



ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

Analysis on the Influence of Rail Pad on Ballasted Railway Track

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A thesis submitted to the school of Graduate Studies of AddisAbaba University
in Partial fulfilment of the degree of Master of Science in Civil Engineering
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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
(Master of Science in Civil Engineering under Railway Engineering)

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Abstract

Railway tracks are commonly constructed with resilient elements (rail pads, ballasts etc.) They serve as the primary means of reducing the level of vibration generated through railway operations, key element in the transfer of wheel-rail forces into the track substructure, filter and transfer the dynamic forces from rails and fasteners to the sleepers. Understanding of the mechanistic behaviour of the rail pad is important in the development of improved track components.

The purpose of this thesis is to study the influence of rail pad on the behaviour of ballasted Railway track under dynamic and static loading. By creating models in the finite element program ABAQUS and ANSYS apdl responses to dynamic and static loading is studied respectively, using a 2D model. Beam on discrete support model is selected for the track; the rail and sleeper are best modelled by Euler-Bernoulli beam element. Spring and dashpot is used for the simulation of rail pads and the connection between the sleeper and subgrade ground. At last the models developed are analysed to study the effect of rail pad and speed on track component under different train location and different method of analysis, using the above Finite Element Packages.

Even though, the main use of Rail pad is to filter the dynamic load coming from train, it has great influence in absorbing the coming static load from train too. Deformed shape, Stress, and displacement due to the static load is presented, the comparison is made due to variation of rail pad type (soft, Hard, Normal, EVA) and K&C value, at the edge and centre point of loading. The Stress, Vertical acceleration and displacement are analysed for a range of K&C value, speed for the dynamic analysis.

For the static load analysed, for the two different point loading, the y (vertical) displacement /deformation/ decreased as both stiffness and damping increased and remain constant when the stiffness increase and damping value remain constant. The value of stress increased when we change the pad from soft to normal, gets reduced for the other twocases i.e. From Normal to hard, and hard to EVA. For the dynamic case the value of Vertical acceleration or vibration increase as both stiffness and damping increased, and also when the speed is increased from 22.22m/s -33.33m/s the acceleration behaves the same. The vertical displacement remains the same for K &C change with in interval of increase provided for both, while it gets reduced for the increase in the speed. The value of stress decreases as the K & C increase and increase while the speed increase.

Keywords: *Resilient, Vibration, Filter, Dynamic load, Rail pad, ABAQUS, ANSYS, Static load, Spring and Dashpot, Speed, Displacement, Acceleration, K&C*

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1. Introduction

1.1 Background

Railway tracks are commonly constructed with resilient elements (rail pads, ballasts etc.) that attenuate the level of impact of railway vehicles on the tracks. They serve as the primary means of reducing the level of vibration generated through railway operations. (Koroma, Hussein, and Owen: 2012)

The rail pad, also known as rail pad assembly, is a key element in the transfer of wheel-rail forces into the track substructure. The rail pad has a fundamental influence in system performance parameters such as track gauge, rail seat inclination, track vertical stiffness, and electrical insulation. This component is also important to the track structure because of its versatility as an engineered product that can be designed with multiple layers, a variety of materials, and optimized geometry. As a result, it can be conceived to achieve specific mechanical properties as it functions within the fastening system, which can ultimately affect the way the track structure responds to the loading demands. (Do Carmo, Edwards, Kernes, Andrawes, Barkan; 2014)

Rail pads are placed on the rail seat to filter and transfer the dynamic forces from rails and fasteners to the sleepers. The high damping value of rail pads reduces excessive high-frequency forces and provides a resiliency between rail and sleeper that helps alleviate rail seat cracking and contact attrition. (A. Remennikov, S. Kaewunruen, 2008)

The understanding of the mechanistic behaviour of the rail pad assembly and how it interacts within the fastening system is important in the development of improved track components. (Do Carmo, Edwards, Kernes, Andrawes, Barkan; 2014)

Even though, Rail pad has a key function to filter the dynamic force from rails, it plays a significant role in the transfer of static force coming from rail which happens at stations and Terminals.

1.2 Problem statement

The load coming from the train; that has greater damage to vehicle part and deterioration to track component is the dynamic load. Even though the effect of dynamic load is high, the static load that a train exerts during its stay at terminals of freight wagon before unloading, and at stations has a significant influence on the damage. This load needs to be damped from rail to nearby structure to reduce the maintenance cost, increase passenger comfort and etc.

Rail pad plays a significant role to reduce this load from its damaging effect. Of the parameters that influence rail pad properties are its stiffness and damping. By simulating the actual track in FEM software; ANSYS and ABAQUS and analysing for the two types of load, result greater detail information on the influence of rail pad. This study has additional advantage to enhance the use of rail pad.

1.3 Scope

The scopes of the research are listed here under

- ❖ Literature review focuses on railway track component function and advantages, Damping in rail pad, stress distribution, track stiffness, rail pad dynamics, model type, parameters and type of rail pad and the modelling aspect, finally, the analysis is made and the comparison and analysis is done only based on the parameters; Displacement, Vertical acceleration, Stress and deformation at different load application point and static and dynamic method analysis by ANSYS apdl and ABAQUS FE software's.
- ❖ modelling and analysing the effect of stiffness, damping, speed value and magnitude in the track,
- ❖ The research doesn't include laboratory test.
- ❖ The thesis will not develop a model to study the lateral dynamic loading
- ❖ It includes no practical benchmark or other simulation model;
- ❖ analysis responses using Finite Element software ANSYS and ABAQUS is interpreted

1.4 Objective

General Objective

The objective of the research is to study the influence of rail pad on the behaviour of ballasted Railway track under dynamic and static loading.

Specific Objective

The specific objectives of the thesis are;

- I. Analyse stress, displacement, and deformation of a track for the static analysis; vertical acceleration and effects analysed for static are to be compared for dynamic analysis;
- II. Compare the effect of speed of a train on track based on the above parameters i.e. stress, displacement, and vertical acceleration

1.5 Methodology

Finite element method of analysis is one of the most powerful and acceptable tool to analyse the dynamic and static behaviour of the track structures. By creating models in the finite element program ABAQUS and ANSYS apdl responses to dynamic and static loading are studied respectively using a 2D model.

The purpose of this thesis is to study the influence of rail pad on the behaviour of ballasted Railway track under dynamic and static loading. Beam on discrete support model is selected for the track; the rail and sleeper are best modelled by Euler-Bernoulli beam element. Spring and dashpot is used for the simulation of railpads and the connection between the sleeper and subgrade ground. Rail on discretely supported model is implemented for different rail pad parameter. In this model any type of irregularity is not included over the length of the track to simulate its effect on track response.

At last the models developed are analysed to study the effect of rail pad and speed on track component under different train location and different method of analysis, using the above Finite Element Packages.

1.6 Structure of the thesis

This thesis presents the whole work in six chapters; *i.e.*

- ❖ Chapter 1 Gives brief Introduction, show the Scope and limitation of the thesis, list the objectives to be addressed, the methodology used, and the structure of the thesis
- ❖ Chapter 2 gives coverage to the literatures reviewed for the work
- ❖ Chapter 3 discusses the modelling aspect and literatures related to the objective
- ❖ Chapter 4 contains result analysis and discussion made for the results obtained from ANSYS and ABAQUS giving concluding points for the next chapter,
- ❖ Chapter 5 Concludes the thesis and recommend the future work to be developed related to this title that this study doesn't cover,
- ❖ Chapter 6 gives list of reference materials reviewed

2. Literature Review

2.1. Railway Track

The purpose of a railway track structure is to provide a stable, safe and efficient guided platform for the train wheels to run at various speeds with different axle loadings. To achieve these objectives, the vertical and lateral alignments of track must be maintained and each component of the structure must perform its desired functions satisfactorily under various axle loads, speeds, environmental and operational conditions.(Esveld, 2001)

Currently, there are 2 types of rail tracks commonly used; (a) Conventional ballasted track, and (b) slab track. Most rail tracks are the traditional ballasted type; however, there are some recent applications of non-ballasted slab tracks. Recent studies indicate that slab tracks may be more cost effective than conventional ballasted tracks when appropriate considerations are given to their life maintenance cost, and the extent of traffic disruption during track maintenance.

The railway track structure provides the necessary conditions for the circulation of trains. There is a great variety of track structures throughout the world of which the ballasted railway track is one of the most common. (Indraratna, Salim, et.al; 2005)

The main type of track used in Ethiopia is the ballasted railway track. Track components are divided into two parts, substructure and superstructure.

2.1.1 Substructure

A railway track sub-structure normally consists of a top layer of railway ballast, an intermediate layer of sub-ballast, and the subgrade, see Figure 2.1.They rest on the subsoil, normally the natural ground. The structure is shown in Figure 2.1 (Huan Feng,2011)

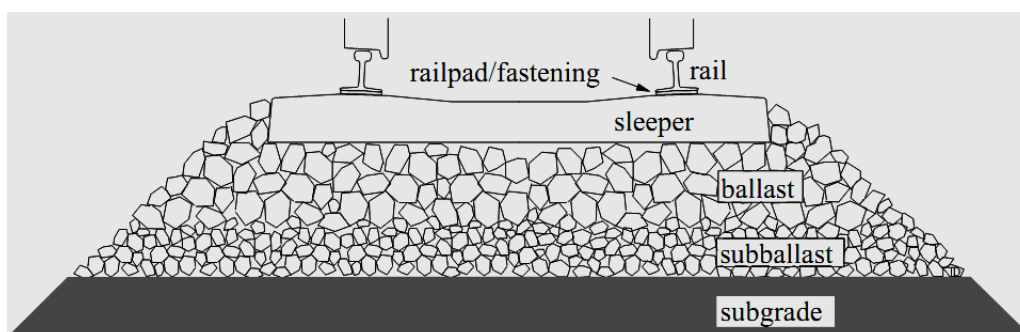


Figure 2.1 Track with its different components: rails, rail pads and fastenings (fastenings not Shown in this figure), sleepers, ballast, sub-ballast, and subgrade. (Huan Feng, 2011)

a) Ballast

The main functions of ballast are to secure track stability in all directions, prevent excessive sleeper movement, absorb static and dynamic loads and provide good drainage of water (Andersson and Berg, 2007). To secure these functions, high quality ballast in terms of good vertical and horizontal load bearing capacity is required (Banverket, 2002). Further, wear resistance, good elastic characteristics and a good drainage capacity are important. To meet these demands it is important to fill the ballast embankment. It is also important to use hard stone materials that carry large pore volumes and are angular. The angularity provides high friction between the grains.

In Sweden, ballast with a particle size of 32 – 64 mm is used to meet these requirements (Andersson Berg, 2007). More finely grained ballast, 11 – 32 mm, is used in railway yards and sometimes at switches and crossings to get a more level surface.

The ballast layer should be at least 300 mm thick below a sleeper and the layer on bridges should be at least 400 mm (Banverket, 2002). The ballast layer should not cover the sleeper surface in order to minimize the risk of ballast spray. It is also important for the sleeper ends to be surrounded with a sufficient amount of ballast to secure the sleeper's lateral stability.

Ballasted tracks are mainly used for local transport of passengers and goods. It consists of rails and sleepers mounted on a ballast bed. The main advantages of ballasted tracks are their low construction costs and inherent vibration damping. The main disadvantage of ballasted tracks is maintenance related since the ballast may degrade and need replacement or realignment that is both costly as well as time consuming. (Ewa Cardas-cinal, 2009)

b) Sub ballast

The main purpose of the sub-ballast is to support the ballast and distribute traffic loads further down in the structure (Banverket, 2002). It also aids in draining the track and prevents small particles from the sub-grade to mix with the ballast. This requires a material with good properties regarding bearing capacity, stiffness and drainage. The employed material is usually sand or gravel with a particle size of 0 – 150 mm that should be frost resistant. (Indraratna, Salim, et.al; 2011)

c) Subgrade

The purpose of the sub-grade is to create a smooth foundation for the layers above. It usually contains filling materials like moraine or blasted stones. The requirements on the sub-grade are to resist settlements and softening, which can cause penetration of ballast, and to resist frost heave. The sub-grade should be located below the frost affected zone (Banverket, 2002).

2.1.2 Super structure

The superstructure consists of rail, rail pad, fastening system and sleeper. All components interact and have different functions in the structure. The track superstructure is shown in Figure 2.2.

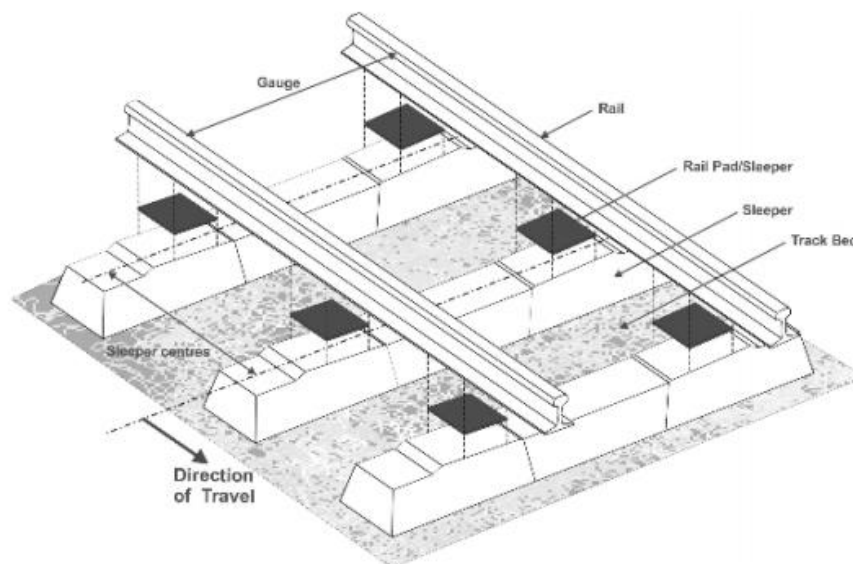


Figure 2.2 Typical Ballasted Railway Track (Remennikov et.al 2008)

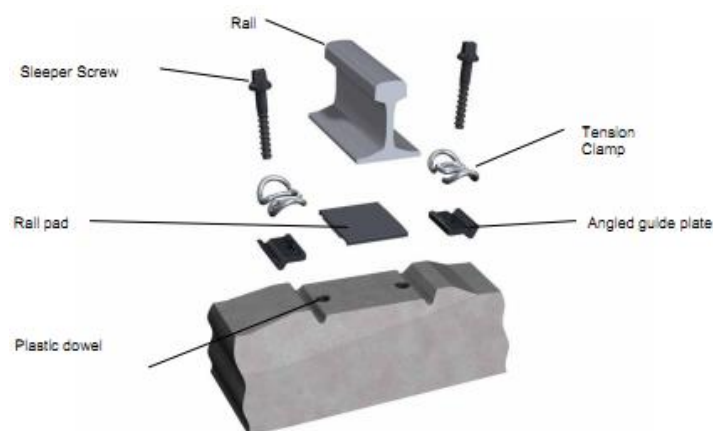


Figure 2.3 Components of a fastening system for ballasted track (Deutsche Bahn AG, Hans-Joerg Terno, 2011)

I. Rail

The functions of the rails are to: distribute dynamic and static loads from vehicle to sleeper, function as a level surface for the wheel to minimize vibrations, offer sufficiently low friction in wheel/rail interaction, and resist thermal expansion (Andersson and Berg, 2007).

To secure the functions, there are high demands on the rails in the form of material hardness, with a high ultimate stress limit (Andersson and Berg, 2007). It should further have a good resistant to wear and good welding abilities. To meet these demands steel is used.

II. Fastening

The fastening system is used to hold the rails onto the sleepers, to ensure fixing of the rails. Depending on the type of the sleeper and geometry of the rail, various kinds of fastening systems are utilised. Besides resisting vertical, lateral, longitudinal and overturning movements of the rail, the fastening system can aid in damping traffic vibrations, and prevent or reduce rail/sleeper attrition. (Heunis, 2011)

III. Rail pad type and function

The rail pad is a rubber or polymer pad that is located between sleeper and rail. Its function is to protect the sleeper from wear and for dampening vibrations; dynamic loads from the rails, resists lateral movements of the rail and electrically insulates the rails from each other. The rail pad is shown in the picture of Figure 2.4 and 2.5.

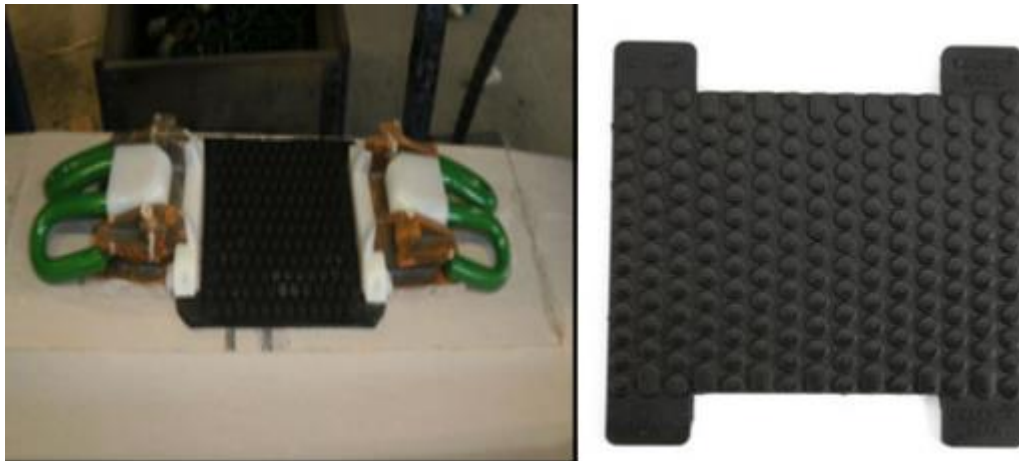


Figure 2.4 Rail pad and clips of type Pandrol Fast clip

From a track dynamic point of view, the railpads play an important role. They influence the overall track stiffness. A soft railpad permits a larger deflection of the rails when the track is

loaded by the train. Hence the axle load from the train is distributed over more sleepers. Besides, since soft railpads can suppress the transmission of high-frequency vibrations down to the sleepers and further down into the ballast, they also contribute to isolate high-frequency vibration.(Heunis, 2011)

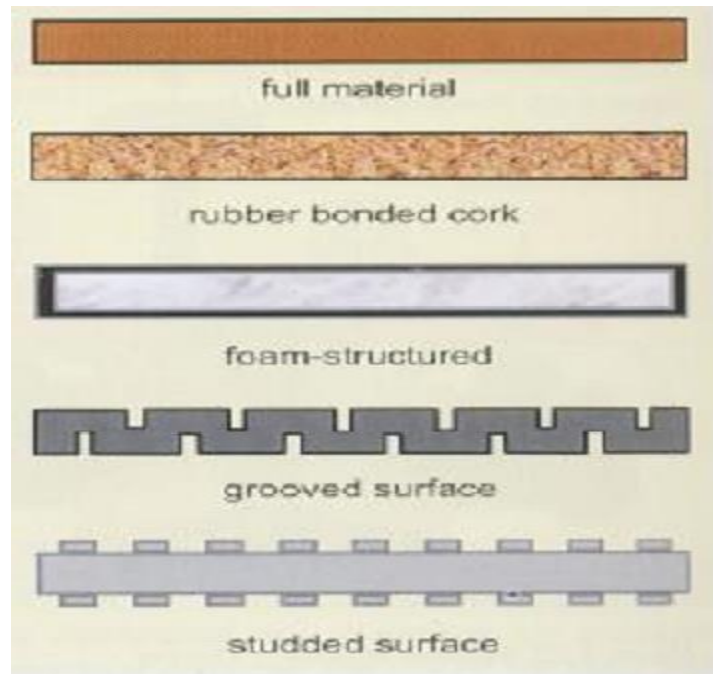


Figure 2.5 Rail pad appearance (Esveld, 2001)

There are many different types of rail pads, such as high-density polyethylene (HDPE) pads, resilient rubber pads and resilient elastomer pads, all of which have different surface profiles.

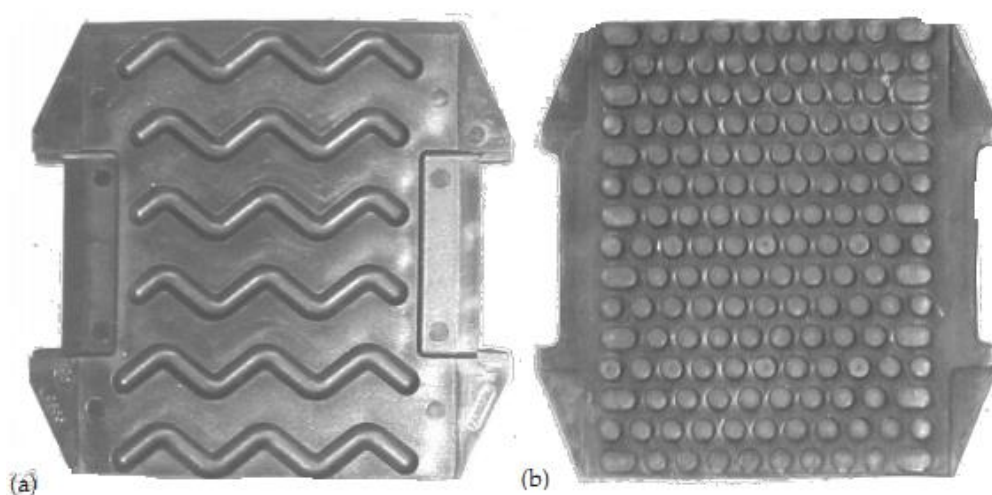


Figure 2.6 Rail pad specimens- (a) HDPE and (b) Studded (Kaewunruen, Remennikov, 2009)

d) Sleeper

Four different types of sleeper materials are available: wood, steel, concrete and polymer.

The functions of a sleeper are to:

- transfer and distribute forces from the rails to the ballast,
- fix the track gauge,
- maintain adequate rail inclination,
- Resist rail movements in all directions (horizontal, vertical and longitudinal).
- Insulate rails electricity(i.e. in concrete , timber)
- Provide a base for rail seats and fastenings.

It is important that the sleepers have a sufficiently regular spacing. Otherwise high stresses arise in the rails and an uneven load distribution to the ballast is created. This will cause faster degradation of the track. Main tracks in Europe usually have a distance of 600 to 650 mm between sleepers, but shorter distances exist (Andersson and Berg, 2007).

Recommended standard values for Mono block sleeper design is a length of 2500 or 2600 mm, a width of maximum 300 mm and height 200 – 230 mm at the rail seat (UIC, 2004). This is for standard gauge tracks and with the aim to reach requirements of acceptable ballast pressure and lateral stability. A standard sleeper should have a minimum weight of 240 kg and the recommended concrete strength class is C50/60 MPa.

2.1.3 Damping in Rail seat pad

Damping is a physical phenomenon occurring in all materials to a lesser or greater degree. In the rail environment, damping is introduced to the system by adding resilient or viscoelastic elements to decrease the vibrations transmitted from the rails to the nearby infrastructure. These materials are difficult to model since they introduce non-linearity to the system. (Ewa Cardas-cinal 2009)

Of the major role given by rail pad, the most important one is damping. They are installed on rail seats in order to attenuate the dynamic content from axle loads and wheel impacts from both regular and irregular train movements. And also, very important because they reduce the interaction force between the rail and the sleepers. (Kaewunruen, Remennikov, 2009).

Further, the rail pads provides a resilience function between rail and sleeper that helps to absorb shock and impact from the wheels to the rails, and reduce a damage of rail supporting point and contact abrasion. (Do Carmo, et al; 2014)

2.1.4 Stress distribution in the track

For simplified analysis, loads can be considered static, constant in direction and magnitude. These loads are typical just dead loads or used in analysing a member at a given moment in time. Dynamic loading is typically the true loading scenario, however is greatly more difficult to model. To describe dynamic loading perfectly every imperfection in the rail structure must be known.

The most commonly thought of track force is the one working in the vertical direction. The vertical force is always perpendicular to the track and is either considered a downward wheel force or an uplift deflection resistance force. As seen in Figure 2.7, the downward force comes at the point directly below the wheel; this wheel force is what causes the deflection in the rail shape and an uplift force away from the wheel load.

The wheel force is a combination of static and dynamic loading. The static load is easily estimated by dividing the entire trainload by the total number of wheels. Static values typically range from 12,000 lbs. (53kN) for light rail passenger to 39,000 lbs. (174 kN) for heavy haul trains in North America. (Selig and Waters, 1994).¹

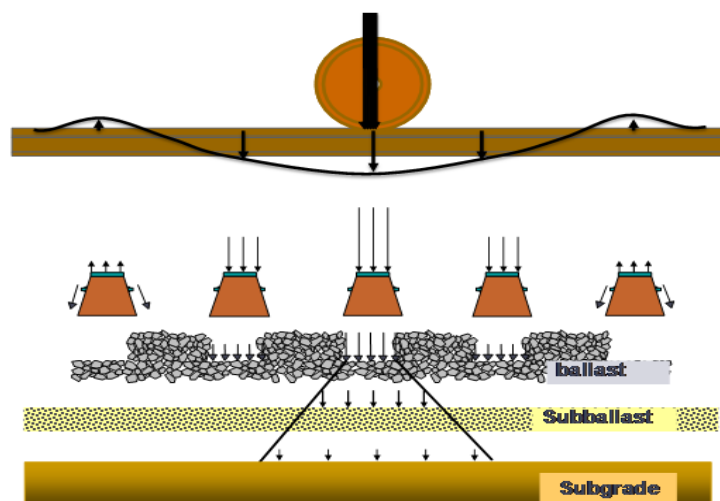


Figure 2.7 Wheel load distribution into the track structure

¹ <http://www.railwaysubstructure.org/railwiki>

2.1.5 Vertical track stiffness

Track stiffness (vertical track load divided by track deflection) is a basic parameter of track design which influences the bearing capacity of track, the dynamic behaviour of passing vehicles and, in particular, the quality of track geometry and the life of track components. (Eric Berggren, 2009)

The most common Way of Track stiffness is that the stiffness (D) presents the proportion between vertical load (Q_t) and track deflection (y) at a given moment (t):

$$D(t) = \frac{Q(t)}{Y(t)}(1)$$

(Leposava puzavac, et.al, 2012)

2.1.6 Influence of Track Stiffness on Track Behaviour under Vertical Load

In general, relatively high track stiffness is beneficial as it provides sufficient track resistance to applied loads and results in decreased track deflection, which reduces track deterioration.

However, very high track stiffness leads to increased dynamic forces in the wheel-rail interface as well as on sleepers and ballast, which may cause wear and fatigue of track components. Also, a particular problem is changes in track stiffness along the track, which causes variations in vehicle-track interaction forces and leads to differential settlement and therefore differential track geometry deterioration and potentially vibration problems. (Eric Berggren, 2009)

2.1.7 Dynamic Analysis of Railway Track using Models

The principal function of a railway track dynamic analysis model is to couple the components of the vehicle and track structure to each other so that their complex interactions are properly represented. Thus the effect of traffic load on stresses, strains and deformations in the components of the railway system may be determined.

Mechanical properties such as mass, moment of inertia, damping and elasticity of the vehicle and track components, form essential input for the models. Many of the components in railway tracks are made of materials with complicated constitutive relationships. Establishing the properties of rail pads, ballast and formation, for example, is particularly difficult due to their non-linear nature. Furthermore, the behaviour of these components could be dependent upon magnitude of load, passage of time or frequency of excitation and pre-load.

In a typical model, the entire system of railway components is divided into three main subsystems: the vehicle; wheel/rail contact; and the track. It is common for the behaviours of these dynamic subsystems to have different relative merit in a given model. But; as stated by Popp et.al “Models should be as simple as possible and as accurate as necessary regarding the task they serve”. David Steffen (2005)

Based on the Objective stated in this thesis i.e. ‘Analysis on the influence of Rail pad on Ballasted Railway Track’ a 2D model is chosen, by which the Vehicle, Track system is modelled in a simple representation. Main focus point is on rail pad and its interaction and influence with other component.

2.2. Rail pad dynamics

2.2.1. Rail pad

The simplest solution to isolate rail track systems is rail pads and rail bearings. An elastic element is introduced between the rail and sleeper (railpad) or slab track (rail bearing). This type of intervention can easily be retrofitted to existing systems but is limited toward the vibration isolation it offers. These materials are usually thin (less than 10 mm) to limit the unwanted static deformation of the rail. Rail pads are always subjected to a preload (due to the fastening mechanisms) and this can have a negative influence on their dynamic properties since their dynamic range is decreased.(Heunis, 2010)

It acts as cushioning material between the rail and sleepers for distribution of vertical load and for dampening vibrations.

Rail pads are effective remedy to prevent cracking of sleepers at rail seat area (Do Carmo, Edwards, Kernes, Andrawes, Barkan; 2014). For lower speed range soft rail pad could absorb the ill effects of impact loading,(Jongyoul, Choi, 2014). Impact could be due to sudden change in profile of rail wheel interface. However, uses of soft rail pads always do not reduce the rail wheel connected force (Huang Feng, 2011). Deterioration of rail pad during service and its effect on track performance could be serious problem. There is no unanimity among world railways regarding the thickness of the rail pads as well as its vertical stiffness. Germany has established a correlation between the weight of the sleepers and the stiffness of the rail pads (Koroma, Hussein, et al, 2012).

As far as ballasted high speed lines are concerned the dynamic rigidity of the rail pads under the rail must not exceed 600 MN/m. Similarly, the total dynamic stiffness of slab track

systems must not exceed 150MN/m (Koroma, Hussein, et al 2012). The stiffness of the existing rubber pads (6 mm thick) being used in Indian railway is between 150 to 250 MN/m. The optimum stiffness of the high speed ballasted track (as an assembly) is found to be 50 MN/m for 200 mph and 78 MN/m for 300 kmph.

2.3. Rail pads and fastenings dynamics

In a track, the rails are fastened onto the sleepers. Usually, rail pads are inserted between the sleepers and the rails. The rail pads provide electrical insulation of the rails and they protect the sleepers from wear. The rail pads also affect the dynamic behaviour of the track.

Grassie et al (1993) compared different track models with measurements and showed that it is important to include the rail pads to obtain an accurate track model. The influence of soft and stiff rail pads on the wheel–rail contact force and the track dynamics has been measured in full-scale experiments with a moving train, as reported by Fermer and Nielsen (1995). In those measurements, the train was equipped with an instrumented wheel set (with and without wheel flats) and the track was also equipped with measurement devices (strain gauges and accelerometers).

Soft rail pads were found to result in lower sleeper acceleration and higher railhead acceleration than the stiff rail pads. Laboratory measurements of stiffness and damping of studded rubber rail pads have been performed by Thompson et al (1999). Also, Fenander(1997) reported these measurements and compared them with field measurements on rail pads. The stiffness of the rail pads was found to be weakly dependent on frequency, but to increase strongly with preload. The damping loss factor of the rail pads was found to be nearly independent of the preload and to increase only slightly with frequency.

The most commonly used physical model of a rail pad is the spring–damper system. The spring is usually assumed to be linear, and the damping is assumed to be proportional to the deformation rate of the rail pad. It should be pointed out, however, that this viscous damping model does not always agree well with measured data (what may give a good fit at one train speed need not give a good fit at another speed).

In practice, the damping is more or less independent of frequency, implying that a constant loss factor model agrees better. As the stiffness of rail pads has been found to increase strongly with pre load (especially for studded rail pads), a more accurate track model should display this behaviour, Wu and Thompson (1999).

Oscarsson (1998) expanded a linear dynamic train – track interaction model to encompass nonlinear stiffness in the rail pads and in the ballast. He undertook an investigation on the influence of the nonlinear rail pad stiffness on rail corrugation growth (the growth of short wavelength periodic irregularities), and he found that a weak nonlinearity of the rail pad stiffness does not influence the corrugation growth very much, whereas a strong nonlinearity may lead to an increased wear of the rail. Also, in Dahlberg (1998), a comparison of linear and nonlinear track models containing linear and nonlinear rail pad stiffness was performed.

A more advanced rail pad model was set up by Fenander (1998). A fractional calculus model of the dynamic behaviour of rail pads was proposed. Earlier, it has been shown that molecular theories for viscoelastic polymers can be used to derive the fractional derivative model and those fractional derivative models accurately describe many viscoelastic materials by use of only few parameters.

The role of the fastenings is normally neglected when investigating track dynamics. The fastenings should keep the rails in place (attached to the sleepers) and prevent the rail from rolling over when lateral forces are applied to it. The stiffness of the fastening is normally much less than that of the rail pad. Therefore, when a wheel set loads the track, only the rail pad stiffness is important. For an unloaded rail, however, the fastening will induce a certain preload (static load) on the rail pad, and, knowing that the rail pad stiffness is nonlinear, this may influence the dynamics of the unloaded track. The dynamic behaviour of rail fasteners at high frequencies has been investigated by Thompson and Verheij (1998).

Rail pad has also an advantage of preventing different failure modes of sleeper. One of the failure modes for concrete cross ties are, rail seat deterioration. Localized crushing of the concrete in the rail seat is one of the potential mechanisms that contribute to this failure mechanism. University of Illinois at Urbana–Champaign is using a matrix-based tactile surface sensor to measure and quantify the forces and pressure distribution acting at the contact interface between the concrete rail seat and the bottom of the rail pad.

Experimentation has shown that a direct relationship existed between rail pad modulus and maximum rail seat pressure. In addition, under a constant vertical load, a direct relationship between the lateral-to-vertical force ratio and the maximum field side rail seat pressure was observed.

Given that all preliminary results indicate that various combinations of pad modulus, track geometry, and lateral-to-vertical force ratio create localized areas of high pressure, crushing

remains a potential mechanism leading to rail seat deterioration. (C. T. Rapp, M. S. Dersch, et al, 2011)

Generally, the dynamic design of tracks relies on the available data, which are mostly focused on the structural condition at a specific toe load. Recent findings show that track irregularities could significantly amplify the loads on railway tracks. This phenomenon gives rise to a concern that the rail pads may experience higher deterioration rate than anticipated in the past. On this ground, an innovative test rig for estimating the dynamic properties of rail pads has been devised at the University of Wollongong.

However, the current numerical models or simulations of railway tracks mostly exclude the time- and frequency-dependent effect on the nonlinear dynamic behaviour of rail pads, although it is evident that deterioration rate has significant influence on the dynamic rail pad properties that affect the dynamic responses of railway tracks.(S.kaewunruen, A. Remmenikov, 2009)

2.4. Rail pad model

The most commonly used physical model of a rail pad is the spring-damper system. The spring can be assumed to be linear, and the damping is assumed to be proportional to the deformation rate of the rail pad. According to the comparison between track models and measurements it's important to include the rail pads to get an accurate track model. In the measurements carried out before, it was found that soft rail pads would result in lower sleeper acceleration and higher railhead acceleration than the stiff rail pads. The stiffness of the fastening is normally much less than that of the rail pad. Therefore, when investing track dynamics the role of the fastenings is normally neglected. Grassie et al (1993). Due to their properties, the common scheme used to model pads is a viscous damping connection. The mass of pads can be neglected.

2.5. The Rail-pad parameters

Rail-pad stiffness and damping properties depend on the type of rail fastening system adopted. Concrete sleepers and ballasted track is the usual configuration for the infrastructure. The values of stiffness and damping are given by rail-pad producers. The vertical stiffness K_p varies between 50 and 300 MN m⁻¹ and the vertical damping C_p varies between 20 and 80 kNs m⁻¹. Silviu et.al (2010)

3. Modelling

3.1. Introduction

Different researchers used several methods (approach) to come with the complete model of railway track and Vehicle. The simplicity of the model depends on the result required. Some of researchers work is listed in short.

3.2. Track Modelling

Castellani (2000) developed an algebraic model for the vibrations generated by urban rail vehicles on floating slab tracks. Castellani (2000) measured the displacement and acceleration of a floating slab track when a locomotive with seven passenger cars travels over it at 90 km/h and compared the results to a numerical simulation. The numerical simulation exhibited a good correlation to the physical set-up up to about 63 Hz. Castellani (2000) also found that a major shortcoming in his model was a description of elastomeric (resilient) materials with frequency dependent behaviour. These materials show strain rate sensitivity and hysteretic energy dissipation.

Zhai and Cai (1997) generated a numerical model for the dynamic interaction between a rail vehicle and a train track. The different components of the system were modelled as springs, dampers and masses with the ballast being modelled as shear springs and dampers. Wheel/rail interaction was modelled with non-linear Hertzian theory and the equations of motion were solved with Newmark's explicit integration scheme. Experimental validation of the model was done through various field tests and the model showed good correlation with measurements conducted on actual train tracks.

Zhai et al. (2004) focused on the damping mechanisms in the ballast of train tracks. They implemented shear damping and stiffness to model the interaction between the particles. This model was then verified by field testing and found to agree well with the measured results. The calculated resonance frequencies were on average lower than the measured values, 70 to 100 Hz compared to 80 to 110 Hz.

Fiala et al. (2007) developed a numerical model that can be used to predict the vibrations and reradiated sound in buildings due to surface rail traffic. This model accounts for a moving vibration source, dynamic soil structure interaction and sound propagation through layered ground. The methods used are explained using a numerical example and the model shows good correlation for relatively stiff soil and direct excitation of the foundation.

Fiala et al. (2007) further states that the dominant frequencies with regards to noise is determined by the acoustic resonance of the room, this acoustic resonance is dependent on the wall absorption and room dimensions. It was also found that base isolation is the most effective solution for noise isolation and that the model is dependent on material properties as well as structural details of the buildings.

Karlström et al. (2006) developed an analytical model to predict ground vibrations caused by railways. The main components involved were rails, sleepers, ground and a rectangular embankment which supports sleepers and rails. There are therefore no rail pads or other elastic components involved and focus is placed on modelling the ground vibrations.

Karlström et al. (2006) drew a comparison between two FEM and analytical modelling methods. Analytical methods offer fast computational times and infinite domains but are rather limited towards geometry and non-linear behaviour. FEM (and other discretization) methods overcome the limitations of analytical approaches but has the disadvantages of struggling with infinite domains, long computational times and a small discretized region (it could only deal with 40 m of track).

The results for the model at speeds of 70 km/h and 200 km/h are compared to simplified models and measured data. Their simulations were found to agree almost exactly with measured data at low speeds and showed good correlation with their measured data at high speed. The simplified models were found to show good correlations with the simulation and measured data up to 1 Hz but differ significantly at higher frequencies.

Cox et al. (2006) designed and manufactured a test rig to evaluate slab track structures for specifically underground railways. Their main aim was to develop a test rig that bridges the gap between full scale and bench top tests with regard to the measurement/comparison of the dynamic properties of various fixation systems. The frequencies they were mostly interested in, was between 40 and 120 Hz as these frequencies are most likely to cause disturbances in surrounding buildings.

A major shortcoming of testing in the field was found to be variables such as train speed as well as soil conditions and therefore a test rig could be better suited for comparison purposes on a shorter timeframe. The track was tested for nine different configurations each using different fasteners and/or rail pads. Cox et al. (2006) found the measured natural frequencies to be higher than in physical systems since their test rig does not include an equivalent to the unsprung mass of the rail vehicle.

An “excitation” model was used to extract parameters for the resilient elements in the tests. The values for dynamic stiffness and damping were adjusted so the response of the model mimicked the measured responses for each different resilient material. This method is limited since only a single dynamic stiffness value can be obtained at a specific frequency. The study found that floating slab tracks perform best when fitted with soft rail fasteners especially in the frequency ranges of concern.

Lombaert et al. (2006) developed a three-dimensional numerical model for normal train track systems and high speed (200 km/h plus) trains. This model was validated against various physical systems. Experiments were used to determine the dynamic characteristics of the soil and track, the transfer functions of the soil, the transfer functions of the track-soil and the vibrations of the track as well as the free field. Further experiments were also conducted to verify the numerical model. The rail pads were modelled as continuous spring-damper connections and no attention was given to complex damping characteristics.

The validation of the numerical model showed relatively good agreement for the track-free field transfer functions, but the numerical model overestimated the response at small distances. Their numerical model for the sleeper response and free field vibrations showed good agreement with the measured data although it has a high dependence on soil properties and a high level of uncertainties. It was also found that a better understanding of the train-track interaction is needed and more field testing is needed (in general this article is not applicable to this work since the speeds involved are much higher and the focus is on the soils transfer properties (vs. the rail pad properties)).

Kaewunruen and Remennikov (2006) conducted a sensitivity analysis to determine the sensitivity of a concrete sleeper to variations in rail pad parameters. Finite element analysis was used and the rail pad stiffness was varied between 0 and 5×10^9 N/m with a maximum rail stiffness of 100×10^6 N/m. Their finite element model incorporated sleeper/ballast interaction and the focus of analysis was on in situ mono-block concrete sleepers. The sleepers’ changes in natural frequencies and dynamic mode shapes were used as comparison between different rail pads. It was found that rail pad stiffness has a nonlinear effect on the effective stiffness of the track system and that it mainly affects the first three vibration modes. High effective stiffness can cause changes in the flexural mode shapes of the track.

Lombaert et al. (2006) developed a three-dimensional numerical model for continuous slab track systems. This model was used to determine the effect of various soils, slab and resilient

slab mat parameters on the vibration transfer characteristics of the system. The main area of interest was the comparison between normal (un-isolated) slab track and floating slab track systems for different (soft and stiff) soils. It was found that floating slab track systems have pronounced responses at low frequencies and is better suited to applications where the frequencies involved are higher.

Lombaert et al. (2006) also stated that the resonance frequency of the slab track system should be as low as possible for minimum transmissibility. This resonance frequency is generally limited by the maximum allowable static rail deflection, and physical systems can have resonance frequencies as low as 8 to 16 Hz.

Vostroukhov and Metrikine (2003) developed an analytical model for a railway track that is supported by viscous-elastic pads. These pads were modelled according to the Kelvin-Voigt model. The main aim of their model was to determine the elastic drag that a high speed train experiences and they found that the elastic drag is comparable to aerodynamic drag at high velocities.

Nielson and Oscarsson (2004) developed a numerical method for simulating the dynamic train-track interaction. This method separates the track properties into linear (associated with the unloaded track) and non-linear (associated with the dynamic loading) contributions. A moving mass model was then employed for simulation purposes. The dynamic properties of the rail pads was determined in laboratory measurements and quantified with a three parameter state-dependent viscoelastic model. They compared this model to field measurements and found good agreement between the two methods.

Picoux and Le Houédec (2005) developed and validated a numerical model for the vibration generated by trains. The main aim was to model vibrations in the soil and a fairly complex three dimensional model was developed. In situ testing was done to verify this model, these tests made use of optical as well as acceleration measurements. It was difficult to compare numerical and measured data since the excitation frequency was quite difficult to determine, but good agreement was found and the model can be used for further analysis purposes.(Eric Berggren, 2009)

Models of track dynamic behaviour may be generally classified into two Categories:

- those that represent the track as a continuously supported rail beam and
- those that represent the track as a discretely supported rail beam.

Continuously supported models of infinite length are based on the beam on elastic foundation theory. The rail, pads, fastenings, sleepers, ballast, sub-ballast and subgrade are components that define the value of the modulus of track elasticity. (Dahlberg, 2004)

Discretely supported models are similar to the continuously supported modelled and often have multiple layers representing the rail pads, sleepers, ballast, sub ballast and subgrade. According to this presentation, a model for the entire railway system can be developed consisting of masses, springs and dampers

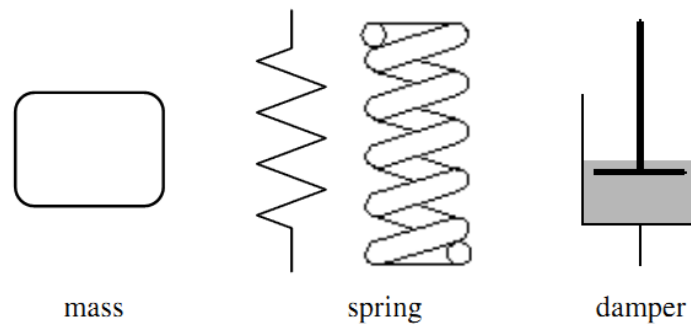


Fig.3.1 Symbols used for the constituents of the model for the rail way system

Continuously Supported Models: Rail as a Beam on Elastic Foundation (BOEF)

Rail as a Beam on Elastic Foundation may be modelled as

- An ordinary Euler – Bernoulli beam (The classical beam theory) or
- A Rayleigh – Timoshenko beam. The Rayleigh – Timoshenko beam theory includes the rotatory inertia of the beam cross section and beam deformations due to the shear force. Also, a longitudinal (axial) force in the rail may be included in these models.

Winkler (1867) used for modelling of the railway system, Euler–Bernoulli beam theory: an infinitely long rail beam resting on a uniform elastic foundation (Beam on Elastic Foundation (BOEF) model).

The rail is modelled as a beam (with bending stiffness EI) which rests on a continuous elastic foundation (Figure 3.1). The elastic foundation represents all track components and is modelled by evenly distributed linear spring stiffness.

The distributed force supporting the beam is then proportional to the beam deflection. Based on this model, the deflection and bending moment of the rail beam due to the moving load can be determined.

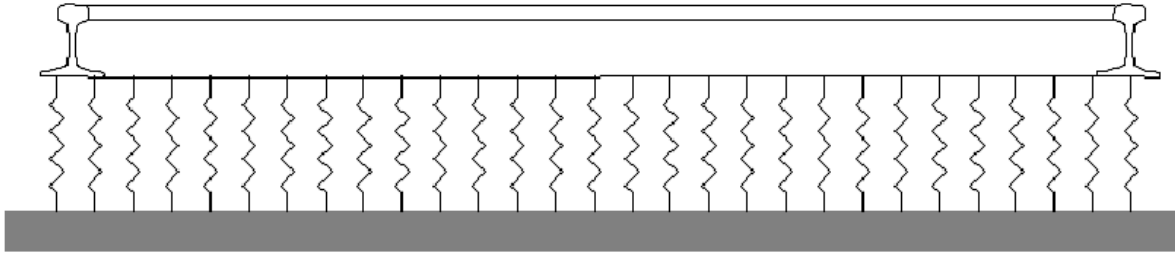


Figure 3.1 Beam on elastic foundation (unloaded)

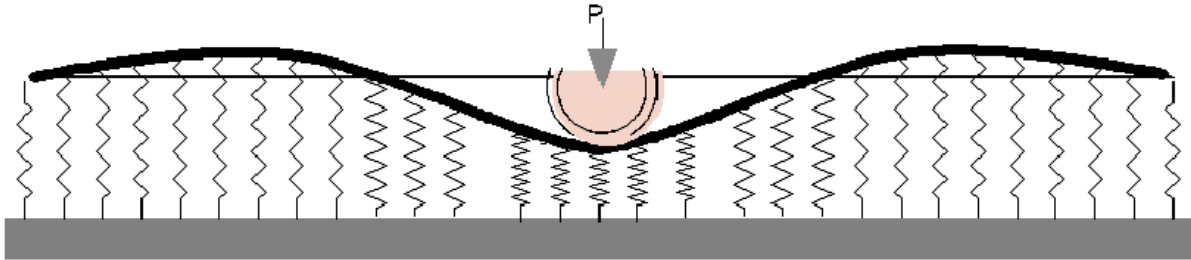


Figure 3.2 Beam on elastic foundation loaded with a point force P from the wheel. The thick line indicates beam (rail) deflection due to the wheel load P.

This is the simplest track model and is still in use for easy and quick track deflection calculations for track design and analysis purposes.

The vertical deflection y , for a given point load P as a function of the distance from the load x , is:

$$y(x) = -\frac{P \cdot \beta}{2u} \cdot e^{-\beta \cdot x} \cdot [\cos(\beta x) + \sin(\beta x)] \quad (2)$$

the bending moment is;

$$M = -\frac{P}{4\beta} \cdot e^{-\beta \cdot x} \cdot [\cos(\beta x) - \sin(\beta x)] \quad (3)$$

Where;

$$\beta = \left(\frac{u}{4EIz} \right)^{1/4} \quad (4)$$

and

- P is the load
- x is the distance along the track
- E is the Young's modulus of elasticity of the rail steel (beam) and
- Iz is the second moment of area of the beam.
- u is the estimate of the track modulus

With this model it is also possible to calculate, for example, the individual sleeper reaction (by multiplying by the sleeper support stiffness and the sleeper deflection), due to the fact that load is proportional to the deflection. The load is shared mostly between three sleepers, with the sleeper beneath the load taking 40-50% of the axle load.

The BOEF method was originally developed for longitudinally sleepereed tracks, and has since been applied to transversely sleepereed track. The BOEF model is generally regarded as the most acceptable method for the analysis of rail foot stress and rail deflections. This model may be used only for static loading of a track on a soft support (i.e. a track with wooden sleepers). Dynamic effects cannot be analysed using this model as it contains no mass.

To simulate this model the function of rail pads, sleepers, and ballast, we have to introduce several continuous layers instead of the single layer.

From the effects of changes in the support (the rail pads), and it was found that a softer rail pad isolates the sleeper more effectively and it will significantly reduce the sleeper strains.

Several deficiencies arise from the BOEF model:

- Sleeper mass and bending flexibility are ignored;
- The shear distortion and rotary inertia of the rail, that are important factors for high frequency vibration, are not accounted by the simple Bernoulli- Euler theory;
- The discrete rail supports from individual sleepers are neglected and replaced by a uniform underlying foundation;
- Impact or impulsive loading associated with high frequency vibration is not considered; and
- Detailed dynamic behaviours of track components are not obtained.

The range and applicability of the one-dimensional theory of beams can be extended by taking account of transverse shear deformations and rotary inertia in the case of vibrating beams. The limits of use the BOEF model are presented in the following table:

Table 1: limits of use BOEF model

Use of the Model	Disadvantages of the Model
Use only for static loading of a track on a soft support (i.e. a track with wooden sleepers)	Dynamic effects cannot be analysed
	The pinned – pinned frequency (when the rail vibrates with nodes at the sleepers) cannot be described

Discretely Supported Models

Modern railway tracks are characterised by accurately positioned continuouswelded rails and sleepers. This makes the mechanical modelling of railway trackrather simple: two straight parallel beams supported by sleepers at equal distances (Fig 3.3)



Figure 3.3 Rail as straight parallel beam supported by sleepers at equal distances

Discretely Supported Models are similar to the continuously supported models but

- They consider the discrete spacing of sleepers.
- they often have multiple layers representing the Rail pads, sleepers, ballast, sub-ballast and subgrade

The most important vibration mode will be like a kind of bending between discrete points or pins, where the rail vibrates with nodes at the sleepers (Figure 3.4). With some simplifying assumptions, this pin-pin vibration resonance occurs at a specific frequency f_{pp} , which can be calculated by: (A.Remennikov, S. Kaewunruen,2008)

$$f_{pp} = \frac{\pi}{2l^2} \sqrt{\frac{EI}{m}} \quad (5)$$

where;

I : distance between two supports [m]

EI: bending stiffness of the rail (static) [Nm²]

m : mass of the rail per unit length [kg/m']

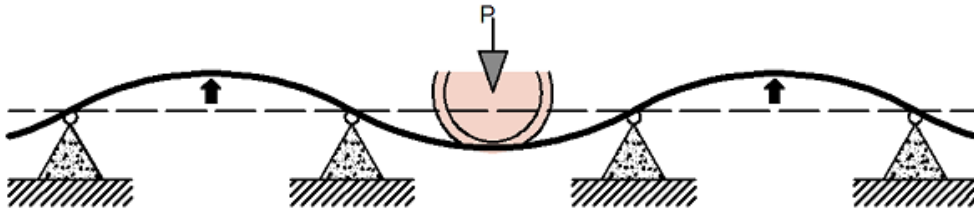


Figure 3.4 Pin Pin Vibration mode

The equation is a general one, based upon an infinitely long beam and perfectly supported at small but rigid points. This assumption doesn't correspond to reality: the rail sits elastically on the sleeper with an elastic rail pad in-between. Also the sleeper seats elastically in the ballast. These and other elements may influence pin-pin resonance. These elements introduce new significant vertical resonances. For most track structures these three vertical resonances are dominant and lie between 30 and 2000 Hz.

Discretely Supported Track Including Ballast and Subgrade

The ballast and subgrade masses influence each other, indicating that a deflection at one point (at one sleeper) will influence the deflection at the neighbouring sleepers.

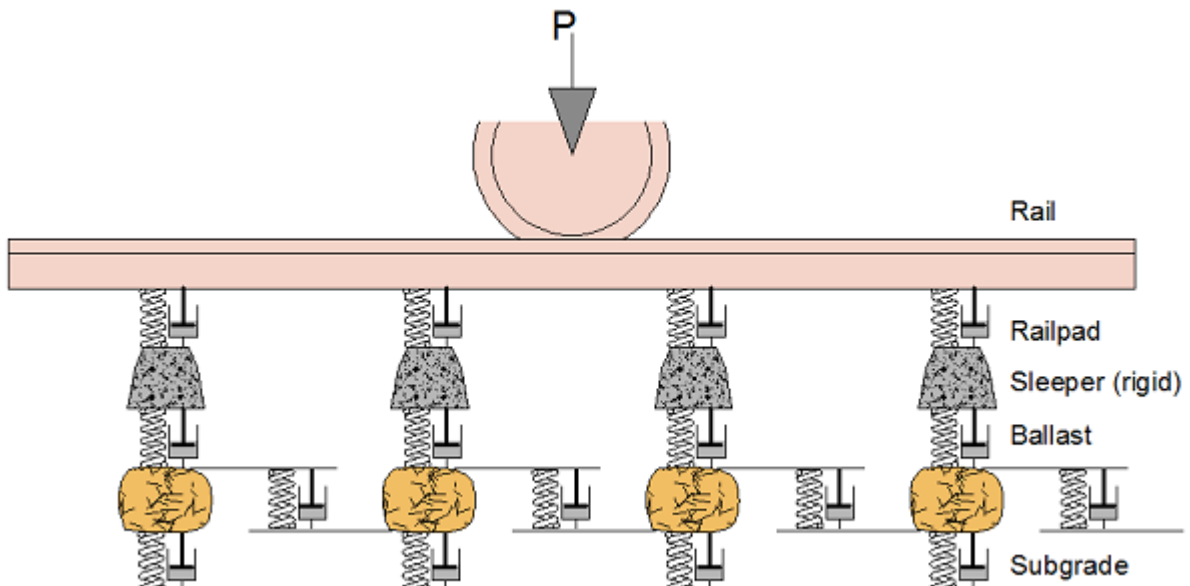


Figure 3.5 Rail on discrete support with rigid masses modelling the sleepers

- Rail is modelled as a beam,
- The rail pads are modelled by spring – damper systems,
- The sleepers are rigid masses,

- The ballast is modelled by spring – damper systems
- The subgrade is modelled by spring – damper systems

By this model, the four resonance vibration modes may be captured:

- Embankment vibration,
- Track-on-the-ballast vibration,
- Rail-on-rail pad vibration, and
- pinned – pinned vibration of the rail (KonstantinosTzanakakis (2013))

Vehicle track model

The modern numerical methods used to simulate the behaviour of train/track system take advantages from many theories: multi-body systems, finite element (FE), discrete element (DE) and analytical methods. The most authors have focused the attention on vehicle behaviour or track behaviour (including rail, rail-pads, sleeper, ballast and ground). Only few of them have treated the coupled system studying the interaction between vehicle and track. The methods used to model vehicle, rail, rail-pads, sleepers, ballast and ground can be summarized here:

- Method adopted to model the vehicle: 2D or 3D multi-body systems;
- Method adopted to model the rail: analytical model (Euler-Bernoulli or Timoshenko), 2D or 3D FEM;
 - Rail-pads: FE models, simple connection (spring/damper couple), analytical models;
 - Sleepers: FE models, rigid elements connected by spring/damper couple, analytical models;
 - Ballast: FE models, DE models, analytical models, rigid elements connected by spring/damper couple;
 - Ground: FE models, analytical models, rigid elements connected by Spring/damper couple.(*Riccardo Ferrara,2013*)

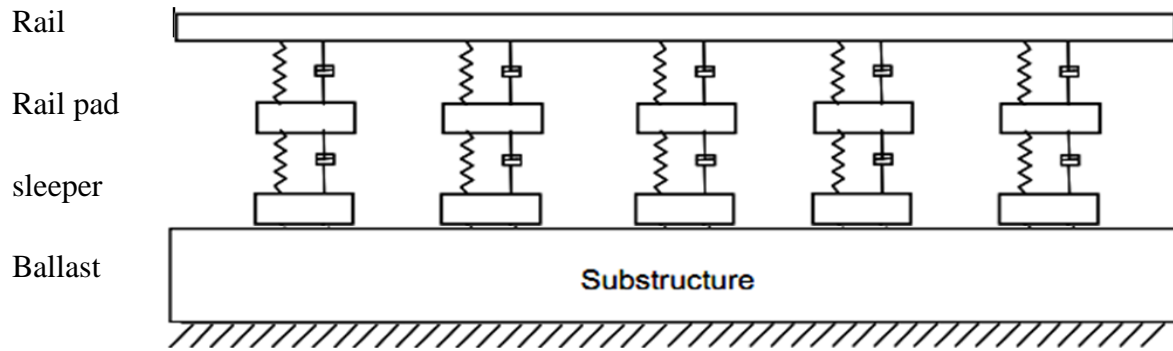


Figure 3.6 Schematic representation of track components

Table 2 Modelling parameter: Laboratory results of drop weight tests on some rail pad types (Esveld, 2001), used for static analysis

Type	Soft	Normal	Hard	EVA
	Fc584	Fc9	Fc846	
Stiffness K [KN/mm]	970	1240	2990	3032
Damping C [KNs/m]	32	34	29	29

Table 3 Common Vertical model Parameters Railway passenger Car Lis (Selected model)
Chinese Manual

Symbol	Name	Unit	Models Ordinary passenger YZ ₂₂
<i>Mc</i>	Body mass	Kg	38,500
<i>Mt</i>	Quality framework	Kg	2,980
<i>Mw</i>	Wheel (unsprung) mass	Kg	1350
<i>Jc</i>	Body nod inertia	Kg.m ²	2.446E6
<i>Jt</i>	Architecture nod inertia	Kg.m ²	3605
<i>Kpz</i>	A Department of County hanging stiffness (per axis)	N/m	2.14E6
<i>Ksz</i>	Two suspension stiffness	N/m	2.535E6

<i>Cpz</i>	A suspension damping (per axis)	N.s/m	4.9e4
<i>Csz</i>	Secondary suspension damping	N.s/m	1.96E5
<i>Lc</i>	Half of the vehicle fixed distance	M	8.4
<i>Lt</i>	Half of the bogie wheelbase	M	1.2
<i>R</i>	Wheel rolling radius	M	0.4575

Table 4 Common Vertical model Parameters Railway passenger Car Lis (Selected model)

Sym.	Parameter	
MC	Car body mass (quarter car)	19400kg
Mt	Bogie mass (half)	500kg
Mw	Wheel mass	500kg
Jt	Bogie mass moment of inertia	176kg-m ²
Ksl	Primary suspension stiffness	788MN/m
Csl	Primary suspension damping	3.5KN-s/m
Ks2	Secondary suspension stiffness	6.11MN/m
Cs2	Secondary suspension damping	158KN-s/m
mr	Rail mass per unit length	60.64kg/m
EI	Rail bending stiffness	6062MN-m ²
Ms	Sleeper mass	118.5kg
Mb	Ballast mass	739kg
Kp	Rail pad stiffness	120MN/m
Kb	Ballast stiffness	182MN/m
Kf	Subgrade stiffness	78.4MN/m
Cp	Rail pad damping	75KN-s/m
Cb	Ballast damping	58.8KN-s/m
Cf	Subgrade damping	31.15KN-s/m
Ls	Sleeper distance	0.6m
N	Number of sleeper	10
	Axle load	25ton
	Young's Modulus Rail	210GN/m ²
	Track stiffness	50MN/m/m

Table 5 Number of Axles and weight per axle of several rolling stock types. (Esveld, 2001)

	Number of Axles	Empty	Loaded
Trams	4	50KN	70KN
Light-Rail	4	80KN	100kN
Passenger Coach	4	100KN	120KN
Passenger motor Coach	4	150KN	170KN
Locomotive	4 or 6	215KN	-----
Freight Wagon	2	120KN	225KN
HeavyHaul(USA Australia)	2	120KN	250-350KN

Parameters used for the Static Analysis

Assumed Case; loading applied by the train at station and terminal

Track type; Ballasted Track,

Sleeper type; pre stressed Concrete

Rail; UIC60

Track length; 1200mm

Axle load selected=250kN

Axle load used=125KN (1/2) load share on one wheel

Load application point

- I. Vertically on the rail at sleeper position
- II. At the centre of sleeper spacing

Obtained result for analysis

- a) Vertical Track Displacement
- b) Longitudinal Stress distribution

3.3. ANSYSModelling for static Analysis

ANSYS is a general-purpose software, used to simulate interactions of all disciplines of Physics, structural, vibration, fluid dynamics, heat transfer and electromagnetic for engineers. ANSYS, which enables to simulate tests or working conditions, enables to test in virtual Environment before manufacturing prototypes of products. Furthermore, determining and improving weak points, computing life and foreseeing probable problems are possible by 3D simulations in virtual environment. ANSYS software offers an unparalleled breadth of

solutions across a broad range of disciplines that can accurately address the fluid, structural, electromagnetic and thermal modelling of any product or process. These solutions are built within the ANSYS Workbench user environment – a single framework enabling you to undertake FEA & CFD simulations quickly and efficiently at both concept and validation stages of design.

ANSYS Multiphysics software offers a comprehensive product solution for both multiphysics and single-physics analysis. The product includes structural, thermal, fluid and both high- and low-frequency electromagnetic analysis. The product also contains solutions for both direct and sequentially coupled physics problems including direct coupled-field elements and the ANSYS multi-field solver.

It is a comprehensive FEA analysis (finite element) tool for structural analysis, including linear, nonlinear and dynamic studies. The engineering simulation product provides a complete set of elements behaviour, material models and equation solvers for a wide range of mechanical design problems. In addition, ANSYS Mechanical offers thermal analysis and coupled-physics capabilities involving acoustic, piezoelectric, thermal–structural and thermo-electric analysis

It also provides a comprehensive set of post-processing tools to display results on models as contours or vector plots to provide summaries of the results (like min/max values and locations). Powerful and intuitive slicing techniques allow the user to get more detailed results over given parts of geometries. All results can be exported as text data or to a spreadsheet for further calculations. Animations are provided for static cases as well as for nonlinear or transient histories. Any result or boundary condition can be used to create customized charts.

3.4. Element Formulation used

Model component	Element type
Rail	Plane 42
Rail pad	COMBIN14
Sleeper	PLANE182
Ballast	COMBIN14

RAIL is modelled as plane 42

PLANE42 is used for 2-D modelling of solid structures. The element can be used either as a plane element (plane stress or plane strain) or as an axisymmetric element. The element is

defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities.

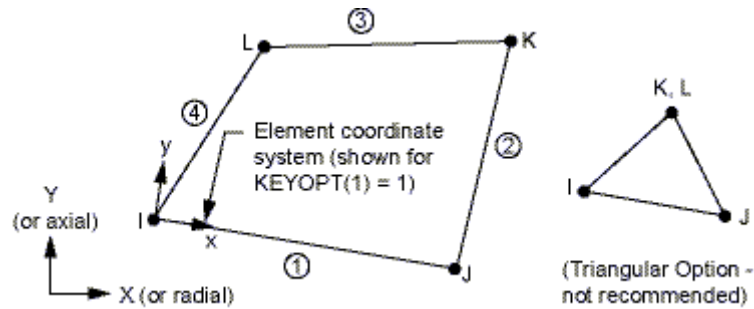


Figure 3.7 PLANE42 Geometry

RAIL PAD is modelled as COMBIN14 spring and Dash pot

COMBIN14 has longitudinal or torsional capability in one, two, or three-dimensional applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node: translations in the nodal x, y, and z directions. No bending or torsion is considered. The torsional spring-damper option is a purely rotational element with three degrees of freedom at each node: rotations about the nodal x, y, and z axes. No bending or axial loads are considered.

The spring-damper element has no mass. Masses can be added by using the appropriate mass element the spring or the damping capability may be removed from the element.

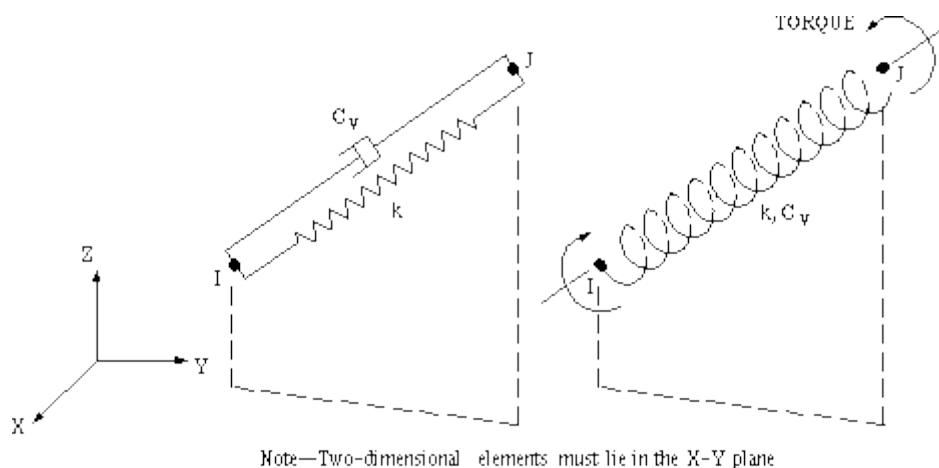


Figure 3.8 COMBIN14 Spring and Damper

SLEEPER is modelled as PLANE182

PLANE182 is used for 2-D modelling of solid structures. The element can be used as either a plane element (plane stress, plane strain or generalized plane strain) or an axisymmetric element. It is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, hyper elasticity, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elasto-plastic materials, and fully incompressible hyper elastic materials. (Ansys Manual, 2009)

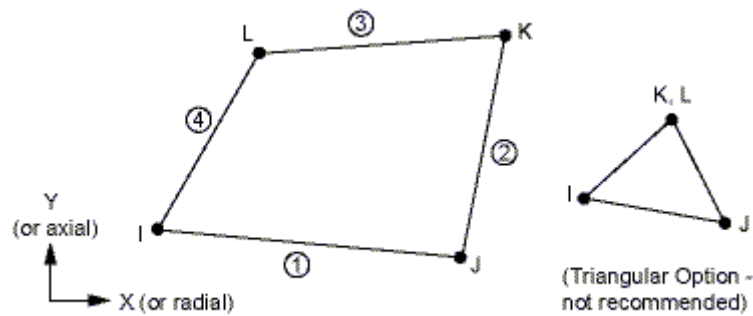


Figure 3.9 PLANE182 Geometry

BALLAST is modelled as Spring and Dashpot by COMBIN14

Model

Model type,2D because it answers the objective correctly, and 3D models require large computer resource and time.

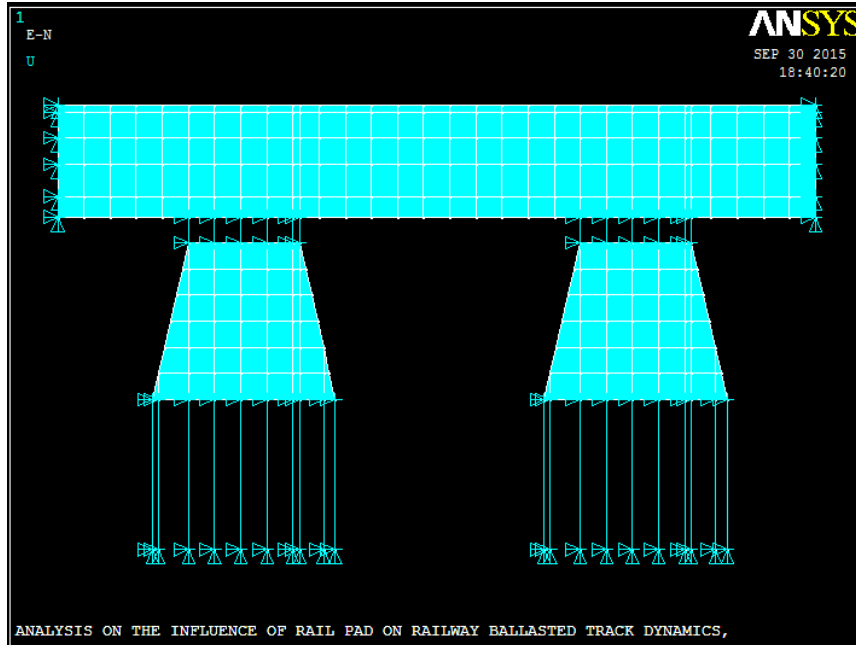


Figure 3.10 ANSYS APDL Model of the Track

Length of Track taken is 1200mm because the aim is to analyse the effect within the span of two sleepers, and half of the span is added on both sides to show that it is continuous.

3.5. ABAQUS Modelling for Dynamic Analysis

a) Modelling Procedures in ABAQUS/CAE

The ABAQUS/CAE environment is divided into different modules, where each module defines a logical aspect of the modelling process; for instance, defining the geometry, defining the material properties, and generating a mesh. The GUI interface generates an input file with all information of the model, to be submitted to the solver, using ABAQUS/Standard or ABAQUS/Explicit routines. The solver performs the analysis and sends the information back to ABAQUS/CAE for evaluation of the results. (Abaqus online documentation, 2010)

b) Models

Most models created in ABAQUS/CAE are assembled from different parts. It always starts with creating different parts separately in the parts module. Different parts may need different material properties, which are defined in the property module. A full range of material properties are available in ABAQUS, such as elastic and plastic behaviour, as well as thermal and acoustic behaviour. The model then is assembled in the assembly model, by combining the different instances originating from different parts. In the step module the analysis is divided

in different analysis step, such as static and dynamic analyses. These can be combined in a way to resemble the physical problem that is to be analysed. The instances in the model will not interact with each other until they have been connected in the interaction module. Connector elements can be defined, to simulate for example spring or dashpot behaviour. The loads acting on the model are defined in the load module, as well as boundary conditions. The load and boundary conditions can be defined to vary over time as well as over different steps. The whole model is then meshed in the mesh module. The meshing techniques vary with the element type and the geometry of the model. (Huang, 2011)

c) Parameters used for the Dynamic Analysis

Assumed Case; Moving train wheel load

Track type; Ballasted Track,

Sleeper type; pre stressed Concrete

Rail; UIC60

Track length; 24000mm

Axle load selected=250kN

Axle load used=125KN (1/2) load share on one wheel

Velocity; 80-120Kmh⁻¹

Rail pad parameter; k=50-300MN/m, C=20-80KNs/mSilviu et al (2010)

Type of Load application; Moving train wheel load

Obtained result for analysis

- a. Longitudinal Stress
- b. Vertical Displacement
- c. Vertical acceleration

Variable parameters applied are;

- Speed of train
- Rail pad stiffness



Figure 3.11 2D Model of the Track by ABAQUS

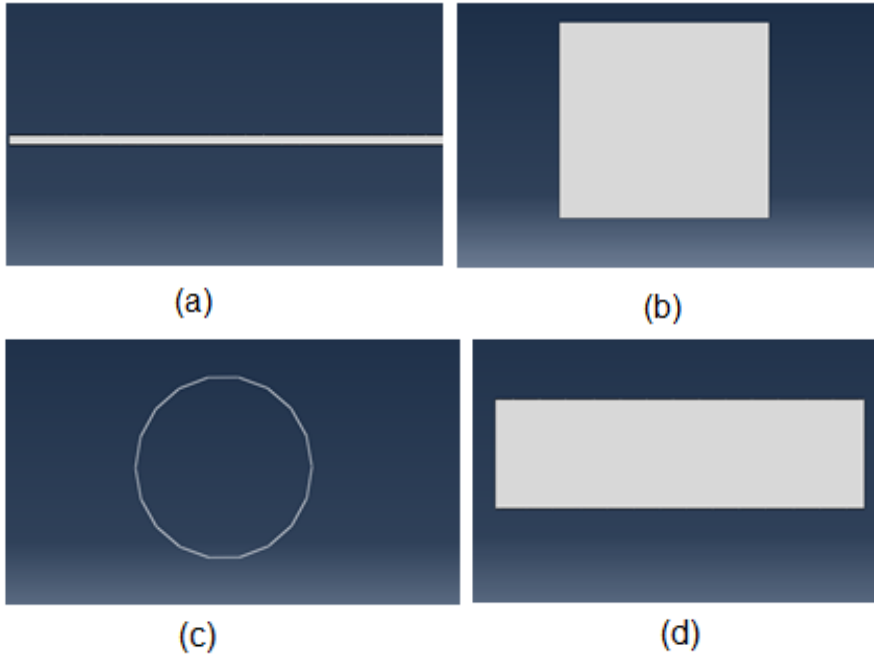


Figure 3.12Part Model a) Rail b) Sleeper c) wheel d) Subgrade

4. Result Analysis and discussion

4.1. Static

The static Load is applied to a track at a section between two sleepers so that the static load simulates a moment at which a train stops at station, and Terminal.

Deformed shape, Stress, and displacement due to the load is presented, the comparison is made due to variation of rail pad type (soft, Hard, Normal, EVA) and respective stiffness and damping value

I. When the load is applied on the head of sleeper

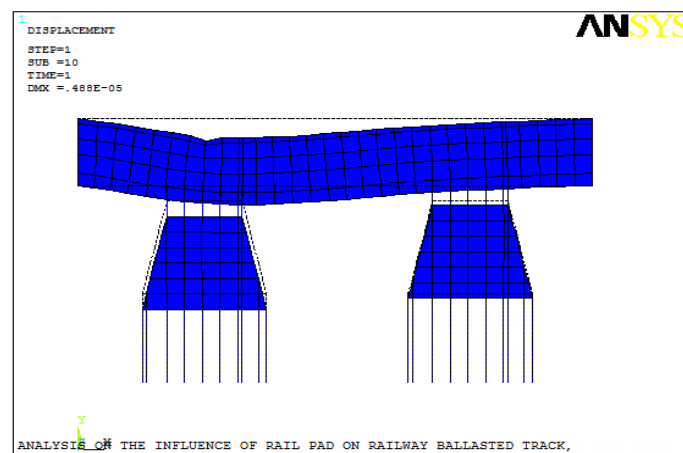


Figure 4.1 Deformation when load is applied at the position of sleeper on rail and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

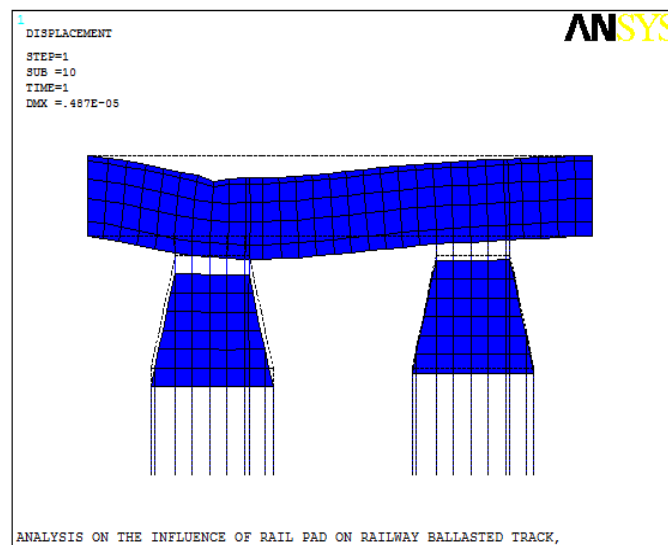


Figure 4.2 Deformation when load is applied at the position of sleeper on rail and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

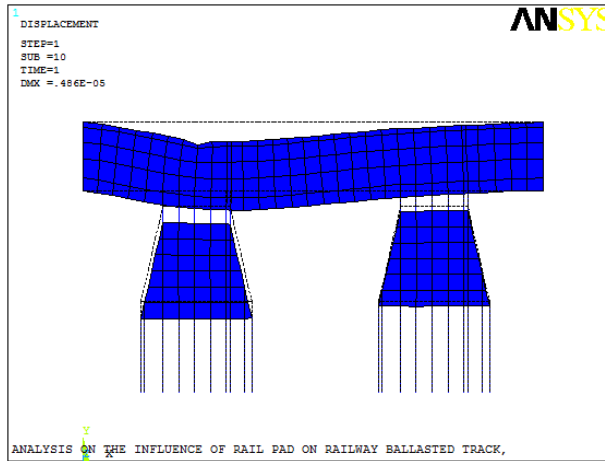


Figure 4.3 Deformation when load is applied at the position of sleeper on rail and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

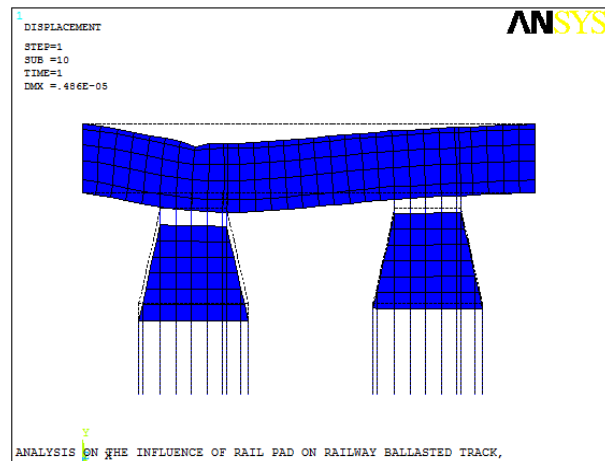


Figure 4.4 Deformation when load is applied at the position of sleeper on rail and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

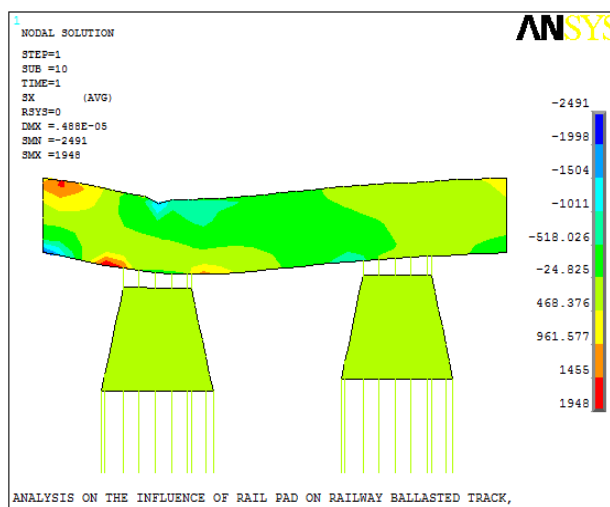


Figure 4.5 Stress when load is applied at the position of sleeper on rail and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

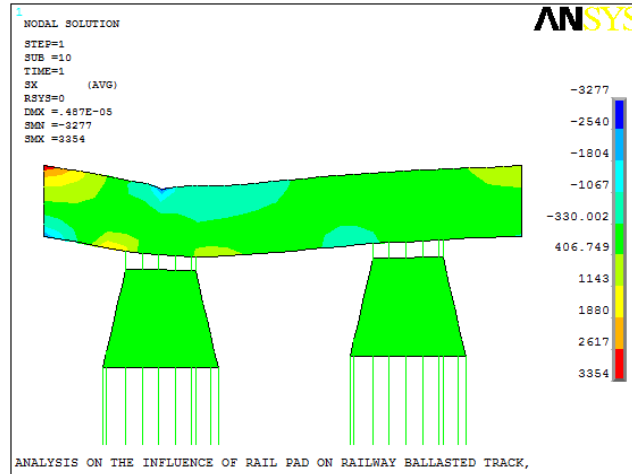


Figure 4.6 stress when load is applied at the position of sleeper on rail and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

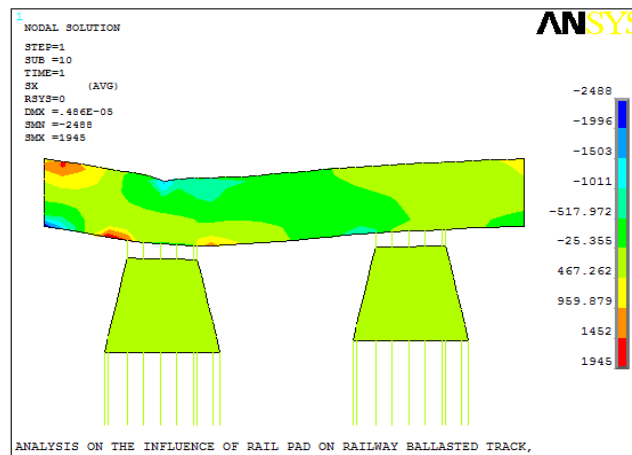


Figure 4.7 Stress when load is applied at the position of sleeper on rail and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

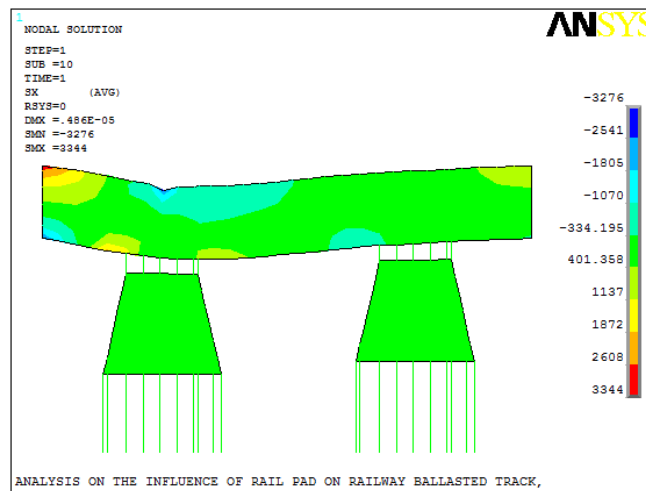


Figure 4.8 Stress when load is applied at the position of sleeper on rail and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

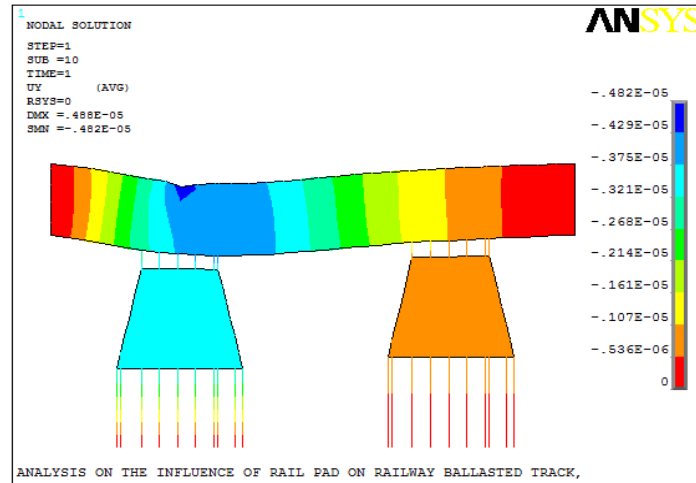


Figure 4.9 Y-displacement when load is applied at the position of sleeper on rail and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

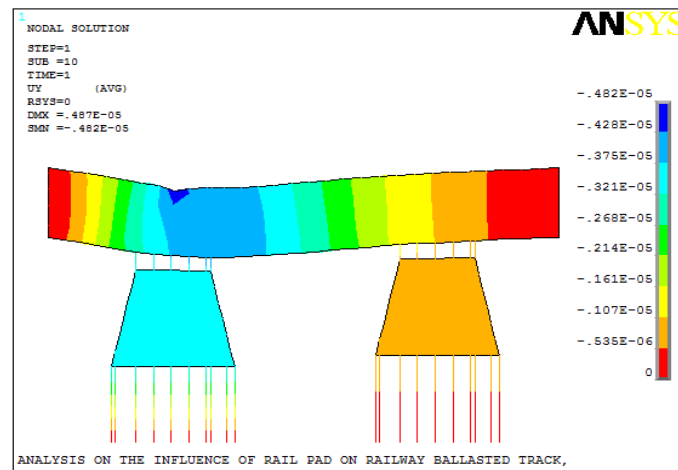


Figure 4.10 Y-Displacement when load is applied at the position of sleeper on rail and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

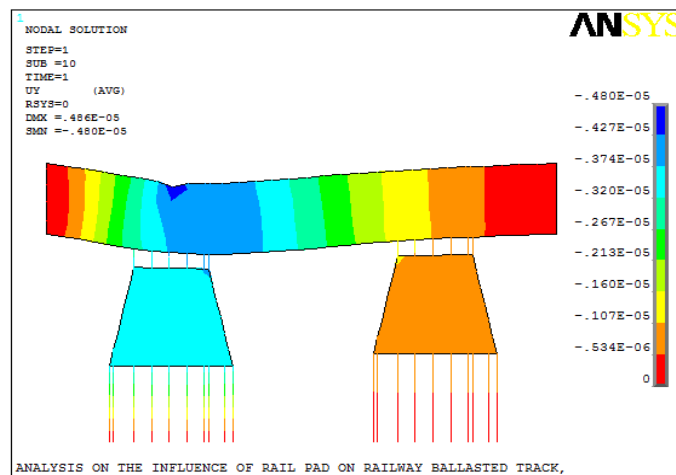


Figure 4.11 Y- Displacement when load is applied at the position of sleeper on rail and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

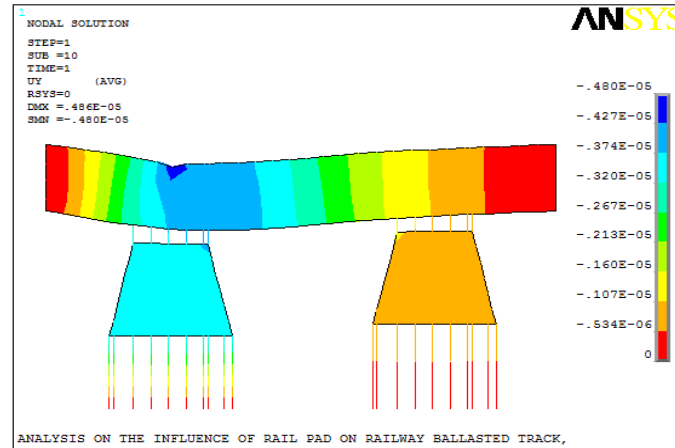


Figure 4.12 Stress when load is applied at the position of sleeper on rail and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

As the result is displayed in the above contour graphs, when the wheel load is applied at edge, and at the centre, the deformation, x-stress and Y-displacement, that comes as a result of rail pad parameter variation shows the effect of rail pad.

The effect share goes from rail to Ballast and to the Vehicle, as well as to Vehicle component. Even though, the main use of Rail pad is to filter the dynamic load coming from train, it has great influence in absorbing the coming static load from train too.

The duration and Magnitude of the load is variable in static and Dynamic case, in dynamic loading the duration is very short while the magnitude is large. But in static loading the time is longer and magnitude is lesser. Static load result greater effect, especially when the train is at its terminal and loaded fully, especially for freight train.

The percentage increase of stiffness (k) and damping (c)

Type	Soft	Normal	Hard	EVA
	Fc584	Fc9	Fc846	
Stiffness K [KN/mm]	970	1240	2990	3032
Damping C [KNs/m]	32	34	29	29

Percentage difference of Stiffness from Soft to Normal, 27.8%, increased

“ “ “ Normal to Hard, 141.1%, increased

“ “ “ Hard to EVA, 1.4%, increased

“ “ Damping from Soft to Normal 6.25%, increased

“	“	“	from Normal to Hard	17.2%, decreased
“	“	“	from Hard to EVA,	No change

For a loading case 1

As the stiffness of the rail pad increase from 970 to 1240MN/m while the damping value increase from 32 to 34kNs/m the deformation gets reduced from 0.488e-5 to 0.487e-5, the stress increases 2491 to 3277 and 1948 to 3354, while the y-displacement doesn't show significant change (i.e. 0.482e-5). When the stiffness value increase from 2990 to 3032MN/m while the damping value remains constant 29kNs/m, the deformation doesn't show significant difference i.e. the stress increase from 2488 to 3276 and the y-deflection and deformation also, doesn't show significant difference 0.48e-5.

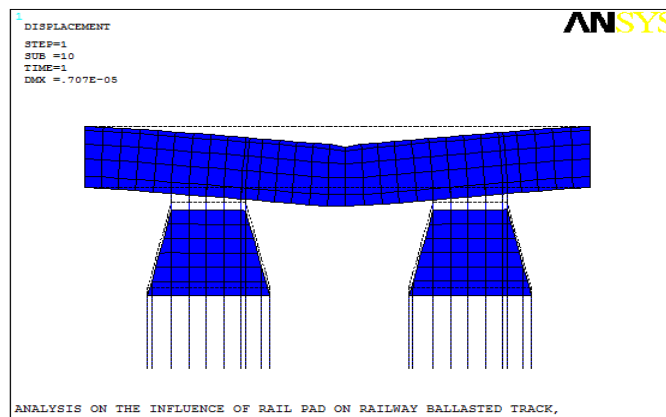


Figure 4.13 Deformation when load is at the centre and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

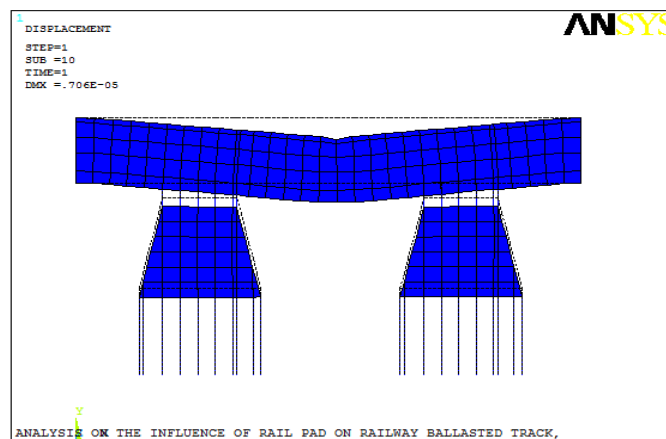


Figure 4.14 Deformation when load is at the centre and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

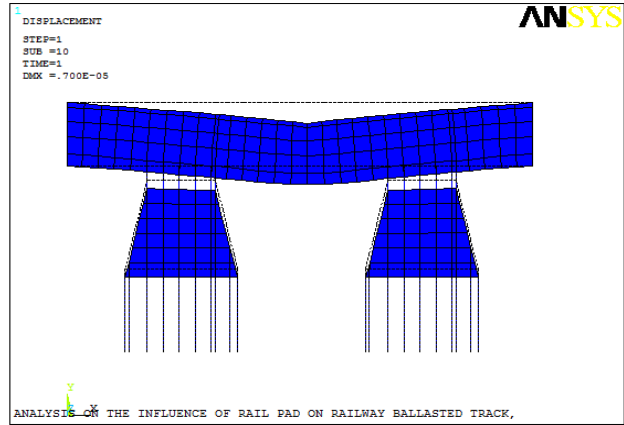


Figure 4.15 Deformation when load is at the centre and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

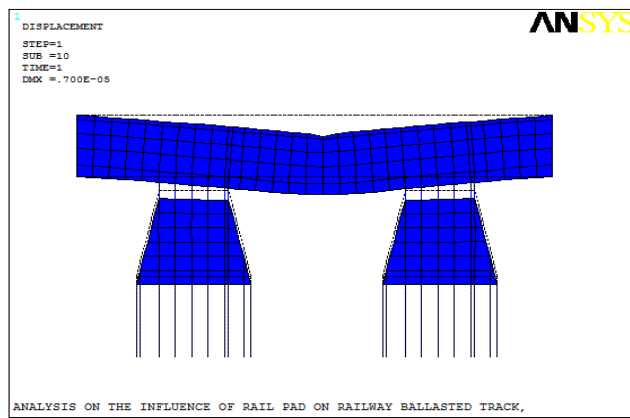


Figure 4.16 Deformation when load is at the centre and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

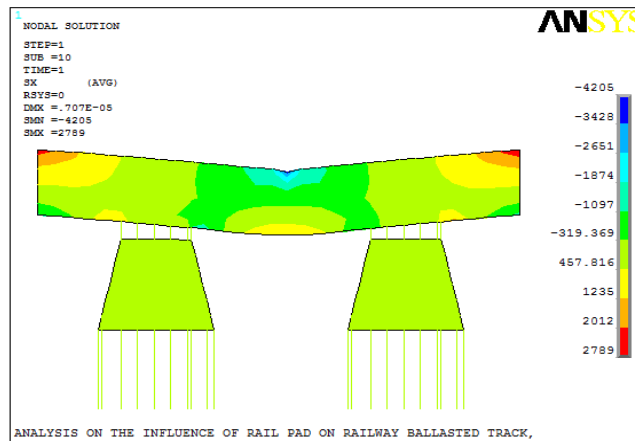


Figure 4.17 Stress when load is applied at the centre and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

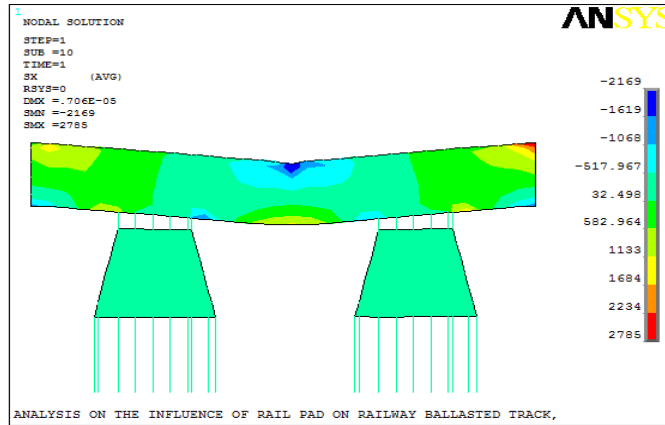


Figure 4.18 Stress when load is applied at the centre and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

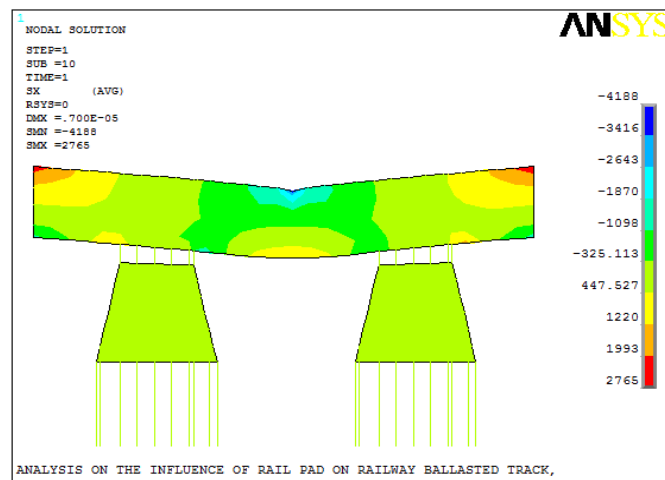


Figure 4.19 Stress when load is applied at the centre and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

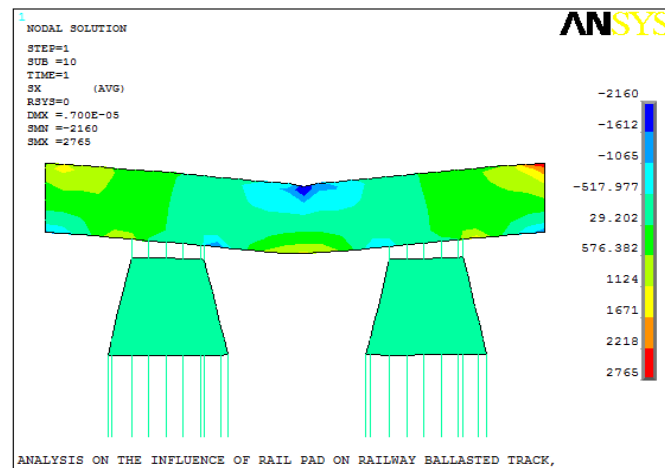


Figure 4.20 Stress when load is applied at the centre and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

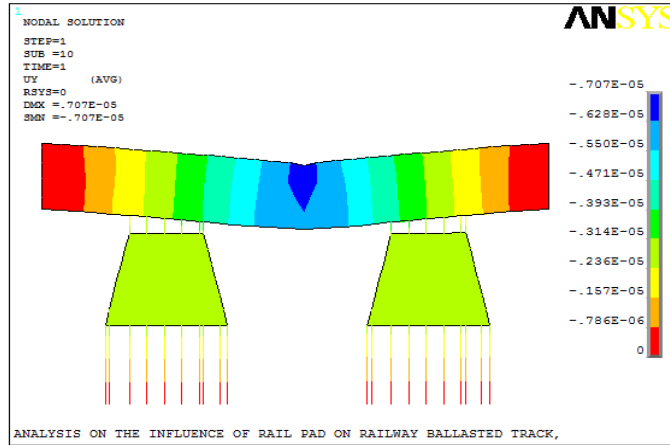


Figure 4.21 Y-displacement when the load is applied at the centre and $k=970\text{kN/mm}$ and $C=32\text{KNs/m}$

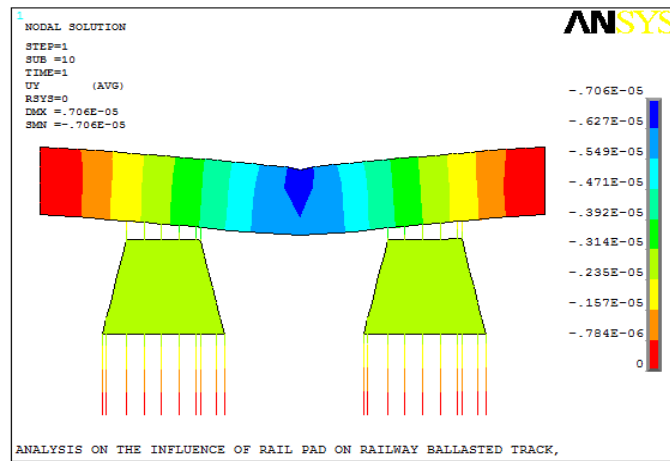


Figure 4.22 Y-displacement when load is applied at the centre and $k=1240\text{kN/mm}$ and $C=34\text{KNs/m}$

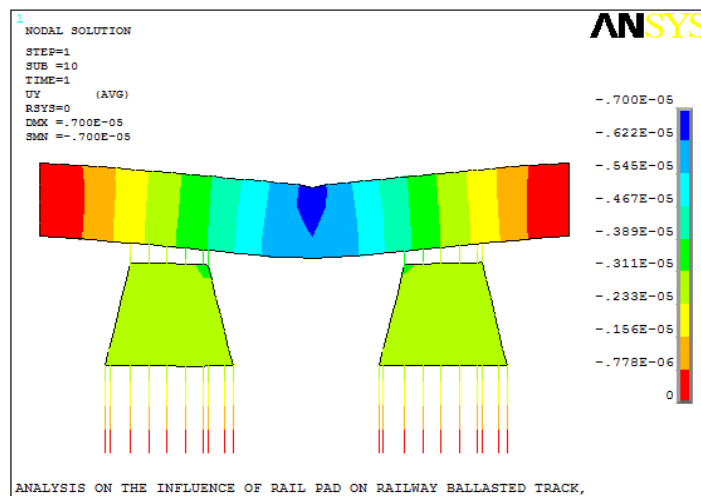


Fig.4.1 Y-displacement when load is applied at the centre and $k=2990\text{kN/mm}$ and $C=29\text{KNs/m}$

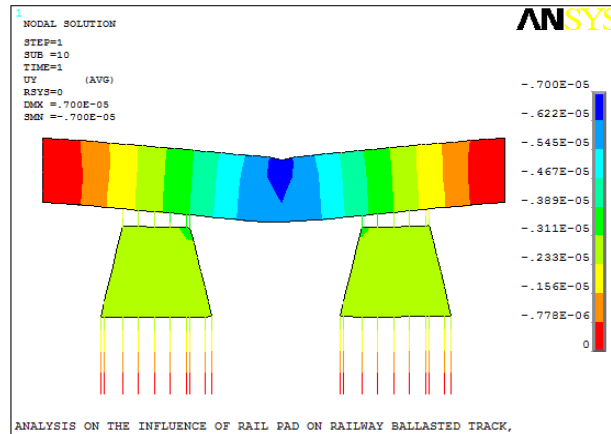


Figure 4.23 Stress when load is applied at the centre and $k=3032\text{kN/mm}$ and $C=29\text{KNs/m}$

II. When the load is applied at the centre

For loading case 2

As the stiffness of the rail pad increase from 970 to 1240MN/m while the damping value decrease from 34 to 32kNs/m the deformation gets reduced from $0.707\text{e-}5$ to $0.706\text{e-}5$, the stress decreases 4205 to 2169 and 2789 to 2785, while the y-displacement remains the same (i.e. $0.707\text{e-}5$). When the stiffness value increase from 2990 to 3032MN/m while the damping value remains constant 29kNs/m, the deformation remains the same i.e. the stress decrease from 4188 to 2160 and the y-deflection remains the same $0.7\text{e-}5$.

From the result of above, soft rail pad ($k=970\text{MN/m}$, $c=32\text{KNS/m}$) result higher displacement from other rail pad type and in both loading case, while; Hard rail pad ($K=2990\text{MN/m}$, $C=29\text{NS/m}$) result lesser displacement.

4.2. Dynamic

Stress, Vertical acceleration and displacement for $K=50,150,250\text{MN/m}$ and $C=20,40,60\text{KNS/m}$

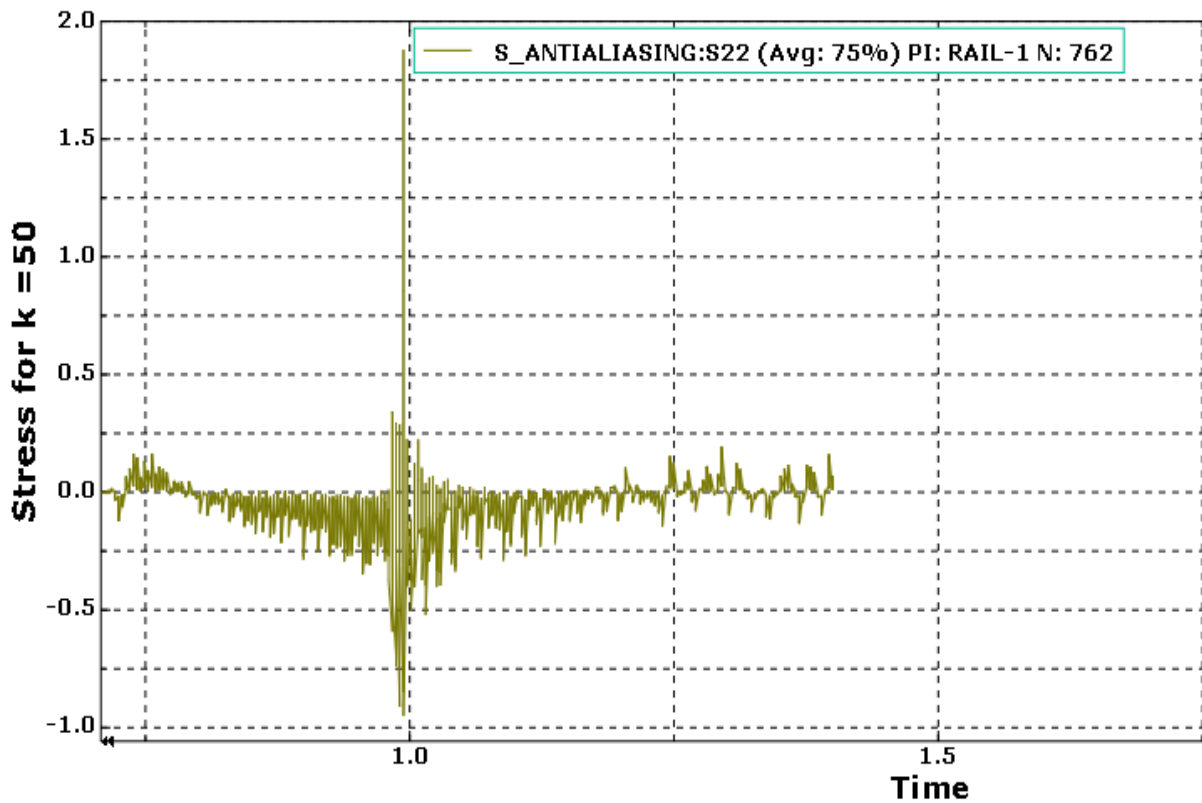


Figure 4.24 S22 stress for $k=50$

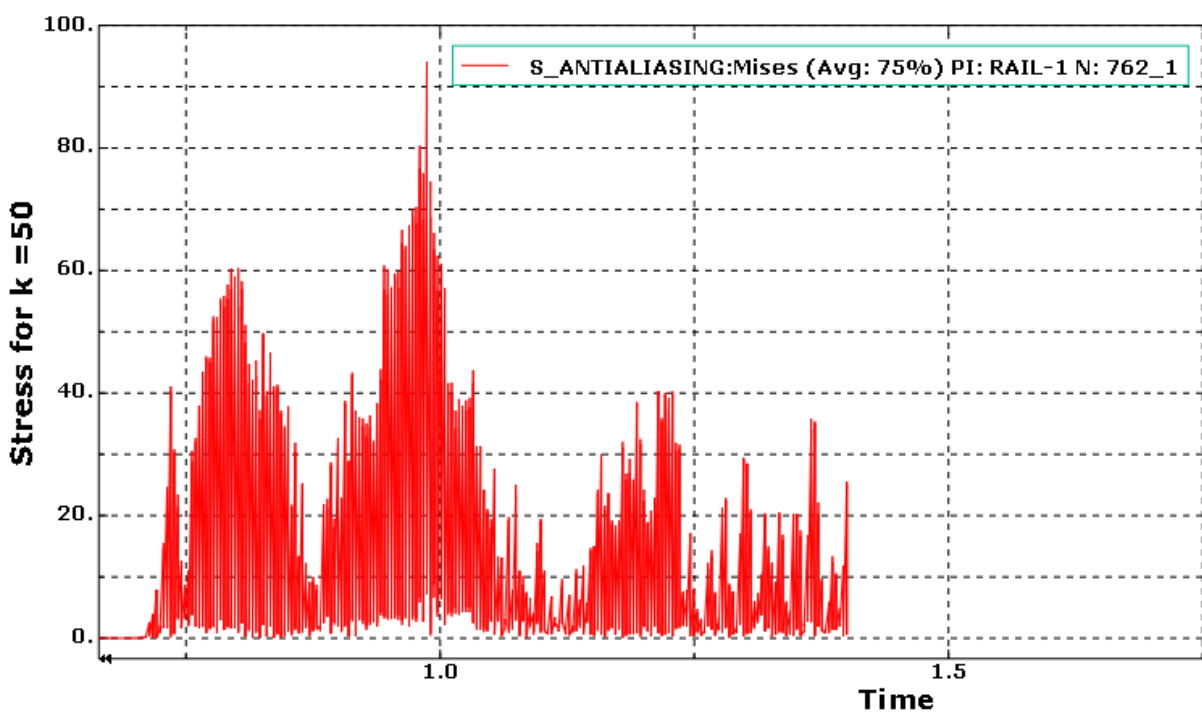


Figure 4.25 Mises stress for $k=50$

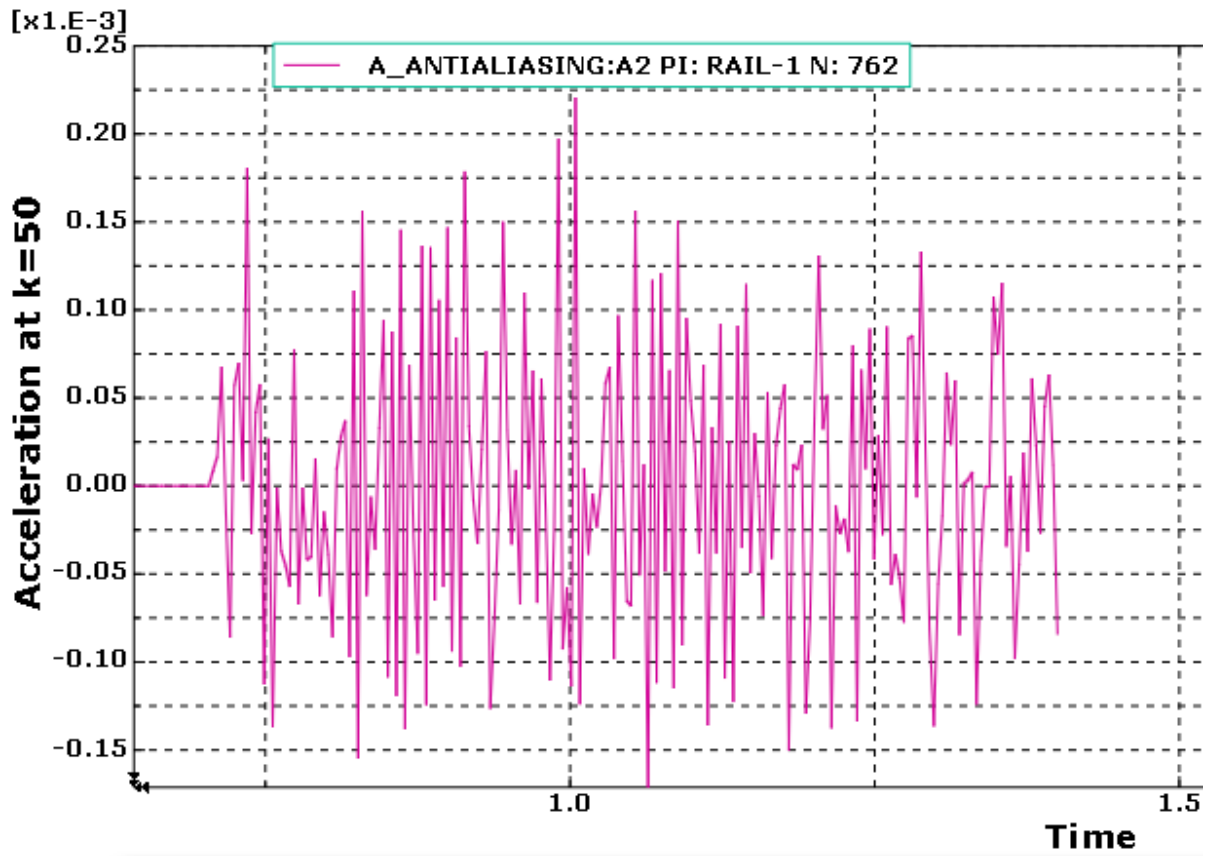


Figure 4.26 Acceleration at k=50

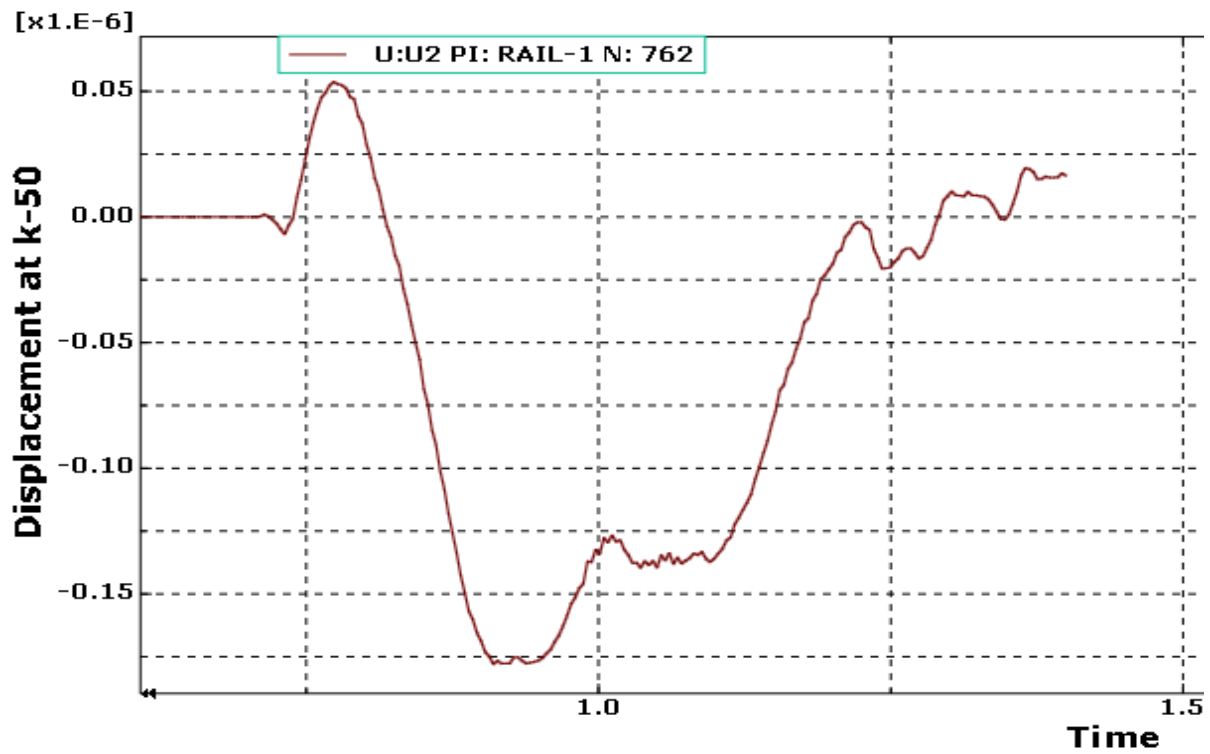


Figure 4.27 Displacement at k=50

For K=150

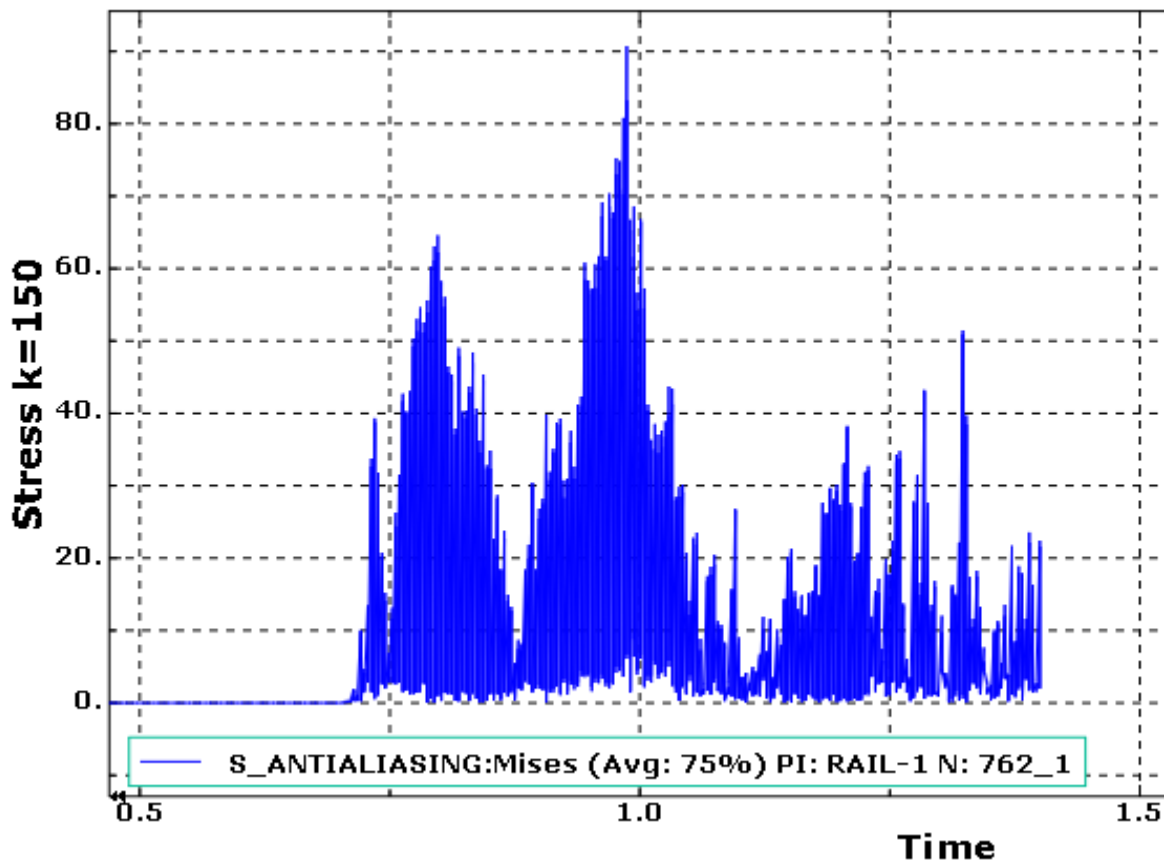


Figure 4.28 Mises stress at k=150

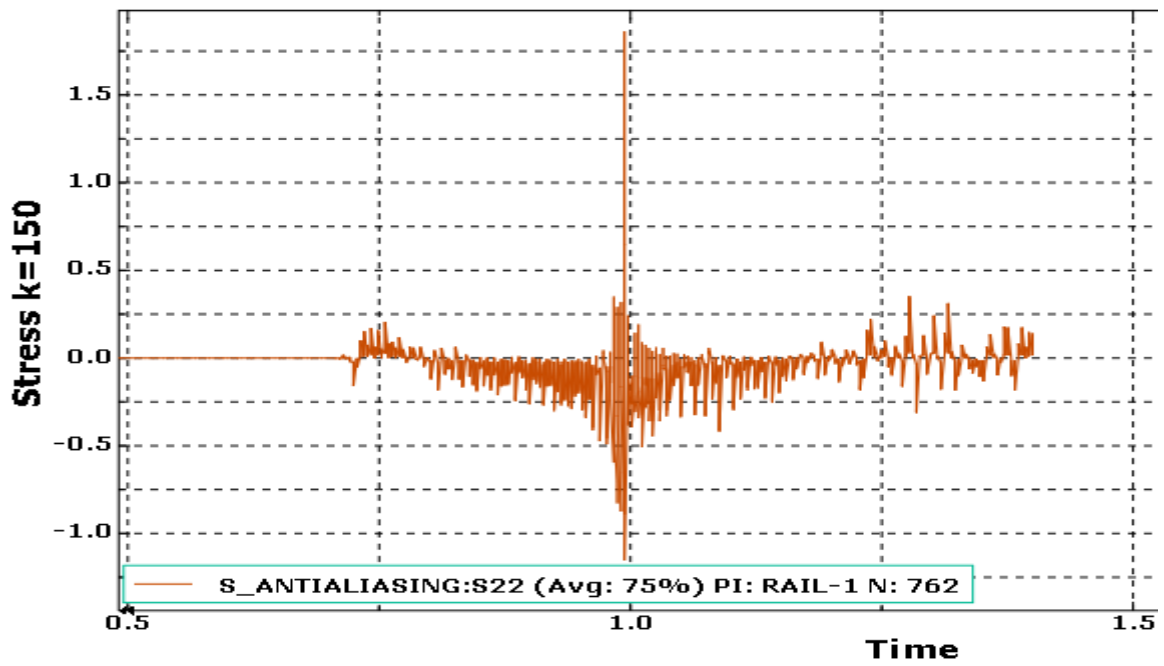


Figure 4.29 S22 Stress at k=150

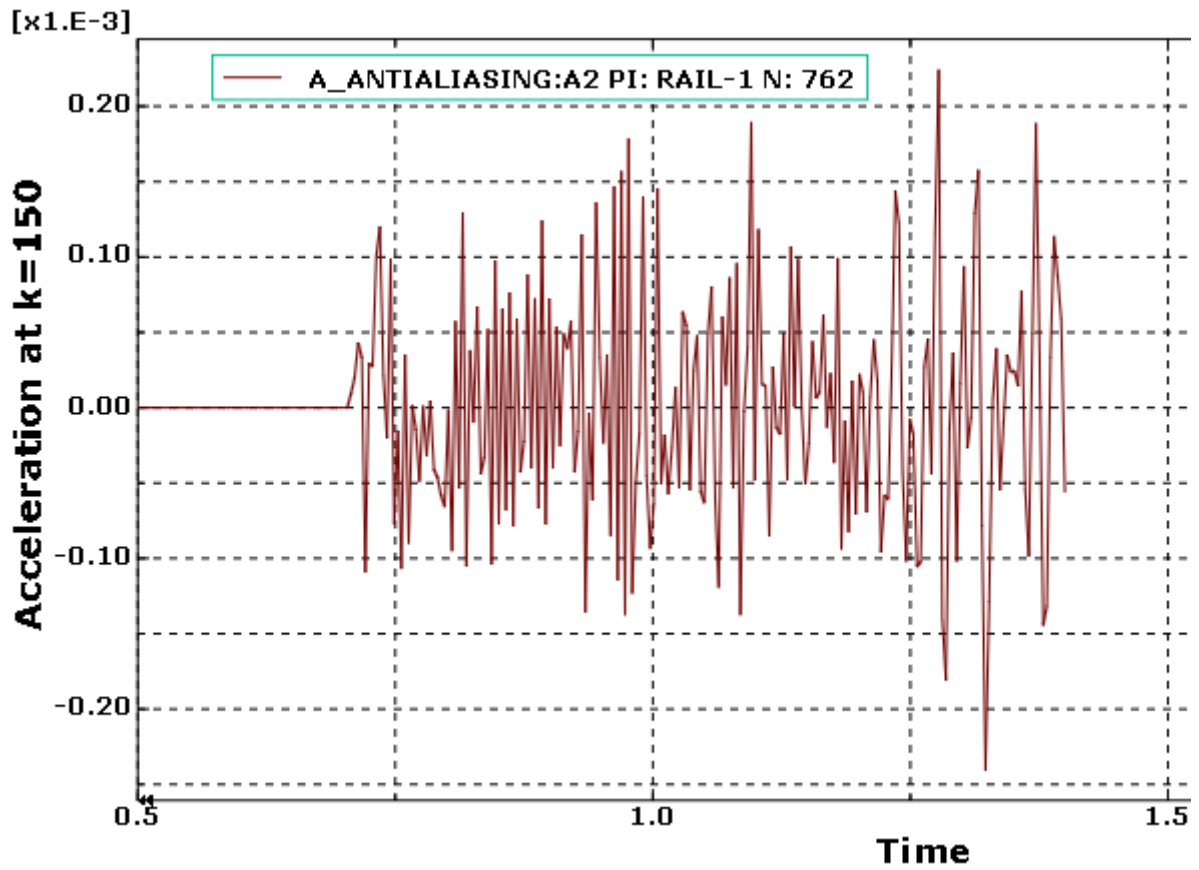


Figure 4.30 Acceleration at k=150

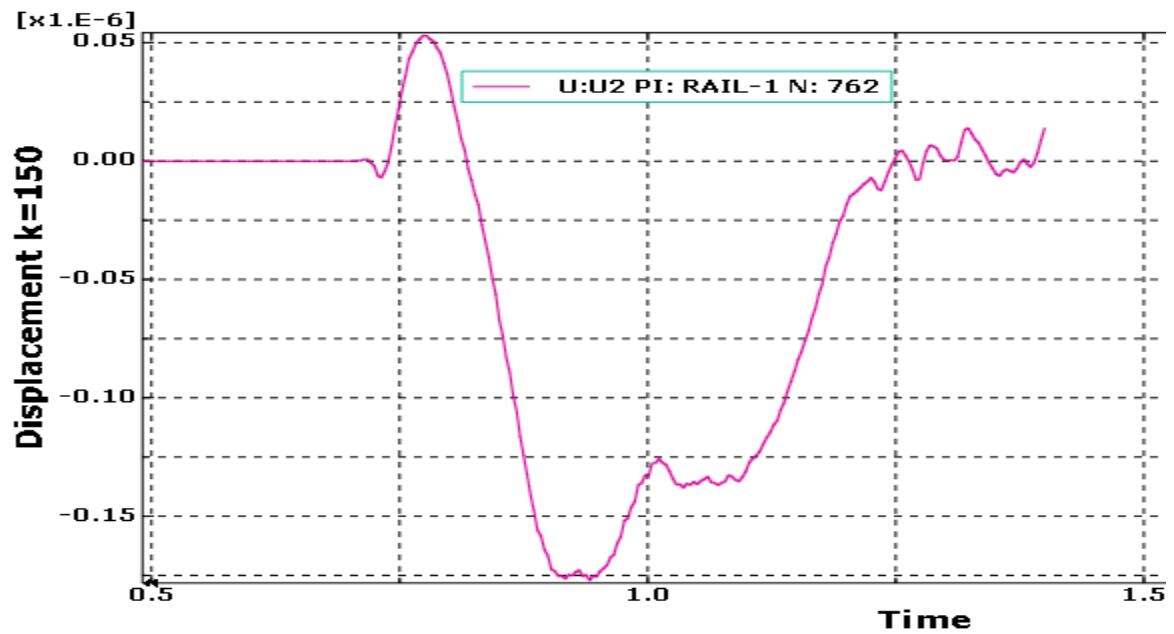


Figure 4.31 Displacement at k=150

For 250

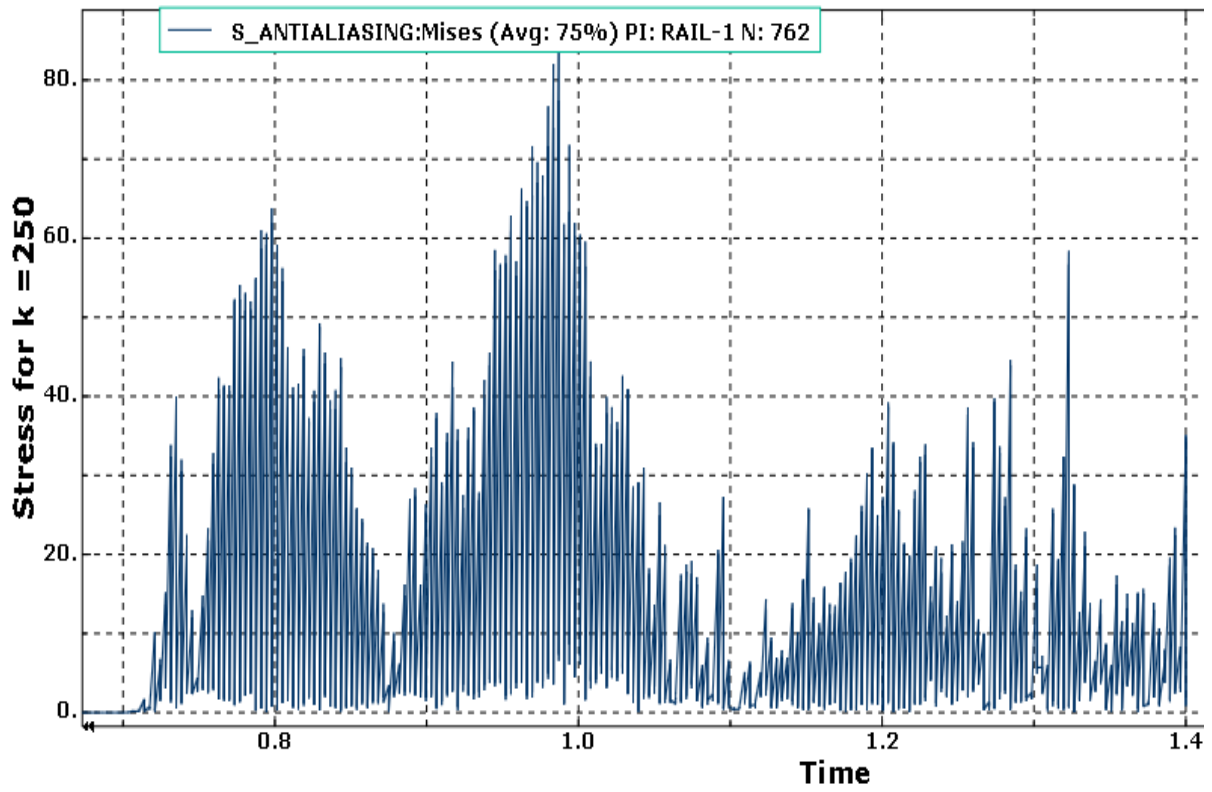


Figure 4.32mises Stress at K=250

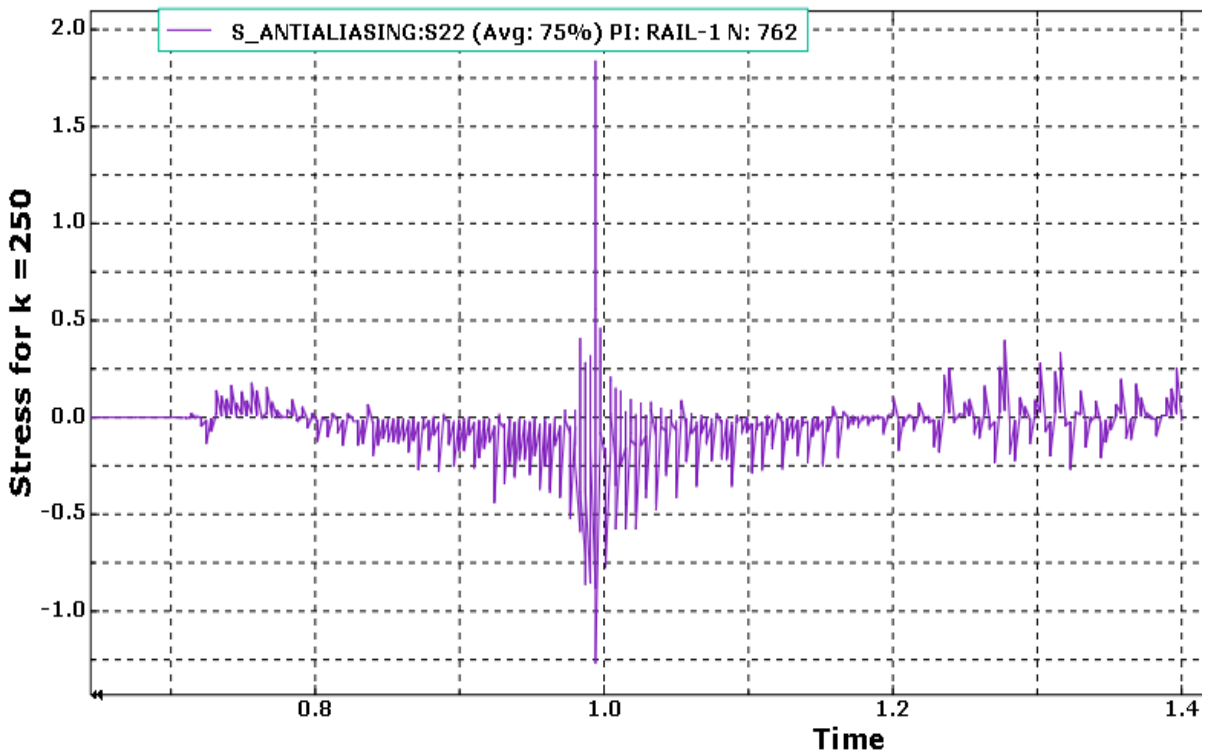


Figure 4.33 S22 Stress at k=250

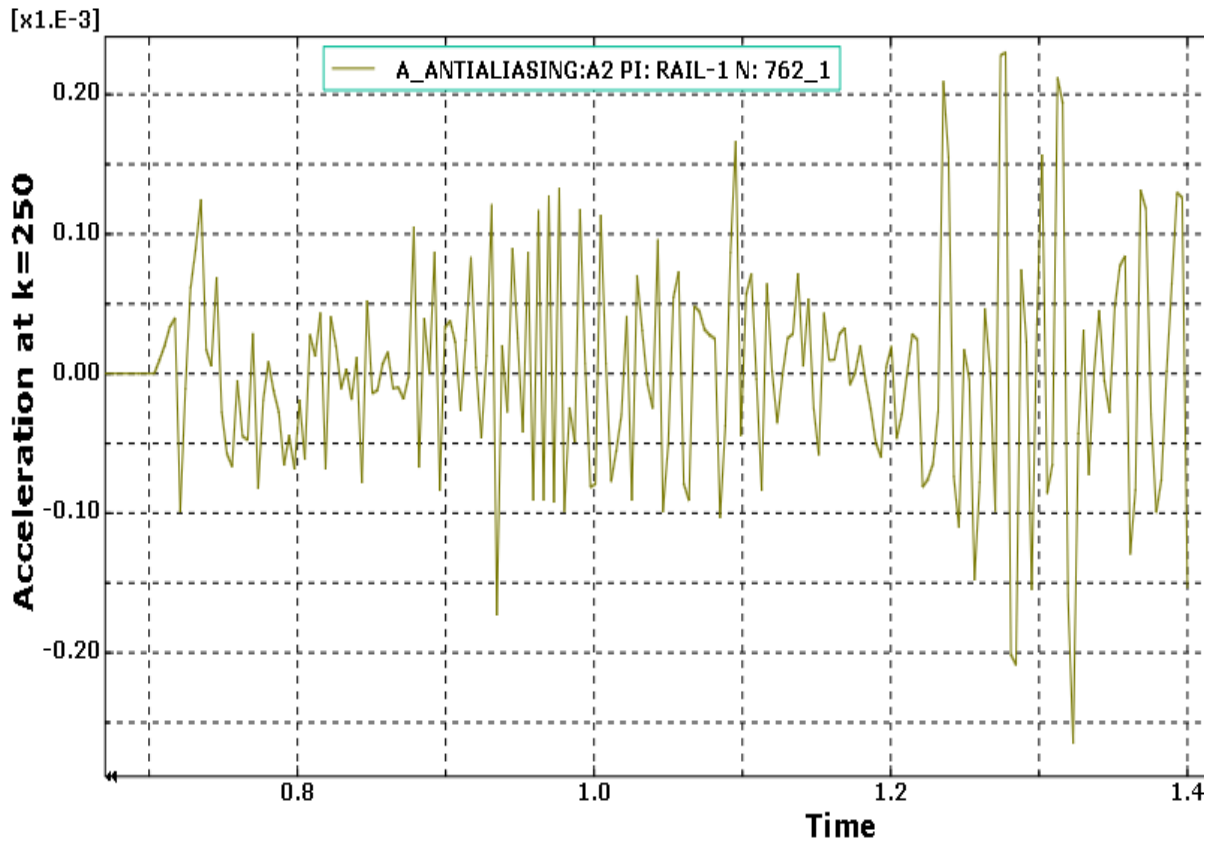


Figure 4.34 Acceleration at k=250

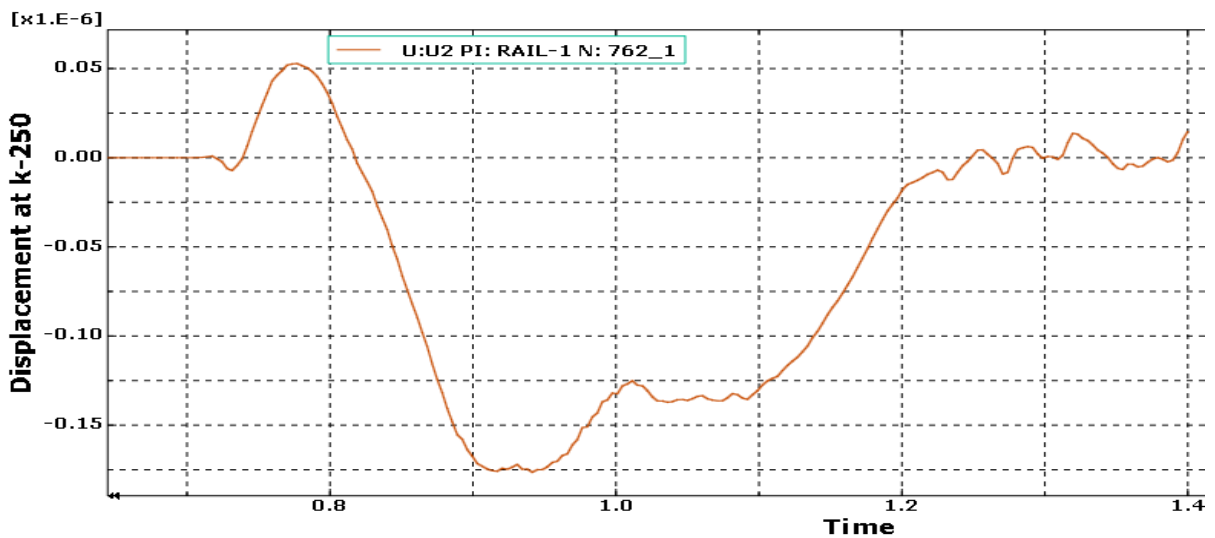


Figure 4.35 Displacement at k=250

The stiffness and damping constant used for the dynamic analysis input range; $k=50-300\text{MN/m}$, $C=20-80\text{KNs/m}$, both taken in an increasing order for the analysis

AS the stiffness and damping increase from 50-250, 20-60, then;

- Stress gets reduced
- Displacement doesn't show perceptible difference
- Acceleration, gets reduced (zakeri,xia;2008)

For V=22.22m/s

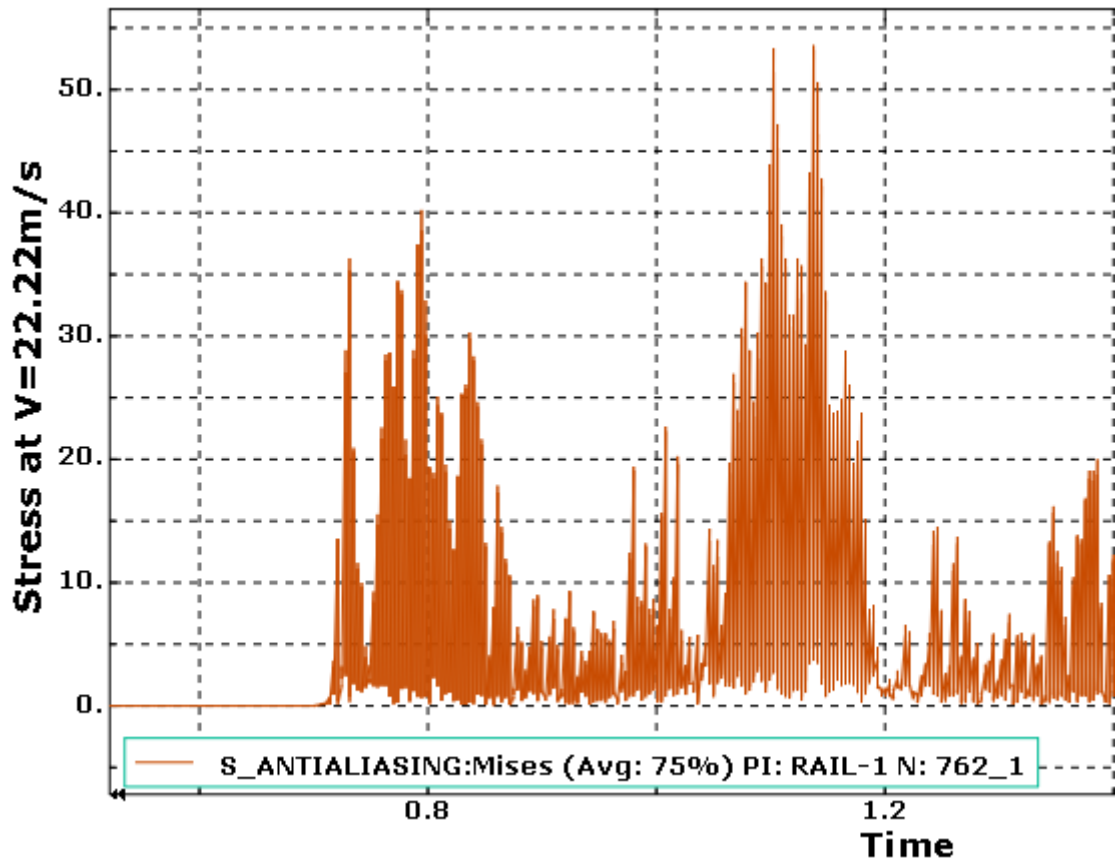


Figure 4.36 Stress at V=22.22m/s

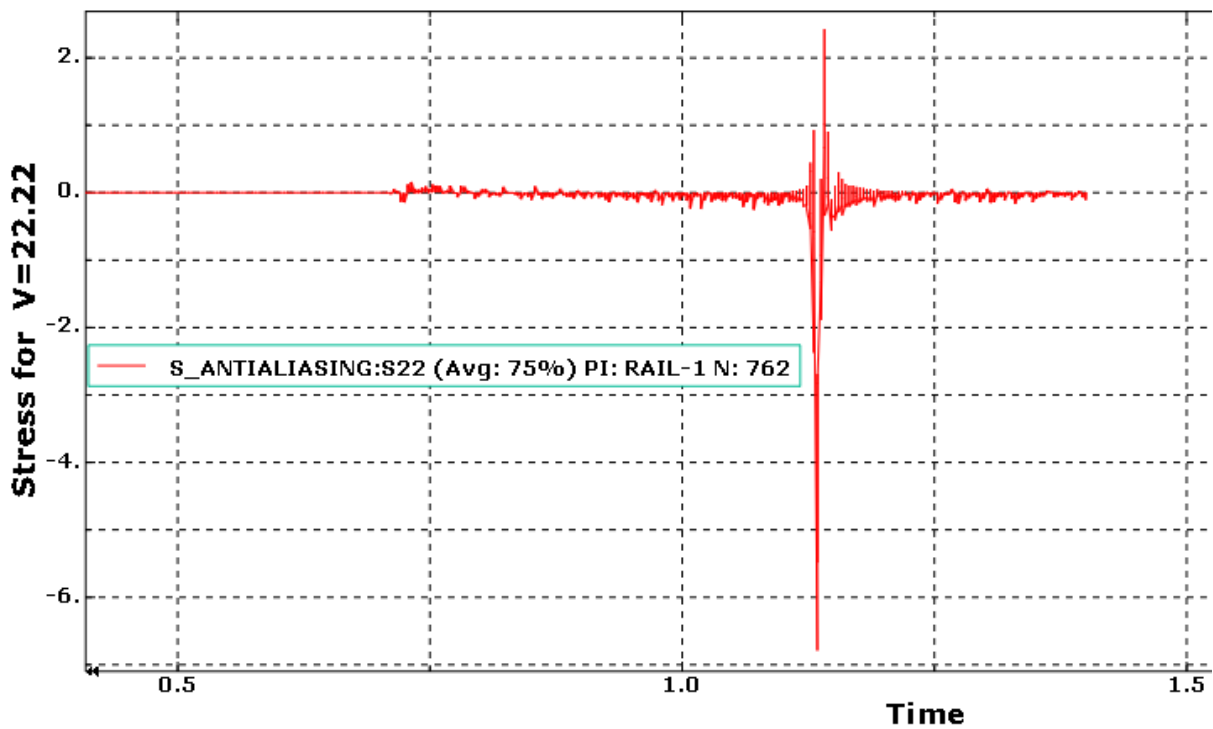


Figure 4.37 S22 stress at V=22.22m/s

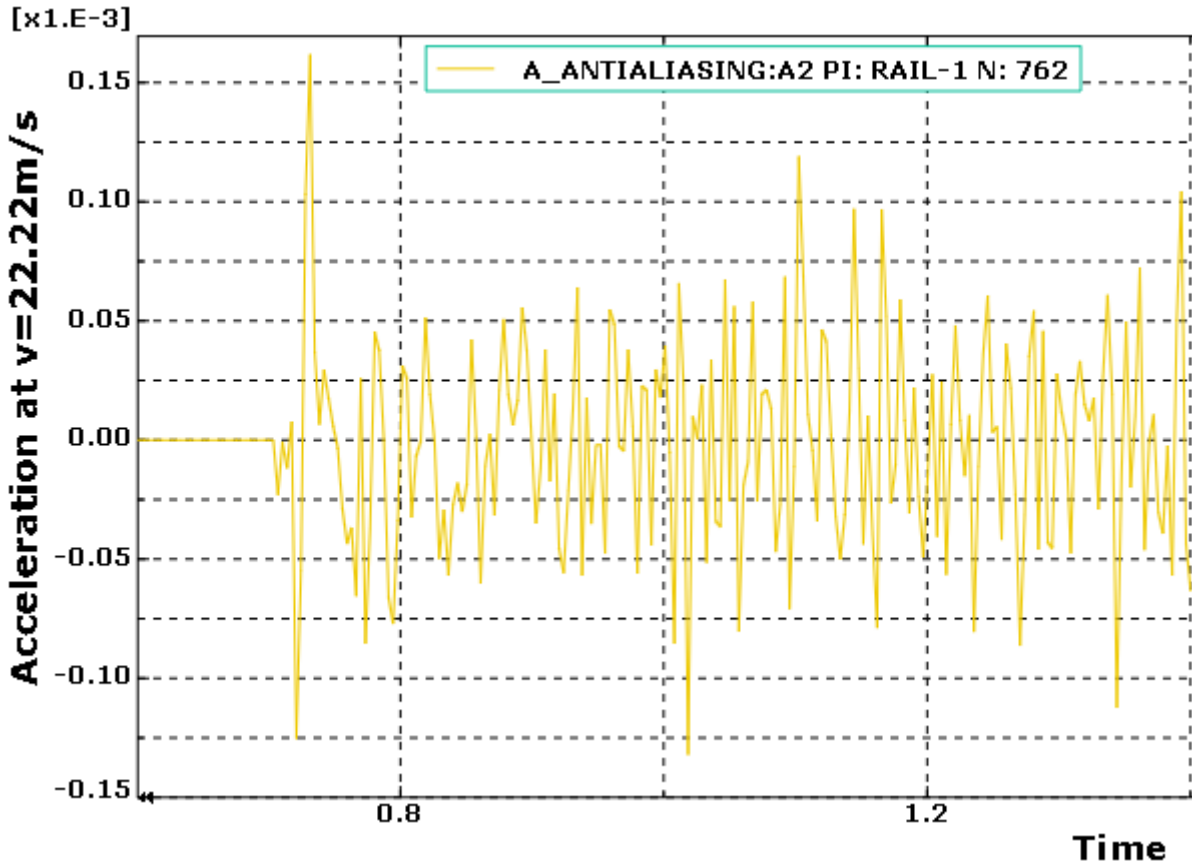


Figure 4.38 Acceleration at V=22.22m/s

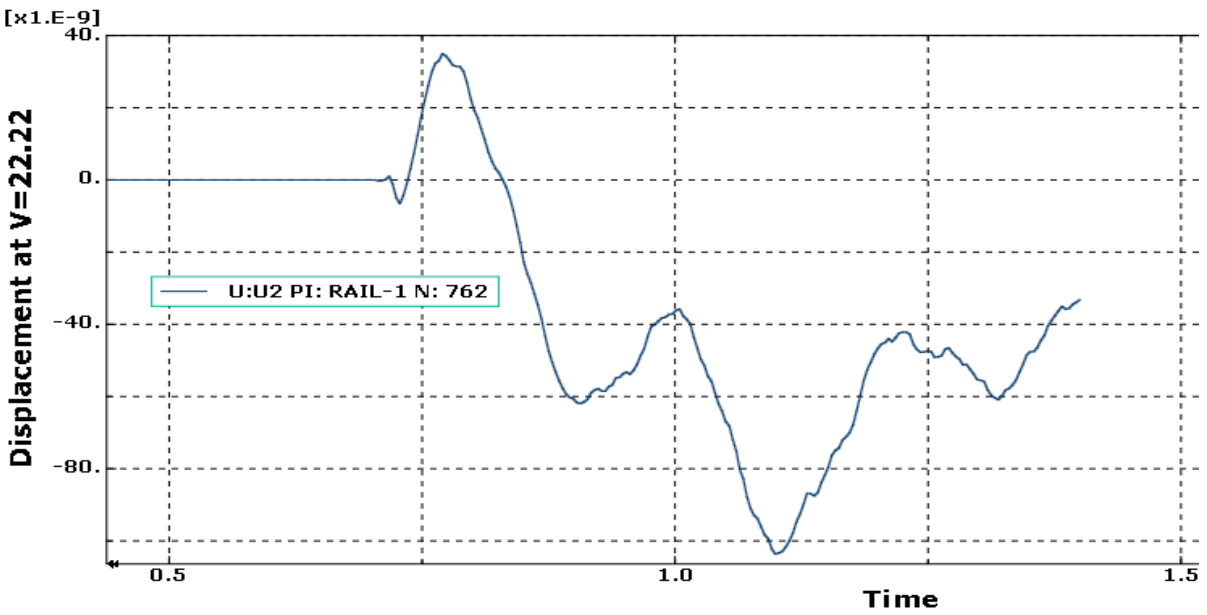


Figure 4.39 Displacement at V=22.22m/s

For V=27.77m/s

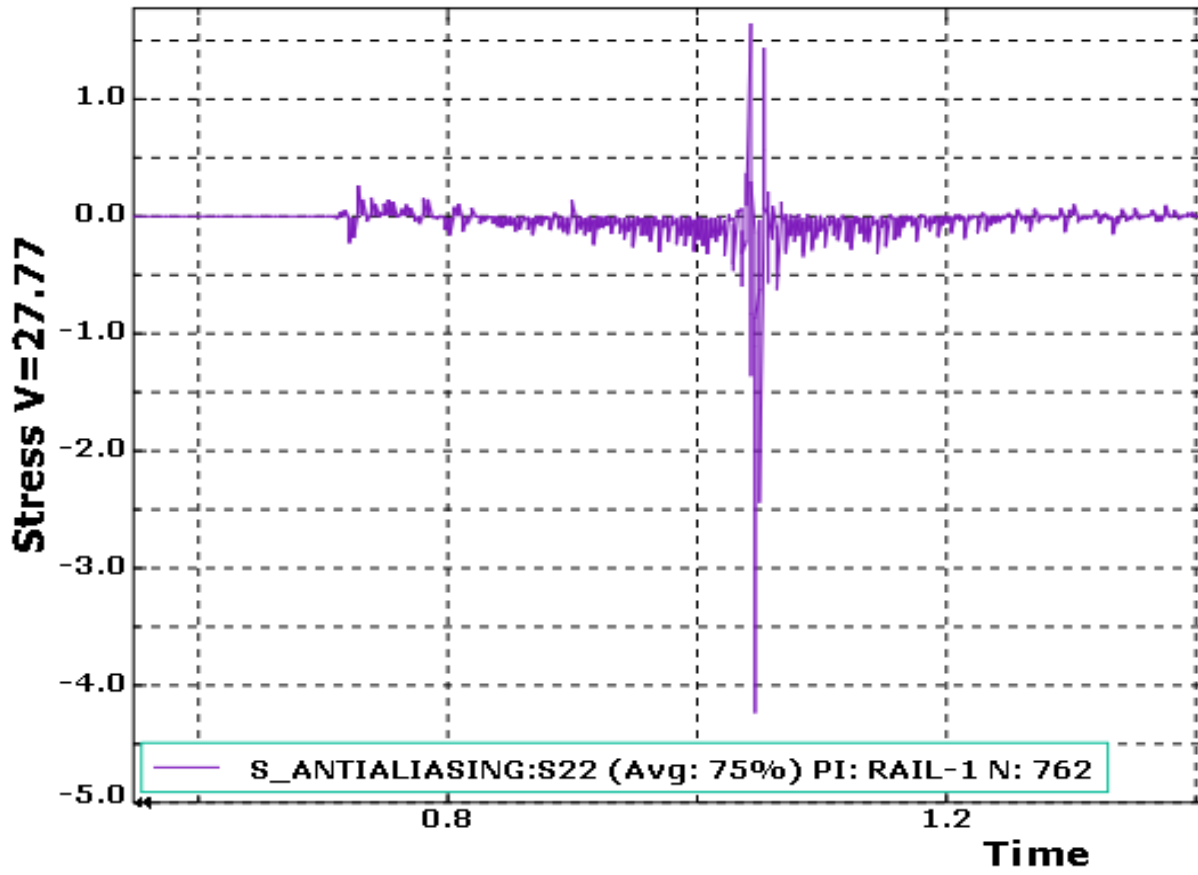


Figure 4.40 S22 stress at V=27.77m/s

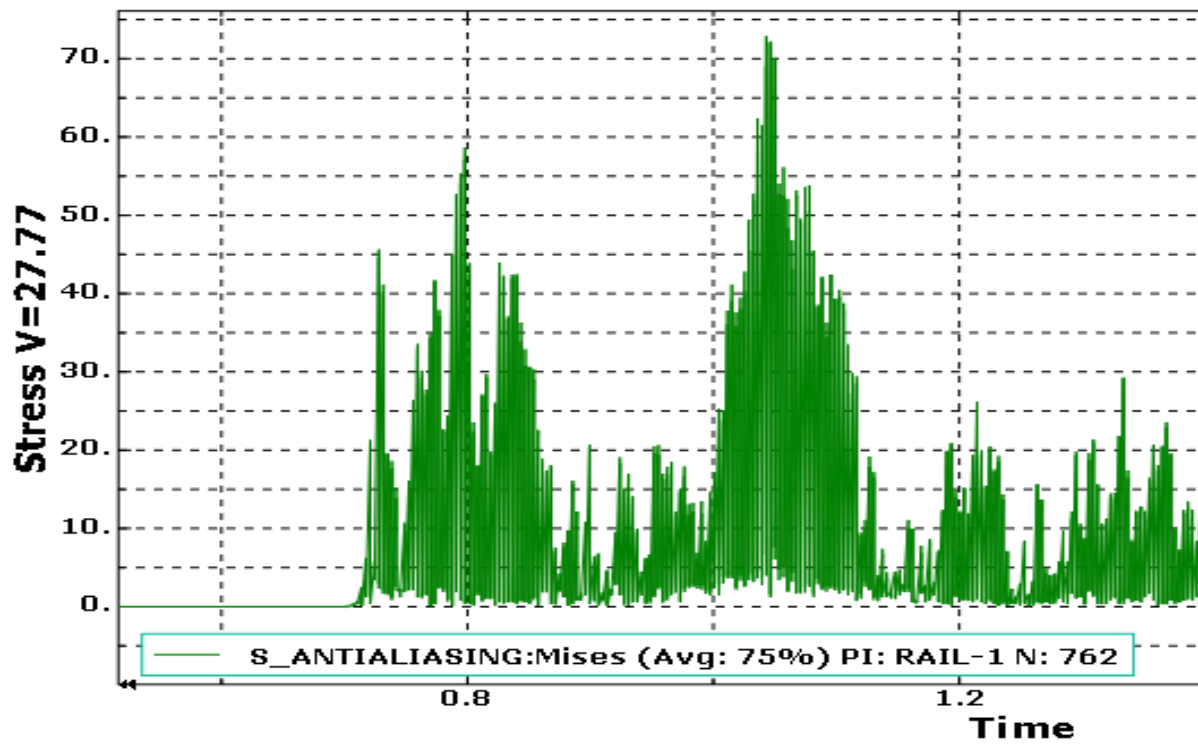


Figure 4.41 Mises stress at V=27.77m/s

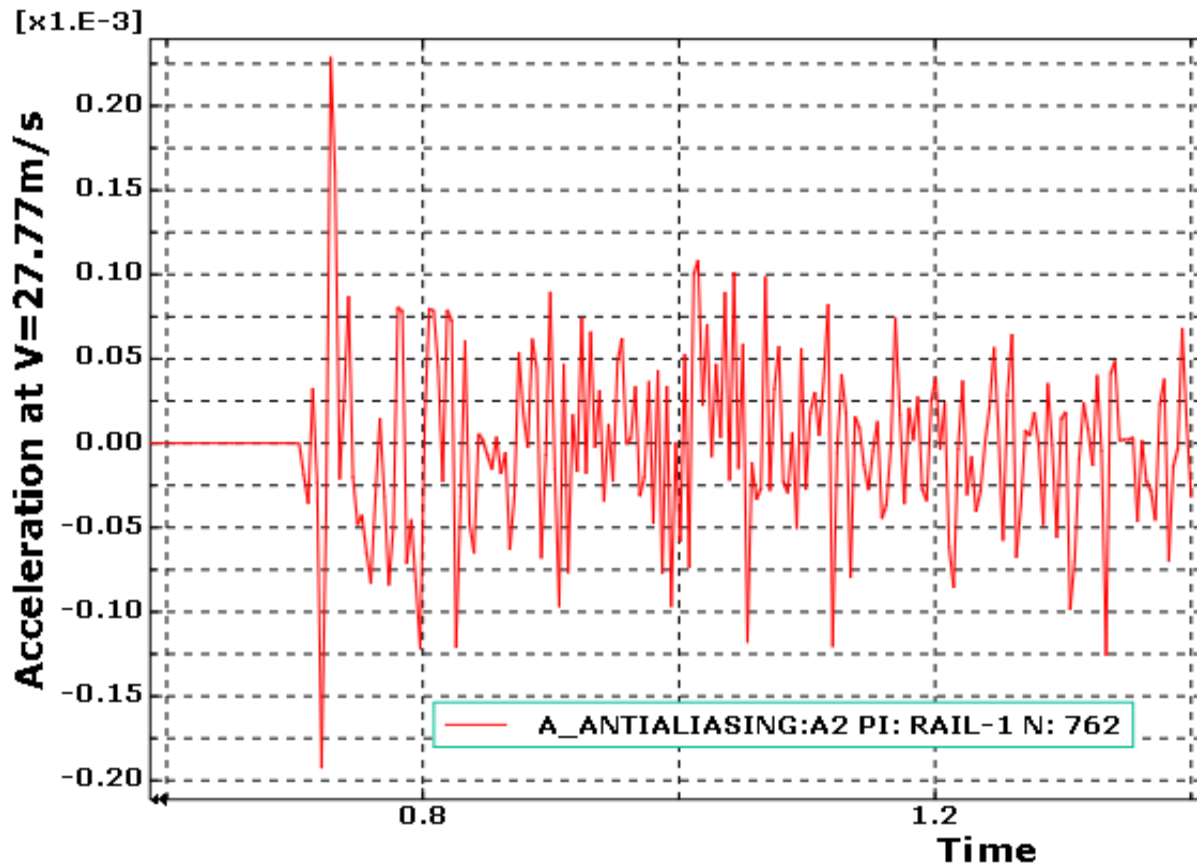


Figure 4.42 Acceleration at V=27.77m/s

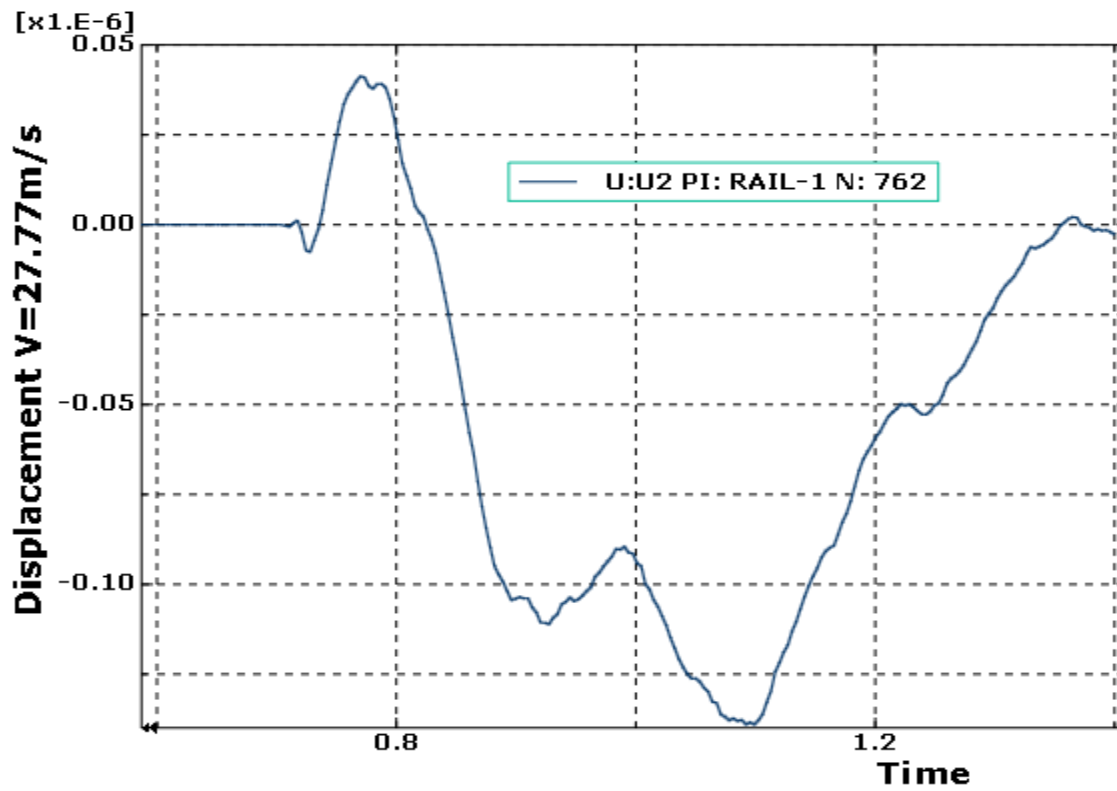


Figure 4.43 Displacement at V=27.77m/s

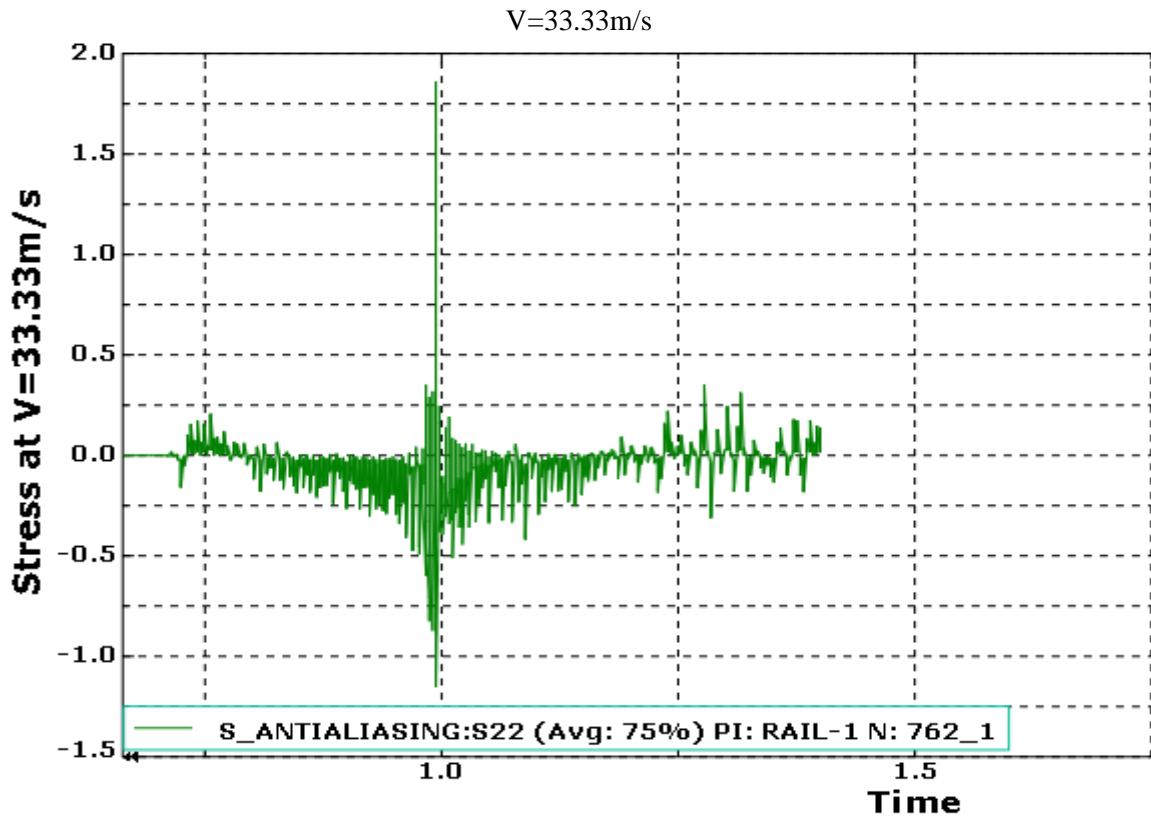


Figure 4.44 S22 Stress at V=33.33m/s

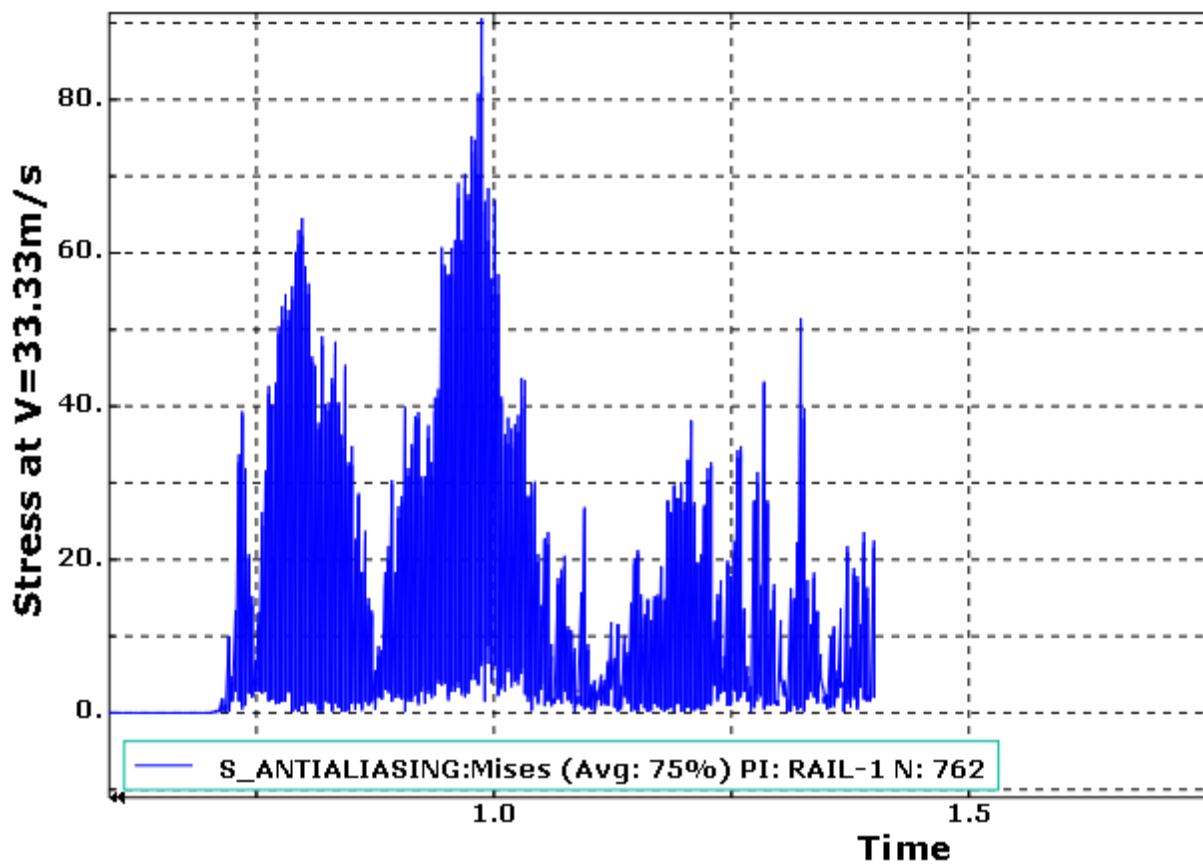


Figure 4.45 Mises Stress at V=33.33m/s

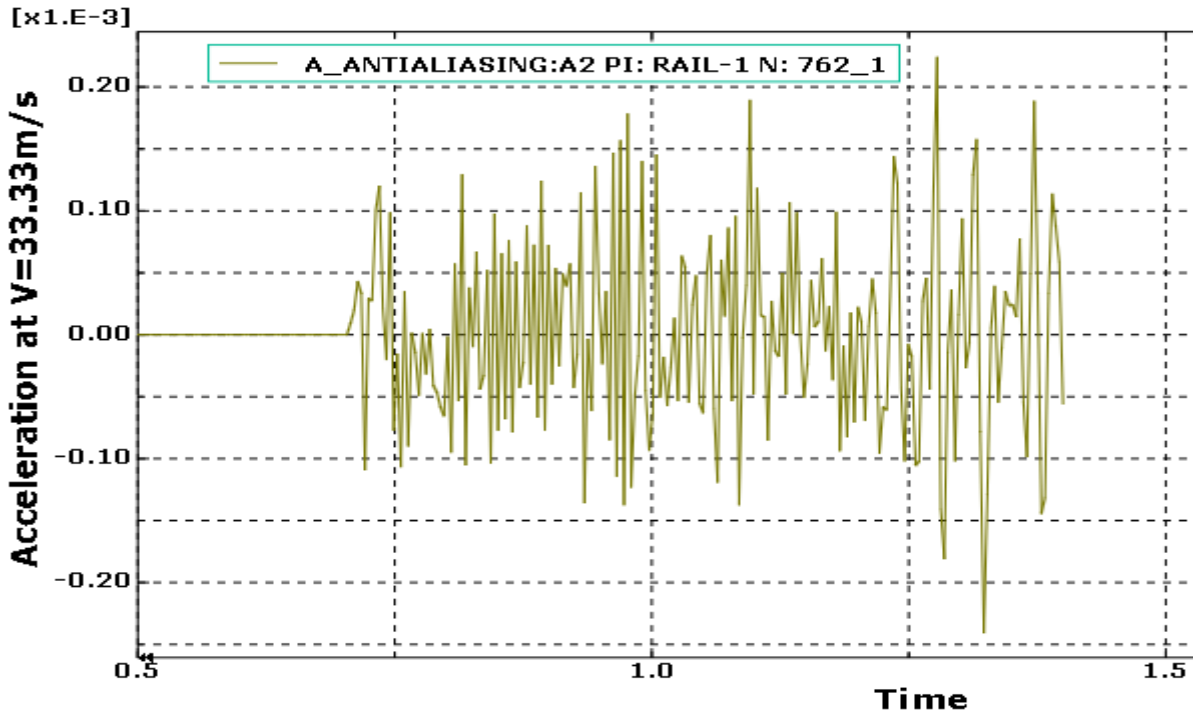


Figure 4.46 Acceleration at V=33.33m/s

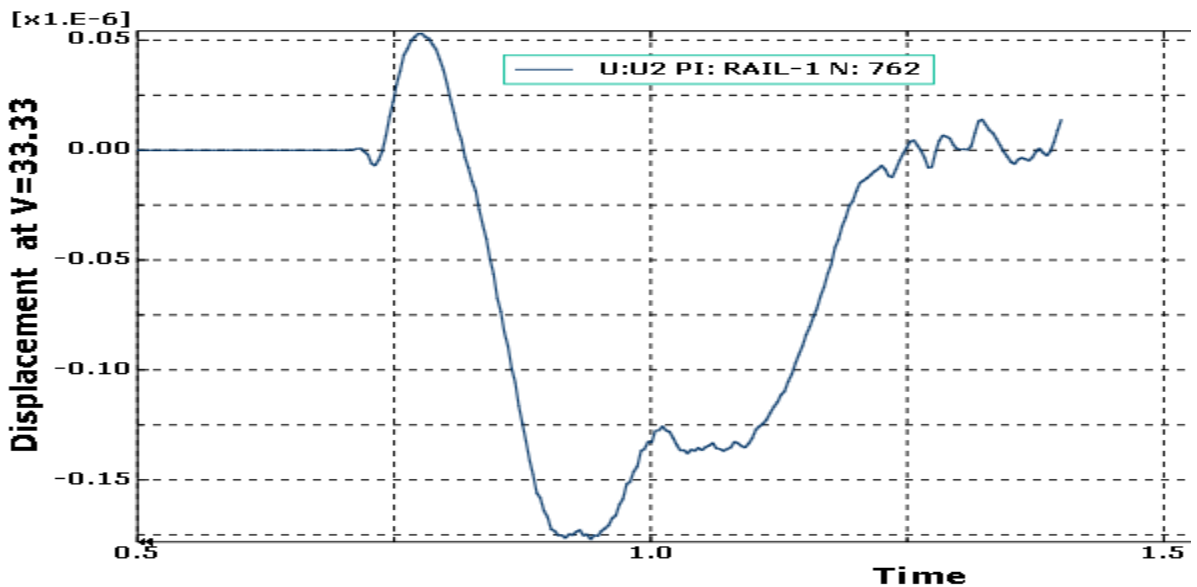


Figure 4.47 Displacement at V=33.33m/s

As the speed of the train increase from 22.22m/s to 33.33m/s, keeping the stiffness constant to $K=150\text{MN/m}$ and $C=40\text{KNs/m}$, vertical acceleration increased, Y displacement increase and the stress magnitude get increased. (Powries, priest,2011)

5. Conclusion and Recommendation

I. Conclusion

The stiffness of Rail pad plays a great role, as shown in the result analysis above, for the static load analysed, for the two different point of loading, the y displacement /deformation/ decreased as both stiffness and damping increased and remain constant when the stiffness increase and damping value remain constant. The value of stress increased when we change the pad from soft to normal, gets reduced for the other two cases i.e. From Normal to hard, and hard to EVA.

For the dynamic case the value of Vertical acceleration or vibration increase as both stiffness and damping increased, and also when the speed is increased from 22.22m/s -33.33m/s the acceleration behaves the same. The vertical displacement remains the same for K & C change with in interval of increase provided for both, while it gets reduced for the increase in the speed. The value of stress decrease as the K & C increase and increase while the speed increase. As Heunis noted a preload (due to the fastening mechanisms), can have a negative influence on their dynamic properties since their dynamic range. (Heunis, 2010)

II. Recommendation

The author of this research recommend the following cases to be studied related to the subject matter investigated for the future researchers,

- The relationship between speed versus rail pad type,
- Examining the relationship between k and C, speed and rail pad stiffness
- under increasing and decreasing K, keeping C constant and Vice versa
- Considering the force type at the beginning of motion/traction

6. Reference

1. Samuel G. Koroma, Mohammed F.M. Hussein, John S. Owen, 2012, The effects of preload and non-linearity on the vibration of railway tracks under harmonic load, University of Nottingham, Nottingham, United Kingdom.
2. Said Daoud, XieGuomei, Liu xiaoming, Equivalent Dynamics Model of Ballasted Track, 2012, Master's degree thesis .ISRN:BTH-AMT-EX-2012/ D-23-SE
3. Thiago B. do Carmo, J. Riley Edwards, Ryan G. Kernes, Bassem O. Andrawes, Chris P. L. Barkan, Laboratory and field investigation of the rail pad assembly Mechanistic behaviour, Proceedings of the 2014 Joint Rail Conference JRC2014, Colorado springs, JRC2014-3784, USA, p.1, 2014
4. Alexander Remennikov, Sakdirat Kaewunruen, Dynamic properties of railway track and its components : a state-of-the-art review, 2008, University of Wollongong, Australia
5. Ryan G. Kernes¹, J. Riley Edwards, Marcus S. Dersch, David A. Lange, and Christopher P. L. Barkan, Investigation of the Dynamic Frictional Properties of a Concrete Crosstie Rail Seat and Pad and its Effect on Rail Seat Deterioration (RSD), Transportation Research Board 91st Annual Meeting, 2011
6. Tore Dahlberg, Railway track settlements - a literature review, Division of Solid Mechanics, IKP, Linköping University, SE-581 83 Linköping, Sweden , 2004.
7. Jungyoul, Choi, Qualitative Analysis for Dynamic Behavior of Railway Ballasted track, M.Sc. Thesis, from Seoul, South Korea 2014.
8. Huan Feng, 3D-models of Railway Track for Dynamic Analysis, 2011, Master's Degree Thesis, TRITA-VBT 11:16 , ISSN 1650-867X , ISRN KTH/VBT-11/16-SE, Stockholm
9. Buddhima Indraratna, PHD, FIEAust., Wadud Salim, PhD, MIEAust Mechanics of Ballasted Rail Track , A Geotechnical perspective, ISBN 0-415-38329-3, Australia
10. José Pedro Freitas da Cunha, Modelling of Ballasted Railway Tracks for High-Speed Trains, 2013, PhD Thesis
11. Alexander Andersson, Hanna Berglund, Johan Blomberg , Oscar Yman, A study on design, construction, monitoring and maintenance procedures to obtain suitable support conditions for railway sleepers, 2013, BSc Thesis, Chalmers University Of Technology, Sweden
12. Clifford F Bonnett, C. Eng., F.I.C. E., F.I. Struct. E, Practical Railway Engineering 2nd Edition, ISBN 1-86094-515-5, 2005, Imperial College Press, London

13. Buddhima Indraratna, PHD, FIEAust.,WadudSalim,PhD, MIEAustCholachat Rujikiatkamjorn, PhD ,Advanced Rail Geotechnology-Ballasted Track , ISBN 978-0-415-66957-3 (hardback) – ISBN 978-0-203-81577-9 (ebook), CRC Press
14. KonstantinosTzanakakis (2013) ,The Railway Track and Its Long Term Behaviour A Handbook for a Railway Track of High Quality ABC ,Athens Greece
15. COENRAAD ESVELD. Modern Railway Track, Second Edition. Delft University of Technology. 2001.
16. Tore Dahlberg, Handbook of Railway Vehicle Dynamics
17. Johannes Jacobus Heunis, Material Model for Rail pads, March 2011,university of Stellenbosch
18. Sharad Kumar Agarwal, Professor of Bridges, Indian Railways Institute of Civil Engineering (IRICEN), ,Dynamic Analysis of High Speed Track:A Parametric Study, Pune, India
19. Ewa Kardas-Cinal, comparative study of running safety and ride comfort of railway vehicle ,Warsaw University of Technology Faculty of Transport, 2009
20. G Lauriks, J Evans, J Förstberg, M Balli and I Barron de Angoiti; UIC comfort tests; Investigation of ride comfort and comfort disturbance on transition and circular curves, Swedish national Road and Transport Research Institute,UIC, 2003.
21. Eric Berggren, Railway Track Stiffness; Dynamic Measurements and Evaluation for Efficient Maintenance, Doctoral Thesis, TRITA AVE 2009:17,ISSN 1651-7660, ISBN 978-91-7415-293-7,Stockholm
22. Sakdirat Kaewunruen, Alexander Remennikov , state dependent properties of rail pads,University of Wollongong, Australia, 2009
23. http://www.railwaysubstructure.org/railwiki/index.php?title=Track_Performance, accessed on 01/01/15
24. Christopher T. Rapp, Marcus S. Dersch, J. Riley Edwards,Christopher P. L. Barkan, Brent Wilson, and Jose Mediavilla, Measuring Rail Seat Pressure Distribution ,Transportation Research Record: Journal of the Transportation Research Board, No. 2374, Transportation Research Board of the National Academies, Washington,in Concrete Crossties,
25. S Kaewunruen RailCorp – Track Engineering, Sydney, NSW AM Remennikov School of Civil, Mining and Environmental Engineering, Faculty of Engineering, University of Wollongong, Wollongong, NSW, State dependent properties of rail pads, Australia

26. Riccardo Ferrara. A Numerical Model to Predict Train Induced Vibrations and Dynamic Overloads. Mechanics of the structures. Universite Montpellier II - Sciences et Techniques du Languedoc; Universitadegli studimediterranea (Reggio de Calabre, Italie), 2013. English.
27. Tse Spartan Azoh, Wolfgang Nzie Bonaventure Djeumako , BertinSohFotsing, Modeling of train track vibrations For maintenance perspectives: application ,European Scientific Journal July 2014 edition vol.10, No.21 ISSN: 1857 – 7881 (Print) e - ISSN 1857- 7431,
28. Silviu NASTAC (PhD); Mihaela PICU,(PhD); “Dunarea de Jos” University of Galati ;Evaluating methods of whole-body-vibration exposure in trains, ISSN 1224-5615; 2010
29. Leposava puzavac, Zdenkapopović, Luka lazarević, Influence of Track Stiffness on Track Behaviour under Vertical Load, University of Belgrade, Faculty of Civil Engineering , Bulevarkralja Aleksandra 73, 11000 Belgrade, Serbia, 2012
30. RajibUIAlamUzzal, Waiz Ahmed and SubhashRakheja, Dynamic Analysis of Railway Vehicle-Track Interactions , Journal of Mechanical Engineering, Vol. ME39, No. 2, December 2008
31. Steffens, David and Murray, Martin, Establishing meaningful results from models of railway track dynamic behaviour, (2005) In Jorge, Cristiano, Eds. Proceedings 8th International Heavy Haul Conference, pages pp. 41-50, Rio de Janeiro, Brazil
32. J. A. Zakeri and H. Xia, Sensitivity analysis of track parameters on train-track dynamic interaction, Journal of Mechanical Science and Technology (2008) 22 1299~1304
33. Chen, Yung-Hsiang; Huang, Yen-Hui (2003), Dynamic Characteristics of Infinite and Finite railways to moving load, Journal of Engineering Mechanics Volume 129 issue 9 [doi 10.1061_(ASCE)0733-9399(2003)129-9(987)]
34. Cai, Z and GP Raymond, Theoretical model for dynamic wheel/rail and track interaction, National Conference Publication, Institution of Engineers, Australia, pp.127-131.
35. Knothe K L and Grassie S L: Modelling of railway track and vehicle-track interaction at high frequencies, Vehicle System Dynamics, Vol 22, 1993.
36. Oscarsson J and Dahlberg T: Dynamic train/track/ballast interaction – computer models and full-scale experiments. Proc 15th IAVSD Symposium on Dynamics of Vehicles on Roads and Tracks, Aug 25-29, 1997, in Budapest, Hungary.
37. Fermer M and Nielsen J C O: Vertical interaction between train and track with softand stiff railpads - full-scale experiments and theory. Proceedings of the Institution ofMechanical Engineers, Part F, Journal of Rail and Rapid Transit,Vol 209(F1), 1995.
38. D. J. Thompson, C. J. C. Jones, T. X. Wu and A de France: The influence of the non-linear stiffness behaviour of rail pads on the track component of rolling noise. Proceedings of the

Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, (1999) 213: 233,
DOI: 10.1243/0954409991531173

39. A. Fenander, Frequency dependent stiffness and damping of rail pad, A division of solid mechanics (charmed) Chalmers University of Technology (1997). Goteborg, Sweden.
40. A. Fenander A fractional derivative rail pad model included in a railway track model, Division of Solid Mechanics, Chalmers University of Technology, S-412 96 Goteborg, Sweden Journal of Sound and Vibration (1998) 212(5), 889-903,
41. Deutsche Bahn AG, Hans-Joerg Terno, State of the art review of mitigation measures on track, RIVAS_UIC_WP3_D3_1_V01-3, 2011
42. T. X. Wu and D. J. Thompson, The effects of local preload on the Foundation stiffness and vertical vibration of railway track, Institute of Sound and Vibration Research, University of Southampton, High field Southampton SO06 0BJ, England, 1999
43. D, J, Thompson, W, J, Van Vliet and J, W, Verheij, Developments of the indirect method for measuring the high frequency dynamic stiffness of resilient elements, Institute of Sound and Vibration Research, University of Southampton, High field, Southampton SO06 0BJ, England, 1998.
44. Ansys Manual, 2009
45. ABAQUS Online Documentation: version 6.12, 2010
46. William Powries and Jeffrey Priest, University of south ampton, 2011