CONSTRUCTION OF THE RADIATION CURVE OF PULSAR CORRESPONDING TO EMISSION FROM MAGNETIC DIPOLE FIELD

By

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Abstract

In this work we derive the equations for the dipole component of pulsar's magnetic field and construct its radiation curve. This is done following spinning separated charges as an alternative model for generating dipolar pulsar magnetic field from Kebede, L.W. 2002. We approach the problem by solving Legendre functions, spherical harmonics theorem (i.e, $Y_{lm}(\theta, \phi)$ for l = m = 1) and Maxwell's equations to find the equations of the magnetic field strength B. The dipolar magnetic field obtained based on this model is proportional to the charge Q and the angular frequency ω of the pulsar (i.e, $B \propto Q\omega$) which has an advantage of accommodating the wide range of neutron stars's (i.e, both pulsars and magnetars) magnetic field. We have taken data for the parameters such as Q and ω for the seven well studied pulsars to check the magnetic field prediction of the model with the observational values, and our result is in close confirmation with the observed strength of the magnetic field for these seven well known pulsars.

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Chapter 1

Introduction

Pulsars are celestial sources of very regularly pulsed radiation and the first discovered at radio wavelengths, pulsars have been seen in virtually every electromagnetic band such as radio, infrared, optical, ultraviolet, X-rays, and gamma-rays [1]. The first pulsar was discovered over 40 years ago , and new ones are still being found today, driven by improvements in hardware (telescopes and computers) and software (search algorithms). These continuing advances have allowed search sensitivities to improve, and have opened up new windows onto exotic pulsars, including high-energy gamma-ray pulsars, very-short-period pulsars (Crab pulsars), and pulsars in tight binary systems. Hewish discovery the pulsar before the Pacini had postulated that a spinning, magnetized neutron star could be the power source in the crab nebula, and an idea independently advanced by Gold [2]. All subsequent evidence has supported the model that pulsars are indeed rotating, highly magnetized neutron stars [3]. The pulsars are highly magnetized rotating neutron stars that built on a basic assumption of neutron star matter is electrically neutral particles composed of mainly heavy nuclei, neutrons, protons, and electrons. The spinning separated charges, which come as a result of plasma diffusion, are the sources for the magnetic fields of neutron stars.

The purpose of this thesis is to show pulsar magnetic fields inherently contain dipolar component magnetic fields, the magnitude of field strength and the dipolar emission of radiation curve of the pulsar. This Chapter 1 and 2 contain a review about the distinctive properties and discovery of the pulsar and some detail review about neutron stars and pulsars. In Chapter 2 we try to mention some basic points about the pulsar magnetic fields and what is currently known about this field. The next two Chapters (Chap. 3 and 4) contain the derivation of the dipolar components of the pulsar magnetic field based the model by kebede, L. W [4]. Results and discussions included in Chapter 5 and finally conclusions included in the last chapter 6.

1.1 Neutron stars and pulsars

1.1.1 Neutron stars

In 1934 Baade and Zwicky proposed the idea of neutron stars, pointing out that they would be at very high density and small radius, and would be much more gravitationally bound than ordinary stars. They also made the remarkably prescient suggestion that neutron stars would be formed in supernova explosions [5]. A neutron star is a type of stellar remnant that can result from the gravitational collapse of a massive star during a supernova event. Such stars are composed almost entirely of neutrons, which are subatomic particles without electrical charge and a slightly larger mass than protons. Neutron stars are very hot and are supported against further collapse because of the Neutron degenerate pressure. Neutron stars have extremely high densities, typically in the order of $10^{14}\ g/cm^3$ ($10^{17}\ kg/m^3$), which is comparable to atomic nucleus density [6]. The mass of a typical isolated neutron star is approximately about 1.4 $M_\odot$ and its equatorial radius is about 10 km [7].

1.1.2 Formation of the neutron stars

Neutron stars are formed when a massive star runs out off fuel and collapses and the very central region of the star is the core collapses, crushing together every proton and electron into form a neutron. Therefore the name implies that the "neutron star". If the core of the collapsing star is roughly between about 1 and 3 solar masses, these newly-created neutrons can stop the collapse, leaving behind a neutron star [8]. Stars with higher masses will continue to collapse into stellar-mass black holes and these stellar remnants measure about 20 km across. One sugar cube of neutron star material would weigh about 1 trillion kilograms (or 1 billion tons) on Earth about as much as a mountain.

In the stellar core, hydrogen is converted into helium through thermonuclear fusion. The thermal energy released from this process creates an outward pressure, which maintains the core in hydrostatic equilibrium and prevents collapse. Once the core's supply of hydrogen is exhausted, this outward pressure is no longer created. The core begins to collapse, causing a rise in both temperature and pressure, which becomes great enough to ignite helium and start the helium-carbon fusion cycle, and this produces sufficient outward pressure to halt the collapse [9].

Many neutron stars are likely undetectable because they simply do not emit enough radiation. However, under certain conditions, they can be easily observed. A handful of neutron stars have been found sitting at the centers of supernova remnants quietly emitting X-rays. More often, though, neutron stars are found spinning wildly with extreme magnetic fields as pulsars or magnetars. From the moment of their birth, stars are spinning and possess a dipole magnetic field [10].

1.1.3 Rotation of the neutron star

Neutron stars rotate extremely rapidly after their creation due to the conservation of angular momentum, like spinning ice skaters pulling in their arms, the slow rotation of the original star's core speeds up as it shrinks. A newborn neutron star can rotate several times a second; sometimes, the neutron star absorbs orbiting matter from a companion star, increasing the rotation to several hundred times per second, reshaping the neutron star into an oblate spheroid. Over time, neutron stars slow down because of their rotating magnetic fields radiate energy, where as the older neutron stars may take several seconds for each revolution. The rate at which a neutron star slows its rotation is usually constant and very small and the strong magnetic field is particularly important when coupled with another aspect of neutron stars rapid rotation [11].

Neutron stars are rapid rotators for exactly the same reason they are strongly magnetized, conservation during collapse of massive stars, in case of conservation of angular momentum. Consider a massive star core rotating with an initial angular velocity ω_i and an initial moment of inertia I_i has angular momentum L_i and as it collapses it must conserve angular momentum, so its angular momentum after collapse is L_f with final angular velocity ω_f and final moment of inertia I_f hence $L_i = L_f = L = I_i\omega_i = I_f\omega_f$. Thus the relations have been derived and observed here. The angular momentum and magnetic flux will be conserved even in case of a supernova explosion. The explosion decreases the star radius dramatically by about 5 orders of magnitude, increasing the angular velocity and the surface magnetic field strength by about 10 orders of magnitude [12].

1.1.4 Pulsars

Pulsars are magnetized rotating neutron stars (NS) emitting a highly focused beam of electromagnetic radiation oriented along the magnetic axis. The misalignment between the magnetic axis and the spin axis leads to a lighthouse effect. As the magnetic axis is inclined with respect to the rotation axis, the pulsar acts like a cosmic light house emitting a radio pulse that can be detected once per rotation period when the beam is directed to the earth [9, 12]. The beam results from the rotational energy of the neutron star, which generates an electrical field from the movement of the very strong magnetic field, resulting in the acceleration of charged particles on the star's surface, and the creation of an electromagnetic beam coming from the poles of the magnetic field. This rotation slows down over time as the electromagnetic radiation is emitted. For some highly rotating pulsars, so-called milliseconds the stability of pulsar period is similar to that achieved by the best terrestrial atomic clocks [13]. Using these astrophysical clocks by accurately measuring the arrival times of their pulses, a wide range of experiments are made possible [14].

Pulsars are astronomical objects, which emit high accurate periodic radiations. In spite of the long observational history and numerous remarkable radiation features, the emission mechanism are not clearly understood. Pulsars are spinning neutron stars that have jets of particles which are moving close to the speed of light streaming out above their magnetic poles and these jets produce very powerful beams of light. Therefore, the beams of light from the jets sweep around as the pulsar rotates.

Neutron stars for which we see such pulses are called "pulsars". The observed magnetic field of pulsars ranges from 10^8 to 10^{15} G and, as already noted, the rotation period ranges from 1 millisecond to 10 seconds and so pulsar generally has shorter period (higher angular velocity) at the beginning of its lifetimes and a longer period at it gets ages [15].

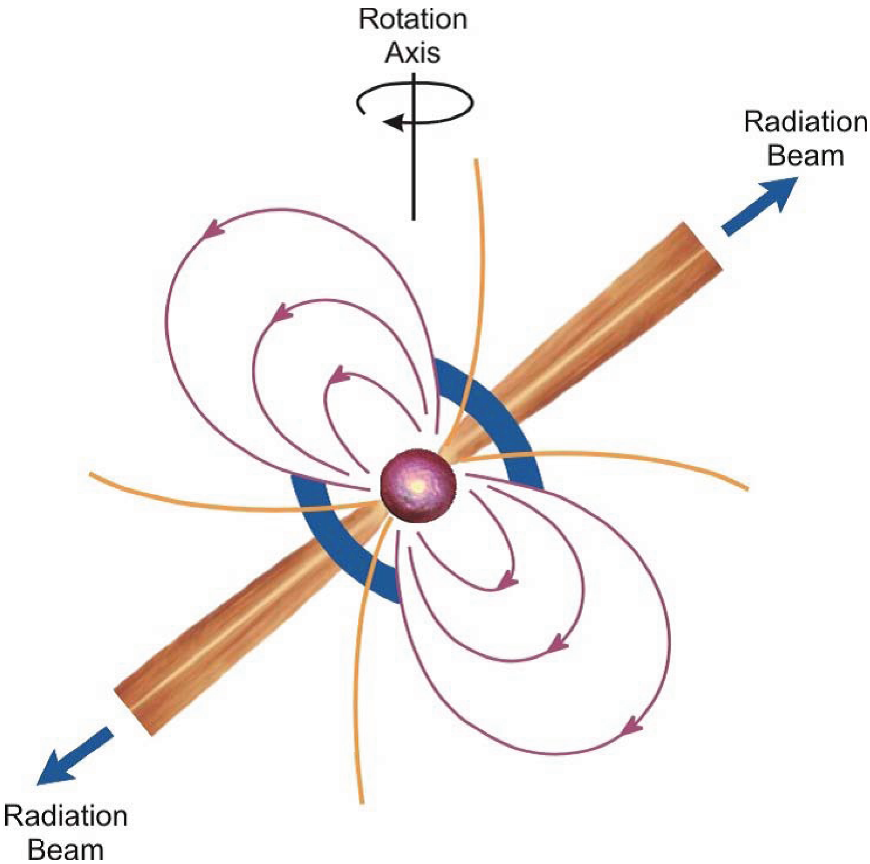

Figure 1.1: A pulsar is a rotating neutron star, which produce the magnetic axis and a rotational axis.

Figure 1.1 shows the fact that the pulsars rotate and emit regular pulses of electromagnetic radiation towards Earth, due to a misalignment of their rotation and magnetic field axes, is what distinguishes pulsars from other NSs. If the pulsar rotates and the magnetic and rotation axes are not perfectly aligned, this beam will sweep through space and, if it happens to pass over the Earth, we will see a pulse of radio waves once per rotation period. These objects are therefore called pulsars [16].

1.1.5 History and discovery of pulsars

The first pulsar was discovered by Jocelyn Bell and Anthony Hewish in 1967 who were actually studying distant galaxies at the time [1]. They were studying interplanetary scintillation of radio galaxies and quasars when they serendipitously noticed extraterrestrial sources emitting regular radio pulses with a period of about 1.3 seconds [3]. Pulsar has been discovered at radio wavelength, emit virtually in every electromagnetic bands (radio, infrared, optics, ultraviolet, x-ray, and gamma-rays). In most cases, the radio band is the best energy band to detect a pulsar, because the instrumental sensitivity compared to flux level is more favorable than in any other wavelengths have been discovered. Ordinary radio pulsars are NSs with magnetic field 10^{12} G and the spin periods between 16 milliseconds and 8.5 seconds [5]. On the other hand, in the optical band less than 10 are known, one of which is the Crab pulsar, the shortest period that discovered at the time, p= 33 ms, the rotation powered pulsars and others EM band, 10 of pulsars have been discovered [8].

1.1.6 Properties of the pulsar

The Sun and many other stars are known to possess roughly dipolar magnetic fields. Stellar interiors are mostly ionized gas and hence good electrical conductors. Charged particles are constrained to move along magnetic field lines and, conversely, field lines are tied to the particle mass distribution. When the core of a star collapses from a size 10^{11} cm to 10^{6} cm, its magnetic flux is conserved and the initial magnetic field strength is multiplied by 10^{10}, the factor by which the cross-sectional area a falls [17]. An initial magnetic field strength of B $\sim 10^{2}$ G becomes B $\sim 10^{12}$ G after collapse, so young NSs should have very strong dipolar fields [18]. The over viewed properties of pulsars explained by the neutron star model [19]. Pulsars are rotating NSs, very small in size and have incredibly strong magnetic fields. Pulsars are born in supernova explosions of massive stars, created in the collapse of the star's core, NSs are the most compact objects next to black holes [20]. From timing measurements of binary pulsars, we determine the masses of pulsars to be typically around 1.35 $M_\odot$, although this range has been expanded recently from 1.2 $M_\odot$ to 2.1 $M_\odot$ [21]. Modern calculations which make use of different equations of states produce results for the size of NSs which are quite similar to the very first calculations by Oppenheimer and Volkov. [22], i.e. a diameter of about 20 km.

Pulsars emitting electromagnetic radiation and in particular magnetic dipole radiation as they essentially represented rotating magnets. Assuming that this is the dominant process of loss in rotational energy and hence responsible for the observed increase in rotation period p, described by $\dot{p}$, we can equate that the corresponding energy out of the dipole to the rate of rotational energy lose. In this way an estimate for the magnetic field strength at the pulsar surface can be formed from the core is $B_s = 3.2 \times 10^{19}\sqrt{p\dot{p}}$ G [11]. Periods for radial oscillations of NSs were predicted to be larger than 1 second and were hence incompatible with the periods of the first discovered pulsars. Finally, another property of pulsars, also most easily observed in the Crab pulsar. It was noticed that the period of pulsars slowly increases, for the Crab pulsar by as much as 36 nanoseconds/day [12]. This property is not expected for the model of an eclipsing binary, where due to the loss of energy one would in fact expect the companions to come closer, reducing the pulse period [13].

Chapter 2

Pulsars magnetic fields

2.1 Introduction

Magnetic fields are most likely the main form of "hair" that allows neutron stars, contrary to black holes, to be distinguished from each other and classified into different groups. Among single neutron stars, we distinguish "classical" pulsars, millisecond pulsars, soft gamma-ray repeaters, anomalous X-ray pulsars, and inactive, thermal X-ray emitters [20].

The neutron star magnetic fields play an essential role by accelerating particles, by channeling these particles or accretion flows, by producing synchrotron emission or resonant cyclotron scattering, and by providing the main mechanism for angular momentum loss from non-accreting stars. Moreover, evidence is mounting that soft gamma-ray repeaters and anomalous x-ray pulsars are really only slightly distinct types of very strongly magnetized neutron stars ("magnetars") in which the magnetic field is the main energy source for the observed radiation [23]. On the other hand, we actually know surprisingly little about neutron star magnetic fields. In particular, most measurements of neutron star magnetic fields are indirect inferences, which are put in doubt both by their inconsistency with other observational evidence and with plausible theoretical models for the physics of their surroundings. Even less is known about the geometry of the magnetic field, its evolution, and its origin, so there is open space for speculation, modeling, and (hopefully) prediction of measurable effects that might test the theoretical ideas.

2.2 Sources of fields

Just recently it has been indicated that plasma density gradients inherent to neutron star matter could lead to large scale plasma diffusion and subsequent charge separation with excess negative charge accumulating in the crust while at the same time, almost the same amount of excess positive charge is left behind at the solid core. Surface magnetic fields are then expected to result from the spinning of these separated charges. In this case, the electron and proton currents resulting from the charge separation probably, all stars at all stages of their evolution have some magnetic field, due to electronic currents circulating in their interiors. Naively, one might expect that such currents should decay over the (microscopic) time scale τ_{coll} in which an average electron transfers its momentum to a more massive particle through a Coulomb (or other) collision. However, any decrease in the current I implies a decrease of the magnetic flux $\phi = cLI$ through the stellar equatorial plane, where c is the speed of light, $L \sim \frac{R}{c}$ is the star's self-inductance, and R is its radius [25].

According to Lenz's law, such a flux decline will induce an emf $\varepsilon = -\frac{1}{c}\frac{d\phi}{dt} = -L\frac{dI}{dt}$ that tends to keep the current going as prescribed by Ohm's law, $\varepsilon = RI$. The resistance R can be estimated in terms of a typical conductivity $\sigma = \frac{n_e e^2 \tau_{coll}}{m_e}$ (where n_e, -e, and m_e denote the electron concentration, charge, and mass). Thus, the star is well described by an electric circuit with an inductance L and a resistance R connected in series, in which the current decays at such a rate that the induced emf is always as strong as required to maintain the instantaneous current against resistive decay. The exponential ("Ohmic") decay time is thus $\tau_{Ohm} = \frac{L}{R} \sim \sigma(\frac{R}{c})^2 = n_e r_e R^2 \tau_{coll}$ where $r_e = \frac{e^2}{(m_e c^2)}$ is the "classical electron radius". Since the electron concentration is typically high, but especially since stellar radii (even in the very compact neutron stars) are large (e.g., compared to typical laboratory scales), in general $\tau_{ohm} \gg \tau_{coll}$ by many orders of magnitude [26]. Thus, stellar magnetic fields can persist for very long times, being effectively "frozen" into the plasma. Essentially the only way of changing the magnetic field configuration is by "deforming the circuit", i. e. by macroscopic displacements of the plasma, which can be thought of as carrying the magnetic flux lines along. In particular, when a star changes its radius, it could be expected to preserve its enclosed magnetic flux, changing the magnetic field strength in inverse proportion to its cross-sectional area [27].

2.3 Pulsar magnetic fields

All stars have weak magnetic fields which are due to the movement of the charged plasma formed during nuclear fusion. A typical star has a magnetic field strength around 100 gauss compared to the Earth which has a magnetic field strength of 0.6 gauss (G). Towards the end of its life, when the nuclear fuel is exhausted, the gravitational force (which is balanced by the nuclear pressure throughout the life of the star) causes the star's density to increase as it undergoes stellar collapse. This increase in density also results in an increase in the star's magnetic field strength as the magnetic field lines are compressed closer together. According to Maxwell's equations, as a magnetized object shrinks by a factor of two, its magnetic field strengthens by a factor of four. In other words the magnetic field strength (B) is inversely proportional to the surface area of the star (B $\sim \frac{1}{r^2}$) and the collapsing star will also spin faster in order to conserve angular momentum [23]. Eventually after the stellar collapse (or supernova as it is commonly known) a very strongly magnetized rotating neutron star called a pulsar with field strengths around 10^{12} G will be left [24]. The magnetic field which is supported by electric currents flowing inside the neutron star, rotates with the neutron star. As a result of this, beams of radio waves shine outward from the neutron star's magnetic poles and sweep through space as it rotates.

The pulsar also blows out a wind of charged particles which carry away energy and angular momentum, causing its rate of spin to decrease gradually. For example, the Crab Nebula pulsar, a remnant of the supernova explosion that was observed in 1054 A.D, now rotates once every 33 milliseconds and is currently slowing down at a rate of about 1.3 mill-seconds every century [2]. The flux being conserved in the collapsing stellar material, Since the interior of the star is superfluid, and the decay time is long compared with the life time of a pulsar . Polar field strengths reaching 10^{12} G occur in young pulsars. While in the older evolved millisecond pulsars it is relatively as low as 10^9 G. The active pulsars in the galaxy have surface magnetic fields in the range of $\sim 26 \times 10^{12}$ G [22]. This is still thousands of times larger than fields attainable in the laboratory. Despite the intensity of the magnetic field, it has very little effect on the structure of the star. Outside the star, however the magnetic field B completely dominates all physical processes, even out weighing gravitation by a very large factor. The magnetic field is directly related to several properties observed in neutron stars. In Non-degenerate stars, like our Sun, and in some white dwarfs, magnetic fields can be directly measured by the Zeeman splitting of spectral lines, or by polarization measurements [26]. Another technique is the Doppler imaging, which consists in analyzing the time-varying profiles of rotating stars, and allows to indirectly inferring the presence of cold spots associated with the strongest magnetized regions of the surface. This direct measure is not possible in neutron stars. The main signature of the magnetic field in neutron stars is the loss of rotational energy due to the electromagnetic torque. Thus, the rotational properties give an estimate of the large-scale dipolar magnetic field.

2.4 Rotation powered pulsars

Rotation-powered pulsars (RPPs) are rapidly rotating, highly magnetized neutron stars, with typical surface magnetic field strengths of $B_s \sim 10^{12}$ G this value is commonly seen in young rotation powered pulsars. Misalignment between the magnetic and rotation axes produces a time dependent magnetic moment, extracting rotational energy from the neutron star.

Emission produced in the vicinity of the magnetic poles appears to pulsate as the radiating region sweeps past the observer's line of sight [17]. As was mentioned earlier, the specific mechanism or mechanisms by which pulsars radiate are uncertain, but must involve charged particle acceleration in the extremely large electric field induced by the time-varying magnetic field. In the case of pure magnetic dipole, all of the rotational energy loss would be converted into magnetic dipole radiation, and the pulsar luminosity would be that of a rotating magnetic dipole. The total luminosity measured from the pulsed emission is much lower than that expected from magnetic dipole, suggesting that a large portion of the energy is carried away from the pulsar by a particle wind, forming the surrounding pulsar wind nebulae observed in many young systems [18].

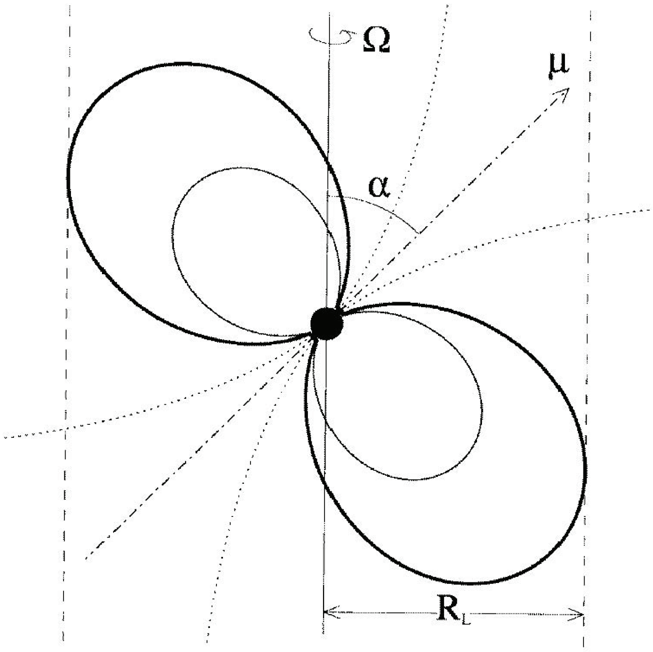

Figure 2.1: The alignment of angle produced between rotational axis and magnetic axis of the dipole radiation powered pulsar.

From the Fig. 2.1 ($\Omega \equiv \omega$) is angular rotational velocity, ($\mu \equiv m$) is the magnetic moment, ($\alpha \equiv \theta$) is the angle between rotational axis and magnetic axis the the pulsar and R_L is its approximate radiation region (zone) radius. Therefore, the beams of light from the jets sweep around as the pulsar rotates. The NSs for such pulses are called "spin-powered pulsars", indicating that the source of energy is the rotation of the magnetized neutron star [19]. A young neutron star with a sufficiently high magnetic field and spin period switches on as a rotation-powered pulsar [21].

Chapter 3

Magnetic dipolar field expansions

3.1 Derivation of the vector potential

To derive the vector potential for uniform spherical charge distribution with a surface charge density σ, we find first the current density. Then the vector $\vec{J}$ is given by

$$\vec{J} = \sigma\delta(r' - R)\vec{v} = \sigma\delta(r' - R)\vec{\omega} \times \vec{r}, \tag{3.1.1}$$

$$\vec{\omega} \times \vec{r} = (\omega\hat{k}) \times (R\sin\theta\cos\phi\hat{i} + R\sin\theta\sin\phi\hat{j} + R\cos\theta\hat{k}) \tag{3.1.2}$$

$\vec{\omega} \times \vec{r} = (-\omega R\sin\theta\sin\phi)\hat{i} + (\omega R\sin\theta\sin\phi)\hat{j} + 0 = \omega R\sin\theta\hat{e_\phi}$,
where the unit vector in the direction of ϕ is $\hat{e_\phi} = (\sin\phi\hat{i} + cos\phi\hat{j})$.
Therefore the current density $\vec{J}$ has only the $\hat{\phi}$ component and the it becomes:

$$\vec{J_\phi} = \sigma\delta(r' - R)\vec{\omega}R\sin\theta\hat{e_\phi} \tag{3.1.3}$$

Now the vector potential is given by

$$\vec{A}_{(r)} = \frac{\mu_o}{4\pi}\int\frac{\vec{J}_{(r)}}{|r - r'|}d^3r' \tag{3.1.4}$$

Since the above Eqn. (3.1.4) is symmetric in ϕ', the x^{th} component of the current does not contribute. This accepts only the y^{th} component, which is A_ϕ. Thus will be

$$\vec{A}_\phi = \frac{\mu_o}{4\pi}\int \sigma\delta\frac{[(r'-R)\vec{\omega}R\sin\theta\cos\phi]}{\mid r-r'\mid}d^3r' \tag{3.1.5}$$

$$\frac{1}{|r-r'|} = 4\pi\sum_{l}^{\infty}\sum_{l=m}^{l}\frac{r_<^l}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')P_l(\cos\theta)Y_{lm}(\theta,\phi), \tag{3.1.6}$$

where $d^3r' = r'^2dr'\sin\theta d\theta d\phi$. Substituting these values in Eqn. (3.1.5) we obtain

$$\vec{A}_\phi = \mu_o\sigma\omega R\int\sum_{1}^{\infty}\sum_{l=m}^{l}\frac{(r_<^l)}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')P_l(\cos\theta)Y_{lm}(\theta,\phi)\delta(r'-R)$$

$$\times\sin\theta\cos\phi(r'^2dr'\sin\theta d\theta d\phi), \tag{3.1.7}$$

but $\sin\theta\cos\phi = -\sqrt{\frac{8\pi}{3}}(\theta,\phi)$, and with $\phi = 0$ and $\sigma(4\pi R^2) =\mid Q_s\mid$. Then the $\vec{A}_\phi$ is

$$\vec{A}_\phi = \frac{\mu_o|Q_s|}{4\pi R}\int\sum_{l=m}^{l}\frac{(r_<^l)}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')P_l(\cos\theta)Y_{lm}(\theta,\phi)\delta(r'-R)$$

$$\times\left(-\sqrt{\frac{8\pi}{3}}\right)Y_{11}(\theta,\phi)r'^2dr'\sin\theta d\theta d\phi \tag{3.1.8}$$

From the normalization and orthogonally relation

$\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta d\theta Y_{lm}^*(\theta',\phi')Y_{lm}(\theta,\phi) = \delta_{l'l}\delta_{m'm}$, where

$$\begin{cases}\delta_{l'l}\delta_{m'm} = 1\ for\ l=1\ and\ m=1\\ \delta_{l'l}\delta_{m'm} = 0\ for\ m\neq 1\ and\ l\neq 1\end{cases} \tag{3.1.9}$$

The integration contributes only for l=1 and m=1 and also from the relation

$P_l^m(\cos\theta) = P_1^1(\cos\theta) = -\sqrt{(1-\cos^2\theta)}\frac{(dp_1\cos\theta)}{d(\cos\theta)} = -\sin\theta\frac{p_1 d(\cos\theta)}{d(\cos\theta)} = -\sin\theta$

$$\vec{A}_\phi = \frac{\mu_o|Q_s|}{4\pi R}\int\sum_{l=m}^{l}\frac{(r_<^l)}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')\sin^2\theta Y_{lm}(\theta,\phi)\delta(r'-R)r'^2dr'$$

$$\times\left(\sqrt{\frac{8\pi}{3}}\right)Y_{11}(\theta,\phi)d\theta d\phi \tag{3.1.10}$$

The integration over delta function result would be comes

$$\vec{A}_\phi = \frac{\mu_o|Q_s|R^2}{4\pi R}\int\sum_{l=m}^{l}\frac{r_<^l}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')\sin^2\theta Y_{lm}(\theta,\phi)\left(\sqrt{\frac{8\pi}{3}}\right)Y_{11}(\theta,\phi)d\theta d\phi \tag{3.1.11}$$

Now the constant terms $\frac{\mu_o}{4\pi} = \frac{1}{c}$. Therefore the vector potential should be

$$\vec{A}_\phi = \frac{|Q_s|\omega R^2}{cR}\int\sum_{l=m}^{l}\frac{r_<^l}{(2l+1)r_>^{l+1}}Y_{lm}^*(\theta',\phi')\sin^2\theta Y_{lm}(\theta,\phi)\left(\sqrt{\frac{8\pi}{3}}\right)Y_{11}(\theta,\phi)d\theta d\phi \tag{3.1.12}$$

3.1.1 The vector potential inside the spheres

The vector potential inside the sphere(crust of star) can be calculated from Eqn. (3.1.12) by substituting $r_< = r$ and $r_> = R$

$$\vec{A}_{\phi in} = -\left(\frac{|Q_s|\omega R}{c}\right)\left(\sqrt{\frac{8\pi}{3}}\right)\sum_{l=m}^{l}\frac{r^l}{(2l+1)R^{l+1}}Y_{lm}(\theta,\phi)\int Y_{lm}^*(\theta',\phi')\sin^2\theta Y_{11}(\theta,\phi)d\theta d\phi \tag{3.1.13}$$

By the Eqn. (3.1.9) the integration holds only for l=1 and m=1,
$\int Y_{lm}^*(\theta',\phi')Y_{11}(\theta,\phi)d\theta' d\phi' = 1$ and $Y_{11}(\theta,\phi) = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$ then we obtain

$$\vec{A}_{\phi in} = \left(\frac{|Q_s|\omega R}{c}\right)\left(\sqrt{\frac{8\pi}{3}}\right)\left(\frac{r^l}{(2l+1)R^{l+1}}\right)Y_{lm}(\theta,\phi) \tag{3.1.14}$$

$$\vec{A}_{\phi in} = -\left(\frac{|Q_s|\omega R}{c}\right)\left(\sqrt{\frac{8\pi}{3}}\right)\left(\frac{1}{3}\right)\left(\frac{r}{R^2}\right)\left(\sqrt{\frac{3}{8\pi}}\right)\sin\theta e^{i\phi} \tag{3.1.15}$$

Therefore the vector potential inside the sphere will be

$$\vec{A}_{\phi in} = -\left(\frac{|Q_s|\omega}{3c}\right)\left(\frac{r}{R}\right)\sin\theta \tag{3.1.16}$$

3.1.2 The vector potential outside spheres

The vector potential outside the sphere (crust) can be derived from Eqn. (3.1.12) by substituting $r_< = R$ and $r_> = r$ thus we have

$$\vec{A}_{\phi out} = \frac{|Q_s|\omega R}{c} \int \sum_{l=m}^{l} \frac{R^l}{(2l+1)r^{l+1}} Y^*_{lm}(\theta', \phi') \sin\theta Y_{lm}(\theta, \phi) \left(\sqrt{\frac{8\pi}{3}}\right) Y_{11}(\theta, \phi) d\theta d\phi \tag{3.1.17}$$

By the Eqn. (3.1.9) the integration holds only for l=1 and m=1 and $\int Y^*_{lm}(\theta', \phi') Y_{11}(\theta, \phi) d\theta' d\phi' = 1$ and $Y_{11}(\theta, \phi) = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$

$$\vec{A}_{\phi out} = -\frac{|Q_s|\omega R}{c} \left(\sqrt{\frac{8\pi}{3}} e^{-i\phi}\right) \left(\frac{1}{3}\right) \left(\frac{R}{r^2}\right) \left(\sqrt{\frac{3}{8\pi}}\right) \sin\theta e^{i\phi} \tag{3.1.18}$$

$$\vec{A}_{\phi out} = -\left(\frac{|Q_s|\omega}{3c}\right) \left(\frac{R^2}{r^2}\right) \sin\theta \tag{3.1.19}$$

The vector potential outside the sphere (crust) should be

$$\vec{A}_{\phi out} = -\left(\frac{|Q_s|\omega}{3c}\right) \left(\frac{R^2}{r^2}\right) \sin\theta \tag{3.1.20}$$

3.2 Derivation of magnetic fields

In this section we will try to derive the dipolar component of pulsar magnetic field. The magnetic field can be derived from the vector potential as

$$\vec{B} = \vec{\bigtriangledown} \times \vec{A} \tag{3.2.21}$$

Now we have only the $\hat{\phi}$ component of the vector potential. Then the magnetic field derived as follow, and can be written as $\vec{B} = \vec{\bigtriangledown} \times \vec{A}_\phi$

$$\vec{\bigtriangledown} \times \vec{A}_\phi = \hat{e}_r \frac{1}{r\sin\theta} \left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\phi}{\partial\phi}\right] + \hat{e}_\theta \left[\frac{1}{r\sin\theta} \frac{\partial\phi_r}{\partial\phi} - \frac{1}{r}\partial(r\frac{A_\phi}{\partial r})\right] + \hat{e}_\phi \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] \tag{3.2.22}$$

But we have only the $\hat{\phi}$ component of the vector potential then the magnetic field will be

$$\vec{B} = \vec{\bigtriangledown} \times \vec{A}_\phi = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi)\right]\hat{e}_\theta \tag{3.2.23}$$

3.2.1 The magnetic field inside the sphere

The magnetic field inside the sphere can be derived by inserting Eqn. (3.1.16) in to Eqn. (3.2.23). Thus

$$\vec{B}_{in} = \vec{\bigtriangledown} \times \vec{A}_{\phi in} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi)\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi)\right]\hat{e}_\theta \tag{3.2.24}$$

$$\vec{B}_{in} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\sin\theta\Big(-\frac{|Q_s|\omega}{3c}\Big)\Big(\frac{r}{R}\Big)\sin\theta\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}r\Big(-\frac{|Q|\omega}{3c}\Big)\Big(\frac{r}{R}\Big)\sin\theta\right]\hat{e}_\theta \tag{3.2.25}$$

$$\vec{B}_{in} = -\Big(\tfrac{|Q_s|\omega}{3c}\Big)\Big(\tfrac{1}{R}\Big)(2\cos\theta)\hat{e}_r + \Big(\tfrac{|Q_s|\omega}{3Rc}\Big)\Big(\tfrac{1}{R}\Big)(2\sin\theta)\hat{e}_\theta$$

$$\vec{B}_{in} = -\Big(\frac{|Q_s|\omega}{3c}\Big)\Big(\frac{1}{R}\Big)\big[(2\cos\theta)\hat{e}_r - \big(2\sin\theta)\hat{e}_\theta\big], \tag{3.2.26}$$

where we have $\hat{e}_r = \sin\theta\hat{j} + \cos\theta\hat{k}$, and $\hat{e}_\theta = \cos\theta\hat{j} - \sin\theta\hat{k}$, by substituting the value of $\hat{e}_r$ and $\hat{e}_\theta$ in the expression $\big[(2\cos\theta)\hat{e}_r - (2\sin\theta)\hat{e}_\theta\big]$ we obtain

$$\begin{aligned}\big[(2\cos\theta)\hat{e}_r - (2\sin\theta)\hat{e}_\theta\big] &= (\cos\theta)(\sin\theta\hat{j} + \cos\theta\hat{k}) - (\sin\theta)(\cos\theta\hat{j} - \sin\theta\hat{k}) \\ &= (\cos^2\theta + \sin^2\theta)\hat{k} \\ &= \hat{k}\end{aligned}$$

Therefore the magnetic inside the sphere [4] will be given as

$$\vec{B}_{in} = -2\Big(\frac{|Q_s|\omega}{3Rc}\Big)\hat{k} \tag{3.2.27}$$

3.2.2 Magnetic field outside the sphere

The magnetic field outside the sphere will be derived by substituting Eqn. (3.1.20) in to Eqn. (3.2.23) given as

$$\vec{B}_{out} = \vec{\bigtriangledown} \times \vec{A}_{out} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_{out})\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_{out})\right]\hat{e}_\theta \tag{3.2.28}$$

$$\vec{B}_{out} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\sin\theta\Big(-\frac{|Q|\omega}{3c}\Big)\Big(\frac{R^2}{r^2}\Big)\sin\theta\right]\hat{e}_r - \frac{1}{r}\left[\frac{\partial}{\partial r}r\Big(-\frac{|Q|\omega}{3c}\Big)\Big(\frac{R^2}{r^2}\Big)\sin\theta\right]\hat{e}_\theta \tag{3.2.29}$$

$$\vec{B}_{out} = -\frac{1}{r}\left[\Big(\frac{|Q_s|\omega}{3C}\Big)\Big(\frac{R^2}{r^2}\Big)2\cos\theta\right]\hat{e}_r + \frac{1}{r}\left[\Big(\frac{|Q_s|\omega}{3c}\sin\theta\Big)\Big(\frac{R^2}{r^2}\Big)\right]\hat{e}_\theta \tag{3.2.30}$$

$$\vec{B}_{out} = -\Big(\frac{|Q_s|\omega}{3c}\Big)\Big(\frac{R^2}{r^3}\Big)\big[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta\big] \tag{3.2.31}$$

Again using the value of $\hat{e}_r$ and $\hat{e}_\theta$ substituting in to the equation $\big[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta\big]$, and simply to obtain as

$$\begin{aligned}\big[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta\big] &= 2\cos\theta(\sin\theta\hat{j} + \cos\theta\hat{k}) + \sin\theta(\cos\theta\hat{j} - \sin\theta\hat{k})\\ &= 3\cos\theta\sin\theta\hat{j} + 2\cos^2\theta\hat{k} - \sin^2\theta\hat{k}\\ &= 3\cos\theta\sin\theta\hat{j} + 2\cos^2\theta\hat{k} - (1-\cos^2\theta)\\ &= 3\cos\theta(\sin\theta\hat{j} + \cos\theta\hat{k}) - \hat{k}\\ &= 3\cos\theta\hat{e}_r - \hat{k}.\end{aligned}$$

Therefore the dipolar component of pulsar magnetic field outside the sphere(crust) of the rotating neutron star (pulsar) [4] should be

$$\vec{B}_{out} = -\Big(\frac{|Q_s|\omega}{3c}\Big)\Big(\frac{R^2}{r^3}\Big)\big[3\cos\theta\hat{e}_r - \hat{k}\big] \tag{3.2.32}$$

3.3 Summary

In this chapter we have derived the magnetic field strength, $\vec{B}$ inside and outside of the sphere's surface (the neutron star) starting from the inside and outside vector potential $\vec{A}$ of the rotating neutron star in ϕ direction. We used the inside the vector potential to find the inside magnetic field strength of the neutron star and similarly for the outside as well. We used to derive the equations or solving the problems by using Legendre functions and equations, and Spherical harmonics theorem (i.e, $Y_{lm}(\theta, \phi)$ for l = m = 1). The inside magnetic field $\vec{B}_{in}$ is proportional to the sine angle θ where as the outside magnetic field $\vec{B}_{out}$ is given as the cosine θ function, where $\theta \equiv \alpha$ is the angle between rotational axis and the magnetic axis that we considered figure 2.1 in Chapter 2. Generally the magnetic field of the rotating neutron stars (pulsars) decreases as we go from the surface of the star.

Chapter 4

Magnetic dipole radiation

4.1 Introduction

Electromagnetic waves are generated by a moving charges or current flowing in medium. Electromagnetic (EM) radiation is made up of electric and magnetic fields that oscillate at right angles to each other [26]. EM radiation propagates at a speed of light c in vacuum. The Poynting vector S is defined as the cross product between the field components, that means $\frac{1}{\mu_o}(\vec{E} \times \vec{B})$, where μ_o is the permeability of the medium through which the radiation passes, E is the electric field, and B is magnetic field. The direction of the vector product S is perpendicular to the plane determined by the vectors E and B.

In this chapter we shall be concerned with the ultimate source of all electromagnetic radiation of moving charges. If this known that the radiation can only be produced if the charges undergo acceleration. The arbitrary motion of a collection of charges will produce radiation which can be described by the dipole expansion. As a result will be both electric and magnetic multi-pole radiation of all orders(except monopole). In this work we will focus on the magnetic dipole radiation. There

are many interesting applications of accelerating charges in general. Electromagnetic waves radiate out to infinity carrying energy with them. A spherical surface magnetic field radiate out at radius r and the total power $P_{(r)}$ passing through this surface is the integral of the poynting vector which is given as:
$P_{(r)} \equiv \oint \vec{S} \cdot d\vec{a} \equiv \frac{1}{\mu_o} \oint (\vec{E} \times \vec{B}) \cdot d\vec{a}$, where $\vec{S} \equiv (\vec{E} \times \vec{B})$. Based on this equation we can derive the power radiation equations and their relations to the magnetic field strength (B) of the rotating NSs.

4.2 The retarded magnetic vector potential

The emission of radiation by localized systems of oscillating surface charges. The approximations are made for fields produced by slowly moving charges. Taking the potentials, fields, charges and currents to vary sinusoidally with time. That is

$$\begin{cases} \rho(x,t) = \rho(x)e^{-i\omega t} \\ \vec{J}(x,t) = \vec{J}(x)e^{-i\omega t}, \end{cases} \tag{4.2.1}$$

where ρ is charge density and $\vec{J}$ is current density respectively,we can express the magnetic dipole vector potential in the Lorenz gauge form as:

$$\vec{A}(\vec{x}) = \frac{\mu_o}{4\pi} \int d^3\vec{x'} \int dt \frac{\vec{J}(\vec{x',t'})}{\mid \vec{x} - \vec{x'} \mid} \delta(t + \frac{\mid \vec{x} - \vec{x'} \mid}{c} - t') \tag{4.2.2}$$

Provided that there is no boundary surface and the Dirac delta function assures the causal behavior of the field with the sinusoidal time dependence in Eqn. (4.2.1) and the solution of the vector potential takes the form

$$\vec{A}(\vec{x}) = \frac{\mu_o}{4\pi} \int \vec{J}(\vec{x'}) \frac{e^{ik(\vec{x} - \vec{x'})}}{\mid \vec{x} - \vec{x'} \mid} d^3\vec{x'}, \tag{4.2.3}$$

where $k = \mid \vec{k} \mid = \frac{\omega}{c}$ is a wave number and $\vec{x}$ and $\vec{x'}$ are the position vectors that describe the vector potential.The magnetic field from Maxwell's equation is given by:

$\vec{H} = \frac{1}{\mu_o}\vec{\nabla} \times \vec{A}$ since $\vec{H} = \frac{1}{\mu_o}\vec{B}$ and we have

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{4.2.4}$$

The electric field outside the source can be written as
$\vec{E} = \frac{iZ_o}{k}\vec{\nabla} \times \vec{H}$, where $Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}$ is the impedance of free space. Given that the vector potentiaal would be written as

$$\vec{A}(x) = \frac{ike^{ikr}}{4\pi\mu_o r} n \times \int \frac{1}{2}\left[x' \times \vec{J}(\vec{x'})d^3(x')\right] = \frac{ike^{ikr}}{4\pi\mu_o r}(\vec{n} \times \vec{m}), \tag{4.2.5}$$

where n is stand for the normal to the field line and m is for the magnetic dipole moment. Giving a current distribution J(x), the field can be determined by calculating the integral in Eqn. (4.2.3). The general principle of the fields in the limit that the source of current is confide to small region, very small as compared to a wavelength of light. If the source dimensions are order of d and the wavelength is $\lambda = \frac{2\pi c}{\omega}$, and if $d \ll \lambda$, then there are three regions of interests. the near zone $d \ll r \ll \lambda$. The intermediate zone $d \ll \lambda \sim r$ and the far (radiation) zone $d \ll \lambda \ll r$ our focus is in the far zone [27]. Since in the far zone ($kr \gg 1$) the exponential in the Eqn. (4.2.3) oscillates rapidly and determines the behavior of the vector potential. In this region it is sufficient to appropriate $\mid \vec{x} - \vec{x'} \mid \cong r - \hat{n} \cdot \vec{x'}$, where $\hat{n}$ is the unit vector in the direction of x. Furthermore, if only the leading term in kr is desired the inverse relation to distance r in the Eqn. (4.2.3) can be replaced by simply r. In this case the vector potential becomes:

$$\lim_{kr \to \infty} \vec{A}(\vec{x}) = \frac{\mu_o}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(\vec{x})e^{-iknx'}d^3(x') \tag{4.2.6}$$

It is easy to show that the fields calculated from Eqn. (4.2.3) and Eqn. (4.2.5) are transverse to the radius vector and fall off as inverse of r (r^{-1}) and thus correspond to radiation fields. If the source dimensions are small compared to a wavelength it is appropriate to expand the integral in Eqn. (4.2.6) in powers of k (wave number) as $e^{-iknx'} = 1 + (-ikn \cdot x') + \frac{(-iknx')^2}{2!} + \frac{(-iknx')^3}{3!} + \frac{(-iknx')^4}{4!} + ... + \frac{(-iknx')^m}{m!}$, which can also be written as

$$e^{-iknx'} = \sum_{m=0}^{\infty} \frac{(-iknx')^m}{m!} \tag{4.2.7}$$

Substituting Eqn. (4.2.7) in to the Eqn. (4.2.6) gives

$$\lim_{kr\to\infty} \vec{A}(\vec{x}) = \hat{e}_\phi \frac{\mu_o}{4\pi} \frac{e^{i(kr-t\omega)}}{r} \sum \frac{(-iknx')^m}{m!} \int \vec{J}(x)(\vec{n} \cdot \vec{x'})^m d^3x' \tag{4.2.8}$$

The magnitude of the m^{th} term is $\frac{1}{m!}\int \vec{J}(x)(\vec{n} \cdot \vec{x'})^m d^3x'$, and since order of magnitude of x' is d and kd is small compared to unity, the successive terms in the expansion of A evidently fall off rapidly with n. Consequently the radiation emitted from the source will come mainly from the first non-vanished term in the expansion (4.2.8).

4.3 Magnetic dipole radiation field

The next term in the expression of Eqn. (4.2.8) (i.e for m=1) leads to a vector potential

$$\vec{A}(x) = \hat{e}_\phi \frac{ik\mu_o}{4\pi} \frac{e^{i(kr-t\omega)}}{r} \int \vec{J}(x)(\vec{n} \cdot \vec{x'})^m d^3x' \tag{4.3.9}$$

This vector potential can be written as the sum of two terms: one which gives a magnetic field and the other be an electric field through the relation for vector quantities, i.e, $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$

$$(\vec{n} \cdot \vec{x'}\vec{J}) = \frac{1}{2}\left[(\vec{n} \cdot \vec{x'})\vec{J} + (\vec{n} \cdot \vec{J}) \times \vec{x'}\right] + \frac{1}{2}\left(\vec{x} \times \vec{J}\right) \times \vec{n} \tag{4.3.10}$$

The second term, ant-symmetric part is recognizable as the magnetization due to the current density $\vec{J}$ and then the magnetization is given by

$$\vec{M} = \frac{1}{2}(\vec{x} \times \vec{J}) \tag{4.3.11}$$

Now considering only the magnetization term, and the vector potential is

$$\vec{A}(x) = \hat{e}_\phi \frac{ik\mu_o}{4\pi}(\vec{n} \times \vec{m})\frac{e^{i(kr-\omega t)}}{r}, \tag{4.3.12}$$

where $\vec{m}$ is the magnetic dipole moment, and given by

$$\vec{m} = \int \frac{1}{2}(\vec{x} \times \vec{J})d^3(x) \tag{4.3.13}$$

Unlike to magnetic field is transverse to the radius vector at a distance r, the electric field has parallel and perpendicular components to $\hat{n}$. That would be written as:

$$\begin{cases} \vec{H} = \frac{ik}{\mu_o}\vec{n} \times \vec{A} \\ \vec{E} = \frac{ik}{\mu_o}(\vec{n} \times \vec{A}) \times \vec{n} \end{cases} \tag{4.3.14}$$

The magnetic dipole fields from Eqn. (4.3.12), and Eqn. (4.3.14) are

$$\vec{H} = \mu_o \frac{k^2}{4\pi}(\vec{n} \times \vec{m}) \times \vec{n}\left(\frac{e^{i(kr-\omega t)}}{r}\right)\hat{e_\theta} \tag{4.3.15}$$

$$\vec{E} = \hat{e_\phi} Z_o(\vec{H} \times \vec{n}) \tag{4.3.16}$$

From the E and H fields the source (where $kr \gg 1$), we notice that E lies in a plane passing through the polar axis, where as H is azimuthal.

$$\vec{E_\phi} = Z_o(\vec{H} \times \vec{n}) \tag{4.3.17}$$

4.4 The Poynting vector

The Poynting vector can be thought of as representing the energy flux in ($\frac{W}{m^2}$) of an electromagnetic field. It is named after its inventor John Henry Poynting. The Poynting vector represents the particular case of an energy flux vector for electromagnetic energy. However, any type of energy has its direction of movement in space, as well as its density, so energy flux vectors can be defined for other types of energy as well. For time-harmonic electromagnetic fields, the average power over time can be found as follows, the pointing vector $\vec{E} \times \vec{H}$ is radial [25]. Thus would be written as

$$\vec{H} = (\mu_o)\frac{k^2}{4\pi}(\vec{n} \times \vec{m}) \times \vec{n}\left(\frac{e^{i(kr-\omega t)}}{r}\right)\hat{e_\theta} \tag{4.4.18}$$

Thus from equation (4.3.16) the electric field for magnetic dipole is

$$\vec{E} = (\mu_o)\left(\frac{-Z_0k^2}{4\pi}\right)(\vec{n} \times \vec{m})\left(\frac{e^{i(kr-\omega t)}}{r}\right)\hat{e_\phi} \tag{4.4.19}$$

The average pointing vector for the magnetic dipole field is

$$\vec{S}_{av} = \frac{1}{2}Re\big[\vec{E} \times \vec{H}^*\big], \tag{4.4.20}$$

where $\vec{H}^*$ is the conjugate of H and E which are given by Eqn. (4.3.14). Therefore, it leads to:

$$\vec{S}_{av} = \frac{1}{2}Re\left[(\mu_o)(\frac{k^2}{4\pi})(\vec{n} \times \vec{m})\frac{e^{i(kr-\omega t)}}{r}\hat{e_\theta}\right] \times \left[(\mu_o)(\frac{Z_ok^2}{4\pi})(\vec{n} \times \vec{m})\frac{e^{i(kr-\omega t)}}{r}\hat{e_\phi}\right] \tag{4.4.21}$$

$$\vec{S}_{av} = \frac{1}{2}(\mu_o)\frac{Z_ok^4}{32\pi^2r^2}\big[(\vec{m} \times \vec{n}) \times \vec{n}\big]^2\hat{e_r} \tag{4.4.22}$$

Inserting $k = \frac{\omega}{c}$, $\lambda = \frac{2\pi}{k}$ and $(\vec{m} \times \vec{n})^2 = m^2 \sin^2\theta$

Then the average pointing vector for the equation is given by

$$\vec{S}_{av} \cong (\mu_o)^2\left(\frac{\omega}{c}\right)^4\left(\frac{Z_om^2\sin^2\theta}{32\pi^2r^2\lambda^2}\right)\hat{e_r} \tag{4.4.23}$$

Therefore the average pointing vector direct proportional to the angular frequency ω^4, magnetic dipole moment m^2 and the square of sine angle θ. Simply the average pointing vector dependence on the above quantities.

4.5 Magnetic dipole radiation power

The magnetic field is in the direction of $\hat{e}_\theta$. So that $\vec{E} \times \vec{B}$ above expressed in Eqn. (4.4.20) is in the direction of $(\hat{e_r})$. Under the approximation for far zone $kr \gg 1$ and again we have the electric field as given in Eqn. (4.3.17). The radial poynting vector is in the k direction. Therefore the poynting vector in Eqn. (4.4.23) will be given as

$$\vec{S}_{av} = \frac{1}{\mu_o}^2 \sqrt{\varepsilon_o \mu o} \left(\frac{\omega}{c}\right)^4 \left(\frac{m^2 \sin^2 \theta}{32\pi^2 r^2}\right) \hat{e}_r \tag{4.5.24}$$

where $Z_o = \sqrt{\frac{\varepsilon_o}{\mu_o}}$ and $\sqrt{\varepsilon_o \mu_o} = \frac{1}{c}$ and then from the above relations we have

$$\vec{S}_{av} = \frac{\mu_o m^2 \omega^4}{32\pi^2 c^3} \left(\frac{\sin^2 \theta}{r^2}\right) \hat{e}_r \tag{4.5.25}$$

The magnetic radiation power is the integral of poynting vector over the NS's surface area. Now it is given by

$$< P >= \int < S_{av} > \cdot da = \int < S_{av} > \cdot r^2 \sin\theta d\theta d\phi \tag{4.5.26}$$

$$< P >= \int \frac{\mu_o m^2 \omega^4}{32\pi^2 c^3} \left(\frac{\sin^2}{r^2}\right) \hat{e_r} \cdot r^2 \sin\theta d\theta d\phi \tag{4.5.27}$$

$< P >= \frac{\mu_o m^2 \omega^4}{32\pi^2 c^3} \int d\varphi \int (\sin^3\theta) d\theta$ and $\sin^3\theta = (1 - \cos^2\theta)\sin\theta$

$$< P >= \frac{\mu_o m^2 \omega^4}{12\pi c^3} \tag{4.5.28}$$

The total average power differentiate over the surface of the sphere as seen in the above Eqn. (4.5.27) is given by:

$$\frac{dP}{d\Omega} = \frac{\mu_o m^2 \omega^4}{32\pi^2 c^3 r^2} \sin^2\theta \tag{4.5.29}$$

Again the average energy flux from the above equation (4.5.25) we have

$$< S_{av} >= \frac{\mu_o m^2 \omega^4}{32\pi^2 c^3} \left(\frac{\sin^2 \theta}{r^2}\right) \hat{e_r} \tag{4.5.30}$$

The average poynting vector at a distance $r \gg \lambda$ of an oscillating magnetic dipole situated at origin. The radial distance from the dipole of the surfaces is proportional to the magnitude of the quantity in the corresponding direction. There is zero field and zero power flow along the axis. This $\frac{1}{r^2}$ dependence result from the fact that the radiation terms for E and for B both varies as the inverse relation to the distance r. Since the energy flow varies as $\sin^2\theta$, it goes to zero along the along the equator of the ring and maximum along the polar plane. The power radiate can be written as

$$P_{rad} = \frac{< S_{av} >}{c} \tag{4.5.31}$$

By substituting the average value of pointing vector in the Eqn. (4.5.31) we obtain

$$P_{rad} = \frac{\mu_o m^2 \omega^4}{32\pi^2 c^4}\left(\frac{\sin^2\theta}{r^2}\right) \tag{4.5.32}$$

If the magnetic dipole is included by some angle $\theta > 0$ from the rotation axis, it emits low frequency ω of electromagnetic radiation. Recall the Larmor formula for radiation from a rotating electric dipole [27].

$$P_{rad} = \frac{2(q\ddot{r}\sin\theta)^2}{3c^3} = \frac{2(\ddot{p})^2}{3c^3}, \tag{4.5.33}$$

where p is perpendicular component of the electric dipole moment. By analogy, the power of magnetic dipole radiation from an inclined magnetic dipole is given as:

$$P_{rad} = \frac{2(\ddot{m})^2}{3c^3} = \frac{2}{3}\frac{m^2\omega^4}{c^3}, \tag{4.5.34}$$

where $\omega = \frac{2\pi}{p}$ is the angular velocity, taken to be perpendicular to the magnetic dipole moment **m**. The radiated power in Eqn. (4.5.34) is derived from a decrease in the rotational kinetic energy, $E = -\frac{1}{2}I\omega^2$, of the pulsar and that is given by

$$P_{rad} = -\frac{dE}{dt} = I\omega\dot{\omega} = \frac{2}{5}MR^2\omega \mid \dot{\omega} \mid, \tag{4.5.35}$$

where the moment of inertia I is taken to be that of a sphere of uniform mass density. By combining Eqn. (4.5.34) and (4.5.35), we have

$$\mathbf{m}^2 = \frac{3}{5}\frac{MR^2 \mid \dot{\omega} \mid c^3}{\omega^3} \tag{4.5.36}$$

Substituting $\omega = \frac{2\pi}{p}$, and $\dot{\omega} = 2\pi(\frac{\dot{p}}{p^2})$, we find

$$\mathbf{m}^2 = \frac{3}{20\pi^2}MR^2 p \mid \dot{p} \mid c^3 \tag{4.5.37}$$

The static magnetic field $\mathbf{B}$ due to magnetic dipole moment $\mathbf{m}$ is

$$\mathbf{B} = \frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{r^3} \tag{4.5.38}$$

So the maximum value of the magnetic dipole moment at radius R is

$$\mathbf{m} = \frac{1}{2}\mathbf{B}R^3 \tag{4.5.39}$$

Inserting Eqn. (4.5.37) in Eqn. (4.5.39), the maximum surface magnetic field is calculated as

$$B^2 = \frac{3Mp|\dot{p}|c^3}{5\pi^2 R^4} = \frac{(3)(2.8\times10^{30})(7.5)(8\times10^{-11})(3\times10^{8})^3}{5\times(3.14)^2(10^4)^4} \approx 1\times10^{30}\ G^2, \tag{4.5.40}$$

where $M = 1.4\ M_{\odot}$ is mass of rotating neutron star, p is its time period of the rotation and R is its radius about 10 km. Therefore we could estimate the the maximum magnetic field strength of rapidly rotating NS (pulsar). Thus, from Eqn. (4.5.40) very strongly magnetized NS magnetic field strength is $B \approx 10^{15}$ G. As it theoretically written in Chaps. 1 and 2 the magnetic field of neutron stars ranges between 10^8 to 10^{15} G and for a very younger NS the magnetic field is about 10^{15} G and especially for magnetars the magnetic field strength, $B \sim 10^{14-16}$ G, [28].

Also from the above relation the power radiate as a function of the magnetic field strength is given by

$$P_{rad} = \frac{B^2 R^6 \omega^4 \sin^2\theta}{6c^3} \tag{4.5.41}$$

θ is the angle of inclination between the dipole magnetic axis and the spinning axis. The maximum power radiated by fast rotating neutron stars, obtained at $\theta = 90^0$ ($\sin 90^0 = 1$) is given by

$$P_{rad} = \frac{B^2 R^6 \omega^4}{6c^3} \approx 10^{24} MW, \tag{4.5.42}$$

for $B = 1.7 \times 10^{15}$ G, $R = 10km = 10^4$ m and $\omega = \frac{2\pi}{p}$=0.837/s

4.6 Summary

In this unit we have drived the the average power flux density and the average power radiate due to the magnetic field generated by the dipolar component of the pulsar (rotating neutron star) magnetic field. We have seen the average energy flux is proportional to the square of the sine angle θ and inversily proportional to the square of the distance r from the surface of the star. The average power radiated or emitted from the pulsar is also proportional to the square of the sine angle θ and inversily proportional to the square of the distance r from the center of the rotating neutron star (pulsar). The average energy flow or the average power radiated is direct proportionality the square of the magnitude magnetic field strength generated from the pulsar, that is, $P_{rad} \propto B^2$. The neutron stars with higher magnetic field strength have the greater rate of energy radiation or higher power radiation emission. For instance, pulsars with strong magnetic fields and the magnetars.

Chapter 5

Results and Discussions

In this work we have derived expression for the net magnetic field strength $\vec{B}$ of the dipolar component of the pulsar inside and outside the sphere's surface, as well as on its surface which is starting from the vector potential $\vec{A}$. We have constructed curve of the pulsar's magnetic field generated by dipolar field with respect to angle variation. From Eqn. (3.2.32) the magnetic field of the dipolar component at the surface of the neutron star (at r = R) be is given by

$$\vec{B} = -\left(\frac{|Q_s|\omega}{3Rc}\right)\left[3\cos\theta\hat{e}_r - \hat{k}\right], \tag{5.0.1}$$

but we have $\left[3\cos\theta\hat{e}_r - \hat{k}\right] = \left[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta\right]$ from Eqn. (3.2.31). Then it gives

$$\vec{B} = -(\frac{|Q_s|\omega}{3Rc})[2\cos\theta\hat{e}_r + \sin\theta\hat{e}_\theta], \tag{5.0.2}$$

and the magnitude of the magnetic field from Eqn. (5.0.2) should be calculated as

$$|\vec{B}| = \left(\frac{|Q_s|\omega}{3Rc}\right)\sqrt{(2\cos\theta\hat{e}_r)^2 + (\sin\theta\hat{e}_\theta)^2} \tag{5.0.3}$$

$$|\vec{B}| = \left(\frac{|Q_s|\omega}{3Rc}\right)\left(\sqrt{4\cos^2\theta + \sin^2\theta}\right) = \left(\frac{|Q_s|\omega}{3Rc}\right)\left(\sqrt{3\cos^2\theta + 1}\right), \tag{5.0.4}$$

where θ the angle between the magnetic axis in the radial direction and rotational axis of the neutron star and $\hat{e}_r$ and $\hat{e}_\theta$ are orthonormal.

Since for typical pulsar R=10 km, $\omega = \frac{2\pi}{p} = 4000\ rad/s$, and $|Q_s| \approx 7 \times 10^{20}$ C [29]. We can solve the surface magnetic field strength by taking the time period *(p)* for typical rotating neutron star, that is for p=7.5 s and for $\theta = 0^0$. Then we have

$$|\vec{B}| = 2\left(\frac{|Q_s|\omega}{3Rc}\right) = 2\frac{(7 \times 10^{20}\ C)(4000\ rad/sec)}{3 \times 10^{12}\ m^2/sec} \approx 2 \times 10^{12}\ G \tag{5.0.5}$$

The magnetic field of the pulsar as the variation of angles is given by

$$|\vec{B}| \approx 10^{12} \times \left(\sqrt{3\cos^2\theta + 1}\right)\ G \tag{5.0.6}$$

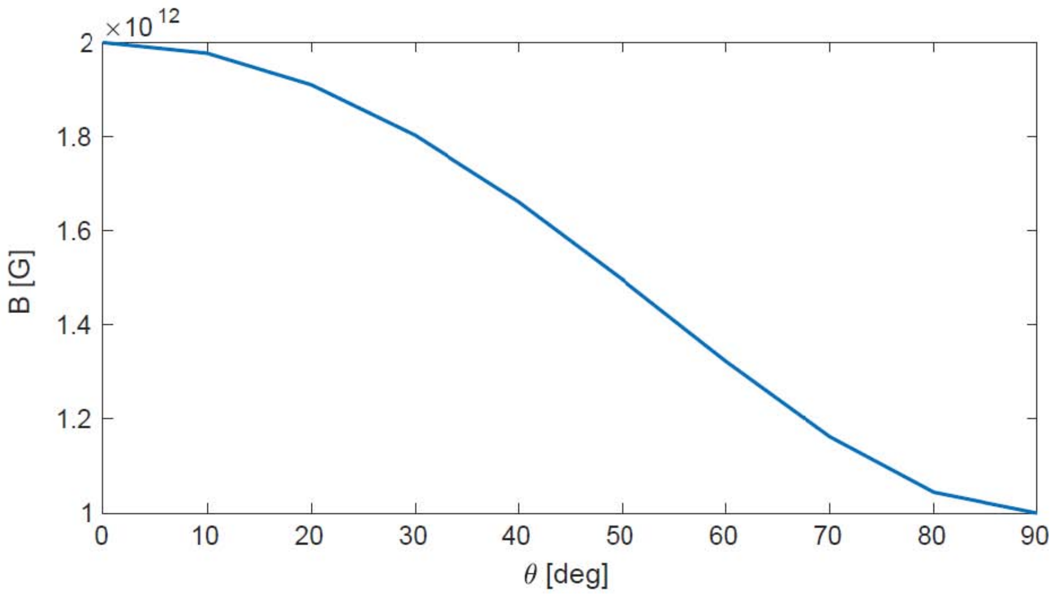

Figure 5.1: Magnetic dipolar component field curve of the pulsar as angle variation.

Figure 5.1 shows the dipole field Eqn. (5.0.5) and (5.0.6) say that a dipole field is parallel to the radial direction over the poles and perpendicular to the radial direction on the equator. It is twice as strong at the pole as at the equator at a fixed radial distance R from the center of the rotating neutron star. On the other hand, the magnetic field of the dipolar component is maximum at ($\theta = 0^0$) and minimum at ($\theta = 90^0$). Then from the results, we have seen that the curve of dipolar component of pulsar magnetic field $\vec{B}$ dropped from 2×10^{12} G to 1×10^{12} G between these angles.

We solved the magnetic field strength by considering the values of different angles. When $\theta = 0^0$ and $\theta = 180^0$ or $\theta = n\pi$, where n = 0, 1, 2, 3,... give the maximum value of B and it is given by

$$|\vec{B}| = 2\left(\frac{|Q_s|\omega}{3Rc}\right), \tag{5.0.7}$$

when $\theta = 90^0$ and $\theta = 270^0$ or for $\theta = (2n-1)\frac{\pi}{2}$, where n = 1, 2, 3,... are give the minimum values of B and it is given by

$$|\vec{B}| = \left(\frac{|Q_s|\omega}{3Rc}\right) \tag{5.0.8}$$

Based on the observational time periods of data and using Eqn. (5.0.7) and (5.0.8) we can calculat and verify the value of the maximum and minimum B for the seven known pulsars such as Crab (p = 0.33-33 ms), PSRB1509-58 (p = 150 ms), Vela (p = 89 ms), PSR B1706-44 (p = 102 ms), PSR B1951+32 (p = 39 ms), Geminga (p = 237 ms), PSR B1055-52 (p = 197 ms) [30]. Where p is spinning time period in milliseconds (ms). For example using the equation of the dipole magnetic field derived above we can verify the values of the field strength B for Crab pulsar for fixed angle, i.e at $\theta = 0^0$ or at $\theta = 180^0$ and its spinning period, p = 0.33 ms to 33 ms. Therefore the magnitude of the magnetic field be simply given by

$$|\vec{B}| = 2\left(\frac{|Q_s|\omega}{3Rc}\right) \tag{5.0.9}$$

And also the estimated value of the magnetic field for the seven known pulsars, such as the Crab pulsar of period $p = 33$ ms is

$$B = 2\frac{(7\times 10^{20})(6000\ rad/sec)}{3\times 10^{12}\ m^2/sec} \approx 2.8\times 10^{12}\ G \tag{5.0.10}$$

Therefore the surface magnetic field for Crab pulsars ranges from 10^{12} *to* 10^{13} G. Similarly for the other six known pulsars, their magnetic fields are listed in the table below

Types pulsar	p (ms)	B$_{min.}(G)$	B$_{max.}(G)$
Crab	0.33-33 ms	3.80×10^{12}	1×10^{13}
PSR B1951+32	39 ms	0.49×10^{12}	1.65×10^{12}
PSR B1509-58	150 ms	4.3×10^{12}	9.71×10^{12}
Vela	89 ms	2.75×10^{12}	3.49×10^{12}
PSR B1706-44	102 ms	1.26×10^{12}	3.25×10^{12}
Geminga	237 ms	0.64×10^{12}	1.69×10^{12}
PSR B1055-52	197 ms	0.55×10^{12}	1.2×10^{12}

Table 5.1: Magnetic fields of the seven known pulsars and their time period.

The table above shows the maximum and minimum magnetic fields of the seven well studied gamma-ray pulsars as they spinning with different periods. The magnetic field from Eqn. (3.2.32) could be rewrite as varying distance r from the surface of the NS.

$$|\vec{B}| = \left(\frac{|Q_s|\omega}{3c}\right)\left(\frac{R^2}{r^3}\right)\sqrt{3\cos^2\theta + 1}, \tag{5.0.11}$$

where at $\theta = 0^0$ and $\theta = 180^0$ the magnetic field strength is

$$|\vec{B}| = 2\left(\frac{|Q_s|}{c}\right)\left(\frac{R^2}{r^3}\right), \tag{5.0.12}$$

and at $\theta = 90^0$ and $\theta = 270^0$ the magnetic field strength is

$$|\vec{B}| = \left(\frac{|Q_s|\omega}{3c}\right)\left(\frac{R^2}{r^3}\right) \tag{5.0.13}$$

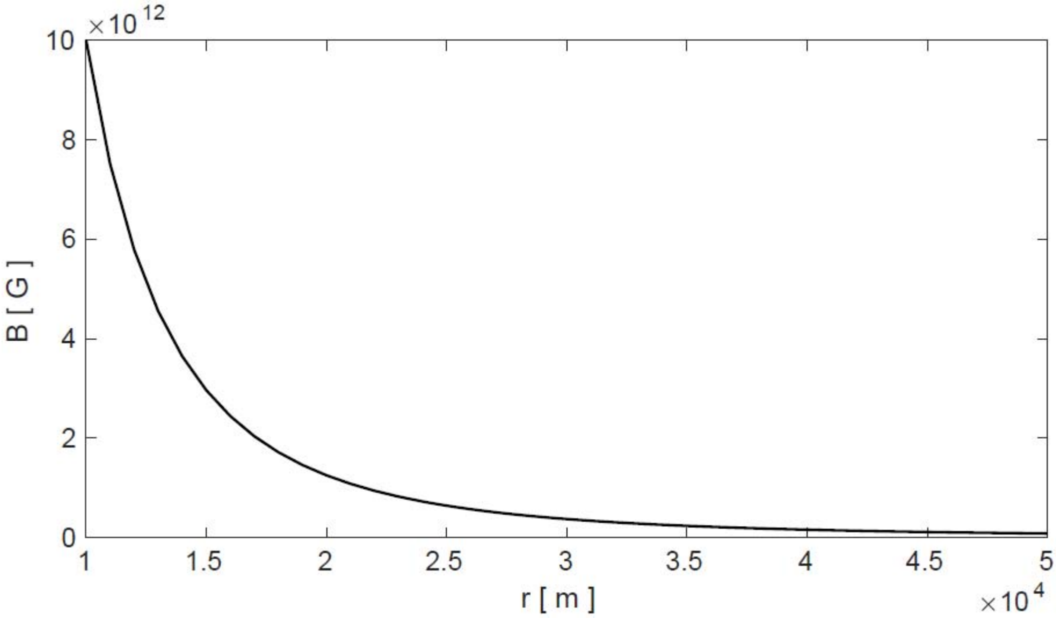

Figure 5.2: The magnetic field vs distance r from center of rotating neutron stars.

Figure 5.2 describes that at any latitude, the magnetic field strength decreases with radial distance as $\frac{1}{r^3}$ for particular angle θ. It has maximum magnetic field at distance or radius, r = R = 10 km and slowly decreases as r goes larger. Also the magnetic field of the dipolar component of the pulsar is inversely proportional to r^3 ($B \propto \frac{1}{r^3}$) from Eqn. (5.0.11) at a fixed angle.

We have derived the power radiation curve of the dipolar component of pulsar and we showed the equation of power radiation arrived at Eqn. (4.5.27), Eqn. (4.5.32) and Eqn. (4.5.34) and we have constructed the power radiation curve of the dipolar components of magnetic field of the pulsar depending on the average energy flux. The average power radiated is derived from Eqn. (4.5.30) and Eqn. (4.5.31) is given as

$$P_{rad} = \frac{\mu_o m^2 \omega^4}{32\pi^2 c^4}\left(\frac{\sin^2\theta}{R^2}\right) \tag{5.0.14}$$

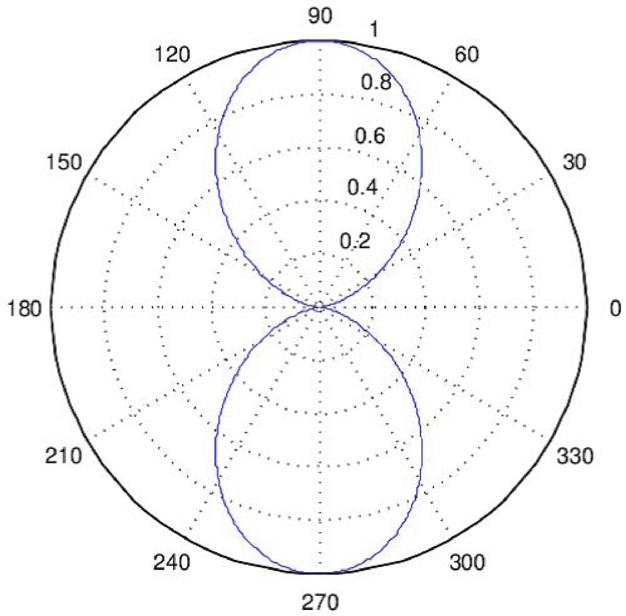

Figure 5.3: the power radiation along the magnetic dipolar component of the pulsar.

Since from Eqn. (4.5.30) $< S_{av} > \propto \sin^2\theta$, the average energy flux is largest at an angle $\theta = 90^0$, and vanish for $\theta = 0^0$ and $\theta = 180^0$. Therefore, figure 5.3 shows that there is no energy radiated along the horizontal axis of the circle, i.e, along the polar axis of the NS and the radiation is largest at poles of the circle, i.e, at the equatorial axis of the NS. Therefore, the power radiation varies with values of sine (θ) and thus at $\theta = 0^0$, the rotational axis and magnetic axis of the NS are aligned the average power radiate is zero and it has maximum values at $\theta = 90^0$ (or the rotation axs and magnetic axis are misaligned by ninety degree). In other word, the power radiation occur between $\theta = 0^0$ and $\theta = 90^0$. Thus the power radiation is maximum at the equator of the rotating neutron star (pulsar).

Chapter 6

Conclusions

In this thesis paper, we constructed the curve of magnetic dipole field versus angle variation and the power radiation emission curve of dipolar magnetic field of the pulsar based on the derived equations of the magnetic fields, the average energy flux radiated and the models used in the theoretical predictions. All current models of pulsar magnetic radiation curves assumed that the pulsar magnetic field emission can be represented by magnetic dipole field. The case of the field should be considered as an approximation to the field which includes the effects of magnetic radiation fields. Therefore, the dipole model is the basic for the others non-dipolar components of the radiation curves of the fields. We showed and derived the magnetic dipolar components of field strength in and outside the rotating NS (the pulsar) and also we calculated its estimate and check value of the surface magnetic field strength for typical pulsar and for seven known pulsars ,i.e it ranges between 10^{12} G to 10^{13} G depending on the spinning period *(p)*. In our study, we used spherical harmonics theorem and Maxwell's equations to solve the equations of the dipolar magnetic fields and the power radiation emission of a rotating neutron stars.

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