DYNAMICS OF COHERENTLY DRIVEN NONDEGRNERATE THREE-LEVEL ATOM IN OPEN CAVITY

Abinet Yeshiwas

A THESIS SUBMITTED TO

THE DEPARTMENT OF PHYSICS

PRESENTED IN PARTIAL FULFILLMENT OF THE THE REQUIREMETS FOR

THE DEGREE OF MASTER OF SCIENCE

ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA
MAY 2014

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared by **Abinet Yeshiwas**, entitled "**Dynamics of Coherently Driven Nondegenerate three-level atom in Open Cavity**" and submitted in partial fulfillment of the requirements for the degree of **Master of Science** complies with the regulations of the University and meets the accepted standards with respect to originality.

Signed by the Examination Committee:

Examiner:______________________Signiture:_______________Date:_________

Examiner:______________________Signiture:_______________Date:_________

Advisor:________________________Signiture:_______________Date:_________

Abstract

We have investigated the squeezing and statistical properties of the light produced by a coherently driven nondegenerate three-level atom in an open cavity coupled to a vacuum reservoir via a single-port mirror. We carry out our analysis by considering the vacuum reservoir to be a noiseless physical entity and by applying the large-time approximation scheme to the quantum Langevin equations.

We have found that the mean photon numbers of light modes a and b are the same in the absence of spontaneous emission and the mean photon number of light mode a is greater than that of light mode b in the presence of spontaneous emission. Our analysis also shows that mode a is in general in chaotic state while mode b is in a chaotic state under certain conditions. It is also found that the superposed light modes are in squeezed state with a maximum quadrature squeezing of 43% for γ = 0 and 46.5% for γ = 0.1 below the coherent vacuum state level.

Acknowledgements

I would like to express my sincere thanks to my advisor and instructor, Dr. Fesseha Kassahun, for his inspirational guidance, critical comments, and unreserved support throughout my work. His very rich experience in the area of Quantum Optics has helped me a lot to use all my efforts in writing this thesis with interest.

I would also like to express my appreciation, love, and respect to my family and friends especially to my mother Fantaye Ayalew and my wife Kiya Dargie for their limitless support.

Addis Ababa University Abinet Yeshiwas

May 2014

Contents

List of figures

Chapter 1
Introduction

The interaction between a three-level atom and light has been a subject of considerable interest of study for many years [1-15]. It has been established that a three-level atom under certain conditions generates squeezed light [1,2,3,4,7,11,12,13,14,15]. In a three-level atom the top, intermediate and bottom levels are denoted by $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively. The atom may be continuously pumped from the bottom to the top level by electron bombardment or coherent light. A three-level atom in the top level may decay to level $|b\rangle$ and then to level $|c\rangle$ due to stimulated emission; or it may decay to the bottom level due to spontaneous emission. It is worth mentioning that the aforementioned processes do not occur simultaneously. Moreover, when the three-level atom makes a transition from the top to the bottom level via the intermediate level, two photons are emitted. If the two photons have the same frequency, then the three-level atom is called degenerate three-level atom otherwise it is called nondegenerate [1,12,13].

The squeezing and statistical properties of the light produced by a nondegenerate three- level atom have been studied by deferent authors [1,2,3,4,7,11,12,13,14]. These studies show that the nondegenerate three-level atom can produce squeezed light under certain conditions. Using the quantum Langevin equation, Lamrot Hailu [1] has shown that the superposition of the light modes produced by a nondegenerate three- level atom is in a squeezed state. In addition, Lu and Zhu [4,7] have considered a nondegenerate three-level laser with the atoms initially prepared in a coherent superposition of the top and bottom levels. They have predicted a maximum squeezing of 50% of the superposed cavity modes.

In this thesis, we seek to study the dynamics of a coherently driven nondegenerate three-level atom, in open cavity coupled to a vacuum reservoir via a single-port mirror. We carry out our analysis by putting the vacuum noise operators in normal order and applying the large-time approximation scheme. In particular we wish to calculate the mean photon number, the variance of the photon number, and the quadrature variance for the individual light modes and for the superposition of the two light modes. And finally we calculate the quadrature squeezing of the superposed light modes.

Chapter 2

Operator Dynamics

We consider here a single nondegenerate three-level atom inside an open cavity which is coupled to a vacuum reservoir. The three-level atom is pumped from the bottom to the top level by coherent light. We carry out our analysis with the assumption that the cavity modes to be at resonance with the transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$, with direct transition between levels $|c\rangle$ and $|a\rangle$ to be dipole forbidden.

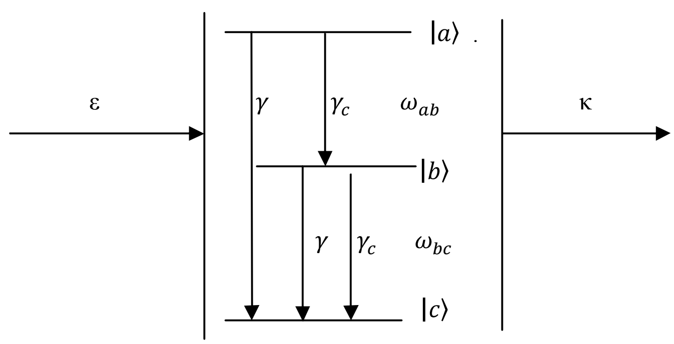

Fig.2.1: Schematic representation of coherently drive nondegenerate three-level atom in an open cavity.

The interaction of a three-level atom with the driving coherent light can be described at resonance by the Hamiltonian

$$\hat{H}' = ig'\left(\hat{\sigma}_c^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_c\right), \tag{2.1}$$

where

$$\hat{\sigma}_c = |c\rangle\langle a| \tag{2.2}$$

is lowering atomic operator, $\hat{a}$ is the annihilation operator for the driving coherent light and g' is the coupling constant between the atom and the coherent light . In order to have a mathematically manageable analysis, it is preferable to replace the operator $\hat{a}$ by a real and constant c-number ε. On account of this, the Hamiltonian can be expressed as

$$\hat{H}' = \frac{i\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c), \tag{2.3}$$

where

$$\Omega = 2\varepsilon g', \tag{2.4}$$

And the interaction of the three-level atom with the cavity modes can be described by the Hamiltonian

$$\hat{H}" = ig\left(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a + \hat{\sigma}_b^\dagger \hat{b} - \hat{b}^\dagger \hat{\sigma}_b\right), \tag{2.5}$$

where

$$\hat{\sigma}_a = |b\rangle\langle a| \tag{2.6}$$

and

$$\hat{\sigma}_b = |c\rangle\langle b| \tag{2.7}$$

are lowering atomic operators, $\hat{a}$ and $\hat{b}$ are the annihilation operators for the cavity mode, and g is the coupling constant between the atom and the cavity modes. On account of Eqs. (2.3) and (2.5), the interaction of the three-level atom with the coherent light and the cavity modes can be described by the Hamiltonian

$$\hat{H} = \frac{i\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c) + ig\left(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a + \hat{\sigma}_b^\dagger \hat{b} - \hat{b}^\dagger \hat{\sigma}_b\right). \tag{2.8}$$

We assume that the cavity mode is coupled to a vacuum reservoir via a single-port mirror. One possible procedure to carry out our analysis is to assume the vacuum reservoir to be a noiseless physical entity. This can be done by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operator does not affect the dynamics of the cavity mode operators. In view of this, we can drop the noise operator and write the quantum Langevin equation for the operators $\hat{a}$ and $\hat{b}$ as

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2}\hat{a} - i\left[\hat{a}, \hat{H}\right] \tag{2.9}$$

and

$$\frac{d\hat{b}}{dt} = -\frac{\kappa}{2}\hat{b} - i[\hat{b}, \hat{H}], \tag{2.10}$$

where κ is the cavity damping constant. Then with the aid of Eq. (2.8), one can easily establish that

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2}\hat{a} - g\hat{\sigma}_a\ , \tag{2.11}$$

$$\frac{d\hat{b}}{dt} = -\frac{\kappa}{2}\hat{b} - g\hat{\sigma}_b. \tag{2.12}$$

Furthermore, the master equation for the three-level atom interacting with a vacuum reservoir is given by

$$\frac{d\hat{\rho}}{dt} = -i\left[\hat{H}, \hat{\rho}\right] + \frac{\gamma}{2}\left(2\hat{\sigma}_b\hat{\rho}\hat{\sigma}_b^\dagger - \hat{\eta}_b\hat{\rho} - \hat{\rho}\hat{\eta}_b\right) + \frac{\gamma}{2}\left(2\hat{\sigma}_c\hat{\rho}\hat{\sigma}_c^\dagger - \hat{\eta}_a\hat{\rho} - \hat{\rho}\hat{\eta}_a\right), \tag{2.13}$$

where

$$\hat{\eta}_a = |a\rangle\langle a|, \tag{2.14}$$

$$\hat{\eta}_b = |b\rangle\langle b|, \tag{2.15}$$

and γ is the spontaneous emission decay constant. With the help of Eq. (2.8), we can put Eq. (2.13) in the form of

$$\frac{d\hat{\rho}}{dt} = \frac{\Omega}{2}\left(\hat{\sigma}_c^{\dagger}\hat{\rho} - \hat{\sigma}_c\hat{\rho} - \hat{\rho}\hat{\sigma}_c^{\dagger} + \hat{\rho}\hat{\sigma}_c\right)$$
$$+g\left(\hat{\sigma}_a^{\dagger}\hat{a}\hat{\rho} - \hat{a}^{\dagger}\hat{\sigma}_a\hat{\rho} + \hat{\sigma}_b^{\dagger}\hat{b}\hat{\rho} - \hat{b}^{\dagger}\hat{\sigma}_b\hat{\rho} - \hat{\rho}\hat{\sigma}_a^{\dagger}\hat{a} + \hat{\rho}a^{\dagger}\hat{\sigma}_a - \hat{\rho}\hat{\sigma}_b^{\dagger}\hat{b} + \hat{\rho}\hat{b}^{\dagger}\hat{\sigma}_b\right)$$
$$+\frac{\gamma}{2}\left(2\hat{\sigma}_c\hat{\rho}\hat{\sigma}_c^{\dagger} - \hat{\eta}_a\hat{\rho} - \hat{\rho}\hat{\eta}_a\right) + \frac{\gamma}{2}\left(2\hat{\sigma}_b\hat{\rho}\hat{\sigma}_b^{\dagger} - \hat{\eta}_b\hat{\rho} - \hat{\rho}\hat{\eta}_b\right). \quad (2.16)$$

Furthermore, applying the relation

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr\left[\frac{d\hat{\rho}}{dt}\hat{A}\right] \quad (2.17)$$

along with Eq. (2.16), we can easily establish that

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -\gamma\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger}\rangle + g\left(\langle\hat{\eta}_b\hat{a}\rangle - \langle\hat{\eta}_a\hat{a}\rangle + \langle\hat{b}^{\dagger}\hat{\sigma}_c\rangle\right), \quad (2.18)$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger}\rangle + g\left(\langle\hat{\eta}_c\hat{b}\rangle - \langle\hat{\eta}_b\hat{b}\rangle - \langle\hat{a}^{\dagger}\hat{\sigma}_c\rangle\right), \quad (2.19)$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_c\rangle + \frac{\Omega}{2}\left(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle\right) + g\left(\langle\hat{\sigma}_b\hat{a}\rangle - \langle\hat{\sigma}_a\hat{b}\rangle\right), \quad (2.20)$$

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = -\gamma\langle\hat{\eta}_a\rangle + \frac{\Omega}{2}\left(\langle\hat{\sigma}_c^{\dagger}\rangle + \langle\hat{\sigma}_c\rangle\right) + g\left(\langle\hat{\sigma}_a^{\dagger}\hat{a}\rangle + \langle\hat{a}^{\dagger}\hat{\sigma}_a\rangle\right), \quad (2.21)$$

$$\frac{d}{dt}\langle\hat{\eta}_b\rangle = -\gamma\langle\hat{\eta}_b\rangle + g\left(\langle\hat{b}^{\dagger}\hat{\sigma}_b\rangle + \langle\hat{\sigma}_b^{\dagger}\hat{b}\rangle - \langle\hat{\sigma}_a^{\dagger}\hat{a}\rangle - \langle\hat{a}^{\dagger}\hat{\sigma}_a\rangle\right), \quad (2.22)$$

$$\frac{d}{dt}\langle\hat{\eta}_c\rangle = \gamma\left(\langle\hat{\eta}_b\rangle + \langle\hat{\eta}_a\rangle\right) - \frac{\Omega}{2}\left(\langle\hat{\sigma}_c^{\dagger}\rangle + \langle\hat{\sigma}_c\rangle\right) - g\left(\langle\hat{b}^{\dagger}\hat{\sigma}_b\rangle + \langle\hat{\sigma}_b^{\dagger}\hat{b}\rangle\right), \quad (2.23)$$

where

$$\hat{\eta}_c = |c\rangle\langle c|. \quad (2.24)$$

The fact that Eq. (2.18) to Eq. (2.23) are nonlinear differential equations makes it impossible to find exact time-dependent solutions of these equations. To get rid of this problem, we apply the large-time approximation to Eqs. (2.11) and (2.12) and get the approximately valid relations

$$\hat{a} = -\frac{2g}{\kappa}\hat{\sigma}_a \tag{2.25}$$

and

$$\hat{b} = -\frac{2g}{\kappa}\hat{\sigma}_b. \tag{2.26}$$

Substitution of Eq. (2.25) and Eq. (2.26) into the aforementioned equations yields

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -\gamma\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^\dagger\rangle - \gamma_c\langle\hat{\sigma}_a\rangle, \tag{2.27}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle - \frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle, \tag{2.28}$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = -\frac{\gamma}{2}\langle\hat{\sigma}_c\rangle + \frac{\Omega}{2}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle) - \frac{\gamma_c}{2}\langle\hat{\sigma}_c\rangle, \tag{2.29}$$

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = -\gamma\langle\hat{\eta}_a\rangle + \frac{\Omega}{2}\left(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle\right) - \gamma_c\langle\hat{\eta}_a\rangle, \tag{2.30}$$

$$\frac{d}{dt}\langle\hat{\eta}_b\rangle = -\gamma\langle\hat{\eta}_b\rangle + \gamma_c(\langle\hat{\eta}_a\rangle + \langle\hat{\eta}_b\rangle), \tag{2.31}$$

$$\frac{d}{dt}\langle\hat{\eta}_c\rangle = \gamma(\langle\hat{\eta}_b\rangle + \langle\hat{\eta}_a\rangle) - \frac{\Omega}{2}\left(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle\right) + \gamma_c\langle\hat{\eta}_b\rangle, \tag{2.32}$$

where

$$\gamma_c = \frac{4g^2}{\kappa} \tag{2.33}$$

is called the stimulated emission decay constant.

And the steady state solutions of Eq. (2.27) - (2.32) are easily found to be

$$\langle\hat{\sigma}_a\rangle = \frac{\Omega}{2(\gamma+\gamma_c)}\langle\hat{\sigma}_b^\dagger\rangle, \tag{2.34}$$

$$\langle\hat{\sigma}_b\rangle = -\frac{\Omega}{(\gamma+\gamma_c)}\langle\hat{\sigma}_a^\dagger\rangle, \tag{2.35}$$

$$\langle\hat{\sigma}_c\rangle = \frac{\Omega}{(\gamma+\gamma_c)}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle), \tag{2.36}$$

$$\langle\hat{\eta}_a\rangle = \frac{\Omega}{2(\gamma+\gamma_c)}\left(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle\right), \tag{2.37}$$

$$\langle\hat{\eta}_b\rangle = \frac{\gamma_c}{(\gamma+\gamma_c)}\langle\hat{\eta}_a\rangle, \tag{2.38}$$

$$\langle\hat{\eta}_c\rangle = -\frac{\Omega}{2\gamma}\left(\langle\hat{\sigma}_c^\dagger\rangle + \langle\hat{\sigma}_c\rangle\right) + \frac{\gamma_c}{\gamma}\langle\hat{\eta}_b\rangle + 1. \tag{2.39}$$

With the help of the completeness relation

$$\hat{\eta}_a + \hat{\eta}_b + \hat{\eta}_c = \hat{I}, \tag{2.40}$$

we see that

$$\langle\hat{\eta}_a\rangle + \langle\hat{\eta}_b\rangle + \langle\hat{\eta}_c\rangle = 1. \tag{2.41}$$

Taking into account Eq. (2.41), we easily get the steady state solutions of Eqs. (2.34) - (2.39) to be

$$\langle\hat{\sigma}_a\rangle = 0, \tag{2.42}$$

$$\langle\hat{\sigma}_b\rangle = 0, \tag{2.43}$$

$$\langle\hat{\sigma}_c\rangle = \frac{\Omega(\gamma_c+\gamma)^2}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}, \tag{2.44}$$

$$\langle\hat{\eta}_a\rangle = \frac{\Omega^2(\gamma_c+\gamma)}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}, \tag{2.45}$$

$$\langle \hat{\eta}_b \rangle = \frac{\Omega^2 \gamma_c}{(\gamma_c+\gamma)^3 + 2\Omega^2 (\gamma_c+\gamma) + \Omega^2 \gamma_c}, \tag{2.46}$$

$$\langle \hat{\eta}_c \rangle = \frac{(\gamma_c+\gamma)^3 + \Omega^2 (\gamma_c+\gamma)}{(\gamma_c+\gamma)^3 + 2\Omega^2 (\gamma_c+\gamma) + \Omega^2 \gamma_c}. \tag{2.47}$$

Chapter 3

Photon Statistics

In this chapter, we present the statistical properties of the light modes produced by a nondegenerate three-level atom, in an open cavity, driven by coherent light. In particular, we calculate the mean and variance of the photon number at steady state.

3.1 The mean photon number

We first seek to calculate the mean photon number of the cavity light modes. For a light mode represented by an operator $\hat{\alpha}$, the mean photon number is expressed by the notation

$$\bar{n} = \langle \hat{\alpha}^\dagger \hat{\alpha} \rangle. \tag{3.1}$$

On account of (3.1) and with the aid of Eqs. (2.25) and (2.26), one can express the mean photon number of light modes a and b in the form

$$\bar{n}_a = \frac{\gamma_c}{\kappa} \langle \hat{\sigma}_a^\dagger \hat{\sigma}_a \rangle \tag{3.2}$$

and

$$\bar{n}_b = \frac{\gamma_c}{\kappa} \langle \hat{\sigma}_b^\dagger \hat{\sigma}_b \rangle, \tag{3.3}$$

with γ_c defined by Eq. (2.33).On account of Eq. (2.6) , we can put (3.2) in the form

$$\bar{n}_a = \frac{\gamma_c}{\kappa} \langle |a\rangle\langle b|b\rangle\langle a| \rangle, \tag{3.4}$$

from which follows

$$\bar{n}_a = \frac{\gamma_c}{\kappa}\langle\hat{\eta}_a\rangle. \tag{3.5}$$

Finally, upon substituting (2.45) into this equation, we have

$$\bar{n}_a = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2(\gamma_c+\gamma)}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right). \tag{3.6}$$

We see that for $\gamma = 0$ the mean photon number for mode a becomes

$$\bar{n}_a = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2+3\Omega^2}\right). \tag{3.7}$$

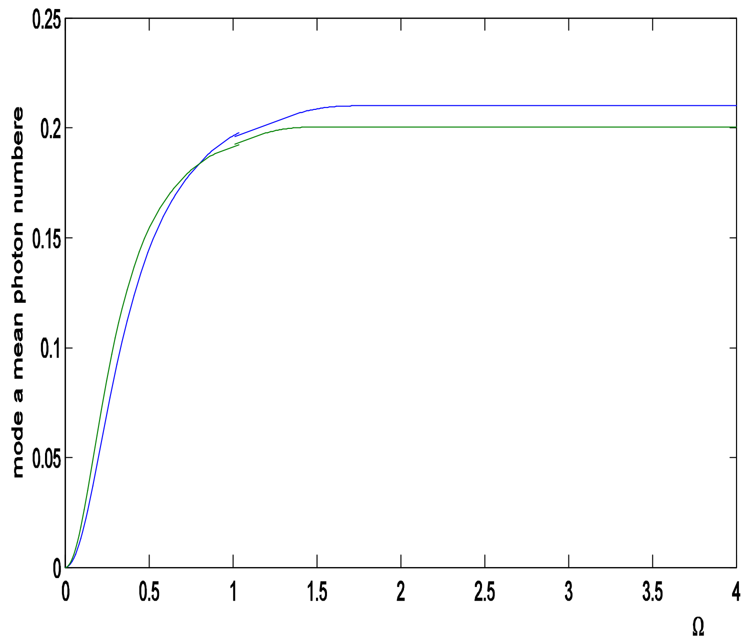

Fig 3.1 Plots of the mean photon number of mode a $(\bar{n}_a)$ *at steady state [Eqs.(3.6) and (3.7)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve).*

The plots in Fig. 3.1 indicate that the mean photon number of mode a is greater for $\gamma = 0$ than that for $\gamma = 0.1$ for $0 < \Omega < 0.77$ and is lesser for $\gamma = 0$ than that for $\gamma = 0.1$ for $0.77 < \Omega < 4$.

Moreover, on account of (2.7), we can write Eq. (3.3) in the form

$$\bar{n}_b = \frac{\gamma_c}{\kappa}\langle|b\rangle\langle c|c\rangle\langle b|\rangle. \tag{3.8}$$

It then follows that

$$\bar{n}_b = \frac{\gamma_c}{\kappa}\langle\hat{\eta}_b\rangle. \tag{3.9}$$

Using Eq. (2.46), we easily find

$$\bar{n}_b = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2\gamma_c}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right). \tag{3.10}$$

And for $\gamma = 0$ the mean photon number for mode b becomes

$$\bar{n}_b = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2+3\Omega^2}\right). \tag{3.11}$$

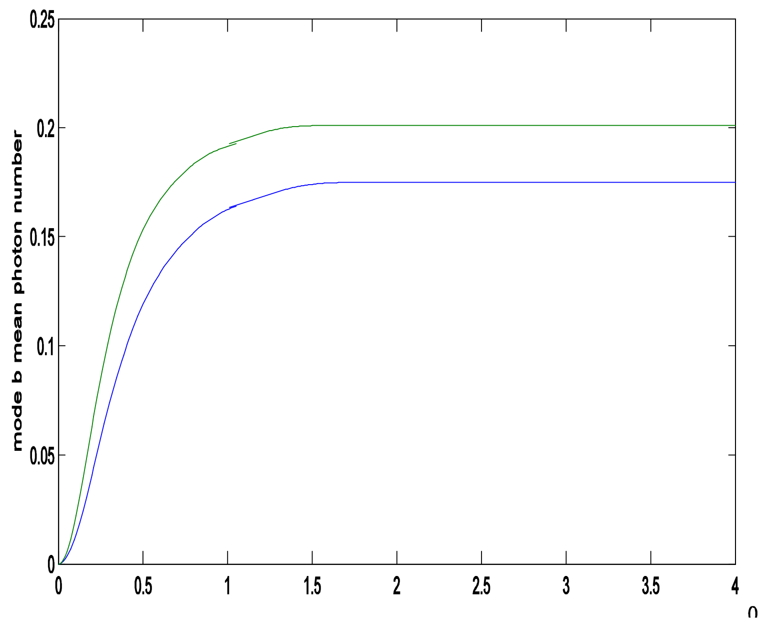

Fig 3.2 Plots of the mean photon number of mode b $(\bar{n}_b)$ *at steady state [Eqs.(3.10) and (3.11)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve).*

It can be seen from the plots in Fig. 3.2 that generally the mean photon number of mode b is greater for $\gamma = 0$ than that for $\gamma = 0.1$

Upon comparing (3.7) and (3.11) and with the help of Fig. 3.1 and Fig. 3.2 we observe that the mean photon numbers of the two light modes have the same value for $\gamma = 0$. However, for $\gamma \neq 0$, $\bar{n}_a > \bar{n}_b$.

In order to determine the mean photon number of the superposed light modes, we take

$$\hat{n} = \hat{n}_a + \hat{n}_b \tag{3.12}$$

to be the number operator for the superposed light modes. Then, the mean photon number of the superposed light modes can be expressed as

$$\bar{n} = \bar{n}_a + \bar{n}_b. \tag{3.13}$$

On account of Eqs. (3.6) and (3.10), we readily get

$$\bar{n} = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2(2\gamma_c+\gamma)}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right). \tag{3.14}$$

And for $\gamma = 0$ the mean photon number becomes

$$\bar{n} = \frac{\gamma_c}{\kappa}\left(\frac{2\Omega^2}{\gamma_c^2+3\Omega^2}\right). \tag{3.15}$$

We can easily see from the plots in Fig. 3.3 that the mean photon number of the superposed light modes is in general greater in the absence of spontaneous emission.

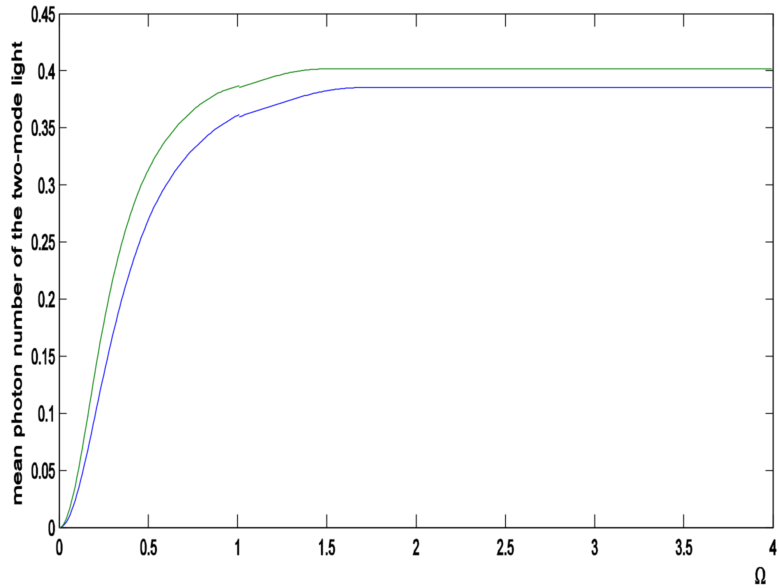

Fig 3.3 Plots of the mean photon number of the superposed light mode $(\bar{n})$ *at steady state [Eqs.(3.14) and (3.15)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) ,and* $\gamma = 0.1$ *(blue curve).*

3.2 Variance of the photon number

The variance of photon number is expressible as

$$(\Delta n)^2 = \langle \hat{n}^2 \rangle - \bar{n}^2 \,. \tag{3.16}$$

In view of the above equation, the variance of photon number for modes a and b can be written as

$$(\Delta n_a)^2 = \langle \hat{a}^\dagger \hat{a}\, \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 \tag{3.17}$$

and

$$(\Delta n_b)^2 = \langle \hat{b}^\dagger \hat{b}\, \hat{b}^\dagger \hat{b} \rangle - \langle \hat{b}^\dagger \hat{b} \rangle^2 . \tag{3.18}$$

In view of the fact that $\hat{a}$ and $\hat{b}$ are Gaussian variables with zero mean, we can rewrite Eqs. (3.17) and (3.18) as

$$(\Delta n_a)^2 = \langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{a} \hat{a}^\dagger \rangle \tag{3.19}$$

and

$$(\Delta n_b)^2 = \langle \hat{b}^\dagger \hat{b} \rangle \langle \hat{b} \hat{b}^\dagger \rangle. \tag{3.20}$$

Employing Eqs. (2.25) and (2.26) and taking into account Eqs. (2.6) and (2.7), we arrive at

$$\langle \hat{a} \hat{a}^\dagger \rangle = \frac{\gamma_c}{\kappa} \langle \hat{\eta}_b \rangle \tag{3.21}$$

and

$$\langle \hat{b} \hat{b}^\dagger \rangle = \frac{\gamma_c}{\kappa} \langle \hat{\eta}_c \rangle. \tag{3.22}$$

Thus with the help of Eqs. (3.5) and Eqs. (3.21), we can express (3.19) as

$$(\Delta n_a)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\eta}_a \rangle \langle \hat{\eta}_b \rangle \tag{3.23}$$

We see that for $\gamma = 0$

$$(\Delta n_a)^2 = \bar{n}_a{}^2, \tag{3.24}$$

where $\bar{n}_a$ is given by Eq. (3.5). This represents the normally ordered variance of the photon number for chaotic light.

Moreover, with the help of Eq. (3.9) and (3.22) we can express (3.20) in the form

$$(\Delta n_b)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\eta}_b \rangle \langle \hat{\eta}_c \rangle. \tag{3.25}$$

Finally with the aid of Eqs. (2.45), (2.46) and (2.47), the variance of the photon number of modes a and b, specified by (3.23) and (3.25) are found to be

$$(\Delta n_a)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left[\frac{\Omega^2(\gamma_c+\gamma)(\Omega^2\gamma_c)}{((\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c)^2}\right] \quad (3.26)$$

and

$$(\Delta n_b)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left[\frac{(\Omega^2\gamma_c)[(\gamma_c+\gamma)^3+\Omega^2(\gamma_c+\gamma)]}{((\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c)^2}\right]. \quad (3.27)$$

And in the absence of spontaneous emission (γ = 0), the variance of the photon number for the two modes becomes

$$(\Delta n_a)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left[\frac{\Omega^4}{({\gamma_c}^2+3\Omega^2)^2}\right] \quad (3.28)$$

and

$$(\Delta n_b)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left[\frac{\Omega^2(\Omega^2+{\gamma_c}^2)}{({\gamma_c}^2+3\Omega^2)^2}\right]. \quad (3.29)$$

The plots in Fig.3.4 show that the variance of the photon number of mode a is zero for, $\gamma = 0$ *, and* $\gamma = 0.1$ for $0 < \Omega < 0.07$. However, this variance is greater for, $\gamma = 0$ than that for $\gamma = 0.1$ for $0.08 < \Omega < 1$. On the other hand, the plots in Fig. 3.5 show that the variance of the photon number of mode b is generally greater in the absence of spontaneous emission.

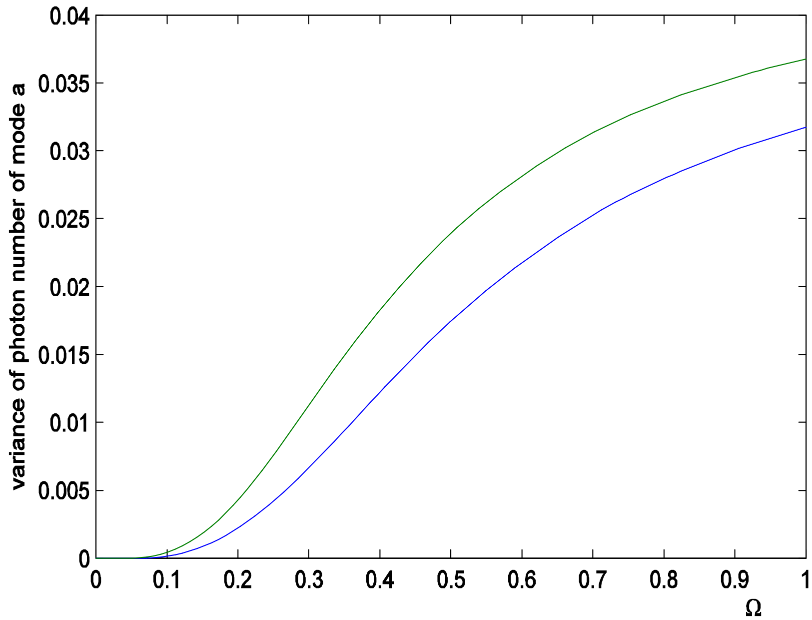

Fig .3.4 Plots of the variance of photon number of mode a $(\Delta n_a)^2$ *at steady state [Eqs.(3.26) and (3.28)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve)*

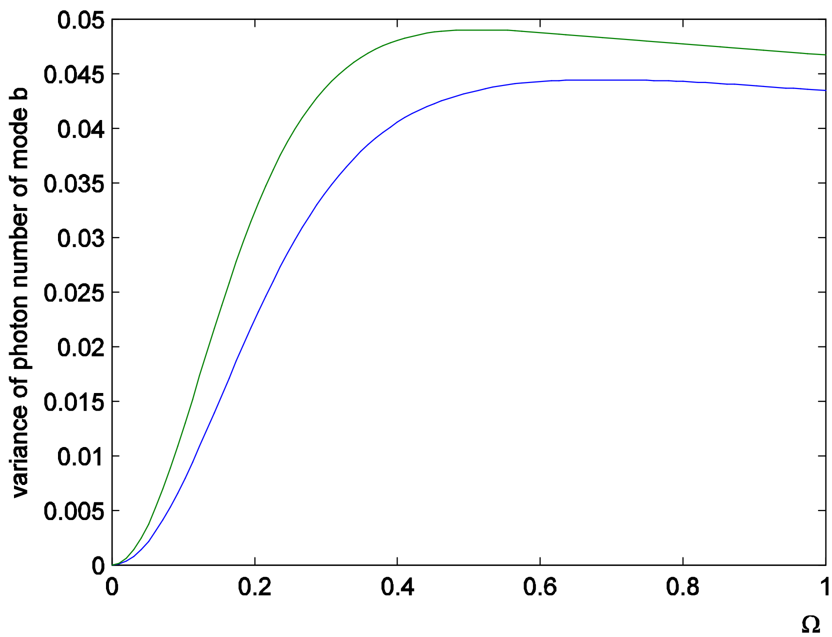

Fig 3.5 Plots of the variance of photon number of mode b $(\Delta n_b)^2$ *at steady state [Eqs.(3.27) and (3.29)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve)*

Furthermore, the variance of photon number for the superposed light modes can be expressed as

$$(\Delta n)^2 = \langle \hat{n}^2 \rangle - \bar{n}^2 \tag{3.30}$$
$$= \langle (\hat{n}_a + \hat{n}_b)^2 \rangle - \bar{n}^2$$

from which follows

$$(\Delta n)^2 = \langle {\hat{n}_a}^2 \rangle + \langle {\hat{n}_b}^2 \rangle + \langle 2\hat{n}_a \hat{n}_b \rangle - \bar{n}^2 . \tag{3.31}$$

On account of Eq. (3.1), we have

$$\langle {\hat{n}_a}^2 \rangle = \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle, \tag{3.32}$$

$$\langle {\hat{n}_b}^2 \rangle = \langle \hat{b}^\dagger \hat{b} b^\dagger \hat{b} \rangle, \tag{3.33}$$

and

$$\langle \hat{n}_a \hat{n}_b \rangle = \langle \hat{a}^\dagger \hat{a} b^\dagger \hat{b} \rangle. \tag{3.34}$$

with the aid of Eqs. (2.6) and (2.7), one can easily establish that

$$\langle {\hat{n}_a}^2 \rangle = \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\eta}_a \rangle, \tag{3.35}$$

$$\langle {\hat{n}_b}^2 \rangle = \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\eta}_b \rangle, \tag{3.36}$$

and

$$\langle \hat{n}_a \hat{n}_b \rangle = 0. \tag{3.37}$$

Hence using Eqs. (3.34), (3.35) and (3.36) we can readily arrive at

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left(\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle\right) - \bar{n}^2 . \tag{3.38}$$

On substituting Eq. (2.45), (2.46), and (3.14), there follows

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left(\frac{\Omega^2(2\gamma_c+\gamma)}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right) - \left(\frac{\gamma_c}{\kappa}\right)^2 \left(\frac{4\Omega^4\gamma_c{}^2+4\Omega^2\gamma_c\gamma+\Omega^4\gamma^2}{[(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c]^2}\right). \quad (3.39)$$

Finally, the variance of photon number for the superposed light modes can be put in the form

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left[\frac{\Omega^4(2\gamma_c{}^2+\gamma^2)+\Omega^2\gamma_c\gamma(7\gamma_c{}^2+5\gamma^2+6\gamma_{c\gamma}+3)+\Omega^2(2\gamma_c{}^4+\gamma^4+3\gamma_c{}^2\gamma^2)}{[(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c]^2}\right] \quad (3.40)$$

And in the absence of spontaneous emission ($\gamma = 0$), we have

$$(\Delta n)^2 = \left(\frac{\gamma_c}{\kappa}\right)^2 \left(\frac{2\Omega^4+2\Omega^2\gamma_c{}^2}{(\gamma_c{}^2+3\Omega^2)^2}\right). \quad (3.41)$$

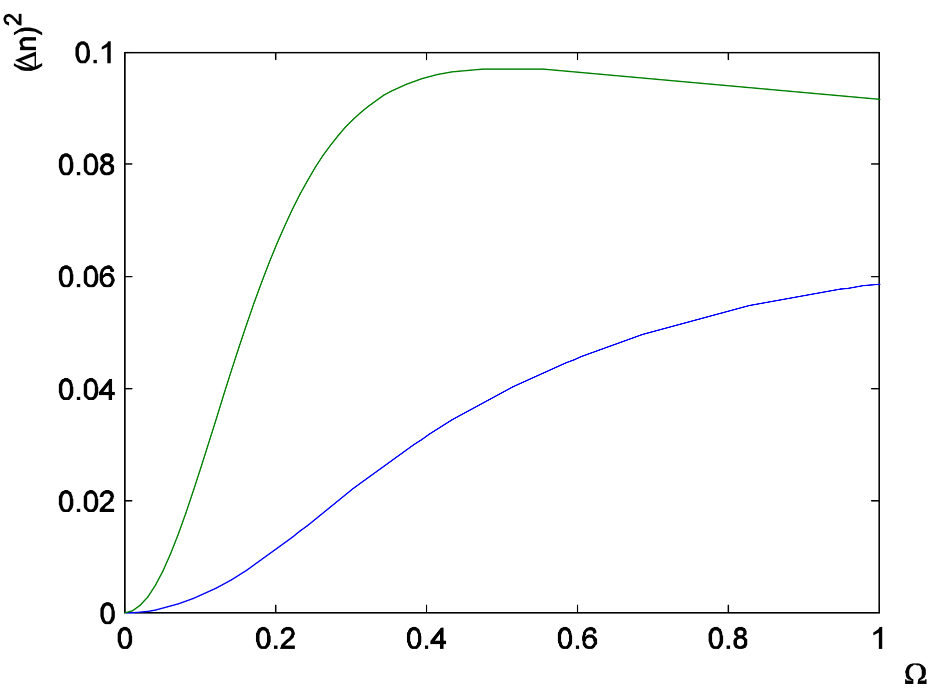

Fig 3.6 Plots of the variance of photon number of the superposed light modes $(\Delta n)^2$ *at steady state [Eqs.(3.40) and (3.41)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) ,and* $\gamma = 0.1$ *(blue curve).*

The plot in Fig. 3.6 clearly indicates that the variance of the photon number of the superposed light modes is greater for $\gamma = 0$ than for $\gamma = 0.1$.

Chapter 4

Quadrature Variance

In this chapter we seek to discuss the variance of the quadrature operators and the quadrature squeezing of the two-mode light.

We define the quadrature operators for mode a by

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a} \tag{4.1}$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{4.2}$$

On account of (2.25) , we have

$$\hat{a}_+ = -\frac{2g}{\kappa}\left(\hat{\sigma}_a^\dagger + \hat{\sigma}_a\right) \tag{4.3}$$

$$\hat{a}_- = -\frac{2g}{\kappa} i\left(\hat{\sigma}_a^\dagger - \hat{\sigma}_a\right). \tag{4.4}$$

With the aid of (2.6), it can readily be established that

$$[\hat{a}_-, \hat{a}_+] = 2i\frac{\gamma_c}{\kappa}(\hat{\eta}_a - \hat{\eta}_b). \tag{4.5}$$

The uncertainty relation for two operators $\hat{A}$ and $\hat{B}$ reads,

$$\Delta A \Delta B \geq \frac{1}{2}\left|\langle[\hat{A}, \hat{B}]\rangle\right|. \tag{4.6}$$

Hence in view of (4.6) along with (4.5), we easily get

$$\Delta a_+ \Delta a_- \geq \frac{\gamma_c}{\kappa}|\langle\hat{\eta}_a\rangle - \langle\hat{\eta}_b\rangle|. \tag{4.7}$$

Substitution of Eqs. (2.45) and (2.46) into (4.7) yields

$$\Delta a_{+}\Delta a_{-} \geq \frac{\gamma_c}{\kappa}\left|\frac{\Omega^2\gamma}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right|. \quad (4.8)$$

And for $\gamma = 0$, we have

$$\Delta a_{+}\Delta a_{-} \geq 0. \quad (4.9)$$

The variance of the quadrature operators for mode a is expressed as

$$(\Delta a_{\pm})^2 = \pm\langle[\hat{a}^{\dagger} \pm \hat{a}]^2\rangle \mp [\langle\hat{a}^{\dagger}\rangle \pm \langle\hat{a}\rangle]^2. \quad (4.10)$$

Introducing (2.42) into (2.25), we see that

$$\langle\hat{a}(t)\rangle = 0. \quad (4.11)$$

It then follows that

$$(\Delta a_{\pm})^2 = \langle\hat{a}^{\dagger}\hat{a}\rangle + \langle\hat{a}\hat{a}^{\dagger}\rangle \pm \langle\hat{a}^{\dagger 2}\rangle \pm \langle\hat{a}^2\rangle. \quad (4.12)$$

Now with the help of (3.5) and (3.21), we arrive at

$$(\Delta a_{+})^2 = (\Delta a_{-})^2 = \frac{\gamma_c}{\kappa}[\langle\hat{\eta}_a\rangle + \langle\hat{\eta}_b\rangle]. \quad (4.13)$$

Finally, upon substituting (2.45) and (2.46), the variance of quadrature operators for mode a becomes

$$(\Delta a_{\pm})^2 = \frac{\gamma_c}{\kappa}\left[\frac{\Omega^2(2\gamma_c+\gamma)}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right]. \quad (4.14)$$

And in the absence of spontaneous emission ($\gamma = 0$), we get

$$(\Delta a_{\pm})^2 = \frac{\gamma_c}{\kappa}\left[\frac{2\Omega^2}{\gamma_c{}^2+3\Omega^2}\right]. \quad (4.15)$$

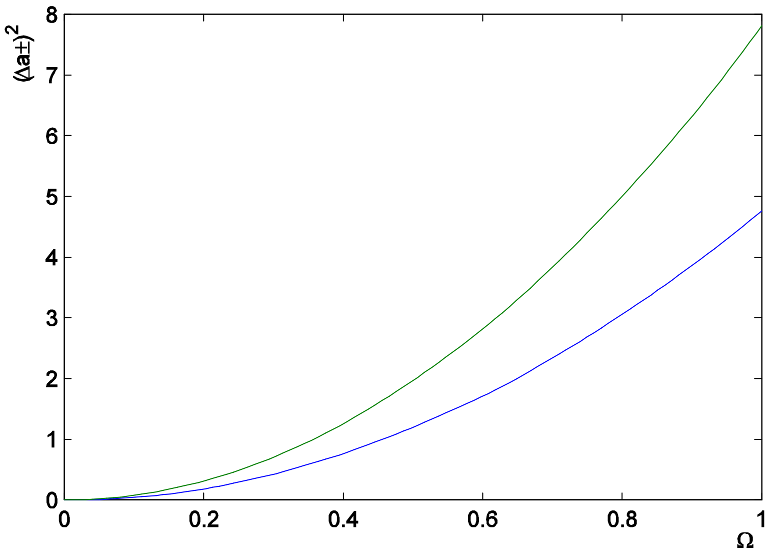

Fig. 4.1 Plots of the quadrature variance of mode a $(\Delta a_{\pm})^2$ at steady state [Eqs.(4.14) and (4.15)] versus Ω for $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ (green curve) , and $\gamma = 0.1$ (blue curve).

We see from Fig. 4.1 that the quadrature variance of mode a is greater for $\gamma = 0$ than that for $\gamma = 0.1$.

In view of Eq. (3.7), one can rewrite the variance of quadrature for mode a, for $\gamma = 0$, in terms of the mean photon number as

$$(\Delta a_{\pm})^2 = 2\bar{n}_a. \tag{4.16}$$

This is the normally ordered quadrature variance for chaotic light.

Following the same procedure, we define

$$\hat{b}_{+} = \hat{b}^{\dagger} + \hat{b} \tag{4.17}$$

and

$$\hat{b}_{-} = i(\hat{b}^{\dagger} - \hat{b}). \tag{4.18}$$

to be the quadrature operators describing the squeezing properties of mode b.

One can also readily establish that

$$\left[\hat{b}_{-}, \hat{b}_{+}\right] = 2i\frac{\gamma_c}{\kappa}(\hat{\eta}_b - \hat{\eta}_c). \quad (4.19)$$

And

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c}{\kappa}|\langle\hat{\eta}_b\rangle - \langle\hat{\eta}_c\rangle|. \quad (4.20)$$

Now substitution of Eqs. (2.46) and (2.47) into (4.20) yields

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c}{\kappa}\left|\frac{\Omega^2\gamma+(\gamma_c+\gamma)^3}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right|. \quad (4.21)$$

And for $\gamma = 0$,

$$\Delta b_{+}\Delta b_{-} \geq \frac{\gamma_c}{\kappa}\left|\frac{{\gamma_c}^2}{{\gamma_c}^2+3\Omega^2}\right|. \quad (4.22)$$

Moreover, the variances of quadrature operators for mode b are expressed as

$$(\Delta b_{+})^2 = (\Delta b_{-})^2 = \frac{\gamma_c}{\kappa}[\langle\hat{\eta}_b\rangle + \langle\hat{\eta}_c\rangle]. \quad (4.23)$$

Thus introducing (2.46) and (2.47), into (4.23) the steady-state variances take the form

$$(\Delta b_{\pm})^2 = \frac{\gamma_c}{\kappa}\left[\frac{\Omega^2(2\gamma_c+\gamma)+(\gamma_c+\gamma)^3}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right]. \quad (4.24)$$

And for $\gamma = 0$,we have

$$(\Delta b_{\pm})^2 = \frac{\gamma_c}{\kappa}\left[\frac{2\Omega^2+{\gamma_c}^2}{{\gamma_c}^2+3\Omega^2}\right]. \quad (4.25)$$

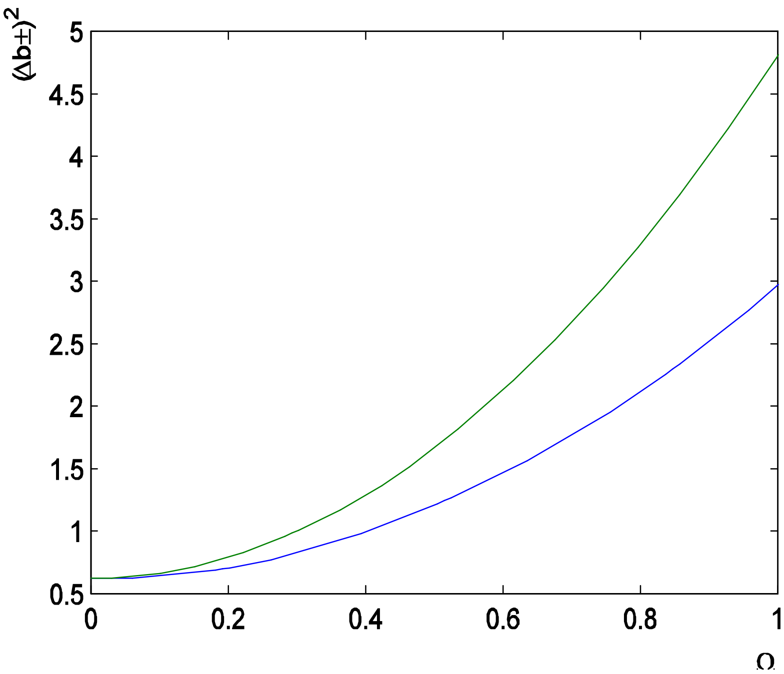

Fig. 4.2 Plots of the quadrature variance of mode b $(\Delta b_{\pm})^2$ *at steady state [Eqs.(4.24) and (4.25)] versus* Ω *for* $\gamma_c = 0.5$, κ $= 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve).*

We observe from Fig. 4.2 that the quadrature variance of mode b is generally greater in the absence of spontaneous emission.

We notice that for $\Omega \gg \gamma_c$, Eq. (4.25) reduces to

$$(\Delta b_{\pm})^2 = \frac{2\gamma_c}{3\kappa}. \tag{4.26}$$

For the same limiting case on, the mean photon number of mode b given by (3.11) takes the form

$$\bar{n}_b = \frac{\gamma_c}{3\kappa}. \tag{4.27}$$

Upon comparing (4.26) and (4.27), we see that

$$(\Delta b_{\pm})^2 = 2\bar{n}_b. \tag{4.28}$$

On account of Eqs. (4.16) and (4.28), it is worth to mention that the light modes produced by a nondegenerate three-level atom in an open cavity in the absence of spontaneous emission are chaotic.

We next seek to calculate the quadrature variance of the superposition of mode a and mode b. To this end, we define the operator representing the superposed light modes by

$$\hat{c} = \hat{a} + \hat{b}. \tag{4.29}$$

We note that

$$\langle \hat{c}^\dagger \hat{c} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle. \tag{4.30}$$

On account of (2.25) and (2.26), we can put (4.29) in the form

$$\hat{c} = -\frac{2g}{\kappa}(\hat{\sigma}_a + \hat{\sigma}_b) \tag{4.31}$$

and

$$\hat{c}^\dagger = -\frac{2g}{\kappa}\left(\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger\right). \tag{4.32}$$

We then define the quadrature operators of the superposed light as

$$\hat{c}_+ = \hat{c}^\dagger + \hat{c} \tag{4.33}$$

and

$$\hat{c}_- = i(\hat{c}^\dagger - \hat{c}). \tag{4.34}$$

Thus, in view of (4.3) and (4.4) and along with (2.25) and (2.26), one can put (4.33) and (4.34) in the form

$$\hat{c}_+ = -\frac{2g}{\kappa}\left(\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger + \hat{\sigma}_a + \hat{\sigma}_b\right) \quad (4.35)$$

and

$$\hat{c}_- = -\frac{2g}{\kappa}i\left(\hat{\sigma}_a^\dagger + \hat{\sigma}_b^\dagger - \hat{\sigma}_a - \hat{\sigma}_b\right). \quad (4.36)$$

One can easily establish that

$$[\hat{c}_-, \hat{c}_+] = 2i\frac{\gamma_c}{\kappa}(\hat{\eta}_a - \hat{\eta}_c). \quad (4.37)$$

It then follows that

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}|\langle\hat{\eta}_a\rangle - \langle\hat{\eta}_c\rangle|. \quad (4.38)$$

Introducing (2.45) and (2.47) into the above expression yields

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}\left|\frac{(\gamma_c+\gamma)^3}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right|. \quad (4.39)$$

And for $\gamma = 0$, we have

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}\left|\frac{{\gamma_c}^2}{{\gamma_c}^2+3\Omega^2}\right|. \quad (4.40)$$

Furthermore, the variances of the quadrature variance for the superposed light modes is given by

$$(\Delta c_\pm)^2 = \langle\hat{c}^\dagger\hat{c}\rangle + \langle\hat{c}c^\dagger\rangle \pm \langle\hat{c}^{\dagger 2}\rangle \pm \langle\hat{c}^2\rangle \mp \langle\hat{c}^\dagger\rangle^2 \mp \langle\hat{c}\rangle^2 - 2\langle\hat{c}^\dagger\rangle\langle\hat{c}\rangle. \quad (4.41)$$

With the help of the expression given by (4.29), we see that

$$\langle\hat{c}\rangle = \langle\hat{a}\rangle + \langle\hat{b}\rangle. \quad (4.42)$$

On account of (2.25) and (2.26) along with (2.42) and (2.43), we easily find

$$\langle\hat{c}\rangle = 0. \quad (4.43)$$

So that on account of (4.43), we have

$$(\Delta c_+)^2 = \langle \hat{c}^\dagger \hat{c} \rangle + \langle \hat{c} c^\dagger \rangle + \langle \hat{c}^{\dagger 2} \rangle + \langle \hat{c}^2 \rangle \tag{4.44}$$

and

$$(\Delta c_-)^2 = \langle \hat{c}^\dagger \hat{c} \rangle + \langle \hat{c} c^\dagger \rangle - \langle \hat{c}^{\dagger 2} \rangle - \langle \hat{c}^2 \rangle \tag{4.45}$$

On account (4.31) and (4.32) along with (2.6) and (2.7) one can easily arrive at

$$(\Delta c_+)^2 = \frac{\gamma_c}{\kappa}[\langle \hat{\eta}_a \rangle + 2\langle \hat{\eta}_b \rangle + \langle \hat{\eta}_c \rangle + 2\langle \hat{\sigma}_c \rangle] \tag{4.46}$$

and

$$(\Delta c_-)^2 = \frac{\gamma_c}{\kappa}[\langle \hat{\eta}_a \rangle + 2\langle \hat{\eta}_b \rangle + \langle \hat{\eta}_c \rangle - 2\langle \hat{\sigma}_c \rangle]. \tag{4.47}$$

Substitution of (2.44), (2.45), (2.46) and (2.47) into the aforementioned equations results in

$$(\Delta c_+)^2 = \frac{\gamma_c}{\kappa}\left[\frac{2\Omega^2(2\gamma_c+\gamma)+(\gamma_c+\gamma)^3+2\Omega(\gamma_c+\gamma)^2}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right] \tag{4.48}$$

and

$$(\Delta c_-)^2 = \frac{\gamma_c}{\kappa}\left[\frac{2\Omega^2(2\gamma_c+\gamma)+(\gamma_c+\gamma)^3-2\Omega(\gamma_c+\gamma)^2}{(\gamma_c+\gamma)^3+2\Omega^2(\gamma_c+\gamma)+\Omega^2\gamma_c}\right]. \tag{4.49}$$

On the basis of (4.48) and (4.49), we observe that the superposed light is in a squeezed state and the squeezing occurs in the minus quadrature.

In the absence of spontaneous emission ($\gamma = 0$), we have

$$(\Delta c_+)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\Omega^2+{\gamma_c}^2+2\Omega\gamma_c}{{\gamma_c}^2+3\Omega^2}\right] \tag{4.50}$$

and

$$(\Delta c_-)^2 = \frac{\gamma_c}{\kappa}\left[\frac{4\Omega^2+\gamma_c{}^2-2\Omega\gamma_c}{\gamma_c{}^2+3\Omega^2}\right]. \qquad (4.51)$$

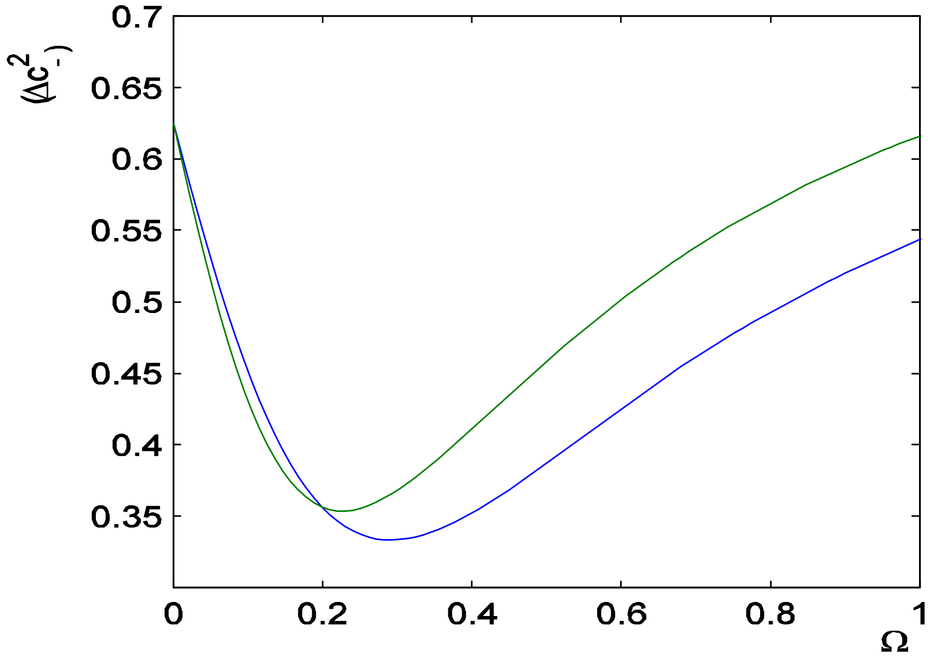

Fig. 4.3 Plots of the quadrature variance of the superposed light modes $(\Delta c_-)^2$ *at steady state [Eqs.(4.49) and (4.51)] versus* Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$ *(green curve) , and* $\gamma = 0.1$ *(blue curve).*

Fig. 4.3 clearly indicates that the quadrature variance of the superposed light modes is lesser in the presence of spontaneous emission.

We see that for $\Omega = 0$ Eqs. (3.14), (4.39), (4.48) and (4.49) take the form

$$\bar{n} = 0 \qquad (4.52)$$

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa} \qquad (4.53)$$

$$(\Delta c_+)^2 = (\Delta c_-)^2 = \frac{\gamma_c}{\kappa} \qquad (4.54)$$

We observe that Eqs. (4.52), (4.53), and (4.54) describe a two mode coherent vacuum state. We intend to calculate the quadrature squeezing of the superposed light modes relative to the quadrature variance of the coherent vacuum-state level given by Eq. (4.54)

We then define the quadrature squeezing of the superposed light modes by

$$S = \frac{(\Delta c_-)_c{}^2 - (\Delta c_-)^2}{(\Delta c_-)_c{}^2}\,. \tag{4.55}$$

Now with the aid (4.49) and (4.54), one can put Eq. (4.55) in the form

$$S = \frac{2\Omega(\gamma_c+\gamma)^2 - \Omega^2\gamma_c}{(\gamma_c+\gamma)^3 + 2\Omega^2(\gamma_c+\gamma) + \Omega^2\gamma_c}. \tag{4.56}$$

And for $\gamma = 0$, we have

$$S = \frac{2\Omega\gamma_c - \Omega^2}{\gamma_c{}^2 + 3\Omega^2}. \tag{4.57}$$

One can also be put (4.57) in the form

$$S = \frac{2\eta - \eta^2}{1 + 3\eta^2}, \tag{4.58}$$

in which $\eta = \frac{\Omega}{\gamma_c}$.

The plots in Fig. 4.3 indicate that for $\gamma = 0$ the maximum quadrature squeezing of the superposed light modes is 43% below the coherent vacuum-state level and this occurs at $\Omega = 0.22$ [1]. We can clearly see that the degree of squeezing of the superposed light modes increase for $0 < \Omega < 0.23$ and then decreases for $0.53 < \Omega < 0.7$.

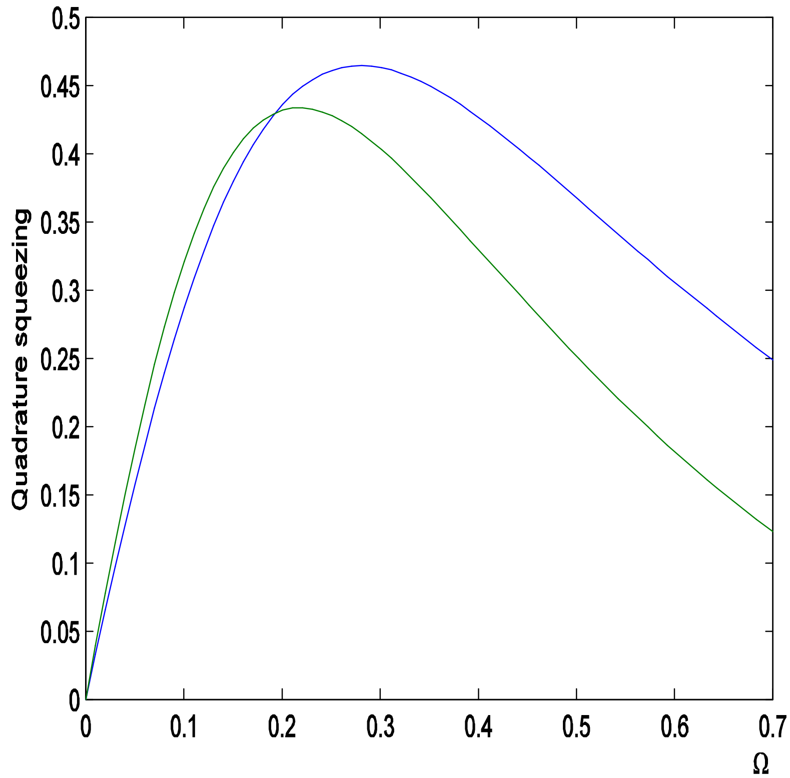

Fig 4.4 Plots of the quadrature squeezing at steady state [Eqs.(4.55) and (4.56)] versus Ω *for* $\gamma_c = 0.5$, $\kappa = 0.8$, $\gamma = 0$*(green curve) , and* $\gamma = 0.1$*(blue curve).*

On the other hand, the maximum quadrature squeezing of the cavity modes is 46.5% below the coherent vacuum-state level for $\gamma = 0.1$ and this occurs at $\Omega = 0.28$. Furthermore, the degree of squeezing for the superposed light modes increases for $0 < \Omega < 0.29$ and then decreases for $0.29 < \Omega < 0.7$.

Chapter 5

Conclusion

In this thesis we have studied the statistical and squeezing properties of the light generated by a coherently driven nondegenerate three-level atom in an open cavity coupled to a vacuum reservoir via a single-port mirror. Applying the large-time approximation scheme, we have obtained the steady state solutions of the quantum Langevin equations. We have carried out our analysis by putting the noise operators associated with the vacuum reservoir in normal order.

Using the steady state solutions of the quantum Langevin equations, we have calculated the mean photon number, the variance of the photon number, and the quadrature variance for the separate light modes and for the superposition of the two light modes. We have seen that the mean photon numbers of the two light modes are the same for $\gamma = 0$ and $\bar{n}_a > \bar{n}_b$ for $\gamma = 0.1$. In addition, our results show that light mode a is in general in chaotic state while light mode b is in a chaotic state for $\gamma_c << \Omega$. Our calculation of the quadrature variance of the superposed light modes indicates that they are in a squeezed state and the squeezing occurs in the minus quadrature.

It so turns out that the maximum quadrature squeezing of the superposed light modes is 43% for $\gamma = 0$ and 46.5% for $\gamma = 0.1$ below the coherent vacuum-state level.

References

[1] Lamrot Hailu (Msc. Thesis, Addis Ababa University, 2013)

[2] Fesseha kassahun, arXiv: 1105.1438v3 [quant-ph] (2012)

[3] Fesseha Kassahun, Fundamentals of Quantum Optics (Lulu, North caroline, 2008)

[4] Darge and K. Fesseha, PMC physics B (2010)

[5] B. Daniel, K. Fesseha opt.commun. 151 (1998) 384-394

[6] M. O. Scully and M. S. Zubairy, Quantum Optics, (Cambridge university press, Cam- bridge, 1997)

[7] N. Lu and S. Y. Zhu, Phys. Rev. A 40, 5735 (1989).

[8] G. J. Milburn and D. F. Walls, Quantum Optics, (springer-verlage, Berlin,1997)

[9] S. M. Barnett and P. M. Radmore, Methods in Theoretical Quantum Optics (Oxford University press, New York, 1997)

[10] S. Tesfa J. Phys. B: At. Mol. Opt. Phys. 41, 145501 (2008).

[11] Driba Hirpho (PhD Dissertation, Addis Ababa University, 2013).

[12] Tewodrose S. Darge (PhD Dissertation, Addis Ababa University, 2010).

[13] Misrak Getahun (PhD Dissertation, Addis Ababa University, 2009).

[14] Eyob Alebachew and K. Fesseha, Opt. Commun. 265, 314 (2006)

[15] K. Fesseha, Phys. Rev. A 63, 33811 (2001)

Declaration

I, the undersigned declare that this thesis is my original work and has not been presented for a degree in any other University. Moreover, I declare that all sources of material used for the thesis have been dully acknowledged.

Name: Abinet Yeshiwas

Signature:————

This thesis has been submitted for examination with my approval as University advisor.

Name: Dr. Fesseha Kassahun

Signature:————-

Place and date of submission

Department of Physics

Addis Ababa University

May 2014