



QUANTUM HALL EFFECT IN TWO DIMENSIONAL ELECTRON GAS

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*For those family and friends of mine who have unlimited
potential but can not attend the school.*

Table of Contents

Table of Contents	vi
List of Figures	vii
Abstract	ix
Acknowledgements	x
Introduction	1
1 CLASSICAL HALL EFFECT	3
1.1 Dynamics of Free Electron Gas	3
1.2 Conductivity and Resistivity Tensors in 2D Channel	5
1.3 Hall Effect	6
1.4 The Hall Coefficient	8
1.5 Hall Voltage of Charge Carriers	8
1.6 Hall Probe	9
2 REDUCED DIMENSIONAL STRUCTURE	10
2.1 MOSFET- Metal Oxide Semiconductor Field Effect Transistor	10
2.2 Superlattice	11
2.3 Liquid Helium Surface	13
2.4 Fermi Gas	13
2.5 Fermi Liquid	14
2.6 Luttenger Liquid	15
3 INTEGRAL QUANTIZED HALL EFFECT	16
3.1 Landau Levels	16
3.2 Degeneracy Of Landau Levels	17
3.3 Integral Quantized Hall Resistance	18
3.4 Integral Quantum Hall Effect in Real Crystal	20
3.4.1 Laughlin-Thought Experiment	21

4	FRACTIONAL QUANTIZED HALL EFFECT	24
4.1	Fractional Quantized Hall Resistance	24
4.2	Quasi-particles in Fractional Quantum Hall Effect	25
4.2.1	Laughlin Wave Function	25
4.2.2	Fractional Charge	27
4.3	The Composite Fermion	28
5	CHIRAL HEAT TRANSPORT IN THE QUANTUM HALL REGIME	31
5.1	Heat Transport	31
5.1.1	Thermal Conductivity	31
5.1.2	Thermoelectric Phenomeana	33
5.2	Chiral Edge Current	34
5.3	Quantized Heat Transport in the IQHE	36
5.4	Quantized Heat Transport in the FQHE	37
5.5	Experimental Implications of Quantized Heat Transport	39
6	CONCLUSION	42
	Appendices	44
	Bibliography	51

List of Figures

1.1	Experimental geometry of Hall effect. The Hall field E_H is established due to charge build-up on opposite faces in the direction normal to the applied electric and magnetic fields, resulting from Lorentz force acting on moving electrons. In steady state, the force $-eE_H$ balances the Lorentz force [6]. . .	7
2.1	Electron energy level diagram of a silicon MOSFET with a positive voltage applied to the aluminum gate [7].	11
2.2	Electron energy level diagram of a GaAs-AlGaAs heterostructure device [7].	12
3.1	Hall resistance and longitudinal resistivity data as a function of the magnetic field for a GaAs/AlGaAs heterostructure at 30mK [19].	20
3.2	Density of states in a 2D electron gas in a strong magnetic field (Ideal crystal) [19].	21
3.3	Density of states in a 2D electron gas in a strong magnetic field. (Real 2D crystal, with impurity and other imperfections)[20].	21
3.4	Geometry for Laughlin's thought experiment. The 2D electron system is wrapped around to form a cylinder. A strong magnetic field B pierces the cylinder every where normal to its surface. A current I circles the loop, giving rise to the Hall voltage V_H and a small magnetic flux ϕ through the loop[21].	22
4.1	The figure shows the stepwise behavior of the transverse resistivity, superimposed with the longitudinal resistance, as a function of magnetic field. The same behavior as in figure 3.1 is seen except now at fractions of ν [25].	25
4.2	(1) single electron in the lowest Landau Level (2) Filled lowest Landau levels, state wave function of is $\Psi(z_1, z_2, \dots, z_n) \sim \prod (z_j - z_k)$ [26].	26

4.3	The lowest Landau Levels for (1) uncorrelated $1/3$ states (2) correlated $1/3$ states, wave function of correlated state is $\Psi(z_1, z_2, z_3, \dots, z_n) \sim \prod (z_j - z_k)^3$ [27].	27
4.4	Only fractional charge of $e/3$, flow along the one-dimensional edges. Upon decreasing the width of the opening (by varying the voltage on the gate electrode shown in the inset) [28].	28
4.5	Composite fermions and their Landau levels [27].	30
5.1	(a) Energy spectrum of a clean integer quantum Hall system as a function of guiding center position X . States near the edges disperse and carry current. By filling them to different Fermi energies $EF^{(L)}$ and $EF^{(R)}$, a net current flows down the sample which yields a quantized Hall resistance R_H . (b) Interpretation in terms of classical orbits: In bulk, electrons orbit around a fixed position and do not carry current. At the edges specular reflection induces current-carrying motion [33].	34
5.2	Schematic representation of edge states for several different quantum Hall fluids at filling factor ν . The integer ν_Q , which specifies the difference between the number of downstream and upstream propagating modes, determines the sign and magnitude of the thermal Hall conductance [35]. . .	39
5.3	Proposed geometry for measurement of the thermal Hall conductance. The heat generated by passing a current between contacts 2 and 3 is detected by two additional point contacts, upstream (1-6) and downstream (4-5) along the edge [35].	40

Abstract

In this work we analyze the modern state of the problem connected with quantum Hall effect (integral and fractional). The quantum Hall effect is a quantum-mechanical version of the Hall effect, observed in two dimensional electron systems subjected to low temperatures ($< 1K$) and strong magnetic fields ($\sim 10T$), in which the Hall conductance (σ_H) takes on the quantized values ($\nu e^2/h$) with ν an integer (integer quantized Hall effect) or a rational fraction (fractional quantized Hall effect), independent of the detail of the sample geometry. The fractional quantum Hall effect is not completely understood at the time being. Recently the idea of chiral heat transport in quantum Hall regime was pushed forward [33], we also analyze thermal transport in the fractional quantum Hall effect (FQHE). As an original part we consider the problem of two electrons in a uniform magnetic field with the account of the center mass: (1) Quantum Mechanical Description and (2) Classical Description.

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Introduction

An electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor. This is most evident in a thin flat conductor. A buildup of charge at the sides of the conductors will balance this magnetic influence, producing an electric field between the two sides of the conductor. The presence of this transverse field is called the Hall field after E. H. Hall who discovered it in 1879. The ratio of the field created to the product of the amount of current density and the magnetic field known as the Hall coefficient (R_H) and is a characteristic of the material of which the element is composed [7].

$$R_H = -1/nec \quad (0.0.1)$$

where n is particle charge density and e is the elementary charge. Chapter one is the discussion of the classical Hall effect.

- Quantum Hall effect(QHE) can be

- (1) Integral quantum Hall effect (chapter 3)
- (2) Fractional quantum Hall effect (chapter 4)

Both integral and fractional quantum Hall effects evolve from the quantization of cyclotron motion of an electron in a two dimensional electron gas(chapter 2) in a perpendicular magnetic field, $\vec{B}$. In the Landau gauge $\vec{A} = [0, Bx, 0]$ the single electron kinetic energy operator

$$\hat{H} = \frac{1}{2m_e}(\hat{p} - \frac{e\vec{A}}{c})^2 \quad (0.0.2)$$

has eigenfunctions

$$\Psi(x, y, z) = (\exp \frac{i}{\hbar}(p_y y + p_z z))(\exp \frac{m_e \omega_c}{2\hbar}(x - x_0)^2) H_n(\sqrt{\frac{m \omega_c}{\hbar}}(x - x_0), \quad (0.0.3)$$

and eigenvalues

$$E_{n,m} = \hbar \omega_c (n + \frac{1}{2}). \quad (0.0.4)$$

Note that a macroscopic number of states, distinguished by quantum number m , share the same kinetic energy. The kinetic energy has discrete allowed values separated by $\hbar \omega_c$ and depends only on the quantum number n . Electrons with the same kinetic energy are said to be in the same Landau level. In (0.0.2) $\hat{p} = \hbar \nabla / i$ is the electron momentum .In (0.0.3) $\omega_c = eB/m_e c$ is the cyclotron frequency, $x_0 = p_y c / eB$ is the initial displacement and $H_n(x)$ represent the Hermite polynomials . The Hall conductance ν takes on the quantized values

$$\sigma = \nu(e^2/h) \quad (0.0.5)$$

where e is the elementary charge and h is Planck's constant. In the integral quantum Hall effect, ν takes on integer values ($\nu = 1, 2, 3$, etc.). In fractional quantum Hall effect ν can occur as a rational fraction ($\nu = 2/7, 1/3, 2/5, 3/5, 5/2$ etc.)[37]

The fractional quantum Hall effect is not completely understood at the time being. Recently the idea of chiral heat transport in quantum Hall regime was pushed forward [33], it is discussed in chapter 5, we also analyze thermal transport in the fractional quantum Hall effect (FQHE), employing a Luttinger liquid model of edge states. The thermal Hall conductance K_H is shown to provide universal characterization of the FQHE state, and reveals nontrivial information about the edge structure. The Lorenz ratio between thermal and electrical Hall conductances violates the free-electron Wiedemann-Franz law, and for some fractional states is predicted to be negative. We will see that thermal transport provide a unique way to detect the presence of the elusive upstream propagating modes, predicted for fractions such as $\nu = 2/3$ and $\nu = 3/5$ [35].

Chapter 1

CLASSICAL HALL EFFECT

1.1 Dynamics of Free Electron Gas

We begin with a discussion of the transport properties associated with free electron gas, since the model serves as a basis for understanding the electron transport behavior of normal metals.

The electrical transport characteristics of solids can be represented by the quantities the electrical conductivity, σ (or its inverse, the electrical resistivity, ρ) defined as constant of proportionality between electrical current density $\vec{j}$ and electric field $\vec{E}$

$$\vec{j} = \sigma \vec{E}. \quad (1.1.1)$$

Note that, for real anisotropic materials, the direction of the current flow need not be in the same direction as the applied field. In this case, the conductivity is second rank tensor and not scalar as assumed in (1.1.1). Ohm's law states that the electrical current I is proportional to the applied potential difference V , i.e.

$$V = IR, \quad (1.1.2)$$

where R is the electrical resistance ($= 1/G$, where G is the conductance) of the sample. Equations (1.1.1) and (1.1.2) are equivalent formulations. Since the current density flowing through a sample of cross-sectional area A is $j = I/A$, and the potential difference

dropped along the length l of the sample is $V = El$, a comparison of (1.1.1) and (1.1.2) shows that the conductivity and resistivity (assumed to be scalars) are related, to the sample resistance via

$$\sigma = \frac{1}{\rho} = \frac{l}{RA}. \quad (1.1.3)$$

Hence the unit of conductivity are $(Ohmsmeter)^{-1}$ or $(\Omega m)^{-1}$.

The equation of motion experienced by a free electron subjected to applied electric and magnetic fields is

$$\vec{F} = m_e \frac{d\vec{v}}{dt} = -e(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}), \quad (1.1.4)$$

where m_e is mass of electron, e is the electron charge.

In such a case, (1.1.4) implies that an applied electric field ($\vec{E}$) and magnetic ($\vec{B}$) should accelerate free electrons without limit. Evidently, this is unphysical, and this is a failing of the simple free electron picture. In real metals, the electron scatters from imperfections in a lattice defects or impurities and, to some extent from other electrons. These scattering events cause a reversal in the electron momentum and hence act like a damping force in equation of motion (1.1.4). If it is assumed, at every scattering event, the extra drift velocity v imparted by fields, is removed on average, with τ being the average time between electron collisions, then (1.1.4) can be modified by the addition of a damping term, $m_e \vec{v}/\tau$, i.e.

$$m_e \left(\frac{d}{dt} + \frac{1}{\tau} \right) \vec{v} = -e \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right). \quad (1.1.5)$$

the quantity τ is also known as the electron relaxation time. when we consider the motion of the system in a uniform magnetic field $\vec{B}$ the equation of motion (1.1.4) is obtained [1]. An important situation is the following: let a static magnetic field $\vec{B}$, lie along the z-axis. Then the component equation of motion are

$$m_e \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_x = -e \left(E_x + \frac{B}{c} v_y \right); \quad (1.1.6)$$

$$m_e \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_y = -e \left(E_y - \frac{B}{c} v_x \right); \quad (1.1.7)$$

$$m_e \left(\frac{d}{dt} + \frac{1}{\tau} \right) v_z = -e E_z. \quad (1.1.8)$$

In the steady state, in a static electric field the time derivatives are zero, so that the drift velocity is

$$v_x = -\frac{e\tau}{m_e} E_x - \omega_c \tau v_x; \quad (1.1.9)$$

$$v_y = -\frac{e\tau}{m_e} E_y + \omega_c \tau v_x; \quad (1.1.10)$$

$$v_z = -\frac{e\tau}{m_e} E_z, \quad (1.1.11)$$

where $\omega_c = eB/m_e c$ is the cyclotron frequency [2].

1.2 Conductivity and Resistivity Tensors in 2D Channel

For the simple case of a free electron gas, the steady state equation of motion given by (1.1.5) predicts a resistivity (inverse of conductivity) that is independent of magnetic field. For real metals, on the other hand a field dependent magnetoresistivity found because of anisotropic fermi surface (the cause of longitudinal magnetoresistance or a none-constancy (energy dependent) of relaxation time τ . The magneto resistivity increases quadratically with magnetic field , and reaches a saturation value at sufficiently high magnetic fields unless the material is compensated equal number of electrons and holes in two bands), in which case the magneto resistivity increases without limit with increasing field [3].

In the presence of a steady magnetic field, the conductivity and resistivity becomes tensors for 2D channel the matrices are respectively: $\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$.

The electrical current density is :

$$\vec{j} = -en_s \vec{v}, \quad (1.2.1)$$

where n_s is the surface density electrons. Thus from (1.1.9) and (1.1.10) we can solve for v_x and v_y , and then using (1.2.1) the current density in the xy plane written as:

$$\vec{j}_x = \left(\frac{\sigma_0}{1 + \omega_c^2 \tau^2}\right) \vec{E}_x - \omega_c \tau \left(\frac{\sigma_0}{1 + \omega_c^2 \tau^2}\right) \vec{E}_y; \quad (1.2.2)$$

$$\vec{j}_y = \omega_c \tau \left(\frac{\sigma_0}{1 + \omega_c^2 \tau^2}\right) \vec{E}_x + \left(\frac{\sigma_0}{1 + \omega_c^2 \tau^2}\right) \vec{E}_y, \quad (1.2.3)$$

where $\sigma_0 = e^2 n_s \tau / m_e$ is Drude formula.

The components of conductivity can be obtained using the phenomenological relation, $j_\alpha = \sigma_{\alpha\beta} E_\beta$, $\alpha, \beta = x, y$, and the above two equations (1.2.2), (1.2.3),

$$\sigma_{xx} = \sigma_{yy} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2}; \sigma_{xy} = -\sigma_{yx} = -\omega_c \tau \frac{\sigma_0}{1 + \omega_c^2 \tau^2}, \quad (1.2.4)$$

Where as the components of resistivity from the relation of conductivity and resistivity tensors [4]: In the case when $\omega_c \tau \gg 1$, as for strong magnetic field and low temperature, the surface conductivity and resistivity components approach the limits

$$\sigma_{xx} = \sigma_{yy} = 0; \sigma_{xy} = -\sigma_{yx} = \frac{n_s e c}{B}; \quad (1.2.5)$$

$$\rho_{xx} = \rho_{yy} = \frac{1}{\sigma_0}; \rho_{xy} = -\rho_{yx} = \frac{B}{n_s e c}. \quad (1.2.6)$$

1.3 Hall Effect

Electron transport in the presence of a magnetic field, as well as electric field, can be described in terms Hall effect in which orthogonal electric and magnetic fields generate in a conductor a secondary electric field, the Hall field, that is normal to both primary fields [5]. Consider the experimental geometry illustrated in the Fig.1.1, in which a constant magnetic field $\vec{B}$ is applied in the z -direction and an electric field $\vec{E}$ is applied along the x -direction. The Lorentz force (1.1.4) causes deflection of the trajectory of the electrons (moving in the $-x$ -direction in the absence of magnetic field) in the y -direction, generating negative and positive space charges on the $-y$ and $+y$ faces of the sample, respectively.

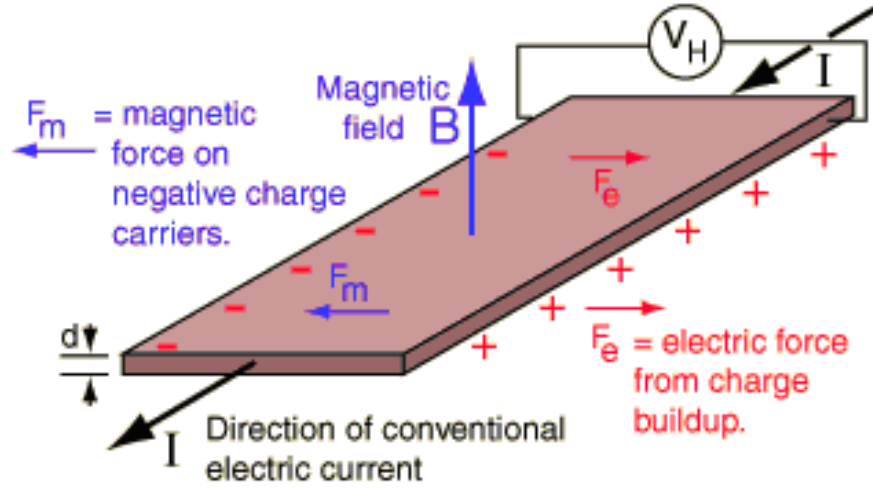


Figure 1.1: Experimental geometry of Hall effect. The Hall field E_H is established due to charge build-up on opposite faces in the direction normal to the applied electric and magnetic fields, resulting from Lorentz force acting on moving electrons. In steady state, the force $-eE_H$ balances the Lorentz force [6].

This charge build-up causes an electric field, the Hall field (after discovery of E. Hall 1879), $\vec{E}_H$, to be established in the $-y$ direction, the associated force $-e\vec{E}_H$ opposing the Lorentz force. In steady state, there is no net force on electrons in the y -direction, and hence no drift-velocity component v_y in that direction; the current flow is thus only in the x -direction. In steady state, the y -component of the equation of motion given by (1.1.10) reduce to

$$0 = -e(E_y - (\frac{B}{c})v_x) \quad (1.3.1)$$

since $v_y = 0$. Thus,

$$E_y = v_x B = -\frac{j_x B}{ne}. \quad (1.3.2)$$

The transverse component of resistivity tensor, ρ_{xy} , given by (1.2.6) is called Hall resistivity [1].

1.4 The Hall Coefficient

The Hall coefficient, R_H is defined by the relation

$$R_H = \frac{E_y}{j_x B} = \frac{\rho_{xy}}{B} = -\frac{1}{nec}. \quad (1.4.1)$$

Therefore, if electrons make up the current, the Hall coefficient is negative, reflecting the charge carried by the electrons, and R_H is inversely proportional to the electron concentration, n . If, on the other hand, holes carry the current, the velocity direction of holes is in opposite direction to that of electrons, i.e. the same as that of the current, and consequently the Lorentz force is unchanged. Consequently, the Hall field for holes is in the opposite direction to that of electrons, and hence the Hall coefficient is correspondingly positive, i.e. the same sign as the charge carried by the holes. Hence, Hall effect measurements can be used to determine the sign of charge carriers in conductors, as well as the carrier concentration. Note that the Hall coefficient will be larger for semiconductors than for metals because carrier concentration in semiconductors is much smaller than conduction-electron density in metals [2].

1.5 Hall Voltage of Charge Carriers

The transverse voltage (Hall voltage) V_H measured in a Hall probe has its origin in the magnetic force on moving charge carrier. The potential difference dropped along the length l of the sample is

$$V_H = E_H l, \quad (1.5.1)$$

where V_H is Hall voltage, E_H is the Hall field.

Using (1.3.2) the Hall voltage can be written as

$$V_H = \frac{IB}{ned}, \quad (1.5.2)$$

where d is thickness of the sample [7].

1.6 Hall Probe

The measurement of large magnetic fields on the order of Tesla is often done by making use of Hall effect. A thin film Hall probe is placed in the magnetic field and the transverse voltage (on the order of microvolt) is measured. Sometimes a thin copper film of thickness d on the order of 100 micrometers is used for a Hall probe. Taking the charge carrier density to be $n = 8.47 \times 10^{28} \text{electrons}/m^3$ for copper, the Hall voltage polarity can be determined. The polarity of the Hall voltage for a copper probe shows that electrons are the charge carriers [8].

Chapter 2

REDUCED DIMENSIONAL STRUCTURE

In bulk metals and semiconductors electrons (or holes) are generally free to move in all three spatial directions. If this freedom is restricted in certain directions the dimensionality of the system becomes reduced. For instance, in a 2-dimensional system the electrons can only move in one plane and may not travel perpendicular to this plane. Examples of artificial structures of material showing quasi-2D behavior are MOSFET, Superlattice [1,9]. And another example of a 2D system is electrons on the surface of liquid helium, here we will concentrate on the 2D semiconductor systems found in MOSFETs, heterojunctions and liquid helium surface. Also included in this chapter are Fermi gas, Fermi liquid and Luttinger liquid.

2.1 MOSFET- Metal Oxide Semiconductor Field Effect Transistor

In MOSFET, inversion layers are formed at the interface between a semiconductor and an insulator or between two semiconductors, with one of them acting as an insulator. The system in which the Quantum Hall Effect (QHE) was discovered has Si for the semiconductor, SiO_2 for the insulator. Figure 2.1 is a schematic side view of silicon MOSFET

showing the aluminum gate, the SiO_2 insulator and the p-type Si crystal substrate. The principle of the inversion layer is quite simple. It is arranged that an electric field perpendicular to the interface attracts electrons from the semiconductor to it. These electrons sit in a quantum well created by this field and the interface. The motion perpendicular to the interface is quantized and thus has a fundamental rigidity which freezes out motional degrees of freedom in this direction. The result is a two-dimensional system of electrons.

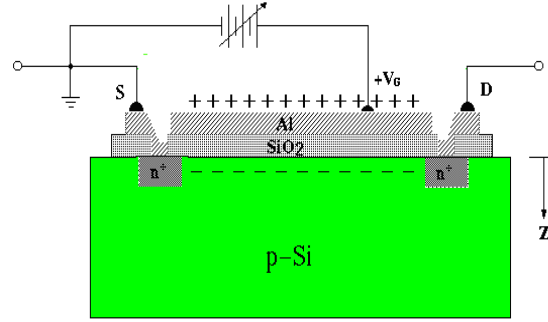


Figure 2.1: Electron energy level diagram of a silicon MOSFET with a positive voltage applied to the aluminum gate [7].

Further, the wavelengths of these electrons are long so that an effective mass approximation with parabolic bands is quite good. The total self-consistent potential seen by the electrons is conveniently described by the picture of "band bending". That is to say, the periodic lattice potential gives rise to energy bands, and the slowly varying electric potential then is regarded as bending these bands. Figure 2.1 also gives the schematic diagram for this process (charge density $\sim 10^{13} cm^{-2}$)[7,9].

2.2 Superlattice

Another type of two-dimensional electron system is formed in the heterostructures of two semiconductors. Using molecular beam epitaxy (MBE) technique, people can grow two semiconductors alternately to form a one dimensional sandwich like structure. Each

layer has a width of about several nanometers. They are called super lattice. They can also be grown by metal-organic chemical vapor deposition (MOCVD). For example, in

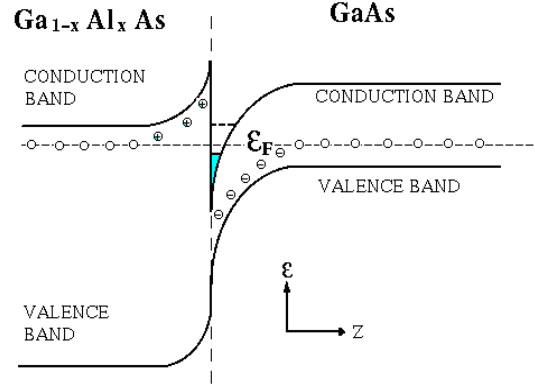


Figure 2.2: Electron energy level diagram of a GaAs-AlGaAs heterostructure device [7].

GaAs – GaAlAs superlattice, a certain controlled number of layers of GaAs is followed by an almost perfectly matched sequence of layers of GaAlAs. The GaAlAs is deliberately doped n-type, which puts mobile electrons into its conduction band. These electrons will migrate to fill the few holes on the top of the GaAs valence band but most of them will end up in states near the bottom of the GaAs conduction band. However, there is a positive charge left on the donor impurities which attracts these electrons to the interface and bends the bands in the process. This is the source of the electric field in this system. The transfer of electrons from GaAlAs to GaAs will continue until the dipole layer formed from the positive donors and the negative inversion layer is sufficiently strong. This dipole layer gives rise to a potential discontinuity which finally makes the Fermi level of the GaAs equal to that of the GaAlAs. Figure 2.2 shows the band structure. (charge density $\sim 10^{11} \text{cm}^{-2}$)[1].

2.3 Liquid Helium Surface

Two-dimensional electron system can also be formed in the vicinity surface of liquid Helium. There exists a potential barrier of about $1eV$ in the surface of liquid Helium which prevents electrons from transmitting into the liquid. On the other hand, the mirror potential attracts the electrons in the surface, resulting a 2D electron system.(charge density $\sim 10^9 cm^{-2}$). Quantum Hall effect has been observed in the first two types [1,7].

2.4 Fermi Gas

Fermi gas, or free electron gas, is a physical model assuming a collection of non-interacting fermions. It is the quantum mechanical version of an ideal gas, for the case of fermionic particles. Electrons in metals and semiconductors can be approximately considered Fermi gases.

The energy distribution of the fermions in a Fermi gas in thermal equilibrium is determined by their density, temperature, and the set of available energy states, via Fermi-Dirac statistics. By the Pauli principle, no quantum state can be occupied by more than one fermion, so the total energy of the Fermi gas at the zero temperature is larger than the product of the number of particles and the single-particle ground state energy. For this reason, the pressure of a Fermi gas is nonzero even at zero temperature, in contrast to that of a classical ideal gas. It is possible to define a Fermi temperature below which the gas can be considered degenerate. This temperature depends on the mass of the fermions and the energy density of states. For metals, the electron gas's Fermi temperature is generally many thousands of kelvins, so they can be considered degenerate. The maximum energy of the fermions at zero temperature is called the Fermi energy. The Fermi energy surface in the momentum space is known as the Fermi surface. Since interactions are neglected by definition, the problem of treating the equilibrium properties and dynamical behavior

of a Fermi gas reduces to the study of the behavior of single independent particles. As such, it is still relatively tractable and forms the starting point for more advanced theories (such as Fermi liquid theory in the interaction) which take into account interactions to some degree of accuracy [10].

2.5 Fermi Liquid

Fermi liquid is a generic term for a quantum mechanical liquid of fermions that arises under certain physical conditions when the temperature is sufficiently low. The interaction between the particles of the many-body system does not need to be small (e.g. electrons in a metal).

The phenomenological theory of Fermi liquids, which was introduced by the Soviet physicist Lev Davidovich Landau in 1956, explains why some of the properties of an interacting fermion system are very similar to those of the Fermi gas (i.e. non-interacting fermions).

Liquid He-3 is a Fermi liquid at low temperatures. He-3 is an isotope of Helium, with 2 protons, 1 neutron and 2 electrons per atom; because there is an odd number of fermions inside the atom, the atom itself is also a fermion. The electrons in a normal metal also form a Fermi liquid.

Similarities to Fermi gas

The Fermi liquid is qualitatively analogous to the non-interacting Fermi gas, in the following sense: The system's dynamics and thermodynamics at low excitation energies and temperatures may be described by substituting the non-interacting fermions with so-called quasiparticles. Physically these may be thought of as being particles whose motion is disturbed by the surrounding particles and which themselves perturb the particles in their vicinity [2,10].

2.6 Luttinger Liquid

Electronic phenomena are much more subtle in one dimension than in three. Whereas in higher dimensions Landau's "Fermi liquid" theory usually applies, it has been known since the 1960s one dimensional is special. There even weak electron-electron interactions destroy the Fermi surface. While the usual approximations for dealing with electron interactions break down, there exists a remarkably simple description of the one dimensional problem known as "Luttinger liquid" theory.

The Tomonaga-Luttinger liquid was first proposed by Tomonaga in 1950. This liquid, more often referred to as simply a Luttinger liquid, is a theoretical model describing interacting electrons (or other fermions) in a one-dimensional conductor. Such a model is necessary as the commonly used Fermi liquid model breaks down for one dimension [10].

Among the physical systems believed to be described by the Luttinger model are:

(1) electrons moving along edge states in the fractional Quantum Hall Effect (2) fermionic atoms in quasi-one-dimensional atomic traps

Attempts to demonstrate Luttinger-liquid-like behavior are the subject of ongoing experimental research in condensed matter physics. The most impressive experimental confirmation of the Luttinger liquid theory has occurred in the study of edge transport in the fractional quantum Hall effect, which is described chapter 5.

Chapter 3

INTEGRAL QUANTIZED HALL EFFECT

3.1 Landau Levels

Let us consider the solution of Schrodinger equation for an electron in a constant magnetic field [11]. The stationary-state Schrodinger equation for free particle in a magnetic field is given by

$$\frac{1}{2m_e}(\hat{p} - \frac{e\vec{A}}{c})^2\Psi(x, y, z) = E\Psi(x, y, z), \quad (3.1.1)$$

where m_e is the electron mass, $\Psi(x, y, z)$ is the electron wave function, E is the energy eigen value, e is the electron charge, $\hat{p} = \hbar\nabla/i$ is the electron momentum, And $\vec{A}$ is the vector potential and $\vec{B} = \nabla \times \vec{A}$.

For the above time independent Schrodinger equation for a free electron in a uniform magnetic field, $\vec{B}$, say along z-direction, we can choose the vector potential as $A_x = 0, A_y = Bx, A_z = 0$ (Landau gauge). Note that at this choice of $\vec{A}$ we get only z-component of magnetic field ($\vec{B}$). Then substituting, the vector potentials and the x, y and z components of the momentum, in (3.1.1), we get the following

$$[\frac{\hat{p}_x^2}{2m_e} + \frac{1}{2m_e}(\hat{p}_y^2 - \frac{eB}{c}x)^2 + \frac{\hat{p}_z^2}{2m_e}]\Psi(x, y, z) = E\Psi(x, y, z). \quad (3.1.2)$$

From (3.1.2) we see that the Hamiltonian does not depend on y and z. This leads us to choose the wave function which has a plane wave dependence on y and z coordinates [12],

i.e.

$$\Psi(x, y, z) = \psi(x) \exp \frac{i}{\hbar} (p_y y + p_z z). \quad (3.1.3)$$

We seek a solution of Schrodinger equation in this form. Substituting (3.1.3) in (3.1.2), the resulting equation is one dimensional Schrodinger equation for simple harmonic oscillator frequency ω_c (the cyclotron frequency) and displacement $(x - x_0)$. Thus harmonic oscillator equation given by

$$-\frac{\hbar^2}{2m_e} \frac{d^2\psi(x)}{dx^2} + \frac{m\omega_c^2}{2} (x - x_0)^2 \psi(x) = \varepsilon \psi(x), \quad (3.1.4)$$

where $\omega_c = eB/m_e c$, $x_0 = p_y c / eB$, $\varepsilon = E - p_z^2 / 2m_e$.

From which, we get the quantized energy and wave function [13]. The quantized energy of the different oscillator levels Landau Levels (the energy eigenvalue of free electron Hamilton in a uniform magnetic field $\vec{B}$ to the z-direction) is obtained as a solution of the the above equation as

$$E = \hbar\omega_c \left(n + \frac{1}{2}\right) + \frac{p_z^2}{2m_e} (n = 0, 1, 2, 3, \dots). \quad (3.1.5)$$

And the corresponding eigne state is

$$\Psi(x, y, z) = \left(\exp \frac{i}{\hbar} (p_y y + p_z z)\right) \left(\exp \frac{m_e \omega_c}{2\hbar} (x - x_0)^2\right) H_n \left(\sqrt{\frac{m\omega_c}{\hbar}} (x - x_0)\right), \quad (3.1.6)$$

where $H_n(x)$ represent the Hermite polynomials [14].

In (3.1.6) the wave function , $\Psi(x, y, z)$, depend on the parameters n, p_z, p_y , where as in (3.1.5) the Landau Level energy, E , depends only on the parameters n, p_z . This implies degeneracy [15].

3.2 Degeneracy Of Landau Levels

As a result of quantization of the electron spectrum, all the electronic states should be accommodated on the Landau levels, which should be, therefore, highly degenerate. The

degeneracy of Landau level, $g(B)$, could be calculated as follows. Since the electrons assumed to be in the sample, x_0 should stay inside the sample of dimensions (L_x, L_y) , $0 < x_0 < L_x$ for maximum magnetic field, $\vec{B}_o$, we have $L_x = p_y c / e B_o$. Again if we assume periodic boundary condition ($s = p_y L_y / h, s = 0, 1, 2, 3, \dots$), where s is the number of states with different p_y . The number of states, s , with the same energy but different p_y can be written as

$$s = \frac{\Phi}{\phi_0} = g(B), \quad (3.2.1)$$

where Φ is the maximum magnetic flux, $\phi_0 = hc/e$ is the quanta of elementary magnetic flux [15].

From (3.2.1) follows the magnetic flux is quantized and can not be smaller than ϕ_0 . Degeneracy of the Landau Levels (LLs), $g(B)$, depends on the external magnetic field, $\vec{B}$, in high magnetic fields LLs are highly degenerate and degeneracy can be even larger than a number of electrons in the sample [16]. An important concept is also the filling ratio ν . It is equal to the number of filled Landau levels, $\nu = n_s / N$, where n_s is the number density of electrons in two dimensions and $N = g(B)/a$ is the degeneracy of the Landau levels per unit area, a [17].

3.3 Integral Quantized Hall Resistance

The classical result of the Hall resistivity is given in (1.2.6) and says that the Hall resistivity is $\rho_{xy} = B / n_s e c = \rho_H$. However, the density number of the current carrying states in each Landau level is $N = g(B)/a = eB/hc$. Therefore, there are ν Landau levels at energy below the Fermi energy completely filled with $n_s = \nu N = \nu eB/hc$ electrons. The Hall resistivity can be obtained [18];

$$\rho_H = \frac{B}{\nu(eB/hc)ec} = \frac{h}{\nu e^2}, (\nu = 1, 2, 3, \dots). \quad (3.3.1)$$

The value of the Hall resistivity (3.3.1) only depends on the fundamental constants of physics: e is elementary electric charge and h is plank's constant. The quantum Hall effect observed in two dimensional electron systems subjected to low temperatures ($< 1K$) and strong magnetic fields, in which value of the Hall resistivity is quantized in units of $h/e^2 (= 25812.807\Omega)$ divided by consecutive integers. This is the integral quantum Hall effect first discovered by Von Kiltizing in 1980 for an inversion layer in Si MOSFET (Metal Oxide Silicon Field Effect Transistor). The quantum Hall resistance is also known as Von Klitizing constant. Due to high precision of the measurement (one part per million), the quantization of Hall resistance now used world wide in national meteorology institutes as a primary standard of resistance. Further more the IQHE is used to find the structure constant since the constant e^2/h is proportional to the fine structure constant in electrodynamics, so the quantum Hall effect provides us an independent way of accurately measuring this constant [1].

The experimental Hall resistance exhibits several plateaus. Figure 3.1 shows the integral quantum Hall effect in GaAs-GaAlAs hetetojunction recorded at 30mK. It also included is longitudinal resistivity. The experiments show that between two adjacent Landau levels the Hall resistance has fixed values (plateau) and the longitudinal resistivity vanishes. Furthermore, corresponding to sharp rises in between quantized Hall plateau, there are sharp picks in the longitudinal resistivity. The zeros and plateaus in the two components of the resistivity tensor are intimately connected and can be understood in terms of Landau levels formed in the magnetic field. In the absence of magnetic field the density of states in 2D is constant as a function of energy, but in the applied magnetic field, the available states clump in to Landau levels separated by cyclotron energy (see Fig.(3.1)), with region of energy between Landau levels where there are no allowed states. As the magnetic field swept the LLs move relative to the Fermi energy. When the Fermi energy lies in a gap between LLs electrons can not move to a new states and so there is no scattering. Thus the transport is dissipation less and the resistance falls to zero.

Where as the Hall resistance can not change from the quantized value for the whole time the Fermi energy is in the gap between LLs , and so plateau results. This picture has assumed a fixed Fermi energy, i.e fixed carrier density, and changing magnetic field [1].

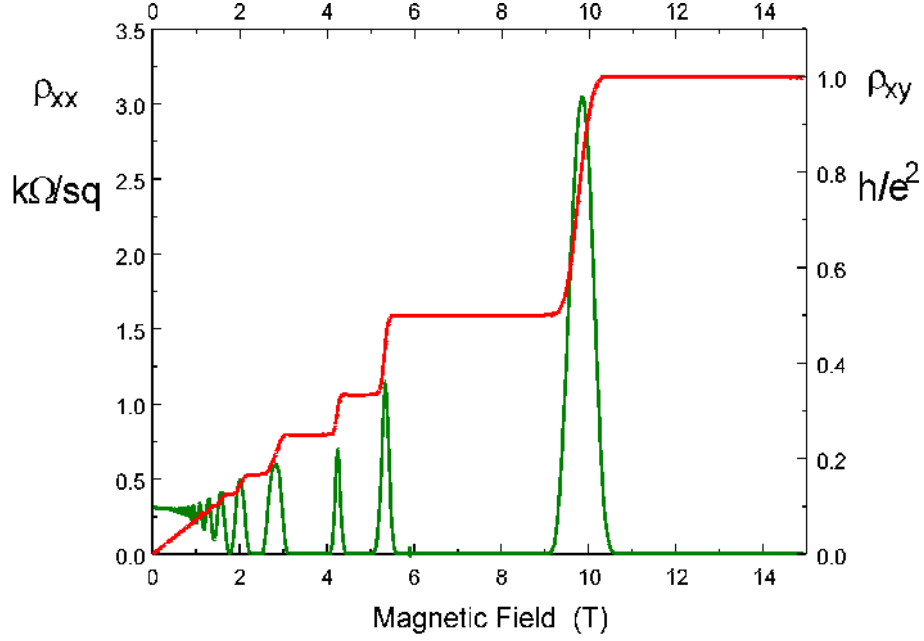


Figure 3.1: Hall resistance and longitudinal resistivity data as a function of the magnetic field for a GaAs/AlGaAs heterostructure at 30mK [19].

3.4 Integral Quantum Hall Effect in Real Crystal

The above discussions of integral quantum Hall effect suggests that the measurements under quantum conditions of temperature and magnetic field the Hall resistance is accurately quantized at $25,813.802$ ohms whether or not the semiconductor is of very high purity and perfection. In real crystals the sharp Landau Levels are broadened (Figure 3.2), but this does not affect the Hall resistivity. Laughlin interpreted the results for real systems as the expression of the general principle of gauge invariance [2].

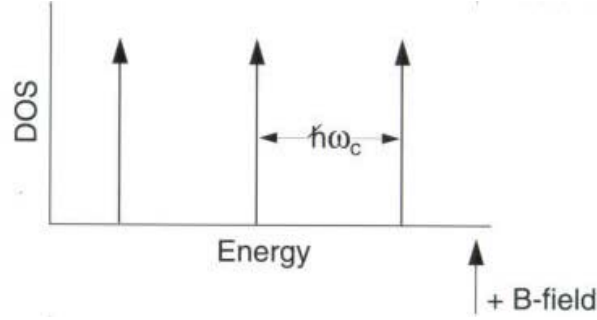


Figure 3.2: Density of states in a 2D electron gas in a strong magnetic field (Ideal crystal) [19].

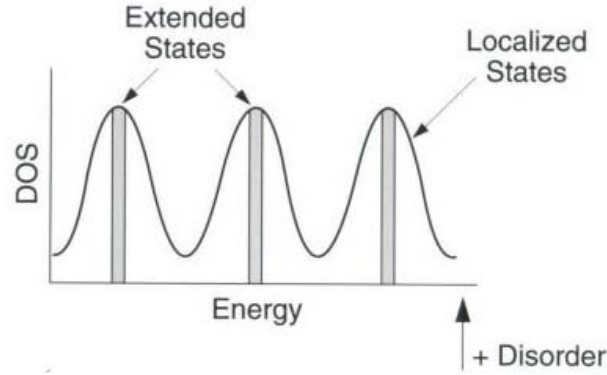


Figure 3.3: Density of states in a 2D electron gas in a strong magnetic field. (Real 2D crystal, with impurity and other imperfections)[20].

3.4.1 Laughlin-Thought Experiment

In Laughlin's thought-experiment the 2D electron system is bent to form a cylinder (Fig 3.4) whose surface is pierced every where by a strong magnetic field normal to the surface. The current I circles the loop. The magnetic field acting on the charge carriers to produce a Hall voltage V_H perpendicular to the current and to $\vec{B}$; that is V_H developed between one edge of the cylinder and the other. The circulating current is accompanied by a small magnetic flux ϕ that threads the current loop. The aim of the thought-experiment is to find the relation between I and V_H . Under the condition of vanishing conductivity σ_{xx} (no energy dissipation) energy is conserved and one can write Faraday's law of induction in a form which relates the current I in the loop to the total energy of the system E with

respect to the magnetic flux ϕ threading the loop

$$I = \frac{\partial E}{\partial \phi}. \quad (3.4.1)$$

The value of I can be found from the variation ΔE of the electronic energy that accompanies a small variation $\Delta\phi$ of the flux.

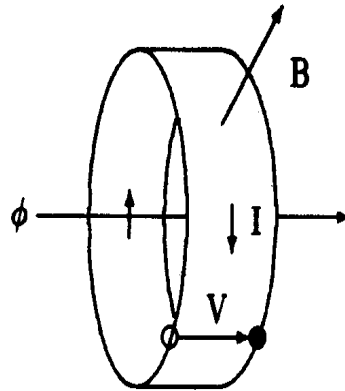


Figure 3.4: Geometry for Laughlin's thought experiment. The 2D electron system is wrapped around to form a cylinder. A strong magnetic field B pierces the cylinder everywhere normal to its surface. A current I circles the loop, giving rise to the Hall voltage V_H and a small magnetic flux ϕ through the loop[21].

Localized and Extended States

The carrier states divided in two classes Localized states and Extended states. It is thought that a perfect conductor gives the strongest effect, but the QHE actually relies on the presence of dirt in the samples. The presence of dirt and disorder can create a background potential landscape, with hills and valleys, in which the electrons move. At low temperature each electron trajectory can be drawn as a contour in the landscape. Most of this contours encircles hills and valleys so don't transfer an electron from one side of the sample to the other, they are localized states. Which are not continuous around the loop. A few states (just one at $T=0$) in the middle of each LL will be extended across the sample and carry the current. The extended states are continuous around the loop. At

higher temperatures the electrons have more energy so more states become delocalized and the width of extended states increases [22]. The two classes of states respond differently to the application of the flux ϕ . To a localized state a change in ϕ looks like a gauge transformation, which can not affect the energy of the state. The extended states enclose ϕ , and there energy may be changed. If the flux varies from 0 to $\phi = hc/e$ (flux quanta), the total change in energy corresponds to a transport of states from one edge to the other with $\Delta E = \nu e V_H$, [23] we have

$$I = \frac{\Delta E}{\Delta \phi} = \left(\frac{\nu e^2}{h} \right) V_H. \quad (3.4.2)$$

Which leads to quantized Hall resistivity (=Hall resistance) $\rho_H = h/\nu e^2$. The integer ν corresponds to the number of filled landau levels if the free electron model is used, but can be in principle any positive integer number.

Chapter 4

FRACTIONAL QUANTIZED HALL EFFECT

4.1 Fractional Quantized Hall Resistance

At sufficiently high magnetic fields, where the lowest Landau level is occupied, the integral quantum Hall Effect does not vanish in this regime. However the Hall resistivity found still to be quantized but now the quantization factor is not integer but fraction, $\nu = n/m$, where $n(< m)$ and m are integers. This is the fractional quantum Hall effect. Laughlin (1982) has formulated a theory for the fractional quantum Hall effect wherein, because of electron-electron interactions, the 2D electron gas becomes an incompressible quantum fluid in which, for a fractional Landau level filling factor, say $\nu = 1/m$, the quasi-particle excitations have a charge Q that is the fraction of electronic charge i.e. $Q = e/m$ [24]. This extraordinary prediction of fractional charge has been confirmed experimentally by measurements of the noise in a current passing through a constriction in the 2D gas subjected to a very large magnetic field (in the fractional quantum Hall effect regime) i.e. between quantum point contacts.

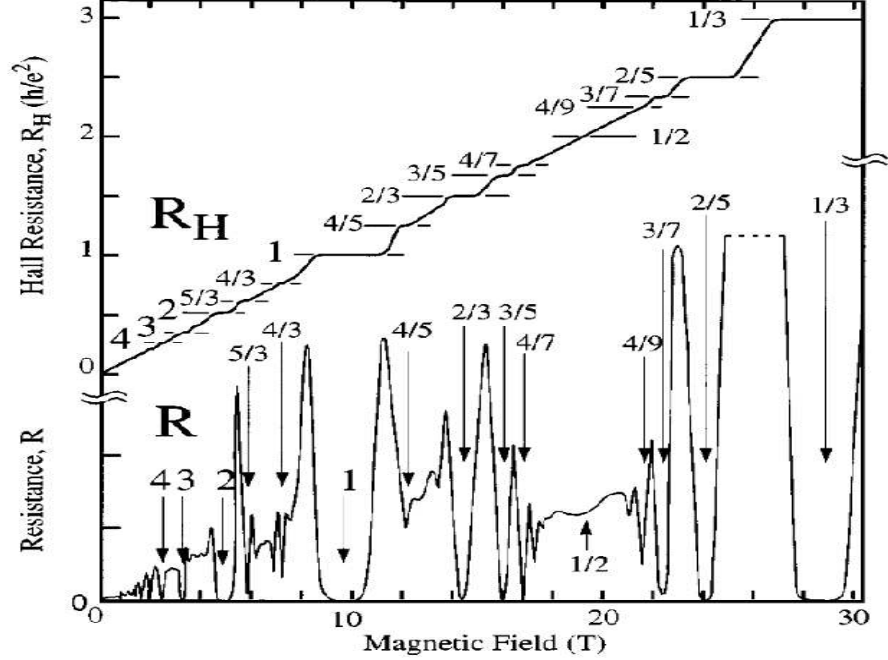


Figure 4.1: The figure shows the stepwise behavior of the transverse resistivity, superimposed with the longitudinal resistance, as a function of magnetic field. The same behavior as in figure 3.1 is seen except now at fractions of ν [25].

4.2 Quasi-particles in Fractional Quantum Hall Effect

4.2.1 Laughlin Wave Function

Fractionally quantized Hall resistance is not possible for non-interacting electrons [21].

By introducing an interaction potential, the Hamiltonian becomes

$$H = \sum_i \left[\frac{1}{2m} \left(\hat{p}_i + \frac{e\vec{A}_i}{c} \right)^2 + V(\vec{r}_i) \right] + \sum_{i>j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} \quad (4.2.1)$$

By restricting our discussion to extreme quantum limit in which the Landau level degeneracy is large enough that electrons can be accommodated within the lowest Landau level,

the single-particle wavefunctions in the lowest Landau level can be written as

$$\Psi_m(z_1) = \frac{z^m}{\sqrt{2^{m+1}m!\pi}} \exp\left(-\frac{|z|^2}{4l_c^2}\right), \quad (4.2.2)$$

where, $z = x + iy$ position of the particle, $l_c = \hbar c / eB$ is the classical cyclotron orbit radius [4].

The FQHE is similar to the IQHE, except that it occurs at fraction values of ν , such that $1/3, 2/3, 2/5$. Electron-electron interaction plays an essential role in the FQHE. In a strong magnetic field such that all electrons are accommodated in the lowest Landau Levels, the kinetic energy of the electrons is quenched so that electron-electron interaction becomes important. The wave function describing $\nu = 1$ state can be written as

$$\Psi(z_1, z_2, z_3, \dots, z_n) = \prod_{j < K}^N (z_j - z_k) \exp\left(-\sum_i \frac{|z_i|^2}{4l_c^2}\right), \quad (4.2.3)$$

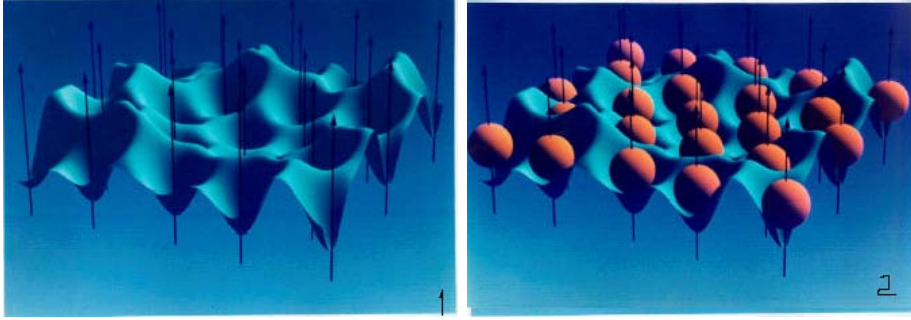


Figure 4.2: (1) single electron in the lowest Landau Level (2) Filled lowest Landau levels, state wave function of is $\Psi(z_1, z_2, \dots, z_n) \sim \prod (z_j - z_k)$ [26].

Any many electron wave function formed entirely within the lowest Landau level must be a product of one electron orbital for each coordinate which are of the form given above [26]. According to Laughlin, the wave function can be written as

$$\Psi_m(z_1, z_2, z_3, \dots, z_n) = \prod_{j < K}^N (z_j - z_k)^m \exp\left(-\sum_i \frac{|z_i|^2}{4l_c^2}\right), \quad (4.2.4)$$

the exponent m must be odd integer. At a series of fractional values of filling factor, the electrons take special configuration that are particularly effective in minimizing the

repulsive coulomb interaction. The ground state at, $\nu = 1/3$ is well described by the following wave function proposed by Laughlin:

$$\Psi_3(z_1, z_2, z_3, \dots, z_n) = \prod_{j < k}^N (z_j - z_k)^3 \exp\left(-\sum_i \frac{|z_i|^2}{4l_c^2}\right) \quad (4.2.5)$$



Figure 4.3: The lowest Landau Levels for (1) uncorrelated $1/3$ states (2) correlated $1/3$ states, wave function of correlated state is $\Psi(z_1, z_2, z_3, \dots, z_n) \sim \prod (z_j - z_k)^3$ [27].

4.2.2 Fractional Charge

Laughlin showed that these ingenious wave functions have as a consequence quasi-particles with fractional charge. The fractional quasi-particles are the positive quasi-holes and the negative quasi-electrons, with equal but opposite charges. The Laughlin wave function for $\nu = 1/3$ allows for quasi-particles with charge $e/3$. The existence of these fractional charges has been verified directly in shot noise experiments by Saminadayar et al in 1997. The fractional quasi-particles are a collective effect involving all the electrons. It is not possible to extract and isolate a $e/3$ quasi-electron from the quantum Hall liquid. They are topological in the same way as vortices. In the Laughlin wave function the quasi-holes are clearly simple vortices. Quasi-particles (Composite fermion) which are electrons with attached flux quanta [21].

Quantum Point Contact

A quantum point contact in a two dimensional electron gas. Based on the theoretical work of C.L. Kane and M.P.A. Fisher, such a device has recently been used to measure the fractional charge, $e/3$, of the Laughlin quasiparticle in the fractional quantum Hall effect. Quasiparticles flow along the one-dimensional edges as indicated by the arrows. (a) In the strong pinch off limit, electrons tunnel from left to right, while (b) with weak pinch-off, fractionally charged quasiparticles tunnel between the top and bottom edges through the electron gas [1].

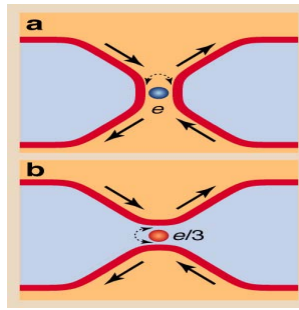


Figure 4.4: Only fractional charge of , $e/3$, flow along the one-dimensional edges. Upon decreasing the width of the opening (by varying the voltage on the gate electrode shown in the inset)[28].

4.3 The Composite Fermion

Composite fermions are a new class of particles discovered in condensed matter physics [29]. A composite fermion is the bound state of an electron and an even number of quantized vortices (often thought of as an electron carrying an even number of magnetic flux quanta). When a two-dimensional electron system is exposed to a strong transverse magnetic field, electrons minimize their interaction energy by capturing an even number of quantized vortices to transform into composite fermions. The complex, strongly correlated liquid of interacting electrons transforms into a simple, weakly interacting gas of composite fermions [30]. (An artistic depiction by Kwon Park.)

Composite fermions are predicted theoretically to explain the remarkable phenomenon of the "fractional quantum Hall effect" (FQHE). Since its inception, the composite fermion concept has been critically examined through a large number of tests, within the FQHE, which have established a close correspondence between the reality and the composite fermion theory.

It is experimentally established that composite fermions:

- Fill a fermi sea (the composite-fermion fermi sea)
- Execute semiclassical cyclotron orbits
- Form Landau levels (called CF-Landau levels)
- Show integral QHE (FQHE of electrons)
- Can be seen in mesoscopic experiments

The composite fermion theory possesses many qualities we desire in a theory [31].

Unification. The composite fermion theory explains all fractions in an equivalent manner. It unifies the fractional and the integral quantum Hall effects, earlier thought to be unrelated, showing that they are both integral quantum Hall effects, only for different particles. Furthermore, the theory also describes states where no FQHE is seen, for example, the compressible states at even denominator fractions [32].

Uniqueness. The composite fermion theory provides a detailed microscopic description of a strongly correlated many body state with no adjustable parameters.

Simplicity. A simple intuitive explanation for the basic phenomenology of the fractional quantum Hall effect is obtained, e.g., for the appearance of certain sequences of odd-denominator fractions and the lack of FQHE at certain even-denominator fractions.

New particle. Strongly interacting particles of one kind often reorganize to form new particles that are weakly interacting. These weakly interacting particles form the basis for describing the physics, and phenomena that looked mysterious earlier become simply explicable as properties of nearly free particles. Composite fermions are the weakly

interacting objects in the lowest Landau level liquid. They embody the profound reorganization that takes place when a collection of two-dimensional electrons is subjected to a strong magnetic field. Any number of magnetic flux may be associated with charge to produce a quasiparticle of the flux/charge composite, see the figure below.

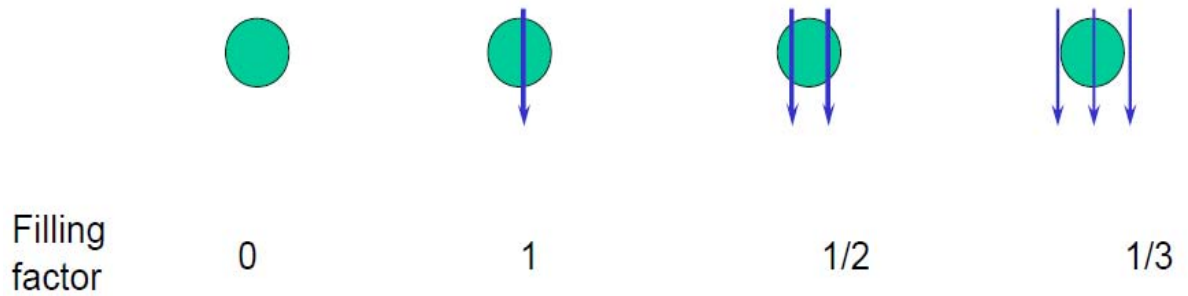


Figure 4.5: Composite fermions and their Landau levels [27].

Chapter 5

CHIRAL HEAT TRANSPORT IN THE QUANTUM HALL REGIME

Measurements of the heat transport at the edges of two-dimensional electron systems appear to provide explanations about the quantum Hall state that have not happen via charge transport. In this section, we discuss first heat transport through metals.[33].

5.1 Heat Transport

5.1.1 Thermal Conductivity

The electrons in solids not only conduct electricity but also conduct Heat, as they transfer energy from a hot junction to a cold junction. Just as the electrical conductivity characterizes the response of a material to an applied voltage, the thermal conductivity likewise characterize the material with regard to heat flow. In fact electrical and thermal conductivity are coupled, since thermal conduction also transports charge and electrical conduction transports energy. Heat flux or thermal current density , J_Q , between two points is proportional to the temperature gradient, i.e

$$J_Q = -K_T \nabla T \tag{5.1.1}$$

where the constant of proportionality, K_T , is the thermal conductivity coefficient. (For the case of non-cubic crystals, K_T is a tensor quantity.)

In the absence of free electrons (i.e for the case of electrical insulator or non-metals), heat is carried by phonos. For the case of metals containing free electrons, the electrons can also transport heat. In fact, this is the dominant contribution to thermal conductivity of metals, since the phonos are now scattered by conduction electrons as well as by phonon scattering, with the result that phonon much smaller than in non-metals, and hence the phonon contribution of K_T is reduced. The electronic thermal conductivity, i.e the heat carried by electrons, can be calculated using Boltzmann transport formulation, which can be used to discuss electrical conductivity, except now the quantity of interest is the heat flux, J_Q . Accordingly, the electronic thermal conductivity for a free electron gas :

$$K_T = \left(\frac{1}{3}\right)(v_F)^2\tau(\varepsilon_F)c_v \quad (5.1.2)$$

where $c_v = (\pi^2/3)(k_B)^2D(\varepsilon_F)T$ is the electronic specific heat per unit volume, $D(\varepsilon_F)$ is the density of states on the Fermi surface. The mean free path is $l = v_F\tau$. Here, the fermi speed $v_F = (2\varepsilon_F/m_e)^{1/2}$ is taken as a measure of the mean electron speed, since it is those electrons at the fermi surface, i.e at the top of the fermi Dirac electron distribution, that are able to take part in the transport process, τ is the collision time. Making use of the expression for c_v and v_F in to (5.1.2) leads to :

$$K_T = \frac{\pi^2n(k_B)^2T\tau}{3m_e}, \quad (5.1.3)$$

where n is the electron density.

The factor $n\tau/m_e$ appears both in the expression for σ_o (the free electron electrical conductivity) and in that K_T (the free electron thermal conductivity). Thus dividing these two quantities eliminates the parameter relating the particular electron gas, there by producing a universal value for the ratio L of thermal and electrical conductivities,

$$L = \frac{K_T}{\sigma_o T} = \frac{\pi^2(k_B)^2}{3e^2}, \quad (5.1.4)$$

known as the Weidemann-Franz law. The Lorenz number L has the theoretical value $2.45 \times 10^{-8} W\Omega K^{-2}$.

The Weidemann-Franz law states that the ratio of thermal and electrical conductivities is simply equal to a universal constant multiplied by the temperature. The Lorenz number, is not generally constant at all temperature. This is because the Weidemann-Franz law is only valid if, in the relaxation time approximation, a particular electron-scattering mechanism causes a degradation of an electrical current in exactly the same way and at the same rate as for thermal current. This is the case of elastic scattering (or more generally for inelastic scattering process where the change in electron energy is much smaller than $k_B T$). Thus the Weidemann-Franz law should be obey at very low temperatures where impurity scattering is dominant, and at high temperatures ($T \gg \theta_D$) in the phonon scattering regime. However, in the intermediate temperature range ($T < \theta_D$) inelastic phonon scattering events can cause a greater degradation of thermal currents than of electrical currents because the electron energy as well as the velocity can be changed. Thus the ratio $L = K_T/\sigma T$ is less than the universal value $2.45 \times 10^{-8} W\Omega K^{-2}$ in the this temperature rang. ($T \ll \theta_D$, Deby temperature) [1].

5.1.2 Thermoelectric Phenomeana

In many metals and semiconductors there exists a coupling between the electrical current and thermal current. This coupling can be appreciated by observing that when electrons carry thermal current, they are also transporting charge and therefore generating electric fields. Thus a temperature gradient in a long, thin bar should be accompanied by an electric field directed opposite to the thermal gradient.

In fact a heat current can be used to induce an electric potential (seebeck or thermoelectric effect) and, conversely, an electric current can be used to move heat (Peltier effect). Thermoelectric behavior can be discussed in terms of Boltzmann transport theory in the same way as for electrical conductivity (electrical current and no thermal flux) and thermal conductivity (thermal flux and no electrical current). Then the electrical current density j and heat flux J_Q can be written in terms of the appropriate 'driving force' and

Onsager coefficients, L^{ij} , as

$$j = L^{11}E + L^{12}(-\nabla T), \quad (5.1.5)$$

$$J_Q = L^{21}E + L^{22}(-\nabla T).$$

The Onsager coefficients, which are tensor quantities in general, they can be evaluated by expressing the charge and heat fluxes via Boltzmann formulation.

$$L^{11} = \sigma_o$$

$$L^{21} = TL^{12} = -\left(\frac{\pi^2}{3e}\right)(k_B)^2 T \sigma' \quad (5.1.6)$$

$$L^{22} = \left(\frac{\pi^2}{3e^2}\right)(k_B)^2 T \sigma$$

where σ' is the energy derivative of the conductivity evaluated at the fermi level : $\sigma' = \partial\sigma(\varepsilon_F)/\partial\varepsilon|_{\varepsilon=\varepsilon_F}$ [34].

5.2 Chiral Edge Current

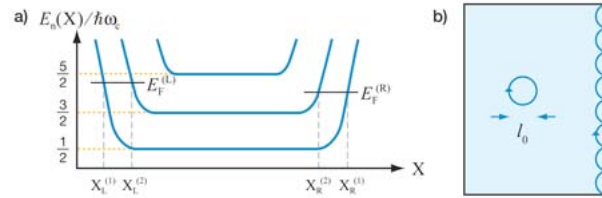


Figure 5.1: (a) Energy spectrum of a clean integer quantum Hall system as a function of guiding center position X . States near the edges disperse and carry current. By filling them to different Fermi energies $E_F^{(L)}$ and $E_F^{(R)}$, a net current flows down the sample which yields a quantized Hall resistance R_H . (b) Interpretation in terms of classical orbits: In bulk, electrons orbit around a fixed position and do not carry current. At the edges specular reflection induces current-carrying motion [33].

Edges play a crucial role in QHE, where persistent local currents lead to the quantization of the Hall effect. This extremely precise quantization, however, limits how much may be learned from charge transport at the edge. But now a new type of measurement,

reported in Physical Review Letters by G. Granger, J. P. Eisenstein, and J. L. Reno at Caltech, involving heat transport at the edge, may provide new insight [33].

In an idealized situation where a sample is very clean. The wave functions are extended parallel to the edge but are confined around a positional quantum number X , called the guiding center coordinate, perpendicular to the edge. States far from the edge have a simple harmonic oscillator spectrum with discrete energy levels called Landau levels. The separation between allowed values of X is tiny and so there is a huge number of allowed states for each Landau level n .

The energies of states with guiding center closer to the edge deviate from the bulk value, generally increasing as X approaches the edge [see Fig. 5.1(a)]. Classically, this corresponds to skipping trajectories involving specular reflection of the electrons [Fig. 5.1(b)], allowing their transport down the length of the system. Thus each edge carries a current, even in equilibrium, and these currents are chiral; they have a unique direction determined only by the relative orientation of the magnetic field and the edge itself.

Edge states play a crucial role in the thermodynamics of the quantum Hall system at low temperatures: they are the only gapless excitations available in the system, at least in the clean limit. These excitations are formed by exciting electrons from just below the Fermi energy at an edge to just above [see Fig. 5.1(a)].

Granger and coauthors have developed a method for probing this property of the edge states. Using very small contacts laid out around the system edges, they injected electrons at a higher temperature than the system bulk, and measured the local electron temperature a distance away from the injection point. The result was that the electrons were warmer than the bulk electrons on only one side of the injection point, down the direction that the hot electrons should be carried by the edge current. This constitutes one of the most direct experimental demonstrations of chiral edge currents to date .

This new method is promising in that it probes properties near the quantum Hall edge that are inaccessible in charge transport experiments. The cooling of the electrons

along the edge must involve inelastic processes, including coupling with lattice phonons and localized electron states. There are also situations in the fractional quantized Hall regime in which several edge states may carry currents in different directions at the same edge; such counterpropagating modes have not been observed in charge transport, most likely due to the effects of disorder when states of opposite chirality are proximate to one another [35].

5.3 Quantized Heat Transport in the IQHE

The connection between quantized electrical transport and the microscopic structure of edge states is of fundamental importance to the quantum Hall effect. The Hall conductance is related to the additional edge current J , which flows when the chemical potential μ of the edge is raised,

$$G_H = e \frac{\partial J}{\partial \mu} = \nu \frac{e^2}{h}, \quad (5.3.1)$$

In the integer quantum Hall effect, the edge states consist of a single noninteracting electron mode for each full Landau level. The contribution of each mode to the Hall conductance has the quantized value e^2/h .

In addition to charge, energy is also transported by quantum Hall edge states. At temperatures well below the quantum Hall gap, the energy moving along the edge cannot easily escape, since there are no bulk current carrying electronic excitations. When the top and bottom edges of a Hall bar are at different temperatures a thermal transport current will then flow. This gives rise to the thermal analogue of the Hall effect, known as the Leduc-Righi effect. One can define a thermal Hall conductance, analogous to (5.3.1) [36],

$$K_H = \frac{\partial J_Q}{\partial T} = \nu_Q \frac{\pi^2 k_B^2}{3h} T, \quad (5.3.2)$$

where J_Q is the thermal current carried by the edge modes. For the free-electron edge modes in the integer quantum Hall effect, each mode contributes a quantized amount to

K_H , of magnitude $K_H = \pi^2 k_B^2 / 3hT$.

The thermal K_H and electrical G_H Hall conductances are thus simply related by the Wiedemann-Franz law for free electrons. A similar quantization of the thermal conductance occurs for a quantum point contact [35].

5.4 Quantized Heat Transport in the FQHE

In the fractional quantum Hall effect (FQHE) the edge modes are no longer free-electron-like, but rather are chiral Luttinger liquids. The charge carried by these modes contributes to the electrical Hall conductance, giving an appropriately quantized fractional value. But as in the integer quantum Hall effect (IQHE), one anticipates that the edge modes will also dominate the transport of heat at low temperatures. In the study thermal transport in the FQHE, by employing a chiral Luttinger liquid model of the edge states, it is found that the FQHE edge modes also contribute a quantized thermal Hall conductance. But in contrast to the IQHE, K_H is no longer related to the Hall conductance G_H , via the Wiedemann-Franz law. Rather, K_H provides an independent quantized characterization of the FQHE state. In fact, the quantized thermal Hall conductance is a universal property of the quantum Hall state, in some ways as fundamental as the electrical Hall conductance, although of course much more difficult to measure. But if measured, K_H would provide a nontrivial test of microscopic edge state theories, as stated below .

For hierarchical FQHE states, multiple propagating modes on a given edge are predicted . But even more interesting is the prediction that for certain filling fractions, such as $\nu = 2/3$ and $\nu = 3/5$, some of the chiral edge channels propagate in the "wrong" direction opposite to that of the classical skipping orbits specified by the sign of the magnetic field. For a clean edge, the "upstream" modes carry electrical charge, and contribute negatively to the Hall conductance. The net electrical Hall conductance is nevertheless positive (and quantized), since the contribution from the "downstream" modes is larger

in magnitude. Unfortunately, the "upstream" modes have not yet been detected experimentally, presumably due to effects of edge state equilibration. It is proposed that these upstream modes can have great effect on the thermal transport. Specifically, for the fraction $\nu = 3/5$ the thermal Hall conductance K_H is predicted to be negative of opposite sign to the electrical Hall conductance, G_H . The upstream modes actually dominate in the thermal transport, so that the net thermal current along the edge is in the opposite direction to the net charge transport current. For $\nu = 2/3$, on the other hand, It is predicted that the thermal Hall conductance vanishes, due to a cancelation between upstream and downstream modes. Rather than being carried ballistically, the heat transport along the edge is predicted to be diffusive, leading to a non vanishing thermal Hall conductivity. These predictions are robust, being valid in the presence of equilibration processes, due to edge impurity scattering. Thus, thermal transport measurements may provide a unique way to establish the existence of the elusive upstream moving channels [35].

Upon combining with (5.3.1) and (5.3.2), the Lorenz ratio can be expressed as

$$L = \frac{K_H}{TG_H} = \frac{\nu_Q}{\nu} L_o, \quad (5.4.1)$$

where $L_o = (\pi^2/3)(k_B/e)^2$ is the free-electron value. Notice that each mode contributes the same amount to the thermal conductance. This leads to a Lorenz ratio for the FQHE which violates the Wiedemann Franz law. Like the Hall conductance ν , the thermal coefficient ν_Q is a robust and universal topological quantity, characterizing the Hall state.

When all of the edge channels move in the same direction, as shown for $\nu = 2$ [Fig. 5.2(a)], K_H is simply a measure of the total number of channels. But when channels move in both directions, there is an exact cancelation between the contribution of the upstream and downstream modes to the thermal Hall conductance. This leads to some striking predictions. For $\nu = 3/5$ there are three modes, two of which move upstream, as sketched in Fig. 5.2(c). This implies a thermal Hall conductance that is negative opposite in sign to the electrical Hall effect. For $\nu = 2/3$ [Fig. 5.2(b)] there is one upstream channel

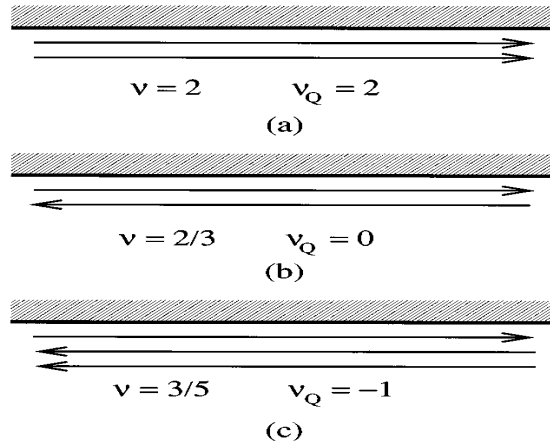


Figure 5.2: Schematic representation of edge states for several different quantum Hall fluids at filling factor ν . The integer ν_Q , which specifies the difference between the number of downstream and upstream propagating modes, determines the sign and magnitude of the thermal Hall conductance [35].

and one downstream channel. Thus, $K_H = 0$: in equilibrium there is no net heat flow along the edge. These results are robust, and will survive in the presence of edge impurity scattering, which serves to equilibrate the various edge modes.

5.5 Experimental Implications of Quantized Heat Transport

As shown above, the thermal Hall conductance K_H contains important information about the structure of the edge excitations in the FQHE. In particular, the sign of K_H is sensitive to the presence of edge modes that propagate "upstream". This section a briefly discussion of the feasibility of measuring the Hall thermal conductance. To see a particular geometry that should at least enable a measurement of the sign of K_H .

To extract K_H requires measuring the thermal current carried by the edge excitations. Although this is clearly much more challenging than measuring charge transport, a recent experiment by Molenkamp et al has demonstrated the feasibility of measuring thermal transport in mesoscopic structures. Specifically, in this experiment the thermal

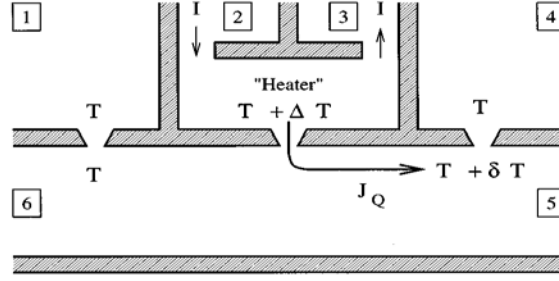


Figure 5.3: Proposed geometry for measurement of the thermal Hall conductance. The heat generated by passing a current between contacts 2 and 3 is detected by two additional point contacts, upstream (1-6) and downstream (4-5) along the edge [35].

conductance of a quantum point contact (in zero magnetic field) was extracted. The trick was using additional point contacts as "thermometers", to measure the local temperature of the electron gas on either side of the point contact. The additional point contacts were biased on the edge of a step between two plateaus, so that they would have a large, temperature-independent thermopower of order 40 mV/K. Then, by measuring the voltage across these additional point contact thermometers the local temperature change was extracted. In Molenkamp experiment, the thermal current was estimated from the change in temperature by estimating the heat capacity of the electron gas. This allowed for a determination of the thermal conductance and Peltier coefficient of the point contact, which agreed favorably with theoretical expectations [35].

It should be possible to adapt this technique to measure the thermal transport of quantum Hall edge states. As a concrete example, consider the geometry sketched in Fig. 5.2. As in the experiment by Molenkamp et al, the sample can be heated locally by driving a small electric current through the electron gas. Specifically, a current between contacts 2 and 3 (see Fig. 5.2) would locally heat the edge of the quantum Hall fluid. Alternatively, it might be possible to heat the edge directly by coupling in a local probe. This local heating will be carried away by the edge states, either raising the temperature of the downstream or upstream edge, depending on the sign of K_H . This temperature change can then be detected by measuring the voltages across the point contact thermometers.

While this measurement might not be suitable to extract the magnitude of K_H , it should be adequate to determine the direction of heat propagation and hence the sign of K_H . For this, one need only detect an asymmetry in the temperature change of the upstream and downstream edges. For integer quantum Hall states, and fractional states at filling factors such as $\nu = 1/3, 2/5$, the heat should flow downstream, resulting in a temperature increase at the downstream thermometer only. On the other hand, for filling $\nu = 3/5$, since K_H is negative, heat flows “upstream“, so the temperature increase should be detectable at the upstream thermometer. For $\nu = 2/3$, the heat is predicted to diffuse along the edge. The temperature increases should then be the same at the upstream and downstream thermometers, but smaller in magnitude.

On sufficiently long length scales, the edge states will thermally equilibrate with the phonons in the substrate. It is thus crucial that the heater and thermometer are closer together than the electron-phonon thermal equilibration length. While this equilibration length has not been measured, we expect it to be quite long. Energy relaxation rates have been measured for two-dimensional electron gases in GaAs in zero magnetic field. At 1 K relaxation times of order 20ns are found, which suggests an equilibration length of upwards of 100nm. At low temperatures, the interaction between electrons and the lattice in GaAs is dominated by the piezoelectric coupling to acoustic phonons and the relaxation rate decreases as the temperature is lowered. Thus, we suspect that lattice thermalization will not be a problem in the proposed edge state experiment.

Chapter 6

CONCLUSION

Review of the present theoretical understanding of quantum Hall effect occurring in 2D electron gas with a strong $\vec{B}$ perpendicular to the system plane. The essential signatures are: resistivity component $\rho_{xx} \rightarrow 0$ as $T \rightarrow 0$, with conductivity component $\sigma_{xy} = \nu e^2/h$ over a finite range of $\vec{B}$ (“Hall plateau“), where ν is an integer or ratio of integers having odd denominator. Defining Landau filling factor as $\nu = n(hc/eB)$, where n is the areal density of carriers, conditions for occurrence are (a) a gap (i.e. a discontinuity in the chemical potential), pinned to fixed filling factor ν , (b) localization of charged excitations for ν . The integral effect is described by the interaction between electrons and impurities, while the fractional effect due to electron-electron correlations. Incompressible quantum fluid states arise when the filling factor is a rational fraction with odd denominator, because of restriction imposed on electron-electron correlations by the constraint that the electrons share the lowest Landau level. The elementary excitations of the incompressible fluid states are quasiholes (quasiparticles) by disorder. In the most prominent cases, these excitations may be described in terms of Laughlin wave function [38].

We also have seen that the thermal conductance of a quantum Hall edge state is universal and quantized: $K_H = \nu_Q K_0$ with $K_0 = 5(\pi^2/3)(k_B^2/h)T$. The integer ν_Q specifies the difference between the number of downstream and upstream edge modes. Moreover, the quantization of K_H was shown to be robust, valid in the presence of interactions and impurity scattering at the edge. Since the additional edge channels come in pairs one

upstream and one downstream they do not change ν_Q . On sufficiently long length scales, so that all edge modes are equilibrated, we see only a single charge mode and a single heat mode. These two modes carry the charge and thermal currents, leading to the quantized conductances. In the FQHE, the thermal Hall conductance K_H contains additional information about the microscopic edge structure, not present in the electrical conductance. For $\nu = 3/5$, we predict that K_H is negative, due to the presence of "upstream" propagating edge modes. A measurement of the sign of K_H would thus provide a critical and nontrivial test of current edge state theories [35].

Appendices

Appendix A

Two electrons in a uniform magnetic field: Quantum Mechanical Description

In this section we develop a method for solving schrodinger equation for two electrons in a uniform magnetic field. These studies appeared important because they led to discovery of a new state, the incompressible electron liquid [4]. The Hamiltonian of two electrons in an interaction potential is given by

$$H = \frac{1}{2m_e}(\hat{p}_1 - \frac{e\vec{A}_1}{c})^2 + \frac{1}{2m_e}(\hat{p}_2 - \frac{e\vec{A}_2}{c})^2 + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}, \quad (6.0.1)$$

where the parameters $\hat{p}_1$ and $\hat{p}_2$ are the momentum of two electrons, $\vec{r}_1$ and $\vec{r}_2$ are the positions of two electrons, $\vec{A}_1$ and $\vec{A}_2$ are the vector potentials of these electrons.

Expanding (6.0.1) will give us

$$H = \frac{1}{2m_e}(\hat{p}_1^2 + \hat{p}_2^2) + \frac{e^2}{2m_e c^2}[\vec{A}_1^2 + \vec{A}_2^2] - \frac{e}{m_e c}[\vec{A}_1 \cdot \vec{p}_1 + \vec{A}_2 \cdot \vec{p}_2] + \frac{e^2}{r}. \quad (6.0.2)$$

Now, we want to reduce the two-body problem into a single-body one. For this purpose, we introduce two new variables as follows:

$$\vec{r} = \vec{r}_1 - \vec{r}_2, \quad (6.0.3)$$

$$\vec{R} = (\vec{r}_1 + \vec{r}_2)/2, \quad (6.0.4)$$

the new coordinates thus correspond to the difference in the electronic positions and the center of mass of the two electrons, respectively.

And the vector potentials are

$$\vec{A}_1 = \frac{1}{2}\vec{H} \times (\vec{R} + \frac{\vec{r}}{2}), \quad (6.0.5)$$

$$\vec{A}_2 = \frac{1}{2}\vec{H} \times (\vec{R} - \frac{\vec{r}}{2}). \quad (6.0.6)$$

Now after appropriate substitution the Hamiltonian (6.0.2) in the new coordinate system is

$$H = \frac{1}{4m_e}P^2 + \frac{1}{2(m_e/2)}p^2 + \frac{e^2H^2}{4m_e c^2}[R^2 + \frac{r^2}{4}] - \frac{eH}{m_e c}[\vec{R} \times \vec{P} + \vec{r} \times \vec{p}] + \frac{e^2}{r}, \quad (6.0.7)$$

where P and p are the center of mass and relative momentum of the electrons respectively.

Schrodinger equation for the two electrons in the new coordinates is given by

$$\hat{H}\Psi(\vec{R}, \vec{r}) = E\Psi(\vec{R}, \vec{r}), \quad (6.0.8)$$

where the wave function depends on the on the center of mass position and relative position of the two electrons.

To decouple the Schrodinger equation, we use the variable technique separation method.

To do this, we assume that the wave function can be separated, i.e.

$$\Psi(\vec{R}, \vec{r}) = \psi(\vec{R})\psi(\vec{r}). \quad (6.0.9)$$

The schrodinger equation after separation of variables

$$[\frac{1}{2(2m_e)}P^2 + \frac{(2m_e)\omega_c^2}{8}R^2 - \omega_c \hat{L}_z^R]\psi(\vec{R}) = E_R\psi(\vec{R}), \quad (6.0.10)$$

and

$$[\frac{1}{2(m_e/2)}p^2 + \frac{2(m_e/2)\omega_c^2}{16}r^2 + \frac{e^2}{r} - \omega_c \hat{L}_z^r]\psi(\vec{r}) = E_r\psi(\vec{r}), \quad (6.0.11)$$

where $\omega_c = eH/m_e c$ is the cyclotron frequency, $\hat{L}_z^R$ and $\hat{L}_z^r$ are the angular momentum of center of mass and relative position of the electrons.

The Hamiltonian H_R (6.0.10) and H_r (6.0.11) coincide with the Hamiltonian of a single particle of mass m_e and charge e , assuming R and r are 2D, the energy eigenvalue of center of mass E_R and relative position E_r can be obtained as

$$E_R = \hbar\omega_c(n_R + \frac{|M| + M + 1}{2}), \quad (6.0.12)$$

$$M = 0, \pm 1, \pm 2, \pm 3, \dots, n_R = 0, 1, 2, \dots$$

$$E_r = \hbar\omega_c(n_r + \frac{|m| + m + 1}{2}). \quad (6.0.13)$$

$$m = 0, \pm 1, \pm 2, \pm 3, \dots, n_r = 0, 1, 2, \dots$$

But we have the term e^2/r which could be considered in the frame work of the perturbation theory as follows. The coulomb energy for the state $n=0$, $m>0$ can be calculated with the help of the perturbation energy [4]

$$E^1 = \langle 0m | e^2/\varepsilon r | 0m \rangle = \frac{e^2}{\varepsilon a_H} \frac{\Gamma(m + 1/2)}{m!}, \quad (6.0.14)$$

where $a_H = \sqrt{\hbar/eH}$ is magnetic length, ε is electric permeability,

at large m : $\Gamma(m)/m! = (m-1)/m! \sim 1/m$, then $E^1 = e^2/\varepsilon a_H m$.

Now the total energy of the two electrons takes the form

$$E = E_R + E_r + E^1. \quad (6.0.15)$$

We may not that the energy of the center of mass must be taken into account.

Appendix B

Two electrons in a uniform magnetic field: Classical Description

In this section we develop a method for solving the equation of motion of two charged particles in a magnetic field. Lets consider a system of two particles with the same mass m_e and positions $\vec{r}_1, \vec{r}_2$ respectively [38]. The equations of motion are

$$m_e \ddot{\vec{r}}_1 = F_1 + \frac{e}{c} \dot{\vec{r}}_1 \times \vec{H}, \quad (6.0.16)$$

$$m_e \ddot{\vec{r}}_2 = F_2 + \frac{e}{c} \dot{\vec{r}}_2 \times \vec{H}, \quad (6.0.17)$$

where F_1 is the electric force exerted on particle 1 by particle 2, and $e\dot{\vec{r}}_1 \times \vec{H}$ is the magnetic force exerted on particle 1, with a similar description of the forces on particle 2.

Now we suppose the following

$$\vec{r} = \vec{r}_1 - \vec{r}_2, \quad (6.0.18)$$

is the relative position ,

$$\vec{R} = (\vec{r}_1 + \vec{r}_2)/2 \quad (6.0.19)$$

is the center of mass position, and

$$U = \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \quad (6.0.20)$$

is the potential of the electrons.

Then, we have

$$F_1 = \frac{\partial U}{\partial r_1} = e^2 \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}, \quad (6.0.21)$$

$$F_2 = \frac{\partial U}{\partial r_2} = e^2 \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}. \quad (6.0.22)$$

So the equations of motion can take the form

$$m_e(\ddot{\vec{R}} + \ddot{\vec{r}}/2) = F_1 + \frac{e}{c}(\dot{\vec{R}} + \dot{\vec{r}}/2) \times \vec{H}, \quad (6.0.23)$$

$$m_e(\ddot{\vec{R}} - \ddot{\vec{r}}/2) = F_2 + \frac{e}{c}(\dot{\vec{R}} - \dot{\vec{r}}/2) \times \vec{H}. \quad (6.0.24)$$

Our aim now is to describe the system by distinguishing the external motions of its parts.

This aim is facilitated by Newton third law,

$$\vec{F}_1 = -\vec{F}_2 \quad (6.0.25)$$

Notice now that, because of (6.0.25) the internal forces cancel.

When we add the two equations

$$\ddot{\vec{R}} = -\omega_c \dot{\vec{R}} \times \vec{h}, \quad (6.0.26)$$

where $\omega_c = eH/m_e c$ is the cyclotron frequency, $\vec{h}$ is a unit vector in the direction of magnetic field, $\vec{H}$.

Equation (6.0.26) can be regarded as an equation of motion for the system as a whole. It is like the equation of motion for a single particle with mass $m_e/2$ and position $\vec{R}$.

For $\vec{\Omega} = \omega_c \vec{h}$, (6.0.26) can be written as

$$\ddot{\vec{R}} = \vec{\Omega} \times \dot{\vec{R}}. \quad (6.0.27)$$

This means that the vector $\vec{R}$ rotates with a constant velocity $\vec{\Omega}$.

The internal structure can be described in terms of the relative position of particles with respect to one another.

When we subtract the two equations

$$\ddot{\vec{r}} = -\omega_c \dot{\vec{r}} \times \vec{h} + 2e^2 \frac{\vec{r}}{m_e r^3}. \quad (6.0.28)$$

Equation (6.0.28) describe the internal motion of the two particle system.

Now lets multiply (6.0.28) by $\dot{\vec{r}}$ in both sides

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} = -\omega_c(\dot{\vec{r}} \times \vec{h}) \cdot \dot{\vec{r}} + 2e^2 \frac{\dot{\vec{r}} \cdot \dot{\vec{r}}}{m_e r^3}, \quad (6.0.29)$$

but, $\dot{\vec{r}} \times \vec{h}) \cdot \dot{\vec{r}} = 0$.

So (6.0.29) can put in the form

$$\frac{d}{dt} \left(\frac{\dot{r}^2}{2} \right) = 2e^2 \frac{\dot{\vec{r}} \cdot \dot{\vec{r}}}{m_e r^3}. \quad (6.0.30)$$

Rearranging this expression gives

$$d\left(\frac{\dot{r}^2}{2}\right) = \frac{2e^2}{m_e r^3} \dot{\vec{r}} \cdot \dot{\vec{r}} dt, \quad (6.0.31)$$

or, in the following form

$$d\left(\frac{\dot{r}^2}{2}\right) = -d\left(\frac{2e^2}{m_e r}\right). \quad (6.0.32)$$

Further it can be written as

$$\frac{\mu}{2} \dot{r}^2 + \frac{e^2}{r} = E = \text{constant}, \quad (6.0.33)$$

where $\mu = m_e/2$ is the center of mass of the system.

We introduce polar coordinates, (r, θ) . Then the energy in terms of the polar coordinates

$$\frac{\mu}{2} [\dot{r}^2 + r^2 \dot{\theta}^2] + \frac{e^2}{r} = E. \quad (6.0.34)$$

Let ω_c is large enough because of the strong magnetic field. For r and θ functions of time, we can write $\theta = \omega_c t + \theta_o$. And (6.0.34) takes the form

$$\dot{r}^2 = \frac{2}{\mu} \left(E - \frac{e^2}{r} - \frac{\mu}{2} r^2 \omega_c^2 \right). \quad (6.0.35)$$

After this to evaluate the time t , we write (6.0.35) as

$$\frac{dr}{dt} = \sqrt{\frac{2}{\mu} \left[E - \frac{e^2}{r} - \frac{\mu}{2} r^2 \omega_c^2 \right]}, \quad (6.0.36)$$

then,

$$\int \frac{dr}{\sqrt{\frac{2}{\mu}[E - \frac{e^2}{r} - \frac{\mu}{2}r^2\omega_c^2]}} = t + c. \quad (6.0.37)$$

Consider the situation when coulomb energy is small (large r). In this case (6.0.37)

$$\int \frac{dr}{\sqrt{\frac{2}{\mu}[E - \frac{\mu}{2}r^2\omega_c^2]}} = t + c. \quad (6.0.38)$$

or, displacement of harmonic oscillator given by

$$r(t) = A \cos(\omega_c t + \theta_o), \quad (6.0.39)$$

where $E = \mu\omega_c^2 A^2/2$, then

$$r(t) = \sqrt{4E/m\omega_c^2} \cos(\omega_c t + \theta_o). \quad (6.0.40)$$

The correction connected with $-e^2/r$ can be calculated by the perturbation method. This was done in Appendix A with the help of quantum mechanical theory of perturbation.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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