DYNAMIC ANALYSIS OF LIQUID CONTAINING CYLINDERICAL TANKS

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A Thesis Submitted to the School of Graduate Studies of Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering (Structures)

Addis Ababa

October, 2005 G.C.

ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

FACULTY OF TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING

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Acknowledgment

First of all I would like to praise Almighty God for helping me accomplish my master's degree. Then I would like to forward my deepest gratitude to Dr. Ing. Adil Zekaria for providing me with all the helps I needed from the inception of the title up to the completion.

I owe a debt of gratitude to all my instructors Dr. Shifferaw Taye, Dr. Girma Zereyohannes, Dr. Asnake Adamu, Dr. Bedlu Habte and the Technology Faculty as a whole for their help during my stay in Addis Ababa University.

I would like to thank my father, Seyoum Asres, for his deepest concern and for all his help and support he has provided me. He has been my backbone in every aspect. I also like to thank C.D.S.Co. Awassa branch Staffs and Rama Construction PLC.

Lastly but not least I would like to thank all my friends, Dagmawi Melak, Dr. Bruck Assgede, Samuel Betseha, Shewaferawe Dinku, Thomas Seyoum, Biruh Habtu, Oniya Yaynu, Fissseha Mengistu and all my friends who were besides me giving me encouragement and practical support during the work. Thank you all.

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P_i	impulsive pressure
P_s,	Convective pressure
P_{i1}	the impulsive pressure acting on rigid tank wall
P_{i2}	the impulsive pressure due to flexibility of tank wall
m_1	impulsive mass
m_2.	convective mass
P_1	equivalent seismic force corresponding to m1
P_2	equivalent seismic force corresponding to m2
P_3	equivalent seismic force corresponding to m3
u_1	acceleration corresponding to m1
u_2	acceleration corresponding to m2
h_1	heights producing bending moment on the shell of the weight w1
h_2	heights producing bending moment on the shell of the weight w2
h_3	heights producing bending moment on the shell of the weight w3
M_1	moments corresponding to P1
M_2	moments corresponding to P2
R	radius of tank
γ	unit weight of liquid
h	height of liquid fill
Ww	weight of liquid
W_1	weight of liquid moving with rigid wall producing impulsive force
W_2	weight of liquid producing the convective force
W_3	weight of liquid used to represent constrained liquid
h_1^0	equivalent height accounting for bottom pressure W1
h_2^0	equivalent height accounting for bottom pressure of weight W2
K	spring stiffness
T	period of oscillating liquid
ω	natural circular frequency of the liquid (rad/sec)
A	maximum acceleration experienced by the sloshing mass
A_0	maximum ground acceleration
A^*	maximum acceleration of the single degree of freedom system

g Acceleration due to gravity

BM bending moment

OTM overturning moment

L length of shell (height)

h,t thickness of shell

H liquid height

(r, z) cylindrical coordinate system used with the center of the base as the origin.

w, u The radial and axial displacement components of a point on the shell middle surface

$\phi(r, z, t)$ velocity potential function

t time

I_0 the modified Bessel function of the first kind of order zero

ρ_l density of liquid

ρ_s density of shell

E the modus of elasticity of the material

ν Poisson's ratio of material

$P(r,z,t)$ the pressure exerted on the tank wall at any time t

ωn natural frequency of shell

D extensional rigidity of shell

K bending rigidity of shell

$U(t)$ strain energy of shell

$T(t)$ kinetic energy of shell

N_z the membrane force resultant along axial direction

N_θ the membrane force resultant along hoop direction

M_z the axial bending moment resultant

εz the normal strain along axial direction

$\varepsilon\theta$ the normal strain along hoop direction

Kz mid surface change in curvature

$[P]$ differential operator

$m(z)$ the mass of the shell per unit area;

$\{\varepsilon\}$ the strain vector

$\{d\}$ the displacement vector

S_{as} the spectral value of the pseudo-acceleration corresponding to the fundamental sloshing frequency

m_s effective sloshing mass for mechanical model

H_s height at which the effective sloshing mass acts

$U(x,t)$ displacement function

$q(t)$ the generalized displacement coordinate

$\phi(x)$ Shape function

$[k_e]$ stiffness of element function

$\{P_e(t)\}$ element force of the finite element assemblage

$[a_e]$ Boolean matrix

$[m_e]$ element mass matrix

$\{u_e\}$ element displacement matrix

N_e the number of elements

Σ denotes the direct assembly procedure for assembling according to the matrix a_e

$[c]$ damping matrix

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Abstract

Liquid containing cylindrical containers are one of those sensitive structures affected by dynamic loads. These liquid containing structures are affected not only by the inertia effect of the shell alone but also by the impulsive as well as convective pressures that are developed by the liquid they contain. There are a number of analysis methods; among them the Housner approximate method is widely used.

In order to study the behavior of ground supported cylindrical tanks caused by dynamic loading a finite element method was undertaken. Using the FEM the flexibility of the shell wall and the liquid properties can be included. Due to the complex nature of theoretical approaches especially when considering the dynamic nature of structure, the use of software becomes very important. To model this system the ANSYS finite element software was used. Different parameters were considered, and the liquid was idealized as a displacement based element. For the analysis the three dimensional as well as two dimensional approaches have been used. And the fluid property such as sloshing and development of hydrodynamic effect were included in these elements.

Ground supported cylindrical tanks having variable height, radius and thickness were analyzed. The results showed that the sloshing frequencies of the Housner approximate method and the FEM are comparable. The two dimensional FEM produced coupled frequencies of the system which have higher frequencies than the sloshing frequencies.

As compared to Housner's simplified method the shear for shallow and slender tanks obtained using the FEM are generally larger, while the bending moments obtained using FEM for slender tanks were smaller. The axial force of FEM was also found out to be significant. The vertical displacement of the free surface obtained using FEM for sloshing and coupled mode were very small as compared to Housner's method.

Key words: Cylindrical tanks; Sloshing; Fluid-Structure interaction

Chapter 1

Introduction

1.1 Problem Background

Cylindrical containers (tanks) have got wide range of use among them being for containment of liquids like water, petroleum products, chemicals of different nature etc. These tanks can be utilized for residential as well as for industrial usage, and their size vary from small to large sized tanks.

Unlike other structures these structures are in contact with liquid and their response under seismic load is quite different. Apart from the hydrostatic pressure the seismic force imparts hydrodynamic pressure. This liquid structure interaction is of interest for the design of cylindrical tanks and due consideration should be given during design of the structure.

A number of studies have been carried out on cylindrical tanks containing liquid. Both mathematical as well as experimental methods were used to model the problem. The greater contribution is attributed to Housner. Based on experimental investigations, he developed simplified analysis method to model the hydrodynamic pressure and determine the resulting forces and overturning moments at the base of the tank.

But during earthquakes that occurred afterwards the tanks designed using Housner's method suffered failure. As a result different solutions were proposed to overcome the above problem. Among the solutions were amplifying the Housner's result and finite element technique to analyze the problem. Due to the complex nature of the problem involved no closed form solution can be computed using analytical techniques.

With the growing numerical technique, efficient computers and also with the development of finite element method the analyses of liquid containing tanks have become possible. Different modeling techniques, which depict the problem, were used.

Among them being the development of liquid finite element model and the use of modeling the shell region using displacement based finite element and the liquid using analytical techniques.

As described above the analysis of liquid containing structures is quite complex. Due to the dynamic effect and the fluid structure interaction the problem becomes complicated. The aim of this paper is to develop a finite element model that includes the effect of the liquid inside the tanks using existing finite element software and to study the behavior of the tanks.

1.2 Objective

The general objective of the thesis is to study the behavior of dynamically loaded ground supported cylindrical tanks giving due consideration to the hydrodynamic forces.

The specific objectives of the thesis are summarized as follows;

1. To review the different methods used for dynamic analysis of liquid containing cylindrical tanks based on previous studies;
2. The development of model that incorporates the effect of liquid inside cylindrical tanks using existing finite element software package;
3. Analysis of the dynamic response of cylindrical tanks of variable aspect ratio and height.

1.3 Methodology

Finite element software with dynamic analysis capability was used to solve the problem. In order to include the interaction problem, finite elements that incorporate the effect of liquid and the shell were used. Both the shell regions and liquid regions are modeled using finite elements.

Chapter 2

Dynamic Analysis of Liquid Containing Cylindrical Tanks

2.1 Introduction

Liquid storage tanks are important structures for storage of water, petroleum and liquid natural gas etc. During seismic activities, due to the flowing nature of the material they contain they are vulnerable to damages. Early developments of seismic response theories considered the cylindrical tanks to be rigid and the flexibility of the tank shell was neglected.

Due to the complex nature of the problem, no closed form solutions are generally available and those considering this effect use simplifying assumptions, neglecting sloshing and flexibility effects.

The problem of liquid containing cylindrical tanks has been of concern for decades. The earliest Investigations (1948) were due to Jacobson and his colleagues [6]. They were concerned with analytical as well as experimental studies of motion of a liquid inside cylindrical and rectangular tanks subjected to horizontal ground motion.

In 1957 Housner investigated the hydrodynamic pressures developed within such tanks when they are subjected to ground motions [1, 6, 9]. The results were presented in the form of equivalent masses together with their locations for representing force and moment effects on the rigid tank due to liquid motion [6].

In 1960 Cooper and in 1963 Abramson presented review articles on the state-of-the-art of liquids in moving containers [1] where much of the work had direct application to space vehicle technology. In 1964 Alaska earthquake caused the first large scale damage to liquid storage tanks and initiated many investigations of the dynamic characteristics of the containers [2]. In 1976 Howard I. Epstein modified the results obtained from

Housner's mechanical model using additional coefficients to account for the flexibility of the containers [1].

During the 1980's much of the works concerning the tanks is attributed to Medhat A. Haroun. In 1982 Medhat A. Haroun and G.W Housner analyzed the problem giving due consideration to the interaction problem [2]. Also in 1983 and 1985 he, along with his colleagues, developed a method of analyzing the problem using analytical and numerical methods. The methods used will be discussed below briefly.

2.2 Review of basic concepts

Structures that are in contact with liquid show a markedly different behavior under seismic loads. During the horizontal seismic excitation of a tank containing fluid, the inertia of the fluid exerts pressure (force per unit area) on the tank walls, which is called the impulsive pressure P_i. In addition to the impulsive pressure the original liquid level will be disturbed by the excitation leading to sloshing. This sloshing effect causes additional convective pressure, P_s, to develop on the tank walls. Fig. 2.1 shows the pressure distribution acting on the tank wall. The convective pressure is independent of the impulsive pressure [9].

The impulsive pressure, which is caused by the inertia forces, is divided into two components. The first is the pressure acting on rigid tank wall P_{i1}, and the second is due to the flexibility of the tank wall, P_{i2}, as shown in Fig 2.1. a & b. The latter one is said to diminish in proportion to the stiffness of the tank. The stiffer the tank the lesser the pressure caused by flexibility of the tank [9].

In the following sections some of the method's that have been used to analyze the interaction problem will be addressed.

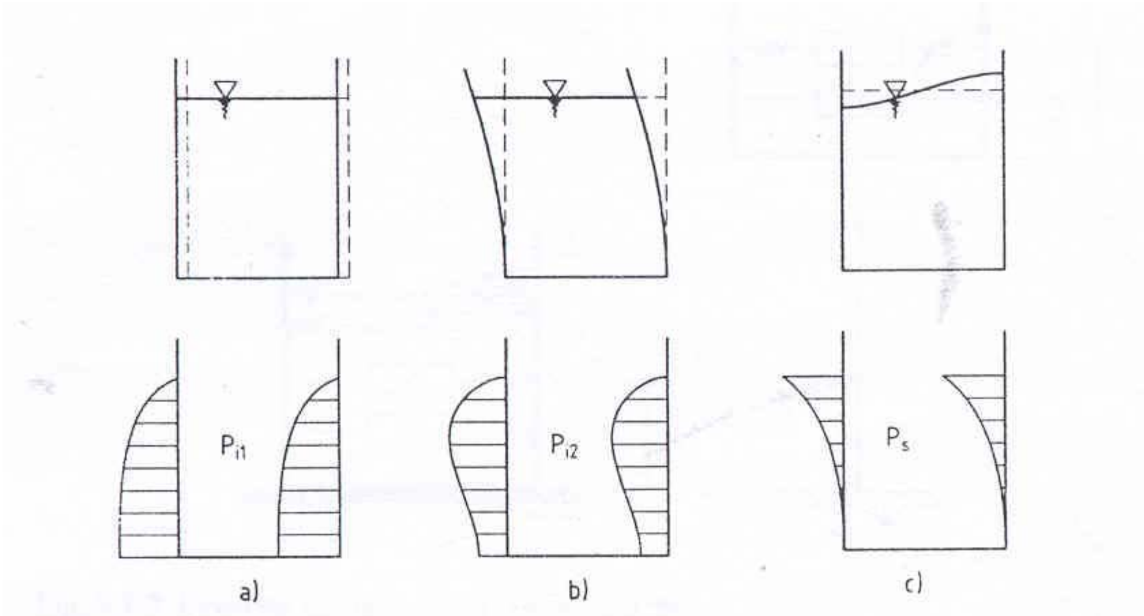


Fig. 2.1: Dynamic fluid pressures acting on the tank wall [9]
(a) Impulsive pressure acting on rigid wall (b) Impulsive pressure acting on flexible wall
(c) Convective pressure

2.3 Housner's Approximate Method

2.3.1 Assumptions

Housner's approximate method is based on the following assumptions:

1. Tanks with constant rectangular or circular sections
2. Flat bottom
3. Purely horizontal seismic excitation
4. Rigid tank walls

Housner's approximate method was based on results from experimental as well as analytical methods. From the results of the investigations he developed a lumped mass approach to model the problem. The horizontal ground acceleration induces impulsive force due to inertia of the liquid and convective force due to sloshing. The liquid mass was split accordingly into the impulsive mass m_1, and the convective mass m_2.

The masses m_1 and m_2 are acted upon by the accelerations u_1 and u_2 and yield equivalent seismic forces P_1 and P_2 respectively. These values multiplied by the heights h_1 and h_2 causes moments M_1 and M_2 on the tank bottom. The masses m_1 and m_2 were determined in such a way that the resulting stresses are similar to the experimental ones caused by the actual liquid in the seismically excited container [9].

The tanks experience the same absolute displacement as the ground due to the assumption that the walls are rigid. The evaluation of impulsive mass depends on the geometry of the tank. Based on the geometry of the tank it is classified as squat and slender tank. This classification helps to determine the contribution of the impulsive and convective mass.

Housner demonstrated that the forces due to the lateral acceleration of the liquid in the tank could be found using the mathematical models shown in figure 2.2 below. He made a cut off between shallow and slender (tall) tanks. The Rationale for this cut off will be discussed below.

2.3.2 Shallow tanks and slender tanks

For a cylindrical storage tank of radius R, containing an incompressible liquid of unit weight γ, filled to depth H, the total weight of liquid is given by

$$W_w = \pi R^2 H \gamma \tag{2.1}$$

A shallow tank is one with height to radius ratio of less or equal to 1.5. The lumped mass model for this case is as shown in Figure 2.2. It consists of a mass of weight, W_1, moving with the rigid tank wall producing the impulsive force. And a mass of weight W_2, producing the convective force.

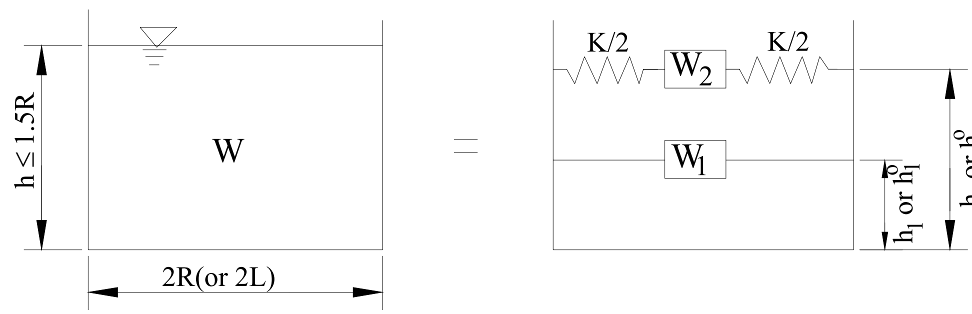


Fig. 2.2: Shallow Tank- Lumped mass approach for H/R $\leq$ 1.5 using Housner's method[9]

The heights of these weights were located on the basis of producing the correct moment about the base. These heights are designated by the letters h_1 and h_2 as shown in Fig 2.2. The heights h_1 and h_2 are used to calculate the bending moments about the base of the structure. The bending moment just above the base is resisted by the shell.

The dynamic pressure distribution on the bottom of the tank that is used to calculate the overturning moment was defined using the same weights. But new equivalent heights were used to account for the bottom pressure. The heights which are used to calculate the overturning moment of the structure are given by h_1^0 and h_2^0 as shown in Fig. 2.2. The expressions for the heights are given by the following equations;

$$h_1 = \frac{3}{8}H \tag{2.2}$$

$$h_2 = H\left[1 - \frac{\cosh(1.84H/R) - 1}{1.84(H/R).\sinh(1.84(H/R))}\right] \tag{2.3}$$

$$h_1^0 = (H/8)\left[\frac{4}{\left[\frac{\tanh(\sqrt{3}.(R/H)}{\sqrt{3}.(R/H)}\right]} - 1\right] \tag{2.4}$$

$$h_2^0 = H\left[1 - \frac{\cosh(1.84H/R) - 2.01}{1.84(H/R).\sinh(1.84(H/R))}\right] \tag{2.5}$$

The period, T, of the oscillating liquid is given by:

$$T = \frac{2\pi}{\omega} \tag{2.6}$$

The value of the natural circular frequency is given by

$$\omega^2 = \frac{1.84g}{R}\tanh(1.84\frac{H}{R}) \tag{2.7}$$

For the case of a tank subjected to one dimensional horizontal earthquake base excitation, the maximum forces exerted on the rigid tank by the two masses are

$$P_1 = \frac{W_1}{g}A_0 \tag{2.8}$$

$$P_2 = \frac{W_2}{g}A \tag{2.9}$$

Where A_0 is the maximum ground acceleration and A is the maximum acceleration experienced by the sloshing mass that may be found from the response spectrum of the earthquake using the period given above. While the weights are computed as

$$W_1 = W_w\left[\frac{\tanh(\sqrt{3}.(R/H)}{\sqrt{3}(R/H)}\right] \tag{2.10}$$

$$W_2 = W_w.0.18.(R/H).\tanh(1.84(H/R) \tag{2.11}$$

Assuming the forces to occur simultaneously, the maximum bending moment is given by

$$BM = P_1h_1 + P_2h_2 \tag{2.12}$$

Also the overturning moments are given by

$$OTM = P_1h_1^0 + P_2h_2^0 \tag{2.13}$$

For the case of tall (slender) tanks, the generation of liquid velocity relative to the tank is restricted to a height of 1.5R from the free surface. Slender tanks are those, whose height to radius ratio is grater than 1.5. The mathematical model corresponding to this case is shown in Fig. 2.3. The case (i.e. height to radius ratio of 1.5) can be visualized as a fixed rigid membrane separating the tank into two regions. An additional mass of weight W_3 is used to represent the constrained liquid.

The maximum forces corresponding to the mass are

$$P_1 = \frac{W_1}{g} A_0 \tag{2.14}$$

$$P_2 = \frac{W_2}{g} A^* \tag{2.15}$$

$$P_3 = \frac{W_3}{g} A_0 \tag{2.16}$$

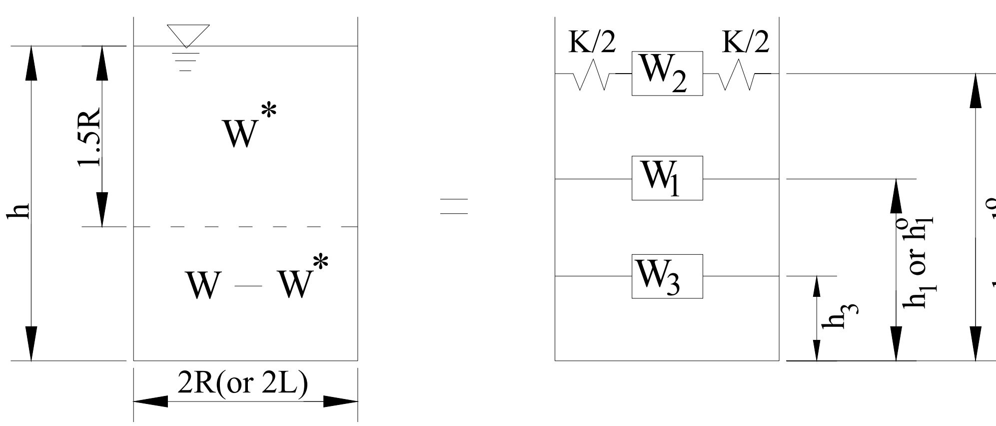


Fig. 2.3: Slender Tank [9]

Where A^* is the maximum acceleration of the single degree of freedom system for period of T*, where T* is given by:

$$T^* = 4.65\left(\frac{R}{g}\right)^{1/2} \tag{2.17}$$

The weights are given by the expression:

$$W_1 = \gamma.(1.5R^3)\Pi.(0.7095) \tag{2.18}$$

$$W_2 = W_w.0.318.(\frac{R}{H})\tanh(1.84(\frac{H}{R})) \tag{2.19}$$

$$W_3 = \gamma(H - 1.5R).R^2.\Pi \tag{2.20}$$

And the bending moment caused by the liquid is given by;

$$BM = P_1h_1 + P_2h_2 + P_3h_3 \tag{2.21}$$

The heights are given by:

$$h_1 = \frac{3}{8}.(1.5R) + (H - 1.5R) \tag{2.22}$$

$$h_2 = H\left[1 - \frac{\cosh(1.84H/R) - 1}{1.84(H/R).\sinh(1.84(H/R))}\right] \tag{2.23}$$

$$h_3 = \frac{H - 1.5R}{2} \tag{2.24}$$

$$h_1^0 = (1.5R/8)\left[\frac{4}{0.7095} - 1\right] + H - 1.5R \tag{2.25}$$

$$h_2^0 = H\left[1 - \frac{\cosh(1.84H/R) - 2.01}{1.84(H/R).\sinh(1.84(H/R))}\right] \tag{2.26}$$

Most of the tanks in the past were designed using the well known simplified Housner approach, and still this method is being used now. The method considers the hydrodynamic pressure on rigid wall and neglects the effect of wall flexibility. Some tanks designed according to this approach suffered serious damages under seismic loading. The 1964 Alaska earthquake caused the first large scale damage to liquid storage tanks [2]. Then it was determined that the actual hydrodynamic pressure was larger than the one computed according to Housner's approach. In order to include the flexibility of the wall additional corrective factors were considered [9] and other analytical as well as numerical methods were developed. The next section gives a brief discussion on these methods.

2.4 Analytical Method

The analytical model used for analysis is based on the governing differential equations of both the liquid and the shell surfaces. Different analytical methods have been devised but the widely used one is that which uses theory of thin shell motion [15].

2.4.1 Tank Geometry and Coordinate System

The tank under consideration is as shown in Fig 2.4. The tank is ground supported, circular, thin walled cylindrical liquid container of radius R, length L and thickness h. The liquid height is defined by H. A cylindrical coordinate system is used with the center of the base as the origin. The radial and axial displacement components of a point on the shell middle surface are denoted by w and u, respectively [15].

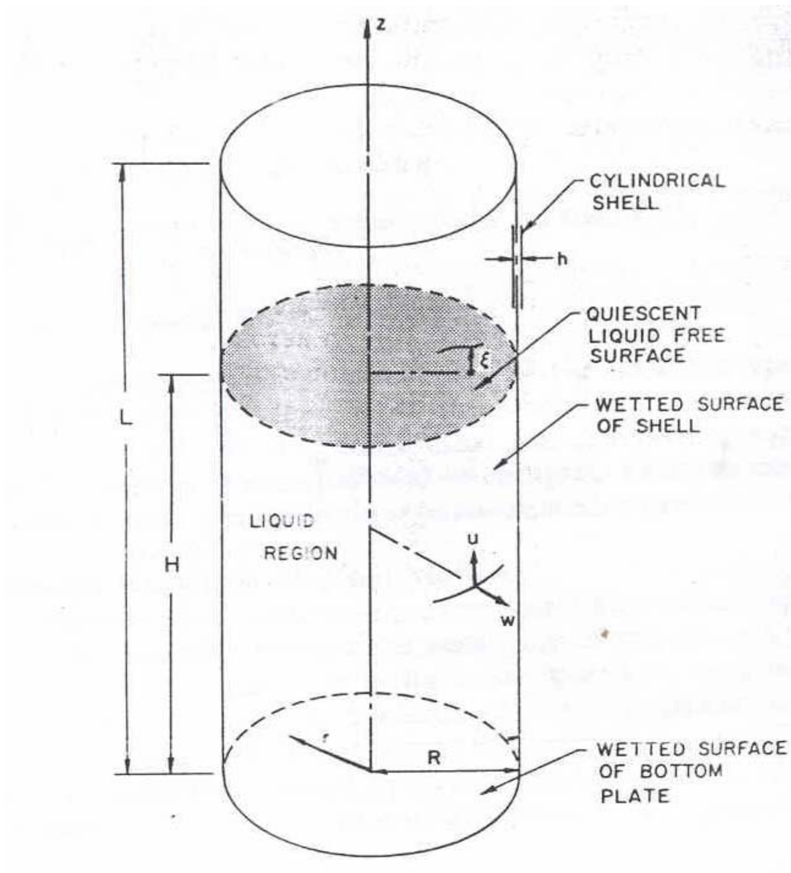


Fig. 2.4: Cylindrical Tank and Coordinate System [2]

2.4.2 Equation Governing Liquid Motion

For the irrotational flow of an incompressible liquid, the velocity potential function, ϕ (r, z, t), satisfies the Laplace equation in the region occupied by the liquid [15].

$$\nabla^2 \phi = 0 \tag{2.27}$$

In addition to being a harmonic function, ϕ must satisfy the proper boundary conditions that can be expressed as follows:

1. At the rigid tank bottom, z=0, the liquid velocity in the vertical direction is zero

$$\frac{\partial \phi}{\partial z}(r,0,t) = 0 \tag{2.28}$$

2. The liquid adjacent to the wall of the elastic shell, r=R, must move radially with a velocity similar to that of the shell

$$\frac{\partial \phi}{\partial r}(R, z, t) = \frac{\partial w}{\partial t}(z, t) \tag{2.29}$$

in which $w(z,t)$ = the shell radial displacement.

3. At the liquid free surface, two boundary conditions must be imposed if sloshing modes are not neglected (i.e. pressure and velocity). If sloshing modes are neglected, only one condition needs to be specified at the surface, namely

$$\frac{\partial \phi}{\partial t}(r, H, t) = 0 \tag{2.30}$$

Therefore the solution that satisfies the above equations needs to be expressed as

$$\phi(r,z,t) = \sum_{i=1}^{\infty} A_i I_0(\alpha_i R)\cos(\alpha_i z) \tag{2.31}$$

in which I_0 = the modified Bessel function of the first kind of order zero.

$$\alpha_i = \frac{(2i-1)\pi}{2H} \tag{2.32}$$

$$A_i = \frac{2\int_0^H \dot{w}(z,t)\cos(\alpha_i z)dz}{\alpha_i H I_0'(\alpha_i R)} \tag{2.33}$$

The pressure distribution, P(r, z, t), can be determined from the Bernoulli equation and is given by;

$$p(r,z,t) = -\rho_l \frac{\partial \phi}{\partial t} + \rho_l g*(H-z) \tag{2.34}$$

2.4.3 Equation Governing Shell Motion

A cylindrical shell undergoing axisymmetrical vibration is governed basically by two differential equations: The first one shown in Eq. 2.35 is of the second order governing the dynamic equilibrium in the axial direction. The second one shown in Eq. 2.36 is of the fourth order governing the dynamic equilibrium in the radial direction [15].

The governing sets of differential equations are based on the formulation from theory for thin shells due to V.V. Novozhilov [15]. The governing equations are expressed as follows:

$$\frac{\partial^2 u}{\partial z^2} + \frac{\upsilon}{r}\frac{\partial w}{\partial z} - \frac{\rho_s h}{D}\frac{\partial^2 u}{\partial t^2} = 0 \tag{2.35}$$

$$K\frac{\partial^4 w}{\partial z^4} + \frac{Eh}{(1-\upsilon^2)}\frac{w}{R^2} + \frac{\upsilon Eh}{(1-\upsilon^2)R}\frac{\partial u}{\partial z} + \rho_s h\frac{\partial^2 w}{\partial t^2} = \begin{cases} p(z,\ t) \\ 0 \end{cases} \tag{2.36}$$

In which E= the modus of elasticity of the material

ν = Poisson’s ratio of the shell

p (z, t) = the pressure exerted on the tank wall at any time t; 0 < z <H

D and K are the extensional and bending rigidities of the shell respectively. The expression for D and K are given by;

$$D = \frac{Eh}{(1-\upsilon^2)} \tag{2.37}$$

$$K = \frac{Eh^3}{12(1-\upsilon^2)} \tag{2.38}$$

If ν is taken equal to zero, the two equations of motion uncouple. Furthermore for the case of empty tank, p (z, t) =0, Eq. 2.35 and Eq. 2.36 show two distinct behaviors of the shell. The first of these equations is similar to that describing the free vibration of a rod undergoing axial vibrations; and therefore the natural frequencies are given by

$$\omega_n = \frac{(2n-1)\pi}{2L}\sqrt{\frac{E}{\rho_s}} \tag{2.39}$$

The second of the governing differential equation due to the uncoupling and empty tank assumption gives the following simplification;

$$\frac{d^4 w}{dz^4} - \beta^4 \omega = 0 \tag{2.40}$$

In which the value of β is given by;

$$\beta^4 = \frac{\rho_s h \varpi^2 - \dfrac{Eh}{R^2}}{\dfrac{Eh^3}{12}} \tag{2.41}$$

It is clear that Eq. 2.40 is precisely the equation governing the free vibration of a uniform flexural beam; therefore the natural frequencies of vibration are given by;

$$\varpi_i = \left(\frac{\dfrac{Eh}{R^2} + \beta_i^4 \dfrac{Eh^3}{12}}{\rho_s h} \right)^{1/2} \tag{2.42}$$

If the bending rigidity of the shell is neglected, the equation reduces to

$$\varpi_i = \left(\frac{\left(\frac{E}{\rho_s} \right)^{1/2}}{R} \right) \tag{2.43}$$

The above equation provides the circular natural frequency of a closed ring undergoing uniform radial expansion. It is of interest to note that ν is not equal to zero in practical cases. But still the equations give reasonable approximation of the natural frequencies of empty tank [15].

2.5 Numerical Method

In the numerical approach, the mass and the stiffness matrices are derived directly from the expression of the potential and kinetic energies.

The strain energy of the shell, including the effect of stretching and bending can be written as,

$$U(t) = \frac{1}{2}\int_0^L \left(N_z \varepsilon_z + N_\theta \varepsilon_\theta + M_z K_z \right) \ \pi \tag{2.44}$$

in which N_z and N_θ = the membrane force resultant; and

M_z = the axial bending moment resultant.

ε_z =the normal strain along axial direction

ε_θ =the normal strain along hoop direction

K_z =mid surface change in curvature

The shell material is assumed to be homogeneous, isotropic and linearly elastic. Therefore, the force and moment resultants can be expressed in terms of the normal strains in the middle surface and; and in terms of the mid surface change in curvature as follows:

$$\{\sigma\} = [D]\{\varepsilon\} \tag{2.45}$$

in which

$$\{\sigma\} = \begin{Bmatrix} N_z \\ N_\theta \\ M_z \end{Bmatrix} \tag{2.45.1}$$

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ K_z \end{Bmatrix} \tag{2.45.2}$$

$$[D] = \frac{Eh}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{h^2}{12} \end{bmatrix} \tag{2.45.3}$$

The generalized strain vector {ε} can be written in terms of the displacement vector {d} as follows:

$$\{\varepsilon\} = [P]\ \{d\} \tag{2.46}$$

in which {P} = a differential operator matrix relating the strain with the displacement,

and

$$\{d\} = \begin{Bmatrix} u \\ w \end{Bmatrix} \tag{2.47}$$

With the aid of the above equations one can write the potential energy expression as

$$U(t) = \pi R \int_0^L \left(([P]\{d\})^T [D]([P]\{d\}) \right) dz \tag{2.48}$$

The kinetic energy of the shell, neglecting rotary inertia, can be expressed as

$$T(t) = \pi R \int_0^L \left(m(z) \{\dot{d}\}^T \{\dot{d}\} \right) dz \tag{2.49}$$

In which m (z) = the mass of the shell per unit area;

{d} = the displacement vector defined by Eq. 2.47; and

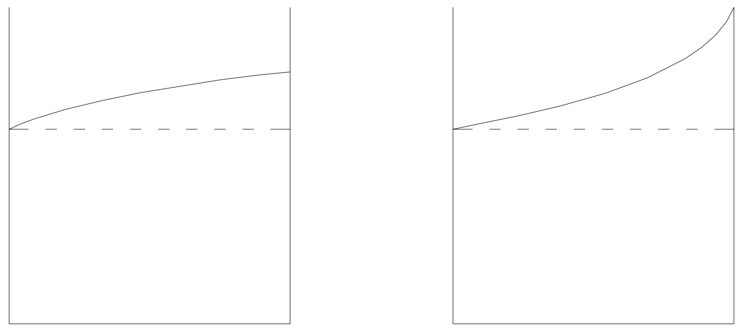
$(\bullet)$ means differentiation with respect to the time, t.

By means of the above equations the shell stiffness matrix and shell mass matrix are derived. And the corresponding stresses are obtained [15, 16].

2.6 Sloshing in cylindrical tanks

When a tank is excited by lateral motion, liquid at the free surface undergoes sloshing. Numerous events of damages to shells of liquid storage tanks have been reported in past earthquakes. Under the assumption of small amplitude sloshing, the theoretically computed overturning moment is negligibly small as compared to the impulsive pressure. Therefore more attention has been given to the evaluation of shell stresses due to impulsive pressure, and due to shell flexibility [20].

Although it is evident that large amplitude sloshing can be excited by the long period component of earthquake motion, only small amplitude theory has been used in practice to calculate sloshing heights. If improper freeboard to the fixed roof system is allowed, sloshing motion creates high localized impact pressures on the roof causing structural damage. Figure 2.5 shows two of the modes associated with liquid sloshing.



(a) Small amplitude oscillations (b) Large amplitude Oscillations

Fig.2.5: Liquid sloshing modes due to fluid motion [20]

For small amplitude oscillations, a linearized version of the free surface gives the natural frequencies and the corresponding modes [20].

For the linearized sloshing theory, the following assumptions are used. The liquid is assumed to be homogenous and incompressible. Based on the following assumption the convective dynamic pressure can be evaluated with reasonable accuracy by considering the tank wall to be rigid [20]. Therefore the maximum convective pressure due to the fundamental sloshing mode is given by;

$$\left|p_s(R,\theta,z)\right|_{max} = 0.837\rho_l R\cos(\theta)\left[\cosh(1.84z/R) \big/ \cosh(1.84H/R)\right]S_{as} \qquad (2.50)$$

In which ρ_l = the mass density of the liquid

S_{as} = the spectral value of the pseudo-acceleration corresponding to the fundamental sloshing frequency

The maximum vertical displacement of the free surface occurs at the junction with the shell and on the excitation axis. It is given by

$$\varepsilon_{max} = 0.837R\frac{S_{as}}{g} \tag{2.51}$$

The widely used mechanical model is employed to compute the effective mass, m_s, due to sloshing. It is given by

$$m_s = 0.455\pi.\,\rho_l R^3 \tanh\left(\frac{1.84H}{R}\right) \tag{2.52}$$

It acts at a height of H_s. The values for this height were presented by Medhat A. Haroun [4] with the corresponding natural frequency for different height to radius ratio.

Chapter 3

Modeling of the System Using Finite Element Method

In Finite element method, modeling of the problem is one of the aspects that govern the accuracy of the solution and efficiency of computational time. In order to model the interaction of liquid containing cylindrical containers, ANSYS finite element software has been used. Elements, modeling and the parameters used are described in this chapter. Before stating about the modeling a brief description about dynamic analysis and finite element will be presented below.

3.1 Introduction

The analysis and design of structures based on the effect of time dependent forces such as earthquake, harmonic motion etc is part of the field of structural dynamics. Different idealizations techniques are used to discretize structures based on the simplicity or complexity of the structural system in order to compute the time dependent forces. In general these methods, that are widely used to compute the time dependent forces of such systems at any instant time, can be categorized into the following methods;

These are;

1. The lumped mass approach
2. The generalized displacement technique
3. The finite element idealization

In the lumped mass approach the structural dynamic problem is formulated for simple structures. These simple structures are those that can be idealized as a system with a concentrated or lumped mass and a massless supporting structure. Linearly elastic as well as inelastic structures subjected to applied force or earthquake induced ground motion can be considered.

The generalized displacement technique starts with the deflected shape of a system that is assumed or shape of coordinates are chosen arbitrarily. But the constraint and continuity of internal displacements are not violated. The displacement function of the system is then given by;

$$U(x,t) = \sum_{r=1}^{\infty} \phi(x)q(t) \tag{3.1}$$

Where q (t) = the generalized displacement coordinate

$\phi(x)$ = Shape function

The finite element method is used to idealize structures with infinite degrees of freedom into an assemblage of finite elements having specified nodal points. Both the principle of lumped mass and generalized coordinate expresses the displacement of any given structure in terms of finite number of displacement coordinates. This method is almost applicable to all system of structural problems. The next section gives brief discussion on this method [8, 14, 18and 19].

3.2 Finite Element Method

An explosive growth of research on the finite element method took place beginning in the early 1960s, leading to the development of finite elements appropriate for idealizing different types of structural continua and their application to practical problems [14].

The finite element method is one of the most important developments in applied mechanics. This method is applicable to wide range of problems. This ranges from assemblage of one dimensional finite elements to a three dimensional complex problems. Also static as well as dynamic analysis can be performed [12, 14].

Finite element provides a more detailed numerical analysis method in order to study the behavior of fluid structure interaction. The liquid mass can be derived in an Eulerian, Lagrangian or ALE (Arbitrary Lagrangian Eulerian) formulation [9, 12, 17].

The Eulerian approach is widely used in fluid mechanics. Here the computational mesh is fixed and the fluid moves with respect to the grid. In the Eulerian approach a velocity potential function is assumed and the behavior of the liquid is described through pressure or velocity variables at the element nodes. But using this configuration it is difficult to describe the structure configuration. Since the structure configuration needs displacement variables [9, 17].

In order to overcome the above complication, Lagrangian elements can be used and the fluid elements use displacement as fluid element variables. In the Lagrangian algorithms, each individual node of the computational mesh follows the associated material particle during motion. These formulations are frequently used in structural mechanics, in combination with both solid and structural (beam, plate, shell) elements. Also it allows easy tracking of free surfaces and interfaces between different materials [9, 17]

ALE(Arbitrary Lagrangian Eulerian) algorithms are particularly useful in flow problems involving large distortions. The key idea in this formulation is the introduction of a computational mesh which can move with a velocity independent of the velocity of the material particles. It is the generalized description of the above two formulations [17].

3.3 Dynamic Analysis of Structural Continua

Any structural continuum with infinite degrees of freedom can be idealized as an assemblage of finite elements with finite number of degrees of freedoms (DOFs). Thus the partial differential equation governing the motion of the structural continuum is reduced to a system of ordinary differential equations. An assemblage of two or three dimensional elements can be used for idealization. The number of elements chosen for the assemblage depends on the accuracy desired. With properly formulated finite elements, the result converges to the exact solution with decreasing element size. Accordingly the larger the number of elements the more accurate the solution obtained. In addition the aspect ratio of the elements should be kept near to one in order to get accurate solutions [14].

Compatibility at nodes does not always ensure compatibility across the element boundaries. To avoid such discontinuities, interpolation functions over the element are assumed in such a fashion that the common boundaries will deform together, such elements are called compatible elements. In dynamic analysis of the finite element method, at each time instant the state of stress within each element is determined from nodal displacement. This is accomplished using interpolation functions, strain displacement relations and constitutive properties of the material [14].

The analysis procedure of the finite element method for the formulation of the equations of motion may be summarized as follows [14]:

1. Idealization of the structure as an assembly of finite element interconnected only at nodes. The nodes need to have defined DOF $\{u\}$.

2. Formulation of the element stiffness matrix $[k_e]$, the element mass matrix $[m_e]$ and the element (applied) force vector $\{P_e(t)\}$ with reference to the DOF of the element. The force-displacement relation and the inertia force-acceleration relation are

$$\{(f_s)_e\} = [k_e]\{u_e\} \tag{3.2}$$

$$\{(f_I)_e\} = [m_e]\{\ddot{u}_e\} \tag{3.3}$$

3. Formulation of the transformation matrix $[a_e]$ that relates the displacements $\{u_e\}$ and forces $\{p_e\}$ for the element to the global displacement $\{u\}$ and forces $\{p\}$ of the finite element assemblage;

$$\{u\} = [a_e{}^T]\{u_e\} \tag{3.4}$$

$$\{p(t)\} = [a_e{}^T]\{p_e(t)\} \tag{3.5}$$

Where $[a_e]$- Boolean matrix consisting of zeros and ones. It locates the elements of $[k_e]$,$[m_e]$and $\{p_e\}$at the proper locations in the mass matrix, stiffness matrix and force vector,

4. Assembly of the element matrices to determine the stiffness and mass matrices and applied force vector for the assemblage of finite elements:

$$[k] = \sum_{i=1}^{N_e}[k_e] \tag{3.6}$$

$$[m] = \sum_{i=1}^{N_e}[m_e] \tag{3.7}$$

$$\{p(t)\} = \sum_{i=1}^{N_e}\{p_e(t)\} \tag{3.8}$$

Where N_{e-} is the number of elements

Σ -denotes the direct assembly procedure for assembling according to the matrix a_e

5. Formulation of the equation of motion for the assemblage

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{p(t)\} \tag{3.9}$$

Where [c] is the damping matrix

6. Solution of the equation of motion and the corresponding frequencies and stresses

3.4 Dynamic Response Computation Methods

The finite element formulation of the equation of motion is that which was shown by Eq. 3.9.The solution to this problem, leads to the determination of modes, natural frequencies and associated stress and displacements.

Complex structures consist of quite a large number of degrees of freedom, and the computational time required for the solution process of this problem becomes quite a lot. Also the memory requirement of the solution process becomes inefficient. In order to avoid such computational problems different eigenvalue solving approaches have been devised. The approaches used to solve this eigenvalue problems and spectrum analysis are discussed in the next section.

3.4.1 Modal Analysis

Modal analysis uses the contribution of individual modes to element forces. In this case the coupled equation of motion is uncoupled into a set of equations and the individual mode contribution is obtained. Additionally, to obtain the vibration properties, natural frequencies and mode shapes of a structure requires solving the eigenvalue problem.

Several mode extraction methods are available, these are Block Lanczos, subspace, PowerDynamics, reduced, unsymmetric, damped and QR damped. After computing the modal results spectrum analysis is undertaken.

3.4.2 Spectrum Analysis

A spectrum analysis is one with in which the results of a modal analysis are used with a known spectrum to calculate displacements and stresses in the model. It is mainly used in place of a time history analysis to determine the response of structures to random or time dependent loading conditions such as earthquakes, wind loads, ocean wave loads, jet engine thrust, and rocket motor vibrations and so on. A spectrum is a graph of spectral values versus frequencies (periods) that captures the intensity and frequency content of time history loads.

3.5 Model Description

3.5.1 Modeling

As mentioned earlier the model of the fluid as well as the shell was done using ANSYS software. Two sets of modeling were used for the problem. The first was using the three dimensional approach, while the second was using the two dimensional approach. Material property is assumed to be homogenous within the material and the different materials were coupled at the interface where they join together. The linear analysis method has also been used.

In the first case of three dimensional modeling the shell was modeled using shell element (Shell 63). This element has 6 DOFS at each node. It has three translational and three rotational DOFs. It has both the bending as well as the membrane capacity. Both inplane as well as normal stresses are permitted. It has four nodes and variable thickness can be input at the nodes, giving a shell with smoothly varying thickness. The element was chosen for its bending capacity as well its capacity of accepting in plane and normal loads. The remaining description of the element is given in appendix A.

The fluid inside the cylindrical tank was modeled using fluid element (Fluid 80). This element is a three dimensional solid element having three translational degree of freedom system at each node. This element has the ability to model contained fluids giving the result of hydrostatic pressure, hydrodynamic pressure as well as fluid structure interaction. The fluid elastic modulus of the element (in the input) should be taken as the bulk modulus of the fluid. Also viscosity of element needs to be given as input. The element description is attached in the appendix B.

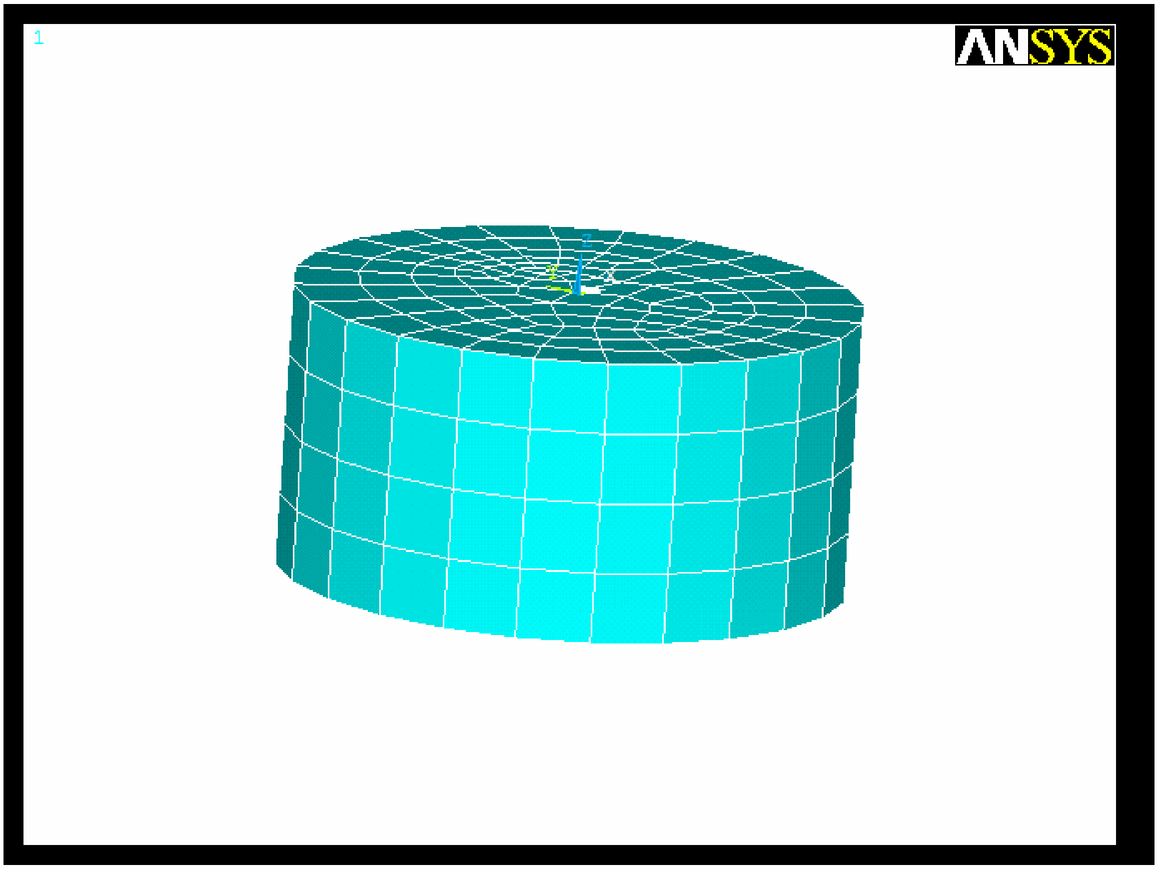


Fig. 3.1: Three Dimensional model of Shell 63 and Fluid 80 with Fixed Base

In the second type of modeling the two dimensional approach has been adopted. In this case the container was modeled using shell elements (Shell 61). The element has two nodes, each having four degrees of freedom at each node. These are translations in the x, y and z directions and a rotation about the nodal z-axis. The element has a linearly varying thickness capacity that may vary between the nodes. The element was chosen for its axsymmetric property, in which modeling of part of the section models the whole structure. The element description is given in appendix C.

The liquid part was modeled using fluid elements (Fluid 81). This element is used to model fluids contained within vessels having no net flow rate. It is defined by four nodes having three degrees of freedom at each node; translation in the nodal x, y and z directions. The element is well suited to calculate hydrostatic pressure and fluid/solid interactions. Acceleration effects like that of sloshing can be included. As in the three dimensional fluid case, the fluid elastic modulus should be the bulk modulus of the fluid. The viscosity should also be given as input. The element description is given in Appendix D.

Both this two dimensional elements are of the axsymmetric harmonic type in which modeling of part of the section idealizes the response of the whole structure. While in the three dimensional modeling, the whole structure was modeled. The two dimensional approach was far more time saving and interpreting the result was easy, since the number of element used is smaller compared to the three dimensional analysis. Also response of the coupled shell-liquid interaction was obtained in this case. While generating models using two or more elements, coupling of the elements becomes important. The next section gives a brief description of coupling of elements.

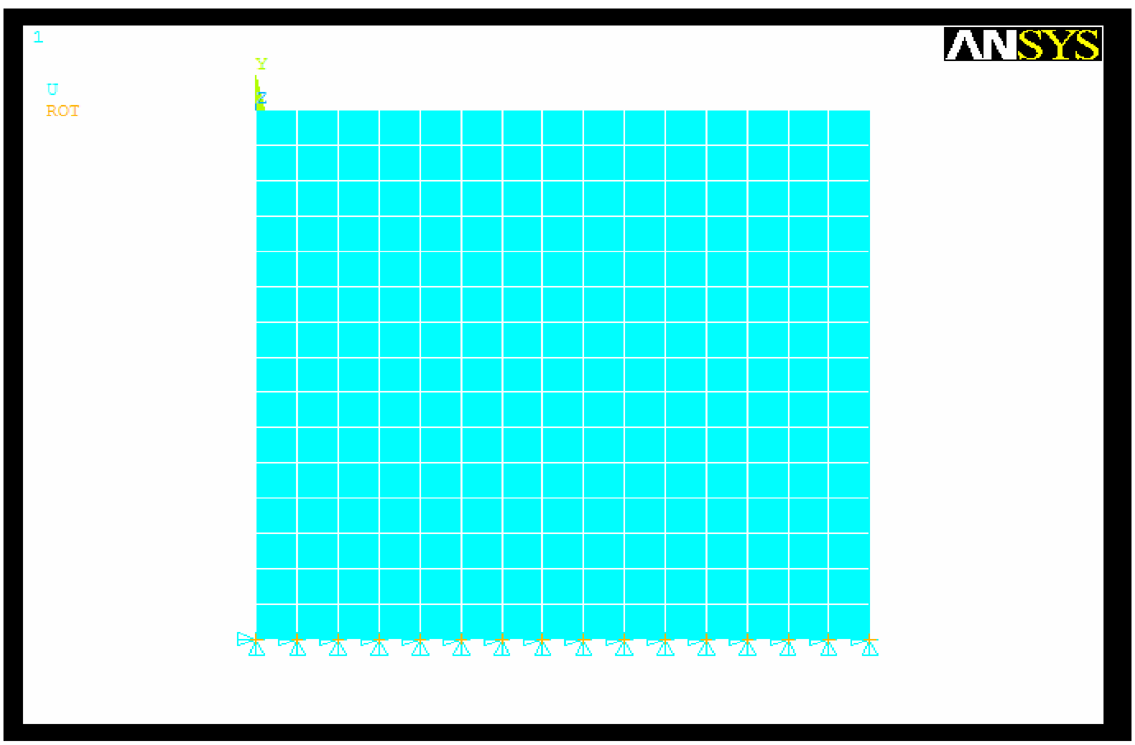


Fig. 3.2: Two Dimensional model of Shell 61 and Fluid 81

3.5.2 Coupling

When generating a model, we typically define the relationship among different degrees of freedom by using elements to connect the nodes. However, there are cases in which we need to model features that cannot be adequately described with elements. In this case special association among nodal degrees of freedom can be established using coupling. When you need to force two or more degrees of freedom (DOFs) to take the same (but unknown) value, you can couple these DOFs together. A set of coupled DOFs contains a prime DOF, and one or more other DOFs. Coupling will cause only the prime DOF to be retained in the matrix equation, and will cause all other DOFs in a coupled set to be eliminated. The value calculated for the prime DOF will then be assigned to all the other DOFs in a couple set.

In the case of using two different elements that are bounded together at a node coupling of the nodes is required. This coupling causes the nodal results to be transferred from one element to the other, so that equilibrium of nodal values will be attained. Coincident nodes in a model can be coupled by generating one coupled set for each specified DOF label at every pair of coincident nodes. If all DOFs are to be coupled for coincident nodes, it will be more efficient to simply merge those nodes together. Coupling operates in the nodal coordinate system of each node; therefore the nodal coordinate systems should be consistent. While coupling a DOF should not appear in more than one couple set.

3.5.3 Parameters

Having modeled the problem of interaction between the shell and the liquid using Shell 61, Shell 63, Fluid 80 and Fluid 81, the following parameters were used to study the interaction. These parameters were selected on the basis of Housner's simplified analysis so as to make comparison of the results and to study the behavior of the interaction problem.

1. Contained liquid (water) inside cylinders having different height and radius were analyzed for the sloshing frequency. The range of heights were 1-5m and the range of radius was also 1-5m;

2. The interaction between a shell having two different thicknesses and the liquid was analyzed;
3. Horizontal base acceleration in one direction was given as dynamic load;

4. Flat bottomed, ground supported containers were used;

5. Response spectrum from E.B.C.S. 8, 1995 GC was used and the soil type A was chosen;

Chapter 4

Analysis of Results

4.1 General

Modeling of the cylindrical tank was undertaken as illustrated in chapter three. After preparing the model that depicts the liquid and the container property, meshing of the elements was undertaken and response spectrum analysis was conducted.

The material properties used for the system are as shown in Table 4.1. The temperature for the water is at about 15^0C and these material properties were held constant throughout the analysis. The parameters for steel were also held constant. In order to study the effect of thickness of the container, values of 25.4 mm and 50 mm were used.

Table 4.1: Material Properties

Material Property	
Water	
Density	1000kg/m3
Bulk modulus	2.0684e9 Pascal
Viscosity	1.13e-3 N.S/m2
Steel	
Density	7850kg/m3
Poisson ratio	0.3
Modulus of elasticity	20.67e10 Pascal

In the analysis, two dimensional and three dimensional approaches have been used. The two cases were used because the three dimensional approach gave frequencies of the sloshing modes only and the coupled effects (liquid-shell frequencies) were not initiated. And these sloshing frequencies were not able to give the bending moment of the system, i.e. the bending moments obtained were very small. Therefore the two dimensional approach was used since it gives the coupled frequency of the system with the corresponding bending moments. As mentioned in the previous chapter response spectrum analysis for the two dimensional and three dimensional modeling has been undertaken. The results obtained are shown in the next sections.

4.2 The three dimensional approach

4.2.1 Comparison of the Natural Frequency of sloshing

The sloshing natural frequencies of the contained water for different height to radius ratio are shown in figures 4.1- 4.6 and the results are compared with that of Housner's. These analyses were conducted using the three dimensional method.

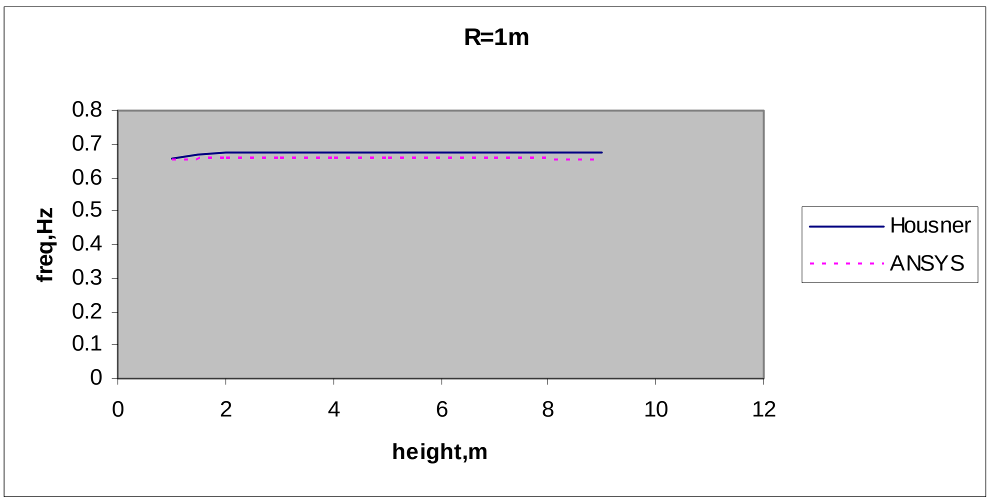


Fig. 4.1 Sloshing frequencies for constant radius of one meter and variable height

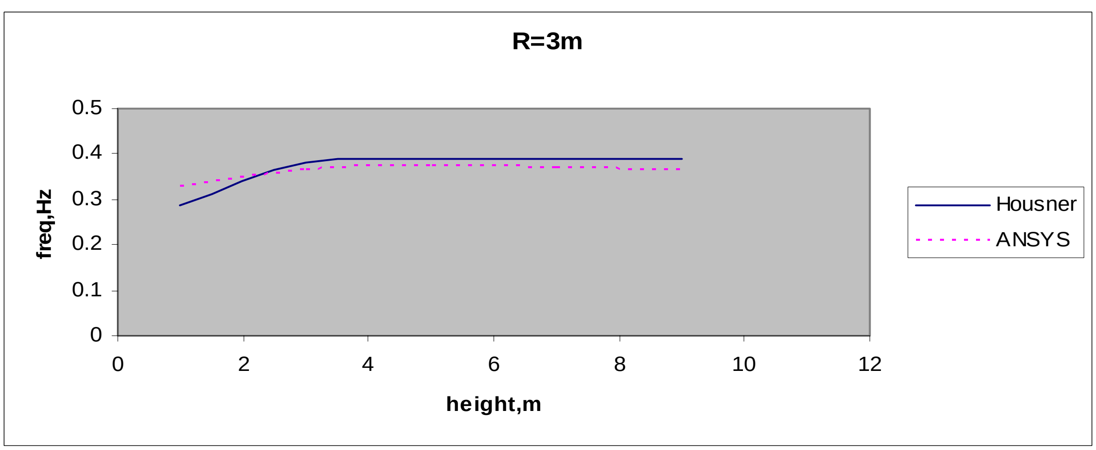


Fig. 4.2 Sloshing frequencies for constant radius of three meter and variable height

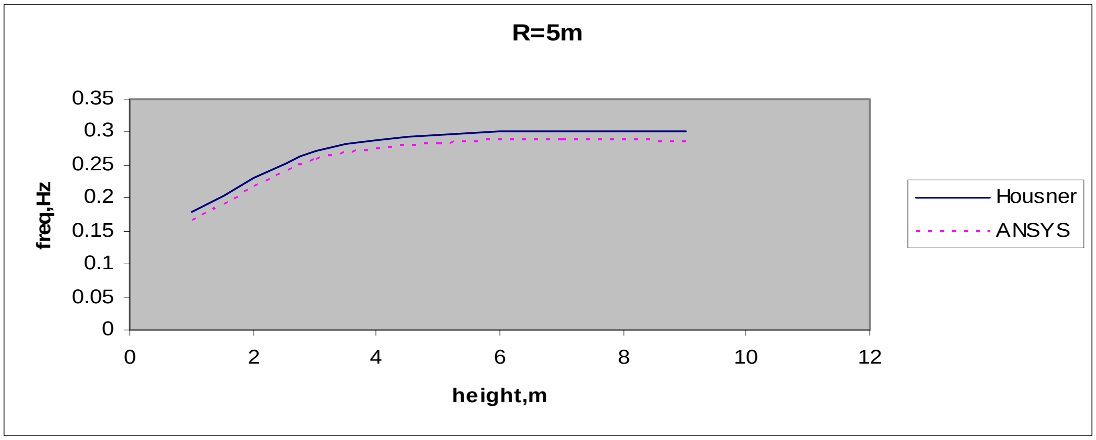


Fig. 4.3 Sloshing frequencies for constant radius of five meter and variable height

For the above cases of constant radius but variable heights the sloshing modes of FEM and that of Housner's approximate method are in close agreement. For the cases of constant radius of one, three and five meters the results are almost similar, as shown in the graphical results above.

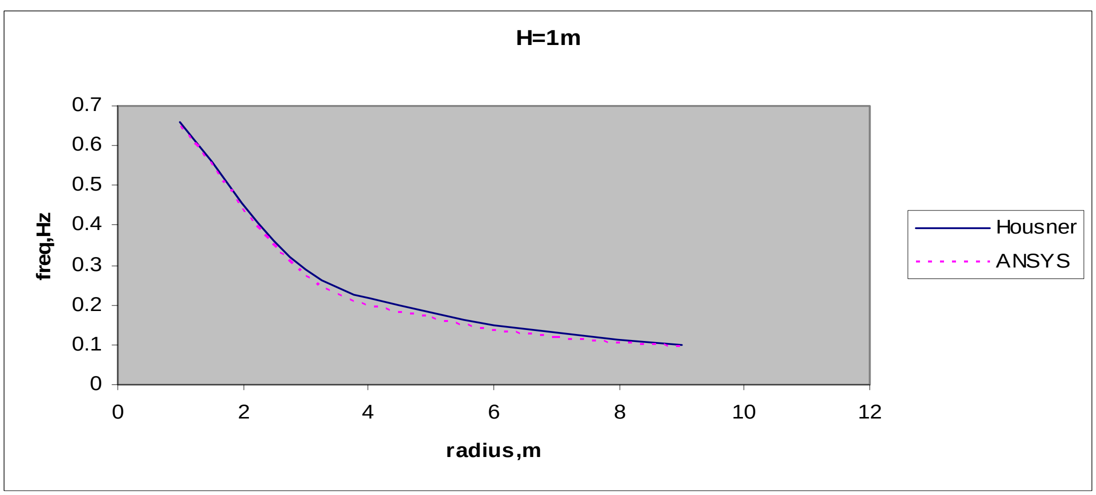


Fig. 4.4 Sloshing frequencies for constant height of one meter and variable radius

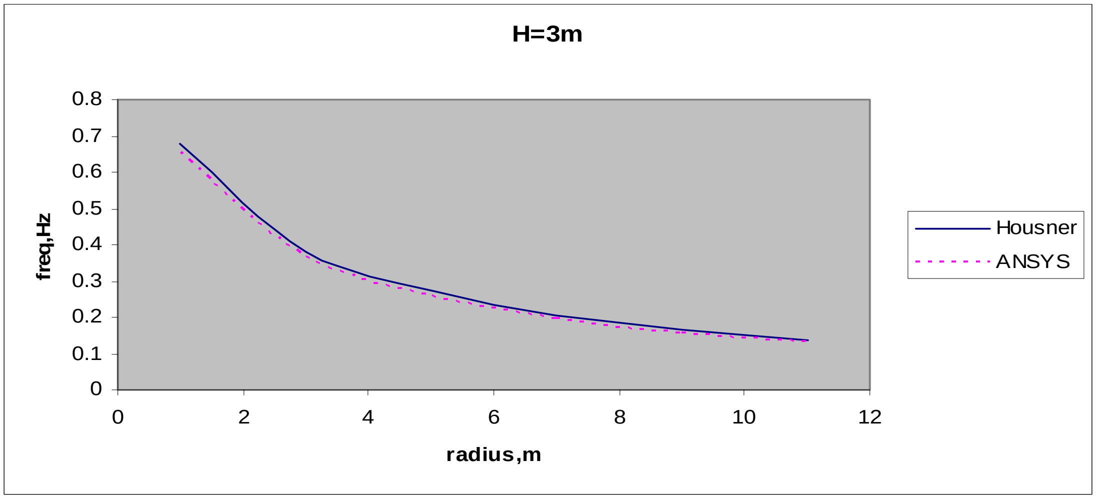


Fig. 4.5 Sloshing frequencies for constant height of three meter and variable radius

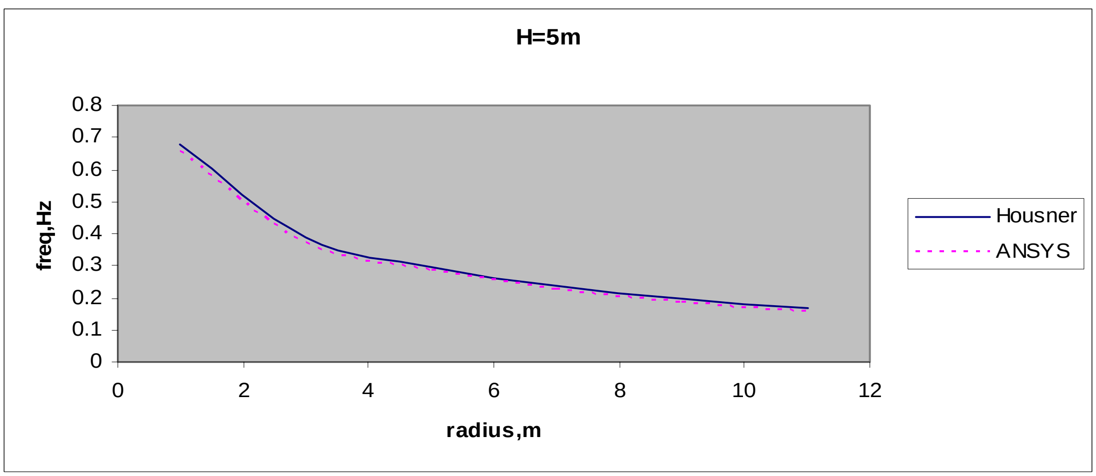


Fig. 4.6 Sloshing frequencies for constant height of five meter and variable radius

Similarly for the constant heights of one, three and five meters shown above graphically the results of FEM and Housner are almost the same.

The graphical results presented above show that the finite element method can produce sloshing modes that are in good harmony with the experimentally obtained results of Housner. Figure 4.7 below shows the sloshing mode shape of a three dimensional cylinder. The model gave results only for the sloshing mode shape and only the deformation of the fluid element was obtained.

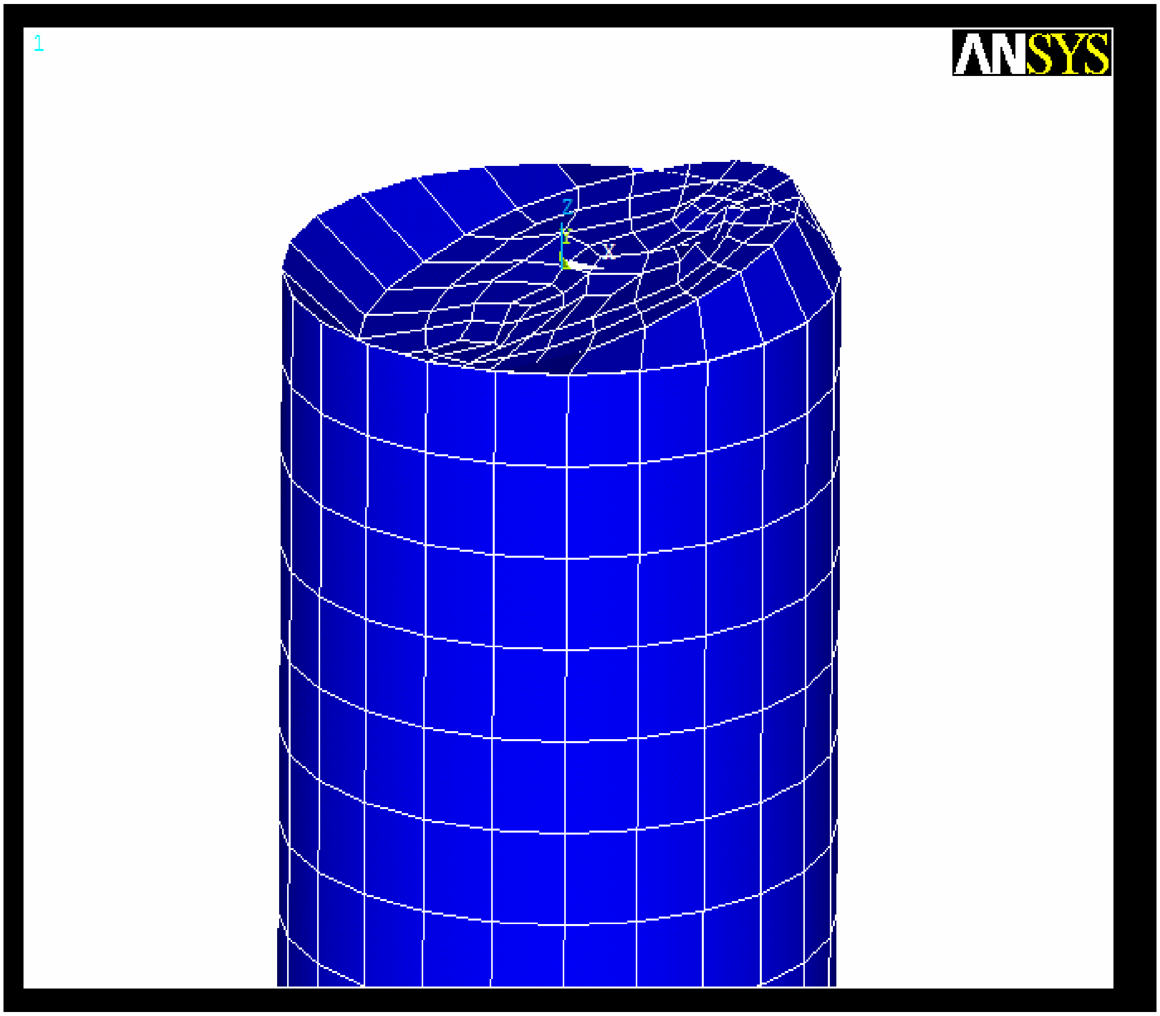


Fig. 4.7: First sloshing mode shape of a cylinder having a height of five meter and radius of one meter as obtained from the 3D finite element model

4.2.2 Comparison of the shear stress

The shear stresses resulting from both Housner's as well as finite element method were compared graphically. These results are shown below for the three dimensional analysis.

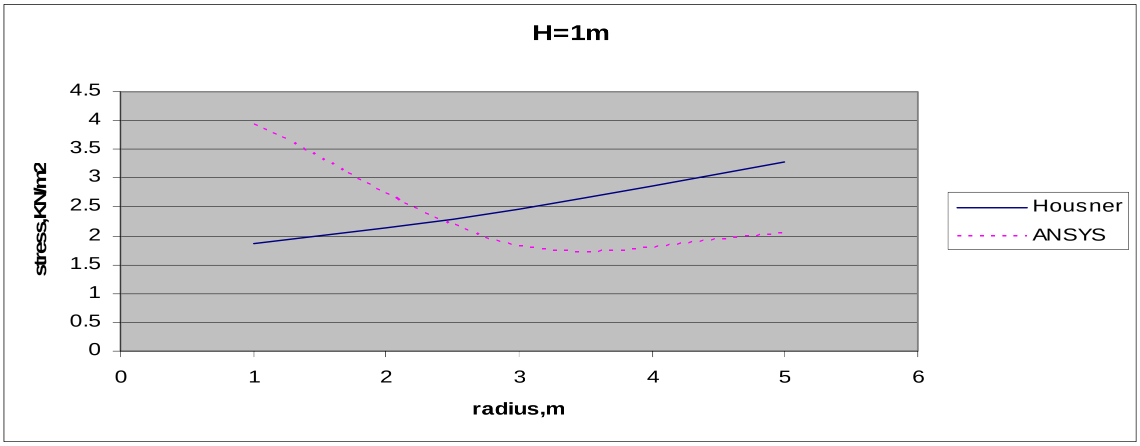


Fig. 4.8 Shear Stress for constant height of one meter and variable radius

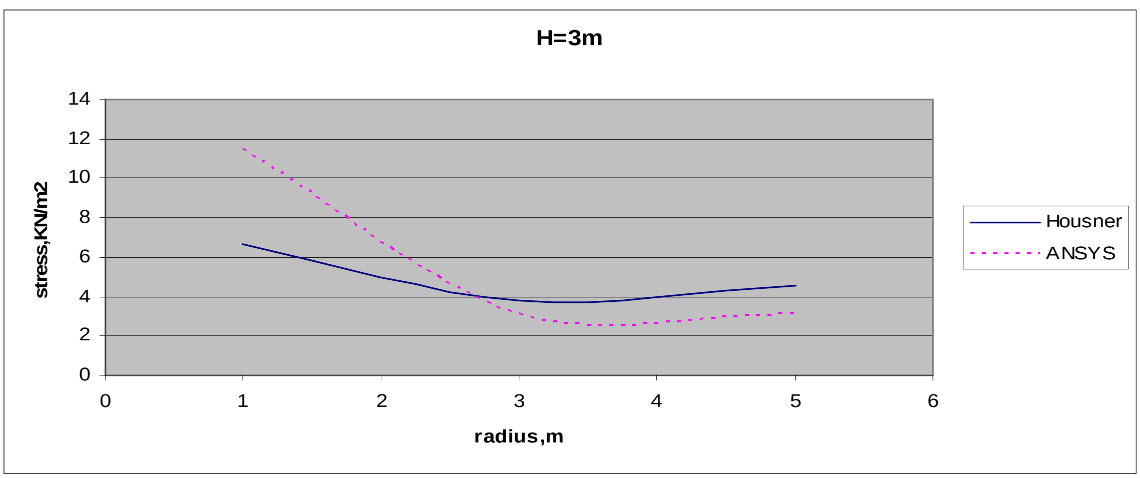


Fig. 4.9 Shear Stress for constant height of three meter and variable radius

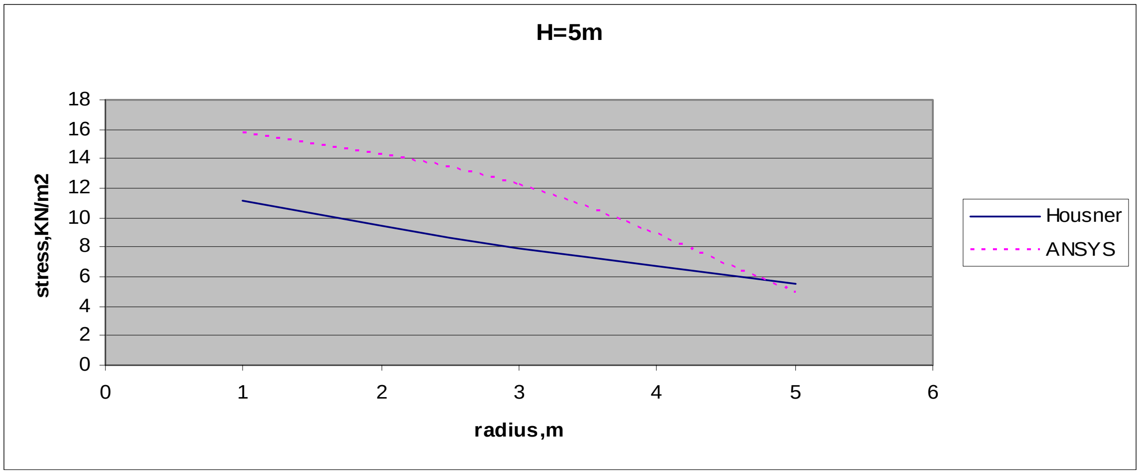


Fig. 4.10 Shear Stress for constant height of five meter and variable radius

From the graphical results for the constant height of one and three meters the FEM result was greater up to radius of 2.5 m and 2.8 m respectively. For the radius exceeding those mentioned earlier the shear stress results of Housner were found to be greater than that of FEM. For the constant height value of five meter the stress results of FEM for all the analyzed cases were found to be greater except near radius value of 5m.

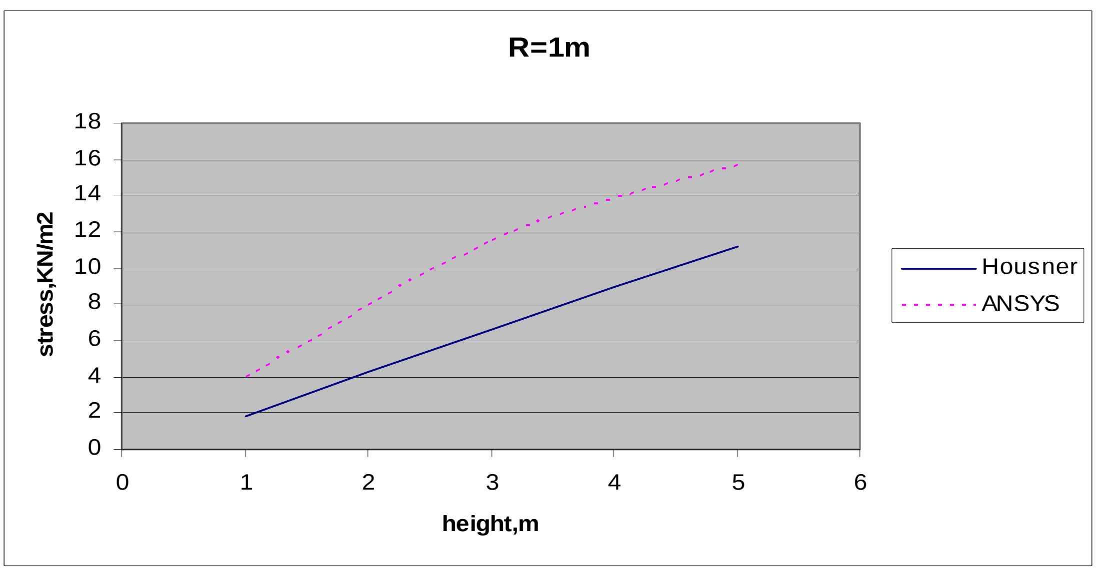


Fig. 4.11 Shear Stress for constant radius of one meter and variable height

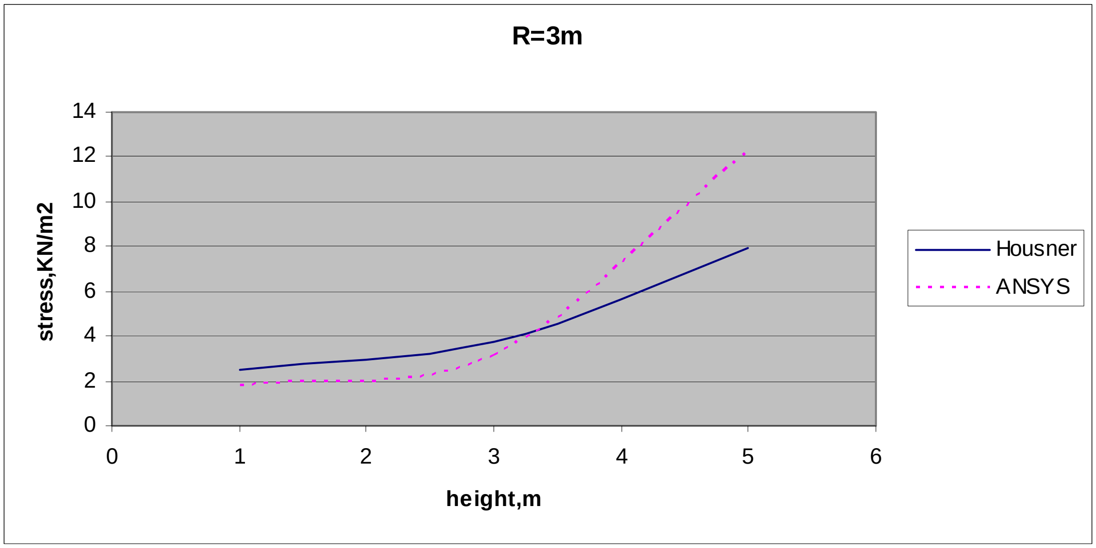


Fig. 4.12 Shear Stress for constant radius of three meter and variable height

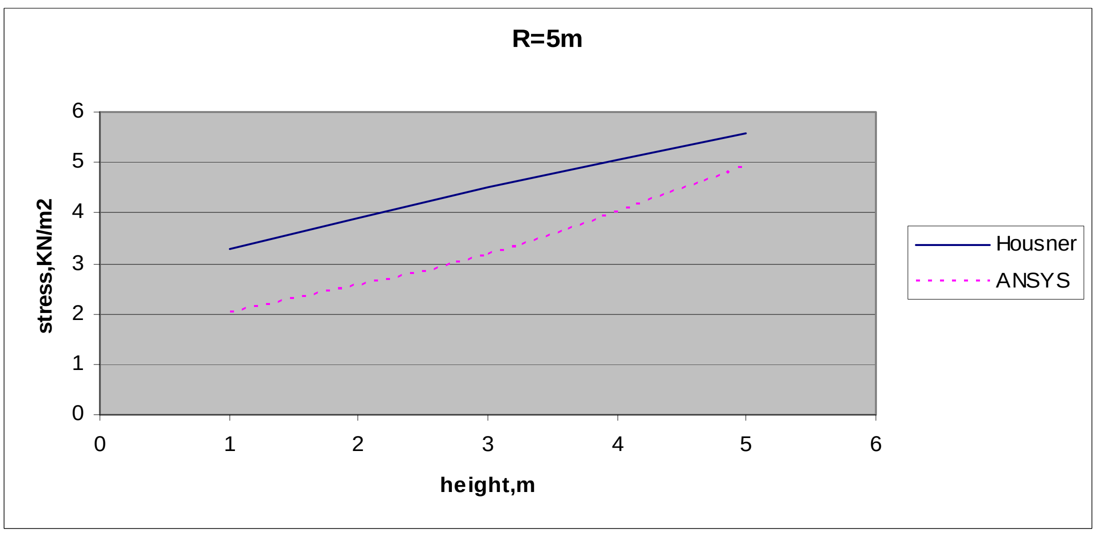


Fig. 4.13 Shear Stress for constant radius of five meter and variable height

For the case of constant radius and variable heights, when the radius was one meter the value of FEM for all the height values were found to be large. On the other hand for the constant radius of five meter the result of Housner was found to be larger. For constant radius of three meter the stress result of FEM was greater up to radius of 3.4m and the Housner result exceeded for values greater than 3.4m.

The bending moments obtained for the three dimensional analysis were very small. The discrepancy of the result was due to the fact that the sloshing modes were very small, and the coupling effect (liquid-shell frequency) was not initiated due to the large number of DOF present. As a result of this a large number of sloshing frequencies were obtained and the liquid-shell coupled frequencies were not obtained. Therefore the two dimensional approach have been considered. The next section gives the result obtained using the two dimensional approach.

4.3 The two dimensional approach

The two dimensional approach has been used to analyze the interaction problem. In this method the elements Shell 61 and Fluid 81 were used. The results obtained are shown below.

4.3.1 Natural Frequencies

The two dimensional analysis gave frequencies of both the sloshing and coupled system. These frequencies are given as output together in one solution run. To show the sequence in which the sloshing and coupled frequencies occur, an output from ANSYS is shown below. This result was for a container having a height of three meter, radius of one meter and thickness of 0.0254m.

Table 4.2: Sloshing frequency for constant height of three meter and radius of one meter from FEM output

Mode no.	1	2	3	4	5	6	7	8
Freq(Hz)	6.69E-02	8.58E-02	0.52134	1.2243	1.8631	2.7336	3.9094	140.11

The results obtained above show that the fundamental sloshing frequencies are much smaller than the ones computed by Housner's approach. This is due to the fact that the motion of the fluid giving rise to smaller sloshing frequencies. For this specific case the third mode gave frequency result comparable to that of the fundamental mode given by Housner. Tables 4.3, 4.4 & 4.5 show the results obtained for cylindrical containers having thickness of 0.0254m and 0.05m.

Table 4.3: Sloshing frequency for constant height of one meter and variable radius

Sloshing Frequency				
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency	Housner Frequency
m	m	Hz	Hz	Hz
1	1	0.62899	0.63018	0.65933
1	3	0.27114	0.27114	0.28859
1	5	0.16653	0.16653	0.17947

For the above case of constant height of one meter, the sloshing frequencies of Housner were found to be greater than that of FEM for both thickness cases. While the frequencies for the two thickness cases were almost the same. Comparing the percentage variation of frequencies for Housner and FEM method, the largest deviation of both 0.0254m and 0.05m thick containers was as high as 7.21%.

Table 4.4: Sloshing frequency for constant height of three meter and variable radius

Sloshing Frequency				
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency	Housner Frequency
m	m	Hz	Hz	Hz
3	1	0.52134	0.52134	0.67617
3	3	0.33513	0.33556	0.38067
3	5	0.24662	0.2467	0.27079

For the constant height of three meter, the sloshing frequencies of Housner's result were found to be greater. And the frequencies for the thickness considered were almost the same. The percentage variation of FEM and Housner's result is as high as 22.898% for both the thickness of 0.0254m and 0.05m. These values showed deviations especially for smaller tank dimensions as shown above.

Table 4.5: Sloshing frequency for constant height of five meter and variable radius

Sloshing Frequency				
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency	Housner Frequency
m	m	Hz	Hz	Hz
5	1	0.51900	0.51402	0.67618
5	3	0.30483	0.30256	0.38955
5	5	0.25501	0.25364	0.29486

For the constant height of five meter, the sloshing frequencies of Housner's result were found to be greater. And the frequencies of FEM for the thickness considered showed a small variation. The percentage variation of FEM and Housner's result is as high as

23.25% for the thickness of 0.0254m, and for the thickness of 0.05m the variation was found to be 23.98%. These values showed deviations especially for smaller tank dimensions as shown above. Fig. 4.14 below shows the sloshing mode shape of a two dimensional model.

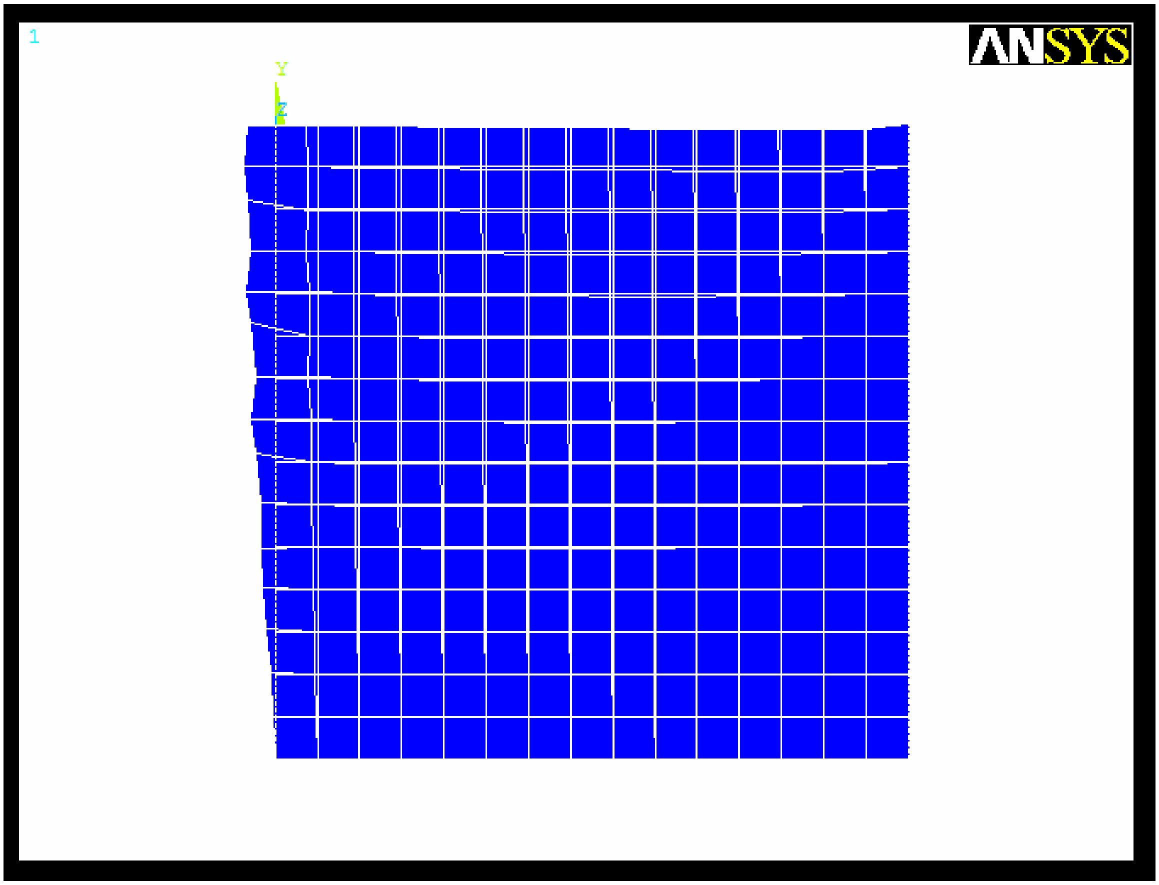


Fig. 4.14: The sloshing mode shape of a cylinder having a height of three meter and radius of three meter

Table 4.6: Coupled frequency for constant height of one meter and variable radius

Coupled frequency			
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency
m	m	Hz	Hz
1	1	749.52	883.34
1	3	459.02	645.68
1	5	350.53	475.69

Table 4.7: Coupled frequency for constant height of three meter and variable radius

Coupled frequency			
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency
m	m	Hz	Hz
3	1	140.11	170.55
3	3	172.81	221.58
3	5	145.49	193.14

Table 4.8: Coupled frequency for constant height of five meter and variable radius

Coupled frequency			
Height	Radius	t=0.0254m Frequency	t=0.05m Frequency
m	m	Hz	Hz
5	1	54.462	66.202
5	3	83.306	106.22
5	5	83.782	110.8

The result of Housner did not provide the coupled frequency (liquid-shell frequencies), which arises as a result of the fluid structure interaction. But using the FEM the coupled frequencies were obtained. The coupled frequencies are larger in magnitude than the sloshing frequencies. These coupled frequencies did not have any trend that could be related with height and radius. Also the result showed that with increased thickness the frequencies increased. Figure 4.15 shows the coupled system mode shape. As shown from the figure the coupled system produced deflection of the container (shell).

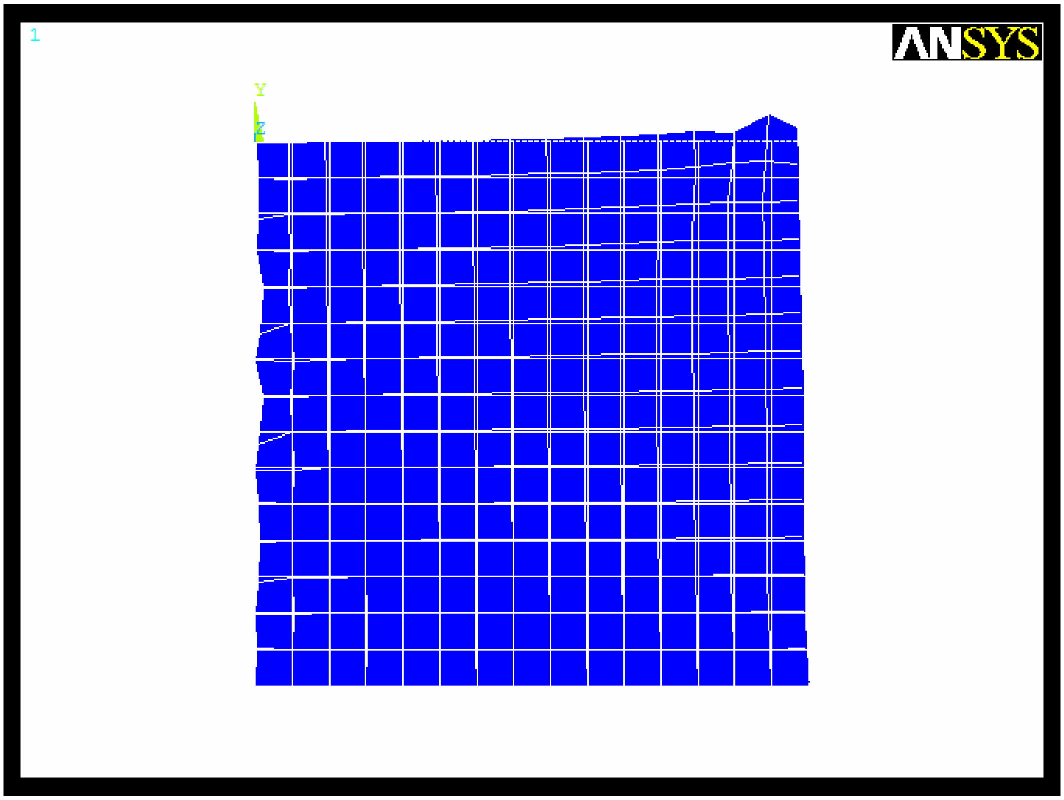


Fig. 4.15: The coupled mode shape of a cylinder having a height of three meter and radius of three meter

4.3.2 Shear Stress Result

The shear stress results obtained using the two dimensional analysis are compared with that of Housner. The results obtained are shown below graphically. The thickness of the container used for this analysis is 0.0254m and variable height and radius have been used for the analysis.

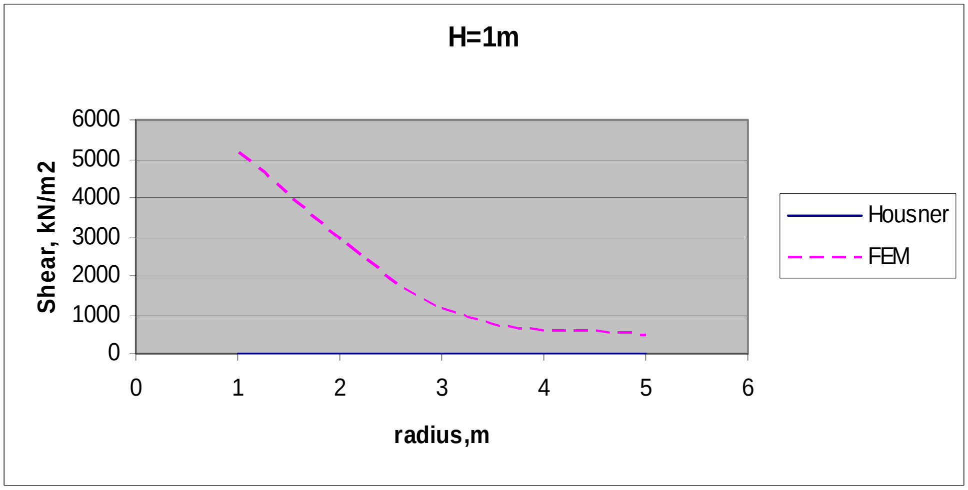


Fig. 4.16 Shear Stress for constant height of one meter and variable radius

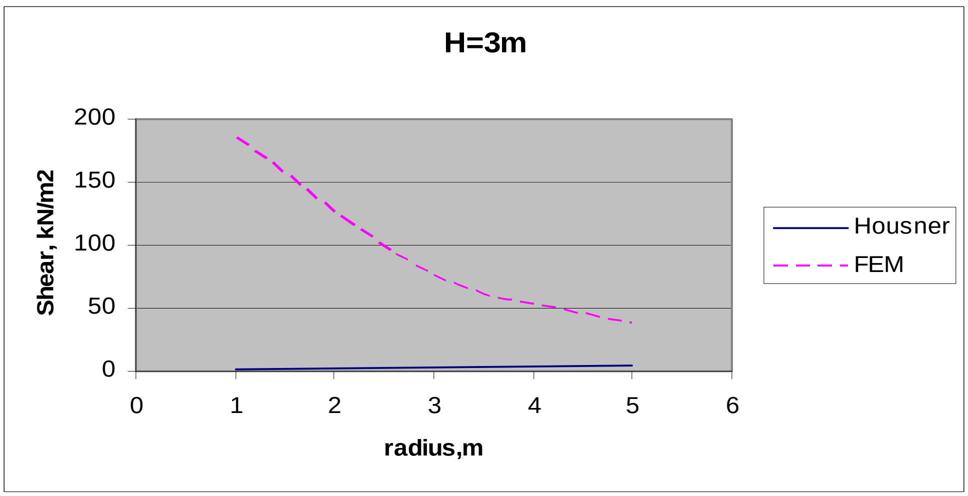


Fig. 4.17 Shear Stress for constant height of three meter and variable radius

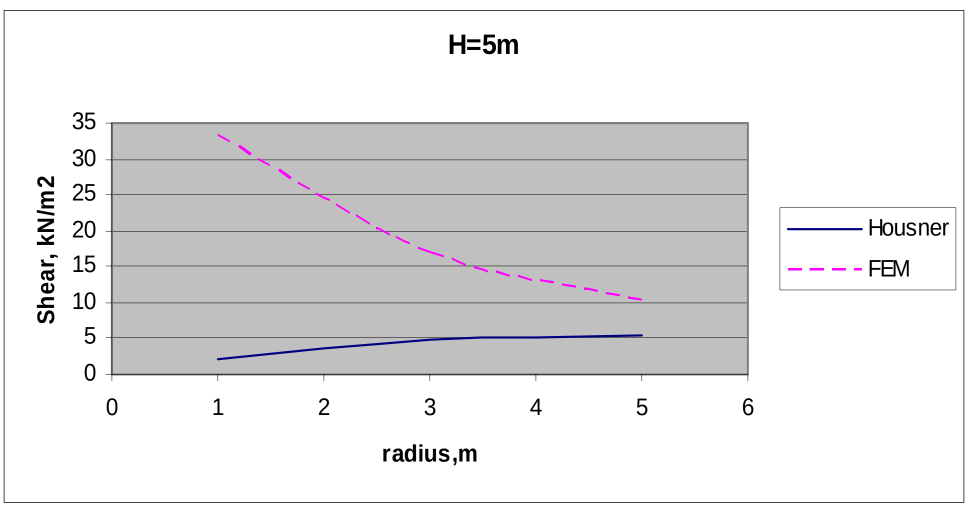


Fig. 4.18 Shear Stress for constant height of one meter and variable radius

From the graphical results above, it was observed that for constant heights of one, three and five meter the result of FEM was larger for all cases of variable heights considered. It was observed that the results of shallow containers had larger discrepancy while for slender containers the results were found to converge.

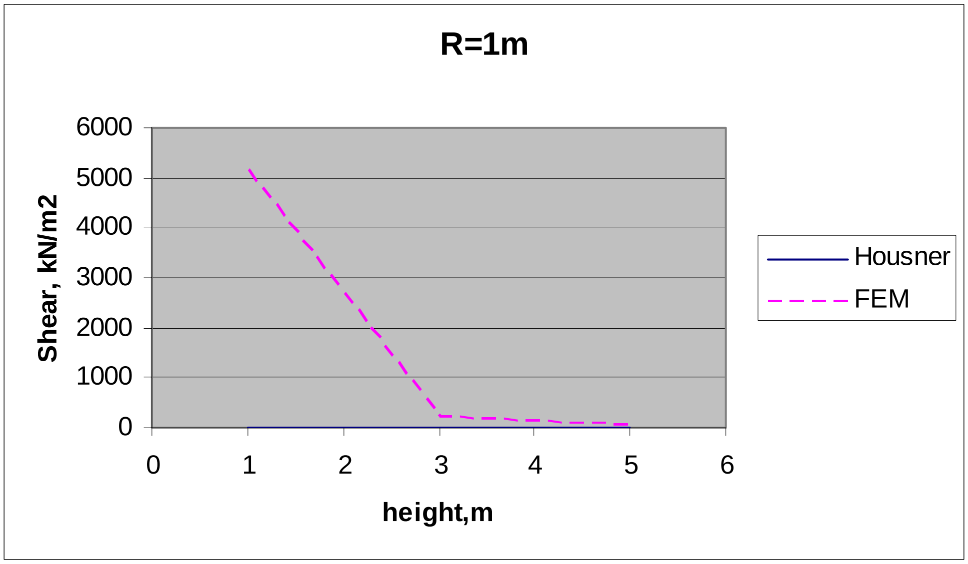


Fig. 4.19 Shear Stress for constant radius of one meter and variable height

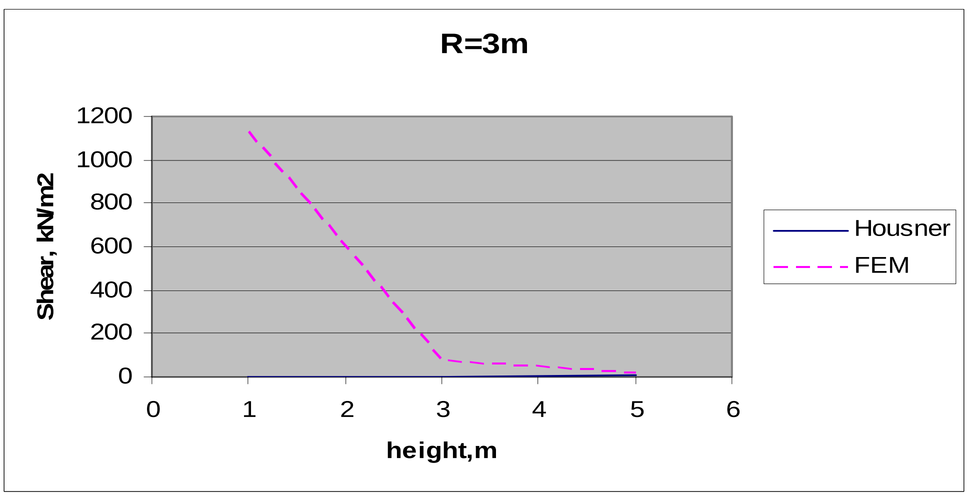


Fig. 4.20 Shear Stress for constant radius of three meter and variable height

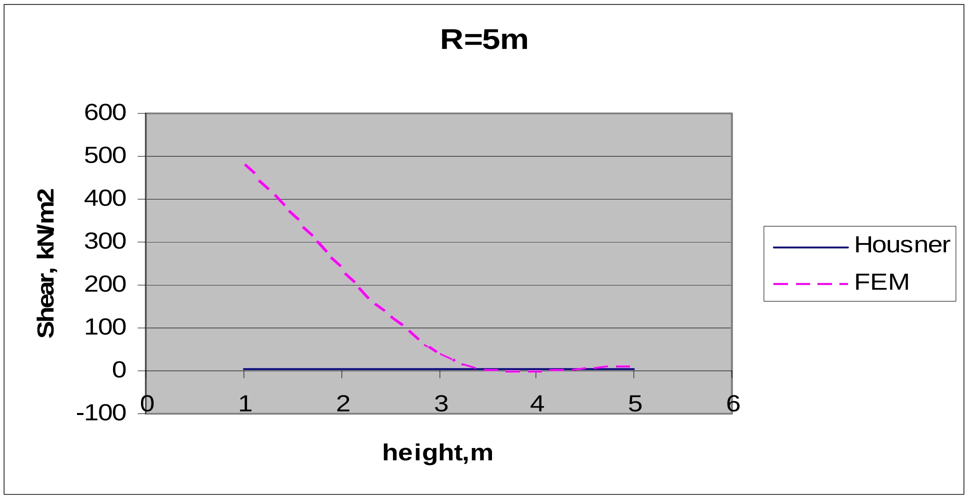


Fig. 4.21 Shear Stress for constant radius of five meter and variable height

In the case of constant radius of one, three and five meter the value of FEM was noticed to be greater for all cases. For both shallow and slender containers the result of FEM was greater, but for slender containers the difference was found out to be relatively small.

4.3.3 Bending Moment Result

The bending moment obtained using the two dimensional analysis are comparable with that of Housner. The results obtained are shown below graphically. The thickness of the container used for this analysis is 0.0254m and variable height and radius have been used for the analysis.

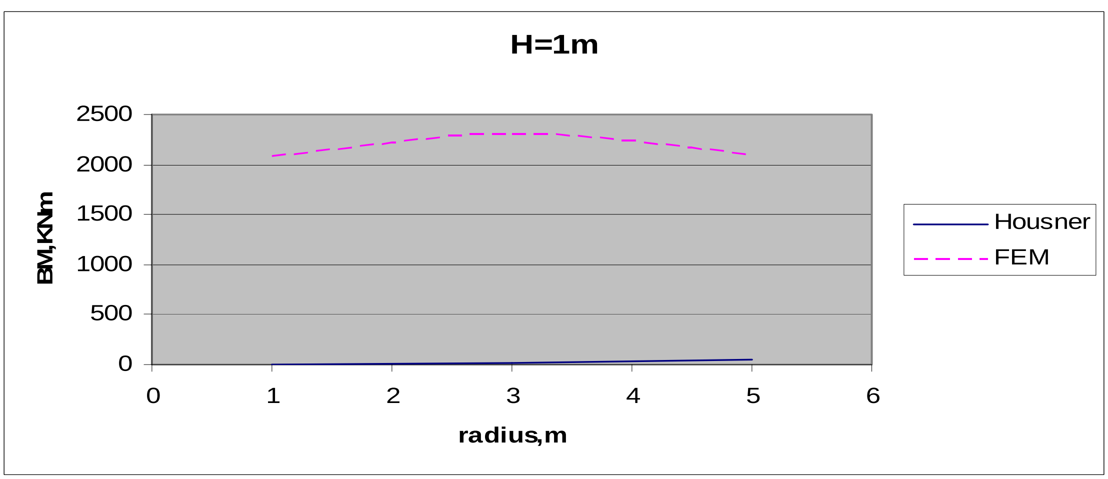


Fig. 4.22 Bending Moment for constant height of one meter and variable radius

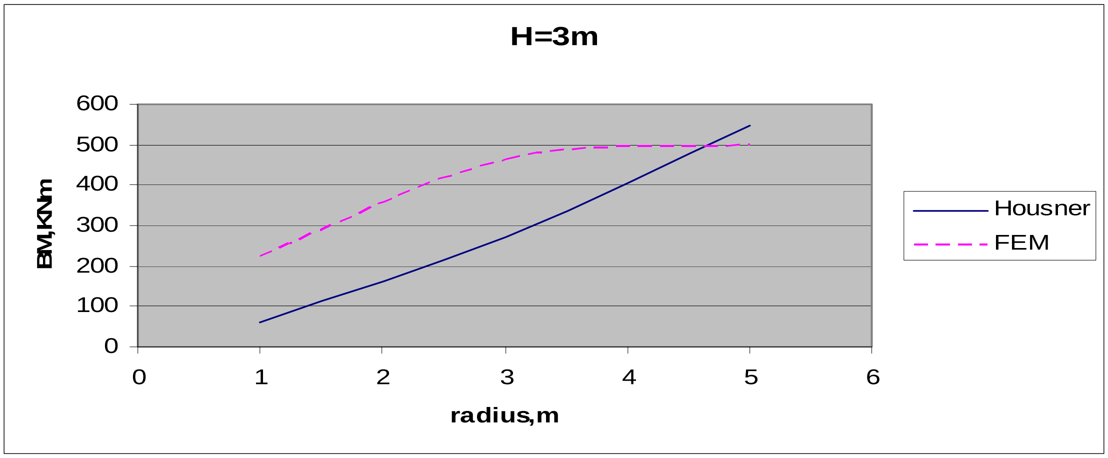


Fig. 4.23 Bending Moment for constant height of three meter and variable radius

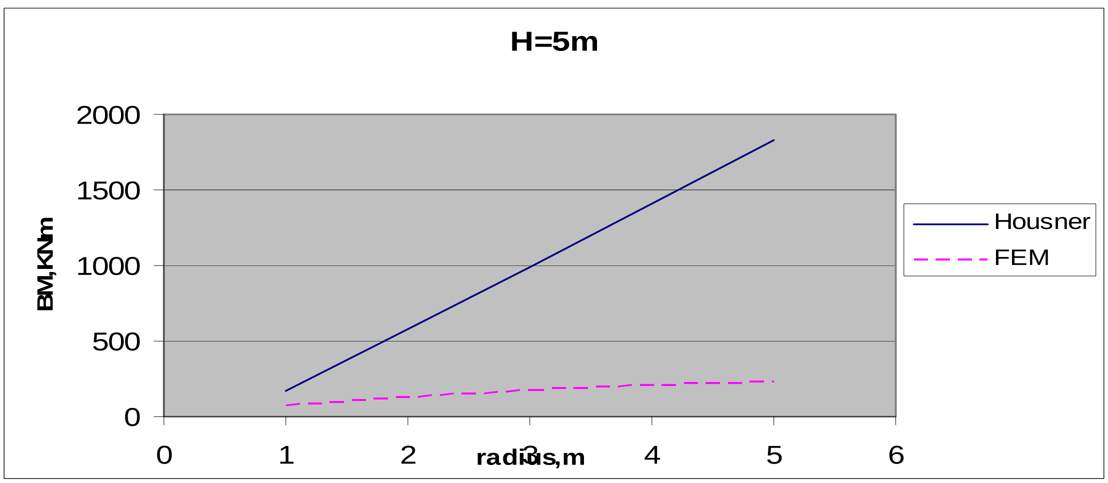


Fig. 4.24 Bending Moment for constant height of five meter and variable radius

From the graphical results above, it was observed that for constant height of one meter the result of FEM was larger for all cases of variable heights considered. For increased height of three meter the result of FEM was greater up to height of 4.5m while the result of Housner was greater above this value. And for height of five meter the result of Housner was greater for all cases.

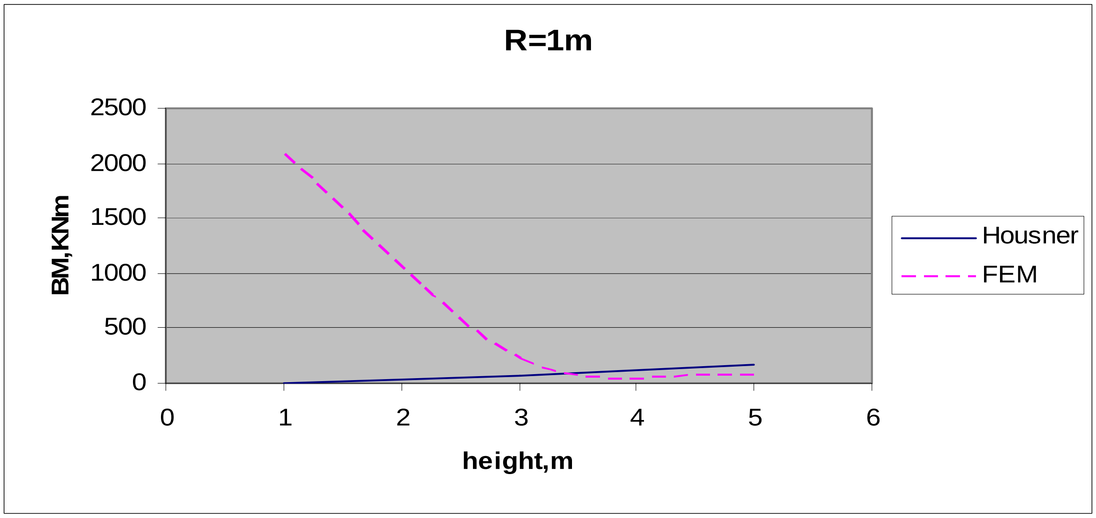


Fig. 4.25 Bending Moment for constant radius of one meter and variable height

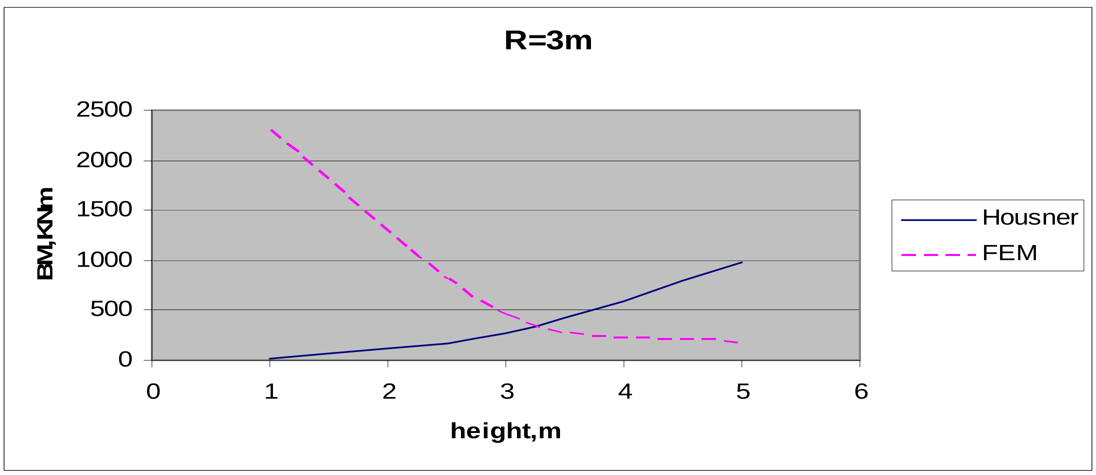


Fig. 4.26 Bending Moment for constant radius of three meter and variable height

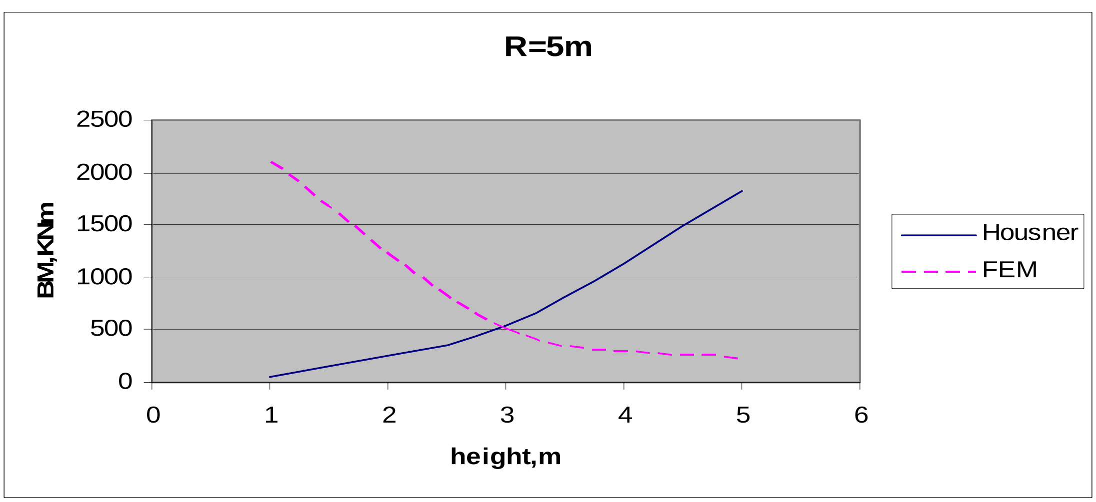


Fig. 4.27 Bending Moment for constant radius of five meter and variable height

In the case of constant radius of one meter and three meter the value of FEM was noticed to be greater up to heights of 3.4m and 3.3m respectively, and for the remaining portion the value of Housner exceeded that of FEM. On the other hand for the constant radius of 5m the results of FEM were found out to be greater up to height of 2.9m and for heights exceeding this value the results from Housner were found out to be greater.

4.3.4 The effect of thickness on the result

In order to study the effect of thickness of the container on the result, thickness values of 0.0254m and 0.05m were used. The output results are as shown below in tabular form.

Table 4.9: Summary of results for constant height of one meter and variable radius

Height m	Radius m	Shear t=2.54cm kN	Shear t=5cm kN	Axial t=2.54cm kN	Axial t=5cm kN	MZ(bending) t=2.54cm kNm	MZ(bending) t=5cm kNm
1	1	3.24E+04	7.29E+04	8.58E+05	1.43E+06	2.08E+03	6.84E+03
1	3	2.12E+04	5.40E+04	9.88E+05	1.82E+06	2.30E+03	8.35E+03
1	5	1.50E+04	3.89E+04	9.06E+05	1.73E+06	2.09E+03	7.74E+03

Table 4.10: Summary of results for constant height of three meter and variable radius

Height m	Radius m	Shear t=2.54cm kN	Shear t=5cm kN	Axial t=2.54cm kN	Axial t=5cm kN	MZ(bending) t=2.54cm kNm	MZ(bending) t=5cm kNm
3	1	3.48E+03	8.09E+03	9.25E+04	1.58E+05	2.24E+02	7.59E+02
3	3	4.24E+03	1.07E+04	1.98E+05	3.56E+05	4.61E+02	1.65E+03
3	5	3.57E+03	9.33E+03	2.15E+05	4.02E+05	4.99E+02	1.84E+03

Table 4.11: Summary of results for constant height of five meter and variable radius

Height m	Radius m	Shear t=2.54cm kN	Shear t=5cm kN	Axial t=2.54cm kN	Axial t=5cm kN	MZ(bending) t=2.54cm kNm	MZ(bending) t=5cm kNm
5	1	1.05E+03	2.43E+03	2.78E+04	4.75E+04	6.73E+01	2.28E+02
5	3	1.58E+03	3.95E+03	7.38E+04	1.32E+05	1.72E+02	6.10E+02
5	5	1.59E+03	4.13E+03	9.58E+04	1.78E+05	2.22E+02	8.17E+02

The above results show that the axial force, shear force and bending moment almost increased with increasing thickness of the container for FEM.

To show the effect of increased thickness, using the Housner method, on the shear force and bending moment a representative value of constant height of three meter and variable radius is shown below.

Table 4.12: Housner result for constant height of three meter and variable radius

Height m	Radius m	t=0.0254m Shear kN	t=0.0254m BM kNm	t=0.05m Shear kN	t=0.05m BM kNm
3	1	41.789	60.751	53.559	78.404
3	3	213.825	271.881	245.922	307.989
3	5	426.639	547.747	480.134	607.928

The above tabular result of Housner showed that with increased thickness the shear force as well as the bending moment values increased. But the sloshing modes for both the cases were constant.

4.3.5 Free Board Requirement

One of the requirements that need consideration in the design of liquid containing structures is freeboard. The freeboard gives the amount of clearance, so that the localized impact pressure will not damage the roof of the structure during motion of the liquid. The tables below show the maximum vertical displacements obtained using FEM and Housner method.

Table 4.13 Freeboard for constant height of one meter and variable radius

Free Board requirement				
Height (m)	Radius (m)	FEM(m) Sloshing	FEM(m) Coupled	Housner (m)
1	1	0.0284	0.0287	0.3845
1	3	0.004	0.004	0.2858
1	5	0.001	0.016	0.4557

Table 4.14 Freeboard for constant height of three meter and variable radius

Free Board requirement				
Height (m)	Radius (m)	FEM(m)		Housner (m)
		Sloshing	Coupled	
3	1	0.024	0.0068	0.5126
3	3	0.0223	0.011	0.4583
3	5	0.0054	0.008	0.5066

Table 4.15 Freeboard for constant height of five meter and variable radius

Free Board requirement				
Height (m)	Radius (m)	FEM(m)		Housner (m)
		Sloshing	Coupled	
5	1	0.0296	0.0308	0.5127
5	3	0.0184	0.0328	0.5329
5	5	0.0175	0.006	0.5260

The above results showed that for all cases considered the freeboard requirement of Housner was much greater than the FEM. The modes considered by FEM were the sloshing and coupled modes. In this paper the case of modal combination has not been considered in the analysis. For one of the analysis results modal combination using SRSS has been used and the following result was obtained.

Table 4.16 Modal Combination of Freeboard for constant height of three meter and radius of one meter

Frequency, Hz	0.521	1.224	2.733	3.91	140.11
Displacement,m	0.0241	0.0724	0.212	0.578	0.0068

The maximum freeboard using Housner's result was obtained as 0.513m and that of finite element approach using SRSS modal combination is 0.62m.

4.3.6 Summary of the Analysis Results

Generalizing the analysis result, Housner's simplified method is experimentally verified analysis method. Comparing the results of FEM with this simplified method therefore would be on the safe side. The analysis result of FEM gave no general trend for the different cases, but the following observations were made.

In the three dimensional approach the sloshing frequencies obtained were almost similar to that obtained using Housner's approximate method. The shear stress results showed that there is a deviation in the values, and the result of FEM was found to be greater for most cases of slender tanks. But the coupled effects (liquid-shell interaction frequencies) were not excited; as a result the bending moment results obtained were not comparable to that of Housner's result.

In the two dimensional approach of the FEM the sloshing frequencies have a slight deviation. The shear force results of FEM were found out to be greater for almost all containers and the values were noticed to decrease with increasing H/R ratio as compared to Housner's result. For smaller radius and height values, higher deviations were noticed. The bending moment results of FEM for shallow containers were found to be greater, whereas for slender containers the results from Housner were found to be greater. Considering the axial force, no mention of these stresses was incorporated in the Housner result, but during the FEM analysis the values of the results were found to be significant. The freeboard obtained for all analysis cases using FEM were smaller as compared to the Housner method.

The above results signify that for shallow and slender containers the shear force results of Housner need to be magnified, while for shallow containers the bending moment result of Housner needs magnification. But the effect of the axial forces developed should be taken into account while designing these structures, since the results obtained using the FEM approach were found to be significant.

Comparing the effect of flexibility on the FEM result to the simplified analysis method of Housner; shallow tanks had rather an amplified bending moment result due to the flexibility effect. But the bending moment results of slender containers from FEM were found out to be smaller. While the shear force obtained using the two dimensional FEM analysis for the liquid-shell coupled frequencies was greater for all cases.

The Housner Approximate Method is a simplified approach of analysis for the dynamic analysis of liquid containing structures. Comparing the time and computational effort required, the Housner method becomes easy to manipulate. Based on the above results the Housner method needs to be amplified for the shear force and bending moment. Also the axial stresses needs to be taken into account when analyzing these containers, which for approximation purpose can be taken to be equal to the shear force developed. The limitation of the finite element method was due to the fact that, the second order effect of the axial load was not considered. Additionally the vertical displacements of the free surface obtained using FEM were very small, while the Housner result for the vertical displacement was conservative and this value needs to be taken. On the other hand modal combination of the results using FEM also gave results comparable to that of Housner.

Chapter 5

Conclusion and Recommendations

5.1 Conclusion

Dynamic analysis of liquid containing cylindrical tanks was undertaken using the finite element method. Three dimensional and two dimensional analyses of the structural interaction problem were conducted. These interaction problems were quite complex and they have been challenging as well as interesting.

Using the ANSYS finite element software, elements depicting the properties of the container and the liquid were selected and coupled. From the results that were obtained using the finite element method and comparing them with the Housner Approximate method the following conclusions can be drawn.

It has been shown that the sloshing frequencies were in agreement with that given by Housner's method. Using FEM the coupled frequencies of the system were initiated using the two dimensional approach, but the Housner Approximate method did not give these frequencies. The coupled frequencies (liquid-shell frequencies) were much larger than the sloshing frequencies. The flexibility of the FEM analysis gave significant variation on shallow tanks. While the results obtained using FEM for slender tanks was smaller as compared to Housner's method.

Comparing results of stresses, the Housner method underestimates the radial shear stress of the shell for most of the cases considered.

The bending moment of the FEM as compared to Housner's, for H/R ratio less than 1.5 produced higher results while for H/R greater than 1.5 the result was found out to be smaller.

The axial forces obtained using FEM are quite significant. The order of magnitude of the force is almost equal to the shear force developed. And they might as well cause secondary effect as well as buckling of the whole system. Therefore due considerations of the axial forces need to be given while designing the structure. The vertical displacements of the free surface obtained using FEM for individual modes were much smaller than Housner's; on the other hand modal combination gave result comparable to Housner.

5.2 Recommendation

The analysis presented above does not consider all the factors associated with the dynamic analysis of liquid containing tanks. Therefore the following recommendations are put forward for further work.

1. Dynamic analysis of closed containers: Containers having differently shaped roofs.

2. Modeling for different base conditions and variable wall thicknesses along the heights.

3. Dynamic analysis based on variable material and fluid properties.

4. Developing model that incorporates the second order effect of the axial load and consideration of modal superposition.

5. Laboratory setup of the interaction problem.

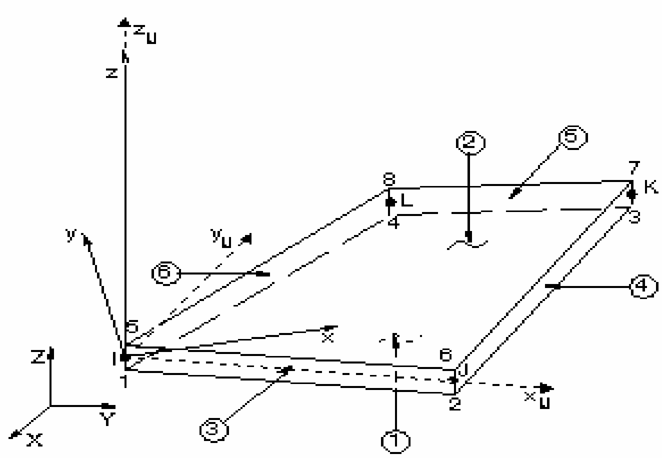
Appendix A

SHELL63

A1. Element Description

SHELL63 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. A consistent tangent stiffness matrix option is available for use in large deflection (finite rotation) analyses.

Fig. A1. SHELL63 Elastic Shell



A2. Input Data

The geometry, node locations, and the coordinate system for this element are shown in Fig. A1. The element is defined by four nodes, four thicknesses, elastic foundation stiffness, and the orthotropic material properties. Orthotropic material directions correspond to the element coordinate directions. The element x-axis may be rotated by an angle THETA (in degrees).

The thickness is assumed to vary smoothly over the area of the element, with the thickness input at the four nodes. If the element has a constant thickness, only TK (I) need be input. If the thickness is not constant, all four thicknesses must be input.

Pressures may be input as surface loads on the element faces as shown by the circled numbers on Fig. A1. Positive pressures act into the element. Edge pressures are input as force per unit length. The lateral pressure loading may be an equivalent (lumped) element load applied at the nodes or distributed over the face of the element. The equivalent element load produces more accurate stress results with flat elements representing a curved surface or elements supported on an elastic foundation since certain fictitious bending stresses are eliminated.

Temperatures may be input as element body loads at the "corner" locations (1-8) shown in Fig. A1.

A3. SHELL63 Input Summary

Element Name

SHELL63

Nodes

I, J, K, L

Degrees of Freedom

UX, UY, UZ, ROTX, ROTY, ROTZ

Real Constants

TK(I), TK(J), TK(K), TK(L), EFS, THETA, RMI, CTOP, CBOT, (Blank), (Blank), (Blank), (Blank), (Blank), (Blank), (Blank), (Blank), (Blank), ADMSUA

Material Properties

EX, EY, EZ, (PRXY, PRYZ, PRXZ or NUXY, NUYZ, NUXZ), ALPX, ALPY, ALPZ, DENS, GXY, DAMP

Surface Loads

Pressures --

face 1 (I-J-K-L) , face 2 (I-J-K-L) , face 3 (J-I), face 4 (K-J), face 5 (L-K), face 6 (I-L)

Body Loads

Temperatures --

T1, T2, T3, T4, T5, T6, T7, T8

Special Features

Stress stiffening, large deflection, Birth and death

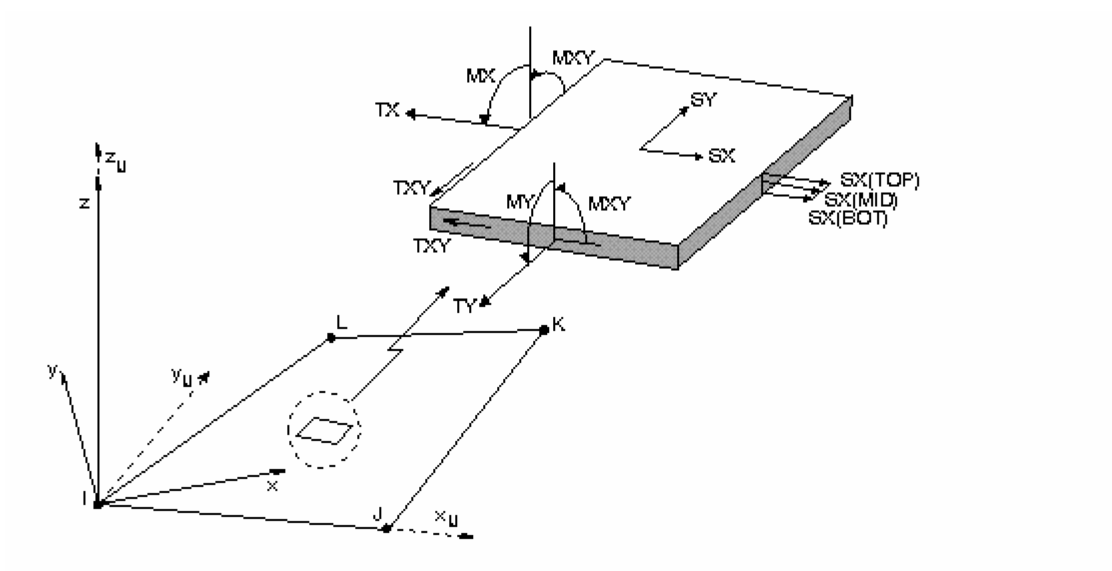
A4. Output Data

The solution output associated with the element is in two forms:

- nodal displacements included in the overall nodal solution
- additional element output as shown in Element output Definitions

Printout includes the moments about the x face (MX), the moments about the y face (MY), and the twisting moment (MXY). The moments are calculated per unit length in the element coordinate system. The element stress directions are parallel to the element coordinate system.

Fig. A2 SHELL63 Stress Output



Appendix B

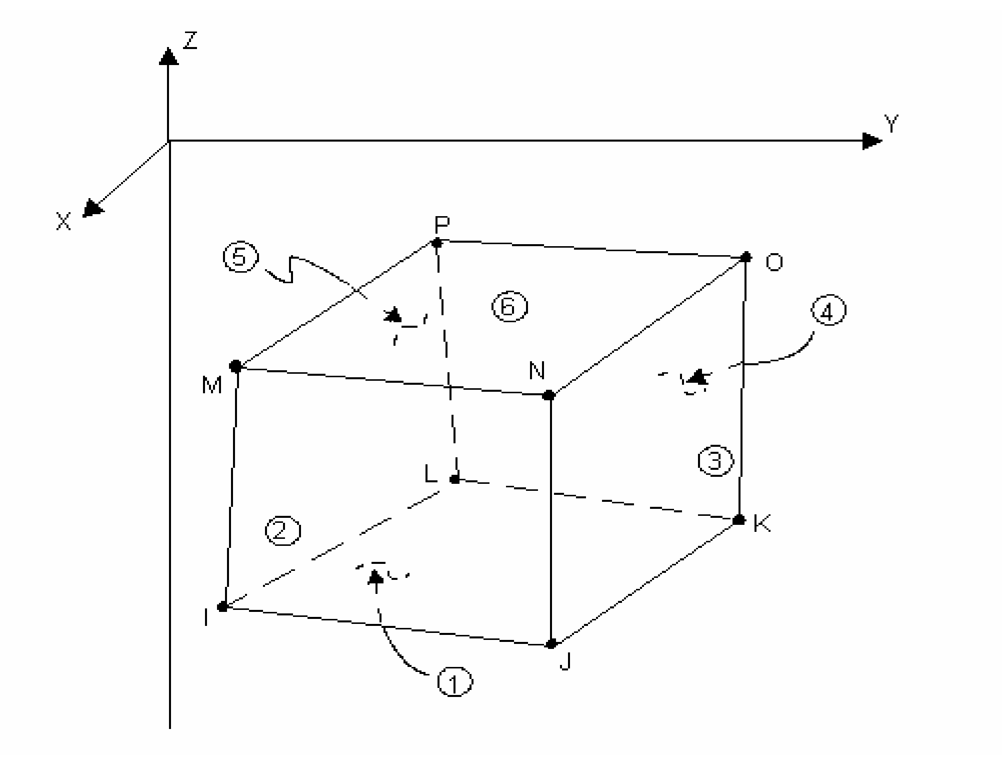
FLUID80

B1. Element Description

FLUID80 is a 3-D structural solid element. The fluid element is used to model fluids contained within vessels having no net flow rate. The fluid element is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. Acceleration effects, such as in sloshing problems, as well as temperature effects, may be included.

The fluid element is defined by eight nodes having three degrees of freedom at each node: translation in the nodal x, y, and z directions. The reduced method is the only acceptable method for modal analysis using the ANSYS fluid element.

Fig. B1. FLUID80 3D Contained Fluid



B2. Input Data

The geometry, node locations, and the coordinate system for this element are shown in Fig. B1.The element input data includes eight nodes and the isotropic material properties. EX, which is interpreted as the "fluid elastic modulus", should be the bulk modulus of the fluid (approximately $2.0684*10^9$ Pascal for water). The viscosity property (VISC) is used to compute a damping matrix for dynamic analyses. A typical viscosity value for water is $1.13*10^{-3}$ $N.S/m^2$

Pressures may be input as surface loads on the element faces as shown by the circled numbers on Fig. B1. Positive pressures act into the element. Temperatures may be input as element body loads at the nodes.

The element also includes special surface effects, which may be thought of as gravity springs used to hold the surface in place. This is performed by adding springs to each node, with the spring constants being positive on the top of the element, and negative on the bottom. Gravity effects must be included if a free surface exists. For an interior node, the positive and negative effects cancel out, and at the bottom, where the fluid must be contained to keep the fluid from leaking out, the negative spring has no effect (as long as all degrees of freedom on the bottom are fixed). These surface springs, while necessary to keep the free surface in place, artificially reduce the hydrostatic motion of the free surface. The error for a tank with vertical walls, expressed as a ratio of the computed answer over the correct answer is 1.0/ (1.0 + (bottom pressure/bulk modulus)), which is normally very close to 1.0. Hydrodynamic results are not affected by this over stiffness.

B3. FLUID80 Input Summary

Element Name

FLUID80

Nodes

I, J, K, L, M, N, O, P

Degrees of Freedom

UX, UY, UZ

Real Constants

None

Material Properties

EX, ALPX, DENS, VISC, DAMP

Surface Loads

Pressures --

face 1 (J-I-L-K), face 2 (I-J-N-M), face 3 (J-K-O-N), face 4 (K-L-P-O), face 5 (L-I-M-P), face 6 (M-N-O-P)

Body Loads

Temperatures --

T(I), T(J), T(K), T(L), T(M), T(N), T(O), T(P)

B4. Output Data

The solution output associated with the element is in two forms:

- degree of freedom results included in the overall nodal solution
- additional element output

The pressure and temperature are evaluated at the element centroid.

Appendix C

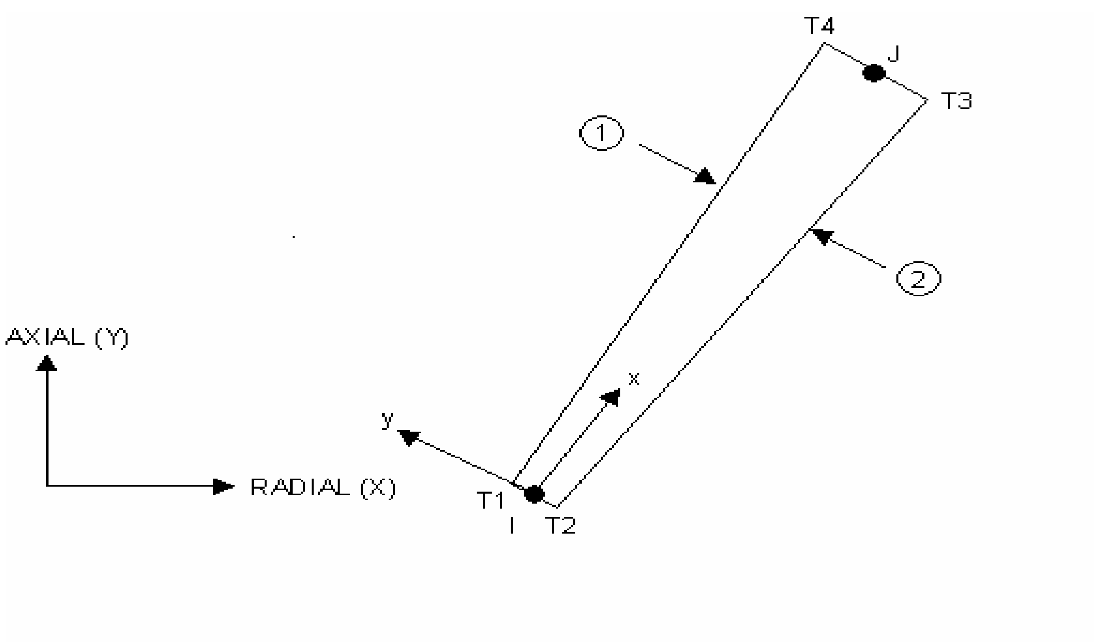
SHELL61

C1. Element Description

SHELL61 has four degrees of freedom at each node: translations in the nodal x, y, and z directions and a rotation about the nodal z-axis. The loading may be axisymmetric or nonaxisymmetric.

Extreme orientations of the conical shell element result in a cylindrical shell element or an annular disc element. The shell element may have a linearly varying thickness.

Fig. C1. SHELL61Axisymmetric-Harmonic Structural Shell



C2. Input Data

The geometry, node locations, and the coordinate system for this element are shown in Fig. C1. The element is defined by two nodes, two end thicknesses, the number of harmonic waves, a symmetry condition, and the orthotropic material properties.

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The material may be orthotropic, with nine elastic constants required for its description. The element loading may be input as any combination of harmonically varying temperatures and pressures. Harmonically varying nodal forces, if any, should be input on a full 360° basis.

The element may have variable thickness. The thickness is assumed to vary linearly between the nodes. If the element has a constant thickness, only TK(I) is required. Real constant ADMSUA is used to define an added mass per unit area.

Harmonically varying pressures may be input as surface loads on the element faces as shown by the circled numbers on Fig. C1. Positive pressures act into the element. The pressures are applied at the surface of the element rather than at the centroidal plane so that some thickness effects can be considered. These include the increase or decrease in size of surface area the load is acting on and (in the case of a non-zero Poisson's ratio) an interaction effect causing the element to grow longer or shorter under equal pressures on both surfaces. Material properties like modulus of elasticity and Poisson ratio are required for this effect.

Harmonically varying temperatures may be input as element body loads at the four corner locations shown in Fig C1. If all other temperatures are unspecified, they default to T1. If only T1 and T2 are input, T3 defaults to T2 and T4 defaults to T1.

C3. SHELL61 Input Summary

Element Name

SHELL61

Nodes

I, J

Degrees of Freedom

UX, UY, UZ, ROTZ

Real Constants

TK(I), TK(J), (TK(J) defaults to TK(I)), ADMSUA

Material Properties

EX, EY, EZ, PRXY, PRYZ, PRXZ (or NUXY, NUYZ, NUXZ), ALPX, ALPZ, DENS, GXZ, DAMP.

Surface Loads

Pressures --

face 1 (I-J), face 2 (I-J)

Body Loads

Temperatures --

T1, T2, T3, T4

Mode Number --

Input mode number

Special Features

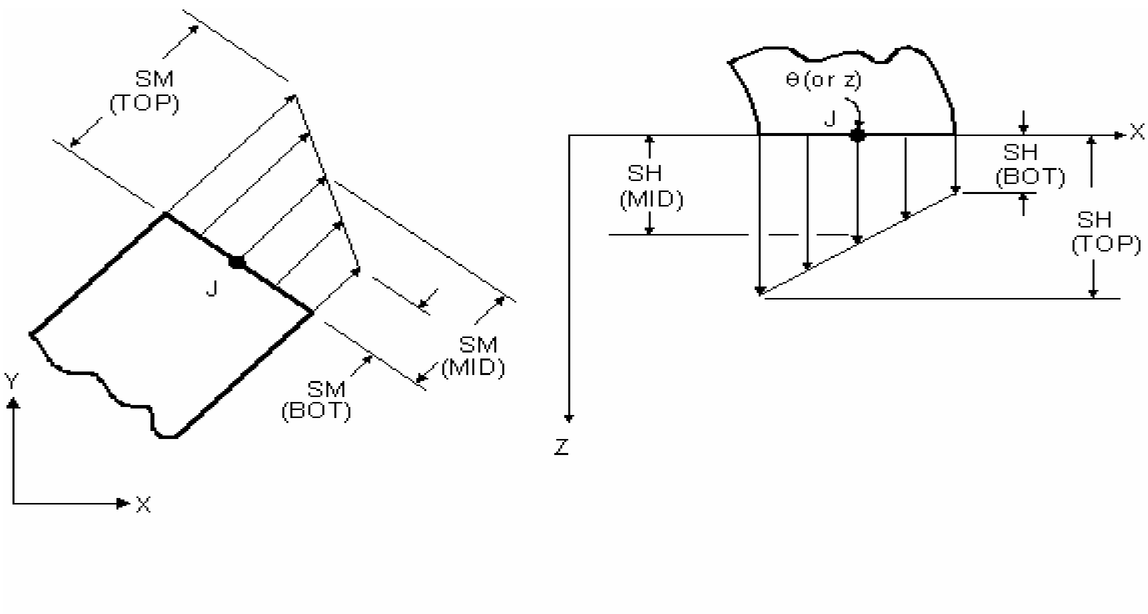
Stress stiffening

C4. Output Data

The solution output associated with the element is in two forms:

- nodal displacements included in the overall nodal solution
- additional element output as shown in Element output Definitions

Fig. C2. SHELL61 Stress Output



Appendix D

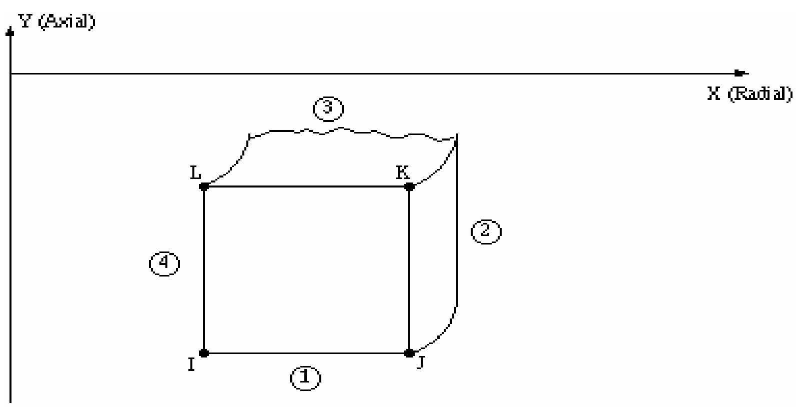
FLUID81

D1. Element Description

The element is used to model fluids contained within vessels having no net flow rate. It is defined by four nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element is used in a structural analysis as an axisymmetric ring element.

The element is a generalization of the axisymmetric version of FLUID79, the two-dimensional fluid element, in that the loading need not be axisymmetric. The fluid element is particularly well suited for calculating hydrostatic pressures and fluid/solid interactions. Acceleration effects, such as in sloshing problems, as well as temperature effects, may be included. The reduced method is the only acceptable method for modal analyses using this fluid element.

Figure D1. FLUID81 Axisymmetric-Harmonic Contained Fluid Element



D2. Input Data

The geometry, node locations, and the coordinate system for this element are shown in Fig. D1. The element input data includes four nodes, the number of harmonic waves, the symmetry condition, and the isotropic material properties.

EX, which is interpreted as the "fluid elastic modulus," should be the bulk modulus of the fluid (approximately 300,000 psi for water). The viscosity property (VISC) is used to compute a damping matrix for dynamic analyses. A typical viscosity value for water is 1.639×10^{-7} lb-sec/in^2. Density (DENS) must be input as a positive number.

Harmonically varying pressures may be input as surface loads on the element faces as shown by the circled numbers on Fig. D1. Positive pressures act into the element. Harmonically varying temperatures may be input as element body loads at the nodes. If all other temperatures are unspecified, they default to T(I).

D3. FLUID81 Input Summary

Element Name
FLUID81

Nodes
I, J, K, L

Degrees of Freedom
UX, UY, UZ

Real Constants
None

Material Properties
EX, ALPX, DENS, VISC, DAMP

Surface Loads

Pressures --

face 1 (J-I), face 2 (K-J), face 3 (L-K), face 4 (I-L)

Body Loads

Temperatures --

T(I), T(J), T(K), T(L)

Mode Number

Input mode number

D4. Output Data

The solution output associated with the element is in two forms:

- degree of freedom results included in the overall nodal solution
- additional element output

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DECLARATION

I, the undersigned, declare that the thesis is my original work, and has not been presented for a degree in any other university and that all sources of materials used for the thesis have been duly acknowledged.

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