length

are necessary data for the analysis.

## 6.2 Study sites and traffic data

The necessary peak hour traffic data was collected on two locations of roundabout and traffic light. These locations were chosen based on the principal of possible representative of the target population of roundabout and traffic light in terms of throughput, geometry and size. The chosen roundabout and traffic light have four legs and the names are as follows (the name being adopted from the area).

- Roundabout with four-legs that approaches: **Gerji Imperial**
- Traffic light with four-legs that approaches: **Olympia Traffic light**

The roundabout (Imperial ring road) and the traffic light road (Olympia road) has moderate vehicle flow and has four legs both are two lane with one canalized right turn lane to the far right (which is not included in our data). There are adequate traffic flows during the morning, noon and night sessions peak hours on both locations. And the roundabout has two circulatory lanes.

The data is collected at peak hours and normal times; it is collected for two hour duration in three periods. The other important parameters for analysis is Geometry; the summarized geometric data measured is shown below.

Table 3: Summery of intersection Geometry

Name of the road	No. legs	No.circulating lane	lane width(m)	No. entry lane	Central island diameter
GERGI-IMPERIAL	4	2	3.8	3	22(m)
OLYMPIA ROAD	4	-	3.7	3	-

Having all the above mentioned data we can proceed to analyze the capacity, since the data collected was empirical there is no need for an additional information which represents the driver's behavior because the data collected are from the real world, which has already taken many factors that influence capacity into account (such as the driver's behavior, pedestrian's behavior etc). The average arrival rate of the roundabout is 0.40, for the traffic light is 0.43 and traffic volumes are assumed to be equal for both streets.

### 6.3 Comparison of traffic light versus roundabout

Let us now compare the roundabout performance with signalized control methods of an intersection. This comparison is our main motive for studying roundabout characteristics. Let us replace the roundabout intersection with traffic lights. For simplicity, we consider the intersection of two-way-stop-controlled (TWSC) streets which are assumed to direct double-lane traffic flow. Basically there are two types of signalization: fixed-time and traffic adaptive. Here we describe the fixed-time method only because this is the only method being employed currently in Addis Ababa. In this control scheme, the traffic flow is controlled by a set of traffic lights which are operated on a fixed-cycle. The lights periodically turn green with a fixed period (cycle length)  $T$ . This period is divided into two parts: in the first part, the traffic light is green for one street (simultaneously red for the other streets and direction) this part lasts for  $T_g$  seconds ( $T_g < T$ ). In the second part, the lights change color and movement is allowed for the vehicles in opposite direction. The second part lasts from ( $T_g$  to  $T$ ). This phenomenon is repeated periodically [16].

Here we compare the performance of the corresponding roundabout with fixed-time signalization strategy. It is possible to count the queue-lengths formed behind the red lights and at the roundabout intersection, and the time-headways between successive cars passing each lane.

According to the graph fig .6.1, the queue length for traffic light dominates but has sharp end point that means in the case of traffic light the vehicles waiting time is less than that of roundabout as for the roundabout the figure shows that the central island makes the vehicle to enter the roundabout maneuver more easily, So this leads

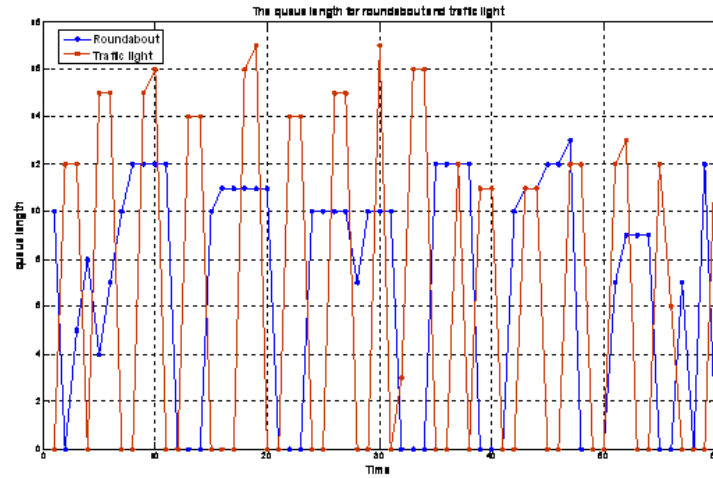


Figure 6.1: The queue length for Roundabout and traffic light

to increase of car density in the central island which correspondingly increases the probability of blocking the in-flow direction due to yielding effect. Blocking effect is the dominant factor that leads to an overall increase of delay.

### 6.3.1 Throughput of Roundabout verses Traffic light

This result can be explained by noting that in sufficiently light traffic states, the approaching cars can easily find the required space gap in the flow of conflicting direction, hence they can enter the roundabout without spending much times whereas in a signalized scheme, they have to wait at the red parts of the signal even if the flow is negligible in the conflicting direction.

According to the above graphs fig.6.2 and fig.6.3 , in relatively light traffic states, characterized by a large average gap (headway), a roundabout shows a better performance and gives rise to lower delays. Conversely, in more congested traffic situations, controlling the intersection by signalized traffic light leads to better results. Our results

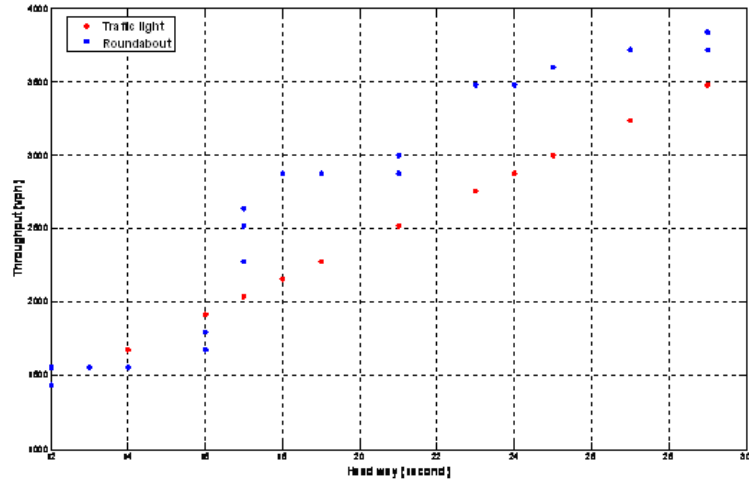


Figure 6.2: Throughput of Roundabout versus Traffic light

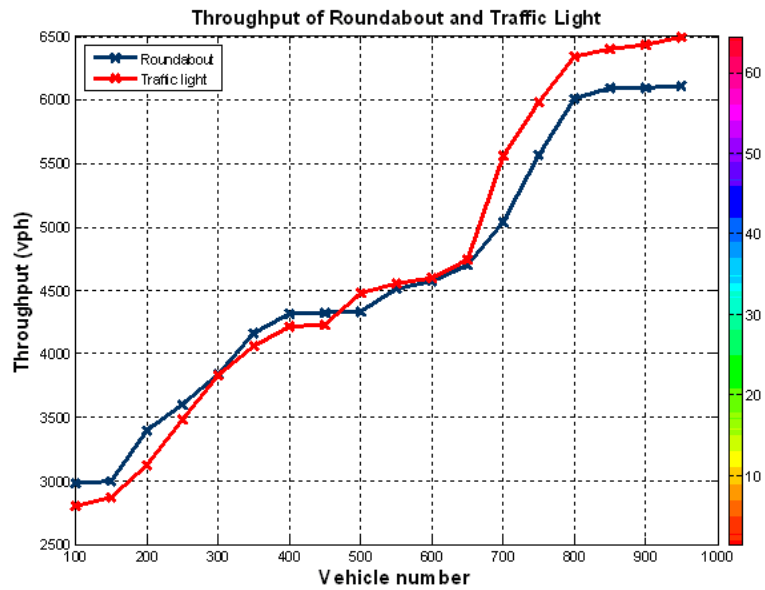


Figure 6.3: Throughput of Roundabout versus Traffic light

gives the critical throughput in which the intersection should be controlled in a self-organized manner is approximately the throughput is between 4000vph and 4500vph, this proves that below certain congestion, the roundabout efficiency is higher than fixed-time signalized junction point. In general, average throughputs for two-lane roundabout are lower than the signalized intersection.

## 6.4 Conclusions and Recommendation

In addition to the features that characterize a modern roundabout, yield-at-entry, deflection and flare, roundabouts often have other important features. It is a solution that is environmentally friendly and requires less in annual maintenance costs since it replaces traffic signals. Nevertheless, the efficiency of roundabouts is still under debate, and many experts believe that signalized intersections show a better performance in most circumstances. To settle this debate, at least to a partial degree, we have tried to quantitatively explore the basic features of roundabout in order to have a better insight into the problem. In this paper we have investigated the characteristics of traffic at an isolated roundabout in the framework of cellular automata. For this purpose, we have developed and analyzed the performance of the various aspects of roundabouts, the most important of which is throughput. Our result shows that overall throughput is significantly affected by driver's behavior.

Two important findings from this research are:

- i)* Throughput increases linearly with arrival rate when the entrance road is not in a saturated situation. It reaches a maximum when arrival rates reach their maximum and it also depends on the turning rates.
- ii)* Throughput decreases as LT rate increases i.e. vehicles on average need to travel longer distances on the roundabout.

The Driver's behavior has an impact on the overall performance of the roundabout and individual roads. Moderate, urgent and conservative behaviors leads to free-flow for all arrival/turning rates considered, whereas radical behavior can lead rapidly to congestion. There is some difference between moderate and urgent behavior with respect to throughput. Conservative behavior leads to decreased throughput. Compared to moderate, urgent behavior has better performance with higher throughput when all arrival rates are  $< 0.25$ . In particular, when  $0.45 > \text{all arrival rates} > 0.25$ , saturation occurs for moderate behavior.

The Addis Ababa roundabout and traffic light analysis results indicate most of the roundabout and traffic lights are over saturated and is common to see at peak hours traffic polices have to regulate the traffic on these junctions since these traffic control mechanisms could not function or regulate the traffic by themselves. As the study revealed, the major problems are related to high traffic flow, high volume of pedestrians and the driver's behavior.

The major conclusion made from our simulation result proves the existence of critical congestion, dominated by the statistics of arrival rate, space gaps and driver's behavior, over which the intersection is made more efficient by signalization approach. In a more realistic situation, the flow can circulate around the central island via an additional lane. The interior lane should be used by those vehicle intending to make left or U-turns while the exterior one should be taken by those drivers who tend to turn right or move straightforward.

In the present case of double-lane circulation, our simulations implies that injection of vehicles from more than two entries leads to global blocking of flows and growing delays. This effect is due to the saturation of circulating flow which hinders the

incoming fluxes i.e. as traffic demand growing causes roundabout traffic to fail, they are converted to other types of intersection.

Nowadays the number of vehicles in our city is increasing rapidly, from our collected data especially at peak hour more than 1500 number of vehicles exist at each intersection per hour, from our result if the number of cars is higher than 500 then the flow at traffic light junction is higher than the roundabout. This implies that junctions which are currently constructed or under construction has to be substituted by traffic light.

Since the collected data for the analysis is very limited, especially peak hour traffic data and number of junctions included in the study cannot be an end by itself. An additional further study is recommended with more data collection. So it is possible to refine the model and to use it as traffic junction improvement.

It is better to separate the pedestrians from vehicles traffic on the roundabout where high pedestrians flow were observed, since they affect the normal traffic flow of the roundabouts. Besides, it is necessary for the safety of the pedestrians too.

The efficiency of the Multi-stream Minimum Acceptable Space (MMAS) Cellular Automata (CA) algorithm that is used to investigate traffic flow at two-lane roundabouts has to be measured.

# Appendix A

## A.1 Operational Analysis Formulas

This appendix presents the assumptions used to develop the capacity forecast relationship for the **Double-Lane Roundabout**

### A.1.1 Traffic operation at a roundabout

A roundabout brings together conflicting traffic streams, allows the streams to safely merge and traverse the roundabout, and exit the streams to their desired directions. The geometrical elements of the roundabout provide guidance to drivers approaching, entering, and traveling through a roundabout.

Drivers approaching a roundabout must slow to a speed that will allow them to safely interact with other users of the roundabout, and to negotiate the roundabout. The width of the approach roadway (the number of lanes) and the density of traffic present on the approach govern this speed. As drivers approach the yield line, they must check for conflicting vehicles already on the circulating roadway and entry determine when it is safe and practical to enter the circulating stream. The widths of the approach

roadway and entry determine the number of vehicles streams that may form side by side at the yield line and govern the rate at which vehicles may enter the circulating roadway. The size of inscribed circle affects the radius of the driver's path, which in turn determines the speed at which drivers travel on the roundabout. The width of the circulating roadway determines the number of vehicles that may travel side by side on the roundabout.

Geometrical elements that affect entry capacity include:

1. Approach half width
2. Entry width
3. Entry angle
4. Average effective flare length.

### A.1.2 Equations

$$\begin{aligned}
 Q_e &= k(F - f_c Q_c) & f_c Q_c &\leq F \\
 &= 0, & f_c Q_c &\geq F
 \end{aligned}$$

Where:  $Q_e$  = entry capacity,

$Q_c$  = circulating flow,

$$k = 1 - 0.00347(\phi - 30) - 0.978\left(\frac{1}{r} - 0.05\right)$$

$$F = 303x_2$$

$$f_c = 0.210t_D(1 + 0.2x_2)$$

$$t_D = 1 + \frac{0.5}{1 + \exp\left(\frac{D-60}{10}\right)}$$

$$x_2 = v + \frac{e-v}{1+2S}$$

$$S = \frac{1.6(e-v)}{l'}$$

Where:  $e$  = entry width,  $m$

$v$  = approach half width,  $m$

$l'$  = effective flare length,  $m$

$S$  = sharpness of flare,  $m/m$

$D$  = inscribed circle diameter,  $m$

$\phi$  = entry angle, degrees

$r$  = entry radius,  $m$

### A.1.3 Parameter assumptions

$$D = 55m$$

$$r_e = 20m$$

$$\phi = 30 \text{degrees}$$

$$v = 8m$$

$$e = 8m$$

$$l' = 40m$$

$$S = \frac{1.6(e-v)}{l'} = 0$$

$$t_D = 1 + \frac{0.5}{1 + \exp\left(\frac{D-60}{10}\right)} = 1.3112$$

$$x_2 = v + \frac{e-v}{1+2S} = 8$$

$$F = 303x_2 = 303(8) = 2424$$

$$f_c = 0.210t_D(1 + 0.2x_2) = 0.7159$$

$$k = 1 - 0.00347(\phi - 30) - 0.978\left(\frac{1}{r} - 0.05\right) = 1$$

#### A.1.4 Final equation

$$Q_e = 2424 - 0.7159Q_c$$

# Appendix B

## B.1 Regression

Table 4: Variables Entered / Removed

Model	Variables Entered	Variable Removed	Method
1	Density <sup>a</sup>	-	Enter

*a*· All requested Variables entered

*b*· Dependent Variable: Speed

Table 5: Model Summary

Model	R	R Square	Adjusted R Square	Std. Error the Estimate
1	.381 <sup>a</sup>	.145	.140	1.99092

*a*· Predictor: (Constant), Density

Table 6: ANOVA<sup>b</sup>

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	111.397	1	111.397	28.104	.000 <sup>a</sup>
Residual	657.987	166	3.964		
Total	769.384	167			

*a*· Predictors: (Constant), Density

*b*· Dependent Variable: Speed

Table 7: Coefficients

	Unstandardized Coefficients		Standardize Coefficients		
Model	B	Std. Error	Beta	t	Sig.
1 (Constant)	0.988	0.440		8.244	.000
Density	-1.002	1.630	-3.81	-5.301	.000

*a*· Dependent Variable: Speed

If a dependent variable is speed and an independent variable density are related by an estimated regression function, then from the coefficients table we can read the constants so that, our estimated regression function given by the equation

$$v(x, t) = 1 - \rho(x, t)$$

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