Sc

THE EFFECTIVENESS OF TEACHING MATHEMATICAL CONCEPTS BY THE INDUCTIVE AND DEDUCTIVE APPROACHES: AN EXPERIMENTAL STUDY IN GRADE ELEVEN OF ENTOTO COMPREHENSIVE SECONDARY SCHOOL

BY

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ABSTRACT

An experimental study was designed to investigate the relative effectiveness of the deductive and inductive approaches in teaching mathematical concepts using specific models. 390 grade eleven students were selected by an achievement test whose content was grade nine and ten mathematics and their average result of semester I of 1992/93 academic year. They were grouped into low-, medium-, and high-achievers on the basis of the above criteria. Each group was divided into two sub groups who did not have significant difference in ability at the beginning. Eight concepts of mathematics were taught for three weeks. A week after the termination of the lessons, a test whose content was the eight concepts taught was administered and 346 students, who attended all the lessons were used for data analysis. The Mann Whitney U-test for larger samples at 0.05 level of significance showed no significant differences between the effectiveness of the two methods and between the two-medium- and the two-high achiever groups. But a significant difference between the two low-achiever groups in favor of the inductive method was found. The overall test results favored the inductive method although there were no statistically significant differences.

CHAPTER ONE

INTRODUCTION

1.1 Background of the study

The process of education should not be haphazard, rather should be planned and organized. Curriculum may be taken broadly as a plan for the educational process. In curriculum planning, there is a continum consisting of planning at the beginning and implementation at the end. It is at the implementation level that the desired behavioral changes on the part of the learner take place. The statement of good behavioral objectives, the selection and organization of content will not bring about the desired behavioral changes in themselves. There are other factors among which is the selection of appropriate teaching methods and procedures.

Method, according to Gage (1988:531), refers to the more formal and self-conscious aspects of teaching; the way in which the teacher interacts with students; the kind of questions posed and the way they are addressed; the pacing of the lesson, the activities students are required to take part in; and the kind of decision making that the teacher allows the students. In general there is no one method which is the weakest nor the best. A method best serves one purpose and fails for others. Results of studies regarding the effectiveness of teaching methods vary from situation to situation. For instance results of studies on the relative effectiveness of the deductive and inductive methods vary from researchers to researchers. Some favored the deductive method, others favored the inductive method, and others concluded that there is no significant difference between the two methods for a certain teaching purpose. Hence there should be a continuous study on the relative effectiveness of teaching methods.

Discovery of ideas, theorems, methods of reasoning, and solving problems by pupils is and should be an objective of the mathematics program at every grade level. This way of learning makes the study of mathematics more interesting and students get meaning out of it. To this end, John Perry, as cited in Kidd (1970:vii), argued that there is hardly any man who may not become a discoverer if favorable conditions are created and if he is given chances of exercising his individuality.

It is true that in every discipline concepts are basis for its further development. This particularly is true of mathematics for it is highly sequential by its nature. The understanding of subsequent concepts is hardly possible if prerequisite concepts are not clearly established. The treatments given to students regarding concept learning should emphasize the process- product approach because providing ready made definitions and procedures will not bring about meaningfulness of the subject and also not interesting. In Addis Ababa high schools, the nature of concept learning is that of deductive in nature, students are not encouraged to discover or more generally the process aspect is neglected[1]. This study, therefore, attempts to compare the relative effectiveness of the deductive and inductive methods of teaching concepts of mathematics at different levels of student characteristics or ability.

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[1] The writer has observed different mathematics lessons during the teaching practice program of Addis Ababa University.

1.2. Statement of the Problem

The purpose of this study was to compare the relative effectiveness of the deductive and inductive strategies in learning concepts of mathematics at different ability levels. Thus the study was conducted to test the following null hypotheses.

H_{01}. There is no significant difference in the test scores of the groups who are taught by the deductive or inductive approach.

H_{02}. There is no significant difference in the test scores of high achievers who are taught by the deductive or inductive approach.

H_{03}. There is no significant difference in the test scores of medium achievers who are taught by the deductive or inductive approach.

H_{04}. There is no significant difference in the test scores of low achievers who are taught by the deductive or inductive approach.

If either of the above null hypotheses was to be rejected as a result of the test, the following alternatives were accepted.

H_{a1}. There is a significant difference in the test scores of the groups who are taught by the deductive or inductive approach.

H_{a2}. There is a significant difference in the test scores of high achievers who are taught by the deductive or inductive approach.

H_{a3}. There is a significant difference in the test scores of medium achievers who are taught by the deductive or inductive approach.

H_{a4}. There is a significant difference in the test scores of low achievers who are taught by the deductive or inductive approach.

The interpretation of these alternative hypotheses will be in accordance with the direction the statistical values take.

1.3. Significance of the Study

It will not be exaggerating if one explains the difficulty of modern life with out mathematics. Students learn mathematics not only to be mathematicians but also they need it in all walks of life. The banker, accountant, economist, carpenter, pilot, etc. will not be successful in their jobs without mathematical knowledge. Scientific and technological advancements will not function in its absence. Kinney and Purdy (1952:1) stressed the importance of mathematics when they say, if we wiped out from the literature and from the minds of chemists, physicists, and engineers the mathematics that underlies our daily life, then in a few hours our systems of communication and transportation would be paralyzed; our industries and utilities would grind to a halt; civilization, as we know it, would cease to exist.

All these extended usefulness of mathematics would be possible if students are equipped with basic concepts, theorems, procedures, and skills of it. Failure to understand important concepts will result in discouragement, absenteeism, and even behavioral problems. A clear improvement in attitudes and even achievement lies in clarifying these concepts.

According to Marks (1967:241), the format of many textbook pages is still of the form, "definition, illustration, practice" from which little discovery is possible. Even if textbooks attempt to emphasize discovery, sometimes, teachers fail to recognize it and use the material in the same old way. But if teachers are to approach and analyze their teaching of concepts with more than common-sense notions, then according to Conney and Davis (1976:217), they should be aware of the nature of the knowledge they are teaching and of appropriate pedagogical approaches for teaching these types of knowledge.

As already mentioned earlier, mathematics lessons, particularly concept learning is exclusively done deductively in Addis Ababa high schools and also sufficient time is not given for concept attainment. But according to Gage (1988:532), variety in teaching is a good thing because it recaptures students' attention, arouse curiosity, prevents boredom, and gives students new ways to learn when old ones may have failed. Furthermore, according to the knowledge of the writer, there is no any study done in Ethiopia on the relative effectiveness of the deductive and inductive methods of teaching concepts of mathematics.

This study is hoped that the results:

- will indicate the better approach for learning concepts of mathematics
- will indicate which kinds of students (low-, medium-, high-achievers) benefit more from which approach
- will contribute to the development of a theory of teaching mathematics in secondary schools of

Ethiopia

- will help to give advise to secondary school teachers to the problem of how to teach concepts of mathematics efficiently, because teachers expect feasible solutions from researchers

1.4. Methods and Procedures

1.4.1. Sources of Data

To compare the effectiveness of the two strategies, grade eleven students of Entoto Government School were used, because the writer believed that students of grades eleven and twelve have a good deal of experience regarding the rigor of mathematics and proficiency of language. But grade twelve students were excluded since they were busy in preparing themselves for E.S.L.C.E., therefore grade eleven students of the aforementioned school are sources of the data.

On the basis of the scores of test II, whose content is grade nine and ten mathematics (see Appendix III), students were grouped into low-, medium-, and high-achievers. But such a test, which is taken to be a criterion for forming such groups, had to be constructed according to test construction criteria. Accordingly, test I, consisting of 40 multiple choice items (see Appendix I) was administered to grade eleven students of Menelik II school to a total number of 252 students. The raw scores ranged from 4 to 35.

To determine the quality of the test discrimination index of items was calculated according the steps suggested by Ebel (1979:258) (see Appendix II). Items 2, 3, 7, 12, 16, 18, 22, 32, 37 and 38 were rejected because of their low discrimination index and items 8, 24, 29, and 33 were subjected to improvement. All the other items, with discrimination index of 0.30 and above, were accepted and are part of test II.

Items 4, 14, and 34, with item discrimination index less or equal to 0.19, according to Ebel (1979:267), could have been rejected automatically. But an item, according to Mehrens and Lehman (1984:196), should not be discarded from a test simply because it has a low discrimination index if it has no weaknesses in its formulation. Thus the three items and item 19 were taken for test II as they are.

The five items, items 8, 24, 29, and 33 were improved as follows.

In item 8, choice (d) was canceled because it gives clue, for two lines can not intersect in two distinct points.

Item 24 was rewritten as follows.

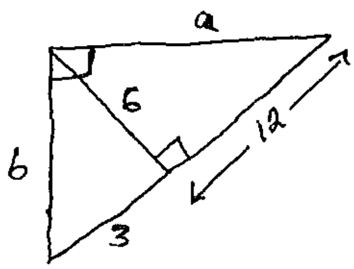
What is the lengths of a and b respectively in the figure below?

a) $3\sqrt{5}$, $6\sqrt{5}$

b) $6\sqrt{5}$, $3\sqrt{5}$

c) 6, $3\sqrt{5}$

d) $3\sqrt{5}$,6



The inclusion of the height, h, as an unknown quantity makes the calculation cumbersome and tedious and hence is assigned a value.

In item 29, choice (c) was canceled because it gives clue for the two graphs indeed intersect at one point, (0,1) and this can not be a good destructor.

In item 33, choice (d) was rewritten as tan θ = tan ρ. Infact, there is nothing wrong with the former formulation of this choice, but it is ambiguous for students were not introduced with the reciprocals of trigonometric ratios.

After rejecting or improving items, test II consisting of 30 multiple choice items (see Appendix III), was administered to total number of 865 grade eleven students of Entoto School. The results ranged from 4 to 26 in raw scores (see Appendix IV). The reliability of the test was calculated using Kuder-Richardson (K-R21') reliability coefficient as cited in Ebel (1979:281). The reliability coefficient, r, of test II was r=0.65.

The test, test II, was a major criterion if not the only one, for grouping students into low-, medium-, and high-achievers because 1) the test was constructed based on the principles of constructing multiple choice tests, 2) the scope of the test was wider (it is of grades 9 and 10 content), 3) item analysis was conducted, and 4) the test was surprised. The other criterion, though not determinant, was the average scores of students of the current year of semester I. The role of this criterion is that students with high scores in test II and extremely low average score, or students with very low score in test II and very high average score are not selected. That is the average score of students was used as a counter check for the formation of reliable groups.

Based on the above criteria three hundred and ninety (390) students: 130 high achievers whose scores ranged from 17-26, 130 medium achievers whose scores ranged from 13-14, and 130 low achievers whose scores ranged from 4-9 were selected and grouped (see Appendix V). Students in the three levels of achievement had to be taught in the two methods- the deductive and inductive methods. Thus students in each level were further divided into two forming a total of six experimental sections as follows.

1. Low achievers I
2. Low achievers II
3. Medium achievers I
4. Medium achievers II
5. High achievers I
6. High achievers II

Assignment in the two groups of each level, i.e., in the two groups of the low-, medium-, and high-achievers was done so that there were no significant differences between them. Each of the six groups were consisted of 65 students which is the average class size in Addis Ababa government schools.

Each group was taught eight concepts. Groups 1,3, and 5 through the deductive method and groups 2,4, and 6 through the inductive method. Concepts taught for all groups consisted of the same number of examples, the same content, the same time of teaching, and the same questions and exercises except that the lessons for groups 1, 3, 5 were prepared deductively and that of 2, 4, 6 inductively (see Appendix VI and Appendix VII respectively). The lessons for all groups were conducted by the writer himself to avoid differences in competence as a result of using different teachers because the intention is to keep all other situations constant except student ability and method of teaching.

The eight concepts were labelled as

a) Distance Formula in a Plane

b) Equation of a circle

c) Slope of a line

d) Reflections

e) Exponential functions

f) Logarithmic functions

g) Arithmetic Progression

h) Geometric Progression

The first three and the last two concepts were from grade twelve mathematics textbook and the other three are from grade eleven mathematics textbook. These concepts were new to students, i.e., the students selected were not taught these concepts before the experiment was conducted. The lessons for these concepts, i.e., the lessons prepared deductively and inductively were prepared according to the following model.

The Taba-Steinhoefel model for teaching concepts of Mathematics[2]

This model emphasizes the formation of concepts and discovery of principles and generalizations. The model encompasses general strategies only, that is, specific steps and procedures are not included.

2 For the details of the models of Taba and Steinhoefel see chapter two of this paper

Learning Concept

Inductively through concept formation	Deductively
a. Providing instances or examples representing the concept	a. Stating the definition of the concept
	b. Stressing the defining attributes of the concept
b. Enumeration (of similarities in the examples)	c. Providing examples representing the concept
c. Grouping regarding common attributes	
d. Labeling or describing the categories using common attributes and their inter-concept relationships	d. Proving that examples posses all of the attributes of the

After each group took the lessons for the eight concepts, using the methods indicated, a test, test III, consisting of 20 items (see Appendix VIII) was administered to all groups on the same day and at the same time to measure their achievements. The content of test III consisted of the eight concepts taught. The correlation between scores on the odd- and even- numbered items (see Appendix IX) was calculated using the rank difference correlation as described in Ebel (1979:220) and was

0.61. However, this correlation coefficient shows only the internal consistency of the test. The reliability of the whole test, according to the Spearman- Brown formula, as cited in Ebel (Ibid:278), is given by $r= 2\ r_s\ /(1+r_s)$, where r_s is the correlation coefficient obtained by splitting the test into halves. Thus for test III, r= 0.76. The results of test III were used for analysis.

1.4.2. Method of Data Analysis

In order to test the four stated two-tailed hypotheses, the scores from test III were arranged in four forms as follows:

a) The scores of students who were taught with the deductive method (groups 1, 3, 5) and with the inductive method (groups 2, 4, 6) were arranged for comparison

b) The scores of the two groups, low achievers I and II who took the lessons through the deductive and inductive methods respectively were arranged for comparison

c) The scores of the two groups, medium achievers I and II who took the lessons through deductive and inductive methods respectively were arranged for comparison

d) The scores of the two groups, high achievers I and II, who took the lessons through deductive and inductive methods respectively were arranged for comparison

Then the Mann-Whitney U-test was employed to test the four two-tailed hypotheses for the scores arranged above with the level of significance of 0.05.

1.5 Limitations of the Study

The study was intended to see differences in achievement as a result of teaching concepts of mathematics through the deductive or inductive methods among students of the same level of academic ability. The experimental lesson was conducted in six sections using the two methods, i.e., high achievers I, medium achievers I, and low achievers I were taught by the deductive method while high achievers II, medium achievers II, and low achievers II by the inductive method.

The results of the study may have been affected by the communications made by students of different groups and taught by different methods because there was no chance to control students from communicating.

The other problem the writer faced in conducting this study was the absence of recent literatures (or researches conducted) on the subject under discussion. Thus the writer was obliged to use those old literatures and furthermore was not able to relate the findings of this study to findings that are currently obtained (if there are any).

1.6 Definition of Terms

The following key terms are defined below and were used in the study accordingly.

1. Concept: An idea or aggregation of ideas that has been acquired as a symbol or generalization for an intangible, for example, the concept of square, circle, etc.

2. Deductive Method: Method of teaching, study, or argument which proceeds from general or universally applicable principles to particular applications of these principles and shows the validity of the conclusions.

3. High Achievers: Those students who have relatively high academic achievement in the pretest which was administered at the beginning of this experimental study and in the average score of semester I.

4. Inductive Method: Method of teaching based on the presentation to the learner of a sufficient number of specific examples to enable him to arrive at a definite rule, principle, or fact.

5. Low Achievers: Those students who have relatively low academic achievement in the pretest which was administered at the beginning of this experimental study and in the average score of semester I.

6. Medium Achievers: Those students who have relatively medium academic achievement in the pretest which was administered at the beginning of this experimental study and in the average score of semester I.

CHAPTER TWO

2. REVIEW OF THE RELATED LITERATURE

In this chapter what educators or scholars contributed to the understanding of key concepts and procedures were reviewed. It consists of three parts. In the first part, the concept of "concept", its characteristics, and consequences of misconceptions were dealt with. In the second part, teaching-learning strategies for developing concepts was dealt with. In particular, the characteristics of the deductive and inductive methods of teaching were stressed.

Finally, researches conducted and results obtained regarding the relative effectiveness of the deductive and inductive methods of teaching were summarized.

2.1 The Teaching-Learning of Concepts

An activity on which teachers of any subject spend much time, and infact, the central objective of any academic instruction, according to Henderson (1967:573), is the teaching of concepts. Lawther (1989:519) further capitalized on the impossibility of successful advancement in the field of human learning without concepts being used as tools of thought. He further noted that "unorganized masses of factual material can neither be comprehended nor retained. They must be classified under their appropriate concepts, and mentally filed for use" (Ibid.). Thus it would be logical to ask what concepts really are.

2.1.1. Characteristics of Concepts

The word "concept" seems to have as many definitions as there are writers on conceptual learning. Some of the definitions given by scholars are as follows. "A concept is an idea consisting of characteristics that are common across objects or events" (Clifford, 1981:279); "a generalization about data which are related" (Lovell, 1964:13); "a symbol that represents a class or group of objects or events with common properties" (Klein, 1987:308); "the characteristics that make certain items examples of a type of thing and that distinguish any and all examples from non examples" (Ehrenberg, 1981:37). Bruner, Goodnow, and Austin, as cited in Rosskopf (1968:79), defined it as "a way of grouping an array of objects or events in terms of those characteristics that distinguish this array from other objects or events in the universe". Gagne summarized general properties common to definitions of a concept as:

1. A concept is an inferred mental process.
2. The learning of a concept requires discrimination of stimulus objects (distinguishing "positive" and "negative" instances")
3. The performance which shows that a concept has been learned consists in the learner being able to place an object in a class.

(Gagne, 1966:83)

A concept, according to Cooney (1976:216) and Ehrenberg (Ibid.), involves a label, a name or an expression to refer to any and all examples of a given concept. Ehrenberg further explained the difference between the concept and the label as a person being able to know the label for a concept without knowing the characteristics of any and all examples and vice versa.

Most explications of a concept, according to Cooney and Henderson, involves two components namely the object language and the meta language (Henderson, 1967:573); a designator expression and the referent of the expression (Cooney, 1976:216). They further explained that the object language or the referents represent the necessary and sufficient conditions for an object to be a concept, and the designatory expression or the meta language represent definition of objects or other expressions which connote the same ideas or denote the same objects. Many natural concepts, however, according to Donahoe (1980:282), do not have common attributes, they rather are described, i.e., they are characterized in terms of family resemblance relationships, fuzzy boundaries, and internal structures.

Smith (1978:340) defined a concept as a combination of things as a synthesis of many examples or things that exemplify the concept, as involving rules, or descriptions (although some concepts do not have convenient rules), and as having titles eventhough not all concepts have titles or names. To understand a concept, according to Klein (1987:309), one must learn what attributes or referents are relevant. He further noted that for each concept, a rule (if there is) defines or describes which objects or events are examples of that particular concept. When the child forms a concept, according to Lovell (1964:12), he has to be able to recognize and differentiate between the properties of the objects, and generalize his findings in respect of any common features he may find.

Ehrenberg (1981:37-38) pinpointed some of the characteristics of a concept as all concepts being abstract since they constitute a generalized mental image of the characteristics that make items examples; that concepts cannot be verified like facts as being "right" or "wrong"; that concepts are hierarchical, i.e., some classes include other classes. Fredrick and Davies, as cited in Rosskopf claimed that anything that is a concept has the following attributes.

1. Psychological meaningfulness
2. Intrinsic, functional, or formal properties
3. Abstractness
4. Inclusiveness
5. Generality
6. Structure
7. Function

(Rosskopf, 1968:78)

Henderson (1967:574) summarized the nature or the characteristics of concepts of mathematics as they are embodied in an abstract theory, they are precise, the referent set is not empty, and formal definitions are used to teach them.

Good (1973:124) defined a concept operationally as "an idea or representation of the common element or attribute by which groups or classes may be distinguished".

2.1.2. The Role of Concepts in the Teaching-Learning Process

To learn a concept, according to Ehrenberg (Ibid.), students must form a clear mental image of how examples differ from non- examples. Concepts, according to Klein (1987:309), enable us to grasp objects that share common properties and respond in a similar manner to each example of a concept.

An outgrowth of "back to basics" movement, according to Chisko (1985:592), made mathematics educators to stress the need for students' understanding of concepts and shifted the emphasis from specific content to process of mathematical thinking. He further stressed the importance of concepts when he wrote "for students to ask, and eventually answer, reasonable questions, they must be able to arrange and analyze information. Systematic thinking is necessary". (Ibid:522)

Feher, as cited in sobel, considered the importance of learning concepts in the field of mathematics as:

> The learning process of going from experience with things, to thinking about things, to abstraction and concept formation, and finally to reorganizing the newly learned concept into the whole structure is, in the field of mathematics, the one that has the promise of permanence of learning to the solution of quantitative problems. The building of concepts is so necessary if we are to develop mathematics as a way of thinking that will serve us in our various life careers.
>
> (Sobel, 1956:425).

Regarding the importance of concepts in problem solving, Rosskopf (1968:80) noted that the first step towards solving a problem is categorization of the problem and for this to be possible, it is necessary to bring into play concepts that have been effective in solving problems of this particular category.

The learning of concepts, as already mentioned earlier, is a mental process. Thus, knowing appropriate facts, algorithms, and procedures, according to Garofalo, is not sufficient to guarantee success in problem solving. He implicitly stressed the role of concepts in solving mathematical problems when he wrote:

> ...the decision one makes and the strategies one uses in connection with the control and regulation of one's actions, the emotions one feels while working on mathematical task, and the beliefs one holds relevant to performance on mathematical tasks, influence the direction and outcome of one's performance.
>
> (Garofalo, 1989:502)

Bruner and others, as cited in Rosskopf, summarized the advantages of categorizing of objects by grouping together things that satisfy the defining characteristics of a class as

> an individual a) reduces the complexity of his environment; b) identifies the objects in a particular universe of discourse; c) reduces the necessity for constant learning; d) gives direction to activity; e) orders and relates classes.
>
> (Rosskopf, 1968:80)

2.1.3 Consequences of Learning Concepts Incorrectly

The most flagrant and frequent mathematical errors made by remedial students in high school, according to Roseman (1985:502) stem from their failure to understand important concepts. Such misconceptions, he further explained, results in discouragement, absenteeism, and behavioral problems. A major reason why students develop negative attitudes towards the learning of mathematics, according to the National Science Foundation (NSF) report (1983), as cited in Stepans(1985:1), is the overwhelming emphasis placed on symbol manipulation.

The "where did that come from?" reaction,either verbalized or not, according to Garofalo (1991:318), follows the presentation of not-so-obvious concepts, formulas, rules and procedures. He further explained that when mathematical concepts and procedures are presented without clarifying the process of obtaining these concepts and procedures, the result is hiding the fact that mathematics is created by people, that it involves intuition, exploring, conjecturing, and reasoning, and that it is purposeful.

Skemp, as cited in Garofalo (1991:319), sees the teaching of meaningless rules as insults to intelligence. Hedrick explicitly stressed the importance of process or meaningful learning in his long statements when he wrote:

> shall we abandon processes and ways of thinking in favor of fact-learning and skill acquisition because these latter seem on the surface the more "practical"? What if the fact,the skill,does not apply to life? What if it does not transfer? May not the process, the way of thinking, transfer to life when the fact, the skill, does not? Which, then, is the more practical-the "practical" one? May not the impractical prove the more practical?
>
> What shall it profit a man if he learns every fact, and acquires every skill of mathematics - if he looses the soul of the subjects?
>
> (Hedrick, 1967:49)

Teaching for understanding, according to Herscovics (1980:572-579), requires that the continuity of mathematical concept be demonstrated to the student during, and prior to, the introduction of the new mathematical symbols used to express the concept. He further elaborated this fact by exemplifying that to some students, translating a word problem into an equation is tantamount to translating it into a language unknown to them.

Educators attributed some of the failures in mathematics instruction due to an instructor assuming either or both of the following. First, assuming that students have understood a mathematical concept at a high level of operational thinking when in reality they have a much lower level of mastery. (Rosskopf, 1968:85). Second, "simplifying" a concept to the point of making it incorrect or misleading because of assuming that students would not understand if they are told all of the details. (Bompart, 1973:431). Bompart further argued that simplification is indeed appropriate, and is often desired, but extreme precision in the initial introduction of a concept is, perhaps, unjustified.

Children with low conceptual levels, according to Hunt as cited in Wright (1977:150), respond uniformly to varying circumstances, have relatively little self control, and depend on others to provide them with conceptual structures. Bompart (Ibid:431-433) illustrated the practice of "teaching concepts incorrectly", some of which are listed below.

1. The equation $X^2 + 1 = 0$ has no solution.
2. The algebraic expression $X^2 - 2$ can not be factored

3. This is the graph of the function
4. A polynomial equation of degree five or greater can not be solved.
5. As n decreases, $\frac{1}{n}$ increases.

2.2 Teaching-Learning Strategies for Developing Concepts

A strategy is defined as "a plan of relatively structured procedures by which specific objectives are achieved" (Hernandez, 1973:607); "a carefully planned calculation and coordination of the specific ends and means necessary to achieve a goal" (Rosskopf, 1968:80). A continuous problem facing educators, according to Koran (1971:300), is the determination of the most effective methods for training large numbers of students having dissimilar patterns of abilities. Infact, educators and researchers do not ask only which is best, Method A or Method B, but also, according to Wright (1977:150), for which students are Methods A and B appropriate?

However, according to Hernandez (1973:607), no one strategy has been shown effective under all conditions. The choice of an instructional strategy depends, according to scholars, upon the emotional and cognitive capacities of the students, the level of readiness and individual learning styles, as well as the teacher's personality. (Hernandez, 1973:611; Rosskopf, 1968:80; Ehrenberg, 1981:42).

Educators labeled the process of concept learning into two phases, namely concept formation and concept acquisition (attainment). (Lakshmi, 1976:27; Tennyson, 1986:44; Joyce, 1985:29). Understanding and comprehension go beyond perception as they involve the integration and organization of the knowledge obtained through perception. Thus concept formation, according to Kuppuswamy (1984:156-157) is at the basis of our thinking as well as reasoning. Concept formation, according to Joyce (Ibid.), is the act by which new categories are formed, it is an act of invention. In concept formation, according to Lakshmi (Ibid.), students learn the concept from the very scratch, i.e, they learn the very attributes of the stimulus. In concept attainment, on the other hand, students' task is only to discover the dimension already learned previously. In concept attainment, the concept already exists. Students in the process of concept attainment, treat various objects, events or ideas as belonging to a class and make equivalent or nearly equivalent responses to all the members of that class. (Tennyson, 1986:44; Kuppuswamy, 1984:156-157; Lakshmi, 1976:27). Joyce (1985:29) further explained the process of concept attainment to be the search for and listing of attributes that can be used to distinguish examples from non examples of various categories.

Polya, as cited in Taback, contended regarding the importance of student's role in learning by saying "what the teacher says in the classroom is not unimportant, but what the students think is a thousand times more important". (Taback, 1992:255). Smith (1978:340-357) suggested possible strategies for developing concepts as guidance versus discovery, proceeding from simple to complex, proceeding from the concrete to the abstract, from rules or generalizations to specific instances (the deductive method), and from specific or particular instances to generalizations or rules (the inductive method). In order to foster growth of word - symbol meanings, Lawther stressed that the following should be kept in mind:

Concepts:

1. develop through numerous specific experiences from every day life
2. grow slowly from the concrete, perceptual level to the abstract, relationship - of-common-elements level
3. must await educational readiness for learning on a higher, abstract level

(Lawther, 1989:520)

He further suggested that the teacher can accelerate the formation of concepts by:

1. introducing the concept with examples in which the basic relationship stand out
2. keeping the early examples as close as possible to the concrete objects and perceptual level of the every day life of the child
3. reducing the number of concepts to be covered, and allowing more time and a greater wealth of experience for each
4. giving regular practice in use, with guidance, motivation, and opportunities for success
5. directing the child's attention to the general traits or aspects by copious illustrations, examples, pictures, and the like
6. helping the child to formulate in verbal symbols an expression of these general traits, or aspects, for the purpose of analysis
7. helping him to discover what symbolic cues in his own repertoire will fuse into the concept if brought together under the focus of his attention.

(Ibid.)

2.2.1. The Deductive Method of Teaching

The deductive method, also called "expository teaching technique" (Hernandez, 1973:608), "reception learning" (Ausubel, 1966:158), the "ruleg" method (Hermann, 1971:22), the traditional method (Lindgren, 1985:218), is defined as "a scientific form of reasoning that proceeds from general statements (such as those found in theories) to statements about specific situations" (Clifford, 1981:686); "a method in teaching that proceeds from rules of generalizations to examples and subsequently to conclusions or to the applications of the generalizations" (Belcastro, 1966:78).

2.2.1.1. Characteristics of the Deductive Method

In the deductive approach, according to Lindgren (1985:215-216), students are presented with a pre-processed information in a final form. He further mentioned the role of teachers being that of expounding and explaining simply because they want students to learn rules before they tackle problems. In the deductive concept learning approach, according to Clifford (1981:686), students learn concepts by having them labelled, defined, and sometimes even exemplified. According to Ausubel (1966:158), the principal content of what is to be learned is presented to the learner in more or less final form.

According to Davis (1951:142) and Hernandez (1973:608), from postulational view point, a mathematical subject is built upon a set of fundamental assumptions or postulates with the aid of definitions and the process is wholly deductive. Educators have tried to evaluate the deductive method of teaching. It has the advantage of stressing the importance of the well-known, familiar theory without theoretical concepts (Lindgren, 1985:218); it takes less time as a result of which wide range of material can be covered since concepts are immediately labelled and defined (Clifford, 1981:690). Its possible drawbacks are that the theories the deductive method requires us to learn may be inappropriate and incomprehensible, furthermore, students learned by this method are likely to feel insecure and to flounder when placed in learning situations in which they receive little direction (Lindgren, 1985:220); it makes students receptive and passive Ausubel, (Ibid.); it does not help students to see mathematics as a creative, dynamic subject. To this end, it would be worthwhile to quote Brutlag.

> In doing mathematics, a deductive proof is usually the last thing done in an investigation. For mathematicians, the deductive argument is the frosting on the cake. But for many students, a deductive argument, given without any personal investigation, is the last nail in the coffin....
> (Brutlag, 1990:611)

2.2.1.2 Model for the Deductive Method of Teaching

The deductive method of teaching, as already described, is a method that proceeds from generalizations, rules, principles to particular instances. Steinhoefel (1976:63-66) developed a model for teaching mathematical concepts. The model consists of deductive reasoning in which the rules, generalizations, definitions are given and the follow-up operations in which students' actions to the given generalization in practicing the patterns. The model is the following.

Teaching Concepts of Mathematics Deductively

Deductive Reasoning

- Stating the principle, law, theory generalization, of a given concept
- Stressing the defining attributes
- Providing examples that represent the concept
- Proving that the examples posses all attributes of the concept defined

Follow-up Operations

- Developing operational patterns for doing the actions: identification, exemplification, application based on the definitions, rules, generalizations
- Practicing the patterns
- Checking whether students master the concept or not
- Deciding whether to reteach or to proceed

2.2.2 The Inductive Method of Teaching

The inductive method, also called the discovery approach and the "common sense" approach (Lindgren, 1985:218), meaningful learning (Ausubel, 1966:158), the egrule method (Hermann, 1971:22), is defined as "the presentation of a loosely structured sequence of specific instances or exemplars from which the learner was to discover and verbalize the concept "(Rizzuto, 1970:270)," a method of teaching based on the presentation to the learner of a sufficient number of specific examples to enable him to arrive at a definite rule, principle, or fact,..." (Belcastro, 1966:78), "a scientific form of reasoning that proceeds from specific observations to general statements that apply to many different situations" (Clifford, 1981:690).

Garofalo claimed the importance of or the significance of the inductive method when he wrote "searching for patterns, generalizations, verifications and related problems must take precedence over rehearsing algorithms" (Garofalo, 1989:504). He further argued that the approach to teaching mathematics that is based on the " here is the procedure, here are few examples, now here are some for practice" method cannot but put students in a position to develop more realistic and healthy beliefs about mathematics. (Ibid:503-504).

2.2.2.1 Characteristics of the Inductive Method

The inductive method, according to Davis (1951:142), forms the basis for intuitive formulation of generalizations, which is characteristic of nearly all scientific investigations.
The inductive strategy, according to Hernandez (1973:610), stresses process implicitly as the students are guided in the process, and explicitly as the lesson is terminated with a discussion of the process itself. It also may lead to development of student's mental set that will predispose him to use this strategy in other problem - solving situations. It requires students to compare and contrast stimuli, and thus, according to Clifford (1981:282), has the advantage of letting the student discover the concept.

The fact that it belongs with learner's experiences, which inturn enable them to arrive at empirically- based conclusions, according to Lindgren (1985:218), makes it closer to the scientific approach. It also has the advantage of facilitating the development of making intelligent guesses about the probable solution when only some of the necessary data are obtained. According to Sobel (1956:426), there is an active student participation in discovering the concept and in applying it.

Its drawback, according to Lindgren (Ibid.), is that due to its overemphasis on experience, it leads learners to ignore or even reject the need to develop the theoretical concepts that are essential for greater understanding of problem situation. Because of its inordinate time-cost, according to Ausubel (1966:162), it is not a feasible means of acquiring a wide range of subject matter.

2.2.2.2. Model for the Inductive Method of Teaching

The main assumptions of this strategy, according to Hernandez, are "a) that a specific process can be taught and b) that the student learns as he interacts with the data presented to him" (Hernandez, 1973:609). Polya, as cited in Taback (1992:253), believed and discussed the aim of teaching to be that of teaching young people to "THINK". Taba, as cited in Joyce (1980:50), developed an inductive teaching strategy for concept formation. Bruner and his associates, as cited in Joyce (1980:41), have identified a selection model for concept attainment, in that an individual, looking over the array of unmarked examples, selects one and inquires whether it is a yes or a no, i.e., the examples are not marked as yes or no. The model for teaching concepts inductively with regard to the formation as well as the attainment of concepts developed by Taba and Bruner and his associates respectively is as follows.

Teaching Concepts Inductively

Concept Formation

	overt activity	covert mental operations	eliciting questions
1.	Enumeration and Listing.	Differentiation (identifying separate items).	what did you see? hear? note?
2.	Grouping.	Identifying common properties, abstracting	What belongs together? On what criteria?
3.	Labelling, categorizing.	Determining the hierarchical order of items super-and sub-ordination.	How would you call these groups? What belongs to what?

Concept Attainment

1. presentation of data and identification of attributes,
   - Teacher presents unlabeled examples
   - Students inquire which examples, including their own, are positive ones
   - Students generate and test hypotheses.
2. Testing attainment of the concept
   - Students identify additional unlabeled examples
   - Students generate examples
   - Teacher confirms hypothesis: names concept and restates definition according to essential attributes
3. Analysis of thinking strategy
   - Students describe thoughts
   - Students discuss role of hypothesis and attributes
   - Students discuss type and number of hypotheses.

2.3 Summary of Research Findings Regarding the Relative Effectiveness of the Inductive and Deductive methods of Teaching

There has been a growing concern as to what methods of instruction are most appropriate for children of all levels of mathematical and academic ability. But a method, according to Henderson (1963:1008), maximizes certain factors and minimizes others. To determine the factors that are maximized and/or minimized by a certain method is the function of research on teaching methods. However, educators and/or researchers had never been and are not in a complete agreement about their findings regarding the effectiveness of certain methods.

In this section research findings on the relative effectiveness of the deductive and inductive methods of teaching is reviewed. The findings seem to be contradictory. However, at the time the study was conducted, the situations were quite different and hence such contradictory results (as they seem to be) are expected. In some studies, as can be seen in subsequent pages, the deductive and inductive methods are found to be equally effective, i.e., there is no significant difference in their effectiveness. But under the same conditions, the deductive method, say, is found to be in favor of certain ability groups while the inductive method is in favor of other ability groups. This should not be taken as a contradictory result because certain ability group may benefit from one method, say, the deductive method while other ability group may benefit from the inductive method eventhough there is no significant difference in the effectiveness of the two methods or eventhough the two methods are found to be equally effective.

Hermann (1969:65-66) reviewed the findings obtained from discovery learning (inductive learning) experiments as compared to the expository (deductive) method of teaching. He did not mention the methods and procedures of the studies but summarized the results as follows.

a) Better transfer is obtained from inductive learning.
b) Inductive learning is relatively more effective as the difficulty of the transfer task increases.
c) Inductive learning is relatively more effective as

the period of time between learning and testing on a transfer task increased.

d) The inductive method is relatively more effective for a low ability groups than for high ability groups.

Hermann, however, pointed that direct comparison of experimental findings is difficult due to differing ideas concerning the nature of discovery learning.

Blaine R. Worthen (1968), as cited in Lindgren (1985:219), compared the expository and the discovery methods in a study involving more than 400 pupils in 16 fifth and sixth-grade classes. Teachers were trained for 20 weeks in teaching arithmetic principles in both methods and spent six weeks teaching two classes, one by the discovery and the other by the expository method. The expository method was found superior as far as initial learning is concerned and the discovery (inductive) method was found to be superior as measured by what students retained five to eleven weeks after instruction ended.

Foord (1964:131-136) constructed two linear programs aiming to teach the pupil how to find areas of rectangles and parallelograms. The programs were based on "ruleg" (deductive) and "guided discovery" methods for the 2nd year and 4th year children. The results show that there is no significant differences in the scores of the two groups, i.e., neither method was found to be significantly superior to the other. But the mean scores revealed that the guided discovery program was more appropriate to the lower I.Q. groups, and the Ruleg (deductive) for the higher I.Q. groups of 4th year children.

Hermann (1971:22-29) clearly specified the Egrule and

Ruleg methods and were conducted to grades five and nine to compare the two methods on grade, intelligence, and instructional effectiveness on concept learning and principle learning using programmed instruction. The results showed that grade nine students performed significantly better than grade five students, and high intelligence students performed significantly better than average intelligence students. Concerning the relative effectiveness of the methods, no significant difference was found.

A review of research comparing inductive and deductive instructional procedures, according to Koran (1971:300-305), does not provide consistent empirical support for the use of one or the other method. Some findings, according Koran, have supported the use of inductive methods, others favor the use of the deductive method, while still other researchers have found the two methods to be about equally effective. Koran designed an experiment to investigate individual differences in learning from inductive and deductive sequences of programmed instruction. 167 upper division university students were selected and were taught concepts selected from statistics and test interpretation and were programmed based on inductive and deductive principles. Results revealed that inductive and deductive instructional procedures were about equally effective in terms of time required to teach the given body of information and in promoting transfer of training. However, subjects in the inductive treatments made significantly more program errors than did subjects in the deductive treatments.

Sobel (1956:425-430) conducted a study to discover whether or not there is any relationship between the learning of certain algebraic concepts and the deductive and inductive methods of teaching. Sobel chose seven algebra classes from a group of six schools at ninth-grade level. The students were grouped into high I.Q. groups and average I.Q. groups. Results showed that there was no significant difference between the effectiveness of the two methods of teaching, i.e., they are equally effective. But the inductive method of teaching was found to be superior to the high I.Q. group. The two methods resulted in no real difference in learning for the average I.Q. groups. Sobel further recommended that the fact that there is no real difference between the two methods for the average I.Q. groups would imply that to this group one method may be used just as well as another. But, Sobel stressed that, consideration should be given to the inductive method of teaching in that it is the one which is claimed to be appropriate by mathematics educators from the point of view of meaningful learning.

Michael, as cited in Henderson (1963:1018-1019), studied the relative effectiveness of inductive and deductive methods for teaching the fundamental operations on real numbers. The design of the study was based on analysis of covariance with an unspecified number of pupils in grade nine. Three criterion tests 1) accuracy of computation, 2) ability to use generalizations, and 3) attitudes toward algebra and toward mathematics in general. The tests were administered before and after the teaching program so that measures of growth could be obtained. Scores in accuracy of computation favored the inductive method eventhough the difference was not significant.

The difference in ability to use generalizations favored the deductive method. The teaching methods produced no significant difference in attitude toward mathematics in general, but a significantly more favorable attitude forward algebra was associated with the inductive method.

Nichols, as cited in Henderson (1963:1017-1018), formed a group of 42 pairs of students in a high school. The members of each group matched in so far as possible as to score on a geometry test, I.Q., age, and sex. Both groups were taught the same 15 "basic geometric principles" (theorems) for the same length of time using the two methods, a "dependent approach", in which students depended on the teacher for statement of assumptions, theorems, definitions, and verbalization of principles. Direct student participation was encouraged in developing new topics to the extent of ascertaining that the students were not "lost", the method of presentation was highly verbal and abstract. The other group was taught by a "structured search approach", in which students discovered every relationship which was presented by the teacher to the dependence group, through a series of concrete experiences, with drawings of geometric figures, and through mensuration. Nichols concluded that the two approaches were equally effective, interms of the criterion test in geometry. For high school freshmen whether of average or superior I.Q., there was no statistically significant differences between students taught by the two approaches in performance on the criterion test.

As already described, in the review of literature, regarding the relative effectiveness of the deductive and inductive methods of teaching, the results do not suggest one dependable method. For different situations, data, time, the results as well are different. This would imply that a continuous research has to be done to determine the relative effectiveness of the deductive and the inductive methods of teaching to existing situations and subjects.

CHAPTER THREE

Report and Analysis of Findings

The intention of this study was to compare the relative effectiveness of the deductive and the inductive methods in teaching concepts of mathematics. Specifically, the study was conducted on different student characteristics, i.e., the low-, medium-, and high-achievers were used as to investigate which level of students were to benefit from the two methods under consideration. The comparison of the two methods was based on the scores of students on test (III) which was administered at the end of the experimental lessons for each level in the study population.

In this chapter, then, test results of the students are presented and analyzed. The Mann-Whitney U-test as cited in Clauss and Ebner (1983:230-236), and Hinkle (1988:572) for samples greater than twenty at 0.05 level of significance was used to either accept or reject the four null hypotheses stated in the first chapter of this paper (see page 3)

3.1. Comparison between the relative effectiveness of the two methods of teaching.

To test the null hypothesis: "There is no significant difference in the test scores of the groups who are taught by deductive or inductive approach", the test results of students who were taught by the deductive and those taught by inductive method is arranged in the following table.

TABLE 1

COMPARISON BETWEEN THE TEST RESULTS OF THE STUDENTS WHO WERE TAUGHT BY THE DEDUCTIVE AND INDUCTIVE METHODS.

(1) i	(2) scores	(3) f_{1i}	(4) f_{2i}	(5) f_i	(6) cf_i	(7) $2cf_i-f_i+1$	(8) (3) (7)	(9) (4) (7)
1	2	7	2	9	9	10	70	20
2	3	10	5	15	24	34	340	170
3	4	13	12	25	49	74	962	888
4	5	14	12	26	75	125	1750	1500
5	6	12	13	25	100	176	2112	2288
6	7	13	15	28	128	229	2977	3435
7	8	13	12	25	153	282	3666	3384
8	9	11	10	21	174	328	3608	3280
9	10	13	13	26	200	375	4875	4875
10	11	12	13	25	225	426	5112	5538
11	12	9	10	19	244	470	4230	4700
12	13	11	9	20	264	509	5599	4581
13	14	7	13	20	284	549	3843	7137
14	15	6	8	14	298	583	3498	4664
15	16	6	5	11	309	608	3648	3040
16	17	6	5	11	320	630	3780	3150
17	18	2	4	6	326	647	1294	2588
18	19	2	2	4	330	657	1314	1314
19	20	3	5	8	338	669	2007	3345
20	21	1	3	4	342	681	681	2043
21	22	1	1	2	344	687	687	687
22	23	1	1	2	346	691	691	691
	Total	173	173	346			56744	63318

f_{1i} - frequency of scores of students who were taught by the deductive method

f_{2i} - frequency of scores of students who were taught by the inductive method

$$f_i = f_{1i} + f_{2i}$$

$R_1 = \frac{1}{2}(56744) = 28372$

$R_2 = \frac{1}{2}(63318) = 31659$

$n_1 = n_2 = 173$

R_1 - sum of ranks of scores of students who were taught by the deductive method

R_2 - sum of ranks of scores of students who were taught by the inductive methods

n_1 - number of students that were taught by the deductive method

n_2 - number of students that were taught by the inductive method

$$u_1 = R_1 - \frac{n_1(n_1 + 1)}{2} = 28372 - 15051 = 13321$$

$$u_2 = R_2 - \frac{n_2 (n_2 + 1)}{2} = 31659 - 15051 = 16608$$

$$\sigma = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} = 930.29$$

$$\mu = \frac{n_1 n_2}{2} = 14964.5$$

$$u = \frac{u_1 - \mu}{\sigma} = \frac{13321 - 14964.5}{930.29} = -1.77 \text{ or}$$

$$u^1 = \frac{u_2 - \mu}{\sigma} = \frac{16608 - 14964.5}{930.29} = 1.77$$

Let $u_\alpha = u$ or u^1

Then $/u_\alpha/ = 1.77 < 1.96$ (the critical value)

Thus, the null hypothesis which states: "There is no significant difference in the test scores of students who were taught by the deductive and inductive approaches" is accepted. That means the students who were taught by the deductive and inductive methods did not significantly vary in acievement in learning the concepts of mathematics.

3.2. Comparison between the scores of the two high achiever groups.

To test the null hypothesis: "There is no significant difference in the test scores of high achievers who are taught by the deductive or inductive approach", the test results of the high achiever students who were taught by the deductive and those taught by inductive method is arranged in the following table.

TABLE 2.

COMPARISON BETWEEN THE SCORES OF THE TWO HIGH ACHIEVERS THAT WERE TAUGHT BY THE DEDUCTIVE AND INDUCTIVE METHODS

(1) i	(2) scores	(3) f_{1i}	(4) f_{2i}	(5) f_i	(6) cf_i	(7) $2cf_i - f_i + 1$	(8) (3) (7)	(9) (4) (7)
1	3	1	1	2	2	3	3	3
2	4	2	3	5	7	10	20	30
3	5	4	3	7	14	22	88	66
4	6	3	2	5	19	34	102	68
5	7	4	3	7	26	46	184	138
6	8	6	4	10	36	63	378	252
7	9	3	3	6	42	79	237	237
8	10	5	5	10	52	95	475	475
9	11	3	5	8	60	113	339	565
10	12	4	3	7	67	128	512	384
11	13	3	3	6	73	141	423	423
12	14	3	3	6	79	153	459	459
13	15	3	3	6	85	165	495	495
14	16	3	3	6	91	177	531	531
15	17	4	2	6	97	189	756	378
16	18	1	2	3	100	198	198	396
17	19	2	2	4	104	205	410	410
18	20	3	5	8	112	217	651	1085
19	21	1	3	4	116	229	229	687
20	22	1	1	2	118	235	235	235
21	23	1	1	2	120	239	239	239
	Total	60	60	120			6964	7556

f_{1i} - frequency of scores of students who were taught by the deductive method

f_{2i} - frequency of scores of students who were taught by the inductive method

$f_i \quad = \quad f_{1i} + f_{2i}$

cf_i - cumulative frequency

$$R_1 = \frac{6964}{2} = 3482$$

$$R_2 = \frac{7556}{2} = 3778$$

$$n_1 = n_2 = 60$$

R_1 - sum of ranks of scores of students who were taught by the deductive method

R_2 - sum of ranks of scores of students who were taught by the inductive method

n_1 - number of students who were taught by the deductive method

n_2 - number of students who were taught by the inductive method

$$u_1 = R_1 - \frac{n_1(n_1 + 1)}{2} = 3482 - 1830 = 1652$$

$$u_2 = R_2 - \frac{n_2(n_2 + 1)}{2} = 3778 - 1830 = 1948$$

$$\sigma = \frac{n_1n_2(n_1 + n_2 + 1)}{12} = 190.53$$

σ - standard deviation

$$\mu = \frac{n_1n_2}{2} = 1800$$

$$u = \frac{u_1 - \mu}{\sigma} = \frac{1652 - 1800}{190.53} = -0.78 \quad \text{or}$$

$$u^1 = \frac{u_2 - \mu}{\sigma} = \frac{1948 - 1800}{190.53} = 0.78$$

Let $u_\alpha = u$ or u^1

Then $/u_\alpha/ = 0.78 < 1.96$ (the critical value)

Thus, the null hypothesis which states: "There is no significant difference in the test scores of high-achievers who were taught by the deductive and inductive approaches" is accepted. That means the high achiever groups did not significantly vary in acievement as a result of learning the concepts of mathematics by the deductive and inductive methods.

3.3 Comparison between the test scores of the two medium achiever groups

To test the null hypothesis: "There is no significant difference in the test scores of medium achievers who are taught by the deductive or inductive approach", the test results of medium achiever students who were taught by the deductive and those taught by inductive method is arranged in the following table.

TABLE 3

COMPARISON BETWEEN THE SCORES OF THE TWO MEDIUM ACHIEVERS THAT WERE TAUGHT BY THE DEDUCTIVE AND INDUCTIVE METHODS

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
i	scores	f_{1i}	f_{2i}	f_i	cf_i	$2cf_i-f_i+1$	(3) (7)	(4) (7)
1	2	1	-	1	1	2	2	0
2	3	4	3	7	8	10	40	30
3	4	5	5	10	18	27	135	135
4	5	6	6	12	30	49	294	294
5	6	5	6	11	41	72	360	432
6	7	4	5	9	50	92	368	460
7	8	4	4	8	58	109	436	436
8	9	3	2	5	63	122	366	244
9	10	4	5	9	72	136	544	680
10	11	6	4	10	82	155	930	620
11	12	3	4	7	89	172	516	688
12	13	5	2	7	96	186	930	372
13	14	2	4	6	102	199	398	796
14	13	2	3	5	107	210	420	630
15	16	3	2	5	112	220	660	440
16	17	2	3	5	117	230	460	690
17	18	1	2	3	120	238	238	476
	Total	60	60	120			7097	7423

f_{1i} - frequency of scores of students who were taught by the deductive method

f_{2i} - frequency of scores of students who were taught by the inductive method

$f_i = f_{1i} + f_{2i}$

cf_i - cumulative frequency

$$R_1 = \frac{7097}{2} = 3548.5$$

$$R_2 = \frac{7423}{2} = 3711.5$$

$$n_1 = n_2 = 60$$

R_1 - sum of ranks of scores of students who were taught by the deductive method

R_2 - sum of ranks of scores of students who were taught by the inductive method

n_1 - number of students who were taught by the deductive method

n_2 - number of students who were taught by the inductive method

$$u_1 = R_1 - \frac{n_1(n_1 + 1)}{2} = 3548.5 - 1830 = 1718.5$$

$$u_2 = R_2 - \frac{n_2(n_2 + 1)}{2} = 3711.5 - 1830 = 1881.5$$

$$\sigma = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} = 190.53$$

σ - standard deviation

$$\mu = \frac{n_1 n_2}{2} = 1800$$

$$u = \frac{u_1 - \mu}{\sigma} = \frac{1718.5 - 1800}{190.53} = -0.43 \text{ or}$$

$$u^1 = \frac{u_2 - \mu}{\sigma} = \frac{1881.5 - 1800}{190.53} = 0.43$$

Let $u_\alpha = u$ or u^1

Then $/u_\alpha/ = 0.43 < 1.96$ (the critical value)

Thus, the null hypothesis which states: "There is no significant difference in the test scores of medium achievers who were taught by the deductive and inductive approaches" is accepted. That means the medium achievers did not significantly vary in achievement as a result of learning the concepts of mathematics using the deductive and inductive methods.

3.4. Comparison between the two low achiever groups

To test the null hypothesis: "There is no significant difference in the test scores of low achievers who are taught by the deductive or inductive approach", the test results of low achiever students who were taught by the deductive and those taught by inductive method is arranged in the following table.

TABLE 4

COMPARISON BETWEEN THE SCORES OF THE TWO LOW ACHIEVERS THAT WERE TAUGHT BY THE DEDUCTIVE AND INDUCTIVE METHODS

(1) i	(2) scores	(3) f_{1i}	(4) f_{2i}	(5) f_i	(6) cf_i	(7) $2cf_i-f_i+1$	(8) (3) (7)	(9) (4) (7)
1	2	6	2	8	8	9	54	18
2	3	5	1	6	14	23	115	23
3	4	6	4	10	24	39	234	156
4	5	4	3	7	31	56	224	168
5	6	4	5	9	40	72	288	360
6	7	5	7	12	52	93	465	651
7	8	3	4	7	59	112	336	448
8	9	5	5	10	69	129	645	645
9	10	4	3	7	76	146	584	438
10	11	3	4	7	83	160	480	640
11	12	2	3	5	88	172	344	516
12	13	3	4	7	95	184	552	736
13	14	2	6	8	103	199	398	1194
14	15	1	2	3	106	210	210	420
	Total	53	53	106			4929	6413

f_{1i} - frequency of scores of students who were taught by the deductive method

f_{2i} - frequency of scores of students who were taught by the inductive method

$f_i \quad = \quad f_{1i} + f_{2i}$

cf_i - cumulative frequency

$$R_1 = \frac{4929}{2} = 2464.5$$

$$R_2 = \frac{6413}{2} = 3206.5$$

$$n_1 = n_2 = 53$$

R_1 - sum of ranks of scores of students who were taught by the deductive method

R_2 - sum of ranks of scores of students who were taught by the inductive method

n_1 - number of students who were taught by the deductive method

n_2 - number of students who were taught by the inductive method

$$u_1 = R_1 - \frac{n_1(n_1 + 1)}{2} = 2464.5 - 1431 = 1033.5$$

$$u_2 = R_2 - \frac{n_2(n_2 + 1)}{2} = 32065.5 - 1431 = 1775.5$$

$$\sigma = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} = 158.26$$

σ - standard deviation

$$\mu = \frac{n_1 n_2}{2} = 1404.5$$

$$u = \frac{u_1 - \mu}{\sigma} = \frac{1033.5 - 1404.5}{158.26} = -2.34 \quad \text{or}$$

$$u^1 = \frac{u_2 - \mu}{\sigma} = \frac{1775.5 - 1404.5}{158.26} = 2.34$$

Let $u_\alpha = u$ or u^1

Then $/u_\alpha/ = 2.34 > 1.96$ (the critical value).

Thus, the null hypothesis is rejected and the alternative which states: "There is a significant difference in the test scores of the low achievers who were taught by the deductive and inductive approaches" is accepted. Since there is a significant difference between the groups in comparison, the hypothesis was interpreted in the direction of the U-test. That is, the low achievers that were taught by the inductive method achieved better than the low achievers who were taught by the deductive method.

CHAPTER FOUR

CONCLUSIONS AND RECOMMENDATIONS

4.1 CONCLUSIONS

The objective of this empirical study was to compare the relative effectiveness of the deductive and inductive methods of teaching concepts of mathematics. The study was conducted on students of different characteristics, i.e., the low -, medium -, and high- achievers of Entoto school of grade eleven. Grouping of students into the low, medium and high achievers was made mainly on the basis of test whose content is grade nine and ten mathematics, and the average score of semester I of the current year. The average score was used as a counter check, i.e., students with high and low test results but extremely low and high average score respectively were not selected for the study population.

Based on these criteria, then, 390 students, 130 high achievers, 130 medium achievers, and 130 low achievers were selected. Students in each level were further divided into two groups as low achievers I and II, medium achievers I and II, and high achievers I and II. The low achievers I, Medium achievers I, and high achievers I were taught by the deductive method, where as the low achievers II, medium achievers II, and high achievers II were taught by the inductive method.

After each group attended the lessons for the concepts selected with the methods indicated a test was administered to measure their achievement.

The test scores of those students who attended all the lessons regularly, 120 high achievers; 120 medium achievers, and 106 low achievers were used for data analysis.

The Mann-Whitney U-test was used to test the two-tailed hypotheses at a 0.05 level of significance.

The test results should be interpreted based on the following conditions. That

a) only limited number of concepts (eight) were selected.

b) Students were taught for only three weeks (three lessons per week).

c) students were not used to learn (or not exposed) to the inductive method

d) the test was administered a week after the termination of the lessons.

On the basis of the analysis of data, the following conclusions are drawn.

1. The deductive and inductive methods of teaching were found to be equally effective in teaching the concepts of mathematics. Several researchers have reported that the two methods of teaching being equally effective though they were using varying situations (Foord, 1964; Hermann, 1971; Koran, 1971). Particularly, this finding is in agreement with the findings of Sobel (1956).

2. The high achievers could equally learn the concepts of mathematics through the deductive as well as inductive methods of teaching.

   This result is in agreement with Nichols in Henderson (1963) although he used different categories of concepts. The independent work (or study) of students in preparing themselves for an examination may have brought about equality of achievement of the high achievers. McLeish, as cited in Gage and Berliner (1988:400) called this situation "the equalization effect".

3. The medium achievers could equally learn the concepts of mathematics through the deductive as well as the inductive methods of teaching. This result is in agreement with the findings of Sobel (1956).

4. The low achievers benefitted from the inductive method more than from the deductive method in learning the concepts of mathematics. Foord (1964) also reported that the guided discovery (inductive method) was more appropriate for the low I.Q. groups than the deductive method.

5. Despite the fact that there were no significant differences in the effectiveness of the two methods for the high and medium- achievers, and for the whole groups, a clear tendency was shown in favor of the inductive method. This was indicated by comparing R_1 and R_2 (see chapter three of this paper).
We found $R_2 > R_1$.

4.2 Recommendations

It is the concern of educators and /or researchers to create favorable situations for the improvement of the teaching learning process. This small study on the comparison of the relative effectiveness of the deductive and inductive methods of teaching will contribute to the improvement of mathematics instruction. In light of the analysis of the data, the following recommendations are given.

1. The results of the test (see chapter three of this paper) were in favor of the inductive method. According to Clauss and Ebner (1983), when $R_2 > R_1$, the effectiveness of the methods is in favor of the method used for group two. Thus mathematics classes should develop a sense of discovery so that students discover certain generalizations themselves rather than being provided ready made. In particular, the inductive method would be used better when situations invite discovery.

2. Institutions for secondary school teachers training, should emphasize those methods which enable teachers to teach concepts effectively. Particularly, the pre- and in-service teachers need be given appropriate methodological assistance that emphasize discovery approaches so that they could use them in their actual teaching for better results.

3. The approach for concept learning in mathematics is a virgin awaiting detailed scientific investigations to provide evidence to the nature and process of concept formation. Thus further study on this area would be advisable by considering different categories of concept learning. In particular, the effectiveness of the deductive and inductive approaches be investigated on a long-term basis and for other objectives such as principle learning, problem solving, etc. in mathematics.

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APPENDIX I

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

TEST I

TIME ALLOWED 2 HOURS

1. NAME ______________________________

2. SECTION ______________________________

3. CHOOSE THE BEST ANSWER AND WRITE THE LETTER OF YOUR CHOICE ON THE SPACE PROVIDED.

------1. Let A = {0,1,2}, B = {1,3,4}. Then $A \Delta B$ is

a) {0,2} c) { 0,2,3,4}
b) {3,4} d) { }

------2. If A = { {1}, 1}, then which one of the following is not true?

a) $\{1\} \in A$ c) $\{ \{1\} \} \subsetneq A$
b) $\{1\} \subsetneq A$ d) $\{ \{1\} \} \in A$

------3. Which one of the following is true about DeMorgan's rule

a) $(A \cap B)' = A' \cap B'$ c) $(A' \cap B')' = A' \cup B'$
b) $(A \cup B)' = A' \cup B'$ d) $(A' \cup B') = A \cap B$

------4. Which one of the following is necessarily true?

A. The sum of any two irrational numbers is an irrational number
B. The product of any two irrational numbers is an irrational number

a) Only A c) Both A and B
b) Only B d) Neither A nor B

------5. Which one of the following is the correct relation?

a) $\sqrt{2} + \sqrt{3} < \sqrt{2+3}$
b) $/-0.394/ < /0.394/$
c) $-6.9431 > -6.8456$
d) $4.631 < 4.632$

------6. Which one of the following points lies on the line given by 2X + 3Y - 6 = 0?

a) (2,3)c) (3,2)
b) (0,2)d) (-3,0)

------7. Suppose that L1 and L2 are two lines in a plane given by

L1: ax+by+c=0
L2: mx+ny+d=0

Under what condition will the two lines be parallel?

a) If an-bm=0c) If cd-bn=0
b) If an+bm=0d) If cd+bn=0

-----8. Suppose that a simultaneous equation does not have a solution. Then which of the following is true?

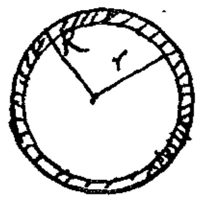
a) The tow lines are perpendicular
b) The two lines are parallel
c) The tow lines are identical
d) The two lines intersect at two distinct points

------9. Which of the following triples can not be the lengths of the side of a triangle?

a) 2,3,6 b) 5,7,8 c) 5,5,9

------10. Given the following figures, the area of the shaded ring is given by

a) $\pi(R-r)$
b) $\pi(R^2-r^2)$
c) $\pi(r^2-R^2)$
d) $\pi(r-R)$



------11. If two plane figures have the same area, then

a) The two plane figures are congruent
b) The two plane figures are similar
c) All of the above
d) None of the above

------12. The number of the folding symmetries of an equilateral triangle is

a) 1 b) 2 c) 3 d) 4

------13. Which one of the following is the right expression for $100-X^2$ when expressed as a product of its linear factors?

a) $(10+x)(10-x)$ c) $(100+x)(100-x)$
b) $(x+10)(x-10)$ d) $(x+100)(x-100)$

------14. The area of a rectangle is 84 cm^2 and one of its sides is 7 centimeters. What is the length of the other side?

a) 8 G) 11 c) 12 d) 14

------15. Given the quadratic formula $X = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

for solving the quadratic equation $ax^2+bx+c=0$, $a \neq 0$, the equation will have two distinct solutions if

a) The discriminant is positive
b) The discriminant is negative
c) The discriminant is zero

------16. If $a > 0$, $b > 0$, then which of the following is not true?

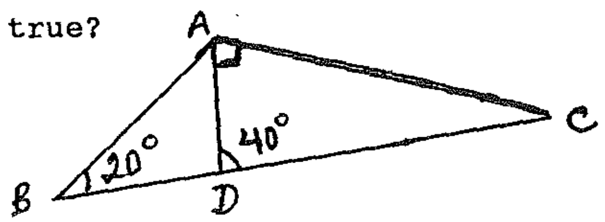
a) $a/b = a^{\frac{1}{2}} \times 1/b^{\frac{1}{2}}$
b) $\sqrt{a/b} = \sqrt{a}/\sqrt{b}$
c) $\sqrt{a/b} = \sqrt{ab}/b$
d) $\sqrt{a/b} = a/\sqrt{ab}$

------17. The truth set for the simultaneous equations given by

$$\begin{cases} x^2 + y^2 = 1 \\ y = 3x + 3 \end{cases} \qquad x, y \in R \text{ is}$$

a) $\{(-1, 3/5), (-4/5, 3/5)\}$ c) $\{(-1,0), (-4/5, 3/5)\}$
b) $\{(-4/5, 0), (-1,0)\}$ d) $\{(-1,-4/5), (0,3/5)\}$

------18. In the figure below $m(ABD)=20$ and $m(ADC)=40$ which of the following is true?



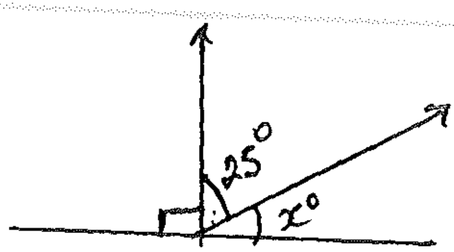
a) AD=BD
b) AB=AC
c) AC=AD
d) AB=BD

------19. A line drawn from a vertex of a triangle and perpendicular to the opposite side is called....

a) altitude b) median
c) side bisector

------20. Find X in the figure below.

a) 25
b) 35
c) 65
d) 75



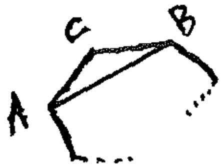
------21. Which of the following relations is possible?

A. Any rhombus is a square
B. Any square is a rectangle

a) Only A
b) Only B
c) Both A and B
d) Neither A nor B

------22. If in the figure, $\overline{AC}$ and $\overline{BC}$ are two adjacent sides of a regular polygon, and $m(\hat{ABC}) = m(\hat{ACB})$, then the number of sides of the polygon is ------- units.

a) 5
b) 6
c) 7
d) 8



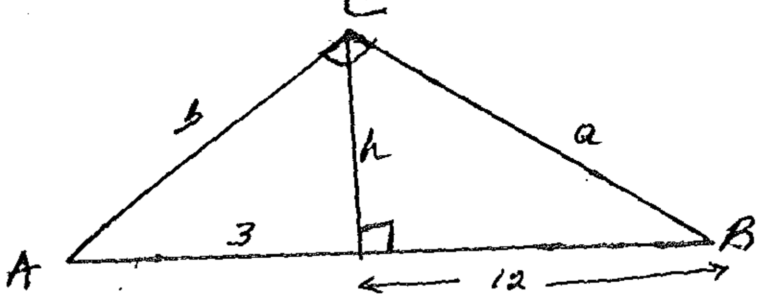
------23. Which of the following relations is true?

A. If two triangles are similar, then they are congruent
B. If two triangles are congruent, then they are similar

a) Only A
b) Only B
c) Both A and B
d) Neither A nor B

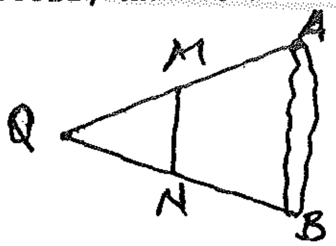
------24. What is the length of a,b, and h respectively in the figure below?

a) $3\sqrt{5}$, $6\sqrt{5}$, 6
b) $6\sqrt{5}$, $3\sqrt{5}$, 6
c) 6, $3\sqrt{5}$, $6\sqrt{5}$
d) $3\sqrt{5}$, 6, $6\sqrt{5}$



------25. In the figure below, QA=100 meters, QM=50 meters, QB=120 meters QN=60 meters, MN= 40 meters. Find AB

a) 70 meters
b) 80 meters
c) 90 meters
d) 100 meters



------26. If a and b are non-zero numbers, how is $\frac{a^{-10} b^{-4}}{a^5}$ rewritten using only positive integral exponents?

a) a^5/b^4a^{10}
b) a^{15}/b^4
c) b^4/a^{15}
d) $1/a^{15}b^4$

------27. The expanded form for 125.6004 using exponents is

a) $1x10^2+2x10^1+5x10^0+6x10^{-1}+4x10^{-4}$
b) $1x10^2+2x10^1+5x10^0+6x10^0+4x10^{-3}$
c) $1x10^2+2x10^1+5x10^0+6x10^{-4}+4x10^{-1}$
d) $1x10^2+2x10^1+5x10^0+6x10^{-3}+4x10^0$

------28. Which one of the following numbers is the greatest of all?

a) $(7^{4/3})^5$
b) $((7^{1/3})^4)^5$
c) $(7^{1/3})^{20}$
d) None of the above

------29. How would the graph of 10^x differ from the graph of $3^x \in R$?

a) The graph of 10^x lies above the graph of 3^x for all values of X
b) The graph of 3^x lies above the graph of 10^x for negative values of x and lies below for positive values of x
c) Both graphs intersect at one point
d) The graph of 3^x lies below the graph of 10^x for negative values of x and lies above for positive values of x.

------30. What is $\log_2(2^{18} \times 2^{27} \times 2^5)$?

a) $2^{18} + 2^{27} + 2^5$

b) $2^{18} \times 2^{27} \times 2^5$

c) 18+27+5

d) 18x27x5

------31. Which one of the following relations is not true for $a,b,x > 0$?

a) If $a<x<b$, then $\log_2 a < \text{Log}_2 x < \log_2 b$

b) The graph of $y = \log_2 x$ has no any sharp corners

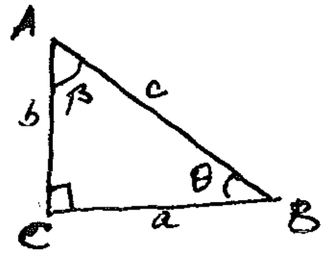
c) $1 < \text{Log}68 < 2$

d) $\log_3 x < \log_2 x$ for all values of x

------32. If $\log 2 \approx 0.3010$ and $\log 1.41 \approx 0.1505$ then what is $\log\sqrt{200}$?

a) 5.44 b) 14.1 c) 1.1505 d) 30.10

------33. Which one of the following relations is not true in the following figure?



a) $\sin\theta = \cos\beta$

b) $\cos\theta = \sin\beta$

c) $\tan\theta = b/a$

d) $\frac{1}{\tan\theta} = \frac{1}{\tan\beta}$

------34. What is the length of one of the perpendicular sides of right-angled triangle if its area is 35 sq. units and the other perpendicular side is 7 units?

a) 5 b) 10 c) 12 d) 14

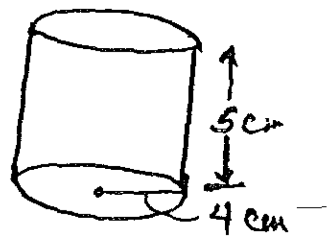
------35. What is the volume of the figure below?

a) $20\,\pi\ \text{cm}^3$

b) $40\,\pi\ \text{cm}^3$

c) $60\,\pi\ \text{cm}^3$

d) $80\ \text{cm}^3$



------36. If $\log_2 32 = x+3$ then the value of x is

a) 2 b) 3 c) 5 d) 8

-----37. Which one of the following is true about the graph of

$y = a^x,\ a > 0,\ a \neq 1,\ x \in R$

a) Y is negative for negative values of X

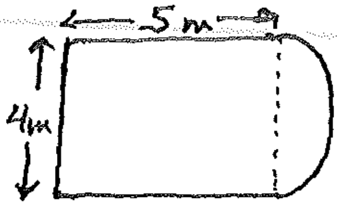
b) Y is zero for at least one value of X

c) The graph has sharp corners

d) It cuts the Y-axis at (0,1)

------38. What is the perimeter of the figure below?

a) $(18+4\pi)$ meters
b) $(14+4\pi)$ meters
c) $(14+\pi)$ meters
d) $(18+\pi)$ meters



------39. How is $a^{3n} + b^{3n}$ expressed as the product of its prime factors?

a) $(a + b)(a^2 + ab + b^2)^n$
b) $(a^n + b^n)(a^{2n} - (ab)^n + b^{2n})$
c) $(a + b)(a^2 - ab + b^2)^n$
d) $(a^n + b^n)(a^{2n} + (ab)^n + b^{2n})$

------40. The truth set for the inequality $(x+3)(x-4)<0$, $x \in R$ is

a) $x<-3$ or $x<4$
b) $x>-3$ or $x>4$
c) $-3<x<4$
d) $x<-3$ or $x>4$

APPENDIX II

Item	Upper group correct response	Lower group correct response	Discrimination index
1	48	15	0.49
2	19	20	-0.01
3	17	15	0.03
4	19	11	0.12
5	36	13	0.34
6	42	13	0.43
7	14	12	0.03
8	27	10	0.25
9	27	6	0.31
10	54	24	0.44
11	34	10	0.35
12	35	25	0.15
13	40	26	0.21
14	40	31	0.13
15	35	14	0.31
16	35	33	0.03
17	39	17	0.32
18	14	4	0.15
19	22	6	0.24
20	55	26	0.43
21	48	23	0.37
22	21	10	0.16
23	49	22	0.40
24	26	10	0.24
25	49	23	0.38
26	38	11	0.40
27	54	24	0.44
28	39	15	0.35
29	27	11	0.24
30	59	35	0.35
31	39	10	0.43
32	11	11	0.00
33	36	21	0.22
34	23	10	0.19
35	34	4	0.44
36	43	18	0.37
37	23	12	0.16
38	21	8	0.19
39	29	8	0.31
40	42	21	0.31

APPENDIX III

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

TEST II

TIME ALLOWED 1:30 HOURS

1. NAME ______________________________

2. SECTION ______________________________

YOUR 3. CHOOSE THE BEST ANSWER AND WRITE THE LETTER OF CHOICE ON THE SPACE PROVIDED.

------1. Let A = {0,1,2}, B = {1,3,4}. Then $A \Delta B$ is

a) {0,2} c) { 0,2,3,4}
b) {3,4} d) { }

------2. Which one of the following is necessarily true?

A. The sum of any two irrational numbers is an irrational number
B. The product of any two irrational numbers is an irrational number

a) Only A c) Both A and B
b) Only B d) Neither A nor B

------3. Which one of the following is the correct relation?

a) $\sqrt{2} + \sqrt{3} < \sqrt{2+3}$
b) /-0.394/ < /0.394/
c) -6.9431 > -6.8456
d) 4.631 < 4.632

------4. Which one of the following points lies on the line given by 2X + 3Y - 6 = 0?

a) (2,3) c) (3,2)
b) (0,2) d) (-3,0)

------5. Suppose that a simultaneous equation does not have a solution. Then which of the following is true?

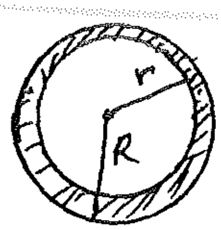
a) The tow lines are perpendicular
b) The two lines are parallel
c) The tow lines are identical
d) The two lines intersect at two distinct points

------6. Which of the following triples can not be the lengths of the side of a triangle?

a) 2,3,6 b) 5,7,8 c) 5,5,9

------7. Given the following figures, the area of the shaded ring is given by

a) $\pi(R-r)$
b) $\pi(R^2-r^2)$
c) $\pi(r^2-R^2)$
d) $\pi(r-R)$



------8. If two plane figures have the same area, then

a) The two plane figures are congruent
b) The two plane figures are similar
c) All of the above
d) None of the above

------9. Which one of the following is the right expression for $100-x^2$ when expressed as a product of its linear factors?

a) $(10+x)(10-x)$
b) $(x+10)(x-10)$
c) $(100+x)(100-x)$
d) $(x+100)(x-100)$

------10. The area of a rectangle is 84 cm^2 and one of its sides is 7 centimeters. What is the length of the other side?

a) 8 b) 11 c) 12 d) 14

------11. Given the quadratic formula $X = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

for solving the quadratic equation $ax^2+bx+c=0$, $a \neq 0$, the equation will have two distinct solutions if

a) The discriminant is positive
b) The discriminant is negative
c) The discriminant is zero

------12. The truth set for the simultaneous equations given by

$$\begin{cases} x^2 + y^2 = 1 \\ y = 3x + 3 \end{cases} \qquad x, y \in R \text{ is}$$

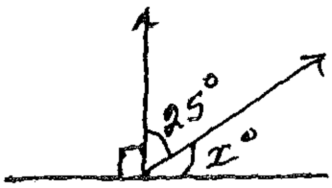
a) $\{(-1, 3/5), (-4/5, 3/5)\}$
b) $\{(-4/5, 0), (-1, 0)\}$
c) $\{(-1, 0), (-4/5, 3/5)\}$
d) $\{(-1, -4/5), (0, 3/5)\}$

------13. A line drawn from a vertex of a triangle and perpendicular to the opposite side is called....

a) altitude
b) median
c) side bisector

------14. Find X in the figure below.

a) 25
b) 35
c) 65
d) 75



-----15. Which of the following relations is possible?

A. Any rhombus is a square

B. Any square is a rectangle

a) Only A c) Both A and B

b) Only B d) Neither A nor B

-------16. Which of the following relations is true?

A. If two triangles are similar, then they are congruent

B. If two triangles are congruent, then they are similar

a) Only A c) Both A and B

b) Only B d) Neither A nor B

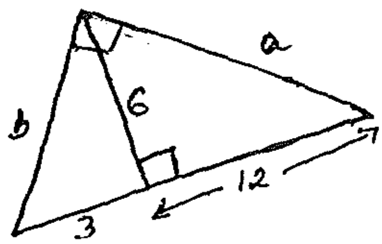
------17. What is the length of a and b respectively in the figure below?

a) $3\sqrt{5}$, $6\sqrt{5}$

b) $6\sqrt{5}$, $3\sqrt{5}$

c) 6, $3\sqrt{5}$

d) $3\sqrt{5}$, 6



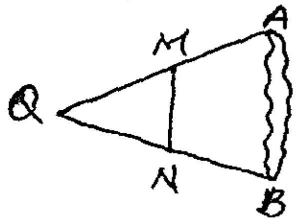
------18. In the figure below, QA=100 meters, QM=50 meters, QB=120 meters QN=60 meters, MN= 40 meters. Find AB

a) 70 meters

b) 80 meters

c) 90 meters

d) 100 meters



------19. If a and b are non-zero numbers, how is $\dfrac{a^{-10}b^{-4}}{a^{5}}$

rewritten using only positive integral exponents?

a) a^5/b^4a^{10}

b) a^{15}/b^4

c) b^4/a^{15}

d) $1/a^{15}b^4$

------20. The expanded form for 125.6004 using exponents is

a) $1x10^2+2x10^1+5x10^0+6x10^0+4x10^{-3}$

b) $1x10^2+2x10^1+5x10^0+6x10^{-1}+4x10^{-4}$

c) $1x10^2+2x10^1+5x10^0+6x10^{-4}+4x10^{-1}$

d) $1x10^2+2x10^1+5x10^0+6x10^{-3}+4x10^0$

------21. Which one of the following numbers is the greatest of all?

a) $(7^{4/3})^5$

b) $((7^{1/3})^4)^5$

c) $(7^{1/3})^{20}$

d) None of the above

------22. How would the graph of 10^x differ from the graph of 3^x $x \in R$?

a) The graph of 10^x lies above the graph of 3^x for all values of X

b) The graph of 3^x lies above the graph of 10^x for negative values of x and lies below for positive values of x

c) The graph of 3^x lies below the graph of 10^x for negative values of x and lies above for positive values of x.

------23. What is $\log_2 (2^{18} \times 2^{27} \times 2^5)$?

a) $2^{18}+2^{27}+2^5$

b) $2^{18} \times 2^{27} \times 2^5$

c) 18+27+5

d) 18x27x5

------24. Which one of the following relations is not true for a,b,x > 0?

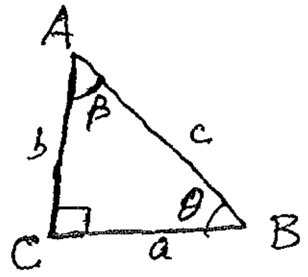
a) If a<x<b, then $\log_2 a < \mathrm{Log}_2 x < \log_2 b$

b) The graph of $y = \log_2 x$ has no any sharp corners

c) 1 < Log 68 < 2

d) $\log_3 x < \log_2 x$ for all values of x

------25. Which one of the following relations is not true in the following figure?



a) $\sin\theta = \cos\beta$

b) $\cos\theta = \sin\beta$

c) $\tan\theta = b/a$

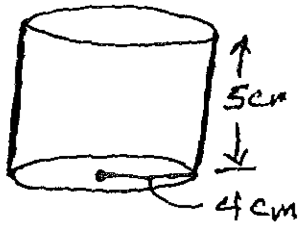
d) $\tan\theta = \tan\beta$

------26. What is the length of one of the perpendicular sides of a right-angled triangle if its area is 35 sq. units and the other perpendicular side is 7 units?

a) 5 b) 10 c) 12 d) 14

------27. What is the volume of the figure below?

a) 20π cm^3
b) 40π cm^3
c) 60π cm^3
d) 80 cm^3



------28. If $\log_2 32 = x+3$ then the value of x is

a) 2 b) 3 c) 5 d) 8

------29. How is $a^{3n} + b^{3n}$ expressed as the product of its prime factors?

a) $(a + b)(a^2 + ab + b^2)^n$
b) $(a^n + b^n)(a^{2n} - (ab)^n + b^{2n})$
c) $(a + b)(a^2 - ab + b^2)^n$
d) $(a^n + b^n)(a^{2n} + (ab)^n + b^{2n})$

------30. The truth set for the inequality $(x+3)(x-4)<0$, $x \in R$ is

a) $x<-3$ or $x<4$
b) $x>-3$ or $x>4$
c) $-3<x<4$
d) $x<-3$ or $x>4$

APPENDIX IV

Score	Frequency
26	2
25	4
24	4
23	9
22	10
21	23
20	17
19	30
18	33
17	35
16	65
15	69
14	77
13	78
12	86
11	71
10	90
9	65
8	39
7	28
6	22
5	4
4	5

APPENDIX V

Test scores and average scores of semester I

High achievers I		High achievers II	
Test score	Average score	Test score	Average score
26	83.5	26	87.3
25	83	25	87.7
25	88.3	25	84.6
24	79	24	88
24	76.8	24	83.8
23	83	23	81
23	80	23	85
23	77.5	23	87.8
23	83.8	23	85.2
22	85	22	81
22	81	22	85
22	80	22	84.7
22	84.5	22	84.5
22	81	22	79
21	83.7	21	79.5
21	77.8	21	80.4
21	80.5	21	82.3
21	79	21	81
21	81	21	82.5
21	79	21	79.4
21	79.8	21	77.3
21	76	21	78.6
20	72	20	75
20	80	20	76.7
20	76.5	20	81
20	75	20	75
20	72	20	77.3
20	73	20	79
20	72	20	78.5
19	68	19	69
19	70	19	71.4
19	66	19	68.7
19	72.3	19	72
19	74	19	71.4
19	69	19	74
19	75	19	75
19	71	19	69.5
19	71.8	19	73.2
19	74	19	70.7
19	76	19	74
19	73	19	73.2
19	71	19	76
18	67	18	70
18	68	18	69
18	70	18	68.4
18	69	18	66.5
18	65.8	18	68.6
18	66.3	18	67
18	69	18	72
18	66.4	18	69.2
18	66	18	68.1
18	65.6	18	69
18	67.2	18	68
18	69.1	18	71.2
18	70	18	66.2
18	66.2	18	65
17	65	17	66
17	66.4	17	65.8
17	67	17	68
17	68.4	17	66.4
17	67.2	17	69
17	66	17	65.3
17	68	17	67.4
17	66	17	67.2
17	65.7	17	66

Medium Achievers I		Medium Achievers II	
Test score	Average score	Test score	Average score
14	60	14	60
14	58	14	59
14	53	14	60
14	59.5	14	53.5
14	59	14	54
14	60	14	58.3
14	55.5	14	51.7
14	56.8	14	60
14	53.5	14	52.3
14	60	14	54
14	60	14	57
14	53	14	60
14	50	14	59
14	55	14	60
14	60	14	54.6
14	58.5	14	51.3
14	57	14	57.1
14	55.8	14	54.8
14	51	14	50.2
14	52.7	14	59.6
14	57.8	14	54.5
14	56.1	14	52.6
14	55.5	14	56
14	52.5	14	56
14	56	14	52
14	58	14	54
14	57	14	51
14	57	14	58.6
14	57	14	60
14	60	14	56
14	50	14	60
14	52.3	14	59.3
14	60	14	59.7
14	60	14	51
13	55	13	52
13	59	13	55
13	59	13	57
13	51	13	58.3
13	56.8	13	52.8
13	59	13	53.1
13	58.3	13	52.3
13	57.6	13	53.1
13	57	13	51
13	53	13	57.6
13	59	13	55
13	51	13	50
13	55.6	13	56
13	52.8	13	51
13	51.1	13	55
13	54.8	13	57.8
13	56.8	13	52
13	53.2	13	50
13	51.5	13	52.3
13	50.8	13	55.5
13	51.3	13	51.8
13	57.8	13	50.8
13	56.3	13	54.1
13	51	13	57
13	50	13	52
13	51	13	60
13	54.1	13	50.5
13	52.3	13	50
13	55.7	13	53.7
13	57	13	50.6

Low Achievers I		Low Achievers II	
Test score	Average score	Test score	Average score
9	44	9	48.5
9	48	9	43.6
9	40.1	9	40.6
9	44.5	9	42.6
9	40	9	43
9	43	9	41
9	47	9	46.5
9	41.6	9	45.5
9	38.3	9	48.8
9	40.5	9	45.3
9	40.1	9	37.5
9	47.3	9	45.6
9	48.7	9	48.8
9	44.3	9	48.6
9	48.1	9	46.1
9	48.5	9	48.3
9	48.1	9	44.3
9	48.8	9	48.5
9	48.3	9	40.8
9	45.7	9	46.5
9	41.8	8	40
8	41	8	42.6
8	47.3	8	44
8	44.3	8	39.6
8	48	8	40.5
8	42.1	8	43.5
8	43	8	43.8
8	43.6	8	47.8
8	47	8	41
8	43.5	8	44.6
8	43.6	8	41.3
8	47.3	8	43.2
8	42.1	8	45.5
8	42.3	8	40.7
8	41.2	8	47.7
8	42.3	8	44.8
8	44.7	8	45.2
8	41.8	7	46
8	45.8	7	47
7	43.5	7	45
7	45.1	7	42
7	47.1	7	41.8
7	48.6	7	40
7	41	7	47.5
7	41.1	7	44
7	37	7	46.5
7	41	7	45.3
7	39	7	39
7	39.6	7	41.5
7	44.8	7	45
7	46	7	42.5
7	42	6	39
7	42.2	6	46.6
6	42	6	42
6	47	6	37
6	40.5	6	39.1
6	42.5	6	38.8
6	39.5	6	40.3
6	46	6	41
6	43	6	43.3
6	44.3	6	45.3
5	32.8	5	32.8
5	33.7	5	33.8
4	33	4	33.3
4	33.3	4	33

Appendix VI

Lesson for the Deductive Group

Concept Label I. <u>DISTANCE FORMULA IN A PLANE</u>

<u>Introduction:</u>

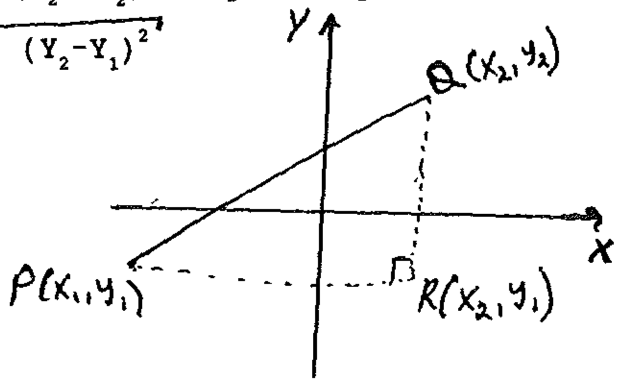
We know that given two points in a plane, we can construct a line segment joining the given points. But how do we determine the distance between the two points? We will determine the distance of a line segment (or the distance between two points) in a plane.

<u>Lesson Development</u>:

<u>Definition</u>:

The distance between two arbitrary points $P(X_1, Y_1)$ and $Q(X_2, Y_2)$ is given by

$PQ = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2}$



ΔPRQ is a right angled triangle. Moreover, $PR = /X_2-X_1/$ and $RQ = /Y_2-Y_1\ /$. Hence applying the Pythagoras theorem on ΔPRQ, we can attain the given formula:

$$(PQ)^2 = (PR)^2 + (RQ)^2$$

$$= (X_2-X_1)^2 + (Y_2-Y_1)^2$$

Hence $PQ = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2}$

<u>Examples</u>

1. Find the distance between the two points.
   a) (1,1), (2,5)

<u>Solution</u>: Let P(1,1) and Q(2,5)

Then $PQ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1^2+4^2} = \sqrt{1+16} = \sqrt{17}$

b) (4,-2) and (-10,1)

<u>Solution</u>: Let P(4,-2), Q(-10,1)

Then $PQ = \sqrt{(-10-4)^2 + (1-(-2))^2} = \sqrt{(-14)^2+3^2} = \sqrt{196+9} = \sqrt{205}$

c) (2,0) and (5,0)

Solution: Let P (2,0) and Q (5,0)

Then $PQ = \sqrt{(5-2)^2 + (0-0)^2} = \sqrt{3^2+0^2} = \sqrt{9+0} = \sqrt{9} = 3$

Alternative: The two points lie on the X-axis.

Then PQ = /5-2/ = /3/ = 3

d) (0,1) and (0,4)

Solution: Let P(0,1) and Q(0,4) Then

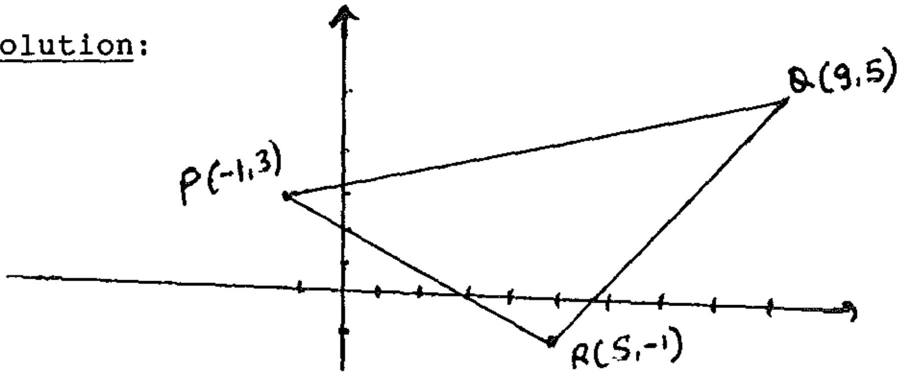
$$PQ = \sqrt{(0-0)^2 + (4-1)^2} = \sqrt{0^2+3^2} = \sqrt{0+9} = \sqrt{9} = 3$$

Alternative:The two points lie on the Y-axis.

Then PQ = /4-1/ = /3/ = 3

2. Determine whether or not the triangle with vertices (9,5), (5,-1), (-1,3) is isosceles.

Solution:



Find PQ, QR, RP then decide whether or not $\triangle PQR$ is isosceles.

Exercise:Find the distance between the following pair of points

1. (7,1) and the origin
2. (5,6) and (7,10)
3. (-5,-2) and (0,6)

Concept label II. Slope of a Line

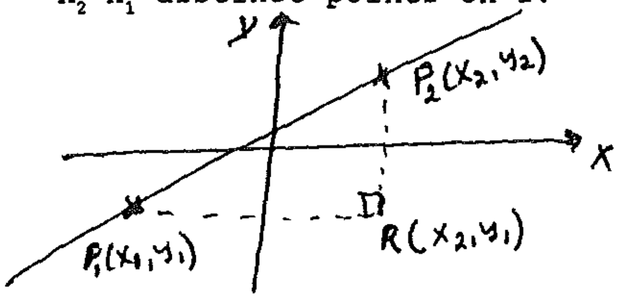
Introduction:

The slope of a line is a measure of the steepness of a line as we go from the left to the right or from the right to the left along the X-axis.

Lesson Development:

In your earlier studies you learnt that the constant ratio $\frac{\text{vertical increase}}{\text{Horizontal increase}}$ to be the slope of a non-vertical line. We define it here as follows:

Definition: The slope of a non-vertical line l is the ratio $\frac{Y_2-Y_1}{X_2-X_1}$, where $P_1(x_1,y_1)$ and $P_2(X_2,Y_2)$ are any two distinct points on l.



As you can see in the figure above, the vertical increase is y_2-y_1 and the horizontal increase is x_2-x_1.

$$\text{Then } \frac{\text{Vertical increase}}{\text{Horizontal increase}} = \frac{y_2-y_1}{x_2-x_1}$$

Examples: Find the slope of the line determined by the pair of points

1. (7,9) and (3,11)

   Solution: Let $P_1(x_1,y_1) = (7,9)$, $P_2(x_2,y_2) = (3,11)$.

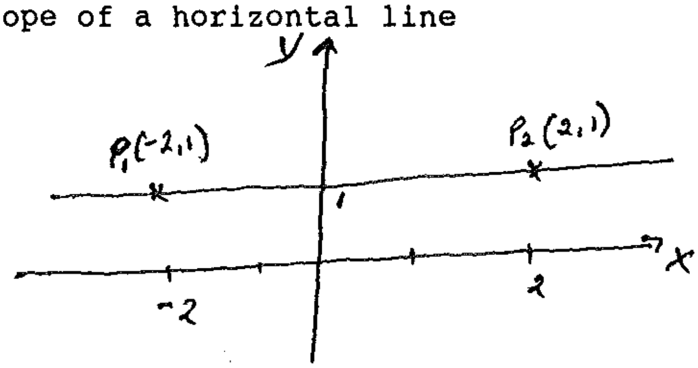
   Then $\frac{y_2-y_1}{x_2-x_1} = \frac{11-9}{3-7} = \frac{2}{-4} = -\frac{1}{2}$

2. (-3,-1) and (2,3)

   Solution: Let $P_1(x_1,y_1) = (-3,-1)$, $P_2(x_2,y_2) = (2,3)$

   Then $\frac{y_2-y_1}{x_2-x_1} = \frac{3-(-1)}{2-(-3)} = \frac{4}{5}$

3. The slope of a horizontal line

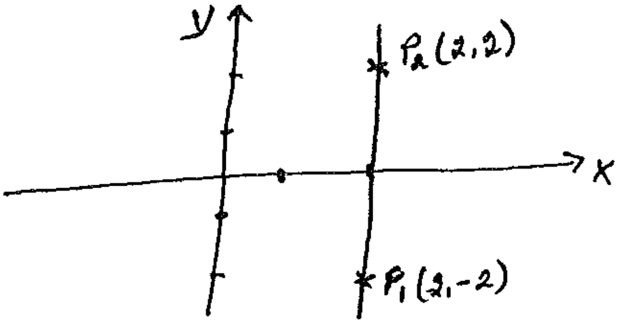


Solution: $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{1-1}{2-(-2)} = \dfrac{0}{4} = 0$

Its vertical increase is 0 and its horizontal increase is 4.

Hence, the slope of a horizontal line is zero.

4. The slope of a vertical line.

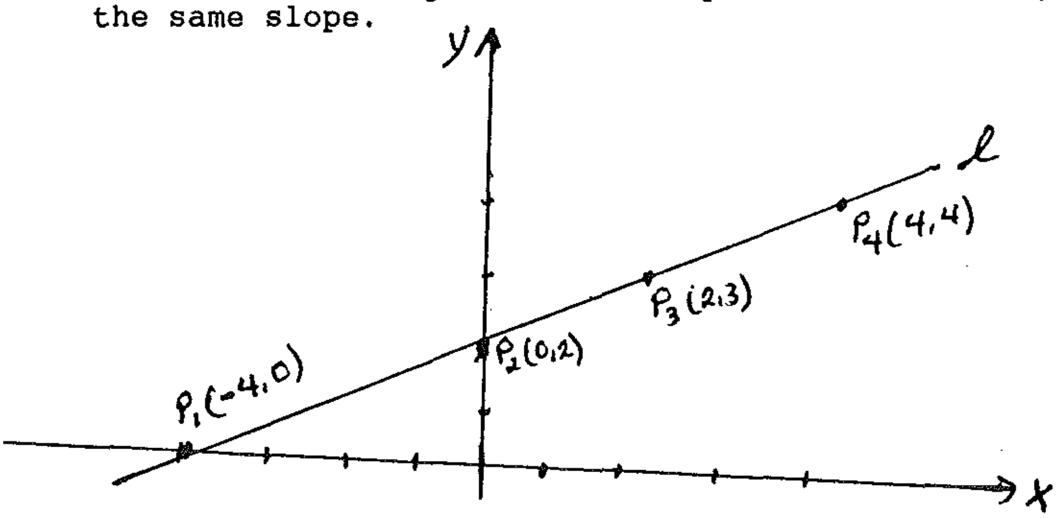


Vertical increase is $2-(-2) = 4$
Horizontal increase is $2-2 = 0$

Hence, $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-(-2)}{2-2} = \dfrac{4}{0}$ But what is $\dfrac{4}{}$?

Therefore, vertical lines do not have a slope.

5. The slope is independent of the choice of points on the line, i.e., for any two distinct points on the line, we get the same slope.



Take P_1 and P_2. Then $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-0}{0-(-4)} = \dfrac{2}{4} = \dfrac{1}{2}$

Take P_3 and P_4. Then $\dfrac{Y_2-Y_1}{X_2-X_1} = \dfrac{4-3}{4-2} = \dfrac{1}{}$

Exercise: Determine the slope of the line determined by the pair of points.

1. (0,3) and (3,0)
2. (6,2) and (-3,2)
3. (3,4) and (3,6)
4. (m,n) and (-n,-m), $m,n \neq 0$

Concept Label III. <u>Reflections</u>

<u>Introduction:</u> Reflection is a rigid motion where the size and shape of an object is not changed as a result of this motion. In particular, how far behind the mirror does an object appear? How does it look like? Such questions will get answer in this lesson.

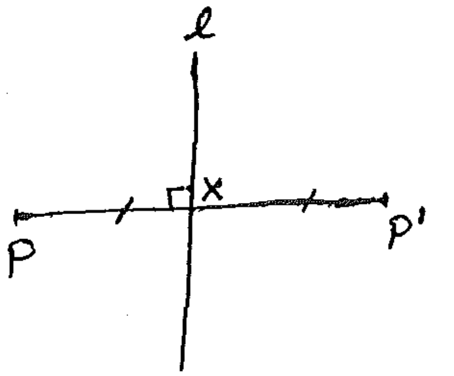
<u>Lesson Development</u>:

<u>Definition</u>: If P is a point not on a line l, then its image in l is the point P^1 such that

a) $\overleftrightarrow{PP^1} \perp l$ i.e. $\overleftrightarrow{PP^1}$ is perpendicular to l

b) If $\overleftrightarrow{PP^1}$ meets l at x, then $PX = P^1x$

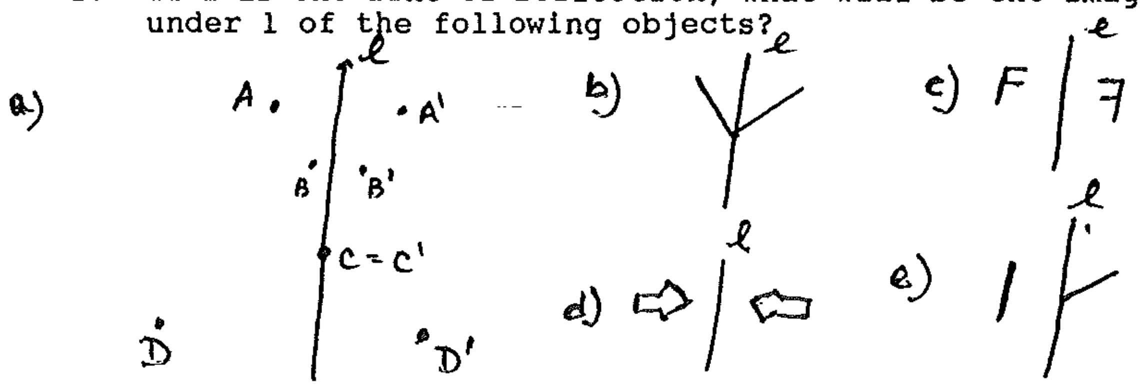
In short, P^1 is the image of P in l if l is the perpendicular bisector PP^1. If P is on l, its image P^1 in l is P itself. In either case, we say that we reflect P in l to get P^1, and we call l the line, or axis, of reflection.



That is
1) $Px = \overline{P^1X}$
2) $l \perp \overline{PP^1}$

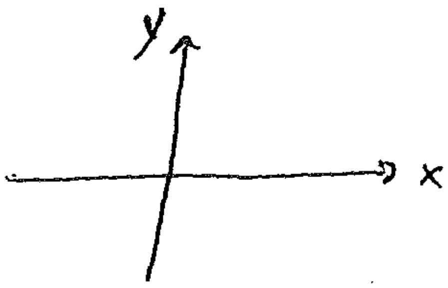
<u>Examples</u>:

1. If l is the line of reflection, what will be the image under l of the following objects?



2. If the plane is reflected in the Y-axis, what is the image of

a) (-2,1) b) (2,4) c) (-2,-5)
d) (0,3)

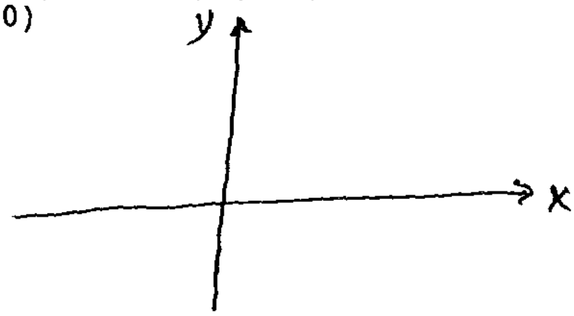


Solution:

a) (2,1) b) (-2,4) c) (2.-5)
d) (0,3)

3. If the plane is reflected in the X-axis, what is the image of

a) (-2,1) b) (2,4) c) (-2,-5)
d) (3,0)

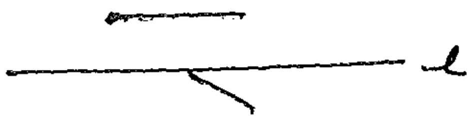


Solution:

a) (-2,-1) b) (2,-4) c) (-2,5)
d) (3,0)

Exercises:

1. If l is the line of reflection, what is the image under l of the following object?



2. Give the equations of the images obtained by reflecting the following lines in the Y-axis.

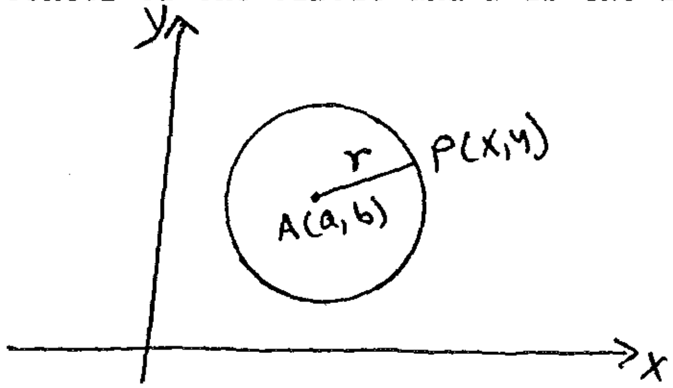
a) x = 2
b) x = -3
c) y = 1

Concept Label IV **Equation of a Circle**

Introduction: In your earlier grades you learnt how to construct a circle and to determine its circumference and area. In this lesson we define or describe circles analytically.

Lesson Development: You remember that a circle is defined as a set of points in a plane all at the same distance from the center. The equation of a circle is given as follows.

Def^{n} Equations for circles are usually written in the form $(x-a)^2 + (y-b)^2 = r^2$, where (a,b) is the center of the circle and r is the radius.



The definition of a circle you already know is equivalent to this form of equation of a circle. It only determines r, the distance between the point on the circle and the center.

Eg. 1. Find the equation of a circle with the given center and radius.

a) (3,4),1 b) (-3.-4),9 c) (0,0), a,a>0

Solution:

a) $(x-3)^2 + (y-4)^2 = 1$
b) $(x+3)^2 + (y+4)^2 = 81$
c) $(x-0)^2 + (y-0)^2 = a^2$ or $x^2 + y^2 = a^2$

2. Identify the center and radius of the circle given by:

Solution

a) $(x-2)^2 + (y-2)^2 = 9$ Center = (2,2), r = 3
b) $(x+4)^2 + (y+7)^2 = 64$ Center = (-4,-7), r = 8

By completing the square method find the center and radius of the circle represented by

a) $x^2+10x+25+y^2 = 16$
b) $x^2+y^2-4x-10y-7 = 0$

Solution:

a) $x^2+10x+25+y^2 = 16$ is equivalent to
$x^2 +10x +y^2 = 16-25$
and to $x^2+10x+25+y^2 = 16-25+25$
and to $(x+5)^2+y^2 = 16$

Hence, $(-5,0)$ is the center and $r = 4$

b) $x^2+y^2-4x-10y-7 = 0$ is equivalent to
$x^2-4x+y^2-10y = 7$
and to $x^2-4x+4+y^2-10y+25 = 7+29$
and to $(x-2)^2 + (y-5)^2 = 36$
Hence $(2,5)$ is the center and $r = 6$

Exercises:

1. Determine the center and radius of the circle given by
a) $(x-\frac{1}{4})^2 + (y+6)^2 = 12$

b) $(x+\sqrt{2})^2 + (y-1)^2 = 1$

2. Write the equation of a circle if the center and the radius are given by
a) $(-1,-1),1$

b) $(-\sqrt{3},\sqrt{3}),\ \sqrt{3}$

3. Determine whether or not the following equations represent a circle, and if it represents find the center and radius

a) $x^2-x+y^2-y+11 = 0$
b) $x^2+12x+y^2-16y+19 = 0$

4. Is $x^2+y^2 = -16$ an equation of a circle? Why?

Concept Label V. Arithmetic Progression

Introduction: An arithmetic progression is a sequence whose graph lies on a straight line. When we count by 1's, 2's, and so on, any term of the sequence is obtained by adding 1, or 2 and so on respectively to the preceding term.

Lesson development:

Definition: An arithmetic progression is a sequence in which each term after the first is obtained by adding a fixed number to the preceding term. If we denote the sequence by $\{A_{(n)}\}$ and the fixed number by d, $A_{(n+1)}=A_{(n)}+d$ for all natural numbers n. The number d is called the common difference.

Examples

1. Write the first 5 terms in counting by 5's beginning with 3.

   Solution
   The first term is 3
   The second term is 3+5 = 8
   The third term is 8+5 = 13

So, 3, 8, 13, 18, 23 are the first five terms. What is the value of d? d=5

2. The first term of an arithmetic progression is 3 and the common difference is 4. Find the tenth term.

   Solution:

   $A_1=3$
   $d=4$

   The terms are 3, 7, 11, 15, 19, 23, 27, 31, 35, 39,...
   Then the tenth term is 39.

Definition: The n^{th} term of an arithmetic progression with first term A_1 and common difference d is given by $A_n=A_1+(n-1)d$.

Examples:

3. Find the 30^{th} term of the sequence 9,11,13,...

Solution

$A_1=9$
$d=2$

Then $A_{30}=9+(30-1)2$
$=9+(29\times2)$
$=\underline{67}$

4. Find the n^{th} term for examples (1) and (2)
   Solution:
   a) 3, 8, 13,...
   $A_1=3$
   $d=5$
   $A_n=3+(n-1)5$
   $=3+5n-5$
   $=\underline{5n-2}$
   b) $A_1=3$
   $d=4$
   $A_n=3+(n-1)4$
   $=3+4n-4$
   $=\underline{4n-1}$

5. The 3^{rd} term of an arithmetic progression is 9 and the common difference is 2. Find the 200^{th} term.

Solution:

$A_3=9$
$d=2$
Then $A_2 =9-2=7$ and $A_1=7-2=5$
That is, $A_1=5$ and $d=2$
$A_{200}=5+(200-1)2$
$=5+(199x2)$
$=5+398$
$=\underline{403}$

Exercises:

1. Write the first 5 terms and find a formula for the n^{th} term obtained in counting by 7's beginning with 5.

2. The 4^{th} and 5^{th} terms of an arithmetic progression are 47 and 52. What is d? Find A_{50}.

Concept Label VI. Geometric Progression

Introduction: In the discussion of the arithmetic progression, we have seen the linear increment of sequences. Here, we will discuss what the exponential increment of sequences, the geometric progressions, look like.

Lesson Development:

Definition: A geometric progression is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed number. If we denote the sequence by $\{G_n\}$ and the fixed number by r, $G_{n+1}=rG_n$ for all natural numbers n. The number r is called the common ratio.

Examples:

1. In the sequence 2, 4, 8, 16,... what is the 5^{th} term? The sixth term? The common ratio?

Solution:

2, 4, 8, 16, 32, 64...and r=2

2. In the sequence 1, 1/3, 1/9, 1/27,...What

is the 5^{th} term? The 6^{th} term? The common ratio?

Solution:

The 6^{th} term is 1/81. The 7^{th} term is 1/243, r=⅓

3. Write the next three terms in the sequence

a) 16, 4, 1,...

b) 0.3, 0.03, 0.003,...

c) What is the common ratio in each case?

Solution:

a) 16, 4, 1, ¼, 1/16, 1/64,... r=¼

b) 0.03, 0.03, 0.003, 0.0003, 0.00003, 0.000003,... r=0.1

Definition: In the general case, the n^{th} term of a geometric progression $\{G_n\}$ with the common ratio r and first term G_1 will be $G_n=r^{n-1}G_1$.

Examples:

4. The first two terms of a geometric progression are 3 and 12 respectively. Find the 5^{th} term.

Solution:

$G_1=3$
$r=4$
Then $G_5=4^4x3=256x3=\underline{768}$

5. Find the 7^{th} term of the sequence 5, 10, 20,...

Solution:

$r = 2$

$G_1 = 15$

$G_7 = 2^6x5 = 64x5 = \underline{320}$

6. The 4^{th} and the 9^{th} terms of a geometric progression are ⅓ and 81 respectively. Find the sequence.

Solution:

$G_4 = ⅓ \qquad G_4 = r^3G_1 = ⅓$

$G_9 = 81 \qquad G_9 = r^8 G_1 = 81$

Then $\frac{G_9}{G_4} = \frac{r^8G_1}{r^3G_1} = \frac{81}{⅓}$

That is $r^5 = 243$
Hence $r = 3$

Then $G_3 = 1/9$, $G_2 = 1/27$, $G_1 = 1/81$

The sequence is: 1/81, 1/27, 1/9, 1/3, 1, 3, 9, 27, 81,...

Exercises:

1. Find the 8^{th} term of the sequence 2, -6, 18, -54,...

2. Find the 4^{th} and 8^{th} terms of the sequence: -0.008, 0.004, 0.2,...

Concept Label VII. The exponential function defined by $a^x, a>0$

Introduction:

In grade 10, you learnt that a^x when $a>0$ and x any real number is called a power with base a and exponent x. Moreover, you remember the five laws of exponents. We use such expressions in an exponential function.

Lesson Development:

Definition:

A function f defined by $f(x)=a^x$ where $a>0$ and $a\neq1$ and x any real number is called an exponential function with base a.

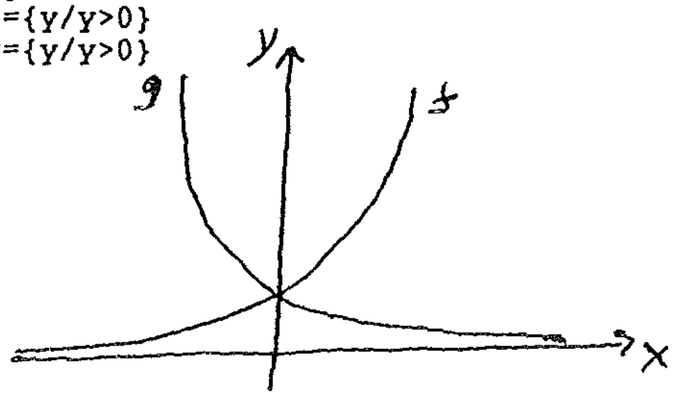
Examples:

1. Determine the domain and range of the following functions and sketch their graphs on the same coordinate axes.

a) $f(x)=2^x$ b $g(x)=2^{-x}$

Solution:

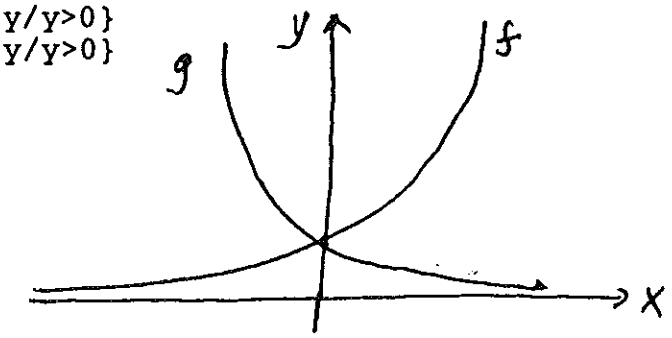
Domain of f = all real numbers
Domain of g = all real numbers
Range of $f=\{y/y>0\}$
Range of $g=\{y/y>0\}$



2. Answer question (1) for a) $f(x)=3^x$ b) $g(x)=(⅓)^x=3^{-x}$

Solution:

Domain of f=all real numbers
Domain of g=all real numbers
Range of f={y/y>0}
Range of g={y/y>0}



3. What is the domain and range of $f(x)=a^x, a>0, 0\neq 1$?

Solution:

Domain of f=all real numbers

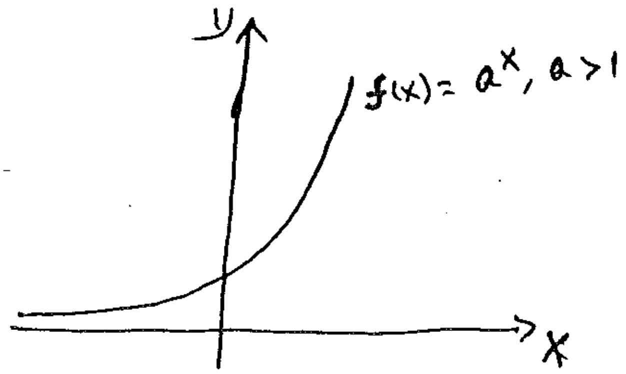
Range of f={y/y>0}

4. If a>1, what is the value of the function $y=a^x$ for

a) x>0
b) x<0
c) x=0
d) what does the graph look like?

Solution:

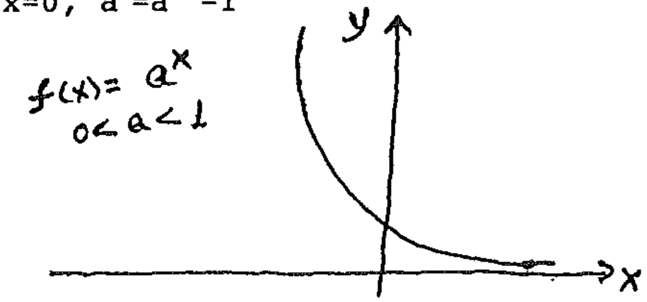
a) If x>0, a^x is large and positive
b) If x<0, a^x is less than one and approaches to zero
c) If x=0, $a^x=a^0=1$
d)



5. Answer question (4) if a<1

Solution:

a) If x>0, a^x is less than one and approaches zero
b) If x<0, a^x is large and positive
c) If x=0, $a^x=a^0=1$
d)



Exercises:

1. Sketch the graphs of $y=10^x$ and $y=3^x$ on the same coordinate axes and compare the values for x<o, x>0, and x=0

2. What is the truth set of the equation $a^x=b^x$ for a>b>1?

Concept Label VIII. The Logarithmic Functions

Introduction:

In grade 10, you learnt about logarithms. Furthermore, in the preceding chapters of grade 11, you learnt that the inverse of a function is again a function provided that the function is a one-to-one correspondence. In this lesson we combine these two concepts and obtain the logarithmic function as an inverse of the exponential function.

Lesson Development:

Consider the exponential function $y=a^x$, $a>0, a\neq 1$. As you remember there is a one-to-one correspondence between x(domain) and a^x (range). Hence the exponential function $f(x)=a^x$ has an inverse, called the logarithmic function.

Definition:

If $a>0, a\neq 1$ and $y=a^x$, then the exponent x is called the logarithm of y to the base a if and only if $a^x=y$. In symbols, $x=\log_a{}^y$ if and only if $a^x=y$.

In other words, the ordered pairs of the logarithmic function defined by $y=\mathrm{Log}_a{}^x$ are the same to that of $y=a^x$ except that the order has been reversed.

Examples:

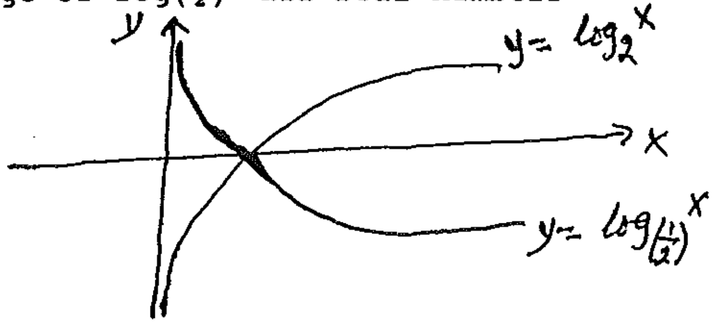
1. $3=\log_2^8$ since $2^3=8$ 3. $⅓=\log_{27}^3$ since $27^{⅓}=3$

2. $2=\log_3^9$ since $3^2=9$

4. Sketch the graphs of $y=\log_2^x$ and $y=\log(\frac{1}{2})^x$ on the same coordinate axes and determine the domain and range of each.

Solution:

Domain of $\log_2^x=\{x/x>0\}$ Domain of $\log(\frac{1}{2})^x=\{x/x>0\}$
Range of $\log_2^x$=all real numbers
Rage of $\log(\frac{1}{2})^x$=all real numbers

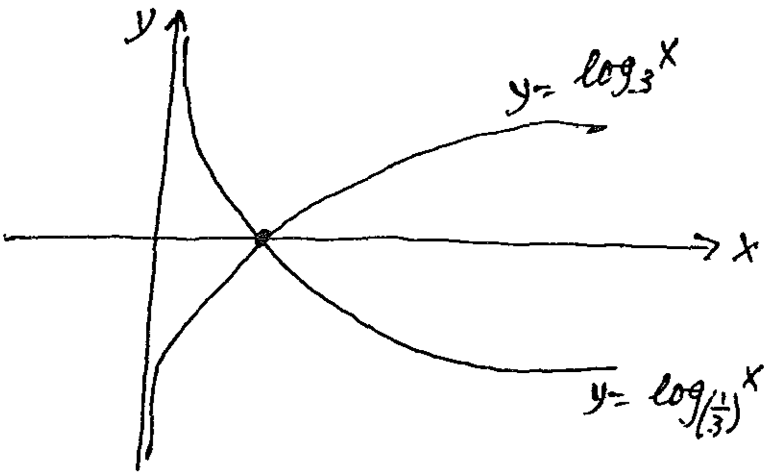


5. Sketch the graphs of $y=\log_3^x$ and $y=\log(⅓)^x$ on the same coordinate axes and determine the domain and rage of each.

Solution:

Domain of $\log_3^x=\{x/x>0\}$ Domain of $\log(⅓)^x$ = $\{x/x>0\}$

Range of $\log_3^x$=all real numbers
Range of $\log(⅓)^x$=all real numbers



6. What is the value of the function $y=\log_a^x$, $a>1$ when

a) $x>1$ b) $x<1$ c) $x=1$

Solution:

a) If $x>1$, $\log_a^x$ is large and positive

b) If $x<1$, $\log_a^x$ is large but negative

c) If $x=1$, $\log_a^x=\log_a^1=0$

7. What is the value of the function $y=\log_a^x$, $a<1$ when

a) $x>1$, b) $x<1$ d) $x=1$

Solution:

a) If $x>1$, $\log_a^x$ is large but negative

b) If $x<1$, $\log_a^x$ is large and positive

c) If $x=1$, $\log_a^x=\log_a^x=\log_a^1=0$

Exercises:

1. Sketch the graphs of $y=\log_{10}^x$ and $y={}_3^x$ on the same coordinate axes and compare the values for $x<1$, $x>1$, and $x=1$

2. Do exercise (1) for $y=\log(1/10)^x$ and $y=\log(½)^x$

3. What is $\log_2^{(-4)}$? Explain!

Appendix 7

Lesson for the Inductive Group

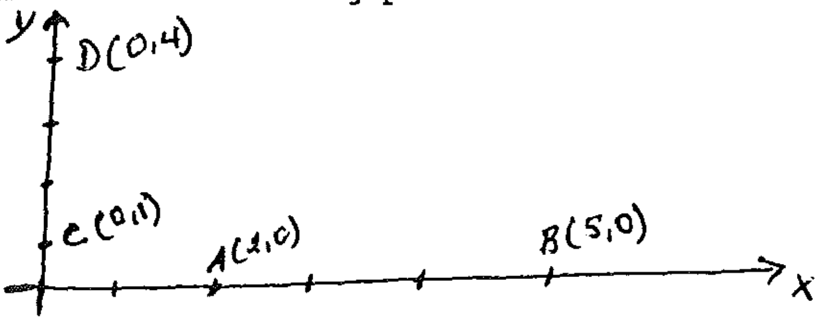
Concept Label I. Distance Formula in a Plane

Introduction:

We know that given two points in a plane, we can construct a line segment joining the two given points. But how do we determine the distance between the two points? We will determine the distance of a line segment (or the distance between two points) in a plane.

Lesson Development:

Consider the following points on the coordinate plane

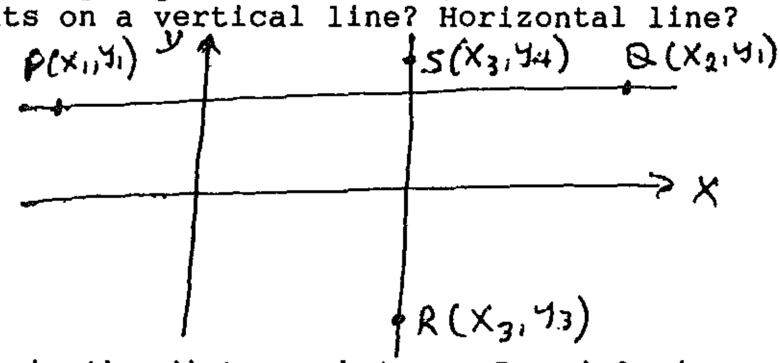


How many units should you go to get B from Point A? What is the length of AB?

Did you find that AB=5-2=3?

How many units should you go to get D from point C? What is the length of CD? Did you find that CD=4-1=3?

Can you generalize, now, the distance between any two points on a vertical line? Horizontal line?

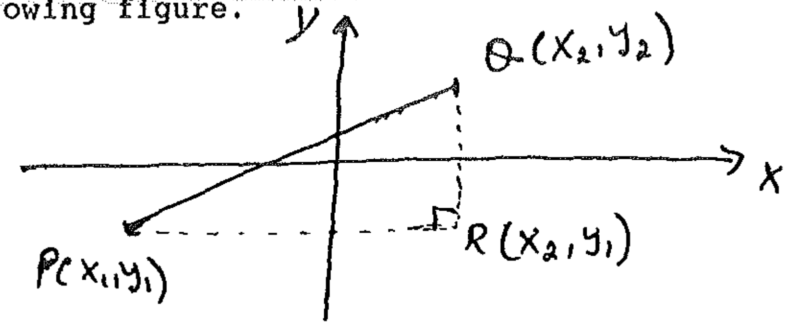


What is the distance between P and Q, i.e., What is the length of PQ? Did you get that $PQ=/x_2-x_1/$?

What is the distance between R and S, i.e., the length of RS?

Did you get that $RS=/y_4-y_3/$?

Now consider any two arbitrary points $p(x_1,y_1)$ and $Q(x_2,y_2)$ in the following figure.



As you can see PQ is neither vertical nor horizontal. To find PQ, form the triangle as shown and determine the coordinates of R.

What kind of triangle is ΔPRQ? What is the length of PR? RQ? observe that PR and RQ are horizontal and vertical lines respectively. So you get $PR=/x_2-x_1/$, $RQ=/y_2-y_1/$ and ΔPRQ is a right-angled triangle.

What does the Pythagoras theorem say to right-angled triangles? Can you apply it to ΔPRQ?

You get $(PQ)^2 = (PR)^2 + (RQ)^2$. substitute the values of PR and RQ

$$= (x_2-x_1)^2 + (y_2-y_1)^2$$

$$\text{Hence } PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

<u>Definition</u>: The distance between two arbitrary points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

<u>Examples</u>:

1. Find the distance between the two points

a) (1,1), (2,5)

<u>Solution</u>: Let P(1,1), Q(2,5)

Then $PQ = \sqrt{(2-1)^2 + (5-1)^2} = \sqrt{1^2+4^2} = \sqrt{1+16} = \sqrt{17}$

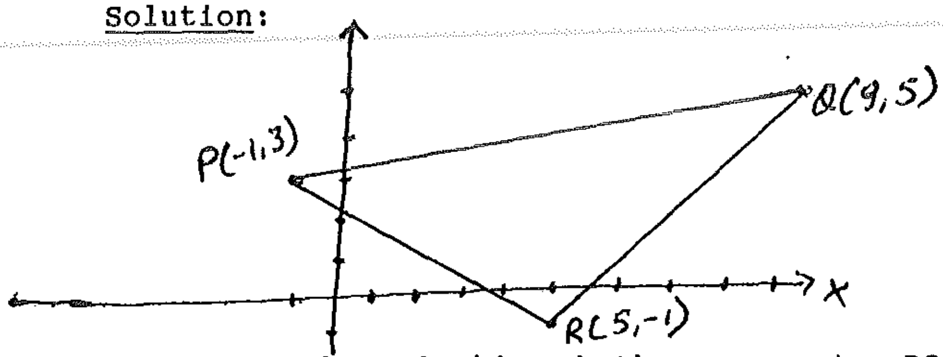
b) (4,-2), (-10,1)

<u>Solution</u> Let P(4,-2) and Q(-10,1)

Then $PQ = \sqrt{(-10-4)^2 + (1-(-2)^2} = \sqrt{(-14)^2+3^2} = \sqrt{196+9} = \sqrt{205}$

2. Determine whether or not the triangle with vertices (9,5), (5,-1), (-1,3) is isosceles

Solution:



Find PQ, QR, RP then decide whether or not $\triangle PQR$ is isosceles

Exercises:

Find the distance between the following points

1. (7,1) and the origin
2. (5,6) and (7,10)
3. (-5,-2) and (0,6)

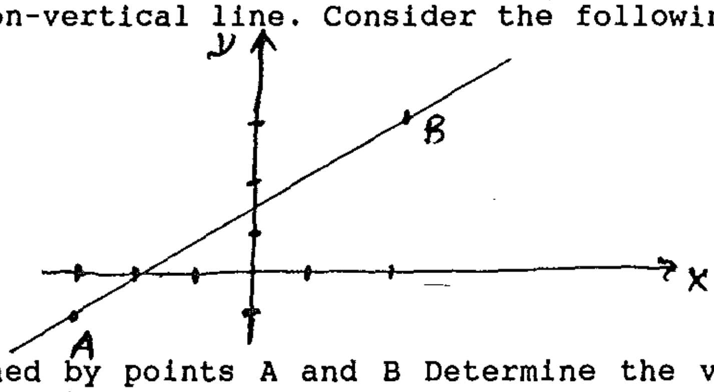
Concept Label II. Slope of a Line

Introduction:

The slope of a line is a measure of the steepness of a line as we go from the left to the right or from the right to the left along the X-axis

Lesson Development:

In your earlier studies you remember that the constant ration $\dfrac{\text{vertical increase}}{\text{Horizontal increase}}$ represents the slope of a non-vertical line. Consider the following line

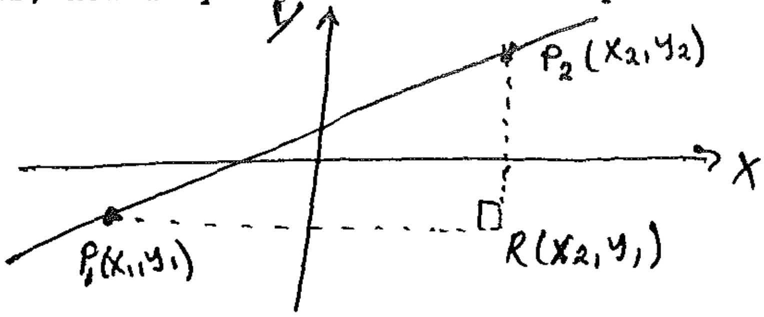


determined by points A and B Determine the vertical and horizontal increases as you go from A to B, Determine the coordinates of A and B. That is A(-3,-1) and B (2,3).

Then the vertical increase is from -1 to 3 on the Y-axis, that is 4 units. The horizontal increase is from -3 to 2 on the x-axis, that is 5 units. Then

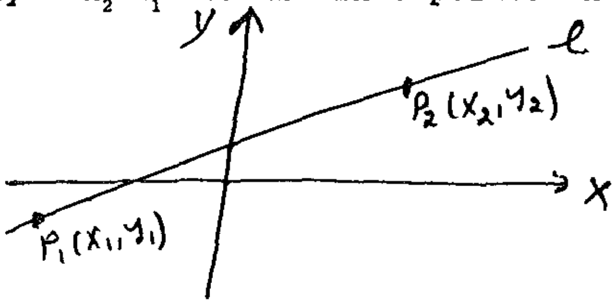
The slope is = $\dfrac{\text{Vertical increase}}{\text{Horizontal increase}}$ = 4/5

In general, if you take any two points on the line as follows, how do you determine the slope?

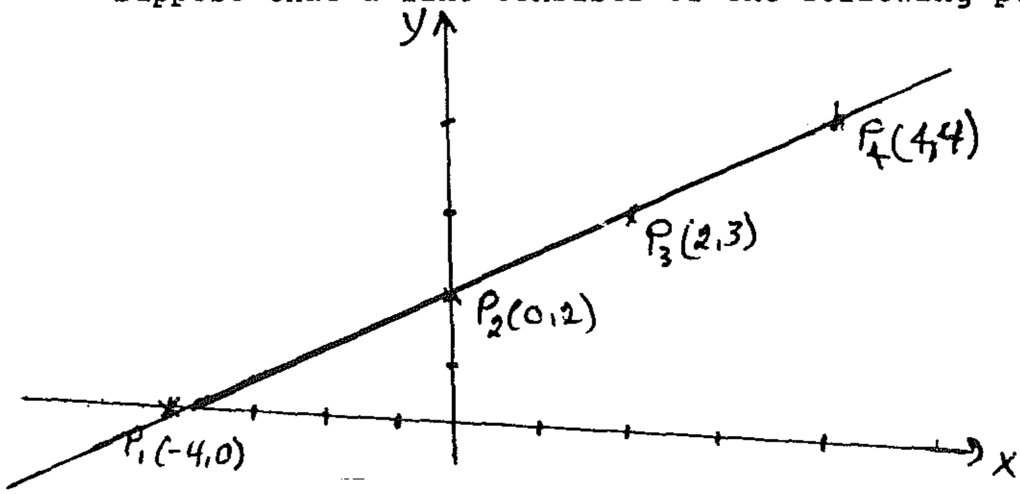


What is the vertical increase? It is y_2-y_1
What is the horizontal increase? It is x_2-x_1
Then slope = $\dfrac{\text{Vertical increase}}{\text{Horizontal increase}} = \dfrac{y_2-y_1}{x_2-x_1}$

Definition: The slope of a non-vertical line l is the ratio $\dfrac{y_2-y_1}{x_2-x_1}$, where $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are any two distinct points on l



Suppose that a line consists of the following points



Take P_1 and P_2 and find the slope. slope = $\dfrac{2-0}{0-(-4)}$ = 2/4 = ½

Take P_3 and P_4 and find the slope, slope = $\dfrac{4-3}{4-2}$ = ½

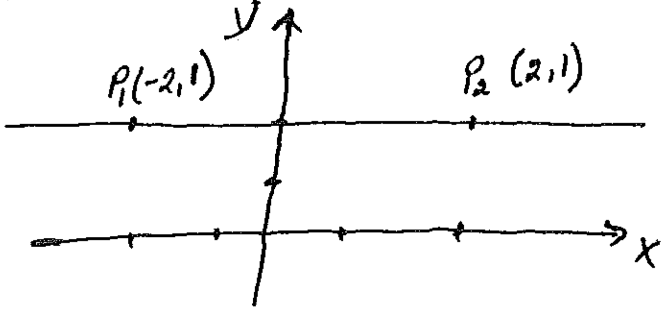
So, do you see that the slope of a line is independent of the choice of points on the line?

Examples: 1. Find the slope of the line determined by (7,9), (3,11)

Solution: Let $P_1(x_1,y_1)=(7,9)$, $P_2(x_2,y_2) = (3,11)$. Then

$$\frac{y_2-y_1}{x_2-x_1} = \frac{11-9}{3-7} = \frac{2}{-4} = \frac{-1}{2}$$

2. Find the slope of a horizontal line l, as shown.

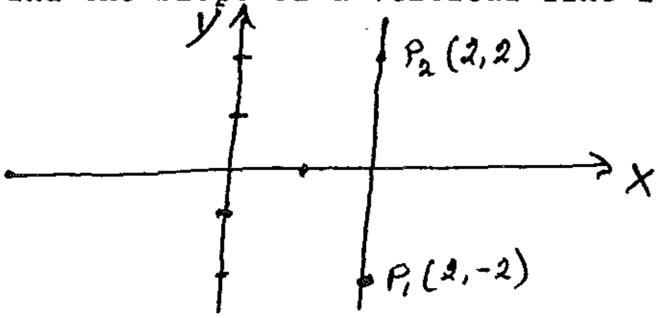


Solution: Take two points $P_1(-2,1)$ and $P_2(2,1)$. Then

$$\frac{y_2-y_1}{x_2-x_1} = \frac{1-1}{2-(-2)} = \frac{0}{4} = 0$$

Since the value of y is fixed and the vertical increase is zero, the slope of a horizontal line is zero.

3. Find the slope of a vertical line l as shown



Solution: Take two points P_1 and P_2 on l. Then

$$\frac{y_2-y_1}{x_2-x_1} = \frac{2-(-2)}{2-2} = \frac{4}{0}$$

So, what is the slope of a vertical line? Why? the horizontal increase of a vertical line is zero. Then we can not talk about the slope of a vertical line.

Exercises: Determine slope of the line determined by the pair of points

1. (0,3) and (3,0)
2. (6,2) and (-3,2)
3. (3,4) and (3,6)
4. (m,n) and (-n,-m), $m,n \neq 0$

Concept Label III. Reflections

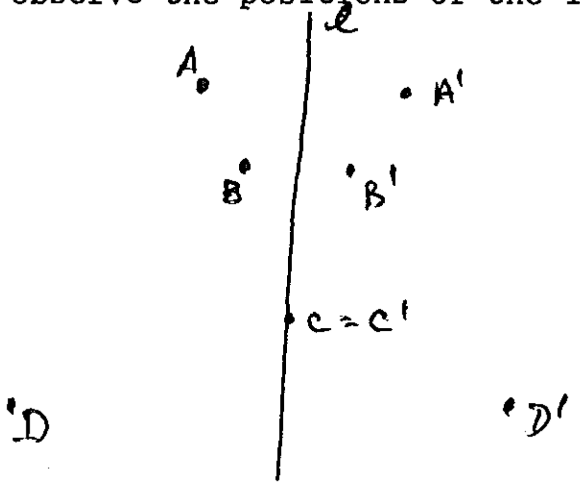
Introduction:

Reflection is a rigid motion where the size and shape of an object is not changed as a result of this motion. In particular, how far behind the mirror does an object appear? How does it look like? Such questions will get answer in this lesson.

Lesson Development:

Assume that you are looking at yourself in a mirror from a certain distance. Where does your image appear? Is it infront of the mirror or behind the mirror? How far is your image from the mirror? Now come closer and touch the mirror with your finger. Where is the tip of your finger in the mirror?

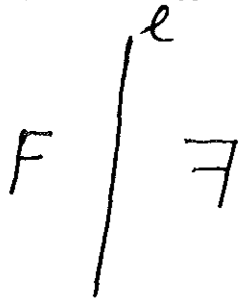
Assume that l is a mirror in the following picture and observe the positions of the images of objects



What is the relation between the distance from A to the mirror and from the mirror to A^1, the image of A? Did you observe the same relation between B,D and B^1,D^1 with respect to l? What is the image of C?

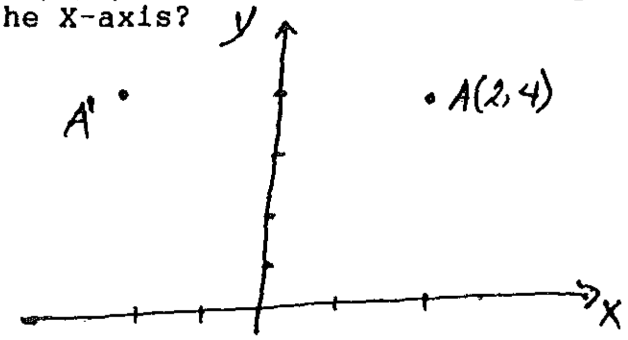
Now, what is the relationship between AA^1 and l. BB^1 and l? DD^1 and l? Did you observe that $AA^1 \perp l$? $BB^1 \perp l$ and $DD1 \perp l$?

What does the image of the following figure look like? How far is it from the mirror?



Did you see that it is found at the same distance as the original object is from the mirror?

Suppose the plane is reflected in th Y-axis, i.e. consider the Y-axis as a mirror in the plane. What is the image of (2,4)? What if the same point is reflected in the X-axis?



Did you observe that in reflecting a point in the Y-axis, only the x-coordinate changes its sign and the y-coordinate is fixed? And also the x-coordinate is fixed in reflecting a point in the X-axis while the y-coordinate changes sign?

<u>Definition</u>: If P is a point not on a line l, then its image in l is the point P^1 such that

a) $\overleftrightarrow{PP^1} \perp l$,i.e. $\overline{PP^1}$ is perpendicular to l

b) If $\overleftrightarrow{PP^1}$ meets l at x, then $PX=P^1X$

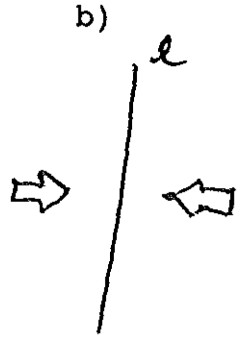
If P is on l, its image P^1 in l is P itself. In either case we say that we reflect P in l to get P^1, and we call l the line, or axis of reflection.

<u>Examples</u>:

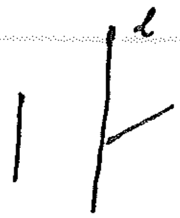
1. If l is the line of reflection, what will be the image under l of the following objects?

a)

b)



c)



2) If the plane is reflected in the Y-axis, what is the image of

a) (-2,1) b) (-2,-5) c) (0,3)

Solution: The images are

a) (2,1) b) (2,-5) c) (0,3)

3) If the plane is reflected in the X-axis, what is the image of

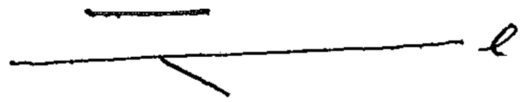
a) (-2,1) b) (-2,-5) c) (3,0)

Solution: The images are

a) (-2,-1) b) (-2,5) c) (3,0)

Exercises:

1. If l is the line of reflection, what is the image under l of the following object?



2. Give the equations of the images obtained by reflecting the following lines in the Y-axis.

a) x=2
b) x=-3
c) y=1

Concept Label IV. Equation of a Circle

Introduction:

In your earlier grades you learnt how to construct a circle and to determine its circumference and area. In this lesson we define or describe circles analytically.

Lesson Development:

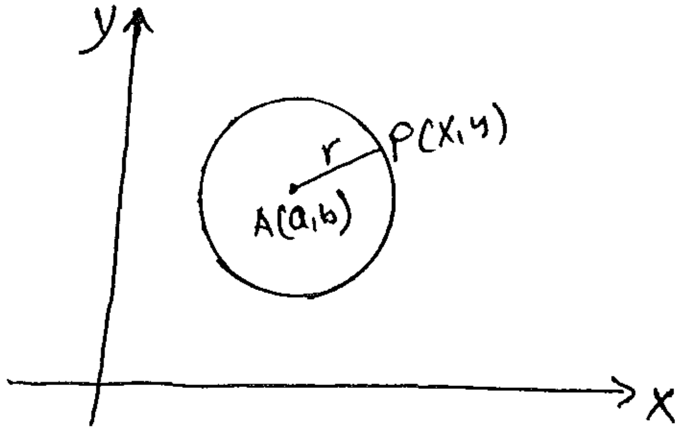
You remember that a circle is a set of points in a plane all at the same distance from the center.

Suppose that (2,2) is a center of a circle and the radius is 3. What are the set of points in a plane which are 3 units long from the center, i.e., (2,2)?

Let (x,y) be any arbitrary point on the circle. What should the distance between (2,2) and (x,y) be? Did you get $(x-2)^2 + (y-2)^2 = 9$? This equation is an equation of a circle. Can you identify the center and radius from the equation?

Suppose the equation of a circle is given by $(x+3)^2 + (y+4)^2=81$. What is the center? The radius? Did you see that (-3,-4) is the center and 9 is the radius?

Definition: Equations of circles are usually written in the form $(x-a)^2 + (y-b)^2= r^2$ Where (a,b) is the center and r the radius.



Examples:

1. Find the equation of a circle with the given center and radius.

   a) (3,4), 1   b) (0,0), a, a>0

Solution:

a) $(x-3)^2 + (y-4)^2=1$

b) $(x-0)^2 + (y-0)^2 = a^2$ or $x^2+y^2 = a^2$

2. Identify the center and radius of the circle given by $(x+4)^2+(y+7)^2 = 64$

   Solution: Center = (-4,-7), r = 8

Unfortunately, equations of circles may not be given in this form usually. We need to convert equations in this form if the equation represents a circle. Suppose we are given.

$x^2+y^2-4x-10y-7=0$ Is it equation of a circle? If so, what are the center and the radius? As you can see it is not in the form we already know. Apply the completing the square method on both x and y. you get.

$x^2+y^2-4x-10y-7=0$ is equivalent to
$x^2-4x \quad +y^2-10y=7$
and to $x^2-4x+4+y^2-10y+25=7+4+25$
and to $(x-2)^2 + (y-5)^2=36$

What does this equation represent? Did you see that the center is (2,5) and the radius is 6?

3. Use the completing the square method to determine whether or not $x^2+10x+25+y^2 = 16$ represents equation of a circle. If so, identify the center and radius.

Solution:

$x^2+10x+25+y^2=16$ is equivalent to

$x^2+10x+y^2 =16-25$

and to $x^2+10x+25+y^2 = 16-25+25$

and to $(x+5)^2 + Y^2 =16$

This is equation of a circle with center (-5,0) and radius 4.

Exercises:

1. Determine the center and radius of the circle given by

   a) $(x-\frac{1}{4})^2+(y+6)^2 =12$ b) $(x+\sqrt{2})^2+(y-1)^2 =1$

2. Write the equation a circle if the center and radius are given by:

   a) (-1,-1),1 d) $(-\sqrt{3}, \sqrt{3}),\sqrt{3}$

3. Determine whether or not the following equations represent a circle, and if so identify the center and radius

   a) $x^2-x+y^2-y+11=0$

   b) $x^2+12x+y2-16+19=0$

4. Is $x^2 + y^2 = -16$ an equation of a circle? Why?

Concept Label V. Arithmetic Progression (A.P.)

Introduction:

An arithmetic progression is a sequence whose graph lies on a straight line. When we count by 1's, 2's, and so on, any term of the sequence is obtained by adding 1 or 2 and so on respectively to the preceding term. The purpose of this lesson is to develop a formula for the general term of an arithmetic progression.

Lesson Development:

Suppose you count by 5's beginning with 3. What are the first five terms?

The first term is 3
The second term is 8=3+5
The third term is 13=8+5=3+5+5
Did you get that the terms are 3, 8, 13, 18, 23...? What is the difference between the first and the second terms? The 4^{th} and the 5^{th} terms? What is the 6^{th} term? How did you get the 6^{th} term? Did you see that the difference between any two consecutive terms is the same? This same difference is called the common difference, d. Thus d=5 in this sequence.

Definition: An arithmetic progression is a sequence in which each term after the first is obtained by adding a fixed number to the preceding term. If we denote the sequence by $\{A_n\}$ and the fixed number by d, A_{n+1} =An+d for all natural numbers n. The number d is called the common difference.

Example:

1. The first therm of an A.P. is 3 and the common difference is 4. Find the 10^{th} term.

Solution:

$A_1 = 3$
$d = 4$

The terms are 3, 7, 11, 15, 19, 23, 27, 31, 35, 39,...

Then the 10^{th} term is 39.

But how do we know the n^{th} term?

Suppose the first term of an A.P. is 9 and the common difference is 2. What is the 30^{th} term?

$$A_1 = 9,\ d=2$$
$$A_2 = A_1+d=9+2$$
$$A_3 = A_2+d=A_1+d+d=A_1+2d=9+(2x2)$$
$$A_4 = A_3+d=A_1+2d+d=A_1+3d=3+(3x2)$$
$$A_5 = A_4+d=A_1+3d+d=A_1+4d=9+(4x2)$$
.
.
.
$$A_{30}=A_1+29d=9+(29x2)=9+58=67$$

In general, if A_1 is the first term and d is the common difference, how do you find the n^{th} term?

$$A_1=A_1$$
$$A_2=A_1+d$$
$$A_3=A_2+d=A_1+d+d=A_1+2d=A_1+(3-1)d$$
$$A_4=A_3+d=A_1+2d+d=A_1+3d=A_1+(4-1)d$$
$$A_5=A_4+d=A_1+3d+d=A_1+4d=A_1+(5-1)d$$
.
.
.
$$A_n= A_1+(n-1)d$$

<u>Definition</u>: The n^{th} term of an A.P. with first term A_1 and common difference d is $A_n=A_1+(n-1)d$

<u>Examples</u>:

2. Find the n^{th} term for

a) $A_1=3$, $d=4$
b) $A_1=3$, $d=5$

<u>Solution</u>: a) $A_n= A_1+(n-1)d$
$=3+(n-1)4$
$=3+4n-4$
$=4n-1$

b) $A_n = A_1+(n-1)d$
$=3+(n-1)5$
$=3+5n-5$
$=5n-2$

3. The 3^{rd} term of an A.P. is 9 and the common difference is 2. find the 200^{th} term.

<u>Solution</u>:

$A_3=9$

$d=2$

Then $A_2=A_3-d=9-2=7$

$A_1=A_2-d=7-2=5$

That is $A_1=5$, $d=2$

$A_n=A_1+n-1)d$

$A_{200}=A_1+(200-1)d$

$=5+199d$
$=5+(199x2)$
$=5+398$
$=403$

Exercises:

1. Write the first 5 terms and find a formula for the n^{th} term obtained in counting by 7's beginning with 5.

2. The 4^{th} and 5^{th} term of an A.P. are 47 and 5 respectively. What is d? find A_{50}.

Concept Label VI Geometric Progression (G.P)

Introduction:

In the discussion of the arithmetic progression, we have seen the linear increment of sequences. Here, we will discuss what the exponential increment of sequences, the geometric progressions, look like.

Lesson Development:

Observe the following sequence 2, 4, 8, 16,... what is the 5^{th} term? The 6^{th} term? What is the ratio of the 2^{nd} term to the 1^{st}? The 4^{th} term to the 3^{rd}? Are the two ratios different? This ratio is called a common ratio.

In the sequence 1, 1/3, 1/9, 1/27,...what is the 5^{th} term? The 6^{th} term,? The common ratio?

Let G_1 be the 1^{st} term and G_2 be the 2^{nd} and the common ratio, r. Then.

Definition: A geometric progression is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed number. If we denote the sequence by $\{G_n\}$ and the fixed number by r, $G_{n+1} = Rg_n$ for all natural numbers n. The number r is called the common ratio.

Examples:

1. Write the next three terms in the sequence

   a) 16, 4, 1,...
   b) 0.3,0.03,0.003,...

Solution:

a) $G_1=16$, $G_2=4$

$$\frac{G_2}{G_1} = \frac{4}{16} = \tfrac{1}{4} = r=\tfrac{1}{4}$$

Then $G_4 = \frac{1}{4}$, $G_5 = 1/16$, $G_6 = 1/64$

Then the sequence is 16, 4, 1, 1/4, 1/16, 1/64,...

b) $G_1 = 0.3$, $G_2 = 0.03$ $G_2/G_1 = 0.03/0.3 = 0.1$
$r= 0.1$

Then $G_4 = 0.0003$, $G_5 = 0.00003$, $G_6 = 0.000003$
Then the sequence is 0.3, 0.03, 0.003, 0.0003, 0.00003, 0.000003,...

How do we determine the n^{th} term?

Suppose that the 1^{st} term of a G.P. is 5 and the common ratio is 2. What is the 7^{th} term?

$$\begin{aligned}
G_1 &= 5 \quad \text{and } r = 2\\
G_2 &= Rg_1 = 2x5\\
G_3 &= Rg_2 = r^2G_1 = 2^2x5\\
G_4 &= Rg_3 = r^3G_1 = 2^3x5\\
G_5 &= Rg_4 = r^4G_1 = 2^4x5\\
&\vdots\\
G_7 &= r^{(7-1)}G_1 = r^6G_1 = 2^6x5 = 320
\end{aligned}$$

In general, if G_1 is the first term and r is the common ratio, we have

$$\begin{aligned}
G_1 &= G_1\\
G_2 &= Rg_1\\
G_3 &= Rg_2 = r^2G_1 = r^{(3-1)}G_1\\
G_4 &= Rg_3 = r^3G_1 = r^{(4-1)}G_1\\
G_5 &= Rg_4 = r^4G_1 = r^{(5-1)}G_1\\
&\vdots\\
G_n &= r^{(n-1)}G_1
\end{aligned}$$

Definition: In the general case, the n^{th} term of a geometric progression with first term G_1 and common ratio r is given by $G_n=^{r(n-1)}G_1$

Examples:

2. The first two terms of a G.P. are 3 and 12 respectively. Find the 5^{th} term.

   Solution:

   $G_1 = 3$ Then $r = \frac{G_2}{G_1} = \frac{12}{3} = 4$
   $G_2 = 12$

   So, $G_3 = 48$ or $G_5 = r^4 G_1 = 4^4 x3$
   $G_4 = 192$ $= 256x3$
   $G_5 = 768$ $= 768$

3. The 4^{th} and the 9^{th} terms of a G.P. are ⅓ and 81 respectively. Find the sequence.

   Solution:

   $G_4 = 1/3$ Then $G_4 = r^3 G_1 = 1/3$

   $G_9 = 81$ $G_9 = r^8 G_1 = 81$

   $\frac{G_9}{G_4} = \frac{r^8 G_1}{r^3 G_1} = \frac{81}{1/3} = 243$

   That is $r^5 = 243$
   Then $r = 3$

   Then $G_3 = G_4/r = (1/3)/3 = 1/9$
   $G_2 = (1/9)/3 = 1/27$
   $G_1 = (1/27)/3 = 1/81$

   Then the sequence is

   1/81, 1/27, 1/9, 1/3, 1, 3, 9, 27, 81,...

Exercises:

1. Find the 8^{th} term of the sequence 2, -6, 18, -54,...
2. Find the 4^{th} and 8^{th} terms of the sequence given by 0.008, 0.004, 0.2,...

Concept Label VII. The Exponential Function Defined by a^x, $a>0$

Introduction: In grade 10, you learnt that a^x when $a>0$ and x any real number is called a power with base a and exponent x. Moreover, you remember the five laws of exponents. We use such expressions in an exponential function.

Lesson Development:

Consider the function $f(x)=2^x$ and $g(x) = 2^{-x}$ you remember that 2^x and 2^{-x} are powers with base 2 and x any real number. Then what is domain of f? of g? did you see that:

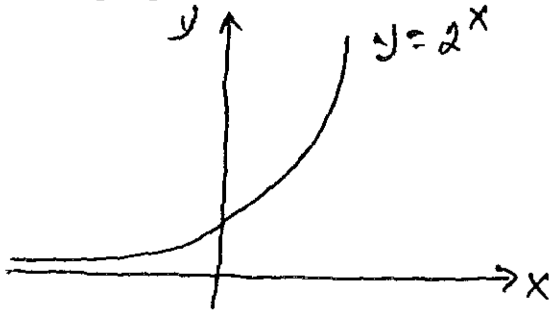
Domain of f is any real number and
Domain of g is any real number?

Is $2^{-x} = (\frac{1}{2})^x$?

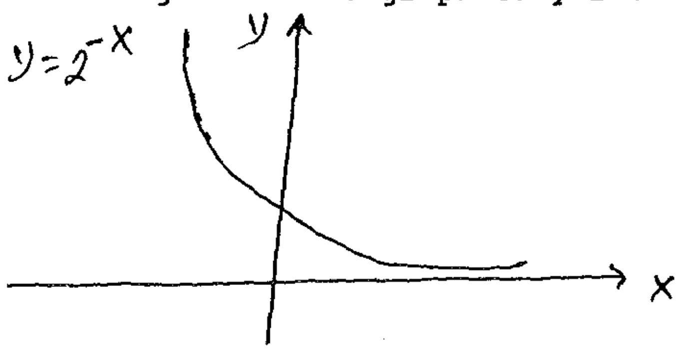
Now, do 2^x and 2^{-x} ever be negative? Zero? What is the value of 2^x if $x = 0$, i.e., What is 2^0?
What is the value of 2^{-x} if $x = 0$?

Did you see that (0,1) is the y-intercept of both 2^x and ~~2^x~~
2^{-x} ?

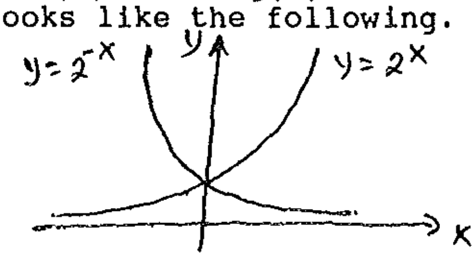
What is the value of 2^x if $x>0$? $x<0$? did you see that 2^x is never negative nor zero? And do you observe that if $x<0$, $2^x<1$ and $2^x>1$ if $x>0$? Do your observations agree to the graph of $y=2^x$?



What is the value of 2^{-x} if $x>0$? $x<0$? Did you see that if $x<0$, $2^{-x} >1$ and $2^{-x}<1$ if $x>0$? Do your observations agree to the graph of $y=2^{-x}$?



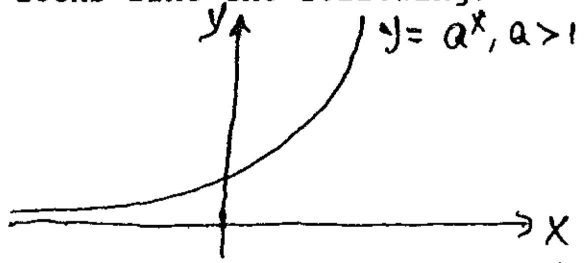
If you draw $f(x)=2^x$ and $g(x)=2^{-x}$ on the same coordinate axes, it looks like the following.



Consider now $y=a^x$, $a>1$

What is the value of a^x if $x>0$? $x<0$? $x=0$? Did you observe that $a^x>1$ if $x>0$, $a^x<1$ if $x<0$, and $a^x=1$ if $x=0$? Did you also observe that a^x is never 0 nor negative?

What does the graph look like? Did you get that the graph looks like the following?



<u>Definition</u>: A function f defined by $f(x)=a^x$, $a>0$, $a\neq1$ and x any real number is called an exponential function with base a.

<u>Examples</u>:

1. Determine the domain and range of the following functions and sketch their graphs on the same coordinate axes.

   a) $f(x) = 3^x$ b) $g(x) = (1/3)^x = 3^{-x}$

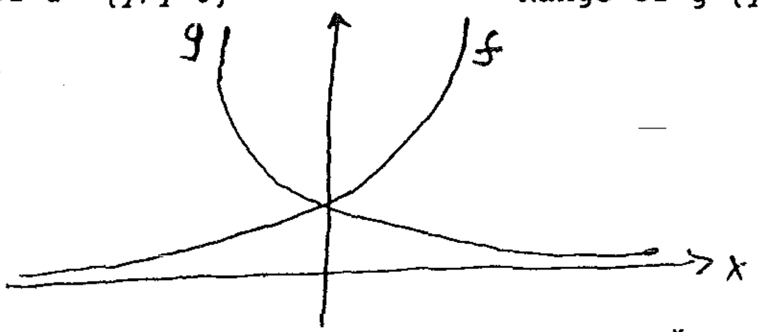
<u>Solution</u>:

Domain of f= all real numbers

Range of f= {y/y>0}

Domain of g= all real numbers

Range of g={y/y>0}



2. What is the domain and range of $f(x)=a^x, a>0$, $a\neq1$?

<u>Solution</u>:

Domain of f= all real numbers
Range of f= {y/y>0}

3. If $a<1$, what is the value of the function $y=a^x$ for

a) $x>0$
b) $x<0$
c) $x=0$
d) what does the graph looks like?

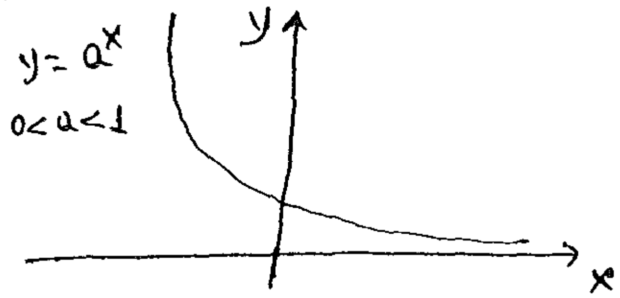
Solution:

a) If $x>0$, a^x is less than one and approaches zero

b) If $x<0$, a^x is large and positive, i.e. $a^x>1$

c) If $x=0$, $a^x=a^0=1$

d)



Exercises:

1. Sketch the graphs of $y=10^x$ and $y=3^x$ on the same coordinate axes and compare the values for $x<0$, $x>0$ and $x=0$

2. What is the truth set of the equation $a^x=b^x$ for $a>b>?1$

Concept Label VIII. The Logarithmic Functions

Introduction:

In grade 10, you learnt about logarithms. Furthermore, in the preceding chapters of grade 11, you learnt that the inverse of a function is again a function provided that the function is a one-to-one correspondence. In this lesson, we combine these two concepts and obtain the logarithmic function as an inverse of the exponential function.

Lesson Development:

Consider the exponential function $y=a^x, a>0, a\neq 1$. You remember that there is a one-to-one correspondence between x(the domain) and a^x(the range). Hence the exponential function $f(x)=a^x$ has an inverse, called the logarithmic function.

You remember that

$3=\log2^8$ because $2^3=8$

$2=\log 3^{9}$ because $3^{2}=9$

$⅓=\log 27^{3}$ because $(27)^{⅓}=3$

$2^3=8$ is an exponential expression. 3 is called the logarithm of 8 in base 2. In other words $(3,2^3)$, $(2, 3^2)$, (⅓, 27⅓) are elements of exponential function. The corresponding elements of the logarithmic function are the same numbers but with reversed order, i.e. $(2^3,3)$, $3^2,3)$, (27⅓, ⅓).

Consider $f(x)=2^x$ and $g(x)=(\frac{1}{2})^x$. You remember these functions. They are 1-1 correspondences. Then they have inverses, $\log 2^x$ and $\log(\frac{1}{2})^x$ respectively. You remember domain of f= range of f^{-1} and range of f=domain of f^{-1}

What is the domain of $f^{-1}(x)=\log 2^x$? $g^{-1}(x)=\log(\frac{1}{2})^x$? what is their range?

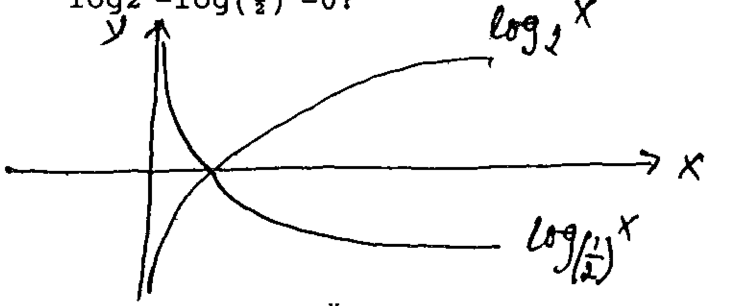
Did you see that $\{x/x>o\}$ is their domain and their range is the set of all real numbers?

What does their graph looks like?

If $x>1$, What is $\log 2^x$? Did you see that $\log 2^x>1$?
What is $\log(\frac{1}{2})^x$? Did you see that $\log(\frac{1}{2})^x<0$?

If $x<1$, What is $\log 2^x$? Did you see that $\log 2^x<0$?
What is $\log(\frac{1}{2})^x$? Did you see that $\log(\frac{1}{2})^x>1$?

If $x=1$, What is $\log 2^x$? $\log(\frac{1}{2})^x$? Did you see that $\log 2^x=\log(\frac{1}{2})^x=0$?



Consider the function $y=\log a^x$, $a>1$. What is the value of y when $x>1$, $x<1$, $x=1$?

Did you observe that if $x>1$, $\log a^x>1$, if $x<1$, $\log a^x<0$ and $\log a^x=0$ if $x=1$?

Definition: If $a>0$, $a\neq 1$ and $y=a^x$, then the exponent x is called the logarithm of y to the base a if and only if $a^x=y$. In symbols, $x=\log a^y$ if and only if $a^x=y$.

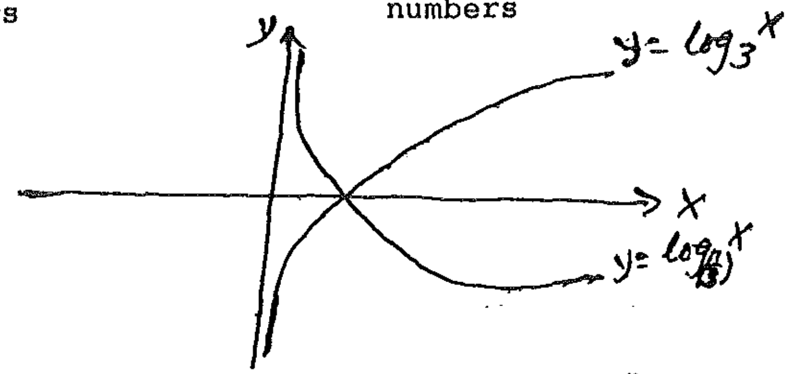
Examples:

1. Sketch the graphs of $y=\log 3^x$ and $y=\log(⅓)^x$ on the same coordinate axes and determine the domain and range of each.

Solution:

Domain of $log3^x$={x/x>0}
Rage of $log3^x$=all real numbers

Domain of $log(⅓)^x$={x/x>0}
Range of $log(⅓)^x$=all real numbers



2. What is the value of the function $y=loga^x$, a<1 when

   a) x>1   b) x<1   c) x=1?

Solution:

a) If x>1, $loga^x$ is large but negative, i.e., $loga^x<0$

b) If x<1, $loga^x$ is large and positive

c) If x=1, $loga^x$ = $loga^1=0$

Exercises:

1. Sketch the graphs of $y=log10^x$ and $y=log3^x$ on the same coordinate axes and compare the values for x<1, x>1, and x=1

2. Do exercise (1) for $y=log(1/10)^x$ and $y=log(⅓)^x$

3. What is $log2^{(-4)}$? Explain!

APPENDIX VIII

Addis Ababa University
School of Graduate Studies
Test III

Name ______________________
Group ________
Section ________

There are two parts in this test booklet.

Part I There are 16 questions. Choose the best answer and write the letter of your choice on the space provided to the left of each question. (16 points)

Part II There are four short answer questions. The points for each question are given in brackets.

PartI

____ 1. Suppose we reflect point P in line l to get its image P. Then, which of the following is not true?
   a. $\overline{PP'}$ is perpendicular to l
   b. If $\overline{PP'}$ meets l at X, then $PX = P'X$
   c. If P is on l, then $P = P'$
   d. $\overline{PP'}$ is parallel to l

____ 2. Suppose that the image of (-3,5) is (3,5). Then the equation of the mirror is
   a. $y = 0$
   b. $x = 0$
   c. $y = 5$
   d. $x = 5$

____ 3. Which one of the following is true about the exponential function $y = a^x$, $a > 0$, $a = 1$?
   a. Its graph cuts the X-axis at (1,0)
   b. It is an increasing function
   c. Its range is the set of positive real numbers
   d. If $x < 0$, $a^x < 0$

____ 4. The truth set of the equation $a^x = b^x$ for $a > b > 1$ is
   a. { 0 }
   b. { 1 }
   c. { 0,1 }
   d. { 0,1,2 }

____ 5. Which one of the following is not true about the logarithmic functions?
   a. Their domain is the set of all real numbers
   b. Their range is the set of all real numbers
   c. Their graphs cut the X-axis at (1,0)
   d. They are inverses of the exponential functions

_____ 6. Which one of the following is not true about the functions $y = \log_4 x$ and $y = \log_{1/4} x$?

a. Both have the same domain
b. Both have the same range
c. Their graphs intersect at (1,0)
d. Both are increasing functions

_____ 7. Which one of the following is necessarily true?

A. $f(x) = \log_a x$ is an increasing function
B. $g(x) = a^x$ is a decreasing function

a. Only A
b. Only B
c. Both A and B
d. Neither A nor B

_____ 8. What kind of triangle is the triangle determined by the three vertices (0,0), (0,3), (4,0)?

a. An isosceles triangle
b. An equilateral triangle
c. A right-angled triangle
d. None of the above

_____ 9. Suppose the distance between two points is zero. Then,

a. the two points are identical
b. the two points lie on the same vertical line
c. the two points lie on the same horizontal line
d. the two points lie on parallel lines

_____ 10. Which one of the following is true about the slope of a line?

a. It is the measure of the steepness of a line
b. It is the ratio of the vertical increase to the horizontal increase of two distinct points on the line
c. It is independent of the choice of points on the line
d. All of the above

_____ 11. If a line has no slope, then

a. the line is parallel to the X-axis
b. the line is parallel to the Y-axis
c. all of the above
d. none of the above

_____ 12. The slope of a line determined by the two points (m,n) and $(-n,-m)$ where $m,n = 0$ is

a. -1
b. 1
c. m/n
d. -m/n

_____ 13. The equation of a circle with then center (-1,-1) and the radius $\frac{1}{2}$ is
   a. $(x-1)^2 + (y-1)^2 = \frac{1}{2}$
   b. $(x-1)^2 + (y-1)^2 = \frac{1}{4}$
   c. $(x+1)^2 + (y+1)^2 = \frac{1}{2}$
   d. $(x+1)^2 + (y+1)^2 = \frac{1}{4}$

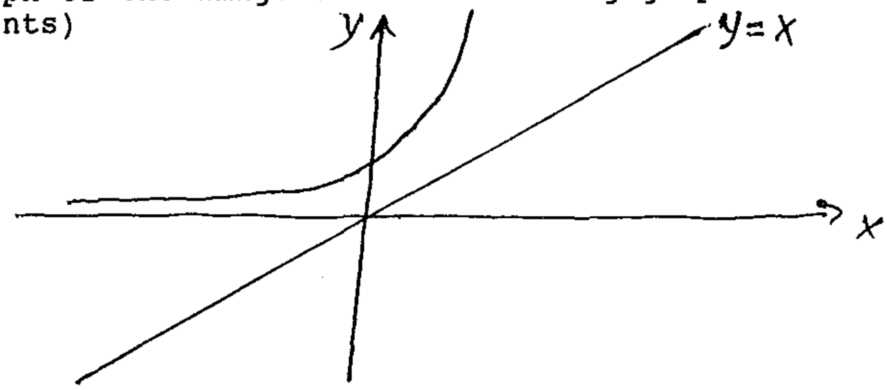
_____ 14. The center and radius of a circle determined by the equation $x^2 - 2x + y^2 + 4y + 6 = 0$ are respectively
   a. (1,-2), 1
   b. (-1,2), 1
   c. (1,-2), $\sqrt{11}$
   d. It is not an equation of a circle

_____ 15. The first and the second terms of an arithmetic progression are 3 and 9 respectively. The n^{th} term is given by
   a. 6n - 3
   b. 6n + 3
   c. 3n + 6
   d. 3n - 6

_____ 16. Suppose that the first term of a geometric progression is 5 and the common ratio is 2. The eighth term is
   a. 64
   b. 256
   c. 640
   d. 1280

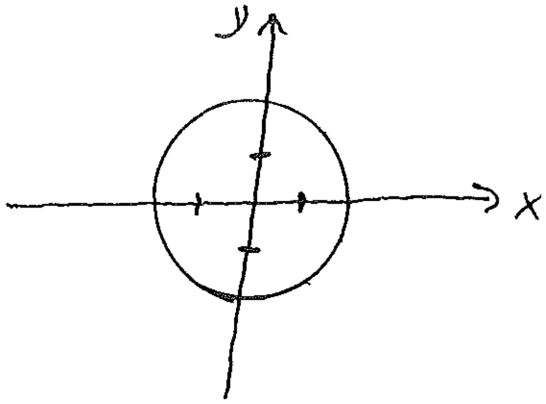
Part II

17. If the line y = x is the line of reflection, what does the graph of the image of the following graph look like? (2 points)



18. The third and the sixth terms of a geometric progression are 1 and 27 respectively. Write the sequence. (2 points)

19. Identify the center, the radius, and write the equation of the following circle. (2 Points)



20. Calculate the distance between the points (-5,-2) and (1,6). (2 points)

APPENDIX IX

The odd- and even- numbered scores of test III.

Item	Score
1	110
2	142
3	141
4	126
5	63
6	78
7	84
8	157
9	126
10	75
11	47
12	111
13	141
14	157
15	94
16	63
17	220
18	221
19	236
20	189