

NUCLEAR FUSION ENERGY



BY

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NUCLEAR FUSION ENERGY

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Chapter One

Introduction

1.1 Nuclear Fusion

Nuclear fusion was first achieved on earth in the early 1930s by bombarding a target containing deuterium, the mass-2 isotope of hydrogen, with high-energy deuterium in a cyclotron.[1] To accelerate the deuterium beam a great deal of energy is required, most of which appeared as heat in the target. As a result, no net useful energy was produced.

In the 1950s the first large-scale but uncontrolled release of fusion energy was demonstrated in the tests of thermonuclear weapons by the United States, the USSR, the United Kingdom, and France. This was such a brief and uncontrolled release that it could not be used for the production of electric power.

In this chapter we briefly discuss fusion reactions. We first define fusion cross section and reactivity, and then present and justify qualitatively the standard parametrization of these two important quantities. Next, we consider a few important fusion reactions, and provide expressions, data, and graphs for the evaluation of their cross sections and reactivities. These results will be used in the following chapters to derive the basic requirements for fusion energy production, as well as to study fusion ignition and burn in a confined fuel.

1.2 Exothermic Nuclear Reactions : fission and fusion

According to Einstein's mass – energy relationship, a nuclear reaction in which the total mass of the final products is smaller than that of the reacting nuclei is exothermic, that is, releases an energy proportional to such a mass difference.

$$Q = (\sum_i m_i - \sum_f m_f) c^2 \quad (1.1)$$

Here the symbol m denotes mass, the subscripts i and f indicate, respectively, the initial and the

final products, and c is the speed of light.

We can identify exothermic reactions by considering the masses and the binding energies of each of the involved nuclei.

The mass m of a nucleus with atomic number Z and mass number A differs from the sum of the masses of the Z protons and $A - Z$ neutrons which build up the nucleus by a quantity ,

$$\Delta m = Zm_p + (A - Z)m_n - m \quad (1.2)$$

where m_p and m_n are the mass of the proton and of the neutron, respectively. For stable nuclei Δm is positive, and one has to provide an amount of energy equal to the binding energy ,

$$B = \Delta m c^2 \quad (1.3)$$

in order to dissociate the nucleus into its component neutrons and protons.

The Q value of a nuclear reaction can then be written as the difference between the final and the initial binding energies of the interacting nuclei:

$$Q = \sum_f B_f - \sum_i B_i \quad (1.4)$$

A particularly useful quantity is the average binding energy per nucleon B/A , which is plotted in Fig. 1.1 as a function of the mass number A . We see that B/A , which is zero for $A = 1$, that is, for the hydrogen nucleus , grows rapidly with A , reaches a broad maximum of 8.7MeV about $A=56$ and then decreases slightly. For the heaviest nuclei $B/A \approx 7.5\text{MeV}$.Notice the particularly high value of B/A for helium nucleus. The symbols D and T indicate deuterium and tritium, that is, the hydrogen isotopes with mass two and three, respectively.

According to the above discussion exothermic reactions occur when the final reaction products have larger B/A than the reacting nuclei. As indicated in Fig. 1.1, this occurs for fission

reactions, in which a heavy nucleus is split into lighter fragments, and for fusion reactions, in which two light nuclei merge to form a heavier nucleus.

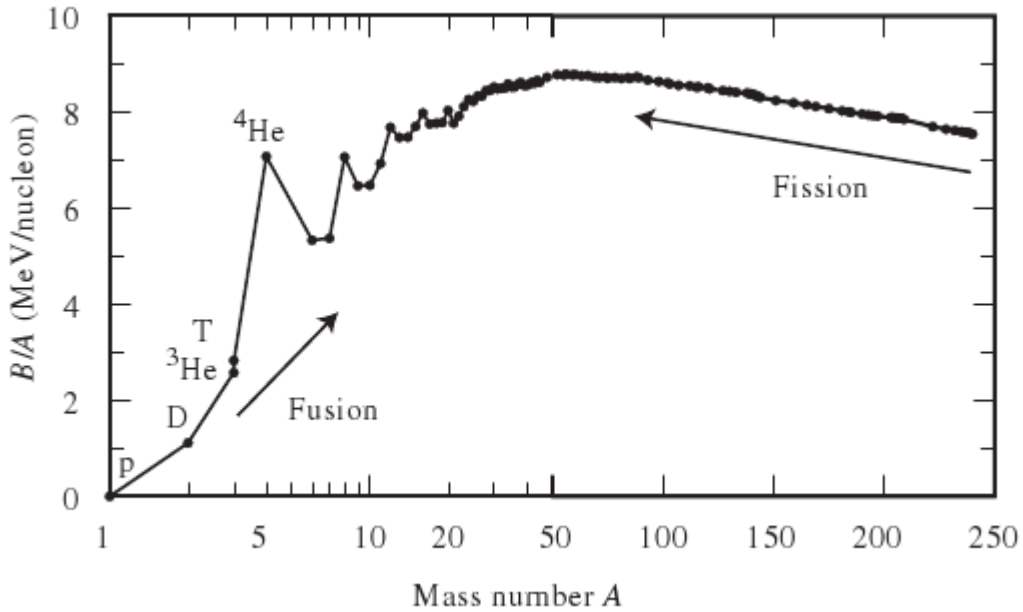


Fig. 1.1 Binding energy per nucleon versus mass number A , for the most stable isobars. For $A = 3$ also the unstable tritium is included, in view of its importance for controlled fusion. Notice that the mass number scale is logarithmic in the range 1–50 and linear in the range 50–250.

1.3 Nuclear Fusion Reactions

All reactions in any fusion cycle must be two-body reactions (two particles in the initial state). Why? This is because the simultaneous collision of three particles is too improbable an event to be significant.

In most fusion reactions two nuclei (X_1 and X_2) merge to form a heavy nucleus (X_3) and a lighter particle (X_4). But occasionally X_4 will be a gamma ray. To express this, we shall use either of the equivalent standard notations

$$X_1 + X_2 \rightarrow X_3 + X_4 \quad , \quad \text{or} \quad X_1(x_2, x_4)X_3 \quad (1.5)$$

Due to conservation of energy and momentum, the energy released by the reaction is distributed among the two fusion products in quantity inversely proportional to their masses . We indicate the velocities of the reacting nuclei in the laboratory system with \mathbf{v}_1 and \mathbf{v}_2 , respectively, and their relative velocity with $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$.

The center-of-mass energy of the system of the reacting nuclei is then

$$\epsilon = \frac{1}{2} m_r v^2 \quad (1.6)$$

where $v = |\mathbf{v}|$, and

$$m_r = \frac{m_1 m_2}{m_1 + m_2} \quad (1.7)$$

is the reduced mass of the system.

1.3.1 Fusion Cross section, Reactivity and Reaction Rate

The most important quantity for the analysis of nuclear reactions is the cross section, which measures the probability per pair of particles for the occurrence of the reaction. To be more specific, let us consider a uniform beam of particles of type ‘1’, with velocity v_1 , interacting with a target containing particles of type ‘2’ at rest. The cross section $\sigma_{12}(v_1)$ is defined as the number of reactions per target nucleus per unit time when the target is hit by a unit flux of projectile particles, that is, by one particle per unit target area per unit time. Actually, the above definition applies in general to particles with relative velocity v , and is therefore symmetric in the two particles, since we have $\sigma_{12}(v) = \sigma_{21}(v)$.

Cross sections can also be expressed in terms of the center-of-mass energy 1.6, and we have

$\sigma_{12}(\epsilon) = \sigma_{21}(\epsilon)$. In most cases, however, the cross sections are measured in experiments in which a beam of particles with energy ϵ_1 , measured in the laboratory frame, hits a target at rest. The corresponding beam-target cross-section $\sigma_{12}^{bt}(\epsilon_1)$ is related to the center-of-mass cross-section $\sigma_{12}(\epsilon)$ by

$$\sigma_{12}(\epsilon) = \sigma_{12}^{bt}(\epsilon_1) \quad (1.8)$$

$$\text{with } \epsilon_1 = \epsilon \frac{(m_1 + m_2)}{m_2} .$$

From now on, we shall refer to center-of-mass cross-sections and omit the indices 1 and 2. If the target nuclei have density n_2 and are at rest or all move with the same velocity, and the relative velocity is the same for all pairs of projectile – target nuclei, then the probability of reaction of nucleus ‘1’ per unit path is given by the product $n_2 \sigma(v)$. The probability of reaction per unit time is obtained by multiplying the probability per unit path times the distance v traveled in the unit time, which gives $n_2 \sigma(v) v$.

Another important quantity is the *reactivity*, defined as the probability of reaction per unit time per unit density of target nuclei. In the present simple case, it is just given by the product σv . In general, target nuclei move, so that the relative velocity v is different for each pair of interacting nuclei. In this case, we compute an *averaged reactivity*

$$\langle \sigma v \rangle = \int_0^\infty \sigma(v) v f(v) dv \quad (1.9)$$

where $f(v)$ is the distribution function of the relative velocities, normalized in such a way that

$\int_0^\infty f(v) dv = 1$. It is to be observed that when projectile and target particles are of the same species, each reaction is counted twice. Both controlled fusion fuels and stellar media are usually mixtures of elements where species ‘1’ and ‘2’, have number densities n_1 and n_2 , respectively. The volumetric reaction rate, that is, the number of reactions per unit time and per unit volume is then given by

$$R_{12} = \frac{n_1 n_2}{1 + \delta_{12}} \langle \sigma v \rangle = \frac{f_1 f_2}{1 + \delta_{12}} n^2 \langle \sigma v \rangle \quad (1.10)$$

Here n_1 and n_2 are number densities (the number of pair of particles), n is the total nuclei

number density and f_1 and f_2 are the atomic fractions of species '1' and '2', respectively. The Kronecker symbol δ_{ij} (with $\delta_{ij} = 1$, if $i = j$ and $\delta_{ij} = 0$ else where) is introduced to properly take into account the case of reactions between like particles.

Equation 1.10 shows a very important feature for fusion energy research: the volumetric reaction rate is proportional to the square of the density of the mixture. For future reference, it is also useful to recast it in terms of the mass density ρ of the reacting fuel .

The reaction rate is proportional to the square of the density

$$R_{12} = \frac{f_1 f_2}{1 + \delta_{12}} \frac{\rho^2}{m_{ave}^2} \langle \sigma v \rangle \quad (1.11)$$

where m_{ave} is the average nuclear mass. Here, the mass density is computed as $\rho = \sum_j n_j m_j = n m_{ave}$ where the sum is over all species, and the very small contribution due to the electrons is neglected. We also immediately see that the specific reaction rate, that is, the reaction rate per unit mass, is proportional to the mass density, again indicating the role of the density of the fuel in achieving efficient release of fusion energy.

1.3.2 Coulomb Barrier

In order to fuse, two positively charged nuclei must come into contact, winning the repulsive Coulomb force. Such a situation is made evident by the graph of the radial behavior of the potential energy of a two nucleon system, shown in Fig. 1.2. The potential is essentially Columbian and repulsive,

$$V_c(r) = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{r_n} \quad (1.12)$$

at distances r greater than

$$r_n \cong 1.44 \times 10^{-13} (A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}}) \text{ cm} \quad (1.13)$$

which is about the sum of the radii of the two nuclei $R_1 + R_2$ (classically it is called the distance of closest approach). In the above equations Z_1 and Z_2 are the atomic numbers, A_1 and A_2 are the mass numbers of the interacting nuclei, and e is the electron charge. At distances $r < r_n$ the two nuclei feel the strong, attractive nuclear force, characterized by a potential well of depth $U_0 = 30 - 40 \text{ MeV}$.

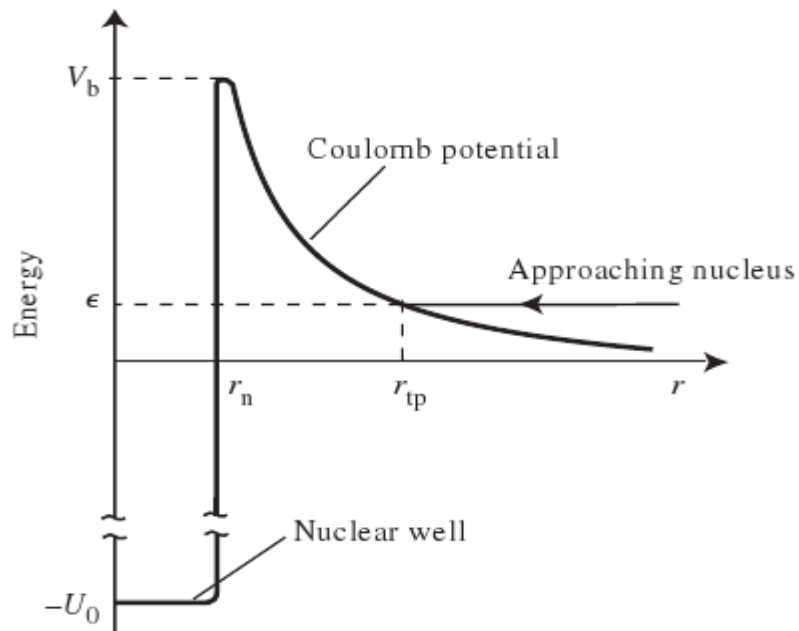


Fig. 1.2 Potential energy versus distance between two charged nuclei approaching each other with center-of-mass energy ϵ . The figure shows the nuclear well, the Coulomb barrier, and the classical turning point.

Using equations 1.12 and 1.13 we find that the height of the Coulomb barrier

$$V_b \simeq V_c(r_n) = \frac{Z_1 Z_2}{(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}})} \text{ MeV} \quad (1.14)$$

is of the order of one million electron-volts (1 MeV).

According to classical mechanics, only nuclei with energy exceeding such a value can overcome the barrier and come into contact. Instead, two nuclei with relative energy $\epsilon < V_b$ can only approach each other up to the classical turning point distance

$$r_{tp} = \frac{Z_1 Z_2 e^2}{(4 \pi \epsilon_0) \epsilon} \quad (1.15)$$

$V_c \sim Z_1 Z_2$ implying that the coulomb barrier is smallest for hydrogen and its isotopes since they contain single positive charge in their nucleus .

Quantum mechanics , however , allows for tunneling a potential barrier of finite extension, thus making fusion reactions between nuclei with energy smaller the height of the barrier possible .Concerning the barrier transparency, we shall see that it is often well approximated by

$$T \approx T_G = \exp\left(-\sqrt{\frac{\epsilon_G}{\epsilon}}\right) \quad (1.16)$$

where $G = \sqrt{\epsilon_G/\epsilon}$ is known as the Gamow factor (after the scientist who first computed it), and

$$\epsilon_G = 2m_r c^2 (\pi \alpha_f Z_1 Z_2)^2 = 986.1 Z_1^2 Z_2^2 A_r \text{ Kev} \quad (1.17)$$

is the Gamow energy, $\alpha_f = e^2/\hbar c = 1/137.04$ is the fine-structure constant commonly used in quantum mechanics, and $A_r = m_r/m_p$. Here m_r is the reduced mass of the two fusing nuclei which have electric charges $Z_1 e$ and $Z_2 e$. Equation 1.16 holds as far as $\epsilon \ll \epsilon_G$, which sets no limitations to the problems we are interested in.

Equations 1.16 and 1.17 show that the chance of tunneling decreases rapidly with the atomic number and mass, thus providing a first simple explanation for the fact that fusion reactions of interest for energy production on earth only involves the lightest nuclei. Also note that the probability of barrier penetration increases as the energy ϵ of the reacting nuclei increases.

1.3.3 Fusion cross section parametrization

A widely used parametrization of fusion reaction cross-sections is

$$\sigma \approx \sigma_{geom} \times T \times \mathfrak{R} \quad (1.18)$$

where σ_{geom} is a geometrical cross-section, T is the barrier transparency, and \mathfrak{R} , is the probability that nuclei come into contact to fuse. The first quantity is of the order of the square of the De-Broglie wavelength of the system:

$$\sigma_{geom} \approx \lambda^2 = \left(\frac{\hbar}{m_r v}\right)^2 \propto \frac{1}{\epsilon} \quad (1.19)$$

where \hbar is the reduced Planck constant and m_r is the reduced mass (equation 1.7) and $\lambda = \hbar / m_r v$ is called the reduced de Broglie wavelength.

The *reaction characteristics* R contains essentially all the nuclear physics of the specific reaction. It takes substantially different values depending on the nature of the interaction characterizing the reaction. It is largest for reactions due to strong nuclear interactions; it is smaller by several orders of magnitude for electromagnetic nuclear interactions; it is still smaller by as many as 20 orders of magnitude for weak interactions. For most reactions, the variation of $R(\epsilon)$ is small compared to the strong variation due to the Gamow factor.

In conclusion, the fusion cross section is often written as

$$\sigma(\epsilon) = \frac{S(\epsilon)}{\epsilon} \exp\left(-\sqrt{\frac{\epsilon_G}{\epsilon}}\right) \quad (1.20)$$

Here the exponential follows from the previous discussion of quantum tunneling and the function $S(\epsilon)$ is called the astrophysical S factor, which for many important reactions is a weakly varying function of the energy.

For example , if we consider the fusion of two protons (which we will see below is an important ingredients of the reactions that power the sun) , at a typical stellar temperature of 10^7 K , we find $\epsilon_G \approx 490\text{KeV}$ and $\epsilon \approx k_B T = 1\text{KeV}$. Hence the probability of fusion is proportional to $\exp(-\sqrt{\epsilon_G/\epsilon}) \approx \exp^{-22} \approx 10^{-9.6}$ which is a large suppression factor and so the actual fusion rate is still extremely slow .[2]

Besides quantum tunneling ,the other reason that fusion occurs at lower temperature than expected is that a collection of nuclei at a given mean temperature, whether in stars or elsewhere, will have a Maxwellian distribution of energies about the mean and so there will be some nuclei with energies substantially higher than the *mean energy* .

Nevertheless, even a stellar temperature of 10^8 K corresponds to an energy of only about 10KeV, so the fraction of nuclei with energies of order 1MeV in such a star would only be of the order of $\exp(-\epsilon/k_B T) = \exp^{-100} = 10^{-43}$, a minute amount.[2]

1.4 Some Important Fusion Reactions

In Table 1.1 we list some fusion reactions of interest to controlled fusion research and to astrophysics. For each reaction the table gives the Q-value, the zero-energy astrophysical factor $S(0)$ and the square root of the Gamow energy (G) . For the cases in which $S(\epsilon)$ is weakly varying these data allow for relatively accurate evaluation of the cross section, using equation 1.20, with $S = S(0)$. [3]

	Q (MeV)	$\langle Q_\nu \rangle$ (MeV)	$S(0)$ (keV barn)	$\epsilon_G^{1/2}$ (keV ^{1/2})
<i>Main controlled fusion fuels</i>				
$D + T \rightarrow \alpha + n$	17.59		1.2×10^4	34.38
$D + D \rightarrow \begin{cases} T + p \\ {}^3\text{He} + n \\ \alpha + \gamma \end{cases}$	4.04 3.27 23.85		56 54 4.2×10^{-3}	31.40 31.40 31.40
$T + T \rightarrow \alpha + 2n$	11.33		138	38.45
<i>Advanced fusion fuels</i>				
$D + {}^3\text{He} \rightarrow \alpha + p$	18.35		5.9×10^3	68.75
$p + {}^6\text{Li} \rightarrow \alpha + {}^3\text{He}$	4.02		5.5×10^3	87.20
$p + {}^7\text{Li} \rightarrow 2\alpha$	17.35		80	88.11
$p + {}^{11}\text{B} \rightarrow 3\alpha$	8.68		2×10^5	150.3
<i>The p-p cycle</i>				
$p + p \rightarrow D + e^+ + \nu$	1.44	0.27	4.0×10^{-22}	22.20
$D + p \rightarrow {}^3\text{He} + \gamma$	5.49		2.5×10^{-4}	25.64
${}^3\text{He} + {}^3\text{He} \rightarrow \alpha + 2p$	12.86		5.4×10^3	153.8
<i>CNO cycle</i>				
$p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$	1.94		1.34	181.0
$[{}^{13}\text{N} \rightarrow {}^{13}\text{C} + e^+ + \nu + \gamma]$	2.22	0.71	—	—
$p + {}^{13}\text{C} \rightarrow {}^{14}\text{N} + \gamma$	7.55		7.6	181.5
$p + {}^{14}\text{N} \rightarrow {}^{15}\text{O} + \gamma$	7.29		3.5	212.3
$[{}^{15}\text{O} \rightarrow {}^{15}\text{N} + e^+ + \nu + \gamma]$	2.76	1.00	—	—
$p + {}^{15}\text{N} \rightarrow {}^{12}\text{C} + \alpha$	4.97		6.75×10^4	212.8
<i>Carbon burn</i>				
${}^{12}\text{C} + {}^{12}\text{C} \rightarrow \begin{cases} {}^{23}\text{Na} + p \\ {}^{20}\text{Ne} + \alpha \\ {}^{24}\text{Mg} + \gamma \end{cases}$	2.24 4.62 13.93		8.83×10^{19}	2769

Table 1.1 Some important fusion reactions and parameters of the cross – section factorization 1.20. The Q – value includes both positron disintegration energy and neutrino energy when relevant . The quantity $\langle Q_\nu \rangle$ is the average neutrino energy . $1\text{barn}=10^{-24}\text{cm}^2$.

For some of the main reactions, Table 1.2 gives the measured cross- sections at $\epsilon=10\text{KeV}$ and $\epsilon=100\text{KeV}$, as well as the maximum value of the cross-section σ_{max} , and the energy ϵ_{max} at which the maximum occurs. Also shown, in parentheses, are theoretical data for the proton - proton and CNO reactions. In the tables and in the following discussion, the reactions are grouped according to the field of interest . [3]

Reaction	σ (10 keV) (barn)	σ (100 keV) (barn)	σ_{max} (barn)	ϵ_{max} (keV)
$D + T \rightarrow \alpha + n$	2.72×10^{-2}	3.43	5.0	64
$D + D \rightarrow T + p$	2.81×10^{-4}	3.3×10^{-2}	0.096	1250
$D + D \rightarrow {}^3\text{He} + n$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$T + T \rightarrow \alpha + 2n$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$D + {}^3\text{He} \rightarrow \alpha + p$	2.2×10^{-7}	0.1	0.9	250
$p + {}^6\text{Li} \rightarrow \alpha + {}^3\text{He}$	6×10^{-10}	7×10^{-3}	0.22	1500
$p + {}^{11}\text{B} \rightarrow 3\alpha$	(4.6×10^{-17})	3×10^{-4}	1.2	550
$p + p \rightarrow D + e^+ + \nu$	(3.6×10^{-26})	(4.4×10^{-25})		
$p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$	(1.9×10^{-26})	2.0×10^{-10}	1.0×10^{-4}	400
${}^{12}\text{C} + {}^{12}\text{C}$ (all branches)		(5.0×10^{-103})		

Table 1.2 Fusion Reactions : Cross sections at centre – of – mass energy of 10KeV and 100KeV , maximum cross - sections σ_{max} and location of the maximum ϵ_{max} . Values in parentheses are estimated theoretically ; all others are measured data

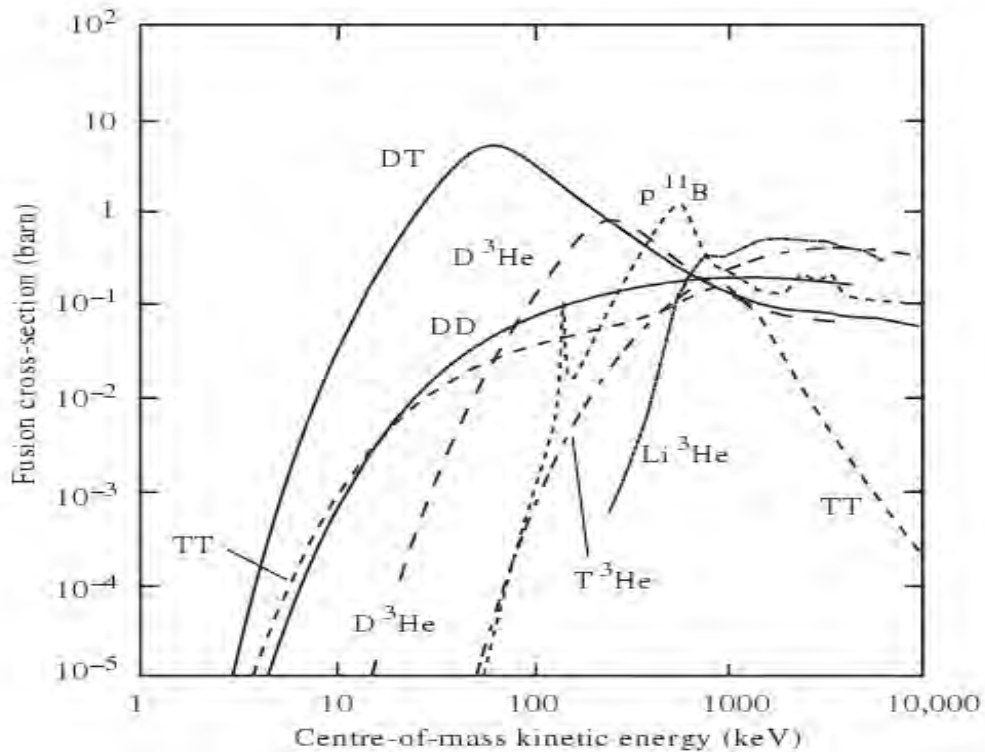


Figure 1.3 Fusion cross sections versus center – of – mass energy for reactions of interest to controlled fusion energy . The curve labeled DD represents the sum of the cross sections of the various branches of the reaction.

1.5 Maxwell-averaged Fusion Reactivities

As we have seen earlier, the effectiveness of a fusion fuel is characterized by its reactivity $\langle \sigma v \rangle$. Both in controlled fusion and in astrophysics we usually deal with mixtures of nuclei of different species, in thermal equilibrium, characterized by Maxwellian velocity distributions

$$f_j(v_j) = \left(\frac{m_j}{2\pi k_B T} \right)^{\frac{3}{2}} \exp\left(\frac{-m_j v_j^2}{2k_B T} \right) \quad (1.21)$$

where the subscript j labels the species, T is the temperature and k_B is Boltzmann constant. The expression for the average reactivity 1.9 can now be written as

$$\langle \sigma v \rangle = \int \int dv_1 dv_2 \sigma_{1,2}(v) v f_1(v_1) f_2(v_2) \quad (1.22)$$

where $v = |\mathbf{v}_1 - \mathbf{v}_2|$ and the integrals are taken over the three-dimensional velocity space. In order to put equation 1.22 in a form suitable for integration, we express the velocities \mathbf{v}_1 and \mathbf{v}_2 by means of the relative velocity and of the center-of-mass velocity

$$\mathbf{v}_c = (\mathbf{v}_1 + \mathbf{v}_2) / (m_1 + m_2) \quad (1.23)$$

where $\mathbf{v}_1 = \mathbf{v}_c + \mathbf{v} m_2 / (m_1 + m_2)$ (1.24)

and $\mathbf{v}_2 = \mathbf{v}_c - \mathbf{v} m_1 / (m_1 + m_2)$ (1.25)

Equation 1.22 then becomes

$$\langle \sigma v \rangle = \frac{(m_1 m_2)^{\frac{3}{2}}}{(2\pi k_B T)^3} \int \int dv_1 dv_2 \exp\left[-\frac{(m_1 + m_2) v_c^2}{2k_B T} - \frac{m_r v^2}{2k_B T} \right] \sigma(v) v \quad (1.26)$$

where m_r is the reduced mass defined by equation 1.7, and the subscripts '1,2' have been omitted. It can be shown [4] that the integral over $dv_1 dv_2$ can be replaced by an integral over

$d\mathbf{v}_c d\mathbf{v}$, so that we can write

$$\langle \sigma v \rangle = \left[\left(\frac{m_1 + m_2}{2\pi k_B T} \right)^{\frac{3}{2}} \int d\mathbf{v}_c \exp\left(-\frac{(m_1 + m_2)v_c^2}{2k_B T} \right) \right] \times \left(\frac{m_r}{2\pi k_B T} \right)^{\frac{3}{2}} \int d\mathbf{v} \exp\left(-\frac{m_r v^2}{2k_B T} \right) \sigma(v) v \quad (1.27)$$

The term in square brackets is unity, being the integral of a normalized Maxwellian, and we are left with the integral over the relative velocity. By writing the volume element in velocity space as $d\mathbf{v} = 4\pi v^2 dv$, and using the definition 1.6 of center-of-mass energy ϵ , we finally get

$$\langle \sigma v \rangle = \frac{4\pi}{(2\pi m_r)^{\frac{1}{2}} (k_B T)^{\frac{3}{2}}} \int_0^\infty \sigma(\epsilon) \epsilon \exp\left(\frac{-\epsilon}{k_B T} \right) d\epsilon \quad (1.28)$$

1.5.1 Gamow Form for Non-Resonant Reactions

Useful and enlightening analytical expressions of the reactivity can be obtained by using the simple parametrization 1.20 of the cross section. In this case the integrand of equation 1.28 becomes

$$y(\epsilon) = S(\epsilon) \exp\left[-\left(\frac{\epsilon_G}{\epsilon} \right)^{\frac{1}{2}} - \frac{\epsilon}{k_B T} \right] = S(\epsilon) g(\epsilon, k_B T) \quad (1.29)$$

An interesting result is obtained for temperatures $T \ll \epsilon_G$ and stems from the fact that the function $g(\epsilon, k_B T)$ is the product of a sharply peaked function, being the product of a decreasing exponential coming from the Maxwell-Boltzmann distribution that vanishes at large energy, $\exp(-\epsilon/k_B T)$ and an increasing exponential originating from the barrier penetrability that vanishes at low energy, $\exp(-\sqrt{\epsilon_G/\epsilon})$ as shown in Fig. 1.4. Since the quantity $S(\epsilon)$ is slowly varying, the behavior of the integrand is dominated by behavior of the exponential term.

In equation 1.29 the term $1/\epsilon$ is conveniently factored out because many nuclear cross

sections have this behavior at low energies.

The importance of the temperature and the Gamow energy is clear. A schematic illustration of the interplay between these effects is shown in figure shown below .

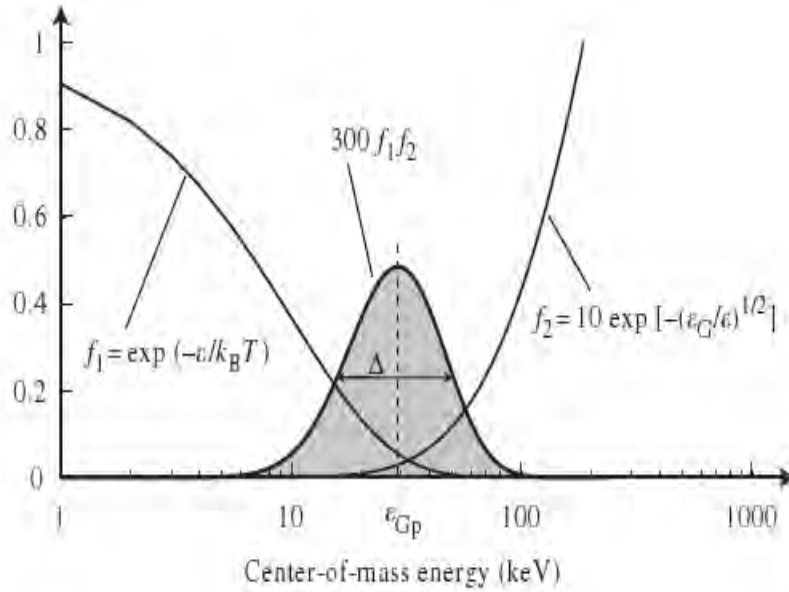


Fig 1.4. Gamow peak for DD reactions at $T = 10\text{KeV}$. Most of the reactivity comes from reaction between nuclei with center – of – mass energy between 15 and 60 KeV.

The reactivity has a maximum at the Gamow peak energy ϵ_{G_p} . The value of ϵ_{G_p} is determined from the location of the maximum of the integrand at $\epsilon = \epsilon_{G_p}$ with $b = \sqrt{\epsilon_G}$:

$$\left[\frac{d}{d\epsilon} (\epsilon/k_B T + b\epsilon^{-1/2}) \right]_{\epsilon = \epsilon_{G_p}} = 0$$

which gives $\epsilon_{G_p} = [1/4\epsilon_G(k_B T)^2]^{1/3}$,where T is the temperature in KeV . This energy is frequently called the most effective energy for thermonuclear reactions. Rewriting the Gamow peak energy as

$$\epsilon_{G_p} = \left(\frac{\epsilon_G}{4k_B T} \right)^{1/3} k_B T = \xi k_B T \tag{1.30}$$

and then substituting the Gamow energy (equation 1.19) we get

$$\xi = 6.2696 (Z_1 Z_2)^{\frac{2}{3}} A_r^{\frac{1}{3}} T^{-\frac{1}{3}} \quad (1.31)$$

To perform the integration we use the saddle - point method , that is , we first expand $y(\epsilon)$ in Taylor series around $\epsilon = \epsilon_{G_p}$, thus writing

$$y(\Delta) \simeq S(\epsilon) \exp \left[-3 \xi + \left(\frac{\epsilon - \epsilon_{G_p}}{\Delta/2} \right)^2 \right] \quad (1.32)$$

with
$$\Delta = \frac{4}{\sqrt{3}} \xi^{\frac{1}{2}} k_B T \quad (1.33)$$

Equation 1.33 shows that most of the contribution to the fusion reactivity comes from a relatively narrow range of energies with width Δ centered around $\epsilon = \epsilon_{G_p}$, in the high energy portion of the velocity distribution function (see figure 1.4).

Using equations 1.29 – 1.33 and with the further assumption of non – exponential behavior of $S(\epsilon)$, we can integrate equation (1.28) to get the reaction rate in the so – called Gamow form

$$\langle \sigma v \rangle = \frac{8}{\pi \sqrt{3}} \frac{\hbar}{m_r Z_1 Z_2 e^2} S_{ave} \xi^2 \exp(-3 \xi) \quad (1.34)$$

Here we have used $\int_0^\infty \exp(-x^2) dx = \sqrt{\pi}/2$, and indicated with S_{ave} an appropriately averaged value of S . In the cases in which S depends weakly on ϵ , one can simply set $S_{ave} = S(0)$. In the following , when distinguishing between S_{ave} and $S(0)$ is not essential , we shall simply use the symbol S .

Inserting the values of the numerical constants equation 1.34 becomes

$$\langle \sigma v \rangle = \frac{6.4 \times 10^{-18}}{A_r Z_1 Z_2} S \xi^2 \exp(-3 \xi) \text{ cm}^3/\text{s} \quad (1.35)$$

where S is in units of Kev barn and ξ is given by equation 1.31. We remark that the Gamow

form is appropriate for reactions which do not exhibit resonances in the relevant energy range. Equation 1.35 can be used to appreciate the low – temperature behavior of the reactivity .

By differentiation we get

$$\frac{d\langle \sigma v \rangle}{\langle \sigma v \rangle} = \frac{-2}{3} + \xi \frac{dT}{T} \quad (1.36)$$

which leads to

$$\langle \sigma v \rangle \propto T^\xi \quad (1.37)$$

where $\xi \gg 1$. A strong temperature dependence is then found when $T \ll 6.27Z_1^2Z_2^2A_r$, making apparent the existence of temperature thresholds for fusion burn, which are increasing functions of the mass of the participating nuclei.

The fusion reaction rate per unit volume (1.11) is obtained by multiplying the reactivity (equation1.35) with the number of pairs of particles per unit volume $n_1n_2/1+\delta_{12}$. [2]

$$R_{12} = \frac{n_1 n_2}{1 + \delta_{12}} \frac{6.4 \times 10^{-18}}{A_r Z_1 Z_2} S \xi^2 \exp(-3 \xi) \quad (1.38)$$

Equation 1.38 clearly shows that the rate of reaction depends very strongly on both the temperature and the nuclear species because of the factor $\xi^2 \exp(-3 \xi)$.A schematic illustration of the combined effect of the two exponential terms on the reaction rate (equation 1.39) can also be seen in figure 1.4 with the reaction rate(in arbitrary units) on the y- axis.

For the DD reactions at a temperature of 10KeV, we have $\epsilon_G = 986\text{KeV}$ and, so that fusion occurs most likely at $\epsilon_{G_p} = 29.2\text{Kev}$ & for pp reaction at $k_B T = 1.7\text{KeV}$, it is 7.2KeV. This most effective energy is greater than ϵ , reflecting the fact that the barrier – penetration factor has favored the selection of particles on the high energy tail of the maxwell- Boltzmann energy distribution. [3]

Chapter Two

Energy Released in Fusion Reactions

Introduction

The energy released in nuclear reactions (fission & fusion) has to do with the relative energy or binding energy per nucleon. The curve of the binding energy per nucleon (fig1.1) has a maximum at $A = 56$, where the nuclei are most tightly bound. This suggests that we can release energy in two ways: below $A=56$ by assembling lighter nuclei into heavier nuclei (fusion), or above $A=56$, by breaking heavier nuclei into lighter nuclei (fission). In either case we "climb the curve of the binding energy" and liberate nuclear energy.

In fusion, the reacting nuclei (often the lightest nuclei, e. g. hydrogen, helium, lithium,) have very low binding energy per nucleon, but as we go up from H to He to Li, the binding energy increases significantly. It is the large difference in binding energy per nucleon between, say deuterium (d) or tritium (t) and alpha (${}^4\text{He}$), which is responsible for the energy released as a result of fusion.

The energy released in fusion per unit mass of material comparable with that released in fission ($\approx 1\text{Mev } u^{-1}$) and, in some cases, exceeds it by a considerable factor. What is fusion energy? It is the energy released during the fusion of atomic nuclei. It is the system's ability to do work or release heat or radiation. The total energy in a system is always conserved, but it can be transferred to another system or charged atom.

2.1 Energy Released in Some Fusion Reactions

Basically the desirable fusion reactions are those which have a high energy release (Q value), or high mass defect or mass difference between reactants and products. It is the difference in binding energies of the reactants and products, which allows for an exothermic fusion reaction.

For the fusion reactions between two nuclei X_1 and X_2 to produce two other nuclei X_3 and X_4 ,

assuming that none of the particles is internally excited, (i. e. each is in its ground state), the energy released or the Q value is given in equation 1.1 .For most applications of fusion, from controlled fusion reactors to solar processes, the reacting particles have energies in the range of 1-10KeV. This initial kinetic energies are thus small compared with the energies released which are of several MeV . Hence the energy released ,& the final total energy of the product particles , will then be equal to the Q – value .

The reaction in which two deuterium nuclei (^2H) fuse together to form an α – *particle* is

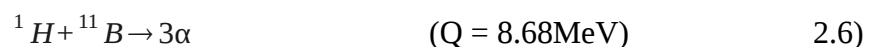
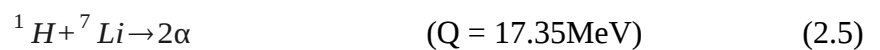
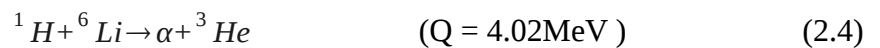
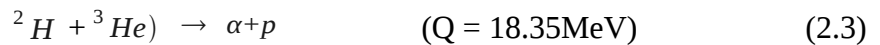


where the γ is essential for energy balance , since ^4He (α) has no excited states. Using $m_D=2.014102\text{u}$, $m_\alpha=4.002603\text{u}$, $c^2=931.10\text{MeV/u}$, the energy released in this reaction (equation 2.1) is 23.85MeV . [5] Of course, the more stable the end product formed , the greater is the energy release in the reaction .

The other important fusion reaction occurs when two tritium nuclei fuse together to produce alpha particle and two neutrons The energy released in this reaction is found to be 11.33Mev.



The fusion reactions between hydrogen isotopes and light nuclei (Helium, Lithium ,Boron) yields (*The energy released in each reaction is given in parentheses*)



2.2 Energy Released in the Sun

Introduction

In 1920 Aston found that the mass of the helium nucleus is smaller than four times the mass of the hydrogen atom. Immediately, Eddington (1920, 1926) observed that the transformation of hydrogen into helium could provide enough power to sustain the sun and, more generally, postulated nuclear reactions as the mechanism powering the stars. However, he was puzzled by the fact that the inferred star temperatures are well below those thought necessary to allow particles to react effectively.

Just at the dawn of wave-mechanics, Gurney and Condon (1929) and, independently, Gamow (1928), computed the probability of tunneling a barrier. Gamow showed that quantum-mechanical tunneling explains observations on α -particle decay. In the following year, Atkinson and Houtermans (1929) used Gamow's result to point out that tunneling opens the way to hydrogen fusion reactions, which could be responsible for the energy production in the stars.

In 1932, Cockcroft and Walton at the Cavendish Laboratory, Cambridge University, directed by Lord Rutherford, were able to produce and detect for the first time a fusion reaction, by bombarding lithium samples with a 100 Kev proton beam generated by an accelerator they had designed and built (Cockcroft and Walton 1932). In the following 2 years, at the same laboratory, the team led by Lord Rutherford and also including Oliphant, Lewis, Hartweck, Kempton, Shire and Crouther, discovered many other fusion reactions between light elements and accelerated protons or deuterons .

In 1937 von Weizsäcker proposed the proton - proton reaction chain as the origin of the sun power. Arguments about the insufficiency of the relevant cross section to account for astronomical observations were put to rest the following year by Bethe and Critchfield (1938), who developed a theory of that reaction, based on the work on β -decay by Fermi and by Gamow and Teller. Soon later, Bethe (1939) developed the theory of the CNO cycle of energy production in stars. In a few years, with important contributions by Bethe, von Weizsäcker,

Gamow, Teller and others, the basics of stellar nucleosynthesis were established. The main reactions were identified, their cross sections approximately computed and the results compared with available data on stellar composition.

This section addresses two basic questions :

1. What are the main energy-releasing reactions that take place in the core of the sun?
2. How much energy is released due to fusion reactions occurring in the sun?

2.3 Fusion Reactions in the Sun's Interior

2.3.1 Factors that determine the dominant reactions

The two factors which determine the most likely nuclear reactions are : the abundance of the reacting species and the reaction probability at the temperatures prevailing in the solar core . For example , hydrogen is by far the most abundant material in the universe – more than 90% of the atoms in the universe are hydrogen , and all but less than 1% of the remainder are helium. (This helium was found during the early stages of the evolution of the universe and not as a result of the latter stellar processes) .

The strong coulomb repulsion between positively charged nuclei increases as the product of their nuclear charges , so only the lightest elements will have appreciable reaction probabilities. The most abundant of these are: H, He, C, N,O,Ne,Mg & Si.

From the work of Bethe in 1938 , we know that two nuclear reaction cycles appear to be most promising in accounting for solar radiation production. Both of these have as end product the burning of four hydrogen nuclei to produce one helium nucleus.

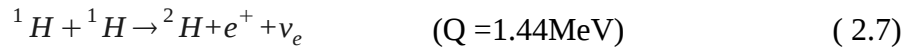
The two different fusion reactions in the sun that lead to the conversion of hydrogen to helium are the proton – proton cycle (pp cycle)and the carbon (CNO) cycle . Our sun's core temperature is about $10^7 K$ and the proton – proton cycle accounts for about 98% of the energy produced. However , if the core temperature were to increase to about $1.5 \times 10^7 K$, the contributions from the pp and CNO cycles would be approximately equal.

2.4 Hydrogen Burning

2.4.1 The proton – proton Cycle(The PP Cycle)

As a cloud of interstellar gas contracts to form a star , its temperature increases as gravitational potential energy is converted into kinetic energy. When the interior heats up to about $10^7 K$, the first thermonuclear reactions begin to take place converting hydrogen into helium in a sequence of reactions known as the *proton – proton cycle*. The PP cycle has more than one branch , but one of these , the PPI cycle , is dominant.

In the first hydrogen – burning reaction (PPI cycle) two protons (1H) fuse via the weak interaction to form the only stable two nucleon system, namely, the deuteron (*the energy released in each step is given in parentheses*)



whose nucleus contains one proton and one neutron. It requires a β -decay to occur during the time the protons are interacting with each other , creating a positron (e^+) and a neutrino. Conservation of charge is maintained in this reaction by the emission of the positron.

The first reaction, being a weak interaction, proceeds at an extremely slow rate and sets the scale for the long lifetime of the sun.

The probability of a β -decay during this short period is extremely low and so the reaction has a very small cross section ; for the formation of deuterium , the cross section (table 1.2) is calculated to be of the order of 10^{-26} barns at 10 KeV energies and 10^{-25} barns at 100 MeV energies . The reaction rate is nevertheless very small and even at high densities at the core of the sun (about 125 g/cm^3 , 7.5×10^{25} protons/ cm^3) , the reaction rate is only about 5×10^{-18} sec per proton . So , what keeps the sun radiating energy? It is the enormous number of reacting protons, of the order of 10^{56} , so that the total reaction rate is of the order of 10^{38} /sec .

Once the deuteron is formed, it very rapidly fuses with more hydrogen to produce light helium (${}^3\text{He}$) and a gamma ray via the electromagnetic interaction:



At this point, no deuterium – deuterium reactions will occur because it is extremely unlikely that a deuteron will collide with another deuteron. Given that the average lifetime of a proton before fusion is about 10^{18} s , we know that only one deuteron is produced per second for every 10^{18} protons and thus it is about 10^{18} times more likely that a deuteron will react with a proton than with another deuteron. Deuterons are thus “cooked” to ${}^3_2\text{He}$ nearly as rapidly as they are formed.

Note that the first two steps must occur twice before the last can take place and the energy released in this case is $E=11.82 \text{ MeV}$.

The reaction between ${}^3_2\text{He}$ and a proton is very unlikely since the nucleus ${}^4_3\text{Li}$ is unstable (unbound) and it breaks up as soon as it is formed: ${}^4_3\text{Li} \rightarrow {}^3_2\text{He} + {}^1_1\text{H}$.

By far the most probable fate of a ${}^3_2\text{He}$ nucleus is to react with another ${}^3_2\text{He}$ (Previously formed by the Proton – proton reaction) to produce α particle (ordinary helium ${}^4_2\text{He}$), two protons and another gamma ray via the nuclear strong interaction:



The relatively large energy release in this reaction is because ${}^4_2\text{He}$ is a doubly magic nucleus and so is very tightly bound. In this reaction chain a total of six protons are involved even though two of them are again released in the final step.

It is also unlikely for ${}^3_2\text{He}$ to react with ${}^2_1\text{H}$ because the density of ${}^2_1\text{H}$ is very low and because the ${}^2_1\text{H}$ is converted to ${}^3_2\text{He}$ very rapidly.

The termination of the proton – proton chain given in Equation 2.9 is not unique , although about 85% of the sun's energy is thought to be produced by reactions ending in that way.

The overall effect of the proton – proton chain of reactions (equations 2.7 - 2.9) is the conversion of four protons into helium nucleus (an alpha particle α)



and the energy released (eqn.1.1) is the difference between the mass of four protons and the mass of an alpha particle plus two positrons : $Q^1 = (4m_p - m_\alpha - 2m_e)c^2$ where the masses involved are here nuclear , not atomic – masses . It turns out to be $24.68 MeV$.

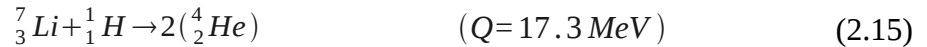
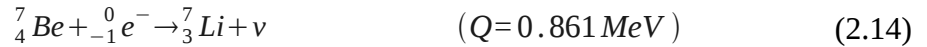
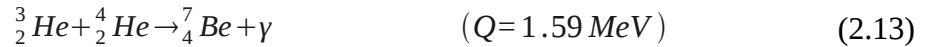
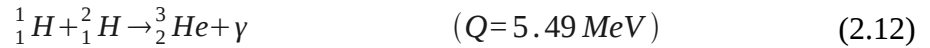
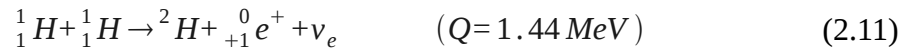
Thus, four protons are burned (consumed) and a helium nucleus along with two positrons, two neutrinos and two gamma rays . It is these gamma rays that are the ultimate source of most of the sun's electromagnetic radiation. Gamma rays have no charge and are just energetic photons of electromagnetic radiation.

Note that very high temperature and high densities are needed for protons to collide, not only with enough energy to fuse, but frequently enough to generate the sun's energy. Since the temperature of the sun is around $10^7 K$, all its material is fully ionized. Matter in this state is referred as plasma. The two positrons produced in the above fusion reactions will annihilate with two of the electrons in the plasma , turning their combined rest masses into gamma ray energy . Thus the energy equivalent of rest masses of four electrons(two positive and two negative) is also made available , or $Q^{11} = 4m_e c^2$. The total energy release for the PPI cycle is then $Q = Q^1 + Q^{11} = [4(m_p + m_e) - (m_\alpha + 2m_e)]c^2 = (4m_p - m_\alpha)c^2$ and yields $26.72 MeV$ (where the masses in the last part are now the customary atomic masses : $m_p = 1.007825u$, and $m_\alpha = 4.002603u$).

However, neutrinos are so penetrating that in essentially all cases they escape from the sun, each carrying energy $0.26 MeV$ on average, without interacting with any solar material.

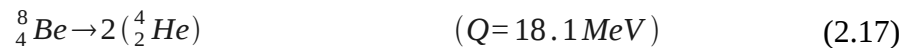
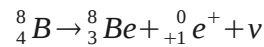
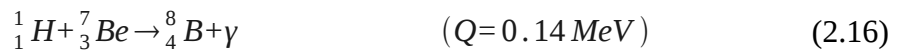
Some are intercepted by the earth , bringing us only direct information about the sun's interior. Subtracting the neutrinos energy (0.52MeV) leaves 26.2MeV per cycle available within the sun. Hence, the net energy released due to the burning of four hydrogen nuclei in the sun's core to form helium nucleus is 26.20 MeV .This is the energy liberated per alpha particle fused from protons . Thus on average, 6.55 MeV of electromagnetic energy is radiated from the sun for every proton consumed in the PPI chain. [3]

The other PPII - chain occur less frequently and so contribute a minor amount to the sun's luminosity. The PPII - chain is (*the energy released in each reaction is given in parentheses*)



The PPII - chain also converts hydrogen to two helium nuclei with the neutrinos taking away as much as 0.86 MeV .

The PPIII - chain involves the same first three steps as PPII - chain and goes on as

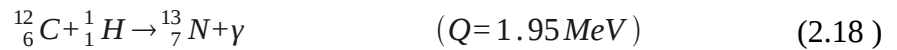


The net result is the formation of two helium nuclei with the neutrinos escaping with 7.2MeV .
The total energy released in the PPIII - chain is 18.1MeV . [6]

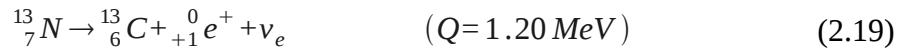
2.4.2 The Carbon cycle (CNO Cycle)

The other interesting cycle, the carbon, or CNO cycle, also plays a role in solar energy generation. This cycle acts as a key source of energy in more massive stars. The carbon cycle, the other chain responsible for energy production and hydrogen burning in stars, also converts hydrogen to helium , but it involves heavier elements carbon, nitrogen,oxygen present in the interior of the sun, requiring ^{12}C nucleus as a catalyst for the process, because it reappears at its end.

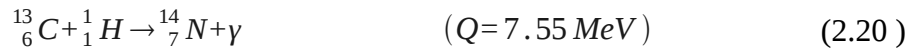
The carbon cycle begins with the reaction between a proton ^1H and ^{12}C nucleus to form ^{13}N nucleus (*the energy released in each step is given in parentheses*) :



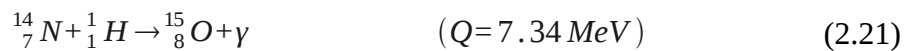
This ^{13}N undergoes beta decay to ^{13}C within a few minutes



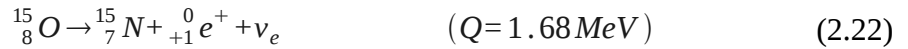
The stable isotope ^{13}C will react with a proton under solar interior conditions to form ^{14}N through



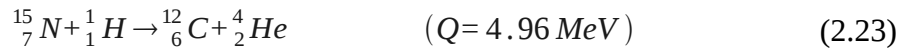
The nitrogen isotope is also stable , and another proton capture turns out to be the only important process, resulting in the formation of oxygen through



The ^{15}O nucleus beta decays within a few minutes to a stable nitrogen isotope through

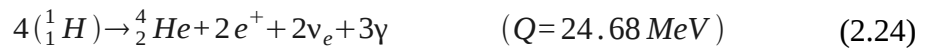


A proton capture on the stable ^{15}N produces two nuclei, ^{12}C and ^4He



The ^{12}C nucleus produced (2.23) is ready to re – enter the chain over and over again in the initial reaction to continue the process .

The final result of the carbon cycle is also the conversion of four hydrogen into helium nucleus



Unlike the first reaction in the pp chain , none of the reactions in the CNO cycle requires a beta decay to occur at the same time as fusion is taking place and so, in principle , the CNO reactions can proceed faster. However, the coulomb barrier also affects the reaction rates and it is much higher between the proton and a carbon, nitrogen, or oxygen nucleus than it is between two protons .So higher temperature is required for the carbon cycle.

As the stellar core heats up , therefore , the pp reactions begins first. Later , when the core temperature has increased sufficiently , more protons have thermal energies able to penetrate the higher coulomb barriers and the CNO reactions become more probable .

Of all possible fusion reactions , hydrogen burning generates the most energy per unit mass of matter(about 7MeV u^{-1}) and it is the source of power which maintains a star's stability for the greater part of its life . In the case of our sun , we can easily estimate its rate of fuel consumption . The flux of solar radiation reaching the earth (solar constant) is $1.4 \times 10^3 \text{W. m}^{-2}$ from which we deduce that the total solar out put is $4 \times 10^{26} \text{W}$, assuming it is radiated isotropically (distributed uniformly in space) . As we have seen , any of the hydrogen – burning

cycles of fusion reactions consumes four protons and liberates about 26MeV of energy .

Thus, to generate its output , we estimate that the sun uses up $(4 \times 10^{26} \text{ J/s}) / (26 \times 1.6 \times 10^{-13} \text{ J}) \approx 4 \times 10^{38}$ protons / sec .This means the sequence of fusion reactions must occur 10^{38} times per second. The sun contains about 7×10^{56} protons and , its present age , has burned about 10% of its original hydrogen .It is expected to continue to shine , as it does now , with little apparent change for about 6 billion years . The number of reactions , is therefore $(1/4 \text{ protons per reactions}) \times (7 \times 10^{56} \text{ protons}) = 2 \times 10^{56}$ reactions .

Each step in the CNO chain need occur only once to convert four protons to α particle . The second and the fifth steps take place because ^{13}N and ^{15}O are unstable isotopes , with half-lives of only a few minutes,(Recall that half life is the time in which half of the original quantity of an isotope has disintegrated into its more stable nuclear form.)

According to Einstein mass - energy relation ship $E= mc^2$, the sun is releasing energy at the expense of its own rest mass ,which is slowly decreasing . Of course , there is no plausible way of converting all the mass into energy. Only a certain fraction of the rest mass energy of the sun can be converted.

Because of the great binding energy of the helium nucleus ,furthermore ,the most abundant source of energy in stellar interiors appears to be the fusion of hydrogen atom into one helium atom.

The fraction of the rest mass of energy of the sun converted into radiation in each of these conversion processes is , fraction converted (f) = $26.73\text{MeV} / (4 \times 931\text{MeV}) = 0.007$ i.e the transmutation of hydrogen into helium will liberate 0.7% of the rest mass of the sun in the form of energy. The interior of the sun covers 10% of the mass of the sun . So $10\% \times 0.007 = 0.07\%$ of the rest mass of the sun is converted into solar energy per reaction.

2.5 Helium Burning

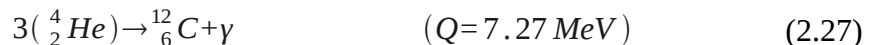
When the hydrogen fuel in the stellar core is exhausted, gravitational contraction compresses the core and raises its temperature to $10^8 K$ and produces high density value (about $10^6 g/cm^3$) needed to penetrate the coulomb barrier so that helium nuclei fusion reactions can take place to form an equilibrium mixture with heavier element ${}^8 Be$ via the reaction



In this reaction two α particles (4He nuclei) combine for a very short time ($\approx 10^{-16} s$) in an unstable beryllium-8 isotope nucleus. Despite the very short lifetime of beryllium-8 nucleus, when the above mentioned values of the density and the temperature have been attained, it manages to take part in the reaction with another helium-4

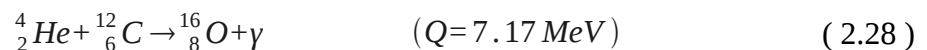


Overall three helium-4 nuclei will form carbon-12



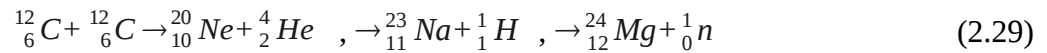
The energy released in this fusion reaction is $7.27 MeV$. This reaction is known as the triple alpha process, the first stage of helium burning. The intermediate beryllium-8 is not very stable, and the back reaction occurs readily. Nevertheless an equilibrium is established where some Beryllium-8 takes part in the second step.

At still higher temperature, the presence of ${}^{12}C$ enables another series of fusion reactions to occur, in addition to the CNO Cycle. Thus, ${}^{16}O$ can be produced via the α - capture reaction



2.6 Beyond Helium Burning

The next stage after helium burning is reached when the carbon /or oxygen core has shrunk until the temperature at the sun's center has reached about 5×10^8 K and there is enough relative kinetic energy available to allow carbon nuclei to interact with each other at a significant rate producing ^{20}Ne , ^{23}Na and ^{23}Mg via the reactions:



Note that two of these reactions produce neutrons and protons which are used to synthesize heavy elements .

Other reactions involving fusion of light nuclei and helium-4 capture can continue to release energy, until the process ends near $A \approx 56$ (i. e. the peak of the binding energy per nucleon curve), beyond which there is no energy gain in fusing nuclei. Note that the energy produced by fusion in the sun increases the pressure inside the sun and prevents its collapse due to gravity [7].

More generally , all the major two particle combinations have unstable ground states:
 $P+P \rightarrow \text{Helium 2 (unstable)} \rightarrow P+P$, $P+\text{Helium}-4 \rightarrow \text{(unstable)} \rightarrow P+\text{Helium}-4$

$\text{He}-4 + \text{He}-4 \rightarrow \text{(unstable)} \text{He}-4 + \text{He}-4$.That is , the nuclear force produces no two particle exothermic reactions in a gas of protons and an alpha particles . Thus one is led to look either for more peculiar reactions between those particles or for reacting with rarer constituents of the gas .

Chapter Three

3.1 Controlled Fusion Reactions

Introduction

As a controlled source of power on earth, fusion has even greater potential than fission and the rewards for harnessing it are great. Light nuclei are more plentiful than fissile nuclei, and there would be much less radioactive waste from a fusion reactor than from a fission reactor. Furthermore, any radioactivity which might be produced would decay away relatively rapidly and there would be no need to store the waste for geological periods of time.

In a fusion reactor, it is intended to generate this energy by heating the reactants. When fusion is driven by heat energy, the process is called thermonuclear fusion.

Thermonuclear fusion is the principle behind the thermonuclear bomb in which detonation of a fission raises the temperature enough to trigger fusion in a core made of light nuclear material and has been achieved in the laboratory, but it requires a temperature of about a 100 million degrees and maintain the required conditions stably and effectively.

3.2 Possibility of Controlled Fusion

The essence of controlling fusion reactions and extracting usable energy is to heat a thermonuclear fuel to temperatures 10^8K (assuming mean kinetic energies $\epsilon = k_B T = 10\text{KeV}$) while simultaneously maintaining a high enough density for a long enough time that the rate of fusion reactions (Deuterium-tritium reaction in this case) will be large enough to generate the desired power. At these temperatures, the atoms must become ionized (for hydrogen, only 13.6eV is needed to strip the electron), and so the fuel is a hot mixture of clouds of positive ions and negative electrons but it is overall electrically neutral. Such a situation is called plasma, and the dynamical equations that govern plasma behavior are beyond the scope of this discussion.[5]

At the temperatures required for fusion, any material container will vaporize and so the central problem is how to contain the plasma for sufficiently long times for the reaction to take place.

There are at present two schemes under investigation for confining the thermonuclear fuel: *Magnetic confinement* and *inertial confinement*. A brief discussion of these will be done in the following section.

3.2.1 Bremsstrahlung Losses

Confinement of plasma is of course not absolute – there will be many ways for plasma to lose energy. The primary mechanism is *bremsstrahlung*, in which the coulomb scattering of two particles produces an acceleration which in turn give rise to the emission of radiation.

The largest accelerations are suffered by the lightest particles (electrons), but because the electrons and ions are approximately in thermal equilibrium, any loss by the electrons is felt also by the ions, which are then less energetic and less successful in penetrating the coulomb barrier.

From electromagnetic theory we know that any moving accelerated charged particle radiates energy in the form of electromagnetic radiation. From electromagnetic theory, the energy radiated per unit time (the power) by an electron experiencing acceleration a is

$$\frac{dE}{dt} = P_{br} = \frac{e^2 a^2}{6\pi\epsilon_0 c^3} \quad (3.1)$$

If the electron is at a distance r from an ion of charge Z , the acceleration is

$$a = \frac{F}{m_e} = \frac{Ze^2}{4\pi\epsilon_0 m_e r^2} \quad (3.2)$$

If τ is the characteristic time during which the ion and electron interact, then the number of ions encountered at a distance r will be

$$(n)(v_e \tau)(2\pi r dr) \quad (3.3)$$

where n is the density of positive ions .

Thus , the contribution to the total energy per unit time, from electrons scattered at impact parameters between positions r & $r+dr$ is

$$dP = \frac{e^2 n}{6\pi\epsilon_0 c^3} \frac{Z^2 e^4 (v_e \tau)(2\pi r dr)}{(4\pi\epsilon_0)^2 (m_e^2 r^4)} \quad (3.4)$$

The characteristic interaction time τ can be estimated as r/v_e and thus

$$dP = \frac{4\pi e^6 Z^2 n}{3(4\pi\epsilon_0)^3 (m_e^2 c^3)} \frac{dr}{r^2} \quad (3.5)$$

Integrating from minimum values to maximum values of r gives the total power radiated by a single electron

$$\int dP = \frac{4\pi e^6 Z^2 n}{(4\pi\epsilon_0)^3 (3m_e^2 c^3)} \int_{r_{\min}}^{r_{\max}} \frac{dr}{r^2} \quad (3.6)$$

and multiplying the result by the density of electrons n_e gives the power per unit volume radiated by the plasma. We can take $r_{\max} \rightarrow \infty$ and for r_{\min} it is tempting to try the distance of closest approach, which for 10KeV electrons turns out to be 144Z Fermi.

If we calculate the quantum mechanical uncertainty in the electron's position , $\Delta p \approx p \approx 100 \text{ KeV}/c$,then Δx is the order of 2000fm. We therefore cannot specify r_{\min} as precisely as 144Z Fermi, and we estimate should take as better estimate $r_{\min} = \hbar/m_e v_e$.

Now carry out the integration, the power per unit volume radiated in bremsstrahlung becomes

$$P_{br} = \frac{4\pi n n_e}{3(4\pi\epsilon_0)^3} \frac{Z^2 e^6 v_e}{m_e c^3 \hbar} \quad (3.7)$$

Continuing the estimate, we put for v_e the velocity corresponding to the mean kinetic energy of the Maxwell- Boltzmann distribution,

$$v_e \simeq \sqrt{\frac{(3k_B T)}{m_e}} \quad (3.8)$$

Evaluating all the numerical coefficients , the final estimate is

$$P_{br} = 0.5 \times 10^{-36} Z^2 n n_e (k_B T)^{\frac{1}{2}} W / m^3 \quad (3.9)$$

where $k_B T$ is in KeV , and $n n_e$ is the product of the densities of the two fusing ions .

The effective energy produced by the fusion process will be reduced by the heat radiated by the hot plasma . The mechanism for this is predominately electron bremsstrahlung. For fixed ion densities, the power loss per unit volume (equation 3.9) due to bremsstrahlung process is proportional to $T^{1/2} Z^2$, where Z is the atomic number of the ionized atoms and T is the temperature in Kelvin. This shows

1. fusion reactions using nuclei other than hydrogen isotopes have substantially greater bremsstrahlung losses as well as generally smaller reaction rates in the Kev region(because of Coulomb barrier)
2. for a plasma with given constituents and at a fixed ion density , there will be a minimum temperature below which the radiation losses will exceed the power produced by fusion. Therefore choose to operate our fusion reactor at a temperature where the power gain from fusion exceeds the bremsstrahlung losses. Other radiation losses , including synchrotron radiation from charged particles orbiting about magnetic field lines , can also be neglected. The fusion reactor will have a net energy gain if the energy released in fusion reactions exceeds the radiation losses and the original energy investment in heating the plasma to the operating

temperature. If we operate at a temperature at 10KeV, the deuterium – tritium fusion gain is greater than the radiation loss, and we can neglect the loss in energy to radiation. [5]

3.2.2 Reactivities for Controlled Fusion Fuels

Graphs of the variation of the reactivity with temperature (in KeV) for several fusion reactions, obtained by numerical integration of equation 1.28 with the best available cross-sections, are shown in Fig. 3.1 for the reactions of interest to controlled fusion. Different reactions peak at different temperatures.

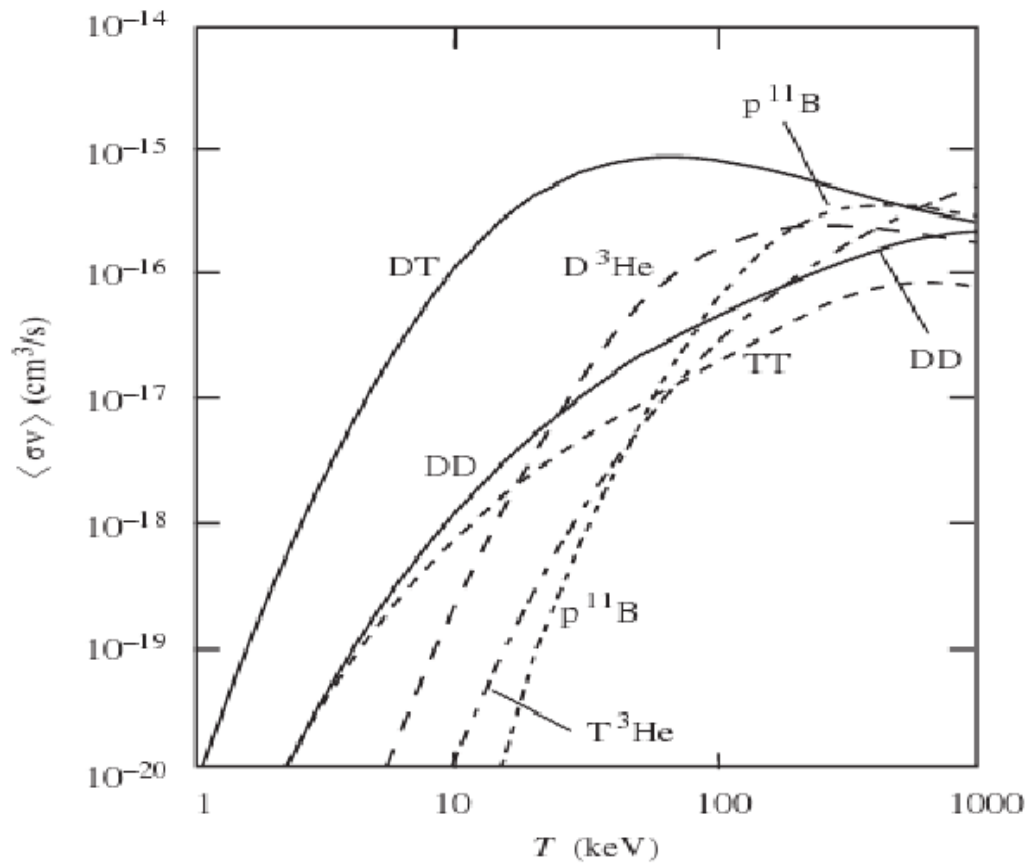


Figure 3.1 Maxwell – averaged reaction reactivity versus temperature for reactions of interest to controlled fusion.

In the figure we see that the deuterium - tritium reaction has the largest reactivity in the whole temperature interval below 400KeV. The deuterium-tritium reaction has a broad maximum(peaks) at about 64keV and becomes less favorable compared with some other

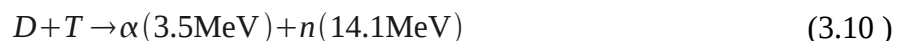
reactions at very high temperatures. However, $k_B T$ in a practical thermonuclear reactor is likely to be between 10KeV and 30KeV and, in this range, the deuterium – tritium is 100 times larger than that of any other reaction at 10- 30KeV and 10 times larger at 500KeV.

The second most probable reaction is deuterium - deuterium at temperatures $T < 25\text{KeV}$, while it is deuterium- Helium-3 for $25 < T < 250\text{KeV}$. The reactivity of proton- Boron-11 equals that of deuterium- helium-3 at temperature about 250KeV and that of deuterium - tritium at about 400KeV. At such very high temperatures other reactions (such as Tritium – ^3He , proton - beryllium-9, deuterium- lithium-6) have reactivity comparable to that of proton-boron-11, but they are less interesting for controlled fusion because the fuels involved either contain rare isotopes or generate radioactivity (Dawson 1981).

3.2.3 Main Controlled Fusion Fuels

First, we consider the reactions between the hydrogen isotopes deuterium and tritium, which are most important for controlled fusion research. Due to $Z=1$, these hydrogen reactions have relatively small values of Gamow energy (ϵ_G) and hence relatively large tunnel penetrability. They also have a relatively large S.

In deuterium- tritium(DT) reactions one atom of deuterium and one atom of tritium combine to form α – *particle* and a neutron.

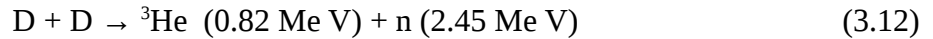


has the largest cross-section, which reaches its maximum (about 5 barn) at the relatively modest energy of 64KeV (see Fig. 1.3). Its $Q_{DT}=17.6\text{MeV}$ is the largest of this family of reactions. It is to be observed that the cross section of this reaction is characterized by a broad resonance for the formation of the compound ^5He nucleus at $\sim 64\text{KeV}$. Therefore, the astrophysical factor S exhibits a large variation in the energy interval of interest.

The energy released in this reaction (equation 3.10) is 17.6MeV. This reaction would likely to show a particularly large energy release of 17.6MeV per reaction . If the incident particles have

negligibly small kinetic energies, alpha particle and neutron share 17.6MeV consistent with linear momentum conservation , and a mono energetic neutron with energy 14.1MeV emerges.

The deuterium - deuterium (DD) reactions



are nearly equal probable. In the 10–100 KeV energy interval, the cross sections for each of them are about 100 times smaller than for deuterium - tritium.

The tritium - tritium (TT) reaction



has cross section comparable to that of deuterium - deuterium. Notice that since the reaction has three products, the energies associated to each of them are not uniquely determined by conservation laws.[1]

3.3 Controlled Fusion Reactors

3.3.1 Lawson's criterion

A preliminary stage on the way to either the break – even or ignition points is to be able to confine a hot , reacting plasma long enough that the nuclear energy produced exceeds the energy required to create the plasma (thermal energy supplied to heat the plasma). This leads to a requirement , known as the Lawson criterion, for the product of plasma density (n) and confinement time (τ). It can be estimated as follows : The fusion energy output(E_f) is

$$E_f = R_{12} Q \tau \quad \text{or} \quad E_f = n_1 n_2 \langle \sigma v \rangle Q \tau \quad (3.14)$$

which is reaction rate(equation 1.11) multiplied by the energy released per reaction(Q value)

times the confinement time during which reaction takes place.

If particles 1 and 2 are different , as they are in a deuterium – tritium (DT) plasma ,and equal numbers of each, we have $n_1=n_2=n/2$ and equation (3.14) become

$$E_f = \frac{1}{4} n^2 \langle \sigma v \rangle Q \tau \quad (3.15)$$

For deuterium – deuterium plasma, there is an additional factor of 2 multiplying the right - hand side of this expression (assuming $n \gg 1$) because each deuteron can interact with all the remaining gas atoms and not just half of them.

There are n ions and n electrons in the plasma and, in equilibrium , each has to be given the same initial , average thermal kinetic energy $(3/2)k_B T$. So , the energy required to create the plasma is

$$E_p = 3nk_B T \quad (3.16)$$

The Lawson criterion requires that $E_f > E_p$. From equations (3.15) and (3.16), this criterion can be written as

$$n \tau > \frac{12k_B T}{\langle \sigma v \rangle Q} \quad (3.17)$$

If a deuterium – tritium plasma is operated at $k_B T = 20 \text{KeV}$, we obtain , from Figure(1.5)we can estimate $\langle \sigma_{dt} v \rangle \approx 4.5 \times 10^{-16} \text{cm}^3 \text{s}^{-1}$. Also from equation (3.10), $Q_{DT} = 17.6 \text{MeV}$ which gives the Lawson criterion

$$n \tau > 3 \times 10^{19} \text{m}^{-3} \text{sec}. \quad (3.18)$$

This means that if, for example , $n = 10^{20} \text{m}^{-3}$, the confinement time must exceed about 0.3s.

A deuterium – deuterium plasma would need to be heated to a higher temperature because of bremsstrahlung losses . Operating at 100KeV taking $\langle \sigma v \rangle \approx 10^{-22} \text{m}^3 \text{s}^{-1}$ leads to a value for

the product

$$n\tau > 3 \times 10^{21} m^{-3} sec. \quad (3.19)$$

to meet the Lawson criterion , which is about 100times larger than that for the deuterium – tritium plasma.

Thus temperature , a very high particle density and a long confinement time all have to be attained simultaneously in an operating reactor , and a quantity called the triple product $n\tau T$ is often used by designers as a measure of the difficulty of meeting a particular target criterion. In our examples, the triple products to meet the Lawson criterion are $6 \times 10^{20} m^{-3} .sec.KeV$ for deuterium – tritium at 20KeV and $3 \times 10^{23} m^{-3} .sec.KeV$ for deuterium – deuterium at 100KeV.

Unfortunately , a thermonuclear reactor that can deliver a net power output over a reasonable time interval is not yet a reality , and many problems must be solved before a successful device is constructed. Let us turn now to an examination of the basic nuclear reactor types and see how close they come to meeting the Lawson Criterion.

At the temperatures required for fusion, any material container will vaporize and so the central problem is how to contain the plasma for sufficiently long times for the reaction to take place . The two main methods used for these purpose are *magnetic confinement fusion (MCF)* and *inertial confinement fusion(ICF)* . Both techniques present enormous technical challenges . In practice most work has been done on magnetic confinement fusion (MCF) and so this method will be discussed in more detail than the inertial confinement method(ICF). [7]

3.3.2 Magnetic Confinement Fusion (MCF)

The only known method of confining plasma for prolonged periods is its thermal insulation by means of a magnetic field . This method was proposed in the USSR in 1950 (I. E. Tamm et al.) and in the USA in 1951(L. Spitzer). The idea is based on the use of pinching , i.e. , transverse compression of the plasma by an electric current passing through it .

In magnetic confinement, the plasma is confined by magnetic fields and heated by electromagnetic fields. Firstly we recall the behavior of a particle of charge q in a uniform magnetic field B , taking the two extreme cases where the velocity v of the particle is (a) at right angles to B and (b) parallel to B . In case (a) the particle traverses a circular orbit of fixed radius (compare the principle of the cyclotron) and in case (b) the path is a helix of fixed pitch along the direction of the field (compare the motion of electrons in a time projection chamber).

Two techniques have been proposed to stop particle losses: magnetic 'mirrors' and a geometry that would ensure a stable indefinite circulation. In the former, it is arranged that the field in a region is greater at the boundaries of the region than in the interior. Then as the particle approaches the boundary, the force it experiences will develop a component that points into the interior where the field is weaker. Thus the particle is trapped and will oscillate between the interior and the boundaries. However, most practical work has been done on case (b) and for that reason we will restrict our discussion to this technique.

The simplest configuration is a toroidal field produced by passing a current through a doughnut-shaped solenoid. In principle, charged particles in such a field would circulate endlessly, following helical paths along the direction of the magnetic field. In practice, the field would be weaker at the outer radius of the torus and the non-uniformity of the field would produce instabilities in the orbits of some particles and hence lead to particle loss. To prevent this effect a second field is added called a poloidal field. It can be achieved using a set of external coils. This produces a current around the axis of the torus and under the combined effect of both fields, charged particles in the plasma execute helical orbits about the mean axis of the torus. The current serves the dual purpose of heating the plasma and confining the particles.

Most practical realizations of these ideas are devices called *tokamaks* (after the Russian acronym for the device), in which the poloidal field generated along the axis of the torus through the plasma itself. The tokamak is at present one of the two most promising candidates for the basic design of a fusion power reactor.

The largest tokamaks currently in operation are JET which is a European collaboration and sited at the culham Laboratory in Berkshire, UK , TFTR(USA) and JT -60 Japan . Figure 3.2 shows a sketch of the main field components of the Joint European Torus (JET) , which is a major project set up by several European countries to study the conditions and and technical requirements for magnetic confinement fusion(MCF) .

The strong , main toroidal field is generated with external coils as shown .The current generating the poloidal field is induced by transformer action on the plasma . The plasma itself forms a single secondary turn. A current pulse in the primary windings of the transformer induces a large current of up to 7MA in the plasma. This current not only generates the required poloidal field , but also provides several megawatts of resistive heating to the plasma . Unfortunately , this form of ohmic plasma heating is not sufficient to raise the temperature high enough for fusion to occur because plasma resistance R decreases with temperature and power input ($i^2 R$) becomes less efficient as the plasma heats up . [7]

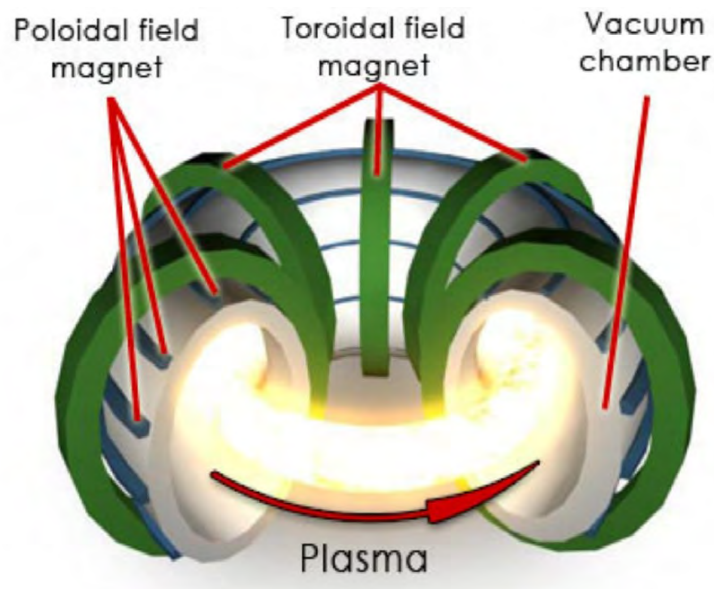


fig.(a)

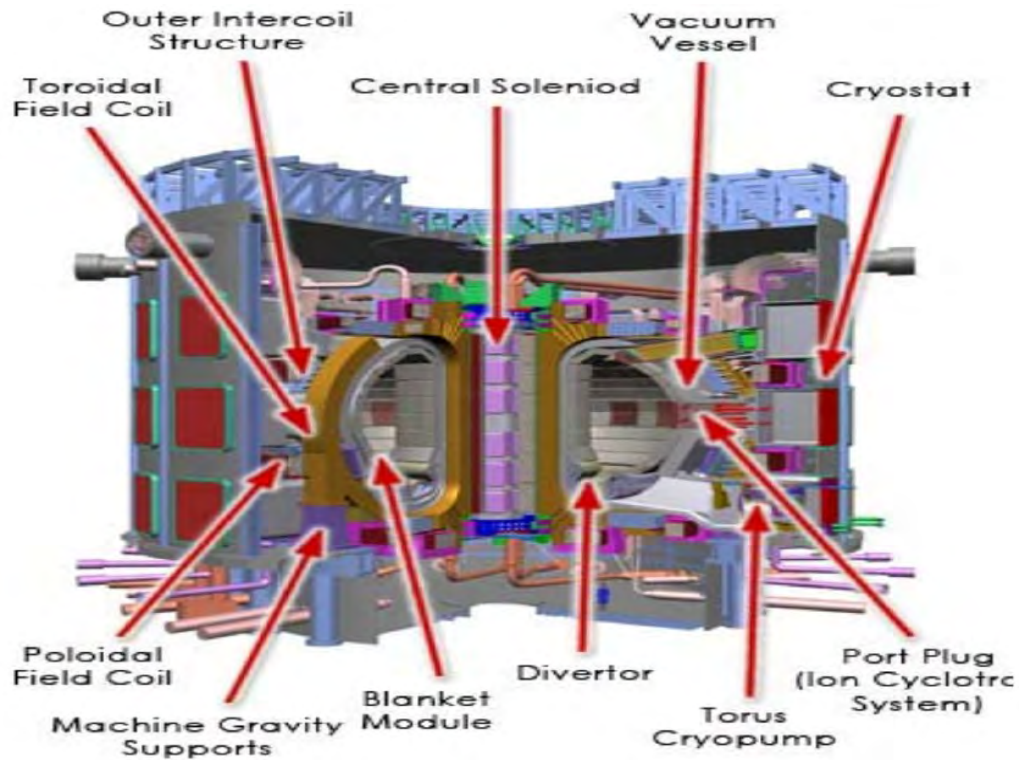


fig. (b)

Figure 3.2 Schematic diagrams showing : (a) the main magnetic field components of the JET tokamak; (b) how these elements are incorporated into the JET device .

In most tokamaks , additional heating is provided by a combination of powerful radio-frequency (r .f) sources and neutral beam injection(NBI). In radio- frequency heating , high power radio – or micro- waves are directed into the plasma , which will absorb energy resonantly if it is delivered at either the electron or ion cyclotron frequencies .In neutral - beam heating , hydrogen or deuterium ions are accelerated to energies up to 100Kev and then neutralized by charge – exchange reactions in a region of hydrogen or deuterium gas before being injected into the plasma . Since these fast atoms are neutral, they are unaffected by the magnetic field until they become ionized or undergo charge exchange during collisions inside the plasma . They then become trapped in the field and transfer their energy to the plasma by collisional energy loss. Neutral – beam injection has produced ion temperatures up to 30Kev in the JET and TFTR tokamaks . It can also be used to refuel the plasma as the deuterium and tritium become depleted .

In the past decade, considerable progress has been made in approaching the basic criteria for achieving ignition in a magnetically confined plasma . Each of the required values of n, τ and T for ignition have been reached , but not all at the same time . The triple product has been increased by several orders of magnitude within the past decade and now is close to meeting the Lawson criterion and about a factor of six from ignition.

In December 1997, a D – T plasma in JET reached a peak fusion power output of 16MW and a power of 10MW was sustained for at least half a second .The ratio of output power to net input power to the plasma was 0.65 (more than double the previous record) and the burning of the D – T fuel was as expected . These are impressive achievements .However, a great deal of work still needs to be done even to reach ignition for the first time and then much more to demonstrate that efficiencies can be improved to the level required for commercial fusion power production.

3.3.3 Inertial Confinement Fusion (ICF)

Difficulties with achieving stable plasma confinement in a magnetic field led to a radical , alternative proposal for obtaining controlled thermonuclear power . As shown in figure 3.2 in inertial confinement fusion(ICF) , a laser pulse of energy is directed from several directions at once on to a small pellet of fusible material – such as a frozen D -T mixture .

The laser pulses are extremely short , typically 10^{-7} - 10^{-9} s. , which is many orders of magnitude shorter than the times associated with the pulsed poloidal current in a tokamak (which could be as long as 1s), but this is compensated for by much higher plasma densities. The energy is delivered with such power that material is heated and violently ejected from the surface . When this happens , material inside the surface is driven inwards , compressing the core and raising its temperature to the point at which fusion occurs at a high rate . The whole process is then repeated in what amounts to a series of micro thermonuclear explosions .

The number of pellets used per unit time need not be high and each one is small . For reference , complete conversion of 1mg of D -T (containing 2.4×10^{20} atoms) liberates about 350MJ of energy , and an ICF reactor might be designed to operate at 10 micro explosions per

second , each consuming 1mg of material .This would generate about 3.5GW of raw power , which would be absorbed in a surrounding thermal blanket and converted into electric power by conventional means .

In ICF , no attempt is made to contain the reacting atoms although the time during which fusion occurs can be increased to some extent by using tampers of higher density materials to slow down the expansion of the ignited core. The problem of containment is replaced with the major technical difficulty of generating and directing sufficient power to trigger fusion. Proposals include the use of high – power , pulsed lasers or intense beams of charged particles to bombard the pellets.

To appreciate some of the technical challenges facing developers of ICF we will make crude estimates of the main parameters . These are the confinement time and the compression factor .The product has to be large enough so that a significant fraction of the material undergoes fusion before the reacting nuclei fly apart . We will assume an equal mixture of deuterons and tritons reacting at $k_B T = 20\text{KeV}$ - the most favorable case .

The confinement time is determined by the relative speed v of the heated ions and the radius r of the compressed fuel pellet . At 20KeV , $v \approx 2 \times 10^6 \text{ m s}^{-1}$ and the radius of a compressed fuel pellet will be no more than about 0.2mm , giving an estimate for the confinement time τ of $r/v \sim 10^{-10} \text{ s}$ (100ps) .

The condition for the product $n\tau$ is roughly the same for ICF as for MCF described above . So , taking the value $n\tau = 3 \times 10^{19} \text{ m}^{-3} \cdot \text{s}$, obtained from Equation (3.17) for the Lawson criterion for deuterium - tritium(D – T) at 20KeV , we obtain $n \approx 3 \times 10^{29} \text{ m}^{-3}$, which is about 10 times the atom density of ordinary liquid or solid hydrogen . However , meeting the Lawson criterion in this example requires only enough energy from fusion to raise the average energy of an atom in the pellet to 20KeV . Very little of the material in the pellet is used up in achieving this.[7]

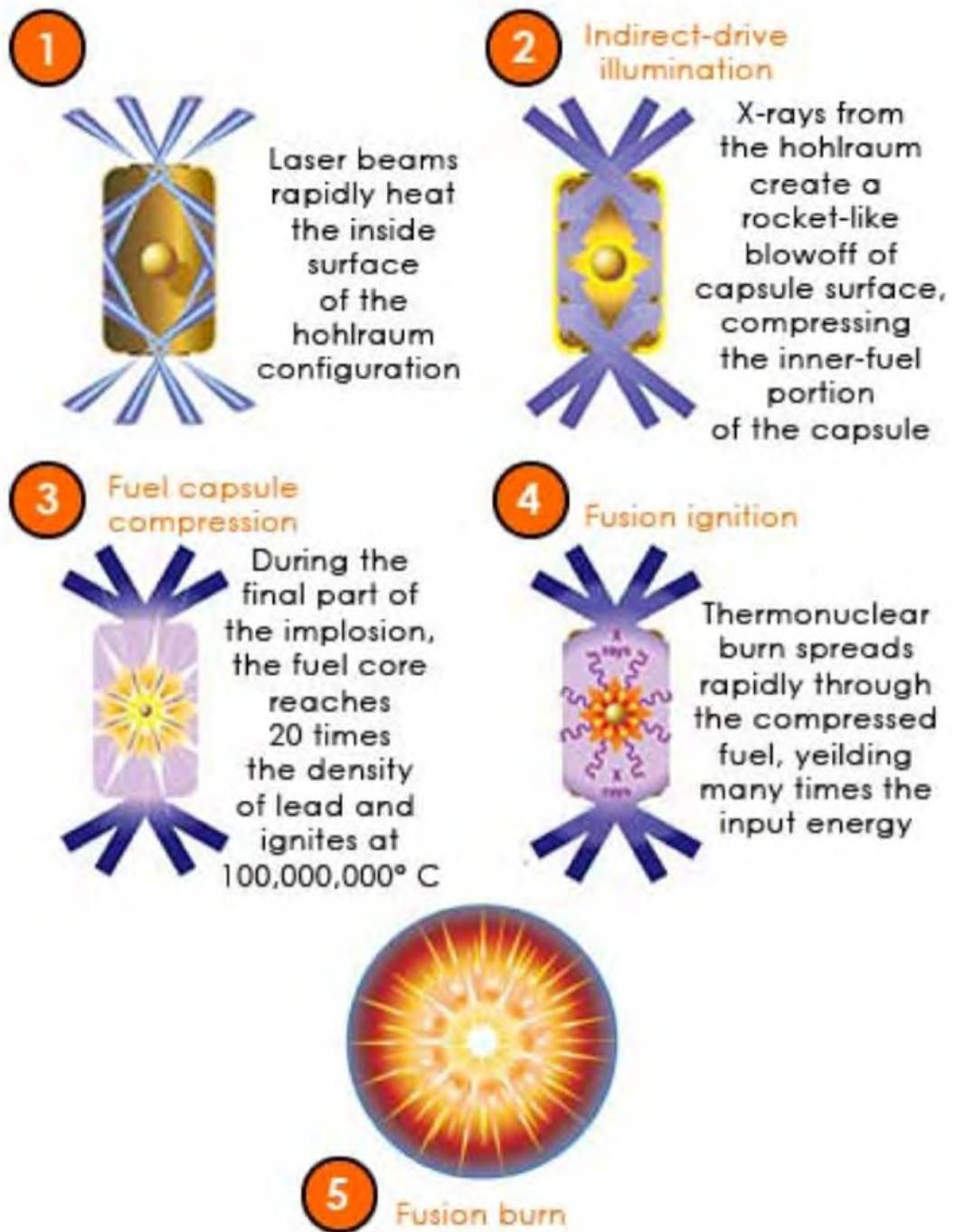


Figure 3.3 Fusion – reactor – , showing laser pulse of energy directed from several directions on to a small pellet of fusible materials .

Chapter Four

4.1 Uncontrolled Fusion Reaction - The Hydrogen Bomb

In Chapter two we have seen that how light nuclei usually hydrogen isotopes deuterium and tritium are fused and produce tremendous amount of energy for the sun and other stars . It is also the same sort of power that is given off by hydrogen bomb, but obviously , the hydrogen bomb is uncontrolled fusion.

Prior to the development of the atomic bomb , the temperatures required to initiate nuclear fusion on earth was unattainable . When it was found that the temperatures inside an exploding atomic bomb are four to five times the temperatures at the center of the sun , thermonuclear bomb was but a step away .

Different fusion reactions are the source of energy for man – made and other types of hydrogen bombs. Great as the energy release is in the hydrogen bomb explosion ,it is very small compared to the amount of energy release by the sun.

Scientists who worked on the first fission(atomic) bomb during world war II were aware of the potential for building an even more powerful bomb that operated on fusion principles .Here is how it would work.

The Joint European Torus(JET) Nuclear Fusion Reaction : The core of the fusion bomb consist of a fission bomb ,such as the one they were then developing .The core could then be surrounded by a casing filled with isotopes of hydrogen. Isotopes of casing are various forms of hydrogen that all have a single proton in their nucleus, but may have zero, one, or two neutrons. The nuclei of the hydrogen isotopes are the proton, the deuteron, and the triton.

Imagine that a device such as the one described here could be exploded. In the first fraction of a second, the fission bomb would explode, releasing huge amount of energy. In fact, the temperature at the heart of the fission bomb would reach a few millions degrees, the only way

that humans know of for producing such high temperatures.

The temperature would not last very long, but in the microseconds that it did exist, it would provide the energy (high temperature) for fusion to begin to occur within the casing surrounding of the fission bomb. Protons, deuterons, and tritons would begin fusing with each other, releasing more energy, and initiating other fusion reactions among other hydrogen isotopes. The original explosion of the fission bomb would have ignited a small star – like reaction in the casing surrounding it.

Note:

➤ *Hydrogen isotopes have been made to undergo fusion in uncontrollable manner in a fission bomb. This is the so called hydrogen bomb.*

➤ *A hydrogen bomb, which is a fusion bomb, cannot explode until it is "ignited"*

We can see that there is no such a thing as a "baby" hydrogen bomb. It cannot be less energetic than its fuel, which is an atomic bomb. The hydrogen bomb is another example of a new discovery applied to destructive rather than constructive purposes. The constructive side of the picture is the controlled release of vast amounts of clean energy.

From military standpoint , the fusion bomb had one powerful advantage over the fission bomb . For technical reasons , there is a limit to the size one can make a fission bomb . But there is no technical limit on the size of a fusion bomb . One simply makes the casing surrounding the fission bomb larger and larger , until there is no longer a way to lift the bomb into the air so that it can be dropped on an enemy.

On August 20,1953, the former Soviet Union(now Russia) announced the detonation of the world's first fusion bomb. It was about 1,000 times more powerful than was the fission bomb that has been dropped on Hiroshima less than a decade earlier. The Soviet(now Russia) and the United States became proficient at manufacturing fusion bombs on an almost assembly – line schedule .

Conclusions

Fusion involves low – mass nuclei whose combined mass is more than the resulting fused massive nucleus. The mass that is given up to form the massive nucleus is converted to energy. Einstein relation $E=mc^2$ tells how much energy(E) can be made from matter with mass m .Since the speed of light squared is a very large value and so a little bit of mass can produce a lot of energy.

At very high temperatures, electrons are stripped from atomic nuclei to form a plasma (ionized gas). Under such conditions, the repulsive electrostatic forces that keep positively charged nuclei apart can be overcome, and the nuclei of select light elements can be brought together to fuse and form other elements. Nuclear fusion of light elements releases tremendous amounts of energy and is the fundamental energy-producing process in stars.

In the cores of the sun and other stars , four hydrogen nuclei, each with the mass of one proton, are fused together to form a single helium nucleus . So the sun produces a tremendous amount of energy and could last for about 10 billion years on hydrogen burning in its core.

On the earth neither the proton – proton nor the carbon cycle offers any hope of practical application, since their several steps require a great deal of time even at high temperatures and densities. The efficiencies of these cycles vary differently with temperature , with the proton – proton cycle predominant at the sun's core whose temperature is about 10^7K .

For a given mass of fuel, the energy released from fusion substantially exceeds the energy released from fission (the neutron-induced splitting of heavy elements such as uranium) and far exceeds (by millions of times) the energy released in chemical reactions (e. g., the burning of coal, gas, or oil).

Some fusion reactions are easier to produce than others. Some products of the various fusion reactions are more desirable than others. The first generation of fusion plants is expected to employ a plasma fuel mixture of the less common isotopes of hydrogen: deuterium and tritium. A helium nucleus and a neutron are the products of this fusion reaction.

By the standard measures of environmental impact (land use, mining, emissions, waste disposal, etc.), fusion should rank as an attractive energy source. With only a small inventory of fusion fuel in the machine at any time, the risk of accidental power excursions is negligible. Advanced fusion fuel cycles can be expected to reduce both the tritium and neutron-activation concerns but will require further development.

Two general approaches to fusion technology have received emphasis: magnetic confinement and inertial confinement. In magnetic fusion, a hot plasma (more than 100 million degrees Celsius) is confined by strong magnetic fields and heated by external sources until the fusion reaction becomes self-sustaining. A "burning" plasma, with sufficient size and density to sustain itself by self-heating, is said to be "ignited." The magnetic field also prevents the hot plasma from contacting the machine structures. In inertial fusion, small fuel pellets are bombarded by intense laser or particle beams to produce an ignited, dense plasma, held by inertia for brief energy-producing pulses. Several distinct variations of these broad approaches are being pursued in a number of laboratories worldwide.

Progress in fusion research begun more than 40 years ago, fusion research, particularly in the last two decades, has made outstanding progress, incorporating state-of-the-art advancements in physics understanding and technological developments. Producing 16 megawatts of deuterium-tritium fusion power in 1997 in the Joint European Torus (JET) and 10.7 megawatts (approaching energy break-even) in the Tokamak Fusion Test Reactor (TFTR) at Princeton Plasma Physics Laboratory in 1994 are important milestones in MFE development.

Experiments continue to be performed in specific tokamak operating around the world to build upon these achievements and to provide increased scientific understanding. Recent advances include the achievement of improved energy confinement and improved methods for handling high heat fluxes.

With the object of sharing the benefits and reducing the costs to each country for a "next step" burning plasma tokamak experiment, Canada, China, the European Community, Japan, the

Russian Federation, and the United States are involved in negotiations to collaborate on the design, construction, and operation of the International Thermonuclear Experimental Reactor (ITER). The programmatic objective of ITER is to demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes. The ITER timetable envisions a 10-year construction phase followed by a 20-year operation phase. ITER will produce about 500 megawatts of fusion energy. [12]

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