



**EVALUATION OF HAMMERHEAD PIER CAP BRIDGE DESIGN
USING STRUT AND TIE MODEL**

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Abstract

The design of D-regions of reinforced concrete beams in the past was based on rules of thumb and past experience. Strut-and-Tie Modeling (STM) is an accepted design method to deal with these D-regions and also for B-regions. It simply idealizes the entire structures, or portions of the structure as a truss where steel bar is placed according to the locations of the tension members.

This report is written to investigate the use of strut-and-tie modeling with AASHTO LRFD provisions to design bridge structure which includes the girder, hammerhead pier caps & pier. A brief background on STM, STM as Standard Design Provision, Euro Code 2 Provisions for STM & a guide to design with STM using AASHTO LRFD is discussed. Design examples of a bridge structure of step-by-step are performed using STM with AASHTO LRFD section 5.6.3 provisions and compared with conventional method. Design example of hammerhead pier cap is also performed according to EURO Code provision to show the difference with AASHTO LRFD provision for STM. The strength of struts and nodes according to AASHTO was checked in this report. Conclusions and Recommendations based on the findings of this report are listed.

The important findings provided with this work are a refined procedure to guide designers using STM. The findings of this work show that STM with AASHTO LRFD & EURO Code 2 provisions can be an applicable design method for both the D-region & B-region of structures. However certain criteria need to be met and the engineer needs to give special consideration to the geometry of their models, crack control and concrete strength.

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1. INTRODUCTION

1.1. Introduction

Strut-and-tie modeling is an analysis and design tool for reinforced concrete elements in which it may be assumed that internal stresses are transferred through a truss mechanism. The tensile ties and compressive struts serve as truss members connected by nodal zones. The internal truss, idealized by the strut-and-tie model, implicitly account for the distribution of both flexure and shear.

The strut-and-tie method has been demonstrated to be a powerful tool for the modeling and design of structural concrete. In the conceptual and the structural design stage, the method allows study of different resistant models and evaluation of the most suitable one. In the analysis stage, this method enables the stresses in the structure to be adequately evaluated, allowing calculation of the amount of reinforcement and verification of the stress level of concrete. Additionally, it is especially useful in identifying the critical zones and organizing the reinforcement layout.

Every structural member can be subdivided in so-called B-regions and D-regions. B stands for BERNOLLI and corresponds to those parts of the member where the assumption of BERNOLLI can be accepted in which beam theory applies (planar sections remain plane after load has applied). D stands for “Discontinuity” and corresponds to the regions where the assumption of BERNOLLI is no longer valid; this is typically the region around: geometrical discontinuity: changes in cross-section, openings, nodes in frames, connections between girders and beams, etc.;& static discontinuity: isolated loads, supports, temperature changes, anchorage of prestressing tendons, etc.

Strut-and-tie modeling (STM) is a versatile, lower-bound (i.e., conservative) design method for reinforced concrete structural components. STM is most commonly used to design regions of structural components disturbed by a load and/or geometric discontinuity. Load and geometric discontinuities cause a nonlinear distribution of strains to develop within the surrounding region. As a result, plane sections can no longer be assumed to remain plane within the region disturbed by the discontinuity. Sectional design methodologies are predicated on traditional beam theory, including the assumption that plane sections remain plane, and are not appropriate for application

to disturbed regions, or D-regions. The design of D-regions must therefore proceed on a regional, rather than a sectional, basis. STM provides the means by which this goal can be accomplished.

Therefore, a procedure for STM development according to AASHTO LRFD design method is presented along with an example using STM for the design of girder, hammerhead pier caps & pier. The purpose of the work presented here is to show how STM represent the flow of forces through a member and how it is an appropriate design method.

1.2. Statement of the Problem

Strut and tie model is considered a rational and consistent basis for designing cracked reinforced concrete structure. It is mainly applied to the zones where the beam theory does not apply, such as geometrical discontinuities, loading points, deep beams and corbels. In strut-and-tie modeling, the internal stresses are transferred through a truss mechanism. The tensile ties and compressive struts serve as truss members connected by nodal zones. The advantages have been thrust into the back ground by several recent developments of design equations based on truss models. In most instances, hammerhead pier cap can be defined as deep reinforced concrete members and therefore, should be designed using the strut-and-tie modeling approach. However, most bridge engineers do not have a broad knowledge on the STM due to the unfamiliarity with the design procedure. Therefore it is likely that, with the formulation of a well-defined strut-and-tie modeling procedure, having the awareness will become more comfortable with the design method and therefore, employ the method more often and consistently. The other problem is using STM without assessing best modeling may not be a better design than the elastic stress analysis. Since the concept of STM is nearly new there is lack of experience in design of concrete structure using STM in our country.

1.3. Objective of the Thesis

The specific objectives of the study are:

- To develop a uniform design procedure for employing strut-and-tie modeling for girder, hammerhead pier cap & pier and compare with the elastic stress analysis for both B and D region.
- To assess best STM for Hammerhead Pier Cap.

1.4. Methodology of the Thesis

The method which will be employed to achieve the objectives of the research in general is

- Literature Review
- Design of hammerhead pier cap bridge using STM and compare with Linear Elastic Analysis Method
- Writing up
 - Analyzing and evaluating the data gained from the above method.
 - It also involves compiling, write up draft and final thesis.

1.5. Scope of Study

In this study T-girder, hammerhead pier caps & pier was designed using the strut-and-tie modeling procedure and the results compared to the results of the sectional design method. By comparing the results, the reduction or increase in the flexural steel and the shear steel can be quantified.

To apply strut and tie method in design of reinforced concrete as the first principle as well as the simplified approach adapted in code of practices.

In addition specific checks on the level of concrete stresses in the member are introduced to ensure sufficient ductile behavior and control of diagonal crack widths at service load level.

1.6. The Importance of the Study

The importance of this study can be summarized as follow;

- The development of strut and tie model theory in analysis and design of reinforced concrete element would be compiled and reported:
- The analysis of strut and tie method is compared with other established method hence the importance of the method is highlighted: and
- The design examples using strut and tie method are illustrated to enhance the understanding of the application of the method and it is also useful to be used as reference in the application.

2. LITERATURE REVIEW

2.1. Introduction

When designing a D-region using STM, the complex flow of forces through a structural component is first simplified into a truss model, known as a strut-and-tie model. A basic two-dimensional strut-and-tie model consists of concrete compression members (i.e., struts) and steel tension members (i.e., ties) interconnected within a single plane. Complexity introduced by loading, boundary conditions, and/or component geometry may occasionally necessitate the development of a three-dimensional strut-and-tie model. The completed model is used by the designer to proportion and anchor the primary reinforcement, and ensure that the concrete has sufficient strength to resist the applied loads. Strut-and-tie modeling can be applied to any structural component with any loading and support conditions. This versatility of STM is a source of both clarity and confusion.

The conventional procedure for designing reinforced concrete structures can be divided into three stages: (a) selecting member dimensions, (b) determining the quantity, placing and anchorage details of reinforcement and ensuring ultimate strength criteria is satisfied and, (c) satisfying member deformation under service loading conditions. Traditionally, STM was utilized only in the aforementioned second design phase (b) to ensure force equilibrium [ACI, 1997]. Using data acquired from testing large-scale bridge structural systems, the study reported herein expands the application of STM to performance evaluation by predicting nonlinear force displacement response using properly formulated models.

As STMs can be formulated for both B- and D-region structural members, the expanded application of this modeling technique implies that the three previously described design phases can be accomplished using a single model. This may ultimately result in a consistent standard for the dimensioning and reinforcement detailing of structural systems.

Later, Schlaich, et al worked to combined individual research conducted on various reinforced concrete elements in such a fashion that Strut-and-Tie modeling could be used for entire structure.

Regardless of STM adopted, the following design criteria must be met

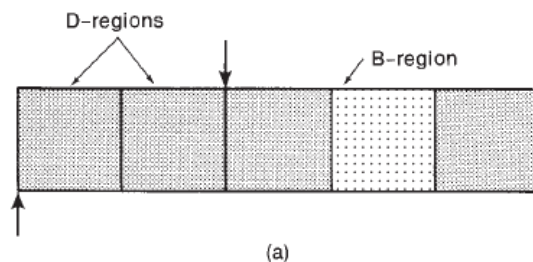
- Limits on bearing stresses and on compressive stresses in struts.
- Satisfactory anchorage and careful detailing of tension tie reinforcements
- Critical examination of nodal zones to determine their maximum capacities
- Provision of adequate crack control reinforcement throughout, to ensure the redistribution of internal stresses after cracking of concrete.

Strut-and-tie modeling (STM) is an accepted design method by AASHTO to deal with D-regions since it simplifies the non-linear stresses into a truss model through a series of struts and ties. A procedure for STM development according to AASHTO LRFD design method is presented along with an example using STM for the design of a Hammerhead Bent Cap Bridge Structure.

2.2. Overview of Strut and Tie Model

Strut-and-Tie Method (STM) has been used for several years in Europe and had been included in the AASHTO LRFD [5] Bridge Specification since 1994, it is a new concept for many structural engineers, recommendation for the use of STM to design reinforced concrete members were discussed by previous researchers. In selecting the appropriate design approach, focused on understanding the internal distribution of forces in a reinforced concrete structure and have defined two specific regions; B-Regions and D-Regions.

St. Venant's principle suggests that the localized effect of a disturbance dies out by about one member-depth from the point of the disturbance. On this basis, D-regions are assumed to extend one member-depth each way from the discontinuity. This principle is conceptual and not precise. However, it serves as a quantitative guide in selecting the dimensions of D-regions.



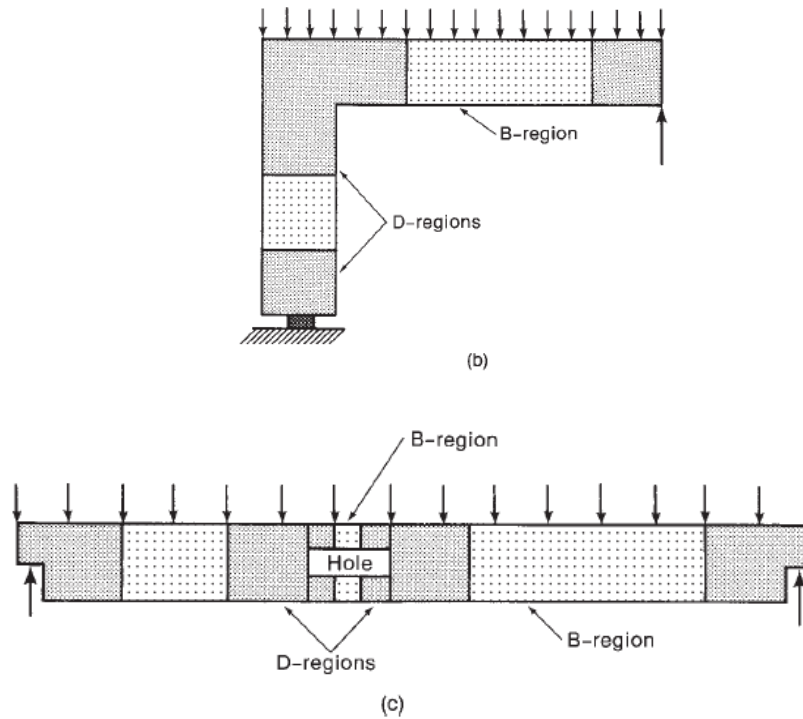


Figure 2-1 : Region and D- Region

A strut-and-tie model for a deep beam is shown in figure 2.2. It consists of two concrete Compressive struts, longitudinal reinforcement serving as a tension tie, and joints referred to as nodes. The concrete around a node is called a nodal zone. The nodal zones transfer the forces from the inclined struts to other struts, to ties and to the reactions.

A strut-and-tie model is a model of a portion of the structure that satisfies the following:

- (a) It embodies a system of forces that is in equilibrium with a given set of loads, and
- (b) The factored-member forces at every section in the struts, ties, and nodal zones do not exceed the corresponding design member strengths for the same sections.

The lower-bound theorem of plasticity states that the capacity of a system of members, supports, and applied forces that satisfy both (a) and (b) is a lower bound on the strength of the actual structure. For the lower-bound theorem to apply, (c) The structure must have sufficient ductility

to make the transition from elastic behavior to plastic behavior that redistributes the factored internal forces into a set of forces that satisfy items (a) and (b).

The combination of factored loads acting on the structure and the distribution of factored internal forces is a lower bound on the strength of the structure, provided that no element is loaded or deformed beyond its capacity. Strut-and-tie models should be chosen so that the internal forces in the struts, ties, and nodal zones are somewhere between the elastic distribution and a fully plastic set of internal forces.

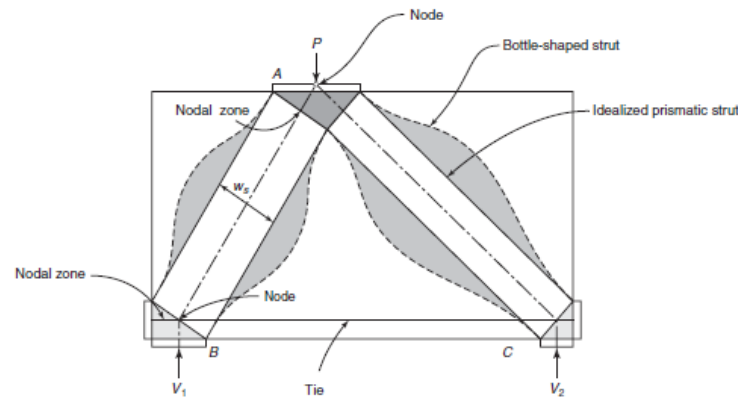


Figure 2-2 : Strut and Tie Model for deep beam

2.3. General Strength of Truss Members

Rules in Selecting Strut-and-Tie Models

In designing using the Strut-and-Tie Method, a Strut-and-Tie Model representing idealized load transfer mechanism in the D-Region under consideration is to be selected. The selected Strut-and-Tie Model should consist of Struts, Ties, and Nodes and has to be in equilibrium with the forces acting on the D-Region. The finite dimensions of Strut-and-Tie Model components, representing the stress fields of Struts, Ties, and Nodes should be considered. Tie stress fields can cross Strut stress fields. To avoid severe strain incompatibility between Struts and Ties, the angle between a Strut and a Tie framing into a Node cannot be smaller than 25 degrees. The Strut-and-Tie Model components must have sufficient capacity to resist the force demand such that

$$P_r = \Phi P_n \quad (2.1)$$

Where:

Φ = resistance factor for tension or compression specified in article 5.5.4.2, as appropriate ($\Phi=0.7$, AASHTO 5.6.3.2-1)

P_r = nominal resistance of Strut, Tie, or Node, and

P_n = factored resistance of the Strut, Tie, or Node.

As previously stated, the truss model is comprised of tension ties, compression struts, and nodal zones. For the adequate design of the reinforced concrete member, the elements of the truss model must be sized. The following sections present the general strength of the tensile ties, compressive struts, and nodal zones.

Strength of Compressive Strut

Struts are the compression members of a strut-and-tie model and represent concrete stress fields whose principal compressive stresses are predominantly along the centerline of the strut. The idealized shape of concrete stress field surrounding a strut in a plane (2-D) member, however, can be prismatic figure (a), bottle-shaped Figure (b), or fan-shaped Figure(c). Struts can be strengthened by steel reinforcement, and if so, they are termed reinforced struts.

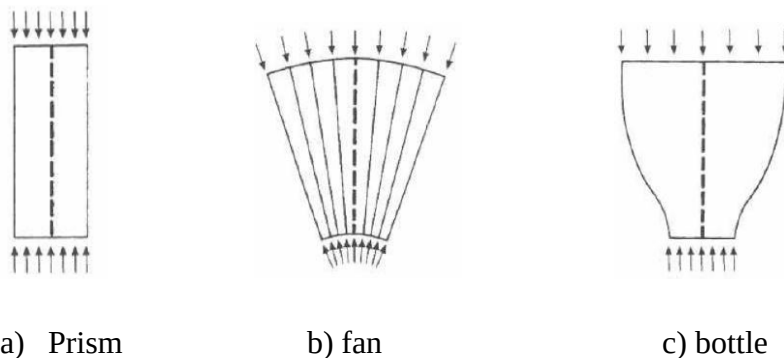


Figure 2-3 : Basic Type of Struts in a 2-D Member: (a) Prismatic (b) Bottle-Shaped

(c) Fan-Shaped

In the design using strut-and-tie models, it is necessary to check that the crushing of the compressive strut does not occur. Struts are the compression members of a Strut-and-Tie Model and represent concrete stress fields represent one dimensional stress fields, which should not exceed the compressive strength of the concrete. Cracking may develop in bottle shaped elements if no crack control reinforcement is used.

AASHTO uses the following formula to limit the compressive stress in the strut the nominal resistance of an unreinforced compressive Strut shall be taken as

$$P_n = f_{cu} A_c \quad (2.2)$$

Where:

P_n = nominal strength of compressive strut (N)

f_{cu} = limiting compressive stress as specified in article 5.6.3.3.3(MPa) and

A_{cs} = effective cross sectional area of strut as specified in article 5.6.3.3.2(mm²)

$$f_{cu} = f'_c / (0.8 + 170\varepsilon_1) \leq 0.85f'_c \quad (2.3)$$

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \alpha_s \quad (2.4)$$

α_s = the smallest angle between the strut and longitudinal ties that run through the strut 45°

ε_s = strain in the steel running through the strut

ε_1 = strain in the steel tie perpendicular to the strut

ASSHTO also recommends for the required amount of crack control reinforcement as follows.,

- The spacing of the bars shall not exceed 300mm
- The ratio of reinforcement area to gross concrete area shall not be less than 0.003 in each direction.

Strength of Tensile Ties

In order to simplify the equilibrium analysis of a strut-and tie model it is often convenient to combine a number of separate and parallel reinforcing bars and represent them as a single tie.

Tension tie reinforcement shall be anchored to the nodal zones by specified embedment length, hooks, or mechanical anchorages. The tension force shall be developed at the inner face of the nodal zone.

The nominal resistance of a tension tie in N shall be taken as;

$$P_n = f_y A_{st} + A_{ps} [f_{pe} + f_y] \quad (2.5)$$

Where:

A_{st} = total area of longitudinal mild steel reinforcement in the tie(mm^2)

A_{ps} =area of prestressing steel (mm^2)

f_y = yield strength of mild steel longitudinal reinforcement(MPa)

f_{pe} = stress in prestressing steel due to prestress after losses(MPa)

Care must be exercised in the strut-and-tie as the real distribution of bars, of the tensile reinforcement and also in the selection of how to distribute and anchor the reinforcement. This becomes apparent due to the ability of the joint or nodal zone to transfer forces between the strut-and-tie is dependent on the surface area of the reinforcement, the height over which it is distributed, the length of the node, and the type of anchorage method that is employed. ACI and AASHTO have provisions, which require the tie reinforcement be distributed over such a height that if the tie were anchored on the far side of the node that the nodal stresses limit value will not be exceeded (Kuchma and Tjhin, 2001).

Strength of Nodal Zones

Nodal zones are assumed to fail by crushing. Anchorage of the tension ties is a matter of design consideration. If a tension tie is anchored in a nodal zone there is a strain incompatibility

between the tensile strains in the bars and the compressive strain in the concrete of the node. This tends to weaken the nodal zone. ASASHTO limits the concrete compressive stress in the node region shall not exceed:

For node regions bounded by compressive struts and bearing areas: $0.85\Phi f'_c$

For node regions anchoring a one-direction tension tie: $0.75\Phi f'_c$

For node regions anchoring tension ties in more than one direction: $0.65\Phi f'_c$

Where:

Φ = the resistance factor for bearing on concrete as specified in article 5.5.4.2

2.4. Shear Concerns in Strut-And-Tie Models

Truss analogy assumes that a pattern of parallel inclined crack forms in region of high shear, indicated in figure 2.4 (Inclined cracking) and that the concrete in between adjacent inclined cracks can carry an inclined compressive force, and hence act like a diagonal strut. This suggests that if transverse stirrups are provided at a regular interval along the beam, truss like action can be achieved whereby the main reinforcement provides longitudinal tension chord and the compressive concrete on the other side of the beam the longitudinal compressive chord. In the analogous truss shown in figure 2.5 (Truss likes action), the transverse reinforcing steel is vertical but clearly truss action can also be achieved with inclined steel stirrups.

A feature of truss method is that the forces in the stirrups and the diagonal strut can be determined using simple statics. For example, in figure 2.6 (analogous truss) the strut is inclined at θ degrees while stirrup is vertical, so that the shear force acting in a cross-section is carried by the vertical component of the diagonal compressive force D : $D\sin\theta=V$.

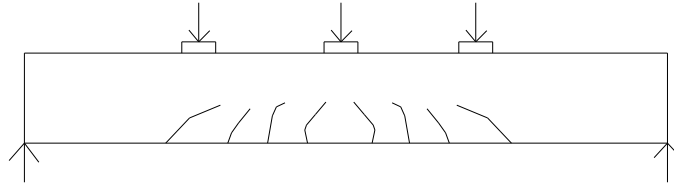


Figure 2-4 : Inclined cracking

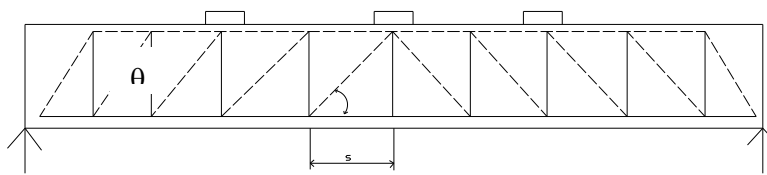


Figure 2-5 : Truss like action

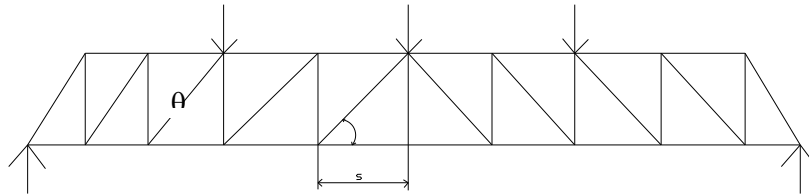


Figure 2-6 : Analogous truss

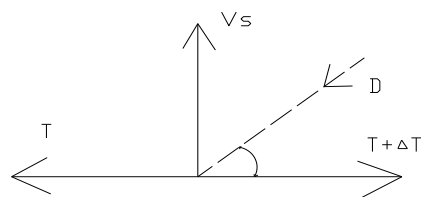


Figure 2-7 : Truss analogy

By considering the joint in figure 2.7(Truss analogy), we can see that the force V_s in the stirrup is equal to the shear force. With the stirrup spacing s and the beam depth d , the number of stirrup n is determine by their spacing s and the angle θ .

$$n = d / s \tan \theta \quad (2.6)$$

In common case, the inclined crack cut n stirrups and these together carry the applied shear force.

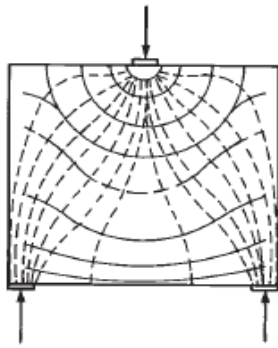
2.5. Layout of Strut and Tie

The development of a strut-and-tie model is typically performed in a two-step process. First, the geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure, elastic stress distribution, based on observed test specimen & load path. Second, the STM is analyzed to determine the forces in the struts and ties.

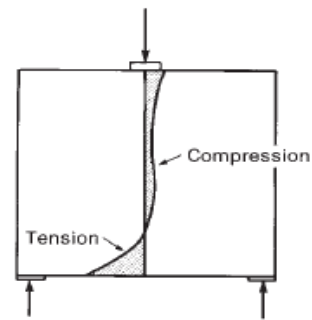
In the development of the strut-and-tie model, the placement of the struts and ties should be representative of the elastic flow of forces within the structural component (Bergmeister et al., 1993; Schlaich et al., 1987). The designer has a few options for determining the proper orientation of the struts and tie: (1) use the locations of the applied loads and boundary forces to develop a logical load path represented by the struts and ties, (2) follow the known cracking pattern of the structure being designed if such information is available (Macgregor and Wight, 2005), or (3) perform a linear elastic finite element analysis to visualize the flow of forces in the component and place the struts and ties accordingly.

2.5.1. Based on Elastic Stress Distribution

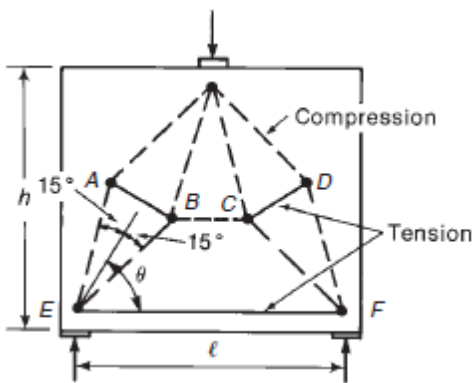
From an elastic analysis, such as finite element analysis, it is possible to derive the stress trajectories in uncracked B- and D- region such as shown in figure 2.8(a) for deep beam.



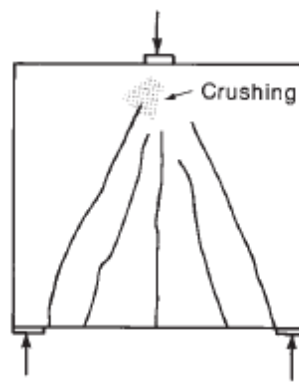
a) Stress trajectories



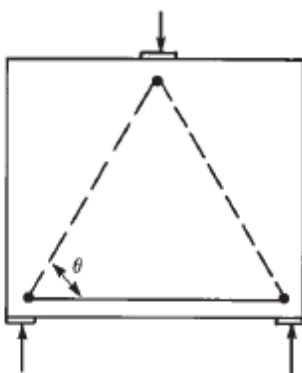
b) distribution of theoretical horizontal elastic stresses at mid span



c) Truss model



d) crack pattern in test



e) Simplified truss

Figure 2-8 : Single-span deep beam supporting a concentrated load.

Factors Affecting the Choice of Strut-and-Tie Models

Guidelines for the choice of strut-and-tie models include the following areas.

Equilibrium

1. The strut-and-tie model must be in equilibrium with the loads. There must be a clearly laid out load path.

Direction of Struts and Ties

2. The strut-and-tie model for a simply supported beam with unsymmetrical applied, concentrated loads consists of an arch, made of straight line segments or a hanging cable, that has the same shape as the bending-moment diagram for the loaded beam as shown in Figure 2.9. This is also true for uniformly loaded beams, except that the moment diagram and the strut-and-tie model have parabolic sections.

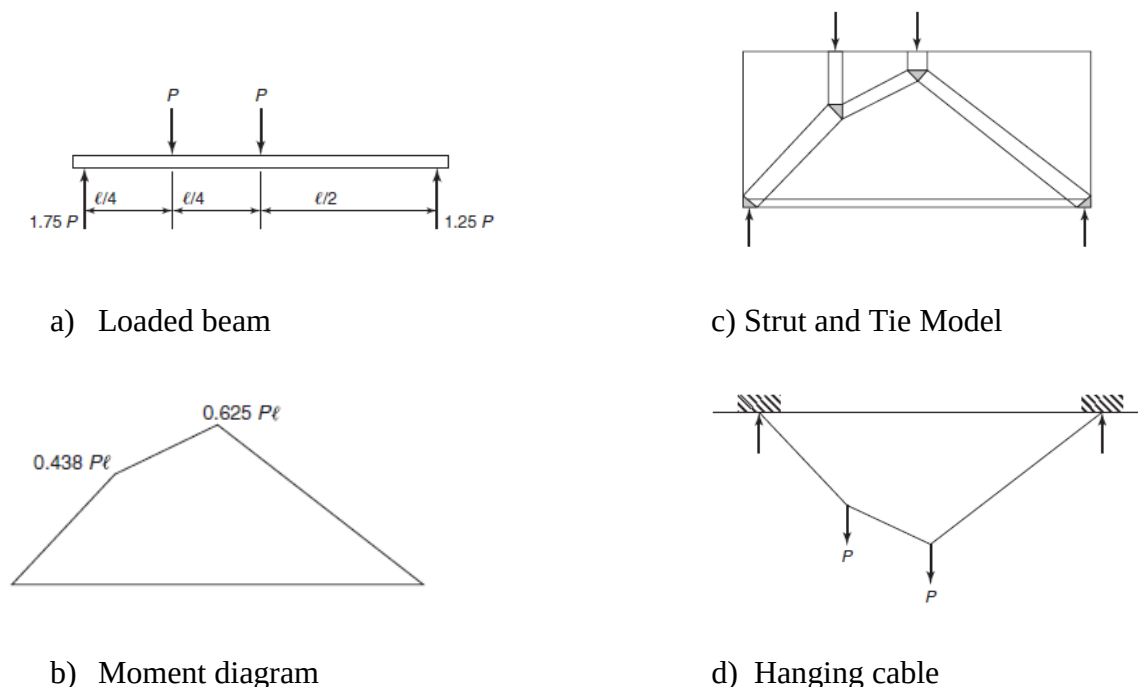


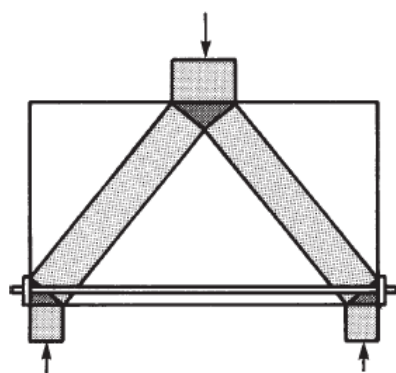
Figure 2-9 : Statically equivalence of various types of structures.

3. The strut-and-tie model should represent a realistic flow of forces from the loads through the D-region to the reactions. Frequently this can be determined by observation. From an elastic stress analysis, such as a finite element analysis, it is possible to derive the stress trajectories in an uncracked D-region, as shown in figure 2.8 (a) for a deep beam. Principal compression stresses act parallel to the dashed lines, which are known as compressive stress trajectories. Principal tensile stresses act parallel to the solid lines, which are called tensile stress trajectories. Such a diagram shows the flow of internal forces and is a useful, but by no means an essential step in laying out a strut-and-tie model. The compressive struts should roughly follow the direction of the compressive stress trajectories, as shown by the refined and simple strut-and-tie models in figure 2.8(c) and (e). Generally, the strut direction should be within $\pm 15^\circ$ of the direction of the compressive stress trajectories.

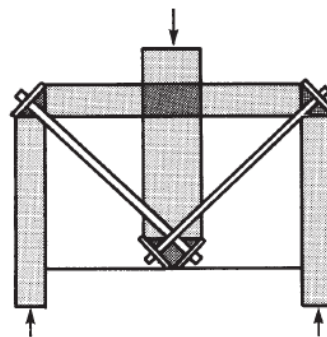
Because a tie consists of a finite arrangement of reinforcing bars which usually are placed orthogonally in the member, there is less restriction on the conformance of ties with the tensile stress trajectories. However, they should be in the general direction of the tension stress trajectories.

4. Ties can cross struts.

5. Struts cannot cross or overlap, as shown in figure 2.10(b), because the width of the individual struts have been calculated assuming they are stressed to the maximum.



a) Correct Strut and Tie Model



b) incorrect Strut and Tie Model

Figure 2-10 : Suitable and unsuitable strut-and-tie models.

6. It generally is assumed that the structure will have enough plastic deformation capacity to adapt to the directions of the struts and ties chosen in design if they are within of the elastic stress trajectories. The crack-control reinforcement is intended to allow the load redistribution needed to accommodate this change in angles.

Ties

7. In addition to generally corresponding to the tensile stress trajectories, ties should be located to give a practical reinforcement layout. Wherever possible, the reinforcement should involve groups of orthogonal bars which are straight, except for hooks needed to anchor the bars.

8. If photographs of test specimens are available, the crack pattern may assist one in selecting the best strut-and-tie model.

Load-Spreading Regions

9. Elastic analyses of members of width b subjected to in-plane loads applied to one edge show that the load-spreading angle is primarily a function of the ratio of the width of the load plate, a , to the width of the member, b . A strut slope of 2-to-1 (parallel to the axis of the load-to-perpendicular to the axis) is conservative for a wide range of cases.

10. Angle θ , between the struts and attached ties at a node, as shown in figure 2.8 (c), should be large (on the order of 45°) and never less than the 25° specified in ACI Code Section A.2.5. The size of compression struts is sensitive to the angle between the strut and the reinforcement in a tie.

Minimum Steel Content

11. The loads will try to follow the path involving the least forces and deformations. Because the tensile ties are more deformable than the compression struts, the model with the least and shortest ties is the best. Thus the strut-and-tie model in figure 2.10(a) is a better model than the

one in figure 2.10(b) because figure 2.10 (a) more closely approaches the elastic stress trajectories in figure 2.8(a).

Schlaich et al. proposes the following criterion for guidance in selecting a good model:

$$SF_i L_i \varepsilon_{mi} = \text{minimum} \quad (2.7)$$

Where:

F_i , L_i and ε_{mi} are the force, length, and mean strain in strut or tie i , respectively. Because the strains in the concrete are small, the struts can be ignored in the summation.

The above formula helps in eliminating the less suitable models but do not give the final solution. For example, both models in figure 2.11 form a stable truss. Their member forces are determined in the usual way for both cases. Not only figure 2.11a uses less tie reinforcement but also reasonable orientation of the ties compared to model figure. 2.11b. Moreover, applying the formula shows the same result in selecting a model, figure 2.11a. In special cases, models with higher steel are selected in order to have practical layout. The above criterion helps in eliminating less desirable models, figure below.

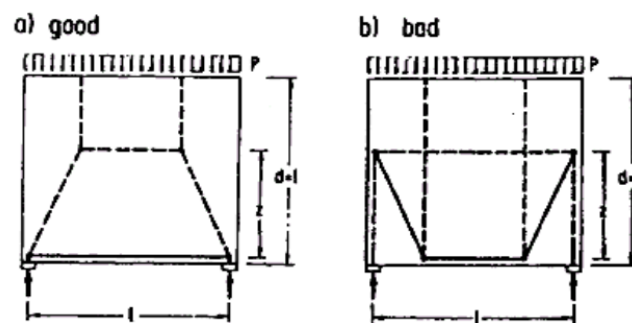


Figure 2-11 : Trial model for deep beam supporting uniform load a) good model with minimum tie b) bad model with considerable tie

Suitable Strut-and-Tie Layouts

12. The finite widths of struts and ties must be considered. The axis of a strut representing the compression zone in a deep flexural member should be located about $a/2$ from the compression

face of the beam where a is the depth of the rectangular stress block, as shown in figure 2.12. Similarly, if hydrostatic nodal zones are used, the axis of a tension tie should be about $a/2$ from the tensile face of the beam. One of the first steps in modeling a beam-like member is to locate the nodes in the strut-and-tie model. This can be done by estimating values for $a/2 \cdot f_{ce} = 0.85\beta_c$.

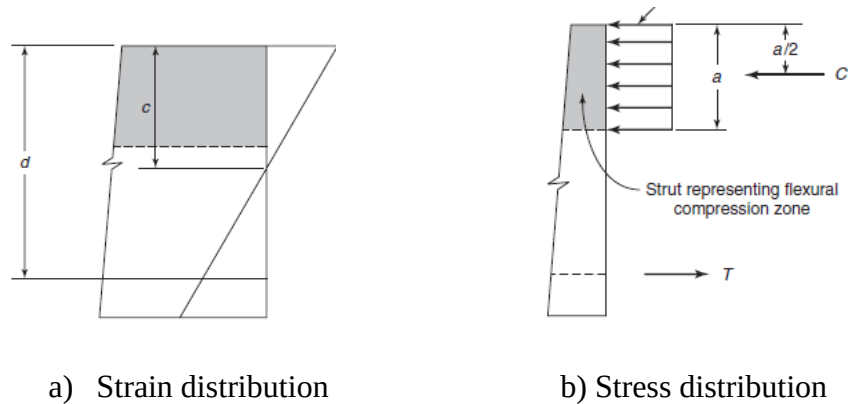


Figure 2-12 : strut representing the compression stress block in a beam

13. Subdividing a nodal zone, assuming each part of the nodal zone can be assigned to a particular force or reaction, makes the truss easier to lay out.
14. Sometimes a better representation of the real stress flow is obtained by adding two possible simple models, each of which is in equilibrium with a part of the applied load provided the struts do not overlap or cross.

2.5.2. Based on Observed Test Specimen

From the observed test specimens, the crack pattern may assist one in selecting the best strut and tie model. Figure 2.8(d) shows the crack pattern in a deep beam. Figure 2.8(c) and 2.8(e) show possible models for this region. If there is a compression strut crosses a zone of cracking in the test specimen, which suggests that this is not correct location for a compression struts.

2.5.3. Based on Load Path

In actual practice, it may be difficult to plot the stress trajectories without using specialized computer programs. This may therefore make the STM application rather difficult. As an alternative to the stress trajectory type of approach to STM a simplified load path method has been proposed. The first step in the load path method, is to ensure that the outer equilibrium of the D-region is satisfied by determining all the forces and reactions (support forces) acting on the region. In a boundary adjacent to a B- region the loads on the D-region are taken from the B- region design, assuming for example that a linear distribution of stresses (P) exist as in figure 2.13. The next step is to subdivide the stress diagram (in this case the linear distribution p) so that loads find their way from one side of the structure to other. For the fig 2.8, the load distribution, p , that is imposed at the top of the beam resisted by two reactions on the underside of the beam: A and B load paths will tend to take the shortest path possible from the applied loads to reactions. (sclaich et al ,1987) Following load path is important techniques that may be used in preparing appropriate STM especially for end anchorages.

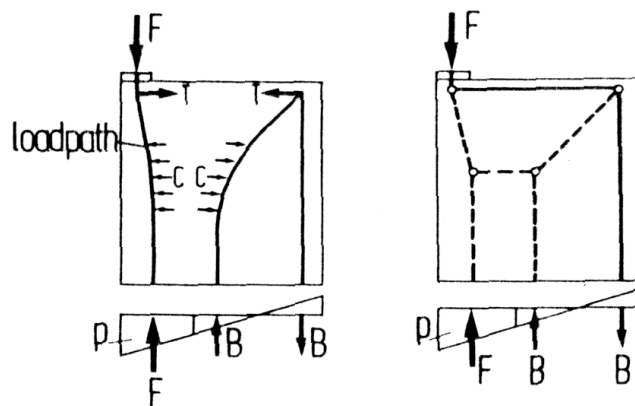


Figure 2-13 : Load paths and strut and tie model

2.6. Deformation Capacity of Structural Concrete in Disturbed Region

The design of structural concrete in disturbed regions such as shear critical members with limited ductility should be based on realistic estimation of the deformation capacity. The deformation capacity of structural concrete, which is mostly controlled by the ultimate comprehensive strain of concrete regardless of whether the reinforcement yields, is estimated on the construction of bending and translational and rotational failure mechanisms for compression struts.

Strut and tie models have been successfully used to detail structural concrete in D-regions, as these models illustrate the explicable force flows within D-regions. Inelastic deformation of shear-dominated walls, potential plastic hinges at the base of columns, and coupling beams in such D-regions are associated with limited ductility and shear strength reduction. In practical designs, it has been recommended that the contribution of concrete to shear strength is reduced for such shear critical members of limited ductility.

To estimate the deformation capacity of deep beam without web reinforcement as a typical type of structural concrete in D-regions, failure mechanisms compatible with their corresponding STM would be considered on the basis of the basic concept of limit analysis. The bending, translational and rotational failure mechanisms of struts in the crushing failure mode are investigated as governing deformation patterns of STMs in D-regions.

On the basis of ACI investigation, the following main conclusions are drawn.

When the deformation limits of struts in failure mechanisms alone are considered to control the deformation capacity of structural concrete in D-regions, the deformation capacity is expressed in terms of a/d , the reinforcement degree of the longitudinal ties, and the ultimate compressive strain of the concrete.

The deformation capacity depending on whether or not each failure mechanism involves yielding of the longitudinal tie. The deformation capacity of simply supported deep beams in flexural-shear failure mode can be explained by the bending mechanisms or the translational mechanisms involving the yielding of the tie. The deformation of coupling beam in a flexural-shear failure

mode, however, can be better explained by the translational mechanism than by the bending mechanism.

For deep beams with web reinforcement, based on an STM relying on a diagonal strut alone predicted a smaller deformation capacity compared to that predicted in the experimental results. To explain the larger deformation capacity due to the web reinforcement for diagonal tension failure and bond failure should be considered.

2.7. Strut and Tie Model versus Sectional Shear Design

The transition from B to D- region in beam is an important topic. It is not rational to perform a sectional shear design in B- region of beams and to deal separately with the D-region by using STM. This leads to discrepancies.

The angle θ for the inclined struts in the web

If a steeper strut angle is assumed in order to achieve complete truss, then higher amount of stirrups are calculated than required for the angle θ corresponding to the shear design. If on the other hand a flatter angle θ is used to achieve a complete truss, then it would lead to fewer amounts of stirrups than required for angle θ based on the shear design, and this would be unsafe.

The truss model provides a consistent transition between the B and D- regions in structural concrete beams. However, the inner lever arm Z and the angle θ must be known. The inner lever arm can be determined from the flexural design of the section with the highest moment and then it can be conservatively assumed to be constant over the length of the beam. The angle θ can be based either on assumptions are made or $45 \geq \theta \geq 22$. There is no guidance for the designer on which angle to select and there is no influence of axial forces of the prestress.

Approximately $\theta=33$ shows that the shear in the medium range which is well below the maximum shear force. However, many cases of beam building lie in the low shear range, and this may result in a very low values of around 20 degree for θ . This leads to higher forces in the tension chords, which is a disadvantage for staggering the longitudinal reinforcement, since

higher forces occur near the support. This should be regarded as the price for using the simple truss model.

On the basis of ACI investigation, the following main conclusions are drawn.

- The truss model is indispensable for designing beams with shear reinforcement and shear design should be based on this concept. Thereby, beam should always contain a minimum amount of stirrups. Using the truss model does not lead to a contradiction to the shear design?
- Using the truss model for designing the whole beam, often called a “full member design” allows a consistent transition from B- to D- region in beams and thus avoids the conflict between a sectional shear design and a separate design of the D- region .when doing such a “full member design”, the angle is θ is often freely selected, with the consequence of often higher amount of stirrup. The intention is to select an angle θ for the struts which would lead to a complete truss between the given supports. However, this is not necessary, because the B- regions with the inclined compression stress fields are always in equilibrium for any length.
- Members without shear reinforcement (i.e., slabs) should be treated separately, because the concrete term V_c for members with stirrups is different from the shear capacity V_{ct} for members without stirrups. The model for members without stirrups is a truss with concrete ties. This model could also be used as an explanation for the load transfer of the term V_c for members with stirrups, but this would lead to more complicated design equations for the forces in the tension chord than is the case for the simple truss model.
- The indirect support of the beam supported by another one is an important and critical D-region, which has led to damage of structures in the past. The total support force has to be transferred by hanger stirrups which must be placed within the intersection of the webs of the two beams. The modeling of the stress fields of the STM gives the guidance that only one arrangement is correct for the stirrups. The anchorage of the longitudinal reinforcement is correct for the stirrups. The anchorage of the longitudinal reinforcement of the indirectly supported beam is a further critical issue which requires careful consideration, because transverse tension in two directions considerably reduces

the bond strength and thus requires far longer anchorage length than for a direct support.

2.8. Application of the Strut and Tie Model

2.8.1. When it is used

Strut-and-tie models are used primarily to design regions near discontinuities or D-regions. Global strut-and-tie models (models used to design an entire structural member) can be used; however, it is best to focus on local strut-and-tie models (models used to design D-regions) since B-regions are more easily designed with conventional methods. A discontinuity in the stress distribution occurs at an abrupt change in the geometry of a structural element (geometric discontinuities), at a concentrated load or reaction (loading or statically discontinuities), or a combination of the two (loading and geometric discontinuities). Consider a simple span beam of depth h with a concentrated load at mid-span (see figure 2.14).

As illustrated this beam has three disturbed regions, one at each end of the beam and one at mid span. According the St. Venant's principle, the disturbed regions at each end of the beam will have a length equal to h , and the region at mid-span will have a length equal to $2h$. If the span of the beam is reduced such that the distance between the applied load and the end reaction is less than $2h$, the disturbed regions overlap. Hence the entire beam will be considered a D-region and the behavior of the beam will be strongly influenced by the disturbed flow of stresses. For this case the strut-and-tie model approach would be appropriate for design. Typical girders used in bridge design have span lengths of $20h$ to $25h$. Therefore, with the exception of the ends of the girders, if there are no geometric discontinuities within the span, the presence of disturbed regions due to loading have little effect on the overall behavior of the member and the localized effects can generally be ignored.

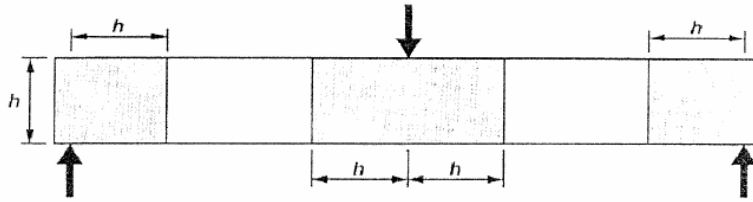


Figure 2-14 : Simple span beam used for discussion above.

For *B-regions*, the AASHTO LRFD Specifications permit the use of either traditional section models or the strut-and-tie model. For regions near significant discontinuities, the use of the strut-and-tie model or some type of inelastic approach should be used. A concentrated load which causes more than 50% of the shear at the face of a support and is closer than $2d$ from the support face is a situation that requires a strut-and-tie model or some type of inelastic approach.

2.8.2. Procedure for Strut and Tie Model

Step 1 – Delineate the D-Regions

As discussed in the previous section, the extent of a D-region can be determined using St. Venant's principle. Using St. Venant's principle, a D-region is assumed to extend a distance equal to the largest cross-sectional dimension of the member away from a geometrical discontinuity or a large concentrated load. The determined B-region/D-region interface is the assumed location where the stress distribution becomes linear again. Using this basic assumption, the D-regions can be delineated. This step is used when it is intended to design only D-region.

Step 2 – Determine the Boundary Conditions of the D-Regions

Once the extent of a D-region has been determined, the bending moments, shear forces, and axial forces must be determined at the B-region/D-region interface from analysis of the B-region. Using B-region analysis, the bending moment, shear force, and axial force are then used to determine the stress distribution at the B-region/D-region interface. The calculated stress distributions at the B-region/D-region interface can then be modeled as equivalent point loads.

The location and magnitude of the equivalent point loads is determined from the stress distributions directly. When determining the boundary conditions on the B-region/D-region interface, it is essential that equilibrium be maintained on the boundary between B- and D regions. If the bulk of the structure falls into a D-region it may be expedite to use a global model of the structure and use the external loads and reactions as the boundary conditions.

Step 3 – Sketch the Flow of Forces

After the stress distributions acting on the B-region/D-region interface have been modeled as equivalent point loads, the flow of forces through the structure should be determined. For most design cases, the flow of forces can easily be seen and sketched by the designer. When the flow of forces becomes too complex to be approximated with a sketch, a finite element analysis can be used to determine the flow of forces through a reinforced concrete structural member. For most D-regions, such efforts are unwarranted since the stress paths can be estimated easily. Another method used by many to determine the flow of forces is the load path method as proposed by Schlaich et al. (1987)

Step 4 – Develop a STM

A STM should be developed to model the flow of forces through the D-region determined in the previous step. The geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure, elastic stress distribution, based on observed test specimen & load path. When developing a STM, try to develop a model that follows the most direct force path through the D-region. Also, avoid orienting struts at small angles when connected to ties. According to Collins and Mitchell, as the angle between a strut and tie decreases, the capacity of the strut also decreases (1986). For this reason, many design specifications specify a minimum angle between struts and ties. It should be noted that the AASHTO LRFD provisions do not specify a minimum angle between struts and ties; however, the limiting strut compressive stress equation defined in the specification is a function of the angle between the strut and tie and decreases as the angle between the strut and tie decreases. A D-region may be subjected to more than one type of loading. It is imperative that a STM be developed and analyzed for each different loading case. On a similar note, for a given load case for a D-region, more than one STM can be developed. Schlaich and Schäfer (1991) suggest that

models with the least and shortest ties are the best. In addition, Schlaich and Schäfer also suggest that two simple model can sometimes be superimposed to develop a more sophisticated model that better models the flow of forces through a D-region.

Step 5 – Calculate the Forces in the Struts and Ties

The strut and tie forces can be calculated knowing the geometry of the developed STM and the forces acting on the D-region. It is desirable to use a computer program to calculate the forces because, often times, the geometry of the STM may need to be modified during the design process which will require the forces in the struts and ties to be recalculated.

Step 6 – Select Steel Area for the Ties

The required amount of reinforcement for each tie can easily be determined by dividing the force in the tie by the product of the yield stress of the steel and resistance factor specified by a design specification. The reinforcement chosen to satisfy the steel requirements must be placed so that the centroid of the reinforcement coincides with the centroid of the tie in the STM. If reinforcement chosen to satisfy the tie requirements cannot fit in the assumed location of the tie the location of the tie in the STM needs to be modified, and the member forces need to be calculated again.

Step 7 – Check Stress Levels in the Struts and Nodes

The stress levels in all of the struts and nodes must be compared to the allowable stress limits given in design specifications. In order to determine the stress levels in the struts and nodes, the geometry of the struts and nodes must first be estimated. The geometry of the struts and nodes can be determined based on the dimensions of bearing pads and the details of reinforcement connected to the struts and nodes.

Accurately determining the geometry of internal struts and nodes not attached to bearing pads and reinforcement is more difficult than finding the geometry of struts and nodes directly in contact with the boundary of the D-region. In the case of internal nodes and struts, it may not be possible to precisely define the strut and node geometry. Brown et al. (2006) explain that this uncertainty is acceptable because force redistribution can take place for internal struts and nodes.

When stresses in struts and nodes are found to be larger than permissible stresses, bearing areas, the reinforcement details, or the overall member geometry of the member can be modified in an effort to increase the overall geometry of the strut and/or node. When changing any or all of these items, the STM will likely need to be modified. If the STM is modified, the member forces need to be calculated again, the ties may need to be redesigned, and then, the stresses in the struts and nodes can be checked again. The concrete strength can be increased if modifying the geometry of the STM or the member itself is not possible.

Step 8 – Detail Reinforcement

Once all the steel chosen for the ties in the STM has been finalized, the anchorage of the reinforcement must be properly detailed in order for it to reach its yield stress prior to leaving nodal zones. In addition, appropriate crack control should be placed in areas that are expected to be subject to cracking. Most design specifications specify a minimum amount of crack control that must be placed in a D-region that has been designed with a STM.

2.9. Strut and Tie Model Method as Standard Design Provision

Although truss models have been used since the turn of century [Ritter, 1899], they have gained increased popularity in recent years. The reason for this increased popularity is that they provide the design of complex regions in reinforced concrete. This increased popularity is due to the fact that the designer can model the flow of the forces with strut and tie, even for complex design situations. Empirically based are not only limited in their applicability, but also provide the designer with insight into the actual behavior. An additional advantage using strut and tie models are that sketching the flow of the force within a member highlights the need for careful details of reinforcement in key regions. STM design provisions consist of rules for defining the dimensions and ultimate stress limits of strut and nodes as well as the requirements for the distribution and anchorage of reinforcement. Guidelines for STM design have been developed for European practice. Although the STM has been included in the Canadian standard for the design of concrete structures in 1984 and AASHTO LRFD bridge specifications since 1994. With the implementation of the AASHTO LRFD [12] Bridge Specifications, bridge designers were presented with a new approach in the design of deep reinforced concrete sections with the strut-

and-tie design method. While strut-and-tie modeling has been employed in the past for various reinforced concrete designs, the introduction of the AASHTO LFRD [12] Specifications marks the first time it is presented as a suggested design procedure. To help U.S. Engineers improve their ability to use STM for analysis and design of concrete members, joint ACI-ASCE committee 318-E, shear and torsion, recently completed a publication that contains a variety of STM examples.

2.10. Euro Code 2 Provisions for Strut and Tie Model

Strut and tie modeling of reinforced concrete structures is used in the design of discontinuous or d-region. For those regions, shear is the critical mode of failure and Euro code 2 allows for its design using strut and tie models (STM). However Euro code 2 provides very little guidance in using strut and ties models, which covers mainly the effective concrete strength provisions for the various strut and tie elements.

Euro Code 2 STM Provisions is as follow

a) Struts

The identification of the axis of the concrete compression struts in a truss model is in general not a difficult problem. It is less evident to determine reasonable values for the dimensions of the cross-section of a strut,

- (1) The design strength for a discrete concrete strut in a region with transverse compressive stress or no transverse stress may be calculated from Expression (Equ. 2.8) (see Figure 2.15)

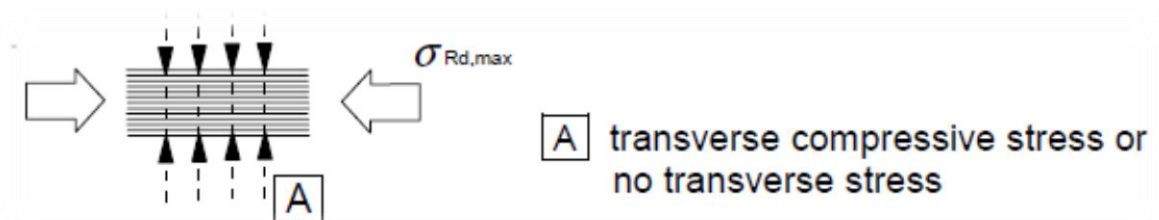


Figure 2-15 : Design strength of concrete struts without transverse tension

$$\sigma_{Rd,max} = f_{cd} \quad (2.8)$$

It may be appropriate to assume higher design strength in regions where multi-axial compression exists.

(2) The design strength for notional concrete struts should be reduced in cracked compression zones and, unless a more rigorous approach is used, may be calculated from Expression (Equ. 2.9) (see Figure 2.16)

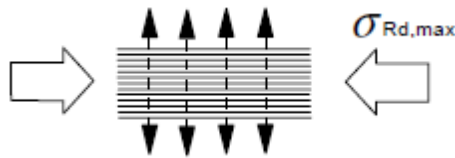


Figure 2-16 : Design strength of concrete struts with transverse tension

$$\sigma_{Rd,max} = 0.6n f_{cd} \quad (2.9)$$

Note: The value of n for use in a Country may be found in its National Annex. The recommended value is given by equation (Equ. 2.10)

$$n = 1 - f_{ck} / 250 \quad (2.10)$$

b) Ties

The design of transverse ties and reinforcement is similar to the conventional method,

c) Nodes

The design values for the compressive stresses within nodes may be determined by:

(1) in compression nodes where no ties are anchored at the node (see Figure 2.17)

$$\sigma_{Rd,max} = k_1 n f_{cd} \quad (2.11)$$

Note: The value of k_1 for use in a Country may be found in its National Annex. The recommended value is 1.

Where

$\sigma_{Rd,max}$ is the maximum stress which can be applied at the edges of the node.


$$\sigma_{\text{Rd,max}} = k_2 n f_{\text{cd}} \quad (2.12)$$


c) In compression - tension nodes with anchored ties provided in more than one direction (see Figure 2.19)

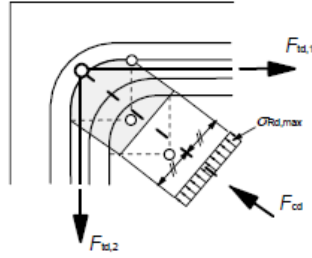


Figure 2-19 : Compression tension node with reinforcement provided in two directions

$$\sigma_{Rd,max} = k_3 n f_{cd} \quad (2.13)$$

Note: The value of k_3 for use in a Country may be found in its National Annex. The recommended value is 0.75.

3. METHODOLOGIES

3.1. Introduction

The scope of this study is to highlight the application of a newer generation strut-and-tie model, which is not in practice at the time of the original design. A 2D strut-and-tie model is developed for the analysis of T-girder, hammerhead pier caps & pier. The lack of familiarity with the procedure has caused most practicing engineers, to avoid implementation of substructure design using STM. This chapter presents a series of four design comparisons performed to illustrate the use of strut-and-tie modeling and to compare these designs with traditional sectional approaches.

The description of the proposed design procedure presents the process of the creation truss model & determining internal truss forces. Sizing the area of ties, checking stresses in the nodal zones and struts. The final section in the design procedure is providing adequate anchorage for steel tie reinforcement and crack control reinforcement.

3.2. Design Calculations

3.2.1. Example 1: Bridge Girder Design using STM

3.2.1.1. Description of Structure

The bridge pier designed in this section is from Ethiopian Road Authority, Tekeze Bridge that have T- girder superstructure. The procedure for the strut and tie modeling of the pier is demonstrated in this section.

Table 3-1: Material Property

Material Property	values
Concrete Density	25kg/m ³
Concrete 28-day Compressive Strength	$f'_c=24\text{MPa}$
Steel Reinforcement Strength	$f_y=400\text{ MPa}$ for bar Diam $\geq 20\text{mm}$ $f_y=276\text{ MPa}$ for bar Diam $< 20\text{mm}$

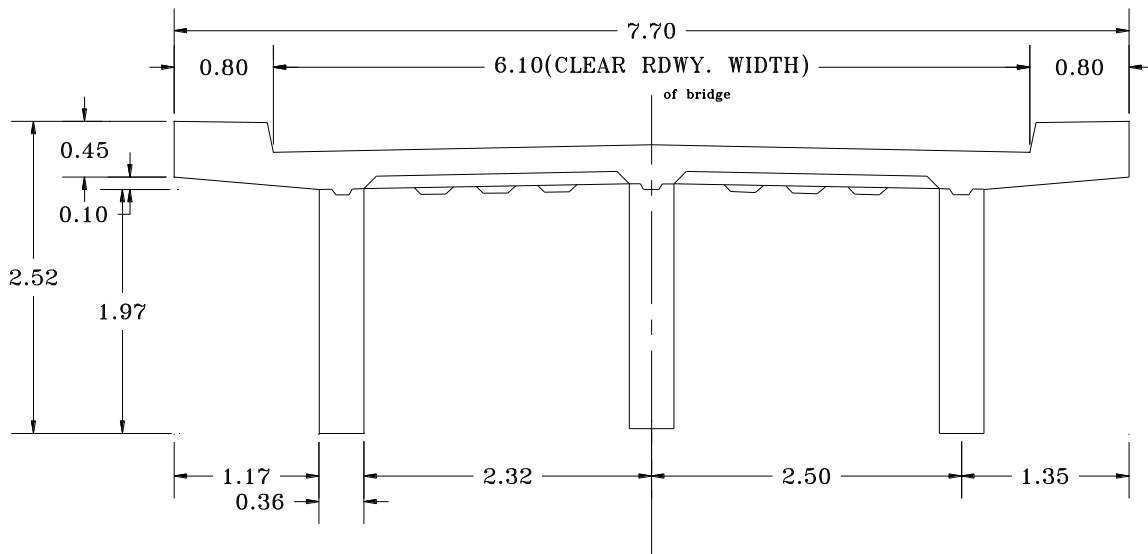


Figure 3-1 : Bridge Girder dimensions typical cross section

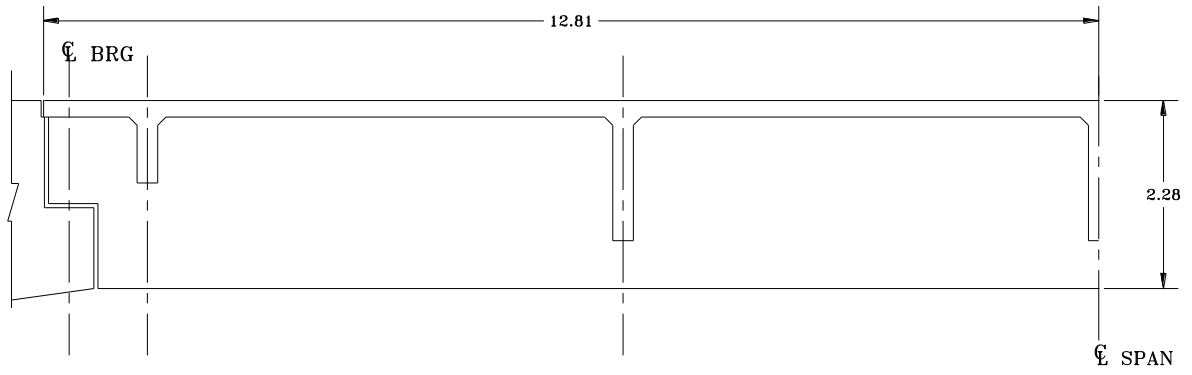


Figure 3-2 : Bridge Girder dimensions longitudinal section

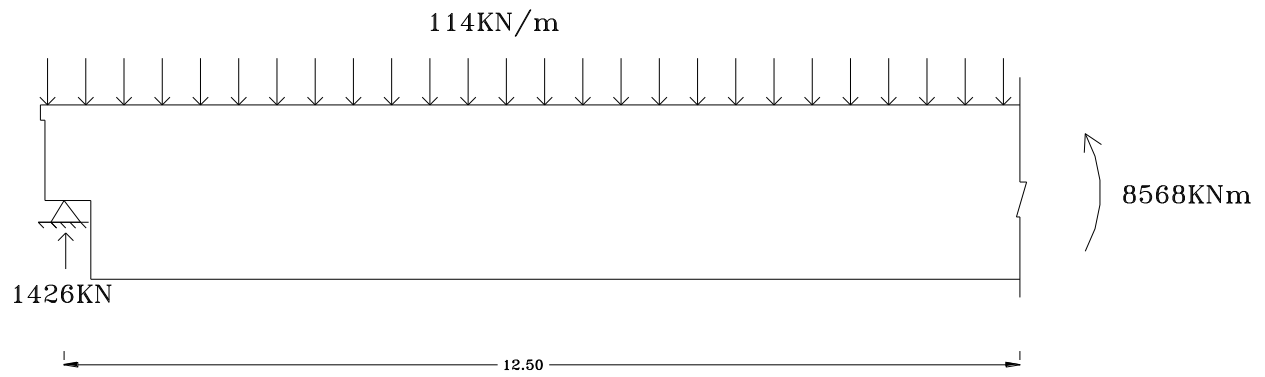


Figure 3-3 : Bridge Girder Loading

3.2.1.2. Design Calculation

1. Delineate D-regions from B-regions

The entire Bridge Girder is considered to be in the B-region since the disturbance factor that occurs at edge would diminish due to the longer span length. As STMs can be formulated for both B- and D-region structural members, it would be a comparison method to design the girder using STM.

2. Determine the Boundary Conditions

The bending and shears stress distribution calculation is shown. The equivalent point loads on the boundary of the D-region will be determined based on the geometry of the STM developed in Steps 4 and the loading on the D-region.

$C \text{ or } T = M / z$, where, $z \approx 0.9d$

Where, f_{cd} = design compressive strength of concrete

z = moment arm

d = effective depth of the section

$T = C = M / z = 8908 / (0.9 * 2.105) = 4702.1 \text{ kNm}$

3. Visualize load path / sketch the flow of stresses

SAP2000 software was used to generate the principle stresses in the girder due to the final loading conditions. It is easy to visualize where the stress distributions are concentrated. The load path also used to visualize with the orientation of struts and ties in the model.

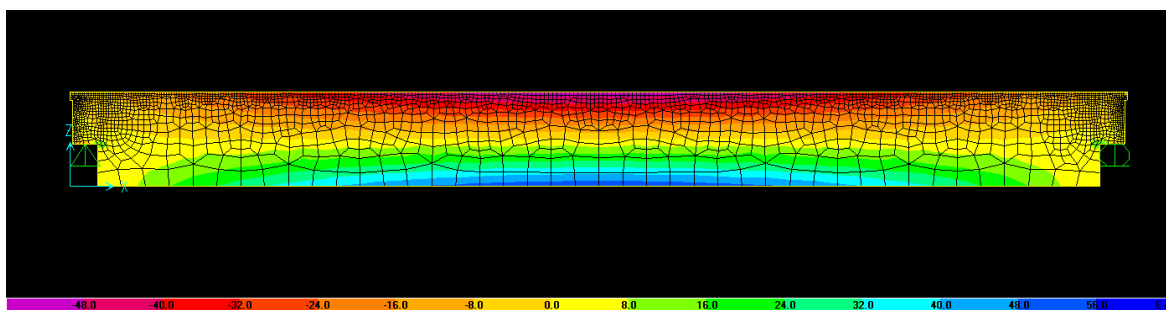


Figure 3-4 : Bridge Girder stress distribution

4. Develop a STM that is compatible with the flow of forces.

In order to determine the flow of forces through the girder, a STM that is typically used for dapped-end beams was used. The geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure & elastic stress distribution. The STM is shown in Figure 3.5. It should be noted that Strut AB was placed at the centroid of the horizontal compression steel determined from the beam design.

Also, Ties IJ were placed at the centroid of the tension steel determined from the beam design. The distributed load on top of the beam has been converted into equivalent point loads based on the truss geometry. Member labels can be seen in the figure.

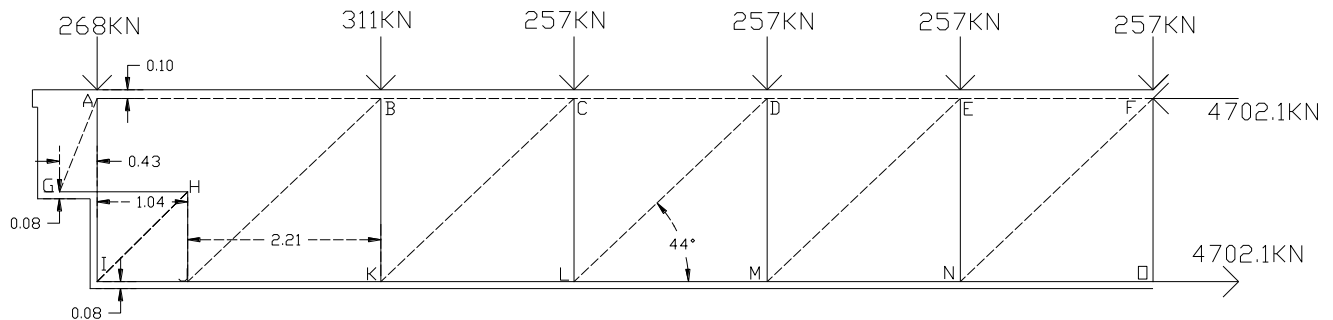


Figure 3-5 : STM

5. Calculate forces in struts and ties.

The computer software SAP2000 was used to compute the forces in truss members. It is convenient to use software for this step since the model geometry can easily be changed and member forces recalculated if necessary. However, it is also a good idea to check some of the members by hand to verify that the model is set up properly using the software. All member forces are shown in Table 3.2

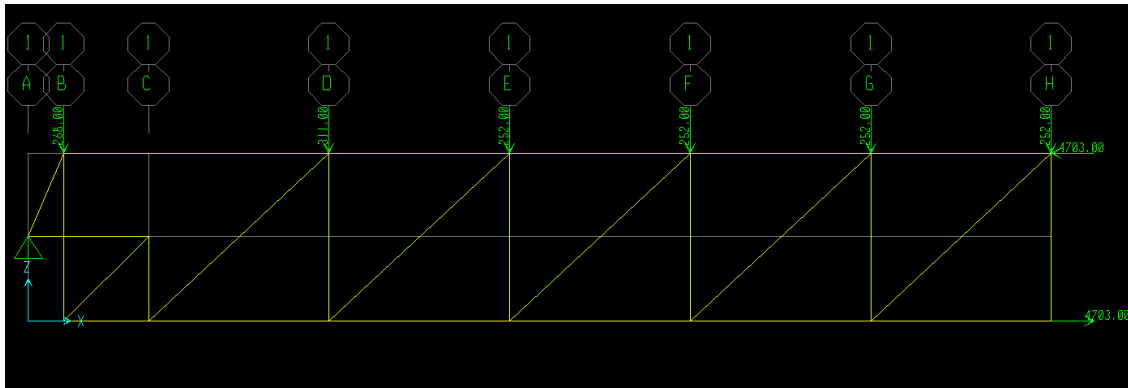


Figure 3-6 : Sap Model for Girder

Table 3-2: Member Force

Members	Tension force(KN)
GH	857
IJ	832
JK	2152
KL	3264
LM	3984
MN	4344
NO	4632
AI	1016
HJ	864
BK	961
CL	577
DM	585
EN	0.3
FO	0.3

Members	Compression force (KN)
AB	944
BC	2248
CD	3272
DE	3984
EF	4416
GA	1595
IH	1072
JB	1601
KC	1376
LD	769
ME	577
NF	321

6. Size the area of ties.

Section 5.6.3.4 of the AASHTO LRFD Specifications was used to determine the required amount of reinforcement for each tie. The required amount of reinforcement for each tie was determined as follows:

$$\Phi P_n \geq P_u \quad (3.1)$$

$$\Phi P_n = \Phi f_y A_{st}, \text{ where } \Phi = 0.9 \quad (3.2)$$

$$A_{st} \geq P_u / \Phi f_y \quad (3.3)$$

Longitudinal bars and stirrups are sized and steel is chosen as shown in table 3.3 based on the calculation shown here:

Example: Ties GH

$$A_{st} \geq 857 / (0.9 * 400) = 2380.56 \text{ mm}^2$$

$$\text{Using } \Phi 32 \text{ bars, with } A_{st} = 804.3 \text{ mm}^2$$

$$\#32 \text{ bars required} = 2380.56 / 804.3 = 3, \text{ use 3\# 32 bars.}$$

Table 3-3 : Longitudinal Steel

Members	Force(Kn)	$A_{s, \text{required}}$	No of bars	Provided no of bars	$A_{s, \text{provided}}$
GH	857	2380.56	3.0	4	3217
IJ	832	2311.11	2.9	4	3217
JK	2152	5977.78	7.4	8	6433.98
KL	3264	9066.67	11.3	14	11259.47
LM	3984	11066.67	13.8	14	11259.47
MN	4344	12066.67	15.0	16	12867.96
NO	4632	12866.67	16.0	16	12867.96

Check the assumed location of the tie centroid for two rows of bars:

$$50(\text{clear cover}) + 12(\text{dia. of stirrup}) + 32*2 (\text{dia. of bar}) + 40(\text{spacing between rows}) = 166\text{mm}$$

So the assumed location of 180mm above the bottom edge of girder is ok.

The vertical ties represent the centroid of stirrups that will be spaced across a “stirrup band”. For ties AI the band spans a distance of $(0.43+1.04/2)=0.95\text{m}$. The total strength of all the stirrups in this band must be greater than the force in AI. Stirrup bands for HJ also span $(1.04+2.21)/2\text{m}=1.625\text{m}$. The calculation for the spacing of the stirrups is shown here based on using $\Phi 12$ 4-legged stirrups.

Tie AI

$$\Phi P_n = \Phi f_y A_{st} (\text{stirrups}) \geq P_u ; \Phi = 0.9$$

$$\# \text{ Stirrups} = P_u / (\Phi f_y A_{st}) = 1016*1000 / (0.9*276*4*113.1) = 9.04$$

The required spacing within the 955 mm band is

$$S \leq 950/9.04 = 105\text{mm}, \text{ use spacing of } 100 \text{ mm}$$

$$A_{st} = 950*(4*113.1)/100 = 4297.8\text{mm}^2$$

Minimum transverse reinforcement in this region

$$A_v = 0.0316*f_c^{0.5}*bS/f_y \quad (3.4)$$

$$A_v = 0.0316*24^{0.5}*360*100/276 = 20.2\text{mm}^2$$

$$4*113.1 > 20.2\text{mm}^2 \quad \text{ok!}$$

Table 3-4 : Stirrup Reinforcement

Members	Force(KN)	Band width (mm)	Spacing	Spacing provided	$A_{s,provided}$
HJ	864	1622.6	105.52173	100	1835.1606
BK	961	2208	129.09811	120	2497.248
CL	577	2208	215.01435	200	2497.248
DM	585	2208	212.07398	200	2497.248

7. Check stresses in the nodal zones and struts.

(a) Check stresses in the struts.

In order to check the capacities of the struts, the area of the struts must be determined. The strut areas were calculated by finding the product of the widths and depths of each of the struts. The strut width, w_s , refers to the dimension of the struts in the plane of the STM. AASHTO LRFD Figure 5.6.3.3.2-1 gives guidance of how to determine the width of struts based on the geometry of bearing pads and tie reinforcement details. The widths of struts IH, JB, KC, LD, ME & NF were determined using AASHTO LRFD Figure 5.6.3.3.2-1(a). A portion of AASHTO LRFD Figure 5.6.3.3.2-1(a) has been reproduced in Figure 3.7. Figure 3.7 shows that the boundaries of the strut are allowed to extend a distance of up to 6 bar diameters beyond a piece of anchored reinforcement. The only time that this rule does not apply is when extending the boundaries of a strut causes it to overlap with another strut or the boundary of the D-region itself. It should also be noted that Figure 3.7 displays an equation that can be used to calculate the strut widths.

Table A.4-3 summarizes the strut width calculations for struts IH, JB, KC, LD, ME & NF.

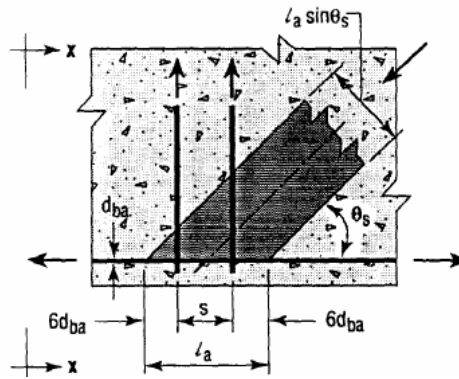


Figure 3-7 : Portion of AASHTO LRFD Figure 5.6.3.3.2-1(a).

Table 3-5 : Calculated strut widths based on Figure 3.7

member	Connecting Tie	θ_s (degrees)	l_a (mm)	$w_s = l_a \sin(\theta_s)$ (mm)
IH	AI	45	192	135.8
JB	HJ	44	192	133.37
KC	BK	44	192	133.37
LD	CL	44	192	133.37
ME	DM	44	192	133.37
NF	EN	44	192	133.37

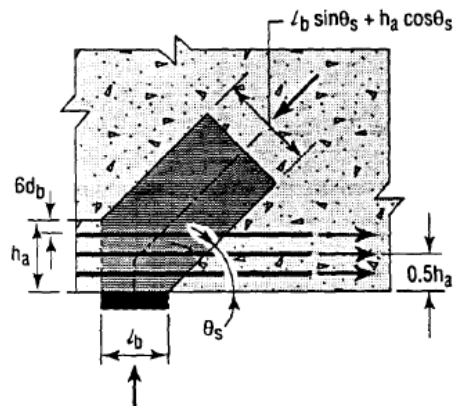


Figure 3-8 : Portion of AASHTO LRFD Figure 5.6.3.3.2-1(b).

Table 3-6 : Calculated strut widths based on Figure 3.8

member	Connecting Tie	θ_s (degrees)	l_b (mm)	$w_s = l_b \sin(\theta_s) + h_b \cos(\theta_s)$ (mm)
GA	AI	68	400	430.81

The widths for struts AB, BC, CD, DE and EF were determined by multiplying the distance from the centroid of the tie to the top of the beam by two (the $6d_b$ rule did not govern). Based on the AASHTO LRFD Figure 5.6.3.3.2-1(a) which shows the $6d_b$ rule, the depth of GA, IH, JB, KC, LD, ME & NF was determined to be 360mm (the full depth of the member) and for rest of the member 2690mm (effective width of t-girder). Table 3.7 summarizes the calculated widths and cross-sectional areas of all the members. The values for ties were included in this table because these values are needed for the node capacity check calculations in Step 7(b).

Table 3-7 : Member effective widths and cross-sectional areas.

Members	Width (mm)	$A_{cs}(\text{mm}^2)$
AB	200	538000
BC	200	538000
CD	200	538000
DE	200	538000
EF	200	538000
GA	430.81	155092
IH	135.8	48888
JB	133.37	48013
KC	133.37	48013
LD	133.37	48013
ME	133.37	48013
NF	133.37	48013

Once the areas of the struts were determined, the capacities of the struts were checked according to Section 5.6.3.3.3 of the AASHTO LRFD. For each strut, the strain in the adjoining tie, ϵ_s was estimated in order to determine the principal tensile strain ϵ_1 in the strut. The strain in a tie can be estimated to be:

$$\epsilon_s = P_{u,tie}/A_{st}E_s$$

After the strain in the tie is determined, the principal tensile strain in the concrete is calculated according to AASHTO LRFD eq. 5.6.3.3.3-2 as:

$$\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002) \cot^2 \alpha_s \quad (3.5)$$

The limiting compressive stress (f_{cu}) in the tie can now be calculated:

$$f_{cu} = f'_c / (0.8 + 170\epsilon_1) \leq 0.85f'_c \quad (3.6)$$

For members connected to more than one tie, the limiting compressive stress in the strut was calculated based on each of the ties separately, and the smaller of the two compressive stresses was taken as the limiting compressive stress for the strut. Finally, the capacity of the strut can be calculated by multiplying the limiting compressive stress with the smallest calculated area of the strut at either end of the strut.

$$\Phi P_n = \Phi f_{cu} A_{cs} \quad (3.7)$$

For compression in a STM, Φ is taken as 0.70 (AASHTO LRFD 5.5.4.2.1). Table 3.8 summarizes the calculations performed for each of the struts. All of the struts were found to have adequate capacity.

Table 3-8 : Summary of strut calculations.

Members	P_u (kN)	Tie Force (kN)	$A_{st,provided}$ (mm ²)	α_s (degree)	ϵ_s	ϵ_l	f_{cu}	$F_{cu,used}$	A_{cs} (mm ²)	ΦP_n (k)	$\Phi P_n > P_u?$
AB	944	1016	4297.80	90	1.1E-06	0.0000	29.99	20.4	538000	7682.64	yes
BC	2248	961	2497.25	90	1.8E-06	0.0000	29.99	20.4	538000	7682.64	yes
CD	3272	577	2497.25	90	1.1E-06	0.0000	29.99	20.4	538000	7682.64	yes
DE	3984	585	2497.25	90	1.1E-06	0.0000	29.99	20.4	538000	7682.64	yes
EF	4416	0	2497.25	90	0.0E+00	0.0000	30.00	20.4	538000	7682.64	yes
GA	1595	857	2412.74	68	1.7E-06	0.0003	28.04	20.4	155092	2214.71	yes
IH	1072	832	2412.74	45	1.6E-06	0.0020	21.04	20.4	48888	698.12	no
		1016	4297.80	45	1.1E-06	0.0020	21.05	20.4	48013	698.12	no
JB	1601	864	3670.32	44	1.1E-06	0.0021	20.60	20.4	48013	685.63	no
		2152	6433.98	44	1.6E-06	0.0021	20.60	20.4	48013	685.63	no
KC	1376	961	2497.25	44	1.8E-06	0.0021	20.60	20.4	48013	685.63	no
		3264	11259.47	44	1.4E-06	0.0021	20.60	20.4	48013	685.63	no
LD	769	577	2497.25	44	1.1E-06	0.0021	20.60	20.4	48013	685.63	no
		3984	11259.47	44	1.7E-06	0.0021	20.60	20.4	48013	685.63	no
ME	577	585	2497.25	44	1.1E-06	0.0021	20.60	20.4	48013	685.63	yes
		4344	12867.96	44	1.6E-06	0.0021	20.60	20.4	48013	685.63	yes
NF	321	4632	12867.96	44	1.7E-06	0.0021	20.60	20.4	48013	685.63	yes

Because the strut capacity is less than required resistance for strut IH, JB, KC & LD; we should increase the strut width at least to the required width since we do have space to increase.

$$w = P / (\Phi * f_{cu} * b_w)$$

$$w_{IH} = 1072 / (0.7 * 20.4 * 360) = 208.5 \text{ mm}$$

$$w_{JB} = 1601 / (0.7 * 20.4 * 360) = 311.5 \text{ mm}$$

$$w_{KC} = 1376 / (0.7 * 20.4 * 360) = 267.7 \text{ mm}$$

$$w_{LD} = 769 / (0.7 * 20.4 * 360) = 149.6 \text{ mm}$$

7(b) – Check the capacity of the nodes.

The capacities of the nodes need to be checked for the forces imposed by the anchored ties, at Node/strut interfaces where the strut capacity was determined to be larger than the allowable node capacity, and for bearing caused by applied loads or boundary loads. Table 3.9 summarizes the locations where node/strut interfaces need to be checked due to the fact the calculated strut limiting compressive stress is larger than the allowable node limiting compressive stress. According to AASHTO LRFD 5.6.3.5, the limiting compressive stresses are $0.85f'_c$, $0.75f'_c$, and $0.65f'_c$ for CCC, CCT, and CTT nodes, respectively.

Table 3-9 : Summary of node capacity calculations.

Node	Type	f_{cu} (MPa)	Adjoining Member	A_{cs} (mm ²)	P_u (KN)	ΦP_n (KN)	$\Phi P_n > P_u$?
A	CCT	18	GA	155,092	1595	1954.2	Yes
B	CCT	18	BC	538,000	2248	6778.8	Yes
C	CCT	18	CD	538,000	3272	6778.8	Yes
D	CCT	18	DE	538,000	3984	6778.8	Yes
E	CCC	20.4	EF	538,000	4416	7682.6	Yes
F	CCC	20.4	EF	538,000	4416	7682.6	Yes
G	CCT	18	Bearing pad	120,000	1425.2	1512.0	Yes

Since the above nodal zone see the largest forces and are satisfactory, there is no need to check the other diagonal struts or node regions. In other words, the required stresses in the other struts and nodes will be significantly less than above nodes.

8. Check the detailing for the anchorage of the ties.

At Node G & I must be properly anchored with a 180-degree or 90-degree hook. Due to the limited height in the dapped section of the beam, a 180-degree hook will be used. According to Section 5.11.2.4.1, the development length, l_{d_h} , for a #32 bar with a hook can be determined as follows:

$$l_{hb} = 100d_b / \sqrt{f'_c} = 100 * 32 / \sqrt{24} = 653.2 \text{ mm} \quad (3.8)$$

At Node H, Tie GH will not have hooks. The required development length is calculated to be:

$$l_{db} = 0.02 A_b f_y / \sqrt{f'_c} \geq 0.06 d_b f_y$$

$$l_{db} = 1313.5 \text{ mm} \geq 768 \text{ mm}$$

Since the reinforcement is placed more than 300 mm of fresh concrete is cast below the reinforcement, the development length must be increased by a factor of 1.4.

$$l_{db} = 1313.5 * 1.4 \text{ mm} = 1839 \text{ mm}$$

It will be assumed that the details of the stirrups chosen will conform to AASHTO LRFD 5.11.2.6.4 and, therefore, the stirrups will be considered properly anchored.

9. Detail the crack control reinforcement

Appropriate crack control must be detailed in order for the design to conform to Section 5.6.3.6 of the AASHTO LRFD.

In the dapped section of the beam, it will be assumed that stirrups spaced at 100mm are used. Based on this spacing, the required stirrup area is calculated to be:

$$A_{st-required} = 0.003 * b * s = 0.003 * 360 * 100 = 108 \text{ mm}^2 \leq 4 * 113.1 \text{ mm}^2$$

For the next section of the beam, it will be assumed that stirrups spaced at 120mm & 200 are used. Based on this spacing, the required stirrup area is calculated to be:

$$A_{st-required} = 0.003 * b * s = 0.003 * 360 * 120 = 129.6 \text{ mm}^2 \leq 4 * 113.1 \text{ mm}^2$$

$$A_{st-required} = 0.003 * b * s = 0.003 * 360 * 200 = 216 \text{ mm}^2 \leq 2 * 113.1 \text{ mm}^2 = 226.2 \text{ mm}^2$$

The figure for reinforcement detail for both STM & conventional method is shown in appendix.

Table 3-10 : Summary of reinforcement of STM & conventional method

	Steel bar using STM	Steel bar using conventional method
Main bottom reinforcement	16 Φ 32	12 Φ 32
Main bottom reinforcement at dapped end	4 Φ 32	3 Φ 32 @ & diagonally 2 Φ 32
Stirrups for the first 1.2m(G-J)	4 legs, Φ 24 @ 100 mm	4 legs, Φ 24 @ 120 mm
Stirrups for the next 4.42m(J-L)	2 legs, Φ 24 @ 120mm	2 legs, Φ 24 @ 120& 180 mm
Stirrups for the last 6.62m(L-O)	2 legs, Φ 32 @ 200mm	2 legs, Φ 32 @ 300mm

3.2.2. Example 2: Hammerhead Pier Cap Design using STM

3.2.2.1. Description of Structure

The bridge pier cap designed in this section is from Ethiopian Road Authority, Tekeze Bridge that have T- girder superstructure design example. The procedure for the strut and tie modeling of the pier cap is demonstrated in this section.

Table 3-11 : Material Properties

Material Property	values
Concrete Density	25kg/m ³
Concrete 28-day Compressive Strength	$f'_c=24\text{MPa}$
Steel Reinforcement Strength	$f_y=400\text{ MPa for bar Diam} \geq 20\text{mm}$ $f_y=300\text{MPa for bar Diam}<20\text{mm}$

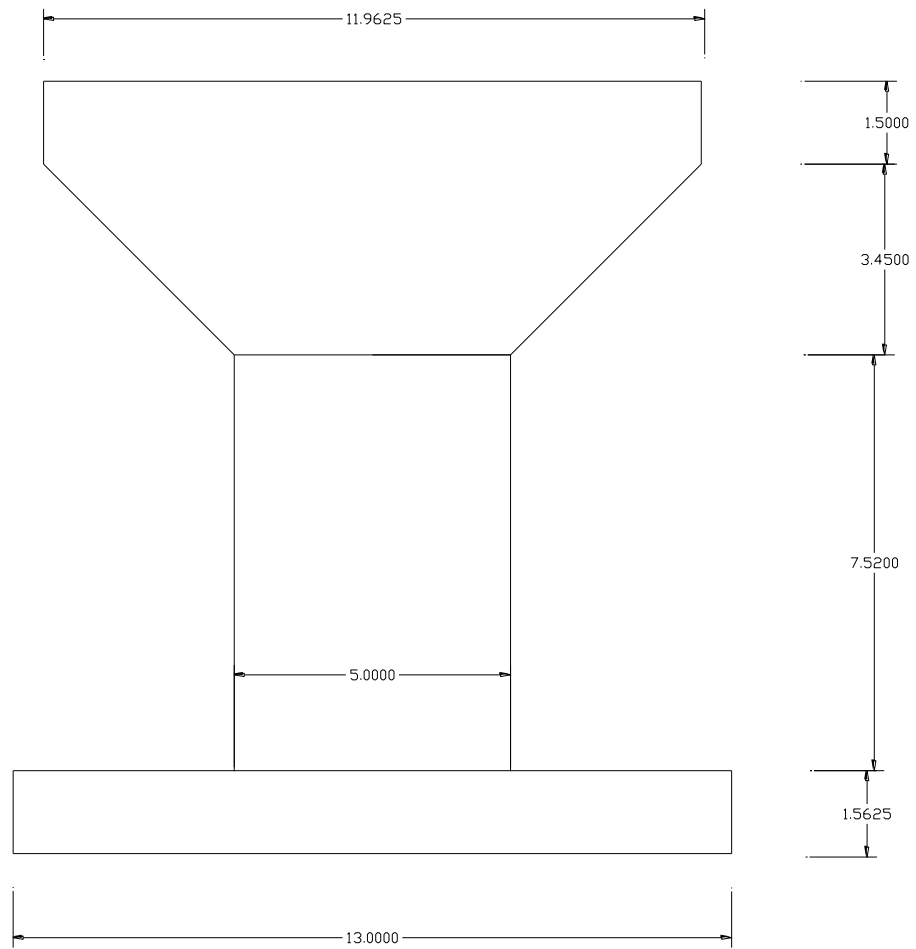


Figure 3-9 : Bridge Pier Dimensions Front Elevation

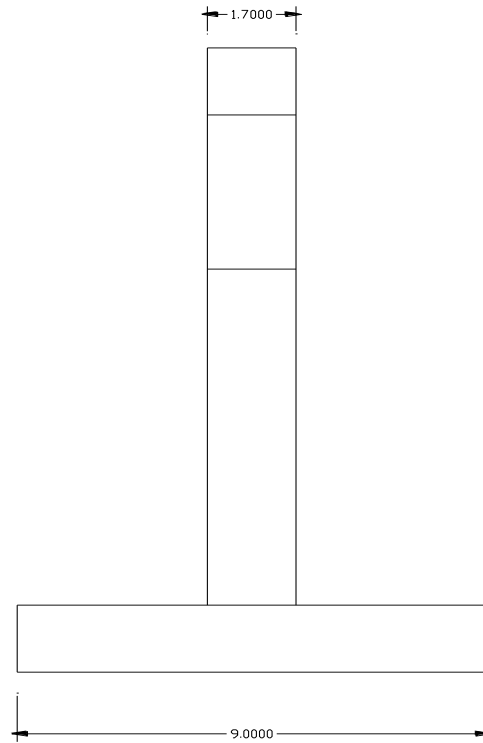


Figure 3-10 : Bridge Pier Dimensions Side Elevation

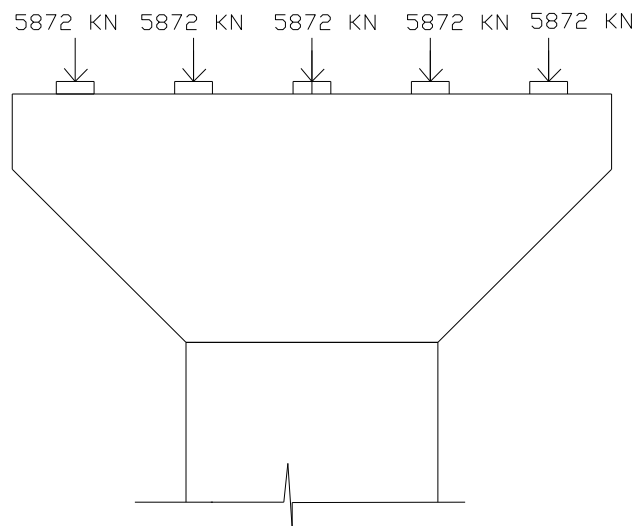


Figure 3-11 : Pier Cap Final Loading

3.2.2.2. Design Calculation (According to ASSHTO LRFD)

1. Delineate D-regions from B-regions

The entire pier cap is considered to be in the D-region since the smallest beam depth dimension is 1.5 m. and the distance between the bearing concentrated loads is 2.35 m, which is less than $2(1.5) = 3\text{m}$. Even if part of the cap were to be considered as a B-region, it would still be reasonable to do the entire design with STM.

2. Determine the Boundary Conditions on the D-region

To generalize the pier cap as a truss, the column under it is considered as three compressive struts. These struts are resisted by three supports. Two prevent displacement in the y-direction only, while the third prevents displacement in the x and y directions. The x-direction location of these struts and corresponding supports depend on the force in each strut and will be determined with the development of the truss model.

3. Visualize load path / sketch the flow of stresses

SAP2000 software was used to generate the principle stresses in the pier cap due to the final loading conditions. It is easy to visualize where the stress distributions are concentrated. The load path also used to visualize with the orientation of struts and ties in the model.

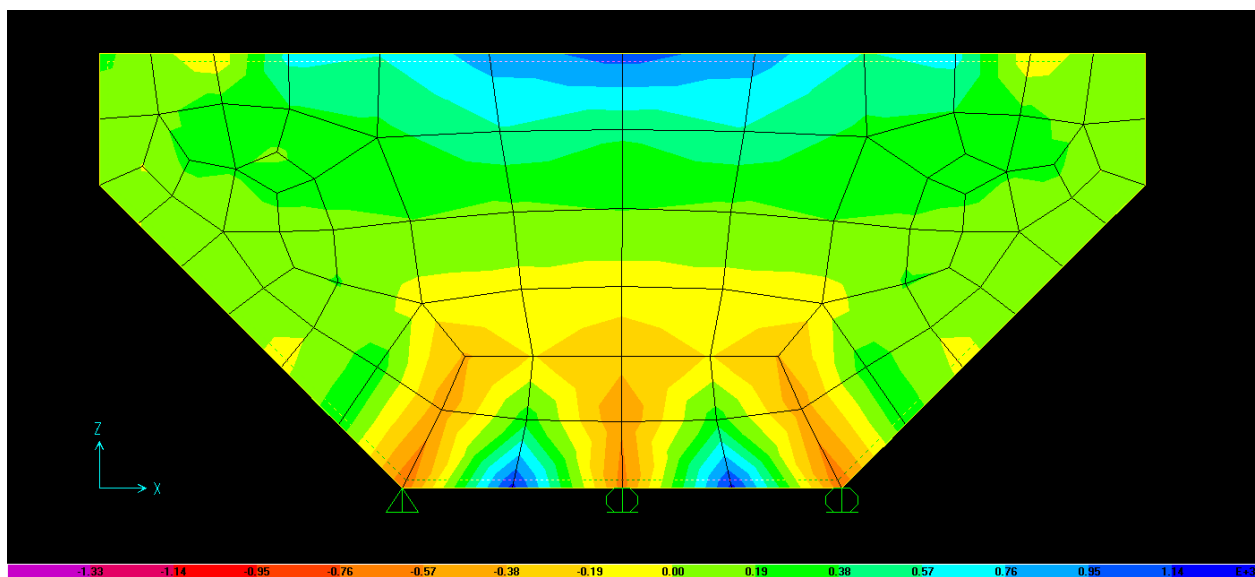


Figure 3-12 : Pier cap stress distribution

4. Develop a STM that is compatible with the flow of forces.

The model used should realistically represent the distribution of stresses from step 3. The geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure & elastic stress distribution. Establishing geometry requires trial and error in which member sizes are assumed, the truss geometry is established, member forces are determined, and the assumed member sizes verified. The column supporting the pier cap can be broken into three compressive struts

To dimension the STM, a sectional analysis of the center of the pier cap was done to get an estimate of the widths of the bottom struts HG and GF (figure 3.14). The analysis was based on the section located 0.15m from the edge of the column since this is the location of nodes H and F. It was estimated that the horizontal tie spanning the top of the pier cap should be placed 150 mm from the top edge of the cap. This accounts for the 50 clear cover, shear ties and two layers of longitudinal steel that is expected based on the sectional analysis. The limiting stress in the concrete is estimated as $0.85f'_c$ since the struts are not anchored to any ties. From the analysis, it was determined that the centerline of the strut should be placed about 150 mm above the bottom of the pier cap. Calculations are shown here:

$$M_u = 5872 * 2.35 = 13799.2 \text{ KNm}$$

$$T = C, A_s F_y = F_{cu} * a * b, \quad b = 1700 \text{ mm}$$

$$M_n = A_s F_y (d - a/2) = 0.85 f'_c a b (d - a/2)$$

Estimate for d

$$\begin{aligned} D &= 4950 (\text{pier cap depth}) - 50 (\text{clear cover}) - 24 (\text{stirrup}) - 32 (\text{bar}) - 20 (\text{half bar spacing}) \\ &= 4824 \text{ mm} \end{aligned}$$

$$M_u = \Phi M_n = \Phi * 0.85 * f'_c * a * b * (d - a/2), \Phi = 0.7$$

$$13,799.2 * 10^6 = 0.7 * 0.85 * 24 * a * 1700 (4824 - a/2)$$

$$a \geq 119.3 \text{ mm}, a = 300 \text{ mm} \leq 12d_b = 32 * 12 = 384 \text{ mm}$$

Therefore, the lines representing the centerline of the bottom struts will be placed at 150mm from the edge of the pier cap and the centerline of the longitudinal ties will be spaced 150mm from the top edge of the beam as shown in figure 3.14.

An alternative truss model is shown in figure 3-13. This model gives more main reinforcement which contradicts with STM principles.

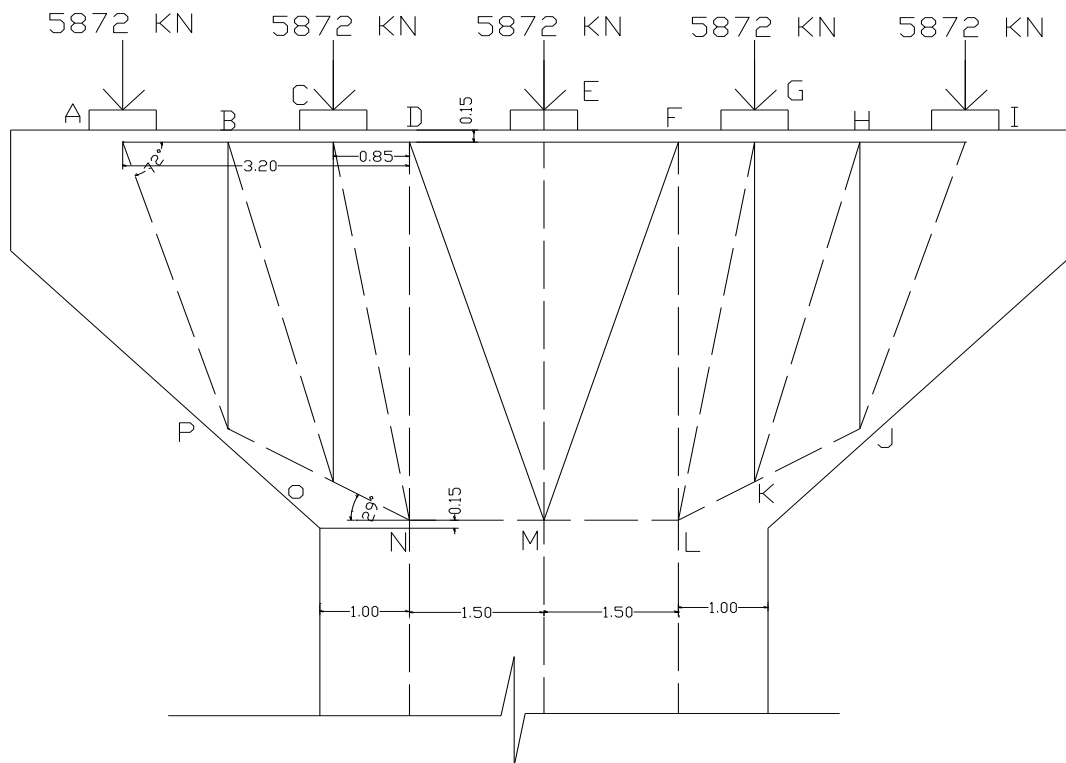


Figure 3-13: STM-1

A more conventional truss model is typically used in STM as shown in figure 3.14. This model more resembles to the flow force. The locations of the struts were chosen to keep their angles with the horizontal ties above 45° and to represent the compression fan stress trajectories as accurately as possible near the applied loads. Ultimately, it is up to the judgment of the designer to come up with a model that will work in the geometry of the pier cap while following AASHTO guidelines.

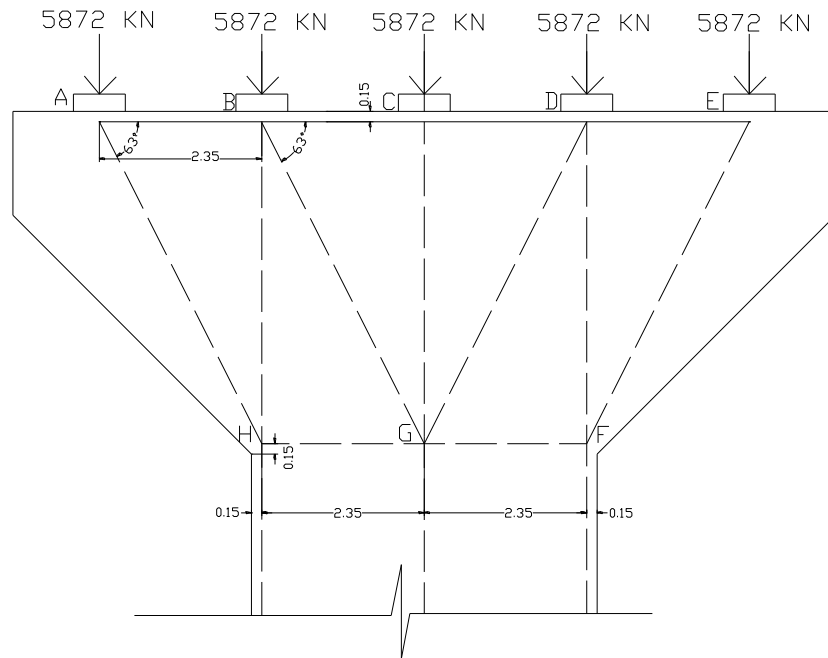


Figure 3-14: STM -2

5. Calculate forces in struts and ties.

The computer software SAP2000 was used to compute the forces in truss members. It is convenient to use software for this step since the model geometry can easily be changed and member forces recalculated if necessary. However, it is also a good idea to check some of the members by hand to verify that the model is set up properly using the software. All member forces are shown in Table 3.12.

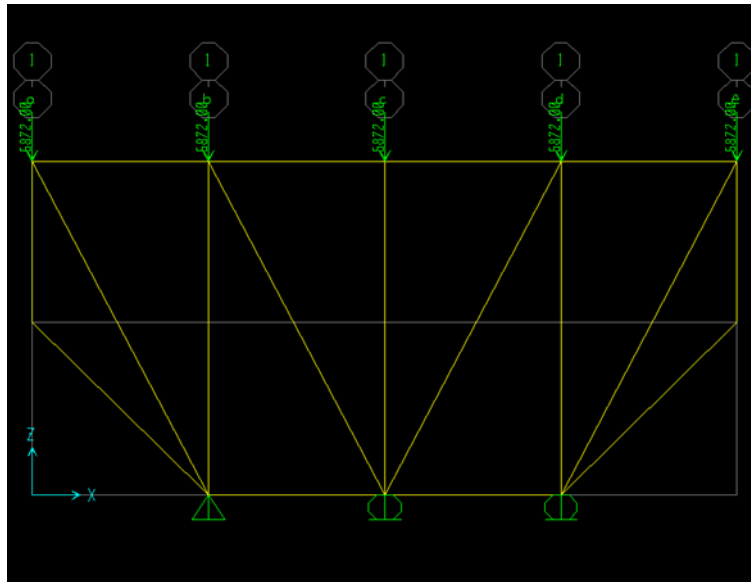


Figure 3-15: Sap Model for pier cap 3

Table 3-12 : Members Force

Members	Tension force(KN)
AB	2,991.9
BC	3,936.7
CD	2,991.9
DE	3,936.7

Members	Compression force (KN)
AH	-6,590.3
HB	-4,016.9
BG	-2,082.1
HG	-2,991.9
CG	-5,872
EF	-6,590.3
FD	-4,016.9
DG	-2,082.1
GF	-2,991.9

6. Size the area of ties.

Longitudinal bars and stirrups are sized and steel is chosen based on the calculation shown here:

Example: Ties AB & DE

$$A_{st} \geq 2,991.9 / (0.9 \times 400) = 8310.8 \text{ mm}^2$$

$$\text{Using } \Phi 32 \text{ bars, with } A_{st} = 804.3 \text{ mm}^2$$

$$\#32 \text{ bars required} = 8310.8 / 804.3 = 10.3, \text{ use } 12\Phi 32 \text{ bars.}$$

Example: Ties BC & CD

$$A_{st} \geq 3,936.7 / (0.9 \times 400) = 10,935.3 \text{ mm}^2$$

$$\text{Using } \Phi 32 \text{ bars, with } A_{st} = 804.3 \text{ mm}^2$$

$$\#32 \text{ bars required} = 10,935.3 / 804.3 = 13.6, \text{ use } 14\Phi 32 \text{ bars}$$

For the ties from B to D, the #32 bars can be placed in two rows of 7 bars.

Check the assumed location of the tie centroid for two rows of bars:

$$\begin{aligned} & 50(\text{clear cover}) + 24(\text{dia. of stirrup}) + 32(\text{dia. of bar}) + 16(\text{half the spacing between rows}) \\ & = 122 \text{ mm} \end{aligned}$$

So the assumed location of 150mm below the top edge of the pier cap is ok.

Check horizontal spacing of longitudinal steel:

$$\begin{aligned} & [\text{Cap width} - 2 \times (\text{clear cover}) - 4 \times (\text{dia of stirrups}) - 7 \times (\text{dia of bars})] / \# \text{ of space} \\ & = (1700 - 2 \times 50 - 4 \times 24 - 7 \times 32) / 6 = 213.3 \text{ mm} \end{aligned}$$

213.3mm is much larger than the minimum spacing requirement of $1.5d_b = 38.4$ Horizontal spacing of the longitudinal steel is not an issue with 7 bars in a row.

Surface reinforcement

$$A_s = 0.004A_c = 0.004 \cdot 3450 \cdot 1600 = 22080 \text{ mm}^2$$

$$\# \text{ Bars} = 222080/452.4 = 48.8, \text{ Provide } 3 \cdot 16 \Phi 24$$

7. Check stresses in the nodal zones and struts.

The bottom cord struts are checked to see if the estimated location of the truss members based on the sectional analysis is ok.

Strut HG & GF

$$P_u = -2,991.9 \text{ KN},$$

There are no ties designed to run through these struts, therefore, ϵ_1 is taken as 0.

$$F_{cu} = 24/0.8 = 30 \leq 0.85 \cdot 24 = 20.4, \text{ use } f_{cu} = 20.4 \text{ MPa}$$

$$\Phi P_n = 0.7 \cdot 20.4 \cdot 1000 \cdot 0.3 \cdot 0.75 = 3,213 \text{ KN} > 2,991.9 \text{ KN} \quad \text{ok!}$$

Nodal Zone B & D

The nodal zones for nodes B and D need to be checked before the struts connecting to them can be checked since the node width, and therefore the strut width, depend on the size of the bearing pad which is 750mm across. These nodes are considered CTT nodes since they connect two compressive struts with 2 tension ties. Therefore, the limiting compressive stress in the concrete nodal zone is taken as:

$$F_{cu} = 0.65 \Phi f'_c = 0.65 \cdot 0.75 \cdot 24 = 11.7 \text{ MPa}$$

$$A_{cs} = P_u / F_{cu} = 5872 / 11.7 = 501880 \text{ mm}^2 < 750 \cdot 750 = 562500 \text{ mm}^2 \quad \text{ok!}$$

Since the bearing area is larger than the required area of concrete to resist the applied load at B & D, the bearing width is adequate. Struts BG and DG will be checked based on the size of node B which is controlled by the bearing width.

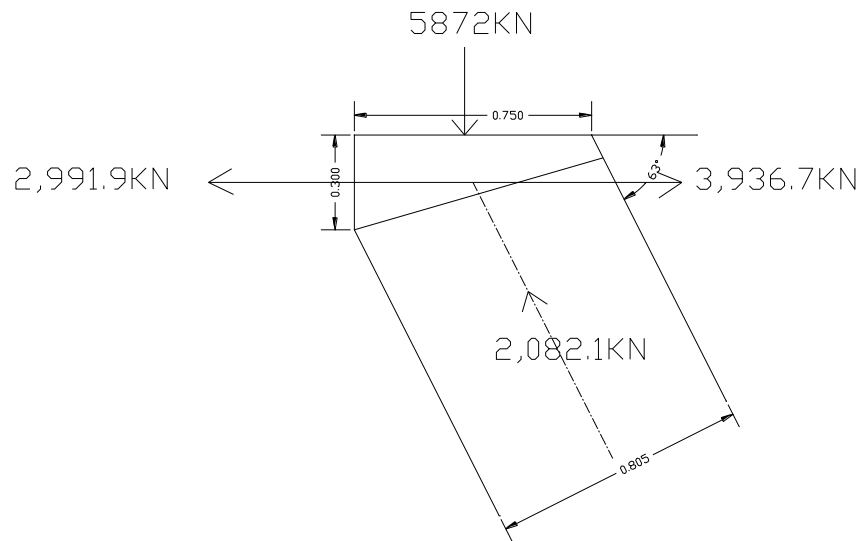


Figure 3-16: Node B

The nodal region and the forces acting on it are shown in figure 3.16. The dimensions of the node are based on the 750mm bearing width, the 150mm distance from the top edge to the center of the longitudinal steel and the following equation for the width of strut BG:

The angle, α_s between strut BG & BC is 63°

$$BG_{\text{width}} = 750\sin(63) + 300\cos(63) = 804.5\text{mm} = 0.8045\text{m}$$

Struts BG & DG

$$P_u = 2,082.1 \text{ KN}$$

The strain through the strut can be taken as the average strain in the two longitudinal ties that cross it which is tie BC and tie CD

Tie AB

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = 8310.8 * 400 / 9,651.6 = 344.4 \text{ MPa}$$

$$\epsilon_s \text{ in tie BC; } f_s/E = 344.4/210 = 0.00164$$

Tie BC

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = 10,935.3 * 400 / 11,260.2 = 388.5 \text{ MPa}$$

$$\epsilon_s \text{ in tie CD ; } f_s/E = 388.5/210 = 0.00185$$

Average strain in these ties is $\epsilon_s = 0.00175$

$$\epsilon_1 = 0.00175 + (0.00175 + 0.002) \cot^2 63 = 0.0027$$

$$f_{cu} = 24 / (0.8 + 170 * 0.0027) \leq 0.85 * 24$$

$$f_{cu} = 19 \leq 20.4, \text{ use } f_{cu} = 19 \text{ MPa}$$

According to AASHTO section 5.6.3.3.2, “The value of A_{cs} shall be determined considering both the available concrete area and the anchorage conditions at the ends of the strut. When a strut is anchored by reinforcement, the effective area may be considered to extend a distance of up to six bar diameters from the anchored bar” where the anchored bars are the longitudinal #32’s. Therefore, the distance of the effective A_{cs} is taken as:

$$A_{cs} = 804.5 * 750 = 603,375 \text{ mm}^2$$

$$\Phi P_n = 0.7 * 19 * 603,375 = 8,024.9 \text{ KN} > 2,082.1 \text{ KN} \quad \text{ok!}$$

Nodal Zone A & E

The nodal zones for nodes A and E need to be checked before the struts connecting to them can be checked since the node width, and therefore the strut width, depends on the size of the bearing pad which is 750mm across. These nodes are considered CCT nodes since they connect two compressive struts with one tension ties. Therefore, the limiting compressive stress in the concrete nodal zone is taken as:

$$F_{cu} = 0.75\Phi f'_c = 0.75*0.75*24 = 13.5\text{MPa}$$

$$A_{cs} = P_u/F_{cu} = 5872/13.5 = 434,962.9\text{mm}^2 < 750*750 = 562,500\text{mm}^2 \quad \text{ok!}$$

Since the bearing area is larger than the required area of concrete to resist the applied load at A& E, the bearing width is adequate. Struts AH and EF will be checked based on the size of node A which is controlled by the bearing width.

Struts AH & EF

$$P_u = 6,590.3 \text{ KN}$$

Tie AB

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = 8310.8*400/9,651.6 = 344.4\text{MPa}$$

$$\epsilon_s \text{ in tie BC; } f_s/E = 344.4/210 = 0.00164$$

$$\epsilon_1 = 0.00164 + (0.00164+0.002) \cot^2 63 = 0.0026$$

$$f_{cu} = 24/(0.8+170*0.0026) \leq 0.85*24$$

$$f_{cu} = 19.4 \leq 20.4, \text{ use } f_{cu} = 19.4\text{MPa}$$

$$A_{cs} = 804.5*750 = 603,375\text{mm}^2$$

$$\Phi P_n = 0.7*19.4*603,375 = 8,193.8\text{KN} > 6,590.3 \text{ KN} \quad \text{ok!}$$

Since the strut AH see the largest forces and are satisfactory, there is no need to check the other diagonal struts. In other words, the required stresses in the other struts and nodes will be significantly less than that of AH.

8. Provide adequate anchorage for steel tie reinforcement.

135° hooks are recommended for the shear stirrups. The hook length for #24 bars is $6d_b = 144\text{mm}$. For the longitudinal steel, anchorage will be provided by 90° hooks. The required development length for #32 bars is 655mm. (AASHTO 5.11.2.4). The length of the hook should

be at least $12d_b = 384\text{mm} \approx 400\text{mm}$. Steel anchorages can be seen in the steel layout drawings in figures 3.17 & 3.18.

9. Provide additional crack control reinforcement.

Check minimum crack control reinforcement in the vertical and horizontal direction for 300mm stirrup spacing:

$$A_{st} = 0.003bS = 0.003 \cdot 1700 \cdot 300 = 1,530\text{mm}^2 < 4 \cdot 452.4 = 1,809.6\text{mm}^2$$

Since 4-legged stirrups are used, it is convenient to use 4 bars of vertical and horizontal crack control reinforcement at the 300mm spacing since they can be easily tied.

The steel layout and spacing for the longitudinal and transverse reinforcement is shown in figure 3.17 for the entire pier cap. The typical cross section is also shown in figure 3.18.

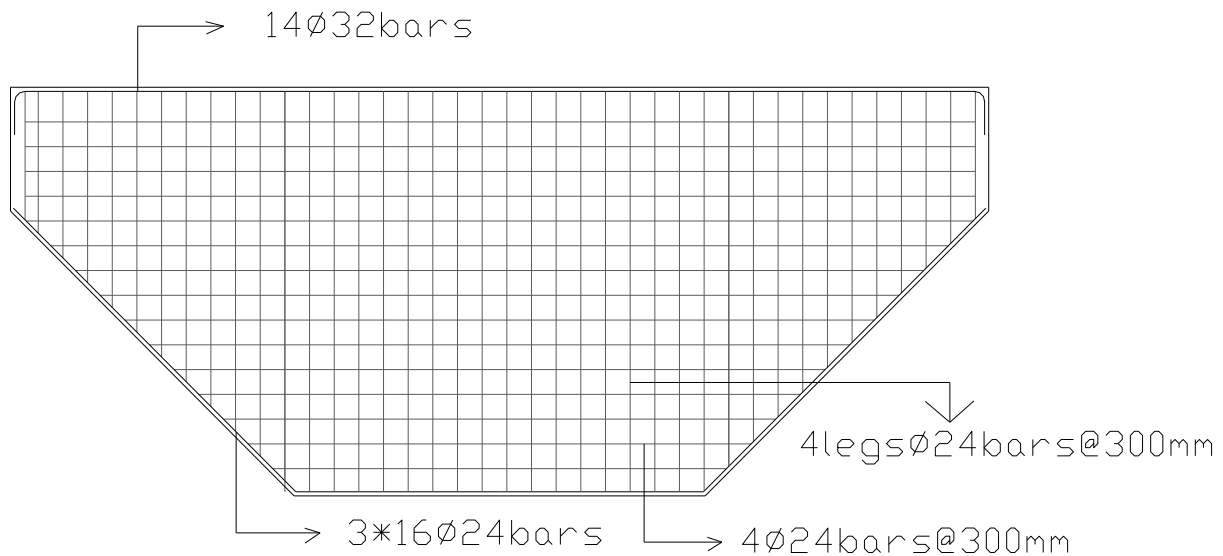


Figure 3-17: Steel Layouts

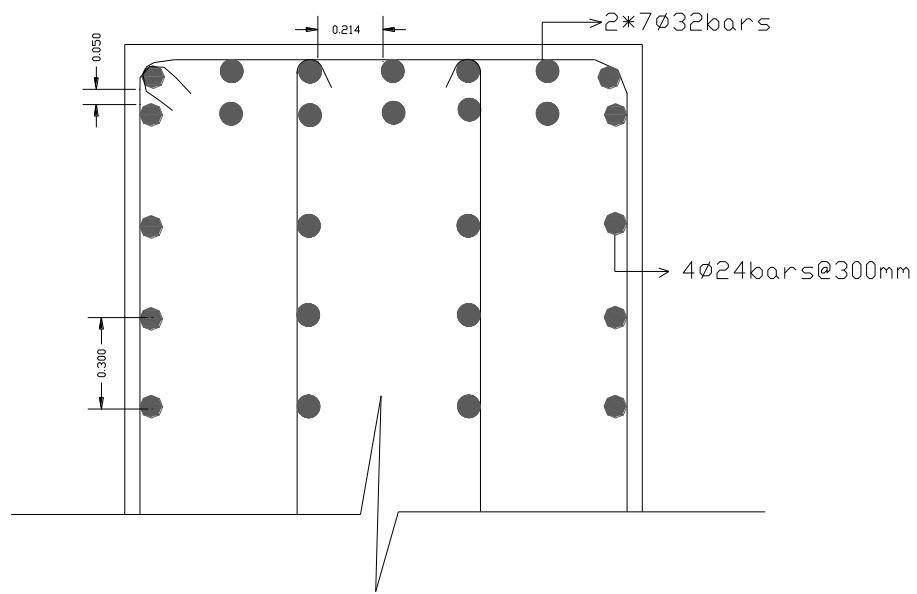


Figure 3-18: Typical Cross Sections

3.2.2.3. Design Calculation (According to EURO Code)

Euro code 2 allows for D- region structures design using strut and tie models. In this example it is intended to show the design of the above example according to euro code provision. Since it is the same process up to step5 we can refer from the above example.

6. Size the area of ties.

Longitudinal bars and stirrups are sized and steel is chosen as shown in table 3.14 based on the calculation shown here.

$$f_{yd} = f_{yk}/1.15 = 400/1.15 = 347.8 \text{ Mpa}$$

$$A_{st} \geq P_n / f_{yd}$$

Example: Ties AB & DE

$$P_n = 2,991.9 \text{ KN}$$

$$A_{st} \geq 2,991.9 / 347.8 = 8,602.4 \text{ mm}^2$$

Using Φ 32 bars, with $A_{st} = 804.3 \text{ mm}^2$

#32 bars required = $8,602/804.3 = 10.7$, use 11 Φ 32 bars.

Example: Ties BC& CD

$$P_n = 3,936.7 \text{ KN}$$

$$A_{st} \geq 3,936.7 / 347.8 = 11,318.9 \text{ mm}^2$$

Using Φ 32 bars, with $A_{st} = 804.3 \text{ mm}^2$

#32 bars required = $11,318.9/804.3 = 14.1$, use 16 Φ 32 bars.

For the ties from B to D, the #32 bars can be placed in two rows of 8 bars.

Check the assumed location of the tie centroid for two rows of bars:

$$50(\text{clear cover}) + 24(\text{dia. of stirrup}) + 32(\text{dia. of bar}) + 16(\text{half the spacing between rows}) = 122\text{mm}$$

So the assumed location of 150mm below the top edge of the pier cap is ok.

Check horizontal spacing of longitudinal steel:

$$= [\text{Cap width} - 2 * (\text{clear cover}) - 4 * (\text{dia of stirrups}) - 8 * (\text{dia of bars})] / \# \text{of space}$$

$$= (1700 - 2 * 50 - 4 * 24 - 8 * 32) / 7 = 178.3\text{mm}$$

178.3mm is much larger than the minimum spacing requirement of $1.5d_b = 38.4$ Horizontal spacing of the longitudinal steel is not an issue with 8 bars in a row.

Surface reinforcement.

$$A_{s,\text{surf}} \geq 0.005 A_{\text{ct,ext}}$$

$$A_{s,\text{surf}} \geq 0.005 * 3450 * 1600 = 27600$$

$$\# \text{ bars} = 27600 / 452.4 = 61, \text{ Provide } 3 * 21 \Phi 24$$

7. Check stresses in the nodal zones and struts.

Strength of strut where there is transverse tension

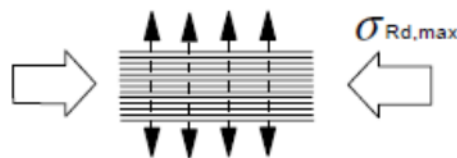


Figure 3-19 : Design strength of concrete struts with transverse tension

$$\sigma_{Rd,\text{max}} = 0.6nf_{cd} \quad (3.9)$$

$$n = 1 - f_{ck} / 250 \quad (3.10)$$

$$\sigma_{Rd,max} = 0.6 * \nu * f_{ck} = 0.6 * (1 - f_{ck} / 250) * f_{cd} = 8.976 \text{ MPa}$$

Strut AH & EF

$$P_u = -6,590.3 \text{ kN}$$

The angle α_s between strut BG & BC is 63°

$$BG_{width} = 750 \sin(63) + 300 \cos(63) = 804.5 \text{ mm} = 0.8045 \text{ m}$$

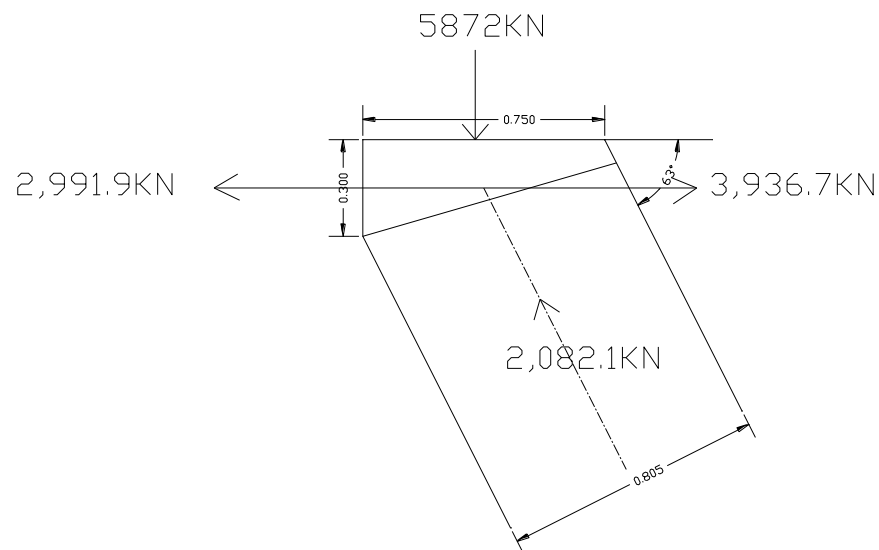


Figure 3-20: Node B

$$\text{Stress in struts} = \text{Force} / \text{Area} = 6,590.3 / 750 * 804.5 = 10.92 \text{ MPa}$$

$$10.92 \text{ MPa} \leq \sigma_{Rd,max} = 8.976 \text{ MPa}$$

ok!

Strut HG & GF

Strength of strut where there is no transverse tension

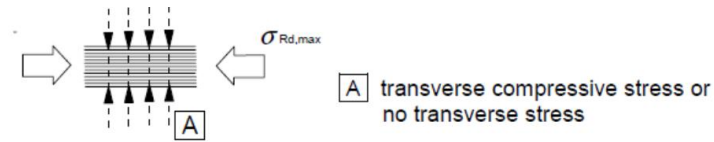


Figure 3-21: Design strength of concrete struts without transverse tension

$$\sigma_{Rd,max} = f_{cd} \quad (3.11)$$

$$\sigma_{Rd,max} = f_{cd} = 0.85f_{ck} / 1.5 = 17 \text{ MPa}$$

$$P_u = -2,991.9 \text{ KN}$$

$$\text{Stress in struts} = 2,991.9 / 300 \times 750 = 13.3 \text{ MPa}$$

$$\text{Stress in struts} = 13.3 \text{ MPa} \leq \sigma_{Rd,max} = 17 \text{ MPa} \quad \text{ok!}$$

Node B

The nodal zones for nodes B and D need to be checked before the struts connecting to them can be checked since the node width, and therefore the strut width, depend on the size of the bearing pad which is 750mm across. These nodes are considered in compression - tension nodes with anchored ties provided in more than one direction, Therefore, the limiting compressive stress in the concrete nodal zone is taken as:(see Figure 3.20).

c) In compression - tension nodes with anchored ties provided in more than one direction (see Figure 3.22)

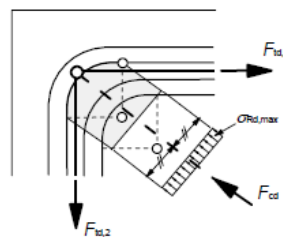


Figure 3-22: Compression tension node with reinforcement provided in two directions

$$\sigma_{Rd,max} = k_3 \eta f_{cd} \quad (3.12)$$

The recommended value of η in EURO Code2 is 0.75

$$\sigma_{Rd,max} = k_3 v' f_{cd}$$

$$\sigma_{Rd,max} = 0.75 * (1 - 24/250) * 17 = 11.22 \text{ MPa}$$

$$\text{Stress in struts BH} = 4,016.9 / (750 * 750) = 7.14 \text{ MPa} \leq \sigma_{Rd,max} = 11.22 \text{ MPa} \quad \text{ok!}$$

$$\text{Stress in struts BG} = 2,082.1 / (804.5 * 750) = 3.45 \text{ MPa} \leq \sigma_{Rd,max} = 11.22 \text{ MPa} \quad \text{ok!}$$

$$\text{Stress at node} = 5872 / 750 * 750 = 10.4 \text{ MPa} \leq \sigma_{Rd,max} = 11.22 \text{ MPa} \quad \text{ok!}$$

8. Provide adequate anchorage for steel tie reinforcement.

135° hooks are recommended for the shear stirrups. The hook length for #24 bars is $5d_b = 120\text{mm}$ (EURO CODE2, 8.5). For the longitudinal steel, anchorage will be provided by 90° hooks (EURO CODE2, 8.4.1). The required development length for #32 bars is 655mm. The length of the hook should be at least $12d_b = 384\text{mm} \approx 400\text{mm}$. Steel anchorages can be seen in the steel layout drawings in figures 3.23&3.24.

9. Provide additional crack control reinforcement.

Check minimum crack control reinforcement in the vertical and horizontal direction for 300mm stirrup spacing:

$$A_{s,surf} \geq 0.005 A_{ct,ext}$$

$$A_{s,surf} \geq 0.005 b S = 0.005 * 1700 * 200 = 1700 \text{ mm}^2 < 4 * 452.4 \text{ mm}^2$$

Since 4-legged stirrups are used, it is convenient to use 4 bars of vertical and horizontal crack control reinforcement at the 200mm spacing since they can be easily tied.

The steel layout and spacing for the longitudinal and transverse reinforcement is shown in figure 3.23 for the entire pier cap. The typical cross section is also shown in figure 3.24.

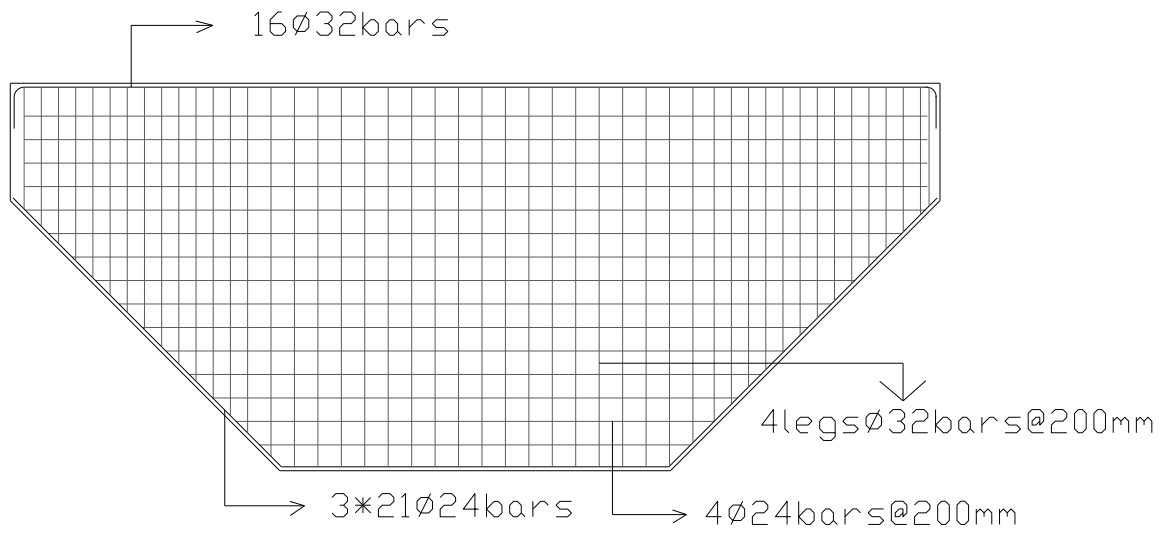


Figure 3-23: Steel Layouts

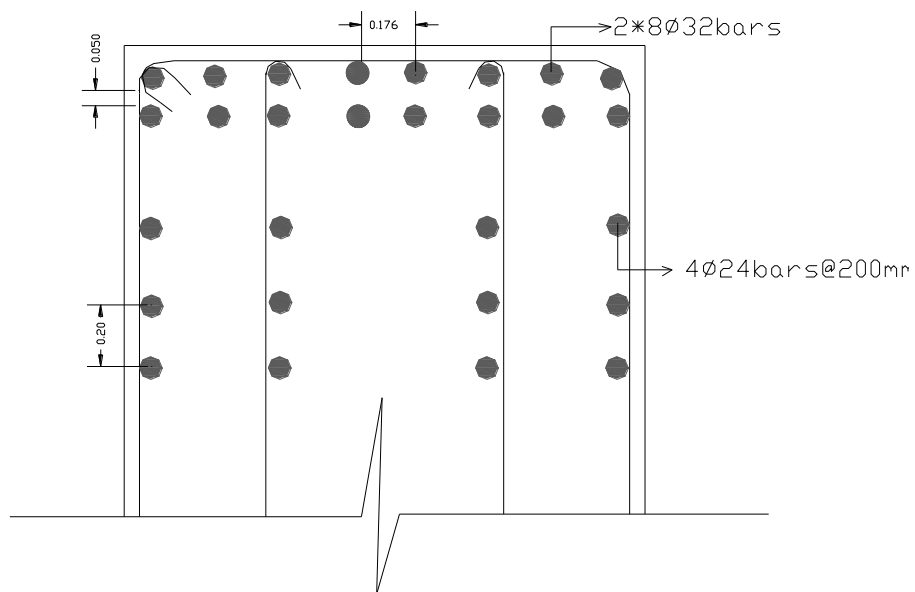


Figure 3-24: typical Cross Section

3.2.2.4. Design Calculation (Conventional Method)

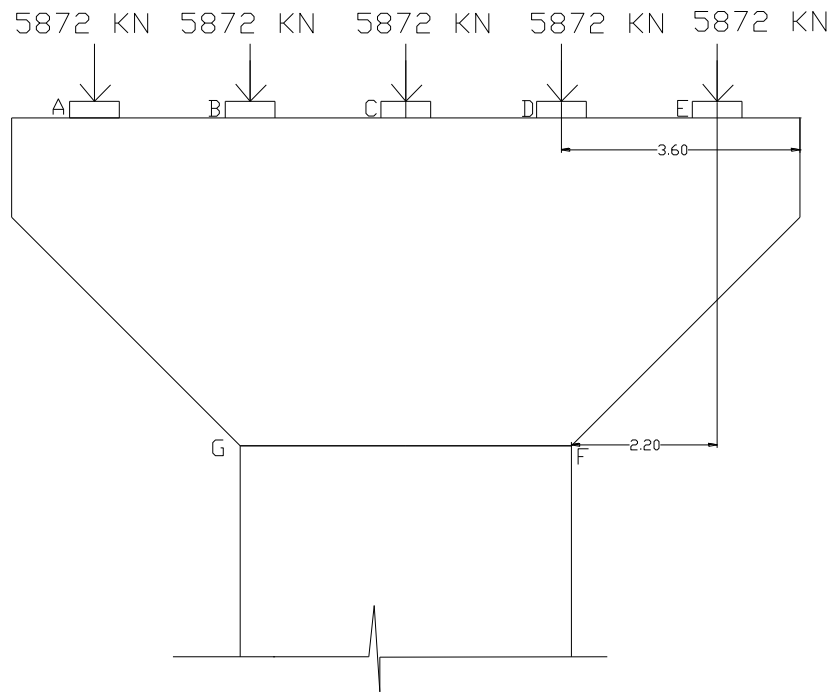


Figure 3-25: Pier cap final loading

Maximum moment occur at point F & G

$$M = 5872 \times 2.2 = 12918.4 \text{ KNm/m}$$

$$d_{\text{provided}} = 4.7 \times 1000 - 32 \times 2 - 50 - 24 = 4562 \text{ mm}$$

Maximum reinforcement is limited by ductility requirement, which is given in AASHTO art5.7.3.3.1.

$$a/d_e \leq 0.42$$

$$a \leq 0.42\beta d, \beta = 0.85 \text{ for } f'_c \leq 28 \text{ Mpa}$$

$$\text{then } a = 0.42 \times 0.85d = 0.357d$$

$$M_u = 0.85\Phi f'_c \cdot a \cdot b \cdot (d - (a/2))$$

$$= 0.85 \times 0.9 \times 0.357 \times (1 - 0.357/2)$$

$$= 0.2244 f'_c b d^2, \Phi = 0.9 \text{ for flexure}$$

$$d = \sqrt{(M_u / (0.2244f'_c b))} = 1187.97 \text{ mm} \leq d_{\text{provided}}$$

Reinforcement

$$a = A_s f_y / 0.85 f'_c b, f_y = 400 \text{ MPa}$$

$$a = 0.357d = 0.357 \times 4562 = 1628.634$$

$$A_s = a \times 0.85 f'_c b / f_y = 9,575.10 \text{ mm}^2$$

$$\rho = M_u / (\Phi f_y b d (d - a/2)) = 0.001032$$

$$\rho_{\min} = 0.03 f'_c / f_y = 0.001800 > \rho \quad \text{Use } \rho_{\min}$$

$$A_s = \rho b d = 13,959.7 \text{ mm}^2$$

$$\text{Area of one bar } (\Phi 32) = 804.3 \text{ mm}^2$$

$$\text{Spacing} = b \times a_s / A_s = 97.95 \text{ mm}$$

$$\text{Number of bars} = b/S = 17.4, \text{ use } 18 \Phi 32 \text{ bars.}$$

Design for Shear

Maximum shear occur at point F

$$V = 5872 \text{ KN}$$

Shear strength

$$V_u \leq \Phi V_n = \Phi (V_c + V_s), \Phi = 0.9$$

Where V_u = factored shear forces at the section

V_n = the nominal shear strength, determined as the lesser of $V_n = V_c + V_s$ or $V_n = 0.25(\sqrt{f'_c})bd_e$

V_c = the nominal shear strength provided by the concrete

V_s = the nominal shear strength provided by the shear reinforcement

Shear strength provided by concrete

$V_n = 0.25(\sqrt{f_c})bd_e = 0.25*(\sqrt{24})*1700*4562 = 9,498.4\text{KN} > V_u = 5,872\text{KN}$, no shear reinforcement is required

Minimum shear reinforcement

$$A_v = 0.083*\sqrt{f_c'}*b_w*S / f_y$$

$$S_{\max} = A_v*f_y / (0.083*\sqrt{f_c'}*b_w)$$

If $V_u < 0.10*f_c' * b_w * d$, $S \leq 0.8d \leq 600\text{mm}$

$$V_u = 5,872\text{KN} < 0.10*f_c' * b_w * d = 18,613\text{KN}$$

$$S \leq 3649.6\text{mm} \leq 600\text{mm}$$

ok!

Surface reinforcement

$$A_s = 0.004A_c = 0.004*3450*1600 = 22080\text{mm}^2$$

$$\# \text{ Bars} = 222080/452.4 = 48.8, \text{ Provide } 3*16 \Phi 24$$

The minimum horizontal crack control reinforcement that must also be provided is calculated based on a spacing of 300mm. as follows:

$$A_{st} = 0.003bS = 0.003*1700*300 = 1530\text{mm}^2 \leq 4*452.4 \text{ mm}^2$$

Table 3-13 : Summary of reinforcement of refined STM according to AASHTO, EURO code & Steel bar using Conventional approach.

	Steel bar using refined STM according to AASHTO	Steel bar using refined STM according to EURO code	Steel bar using Conventional approach.
Main top reinforcement	14 Φ 32	16 Φ 32	18 Φ 32
Horizontal crack control reinforcement	4 Φ 24 @ 300 mm	4 Φ 24 bars @ 200mm	4 Φ 24 @ 140 mm
Stirrups & vertical crack control reinforcement	4 legs , Φ 24 bars @ 300mm	4 legs , Φ 24 bars @200mm	4 legs , Φ 24 bars @300mm

3.2.3. Example 3: Hammerhead Pier Cap Design

3.2.3.1. Description of Structure

The bridge pier designed in this section is from AACRA, Rwanda over pass Bridge that have box- girder superstructure design example. The procedure for the strut and tie modeling of the pier is demonstrated in this section.

Table 3-14 : Material Properties

Material Property	values
Concrete Density	25kg/m ³
Concrete 28-day Compressive Strength	$f'_c=24\text{MPa}$
Steel Reinforcement Strength	$f_y=400\text{MPa}$

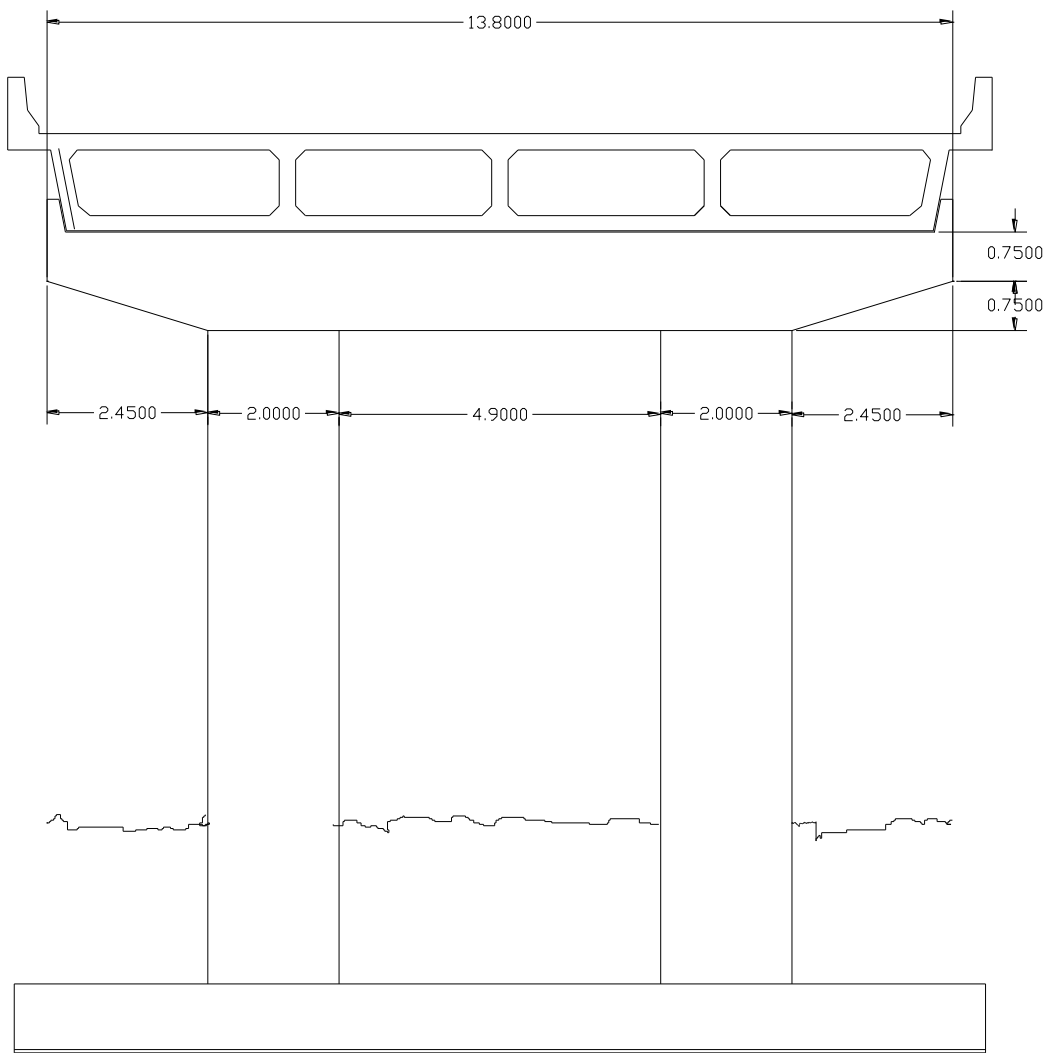


Figure 3-26: Bridge Pier Dimensions Front Elevation

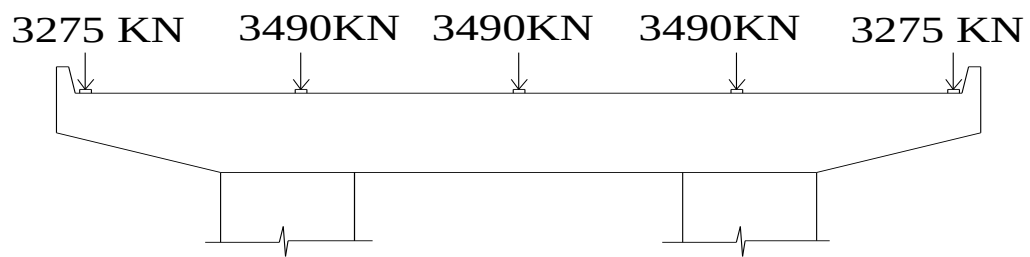


Figure 3-27: Pier Cap Final Loading

3.2.3.2. Design Calculation

1. Delineate D-regions from B-regions

Figure 3.26 shows a pier cap-beam as an alternative to the hammerhead pier cap shape. These are generally fairly deep beams composed of mostly or entirely B-regions.

2. Determine the Boundary Conditions on the D-region

To generalize the pier cap as a truss, the column under it is considered each as two compressive struts.

3. Visualize load path / sketch the flow of stresses

SAP2000 software was used to generate the principle stresses in the pier cap due to the final loading conditions. It is easy to visualize where the stress distributions are concentrated. The load path also used to visualize with the orientation of struts and ties in the model.

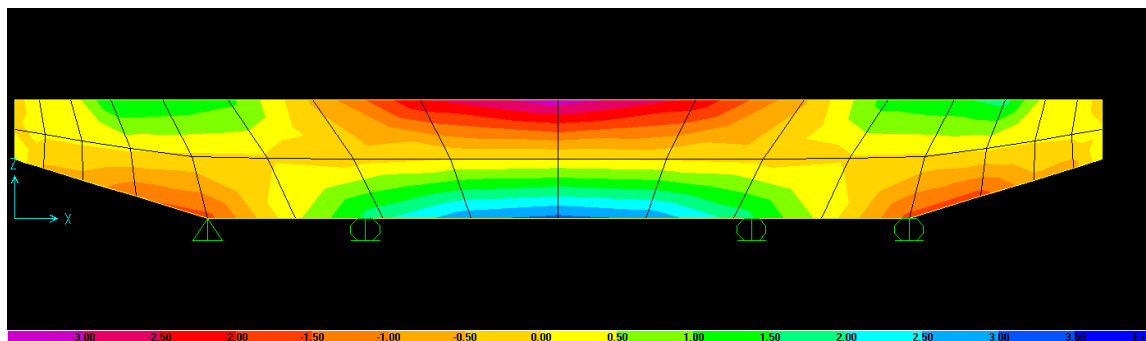


Figure 3-28: Pier cap stress distribution

4. Develop a STM that is compatible with the flow of forces.

The model used should realistically represent the distribution of stresses from step 3. The geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure & elastic stress distribution. The column supporting the pier cap can be broken into two compressive struts. Each strut takes approximately the same load from bearing. At a distance below the column connection to the pier cap, the compressive stresses in the column are uniform. Therefore, the widths of these struts are calculated based on

the percentage of the load they carry to the total load in the column. Each strut is 1 m wide. Therefore, the centerlines of the two struts are located a distance of $1/2 = 0.5\text{m}$ from the edge of the column.

It was estimated that the horizontal tie spanning the top of the pier cap should be placed 150 mm from the top edge of the cap. This accounts for the 50mm clear cover, shear ties and two layers of longitudinal steel that is expected based on the sectional analysis. The limiting stress in the concrete is estimated as $0.85f'_c$ since the struts are not anchored to any ties. From the analysis, it was determined that the centerline of the strut should be placed about 160 mm above the bottom of the pier cap. Calculations are shown here:

$$M_u = 3275 \times 2.53 = 8285.75 \text{ KNm}$$

$$T = C, A_s F_y = F_{cu} * a * b \quad b = 800 \text{ mm}$$

$$M_n = A_s F_y (d - a/2) = 0.85 f'_c a b (d - a/2)$$

Estimate for d

$$D = 1500 (\text{pier cap depth}) - 50 (\text{clear cover}) - 24 (\text{stirrup}) - 32 (\text{bar}) - 20 (\text{half bar spacing}) = 1374 \text{ mm}$$

$$M_u = \Phi M_n = \Phi * 0.85 * f'_c * a * b * (d - a/2), \Phi = 0.7$$

$$8285.75 * 10^6 = 0.7 * 0.85 * 24 * a * 1500 (1374 - a/2)$$

$$a \geq 318.5 \text{ mm}, a = 320 \text{ mm} \leq 12d_b = 32 * 12 = 384 \text{ mm}$$

Therefore, the lines representing the centerline of the bottom struts will be placed at 160mm from the edge of the pier cap and the centerline of the longitudinal ties will be spaced 150mm from the top edge of the beam as shown in figure 3.29.

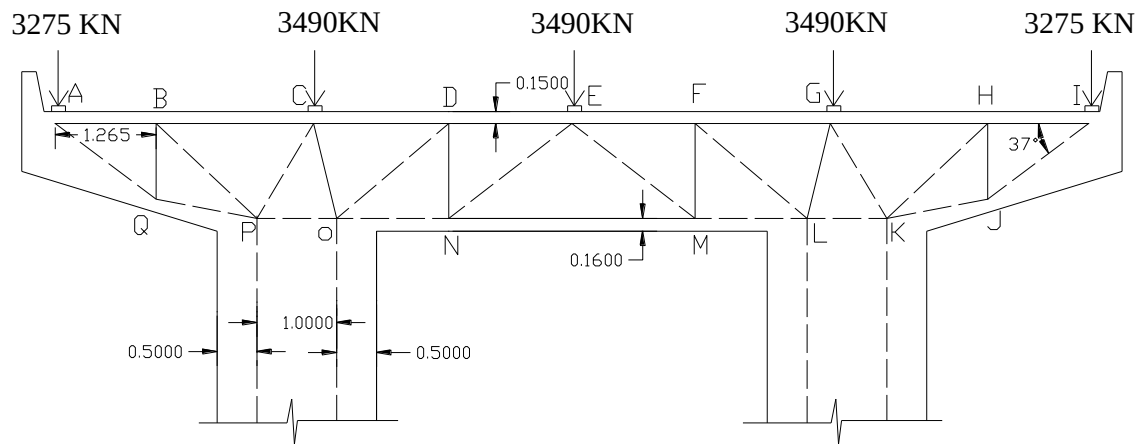


Figure 3-29: STM

5. Calculate forces in struts and ties.

The computer software Sap was used to compute the forces in truss members. All member forces are shown in table 3.15.

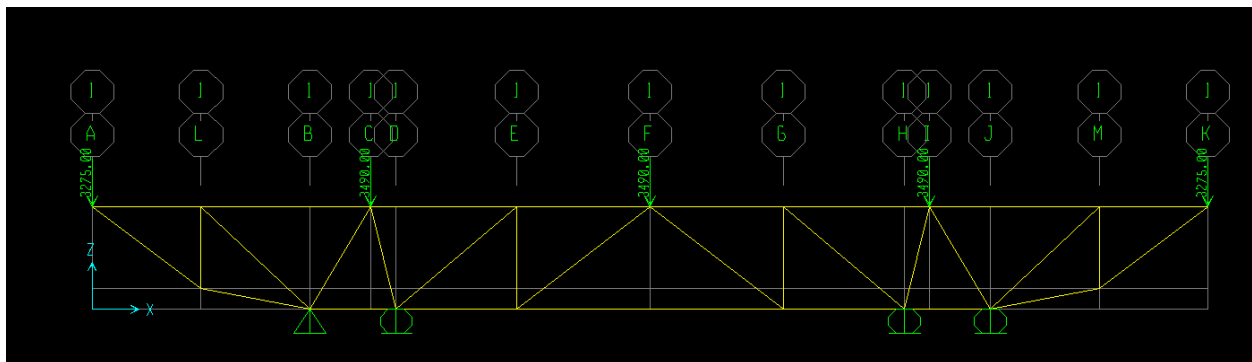


Figure 3-30: Sap Model for pier cap

Table 3-15 : Members Force

Members	Tension force(KN)
AB	3881
BC	6244
CD	3259
DE	1225
BQ	1680
CO	980
DN	1328
NM	525
IH	3881
HG	6244
GF	3259
FE	1225
HJ	1680
GL	980
FM	1328

Members	compression force(KN)
AQ	-4749
QP	-4332
PO	-3745
ON	-1469
BP	-2953
PC	-5134
DO	-2310
EN	-2283
IJ	-4749
JK	-4332
KL	-3745
LM	-1469
HK	-2953
GK	-5134
FL	-2310
EM	-2283

6. Size the area of ties.

Longitudinal ties are sized and steel is chosen as shown below:

Example: Ties AB ,BC,GH& HI

$$A_{st} \geq 6244 / (0.9 * 400) = 17344.4 \text{ mm}^2$$

Using Φ 32 bars, with $A_{st} = 804.3 \text{ mm}^2$

#32 bars required = $17344.4 / 804.3 = 21.6$, use 24# 32 bars.

Example: Ties CD, DE, EF, & FG

$$A_{st} \geq 3259 / (0.9 * 400) = 9053 \text{ mm}^2$$

Using Φ 32 bars, with $A_{st} = 804.3 \text{ mm}^2$

#32 bars required = $9053 / 804.3 = 11.3$ use 12# 32 bars.

Example: Tie NM

$$A_{st} \geq 425 / (0.9 * 400) = 1250 \text{ mm}^2$$

Using Φ 20 bars, with $A_{st} = 314.2 \text{ mm}^2$

#32 bars required = $1250 / 314.2 = 3.97$ use 4# 20 bars.

For the ties from A to D & F to I, the #32 bars can be placed in two row 12 Φ 32 bar.

Check the assumed location of the tie centroid for two rows of bars:

$$50(\text{clear cover}) + 24(\text{dia. of stirrup}) + 32(\text{dia. of bar}) + 16(\text{spacing between rows}) = 122 \text{ mm}$$

So the assumed location of 150mm below the top edge of the pier cap is ok.

Check horizontal spacing of longitudinal steel:

$$[\text{Cap width} - 2 * (\text{clear cover}) - 4 * (\text{dia of stirrups}) - 11 * (\text{dia of bars})] / \# \text{ of space}$$

$$= (1500 - 2 * 50 - 4 * 24 - 12 * 32) / 11 = 83.64 \text{ mm}$$

83.64mm is much larger than the minimum spacing requirement of $1.5d_b = 38.4$ Horizontal spacing of the longitudinal steel is not an issue with 8 bars in a row.

The vertical ties represent the centroid of stirrups that will be spaced across a “stirrup band”. For ties BQ and HJ the band spans a distance of $1.265 / 2 = 0.6325 \text{ m}$ in each direction. The total strength of all the stirrups in this band must be greater than the force in BQ and HJ respectively. The calculation for the spacing of the stirrups is shown here based on using $\Phi 20$ 4-legged stirrups:

Tie BQ and HJ

$$\Phi P_n = \Phi f_y A_{st} (\text{stirrups}) \geq P_u ; \Phi = 0.9$$

$$\# \text{ Stirrups} = P_u / (\Phi f_y A_{st}) = 1680 \times 1000 / (0.9 \times 400 \times 4 \times 314.2) = 3.71$$

The required spacing within the 1265mm band is

$$S \leq 1265 / 3.71 = 340 \text{ mm, use spacing of 300 mm, } \Phi 20$$

$$A_{st} = 1265 \times (4 \times 314.2) / 300 = 5299.5 \text{ mm}^2$$

Minimum transverse reinforcement in this region

$$A_v = 0.0316 \times f_c^{0.5} \times b_v S / f_y$$

$$A_v = 0.0316 \times 24^{0.5} \times 1500 \times 300 / 400 = 174.2 \text{ mm}^2 \times 4 \times 314.2 > 174.2 \text{ mm}^2 \quad \text{ok!}$$

7. Check stresses in the nodal zones and struts.

The bottom cord struts are checked to see if the estimated location of the truss members based on the sectional analysis is ok.

Strut PO & LK

$$P_u = -3745 \text{ KN, } \epsilon_1 = 0$$

$$f_{cu} = 24 / 0.8 = 30 \leq 0.85 \times 24 = 20.4, \text{ use } f_{cu} = 20.4 \text{ MPa}$$

$$\Phi P_n = 0.7 \times 20.4 \times 1000 \times 0.32 \times 1.5 = 6854.4 \text{ KN} > 3745 \text{ KN} \quad \text{ok!}$$

Strut QP & KJ

$$P_u = -4332 \text{ KN}$$

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = [1680/(0.9*400)] * 400/5299.5 = 0.352 \text{ MPa}$$

$$\epsilon_s \text{ in tie CO; } f_s/E = 0.352/210 = 0.00168 \text{ m/m}$$

The angle, α s between strut QP& QB is 79°

$$\epsilon_1 = 0.00168 + (0.00168 + 0.002) \cot^2 279 = 0.00182$$

$$f_{cu} = 24/(0.8 + 170 * 0.00182) \leq 0.85 * 24$$

$$f_{cu} = 21.64 \text{ MPa} \leq 20.4 \text{ MPa, use } f_{cu} = 20.4$$

$$\Phi P_n = 0.7 * 20.4 * 1000 * 0.32 * 1.5 = 6854.4 \text{ KN} > 4332 \text{ KN} \quad \text{ok!}$$

Nodal Zone C,E& G

The nodal zones for nodes B need to be checked before the struts connecting to them can be checked since the node width and therefore the strut width, depend on the size of the bearing pad which is 400mm across. These nodes are considered CTT nodes since they connect one compressive strut with 2 or more tension ties. Therefore, the limiting compressive stress in the concrete nodal zone is taken as:

$$F_{cu} = 0.65 \Phi f'_c = 0.65 * 0.75 * 24 = 11.7 \text{ MPa}$$

$$A_{cs} = P_u / F_{cu} = 3490 / 11.7 = 298,290 \text{ mm}^2 < 400 * 400 = 160,000 \text{ mm}^2$$

Since the bearing area is less than the required area of concrete to resist the applied load at C & G, the bearing width is not adequate and increases the concrete strength. Try bearing area $450 \times 450 = 202,500 \text{ mm}^2$

$$F_{cu} = 0.65 \Phi f'_c = 0.65 * 0.75 * 40 = 19.5 \text{ MPa}$$

$$A_{cs} = P_u / F_{cu} = 3490 / 19.5 = 178,975 \text{ mm}^2 < 450 * 450 = 202,500 \text{ mm}^2$$

Struts AQ and IJ will be checked based on the size of node A & I which is controlled by the bearing width.

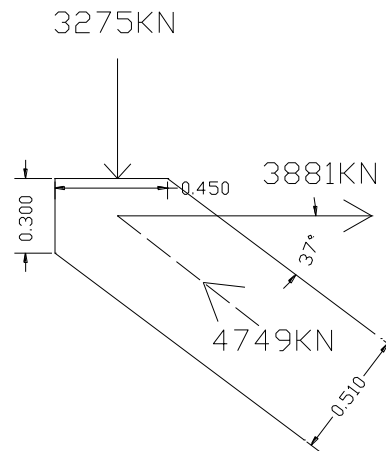


Figure 3-31: Nodes A

The nodal region and the forces acting on it are shown in figure 3.31. The dimensions of the node are based on the 450mm bearing width, the 150mm distance from the top edge to the center of the longitudinal steel and the following equation for the width of strut AQ:

$$AQ_{\text{width}} = 450 \sin (37) + 300 \cos (37) = 510.1 \text{ mm}$$

Struts AQ & IJ

$$P_u = 4749 \text{ KN}$$

The strain through the strut can be taken as the strain in the longitudinal ties that cross it which is tie AB.

Tie AB

$$f_s = (A_{\text{st,required}}/A_{\text{st,provided}}) * f_y = 17344.4 * 400 / 19303.2 = 359.41 \text{ MPa}$$

$$\epsilon_s \text{ in tie AB ; } f_s/E = 359.41/210 = 0.00171$$

The angle, α s between strut AQ & AB is 37°

$$\epsilon_1 = 0.00171 + (0.00171 + 0.002) \cot^2 37 = 0.0082$$

$$f_{cu} = 40 / (0.8 + 170 * 0.0082) \leq 0.85 * 40$$

$$f_{cu} = 18.17 \leq 34, \text{ use } f_{cu} = 18.17$$

$$A_{cs} = 390.61 * 1400 = 546,854 \text{ mm}^2$$

$$\Phi P_n = 0.7 * 18.17 * 546,854 = 6955.4 \text{ kN} > 4749 \text{ kN}$$

ok!

Struts CP & GK

Struts CP & GK will be checked based on the size of node C & G which is controlled by the bearing width.

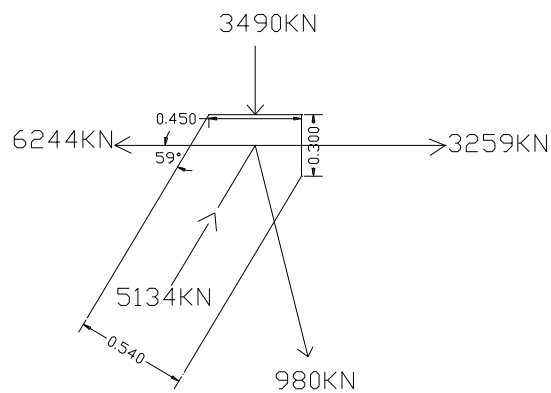


Figure 3-32: Node C

$$CP_{width} = 450 \sin (59) + 300 \cos (59) = 540.2 \text{ mm}$$

$$P_u = 5134 \text{ kN}$$

The strain through the strut can be taken as the average strain in the two longitudinal ties that cross it which is tie BC and tie CD.

Tie BC

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = 17344.4 * 400 / 19303.2 = 359.41 \text{ MPa}$$

$$\epsilon_s \text{ in tie BC ; } f_s/E = 359.41/210 = 0.00171$$

Tie CD

$$f_s = (A_{st,required}/A_{st,provided}) * f_y = 9053 * 400 / 9651.6 = 375.2 \text{ MPa}$$

$$\epsilon_s \text{ in tie BC ; } f_s/E = 375.2/210 = 0.00179$$

$$\epsilon_s = 0.00175$$

The angle, α_s between strut AQ & AB is 59°

$$\epsilon_1 = 0.00175 + (0.00175 + 0.002) \cot^2 59 = 0.0031$$

$$f_{cu} = 40 / (0.8 + 170 * 0.0031) \leq 0.85 * 40$$

$$f_{cu} = 30.1 \leq 34, \text{ use } f_{cu} = 30.1$$

$$A_{cs} = 540.2 * 1400 = 756,331.4 \text{ mm}^2$$

$$\Phi P_n = 0.7 * 30.1 * 756,331.4 = 15,935 \text{ kN} > 5134 \text{ kN} \quad \text{ok!}$$

Since the nodal zone C and strut CP see the largest forces and are satisfactory, there is no need to check the other diagonal struts or node regions. In other words, the required stresses in the other struts and nodes will be significantly less than that of CP and C respectively.

8. Provide adequate anchorage for steel tie reinforcement.

135° hooks are recommended for the shear stirrups. The hook length for #20 bars is $6d_b = 120 \text{ mm}$. For the longitudinal steel, anchorage will be provided by 90° hooks. The required development length for #32 bars is 655 mm. (AASHTO 5.11.2.4). The length of the hook should be at least $12d_b = 384 \text{ mm} \approx 390 \text{ mm}$. Steel anchorages can be seen in the steel layout drawings in figures 3.26 through 3.28.

9. Provide additional crack control reinforcement.

Check minimum crack control reinforcement in the vertical direction for 300 stirrup spacing:

$$A_{st} = 0.003bS = 0.003*1500*300 = 1350\text{mm}^2 > 4*314.2\text{mm}^2$$

Since spacing of 300mm doesn't satisfy the minimum crack control reinforcement it needs a correction.

$$A_{st} = 0.003bS = 0.003*1500*200 = 900\text{mm}^2 < 4*314.2\text{mm}^2$$

The minimum horizontal crack control reinforcement that must also be provided is calculated based on a spacing of 250mm. as follows:

$$A_{st} = 0.003bS = 0.003*1500*250 = 1125\text{mm}^2$$

Since 4-legged stirrups are used, it is convenient to use 4 bars of horizontal crack control reinforcement at the 250mm spacing since they can be easily tied to the stirrups.

Use 4 #20 bars. $A_{st,provided} = 4(314.2) = 1256.8\text{mm}^2$

The steel layout and spacing for the longitudinal and transverse reinforcement is shown in figure 3.33 for the entire pier cap. The different cross sections where the longitudinal steel varies are also shown in figure 3.34& 3.35.

Table 3-16 : Summary of reinforcement of refined STM & conventional approach

	Steel bar using refined STM	Steel bar using Conventional approach.
Main top reinforcement	24Φ32	20Φ32
	12Φ32	20Φ32
Horizontal crack control reinforcement	4Φ20 @ 250 mm	8Φ20 @ 200 mm
Stirrups & vertical crack control reinforcement	4 legs Φ20 bars @ 200mm	4 legs Φ14 bars @ 200mm

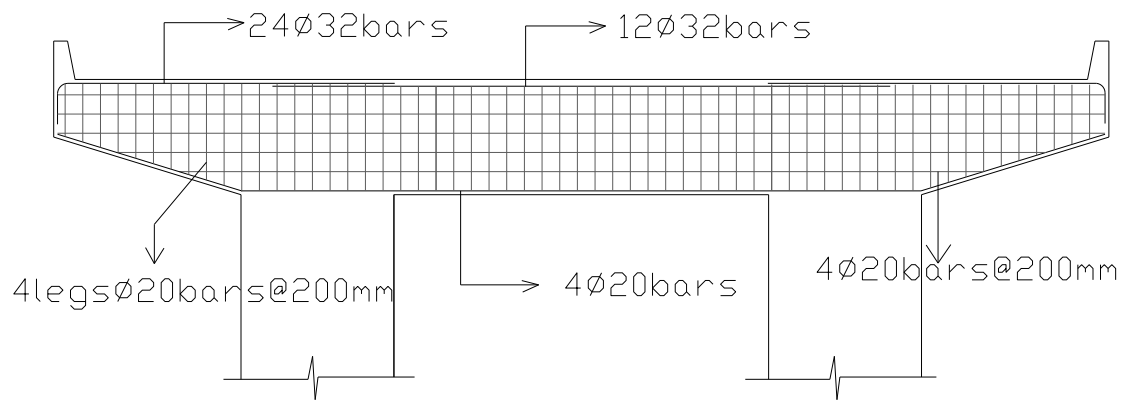


Figure 3-33: Steel Layouts

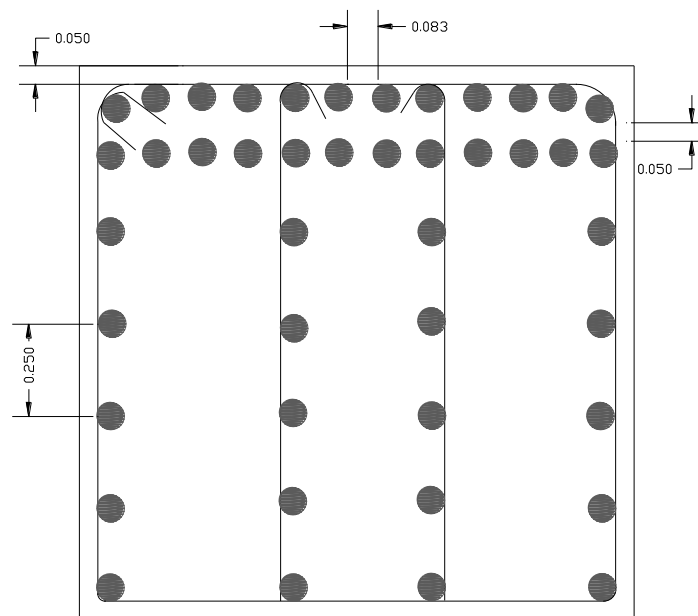


Figure 3-34: Cross Sections for A-C & G-I

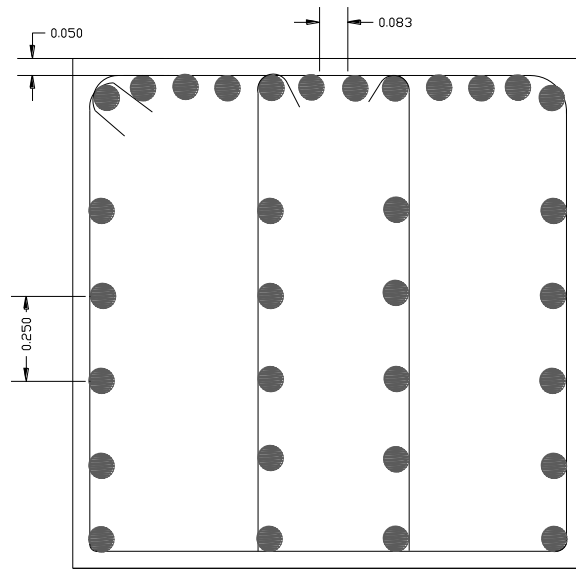


Figure 3-35: Cross Sections for C-G

3.2.4. Example 4: Pier Design

3.2.4.1. Description of Structure

The bridge pier designed in this section is from Ethiopian Road Authority, Tekeze Bridge .The procedure for the strut and tie modeling of the pier is demonstrated in this section.

Table 3-17 : Material Properties

Material Property	values
Concrete Density	25kg/m ³
Concrete 28-day Compressive Strength	$f'_c=24\text{MPa}$
Steel Reinforcement Strength	$f_y=400\text{MPa}$

Axial load: $N= 31842 \text{ KN}$

$M_x = 15561 \text{ KNm}$

$M_y = 6213 \text{ KNm}$

Pier dimension = 5mx1.7m

$d1 = 5000-50-32-12 = 4906\text{mm}$

$d2 = 1700-50-32-12 =1606\text{mm}$

The Strut-and-tie modeling approach is based on axial forces and therefore to convert the actual boundary actions to Strut-and-Tie Model assumptions, the effect of moment has to be converted to an equivalent static action. To this end, the following expression using the force-couple approach will yield the desired result.

The alternative assumption may include

i) $C \text{ or } T = M/ z$, where, $z \approx 0.9d$

Where, f_{cd} =design compressive strength of concrete

x = the depth of compressive concrete

z = moment arm

d = effective depth of the section

$$C_x = M_x/z = 15561 / (0.9 \times 4906) = 3525 \text{ KNm}$$

$$C_y = M_y/z = 6213 / (0.9 \times 1606) = 4299 \text{ KNm}$$

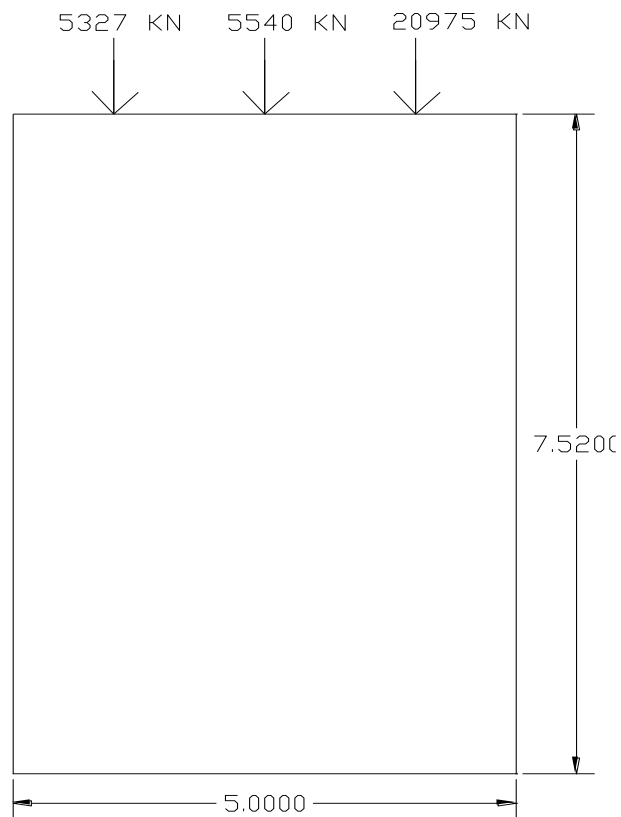


Figure 3-36: The boundary force on the Column

3.2.4.2. Design calculation

1. Delineate D-regions from B-regions

The entire pier is considered to be in the D-region since the width of the pier is wider compared to the length.

2. Determine the Boundary Conditions on the D-region

To generalize the pier as a truss, the column under it is considered as three compressive struts. These struts are resisted by three supports. Two prevent displacement in the y-direction only, while the third prevents displacement in the x and y directions. The x-direction location of these struts and corresponding supports depend on the force in each strut and will be determined with the development of the truss model.

3. Visualize load path / sketch the flow of stresses

SAP2000 software was used to generate the principle stresses in the pier cap due to the final loading conditions. It is easy to visualize where the stress distributions are concentrated. The load path also used to visualize with the orientation of struts and ties in the model.

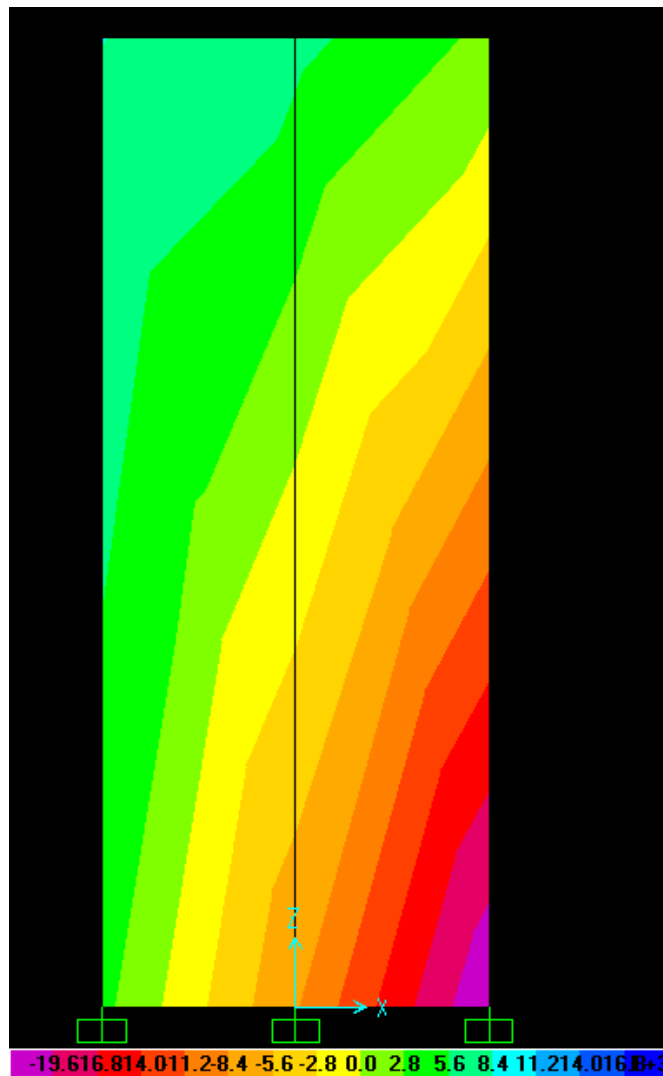


Figure 3-37: Pier stress distribution

The model used should realistically represent the distribution of stresses from step 3 and to make a compatible flow force from the pier cap designed on the above section. The geometry of the STM is determined using knowledge of the locations of the applied loads, boundary forces, geometry of the structure & elastic stress distribution.



5. Calculate forces in struts and ties.

The computer software SAP2000 was used to compute the forces in truss members. All member forces are shown in Table 3.18.

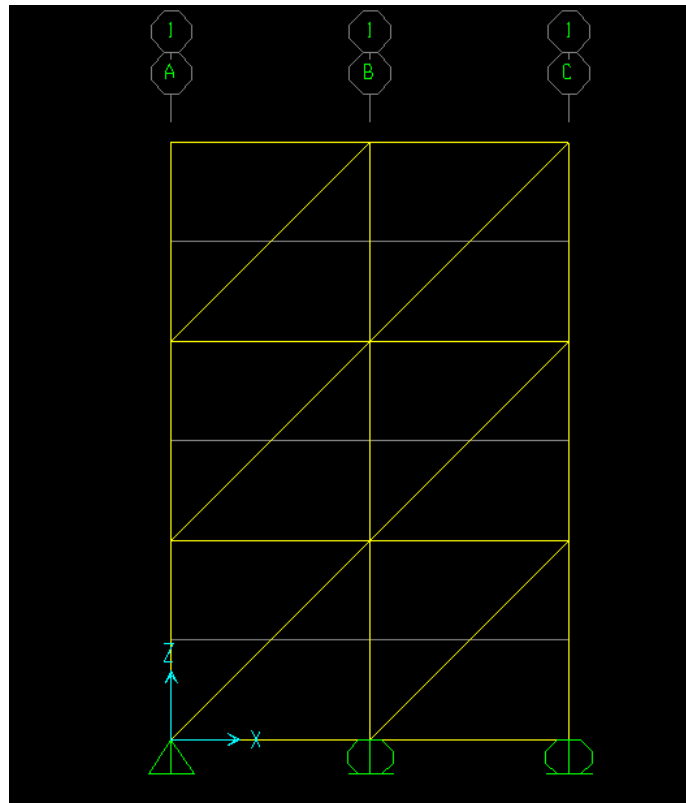


Figure 3-39: Sap Model for pier

Table 3-18 : Members Force

Compression Members	Force (KN)
AB	-5301
BC	-3663
CD	-2816
EF	-7176
FG	-9692
GH	-11355
IJ	-19894
JK	-18446
KL	-17626
FI	-2375
GJ	-1242
HK	-1212
CG	-823
DH	-800

Tension Members	Force (KN)
AE	24
EI	1662
BF	1620
FJ	864
GK	836
HL	22
BE	2276
CF	1160
DG	1108

6. Size the area of ties and longitudinal bars.

Tie BE

$$2276 \cdot \cos(59) = 1173$$

$$A_{st} \geq 1173 / (0.9 \cdot 400) = 3257 \text{ mm}^2$$

$$\text{Using } \Phi 12 \text{ bars, with } A_{st} = 113.1 \text{ mm}^2$$

$$\#32 \text{ bars required} = 3257 / 113.1 = 28.8 \approx 29$$

$$\text{Spacing, } 2520 / 29 = 87 \text{ mm}$$

Use 12 Φ c/c 80 mm for the top 2.52 m

Tie CF

Horizontal force, $1160 \cdot \cos(59) = 598 \text{ kN}$

$$A_{st} \geq 598 / (0.9 \cdot 400) = 1660 \text{ mm}^2$$

Using Φ 12 bars, with $A_{st} = 113.1 \text{ mm}^2$

$$\#32 \text{ bars required} = 1660 / 113.1 = 14.7 \approx 15$$

$$\text{Spacing, } 2500 / 15 = 166 \text{ mm}$$

Use 12 Φ c/c 160 mm for the middle 2.5 m of the column

Tie DG

$$1108 \cdot \cos(59) = 571$$

$$A_{st} \geq 571 / (0.9 \cdot 400) = 1586 \text{ mm}^2$$

Using Φ 12 bars, with $A_{st} = 113.1 \text{ mm}^2$

$$\#32 \text{ bars required} = 1586 / 113.1 = 14.01 \approx 15$$

$$\text{Spacing, } 2500 / 15 = 166 \text{ mm}$$

Use 12 Φ c/c 160 mm for the bottom 2.5 m of the column

Tie EI

$$A_{st} \geq 1662 / (0.9 \cdot 400) = 4617 \text{ mm}^2$$

Using Φ 12 bars, with $A_{st} = 113.1 \text{ mm}^2$

$$\# \text{ Bars required} = 4617 / 113.1 = 40.8 \approx 41$$

$$\text{Spacing, } 1260 / 42 = 30.73 \text{ mm}$$

Use 12 Φ c/c 30mm for the above 1.26m of the column.

The minimum ratio of tie reinforcement to total volume of concrete core

$$r \geq 0.45(A_g/A_c - 1) f'_c/f_{yk}$$

Where

A_g = gross area of concrete section (mm^2)

A_c = area of core measured to the outside diameter of the spiral (mm^2)

f'_c = specified strength of concrete

f_{yk} = specified yield strength of tie reinforcement

$$r \geq 0.45(5*1.7/4.9*1.6 - 1) 24/400$$

$$r \geq 0.00051$$

$$A_{st} \geq 0.00051*5*1.7 = 4684 \text{ mm}^2$$

Bar BE

$$2276*\sin(59) = 1951$$

$$A_{st} \geq 1951 / (0.9*400) = 5420 \text{ mm}^2$$

Using Φ 32bars, with $A_{st} = 804.3 \text{ mm}^2$

$$\#32 \text{ bars required} = 5420/804.3 = 6.7 \approx 8$$

The minimum area of reinforcement for compression components

$$A_s f_y / (A_g f'_c) \geq 0.135 \quad (3.13)$$

A_s = area of reinforcement (mm^2)

f_y = specified yield strength of reinforcing bars(MPa)

A_g = gross area of section (mm^2)

f'_c = specified compressive strength of concrete (MPa)

$$A_s \geq 0.135 * (A_g f'_c) / f_y$$

$$A_s \geq 0.135 * (5000 * 1700 * 24) / 400 = 68850 \text{mm}^2$$

Using Φ 32 bars, with $A_{st} = 804.3 \text{mm}^2$

$$\# \text{ bars required} = 68850 / 804.3 = 85.6 \approx 86$$

We will check of the section for the worst boundary force condition, with maximum force for the reduce area.

$$\sigma = P/A = (13151 + 3525 + 4299) / (1.7 * 5) = 2.468 \text{Pa} < 24 \text{MPa} = f'_{cd}$$

7. Check stresses in the nodal zones and struts.

The bottom cord struts are checked to see if the estimated location of the truss members based on the sectional analysis is ok.

Strut DH & CG

$$P_u = -823 \text{KN}, \epsilon_1 = 0$$

$$F_{cu} = 24 / 0.8 = 30 \leq 0.85 * 24 = 20.4, \text{ use } f_{cu} = 20.4$$

$$\Phi P_n = 0.7 * 20.4 * 1000 * 0.2 * 1.7 = 4855 \text{KN} > 823 \text{KN} \quad \text{ok!}$$

Strut IJ

$$P_u = -19894 \text{KN}$$

$$\Phi P_n = 0.7 * 20.4 * 1000 * 2 * 1.7 = 48552 \text{KN} > 19894 \text{KN} \quad \text{ok!}$$

Strut FI

$$P_u = -2375 \text{ KN}$$

$$\Phi P_n = 0.7 * 20.4 * 1000 * 0.2 * 1.7 = 4855.2 \text{ KN} > 2375 \text{ KN}$$

ok!

Table 3-19 : Summary of reinforcement of STM & conventional method

	<i>Steel bar using refined STM</i>	<i>Steel bar using conventional method</i>
<i>Ties</i>	- 12 Φ c/c 30mm for the above 1.26m of the column. - 12 Φ c/c 80 mm for the next 1.26 m of the column - 12 Φ c/c 160 mm for rest of the column	- Φ 12 c/c 200
<i>Longitudinal reinforcement</i>	- 86 Φ 32	- 110 Φ 32

CONCLUSION

Strut-and-Tie Model is a useful tool for structural engineers. As current practice is more and more relying on computer, this will made the designer slowly forgetting first principle. STM is providing a way in engineering visualization, allowing consistent design. It is creating opportunities, to modify finite element programs to come up with Strut-and-Tie model and investigate alternative solution. Although STM creation requires creativity and judgment, the designer needs to think about following the stress trajectories fairly closely especially at supports and locations of concentrated loads. Appropriate strut-to-tie angles also need to be considered to help limit the principle strain in the concrete.

The design study compares the reinforcing requirements of the original design with the results obtained in the strut-and-tie modeling method. Based on the results of the design study and the procedure used in the modeling, recommendations are proposed for employing the strut-and-tie model. As was shown earlier, modeling of the two hammerhead pier caps, girder & pier using STM has resulted in a close checking of the structure, demonstrating its validity and practical application.

In this check, it has been demonstrated that the reinforcement quantities obtained for the first and third example are a little bit higher than conventional approach for both the main bar and the stirrups. This is because the presence of disturbed region is diminished due to the length of the girder, and the pier cap is fairly deep beam but STM can be as an alternative method of analysis, it may also be used as a counter check to other types of analysis.

The main Reinforcement quantities obtained for the second and last example are lower than conventional method. This is true because the STM method uses the lower bound principle which gives a conservative result for especially for D-regions. The amount of stirrups obtained is higher in all case.

This work presents a clear and concise procedure for utilizing the strut-and-tie model for the analysis and design of hammerhead piers caps, girder & pier.

RECOMMENDATIONS

The following is a list of recommendations for further research and adaptations to strut and-tie modeling to establish more consistent designs based on the conclusions of this report.

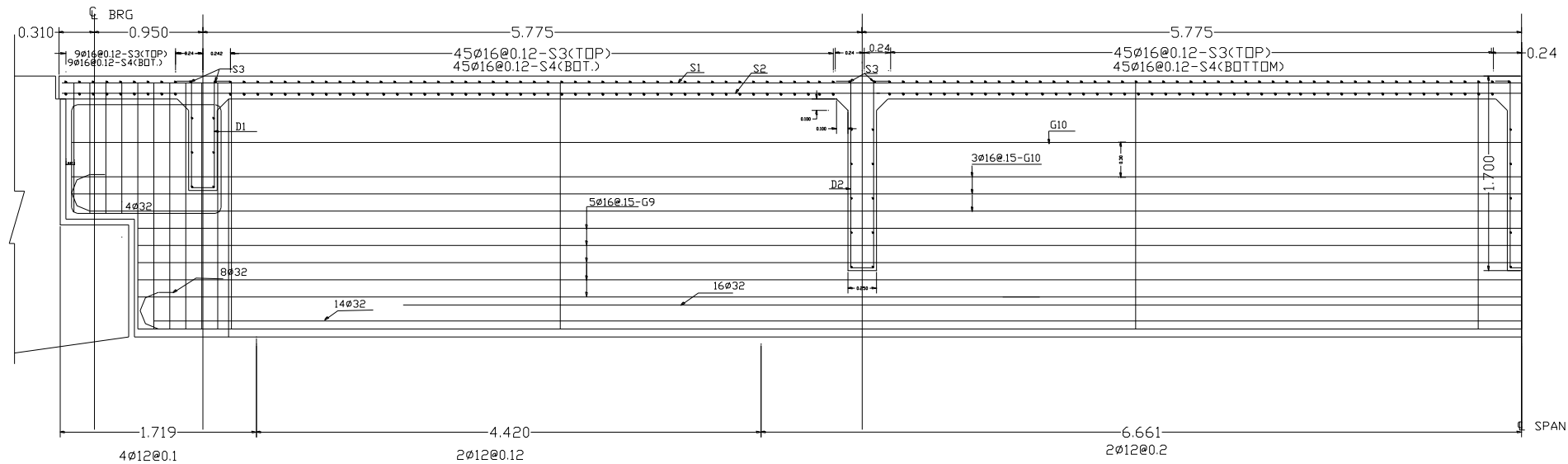
- STM can be as an alternative method of analysis for B- regions; it may also be used as a counter check to other types of analysis. Being familiar with STM especially for those D- regions.
- Provide more guidance to create appropriate truss models and it would be better if excel sheet is prepared such as for checking strut and node strength, reinforcement calculation so that one can easily design structures using STM. Computer aided in application of STM will help to establish many possibilities of solution.
- Longitudinal bar of pier obtained by STM need to be checked by conventional method since the nature of truss model for pier gives more of for design of ties.
- Provide clarification or changes to account for situations where ties cross the end of a strut from multiple directions.
- Provide clarification on STM in Euro code 2 since it provides very little guidance in using strut and ties models, which covers mainly the effective concrete strength provisions for the various strut and tie elements.

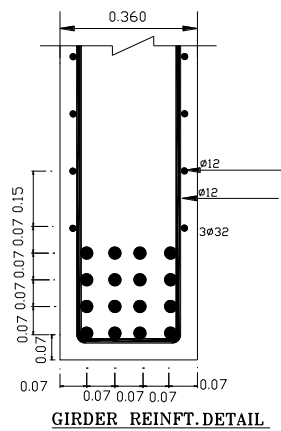
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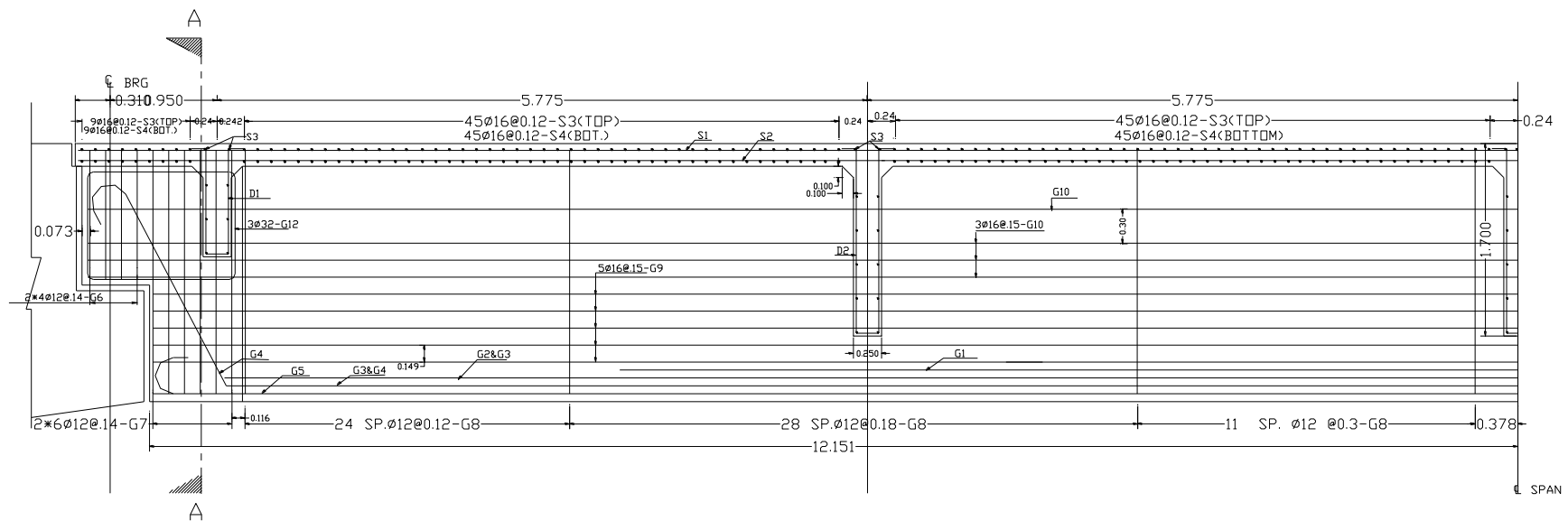
APPENDIX - BRIDGE GIRDER REINFORCEMNT DETAIL

Bridge Girder Detail using STM





Bridge Girder Reinforcement Detail using Conventional method



BILL OF MATERIALS FOR ONE					
SUSPENDED SPAN					
BAR	SIZE	NUMBER	LENGTH	TOTAL LENGTH	SHAPE
C1	12	10	25.52	255.2	—
C2	12	192	2.53	500.94	—
D1	12	20	2.25	45	—
D2	12	30	3.65	109.5	—
D3	12	42	5.28	221.76	—
G1	Y32	9	16.05	144.45	—
G2	Y32	6	20	120	—
G3	Y32	6	23.07	138.42	—
G4	Y32	6	27.9	167.4	—
G5	Y32	9	25.14	226.26	—
G6	12	48	2.75	132	—
G7	12	72	4.81	346.32	—
G8	12	225	4.92	1107	—
G9	16	30	24.2	726	—
G10	16	24	25.4	609.6	—
G11	Y32	18	4.56	82.08	—
S1	12	18	25.52	459.36	—
S2	16	30	25.52	765.6	—
S3	16	206	7.6	1565.6	—
S4	16	196	6	1176	—
G5'	Y32	9	25.74	231.66	—
G9'	16	30	24.8	744	—

