SOLAR RADIATIVE TRANSFER AND GLOBAL CLIMATE MODELLING

By

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Abstract

In this project we have studied Solar radiative transfer and global climate modelling. We consider a parallel beam of electromagnetic radiation incident on a slab of thickness ds. As this radiation passes through the atmosphere it interacts and is being absorbed by the atmospheric gases, the continental lands and oceans and large water bodies. In this project work, we describe the optical depth or optical thickness, the intensity or spectral radiance, absorption, reflection, scattering of particles by applying Beer's law or Lambert's law. The intensity of the incident radiation decreases exponentially as a function of solar zenith angle, fraction of absorbers in the atmosphere, the scattering coefficient, scale height, altitude and density of the atmosphere. The absorption and emission of radiation by the atmospheric gases causes climate changes. More over the major climate system components (atmosphere, land surface, ocean, and sea ice), and their interactions are described using the basic equations that govern the atmosphere. These can be formulated as a set of seven equations with seven unknowns such as the three velocity components (u, v, w), the pressure p, the temperature T, the specific humidity q and the density ρ. These equations, solved numerically by computers, are used to produce our weather forecasts. While there are details about these equations which are uncertain (for example, how we parameterize processes smaller than the grid size of the models), the equations for the most part are accepted as fact. The radiation that passes through the atmosphere is absorbed by the lands and ocean surfaces and is re-emitted as infrared back into the atmosphere so as to heat it and this causes climatic changes.

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Chapter 1

INTRODUCTION

Radiative transfer describes how radiation is transformed along its path through absorption, emission, and scattering [1, 2]. Radiative transfer codes are a key component of weather and climate models, and they cover a large range of varying complexities [1, 3, 4].

In most cases in astronomy we can regard radiation as a particle phenomenon. In this picture light consists of photons moving with the light speed along straight lines through space [5, 6, 7]. They can be created and destroyed by interaction with matter. Hot gases or dust clouds can cool by emitting copious amounts of photons. It is typically this kind of radiation that we observe with our telescopes [2]. But the radiation that is emitted in one region can be absorbed by other matter, which can thus be radiatively heated [8, 9].

A serious interpretation of observations therefore often forces us to learn about the emission, absorption and transport of radiation inside our objects of interest [2, 10, 11]. The theory of radiative transfer (also called radiation transport) is the theory of how radiation and matter interact based on the particle description of light will be discussed in this study [2, 12, 13].

We also focused on a climate modelling. A global climate model(GCM) is a complex mathematical representation of the major climate system components and their interactions [3, 14, 15]. Earth's energy balance between the four components is the key to long-term climate prediction. The main climate system components treated in a climate model are: The atmospheric component, the land surface component, the ocean component, the sea ice component [3, 16, 17]. In addition large numbers of observations are needed to test the validity of the models in order to gain confidence in the conclusions derived from their results. Many climate models have been developed to perform climate projections, i.e. to simulate and understand climate changes in response to the emission of greenhouse gases and aerosols [3]. Models can be formidable tools to improve our knowledge of the most important characteristics of the climate system and of the causes of climate variations [18, 19].

This study focuses overwhelmingly on the treatment of solar radiative transfer within GCMs. Hence, the purpose of this study is to briefly review methods for computing solar radiative transfer in GCMs and to speculate on both deficiencies in these methods and how far we should be going to address these deficiencies [20, 21].

This project is organized in to six chapters. We first note in Chapter one that the atmosphere behaves on an introduction part of solar radiative transfer and global climate modelling. Some basic concepts of atmospheric radiation such as radiometric quantities, atmospheric scattering, radiance temperature, solar constant, zenith angle and albedo are present in Chapter two. In Chapter three we derive the radiative intensity, optical depth and transmissivity of radiation transfer through the atmosphere. In Chapter four we introduce basic climate modelling theory, types and uses of models, components of climate modelling specially in atmosphere. In Chapter five we make some further simplifications about radiation, which allow as to set up simple models of two types of atmospheric waves;long wave and short wave fluxes. Finally in Chapter six we discuss summary and conclusions of the study.

Chapter 2

BASIC CONCEPTS OF ATMOSPHERIC RADIATION

2.1 Introduction

Radiation is the process through which energy moves through space from the source, without any material medium. Radiation sources are generally collections of matter or devices that convert other forms of energy into radiative energy [4, 5]. In some cases the energy to be converted is stored with in the object like the Sun and radioactive materials. In other cases the radiation source is only an energy converter, and other forms of energy must be applied in order to produce radiation. Most forms of radiation can penetrate through a certain amount of matter. But in most situations, radiation energy is eventually absorbed by the material and converted into another form of energy. Solar radiation is composed of electromagnetic waves that travel through space. Electromagnetic waves are formed when an electric field couples with a magnetic field. The electric and magnetic fields of an electromagnetic wave are perpendicular to each other and to the direction of wave. Electromagnetic energy is a form of energy that is transferred by radiation from all things in nature. Electromagnetic radiation spectrum comprises a broad band of wavelengths that travels from its source through space in the form of harmonic waves at the uniform speed of light in vacuum ($3 \times 10^8 ms^{-1}$) [4, 6].

2.2 Atmospheric scattering

Scattering is the process by which a particle in the path of an electromagnetic wave abstracts energy from the incident wave and reradiates that energy in all directions. In scattering, no energy transformation takes place, but only a change in the spatial distribution of the energy. Sunlight coming into the atmosphere can be scattered in any direction as it passes through the air. This diffuses the light and spreads it out in all directions so that it does not appear just as a single, straight beam [6].

In the atmosphere, the particles responsible for scattering cover the sizes from gas molecules ($10^{-8} cm$) to large raindrops and hail particles of about 1 cm. The relative intensity of the scattering pattern depends mainly on the ratio of particle size to wavelength of incident wave [1, 2]. There are three different types of scattering: Rayleigh scattering, Mie scattering, and nonselective scattering.

2.2.1 Rayleigh scattering

This occurs when the particles causing the scattering are smaller in size than the wavelengths of radiation in contact with them. This type of scattering is therefore wavelength-dependent. As the wavelength decreases, the amount of scattering increases. Because of Rayleigh scattering, the sky appears blue. This is because blue light is scattered around four times as much as red light, and UV light is scattered about 16 times as much as red light. This is the origin of the blue colour of the sky, the yellow colour of the Sun, and the orange colour of the sunset [1].This type of scattering Mainly consists of scattering from atmospheric gases. Cloud drops can Rayleigh-scatter microwaves, which is the physical basis of cloud radar. As Rayleigh scattering is characterized by symmetry between forward and backward scattering Rayleigh scattering decreases with increasing wavelength of the incident beam according to:

$$\frac{N_{sc}}{N_o} \propto \lambda^{-4}, \tag{2.2.1}$$

where N_o and N_{sc} are the total intensities of the incident and scattered radiation. This relation explains why the sky appear blue under cloudless conditions. Rayleigh scattering is also responsible for the albedo of the clear-sky atmosphere. It can be shown that the planetary albedo of a purely Rayleigh atmosphere is ~ 0.2 (assuming a surface albedo of 0.16), which is somewhat higher than the observed clear-sky planetary albedo of 0.17 [8, 13].

2.2.2 Mie scattering

It is caused by pollen, dust, smoke, water droplets, and other particles in the lower portion of the atmosphere. The dependence of Mie scatter on the wavelength of the incident beam is more complex. As a rule of thumb:

$$\frac{N_{sc}}{N_o} \sim \lambda^{-13}, \tag{2.2.2}$$

It occurs when the particles causing the scattering are larger than the wavelengths of radiation in contact with them. Mie scattering is responsible for the white appearance of the clouds. It is highly directional, with most radiation being scattered in the forward direction. Scattering of sunlight by cloud drops is an example of Mie scattering. So, Mie scattering is the reason we can actually see clouds. The fact that most scattering is in the forward direction makes the clouds white, unless they are deep. Light rays in clouds typically experience multiple scattering events and in most clouds they are backscattered enough to make clouds white when viewed from the top [1, 8].

2.2.3 Nonselective scattering

It is the last type of scattering. It occurs in the lower portion of the atmosphere when the particles are much larger than the incident radiation. This type of scattering is not wavelength-dependent and is the primary cause of haze. Nonselective scattering gets its name from the fact that all wavelengths are scattered about equally. This type of scattering causes fog and clouds to appear white to our eyes because blue, green, and red light are all scattered in approximately equal quantities (blue+green+red light = white light) [8, 13].

2.3 Radiometric quantity

Radiometry deals with quantities associated with the radiation field, and the interaction coefficients deals with quantities associated with the interaction of radiation and matter [6, 7]. The whole discipline of optical measurement techniques can be roughly subdivided into photometry and radiometry. Where as photometry focuses on determining optical quantities that are closely related to the sensitivity of the human eye. Radiometry deals with the measurement of energy per time (power, given in watts) emitted by light sources or impinging on a particular surface. Thus, the units of all radiometric quantities are based on watts (W). Symbols for radiometric quantities are denoted with the subscript "e" for "energy". Similarly, radiometric quantities given as a function of wavelength are labelled with the prefix "spectral" and the subscript "λ" (for example spectral radiant power F_λ) [6]. Commonly used radiometric quantities are the following;

2.3.1 Radiant flux or radiant power

Radiant power F_e is defined as the total power of radiation emitted by a source (lamp, light emitting diode, etc.), transmitted through a surface, or impinging upon a surface and is measured in watts (W). The definitions of all other radiometric quantities are based on radiant power. If a light source emits uniformly in all directions, it is called an isotropic light source [5, 6].

Radiant power characterizes the output of a source of electromagnetic radiation only by a single number and does not contain any information on the spectral distribution or the directional distribution of the lamp output. The more important radiation field characteristics is flux of radiation, which is understood to be the quantity of radiant energy of the wavelength λ per unit area per unit time. It should be noted that there are many terminological variants for the quantity. Which we here call radiant flux. Among other names, it is also called irradiance, radiant emittance, and energetical illumination [7]. It is defined as the rate of energy transfer by electromagnetic radiation.

$$F = \frac{dE_\lambda}{dt}, \tag{2.3.1}$$

and its unit is J/s called Watt. Where, E is radiant energy. The sun's radiant flux is $F = 3.9 \times 10^{26} W$.

2.3.2 Monochromatic intensity

It is the energy transferred by electromagnetic rradiation in a specified direction per unit area per unit time normal to the direction considered at a given wave length is known as monochromatic intensity.

$$I_\lambda = \frac{dE_\lambda}{dtdAd\Omega}, \tag{2.3.2}$$

where Ω is solid angle and

$$d\Omega = \sin(\theta) d\theta d\psi. \tag{2.3.3}$$

The integral of monochromatic intensity over a finite area of the electromagnetic spectrum is known as intensity or radiance.

$$I = \int_{\lambda_1}^{\lambda_2} I_\lambda d\lambda = \int_{v_1}^{v_2} I_v dv. \tag{2.3.4}$$

Both I_λ and I_v are called monochromatic intensity, even though they are expressed in different units. They have the relation as

$$I_v = \lambda^2 I_\lambda. \tag{2.3.5}$$

2.3.3 Monochromatic irradiance

It is the rate of energy transfer per unit area by radiation with a given wavelength through a plane surface. Irradiance covers an infinitesimal wavelength interval of the electromagnetic spectrum. The unit used for the measurement of irradiance is $Wm^{-2}nm^{-1}$. Let the radiation be incident normal to a horizontal plane, then the monochromatic intensity can be:

$$F_\lambda = \frac{dE_\lambda}{dtdAd\lambda} = \int I_\lambda cos(\theta)d\Omega, \tag{2.3.6}$$

but

$$I_\lambda(\lambda, T) = \frac{hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda KT}} - 1}. \tag{2.3.7}$$

2.3.4 Irradiance

The irradiance E_e is the amount of radiant power impinging upon a surface per unit area [5]. In detail, the differential (radiant) power dF_e upon the (differential) surface element dA is given by

$$dF_e = E_e dA. \tag{2.3.8}$$

Generally, the surface element can be oriented at any angle towards the direction of the beam. However, irradiance is maximized when the surface element is perpendicular to the beam:

$$dF_e = E_{e,n}\, dA_n. \tag{2.3.9}$$

Note that the corresponding area element dA_n, which is oriented perpendicular to the incident beam(n means normal), is given by

$$dA_n = \cos(\vartheta)dA \tag{2.3.10}$$

with (ϑ) denoting the angle between the beam and the normal of dA, we get

$$E_e = E_{e,n} \times \cos(\vartheta), \tag{2.3.11}$$

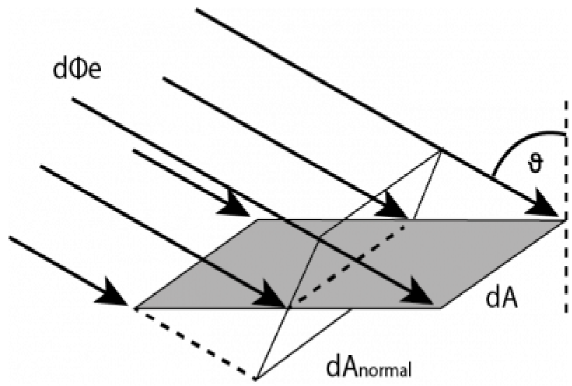

Figure 2.1: Irradiance is defined as incident radiant power dF_e per surface area element dA [32]

The unit of irradiance is W/m^2.

2.4 Radiance or brightness temperature

It is the temperature measured by a satellite instrument, usually detected in terms of radiance, but converted into a temperature through Planck's function at a given wavelength. Planck's law provides the unique description of the monochromatic irradiance (E_λ) of radiation from a black body as a function of its temperature T [4]. The temperature T can be obtained from Planck's function as

$$E_\lambda(t) = B_\lambda(t) = \frac{2hc^2}{\lambda^5} \frac{1}{exp\left(\frac{hc}{\lambda kT}\right) - 1} \tag{2.4.1}$$

where, k is Boltzmann constant, c is the velocity of light in a vacuum. Equation (2.4,1)can be written as

$$E_\lambda(t) = \frac{c_1}{\lambda^5}\left(\frac{1}{e^{\frac{c_2}{\lambda T}} - 1}\right), \tag{2.4.2}$$

where $c_1 = 2hc^2$ and c_2=hc/k. Here,

$$e^{\frac{c_2}{\lambda T}} - 1 = \frac{c_1}{\lambda^5 E_\lambda}, \tag{2.4.3}$$

$$e^{\frac{c_2}{\lambda T}} = 1 + \frac{c_1}{\lambda^5 E_\lambda}, \tag{2.4.4}$$

$$\frac{c_2}{\lambda T} = \ln[1 + \frac{c_1}{\lambda^5 E_\lambda}], \tag{2.4.5}$$

$$T = \frac{c_2}{\lambda \ln[1 + \frac{c_1}{\lambda^5 E_\lambda}]}, \tag{2.4.6}$$

where, T is called the brightness temperature or radiance temperature or equivalent black body temperature, λ is the wavelength, E_λ is the emitted radiation at wavelength λ, and c_1 and c_2 are empirical constants, respectively.

2.5 Solar constant

The solar energy reaching the periphery of the Earths atmosphere is known as the solar constant, and is considered to be constant for all practical purposes. The solar constant is defined as the amount of solar radiation incident, per unit area and time, on a surface which is perpendicular to the radiation and is situated at the outer limit of the atmosphere, the Earth being at its mean distance from the Sun.

The solar constant consists of all types of solar radiation, and is not just confined to visible light. Because of the difficulty in obtaining accurate measurements, the exact value of the solar constant is not known with certainty. It is measured by satellite to be roughly 1,366 Wm^{-2}. The actual value of the energy varies with several factors, the most important being the Earths distance from the Sun, which changes because of the Earths elliptical orbit. It fluctuates by about 6.9 percent during a year from 1,412Wm^{-2} in early January to 1,321 Wm^{-2} in early July, due to the Earths varying distance from the Sun, and by a few parts per thousand from day to day [8, 9].

2.6 Albedo

It is known as surface reflectivity of solar radiation. It is defined as the ratio of reflected to incident electromagnetic radiation. Albedo is a measure of surface diffuse reflectivity, which has a non dimensional value, normally represented in percent. The term has its origins from a Latin word albus, meaning white. It is quantified as the percentage of solar radiation of all wavelengths reflected by a body or surface to the amount incident upon it. An ideal white body has an albedo of 100 percent, where as the albedo is 0 percent in the case of an ideal black body [4]. The albedo of the

Surface	Details	Albedo (in %)
Soil	Wet–dry	4–40
Sand	—	15–45
Grass	Long–short	16–26
Agricultural crops	—	18–25
Tundra	—	18–25
Forests	Desiduous	15–20
	Coniferous	5–15
Water	Small zenith angle	3–10
	Large zenith angle	10–100
Snow	Old/fresh	40–95
Ice	Sea	30–45
	Glacier	20–40
Clouds	Thick	60–90
	Thin	30–50

Table 2.1: Albedo of various surfaces [4]

Earths surface varies with the type of material that covers it [8, 9]. Surface reflectance values show large geographic variation. Mean annual albedo values differ considerably between the equator and the poles, largely due to the presence of snow and ice-covered surfaces along with cloudy skies in high latitudes, which greatly increases albedo values in those areas. The reflectance properties of the surface change from one season to another throughout the high latitudes, snow cover and ice extent reach maximum values during the cold seasons, significantly increasing the surface reflectance values [10, 11].

2.7 Zenith angles

The solar zenith angle is the angle between the suns rays and the vertical direction. It is closely related to the solar altitude angle, which is the angle between the suns rays and a horizontal plane. Since these two angles are complementary, the cosine of either one of them equals the sine of the other. For any non-zero zenith angle the radiative energy flux hitting the top of the atmosphere is distributed over a larger area. The zenith angle for sunlight is a function of latitude, season and time of day. At any time, the insolation S equals,

$$S = S_0 \left(< r_E > / r_E(t)\right)^2 \cos(\theta) \tag{2.7.1}$$

with $r_E(t)$ the distance from the Earth to the Sun as a function of time, $< r_E >$ the average distance from the Earth to the Sun, and (θ) the zenith angle. The $\cos(\theta)$ factor accounts for the projection of the incident rays onto the surface. With this equation, the insolation averaged over a day can be calculated as a function of latitude and season; this requires standard astronomical equations for the declination of the Sun (the angle between the rays of the Sun and the plane of the Earth's equator) and the distance between the Earth and the Sun as a function of day number. It can be seen that the insolation vanishes in the polar nights. The maximum insolation is achieved at the poles in the summer hemispheres. This is due to the fact that, although the solar zenith angle increases when going poleward, the length of the day also increases in the summer hemisphere [1, 10]. In the winter hemisphere there is a strong insolation contrast between the equator and the pole; in the summer hemisphere there is a much weaker, indeed reversed, insolation contrast. The average of this field over all latitudes equals $S_0/4 = 342Wm^{-2}$. The zenith angle varies during the day when the sun traverses the sky, with a zenith angle of $\pi/2$ at sun rise and sunset and a minimum zenith angle at local noon.

Chapter 3

RADIATION TRANSFER THROUGH THE ATMOSPHERE

3.1 Radiative intensity

In radiometry, radiant intensity is the radiant flux emitted, reflected, transmitted or received, per unit solid angle, and spectral intensity is the radiant intensity per unit frequency or wavelength, depending on whether the spectrum is taken as a function of frequency or of wavelength. These are directional quantities. Radiant intensity is distinct from irradiance and radiant exitance, which are often called intensity in branches of physics other than radiometry. In radio-frequency engineering, radiant intensity is sometimes called radiation intensity [1].

Consider a unit sphere drawn around a fixed point. Any radiation coming from that point will pass through the unit sphere at a location which can be identified with the usual zenith angle (θ) and azimuth angle (ϕ). The zenith angle is the angle between the ray and the local vertical. If we identify a small solid angle

$$d\Omega = sin(\theta) d\phi d\theta, \tag{3.1.1}$$

around this direction we can identify a small part dF of the radiative flux that passes through this solid angle. The radiative intensity I is now defined as the flux per unit solid angle in the given direction, in equations

$$I d\Omega = dF, \tag{3.1.2}$$

$$I = dF/d\Omega. \tag{3.1.3}$$

The unit of flux F is Wm^{-2}, so the unit of intensity I is $Wm^{-2}sr^{-1}$ with solid angles measured in steradians (sr). The intensity is normally a function of the direction [12]. Isotropic radiation is constant in all directions. Black body emission is an example of isotropic radiation. The flux and the intensity are related. Let us calculate the upward flux due to radiation of intensity $I(\phi,\ \theta)$; that is, the flux through a surface that is perpendicular to the upward direction $\theta = 0$. The upward flux F is the integral of the upward component of the intensity I over all directions. The upward component of $I(\phi,\ \theta)$ is $I(\phi,\ \theta)\cos(\theta)$. The upward flux F therefore is

$$F = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I(\phi, \theta) \cos(\theta) \sin(\theta) d\theta d\phi. \tag{3.1.4}$$

We only integrate over zenith angles θ up to $\pi/2$, as larger zenith angles correspond to downward radiation. We can generalize the above expression for flux in any direction; that is, flux through surfaces with any orientation [11]. For isotropic radiation, I is not a function of direction and we can readily evaluate the double integral. We find

$$F = \pi I. \tag{3.1.5}$$

As black body radiation is isotropic, the Planck law, can also be expressed in terms of radiative intensity [2, 8]. The difference between the expressions for black body intensity and flux is a factor π, as above, but remember that the black body intensity has units of $Wm^{-2}sr^{-1}$. The solar constant S_0 can be related to the radiation intensity coming from the Sun. If the radiative flux at the surface of the Sun is σT_S^4, then its intensity will be $I = \sigma T_S^4/\pi$, assuming isotropic radiation. The Sun subtends a solid angle of about $\pi R_S^2/r_E^2$ in the sky. The flux hitting the surface normal to the solar radiation will therefore be $\sigma T_S^4 R_S^2/r_E^2$, which is the definition of the solar constant S_0,

$$4\pi r_E^2 S_0 = 4\pi R_S^2 \sigma T_S^4, \tag{3.1.6}$$

This calculation is only valid for small values of R_S/r_E. A more accurate calculation demonstrates the equivalent result for finite values of R_S/r_E if the radiating sphere is directly overhead [8, 19].

From the geometry in Fig. (3.1) we can see that the flux hitting the surface from

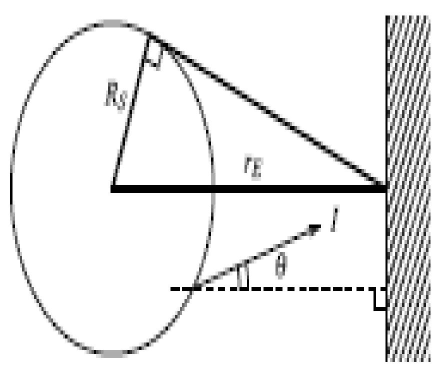

Figure 3.1: Geometry used to calculate the flux from radiating sphere of radius R_S at a distance r_E [1].

isotropic radiation from a sphere equals

$$F = \int_0^{2\pi} \int_0^{\sin^{-\frac{R_S}{r_E}}} I \cos(\theta) \sin(\theta) d\theta d\phi, \tag{3.1.7}$$

where, $I\cos(\theta)$ is the intensity component perpendicular to the surface and the integral is over the solid angle subtended by the sphere. This integral is

$$F = \pi I \frac{R_S^2}{r_E^2}. \tag{3.1.8}$$

For the Sun we have $\pi I = \sigma T_s^4$ so we arrive again at the definition of the solar constant. The special angular distribution of radiation intensity $I_{\nu(r,\ \Omega)}$ in studied medium satisfies the so-called radiative transfer equation, quite various approaches can be used for deriving this equation. It can be obtained using rigorous methods of statistical physics, by using the boltzmann equation for radiative transfer of photon gas [1, 10].

3.2 Beer's law

Bear's law also known as bouguer's law or lambert's law describes the decrease in intensity of a radiation as it passes through an absorbing and scattering media with path length [11]. Consider a parallel beam of EMR incident on a slab of thickness ds, I_λ is the incident monochromatic intensity at wave length λ, dI_λ is the intensity that is removed from the out going radiation by extinction. Absorbtion and scattering are both extinction. After passing through the layer the monochromatic intensity radiation is decreased by

$$dI_\lambda = -I_\lambda \rho r k_\lambda ds. \tag{3.2.1}$$

where ρ is the density of air, r is the mass of the absorbing gas per unit mass of air, and k_λ is the mass absorption or scattering coefficient.

$$I_\lambda(transmitted) = I_\lambda(incident) - dI_\lambda(absorbed\ or\ scatterd). \tag{3.2.2}$$

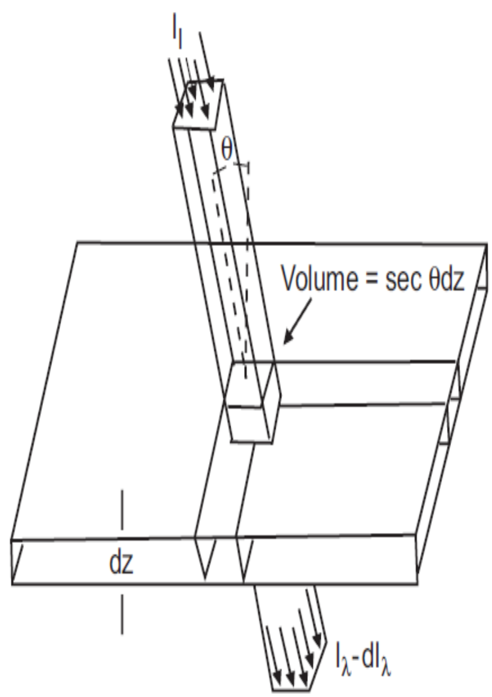

Figure 3.2: Depletion of incoming beam of parallel radiation on passing through a slab of absorbing material [27]

Let n be the number of absorbers or scatters per unit volume(air number density), $\sigma_s(\lambda)$ is the scattering cross-section at wave length λ and $\sigma_a(\lambda)$ is the absorption cross-section at wave length λ.

$$\sigma(\lambda) = \sigma_a(\lambda) + \sigma_s(\lambda), \tag{3.2.3}$$

$$\varepsilon_s(\lambda) = the\ scattering\ coefficient = n\sigma_s(\lambda), \tag{3.2.4}$$

$$\varepsilon_a(\lambda) = the\ absorption\ coefficient = n\sigma_a(\lambda). \tag{3.2.5}$$

Dimensions of $\varepsilon_a(\lambda)$ and $\varepsilon_s(\lambda)$ is per length $(1/m)$. $\varepsilon_a(\lambda)$ and $\varepsilon_s(\lambda)$ can interpreted as the probability of scattering and absorption per unit path length. The incident beam (radiation) seas the effective number of scatters or absorbers dN=N $\sigma(\lambda)$ ds, its dimension is number. Absorption or scattering is proportional to these numbers. dI_λ is absorbed intensity, $dI_\lambda \propto I_\lambda$(incident intensity), $dI_\lambda \propto n\sigma ds$(effective number of scatters or absorbers), ds path length.

Number density of scatters or absorbers(molecules or aerosols)

$$\sigma_\lambda = \sigma_s(\lambda) + \sigma_a(\lambda), \tag{3.2.6}$$

$$dI_\lambda = -I_\lambda n\sigma ds. \tag{3.2.7}$$

The minus sign indicates the radiant energy removed or decreases.

$$\frac{dI_\lambda}{I_\lambda} = -n\sigma(\lambda)ds = -\varepsilon(\lambda)ds, \tag{3.2.8}$$

$$\varepsilon(\lambda) = \varepsilon_s(\lambda) + \varepsilon_a(\lambda), \tag{3.2.9}$$

$$\varepsilon(\lambda) = n\sigma_s(\lambda) + n\sigma_a(\lambda), \tag{3.2.10}$$

This is effective or total extinction (absorption + scattering).

$$\int_{I_0(\lambda)}^{I(\lambda)} \frac{dI_\lambda}{I_\lambda} = -\int_0^L \varepsilon(\lambda)ds, \tag{3.2.11}$$

$$\ln\frac{I(\lambda)}{I_0(\lambda)} = -\varepsilon(\lambda)L, \tag{3.2.12}$$

$$I(\lambda) = I_0(\lambda)e^{-\varepsilon(\lambda)L}, \tag{3.2.13}$$

where, L is total path length traversed by the radiation, $I_0(\lambda)$ is incident intensity and $I(\lambda)$ is transmitted intensity. For the absorption the atmosphere is a mixture of different absorbers with absorption cross-section $\sigma_i(\lambda)$ and concentration n_i. Total absorption coefficient is

$$\varepsilon_a(\lambda) = \sum_i \sigma_i(\lambda)n_i. \tag{3.2.14}$$

The incident radiation is attenuated(decreased)by the extinction. The solar radiation that is incident at the top of the atmosphere is attenuated by the absorbing and scattering constituent of the atmosphere. Total scattering

$$\varepsilon_s(\lambda) = \varepsilon_R(\lambda) + \varepsilon_M(\lambda), \tag{3.2.15}$$

$$\varepsilon(\lambda) = \varepsilon_a(\lambda) + \varepsilon_s(\lambda), \tag{3.2.16}$$

$$\varepsilon_\lambda = \sum_i \sigma_i(\lambda) n_i + \varepsilon_R(\lambda) + \varepsilon_M(\lambda). \tag{3.2.17}$$

The intensity(spectral radiance) the incident radiation or incoming radiation after traversing a distance L through a medium is

$$I(\lambda) = I_0(\lambda) e^{-L\left[\sum_i \sigma_i(\lambda) n_i + \varepsilon_R(\lambda) + \varepsilon_M(\lambda)\right]}. \tag{3.2.18}$$

For the atmosphere, $I_0(\lambda)$ is the solar radiation that strikes the top of the atmosphere. This solar radiation is continuously attenuated until it reaches the surface of the earth.

$$\varepsilon_R(\lambda) = \sigma_R(\lambda) n_R, \tag{3.2.19}$$

where, $\sigma_R(\lambda)$ is the Rayleigh scattering cross-section and n_R is the number density of particles responsible for Rayleigh scattering.

$$\varepsilon_M(\lambda) = \sigma_M(\lambda) n_M, \tag{3.2.20}$$

where, $\sigma_M(\lambda)$ is the Mie scattering cross-section and n_M is the number density of particles responsible for Mie scattering. We can also use mass density rather than number density

$$\rho = nm \tag{3.2.21}$$

$$n = \rho / m, \tag{3.2.22}$$

$$n_i = \rho_i/m_i, \tag{3.2.23}$$

where, ρ is mass density, n is number density, m is mass of individual particles. After passing through a layer the monochromatic intensity of radiation is decreased by

$$dI_\lambda = -I_\lambda \rho r k_\lambda ds, \tag{3.2.24}$$

$$\rho r = n, \tag{3.2.25}$$

$$r = 1/m, \tag{3.2.26}$$

where, r is mass of the absorbing gas per unit mass of air k_λ is mass absorption or scattering coefficient ρ is density of the air.

$$\frac{dI_\lambda}{I_\lambda} = -k_\lambda \rho r ds. \tag{3.2.27}$$

For the atmosphere, the mass density depends on altitude.

$$P = \rho RT, \tag{3.2.28}$$

that means

$$\rho = \frac{P}{RT}, \tag{3.2.29}$$

$$\rho(z) = \frac{P(0)}{RT} e^{\frac{-z}{H}} = \rho(0) e^{\frac{-z}{H}}, \tag{3.2.30}$$

where, H is scale height, R is universal gas constant, T is temperature and $p(0)$ is pressure at sea level. Then

$$\frac{dI_\lambda}{I_\lambda} = -k_\lambda r \rho(0) e^{\frac{-z}{H}}. \tag{3.2.31}$$

The atmosphere considered as flat and horizontally homogeneous.

$$\int_{I_\infty}^{I(\lambda,z)} \frac{dI(\lambda)}{I_\lambda} = -k_\lambda r \rho(0) \int_z^\infty e^{\frac{-z}{H}} dz, \tag{3.2.32}$$

$$\int_{I_\infty}^{I(\lambda,z)} \frac{dI(\lambda)}{I_\lambda} = -k_\lambda r\rho(0)[-H]e^{\frac{-z}{H}}/_z^\infty, \tag{3.2.33}$$

$$\int_{I_\infty}^{I(\lambda,z)} \frac{dI(\lambda)}{I_\lambda} = k_\lambda r\rho(0)H\left[0 - e^{\frac{-z}{H}}\right], \tag{3.2.34}$$

$$\ln\frac{I_{\lambda(z)}}{I_{\lambda(\infty)}} = -k_\lambda r\rho(0)He^{\frac{-z}{H}}, \tag{3.2.35}$$

$$I_\lambda(z) = I_\lambda(\infty)e^{-k_\lambda r\rho(0)He^{\frac{-z}{H}}}, \tag{3.2.36}$$

where, $I_\lambda(\infty)$ is intensity out side of the atmosphere.

3.3 Optical depth and transmissivity

Optical depth or optical thickness is the natural logarithm of the ratio of incident to transmitted radiant power through a material. Thus, the larger the optical depth, the smaller the amount of transmitted radiant power through the material. Transmissivity is a measure of the capacity of a material to transmit radiation (the ratio of the amounts of energy transmitted and received) [11]. After passing through a layer of atmosphere, the monochromatic intensity of radiation is decreased by

$$dI_\lambda = -I_\lambda n\sigma(\lambda)ds, \tag{3.3.1}$$

the decrease is due to absorption and scattering. n is effective number of scatters or absorbers.

$$\rho_{eff} = \rho\frac{M_{absorber}}{M_{air}} = n_{eff}m = \rho r, \tag{3.3.2}$$

$$r = \frac{M_{absorber}}{M_{air}}, \tag{3.3.3}$$

where, $M_{absorber}$ is the mass of the absorbers in a given mass of the air (M_{air}) and ρ is mass density of the air.

$$n_{eff} = \frac{\rho r}{m}, \tag{3.3.4}$$

$$dI_\lambda = -I_\lambda \frac{\rho r}{m} \sigma ds, \tag{3.3.5}$$

$$dI_\lambda = -I_\lambda k_\lambda \rho r ds, \tag{3.3.6}$$

where, $k_\lambda = \sigma(\lambda)/m$ is mass absorption or scattering coefficient.

If the radiation is incident at some angle θ, then the radiation has traversed a distance ds, through the air at an angle θ.

$$dz = ds \cos(\theta) \tag{3.3.7}$$

,

$$ds = dz/\cos(\theta) = -\sec(\theta)dz. \tag{3.3.8}$$

ds and dz are opposite, because as we see in fig (3.2) the radiation is moving down ward so, ds is negative and dz is up ward(positive). Substituting equations (3.3.8) in to (3.3.6) we have:

$$dI_\lambda = I_\lambda k_\lambda \rho r \sec(\theta) dz, \tag{3.3.9}$$

$$\int_z^\infty \frac{dI_\lambda}{I_\lambda} = \int_z^\infty k_\lambda \rho r \sec(\theta) dz, \tag{3.3.10}$$

$\rho = \rho(z)$,

$$\ln\left[\frac{I_\lambda(\infty)}{I_\lambda(z)}\right] = \sec(\theta) \int_z^\infty k_\lambda \rho(z) r dz, \tag{3.3.11}$$

$$\tau_\lambda = \int_z^\infty k_\lambda \rho(z) r dz = k_\lambda \rho(0) r \int_z^\infty e^{\frac{-z}{H}} dz = k_\lambda \rho(0) r H e^{\frac{-z}{H}}, \tag{3.3.12}$$

is called the normal optical depth or optical thickness of the atmosphere. It is a measure of the cumulative depletion that a beam of radiation directed straight downward with zero zenith angle would experience in passing through the layer.

$$\rho(z) = \rho(0) e^{\frac{-z}{H}}, \tag{3.3.13}$$

is the vertical density profile of the atmosphere,

τ_λ is dimension less because their dimensions cancel out each other.

$$\ln\left[\frac{I_\lambda(\infty)}{I_\lambda(z)}\right] = \sec(\theta)\tau_\lambda, \tag{3.3.14}$$

$$I_\lambda(z) = I_\lambda(\infty)e^{-\sec(\theta)\tau_\lambda}, \tag{3.3.15}$$

$$I_\lambda(z) = I_\lambda(\infty)t_\lambda, \tag{3.3.16}$$

$$t_\lambda = e^{-\sec(\theta)\tau_\lambda}. \tag{3.3.17}$$

is the transmissivity of the layer.

$$\tau = k_\lambda \rho(0) r H e^{\frac{-z}{H}}, \tag{3.3.18}$$

$$t_\lambda = e^{-\sec(\theta)k_\lambda\rho(0)rHe^{\frac{-z}{H}}}, \tag{3.3.19}$$

where, H depends on temperature, because $H = RT/g$. In the absence of scattering, the monochromatic absorptivity of the layer of the atmosphere is

$$a_\lambda = 1 - t_\lambda, \tag{3.3.20}$$

$$a_\lambda = 1 - e^{-\sec(\theta)\tau_\lambda}. \tag{3.3.21}$$

Absorption and emission of infrared radiation in the absence of scattering

Equation for the transfer infrared radiation through the gaseous medium(atmosphere).

$$dI_\lambda(absorption) = I_\lambda k_a(\lambda)\rho r \sec\theta dz, \tag{3.3.22}$$

$$dI_\lambda(absorption) = -I_\lambda a_\lambda, \tag{3.3.23}$$

where, $k_a(\lambda)$ is mass absorption coefficient, a_λ is the absorptivity of the layer.

$$dI_\lambda(emission) = B_\lambda(T)\varepsilon_\lambda, \tag{3.3.24}$$

where, ε_λ is emissivity of black body. $B_\lambda(T)$ is radiation emitted by a black body. The net change(decrease or increase)of the radiation after passing through a layer ds of the atmosphere is

$$dI_\lambda = -(I_\lambda a_\lambda - B_\lambda \varepsilon_\lambda), \tag{3.3.25}$$

since $a_\lambda = \varepsilon_\lambda$ for a black body

$$dI_\lambda = -(I_\lambda - B_\lambda)a_\lambda, \tag{3.3.26}$$

but $a_\lambda = k_a(\lambda)\rho r \sec(\theta)dz$,

$$dI_\lambda = [I_\lambda - B_\lambda(T)]\, k_a(\lambda)\rho r \sec(\theta)dz, \tag{3.3.27}$$

$$\int_{I_\lambda(z)}^{I_\lambda(\infty)} \frac{dI_\lambda}{I_\lambda - B_\lambda} = \int_Z^\infty k_a(\lambda)\rho r \sec(\theta)dz, \tag{3.3.28}$$

but $\rho = \rho(o)e^{-z/H}$.

For isothermal atmosphere we have

$$\int_{I_\lambda(z)}^{I_\lambda(\infty)} \frac{dI_\lambda}{I_\lambda - B_\lambda} = \int_Z^\infty k_a(\lambda)\rho(o) r \sec(\theta)e^{\frac{-z}{H}}dz, \tag{3.3.29}$$

$$\ln\left[\frac{I_\lambda(\infty) - B_\lambda}{I_\lambda(z) - B_\lambda}\right] = k_a(\lambda)\rho(o) r \sec(\theta)He^{\frac{-z}{H}}, \tag{3.3.30}$$

$$\ln\left[\frac{I_\lambda(z) - B_\lambda}{I_\lambda(\infty) - B_\lambda}\right] = -k_a(\lambda)\rho(o) r \sec(\theta)He^{\frac{-z}{H}}, \tag{3.3.31}$$

$$I_\lambda(z) = B_\lambda(T) + [I_\lambda(\infty) - B_\lambda]\, e^{-k_a(\lambda)\rho(o) r \sec(\theta)He^{\frac{-z}{H}}}. \tag{3.3.32}$$

$I_\lambda(\infty)$ is the incident monochromatic intensity at the top of the atmosphere.

Chapter 4

BASIC CLIMATE MODELLING THEORY

4.1 Global climate modelling

Climate models are important tools for improving our understanding and predictability of climate behavior on seasonal, annual, decadal, and centennial time scales. Models investigate the degree to which observed climate changes may be due to natural variability, human activity, or a combination of both. Their results and projections provide essential information to better inform decisions of national, regional, and local importance, such as water resource management, agriculture, transportation, and urban planning [13, 14].

A global climate model (GCM) is a complex mathematical representation of the major climate system components (atmosphere, land surface, ocean, and sea ice), and their interactions. Earth's energy balance between the four components is the key to long-term climate prediction. Increasing climate model accuracy involves continually improving its completeness, correctness, and resolution. This includes the addition of new processes that represent components such as land and ocean carbon cycles, interactions between cloud droplets and aerosols, and ice sheets [3, 9]. In general terms, a climate model could be defined as a mathematical representation of the climate system based on physical, biological and chemical principles (Fig. 4.1).

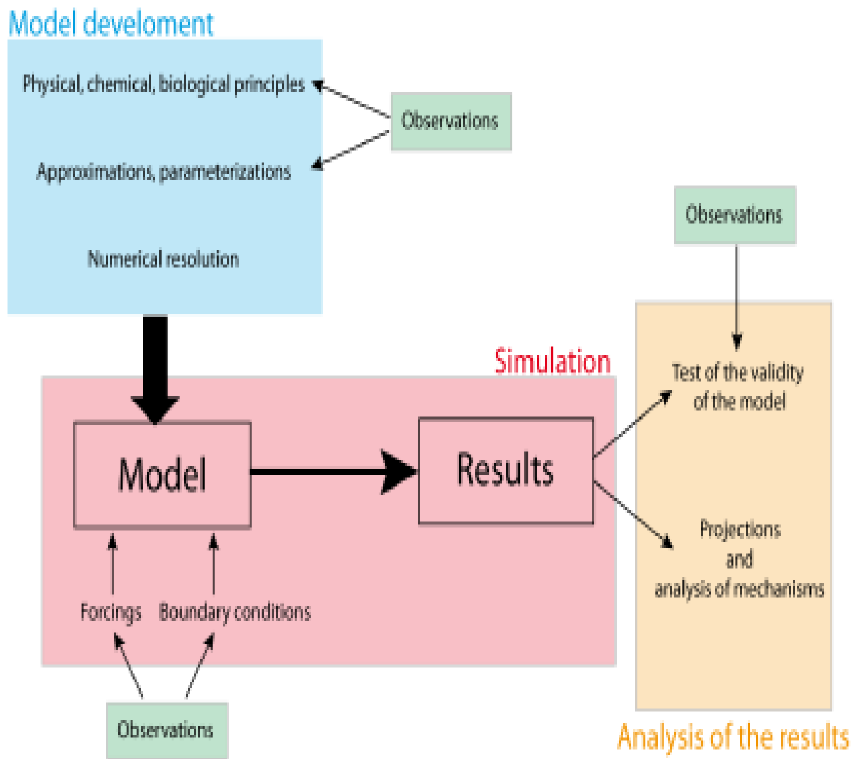

Figure 4.1: Represntations of the development and use of climate model [3]

The equations derived from these laws are so complex that they must be solved numerically. As a consequence, climate models provide a solution which is discrete in space and time, meaning that the results obtained represent averages over regions, whose size depends on model resolution, and for specific times.

There are two types of processes with in climate models that are used today: simulated and parameterized. Simulated processes are larger than grid-scale and based on bedrock scientific principles (conservation of energy, mass, and momentum).

An example of a simulated process is one that represents tropical cyclones and storm activity. Parameterized processes represent more complex processes that are smaller than grid scale (so, cannot be physically represented) using simpler processes. Their formulations are guided by fundamental physical principles, but also make use of observational data. An example of a parameterized process is one that represents cloud and aerosol composition [16].

4.2 Uses of climate models

GCMs are critical tools that enable us to improve the understanding and prediction of atmosphere, ocean, and climate behavior. Models allow us to determine the distinct influence of different climate features by providing a way of exploring climate sensitivities with experiments that cannot be performed on the actual Earth. Changes can be made to one feature in a climate model, such as warming or cooling ocean surface temperatures, to discern the impact those changes have on the climate. The uses for climate modelling also include diagnosis and prognosis. An example of a diagnostic use is detection and attribution. Detection and attribution require first demonstrating that a detected change is statistically significant, and then attributing this change to unnatural causes such as the role of anthropogenic forcing in 20^{th} century climate change. Prognostic climate modelling predicts future climate, such as global warming trends, using current or historic data (ocean structure, radiative forcing, etc) as a basis [15, 16].

Climate models use quantitative methods to simulate the interactions of the atmosphere, oceans, land surface, and ice. They are used for a variety of purposes from study of the dynamics of the weather and climate system to projections of future climate. The models are based mainly on the laws of physics, but also empirical techniques which use, for example, studies of detailed processes involved in cloud formation. The most sophisticated computer models simulate the entire climate system. As well as linking the atmosphere and ocean, they also capture the interactions between the various elements, such as ice and land. Climate models have been used successfully to reproduce the main features of the current climate; the temperature changes over the last hundred years, and the main features of the Holocene (6,000 years ago) and Last Glacial Maximum (21,000) years ago [23].

Atmospheric models calculate winds, heat transfer, radiation, relative humidity, and surface hydrology within each grid and evaluate interactions with neighboring points [16, 17]. GCMs have been used for a range of applications, including investigating interactions between processes of the climate system, simulating evolution of the climate system, and providing projections of future climate states under scenarios that might alter the evolution of the climate system. The most widely recognized application is the projection of future climate states under various scenarios of increasing atmospheric carbon dioxide (CO_2).

4.3 Types of climate models

Climate models can be classified by the basic physical processes included into the consideration,

a) Energy Balance Models (EBMs) Zero or one-dimensional models predicting the variation of the surface (strictly sea level) temperature as a function of the energy balance of the earth with latitude. This can be used to investigate sensitivity of climate systems to external changes and interpret results from complex models. (e.g., allow to change the albedo by latitude) calculate a balance between the incoming and outgoing radiation of the planet.

b) Radiative-Convective climate Models are 1-D with respect to altitude and compute the vertical (usually globally averaged) temperature profile by explicit modelling of radiative processes and 'convective adjustment' which re-establishes a predetermined lapse rate. RC models study the effects of changing atmospheric composition and investigate likely relative influences of different external and internal forcings.

c) Statistical Dynamical(SD) Models are 2-D models that deal explicitly with surface processes and dynamics in a zonally averaged frame work and have vertically resolved atmosphere. SD models used to make simulations of the chemistry of stratosphere and mesosphere.

d) Global Circulation Models (GCMs) where the 3-dimensional nature of the atmosphere and/or ocean is incorporated. The term GCM was introduced because one of the first goals of these models is to simulate the three dimensional structure of winds and currents realistically. Vertical resolution is generally finer than horizontal resolution. Includes Atmospheric General Circulation Models (AGCMs) and Ocean General Circulation Models (OGCMs) and the coupled AOGCM [3, 18].

EMICs (Earth Models of Intermediate Complexity) are located between those two extremes. They are based on a more complex representation of the system than EBMs but include simplifications and parameterizations for some processes that are explicitly accounted for in GCMs. The EMICs form the broadest category of models [24]. Some of them are relatively close to simple models, while others are slightly degraded GCMs.

When employed correctly, all the model types can produce useful information

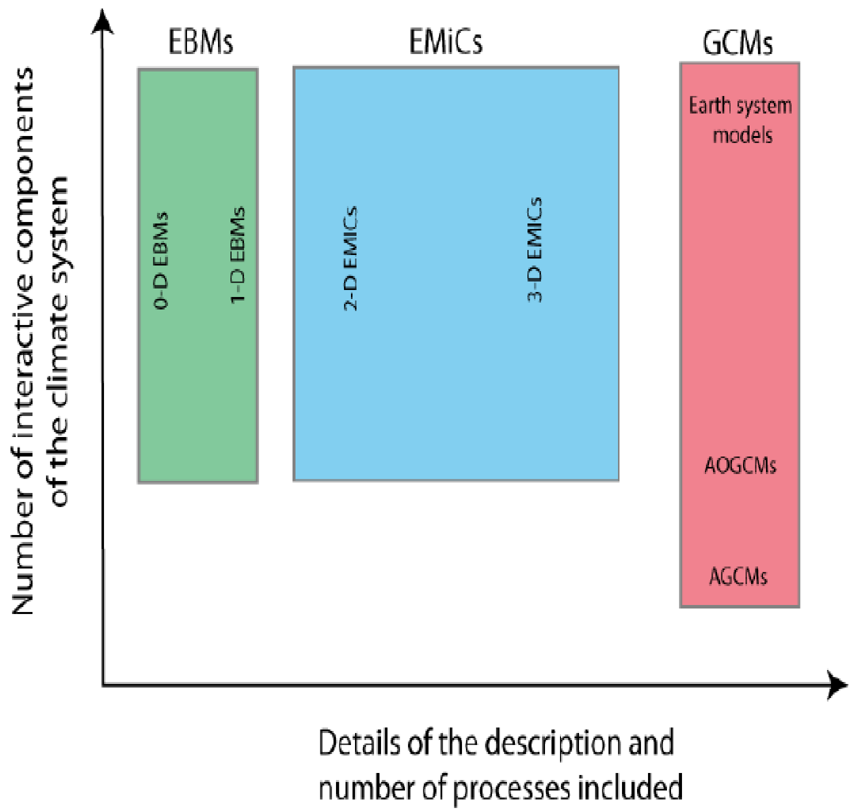

Figure 4.2: Types of climate model [24]

on the behavior of the climate system. There is no perfect model, suitable for all purposes. This is why a wide range of climate models exists, forming what is called the spectrum or the hierarchy of model. Depending on the objective or the question, one type of models could be selected. The best type of model to use depends on the objective or the question. On the other hand, combining the results from various types of models is often the best way to gain a deep understanding of the dominant processes in action [3, 9, 16]. In the majority of climate studies, at least the physical behavior of the atmosphere, ocean and sea ice must be represented. In addition, the terrestrial and marine carbon cycles, the dynamic vegetation and the ice sheet components are more and more regularly included, leading to what are called Earth-system models.

4.4 Components of a climate model

4.4.1 Atmosphere

The atmospheric component, which simulates clouds and aerosols, and plays a large role in transport of heat and water around the globe. The basic equations that govern the atmosphere can be formulated as a set of seven equations with seven unknowns: the three components of the velocity (components u, v, w), the pressure p, the temperature T, the specific humidity q and the density ρ. The seven equations, written for the atmosphere, are:

(1) Newtons second law (momentum balance, i.e.force equals mass times acceleration),

$$\vec{F} = m\vec{a}. \tag{4.4.1}$$

(2)

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\vec{\nabla}p - \vec{g} + \frac{\vec{F_{fric}}}{m} - 2\vec{\Omega} \times \vec{v}. \tag{4.4.2}$$

(3) In this equation, d/dt is the total derivative, including a transport term,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}, \tag{4.4.3}$$

$\vec{g}$ is the apparent gravity vector (i.e. taking the centrifugal force into account), $\vec{F_{fric}}$ is the force due to friction, and $\vec{\Omega}$ is the angular velocity vector of the Earth (the last term is the Coriolis force),

(4) The continuity equation or the conservation of mass

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}.(\rho\vec{v}). \tag{4.4.4}$$

(5) The conservation of the mass of water vapour

$$\frac{\partial \rho q}{\partial t} = -\vec{\nabla}.(\rho \vec{V} q) + \rho(E - C), \tag{4.4.5}$$

where E and C are evaporation and condensation respectively

(6) The first law of thermodynamics (the conservation of energy).

$$Q = C_p \frac{dT}{dt} - \frac{1}{\rho}\frac{dP}{dt}, \tag{4.4.6}$$

where Q is heating rate per unit mass C_p the specific heat

(7) The equation of state $P = \rho RT$. Before these equations are used in models some standard approximations have to be performed.

Unfortunately, these seven equations do not form a closed system. First, the frictional force and the heating rate must be specified. Computing the heating rate, in particular, requires a detailed analysis of the radiative transfer in the atmosphere, accounting for both the long wave and the shortwave radiation in the atmospheric columns as well as of the heat transfers associated with evaporation, condensation and sublimation. The influence of clouds in these processes is usually a source of considerable uncertainty. This part of the model is commonly referred to as the model physics while the calculation of the transport is called the dynamics.

Climate modelling through radiative-convective models

Radiative convective models have also been used extensively in studies concerned with stellar and planetary atmospheres. Radiative equilibrium temperature can not be sustained in the lower regions of the atmosphere. The lower regions the vertical gradient of the radiative equilibrium temperature, dT/dz(-dT/dz) is known as the lapse rate), is negative and so steep that the gradient of potential temperature $d\theta/dz$

is negative [14]. For a gray atmosphere in radiative equilibrium the long wave radiative flux is approximately given by

$$q_T = \frac{4\sigma}{3}\frac{dT^4}{d\tau}, \tag{4.4.7}$$

$$d\tau = -K\rho dz, \tag{4.4.8}$$

where, τ is the optical depth, K the absorption coefficient, σ Stefan Boltzman constant and ρ the density. The optical depth is measured from the top of the atmosphere.i.e., $\tau = 0$ at $z = \infty$. For simplicity of presentation the absorption of solar radiation by the atmosphere has been neglected. As a result the long wave radiative flux q_T is required to be is a positive constant in order to satisfy the condition radiative equilibrium. If we assume hydrostatic equilibrium so that $\tau = KP/g$ where, p is the pressure,

$$\frac{-dT}{dz} = \frac{g}{4R}\frac{q_T\tau}{q_T\tau + C_1}, \tag{4.4.9}$$

Where, c_1 is constant to be determined from the boundary conditions. We immediately note from equation(4.4.9) that -dT/dz increases with τ and $dT/dz \rightarrow 0$ as $\tau \rightarrow 0$. In the atmosphere the optical depth generally decreases exponentially with increasing altitude, and hence $dT/dz \rightarrow 0$ for large value of z.

The gradient of θ and T are related by

$$\frac{d\theta}{dz} = \frac{\theta}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right). \tag{4.4.10}$$

The atmosphere is unstable when

$$\frac{d\theta}{dz} < 0, \tag{4.4.11}$$

i.e when

$$-\frac{dT}{dz} > \Gamma, \tag{4.4.12}$$

where

$$\Gamma = \frac{g}{c_p}, \tag{4.4.13}$$

is the adiabatic temperature gradient.

When the lapse rate $(-dT/dz)$ is greater than Γ, the atmosphere is called super adiabatic while the converse situation is called sub adiabatic.

The underlying principles and assumptions in the model formulation are best explained from consideration of the thermodynamic energy equation given by

$$\frac{\partial}{\partial t}\rho c_p T + \nabla.\rho c_p V T + \frac{\partial}{\partial z}\rho c_p W T + \rho g W = Q + Q_F, \tag{4.4.14}$$

where, V is the horizontal velocity vector, W is the vertical velocity, c_p specific heat at constant pressure, g acceleration due to gravity, T atmospheric temperature, Q net radiative heating and Q_F is the frictional heating. Hence, the earth's atmosphere model is used only in a horizontally averaged sense. up on averaging (4.4.14) with respect to latitude and longitude and denoting the averaged quantities with angle bracket's we have

$$\frac{\partial < \rho C_p T >}{\partial t} = -\frac{d}{dz}(q_T + q_s + q_c), \tag{4.4.15}$$

where, $q_c =< \rho c_p W T >$ is the convective flux. $< \rho g W >= 0$ to satisfy mass conservation. By integrated the above equation to yield

$$q_T(z) + q_s(z) + q_c(z) = const = 0, \tag{4.4.16}$$

where, q_T is net long wave radiative flux, q_s is net solar radiative flux and q_c is the convective flux. With the boundary condition

$$q_T(\infty) + q_s(\infty) = 0, \tag{4.4.17}$$

$$q_c(\infty) = 0. \tag{4.4.18}$$

Since W should vanish at the top of the atmosphere, $q_c(\infty) = 0$. The quantity $q_T(\infty)$ the thermal radiation emitted to space by the earth atmosphere system, and $q_s(\infty)$ is the difference between the incoming solar radiation and the solar radiation which is reflected to space by the system i.e $q_s(\infty)$ is the total solar radiation absorbed by the planet as a whole.

q_c can be formulated in terms of a fickian type diffusion equation

$$q_c = -\rho c_p K_H \left(\frac{dT}{dz} + \Gamma \right), \tag{4.4.19}$$

when

$$\frac{dT}{dz} + \Gamma \leq 0. \tag{4.4.20}$$

$$q_c = 0, \tag{4.4.21}$$

when

$$\frac{dT}{dz} + \Gamma > 0. \tag{4.4.22}$$

where, K_H is the thermal diffusivity [14, 17]. The heat flux is assumed to be proportional to the gradient of the potential temperature:$\left(\frac{d\theta}{dz} \propto \left[\frac{dT}{dz} + \Gamma\right]\right)$. This theory of free convection[25] was originated by priestly [1959]. By invoking similarity arguments, priestly showed that K_H was given by

$$K_H = 1.32 z^2 \left(\frac{\left|(\frac{dT}{dz}) + \Gamma\right| g}{T} \right)^{\frac{1}{2}}. \tag{4.4.23}$$

The region where the radiative equilibrium lapse rate was super adiabatic, the temperature gradient was almost adiabatic. $(\frac{dT}{dz}) + \Gamma \approx 0$.

At altitude z in a vertical column model the net flux of long wave radiation for wave number interval $\Delta\nu_t$ is given by

$$q_t(z) = \pi B_t(0) T_t(z, 0) + \int_0^\infty \pi B_t(z') dT_t(z', z), \tag{4.4.24}$$

here, $\pi B_t(z)$ is the spectrally averaged value of the planck function.

$$\pi B_t(z) = \frac{1}{\Delta\nu_t}\int_{\Delta\nu_t} d\nu \pi B_\nu(z), \tag{4.4.25}$$

where $\pi B_\nu(z)$ is the planck function for wave number ν and temperature T(z); $T_t(z,\ z')$ is the spectrally averaged flux transmissivity:

$$T_t(z,\ z') = \frac{1}{\Delta\nu_t}\int_{\Delta\nu_t} d\nu T_\nu(z,\ z'), \tag{4.4.26}$$

where, $T_\nu(z,\ z')$ is the monochromatic flux transmissivity between levels z and z' of the model atmosphere. The flux transmissivity is related to the transmissivity $\tau(z,\ z',\ \mu)$ by

$$T_t(z,\ z') = 2\int_0^1 d\mu\mu\tau(z,\ z',\ \mu), \tag{4.4.27}$$

where, μ is the cosine of the angle between the ray path and the vertical. The total net long-wave radiative flux is obtained by summing the contributions from each spectral interval. The total net long-wave radiative flux is given by

$$q_T(z) = \sum_t q_t(z)\Delta\nu_t. \tag{4.4.28}$$

Because the planck function changes slowly with wave number for the temperature of the earth's atmosphere,(4.4.21)is valid for rather large spectral intervals.

4.4.2 Ocean

The ocean component, which simulates current movement and mixing, and biogeochemistry, since the ocean is the dominant reservoir of heat and carbon in the climate system. The major equations that govern the ocean dynamics are based on the same principles as the equations for the atmosphere. The only significant difference is that the equation for the specific humidity is not required for the ocean, while a new equation for the salinity needs to be introduced. The equation of state is also fundamentally different. Unlike the atmosphere, there is no simple law for the ocean and the standard equation of state is expressed as a function of the pressure, the temperature and the salinity as a long polynomial series. It is much easier to compute the heating rate in the ocean than in the atmosphere. In addition to the heat exchanges at the surface, the only significant heat source in the ocean is the absorption of solar radiation. This is taken into account in the model through an exponential decay of the solar irradiance. The situation for salinity is even more, as there is no source or sink of salinity inside the ocean. The equations governing these two variables are thus relatively simple:

$$\frac{dT}{dt} = F_{sol} + F_{diff}, \tag{4.4.29}$$

$$F_{diff} = \frac{dT}{dt} - F_{sol}. \tag{4.4.30}$$

Where, F_{sol} is the absorption of solar radiation in the ocean and F_{diff} is a diffusion term. Equation (4.4.29) does not apply to the in situ temperature, but the potential temperature in order to account for the effect of the compressibility of seawater. The difference between those two temperatures is relatively low in the upper ocean, but it can reach several tenths of a degree in the deeper layers, an important difference in areas where the gradients are relatively small. As small-scale processes tend to mix properties, they are generally modelled as a diffusion term F_{diff} [26]. However, all small-scale processes cannot be represented by a diffusion term. For instance, dense water formed at high latitudes can flow down the slope in narrow boundary currents called overflows. They have a strong influence on the water mass properties but could not be represented on the model grid scale. In this case, a parameterization of their effects as a transport process rather than a diffusion term appears more appropriate.

4.4.3 Sea ice

The sea ice component, which modulates solar radiation absorption and air-sea heat and water exchanges. The physical processes governing the development of sea ice can be conceptually divided into two parts (Fig. 4.3). The first one covers the thermodynamic growth or decay of the ice, which depend on the exchanges with the atmosphere and the ocean. For those processes, the horizontal heat conduction through ice, can be safely neglected because the horizontal scale is much larger than the vertical one. The thermodynamic code of a sea ice model is thus basically one-

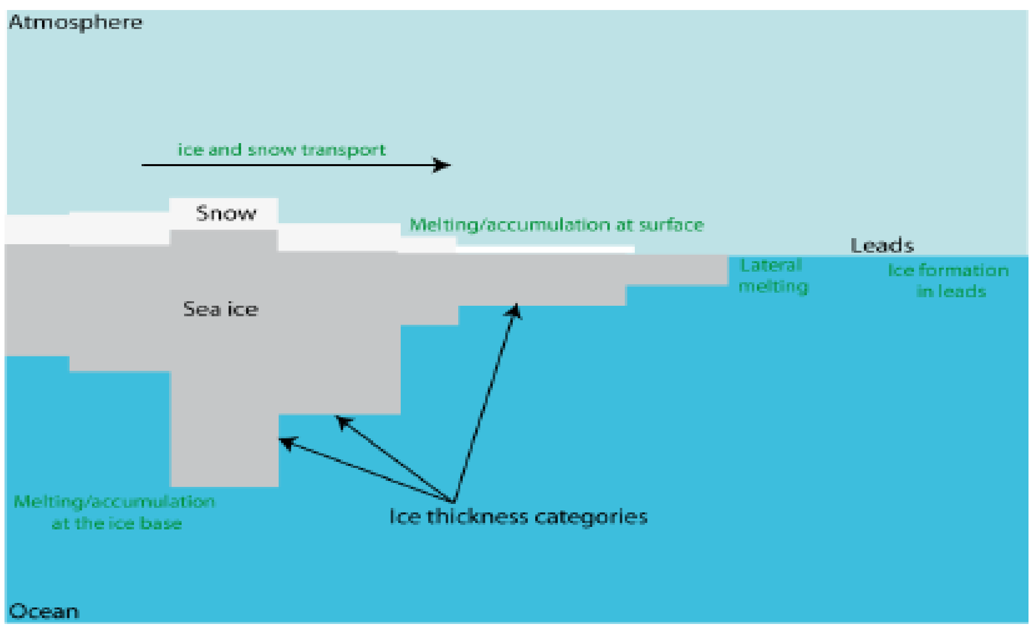

Figure 4.3: The main process that have taken in to account in a sea ice model [3]

dimensional over the vertical, and the conduction equation can be written:

$$\rho_c C_{pc} \frac{\partial T_c}{\partial t} = K_c \frac{\partial^2 T_c}{\partial Z^2}, \tag{4.4.31}$$

where, ρ_c, C_{pc}, and K_c are the density, specific heat and thermal conductivity, and T_c is the temperature. The subscript c stands for either ice (i) or snow (s). Equation (4.4.31) can be written as

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial z^2}, \tag{4.4.32}$$

where

$$D = \frac{K_c}{\rho_c C_{pc}}, \tag{4.4.33}$$

is the thermal diffusivity. The equation to be solved is

$$D\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}. \tag{4.4.34}$$

From the form of this equation, it is obvious that a solution could be found in which T(z, t) is solely a function of z^2/t, or, for that matter, z/t^2. Thus, let $u = z/t^{1/2}$ and you will see that equation(4.4.31) reduces to the second order total differential equation.

$$D\frac{d^2 T}{du^2} = -\frac{u}{2}\frac{dT}{du}, \tag{4.4.35}$$

Let

$$T' = \frac{dT}{du}, \tag{4.4.36}$$

and it becomes even easier a first order equation.

$$D\frac{dT'}{du} = -\frac{1}{2}uT' \tag{4.4.37}$$

Up on integration we obtain

$$\ln T' = -\frac{u^2}{4D} + \ln A. \tag{4.4.38}$$

Where $\ln A$ is an integration constant to be determined by the initial and boundary conditions. That is,

$$T' = Aexp[-u^2/(4D)]. \tag{4.4.39}$$

We have to integrate again, with respect to u.

$$T = A \int exp[-u^2/(4D)]du. \tag{4.4.40}$$

When studying the large-scale dynamics of sea ice, the ice is modelled as a two dimensional continuum. This hypothesis works if, in a model grid box, a large number of ice floes of different sizes and thicknesses are present as well as the leads. Newton's second law then gives

$$m\frac{d\vec{u_i}}{dt} = \vec{\tau_{ai}} + \vec{\tau_{wi}} - mf\vec{e_z}\times\vec{u_i} - mg\vec{\nabla}\eta + \vec{F_{int}}, \tag{4.4.41}$$

where, m is the mass of snow and ice per unit area, $\vec{u_i}$ is the ice velocity. $\vec{\tau_{ai}}$ and $\vec{\tau_{wi}}$ are the forces per unit area from the air and water. $\vec{F_{int}}$ is the force per unit area due to internal interactions and f, $\vec{e_z}$, g and η are respectively the Coriolis parameter, a unit vector pointing upward, the gravitational acceleration, and the sea-surface elevation. The first two terms on the right hand side represent the interactions with the ocean and the atmosphere. The third term is the Coriolis force and the forth term the force due to the oceanic tilt. The internal forces $\vec{F_{int}}$ are a function of ice thickness and concentration, providing a strong link between dynamics and thermodynamics, while the velocity obtained from equation (4.4.41) is used in the computation of the transport of the model state variables such as the ice thickness, the concentration of each ice thickness category and the internal sea ice temperature and salinity. If we ignore the terms independent of time we have,

$$\frac{d\vec{u_i}}{dt} = -f\vec{e_z}\times\vec{u_i}, \tag{4.4.42}$$

$$\int_{u_i(o)}^{u_i(t)} \frac{d\vec{u_i}}{\vec{e_z}\times\vec{u_i}} = -\int_0^t f dt, \tag{4.4.43}$$

$$\ln\left(\frac{u_i(t)}{u_i(o)}\right) = -ft, \tag{4.4.44}$$

$$u_i(t) = u_i(o)exp(-ft). \tag{4.4.45}$$

4.4.4 Land surface

The land surface component, which simulates surface characteristics such as vegetation, snow cover, soil water, rivers, and carbon storing. As with sea ice, horizontal heat conduction and transport in soil can be safely neglected. Therefore, thermodynamic processes are only computed along the vertical in a similar way to Equation (4.4.27). In the first generation of land surface models, only one soil layer was considered. Soil temperature can then be computed from the energy balance at the surface:

When a snow layer is present, the computation of the development of the snow depth, density and concentration is part of the surface energy balance. This is very important for the albedo, whose parameterization as a function of the soil characteristics (snow depth, vegetation type, etc) is a crucial element of surface models. The latent heat of fusion times the evaporation rate ($L_f E$), as is classically done in surface models. This evaporation rate depends on the characteristics of the soil and the vegetation cover as well as on the water availability [7, 16, 17].

Chapter 5

RADIATION AND SIMPLE CLIMATE MODELS

5.1 Planetary radiative equilibrium and radiative energy distribution

Radiation is a key factor controlling the Earth's climate. Radiation equilibrium at the top of the atmosphere (TOA) represents the fundamental mode of the climate system[17]. Planetary radiative equilibrium (over the entire planet and long time interval): TOA incoming radiation $\equiv$ TOA outgoing IR radiation

TOA incoming radiation = incoming solar radiation -reflected solar radiation

TOA outgoing IR radiation = outgoing IR radiation (emitted by the atmosphere-surface system)

5.2 Radiative and radiative-convective equilibrium models

Climate models can be classified by their dimensions:

Zero Dimensional Models (0-D): consider the Earth as a whole (no change by latitude, longitude, or height)

One Dimensional Models (1-D):allow for variation in one direction only (e.g., resolve the Earth into latitudinal zones or by height above the surface of the Earth)

Two Dimensional Models (2-D): allow for variation in two directions at once (e.g., by latitude and by height)

Three Dimensional Models (3-D) allow for variation in three directions at once i.e., divide the earth-atmosphere system into domains, each domain having its own independent set of values for each of the climate parameters used in the model[19, 20].

In a real atmosphere, solar heating rates do not equal to IR cooling rates. This imbalance is the key driver of atmospheric dynamics. Let's consider a hypothetical motionless atmosphere, radiative transfer processes only. Then the climate state (temperature profile) is determined by the radiative equilibrium. The radiative equilibrium climate model is a model that predicts the atmosphere temperature profile of an atmosphere in radiative equilibrium

$$\frac{dF_{net}}{dZ} = 0. \tag{5.2.1}$$

Under the "gray atmosphere" assumption, we can solve for the temperature profile analytically. Long wave flux profile:

$$F^{\uparrow}(\tau) = F_{sun}\left(1 + \frac{3}{4}\tau\right). \tag{5.2.2}$$

and

$$F^{\downarrow}(\tau) = F_{sun}\left(\frac{3}{4}\tau\right). \tag{5.2.3}$$

where

$$F_{sun} = (1 - \bar{r})F_0/4. \tag{5.2.4}$$

Atmosphere black body emission and temperature profiles:

$$B(\tau) = \frac{F_{sun}}{2\pi}\left(1 + \frac{3}{2}\tau\right), \tag{5.2.5}$$

and

$$T^4(\tau) = T_e^4\left(\frac{1}{2} + \frac{3}{4}\tau\right). \tag{5.2.6}$$

Surface temperature is discontinuous with the atmosphere (hotter):

$$B_s = B(\tau^\star) + \frac{F_{sun}}{2\pi} \tag{5.2.7}$$

and

$$T_s^4 = T_e^4\left(1 + \frac{3}{4}\tau^\star\right). \tag{5.2.8}$$

Implications: Greenhouse effect larger $\tau^\star$ increases surface temperature

Runaway greenhouse effect $\tau^\star$ increases $\Rightarrow T_s$ increases

Positive feedback higher temperature $\Rightarrow$ greenhouse gases

Eddington gray radiative equilibrium temperatures; If one wants to have the temperature profile in terms of height, one needs to relate optical depth to height[11,17]. Assume that an absorber has the exponential profile

$$\rho_a = \rho_0 exp\left(\frac{-Z}{H_a}\right), \tag{5.2.9}$$

So the profile of optical depth is

$$\tau(z) = \bar{K}_a\rho_0 H_a exp\left(\frac{-Z}{H_a}\right) = \tau^\star exp\left(\frac{-z}{H_a}\right). \tag{5.2.10}$$

Temperature profile

$$T^4(z) = T_e\left(1 + \frac{3}{4}\tau^\star exp\left(\frac{-z}{H_a}\right)\right). \tag{5.2.11}$$

Lapse rate

$$\frac{dT}{dz}(z) = \frac{-3}{8}\frac{\tau^\star}{1+\frac{3}{2}\tau^\star}\frac{T(z)}{H_a}exp\left(\frac{-z}{H_a}\right). \tag{5.2.12}$$

Implications: Low $\tau^* \Rightarrow$ stable atmosphere

Smaller scale height Ha of the absorber causes steeper lapse rate

Steepest lapse rate near the surface (z=0)

5.3 Simple energy balance models

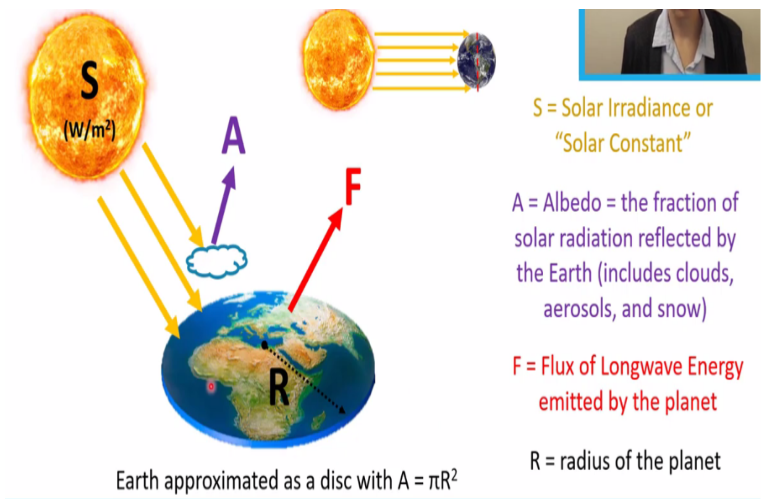

Figure 5.1: Zero dimensional energy balance model [31]

Let's estimate the effective temperature assuming that the Earth is in the radiative equilibrium. But in this case we define albedo A=$\bar{r}$ and solar constant S=F_{s0}. The sun emits $F_s = 6.2x10^{-7}W/m^2$ (a black body with about T= 5800K). From the energy conservation law, we have

$$F_s 4\pi R_s^2 = F_{s0} 4\pi D_O^2, \tag{5.3.1}$$

where R_S is the radius of the sun $(6.96x10^5 km)$;F_{s0} is the solar flux reaching the top of the atmosphere (called the solar constant about 1368 W/m^2) at the average distance of the Earth from the sun, $D_0 = 1.5x10^8$ km. Thus we have

$$F_{s0} = F_s R_s^2 / D_0^2. \tag{5.3.2}$$

If the instantaneous distance from the Earth to sun is D, then the total sun energy flux F_0 reaching the Earth is

$$F_0 = F_{s0}(D_0/D)^2. \tag{5.3.3}$$

The total sun energy intercepted by the cross section of the Earth is $F_{s0}\pi R_e^2$ where R_e is the radius of the Earth. This energy is spread uniformly over the entire planet (with surface area $(4\pi R_e^2)$. Thus the amount of received energy per unit surface becomes

$$F_{s0}\pi Re^2/4\pi R_e^2 = F_{s0}/4. \tag{5.3.4}$$

Therefore, the total energy $Q_{in}(inW/m^2)$ absorbed by the earth-atmosphere system is:

$$Q_{in} = (1 - \bar{r})F_{s0}/4, \tag{5.3.5}$$

where r is the spherical (or global) albedo. Spherical albedo of the earth is about 0.3. Assuming that the Earth is a black body with temperature T_e, we have:

$$Q_{out} = F_b = \sigma_B T_e^4, \tag{5.3.6}$$

where σ_B is the Stefan-Boltzmann constant [8, 18]. From the balance of incoming and outgoing energy, the effective temperature of the Earth is:

$$Q_{in} = Q_{out}, \tag{5.3.7}$$

$$F_{so}(1 - \bar{r})/4 = \sigma_B T_e^4, \tag{5.3.8}$$

$$T_e^4 = F_{s0}(1 - \bar{r})/4\sigma_B, \tag{5.3.9}$$

$$T_e = 255K = -18^0C, \tag{5.3.10}$$

is very low. T_e is much lower than the global average surface temperature (about 288 K). Because we didn't include the greenhouse effect and ignore the temperature structure.

Planet	Relative distance to the sun with respect to the Earth	Global albedo	T_e(K)
Mercury	0.39	0.06	441
Venus	0.72	0.78	226
Earth	1	0.3	255
Mars	1.52	0.17	217
Jupiter	5.2	0.45	106

Table 5.1: Effective temperature of some planets in the radiative equilibrium [19]

Up welling flux at the surface $F_s^{\uparrow} = \sigma T_s^4$ Up welling flux $F_{TOA}^{\uparrow}$ at the top of the atmosphere (TOA): from satellite observations The difference between the up welling fluxes at the surface and TOA gives a measure of greenhouse effect G:

$$G = \sigma T_s^4 - F_{TOA}^{\uparrow}. \tag{5.3.11}$$

NOTE: G is the amount of heat (e.g.,measured in Watts) per unit area of the Earth. A reasonable estimate of the greenhouse effect is

$F_{TOA}^{\uparrow} = 235W/m^2$,

$T_s^4 = 288K$,

$\sigma T_s^4 = 390W/m^2, \;\; G = 390 - 235 = 155W/m^2$.

5.3.1 Simple model of greenhouse effect (single layer gray energy balance model)

Let's include the atmosphere assuming that it emits (absorbs) as a gray body. Emissions of greenhouse gases alter atmospheric opacity [21].

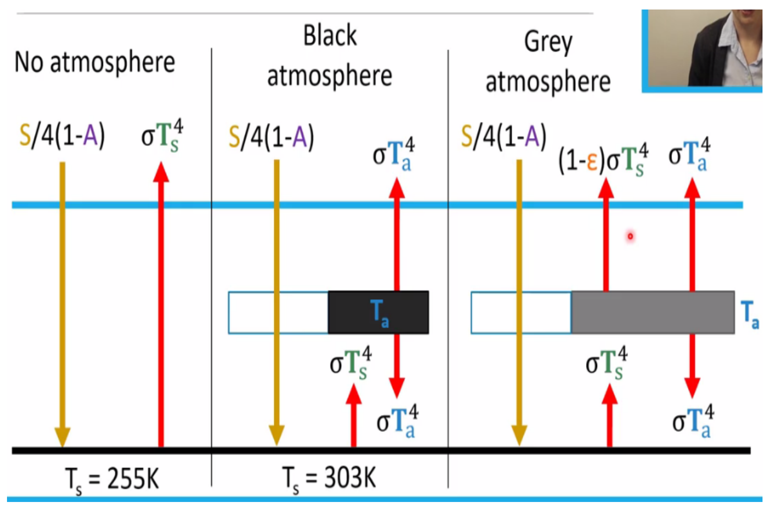

Figure 5.2: Relation ship between body's temperature with no atmosphere, a black body atmosphere and in a gray body atmosphere [31]

The observed temperature 33 K higher than effective temperature. The earth is not a perfect black body some emitted energy is re-absorbed by the atmosphere [31]. $F = \sigma \varepsilon T_e^4$, ε is emissivity factor.

$$T_e^4 = F_{so}(1 - \bar{r})/4\varepsilon\sigma, \tag{5.3.12}$$

With an emissivity of 0.61, we get $T_e =\sim 288$ K.

Observed surface temperature T_s=288 K.

Energy balance at the top of atmosphere

$$\frac{F_{So}}{4}(1-\bar{r}) = \sigma T_a^4. \tag{5.3.13}$$

but $T_a = T_e = 255K$.

Energy balance at the surface

$$\sigma T_s^4 = \frac{F_{So}}{4}(1-\bar{r}) + \sigma T_a^4, \tag{5.3.14}$$

$$T_s = \left(\frac{F_{So}(1-\bar{r})}{4\sigma} + T_a^4\right)^{1/4} = 303K. \tag{5.3.15}$$

Which is much warmer than the observed temperature. We have a gray atmosphere not a perfect black body. Zero dimensional Energy balance models are a simple but use full tool for estimating the balance of energy from incoming solar and out going terrestrial radiation [31].

Assume that the atmosphere does not absorb solar radiation - all is absorbed at the surface. TOA balance:

$$\frac{F_0}{4}(1-\bar{r}) = (1-\varepsilon\sigma T_s^4) + \varepsilon\sigma T_a^4. \tag{5.3.16}$$

Surface balance:

$$\frac{F_0}{4}(1-\bar{r}) + \varepsilon\sigma T_a^4 = \sigma T_s^4, \tag{5.3.17}$$

where σ is the Stefan-Boltzmann constant and ε is the emissivity of the atmosphere.

$$T_s^4 = \frac{F_0(1-\bar{r})}{2\varepsilon(2-\varepsilon)}, \tag{5.3.18}$$

for $\varepsilon = 0.6$,

$\Rightarrow T_s = 278K$.

NOTE: increasing for ε increases T_s. This is so-called "runaway greenhouse effect" : warmer $T_s \Rightarrow$ more evaporation $\Rightarrow$ more water vapor $\Rightarrow$ higher emissivity $\Rightarrow$ warmer T_s. Because we assume that atmosphere is a gray body.

5.3.2 Simple model of greenhouse effect (single black layer with spectral window energy balance model)

A more realistic way to deal with partial long wave transparency of the atmosphere is to assume that a fraction of the spectrum is clear. Assume that

$$f = \frac{1}{\sigma T^4} \int_{v_1}^{v_2} B_v(T) dv, \tag{5.3.19}$$

is the fraction of long wave (LW) spectrum which is completely transparent and the remainder of the LW spectrum is black. Surface is black and no SW absorption by the atmosphere. TOA balance:

$$\frac{F_0}{4}(1 - \bar{r}) = f\sigma T_s^4 + (1 - f)\sigma T_a^4. \tag{5.3.20}$$

Surface balance:

$$\frac{F_0}{4}(1 - \bar{r}) + (1 - f)\sigma T_a^4 = \sigma T_s^4. \tag{5.3.21}$$

Thus we can express T_s and T_a via T_e

$$T_s = \left(\frac{2}{1+f}\right)^{\frac{1}{4}} T_e, \tag{5.3.22}$$

and

$$T_a = \left(\frac{1}{1+f}\right)^{\frac{1}{4}} T_e. \tag{5.3.23}$$

Limits: No window f=0 $\Rightarrow$

$$T_s = 2^{\frac{1}{4}} T_e, \tag{5.3.24}$$

and

$$T_a = T_e, \tag{5.3.25}$$

All window f=1 $\Rightarrow$

$$T_s = T_e, \tag{5.3.26}$$

and

$$T_a = T_e/2^{\frac{1}{4}}, \tag{5.3.27}$$

Earth: f is about 0.3 $\Rightarrow$ T_s =284 K and T_a = 239K.

5.3.3 Derivation of the Eddington gray radiative equilibrium

If a fluctuating variable exhibits excessive intermittency it can fall into a grey zone between internal and external. For instance, desert dust storms, which can alter the vertical distribution of radiation significantly over large regions, are highly intermittent. Another grey variable is human activity. On one hand, climate determines greatly the distribution of human settlement and activity. It has been known for many decades that human activity can directly impact climate, as exemplified in studies of desertification, deforestation and greenhouse gas emissions [27, 28, 29]. Desertification and deforestation result in direct alterations to surface albedo, surface roughness, and evapotranspiration. This has direct impacts on local partition of energy available for heat and water fluxes. These alter circulation and moisture patterns and hence local and global climate. Since the atmosphere is gray (all wavelength are equivalent), one can write the wavelength integrated thermal emission radiative transfer equation (no scattering)

$$\mu \frac{dI}{d\tau} = I - B, \tag{5.3.28}$$

where, I is the integrated radiance ($Wm^{-2}st^{-1}$), τ increases downward, and $\mu > 0$ in the upward direction.

Note that deriving the variation of B with the optical depth τ is equivalent to determining the temperature profiles since the black body emission is a function of temperature only.

Using the Eddington approximation, the net flux (positive upward) becomes

$$F_{net} = 2\pi \int_{-1}^{1} I\mu d\mu = \frac{4\pi}{3} I_1. \tag{5.3.29}$$

The radiative equilibrium assumption implies that F_{net} and I_1 is constant with optical depth. Integrating the above radiative transfer equation over $d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu d\mu = 2\pi \int_{-1}^{1} I\mu d\mu - 2\pi \int_{-1}^{1} B d\mu, \tag{5.3.30}$$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B. \tag{5.3.31}$$

Under the radiative equilibrium assumption, we have

$$I_0 = B. \tag{5.3.32}$$

Integrating the radiative transfer equation over $\mu d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu^2 d\mu = 2\pi \int_{-1}^{1} I\mu d\mu - 2\pi \int_{-1}^{1} B\mu d\mu. \tag{5.3.33}$$

Since B is isotropic the last term drops out leaving

$$\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net} = \frac{4\pi}{3} I_1, \tag{5.3.34}$$

$$\frac{dB}{d\tau} = I_1. \tag{5.3.35}$$

Thus, the solution for B is simply a linear function of optical depth:

$$B(\tau) = B(0) + I_1 \tau, \tag{5.3.36}$$

Constants B(0) and I_1 need to be determined from the boundary conditions. Top of the atmosphere: First boundary condition: no thermal down welling flux

$$F^{\downarrow}(0) = 2\pi \int_{-1}^{0} I\mu d\mu = \pi B(0) - \frac{2\pi}{3} I_1 = 0. \tag{5.3.37}$$

so we have

$$I_1 = \frac{3}{2} B(0), \tag{5.3.38}$$

or

$$F_{net} = 2\pi B(0). \tag{5.3.39}$$

Second boundary condition: up welling long wave flux is equal to the absorbed solar flux F_{sun}:

$$F^{\uparrow}(0) = 2\pi \int_{0}^{1} I\mu d\mu = \pi B(0) + \frac{2\pi}{3} I_1 = F_{sun}, \tag{5.3.40}$$

Recall that the absorbed solar flux F_{sun} is

$$F_{sun} = (1 - \bar{r})(F_0/4), \tag{5.3.41}$$

Putting in

$$I_1 = \frac{3}{2} B(0), \tag{5.3.42}$$

gives $F_{sun} = 2\pi$ B(0)=F_{net}. So now we have the B(0) and I_1 and thus the atmosphere Planck function profile is determined

$$B(\tau) = \frac{F_{net}}{2\pi}(1 + \frac{3}{2}\tau). \tag{5.3.43}$$

The final step is to apply the boundary condition at the surface to obtain the surface temperature T_s. This boundary condition is that the emitted flux by the surface equals to the sum of the down welling shortwave and long wave flux at the black surface:

$$F_{sun} + F^{\downarrow}(\tau^{\star}) = \pi B, \tag{5.3.44}$$

where

$$F^{\downarrow}(\tau^{\star}) = \pi B(\tau^{\star}) - \frac{2\pi}{3} I_1, \tag{5.3.45}$$

Using

$$F_{sun} = \frac{4\pi}{3} I_1, \tag{5.3.46}$$

gives the emission from the surface

$$B_s = B\left(\tau^{\star}\right) + \frac{F_{sun}}{2\pi}, \tag{5.3.47}$$

which is discontinuous with the atmospheric emission. It can be expressed in terms of temperature by

$$T^4(\tau) = T_e^4 \left(\frac{1}{2} + \frac{3}{4}\tau\right), \tag{5.3.48}$$

where

$$\sigma T^4 = F_{sun}, \tag{5.3.49}$$

$$T_{top}^4 = \frac{1}{2} T_e^4, \tag{5.3.50}$$

$$T_s^4 = T_e^4 \left(1 + \frac{3}{4}\tau^{\star}\right). \tag{5.3.51}$$

For $F_0 = 1366 W/m^2$ and $\bar{r} = 0.3$:[30] $T_e = 255K$ and a "top"temperature $T_t = 214$ K. Assuming a global averaged surface air temperature of $T(\tau^{\star}) = 288$ K gives a gray body optical depth of $\tau^{\star} = 1.5$, and a surface skin temperature of $T_s = 308$ k.

Chapter 6

SUMMARY AND CONCLUSION

In this study we have seen the solar radiative transfer and global climate modelling. Radiative transfer or radiation transport is the theory of how radiation and matter interact based on the particle description of light. Virtually all the exchange of energy between the Earth and the rest of the universe takes place by radiation transfer. Radiation transfer is also a major way of energy transfer between the atmosphere and the underlying surface and between different layers of the atmosphere. All objectives radiate energy, not merely at one single wavelength but over a wide range of different wavelengths. The hotter the objective, the shorter the wavelength of the peak radiation.

The specific intensity of radiation is the energy flux per unit time, unit frequency, unit solid angle and unit area normal to the direction of propagation. According to beer lambert's law the monochromatic intensity of radiation is decreased by dI_λ= -$I_\lambda k_a \sec\theta$ dz, where k_a is the absorption coefficient. Using such a relation we have determine the optical depth and the transmission through the path ds. Assuming that the sun is directly over head the appropriate optical path for each frequency is the optical depth, measured vertically down wards from the top of the atmosphere (taken to be $z = \infty$). $\tau_\lambda = k_a \rho(o) H r e^{-z/H}$ shows how the optical depth increases as the solar radiation penetrates down wards i.e as z decreases and $t_\lambda = e^{-\sec(\theta)\tau_\lambda}$ is the transmissivity of the layer. In general, we have discussed the solar zenith angle, the fraction of absorbers in the atmosphere, the absorption (scattering) coefficient, the scale height, the altitude and the density of the atmosphere are all the factors that affect the incident intensity of radiation.

We have also seen the climate modelling and their uses. Global climate models have been used for a range of applications, including investigating interactions between processes of the climate system, simulating evolution of the climate system, and providing projections of future climate states under scenarios that might alter the evolution of the climate system. Finally we have discussed the main climate system components treated in a climate model that are: The atmospheric component, which simulates clouds and aerosols, and plays a large role in transport of heat and water around the globe. The land surface component, which simulates surface characteristics such as vegetation, snow cover, soil water, rivers, and carbon storing. The ocean component, which simulates current movement and mixing, and biogeochemistry, since the ocean is the dominant reservoir of heat and carbon in the climate system and the sea ice component, which modulates solar radiation absorption and air-sea heat and water exchanges. Similarly we have determined the relation ship between the surface and equilibrium temperature of the atmosphere using the derivations of the eddington gray radiative equilibrium in a simple climate models. Then the climate state (temperature profile) is determined by the radiative equilibrium. The radiative equilibrium climate model is a model that predicts the atmosphere temperature profile of an atmosphere in radiative equilibrium.

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