ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES
FACULTY OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING

EVALUATION OF APPROXIMATE METHODS FOR THE DESIGN

OF

BIAXIALLY LOADED REINFORCED CONCRETE COLUMNS

TEFERA DESTA

May 1999

EVALUATION OF APPROXIMATE METHODS FOR THE DESIGN OF BIAXIALLY LOADED REINFORCED CONCRETE COLUMNS

A Thesis presented to the School of Graduate Studies, Addis Ababa University in Partial Fulfillment of the Requirements for the Degree Master of Science in Civil Engineering

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NOTATIONS

A	Area
A_c	Gross concrete area
A_s	Area of tension steel
A'_s	Area of compression steel
$A_{s,tot}$	Total steel area
a	Location of compressive force in the compressed zone of concrete from the most compressed fiber
b	Width of cross-section
b'	Concrete cover to the center of reinforcement steel along the width
d	Effective depth of cross-section
e_y	Eccentricity of normal load along y-axis
e_z	Eccentricity of normal load along z-axis
f_c	compression stress in concrete
f_c'	Cylinder strength of concrete
f_{cu}	Cube strength of concrete
f_{cd}	Design strength of concrete in compression
f_s	Tensile stress in steel
f_s'	Compression stress in steel
f_y	Yield stress of steel
f_{yd}	design stress of steel both in tension and compression
h	Overall depth of cross-section
h'	Concrete cover to center of reinforcement steel along depth
k	Relative eccentricity ratio
k_a, k_d	Factors for the location of compressive force in the compressed zone of concrete
M	bending moment
M_y	Bending moment about y-axis
Mz	Bending moment about z-axis
M_{uy}	Ultimate uniaxial moment capacity about y-axis
M_{uz}	Ultimate uniaxial moment capacity about z-axis
m_y	relative moment about y-axis
m_z	relative moment about z-axis
m_{uy}	Relative ultimate uniaxial moment capacity about y-axis

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m_{uz}	Relative ultimate uniaxial moment capacity about z-axis
N	Normal force
N_c	Compressive force on compressed zone of concrete
N_s	Tensile force in tension steel
N'_s	Compression force in compression steel
N_y	Normal force acting with uniaxial eccentricity e_y
N_z	Normal force acting with uniaxial eccentricity e_z
N_o	Concentric axial load capacity of cross-sections
n	Relative normal force
n_y	Relative normal force with uniaxial eccentricity e_y
n_z	Relative normal force with uniaxial eccentricity e_z
n_o	Relative normal force with out eccentricities
x	Compressed depth of concrete
y	Major axis of cross-section
z	Minor axis of cross-section
α	Exponent
β	Biaxial bending design constant
ω	mechanical reinforcement ratio
ε	Strain
ε_c	Compression strain in concrete
ε_s	Tensile strain in steel
ε'_s	Compression strain in steel
α_c, α_d	Stress factors for compressive force in concrete
γ, η	Moment magnification factors

ABSTRACT

The ultimate capacity of reinforced concrete sections under normal force and biaxial bending moments can be represented by a three dimensional interaction surface in terms of the normal force and biaxial moments. The surface can be conveniently represented by a family of curves either on a plane of constant normal force relating the two moment components or as interaction diagram on a plane of constant angle relating the normal force and the resultant of the two moment components. However, the curves can not be described by exact and closed form mathematical expressions due to a wide variety of parameters involved in the determination of ultimate biaxial moment capacity of cross-sections with normal force. Thus the systematic generation of such curves to be used for design involves obtaining sufficient number of suitable points by iteration which normally requires use of computer programs. In the absence of such facilities, the use of approximate methods becomes mandatory. There are different approximate methods adopted in different codes for the design of biaxially loaded reinforced concrete columns and the purpose of this thesis work is to assess and evaluate the proximity of some of these approximate methods with the exact solution. The approximate methods evaluated include those recommended by EBCS-2 [1], CP110 [2,3] and ACI [4,5].

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Tefera Desta

1. INTRODUCTION

Reinforced concrete columns are mostly vertical structural members that are loaded chiefly in compression. However, almost all columns are subjected to moments in addition to axial loads, as a result of the load not being centered on the column or the columns resisting a portion of the unbalanced moments at the ends of the beams supported by the columns. When a column is subjected to normal force accompanied by bending moment about one of the principal axes of the cross-section, it is called uniaxially eccentrically loaded column. If on the other hand columns support axial force and bending moments about the two principal axes of the cross-section, they are called biaxially loaded columns. Such columns are not unusual in reinforced concrete buildings and are not restricted to only corner column in a frame. For example, reinforced concrete columns in moment resisting frames, such as the grid frames in both directions, are chosen as the lateral force resisting system in seismic regions and thus are subjected to biaxial bending and must be designed accordingly. In recognition of the common occurrence of biaxial bending, modern codes recommend that bending moments resulting from minimum eccentricity and second order effects about the other axis need also be considered in the design of even uniaxially loaded columns.

The capacity of reinforced concrete columns with a given dimension of cross-section and amount of reinforcement can be expressed as interaction diagram in terms of normal force and bending moment about either axis for the case of uniaxial bending. The corresponding diagram for axial load and bending about the major and minor axes of the cross-section is a surface. The expression of the surface by convenient curves to be used for design necessitates obtaining suitably spaced points which involve double iteration, a much more difficult task. Therefore, analysis/design of reinforced concrete sections for normal force with biaxial moments normally requires use of computer programs and designers opt to use approximate methods in the absence of such facilities.

There are different approximate methods for the design of biaxially loaded reinforced concrete columns and the purpose of this thesis work is to assess or evaluate some of them. The applicability or suitability of the methods is compared with more rigorous solutions obtained using computer program [6] for the preparation of interaction diagrams for biaxially loaded rectangular reinforced concrete sections. For this purpose, a theoretical review of general cross-section analysis for reinforced concrete members under normal force with bending moments at the ultimate limit state is presented in chapter 2. In chapter 3, some commonly used approximate methods for the design of biaxially loaded reinforced concrete sections are briefly explained. Of these, the approximate methods of design according to CP110 [2,3] and ACI [4,5] which are based on approximating the load contours for biaxial moment capacity of cross-sections at constant normal force in terms of interaction equation are further investigated in chapter 4. The Ethiopian building code, EBCS-2 [1] also allows an approximate method of design in which biaxially loaded rectangular reinforced concrete column can be designed for the given normal force and equivalent uniaxial bending moment acting about one of the principal axis of the cross-section. This approach of design is supposed to be normally conservative, however, the extent to which it lies on the conservative or possibly on the unconservative side is investigated in chapter 5 and lastly conclusions and recommendations are given in chapter 6.

2. THEORETICAL REVIEW FOR ANALYSIS OF REINFORCED CONCRETE SECTIONS

2.1 General

In the final stage of a design process, individual members are proportioned to safely resist the internal forces (the design values of the internal normal force, bending moments and shear forces) resulting from loading or imposed deformations recommended by building code authorities. The proportioning of such members using the limit state design procedure ensure acceptable levels of safety against occurrence of the different limit states. The usual approach will be to design on the basis of the most critical limit state and then to check that the remaining limit states will not be reached. For a building structure for example, primary attention is placed on the ultimate limit state with the serviceability limit state being checked after initial design is completed.

Presently, most reinforced concrete structures are designed for moments, shears and normal forces that are determined by elastic analysis, optionally followed by a limited amount of plastic moment redistribution. On the other hand, the actual proportioning of members is done by the methods of the ultimate limit state, with the recognition that inelastic section and member response would result upon over loading. This seems to be an inconsistent approach to the total analysis-design process, although it can be shown to be both safe and conservative and therefore recommended as one of the possible approaches (analysis-design) by many codes including the Ethiopian code EBCS-2[1].

In this chapter, a theoretical review is made pertaining to one aspect of the analysis-design process namely analysis of reinforced concrete sections which is some how involved because it is typically characterized by non-linearity's as a result of the non-linear stress-strain relationship of the constituent materials and cracking of the section once the tensile strength of concrete has been exceeded.

2.2 Basic Assumptions

Determination of the ultimate capacity of reinforced concrete sections is based up on the following assumptions.

1. Plane sections remain plane implying that there is a linear variation of strain across the cross-section and the strain at any point on the cross-section is proportional to the distance from the neutral axis
2. There is no bond slip between steel and concrete, that is, the strain in the reinforcement steel and concrete at the same location are equal.
3. Concrete is weak in tension and its tensile strength can be neglected
4. The idealized stress-strain relationships for steel and concrete are according to EBCS-2 [1] and as shown in fig.2.1 (a) and (b).

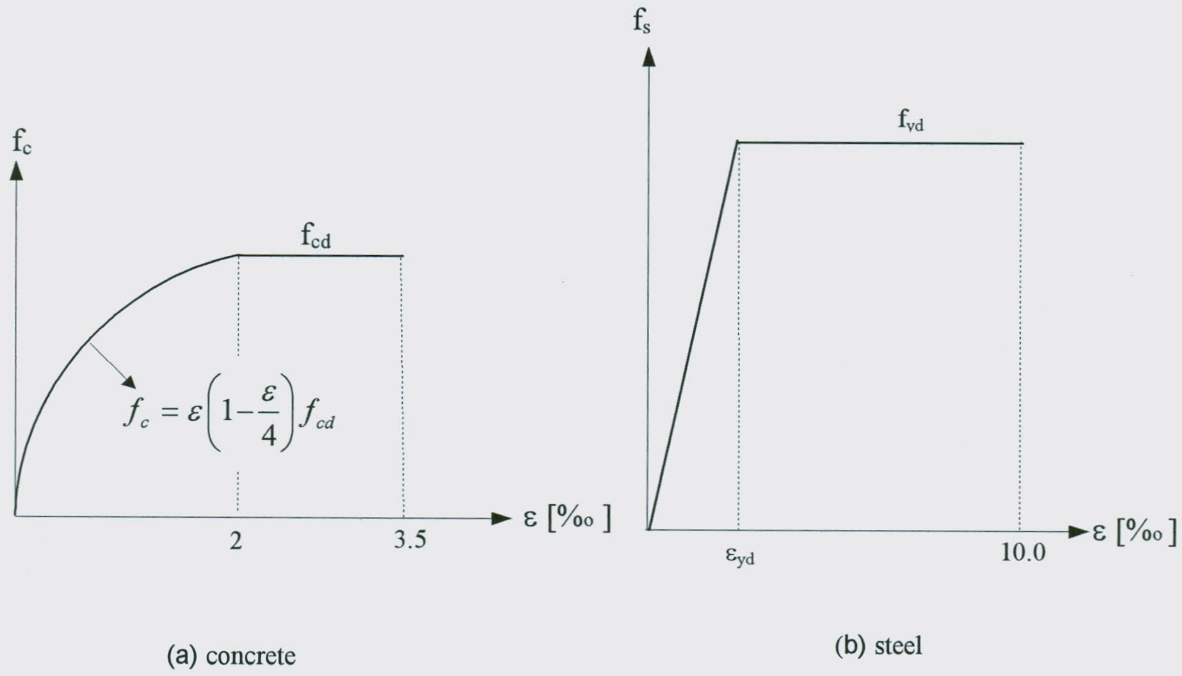


Fig.2.1 stress-strain diagram for steel and concrete[1]

2.3 Ultimate Limit States for Reinforced Concrete Sections

According to EBCS-2 [1], a reinforced concrete section is in the ultimate limit state if the strain distribution over the cross-section lies in one of the five zones of strain profiles shown in fig.2.2

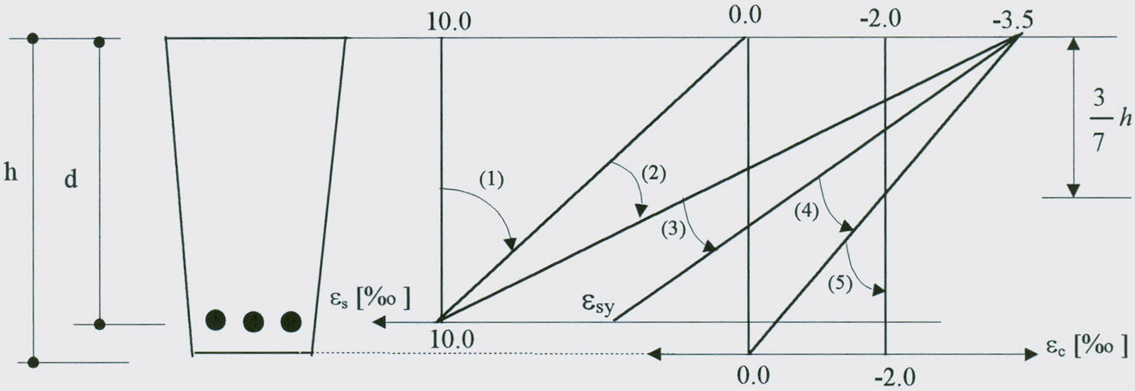


Fig.2.2: strain distribution over the cross-section at ultimate limit state[1]

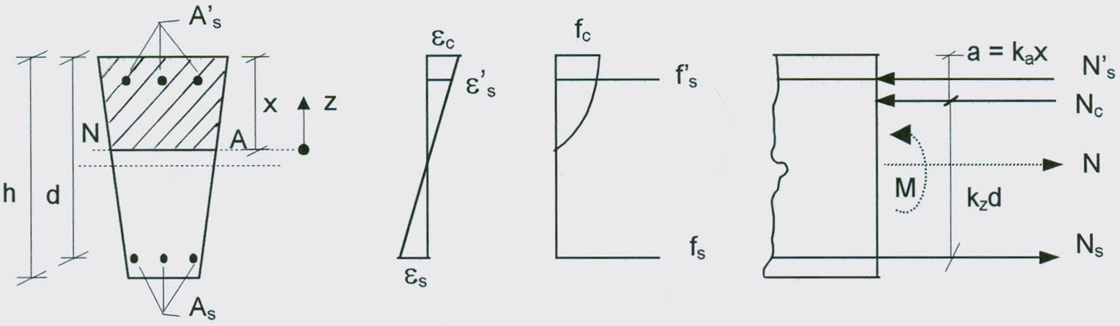
Strain profiles in zone (1) correspond to a situation where the whole cross-section is subjected to tension. The three zones of strain profiles [(2), (3), (4)] have one common feature namely the neutral axis lies with in the cross-section where as zone (5) corresponds to a condition in which the whole cross-section is under compression and hence the neutral axis lies outside of the cross-section, a case of predominant compressive normal force with significant bending moment. In general there are various possibilities for the location of the neutral axis and each location corresponds to a particular combination of internal normal force and bending moments representing the ultimate capacity of the section.

2.4 Stress Resultant

Each strain profile in the five zones corresponds to a particular combination of ultimate internal forces (normal force and bending moment) which are determined as the stress resultants in the concrete and reinforcement steel. The determination of stress resultants with the locations of the point of applications is one of the most important steps in the analysis of reinforced concrete sections and can broadly be divided in to two cases based on the location of the neutral axis.

Case- I: The neutral axis lies within the cross-section

(a): $\varepsilon_c \leq 2‰$



(a) Cross-section (b) Strain (c) Stress (d) Resultant

Fig.2.3

For the strain distribution shown in fig.2.3(b), the strain at a distance z from the neutral axis can be expressed as:

$$\varepsilon = \frac{\varepsilon_c}{x} z \qquad (2.1)$$

The stress in the concrete at the corresponding point can be determined from the idealized stress-strain relationship for concrete

$$f_c = \varepsilon\left(1-\frac{\varepsilon}{4}\right) f_{cd} = \frac{\varepsilon_c}{x} z\left(1-\frac{\varepsilon_c}{4x} z\right) f_{cd} \qquad (2.2)$$

The resultant normal force on the compressed concrete zone can be obtained by integrating the stress distribution over the compressed area.

$$N_c = \int_0^{z=x} f_c\, b_z\, dz \qquad (2.3)$$

Where b_z is the width of cross-section at distance z from the neutral axis.

Inserting equation (2.2) in to equation (2.3) and performing the integration over the given limits, the total compressive force on the compressed zone of concrete for sections with constant width b can be expressed as follows after rearranging terms.

$$N_c = \alpha_c \, b \, x \, f_{cd} \qquad (2.4a)$$

Where:

$$\alpha_c = \frac{\varepsilon_c}{12}(6 - \varepsilon_c) \qquad (2.4b)$$

The location of N_c from the most compressed edge, a, can be determined by finding the centroid of the stress distribution with reference to the same point.

$$a = x - \frac{1}{N_c}\int_0^{z=x} f_c \, b_z \, y \, dz \qquad (2.5)$$

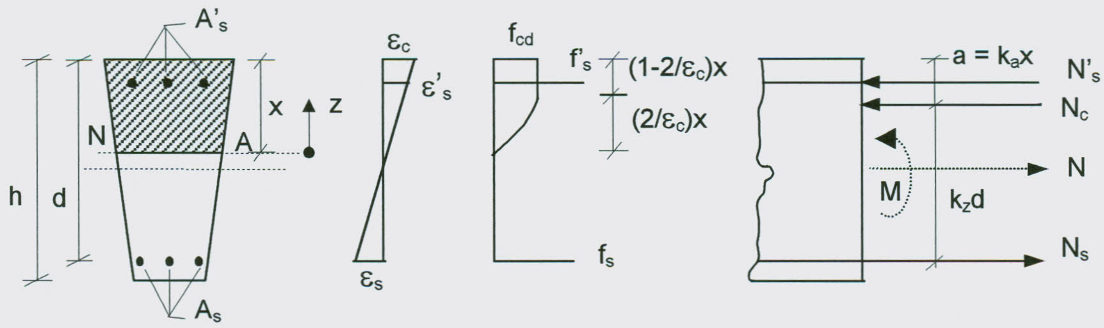
Substituting the value of N_c from equation (2.4) and the expression for f_c from equation (2.2) in to equation (2.5) and performing the integration over the given limits, the location of N_c from the most compressed edge for sections with constant width b can be expressed as follows:

$$a = k_a \, x \qquad (2.6a)$$

Where:

$$k_a = \frac{8 - \varepsilon_c}{4(6 - \varepsilon_c)} \qquad (2.6b)$$

(b) $\varepsilon_c > 2‰$



(a) Cross-section (b) Strain (c) Stress (d) Resultant

Fig.2.4

In this case, the compressive stress distribution on concrete has two components: rectangular and parabolic, as shown in fig.2.4. The parabolic part can still be expressed by equation (2.2) and the resultant compressive force in the compression zone is determined by integrating the stress distribution over the compressed concrete area. For sections with constant width, this total force can be determined from the following:

$$N_c = b\left(1-\frac{2}{\varepsilon_c}\right)x f_{cd} + b f_{cd} \int_0^{z=2x/\varepsilon_c} \frac{\varepsilon_c}{x} z\left(1-\frac{\varepsilon_c}{4x} z\right)dz$$

After performing the integration over the given limits and rearranging terms, N_c can be expressed with the same form of expression as equation (2.4a) but different coefficient α_c as follows:

$$N_c = \alpha_c b x f_{cd} \quad (2.7a)$$

Where:

$$\alpha_c = \frac{3\varepsilon_c - 2}{3\varepsilon_c} \quad (2.7b)$$

The location of N_c from the most compressed fiber can be obtained using the same procedure as in the previous case. Thus the distance of the point of application of the compressive force in the concrete N_c from the outer most concrete fibers under compression is given by:

$$a = k_a x \quad (2.8a)$$

Where:

$$k_a = \frac{\varepsilon_c(3\varepsilon_c - 4) + 2}{2\varepsilon_c(3\varepsilon_c - 2)} \quad (2.8b)$$

The equations for stress factor α_c and relative distance of the point of application of the compressive force N_c as a function of ε_c are represented diagrammatically for ease of application as shown in fig.2.5.

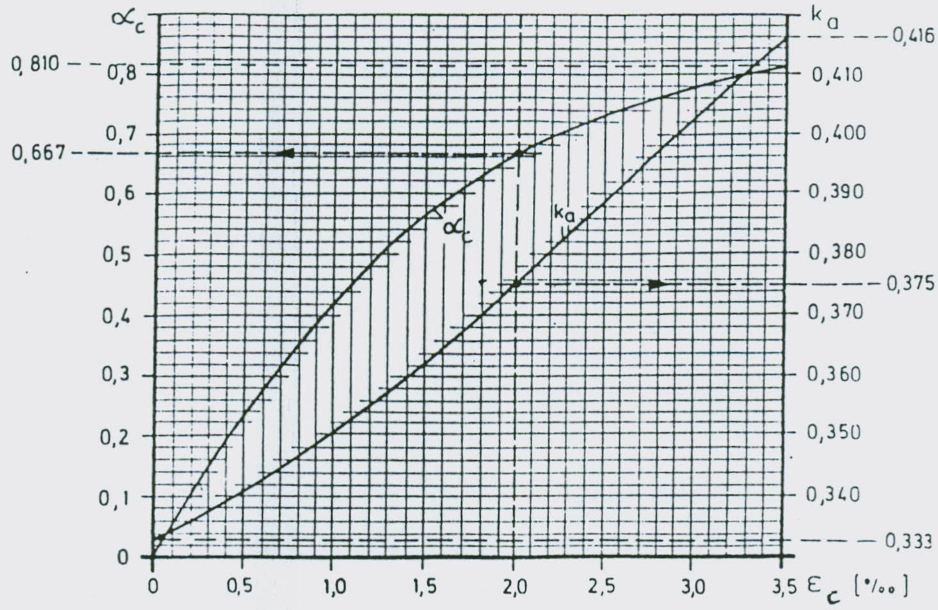


Fig.2.5 variation of α_c and k_a with ε_c[7]

Case-II the neutral axis lies outside the cross-section.

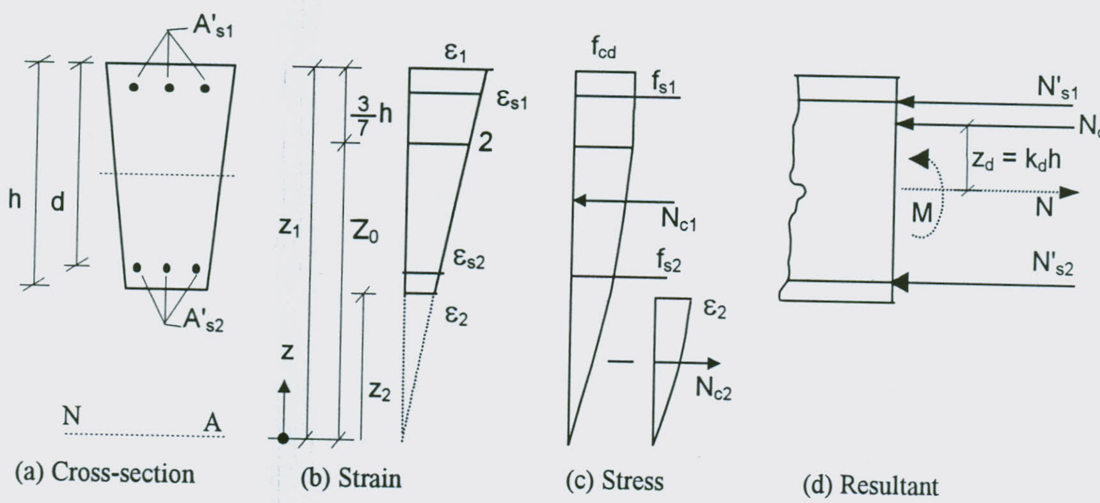


Fig.2.6

In this case the whole cross-section is under compression and the neutral axis lies outside of the cross-section. The strain distribution for this case is such that a fiber located at 3h/7 from the most compressed edge has a strain level of 2‰ as shown in fig.2.6 (b). With this strain distribution, the strain at the less compressed edge ε_2, its distance from the neutral axis z_2, the corresponding distances z_1 and z_o of the most compressed edge and the concrete fiber with strain level equal to 2‰ respectively, can be determined from the stress distribution shown in fig 2.6(b). They are given by:

$$\varepsilon_2 = \frac{14 - 4\varepsilon_1}{3} \qquad (2.9a)$$

$$z_1 = \frac{3\varepsilon_1}{7(\varepsilon_1 - 2)}h \qquad (2.9b)$$

$$z_2 = \frac{14 - 4\varepsilon_1}{7(\varepsilon_1 - 2)}h \qquad (2.9c)$$

$$z_0 = \frac{6}{7(\varepsilon_1 - 2)}h \qquad (2.9d)$$

The stress resultant Nc can be determined by integrating the compressive stress over the whole cross-section

$$N_c = N_{c1} - N_{c2} = \int_0^{z=z_1} f_c b_z \, dy - \int_0^{z=z_2} f_c b_z \, dz \qquad (2.10)$$

By performing the integration in the same way as in the previous cases, it can be shown that the resultant compressive force on the concrete for a section with constant width b can be expressed as follows:

$$N_c = \alpha_d \, b h f_{cd} \qquad (2.11a)$$

Where:

$$\alpha_d = \frac{1}{189}\left(125 + 64\varepsilon_1 - 16\varepsilon_1^2\right) \qquad (2.11b)$$

The location of N_c, in this case from the centroid of the cross-section, can be determined with similar procedure as in the previous cases and found to be as follows:

$$z_d = k_d h \qquad (2.12a)$$

Where:

$$k_d = \frac{40}{7}\left[\frac{(\varepsilon_1 - 2)^2}{125 + 64\varepsilon_1 - 16\varepsilon_1^2}\right] \qquad (2.12b)$$

Similar to case (a), the coefficients for stress factor α_d to compute the total compressive force on concrete and k_d for its location are represented diagrammatically and shown in fig.2.7.

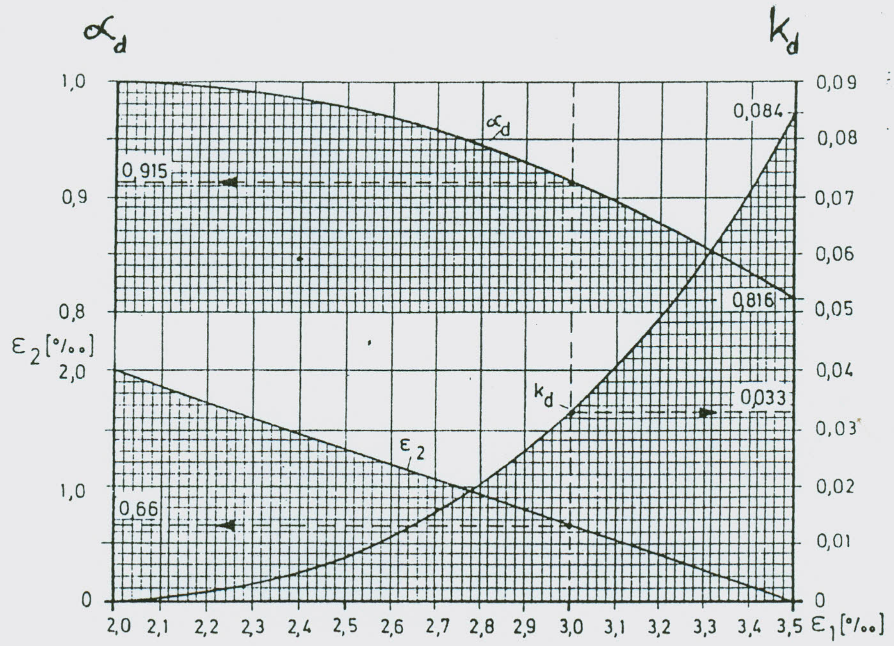


Fig.2.7 variation of α_d and k_d with ε_1 [7]

After determining the total compressive force on concrete and its point of application; the next step is to compute the forces in the reinforcement steel, which are easily determined by using the strain distribution diagram and idealized stress-strain relation ship for steel.

Once the internal forces in the steel and concrete with their locations are calculated, the capacity of the given cross-section at ultimate limit states can be determined by summing up all the forces in the axial direction and their bending moments about the centroid of the cross-section.

2.5 Compression with Uniaxial Bending

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to the load not being centered on the column, or may result from the column resisting a portion of the unbalanced moments at the ends of the beams supported by the column. Since reinforced concrete columns are normally framed in two orthogonal directions, they are subjected to biaxial bending. . Uniaxial bending is a special case where eccentricity of the loading is along one or the other axis and the neutral axis is therefore parallel to the axis of bending.

The capacity of cross-sections under compression with uniaxial bending at the ultimate limit state is best expressed by plotting the axial load N-versus the bending moment M at failure called N-M interaction diagram. Interaction diagrams for reinforced concrete cross-sections can in general be plotted by assuming a series of strain distribution at failure in different zones of the strain profile shown in fig.2.2 and computing the corresponding values of N and M. One pair of such values represent the coordinates of a particular point on the interaction diagram. Fig.2.8 shows an interaction diagram for a rectangular reinforced concrete cross-section having $\omega = 0.4$ and $f_y = 460MPa$, in a non-dimensionalized form, which can be plotted by choosing sufficient number of strain distributions in the ultimate limit state and determining the corresponding stress resultants as described in article 2.4

In this approach, the strains could be varied in a controlled manner, however, the corresponding points on the interaction diagram can not be expected to be evenly spaced because the internal forces are not linearly related to strain. The points could therefore be clustered together at some regions while dispersed far apart at other regions, which make it unsuitable for systematic generation of interaction charts. A preferred procedure would therefore be to vary the normal force N in a controlled manner and determine the associated ultimate bending moment; a much more difficult task requiring iteration [8,9]. The points so obtained are evenly spaced and result in a smooth or best-fit plot of the interaction diagram. The uniaxial design charts included in appendix A are prepared using the computer program[6] based on this procedure.

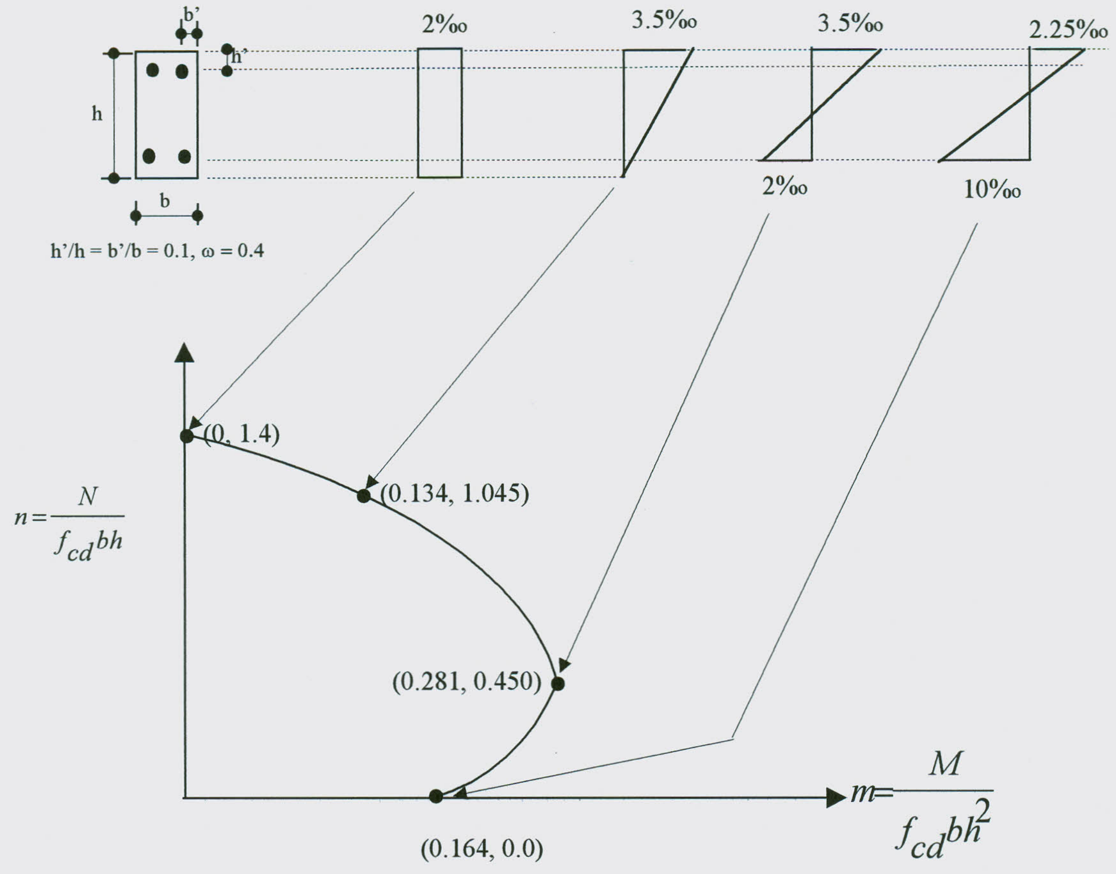


Fig.2.8 strain distribution over the cross-section and corresponding points on the interaction diagram

2.6 Compression with Biaxial Bending

When the normal force is acting with eccentricities along both principal axes or accompanied by bending moments in the two principal planes, the neutral axis will not be parallel to one of the principal axes of the cross-section and is in general inclined to the principal axes. Thus the location of the neutral axis is defined either by locating two points, one on each axis through which the neutral axis passes (called intercepts of the neutral axis) or by locating one of the intercepts and defining its angle of inclination from either of the principal axes. The determination of the location of the neutral axis is therefore much more laborious than that for members under uniaxial bending and represents one of the difficulties involved in the analysis of cross-sections subjected to normal force and biaxial moments. For a given cross-section, each location of the neutral axis and one set of a strain profile corresponds to a particular set of internal forces; normal force N and bending moments M_y, M_z acting about the major and minor axes of the cross-section respectively, which under combined action would cause failure of the section. This set of internal forces represents the coordinate of a point on a three-dimensional interaction surface called failure surface. The failure surface for a given cross-section represents possible combination of normal force N and biaxial moments M_y, M_z as its capacity at ultimate limit state.

The failure surface for a given cross-section can be represented in convenient form for design by a family of curves either on a plane of constant normal force relating the two moment components called the load contours or as an interaction diagram relating the normal force N and the resultant moment M on a plane of constant angle defining the proportion of the two moment components. The biaxial design charts in reference [10], some of which are reproduced at different cover ratios and included in appendix B, are of the first type. The planes of constant normal force are defined by the relative normal force n, which varies with an increment of 0.2. The curves on such plane of constant normal force represent possible combination of moment components m_y, m_z in relative terms as the capacity of cross-sections at the specified normal force level. Biaxial charts of the second type are given in reference [11] one of which has been copied and included at the end of appendix B to demonstrate the features of such biaxial charts.

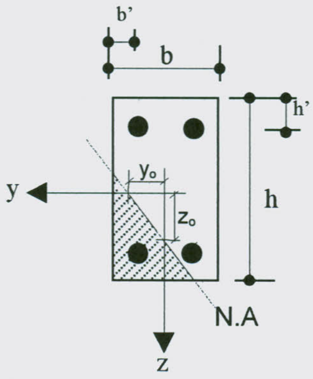
The ultimate moments (m_y, m_z) corresponding to a given normal load level and the angle relating the moments are determined using the iterative procedure described in references [6, 8]. Both parameters namely the internal normal force and the angle relating the moments could be varied in a controlled manner allowing the systematic generation of the biaxial charts [10].

For the purpose of illustration, the ultimate biaxial moment capacity (M_y, M_z) of a reinforced concrete cross-section shown in fig.2.9 (a) with the given reinforcement ratio and material properties have been determined at a specified normal load level ($n = 0.2$) and various angles relating the moment components using the computer program [6]. The results are summarized in table 2.1 and shown also in fig.2.9 (b).

Table 2.1

Point	M_y (kNm)	M_z (kNm)	Arctan(M_z/ M_y) (^{o})	y_o (m)	z_o (m)
1	114.00	0.00	0.000	∞	-0.0929
2	86.88	25.08	16.104	0.0394	-0.0504
3	50.16	43.44	40.896	0.0252	-0.0788
4	28.42	53.064	61.831	0.0255	-0.1350
5	0.00	57.00	90.000	0.0464	∞

Cross-section

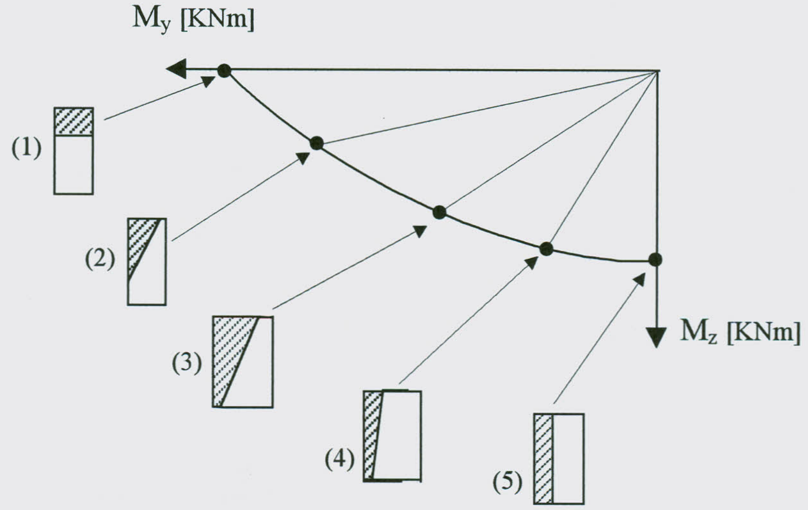


Geometric and Material data

$b = 200$ mm
$h = 400$ mm
$b' = 20$ mm
$h' = 40$ mm

$f_{cd} = 15$ N/mm^2
$f_{yd} = 550$ N/mm^2
$\omega = 0.4$
$n = 0.2 \Rightarrow N = nf_{cd}bh$
$\Rightarrow N = 240$KN

(a)



(b)

Fig.2.9 Biaxial interaction diagram for a given cross-section

3. APPROXIMATE METHODS FOR THE DESIGN OF BIAXIALLY LOADED RECTANGULAR REINFORCED CONCRETE COLUMNS

3.1 General

As explained in chapter 2, a great deal of computational work is involved in determining the ultimate capacity of reinforced concrete sections under normal force and biaxial bending making simple and exact analytical formulations for manual application for the design of biaxially loaded cross-sections very difficult. Thus analysis and design of reinforced concrete sections for normal force and biaxial bending normally requires the use of computer programs and in the absence of such facilities, designers opt to use approximate methods.

There are different approximate methods for the design of biaxially loaded reinforced concrete sections adopted by different codes. Most of the approximate methods which are in common use can in general be grouped in to three categories each of which are briefly explained in the following sections. In the first category, cross-sections are checked for uniaxial bending about each principal axes of the cross-section separately. This is usually employed for a case of small ratio of relative eccentricity, that is, there is significant bending about one of the principal axis of the cross-section. The approximate methods in the second group are based on approximating the failure surface, which represent the ultimate capacity of a cross-section under normal force and biaxial bending. In the third category of approximate methods, the biaxial moments are converted in to an equivalent uniaxial moment allowing design of the cross-section for the given normal force and the equivalent uniaxial bending moment acting about one of the principal axis of the cross-section.

3.2 Approximations for Small Ratios of Relative Eccentricities

One possible way of approximations for the design of biaxially loaded rectangular columns in a condition where the ratio of relative eccentricities along the major and minor axes of the cross-section is relatively small is to check the capacity of the cross-section for the given normal force and bending moments about the major and minor axes separately.

Different codes allow this separate check for uniaxial bending and the provisions of the German code, DIN 1045-1: 1997-02[12], and the Ethiopian code of practice, EBCS-2 [1], are briefly presented in the following sections.

3.2.1 DIN 1045-1: 1997-02 Provision

According to the German code [12], separate checks in the directions of the principal axes are possible for rectangular columns provided the following conditions are satisfied:

$$\frac{e_z/h}{e_y/b} \leq 0.2$$

$$or \quad \frac{e_y/b}{e_z/h} \leq 0.2$$

Where e_y and e_z are load eccentricities according to first order theory in the direction corresponding to the sides of the cross-section b and h respectively. This means that the point of application of the design value of the axial force N lies within the shaded area in the cross-section shown in fig.3.1. If the above condition is not satisfied, the code insists that a more rigorous solution must be used

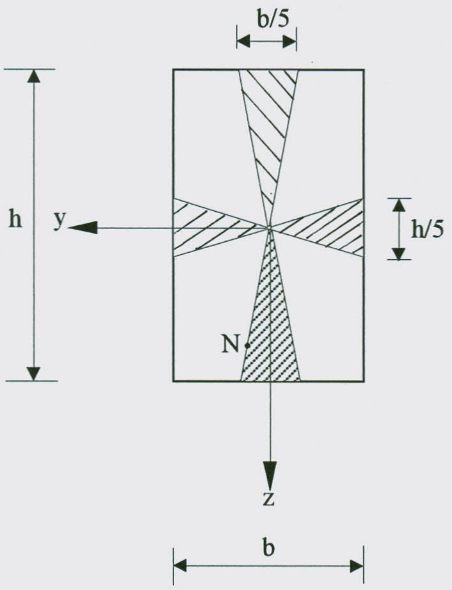


Fig.3.1

For the case where $e_z \geq 0.2h$ separate checks are still allowed provided the check for bending about the weaker axis (z axis in fig.3.1) is based on a reduced thickness x as shown in fig.3.2. The value of x corresponds to the depth of compressed zone of the cross-section due to the given normal force and bending moment about the stronger axis allowing for additional eccentricity according to the code [12]

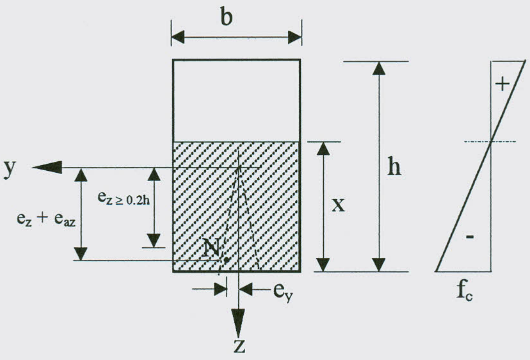


Fig.3.2

3.2.2 EBCS - 2 Provision

EBCS-2 [1] allows the following approximation for the design of rectangular reinforced concrete columns under biaxial bending in the absence of more accurate methods. Columns of rectangular cross-section, which are subjected to biaxial moments, may be checked separately for uniaxial bending in each respective direction if the ratio of the relative eccentricity is less than or equal to 0.2. This separate check for uniaxial bending is also equally applicable to biaxial bending in general, provided that the central one third parts of the effective lengths of the buckled column in the principal directions do not overlap as shown in fig.3.3

However, these two conditions will not often be fulfilled and practically most of biaxially loaded columns do not satisfy these conditions. In such cases the code[ ] provides that the column can be designed for the given normal force and equivalent uniaxial bending moment corresponding to the larger relative eccentricity as described in section 3.4.3.

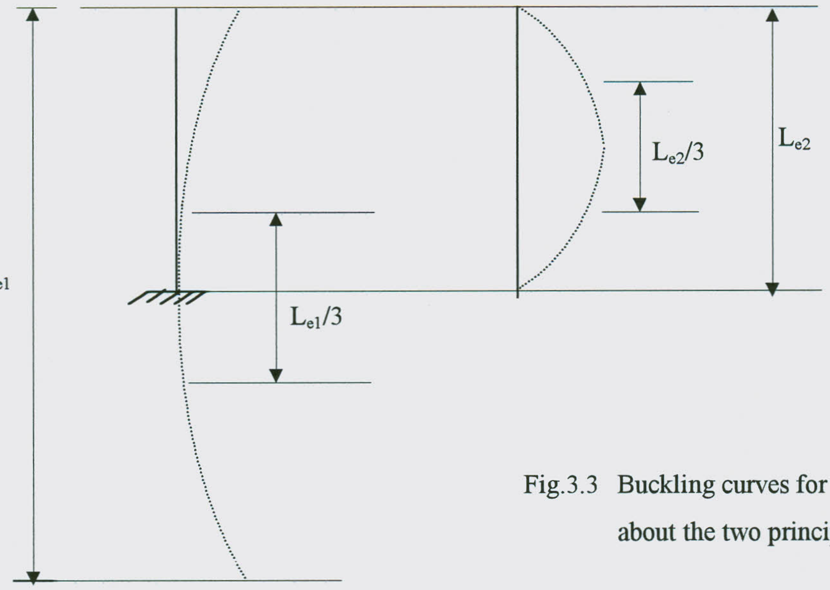


Fig.3.3 Buckling curves for bending about the two principal axes

3.3 Approximations Based on Simplified Mathematical Expressions for Approximation of the Failure Surface.

3.3.1 The Failure Surface

The capacity of cross-sections at ultimate limit state under normal force with biaxial moments about the two principal axes can be represented by a three dimensional interaction surface called failure surface in terms of the normal force and bending moments about the two axes. Typical failure surface for a reinforced concrete section is shown in fig.3.4

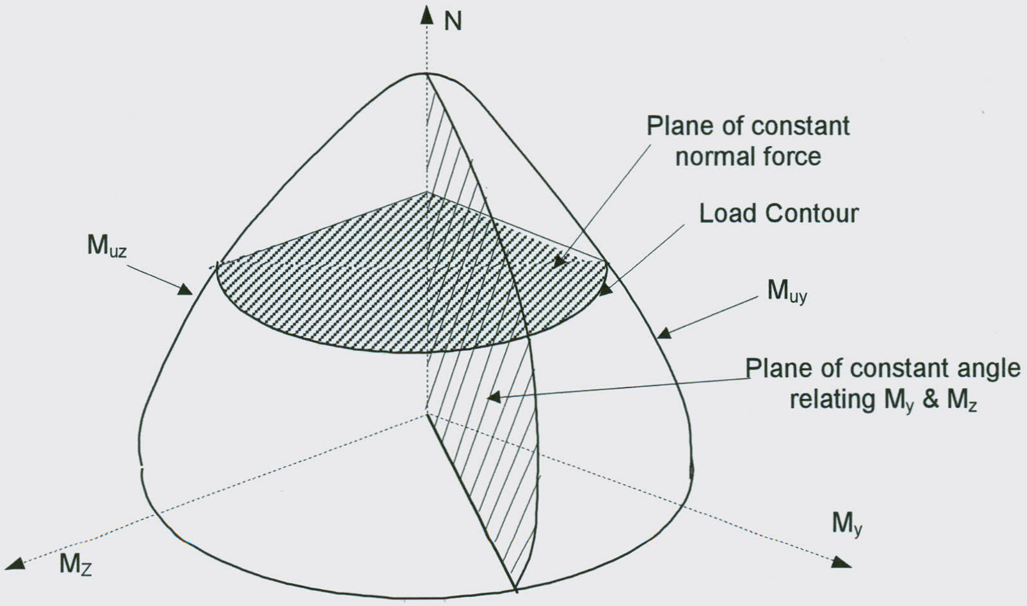
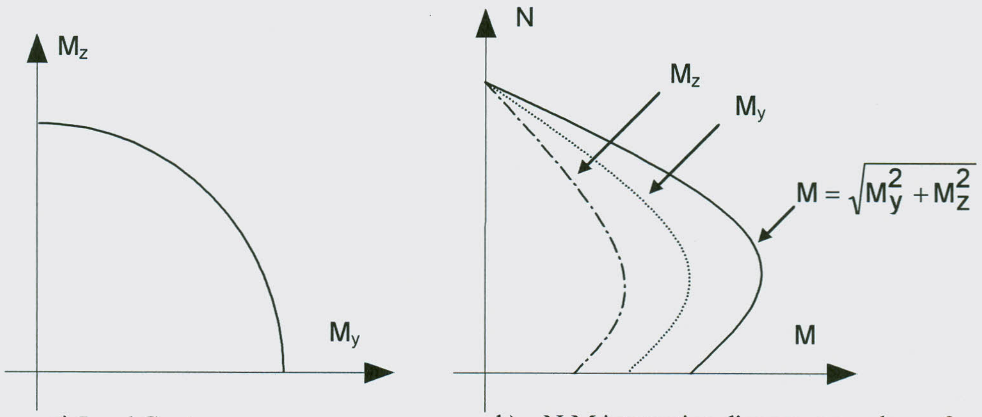


Fig.3.4: Failure surface [N, M_y, M_z]

The surface is found to be sensitive to the geometry or dimension of the cross-section, amount, location and distribution of reinforcement and stress-strain characteristics of concrete and steel. Due to such wide variety of factors, the shape of the failure surface for reinforced concrete sections in general is too complex making simple and exact mathematical formulations for its expression very difficult. Therefore some approximate methods in the determination of the capacities of cross-sections under normal force with biaxial moments are based on approximation of the failure surface which can be conveniently represented by a family of curves either on planes of constant normal force relating the two moment components or as interaction diagram on planes of constant angles relating the normal force and the resultant moment as shown in fig.3.5. Hence approximation of the failure surface has directly to do with derivation of approximate expressions for these curves.



a) Load Contour

b) N-M interaction diagram on a plane of constant angle relating M_y & M_z

Fig.3.5 Curves on the failure surface

Pannell[13] tried to relate the interaction curves relating the normal force and the resultant moment on any radial plane to the corresponding uniaxial interaction diagrams on the major plane. G.A. Hartley[14] suggested that the curves on each plane of constant normal force relating the two moment components can be expressed in the form of exponential functions in polar coordinate. Bressler[15] proposed that the curves on the plane of constant normal force can approximately be expressed in the form of simple interaction equation. This idea has found much acceptance [16,17,18] and become the basis of the approximate interaction equation for biaxial bending in many codes [2,3,4,5] some of which are briefly described in the following sections.

3.3.2 Interaction Equation

If a horizontal section is taken through the failure surface shown in fig.3.4, that is, on a plane of constant normal force, the boundary of the surface on this plane traces a curve called load contour and represents possible combinations of the moment components M_y and M_z at the given normal force leading to failure of the cross-section. The shape of the curve (load contour) is sensitive to the dimension of the cross-section, strength and elastic properties of concrete and steel, amount and arrangement of reinforcing steel, concrete cover and the level of normal force. Therefore a generalized expression for such curves at various levels of the normal force for a reinforced concrete section subjected to normal force and biaxial moments can not be easily derived. When the capacities of a given cross-section at ultimate limit states are expressed in terms of dimensionless parameters N/N_o, M_y/M_{uy} and M_z/M_{uz}, the failure surface assumes a shape shown in fig.3.6 and as initially suggested by Bressler[15], the curves on a horizontal plane (constant plane of normal force) can in general be approximated in the form of simple interaction equation as follows:

$$\left(\frac{M_y}{M_{uy}}\right)^{\alpha_1} + \left(\frac{M_z}{M_{uz}}\right)^{\alpha_2} = 1.0 \qquad (3.1)$$

Where:

M_{uy} and M_{uz} are the capacities of the cross-section for uniaxial bending moment about the major and minor axes respectively at the specified constant normal force.

The exponent α_1 and α_2 depend on the column dimensions, amount and arrangement of reinforcement, and material strengths. Bresler[15] suggests to use the same value for α_1 and α_2, that is, $\alpha_1 = \alpha_2 = \alpha$ and the interaction equation (3.1) is rewritten using one exponent α as given by equation (3.2).

$$\left(\frac{M_y}{M_{uy}}\right)^{\alpha} + \left(\frac{M_z}{M_{uz}}\right)^{\alpha} = 1.0 \qquad (3.2)$$

According to Bresler[15] calculated values of α vary from 1.15 to 1.55. For practical purposes, α can be taken as 1.5 for rectangular section and between 1.5 and 2.0 for square sections [19].

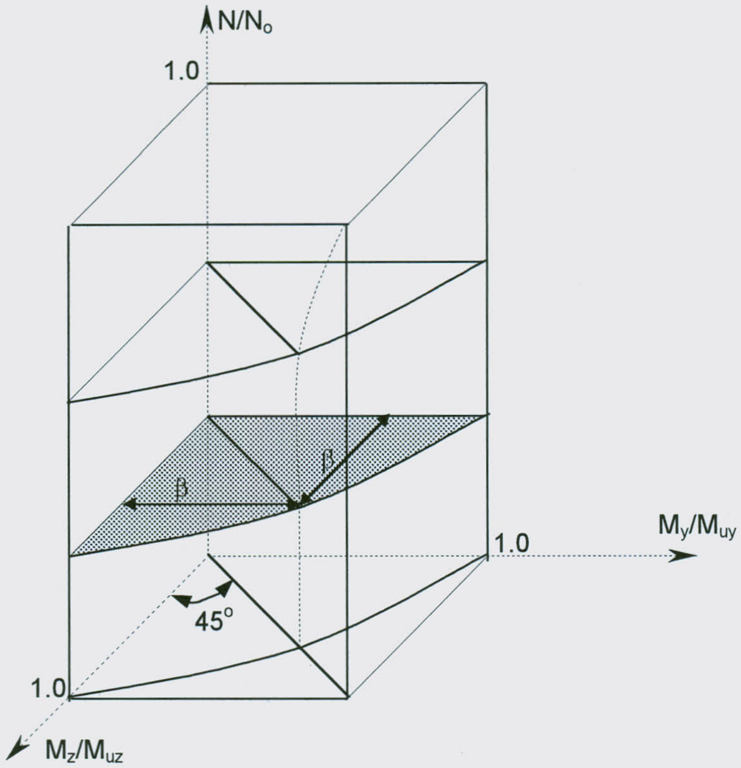


Fig.3.6 Failure surface [N/N_o, M_y/M_{uy}, M_z/M_{uz}]

3.3.2.1 CP110 Design Procedure

When a cross-section is subjected to ultimate normal force N and biaxial moments M_y and M_z acting about the major and minor axes of the cross-section respectively, CP110[2,3] recommends that the section is adequate to resist the applied normal force and biaxial moments if the inequality in equation (3.3) is satisfied. In the limit, the inequality can be equated to 1.0 in which case equation (3.3) reduces to the interaction equation (3.2).

$$\left(\frac{M_y}{M_{uy}}\right)^{\alpha} + \left(\frac{M_z}{M_{uz}}\right)^{\alpha} \leq 1.0 \qquad (3.3)$$

Where

M_{uy} and M_{uz} are the same as defined in equation (3.2), but the exponent α in this case is mainly related to the normal force and given by equation (3.4)

$$\alpha = \frac{1}{3}\left(2 + 5\frac{N}{N_o}\right) \qquad (3.4)$$

Where

N_o represents the capacity of the cross-section under concentric normal force and can be determined from

$$N_o = A_c f_{cd} + A_s f_{yd} = A_c f_{cd}(1+\omega)$$

With these α can be expressed as:

$$\alpha = \frac{1}{3}\left(2 + \frac{5n}{1+\omega}\right) \tag{3.5}$$

Where: $$n = \frac{N}{f_{cd} A_c}$$

$$\omega = \frac{A_s f_{yd}}{A_c f_{cd}}$$

Note that the concrete displaced by the reinforcing steel is neglected in the formulation of equation (3.5), and this results in relatively lower value of α whose effect on the capacity of the cross-section is to be on the conservative side.

3.3.2.2 ACI- Design Procedure

In contrast to CP110 [2,3], A.L. Parme[18] defined the exponent α to be used in the interaction equation (3.2) in less convenient but more meaning full form as given by equation (3.6).

$$\alpha = \frac{\log 0.5}{\log \beta} \tag{3.6}$$

β in equation (3.6) is called the biaxial bending design constant and equals the ordinate of a point on the non-dimensionalized contours on a plane of constant normal force (fig.3.6) at which the two moment components in non-dimensional terms are equal in magnitude. This therefore necessitates the determination of one point on the constant load contours using exact approaches. With this definition β ranges from 0.5 where the curve relating the two moment components in the form of interaction equation becomes a straight line to 1.0 in which the interaction equation (3.2) describes two lines each of which is parallel to one of the axes of the cross-section. This approach seems to give more accurate results as the α-values are based on a parameter β which represents an actual point on the constant load contour at which the component moment capacities (related to the respective uniaxial moment capacities) about both axes are equal. This point is essentially located at 45^0 from either of the principal axis of the cross-section and once this point is obtained, the curves relating the biaxial moment capacity (in non-dimensional terms) of cross-sections at constant normal force using the interaction equation (3.2) can be approximated by straight lines as shown in fig.3.7 to be used for design with trial and adjustment procedure [18]. For this purpose, A.L. Parme[18] computed values of β for different cross-sections and the results are presented diagrammatically in which the biaxial bending design constant β is mainly plotted against the normal force and the effects of such other factors as amount, arrangement and strength of reinforcement steel are also included. This procedure for the design of biaxially loaded rectangular reinforced concrete sections is also adopted in ACI [5] as a standard method of design and the same diagrams for values of β are available in ACI-design hand book [5].

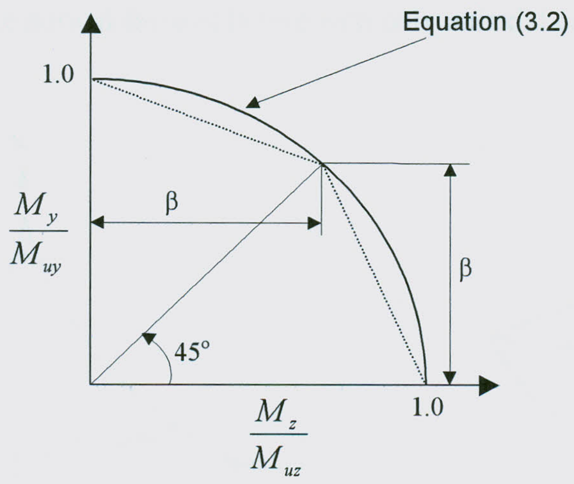


Fig.3.7 Interaction curves

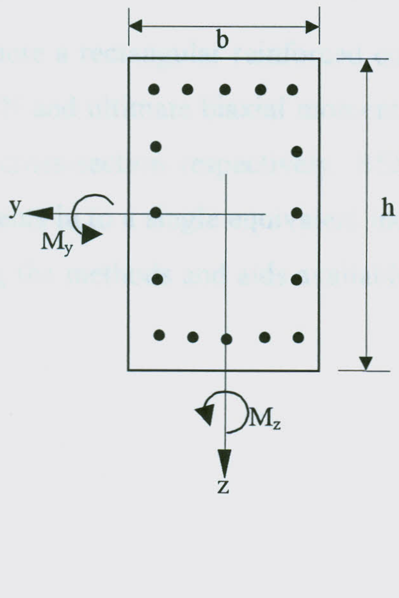
As stated in reference[20], this reciprocal load equation is supposed to represent a good approximation for axial load range of N/N_0 = 0.1 to 0.9 which in relative terms is $n/(1+\omega)$ = 0.1 to 0.9. Especially when the normal force level n is greater than the balanced condition for the uniaxial cases, the reciprocal load equation is believed to give satisfactorily accurate results[5].

3.4 Approximations Based on Equivalent Uniaxial Bending

Another approximate procedure for the design of biaxially loaded rectangular reinforced concrete columns which is in common use is to replace the biaxial eccentricities by an equivalent eccentricity and design the column for uniaxial bending and axial load. This approach of approximation in the design of biaxially loaded reinforced concrete columns is more attractive because it can be used directly for design where as the approximate methods explained in article 3.3.2 can be used for checking a section which has already been detailed or used for design only in an iterative sense with trial and adjustment procedure. In this regard, there are various approximations for determining the equivalent uniaxial bending moment suggested by different authors and adopted in different codes, some of which are presented in the following sections.

3.4.1 MacGregor's Recommendation

MacGregor[23] suggests the following empirical equation for the computation of the equivalent uniaxial eccentricity of load to be used in the design of columns.



$$e_y = \frac{M_z}{N}$$

$$e_z = \frac{M_y}{N}$$

Fig.3.9

The reciprocal load equation is not suitable for design. It is more or less used as checking criteria and can not lead to a direct design procedure. However, to assist the process of checking, it can be expressed in non-dimensional terms (relative normal forces) by dividing all the loads by the same non-dimensionalizig parameter $A_c f_{cd}$ as follows

$$\frac{1}{n} = \frac{1}{n_y} + \frac{1}{n_z} - \frac{1}{n_0} \qquad (3.8)$$

Where: $n = \frac{N}{A_c f_{cd}}$; $n_y = \frac{N_y}{A_c f_{cd}}$

$n_z = \frac{N_z}{A_c f_{cd}}$; $n_0 = \frac{N_0}{A_c f_{cd}}$

The concentric axial load capacity N_0 is given by

$$N_0 = A_c f_{cd} + A_s f_{yd} \quad , \quad A_s f_{yd} = \omega A_c f_{cd}$$

$$= A_c f_{cd} (1 + \omega)$$

$$\therefore \; n_0 = \frac{N_0}{A_c f_{cd}} = \frac{A_c f_{cd} (1+\omega)}{A_c f_{cd}} = 1 + \omega$$

With this equation (3.8) reduces to

$$\frac{1}{n} = \frac{1}{n_y} + \frac{1}{n_z} - \frac{1}{1+\omega} \qquad (3.9)$$

Where:

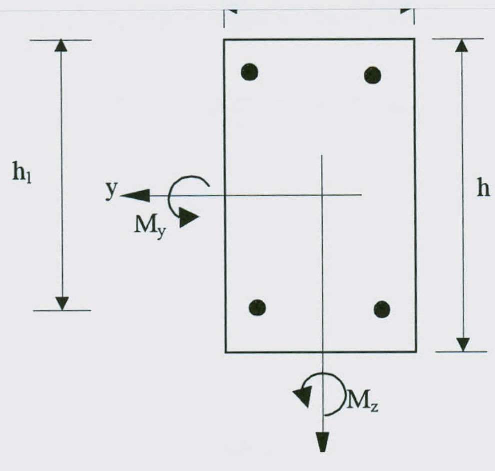
n= the relative normal force with biaxial eccentricity e_y and e_z

n_y = the relative normal force at failure with only uniaxial eccentricity e_y

n_z = the relative normal force at failure with only uniaxial eccentricity e_z

ω = mechanical reinforcement ratio

Note that the area of concrete displaced by the reinforcing steel is not taken in to account in the formulation of equation (3.9) and simply the gross concrete area is used.



$$e_y = \frac{M_z}{N}$$

$$e_z = \frac{M_y}{N}$$

3.4.3 EBCS - 2 Provision

The EBCS-2 [1] allows an approximate method of design in which a rectangular reinforced concrete columns under biaxial bending can in general be designed for the given normal force and an equivalent uniaxial bending moment computed using the equivalent uniaxial eccentricity e_{eq} given by equation (3.13) corresponding to the larger relative eccentricity in the absence of more accurate methods.

$$e_{eq} = e_{tot}(1 + k\gamma) \qquad (3.13)$$

Where:

e_{tot} denotes the total eccentricity allowing for initial imperfections and second order effects in the direction of the larger relative eccentricity

k denotes the relative eccentricity ratio

γ is a factor which depends on the relative normal force and according to EBCS-2 [1] it is given as in table 3.1

Table3.1

n	0	0.2	0.4	0.6	0.8	≥ 1.0
γ	0.6	0.8	0.9	0.7	0.6	0.5

For this approximate method, one-fourth of the total reinforcement must either be distributed along each face of the column or concentrated at each corner. Thus the application of this method is limited to only these two arrangements of reinforcement.

4. EVALUATION OF THE APPROXIMATE METHOD OF DESIGN BASED ON INTERACTION EQUATION ACCORDING TO CP110 AND ACI

4.1 Comparison of cross-section Capacities

The approximate methods of design based on interaction equation explained in article 3.3.2 do not lead to a direct design procedure, rather they can be used for design with trial and adjustment procedure or checking the adequacy of a given cross-section under a specified normal force and biaxial moments. Thus it is not possible to develop a direct design procedure or generate design charts using these approximations. However, for the purpose of assessing or investigating the applicability or proximity of the methods to the exact solution, theoretical curves or load contours can be plotted for different cross-sections using the interaction equation (3.2) which is rewritten here in terms of the relative moments as given by equation (4.1).

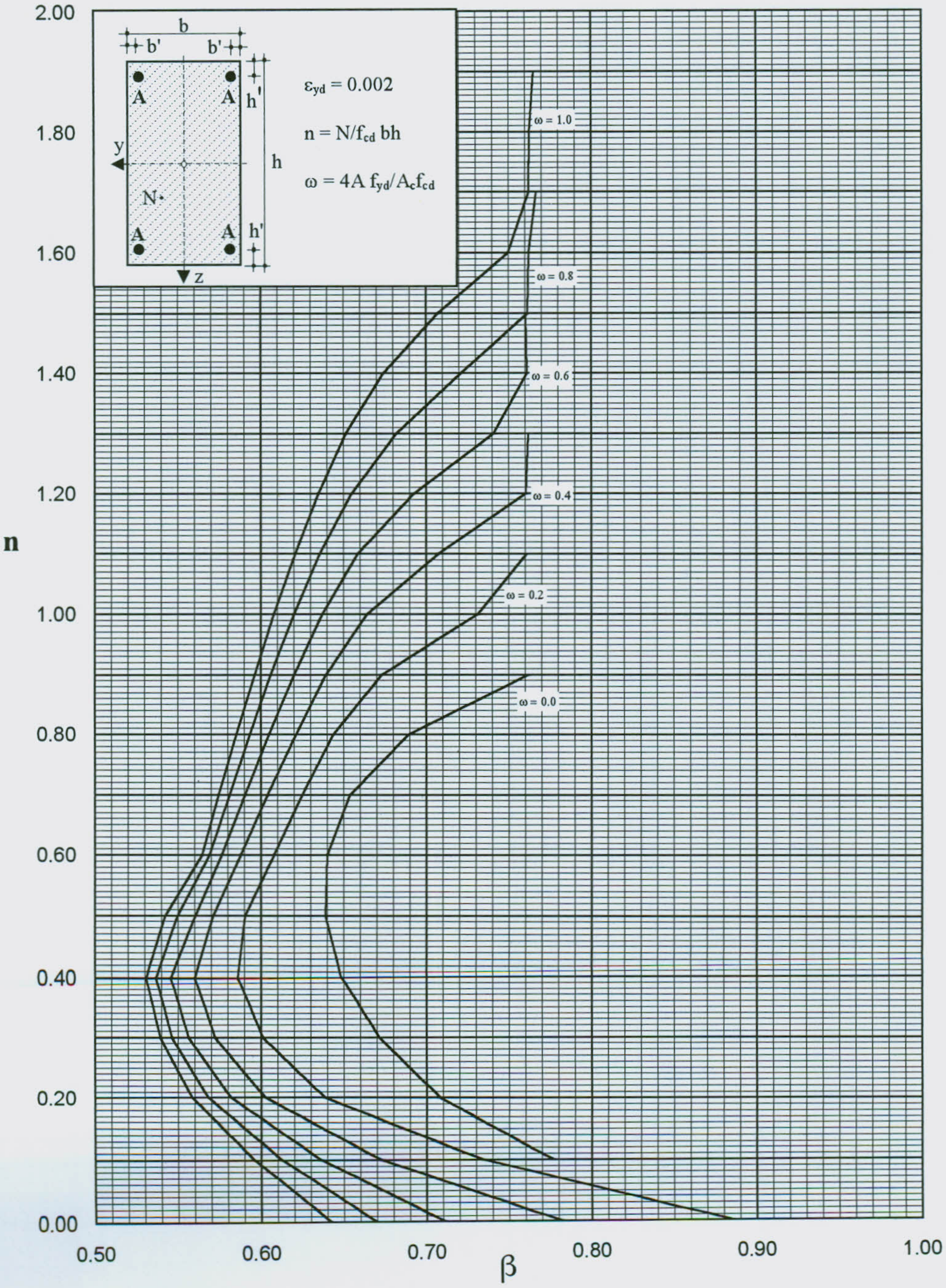
$$\left(\frac{m_y}{m_{uy}}\right)^{\alpha} + \left(\frac{m_z}{m_{uz}}\right)^{\alpha} = 1.0 \qquad (4.1)$$

Where:

$$m_y = \frac{M_y}{f_{cd}bh^2} \quad ; \quad m_{uy} = \frac{M_{uy}}{f_{cd}bh^2}$$

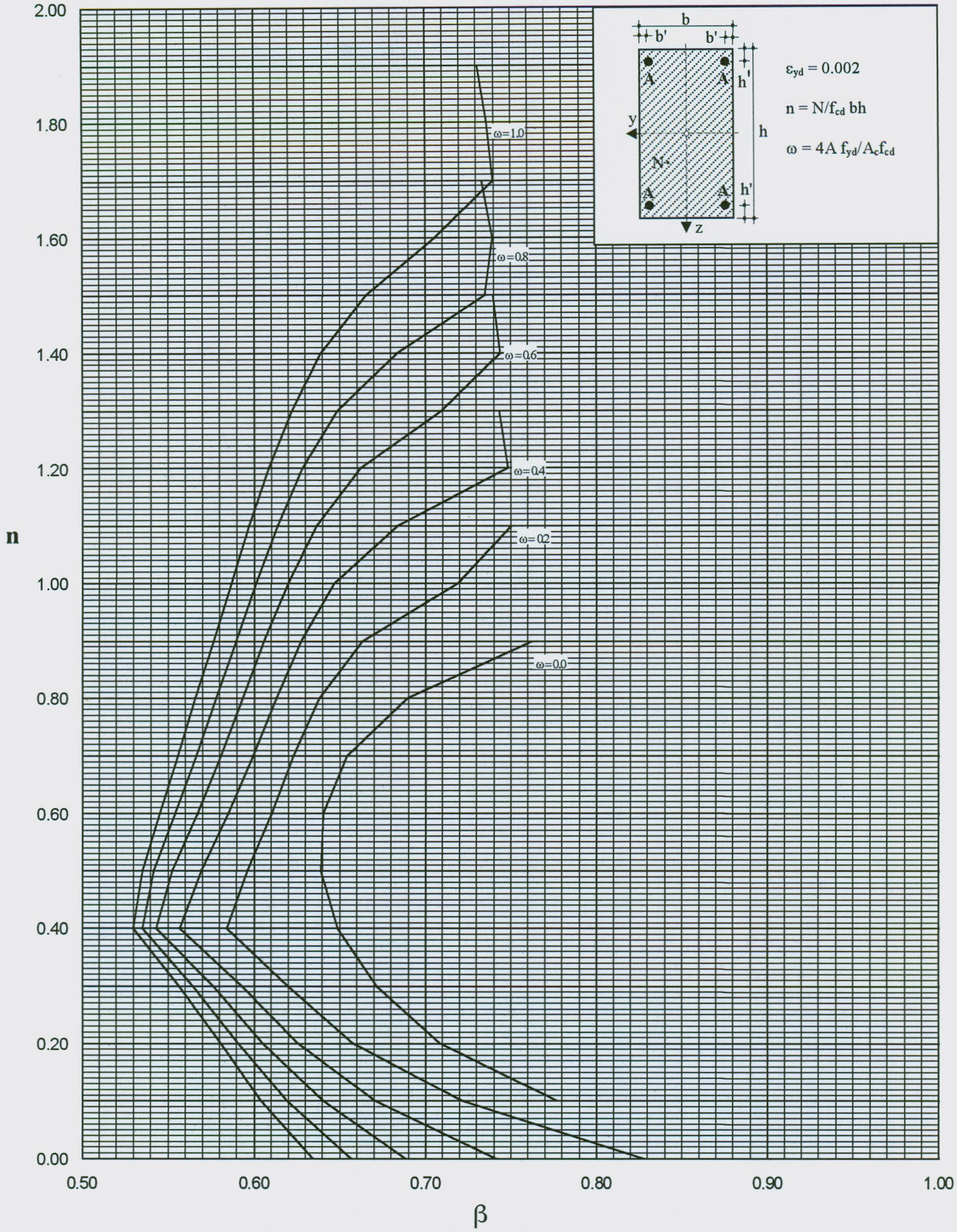
$$m_z = \frac{M_z}{f_{cd}hb^2} \quad ; \quad m_{uz} = \frac{M_{uz}}{f_{cd}hb^2}$$

The nature of such curves depends on the values of the exponent α which is defined in article3.3.2 and according to CP110, it is primarily related to the normal force and equation (3.5) is used for the computation of α. Where as according to ACI the exponent α is defined in terms of the biaxial bending design constant β as given by equation (3.6). As explained earlier the values of the biaxial bending design constant β is given in a diagrammatic form for various cross-sections in references [5] and [18]. For the purpose of evaluating the merits of the procedure however, one requires exact values of β, which are difficult to read from the available diagrams in the literatures [5,18]. Therefore values of β are computed using the definition given in article 3.3.2.2 in accordance with EBCS-2 [1] for three patterns of reinforcement and cover ratios of 0.1 and 0.2 using the computer program [6]. The results are given in fig.4.1 where the value of β is plotted against the relative normal force n for different mechanical reinforcement ratio ω.



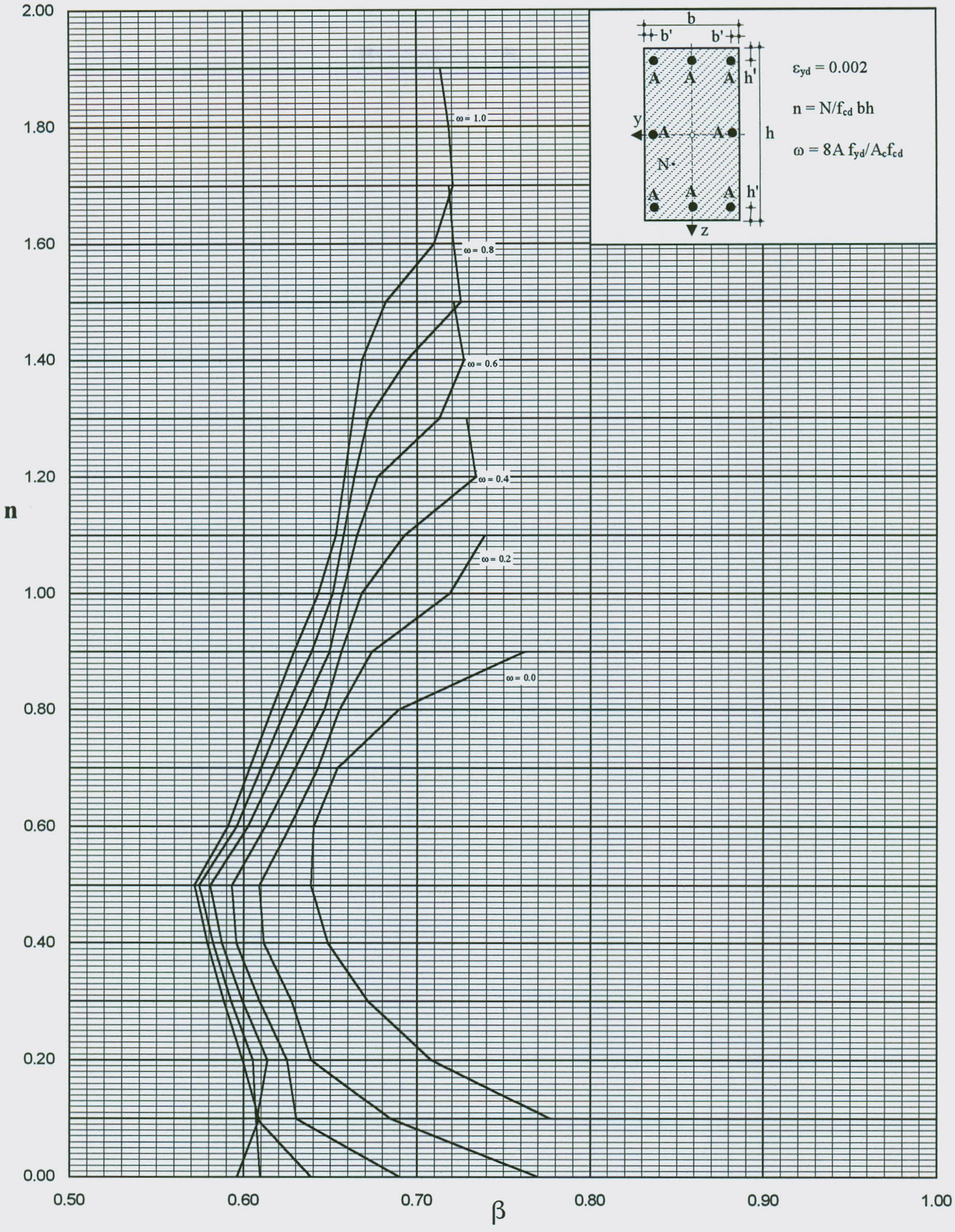
(a) h'/h = b'/b = 0.10

Fig.4.1 biaxial bending design constant



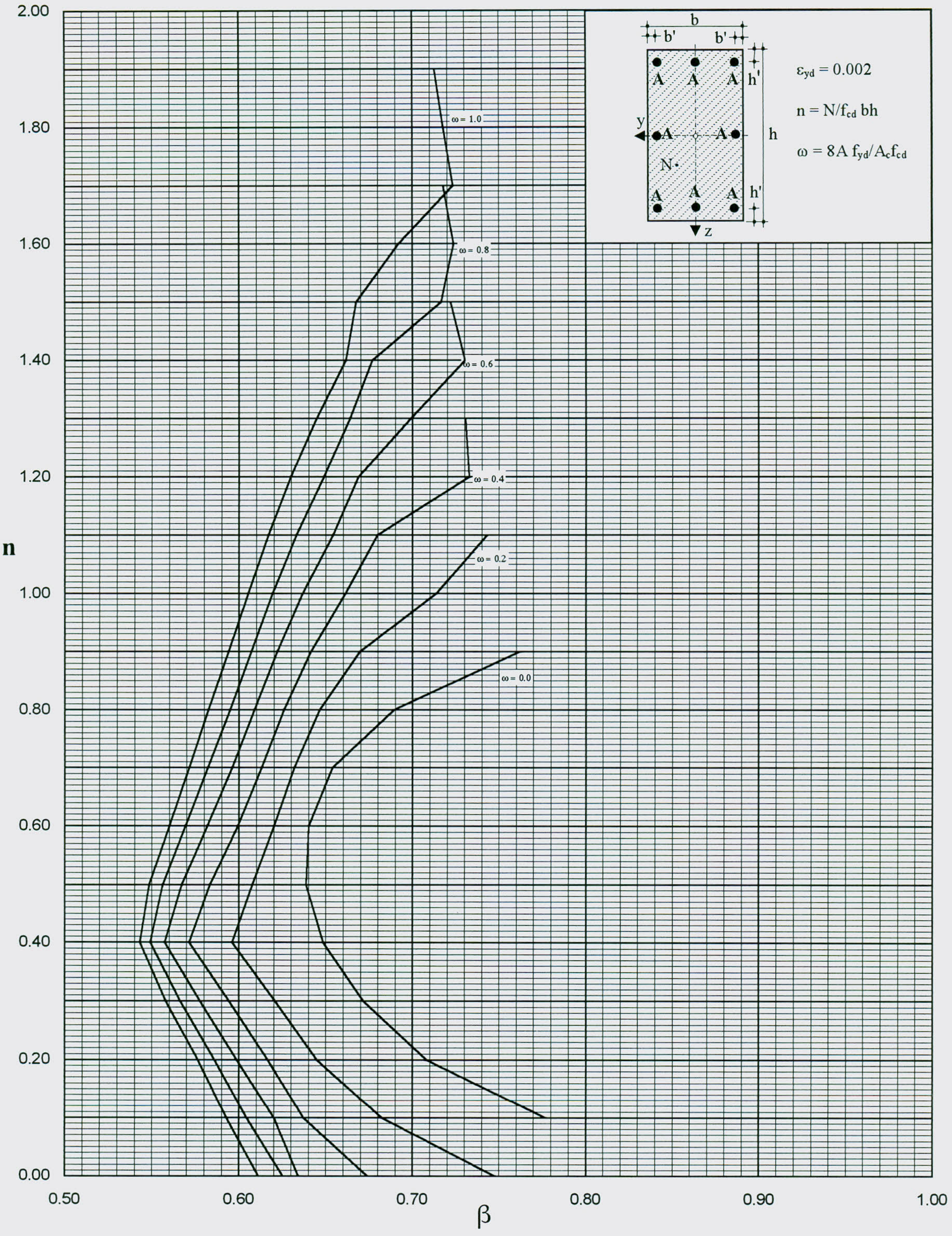
(b) h'/h = b'/b = 0.2

Fig.4.1 (Continued…)



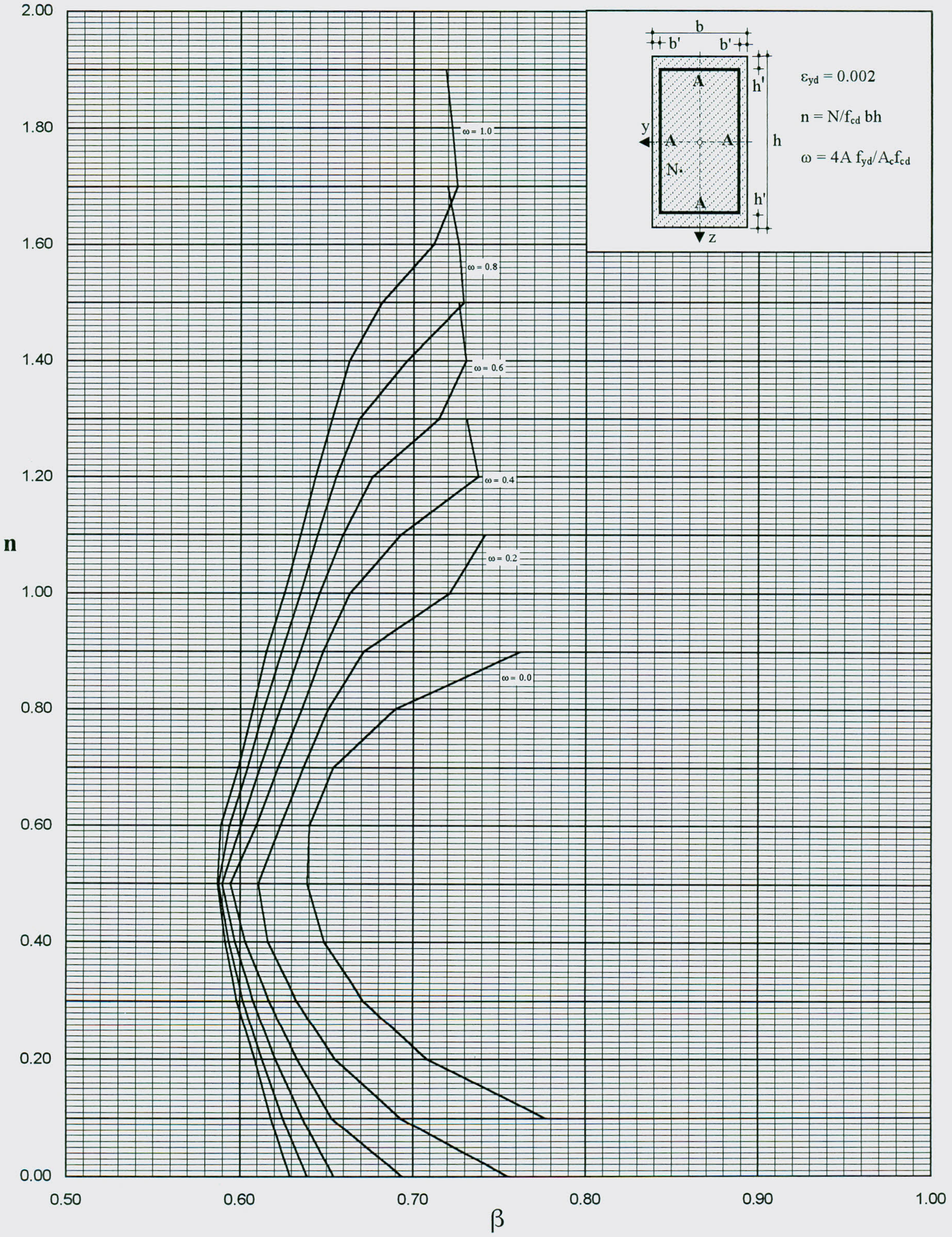
(c) h'/h = b'/b = 0.1

Fig.4.1 (Continued…)



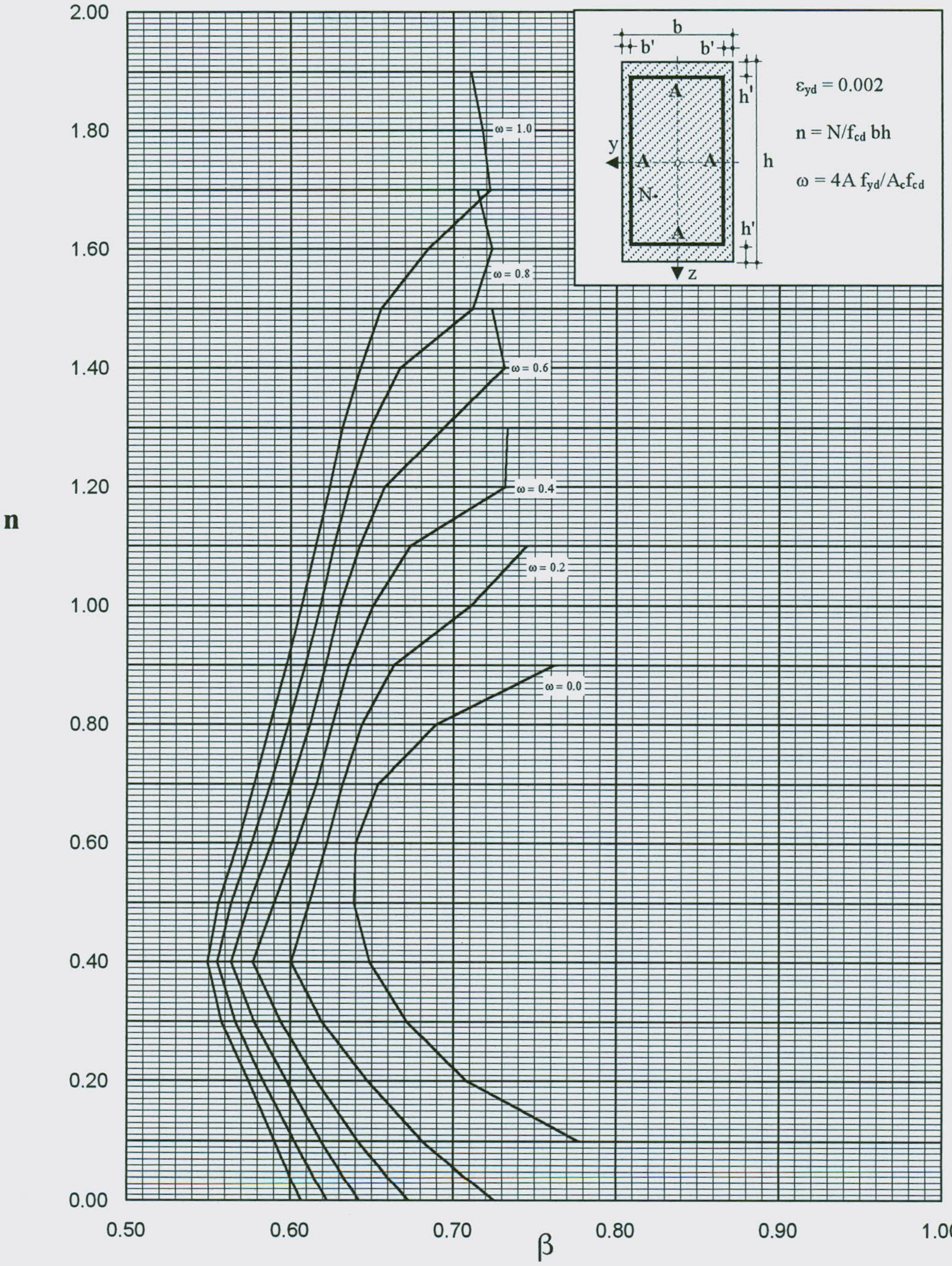
(d) h'/h = b'/b = 0.20

Fig.4.1 (Continued…)



(e) h'/h = b'/b = 0.10

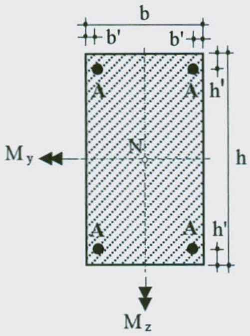
Fig.4.1 (Continued…)



(f) h'/h b'/b = 0.2

Fig.4.1 (Continued…)

The theoretical curves for the capacity of cross-sections under biaxial bending at different levels of normal force obtained using the two approaches (CP110 and ACI) are compared with the exact solution as shown in fig.4.2.The effect of other factors like amount, distribution and location of the reinforcement steel are taken in to account by varying the respective parameters as indicated in the figures.



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 4A$

$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

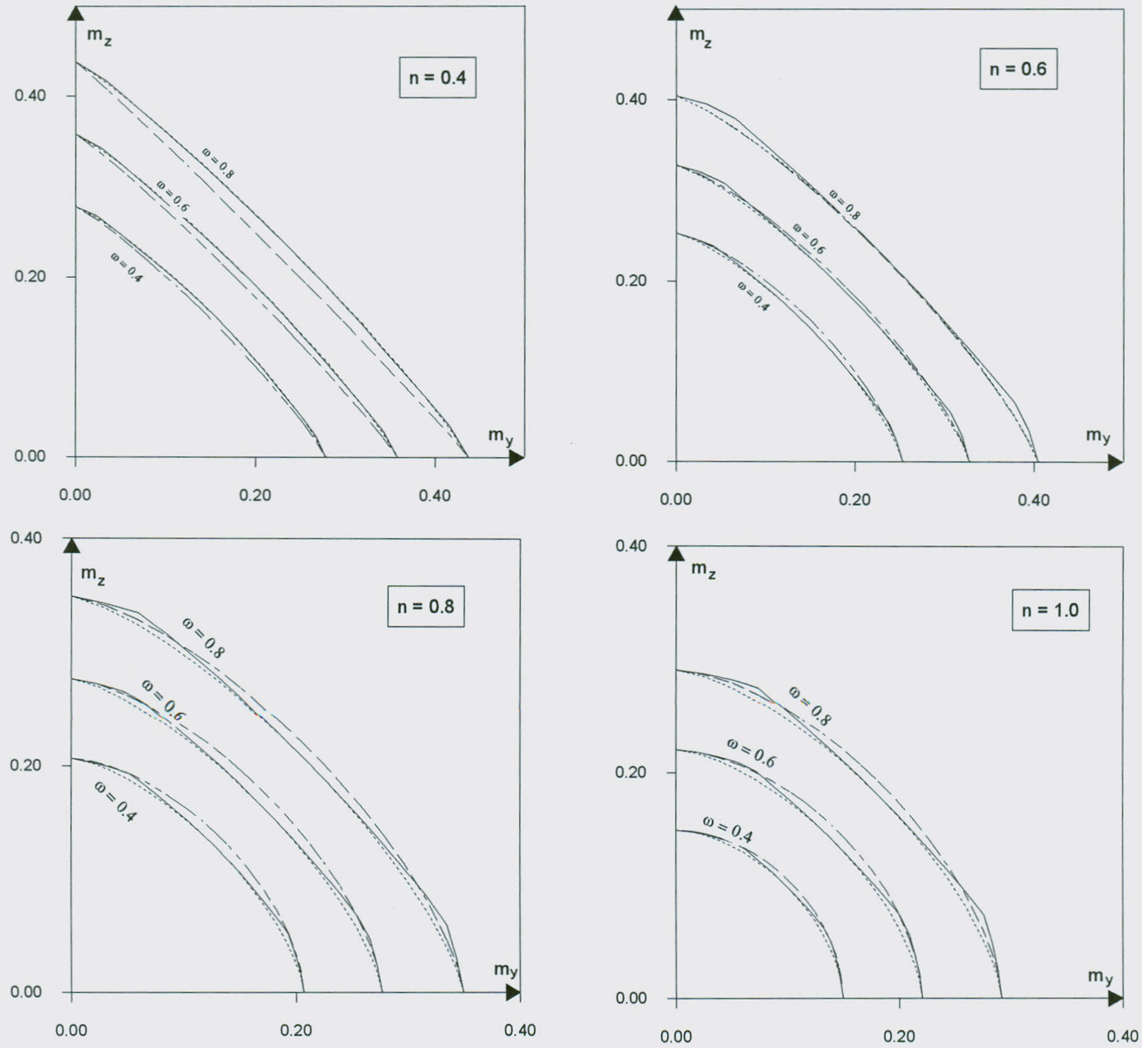
$m_z = M_z / hb^2 f_{cd}$

$n = N / bh\ f_{cd}$

–·–·–·– CP110

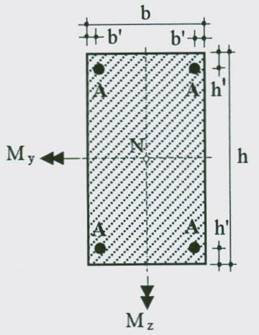
·············· ACI

——— Exact



(a) h'/h = b'/b = 0.10

Fig.4.2 comparison of capacities of cross-sections



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 4A$

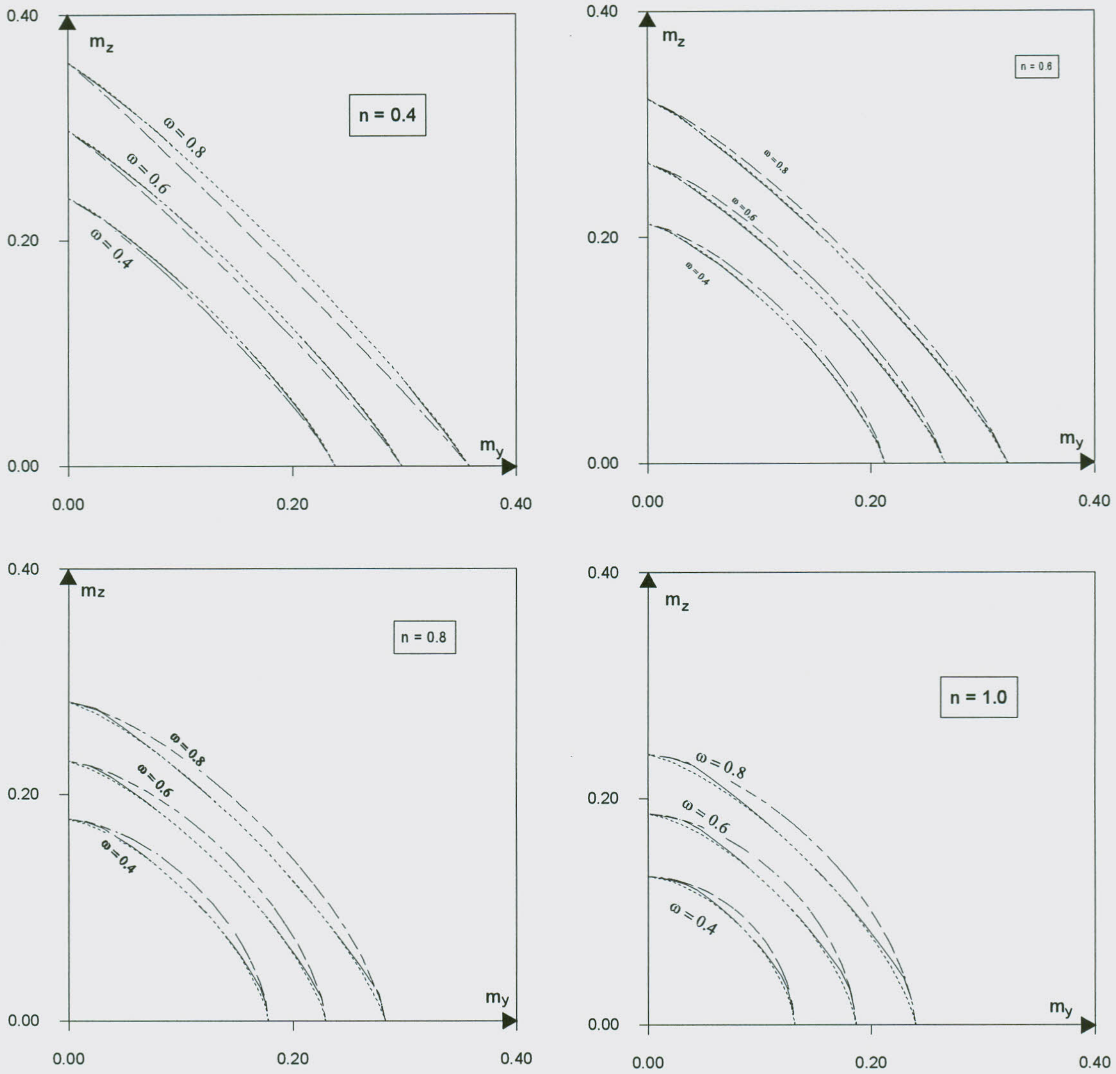
$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

$m_z = M_z / hb^2 f_{cd}$

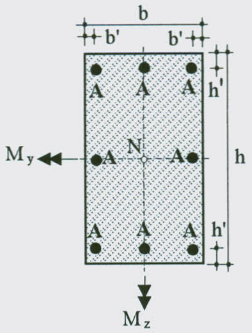
$n = N / bh\ f_{cd}$

-·-·-·- CP110
·········· ACI
——— Exact



(b) h'/h = b'/b = 0.20

Fig.4.2 (continued…)



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 8A$

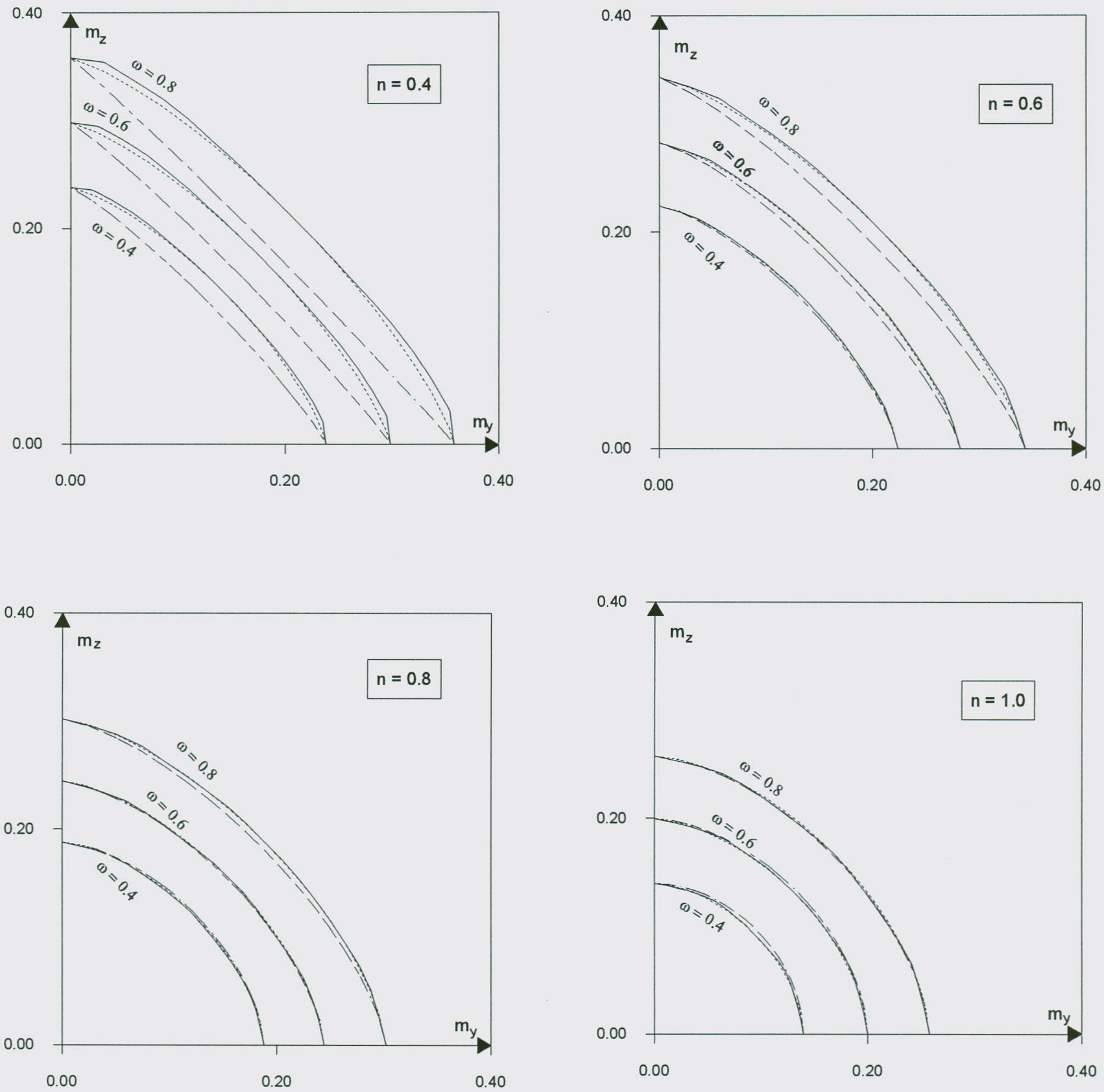
$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

$m_z = M_z / hb^2 f_{cd}$

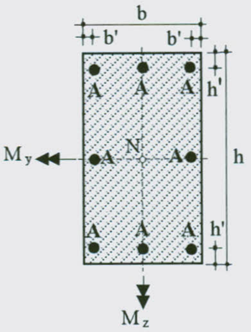
$n = N / bh\ f_{cd}$

-·-·-·- CP110
········ ACI
——— Exact



(c) h'/h = b'/b = 0.10

Fig.4.2 (continued…)



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 8A$

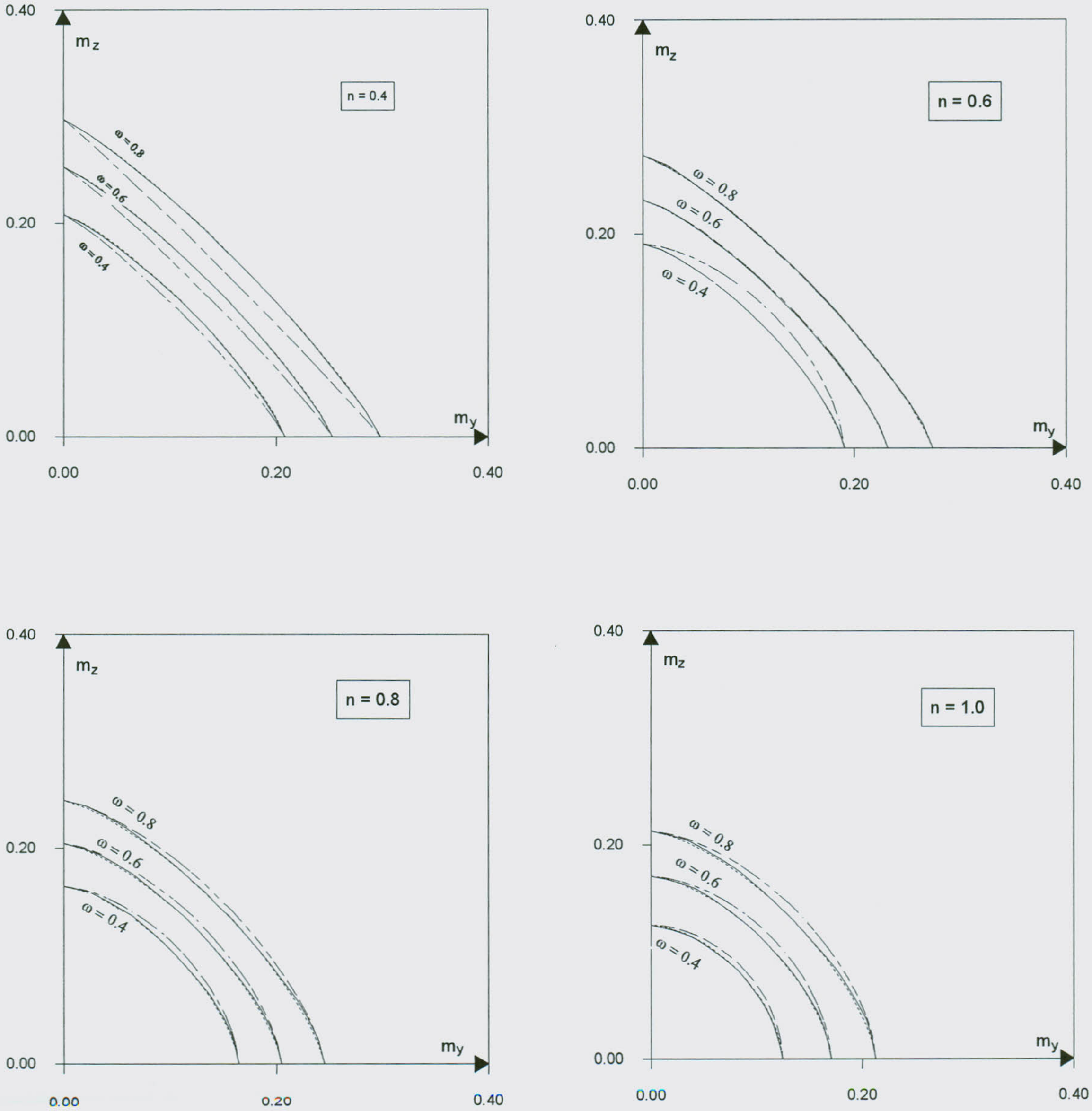
$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

$m_z = M_z / hb^2 f_{cd}$

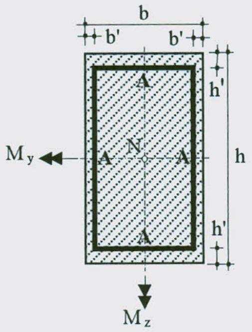
$n = N / bh\ f_{cd}$

-.-.-.- CP110
.... ACI
——— Exact



(d) $h'/h = b'/b = 0.20$

Fig.4.2 (continued…)



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 4A$

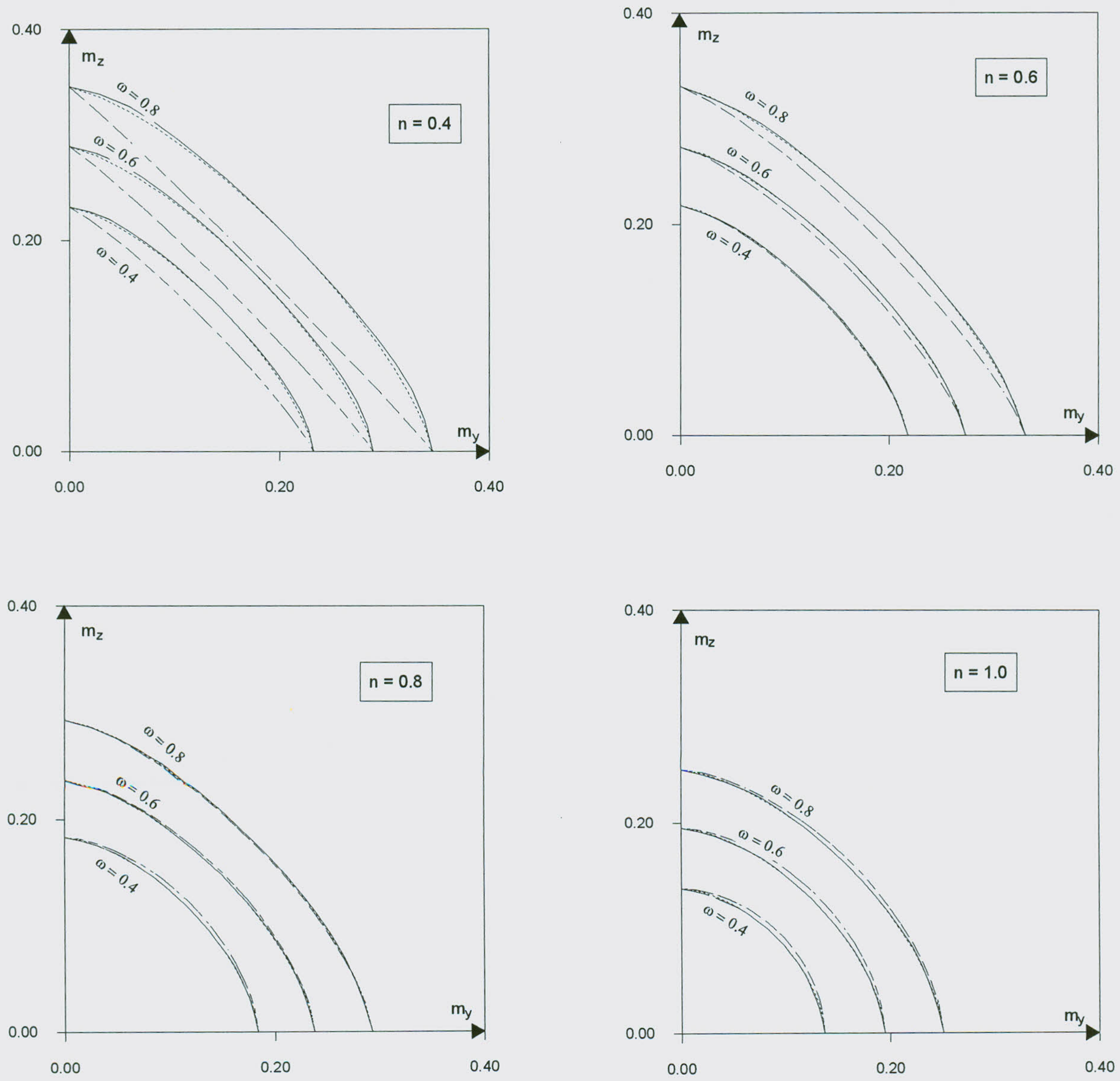
$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

$m_z = M_z / hb^2 f_{cd}$

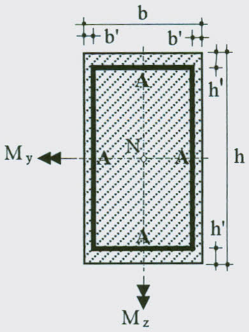
$n = N / bh\ f_{cd}$

-·-·-·- CP110
········ ACI
——— Exact



(e) h'/h = b'/b = 0.10

Fig.4.2 (continued…)



$\varepsilon_{yd} = 0.002$

$A_{s,tot} = 4A$

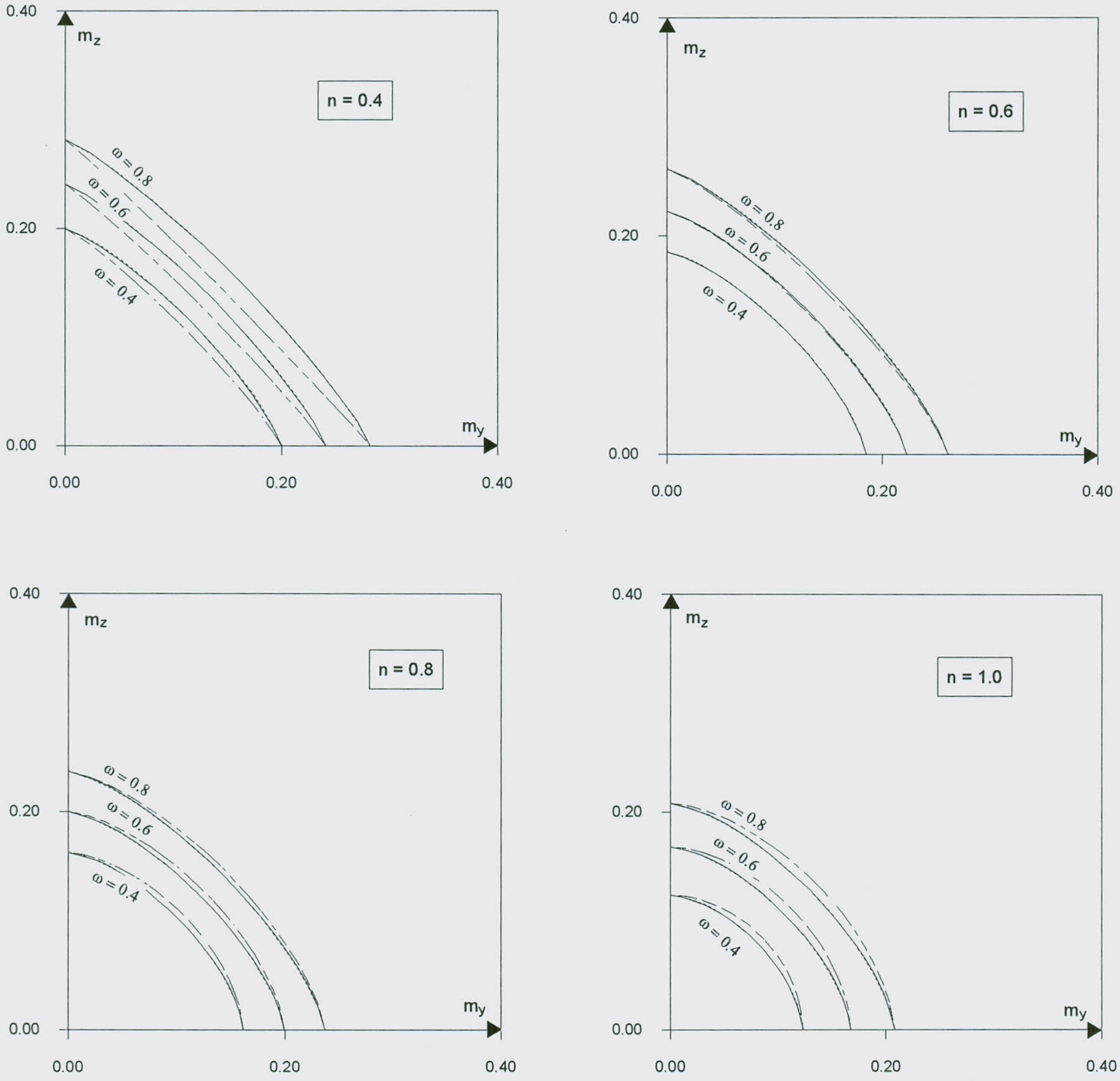
$\omega = A_{s,tot}\ f_{yd} / bh\ f_{cd}$

$m_y = M_y / bh^2 f_{cd}$

$m_z = M_z / hb^2 f_{cd}$

$n = N / bh\ f_{cd}$

-·-·-· CP110
.... ACI
——— Exact



(f) h'/h = b'/b = 0.20

Fig.4.2 (continued…)

4.2 Discussion of Results

As it can be seen in fig.4.2, expression for the capacity of rectangular reinforced concrete sections under normal force and biaxial moments in terms of simple interaction equation according to ACI [5] represents a good approximation under a wide variety of parameters. The effects of normal force, amount, location and distribution of reinforcement are taken in to account by varying the respective parameters and in most of the cases investigated, this approximation is found to be very close to the exact solution. However, for the case of bars arranged at the four corners a slight deviation (on the conservative side) is observed at relatively larger normal force level ($n = 0.8$, 0.1) with a condition that there is significant bending moment about one of the principal axes of the cross-section. In all of other cases, the theoretical curves obtained using this approximation are close enough to the exact solution. As the arrangement of reinforcement becomes more uniformly distributed on all four sides of the cross-section, the curves get closer and closer to the exact solution and thus it can be concluded that the use of this approximation for the design of biaxially loaded rectangular reinforced concrete columns with uniform distribution of reinforcement gives satisfactorily accurate results.

On the other hand, approximating the biaxial moment capacity of a given cross-section in terms of interaction equation at different levels of normal force according to CP110[2,3] is found to be close enough to the exact solution only for arrangement of bars with more or less uniform distribution on all four faces and moderate levels of normal force ($n = 0.6$, 0.8). For the cases of arrangement of reinforcement with eight bars and uniform distribution of bars on all four sides of the cross-section, a wide discrepancy is observed between the interaction curves obtained using this approximation and exact solution at lower normal force level ($n \leq 0.4$) although the deviation is on the conservative side. Considerable differences are also observed at larger levels of normal force ($n \geq 0.8$) for reinforcement pattern with four bars at each corner. The deviation in such cases lies on the unconservative side contrary to the most common trend that approximate methods normally give at least conservative results. The deviation tends to decrease as the arrangement of bars tends to be more uniformly distributed on all four faces of the cross-section. As indicated in fig.4.2, in addition to the arrangement of reinforcement and level of normal force, other factors like amount of reinforcement and concrete cover are also found to have significant effects. As a result, this method is not versatile under wide variety of parameters. However, out of the result of investigations made so far and shown in fig.4.2, the method is found to give close enough results to the exact solution at moderate levels of normal force ($n = 0.6, 0.8$) for reinforcement pattern with uniform distribution on all four sides of the cross-section.

5 EVALUATION OF THE APPROXIMATE METHOD OF DESIGN ACCORDING TO EBCS-2

5.1 General

As explained in section 3.4.3, the Ethiopian building code, EBCS-2 [1] allows an approximate method of design in which a biaxially loaded rectangular reinforced concrete column can in general be designed for the given normal force and uniaxial bending moment computed using the equivalent eccentricity of load given by equation (3.13). The eccentricities in equation (3.13) correspond to the final design moments including second order effects. Therefore the investigation for suitability of this method can be started at a stage where one obtains the final design moments M_y, M_z including second order effects either by performing second order elastic analysis or adding the second order moments calculated separately according to EBCS-2 [1] to the first order moments. Thus the assessment for suitability of this approximate method for the design of rectangular reinforced concrete columns involves only cross-section analysis and hence equation (3.13) can be expressed in terms of moments (in relative or non-dimensional terms) as follows; assuming that $m_y > m_z$ and m_y corresponds to the dimension h of the cross-section:

$$m_{eq} = m_y (1 + k\gamma) \qquad (5.1)$$

Where:

$$m_y = \frac{M_y}{f_{cd} A_c h} = n e_z / h \quad ; \quad m_{eq} = \frac{M_{eq}}{f_{cd} A_c h} = n e_{eq} / h$$

$$k = \frac{e_y / b}{e_z / h} = \frac{m_z}{m_y} \quad ; \quad n = \frac{N}{f_{cd} A_c}$$

γ is a factor depending on the relative normal force n, and is given in table 3.1

5.2 Comparison Based on Steel Requirement

For the purpose of assessing suitability of this approximate method, a number of column cross-sections under different combinations of normal force and bending moments about the major and minor axes of the cross-sections are considered. The exact amount of reinforcement required for a given dimension of cross-section under a particular combination of normal force and biaxial moments is obtained using the computer program [6]. The equivalent uniaxial bending moment is determined using equation (5.1) and the amount of reinforcement required for the given dimension of cross-section under the same normal force with the equivalent uniaxial bending moment is determined again using the rigorous solution. To facilitate more accurate reading of the chart values, uniaxial design charts with finer increment of ω, mechanical reinforcement ratio, are prepared using the computer program [6] for different cover ratios and included in appendix A.

The code [1] recommends that the reinforcement obtained using this approximate method must be either concentrated at the corners or equally distributed on all four sides of the column cross-section. Therefore the application of this method is limited to only these two arrangements of reinforcement and thus the assessment for its suitability is made accordingly. However, its proximity to the exact solution still depends on other parameters like relative eccentricity ratio defining the proportion of the two moment components, the level of normal force and concrete cover. Sufficient data for comparison of amount of reinforcement that takes the effect of all parameters enumerated above in to account are generated and given in tables 5.1 and 5.2 for bars concentrated at the corners and uniformly distributed on all four sides of the cross-section respectively.

Table 5.1 Data for comparison of amount of reinforcement

With bars concentrated at the corners

(a) h'/h = b'/b = 0.5

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔA_s (%)
0.2	0.2447	0.0489	0.1998	0.2838	0.400	0.455	13.75
0.2	0.2381	0.0638	0.2680	0.2891	0.400	0.465	16.25
0.2	0.2239	0.0815	0.3640	0.2891	0.400	0.465	16.25
0.2	0.2091	0.0975	0.4663	0.2871	0.400	0.462	15.50
0.2	0.1950	0.1126	0.5774	0.2851	0.400	0.458	14.50
0.2	0.1813	0.1269	0.6999	0.2828	0.400	0.455	13.75
0.2	0.1678	0.1408	0.8391	0.2804	0.400	0.445	11.25
0.2	0.1543	0.1543	1.0000	0.2777	0.400	0.440	10.00
0.4	0.2656	0.0531	0.1999	0.3134	0.400	0.435	8.75
0.4	0.2532	0.0679	0.2682	0.3143	0.400	0.437	9.25
0.4	0.2376	0.0865	0.3641	0.3155	0.400	0.439	9.75
0.4	0.2227	0.1038	0.4661	0.3161	0.400	0.440	10.00
0.4	0.2082	0.1202	0.5773	0.3164	0.400	0.440	10.00
0.4	0.1940	0.1358	0.7000	0.3162	0.400	0.440	10.00
0.4	0.1798	0.1509	0.8393	0.3156	0.400	0.440	10.00
0.4	0.1655	0.1655	1.0000	0.3145	0.400	0.439	9.75
0.6	0.2565	0.0513	0.2000	0.2924	0.400	0.440	10.00
0.6	0.2470	0.0662	0.2680	0.2933	0.400	0.442	10.50
0.6	0.2323	0.0846	0.3642	0.2915	0.400	0.440	10.00
0.6	0.2181	0.1017	0.4663	0.2893	0.400	0.435	8.75
0.6	0.2042	0.1179	0.5774	0.2867	0.400	0.434	8.50
0.6	0.1904	0.1333	0.7001	0.2837	0.400	0.420	5.00
0.6	0.1765	0.1481	0.8391	0.2802	0.400	0.410	2.50
0.6	0.1625	0.1625	1.0000	0.2763	0.400	0.405	1.25
0.8	0.2129	0.0426	0.2001	0.2385	0.400	0.440	10.00
0.8	0.2083	0.0558	0.2679	0.2418	0.400	0.450	12.50
0.8	0.1985	0.0722	0.3637	0.2418	0.400	0.450	12.50
0.8	0.1867	0.0871	0.4665	0.2390	0.400	0.442	10.50
0.8	0.1750	0.1010	0.5771	0.2356	0.400	0.435	8.75
0.8	0.1633	0.1143	0.6999	0.2319	0.400	0.430	7.50
0.8	0.1514	0.1271	0.8395	0.2277	0.400	0.418	4.50
0.8	0.1394	0.1394	1.0000	0.2230	0.400	0.410	2.50
1	0.1551	0.0310	0.1999	0.1706	0.400	0.430	7.50
1	0.1528	0.0410	0.2683	0.1733	0.400	0.437	9.25
1	0.1490	0.0542	0.3638	0.1761	0.400	0.442	10.50
1	0.1439	0.0671	0.4663	0.1775	0.400	0.446	11.50
1	0.1348	0.0779	0.5779	0.1738	0.400	0.440	10.00
1	0.1258	0.0881	0.7003	0.1698	0.400	0.427	6.75
1	0.1167	0.0979	0.8389	0.1656	0.400	0.415	3.75
1	0.1075	0.1075	1.0000	0.1613	0.400	0.405	1.25

Table 5.1 (continued….)

(b) h'/h = b'/b = 0.10

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.2274	0.0455	0.2001	0.2638	0.400	0.460	15.00
0.2	0.2218	0.0594	0.2678	0.2693	0.400	0.475	18.75
0.2	0.2087	0.0760	0.3642	0.2695	0.400	0.475	18.75
0.2	0.1952	0.0910	0.4662	0.2680	0.400	0.470	17.50
0.2	0.1821	0.1052	0.5777	0.2663	0.400	0.465	16.25
0.2	0.1694	0.1186	0.7001	0.2643	0.400	0.460	15.00
0.2	0.1569	0.1317	0.8394	0.2623	0.400	0.455	13.75
0.2	0.1443	0.1443	1.0000	0.2597	0.400	0.450	12.50
0.4	0.2476	0.0495	0.1999	0.2921	0.400	0.435	8.75
0.4	0.2365	0.0634	0.2681	0.2936	0.400	0.440	10.00
0.4	0.2223	0.0809	0.3639	0.2951	0.400	0.442	10.50
0.4	0.2087	0.0973	0.4662	0.2963	0.400	0.443	10.75
0.4	0.1953	0.1128	0.5776	0.2968	0.400	0.445	11.25
0.4	0.1821	0.1275	0.7002	0.2969	0.400	0.445	11.25
0.4	0.1689	0.1417	0.8390	0.2964	0.400	0.443	10.75
0.4	0.1555	0.1555	1.0000	0.2955	0.400	0.443	10.75
0.6	0.2349	0.0470	0.2001	0.2678	0.400	0.440	10.00
0.6	0.2249	0.0603	0.2681	0.2671	0.400	0.438	9.50
0.6	0.2119	0.0771	0.3639	0.2659	0.400	0.436	9.00
0.6	0.1992	0.0929	0.4664	0.2642	0.400	0.430	7.50
0.6	0.1866	0.1078	0.5777	0.2621	0.400	0.425	6.25
0.6	0.1741	0.1219	0.7002	0.2594	0.400	0.418	4.50
0.6	0.1615	0.1355	0.8390	0.2563	0.400	0.408	2.00
0.6	0.1486	0.1486	1.0000	0.2526	0.400	0.400	0.00
0.8	0.1974	0.0395	0.2001	0.2211	0.400	0.440	10.00
0.8	0.1929	0.0517	0.2680	0.2239	0.400	0.450	12.50
0.8	0.1822	0.0663	0.3639	0.2220	0.400	0.445	11.25
0.8	0.1716	0.0800	0.4662	0.2196	0.400	0.439	9.75
0.8	0.1610	0.0929	0.5770	0.2167	0.400	0.430	7.50
0.8	0.1503	0.1052	0.6999	0.2134	0.400	0.419	4.75
0.8	0.1394	0.1170	0.8393	0.2096	0.400	0.410	2.50
0.8	0.1284	0.1284	1.0000	0.2054	0.400	0.395	-1.25
1.0	0.1445	0.0289	0.2000	0.1590	0.400	0.428	7.00
1.0	0.1423	0.0381	0.2677	0.1613	0.400	0.432	8.00
1.0	0.1385	0.0504	0.3639	0.1637	0.400	0.440	10.00
1.0	0.1323	0.0617	0.4664	0.1632	0.400	0.438	9.50
1.0	0.1242	0.0717	0.5773	0.1601	0.400	0.430	7.50
1.0	0.1160	0.0812	0.7000	0.1566	0.400	0.420	5.00
1.0	0.1076	0.0903	0.8392	0.1527	0.400	0.410	2.50
1.0	0.0991	0.0991	1.0000	0.1487	0.400	0.400	0.00

Table 5.1 (continued….)

(c) h'/h = b'/b 0.15

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs(%)
0.2	0.2103	0.0421	0.2002	0.2440	0.400	0.475	18.75
0.2	0.2057	0.0551	0.2679	0.2498	0.400	0.496	24.00
0.2	0.1935	0.0704	0.3638	0.2498	0.400	0.496	24.00
0.2	0.1812	0.0845	0.4663	0.2488	0.400	0.495	23.75
0.2	0.1693	0.0977	0.5771	0.2475	0.400	0.490	22.5
0.2	0.1576	0.1104	0.7005	0.2459	0.400	0.485	21.25
0.2	0.1460	0.1225	0.8390	0.2440	0.400	0.475	18.75
0.2	0.1344	0.1344	1.0000	0.2419	0.400	0.470	17.50
0.4	0.2294	0.0459	0.2001	0.2707	0.400	0.437	9.25
0.4	0.2196	0.0588	0.2678	0.2725	0.400	0.445	11.25
0.4	0.2069	0.0753	0.3639	0.2747	0.400	0.450	12.50
0.4	0.1946	0.0907	0.4661	0.2762	0.400	0.455	13.75
0.4	0.1824	0.1053	0.5773	0.2772	0.400	0.458	14.50
0.4	0.1703	0.1192	0.6999	0.2776	0.400	0.460	15.00
0.4	0.1580	0.1326	0.8392	0.2773	0.400	0.460	15.00
0.4	0.1455	0.1455	1.0000	0.2765	0.400	0.460	15.00
0.6	0.2132	0.0426	0.1998	0.2430	0.400	0.438	9.50
0.6	0.2045	0.0548	0.2680	0.2429	0.400	0.438	9.50
0.6	0.1930	0.0702	0.3637	0.2421	0.400	0.436	9.00
0.6	0.1817	0.0847	0.4662	0.2410	0.400	0.430	7.50
0.6	0.1704	0.0984	0.5775	0.2393	0.400	0.424	6.00
0.6	0.1590	0.1113	0.7000	0.2369	0.400	0.420	5.00
0.6	0.1475	0.1238	0.8393	0.2342	0.400	0.410	2.50
0.6	0.1358	0.1358	1.0000	0.2309	0.400	0.400	0.00
0.8	0.1826	0.0365	0.1999	0.2045	0.400	0.440	10.00
0.8	0.1768	0.0474	0.2681	0.2052	0.400	0.445	11.25
0.8	0.1673	0.0609	0.3640	0.2038	0.400	0.440	10.00
0.8	0.1577	0.0736	0.4667	0.2019	0.400	0.435	8.75
0.8	0.1481	0.0855	0.5773	0.1994	0.400	0.425	6.25
0.8	0.1384	0.0969	0.7001	0.1965	0.400	0.420	5.00
0.8	0.1284	0.1077	0.8388	0.1930	0.400	0.410	2.50
0.8	0.1182	0.1182	1.0000	0.1891	0.400	0.390	-2.50
1.0	0.1349	0.0270	0.2001	0.1484	0.400	0.430	7.50
1.0	0.1327	0.0356	0.2683	0.1505	0.400	0.437	9.25
1.0	0.1289	0.0469	0.3638	0.1523	0.400	0.440	10.00
1.0	0.1219	0.0568	0.4660	0.1503	0.400	0.434	8.50
1.0	0.1145	0.0661	0.5773	0.1476	0.400	0.425	6.25
1.0	0.1070	0.0749	0.7000	0.1445	0.400	0.420	5.00
1.0	0.0994	0.0834	0.8390	0.1411	0.400	0.405	1.25
1.0	0.0915	0.0915	1.0000	0.1373	0.400	0.395	-1.25

Table 5.1 (continued….)

(d) h'/h =b'/b =0.20

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.1911	0.0382	0.1999	0.2217	0.4000	0.485	21.25
0.2	0.1865	0.0500	0.2681	0.2265	0.4000	0.500	25.00
0.2	0.1755	0.0639	0.3641	0.2266	0.4000	0.500	25.00
0.2	0.1649	0.0769	0.4663	0.2264	0.4000	0.500	25.00
0.2	0.1545	0.0892	0.5773	0.2259	0.4000	0.497	24.25
0.2	0.1442	0.1010	0.7004	0.2250	0.4000	0.495	23.75
0.2	0.1338	0.1123	0.8393	0.2236	0.4000	0.490	22.50
0.2	0.1232	0.1232	1.0000	0.2218	0.4000	0.480	20.00
0.4	0.2172	0.0434	0.1998	0.2563	0.4000	0.480	20.00
0.4	0.2016	0.0540	0.2679	0.2502	0.4000	0.460	15.00
0.4	0.1896	0.0690	0.3639	0.2517	0.4000	0.448	12.00
0.4	0.1780	0.0830	0.4663	0.2527	0.4000	0.450	12.50
0.4	0.1665	0.0961	0.5772	0.2530	0.4000	0.450	12.50
0.4	0.1552	0.1087	0.7004	0.2530	0.4000	0.450	12.50
0.4	0.1439	0.1207	0.8388	0.2525	0.4000	0.450	12.50
0.4	0.1324	0.1324	1.0000	0.2516	0.4000	0.448	12.00
0.6	0.1930	0.0386	0.2000	0.2200	0.4000	0.430	7.50
0.6	0.1856	0.0497	0.2678	0.2204	0.4000	0.431	7.75
0.6	0.1756	0.0639	0.3639	0.2203	0.4000	0.431	7.75
0.6	0.1655	0.0772	0.4665	0.2195	0.4000	0.430	7.50
0.6	0.1554	0.0897	0.5772	0.2182	0.4000	0.425	6.25
0.6	0.1451	0.1016	0.7002	0.2162	0.4000	0.415	3.75
0.6	0.1347	0.1130	0.8389	0.2138	0.4000	0.410	2.50
0.6	0.1240	0.1240	1.0000	0.2108	0.4000	0.400	0.00
0.8	0.1684	0.0337	0.2001	0.1886	0.4000	0.445	11.25
0.8	0.1622	0.0435	0.2682	0.1883	0.4000	0.442	10.50
0.8	0.1538	0.0560	0.3641	0.1874	0.4000	0.440	10.00
0.8	0.1452	0.0677	0.4663	0.1858	0.4000	0.435	8.75
0.8	0.1365	0.0788	0.5773	0.1838	0.4000	0.420	5.00
0.8	0.1276	0.0893	0.6998	0.1812	0.4000	0.410	2.50
0.8	0.1184	0.0994	0.8395	0.1780	0.4000	0.400	0.00
0.8	0.1091	0.1091	1.0000	0.1746	0.4000	0.390	-2.50
1.0	0.1264	0.0253	0.2002	0.1391	0.4000	0.430	7.50
1.0	0.1241	0.0333	0.2683	0.1407	0.4000	0.438	9.50
1.0	0.1192	0.0434	0.3641	0.1409	0.4000	0.438	9.50
1.0	0.1126	0.0525	0.4663	0.1389	0.4000	0.430	7.50
1.0	0.1059	0.0611	0.5770	0.1365	0.4000	0.420	5.00
1.0	0.0991	0.0694	0.7003	0.1338	0.4000	0.410	2.50
1.0	0.0920	0.0772	0.8391	0.1306	0.4000	0.400	0.00
1.0	0.0847	0.0847	1.0000	0.1271	0.4000	0.390	-2.50

Table 5.2 Data for comparison of amount of reinforcement with uniform distribution of bars

(a) h'/h =b'/b =0.05

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.2145	0.0429	0.2	0.2488	0.400	0.470	17.50
0.2	0.2087	0.0559	0.2678	0.2534	0.400	0.480	20.00
0.2	0.1999	0.0728	0.3642	0.2581	0.400	0.492	23.00
0.2	0.1903	0.0887	0.4661	0.2613	0.400	0.505	26.25
0.2	0.1797	0.1037	0.5771	0.2627	0.400	0.510	27.50
0.2	0.1682	0.1178	0.7004	0.2624	0.400	0.508	27.00
0.2	0.1563	0.1311	0.8388	0.2612	0.400	0.505	26.25
0.2	0.1439	0.1439	1.0000	0.259	0.400	0.496	24.00
0.4	0.2336	0.0467	0.1999	0.2756	0.400	0.490	22.50
0.4	0.2261	0.0606	0.268	0.2806	0.400	0.504	26.00
0.4	0.2145	0.0781	0.3641	0.2848	0.400	0.515	28.75
0.4	0.2023	0.0943	0.4661	0.2872	0.400	0.521	30.25
0.4	0.19	0.1097	0.5774	0.2887	0.400	0.530	32.50
0.4	0.1775	0.1243	0.7003	0.2894	0.400	0.530	32.50
0.4	0.1648	0.1383	0.8392	0.2893	0.400	0.530	32.50
0.4	0.1517	0.1517	1.0000	0.2882	0.400	0.528	32.00
0.6	0.2188	0.0438	0.2002	0.2495	0.400	0.441	10.25
0.6	0.2123	0.0569	0.268	0.2521	0.400	0.452	13.00
0.6	0.2022	0.0736	0.364	0.2537	0.400	0.460	15.00
0.6	0.1913	0.0892	0.4663	0.2537	0.400	0.460	15.00
0.6	0.18	0.1039	0.5772	0.2527	0.400	0.458	14.50
0.6	0.1682	0.1178	0.7004	0.2507	0.400	0.450	12.50
0.6	0.1563	0.1311	0.8388	0.2481	0.400	0.440	10.00
0.6	0.1439	0.1439	1.0000	0.2446	0.400	0.430	7.50
0.8	0.1858	0.0376	0.2024	0.2084	0.400	0.440	10.00
0.8	0.1811	0.0485	0.2678	0.2102	0.400	0.445	11.25
0.8	0.1738	0.0632	0.3636	0.2117	0.400	0.455	13.75
0.8	0.1653	0.0771	0.4664	0.2116	0.400	0.455	13.75
0.8	0.1561	0.0901	0.5772	0.2102	0.400	0.445	11.25
0.8	0.1463	0.1025	0.7006	0.2078	0.400	0.440	10.00
0.8	0.136	0.1142	0.8397	0.2045	0.400	0.430	7.50
0.8	0.1253	0.1253	1.0000	0.2005	0.400	0.420	5.00
1.0	0.1385	0.0277	0.2	0.1524	0.400	0.420	5.00
1.0	0.1356	0.0363	0.2677	0.1538	0.400	0.428	7.00
1.0	0.1311	0.0477	0.3638	0.155	0.400	0.430	7.50
1.0	0.1257	0.0586	0.4662	0.155	0.400	0.430	7.50
1.0	0.1196	0.069	0.5769	0.1541	0.400	0.428	7.00
1.0	0.1126	0.0788	0.6998	0.152	0.400	0.420	5.00
1.0	0.105	0.0881	0.839	0.1491	0.400	0.415	3.75
1.0	0.0968	0.0968	1.0000	0.1452	0.400	0.400	0.00

Table 5.2 (continued…)

(b) h'/h = b'/b 0.10

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.1994	0.0399	0.2001	0.2313	0.400	0.470	17.50
0.2	0.1941	0.0520	0.2679	0.2357	0.400	0.480	20.00
0.2	0.1856	0.0676	0.3642	0.2397	0.400	0.498	24.50
0.2	0.1763	0.0822	0.4663	0.2421	0.400	0.504	26.00
0.2	0.1661	0.0959	0.5774	0.2428	0.400	0.510	27.50
0.2	0.1555	0.1089	0.7003	0.2426	0.400	0.510	27.50
0.2	0.1445	0.1213	0.8394	0.2415	0.400	0.500	25.00
0.2	0.1331	0.1331	1.0000	0.2396	0.400	0.498	24.50
0.4	0.2170	0.0434	0.2000	0.2561	0.400	0.482	20.50
0.4	0.2093	0.0561	0.2680	0.2598	0.400	0.500	25.00
0.4	0.1983	0.0722	0.3641	0.2633	0.400	0.510	27.50
0.4	0.1869	0.0872	0.4666	0.2654	0.400	0.522	30.50
0.4	0.1755	0.1013	0.5772	0.2667	0.400	0.522	30.50
0.4	0.1638	0.1147	0.7002	0.2670	0.400	0.522	30.50
0.4	0.1519	0.1275	0.8394	0.2667	0.400	0.522	30.50
0.4	0.1398	0.1398	1.0000	0.2656	0.400	0.522	30.50
0.6	0.2025	0.0405	0.2000	0.2309	0.400	0.450	12.50
0.6	0.1962	0.0526	0.2681	0.2330	0.400	0.460	15.00
0.6	0.1866	0.0679	0.3639	0.2341	0.400	0.462	15.50
0.6	0.1764	0.0823	0.4666	0.2340	0.400	0.462	15.50
0.6	0.1659	0.0958	0.5775	0.2330	0.400	0.460	15.00
0.6	0.1551	0.1086	0.7002	0.2311	0.400	0.445	11.25
0.6	0.1441	0.1209	0.8390	0.2287	0.400	0.440	10.00
0.6	0.1326	0.1326	1.0000	0.2254	0.400	0.425	6.25
0.8	0.1739	0.0348	0.2001	0.1948	0.400	0.440	10.00
0.8	0.1695	0.0454	0.2678	0.1967	0.400	0.450	12.50
0.8	0.1625	0.0591	0.3637	0.1980	0.400	0.454	13.50
0.8	0.1543	0.0720	0.4666	0.1975	0.400	0.452	13.00
0.8	0.1455	0.0840	0.5773	0.1959	0.400	0.442	10.50
0.8	0.1363	0.0954	0.6999	0.1935	0.400	0.438	9.50
0.8	0.1267	0.1063	0.8390	0.1905	0.400	0.428	7.00
0.8	0.1168	0.1168	1.0000	0.1869	0.400	0.412	3.00
1.0	0.1306	0.0261	0.1998	0.1437	0.400	0.422	5.50
1.0	0.1278	0.0342	0.2676	0.1449	0.400	0.428	7.00
1.0	0.1233	0.0449	0.3642	0.1458	0.400	0.430	7.50
1.0	0.1183	0.0552	0.4666	0.1459	0.400	0.430	7.50
1.0	0.1123	0.0649	0.5779	0.1448	0.400	0.428	7.00
1.0	0.1056	0.0740	0.7008	0.1426	0.400	0.420	5.00
1.0	0.0984	0.0826	0.8394	0.1397	0.400	0.410	2.50
1.0	0.0908	0.0908	1.0000	0.1362	0.400	0.400	0.00

Table 5.2 (continued…)

(c) h'/h = b'/b = 0.15

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.1841	0.0368	0.1999	0.2135	0.400	0.465	16.25
0.2	0.1791	0.0480	0.2680	0.2175	0.400	0.480	20.00
0.2	0.1712	0.0623	0.3639	0.2210	0.400	0.490	22.50
0.2	0.1626	0.0758	0.4662	0.2232	0.400	0.500	25.00
0.2	0.1533	0.0885	0.5773	0.2241	0.400	0.504	26.00
0.2	0.1436	0.1006	0.7006	0.2241	0.400	0.504	26.00
0.2	0.1337	0.1122	0.8392	0.2235	0.400	0.500	25.00
0.2	0.1233	0.1233	1.0000	0.2219	0.400	0.495	23.75
0.4	0.1997	0.0399	0.1998	0.2356	0.400	0.475	18.75
0.4	0.1923	0.0515	0.2678	0.2387	0.400	0.495	23.75
0.4	0.1820	0.0663	0.3643	0.2417	0.400	0.510	27.50
0.4	0.1716	0.0800	0.4662	0.2436	0.400	0.518	29.50
0.4	0.1612	0.0931	0.5775	0.2450	0.400	0.520	30.00
0.4	0.1507	0.1055	0.7001	0.2457	0.400	0.522	30.50
0.4	0.1400	0.1175	0.8393	0.2458	0.400	0.522	30.50
0.4	0.1291	0.1291	1.0000	0.2453	0.400	0.520	30.00
0.6	0.1872	0.0374	0.1998	0.2134	0.400	0.455	13.75
0.6	0.1813	0.0486	0.2681	0.2153	0.400	0.464	16.00
0.6	0.1726	0.0628	0.3638	0.2166	0.400	0.470	17.50
0.6	0.1635	0.0763	0.4667	0.2169	0.400	0.470	17.50
0.6	0.1540	0.0889	0.5773	0.2162	0.400	0.466	16.50
0.6	0.1443	0.1010	0.6999	0.2150	0.400	0.464	16.00
0.6	0.1341	0.1126	0.8397	0.2129	0.400	0.455	13.75
0.6	0.1237	0.1237	1.0000	0.2103	0.400	0.440	10.00
0.8	0.1631	0.0326	0.1999	0.1827	0.400	0.448	12.00
0.8	0.1589	0.0426	0.2681	0.1845	0.400	0.450	12.50
0.8	0.1521	0.0554	0.3642	0.1853	0.400	0.460	15.00
0.8	0.1446	0.0674	0.4661	0.1850	0.400	0.460	15.00
0.8	0.1366	0.0788	0.5769	0.1839	0.400	0.450	12.50
0.8	0.1282	0.0897	0.6997	0.1820	0.400	0.440	10.00
0.8	0.1193	0.1001	0.8391	0.1794	0.400	0.430	7.50
0.8	0.1100	0.1100	1.0000	0.1760	0.400	0.420	5.00
1.0	0.1237	0.0247	0.1997	0.1361	0.400	0.424	6.00
1.0	0.1210	0.0324	0.2678	0.1372	0.400	0.430	7.50
1.0	0.1167	0.0425	0.3642	0.1380	0.400	0.435	8.75
1.0	0.1116	0.0521	0.4668	0.1377	0.400	0.435	8.75
1.0	0.1061	0.0612	0.5768	0.1367	0.400	0.430	7.50
1.0	0.1000	0.0700	0.7000	0.1350	0.400	0.420	5.00
1.0	0.0933	0.0783	0.8392	0.1325	0.400	0.410	2.50
1.0	0.0862	0.0862	1.0000	0.1293	0.400	0.400	0.00

Table 5.2 (continued…)

(d) h'/h = b'/b

n	m_y	m_z	k	m_{eq}	ω	ω'	ΔAs (%)
0.2	0.1678	0.0336	0.2002	0.1947	0.400	0.470	17.50
0.2	0.1631	0.0437	0.2679	0.1981	0.400	0.482	20.50
0.2	0.1553	0.0565	0.3638	0.2005	0.400	0.500	25.00
0.2	0.1468	0.0685	0.4666	0.2016	0.400	0.502	25.50
0.2	0.1380	0.0797	0.5775	0.2018	0.400	0.502	25.50
0.2	0.1290	0.0903	0.7000	0.2012	0.400	0.500	25.00
0.2	0.1198	0.1005	0.8389	0.2002	0.400	0.495	23.75
0.2	0.1102	0.1102	1.0000	0.1984	0.400	0.485	21.25
0.4	0.1798	0.0360	0.2002	0.2122	0.400	0.460	15.00
0.4	0.1728	0.0463	0.2679	0.2145	0.400	0.470	17.50
0.4	0.1632	0.0594	0.3640	0.2167	0.400	0.485	21.25
0.4	0.1537	0.0717	0.4665	0.2182	0.400	0.490	22.50
0.4	0.1442	0.0833	0.5777	0.2192	0.400	0.495	23.75
0.4	0.1347	0.0943	0.7001	0.2196	0.400	0.500	25.00
0.4	0.1250	0.1048	0.8384	0.2193	0.400	0.495	23.75
0.4	0.1150	0.1150	1.0000	0.2185	0.400	0.495	23.75
0.6	0.1713	0.0343	0.2002	0.1953	0.400	0.450	12.50
0.6	0.1656	0.0444	0.2681	0.1967	0.400	0.465	16.25
0.6	0.1573	0.0572	0.3636	0.1973	0.400	0.465	16.25
0.6	0.1487	0.0693	0.4660	0.1972	0.400	0.465	16.25
0.6	0.1398	0.0807	0.5773	0.1963	0.400	0.460	15.00
0.6	0.1307	0.0915	0.7001	0.1948	0.400	0.450	12.50
0.6	0.1213	0.1018	0.8392	0.1926	0.400	0.440	10.00
0.6	0.1117	0.1117	1.0000	0.1899	0.400	0.430	7.50
0.8	0.1524	0.0305	0.2001	0.1707	0.400	0.450	12.50
0.8	0.1481	0.0397	0.2681	0.1719	0.400	0.455	13.75
0.8	0.1413	0.0514	0.3638	0.1721	0.400	0.455	13.75
0.8	0.1339	0.0624	0.4660	0.1713	0.400	0.450	12.50
0.8	0.1261	0.0728	0.5773	0.1698	0.400	0.445	11.25
0.8	0.1180	0.0826	0.7000	0.1676	0.400	0.438	9.50
0.8	0.1096	0.0920	0.8394	0.1648	0.400	0.420	5.00
0.8	0.1009	0.1009	1.0000	0.1614	0.400	0.405	1.25
1.0	0.1169	0.0234	0.2002	0.1286	0.400	0.425	6.25
1.0	0.1142	0.0306	0.2680	0.1295	0.400	0.430	7.50
1.0	0.1099	0.0400	0.3640	0.1299	0.400	0.430	7.50
1.0	0.1049	0.0489	0.4662	0.1294	0.400	0.425	6.25
1.0	0.0992	0.0573	0.5776	0.1279	0.400	0.420	5.00
1.0	0.0932	0.0652	0.6996	0.1258	0.400	0.410	2.50
1.0	0.0867	0.0728	0.8397	0.1231	0.400	0.400	0.00
1.0	0.0800	0.0800	1.0000	0.1200	0.400	0.390	-2.50

5.3 Discussion of Results

The data for comparison of amount of reinforcement is investigated statistically and the Gaussian distribution for ΔA_s, expressed in percent, is shown in fig.5.1. From the distribution, it can be concluded that the approximate method of design [1] gives mostly conservative results. With regard to arrangement of bars, the pattern of reinforcement with uniform distribution is found to give more conservative results than that of the case of four-bar arrangement. Based on the results of investigation of the data considered in this thesis work, the approximate method is found to give on the average 10% more reinforcement with a standard deviation of 6% for the four-bar arrangement and the corresponding values for reinforcement pattern with uniform distribution on all four sides of the cross-section are found to be 16% and 9% respectively.

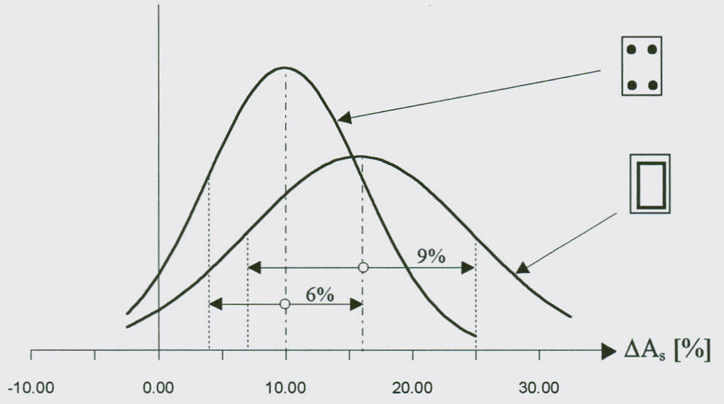
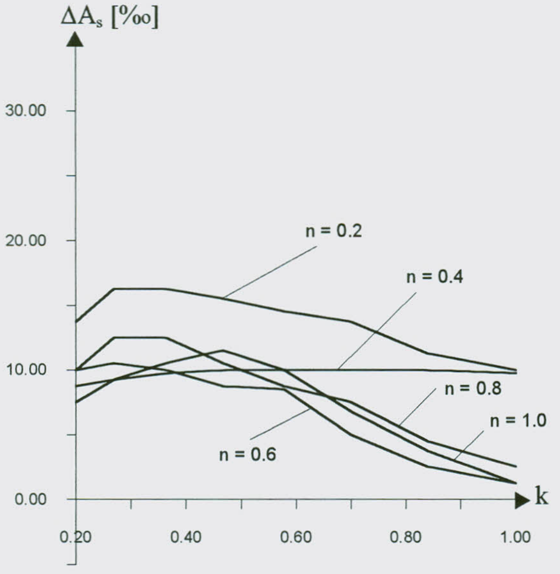
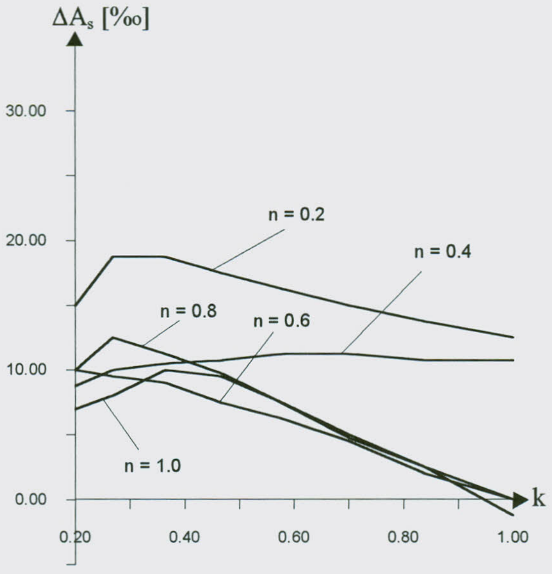


Fig.5.1 Gaussian distribution of data for comparison of amount of reinforcement

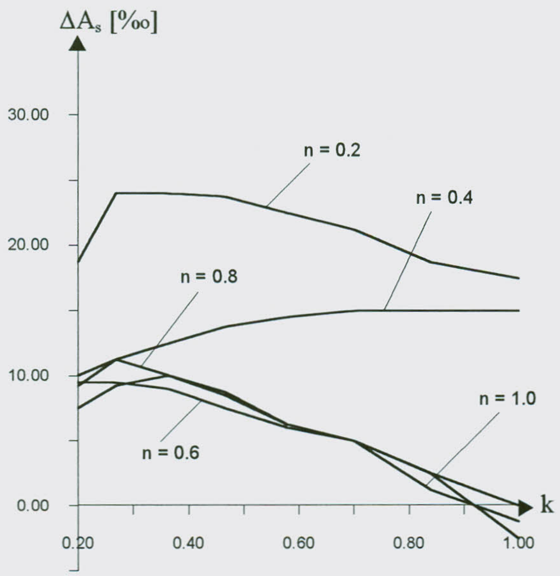
The distribution also shows that there are few cases in which the approximate method[1] becomes unconservative and out of the data considered, a maximum difference in the amount of reinforcement of –2.5% is observed. Therefore it becomes necessary to further investigate the general trend of the approximate method with respect to parameters of loading and other factors such as concrete cover and reinforcement patterns to state the conditions in which the approximate method tends to become unconservative. For this purpose, the data in tables 5.1 and 5.2 are presented graphically where ΔA_s is plotted against the relative eccentricity ratio k as shown in figures 5.2 and 5.3 for the two patterns of reinforcement.



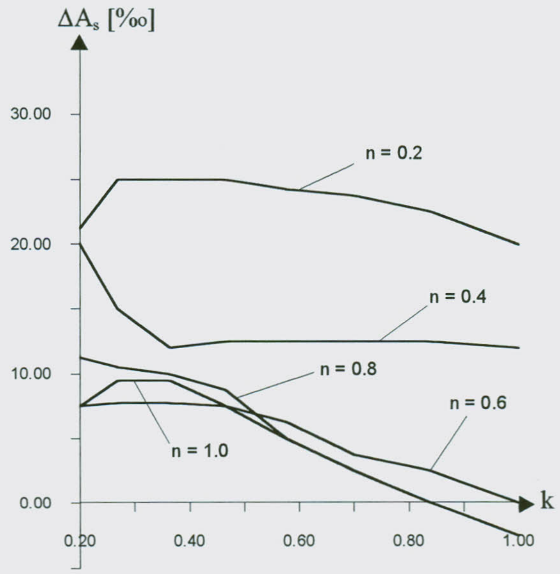
(a) h'/h = b'/b = 0.05



(b) h'/h = b'/b = 0.10

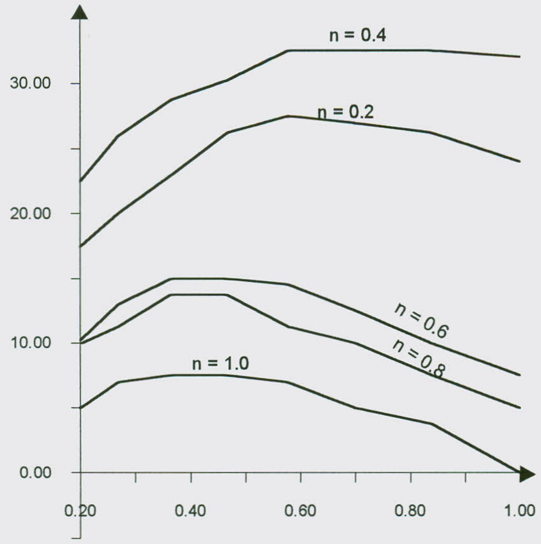


(c) h'/h = b'/b = 0.15

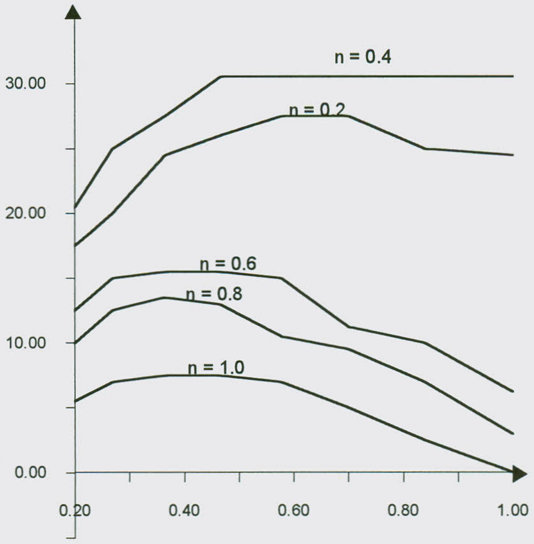


(d) h'/h = b'/b = 0.20

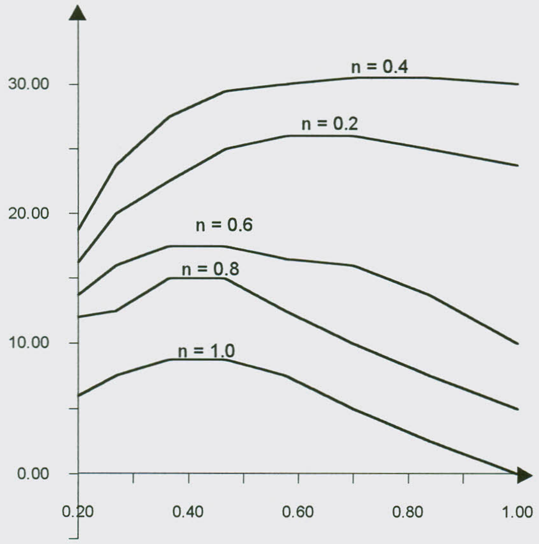
Fig.5.2 Comparison of amount of reinforcement with bars concentrated at each corner



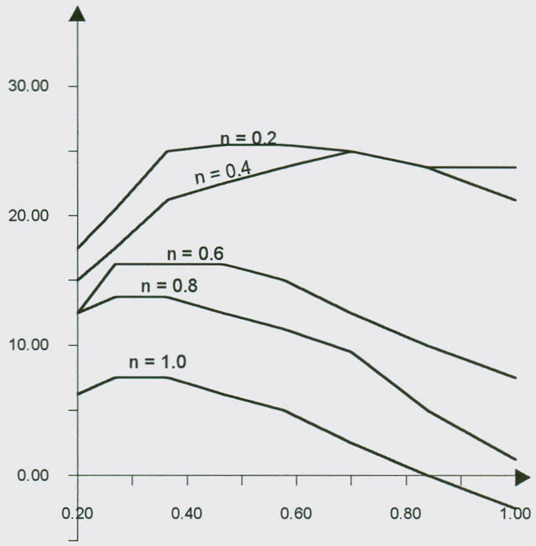
(a) h'/h = b'/b = 0.05



(b) h'/h = b'/b = 0.10



(c) h'/h = b'/b = 0.15



(d) h'/h = b'/b = 0.20

Fig.5.3 Comparison of amount of reinforcement with uniform distribution of bars

As indicated in figures 5.2 and 5.3, the curves relating the change in the amount of reinforcement to the relative eccentricity ratio at moderate and larger normal force levels ($n \geq 0.6$) have a common feature, namely the change in the amount of reinforcement decreases significantly as the relative eccentricity ratio approaches unity. Especially for the four-bar reinforcement pattern (fig.5.2), the approximate method tends to become unconservative at larger levels of normal force ($n \geq 0.8$) when the relative eccentricity ratio approaches 1.0. This trend becomes more significant as the concrete cover to reinforcement bars becomes larger and larger.

On the other hand for arrangement of bars with uniform distribution, the approximate method is found to give conservative values in most of the cases as shown in fig.5.3. The change in the amount of reinforcement (ΔA_s) decreases with increase in the normal force level except at $n = 0.4$ where the largest change in the amount of reinforcement is observed. This may be attributed to the fact that the coefficient reflecting the effect of normal force in equation (5.1) for the computation of equivalent uniaxial moment is the greatest of all. For this arrangement of bars, although the concrete cover to reinforcement seems not to have noticeable effects, it is observed that the change in the amount of reinforcement decreases with increase in concrete cover at lower normal force levels. As indicated in fig.5.3, the change in the amount of reinforcement (ΔA_s) decreases as the relative eccentricity ratio approaches 1.0 at moderate to larger normal force levels. The approximate method with uniform distribution of bars tends to become on the unconservative side at larger normal force ($n \geq 1.0$) when the relative eccentricity ratio is approaching 1.0; however, it gives safer values as compared to the four-bar reinforcement pattern in most of the cases investigated

6. CONCLUSIONS AND RECOMMENDATIONS

Analysis/design of biaxially loaded reinforced concrete columns based on a rigorous approach involves a great deal of computational effort and therefore normally requires the use of computers and relevant software's. In the absence of such facilities and also to expedite the analysis/design process, simple approximate methods are widely used [1,2,3,5,12]. Where as the different code use quite different approaches for the development of the approximate methods, there is no indication in the codes and/or literature as to the relative merits of the different approaches or even the extent to which the respective approximate methods may lie on the conservative or unconservative sides. The purpose of this thesis work was therefore to evaluate different approximate methods. Accordingly, the approximate methods for the design of biaxially loaded rectangular reinforced concrete sections according to CP110 [2,3] and ACI [5], which are based on approximating load contours at constant normal force in terms of interaction equation, and that of EBCS-2 [1] provision are evaluated in this thesis work. The evaluation has been effected through comparison of the results of the approximate methods to that of rigorous method [6,10]. Based on the results of investigation, the following conclusions and recommendations are made regarding these approximate methods.

- The expression for biaxial moment capacity of cross-sections under constant normal force in terms of interaction equation according to ACI [5] is found to represent a very good approximation and has close proximity to the exact solution for cross-sections having symmetry about both axes. This is so because the nature of theoretical curves obtained from the interaction equation is entirely controlled by the value of the exponent α. In this case, α is expressed in terms of the biaxial bending design constant β which is equal to either of the moment components in relative terms (M_y/M_{uy} or M_z/M_{uz}) at 45° from either of the principal axes. Thus the effect of all parameters or factors influencing the nature of load contours for cross-sections can be taken in to account through the biaxial bending design constant β. In addition to its proximity, the approximate method is found to be on the conservative side in all of the cases investigated. Therefore it can be concluded that the approximate method for the design of biaxially loaded rectangular reinforced concrete sections based on approximating the load contours in terms of interaction equation according to ACI [5] gives satisfactorily accurate results for cross-sections having symmetry about both axes.

- The approximation for biaxial moment capacity of cross-sections in terms of interaction equation according to CP110 [2,3] is found to be close enough to the exact solution under only few conditions and in the majority of the cases investigated, considerable discrepancies are observed between the interaction curves and exact solution [10]. The deviation in most of the cases, especially at larger normal force levels, is on the unconservative side. The method is found to give close enough results to the exact solution at moderate levels of normal force ($n = 0.6, 0.8$) for reinforcement patterns with uniform distribution on all four sides of the cross-section.

- The approximate method of design for biaxially loaded rectangular reinforced concrete columns according to EBCS-2 [1] in which the biaxial moments are converted in to equivalent uniaxial bending moment is found to give mostly conservative results. It is only in very few cases that slightly unconservative results with a maximum difference of –2.5% in the amount of reinforcement are observed. In the majority of the cases investigated, the amount of reinforcement obtained using the approximate method is greater than that of the exact solution with an average value for ΔA_s of 10% and 16% for reinforcing bars concentrated at the corners and equally distributed on all four sides of the cross-section respectively. These figures represent merely an average value and as much as 25% and 32.5% for the change in the amount of reinforcement are obtained with the respective arrangement of reinforcement. The results tend to lie on the unsafe side as the relative eccentricity ratio approaches 1.0, at larger normal force levels ($n \geq 0.8$) and greater cover ratios, the more so, for the four-bar reinforcing pattern.

- In this thesis work, the investigation for suitability of the approximate methods for the design of biaxially loaded rectangular reinforced concrete columns according to CP110 [3] and ACI [5] is made only for sections having double axes of symmetry and it is recommended that proximity of the approximate methods to the exact solution for sections with single axis of symmetry need be further investigated.

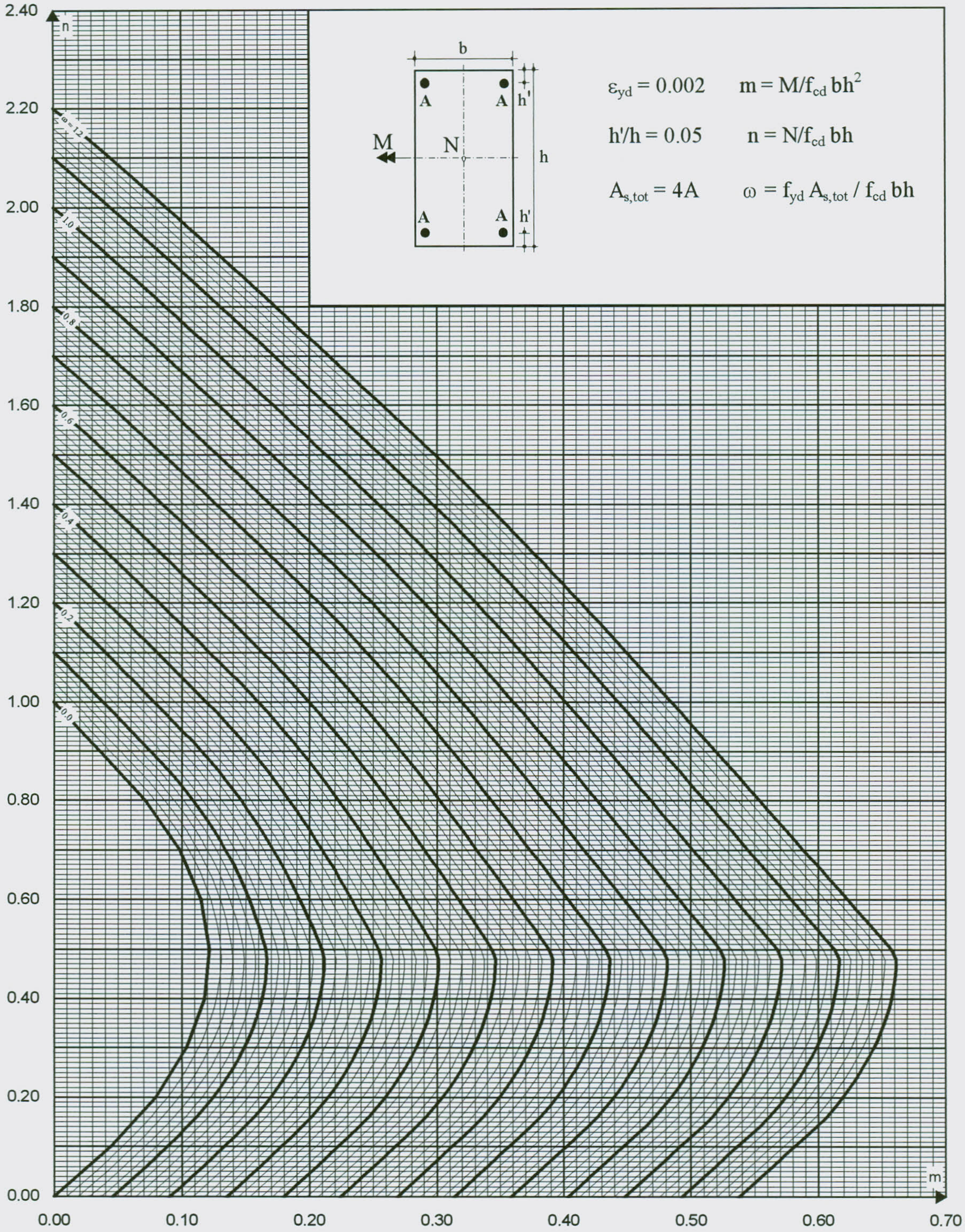
- The approximate methods according to CP110 [3] and ACI [3] can be used for design only in an iterative sense, for calculating the capacities of trial design with varying reinforcement until a satisfactory arrangement is found. The approximate methods according to BS8110 [3] and EBCS-2 [1] on the other hand, which are based on converting the biaxial moments in to an equivalent uniaxial moment, can be directly used for design once the equivalent uniaxial moment is determined using the design aids available for uniaxial bending [10]. This can be regarded as one attractive feature of such types of approximate methods, however, the extent to which the approximate method according to BS8110 [3] lies on the conservative or possibly on the unconservative side is not known and therefore it is recommended that the proximity of this method to exact solution be further investigated as well.

APPENDIX

Appendix A

UNIAXIAL CHARTS

Uniaxial Chart No.1



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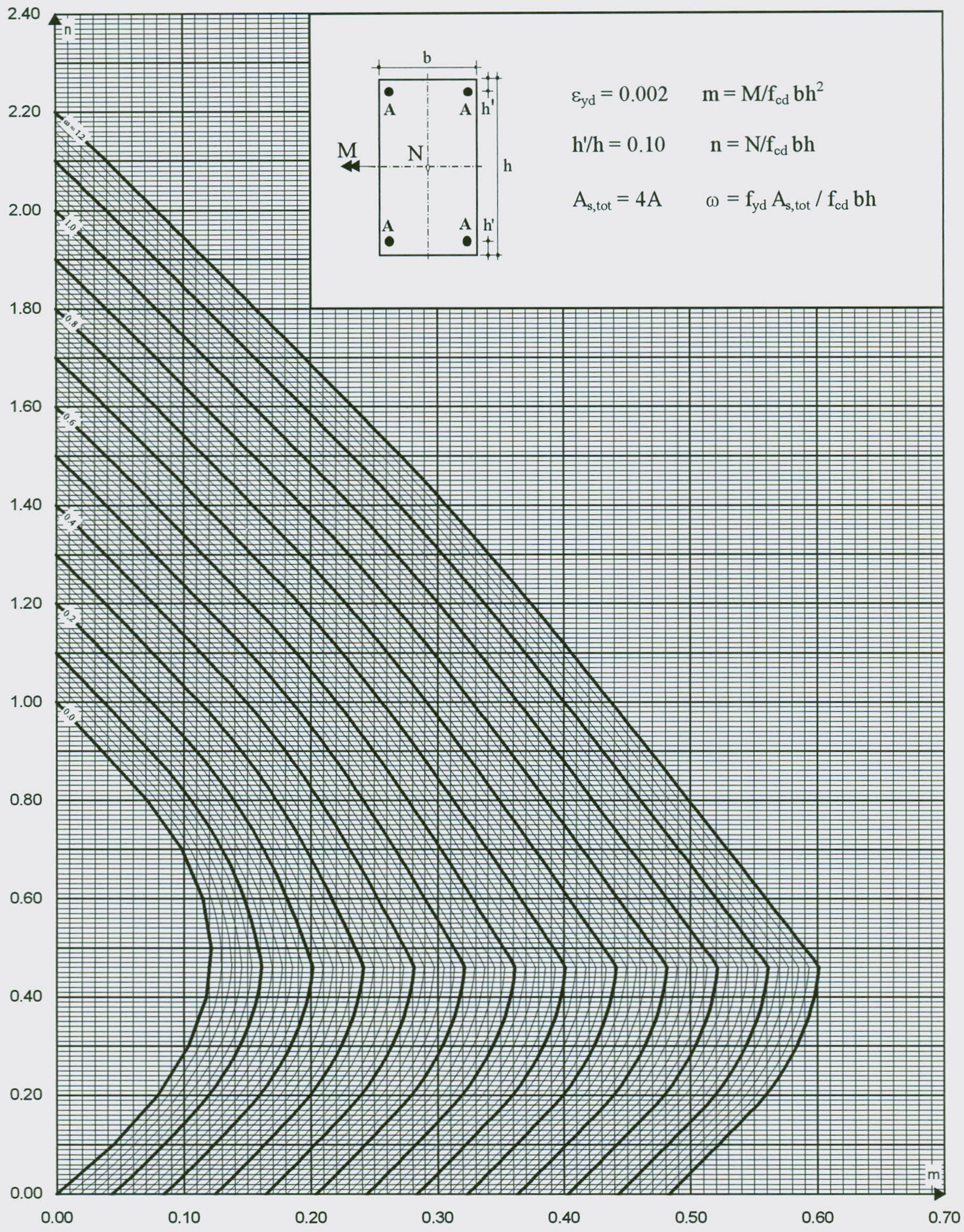
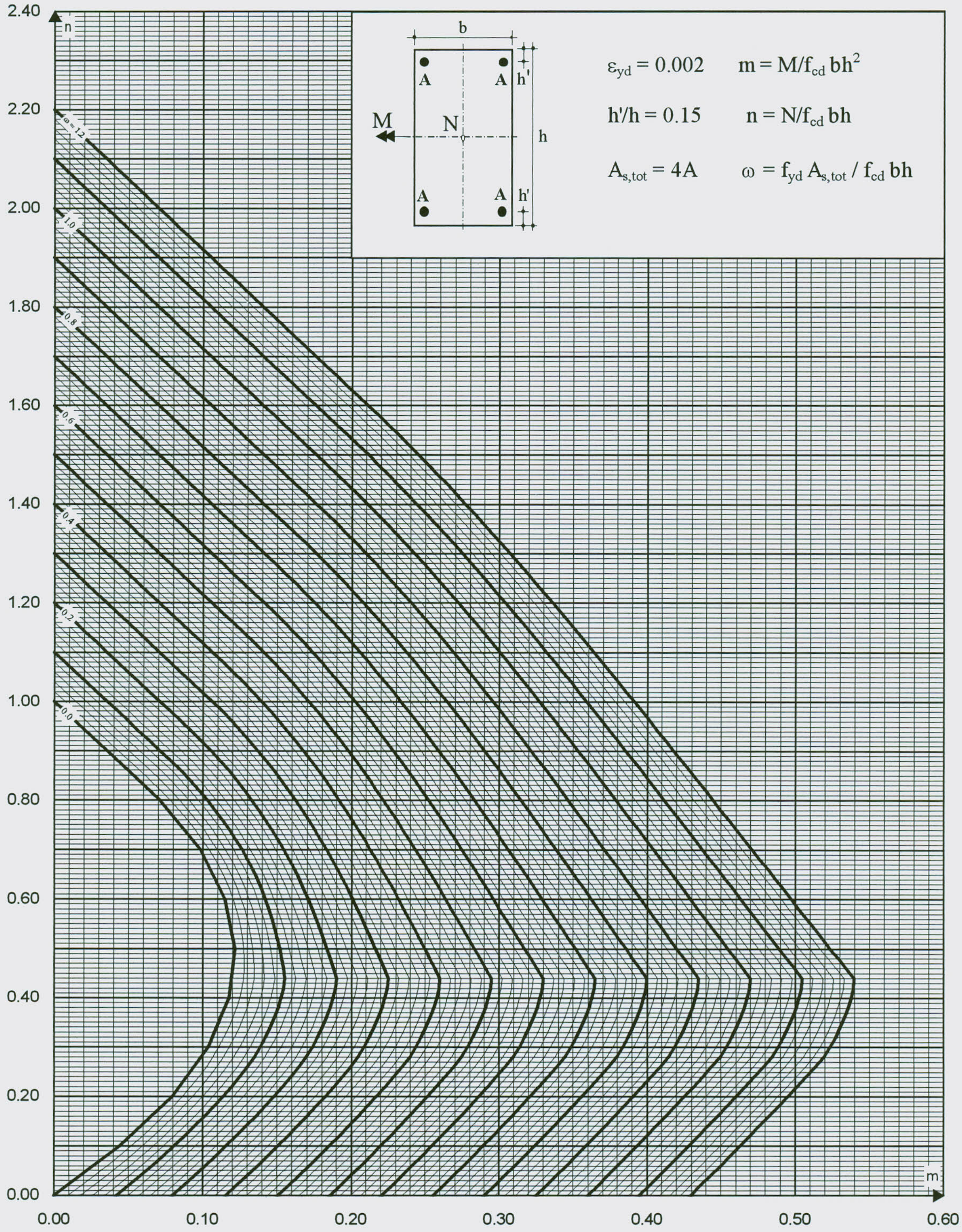


Uniaxial Chart No.3



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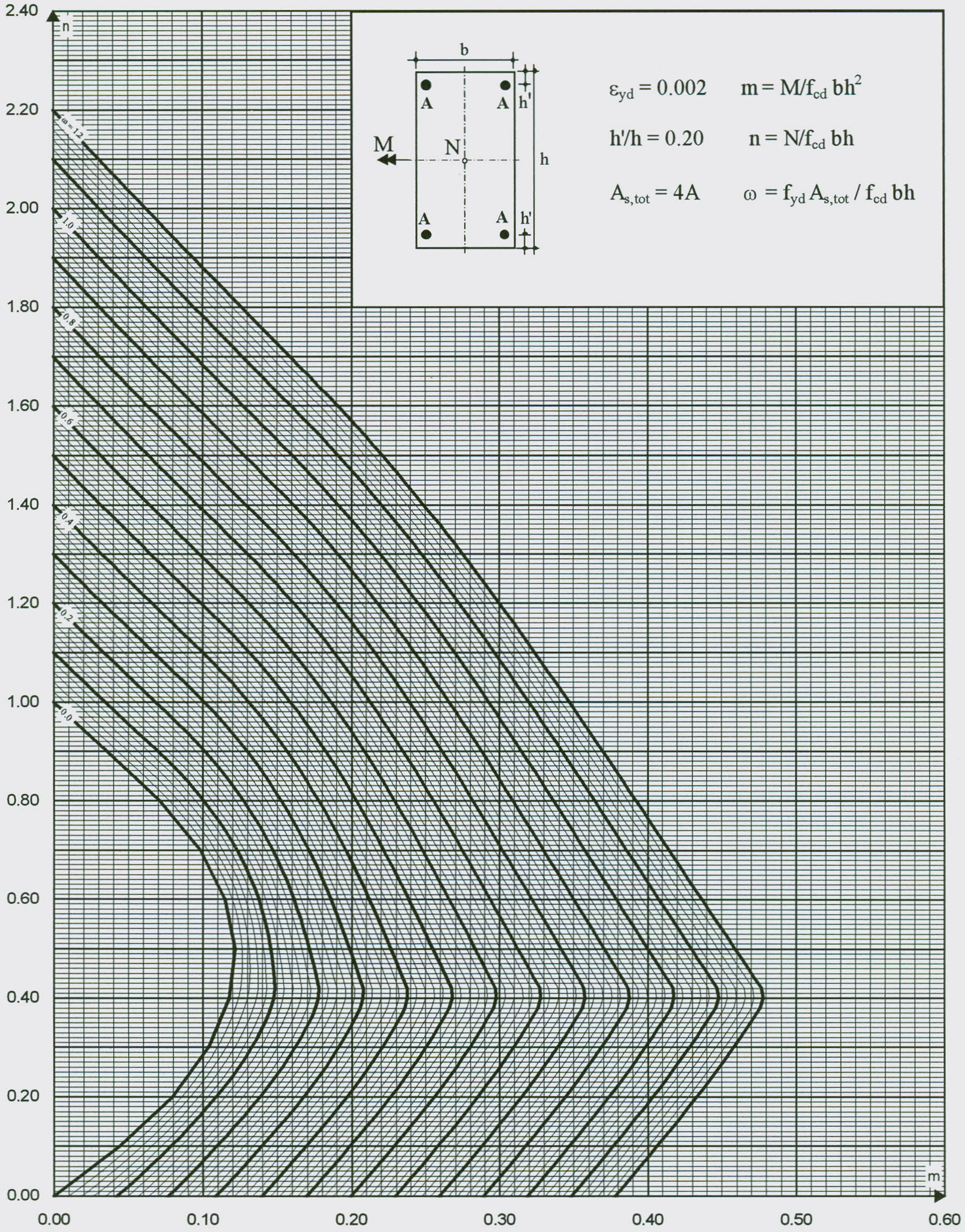

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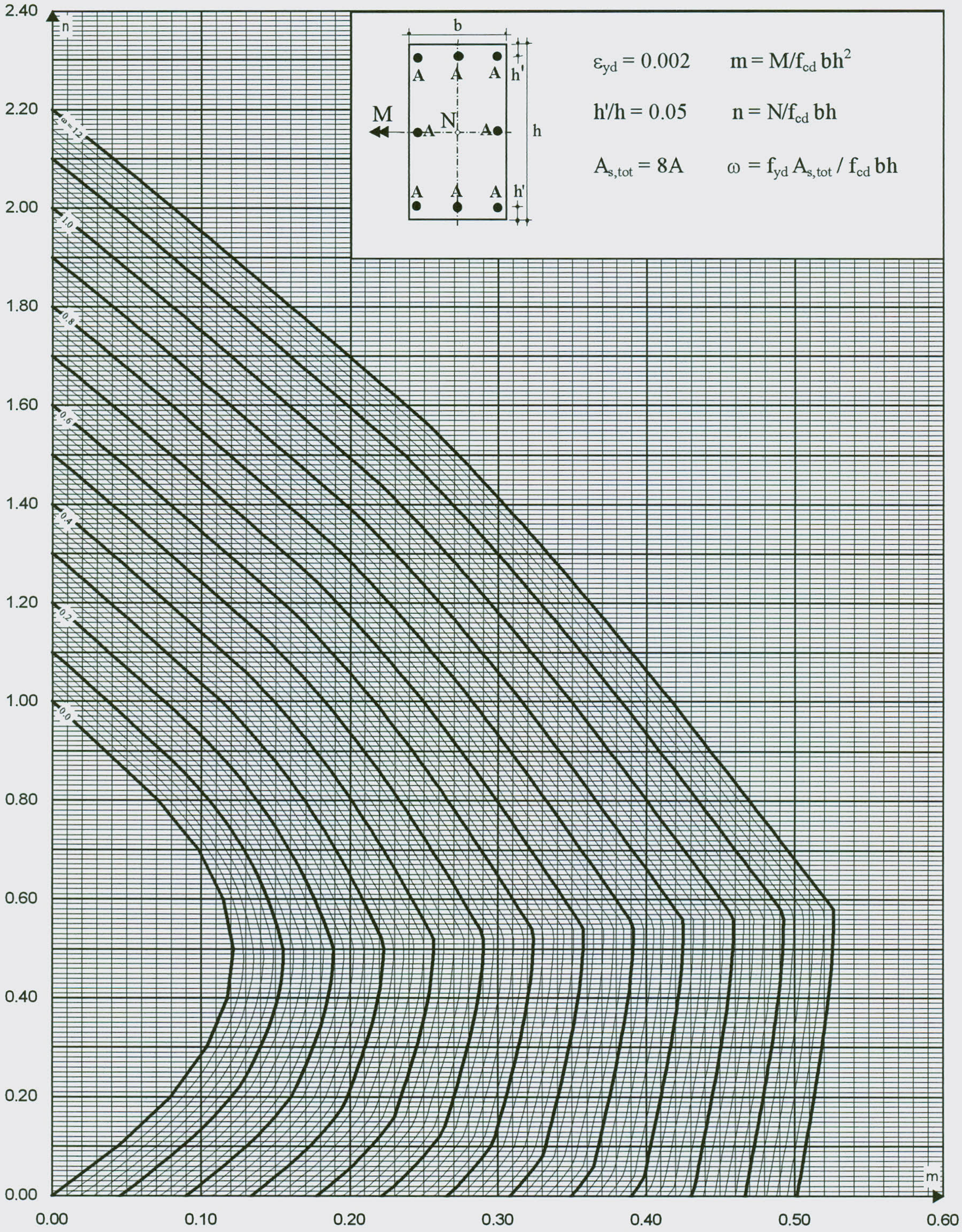

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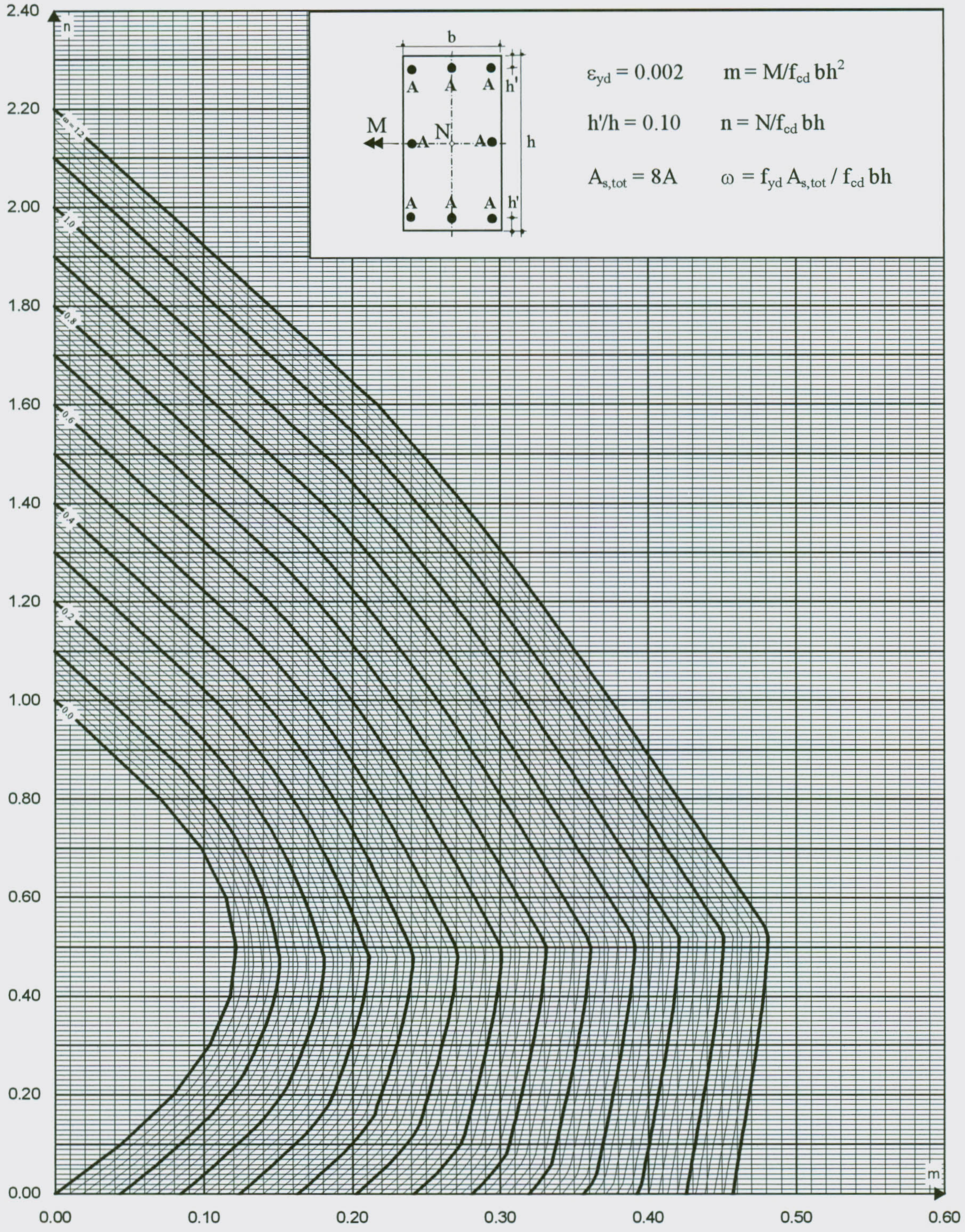

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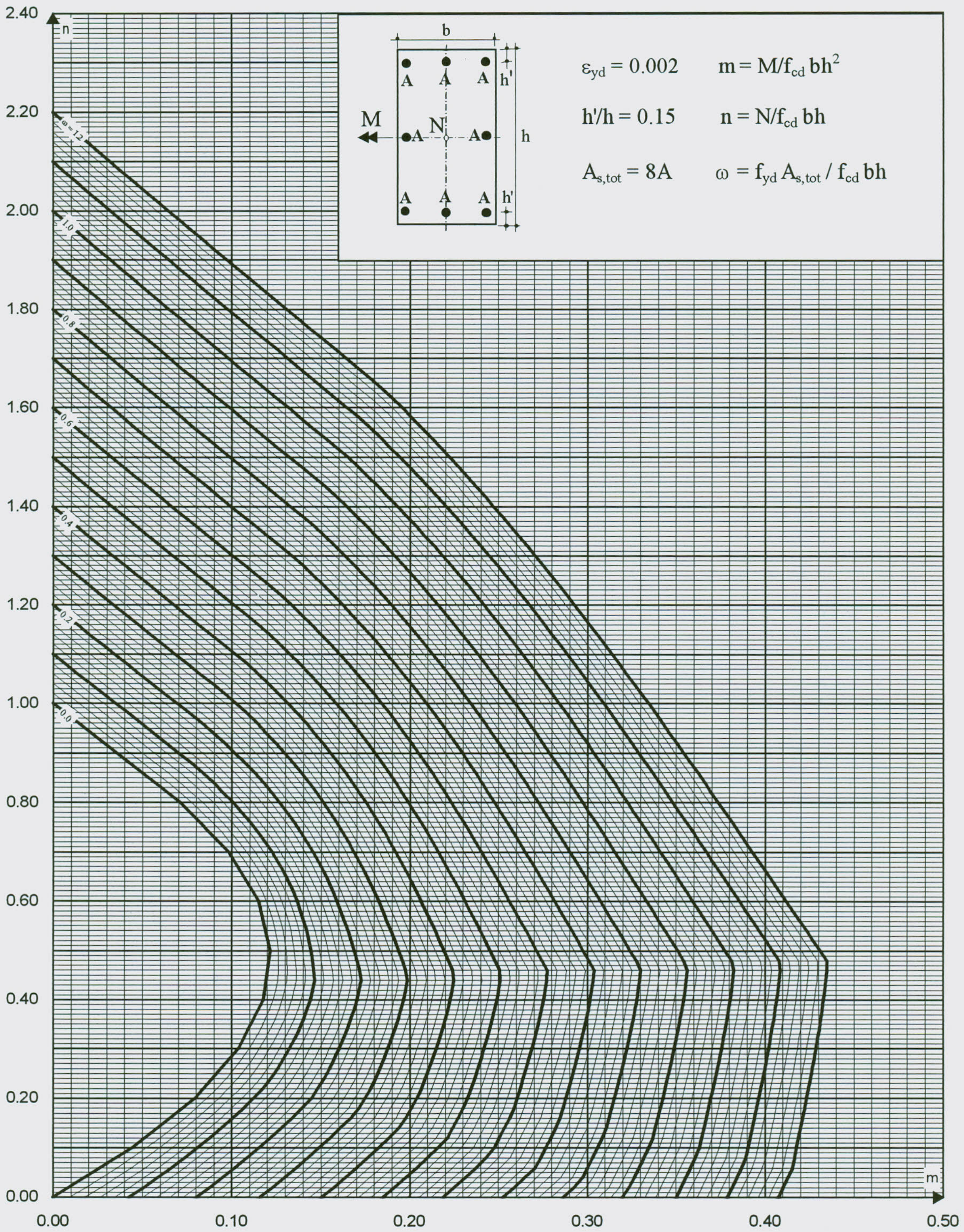

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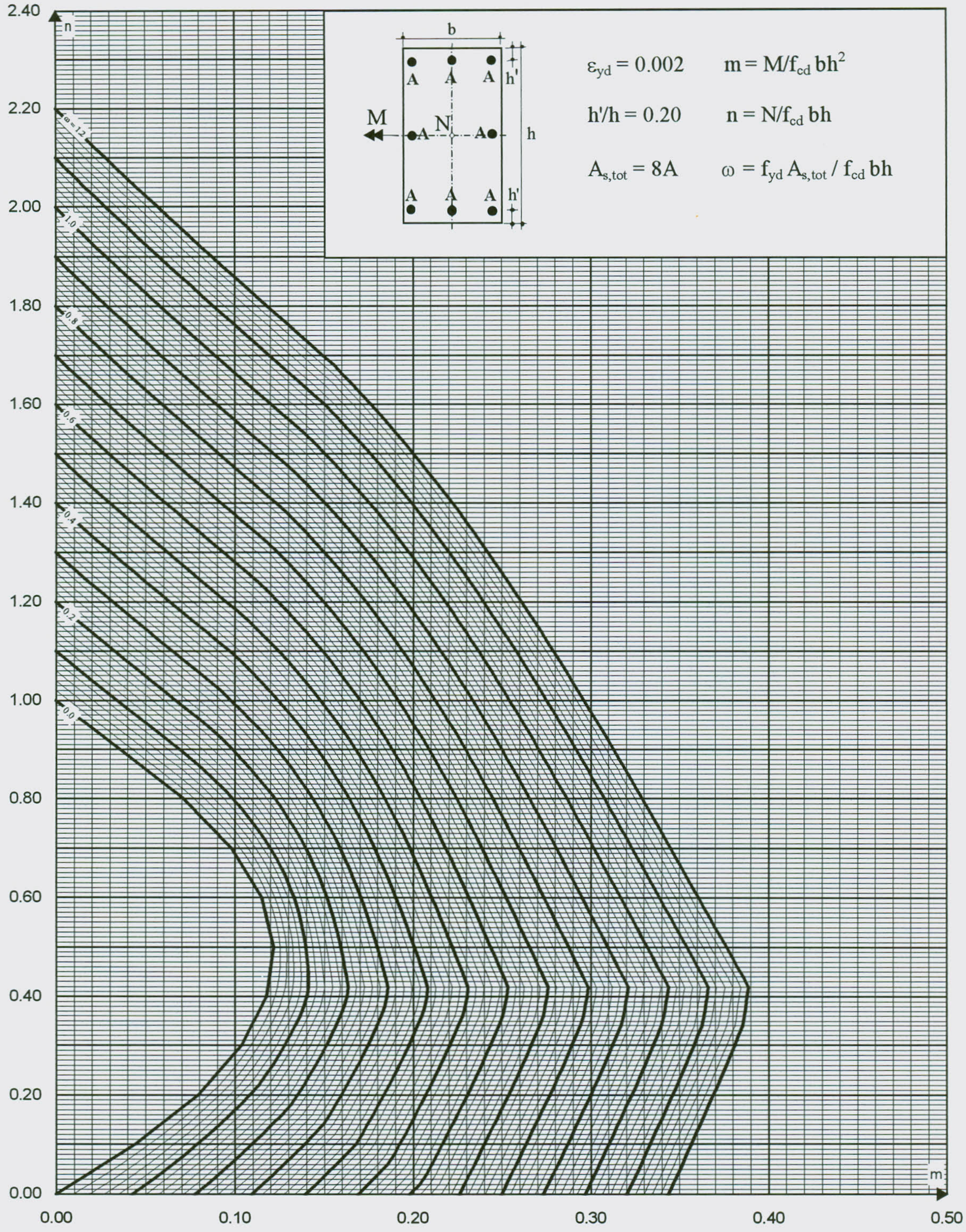
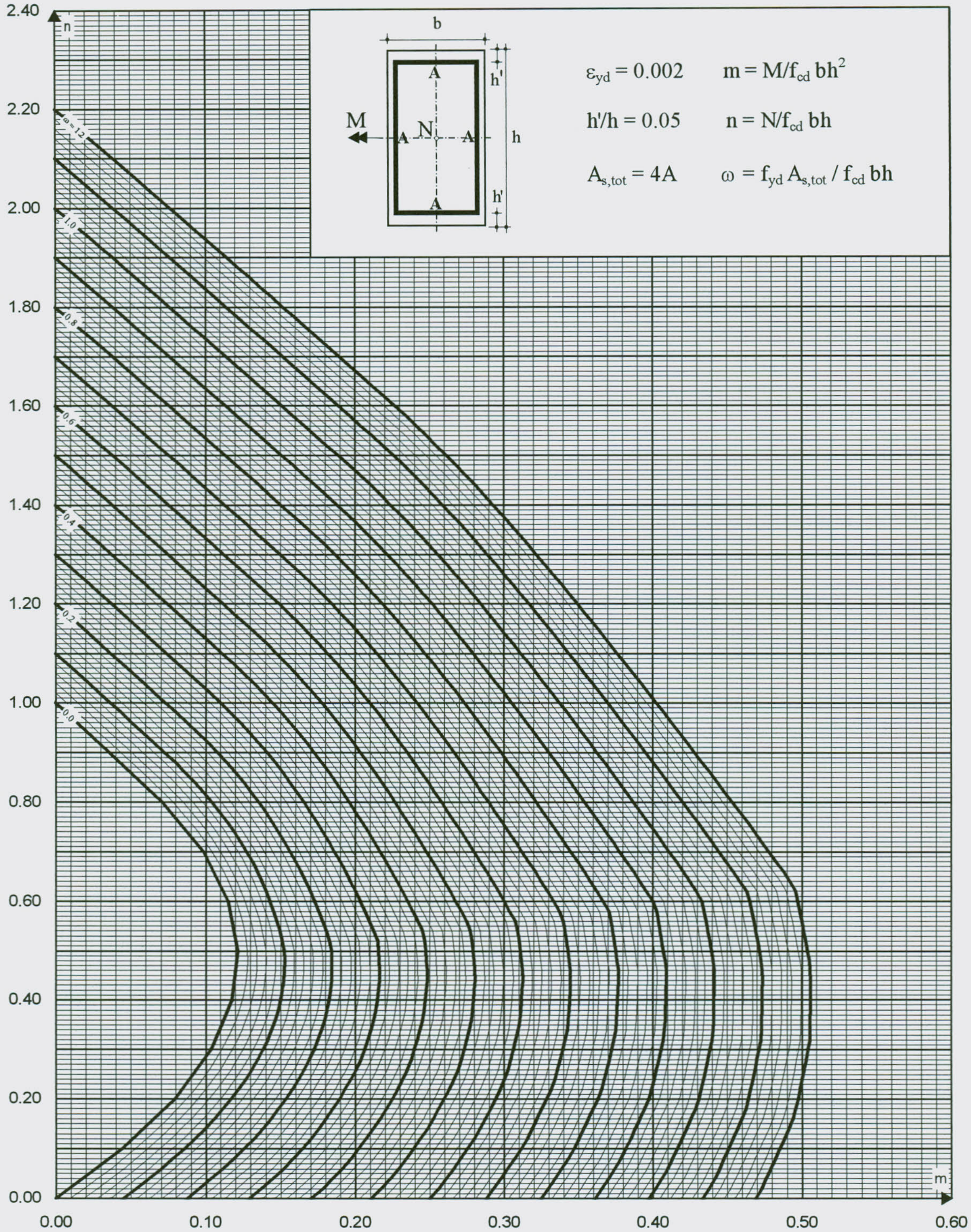


Uniaxial Chart No.9



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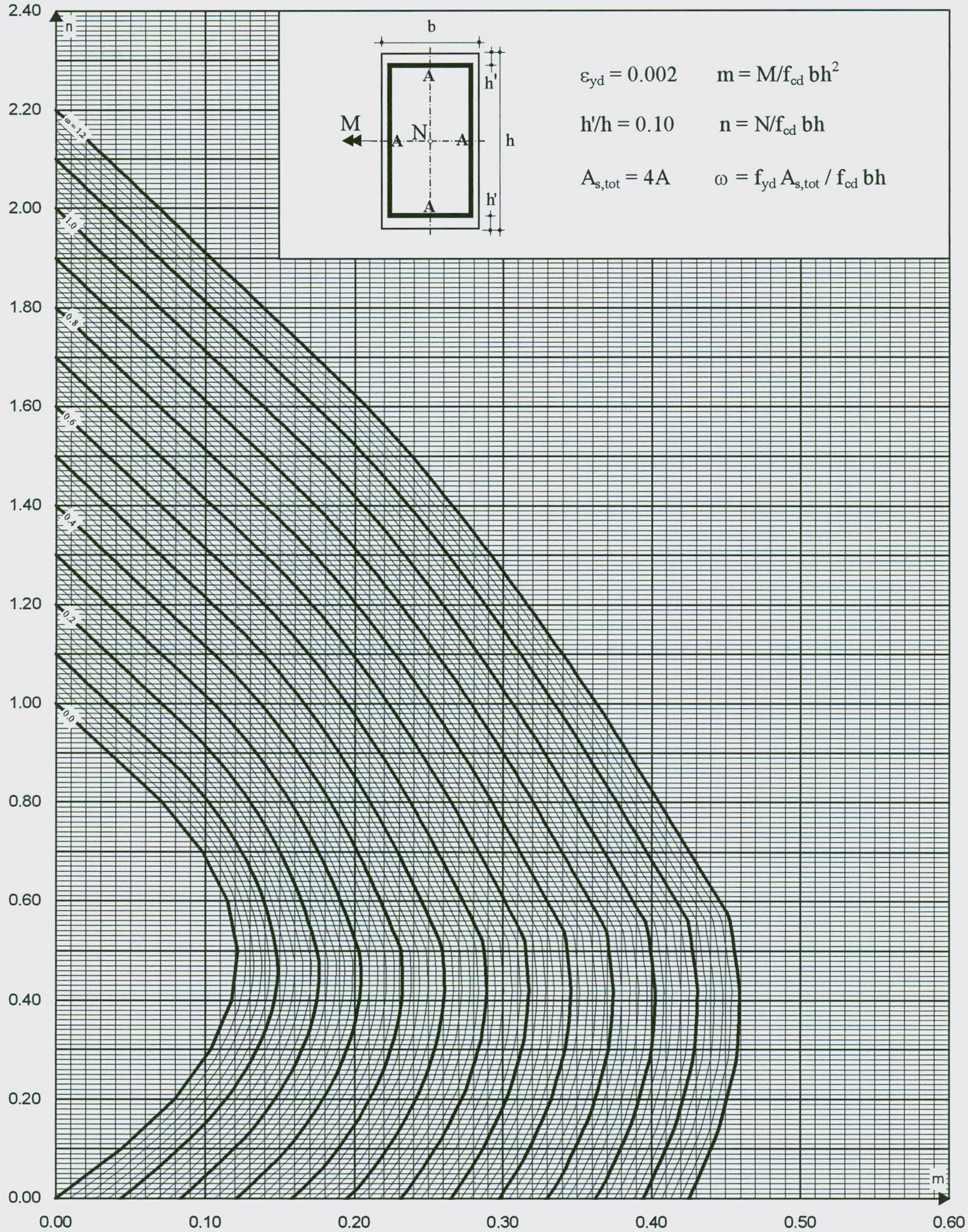
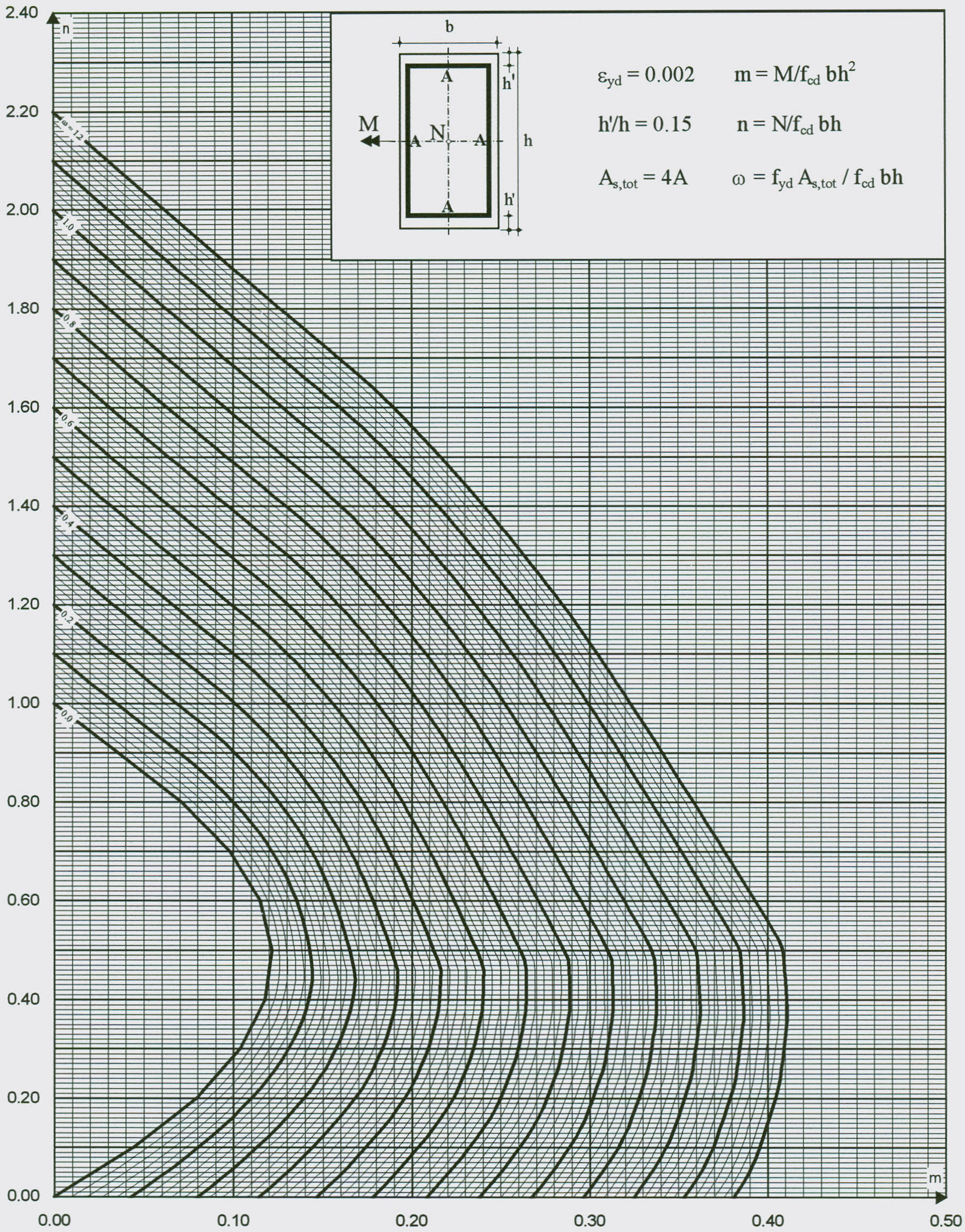


Uniaxial Chart No.11



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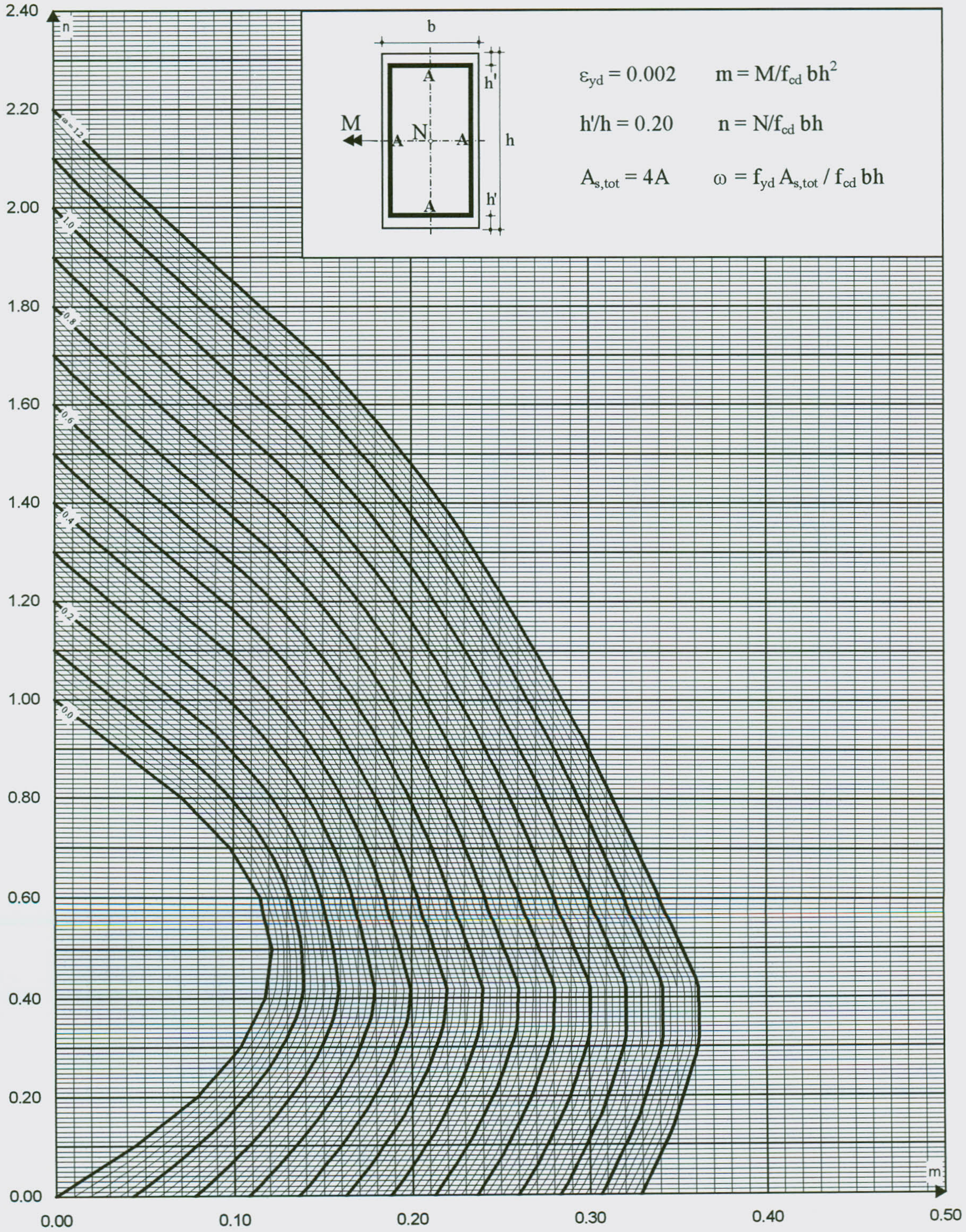
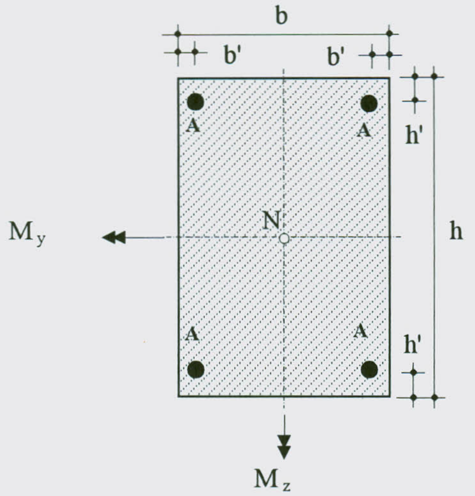


Appendix B

BIAXIAL CHARTS

Biaxial Chart No. 1



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.05$

$A_{s,tot} = 4A$

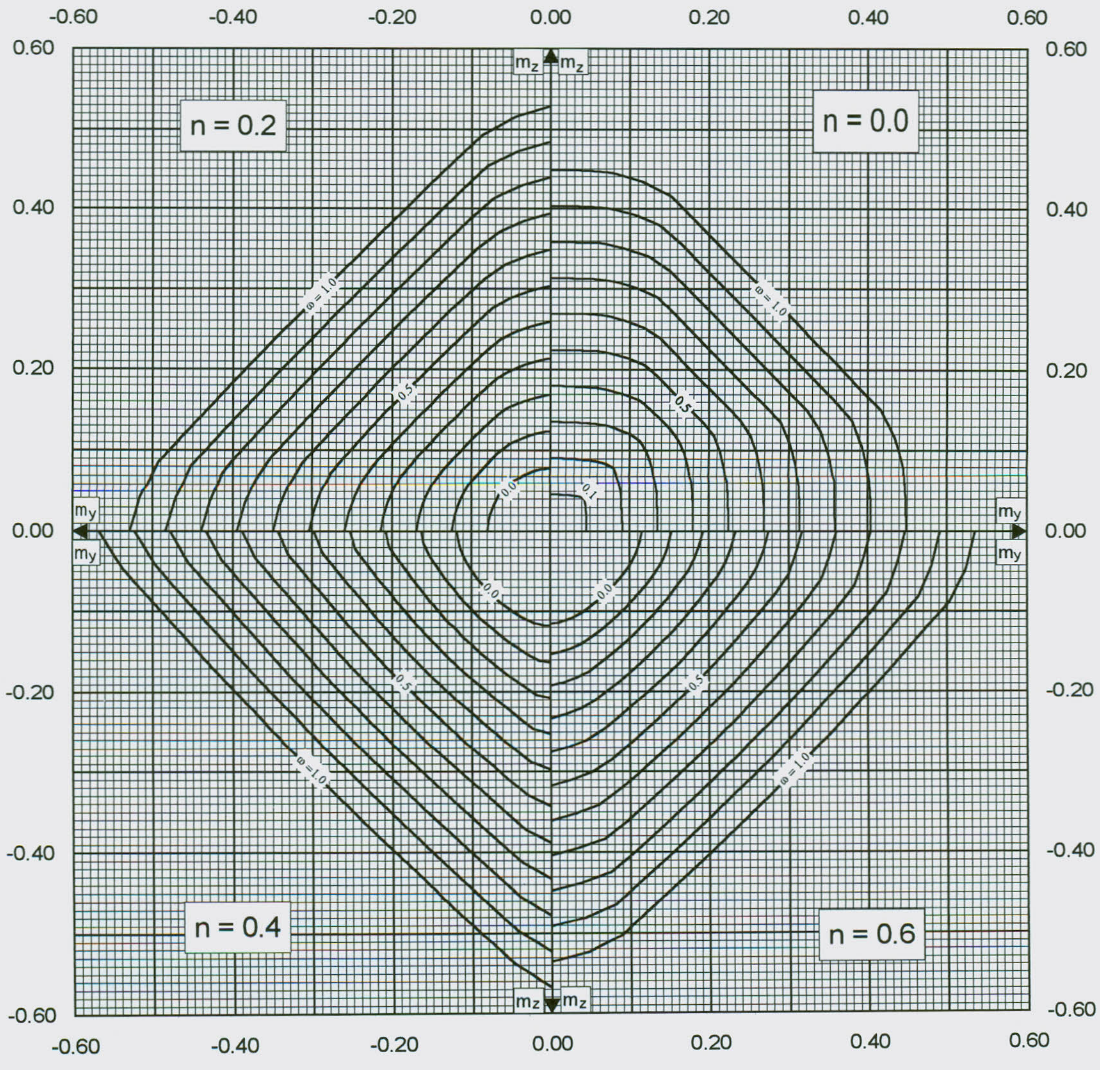
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

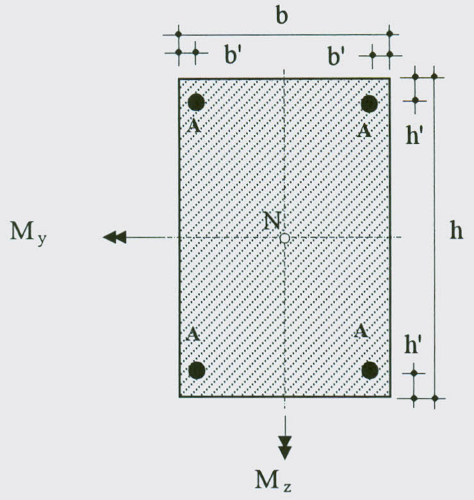
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\ f_{yd} / A_c\, f_{cd}$



Biaxial Chart No. 2



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.05$

$A_{s,tot} = 4A$

$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

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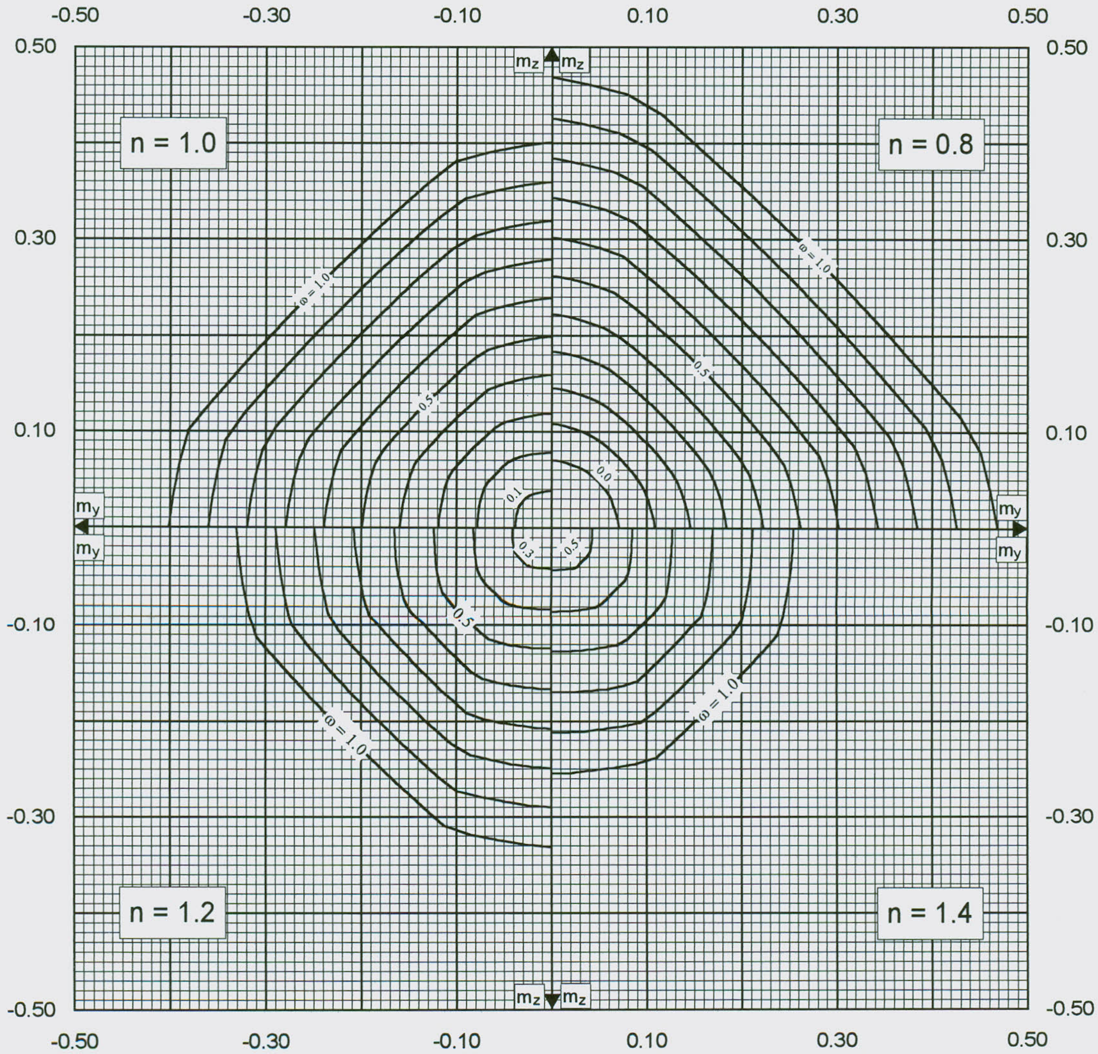
$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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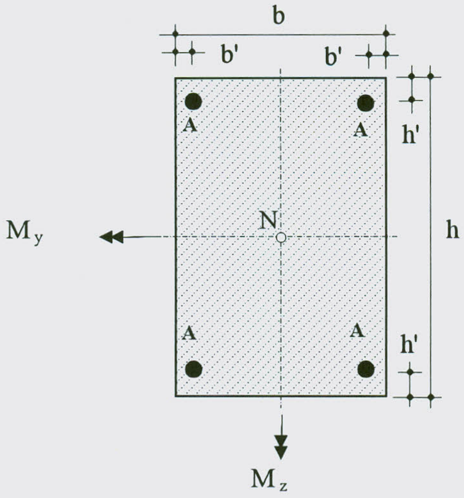


$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

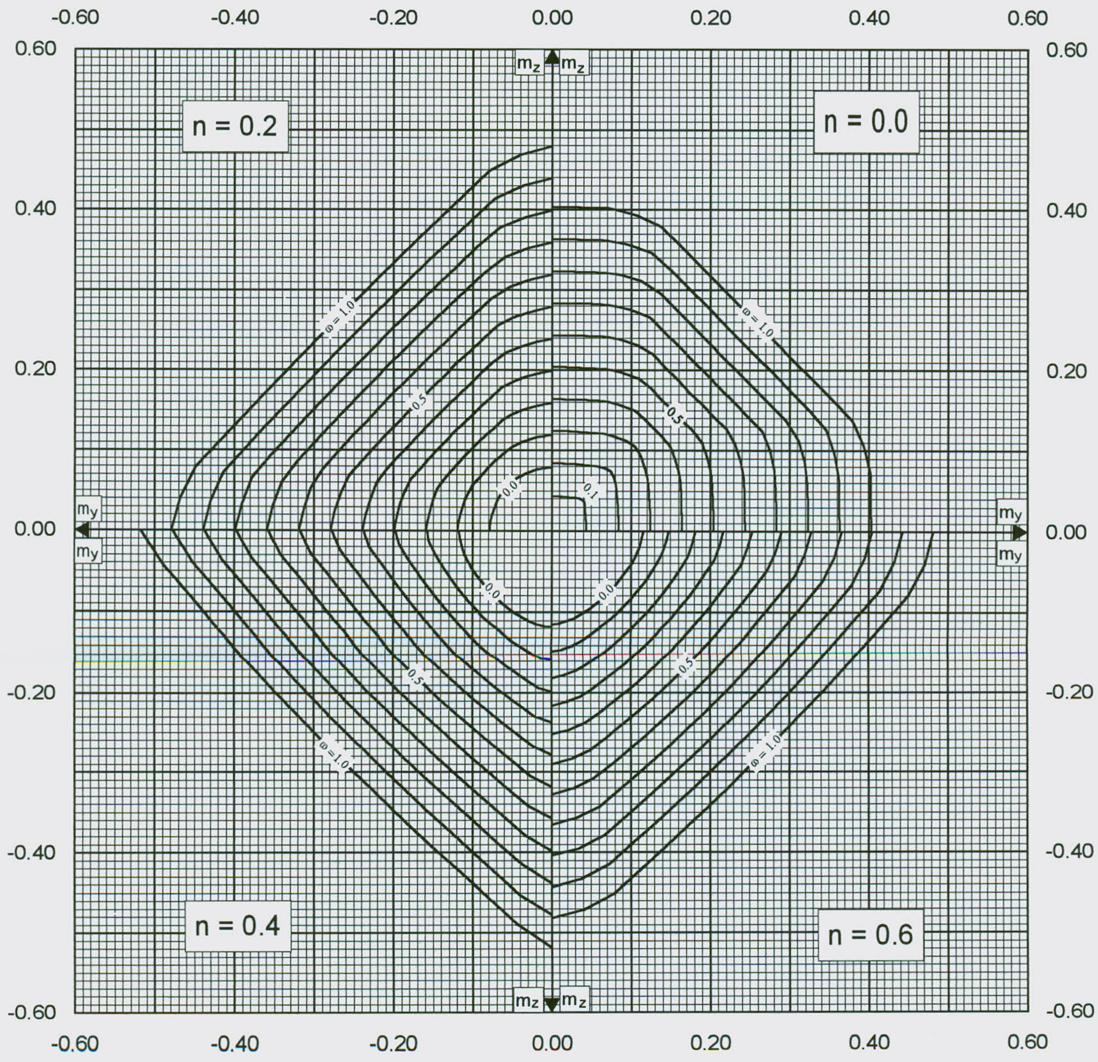
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

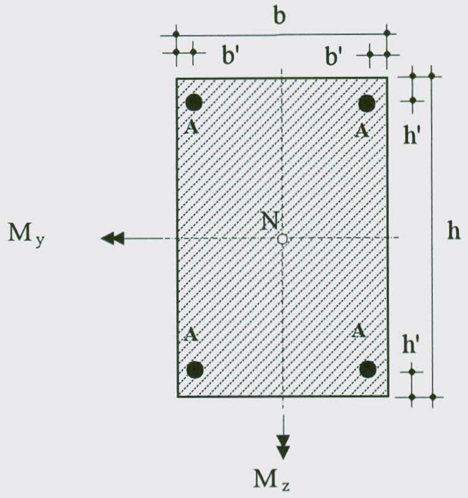
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\ f_{yd} / A_c\, f_{cd}$



Biaxial Chart No. 4

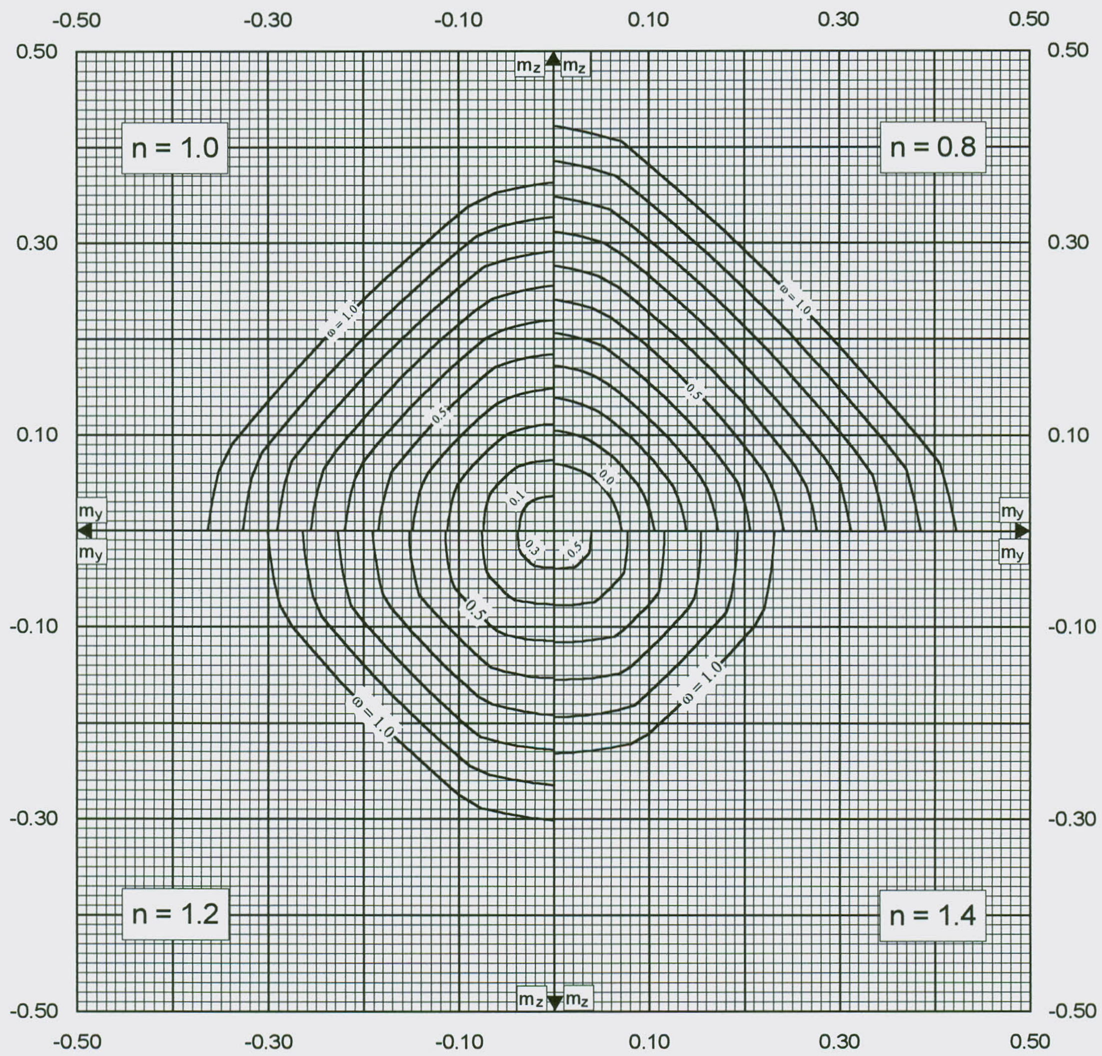


$\varepsilon_{yd} = 0.002$ $m_y = M_y / A_c\, h\, f_{cd}$

$h'/h = b'/b = 0.10$ $m_z = M_z / A_c\, b\, f_{cd}$

$A_{s,tot} = 4A$ $n = N / A_c\, f_{cd}$

$A_c = bh$ $\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$



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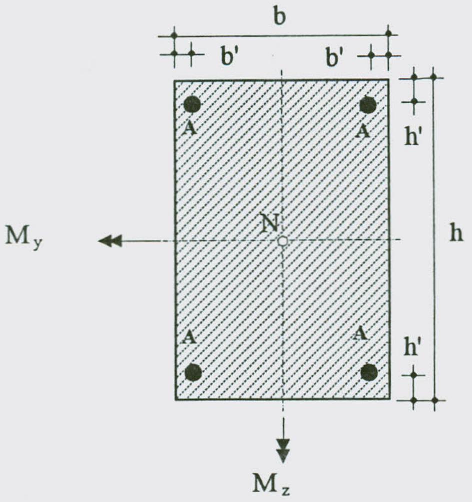

$\varepsilon_{yd} = 0.002$

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$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

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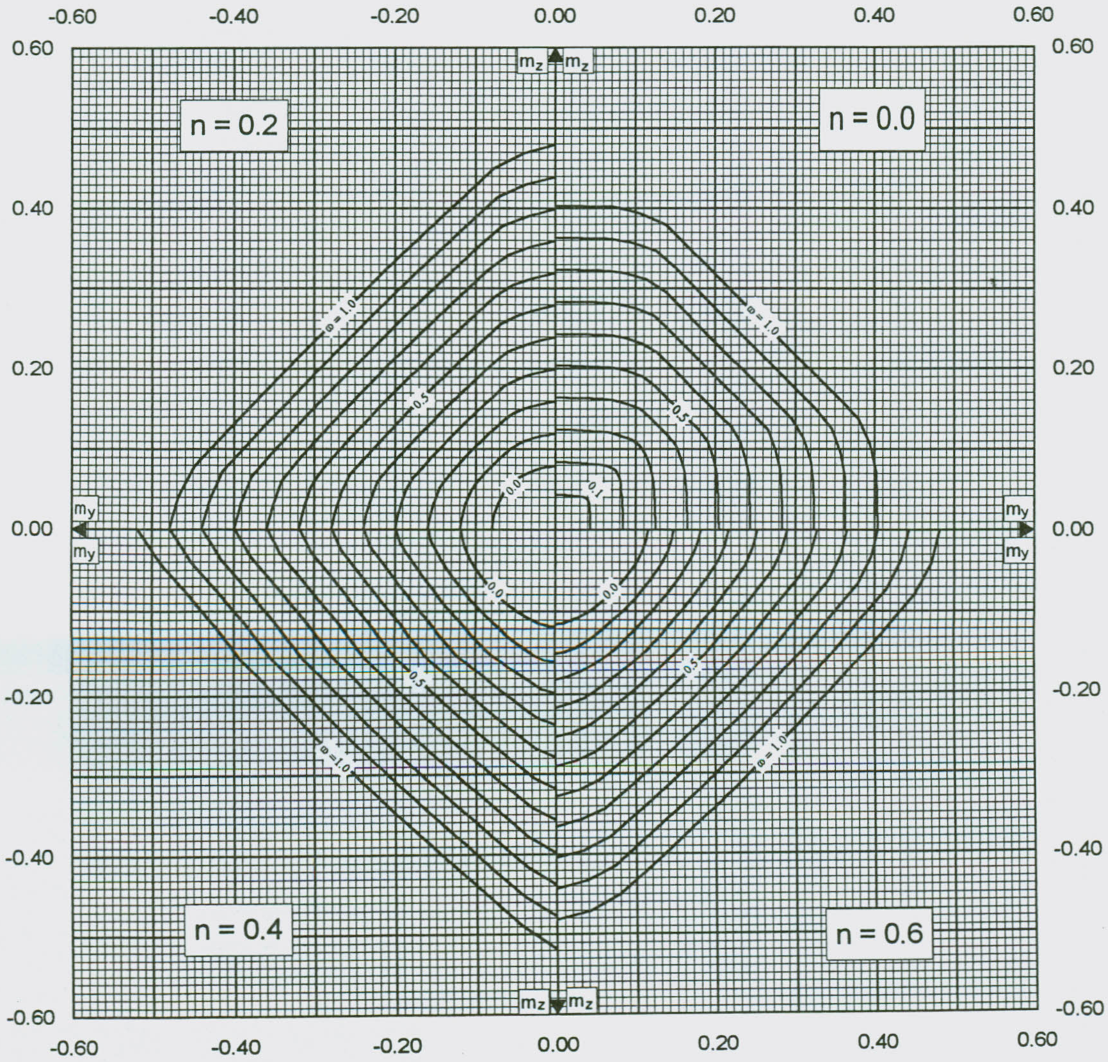
$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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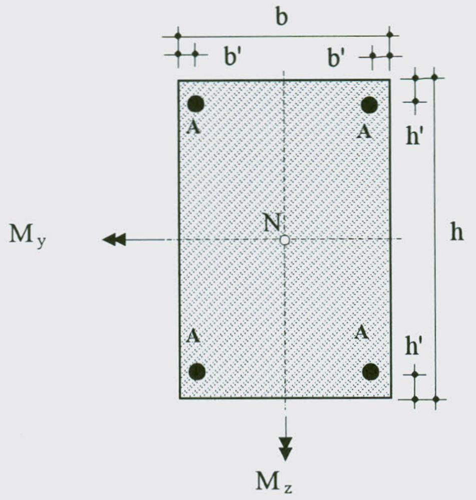


$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.15$

$A_{s,tot} = 4A$

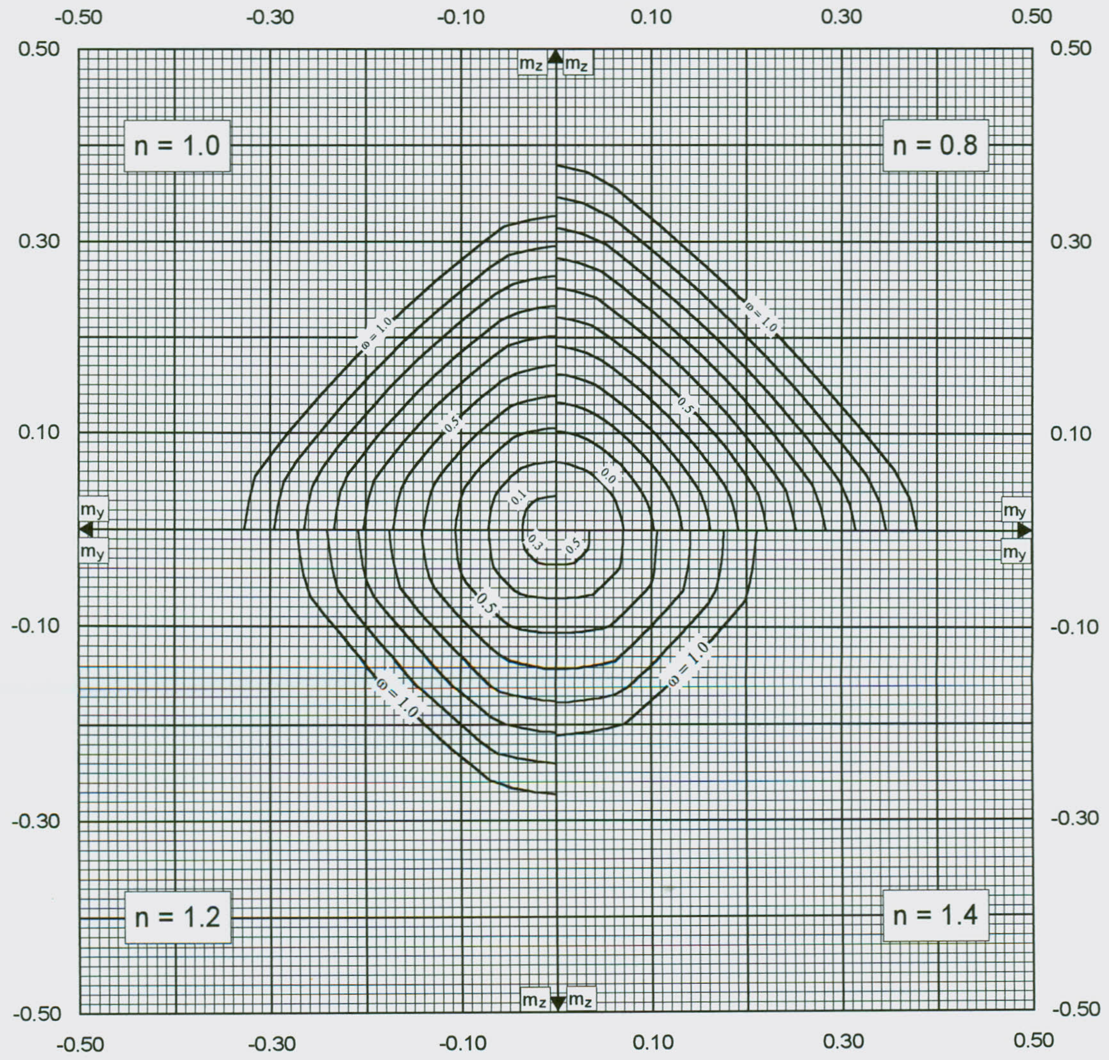
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$



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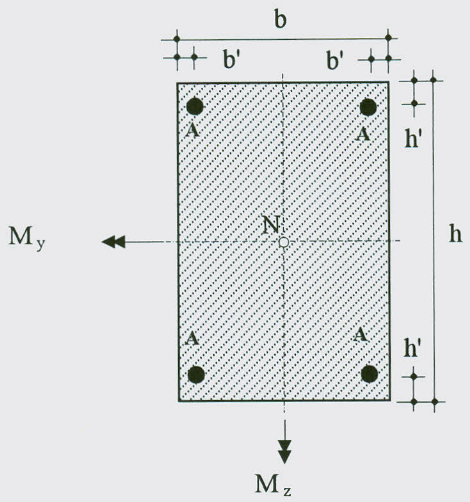

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$A_{s,tot} = 4A$

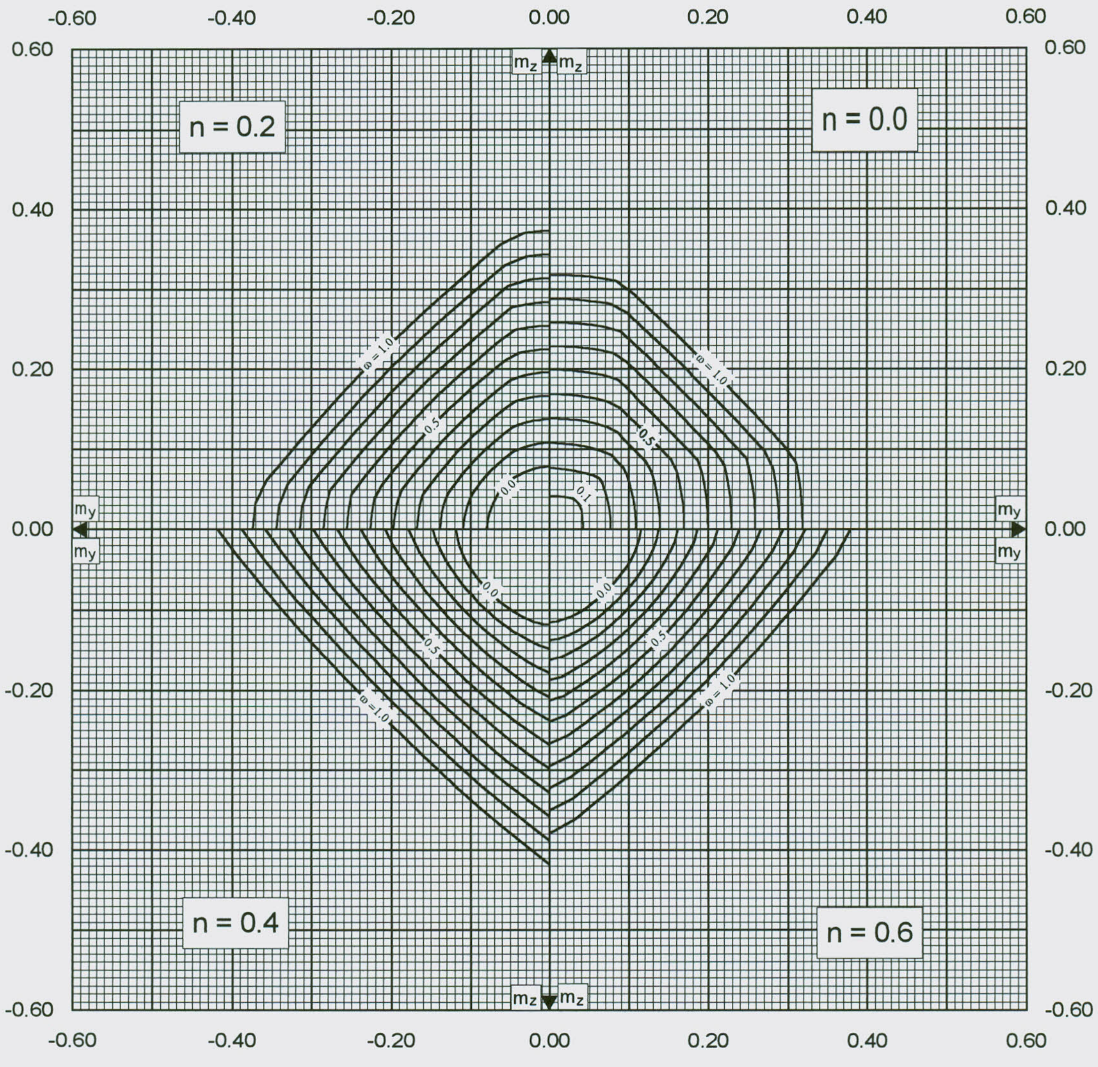
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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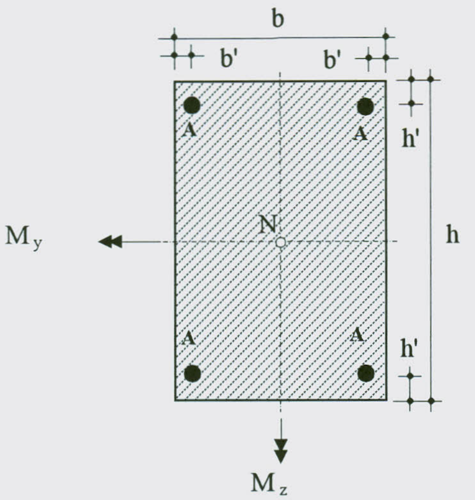

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$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

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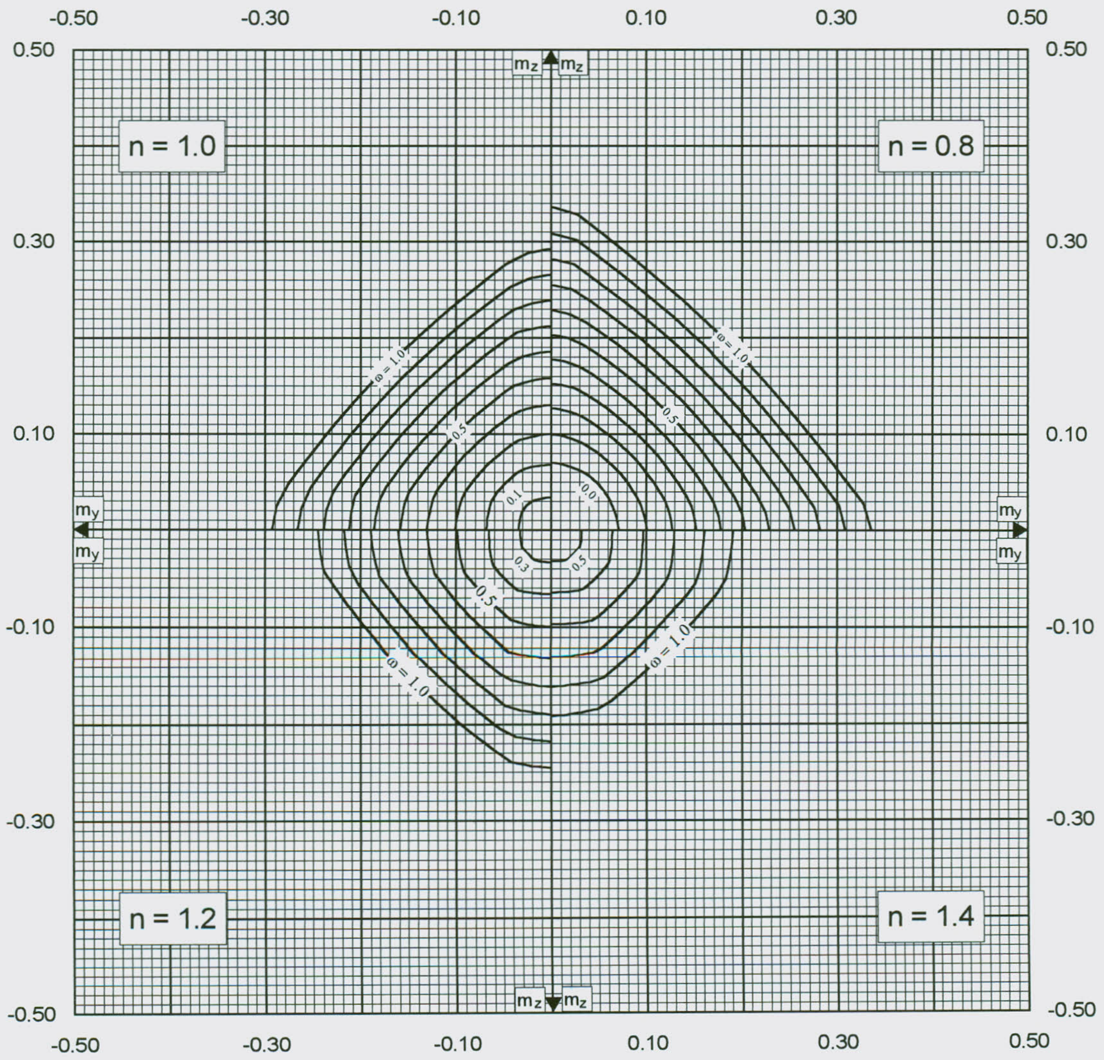
$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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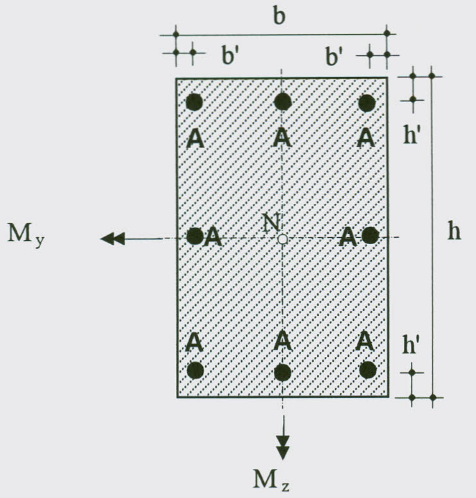

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$A_{s,tot} = 8A$

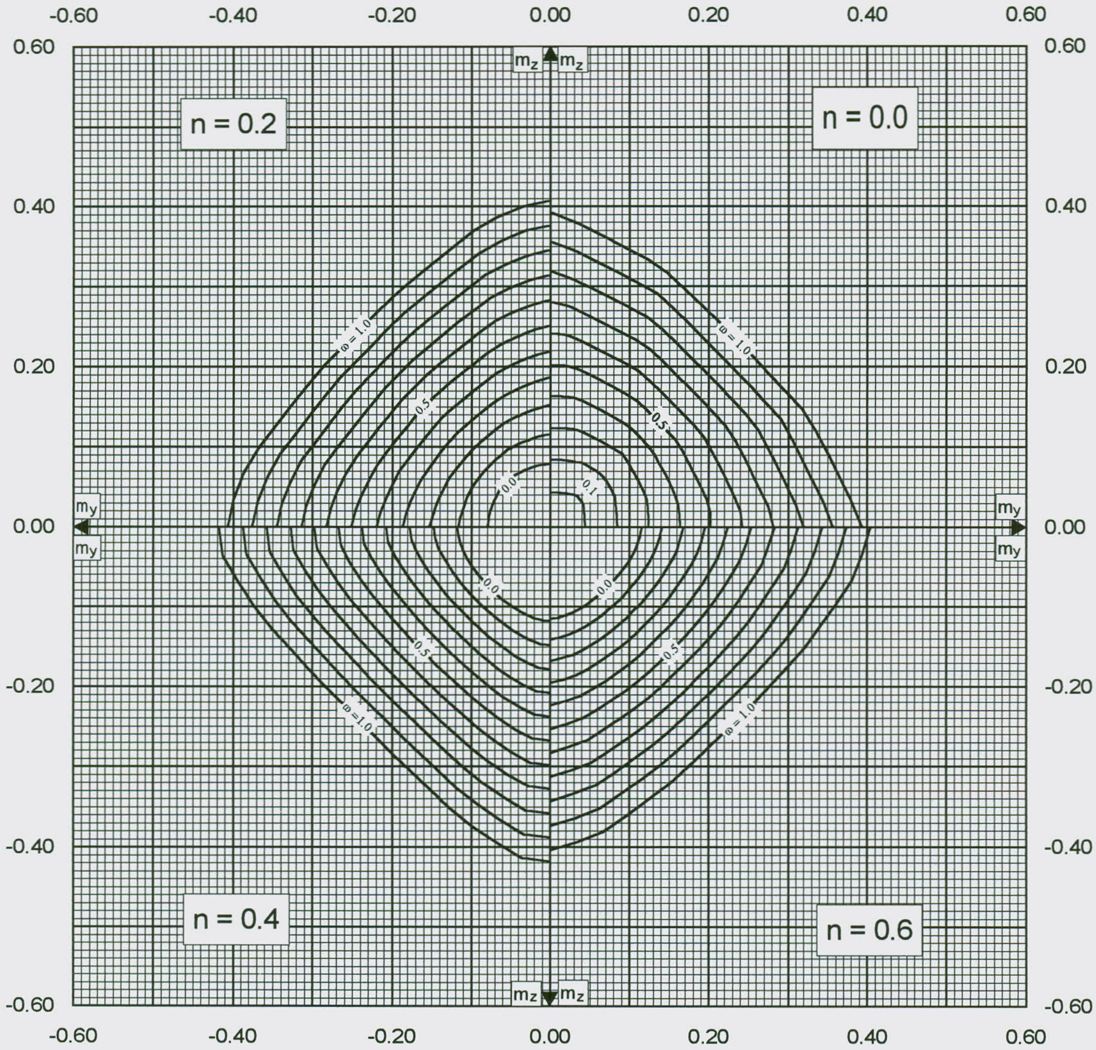
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\ f_{yd} / A_c\, f_{cd}$



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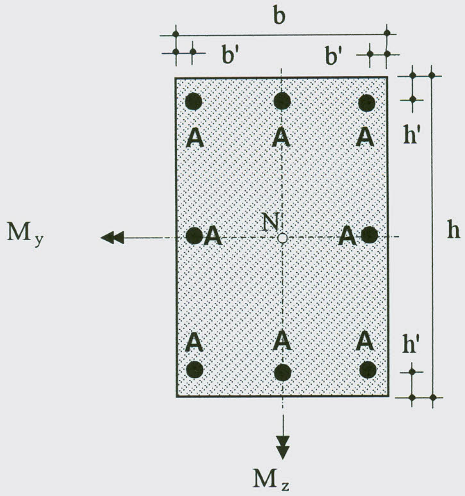


$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 8A$

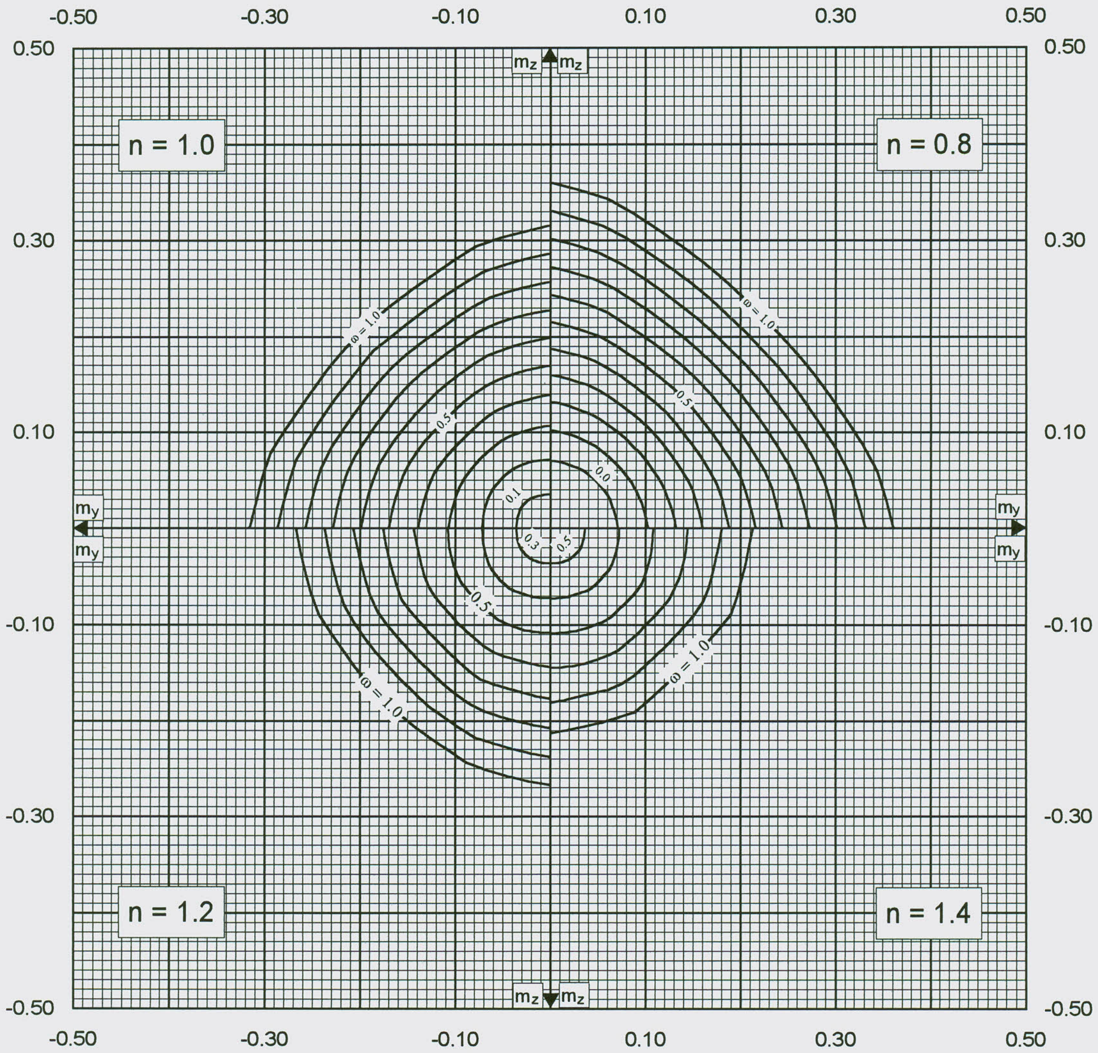
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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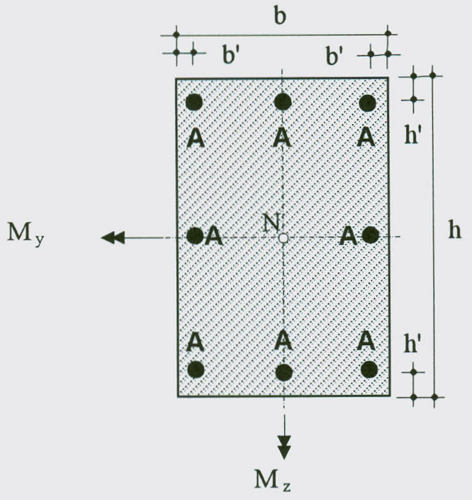


$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.20$

$A_{s,tot} = 8A$

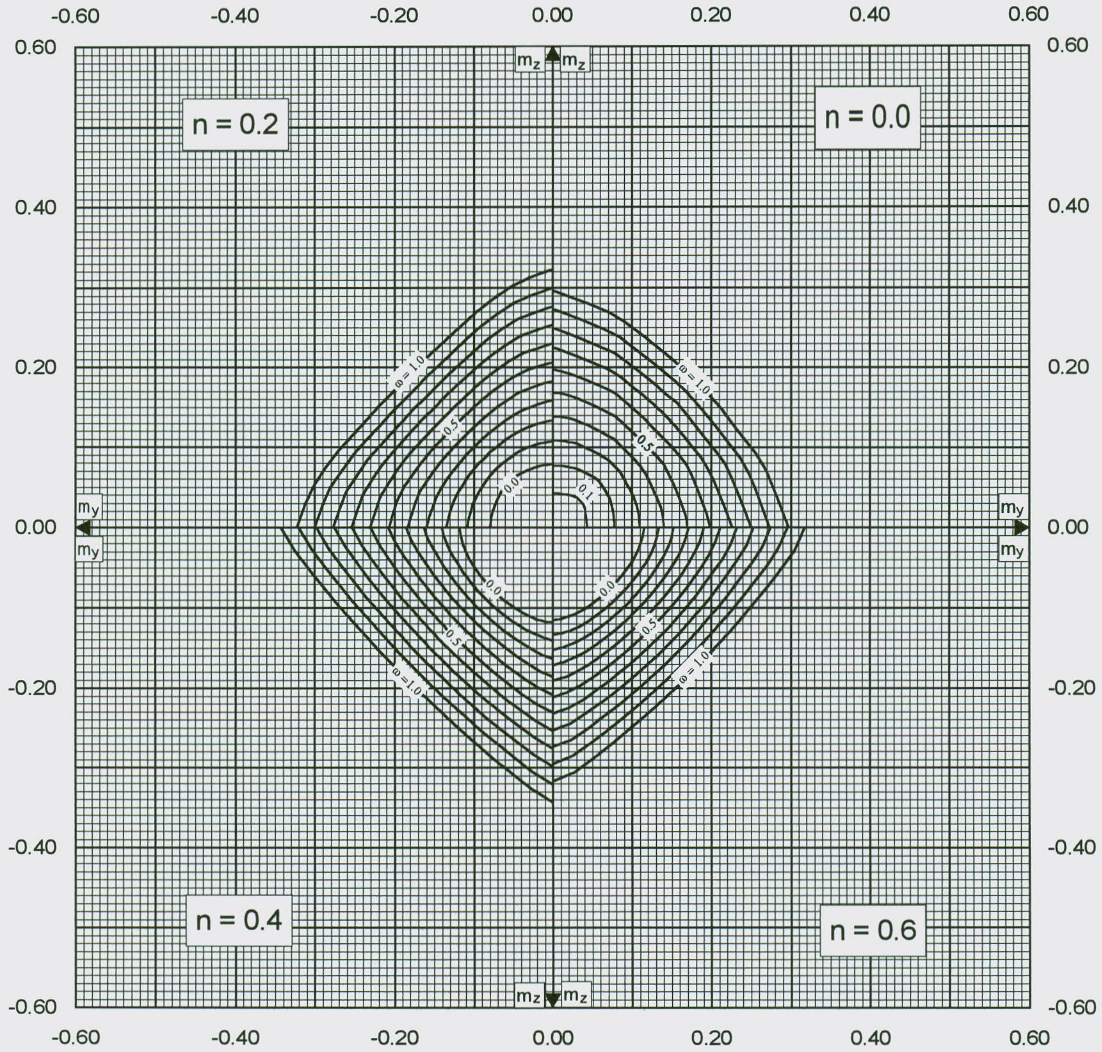
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



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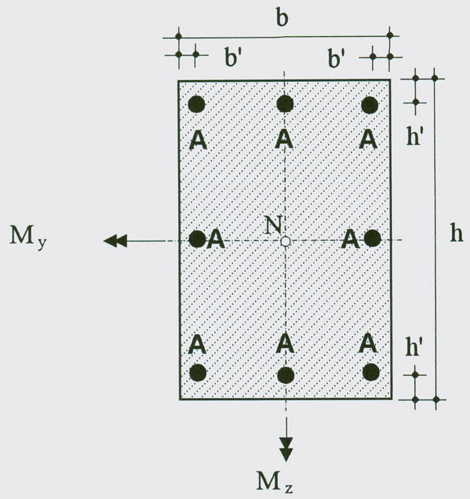

$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.20$

$A_{s,tot} = 4A$

$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

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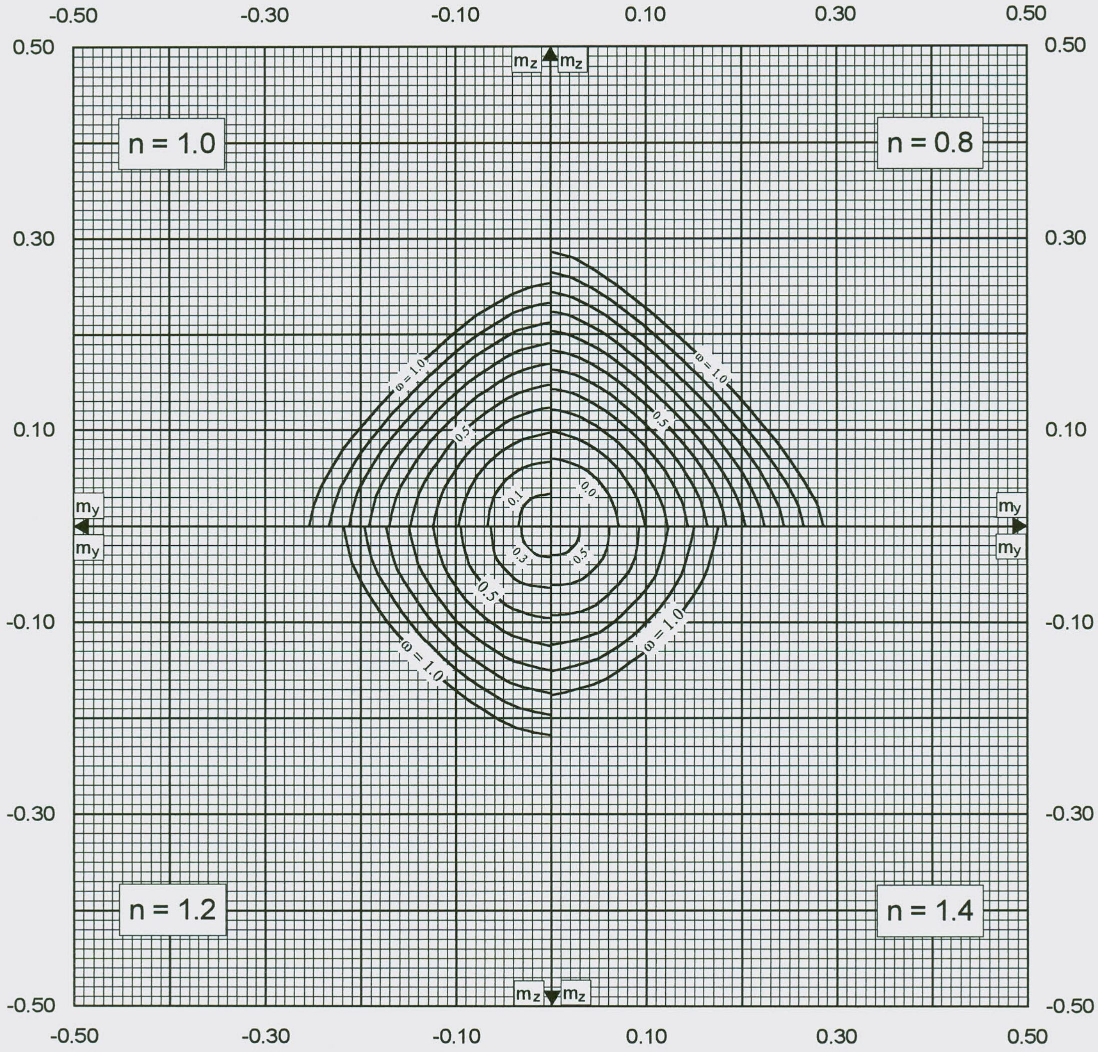
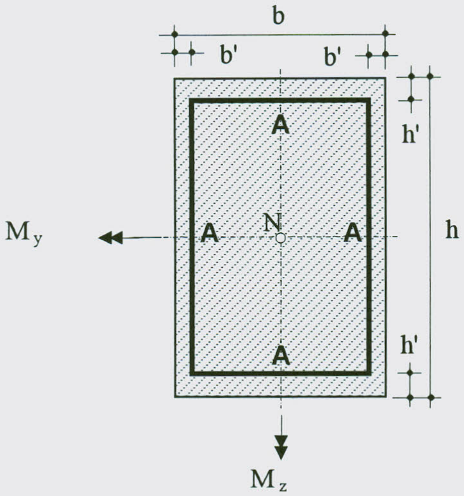


Biaxial Chart No. 13



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.05$

$A_{s,tot} = 4A$

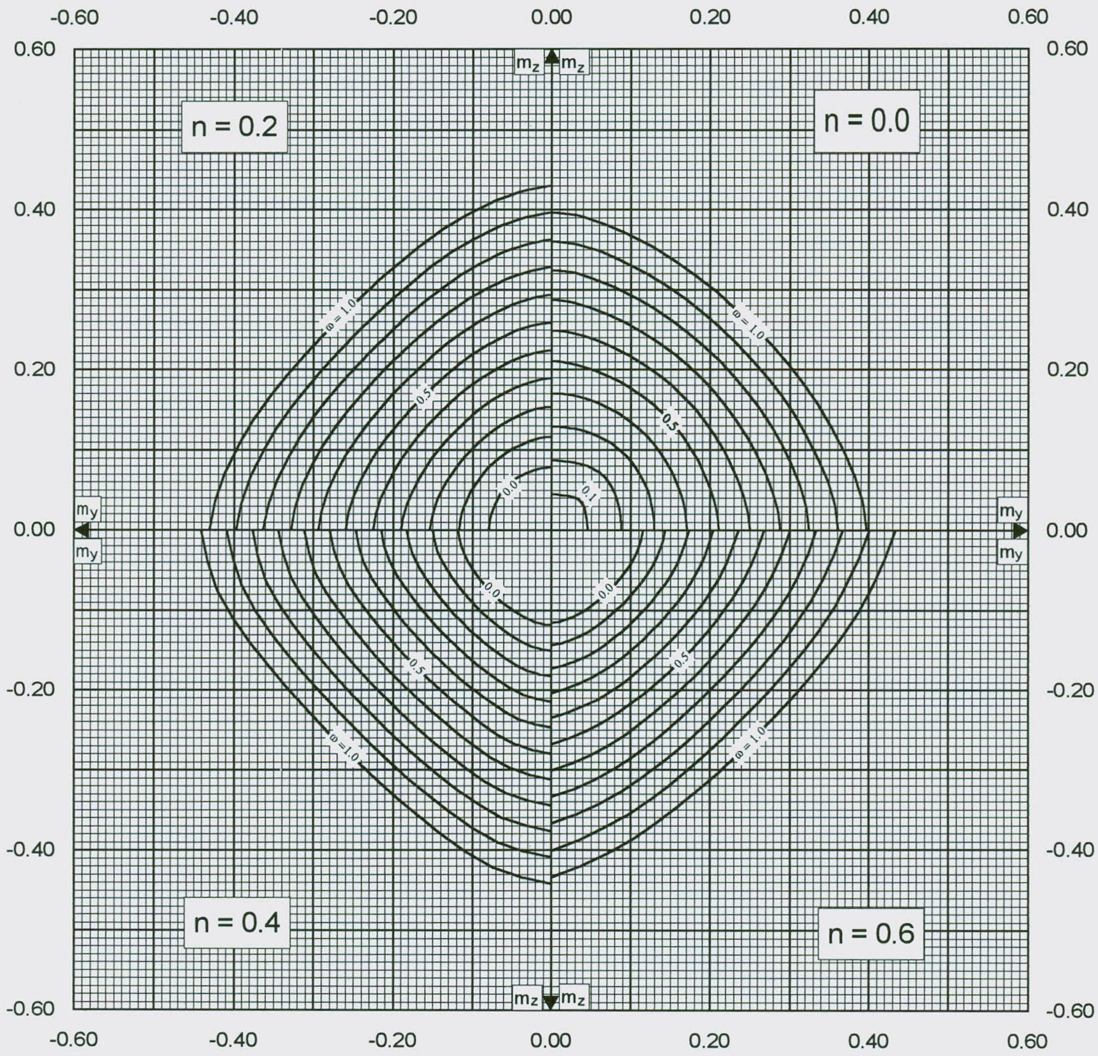
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

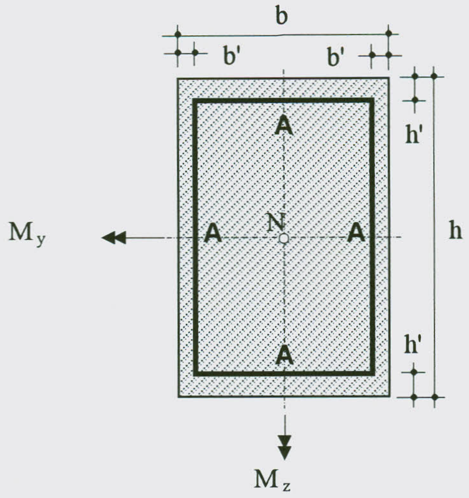
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.14



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.05$

$A_{s,tot} = 4A$

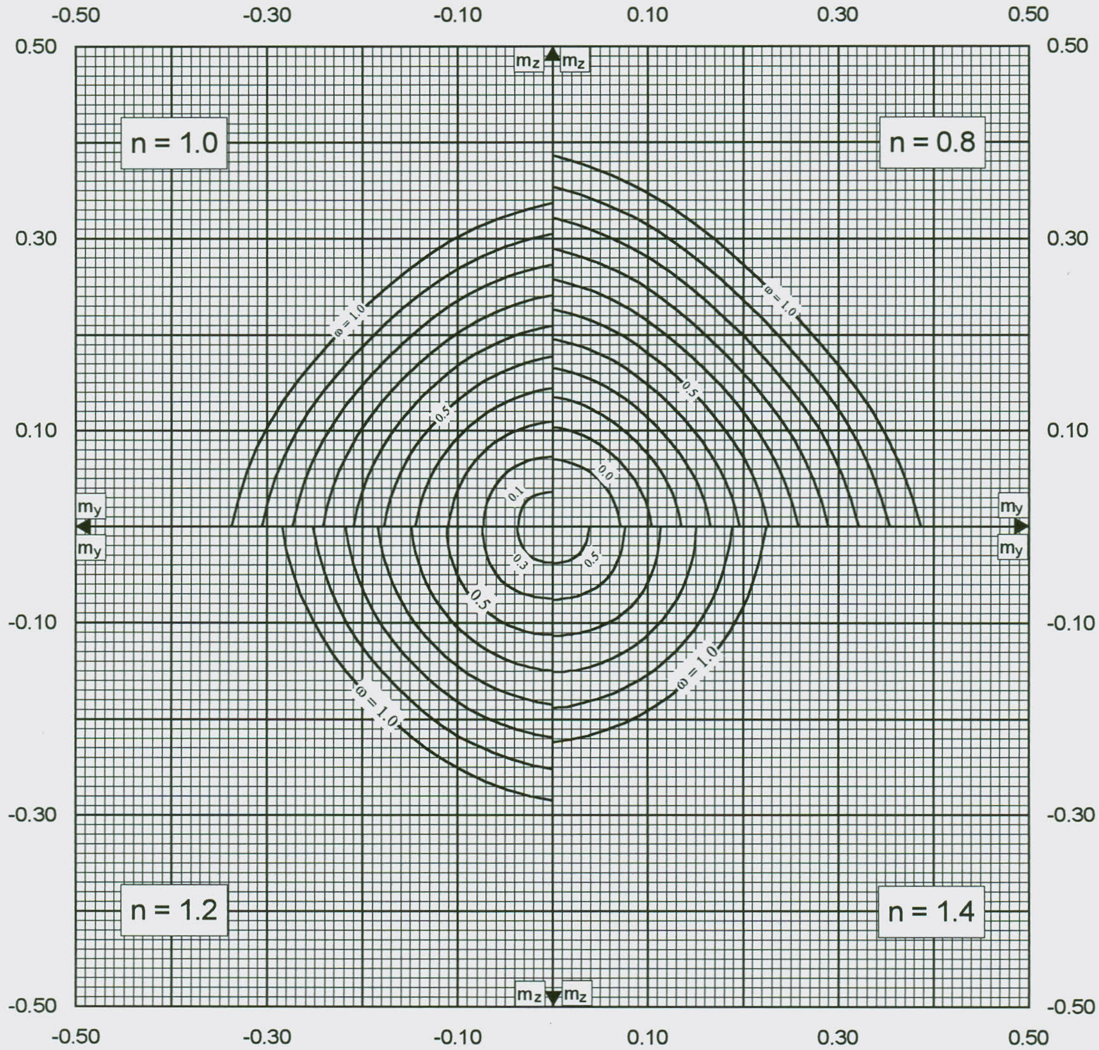
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

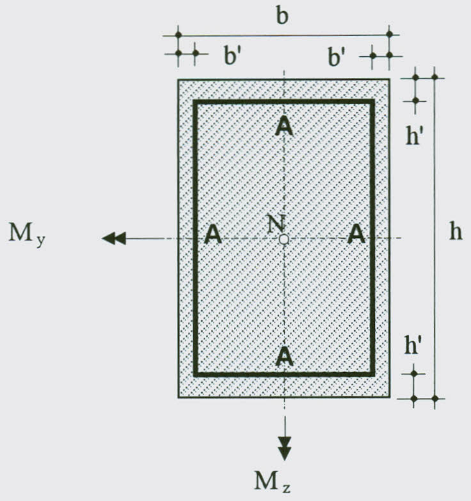
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.15

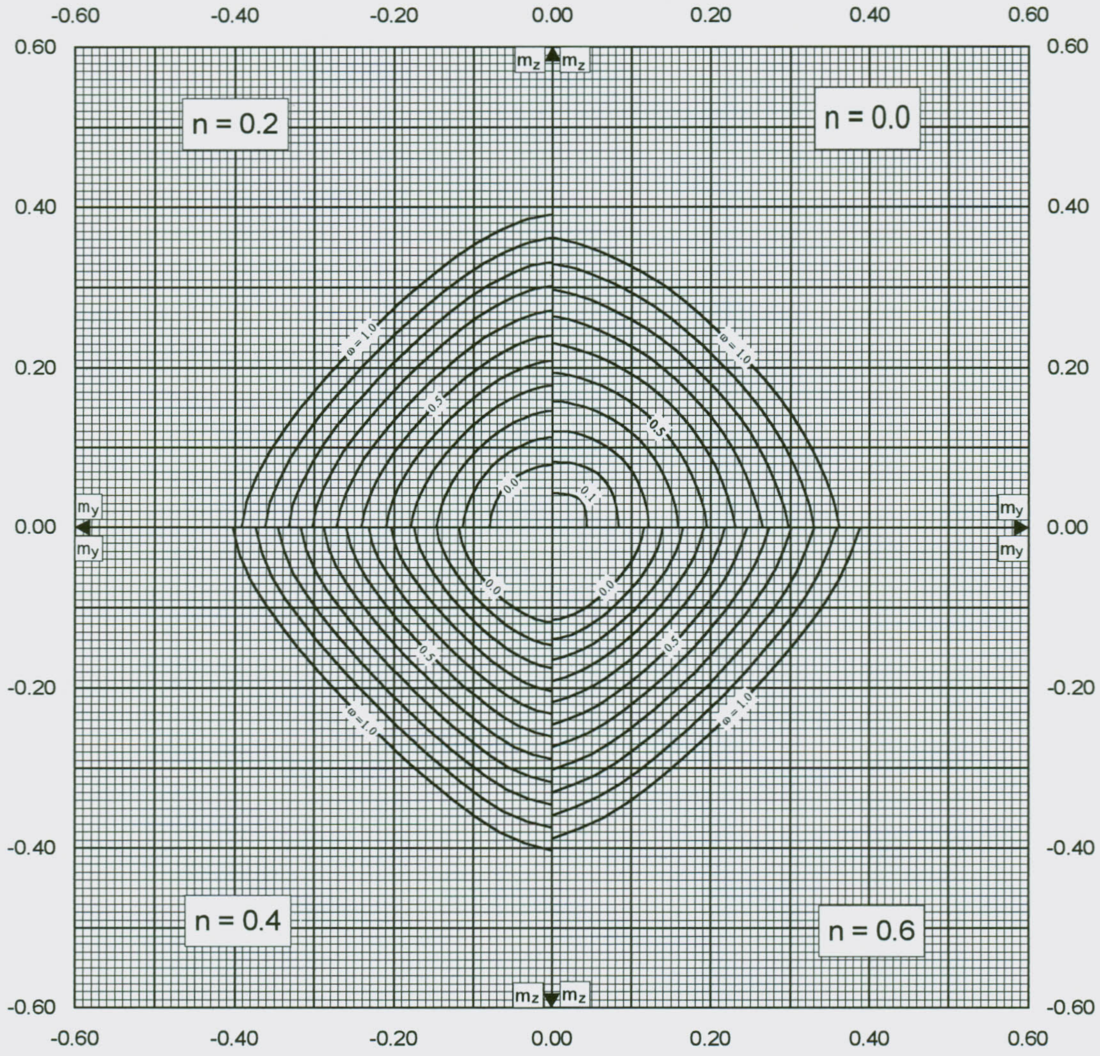


$\varepsilon_{yd} = 0.002$ $m_y = M_y / A_c\, h\, f_{cd}$

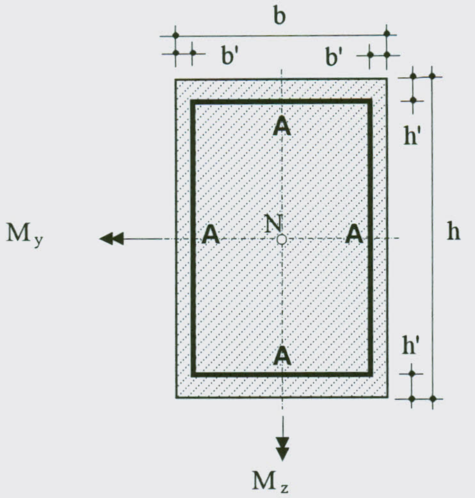
$h'/h = b'/b = 0.10$ $m_z = M_z / A_c\, b\, f_{cd}$

$A_{s,tot} = 4A$ $n = N / A_c\, f_{cd}$

$A_c = bh$ $\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.16



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.10$

$A_{s,tot} = 4A$

$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

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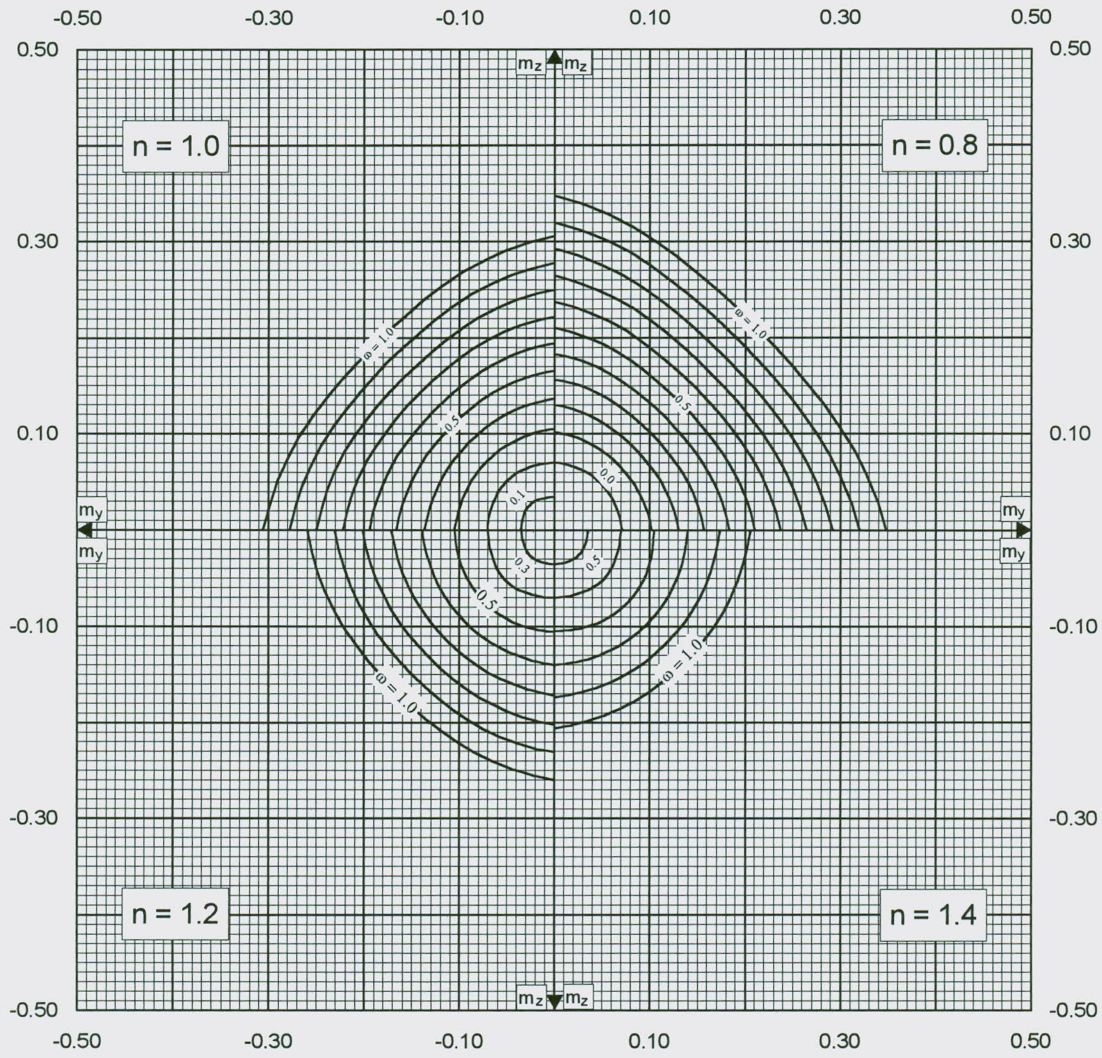
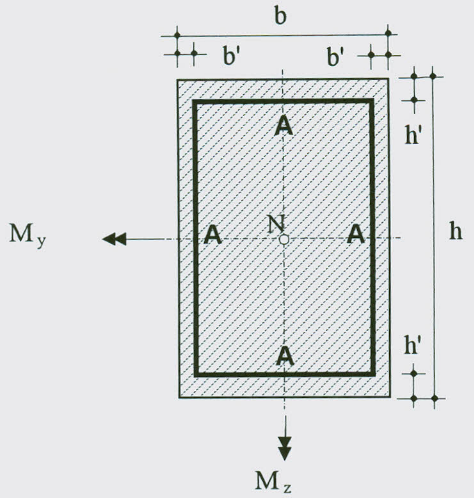


Biaxial Chart No.17



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.15$

$A_{s,tot} = 4A$

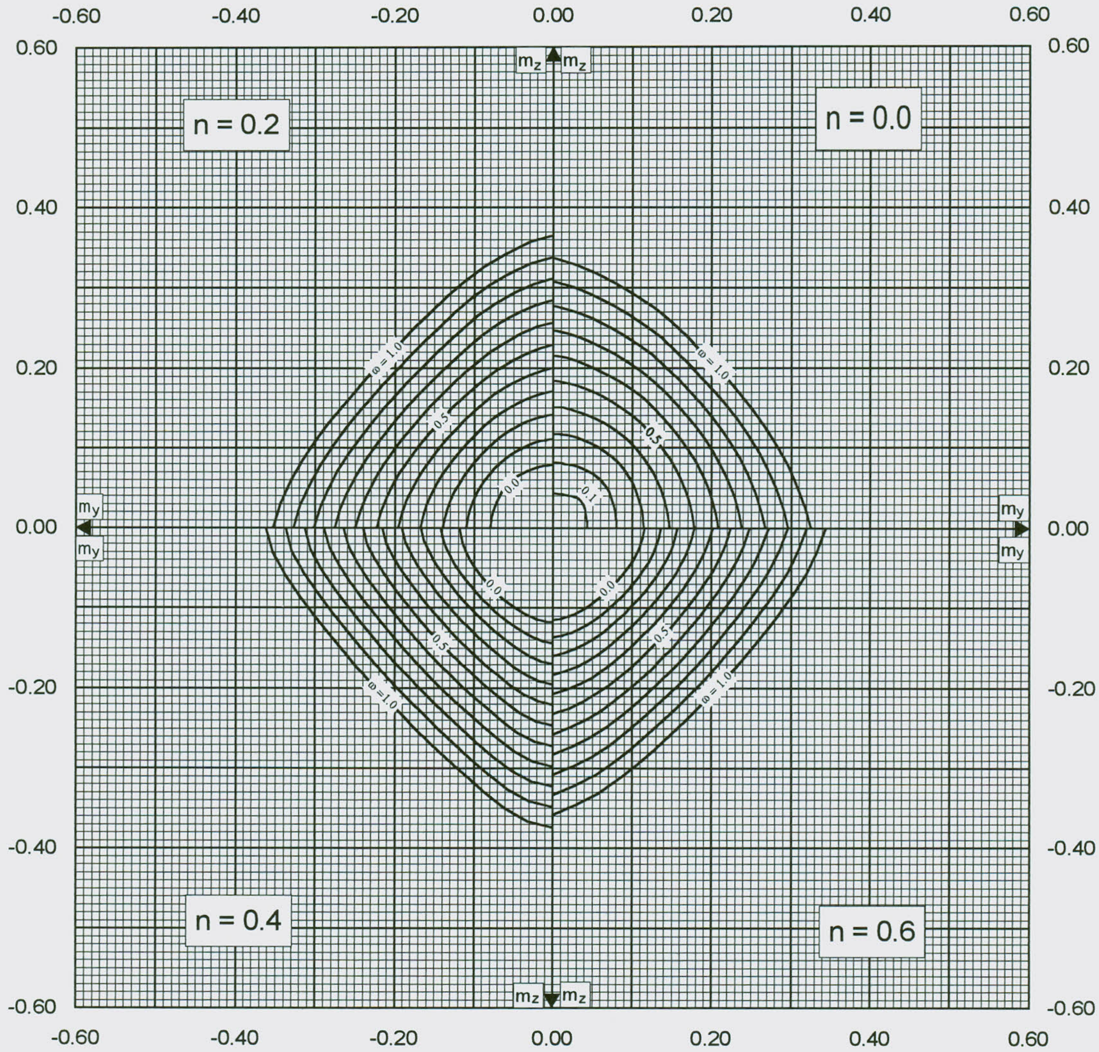
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

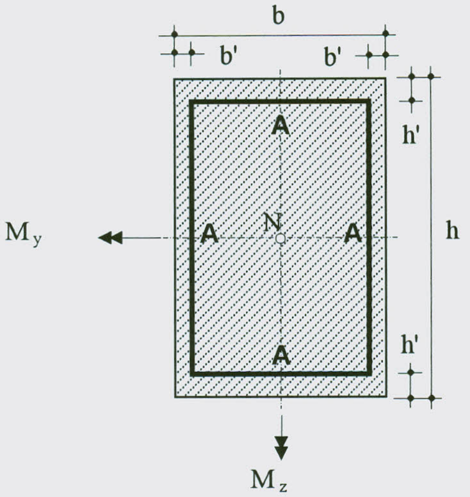
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\, f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.18



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.15$

$A_{s,tot} = 4A$

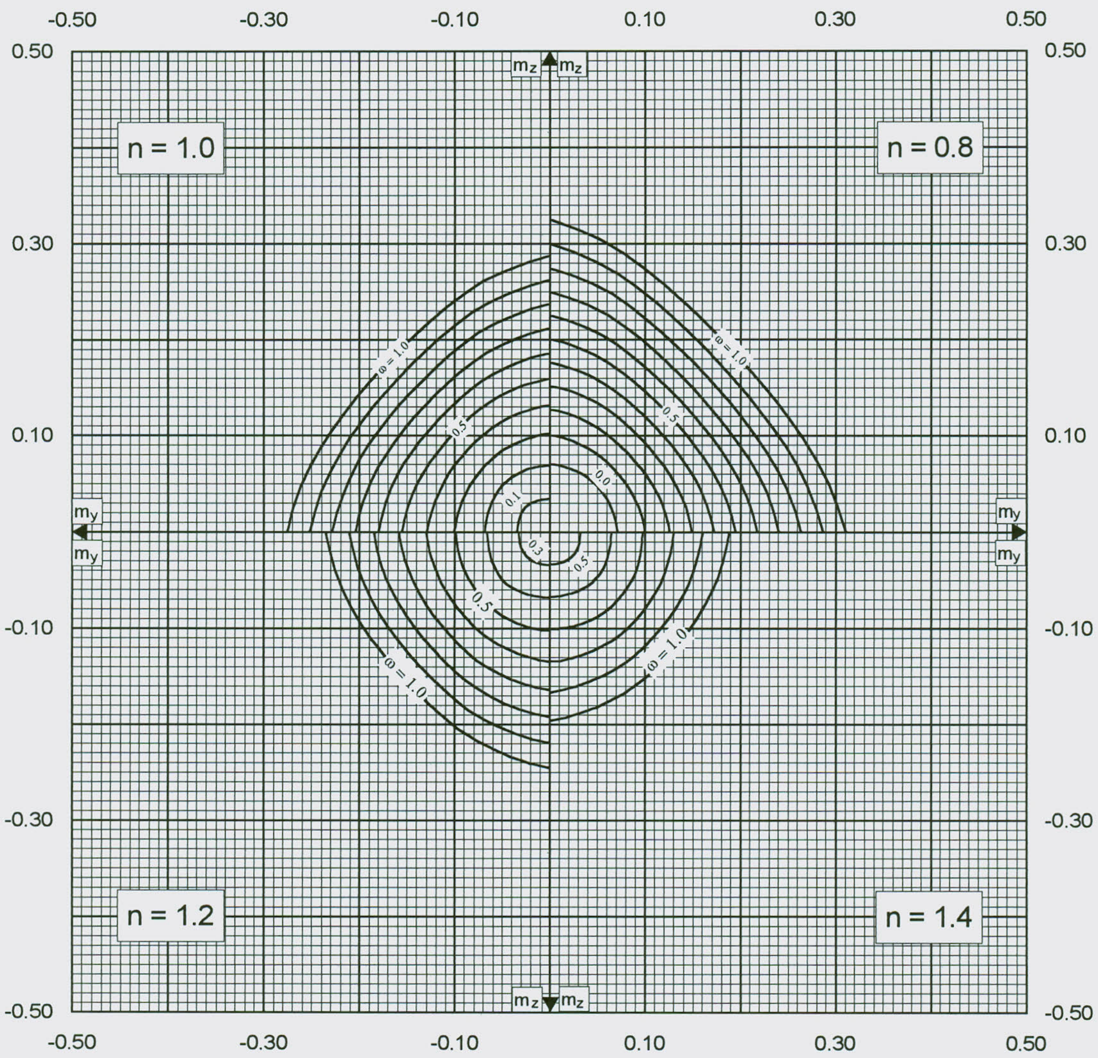
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

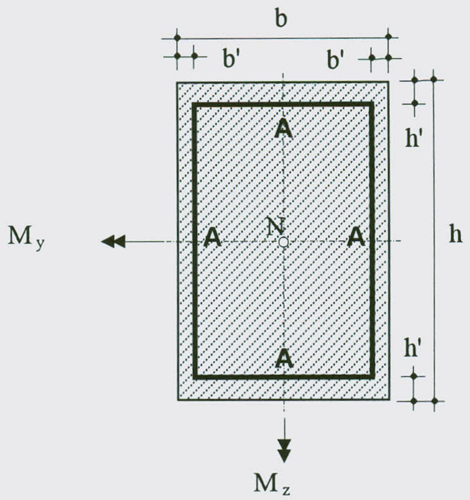
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.19



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.20$

$A_{s,tot} = 4A$

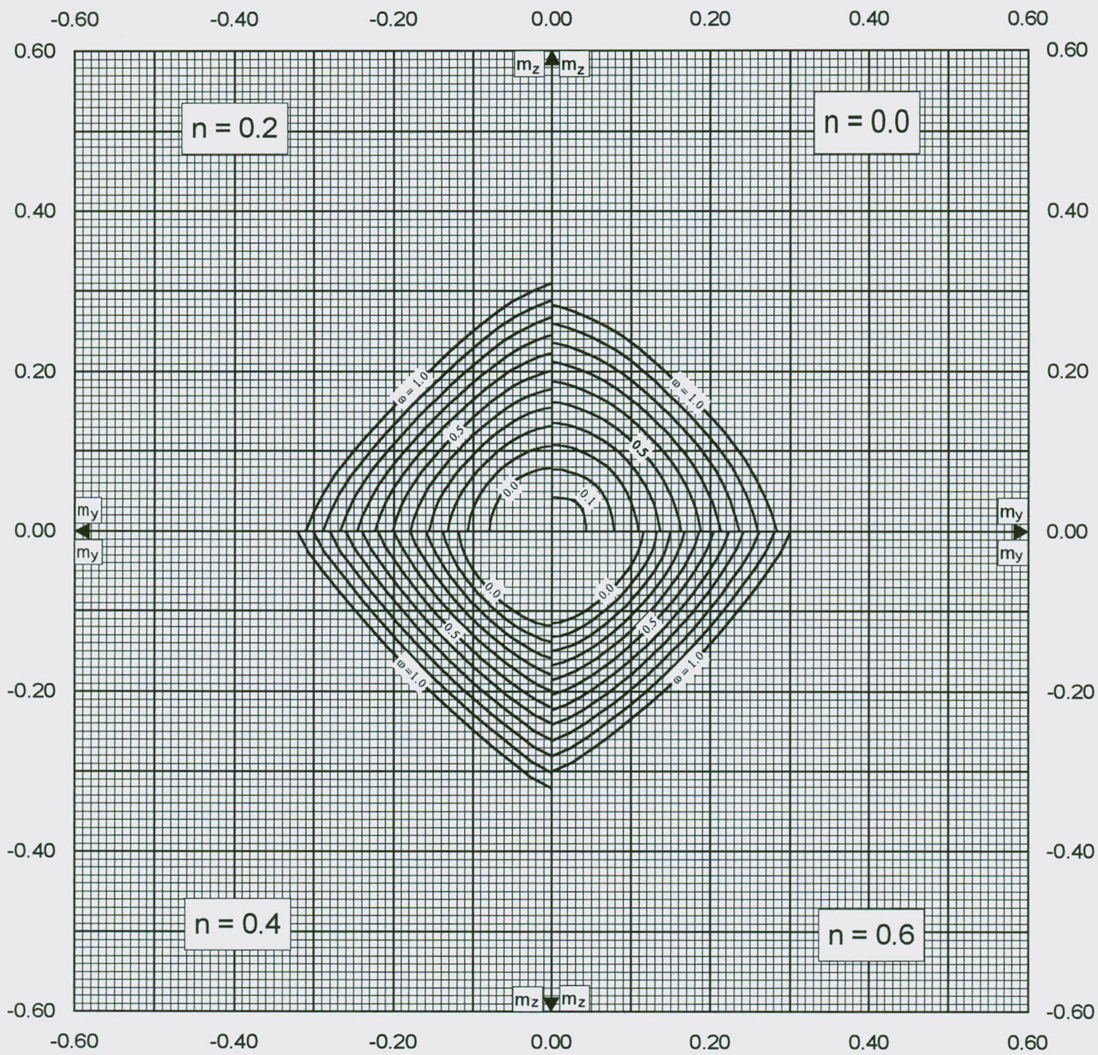
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

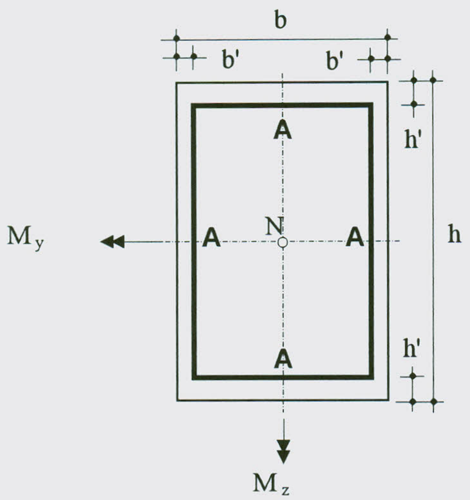
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



Biaxial Chart No.20



$\varepsilon_{yd} = 0.002$

$h'/h = b'/b = 0.20$

$A_{s,tot} = 4A$

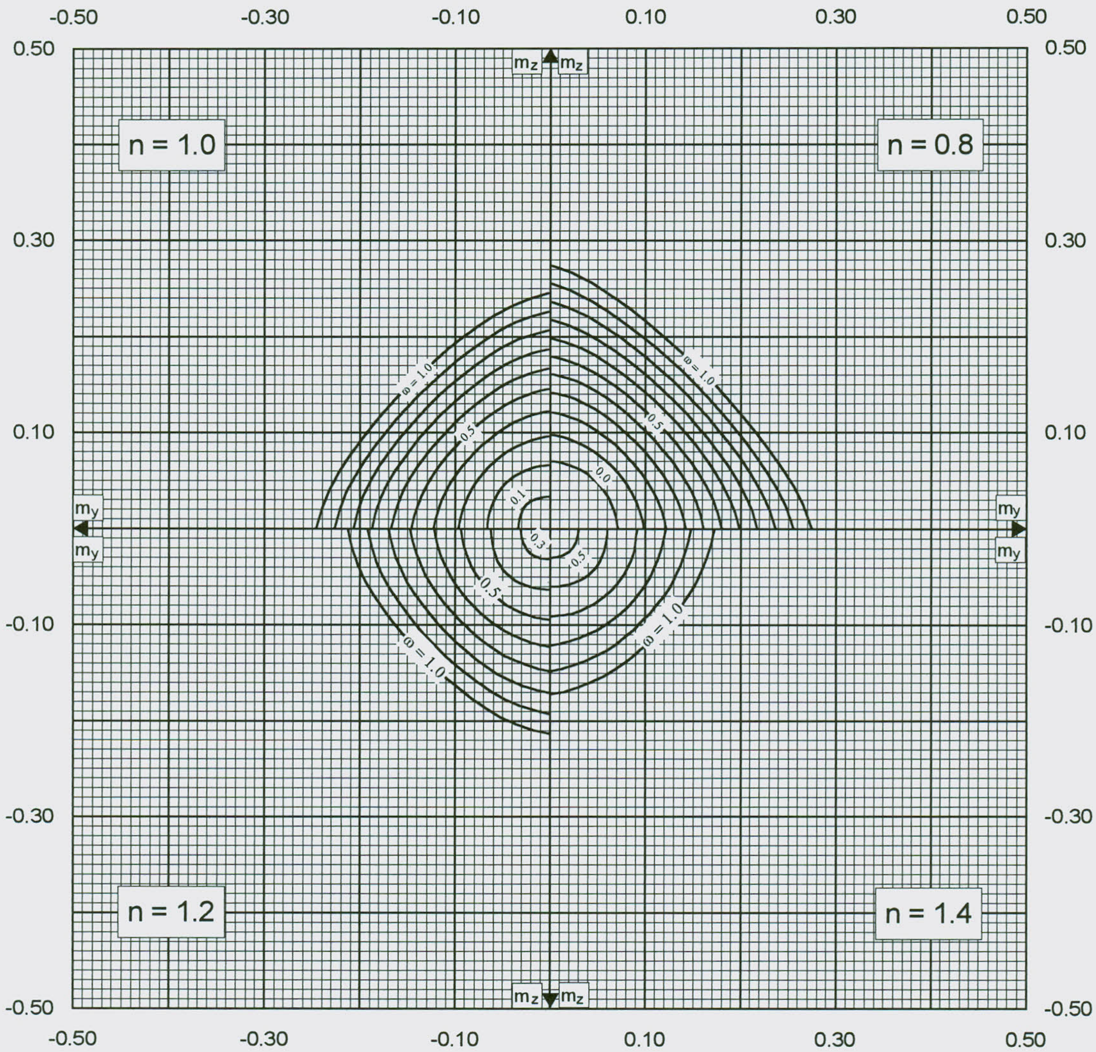
$A_c = bh$

$m_y = M_y / A_c\, h\, f_{cd}$

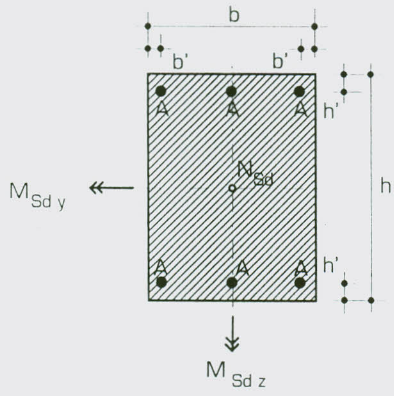
$m_z = M_z / A_c\, b\, f_{cd}$

$n = N / A_c\, f_{cd}$

$\omega = A_{s,tot}\; f_{yd} / A_c\, f_{cd}$



Bemessung für zweiachsig ausmittigen Druck **Tafel 28**



$\varepsilon_{yd} = 0{,}002391$

$\frac{h'}{h} = \frac{b'}{b} = 0{,}20$

$A_{s,tot} = \bar{\rho}_{o,tot} \frac{f_{cd}}{15} A_c = 6A$

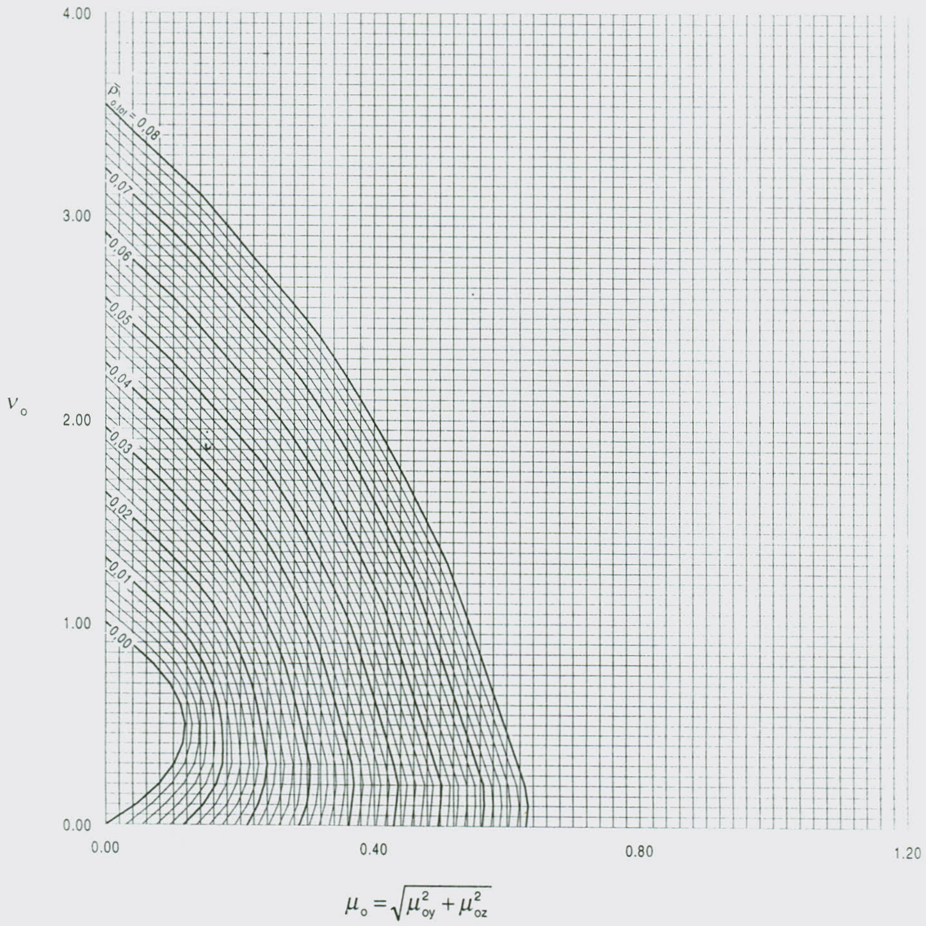
$\nu_o = \frac{N_{Sd}}{A_c f_{cd}}$

$\mu_{oy} = \frac{M_{Sdy}}{A_c h f_{cd}}$

$\mu_{oz} = \frac{M_{Sdz}}{A_c b f_{cd}}$

$\arctan\left(\frac{\mu_{oz}}{\mu_{oy}}\right) = 15^0$

$\frac{\mu_{oz}}{\mu_{oy}} = 0{,}27$



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DECLARATION

I, the undersigned, hereby declare that this thesis is my original work carried out under the supervision of Dr.-Ing. Girma Zerayohannes, has not been presented as a thesis in any other university and that all sources for this thesis are duly acknowledged.

Tefera Desta

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Ref. No.: E-120/91 BC/ak

To: The School of Graduate Studies

From: Bayou Chane (Dr.)
Chairman
Department of Civil Engineering

Subject: **Completion of M.Sc. Thesis**

Tefera Desta has successfully presented and defended his M.Sc. thesis entitled "Evaluation of Approximate Methods for the Design of Biaxially Loaded Reinforced Concrete Columns" on June 03, 1999. He has also incorporated the comments forwarded by the Examiners.

Thank you.