

# **SQUEEZING IN SECOND HARMONIC GENERATION**

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by

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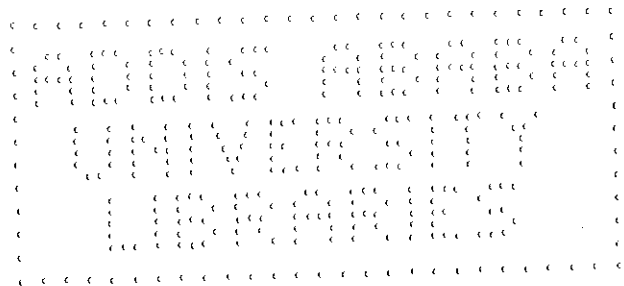
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# 1. Introduction

Squeezing is one of the properties of light which cannot be explained classically. This property of light involves the reduction of one of quadratures below the vacuum level at the expense of increased fluctuations in the other quadrature without violation of the uncertainty principle.

Squeezed light can be generated in various physical processes such as parametric oscillation [1-4], four wave mixing [5-8], resonance fluorescence [9-11] and second harmonic generation [12,13].

Collet and Walls [14] have obtained the squeezing spectra for the output fields of several intracavity nonlinear optical systems and have showed that at the critical points, such as, the turning points for optical bistability, the threshold for parametric oscillation and the self-pulsing instability in second harmonic generation, perfect squeezing in the output field is possible.

Perfect squeezing in one quadrature must be associated with an infinite uncertainty in the other and hence can only occur at a critical point. At a critical point the squeezing spectrum for one quadrature also diverges for a certain frequency [14].

In degenerate second harmonic generation a light mode of frequency  $\omega_0$  (fundamental mode) interacts with a nonlinear medium and is up converted into a light mode of frequency  $2\omega_0$  (harmonic mode). The nonlinear medium is placed inside a cavity driven by a coherent light of frequency  $\omega_0$ .

The squeezing properties of the fundamental and harmonic modes have been extensively studied [14-22]. In particular Collet and Walls [14] showed that optimum squeezing occurs in the vicinity of the self-pulsing threshold.

Pereira, Xiao, Kimble and Hall [16] employed the process of intracavity frequency doubling to generate a squeezed beam of light by reflection of a laser from a nonlinear cavity and they observed noise reductions of 13%.

Recently there has also been an interest in the process of nondegenerate second harmonic generation in a cavity as a source of squeezed light. In nondegenerate second harmonic generation, two modes with frequency  $\omega_1$  and  $\omega_2$  combine to create a second harmonic mode of frequency  $\omega$ , with  $\omega = \omega_1 + \omega_2$ . This process has been investigated theoretically by several authors [23-25].

One of the main objectives of this thesis is to calculate the output squeezing spectra and the mean photon numbers for the fundamental and harmonic modes involved in degenerate second harmonic generation inside a cavity coupled to squeezed vacuum reservoirs. We also calculate the squeezing spectra for the fundamental and harmonic modes involved in nondegenerate second harmonic generation inside a cavity coupled to ordinary vacuum reservoirs.

The arrangement of this thesis is as follows: In chapter two we present our analysis of degenerate second harmonic generation. This involves derivation of linearized Langevin equations of evolution for the fundamental and harmonic modes. The solution of the resulting equations are used to calculate the squeezing spectra, the quadrature variances and the mean photon numbers of these two modes. In chapter three, we obtain the linearized Langevin equations of evolution for both the fundamental and harmonic modes of nondegenerate second harmonic generation. We then obtain the quadrature operators in frequency space for the sum and difference of these two modes. Finally, these results are used to calculate the squeezing spectra.

## 2. Degenerate Second Harmonic Generation

In degenerate second harmonic generation a light mode of frequency  $\omega_0$  (fundamental mode) interacts with a nonlinear medium and is up converted into a light mode of frequency  $2\omega_0$  (harmonic mode). The nonlinear medium is placed inside a cavity driven by a coherent light of frequency  $\omega_0$ . Here we seek to consider the case for which these modes of light are coupled to squeezed vacuum reservoirs via a single port mirror. We seek here to calculate the squeezing spectra for both the fundamental and second harmonic modes. In addition, we also obtain the quadrature fluctuations of these two modes, with the aid of which their mean photon numbers are determined.

### 2.1 Linearized Langevin Equations

We carry out our analysis using linearized Langevin equations. To this end, the quantum system under consideration is describable in the interaction picture by the Hamiltonian ( $\hbar = 1$ )

$$H = i\varepsilon(a^\dagger - a) + i\frac{\kappa}{2}(b^\dagger a^2 - ba^{\dagger 2}), \quad (2.1)$$

where  $a(b)$  is the annihilation operator for the light mode of frequency  $\omega_0(2\omega_0)$ ,  $\kappa$  is the coupling constant and  $\varepsilon$  is proportional to the amplitude of the coherent driving light. The quantum Langevin equation for the operator  $a(t)$  is given by

$$\frac{d}{dt}a(t) = -i[a(t), H(t)] - \frac{\gamma_a}{2}a(t) + F_a(t), \quad (2.2)$$

where  $\gamma_a$  is the cavity damping rate for the fundamental mode and  $F_a(t)$  is a noise operator with vanishing mean and satisfying the correlation functions

$$\langle F_a(t)F_a(t') \rangle = e^{i\theta_a} M_a \gamma_a \delta(t - t'), \quad (2.3a)$$

$$\langle F_a^\dagger(t)F_a(t') \rangle = N_a \gamma_a \delta(t-t'), \quad (2.3b)$$

$$\langle F_a(t)F_a^\dagger(t') \rangle = (N_a + 1) \gamma_a \delta(t-t'). \quad (2.3c)$$

For a squeezed vacuum reservoir

$$N_a = \sinh^2(r_a), \quad (2.4a)$$

$$M_a = \cosh(r_a) \sinh(r_a), \quad (2.4b)$$

with  $r_a$  being the squeeze parameter.

We note that

$$[a(t), H(t)] = i(\varepsilon - \kappa a^\dagger(t)b(t)), \quad (2.5)$$

and substitution of (2.5) into Eq.(2.2) yields

$$\frac{d}{dt}a(t) = \varepsilon - \kappa a^\dagger(t)b(t) - \frac{\gamma_a}{2}a(t) + F_a(t). \quad (2.6)$$

In addition, following a similar procedure, one can readily verify that

$$\frac{d}{dt}b(t) = \frac{\kappa}{2}a^2(t) - \frac{\gamma_b}{2}b(t) + F_b(t), \quad (2.7)$$

in which  $\gamma_b$  is the cavity damping rate for the second harmonic mode and  $F_b(t)$  is a noise operator with vanishing mean and satisfying the correlation functions

$$\langle F_b(t)F_b(t') \rangle = e^{i\theta_b} M_b \gamma_b \delta(t-t'), \quad (2.8a)$$

$$\langle F_b^\dagger(t)F_b(t') \rangle = N_b \gamma_b \delta(t-t'), \quad (2.8b)$$

$$\langle F_b(t)F_b^\dagger(t') \rangle = (N_b + 1) \gamma_b \delta(t-t'). \quad (2.8c)$$

For a squeezed vacuum reservoir

$$N_b = \sinh^2(r_b), \quad (2.9a)$$

$$M_b = \cosh(r_b) \sinh(r_b), \quad (2.9b)$$

with  $r_b$  being the squeeze parameter.

Next we seek to obtain the steady state solutions of the quantum Langevin Eqs. (2.6) and (2.7). Taking the expectation values of these equations, we see that

$$\frac{d}{dt} \langle a(t) \rangle = \varepsilon - \kappa \langle a^\dagger(t) b(t) \rangle - \frac{\gamma_a}{2} \langle a(t) \rangle + \langle F_a(t) \rangle, \quad (2.10a)$$

$$\frac{d}{dt} \langle b(t) \rangle = \frac{\kappa}{2} \langle a^2(t) \rangle - \frac{\gamma_b}{2} \langle b(t) \rangle + \langle F_b(t) \rangle. \quad (2.10b)$$

Using the property of the noise operators

$$\langle F_a(t) \rangle = \langle F_b(t) \rangle = 0, \quad (2.11)$$

we find at steady state

$$\varepsilon - \kappa \langle a^\dagger b \rangle - \frac{\gamma_a}{2} \langle a \rangle = 0, \quad (2.12a)$$

$$\frac{\kappa}{2} \langle a^2 \rangle - \frac{\gamma_b}{2} \langle b \rangle = 0. \quad (2.12b)$$

Now applying the semiclassical approximation

$$\langle a^\dagger b \rangle = \langle a^\dagger \rangle \langle b \rangle = \alpha_0^* \beta_0 \quad (2.13a)$$

and

$$\langle a^2 \rangle = \alpha_0^2, \quad (2.13b)$$

one gets

$$\varepsilon_1^* \varepsilon_2 + \frac{\gamma_a}{2} \varepsilon_1 - \kappa \varepsilon = 0, \quad (2.14a)$$

$$\varepsilon_1^2 = \gamma_b \varepsilon_2, \quad (2.14b)$$

where

$$\langle a \rangle = \alpha_0, \quad (2.15a)$$

$$\langle b \rangle = \beta_0, \quad (2.15b)$$

$$\epsilon_1 = \kappa \alpha_0, \quad (2.15c)$$

$$\epsilon_2 = \kappa \beta_0. \quad (2.15d)$$

Upon multiplying (2.14a) by  $\epsilon_1^*$ , we have

$$\epsilon_1^{*2} \epsilon_2 + \frac{\gamma_a}{2} \epsilon_1^* \epsilon_1 - \kappa \epsilon_1^* \epsilon = 0, \quad (2.16)$$

so that employing the complex conjugate of (2.14b) in (2.16), one finds

$$\gamma_b \epsilon_2^* \epsilon_2 + \frac{\gamma_a}{2} \epsilon_1^* \epsilon_1 - \kappa \epsilon_1^* \epsilon = 0. \quad (2.17a)$$

Upon subtracting this equation from its complex conjugate, we get

$$\kappa \epsilon (\epsilon_1 - \epsilon_1^*) = 0, \quad (2.17b)$$

from which follows

$$\epsilon_1 = \epsilon_1^*. \quad (2.17c)$$

With the aid of (2.14b) and (2.17c), one can easily show that

$$\epsilon_2 = \epsilon_2^*. \quad (2.17d)$$

Employing the square of (2.14a) along with (2.14b), (2.17c) and (2.17d), one arrives at

$$\gamma_b \epsilon_2^3 + \gamma_a \gamma_b \epsilon_2^2 + \frac{1}{4} \gamma_a^2 \gamma_b \epsilon_2 = (\kappa \epsilon)^2. \quad (2.18)$$

We now proceed to linearize Eqs. (2.6) and (2.7) about the steady state solutions. To achieve this, we write the mode operators  $a(t)$  and  $b(t)$  as

$$a(t) = \alpha_0 + A(t), \quad (2.19a)$$

$$b(t) = \beta_0 + B(t), \quad (2.19b)$$

in which  $A(t)$  and  $B(t)$  represent small variations about the steady state values. Applying Eqs. (2.6), (2.7), (2.12a), (2.12b), (2.15c), (2.15d), (2.19a), (2.19b), and neglecting second-order terms in  $A(t)$  and  $B(t)$ , we get a linearized equations of evolution for the fluctuation operators

$$\frac{d}{dt}A(t) = -\frac{\gamma_a}{2}A(t) - \epsilon_2 A^\dagger(t) - \epsilon_1 B(t) + F_a(t), \quad (2.20a)$$

$$\frac{d}{dt}B(t) = -\frac{\gamma_b}{2}B(t) + \epsilon_1 A(t) + F_b(t). \quad (2.20b)$$

In order to decouple these differential equations, we introduce new operators defined by

$$A_\pm(t) = A^\dagger(t) \pm A(t), \quad (2.21a)$$

$$B_\pm(t) = B^\dagger(t) \pm B(t), \quad (2.21b)$$

so that combining Eqs. (20) and (21), we obtain

$$\frac{d}{dt}A_\pm(t) = -\left(\frac{\gamma_a}{2} \pm \epsilon_2\right)A_\pm(t) - \epsilon_1 B_\pm(t) + E_\pm(t), \quad (2.22a)$$

$$\frac{d}{dt}B_\pm(t) = -\frac{\gamma_b}{2}B_\pm(t) + \epsilon_1 A_\pm(t) + F_\pm(t), \quad (2.22b)$$

in which

$$E_\pm(t) = F_a^\dagger(t) \pm F_a(t), \quad (2.23a)$$

$$F_\pm(t) = F_b^\dagger(t) \pm F_b(t). \quad (2.23b)$$

Now multiplying Eq. (2.22a) by  $-\gamma_b/2$  and (2.22b) by  $\epsilon_1$  and adding the resulting equations gives

$$\epsilon_1 \frac{d}{dt}B_\pm(t) = \frac{\gamma_b}{2} \frac{d}{dt}A_\pm(t) + \left[\frac{\gamma_b}{2}\left(\frac{\gamma_a}{2} \pm \epsilon_2\right) + \epsilon_1^2\right] A_\pm(t) - \frac{\gamma_b}{2}E_\pm(t) + \epsilon_1 F_\pm(t). \quad (2.24)$$

On differentiating Eq.(2.22a) with respect to time and taking into account (2.24), we arrive at

$$\frac{d^2}{dt^2}A_\pm(t) + \left[\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2}\right] \frac{d}{dt}A_\pm(t) + \left[\frac{\gamma_b}{2}\left(\frac{\gamma_a}{2} \pm \epsilon_2\right) + \epsilon_1^2\right] A_\pm(t) =$$

$$\frac{d}{dt}E_{\pm}(t) + \frac{\gamma_b}{2}E_{\pm}(t) - \epsilon_1 F_{\pm}(t). \quad (2.25)$$

To obtain the solution of this differential equation, one can write [26]

$$\left(\frac{d}{dt} + \lambda_{\pm}\right)\left(\frac{d}{dt} + \eta_{\pm}\right)A_{\pm}(t) = \frac{d}{dt}E_{\pm}(t) + \frac{\gamma_b}{2}E_{\pm}(t) - \epsilon_1 F_{\pm}(t), \quad (2.26)$$

where on account of (2.25) and (2.26)

$$\lambda_{\pm} = \frac{1}{2} \left[ \frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2} \right] + \sqrt{\frac{1}{4} \left[ \frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2} \right]^2 - \left[ \frac{\gamma_b}{2} \left( \frac{\gamma_a}{2} \pm \epsilon_2 \right) + \epsilon_1^2 \right]}, \quad (2.27a)$$

$$\eta_{\pm} = \frac{1}{2} \left[ \frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2} \right] - \sqrt{\frac{1}{4} \left[ \frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2} \right]^2 - \left[ \frac{\gamma_b}{2} \left( \frac{\gamma_a}{2} \pm \epsilon_2 \right) + \epsilon_1^2 \right]}. \quad (2.27b)$$

Now setting

$$\frac{d}{dt}A_{\pm}(t) + \eta_{\pm}A_{\pm}(t) = Y_{\pm}(t), \quad (2.28a)$$

we have

$$\frac{d}{dt}Y_{\pm}(t) + \lambda_{\pm}Y_{\pm}(t) = \frac{d}{dt}E_{\pm}(t) + \frac{\gamma_b}{2}E_{\pm}(t) - \epsilon_1 F_{\pm}(t). \quad (2.28b)$$

The solution of this equation can be expressed as

$$Y_{\pm}(t) = Y_{\pm}(0)e^{-\lambda_{\pm}t} + \int_0^t dt' e^{-\lambda_{\pm}(t-t')} \left[ \frac{d}{dt'}E_{\pm}(t') + \frac{\gamma_b}{2}E_{\pm}(t') - \epsilon_1 F_{\pm}(t') \right]. \quad (2.29a)$$

we note that

$$\int_0^t e^{-\lambda_{\pm}(t-t')} \frac{d}{dt'}E_{\pm}(t') dt' = E_{\pm}(t) - E_{\pm}(0)e^{-\lambda_{\pm}t} - \int_0^t \lambda_{\pm} E_{\pm}(t') e^{-\lambda_{\pm}(t-t')} dt', \quad (2.29b)$$

so that using this result in (2.29a), one readily gets

$$\begin{aligned} Y_{\pm}(t) &= [Y_{\pm}(0) - E_{\pm}(0)] e^{-\lambda_{\pm}t} + E_{\pm}(t) \\ &+ e^{-\lambda_{\pm}t} \int_0^t [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\lambda_{\pm}t'} dt'. \end{aligned} \quad (2.29c)$$

Moreover, with the aid of (2.29c), the solution of Eq. (2.28a) can be written as

$$A_{\pm}(t) = A_{\pm}(0)e^{-\eta_{\pm}t} + \left[ \frac{Y_{\pm}(0) - E_{\pm}(0)}{\eta_{\pm} - \lambda_{\pm}} \right] (e^{-\lambda_{\pm}t} - e^{-\eta_{\pm}t}) + \int_0^t E_{\pm}(t') e^{-\eta_{\pm}(t-t')} dt'$$

$$+e^{-\eta_{\pm}t} \int_0^t e^{(\eta_{\pm}-\lambda_{\pm})t''} dt'' \int_0^{t''} [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\lambda_{\pm}t'} dt'. \quad (2.30a)$$

Evaluating the double integral applying integration by parts, one easily finds

$$\begin{aligned} & e^{-\eta_{\pm}t} \int_0^t e^{(\eta_{\pm}-\lambda_{\pm})t''} dt'' \int_0^{t''} [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\lambda_{\pm}t'} dt' \\ &= \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ e^{-\lambda_{\pm}t} \int_0^t [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\lambda_{\pm}t'} dt' \right. \\ & \quad \left. - e^{-\eta_{\pm}t} \int_0^t [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\eta_{\pm}t'} dt' \right]. \end{aligned} \quad (2.30b)$$

Now combination of (2.30a) and (2.30b) leads to

$$\begin{aligned} A_{\pm}(t) &= \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ [(\eta_{\pm} - \lambda_{\pm})A_{\pm}(0) - Y_{\pm}(0) + E_{\pm}(0)] e^{-\eta_{\pm}t} \right. \\ & \quad \left. + [Y_{\pm}(0) - E_{\pm}(0)] e^{-\lambda_{\pm}t} \right. \\ & \quad \left. + e^{-\lambda_{\pm}t} \int_0^t [(\gamma_b/2 - \lambda_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\lambda_{\pm}t'} dt' \right. \\ & \quad \left. - e^{-\eta_{\pm}t} \int_0^t [(\gamma_b/2 - \eta_{\pm})E_{\pm}(t') - \epsilon_1 F_{\pm}(t')] e^{\eta_{\pm}t'} dt' \right]. \end{aligned} \quad (2.31)$$

Furthermore, from (2.22) and (2.28a), we note that

$$Y_{\pm}(0) = [\eta_{\pm} - (\gamma_a/2 \pm \epsilon_2)] A_{\pm}(0) - \epsilon_1 B_{\pm}(0) + E_{\pm}(0), \quad (2.32)$$

so that on substituting this result into (2.31), we have

$$\begin{aligned} A_{\pm}(t) &= \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ \left( [(\gamma_a/2 \pm \epsilon_2) - \lambda_{\pm}] A_{\pm}(0) + \epsilon_1 B_{\pm}(0) \right) e^{-\eta_{\pm}t} \right. \\ & \quad \left. + \left( [\eta_{\pm} - (\gamma_a/2 \pm \epsilon_2)] A_{\pm}(0) - \epsilon_1 B_{\pm}(0) \right) e^{-\lambda_{\pm}t} \right. \\ & \quad \left. + \int_0^t E_{\pm}(t') \left[ \left( \frac{\gamma_b}{2} - \lambda_{\pm} \right) e^{-\lambda_{\pm}(t-t')} - \left( \frac{\gamma_b}{2} - \eta_{\pm} \right) e^{-\eta_{\pm}(t-t')} \right] dt' \right. \\ & \quad \left. + \int_0^t \epsilon_1 F_{\pm}(t') \left[ e^{-\eta_{\pm}(t-t')} - e^{-\lambda_{\pm}(t-t')} \right] dt' \right]. \end{aligned} \quad (2.33a)$$

Using (2.27a) and (2.27b), we see that

$$\gamma_a/2 \pm \epsilon_2 - \lambda_{\pm} = -(\gamma_b/2 - \eta_{\pm}),$$

$$\eta_{\pm} - (\gamma_a/2 \pm \epsilon_2) = (\gamma_b/2 - \lambda_{\pm}),$$

we can rewrite Eq. (2.33a) in the form of

$$\begin{aligned} A_{\pm}(t) = & \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ [(\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}t} - (\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}t}] A_{\pm}(0) \right. \\ & + \epsilon_1 [e^{-\eta_{\pm}t} - e^{-\lambda_{\pm}t}] B_{\pm}(0) \\ & + \int_0^t E_{\pm}(t') [(\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}(t-t')} - (\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}(t-t')}] dt' \\ & \left. + \int_0^t \epsilon_1 F_{\pm}(t') [e^{-\eta_{\pm}(t-t')} - e^{-\lambda_{\pm}(t-t')}] dt' \right]. \end{aligned} \quad (2.33b)$$

Upon setting

$$f_{\pm}(t, t') = \frac{\epsilon_1}{\eta_{\pm} - \lambda_{\pm}} [e^{-\eta_{\pm}(t-t')} - e^{-\lambda_{\pm}(t-t')}] , \quad (2.34a)$$

$$g_{\pm}(t, t') = \frac{1}{\eta_{\pm} - \lambda_{\pm}} [(\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}(t-t')} - (\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}(t-t')}] , \quad (2.34b)$$

$$g'_{\pm}(t, t') = \frac{1}{\eta_{\pm} - \lambda_{\pm}} [(\gamma_b/2 - \lambda_{\pm})e^{-\eta_{\pm}(t-t')} - (\gamma_b/2 - \eta_{\pm})e^{-\lambda_{\pm}(t-t')}] , \quad (2.34c)$$

we have

$$\begin{aligned} A_{\pm}(t) = & -g_{\pm}(t)A_{\pm}(0) + f_{\pm}(t)B_{\pm}(0) \\ & - \int_0^t g_{\pm}(t, t')E_{\pm}(t')dt' + \int_0^t f_{\pm}(t, t')F_{\pm}(t')dt', \end{aligned} \quad (2.35)$$

Combination of (2.15c), (2.15d), (2.19a), (2.19b), (2.21a) and (2.21b) yields

$$A_{\pm}(0) = \left[ a^{\dagger}(0) \pm a(0) - (1 \pm 1) \frac{\epsilon_1}{\kappa} \right], \quad (2.36a)$$

$$B_{\pm}(0) = \left[ b^{\dagger}(0) \pm b(0) - (1 \pm 1) \frac{\epsilon_2}{\kappa} \right]. \quad (2.36b)$$

Then in view of these expressions, we see that

$$\begin{aligned} A_{\pm}(t) = & -g_{\pm}(t) \left[ (a^{\dagger}(0) \pm a(0)) - (1 \pm 1) \frac{\epsilon_1}{\kappa} \right] \\ & + f_{\pm}(t) \left[ (b^{\dagger}(0) \pm b(0)) - (1 \pm 1) \frac{\epsilon_2}{\kappa} \right] \end{aligned}$$

$$- \int_0^t g_{\pm}(t, t') E_{\pm}(t') dt' + \int_0^t f_{\pm}(t, t') F_{\pm}(t') dt'. \quad (2.37a)$$

This can be put in the form [26]

$$\begin{aligned} A_{\pm}(t) &= \left[ \frac{1 \pm 1}{k} \right] [\epsilon_1 g_{\pm}(t) - \epsilon_2 f_{\pm}(t)] - g_{\pm}(t) [a^{\dagger}(0) \pm a(0)] \\ &+ f_{\pm}(t) [b^{\dagger}(0) \pm b(0)] - \int_0^t g_{\pm}(t, t') E_{\pm}(t') dt' + \int_0^t f_{\pm}(t, t') F_{\pm}(t') dt'. \end{aligned} \quad (2.37b)$$

On the other hand, the solution of (2.22b) can be expressed as

$$B_{\pm}(t) = B_{\pm}(0) e^{-\frac{\gamma_b}{2} t} + e^{-\frac{\gamma_b}{2} t} \int_0^t [\epsilon_1 A_{\pm}(t') + F_{\pm}(t')] e^{\frac{\gamma_b}{2} t'} dt', \quad (2.38a)$$

so that with the aid of (2.37b), one obtains

$$\begin{aligned} B_{\pm}(t) &= B_{\pm}(0) e^{-\frac{\gamma_b}{2} t} + \epsilon_1 e^{-\frac{\gamma_b}{2} t} \left[ \left[ \frac{1 \pm 1}{k} \right] \int_0^t (\epsilon_1 g_{\pm}(t'') - \epsilon_2 f_{\pm}(t'')) e^{\frac{\gamma_b}{2} t''} dt'' \right. \\ &- (a^{\dagger}(0) \pm a(0)) \int_0^t g_{\pm}(t'') e^{\frac{\gamma_b}{2} t''} dt'' + (b^{\dagger}(0) \pm b(0)) \int_0^t f_{\pm}(t'') e^{\frac{\gamma_b}{2} t''} dt'' \\ &- \left. \int_0^t e^{\frac{\gamma_b}{2} t''} dt'' \int_0^{t''} g_{\pm}(t'', t') E_{\pm}(t') dt' + \int_0^t e^{\frac{\gamma_b}{2} t''} dt'' \int_0^{t''} f_{\pm}(t'', t') F_{\pm}(t') dt' \right] \\ &+ e^{-\frac{\gamma_b}{2} t} \int_0^t F_{\pm}(t') e^{\frac{\gamma_b}{2} t'} dt'. \end{aligned} \quad (2.38b)$$

Using the same technique as in the previous case, we find

$$\begin{aligned} \int_0^t e^{\gamma_b/2 t''} dt'' \int_0^{t''} g_{\pm}(t'', t') E_{\pm}(t') dt' &= \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ e^{(\gamma_b/2 - \eta_{\pm}) t} \int_0^t E_{\pm}(t') e^{\eta_{\pm} t'} dt' \right. \\ &- \left. e^{(\gamma_b/2 - \lambda_{\pm}) t} \int_0^t E_{\pm}(t') e^{\lambda_{\pm} t'} dt' \right] \end{aligned} \quad (2.39a)$$

and

$$\begin{aligned} \int_0^t e^{\gamma_b/2 t''} dt'' \int_0^{t''} f_{\pm}(t'', t') F_{\pm}(t') dt' &= \frac{\epsilon_1}{\eta_{\pm} - \lambda_{\pm}} \left[ \frac{e^{(\gamma_b/2 - \eta_{\pm}) t}}{\gamma_b/2 - \eta_{\pm}} \int_0^t F_{\pm}(t') e^{\eta_{\pm} t'} dt' \right. \\ &- \left. \frac{e^{(\gamma_b/2 - \lambda_{\pm}) t}}{\gamma_b/2 - \lambda_{\pm}} \int_0^t F_{\pm}(t') e^{\lambda_{\pm} t'} dt' - \frac{\eta_{\pm} - \lambda_{\pm}}{(\gamma_b/2 - \lambda_{\pm})(\gamma_b/2 - \eta_{\pm})} \int_0^t F_{\pm}(t') e^{\frac{\gamma_b}{2} t'} dt' \right]. \end{aligned} \quad (2.39b)$$

Hence substituting (2.39a) and (2.39b) into (2.38b), we easily obtain

$$\begin{aligned}
B_{\pm}(t) &= \frac{1}{\eta_{\pm} - \lambda_{\pm}} \left[ \epsilon_1 \left( \frac{1 \pm 1}{\kappa} \right) \epsilon_1 (e^{-\eta_{\pm} t} - e^{-\lambda_{\pm} t}) \right. \\
&- \epsilon_2 \left( \frac{1 \pm 1}{\kappa} \right) [(\gamma_b/2 - \lambda_{\pm}) e^{-\eta_{\pm} t} - (\gamma_b/2 - \eta_{\pm}) e^{-\lambda_{\pm} t}] \\
&- (a^{\dagger}(0) \pm a(0)) \epsilon_1 (e^{-\eta_{\pm} t} - e^{-\lambda_{\pm} t}) \\
&+ (b^{\dagger}(0) \pm b(0)) [(\gamma_b/2 - \lambda_{\pm}) e^{-\eta_{\pm} t} - (\gamma_b/2 - \eta_{\pm}) e^{-\lambda_{\pm} t}] \\
&+ \int_0^t [(\gamma_b/2 - \lambda_{\pm}) F_{\pm}(t') - \epsilon_1 E_{\pm}(t')] e^{-\eta_{\pm}(t-t')} dt' \\
&\left. - \int_0^t [(\gamma_b/2 - \eta_{\pm}) F_{\pm}(t') - \epsilon_1 E_{\pm}(t')] e^{-\lambda_{\pm}(t-t')} dt' \right]. \tag{2.40a}
\end{aligned}$$

In view of (2.34a), (2.34b) and (2.34c), Eq. (2.40a) turns out to be [26]

$$\begin{aligned}
B_{\pm}(t) &= \left[ \frac{1 \pm 1}{\kappa} \right] [\epsilon_1 f_{\pm}(t) - \epsilon_2 g'_{\pm}(t)] - f_{\pm}(t) [a^{\dagger}(0) \pm a(0)] \\
&+ g'_{\pm}(t) [b^{\dagger}(0) \pm b(0)] - \int_0^t f_{\pm}(t, t') E_{\pm}(t') dt' + \int_0^t g'_{\pm}(t, t') F_{\pm}(t') dt'. \tag{2.40b}
\end{aligned}$$

## 2.2 Squeezing Spectrum

In this section we wish to calculate the squeezing spectrum for the low-and- high frequency modes. To this end, we recall that the output squeezing spectrum is defined by

$$S_{a_{\pm}}(w) = \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau} \langle a_{\pm}^{out}(t + \tau), a_{\pm}^{out}(t) \rangle_{ss}, \tag{2.41}$$

in which

$$\langle a_{\pm}^{out}(t + \tau), a_{\pm}^{out}(t) \rangle = \langle a_{\pm}^{out}(t + \tau) a_{\pm}^{out}(t) \rangle - \langle a_{\pm}^{out}(t + \tau) \rangle \langle a_{\pm}^{out}(t) \rangle. \tag{2.42a}$$

The output and input operators are related by

$$a_{\pm}^{out}(t) = \sqrt{\gamma_a} a_{\pm}(t) - a_{\pm}^{in}(t), \tag{2.42b}$$

where  $a_{\pm}(t)$  is the intracavity mode.

The intracavity quadrature operators  $a_+(t)$  and  $a_-(t)$  are defined by

$$a_+(t) = a^\dagger(t) + a(t), \quad (2.43a)$$

$$a_-(t) = i(a^\dagger(t) - a(t)) \quad (2.43b)$$

and the input quadrature operators  $a_+^{in}(t)$  and  $a_-^{in}(t)$  are defined by

$$a_+^{in}(t) = \frac{1}{\sqrt{\gamma_a}}(F_a^\dagger(t) + F_a(t)) = \frac{1}{\sqrt{\gamma_a}}E_+(t), \quad (2.44a)$$

$$a_-^{in}(t) = \frac{i}{\sqrt{\gamma_a}}(F_a^\dagger(t) - F_a(t)) = \frac{i}{\sqrt{\gamma_a}}E_-(t). \quad (2.44b)$$

On employing (2.42b) along with (2.44) and taking into account the properties of the noise operators defined by (2.3), one easily finds

$$\begin{aligned} \langle a_{\pm}^{out}(t+\tau), a_{\pm}^{out}(t) \rangle &= \pm 2(\cos \theta_a M_a \pm N_a \pm 1/2)\delta(\tau) + \gamma_a \langle a_{\pm}(t+\tau), a_{\pm}(t) \rangle \\ &\quad - \sqrt{\gamma_a} [\langle a_{\pm}(t+\tau) a_{\pm}^{in}(t) \rangle + \langle a_{\pm}^{in}(t+\tau) a_{\pm}(t) \rangle]. \end{aligned} \quad (2.45)$$

Recalling that

$$a_+(t) = 2\alpha_0 + A_+(t), \quad (2.46a)$$

$$a_-(t) = iA_-(t) \quad (2.46b)$$

and taking into account (2.44), we see that

$$\begin{aligned} \langle a_{\pm}^{out}(t+\tau), a_{\pm}^{out}(t) \rangle &= \pm 2(\cos \theta_a M_a \pm N_a \pm 1/2)\delta(\tau) \pm \gamma_a \langle A_{\pm}(t+\tau), A_{\pm}(t) \rangle \\ &\quad \mp [\langle A_{\pm}(t+\tau) E_{\pm}(t) \rangle + \langle E_{\pm}(t+\tau) A_{\pm}(t) \rangle]. \end{aligned} \quad (2.47)$$

We consider the case for which the intracavity modes are initially in vacuum states. We then note that

$$\langle a(0) \rangle = \langle a^\dagger(0) \rangle = \langle b(0) \rangle = \langle b^\dagger(0) \rangle = 0. \quad (2.48a)$$

$$\langle a(0)b(0) \rangle = \langle a(0)b^\dagger(0) \rangle = \langle a^\dagger(0)b^\dagger(0) \rangle = \langle a^\dagger(0)b(0) \rangle = 0. \quad (2.48b)$$

$$\langle (a^\dagger(0) \pm a(0))^2 \rangle = \langle (b^\dagger(0) \pm b(0))^2 \rangle = \pm 1. \quad (2.48c)$$

Now applying (2.37b) along with (2.3), (2.23) and (2.48) one readily obtains

$$\langle A_\pm(t) \rangle = \left[ \frac{1 \pm 1}{k} \right] [\epsilon_1 g_\pm(t) - \epsilon_2 f_\pm(t)], \quad (2.49a)$$

$$\langle A_\pm(t + \tau) \rangle = \left[ \frac{1 \pm 1}{k} \right] [\epsilon_1 g_\pm(t + \tau) - \epsilon_2 f_\pm(t + \tau)] \quad (2.49b)$$

and

$$\langle A_\pm(t + \tau) E_\pm(t) \rangle = - \int_0^{t+\tau} g_\pm(t + \tau, t') \langle E_\pm(t') E_\pm(t) \rangle dt', \quad (2.50a)$$

$$\langle E_\pm(t + \tau) A_\pm(t) \rangle = - \int_0^t g_\pm(t, t') \langle E_\pm(t + \tau) E_\pm(t') \rangle dt', \quad (2.50b)$$

$$\begin{aligned} \langle A_\pm(t + \tau) A_\pm(t) \rangle &= \left[ \frac{1 \pm 1}{k} \right]^2 [\epsilon_1 g_\pm(t + \tau) - \epsilon_2 f_\pm(t + \tau)] [\epsilon_1 g_\pm(t) - \epsilon_2 f_\pm(t)] \\ &\quad \pm [g_\pm(t + \tau) g_\pm(t) + f_\pm(t + \tau) f_\pm(t)] \\ &\quad + \int_0^{t+\tau} \left[ \int_0^t g_\pm(t + \tau, t') g_\pm(t, t'') \langle E_\pm(t') E_\pm(t'') \rangle dt'' \right] dt' \\ &\quad + \int_0^{t+\tau} \left[ \int_0^t f_\pm(t + \tau, t') f_\pm(t, t'') \langle F_\pm(t') F_\pm(t'') \rangle dt'' \right] dt'. \end{aligned} \quad (2.50c)$$

From (2.11) and (2.23) we note that

$$\langle E_\pm(t') E_\pm(t'') \rangle = 2\gamma_a (\cos \theta_a M_a \pm N_a \pm 1/2) \delta(t' - t''), \quad (2.51a)$$

$$\langle F_\pm(t') F_\pm(t'') \rangle = 2\gamma_b (\cos \theta_b M_b \pm N_b \pm 1/2) \delta(t' - t''). \quad (2.51b)$$

On account of these relations, (2.34) and (2.37b), expression (2.50) takes the form

$$\begin{aligned} \langle A_\pm(t + \tau) E_\pm(t) \rangle &= \frac{-2\gamma_a (\cos \theta_a M_a \pm N_a \pm 1/2)}{\eta_\pm - \lambda_\pm} [(\gamma_b/2 - \eta_\pm) e^{-\eta_\pm \tau} \\ &\quad - (\gamma_b/2 - \lambda_\pm) e^{-\lambda_\pm \tau}], \end{aligned} \quad (2.52a)$$

$$\langle E_{\pm}(t + \tau)A_{\pm}(t) \rangle = 0, \quad (2.52b)$$

$$\begin{aligned} \langle A_{\pm}(t + \tau)A_{\pm}(t) \rangle &= \frac{1}{(\eta_{\pm} - \lambda_{\pm})^2} \left[ \pm [(\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}(t+\tau)} \right. \\ &\quad \left. - (\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}(t+\tau)}] [(\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}t} - (\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}t}] \right. \\ &\quad \left. \pm \epsilon_1^2 [e^{-\eta_{\pm}(t+\tau)} - e^{-\lambda_{\pm}(t+\tau)}] [e^{-\eta_{\pm}t} - e^{-\lambda_{\pm}t}] \right. \\ &\quad \left. + \left[ \frac{(1 \pm 1)\epsilon_1}{\kappa} \right]^2 [(\epsilon_2 - \gamma_b/2 + \lambda_{\pm})e^{-\lambda_{\pm}(t+\tau)} - (\epsilon_2 - \gamma_b/2 + \eta_{\pm})e^{-\eta_{\pm}(t+\tau)}] \right. \\ &\quad \left. [(\epsilon_2 - \gamma_b/2 + \lambda_{\pm})e^{-\lambda_{\pm}t} - (\epsilon_2 - \gamma_b/2 + \eta_{\pm})e^{-\eta_{\pm}t}] \right. \\ &\quad \left. + \gamma_a(\cos \theta_a M_a \pm N_a \pm 1/2) \left[ (\gamma_b/2 - \eta_{\pm})e^{-\eta_{\pm}\tau} \left[ \left( \frac{\gamma_b/2 - \eta_{\pm}}{\eta_{\pm}} \right) (1 - e^{-2\eta_{\pm}t}) \right. \right. \right. \\ &\quad \left. \left. - 2 \left( \frac{\gamma_b/2 - \lambda_{\pm}}{\eta_{\pm} + \lambda_{\pm}} \right) (1 - e^{-(\eta_{\pm} + \lambda_{\pm})t}) \right] \right. \\ &\quad \left. + (\gamma_b/2 - \lambda_{\pm})e^{-\lambda_{\pm}\tau} \left[ \left( \frac{\gamma_b/2 - \lambda_{\pm}}{\lambda_{\pm}} \right) (1 - e^{-2\lambda_{\pm}t}) - 2 \left( \frac{\gamma_b/2 - \eta_{\pm}}{\eta_{\pm} + \lambda_{\pm}} \right) (1 - e^{-(\eta_{\pm} + \lambda_{\pm})t}) \right] \right] \\ &\quad \left. + \gamma_b(\cos \theta_b M_b \pm N_b \pm 1/2) \epsilon_1^2 \left[ e^{-\eta_{\pm}\tau} \left[ \frac{1}{\eta_{\pm}} (1 - e^{-2\eta_{\pm}t}) - \frac{2}{\eta_{\pm} + \lambda_{\pm}} (1 - e^{-(\eta_{\pm} + \lambda_{\pm})t}) \right] \right. \right. \\ &\quad \left. \left. + e^{-\lambda_{\pm}\tau} \left[ \frac{1}{\lambda_{\pm}} (1 - e^{-2\lambda_{\pm}t}) - \frac{2}{\eta_{\pm} + \lambda_{\pm}} (1 - e^{-(\eta_{\pm} + \lambda_{\pm})t}) \right] \right] \right]. \quad (2.52c) \end{aligned}$$

Now with the aid of (2.49) along with (2.34) and (2.52c) it can be verified that at steady state

$$\begin{aligned} \langle A_{\pm}(t + \tau), A_{\pm}(t) \rangle_{ss} &= \frac{\gamma_a(\cos \theta_a M_a \pm N_a \pm 1/2)}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm}\tau}}{\lambda_{\pm}} (\frac{\gamma_b^2}{4} - \lambda_{\pm}^2) - \frac{e^{-\eta_{\pm}\tau}}{\eta_{\pm}} (\frac{\gamma_b^2}{4} - \eta_{\pm}^2) \right] \\ &\quad + \frac{\epsilon_1^2 \gamma_b(\cos \theta_b M_b \pm N_b \pm 1/2)}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm}\tau}}{\lambda_{\pm}} - \frac{e^{-\eta_{\pm}\tau}}{\eta_{\pm}} \right], \quad (2.53) \end{aligned}$$

so that substitution of (2.52a), (2.52b) and (2.53) into expression (2.47) yields

$$\begin{aligned} \langle a_{\pm}^{out}(t + \tau), a_{\pm}^{out}(t) \rangle_{ss} &= \pm 2(\cos \theta_a M_a \pm N_a \pm 1/2) \delta(\tau) \\ &\quad \pm \frac{\gamma_a^2(\cos \theta_a M_a \pm N_a \pm 1/2)}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm}\tau}}{\lambda_{\pm}} (\frac{\gamma_b^2}{4} - \lambda_{\pm}^2) - \frac{e^{-\eta_{\pm}\tau}}{\eta_{\pm}} (\frac{\gamma_b^2}{4} - \eta_{\pm}^2) \right] \\ &\quad \pm \frac{\epsilon_1^2 \gamma_a \gamma_b(\cos \theta_b M_b \pm N_b \pm 1/2)}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm}\tau}}{\lambda_{\pm}} - \frac{e^{-\eta_{\pm}\tau}}{\eta_{\pm}} \right] \end{aligned}$$

$$\pm \frac{2\gamma_a(\cos \theta_a M_a \pm N_a \pm 1/2)}{\eta_{\pm} - \lambda_{\pm}} \left[ \left( \frac{\gamma_b}{2} - \eta_{\pm} \right) e^{-\eta_{\pm} \tau} - \left( \frac{\gamma_b}{2} - \lambda_{\pm} \right) e^{-\lambda_{\pm} \tau} \right]. \quad (2.54)$$

Then employing (2.54) in (2.41) and carrying out the integration, taking into account the stationarity property, one easily gets

$$S_{a\pm}(\omega) = \pm 2(\cos \theta_a M_a \pm N_a \pm 1/2) \pm \frac{2\epsilon_1^2 \gamma_a \gamma_b (\cos \theta_b M_b \pm N_b \pm 1/2)}{(\omega^2 + \lambda_{\pm}^2)(\omega^2 + \eta_{\pm}^2)} \\ \pm \frac{2\gamma_a(\cos \theta_a M_a \pm N_a \pm 1/2)}{(\omega^2 + \lambda_{\pm}^2)(\omega^2 + \eta_{\pm}^2)} \left[ \omega^2(\gamma_a + \gamma_b - 2(\eta_{\pm} + \lambda_{\pm})) + \frac{\gamma_a \gamma_b^2}{4} - \gamma_b \eta_{\pm} \lambda_{\pm} \right]. \quad (2.55)$$

Furthermore, with the aid of (2.4) and (2.9) we see that

$$\pm 2(\cos \theta_a M_a \pm N_a \pm 1/2) = (1 \pm \cos \theta_a) \frac{e^{2r_a}}{2} + (1 \mp \cos \theta_a) \frac{e^{-2r_a}}{2} = R_{\pm}(r_a, \theta_a), \quad (2.56a)$$

$$\pm 2(\cos \theta_b M_b \pm N_b \pm 1/2) = (1 \pm \cos \theta_b) \frac{e^{2r_b}}{2} + (1 \mp \cos \theta_b) \frac{e^{-2r_b}}{2} = R_{\pm}(r_b, \theta_b). \quad (2.56b)$$

Finally, on account of (2.27) and (2.56), the expression for the squeezing spectrum takes the form [26]

$$S_{a\pm}(\omega) = \left[ 1 \mp \left[ \frac{2\gamma_a \epsilon_2 (\omega^2 + \frac{\gamma_b^2}{4}) \pm \gamma_a \gamma_b \epsilon_1^2 (1 - \frac{R_{\pm}(r_b, \theta_b)}{R_{\pm}(r_a, \theta_a)})}{[\frac{\gamma_b}{2}(\frac{\gamma_a}{2} \pm \epsilon_2) + \epsilon_1^2 - \omega^2]^2 + \omega^2 (\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2})^2} \right] \right] R_{\pm}(r_a, \theta_a). \quad (2.57)$$

where the + and - refer to the squeezed and unsqueezed quadrature operators, respectively. In addition, following a similar procedure, it can be readily established that for the high frequency mode

$$\langle B_{\pm}(t + \tau), B_{\pm}(t) \rangle_{ss} = \frac{\epsilon_1^2 \gamma_a (\cos \theta_a M_a \pm N_a \pm \frac{1}{2})}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm} \tau}}{\lambda_{\pm}} - \frac{e^{-\eta_{\pm} \tau}}{\eta_{\pm}} \right] \\ + \frac{\gamma_b (\cos \theta_b M_b \pm N_b \pm 1/2)}{(\eta_{\pm} - \lambda_{\pm})^2} \left[ \frac{e^{-\eta_{\pm} \tau}}{\eta_{\pm}} (\gamma_b/2 - \lambda_{\pm})^2 + \frac{e^{-\lambda_{\pm} \tau}}{\lambda_{\pm}} (\gamma_b/2 - \eta_{\pm})^2 \right. \\ \left. - \frac{2(\gamma_b/2 - \lambda_{\pm})(\gamma_b/2 - \eta_{\pm})}{\eta_{\pm} + \lambda_{\pm}} (e^{-\eta_{\pm} \tau} + e^{-\lambda_{\pm} \tau}) \right], \quad (2.58)$$

$$\langle B_{\pm}(t + \tau) F_{\pm}(t) \rangle_{ss} = - \frac{2\gamma_b (\cos \theta_b M_b \pm N_b \pm 1/2)}{\eta_{\pm} - \lambda_{\pm}} \left[ (\gamma_b/2 - \eta_{\pm}) e^{-\lambda_{\pm} \tau} \right. \\ \left. - (\gamma_b/2 - \lambda_{\pm}) e^{-\eta_{\pm} \tau} \right], \quad (2.59a)$$

$$\langle F_{\pm}(t + \tau)B_{\pm}(t) \rangle = 0. \quad (2.59b)$$

In view of these results and (2.51), we have

$$\begin{aligned} \langle b_{\pm}^{out}(t + \tau), b_{\pm}^{out}(t) \rangle_{ss} &= R_{\pm}(r_b, \theta_b) \delta(\tau) + \gamma_b \langle B_{\pm}(t + \tau), B_{\pm}(t) \rangle_{ss} \\ &\mp [\langle B_{\pm}(t + \tau)F_{\pm}(t) \rangle_{ss} + \langle F_{\pm}(t + \tau)B_{\pm}(t) \rangle_{ss}], \end{aligned} \quad (2.60a)$$

so that on account of (2.58) and (2.59) we have

$$\begin{aligned} \langle b_{\pm}^{out}(t + \tau), b_{\pm}^{out}(t) \rangle_{ss} &= R_{\pm}(r_b, \theta_b) \delta(\tau) + \frac{R_{\pm}(r_b, \theta_b) \gamma_b}{(\eta_{\pm} - \lambda_{\pm})^2} \left[ \frac{e^{-\eta_{\pm} \tau}}{\eta_{\pm}} (\gamma_b/2 - \lambda_{\pm}) [\gamma_b/2 (\gamma_b/2 - \lambda_{\pm}) \right. \\ &\quad \left. - \eta_{\pm} (\eta_{\pm} - \lambda_{\pm})] + \frac{e^{-\lambda_{\pm} \tau}}{\lambda_{\pm}} (\gamma_b/2 - \eta_{\pm}) [\gamma_b/2 (\gamma_b/2 - \eta_{\pm}) + \lambda_{\pm} (\eta_{\pm} - \lambda_{\pm})] \right. \\ &\quad \left. - 2 \frac{\gamma_b/2 (\gamma_b/2 - \lambda_{\pm}) (\gamma_b/2 - \eta_{\pm})}{\eta_{\pm} + \lambda_{\pm}} (e^{-\lambda_{\pm} \tau} + e^{-\eta_{\pm} \tau}) \right] \\ &\quad + \frac{\gamma_a \gamma_b}{2} \frac{\epsilon_1^2 R_{\pm}(r_a, \theta_a)}{\eta_{\pm}^2 - \lambda_{\pm}^2} \left[ \frac{e^{-\lambda_{\pm} \tau}}{\lambda_{\pm}} - \frac{e^{-\eta_{\pm} \tau}}{\eta_{\pm}} \right]. \end{aligned} \quad (2.60b)$$

Hence the squeezing spectrum at steady state turns out to be

$$\begin{aligned} S_{b\pm}(\omega) &= \left[ 1 + \left[ \frac{\gamma_b [\gamma_b (\eta_{\pm} + \lambda_{\pm} - \gamma_b/2) - 2\eta_{\pm} \lambda_{\pm}] [\eta_{\pm} + \lambda_{\pm} - \gamma_b/2] + \gamma_a \gamma_b \epsilon_1^2 \frac{R_{\pm}(r_a, \theta_a)}{R_{\pm}(r_b, \theta_b)}}{(\omega^2 + \lambda_{\pm}^2)(\omega^2 + \eta_{\pm}^2)} \right] \right] \\ &\quad \times R_{\pm}(r_b, \theta_b), \end{aligned} \quad (2.61a)$$

Now on substituting the explicit forms of  $\eta_{\pm}$  and  $\lambda_{\pm}$ , there follows [26]

$$S_{b\pm}(\omega) = \left[ 1 \mp \left[ \frac{2\epsilon_2 \epsilon_1^2 \gamma_b \pm \gamma_a \gamma_b \epsilon_1^2 (1 - \frac{R_{\pm}(r_a, \theta_a)}{R_{\pm}(r_b, \theta_b)})}{[\gamma_b/2 (\gamma_a/2 \pm \epsilon_2) + \epsilon_1^2 - \omega^2]^2 + \omega^2 [\gamma_a/2 \pm \epsilon_2 + \gamma_b/2]^2} \right] \right] R_{\pm}(r_b, \theta_b). \quad (2.61b)$$

On account of the relation (2.14b) along with (2.56a) and (2.56b) with  $\theta_a = \theta_b = \pi$ , expressions (2.57) and (2.61b) take the form

$$S_{a\pm}(\omega) = \left[ 1 \mp \left[ \frac{2\gamma_a \epsilon_2 [(\omega^2 + \gamma_b^2/4) \pm \gamma_b^2/2 (1 - e^{\mp 2(r_b - r_a)})]}{[\gamma_b/2 (\gamma_a/2 + \epsilon_2 (2 \pm 1)) - \omega^2]^2 + \omega^2 (\frac{\gamma_a \pm \gamma_b}{2} \pm \epsilon_2)^2} \right] \right] e^{\mp 2r_a}, \quad (2.62a)$$

$$S_{b\pm}(\omega) = \left[ 1 \mp \left[ \frac{2\gamma_b^2 \epsilon_2 [\epsilon_2 \pm \gamma_a/2 (1 - e^{\mp 2(r_a - r_b)})]}{[\gamma_b/2 (\gamma_a/2 + \epsilon_2 (2 \pm 1)) - \omega^2]^2 + \omega^2 (\frac{\gamma_a \pm \gamma_b}{2} \pm \epsilon_2)^2} \right] \right] e^{\mp 2r_b}. \quad (2.62b)$$

We see that for ordinary vacuum reservoir (2.62a) and (2.62b) reduce to

$$S_{a\pm}(\omega) = 1 \mp \frac{2\epsilon_2\gamma_a(\omega^2 + \frac{\gamma_b^2}{4})}{[\gamma_b/2(\gamma_a/2 \pm \epsilon_2) + \epsilon_1^2 - \omega^2]^2 + \omega^2(\gamma_a/2 \pm \epsilon_2 + \gamma_b/2)^2}, \quad (2.63a)$$

$$S_{b\pm}(\omega) = 1 \mp \frac{2\epsilon_2\epsilon_1^2\gamma_b}{[\gamma_b/2(\gamma_a/2 \pm \epsilon_2) + \epsilon_1^2 - \omega^2]^2 + \omega^2(\gamma_a/2 \pm \epsilon_2 + \gamma_b/2)^2}. \quad (2.63b)$$

The squeezing spectra defined by (2.63a) and (2.63b) are in complete agreement with the results found by Walls and Milburn [14].

We now proceed to determine the squeezing spectra at the critical point. At the critical point

$$\epsilon_2 = \frac{\gamma_a}{2} + \frac{\gamma_b}{2}. \quad (2.64a)$$

and the critical frequency is

$$\omega_c^2 = \epsilon_1^2 + \gamma_b/2(\gamma_a/2 - \epsilon_2), \quad (2.64b)$$

so that substituting (2.64a) and (2.14b) into (2.64b), one obtains

$$\omega_c^2 = \frac{\gamma_b}{4}(2\gamma_a + \gamma_b). \quad (2.64c)$$

On account of (2.64a) and (2.14b), the squeezing spectra for the squeezed quadratures for the low and high frequency modes at the critical point turn out to be

$$S_{a+}(w) = \left[ 1 - \left[ \frac{\gamma_a(\gamma_a + \gamma_b) [(\omega^2 + \gamma_b^2/4) + \gamma_b^2/2(1 - e^{-2(r_b - r_a)})]}{[\gamma_b(\gamma_a + \gamma_b) - \gamma_b^2/4 - \omega^2]^2 + \omega^2(\gamma_a + \gamma_b)^2} \right] \right] e^{-2r_a}. \quad (2.65)$$

$$S_{b+}(w) = \left[ 1 - \left[ \frac{\gamma_b^2/2(\gamma_a + \gamma_b) [\gamma_a + \gamma_b + \gamma_a(1 - e^{-2(r_a - r_b)})]}{[\gamma_b(\gamma_a + \gamma_b) - \gamma_b^2/4 - \omega^2]^2 + \omega^2(\gamma_a + \gamma_b)^2} \right] \right] e^{-2r_b}. \quad (2.66)$$

At the critical point and critical frequency the squeezing spectrum for the unsqueezed quadrature diverges. However, for the squeezed quadrature we have

$$S_{a+}(\omega_c) = \left[ 1 - \left[ \frac{\gamma_a [\gamma_a + \gamma_b [2 - e^{-2(r_b - r_a)}]]}{(\gamma_a + \gamma_b)^2} \right] \right] e^{-2r_a}. \quad (2.67a)$$

Similarly, one finds

$$S_{b+}(\omega_c) = \left[ 1 - \left[ \frac{\gamma_b [\gamma_b + \gamma_a [2 - e^{-2(r_a - r_b)}]]}{(\gamma_a + \gamma_b)^2} \right] \right] e^{-2r_b}. \quad (2.67b)$$

For an ordinary vacuum, expressions (2.67a) and (2.67b) reduce to

$$S_{a+}(\omega_c) = 1 - \frac{\gamma_a}{\gamma_a + \gamma_b}, \quad (2.68a)$$

$$S_{b+}(\omega_c) = 1 - \frac{\gamma_b}{\gamma_a + \gamma_b}. \quad (2.68b)$$

These results are in complete agreement with the results obtained by Walls and Milburn [14].

If the damping rate for the high frequency mode is much smaller than that of low frequency mode ( $\gamma_b \ll \gamma_a$ ), perfect squeezing is approached in the low frequency mode at  $\omega_c = \pm\sqrt{2\gamma_a\gamma_b}$ . On the other hand, if  $\gamma_b \gg \gamma_a$ , perfect squeezing is approached in the high frequency mode at  $\omega_c = \pm\gamma_b$ . The critical frequency is not, in fact, the one giving the greatest squeezing except in the above limiting cases [14].

### 2.3 The Mean Photon Number

In this section we seek to find the mean photon number of both the low-frequency and high-frequency modes. To this end, recalling that

$$a_+(t) = a^\dagger(t) + a(t)$$

and

$$a_-(t) = i(a^\dagger(t) - a(t)),$$

we have

$$(\Delta a_+(t))^2 = \langle (a_+(t))^2 \rangle - \langle a_+(t) \rangle^2, \quad (2.69a)$$

$$(\Delta a_-(t))^2 = \langle (a_-(t))^2 \rangle - \langle a_-(t) \rangle^2. \quad (2.69b)$$

Applying the definitions of the quadrature operators, we have

$$\langle (a_+(t))^2 \rangle = \langle (a^\dagger(t))^2 \rangle - \langle (a(t))^2 \rangle + 2\langle a^\dagger(t)a(t) \rangle + 1, \quad (2.70a)$$

$$\langle (a_+(t))^2 \rangle = \langle (a^\dagger(t))^2 \rangle - \langle (a(t))^2 \rangle + 2\langle a^\dagger(t) \rangle \langle a(t) \rangle + 1. \quad (2.70b)$$

Hence using (2.70a) and (2.70b) in Eq. (2.69a) and (2.69b), respectively one gets

$$(\Delta a_+(t))^2 = (\Delta a^\dagger(t))^2 + (\Delta a(t))^2 + 2\langle a^\dagger(t)a(t) \rangle - 2\langle a^\dagger(t) \rangle \langle a(t) \rangle + 1, \quad (2.71a)$$

$$(\Delta a_-(t))^2 = -(\Delta a^\dagger(t))^2 - (\Delta a(t))^2 + 2\langle a^\dagger(t)a(t) \rangle - 2\langle a^\dagger(t) \rangle \langle a(t) \rangle + 1. \quad (2.71b)$$

Adding (2.71a) and (2.71b) gives us

$$\langle a^\dagger(t)a(t) \rangle = \frac{1}{4} \left[ (\Delta a_+(t))^2 + (\Delta a_-(t))^2 + 4\langle a^\dagger(t) \rangle \langle a(t) \rangle - 2 \right]. \quad (2.72a)$$

Similarly, one finds for the high frequency mode

$$\langle b^\dagger(t)b(t) \rangle = \frac{1}{4} \left[ (\Delta b_+(t))^2 + (\Delta b_-(t))^2 + 4\langle b^\dagger(t) \rangle \langle b(t) \rangle - 2 \right]. \quad (2.72b)$$

Next we wish to obtain the variance of the intracavity quadrature operators for these two modes. recalling that

$$a_+(t) = 2\alpha_0 + A_+(t),$$

$$a_-(t) = iA_-(t),$$

one obtains

$$\langle a_\pm(t), a_\pm(t) \rangle = \pm \langle A_\pm(t), A_\pm(t) \rangle. \quad (2.73a)$$

Now setting  $\tau = 0$  in (2.53), one finds

$$\begin{aligned} \langle a_\pm(t), a_\pm(t) \rangle_{ss} &= \pm \frac{\gamma_a (\cos \theta_a M_a \pm N_a \pm \frac{1}{2})}{(\eta_\pm - \lambda_\pm)^2} \left[ \frac{1}{\lambda_\pm} \left( \frac{\gamma_b^2}{4} - \lambda_\pm^2 \right) - \frac{1}{\eta_\pm} \left( \frac{\gamma_b^2}{4} - \eta_\pm^2 \right) \right] \\ &\quad \pm \frac{\epsilon_1^2 \gamma_b (\cos \theta_b M_b \pm N_b \pm \frac{1}{2})}{\eta_\pm^2 - \lambda_\pm^2} \left[ \frac{1}{\lambda_\pm} - \frac{1}{\eta_\pm} \right]. \end{aligned} \quad (2.73b)$$

Employing (2.73b) along with (2.27) and (2.56), one obtains for the low frequency mode

$$\Delta a_{\pm ss}^2 = \left[ \frac{\frac{\gamma_a \gamma_b}{4} (\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2}) + \frac{1}{2} (\gamma_a + \gamma_b) \epsilon_1^2 - \epsilon_1^2 \frac{\gamma_b}{2} (1 - \frac{R_{\pm}(r_b, \theta_b)}{R_{\pm}(r_a, \theta_a)})}{[\frac{\gamma_b}{2} (\frac{\gamma_a}{2} \pm \epsilon_2) + \epsilon_1^2] [\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2}]} \right] R_{\pm}(r_a, \theta_a), \quad (2.74)$$

similarly, for the high frequency mode, we have

$$\Delta b_{\pm ss}^2 = \left[ \frac{\frac{\gamma_b}{2} (\frac{\gamma_a}{2} \pm \epsilon_2) (\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2}) + \frac{1}{2} (\gamma_a + \gamma_b) \epsilon_1^2 - \frac{\gamma_a}{2} \epsilon_1^2 (1 - \frac{R_{\pm}(r_a, \theta_a)}{R_{\pm}(r_b, \theta_b)})}{[\frac{\gamma_b}{2} (\frac{\gamma_a}{2} \pm \epsilon_2) + \epsilon_1^2] [\frac{\gamma_a}{2} \pm \epsilon_2 + \frac{\gamma_b}{2}]} \right] R_{\pm}(r_b, \theta_b). \quad (2.75)$$

Furthermore, taking into account (2.74), (2.14b) and (2.56) with  $\theta_a = \theta_b = \pi$ , we find that the variance of the intracavity quadrature operators for these modes have the form

$$\Delta a_{\pm ss}^2 = \left[ \frac{\gamma_a/2 [\frac{\gamma_a + \gamma_b}{2} + \epsilon_2(2 \pm 1)] + \gamma_b \epsilon_2 e^{\mp 2(r_b - r_a)}}{[\gamma_a/2 + \epsilon_2(2 \pm 1)] [\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2]} \right] e^{\mp 2r_a}, \quad (2.76a)$$

$$\Delta b_{\pm ss}^2 = \left[ \frac{(\gamma_a/2 \pm \epsilon_2) (\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2) + \epsilon_2 [\gamma_b + \gamma_a e^{\mp 2(r_a - r_b)}]}{[\gamma_a/2 + \epsilon_2(2 \pm 1)] [\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2]} \right] e^{\mp 2r_b}, \quad (2.76b)$$

For ordinary vacuum reservoir, these equations reduce to

$$\Delta a_{\pm ss}^2 = \frac{\gamma_a/2 [\frac{\gamma_a + \gamma_b}{2} + \epsilon_2(2 \pm 1)] + \gamma_b \epsilon_2}{[\gamma_a/2 + \epsilon_2(2 \pm 1)] [\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2]} \quad (2.77a)$$

and

$$\Delta b_{\pm ss}^2 = \frac{(\gamma_a/2 \pm \epsilon_2) (\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2) + \epsilon_2 (\gamma_a + \gamma_b)}{[\gamma_a/2 + \epsilon_2(2 \pm 1)] [\frac{\gamma_a + \gamma_b}{2} \pm \epsilon_2]}, \quad (2.77b)$$

so that at the critical point the variance of the squeezed quadrature operators turn out to be

$$\Delta a_{+ss}^2 = \left[ \frac{2\gamma_a + \gamma_b e^{-2(r_b - r_a)}}{4\gamma_a + 3\gamma_b} \right] e^{-2r_a}, \quad (2.78a)$$

$$\Delta b_{+ss}^2 = \left[ \frac{2\gamma_a [1 + 1/2 e^{-2(r_a - r_b)}] + \gamma_b}{4\gamma_a + 3\gamma_b} \right] e^{-2r_b}. \quad (2.78b)$$

But  $\Delta a_{-ss}^2$  and  $\Delta b_{-ss}^2$  diverge. It is easy to check that for  $r_a = r_b = r$  expressions (2.78a) and (2.78b) reduce to

$$\Delta a_{+ss}^2 = \frac{2\gamma_a + \gamma_b}{4\gamma_a + 3\gamma_b} e^{-2r}, \quad (2.79a)$$

$$\Delta b_{+ss}^2 = \frac{3\gamma_a + \gamma_b}{4\gamma_a + 3\gamma_b} e^{-2r}. \quad (2.79b)$$

If we consider the case for which  $\gamma_a = \gamma_b = \gamma$ , we have

$$\Delta a_{+ss}^2 = \frac{3}{7} e^{-2r}, \quad (2.80a)$$

$$\Delta b_{+ss}^2 = \frac{4}{7} e^{-2r}. \quad (2.80b)$$

These results are independent of the driving field amplitude and the cavity damping rates.

For an ordinary vacuum reservoir

$$\Delta a_{+ss}^2 = \frac{3}{7}, \quad (2.81a)$$

$$\Delta b_{+ss}^2 = \frac{4}{7}. \quad (2.81b)$$

On account of (2.72a), (2.76a), (2.15a) and (2.15c), the mean photon number for the low frequency mode turns out to be

$$\begin{aligned} \langle a^+ a \rangle_{ss} &= \frac{1}{4} \left[ \frac{\left[ \gamma_a/2 \left[ \frac{\gamma_a + \gamma_b}{2} + 3\epsilon_2 \right] + \gamma_b \epsilon_2 e^{-2(r_b - r_a)} \right]}{[\gamma_a/2 + 3\epsilon_2] \left[ \frac{\gamma_a + \gamma_b}{2} + \epsilon_2 \right]} e^{-2r_a} \right. \\ &\quad \left. + \frac{\left[ \gamma_a/2 \left[ \frac{\gamma_a + \gamma_b}{2} + \epsilon_2 \right] + \gamma_b \epsilon_2 e^{2(r_b - r_a)} \right]}{[\gamma_a/2 + \epsilon_2] \left[ \frac{\gamma_a + \gamma_b}{2} - \epsilon_2 \right]} e^{2r_a} + 4 \frac{\epsilon_1^2}{\kappa^2} - 2 \right]. \end{aligned} \quad (2.82a)$$

Moreover, using (2.72b), (2.76b), (2.15b) and (2.15d) we have

$$\begin{aligned} \langle b^+ b \rangle_{ss} &= \frac{1}{4} \left[ \frac{\left[ (\gamma_a/2 + \epsilon_2) \left( \frac{\gamma_a + \gamma_b}{2} + \epsilon_2 \right) + \epsilon_2 \left[ \gamma_b + \gamma_a e^{-2(r_a - r_b)} \right] \right]}{[\gamma_a/2 + 3\epsilon_2] \left[ \frac{\gamma_a + \gamma_b}{2} + \epsilon_2 \right]} e^{-2r_b} \right. \\ &\quad \left. + \frac{\left[ (\gamma_a/2 - \epsilon_2) \left( \frac{\gamma_a + \gamma_b}{2} - \epsilon_2 \right) + \epsilon_2 \left[ \gamma_b + \gamma_a e^{2(r_a - r_b)} \right] \right]}{[\gamma_a/2 + \epsilon_2] \left[ \frac{\gamma_a + \gamma_b}{2} - \epsilon_2 \right]} e^{2r_b} + 4 \frac{\epsilon_2^2}{\kappa^2} - 2 \right]. \end{aligned} \quad (2.82b)$$

It is easy to see that at the critical point, the mean photon number of these two modes diverges. The method of deriving the squeezing spectrum from the linearized equations of motion breaks down at the critical point because the fluctuations in some quadrature diverge at this critical point, rendering the linearization procedure invalid. However, the squeezing spectrum so derived at critical point is still useful; it gives an upper limit to the amount of squeezing possible and also is a measure of the squeezing that will be observed in the region where the linearization procedure is valid [25].

We now consider the case for which

$$\gamma_a = \gamma_b = \gamma, \quad (2.83a)$$

$$r_a = r_b = r. \quad (2.83b)$$

With the aid of (2.83a) and (2.83b), one can write Eqs. (2.82a) and (2.82b) in the form

$$\begin{aligned} \langle a^\dagger a \rangle_{ss} &= \frac{1}{4} \left[ \frac{\gamma/2(\gamma + 3\epsilon_2) + \gamma\epsilon_2}{(\gamma/2 + 3\epsilon_2)(\gamma + \epsilon_2)} e^{-2r} \right. \\ &\quad \left. + \frac{\gamma/2(\gamma + \epsilon_2) + \gamma\epsilon_2}{(\gamma/2 + \epsilon_2)(\gamma - \epsilon_2)} e^{2r} + 4\frac{\gamma\epsilon_2}{\kappa^2} - 2 \right] \end{aligned} \quad (2.84a)$$

$$\begin{aligned} \langle b^\dagger b \rangle_{ss} &= \frac{1}{4} \left[ \frac{(\gamma/2 + \epsilon_2)(\gamma + \epsilon_2) + 2\gamma\epsilon_2}{(\gamma/2 + 3\epsilon_2)(\gamma + \epsilon_2)} e^{-2r} \right. \\ &\quad \left. + \frac{\gamma/2 - \epsilon_2)(\gamma - \epsilon_2) + 2\gamma\epsilon_2}{(\gamma/2 + \epsilon_2)(\gamma - \epsilon_2)} e^{2r} + 4\frac{\epsilon_2^2}{\kappa^2} - 2 \right]. \end{aligned} \quad (2.84b)$$

Combining (2.72a), (2.76a), (2.15a) and (2.15c), the mean photon number for the low-frequency mode can be expressed as

$$\begin{aligned} \langle a^\dagger a \rangle_{ss} &= \frac{1}{4} \left[ \frac{\gamma(\gamma + 5\epsilon_2)(-2M + 2N + 1)}{(\gamma + \epsilon_2)(\gamma + 6\epsilon_2)} + \frac{\gamma(\gamma + 3\epsilon_2)(2M + 2N + 1)}{(\gamma - \epsilon_2)(\gamma + 2\epsilon_2)} \right. \\ &\quad \left. + \frac{4\gamma\epsilon_2}{\kappa^2} - 2 \right]. \end{aligned} \quad (2.85a)$$

Moreover, Using (2.72b), (2.76b), (2.15b) and (2.15d) we have

$$\langle b^\dagger b \rangle_{ss} = \frac{1}{4} \left[ \frac{(\gamma^2 + 2\epsilon_2^2 + 7\gamma\epsilon_2)(-2M + 2N + 1)}{(\gamma + \epsilon_2)(\gamma + 6\epsilon_2)} + \frac{(\gamma^2 + 2\epsilon_2^2 + \gamma\epsilon_2)(2M + 2N + 1)}{(\gamma - \epsilon_2)(\gamma + 2\epsilon_2)} \right]$$

$$+4 \frac{\epsilon_2^2}{\kappa^2} - 2 \Big]. \quad (2.85b)$$

For ordinary vacuum reservoirs Eqs. (2.85a) and (2.85b) therefore reduce to

$$\langle a^\dagger a \rangle_{ss} = \frac{2\epsilon_2^2(\gamma^2 + 3\gamma\epsilon_2 + 3\epsilon_2^2)}{(\gamma + \epsilon_2)(\gamma - \epsilon_2)(\gamma + 6\epsilon_2)(\gamma + 2\epsilon_2)} + \frac{\gamma\epsilon_2}{\kappa^2}, \quad (2.86a)$$

$$\langle b^\dagger b \rangle_{ss} = \frac{2\epsilon_2^3(4\epsilon_2 + 3\gamma)}{2(\gamma + \epsilon_2)(\gamma + 6\epsilon_2)(\gamma - \epsilon_2)(\gamma + 2\epsilon_2)} + \frac{\epsilon_2^2}{\kappa^2}. \quad (2.86b)$$

Upon setting  $\gamma_a = \gamma_b = \gamma$  in Eq. (2.18), one gets

$$\epsilon_2 = \frac{\kappa^2 \epsilon^2}{\gamma(\epsilon_2^2 + \gamma\epsilon_2 + \frac{1}{4}\gamma^2)}. \quad (2.87)$$

Substituting this result into (2.86a) and (2.86b) yields

$$\langle a^\dagger a \rangle_{ss} = \frac{2\epsilon_2^2(\gamma^2 + 3\gamma\epsilon_2 + 3\epsilon_2^2)}{(\gamma + \epsilon_2)(\gamma - \epsilon_2)(\gamma + 6\epsilon_2)(\gamma + 2\epsilon_2)} + \frac{\epsilon^2}{\epsilon_2^2 + \gamma\epsilon_2 + \frac{1}{4}\gamma^2} \quad (2.88a)$$

and

$$\langle b^\dagger b \rangle_{ss} = \frac{2\epsilon_2^3(4\epsilon_2 + 3\gamma)}{2(\gamma + \epsilon_2)(\gamma + 6\epsilon_2)(\gamma - \epsilon_2)(\gamma + 2\epsilon_2)} + \frac{\kappa^2 \epsilon^4}{(\gamma\epsilon_2^2 + \gamma^2\epsilon_2 + \frac{1}{4}\gamma^3)^2}. \quad (2.88b)$$

When  $\kappa = 0$ , these equations reduce to

$$\langle a^\dagger a \rangle_{ss} = \frac{\epsilon^2}{(\gamma/2)^2}, \quad (2.89a)$$

$$\langle b^\dagger b \rangle_{ss} = 0. \quad (2.89b)$$

### 3. Nondegenerate Second Harmonic Generation

In the process of non-degenerate second harmonic generation, two modes with frequency  $\omega_1$  and  $\omega_2$  combine in a nonlinear medium placed inside a cavity to create a second harmonic mode of frequency  $\omega$  with  $\omega = \omega_1 + \omega_2$ . In this chapter we analyze the squeezing spectra for the sum and difference of the fundamental modes.

#### 3.1 Linearized Langevin Equations

The Hamiltonian describing this process in the interaction picture is given by

$$H(t) = i\varepsilon_1(a_1^\dagger - a_1) + i\varepsilon_2(a_2^\dagger - a_2) + i\kappa(a_1 a_2 a_3^\dagger - a_1^\dagger a_2^\dagger a_3), \quad (3.1)$$

where  $a_1$  and  $a_2$  are the annihilation operators for the two subharmonic (fundamental) modes of frequency  $\omega_1$  and  $\omega_2$ , respectively and  $a_3$  is the annihilation operator for the light mode of frequency  $\omega$ ,  $\kappa$  is the coupling constant and  $\varepsilon_1$  and  $\varepsilon_2$  are proportional to the amplitude of the coherent driving fields.

The quantum Langevin equations of evolution for the three cavity modes  $a_1$ ,  $a_2$  and  $a_3$  are given by

$$\frac{d}{dt}a_1(t) = -i[a_1(t), H(t)] - \frac{\gamma_1}{2}a_1(t) + F_1(t), \quad (3.2a)$$

$$\frac{d}{dt}a_2(t) = -i[a_2(t), H(t)] - \frac{\gamma_2}{2}a_2(t) + F_2(t), \quad (3.2b)$$

$$\frac{d}{dt}a_3(t) = -i[a_3(t), H(t)] - \frac{\gamma_3}{2}a_3(t) + F_3(t), \quad (3.2c)$$

where  $F_j$  are the noise operators and  $\gamma_j$  are cavity damping rates with  $j=1, 2, 3$ .

We note that

$$[a_1(t), H(t)] = i(\varepsilon_1 - \kappa a_2^\dagger a_3), \quad (3.3a)$$

$$[a_2(t), H(t)] = i(\varepsilon_2 - \kappa a_1^\dagger a_3), \quad (3.3b)$$

$$[a_3(t), H(t)] = i\kappa a_1 a_2. \quad (3.3c)$$

Substituting (3.3a), (3.3b) and (3.3c) into (3.2a), (3.2b) and (3.2c), respectively we have

$$\frac{d}{dt}a_1(t) = \varepsilon_1 - \kappa a_2^\dagger(t)a_3(t) - \frac{\gamma_1}{2}a_1(t) + F_1(t), \quad (3.4a)$$

$$\frac{d}{dt}a_1^\dagger(t) = \varepsilon_1^* - \kappa a_2(t)a_3^\dagger(t) - \frac{\gamma_1}{2}a_1^\dagger(t) + F_1^\dagger(t), \quad (3.4b)$$

$$\frac{d}{dt}a_2(t) = \varepsilon_2 - \kappa a_1^\dagger(t)a_3(t) - \frac{\gamma_2}{2}a_2(t) + F_2(t), \quad (3.4c)$$

$$\frac{d}{dt}a_2^\dagger(t) = \varepsilon_2^* - \kappa a_1(t)a_3^\dagger(t) - \frac{\gamma_2}{2}a_2^\dagger(t) + F_2^\dagger(t), \quad (3.4d)$$

$$\frac{d}{dt}a_3(t) = \kappa a_1(t)a_2(t) - \frac{\gamma_3}{2}a_3(t) + F_3(t), \quad (3.4e)$$

$$\frac{d}{dt}a_3^\dagger(t) = \kappa a_1^\dagger(t)a_2^\dagger(t) - \frac{\gamma_3}{2}a_3^\dagger(t) + F_3^\dagger(t). \quad (3.4f)$$

Assuming that  $\gamma_1 = \gamma_2 = \gamma$  and  $\varepsilon_1 = \varepsilon_2 = \varepsilon$ , Eqs. (3.4) can be rewritten as

$$\frac{d}{dt}a_1(t) = \varepsilon - \kappa a_2^\dagger(t)a_3(t) - \frac{\gamma}{2}a_1(t) + F_1(t), \quad (3.5a)$$

$$\frac{d}{dt}a_1^\dagger(t) = \varepsilon^* - \kappa a_2(t)a_3^\dagger(t) - \frac{\gamma}{2}a_1^\dagger(t) + F_1^\dagger(t), \quad (3.5b)$$

$$\frac{d}{dt}a_2(t) = \varepsilon - \kappa a_1^\dagger(t)a_3(t) - \frac{\gamma}{2}a_2(t) + F_2(t), \quad (3.5c)$$

$$\frac{d}{dt}a_2^\dagger(t) = \varepsilon^* - \kappa a_1(t)a_3^\dagger(t) - \frac{\gamma}{2}a_2^\dagger(t) + F_2^\dagger(t), \quad (3.5d)$$

$$\frac{d}{dt}a_3(t) = \kappa a_1(t)a_2(t) - \frac{\gamma_3}{2}a_3(t) + F_3(t), \quad (3.5e)$$

$$\frac{d}{dt}a_3^\dagger(t) = \kappa a_1^\dagger(t)a_2^\dagger(t) - \frac{\gamma_3}{2}a_3^\dagger(t) + F_3^\dagger(t). \quad (3.5f)$$

Now we seek to determine the equation of evolution of the quadrature operators for the harmonic mode and the sum and difference of the fundamental modes. To this end, taking the expectation value of these equations, we see that

$$\frac{d}{dt}\langle a_1(t) \rangle = \varepsilon - \kappa \langle a_2^\dagger(t)a_3(t) \rangle - \frac{\gamma}{2}\langle a_1(t) \rangle + \langle F_1(t) \rangle, \quad (3.6a)$$

$$\frac{d}{dt}\langle a_1^\dagger(t) \rangle = \varepsilon^* - \kappa \langle a_2(t)a_3^\dagger(t) \rangle - \frac{\gamma}{2}\langle a_1^\dagger(t) \rangle + \langle F_1^\dagger(t) \rangle, \quad (3.6b)$$

$$\frac{d}{dt}\langle a_2(t) \rangle = \varepsilon - \kappa\langle a_1^\dagger(t)a_3(t) \rangle - \frac{\gamma}{2}\langle a_2(t) \rangle + \langle F_2(t) \rangle, \quad (3.6c)$$

$$\frac{d}{dt}\langle a_2^\dagger(t) \rangle = \varepsilon^* - \kappa\langle a_1(t)a_3^\dagger(t) \rangle - \frac{\gamma}{2}\langle a_2^\dagger(t) \rangle + \langle F_2^\dagger(t) \rangle, \quad (3.6d)$$

$$\frac{d}{dt}\langle a_3(t) \rangle = \kappa\langle a_1(t)a_2(t) \rangle - \frac{\gamma_3}{2}\langle a_3(t) \rangle + \langle F_3(t) \rangle, \quad (3.6e)$$

$$\frac{d}{dt}\langle a_3^\dagger(t) \rangle = \kappa\langle a_1^\dagger(t)a_2^\dagger(t) \rangle - \frac{\gamma_3}{2}\langle a_3^\dagger(t) \rangle + \langle F_3^\dagger(t) \rangle. \quad (3.6f)$$

Using the property of the noise operators defined by (2.11), we find at a steady state

$$\varepsilon - \kappa\langle a_2^\dagger(t)a_3(t) \rangle - \frac{\gamma}{2}\langle a_1(t) \rangle = 0, \quad (3.7a)$$

$$\varepsilon^* - \kappa\langle a_2(t)a_3^\dagger(t) \rangle - \frac{\gamma}{2}\langle a_1^\dagger(t) \rangle = 0, \quad (3.7b)$$

$$\varepsilon - \kappa\langle a_1^\dagger(t)a_3(t) \rangle - \frac{\gamma}{2}\langle a_2(t) \rangle = 0, \quad (3.7c)$$

$$\varepsilon^* - \kappa\langle a_1(t)a_3^\dagger(t) \rangle - \frac{\gamma}{2}\langle a_2^\dagger(t) \rangle = 0, \quad (3.7d)$$

$$\kappa\langle a_1(t)a_2(t) \rangle - \frac{\gamma_3}{2}\langle a_3(t) \rangle = 0, \quad (3.7e)$$

$$\kappa\langle a_1^\dagger(t)a_2^\dagger(t) \rangle - \frac{\gamma_3}{2}\langle a_3^\dagger(t) \rangle = 0. \quad (3.7f)$$

Now employing the semiclassical approximation  $\langle a_i a_j \rangle = \langle a_i \rangle \langle a_j \rangle$  and setting

$$\langle a_j \rangle = \alpha_j, \quad (3.8)$$

with  $j=1, 2, 3$  one gets

$$\varepsilon - \kappa\alpha_2^*\alpha_3 - \frac{\gamma}{2}\alpha_1 = 0, \quad (3.9a)$$

$$\varepsilon^* - \kappa\alpha_2\alpha_3^* - \frac{\gamma}{2}\alpha_1^* = 0, \quad (3.9b)$$

$$\varepsilon - \kappa\alpha_1^*\alpha_3 - \frac{\gamma}{2}\alpha_2 = 0, \quad (3.9c)$$

$$\varepsilon^* - \kappa\alpha_1\alpha_3^* - \frac{\gamma}{2}\alpha_2^* = 0, \quad (3.9d)$$

$$\kappa\alpha_1\alpha_2 - \frac{\gamma_3}{2}\alpha_3 = 0, \quad (3.9e)$$

$$\kappa\alpha_1^*\alpha_2^* - \frac{\gamma_3}{2}\alpha_3^* = 0. \quad (3.9f)$$

Applying the same technique as in the previous case, we find

$$\varepsilon = \varepsilon^*, \quad (3.10a)$$

$$\alpha_1 = \alpha_1^*, \quad (3.10b)$$

$$\alpha_2 = \alpha_2^*, \quad (3.10c)$$

$$\alpha_3 = \alpha_3^*. \quad (3.10d)$$

We use the linearization procedure about the steady state values

$$a_j = \alpha_j + A_j(t), \quad (3.11)$$

in which  $A_j(t)$  represent small variations about the steady state values with  $j=1, 2, 3$ . On account of (3.10) and (3.11), one can rewrite Eq. (3.5) as

$$\frac{d}{dt}A_1(t) = -\frac{\gamma}{2}A_1(t) - \kappa\alpha_3A_2^\dagger(t) - \kappa\alpha_2A_3(t) + F_1(t), \quad (3.12a)$$

$$\frac{d}{dt}A_1^\dagger(t) = -\frac{\gamma}{2}A_1^\dagger(t) - \kappa\alpha_3A_2(t) - \kappa\alpha_2A_3^\dagger(t) + F_1^\dagger(t), \quad (3.12b)$$

$$\frac{d}{dt}A_2(t) = -\kappa\alpha_1A_3(t) - \kappa\alpha_3A_1^\dagger(t) - \frac{\gamma}{2}A_2(t) + F_2(t), \quad (3.12c)$$

$$\frac{d}{dt}A_2^\dagger(t) = -\kappa\alpha_1A_3^\dagger(t) - \kappa\alpha_3A_1(t) - \frac{\gamma}{2}A_2^\dagger(t) + F_2^\dagger(t), \quad (3.12d)$$

$$\frac{d}{dt}A_3(t) = \kappa\alpha_1A_2(t) + \kappa\alpha_2A_1(t) - \frac{\gamma_3}{2}A_3(t) + F_3(t), \quad (3.12e)$$

$$\frac{d}{dt}A_3^\dagger(t) = \kappa\alpha_1A_2^\dagger(t) + \kappa\alpha_2A_1^\dagger(t) - \frac{\gamma_3}{2}A_3^\dagger(t) + F_3^\dagger(t). \quad (3.12f)$$

It proves to be convenient to introduce new operators  $A_+$  and  $A_-$  defined by

$$A_\pm(t) = \frac{1}{\sqrt{2}}(A_1(t) \pm A_2(t)). \quad (3.13)$$

In view of (3.13), Eq. (3.12) is found to be of the form

$$\frac{d}{dt}A_{\pm}(t) = -\frac{\gamma}{2}A_{\pm}(t) - \kappa\alpha_3 A_{\pm}^{\dagger}(t) \mp \kappa\beta_{\pm}A_3(t) + \sqrt{\gamma}A_{\pm}^{in}(t), \quad (3.14a)$$

$$\frac{d}{dt}A_3(t) = -\frac{\gamma_3}{2}A_3(t) + \kappa(\beta_+A_+ - \beta_-A_-) + \sqrt{\gamma_3}A_3^{in}(t), \quad (3.14b)$$

in which,

$$\beta_{\pm} = \frac{1}{\sqrt{2}}(\alpha_1 \pm \alpha_2). \quad (3.15a)$$

$$F_1(t) \pm F_2(t) = \sqrt{2\gamma}A_{\pm}^{in}(t). \quad (3.15b)$$

Subtracting (3.9c) from (3.9a) and taking into account (3.10), we have

$$(\kappa\alpha_3 - \gamma/2)(\alpha_1 - \alpha_2) = 0,$$

so that, in view of (3.15a)

$$\beta_-(\kappa\alpha_3 - \gamma/2) = 0,$$

from which follows for  $\beta_- \neq 0$

$$\alpha_3 = \frac{\gamma}{2\kappa}. \quad (3.15c)$$

Substituting (3.15c) into (3.14a), we see that

$$\frac{d}{dt}A_{\pm}(t) = -\frac{\gamma}{2} \left[ A_{\pm}(t) \pm A_{\pm}^{\dagger}(t) \right] \mp \kappa\beta_{\pm}A_3(t) + \sqrt{\gamma}A_{\pm}^{in}(t), \quad (3.15d)$$

$$\frac{d}{dt}A_3(t) = -\frac{\gamma_3}{2}A_3(t) + \kappa(\beta_+A_+ - \beta_-A_-) + \sqrt{\gamma_3}A_3^{in}(t), \quad (3.15e)$$

The equations for the fluctuation operators of (3.15d) and (3.15e) simplify when written in terms of the quadrature operators defined by

$$X_{\pm}(t) = A_{\pm}^{\dagger}(t) + A_{\pm}(t), \quad (3.16a)$$

$$Y_{\pm}(t) = i(A_{\pm}^{\dagger}(t) - A_{\pm}(t)), \quad (3.16b)$$

$$X_3(t) = A_3^\dagger(t) + A_3(t), \quad (3.16c)$$

$$Y_3(t) = i(A_3^\dagger(t) - A_3(t)). \quad (3.16d)$$

Upon differentiating (3.16) w.r.t time, we obtain

$$\frac{d}{dt}X_\pm(t) = \frac{d}{dt}A_\pm^\dagger(t) + \frac{d}{dt}A_\pm(t) \quad (3.17a)$$

$$\frac{d}{dt}Y_\pm(t) = i \left[ \frac{d}{dt}A_\pm^\dagger(t) - \frac{d}{dt}A_\pm(t) \right], \quad (3.17b)$$

$$\frac{d}{dt}X_3(t) = \frac{d}{dt}A_3^\dagger(t) + \frac{d}{dt}A_3(t) \quad (3.17c)$$

$$\frac{d}{dt}Y_3(t) = i \left[ \frac{d}{dt}A_3^\dagger(t) - \frac{d}{dt}A_3(t) \right]. \quad (3.17d)$$

Now combination of (3.15d), (3.15e) and (3.17) leads to

$$\frac{d}{dt}X_\pm(t) = -\frac{\gamma}{2}(1\pm 1)X_\pm(t) \mp \kappa\beta_\pm X_3(t) + \sqrt{\gamma}X_\pm^{in}(t), \quad (3.18a)$$

$$\frac{d}{dt}Y_\pm(t) = -\frac{\gamma}{2}(1\mp 1)Y_\pm(t) \mp \kappa\beta_\pm Y_3(t) + \sqrt{\gamma}Y_\pm^{in}(t), \quad (3.18b)$$

$$\frac{d}{dt}X_3(t) = -\frac{\gamma_3}{2}X_3(t) + \kappa[\beta_+X_+(t) - \beta_-X_-(t)] + \sqrt{\gamma_3}X_3^{in}(t), \quad (3.18c)$$

$$\frac{d}{dt}Y_3(t) = -\frac{\gamma_3}{2}Y_3(t) + \kappa[\beta_+Y_+(t) - \beta_-Y_-(t)] + \sqrt{\gamma_3}Y_3^{in}(t). \quad (3.18d)$$

Furthermore, the linearized equations can be written in the form

$$\frac{d}{dt}X(t) = A_x X(t) + \Gamma X^{in}(t), \quad (3.19a)$$

$$\frac{d}{dt}Y(t) = A_y Y(t) + \Gamma Y^{in}(t), \quad (3.19b)$$

where

$$X = \begin{bmatrix} X_+ \\ X_- \\ X_3 \end{bmatrix}, \quad (3.20a)$$

$$Y = \begin{bmatrix} Y_+ \\ Y_- \\ Y_3 \end{bmatrix}, \quad (3.20b)$$

$$X^{in} = \begin{bmatrix} X_+^{in} \\ X_-^{in} \\ X_3^{in} \end{bmatrix}, \quad (3.20c)$$

$$Y^{in} = \begin{bmatrix} Y_+^{in} \\ Y_-^{in} \\ Y_3^{in} \end{bmatrix}, \quad (3.20d)$$

and the matrices  $A_x$  and  $A_y$  are defined by

$$A_x = \begin{bmatrix} -\gamma & 0 & -\kappa\beta_+ \\ 0 & 0 & \kappa\beta_- \\ \kappa\beta_+ & -\kappa\beta_- & -\gamma_3/2 \end{bmatrix}. \quad (3.20e)$$

$$A_y = \begin{bmatrix} 0 & 0 & -\kappa\beta_+ \\ 0 & -\gamma & \kappa\beta_- \\ \kappa\beta_+ & -\kappa\beta_- & -\gamma_3/2 \end{bmatrix}. \quad (3.20f)$$

Moreover,  $\Gamma$  is given by

$$\Gamma = \begin{bmatrix} \sqrt{\gamma} & 0 & 0 \\ 0 & \sqrt{\gamma} & 0 \\ 0 & 0 & \sqrt{\gamma} \end{bmatrix}.$$

(3.20g)

## 3.2 The Squeezing Spectrum

We now proceed to calculate the squeezing spectra for the sum and difference of the of the fundamental modes. To this end, we make a Fourier transform of Eq. (3.19) and solve for  $X(\omega)$  and  $Y(\omega)$ . The Fourier transform of  $X(t)$  and  $Y(t)$  are given by

$$X(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} X(\omega) d\omega, \quad (3.21a)$$

$$Y(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} Y(\omega) d\omega, \quad (3.21b)$$

Substituting (3.21a) and (3.21b) into (3.19a) and (3.19b) respectively yields

$$\int_{-\infty}^{\infty} d\omega e^{-i\omega t} [A_x X(\omega) + \Gamma X^{in}(\omega) + i\omega X(\omega)] = 0, \quad (3.22a)$$

$$\int_{-\infty}^{\infty} d\omega e^{-i\omega t} [A_y Y(\omega) + \Gamma Y^{in}(\omega) + i\omega Y(\omega)] = 0. \quad (3.22b)$$

We then see that

$$X(\omega) = -(A_x + i\omega I)^{-1} \Gamma X^{in}(\omega), \quad (3.23a)$$

$$Y(\omega) = -(A_y + i\omega I)^{-1} \Gamma Y^{in}(\omega), \quad (3.23b)$$

where  $I$  is a unit matrix. The output operators are related to the input operators by

$$X^{out}(\omega) = \Gamma X(\omega) - X^{in}(\omega), \quad (3.24a)$$

$$Y^{out}(\omega) = \Gamma Y(\omega) - Y^{in}(\omega). \quad (3.24b)$$

In view of these equations, we have

$$X^{out}(\omega) = -[\Gamma(A_x + i\omega I)^{-1} \Gamma + I] X^{in}(\omega), \quad (3.25a)$$

$$Y^{out}(\omega) = -[\Gamma(A_y + i\omega I)^{-1} \Gamma + I] Y^{in}(\omega). \quad (3.25b)$$

We now proceed to find out  $X_+^{out}(\omega)$ ,  $X_-^{out}(\omega)$  and  $X_3^{out}(\omega)$ . To this end, setting

$$A = A_x + i\omega I, \quad (3.26a)$$

$$A' = A_y + i\omega I. \quad (3.26b)$$

Now applying (3.20e) and (3.26a),

$$A = \begin{bmatrix} -\gamma + I\omega & 0 & -\kappa\beta_+ \\ 0 & i\omega & \kappa\beta_- \\ \kappa\beta_+ & -\kappa\beta_- & -\gamma_3/2 + i\omega \end{bmatrix}. \quad (3.27)$$

To find the inverse of the matrix A, one can use Cayley-Hamilton theorem.

The characteristic equation of this matrix can be found by setting  $\det(A - \lambda I) = 0$

$$\det A = \begin{vmatrix} a - \lambda & 0 & -b \\ 0 & c - \lambda & d \\ b & -d & e - \lambda \end{vmatrix}, \quad (3.28)$$

in which

$$a = -\gamma + i\omega, \quad (3.29a)$$

$$b = \kappa\beta_+, \quad (3.29b)$$

$$c = i\omega, \quad (3.29c)$$

$$d = \kappa\beta_-, \quad (3.29d)$$

$$e = -\frac{\gamma_3}{2} + i\omega. \quad (3.29e)$$

Therefore, the characteristic equation can be written as

$$\lambda^3 + \beta_1\lambda^2 + \beta_2\lambda + \beta_3 = 0, \quad (3.30)$$

where

$$\beta_1 = -[a + c + e], \quad (3.31a)$$

$$\beta_2 = -[ce + a(c + e) + b^2 + d^2], \quad (3.31b)$$

$$\beta_3 = - [ace + ad^2 + b^2c]. \quad (3.31c)$$

By the Cayley-Hamilton theorem, a matrix must satisfy its own characteristic equation.

Thus we have

$$\beta_3 I + \beta_2 A + \beta_1 A^2 + A^3 = 0, \quad (3.32a)$$

and there follows

$$I = -\frac{\beta_2}{\beta_3} A - \frac{\beta_1}{\beta_3} A^2 - \frac{1}{\beta_3} A^3. \quad (3.32b)$$

This equation leads to

$$A^{-1} = -\frac{1}{\beta_3} [\beta_2 I + \beta_1 A + A^2]. \quad (3.33)$$

Finally, combining (3.28), (3.29), (3.30) and (3.32), we see that

$$A^{-1}(\omega) = \begin{bmatrix} \mu_1(\omega) & \mu_2(\omega) & \mu_3(\omega) \\ \mu_2(\omega) & \mu_4(\omega) & -\mu_5(\omega) \\ -\mu_3(\omega) & \mu_5(\omega) & \mu_6(\omega) \end{bmatrix}, \quad (3.35)$$

in which

$$\mu_1(\omega) = \frac{1}{D_x} \left[ -i\frac{\gamma_3}{2}\omega - \omega^2 + (\kappa\beta_-)^2 \right], \quad (3.36a)$$

$$\mu_2(\omega) = \frac{1}{D_x} \kappa^2 \beta_+ \beta_-, \quad (3.36b)$$

$$\mu_3(\omega) = \frac{1}{D_x} i\kappa\beta_+\omega, \quad (3.36c)$$

$$\mu_4(\omega) = \frac{1}{D_x} \left[ -i(\gamma + \frac{\gamma_3}{2})\omega - \omega^2 + (\kappa\beta_+)^2 + \frac{\gamma\gamma_3}{2} \right], \quad (3.36d)$$

$$\mu_5(\omega) = \frac{1}{D_x} [i\kappa\beta_-\omega - \kappa\beta_-\gamma], \quad (3.36e)$$

$$\mu_6(\omega) = \frac{1}{D_x} [-i\gamma\omega - \omega^2], \quad (3.36f)$$

with

$$D_x = i\omega \left[ (\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2 + \frac{\gamma\gamma_3}{2} \right] + \omega^2 \left( \gamma + \frac{\gamma_3}{2} \right) - \gamma(\kappa\beta_-)^2. \quad (3.36g)$$

Taking into account (3.20g), (3.35) and carrying out the matrix multiplication, we arrive at

$$\Gamma A^{-1} \Gamma + I = \begin{bmatrix} (\gamma\mu_1 + 1) & \gamma\mu_2 & \sqrt{\gamma\gamma_3}\mu_3 \\ \gamma\mu_2 & (\gamma\mu_4 + 1) & -\sqrt{\gamma\gamma_3}\mu_5 \\ -\sqrt{\gamma\gamma_3}\mu_3 & \sqrt{\gamma\gamma_3}\mu_5 & (\gamma_3\mu_6 + 1) \end{bmatrix}. \quad (3.37)$$

Substituting this result into Eq. (3.25a) and using (3.20a), we have

$$\begin{bmatrix} X_+^{out}(\omega) \\ X_-^{out}(\omega) \\ X_3^{out}(\omega) \end{bmatrix} = \begin{bmatrix} (\gamma\mu_1 + 1) & \gamma\mu_2 & \sqrt{\gamma\gamma_3}\mu_3 \\ \gamma\mu_2 & (\gamma\mu_4 + 1) & -\sqrt{\gamma\gamma_3}\mu_5 \\ -\sqrt{\gamma\gamma_3}\mu_3 & \sqrt{\gamma\gamma_3}\mu_5 & (\gamma_3\mu_6 + 1) \end{bmatrix} \begin{bmatrix} X_+^{in}(\omega) \\ X_-^{in}(\omega) \\ X_3^{in}(\omega) \end{bmatrix}. \quad (3.38)$$

It then follows that

$$X_+^{out}(\omega) = - [(\gamma\mu_1 + 1)X_+^{in}(\omega) + \gamma\mu_2 X_-^{in}(\omega) + \sqrt{\gamma\gamma_3}\mu_3 X_3^{in}(\omega)], \quad (3.39a)$$

$$X_-^{out}(\omega) = - [\gamma\mu_2 X_+^{in}(\omega) + (\gamma\mu_4 + 1)X_-^{in}(\omega) - \sqrt{\gamma\gamma_3}\mu_5 X_3^{in}(\omega)], \quad (3.39b)$$

$$X_3^{out}(\omega) = - [-\sqrt{\gamma\gamma_3}\mu_3 X_+^{in}(\omega) + \sqrt{\gamma\gamma_3}\mu_5 X_-^{in}(\omega) + (\gamma_3\mu_6 + 1)X_3^{in}(\omega)]. \quad (3.39c)$$

Substituting the explicit forms of  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$ ,  $\mu_4$ ,  $\mu_5$  and  $\mu_6$  into (3.39), we obtain

$$X_+^{out}(\omega) = -\frac{1}{D_x} \left[ (i\omega [(\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2] + \frac{\gamma_3}{2}\omega^2) X_+^{in}(\omega) + \gamma\kappa^2\beta_+\beta_- X_-^{in}(\omega) + i\sqrt{\gamma\gamma_3}\kappa\beta_+\omega X_3^{in}(\omega) \right], \quad (3.40a)$$

$$X_-^{out}(\omega) = -\frac{1}{D_x} \left[ \left[ i\omega((\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2 - \gamma^2) + \gamma((\kappa\beta_+)^2 - (\kappa\beta_-)^2 + \frac{\gamma\gamma_3}{2}) + \omega^2\frac{\gamma_3}{2} \right] X_-^{in}(\omega) + \gamma\kappa^2\beta_+\beta_- X_+^{in}(\omega) + (i\omega - \gamma)\sqrt{\gamma\gamma_3}\kappa\beta_- X_3^{in}(\omega) \right], \quad (3.40b)$$

$$X_3^{out}(\omega) = -\frac{1}{D_x} \left[ -i\kappa\beta_+\omega\sqrt{\gamma\gamma_3} X_+^{in}(\omega) + (i\omega - \gamma)\kappa\beta_- \sqrt{\gamma\gamma_3} X_-^{in}(\omega) - i\omega(\gamma + \omega)\gamma_3 X_3^{in}(\omega) \right]. \quad (3.40c)$$

Following a similar procedure, one can readily verify that

$$\begin{aligned}
Y_+^{out}(\omega) = & -\frac{1}{D_y} \left[ [i\omega((\kappa\beta_+)^2 + (\kappa\beta_-)^2) - \omega^2 - \gamma^2] \right. \\
& \left. + \gamma((\kappa\beta_-)^2 - (\kappa\beta_+)^2 + \frac{\gamma\gamma_3}{2}) + \omega^2 \frac{\gamma_3}{2} \right] Y_+^{in}(\omega) \\
& + \gamma\kappa^2\beta_+\beta_- Y_-^{in}(\omega) + (i\omega - \gamma)\sqrt{\gamma\gamma_3}\kappa\beta_+ Y_3^{in}(\omega) \Big], \tag{3.41a}
\end{aligned}$$

$$\begin{aligned}
Y_-^{out}(\omega) = & -\frac{1}{D_y} \left[ (i\omega [(\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2] + \frac{\gamma_3}{2}\omega^2) Y_-^{in}(\omega) \right. \\
& \left. + \gamma\kappa^2\beta_+\beta_- Y_+^{in}(\omega) - i\sqrt{\gamma\gamma_3}\kappa\beta_- \omega Y_3^{in}(\omega) \right], \tag{3.41b}
\end{aligned}$$

$$\begin{aligned}
Y_3^{out}(\omega) = & -\frac{1}{D_y} \left[ i\omega\kappa\beta_- \sqrt{\gamma\gamma_3} Y_-^{in}(\omega) - \sqrt{\gamma\gamma_3}(i\omega - \gamma)\kappa\beta_+ Y_+^{in}(\omega) \right. \\
& \left. + \left[ i\omega((\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2 - \frac{\gamma\gamma_3}{2}) + \gamma(\omega^2 - (\kappa\beta_+)^2) - \omega^2 \frac{\gamma_3}{2} \right] Y_3^{in}(\omega) \right], \tag{3.41c}
\end{aligned}$$

with

$$D_y = i\omega \left[ (\kappa\beta_+)^2 + (\kappa\beta_-)^2 - \omega^2 + \gamma \frac{\gamma_3}{2} \right] + \omega^2 \left( \gamma + \frac{\gamma_3}{2} \right) - \gamma(\kappa\beta_+)^2. \tag{3.41d}$$

We now assume that the harmonic loss is negligible, we set  $\gamma_3 = 0$ . This leads to

$$\beta_- = \beta_+ = \frac{\beta_0}{\sqrt{\gamma}}, \tag{3.42a}$$

where

$$\beta_0 = \frac{2\varepsilon}{\sqrt{2\gamma}}. \tag{3.42b}$$

Hence it is easy to check that

$$X_+^{out}(\omega) = -\frac{1}{D} \left[ i\omega \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right) X_+^{in}(\omega) + \kappa^2\beta_0^2 X_-^{in}(\omega) \right], \tag{3.43a}$$

$$X_-^{out}(\omega) = -\frac{1}{D} \left[ i\omega \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right) X_-^{in}(\omega) + \kappa^2\beta_0^2 X_+^{in}(\omega) \right], \tag{3.43b}$$

$$Y_+^{out}(\omega) = -\frac{1}{D} \left[ i\omega \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right) Y_+^{in}(\omega) + \kappa^2\beta_0^2 Y_-^{in}(\omega) \right], \tag{3.43c}$$

$$Y_-^{out}(\omega) = -\frac{1}{D} \left[ i\omega \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right) Y_-^{in}(\omega) + \kappa^2\beta_0^2 Y_+^{in}(\omega) \right], \tag{3.43d}$$

with

$$D = i\omega \left[ \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right] + \gamma \left[ \omega^2 - \frac{\kappa^2\beta_0^2}{\gamma} \right]. \quad (3.43e)$$

We now proceed to obtain a closed form expression for the output squeezing spectra in frequency space. To this end, we note that

$$X_i^{out}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} X_i^{out}(t) dt, \quad (3.44a)$$

$$X_i^{out}(\omega') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega' t'} X_i^{out}(t') dt', \quad (3.44b)$$

with  $i = +, -$ . With the aid of these expressions, we have at steady state

$$\langle X_i^{out}(\omega'), X_i^{out}(\omega) \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(\omega+\omega')t+i\omega'(t'-t)} \langle X_i^{out}(t'), X_i^{out}(t) \rangle_{ss} dt dt'. \quad (3.45a)$$

Setting  $\tau = t' - t$ , we find

$$\langle X_i^{out}(\omega'), X_i^{out}(\omega) \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(\omega+\omega')t} e^{i\omega'\tau} \langle X_i^{out}(t+\tau), X_i^{out}(t) \rangle_{ss} dt d\tau. \quad (3.45b)$$

We assume that  $\langle X_i^{out}(t+\tau), X_i^{out}(t) \rangle_{ss}$  depends only on  $\tau$  and carrying out integration over  $t$ , we have

$$\langle X_i^{out}(\omega'), X_i^{out}(\omega) \rangle = \int_{-\infty}^{\infty} e^{i\omega'\tau} \langle X_i^{out}(t+\tau), X_i^{out}(t) \rangle_{ss} \delta(\omega + \omega') d\tau. \quad (3.46a)$$

Replacing  $\omega'$  by  $-\omega'$  and integrating this over  $\omega'$  leads to

$$\int_{-\infty}^{\infty} \langle X_i^{out}(\omega'), X_i^{out}(\omega) \rangle d\omega' = \int_{-\infty}^{\infty} e^{-i\omega\tau} \langle X_i^{out}(t+\tau), X_i^{out}(t) \rangle_{ss} d\tau. \quad (3.46b)$$

From this result, we see that the squeezing spectrum is expressible as

$$S_{X_i}(\omega) = \int_{-\infty}^{\infty} \langle X_i^{out}(\omega'), X_i^{out}(\omega) \rangle d\omega'. \quad (3.47a)$$

one then can write

$$S_{X_+}(\omega) = \int_{-\infty}^{\infty} \langle X_+^{out}(\omega'), X_+^{out}(\omega) \rangle d\omega', \quad (3.47b)$$

$$S_{X_-}(\omega) = \int_{-\infty}^{\infty} \langle X_-^{out}(\omega'), X_-^{out}(\omega) \rangle d\omega'. \quad (3.47c)$$

Recalling that

$$\langle X_+^{out}(\omega'), X_+^{out}(\omega) \rangle = \langle X_+^{out}(\omega') X_+^{out}(\omega) \rangle - \langle X_+^{out}(\omega') \rangle \langle X_+^{out}(\omega) \rangle \quad (3.47d)$$

and on account of (3.43a), we see that

$$\langle X_+^{out}(\omega) \rangle = -\frac{1}{D} \left[ i\omega \left( \frac{2\kappa^2 \beta_0^2}{\gamma} - \omega^2 \right) \langle X_+^{in}(\omega) \rangle + \kappa^2 \beta_0^2 \langle X_-^{in}(\omega) \rangle \right]. \quad (3.48)$$

One can write

$$\langle X_+^{in}(\omega) \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} \langle X_+^{in}(t) \rangle dt. \quad (3.49)$$

Taking into account (3.15b) and (3.16a), we have

$$X_{\pm}^{in}(t) = \frac{1}{\sqrt{2\gamma}} (F_1^{\dagger}(t) \pm F_2^{\dagger}(t) + F_1(t) \pm F_2(t)), \quad (3.50a)$$

It then follows that

$$\langle X_{\pm}^{in}(t) \rangle = 0. \quad (3.50b)$$

And hence

$$\langle X_{\pm}^{in}(\omega) \rangle = 0. \quad (3.50c)$$

Therefore, on account of this result, the correlation function (3.47d) reduces to

$$\langle X_+^{out}(\omega'), X_+^{out}(\omega) \rangle = \langle X_+^{out}(\omega') X_+^{out}(\omega) \rangle. \quad (3.51a)$$

Substitution of (3.43a) into this expression yields

$$\begin{aligned} \langle X_+^{out}(\omega'), X_+^{out}(\omega) \rangle &= \frac{1}{D(\omega)D(\omega')} \left[ -\omega\omega' \left( \frac{2\kappa^2 \beta_0^2}{\gamma} - \omega^2 \right) \left( \frac{2\kappa^2 \beta_0^2}{\gamma} - \omega'^2 \right) \langle X_+^{in}(\omega') X_+^{in}(\omega) \rangle \right. \\ &\quad \left. + i\omega\kappa^2 \beta_0^2 \left( \frac{2\kappa^2 \beta_0^2}{\gamma} - \omega^2 \right) \langle X_+^{in}(\omega') X_-^{in}(\omega) \rangle \right. \\ &\quad \left. + i\omega'\kappa^2 \beta_0^2 \left( \frac{2\kappa^2 \beta_0^2}{\gamma} - \omega'^2 \right) \langle X_-^{in}(\omega') X_+^{in}(\omega) \rangle + (\kappa^2 \beta_0^2)^2 \langle X_-^{in}(\omega') X_-^{in}(\omega) \rangle \right]. \quad (3.51b) \end{aligned}$$

Applying (3.50a), one can write

$$\begin{aligned}
\langle X_{\pm}^{in}(t')X_{\pm}^{in}(t) \rangle &= \frac{1}{2\gamma} \left[ \langle F_1^{\dagger}(t')F_1^{\dagger}(t) \pm F_1^{\dagger}(t')F_2^{\dagger}(t) + F_1^{\dagger}(t')F_1(t) \pm F_1^{\dagger}(t')F_2(t) \right. \\
&\quad \pm F_2^{\dagger}(t')F_1^{\dagger}(t) + F_2^{\dagger}(t')F_2^{\dagger}(t) \pm F_2^{\dagger}(t')F_1(t) + F_2^{\dagger}(t')F_2(t) \\
&\quad + F_1(t')F_1^{\dagger}(t) \pm F_1(t')F_2^{\dagger}(t) + F_1(t')F_1(t) \pm F_1(t')F_2(t) \\
&\quad \left. \pm F_2(t')F_1^{\dagger}(t) + F_2(t')F_2^{\dagger}(t) \pm F_2(t')F_1(t) + F_2(t')F_2(t) \right]. \quad (3.52a)
\end{aligned}$$

In view of the correlation functions defined by (2.8a), (2.8b) and (2.8c) for ordinary vacuum reservoirs and using the fact that the two subharmonic modes are not correlated, we have

$$\langle X_{\pm}^{in}(t')X_{\pm}^{in}(t) \rangle = \delta(t-t'), \quad (3.52b)$$

$$\langle X_{\pm}^{in}(t')X_{\mp}^{in}(t) \rangle = 0, \quad (3.52c)$$

so that on taking into account (3.22a) and (3.52) we have

$$\langle X_{\pm}^{in}(\omega')X_{\pm}^{in}(\omega) \rangle = \delta(\omega + \omega'), \quad (3.53a)$$

$$\langle X_{\pm}^{in}(\omega')X_{\mp}^{in}(\omega) \rangle = 0. \quad (3.53b)$$

In view of the above results

$$\begin{aligned}
\langle X_{+}^{out}(\omega'), X_{+}^{out}(\omega) \rangle &= \frac{1}{D(\omega)D(\omega')} \left[ -\omega\omega' \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right) \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega'^2 \right) \right. \\
&\quad \left. + (\kappa^2\beta_0^2)^2 \right] \delta(\omega + \omega') \quad (3.54a)
\end{aligned}$$

$$\begin{aligned}
\langle X_{-}^{out}(\omega'), X_{-}^{out}(\omega) \rangle &= \frac{1}{D(\omega)D(\omega')} \left[ -\omega\omega' \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right) \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega'^2 - \gamma^2 \right) \right. \\
&\quad \left. + (\kappa^2\beta_0^2)^2 \right] \delta(\omega + \omega'). \quad (3.54b)
\end{aligned}$$

Using (3.47a)

$$S_{X_{+}}(\omega) = \int_{-\infty}^{\infty} \frac{1}{D(\omega)D(\omega')} \left[ -\omega\omega' \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right) \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega'^2 \right) \right.$$

$$+(\kappa^2\beta_0^2)^2] \delta(\omega + \omega')d\omega', \quad (3.54c)$$

$$S_{X_-}(\omega) = \int_{-\infty}^{\infty} \frac{1}{D(\omega)D(\omega')} \left[ -\omega\omega' \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right) \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega'^2 - \gamma^2 \right) \right. \\ \left. + (\kappa^2\beta_0^2)^2 \right] \delta(\omega + \omega')d\omega'. \quad (3.54d)$$

Upon replacing  $\omega'$  by  $-\omega'$  in (3.54c) and (3.54d) and carrying out the integration, the squeezing spectrum turns out to be

$$S_{X_+}(\omega) = \frac{1}{D^2(\omega)} \left[ \omega^2 \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right)^2 + (\kappa^2\beta_0^2)^2 \right], \quad (3.55a)$$

$$S_{X_-}(\omega) = \frac{1}{D^2(\omega)} \left[ \omega^2 \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right)^2 + (\kappa^2\beta_0^2)^2 \right]. \quad (3.55b)$$

Similarly

$$S_{Y_+}(\omega) = \frac{1}{D^2(\omega)} \left[ \omega^2 \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 - \gamma^2 \right)^2 + (\kappa^2\beta_0^2)^2 \right], \quad (3.56a)$$

$$S_{Y_-}(\omega) = \frac{1}{D^2(\omega)} \left[ \omega^2 \left( \frac{2\kappa^2\beta_0^2}{\gamma} - \omega^2 \right)^2 + (\kappa^2\beta_0^2)^2 \right]. \quad (3.56b)$$

Setting

$$\sigma = \frac{\omega}{\omega_0}, \quad (3.57a)$$

$$\lambda = \frac{\gamma^2}{\omega_0^2}, \quad (3.57b)$$

and

$$\omega_0 = \frac{\kappa\beta_0}{\sqrt{\gamma/2}}, \quad (3.57c)$$

Eq. (3.56) can be rewritten as

$$S_{X_+}(\omega) = S_{Y_-}(\omega) = 1 - \frac{\lambda\sigma^2(\sigma^2 - 1)}{\sigma^2(\sigma^2 - 1)^2 + \lambda(\sigma^2 - 1/2)^2}, \quad (3.58a)$$

$$S_{X_-}(\omega) = S_{Y_+}(\omega) = 1 + \frac{\lambda\sigma^2(\sigma^2 + \lambda - 1)}{\sigma^2(\sigma^2 - 1)^2 + \lambda(\sigma^2 - 1/2)^2}. \quad (3.58b)$$

It is obvious that squeezing occurs in  $S_{X_+}(\omega)$  and  $S_{Y_-}(\omega)$  when  $\sigma > 1$  or  $\omega > \omega_0$  and squeezing occurs in  $S_{X_-}(\omega)$  and  $S_{Y_+}(\omega)$  when  $\sigma^2 < (1 - \lambda)$ . It can be seen that a large amount of squeezing (close to ideal) can be achieved with  $S_{X_+}(\omega)$  and  $S_{Y_-}(\omega)$  for  $\lambda \gg 1$ .

It is interesting to note that at the demarcation frequency  $\omega = \omega_0$  or  $\sigma = 1$

$$S_{X_+}(\omega) = S_{Y_-}(\omega) = 1.$$

$$S_{X_-}(\omega) = S_{Y_+}(\omega) = 1 + \frac{4\gamma^2}{\omega_0^2}.$$

Hence, no squeezing exists under this consideration.

## CONCLUSION

We have presented a derivation of the linearized Langevin equations for the fundamental and harmonic modes involved in degenerate second harmonic generation inside a cavity coupled to two independent squeezed vacuum reservoirs. The solutions of the resulting equations have been used to calculate the squeezing spectra, the quadrature variances and the mean photon numbers of these two modes.

We have seen that the effect of the reservoirs is to increase the degree of squeezing. We have also considered the squeezing spectra at both the critical point and critical frequency and for ordinary vacuum reservoirs. We have seen that for this case when  $\gamma_b \ll \gamma_a$ , perfect squeezing is approached in the low frequency mode at  $\omega_c = \pm\sqrt{2\gamma_a\gamma_b}$  and if  $\gamma_b \gg \gamma_a$ , perfect squeezing is approached in the high frequency mode at  $\omega_c = \pm\gamma_b$ . Moreover, we have found that for  $r_a = r_b = r$ ,  $\gamma_a = \gamma_b = \gamma$  and at the critical point the variance of the intracavity quadrature operators for the two modes are independent of the driving field amplitudes and the cavity damping rates. In addition, the mean photon numbers of the two modes diverge at the critical point. This shows that the linearization method breaks down at the critical point.

We also have obtained the linearized Langevin equations for both the fundamental modes as well as the harmonic mode of nondegenerate second harmonic generation. We have then found the explicit expressions for the quadrature operators in frequency space for the sum and difference of the fundamental modes. With the aid of these results, we have calculated the squeezing spectra.

For this case we have shown that squeezing occurs in  $X_+(\omega)$  and  $Y_-(\omega)$  when  $\sigma > 1$  or  $\omega > \omega_0$  and in  $X_-(\omega)$  and  $Y_+(\omega)$  when  $\sigma^2 < 1 - \lambda$ . We have also seen that a large

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