MAGNETIC FIELD DECAY DUE TO GRAVITATIONAL RADIATION IN NEUTRON STAR

A THESIS SUBMITTED TO
THE GRADUATE PROGRAMS OF
ADDIS ABABA UNIVERSITY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN PHYSICS

BY
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ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA
NOVEMBER 2017

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Department: **Physics**

Degree: **M.Sc.** Convocation: **November** Year: **2017**

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THIS WORK IS DEDICATED TO MY FAMILY.

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Acknowledgements

First of all, I would like to thank the almighty **GOD** for letting me to accomplish this study. Secondly, I would like to express my sincere gratitude to thank my advisor Dr. Remudin Reshid for his unlimited and constructive guidance, advice, suggestions, comments his scientific excitement, integral view on research and overly enthusiasm, which have made a deep impression on me. I would further like to thank my late advisor and instructor Dr. Legesse Wetro for his encouragement and gave me valuable comments and suggestions on my thesis. My strongest thanks goes to my Mother W/ro Hirut Desta, my Brothers and Sisters for they are the heros of my success in financial support and encouragement. I would like to give my special thanks and grateful to my wife Bizuye Birlew and to my daughter Hiyab Tesfay for their kindheartedly encouragement and entertainment. I would also need to thank all my instructors and physics department staff members. Finally, I would like to express my thanks to all my friends especially Dr. Tadesse Desta for their love and support.

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November, 2017.

Abstract

In this thesis, magnetic field decay due to gravitational radiation in neutron star (NS) is theoretically studied. By considering the influence of the "dynamo mechanism"on the generation of a strong magnetic field in NS. A NS formed as a large star die in a type II supernova, and their magnetic fields during formation is 10^{12} Gauss and 10^{15} Gauss for both magnetars and pulars, respectively. But their magnetic field is decreased. Thus, this study focuses on by how much does the magnetic field decay using the mathematical calculation of decay law equation. Based on our findings, we have two results. The first result shows that, the magnetic field of a rotating NS depends only on the decaying time since the moment of inertia is constant. The second result also shows, the magnetic field of a rotating NS depends on the decaying time and rotational frequency since moment of inertia varies as a function of rotational frequency. The variation in moment of inertia causes to the variation in braking index as a function of rotational frequency. Thus, the quadrapole magnetic field is decaying for positive $\dot{\Omega}$ and increased for negative $\dot{\Omega}$, respectively.

Introduction

Our universe consists of elementary particles (electron, proton, neutron, positron, neutrino, and photon, etc.) and massive bodies (stars, galaxies, and so on). Stars are formed in molecular clouds in the interstellar medium, which consist mostly of molecular hydrogen (primordial elements made a few minutes after the beginning of the universe) and dust. The dust originates from the cool surfaces of supergiants, massive stars in a late stage of stellar evolution [1].

Grains of carbon or the night sky contains myriads of stars including those in the milky way, which is a side view of our galaxy looking along the plane of the disc. Our galaxy includes about 10^{11} stars [2]. Beyond our galaxy are billions of other galaxies. The nearest star is Proxima Centauri it is about 4 light years away and the nearest large galaxy is Andromeda which is about 2 million light year away. Our galactic disc has a diameter of about 100,000 light years [3].

Basic factors in the formation of stellar objects are gravity, dust, gas pressure, rotation, magnetic fields, winds and radiation from nearby young stars and radiative shock waves [4]. Stars are believed to begin their life as collapsing masses of hydrogen gas (protostars). As they contract, they heat up (potential energy is transformed to kinetic energy). When the temperature reaches about 10 million degrees, nuclear fusion begins and forms heavier elements (nucleosynthesis), mainly helium at first [5]. The energy released during these reactions heats the gas so its outward pressure balances the inward gravitational force, and the young star stabilizes as a main sequence star. The tremendous luminosity of stars comes from the energy released during these thermonuclear reactions. This thermonuclear fusion provides the energy for the great furnace, which drives stars through various stage of combustion-hydrogen, helium, carbon, neon, oxygen, silicon, and magnesium [6]. After billions of years, as helium is collected in the core and hydrogen is used up, the core contracts and heats further. The envelope expands and cools, and the star becomes a red giant (larger in diameter, redder in color) [7]. The next stage of stellar evolution depends on the mass of the star, which may have lost much of its original mass as its outer envelope ejected into space as a result of star's explosion [7].

The explosion of the star is divided into two, namely, nova and supernova. The term nova refers to a class of exploding stars whose luminosity temporarily increases from several thousand to as much as 100,000 times its original level [8]. Most novas are thought to involve special double-star systems (close binaries). The term supernova refers to a completely different phenomenon: a supernova is any of the class of violently exploding stars whose luminosity after eruption suddenly increases millions or billions of times its normal level [8].

Neutron stars (NSs) are born due to the supernova explosion [9]. NSs differ from normal stars in two fundamental ways. First, since they do not burn nuclear fuel, they cannot support themselves against gravitational collapse by generating thermal pressure [10]. Instead, NSs are supported largely by the pressure of degenerate neutrons [10]. The second characteristic distinguishing NSs from normal stars is their exceedingly small size. Relative to normal stars of comparable mass, NSs have much smaller radii and hence, much stronger surface gravitational fields [11].

A NS is the remaining core of a massive star. Once it has exploded millions of NSs populate our galaxy. Made almost entirely of neutrons (subatomic particles with no electric charge), these stellar corpses concentrate more than the mass of our Sun with in a sphere of about 20 kilometer's in diameter.

In this thesis we will consider the complex magnetic fields of a compact star called NS. Observations and theory suggest that complex quadrapole magnetic fields prevail near the surface of NSs and play an important role in the physics of rotation powered pulsars [10].
Objectives of the thesis is driving decay law equation for magnetic quadrupole radiation in NS for both constant moment of inertia and moment of inertia as a function of rotational frequency. And modeling (simulating) the decay curve of this quadrupole field in 2-dimension.
The thesis is organized as follows. Chapter one deals with formation, birth, classification and nature of NSs. Chapter two deals with the source of magnetic fields in neutron stars. Chapter three deals with power radiated due to gravitational quadrupole radiation, magnetic field decay due to gravitational quadrupole radiation when moment of inertia is constant and when a moment of inertia varies as function of rotational frequency. Chapter four deals with result and discussion. Conclusion is given in chapter five.

Chapter 1

Neutron Stars

1.1 Neutron star formation

Neutron stars are one of the possible end state for a massive star. Neutron stars are formed as a large star dies in a type II supernova [9]. A type II supernova occurs when the iron core of a super giant star collapses to the density of an atomic nucleus (a few hundred million tons per cubic centimeter). At such tremendously high densities, protons and electrons fuse together to form neutrons. Hence the name “NSs”. The supernova blows off much of the star, and you are left with the collapsed iron core of a star (this happens to stars that have a mass about 8 to about 25 times the mass of the Sun) [12]. The remaining iron core collapses down to about 20 kilometers wide, but it still maintains a mass between 1.5 to 5 times the mass of our Sun. Neutron stars are typically about 20 kilometers or so, about the size of a decent sized city. Neutron stars are immensely dense and strong, a teaspoon sized NS matter would weigh about 100 million tons. The crust of the NS would also be about 100 billion times stronger than steel [13].

In general NSs come from two possible evolutionary scenarios, as shown in Fig. 1.1 i.e

1. Stars starting in the upper mass range eventually becomes supernova of type II or type Ib. They shed their outer layers, and leave a rapidly spinning NS (a pulsar) as a final object [14].

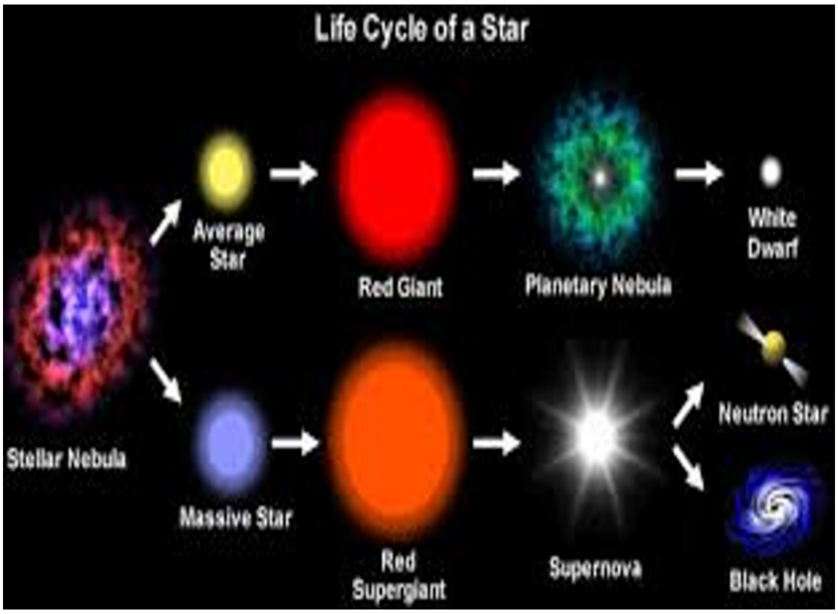

Figure 1.1: Neutron star formation

2. Stars of the lower mass range becomes White dwarfs. Those in a binary system may accumulate mass, and becomes supernova type Ia and perhaps a NS remains [14].

1.2 The birth of a Neutron Star

In cores larger than the Chandrasekhar limit (typically quoted as 1.4 $M_\odot$), electron degeneracy pressure will be generate, but the electrons will become ultra-relativistic. Ultra-relativistic electrons provide a pressure that has the same scaling as the gravitational pressure of the collapsing star, which are unable to reach equilibrium state. This marks the beginning of a NS [15].

As the core continues to collapse, the matter within it will continue to heat up due to the release of gravitational potential energy. Enough free energy is available that the following inverse beta decay reaction can occur.

$p^+ + e^- + 1.36\ MeV \rightarrow n + v_e{}^-$ [16].

Ordinarily neutrons generated in this fashion would be unstable, and the neutron would turn back into an electron and a proton within ~ 10 minutes via beta decay. The degenerate electron gas in the star has filled all of the available electron states in the core. There is no electrons of energies ≤ 1.36 MeV can be formed, which makes the neutrons stable [16].

1.3 Classification of Neutron Stars

Two other physical properties characterizes a NS, their fast rotation (or spin) and their strong magnetic field. Astronomers have found different classes of NSs based on these properties. Some of them are the fastest spinning stars in the universe (up to thousands of revolutions per second); they are named "pulsars" as they generate regular pulses of electromagnetic radiation including radio, visible, X-ray and gamma-ray wavelengths [7].

Another class of NS is known as a "magnetar", due to their ultra-strong magnetic field. Their magnetic field intensity is indeed about 10^{15} Gauss, a thousand times more than an ordinary NS [17].

Their typical polar surface magnetic fields $10^{12} - 10^{13}$ Gauss and $10^{14} - 10^{15}$ Gauss, of pulsars and magnetars respectively. In addition to this, astronomers can estimate the age of a supernova remnant (SNR) which is, how long ago the supernova occurred. This is achieved by measuring the properties of SNR, such as its size and the rate of expansion [18]. Therefore, the burster remains radio "loud" (loud or loudness is where the ratio between the radio to optical flux is greater than 10) for approximately 10 million years in the case of pulsars [7] and where as the burster eject bursts of high-energy X-rays or gamma-rays for 10,000 years in the case of magnetars [17]. Magnetars are subdivided into soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) [2]. Pulsars are also subdivided into Crab pulsar, Vela pulsar, Millisecond pulsar, etc [17].

1.4 Nature of Pulsars and Magnetars

1.4.1 Pulsars

Let us start with idea of skater to express the general theory of rotation (pulsar spin). So if the skater goes into a spin. Typically, a skater starts off with arms and one leg spread out. As we draws our arms and leg in towards our body, we starts spinning faster and faster. This is an example of the law of angular momentum, which a complex way to understand. When something spins and shrinks in size, but keeps the same mass, the object spins faster. The same happens with a NS. The star has shrunk in size, but maintained its mass, so it spins very fast. The record speed is 1122 rotations in one second, with the record slowest speed is one rotation in 4.308 seconds [19].

Now, a NS has a solid core and a "liquid" mantle. Its crust is only about an inch thick, but the solid core and "liquid" mantle gives the star a magnetic field, and with its mass and density, the field is about a trillion times as strong as Earth's [20]. An initial magnetic field strength of a pulsar is $B \sim 100$ Gauss [21]. Neutron stars emit high-energy beams at its North and South magnetic poles as shown in Fig. (1.2), which is usually made from material from a companion star. If these beams are pointed at Earth, the NS rotates, they seem to pulse. So a pulsating NS is known as pulsar. Generally all pulsars are NSs, but not all NSs are pulsars. All depends on which way its energy beams are pointing.

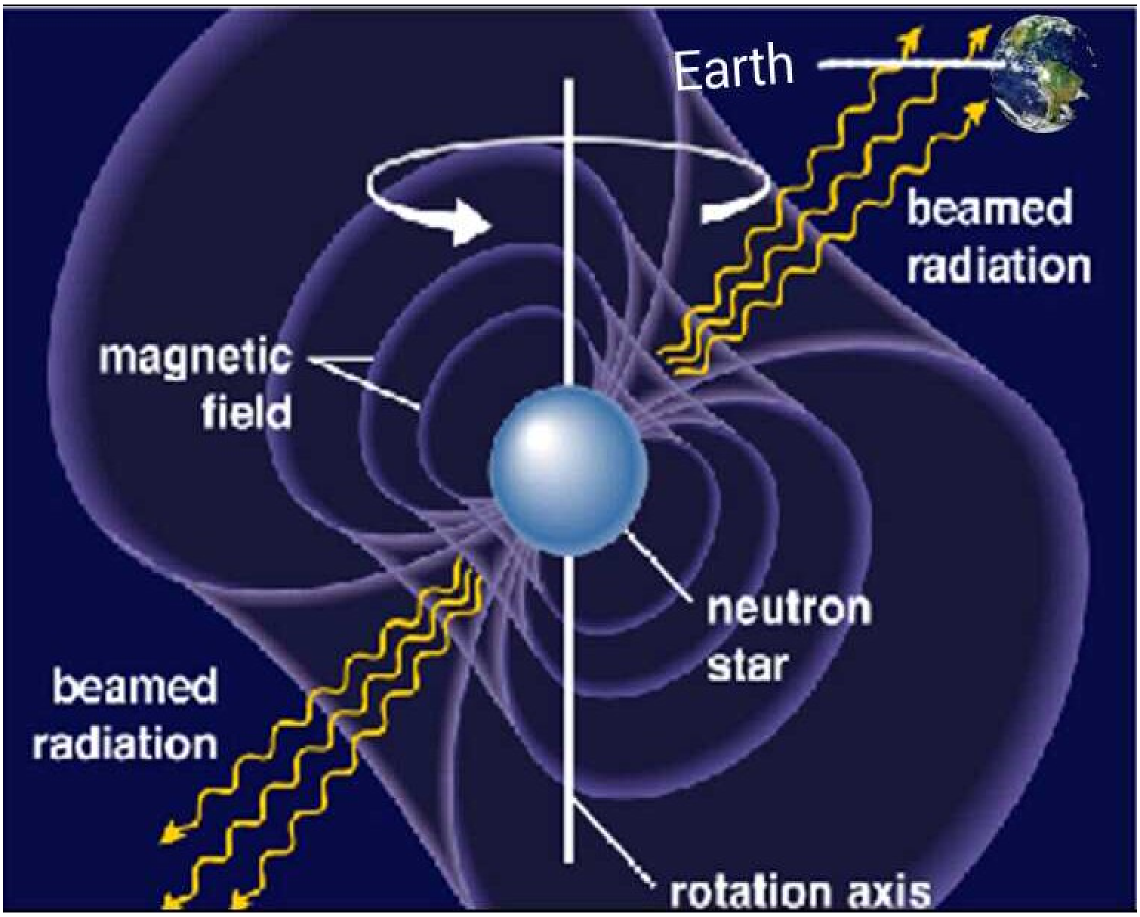

Figure 1.2: Neutron stars emit high-energy beams

1.4.2 Magnetars

It is believed that magnetars are a type of NS that were made during a supernova explosion, similar to that of a pulsar [9]. They are one of the most dense objects in the universe. It is theorized that the dynamo mechanism may be the reason to their formation. Basically, if the spin, temperature and the magnetic field of a NS are with in the right ranges it can convert the heat and rotational energy into very strong magnetic energy [22].

Although NSs and a magnetar are similar in formation they hold very different characteristics which sets them apart from each other. For instance magnetars rotate at a very slower rate, usually once every 8 to 10 seconds as opposed to one or more rotations a second for NSs [23]. Another difference between a magnetar and a NS is that a magnetar emits a steady glow of x-rays with more radiant power than could be supplied by the rotation of a NS [17]. The magnetic fields made by a magnetar are about 1,000 trillion that of the earth's magnetic field and can reach surface temperatures of 18 million degrees Fahrenheit [24].

Chapter 2

The source of magnetic fields in neutron stars

2.1 Dynamo Mechanism

The magnetic field of a rotating body of conducting gas or liquid develops self-amplifying electric currents, and thus a self-generated magnetic field, due to a combination of differential rotation (different angular velocity of different parts of body), coriolis forces and induction [25]. The distribution of currents can be quite complicated, with numerous open and closed loops, and thus the magnetic field of these currents in their immediate vicinity is also quite twisted. At large distances; however, the magnetic fields of currents flowing in opposite directions cancel out and only a net dipole field survives, slowly diminishing with distance [2]. Because the major currents flow in the direction of conductive mass motion (equatorial currents), the major component of the generated magnetic field is the dipole field of the equatorial current loop, thus producing magnetic poles near the geographic poles of a rotating body [26].

The magnetic fields of all celestial bodies are often aligned with the direction of rotation, with notable exceptions such as certain pulsars [27]. Another feature of this dynamo model is that the currents are AC rather than DC. Their direction, and thus the direction of the magnetic field they generate, alternates more or less periodically, changing amplitude and reversing direction, although still more or less aligned with the axis of rotation [28]. If the gas or liquid is very viscous (resulting in turbulent differential motion), the reversal of the magnetic field may not be very periodic [29].

2.1.1 Surface activity

Starspots are regions of intense magnetic activity on the surface of a star (On the Sun they are termed sunspots). These form a visible component of magnetic flux tubes that are formed with in a star's convection zone. Due to the differential rotation of the star, the tube becomes curled up and stretched, inhibiting convection and producing zones of lower than normal temperature [30]. Coronal loops often form above starspots, forming from magnetic field lines that stretch out into the corona. These in turn serve to heat the corona to temperatures over a million kelvins [31].

The magnetic fields linked to starspots and coronal loops are linked to flare activity, and the associated coronal mass ejection. The plasma is heated to tens of millions of kelvins, and the particles are accelerated away from the star's surface at extreme velocities [32].

Surface activity appears to be related to the age and rotation rate of main-sequence stars. Young stars with a rapid rate of rotation exhibit strong activity. By contrast middle-aged, Sun-like stars with a slow rate of rotation show low levels of activity that varies in cycles. Some older stars display almost no activity, which may mean they have entered to quiet [33].

2.1.2 Magnetosphere

Magnetosphere is a region of space surrounding a star that is dominated by the stars magnetic field so that charged particles are trapped on it. A star with a magnetic field will generate a magnetosphere that extends outward into the surrounding space [34]. Field lines from this field originate at one magnetic pole on the star then end at the other pole, forming a closed loop. The magnetosphere contains charged particles that are trapped from the stellar wind, which then move along these field lines. As the star rotates, the magnetosphere rotates with it, dragging along with it charged particles [34].

As stars emit matter with a stellar wind from the photosphere, the magnetosphere creates a torque on the ejected matter [35]. This results in a transfer of angular momentum from the star to the surrounding space, causing a slowing of the stellar rotation rate [35]. Rapidly rotating stars have a higher mass loss rate, resulting in a faster loss of momentum. As the rotation rate slows, so as a result of the angular deceleration. By this process, a star will gradually approach, but never quite reach, the state of zero rotation [35].

After some massive stars have ceased thermonuclear fusion, a portion of their mass collapses into a compact body of neutrons called a NS [27]. These bodies retain a significant magnetic field from the original star, but the collapse in size causes the strength of this field to increase rapidlly [28]. The rapid rotation of these collapsed NSs results in a pulsar, which emits a narrow beam of energy that can periodically point toward an observer. Compact and fast-rotating astronomical objects have extremely strong magnetic fields [12]. The magnetic field of a newly born fast-spinning NS is so strong (up to 10^{12} Gauss) that it electromagnetically radiates enough energy to quickly (in a matter of few million years) slow down the star rotation by 100 to 1000 times [2]. The existence of such stars was confirmed in 1998 with the measurement of the star SGR 1806-20 [36]. The magnetic field of this star has increased the surface temperature to 18 million K and it releases enormous amounts of energy in gamma ray bursts [36].

Chapter 3

Gravitational Radiation

Disturbance in a gravitational field propagates out ward at a speed of light. The propagating disturbance represents a gravitational wave. Any gravitational wave can be decomposed into a superposition of plane waves [38]. In linearized theory(it is an approximation scheme in general relativity in which the non-linear contributions from the spacetime metric are ignored) one can consider a localized source of gravitational waves in steady oscillation, radiating a periodic wave. But the exact theory stand on that the energy of the sources decreases, to counter balance the energy carried off by the radiation which has quadrupole nature. This quadrupole system is used to approximate the gravitational radiation in the non-relativistic case [37].

An axis symmetric object rotating rigidly about its symmetry axis has no time varying quadrupole (or higher) moment, and hence does not radiate gravitational waves. The axis symmetric objects rotating rigidly about its symmetry axis means ($I_1 = I_2$), due to this the quadrupole moment becomes constant and no gravitational waves is emitted.

If the principal moments of inertia of an object are I_1, I_2 and I_3 then radiation will be produced if it rotates about the principal axis $\hat{e_3}$ and is non axis symmetric ($I_1 \neq I_2$). Alternatively, it can radiate if it is axis symmetric ($I_1 = I_2$), but the rotation axis is not the symmetry axis $\hat{e_3}$ [12]. The general case would be a nonsymmetric object rotating about an arbitrary axis. A possible physical application would be a pulsar where the rigid crust can support a "mountain" [12].

3.1 The power radiated due to gravitational quadrupole radiation

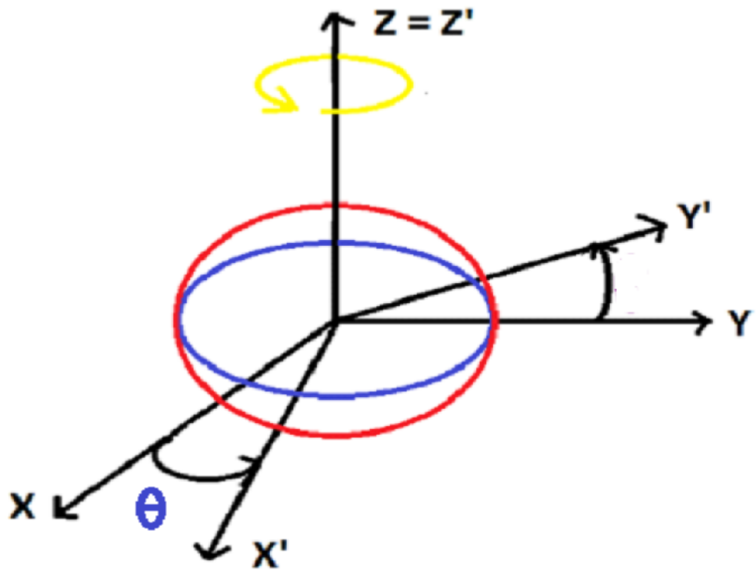

Figure 3.1: Rotation of NS by angle $\Theta = \Omega t$ rotating rigidly about z-axis

$$X' = R_{ij} x \tag{3.1.1}$$

where

$$R_{ij} = \begin{pmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.1.2}$$

is 3×3 transformation (rotation) matrix about z-axis or $\hat{e_3}$.

where $\Theta = \Omega t$ and $\Omega =$ constant.

If the NS is rotating rigidly about z-axis with angular velocity as shown in Fig. (3.1), then one can transform the coordinate from S frame to S'

frame or vice versa.

$$x_1 = x'_1 \cos\theta - x'_2 \sin\theta. \\ x'_1 = x_1 \cos\theta + x_2 \sin\theta. \tag{3.1.3}$$

$$x_2 = x'_1 \sin\theta + x'_2 \cos\theta. \\ x'_2 = x_1 \sin\theta - x_2 \cos\theta. \tag{3.1.4}$$

$$x_3 = x'_3. \tag{3.1.5}$$

The moment of inertia in fixed coordinate system (S frame) or the second moment of mass distribution is [12]

$$I_{ij} = \int x'_i x'_j \rho(x') d^3x. \tag{3.1.6}$$

Where $\rho(x')$ is the rest mass density in S' frame.

In the study of stars rotation where there is a rotational symmetry about an axis. which we call the x'_3 axis. Then all the "off diagonal" terms becomes zero.

$$I_{12} = I_{13} = I_{23} = 0. \tag{3.1.7}$$

The moment of inertia of a rotating star or the components of the star's reduced quadrupole moment is defined as [12]

$$I'_{ij}(t) = \int x_i x_j \rho(x') d^3x'. \tag{3.1.8}$$

Using this equation the components of $I'_{ij}(t)$ can be derived, i.e,

$$\begin{aligned} I'_{11}(t) &= \int x_1 x_1 \rho(x') d^3x', \\ &= \int (x'_1 \cos\theta - x'_2 \sin\theta)(x'_1 \cos\theta - x'_2 \sin\theta)(\rho(x') d^3x'), \\ &= \int (x'^2_1 \cos^2\theta - 2x'_1 x'_2 \cos\theta \sin\theta + x'^2_2 \sin^2\theta)\rho(x') d^3x', \\ &= (I_{11} \cos^2\theta - 2I_{12} \cos\theta \sin\theta + I_{22} \sin^2\theta), \end{aligned} \tag{3.1.9}$$

Since I_{12}=0, Eqn. (3.1.9) reduces to

$$I'_{11}(t) = I_{11} \cos^2\theta + I_{22} \sin^2\theta, \tag{3.1.10}$$

Using trigonometric identities $\cos^2\theta = (\frac{1+\cos 2\theta}{2})$ and $\sin^2\theta = (\frac{1-\cos 2\theta}{2})$ this further reduces to

$$I'_{11}(t) = \frac{1}{2}(I_{11} + I_{22}) + \frac{1}{2}(I_{11} - I_{22})\cos 2\theta. \qquad (3.1.11)$$

and also

$$\begin{aligned}
I'_{12}(t) &= \int x_1 x_2 \rho(x') d^3x', \\
&= \int (x'_1 \cos\theta - x'_2 \sin\theta)(x'_1 \sin\theta + x'_2 \cos\theta), \\
&= \int (x'^2_1 \cos\theta \sin\theta + x'_1 x'_2 \cos^2\theta - x'_2 x'_1 \sin^2\theta - x'^2_2 \sin\theta \cos\theta)\rho(x')d^3x', \\
&= \int (x'^2_1 - x'^2_2)\cos\theta \sin\theta + x'_1 x'_2(\cos^2\theta - sin^2\theta)\rho(x')d^3x', \\
&= (I_{11} - I_{22})\cos\theta \sin\theta,
\end{aligned} \qquad (3.1.12)$$

using $2\cos\theta\sin\theta = \sin 2\theta$ this reduces to

$$I'_{12}(t) = \frac{1}{2}(I_{11} - I_{22})\sin 2\theta. \qquad (3.1.13)$$

following similar procedure give

$$I'_{13}(t) = I'_{23}(t) = I'_{31}(t) = I'_{32}(t) = 0. \qquad (3.1.14)$$

$$I'_{21}(t) = \frac{1}{2}(I_{11} - I_{22})\sin 2\theta. \qquad (3.1.15)$$

$$I'_{22}(t) = \frac{1}{2}(I_{22} - I_{11}) + \frac{1}{2}(I_{22} - I_{11})\cos 2\theta. \qquad (3.1.16)$$

as well as

$$I'_{33}(t) = I_{33}. \qquad (3.1.17)$$

Then using $\Theta = \Omega t$ we have

$$I'_{11}(t) = \frac{1}{2}(I_{11} + I_{22}) + \frac{1}{2}(I_{11} - I_{22})\cos 2\Omega t. \qquad (3.1.18)$$

$$I'_{12}(t) = \frac{1}{2}(I_{11} - I_{22})\sin 2\Omega t. \qquad (3.1.19)$$

$$I'_{21}(t) = \frac{1}{2}(I_{11} - I_{22})\sin 2\Omega t. \qquad (3.1.20)$$

$$I'_{22}(t) = \frac{1}{2}(I_{22} - I_{11}) + \frac{1}{2}(I_{22} - I_{11})\cos 2\Omega t. \qquad (3.1.21)$$

$I'_{13}(t) = I'_{23}(t) = I'_{31}(t) = I'_{32}(t) = 0$ and

$$I'_{33}(t) = I_{33}. \tag{3.1.22}$$

In general relativity the generation of gravitational waves is given quantitatively by combining the third time derivative of the quadrapole moment described above, with the appropriate coupling constant. The latter can only depends on the constant G and c (for classical waves) and by dimentional analisis this constant must have the form $\frac{G}{c^5}$. The total power radiated by a rotating NS is given by [12],

$$P_{GW} = -\frac{1}{5}\frac{G}{c^5}(\frac{\partial^3 I'_{ij}}{\partial t^3}\frac{\partial^3 I'_{ij}}{\partial t^3}). \tag{3.1.23}$$

From the above we have the values of $I'_{ij}(t)$; so let us derivated that, at $\theta = \Omega t$

$$I'_{11} = \frac{1}{2}(I_{11} - I_{22}) + \frac{1}{2}(I_{11} - I_{22})\cos 2\Omega t, \tag{3.1.24}$$

$$\frac{\partial I'_{11}}{\partial t} = 0 + \frac{1}{2}(2\Omega)(I_{11} - I_{22})(-\sin 2\Omega t), \tag{3.1.25}$$

$$\frac{\partial^2 I'_{11}}{\partial t^2} = \frac{1}{2}(2\Omega)^2(I_{11} - I_{22})(-\cos 2\Omega t), \tag{3.1.26}$$

$$\frac{\partial^3 I'_{11}}{\partial t^3} = \frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\sin 2\Omega t. \tag{3.1.27}$$

following similar procedure give

$$\frac{\partial^3 I'_{12}}{\partial t^3} = \frac{\partial^3 I'_{21}}{\partial t^3} = -\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\cos 2\Omega t. \tag{3.1.28}$$

Similarly

$$I'_{13} = I'_{23} = I'_{31} = I'_{32} = 0 \text{ and}$$

$$I'_{33} = I_{33}. \tag{3.1.29}$$

So that their 3^{rd} derivation becomes zero for Eqn. (3.1.29).

Similarly

$$\frac{\partial^3 I'_{22}}{\partial t^3} = -\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\sin 2\Omega t. \tag{3.1.30}$$

Then the power radiated becomes

$$P_{GW} = -\frac{1}{5}\frac{G}{c^5}\left(\frac{\partial^3 I'_{11}}{\partial t^3}\frac{\partial^3 I'_{11}}{\partial t^3} + \frac{\partial^3 I'_{12}}{\partial t^3}\frac{\partial^3 I'_{12}}{\partial t^3} + \frac{\partial^3 I'_{21}}{\partial t^3}\frac{\partial^3 I'_{21}}{\partial t^3} + \frac{\partial^3 I'_{22}}{\partial t^3}\frac{\partial^3 I'_{22}}{\partial t^3}\right),$$
$$= -\frac{1}{5}\frac{G}{c^5}[(\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})(\sin 2\Omega t))^2 + (-\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\cos 2\Omega t)^2$$
$$+ (-\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\cos 2\Omega t)^2 + (-\frac{1}{2}(2\Omega)^3(I_{11} - I_{22})\sin 2\Omega t)^2], \tag{3.1.31}$$

$$P_{GW} = -\frac{32}{5}\frac{G\Omega^6}{c^5}(I_{11} - I_{22})^2, \quad = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2. \tag{3.1.32}$$

where $\epsilon = \frac{(I_{11}-I_{22})}{I}$ is the stellar equatorial gravitational oblateness and the negative sign indicates the power dissipation.

3.2 Magnetic field decay due to gravitational quadrupole radiation

As described in Chapter one, the measured magnetic fields at the poles and the decaying time are $10^{12} - 10^{13}$ Gauss and 10 million years [7], respectively in the case of radio pulsars where as $10^{14} - 10^{15}$ Gauss and 10,000 years [17], respectively in the case of magnetars. From the astronomical data one can observe that, the pulsar remains radio "loud" for a longer time than magnetars and the stronger the original magnetic field of the NS is the shorter will be the decaying time or vice versa. There fore, we can conclude that the magnetic field is a non linear time dependent variable. This approach leads to formulate (derive) the fundamental relationship between magnetic field and decaying time.

Since the magnetic field is a non linear time dependent variable, then one can apply the spin down law for magnetic field as [12]

$$\dot{B}(t) = -kB(t)^n. \tag{3.2.1}$$

where n is the braking index, magnetic field decay due to gravitational quadrupole radiation can be derived in two ways.

¡. when the moment of inertia (I) is constant.

¡¡. when the moment of inertia (I) varies with the rotational frequency (Ω).

3.2.1 Magnetic field decay due to gravitational quadrupole radiation when the moment of inertia is constant

When the moment of inertia is constant, the braking index (n) is written interms of rotational frequency as [12];

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}. \tag{3.2.2}$$

As a star emits radiation, the energy is extracted from its rotation. By observing the luminosity of a star, we can estimated a rate at which star is slowing down. The rotational energy of a uniform spinning sphere is;

$$E_{rot} = \frac{1}{2}I\Omega^2. \tag{3.2.3}$$

And the first time derivative of rotational energy is equal to power radiation. From this the power radiated for gravitational quadrapole become's;

$$P_{GW} = \frac{d}{dt}(\frac{1}{2}I\Omega^2) = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2. \tag{3.2.4}$$

$$I\dot{\Omega}\Omega = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2, \tag{3.2.5}$$

Depending on the above relation let us derive $\dot{\Omega}$ and $\ddot{\Omega}$.

Then $\dot{\Omega}$ becomes

$$\dot{\Omega} = -\frac{32}{5}\frac{G\Omega^5}{c^5}I\epsilon^2. \tag{3.2.6}$$

following similar steps $\ddot{\Omega}$ becomes

$$\ddot{\Omega} = -32\frac{G\Omega^4\dot{\Omega}}{c^5}I\epsilon^2. \tag{3.2.7}$$

Now using the expression for $\dot{\Omega}$ and $\ddot{\Omega}$ obtained above and Eqn. (3.2.2) we have;

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 5. \tag{3.2.8}$$

Using Eqn. (3.2.1) and n = 5 for pure quadrupole radiation.

$$\begin{aligned} \frac{dB(t)}{dt} &= -kB(t)^5, \\ \frac{dB(t)}{B(t)^5} &= -kdt, \end{aligned} \tag{3.2.9}$$

Integrating Eqn. (3.2.9)

$$\int \frac{dB(t)}{B(t)^5} = \int -kdt, \tag{3.2.10}$$

$$-\frac{1}{4}B(t)^{-4} = -kt + c, \tag{3.2.11}$$

Now let us solve the value of the constant (c) of integration using the initial condition $B(t=0) = B_0$ we obtain;

$$-\frac{1}{4}B(0)^{-4} = -\frac{1}{4}B_0{}^{-4} = c, \tag{3.2.12}$$

hence Eqn. (3.2.11) becomes;

$$\frac{1}{4}B(t)^{-4} = kt + \frac{1}{4}B_0^{-4}, \tag{3.2.13}$$

Using Eqn. (3.2.4) the power radiated for gravitational quadrapole radiation in constant moment of inertia is,

$$P_{Gw} = I\dot{\Omega}\Omega = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2, \tag{3.2.14}$$

from this $\dot{\Omega}$ becomes

$$\dot{\Omega} = -\frac{32}{5}\frac{G\Omega^5}{c^5}I\epsilon^2, \tag{3.2.15}$$

The spin down expressed as a power law in terms of rotational frequency for n = 5 is [12];

$$\dot{\Omega} = -k\Omega^5, \tag{3.2.16}$$

Insert Eqn. (3.2.15) in Eqn. (3.2.16) then we obtain:

$$\dot{\Omega} = -k\Omega^5 = -\frac{32}{5}\frac{G\Omega^5}{c^5}I\epsilon^2, \tag{3.2.17}$$

From this let's solve the value of k i.e,

$$k = \frac{32}{5}\frac{G}{c^5}I\epsilon^2. \tag{3.2.18}$$

Let us insert Eqn. (3.2.18) in to Eqn. (3.2.13),

$$-\frac{1}{4}B(t)^{-4} = -(\frac{32}{5}\frac{G}{c^5}I\epsilon^2)t - \frac{1}{4}B_0^{-4}, \tag{3.2.19}$$

$$B(t)^{-4} = 4(\frac{32}{5}\frac{G}{c^5}I\epsilon^2)t + B_0^{-4}, \tag{3.2.20}$$

Then the general decay law or the magnetic field as a function of time in constant moment of inertia due to gravitational quadrapole radiation becomes

$$B(t) = [(\frac{128}{5}\frac{G}{c^5}I\epsilon^2)t + B_0^{-4}]^{-\frac{1}{4}}. \tag{3.2.21}$$

where $t = 0$ $B(t) = B_0$, but at $t > 0$ since $[(\frac{128}{5}\frac{G}{c^5}I\epsilon^2)t] \gg \mathrm{B}_0^{-4}$ Eqn. (3.2.21) reduced to:

$$B(t) = [(\frac{128}{5}\frac{G}{c^5}I\epsilon^2)t]^{-\frac{1}{4}}. \tag{3.2.22}$$

Based on Eqns. (3.2.21) and (3.2.22), We can conclude that for gravitational quadrupole radiation emitting, the magnetic field B(t) of a rotating NS is inversely proportional to the fourth root of the decaying time (t) and moment of inertia of a rotating NS (I).

3.2.2 Magnetic field decay due to gravitational quadrupole radiation when a moment of inertia varies as function of rotational frequency

When a moment of inertia varies as a function of rotational frequency then the braking index varies as a function of rotational frequency. we can know derive the braking index interms of rotational frequency. The slowing down of a rotating NS is proportional to the spin down expressed as a power law in terms of rotational frequency as was shown in Eqn. (3.2.16) i.e,

$$\dot{\Omega} = -k\Omega^n. \tag{3.2.23}$$

For the gravitational quadrapole model, using Eqn. (3.2.2) the braking index as a function of rotational frequency is,

$$n(\Omega) = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2}. \tag{3.2.24}$$

From the power radiated for gravitational quadrapole radiation $\dot{\Omega}$ and $\ddot{\Omega}$ can be derived as follows:

$$P_{GW} = \frac{d}{dt}(\frac{1}{2}I\Omega^2) = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2. \tag{3.2.25}$$

letting $-\frac{32}{5}\frac{G}{c^5}$ = C, we have

$$\frac{d}{dt}(\frac{1}{2}I\Omega^2) = C\Omega^6 I^2\epsilon^2, \tag{3.2.26}$$

$$\frac{1}{2}[\dot{I}\Omega^2 + 2I\dot{\Omega}\Omega] = C\Omega^6 I^2\epsilon^2, \tag{3.2.27}$$

Using the relation

$$\dot{I} = I'\dot{\Omega},\ \dot{I}' = I''\dot{\Omega} \text{ and}$$

$$\ddot{I} = I''\dot{\Omega}^2 + I'\ddot{\Omega}, \tag{3.2.28}$$

Eqn. (3.2.27) can be re-written as:

$$\frac{1}{2}[I'\dot{\Omega}\Omega^2 + 2I\dot{\Omega}\Omega] = C\Omega^6 I^2\epsilon^2, \tag{3.2.29}$$

From this $\dot{\Omega}$ becomes

$$\dot{\Omega} = \frac{2C\Omega^5 I^2\epsilon^2}{I'\Omega + 2I}. \tag{3.2.30}$$

following similar steps $\ddot{\Omega}$ becomes

$$\ddot{\Omega} = \frac{10C\Omega^4\dot{\Omega}I^2\epsilon^2 + 4C\Omega^5 II'\dot{\Omega}\epsilon^2 + 4C\Omega^5 I^2\epsilon\dot{\epsilon} - \dot{\Omega}^2(I''\Omega + 3I')}{I'\Omega + 2I}. \tag{3.2.31}$$

Now using the expression for $\dot{\Omega}$ and $\ddot{\Omega}$ obtained above we have

$$n(\Omega) = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = \frac{10C\Omega^5\dot{\Omega}I^2\epsilon^2 + 4C\Omega^6 II'\dot{\Omega}\epsilon^2 + 4C\Omega^6 I^2\epsilon\dot{\epsilon} - \dot{\Omega}^2(I''\Omega^2 + 3I'\Omega)}{\dot{\Omega}^2(I'\Omega + 2I)}, \tag{3.2.32}$$

where $\dot{\Omega}(I'\Omega + 2I) = 2C\Omega^5 I^2\epsilon^2$

$$n(\Omega) = \frac{5\dot{\Omega}^2(I'\Omega + 2I) + \frac{2I'\Omega}{I}\dot{\Omega}^2(I'\Omega + 2I) + \frac{2\Omega\dot{\epsilon}}{\dot{\Omega}\epsilon}\dot{\Omega}^2(I'\Omega + 2I) - \dot{\Omega}^2(I''\Omega^2 + 3I'\Omega)}{\dot{\Omega}^2(I'\Omega + 2I)}, \tag{3.2.33}$$

$$n(\Omega) = 5 + [\frac{2\Omega I'}{I}] + [\frac{2\Omega\dot{\epsilon}}{\dot{\Omega}\epsilon}] - [\frac{I''\Omega^2 + 3I'\Omega}{I'\Omega + 2I}]. \tag{3.2.34}$$

This equation is reduced to:

$n(\Omega) = 5 + n'(\Omega)$ where $n'(\Omega) = [\frac{2\Omega I'}{I}] + [\frac{2\Omega\dot{\epsilon}}{\dot{\Omega}\epsilon}] - [\frac{I''\Omega^2+3I'\Omega}{I'\Omega+2I}]$.

Using our idea moment of inertia varies with rotational frequency, it is possible to derive magnetic field decay law.

From Eqn. (3.2.1) and n = $n(\Omega)$

$$\begin{aligned} \frac{dB(t)}{dt} &= -kB(t)^{n(\Omega)}, \\ \frac{dB(t)}{B(t)^{n(\Omega)}} &= -kdt, \end{aligned} \tag{3.2.35}$$

By integrating it we get,

$$-\frac{1}{n(\Omega) - 1}B(t)^{1-n(\Omega)} = -kt + c, \tag{3.2.36}$$

By calculating the value of the constant (c) of integration using the initial condition $B(t = 0) = B_0$ Eqn. (3.2.36) becomes,

$$\frac{1}{n(\Omega) - 1}B(t)^{1-n(\Omega)} = kt + \frac{1}{n(\Omega) - 1}B_0^{1-n(\Omega)}, \tag{3.2.37}$$

From power radiated for gravitational quadrapole radiation when moment of inertia varies as a function of rotational frequency.

$$P_{GW} = \frac{d}{dt}(\frac{1}{2}I\Omega^2) = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2, \tag{3.2.38}$$

$$\frac{1}{2}(\dot{I}\Omega^2 + 2I\dot{\Omega}\Omega) = -\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2, \tag{3.2.39}$$

The expression for $\dot{\Omega}$ becomes

$$\dot{\Omega} = -\frac{1}{I\Omega}[\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2 + \frac{1}{2}\dot{I}\Omega^2]. \tag{3.2.40}$$

Using this the spin down expressed as a power law in terms of rotational frequency is,

$$\dot{\Omega} = -k\Omega^{5+n'(\Omega)} = -\frac{1}{I\Omega}[\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2 + \frac{1}{2}\dot{I}\Omega^2], \tag{3.2.41}$$

From this let us solve for k i.e,

$$k = \frac{1}{I\Omega^{6+n'(\Omega)}}[\frac{32}{5}\frac{G\Omega^6}{c^5}I^2\epsilon^2 + \frac{1}{2}\dot{I}\Omega^2], \tag{3.2.42}$$

$$k = \frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{\dot{I}}{I\Omega^4\Omega^{n'(\Omega)}}, \tag{3.2.43}$$

Now using $\dot{I} = I'\dot{\Omega}$ we obtain

$$k = \frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{I'\dot{\Omega}}{I\Omega^4\Omega^{n'(\Omega)}}. \tag{3.2.44}$$

Substituting Eqn. (3.2.44) in to Eqn. (3.2.37) gives

$$\frac{1}{n(\Omega)-1}B(t)^{1-n(\Omega)} = (\frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{I'\dot{\Omega}}{I\Omega^4\Omega^{n'(\Omega)}})t + \frac{1}{n(\Omega)-1}B_0^{1-n(\Omega)}, \tag{3.2.45}$$

$$B(t)^{1-n(\Omega)} = (n(\Omega)-1)(\frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{I'\dot{\Omega}}{I\Omega^4\Omega^{n'(\Omega)}})t + B_0^{1-n(\Omega)}, \tag{3.2.46}$$

The expression for magnetic field decay of NSs via gravitational quadrapole radiation for the case where the moment of inertia varies with the rotational frequency of the NSs becomes;

$$B(t) = [(n(\Omega)-1)(\frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{I'\dot{\Omega}}{I\Omega^4\Omega^{n'(\Omega)}})t + B_0^{1-n(\Omega)}]^{\frac{1}{1-n(\Omega)}}. \tag{3.2.47}$$

The variation in moment of inertia causes to the variation in braking index as a function of rotational frequency. Based on the variation in braking index the decay law has been obtained in Eqn. (3.2.47). From this result we can summerized the magnetic field of NSs is decaying due to the power radiated from the star, the rotational frequency of the NS and the decaying time.

Chapter 4

Result and Discussion

We have found the magnetic field decay equation. So in our derivation of the decay law of NSs due to gravitational quadrapole radiation for the case moment of inertia is constant has been obtained in Eqn. (3.2.21) as, $B(t) = [(\frac{128}{5}\frac{G}{c^5}I\epsilon^2)t + B_0^{-4}]^{-\frac{1}{4}}$.

Depending on this finding we provide plot for the magnetic field decay using all the constants from the appendix and the decaying time for both magnetars and pulsars to compare the values obtained from this expression with observational result.

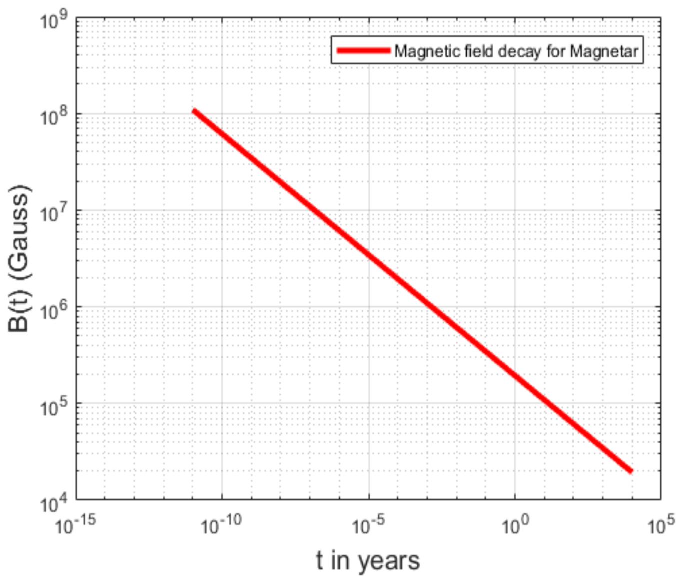

Figure 4.1: This graph illustrates the magnetic field for constant moment of inertia versus decaying time diagram for generic Magnetar.

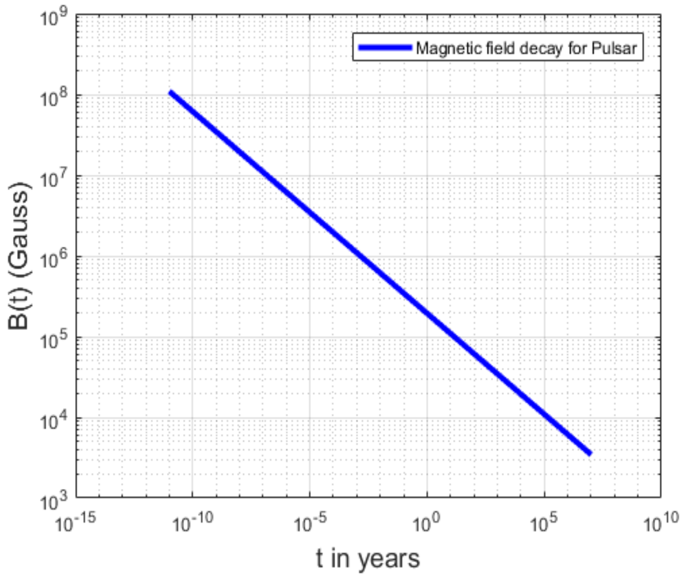

Figure 4.2: This graph illustrates the magnetic field for constant moment of inertia versus decaying time diagram for generic Pulsar.

Figs. (4.1) and (4.2) shows magnetic field decay of NSs (magnetars and pulsars, respectively) due to gravitational quadrapole radiation from Eqn. (3.2.21) for special case where we take the moment of inertia is constant. The above two plots clearly show how the magnetic field of both magnetars and pulsars are decaying. Both plots are indicative of the magnetars and pulsars magnetic field is decaying rapidly in the first milisecond. But as the time is increased the magnetic field of both magnetars and pulsars is decaying slowly.

Our result for magnetic field decay in constant moment of inertia is generallized from the result in Fig. (4.3). So the plot indicates that the magnetic field is decaying for both magnetars and pulsars. But in the first three days the magnetars are decaying more rapidly than the pulsars.

The decay law for magnetic field due to gravitational quadrapole radiation of NS for the case where the moment of inertia varies as a function of frequency has been obtained in Eqn. (3.2.47) as,

$$B(t) = [(n(\Omega) - 1)(\frac{32}{5}\frac{G}{c^5\Omega^{n'(\Omega)}}I\epsilon^2 + \frac{1}{2}\frac{I'\dot{\Omega}}{I\Omega^4\Omega^{n'(\Omega)}})t + B_0^{1-n(\Omega)}]^{\frac{1}{1-n(\Omega)}}.$$

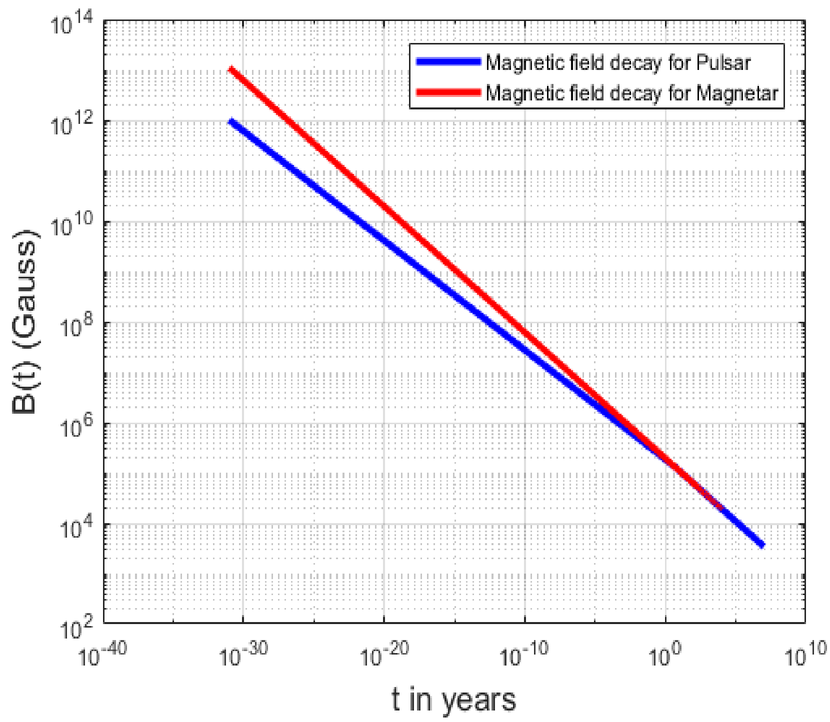

Figure 4.3: This graph illustrates the magnetic field for constant moment of inertia versus decaying time diagram for both Pulsars and Magnetars.

If we consider pure quadrapole radiation the braking index n becomes 5, then the quadrapole magnetic field for moment of inertia varies as a function of rotational frequency is reduced to:

$B(t) = [(\frac{128}{5}\frac{G}{c^5}I\epsilon^2 + \frac{2I'\dot{\Omega}}{I\Omega^4})t + B_0^{-4}]^{\frac{1}{-4}}$.

Using observed data for rotational frequency and time derivation of rotational frequency for different pulsars we have ploted graphs for the magnetic field decay law in moment of inertia varies as a function of rotational frequency.

Fig. (4.4) shows magnetic field decay of pulsars due to gravitational quadrapole radiation from Eqn. (3.2.47) for special case where we take the moment of inertia varies as a function of rotational frequency. The plots in Fig. (4.4) clearly shows by how much the magnetic field is decaying for pulsars their $\dot{\Omega}$ is positive. Both plots are indicative of the pulsars magnetic field is decaying rapidly in the first milisecond. But as the time is increased the magnetic field of pulsars is decaying slowly relative to the first milisecond.

Fig. (4.5) shows magnetic field decay of pulsars due to gravitational quadrapole radiation from Eqn. (3.2.47) for special case where we take the moment of inertia varies as a function of rotational frequency. The plots in Fig. (4.5) clearly shows by how much the magnetic field is increased for pulsars their $\dot{\Omega}$ is negative. Both plots are indicative of the pulsars magnetic field is increased. As we can see the magnetic field is negative, this indicates the polarity of the field.

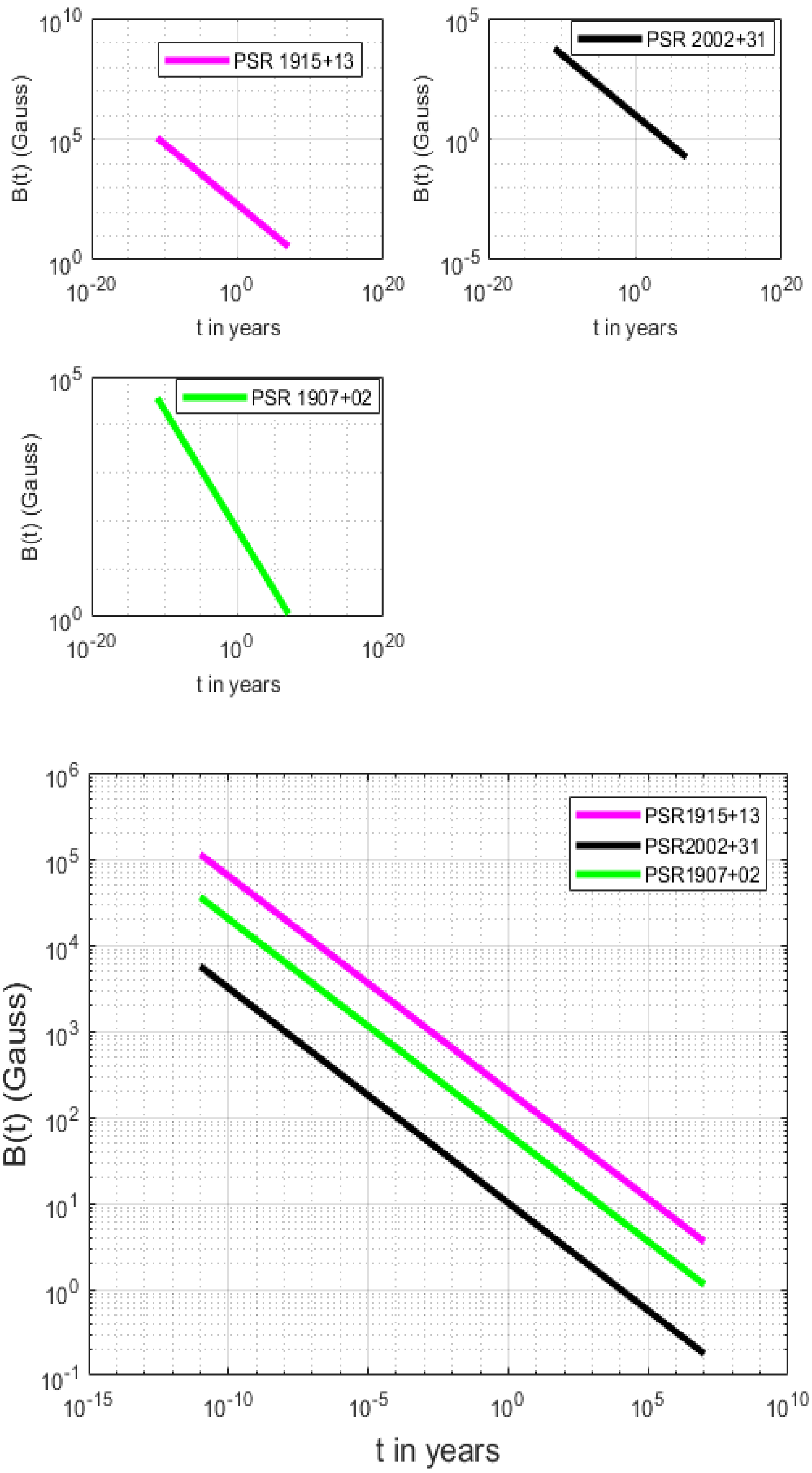

Figure 4.4: These graphs illustrates the magnetic field versus decaying time in the case variable moment of inertia for some pulsars which have positive $\dot{\Omega}$.

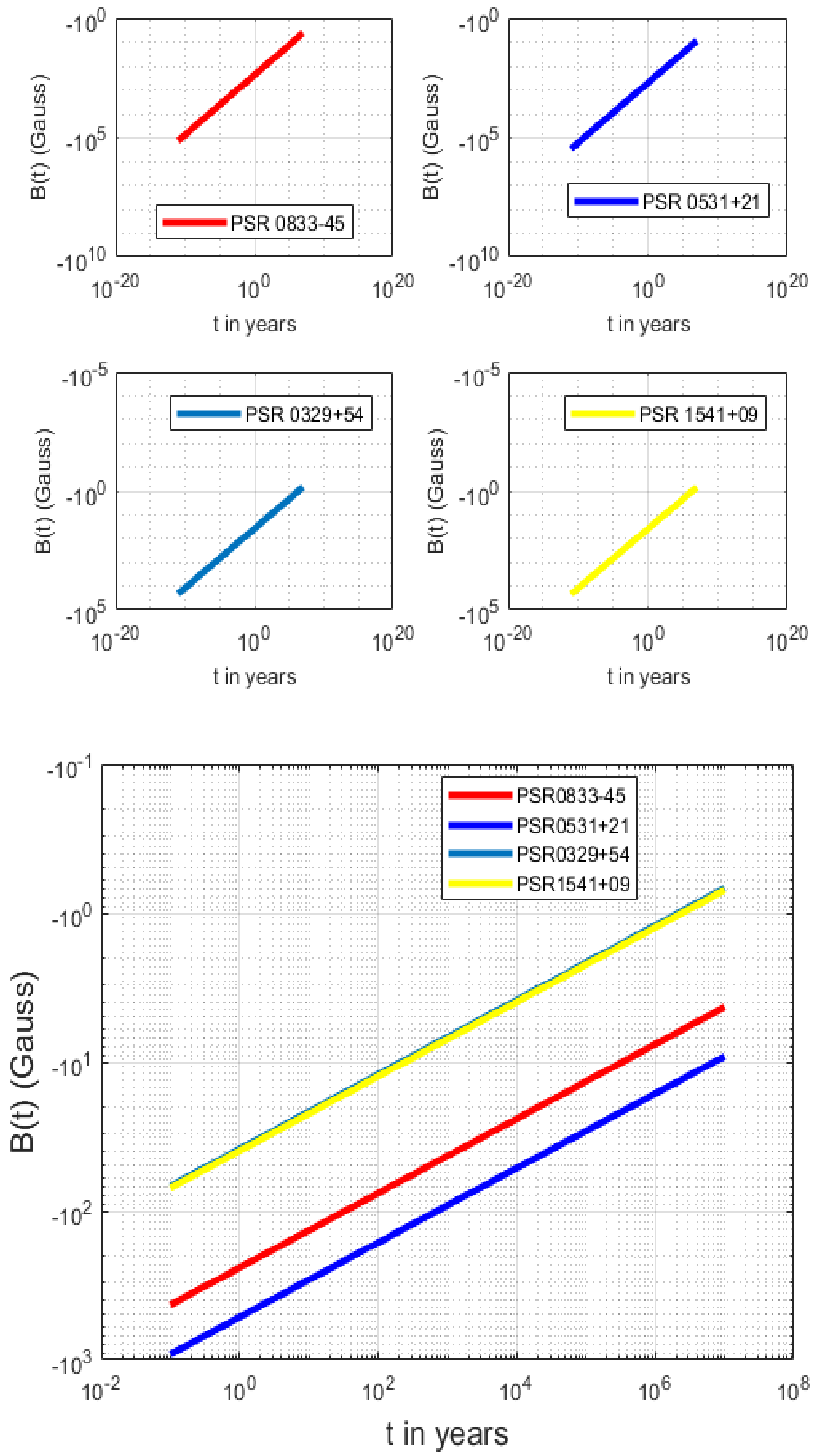

Figure 4.5: These graphs illustrates the magnetic field versus decaying time in the case variable moment of inertia for some pulsars which have negative $\dot{\Omega}$.

Chapter 5

Conclusion

In this thesis we have calculated quadrapole magnetic field of a rotating neutron star (NS) from power radiated and spin down law of the rotating NS, when a moment of inertia is constant and moment of inertia varies as a function of rotational frequency. Based on our derivation, The quadrapole magnetic field indicated in Eqns. (3.2.21) and (3.2.47) shows, the NSs magnetic field is decaying rapidly in the first milisecond in the case of constant moment of inertia and in the case when moment of inertia varies as a function of rotational frequency with positive $\dot{\Omega}$. But as the decaying time increase the magnetic field of NSs decaying slowly relative to the first milisecond.

So our decay law shows that in the first milisecond the NSs magnetic field drops immediately to 10^8 G. When the time is increased from a milisecond to ten thousand years, both magnetars and pulsars magnetic field decaying by four order. As the time increases from a milisecond to ten million years, pulsars magnetic field is decaying by five order and six order in the case when moment of inertia is constant and in the case when moment of inertia varies as a function of rotational frequency with positive $\dot{\Omega}$, respectively.

In the case when moment of inertia varies as a function of rotational frequency with negative $\dot{\Omega}$ the pulsars magnetic field is increased rather than decaying. The value of this magnetic field is negative. Thus, the negative sign indicates the polarity of the pulsars magnetic field.

The variation in moment of inertia causes to the variation in braking index as a function of rotational frequency. Thus, when the braking index varies from the value for pure quadrapole radiation depending on the rotational frequency of the NS, the quadrapole magnetic field is decaying.

We studied about magnetic field decay of NSs due to gravitational quadrapole radiation in a special case the moment of inertia varies as a function of rotational frequency. However, in our case the rotational frequency is constant of position.

We suggest that further research in magnetic field decay due to gravitational radiation in NSs when the rotational frequency varies with position is likely to studied.

Appendix

The following constants and abbreviations have been used in this paper for conversion or direct application.

A. Constants

1. Solar mass ($M_{\odot}$) = 1.99×10^{30} kg $\approx 2 \times 10^{30}$ kg
2. Solar radius($R_{\odot}$) = 6.96×10^{8} m $\approx 7 \times 10^{8}$ m
3. c = 3×10^{8} m/s is speed of light
4. 1T (Tesla) = 10^{4} G(Gauss) are units of magnetic field in mks and cgs systems
5. 1 lightyears = 9.45×10^{15} m
6. 1 ev = 1.602×10^{-19} joules
7. Gravitational constant(G) = 6.67×10^{-11} m^{3}kg^{-1}sec^{-2}
8. I = 10^{38} kgm^{2}
9. $B_{0P} = 10^{12}$ Gauss
10. $B_{0M} = 10^{15}$ Gauss
11. $t_M = 10^{4}$ year
12. $t_P = 10^{7}$ year
13. $\epsilon = 10^{-4} - 10^{-5}$

This table has contained frequency and another observational datas of seven pulsars.

Name of pulsar	Ω (Hz)	$\dot{\Omega}$ Hzs^{-1}	B_0 G	I' $\frac{G}{Hz}$	Ref
PSR 0531+21	30.18	-3.848E-10	7.6E12	3.313E36	[41]
PSR 0833-45	11.21	-1.57E-11	6.8E12	8.92E36	[42], [40]
PSR 0329+54	1.4	-4.02E-15	0.45E12	71.428E36	[39]
PSR 2002+31	0.4737	1.39E-14	4.3E12	211.104E36	[43]
PSR 1915+13	5.138	1.30E-14	1.6E12	19.462E36	[43], [44]
PSR 1907+02	2.021	3.277E-15	2.1E12	49.48E36	[43], [45]
PSR 1541+09	1.336	-7.682E-16	0.12E12	74.85E36	[43]

B. Abbreviations

1. NSs is read as neutron stars.
2. SNR is read as supernova remnant.
3. SGRs is read as soft gamama repeaters.
4. APXs is read as Anomalous X-ray Pulsars.
5. $B_P(t)$ is read as Magnetic field of a pulsar.
6. $B_M(t)$ is read as Magnetic field of a magnetar.
7. B_{0P} is read as initial Magnetic field of a pulsar.
8. $\dot{\Omega}_P$ is read as time derivative angular frequency of a pulsar.
9. Ω_P is read as angular frequency of a pulsar.
10. t_P is read as decaying time of a pulsar.
11. B_{0M} is read as initial Magnetic field of a magnetar.
12. $\dot{\Omega}_M$ is read as time derivative angular frequency of a magnetar.
13. Ω_M is read as angular frequency of a magnetar.
14. t_M is read as decaying time of a magnetar.

Bibliography

[1] G. S. Bisnovatyi-Kogan, (2011) spse.**book**.B.

[2] Duncan. Robert c.(2003). Vol. **288** “Magnetars, Soft Gamma Repeaters, and very strong Magnetic fields”.

[3] M. A. Seeds, Michael A.(1995) Vol. 3 hetu.**book**.S.

[4] Urpin V. A., Levshakov S. A. Iakovlev D. G. (1986), MNRAS, **219**, 703

[5] Giancoli, D. C., (2005), ppwa.**book**.G 6th ed.

[6] A. S. Eddington, Observatory (1920) Vol. **28**.

[7] Baade W. Zwicky F. (1934), Phys. Rev., **46**, 76

[8] Payne-Gaposchkin, C. (1957) Vol. **126** Sci.**126**.1350S.

[9] Lattimer J. M. Prakash M. (2001), ApJ, **550**, 426.

[10] Camenzind Max, (2007), coaw.**book**.C.

[11] $http//magine.gsfc.nasa.gov/science/object/neutron_{s}tars.htm$ 13.07.17

[12] Stuart L. Shapiro, Saul A. Teukolsky ; Black Holes, White Dwarfs and Neutron Stars **“the physics of compact objects”.**

[13] J.-L. Tassoul, (2000) Vol. **36** stro.**book**.T.

[14] de Boer, Klaas; Seggewiss, Wilhelm, (2008), sse.**book**.D.

[15] Dr. rer. nat. (Tubingen,2010) , "Magnetzed Neutron Star".

[16] Alfred Whitehead Physics 518, Fall 2009.

[17] Michel, F. C. 1991 Chicago, IL, Press, 1991, **533** p.

[18] Leahy, Denis; Green, Kalie; Tian, Wenwu (2014) Vol. **438** kes **73** and AXP **1E** 1841-045

[19] Erik Kuulker. Europian space agency observed data

[20] http://imagine.gsfc.nassa.gov/index.html. 28.06.16

[21] http://www.cv.nrao.edu/course/astr534/Pulsars.html 11.17.2010

[22] Gaensler, Bryan M.; Slane, Patrick O 2006 ARAA.**44**.17G .

[23] I. Bombaci and B. Datta, Astrophysics. J. **520**,L69(200); [arXiv:astro-ph/0001478].

[24] Mc Gillivray, D., 1987, Physics and Astronomy (Macmillan).

[25] Bonnano. A., Rezzolla. L., Urpin.v 2003 AA.**410L**.33B.

[26] Brandeburg, Axel; Nordund,Ake; Stein, Robert F.; Torkelsson, Ulf (1995) Vol. **446** ApJ.**446**.741B.

[27] Rezzolla. Luciano; Ahmedov, Bobomurat J. 2004 MNRAS.**352**.1161R.

[28] Goldreich P. Julian W. H. (1969), ApJ,**157**, 869.

[29] Spruit H. C. AA.**349**..189S.

[30] Sherwood, Jonathan (December 3, 2002). "Dark Edge of Sunspots Reveal Magnetic Melee".

[31] Hudson, H. S.; Kosugi, T. (1999) Sci.**285**.849H.

[32] Hathaway, David H. (January 18, 2007) AAS.**210**.9901H.

[33] Berdyugina, Svetlana V. (2005) LRSP.**2**.8B.

[34] Harpaz, Amos (1994) stev.**book.**H.

[35] Nariai, Kyoji (1969) ApSS.**3.**150N.

[36] Isbell, D.; Tyson, T. May 20, 1998. "Strongest Stellar Magnetic Field yet Observed Confirms Existence of Magnetars".

[37] Misner, Thorne, Wheeler **"Gravitation".**

[38] Zimmermann, M ; Szedenits, E, Jr. 1979 PhRvD.**20**.351Z.

[39] Demiaski, Prszyski, 1979, Nature,**282**, 383.

[40] Downs, G. S. 1981, Astrophys. J.,**249**, 687.

[41] Groth, J. 1975, Astrophys. J. Suppl. Ser.,**29**, 453.

[42] Lyne, A. G., Pritchard, R. S., and Smith, F. G. 1993, MNRAS, **265**, 1003

[43] Gullahorn G. E., Rankin J. M., 1982, ApJ,**260**, 520

[44] Biemond Jacob 2004 astro.**ph**.1468B.

[45] Nowakowski, L.; Usowicz, J.; Kepa, A.; Wolszczan, A. 1982 AA.**116**.158N.

Declaration

I, here by declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of materials have been duly acknowledged.

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