A computer program for generating design charts of L-shaped short columns on the basis of EBCS 2 - 1995

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MSc Thesis

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NOTATIONS

A_s	-	Area of steel reinforcement
A_c	-	Area of concrete cross section
f_c	-	Compressive strength of the concrete
f_{cd}	-	Compressive Design strength of concrete
f_{ck}	-	Characteristic compressive strength of concrete
γ_c	-	Partial safety factor for concrete
ε_c	-	Stain in concrete fiber
ε_{cu}	-	Maximum compressive strain in concrete
f_s	-	Tensile strength of steel
f_{yk}	-	Characteristic strength of reinforcement
γ_s	-	Partial safety factor for steel
f_{yd}	-	Design strength of reinforcement
ε_s	-	Strain of reinforcement
d_p	-	Concrete cover
b	-	Width of the cross-section
e	-	Eccentricity
h	-	Depth of the cross-section
M_{nx}	-	Nominal maximum bending capacity of a section about x axis
M_{ny}	-	Nominal maximum bending capacity of a section about y-axis
M_{xd}	-	Design bending moment about global x -axis
M_{yd}	-	Design bending moment about global y-axis
M_{cx}	-	Ultimate bending capacity of *concrete* section about x-axis
M_{cy}	-	Ultimate bending capacity of *concrete* section about y-axis
M_{sx}	-	Ultimate bending capacity of *steel bars* about x -axis
M_{sy}	-	Ultimate bending capacity of *steel bars* about y-axis
M_u	-	Ultimate bending moment
P_u	-	Ultimate axial load
P_n	-	Nominal maximum compressive axial force capacity of a section
N_c	-	Ultimate Axial force capacity of concrete in the section.

N_s	-	Ultimate Axial force capacity of reinforcement bars in the section
dA	-	Differential area in the compressive zone of the cross-section
E_s	-	Elastic modulus of reinforcement steel
A_{sj}	-	Area of steel bar 'j' at which tensile stress acts
A'_{sj}	-	Area of steel bar 'j' at which compressive stress acts
f_{sj}	-	Tensile stress acting on steel bar 'j'
f_{csj}	-	compressive stress acting on steel bar 'j'
x, y	-	Local coordinate axis
X, Y	-	Global coordinate axis
α_1, α_2	-	Load-contour method coefficients
θ	-	Angle of inclination of the neutral axis with the global X- axis towards the left.

ABSTRACT

In this thesis a program is developed to generate design charts of L-shaped reinforced concrete column on the basis of EBCS-2, 1995. Design chart can be prepared for any cross-sectional dimension of columns with possible arrangements of reinforcement bars. The program will have also freedom for the choice or different reinforcement and concrete grades. The section is always bi-axial and strength is computed by assuming different position of the neutral axis of given orientation. Strain compatibility and equilibrium equations are applied for the section capacity determination according to the Ethiopian Building Code Standard.

Green's theorem is used to convert the double integral over the area in to line integral. The Gaussian Integration is also used to convert the double integral over the compressive zone in to a line integral along the perimeter of the compressive zones. Finally numerical method is used to convert the line integral in to simpler mathematical expression for programming. The principles of coordinate geometry are used to calculate the intersection point of two lines, the distance between to points and other important parameters to be incorporated in the program. Although equivalent rectangular stress-distribution due to Whitney can be used with out much loss in accuracy, the parabolic-rectangular stress distribution is used for the analysis of the concrete section as is primarily recommended by EBCS-2, 1995.

The design chart can be generated for all possible reinforcement ratios and any possible orientation of the neutral axis. It should however be noted that the Ethiopian building code standard limitations on the reinforcement ratio, section dimension, amount of reinforcement bars should be maintained for good design. The program is written in visual Fortran programming language; however, the program can be run from Microsoft visual C++ window without any change in the algorithm. The output of the program will be automatically exported and saved in to a text file that can be opened using Microsoft office excel program to generate the charts. The chart generated is finally verified with registered version of Alashki's program for accuracy and satisfactory results are obtained.

CHAPTER ONE

1. INTRODUCTION

1.1 Rationale of the research

Engineering has developed from observations of the ways natural and constructed systems react and from the development of empirical equations that provide bases for design. Civil engineering is the broadest of the engineering fields, partly because it is the oldest of all engineering fields. In modern usage, civil engineering is a broad field of engineering that deals with the planning, construction, and maintenance of fixed structures, or public works, as they are related to earth, water, or civilization and their processes. Civil engineering is still an umbrella term, comprised of many related specialties like structural engineering that studies about the analysis and design of structural systems. One of the frequently encountered structural elements in many of the civil engineering structures is column.

Columns are structural elements primarily required to resist axial forces. However, a column is seldom subjected to axial forces only and moments about one or two axes are also usually present. For practical purposes column cross-sections are normally symmetrical about the axis of bending. However, there are some cases where sections are asymmetric as the common type of cross-sections may waste the very scarce room spaces, may be expensive, may have superfluous edges on the usable rooms, and may present offensive architectural view for dwellers. L-shaped columns can however solve such problems if used at corner points of buildings.

Next to rectangular and circular shapes, L-sections may be the most frequently encountered reinforced concrete columns, since they can be used at outside and re-entrant building corners. Nevertheless information for their analysis and design is not generally available to structural engineers either in working stress or ultimate strength theories. [Mallikarjuna & P. Mahadevappa, 1991] Although it is possible to drive a family of equations to evaluate the strength of columns subjected to a combined bending and axial loads, these equations require much computations to generate. The problem will be more compounded in the case of L-shaped reinforced concrete columns cross-sections as they are unsymmetrical about both horizontal and vertical axes.

Generally, to reduce the required number of trials and to obtain a satisfactory design, we use design charts. These are plots of axial load versus bending moment on an orthogonal axis. In this thesis, the Ethiopian Building Code Standard - Structural Use of Concrete (EBCS 2-1995) is chiefly used as a guideline.

In earlier days, a number of researches have focused towards the development of design charts for rectangular and circular shaped reinforced concrete columns. They have also contributed a lot for the development of building codes and standards that are used at the moment. As to my knowledge, most codes however don't have the provision for the design of L-shaped reinforced concrete columns.

These days, the discoveries of personal computers and programming softwares have greatly helped researchers in the field to solve complicated and difficult problems with the required accuracy in short period of time. It also makes possible the selection of a number of alternative sections, all of which have been subjected to comprehensive checks to ensure that they meet the relevant requirements. Researches have still continued in different institutions to generate simple and user friendly programs that solves complicated civil engineering problems.

Different academicians and practitioners in Addis Ababa University have been also developing programs and design aids including [Girma (Dr.-Ing.); 1995], [Diriba; 1996], [Kibrealem; 2005], and [Asnakew; 2009]. However, developing a program for design aid of L-shaped columns on the basis of the Ethiopian Building Code Standard is still an unsolved problem.

Therefore, it is with these underlying facts that this topic has been selected for research. The study attempts to produce a program for generating design charts of L-shaped short reinforced concrete column subjected to axial load and bending moments. The fundamental principles like equilibrium of forces, compatibility of strains and known stress-strain relationships are used to set up the necessary equations for design chart generation based on EBCS 2, 1995. A Visual Fortran programming language is used for the generation of the design charts; however, Microsoft visual C++ can also be used to run the program without any algorithm change.

1.2 Research Goals

This research is aimed at producing a program for generating design charts of L-shaped reinforced concrete columns subjected to axial load and bending moments in accordance with the basics of EBCS-2, 1995. The final output of the research will be used as a design aid for any design institutions, a teaching material for colleges and universities and an input for further research and development in the field. This will definitely fill some gap that structural engineers face to design L-shaped reinforced concrete columns based on the Ethiopian Building Code Standard: EBCS-2, 1995.

1.3 Methodology of the research

L-shaped reinforced concrete corner columns are mostly subject to axial load and biaxial bending moment. Construction of an interaction surface for a given reinforced concrete column would appear to be an obvious extension of uniaxial bending analysis. The research assumes an equal leg and uniform thickness cross-section. The maximum compressive strain in the concrete, the maximum tensile and compressive strain in the reinforcement bars are taken from the Ethiopian Building Code Standard. Rectangular-parabolic stress-strain diagram is assumed for the analysis of the concrete section capacity as recommended by the Ethiopian Building Code Standard. Linear strain distribution is also assumed to calculate the capacity within the reinforcement bars.

For a selected inclination angle of the neutral axis, successive choices of neutral axis distance could be taken. For each, using strain compatibility and stress-strain relations to establish bar forces and the concrete compressive resultant, then using equilibrium equations to find P_n, M_{nx} and M_{ny}, one can determine a single point on the interaction surface. Repetitive calculations, easily done by computer, will establish sufficient points to define the surface.

Note that the failure surface can be described either by a set of curves defined by radial plane passing through the P_n axis or by asset of curves defined by horizontal plane intersections, each for a constant P_n, defining load contours [Nilson, Darwin & Dolan; 2004]. The radial plane method is used for this thesis as it is simpler to program than the load contours in addition to some major reasons. Visual Fortran programming language is used for the thesis and the analysis out puts are then exported to Microsoft Office Excel program for generating the chart.

1.4 Organization of the research

The research is organized in such a way that it can systematically convey the works undertaken and be clear and consistent in flow simultaneously, so that the reader can easily grasp the required targets. The subsequent topics discuss literatures reviewed from different sources which are selected to be of vital importance to fully understand the work.

The next chapter discusses the formulation of the equations. Different mathematical formulations like integration, Gaussian Quadrature and Green's theorem are used to formulate the equations. The double integral equations will be converted to line integral for simplification purpose in programming. The underlying philosophy as to the requirements, scope and limitation of the formulations are given in the same chapter.

The results of the work are also well discussed in the fourth chapter with design examples and program guidelines for the users. Conclusions and recommendations are made based on the results of the work in the fifth chapter.

Finally, sample design chart, programming logic flow charts and program codes are presented as an appendix after all the references used for the work are illustrated.

CHAPTER TWO

2. *LITRATURE REVIEW*

2.1 Limit States

The limit states can be placed in two categories:-

1. The ***Ultimate Limit States*** are those associated with collapse, or with other forms of structural failure which may endanger the safety of people. States prior to structural collapse which, for simplicity, are considered in place of collapse itself are also treated as ultimate limit states.

2. The ***Serviceability Limit States*** correspond to states beyond which specified service requirements are no longer met.[EBCS-2; 1995]

In the calculation of reinforced concrete cross sections capacities in the ultimate limit states for flexure and axial load, the following five basic assumptions shall be made according to EBCS 2-1995:-

1. *Plane sections remain plane*, which means the strain distribution in the concrete in compression and the strain in the reinforcement whether in tension or compression are linear.

2. The reinforcement is subjected to the same variations in strain as the adjacent concrete, which means that *there is no bond slip between the reinforcement and the concrete*.

3. The tensile strength of the concrete is negligible and is disregarded in computations.

4. The maximum compressive strain in the concrete is taken to be 0.0035 with bending (simple or compound) and 0.02 in axial compressions.

5. The maximum tensile strain in the reinforcement is taken to be 0.01.

Parabolic-rectangular stress-strain curve of concrete is used for the calculation of the section capacity of the concrete and elasto-plastic stress-strain diagram for the calculation of section capacity of ordinary steel.

The stress-strain relationship for the concrete in compression and for reinforcement both in tension and compression are as shown in the Fig. 2.1 and Fig. 2.2 below.

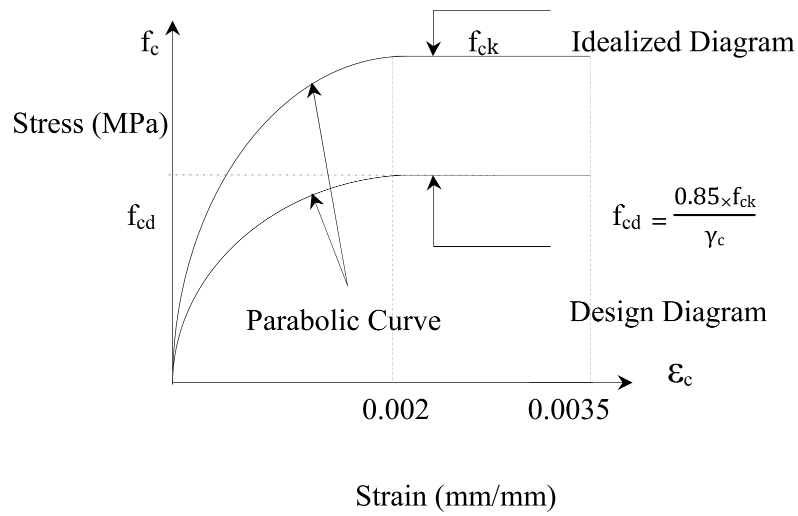


Fig. 2.1 Parabolic-Rectangular Stress-Strain diagram for Concrete in Compression

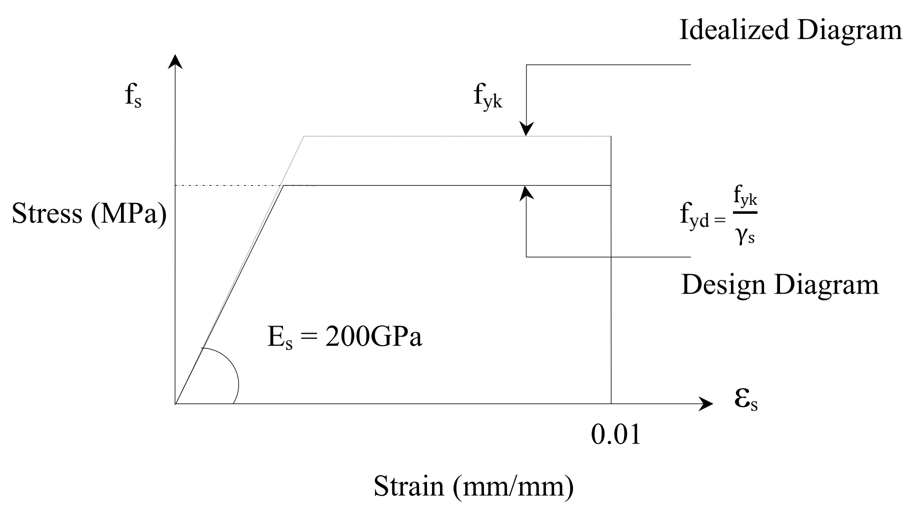


Fig. 2.2 Stress-Strain diagram for reinforcing steel

2.2 Column Interaction Diagram

The cross sectional dimensions of a column are generally considerably less than its height. Columns support vertical loads from floors and roofs and transmit those loads to the foundations. If a column is loaded to failure the reinforcement is likely to reach its yield strength before the concrete fails in compression (Fig. 2.3), this statement remains valid for the material strengths generally used in buildings as well as for higher concrete strengths where the strain coinciding with the maximum stress increases as the concrete strength increases. If high strength reinforcement is used the concrete may reach its maximum stress before the reinforcement yields. [Park and Paulay; 1984]

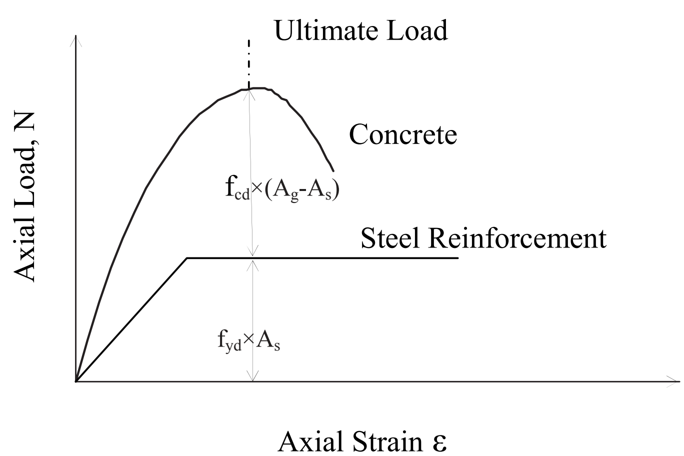


Fig. 2.3 Axial load-Strain response for steel and concrete in axially loaded column

When a symmetrical column is subjected to a concentric axial load, longitudinal strain develops uniformly across the section. Because the steel and the concrete are bonded together, the strain in the concrete and steel are equal. For any given strain, it is possible to compute the stresses in the concrete and steel using the stress-strain curves for the two materials. [MacGregor; 1997]

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to load not being centered on the column, or may result from columns resisting a portion of the unbalanced moment at the ends of the beams supported by the columns.

Structural column failure is of major significance in terms of economic as well as human loss. Thus extreme care needs to be taken in column design, with a higher reserved strength than in the case of beams and other horizontal structural elements, particularly since compression failure provides little visual warning. The analysis of reinforced concrete sections are characterized by material non-linearity arising from the non-linear stress-strain relationships and the cracking of the cross-sections. As a result, the determination of strain distribution for given internal forces necessitates the application of numerical methods accompanied by iterations.

The section strain distribution for the general case of bi-axial bending and axial load can be determined by the direction angle of the resultant curvature and the strains at two characteristic fibers of the cross-section, the greatest compressive and tensile strain for cracked section or the greatest and smallest compressive strain for uncracked sections. The design of a reinforced concrete column is essentially one of trial and error. Design consists of finding a cross section that will support satisfactorily both an axial load and bending moment in addition to the serviceability requirements. However, since the capacity of a section to carry axial load is dependent on the magnitude of the moment that is acting, there are no closed form solutions for determining a section uniquely. The problem will be compounded when the cross section is unsymmetrical.

An interaction diagram is a plot of the failure load and failure moment for a given column for the full rage of eccentercities from zero to maximum point as shown in Fig. 2.4 below. Failure of column could occur as a result of material failure by initial yielding of the steel at the tension face or initial crushing of the concrete at the compression face, or by loss of lateral structural stability (i.e., through buckling).

If the column fails due to initial material failure, it is classified as a short or non-slender column. As the length of the column increases, the probability that failure will occur by buckling also increases. In the case of design, where the aim is to determine the required area of reinforcement design chart will be used.

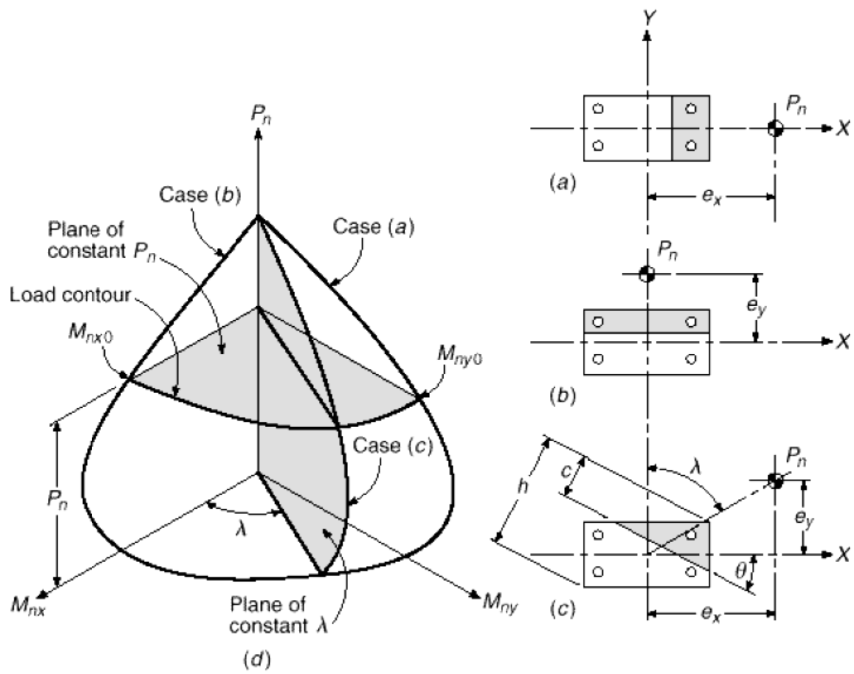


Fig. 2.4 Interaction Diagram for compression plus biaxial bending

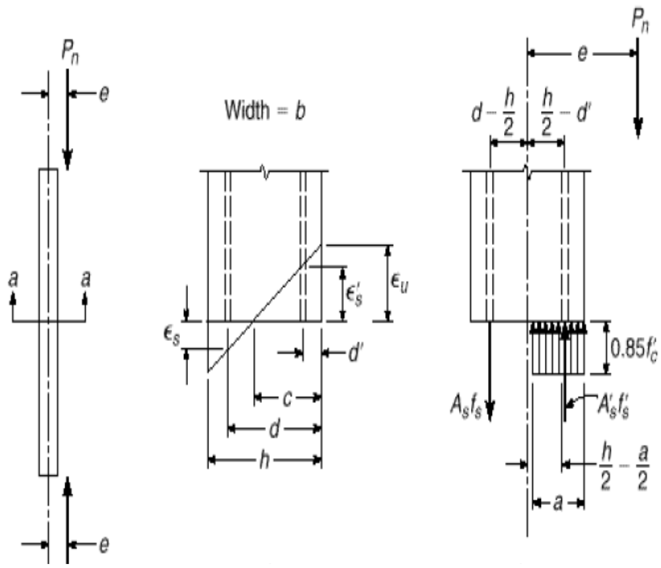
(a) Uniaxial Bending about Y-axis
(b) Uniaxial bending about X-axis
(c) Bi-axial bending about diagonal axis
(d) Interaction Surface

There are two commonly used methods of section analysis in compression members with bending moments whose trial section dimension and reinforcement area are known. Those are the exact method and the approximate method. [Nilson, Darwin & Dolan; 2004]

2.2.1 Exact Method

Limiting the maximum compressive strain of concrete depending on the position of the neutral axis, the method assumes a strain distribution governed by strain compatibility equation i.e. the steel strain at any location are the same as the strains in the adjacent concrete.

Corresponding stresses, compressive parabolic-rectangular on concrete and stresses on steel governed by Hooke's law till it yields are computed from their corresponding stress-strain curves. Applying strain compatibility and the two equilibrium equations, the stress resultants; compression and bending moments that will give the assumed strain values and patterns are determined as shown in Fig. 2.5 below.



(a) (b) (c)

Fig. 2.5 Column Subjected to eccentric compression

(a) Loaded Column
(b) Strain distribution at section a-a
(c) Stresses and forces at nominal strength

The distance 'e' in the above Fig. is referred as eccentricity of the load P_n. In the construction of strength interaction diagrams for a given cross-section, full ranges of eccentricities from the smallest value to infinity are considered. For any eccentricity, there is a unique pair of values: P_n and M_n (for compression plus uniaxial bending) and unique triple values: P_n, M_{nx} and M_{ny} (for compression plus biaxial bending) that will produce the state of incipient failure [Nilson, Darwin & Dolan; 2004]

2.2.2 Approximate Method

An exact mathematical formula to describe a failure surface is impractical because of the number of parameters affecting the surface shape. Therefore, different methods of failure surface approximations have been suggested such as:

1. The Bresler reciprocal load method
2. The Bresler load contour method
3. The PCA load contour method
4. The Weber design chart method etc.

Approximate methods are frequently employed for both design and analysis purposes. The most commonly used approximate methods are based on the concept of failure surfaces introduced by Bresler. He has presented load contours and reciprocal load methods which leads to an interaction equation based on a failure surface.

2.2.2.1 The Bresler Load Contours Method

The load contour method is based on representing the failure surface of Fig. 2.6 below by a family of curves corresponding to constant values of P_n.

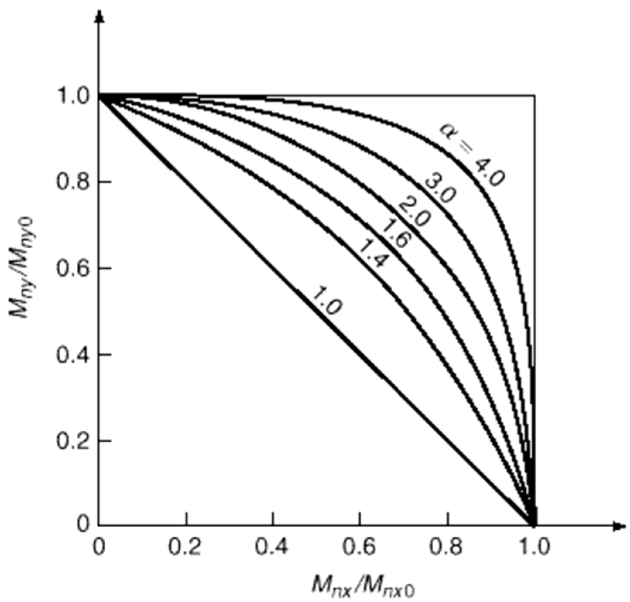


Fig. 2.6 Interaction contours at constant P_n for varying α

The general form of these curves can be approximated by a non-dimensional interaction equation:

…………………………………………………………………………… (2.1)

Where $M_{nx} = P_n \times e_y$

$M_{nxo} = M_{nx}$ when $M_{ny} = 0$

$M_{ny} = P_n \times e_x$

$M_{nyo} = M_{ny}$ when $M_{nx} = 0$

α_1 and α_2 are exponents depending on column dimension, amount and distribution of steel reinforcement, stress-strain characteristic of steel and concrete, amount of concrete cover and size of lateral ties or spiral. [Nilson, Darwin & Dolan; 2004] When $\alpha 1 = \alpha 2 = \alpha$, the shape of such interaction contours are as shown in Fig. 2.6. Calculations reported by Bresler indicate that α falls in the range from 1.15 to 1.55 for square and rectangular columns. Values near the lower end of that range are the more conservative. [Nilson, Darwin & Dolan; 2004]

2.2.2.2 The Bresler Reciprocal Load Method

A simple approximate design method developed by Bresler has been satisfactorily verified by comparison with results of extensive tests and accurate calculations [Nilson, Darwin & Dolan; 2004]. It is noted that the column interaction diagram surface of Fig. 2.4 can, alternatively, be plotted as a function of the axial load P_n and eccentricity $e_x = \frac{M_{ny}}{P_n}$ and $e_y = \frac{M_{nx}}{P_n}$ as shown in the Fig. 2.7.

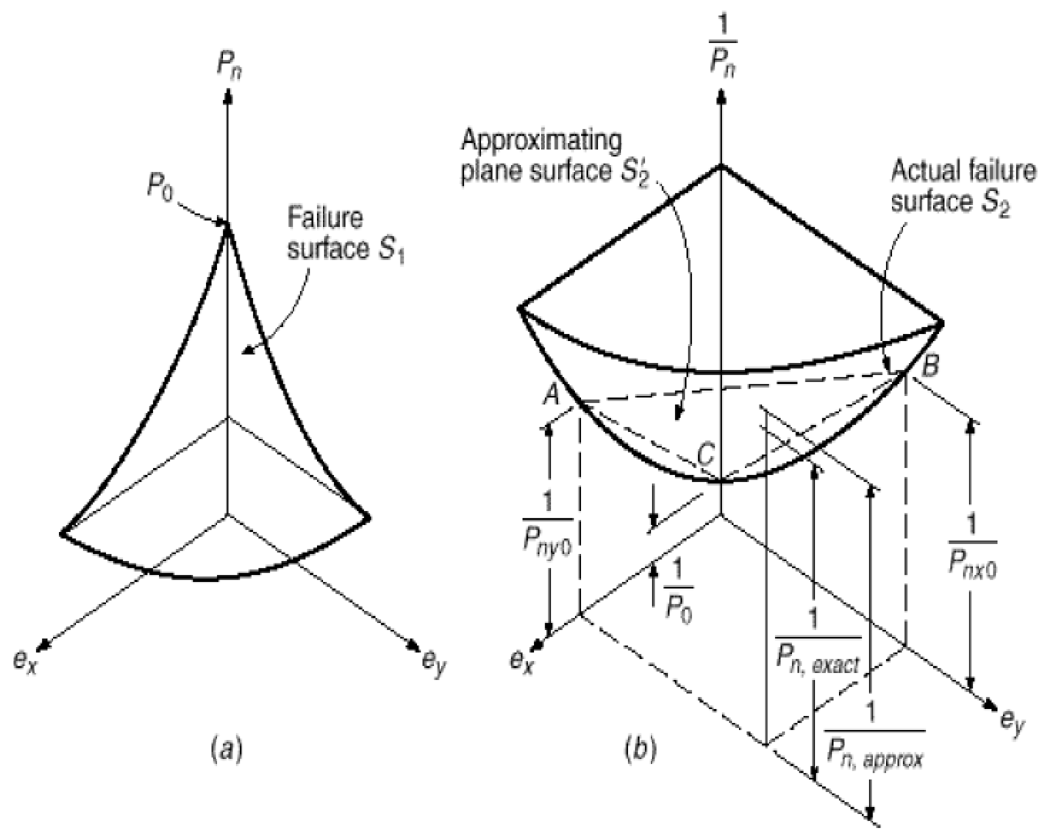


Fig. 2.7 Interaction surfaces for reciprocal load method

The surface S_1 can be transformed in to an equivalent failure surface S_2 where e_x and e_y are plotted against $\frac{1}{P_n}$ rather than P_n. thus, $e_x = e_y = 0$ corresponds to the inverse of the capacity of the column if it were concentrically loaded.

$$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_o} \quad \text{.......} \quad (2.2)$$

Here

P_n = approximate value of nominal load in biaxial bending with eccentricities e_x and e_y

P_{nxo} = nominal load when only eccentricity e_y is present (e_x= 0)

P_{nyo} = nominal load when only eccentricity e_x is present (e_y= 0)

P_o = nominal load for concentrically loaded columns

2.2.2.3 The PCA load contour method

Also called Parme - Gowens load contour method, this method has been developed as an extension of the Bresler load contour method in which in the Bresler interaction equation (2.1) is taken as the basic strength criterion. In this approach, a point on the load contour is defined in such a way that the biaxial moment strength M_{nx} and M_{ny} are the same ratio as the uniaxial moment strength M_{nxo} and M_{nyo},

$$\frac{M_{ny}}{M_{nx}} = \frac{M_{nyo}}{M_{nx0}} = \beta \quad \text{.......} \quad (2.3)$$

The actual value of β depends on the ratios of P_n to P_o as well as the material and cross sectional properties, with the usual range of values between 0.55 and 0.70. [Chen W. and etal; 1999] Substituting equation (2.3) into equation (2.1) for $\alpha 1 = \alpha 2 = \alpha$, gives:

$$\left(\frac{\beta M_{nxo}}{M_{nxo}}\right)^{\alpha} + \left(\frac{\beta M_{nyo}}{M_{nyo}}\right)^{\alpha} = 1 \quad \text{.......} \quad (2.4)$$

$$2\beta^{\alpha} = 1 \text{ and } \alpha = \frac{\log 0.5}{\log \beta} \quad \text{.......} \quad (2.5)$$

$$\text{Thus, } \left(\frac{M_{nx}}{M_{nxo}}\right)^{\frac{\log 0.5}{\log\beta}} + \left(\frac{M_{ny}}{M_{nyo}}\right)^{\frac{\log 0.5}{\log\beta}} = 1 \quad \text{.......} \quad (2.6)$$

2.3 Characteristics of Interaction Diagram

Each point on the interaction diagram represents a combination of moment and axial force at which failure occurs. A section will therefore be safe against failure for all combination of M and N that fall inside the curve and the section will fail for a combination of axial load and bending moment falling on or outside the curve. A number of important points in reference to Fig. 2.8 can be identified on a typical interaction diagram as indicated below: -

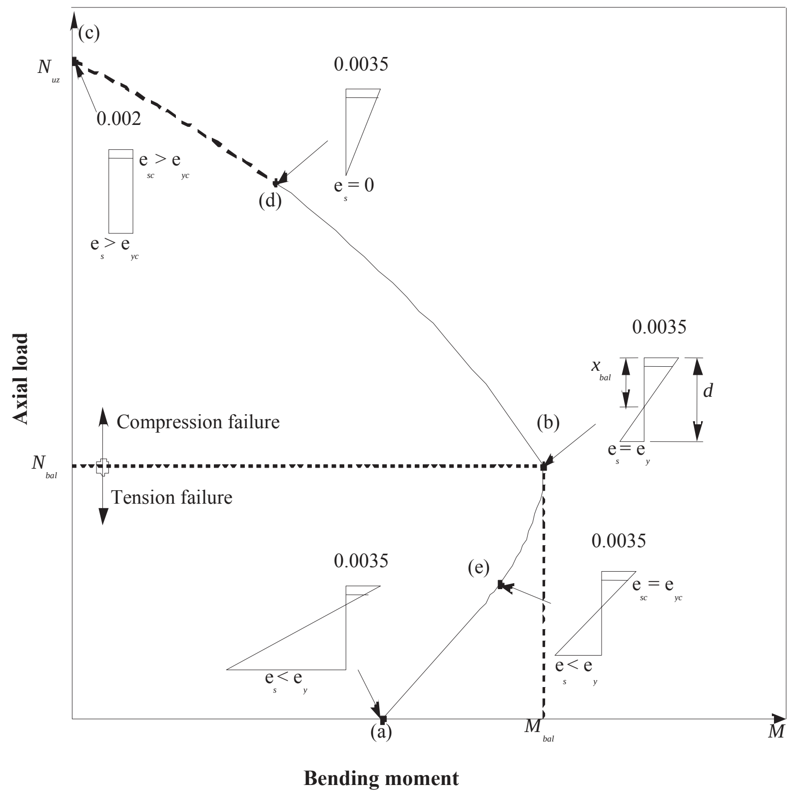


Fig. 2.8 Typical Moment-Axial Load (M-N) interaction Diagram

2.3.1 Pure bending

For pure bending N = 0 and the behavior represents that of a beam in bending. For the reinforcement ranges normally used, the reinforcement will have yielded by the time the ultimate strain in concrete is reached. It is interesting to note that the presences of a small axial load will generally increases the moment capacity of the beam. This can be easily visualized in the above interaction diagram.

2.3.2 Balance point

The balance failure point is defined as the point at which the concrete reaches its ultimate strain $\varepsilon_{cu} = 0.0035$ at the same time the tension reinforcement yields. The neutral axis depth, X_{bal}, at the balance point can be determined from

$$X_{bal} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + |e_y|}\right) \times d \quad \ldots (2.7)$$

Here d is the depth of the tension reinforcement. The strain in compression reinforcement should be calculated to determine the stress since it may have yielded or not. For combination of N and M that falls on the interaction diagram below the balance point, the failure point is ductile with the reinforcement yielding before the concrete fails in compression. For combination of N and M above the balance point the failure mode is brittle where the concrete crushes without yielding of the tension reinforcement.

2.3.3 Pure axial compression

At point (c) the column is subjected to an axial force only with M = 0. The capacity of the section is equal to N given by the equation

$$P_u = f_{cd} \times A_c + f_{yd} \times A_s \quad \ldots (2.8)$$

Note that the tension reinforcement yields in compression for this case. This strength cannot normally be attained in a structure because almost always there will be moments present and any moment leads to a reduction in the axial load capacity. Such moments or eccentricities arise from unbalanced moments in the beams, misalignment of columns from floor to floor, uneven compaction of concrete across the width of the section, or misalignment of reinforcement . . .

2.3.4 Zero strain in tension reinforcement

Moving from point (b) to point (c) on the interaction diagram it can be seen that the neutral axis increases from X_{bal} to infinity as N increases. The strain in the tension reinforcement changes from yielding in tension to yielding in compression, passing through zero at point (d). Moving form point (d) to point (c) the neutral axis will at some point fall outside the section and the strain distribution will change from triangular to uniform.

It is also worth noting that between points (b) and (c) an increase in axial load N will lead to a smaller moment capacity M at failure. On the other hand below the balance point an increase in N will increase the moment capacity of the section. If the tensile strength of the concrete is ignored in the calculation, this represents the onset of cracking on the bottom face of the section. All points lower than this in the interaction diagram represents cases in which the section is partially cracked.

2.3.5 Yielding of compression reinforcement

As the axial force N increases and the neutral axis x increases, the strain in compression reinforcement will often change from elastic to yielding. This will clearly be influenced by the strength of the reinforcement and its position within the section. This point will typically correspond to a change in slope of the interaction diagram as shown at point (e). At this point the reinforcement has been strained to several times the yield strain before the concrete reaches its crushing strain, this implies ductile behavior.

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CHAPTER THREE

3. EQUATIONS FORMULATION

3.1 EBCS 2-1995 Requirements

The Ethiopian building code standard specifies some requirements that must be satisfied for designing columns in addition to the five basic assumptions set out in the first section of the second chapter. The sign convention used is compressive forces and stresses are taken as positive and tensile forces and stresses are taken as negative. Similarly clock-wise moments are taken as negative and counter clock-wise moments are taken as positive. Moreover the following limitations and requirement should be satisfied as per EBCS-2, 1995.

- All reinforced columns must have a minimum percentage of reinforcement to resist temperature, shrinkage, creep, accidental loads, and to cope up with the normal tolerance such as minimum eccentricity allowed in construction. Similarly the maximum amount of steel that can be placed without causing problem in construction and to maintain the bonded action of the member is also to be adhered. The limits of the reinforcement are:

$$0.008 \leq \frac{A_s}{A_g} \leq 0.08$$

Here A_s is the area of longitudinal reinforcement (mm^2)

A_g is the gross area of concrete section (mm^2)

- The minimum amount of reinforcement is further controlled by the minimum diameter and number of main bars. The main bars in the columns refer to longitudinal bars. The limits are:

$$\begin{cases} \text{Ø} \geq 12mm \\ n \geq 4 \text{ pcs [Rectangular]}, n \geq 6 \text{ pcs [circular]} \end{cases}$$

Where Ø is the diameter of longitudinal bars in mm

n is the number of reinforcement bars

- The minimum lateral dimension of a column shall be at least 150mm
-

- The maximum length to width ratio is less than 4 otherwise the columns should be treated as a wall.

- The minimum reinforcement cover shall be 15mm for mild exposure, 25mm for moderate exposure and 50mm for severe exposure.

- The clear horizontal and vertical spacing between bars shall be at least equal to the largest of the following values:
  - 20mm
  - The diameter of the largest bar or effective diameter of the bundle
  - The maximum size of the aggregate plus 5mm

There are generally two possible loading cases on columns:-

- Axial load + uniaxial bending: -

In this case the column cross-section is subjected to an axial load and a bending moment in one direction. This will happen mostly in case of side columns.

- Axial load + biaxial bending: -

In this case the column cross-section is subjected to an axial load and bending moments in two orthogonal directions. L-shaped reinforced concrete columns are mostly subjected to an axial load and bi-axial bending moment as they are located at corner points. The thesis deals with axial load plus biaxial bending moment as the cross-section is mostly biaxial.

3.2 Equations Formulation

Infinitesimal area ‘dA’, in Fig. 3.1 at ‘y’ perpendicular distance from the x-axis in the compression zone of the section is considered. The local axis is inclined at an angle of ‘θ’ from the global coordinate. The stress resultant on ‘dA’ and on the whole compression area is determined as follows using integral calculus:

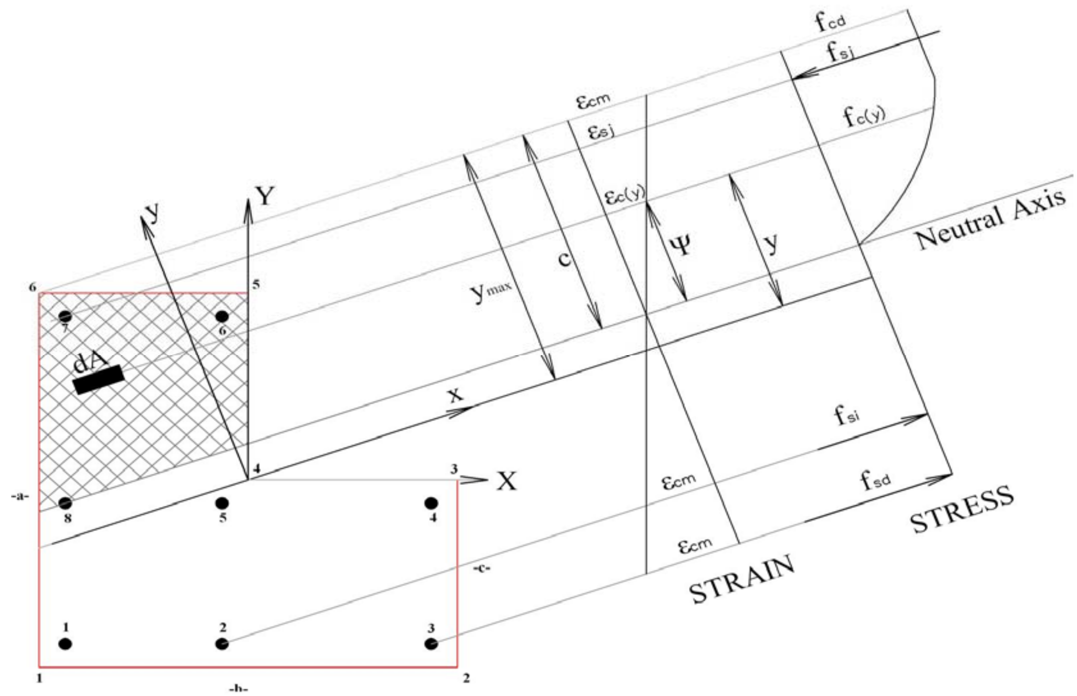


Fig. 3.1 L-shaped column section and arrangement of stress- strain curves

$$dN_c = f_c(y)dA \quad\quad N_c = \iint_A f_c(y)dA \quad (3.1)$$

$$dM_{xc} = -yf_c(y)dA \quad\quad M_{xc} = -\iint_A yf_c(y)dA \quad (3.2)$$

$$dM_{yc} = xf_c(y)dA \quad\quad M_{yc} = \iint_A xf_c(y)dA \quad (3.3)$$

The stress resultant in the reinforcement bars is also calculated simply as follows:

$$N_s = \sum_{j=1}^{N} A_{sj}f_{sj} \quad (3.4)$$

$$M_{xs} = \sum_{j=1}^{N} y_{sj}A_{si}f_{sj} \quad (3.5)$$

$$M_{ys} = \sum_{k=0}^{n} x_{sj}A_{sj}f_{sj} \quad (3.6)$$

To account the area of concrete replaced by reinforcement bars in the compression zone of the cross section, the stress resultant $f_c(y)$ should be deducted from f_{sj}. The total stress resultant about the global axis will be found by summing up the contribution of concrete & steel stress resultants and transforming by the angle (-θ)

$$P_n = N_c + N_s \quad (3.7)$$

$$M_{Xn} = (M_{xc} + M_{xs}) \sin(-\theta) + (M_{yc} + M_{ys}) \cos(-\theta) \quad (3.8)$$

$$M_{Yn} = -(M_{xc} + M_{xs}) \cos(-\theta) + (M_{yc} + M_{ys}) \sin(-\theta) \quad (3.9)$$

Where:-

$f_c(y)$	-	Compressive stress on the concrete at location y from the local axis
x and y	-	Coordinates with respect to the local axis
X and Y	-	Coordinates with respect to the global axis
A_{sj}	-	Area of the reinforcement bar j
f_{sj}	-	Stress on the reinforcement bar j
θ	-	Clock-wise angle that the local axis makes from the global axis
M_{xc} & M_{yc}	-	Moment stress resultant of concrete about the local centroidal axis
M_{xs} & M_{ys}	-	Stress resultant of reinforcement bars about the local centroidal axis
M_{Xn} & M_{Yn}	-	Moment capacities of the section about the centroidal global axis

The strain at different location on the cross section depends on the position of the neutral axis and summarized in the following relations from EBCS 2-1995 curves of strain in concrete for the ultimate limit state:

$$\varepsilon_{cm} = \frac{0.01}{(\frac{d}{c}-1)} \text{ for } k_x < \frac{7}{27} \quad (3.10)$$

$$\varepsilon_{cm} = \varepsilon_{cu} = \begin{cases} 0.0035 \text{ for } \left(\frac{7}{27}\right) \leq k_x \leq 1 & (3.11) \\ \frac{0.002}{(1-\frac{3d}{7c})} \text{ for } k_x > 1 & (3.12) \\ 0.002 \text{ for } k_x = \infty & (3.13) \end{cases}$$

Since the strain at distance y from the neutral axis is the same in both the steel and the concrete, the following relation can be derived:

$$\varepsilon_s(y) = \varepsilon_c(y) = \frac{\varepsilon_{cu}(\psi)}{c} \quad \text{.........} (3.14)$$

Where $k_x = \frac{c}{d}$ - c & d are NA & tension reinforcements depths respectively

$f_{cd} = \frac{0.67 \times f_{cu}}{\gamma_c}$ - where $\gamma_c = 1.50$ for concrete

$f_{yd} = \frac{f_{yk}}{\gamma_s}$ - where $\gamma_s = 1.15$ for steel

$\psi = c + y - y_{max}$ - shown on Fig. 3.1

ε_{cm} - maximum strain in the concrete

For concrete, if $\varepsilon_c(y) \leq 0.002$, the compressive strength will be $f_c(y) = 1000\varepsilon_c(y)[250\varepsilon_c(y)\text{-}1]f_{cd}$ (parabolic curve) and full design strength of the concrete will be attained for the strain range $0.002 \leq \varepsilon_c(y) \leq 0.0035$ (rectangular curve). On the other hand, for the steel, if $|\varepsilon_s(y)| \leq \varepsilon_{yd,}$ the stress will be $f_s(y) = E_s\varepsilon_s(y)$ and the design yield strength will be attained when $|\varepsilon_s(y)| > \varepsilon_{yd}$.

When the reinforcing steel is under compression, the area of concrete replaced by the reinforcement should be accounted. In this way the contribution of the concrete and steel reinforced bars is calculated. The double integrals in equations (3.1), (3.2), (3.3) are not easily utilized without simplifications. The following two sub topics will deal with the simplification of the double integral in to line integral.

3.2.1 Green's Theorem

Green's theorem transforms the double integration over an area A in to a line-integration along the closed line L that encloses the area. It is formally stated as follows:

$$\iint\limits_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \oint\limits_L Pdx + \oint\limits_L Qdy \quad \dots \dots (3.15)$$

Here P and Q are two functions of x and y, A is the area of integration and L is closed curve that encloses the area A. The theorem can be applied to the biaxial problem, as it is formulated in equations (3.1), (3.2) and (3.3). If we let

$$P = 0 \quad (3.16)$$

$$Q = \frac{1}{(r+1)} \times x^{r+1} \times y^{s} \times f_c(y) \quad (3.17)$$

Where r and s in the above equations are non-negative integers, with this definition of P and Q equation (3.15) becomes

$$\iint_A (x^r)(y^s) f_c(y)\,dx\,dy = \frac{1}{r+1} \oint_L (x^{r+1})(y^s)\, f_c(y)\,dy \quad (3.18)$$

The stress resultant R, of the concrete compressive zone (N_c or M_{xc} or M_{yc}), is thus reduced to the line integral

$$R = \frac{1}{r+1} \oint_L (x^{r+1})(y^s) f_c(y)\,dy \quad (3.19)$$

This is the basic equation of the present formulation of the biaxial problem, depending on the values of r and s, the left-hand side of equation (3.19) represents:

- The axial force N_c (equation 3.1), for r = 0 and s = 0
- The bending moment M_{xc} (equation 3.2), for r = 0 and s = 1
- The bending moment M_{yc} (equation 3.3), for r = 1 and s = 0

More important section properties can be formulated from the above relations. If the function $f_c(y) = 1$ then the left hand side of equation (3.19) represents:

- The area A for r = 0 and s = 0
- The area moment S_x for r = 0 and s = 1
- The area moment S_y for r = 1 and s = 0
- The moment of inertia I_x for r = 2 and s = 0
- The moment of inertia I_y for r = 0 and s = 2
- The product of inertia I_{xy} for r = 1 and s = 1

The right-hand side of equation (3.19) is a line-integration along the sides of the integration area A and can be written as:

$$\frac{1}{r+1}\oint_L (x^{r+1})(y)^s f_c(y)dy = \frac{1}{r+1}\sum_L S_L \quad (3.20)$$

$$\text{where } S_L = \int_L (x^{r+1})(y^s) f_c(y)dy \quad (3.21)$$

is the integral along the 'L' side of the closed polygon that encloses the compressive zone of the section. The line-integral of equation (3.21) is evaluated along the sides of the compressive zone. The sides are defined by the xy- coordinates of the end points as shown in the Fig. 3.2 below.

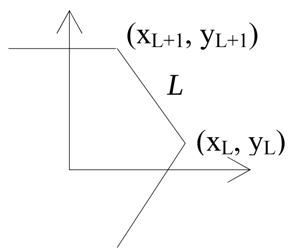


Fig. 3.2 Definition of side L

The equation of side L is

$$x = \alpha_L + \beta_L y \quad (3.22)$$

$$\alpha_L = x_L - \beta_L y_L \quad (3.23)$$

$$\beta_L = \frac{(x_{L+1} - x_L)}{(y_{L+1} - y_L)} \quad (3.24)$$

where (x_L, y_L) and (x_{L+1}, y_{L+1}) are starting and ending points of side 'L' respectively. With equation (3.18), the line integral in equation (3.21) becomes

$$S_L = \int_L (\alpha_L + \beta_L y)^{(r+1)}(y)^s f_c(y)dy = \int_L G_L(y)dy \quad (3.25)$$

$$G_L(y) = (\alpha_L + \beta_L y)^{r+1}(y^s) f_c(y) \quad (3.26)$$

The numerical evaluation of equation (3.25) takes the form shown in equation (3.23) below:

3.2.2 Gaussian Quadrature

Throughout the engineering fields, there are countless applications for integral calculus. Sometimes, the evaluation of expressions involving these integrals can become daunting, if not indeterminate. For this reason, a wide variety of numerical methods have been developed to simplify the integral. Here, we will use the Gauss Quadrature Rule of integral approximation.

$$S_L = \int_L G_L(y)dy \cong \frac{(y_{L+1} - y_L)}{2} \sum_{i=1}^{Ng} w_i G_L(y_i) \quad (3.27)$$

where N_g is the order of the Gaussian Quadrature (in this study $N_g = 3$), w_i is the i-th weight, and y_i is sample point at which the function $f_c(y)$ is evaluated. The coefficients w_l, w_2 and w_3, and the function arguments y_1, y_2 and y_3 are calculated by assuming the formula gives exact expressions for integrating. The following table shows the values of the weighting factor and function arguments for three Gaussian points.[A. Kaw and M. Keteltas; 2009]

Table 3.1 Weighting factors 'w' and function arguments 'y' used in Gauss Quadrature

Points	Weighting Factor	Function Argument
3	$w_1 = 0.555555555555556$	$y_1 = 0.577350269189626$
	$w_2 = 0.888888888888889$	$y_2 = 0.774596669241483$
	$w_3 = 0.347854845137454$	$y_3 = 0.339981043584856$

In this way the double integration is reduced to the numerical evaluation of a few line integrals. The number of line integrations is equal to the number of the sides of the section in the compressive zone. Each of these integral involves typically three sample points thus the number of computer operations is very small.

3.3 Reinforcement bar and column section coordinate

The program is written to be very flexible for all possible arrangement of reinforcement bar and cross-section shapes. There are six coordinates of the cross section and eight reinforcement bars of similar diameter in the Fig. shown below.

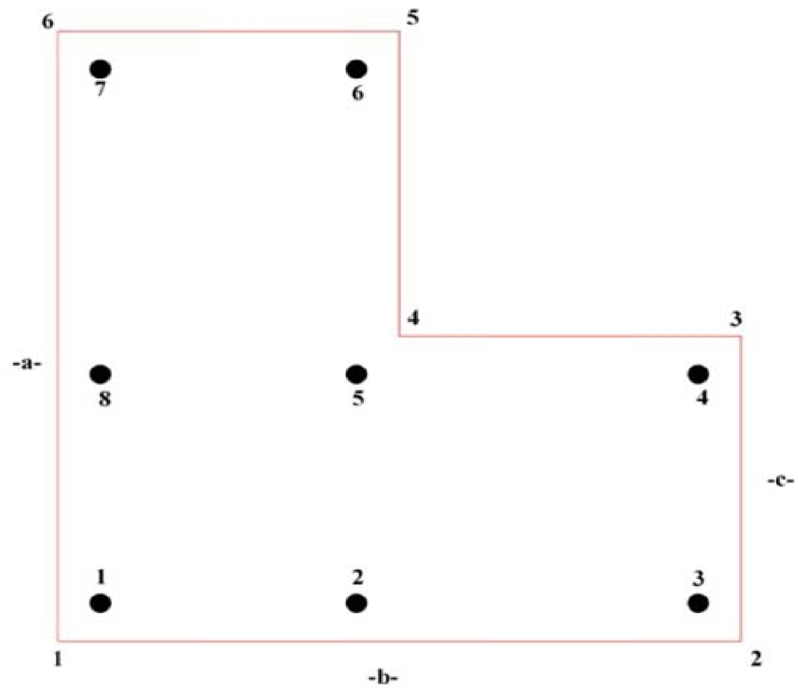


Fig. 3.3 Dimension of the cross section and designation of points

The two tables show the section coordinate and reinforcement bar coordinates. The concrete section dimensions are as shown below in Fig. 3.3 for easy reference to identify location and reinforcement bar coordinates.

The lower left corner of the cross-section is taken as the reference origin to assign section dimension and reinforcement locations. Here d_p means the concrete cover.

Table 3.2 Coordinates of reinforcement bars

No	Reinforcement bars	X-coordinate	Y-coordinate
1	Rebar -1	d_p	d_p
2	Rebar -2	$c - d_p$	d_p
3	Rebar -3	$b - d_p$	d_p
4	Rebar -4	$b - d_p$	$c - d_p$
5	Rebar -5	$c- d_p$	$c- d_p$
6	Rebar -6	$c - d_p$	$a - d_p$
7	Rebar -7	d_p	$a - d_p$
8	Rebar -8	d_p	$c - d_p$

Table 3.3 Coordinates of corner points of column cross section

No	Section Coordinate	X-coordinate	Y-coordinate
1	Corner - 1	0	0
2	Corner - 2	b	0
3	Corner - 3	b	c
4	Corner - 4	c	c
5	Corner - 5	c	a
6	Corner - 6	0	a

3.4 Stress Resultants

Now we have reached at appoint that we can summarize the formulas stated in the above equation to incorporate in the programming languages in simpler manner for both the axial load and the bending moments. The formulae stated in equation above are now summarized for the section under consideration the following ways:

For concrete:

- N_c, when r = 0 and s = 0

$$N_c = \sum_{n} \frac{(y_{L+1} - y_L)}{2} \times \sum_{i=1}^{Ng} w_i\,(\alpha_L + \beta_L\, y_i) \times f_c\,(y_i) \quad (3.28)$$

- M_{xc}, when r = 0 and s =1

$$M_{xc} = -\sum_{n} \frac{(y_{L+1} - y_L)}{2} \times \sum_{i=1}^{Ng} w_i(\alpha_L + \beta_L\, y_i) \times y_i \times f_c\,(y_i) \quad (3.29)$$

- M_{yc} when r =1 and s =0

$$M_{yc} = \frac{1}{2}\sum_{n} \frac{(y_{L+1} - y_L)}{2} \times \sum_{i=1}^{Ng} w_i(\alpha_L + \beta_L y_i)^2 \times f_c(y_i) \quad (3.30)$$

For Steel:

$$N_s = \sum_{j=1}^{N} A_{sj} f_{sj} \quad (3.31)$$

$$M_{xs} = -\sum_{j=1}^{N} y_{sj} A_{sj} f_{sj} \quad (3.32)$$

$$M_{ys} = \sum_{j=1}^{N} x_{sj} A_{sj} f_{sj} \quad (3.33)$$

Total is found by summing up the two in the following way:

$$N = N_s + N_c \quad (3.34)$$

$$M_x = M_{xc} + M_{xs} \quad (3.35)$$

$$M_y = M_{yc} + M_{ys} \quad (3.36)$$

3.5 Design Chart Generation Steps

The following seven steps are the procedures used in the generation of design charts for L-shaped reinforced concrete columns:-

1. Select characteristic strength values for concrete C- , reinforcement bars S-, and section dimensions according to the assignment shown in Fig. 3.3 above taking the lower left corner of the cross-section as the origin

2. Select a value for reinforcement ratio, $\rho = A_s / A_g$ assign the area of reinforcement of equal diameter for the given location

3. Select a value for θ, the clock-wise angle that the neutral axis makes with the global X-axis. in choosing the angle θ the trigonometric functions should apply.

4. Calculate strains ε_s and stresses f_s in the reinforcement using strain compatibility relations

5. Calculate N and M using strain compatibility and sign conventions stated above

6. Repeat steps 3 to 5 for different values of c. This produces one curve for a given A_s / A_g on the interaction diagram

7. Repeat steps 2 to 6 for different values of A_s / A_g to generate the complete M-N interaction design chart

CHAPTER FOUR

4. RESULTS OF THE THESIS

4.1 Results

Based on the out put of the program the following are visualized:

- The designed program has efficiently executed the assignment in a faction of seconds with higher degree of accuracy as checked by version 4.1 of the Alashki's registered software that is used to generate failure surface, design section and check section in different design codes both at the ultimate and serviceability limit state.

- The shape of the design chart looks like the design chart of rectangular and circular reinforced concrete column cross section with equally spaced failure surfaces for the given cross section and reinforcement arrangement.

- At lower reinforcement ratio (ρ = 0.008), the balanced point occurs at higher axial load point than at higher reinforcement ratio (ρ = 0.08). In other words the balanced point moves down as the reinforcement ratio increases which gives the failure surface at higher reinforcement ratio to be straighter.

4.2 User's Guide

To use the program the user should follow the following steps:

- The user should first choose the concrete and reinforcement grades to be used for the L-shaped reinforced concrete design.

- The user should also input dimensions of the reinforced concrete cross-section that is going to be used for the design

- The user should input reinforcement bar and corner coordinates of the cross-section in counter-clock wise direction taking the lower left corner point as the origin. The user should also input the inclination angle of the neutral axis from the positive global X-axis

- Assume different reinforcement ratios in the range ρ = 0.008 to ρ = 0.08 to find the total reinforcement area and the area of single reinforcement bar to be used as input for the program.

- Then run the program to get the coordinate points on the failure surface of the reinforce concrete column in text format for that particular reinforcement ratio

- Open the text format data in Microsoft excel program by choosing the file type that best fits the data as d*elimited by space* to get the data in separate cells

- Generate the chart using the scatter chart option and limit the minimum axes values to be fixed to be zero

- Have a look at the chart and if the upper part of the chart swings back, delete some data on the top of the coordinates and substitute the data with pure axial compression capacity of the coordinate by your self or extrapolate the chart

- Change the reinforcement ratio and generate another data for the plot and incorporate those data in the same chart

- Finalize the chart to be ready for use by assigning the reinforcement ratios as chart label as they will be required for deign

4.3 Design Examples

Design an L-shaped reinforced concrete corner column with the following design loads from analysis [N_{sd} = 425KN, M_{xd} = 140KN-m and M_{yd} = 140KN-m]. The section should have no edge exposed to the interior of the building having HCB wall.

Solution:

- Assume concrete grade C-25 and reinforcing steel grade S-300
- Since the section should not have edges in the interior of the room, the maximum thickness shall not exceed the thickness of the HCB wall i.e. 200mm therefore lets also assume a column section with a = b = 400mm and c = 200mm based on the designation in Fig. (3.3).

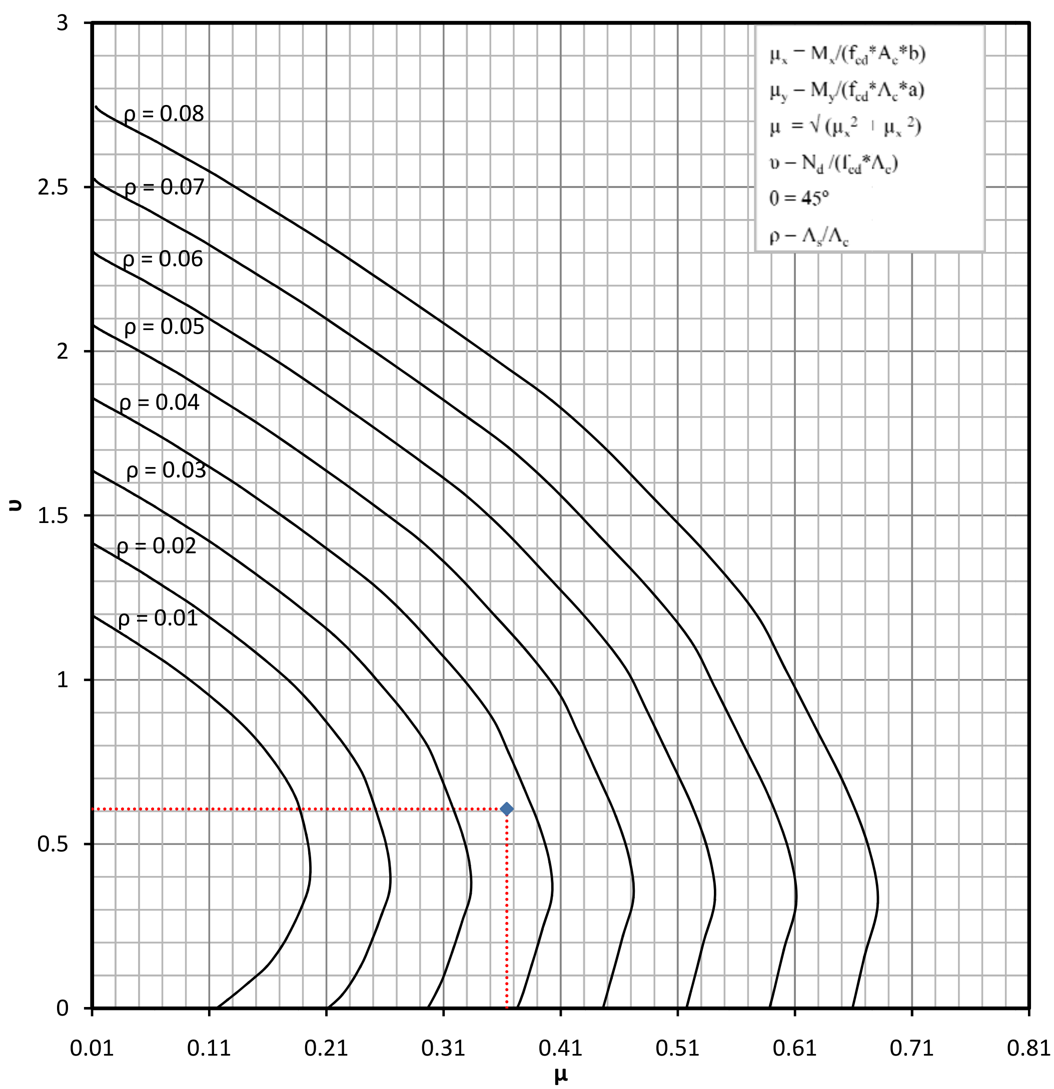
- The same amount and locations of reinforcement bars as positioned Fig. (3.3) is assumed.

- Design loads are: N_{sd} = 425KN, M_{xd} = 140KN-m and M_{yd} = 140KN-m

- Determine θ from the given moment as

- — 45º

- Generate the design chart for the assumed section, reinforcement location and material properties following the steps shown in the user guideline using the program



- Calculate μ and υ using the formulas depicted on the design chart generated above as follows to find the reinforcement ratio from the chart

$$\upsilon = \frac{N_{sd}}{A_c \times f_{cd}} = \frac{425*1000}{(400*400-200*200)*11.33} = 0.607$$

$$\mu_x = \frac{M_{xd}}{A_c \times f_{cd} \times b} = \frac{140 \times 1000000}{(400 \times 400\text{-}200 \times 200) \times 11.33 \times 400} = 0.2574 \text{ and}$$

$$\mu_y = \frac{M_{yd}}{A_c \times f_{cd} \times h} = \frac{140 \times 1000000}{(400 \times 400\text{-}200 \times 200) \times 11.33 \times 400} = 0.2574 \text{ which gives}$$

$$\mu = \sqrt{(\mu_x{}^2 + \mu_y{}^2)} = \sqrt{(0.2574^2 + 0.0.2574^2} = 0.3640$$

- ρ when read from the graph will be approximately 0.036

- Therefore $A_s = \rho \times A_c = 0.036 \times (400 \times 400 - 200 \times 200) = 4320\text{mm}^2$

- As the number of reinforcement bars are eight and each bar has the same area A_s will be $A_s = \frac{4320\text{mm}^2}{8} = 540\text{mm}^2$

- therefore 8Φ26mm bars at the locations indicated

In this way any deign problem with regard to L-shaped columns can be solved in short period of time and greater accuracy using the program generated.

CHAPTER FIVE

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

- The cross-section is unsymmetrical in both axes and there is no case by which the cross section is subjected to uniaxial bending moment only even at 0º, 90º, 180º… where all symmetrical cross-sections are under uniaxial bending. Moreover, L-shaped reinforced concrete columns sections are used at corner points where there are moments from beams in the orthogonal directions. Therefore only biaxial case is considered for this thesis.

- The width, depth and thickness of the cross-section, the grade of steel reinforcement and class of the concrete, the location of the reinforcement bars and concrete cover dimensions can be varied as the program allows such changes to be incorporated without major changes on the algorithm.

- Readily available any shaped reinforced concrete column design charts can be generated using the very same program with some modifications.

- The program is developed for normal grade of concrete and steel specified in the Ethiopian building code standard EBCS 2-1995 only.

5.2 Recommendations

- The program is made in such away that it is flexible to conform to any arbitrary reinforced concrete column cross sections (except circular column) to serve as an input for further research and development in the field of columns design.

- There are cases by which columns in seismic zones encounter tensile forces especially in high rise buildings. Moreover codes lacks the provision for tension forces plus bending moment even in symmetrical column cross -sections. Therefore the program can be extended further to include tensile forces and bending moment for completeness.

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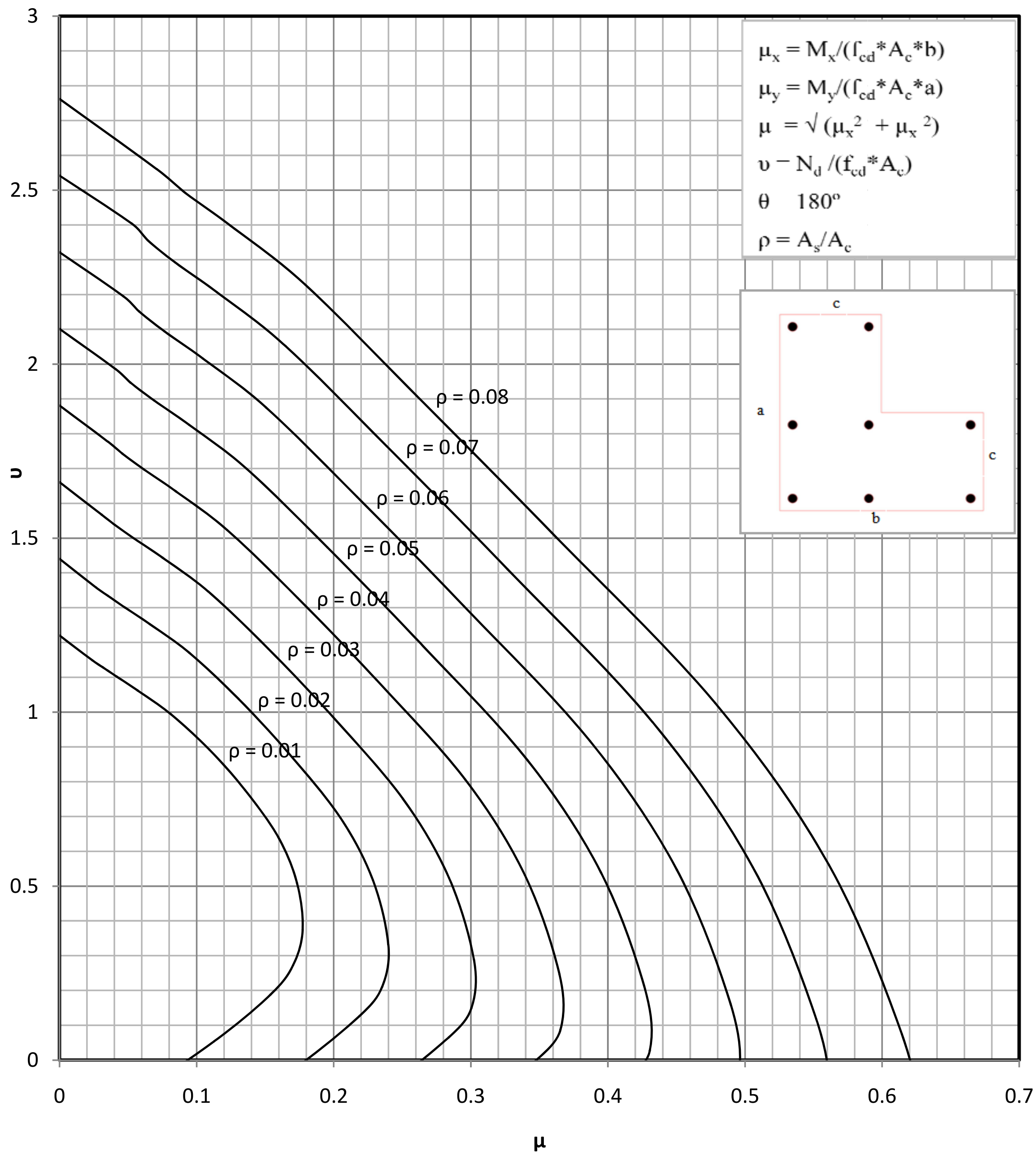
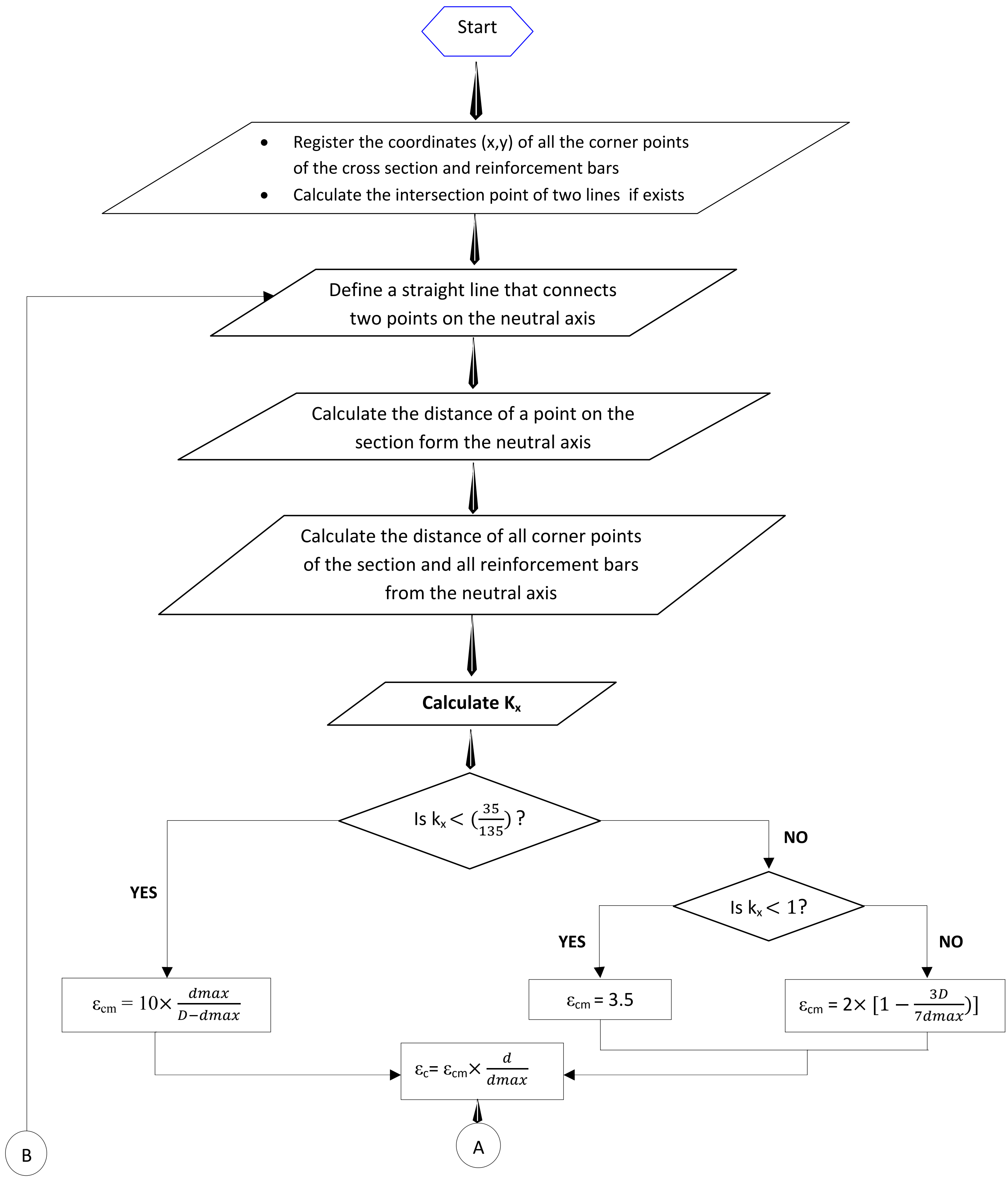
A P P E N D I C E S

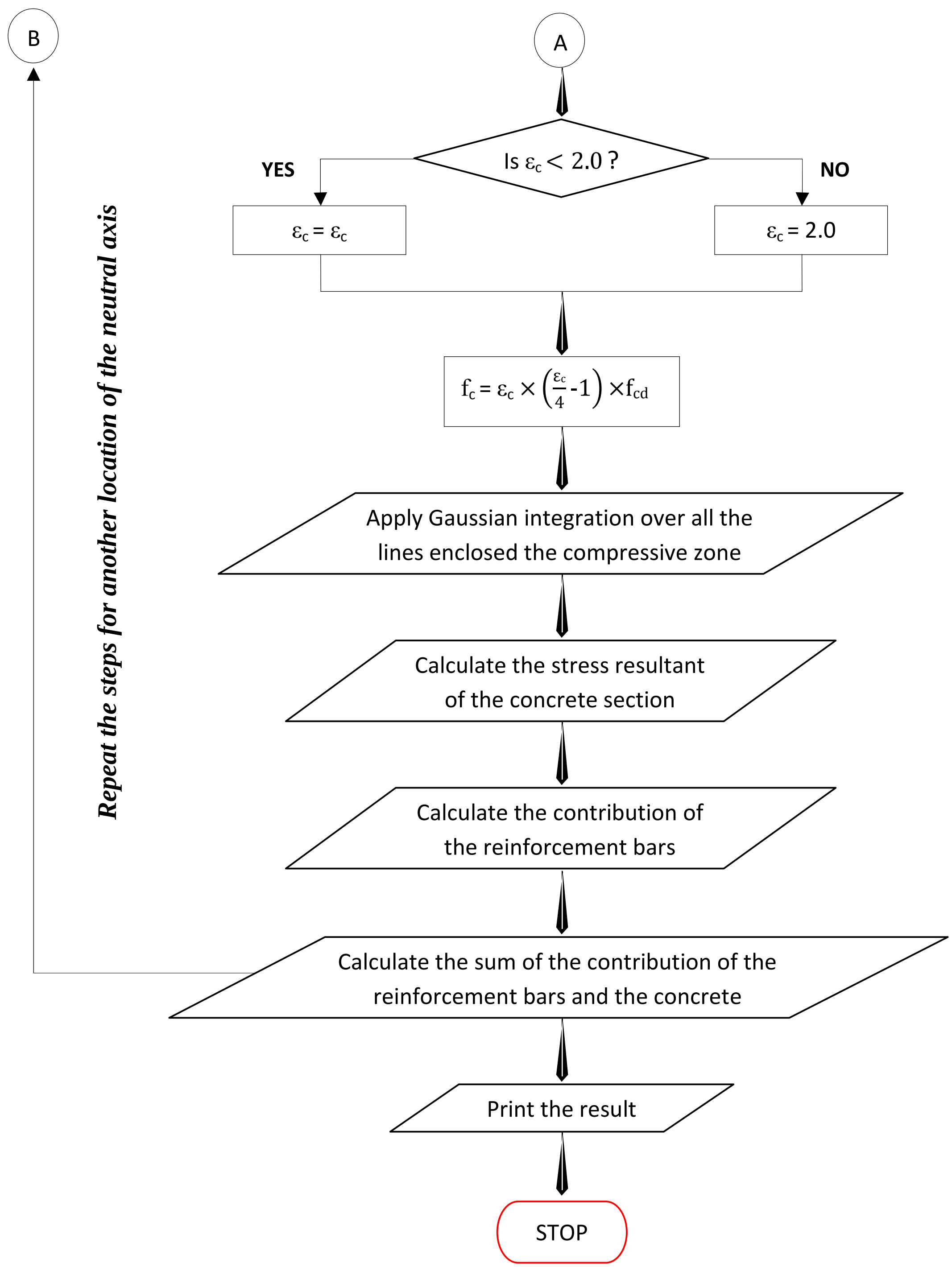
APPENDIX A – SAMPLE DESIGN CHART



The above sample design chart is generated for L-shaped reinforce concrete column of section dimension a = b = –   = 200 mm and eight reinforcement bars at the location indicated.

APPENDIX B – LOGIC FLOW CHART





All the steps shown in the above flow chart shall be repeated for different reinforcement ratio in the range $\rho = 0.008$ to $\rho = 0.08$ to generate as many envelopes as possible.

DECLARATION

I declare that this research is my original work and has not been presented for a degree in any other university, and that all sources of materials used by the research have been duly acknowledged.

Confirmed by:

Melaku Mohammed

Dr.-Ing. Girma Zerayohannes
(Advisor)

APPENDIX C – COMPUTER CODE

C-1 Main Program

```
PROGRAM FINAL

USE COORDINATE
IMPLICIT NONE

REAL FCU, FCK, FCD, FYK, FYD, H, C, S, AREA, RHO, AST, AS., MXX, MYY, P, THETA, ANGLE, DIAM
REAL, PARAMETER:: PI = 3.14159265358979323846264338327950
RECORD /NAXIS/ NA
REAL XNA,NC,MX,MY,Q,TX,TNC,TMX,TMY,TM,NU,MXU,MYU,FCD1,AC,H,T,DP,RATIO

        WRITE (*,*) 'ENTER THE GRADE OF CONCRETE C - [MPA]:'
        READ (*,*) FCU
        FCK = FCU*1000/1.25
        FCD = 0.85*FCK/1.5
        WRITE (*,*) 'ENTER THE GRADE OF REINFORCEMENT S - [MPA]:'
        READ (*,*) FYK
        FYD = FYK*1000/1.15
 100    CONTINUE
        WRITE (*,*) 'ENTER THE DEPTH OF THE SECTION H [MM]:'
        READ (*,*) H
        WRITE (*,*) 'ENTER THE THICKNESS OF THE SECTION T [MM]:'
        READ (*,*) C
        S=B/C
        IF (H .LE. C) THEN
        WRITE (*,*) '********************************************************************************'
        WRITE (*,*) ' '
        WRITE (*,*) ' WARNING!'
        WRITE (*,*) ' PLEASE ENTER APPROPARIATE DIMENSION FOR THE CROSS-SECTION !'
        WRITE (*,*) ' '
        WRITE (*,*) '*******************************************************************************'
GO TO 100
        ELSE IF (S .GE. 4) THEN
        WRITE (*,*) '*******************************************************************************'
        WRITE(*,*) ' '
        WRITE(*,*) ' WARNING!'
        WRITE(*,*) ' PLEASE ENTER APPROPARIATE DIMENSION FOR THE CROSS-SECTION !'
        WRITE (*,*) ' '
        WRITE (*,*) '*******************************************************************************'
GO TO 100
        ELSE IF ((H .LT. 150).OR. (C.LT.150)) THEN
        WRITE (*,*) '********************************************************************************'
        WRITE (*,*) ' '
        WRITE (*,*) ' WARNING!'
        WRITE (*,*) ' PLEASE ENTER APPROPARIATE DIMENSION FOR THE CROSS-SECTION !'
        WRITE (*,*) ' '
        WRITE (*,*) '***********************************************************************************'
GO TO 100
```

```
        ELSE
        AREA = (H**2-(H-C) **2)/1000000
        ENDIF
200     CONTINUE
        WRITE (*,*) ' ENTER THE REINFPORCEMENT RATIO '
        READ (*,*), RHO
        IF ((RHO .LT. 0.008).OR. (RHO.GT.0.08))THEN
        WRITE (*,*) '***************************************************************************'
        WRITE (*,*) ' '
        WRITE (*,*) ' WARNING!'
        WRITE (*,*) ' PLEASE ENTER APPROPARIATE REINFORCEMENT RATIO !'
        WRITE (*,*) ' '
        WRITE (*,*) '***************************************************************************'
GO TO 200
        ELSE
        AST = RHO*AREA
        AS = AST/8
        DIAM =1000* SQRT (4*AS/PI)
        ENDIF
        WRITE (*,*) 'ENTER THE DESIGN LOADS P [KN], MX [KN-M] AND MY [KN-M]'
        READ (*,*) P, MXX, MYY
        THETA = ATAN (MYY/MXX)
        ANGLE = THETA*180/PI
        WRITE (*,*) DIAM
  300   CONTINUE
        WRITE (*,*) ' ENTER THE THICKNESS OF THE CONCRETE COVER DP [MM]: '
        READ (*,*), DP
        DP = DP/1000
        IF ((DP.LT.0.015).OR. (DP.GT.0.025)) THEN
        WRITE (*,*) '  '
        WRITE (*,*) '**********************************************************'
        WRITE (*,*) ' WARNING !!!'
        WRITE (*,*) ' PLEASE ENTER PROPER DIMENSION FOR THE CROSS-SECTION !'
        WRITE (*,*) ''
        WRITE (*,*) '**********************************************************'
 GO TO 300
        ELSE
        ENDIF

  OPEN (FILE='RESULT.TXT', UNIT=1)
        CALL FORM_SHAPE (NA, H, T, DP)
                NA.C1.X = -100
                NA.C2.X = 100
                NA.ABOVE_C.X = 0
                NA.ABOVE_C.Y = 100
                XNA = -2E3
                AC = H**2 - (H-T) **2
                NU = FCD * AC
                MXU = NU*H
                MYU = NU*H
```

```
DO WHILE (XNA <= 1.35)
                NA.C1.Y = XNA
                NA.C2.Y = XNA
                CALL CALCULATE_CAPACITY (NA, Q, NC, MX, MY)
                TX = XNA
                TMX = MX/MXU
                TMY = MY/MYU
                TNC = NC/NU
                TM = SQRT (TMX**2 + TMY**2)

WRITE(*,*) "X  =  ",  TX , '    ',TMX ,'    ' ,TMY, '    ' ,TNC !TMX,TMY,
        WRITE (1,*) TM, TNC! TMX, TMY,
                IF (ABS (XNA) < 2.0 * NA.D) THEN
                        XNA = XNA + 0.001
                        ELSE
                        XNA = XNA + ABS (XNA) / 2
                ENDIF
                ! EXIT
        ENDDO
        CLOSE ()
END
```

C-2 *Coordinate Module*

```
MODULE COORDINATE

! DEFINE COORDINATE POINT AND AXIS
STRUCTURE /POINT/
  REAL X
  REAL Y
END STRUCTURE

STRUCTURE /NAXIS/
INTEGER NCORNERS, NBARS
RECORD /POINT/ CORNERS (10), /POINT/ BARS (10), /POINT/ O_CORNERS (10), /POINT/O_BARS (10)
RECORD /POINT/ C1, /POINT/ C2, /POINT/ ABOVE_C
REAL ANGLE, DMAX, DMIN, D
END STRUCTURE

CONTAINS
SUBROUTINE FIND_FURTHEST (NA)
   RECORD /NAXIS/ NA
   RECORD /POINT/ CMIN, CMAX, IC
           REAL D, DMIN1
           INTEGER I
           LOGICAL ABOVE
   DMIN1 = 1E16
   ABOVE = NA_ABOVE (NA, NA.ABOVE_C)
           NA.DMAX = 0
           NA.DMIN = 0
           DO I = 1, NA.NCORNERS
              D = CALCD (NA, NA.CORNERS (I))
                      IF (ABOVE .EQ. NA_ABOVE (NA, NA.CORNERS (I))) THEN
                          IF (D .GT. NA.DMAX) THEN
                                  NA.DMAX = D
                                  CMAX = NA.CORNERS (I)
                                END IF
                                IF (D .LT. DMIN1) THEN
                                  DMIN1 = D
                                END IF
                      ELSE
          IF (D .GT. NA.DMIN) THEN
                                  NA.DMIN = D
                                  CMIN = NA.CORNERS (I)
                                END IF
                      END IF
           END DO
           IF (INT_INTERSECT (NA.C1, NA.C2, CMIN, CMAX, IC)) THEN
              NA.D = NA.DMAX + NA.DMIN
           ELSE
              NA.DMIN = DMIN1
              NA.D = NA.DMAX - NA.DMIN
           END IF
END SUBROUTINE
```

```
SUBROUTINE BAR_CONTRIBUTION (NA, NC, MX, MY)
   RECORD /NAXIS/ NA
   REAL NC, MX, MY
           REAL DR, ECM, ES, FS
           LOGICAL ABOVE

   ABOVE = NA_ABOVE (NA, NA.ABOVE_C)
                      ! GET ECM
           ECM = GETECM (NA)

           ! CONTRIBUTION OF EACH BAR
           DO I = 1, NA.NBARS
                      DR = CALCD (NA, NA.BARS (I))
                      IF (ABOVE .EQ. NA_ABOVE (NA, NA.BARS (I))) THEN
                          DR = DR      ! DR = DR WAS NOT PRESENT.
                      ELSE
                          DR = -DR
                      ENDIF
                      ES = ECM * DR / NA.DMAX
                      FS = ESM * ES / 1000
                      IF (ES .LT. 0) THEN
                         IF (FS .LE. -FYD) FS = -FYD
                      ELSE
                         IF (FS .GT. FYD) THEN
                                 FS = FYD - GETG (NA, NA.BARS (I))
                                 ELSE
            FS = FS - GETG (NA, NA.BARS (I))
                                 ENDIF
                      ENDIF

                      NC = NC + AS * FS
                      MX = MX + AS * FS * NA.BARS (I).Y
                      MY = MY + AS * FS * NA.BARS (I).X
           ENDDO
END SUBROUTINE

! CALCUATE SECTION CAPACITY
SUBROUTINE CALCULATE_CAPACITY (NA, Q, NC, MX, MY)
   RECORD /NAXIS/ NA
   REAL Q, NC, MX, MY
           REAL X, Y
           LOGICAL USENA
           INTEGER I
   Q = NA.ANGLE
           ! ROTATE
           DO I = 1, NA.NCORNERS
                      X = NA.O_CORNERS (I).X
                      Y = NA.O_CORNERS (I).Y
                      NA.CORNERS (I).X = (X * COS (Q) + Y * SIN (Q))
                      NA.CORNERS (I).Y = (-X * SIN (Q) + Y * COS (Q))
           END DO
```

```
   DO I = 1, NA.NBARS
                  X = NA.O_BARS (I).X
                  Y = NA.O_BARS (I).Y
                  NA.BARS (I).X = (X * COS (Q) + Y * SIN (Q))
                  NA.BARS (I).Y = (-X * SIN (Q) + Y * COS (Q))

          ENDDO
   ! FURTHEST DISTANCES
          CALL FIND_FURTHEST (NA)
          ! CONCRETE
          USENA = .TRUE.
          NC = INTEGRATE (ENC, NA.CORNERS, NA.NCORNERS, NA, USENA)
          MX = INTEGRATE (EMX, NA.CORNERS, NA.NCORNERS, NA, USENA)
          MY = INTEGRATE (EMY, NA.CORNERS, NA.NCORNERS, NA, USENA)

          ! STEEL
          CALL BAR_CONTRIBUTION (NA, NC, MX, MY)

          ! ROTATE
          X =-MX! NEGATIVE IS ADDED
          Y = MY
          MX = (X * COS (-Q) + Y * SIN (-Q))
          MY = (-X * SIN (-Q) + Y * COS (-Q))

END SUBROUTINE

SUBROUTINE FORM_SHAPE (NA, H, T, DP)
   RECORD /NAXIS/ NA
   REAL H, T, DP, A, SX, SY
          INTEGER I
          RECORD /POINT/ CENTROID
          LOGICAL USENA

          ! CORNERS
          NA.NCORNERS =7
          NA.CORNERS (1).X = 0
          NA.CORNERS (1).Y = 0
          NA.CORNERS (2).X = H
          NA.CORNERS (2).Y = 0
          NA.CORNERS (3).X = H
          NA.CORNERS (3).Y = T
          NA.CORNERS (4).X = T
          NA.CORNERS (4).Y = T
          NA.CORNERS (5).X = T
          NA.CORNERS (5).Y = H
          NA.CORNERS (6).X = 0
          NA.CORNERS (6).Y = H
          NA.CORNERS (7).X = 0
          NA.CORNERS (7).Y = 0


          USENA = .FALSE.
```

```
        A = INTEGRATE (EAREA, NA.CORNERS, NA.NCORNERS, NA, USENA)
        SX = INTEGRATE (ESX, NA.CORNERS, NA.NCORNERS, NA, USENA)
        SY = INTEGRATE (ESY, NA.CORNERS, NA.NCORNERS, NA, USENA)
        CENTROID.X = SY / A
        CENTROID.Y = SX / A

        DO I = 1, NA.NCORNERS
                NA.CORNERS (I).X = NA.CORNERS (I).X - CENTROID.X
                NA.CORNERS (I).Y = NA.CORNERS (I).Y - CENTROID.Y
        END DO

        NA.NBARS = 8
        NA.BARS (1).X = NA.CORNERS (1).X + DP
        NA.BARS (1).Y = NA.CORNERS (1).Y + DP
        NA.BARS (2).X = NA.CORNERS (1).X + T - DP
        NA.BARS (2).Y = NA.CORNERS (1).Y + DP
        NA.BARS (3).X = NA.CORNERS (1).X + H - DP
        NA.BARS (3).Y = NA.CORNERS (1).Y + DP
        NA.BARS (4).X = NA.CORNERS (1).X + H - DP
        NA.BARS (4).Y = NA.CORNERS (1).Y + T - DP
        NA.BARS (5).X = NA.CORNERS (1).X + T - DP
        NA.BARS (5).Y = NA.CORNERS (1).Y + T - DP
        NA.BARS (6).X = NA.CORNERS (1).X + T - DP
        NA.BARS (6).Y = NA.CORNERS (1).Y + H - DP
        NA.BARS (7).X = NA.CORNERS (1).X + DP
        NA.BARS (7).Y = NA.CORNERS (1).Y + H - DP
        NA.BARS (8).X = NA.CORNERS (1).X + DP
        NA.BARS (8).Y = NA.CORNERS (1).Y + T - DP

        DO I = 1, NA.NCORNERS
                NA.O_CORNERS (I) = NA.CORNERS (I)
        ENDDO
        DO I = 1, NA.NBARS
                NA.O_BARS (I) = NA.BARS (I)
        ENDDO
END SUBROUTINE

! SUM OF TWO COORDINATE POINTS
FUNCTION INT_SUM (C1, C2)
        RECORD /POINT/C1, C2, C, INT_SUM
        C.X = C1.X + C2.X
        C.Y = C1.Y + C2.Y
        INT_SUM = C
END FUNCTION

! PRODUCT OF A COORDINATE POINT WITH A CONSTANT
FUNCTION INT_PRODUCT (C1, K)
        RECORD /POINT/ C1, C, INT_PRODUCT
        REAL K
        C.X = C1.X * K
        C.Y = C1.Y * K
        INT_PRODUCT = C
```

```
END FUNCTION

! DIFFERENCE BETWEEN TWO COORDINATE POINTS
FUNCTION INT_DIFFERENCE (C1, C2)
        RECORD /POINT/C1, C2, C, INT_DIFFERENCE
         REAL K
        K = -1.00
        C = INT_PRODUCT (C2, K)
   I    NT_DIFFERENCE = INT_SUM (C1, C)
END FUNCTION

! SLOPE OF THE LINE CONNECTING POINTS
FUNCTION INT_SLOPE (C1, C2)
        REAL INT_SLOPE
        RECORD /POINT/ C1, C2
        NT_SLOPE = (C2.Y-C1.Y)/ (C2.X-C1.X)
END FUNCTION

! DOT PRODUCT OF TWO COORDIANTE POINTS
FUNCTION INT_DOT_PRODUCT (C1, C2)
        REAL INT_DOT_PRODUCT
        RECORD /POINT/ C1, C2
        INT_DOT_PRODUCT = C1.X * C2.X + C1.Y * C2.Y
END FUNCTION

! THE INVERSE OF A NUMBER
FUNCTION INT_INVERSE (K)
        REAL K, INT_INVERSE
        INT_INVERSE = 1/K
END FUNCTION

! UNIT VECTOR
FUNCTION INT_UNIT_VECTOR(C)
         RECORD /POINT/ C, INT_UNIT_VECTOR
        REAL D, MAG
        MAG = INT_DOT_PRODUCT(C, C)
        D = INT_INVERSE (SQRT (MAG))
        INT_UNIT_VECTOR = INT_PRODUCT(C, D)
END FUNCTION

! PROJECTION OF THE LINE
FUNCTION INT_PROJECTION(C, C1, C2)
        REAL DIS, LO
        RECORD /POINT/ E, V, R, C1, C2, C, INT_PROJECTION
        DIS = SQRT ((C1.X - C2.X) **2 + (C1.Y - C2.Y) **2)
        E = INT_DIFFERENCE (C2, C1)
        E = INT_UNIT_VECTOR (E)
        V = INT_DIFFERENCE(C, C1)
        LO = INT_DOT_PRODUCT (V, E)
        INT_PROJECTION = INT_SUM (C1, INT_PRODUCT (INT_DIFFERENCE (C2, C1), (LO/DIS)))
END FUNCTION
```

```
! POINTS OF INTERSECTIONS
FUNCTION INT_INTERSECT (C1, C2, C3, C4, IC)
        LOGICAL INT_INTERSECT
        RECORD /POINT/ C1, C2, C3, C4, IC
        REAL SLOPE1, SLOPE2, X, Y
        IF (C1.X.NE.C2.X) THEN
        SLOPE1 = INT_SLOPE (C1, C2)
        END IF
        IF (C3.X.NE.C4.X) THEN
        SLOPE2 = INT_SLOPE (C3, C4)
        END IF
        IF (C1.X.EQ.C2.X) THEN
                X = C1.X;
                Y = SLOPE2 * X + (C3.Y - SLOPE2 * C3.X);
        ELSEIF (C3.X.EQ.C4.X) THEN
                X = C3.X;
                Y = SLOPE1 * X + (C1.Y - SLOPE1 * C1.X);
        ELSE
        IF (SLOPE1.EQ.SLOPE2) THEN
                INT_INTERSECT = .FALSE.
        RETURN
        ENDIF
                X = ((C3.Y - SLOPE2 * C3.X) - (C1.Y - SLOPE1 * C1.X))/ (SLOPE1 - SLOPE2);
                Y = SLOPE1 * X + (C1.Y - SLOPE1 * C1.X);
        ENDIF

  IF    (((X .GE. C1.X .AND. X .LE. C2.X) .OR. (X .GE. C2.X .AND. X .LE. C1.X)) .AND. &
        ((Y .GE. C1.Y .AND. Y .LE. C2.Y) .OR. (Y .GE. C2.Y .AND. Y .LE. C1.Y)) .AND. &
        ((X .GE. C3.X .AND. X .LE. C4.X) .OR. (X .GE. C4.X .AND. X .LE. C3.X)) .AND. &
        ((Y .GE. C3.Y .AND. Y .LE. C4.Y) .OR. (Y .GE. C4.Y .AND. Y .LE. C3.Y)) ) THEN
    IC.X = X;
                IC.Y = Y;
                INT_INTERSECT = .TRUE.;
                RETURN
                END IF
                INT_INTERSECT = .FALSE.;
END FUNCTION

FUNCTION NA_ABOVE (NA, C)
  LOGICAL NA_ABOVE
        RECORD /NAXIS/ NA
        RECORD /POINT/ C, /POINT/ IC
        IC = INT_DIFFERENCE (NA.C2, NA.C1)
        IC = INT_PRODUCT(IC, ((C.X - NA.C1.X) / (NA.C2.X - NA.C1.X)))
        IC = INT_SUM(IC, NA.C1)
        NA_ABOVE = (C.Y .GT. IC.Y)
END FUNCTION

FUNCTION CALCD (NA, C)
        RECORD /NAXIS/ NA
        REAL D, CALCD
        RECORD /POINT/ C, CR
```

```
        CR = INT_PROJECTION(C, NA.C1, NA.C2)
        CALCD = SQRT ((CR.X-C.X) **2+ (CR.Y-C.Y) **2)
END FUNCTION

FUNCTION GETF (NA, EC)
        RECORD /NAXIS/ NA
        REAL EC, GETF
        IF (EC .GT. 2) EC = 2
        GETF = EC * (1 - EC / 4) * FCD
END FUNCTION

FUNCTION GETECM (NA)
        RECORD /NAXIS/ NA
        REAL KX, ECM, GETECM
        KX = NA.DMAX / NA.D
        IF (KX .LT. (3.5 / 13.5)) THEN
        ECM = 10 * NA.DMAX / (NA.D - NA.DMAX)
        ELSE
        IF (KX .LT. 1) THEN
        ECM = 3.5
        ELSE
        ECM = 2.0 / (1 - (3 * NA.D) / (7 * NA.DMAX))
        ENDIF
        END IF
        GETECM = ECM
END FUNCTION

FUNCTION GETG (NA, C)
        RECORD /NAXIS/ NA
        RECORD /POINT/ C
        REAL D, EPHI, ECM, SIGMA, GETG
        ECM = GETECM (NA)
        D = CALCD (NA, C)
        EPHI = (D / NA.DMAX) * ECM
        SIGMA = GETF (NA, EPHI)
        GETG = SIGMA
END FUNCTION

FUNCTION INTEGRATE (TYP, C, N, NA, USENA)
        INTEGER TYP, N, I, L, R, S
        LOGICAL ABOVE, USENA
        RECORD /POINT/ C (30), IC, PREV_C, RC
        RECORD /NAXIS/: NA
        REAL  ALPHA,BETA,E,EW,H,G,Y,TOTAL,LTOTAL,GS,INTEGRATE
         IF (USENA) ABOVE = NA_ABOVE (NA, NA.ABOVE_C)
        R = RA (TYP)
        S = SA (TYP)
        TOTAL = 0
        PREV_C = C (1)
        DO I = 2, N
        LTOTAL = 0
                IC = C (I)
```

```
            IF (USENA) THEN
               IF (INT_INTERSECT (NA.C1, NA.C2, C (I - 1), C (I), IC)) THEN
                     IF (ABOVE == NA_ABOVE (NA, C (I))) THEN
                           PREV_C = IC
                           IC = C (I)
                     ENDIF
                  ELSE
                     IF (ABOVE /= NA_ABOVE (NA, C (I))) THEN
                           PREV_C = IC
                           GOTO 10
                     ENDIF
               ENDIF
            ENDIF

            IF (IC.Y.EQ.C (I - 1).Y) THEN
                  PREV_C = IC
                  GOTO 10
            ENDIF
            BETA = (IC.X - PREV_C.X) / (IC.Y - PREV_C.Y)
            ALPHA = PREV_C.X - BETA * PREV_C.Y
            H = (IC.Y + PREV_C.Y) / 2
            G = (IC.Y - PREV_C.Y) / 2
            ! GAUSS INTEGRATION
            DO L = 1, RULE
                  E = GAUSS_POINTS (RULE, L)
                  EW = GAUSS_WEIGHTS (RULE, L)
                  Y = H + G * E
                  RC.X = ALPHA + BETA * Y
                  RC.Y = Y
                  GS = ((ALPHA + BETA * Y) ** (R + 1)) * (Y**S)
                  GS = GS * G
                  IF (USENA .AND. (TYP .LE. EMY)) GS = GS * GETG (NA, RC)
                  LTOTAL = LTOTAL + EW * GS
            ENDDO
            TOTAL = TOTAL + (1.0 / (R + 1)) * LTOTAL
        PREV_C = IC
! END LOOP
10 CONTINUE
   ENDDO
       INTEGRATE = TOTAL
END FUNCTION
END MODULE
```