GLASHOW-WEINBERG-SALAM MODEL FOR ELECTROWEAK INTERACTIONS OF LEPTONS WITH SIMPLE APPLICATIONS

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Abstract

We have reviewed the GWS theory of weak and electromagnetic interaction among leptons and study their property. Thus electroweak interaction emits intermediate vector bosons (W and Z), just as photon is emitted in the case of QED. Intermediate vector bosons have to be massive so that we have reviewed their masses $M_W = \frac{1}{2}g\nu$ and $M_Z = \frac{1}{2}\nu\sqrt{g^2 + g'^2}$ in the frame work of Higgs mechanism which is applied to break unified $SU(2) \times U(1)$ symmetry spontanously. The Lagrangian that describes the complete interaction and the decay of W^- boson is finally presented in this thesis. .

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Chapter 1

INTRODUCTION

This thesis is devoted to study the GWS model for electroweak interaction of leptons that was developed by Glashow,Weinberg and Salam. It is minimal model of electroweak interaction with respect to the number of required fields and it is characterized by the representation of the weak interaction universality(the interaction at which all the weak currents couple with the same strength) as a symmetry under weak isospin $S(U)_W$ and weak $U(1)$ hypercharge transformation[15] .The electroweak theory that were able to unify the weak interaction with the electromagnetic interaction by combining the $SU(2)$ weak isospin group with the $U(1)$ singlet that represent QED,the product is $SU(2)\times U(1)$ which is the conclusion of Glashow,Weinberg and Salam in 1960's. In order to generate the mass of the weak gauge bosons or weak isovector triplet W^+, W^-, Z^0[4], the Higgs mechanism for the spontanous symmetry breaking of this unified ($S(U)_W \times U(1)_W$) symmetry is used in such a way that the electromagnetic gauge group $U(1)_{EM}$ remains conserved as residual symmetry[1]. The higgs sector of the GWS model is needed to give mass to the gauge boson is described in this thesis.

1.1 The standard model

The Standard Model unifies the strong force of QCD and the electroweak force which results from the unification of QED and weak interaction theory.A quantitative description of the Standard Model can involve quantum field theory, group theory.Our objective in this thesis is to give a descriptive account of the development of the Standard Model with emphasis on the unification framework and directions it may lead for future theoretical extensions.

1.2 Constituents and Mediators of the Standard Model

According to the Standard Model, all matter in the universe is made up of a dozen fermions (and their antiparticles) six quarks (u, d, s, c, t, b) and six leptons $(e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau)$ that come paired up as doublets of three generations. For each quark flavor there are three colors.All the quarks and lepton are fermions.In particular, those with spin 1 are called gauge bosons. The Standard Model requires the W^+, W^-, Z, and γ spin-one gauge bosons to mediate the electroweak force and eight gluons to mediated the strong or color force.The constituent particles of the Standard Model and some of their properties are listed in the chart bellow. The gauge bosons and the forces they mediate are listed in Table 1.1. Gravity is included for completeness even though it is not part of the Standard Model. As shown in the chart bellow,the matter particles consist of three

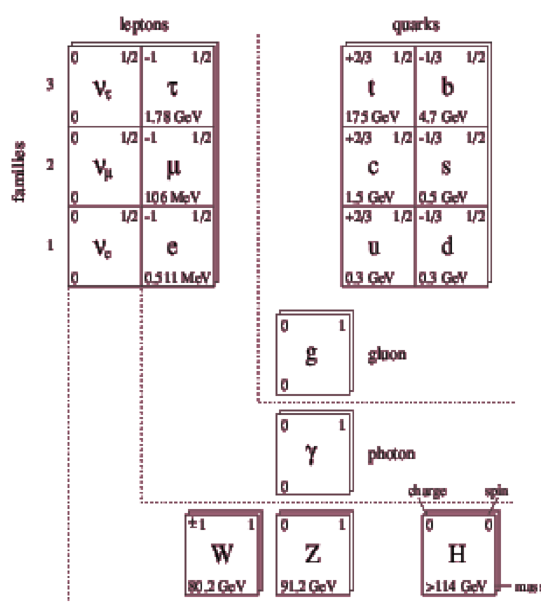

Figure 1.1: "This chart shows the particles of the standard model".

families of fermions (quarks and leptons). The gauge particles are arranged such as to show the hierarchy of interactions (by means of the dotted lines): eight gluons, differing in the color quantum numbers, mediate the strong interaction and couple only to quarks; the photon is responsible for the electromagnetic interaction and couples to quarks and charged leptons;the W and Z particles mediate the weak interaction and couple to all fermions. Bosons of the same row also couple among themselves and the charged Ws couple to the photon in addition. By definition the Higgs particle (shown in the lower right corner) couples directly only to particles which have a nonzero mass[4]".

interaction	exchange boson	spin	mass
strong	8 gluons (g)	1	0
electromagnetic	γ	1	0
weak	$W^{\pm}$	1	80.4 GeV
	Z	1	91.2 GeV
gravity	graviton?	2	0
	Higgs	0	?

Table 1.1: Comparison of Fundamental Forces.

1.3 Weak Isospin and W Boson

The grouping of leptons into pairs of three generations suggests that the leptons of each generation are weak isospin doublets in analougy with the fact that proton and neutron are isospin doublets that are indistinguishable by the strong force. An isospin group with two members satisfies a symmetry representation called $SU(2)$.Consequently, for the weakly interacting isospin group $SU(2)$ there must be three gauge bosons. These can be identified as the weak isotriplet vector W^+, W^- and Z^0 bosons.

Chapter 2

Gauge Theories:QED And Yang Mills Theory

2.1 Gauge Theory

This section introduces the gauge theories that are now believed to underlie all interaction of elementary particles. We begin with the principle of Lagrangian formulation,global and local gauge invariance of the Lagrangian.

2.2 Local Gauge Invariance

Notice that the Dirac Lagrangian

$$L = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi \tag{2.2.1}$$

is invariant unnder the transformation

$$\psi \to e^{i\theta}\psi \tag{2.2.2}$$

This is "global gauge transformation" (where θ is any real number), for then $\bar{\psi} \to \bar{\psi}e^{-i\theta}$ and in the combination $\bar{\psi}\psi$ the exponential factors cancel out. We call (2.2.2) a global gauge transformation. What if the phase factor is diffferent at different space-time points, that is what if θ is a function of x^{μ}.

$$\psi \to e^{i\theta(x)}\psi \tag{2.2.3}$$

(Local gauge transformation Lagrangian is not invariant under such a local transformation,we pick out extra term from the derivative of θ

$$\partial_\mu(e^{i\theta}\psi) = i(\partial_\mu\theta)e^{i\theta}\psi + e^{i\theta}\partial_\mu\psi$$

So that

$$L \to L - (\partial_\mu\theta)\bar{\psi}\gamma^\mu\psi \tag{2.2.4}$$

it is convenient to pull out the factor of $-q$ out of θ, by letting $\lambda(x) = -\frac{\theta(x)}{q}$, where q is the charge of the particle involved, in terms of λ,then

$$L \to L + (q\bar{\psi}\gamma^\mu\psi)\partial_\mu\lambda \tag{2.2.5}$$

under the local gauge transformation

$$\psi \to e^{iq\lambda(x)}\psi \tag{2.2.6}$$

The most important point comes when we demand that the complete Lagrangian be invariant under local gauge transformation, since the free Lagrangian (2.2.1) is not locally invariant, we are obliged to add something, in order to soak up the extra term in (2.2.5). Suppose

$$L = [i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi]\psi - (q\bar{\psi}\gamma^\mu\psi)A_\mu \tag{2.2.7}$$

where A_μ some new field called "gauge field" which transforms under local gauge transformation according to the rule.

$$A_\mu \to A_\mu + \partial_\mu\lambda \tag{2.2.8}$$

This new improved Lagrangian is now invariant under local gauge transformation, the price we have to pay is introduction of new vector field that couples to ψ through the last term in equuation (2.2.7) In generalization, If in the original free Lagrangian we replace every derivative (∂_μ) by so called " covariant derivative"

$$D_\mu \equiv \partial_\mu + iqA_\mu \tag{2.2.9}$$

the transformation (2.2.8) will cancel the offending term and then we can obtain that

$$D_\mu \to e^{iq\lambda} D_\mu \psi \tag{2.2.10}$$

and the invariance of Lagrangian is restored.The substitution of D_μ for ∂_μ, then is a simple device for converting a globally invariant Lagrangian in to locally invariant one, we call this "minimal coupling rule " this is what we used to generate the extra term in (2.2.7). But the covariant derivative introduces a new vector field A_μ which requires its own free Lagrangian. Later on in 1954 Yang and Mills applied the same strategy (insisting that a global invariance hold locally) to the group $SU(2)$, and later on again the idea was extended to color $SU(3)$ producing chromodynamics.In the standard model all of the fundamental interactions are generated in this way.

2.3 YANG-MILLS THEORY

Supposing that we have two spin $\frac{1}{2}$ fields, ψ_1 and ψ_2. The Lagrangian in the absence of any interaction, is

$$L = [i\bar{\psi}_1\gamma^\mu\psi_1\partial_\mu\psi_1 - m_1\bar{\psi}_1\psi_1] + [i\bar{\psi}_2\gamma^\mu\partial_\mu\psi_2 - m_2\bar{\psi}_2\psi_2] \tag{2.3.1}$$

Is just the sum of two Dirac Lagrangians.We can write this in more compact form by combining ψ_1 and ψ_2 in to a two component column vector:

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2.3.2}$$

ψ_1 and ψ_2 are four component Dirac spinors, and it is preferable to use the double-index notations:$\psi_{\alpha,i}$,where $\alpha = 1, 2$ identifies the particle and $i = 1, 2, 3, 4$ labels the spinor components.However presently we are concerned with the particle index. The adjoint spinor is then

$$\bar{\psi} = \begin{pmatrix} \bar{\psi}_1 & \bar{\psi}_2 \end{pmatrix} \tag{2.3.3}$$

And the Lagragian becomes

$$L = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \bar{\psi}M\psi \tag{2.3.4}$$

Where $M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$ is the mass matrix.If two masses are equal, $m_1 = m_2 = m$ then the Lagrangian be

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi \tag{2.3.5}$$

This is similar to that of single particle Dirac Lagrangian. ψ is now a two-element column vector, and L admits a more general global invariance than before

$$\psi \to U\psi \tag{2.3.6}$$

Where U is 2×2 unitary matrix $U^+U = 1$ for this transformation

$$\bar{\psi} \to \bar{\psi}U^+ \tag{2.3.7}$$

and hence the combination $\bar{\psi}\psi$ is invariant. So any unitary matrix is written in the form

$$U = e^{iH} \tag{2.3.8}$$

where H is hermitian $(H^+ = H)$. The most general hermitian 2×2 matrix can be expressed in terms of four real numbers,a_1, a_2, a_3 and θ

$$H = \theta 1 + \tau.a \tag{2.3.9}$$

The dot product is convenient short hand for $\tau_1 a_1 + \tau_2 a_2 + \tau_3 a_3$, where τ_1, τ_2, τ_3 are pauli matrices and 1 is 2×2 unit matrix. Thus any unitary 2×2 matrix can be expressed as a product $U = e^{i\theta} e^{i\tau.a}$ Now we shall concentrate on the transformation of the form

$$\psi \to e^{i\tau.a}\psi \tag{2.3.10}$$

[global $SU(2)$ transformation]. Generalizing the terminiology we say that the Lagrangian (2.3.5) invariant under global $SU(2)$ gauge transformation. What Yang and Mills did was to promote this global invariance to the status of a local invariance.To do this the first step is letting the parameter a be function of x^μ, so let $\lambda(x) \equiv -\frac{a(x)}{q}$ where q is the coupling constant analogous to electric charge.

$$\psi \to S\psi, \tag{2.3.11}$$

where $S \equiv e^{-iq\tau.\lambda(x)}$ is local $SU(2)$ transformation. As it stands L is not invariant under such a transformation,for the derivative picks up an extra term:

$$\partial_\mu \psi \to S\partial_\mu \psi + (\partial_\mu S)\psi \tag{2.3.12}$$

Now we replace the (∂_μ) in L by a "covariant derivative" in order to ensure the local invariance, by taking in to account the structure of the equation:

$$D_\mu \equiv \partial_\mu + iq\tau.A_\mu \tag{2.3.13}$$

and assigne to the gauge field A_μ in this time it takes three of them, a transformation rules such that

$$D_\mu \psi \to S(D_\mu \psi) \tag{2.3.14}$$

For the Lagrangian (2.3.5) will clearly be invariant. It is possible to deduce the transformation for A_μ from (2.3.14), that $A_\mu \to A'_\mu$, where A'_μ is given by

$$\tau.A'_\mu = S(\tau.A_\mu)S^{-1} + \frac{i}{q}(\partial_\mu S)S^{-1} \tag{2.3.15}$$

Transformation of non-Abelian field under local gauge transformation. It is straight forward to show. But S and S^{-1} in the first term can not be brought together, because they do not commute with $\tau.A_\mu$.

2.4 The Gauge-Covariant Formulation Of GWS Theory:Weak Isospin And Weak Hypercharge

In the theory of weak interaction the leptons are divided into left-handed isodoublets ($T = \frac{1}{2}$,$T_3 = \pm\frac{1}{2}$)

$$L_l = \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L = \frac{1-\gamma_5}{2}\begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}, \tag{2.4.1}$$

and right handed isosinglets $T = 0$ is

$$R_l = (\psi_l)_R = \frac{1-\gamma_5}{2}\psi_l \tag{2.4.2}$$

Where l runs over e, μ, τ,starting from (2.4.1) and (2.4.2) we demand the invariance of the theory with respect to gauge transformation, that is,

$$\begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L \rightarrow \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L^{'} = e^{ia(x).\widehat{T}} \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L$$

$$= e^{ia(x).\frac{\widehat{\tau}}{2}} \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}_L \equiv \widehat{U}_2 L_l \tag{2.4.3}$$

$$(\psi_l)_R \rightarrow (\psi_l)_R^{'} = e^{ia(x)\widehat{Y}} (\psi_l)_R \equiv \widehat{U}_1 R_l \tag{2.4.4}$$

Here $\widehat{Y} = Y$ is the generator (constant) of $U(1)$ group. By the relation

$$\bar{L}_l \gamma^\mu i D_\mu L_l + \bar{R}_l \gamma^\mu i D_\mu R_l \tag{2.4.5}$$

for the kinetic energy term of the Lagrangian density. We can find the gauge invariance with respect to (2.4.3) if we replace ∂_μ by the covariant derivative $\widehat{D}_\mu$,where

$$\widehat{D}_\mu = \partial_\mu - ig\widehat{T}.A_\mu - i\frac{g^{'}}{2}\widehat{Y}B_\mu \tag{2.4.6}$$

where $\widehat{T}_1, \widehat{T}_2, \widehat{T}_3$ are three generators of the $SU(2)$ group, $\widehat{Y}$ is the generator of $U(1)$ group. In order to ensure the gauge invariance,we have to introduce gauge transformation

$$A_\mu.\widehat{T} \rightarrow U_2 A_\mu.\widehat{T} U_2^{-1} + \frac{i}{g} U_2(\partial_\mu U_2^{-1}), B_\mu \rightarrow B_\mu^{'} = B_\mu + \frac{2i}{g} U_1(\partial_\mu U_1^{-1}) \tag{2.4.7}$$

Here U_1 and $\widehat{U}_2$ are $U(1)$ and $U(2)$ transformation respectively.The new degree of freedom connected to gauge field A_μ and B_μ have to be supplemented by gauge invariant kinetic energy term in the Lagrangian density. This is done by using field strength tensors.

$$\widehat{F}_{\mu\nu} = F_{\mu\nu}.\widehat{T} = \widehat{D}_\mu(A_\nu.\widehat{T}) - \widehat{D}_\nu(A_\mu.\widehat{T})$$

$$= (\partial A_\mu - \partial_\mu A_\mu + gA_\mu \times A_\nu).\widehat{T} \Rightarrow F_{\mu\nu} = \partial A_\mu - \partial_\mu A_\mu + gA_\mu \times A_\nu \tag{2.4.8}$$

And

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.4.9}$$

with gauge-transformation properties

$$\widehat{F}_{\mu\nu} = \widehat{F}'_{\mu\nu} = \widehat{U}_2\widehat{F}_{\mu\nu}U_2^{-1} = \widehat{U}_2(F_{\mu\nu}.\widehat{T})\widehat{U}_2^{-1}, B_{\mu\nu} \to B'_{\mu\nu} = B_{\mu\nu} \tag{2.4.10}$$

respectively. The gauge invariant kinetic energies of the gauge fields A_μ and B_μ are given by

$$L'_A = -\frac{1}{2}Tr\{(F_{\mu\nu.\widehat{T}})(F^{\mu\nu}.\widehat{T})\} = -\frac{1}{4}F_{\mu\nu}.F^{\mu\nu} \tag{2.4.11}$$

and

$$L'_B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.4.12}$$

In summarizing the result

$$L' = \bar{L}_l\gamma^\mu i\widehat{D}_\mu L_l + \bar{R}_l\gamma^\mu i\widehat{D}_\mu R_l - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.4.13}$$

In explicite representation

$$L' = i\bar{L}_l\gamma^\mu[\partial_\mu - ig\widehat{T}.A_\mu - i\frac{g'}{2}\widehat{Y}B_\mu]L_l + i\bar{R}_l\gamma^\mu[\partial_\mu - ig\widehat{T}.A_\mu - i\frac{g'}{2}\widehat{Y}B_\mu]R_l$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.4.14}$$

With the help of table 1.2, we can easily verify by taking appropriate quantum numbers

Fermion	T	T_3	Y	Q
ν_e, ν_μ, ν_τ	1/2	1/2	−1	0
e_L, μ_L, τ_L	1/2	−1/2	−1	−1
e_R, μ_R, τ_R	0	0	−2	−1
u_L, c_L	1/2	1/2	1/3	2/3
$(d_C)_L, (s_C)_L$	1/2	−1/2	1/3	−1/3
u_R, c_R	0	0	4/3	2/3
$(d_C)_R, (s_C)_R$	0	0	−2/3	−1/3

Table 2.1: Quantum numbers of weak isospin and hypercharge of leptons and quarks

T and Y for L_e and R_e respectively, At this stage the gauge fields still massless,However some of them acquire a mass, since we know a vector bosons must be massive. We shall achieve this in the next chapter by applying the Higgs mechanism.

2.5 The Standard model is a Gauge Theory

At present the standard model of elementary particle (leptons and quarks) theory describes three of the four known forces of nature, That are; electromagnetic, weak and strong interaction,but not gravitation.

It is based on quantum fields that are generalization of Yang-Mills gauge fields. Yang-Mills theories are based on the principle of local gauge symmetry,that are the symmetry to be present in the physical system already built the Lagrangian density of the system. The electroweak part of the standard model is based on the gauge group $SU(2)_L \times U(1)_Y$ which consists of two sets of gauge transformations with gauge coupling g and $g^{'}$. $U(1)_Y$ is the gauge group of the weak hypercharge,and $SU(2)_L$ is the gauge group of the weak isospin, where the index L reminds one that only the left handed fields participate in the weak interaction (parity violation). The gauge fields associated with the gauge groups are identified with the mediators of the interactions between matter particles. These objects are called gauge bosons and in order not to violate gauge invariance they should all be massless in contradiction to what is required by the experiment. The way massive gauge bosons can be introduced into a gauge theory without spoiling the gauge symmetry will be explained in the introduction section for the simple case of the U(1) gauge group.This can then be easily extended to the $SU(2) \times U(1)$ group.This group is the gauge group of GWS theory, because it is a correct theory that describes the electroweak interaction up the currently tested energies. Electroweak interaction among particles, leptons and quarks is a theory of GWS which is recognized as one of the pillars of the standard model[15].

Chapter 3

GLASHOW-WEINBERGE-SALAM MODEL FOR LEPTONS

We can formulate the GWS theory of weak and electromagnetic interaction among leptons and study their properties.The starting point is as follows. There exist chagrged and neutral weak currents, the charged current contains only couplings between left-handed leptons, the bosons W^+, W^- and Z^0 mediating the weak interaction must be very massive, massles bosons which recieve masses through the Higgs mechanism. In order to fulfill these we introduce two vector fields, one isospin triplet $A^i_\mu (i = 1, 2, 3)$ and one singlet B_μ which should finally result as fields of physical particles W^+, W^-, Z^0 and the photon through the symmetry breaking induced by the Higgs mechanism which is part of the discussion in the next section. The leptonic fields have to be distinguished according to their helicity. Every fermion generation (e, μ, τ) contain two related left- handed lepptons.These form an "isospin" doublet of left- handed leptons, denoted by $L_i (i = e, \mu, \tau)$:

$$L_e = \frac{1-\gamma_5}{2}\begin{pmatrix}\psi_{\nu_e}\\ \psi_e\end{pmatrix}, L_\mu = \frac{1-\gamma_5}{2}\begin{pmatrix}\psi_{\nu_\mu}\\ \psi_\mu\end{pmatrix}, L_\tau = \frac{1-\gamma_5}{2}\begin{pmatrix}\psi_{\nu_\tau}\\ \psi_\tau\end{pmatrix} \tag{3.0.1}$$

There are also right-handed components of the charged massive leptons. A right-handed neutrino does not exist, therefore right-handed leptons can be represented by singlets:

$$R_e = (\frac{1+\gamma_5}{2})\psi_e, R_\mu = (\frac{1+\gamma_5}{2})\psi_\mu, R_\tau = (\frac{1+\gamma_5}{2})\psi_\tau \tag{3.0.2}$$

Now we consider various current successively, that is charged,neutral and electromagnetic,here we are showing only the first generation, and we can treat the others in the same manner. In our calculation we are going to use the property of γ_5 anticommuting with every γ^α and we further utilize the property that $(\gamma_5)^2 = 1$. This yields for example, $(1-\gamma_5)^2 = 2(1-\gamma_5)$ and $\gamma^\alpha(1-\gamma_5) = \frac{1}{2}\gamma^\alpha(1-\gamma_5)^2 = \frac{1}{2}(1+\gamma_5)\gamma^\alpha(1-\gamma_5) = 2\frac{1+\gamma_5}{2}\gamma^\alpha\frac{1-\gamma_5}{2}$ Finally we apply the relation (3.0.1) that is

$$\bar{L}_e = \begin{pmatrix} \bar{\psi}_{\nu_e} & \bar{\psi}_e \end{pmatrix} \frac{1+\gamma_5}{2} \tag{3.0.3}$$

The charged currents have the form

$$\begin{aligned} J_-^{(e)\alpha} &= \bar{\psi}_e\gamma^\alpha(1-\gamma_5)\psi_{\nu_e} = 2\bar{\psi}_e\frac{1+\gamma_5}{2}\gamma^\alpha\frac{1-\gamma_5}{2}\psi_{\nu_e} \\ &= 2\bar{L}_e\gamma^\alpha \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} L_e = 2\bar{L}_e\gamma^\alpha\widehat{T}_- L_e \end{aligned} \tag{3.0.4}$$

$$J_+^{(e)\alpha} \equiv (J_-^{(e)\alpha})^\dagger = 2\bar{L}_e\gamma^\alpha\widehat{T}_+ L_e \tag{3.0.5}$$

with $\widehat{T}_\pm = \widehat{T}_1 \pm i\widehat{T}_2$ and $(\widehat{T}_-)^+ = \widehat{T}_+$. The electromagnetic current exists only for e^-, μ^-, τ^-. so it is given by

$$\begin{aligned} J_{EM}^{(e)\alpha} &= \bar{\psi}_e\gamma^\alpha\psi_e = \frac{1}{2}\bar{\psi}_e\gamma^\alpha(1-\gamma_5)\psi_e + \frac{1}{2}\bar{\psi}\gamma^\alpha(1+\gamma_5)\psi_e \\ &= \bar{\psi}_e\frac{1+\gamma_5}{2}\gamma^\alpha\frac{1-\gamma_5}{2}\psi_e + \bar{\psi}_e\frac{1-\gamma_5}{2}\gamma^\alpha\frac{1+\gamma_5}{2}\psi_e \\ &= \bar{L}_e\gamma^\alpha \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} L_e + \bar{R}_e\gamma^\alpha R_e \\ &= \bar{L}_e\gamma^\alpha(\frac{1}{2} - \widehat{T}_3)L_e + \bar{R}_e\gamma^\alpha R_e \end{aligned} \tag{3.0.6}$$

This electromagnetic current splits up in to a part $-\bar{L}_e\gamma^\mu T_3 L_e$ belonging to isotriplet. and the other part $\frac{1}{2}\bar{L}_e\gamma^\alpha L_e + \bar{R}_e\gamma^\alpha R_e$ representing an isosinglet current We may expect the neutral current $J_0^{(e)\alpha}$ to have similar form , so that we can rearrange the currents in to isotriplets and isovector gauge field A_μ,

$$\bar{L}_e\gamma^\alpha T L_e \tag{3.0.7}$$

and an iso singlet to be associated with the gauge field B_μ,

$$\frac{1}{2}\bar{L}_e\gamma_\alpha L_e + \bar{R}_e\gamma_\alpha R_e \tag{3.0.8}$$

These currents are minimally coupled to the the corresponding gauge fields:

$$L_{int}^{(e)} = g(\bar{L}_e\gamma^\alpha T L_e).A_\alpha - g^{'}[\frac{1}{2}(\bar{L}_e\gamma^\alpha L_e) + (\bar{R}_e\gamma^\alpha R_e)]B_\alpha \tag{3.0.9}$$

This equation characterizes the structure of ineraction as demanded by our general group-theoretical consideration concerning the two gauge fields A_α and B_α. The real(physical) photon does not couple to the singlet current (3.0.8), but to J_{EM}^α (3.0.6). Therefore it has to be represented by a mixture of B_μ and A_μ^3 fields.

$$A_\mu = cos\theta B_\mu + sin\theta A_\mu^3 \tag{3.0.10}$$

the combination is orthogonal to A_μ,

$$Z_\mu = -sin\theta B_\mu + cos\theta A_\mu^3 \tag{3.0.11}$$

Which describe the (physical) neutral intermediate boson of weak interactions. The inverse of (3.0.10) and (3.0.11) are

$$B_\mu = cos\theta A_\mu - sin\theta Z_\mu, A_\mu^3 = sin\theta B_\mu + cos\theta Z_\mu \tag{3.0.12}$$

The mixing angle θ is called the Weinberge angle, finally we can write

$$W_\mu^{(\pm)} = \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2) \tag{3.0.13}$$

where $A_\mu^1 = W_\mu^1$ and $A_\mu^2 = W_\mu^2$, the field W_μ describes an incoming negative or an out going positive W boson. by inserting (3.0.12) and (3.0.13) in to Lagrangian density $L_{int}^{(e)}$, we can easily obtain

$$\begin{aligned} L_{int}^{(e)} &= \frac{g}{\sqrt{2}}\bar{L}_e\gamma^\alpha(\widehat{T}_- W_\alpha^{(-)} + \widehat{T}_+ W_\alpha^{(+)})L_e \\ &+[gcos\theta\bar{L}_e\gamma^\alpha\widehat{T}_3 L_e + g^{'}sin\theta(\frac{1}{2}\bar{L}_e\gamma^\alpha L_e + \bar{R}_e\gamma^\alpha R_e)]Z_\alpha \\ &+[-g^{'}cos\theta(\frac{1}{2}\bar{L}e\gamma^\alpha L_e + \bar{R}_e\gamma^\alpha R_e) + gsin\theta\bar{L}_e\gamma^\alpha\widehat{T}_3 L_e]A_\alpha \\ &\equiv \frac{g}{2\sqrt{2}}(J_-^{(e)\alpha}W_\alpha^{(-)} + J_+^{(e)\alpha}W_\alpha^{(+)} + J_0^{(e)\alpha}Z_\alpha) - eJ_{EM}^{(e)\alpha}A_\alpha \end{aligned} \tag{3.0.14}$$

with

$$J_{-}^{(e)\alpha} = 2\bar{L}_e\gamma^\alpha\widehat{T}_{-}L_e, J_{+}^{(e)\alpha} = 2\bar{L}_e\gamma^\alpha\widehat{T}_{+}L_e,$$

$$J_0^{(e)\alpha} = 2\sqrt{2}[cos\bar{L}_e\gamma^\alpha\widehat{T}L_e + \frac{g^{'}}{g}sin(\frac{1}{2}\bar{L}_e\gamma^\alpha L_e + \bar{R}_e\gamma^e R_e)],$$

$$-eJ_{EM}^{(e)\alpha} = -e[\bar{L}_e\gamma^\alpha(\frac{1}{2} - \widehat{T}_3)L_e + \bar{R}_e\gamma^\alpha R_e]$$

$$= -[\bar{L}_e\gamma^\alpha(\frac{g^{'}cos\theta}{2} - gsin\theta\widehat{T}_3)L_e + g^{'}cos\theta\bar{R}_e\gamma^\alpha R_e\alpha]$$

Equation (3.0.14) contains the explicite form of the neutral weak current in the frame work of GSW model. Using the relation $e = gsin\theta = g^{'}cos\theta$ we can write

$$J_0^{(\alpha)\alpha} = 2\sqrt{2}[cos\theta\bar{L}_e\gamma^\alpha\widehat{T}_3L_e + \frac{g^{'}}{g}sin\theta(\frac{1}{2}\bar{L}_e\gamma^\alpha L_e + \bar{R}_e\gamma^\alpha R_e)]$$

$$= \frac{2\sqrt{2}}{cos\theta}[\bar{L}_e\gamma^\alpha(cos^2\widehat{T}_3 - \frac{1}{2}sin^2\theta)L_e + sin^2\theta\bar{R}_e\gamma^\alpha R_e]$$

$$= \frac{\sqrt{2}}{cos\theta}[\bar{L}_e\gamma^\alpha \begin{pmatrix} 1 & 0 \\ 0 & -cos2\theta \end{pmatrix} L_e + 2sin^2\theta\bar{R}_e\gamma^\alpha R_e] \tag{3.0.15}$$

We can write interms of left-handed isodoublets and right-handed isosinglets spinors, so that L_l and R_l have been expressed in terms of leptons and neutrino spinors ψ_l and ψ_{ν_l} where $l = e^-, \mu^-, \tau^-$ then the neutral weak current then becomes,

$$J_0^{(e)\alpha} = \frac{\sqrt{2}}{cos\theta}[\bar{\psi}_{\nu_l}\gamma^\alpha(1-\gamma_5)\psi_{\nu_l} - (1-2sin^2\theta)\bar{\psi}l\gamma^\alpha(1-\gamma_5)\psi_l + 2sin^2\theta\bar{\psi}_l\gamma^\alpha(1+\gamma_5)\psi_l]$$

$$= (\sqrt{2}cos\theta)^{-1}[\bar{\psi}_{\nu l}\gamma^\alpha(1-\gamma_5)\psi_{\nu l} - (1-sin^2\theta)\bar{\psi}_l\gamma^\alpha(1-\gamma_5)\psi_l + 2sin^2\theta\bar{\psi}_l(1+\gamma_5)\psi_l] \tag{3.0.16}$$

In the last expressions we have generated the three weak currents,two of them are charged and one neutral, so that they are making coupled with two vector bosons of charged current of weak interaction and one neutral gauge boson. since all the gauge bosons of these weak interactions that mediating the interactions had to be massive through Higgs mechanism which is points of discussion in the following section.

3.1 The Higgs Mechanism

In particle mechanism,this mechanism is a process in which gauge bosons in a gauge theory can aquire non-vanishing masses through absorption of Goldstone bosons arising in spontanous symmetry breaking. The implementation of the mechanism adds an extra Higgs field to the gauge theory. The spontanous symmetry breaking of the underlying local symmetry triggers convertion of components of this Higgs field to Goldstone bosons which interact with some of the other fields in the theory, so as to produce mass terms for some of the gauge bosons.This mechanism may also leave behind elementary scalars (spin-0) particles,known as Higgs bosons. In standard model,the phrase "Higgs mechanism" refers specifically to the generation of masses for the $W^{\pm}$ and Z^0 weak gauge bosons through electro weak symmetry breaking. The mechanism is as follows. Let

$$\phi(x) = \frac{(\phi_1(x) + i\phi_2(x))}{\sqrt{2}} \tag{3.1.1}$$

be a complex scalar field and $A_\mu(x)$ be the massless gauge field of the $U(1)$ gauge group,then the Lagrangian of the theory is taken to be

$$L = (D_\mu\phi)^*(D_\mu\phi) - V(\mid \phi \mid) - \frac{1}{4}F_{\mu\nu}F^{\nu\mu\nu} \tag{3.1.2}$$

Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the antisymmetric tensor of the gauge field and $D_\mu = \partial_\mu - igA_\mu$ is the covariant derivative that ensures invariance of the Lagrangian under the local gauge transformation.

$$\phi \rightarrow \phi^{'} = e^{ig\chi(x)}\phi$$

$$A_\mu \rightarrow A_\mu^{'} = A_\mu - \partial_\mu\chi(x) \tag{3.1.3}$$

An explicite mass term of the form $\frac{m^2}{2}.A_\mu A^\mu$, on the other hand would break local gauge invariance and is there fore not allowed in $\pounds$. The potential V in (1.5.2) has the the form

$$V(\mid \phi \mid) = -\mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2, \tag{3.1.4}$$

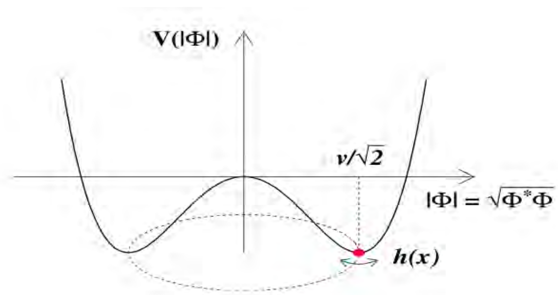

Figure 3.1: A graphical illustration of so called Higgs potential.For a complex scalar field ϕ there exists an infinite number of minima of a potential V,as indicated by the dashed ellipse.The real field $h(x)$ can be expressed as an excitation of ϕ around the value at its minimum.

where the parameter $-\mu^2$ and λ are chosen to be positive(this choice result in the potential to have the form shown in figure 1.2). To find the ground state of the field ϕ one has too calculate the minimum of the potential. The absolute value of ϕ at the minimum is $\mid \phi \mid = \sqrt{\frac{\mu^2}{2\lambda}} =: \frac{\nu}{\sqrt{2}}$. The constant $\frac{\nu}{\sqrt{2}}$ is called the vacuume expectation value of the field ϕ. Since ϕ is complex the ground state is degenerate and the set of minima is described by $\phi_1^2 + \phi_2^2 = \nu^2$. One of these posible ground states has to be selected as the ground state (vacuume) of the theory. Without loss of generality it will be taken as the real field $\phi_{g.s} := \frac{\nu}{\sqrt{2}}$. Whereas the Lagrangian of the theory is invariant under the gauge transformation, the ground state is not. This is commonly referred to as spontaneous symmetry breaking. In the next step the fields are investigated in the vicinity of the ground state. Let the field ϕ be expanded as follows:

$$\phi(x) = \phi_{g.s} + \frac{1}{\sqrt{2}}[h(x) + i\xi(x)]. \tag{3.1.5}$$

When this is substituted in to (1.5.2) we can obtain the following expression

$$L = \frac{1}{2}(\partial_\mu - igA_\mu)(\nu + h - i\xi)(\partial^\mu + igA^\mu)(\nu + h + i\xi) - V(h,\xi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{3.1.6}$$

With the potential

$$V(h,\xi) = \frac{\mu^2}{2}[(\nu + h)^2 + \xi^2] - \frac{\lambda}{4}[(\nu + h)^2 + \xi^2]^2. \tag{3.1.7}$$

The masses of the field quanta ($\frac{m^2}{2}$,to be precise) are given by the coefficients in front of the quadratic terms of the fields. The first term on the right hand side of (1.5.5) contains the expression $\frac{g^2\nu^2}{2}A_\mu A^\mu$, which can be interpreted as a massive gauge boson with mass

$$m_A = g\nu. \tag{3.1.8}$$

To second order in the fields (1.5.6) can be written as $V^{(2)} = \text{const} + \frac{2\lambda\nu^2}{2}h^2$, which yields a scalar boson with mass

$$m_h = \sqrt{2\lambda}\nu \tag{3.1.9}$$

Since all terms proportional to ξ^2 cancel this scalar particle is massless. To summarize: for a simple parabolic potential $V = \mu^2\phi^*\phi$ in (1.5.1) the theory contains two massive scalar particles ϕ_1 and ϕ_2 and a massless gauge boson A_μ. If the potential is deformed (by whatever mechanism) to the form shown in Fig 1.2, the mass of one of the scalar particles shifts to the gauge boson. This is generalized in the Goldstone theorem which states that for each spontaneously broken dimension of the gauge symmetry group there exists a massless scalar field, called a Goldstone boson. Whereas the h field, called Higgs field after one of its inventors, is a real field and should be observable in experiments, the ξ field (and the Goldstone bosons in general) can be eliminated from the Lagrangian by a special gauge transformation. In other words, the Goldstone bosons are unphysical states corresponding to the gauge freedom of the theory. This section will be concluded by listing some important properties of the dynamical Higgs field (not to be mixed up with the constant field ν that pervades space-time) and in the next section it will be discussed how the Higgs mechanism works in the context of the gauge group of the unified electroweak theory.

The Higgs field cannot be gauged away and should therefore be observable as a physical particle H.

The mass of the Higgs particle is given through the parameters ν and λ. Whereas ν is determined by the gauge boson mass and the generalized charge quantum number of the theory, the parameter λ of the Higgs potential is unknown. As a consequence the Higgs boson mass is not predicted by the theory.

There are self-interaction terms of the Higgs boson (e.g. four h fields with the coupling λ).

The number of degrees of freedom of the fields is conserved. A massless gauge boson has two degrees of freedom (the transverse polarization states) and a complex scalar field also has two.When the Higgs mechanism comes into action we are left with a massive gauge boson with three polarization states and one real Higgs boson.

3.2 Application of Higgs Mechanism for the $S(U) \times U(1)$ gauge group of GWS model

The first attempt (Schwinger 1958) to unify weak and the electromagnetic interaction used to the $SU(2)$ group.This group was chosen to accommodate the three known gauge bosons, two vector bosons of charged current of the weak interaction and the photon. Having one dimension more than the $SU(2)$ group alone,and this gauge theory predicted another neutral gauge boson,known as the neutral current of weak interaction. Since all the gauge bosons of the weak interaction had to be massive, in a theory with three dimensional spontanous symmetry breaking,by introducing three Goldstone bosons. In the GWS model this is achieved by breaking $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$, where $U(1)_{em}$ is the gauge group of the electric charge(with the massless photon as the gauge boson).The mechanism is as follows,but the final step is to study the spontanous breaking of a local gauge symmetry. Here we take a simplest example:a $U(1)$ gauge symmetry. In the following section we extend to the discussion to $SU(2)$ .First, we must make our lagrangian,

$$L = (\partial_\mu \phi)^*(\partial_\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 \tag{3.2.1}$$

is invariant under U(1) local gauge transformation ,

$$\phi \rightarrow e^{i\alpha(x)}\phi \tag{3.2.2}$$

this requires ∂_μ to be replaced by the covariant derivative

$$D_\mu = \partial_\mu - ieA_\mu \tag{3.2.3}$$

where the gauge field transforms as

$$A_\mu \rightarrow A_\mu + (1/e)\partial_\mu \alpha \tag{3.2.4}$$

The gauge lagrangian is thus

$$L = (D_\mu \phi)^*(D_\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2,$$

$$L = D_\mu^* \phi^*(D_\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2,$$

$$L = (\partial_\mu + ieA_\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3.2.5}$$

Here we take $\mu^2 < 0$, since we want to generate masses by spontanous symmetry breaking. We repeat by now a familiar procedure of translating the field ϕ to a true ground state.

$$\phi(x) = \sqrt{\frac{1}{2}}[\nu + \eta(x) + \xi(x)] \tag{3.2.6}$$

substituting equation (3.2.6) in to (3.2.5) we can obtain the Lagrangian

$$L^{'} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - \nu^2 \lambda \eta^2 + \frac{1}{2}e^2 \nu^2 \xi A^\mu - e\nu A_\mu \partial^\mu \xi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + interaction\ term \tag{3.2.7}$$

The particle spectrum of $L^{'}$ appears to be massless Goldston boson ξ, a massive scalar η, and more crucially a massive vector A_μ for which we have sought so long. From equation (3.2.7) we have $m_\xi = 0, m_\eta = \sqrt{2\lambda}\nu$ and $m_A = e\nu$. We have dynamically generated a mass for the gauge field, but still we have a problem of occurence of a massless Goldstone boson. To handle this problem, we can find a particular gauge transformaton which will elliminate a field from Lagrangian. A clue is to note that

$$\phi = \sqrt{\frac{1}{2}}(\nu + \eta + i\xi) \simeq \sqrt{\frac{1}{2}}(\nu + \eta)e^{\frac{i\xi}{\nu}} \tag{3.2.8}$$

to lowest order in ξ. This suggests that we should substitute a different set of real fields h, θ, A_μ, where

$$\phi \rightarrow \sqrt{\frac{1}{2}}(A_\mu + h(x))e^{\frac{i\theta(x)}{\nu}}$$
$$A_\mu \rightarrow A_\mu + \frac{1}{e\nu}\partial_\mu\theta \tag{3.2.9}$$

in to the original Lagrangian (3.1.7). This is a particular choice of gauge,$\theta(x)$ chosen so that h is real. we there fore anticipate that the theory will be independent of θ. In deed, we obtain

$$L^{''} = \frac{1}{2}(\partial_\mu h)^2 - \lambda\nu^2 h^2 + \frac{1}{2}e^2\nu^2 A_\mu^2 - \lambda\nu h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2 A_\mu^2 h^2 + \nu e^2 A_\mu^2 h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3.2.10}$$

The Goldston boson actually does not appear in the theory. That is, the apparent extra degree of freedom is spurios, because it corresponds only to the freedom to make a gauge transformation. The unphysical Goldstone field have already been gauged away. The Lagrangian describes two interacting massive particles, a vector gauge boson A_μ, and a massive scalar h, which is called Higgs particle. The unwanted massless Goldstone boson has been turned in to the badly needed longitudinal polarization of the massive gauge particle. This is called " Higgs Mechanism" as mentioned in section 3.1.

3.3 Spontanous Breaking of Local SU(2) Gauge Symmetry

. It is necessary to repeat the procedure for an SU(2) gauge symmetry, because it provides preparation for the discussion of the gauge symmetries of the electroweak interaction. We take the lagrangian

$$L = (\partial_\mu \phi)^+ (\partial^\mu \phi)) - \mu^2 \phi^+ \phi - \lambda(\phi^+ \phi)^2 \tag{3.3.1}$$

where ϕ is an SU(2) doublet of complex scalar field:-

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \sqrt{\frac{1}{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{3.3.2}$$

L is invariant under global SU(2) phase trasformations

$$\phi \to \phi^{'} = e^{i\alpha_a \frac{\tau_a}{2}} \phi \tag{3.3.3}$$

To achieve local,that is, $\alpha_a(x), SU(2)$ invariance of $\pounds$,we follow the step,and we replace ∂_μ by the covariant derivative

$$D_\mu = \partial_\mu + ig\frac{\tau}{2} W^a_\mu \tag{3.3.4}$$

and three gauge fields, $W^a_\mu(x)$ with a=1,2,3. Under an infinitesimal gauge transformation

$$\phi(x) \to \phi^{'}(x) = (1 + i\alpha(x).\frac{\tau}{2})\phi(x) \tag{3.3.5}$$

the three gauge fields transform as

$$W_\mu \to W_\mu - \frac{1}{g}\partial_\mu \alpha - \alpha \times W_\mu \tag{3.3.6}$$

The $\alpha \times W_\mu$ occurs because W_μ is an SU(2) vector.The gauge invariance Lagrangian is then

$$L = (\partial_\mu \phi + ig\frac{\tau}{2}.W_\mu \phi)^+ (\partial^\mu \phi + ig\frac{1}{2}\tau.W^\mu \phi) - V(\phi) - \frac{1}{4} W_{\mu\nu}.W^{\mu\nu} \tag{3.3.7}$$

with

$$V(\phi) = \mu^2 \phi^+ \phi + \lambda(\phi^+ \phi)^2 \tag{3.3.8}$$

And we have added the kinetic energy term of the gauge fields with

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - gW_\mu \times W_\nu \tag{3.3.9}$$

The last term in (3.3.9) and (3.3.6),arises from non-Abelian character of the group,it occurs because $\tau' s$ do not commute with each other.If $\mu^2 > 0$, the Lagrangian equation (3.3.7) describes a system of four scalar particles ϕ_i of (3.3.2) each of mass μ^2 interacting with three massless bosons W_μ^a. We are interested in the case $\mu^2 < 0$ and $\lambda > 0$.The potential $V(\phi)$ of (3.3.8) then has its minimum at a finite value of $|\phi|$ where

$$(\phi)^+\phi \equiv \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda} \tag{3.3.10}$$

This points at which $V(\phi)$ is minimized is invariant under the SU(2)transformations. We must expand $\phi(x)$ about a particular minimum,So we can choose

$$\phi_1 = \phi_2 = \phi_4 = 0, \phi_3^2 = -\frac{\mu^2}{\lambda} \equiv \nu^2 \tag{3.3.11}$$

The effect is equivalent to the spontanous breaking of the SU(2) symmetry. We now expand $\phi(x)$ about this particular vacuume

$$\phi_0 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{3.3.12}$$

The result is that, due to gauge invariance,we can simply substitute the expansion

$$\phi(x) = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ \nu + h(x) \end{pmatrix} \tag{3.3.13}$$

in to the lagrangian (3.3.7). That is,of the four scalar fields,the only one that remains is h(x) (the Higg's field).The argument is as follows, proceding in analogy to (1.9.9), we parametrize fluctuations from the vacuume ϕ_0 interms of four fields $\theta_1, \theta_2, \theta_3$ and h , using the form

$$\phi(x) = e^{i\tau.\frac{\theta(x)}{\nu}} \begin{pmatrix} 0 \\ \frac{\nu + h(x)}{\sqrt{2}} \end{pmatrix} \tag{3.3.14}$$

To verify that this is perfectly general, we examine a small perturbations,So we can find that

$$\phi \backsimeq \sqrt{\frac{1}{2}} \begin{pmatrix} 1+i\frac{\theta_3}{\nu} & i(\frac{\theta_1-i\theta_2}{\nu}) \\ i(\frac{\theta_1+i\theta_2}{\nu}) & (1-i\frac{\theta_3}{\nu}) \end{pmatrix} \begin{pmatrix} 0 \\ \nu+h(x) \end{pmatrix}$$

$$\backsimeq \sqrt{\frac{1}{2}} \begin{pmatrix} \theta_2+i\theta_1 \\ \nu+h-i\theta_3 \end{pmatrix} \tag{3.3.15}$$

We can see that the four fields are indeed dependent and fully parametrized the deviation from the vacuume ϕ_0.Now the Lagrangian is locally SU(2) invariant[4]. There fore, we can gauge the three massless Goldstone boson fields $\theta(x)$ of (3.3.14). The Lagrangian will then contain no trace of $\theta(x)$, hence we arrive at the result (3.3.13).To determine the masses generated for the gauge bosons W_μ^a. The relevant term of of (3.3.7) is then be

$$|ig\frac{\tau}{2}.W_\mu\phi|^2 = (ig\frac{\tau}{2}.W_\mu\phi)^+(ig\frac{\tau}{2}.W_\mu\phi)$$

$$= \frac{g^2}{8}| \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & \mathcal{W}_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix} |^2$$

$$= \frac{g^2}{8} | \begin{pmatrix} W_\mu^3 & W_\mu^- \\ W_\mu^+ & W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix} |^2$$

$$= \frac{g^2\nu^2}{8}[(W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2] \tag{3.3.16}$$

We compare this terms with a typical mass term of boson,$\frac{1}{2}M^2B_\mu^2$ and find $M = \frac{1}{2}g\nu$. That is the Lagrangian describes three massive gauge fields and one massive scalar h. In summary, the gauge fields have "eaten up" the Goldstone bosons and become massive. The scalar degrees of freedom become the longitudinal polarization of the massive vector bosons. This is another example of Higgs Mechanism. The higgs mechanism has enabled us to avoid massless particles. Remember that the basic problem is not to generate masses,but to incorporate the mass of weak boson while still preserving the renormalizability of the theory. It is widely belived that gauge principles may generate the structure of all particle interaction. The standard model for weak and electromagnetic interaction is constructed from the gauge theory with four gauge fields, the photon and massive bosons ($W^{\pm}$ and Z^0).We generate the masses of the gauge fields by spontanous symmetry breaking ensuring that one of them (the photon) remains massless.

3.4 Choice of The Higgs Field

Now we want to formulate the higgs mechanism so that the W^+ or W^- and Z^0 become massive and the photon remains massless. To do this we introduce four scalar fields ϕ_i. We have to add L_1 and $SU(2) \times U(1)$ gauge invariant Lagrangian for the scalar fields.

$$L_2 = \mid (i\partial_\mu - gT.W_\mu - g^{'}\frac{Y}{2}B_\mu)\phi \mid^2 - V(\phi) \tag{3.4.1}$$

To keep L_2 gauge invariant, the ϕ_i must belongs to $SU(2) \times U(1)$ multiplets. The most economical choice is to arrange four fields in an isospin doublet with weak hypercharge $Y = 1$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$$

with

$$\begin{aligned} \phi^+ &\equiv \frac{(\phi_1 + i\phi_2)}{\sqrt{2}} \\ \phi_0 &\equiv \frac{(\phi_3 + i\phi_4)}{\sqrt{2}} \end{aligned} \tag{3.4.2}$$

This is infact the choice originally made in 1967 by Weinberge.It complete the specification of the standard model of the electroweak interactions. It is also called "Weinberge-Salam Model" To generate gauge boson masses,we use familiar higgs potential $V(\phi)$. Where $V(\phi) = \mu^2\phi^+\phi + \lambda(\phi^+\phi)^2$ with $\mu^2 < 0$ and $\lambda > 0$ and choosing the vacuume expectation value,ϕ_0,of $\phi(x)$. The appropriate choice is

$$\phi_0 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{3.4.3}$$

Any choice of ϕ_0 which breaks a symmetry operation will generate a mass for the corresponding gauge bosons, However, if the vacuume ϕ_0 is still left-invariant by some sub group of gauge transformations, then the gauge bosons associated with this sub group will remain massless.Now, the choice ϕ_0 with $T = \frac{1}{2}, T_3 = -\frac{1}{2}$ and $Y = 1$ breaks both $SU(2)$ and $U(1)_Y$ gauge symmetries. But ϕ_0 is neutral. The $U(1_{em})$ symmetry with generator $Q = T_3 + \frac{Y}{2}$ remains unbrocken. That is $Q\phi_0 = 0$ So that

$$\phi_0 \to \phi^{'} = e^{i\alpha(x)Q}\phi_0 = \phi_0 \tag{3.4.4}$$

For any value of $\alpha(x)$. The vacuume is that remains invariant under $U(1)_{em}$ transformations,and the photon remains massless. Out of four $SU(2)\times U(1)$ generators T, Y,only the combination Q obeys relation of Gell-Mann[1]. The other three break the symmetry and generate massive gauge bosons. The higgs field has to carry both isospin and hypercharge in order to give mass to the gauge bosons of $SU(2) \times U(1)_Y$.

3.5 Masses Of The Gauge Bosons

The gauge boson masses are identified by substituting the expectation value ϕ_0 for $\phi(x)$ in the Lagrangian L_2. The relevant term in (3.2.1) is

$$\mid (-ig\frac{\tau^a}{2}.W_\mu^a - i\frac{g^{'}}{2}B_\mu)\phi \mid^2$$

Where $a = 1, 2, 3$ and τ^1, τ^2, τ^3 are the pauli's matrices,given by $\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, so that the above expression becomes

$$= \frac{1}{8} \mid \begin{pmatrix} gW_\mu^3 + g^{'}B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g^{'}B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \mid^2$$

$$= \frac{1}{8} \mid \begin{pmatrix} gW_\mu^3 + g^{'}B_\mu & g(W_\mu^-) \\ g(W_\mu^+) & -gW_\mu^3 + g^{'}B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \mid^2$$

$$= \frac{1}{8}\nu^2 g^2[(W_\mu^1)^2 + (W_\mu^2)^2] + \frac{1}{8}\nu^2(g^{'}B_\mu - gW_\mu^3)(g^{'}B_\mu - gW_\mu^3)$$

$$= (\frac{1}{2}\nu g)^2 W_\mu^+ W_\mu^- + \frac{1}{8}(W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg^{'} \\ -gg^{'} & g^{'2} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \tag{3.5.1}$$

Since $W_\mu^\pm = \sqrt{\frac{1}{2}}(W_\mu^1 \pm iW_\mu^2)$, comparing the first term with the mass term expected for a charged bosons, so that we have

$$M_W = \frac{1}{2}\nu g \tag{3.5.2}$$

The remaining term is off-digonal in the W_μ and B_μ basis;

$$\frac{1}{2}[g^2(W_\mu^3)^2 - 2gg^{'}W_\mu^3 B_\mu + g^{'2}B_\mu^2]$$

$$\frac{1}{8}\nu^2[gW_\mu^3 - g^{'}B_\mu]^2 + 0[g^{'}W_\mu^3 + g^{'}B_\mu]^2 \tag{3.5.3}$$

Now the physical field $Z\mu$ and A_μ diagonalize the mass matrix, so that (3.5.3) should be identified with $\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2$ where $\frac{1}{2}$ is appropriate for a mass term of a neutral vector boson. So on normalizing the fields,we have

$$A_\mu = \frac{g^{'}W_\mu^3 + gB_\mu}{\sqrt{g^2 + g^{'2}}} \tag{3.5.4}$$

with $M_A = 0$

$$Z_\mu = \frac{gW_\mu^3 - g^{'}B_\mu}{\sqrt{g^2 + g^{'2}}} \tag{3.5.5}$$

with $M_Z = \frac{1}{2}\nu\sqrt{g^2 + g^{'2}}$. We can express these notation by using the notation(3.5.10), from this

$$\frac{g^{'}}{g} = tan\theta_W \tag{3.5.6}$$

Interms of θ_W,(3.5.4) and (3.5.5) become

$$A_\mu = cos\theta_W B_\mu + sin\theta_W W_\mu^3, Z_\mu = -sin\theta_W B_\mu + cos\theta_W W_\mu^3$$

From (3.5.2) and (3.5.5),we have that

$$\frac{M_W}{M_Z} = \frac{1}{sec\theta_W} = cos\theta_W \tag{3.5.7}$$

The inequality $M_Z \neq M_w$ is due to the mixing between the W_W and B_μ fields.The mass eigen states are automatically a massless photon (A_μ) and a massive Z_W field with

$M_Z > M_W$.In the limit $\theta_W = 0$,we see that $M_W = M_Z$. The model was constructed with the requirement that the photon be massless, and so the result $M_A = 0$ is just the consistency check on our calculation and not the prediction. However, the result (3.5.7) for $\frac{M_W}{M_Z}$ is a prediction of the standard model (with its Higgs doublet)[1].

Chapter 4

The Lagrangian Formulation of GWS Model for leptons

The lagrangian recognizes it self after the Higgs boson aquires a vacuume expectation value (being massive). Thus the Lagrangian of GWS that best describing the electromagnetic and weak interaction of leptons.So we can write the closed form of the Lagrangian by breaking up in to several components as follows.

$$L_{SW} = L_{SW}^{(2)} + \sum_{l=e,\mu,\tau} L_{SW}^{(3L)} + L_{SW}^{(3B)} + L_{SW}^{4B} + L_{int}^{HG} + L_{SW}^{H} \tag{4.0.1}$$

These complete Lagrangian consists of the following parts

(a) Free fields(massive vector bosons,photons,leptons):

$$L_{SW}^{2} = -\frac{1}{2}(\partial_\mu W_\nu^{(+)} - \partial_\nu W_\mu^{+})(\partial^\mu W^{(-)\nu} - \partial^\nu W^{(-)\mu}) + M_W^2 W_\mu^{(+)} W^{(-)\mu} - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu)$$
$$+\frac{1}{2}M_Z^2 Z_\mu Z^\mu - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \sum_{l=e,\mu,\tau}[\bar{\psi}_{\nu_l} i\gamma^\mu \partial_\mu \frac{(1-\gamma_5)}{2}\psi_{\nu_l} + \bar{\psi}_l(i\gamma^\mu\partial_\mu - m_l)\psi_l] \tag{4.0.2}$$

(b) $L_{SW}^{(3L)}$,The lepton boson interaction

$$L_{SW}^{(3L)} = \frac{g}{2\sqrt{2}}\underbrace{[\bar{\psi}_l\gamma^\mu(1-\gamma_5)\psi_{\nu_l}W_\mu^{(-)} + \bar{\psi}_{\nu_l}\gamma^\mu(1-\gamma_5)\psi_l W_\mu^{(+)}]}_{a}$$
$$+\frac{g}{4cos\theta}\underbrace{[\bar{\psi}_{\nu_l}\gamma^\mu(1-\gamma_5)\psi_{\nu_l} - \bar{\psi}_l\gamma^\mu(1-4sin^2-\gamma_5)\psi_l]Z_\mu}_{b} - e\underbrace{\bar{\psi}_l\gamma^\mu\psi_l A_\mu}_{c} \tag{4.0.3}$$

The Feynman diagram will be

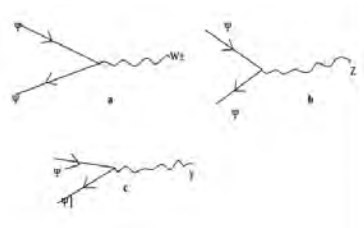

Figure 4.1: The vertex factor for $a = \frac{g}{2\sqrt{2}}\gamma^{\mu}(1-\gamma_5)$, for $b = \frac{g}{4cos\theta}\gamma^{\mu}(1-\gamma_5)$ and for $c = -e\gamma^{\mu}$ respectively

(c) Third order interaction of vector bosons:

$$L^{(3B)}_{SW} = igcos\theta[(\partial_\mu W^{(-)}_\nu - \partial_\nu W^{(-)}_\mu)W^{(+)\mu}Z^\nu - (\partial_\mu W^{(+)}_\nu - \partial_\nu W^{(+)}_\mu)W^{(-)\mu}Z^\nu]$$
$$-ie(\partial_\mu W^{(-)}_\nu - \partial_\nu W^{(-)}_\mu)W^{(+)\mu}A^\nu + ie(\partial_\mu W^{(+)}_\nu - \partial_\nu W^{(+)}_\mu)W^{(-)\mu}A^\nu$$
$$+igcos\theta(\partial_\mu Z_\nu - \partial_\nu Z_\mu)W^{(+)\mu}W^{(-)\nu} - ie(\partial_\mu A_\nu - \partial_\nu A_\mu)W^{(+)\mu}W^{(-)\nu} \quad (4.0.4)$$

(d) Fourth order interactions of vector bosons:

$$L^{(4B)}_{SW} = -g^2cos^2\theta\,\underbrace{(W^{(+)}_\mu W^{(-)\mu}Z_\nu Z^\nu - W^{(+)}_\mu W^{(-)}_\nu Z^\mu Z^\nu)}_{1} - e^2\,\underbrace{(W^{(+)}_\mu W^{(-)\mu}A_\nu A^\nu - W^{(+)}_\mu W^{(-)}_\nu A^\mu A^\nu)}_{2}$$
$$+egcos\theta\,\underbrace{(2W^{(+)}_\mu W^{(-)\mu}Z_\mu A^\nu - W^{(+)}_\mu W^{(-)}_\nu Z^\mu A^\nu - W^{(+)}_\mu W^{(-)}_\nu Z^\nu A^\mu)}_{3}$$
$$+g^2\,\underbrace{(W^{(+)}_\mu W^{(-)\mu}W^{(+)}_\nu W^{(-)\nu} - W^{(-)}_\mu W^{(-)\mu}W^{(+)}_\nu W^{(+)\nu})}_{4} \quad (4.0.5)$$

The Feynman diagram will be

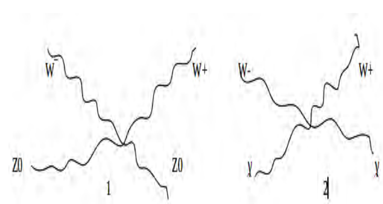

Figure 4.2: The vertex factor is $-g^2cos^2\theta$ and $-e^2$ respectively

(e) The Higgs-Gauge boson interaction results from minimal coupling terms in (3.4.1)

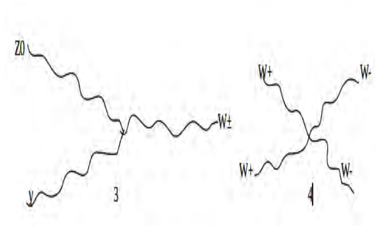

Figure 4.3: The vertex factor is $egcos\theta$ and g^2 respectively

$$L_{int}^{HG} = \mid (i\partial_\mu + g\widehat{T}.A_\mu + \frac{g^{'}}{2}B_\mu Y)\phi \mid^2 - \mid \partial_\mu \phi \mid^2 \tag{4.0.6}$$

By using (3.0.12) we have

$$gT^3A_\mu^3 + \frac{g^{'}}{2}B_\mu = \frac{1}{2}\begin{pmatrix} gA_\mu^3 + g^{'}B_\mu & 0 \\ 0 & -gA_\mu^3 + g^{'}B_\mu \end{pmatrix}$$

$$= \begin{pmatrix} eA_\mu + g\frac{cos2\theta}{2cos\theta} & 0 \\ 0 & -\frac{g}{2cos\theta}Z_\mu \end{pmatrix} \tag{4.0.7}$$

Again by using (3.0.13) and (4.0.7) the gauge field terms can be expressed as the following isospin matrix

$$g\widehat{T}.A_\mu + \frac{g^{'}}{2}B_\mu Y = gT^3A_\mu^3 + \frac{g^{'}}{2}B_\mu + \frac{g}{\sqrt{2}}(T_+W_\mu^+ + T_-W_\mu^-)$$

$$= \begin{pmatrix} eA_\mu + g\frac{cos2\theta}{2cos\theta} & \frac{g}{\sqrt{2}}W_\mu^+ \\ \frac{g}{\sqrt{2}}W_\mu^- & -\frac{g}{2cos\theta}Z_\mu \end{pmatrix} \tag{4.0.8}$$

Writing out the square in (4.0.6) we can find that the mixed terms involving $\partial_\mu\phi$ cancel each other because of the factor i.Then the remaining term by using (3.3.13) becomes

$$L_{int}^{HG} = \frac{1}{2}(\nu + h)^2[(g\widehat{T}.A_\mu + \frac{g^{'}}{2}B_\mu Y)\begin{pmatrix} 0 \\ 1 \end{pmatrix}]^+[(g\widehat{T}.A^\mu + \frac{g^{'}}{2}B^\mu Y)\begin{pmatrix} 0 \\ 1 \end{pmatrix}]$$

$$= \frac{1}{2}(\nu + h)^2 \begin{pmatrix} \frac{g}{\sqrt{2}}W_\mu^+ \\ -\frac{g}{2cos\theta}Z_\mu \end{pmatrix}^+ \begin{pmatrix} \frac{g}{\sqrt{2}}W_\mu^+ \\ -\frac{g}{2cos\theta}Z_\mu \end{pmatrix}$$

$$= \frac{g^2}{8}H^2(\frac{1}{cos^2\theta}Z_\mu Z^\mu + 2W_\mu^+W^{+\mu}) \tag{4.0.9}$$

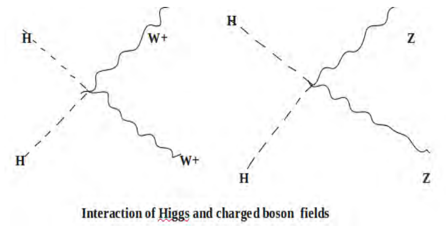

Figure 4.4: The vertex factor is $\frac{g^2}{4}$ and $\frac{g^2}{8cos^2\theta}$ respectively

(f) The Higgs self interaction

$$L_H = \underbrace{-\frac{gm_H^2}{4m_W}H^3}_{1} - \underbrace{\frac{g^2m_H^2}{32m_W^2}H^4}_{2} \tag{4.0.10}$$

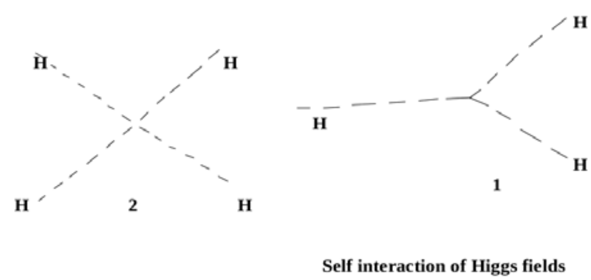

Figure 4.5: for 2,1, the vertex factor is $-\frac{g^2m_H^2}{32m_W^2}$ and $-\frac{gm_H^2}{4m_W}$ respectively

The complete Lagrangian which describes the dynamics of the interaction is:

$$L_{SW} = L_{SW}^{(2)} + \sum_{l=e,\mu,\tau} L_{SW}^{(3L)} + L_{SW}^{(3B)} + L_{SW}^{4B} + L_{int}^{HG} + L_{SW}^{H} \tag{4.0.11}$$

Where

(a) $L_{SW}^{(2)}$ describes the part of the free boson and lepton fields

(b) $\sum_{l=e,\mu,\tau} L_{SW}^{(3L)}$ represents the coupling between the leptons of the generation $l = e, \mu, \tau$ and the intermediate bosons.

(c) $L_{SW}^{(3B)}$ is the third order term of the bosonic fields describing their self-coupling.

(d) L_{SW}^{4B} is the fourth order term of bosonic fields describing self-coupling.

(e) L_{int}^{HG} is the Higgs-gauge boson interaction (the coupling between Higgs field and gauge bosons)

(f) Finally L_{SW}^{H} contains the Higgs three and four point self-interaction term.

Thus the Lagrangian density that contains a large number of terms; its conceptual simplicity, however, originates from the underlying gauge principle of the $SU(2)\times U(1)$ group. We now apply this frame work to study decay of W^- boson.

4.1 Decay of the Charged Boson W^-

In this section we first discuss decay of the negatively charged boson W^-, induced by the term in $L_{SW}^{(3L)}$ of (4.0.11). The lowest order scattering matrix element is:

$$S(W^- \to l\bar{\nu}_l) = -i\int d^4(x)\frac{(-g)}{2\sqrt{2}}\bar{\psi}_{\nu l}(x)\gamma^\mu(1-\gamma^5)\psi_{\nu l}(x)W_\mu^{(-)}(x) \tag{4.1.1}$$

With the notation of figure (4.6 b) the wave function of the incoming and out going

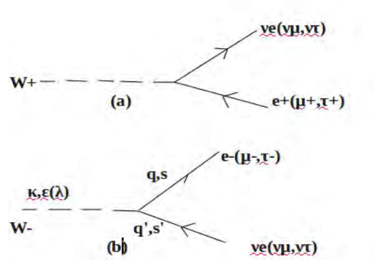

Figure 4.6: Leptonic decay mechanism of intermediate bosons

particles are

$$W_\mu^{(-)}(x) = (2Vk_0)^{-\frac{1}{2}}\varepsilon_\mu(k,\lambda)e^{-ik.x}, \psi_l(x) = (2Vq_0)^{-\frac{1}{2}}u_l(q,s)e^{-iq.x},$$

$$\psi_{\nu l}(x) = (2Vq_0^{'})^{-\frac{1}{2}}\upsilon_\nu(q^{'},s^{'})e^{+iq^{'}.x} \tag{4.1.2}$$

$\varepsilon_\mu(k,\lambda)$ is the polarization vector of the $W^{(-)}$ boson.All we need to know about ε_μ is the summation over all three direction of polarization λ yields.

$$\sum_{\lambda=1}^{3}\varepsilon_\mu(k,\lambda)\varepsilon_\nu(k,\lambda)=-g_{\mu\nu}+\frac{k_\mu k_\nu}{M_{W^2}} \tag{4.1.3}$$

Now we have to follow to perform the space-time integration

$$\int d^4x e^{(iq.x+iq^{'}.x-ik.x)}=(2\pi)^4\delta^4(q+q^{'}-k) \tag{4.1.4}$$

Squaring the matrix element and integrating over final states,averaging over the polarization of the incoming boson and finally dividing by the time T yields the decay rate.

$$W=\frac{1}{T}V\int\frac{d^3q}{(2\pi)^3}V\int\frac{d^3q^{'}}{(2\pi)^3}\frac{1}{3}\sum_{\lambda}\sum_{s,s^{'}}\mid S(W^{(-)}\rightarrow l\bar{\nu}_l)\mid^2$$

$$=\frac{g^2}{8}\frac{1}{(2\pi)^2}\int\frac{d^3q}{2q_0}\int\frac{d^3q^{'}}{2q_0^{'}}\frac{1}{2k_0}\frac{1}{3}\sum_{\lambda,s,s^{'}}\mid M_{\lambda ss^{'}}\mid^2\delta^4(q+q^{'}-k) \tag{4.1.5}$$

with

$$M_{\lambda ss^{'}}=\varepsilon_\mu(k,\lambda)\bar{u}_l(q,s)\gamma^\mu(1-\gamma_5)\upsilon_\nu(q^{'},s^{'}) \tag{4.1.6}$$

Using the rules for calculating traces of Dirac spinors and γ matrices we get the following result:

$$\sum_{\lambda,s,s^{'}}\mid M_{\lambda ss^{'}}\mid^2=\sum_{\lambda}\varepsilon_\mu(k,\lambda)\varepsilon_\nu(k,\lambda)\{\sum_{s,s^{'}}\bar{u}_l(q,s)\gamma^\mu(1-\gamma_5)\times\upsilon_\nu(q^{'},s^{'})\bar{\upsilon}_\nu(q^{'},s^{'})\gamma^\nu(1-\gamma_5)u_l(q,s)\}$$

$$=(-g_{\mu\nu}+\frac{k_\mu k_\nu}{M_W^2})Tr\{(p/+m_l)\gamma^\mu(1-\gamma_5)p^{'}/\gamma^\nu(1-\gamma_5)\} \tag{4.1.7}$$

The Dirac trace becomes

$$Tr\{(p/+m_l)\gamma^\mu(1-\gamma_5)p^{'}/\gamma^\nu(1-\gamma_5)\}=2Tr\{(p/+m_l)\gamma^\mu p^{'}/\gamma^\nu(1-\gamma_5)\}$$

$$=2q_\alpha q_\beta^{'}Tr\{\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu(1-\gamma_5)\}=8q_\alpha q_\beta^{'}(g^{\alpha\mu}g^{\beta\nu}+g^{\alpha\nu}g^{\beta\mu}-g^{\alpha\beta}g^{\mu\nu}+i\varepsilon^{\alpha\mu\beta\nu})$$

$$=8[q^\mu q^{'\nu}+q^\nu q^{'\mu}-(q.q^{'})g^{\mu\nu}+i\varepsilon^{\alpha\mu\beta\nu}q_\alpha q_\beta^{'}] \tag{4.1.8}$$

Here we have used (4.1.3), and one way we may wonder why we are allowed to sum over the neutrino spin $s^{'}$ in (4.1.7), although we saw that only left handed neutrinos exists.The reason is simply that the $V-A$ coupling $\gamma^{\mu}(1-\gamma^{5})$ vanishes when applied to the right-handed neutrinos:$(1-\gamma_5)(1+\gamma_5)\upsilon_\nu = 0$. Inserting the result of the Dirac trace in to (4.1.7) we can see that the last term does not contribute, because (4.1.3) is symmetric with respect to indices μ and ν,where as $\varepsilon^{\alpha\mu\nu}$ is totally antisymmetric.From the energy-momentum relation $k^2 = M_W^2$ we find that:

$$\sum_{\lambda s s'} \mid M_{\lambda s s'} \mid^2 = \frac{8}{M_W^2}[2(q.k)(q^{'}.k) + M_W^2(q.q^{'})] \tag{4.1.9}$$

Because of the delta function in (4.1.5) the second term can be expressed in terms of the rest masses:

$$M_W^2 = k^2 = (q+q^{'})^2 = q^2 + q^{'2} + 2(q-q^{'}) = m_l + 2(q.q^{'}) \tag{4.1.10}$$

If we now insert all terms in to (4.1.5) we obtain the decay rate

$$W = \frac{g^2}{48\pi^2}\frac{1}{k_0 M_W^2}\int\frac{d^3q}{q_0}\int\frac{d^3q^{'}}{q_0^{'}}\delta^4(q+q^{'}-k)\times[(q.k)(q^{'}.k)+\frac{1}{4}M_W^2(M_W^2-m_l^2)]. \tag{4.1.11}$$

For the calculation of the momentum-space integral we refer to results obtained in the following equation:

$$k^\alpha k^\beta\int\frac{d^3q}{q_0}\int\frac{d^3q^{'}}{q_0^{'}}q_\alpha q_\beta^{'}\delta^4(q+q^{'}-k) = \frac{\pi}{6}(1-\frac{m_l^2}{k^2})^2[k^4(1-\frac{m_l^2}{k^2})+2(1+2\frac{m_l^2}{k^2})k^4]\Theta(k^2-m_l^2)$$

$$= \frac{\pi}{2}M_W^4(1-\frac{m_l^2}{M_W^2})^2(1+\frac{m_l^2}{M_W^2})\Theta(M_W-m_l) \tag{4.1.12}$$

The second part can be done in the following way. We integrate in the rest frame of the W^- boson, where $k_\alpha = (M_W, 0)$, yielding

$$\int\frac{d^3q}{q_0}\int\frac{d^3q^{'}}{q_0^{'}}\delta(q+q^{'})\delta(q_0+q_0^{'}-M_W), = \int\frac{d^3q}{q_0\mid q\mid}\delta(q_0+\mid q\mid -M_W)$$

$$= 4\pi\int_0^\infty\frac{\mid q\mid d\mid q\mid}{\sqrt{q^2+m_l^2}}\delta(\sqrt{q^2+m_l^2}+\mid q\mid -M_W), = 4\pi\int_{m_l}^\infty\frac{dx}{2x^2}(x^2-m_l^2)\delta(x-M_W)$$

$$= 2\pi(1-\frac{m_l^2}{M_W^2})\Theta(M_W-m_l) \tag{4.1.13}$$

Where we have substituted $x =\mid q \mid + \sqrt{q^2 + m_l^2}$. By inserting (4.1.12) and (4.1.13) in to (4.1.11) we obtain the final result of the decay rate of th W^- in its rest frame:

$$W = \frac{1}{48\pi} g^2 M_W (1 - \frac{m_l^2}{M_W^2})^2 (1 + \frac{m_l^2}{2M_W^2}) \Theta(M_W - m_l). \tag{4.1.14}$$

If we use the connection between g and the Fermi coupling constant G by $\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ and further exploite the fact that all known leptons are much lighter than the intermediate boson, the result is

$$W(W^- \to l\bar{\nu}_l) \approx \frac{G}{6\pi\sqrt{2}} M_W^3 \approx 225 MeV \approx 3.5 \times 10^{23} s^{-1} \tag{4.1.15}$$

Where we have taken the value of G as that $G = (1.16637 \pm 0.00002) \times 10^{-11} MeV^{-2}$. Taking in to account the three leptonic decay chanels $l = e, \mu, \tau$ we can estimate the life time of the charged intermediate bosons:

$$\tau_w \leq \frac{1}{3W} \approx \frac{2\pi\sqrt{2}}{GM_W^3} \approx 10^{-24} s \tag{4.1.16}$$

This is remarkably long time for elementary particle with the mass of more than $80 GeV$, but it is far too short to be measured directly.

Chapter 5

CONCLUSION

S.Glashow, S.Weinberg and A.Salam proposed a gauge theory based on the frame work of group of transformations generated by the weak isospin and hypercharge generators associated with the gauge group of the unified weak and electromagetic interactions $(SU(2)_L \otimes U(1)_Y)$[2]. The $SU(2)_L$ group provides, a triplets of gauge fields, $W_\mu = W_\mu^1, W_\mu^2, W_\mu^3$ coupling to a weak isospin T and describes a weak interaction between left-handed fermions and right-handed antifermions[9]. The $U(1)_Y$ group results in a singlet field B_μ and couples to weak hypercharge Y. As a result of gauge symmetry,each family of fermions can be classified in to left-handed doublets of weak isospin or right-handed singlets.The electro-weak theory which is based on spontanously brocken $SU(2)_L \times U(1)_Y$ gauge invariance is known as GWS model. We have reviewed Glashow-Weinberge-Salam model for leptons starting with a massless theory which is a requirement of gauge invariance and the massless gauge fields acquire mass by applying Higgs mechanism. Higgs mechanism refers specifically to the generation of masses for the $W^\pm$ and Z_0 weak gauge bosons through electro weak symmetry breaking. Finally the Lagrangian of GWS model for leptons are reviewed, and as an application we study the decay of W^- boson[2].

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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