THERMODYNAMIC AND STATISTICAL DESCRIPTION OF THERMAL PLASMA

By

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The undersigned hereby certify that they have read and recommend to the College of Natural and Computational Sciences for acceptance a project entitled "**Thermodynamic and Statistical Description of Thermal Plasma**" by **Almaz Demsis** in partial fulfillment of the requirements for the degree of **MSc. in Physics (Statistical Physics)**.

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Abstract

In this project, we study the thermodynamic and statistical properties of thermal plasma using the laws of thermodynamics and statistical physics. We studied plasma as a fourth state of matter. We find the equations of electric potential of a test particles by using poisson equation. According to this criterion, the typical energy of interaction of charged particles in a plasma or interaction energy at the average distance between charged particles is small compared with the internal energy of particles. In addition to this, we study about thermodynamic properties of plasma so the internal is small compered with the kinetic energy of charged particles of an ideal plasma. Moreover, we studied the internal partition function of atomic species is obtained as the sum over the quantum states of atoms.

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Chapter 1

Introduction

Plasma consists of electrically charged particles that respond collectively to electromagnetic forces. The charged particles are usually clouds or beams of electrons, ions, and neutrals or a mixture of electrons ions, and neutrals but also can be charged grains or dust particles. Plasma is also created when a gas is brought to a temperature that is comparable to or higher than that in the interior of stars. At these temperatures, all light atoms are stripped of their electrons and the gas is reduced to its constituent parts: positively charged bare nuclei and negatively charged free electrons. The name plasma is also properly applied to ionized gases at lower temperatures where a considerable fraction of neutral atoms or molecules are present. However, the subject matter in this book is concerned primarily, but not exclusively, with energetic or highly ionized plasmas [1].

The earths ionosphere and magnetosphere constitute a cosmic plasma system that is readily available for extensive and detailed in situ observation and even active experimentation. Its usefulness as a source of understanding of cosmic plasmas is enhanced by the fact that it contains a rich variety of plasma populations with densities ranging from more than $10^6 cm^{-3}$ to less than $10^{-2} cm^{-3}$, and temperatures from about 0.1 eV to more than 10keV [1,2].

The space environment around the various planetary satellites and rings in the solar system is filled with plasma such as the solar wind, solar and galactic cosmic rays (high energy charged particles) and particles trapped in the planetary magnetospheres. The first in situ observations of plasma and energetic particle populations in the magnetospheres of Jupiter, Saturn, Uranus, Neptune, and Titan were made by the Voyager 1 and 2 spacecraft from 1979 to 1989. Interplanetary spacecraft have identified magnetospheres around Mercury, Venus, Jupiter, Saturn, Uranus and Neptune [2-4].

On the earth, plasmas are found with dimensions of microns to meters that is, sizes spanning six orders of magnitude. The magnetic fields associated with these plasmas range from about 0.5 (the earths ambient field) to mega gauss field strengths. Plasma lifetimes on earth span 1219 orders of magnitude: Laser produced plasmas have properties measurable in picoseconds, pulsed power plasmas have nanosecond to microsecond lifetimes and magnetically confined fusion oriented plasmas persist for appreciable fractions of a second. Quiescent plasma sources, including fluorescent light sources, continuously produce plasmas whose lifetimes may be measured in hours, weeks or years, depending on the cleanliness of the ionization system or the integrity of the cathode and anode discharge surfaces [5-8].

Plasma physics studies the equilibrium and dynamics of globally neutral collections of charged particles, where the interactions between particles and the self-consistent long-range electromagnetic fields dominate over the Coulomb force between nearest neighbors. Plasmas may be thought of as the fourth state of matter in the sense that they display a distinguishing set of characteristics clearly separating them from the traditionally considered physical states of solid, liquid and gaseous matter. Just as the solid to liquid and liquid to gas phase-transitions generally occur in materials as heat is added, so a transition to the plasma state requires additional energy: once the temperature becomes sufficient to ionize atoms, a gas made up of positively charged ions and electrons is formed. Such charged particles may move about freely, generating electric currents and charge densities which modify the electric and magnetic fields present in the system. However, though plasmas are dubbed on the fourth state of matter. The formation of the plasma state does not occur as a classical phase-transition, rather the properties of the gas change gradually with increasing ionization [4,10].

As the temperature of a material is raised, its state changes from solid to liquid and then to gas. If the temperature is elevated further, an appreciable number of the gas atoms are ionized and a high temperature gaseous state is achieved, in which the charge numbers of ions and electrons are almost the same and charge neutrality is satisfied on a macroscopic scale [1-11].

When ions and electrons move these charged particles interact with the Coulomb force which is a long range force and decays only as the inverse square of the distance r between the charged particles. The resulting current flows due to the motion of the charged particles and Lorentz interaction takes place. Therefore many charged particles interact with each other by long range forces and various collective movements occur in the gaseous state. In typical cases, there are many kinds of instabilities and wave phenomena. The word plasma is used in physics to designate this high temperature ionized gaseous state with charge neutrality and collective interaction between the charged particles and waves [12-13].

This project is organized in to 5 chapters. In chapter 1 we discuses about Distribution of particle density in external field, The concept of a plasma, Classification of plasma types and Debye shielding. On chapter 2 we discuses about Velocity Space Distribution Function, Plasma Frequency. Debye Length and Thermodynamic parameters of plasma. In chapter 3 we discuses about Partition Function. And in chapter 4 we discuses about Case Study: Oxygen.

Chapter 2

The concept of a plasma

The rapid development of plasma physics in the fifties and sixties was stimulated by research directed to achieving controlled thermonuclear fusion and the magneto hydrodynamic conversion of thermal into electrical energy. However, it would be erroneous to connect the development of this new branch of physics only with technical applications. Mankind's interest in the study of near Earth space is the planets of the solar system and a large variety of astrophysical objects stimulated by the creation of sophisticated space observatories has led to the realization that plasma is the natural state of most of the matter in the universe.

An ionized gas in which all or a considerable number of atoms have lost one or several of their electrons and turned into a mixture of free electrons and positive ions is called plasma. Such ionization can take place under various conditions. For example, in the interiors of stars it happens due to the heating of matter to temperatures that are enormous on the scale of those available on the Earth. The ionization of planetary atmospheres or a gas in the vicinity of stars takes place under the action of ultraviolet emission of the sun or stars, respectively. Though the plasma temperature is low in these cases, recombination is a slow process in such rarefied plasma and thus ionization is maintained over a long period of time.

The plasma envelopes of neutron stars consist not of electrons and ions but of electrons and positrons that are the consequence of pair creation in extremely strong electric fields of rapidly rotating neutron stars (the rotation period ranging from few hundreds of a second to many hundreds of seconds and higher) with a magnetic field on the order of 10^{10} to 10^{12}. In spite of large differences among naturally occurring plasmas, their behavior can be described by general physical laws. These plasmas are ensembles of particles interacting with one another through Coulomb forces.

The methods used to describe collective particle interaction in a plasma have already been verified in a large number of both laboratory and astrophysical applications and serve as a sound basis for all modern plasma research. Therefore, it seems to us, this is an appropriate time for basic plasma physics and its main applications to be compiled in an encyclopedic edition with comprehensive coverage. However, before discussing the contents of such an edition, it may be in order to define plasma more accurately and offer a scheme for its classification. Strong electric forces attracting opposite charges in a plasma provide its quasi neutrality, i.e. the approximate equality of electron and ion concentrations.

Any separation of charges due to the displacement of a group of electrons relative to the ions produces electric fields that tend to compensate the disturbance. In order to estimate the strength of this electric field, assume that there is complete separation of charges within a plane slab of plasma of width x that is, inside this region all charges have the same sign.

The electric field satisfies Poisson's equation $\vec{\nabla}.\vec{E} = \rho/\varepsilon_0$, where $\rho = ne$ is the electric charge density in the plasma slab considered and n is the concentration of charged particles. Therefore, if n is constant the electric field is given by $E = \frac{nex}{\varepsilon_0}$ in one dimension. In the absence of external forces and as a result of any spontaneous fluctuation, the particle potential energy cannot exceed the particle thermal energy $K_B T$ in order of magnitude. In other words, significant charge separation can take place only over a region with the linear dimension $x \sim \lambda_D = (2\varepsilon_0 k_B T/ne^2)^{1/2}$. The physical meaning of the quantity λ_D can be made more precise by considering the screening of an electric field in a plasma.

Suppose that a test point charge q is introduced into a plasma. At sufficiently small distance r from the charge the electric potential is $q/4\pi\epsilon_0 r$. However, at large distance the behavior of the potential is different because of the polarization of plasma caused by the field of the test charge.

When statistical equilibrium is established, the spatial distribution of electrons and ions in the vicinity of the test charge is obtained from Boltzmann's distribution $n = n_0 \exp(-U/k_B T)$. Here U is the electrical potential energy of a particle in the test-charge field. The concentration of oppositely charged particles is higher in the vicinity of the test charge where the absolute value of the ratio $U/k_B T$ is relatively high. This leads to a screening of the test-charge field. The spatial profile of the potential q, of the point charge is found by solving Poisson's equation assuming a Boltzmannian distribution of charges in the electric field.

In a classical ideal plasma the potential energy of the particles at the average distance $r \sim n^{-1/3}$ from the test charge is much smaller than their kinetic energy. Therefore, by expanding the exponents on the right-hand side of this equation with respect to its small argument, the solution is given by

$$\phi = \frac{q}{4\pi\epsilon_0 r}\exp(-r/\lambda_D) \tag{2.1}$$

Thus at great distance from the charge q the potential decreases exponentially, and the region where a significant electric field exists near q is limited by a sphere with radius of the order of λ_D. It was Debye who first introduced this screening length in the theory of electrolytes; it was incorporated later into plasma physics. While the Debye radius characterizes the space scale of uncompensated regions of charge, the time scale for charge compensation in these regions can be characterized by the value $\tau \sim \lambda_D/\upsilon_{Te} \approx (2m_e\varepsilon_0/ne^2)^{1/2}$ where υ_{Te} is the thermal velocity of the fastest particles, i.e. the electrons.

The higher the plasma density the smaller the space and time scales of charge un compensation. With in the region occupied by cold and dense plasma quasi neutrality violations occur only in sufficiently small volumes. In hot and rarefied plasma the Debye length can become sufficiently greater than the dimensions of the region occupied by plasma. As electrons and ions move independently from each other, there is no immediate equalization of electron and ion concentrations.

The concept of the Debye radius can also be used for a more accurate definition of plasma as a special state of matter. The ensemble of freely moving charged particles of both signs, i.e. ionized gas can be considered as plasma if the Debye length is small compared with the dimensions of the volume occupied by the gas.

2.1 Classification of plasma types

Plasmas are often described as the fourth state of matter, along gases, liquids and solids, a definition which does little to illuminate their main physical attributes. In fact, a plasma can exhibit behavior characteristics of all three of the more familiar states, depending on its density and temperature, so we obviously need to look for other distinguishing features. A simple textbook definition of a plasma would be: a quasi-neutral gas of charged particles showing collective behavior. This may seem precise enough, but the rather fuzzy-sounding terms of quasi-neutrality and collectivity require further explanation. The first of these quasi-neutrality is actually just a mathematical way of saying that even though the particles making up a plasma consist of free electrons and ions, their overall charge densities cancel each other in equilibrium. So if n_e and n_i are respectively, the number densities of electrons and ions with charge state Z , then these are locally balanced i.e.

$$n_e \simeq Zn_i \tag{2.2}$$

The second property collective behavior arises because of the long-range nature of the $1/r$ Coulomb potential, which means that local disturbances in equilibrium can have a strong influence on remote regions of the plasma. In other words, macroscopic fields usually dominate over short-lived microscopic fluctuations and a net charge imbalance $\rho = e(Zn_i - n_e)$ will immediately give rise to an electrostatic field according to Gausss law,

$$\vec{\nabla}.\vec{E} = \rho/\varepsilon_0 \tag{2.3}$$

Likewise, the same set of charges moving with velocities v_e and v_i will give rise to a current density $J = e(Zn_iv_i - n_ev_e)$. This in turn induces a magnetic field according to Amperes law,

$$\nabla x B = \mu_0 J \tag{2.4}$$

to substitute the value of J

$$\nabla x B = \mu_0 e(Zn_iv_i - n_ev_e) \tag{2.5}$$

It is these internally driven electric and magnetic fields that largely determine the dynamics of the plasma, including its response to externally applied fields through particle or laser beams as for example, in the case of plasma based accelerator schemes. Now that we have established what plasmas are it is natural to ask where we can find them. In fact they are rather ubiquitous: in the cosmos, 99 percent of the visible universe including stars, the inter stellar medium and jets of material from various astrophysical objects is in a plasma state.

2.2 Distribution of particles

In the presence of other charged particles like in a plasma, the charge densities of ions and electrons and the potential experienced by the test charge is changed. That is, we consider a

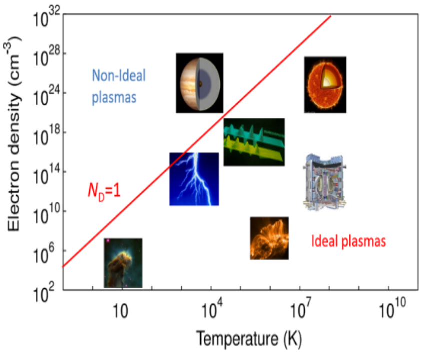

Figure 2.1: Examples of plasma types in the density temperature plane [6].

quasi-neutral plasma, so that the Coulomb field of a charged particle changes the distribution of the surrounding charged particles and as a result, the coulomb field of this particle is shielded at some distance from it. For definiteness, let us consider a positively charged plasma particle as a test case and assume that charged particles in the plasma have charge $\pm e$ the electron charge. The charge density associated with the central singly and positively charged ion as ρ has a charge density given by $\rho_{test} = e\delta(r)$ and the surrounding ions and electron densities, $\rho e = (-e)n_e$ and $\rho_i = (e)n_i$, respectively. If a gas of electrons and a single type of ion ion, the net charge density, ρ, is given by:

$$\rho = \rho_{test} + \rho e + \rho_i = e(n_i(r) - n_e(r) + \delta(r)) \tag{2.6}$$

where n_i and n_e are the number densities of positively (ions) and negatively (electrons) charged plasma particles, respectively. In the presence of other charged particles in the plasma, the electric field is given by Maxwells equation

$$\overrightarrow{\nabla}.\overrightarrow{E} = \rho/\epsilon_0 \tag{2.7}$$

And

$$\overrightarrow{E} = -\overrightarrow{\nabla}\phi \tag{2.8}$$

Using Eq.(2.7) into Eq.(2.6), the electric potential ϕ of a test particle is determined by the poisson equation

$$\nabla^2\phi = -\rho/\epsilon_0 \tag{2.9}$$

The charge density ρ is given by Eq.(2.5). Therefore the poisson equation becomes:

$$\nabla^2\phi(r) = \frac{e(n_i(r) - n_e(r) + \delta(r))}{\epsilon_0} \tag{2.10}$$

One fundamental property of plasmas is charge neutrality. Plasmas shield electric potentials applied to the plasma. When a probe is inserted into a plasma and a positive (negative) potential is applied, the probe attracts (repels) electrons and the plasma tends to shield the electric disturbance. Let us estimate the shielding length. Assume that there is a small perturbation in the ion n_i and electron density n_e and potential ϕ. Since the electrons are in the Boltzmann distribution with electron temperature T_e, the electron density n_e and ion density n_i and ion temperature T_i are given by the Boltzmann distribution. For simplicity, we take the temperature T of all the plasma components to be identical ($T_e = T_i = T$) and has equal densities of electrons and singly charged positive ions $n_e = n_i = n_0$. When there is a varying potential ϕ, the densities of electrons and ions is affected by it. If electrons are in thermal equilibrium, they will adopt a Boltzmann distribution of density and thus n_i and n_e become

$$n_i = n_0 \exp(-e\phi/\kappa_B T) \tag{2.11}$$

And

$$n_e = n_0 \exp(e\phi/\kappa_B T) \tag{2.12}$$

where n_0 is the average number density of charged particles of the plasma and T is the plasma temperature, i.e. the temperature of charged particles. This is because each electron, regardless of velocity possesses a potential energy $e\phi$. This is one elementary example of the general principle of plasmas requiring a self consistent solution of Maxwells equations of electrodynamics plus the particle dynamics of the plasma. Thus, Poison Boltzmann equation and becomes:

$$\nabla^2\phi = \left(\frac{2en_0}{\epsilon_0}\right)\sinh(e\phi/\kappa_B T) - \frac{e\delta(r)}{\epsilon_0} \tag{2.13}$$

Let us estimate the shielding length. This equation is valid at large distances from a test particle compared with the average distance between charged particles $n_0^{-1/3}$. At small distances from a test particle, where other charged particles are absent, the right hand side of the poisson equation is zero. Under the hypothesis that the thermal energy is larger than the electrostatic energy, i.e. for distances where $e\phi \ll \kappa_B T$, using Taylors series, we get

$$\exp(\pm e\phi/\kappa_B T) = 1 + \pm e\phi/\kappa_B T + ... \tag{2.14}$$

And

$$\sinh(e\phi/\kappa_B T) = e\phi/\kappa_B T + ... \tag{2.15}$$

Using this, Poison-Boltzmann equation and becomes:

$$\nabla^2\phi = \left(\frac{2n_0e^2}{\epsilon_{0\kappa_B T}}\right)\phi - \frac{e\delta(r)}{\epsilon_0} \tag{2.16}$$

And

$$\nabla^2\phi = \frac{1}{\lambda_D^2}\phi - \frac{e\delta(r)}{\epsilon_0} \tag{2.17}$$

where the constant λ_D is called the Debye-Huckl radius or Debye Length given by

$$\lambda_D = \sqrt{\frac{\epsilon_{0\kappa_B T}}{2n_0e^2}} \tag{2.18}$$

Taking into account that the electric potential of the particle doesn't depend on angle, we have:

$$\frac{1}{r}\frac{d^2}{dr^2}(r\phi) = \frac{1}{\lambda_D^2}\phi - \frac{e\delta(r)}{\epsilon_0} \tag{2.19}$$

And

$$\frac{d^2}{dr^2}(r\phi) = \frac{1}{\lambda_D^2}\phi r - \frac{er\delta(r)}{\epsilon_0} \tag{2.20}$$

Looking for a spherical symmetric solution that vanishes for $r \to \infty$ and transformed into Eq.(2.1) at small distances r, we get the well-known Debye potential, given by

$$\phi = \left(\frac{A}{r}\right) exp(-r/\lambda_D) \tag{2.21}$$

where A is a constant. In nuclear physics, this is also known as Yukawa potential, to model the Strong interaction. To determine the constant A, we should remember that we are considering the potential around an ion having charge e, and noticing that at short range $(r \ll \lambda_D)\phi$ behaves like the coulomb potential ϕ_0, the vacuum electric potential given by Eq. (2.1). We have $A = \frac{e}{4\pi\epsilon_0}$. Using this we obtain the Debye potential

$$\phi = \left(\frac{e}{4\pi r\epsilon_0}\right)\exp(-r/\lambda_D) \tag{2.22}$$

The corresponding electric field $\overrightarrow{E}$ becomes

$$\overrightarrow{E} = -\overrightarrow{\nabla}\phi \tag{2.23}$$

$$E = -\frac{\partial \phi}{\partial r} \tag{2.24}$$

$$E = \left(\frac{e}{4\pi r\epsilon_0}\right)(1/r + \frac{1}{\lambda_D})\exp(-r/\lambda_D) \tag{2.25}$$

Debye Length characterizes a distance of shielding of electric fields, or it is a measure of shielding distance of electrostatic potential, in the plasma and is one of the basic plasma parameters. By definition, we call an ionized gas a plasma if the Debye Huckle radius of this system is small compared to its dimension. This characteristic shielding of the particle field takes place if the shielding distance λ_D is large compared with the average distance between charged particles in the plasma $n_0^{-1/3}$. Omitting numerical factors, we obtain this criterion in the form

$$\lambda_D \gg n_0^{-1/3} \tag{2.26}$$

And

$$\frac{2n_0^{1/3}e^2}{\epsilon_0\kappa_B T} \ll 1 \tag{2.27}$$

According to this criterion, the typical energy of interaction of charged particles in a plasma or interaction energy at the average distance between charged particles is $\frac{n_0^{1/3}e^2}{\epsilon_0}$ small compared with the thermal energy of particles ($\sim \kappa_B T$). If this criterion is valid, most of the time the charged particles of the plasma are free. This relation is the criterion for an ideal plasma that is similar to a gas. Assume that the ions are in uniform density ($n_i = n_0$) and there is small perturbation in electron density n_e or potential ϕ. Since the electrons are in Boltzmann distribution usually, the electron density n_e becomes

$$n_i = n_0 \tag{2.28}$$

$$n_e = n_0\exp(e\phi/\kappa_B T_e) \tag{2.29}$$

$$n_e \approx n_0(1 + e\phi/\kappa_B T_e) \tag{2.30}$$

$$\nabla^2\phi = \left(\frac{2n_0e^2}{\epsilon_0\kappa_B T_e}\right) - \frac{e\delta(r)}{\epsilon_0} \tag{2.31}$$

This equation can be written as follows

$$\nabla^2\phi = \frac{1}{\lambda_D^2}\phi - \frac{e\delta(r)}{\epsilon_0} \tag{2.32}$$

And

$$\frac{d^2}{dr^2}(r\phi) = \frac{1}{\lambda_D^2}(r\phi) - \frac{er\delta(r)}{\epsilon_0} \tag{2.33}$$

where λ_D is called the Debye-Huckl radius and is given by Eq.(2.17). The solution of this equation, which is transformed into Eq.(2.1) at small distances r, has the form:

$$\phi = \phi_0 \exp(-r/\lambda_D) \tag{2.34}$$

where

$$\phi_0 = \frac{q}{4\pi\epsilon_0 r} \tag{2.35}$$

Here again ϕ_0 is the vacuum electric potential.

2.2.1 Distribution of particle density in external field

The Boltzmann formula which is given $\bar{n_i} = C\exp\left(-\frac{\varepsilon_i}{k_BT}\right)$ allows us to analyze the distribution function of particles in external field. Let us consider a field influence on the particle distribution. We consider a quasi neutral plasma, so the coulomb field of a charged particle changes the distribution of the surrounding particles. As a result, the coulomb field of this particle is shielded at some distance from it. For definiteness, let us consider a positively charged plasma particle as a test case and assume that charged particles in the plasma have a charge $\pm e$ the electron charge. Then in a vacuum the electric potential ϕ of a test charged particle on a distance r from it is equal to

$$\phi = \frac{e}{4\pi\epsilon_0 r} \tag{2.36}$$

In the presence of other charged particles in the plasma, the electric potential of a test particle is determined by the poisson equation

$$\nabla^2\phi = e(N^- - N^+)/\epsilon_0 \tag{2.37}$$

where N^- and N^+ are the number densities of negatively and positively charged plasma particles, which according to the Boltzmann formula $\bar{n}_i = C\exp\left(-\frac{\varepsilon_i}{k_BT}\right)$ are equal to

$$N_{ne} = N_0 \exp\left(\frac{e\phi}{k_BT}\right) \tag{2.38}$$

And

$$N_{ni} = N_0 \exp\left(\frac{-e\phi}{k_BT}\right) \tag{2.39}$$

where N_0 is the average number density of charged particles of the plasma and T is the plasma temperature, i.e the temperature of charged particles. Thus the poisson equation takes the form

$$\nabla^2\phi = 2eN_0 \sinh\left(\frac{e\phi}{k_BT}\right)/\epsilon_0 \tag{2.40}$$

This equation is valid at large distance from a test particle compared with the average distance between charged particles $N_0^{-1/3}$. At small distances from a test particle, where other charged particles are absent, the right hand side of the poisson equation is zero, and the coulomb electric potential of this particle is given by the formula $\phi = \frac{e}{4\pi\epsilon_0 r}$. Taking in to account that the electric potential of the particle does not depend on angle, we have for distance where $e\phi \ll k_BT$

$$\frac{1}{r}\frac{d^2}{dr^2}(r\phi) = \frac{2N_0e^2}{\epsilon_0 k_BT}\phi \tag{2.41}$$

The solution of this equation, which is transformed in to $\phi = \frac{e}{4\pi\epsilon_0 r}$ at small distance r, has the form

$$\phi = \frac{e}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right), \tag{2.42}$$

where $\lambda_D = \sqrt{\frac{\epsilon_0 k_BT}{2N_0e^2}}$. The value λ_D is called the Debye Huckel radius. It characterizes a distance of shielding of electric field in the plasma and is one of the basic plasma parameters. By definition we call an ionized gas a plasma if the Debye Huckel radius of this system is small compared to its dimension. This characteristic of the particle field takes place if the shielding distance r_D is large compared with the average distance between charged particles in the plasma $N_0^{-1/3}$, omitting numerical factors, we obtain this criterion in the form

$$\frac{e^2N_0^{1/3}}{k_BT} \ll 1 \tag{2.43}$$

According to this criterion, the typical energy of interaction of charged particles in plasma or the interaction energy at the average distance between charged particles $e^2N_0^{1/3}$ is small compared with thermal energy of the particle($\sim k_B T$). If this criterion is valid, most of the time the charged particles of the plasma are free. This relation is the criterion for an ideal plasma that similar to a gas.

2.3 Fluctuation in a plasma

We consider an idea quasi neutral plasma that along with neutral particle contains electrons and ions of identical charge density (for simplicity, we assume the ion charge to be equal to that of the electron). The mean potential energy of a test charged particle or a typical interaction potential energy between nearest charged particles is small compared with the typical kinetic energy of this charged particle, which is the definition of an ideal plasma, which in turn is similar to a gas of electrons and ions. The number density of neutral particles of an ideal plasma can exceed that of charged particles, but some properties of this plasma would be determined by the charged particles due to the long-range interaction between them. One of the properties of an ideal plasma is the screening of field in it due to the displacement of charged particles under the action of these fields. A typical distance for this screening is the Debye-Huckel radius $\phi = \frac{e}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right)$. The location of a large number of charged particles in a sphere of the Debye-Huckel radius can also be the definition of an ideal plasma. Displacement of charged particles in an ideal plasma create a plasma potential that causes an energy change for a charged particle inserted in a plasma through its boundary. Our goal now is to determine the average plasma potential together with the distribution function over the plasma potentials. According to formula $\phi = \frac{e}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right)$ the potential interaction energy for a test ion with other ion is equal to

$$U = e\phi = \frac{e^2}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right) \tag{2.44}$$

So that the mean potential energy of a test ion in an ideal plasma is

$$\bar{U} = \int_0^\infty \frac{e^2}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right)\left[N_0\exp\left(-\frac{e\phi}{T}\right) - N_0\exp\left(\frac{e\phi}{T}\right)\right].4\pi r^2 dr = -\frac{e^2}{2\lambda_D}, e\phi \ll T \tag{2.45}$$

where we assume the temperatures of electrons and ions to be identical. For an ideal plasma, if the criterion $\frac{e^2 N_0^{1/3}}{T} \ll 1$ is fulfilled the mean potential energy of the above equation of a charged plasma particle is small compared with its thermal energy that is

$$\bar{U} \ll k_B T \tag{2.46}$$

Note that this potential energy is identical for positively and negatively charged particles (for ions and electrons). Using the same method one can find the mean square for the ion or electron potential energy

$$\bar{U^2} = \int_0^\infty \frac{e^4}{r^2} \exp\left(-\frac{2r}{\lambda_D}\right) \left[N_0 \exp\left(-\frac{e\phi}{T}\right) + N_0 \exp\left(\frac{e\phi}{T}\right)\right] .4\pi r^2 dr, \ = 4\pi N_0 e^4 \lambda_D = \frac{T}{2}\frac{e^2}{\lambda_D}, e\phi \ll T \tag{2.47}$$

And

$$\frac{\bar{U^2}}{T^2} \sim \frac{e^2}{\lambda_D T} \ll 1 \tag{2.48}$$

For an ideal plasma. Next

$$\frac{\bar{U^2}}{\bar{U}^2} = (4N_0/\epsilon_0)\lambda_D^3 \ggg 1 \tag{2.49}$$

According to definition of ideal plasma, because many charged particles located inside a sphere of radius λ_D.

2.3.1 Plasma oscillations

This definition was given by Langmuir, who coined the word "plasma" and made the first attempt at its theoretical description. Finally, one more important quantity, v_{iz}, the characteristic frequency of plasma oscillations, is necessary for the classification of different kinds of plasmas. Although very many different types of waves and oscillations are easily excited in plasma, the oscillations caused by macroscopic violation of quasi neutrality are the ones that characterize plasma as an elastic medium. For simplicity, again consider the case of a charge separation in a plane plasma slab where all electrons are displaced in this slab by distance x. The "restoring" force makes the electrons move according to the equation

$$m_e \ddot{x} = -eE_x = -(ne^2 x)/\epsilon_0 \tag{2.50}$$

It thus follows that neutralization of the excess charge is accompanied by oscillations with the frequency

$$\omega_{pe} = (ne^2/(\epsilon_0 m_e))^{1/2} \tag{2.51}$$

These are the so-called Langmuir oscillations. Due to their great mass the ions essentially do not take part in these oscillations. Unlike sound waves in an ordinary gas where the elastic force is represented by the pressure gradient, in plasma the main role is played by the electric fields due to uncompensated charges. Langmuir oscillations may propagate in plasma in the form of waves with frequency $\omega = \omega_{pe}$ which does not depend on the wavelength in the limit of the long wavelength approximation used above. If the wavelength is small one has to take into account the restoring force caused by plasma compression in the wave. Then the square of the sound velocity should be added to the expression for the squared phase velocity:

$$\frac{\omega^2}{k^2} = \frac{\omega_{pe}^2}{k^2} + \frac{\partial \rho_e}{\partial P_e} \tag{2.52}$$

Here, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, ρ_e is the density of the electron gas ($\rho_e = n_e m_e$) and P_e is its pressure. Taking into account that the ratio of specific heat is equal to 3 in the one-dimensional case considered here, this expression can be rewritten in the from derived by Vlasov from the kinetic equation for electrons:

$$\omega^2 = \omega_{pe}^2 + 3K^2 k_B T_e/m_e \tag{2.53}$$

The second term is smaller than the first one since one can refer to collective plasma behavior, including plasma oscillations, only in the case of wavelengths long compared with the Debye length, so that the phase velocity is much greater than the thermal velocity. In the opposite case it becomes necessary to take into account the influence of the resonant interaction of waves with plasma particles so-called Landau resonance: $(w = k * v)$. Plasma properties are complicated if neutral atoms and molecules coexist with charged particles, i.e., when the plasma is not fully ionized. The degree of plasma ionization is the ratio of the number of charged particles to the initial number of atoms. It is attained by the competition of ionization (destruction of atoms) and its inverse process, recombination, i.e. reunion of electrons and ions into neutral particles. For plasma in thermodynamic equilibrium the degree of ionization does not depend on the details of these processes and in principle, may be established in a purely thermodynamic way. The laws of thermodynamics have the simplest form for the plasma obeying the ideal gas equation, i.e. the case when the kinetic energy of charged particles by far exceeds their interaction energy. Consider a singly ionized plasma. According to the general principles of statistical physics, the ratio of the probabilities of an electron being in the states with energies w_1 and w_2 at a given temperature T is $(g_1/g_2)\exp[(w_2 - w_1)/k_B T]$.

Here g_1 and g_2 denote the quantum weights of the respective states. The ionization degree of a gas, i.e. the ratio of the number of free electrons to that of neutral atoms, is represented by this expression with the condition $w_1 - w_2 = I$ where I is the ionization energy. In this case g_1 is the number of the elementary quantum cells in the phase space of the free electron and g_2 is the quantum weight of the stationary energy level in the atom. If for simplicity, we neglect the excited levels in the atom and suppose that the ground state is not degenerate then I is the ionization energy and $g_2 = 1$. Free electrons have a continuous energy spectrum. The quantum weight of free states roughly equals the phase space volume for an electron with average thermal momentum $p = (2m_e k_B T)^{1/2}$, divided by the elementary phase volume $(2\pi\hbar)^3$

$$g_1 = (2m_e k_B T)^{2/3} V_0/(2\pi\hbar)^3 \tag{2.54}$$

Here, V_0 is the geometrical volume per electron, i.e. $V_0 = 1/n_e$ Hence,

$$g_1 = (2m_e k_B T)^{2/3}/n_e(2\pi\hbar)^3 \tag{2.55}$$

Using this result, one obtains the so-called Saha equation for ionization versus temperature:

$$\frac{n_e}{n_i} = \frac{g_1}{g_2}\exp(-I/k_B T) \approx \frac{(2m_e k_B T)^{2/3}}{n_e(2\pi\hbar)^3}\exp(-I/k_B T) \tag{2.56}$$

The formula may be rewritten in a way more convenient for the calculation of n_e/n_i (in a weakly ionized plasma):

$$\frac{n_e}{n_i} = \left[(2m_e k_B T)^{3/4}/n_e^{1/2}(2\pi\hbar)^{3/2}\right]\exp(-I/2k_B T) \tag{2.57}$$

From the Saha equation it follows that the lower the gas density, the higher its ionization degree. At densities much lower than the condensed matter density, the degree of ionization may be high even if the temperature $k_B T \ll I$. At very low densities, however, thermodynamic equilibrium is much more difficult to achieve due to the scarcity of particle collisions. To determine the extent of ionization in a plasma state far from thermodynamic equilibrium it is necessary to consider the details of the collision processes leading to ionization and recombination.

So far we have considered characteristics, such as density and temperature of a plasma in equilibrium. We can also ask how fast the plasma will respond to an external disturbance, which could be due to electromagnetic waves (e.g. a laser pulse) or particle beams. Consider a quasi-neutral plasma slab in which an electron layer is displaced from its initial position by a distance d, as illustrated in Fig(2.2). This creates two capacitor plates with surface charge $\sigma = en_e\delta$, resulting in an electric field

$$E = \frac{\sigma}{\varepsilon_0} = \frac{en_e\delta}{\varepsilon_0} \tag{2.58}$$

The electron layer is accelerated back towards the slab by this restoring force according to

$$m_e\frac{d\nu}{dt} = -m_e\frac{d^2\delta}{dt^2} = -eE = \frac{e^2n_e\delta}{\varepsilon_0} \tag{2.59}$$

Or

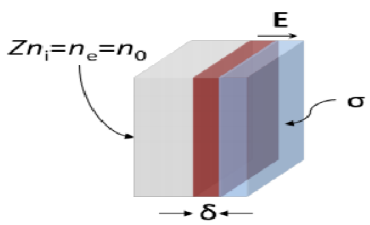

Figure 2.2: Slab or capacitor model of an oscillating electron laye [16].

$$\frac{d^2\delta}{dt^2} + \omega_p^2\delta = 0 \tag{2.60}$$

where

$$\omega_p \equiv \left(\frac{e^2n_e}{\varepsilon_0 m_e}\right)^{1/2} \simeq 5.6 * 10^4 \left(\frac{n_e}{cm^{-3}}\right)^{1/2} s - 1 \tag{2.61}$$

is the electron plasma frequency. For a plasma of temperature T_e, the response time to recover quasi-neutrality is just the ratio of the Debye length to the thermal velocity $v_{et} = \sqrt{k_B T_e/m_e}$ that is,

$$\frac{\lambda_D}{\nu_{et}} = \left(\frac{\varepsilon_0 k_B T_e}{e^2 n_e}.\frac{m}{k_B T_e}\right)^{1/2} = \omega_p^{-1} \tag{2.62}$$

If the plasma response time is shorter than the period of a external electromagnetic field such as a laser, then this radiation will be shielded out. To make this statement more quantitative, consider the ratio

$$\frac{\omega_p^2}{\omega^2} = \frac{e^2n_e}{\varepsilon_0 m_e}.\frac{\lambda^2}{4\pi^2c^2} \tag{2.63}$$

Setting this to unity defines the wavelength λ_μ for which $n_e = n_c$, or

$$n_c \simeq 10^{21} \lambda_\mu^{-2} cm^{-3} \tag{2.64}$$

Radiation with wavelength $\lambda > \lambda_\mu$ will be reflected. In the pre-satellite/cable era, this property was exploited to good effect in the transmission of long wave radio signals, which utilizes reflection from the ionosphere to extend the range of reception. Typical gas jets have $P \sim 1$ bar and $n_e = 10^{18}10^{19} cm^{-3}$, and the critical density for a glass laser is $n_c(1) = 10^{21} cm^{-3}$.

Gas-jet plasmas are therefore under dense, since $\omega^2/\omega_p^2 = n_e/n_c \ll 1$. In this case, collective effects are important if $\omega_p \tau_{int} > 1$, where τ_{int} is some characteristic interaction time, such as the duration of a laser pulse or particle beam entering the plasma. For example, if $\tau_{int} = 100 fs$ and $n_e = 10^{17} cm^{-3}$, then $\omega_p \tau_{int} = 1.8$ and we will need to consider the plasma response on the interaction time scale. Generally this is the situation we seek to exploit in all kinds of plasma applications, including short wavelength radiation, nonlinear refractive properties, generation of high electric/magnetic fields and of course, particle acceleration. First of all, we need to know how fast the electrons and ions actually move. For equal ion

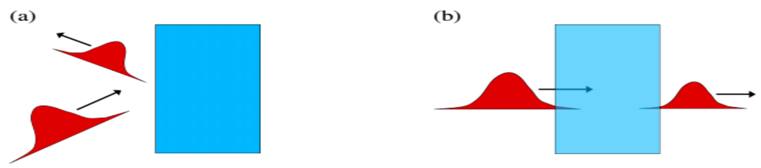

Figure 2.3: Over dense plasma, with $\omega < \omega_p$, showing mirror like behavior. (b) Under dense plasma, with $\omega > \omega_p$, which behaves like a nonlinear refractive medium [7].

and electron temperatures ($T_e = T_i$), we have

$$\frac{1}{2} m_e v_e^2 = \frac{1}{2} m_i v_i^2 = \frac{3}{2} k_B T_e \tag{2.65}$$

Let us consider the case where a small perturbation occurs in a uniform plasma and the electrons in the plasma move due to the perturbation. It is assumed that the ions do not move because they have much greater mass than the electrons. Due to the displacement of electrons, electric charges appear and an electric field is induced. The electric field is given by

$$\epsilon_0 \nabla \cdot E = -e(n_e - n_0) \tag{2.66}$$

Electrons are accelerated by the electric field:

$$m_e \frac{dv}{dt} = -eE \tag{2.67}$$

Due to the movement of electrons, the electron density changes:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \nu) = 0 \tag{2.68}$$

Writing $n_e - n_0 = n_1$ and assuming $|n_e| \ll n_0$, we find $\epsilon_0 \nabla . E = -en_1$, $m_e \frac{\partial v}{\partial t}$, $\frac{\partial n_e}{\partial t} + n_e \nabla . v = 0$ For simplicity, the displacement is assumed to be only in the x direction and sinusoidal with angular frequency ω:

$$n_1(x,t) = n_1 \exp(ikx - i\omega t) \tag{2.69}$$

The time derivative $\partial/\partial t$ is replaced by $-i\omega$ and $\partial/\partial x$ is replaced by ik .

The electric field has only the x component E. Then $ik\epsilon_0 E = -en_1$, $-i\omega m_e v = -eE$ and $-i\omega n_1 = -ikn_e v$. So that we find

$$\omega^2 = \frac{n_e e^2}{\epsilon_0 m_e} \tag{2.70}$$

This wave is called the electron plasma wave or Langmuir wave and its frequency is called the electron plasma frequency $\prod_e$

$$\prod_e = \left(\frac{n_e e^2}{\epsilon_0 m_e}\right)^2 = 5.64 * 10^{11} \left(\frac{n_e}{10^{20}}\right) rad/s \tag{2.71}$$

The following relation holds between the plasma frequency and the Debye length λ_D

$$\lambda_D \prod_e = \left(\frac{k_B T_e}{m_e}\right)^{1/2} = v_B T_e = 4.19 * 10^5 \left(\frac{k_B T_e}{e}\right)^{1/2} m/s \tag{2.72}$$

2.4 Neutral gas consisting of charged particles plasma

Consider a special kind of real gas plasma. In contrast to the van der Waals gas particles, those forming plasma possess an electrical charge. Assume that a gas of volume V and the total number of particles $N = \sum N_i$ consists of positively and negatively charged ions. Here,

N_i is the number of ions of the i^{th} type, Z_{ie} is the electrical charge of ions of the same type, e is the absolute magnitude of the charge of an electron and Z_i takes on values $Z_i = 1, 2, ...$ In the particular case of high temperatures and for a completely ionized atomic gas, Z_i can take on values $Z_1 = Z_{ion} = +1$, $Z_2 = Z_{el} = -1$, i.e. in this case plasma consists of positively charged ions and electrons. As a whole, a gas toughs to be neutral. The condition of neutrality can be written down in the form

$$\sum Z_{ie}N_0 = 0, or \sum Z_{ie}n_{io} = 0 \tag{2.73}$$

where $n_{0i} = N_i/V$ is the mean concentration of the i^{th} ion in the case where the interaction between particles is disregarded and it is supposed that all particles of the i^{th} type are uniformly distributed over the entire volume. Also assume that a completely ionized gas plasma is sufficiently rarefied, i.e. the mean distance r between ions is such that the energy of the Coulomb interaction is much less than the energy of the thermal motion k_BT

$$\frac{Z_i^2e^2}{r} \ll k_BT \tag{2.74}$$

If the concentration of all ions in the plasma is denoted by $n_0 = \sum n_{io}$, then $r \approx n_0^{-1/3}$. Hence, the condition of rarefaction ($\frac{Z_i^2e^2}{r} \ll k_BT$) can be rewritten in the form

$$n_0 \ll \left(\frac{k_BT}{Z_i{}^2e^2}\right)^3 \tag{2.75}$$

We derive the equation of state of rarefied plasma and calculate some thermodynamic quantities satisfying conditions ($\frac{Z_i^2e^2}{r} \ll k_BT$) and ($n_0 \ll \left(\frac{k_BT}{Z_i{}^2e^2}\right)^3$). However, it is necessary to note that the method applied to the van der Waals gas cannot be applied to plasma. This is due to the fact that in plasma the long range acting Coulomb interaction exists between ions:

$$u_{ik} = \frac{Z_iZ_ke^2}{r} \sim \frac{1}{r} \tag{2.76}$$

From this fact two conclusions follow: first, because the radius of action of each ion is large and plasma is rarefied not one but many ions are found in the sphere of action of each ion; second, in the case of such an interaction the integral entering into the expression of the second virial coefficient ($B = 4\upsilon_0 - \frac{2\pi}{k_BT}\int_0^\infty |u(r)|r^2dr$) diverges in the upper limit:

$$\int^\infty |u_{ik}|r^2dr = Z_iZ_ke^2\int^\infty rdr \to \infty \tag{2.77}$$

Therefore, in order to find the free energy and the equation of state of plasma, we use the Helmholtz relationship

$$E = -T^2 \frac{\partial}{\partial T}\left(\frac{F}{T}\right) \tag{2.78}$$

Hence, by knowing the total energy and the free energy can be calculated by the formula

$$F = -T \int \frac{E}{T^2} dT \tag{2.79}$$

The total energy of plasma can be presented as the sum

$$E = E_{id} + E_{coul} \tag{2.80}$$

where, E_{id} is the energy without considering the interaction between ions (an ideal gas) and E_{coul} is the energy of the Coulomb interaction

$$E_{coul} = \frac{1}{2} \sum Z_i e N_i \phi_i(r) \tag{2.81}$$

where $\phi_i(r)$ is the potential of the Coulomb field around the i^{th} ion at the point r created by all the ions except the given one. If we substitute the expression of energy ($E = E_{id} + E_{coul}$) into ($F = -T \int \frac{E}{T^2} dT$), we get

$$F = F_{id} + F_{coul} \tag{2.82}$$

where

$$F_{coul} = -T \int \frac{E_{coul}}{T^2} dT \tag{2.83}$$

is the free energy corresponding to the Coulomb interaction. Thus, the problem of finding the free energy is reduced to finding E_{coul} or the potential $\phi_i(r)$. In order to determine the explicit form of $\phi_i(r)$, we use the screening method proposed by Debye and Heckle in [1923] in the theoretical studies of the properties of electrolytes. The essence of this method is the fact that in a system of charged particles (in our case, plasma) the potential created at the point of an arbitrary ion being found can be presented in the form

$$\phi_i(r) = \left[\phi(r) - \frac{Z_i e}{4\pi\epsilon_0 r}\right] \tag{2.84}$$

where $\phi(r)$ is the potential created by all charges at the point r and $Z_i e/r$ is the potential created by the given ion at the point r. The potential $\phi(r)$ can be found from the known Poisson equation

$$\nabla^2 \phi(r) = -\rho(r)/\epsilon_0 \tag{2.85}$$

where

$$\rho(r) = \sum Z_i e n_i(r) \tag{2.86}$$

is the density of charges at the point r, and $n_i(r)$ is the concentration of all ions of the i^{th} type around the point r. From the two latter expressions, we get for the Poisson equation

$$\nabla^2 \phi(r) = -\sum Z_i e n_i(r)/\epsilon_0 \tag{2.87}$$

The concentration $n_i(r)$ on the right-hand side of this equation is distinct from n_{oi}, because $n_i(r)$ is the concentration with regard to the interaction between ions. As a result of the interaction between ions, they are non-uniformly distributed over the entire volume, and each ion is surrounded by an ion with the opposite charge, and the distribution resembles a mosaic. The cause of distinction of $n_i(r)$ from n_{oi} lies in the fact that each ion possesses the potential energy $Z_i e\phi(r)$ and the distribution of charges in the potential field $\phi(r)$ can be described with the aid of the Boltzmann statistics:

$$n_i(r) = n_{oi} \exp\left[-\frac{Z_i e\phi(r)}{k_B T}\right] \tag{2.88}$$

From Poisson equation

$\nabla^2\phi(r) = -\sum Z_i e n_i(r)/\epsilon_0$ and $n_i(r) = n_{oi}\exp\left[-\frac{Z_i e\phi(r)}{k_B T}\right]$, it follows that to calculate the potential $\phi(r)$ it is necessary to know the concentration $n_i(r)$ and conversely in order to calculate $n_i(r)$ it is necessary to know $\phi(r)$.

Thus, a certain self contradictory problem arises. Therefore, it is necessary to simultaneously solve equations $\nabla^2\phi(r) = -\sum Z_i e n_i(r)/\epsilon_0$ and $n_i(r) = n_{oi}\exp\left[-\frac{Z_i e\phi(r)}{k_B T}\right]$. If we take into account the inequality $Z_i e\phi(r) \ll k_B T$, equivalent to the condition of the rarefied nature of the plasma ($\frac{Z_i^2 e^2}{r} \ll k_B T$), in $n_i(r) = n_{oi}\exp\left[-\frac{Z_i e\phi(r)}{k_B T}\right]$, we get

$$n_i(r) = n_{oi}\left[1 - \frac{Z_i e\phi(r)}{k_B T}\right] \tag{2.89}$$

After substituting (2.88) into (2.86), the Poisson equation takes the form

$$\nabla^2\phi(r) = -\sum Z_i e n_{oi}/\epsilon_0 + \frac{e^2}{\epsilon_0 k_B T}\left(\sum Z_i^2 n_{oi}\right)\phi(r) \tag{2.90}$$

Taking into account the condition of neutrality of the plasma (2.89) and introducing the notation

$$r_0^{-2} = \frac{e^2}{\epsilon_0 k_B T}\left(\sum Z_i^2 n_{oi}\right) \tag{2.91}$$

the Poisson equation can be simplified to

$$\nabla^2\phi(r) - r_0^{-2}\phi(r) = 0 \tag{2.92}$$

Because in plasma the electrostatic potential close to each ion possesses spherical symmetry, it is more convenient to write equation (2.91) down in the spherical coordinate system. In this case, the potential depends only on the quantity of the radius vector and it is sufficient to consider the radial part of the operator ∇^2 :

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{d\phi}{dr}\right) = r_0^{-2}\phi(r) \tag{2.93}$$

It is easy to show that the function satisfies this equation.

$$\phi(r) = \frac{A}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right) \tag{2.94}$$

The constant A entering here can be found from the condition

$$\lim_{r\to 0}\phi(r) = \frac{Z_i e}{4\pi\epsilon_0 r} \tag{2.95}$$

to fulfil which it is necessary to have $A = Z_i e$. Then the potential created by any point charge $Z_i e$ at the distance r is

$$\phi(r) = \frac{Z_i e}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{\lambda_D}\right) \tag{2.96}$$

The potential $\phi(r)$, pre-assigned by formula (2.94), is called the screening Coulomb potential of a point charge, and r_0 is the Debye screening radius. If in the volume only one isolated ion is found, the potential created by it at a distance r is

$$\phi_0(r) = \frac{Z_i e}{4\pi\epsilon_0 r} \tag{2.97}$$

If a point charge is surrounded (screened) by ions with opposite charges, its field weakens, and at the point $r = r_0$ the potential decreases $e = 2.74$ times. Thus, if we substitute expression (2.95) into (2.83), for the potential $\phi_i(r)$ we get

$$\phi_i(r) = \lim_{r\to 0}\left[\frac{z_i e}{4\pi\epsilon_0 r}\exp\left(-\frac{r}{r_0}\right) - \frac{Z_i e}{4\pi\epsilon_0 r}\right] \tag{2.98}$$

If in this limiting case we expand the exponent in powers of $r/r_0 \ll 1$ and restrict ourselves to the first two terms, the potential sought has the appearance

$$\phi_i(r) = \frac{Z_i e}{4\pi\epsilon_0 r_0} \tag{2.99}$$

Substituting this expression as well as (2.90) in (2.98), we get for the energy of plasma at the expense of the Coulomb interaction in (2.80) can be

$$E_{coul} = -e^3 \left(\frac{\pi}{k_B T}\right)^{1/2} \frac{1}{V^{1/2}} \left(\sum Z_i^2 N_i\right)^{3/2} \tag{2.100}$$

If we introduce the notation

$$q_0 = \left(\sum Z_i^2 e^2 N_i\right)^{1/2} \tag{2.101}$$

with the dimensionality of an electrical charge, the energy of the Coulomb interaction of plasma takes the form

$$E_{coul} = -\left(\frac{\pi}{k_B T V}\right) q_0^3 \tag{2.102}$$

For the total energy of plasma, from (2.101) and (2.79) we get

$$E = E_{id} - \left(\frac{\pi}{k_B T V}\right) q_0^3 \tag{2.103}$$

where $E_{id} = 3k_B T N/2$, and $N = \sum N_i$ is the total number of all the ions in plasma. If we take into account the energy of the Coulomb interaction (2.102) in (2.81) and integrate with respect to T and then insert the obtained expression in (2.81), we have for the free energy of the plasma

$$F = F_{id} - \frac{2}{3}\left(\frac{\pi}{k_B T V}\right) q_0^3 \tag{2.104}$$

This expression enables the calculation of all thermodynamic functions and coefficients.

Chapter 3

Thermodynamic properties of plasma

3.1 Thermodynamic parameters of plasma

In the previous chapter, we analyzed systems of atomic particles with a short range interaction between them, where only the nearest neighbors interact. Let us consider a system with a long rang interaction of particles. In a plasma the long rang coulomb interaction occurs between charged particle and we consider a weak ionized gas that contains atoms or molecules of number density N and electrons and positive ions whose average number density is N_0. Although the interaction between charged particle in this system is weak, it is important for some properties of this system. For simplicity, we take the temperature T of all the plasma components to be identical which gives the internal energy per unit volume of this system if we neglect the interaction between particles of the plasma.

$$E' = \frac{3}{2}k_B T(N + 2N_0) \tag{3.1}$$

so that the specific internal energy of the plasma equals

$$E = \frac{3}{2}k_B T(N + 2N_0) + 2N_0\overline{e\phi} \tag{3.2}$$

where $\overline{e\phi}$ is the average energy of interaction between charged particles per particles. We assume the plasma to be an ideal one, i.e. its parameters satisfy the criterion $\frac{e^2 N_0^{1/3}}{k_B T} \ll 1$ this will be come

$$\frac{N_o e^6}{(k_B T)^3} \ll 1 \tag{3.3}$$

In this case fields in the plasma are shielded by charged plasma particles and the coulomb long rang interaction of charged particles acts independently of the short rang interaction involving neutral particles. The electric potential of a charged particle is given by the formula

$$\phi = \frac{e}{4\pi\epsilon_0 r} \exp\left(-\frac{r}{r_D}\right) \tag{3.4}$$

Where r_D is called the Debye-Huckel radius, which characterizes the typical distance of screening of the field created by this charge. In the case of an ideal plasma the Debye-Huckel radius is large compared with the typical distance between the nearest charged particles, so that many charged particles partake in the charge shielding. To determine the contribution of the interaction between charged particles to the plasma energy, the mechanism of this interaction consists of a shift in the surrounding charged particles under the action of the field of a test charged particle. Then the average interaction energy per charged particle is equal to

$$\overline{e\phi} = \int e\phi \left[N_0 \exp\left(-\frac{e\phi}{k_B T}\right) - N_0 \exp\left(\frac{e\phi}{k_B T}\right)\right] dr \tag{3.5}$$

We account for the Boltzmann distribution of surrounding charged particles in the field of a test particle and assume that many particles take part in the shielding of a charged particle, as occurs in an ideal plasma. For an ideal plasma large r gives the main contribution to this integral, and the exponent may be expanded over a small parameter $e\phi \ll k_B T$. Then we have

$$\overline{e\phi} = -2N_0 \int \frac{(e\phi)^2}{(k_B T)} dr \tag{3.6}$$

where $e\phi = \frac{e^2}{4\pi\epsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right)$, substituting this we get

$$\overline{e\phi} = -2N_0 \int \frac{(e\phi)^2}{k_B T} dr = -\frac{4N_0}{\epsilon_0 k_B T} \int_o^\infty \frac{e^2}{r^2} \exp\left(-\frac{2r}{\lambda_D}\right) r^2 dr \tag{3.7}$$

substituting the value of λ_D from the above equation at the end and integrating it we get that

$$\overline{e\phi} = -2N_0 \int \frac{(e\phi)^2}{(k_B T)} dr = -\frac{4N_0}{\epsilon_0 k_B T} \int_o^\infty \frac{e^2}{r^2} \exp\left(-\frac{2r}{\lambda_D}\right) r^2 dr = -\frac{e^2}{2\lambda_D} \tag{3.8}$$

where we use the formula $\phi = \frac{e}{4\pi\epsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right)$ for the electric potential of a test charged particle and the the expression $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{2N_0 e^2}}$ for the Debye-Huckel radius substituting this relation in the formula $E = \frac{3}{2} k_B T(N + 2N_0) + 2N_0\overline{e\phi}$ we get for the internal energy E of the ideal plasma

$$E = E_{id} + E_{int} \tag{3.9}$$

where

$$E_{id} = \left(\frac{3}{2} k_B T N + 3 k_B T N_0 \right) V \tag{3.10}$$

so the internal energy is

$$E_{int} = -N_0 V \frac{e^2}{\lambda_D} = -k_B T N_0 V \sqrt{\frac{4 N_0 e^6}{\epsilon_0 (K_B T)^3}} \tag{3.11}$$

The term due to the interaction of charged particles E_{int} is small compared with the kinetic energy of charged particles of an ideal plasma $3K_B T N_0 V$. This corresponds to the criterion $\frac{N_o e^6}{(K_B T)^3} \ll 1$ for an ideal plasma, so that interactions relatively small. Hence the last term of this formula can be considered as the following term in the expansion of this value over the small parameter of $\frac{N_o e^6}{(K_B T)^3} \ll 1$. Evidently the same operation may be fulfill for other thermodynamic parameters of an ideal plasma. We have for the free energy

$$F = F_{id} + F_{int} \tag{3.12}$$

And use the relation for Helmholtz free energy and its differential

$$F = E - TS, dF = -SdT - pdV \tag{3.13}$$

$$E = F + TS = F - T \left(\frac{\partial F}{\partial T} \right)_V \tag{3.14}$$

Because this relation is valid for each term of the expansion over the small parameter and $E_{int} \sim F \sim T^{-\frac{1}{2}}$ from this it follows

$$F_{int} = \frac{2}{3} E_{int} \tag{3.15}$$

Representing the pressure of an ideal plasma in the form

$$p = p_{id} + p_{int} \tag{3.16}$$

We find the additional term from formula $F = E - TS$, $dF = -SdT - pdV$

$$p_{int} = -\left(\frac{\partial F_{int}}{\partial V} \right)_{T,n} \tag{3.17}$$

where n is the number of charged particles in a given volume. Because $\lambda_D \sim N_0^{\frac{-1}{2}} \sim \sqrt{V}$ we have $F_{int} \sim V^{\frac{-1}{2}}$ so that

$$p_{int} = \frac{F_{int}}{2V} = \frac{E_{int}}{3V} = -\frac{e^2 N_0}{3\lambda_D} = -\frac{T}{24\pi \lambda_D^3} \tag{3.18}$$

And the pressure of an ideal plasma has the form

$$p = NT + 2N_0T - \frac{e^2 N_0}{3\lambda_D} = NT + 2N_0T\left(1 - \frac{1}{48\pi N_0 \lambda_D^3}\right) \tag{3.19}$$

The correction to the entropy ($S = S_{id} + S_{int}$) of an ideal plasma is equal to

$$S_{int} = \frac{E_{int} - F_{int}}{T} = \frac{E_{int}}{3T} = -\frac{V}{24\pi\lambda_D^3} \tag{3.20}$$

The chemical potential of an ideal plasma which has the standard from

$$\mu = \mu_{id} + \mu_{int} \tag{3.21}$$

So that μ_{id} is the chemical potential of an ideal mixture and the correction μ_{int} takes into account the interaction of charged particles of an ideal plasma. We take this correction from the relation

$$\mu = \left(\frac{\partial E}{\partial n}\right)_{S,V} = \left(\frac{\partial F}{\partial n}\right)_{T,V} = \left(\frac{\partial H}{\partial n}\right)_{S,p} = \left(\frac{\partial G}{\partial n}\right)_{T,p} \tag{3.22}$$

We get

$$\mu_{int} = -\left(\frac{\partial G_{int}}{\partial n}\right)_{T,p} \tag{3.23}$$

where n is the number of charged particles of the plasma. It is necessary to take into account that we start from the parameters of a noninteracting plasma, so that it is located in a volume V and has a pressure p. Because the interaction of charged particles charges the plasma pressure, we have according to the Gibbs free energy and its differentials $G = E - TS + pV$, $dG = -SdT + Vdp$. So

$$G_{int}(p = p_{int}) = F_{int}V = -(n_i + n_e)\frac{e^2}{3\lambda_D} = -\frac{\sqrt{4\pi e^3}}{3\sqrt{V}}(n_i + n_e)^{\frac{3}{2}} \tag{3.24}$$

where n_i and n_e are the numbers of electrons and ions in the plasmas.

3.2 Thermodynamics of planetary plasmas

All the concepts (cutoff criteria and level completion for atomic species, state to-state calculations of molecular partition function, non ideal Debye Huckel effects) described in detail in the previous chapters have been implemented in an equilibrium computer program to produce the results here presented. The relevant data can be used, with a fair amount of confidence, in fluid dynamic equilibrium plasma codes. For air plasma, we report also a comparison with existing data.

Thermodynamic properties of high temperature equilibrium plasmas have been calculated in a wide pressure ($0.01 \div 100bar$) and temperature range ($100 \div 50,000K$). The thermodynamic properties have been obtained by using a self-consistent approach including Debye corrections. The partition functions of the species are expressed as the product of the translational and internal contributions

$$Q_s = Q_s^{tr} Q_s^{int} \tag{3.25}$$

As a consequence, the mean energy is given by the sum of the two contributions

$$\overline{U_s} = \overline{U}_s^{tr} \overline{U}_s^{int} \tag{3.26}$$

The translational partition function and the associated energy are given by

$$Q_s^{tr} = \frac{N k_B T}{P} \left(\frac{m_s k_B T}{2\pi\hbar} \right)^{3/2} \tag{3.27}$$

And

$$\overline{U}_s^{tr} = \frac{3}{2} RT \tag{3.28}$$

While the internal partition function and the internal energy are calculated as the sum over atomic or molecular levels

$$Q_s^{tr} = \sum g_{si} \exp\left(-\frac{\varepsilon_{si}}{k_B T} \right) \tag{3.29}$$

And

$$\overline{U}_s^{int} = \frac{N_a}{Q_s^{tr}} \sum g_{si} \varepsilon_{si} \exp\left(-\frac{\varepsilon_{si}}{k_B T} \right) \tag{3.30}$$

For all molecules but H_2, $n_{max,s}$ has been restricted to the bound electronic states originating by the spin and angular momentum coupling of valence electrons. Each bound state supports a finite number of re-vibrational levels. The internal partition function is calculated as

$$Q_s^{int} = \frac{1}{\delta} \sum \sum \sum g_{s,nvj} \exp\left(-\frac{\varepsilon_{s,nvj}}{k_B T} \right) \tag{3.31}$$

where n runs over n_s^m the electronic states and v and J are, respectively, the vibrational and rotational quantum numbers. The statistical weight of a (nvj) state depends only on the electronic and rotational contribution

$$g_{s,nvj} = g_{s,n}^{el} (2j + 1) \tag{3.32}$$

It must be pointed out that the Griem cutoff depends on the plasma composition making necessary a self-consistent solution of the problem. The formation of the plasma potential induces is in the system of real gas properties.

3.3 Plasma characteristics

3.3.1 Velocity space distribution function

In a plasma, electrons and ions move with various velocities. The number of electrons in a unit volume is the electron density n_e and the number of electrons $dn_e(\upsilon x)$ with the x component of velocity between υ_x and $\upsilon_x + d\upsilon_x$ is given by

$$dn_e(\upsilon_x) = f_e(\upsilon_x)d\upsilon_x \tag{3.33}$$

Then $f_e(\upsilon_x)$ is called the electron velocity space distribution function. When electrons are in a thermal equilibrium state with electron temperature K_BT_e, the velocity space distribution function is the Maxwell distribution:

$$f_e(\upsilon_x) = n_e \left(\frac{\beta}{2\pi}\right)^{1/2} \exp\left(-\frac{\beta \upsilon_x^2}{2}\right) \tag{3.34}$$

where $\beta = \frac{m_e}{k_BT_e}$ From the definition, the velocity space distribution function satisfies

$$n_e = \int_{-\infty}^{\infty} f_e(\upsilon_x d\upsilon_x) \tag{3.35}$$

The Maxwell distribution function in the three-dimensional velocity space is given by

$$f_e(\upsilon_x, \upsilon_y, \upsilon_z) = n_e \left(\frac{m_e}{2\pi k_BT_e}\right)^{3/2} \exp\left[-\frac{m_e(\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2)}{2k_BT_e}\right] \tag{3.36}$$

The ion distribution function is defined in the same way as for the electron. The mean of the squared velocity υ_x^2 is given by

$$\upsilon_T^2 = \frac{1}{n}\int_{-\infty}^{\infty} \upsilon_x^2 f(\upsilon_x d\upsilon_x) = \frac{k_BT}{m} \tag{3.37}$$

The pressure p is $p = nk_BT$. The particle flux in the x direction per unit are $\Gamma_{+,x}$ is given by

$$\Gamma_{+,x} = \frac{1}{n}\int_{-\infty}^{\infty} \upsilon_x f(\upsilon_x d\upsilon_x) = n\left(\frac{k_BT}{2\pi m}\right)^{1/2} \tag{3.38}$$

Chapter 4

Partition function

The internal partition function of atomic species is obtained as the sum over the quantum states of atoms. We have to consider that different states can have the same energy (degeneracy) that can be grouped together giving a single term, considering the statistical weight $g_{s,\ell}$ of the ℓ^{th} level of the s^{th} species. Therefore, we have to introduce the statistical weight g_i as the number of states with the same energy and therefore for a single molecule, the equilibrium distribution (N_i/N), where N_i are discriminated only by their energy independently of its molecular state, where N_i is the number of particles with energy ε_i the partition function Q and the energy become

$$\frac{N_i}{N} = \frac{1}{Q} g_i e^{-\beta\varepsilon_i} \tag{4.1}$$

$$Q = \sum g_i e_i^{-\beta\varepsilon_i} \tag{4.2}$$

$$U = \frac{N}{Q} \sum g_i \varepsilon_i e^{-\beta\varepsilon_i} \tag{4.3}$$

We must point out that sum in the partition function is divergent, because the number of levels is infinite while the exponential goes to a finite value. This problem follows from considering an isolated atom in an infinite space. The presence of other particles and the confinement of the atom in a finite volume limits the number of bound states. Therefore, we will change

$$Q = \sum g_i e_i^{-\beta\varepsilon_i} \tag{4.4}$$

as

$$Q_s^{int} = \Sigma g_{s,i} e^{\frac{\varepsilon_{s,i}}{KT}} \tag{4.5}$$

being N_m the maximum number of levels. The level energies are usually referred to the ground state of the species. For a multi-species system, we need a common reference of energy, therefore to the energy levels we should add what is called the formation energy ε_s^f and the level energy is written as $\varepsilon'_{s,i} = \varepsilon_{s,i} + \varepsilon_s^f$, where $\varepsilon_{s,1} = 0$. Defining as Q_s^{int} and $Q_s^{\prime int}$ the partition function calculated respectively, using $\varepsilon_{s,i}$ and $\varepsilon'_{s,i}$.

As a matter of fact, the atomic levels entering in the partition function are calculated or measured considering an isolated atom, such that the energy of the levels is not perturbed by the environment. On the other hand, we present some results obtained considering the influence of the plasma on the energy levels by solving the Schrodinger equation for the atomic hydrogen in the presence of the Debye potential. In the evaluation of the electronic partition functions Q_{ej} of a species j

$$Q_{ej} = \sum g_{nj} \exp \frac{E_{nj}}{k_B T} \tag{4.6}$$

It is necessary to resort to some cut-off criterion of the partition function in order to avoid the divergence of the $\sum$ 12. The following criteria have most extensively been used in recent times. The presence of other particles and the confinement of the atom in a finite volume limits the number of bound states. Therefore it is given by

$$Q_s^{int} = \sum g_{s,i} e^{\frac{\varepsilon_{s,i}}{k_B T}} \tag{4.7}$$

usually referred to the ground state of the species. In the astrophysical community as well as in dense plasma physics other formulations of electronic partition functions are used. In particular, the Plank Larkin(PL) partition function is widely used in the characterization of high pressure high temperature non ideal plasmas. In this case, the partition function for the atomic hydrogen is given by the following equation

$$Q_H^{PL} = e^{\frac{I_H}{k_B T}} \sum g_n \left(e^{\frac{\varepsilon_n}{k_B T}} - 1 - \frac{\varepsilon_n}{k_B T} \right) \tag{4.8}$$

where $\varepsilon_n = I_H - \varepsilon_n = \frac{I_H}{n^2}$ and $g_n = 2n^2$. This equation is well known in the astrophysical literature as well as in the formulation of Equation of State (EOS) for high temperature high pressure plasmas.

However, this partition function gives incorrect population of highly excited states. To overcome this point, introduces the following partition function (always for atomic hydrogen).

$$Q_s^v = e^{\frac{I_s}{k_B T}} \sum w_{sn} e^{\frac{\varepsilon_{sn}}{k_B T}} \tag{4.9}$$

where w_n is the occupation probability of then i^{th} quantum state which in the grand canonical ensemble assumes the form

$$w_{sn} = \exp[-\frac{4\pi}{3}(f_e + f_i) r_n^3] \tag{4.10}$$

where r_n is the Bohr radius of then i^{th} quantum state. Moreover, rewrite the equation as

$$Q_S^V = e^{\frac{I_s}{k_B T}} \sum w_{sn} (e\frac{\varepsilon_{sn}}{k_B T} - 1 - \frac{\varepsilon_{sn}}{k_B T}) + e^{\frac{I_s}{k_B T}} \sum w_{sn} \frac{\varepsilon_{sn}}{k_B T} \tag{4.11}$$

Note that $w_n = 1$ is justified because the main contribution of this component is made by bound states with orbit dimension less than the Landau length $L_n = \frac{q_e^2}{4\pi\epsilon_0 k_B T}$. Values of partition function of atomic hydrogen obtained in combination with the occupation probability

$$w_{sn} = \exp\left[-\left(\frac{\Delta I_H}{I_H - \varepsilon_n}\right)\right] \tag{4.12}$$

where ΔI_H is the lowering of the ionization potential of the hydrogen atom and n is the energy of the n^{th} level. In turn, ΔI_H is expressed according the neighbor approximation, i.e.

$$\Delta I_H = -C\frac{(z+1)q_e^2}{4\pi\epsilon_0 R_0} \tag{4.13}$$

where C is a constant and $R_0 = \sqrt[3]{3\pi N\star/4}$ with $N\star = N_H + N_{H+}$.

Chapter 5

Oxygen plasma

We consider a pure oxygen plasma composed mainly by O_2, O, O^2, O^3 , O^4 , and electrons, and including also the minority species O_2^+ , O_2^-, O^- . We write a set of equilibrium constants as well as the condition of electro neutrality and the Dalton law for the total pressure. The equilibrium constants are calculated, following the statistical thermodynamics, from the relevant partition functions, in the case of atomic species, depend on electron and ionic species densities and temperature (Griem criterion) and on the pressure and temperature (Fermi criterion).

On the other hand, the internal partition functions, calculated using the ground state method, are constant, having $Q^{int} = g_1$. Note also that in the case of atomic species we use a complete set of energy levels, including observed and missing ones, these last obtained by using semi empirical methods based on Ritz and Ritz Rydberg equations. We start examining the properties of selected atomic species obtained by using the Griem and Fermi cutoff criteria. These data should be compared with the ground state values that for the O, O^+ and O^{2+} assume for $T > 2000K$ the values $Q^{int}[O^{+3}(P)] = 9$, $Q^{int}[O^{+4}(S)] = 4$, $Q^{int}O^{+2}(P) = 6$, first and second derivatives being null.

Electronic partition functions, their first and second logarithmic derivatives and internal specific heats are reported as a function of temperature for the three species O, O^+ ,and O^{2+} in Fig, 5.1 at $1bar$ (left) and at $100bar$ (right). In both cases, the two cutoff criteria give different values of partition function, these differences propagating on the first and second logarithmic derivatives as well as in the specific heats.

In particular, the Fermi criterion includes in the calculation more levels than the Griem criterion, with the consequence of larger partition functions. Note also that, due to the energy range of electronic levels, the partition function of the different species increases in well-defined temperature ranges, without significant overlapping between each other. This aspect is better evidenced in the first and second logarithmic derivatives which present well distinct maxima.

The values of the second logarithmic derivative calculated according to Fermi criterion overcome the corresponding Griem values up to the maximum, the opposite occurring in the decreasing region. This behavior is reflected on the specific heat of the single species which in any case presents the trend characteristic of a system containing a finite number of excited levels, i.e., the internal specific heat after the maximum asymptotically reaches a zero value.

The large influence of electronic excitation on the specific heat can be understood by reminding that the corresponding values for the ground state are null, independently of temperature and the reduced translational contribution is $\frac{5}{2}$. Let us now examine the behavior

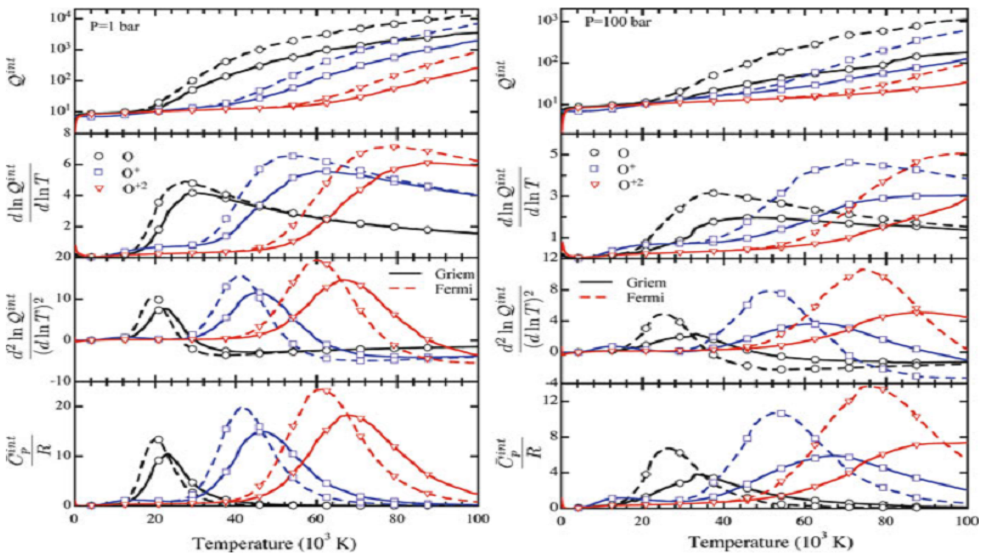

Figure 5.1: Self-consistent atomic partition function, its temperature derivatives and internal specific heat as a function of the temperature for O, O^+, O^{+2} for $P = 1bar$ (left) and $P = 100bar$ (right): comparison between Griem and Fermi cutoff [12].

of the enthalpy variation for the processes

$$O \rightleftharpoons O^+ + e^- O^+ \rightleftharpoons O^{+2} + e^- O^{+2} \rightleftharpoons O^{+3} + e^- \tag{5.1}$$

We can write, respectively,

$$\Delta\overline{H}_1 = 5/2RT + \overline{U}_{O+}^{int} - \overline{U}_{0}^{int} + I_0 \Delta\overline{H}_2 = 5/2RT + \overline{U}_{O+2}^{int} - \overline{U}_{O+}^{int} + I_{0+} \Delta\overline{H}_3 = 5/2RT + \overline{U}_{O+3}^{int} - \overline{U}_{O+2}^{int} + I_{0+2} \tag{5.2}$$

where I_s represents the first, second and third ionization potentials of oxygen respectively. Values of ΔH_i for the ionization reactions calculated at different pressures according to the three cutoff criteria have been plotted as a function of temperature in Fig bellow While

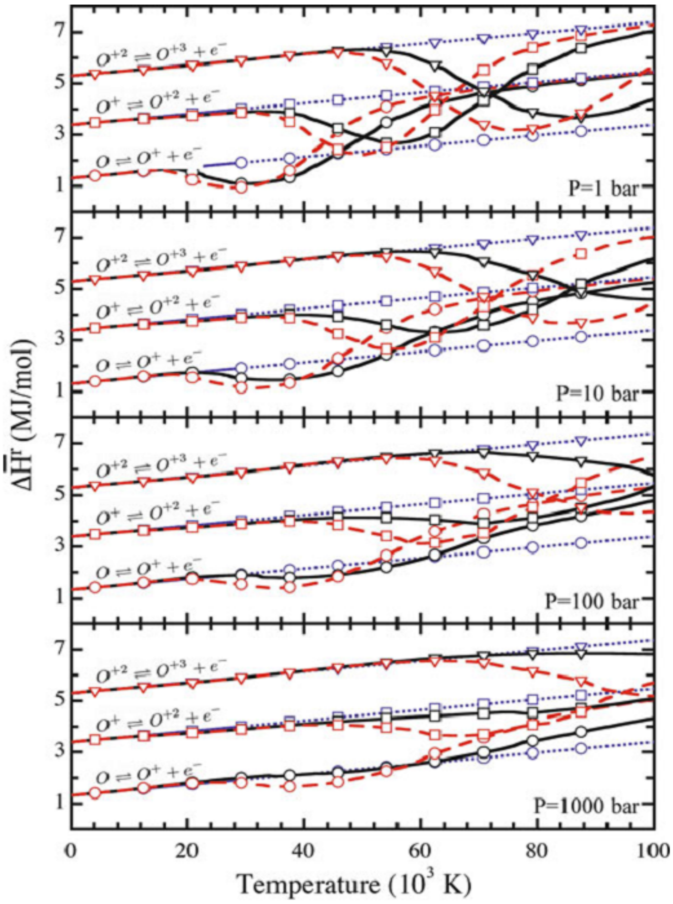

Figure 5.2: Reaction enthalpy of the ionization processes of oxygen and its first and second ions calculated for $P = 1,\ 10,\ 100,\ 1,000 bar$. Comparison between Griem and Fermi cutoff and ground state model [12].

ΔH_i calculated in the ground state approach linearly increases as a function of temperature, results obtained according to Griem and Fermi cutoff criteria initially increases, as in the ground state case, but as the temperature grows ΔH_i starts decreasing up to a minimum value, after which it increases again. Note that in all the cases the ΔH_i calculated inserting the electronic excitation asymptotically converge at very high temperature to the ΔH_{i+1}

calculated without considering the electronic excitation, i.e

$$\Delta H_1 = \frac{2}{5}RT + \overline{U}^{int}_{O+} - \overline{U}^{int}_{O} + I_0 \rightarrow T\frac{5}{2}RT + I_{0+} = \Delta \bar{H}_2(ground) \qquad (5.3)$$

As a matter of fact, at very high temperature we have that $\bar{U}^{int}_{O+} \approx I_{O+}$ and $\bar{U}^{int}_{O} \approx I_O$, because the level energies approach the ionization,

$$I_0 + \bar{U}^{int}_{0+} - \bar{U}^{int}_{O} \rightarrow I_{0+} \qquad (5.4)$$

This is the consequence of taking into account the excitation of both O and O^+ which occurs in different temperature ranges. Beyond the minimum of $\Delta \bar{H}_1$, the electronic excitation of the ionized species $\bar{U}^{int}_{O+}$ begins to contribute to $\Delta \bar{H}$ while the term $\bar{U}^{int}_{O}$ is now decreasing with increasing temperature (compare Fig 5.1). A similar behavior is found for the other ionization reactions, although shifted on the temperature axis. Figures 5.3-5.5 report the

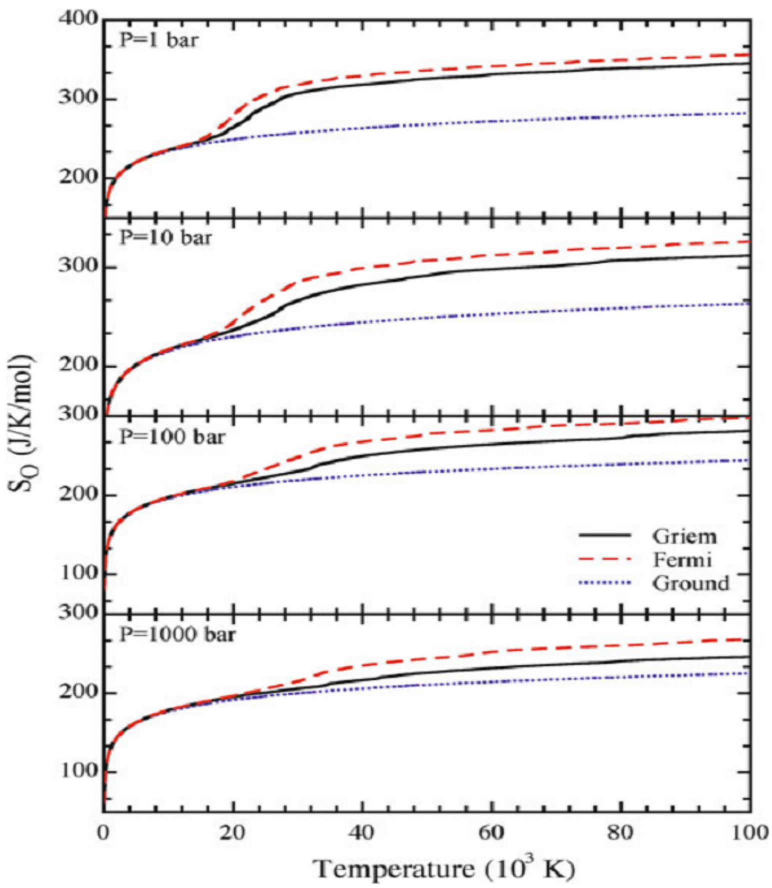

Figure 5.3: Molar entropy of oxygen atom for different pressures and for different cutoff criteria [12].

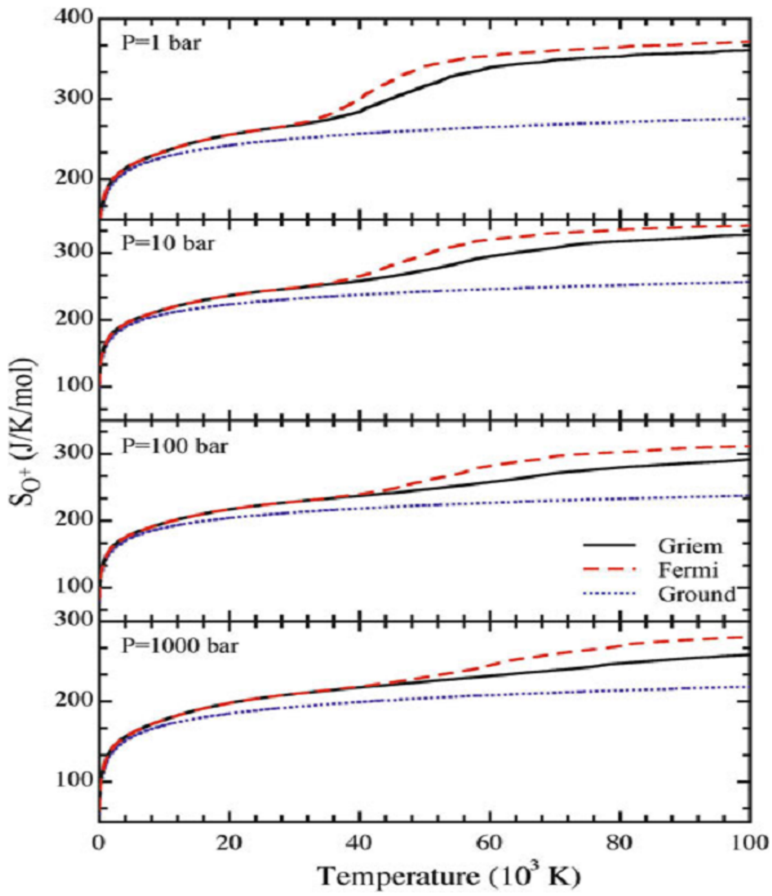

Figure 5.4: Molar entropy of O^+ for different pressures and for different cutoff criteria [12].

entropy of the O, O^+ ,and O^{+2} species as a function of temperature at different pressures calculated according to the ground state method and to Griem and Fermi cutoff criteria. The differences between the three methods reflect the trend of the corresponding electronic partition function and of its first logarithmic derivative. The contribution of the electronic states is well evident in the different plots when Griem and Fermi start deviating from the corresponding values calculated using the Ground state method. In any case, the trend of the entropy for the different species monotonically increases passing from ground to Griem and Fermi methods following the corresponding increase of the electronic contribution.

The dependence of the total thermodynamic quantities on cutoff criterion results from the combination of the single species properties and plasma composition. To analyze this contribution, we report the dependence of the molar fractions of the majority species of the oxygen plasma. Fig.5.6 reports the molar fractions of majority species (left) and their

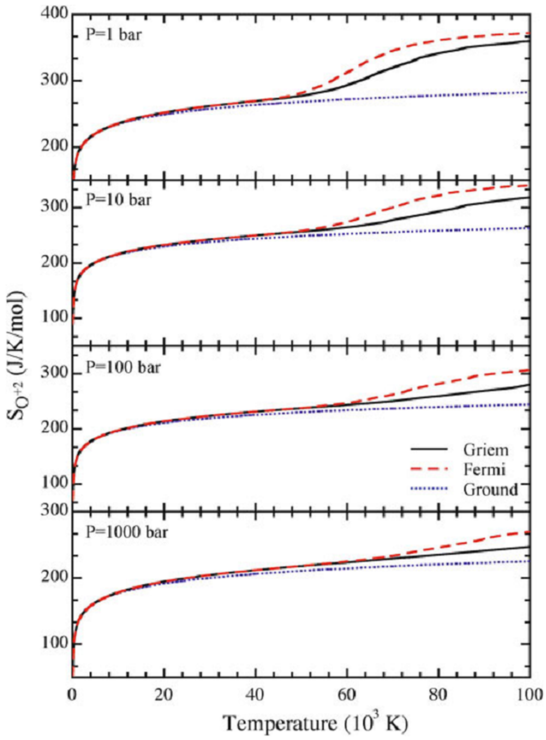

Figure 5.5: Molar entropy of O^{2+} for different pressures and for different cutoff criteria [12].

derivatives for O and O^{+2} (right) as a function of temperature at different pressures comparing the different cutoff criteria. The selected derivatives are representative of the dissociation and first ionization processes, corresponding, respectively, to the maximum and the minimum in O derivative, and the second and third ionizations (maximum and minimum in O^{+2} derivative). We can observe that the dissociation weakly depends on the chosen cutoff, while the three ionization processes are strongly influenced, showing large differences between the Ground and Fermi/ Griem results. This behavior can be used to understand the trend of reactive contribution to the total specific heat.

Let us now examine the thermodynamic properties of plasma mixture. We start with the behavior of the total entropy (Fig.5.7) of oxygen plasma as a function of temperature at different pressures calculated with the different methods. In general, the Fermi criterion presents larger entropy values compared to Griem and ground state methods, the differences not exceeding 10 More complicated is the situation for the reactive contribution as reported in Fig.5.8, which however can be understood by using the derivatives of the molar fractions. The dissociation regime is not affected by the cutoff of electronic partition function as confirmed

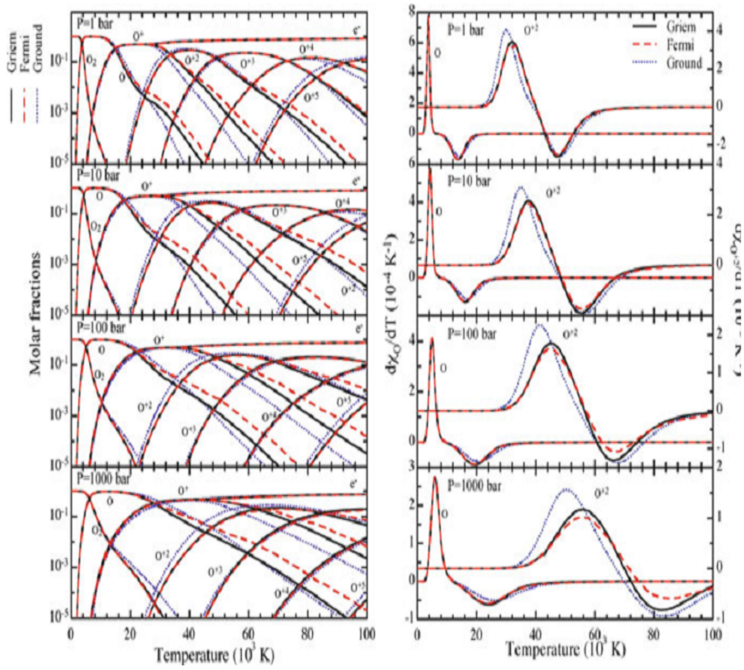

Figure 5.6: Molar fractions of the majority species in the oxygen plasma (left) and its derivative (right), for O and O^{+2} , as a function of the temperature: comparison between Griem and Fermi cutoff criteria and Ground state method [12].

by the results. The ionization regimes are strongly affected by the chosen cutoff criterion. In particular, the second ionization peak follows the history of the derivative of O^{+2} species. The compensation between Fermi, Griem, and Ground state methods occurs only in the dissociation and first ionization regimes while large differences are observed for the second, third, and fourth ionization reactions, these differences increasing at high pressure. At $1000bar$, we observe the largest deviations between the three methods.

The behavior of the different contributions (C_{pf}, C_{pr}) of the specific heat as well as the total specific heat (c_p) are reported in Figs.5.9 In particular, Fig.5.9 (left) reports the frozen specific heat as a function of temperature for the different pressures. In this case, the differences between the three methods can reach at high pressure a factor larger than 2, the values calculated by using the Fermi criterion overcoming in any case the values obtained by Griem and Ground state methods.

$$c^{int} = \frac{1}{M} \sum n_s \bar{C}_s^{int} \tag{5.5}$$

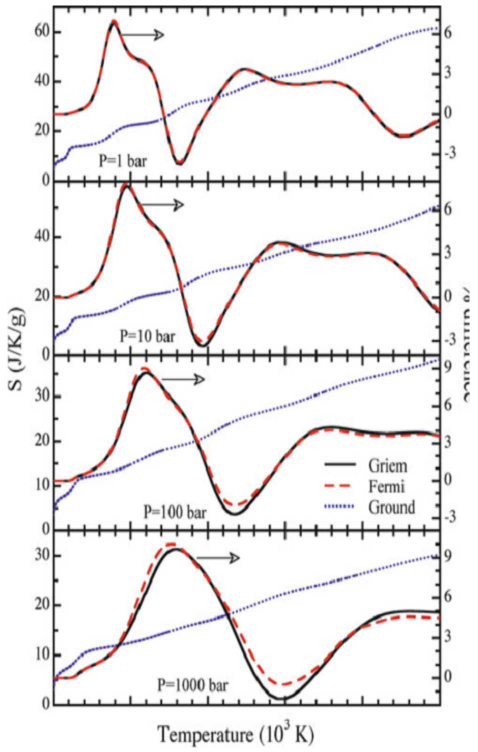

Figure 5.7: Entropy, calculated using the Ground state method, and its relative difference with respect to calculations using Griem and Fermi cutoff criteria [12].

and the frozen and total specific heat, being n_s the number of moles of the s^{th} species and M the total mass. Fig.5.10 reports $c^{int} = c_{pf}$ and $c^{int} = c_p$ calculated according to Fermi and Griem methods. From this figure, we can appreciate that the ratio $c^{int} = c_{pf}$ calculated with the Fermi criterion can exceed by more than a factor of 2 the corresponding ratio obtained by the Griem criterion, being in any case a large fraction of the frozen specific heat. On the other hand, the ratio $c^{int} = c_p$ is less affected by the presence of electronic states, still representing however a large fraction of the total specific heat. In any case, the importance of electronic excitation is emphasized taking into account that c^{int} for the ground state method is zero in the ionization regimes.

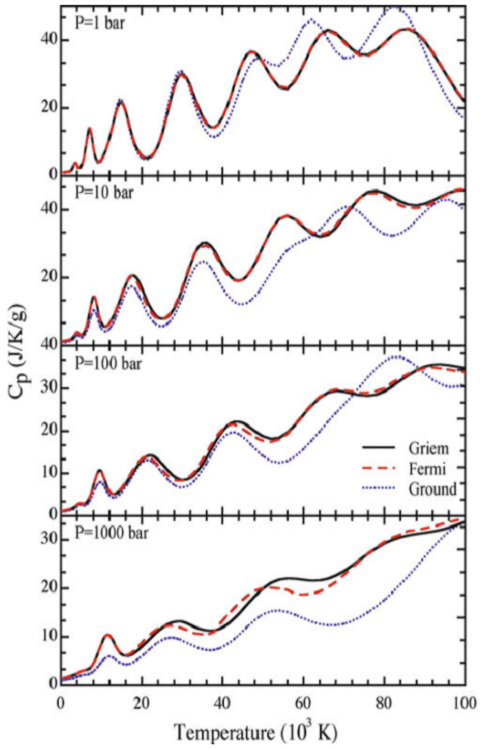

Figure 5.8: Constant pressure specific heat calculated with different cutoff criteria [12].

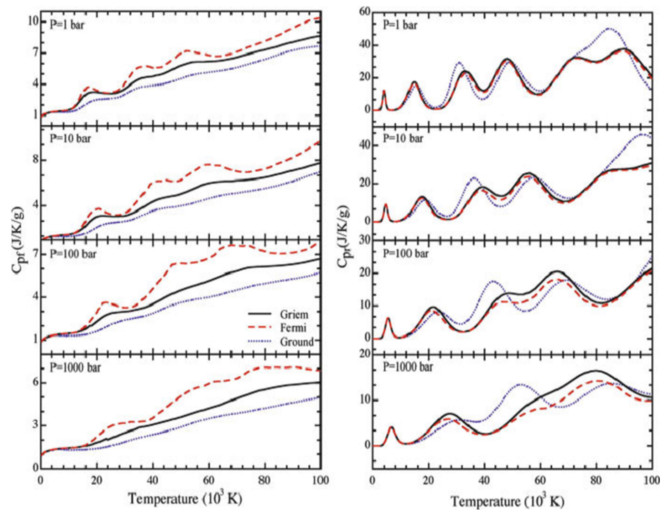

Figure 5.9: Constant pressure specific heat calculated with different cutoff criteria: frozen (left)and reactive (right) contributions [12].

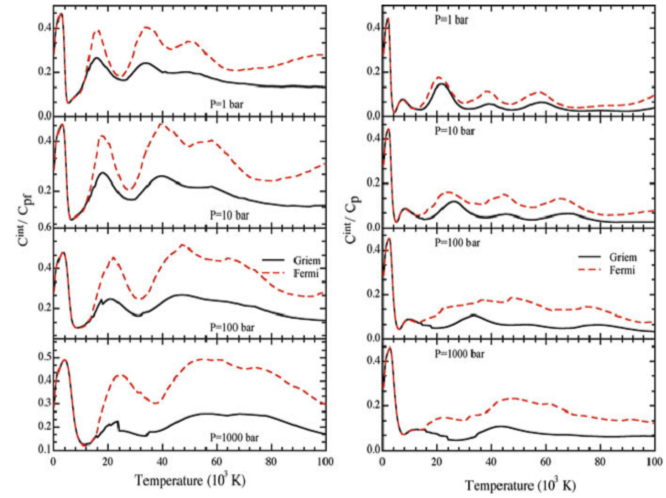

Figure 5.10: $c^{int}/c_{pf}(left)$ and $c^{int} = c_p$(right) as a function of the temperature for different pressures and calculated with different cutoff criteria [12].

Chapter 6

Conclusion

The concept of the Debye radius can also be used for a more accurate definition of plasma as a special state of matter. The ensemble of freely moving charged particles of both signs, i.e. ionized gas can be considered as plasma if the Debye length is small compared with the dimensions of the volume occupied by the gas.

By definition, we call an ionized gas a plasma if the Debye Huckle radius of this system is small compared to its dimension. This characteristic shielding of the particle field takes place if the shielding distance λ_D is large compared with the average

According to this criterion, the typical energy of interaction of charged particles in a plasma or interaction energy at the average distance between charged particles is $\frac{n_0^{1/3} e^2}{\epsilon_0}$ small compared with the thermal energy of particles ($\sim \kappa_B T$). According to definition of ideal plasma, because many charged particles located inside a sphere of radius r_D.

If the wavelength is small one has to take into account the restoring force caused by plasma compression in the wave. Then the square of the sound velocity should be added to the expression for the squared phase velocity.

To determine the extent of ionization in a plasma state from thermodynamic equilibrium it is necessary to consider the detail of the collision processes leading to ionization and recombination.

An ionized gas in which all or a considerable number of atoms have lost one or several of their electrons and turned into a mixture of free electrons and positive ions is called plasma.

Due to the displacement of electrons, electric charges appears and an electric field is induced.

The behavior of the total isentropic coefficient, i.e. including the reactive contributions to the total specific heats, follows the trend of frozen isentropic coefficient in the presence of electronic excitation i.e. the electronic excitation behaves, as already pointed out, like a chemical reaction even though hidden by the ionization reaction in the total isentropic coefficient.

The large influence of electronic excitation on the specific heat can be understood by reminding that the corresponding values for the ground state are null, independently of temperature and the reduced translational contribution is $\frac{5}{2}$.

We can observe that the dissociation weakly depends on the chosen cutoff, while the three ionization processes are strongly influenced, showing large differences between the Ground and Fermi/ Griem results.

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DECLARATION

I, hereby declare that this project is my original work and has not been presented for a degree in any other university, and that all sources of materials have been duly acknowledged.

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