ADDIS ABABA UNIVERSITY
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APPLICATION OF BALANCED INCOMPLETE BLOCK DESIGN IN PPS SAMPLING WITHOUT REPLACEMENT

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Title of Research **APPLICATION OF BALANCED INCOMPLETE BLOCK DESIGN IN PPS SAMPLING WITHOUT REPLACEMENT**

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A B S T R A C T

In this study a procedure for selecting a sample of size n with probability proportional to size without replacement was proposed. This procedure used the combinatorial properties of balanced incomplete block design.

An application of this procedure has been illustrated by using secondary data, obtained from National Labour Survey conducted in 2005 by Central Statistical Agency of Ethiopia.

Further, the variance of the estimated total unemployed population was compared by using Horvitz-Thompson (1952) and Sen –Yates- Grundy (1953) estimators. The result suggests that, variance due to Sen-Yates –Grundy is better estimator than Horvitz-Thompson .

In addition, I would like forward that up to date investigation in the work of application of balanced incomplete block design in pps sampling without replacement at national and/or regional level is important in minimizing cost at large.

Researchers who try to examine the application of BIBD in pps sampling without replacement in the further have to take into account those variables other than unemployed population

T A B L E O F C O N T E N T S

DECLARATION

I, undersigned, declare that the thesis is my original work, has not been presented for a degree in any university and all sources of material used for the thesis have been acknowledged.

Name: **Yigrem Tefera**

Signature: --------------------

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Date: **July 2007**

This thesis has been submitted for examination with my approval as a University advisor.

Dr. M.K. Sharma

CHAPTER ONE

INTRODUCTION

This chapter deals with some of the numerous uses of experimental designs in survey sampling that were applied *in selection and estimation that have been produced for cluster units and others.*

Use of experimental designs in survey sampling dates back to Frankel and Stock (1942). They used Latin square designs to increase the effective depth of stratification in the selection of primary sampling units (clusters) when the number of sample clusters is small.

Mahalanobis (1944) advocated the use of interpenetrating sub samples to measure interviewer variability and to estimate total variance (response variance plus sampling variance). This is an example of a completely randomized design. Fellegi (1964) used cross-over designs to measure the components of response variance.

Chakrabarti (1963) did pioneer work in the use of balanced incomplete block designs (BIBD) for drawing samples with the same first-and second- order inclusion probabilities, π_i and π_{ij}, as simple random sampling (SRS), i.e. $\pi_i = \frac{n}{N}$ and $\pi_{ij} = \frac{n(n-1)}{N(N-1)}$, where n and N denote the sample and population sizes, respectively. This approach ensures variance equivalence with SRS and yet leads to support size (number of samples s with probability of selection p(s)>0) smaller than the support size $\binom{N}{n}$ for SRS. The number of treatments v=N, the plot size K=n and the number of blocks b, is the support size.
Raghavarao and Singh (1975) extended Chakrabarti,s(1963) work to more complex sampling. They applied two associate class partially balanced incomplete block designs (PBIB) to cluster sampling. Singh, Raghavarao and

Federer (1976) extended this work to multidimensional cluster sampling by using higher associate class PBIB designs.

It often happens in practice that certain samples, s, are known to be nonpreferred (for example, the units in s may be too wide spread, thus increasing the travel cost). It is desirable to minimize the probability of selecting a nonpreferred sample and at the same time ensures variance equivalence to SRS or to a more general design. Most of the literature on controlled sampling used various incomplete block designs to construct designs with minimum support size (i.e., minimum number of distinct blocks) and then identify maximum number of distinct blocks with the nonpreferred samples. One of the b blocks is then selected at random and the units in it form the sample. Avadhani and Sukhatme (1973) applied BIBD to controlled sampling, but the application readily follows from Chakrabarti (1963) results. Das and Mohanty (1973) also suggested some schemes for selecting probability proportional to size sampling without replacement.

Gupta et al. (1982) and Kumar et al. (1985) proposed a family of inclusion probability proportional to size (IPPS) sampling schemes based on single balanced incomplete block designs and two balanced incomplete designs.

1.1General objectives

The main purpose of this study is to present probability proportional to size sampling scheme without replacement by using combinatorial properties of the balanced incomplete block design and to illustrate its use by using secondary data, obtained from National Labour Survey conducted in 2005 by Central Statistical Agency of Ethiopia. The scheme facilitates the calculation of the inclusion probability π_{ij} of two units i^{th} and j^{th} of sample size n from a finite population.

1.2 *Specific objectives*

- To show the use of BIBD in probability proportion to size sampling
- To find out the unemployed population by using Horvitz-Thompson estimator
- Find out estimator of its variance and its standard error by using Horvitz- Thompson and Sen-Yates-Grundy equations
- To compare the values of estimator and its standard error in both cases.

CHAPTER T WO

RELATION BETWEEN DESIGN OF EXPERIMENT AND SAMPLE SPACE

This chapter discuses definition of balanced incomplete block design, sample space with required characteristics, use of experimental design and balanced incomplete block design in the practical use as sample space in section 2.1,2.2 , 2.3 and 2.4, respectively.

2.1 Definition: -A balanced incomplete block design (BIBD) is an arrangement of v treatments into b blocks such that

1. Each block contains k (<v) distinct treatments
2. Each treatments appears in r blocks
3. Every pair of treatment appears together in λ blocks.

The integers v, b, r, k, λ are called the parameters of the BIB design. These parameters are not independent and are related by the following identities**:**

$$v\,r = b\,k\,,$$

$$\lambda(v-1) = r(k-1)$$

Table of balanced incomplete designs with from 3 to 15 replications is given in APPENDIX I**.** This appendix provides 214 different BIBD according to your varietal trials v=N and sample size n=k

2.2 Sample space

Sampling designs always envisage a sample space. A sample space consists of a number of samples of specified sizes from which one or more samples are drawn with given probabilities. More precisely, a sample space for drawing a sample of size n in an objective manner from a population of N units, consists of, say S groups of N units each of size n. When these groups satisfy certain requirements, different sampling techniques arise from different characteristics of these S groups of units. For example, in simple random sampling without replacement the possible $\binom{N}{n}$ combinations constitute the S groups. No unit is repeated in any group. Each unit occurs the same number of times in the totality of S groups, so also each pair of units occurs together in a constant number of groups. But in the sample space corresponding to systematic sampling though the k groups where $N = nk$ constitutes the sample space possesses all the characteristics of the sample space for the simple random sampling, it does not possess the last characteristics viz, the number of groups in which any pair of units occur together is not the same from pair to pair and some pairs do not occur at all in any of the samples in the space. An important problem in sample survey technique is to obtain a sample space from which samples can be drawn without repetition such that the probabilities of inclusion of the units of the population in a sample is proportion to given sizes of the units .The sample space accordingly is required to have samples in none of which the units should be repeated and the i^{th} unit in the population should occur in the space r_i times (i =1,2…N), where r_i's are prefixed numbers. Further, there should be at least one sample in which any given pair of units occurs together. If a sample is drawn at random from such a sample space unbiased estimate of mean and variance are possible.

We thus see that sample survey designs ultimately turn out to be a problem of determining a suitable collection of samples possessing requisite characteristics and then drawing one of these samples at random.

It is known that occurrence of each of the units in the population a constant number of times in the sample space ensures an unbiased estimation of the population mean from a sample drawn at random from the sample space (Chakravarty M .C., 1963). Like wise, occurrence of each pair of units in a constant number of samples or at least once for certain sampling techniques ensures unbiased estimability of the variance of estimate of population mean.

As in systematic sampling some pairs of units do not occur together in any sample in the space, unbiased estimate of the variance of estimate of population mean is not available through this technique.

The concept of unbiasedness with reference to sample space can be viewed as follows. If the mean of a sample means from all the samples in a sample space gives the population mean, then a random sample from such a space provides an unbiased estimate of the population mean.

If $\bar{Y}_i$ denoted the mean of the i^{th} sample in the space and $\bar{Y}$ the population mean, the average of squares of deviations $\left(\bar{Y}_i - \bar{Y}\right)$ for all possible samples in the space gives the variance of the estimated mean.

That is, $\operatorname{var}\left(\bar{Y}_i\right) = V = \frac{\sum_{i=1}^{s}\left(\bar{Y}_i - \bar{Y}\right)^2}{S}$. A quadratic function S_i^2 of sample observations will give an unbiased estimate of this variance if the average of s_i^2 for all the samples in the space is equal to V.

The definition of unbiased mean and variance generally agree with those arrived at from consideration of inclusion probabilities of the units and their pairs.

2.3 Designs of Experiments and their uses

Design of experiments had its origin in supplying layout plans of experiments for comparison among a number of experimental treatments in regard to some of their responses when these are applied to a set of experimental units under certain conditions. Consideration of precision of such comparison led to the necessity of suitably grouping the experimental units and then allotting the treatments to these groups. When it became necessary to make such groups incomplete in the sense that all the treatments are not accommodated in a group, combinatorial problems arose. Mathematically minded Statisticians like Fisher, Yates, Finney, Bose, Nair, Kishen among others seized the opportunity and made significant contributions in the area of construction of designs in presence of incompleteness of treatment groups. While Fisher and Yates restricted them selves by considerations of utility of designs for solving specific problems, other workers were guided and actuated more by considerations of mathematical tractability of the problems and curiosity, keeping utility considerations relatively unimportant. As a result great progress was made in evolving diverse methodology for filling the void due to incompleteness by balancing. Soon it was realised that these techniques could be applied profitably to solve problems in many other fields. Thus, Fractional factorials could provide efficient error correcting codes used in communication engineering.

Fractional factorials could also be used for industrial investigations and quality control activities by providing what are known as main effect plans. Balanced incomplete block designs could be used to provide schemes of weighting of objects in groups through which estimates of weight of individual items are obtainable with more precision than when they are weighted individually. Such designs have also been utilised for providing designs for collection of observations for exploring response surfaces.

2.4 Design of experiments and sample space

Another important area where the principles of construction of experimental designs can be adopted is the Sampling Designs. We have discussed earlier the requirements for a sample space. The following discussion shows how certain designs for experiments can provide efficient sample spaces.

In the discipline of Designs of experiments there is an important series of designs known as incomplete block designs for varietals trails. These designs are very much similar to sample spaces. A correspondence can be set-up between

(i) number of treatments in a varietals trial and population size, N in sample survey

(ii) number of blocks b in an incomplete block design and number of Groups S in a sample space

(iii) replications r_i of the units in incomplete block designs and the number of times each unit occurs in the sample space and finally,

(iv) the number of blocks in which the i^{th} and j^{th} treatments occur together, say λ_{ij} and the number of samples in which two given units occur together. From such correspondence, it is evident that any balanced incomplete block design can be used as a sample space. So BIBD design can be considered as a sample space for pps sampling without replacement. Note that, the BIB designs does not exist for all v=N and k=n with total number of blocks as $\binom{N}{n}$. Therefore, the sampler is left with a choice of taking not all possible samples in case of BIB design.

For unbiased estimation of variance it is, however, necessary that none of the λ_{ij}'s zero. Further, the designs should have the $r_i's$ fixed in advance. For many of the sampling techniques $r_i's$ should have a constant value. One main point of difference between an incomplete block design for experiments and a sample space for drawing a sample is that while all the groups or blocks are used for an experiment, only one group is used for the purpose of sampling. Further, enumeration of the different groups is necessary for an experimental design but such enumeration is not necessary for sampling. As a matter of fact for sample space one of sampling purpose such enumeration has to be avoided and a procedure is to be evolved such that without actual enumeration of the samples in the sample space one of the samples can be drawn at random. Special efforts are needed to fulfil this latter requirement.

CHAPTER THREE

Method of selection and Estimator

This chapter presents definition of sample design, two methods of sample selection and an estimator of the population total with its variance. In addition unbiased sample estimators of the variance of the estimator of population total will be assessed.

3.1 Sample Design

We consider a finite population of N units

$u_1, u_2, \ldots, u_N$ (3.1.1)

A sample 's' from (3.1.1) is defined as an ordered set of units from (3.1.1). A sample design is a set 'S' of samples 's' from (3.1.1) with a probability measure 'P' defined on it, and is denoted by

D=D (S, P) (3.1.2)

For any unit u_i of (3.1.1), let

$$\sum \pi_i = \sum_{u_i \varepsilon\, s} p(S) = n \quad for\ 1 \le i \le N \qquad (3.1.3)$$

Where the summation extends overall sample 's' of (3.1.2) which contains u_i at least once. Similarly, for any pair u_i and u_j, let

$$\sum \pi_{ij} = \sum_{u_i\, u_j \varepsilon\, s} P(s) = n(n-1)\pi_i, \quad \sum_i \sum_{j>i} \pi_{ij} = \frac{1}{2} n(n-1) \qquad (3.1.4)$$

π_i is called the inclusion probability of u_i and π_{ij} the joint inclusion probability of the pair u_i and u_j.Clearly, these are the probabilities that a sample 's' from (3.1.2) contains the unit, the pair of units u_i and u_j,respectively.

An unbiased estimator of the population total

$$Y=\sum_{i=1}^{N} y_i \tag{3.1.5}$$

as proposed by Horvitz and Thompson, is then given by

$$\hat{Y}_{HT} = \sum_{i\varepsilon s} \frac{y_i}{\pi_i} \tag{3.1.6}$$

Where the summation extends over all distinct units u_i belonging to s (i.e., we ignore repetitions).

$\hat{Y}_{HT}$ is one of the best –known general estimate of the population total for unequal -probability sampling without replacement.

Because from evidence of Rao and Bayless (1969) compared 10 unequal-probability methods in 20 natural populations found in books and papers on sampling, with N ranging from 9 to 35.They confined themselves to methods

(a) known to have smaller variances than $\hat{Y}_{ppz}$ (probability proportional to size with replacement)
(b) providing a positive unbiased variance estimators

The result suggests that $\hat{Y}_{HT}$ is the best estimate of the population total for unequal-probability sampling without replacement.

The variance of $\hat{Y}_{HT}$ is given by

$$V(\hat{Y}_{HT}) = \sum_{i=1}^{N} y_i^2 \frac{1-\pi_i}{\pi_i} + 2\sum_{i}^{N}\sum_{j>i} y_i y_j \left[\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}\right] \tag{3.1.7}$$

Unbiased estimator of (3.1.7) is

$$V_{HT}\left(\hat{Y}_{HT}\right) = \sum_{i=1}^{n}(1-\pi_i)\left[\frac{y_i}{\pi_i}\right]^2 + \sum_{i=1}^{n}\sum_{j\neq 1}\left[\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right] \tag{3.1.8}$$

An unbiased estimator (3.1.7) is also given by Sen-Yates-Grundy (1953) as follows

$$v_{syg}\left(\hat{Y}_{HT}\right) = \sum_{i}\sum_{j}\left(\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i}-\frac{y_i}{\pi_j}\right)^2 \tag{3.1.9}$$

Where $i, j\varepsilon s$ and $i \neq j$ for both equations (3.1.8) and (3.1.9)

3.2 *The Cumulative Total Method*

In most applications the cluster units (e.g., countries, cities and city blocks) contain different numbers of elements or subunits (aerial units, households, persons). This section deals with some of the numerous methods of sample selection for cluster units of unequal sizes.

Let M_i be the number of elements in the i^{th} cluster unit.

Let $M_o = \sum M_i =$ total number of elements in the population.

If all the M_i are known, another technique developed by Hansen and Hurwitz (1943), is to select the units with proportional to their sizes M_i. One method of selecting a single unit is illustrated in the following small population of N=7cluster units.

Unit	Size M_i	$\sum M_i$	Assigned Range
1	3	3	1-3
2	1	4	4
3	11	15	5-15
4	6	21	16-21
5	4	25	22-25
6	2	27	26-27
7	3	30	28-30

The cumulative sums of the M_i are formed. To select a unit, draw a random number between 1 and M_o =30. Suppose that this 19.In the sum, number 19 falls in unit 4, which covers the range from numbers 16 to 21, inclusive. With this method of drawing, the probability that any unit is selected is proportional to its size.

Now consider n>1. Assume at present that sampling is with replacement .To select a second unit by the cumulative method, draw a new random number between 1 and 30.However, unlike sampling without replacement, we do not forbid the selection of unit 4 a second time. With this rule, the probabilities of selection remain proportional to sizes at each draw. An advantage of selection with replacement is that the formulas for the true and estimated variances of the estimates are simple.

In sampling without replacement, on the other hand, keeping the selection probabilities proportional the chosen sizes is more difficult and sooner or later becomes impossible as n increases.

3.3 The Lahiri's Method

The cumulative total method of selecting a unit is convenient when N is only moderate, or in stratified sampling when the N_h are moderate or small, but the cumulation of the Mi can be time-consuming with N large (e.g., N=20000). For this case, Lahiri (1951) has given an alternative method that avoids the cumulation.Let $M_{\max}$ be the largest of M_i. Draw a random number between 1 and N; suppose this is i. Now draw another random number m between 1 and $M_{\max}$. If m is les than or equal to M_i, the i^{th} unit is selected. If not, try another pair of random numbers. Naturally, this method involves the fewest rejections when Mi do not differ too much in size.

3.4 The Horvitz-Thompson Estimator

Much of sampling with unequal probabilities without replacement work was produced for extensive surveys in which the cluster units had first been stratified by some other principle (e.g., geographical location) into a substantial number of relatively small strata, only a small number of cluster units being drawn from each stratum. The case n_h =2,which provides one degree of freedom from each stratum for estimating sampling errors, is of particular interest.

Suppose that two units are to be drawn from a stratum. The first unit is drawn with probabilities Z_j , proportional to some measure of size. Let the i^{th} unit be selected. If we follow the most natural method, at the second draw one of the remaining units is selected with assigned probabilities $\frac{Z_j}{1-Z_i}$.

Hence the total probability π_i that the i^{th} unit will be selected at either the first or the second draw is

$$\pi_i = Z_i + \sum_{j\neq i}^{N} \frac{Z_j Z_i}{(1-Z_j)} = Z_i\left(1+\sum_{j\neq i}^{N}\frac{Z_j}{1-Z_j}\right)$$

$$= Z_i\left(1 + A - \frac{Z_i}{1-Z_i}\right) \quad (3.4.1)$$

Where A= $\sum \frac{Z_j}{1-Z_j}$ taken over all N units.

A sample of n units is selected, without replacement, by some method. Let

$\pi_i =$ Probability that the i^{th} unit is in the sample

π_{ij} = Probability that the i^{th} and j^{th} units are in the sample

The following relations hold:

$$\sum_{i=1}^{N}\pi_i = n, \sum_{j\neq i}^{N}\pi_{ij} = (n-1)\pi_i \ , \sum_{i}^{N}\sum_{j>i}\pi_{ij} = \frac{1}{2}(n-1)n \quad (3.4.2))$$

To establish the second relation, let P(s) denote the probability of a sample consisting of n specified units. Then $\pi_{ij} = \sum P(s)$ over all samples containing the i^{th} and j^{th} units, and $\pi_i = \sum P(s)$ over all samples containing the i^{th} unit. When we take $\sum \pi_{ij}\ for j \neq i$, every P(s) for a sample containing the i^{th} unit is counted (n-1) times in the sum, since there are (n-1) other values of j in the sample .This proves the second relation. The third relation follows from the second.

The Horvitz-Thompson (1952) estimator of the population total is

$$\hat{Y}_{HT} = \sum_{i=1}^{N} \frac{y_i}{\pi_i} \qquad (3.4.3)$$

Where y_i is the measurement for the i^{th} unit.

THEOREM: - If π_i >0, ($i = 1,2,..., N$)

$$\hat{Y}_{HT} = \sum_{i}^{n} \frac{y_i}{\pi_i}$$

is unbiased estimator of Y, with variance

$$v\left(\hat{Y}_{HT}\right) = \sum_{i=1}^{N} \frac{(1-\pi_i)}{\pi_i} y_i^{\ 2} + 2\sum_{i=1}^{N}\sum_{j>i}^{N} (\pi_{ij} - \pi_i \pi_j)(y_i y_j)\frac{1}{\pi_i \pi_j} \qquad (3.4.4)$$

Where π_{ij} is the probability that units i and j both are in the sample.

Proof: -Let t_i ($i = 1,2,..., N$) be a random variable that takes the value 1 if the t_i unit is drawn and zero otherwise. Then t_i follows the binomial distribution for a sample of size 1, with probability π_i. Thus

$$E(t_i) = \pi_i, V(t_i) = \pi_i(1-\pi_i) \qquad (3.4.5)$$

The value of $\text{cov}(t_i t_j)$ is also required. Since $t_i t_j$ is only 1 if both units appear in the sample,

$$\text{cov}(t_i t_j) = E(t_i t_j) - E(t_i)E(t_j) = \pi_{ij} - \pi_i \pi_j \qquad (3.4.6)$$

Hence, regarding the y_i as fixed and the t_i as random variables,

$$E\left(\hat{Y}_{HT}\right) = E\left(\sum_{i=1}^{N} \frac{y_i t_i}{\pi_i}\right) = \sum_{i=1}^{N} y_i = Y \qquad (3.4.7)$$

$$v\left(\hat{Y}_{HT}\right) = \sum_{i}^{N} (\frac{y_i}{\pi_i})^2 v(t_i) + 2\sum_{i}^{N}\sum_{j>i}^{N} (\frac{y_i}{\pi_j}\frac{y_j}{\pi_j}) \text{cov}(t_i t_j) \qquad (3.4.8)$$

$$= \sum_{i}^{N} \frac{(1-\pi_i)}{\pi_i} y_i^{\ 2} + 2\sum_{i}^{N}\sum_{j>i}^{N} \frac{(\pi_{ij} - \pi_i \pi_j)}{\pi_i \pi_j} y_i y_j \qquad (3.4.9)$$

This proves the theorem.

This variance may be expressed in another form by using the first two of the relations in (3.4.2). These give

$$\sum_{j\neq i}\left(\pi_{ij}-\pi_i\pi_j\right)=(n-1)\pi_i-\pi_i\left(n-\pi_i\right)=-\pi_i\left(1-\pi_i\right) \qquad (3.4.10)$$

Substituting for $\left(1-\pi_i\right)$ in the above,

$$\sum_i^N\frac{\left(1-\pi_i\right)}{\pi_i}y_i^{\,2}=\sum_i^N\sum_{j\neq i}^N\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{y_i}{\pi_i}\right)^2=\sum_i^N\sum_{j>i}^N\left(\pi_i\pi_j-\pi_{ij}\right)\left(\left[\frac{y_i}{\pi_i}\right]^2+\left[\frac{y_j}{\pi_j}\right]^2\right) \qquad (3.4.11)$$

Hence,

$$v\left(\hat{Y}_{HT}\right)=\sum_i^N\sum_{j>i}^N\left(\pi_i\pi_j-\pi_{ij}\right)\left(\left[\frac{y_i}{\pi_i}\right]^2+\left[\frac{y_j}{\pi_j}\right]^2-2\frac{y_i}{\pi_i}\frac{y_j}{\pi_j}\right) \qquad (3.4.12)$$

$$=\sum_i^N\sum_{j>i}^N\left(\pi_i\pi_j-\pi_{ij}\right)\left(\left(\frac{y_i}{\pi_i}\right)-\left(\frac{y_j}{\pi_j}\right)\right)^2 \qquad (3.4.13)$$

Corollary: -From above, using the t_i method, an unbiased sample estimator of $v\left(\hat{Y}_{HT}\right)$ is seen to be

$$v_1\left(\hat{Y}_{HT}\right)=\sum_I^n\left(1-\pi_i\right)\left[\frac{y_i^{\,2}}{\pi_i^{\,2}}\right]+2\sum_i^n\sum_{j>i}^n\left(\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right)y_iy_j \qquad (3.4.14)$$

Provided that none of the π_{ij} in the population vanishes.

A different sample estimator has been given by Yates and Grundy (1953) and by Sen (1953)

$$v_2\left(\hat{Y}_{HT}\right)=\sum_i^n\sum_{j>i}^n\frac{\left(\pi_i\pi_j-\pi_{ij}\right)}{\pi_{ij}}\left[\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right]^2. \qquad (3.4.15)$$

From above, this estimator is with the same restriction on the π_{ij} .

Since the term $(\pi_i \pi_j - \pi_{ij})$ often varies widely, being sometimes negative, v_1 *and* v_2 tend to be unstable quantities. Both estimators can assume negative values for some sample selection methods. Rao and Singh (1973) compared the coefficients of variation of v_1 *and* v_2 in sample s of n=2 from 34 small natural populations found in books and papers on sample surveys, using Brewer's sample selection method, for which $\pi_i = 2Z_i$ as desired.

The estimator v_2 was considerably more stable as well as being always $\geq$0 for this method, while v_1 frequently took negative values.

CHAPTER FOUR

Suggested schemes and inclusion probabilities

In this chapter we will present two schemes, which follow the principles of probability proportional to size sampling. In addition, the property of each scheme will be assessed briefly.

Further, each schemes verifies that if a single block of sample of n units is selected from BIBD (sample space) with probability proportional to size without replacement, by some method, relations (4.1) and (4.2) below holds.

Note: - This statement is equivalent to if a sample of size n is selected from N distinct and identifiable units with probability proportional to size without replacement.

Let $\pi_i =$ Probability that the i^{th} unit is in the sample

π_{ij} = Probability that the i^{th} and j^{th} units are in the sample

The following relation holds for each of the schemes:

$$\sum_{i=1}^{N} \pi_i = n, \sum_{j \neq i}^{N} \pi_{ij} = (n-1)\pi_i , \sum_{i}^{N} \sum_{j>i} \pi_{ij} = \frac{1}{2}(n-1)n \quad (4.1)$$

Then, the Horvitz-Thompson (1952) estimator of the population total is

$$\hat{Y}_{HT} = \sum_{i=1}^{N} \frac{y_i}{\pi_i} \quad (4.2)$$

Where y_i is the measurement for the i^{th} unit.

Scheme I

I. Consider a BIBD $D(v = N, b, r, k = n, \lambda)$, the parameters have their usual meaning.

As mentioned earlier that the blocks of the B.I.B design constitute the group of samples with probability of selection of i^{th} unit and $(i, j)^{th}$ unit respectively $\pi_i = \frac{r}{b}$ *and* $\pi_{ij} = \frac{\lambda}{b}$. This scheme has the following properties.

4.1 Properties

Theorem 1: -The inclusion probability π_i $(i = 1,2,...,N)$ for i^{th} unit in a sample of size n is $\frac{r}{b}$ and $\sum_{i=1}^{N} \pi_i = n$

Proof: - Following the property of B.I.B design, we will prove $\sum_{i=1}^{N} \pi_i = n$

Since $\pi_i = \frac{r}{b}$ and there are v units in the population as sampling units. So taking summation over π_i, we gets

$$\sum \pi_i = \frac{vr}{b} = \frac{bk}{b} = k = n \qquad (4.1.1)$$

Theorem 2: - The inclusion probability for a pair of units i and j in the sample of size n is $\sum_{j} \pi_{ij} = \sum_{u_i, u_j \varepsilon\ s} P(s) = (n-1)\pi_i$

$$\text{and} \qquad \sum_{i} \sum_{j<i} \pi_{ij} = \frac{1}{2}(n-1)n$$

Proof: -Following the property of B.I.B design, we know that any pair of two units occur in λ blocks. Hence inclusion probability of (i, j) unit is $\frac{\lambda}{b}$.

For any given unit, say i, will appear in sample block in (k-1) pairs. So we can say i^{th} unit will appear in a BIB design (in a sample space) with $r(k-1)$ pairs. Hence

$$\sum_{j}\pi_{ij} = \frac{r(k-1)}{b} = \pi_i(k-1) = \pi_i(n-1). \qquad (4.1.2)$$

Since the total number of pairs $\frac{v(v-1)}{2}$ can be formed out of v units.

$$\begin{aligned} Hence \quad \sum_{i}\sum_{j<i}\pi_{ij} &= \frac{v(v-1)\lambda}{2b} = \frac{vr(k-1)}{2b} \\ &= \frac{1}{2}k(k-1) \qquad (4.1.3) \\ &= \frac{1}{2}n(n-1) \end{aligned}$$

Note: - This scheme has drawback because the probabilities are not proportional to the sizes of the units, although it obeys the properties. So we will present another scheme, which takes into account the weights of the units.

Let y_i and x_i denote, respectively, the values of the character of interest and the auxiliary character of the i^{th} unit of a population consisting of N distinct and identifiable units (i=1, 2, N). Let $p = \frac{X_i}{X}$, where $X = \sum_{i=1}^{N} X_i$

The proposed scheme is the following:

Scheme II

Step1.

Consider a BIB design $D(v = N, b, r, k = n, \lambda)$, the parameters have their usual meaning Das and Giri (1986)

Step 2.

Associate with each treatment (unit) in each block a number equal to its size and then form size total for each block.

Step 3.

Now select one block by using sampling with probability proportional to size either by cumulative total method Hansen and Hurwitz or Lahiri's method Murthy (1967) described in Chapter 3. In both cases the probability of selecting i^{th} block is total of i^{th} block divided by rX i.e., $\frac{total\ \ of\ i^{th}block}{rX}$

Step 4.

The elements of the selected block constitute the required sample size. The scheme described has the following properties.

4.2 Properties

Theorem 3: - The inclusion probability π_i (i=1,2,…,N) for i^{th} unit in a sample of size n is

$$\pi_i = \frac{(rx_i + \lambda(X - x_i))}{rX} \tag{4.2.1}$$

$$Hence \quad \sum_{i=1}^{v} \pi_i = n$$

Proof: - Following the properly of BIB design we know that the i^{th} *unit* occurs with other units in r blocks and occurs with any other unit in λ block. Hence the inclusion probability of i^{th} unit in a sample of size n is

$$\pi_i = \left(\frac{rx_i + \lambda \sum_{j \neq i} x_j}{rX} \right) \tag{4.2.2}$$

$$= \left(\frac{rx_i + \lambda(X - x_i}{rX} \right), \tag{4.2.3}$$

Now taking summation over π_i, for all v=N units, we get

$$\sum_{i=1}^{v} \pi_i = \left(\frac{r\sum_{i=1}^{v} x_i + \lambda vX - \lambda \sum_{i=1}^{v} x_i}{rX} \right) \tag{4.2.4}$$

$$= \left(\frac{rX + \lambda vX - \lambda X}{rX} \right) \tag{4.2.5}$$

$$= \left(\frac{rX + \lambda(v-1)X}{rX} \right) \tag{4.2.6}$$

$$= \left(\frac{rX + r(k-1)X}{rX} \right) \tag{4.2.7}$$

$$= k = n \tag{4.2.8}$$

Theorem 4: - The inclusion probability for a pair of units i^{th} and j^{th} in the sample of size n is

$$\sum_{j=i}^{v-1} \pi_{ij} = \sum_{u_i, u_j \varepsilon\ s} P(s) = (n-1)\pi_i$$

$$and \qquad \sum_i \sum_{j<i} \pi_{ij} = \frac{1}{2} n(n-1)$$

Proof: - For any given unit, say i, will appear in a sample block in (k-1) pair. So the i^{th} unit will appear in a BIB design (in a sample space) with other units in r blocks. So the i^{th} unit will appear in total r (k-1) pairs. Hence inclusion probability of i^{th} *and* j^{th} units in a sample blocks is

$\pi_{ij} = \frac{1}{rX}(x_i + x_j + \sum_{j' \neq i, j} x_{j'}),$ is the sum of those $x_{j'}$ units, which occur with, i^{th} and j^{th} treatments. (4.2.9)

Now taking summation over j

$$\sum_{j=1}^{v-1} \pi_{ij} = \left(\frac{r(k-1)x_i + \lambda(k-1)\sum_{j \neq i} x_j}{rX} \right) \tag{4.2.10}$$

$$= (k-1)\left(rx_i + \lambda \sum_{j \neq i} x_j \right) \frac{1}{rX} \tag{4.2.11}$$

$$= (k-1)\pi_i = (n-1)\pi_i \tag{4.2.12}$$

In sampling block, we get $\left(\frac{k(k-1)}{2} \right)$ pair of sampling units, say (i, j) where $i < j = 1,2,...,v.$ So taking summation of their probabilities over (i, j), we get

$$\sum_i \sum_{j<i} \pi_{ij} = \frac{k(k-1)}{2} \; \frac{rX}{rX} \tag{4.2.13}$$

$$= \left(\frac{(k-1)k}{2} \right) = \left(\frac{(n-1)n}{2} \right) \tag{4.2.14}$$

CHAPTER FIVE

Estimation

This chapter presents the estimation of unemployed male, female and total Population. In addition the corresponding unbiased variance estimates calculated by using Sen-Yates-Grundy and Horvitz-Thompson equations, respectively.

The application of balanced incomplete block design in probability proportional to size sampling without replacement will be illustrated using secondary data's obtained from National labour Survey conducted in 2005 and population projection (same year) by Central Statistical Agency of Ethiopia. These data are of qualitative type. The characters of interests are unemployed males, females and total population of Addis Ababa city.

Notation,

Let us consider a universe (Addis Ababa city) of size 10 sub cities with the following characteristics:

We consider a finite population of N=10 units

$u_1, u_2, \ldots, u_{10}$

Let y_i and x_i denote, respectively, the values of the character of interest and the auxiliary character for the i^{th} sub-city of a population consisting of ten distinct and identifiable sub-cities (i=1…10).

The information on the number of males and females as well as the number of estimated unemployed population of males and females for each of the ten sub-cities $i, i = 1,2,...,10$ are given in Table 1..

Table 1: -Number of males and females as well as the number of estimated unemployed population of males and females in the ten sub-cities1, 2...10

U_i	Sub-city	Population			Unemployed		
		Male	Female	Total	Male	Female	Total
1	Arada	150420	164647	315067	11322	16608	27930
2	Addis Ket	189844	197857	387701	15072	17158	32230
3	Lideta	152518	165487	318005	13185	17786	30971
4	Kirkose	176036	190746	366782	15810	21225	37035
5	Yeka	133375	151750	285125	13025	20949	33974
6	Bole	106593	121435	228028	8957	15947	24904
7	AkakiKaliti	67955	73322	141277	4884	6771	11655
8	NifasS.La	131336	143787	275123	10561	18058	28619
9	KolfeKera	127107	132116	259223	11952	21927	33879
10	Gulele	151815	158850	310665	10814	15248	26062
	Total	1386999	1499997	2886996	115582	171677	287259

Source: -Central Statistical Agency

Consider now Balanced Incomplete Block Design (from appendix **I**) *that is listed in* number 27 with parameters V=10,r=9,b=18,k=5, λ =4 of (Residual 30) for the estimation of unemployed population by using Horvitz-Thompson estimator in ten sub-cities.

A correspondence can be set-up between

(i) number of treatments in a varietals trial and population size, N in sample survey i.e. v=N=10

(ii) number of blocks b in an incomplete block design and number of Groups S in a sample space , i.e., b=S=18

(iii) replications r_i of the units in an incomplete block designs and the number of times each unit occurs in the sample space i.e. r_i=r=9

(iv) the number of blocks in which the i^{th} and j^{th} treatments occur together, say λ_{ij} and the number of samples in which two given units occur together i.e., $\lambda_{ij}=\lambda$=4,where λ_{ij}>0 for all pairs i^{th} and j^{th} units.

From such correspondence, it is evident that balanced incomplete block design with parameters V=10,r=9,b=18,k=5,λ=4 can be used as a sample space. So this BIBD design can be considered as a sample space for pps sampling without replacement.

Note: -by applying block section method on D$\left(v=b=19, r=k=9, \lambda=4\right)$, which is listed in number 30 of Appendix I., we can obtain balanced incomplete block design D ($v=10, b=18, r=9, k=5, \lambda=4$)

BY block section we method refers to by which we delete from a symmetric design one block and its elements from all other blocks.

Definition: -A balanced incomplete block design is called symmetric if

(i) number of blocks equal to number of treatments i.e. ,v=b

(ii) consequently, number of replication equals number of block size i.e. r=k

To construct a balanced incomplete design with,
$v=10, b=18, r=9, k=5$, $\lambda=4$, consider a series of symmetric BIB design with
$r=\frac{v(v-1)}{2}$, since the design is symmetric, we have

$$\lambda\left(v-1\right)=r\left(r-1\right)=\left(\frac{v-1}{2}\right)\left(\frac{v-3}{2}\right)$$

Thus $v=4\lambda+3$. If we take $\lambda=t-1$, the parameters of the designs under considerations arise $v=4t-1=b, r=2t-1=k, \lambda=t-1$ (5.1)

Theorem: - If $v = 4t - 1$ is a prime or prime power; the initial block (5.1) provides a solution of the BIB design with parameters $v = 4t - 1 = b, r = 2t - 1 = k, \lambda = t - 1$

Note: - A solution of the BIB design with parameters given by (5.1) is provided by the initial block ($x, x^3, ..., x^{4t-3}$), where as before $v = 4t - 1$ is a prime or prime power and x is a primitive element of $GF\left(p^n\right)$.

A field containing a finite number of elements is called a Galois Field and is denoted by GF(s). A Galois Field can always constructed when s= p^n , where p is a prime and n is a positive integer.

Since $x = 2$ *is a primitive element of* 19 *that is* $2^{19-1} = 1$, is a prime element of $GF(19)$, a solution of the design is given by the initial block (2,8,13,14,18,15,3,12,10).

The difference arising out of this block can be exhibited as in Table 2.

Table 2: - The plan and layouts of BIB design D ($v = 19 = b, r = k = 9, \lambda = 4$).

B1	2	8	**13**	**14**	18	**15**	**3**	12	10
B2	**3**	**9**	**14**	**15**	**0**	**16**	**4**	**13**	**11**
B3	**4**	10	**15**	**16**	1	17	5	**14**	12
B4	5	**11**	**16**	17	2	18	6	**15**	**13**
B5	6	12	17	18	**3**	**0**	7	**16**	**14**
B6	7	**13**	18	**0**	**4**	1	8	17	**15**
B7	8	**14**	**0**	1	5	2	**9**	18	**16**
B8	**9**	**15**	1	2	6	**3**	10	**0**	17
B9	10	**16**	2	**3**	7	**4**	**11**	1	18
B10	**11**	17	**3**	**4**	8	5	12	2	**0**
B11	12	18	**4**	5	**9**	6	**13**	**3**	1
B12	**13**	**0**	5	6	10	7	**14**	**4**	2
B13	**14**	1	6	7	**11**	8	**15**	5	**3**
B14	**15**	2	7	8	12	**9**	**16**	6	**4**
B15	**16**	**3**	8	**9**	**13**	10	17	7	5
B16	17	**4**	**9**	10	**14**	**11**	18	8	6
B17	18	5	10	**11**	**15**	12	**0**	**9**	7
B18	**0**	6	**11**	12	**16**	**13**	1	10	8
B19	1	7	12	**13**	17	**14**	2	**11**	**9**

Note: - deleted treatments are in bold face.

Consider now the above symmetric BIB design D ($v = 19 = b, r = k = 9, \lambda = 4$). Chose any block of D and delete from D the chosen block and all treatments contained in the chosen block. Call the design containing the remaining blocks as D_1. Since one block of D is deleted, D_1 has $b_1 = v - 1$ blocks. Also, since k treatments are deleted, $v_1 = (v - k)$ treatments are there in D_1.

THEOREM: -For a symmetric BIB design with parameters $v = b, r = k, \lambda$, any two blocks have exactly λ treatments in common.

From this theorem, the number of treatments common between the deleted block and the remaining blocks of D is λ. Thus, λ treatments are deleted from each of the $v-1$ blocks of D to obtain D_1. The block size of the design D_1 is thus $k_1 = k - \lambda$. Finally, the replication of treatments and pairs of treatments not appearing in the deleted block remain unaltered. Hence, in D_1 each treatment occurs in r_1=k blocks and each pair of treatments occur together in $\lambda_1 = \lambda$ blocks of D_1. Thus, D_1 is a BIB design with parameters

$$v_1 = v - k, b_1 = v - 1, r_1 = k, k_1 = k - \lambda, \lambda_1 = \lambda$$

This process of getting a BIB design from symmetric BIB design is called the process of block section (Bose, 1939) and D_1 is called the residual of D.

Applying the method of block section to BIB design with parameters $v = 19 = b, r = k = 9, \lambda = 4$, we obtain BIB design with

$$v_1 = v - k = 19 - 9 = 10, b_1 = v - 1 = 19 - 1 = 18, r_1 = k = 9, k_1 = k - \lambda = 9 - 4 = 5, \lambda_1 = 4$$

Choosing the 2^{nd} block to be the one that is deleted, the residual design has the following block contents.

The plan and layout of BIB design with parameters $v = 10, b = 18, r = 9, k = 5, \lambda = 4,$ is given in Table 3.

Notations

Treatments	Assigned no.	Treatments	Assigned no.
1	1	8	6
2	2	10	7
5	3	12	8
6	4	17	9
7	5	18	10

Table 3: -The plan and layouts of BIB design $(v=10, b=18, k=5, r=9, \lambda=4)$

B1	2	6	10	8	7
B2	7	1	9	3	8
B3	3	9	2	10	4
B4	4	8	9	10	5
B5	5	10	1	6	9
B6	6	1	3	2	10
B7	1	2	4	7	9
B8	7	2	5	1	10
B9	9	6	3	8	2
B10	8	10	3	4	1
B11	3	4	7	5	2
B12	1	4	5	6	3
B13	2	5	6	8	4
B14	6	7	9	5	3
B15	9	7	10	6	4
B16	10	3	7	8	5
B17	4	8	1	7	6
B18	1	5	8	9	2

The plan and lay out of BIBD $(v=10, b=18, k=5, r=9, \lambda=4)$i.e., probability sampling scheme design along with each treatment (unit) in each block with sampling units is given in Table 4.

Table 4: - Plan (probability sampling design) with sampling units

B. No	Plan					Corresponding Xi's with sampling units					Sum,	P(s)
1	2	6	10	8	7	387,701	228,028	310,665	275,123	141,277	1342,794	.05167
2	7	1	9	3	8	141,277	315,067	259,223	318,005	275,123	1,308,695	.05036
3	3	9	2	10	4	318,005	259,223	387,701	310,665	366,782	1,642,376	.06321
4	4	8	9	10	5	366,782	275123	259223	310665	285125	1496918	.05761
5	5	10	1	6	9	285,125	310,665	315,067	228,028	259,223	1,398,108	.05380
6	6	1	3	2	10	228,028	315,067	318,005	387,701	310,665	1,559,466	.06001
7	1	2	4	7	9	315,067	387,701	366,782	141,277	259,223	1,470,050	.05656
8	7	2	5	1	10	141,277	387,701	285,125	315,067	310,665	1,439,835	.05542
9	9	6	3	8	2	259,223	228,028	318,005	275,123	387,701	1,468,080	.05650
10	8	10	3	4	1	275,123	310,665	318,005	366,782	315,067	1,585,642	.06102
11	3	4	7	5	2	318,005	366,782	141,277	285,125	387,701	1,498,890	.05768
12	1	4	5	6	3	315,067	366,782	285,125	228,028	318,005	1,513,007	.05833
13	2	5	6	8	4	387,701	285,125	228,028	275,123	366,782	1,542,759	.05936
14	6	7	9	5	3	228,028	141,277	259,223	285,125	318,005	1,231,658	.04740
15	9	7	10	6	4	259,223	141,277	310,665	228,028	366,782	1,305,975	.05026
16	10	3	7	8	5	310,665	318,005	141,277	275,123	285,125	1,330,195	.05118
17	4	8	1	7	6	366,782	275,123	315,067	141,277	228,028	1,326,277	.05105
18	1	5	8	9	2	315,067	2851,25	275,123	259,223	387,701	1,522,239	.05858
	Total										25982964	1

The first order inclusion probability by using equation (4.2.3) for sub-city under the probability design is given in Table 5

Table 5: -The first order inclusion probabilities for each sub-city under the probability design

Sub-city i	1	2	3	4	5	6	7	8	9	10
π_i	.5021	.5161	.5027	.5120	.4963	.4854	.4986	.4943	.4913	.5012

$$\sum_{i=1}^{10} \pi_i = 5 \quad =n$$

The second order inclusion probability by using equation (4.2.9) for sub city under the probability design is given in Table 6

Table 6: -The second order inclusion probabilities for the sub-city under the probability design

	1	2	3	4	5	6	7	8	9	10
1		.2436	.2403	.2398	.2357	.2103	.2167	.227	.2257	.2405
2			.2449	.2506	.2426	.2375	.2259	.237	.2428	.2425
3				.2469	.2157	.2196	.2086	.2127	.2147	.2347
4					.2357	.2117	.2206	.2336	.2295	.2266
5						.2147	.2135	.2126	.2136	.2136
6							.1686	.2069	.2037	.2146
7								.1985	.1785	.1897
8									.2235	.2205
9										.2135

$$\sum_{i=1}^{10} \sum_{j>i} \pi_{ij} = 10 = \frac{n(n-1)}{2}$$

Method of selecting a single unit (block)

Since all the M_i are known, Hansen and Hurwitz (1943) developed a technique to select the unit with probability proportional to the chosen size. This method of selecting a unit is called the cumulative total method. The cumulative sum of the M_i are first formed, then to select a unit, draw a random number between 1 to and M_o =25982964.Suppose that this is 14210341.In the sum, number 14210341 falls in unit 10,which covers the range from numbers 13126323 to 14711964,inclusive. With this method of drawing, the probability that any unit is selected is proportional to its size .In addition, this method of selecting a unit convenient since (N=18) is small.

The selection procedure is given in Table 7

Table 7: -Selection procedures

Block NO.	Plan	Size M_i	$\sum M_i$	Assigned RANGE
1	2,6,10,8,7	1342794	1342794	1-1342794
2	7,1,9,3,8	1308695	2651489	1342795-2651489
3	3,9,2,10,4	1642376	4293865	2651490-4293865
4	4,8,9,10,5	1496918	5790783	4293866-5790783
5	5,10,1,6,9	1398108	7188891	5790784-7188891
6	6,1,3,2,10	1559466	8748357	7188892-8748357
7	1,2,4,7,9	1470050	10218407	8748358-10218407
8	7,2,5,1,10	1439835	11658242	10218408-11658242
9	9,6,3,8,2	1468080	13126322	11658243-13126322
10	8,10,3,4,1	1585642	14711964	13126323-14711964
11	3,4,7,5,2	1498890	16210854	14711965-16210854
12	1,4,5,6,3	1513007	17723861	16210855-17723861
13	2,5,6,8,4	1542759	19266620	17723862-19266620
14	6,7,9,5,3	1231658	20498278	19266621-20498278
15	9,7,10,6,4	1305975	21804253	20498279-21804253
16	10,3,7,8,5	1330195	23134448	21804254-23134448
17	4,8,1,7,6	1326277	24460725	23134449-2226072
18	1,5,8,9,2	1522239	25982964	24460726-25982964

Five sub-cities are selected using probability-sampling design in Table 1.The information about the size of populations as well as the number of estimated unemployed populations for the sub-cities in the sample is given in Table 8

Table 8: -Size of population and number of estimated unemployed population in the selected sub-cities 1, 3, 4, 8 and 10.

Sub-city i	Number of Populations X_i ' s	Number of unemployed Populations Y_i ' s
1	315067	27930
3	318005	30971
4	366782	37035
8	275123	28619
10	310665	26062

The Horvitz-Thompson estimate of the total number of unemployed population in ten sub-cities by using equation (3.1.6)

$\hat{Y}_{HT} = \sum_{i \varepsilon s} \frac{y_i}{\pi_i}, s = (1,3,4,8,10)$, Where the summation extends over all distinct units U_i belonging to s (i.e. we ignore repetitions)

is $\hat{Y}_{HT} = \frac{27930}{.5021} + \frac{30971}{.5027} + \frac{37035}{.512} + \frac{28619}{.4943} + \frac{26062}{.5012}$ =299466.9029

The Horvitz-Thompson estimator of the total number of unemployed populations in 10 sub–cities is about 299467.

Computation of var($\hat{Y}_{HT}$):-To compute the value of var($\hat{Y}_{HT}$) for our data using probability proportional to size (pps) without replacement sampling, as long as , $0 \le \pi_i \le 1$ *and* $\sum_{i}^{N} \pi_i = n$,is given by

$$V\left(\hat{Y}_{HT}\right)=\sum_{i=1}^{N}\frac{y_i^{\,2}\left(1-\pi_i\right)}{\pi_i}+2\sum_{i}^{N}\sum_{j>i}\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j}y_i y_j \qquad (5.2)$$

Or, in a more compact form,

$$\operatorname{var}(\hat{Y}_{HT})=\sum_{i}^{N}\sum_{j>i}^{N}(\pi_i\pi_j-\pi_{ij})\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2 \qquad (5.3)$$

Where the summation extends over all distinct units U_i belonging to s (i.e., we ignore repetitions)

Equation (5.2) is due to Narian (1951) and Horvitz and Thompson (1952), and where π_{ij} the inclusion probability of both elements i *and* j. Equation (5.3) is due to Yates and Grundy (1953)

It can be seen in equation (5.2) that if the π_i's are proportional to the Y's the squared term will be zero and hence $\operatorname{var}\left(\hat{Y}_{HT}\right)$=0. Since the Y_i are unknown and usually the object of a sampling investigation any way, it will not be possible to fully utilize this principle of optimisation. However, let us assume that we know a set of M_i which is in some degree proportional to Y_i. We might then expect some reduction in $\operatorname{var}\left(\hat{Y}_{HT}\right)$ if the π_i can be made proportional to the M_i.

The effectiveness will of course, depend on the nature of the relationship between π_i fixed, it may be possible to alter the π_{ij}'s so that $\operatorname{var}\left(\hat{Y}_{HT}\right)$ is reduced. It may be noted that considerable flexibility exists in the valued the π_i *and* π_{ij} may take on. It is necessary, however, that

$\sum_{i}^{N}\pi_i = n, \sum_{i}^{N}\sum_{j>i}\pi_{ij} = \pi_i\left(\frac{n-1}{2}\right)$, and that $\pi_i > 0$ in order that unbiased estimate of Y can be obtained

1) Estimating $Var(\hat{Y}_{HT})$ from a pps without replacement sample

An unbiased estimator of (5.2) for pps without replacement sampling, has been given by Yates and Grundy (1953) and by Sen (1953) is provided by the estimator

$$V\left(\hat{Y}_{HT}\right) = \sum_{i}^{n}\sum_{j>i}^{n}\left(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 \qquad (5.4)$$

Where $i \ and \ j \ \varepsilon \ s$

The computations of $\text{var}\left(\hat{Y}_{HT}\right)$ by using Sen-Yates- Grundy equation are shown in Table 9

Table 9: - Computations of var($\hat{Y}_{HT}$) by using Sen-Yates- Grundy equation

i	j	Y_i	Y_j	π_i	π_j	$\frac{Y_i}{\pi_i}$	$\frac{Y_i}{\pi_j}$	$\frac{Y_i}{\pi_i} - \frac{Y_i}{\pi_j}$	$\left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2$
1	3	27930	30971	.5021	.5027	55626.36925	61609.30973	-5982.940	35795576.79
	4	27930	37035	.5021	.512	55626.36925	72333.98438	-16707.61	279144403.3
	8	27930	28619	.5021	.4943	55626.36925	57898.03763	-2271.668	5160477.29
	10	27930	26062	.5021	.5012	55625.36925	51999.20192	3627.167	13156342.84
3	4	30971	37035	.5027	.512	61609.30973	72333.98438	-10724.61	115018646.3
	8	30971	28619	.5027	.4943	61609.30973	57898.03763	3711.272	13773540.6
	10	30971	26062	.5027	.5012	61609.30973	51999.20192	9610.107	92354172.12
4	8	37035	28619	.512	.4943	72333.98438	57898.03763	114435.94	208396558.6
	10	37035	26062	.512	.5012	72333.98438	51999.20192	20334.78	413503377.7
8	10	28619	26062	.4943	.5012	57898.03763	51999.20192	5898.835	34796262.73

Table 9 (Cont'd)

i	j	$\pi_i\pi_j$	π_{ij}	$\pi_i\pi_j - \pi_{ij}$	$(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}})$	$(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}})\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$
1	3	.2524	.2403	.0121	.0503	1800517.513
	4	.2570	.2398	.0172	.0717	20014653.72
	8	.2481	.227	.0211	.0929	479408.3346
	10	.2516	.2405	.0111	.0461	606507.4049
3	4	.2573	.2469	.0104	.0421	4842285.009
	8	.2484	.2127	.0357	.1678	2311200.113
	10	.2519	.2347	.0172	.0732	6760325.399
4	8	.2530	.2336	.0194	.083	17296914.36
	10	.2566	.2266	.03	.1323	54706496.87
8	10	.2477	.2205	.0272	.1233	4290379.195
Total						113108687.9

$$V_{SYG}\left(\hat{Y}_{HT}\right) = \sum_{i}^{n}\sum_{j>i}^{n}\left(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$$, where i and j εs

$$\text{var}_{syg}\left(\hat{Y}_{HT}\right) = 113108687.9 \quad and$$

$$Sd_{syg}\left(\hat{Y}_{HT}\right) = 10635.25683$$

2) Estimating $Var(\hat{Y}_{HT})$ from a pps without replacement sample

An unbiased estimator of (5.2) for pps without replacement sampling, has been given by Horvitz and Thompson is provided by the estimator

$$Var_{HT}\left(\hat{Y}_{HT}\right) = \sum_{i}^{n}\left(1 - \pi_i\right)\left(\frac{y_i}{\pi_i}\right)^2 + 2\sum_{i}^{n}\sum_{j>i}^{n}\left[\frac{\pi_{ij} - \pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right]y_i y_j \qquad (5.5)$$

Where i and j ε s

The computations of $\text{var}\left(\hat{Y}_{HT}\right)$ by using Horvitz-Thompson equation are shown in Table10

Table 10: - Computations of var($\hat{Y}_{HT}$) by using Horvitz-Thompson equation

Element i	π_i	$1-\pi_i$	π_i^2	$\frac{1-\pi_i}{\pi_i^2}$	y_i	y_i^2	$(\frac{1-\pi_i}{\pi_i^2})y_i^2$
1	.5021	.4979	.25210441	1.974975368	27930	780084900	1540648462
3	.5027	.4973	.25270729	1.967889411	30971	959202841	1887605114
4	.512	.488	.262144	1.861572266	37035	1371591225	2553316185
8	.4943	.5057	.24433249	2.06972065	28619	919047161	1695198822
10	.5012	.4988	.25120144	1.985657407	26062	67997844	1348713799
Total							9025482382

Table10 (Cont'd)

i	j	Y_i	Y_j	π_i	π_j	$\pi_i\pi_j$	$\pi_{ij}-\pi_i\pi_j$	$\pi_i\pi_j\pi_{ij}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\ y\ y_j$
1	3	27930	30971	.5021	.5027	.2524	-0.0121	0.0606	-0.1996	-172657998
	4	27930	37035	.5021	.512	.257	-0.0172	.0616	-0.2792	-288801004
	8	27930	28619	.5021	.4943	.2481	-0.0211	.0563	-0.3747	-299508452.6
	10	27930	26062	.5021	.5012	.2516	-0.0111	.0605	-0.1834	-133498998.4
3	4	30971	37035	.5027	.512	.2573	-0.0104	.0635	-0.1637	-187765698.2
	8	30971	28619	.5027	.4943	.2484	-0.0357	.0528	-0.6761	-599267353
	10	30971	26062	.5027	.5012	.2519	-0.0172	.0591	-0.2910	-234885364.8
4	8	37035	28619	.512	.4943	.253	-0.0194	.0591	-0.3282	-347860711.1
	10	37035	26062	.512	.5012	.2566	-0.03	.0581	-05163	-498335945.6
8	10	28619	26062	.4943	.5012	.2477	-0.0272	.0546	-0.4981	-371517039.1
Total										-3134098565

$$Var_{HT}\left(\hat{Y}_{HT}\right) = \sum_{i}^{n} (1-\pi_i)\left(\frac{y_i}{\pi_i}\right)^2 + 2\sum_{i}^{n}\sum_{j>i}^{n}\left[\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right]y_i y_j$$

where $i \ and \ j \ \varepsilon \ s$

=9025482382+2(-3134098565)

=2757285252

Therefore,

$$Var_{HT}\left(\hat{Y}_{HT}\right) = 2757285252$$

and $$Sd_{HT}\left(\hat{Y}_{HT}\right) = 52509.85862$$

Consider now the information given in Table 11 on the number of males and females as well as the number of unemployed population male and female are available for the selected sub-cities 1, 3, 4, 8 and 10 .The problem is now to estimate the total numbers of unemployed males and females in the ten sub-cities.

Table11: -Number of males and females and number of unemployed males and females in the selected sub-cities 1,3,4,8 and 10

Sub –cities	Population		Unemployed	
	MALE	Female	MALE	Female
1	150420	164647	11322	16608
3	152518	165487	13185	17786
4	176036	190746	15810	21225
8	131336	143787	10561	18058
10	151815	158850	10814	15248

The Horvitz-Thompson estimate of the total number of unemployed males and_females in ten sub-cities are

$$\hat{Y}M_{HT} = \frac{11322}{.5021}+\frac{13185}{.5027}+\frac{15810}{.512}+\frac{10561}{.4943}+\frac{10814}{.5012} = 122598.3506$$

$$\hat{Y}F_{HT} = \frac{16608}{.5021}+\frac{17786}{.5027}+\frac{21225}{.512}+\frac{18058}{.4943}+\frac{15248}{.5012} = 176868.5523$$, respectively.

Hence, 122598.3506+176868.5523

=299466.9029

The computations of $\mathrm{var}\left(\hat{Y}F_{HT}\right)$ by using Sen-Yates-Grundy equation are shown in Table 12

Table 12: -Computation of $\mathrm{var}\left(\hat{Y}F_{HT}\right)$ by using Sen-Yates-Grundy equation

i	j	Y_i	Y_j	π_i	π_j	$\frac{Y_i}{\pi_i}$	$\frac{Y_i}{\pi_j}$	$\frac{Y_i}{\pi_i}-\frac{Y_i}{\pi_j}$	$\left(\frac{Y_i}{\pi_i}-\frac{Y_j}{\pi_j}\right)^2$
1	3	16608	17786	.5021	.5027	33077.0762	35380.9429	-2303.8667	530780.1
	4	16608	21225	.5021	.512	33077.0762	41455.0781	-8378.0019	70190915.8
	8	16608	18058	.5021	.4943	33077.0762	36532.4701	-3455.3939	11939747
	10	16608	15248	.5021	.5012	33077.0762	30422.9848	2654.0914	7044201.16
3	4	17786	21225	.5027	.512	35380.9429	41455.0781	-6074.1351	36895117.2
	8	17786	18058	.5027	.4943	35380.9429	36352.4701	-1151.5272	1326014.89
	10	17786	15248	.5027	.5012	35380.9429	30422.9848	4957.98511	24581348.5
4	8	21225	18058	.512	.4943	41455.0781	36532.4701	4922.6080	24232069.82
	10	21225	15248	.512	.5012	41455.0781	30422.9848	11032.0933	121707083.2
8	10	18058	15248	.4943	.5012	36532.4701	30422.9848	6109.4853	37325811.36

Table 12(Cont'd)

i	j	$\pi_i\pi_j$	π_{ij}	$\pi_i\pi_j - \pi_{ij}$	$(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}})$	$(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}})\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$
1	3	.2524	.2403	.0121	.0503	266982.4291
	4	.2570	.2398	.0172	.0717	5032688.666
	8	.2481	.227	.0211	.0929	11092002.498
	10	.2516	.2405	.0111	.0461	324737.6735
3	4	.2573	.2469	.0104	.0421	1553284.435
	8	.2484	.2127	.0357	.1678	222505.2989
	10	.2519	.2347	.0172	.0732	1799354.712
4	8	.2530	.2336	.0194	.083	2011261.795
	10	.2566	.2266	.03	.1323	16101847.11
8	10	.2477	.2205	.0272	.1233	4602272.541
Total						32024137.16

$$V_{SYG}\left(\hat{YF}_{HT}\right) = \sum_{i}^{n}\sum_{j>i}^{n}\left(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$$, where i *and* $j \; \varepsilon \, s$

$$\text{var}_{syg}\left(\hat{YF}_{HT}\right) = 33024137.16 \quad and$$

$$Sd_{syg}\left(\hat{YF}_{HT}\right) = 5746.6631$$

The computations of $\text{var}\left(\hat{YF}_{HT}\right)$ by using Horvitz-Thompson equation are shown in Table 13

Table 13: - Computation of $\text{var}\left(\hat{YF}_{HT}\right)$ by using Horvitz-Thompson equation are shown

Element i	π_i	$1-\pi_i$	π_i^{2}	$\frac{1-\pi_i}{\pi_i^{2}}$	y_i	y_i^{2}	$(\frac{1-\pi_i}{\pi_i^{2}})y_i^{2}$
1	.5021	.4979	.25210441	1.974975368	16608	275825664	544748892.4
3	.5027	.4973	.25270729	1.967889411	17786	316341796	622525670.5
4	.512	.488	.262144	1.861572266	21225	450500625	838639469.1
8	.4943	.5057	.24433249	2.06972065	18058	326091364	674918029.8
10	.5012	.4988	.25120144	1.985657407	15248	232501504	461668333.6
Total							3142500395

Table 13(Cont'd)

i	j	Y_i	Y_j	π_i	π_j	$\pi_i\pi_j$	$\pi_{ij}-\pi_i\pi_j$	$\pi_i\pi_j\pi_{ij}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\ y\ y_j$
1	3	16608	17786	.5021	.5027	.2524	-0.0121	0.0606	-0.1996	-58959821.64
	4	16608	21225	.5021	.512	.257	-0.0172	.0616	-0.2792	-98419340.16
	8	16608	18058	.5021	.4943	.2481	-0.0211	.0563	-0.3747	-112375251.8
	10	16608	15248	.5021	.5012	.2516	-0.0111	.0605	-0.1834	-46443992.99
3	4	17786	21225	.5027	.512	.2573	-0.0104	.0635	-0.1637	-61798035.05
	8	17786	18058	.5027	.4943	.2484	-0.0357	.0528	-0.6761	-217149519.4
	10	17786	15248	.5027	.5012	.2519	-0.0172	.0591	-0.2910	-78919470.05
4	8	21225	18058	.512	.4943	.253	-0.0194	.0591	-0.3282	-125792840.6
	10	21225	15248	.512	.5012	.2566	-0.03	.0581	-05163	-167094712.4
8	10	21225	15248	.4943	.5012	.2477	-0.0272	.0546	-0.4981	-137151030.1
Total										-1104104014

$$Var_{HT}\left(\hat{YF}_{HT}\right) = \sum_{i}^{n} (1-\pi_i)\left(\frac{y_i}{\pi_i}\right)^2 + 2\sum_{i}^{n}\sum_{j>i}^{n}\left[\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right]y_i y_j$$

where $i\ and\ j\ \varepsilon\ s$

=3142500395-2(1104104014)

=934392367

Therefore,

$$Var_{HT}\left(\hat{YF}_{HT}\right) = 934392367$$

and $Sd_{HT}\left(\hat{YF}_{HT}\right) = 30566.1964$

The computations of $\mathrm{var}\left(\hat{YM}_{HT}\right)$ by using Sen-Yates-Grundy equation are shown

are shown in Table 14

Table 14: -Computation of $\mathrm{var}\left(\hat{YM}_{HT}\right)$ by using Sen-Yates-Grundy equation

i	j	Y_i	Y_j	π_i	π_j	$\frac{Y_i}{\pi_i}$	$\frac{Y_i}{\pi_j}$	$\frac{Y_i}{\pi_i}-\frac{Y_i}{\pi_j}$	$\left(\frac{Y_i}{\pi_i}-\frac{Y_j}{\pi_j}\right)^2$
1	3	11322	13185	.5021	.5027	22549.2924	26228.3668	-3679.073	13535584.76
	4	11322	15810	.5021	.512	22549.2924	30878.9062	-8329.613	69382457.73
	8	11322	10561	.5021	.4943	22549.2924	21365.5674	118.7255	1401206.059
	10	11322	10814	.5021	.5012	22549.2924	21576.217	973.0759	946876.7072
3	4	13185	15810	.5027	.512	26228.3668	30878.9062	-4650.539	21627516.71
	8	13185	10561	.5027	.4943	26228.3668	21365.5674	4862.7994	23646818
	10	13185	10814	.5027	.5012	26228.3668	21576.217	4652.1498	21642497.72
4	8	15810	10561	.512	.4943	30878.9062	21365.5674	9513.339	90503615.93
	10	15810	10814	.512	.5012	30878.9062	21576.217	9302.6892	86540026.35
8	10	10561	10814	.4943	.5012	21365.5674	21576.217	-210.6498	44373.2539

Table 14(Cont'd)

i	j	$\pi_i \pi_j$	π_{ij}	$\pi_i \pi_j - \pi_{ij}$	$\left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right)$	$\left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2$
1	3	.2524	.2403	.0121	.0503	680839.9134
	4	.2570	.2398	.0172	.0717	4974722.219
	8	.2481	.227	.0211	.0929	130172.0429
	10	.2516	.2405	.0111	.0461	43651.0162
3	4	.2573	.2469	.0104	.0421	910518.4532
	8	.2484	.2127	.0357	.1678	3967936.06
	10	.2519	.2347	.0172	.0732	1584230.836
4	8	.2530	.2336	.0194	.083	7511800.122
	10	.2566	.2266	.03	.1323	11449245.49
8	10	.2477	.2205	.0272	.1233	5471.2222
Total						31258587.38

$$V_{SYG}\left(\hat{YM}_{HT}\right) = \sum_{i}^{n}\sum_{j>i}^{n}\left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 \text{, where } i \text{ and } j \ \varepsilon\, s$$

$$\text{var}_{syg}\left(\hat{YM}_{HT}\right) = 31258587.38 \ \textit{and}$$

$$Sd_{syg}\left(\hat{YM}_{HT}\right) = 5590.9397$$

The computations of $\text{var}\left(\hat{YM}_{HT}\right)$ by using Horvitz-Thompson equation are shown in Table 15

Table 15: - Computation of $\operatorname{var}\left(\hat{YM}_{HT}\right)$ using Horvitz-Thompson equation

Element i	π_i	$1-\pi_i$	π_i^2	$\frac{1-\pi_i}{\pi_i^2}$	y_i	y_i^2	$(\frac{1-\pi_i}{\pi_i^2})y_i^2$
1	.5021	.4979	.25210441	1.974975368	11322	128187684	253167518.4
3	.5027	.4973	.25270729	1.967889411	13185	173844225	342106209.5
4	.512	.488	.262144	1.861572266	15810	249956100	465311343.4
8	.4943	.5057	.24433249	2.06972065	10561	111534721	230845715.2
10	.5012	.4988	.25120144	1.985657407	10814	116942596	232207931.9
Total							1523638718

Table 15(Cont'd)

i	j	Y_i	Y_j	π_i	π_j	$\pi_i\pi_j$	$\pi_{ij}-\pi_i\pi_j$	$\pi_i\pi_j\pi_{ij}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}$	$\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\ y\ y_j$
1	3	11322	13185	.5021	.5027	.2524	-0.0121	0.0606	-0.1996	-29796401.77
	4	11322	15810	.5021	.512	.257	-0.0172	.0616	-0.2792	-49977028.94
	8	11322	10561	.5021	.4943	.2481	-0.0211	.0563	-0.3747	-44803494.26
	10	11322	10814	.5021	.5012	.2516	-0.0111	.0605	-0.1834	-22454782.21
3	4	13185	15810	.5027	.512	.2573	-0.0104	.0635	-0.1637	-34124058.95
	8	13185	10561	.5027	.4943	.2484	-0.0357	.0528	-0.6761	-94144751.34
	10	13185	10814	.5027	.5012	.2519	-0.0172	.0591	-0.2910	-41491533.69
4	8	15810	10561	.512	.4943	.253	-0.0194	.0591	-0.3282	-54799360.36
	10	15810	10814	.512	.5012	.2566	-0.03	.0581	-05163	-88271470.24
8	10	15810	10814	.4943	.5012	.2477	-0.0272	.0546	-0.4981	-56886334.36
Total										-516749216.1

$$Var_{HT}\left(\hat{YM}_{HT}\right) = \sum_{i}^{n} \left(1-\pi_i\right)\left(\frac{y_i}{\pi_i}\right)^2 + 2\sum_{i}^{n}\sum_{j>i}^{n}\left[\frac{\pi_{ij}-\pi_i\pi_j}{\pi_i\pi_j\pi_{ij}}\right]y_i y_j$$

where $i \ and \ j \ \varepsilon \ s$

=1523638718-2(516749216.1)

=490140285.8

Therefore,

$$Var_{HT}\left(\hat{YM}_{HT}\right) = 490140285.8$$

and

$$Sd_{HT}\left(\hat{YM}_{HT}\right) = 22139.1121$$

CHAPTER SIX

Conclusion

In Table 16 we observe that SE $_{HT}\left(\hat{Y}_{HT}\right)$ is 4.9373 times larger than SE $_{SYG}\left(\hat{Y}_{HT}\right)$ and V $_{HT}\left(\hat{Y}_{HT}\right)$ is 24.3773 times larger than $V_{SYG}\left(\hat{Y}_{HT}\right)$.

Smaller the numerical value of $V\left(\hat{Y}_{HT}\right)$ is better in the closeness of $\hat{Y}_{HT}$ to Y. The numerical values $V_{SYG}\left(\hat{Y}_{HT}\right)$ are very less than V $_{HT}\left(\hat{Y}_{HT}\right)$. SO $V_{SYG}\left(\hat{Y}_{HT}\right)$ is better estimate than V$\left(\hat{Y}_{HT}\right)$.

The numerical values of variance, standard error and their ratios of $\hat{Y}_{HT}$ by using Horvitz-Thompson and Yates are given in Table 16

Table 16 - Variance, standard errors and their ratios for estimated total number of unemployed population

	HT	SYG	Ratio= $\frac{HT}{SYG}$
Estimated variance	**2757285252**	**113108687.9**	**24.3773**
Standard error	**52509.85862**	**10635.25683**	**4.9373**

In Table 17 we observe that SE $_{HT}\left(\hat{YF}_{HT}\right)$ is 5.3189 times larger than SE $_{SYG}\left(\hat{YF}_{HT}\right)$ and V $_{HT}\left(\hat{YF}_{HT}\right)$ is 28.2911 times larger than $V_{SYG}\left(\hat{YF}_{HT}\right)$. In Table 18 SE $_{HT}\left(\hat{YM}_{HT}\right)$ is

3.9598 times of $SE_{SYG}\left(\hat{YM}_{HT}\right)$ and $V_{HT}\left(\hat{YM}_{HT}\right)$ is 15.6801 times of $V_{SYG}\left(\hat{Y}M_{HT}\right)$. Again, the discrepancies in the numerical values of V_{HT} and V_{SYG}, for both $\hat{Y}M_{HT}$ *and* $\hat{Y}F_{HT}$, to make it hard for a meaningful interpretation of our findings.

The numerical values of variance, standard error and their ratios of $\hat{Y}F_{HT}$ (estimate of unemployed females) by using Horvitz-Thompson and Yates are given in Table 17

Table 17 - Variance, standard errors and their ratios for estimated total number of unemployed females

	HT	SYG	Ratio= $\frac{HT}{SYG}$
Estimated variance	**934292367**	**33024137.16**	**28.2911**
Standard error	**30566.1964**	**5746.6631**	**5.3189**

The numerical values of variance, standard error and their ratios of $\hat{YM}_{HT}$ (estimate of unemployed males) by using Horvitz-Thompson and Yates are given in Table 18

Table 18:Variance, standard errors and their ratios for estimated total number of unemployed males

	HT	SYG	Ratio= $\frac{HT}{SYG}$
Estimated variance	**490140285.8**	**31258587.38**	**15.6801**
Standard error	**22139.1121**	**5590.9397**	**3.9598**

By observing the above results, we may conclude that our suggested scheme II may be successfully used in probability proportional to size without replacement sampling to estimate population total, variance and its standard error by using balanced incomplete block designs given in appendix **I.**

H orvitz-Thompson vs Sen-Yates-Grundy estimators

Which one of $V_{HT}(\hat{Y}_{HT})$ and $V_{SYG}(\hat{Y}_{HT})$ is more reliable? It is known that both are equal for the simple random sampling without replacement design and stratified simple random sampling design (Remark 2.8.4, page 47, Samndal, Swensson and Wretman). For general probability sampling design, there is no definitive result on reliability. Rao and Singh (1973) gave empirical evidence on overall superiority of $V_{SYG}(\hat{Y}_{HT})$ over $V_{HT}(\hat{Y}_{HT})$ for the Brewer's probability sampling design with the sample size two. Considering five artificial populations and 34 natural populations that are known, Rao and Singh (1973) observed that the gains in efficiency of $V_{SYG}(\hat{Y}_{HT})$ over$V_{HT}(\hat{Y}_{HT})$ are enormous for several of the populations. For the populations with the $V_{SYG}(\hat{Y}_{HT})$ less efficient, the losses in efficiency are small. There is indeed another criterion of non-negativity numerical values of $V_{HT}(\hat{Y}_{HT})$ and $V_{SYG}(\hat{Y}_{HT})$. The overall performance of $V_{SYG}(\hat{Y}_{HT})$ is much better than $V_{HT}(\hat{Y}_{HT})$ under the non-negative criterion. Rao and Singh (1973) proved that $V_{HT}(\hat{Y}_{HT})$ is the unique "hyper-admissible" estimator in a wide class of unbiased estimators of $V(\hat{Y}_{HT})$. But this strength of $V_{HT}(\hat{Y}_{HT})$ has been interpreted as the evidence on the weakness of the '"hyper-admissibility" criterion.

Discussions

In the numerical values of $V_{HT}(\hat{Y}_{HT})$ are strikingly different from the numerical values of $V_{SYG}(\hat{Y}_{HT})$. Considering the fact that $V_{HT}(\hat{Y}_{HT})$ and $V_{SYG}(\hat{Y}_{HT})$ are estimators of the same quantity $V(\hat{Y}_{HT})$ and yet we just cannot discard one over the other, we face an embarrassing reality of the statistical world. We have not gone to an extreme like Professor D. Basu for creating the embarrassment of the circus statistician in his famous elephant example.

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APPENDIX I

Balanced Incomplete Block Designs

With From 3 to 15 Replications

The following table gives one or more solutions for a block designs $D(v,b,r,k,\lambda)$ with 3.$\leq r \leq 20$ *and take* $k \leq \frac{v}{2}$. In every case where a solution is known, at least one solution is given. The only parameters omitted are those such as $D(7,14,6,3,2)$, which exist trivially as multiple of known designs, in this case taking $D(7,7,3,3,1)$ twice. Taking $D(v,b,r,k,\lambda)$ for t times gives a $D(v,tb,tr,k,t\lambda)$. These parameters are given in general in (12.2.2) and (12.2.3). In number 13 and 31, the complete design is given. In every other case, base blocks are given with respect to some Abelian group of automorphisms. In some instance s Boses's notation for the mixed differences approach is used as given in section 15.3.Number 88 is a good instance of Rao's notation, in which ((x,y)(mode (5,7)) means that all residues modulo 5 are to be added to x and all modulo 7 to y, but (x,y) (mod (-, 7)) means that x is to be fixed and all residues modulo 7 are to be added to y. In every case, ∞ means an element fixed by the automorphism group. When there is no design, reference is made to the appropriate theorems. In a number of cases, designs are described as residual designs of others. This is the process described in section 10.1, by which we delete from a symmetric design one block and its element from all other blocks.

Beyond r=20, a few symmetric designs are given. These are due to W.G. Bridges , Tran van Trung, Z.Janko ,V.Toncheo ,and A.L Brouwer and H. A. Wilbrink .

Number	v	b	r	k	λ	Solution
1	7	7	3	3	1	1, 2, 4 (mod7). PG (2, 2).
2	9	12	4	3	1	Residual of 3. EG (2, 3)
3	13	13	4	4	1	0, 1, 3, 9(mod11). PG (2, 3)
4	6	10	5	3	2	Residual of 5
5	11	11	5	5	2	1.2.3.4.5.9(mod13). TypeQ (See.11.6)
6	16	20	5	4	1	Residual of 7. EG (2, 4)
7	21	21	5	5	1	3, 6, 7, 12, 14(mod21). PG (2, 4)
8	10	15	6	4	2	Residual of 10
9	13	26	6	3	1	[1, 3, 9]; [2, 5, 6] (mod13) Theorem 15.3.4
10	16	16	6	6	2	[(1,0,0,0) (0,1,0,0). (0, 0, 1, 0)(0,0,0,1) (1,1,0,0)(0,0,1,1)](mod (2,2,2,2)).
11	25	30	6	5	1	Residual of 12 EG (2, 5).
12	31	31	6	6	1	1, 5, 11, 24, 25, 27(mod 31).
13	8	14	7	4	3	1,2,3,4; 1,2,7,8; 1,3,6,8; 5,6,7,8; 3,4,5,6,7; 2,4,5,7; 1,2,5,6; 1,3,5,7;1,4,6,7 2,3,5,8; 3.4.7.8; 2.4.6.8; 1,4, 5,8; 2,3.6.,7.
14	15	35	7	3	1	$[1_2,4_2,0_2,];[2_1,3_1,0_2];[1_2,4_2,0_2];[2_2,3_2,0_2];[0_1,0_2,0_3]$ $[1_3,4_2,0_1]$; $[2_3,3_3,0_1$](mod5)
15	15	21	7	5	2	Does not exist. Would be residual of 17, (Theorem 16. 1.3)
16	15	15	7	7	3	0, 1, 2, 4, 5, 8, 10(mod 15.) Type T (sec.11.6)
17	22	22	7	7	2	Does not exist. (Theorem 10.3.1.)
18	36	42	7	6	1	Does not exist. Would be residual of 19. (Theorem 12.3.3)
20	9	18	8	4	3	[0,1,2,4]: [0,1,4,6](mod9).
21	21	28	8	6	2	Does not exist. Would be residual of 23. (Theorem 16.1.3.)
22	25	50	8	4	1	[(0,0),(1,0),(0,1),(4,4)(mod(5,5) (0,0),(2,0),(0,2),(3,3)(mod(5,5)
23	29	29	8	8	2	Does not exist. (Theorem 10.3.1.)
24	49	56	8	7	1	Residual design of 25 .EG (2, 7)
25	57	57	8	8	1	1, 6, 7, 9, 19, 38, 42, 49(mod57). PG (2, 7)
26	10	30	9	3	1	(∞, 0, 5), (0, 1, 4), (0, 2, 3), (0, 2, 7) mod9). In (15.3.1)
27	10	18	9	5	4	Residual of 30
28	16	24	9	6	3	Residual of 31.Nonresidual solution. In (16.1.19)
29	19	57	9	3	1	(1, 6, 7, 11), (2, 14, 3), (4, 9, 6) mod19) (Theorem 15.3.4)
30	19	19	9	9	4	1, 4, 5, 6, 7, 9, 11, 16, 17(mod19) TypeQ (section 11.6).
31	25	25	9	9	3	a b c d e f g h i n x e q l k o g w b h j e s p n l u o u l f k l p r e c g o m j e v p y p l g o t s w x h d m x c h j u w r q a b p u v c y t e d v u w q s o f r c s a a w b v n

						f q t j n m l e s s k d h c v y t l g r m l d a q s p t j n g a d o u k h t r k q b m e o u n h l v y r q g I y p w b n k d m v w a t I u l m e J l w y g r t f b w p k v f h a j q K s u x m g f b v x f v r p t e n d I r q b x o d I j y e l s r x j k a M o f n y l h a x
32	28	63	9	4	1	Elements ∞ and (x,y,z)(mod3,3,3)) Base [0,2,1), (0,0,1)(1,1,2)(1,1,0)] [(0,2,0)(1,2,2,)(0,0,2)(1,0,0,)] [∞, (0,1,1,)(1,1,1)(2,1,1,)] All mod (3, 3, 3,). Adding (0, 0, 0)(1, 0, 0,) and (2, 0, 0) to the base gives a complete replication with last block fixed.(Also The.15.3.6)
33	28	36	9	7	2	Residual of 34
34	37	37	9	4	1	1, 7, 9, 10, 12, 16, 26, 33, 34(mod 37) Type B (the.11.6.5)
35	46	49	9	6	1	solution unknown
36	64	72	9	8	1	Residual of 37EG (2, 8)
37	73	73	9	9	1	1, 2, 4, 8, 16, 32, 37, 55, 64(mod73) PG (2, 8)
38	21	70	10	3	1	(0, 1, 13) (0, 4, 10) (0, 16, 19) (mod21) and (0, 7, 14)(mod21) and (0, 7, 4),mod 21) Per.7 Theorem 15.3.3.Als0 (15.4.16)
39	21	30	10	7	3	Residual of40
40	31	31	10	10	3	[1_1, 6_1, 2_2, 5_2, 3_3 ,4_3,3_4,5_4 ,6_4, ∞] (mod 7) [2_1,5_1,3_2,4_2,1_2,6_3,3_4,5_4,6_4, ∞_2](mod 7). [3_1,4_1,1_2,6_2,2_3,5_3,3_4,5_4,6_4, ∞_3 (mod 7). [1_1,2_1,4_1,1_2,2_2,4_2,,1_3,2_3,4_3,0_4](mod 7) [0_1,1_1,2_1,3_1,4_1,5_2,6_2, $\infty_1\infty_2\infty_3$] [0_2,1_2,2_2,3_2,4_2,5_2,6_2, $\infty_1\infty_2\infty_3$]

						$[0_3,1_3,2_3,3_3,4_3,5_3,6_3,\ \infty_1\infty_2\infty_3]$
41	36	45	10	8	2	Does not exist would be residual of 43.The.16.1.3
42	41	82	10	5	1	[1,37,16,18,10]; [8,9,5,21,39](mod 41. In 15.3.12)
43	46	46	10	10	2	Does not exist. (Theorem 10.3.1.)
44	51	85	10	6	1	Solution unknown.
45	81	90	10	9	1	Residual of 46.EG (2, 9)
46	91	91	10	10	1	0,1,3,9,27,49,56,61,77,8(mod 91). PG (2, 9)
47	12	44	11	3	2	[0,1,3];[4,5,9];2,8,6];[∞ ,7,10](mod11).Second solution:[0.1.3]:
						[0,1,4];[0,2,6][∞ ,0,5](mod11).Also (15.3.17)
48	12	33	11	4	3	[0,1,3,7]; 2,4,9,10];[∞ ,5,6,8](mod 11]
49	12	22	11	6	5	[0,1,3,7,8,10];[∞ ,0,5,6,8,10](mod 11).
50	23	23	11	11	5	1,2,4,6,8,9,12,13,16,18(mod 23.Type Q (section 11.6).
51	45	99	11	5	1	Elements (x,y,z)(mod(3,3,4)).
						[(0,1,0),(0,2,0),(1,0,2),(2,0,2),(0,0,1)](mod (3, 3, 5).
						[(2,1,0),(1,2,0),(2,2,2),(1,1,2)(0,0,1)](mod (3, 3, 5))
						{(0,0,0),(0,0,1),(0,0,2),0,0,3),0,0,4)](mod(3,3,-))
52	45	55	11	9	2	Residual of 53.
53	56	56	11	11	2	See Section 15.8.5.
54	100	110	11	10	1	Solution unknown. Residual of 55.EG (2, 10).
55	111	`111	11	11	1	solution unknown PG (2, 10)
56	13	26	12	6	5	[0,1,3,6,7,11];[0,1,2,3,7,11](mod 13).
57	19	57	12	4	2	[0,1,3,12]:[0,1,5,13]:[0,4,6,9}mod 19).
58	21	42	12	6	3	[0,2,10,15,19,20]:[0,3,7,9,10,16](mod 21).
59	22	33	12	8	4	Solution unknown
60	25	100	12	3	1	[0,1,3];[0,4,13];[0,5,11];[0,7,17](mod 25).
						A second solution;
						[(0,1),(4,1),(1,3)];[(1,0),(3,3),(1,2)];
						[(3,2),(2,1),(0,2)];[(1,1),(2,4),(2,0)](mod(5,5)
						Residual of 64
61	33	44	12	9	3	Residual of 64
62	34	34	12	12	4	Does not exist. (Theorem 10.3.1)
63	37	111	12	4	1	Elements (x,y)x= 0,1,2,(mod3)y = 0…10(mod 11). Also
						y=∞ and (x,y)=(∞,∞)
						[(0,0),(0,1),(1,2),(1,5)]:[(0,1)(0,3)(0,8)(1,0)].
						{(0,∞), (0, 7), (1, 5), (2, 1)] (mod (3, 11)).
						$[(\infty,\infty(0,0)(1,0)(2,0)(mod(-,11))]$
			`			$[(0,\infty),(1,\infty)(2,\infty),(\infty,\infty)]$
64	45	45	12	12	3	Case t=3 of Sane's(15,7,3).
65	55	66	12	10	2	Does mot exist. Would be residual of 67.(Theorem 16.1.3)
66	67	122	12	6	1	Solution unknown.
67	67	67	12	12	2	Does not exist (The. 10.3.1).
68	121	132	12	11	1	Residual of 69.EG (2, 11)
69	133	133	12	12	1	1,8,9,11,25,37,69,88,94,99,103,121(mod 133). PG (2, 11)
70	27	117	13	3	1	[0,1,22]:[0,2,8][0,3,14];[0,7,17](mod 26).
						[[,0,13](mod26)period 13.Lines in EG(3,3).
71	27	39	13	9	4	Residual of 75. Planes in EG (3, 3).
72	27	27	13	13	6	[(0,0,1,),(1,0,0,),(1,2,0),(1,1,1),(2,0,2),(1,1,0)

						(1,0,2),(0,2,0),(0,2,1),(1,2,1),(2,1,1,),(0,2,2)
						(2, 2, 1)](mod (3,3,3)). Type q (section 11.6).
73	40	130	13	4	1	[0,1,26,32];[0,7,19,36];[0,3,16,38](mod 40).
						[0, 10, 20, 30](mod 40) period 10. Lines in PG (3, 3).
74	40	52	13	10	3	Solution unknown.
75	40	40	13	13	4	1,2,3,5,6,9,14,15,18,20,25,27,25(mod 40).Planes in PG(3,3).
76	53	53	13	13		Does not exist. (Theorem 10.3.1.)
77	66	143	13	6	1	A fixed other subscripts $i \rightarrow i+1 (\mathrm{mod} 13)$ I
						Base blocks. A Bo Co Yo Zo
						$B_3 B_{10} D_5 D_6 Y_0 Y_2 C_9 C_4 B_2 B_{5_5} Y_0 Y_6 D_1 D_{12} C_6 C_2 Y_0 Y_5$
						$B_4 B_6 C_1 D_{11} Y_0 Z_1 C_{12} C_5 D_8 B_7 Y_0 Z_{_3 3} D_{10} D_2 B_{11} C_8 Y_0 Z_9$
						$C_{11} D_3 D_9 D_{12} Z_0 Z_1 D_7 B_9 B_1 B_{10} Z_1 Z_3 B_8 C_1 C_3 C_4 Z_0 Z_9$
						$Y_2 Y_6 Y_5 Z_4 Z_{12} Z_{10}$
78	66	78	13	11	2	Residual of 79.
79	79	79	13	13	2	See section 15.8.6.
80	144	156	13	12	1	Solution unknown. Residual of 81.EG (2, 12).
81	157	157	13	13	1	Solution unknown. PG(2,12).
82	15	42	14	5	4	[0,1,4,9,11];[0,1,4,10,12];[[,0,1,2,7](mod 14). Double unsolvable case 15
						$[\infty, 0_0, 0_1, 1_1, 2_1, 4_1];[\infty, 0_1, 0_0, 6_0, 5_0, 3_0]$
83	15	53	14	6	5	$[1_0, 2_0, 4_0, 0_1, 1_1, 3_1];[2_0, 3_0, 5_0, 0_1, 1_1, 3_1]$
						$[0_0, 4_0, 5_0, 0_1, 1_1, 3_1](\mathrm{mod}\, 7)$
						$[0_0, 3_0, 9_0, 10_0];[0_0, 0_1, 2_1, 7_1];[0_0, 0_1, 9_1, 10_1]$
84	27	77	14	4	2	$[0_0, 2_0, 5_1, 8_1{}^1];[0_0, 3_0, 4_1, 7_1];[0_0, 4_0{}^0, 3_1, 9_1]$
						$[0_0, 5_0, 2_1, 6_1](\mathrm{mod} 11)$
85	22	44	14	7	4	(1,2,3,7,13,16,21);(1,3,4,5,11,16,20)
86	29	58	14	7	3	[1,7,16,20,23,24,25];[2,3,11,17,19,21](mod 29)
87	36	84	14	6	2	[0,1,3,5,11,23];[0,5,8,9,18,24](mod 35).
						And [∞ , 0, 7, 14, 21, 28] (mod 35) period 7 taken twice.
						Double the unsolvable case 18
88	43	86	14	7	2	Elements (x,y)x=0,…4(mod 5)and x= ∞ y=0..6.(mod 7) and y= ∞
						[(∞ , 0), (0,1), (0,6),(1,5),(1,2),(2,3),(,2,4)](mod (5,7)).
						[(∞ , 0), (0, 1), (0, 6), (3, 5), (3, 2), (1, 3) (1, 4)](mod (5, 7)).
						[(∞ ,0),(∞ , ∞),(0,0),(1,0),(2,0),(3,0,),(4,0,)mod(-,7)] taken twice.
						$[(\infty,0),(\infty,1),(\infty,2),(\infty,3),(\infty,4),(\infty,5),(\infty,6),$
						*taken twice.Double the unsolvable case*19
89	78	91	14	12	2	Does not exist. Would be residual of 91.(Theorem 16.1.3).
90	85	170	14	7	1	Solution unknown.
91	92	92	14	14	2	Does not exist. (Theorem 10, 3, 1)
92	11	55	15	3	3	[0,1,3]; [0,1,5]; [0,2,7]; [0,1,8]; [0,3,5](mod 13).
93	13	39	15	5	5	[0,1,2,4,8];[0,1,3,6,12];[0,2,5,6,10](mod 13).

94	16	80	15	3	2	[0,1,3];[0,3,8];[0,2,12];[0,1,7];[0,4,9](mod 16). Second solution; $[0_1,1_1,2_1];[0_1,2_1,5_1];[0_0,7_0,0_1];$ $[1_0\ ,6_0,0_1];[2_0,5_0,0_1];[3_0,4_0,0_0];$ $[1_0,7_0,0_1];[2_0,6_0,0_1];[3_0,5_0,0_1];$ $[0_0,0_1,4_1];[\mathrm{mod}\,8]$
95	16	48	15	5	4	[0,1,2,4,7];[0,1,8,5,10];[0,1,3,7,11](mod 16).
96	16	40	15	6	5	[0,1,3,5,9,12];[0,1,2,3,6,12](mod 160. [0, 8, 1, 9, 2, 10](Mod16) period 8
97	16	30	15	8	7	[∞, 0,1,2,7,9,12,13]; [3,4,5,6,8,10,11,14](mod 15).
98	21	35	15	9	6	$[0_0,1_0,2_0,4_0,0_1,1_1,2_1,4_1,2_2,];$ $[0_0,6_0,5_0,3_0,6_2,4,_2 3_2,0_1];$ $[1_0,6_1,5_1,3_1,6_2,4_2,3_2,2_2,0_0];$ $[4_0,1_0,3_0,0_1,2_1,6_1,4_2,1_2,2_2];$ $[0_0,2_0,6_0,4_1,1_{,1}3_1,4_2,1_2,2_2](\mathrm{mod}\,7)$
99	26	65	15	6	3	$[\infty,(0,0),(1,3),(2,1),(3,4),(4,2)(\mathrm{mod}(-,5))$ $,[\infty,(0,0),(1,2),(2,4),(3,1),(4,3)](\mathrm{mod}(-,5)).$ $[\infty,(0,0),(1,0),(2,0),(3,0),(4,0)]\,\mathrm{mod}(-,5))$ $.[(0,1),(0,4),(1,2)(1,3),(2,1)(2,4)]\,\mathrm{mod}(5,5)$ $[(0,1),(0,4)(2,2),(2,3),(3,2),(3,3)]\,\mathrm{mod}(5,5)]$
100	28	42	15	10	5	Solution unknown.
101	31	155	15	3	1	[0,1,18];[0,2,5,];[0,4,10];[0,8,20];[0,9,16](mod 31). Lines in PG (4, 2) STEINER TRIPLE SYSTM
102	31	93	15	5	7	1,2,4,8,16];[3,6,12,24,17];[9,8,5,10,20](mod13).
103	31	31	15	15	7	1,2,4,5,7,8,9,10,14,16,18,19,20,25,28,(mod 31). TypeQ (Section 11.6). 1,2,3,4,6,8,12,15,16,17,23,24,27,29,30(mod 31). Type H6 (section 11.).
104	36	36	15	15	6	(0,1),(0,2),(0,3),(0,4),(0,5),(1,0),(2,0),(3,0),(4,0),(5,0),(1,1),(2,2) (3,3), (4,4),(5,5)(mod(6,6)).
105	43	43	15	15	2	Solution unknown.
106	46	69	15	10	3	Solution unknown.
107	56	70	15	12	31	Residual of 109.
108	61	183	15	5	1	[1,9,20,58,34];[4,36,19,49,14];[16,22,15,13,56](mod 61).
109	71	71	15	15	3	Two nonisomorphic designs with the same residual design with respect to block 71. 71: 57,58,59,60,61,62,63,64,65,66,67,68,69,70,71. Other blocks from bases 1: 1 10 15 16 17 22 33 35 41 42 47 49 57 58 59 2: 1 2 4 8 9 11 15 16 18 29 30 31 60 61 62 Collinations.

$$\alpha = (1,8)(2,11)(3,14)(4,9)(5,13)(6,12)(7,10)$$
$$(15,30)(16,29)(17,35)(18,31)(19,34)(20,33)(21,32)(22,51)$$
$$(23,50)(24,54)(25,52)(26,56)(27,74)$$
$$(28,53)(36,44)(37,43)(38,47)(39,49)(40,45)(41,48)(42,46)$$
$$(57,67)(58,64)(59)(60)(61)(62)(63,70)(65)(66,71)(68)(69)$$
$$\beta = (1,2,3,4,5,6,7)(8,15,22,29,36,43,50)$$
$$(9,17,25,33,41,49,51)(10,18,26,34,42,44,52)$$
$$(11,19,27,35,37,45,53)(12,20,28,30,38,46,54)$$
$$(13,21,23,31,39,47,55)(14,16,24,32,40,48,56)$$
$$Firstdesign:(62)(57,70,59,64,65,60,71)$$
$$(58,68,61,66,67,63,69)$$
$$Secoddesign:(66)(57,71,58,67,64,6370)$$
$$(59,65,62,61,68,60,69)$$

110	76	190	15	6	1	Subscripts (mod19), Base blocks. $A_0 B_0 B_1 B_3 B_{14} D_{10}$ $A_0 B_{13} C_0 C_2 C_3 C_7$ $A_0 C_{15} D_0 D_2 D_{11} D_{14}$ $A_1 A_3 B_0 B_7 C_0 D_2$ $A_2 A_7 B_{14} C_0 C_{11} D_0$ $A_{11} A_{14} B_0 C_3 D_0 D_1$ $A_1 A_7 A_{11} B_3 C_2 D_{14}$ $B_7 B_{16} C_{11} C_{17} D_1 D_5$ $B_{13} B_{17} C_5 C_{15} D_{10} D_{16}$
111	91	195	15	7	1	[0,10,27,28,31,43,50];[0,11,20,25,49,55,57](mod 91). [0, 13, 26, 39, 52, 65, 78](mod 91) period 13.
112	91	105	15	13	2	Does not exist. Would be residual of 113.(Theorem 16,1,3).
113	106	106	15	2	2	Does not exist.(Theorem 10.3.1.)
114	136	204	15	10	1	Solution unknown.
115	196	210	15	14	1	Does not exist. Would be residual of 116. (Theorem 12.3.3.)
116	211	211	15	15	1	Does not exist. (Theorem 10,3,1.)
117	33	176	16	3	1	(0,1,10)(0,4,7)(0,13,31)(0,16,28)(0,19,25)(mod 33) and (0,11,22)(mod 33) period 11.
118	49	196	16	4	1	(0,1,10,36)(0,3,8,20)(0,4,6,25)(0,2,18,34)(mod 49).
119	17	68	16	4	3	(0,1,3,7)(0,1,5,7)(0,1,6,9)(0,2,5,9)(mod 17).
120	65	208	16	5	1	(00,01,02,03,04)(mod 13)(10,120,51,81,03)(20,110,31,

						101,103) (40, 90, 61, 61, 71, 03) (mod 13, 5)
121	81	216	16	6	1	Solution unknown.
122	21	56	16	6	4	Base block 1, 3, 7,11,14,18 Automorphisms α =(1,10,3,11,7,6,2)(4,13,20,16,15,8,14)(5,12,17,21,19,9,18) β =(1)(2,9)(3,7)(4,6)(5,8)(10,20)(11,18)(12,19)(13,21)(14)(15)(16) (17) Ovals in a plane of order 4.
123	113	226	16	8	1	Solution unknown.
124	29	58	16	8	4	(0,1,7,16,20,23,24,25)(0,2,3,11,14,17,19,21)(mod 29.
125	17	34	16	8	7	(0,1,3,7,8,12,14,15)(0,2,3,4,7,8,9,11)(mod 17.
126	145	232	16	10	1	Solution unknown.
127	25	40	16	10	6	Residual of 139
128	33	48	16	11	5	Residual of 138.
129	177	236	16	12	1	Solution unknown.
130	45	60	16	12	4	Residual of 137.
131	65	80	16	13	2	Solution unknown. Residual of 136.
132	105	120	16	14	2	Solution unknown. Residual of 135.
133	225	240	16	15	1	Solution unknown. Residual of 134.
134	241	241	16	16	1	Solution unknown. Plane of order 15.
135	121	121	16	16	2	Solution unknown.
136	81	81	16	16	3	Solution unknown.
137	61	61	16	16	4	Base blocks 10 11 12 19 20 21 28 29 30 37 38 39 46 47 48 49 13 14 15 22 23 24 31 32 33 40 41 42 46 47 48 49 16 17 18 25 26 27 34 35 36 43 44 45 46 47 48 49 10 13 16 19 22 25 28 31 34 37 40 43 50 51 52 53 11 14 17 20 23 26 29 32 35 38 41 44 50 51 52 53 12 15 18 21 24 27 30 33 36 39 42 45 50 51 52 53 12 13 17 19 23 27 28 32 36 39 40 44 54 55 56 57 10 14 18 20 24 25 29 33 34 37 41 45 54 55 56 57 11 15 16 21 22 26 30 31 35 38 42 43 54 55 56 57 11 13 18 19 24 26 28 33 35 38 40 45 58 59 60 61 12 14 16 20 22 27 29 31 36 39 41 43 58 59 60 61 10 15 17 21 23 25 30 32 34 37 42 44 58 59 60 61 Automorphisms (1, 14, 19, 28, 41)(2, 15, 20, 29, 42)(3, 13, 21, 30, 40)(4, 17, 23, 31, 44) (5, 18, 23, 32, 45)(6, 16, 24, 33, 43)(7, 11, 25, 34, 38)(8, 12, 26, 35, 39) (9, 10, 27, 36, 37)(46)(47, 54, 51, 53, 57)(48, 58, 56, 55, 59) (49, 50, 61, 52) Also a final block B: 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61
138	49	49	16	16	5	Base blocks 1,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19 2,5,6,7,8,9,25,26,27,28,29,30,31,32,33,34 1,2,10,11,15,18,20,25,27,31,32,35,40,41,47,49 3,4,5,12,13,16,19,27,28,31,33,39,40,44,47,40 5,10,11,15,17,21,28,29,30,33,35,36,38,39,43,47 Automorphisms (1)(33, 4)(5,11,17,8,14,15,6,12,18,9,10,16,7,13,19) (20,26,32,23,29,30,21,27,33,24,25,31,22,28,34)

						(35,41,47,38,44,45,36,42,48,39,40,46,37,43,49)
139	41	41	16	16	6	See Section 17.6.
140	18	102	17	3	2	(∞, 10,8),(0,1,3)(0,1,7)(0,4,10)(0,4,9(mod 17).
141	52	221	17	4	1	(0,13,26,39)(mod 52) period 13(0,3,5,19)(0,6,7,24) (0, 4, 12, 27)(0, 9, 20, 30)(mod 52).
142	35	119	17	5	2	(00,01,02,03,04)(00,01,02,03,04)(mod 7, -) (10, 60, 31, 41, 03)(30, 40, 21, 51, 03)(20, 50, 61, 11, 03)(mod(7, 5).
143	18	51	17	6	5	(∞ , 0,2,8,9,12)(0,1,2,4,7,15)(0,1,3,7,8,12)(mod 17).
144	35	85	17	7	3	Solution unknown
145	120	255	17	8	1	Automorphisms on points 0…119 (m,m+1,m+3,m+4,m+5,m+6,m+7,m+8,m+9,m+10,m+11 m+12, m+13, m+14).m+0, 15, 30, 45,60,75,90,105 Base blocks (0,15,30,45,60,75,90,105) (0,7,31,32,50,56,108,113) (0,21,43,57,71,78,100,107) (15,17,36,40,48,56,102,103) (0,27,41,54,67,81,95,109) (15,19,47,59,81,82,106,115) (0,1,63,65,84,88,111,119) (15,20,31,33,66,69,118,119) (0,2,18,26,35,38,76,82) (15,21,63,73,76,89,95,99) (0,3,22,23,99,104,115,117) (30,35,52,56,64,72,81,84) (0,4,17,29,51,52,61,70) (30,36,78,88,91,104,110,114) (0,5,34,42,73,74,92,98) (45,55,62,66,97,104,106,109) (0, 6, 46, 59, 77, 85, 94, 97)
146	18	34	17	9	8	(∞ , 1,2,4,8,9,13,15,16)(0,1,4,8,9,13,15,16)(mod 17).
147	52	68	17	13	4	Residual of 152.
148	120	136	17	15	2	Residual of 151. Does not exist.
149	256	272	17	16	1	Residual of 150.
150	273	273	17	17	1	(1,2,4,8,16,32,64,91,117,128,137,182,195,205,234,239,256) (mod 273). Plane of order 16.
151	137	137	17	17	2	Does not exist. Theorem 10.3.1.
152	69	69	17	17	4	Points $W, X, Y, Z, A_i, B_i, C_i, D_i, E_i, i = 0,\ldots,12 by Automorphisms$
153	35	35	17	17	8	(0,1,3,4,7,9,11,12,13,14,16,17,21,27,28,29,33)(mod 35).
-154	37	222	18	3	1	(0,1,8)(0,2,14)(0,3,19)(0,4,13)(0,5,15)(0,6,17)(mod 37).
155	91	273	18	6	1	(0,1,3,7,25,38)(0,5,20,32,46,75)(0,8,17,47,57,80)(mod 91).
156	46	138	18	6	2	Points A and 0, 1…44 Automorphisms

$$i \to i + 1 \bmod (13) \quad all \quad po\text{int}\, s$$

Baseblocks

$WXYZA_0 A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 A_{10} A_{11} A_{12}$

$WXYZB_0 B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10} B_{11} B_{12}$

$WXYZC_0 C_1 C_2 C_3 C_4 C_5 C_6 C_7 C_8 C_9 C_{10} C_{11} C_{12}$

$WXYZD_0 D_1 D_2 D_3 D_4 D_5 D_6 D_7 D_8 D_9 D_{10} D_{11} D_{12}$

$WA_1 A_3 A_9 B_2 B_5 B_6 C_4 C_{10} C_{12} D_7 D_8 D_{11} E_0 E_4 E_{10} B_{12}$

$XA_2 A_5 A_6 B_4 B_{10} B_{12} C_7 C_8 C_{11} D_1 D_3 D_9 E_0 E_4 E_{10} E_{121}$

$YA_0 A_{10} A_{12} B_7 B_8 B_{11} C_1\, C_2 C_9 D_2 D_5 D_6 E_0 E_4 E_{10} E_{12}$

$ZA_7 A_9 A_{11} B_1 B_3 B_9 C_2 C_5 C_6 D_4 D_{10} D_{12} E_0 E_4 E_{10} E_{12}$

$A_0 A_1 A_3 A_9 B_0 B_1 B_3 B_9\, C_0 C_1 C_3 C_9 D_0 D_1 D_3 D_9 E_0$

$$\beta = (A)(0,1,5)(2,8,7)(3,4,6)(9,10,14)(11,17,16)(12,13,15)$$
$$(18,19,23)(20,26,25)(21,22,24)(27,28,32)(29,35,34)$$
$$(30,31,33)(36,37,41)(38,44,43)(39,40,42)$$
$$\gamma = (A)(0,2,6)(1,8,3)(4,5,7)(9,11,15)(10,17,12)$$
$$(13,14,16)(18,20,24)(19,26,21)(22,23,25)$$
$$(27,29,33)(28,35,30)(31,32,34)(36,38,42)(37,4439)$$
$$(40,41,43)$$
$$\alpha = (A)(o)(1,2,3,4,5,6,7,8)(9)(10,11,12,13,14,15,16,17)$$
$$(18)(19,20,21,22,23,24,25,26)(28,29,30,31,32,33,34,35$$
$$(36)(37,38,39,40,41,42,43,44)$$

Base blocks

1,5,11,21,34,42	72 images under (α, β, γ)
9, 10, 20,21,31,34	9 images under (β, γ)
9, 16, 19,26,40,42	9 images under (β, γ)
9, 17, 28,29,41,43	9 images under (β, γ)
18, 25, 31,33,38,41	9 images under (β, γ)
9, 10, 11,12,15,17	3 images under (β)
18, 19, 20,23,25,26	3 images under (γ)
27, 28, 31,33,34,35	3 images under (β)
36, 37, 38,39,40,43	3 images under (β)

Also blocks

						$B_i : A, 9J + i,\ j = 0,1,2,3,4,\ i = 0,...,8 each B_i takes twice$.
157	19	57	18	6	5	(0,6,7,10,11,17)(0,1,3,12,14,15)(0,2,5,6,9,11)(mod 19).
158	145	290	18	9	1	Solution unknown.
159	49	98	18	9	3	Solution unknown.
160	55	99	18	10	3	Solution unknown.
161	100	150	18	12	6	Solution unknown.
162	34	51	18	12	6	Solution unknown. Residual of nonexistent 169.
163	85	102	18	15	3	Solution unknown.
164	136	153	18	16	2	Solution unknown. Residual of 167.
165	289	306	18	17	1	Residual of 166.
166	307	307	18	18	1	(0,1,3,30,37,50,55,76,98,117,129,133,157,189,199,222, 293,299)(mod 307). Plane of order 17.
167	154	154	18	18	2	Solution unknown.
168	103	103	18	18	3	Does not exist. Theorem 10.3.1.
169	52	52	18	18	6	Does not exist. Theorem 10.3.1
170	39	247	19	3	1	(0, 13, 26) (mod 39) period 13 (0, 1, 25)(0, 4, 22)(0, 7, 19)(0, 10, 16)(0, 31, 34)(mod 39).
171	20	95	19	4	3	(,0,1,6)(0,1,3,7)(0,1,8,11)(0,2,5,9)(0,2,6,11)(mod 19)
172	20	76	19	5	4	(,0,2,3,7)(0,1,4,9,11)(0,2,3,7,13)(0,4,5,7,13)(mod 19)
173	96	304	19	6	1	Points (h,j)(mod(48,2)) Base blocks B_i ={(o,I),(8,I),(16,I),(24,I),(32,I),(40,I),(mod (48,2))period 8 C_1 = {(0,0),(1,0),(3,0),(13,0),(28,0),(0,1)}(mod 48,-) C_2 = {(0,0),(4,0),(11,0),(17,1),(36,1),(38,1),(38,1)}(mod 48-1) C_3 = {(0,0),(5,0),(19,0),(1,1),(24,1),(42,1)}(mod 48, -1) C_4 ={(0.0),(9,0),(26,0),(4,1),(7,1),(40,1)}(mod 48, - 1) C_5 = {(0,0),(6,0),(8,1),(9,1),(18,1),(22,1)}(mod 48, -) C_6 = {(0,0),(6,0),(8,1),(9,1),(18,1),(22,1)} (mod 48, -) C_7 = {(0,0),(18,0),(11,1),(28,1),(33,1),(39,1)} (mod 48, -)
174	153	323	19	9	1	(000,100,210,220,200,120,110,010) (mod (-, -, 7)) (000, 011, 0116, 219, 218, 0213, 024, 1215, 122)(mod (3, 3, 17)) (000, 019, 018, 2113, 214, 0215, 022, 1216, 121)(mod (3, 3, 17))
175	20	38	19	10	9	Residual of 184.
176	39	57	19	13	6	Unknown Residual of nonexistent 183.
177	96	144	19	16	3	Unknown Residual of 182.
178	153	171	19	17	2	Does not exist. Residual of 181.
179	324	342	19	18	1	Unknown. Residual of 180.
180	343	343	19	19	1	Solution unknown. Plane of order 18.
181	172	172	19	19	2	Does not exist. Theorem 10.3.1.
182	115	115	19	19	3	Solution unknown.
183	58	58	19	19	6	Does not exist. Theorem 10.3.1

184	39	39	19	19	9	

$$\begin{pmatrix}0_0,1_0,4_0,5_0,6_0,7_0,9_0,11_0,16_0,17_0\\ ,2_1,3_1,8_1,10_1,12_1,13_1,14_1,15_1,18_1\end{pmatrix}$$

$$\begin{pmatrix}\infty,1_0,4_0,5_0,6_0,7_0,9_0,11_0,16_0,17_0,\\ 1_1,4_1,5_1,6_1,7_1,9_1,11_1,16_1,17_1\end{pmatrix}(\mathrm{mod}(19))$$

$$and\ \begin{pmatrix}0_1,1_1,2_1,3_1,4_1,6_1,7_1,8_1,9_1,\\ 0_1,11_1,12_1,13_1,14_1\ ,15_1,16_1,17_1,18_1\end{pmatrix} fixed$$

185	61	305	20	4	1	(0,3,18,23)(0,4,6,33)(0,7,8,24)(0,9,19,30)(0,13,25,39)(mod 61)
186	31	155	20	4	2	(0,1,8,11)(0,1,13,17)(0,2,11,14)(0,5,7,13)(0,5,9,15)(mod 31)
187	21	105	20	4	3	(0,2,3,7)(0,3,5,9)(0,1,7,11)(0,2,8,11)(0,1,9,14)(mod 21)
188	11	155	20	4	6	(0,1,8,9)(0,2,5,)(0,2,3,5)(0,4,5,9)(mod 11)
189	81	324	20	5	1	(0,0,0,1)(2001,0021,1202,0111)(0010,1102,1210,1201,1110) (2211, 1222, 2021, 1100)(0202, 1101, 1212, 1210, 0211)(mod(3.3.3.3))
190	17	68	20	5	5	(0,1,4,13,16)(0,3,5,12,14)(0,2,8,9,15)(0,6,7,10,11)(MOD 17)
191	51	170	20	6	2	Points (h,i)(mod 5,10)and A Blocks (0,0)(0,1)(0,2)(1,3)(3,6)(4,5)(mod 5,10) (0, 0)(0, 2)(0, 6)(1, 1)(1, 8)(3, 3)(mod 5, 10) (0, 0)(0, 5)(2, 1)(2, 4)(3, 8)(4, 2)(Mod 5, 10) and blocks B A (i,j),=0,1,2,3,4 and J=0…..9each taken twice.
192	21	70	20	6	5	(2,4,5,6,10,15)(0,4,7,8,10,20,)(2,4,5,10,16,19)(mod 21and (1, 6, 8, 13, 15, 20)(Mod 21) period 7
193	36	90	20	8	4	Solution u unknown.
194	181	362	20	10	1	Solution unknown.
195	61	122	20	10	3	Solution unknown
196	46	92	20	10	4	Solution unknown.
197	37	74	20	10	5	(0,1,7,9,10,12,16,26,33,34)(0,2,14,15,18,20,24,29,31,32) (Mod 37)
198	21	42	20	10	9	Derived design of symmetric (43, 21, 10) (design given by (0,3,5,8,9,10,12,13,14,15,16,20,22,23,24,30,34,35,37,39,40(mod 43)
199	111	185	20	12	2	Solution unknown.
200	45	75	20	12	5	Solution unknown.
201	141	188	20	15	2	Solution unknown.
202	57	76	20	15	5	Solution unknown.Residual of nonexistent 210.
203	36	48	20	15	8	Automorphisms

$$\alpha=(1,2,3,4)(5,6,7,8)(9,10,11,12)(13,14,15,16)(17,18,19,20)$$
$$(21,22,23,24)(25,26,27,28)(29,30,31,32)(33,34,35,36)$$
$$\beta=(1)(2)(3)(4)(5,13,17,29,9,25,33,21)(6,14,18,30,10,26,34,22)$$
$$(7,5,19,31,11,27,35,23)(8,16,20,32,12,28,36,24)$$

Base blocks

						2,3,4,6,7,8,10,11,12,13,17,21,25,29,33
						1,5,9,14,15,16,18,19,20,22,23,24,25,29,33
204	76	95	20	16	4	Residual of 209.
205	171	190	20	18	2	Unknown. Residual of 208.
206	361	380	20	19	1	Residual of 207.
207	381	381	20	20	1	(0,1,19,28,96,118,151,153,176,202,240,254,290,296,300 307,337,361,366,369) (mod 381)
208	191	191	20	20	2	Solution unknown.
209	96	96	20	20	4	Case t=4 of sane's (15, 7, 33).
210	77	77	20	20	5	Does not exist. Theorem 10.3.1.
211	71	71	21	21	6	Points and 1 = 1,2…..10,j = 0,1…..(mod 7)

Collinations.

$$\rho = (\infty)(I_O, I_1, ..., I_6), I = 1,2,...,10$$

$$\sigma = (\infty)(K_O)(K_1, K_2, K_4)(K_3, K_6, K_5), K = 1,2,3,10$$

$$(4_i, 5_{2i}, 6_{4i})(7_i, 8_{2i}, 9_{4i}), i = 0,1,..,6$$

Basic blocks

$$B_1 = 1_O 1_1 1_2 1_3 1_4 1_5 1_6 2_O 2_1 2_2 2_3 2_4 2_5 2_6 3_O 3_1 3_2 3_3 3_4 3_5 3_6$$

$$B_2 = 1_O 2_O 3_0 3_1 3_2 \; 3_4 4_0 5_0 6_0 7_0 7_2 7_3 8_4 8_5 8_6 9_O 9_1 9_5 10_3 10_1 10_2 10_4$$

$$B3 = 2_1 2_2 2_4 3_1 3_2 3_4 4_3 4_4 4_5 5_6 5_1 5_3 6_5 6_2 6_6 7_3 7_6 8_6 8_5 9_5 9_3$$

$$B4 = 1_0 1_3 1_6 1_5 2_0 3_0 4_1 4_5 5_2 5_3 6_4 6_6 7_3 7_4 7_5 8_6 8_1 8_3 9_5 9_2 9_6$$

$$B5 = 1_3 1_6 1_5 3_1 3_2 3_4 4_1 4_3 4_6 5_2 5_6 5_5 6_4 6_5 6_3 7_0 8_0 9_0 10_3 10_5 10_6$$

$$B6 = \infty 1_3 1_5 2_1 2_4 3_2 3_3 4_0 4_2 4_4 4_6 5_0 5_3 7_1 7_4 7_6 8_2 8_3 9_5 10_1 10_2$$

$$B7 = 1_1 1_4 2_0 2_1 2_5 3_3 4_0 4_1 4_5 5_2 5_6 6_0 8_3 8_5 9_0 9_3 9_4 9_5 10_4 10_5 10_6$$

212	78	78	22	22	6	The Tonchev design

1 2 6 8 16 22 30 34 36 37 38 42 44 46 47 51 59 64 67 70 75 76
1 4 10 18 22 31 32 36 37 39 40 46 48 49 53 59 61 65 69 77 78
3 6 12 20 22 31 33 34 39 41 42 44 48 50 55 61 63 67 71 72 73
1 5 7 14 15 22 33 35 36 37 41 43 45 46 50 57 58 63 66 69 74 75
2 10 12 13 15 22 25 27 28 34 39 49 57 60 63 64 69 70 71 74 76 78
4 8 12 14 17 22 23 27 29 36 41 44 52 58 59 62 65 66 71 73 76 78
6 9 10 14 19 22 24 25 29 31 43 46 54 60 61 64 66 67 68 73 75 78
5 10 16 17 21 22 23 24 28 32 33 39 41 46 50 51 52 53 56 64 68 76
7 12 16 18 19 22 23 25 26 34 35 41 43 45 48 51 53 54 55 59 70 78
2 14 18 20 21 22 25 27 28 30 36 38 43 47 50 53 55 56 57 61 65 73
4 9 15 16 20 22 23 27 29 31 32 38 40 45 49 51 52 55 57 6 3 67 75
1 4 7 8 9 19 24 25 28 33 35 44 48 49 50 52 57 61 65 67 70 76
2 3 6 10 11 21 23 26 27 30 35 44 45 46 50 52 54 63 65 67 69 78
1 4 5 12 13 16 25 28 29 30 32 45 46 47 48 54 56 58 67 69 7 73
2 3 4 8 13 20 23 24 25 31 33 35 36 38 41 51 54 64 69 73 74 77
4 5 6 8 10 15 25 26 27 30 31 33 35 40 43 53 56 59 71 72 75 76
1 6 7 10 12 17 27 28 29 30 32 33 35 38 42 51 55 61 66 74 77 78

1 2 3 12 14 19 23 24 29 30 32 34 35 37 40 53 57 63 68 72 73 76
2 4 7 10 14 16 24 26 29 38 39 41 42 44 45 56 57 59 60 61 69 72
2 4 6 9 12 18 24 26 28 37 40 41 43 46 47 51 52 61 62 63 71 74
1 4 6 11 14 20 23 26 28 38 39 42 43 48 49 53 54 58 63 64 66 76
7 8 13 15 16 17 24 26 30 31 34 38 46 49 50 53 58 61 62 63 73 78
2 8 10 17 18 19 26 28 32 33 36 40 44 45 48 55 58 60 63 64 73 75
4 10 12 19 20 21 23 28 31 34 35 42 46 47 50 57 58 59 60 62 75 77
6 12 14 15 16 21 23 25 30 33 36 37 45 48 49 52 60 61 62 64 72 77
3 4 6 16 17 19 29 36 43 50 53 55 56 60 62 63 67 69 70 74 76 77
1 5 6 18 19 21 24 31 38 45 51 55 57 58 62 64 65 69 71 72 76 78
1 3 7 16 20 21 26 33 40 47 52 53 57 59 60 64 66 67 71 73 74 78
6 10 13 16 18 20 24 27 30 35 36 39 40 41 48 57 58 62 66 67 68 70
1 8 12 15 18 20 26 29 30 31 32 41 42 43 50 52 60 64 65 68 69 70
3 10 14 15 17 20 24 28 32 33 34 37 38 43 45 54 59 62 65 67 70 71
5 9 12 15 17 19 23 26 34 35 36 38 39 40 47 56 61 64 65 66 67 69
5 10 11 12 14 18 31 35 37 38 44 49 51 52 53 56 58 6067 70 73 74
7 9 12 13 20 30 33 39 40 44 46 51 53 55 56 57 62 64 6771 77 78
2 8 9 11 14 15 32 35 41 42 46 48 53 55 56 57 62 64 67 71 77 78
4 13 14 15 18 19 26 27 33 37 39 42 45 47 50 51 67 68 73 76 77 78
6 8 9 17 20 21 28 29 35 37 39 41 45 47 49 53 69 70 72 73 75 78
1 10 11 15 16 19 23 24 30 39 41 45 47 49 53 69 70 72 73 75 78
3 12 13 17 18 21 25 26 32 38 41 43 44 46 49 57 66 67 72 75 76 77
2 3 7 9 17 22 30 31 35 38 394 43 45 47 48 52 58 60 68 71 76 77
2 4 5 11 19 22 30 32 33 38 40 41 47 49 50 54 60 62 66 70 72 78
4 6 7 13 21 22 32 34 35 40 42 43 44 45 49 56 62 64 65 68 73 74
1 9 11 12 21 22 24 26 27 33 38 48 56 59 62 63 68 69 70 73 75 77
3 11 13 14 16 22 26 28 29 35 40 50 51 58 61 64 65 70 72 75 77
5 8 9 13 18 22 23 24 28 30 42 45 53 59 60 63 65 6667 72 74 76
7 8 10 11 20 22 23 25 26 32 37 47 55 58 61 62 67 68 69 72 74 76
6 11 1517 18 22 24 25 29 33 34 40 42 44 47 52 53 54 57 58 69 77
1 13 1719 20 22 24 26 27 35 36 37 42 46 49 52 54 55 56 60 71 72
3 8 15 19 21 22 26 28 29 30 31 37 39 44 48 51 54 56 57 62 66 74
3 6 7 8 14 18 23 24 27 32 34 47 48 49 50 51 56 60 66 69 71 75
1 2 5 9 10 20 25 26 29 34 36 44 45 49 50 51 53 62 66 68 71 77
3 4 7 11 12 15 24 27 28 31 36 44 45 46 47 53 55 64 66 68 70 72
2 5 6 13 14 17 23 26 29 31 33 46 47 48 49 55 57 59 65 68 70 74
3 4 5 9 14 21 24 25 26 30 32 34 36 39 42 52 55 58 70 74 75 78
5 6 7 9 11 16 26 27 28 31 32 34 36 37 41 54 57 60 65 73 76 77
1 2 7 11 13 18 23 28 29 31 33 34 36 39 43 52 56 62 67 72 75 78
1 3 6 9 13 15 23 25 28 37 38 40 41 44 50 55 56 58 59 60 68 78
1 3 5 8 11 17 23 25 27 39 40 42 43 45 46 51 57 60 61 62 70 73
3 5 7 10 13 19 25 27 29 37 38 41 42 47 48 52 53 62 63 64 65 75
2 5 7 8 12 21 24 27 29 37 39 40 43 49 50 54 55 58 59 64 67 77
1 9 14 16 17 18 25 27 31 32 35 39 44 47 50 54 59 62 63 64 72 74
3 9 11 18 19 20 27 29 30 33 34 41 45 46 49 56 58 59 61 64 74 76
5 11 13 15 20 21 24 29 32 35 36 43 44 47 48 51 59 60 61 74 7678
2 3 5 15 16 18 28 35 42 45 52 54 55 59 61 62 66 68 69 73 75 76
4 5 7 17 18 20 23 30 37 44 54 56 57 61 63 64 68 70 71 75 77 78
2 6 7 15 19 20 25 32 39 46 51 52 56 58 59 63 65 66 70 72 73 77
1 2 4 15 17 21 27 34 41 48 51 53 54 58 60 61 65 67 68 72 74 75
7 11 14 17 19 21 25 28 30 31 36 40 41 42 49 51 59 63 67 68 69 71
2 9 13 16 19 21 23 27 31 32 33 37 42 43 44 53 58 61 66 69 70 71

4 8 11 16 18 21 25 29 33 34 35 37 38 39 46 55 60 63 65 66 68 71
4 9 10 11 13 17 30 34 37 43 48 50 51 52 55 57 59 64 66 69 72 73
6 8 11 12 13 19 32 36 38 39 45 50 52 53 54 57 59 61 68 71 74 75
1 8 10 13 14 21 31 34 40 41 45 47 52 54 55 56 61 63 66 70 76 77
3 8 9 10 12 16 33 36 42 43 47 49 51 54 56 57 58 63 65 68 72 78
5 8 14 16 19 20 27 28 34 38 40 43 44 46 48 52 68 69 72 74 77 78
7 9 10 15 18 21 23 29 36 38 40 42 46 48 50 54 70 71 72 73 74 76
2 11 12 16 17 20 24 25 31 37 40 42 45 48 50 56 65 66 74 75 7678
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

213 70 70 24 24 8 points $I_j, I = 1,2,\ldots,10,\ j = 0,1,\ldots 6(\mathrm{mod}(7))$

Collinations

$$\rho = (I_0, I_1, I_2, I_3, I_4, I_5, I_6),$$
$$I = 1,2,\ldots,10$$
$$\mu = (K_0)(K_1, K_2, K_4)(K_3, K_6, K_5),$$
$$K = 1,2,3,10$$
$$(4_i, 5_{2i}, 6_{4i})(7_i, 8_{2i}, 9_{4i}),$$
$$i = 0,1,\ldots,6$$
$$\tau = (3j)(10j)(1j,2j)(4j,7j)(5j,8j)6(6j,9j),$$
$$j = 0,1,\ldots,6$$

Basicblocks

$$B_1 = 1_0 1_1 1_2 1_4 2_0 2_1 2_2 2_4 3_0 3_3 3_6 3_5$$
$$4_1 4_6 5_2 5_5 6_3 6_4 7_1 7_6 8_2 8_5 9_3 9_4$$
$$B_2 = 10_0 10_3 10_6 10_5 1_0 1_3 1_6 1_5 2_0 2_3 2_6 2_5$$
$$4_0 4_1 5_0 5_1 6_0 6_4 7_0 7_1 8_0 8_2 9_0 9_4$$
$$B_3 = 10_0 10_3 10_6 10_5 2_0 2_1 2_2 2_4 3_0 3_1 3_2 3_4$$
$$4_0 4_6 5_0 5_5 6_0 6_3 7_2 7_5 8_4 8_3 9_1 9_6$$
$$B_4 = 10_0 10_2 1_3 1_4 2_0 2_5 3_0 3_1 4_0 4_1 4_3 4_5$$
$$5_1 5_2 5_3 5_5 6_2 6_3 7_0 7_3 7_4 7_6 9_1 9_5$$

214 66 66 26 26 10

$Let A = \lfloor a_{ij} \rfloor, i, j = 0,1,\ldots,10$ *and let* A *be the* $0-1$ *circulant with* 1;sin *column* 1,3,4,5,9 *in row* 0.

Then $AA^T = A^T A = 3I + 2J$ *and* $A + A^T + I = J$

The followings incidence matrix of the(66,26,10) *design*:

$$\begin{bmatrix} IAAAAA \\ A^T IA^T A^T AA \\ A^T A^T IAAA^T \\ A^T A^T AIA^T A \\ A^T AAA^T IA^T \\ A^T AA^T AA^T I \end{bmatrix}$$

Appendix II

Measurement of Unemployment

The measurement of unemployment is based on the following three criteria that must be satisfied simultaneously: ”without work “,”currently available for work” and “seeking work” (ILO, 1983).

The standard definition of unemployment is based on the “seeking work” criterion that can be interpreted as activity or efforts undertaken by non-working persons during a specified reference period or prior to it in order to find a job (i.e., paid or self employment). The specific steps may include registration at public or private employment exchange, application to employers, cheeking at work sites, farms, factory gates, market or other assembly places, placing or answering news papers advertisements, seeking assistance of friends or relatives, looking for land, building, machinery or equipment to establish own enterprise, arrange for financial resources, applying for work permits and licences, etc. However, in situation where the conventional means of seeking work are limited of relevance, where labour market is largely unorganised or of limited scope, where labour absorption is, at the time inadequate or where the labour force is largely self employed, the above standard definition of unemployment with its emphasis on seeking work criterion might be restrictive and might fully capture the prevailing employment situations in many developing countries including Ethiopia.

Hence, the International Standards introduced provisions, which allows for relaxation of the seeking work criterion in certain situations. The provision are two types, namely, partial relaxation completely relaxation.

Under partial relaxation, the definition of unemployment includes discouraged persons and future start and lays offs in addition to persons satisfying the standard definitions. Discouraged job seekers are those who want a job but did not take any active step to search for work because they believe that they cannot find one. Future start are those persons without work who have make arrangements to take up paid employment or to undertake self-employment activity at a share has been suspended by the employer for a specified or unspecified period at the end of which the person concerned has a recognized right or recognized expectation to cover with the employer (ILO, 1990 as quoted from OECD, 1983).

Under the completely relaxed definition, unemployment includes persons without work and those who are available for work, including those who were not seeking work. That is, the seeking work criterion is complexly relaxed and unemployment is based on the “without work” and “availability” criterion only. The availability in this situation is tested by taking asking the willingness to take up work for wage of salary in locality prevailing terms, or readiness undertaken self-employment activity; given the necessary resources and facilities. It should note that fulltime students are considered as available only if they are ready to with draw from their studies in order to accept a job.

The National Labour Survey unemployment data in the standard partially relaxed and completely relaxed option of measurements. After through evaluation and assessment of the results obtained using the three alternative and complementary measures; the rates obtained using completely relaxed definition was found most plausible and hence selected for reporting.

In the survey, those persons aged ten years and over who did not work at least four hours or did not have a job to return to, were asked to respond to whether they were available or willing to work if job was found during the coming one month. Those who respond “yes” answer to this question were further tested whether they were ready to make a job under prevailing condition.