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COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF STATISTICS

Modeling and Forecasting of Exchange Rate Volatility of Ethiopia

A thesis submitted to Department of statistics, College of Natural Sciences, Addis Ababa University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Statistics

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Adviser:Prof.M.k.Sharma

June, 2020
Addis Ababa, Ethiopia

DECLARATION

DECLARATION

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of materials used for the thesis have been duly acknowledged

Declared by:
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Confirmed by Advisor:
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Abstract

Modeling and forecasting of volatile data have become the area of interest in financial time series.This study was conducted to apply a hybrid model between Autoregressive Integrated Moving Average (ARIMA) model and Generalized Autoregressive Conditional Heterocedasticity (GARCH) family model. Symmetric (GARCH) and asymmetric(EGARCH) models are used in this study.The time series data used in this study consist of average monthly Ethiopian Birr/ USA Dollar exchange rate from January 2001 to February 2020 obtained from National bank of Ethiopia.The data were converted to returns to enhance their statistical properties and the returns was used to fit a mean and the variance equation.The parameters for ARIMA models were estimated using Ordinary Least Squares Estimation (OLS) method. For hybrid ARIMA-GARCH and ARIMA-EGARCH, the parameters were estimated by using Maximum Likelihood Estimation (MLE).The ARCH LM test indicate that presence of conditional heteroscedasticity in the ARIMA model. The performance of modeling and forecasting of hybrid ARIMA- GARCH type models have been investigated based on forecasting performance criteria such as MSE, MAE and MAPE tests. The data are divided into two parts where 89% of the data is used as in-sample((build the model) period taken from January 2001 until January 2018. while the rest of data (11%) were used for the out-sample period taken from February 2018 until February 2020.The modeling performance of the hybrid models are evaluated using AIC. Results showed that,hybrid ARIMA (3,1,3)-EGARCH(3,1) model was found to be perform better in modeling and forecasting the volatility of monthly exchange rate return compared to hybrid ARIMA(3,1,3)-GARCH (1,1) model. The processes of modeling and forecasting was done by using Eviews statistical software.

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Acronyms

ACF	Autocorrelation Function
ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
AME	Absolute Mean Error
AR	Autoregressive Process
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integrated Moving Average
BIC	Bayesian Information Criterion
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
JB	Jarque - Bera
LM	Lagrange Multiplier
MA	Moving Average
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MaxAPE	Maximum Absolute Percentage Error
PACF	Partial Autocorrelation Functions
RMSE	Root Mean Square Error
SSE	Sum of Squares
SBIC	Schwarz Bayesian Information Criterion
TIC	Theil Inequality Coefficient

1 Introduction

1.1 Background

Modeling and forecasting rate of exchange volatility has become a important aspect and task in financial data series and the economy.Because of the volatility of exchange rate is the measurement of the danger for investment, asset allocation and provides essential information for investors to make the proper decisions.It has obtained great importance to promote participants, policy makers in order to completely understand the changes and therefore the financial stability of an economy(Poon and Granger, (2003)

Exchange rate could be a major variable within the general policy making both in developed and developing economies as its appreciation or depreciation affects the performance of various macroeconomic variable and it's attainment of macroeconomic objective can't be over emphasized. supported the its importance, each and each country pays most attention to the appropriateness of their exchange policy*(Nwude, 2012; Ajao and Igbokoyi, 2013)*.

Modeling exchange rate volatility has gained a great importance particularly after the downfall of the Bretton Woods agreement when major industrial countries has changes floating exchange rate to fixed exchange rate system .An extensive debate about the topic of exchange rate volatility has potential influence on welfare, inflation, international trade, profitability and risk management and investment analysis (Suliman, 2012).

International currency is demanded for buying and selling of products and services. The rate of exchange is that the rate at which one currency will exchange for other units of international currency. Its vary over time looking on many factors. the exchange-rate fluctuation is sometimes treated as a risk, the next risk will cause a better cost for risk-averse investors, due to result in fewer jobs created. rate of exchange system itself is taken into account a financial asset, its important to grasp the accurate of the forecasting of the volatility in financial markets which is critical with respect to "investment, financial risk management and monetary policy making" (Poon and Granger(2003)).

Volatility of in the exchange rate is the dispersion of price among the two different countries' currencies are traded . Exchange rate volatility is a main source of concern for macroeconomic policy makers due to it is similarity with risk and uncertainty. It is well-documented that internationally-oriented countries are particularly sensitive to foreign exchange rate volatility *(Kamal Y, et al.(2012))*

Exchange rate volatility is an implications for the financial system of a country especially the tradable sector. Changes in exchange rates have pervasive effects, with consequences for prices, incomes, interest rates, manufacturing levels, and job opportunities, and thus with direct or indirect repercussions for the welfare of virtually all economic participant Shipanga (2009).

The exchange rate of the Ethiopian Birr against, the US dollar is required a large number of Ethiopian Birr to purchase one US dollar. The huge and unpredictable changes in exchange rates present a major concern for macroeconomic stabilization policy. The changes in exchange rates can have effects on economic conditions, policy makers want to determine what can be done to limit rate exchange variability (Geda, 2006)

Countries have a choice between two types of exchange rate system fixed and floating. A fixed exchange rate system is one that is controlled by the government or monetary authority . Countries decrease or increase their currencies in line with changes in the economic fundamentals. The other is that the floating rate system where the rate of exchange is set by the forces of demand and provide within the exchange market. (Bordo, 2003).

Ethiopia was developing different types exchange rate systems. For example, in the Empiral and Derge regime, a fixed exchange rate system was introduced with massive appreciation and devaluation between the times in the period. In May 1993, the auction exchange rate was introduced. Finally, the auction system was replaced by daily interbank exchange rate system in 2001. Due to this, the demand and supply factors were given more latitude in the determination of the exchange rate. However, the National Bank of Ethiopia would manage the exchange rate pressure basically though reserve requirement.(Haile Kibret,(1994)).

The World Bank 5^{th} Ethiopia economic update, December 2016, states that theexchange rate has increased in their cumulative terms by 84 percent since the nominal devaluation in October 2010. The speed of appreciation has bogged down over the past 6 months. Because of a relative decrease within the rate of domestic currencies, and increase in the U.S. dollar relative to other currencies since January 2016. The Ethiopian Birr still remains overvalued, which is hurting international competitiveness.

Financial analysts are used the time series econometrics models to clarify behavior of exchange rate returns and its volatility because of unexpected events that occurred like non-constant variance within the financial markets and uncertainties in prices and returns *(Ramzan et al. 2012).* Conditional heteroscedastic models are the foremost common and regularly applied models for exchange rate series (Zakaria and Abdalla, 2012).

Hence, this study aims to investigate performances of hybrid Box- Jenkins ARIMA models with the GARCH family models.

1.2 Statement of the problem

Volatility of exchange rate market affects both investment decisions and planning. The exchange rate volatility greatly affects Foreign Direct Investment (FDI), exports and imports, debt payments of countries, and government policy and international investment portfolios. The timely volatility forecasting is vital to understand economic and financial condition of a country.Even central bank interventions do not have a significant effect on the Exchange rates market, because the interventions are small compared to foreign exchange activity *(Roth,et al., 2009).* Exchange rate volatility is a swing or fluctuation over a period of time in the exchange rate. The Ethiopian economy is very sensitive to fluctuations in the Birr/USD exchange rate given the fact that we generally import in US dollars.For accurate modeling and forecasting of volatility many financial institutions are invest in foreign exchange rate market .

Kidane,(1999) studied the effect of devaluation on the Ethiopian macroeconomics performance.The study showed that the Birr exchange rate was pegged to US dollar and lead to over evaluation of Ethiopian currency. But the kidane, study did not sufficient express the volatility of exchange rate return in a model. The purpose of this paper is to model and forecast exchange rate volatility between the ethiopian Birr and the US dollar using hybrid Box-Jenkins ARIMA models with the GARCH family models.

1.3 Objective of the study

- The objective of this study is to model exchange rate volatility between the Ethiopia Birr and United States Dollar using ARIMA-GARCH and ARIMA-EGARCH model.

1.4 Significance of the Study

Exchange rate volatility is a measure of uncertainty or shock of the economic environment. Due to this, high variability in the exchange rate is directly linked to frequent changes in demand for domestic currency, caused mainly by the entry and exit of exchange rate. The forecasting accuracy of exchange rates play a great role because substantial amount of trading passes through the currency exchange market. The forecasting of exchange rate can provide useful information which helps the financial institutions, policy makers and investors to make correct decisions in order to mitigate losses in financial market. Forecasting the volatility of foreign exchange rate has a great importance for international traders' export and import market system .

1.5 Organization of the study

The thesis is organized into the following chapters. Chapter one is introduction of the thesis. This chapter addresses the thesis objectives, significance, and background of the study. Chapter 2 reviews the literature with emphasis on the statistical tools relevant to modeling the exchange rate and its volatility. Chapter 3 explains the methodology applied in building hybrid ARIMA with GARCH models and estimating their parameters. Chapter 4 presents the results of analysis. Chapter 5 provides conclusion and recommendations

2 LITERATURE REVIEW

2.1 General Overview of Volatility

The characteristic of economic asset is its return, which is usually considered to be a random variable.The spread of outcomes of this variable, called asset volatility, which is plays a vital role in numerous financial applications. volatility is the measurement of dispersion among the costs of economic time series data.The variance could be a measure of the dispersion of the density function around its mean. The standard deviation, σ, which is that the square root of the variance, is the most common measure of dispersion for a random variable **(Alexander, 2001)**
Volatility is the spread of all outcomes of an uncertain variable. In finance, we are interested in the outcomes of the returns. It is a quantified measure of market risk. ***(Shah,et al.(2015))***
Bollerslev (1986) generalized the ARCH model and named the model as the Generalized ARCH (GARCH) model which allows for a more flexible lag structure and it bears much resemblance to the extension of the standard times series autoregressive (AR) process to the general autoregressive moving average (ARMA) process.

2.2 Reviews on ARIMA models

Paul et al. (2013) studied an application of ARIMA model for forecasting average daily of price index in Bangladesh. Some of the significant factors for investors to invest their money in capital market are the company's information analysis and prediction as well as divided declaration. An appropriate model is necessary for the share markets which enable the investors to forecast the prices in the future. The study highlighted ARIMA model was used to forecast average daily price index of Square Pharmaceuticals Limited. The data used in the study obtained from Dhaka Stock Exchange (DSE) for the year 2011. Based on the model selection criteria ARIMA (2,1,2) is chosen as the best model for forecasting SPL data series
Weisang and Awazu (2008) presented three ARIMA models which used macroeconomic indicators to model the USD/EUR exchange rate. The study covered the time period from January 1994 to October 2007, the monthly USD/EUR exchange rate was modeled by a linear relationship among the its value. From the finding ARIMA (1,1,1) is the most suitable model for the prediction of the time series of USD/EUR exchange rate.
Pan et al. (2016) proposed an ARIMA model for forecasting the number of patient for epidemic disease in China. Instead of using grey forecast that is appropriate for small sample and lack of information data, ARIMA models are superior since it can deal with substantial amount of sample and correlative time series data. Daily epidemic disease patient data was is used in study taken from January until August 2014.The results showed ARIMA (7,1,0) was appropriate for forecasting the data series . It can be concluded that ARIMA model is the most appropriate model in forecasting the number of patient for epidemic disease in the current study.
Okyere and Kyei (2014) build ARIMA model in modeling producer price rates in Ghana.The study used monthly producer price inflation rates as real data taken from

January 2009 until December 2013 obtained from the website of Ghana Statistical Service. Results showed that based on the model selection criteria ARIMA (1,1,1) was the appropriate model in modeling the producer price inflation rates of Ghana.The results also confirmed there no heteroscedasticity and serial correlation is present in the series. Hence, it can be concluded that ARIMA (1,1,1) was the appropriate model for forecasting the producer price inflation rates of Ghana.

2.3 Reviews on GARCH family models

Bouoiyour and Selmi (2012) examined modeling exchange volatility in Egypt using GARCH models with monthly data taken from 1994 up to 2009. Results showed that volatility clustering present in the series .They considered the leverage effect noticed a tendency to a long memory which by itself could be a source of an explosive process.
Antonakakis and Darby (2012) used daily data taken from November 8, 1993 until December 29, 2000 for four exchange rates against the US dollar, such as the Botswana pula, Chilean peso, Cyprus pound and Mauritius rupee. Bulid the ARCH, GARCH, EGARCH, and the HYGARCH models, the result showed that the IGARCH model gives correct results for out-of-sample forecast for all the exchange rates.
Bala and Asemota (2013) Build GARCH model for exchange rate volatility . They used monthly exchange rate return series for the naira against the US dollar, British pound, and euro. It was showed that most of the models rejected due to the existence of a leverage effect, except for those with volatility breaks. Results improved when the volatility models considered ,corp-orating significant events in the GARCH models was suggested.

2.4 Reviews on hybrid ARIMA-GARCH (symmetric) models

Babu and Reddy (2014) Applied a linear hybrid ARIMA-GARCH model that can maintain the accuracy of prediction while preserving data trend across the forecast horizon. Volatility clustering and fat tail distribution are general characteristics of highly volatile time series data. The proposed model is applied on the chosen NSE India data sets to get multi-ahead prediction and compared the performances with ARIMA, GARCH, wavelet-ARIMA and ANN models. The results obtained are utilized using error performance measures. From the observation, the proposed model performs better in terms of prediction accuracy and preserving data trend compared to the others model.
Nguyen and Bui (2015) used hybrid ARIMA and GARCH model to forecast volatility of VN-Index and compared results with the two single models. The results showed that hybrid ARIMA(1,1,1) - GARCH(1,1) model provide better prediction than the single models of ARIMA(1,1,1) and GARCH(1,1).
Shahla Ramzan et al.(2012) used ARCH family of models for modeling and forecasting exchange rate dynamics in Pakistan for the time period taken from July 1981 up to May 2010 and ARMA(1,1)- GARCH (1,2) was found to be appropriate to remove the persistence in volatility and ARMA(1,1)-EGARCH(1,2) successfully overcame the leverage effect.
Yaziz et al. (2013) applied the performance of hybrid ARIMAGARCH modelling in forecasting gold price. The daily gold price data series taken from 26th November 2005

until 18th January 2016 was used in the study The data are divided into two parts where for the first 35 observations were used as in-sample period while the rest of the five(5) observations as out-of-sample period. The ARIMA (1,1,1)-GARCH (0,2) model was significant at 1% significance level and satisfied the diagnostic checking using Ljung Box Q-statistic, ARCH-LM test and normality analysis on the residuals of the model.The trend of forecast prices follows closely the actual data including the simulation part of five days out-sample period. The result showed that hybrid ARIMA-GARCH model has improved the estimating and forecasting accuracy by five fold compared to the ten last selected forecasting methods in the literature.

2.5 Reviews on hybrid ARIMA-GARCH (asymmetric) models

Mutunga et al. (2015) Applied asymmetric GARCH model for Poor's 500 (USA) and Nikkei 225 (Japanese) taken from the 2nd January 2008 until 31st May 2011. The estimated models are EGARCH (1,1) and GJR-GARCH (1,1) model. The result showed that MSE and MAE values for EGARCH (1,1) model are lower for the both data sets except for one case where MAE for the S and P 500 index data set is lower using GJR-GARCH model. Finally the results showed that first order EGARCH model outperforms first order GJR-GARCH model under the EF approach in forecasting return volatility.
Olowe (2009) Applied GARCH model for the Naira/US Dollar exchange rates on a sample of monthly data taken from 1970 up to 2007. The result show that the Asymmetric Power ARCH and the Threshold Symmetric GARCH are the best fitted models .
Milton et al.(2014), Applied hybrid ARIMA-GARCH model for the Leones/USA dollars exchange rate return, under the normal and non-normal (student's t and skewed Student t) error distributions. Results showed that ARMA-GARCH and ARMA-EGARCH model best fits under the non-normal distribution and the ARMA-GJR model using the skewed Student t- error distribution is better to forecast the Sierra Leone exchange rate.

3 DATA AND STATISTICAL METHODOLOGY

3.1 Source of Data

The time series data used in this study consist of average monthly Ethiopian Birr/ USA Dollar exchange rate from January 2001 to February 2020 obtained from National bank of Ethiopia.

3.2 Definition of Time Series Data

The time series can be defined as a set of observations that are generated sequentially among the time for a specific phenomena any time series is associated with an ordered data through ordered times that is data are correlated.
There are two types of time series models, firstly, univariate time series models, such as, univariate Box-Jenkins models and exponential smoothing models, secondly, multivariate time series models, such as transfer function and intervention analysis models.Time series models have been widely used in the construction of forecasting models, to achieve accurate forecasting that helps in development, planning and decision making.

3.3 Characteristics of Financial Time Series

The common characteristics of financial time series data are:

3.3.1 Volatility Clustering

It refers to the phenomenon that large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes.The volatility clustering is a type of heteroskedasticity and accounts for some of the excess kurtosis typically observed in the distribution of a financial time series.

3.3.2 A leptokurtic

A leptokurtic is the distribution of a kurtosis value greater than that of a standard, normal distribution which gives the distribution a high peak, a thin midrange and fat (heavy) tails.

3.3.3 A platykurtic

A platykurtic means that the distribution has a kurtosis value less than that of a standard normal distribution.

3.3.4 Mesokurtic Distributions

Mesokurtic distributions means that the distribution has a kurtosis value equals to that of standard normal distribution.

3.3.5 Leverage Effects

The leverage effect refers to the tendency of volatility to increase if the previous days returns are negative.For time series exhibiting leverage effects, asymmetric GARCH models should be applied because the asymmetry cannot be captured by symmetric GARCH model

3.4 Stationary and Unit Root Test

A stationary process has the property that the first and second moments do not change over time. It is an important property to define a time series process. Stationarity may be weak or strong (strict).That is the joint distribution of$\{Y_1, Y_2...., Y_t\}$ is identical to that of $\{Y_{1+h}, Y_{2+h}...., Y_{t+h}\}$, where h is an arbitrary positive integers. which means strict Stationarity requires that the joint distribution of $\{Y_1, Y_2...., Y_t\}$ is invariant under time shift. This is a very strong condition that is hard to verify empirically. A stochastic process$\{Y_t\}$ is said to be weakly stationary or covariance stationary if both the mean of Y_t and the covariance between $Y_t$and Y_{t-h} are time invariant, where h is an arbitrary integer. If one of these properties or characteristics is violated the time series process $\{Y_t\}$ is non stationary and we have to handle by applying differencing for appropriate number of times or other methods (Tsay, 2010)).

3.4.1 Unit Root Test

The assumption of stationarity is somewhat an unrealistic situation in most macroeconomic variables. Trivially, a non-stationary process arises when one of the conditions for stationarity does not hold. The widely used unit-root tests are the Augmented Dickey-Fuller (ADF) and Phillips- Perron (PP). The ADF test is used to tests whether a unit root is present in an autoregressive model or not. It is developed by the Dickey and Fuller (1979). In its simplest form it considers an autoregressive of order one AR(p) process with drift given by:

$$y_t = \rho y_{t-1} + \epsilon_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

where ρ is a parameter to be estimated and ϵ_t is assumed to be white noise.If $|\rho| \geq 1, \{Y_t\}$ is a non-stationary series and the variance of Y increases with time and approaches infinity. On the other hand, if $|\rho| < 1, \{Y_t\}$ is a stationary series.series. Thus, the hypothesis of (trend) Stationarity can be evaluated by testing whether the value of ρ is strictly less than one

$$H_0 : \rho = 1 \text{ (the series is not stationary)}$$
$$H_1 : \rho < 1 \text{ (the series is stationary)}$$

Augmented Dickey-Fuller(ADF) Unit Root Test

The standard Dickey-Fuller test is conducted in the following manner: from equation (1) we have:

$y_t - y_{t-1} = (\rho - 1)y_{t-1} + \epsilon_t$.. [2]

This can be rewritten as:

$\Delta y_t = \pi y_{t-1} + \epsilon_t$,where $\pi = \rho - 1$ The null and alternative hypothesis can be rewritten

as
$H_0 : \pi = 0$ vs $H_1 : \pi < 0$ .... [3]
The test statistic is the conventional t-ratio for π: $t_\pi = \frac{\hat{\pi}}{se(\hat{\pi})}$

where $\hat{\pi}$ is the OLS estimate of π and se($\hat{\pi}$) is the standard error of π.
Dickey and Fuller (1979) showed that, under the null hypothesis of a unit root, this statistic does not follow the conventional Student's t-distribution, and they obtained asymptotic results and simulated critical values for various tests and sample sizes. MacKinnon (1996) obtained a much larger set of simulations than those tabulated by Dickey and Fuller.

Phillips and Perron (PP) Unit Root Tests

Phillips and Perron (1988) developed a number of unit-root tests that have become popular in the analysis of time series . The Phillips-Perron (PP) unit-root tests differ from the ADF tests mainly in serial correlation and heteroskedasticity in the errors. The ADF tests is use a parametric auto-regression to approximate the ARIMA structure of the errors within test of the regression, the PP tests ignore any serial correlation in the test regression. The test regression in the PP tests is given by:
$y_t = \alpha + \beta_+ \pi y_{t-1} + \epsilon_t$ ....[4]

where,ϵ_t is a stationary process and may be heteroskedastic. The PP used to tests the serial correlation and heteroskedasticity in the errors ϵ_t of the test regression by directly modifying the Dickey - Fuller test statistic. The modified test statistics, t_π and $T_{\hat{\pi}}$

$$t_{\pi=0} = \frac{\hat{\rho}}{se(\hat{\rho})} \text{ and } T_{\hat{\pi}} = \frac{\hat{\pi}}{se(\hat{\pi})}$$

where $\hat{\rho}$ and $\hat{\pi}$ are the OLS estimates of ρ and π and se($\hat{\rho}$) and se($\hat{\pi}$) are the standard errors of ρ and π respectively
These modified statistics, denoted Z_t and Z_π are given by

$$\begin{aligned} Z_t =& \Big(\frac{\hat{\sigma}^2}{\hat{\tau}^2}\Big)^{\frac{1}{2}}.t_{\pi=0} - \frac{1}{2}\frac{\big(\hat{\tau}^2 - \hat{\sigma}^2\big)}{\hat{\tau}^2} \\ Z_\pi =& T_{\hat{\pi}} - \frac{1}{2}\Big(\frac{T^2.se(\hat{\pi})}{\hat{\sigma}^2}\Big)\big(\hat{\tau}^2 - \hat{\sigma}^2\big) \end{aligned}$$

The terms $\hat{\tau}^2$ and $\hat{\sigma}^2$ are consistent estimates of the variance parameters:

$$\hat{\sigma}^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E(\epsilon_t)^2$$
$$\hat{\tau}^2 = \lim_{T\to\infty} \sum_{t=1}^{T} E(T^{-1}S_T^2)$$

where ,$S_T^2 = \sum_{t=1}^{T} \epsilon_t$The sample variance of the least square residual $\hat{\epsilon}_t$ is a consistent estimate of σ^2 and the Newey-West (1987) long-run variance estimate of ϵ_t is a consistent estimate of τ

3.5 Box Jenkins Models

Box and Jenkin (1976) introduce a methodology to analyze the time series and used methodology to forecast a specific phenomenon in future.Forecast models are divided to seasonal and non-seasonal models. The non-seasonal models used to represent two types of series stationary and non-stationary series and some of these models are:

1. Autoregressive models
2. Moving average models
3. Autoregressive and moving average models
4. Autoregressive integrated moving average models

3.5.1 Autoregressive Model

In the autoregressive model the current value y_t in the time series is expressed as a linear combination of the previous values, and an unexplained portion We assume that the values are taken as the deviation from its mean i.e we can define the Autoregressive model of order p is denoted by .

$Y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + ... + \theta_p y_{t-p} + \epsilon_t$..[5]

where θ_j(j=1,2,...p) is the jth autoregressive parameter and ϵ_t is a white noise series with mean zero and finite variance σ^2 .We can also write the above model in term of B,where B is a back shift order:
$\theta(B)Y_t = \epsilon_t$ where $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - ... - \theta_p B^p$ is a polynomial in B of order p.

3.5.2 Moving Average Model

In the moving average model the first current value of the time series is expressed as a linear combination of the current and previous errors in the moving average model of order q denoted by *MA(q)* the current observation y_t is expressed as a linear combination of the random disturbances going back periods.A time series Y_t is said to be a moving average process of order q(abbreviated MA(q)) .

$$Y_t = \phi_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + ... + \phi_q \epsilon_{t-q} \text{...[6]}$$

where ϕ_j(j=1,2,...q) is the jth autoregressive parameter and ϵ_t is a white noise series with mean zero and finite variance σ^2.We can also write the above model in term of B as follows
$Y_t = \phi(B)\epsilon_t$ where $\phi(B) = 1 + \phi_1 B + \phi_2 B^2 + ... + \phi_q B^q$ is a polynomial in B of order q

3.5.3 Autoregressive Moving Average Model

In applications,of the AR or MA models become cumbersome due to one may need a higher-order model with many parameters to adequately describe the structure of the series. To overcome this problem, the autoregressive moving-average (ARMA) models are introduced *(Box, et al (1994))*. An ARMA model mixed the ideas of AR and MA models into compact form so that the number of parameters used is kept small. A mixed autoregressive moving average model with p autoregressive terms and q moving average terms (abbreviated *(ARMA)*) can be written as:

$$\begin{aligned} Y_t =& \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + ... + \theta_p y_{t-p} \\ &+ \epsilon_t - \phi_1 \epsilon_{t-1} - \phi_2 \epsilon_{t-2} - ... - \phi_q \epsilon_{t-q} \text{..[7]} \end{aligned}$$

3.5.4 Autoregressive Integrated Moving Average Model

Many time series are non-stationary and we cannot apply stationary AR, MA or ARMA processes directly. The reason for handling non-stationary series is to apply differencing so as to make them stationary. Differencing of a series can transform a nonstationary series to a stationary series. One advantage of differencing over de-trending to remove trend is that no parameters are estimated in the differencing operation. One disadvantage, however, is that differencing does not yield an estimate of the stationary process Y_t . If an estimate of $Y_t$is essential, then de-trending may be more appropriate. If the goal is to make the data stationary, then differencing may be more appropriate.
The first difference operator $\Delta$is defined by:
$\Delta = y_t - y_{t-1} \implies \Delta y_t = (1 - B)y_t$
so that Δ an be expressed in terms of the backward shift operator B. In general, higher order differencing can be expressed as:
$\Delta^n y_t = (1 - B)^n y_t$ ....[8]
A series that is stationary without a differencing is said to be integrated of order zero I (0), and a series which is stationary after being difference d times is said to be integrated I(d).
If the original data series is differenced d times before fitting an ARMA(p,q) process, then the model for the original undifferenced series is said to be an ARIMA (p,d,q) process

3.6 Building ARIMA Family Models

General Box-Jenkins applying a three-stage method aimed at selecting an appropriate ARIMA model for the purpose of estimating and forecasting a univariate time series. Three stages are: (a) identification, (b) estimation, and (c) diagnostic checking

3.6.1 Identification of the Model

The first step in ARIMA modeling is identification of the appropriate model. ARIMA procedures are mainly based on the value of the autocorrelation function (ACF) and the partial autocorrelation function (PACF), which are important to check diagnostic of the series.
A comparison of the sample ACF and PACF to those of various theoretical ARIMA processes may suggest several plausible models. A common stationarity inducing transformation is to take logarithms and then first difference of the series.
Once we have achieved stationarity, the next step is identify the p and q orders of the ARIMA model.
The Autocorrelation Function (ACF)
An important tool that helps in detecting stationarity and identifying models for time-series data is the autocorrelation coefficient .The autocorrelation coefficient is used to measure the strong relation between the value of series in different time period,the mathematical syntax for autocorrelation function is

$$\rho_k = \frac{cov(x_t, x_{t+k})}{\sqrt{var(x_t)}\sqrt{var(x_{x+k})}} = \frac{\Upsilon_k}{\Upsilon_0} \quad , k = 1, 2, ..., \frac{n}{2} \quad ....(9)$$

So the variance of stationary series is fixed and equal for all different time period and estimated as: $\Upsilon_k = \frac{\hat{\Upsilon}_k}{\Upsilon_0}$ where ,$\hat{\Upsilon}_k$ is variance of series observation X_t

The partial Autocorrelation Function (PACF):

A second important tool used in identifying models of time series is the partial autocorrelation .The partial autocorrelation coefficient of order measures the correlation between values k periods apart when the effect of time lags 1,2,.....,k-1 is kept constant. When the partial autocorrelation coefficient is looked at as a function of k it is called the partial autocorrelation function (PACF) to estimate the partial autocorrelation of order k, fit an autoregressive model of order AR(k)

Model selection criteria

A model selection criterion is used to determine the approximating model from a set of estimated models. In time series analysis there may be several adequate models that can fit a given data set. The traditional model selection criteria such as the Akaike information criterion (AIC)proposed by Akaike (1974) and the Schwartz Bayesian information criterion (SBIC) proposed by Schwartz (1978) were employed to identify the optimal lag specification for the model.

Akaike's Information Criterion (AIC)

Akaike (1974) proposed the goodness of fit for ARMA (p, q) by balancing the error of the fit against the number of parameters in the model. It is defined by:

$$AIC = -2\log L_k + 2k \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [10]$$

where Lk is the maximized log-likelihood and k = 1+p+q, is the number of parameters in the model.The most appropriate model is the one which minimizes the AIC, and there is no requirement for the models to be nested.

Bayesian Information Criterion (BIC)

Schwarz (1978), introduced the BIC and is defined

$$BIC = -2\log L_k + K\log T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [11]$$

Different simulation studies have tended to identify that BIC does well at getting the correct order in large samples, whereas AIC tends to be superior in smaller samples where the relative number of parameters is large (McQuarrie and Tsai (1998)) as pointed out by Shumway and Stoffer (2010).

3.6.2 Parameters Estimation

After the degree of differencing has been determined, we proceed to identify the autoregressive and moving average orders based on the sample autocorrelations and sample partial autocorrelations. After choosing the most appropriate model, the model parameters are estimated using several estimation procedures. In general, nonlinear estimation procedure is used to estimate ARIMA model parameters to maximize the likelihood function with respect to the parameters. That is, the maximum likelihood estimation method is used to test the parameters of the ARIMA model.

3.6.3 Diagnostic checking

Once a model has been identified and the parameters estimated, diagnostic checks are then applied to the fitted model. These include tests of serial correlation and normality of the residuals

3.6.3.1 Ljung-Box test of serial correlation

Before an estimated model is used for statistical inference, the residuals must be examined for the presence of serial correlation. The Q-statistic is often used as a test of whether the series is white noise. The Q-statistic at lag m is a test statistic for the null hypothesis that there is no autocorrelation up to order m and is computed as::

$$Q = T(T+2)\sum_{j=m}^{m}\frac{\hat{\rho}_j}{T-j} \text{...} [12]$$

where $\hat{\rho}_j$ is the j-th lag autocorrelation and T is the number of observations. If the series is not based upon results of ARIMA estimation, then under the null hypothesis, Q is asymptotically distributed as chi-square with degrees of freedom equal to (m-k), where k denotes the number of parameters.

3.6.3.2 Jarque-Bera Test of Normality

One of the most commonly applied tests for normality is the Jarque-Bera (JB) test. JB uses the property of a normally distributed random variable that the entire distribution is characterized by the first two moments such as the mean and the variance.
Jarque and Bera (1981) formalize these ideas by testing whether the coefficients of skewness and excess kurtosis are jointly zero. If $\hat{\mu_3}$ and $\hat{\mu_4}$ are estimates of the third and the fourth central moments, respectively, $\overline{y}$ is the sample mean and $\hat{\sigma^2}$ is an estimate of the second central moment, the skewness and kurtosis coefficients are computed as:

$$\hat{b_1} = \frac{\hat{\mu_3}}{\hat{\sigma^3}} = \frac{1}{T}\frac{\sum_{i=1}^{T}(y_i-\overline{y})^3}{\{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^2\}^{\frac{3}{2}}}$$

$$\hat{b_2} = \frac{\hat{\mu_4}}{\hat{\sigma^4}} = \frac{1}{T}\frac{\sum_{i=1}^{T}(y_i-\overline{y})^4}{\{\frac{1}{T}\sum_{i=1}^{T}(y_i-\overline{y})^2\}^{2}}$$

The JB test statistic is defined as:
JB= T$\left[\frac{\hat{b_1}}{6}+\frac{\hat{b_2}}{24}\right]$.. [13]
where T is the sample size.The null hypothesis this test statistic is follows a chi-square distribution with two degrees of freedom (Brooks, 2002)

3.6.3.2 Correlogram Squared Residuals

In a hybrid model, the existence of serial correlation can be determined using correlogram squared residuals. The hypotheses to be tested are as follows:
H_0 **:** There is no serial correlation Vs H_1**:** There is serial correlation

3.7 Volatility Testing

The volatility is the measurement of variation among the prices of financial time series data.In the modelling of heteroscedasticity data, volatility is an important aspect. If volatility clustering is present in the data, ARCH or GARCH family models can be used.

Volatility clustering can be defined as large changes that tend to be followed by large changes, of either signs, and small changes that tend to be followed by small changes, also of either signs.

3.8 Autoregressive Conditional Heteroscedasticity Models

The ARCH models were introduced by Engle (1982) and they are specifically designed to model and forecast conditional variances.To generate the autoregressive conditional heteroskedasticity process the conditional variance of the error term is expressed as a function of its past values squares. Let σ_t^2 denote the variance conditional on information at time (t-1). Then the ARCH (p) model can be expressed as :

$$\sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \alpha_2\epsilon_{t-2}^2 + ... + \alpha_p\epsilon_{t-p}^2 \qquad [14]$$

The major assumption behind the least square regression is homoscedasticity i.e constancy of variance. If this condition is violated, the estimates will still be unbiased but they will not be minimum variance estimates. The standard error and confidence intervals calculated in this case become too narrow, giving a false sense of precision. ARCH and related models handle this by modeling volatility itself in the model and thereby correcting the deficiencies of least squares model (Dhamija, 2010).

3.8.1 Testing for ARCH effects

If there is no ARCH effects,the model is linear and individual coefficient estimator should not be significantly different from zero and the model should be insignificant.Before estimating a full ARCH model for financial time series, it is usually a good practice to test for the presence of ARCH effect in the residuals. The Box-Jenkins (1976) approach is based on the assumption that the residuals are homoscedastic (remain constant over time) for ARMA or ARIMA models. Thus, the presence of ARCH effect (whether or not volatility varies over time) has to be tested in series through the squared residuals of the series (Tsay, 2010).

ARCH-LM Test

The presence of heteroscedasticity in the time series data can be checked using ARCH-LM test. Engle (1982) proposed Lagrange Multiplier tests to verify whether the standardized residuals exhibit ARCH effect or not in the time series data.

$$H_0 : \text{No ARCH effects VS } H_1 : \text{ARCH effects exist}$$

The test statistic for LM test is defined as: $LM = TR^2$
where T= number of observation and R^2= coefficient of determination
The null hypothesis is rejected if p-value of the LM statistic is less than $\alpha = 0.05$.Rejecting of null hypothesis indicates that there is an ARCH effect in the time series data.

3.9 GARCH family Models

GARCH-type model can be divided into two categories which are symmetric and asymmetric. The difference between symmetric and asymmetric model is related to their effect of sign on volatilities. In the symmetric models, the conditional variance only depends on the magnitude but not in the sign of the underlying asset, while in the asymmetric models, the sign whether positive or negative with the same magnitude of shocks have different effect on volatility Ahmed and Sulliman,(2011).

3.9.1 Symmetric GARCH Models

There are different symmetric GARCH models.In this study, we will only focus on GARCH model. Engle (1982) first developed Autoregressive Conditional Heteroscedasticity (ARCH) model.

GARCH (m,r) Models

The GARCH (m, r) process is defined as:

$$\sigma_t^2 = \alpha_o + \sum_{j=1}^{m} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 \quad \text{...}[15]$$

where, the restrictions $\alpha_o >$,$\alpha_j \geq 0$,$\beta_j \geq 0$ ensure that the variance is always greater than zero and the restriction $(\sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{r} \beta_j < 1$ is necessary and sufficient condition for the stability of the conditional variance equation (Cryer and Chan, (2008)).{vt} is i.i.d. random variable that is independent of past realization ϵ_{t-j} and the conditional and unconditional means of ϵ_t are equal to zero.The parameter α_o is generally interpreted as long-term volatility to which the system converges. On the other hand, the ARCH term $\alpha_j \epsilon_{t-j}^2$ reflects the effect of lagged shocks on the volatility at time t. Moreover, the GARCH term β_j σ_{t-j}^2 measures the effect of past-expected variance on the current volatility.

3.9.2 Asymmetric Model

Symmetric models perform well in capturing volatility clustering and leptokurtosis of financial returns. But as their distribution is symmetric, they fail in modelling leverage effect Narsoo(2015). Due to the limitation of symmetric model the various asymmetric models were developed such as Exponential GARCH (EGARCH), Threshold GARCH (TGARCH) and Power ARCH (PGARCH). This study only use EGARCH for capturing the asymmetric phenomena.

Exponential GARCH (EGARCH) Models

This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. To overcome some weaknesses of the GARCH model in handling financial time series, Nelson(1991) proposes the exponential GARCH (EGARCH) model. In particular, it allows for asymmetric effects between positive and negative returns.

An EGARCH(m, r) variance equation with explanatory variables is given by (Lee and Brorsen, 1997; Ling and McAleer, 2000):

$$\log \sigma_t^2 = \alpha_0 + \sum_{j=1}^{m} \alpha_j \{ |\frac{\epsilon_{t-i}}{\sigma_{t-i}}| - \gamma_i \frac{\epsilon_{t-i}}{\sigma_{t-i}} \} + \sum_{j=1}^{r} \beta_j \log(\sigma_{t-j}^2) \quad \text{..}[16]$$

where, α_o is the mean of the volatility ,α_j is the size effect, β_j the dgree of the volatility and γ_i is signifies the leverage effect of ϵ_{t-i}.Bad news can have a larger impact on volatility;

again, we expect γ_i to be negative in real applications.
There are some advantages in using a EGARCH model over a GARCH model:

1. EGARCH model allows positive and negative shocks to have a different impact on volatility

2. EGARCH model allows large shocks to have a greater impact on volatility than the standard GARCH model.

3. EGARCH model also ensures that the variance is always positive even if the parameters are negative

3.10 Hybrid Models

Hybrid model is a forecasting technique that combines two or more individual methods. In this study, hybridization of ARIMA and GARCH family models was used. ARIMA model is used to model the linear data of time series. The residuals of this linear model will contain only the nonlinear data since ARIMA model cannot capture the nonlinear structure of the data. The GARCH family approach is used to model the nonlinear patterns of the residuals series from the fitted ARIMA models. Hence, the hybridization of ARIMA model with GARCH family error components is applied to analyze and forecast the time series data. The primary stages in building hybrid models is the same with ARIMA methodology which it consists of model identification, parameter estimation, diagnostic checking and forecasting.

3.10.1 General Equation of Hybrid Models

The equation of hybrid ARIMA (p,d,q)- GARCH- EGARCH(p,q) model can be written as followed where it comprises of two equations. Mean equation comes from ARIMA model while the GARCH family models contain variance equation.

3.10.2 Hybrid ARIMA-GARCH Model

In order to use a hybrid ARIMA-GARCH model, two stages should be applied. In the first stage, we use the most appropriate ARIMA model that fits on stationary and linear time series data and the residuals of the linear model will contain the non-linear part of the series. the last one is the GARCH model which contain non-linear residuals patterns.
Mean Equation:
The equation of ARIMA (p,d,q) can be written as:

$$(1-\phi_1\beta-\phi_2\beta^2-....-\phi_p\beta^p)(1-\beta)^dY_t$$
$$=\sigma+(1-\theta_1\beta-\theta_2\beta^2-...-\theta_q\beta^q)\varepsilon_t \qquad ....[17]$$

where

$$\begin{aligned}\phi_p(\beta) =&1-\phi_1\beta-\phi_2\beta^2-....-\phi_p\beta^p\\ \theta_q(\beta) =&1-\theta_1\beta-\theta_2\beta^2-...-\theta_q\beta^q\end{aligned}$$

$$\begin{aligned}&(1-\beta)^d = d^{th}\ difference\\ &\beta = backward\ shift\ operator\\ &\varepsilon_t = Error\ term\ at\ time\ t\\ &\sigma = constant\end{aligned}$$

Variance Equation:
The general equation of GARCH model can be written as:

$$\sigma_t^2 =\alpha_o+\sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2+\sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad ....[18]$$

where

$$\begin{aligned}\sigma_t^2 &= \text{the estimated conditional variance}\\ \varepsilon_t^2 &= \text{the past squared return}\\ \sigma_{t-j}^2 &= \text{the past of conditional variance}\end{aligned}$$

Estimation of Hybrid ARIMA-GARCH Model
The hybrid ARIMA-GARCH model is a non linear time series model which mixed a linear ARIMA model with the conditional variance of a GARCH model. The estimation procedure of ARIMA and GARCH models are based on maximum likelihood method. Parameters' estimation in logarithmic likelihood function is done through nonlinear Marquardt's algorithm (Marquardt, 1963). The logarithmic likelihood function has the following equation.

$$\log L\big[(y_t);\theta\big] = \sum_{t=1}^{T}\{\log\big[D(Z_t,(\theta)),\mu\big]-\frac{1}{2}\log\big[\sigma_t^2(\theta)\big]\} \quad ....[19]$$

where θ is the vector of the parameters that have to be estimated for the conditional mean, conditional variance, and density function, $z_t$denoting their density function, $\big[D(Z_t,(\theta)),\mu\big]$ is the log-likelihood function of, for $[y_t,\theta]$ a sample of T observation.

3.10.3 Hybrid ARIMA-EGARCH Model

Financial time series data have a certain stylized patterns like fat tails, volatility clusterings, leverage effect and long memory. Symmetric models such as GARCH model performs well in capturing volatility clusterings of financial time series data but as their distribution is symmetric, they fail in modelling leverage effect. Since EGARCH model can handle leverage effect in the data, a hybrid ARIMA-EGARCH model is considered as an appropriate model for the exchange rate series. Therefore, the potential of hybrid ARIMA-EGARCH model is investigated in this study. Mean Equation:
The general equation of ARIMA (p,d,q) can be written as:

$$\begin{aligned}&(1-\phi_1\beta-\phi_2\beta^2-....-\phi_p\beta^p)(1-\beta)^dY_t\\ =&\sigma+(1-\theta_1\beta-\theta_2\beta^2-...-\theta_q\beta^q)\varepsilon_t \quad ....[20]\end{aligned}$$

Variance Equation:

$$\log\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i|\frac{\epsilon_{t-i}}{\sigma_{t-i}}| + \sum_{i=1}^{q}\gamma_i\frac{\epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p}\beta_j\log(\sigma_{t-j}^2) \quad \text{..[21]}$$

where α_0 is mean of the volatility ,α_i is the size effect, γ_i is the sign effect(leverage effect) and β_j is the degree of volatility persistence

3.11 Forecasting evaluation and Accuracy Criteria

The statistical measures such as MSE or MAE depend also upon how far the forecast is away from the actual value. Mean absolute percentage error (MAPE) is pretty useless for any series when the actual series, can take on values close to zero. This arises since MAPE divides the forecast error by the actual value, so if the value is much smaller than the forecast error, the MAPE will blow up, while if the actual value is ever zero, the MAPE will by construction rise to infinity. It is true that MSE penalizes large forecast errors disproportionately more than small errors since the forecast errors are squared.
The accuracy of a forecasting model depends on how close the forecast values (f_t) , and the actual values (Y_t). In practice, the difference between the actual and the forecast values is defined as the forecast error, $e_t = Y_t - f_t$ and T is the size of the sample.Also, $\overline{y} = \frac{1}{T}\sum_{t=1}^{T} Y_t$ is the test mean and $\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(Y_t - \overline{y}\right)^2$ is the test variance.
The following statistical summary measures of a model's forecast accuracy are defined using absolute errors:

1. **Mean absolute error(MAE)**
   It measures the average absolute deviation of forecasted values from original.
   It is desirable that for a good forecast the obtained MAE should be small...
   ones.
   MAE= $\frac{1}{T}\sum_{t=1}^{T}|e_t|$ .... [22]

2. **Mean absolute percentage error(MAPE)**
   Mean Absolute Percentage Error (MAPE) is a measure based on percentage errors.
   It is desirable that for a good forecast the obtained MAPE should be small.
   MAPE $=\frac{1}{T}\sum_{t=1}^{T}\left|\frac{e_t}{Y_t}\right| \times 100$ .... [23]

The following statistical summary measures of a model's forecast accuracy are defined using the squared errors:

3. **Mean square error(MSE)**
   It is a measure of average squared deviation of forecasted values.
   MSE= $\frac{1}{T}\sum_{T=1}^{T} e_t^2$ .... [24]

4. **Root mean square error (RMSE)**
   Root Mean Square Error (RMSE) measures the difference between the true values and estimated values, and accumulates all these difference together as a standard for the predictive ability of a model. The criterion is the smaller value of the RMSE, the better the predicting ability of the model.
   RMSE= $\frac{1}{T}\sqrt{\sum_{t=1}^{T} e_t^2}$ .... [25]

5. **Theil's U-statistics**

is another statistical measure of forecast accuracy. Theil's, U can be estimated as:

U= $\frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T} e_t^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} f_t^2}\sqrt{\frac{1}{T}\sum_{t=1}^{T} Y_t^2}}$

If U = 0 means a perfect fit. On the other hand, if U = 1 the performance is not good.

4 Results

4.1 Descriptive Analysis

The data used in this thesis were the average monthly Exchange Rate of Ethiopia covered the period from 01/01/2001 to 01/02/2020 obtained from National Bank of Ethiopia. Summary statistics for the monthly exchange rate data (ETB/USD) are displayed in Table 1 below.

Sample size	230
Mean	16.05098
Maximum	32.35000
Minimum	8.434100
Std. Dev.	7.199694
Skewness	0.492216
Kurtosis	5.987410
Jarque Bera	19.11342
Probability	0.000071

Table 1: Summary statistics for the monthly exchange rate data (ETB/USD)

Table 1. Shows that the mean value of the exchange rate data is (16.05098). The minimum and maximum monthly exchange rate data is (8.434100) and (32.35000) respectively .There is also evidence of positive skewness.This implies that the series is flatter to the right(non symmetric).The kurtosis value(5.987410) is greater than 'three' this implying that the distribution of the exchange rate data is leptokurtic. The result, also showed that, the series is not normally distributed. the empirical result is consistent with the Jarque-Bera (JB) test from above table is used to assess whether the given series is normally distributed or not. The null hypothesis is that the exchange rate series has normally distributed. Results of JB test show that the null hypothesis is rejected for the data series indicate that the observed series are not normally distributed.

Figure 1 below shows that the monthly exchange rate series, from 01/01/2001 to 01/02/2020 .It shows that unconditional mean and variances are changing over time and the series has an increasing trend over time. The changing mean and variance of exchange rate over time are signs of the non-Stationarity of the series.

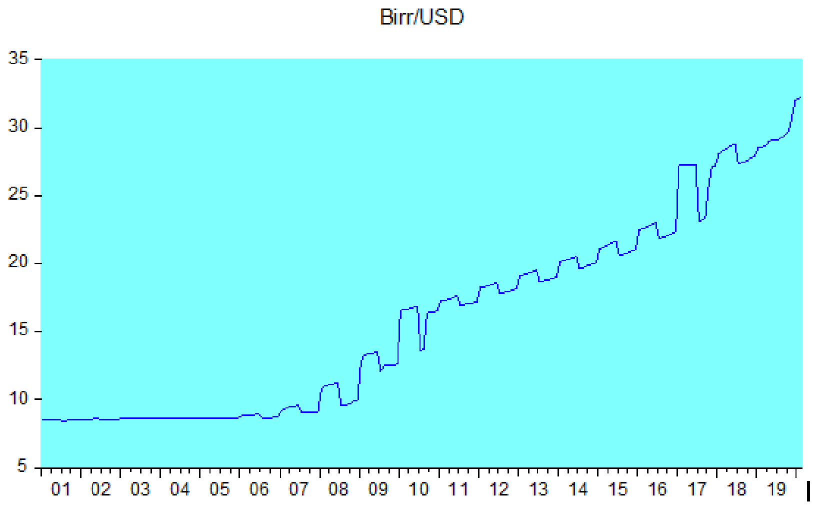


Figure 1: Monthly exchange rate series from January 2001 to February 2020

4.1.1 Testing for Stationarity

The time series under consideration should be checked for stationarity before estimating the model. The variables have to be tested for the presence of unit root(s) and the order of integration of each series has to be determined. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to test the null hypothesis of a unit root against the alternative hypothesis of not unit root.
Table 2. shows the result of ADF test of original exchange rate series.

Test	constant	p-value	constant + trend	p-value
ADF	2.890718	0.8932	-1.494824	0.7488
		Critical values		
1%	-3.659627		-3.899740	
5%	-2.674317		-3.430104	
10%	-2.573656		-3.138608	

Table 2: ADF test on exchange rate series.

Test	constant	p-value	constant + trend	p-value
PP	1.385183	0.9990	-2.689353	0.2421
		Critical values		
1%	-3.458845		-3.998635	
5%	-2.873974		-3.429570	
10%	-2.573472		-3.138293	

Table 3: PP test on exchange rate series.

According to Tables 2 and 3 the exchange rate series are non stationary, because the observed statistic values for both ADF and PP tests are greater than their corresponding critical values. We do not reject the null hypothesis of the presence of unit root in the series. Acceptance of null hypothesis implies that unit root existed. Thus, it is confirmed that the series is not stationary.
To transform the non stationary exchange rate series, we calculate the exchange rate returns as:

$$r_t = \log(x_t) - \log(x_{t-1}) = \log\left(\frac{x_t}{x_{t-1}}\right)$$

where x_t is the exchange rate at time t , x_{t-1} the exchange rate at time t_{-1} and r_t denotes the returns
Both ADF and PP tests below confirm that the exchange rate return series is stationary

Test	constant	p-value	constant + trend	p-value
ADF	-15.89360	0.0000	-16.05994	0.0000
		Critical values		
1%	-3.459627		-3.999740	
5%	-2.874317		-3.430104	
10%	-2.573656		-3.138608	

Table 4: ADF test on exchange rate return series.

Test	constant	p-value	constant + trend	p-value
PP	-14.41425	0.0000	-14.63835	0.0000
		Critical values		
1%	-3.458973		-3.998815	
5%	-2.874029		-3.429657	
10%	-2.573502		-3.138345	

Table 5: PP test for the exchange rate returns series.

Tables 4 and 5 show that the null hypothesis of unit root would be rejected, implying that exchange rate return is stationary.

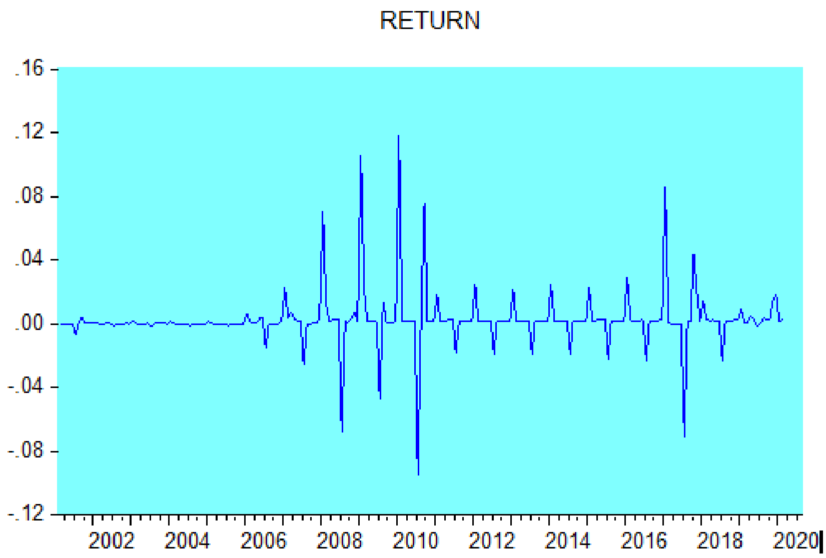


Figure 2: Time series plot of returns

The time series plot of the transformed data that is named exchange rate returns is shown in Figure 2. The plot of the returns which fluctuates about a fixed point indicating stationarity in mean and variance of the series and large changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes,this indicate there is volatility in the series

4.2 ARIMA Model

We include ARIMA model in the presence of volatility, because of ARIMA model take into account the mean behavior of a time series. Box-Jenkins modeling of a stationary time series involves the following four steps: model identification, model estimation, diagnostic checking and forecasting.We have to identify a good-fit model, estimate its parameters, apply diagnostic checking for the residuals and forecasting

4.2.1 Model Identification

The plots of the ACF and PACF are can be used to determine the order of autoregressive terms and moving average terms. Since correlogram of retun sieres of exchange rate Figure *A1* (in the Appendix) does not give much help in identifying an appropriate model. We observe from the ACF and PACF that there are insignificant spikes for all of the lags.In most applications lower order ARIMA models are often considered. In this study, AR (0-3) and MA (0-3) were considered since the return series show insignificant spikes for all of the lags.Several ARIMA(p,d,q) models have been suggested with the objective of identifying which of these models is adequate to fit the exchange return series, the suggested ARIMA models and their corresponding AIC,BIC values are stated as follows:

ARIMA(p,d,q)	AIC	BIC
ARIMA (1,1,0)	-5.103787	-5.072433
ARIMA (0,1,1)	-5.108761	-5.077509
ARIMA(0,1,2)	-5.108244	-5.061365
ARIMA (0,1,3)	-5.190449	-5.127943
ARIMA (1,1,1)	-5.197581	-5.151599
ARIMA (1,1,2)	-5.165711	-5.118679
ARIMA(1,1,3)	-5.135597	-5.057210
ARIMA (2,1,0)	-5.097501	-5.050315
ARIMA (2,1,1)	-5.142115	-5.079200
ARIMA (2,1,2)	-5.194805	-5.116160
ARIMA (2,1,3)	-5.185727	-5.091353
ARIMA(3,1,0)	-5.084067	-5.020944
ARIMA (3,1,1)	-5.132953	-5.054050
ARIMA (3,1,2)	-5.173514	-5.096387
ARIMA(3,1,3)	-5.245790	-5.137811

Table 6: ARIMA Model Identification

As discussed in the methodology part, a model with small AIC and BIC is preferable. Based on these selection criteria, ARIMA(3,1,3) model is selected as the mean equation for the price return series of exchange rate since AIC may favour over-parameterized models than BIC.

4.2.2 Parameter Estimation

Once an appropriate order of ARIMA model has been identified, the next step is estimating model parameters.The parameters can be estimated by using Ordinary Least Squares (OLS) method.

Variables	Coefficients	Std. error	Statistic	P-Value
Constant	0.002399	0.001412	1.69850	0.0909
AR(1)	0.028622	0.104757	0.273224	0.7849
AR(2)	-0.255056	0.100203	-2.545396	0.0116
AR(3)	-0.470382	0.105216	-4.470649	0.0000
MA(1)	0.056928	0.067729	0.840523	0.4016
MA(2)	0.155928	0.066539	2.343401	0.0200
MA(3)	0.844253	0.068916	12.25046	0.0000

Table 7: Parameter estimation of ARIMA (3,1,3) model

From Table 7, the estimated parameters of the Autoregressive and the moving average term at second and third lags are significant at 5% significance level .
the equation of the fitted ARIMA model is given by:

$$Y_t = 0.002399 + 0.028622Y_{t-1} - 0.255056Y_{t-2} - 0.470382Y_{t-3} + 0.056928\epsilon_{t-1} + 0.155928\epsilon_{t-2} + 0.844253\epsilon_{t-3} + \epsilon_t$$

4.2.3 Diagnostic checking

Before we consider the fitted model as a good-fit and interpret its findings, it is essential to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data. This process is conducted by using Jarque-Bera test, Breusch-Godfrey Serial Correlation LM Test and ARCH-LM test on the residuals of the model.

4.2.3.1 Breusch-Godfrey Serial Correlation LM Test

The presence of serial correlation in the residuals was tested using Breusch-Godfrey Serial Correlation LM Test.

Statistic lag	1	2
F-statistic	0.074754	0.051001
	(0.7848)	(0.9508)
X^2- statistic	0.076336	0.206666
	(0.7823)	(0.9487)

Note: Values inside parenthesis are p-values

Table 8: Serial Correlation LM Test on ARIMA(3,1,3) residualsl

The Breusch-Godfrey serial correlation LM test results in Table 8. provide an evidence of absence of serial correlation in the residuals of the mean equation, since the null hypothesis is not rejected at 5% level of significance .which implies that there is no serial correlation up to lag 2.

4.2.3.2 Test for normality

The Jarque-Bera test has been applied to investigate whether the residuals of the fitted model (mean equation) are normally distributed. The results are reported in Figure A_3 (Appendix A).The Jarque-Bera statistic is not significant, and there is no significant evidence to reject the null hypothesis of normality. The result indicate that the residuals of the fitted model are normally distributed for the series.

4.2.3.3 ARCH-LM Test

The residuals of ARIMA (3,1,3) are tested for ARCH effects using ARCH-LM test. This test used to determine whether the standardized residuals exhibit ARCH behavior or not.

F-statistic	10.33218	Probability	0.0003
Obs*R-squared	27.49525	Probability	0.0005

Table 9: ARCH-LM Test for residuals of ARMA(3,1,3) Models

As can be seen from the Table 9. the null hypothesis of no ARCH effect in the of residuals from the mean equations for the exchange rate return is rejected. The test indicates that the respective return series under consideration have a non-constant variance (heteroscedasticity) and need to be modelled using ARCH family models.
As plotted in Figure 3, below there also exist clear volatility clustering in the residuals

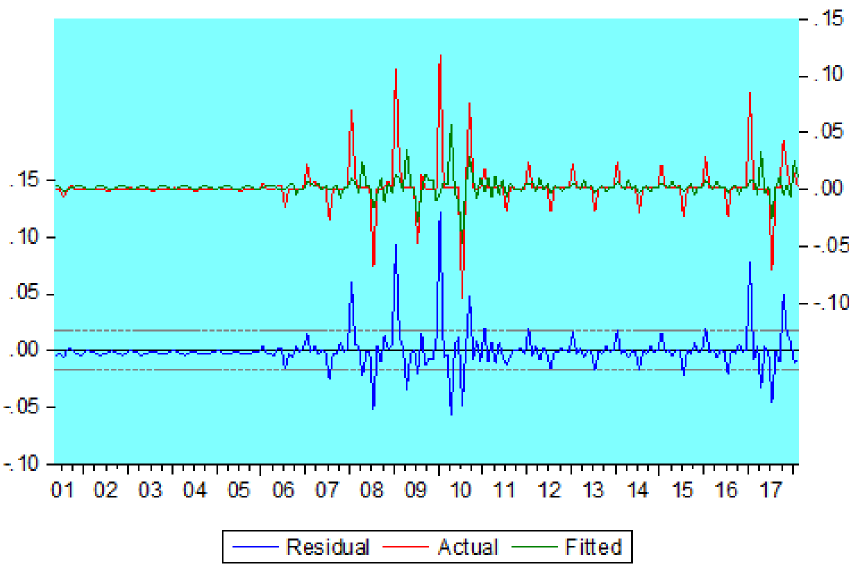


Figure 3: Volatility clusterings in the residuals for ARIMA (3,1,3)

4.2.3.4 Forecasting of ARIMA(3,1,3) model

One of the fundamental applications of time series analysis or developing a time series model is forecasting.

Evaluation of in-sample forecast

The forecasting performance of a model can be examined by the standardized statistical tools such as root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil's inequality (U). The in-sample forecast accuracy measures are given in Table 10 below.

Accuracy measures	Exchange rate return
RMSE	0.01725
MAE	0.008021
MAPE	3.8926
U	0.00015
Bias proportion	0.000000
Variance proportion	0.003761
Covariance proportion	0.516237

Table 10: In-sample forecast measures for ARIMA(3,1,3)

The value of Theil's inequality coefficient is close to zero. Besides that the bias and the variance proportion are also close to zero. This indicate that the forecasting performance of the model is good.

Out-of-sample forecasting

The out-of-sample forecast is performed from February 2018 to February 2020 generating 25 observations. The out-of-sample forecast graph is displayed in Figure 4. below. We can see that there is fluctuation in the exchange rate return

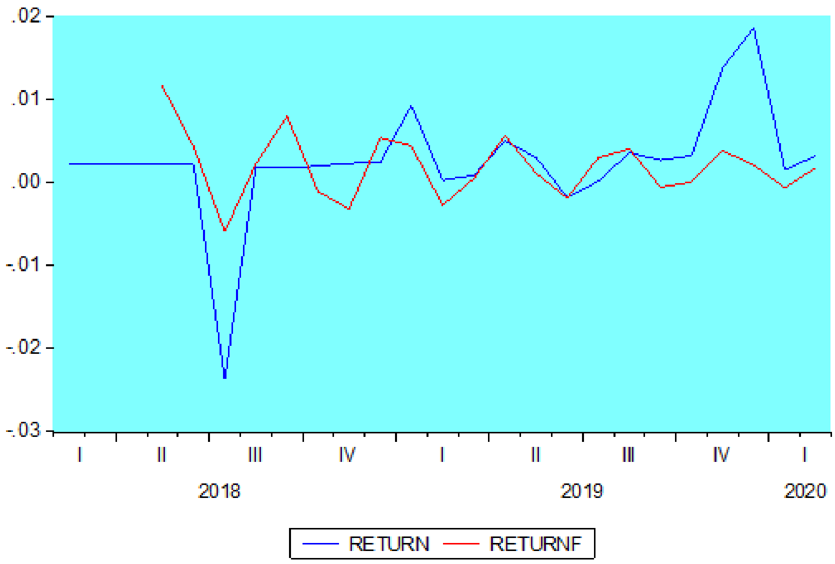


Figure 4: Out-of-sample forecast graph of ARIMA(3,1,3) model

4.3 Hybrid ARIMA-GARCH Model

The presence of volatility clustering and ARCH effect in the residuals ARIMA(3,1,3) show that it is important to apply a GARCH family model.A hybrid ARIMA-GARCH model is proposed to handle heteroscedasticity. Since GARCH models can be categorized into symmetric and asymmetric models, the current study aims to compare which of the two models,ARIMA-GARCH and ARIMA-EGARCH, is more appropriate for modelling and forecasting exchange rate volatility.

4.3.1 Model Identification

If an ARCH effect is found to be significant, the PACF of the squared residuals is helpful to determine the order of ARCH model.The modeling procedure of ARCH models can also be used to build a GARCH model. However, specifying the order of hybrid ARIMA-GARCH model is not easy. Often lower order ARIMA- GARCH models are used in most applications. Computing different statistics such as Akaike Information Criterion (AIC) and Bayesian information or Schwarz Criterion (BIC) in order to choose the more appropriate model is helpful. the GARCH model is denoted by GARCH (p,q) where q is the ARCH order while p is the GARCH order.

Table 11. Hybrid ARIMA-GARCH models

ARIMA (p,d,q)-GARCH(p,q)	Error Distribution	AIC	BIC
ARIMA (3,1,3)-GARCH(0,1)	Normal distribution	-5.290219	-5.151389
	Student's t-distribution	-7.603700	-7.449444
ARIMA (3,1,3)-GARCH(1,0)	Normal distribution	-5.535495	-5.396665
	Student's t-distribution	-7.157857	-7.003601
ARIMA (3,1,3)-GARCH(1,1)	Normal distribution	-6.279078	-6.124822
	Student's t-distribution	-7.936986	-7.767305
ARIMA (3,1,3)-GARCH(1,2)	Normal distribution	-6.349146	-6.179464
	Student's t-distribution	-6.797242	-6.612135
ARIMA (3,1,3)-GARCH(1,3)	Normal distribution	-5.741947	-5.556840
	Student's t-distribution	-5.833270	-5.632738
ARIMA (3,1,3)-GARCH(2,0)	Normal distribution	-5.521534	-5.367278
	Student's t-distribution	-8.163604	-7.993923
ARIMA (3,1,3)-GARCH(2,1)	Normal distribution	-6.367320	-6.197638
	Student's t-distribution	-7.915541	-7.700434
ARIMA (3,1,3)-GARCH(2,2)	Normal distribution	-6.438266	-6.253159
	Student's t-distribution	-5.769849	-5.569317
ARIMA (3,1,3)-GARCH(2,3)	Normal distribution	-5.603311	-5.402778
	Student's t-distribution	-5.549888	-5.333930
ARIMA (3,1,3)-GARCH(3,0)	Normal distribution	-5.507424	-5.337742
	Student's t-distribution	-8.138107	-7.953000
ARIMA (3,1,3)-GARCH(3,1)	Normal distribution	-6.411286	-6.226179
	Student's t-distribution	-7.403797	-7.203265
ARIMA (3,1,3)-GARCH(3,2)	Normal distribution	-6.550088	-6.349556
	Student's t-distribution	-5.929248	-5.713290
ARIMA (3,1,3)-GARCH(3,3)	Normal distribution	-5.906107	-5.690149
	Student's t-distribution	-5.590619	-5.359235

Table 11: Hybrid ARIMA-GARCH models

Based on Table 11, the hybrid ARIMA (3,1,3)-GARCH (1,1) under student's t-distributional is found to be the most appropriate model because it has the smallest value of AIC. Therefore,the hybrid ARIMA (3,1,3)-GARCH (1,1) will be considered for the next diagnostic checking.

Parameter	Variables	Coefficients	Std. error	Statistic	P-Value
Mean Equation	Constant	0.000354	7.39E-05	4.785912	0.0000
	AR(1)	0.020120	0.124781	0.161242	0.8719
	AR(2)	0.010452	0.166893	0.062624	0.9501
	AR(3)	-0.352783	0.071538	4.931425	0.0000
	MA(1)	0.114444	0.092886	1.232085	0.2179
	MA(2)	0.178858	0.076644	2.333617	0.0196
	MA(3)	-0.470192	0.074596	6.303184	0.0000
variance equation	Constant	2.99E-07	1.84E-07	1.621230	0.1050
	ARCH(-1)	0.614570	0.178295	3.446921	0.0006
	GARCH(-1)	0.2386119	0.065392	10.49235	0.0000

Table 12: Parameter estimation of hybrid ARIMA(3,1,3)-GARCH(1,1)

The ARCH and GARCH coefficients are statistically significant shows the existence of volatility clustering in exchange rate return series. Also the significant of ARCH and GARCH term shows that lagged conditional variance and lagged squared disturbance have an impact on conditional variance.This shows that the current monthly price volatility of exchange rate return was affected by its 1-month lagged shocks.Similarly,the current month price volatility of exchange rate return was affected by its 1 month lagged price volatility.

Moreover, the table shows that the sum of ARCH and GARCH, (persistence coefficient) term is very close to one indicate the volatility shocks is quite persistent. The appearance of this show that the shock to volatility is high, as well as the variance, is not stationary under the GARCH model. The ARCH -LM test shows that there is no additional ARCH effect meaning that is variance equation is well specified.

The mean equation of the return series is depend on a function of the conditional variance.Then equation of ARMA (3,l,3)-GARCH (1,1) model can be written as:

Mean Equation

$$\begin{aligned} Y_t =&0.000354 + 0.020120Y_{t-1} + 0.010452Y_{t-2} - 0.352783Y_{t-3} \\ &+ 0.11444\epsilon_{t-1} + 0.178858\epsilon_{t-2} - 0.470192\epsilon_{t-3} + \epsilon_t \end{aligned}$$

Variance Equation

$$\sigma_t^2 = 2.99E - 07 + 0.614570\epsilon_{t-1} + 0.2386119\sigma_{t-1}^2$$

4.3.2 Diagnostic Checking

After the model has been identified and the parameters are estimated, model diagnostic is done to check the adequacy of hybrid ARIMA (3,1,3)-GARCH (1,1) model. This process is conducted by using correlogram of squared residuals and ARCH-LM test on the residuals of the model.

4.3.2.1 ARCH-LM Test

The residuals of ARIMA (3,1,3)-GARCH (1,1) are tested for ARCH effects using ARCH-LM test. This test will determine whether the standardized residuals fulfilled ARCH properties or not.

F-statistic	0.263745	Prob(1,199)	0.6081
Obs*R-squared	0.266043	prob.chi-square(1)	0.6060

Table 13: ARCH-LM test for ARIMA (3,1,3)-GARCH (1,1)

From Table 13,the hypothesis of ARCH effects do not exist cannot be rejected because the p-value of 0.6081 is greater than significance level of 5% , It indicates that there is no conditional heteroscedasticity left in the series .

4.3.2.2 Correlogram Squared Residuals

The hybrid model is then tested for serial correlation using Correlogram Squared Residuals.

$$H_0 : \text{There is no serial correlation vs } H_1 : \text{There is serial correlation}$$

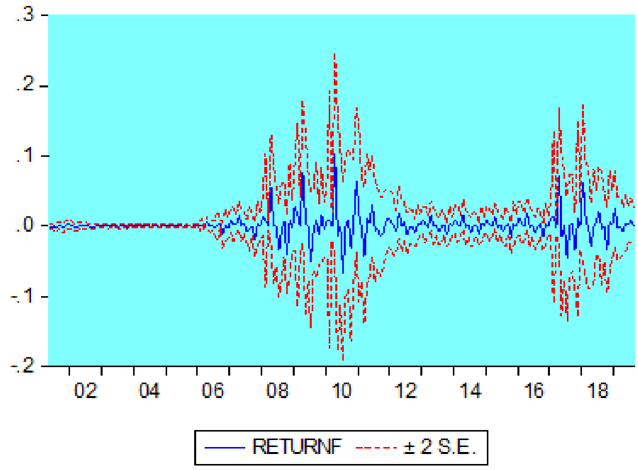
Lags	ACF	PACF	Prob
1	-0.029	-0.029	0.661
2	-0.049	-0.050	0.694
3	0.061	0.058	0.667
4	-0.063	-0.062	0.651
5	-0.065	-0.063	0.635
6	0.075	0.063	0.583
7	-0.058	-0.054	0.603
8	-0.056	-0.050	0.626
9	0.053	0.031	0.655
10	-0.032	-0.026	0.718
11	-0.044	-0.036	0.754
12	0.136	0.113	0.457
13	-0.040	-0.032	0.507
14	-0.059	-0.043	0.522
15	0.042	0.010	0.565
16	-0.062	-0.050	0.568
17	-0.057	-0.039	0.581
18	0.078	0.041	0.545
19	-0.053	-0.040	0.566
20	-0.056	-0.039	0.579
21	0.054	0.014	0.595
22	-0.047	-0.037	0.621
23	-0.062	-0.047	0.620
24	0.209	0.166	0.148

Table 14: Correlogram of Standardized Residuals Squared

From Table 14 , provides ACF and PACF of the squared standardized residuals for the hybrid model. Results showed that values of residual are very close to zero indicating that the model is adequate. That indicate the null hypothesis of no serial correlation cannot be rejected using significance level of 5% .

4.3.3 Forecasting

One of the fundamental uses of developing GARCH family model is forecasting. In this section we make in-sample forecasts based on the fitted ARIMA (3,1,3)-GARCH(1,1) model with Student's t-distribution assumption for residuals



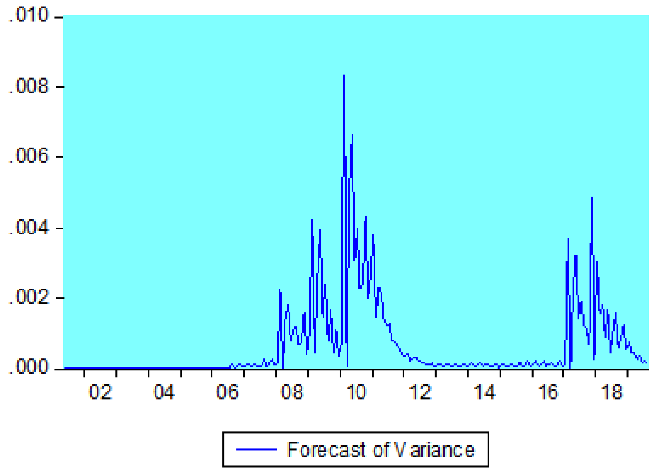


Figure 5: In-Sample Forecasting Results of ARIMA (3,1,3)-GARCH (1,1)

From Figure 5,shows that the forecasts are within plus and minus two standard error bands or, in other words, they lie within 95% confidence interval. So, it can be concluded that the model is satisfactory. The forecasted conditional variance of exchange rate return has two large spikes around 2008-2012 and 2017-2018. These large spikes indicate devaluation of Ethiopian birr against the USD.

4.4 Hybrid ARIMA- EGARCH Model

Financial time series data are known to exhibit certain stylized patterns such as fat tails, volatility clusterings, leverage effect and long memory. Symmetric models such as GARCH model perform well in capturing volatility clusterings of financial time series data but as their distribution is symmetric, they fail in modeling leverage effect. Since EGARCH model can handle leverage effect in the data, a hybrid ARIMA-EGARCH model is considered as an appropriate model for the exchange rate series.

4.4.1 Model Identification

ARIMA (p,d,q)-EGARCH(p,q)	Error Distribution	AIC	BIC
ARIMA (3,1,3)-EGARCH(1,0)	Normal distribution	-5.320025	-5.164269
	Student's t-distribution	-8.077453	-7.906122
ARIMA (3,1,3)-EGARCH(0,1)	Normal distribution	-5.410059	-5.254303
	Student's t-distribution	-8.003296	-7.831965
ARIMA (3,1,3)-EGARCH(1,1)	Normal distribution	-5.417994	-5.246663
	Student's t-distribution	-8.060046	-7.873139
ARIMA (3,1,3)-EGARCH(1,2)	Normal distribution	-5.390449	-5.203542
	Student's t-distribution	-7.905440	-7.702957
ARIMA (3,1,3)-EGARCH(1,3)	Normal distribution	-5.207355	-5.004873
	Student's t-distribution	-7.911075	-7.693017
ARIMA (3,1,3)-EGARCH(2,0)	Normal distribution	-5.521534	-5.367278
	Student's t-distribution	-8.075474	-7.888567
ARIMA (3,1,3)-EGARCH(2,1)	Normal distribution	-5.422521	-5.235614
	Student's t-distribution	-8.219363	-8.16880
ARIMA (3,1,3)-EGARCH(2,2)	Normal distribution	-5.460075	-5.257592
	Student's t-distribution	-7.877116	-7.659058
ARIMA (3,1,3)-EGARCH(2,3)	Normal distribution	-6.214161	-5.996103
	Student's t-distribution	-7.905308	-7.671674
ARIMA (3,1,3)-EGARCH(3,0)	Normal distribution	-5.580734	-5.393827
	Student's t-distribution	-8.057423	-7.854941
ARIMA (3,1,3)-EGARCH(3,1)	Normal distribution	-5.548828	-5.346346
	Student's t-distribution	-8.596238	-8.380280
ARIMA (3,1,3)-EGARCH(3,2)	Normal distribution	-6.942920	-6.724862
	Student's t-distribution	-7.881885	-7.648252
ARIMA (3,1,3)-EGARCH(3,3)	Normal distribution	-7.282927	-7.049294
	Student's t-distribution	-8.394289	-8.145080

Table 15: Hybrid ARIMA-EGARCH models

Based on Table 15, hybrid ARIMA (3,1,3)-EGARCH (3,1) under Student's t-distributional assumption is the most appropriate model because it has the smallest value of AIC . Therefore, hybrid ARIMA (3,1,3)-EGARCH (3,1) will be considered for the next diagnostic checking.

Parameter estimation of the hybrid ARMA (3,1,3)-EGARCH(3,1)model

Parameter	Variables	Coefficients	Std. error	Statistic	P-Value
Mean Equation	Constant	0.001507	7.29E-05	20.67923	0.0000
	AR(1)	0.327234	0.823154	0.397537	0.0000
	AR(2)	0.290263	1.100737	0.263699	0.7920
	AR(3)	0.32708	0.567794	0.576063	0.5646
	MA(1)	-0.329689	0.823104	-0.400543	0.6888
	MA(2)	-0.288163	1.103294	0.261184	0.0032
	MA(3)	-0.324056	0.570495	-0.568026	0.0000
variance equation	α_0	-17.58222	0.596695	-29.46603	0.0000
	α_1	-0.724185	0.134208	-5.395982	0.0000
	γ	0.624389	0.117147	5.329968	0.0000
	β_1	0.168642	0.014050	12.00270	0.0000
	β_2	0.133607	0.010992	12.15532	0.0000
	β_3	-0.944760	0.008565	-110.3085	0.0000

Table 16: Parameter estimation of the hybrid ARMA (3,1,3)-EGARCH(3,1)model

From Table 16,the value of γ for hybrid ARIMA(3,1,3)-EGARCH (3,1) is 0.624389. $\gamma > 0$ means that there are no long leverage effects in the series and since $\gamma \neq 0$. This means the presence of asymmetry of positive shocks on the conditional variance.The asymmetric (leverage) parameter was significant but positive indicating the absence of leverage effects. Leverage effects is said to exist in the EGARCH model if the coefficient of the leverage parameter is significant and negative.The significance of the ARCH and GARCH parameters in the EGARCH model indicates that previous periods squared residual and previous periods variance of the residual have an influence on current variance of the residuals.
The equation of ARMA (3,l,3)-EGARCH (3,1) model can be written as:
Mean Equation

$$\begin{aligned} Y_t =& 0.001507 + 0.0327234 Y_{t-1} - 0.290263 Y_{t-2} - 0.32708 Y_{t-3} \\ & - 0.329689 \epsilon_{t-1} - 0.288163 \epsilon_{t-2} - 0.324056 \epsilon_{t-3} + 0.839454 \epsilon_{t-3} + \epsilon_t \end{aligned}$$

Variance Equation

$$\begin{aligned} \log(\sigma_t^2) =& - 17.58222 - 0.724185 \left|\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right| + 0.624389 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + 0.168642 \ln(\sigma_{t-1}^2) \\ & + 0.168642 \ln(\sigma_{t-2}^2) - 0.94476 \ln(\sigma_{t-3}^2) \end{aligned}$$

4.4.2 Diagnostic Checking

After the model has been identified and the parameters are estimated, diagnostic checking is undertaken to check for the adequacy of hybrid ARIMA (3,1,3)-EGARCH (3,1) model. This process is conducted by using correlogram of squared residuals and ARCH-LM test on the residuals of the model.

4.4.2.1 ARCH-LM Test

The residuals of ARIMA (3,1,3)-EGARCH (3,1) are tested for ARCH effects using ARCH-LM test. This test will determine whether the standardized residuals fulfilled ARCH properties or not.

F-statistic	0.007505	Prob(1,198)	0.9310
Obs*R-squared	0.007574	prob.chi-square(1)	0.9306

Table 17: ARCH-LM for ARIMA (3,1,3)-EGARCH (3,1)

From Table 17, it can be implied that the null hypothesis the nonexistence of ARCH effects cannot be rejected because the p-value of 0.9310 is greater than significance level of 5%. It indicates that there is no conditional heteroscedasticity left in the data series.

4.4.2.2 Correlogram Squared Residuals

The hybrid model is then tested for serial correlation using Correlogram Squared Residuals. The hypotheses to be tested as follows:

$$H_0 : \text{There is no serial correlation}$$
$$H_1 : \text{There is serial correlation}$$

Lags	ACF	PACF	Prob
1	-0.030	-0.030	0.667
2	-0.008	-0.009	0.906
3	0.153	0.153	0.167
4	-0.027	-0.019	0.266
5	-0.018	- 0.018	0.382
6	0.066	0.043	0.400
7	-0.014	-0.005	0.511
8	0.048	0.054	0.565
9	0.046	0.033	0.617
10	-0.019	-0.013	0.699
11	-0.019	-0.034	0.770
12	0.154	0.145	0.405
13	-0.015	-0.001	0.4881
14	-0.013	-0.011	0.557
15	-0.015	-0.067	0.628
16	-0.019	-0.014	0.691
17	-0.020	-0.013	0.747
18	0.034	0.032	0.786
19	-0.019	-0.010	0.830
20	0.174	0.171	0.457
21	-0.015	-0.025	0.517
22	-0.019	-0.008	0.574
23	-0.020	-0.064	0.627
24	0.014	0.006	0.680

Table 18: Correlogram of Standardized Residuals Squared.

Table 18. Indicate the ACF and PACF of the squared standardized residuals for the hybrid model. Results showed that values of residual are very close to zero indicating that the model is adequate. That indicate null hypothesis of no serial correlation cannot be rejected using significance level of 5%.

4.4.3 Forecasting

One of the fundamental uses of developing GARCH family model is forecasting. In this section we make in-and out-sample forecasts based on the fitted ARIMA (3,1,3)-EGARCH(3,1) model with Student's t-distribution assumption for residuals.

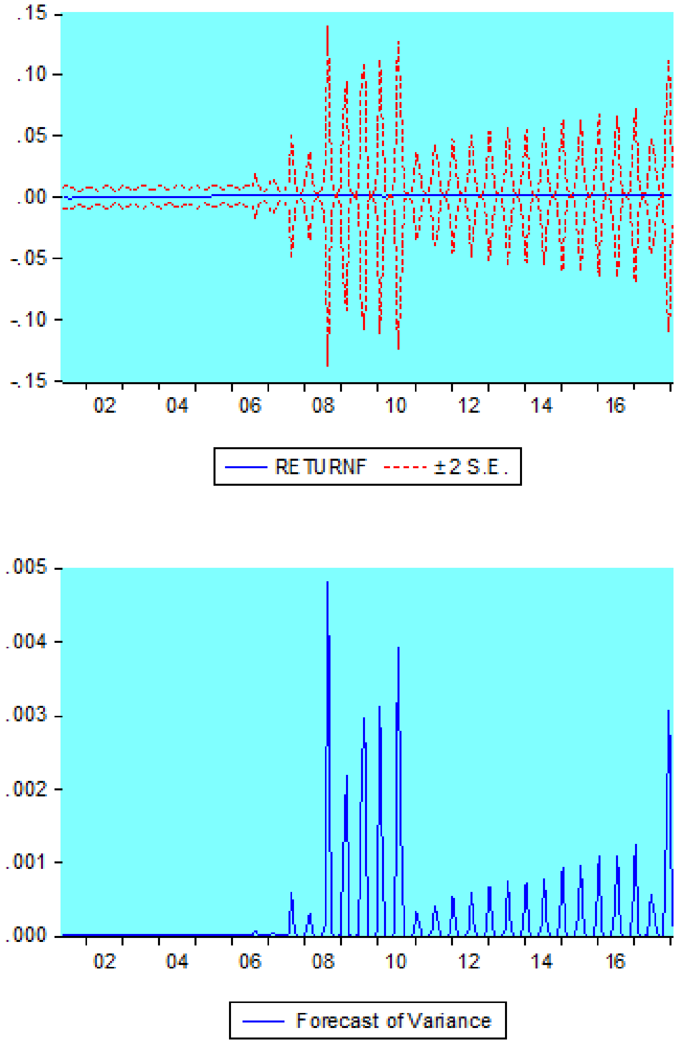


Figure 6: In-Sample Forecasting of ARIMA(3,1,3)-EGARCH (3,1)

Figure 6. Shows that the forecasts are lie within plus and minus two standard error bands or in other words,they lie within 95% confidence interval. So, it can be concluded that the model is satisfactory.

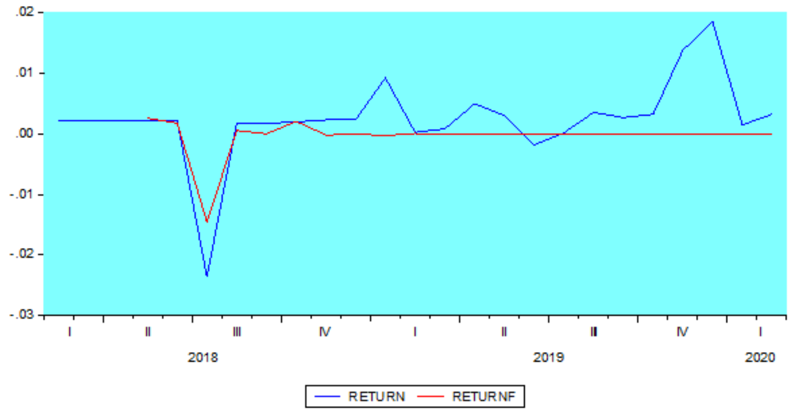


Figure 7: Out-Sample Forecasting of ARIMA (3,1,3)-EGARCH (3,1)

4.5 Performances of Hybrid Models

Accuracy criteria is used to measure the performances of modeling and forecasting of the best selected model. The model with the smallest value of AIC and BIC is the best model that fit the data while the model with the smaller value of RMSE, MAE and MAPE are used as the most appropriate forecasting model.

Evaluation Criteria	ARIMA (3,1,3) -GARCH (1,1)	ARIMA (3,1,3) -EGARCH (3,1)
AIC	-7.936986	-8.596238
BIC	-7.767305	-8.380280

Table 19: Comparison of AIC and BIC values for both hybrid model

Based on Table 19, the AIC ad BIC values for hybrid ARIMA (3,1,3)-EGARCH (3,1) are smaller that hybrid ARIMA (3,1,3)-GARCH (1,1) model. Results showed that hybrid ARIMA (3,1,3)-EGARCH (3,1) model is the more appropriate model in modeling monthly exchange rate data compared to hybrid ARIMA (3,1,3)-GARCH (3,1) model.

Type of sample	Evaluation Criteria	ARIMA (3,1,3)-GARCH (1,1)	ARIMA (3,1,3)-EGARCH (3,1)
In-sample	RMSE	0.018778	0.017245
	MAE	0.009862	0.006587
	MAPE	906.2357	250.6284
Out-sample	RMSE	0.006770	0.023438
	MAE	0.003832	0.003217
	MAPE	206.5204	150.8123

Table 20: Comparison of Forecasting Performances

The results in Table 20. show that the values of RMSE, MAE and MAPE for hybrid ARIMA(3,1,3)-GARCH (1,1) and hybrid ARIMA (3,1,3)-EGARCH (3,1).The comparison of the forecasting performance indicate that the more appropriate model is ARIMA(3,1,3)-EGARCH(3,1). Therefore, ARIMA(3,1,3)-EGARCH(3,1) model captures the volatility and leverage effect of the exchange rate returns and its forecast performance is better than that of ARIMA(3,1,3)-GARCH(1,1) models. The values of RMSE, MAE and MAPE for both in-sample and out-sample forecasting of ARIMA (3,1,3)-EGARCH (3,1) are smaller than ARIMA (3,1,3)- GARCH (1,1). Therefore, we can be concluded that hybrid ARIMA (3,1,3)-EGARCH (3,1) is the most appropriate model for forecasting the monthly exchange rate data.

5 DISCUSSION

The study was aimed to modeling and forecasting of exchange rate volatility of Ethiopia. We applied the two models which are ARIMA model and GARCH family models.
The parameters for ARIMA models were estimated using OLS method.For hybrid ARIMA-GARCH and ARIMA-EGARCH, the parameters were estimated using MLE . MLE is suitable for both linear and nonlinear models while OLS can minimize the sum of squared errors (SSE).The diagnostic checking was conducted on the residuals of the model to validate the goodness of fit using Serial Correlation Test and Heteroscedasticity test.
Based on the result, the mean equation for exchange rate return series ARIMA (3,1,3) was selected among sixteen combinations since it had the smallest AIC. The Lagrange Multiplier (LM) test for ARCH effects in the squared residuals shows that there is a dependency between the squared residuals from mean model (there is ARCH effect or volatility varies over time) that has need to model GARCH family models.
After specifying the order and the type of Hybrid ARIMA- GARCH (symmetric) family model,statistical tests were made and Hybrid ARIMA(3,1,3)- GARCH(1,1) was selected as the best model to describe the volatility of exchange rate return since it had small AIC and symmetric effect was statistically significant at 5% level of significance under student's t, error distribution. The results also show that the coefficient of 1 month lagged shocks (i.e.ARCH (-1)term were positive and statistically significant at the 5% level. This shows that the current monthly price volatility of exchange rate was affected by its 1-month lagged shocks.Similarly, GARCH (-1) term is statistically significant at the 5% significance level. This indicate that the current month price volatility of exchange rate was affected by its 1 month lagged price volatility.
similarly the the order and the type of Hybrid ARIMA- EGARCH family model,statistical tests were made and Hybrid ARIMA(3,1,3)-EGARCH(3,1) was selected as the most appropriate model since it had small AIC and BIC and asymmetric effect was statistically significant at 5% level of significance under student's t, error distribution.
Additionally, the ARIMA(3,1,3)- EGARCH (3,1) result shows that having no leverage effect in exchange rate return since the value of $\gamma > 0$ point out that positive shocks involve a higher next period conditional variance Negative shocks of the same enormity.
In this study, the performances of modeling for hybrid ARIMA-GARCH and ARIMA-EGARCH models were compared using AIC values.The most appropriate forecasting model was selected based on the smallest values of RMSE, MAE and MAPE for both of in-sample and out-sample data.

6 Conclusion and Recommendation

6.1 Conclusion

Modeling and forecasting the volatility of returns series in exchange rate has become fertile field of empirical research in financial markets.
This study examined the capabilities and potential of hybridization between ARIMA and GARCH family models.Monthly data of U.S. Dollar exchange rate against Ethiopia exchange rate (USD/ETB) for the time period taken from January 2001 up to Febru-

ary 2020 was obtained from the National Bank of Ethiopia. We considered two models which are ARIMA model and GARCH family models. For GARCH family models, we used symmetric model (GARCH) and asymmetric(Exponential) model (EGARCH) as hybridization with ARIMA model. Eventhough ARIMA models are able to handle non-stationary data, it cannot capture volatility in the series on the other hand GARCH family models can handle volatile in the series .Meanwhile, symmetric GARCH model cannot capture leverage effect in the series.
The conditional mean equation was estimated using ARIMA model and the conditional variance equation using hybridization ARIMA- GARCH family models with normal and student's t distribution,error distributions After examining different competitive models, we came to the conclusion that ARIMA(3,1,3) model provides the better model for the exchange rate return
The results had shown that hybrid ARIMA (3,1,3)-EGARCH (3,1) is the most appropriate model for modeling exchange rate data since it has the smallest value of AIC compared to hybrid ARIMA (3,1,3)- GARCH (1,1) model.the forecasting performances, of the models were evaluated by using RMSE, MAE and MAPE.
Finally, In -sample and out-of-sample forecasts were made from January 2001 to January 2018 and February 2018 to February 2020, respectively, forecasts are made using ARIMA (3,1 3) and hybrid ARIMA(3,1,3)-EGARCH(3,1) models. The in-sample forecast accuracy using root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and Theil's U statistics indicates that the estimated ARIMA model is good enough to describe the exchange rate. Out-of-sample forecasts are made from February 2018 to February 2020. The result indicates that the return exchange rate has fluctuated over the forecast periods. The empirical results showed that the conditional variance (volatility) is a hotheaded process in Ethiopia currency while it is quite for Ethiopian currency against the US dollar which is required having the mean-reverting variance process.

6.2 Recommendation

The Results of this study, the following recommendations are drawn:

- Since there is a uncertainty in the exchange market.Therefore, the government bodies ,policy makers and concerned bodies should give attention
- Future studies can consider using other family of GARCH models such as Integrated GARCH (IGARCH), Threshold GARCH (TARCH) and other non linear time series approaches (artificial neural network (ANN)) models.

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Appendix . A

Figure A1:Correlogram of monthly exchange rate return up to 30 logs

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|* |</td><td>.|* |</td><td>1</td><td>0.208</td><td>0.208</td><td>7.8806</td><td>0.005</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>0.000</td><td>-0.045</td><td>7.8806</td><td>0.019</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>0.017</td><td>0.028</td><td>7.9355</td><td>0.047</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>-0.048</td><td>-0.060</td><td>8.3621</td><td>0.079</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>0.008</td><td>0.034</td><td>8.3754</td><td>0.137</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>6</td><td>0.002</td><td>-0.011</td><td>8.3763</td><td>0.212</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>7</td><td>0.016</td><td>0.023</td><td>8.4258</td><td>0.297</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>0.000</td><td>-0.013</td><td>8.4258</td><td>0.393</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>9</td><td>-0.010</td><td>-0.004</td><td>8.4447</td><td>0.490</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>-0.012</td><td>-0.012</td><td>8.4727</td><td>0.583</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>11</td><td>-0.008</td><td>-0.000</td><td>8.4839</td><td>0.669</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>12</td><td>-0.026</td><td>-0.027</td><td>8.6129</td><td>0.736</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>13</td><td>0.024</td><td>0.037</td><td>8.7235</td><td>0.793</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.047</td><td>0.032</td><td>9.1496</td><td>0.821</td></tr>
<tr><td>.|** |</td><td>.|** |</td><td>15</td><td>0.264</td><td>0.265</td><td>22.947</td><td>0.085</td></tr>
<tr><td>.|** |</td><td>.|* |</td><td>16</td><td>0.216</td><td>0.115</td><td>32.218</td><td>0.009</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>0.025</td><td>-0.022</td><td>32.344</td><td>0.014</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>18</td><td>0.074</td><td>0.080</td><td>33.445</td><td>0.015</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>19</td><td>0.164</td><td>0.179</td><td>38.892</td><td>0.005</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>20</td><td>-0.111</td><td>-0.188</td><td>41.406</td><td>0.003</td></tr>
<tr><td>.|. |</td><td>.|* |</td><td>21</td><td>0.006</td><td>0.076</td><td>41.412</td><td>0.005</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>22</td><td>0.002</td><td>-0.033</td><td>41.413</td><td>0.007</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>23</td><td>0.023</td><td>0.062</td><td>41.523</td><td>0.010</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>24</td><td>0.053</td><td>0.012</td><td>42.116</td><td>0.013</td></tr>
<tr><td>.|. |</td><td>.|* |</td><td>25</td><td>0.047</td><td>0.078</td><td>42.581</td><td>0.016</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>26</td><td>-0.001</td><td>-0.045</td><td>42.581</td><td>0.021</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>27</td><td>-0.005</td><td>0.053</td><td>42.587</td><td>0.029</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>28</td><td>-0.004</td><td>-0.027</td><td>42.589</td><td>0.038</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>29</td><td>-0.017</td><td>-0.017</td><td>42.653</td><td>0.049</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>30</td><td>-0.018</td><td>-0.117</td><td>42.722</td><td>0.062</td></tr>
</table>

Figure A2: Parameter Estimation of ARIMA (3,1,3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002399	0.001412	1.698505	0.0909
AR(1)	0.028622	0.104757	0.273224	0.7849
AR(2)	-0.255056	0.100203	-2.545396	0.0116
AR(3)	-0.470382	0.105216	-4.470649	0.0000
MA(1)	0.056928	0.067729	0.840523	0.4016
MA(2)	0.155928	0.066539	2.343401	0.0200
MA(3)	0.844253	0.068916	12.25046	0.0000

R-squared	0.152925	Mean dependent var	0.002428
Adjusted R-squared	0.129063	S.D. dependent var	0.018530
S.E. of regression	0.017292	Akaike info criterion	-5.245790
Sum squared resid	0.063693	Schwarz criterion	-5.137811
Log likelihood	584.0369	Hannan-Quinn criter.	-5.202185
F-statistic	6.408908	Durbin-Watson stat	2.000001
Prob(F-statistic)	0.000003		

Inverted AR Roots	.34-.77i	.34+.77i	-.66
Inverted MA Roots	.43+.86i	.43-.86i	-.91

□

Figure A3:: Jarque-Bera normality test for the residuals of ARIMA (3,1,3) model

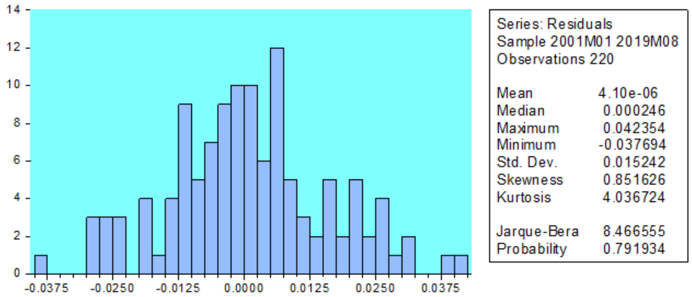

Figure A4: Serial correlation Breush-Godfrey LM test for residuals of ARIMA(3,1,3) model

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.051001	Prob. F(2,193)	0.9503
Obs*R-squared	0.105232	Prob. Chi-Square(2)	0.9487

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 06/01/20 Time: 12:46
Sample: 2001M04 2018M01
Included observations: 202
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.61E-06	0.001567	-0.001029	0.9992
AR(1)	0.020732	0.243988	0.084971	0.9324
AR(2)	0.015220	0.197463	0.077078	0.9386
AR(3)	0.004274	0.141736	0.030156	0.9760
MA(1)	-0.002626	0.057287	-0.045845	0.9635
MA(2)	0.001410	0.051168	0.027554	0.9780
MA(3)	-0.001517	0.055261	-0.027452	0.9781
RESID(-1)	-0.026466	0.218169	-0.121307	0.9036
RESID(-2)	-0.028848	0.173573	-0.166202	0.8682

R-squared	0.000521	Mean dependent var	4.64E-05
Adjusted R-squared	-0.040908	S.D. dependent var	0.017250
S.E. of regression	0.017599	Akaike info criterion	-5.198418
Sum squared resid	0.059777	Schwarz criterion	-5.051020
Log likelihood	534.0402	Hannan-Quinn criter.	-5.138780
F-statistic	0.012574	Durbin-Watson stat	2.008927
Prob(F-statistic)	1.000000		

Figure A5: parameters Estimation of ARIMA(3,1,2)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002562	0.000623	4.111972	0.0001
AR(1)	0.010364	0.128104	0.080904	0.9356
AR(2)	0.473274	0.120778	3.918527	0.0001
AR(3)	-0.026700	0.085793	-0.311210	0.7560
MA(1)	0.057596	0.108198	0.532324	0.5951
MA(2)	-0.788604	0.088943	-8.866422	0.0000

R-squared	0.118532	Mean dependent var	0.002446
Adjusted R-squared	0.097343	S.D. dependent var	0.018784
S.E. of regression	0.017847	Akaike info criterion	-5.186352
Sum squared resid	0.066249	Schwarz criterion	-5.091979
Log likelihood	560.9396	Hannan-Quinn criter.	-5.148217
F-statistic	5.593992	Durbin-Watson stat	2.010990
Prob(F-statistic)	0.000074		

Inverted AR Roots	.66	.06	-.71
Inverted MA Roots	.86	-.92	

Figure A6: parameters Estimation of ARIMA(3,1,1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002537	0.000590	4.299747	0.0000
AR(1)	0.809732	0.114052	7.099676	0.0000
AR(2)	-0.115188	0.088664	-1.299147	0.1953
AR(3)	-0.071319	0.077991	-0.914452	0.3615
MA(1)	-0.826982	0.093614	-8.833983	0.0000

R-squared	0.065831	Mean dependent var	0.002446
Adjusted R-squared	0.047952	S.D. dependent var	0.018784
S.E. of regression	0.018329	Akaike info criterion	-5.137629
Sum squared resid	0.070210	Schwarz criterion	-5.058985
Log likelihood	554.7263	Hannan-Quinn criter.	-5.105849
F-statistic	3.682048	Durbin-Watson stat	2.021159
Prob(F-statistic)	0.006397		

Inverted AR Roots	.51-.27i	.51+.27i	-.21
Inverted MA Roots	.83		

Figure A7: parameters Estimation of ARIMA(3,1,0)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002427	0.001167	2.080509	0.0387
AR(1)	0.040312	0.068016	0.592685	0.5540
AR(2)	-0.087151	0.067813	-1.285156	0.2001
AR(3)	-0.025955	0.068021	-0.381570	0.7032

R-squared	0.009977	Mean dependent var	0.002428
Adjusted R-squared	-0.003773	S.D. dependent var	0.018530
S.E. of regression	0.018564	Akaike info criterion	-5. 084067
Sum squared resid	0.074442	Schwarz criterion	-5. 020944
Log likelihood	566.8836	Hannan-Quinn criter.	-5.092207
F-statistic	0.725576	Durbin-Watson stat	2.000894
Prob(F-statistic)	0.537716		

Inverted AR Roots	.12-.35i	.12+.35i	-.20

Figure A8: parameters Estimation of ARIMA(2,1,0)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002417	0.001189	2.031962	0.0434
AR(1)	0.042673	0.067463	0.632536	0.5277
AR(2)	-0.088184	0.067463	-1.307140	0.1925

R-squared	0.009302	Mean dependent var	0.002417
Adjusted R-squared	0.000213	S.D. dependent var	0.018488
S.E. of regression	0.018486	Akaike info criterion	-5. 097501
Sum squared resid	0.074498	Schwarz criterion	-5. 050315
Log likelihood	569.8780	Hannan-Quinn criter.	-5.111492
F-statistic	1.023436	Durbin-Watson stat	2.004633
Prob(F-statistic)	0.361078		

Inverted AR Roots	.02+.30i	.02-.30i

Figure A9: parameters Estimation of ARIMA(2,1,1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002521	0.000588	4.287297	0.0000
AR(1)	0.837651	0.101782	8.229880	0.0000
AR(2)	-0.169267	0.071121	-2.380005	0.0182
MA(1)	-0.843586	0.082679	-10.20315	0.0000

R-squared	0.061135	Mean dependent var	0.002417
Adjusted R-squared	0.048155	S.D. dependent var	0.018488
S.E. of regression	0.018037	Akaike info criterion	-5. 097501
Sum squared resid	0.070600	Schwarz criterion	-5. 050315
Log likelihood	575.8161	Hannan-Quinn criter.	-5.149971
F-statistic	4.710042	Durbin-Watson stat	2.021776
Prob(F-statistic)	0.003311		

Inverted AR Roots	.50	.34
Inverted MA Roots	.84	

Figure A10: parameters Estimation of ARIMA(2,1,2)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002538	0.000609	4.169088	0.0000
AR(1)	0.010718	0.121531	0.088194	0.9298
AR(2)	0.474020	0.118817	3.989507	0.0001
MA(1)	0.046896	0.087012	0.538959	0.5905
MA(2)	-0.788957	0.086299	-9.142141	0.0000

R-squared	0.117254	Mean dependent var	0.002417
Adjusted R-squared	0.100907	S.D. dependent var	0.018488
S.E. of regression	0.017530	Akaike info criterion	-5. 194805
Sum squared resid	0.066380	Schwarz criterion	-5. 116160
Log likelihood	582.6267	Hannan-Quinn criter.	-5.196347
F-statistic	7.172761	Durbin-Watson stat	1.979783
Prob(F-statistic)	0.000019		

Inverted AR Roots	.69	-.68
Inverted MA Roots	.87	-.91

Figure A11: parameters Estimation of ARIMA(1,1,2)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002518	0.000598	4.209154	0.0000
AR(1)	0.638167	0.140211	4.551491	0.0000
MA(1)	-0.654759	0.147233	-4.447078	0.0000
MA(2)	-0.171568	0.078473	-2.186333	0.0299

R-squared	0.057305	Mean dependent var	0.002407
Adjusted R-squared	0.044332	S.D. dependent var	0.018447
S.E. of regression	0.018033	Akaike info criterion	-5. 165711
Sum squared resid	0.070894	Schwarz criterion	-5. 118679
Log likelihood	578.4627	Hannan-Quinn criter.	-5.150587
F-statistic	4.417264	Durbin-Watson stat	1.988856
Prob(F-statistic)	0.004872		

Inverted AR Roots	.64	
Inverted MA Roots	.86	-.20

Figure A12: parameters Estimation of ARIMA(3,1,3)-GARCH(1,1)

GARCH = C(8) + C(9)*RESID(-1)^2 + C(10)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000354	7.39E-05	4.785912	0.0000
AR(1)	0.020120	0.124781	0.161242	0.8719
AR(2)	0.010452	0.166893	0.062624	0.9501
AR(3)	-0.352783	0.071538	4.931425	0.0000
MA(1)	0.114444	0.092886	1.232085	0.2179
MA(2)	0.178858	0.076644	2.333617	0.0196
MA(3)	-0.470192	0.074596	6.303184	0.0000
		Variance Equation		
C	2.99E-07	1.84E-07	1.621230	0.1050
RESID(-1)^2	0.614570	0.178295	3.446921	0.0006
GARCH(-1)	0.2386119	0.065392	10.49235	0.0000
T-DIST. DOF	2.272123	0.124109	18.30743	0.0000

R-squared	-0.018875	Mean dependent var	0.002429
Adjusted R-squared	-0.047441	S.D. dependent var	0.018487
S.E. of regression	0.018921	Akaike info criterion	-7.936986
Sum squared resid	0.076611	Schwarz criterion	-7.767305
Log likelihood	882.8543	Hannan-Quinn criter.	-7.821789
Durbin-Watson stat	1.966081		

Inverted AR Roots	.82	-.41+.75i	-.41-.75i
Inverted MA Roots	.78	-.42-.76i	-.42+.76i

Figure A13: ARCH test for the squared residuals of ARIMA(3,1,3)-GARCH(1,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.860382	Prob. F(12,177)	0.5883
Obs*R-squared	10.47204	Prob. Chi-Square(12)	0.5746

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.816039	0.777174	2.336723	0.0206
WGT_RESID^2(-1)	-0.027700	0.074320	-0.372716	0.7098
WGT_RESID^2(-2)	-0.034609	0.074293	-0.465850	0.6419
WGT_RESID^2(-3)	-0.006054	0.074309	-0.081467	0.9352
WGT_RESID^2(-4)	-0.025716	0.074306	-0.346083	0.7297
WGT_RESID^2(-5)	-0.028449	0.074280	-0.382997	0.7022
WGT_RESID^2(-6)	0.122111	0.074274	1.644057	0.1019
WGT_RESID^2(-7)	-0.031100	0.074273	-0.418728	0.6759
WGT_RESID^2(-8)	-0.030936	0.074281	-0.416475	0.6776
WGT_RESID^2(-9)	-0.012844	0.074293	-0.172889	0.8629
WGT_RESID^2(-10)	-0.027774	0.074305	-0.373777	0.7090
WGT_RESID^2(-11)	-0.033468	0.074290	-0.450496	0.6529
WGT_RESID^2(-12)	0.149570	0.074312	2.012725	0.0457

R-squared	0.055116	Mean dependent var	1.839867
Adjusted R-squared	-0.008944	S.D. dependent var	8.104586
S.E. of regression	8.140748	Akaike info criterion	7.097609
Sum squared resid	11730.11	Schwarz criterion	7.319774
Log likelihood	-661.2729	Hannan-Quinn criter.	7.187605
F-statistic	0.860382	Durbin-Watson stat	1.992458
Prob(F-statistic)	0.588348		

Figure A14: Correlogram of Standardized Residuals Squared ARIMA(3,1,3)-GARCH(1,1)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|. |	1	-0.029	-0.029	0.1928	0.661
.|. |	.|. |	2	-0.049	-0.050	0.7306	0.694
.|. |	.|. |	3	0.061	0.058	1.5654	0.667
.|. |	.|. |	4	-0.063	-0.062	2.4663	0.651
.|. |	.|. |	5	-0.065	-0.063	3.4244	0.635
.|* |	.|. |	6	0.075	0.063	4.7004	0.583
.|. |	.|. |	7	-0.058	-0.054	5.4711	0.603
.|. |	.|. |	8	-0.056	-0.050	6.1887	0.626
.|. |	.|. |	9	0.053	0.031	6.8312	0.655
.|. |	.|. |	10	-0.032	-0.026	7.0758	0.718
.|. |	.|. |	11	-0.044	-0.036	7.5358	0.754
.|* |	.|* |	12	0.136	0.113	11.863	0.457
.|. |	.|. |	13	-0.040	-0.032	12.247	0.507
.|. |	.|. |	14	-0.059	-0.043	13.060	0.522
.|. |	.|. |	15	0.042	0.010	13.487	0.565
.|. |	.|. |	16	-0.062	-0.050	14.413	0.568
.|. |	.|. |	17	-0.057	-0.039	15.197	0.581
.|* |	.|. |	18	0.078	0.041	16.686	0.545
.|. |	.|. |	19	-0.053	-0.040	17.360	0.566
.|. |	.|. |	20	-0.056	-0.039	18.125	0.579
.|. |	.|. |	21	0.054	0.014	18.851	0.595
.|. |	.|. |	22	-0.047	-0.037	19.399	0.621
.|. |	.|. |	23	-0.062	-0.047	20.357	0.620
.|* |	.|* |	24	0.209	0.166	31.197	0.148
.|. |	.|. |	25	-0.057	-0.045	32.003	0.158
.|. |	.|. |	26	-0.060	-0.033	32.919	0.164
.|. |	.|. |	27	0.061	0.013	33.874	0.170
.|. |	.|. |	28	-0.048	-0.023	34.462	0.186
.|. |	.|. |	29	-0.063	-0.033	35.492	0.189
.|. |	.|. |	30	0.037	-0.026	35.843	0.213
.|. |	.|. |	31	-0.041	-0.019	36.270	0.236
.|. |	.|. |	32	-0.036	-0.008	36.613	0.263
.|. |	.|. |	33	0.057	0.007	37.474	0.271
.|. |	.|. |	34	-0.042	-0.031	37.949	0.294
.|. |	.|. |	35	-0.046	-0.021	38.505	0.314
.|* |	.|. |	36	0.085	0.011	40.432	0.281

Figure A15: parameters Estimation of ARIMA(3,1,3)-EGARCH(3,1)

LOG(GARCH) = C(8) + C(9)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(10)
*RESID(-1)/@SQRT(GARCH(-1)) + C(11)*LOG(GARCH(-1)) + C(12)
*LOG(GARCH(-2)) + C(13)*LOG(GARCH(-3))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001507	7.29E-05	20.67923	0.0000
AR(1)	0.327234	0.823154	0.397537	0.0000
AR(2)	0.290263	1.100737	0.263699	0.7920
AR(3)	0.327086	0.567794	0.576063	0.5646
MA(1)	-0.329689	0.823104	-0.400543	0.6888
MA(2)	-0.288163	1.103294	-0.261184	0.0032
MA(3)	-0.324056	0.570495	-0.568026	0.0000
Variance Equation				
C(8)	-17.58222	0.596695	-29.46603	0.0000
C(9)	-0.724185	0.134208	-5.395982	0.0000
C(10)	0.624389	0.117147	5.329968	0.0000
C(11)	0.168642	0.014050	12.00270	0.0000
C(12)	0.133607	0.010992	12.15532	0.0000
C(13)	-0.944760	0.008565	-110.3085	0.0000
snipsnT-DIST. DOF	2.178149	0.087362	24.93257	0.0000

R-squared	-0.003315	Mean dependent var	0.002428
Adjusted R-squared	-0.031578	S.D. dependent var	0.018530
S.E. of regression	0.018820	Akaike info criterion	-8.596238
Sum squared resid	0.075441	Schwarz criterion	-8.380280
Log likelihood	959.5862	Hannan-Quinn criter.	-8.509029
Durbin-Watson stat	1.912293		

Inverted AR Roots	.97	-.32-.48i	-.32+.48i
Inverted MA Roots	.97	-.32-.48i	-.32+.48i

Figure A16: ARCH test for the squared residuals of ARIMA(3,1,3)-EGARCH(3,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.016263	Prob. F(12,177)	1.0000
Obs*R-squared	0.209255	Prob. Chi-Square(12)	1.0000

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	56.69839	38.98841	1.454237	0.1477
WGT_RESID^2(-1)	-0.011650	0.075159	-0.155001	0.8770
WGT_RESID^2(-2)	-0.011570	0.075159	-0.153941	0.8778
WGT_RESID^2(-3)	-0.009519	0.075159	-0.126651	0.8994
WGT_RESID^2(-4)	-0.011384	0.075159	-0.151472	0.8798
WGT_RESID^2(-5)	-0.026614	0.188927	-0.140868	0.8881
WGT_RESID^2(-6)	-0.021274	0.188926	-0.112606	0.9105
WGT_RESID^2(-7)	-0.023362	0.188922	-0.123657	0.9017
WGT_RESID^2(-8)	-0.024754	0.188923	-0.131025	0.8959
WGT_RESID^2(-9)	-0.021941	0.188924	-0.116137	0.9077
WGT_RESID^2(-10)	-0.023224	0.188921	-0.122930	0.9023
WGT_RESID^2(-11)	-0.023569	0.188922	-0.124757	0.9009
WGT_RESID^2(-12)	-0.025756	0.188921	-0.136333	0.8917

R-squared	0.001101	Mean dependent var	50.98876
Adjusted R-squared	-0.066621	S.D. dependent var	490.7346
S.E. of regression	506.8176	Akaike info criterion	15.36015
Sum squared resid	45464943	Schwarz criterion	15.58231
Log likelihood	-1446.214	Hannan-Quinn criter.	15.45014
F-statistic	0.016263	Durbin-Watson stat	2.000281
Prob(F-statistic)	1.000000		

Figure A17: Correlogram of Standardized Residuals Squared ARIMA(3,1,3)-EGARCH(3,1)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|. |	1	-0.030	-0.030	0.1854	0.667
.|. |	.|. |	2	-0.008	-0.009	0.1982	0.906
.|* |	.|* |	3	0.153	0.153	5.0616	0.167
.|. |	.|. |	4	-0.027	-0.019	5.2169	0.266
.|. |	.|. |	5	-0.018	-0.018	5.2848	0.382
.|. |	.|. |	6	0.066	0.043	6.2065	0.400
.|. |	.|. |	7	-0.014	-0.005	6.2506	0.511
.|. |	.|. |	8	0.048	0.054	6.7368	0.565
.|. |	.|. |	9	0.046	0.033	7.1964	0.617
.|. |	.|. |	10	-0.019	-0.013	7.2776	0.699
.|. |	.|. |	11	-0.019	-0.034	7.3553	0.770
.|* |	.|* |	12	0.154	0.145	12.521	0.405
.|. |	.|. |	13	-0.015	0.001	12.574	0.481
.|. |	.|. |	14	-0.013	-0.011	12.610	0.557
.|. |	*|. |	15	-0.015	-0.067	12.662	0.628
.|. |	.|. |	16	-0.019	-0.014	12.743	0.691
.|. |	.|. |	17	-0.020	-0.013	12.831	0.747
.|. |	.|. |	18	0.034	0.032	13.089	0.786
.|. |	.|. |	19	-0.019	-0.010	13.166	0.830
.|* |	.|* |	20	0.174	0.171	20.020	0.457
.|. |	.|. |	21	-0.015	-0.025	20.068	0.517
.|. |	.|. |	22	-0.019	-0.008	20.149	0.574
.|. |	.|. |	23	-0.020	-0.064	20.245	0.627
.|. |	.|. |	24	0.014	0.006	20.290	0.680
.|. |	.|. |	25	-0.021	-0.010	20.392	0.726
.|. |	.|. |	26	0.010	0.007	20.417	0.771
.|. |	.|. |	27	-0.018	-0.012	20.495	0.809
.|. |	.|. |	28	-0.020	-0.029	20.593	0.842
.|. |	.|. |	29	-0.021	-0.024	20.693	0.870
.|. |	.|. |	30	-0.008	-0.010	20.707	0.897
.|. |	.|. |	31	-0.021	0.000	20.809	0.917
.|. |	.|. |	32	0.025	-0.025	20.956	0.933
.|. |	.|. |	33	-0.019	-0.010	21.047	0.947
.|. |	.|. |	34	-0.022	-0.013	21.161	0.958
.|. |	.|. |	35	-0.023	0.007	21.286	0.967
.|. |	.|. |	36	-0.017	-0.014	21.356	0.975