ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
FACULTY OF TECHNOLOGY
CIVIL ENGINEERING DEPARTMENT

USE OF COMPOSITE CONCRTE SLAB SYSTEM USING HOLLOW BLOCKS AND PRECAST SLAB/BEAM MEMBER

A thesis submitted to
The school of graduate studies of Addis Ababa University in partial fulfillment of the
requirement for the degree of Master of Science in Structural Engineering

Advisor
Adil Zekaria (Dr-Ing)

By
Merid Belay
June 2004

DECLARATION

I, THE UNDERSIGNED DECLARE THAT THIS THESIS IS MY ORIGINAL WORK, IT HAS NOT BEEN PRESENTED FOR A DEGREE IN ANOTHER UNIVERSITY AND THAT ALL SOURCES OF MATERIAL USED FOR THIS THESIS HAS BEEN ACKNOWLEDGE.

Name:	Merid Belay
Signature:	____________________
Place:	Faculty of Technology A.A.U
Date of Submission:	July 13,2004

Table of Content

Pages

7. Appendix

1. INTRODUCTION

1.1 GENERAL

Housing is needed by many people in all parts of the country. Low-income persons usually cannot afford to pay for the labour to build their houses and they have difficulty in doing the building themselves because many of the construction methods and standard practices are too complicated. Simplification of the construction methods therefore seems to be a logical way to reduce the cost of house.

Moreover the increasing cost of construction materials and the time taken are the other inputs, which make the cost of construction to be expensive, and therefore the other way of reducing the cost of construction is to decrease the material and labour inputs.

One of the temporary construction material inputs is the formwork. Formwork is a temporary ancillary construction used as a mould for structure, in which concrete is placed, hardens and matures. The construction of formwork involves considerable expenditure of time and material. The cost of formwork may be up to 20 to 25% of cost of structure in building work and even higher in bridges. In order to reduce this expenditure, it is necessary to design to economical types of formwork and mechanize its construction. When the concrete has reached a certain required strength, the formwork is no longer needed and is removed, in the case of slab the formwork should be remain as it is for 14 days, this condition obstructs activities to be performed at lower level and affects the total time required for the construction. On the other hand the use of wood for use of formwork is a common practice in our country, and this significantly affects the countries limited source of wood. Therefore decreasing the use of formwork in the construction industry reduces the expenditure, time of construction and generally contributes in the reduction of deforestation.

This study uses a simplified precast beam slab system, which reduces the use of formwork from the usual way of construction technique.

1.2 OBJECTIVE AND SCOPE OF THE STUDY

The main objective of this study is to determine economical size and maximum span of the composite element for different combination of concrete topping and support conditions, for use in the construction of small buildings.

In addition to the above, analysis of the hollow concrete blocks, which are used in combination to the precast will be done to identify the weakest point on the hollow block for use as a load bearing structure.

Finally analytical results obtained from this study are compared with those obtained from the laboratory test by the research group of Civil Engineering Department of Technology Faculty, AAU.

2. SIZE OF COMPOSITE ELEMENT

2.1 DETERMINATION OF SIZE OF PRECAST ELEMENT

2.1.1 THICKNESS OF PRECAST ELEMENT

The minimum thickness of the precast element is determined based on, the diameter of the reinforcing bar and the minimum concrete cover for slab reinforcement for moderately exposed structure as stated on EBCS-2 Table 9.2.,therefore there are different thicknesses for different diameter of reinforcing bar used.

For example

for ϕ 10mm reinforcing bar;

Top Cover	= 15 mm
Bottom Cover	= 15 mm
Reinforcement Diameter	= 10 mm
Total thickness	= **40 mm**

The minimum thickness, which can be used for the precast element, is 40mm, and can be increased to any size by considering practical and any other requirements. But keeping the weight of the precast element as low as possible is advantageous for its erection. Increasing the thickness of the precast will increase the self-weight and makes it heavy to be lifted for erection, and also increase the reinforcement required at the initial stage, without having positive effect on the strength for the case of no intermediate support.

2.1.2 WIDTH OF PRECAST ELEMENT

The minimum width of precast element is also determined based on the concrete cover for moderate exposure and the minimum spacing of reinforcing bars in the precast element.

For example

for the case of using ϕ 10mm reinforcing bar;

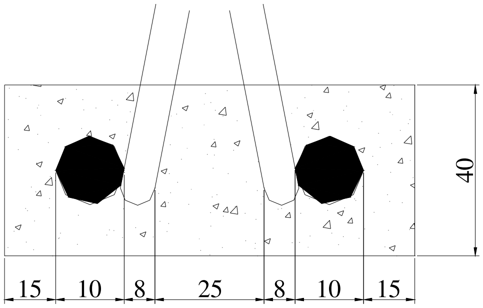


FIG 2.1 Reinforcement arrangement of precast element

Side Concrete cover = 2x15mm =30mm

Reinforcement diameter = 2x10 mm =20mm

Minimum spacing between reinforcement

(Maximum aggregate size +5mm) = 20+5mm =25mm

Maximum stirrup size = 2x8mm =16mm

Total width = =**91mm**

The minimum width of the precast element with ϕ 10mm reinforcing bar is 91mm but as it is mentioned above for the thickness, it is possible to increase the width to any practical size by considering practical and other requirements.

The maximum width of the element also has no positive effect on the strength of the precast element, therefore keeping the width minimum will make the precast light and easy for erection.

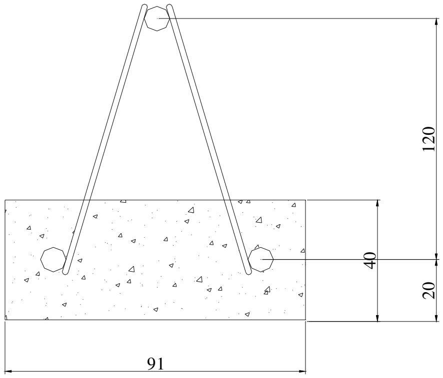


FIG 2.2 Depth and Width of Precast element

Therefore for each diameter of reinforcement the thickness and width of precast element are determined as per the above calculation.

2.2. SIZE OF HOLLOW CONCRETE BLOCKS

Three different pre-determined size of hollow concrete block of width of 200mm, length 540mm and depths 160mm for small span, 200mm for medium, and 240mm for large span slabs are used in this study.

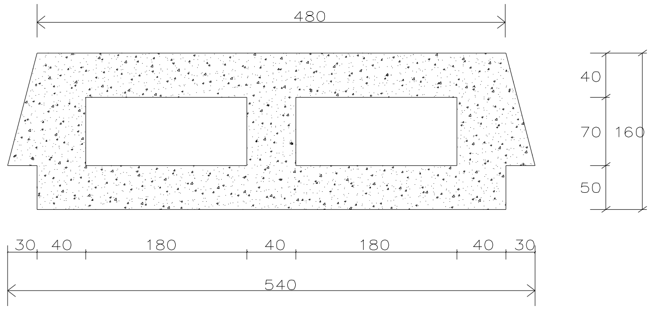


FIG 2.3 Hollow Concrete Block for Small span slabs.

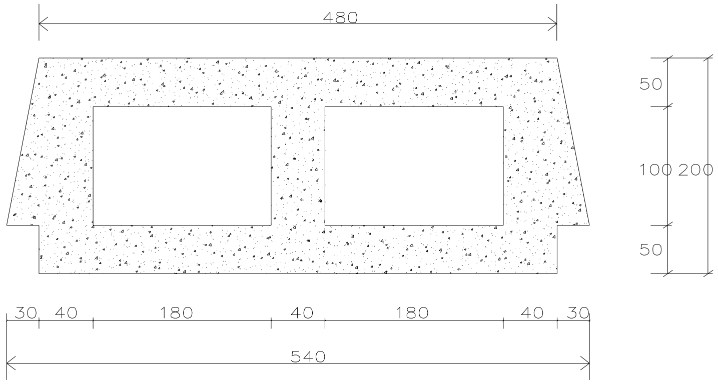


FIG 2.4 Hollow Concrete Block for Medium span slabs.

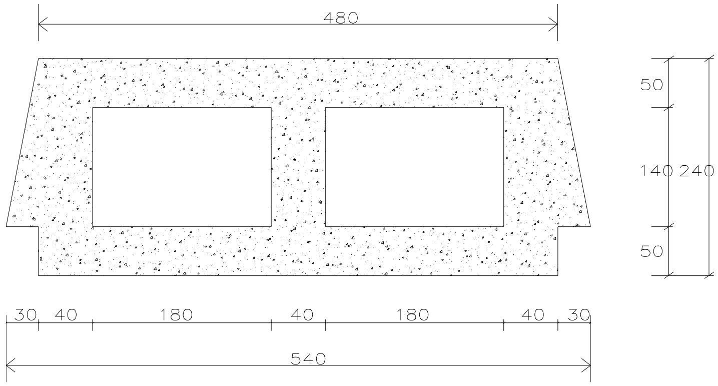


FIG 2.5 Hollow Concrete Block for Large span slabs.

2.3 SIZE OF REINFORCEMENT

The size of reinforcement used in this study ranges from ϕ 8 to ϕ 28mm diameter depends on the maximum geometric ratio of reinforcement ρ of 4% as per EBCS-2 1995 recommendation.

For 160mm deep HCB without concrete topping

$$d_{eff} = 160 - 15 - 12 = 133mm$$

$$b_w = 119mm$$

$$A_{s\max} = 0.04 \times 133 \times 119 = 633.1mm^2$$

which is less than 2 ϕ 24 (904.8mm^2)

Therefore the maximum reinforcement to be used for 160mm deep HCB without concrete topping is 2 ϕ 20 (628.3mm^2)

Similarly maximum diameter of reinforcements are determined and used for their respective depth and width as detailed on Appendix D

2.4 MAXIMUM SPAN OF COMPOSITE STRUCTURE

In the determination of the maximum span of the composite structure, four stages of constructions are studied, these are

Erection Stage:- This is the stage at which the precast element is lifted from its position for erection with the help of simple pulley or labour by supporting the element at its edge as it shown on Fig. 2.7 below.

Block Laying (Initial) Stage:- This is the stage at which the precast element is placed in position and the concrete block is arranged on the precast element.

Concrete Pouring (Intermediate) Stage:-This is the stage at which precast element and HCB are placed in position and cast in situ concrete is poured but not strong enough to carry the weight of the composite structure. This stage is the critical stage for design of the precast element.

Working (Final) Stage:- This is the stage at which concrete is poured and hardened and the slab is at its working condition.

2.4.1 MODELING OF PRECAST ELEMENT

The precast element is modeled like a space truss with the reinforcement employed in the precast element as its members at erection, initial and intermediate stage. This is because the concrete part of the precast element is under tension zone and can be neglected at the above mentioned stages.

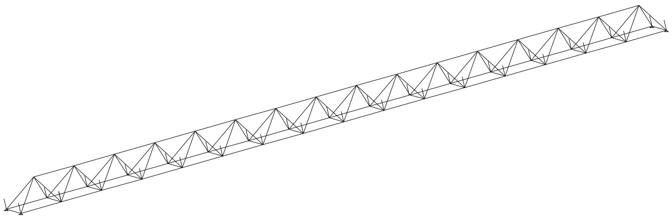


FIG 2.6 Space Truss Model of Precast Element

For the case at working or final stage the precast element is modeled together with the cast in situ concrete and act together as a composite structure.

2.4.2 LOAD ON THE PRECAST ELEMENT

The precast element is loaded differently in the different stages, namely, at erection stage, at block laying (initial) stage, and at intermediate stage. The loads for the case of using ϕ 10mm diameter reinforcement are as follows:

Case I Erection Stage

At this stage the only load to be considered is the self-weight of precast element.

Dead Load

Weight of Precast element = 0.04x0.091x25 = 0.091kN/m

The self weight of the precast element is changed to joint load to be applied on the truss model.

$$G_j = \frac{0.091 \times 0.2}{2} = 0.0091kN$$

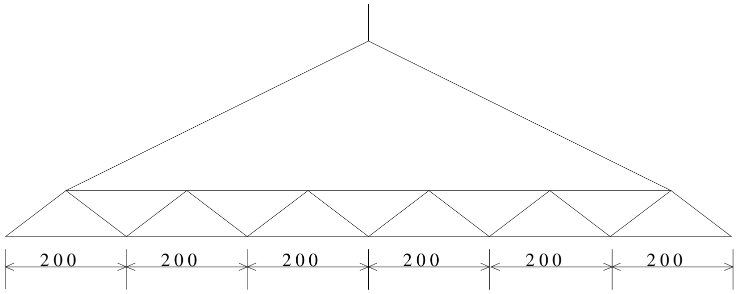


FIG 2.7 Precast Model at Erection Stage

Case II Block Laying (Initial) Stage

The load to be considered at this stage is the self weight of the precast element, the weight of the HCB and the weight of the block layer(live load).

Dead Load

Weight of Precast element = 0.04x0.091x2 = 0.09kN/m

Weight of Concrete hollow block = 14x0.16x0.4 = <u>0.90kN/m</u>

Total Dead load G per m = **<u>0.99kN/m</u>**

Changing the distributed load to joint load

Typical Joint load

$$G_j = \frac{0.99 \times 0.2}{2} = 0.1kN$$

Live Load

The live load to be considered at this stage is the weight of block layers which is assumed to be an 80Kg person standing at the mid span of the precast element.

$$Q_j = \frac{80 \times 10}{1000} = 0.8kN$$

The load is divided to four joints, which are near to the mid span to create the maximum deflection.

Mid span Joint live load

$$Q_j = \frac{0.8}{4} = 0.2kN$$

Case III Concrete Pouring (Intermediate) Stage.

The load to be considered at this stage of construction is the weight of fresh concrete plus the load at case II.

Dead Load

Weight of Precast element =0.04x0.091x25 = 0.09kN/m
Weight of Concrete hollow block = =0.90kN/m
Weight of cast in situ concrete=0.5x(0.091+0.031)x0.12x25=0.18kN/m
Total = **1.17kN/m**

Dead Load on joint

$$G_j = \frac{1.17 \times 0.2}{2} = 0.117kN$$

Live Load

The live load to be considered at this stage is the weight of mason who vibrates and smoothen the fresh concrete, which is assumed to be an 80 kg person.

Live load at mid span joints

$$Q_j = \frac{0.8}{4} = 0.2kN$$

The dead load, which is calculated for 160mm deep HCB will be increased if 5cm concrete topping above the HCB is used, therefore the total dead load for intermediate stage will be

$$G_{j2} = 1.17kN/m + 0.05m \times 0.6m \times 25kN/m^3 = 1.92kN/m$$

Dead Load on joint

$$G_{j2} = \frac{1.92 \times 0.2}{2} = 0.2kN$$

2.4.3 LOAD ON THE COMPOSITE STRUCTURE.

Case IV Working (Final) Stage.

Dead Load

Weight of precast element		=	0.09 kN/m
Weight of concrete blocks		=	0.90 kN/m
Weight of cast insitu concrete		=	0.18 kN/m
Floor finish (Terazzo)=	23x0.02x0.60	=	0.28kN/m
Cement Screed=	23x0.05x0.60	=	0.69kN/m
Ceiling plaster=	23x0.02x0.60	=	0.28kN/m

Partition load =(assume 1kN/m^2) = 0.60kN/m

Total Dead Load = **3.01kN/m**

In the case of 5cm topping

Additional load

Weight of 5cm topping= 0.05x0.6x25 = 0.75kN/m

Total Dead Load = **3.76kN/m**

Live Load

$Q = 2.0kN/m^2$ (for domestic and residential activities)

$= 2.0 \times 0.60$

= 1.2kN/m

Design Load with no concrete topping

$$F_d = 1.3 \times 3.01 + 1.6 \times 1.2$$

$$= 5.83 kN/m$$

Design Load (with 5cm concrete topping)

$$F_d = 1.3 \times 3.76 + 1.6 \times 1.2$$

$$= 6.81 kN/m$$

2.4.4 ANALYSIS OF PRECAST ELEMENT

2.4.4.1. General

The precast element is analyzed for all the cases mentioned above using SAP2000N based on the above loads and cases.

2.4.4.2 Constants and assumptions

In this study the following materials are used

- **Concrete C-25**

According to EBCS 2-1995, the design strength of concrete is gives as

$$f_{cd} = \frac{0.85 \times f_{ck}}{\gamma_c} \text{ in compression, and}$$

$$f_{cd} = \frac{0.85 \times 20}{1.5} = \mathbf{11.33\ MPa}$$

$$f_{ctd} = \frac{f_{ctk}}{\gamma_c} \qquad \text{in tension}$$

$$f_{ctd} = \frac{1.50}{1.5} = \mathbf{1.0\ MPa}$$

$$E_{cm} = 29GPa$$

- **Steel S-300**

The design strength of steel in tension and compression is given by

$$f_{yd} = \frac{f_{yk}}{\gamma_s}$$

$$f_{yd} = \frac{300}{1.15} = \mathbf{260.87MPa}$$

$$E_s = 200GPa$$

2.4.4.3 Determination of Maximum Span

The maximum span of precast element is determined with an iterative procedure, by taking a given size of reinforcement with a trial span length until the span fulfills the serviceability and strength requirements by increasing the span successively with an increment of 0.2m. For the case of diameter 10mm reinforcement the following is calculated:

2.4.4.3.1 Without Intermediate Support

Load combination considered

1) $COMBO1 = 1.3G + 1.6Q$ (For Ultimate Limit state)

2) $COMBO2 = G + Q$ (For Serviceability Limit State)

Using 160mm Deep Hollow Concrete Block

i) With out concrete topping

From all the possible loading stages discussed above, the one at the intermediate stage is the critical load condition and therefore it is used for determination of the maximum span based on the above requirements.

Try L= 2.60m

a) Serviceability Requirement at intermediate stage

As per EBCS-2 1995 Eq 5.1 the final deflection of all horizontal members shall not in general exceed the value

$$\delta = \frac{L}{200}$$

Reinforcement used for the analysis

Top Reinforcement bar	= 1 ϕ 12mm
Bottom reinforcing bars	= 2 ϕ 10mm
Diagonal reinforcing bars	= ϕ 8mm c/c 200mm

Using SAP2000 non linear version the analysis is made and the following deflection is obtained (See appendix B)

$$\delta = 5.1mm < \frac{2600}{200} = 13.0mm$$

Therefore the deflection requirement at intermediate stage with the assumed diameter and span is satisfied.

b) **Strength Requirements at intermediate stage.**

The capacity of the above reinforcing bars is calculated as truss members which can carry axial tension and compression forces according to EBCS 3-1995. The tensile and buckling resistances of the members are calculated based on the formula

$$N_{t,RD} = \frac{A \times F_y}{\gamma_{M1}} \qquad \text{for tension}$$

$$N_{b,RD} = \frac{\chi \times \beta_A \times A \times F_y}{\gamma_{M!}} \qquad \text{for buckling}$$

Where

χ - reduction factor for the relevant buckling mode and can be obtained from

$\gamma_{M1} = 1.1$ (partial safety factor for resistance of members for buckling)

$$\chi = \frac{1}{\phi + (\phi^2 - \bar{\lambda}^2)^{0.5}} \quad \text{but } \chi \leq 1$$

Where

$$\phi = 0.5(1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2)$$

α = an imperfection factor obtained from Table 4.8 of EBCS-3

$$\beta_A = \frac{A}{A_{eff}} = 1.0$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1}\beta_A^{0.5}$$

λ is the slenderness for the relevant buckling mode $= \frac{l_e}{r}$

$$\lambda_1 = 93.9\varepsilon = 93.9\sqrt{\frac{235}{f_y}} = 93.9 \times \sqrt{\frac{235}{300.0}} = 89.12$$

Maximum compression and tension forces on top and bottom reinforcements obtained from SAP2000 analysis result (see appendix B) respectively.

P_{maxc}= 17.29kN (top bar)

P_{maxt}= 3.29kN (bottom bars)

Compression resistance of ϕ 12 reinforcement

With length	L=0.2m
Area	A=113.1mm^2
Moment of Inertia	I=1017.88mm^4
Radius of gyration	$r = \sqrt{I/A}$ =3.0
Slenderness	$\lambda = \frac{l}{r} = \frac{200}{3} = 66.7$
	$\lambda_1 = 93.9\sqrt{\frac{235}{300.0}} = 83.12$
	$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{66.7}{83.12} = 0.8$

Buckling curve for round solid bars is curve c (Table 4.11 EBCS-3 1995)

Imperfection factor α for curve c = 0.49 (Table 4.8 EBCS-3 1995)

$$\phi = 0.5(1 + 0.49(0.8 - 0.2) + 0.8^2) = 0.97$$

$$\chi = \frac{1}{0.97 + (0.97^2 - 0.8^2)^{0.5}} = 0.66$$

$$N_{b,RD} = \frac{0.66 \times 1.0 \times 113.1 \times 300}{1.1} = 20.36kN > 17.29\text{kN}$$

Therefore the bar is sufficient to withstand the compression force.

Resistance of ϕ 10 for tension (Bottom bar)

$$N_{t,RD} = \frac{78.5 \times 300}{1.1} = 21.41kN > 3.29kN$$

Therefore the bar is sufficient to withstand the tension force.

Maximum Tension and Compression force on diagonal reinforcement

Compression

P_{maxc}= 1.65kN <3.86kN

Tension

P_{maxt}= 1.65kN < 13.71kN

Therefore all the members satisfy the requirements for compression and tension failure.

c) Strength Requirements at Final stage

With the above span the member is checked for flexural and shear failure with a load at its working or final stage

System

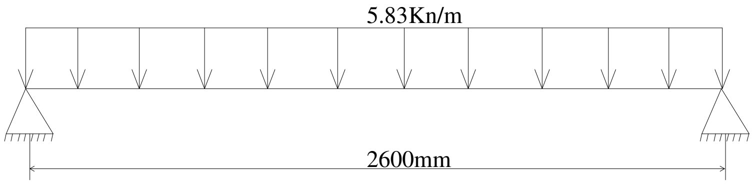


Fig 2.8 Loading of composite element at its Working stage

$$M_{\max} = \frac{F_d \times l^2}{8} = \frac{5.83 \times (2.6)^2}{8} = 4.93KNm$$

$$d = 160 - 15 - 5 = 140mm$$

Using the general design chart No.1 and Table No.1a of EBCS 2: 1995 part 2

$$K_m = \frac{\sqrt{M_{\max}/b}}{d} = \frac{\sqrt{4.93/0.091}}{0.14} = 52.57mm$$

$$K_s = 4.48$$

$$A_{s1} = \frac{K_s M}{d} = \frac{4.48 \times 4.93}{0.140} = 157.8mm^2$$

Use 2 ϕ 10 (157.08mm^2)bottom reinforcement

This shows that 2 ϕ 10 reinforcements are just sufficient for the span of 2.6m.

Check for Shear

Maximum shear force

$$\mathrm{V_{sd}} = \frac{5.83 \times 2.6}{2} = 7.58KN$$

Concrete Shear resistance

$$V_c = 0.25 f_{ctd} K_1 K_2 b_w d$$

Where

$$K_1 = 1 + 50\rho = 1 + 50 \times (\frac{157.1}{91 \times 140}) = 1.62 \le 2.0$$

$$K_2 = 1.6 - d = 1.6 - 0.140 = 1.46 \ge 1.0$$

$$V_c = 0.25 \times 1.0 \times 10^3 \times 1.62 \times 1.46 \times 0.091 \times 0.14$$

$$= 7.53KN = V_{sd}$$

No shear reinforcement is required but inclined shear reinforcement is provided for tying of the top reinforcement with the precast element

d) Serviceability Requirement at working stage.

Check for Deflection at working stage

$$\delta_i = \frac{5}{48} L^2 \frac{M_{cr}}{E_{cm} I_i}$$

$$\delta_{ii} = \frac{5}{48} L^2 \frac{M_k - M_{cr}}{0.75 E_s A_s z(d-x)}$$

$$\delta_{\max} = \frac{5}{48} \times L^2 \frac{M_K}{E_s A_s z(d-x)}$$

$$M_{cr} = 1.7 f_{ctk} Z$$

where

δ_i	is the deflection due to the theoretical cracking moment M_{cr} acting on the uncracked transformed section
δ_{ii}	is the deflection due to the balance of the applied moment over and above the cracking value and acting on a section with an equivalent stiffness of 75% of the cracked value.
$\delta_{\max}$	is the deflection of fully cracked section.
E_{cm}	is the short term elastic modulus of concrete
I_i	is the moment of inertia of the uncracked transformed section
E_s	is the modulus of Elasticity of steel
A_s	is the area of tension reinforcement
M_k	is the maximum applied moment at mid span due to sustained characterstic loads
z	is the internal lever arm at the section of maximum moment
x	is the neutral axis depth at the section of maximum moment
L	is the span

For

$K_x = 0.35$ x=49.0mm

$K_z = 0.85$ z=119.0mm

$$I_i = \frac{91 \times 160^3}{12} + 91 \times 160 \times 31^2 + 5.9 \times 157.1 \times 91^2 + 5.9 \times 113 \times (49-21)^2$$

$=5.33 \text{x} 10^7$ mm^4

$$Z = \frac{53.3 \times 10^6}{111.0}$$

$=4.8 \text{x} 10^5$mm^3

$$M_{cr} = 1.7 \times 1.5 \times 0.48 \times 10^6$$

=1.22 kN-m

$$\delta_{\max} = \frac{5}{48} \times 2600^2 \frac{3.56 \times 10^6}{200 \times 10^3 \times 157.1 \times 119(140-49)}$$

=7.37mm

$$\delta_i = \frac{5}{48} 2600^2 \frac{1.22 \times 10^6}{29 \times 10^3 \times 53.3 \times 10^6}$$

=0.56mm

$$\delta_{ii} = \frac{5}{48} 2600^2 \frac{(3.56-1.22) \times 10^6}{0.75 \times 200 \times 10^6 \times 157.1 \times 119(140-49)}$$

=6.45mm

$$\delta_i + \delta_{ii} = 0.56 + 6.45 = 7.01mm < 7.37mm$$

The total deflection of 7.01mm is less than the allowable which is

$$\delta = \frac{2600}{200} = 13mm$$

Therefore, for the trial span of 2.6m, 2 ϕ 10 bottom bars, 1 ϕ 12 top bar and ϕ 8 c/c 200 diagonal bars, satisfy all the requirement of serviceability and ultimate limit state and increasing the span would require a higher diameter of reinforcements.

With similar iterative procedure used above the maximum span which can be used with different size of diameters are calculated and tabulated below.

The maximum span which can be used for 160mm deep HCB without concrete topping is 4.4m.

Table 2.1 Maximum Span for 160mm Deep HCB without Concrete Topping

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	2.60
12	95	42	3.00
14	99	44	3.60
16	103	46	4.00
20	111	50	4.40

ii) With 50mm concrete topping

For ϕ 10mm reinforcement

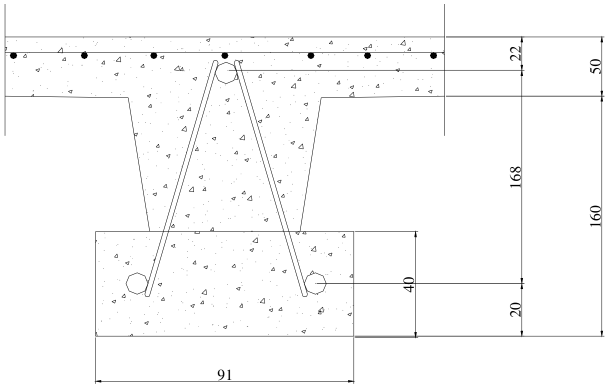


Fig 2.9 Composite structure with concrete topping

Try span of 3.0m

Reinforcement used for the analysis

Top Reinforcement bar = 1 ϕ 14mm
Bottom reinforcing bars = 2 ϕ 10mm
Diagonal reinforcing bars = ϕ 8mm c/c 200mm

a) Serviceability Requirement at intermediate stage

The deflection obtained from SAP200N analysis (Refer appendix B)

$$\delta = 3.99mm < \frac{3000}{200} = 15.0mm$$

Therefore the deflection is with in the allowable limit.

b) Strength Requirements at intermediate stage.

Maximum Compression force on top reinforcement

P_{maxc}= 23.27kN < 30.75kN (Refer appendix B)

Maximum Tension force on the bottom reinforcement

P_{maxt}= 4.14kN < 21.42kN (Refer appendix B)

Maximum Tension and Compression force on diagonal reinforcement

Compression

P_{maxc}= 2.7kN < 3.86kN

Tension

P_{maxt}= 2.7kN < 13.71kN

c) Strength Requirements at Final stage

Checking the member at its working stage

System

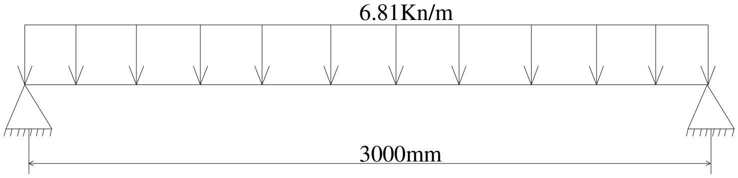


Fig 2.10 Loading of Composite structure at its Working Stage with 5cm Concrete Topping

$$M_{\max} = \frac{q_d \times l^2}{8} = \frac{6.81 \times (3.0)^2}{8} = 7.66KNm$$

$$d = 210 - 15 - 5 = 190mm$$

Using Table No.1a of EBCS 2: 1995 part 2

$K_m = 20.55$ **< 57.83**

$K_x = 0.095$

$x = K_x \times d$

$x = 0.095 \times 190 = 18.05mm < 50mm$

Therefore the section is rectangular with b=600mm

$$K_z = 0.967$$

$$z = K_z \times d$$

$$z = 0.967 \times 190 = 183.73mm$$

$$K_s = 3.96$$

$$A_s = 190.73mm^2$$

Requires 2 ϕ 12 (226.2mm^2) >2 ϕ 10 (157.1mm^2) bottom reinforcement.

The diameter of the reinforcement required for this trial span is greater than the initial ϕ 10mm and therefore by decreasing the next lower span of 2.8m recalculation is made and the following result is found for each of the different diameters.

Table 2.2 Maximum Span for 160mm Deep HCB without 50mm Concrete Topping.

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	2.80
12	95	42	3.80
14	99	44	4.40
16	103	46	5.00
20	111	50	6.00

2.4.4.3.2 With Intermediate Support

Introducing an intermediate support will decrease the top reinforcement required at initial, and intermediate stages, but will not help to increase the span, since the governing stage for maximum span is the final or working stage at which the intermediate support would no more exist.

With similar procedure used for the case of 160mm deep HCB, the maximum span which can be used for 200mm and 240mm deep HCBs are tabulated below.

Table 2.3 Maximum Span for 200mm Deep HCB without Concrete Topping

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	2.90
12	95	42	3.30
14	99	44	3.80
16	103	46	4.00
20	111	50	5.20

Table 2.4. Maximum Span for 200mm Deep HCB with 50mm Concrete Topping

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	3.20
12	95	42	4.00
14	99	44	4.60
16	103	46	5.20
20	111	50	6.50
24	119	54	6.60

Table 2.5. Maximum Span for 240mm Deep HCB without Concrete Topping

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	3.10
12	95	42	3.60
14	99	44	4.20

16	103	46	4.60
20	111	50	6.00
24	119	54	6.70

Table 2.6. Maximum Span for 240mm Deep HCB without 50mm Concrete Topping

Diameter of Reinforcing Bar[mm]	Size of Precast		Maximum Span[m]
	Width [mm]	Depth[mm]	
10	91	40	3.40
12	95	42	4.20
14	99	44	5.00
16	103	46	5.60
20	111	50	6.90
24	119	54	7.50
28	127	58	8.50

Table 2.7. Summary of Maximum Span for Different Combinations.

Depth of HCB and Thickness of Concrete topping [mm]	Minimum Size of Precast		Maximum Span[m]	Reinforcement required		
	Width [mm]	Depth [mm]		Top	Bottom	Diagonal
160+0	111	50	4.40	1 ϕ 20	2 ϕ 20	ϕ8 c/c 200
160+50	111	50	6.00	1 ϕ 12	2 ϕ 20	ϕ8 c/c 200
200+0	111	50	5.20	1 ϕ 20	2 ϕ 20	ϕ8 c/c 200
200+50	119	54	6.60	1 ϕ 12	2 ϕ 24	ϕ8 c/c 200
240+0	119	54	6.70	1 ϕ 24	2 ϕ 24	ϕ8 c/c 200
240+50	127	58	8.50	1 ϕ 14	2 ϕ 26	ϕ8 c/c 200

2.5 ECONOMICAL SPAN OF COMPOSITE STRUCTURE

2.5.1 Cost of Composite Structure

The cost of each of the maximum span for each combination of diameter and depth are determined based on the current market price of the country, the break down for building up of the unit price for, the following basic items are annexed for reference on appendix A

Concrete	600.00 Birr/m^3
Reinforcement	6.50 Birr/kg
Form work	50.00 Birr/m^2
HCB and precast laying	40.00 Birr/m2

2.5.1.1 Cost of reinforcement

The cost of reinforcement of the composite structure is calculated for each of the maximum span calculated above, and includes the top reinforcement, the bottom reinforcement and diagonal reinforcement and mesh for the case with concrete topping.

2.5.1.2 Cost of precast element

The cost of precast element includes the cost of the initial cost for the mould and cost of concrete.

2.5.1.3 Cost of HCB

The cost of the hollow block does not include the cost of the initial cost for fabrication, and only include the labour and material cost based on the cost of hollow concrete blocks used for ribbed slabs.

2.5.1.4 Cost of Cast in situ concrete.

This cost includes the cost of labor and material for production of the concrete.

2.5.1.5 Cost of Labor for precast and block laying

This cost includes the cost of labour required for laying of precast element and hollow concrete blocks.

Based on the above, the following costs are calculated for each span.

Table 2.9 Cost per m^2 for 160mm Deep HCB without Concrete Topping.

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
2.6	91	40	112.27
3.0	95	42	122.32
3.6	99	44	133.98
4.0	103	46	142.69
4.4	111	50	172.95

Table 2.10 Cost per m^2 for 160mm Deep HCB with 50mm Concrete Topping

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
2.8	91	40	162.02
3.8	95	42	156.92
4.4	99	44	163.93
5.0	103	46	172.16
6.0	111	50	195.15

Table 2.11 Cost per m^2 for 200mm Deep HCB without Concrete Topping

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
2.9	91	40	118.88

3.3	95	42	129.09
3.8	99	44	136.90
4.4	103	46	149.78
5.2	111	50	180.35

Table 2.12 Cost per m^2 for 200mm Deep HCB with 50mm Concrete Topping

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
3.2	91	40	167.64
4.0	95	42	173.25
4.6	99	44	183.42
5.2	103	46	191.85
6.5	111	50	212.13
6.6	119	54	237.33

Table 2.13 Cost per m^2 for 240mm Deep HCB without Concrete Topping

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
3.1	91	40	123.40
3.6	95	42	133.77
4.2	99	44	145.75
4.6	103	46	154.78
6.0	111	50	185.68
6.7	119	54	222.98

Table 2.14 Cost per m^2 for 240mm Deep HCB with 50mm Concrete Topping

Maximum span [m]	Minimum Size of Precast		Cost/m^2 Birr
	Width[mm]	Depth[mm]	
3.4	91	40	161.63

4.2	95	42	170.46
5.0	99	44	177.74
5.6	103	46	186.40
6.9	111	50	207.09
7.5	119	54	235.92
8.5	127	58	265.48

2.5.2 Determination of Economical span and combinations.

Similarly the cost per m^2 for each of the combinations are calculated with a possible range of spans as tabulated on table 2.15 next page, then from the possible cost and span combinations the one with minimum cost is selected as an economical combination for that particular range of span.

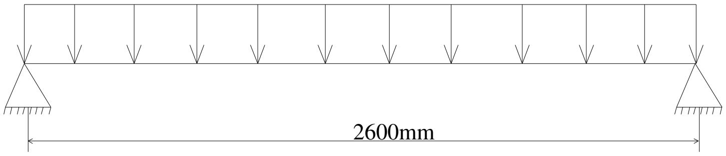
Table 2.16 Summary of Economical span and combinations

Range of span [m]	Depth [mm]	Topping [mm]	Minimum Size of Precast		Cost/m^2 [Birr]
			Depth[mm]	Width[mm]	
Up to 2.6	160	0	40	91	112.27
2.6-2.9	200	0	40	91	118.88
2.9-3.0	160	0	42	95	122.32
3.0-3.1	240	0	40	91	123.40
3.1-3.3	200	0	42	95	129.09
3.3-3.6	240	0	42	95	133.77
3.6-3.8	200	0	44	99	136.90
3.8-4.2	160	0	46	103	142.69
4.2-4.6	200	0	46	103	149.78
4.6-5.0	160	50	50	111	172.16
5.0-5.2	240	0	50	111	180.35
5.2-6.0	240	0	54	119	185.68
6.0-6.9	240	50	50	111	207.09
6.9-7.5	240	50	54	119	235.92
7.5-8.5	240	50	58	127	265.48

3. DESIGN OF SOLID AND RIBBED SLAB FOR COSTCOMPARISON

3.1 DESIGN OF ONE WAY SOLID SLAB

Solid one way slab of span equal to the maximum span of the composite structure are analyzed and designed for cost comparison purpose. Its design for the case of 2.6m span is as follows.



Effective depth of slab from serviceability requirement

$$d = (0.4 + 0.6\frac{f_{yk}}{400})\frac{L_e}{\beta_a}$$

$$d = (0.4 + 0.6\frac{300}{400})\frac{2600}{20} = 110.5mm$$

Total Depth

$$D_{tot} = 110.5 + 15 + 4 = 129.5mm$$

Take depth of slab=130mm

Load on Slab

Dead Load (G)

Self weight =	0.13x25	=	3.25 kN/m^2
Ceiling plaster=	0.02x23	=	0.46 kN/m^2
Terrazzo floor =	0.02x23	=	0.46 kN/m^2
Cement mortar=	0.05x23	=	1.15 kN/m^2
Partition load =		=	1.00 kN/m^2
Total dead Load (G)		=	**6.32 kN/m^2**

Live Load (Q)

$Q = 2.0\ kN/m^2$ (for domestic and residential activities)

Design Load (F_d)

$F_d = 1.3x6.32+1.6x2.0=11.4\ kN/m^2$

$$M = \frac{F_d \times l^2}{8}$$

$$M = \frac{11.4 \times 2.6^2}{8} = 9.6kNm$$

$$K_m = \frac{\sqrt{M / b}}{d}$$

$$K_m = \frac{\sqrt{9.6 / 1.0}}{0.111} = 28.0$$

$$K_s = 4.06$$

$$A_s = \frac{K_s M}{d}$$

$$A_s = \frac{4.06 \times 9.6}{0.111} = 353.0mm^2$$

$$Spacing = \frac{1000 \times 50.3}{353} = 142.5mm$$

Maximum spacing = 2D=260mm

Use ϕ 8 c/c 140mm

Distribution bar=0.2x353=50.6mm^2

Use ϕ 8c/c 260mm

The design for each span is done in a similar manner as above and the cost is tabulated on table 8.

Table 3.1 Cost of Solid slab per m^2 for different spans.

Span[m]	Depth of slab[mm]	Main reinforcement	Distribution reinforcement	Cost/m^2 Birr
2.60	130	Φ 8c/c 140	Φ 8c/c 260	178.32
2.90	150	Φ 8c/c 120	Φ 8c/c 300	192.49
3.00	150	Φ 8c/c 120	Φ 8c/c 300	193.02
3.10	160	Φ 8c/c 110	Φ 8c/c 320	202.13
3.30	160	Φ 8c/c 100	Φ 8c/c 320	206.63
3.60	180	Φ 8c/c 90	Φ 8c/c 350	222.13
3.80	190	Φ 8c/c 80	Φ 8c/c 350	231.95
4.20	200	Φ 8c/c 70	Φ 8c/c 350	246.95
4.60	220	Φ 8c/c 60	Φ 8c/c 350	267.32
5.00	240	Φ 8c/c 50	Φ 8c/c 350	296.26
5.20	250	Φ 10c/c 80	Φ 8c/c 350	301.41
6.00	280	Φ 10/c 60	Φ 8c/c 350	341.55
6.90	320	Φ 10c/c 50	Φ 8c/c 350	396.76
7.50	350	Φ 12c/c 60	Φ 8c/c 350	435.00
8.50	390	Φ 14c/c 70	Φ 8c/c 350	501.31

3.2 DESIGN OF RIBBED SLAB

The design of ribbed slab is done for each of the spans and the calculation for the design of span 2.6m is as follows, the cost of ribbed slab is also calculated for all cases with different depth of ribbed block with 50mm thick concrete topping for each of the maximum spans calculated for composite element.

Dead load

5cm concrete topping = 0.05x25 = 1.25 kN/m^2
Weight of 16cm deep of HCB = 0.16x14 = 2.24 kN/m^2
Ceiling plaster = 0.02x23 = 0.46 kN/m^2
Floor finish = 0.02x23 = 0.46 kN/m^2

Cement mortar = 0.05x23 = 1.15 kN/m^2

Partition load = = 1.00 kN/m^2

Own weight of rib = 0.1x25x0.16 = 0.40 kN/m

Total dead load/m = 6.56x0.4+0.4 = **3.02 kN/m**

Live Load

Q=2.0 kN/m^2

Live load/m= 2.0x0.4=0.8kN/m

Design load = F_d=1.3x3.02+1.6x0.8= 5.2kN/m

$$M = \frac{F_d \times l^2}{8}$$

$$M = \frac{5.20 \times 2.6^2}{8} = 4.4kNm$$

$$K_m = \frac{\sqrt{M / b}}{d}$$

$$K_m = \frac{\sqrt{4.4 / 0.4}}{0.191} = 17.4$$

$$K_s = 3.95$$

$$A_s = \frac{K_s M}{d}$$

$$A_s = \frac{3.95 \times 4.4}{0.191} = 91.0mm^2$$

Use 2 ϕ 8 (span reinforcement)

Minimum reinforcement for topping

$$\rho_{\min} = \frac{0.5}{f_{yk}}$$

$$\rho_{\min} = \frac{0.5}{300} = 0.0017$$

$$A_{s\min} = \rho_{\min} bd$$

$$A_{s\min} = 0.0017 \times 1000 \times 50 = 83.3mm^2$$

Use ϕ 6 c/c 200mm (both direction)

In a similar procedure, the design of other spans are done and tabulated on tables below.

Table 3.2 Cost per m^2 of 160mm Ribbed Slab with 50mm Concrete Topping.

Span [m]	Main reinforcement	Mesh reinforcement	Cost/m^2 Birr
2.60	2 Φ 8	Φ 6c/c 200	193.17
2.90	2 Φ 10	Φ 6c/c 200	200.32
3.00	2 Φ 10	Φ 6c/c 200	200.31
3.10	2 Φ 10	Φ 6c/c 200	200.29
3.30	2 Φ 10	Φ 6c/c 200	200.26
3.60	2 Φ 12	Φ 6c/c 200	209.02
3.80	2 Φ 12	Φ 6c/c 200	209.02
4.20	2 Φ 14	Φ 6c/c 200	219.40

Table 3.3 Cost per m^2 of 200mm Ribbed slab with 50mm Concrete Topping.

Span [m]	Main reinforcement	Mesh reinforcement	Cost/m^2 Birr
2.60	2 Φ 8	Φ 6c/c 200	199.17
2.90	2 Φ 8	Φ 6c/c 200	199.11
3.00	2 Φ 10	Φ 6c/c 200	206.31
3.10	2 Φ 10	Φ 6c/c 200	206.29
3.30	2 Φ 10	Φ 6c/c 200	206.26
3.60	2 Φ 10	Φ 6c/c 200	206.23
3.80	2 Φ 12	Φ 6c/c 200	215.02
4.20	2 Φ 12	Φ 6c/c 200	214.99
4.60	2 Φ 14	Φ 6c/c 200	225.37
5.00	2 Φ 16	Φ 6c/c 200	237.37
5.20	2 Φ 10	Φ 6c/c 200	237.36

Table 3.4 Cost per m^2 of 240 mm Ribbed Slab with 50mm Concrete topping

Span [m]	Main reinforcement	Mesh reinforcement	Cost/m^2 Birr
2.60	2 Φ 8	Φ 6c/c 200	205.17
2.90	2 Φ 8	Φ 6c/c 200	205.11
3.00	2 Φ 8	Φ 6c/c 200	205.09
3.10	2 Φ 10	Φ 6c/c 200	212.29
3.30	2 Φ 10	Φ 6c/c 200	212.26
3.60	2 Φ 10	Φ 6c/c 200	212.23
3.80	2 Φ 12	Φ 6c/c 200	221.02
4.20	2 Φ 12	Φ 6c/c 200	220.99
4.60	2 Φ 14	Φ 6c/c 200	231.37
5.00	2 Φ 14	Φ 6c/c 200	231.35
5.20	2 Φ 14	Φ 6c/c 200	231.34
6.00	2 Φ 20	Φ 6c/c 200	272.18

3.3 COST COMPARISON

The cost comparison of the three types of slab systems, namely ribbed slab, solid slab and the composite beam/slab system is done only on the basis of the cost of the slab, but it is clear that the cost of the structure depends not only on the cost of the slab but also on cost of the frames (beam and columns), and cost of foundation but this study does not include the cost effect of these items and also the effect of time saving.

Based on the above costs of different span of slabs, the following table compares percentage saving of Composite beam/slab system with one way solid slab and ribbed slab.

As one can see from the comparison table, the percentage saving from ribbed slab construction decrease as the span increase, and this is due to the increase in length of the additional reinforcement required for protection of crack due to rotation of simply supported edge.

Where as the trend of percentage saving for the case of solid slab increase when the span increase, and this is due to the increase in depth of the solid slab for deflection control.

4.0 ANALYTICAL INVESTIGATION OF HCB

4.1 GENERAL

The Hollow Concrete Block (HCB) are elements used in combination with the precast element and in situ concrete to form the composite structure.

The HCB used in this study was manufactured from scoria and pumice being used as coarse and fine aggregates, respectively.

The strength of HCB depends on the proportion of the ingredients, and should have sufficient strength to sustain the applied and the dead loads. Therefore it is necessary to check the strength of this element used as part of the composite structure.

With this in mind analytical investigation on the HCB is made, at two different stages: one at the stage of its laying at which the block layers stand at the middle of the block and the other at its working stage.

4.2 MATERIAL PROPERTIES AND ASSUMPTIONS

4.2.1 MATERIAL PROPERTIES

Material properties of the HCB are taken from a result obtained by laboratory investigation by the research group of the Civil Engineering Department of AAU. This group conducted experimental investigation on the design of the appropriate mix for HCB as well as the precast element. After performing number of tests on different mix proportion of the aggregate the one which is suitable for production of the HCB are selected and has the following properties.

Average unit weight (γ) = $\frac{5.36\times 9.81}{0.15^3}=15.6\text{kN/m}^3$

Average cube crushing strength = $\frac{13.3+12.45}{2}=12.88\text{MPa}$

Modulus of Elasticity (E_{cm}) = $9.5(f_{ck} + 8)^{1/3}$

It has been found that, the splitting tensile strength of lightweight concrete can be considerably lower (up to 30%) than that of ordinary concrete of the same compressive strength. However, for moist cured lightweight aggregate concrete the relationship between tensile and compressive strength is of the same order as for ordinary concrete. [ ]

A number of empirical formulas relating the splitting tensile strength of lightweight concrete to its compressive strength has been suggested , the one selected for this study is justified by conducting number of tests and analytical investigation using local materials by Daniel Abebayehu on his thesis[ ].

$$f_{ct} = 0.23\sqrt[3]{f_{cu}^2}$$

Where $f_{ct} -$ is splitting tensile strength in MPa.

$f_{cu} -$ is Compressive cube strength in MPa.

One can see, from the above equation that it is similar with the one stated on EBCS-2 1995, for ordinary concrete.

4.2.2 ASSUMPTIONS

In the analysis of the HCB the following basic assumptions are used,

- The cube strength test results obtained from laboratory investigation have been taken to be equal to the characteristic compressive strength of the material,

- The lightweight aggregate used is moist and cured so that the relationship between tensile and compressive strength is of the same order as for ordinary concrete.

Based on the above assumptions and EBCS-2, 1995 recommendations, the characteristic strength in compression and tension are:

$$f_{ck(cylinder)} = 0.8 f_{ck(cube)}$$

$$f_{ck} = 0.8 \times 12.88 = 10.3 MPa$$

$$f_{ctk} = 0.23 \times 10.3^{2/3}$$

$$f_{ctk} = 1.09 Mpa$$

Modulus of Elasticity

$$E_{cm} = 9.5(10.3 + 8)^{1/3}$$

=25.03GPa

The design strength values are

In compression;

$$f_{cd} = \frac{0.85 f_{ck}}{\gamma} \qquad (4.1)$$

$$f_{cd} = \frac{0.85 \times 10.3}{1.5} = 5.84 MPa$$

In tension;

$$f_{ctd} = \frac{f_{ctk}}{\gamma} \qquad (4.2)$$

$$f_{ctd} = \frac{1.09}{1.5} = 0.73 MPa$$

Therefore the compressive and tensile stress due to design load on the block should not exceed f_{cd} and f_{ctd} respectively for its use as a load bearing structure.

4.2.3 SIZE OF HCB USED IN THIS STUDY

The following figures show the dimension of the HCB used in this study.

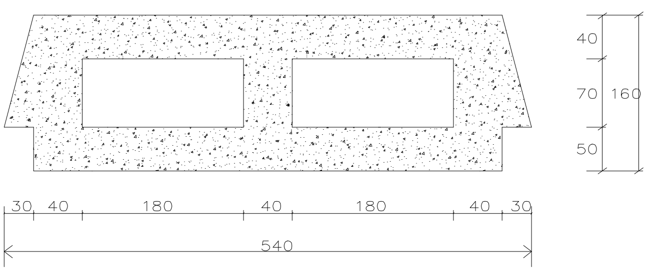


Fig. 4.1a HCB With depth of 160mm (to be used in small span slab)

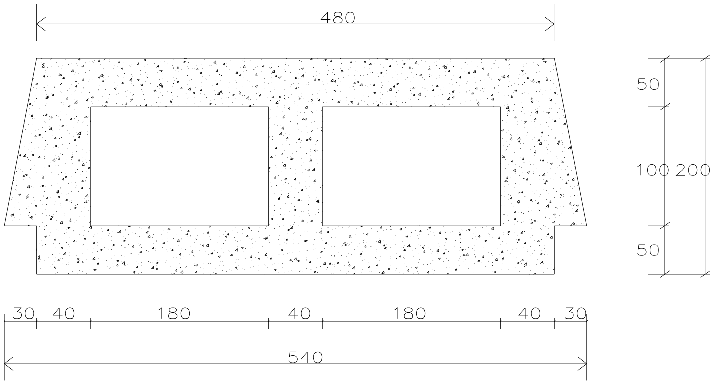


Fig 4.1b HCB With depth of 200mm (to be used in medium span slab)

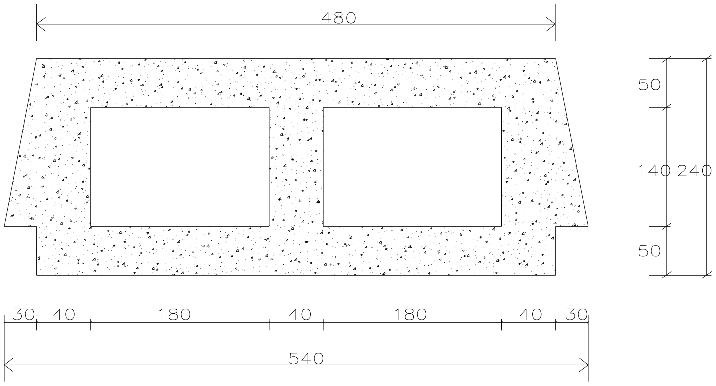


Fig 4.1c HCB With depth of 240mm (to be used in Large span slab)

Fig 4.1 Two-Celled Hollow Concrete Blocks

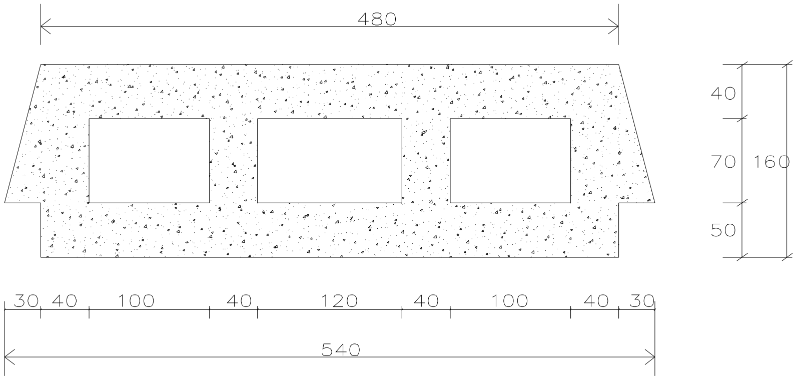


Fig 4.2a HCB With depth of 160mm (to be used in small span slab)

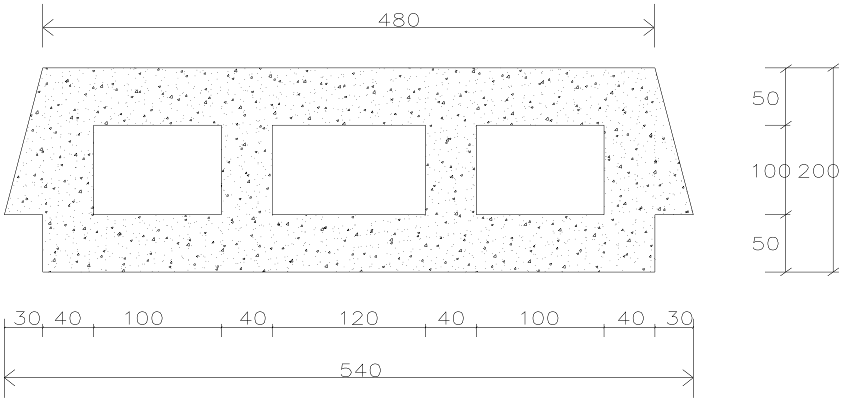


Fig 4.2b HCB With depth of 200mm (to be used in medium span slab)

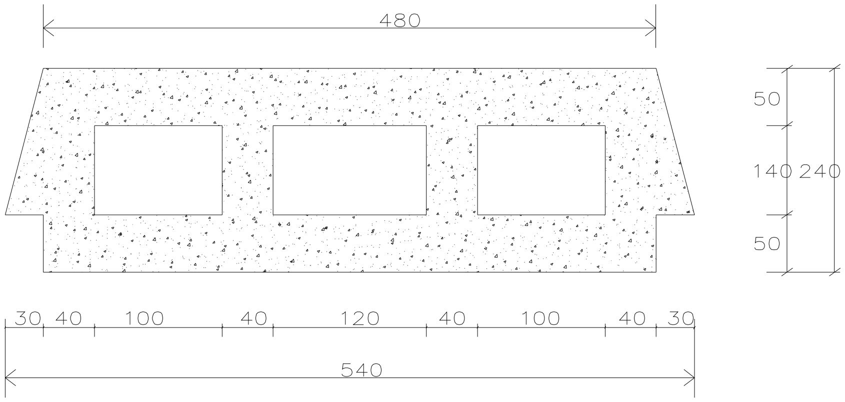


Fig 4.2c HCB With depth of 240mm (to be used in large span slab)

Fig.4.2 Three-Celled Hollow Concrete Block

4.3 LOADS ON THE HOLLOW BLOCKS.

The analysis of the HCB is done using finite solid element model, with element size of 1cm^3.The blocks are analysed for two different surface finishing conditions, one with 50mm concrete topping and the other without concrete topping.

In both cases, three loading conditions were selected to simulate the expected load in the HCB, at initial or block laying stage , at intermediate or concrete pouring stage and at final or working stage.

The loads on the blocks are applied as a surface load on the top surface of the HCB.

4.3.1 WHEN HCB USED IN COMBINATION WITH 5cm CONCRETE TOPPING

a) Case 1. Initial or Block laying Stage.

During initial or block laying stage, in addition to its own weight, an 80kg person is assumed to stand at the center of the block on an area of 80x200mm,

Dead Load

Self weight of the block = front area of the block x 0.2m x 15.6kN/m^3

Live Load

Q_1 = 80kg on an area of 8cmx20cm

$$= \frac{80\times 9.81}{0.2\times 0.08\times 1000} = 49.1 kN/m^2$$

b) Case 2. Intermediate or Concrete pouring Stage.

At the intermediate or concrete pouring stage the weight of the 50mm concrete topping is considered, in addition to the weight of the block, and a person standing at the center.

Dead Load

Total dead load G_1 = 0.05x25=1.25kN/m^2 + self weight

Live Load

$$\mathbf{Q_1} = \frac{80\times 9.81}{0.2\times 0.08\times 1000} = 49.1 kN/m^2$$

c) Case 3. Final or Working Stage.

At this stage, i,e with 50mm concrete topping, all the loads are carried by the precast element in combination with the cast in situ concrete, and the block is not used as a structural element and will not be analysed.

4.3.2 WHEN HCB USED WITHOUTCONCRETE TOPPING

a) Case 1. Initial or Block laying Stage.

The same loading condition is used as the above case

Dead Load

Self weight of the block = front area of the block x 0.2m x 15.6kN/m^3

Live Load

Q_1 = 80kg on an area of 8cmx20cm

$$= \frac{80 \times 9.81}{0.2 \times 0.08 \times 1000} = 49.1 kN/m^2$$

b) Case 2. Intermediate or Concrete pouring Stage.

Since no concrete topping is used at this stage, there is no surface load to be applied on the top surface of the HCB.

c) Case 3 Final or Working Stage.

At the last or working condition, in addition to the weight of the HCB, floor finishing and partition loads are considered.

Dead Load

Floor finishing (Terrazzo) = 0.46kN/m^2
5cm cement sand mortar = 1.15kN/m^2
Ceiling Plaster = 0.46kN/m^2
Partition loads = 1.0kN/m^2

Total Load **G_2** = $3.07kN/m^2$ + own weight

Live Load.

At this stage, a variable or live load of $2.0kN/m^2$, or an 80kg person standing at the center, whichever create the critical case is assumed to act on the top surface of HCB.

Q_1 = 80kg on an area of 8cmx20cm

$$= \frac{80\times 9.81}{0.2\times 0.08\times 1000} = 49.1kN/m^2$$

$Q_2 = 2.0kN/m^2$ (for domestic and residential buildings)

The critical load cases for the two stages are: case 2, for the condition of HCB used in combination with 5cm concrete topping and case 3, for the condition without concrete topping.

LOAD COMBINATIONS

1. $COMBO1 = 1.3G_1 + 1.6Q_1$ (for Intermediate stage)
2. $COMBO2 = 1.3G_2 + 1.6Q_1$ (for final stage)
3. $COMBO3 = 1.3G_2 + 1.6Q_2$ (for final stage)

4.4 DESCRIPTION OF STRESSES.

4.4.1 STATE OF STRESS AT A POINT

A solid body of arbitrary size, shape, and material acted upon by an equilibrium system of body forces and/or surface forces will respond so as to develop a system of internal forces also in equilibrium at any point within the body. If the body were cut by an arbitrary plan, these internal forces would in general be distributed continuously over the cut surface and would vary across the surface in both direction and intensity. Furthermore, the internal force distribution would also be a function of the orientation of the plan chosen for investigation.

Stress is the term used to define the intensity and direction of the internal force acting at a given point on a particular plane. A complete description of the magnitude and directions of stress on all possible planes through a point constitute the state of stress at the point.[ ]

Complete definition of the state stress at a point may be effected by considering all the components of stress that can occur on the faces of a infinitesimal cube of material placed on an arbitrary right-handed Cartesian coordinate system. All components of stress may be expressed as either normal stress, normal to the faces of the cube , or shear stress, parallel to the faces of the cube. Since the purpose here is to investigate the stress acting on planes that go precisely through a given point, the limiting values of stresses when the infinitesimal element dimensions approach zero are the only values of interest.[ ]

Normal stresses will be designated by the symbol σ, and shearing stresses by the symbol τ. The conventions used for subscript notation are the usual ones, defined as follow:

1. For normal stresses a single subscript will be used to correspond to the direction of the outward drawn normal to the plan on which it acts.
2. For shearing stress two subscripts will be used, the first of which indicates the direction of the plane on which it acts and the second subscript to indicate the direction of the shear stress in the plane.
3. Normal stress will be called positive (+) when they produce tension and negative (-) when they produce compression.
4. Shear stresses will be called positive (+) if they are in the direction of an axis whose sign is the same as the sign of the axis in the direction of the outward drawn normal to the plane on which the stress act.

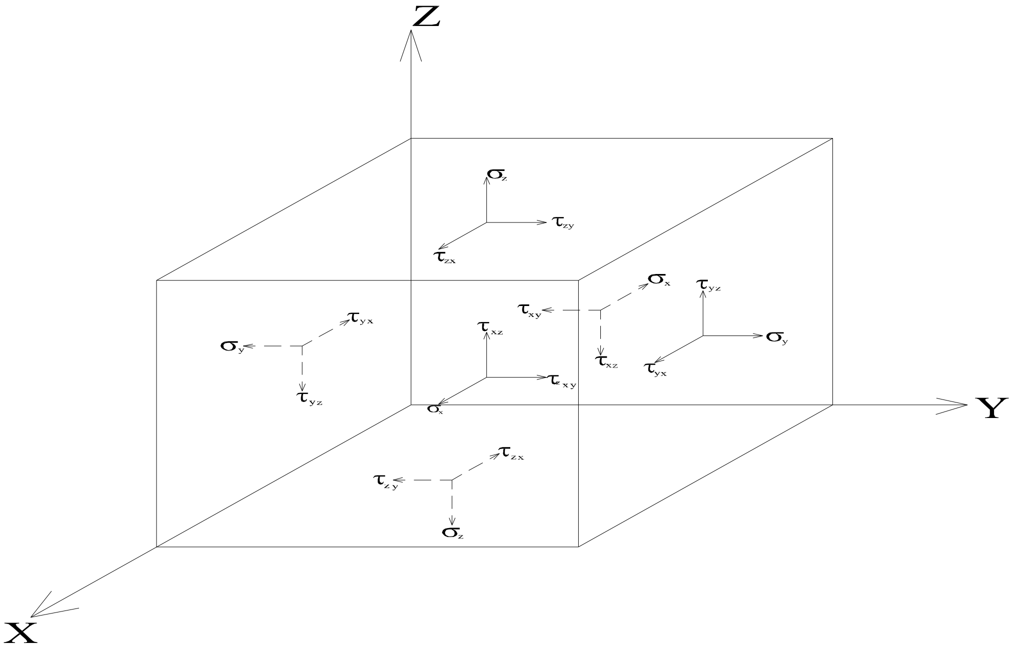


Fig.4.3 Complete definition of the state of stress at a point.

The six apparent shear stress components required in a complete description of the state of stress at a point are reduced to three distinct stresses on the basis on moment equilibrium. Consequently, to completely define the most general state of stress at a point requires the specification of six components of stress, three normal stress , σ_x, σ_y, and σ_z, and three shearing stresses , τ_{xy}, τ_{yz} and τ_{xz}.[ ]

4.4.2 PRINCIPAL NORMAL STRESS

Principal normal stresses, sometimes called simply principal stresses, are the normal stresses that occur on planes where the shearing stresses are zero. The planes on which the principal normal stress act are called principal planes. The principal normal stresses also are local extremes of stress that include the maximum value of normal stress that can occur on any plane through the point. Since failure moduli are often related to the principal stresses or to the maximum principal stress, it will be of interest to develop a means of determining the principal normal stresses. [ ]

The expression for general stress cubic equation for triaxial state of stress will be

$$\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2)$$
$$- (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{xz}\tau_{yz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2) = 0 \tag{4.3}$$

4.4.3 PRINCIPAL SHEARING STRESS

Principal planes are planes upon which the shearing stresses are zero. Thus, the resultant stress on the principal planes is entirely accounted for by the normal stress component. On any other plane through the point the resultant stress will in general have both a normal stress component and a shearing stress component. It is clear that on at least one of these other planes the shearing stress will attain a maximum value. These extreme values of shearing stress are called planes of principal shear.

The magnitude of principal shearing stresses is equal to half the difference between the maximum and minimum principal normal stress, that means [ ]

$$\tau_1 = \pm 1/2(\sigma_2 - \sigma_3) \tag{4.4}$$

$$\tau_2 = \pm 1/2(\sigma_1 - \sigma_3) \tag{4.5}$$

$$\tau_3 = \pm 1/2(\sigma_1 - \sigma_2) \tag{4.6}$$

4.4.4 COMBINED STRESS THEORIES OF FAILURE

The result obtained from the analysis are evaluated based on the following theories:

4.4.4.1 MAXIMUM NORMAL STRESS THEORY (RANKINE'S THEORY)

In words, the maximum normal stress theory, proposed by Rankine, may be expressed as follows:

> ***Failure is predicted to occur in the multiaxial state of stress when the maximum principal normal stress becomes equal to or exceeds the maximum normal stress at the time of failure in a simple uniaxial stress test using a specimen of the same material.***

This theory may be mathematically formulated from the word statement above as follows

Failure is predicted by the maximum normal stress theory to occur if

$$\begin{aligned} (\sigma_1, \sigma_2, \sigma_3) &\geq \sigma_t \\ (\sigma_1, \sigma_2, \sigma_3) &\leq \sigma_c \end{aligned} \qquad (4.7)$$

Where σ_1, σ_2 and σ_3 are the principal normal stresses; σ_t is the uniaxial failure strength in tension; and σ_c is the uniaxial failure strength in compression. It is important to note that failure is predicted to occur if any one expression of the above is satisfied.[ ]

4.4.4.2 MAXIMUM SHEARING STRESS THEORY (TRESCA-GUEST THEORY)

In words, the maximum shearing stress theory, proposed by Tresca, may be expressed as follows:

> ***Failure is predicted to occur in the multiaxial state of stress when the maximum shearing stress magnitude becomes equal to or exceeds the maximum shearing stress magnitude at the time of failure in a simple uniaxial stress test using a specimen of the same material.***

If the maximum normal stress at the time of failure in the simple uniaxial test is σ_f, the corresponding failure value of principal shearing stress τ_f for the uniaxial test is

$$\tau_f = \frac{\sigma_f}{2} \qquad (4.8)$$

With this observation in mind the maximum shearing stress theory may be mathematically formulated from the word statement as follows

Failure is predicted by the maximum shearing stress theory to occur if

$$(|\tau_1|, |\tau_2|, |\tau_3|) \geq |\tau_f| \qquad (4.9)$$

We can formulate the maximum shearing stress theory in terms of principal normal stress as follows:
Failure is predicted by the maximum shearing stress theory to occur if

$$\begin{aligned} |\sigma_1 - \sigma_2| &\geq |\sigma_f| \\ |\sigma_2 - \sigma_3| &\geq |\sigma_f| \qquad (4.10) \\ |\sigma_3 - \sigma_1| &\geq |\sigma_f| \end{aligned}$$

Where σ_1, σ_2 and σ_3 are the principal normal stresses; σ_f is the uniaxial failure strength in tension. It is important to not that failure is predicted to occur if any one expression of the above is satisfied.[ ]

4.4.5 FAILURE PRIDICTION ON HOLLOW CONCRETE BLOCK (HCB)

Blocks are loaded with a live and dead load as detailed on item 4.3. and analyzed using SAP2000 finite element software. Based on the above theories and the analysis results, failure prediction is assessed as shown below.

4.4.5.1 ANALYSIS RESULT

Out of the three combinations discussed previously, combination two is the critical one which creates high stress on the HCB.

a) For Two Celled 160mm Deep HCB

As one can see from the stress contour, maximum stresses are near the junction of internal face of external web and internal face of the bottom shell and has a magnitude of:

```
S O L I D   E L E M E N T   S T R E S S E S FOR ELEMENT 4367

 COMB   COMB2 ------------------ MAX

   JOINT          S11          S22          S33          S12          S13          S23
    4667     0.000180    -2.05E-05     1.06E-05     7.11E-06     0.000152     6.92E-06
    4668     0.000176     2.82E-05     0.000321     3.02E-07     0.000145     6.46E-06
    4722     0.000210    -3.26E-05     5.14E-07     1.20E-05     0.000152     1.18E-05
    4723     0.000221     3.12E-05     0.000372     5.15E-06     0.000159     1.61E-06
    5822     0.000726     4.07E-05     9.27E-06    -1.01E-06     0.000140    -4.41E-06
    5823     0.000716     6.37E-05     0.000314     5.80E-06     0.000134     1.78E-05
    5877     0.000793     3.77E-05     8.28E-06    -5.85E-06     0.000164     4.36E-07
    5878     0.000797     7.57E-05     0.000373     9.52E-07     0.000171     1.29E-05
```

After averaging of joint stresses, one can find the stress at the top face of the element as.

$S11 = 0.758MPa$ $S12=0.00MPa$

$S22=0.054MPa$ $S13=0.15MPa$

$S33=0.176MPa$ $S23=0.007MPa$

With the above value the following principal stresses are calculated based on equations from 4.3 to 4.6 as

$\sigma_1=0.795MPa$ $\tau_1=0.042Mpa$

$\sigma_2=0.139Mpa$ $\tau_2=0.370Mpa$

$\sigma_3=0.055Mpa$ $\tau_3=0.328Mpa$

From the Theory of Normal stress;

To predict tensile or compression failure on the HCB, one of the conditions on eq.4.7 should be satisfied, and therefore,

$$(\sigma_1 \text{ Vs } \sigma_t), \quad 0.795 \geq 0.73MPa$$

The principal stress σ_1 is greater than the tensile capacity of the HCB while the other stresses, σ_2 and σ_3 are below the limiting value.

This result indicates that tensile failure of the HCB at the junction is possible.

From maximum shearing stress theory

To predict shear failure on the HCB, one of the conditions on eq.4.9 should be satisfied, and therefore,

$$(\tau_2 \text{ Vs } \tau_f),\ 0.37 \geq 0.365 MPa$$

The maximum shearing stress τ_2 is greater than the shearing resistance of the HCB, while the other stresses, τ_1 and τ_3 are below the limiting value.

This result indicates that shear failure of the HCB at the junction is also possible.

Therefore one can conclude that, two celled HCB with the dimension indicated on Fig 4.1a needs improvement to be used as load carrying structure.

b) For Three Celled 160mm Deep HCB

```
S O L I D   E L E M E N T   S T R E S S E S for ELEMENT 4367

 COMB   COMB2 ------------------ MAX

   JOINT         S11         S22         S33         S12         S13         S23
    4667    8.66E-05   -1.56E-05    6.49E-06    6.28E-06    0.000132    2.53E-06
    4668    8.64E-05    2.20E-05    0.000266    5.22E-07    0.000127    9.90E-06
    4722    0.000111   -2.35E-05    1.94E-07    1.20E-05    0.000126    8.24E-06
    4723    0.000119    2.22E-05    0.000292    6.24E-06    0.000132    4.19E-06
    5822    0.000449    2.98E-05    6.46E-06    1.29E-05    0.000126   -3.48E-06
    5823    0.000441    3.70E-05    0.000259    1.87E-05    0.000121    1.59E-05
    5877    0.000504    2.96E-05    7.84E-06    7.22E-06    0.000132    2.24E-06
    5878    0.000505    4.48E-05    0.000292    1.30E-05    0.000138    1.02E-05
```

Average stress.

S11 =0.45MPa S12=0.013MPa

S22=0.035MPa S13=0.129MPa

S33=0.141MPa S23=0.004MPa

Principal stresses:

σ_1=0.520MPa τ_1=0.021Mpa

σ_2=0.087Mpa τ_2=0.238Mpa

σ_3=0.044Mpa τ_3=0.217Mpa

From the Theory of Normal stress;

To predict tensile or compression failure on the HCB, one of the conditions on eq.4.7 should be satisfied, and therefore,

$$(\sigma_{1,}\ \sigma_{2,}\ \sigma_3 \text{Vs } \sigma_t\ ),\quad 0.52,0.087,0.044 \le 0.73MPa$$

All the principal stresses are below the limiting value.

This result indicates that no tensile and compression failure is predictable on the HCB.

From maximum shearing stress theory

To predict shear failure on the HCB, one of the conditions on eq.4.9 should be satisfied, and therefore,

$$(\tau_1\ ,\tau_{2,}\tau_3 \text{ Vs } \tau_f\ )\ \ 0.021,0.238,0.217 \ge 0.365MPa$$

All shearing stress are below the limiting value.

This result indicates that no shear failure of the HCB is possible.

Therefore one can conclude that, three celled HCB with the dimension indicated on Fig 4.2a can be used as load carrying structure.

With similar procedure used above the result for all other cases are attached on Appendix E.

4.5 VERIFICATION OF RESULT OBTAINED FROM LABORATORY INVESTIGATION.

4.5.1 LABORATORY TESTING.

Samples produced in the laboratory with the same dimension as the one used in the previous analysis are tested in the laboratory by a research group in AAU Technology Faculty Department of Civil Engineering, to determine their maximum uniformly distributed and point load carrying capacity, and the result obtained are verified with this analysis result as discussed below.

4.5.2 VERIFICATION OF RESULTS.

Most of the cracks which occured on the block at laboratory (see appendix) are similar with possible cracks resulted from stresses S11, and S33 obtained from the analysis, most of the cracks occur near the junction and the results show that the blocks needs some improvement at these points.

4.5.2.1 WITH UNIFORMLY DISTRIBUTED LOAD

From the laboratory investigation result, the first crack is formed at a uniform load of 6.6kN and 6.67kN on the two and three celled HCB respectively.

$$G = \frac{6.6}{0.2 \times 0.48} = 68.75kN/m^2 \quad \text{(For two celled HCB)}$$

$$G = \frac{6.67}{0.2 \times 0.48} = 69.48kN/m^2 \quad \text{(For three celled HCB)}$$

a) STRESS IN THE DIRECTION OF LOCAL AXIS 1-1 (S11)

When this load is applied on the analysis models, maximum stress of S11 occur at the corner of the blocks with a stress ranging from 0.9 to 1.35MPa, , which are a bit greater than the characteristic tensile strength of the block, therefore the analysis verifies that the cracks formed at the bottom corner and at the mid span of the block for two celled HCB are a result of stress S11.

For the case of three celled HCB, the first crack were formed at a uniform load of 6.67kN, and this load create a maximum stress of S11 with rang of 1.05 to 1.40MPa which is also greater than the tensile strength of the block.

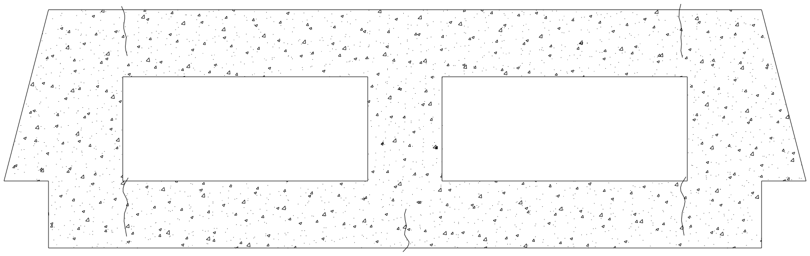


Fig. 4.4 Possible Crack on two-celled HCB due to S-11

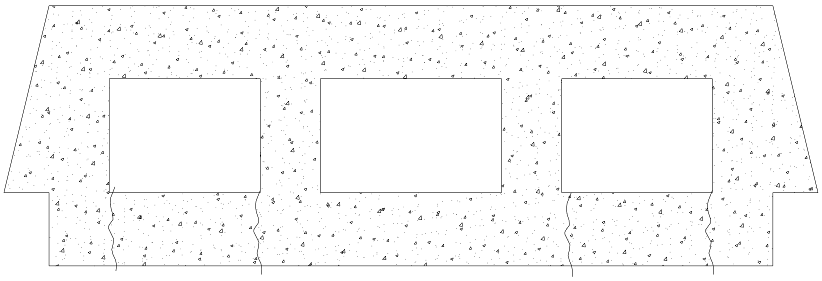


Fig. 4.5 Possible Crack on three-celled HCB due to S-11

b) STRESS IN THE DIRECTION OF LOCAL AXIS 2-2 (S22)

No patterned maximum stress of S22 occurs on the blocks; therefore this stress is not the cause of cracks

c) STRESS IN THE DIRECTION OF LOCAL AXIS 3-3 (S33)

From the stress contour of S33, maximum stresses occur at the junction of external webs and internal face of the bottom shell. This stress is also a tensile stress, with a range of 0.7 to 1.05Mpa for the case of two celled HCB and with a range of 0.84 to 1.12 for the case of three celled HCB, and this stress are nearly equal to the tensile capacity of the HCB and could be the cause of crack to the block as indicated on the figure below.

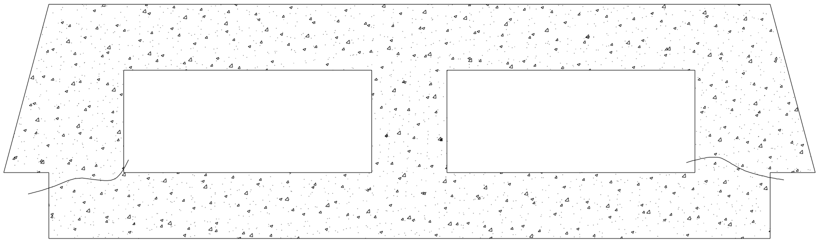


Fig. 4.6 Possible Crack on two-cells HCB due to S-33

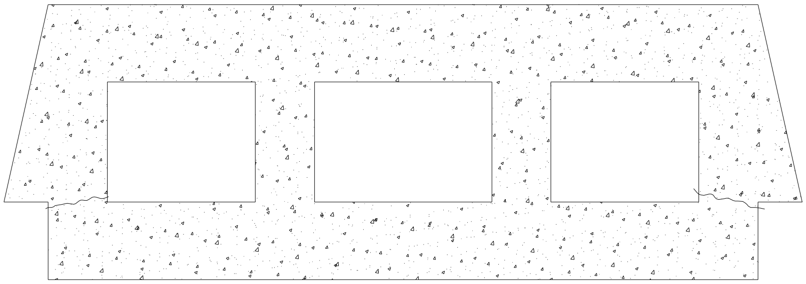


Fig. 4.7 Possible Crack on three-cells HCB due to S-33

d) STRESS ON PLANE 1-2,1-3 and 2-3

No patterned maximum plane stress occurs on the blocks; therefore these stresses are not the cause of cracks observed during laboratory investigation.

4.5.2.2 WITH POINT LOAD

The point load, at which the block failed, is applied on an area of 8cm x 20cm.

From the laboratory investigation result, the failure loads are

$$G = \frac{5.35}{0.08 \times 0.2} = 334.4 kN/m^2 \text{ For two celled blocks and}$$

$$G = \frac{6.80}{0.08 \times 0.2} = 425.0 kN/m^2 \text{ For three celled blocks}$$

This forces form a maximum stress of S11 at the bottom face of the top shell near the webs junction ,at the bottom face of bottom shell and at the junction of external webs and bottom shell with value range of 1.95 to 2.6MPa for the case of two celled HCB.

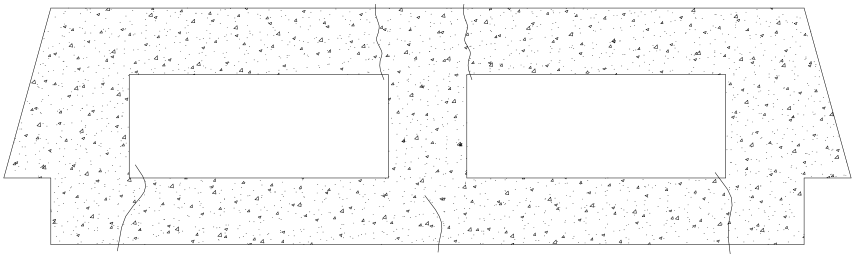


Fig. 4.8 Possible Crack on three-cells HCB due to S-11 (due to point load)

Similarly maximum stress S11 is formed at the bottom face of the bottom shell with value range of 1.8 to 2.4MPa for the case of three celled HCB.

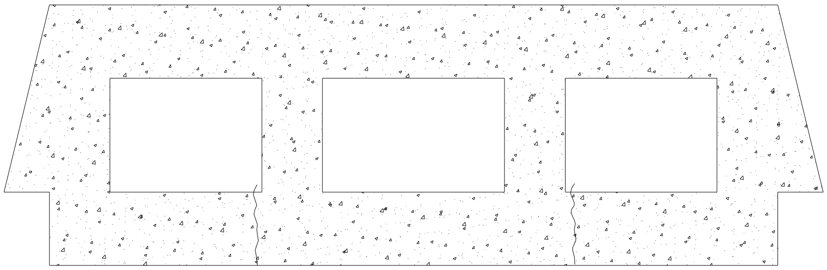


Fig. 4.9 Possible Crack on three-cells HCB due to S-11(due to point load)

These ranges are all greater than the tensile capacity of the HCB which is 1.09MPa, this high difference between the tensile capacity and analysis result could be a result of loading of the block in the analysis and at the laboratory.

However both results, which are found in the laboratory and in the computation show that failures or high stresses are near the sharp corners and it is better to eliminate these sharp corners to improve the carrying capacity of the blocks.

By eliminating the sharp corners as it is shown in the fig below, which decrease the opening area of the HCB by 28%, it is found that the carrying capacity of the block is increased by 50% as it is found in the analysis result attached.

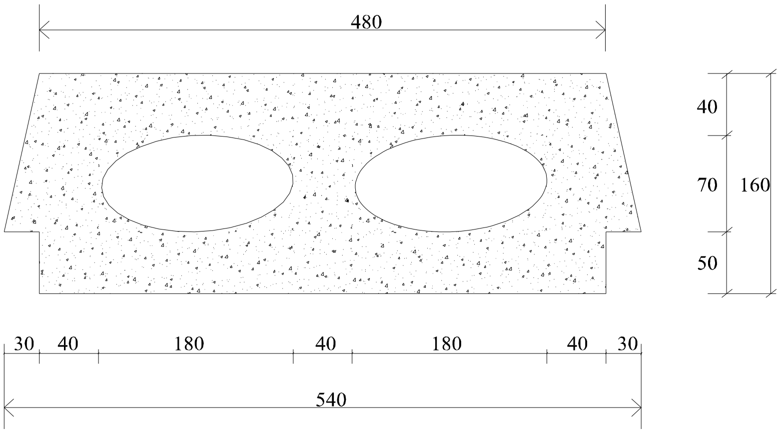


Fig. 4.10 Proposed Dimension of HCB

5.0 CONCLUSIONS AND RECOMMENDATION

5.1 CONCLUSTIONS.

5.1.1 ON COMPOSITE STRUCTURE.

The use of precast beam/slab system is a system in which simplification of construction method is used and also saves the country's limited timber source by avoiding the use of formwork as a main construction material.

In this study other factors which affect the cost of the total construction, such as the cost of the frame (beams and columns), cost of foundation and the effect of time are not considered in evaluation of saving of this type of construction.

Based on this analytical study, one can conclude that, the use of precast beam/slab system is shown to be more economical when used for small span buildings, but can be used up to 8.5m span with out excessive deflection and failure, the saving of this system is about 42 and 37% over ribbed and solid slabs respectively, however this saving decrease as the span increases to8.5m which is the maximum span used in this study.

5.1.2 ON HOLLOW CONCRETE BLOCKS

Analytical investigation shows that, 160mm deep HCB with two cells has carrying capacity less than the tensile capacity of the HCB and should not be used without further improvement, but all other HCBs can be used for the same, with three celled HCB of higher strength,

Moreover the same study shows that high stress concentrations are observed at the junction point the shell and the web , and this stress can be reduced by 50% if these sharp corners are eliminated.

And finally it is observed that the results obtained from laboratory investigation shows higher values of failure load than the analytical result and these might be a result of assumptions made for the material properties, support conditions and, loading conditions used.

5.2 RECOMMENDATIONS.

This study is made only on the theoretical or analytical bases, further studies should be done in the laboratory, and also a comprehensive study of cost saving in buildings by considering all factors, such as cost of two way slabs, frames (beams and columns), foundations and the effect of time saving on the construction of such type of slab shall be done.

6. REFERENCES

[1] "Ethiopian Building Code Standard, Structural Use of Concrete" EBCS-2. Ministry of Work and Urban Development, Addis Ababa, Ethiopia,1995

[2] ";Ethiopian Building Code Standard, Basis of Design and Actions on Structures" EBCS-1. Ministry of Work and Urban Development, Addis Ababa, Ethiopia, 1995

[3] ";Ethiopian Building Code Standard, design of Steel Structures" EBCS-3. Ministry of Work and Urban Development, Addis Ababa, Ethiopia ,1995

[4] Varghese, P.C.,"Limit State Design of Reinforced Concrete",Prentice-Hall of India Private limited, New Delhi-110 001,1999

[5] Dr.Ramchandra," Limit State design", Standard Book House,New Delhi, 1990

[6] "Ethiopian Building Code Standard, Design Aids for Reinforced Concrete Sections on the Basis of EBCS-2" Ministry of Work and Urban Development, Addis Ababa, Ethiopia,1997

[7] J.A.Collins " Failure of Materials in Mechanical Design" A Wiley-interscience publication

[8] SAP2000 Manual

[9] CEB/FIP Manual of Design and Technology for Light Weight Aggregate Concrete, Long-man, 1977

[10] Daniel Abebayhu, "Studies on Pumice Blocks and Scoria Bricks at the Basis of Standardization", Masters Thesis, Dec 1998

[11] ES 87;2001 Clay Bricks-Hollow clay bricks and beam tiles-specification, Ethiopian standard, second edition, 2001

[12] Vol 16,Concrete and Concrete Products, Ethiopian Authority for Standardization, Addis Ababa, September 1990

[13] James G.MacGregor, "Reinforced Concrete Mechanicsand Design" PRENTICE Hall, upper saddle River, New Jersey, 3rd Edition 1997

[14]

[15] Arthur H.Nilson , George Winter, "Design of Concrete Structures"McGraw-Hill 10th edition 1986