ADDIS ABABA UNIVERSITY
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SCHOOL OF MECHANICAL AND INDUSTRIAL ENGINEERING

EFFECT OF CHANGE OF SPUR GEAR TOOTH PARAMETERS ON BENDING AND CONTACT STRESSES

A thesis submitted to the school of Graduate Studies of Addis Ababa institute of Technology in partial fulfillment of the Requirements for the Degree of Masters of Science in Mechanical Engineering (Mechanical Design Stream)

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October, 2014

Addis Ababa, Ethiopia

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Addis Ababa Institute of Technology

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ACKNOWLEDGMENTS

I would like to express my sincere appreciation and gratitude to Dr. Daniel Tilahun for being my advisor and for his valuable guidance, continuous scheduled follow up and useful comments during my thesis work.

I am also thankful to Behailu Mamo (staff in school of Mechanical and Industrial Engineering, AAiT) for his invaluable CATIA training.

In addition, I acknowledge the staffs in the school of Mechanical and Industrial Engineering, AAiT, who have provided me with the most cooperative assistance to my research work.

Last but not least, a big appreciation is also dedicated to my beloved family and friends who have given me encouragement and moral support throughout the duration of my study.

ABSTRACT

The bending and surface stress of the gear tooth is considered to be one of the main contributors for the failure of the gear in a gear set. After the investigation of shot peening to increase the tooth bending strength and surface durability in gears, the surface roughness generated during shot peening leads to macro and micropitting is now considered the dominant restriction on gear life and performance. Thus, analysis of stresses has become popular as an area of research on gears to minimize or to reduce the failures and for optimal design of gears. This thesis investigated the effect of tooth parameters monitoring the stresses induced on spur gear by optimizing face width, root fillet radius, and number of teeth relative to weight of spur gear set. The involute profile of Spur gear has been modeled and the simulation was carried out for the bending and contact stresses. To estimate bending and contact stresses, 3D models were generated by modeling software CATIA V5r16, simulation was done by finite element software package ANSYS 12.0, and optimization was done using Design Expert Dx7 numerical optimization method. Analytical method of calculating gear bending stresses uses Lewis and AGMA bending equation. For contact stresses Hertz contact equation are used. The Study was conducted by varying the face width, number of teeth and root fillet radius to find its effect on the bending and contact stress of spur gear. It was therefore observed that the maximum bending stress and contact stress decreases with increasing face width, number of teeth and root fillet radius relative to spur gear set weight. Using the Design expert software Dx7 the optimal points were selected at face width 37.24mm, root fillet radius 3mm, and number of teeth 22.At these value contact stress was reduced from 389.31Mpa to 294.56Mpa (24.34%) and bending stress was reduced from 105.14Mpa to 49.65Mpa (52.80%).So that, it is recommendable to use optimal values of tooth parameters during design work to reduce stresses. It means stress reduction results in better tooth root load capacity, micropitting resistance, prolongs gear service life.

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LIST OF SYMBOLES	NAMES
a	Contact width
AGMA	American Gear Manufacturer Association
b	Face width of spur gear
d_1	Pitch diameter of gear
d_2	Pitch diameter of pinion
d_b	Base circle diameter
d_o	Outer circle diameter
d_p	Pitch circle diameter
d_r	Root circle diameter
E_1	Modulus elasticity of gear
E_2	Modulus elasticity of pinion
F	Total applied force
F_r	Radial applied force
F_t	Tangential applied force
h	Total height of tooth
h_a	Addendum
h_d	Dedendum
J	Geometry factor of gear
K_a	Application factor
K_m	Load distribution factor
K_s	Size factor
K_v	Dynamic factor
L	Contact length of two cylinders
M	Applied Moment
m	Normal metric module
p	Diametrial pitch
P_{max}	Maximum pressure
r_1	Pitch radius of gear
r_2	Pitch radius of pinion

R_1	Radius of curvature for gear
R_2	Radius of curvature for pinion
r	Radius of involute cutve at arbitrary point
r_f	Root fillet radius
t	Thickness of tooth
T	Applied torque
v_1	Poisson's ratio of gear material
v_2	Poisson's ratio of pinion material
w	Weight of spur gear set
x	Horzontal coordinates of involute curve
y	Vertical coordinates of involute curve
Y	Modified lewis form factor
Z_1	Number of gear tooth
Z_2	Number of pinion tooth
α	Pressure angle
θ	The angle between r_p and r
θ_1	The angle between r_b and r_p
ø	The angle between r_b and r
τ_{max}	Maximum shear stress
σ_b	Bending stress
σ_H	Hertzian contact stress
σ_{von}	Maximum Von Mises stress
ρ_1	Density of gear material
ρ_2	Density of pinion material

CHAPTER ONE

INTRODUCTION

1.1 Background

Gear transmission systems play an important role in many industries. The knowledge and understanding of gear behavior in mesh such as stress distribution, work condition and distortion is critical to monitoring and controlling the gear transmission system.

A pair of teeth in action is generally subjected to two types of cyclic stresses: bending stresses inducing bending fatigue and contact stress causing contact fatigue. Both these types of stresses may not attain their maximum values at the same point of contact. However, combined action of both of them is the reason of failure of gear tooth leading to fracture at the root of a tooth under bending fatigue and surface failure, like pitting or flaking due to contact fatigue. These types of failures can be minimized by careful analysis of the problem during the design stage and creating proper tooth surface profile, optimal teeth parameters with proper manufacturing methods studied by David et al. (2013).

Gears are often required to operate at high torque and speed, while remaining competitively priced and highly reliable. Presently, there is a great challenge in front of automobile industries and wind turbine gearboxes, to overcome failures existing in gears Lihua, (2008). Gears have wide variety of applications. They form the most important components in power transmission systems. Their applications vary from watches to very large mechanical units like the lifting devices and automotive. Gears generally fail when the working stress exceeds the maximum permissible stress. Advances in engineering technology in recent years have brought demands for gear tooth, which can operate at every increasing load capacities and speeds. Therefore, it is essential to determine the maximum stress that a gear tooth is subjected under specific loading. Analysis of gears is carried out so that these can prevented from failure Dadhaniya et al. (2012).Designing highly loaded spur gears for power transmission systems that are both strong and quiet requires analysis methods that can easily be implemented and also provide information on contact and bending stresses. The finite element method is capable of providing this information, but the time needed to create such geometry is large. In order to reduce the modeling time, a preprocessor method that creates the geometry needed for a finite element analysis may be used. Modeling software's like Pro Engineer, Solid Works, CATIA and many more are the best option available to create complex geometry for analysis done by Shreyash D, (2010).

Nowadays computers are becoming more and more powerful, and that is the reason why people tend to use numerical approach to develop theoretical models to predict the effects. Numerical methods can potentially provide more accurate solutions since they normally require much less restrictive assumptions. However the important thing is to choose the correct model and the solution methods to get the accurate results and also reasonable computational time.

In this thesis first the solid model of the spur gear is made with relations and equations modeling option in CATIA V5r16. After the modeling of spur gear the assembly is created of two spur gears in contact. The contact is defined at the pitch circle radius with the appropriate center distance between the two gears. Then the whole assembly is imported in ANSYS Workbench 12.0 for contact and bending stress analysis and to reduce computation time, all but one tooth was removed via extrusion is imported for bending stress analysis. The results of ANSYS 12.0 are then compared with the AGMA equations and Hertzian theory for the specified gear set. Although gears have gained wide range of acceptance in all kinds of applications they have to resist the bending and fatigue stresses produced due to the cyclic load during power transmission. Shot peening is the process of cold forming the surface of a part by means of the impacts off a propelled stream of round hardened steel shot particles. The result of this process is a uniformly dimpled surface, the roughness being determined by the shot size, peening intensity and the hardness of the part being peened.

However, shot peening to increase the tooth bending strength and surface durability in gears, the surface roughness due to shot diameter leads micro-pitting on gear surface is now considered the dominant restriction on gear life and performance. Micropitting is frequently a primary failure mode responsible for initiating other secondary failure modes such as macropitting, scuffing, bending fatigue, and polishing. Localized plastic deformation at asperities changes residual stresses and Cracks form quickly under asperities. Failure analyses also show that shot peened gears have low micropitting resistance because of increased surface roughness; however, grinding improves micropitting resistance Shuanguen Sheng, (2010).

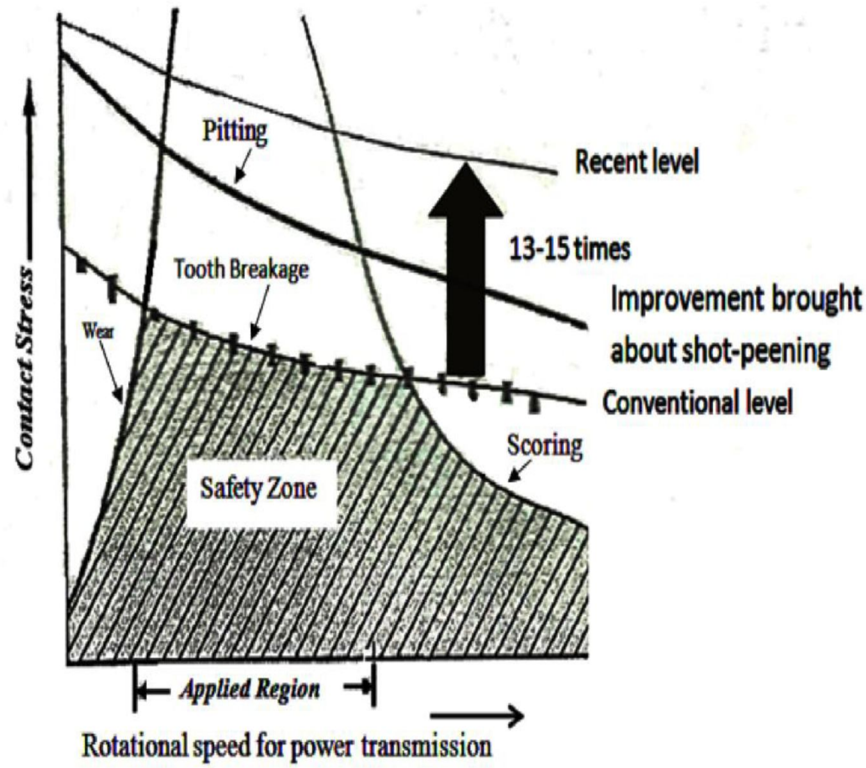


Figure 1.1 Failure of gears with respect to contact stress and rotational speed (Suzuki Y 2004).

1.2 Statement of the problem

Although gear manufacturing has achieved lots of advancement during its evolution, however the failure of gear due to bending and contact stress still remained a challenge for designer and manufacturers. The bending stress is highest at the fillet and can caused breakage or fatigue failure of tooth in root region. Then the failure can propagate from the surface (usually from the pitch circle) to other parts of gear. Whereas contact stresses are on the side of the tooth may causes scoring wear, pitting fatigue.

In order to increase the bending fatigue strength at the tooth root fillet of gears, Shot peening is applied to enhance the fatigue life by increasing surface hardening and inducing compressive surface residual stress. But surface roughness induced during shot peening process and subsurface damage is the main problem for transmission gears which results in pitting and initiate cracks at tooth root (Shuanguen Sheng, 2010).

Hence, this research work focused on analyzing the effect of changing root fillet radius and faces width to increase the tooth strength in spur gear and minimize shot peening effect. In addition, the effect of teeth number on stress intensity was analyzed.

1.3 Objective

1.3.1 General objective

The general objective of this study is to analyze effect of change of tooth root fillet radius, number of teeth and face width on stresses of involute spur gear.

1.3.2 Specific Objective

The specific objectives of the research will be to:

- Model and simulate spur gear using CATIA and ANSYS Workbench.
- Analyses analytical contact and bending Stress of spur gear using Hertzian theory and AGMA standards.
- Comparing the results obtained from FEM with AGMA standards and Hertzian contact theory.
- Analyze effect of spur gear set weight based on change of face width and numbers of teeth.

1.4 Methodology

The methods employed to achieve the above objective are

a) **Literature Survey**

Literature survey of relevant material on stress analysis of gear has been done. The literatures available are from electronic media, journals, and books. In addition, secondary data are referred from previous related research studies, existing statistical data, etc.

b) **Modeling and FEM analysis**

The data collected from the analytical and AGMA standards is analyzed using CATIA and ANSYS Work bench.

c) **Numerical optimization**

Using Design Expert software DX7, optimum values of face width, number of teeth, and root fillet radius selected relative to the weight of spur gear set.

1.5 Organization of the thesis

This thesis is organized in to five chapters. In the first chapter introduction, back ground and objective to be achieved are discussed.

In chapter two, a review of literature appropriate to this thesis is stated. In chapter three material, dimension, conditions, and analytical and FEM analysis used were discussed.

In chapter four results from FEM and analytical method at different face width, root fillet radius, and number of teeth are analyzed and discussed.

Finally, in chapter five conclusion, recommendation and future works were presented.

CHAPTER TWO

LITERATURE REVIEW

There are several kinds of stresses present in loaded and rotating gear teeth. Bending stress and contact stress (Hertz stress) calculation is the basic of stress analysis. The drawback in the stress analysis of the gear tooth using this equation doesn't give the desired results at desired points on the tooth profile. Therefore, investigators, analyzing the gear tooth for stresses, have done several studies.

2.1 Methods used to analyses stresses of gears

In the middle of the 20th century, most gear designs were based upon Lewis original bending equation. Lewis based his analysis on a cantilever beam and assumed that failure will occur at the weakest point of this beam. However, failure due to flexural stresses on bodies with changing or asymmetrical cross-sections was proved inaccurate by Dolan and Broghamer (1942). Their approach used photoelastic experiments to visualize the stress concentrations due to the fillets at the base of spur gears. By these visualization techniques they were able to predict more accurately at what stress levels gears will fail due to high bending stresses. Much earlier work was done using photoelastic experiments to design spur gears based on the stresses observed at the most critical points. Shimamura and Noguchi (1965) show, the use of photoelastic experiment is rare due to the high cost of the equipment and it requires experience and skills to determine the gear stresses. Although this method is useful in determining static stresses in spur gears, the photoelastic trend has become more popular toward its usage in gear dynamic analysis**.**
Many of the first attempts of FEA on spur gears were developed in 2D to simplify the solution. Later, Vijayarangan and Ganesan (1993) have developed 3D spur gear models to analyze bending stress of spur gear by applying FEA. The advantages of this method over experimental techniques are competitive cost effectiveness and repeatable results. The accuracy of the FEA solution can be assessed by verifying that the FEA results correspond closely to experimental results. Hence, the validity of the FEA setup and technique applied can be confirmed.

Jinliang Zhang et al (2007), used tooth contract analysis loaded tooth contact analysis and finite element method to analyze the meshing behavior, tooth surface contract stress, maximum tensile, bending stress and maximum compressive bending stress. The modified pitch cone method is first presented and verified in the gear research center of Dong Feng vehicle-bridge Co., Ltd.

A recent study by Kahraman et al. (2003) considers each gear as a deformable body and meshes them to predict load, stress and deformations. Based on these results presented, a deformable body analysis with a thin rim is necessary.

Mao K (2007) developed an advanced non-linear finite element method, which has been successfully used to accurately simulate gear contact behavior under real load conditions. For gears the stresses were firstly computed by the 2D FEM and the formulae were drawn allowing a simple calculation of maximum tooth root stresses.

Simon and Vilmos (2000) performed stress analysis in hypoid gears, by using finite element method, in order to develop simple equations for the calculation of tooth deflections and stresses. He developed a method for the automatic finite element discretization of the pinion and the gear. The full theory of mismatched hypoid gears was applied for the determination of the nodal point coordinates on the teeth surfaces. He developed a corresponding computer program. With the help of this program the influence of design parameters and load position on tooth deflections and fillet stresses is investigated. On the basis of results, which were obtained by performing a big number of computer runs, equations for the calculation of tooth deflections and fillet stresses were derived. Moriwaki et al. (1993) developed a technique named Global Local Finite Element Analysis (GLFEM) and applied it to a gear tooth for its stress analysis. He realized that for doing the stress analysis of the gear using the finite element analysis, a load acts at a point on the tooth profile and for that a fine subdivision is required at the applied load point. In GLFEM, no fine subdivision is required for the analysis. This method also guarantees an easy determination of the critical section.

Wei Z (2004) investigates the characteristics of an involute gear system including contact stresses, bending stresses, and the transmission errors of gears in mesh. To estimate transmission error in a gear system, the characteristics of involute spur gears were analyzed by using the finite element method. The contact stresses were examined using 2-D FEM models. The bending stresses in the tooth root were examined using a 3-D FEM model. The results of the two dimensional FEM analyses from ANSYS are presented. These stresses were compared with the theoretical values. Bharat Gupta and Abhishek Choubey (2012) explains that Current analytical methods of calculating gear contact stresses use Hertz's equations, which were originally derived for contact between two cylinders. So for CONTACT STRESSES it's necessary to develop and to determine appropriate models of contact elements, and to calculate contact stresses using

ANSYS and compare the results with Hertzian theory. The finite element method is proficient to supply this information but the time required to generate proper model is a large.

Tsay and Fong (1991) applied the tooth contact analysis technique (TCA) and finite element method (FEM) to gear contact and stress analysis. In their study, a mathematical model for pinion and gear involutes teeth is assumed. The geometry of the gears is described by parameters of manufacturing. Computer simulations of the conditions of gear meshing including the axes misalignment and center distance variation are performed. Their paper showed that the locations of bearing contact and contact pattern of mating tooth surfaces are determined by TCA techniques. The results of the TCA provide the location and the direction of applied loads for the computer aided FEM stress analysis, by applying the given mathematical model and TCA techniques. A three-dimensional stress analysis for this type of gearing was investigated by Von-Mises stress contour distribution.

Ruben et al. (2005) study the stresses in the contact zone among a couple of spur gears realized using the finite elements method. The analysis is done by using a plane model involving the contact between two teeth. The geometry is defined according to the standards of the American Gear Manufacturers Association (AGMA). The results obtained are compared with the value given by two others approaches. The first is the theory of Hertz when it is applied to two curved segments in contact. The second approach is the AGMA procedure for calculating contact stresses in spur gear. The results obtained are very similar either using FEM and Hertz´s theory. The contact pressure obtained by FEM is lower than the one obtained by means of Hertz´s theory, this fact reflects the influence of the geometry profile which allows the occurrence of contact zones with greater area, and therefore lower pressures. The ability of the FEM for the simulation of mechanical spur gears contact have been proved, presenting estimates of contact pressure and stress states with similar results and tendencies to those obtained by the contact theory of plane models of Hertz and AGMA.The contact pressure and stress state are highest for higher points on the involute and lower were a single pair of teeth assumes the full load transmitted, and minimal for the contact at the pitch point.

Yi-Cheng Chen and Chung Biau (2006) investigate the contact stress and bending stress of a helical gear set with localized bearing contact, by means of finite element analysis (FEA). The proposed helical gear set comprises an involute pinion and a double crowned gear. Mathematical models of the complete tooth geometry of the pinion and the gear have been derived based on the theory of gearing. In this study, finite element stress analysis was performed to investigate the contact stress and the bending stress of a modified helical gear set comprising an involute pinion and a modified helical gear.

Ali Raad Hassan (2009) explains that Contact stress analysis between two spur gear teeth was considered in different contact positions, representing a pair of mating gears during rotation. A programme has been developed to plot a pair of teeth in contact. This programme was run for each 3° of pinion rotation from the first location of contact to the last location of contact to produce 10 cases. Each case was represented a sequence position of contact between these two teeth. The programme gives graphic results for the profiles of these teeth in each position and location of contact during rotation. Finite element models were made for these cases and stress analysis was done. The results were presented and finite element analysis results were compared with theoretical calculations, wherever available. The presentation dealt with contact stress, considering contact ratio, approach angle, recess angle, contact and length of contact. The stress was more than the correct value of contact stress obtaining from approximating tools. This search was certainly not easy and cannot be carried out without the use of finite element analysis. To apply finite element method in contact stress a special technique was used rather the regular elements, to distinguish between the contact regions which were in two parts. One was the first body named target region and the other body was named contact region. For target region, target elements were used and in contact region contact elements were used. ANSYS software presents a significant technique for this purpose which was used here.

Vivek Karaveer and Ashish Mogrekar (2013) present the stress analysis of mating teeth of spur gear to find maximum contact stress in the gear teeth. The contact stress in the mating gears is the key parameter in gear design. The results obtained from Finite Element Analysis (FEA) are compared with theoretical Hertzian equation values. For the analysis, steel and grey cast iron are used as the materials of spur gear. The spur gears are sketched, modeled and assembled in ANSYS Design Modeler. As Finite Element Method (FEM) is the easy and accurate technique for stress analysis, FEA is done in finite element software ANSYS 14.5. Also deformation for steel and grey cast iron is obtained as efficiency of the gear depends on its deformation. The results show that the difference between maximum contact stresses obtained from Hertz equation and Finite Element Analysis is very less and it is acceptable. The deformation patterns of steel and grey cast iron gears depict that the difference in their deformation is negligible. Here the theoretical maximum contact stress is calculated by Hertz equation. Also the finite element analysis of spur gear is done to determine the maximum contact stress by ANSYS 14.5. It was found that the results from both Hertz equation and Finite Element Analysis are comparable. From the deformation pattern of steel and grey cast iron, it could be concluded that difference between the maximum values of steel and grey CI gear deformation is very less.

Negash (2007) investigates the characteristics of an involute helical gear system mainly focused on bending and contact stresses using analytical and finite element analysis. To estimate the bending stress, three-dimensional solid models for different number of teeth are generated by Pro/Engineer and the numerical solution is done by ANSYS. The analytical investigation is based on Lewis stress formula. This thesis also considers the study of contact stresses induced between two gears using 2D model and the analysis was carried out on the equivalent contacting cylinders. The results obtained from ANSYS are presented and compared with theoretical values. In this work a parametric study is conducted by varying the face width and helix angle to study their effect on the bending stress of helical gear.

2.2 Models for optimal design of gear sets

Several approaches to the models for optimum design of gears have been presented in the recent literature. Spitas et al. (2005) studied the idea of spur gear teeth with circular instead of the standard trochoidal root fillet and it was investigated numerically using BEM. The strength of these new teeth was studied in comparison to the standard design by discretizing the tooth boundary using isoparametric boundary elements. In order to facilitate the analysis the teeth were treated as non-dimensional assuming unitary loading normal to the profile at their Highest Point of Single Tooth Contact. It was demonstrated that the novel circular design surpasses the existing trochoidal design of the spur gear tooth fillet in terms of fatigue endurance without affecting the pitting resistance. The proposed geometry does not produce undercut teeth even for small number of teeth. Similarly, S.Sankar (2011) using circular root fillet instead of standard trochoidal root fillet in gear and concluded that the tooth deflection in the circular root fillet is less when compared to the trochoidal root fillet, further there is appreciable reduction in bending stress and contact shear stress for circular root fillet design in comparison to that of trochoidal root fillet design.

Hiremagalur et al. (2004) studied the effect of backup ratio in spur gear root stresses analysis and design. Backup ratio was considered important in understanding rim failures that start at the tooth root. Here an analytical approach, based on theory of elasticity, was used to provide a computational formulation for root stress calculations in spur gears.

Guigand et al. (2005) gave a method for simulating loaded face gear meshing. It simulated the loaded behavior of face gear meshing and provides information such as the instantaneous pressure distribution across the entire width of the teeth in contact and loaded transmission error. Analytical simulations were used to define the geometry and study the unloaded kinematics. The aim was to obtain the best possible loading of face gear meshing in order to avoid line contact sensitivity due to misalignment.

Mallesh et al. (2009) studied the effect of bending stress at the critical section for different pressure angle on the drive side along with the profile shift is studied. Due to positive profile shift, the thickness of tooth at the root is increases, resulting in greater load carrying capacity of the teeth. Profile shift varied from 0 to 0.5 and found lowest bending strength at critical section with profile shift value of 0.5; drive side pressure angle is also varied from 20 to 30 degree and found lowest bending strength at critical section with 30 degree pressure angle. It has been found that implementation of positive profile shift and pressure angle modification reduces bending stress considerably.

Senthil et al. (2012) have done the optimization of the asymmetric spur gear drive is carried out using an iterative procedure on the calculated maximum fillet stresses through FEM for different rack cutter shifts and finally the optimum values of rack cutter shifts are suggested for the given center distance and the speed ratio of an asymmetric gear drive. Comparisons have also been made successfully with the results of the AGMA and the ISO codes for symmetric gears to justify the results of the finite element method pertaining to this study.

Tucker and Estrin (1980) look at the gear mesh parameters, such as addendum ratios and pressure angles and outline the procedures for varying a standard gear mesh to obtain a more favorable gear set. Gay (1970) considers gear tip scoring and shows how to modify a standard gear set to bring this mode of failure into balance with the pitting fatigue mode. In order to obtain an optimal design he adjusts the addendum ratios of the gear and pinion. The basic approach available is to check a given design to verify its acceptability to determine the optimal size of a standard gear mesh by Osman (1978). With the object of minimizing size and weight, optimization methods are presented for the gearbox design by Kamentskaya (1975).In 2001, Chong and Bar demonstrated a multi objective optimal design of cylindrical gear pairs for the reduction of gear size and meshing vibration. The results of the relation between the geometrical volume and the vibration of a gear pair were analyzed, in addition a design method for cylindrical gear pairs to balance the conflicting objectives by using a goal programming formulation was proposed. The design method reduces both the geometrical volume and the meshing vibration of cylindrical gear pairs while satisfying strength and geometric constraints.

CHAPTER THREE

METHODS AND CONDITIONS

3.1 Material

With improvements in steel making, there have been significant advances in the development of steels for fatigue-sensitive automotive applications, such as gears and shafts. Automotive design requirements that demand smaller and lighter components without sacrificing torque or force capacity have led to the need for components with significantly improved fatigue performance. To satisfy these needs, material/processing combinations that concentrate on surface microstructure have been developed. In this thesis work the material used is SCM 420 alloy steel, SCM 420 steel is an alloy containing chromium and molybdenum. Since it is commonly used in vehicle technology, this improved alloy steel used for manufacture of gears which has improved strength and toughness while exhibiting enhanced machinability. This material suitably used for producing gears for automotive transmissions, it can also be used for gears other than the transmission gears. Improvements in fatigue performance in components are derived primarily by decreasing the surface cyclic tensile stress or by increasing the surface yield stress, thereby increasing the resistance to fatigue crack nucleation.

Performance Requirements

Transmission gears require:

- Hard wears resistant surfaces
- Resistance to tooth root bending fatigue
- Resistance to surface fatigue which leads to pitting
- A tough core
- Dimensional accuracy for smooth meshing and reduced (noise, vibration and harshness)
- Transmission of higher loads without increasing size and weight

Table 3.1 Properties of SCM 420 Alloy structural steel under study

Sr.no	Mechanical property of SCM 420 Alloy structural steel	Symbol	Values
1	Modulus of Elasticity	E	206 Gpa
2	Poisson’s Ratio	v	0.3

3.2 Dimension

For this particular study, spur gear with the following parameters was considered.

Table 3.2 Geometrical parameters for spur gear

Sr.no	Input parameters	Symbol	values
1	Normal Module	m	4mm
2	Number of gear teeth	Z_1	20,23,25
3	Number of pinion teeth	Z_2	20,23,25
4	Pitch circle diameter of gear	d_1	80mm,92mm,100mm
5	Outer circle diameter of gear	d_o	88mm,100mm,108mm
6	Base circle diameter of gear	d_b	75.18mm,86.45mm,94mm
7	Root circle diameter of gear	d_r	70mm,82mm,90mm
8	Pitch circle diameter of pinion	d_2	80mm,92mm,100mm
9	Outer circle diameter of Pinion	d_o	88mm,100mm,108mm
10	Base circle diameter of Pinion	d_b	75.18mm,86.45mm,94mm
11	Root circle diameter of Pinion	d_r	70mm,82mm,90mm
12	Normal Pressure angle	α	20°
13	Applied Torque	T	100N.m
14	Face width for pinion and gears	b	30mm,35mm,40mm,45mm
15	Addendum	h_a	1*m
16	Dedendum	h_d	1.25*m
17	Root fillet radius	r_f	1.5mm,2mm,2.5mm,3mm

3.3 Conditions

Hertz's model of contact stresses and AGMA stress equations are based on the following simplifying assumptions

- The materials in contact are homogeneous and the yield stress is not exceeded,
- Contact stress is caused by the load which is normal to the contact tangent plane which effectively means that there are no tangential forces acting between the solids,
- The contact area is very small compared with the dimensions of the contacting solids,
- The contacting solids are at rest and in equilibrium,

- The effect of surface roughness is negligible.
- The effect of radial load is ignored
- It is assumed that at any time only one pair of teeth is in contact and takes total load.
- It considers only static loading
- No teeth are pointed.
- There is no interference between the tips and root fillets of mating teeth
- The contact ratio is between 1 and 2.
- There is nonzero backlash.
- The friction forces are neglected.

3.4 Analytical and FEM analysis of bending and contact stresses

3.4.1 Analytical analysis of bending and contact stresses

The basis for bending stress analysis of spur gears is based upon Wilfred Lewis' original formulation. His theory began with the basis that a spur gear can be simplified as a short beam which is subjected to both tension and compression. The fig 3.1 below illustrates a beam which is subjected to a bending moment at each end with a bending moment M, and the distance from the free edge of the beam to the neutral axis c, the stress is are related by Equation 3.1

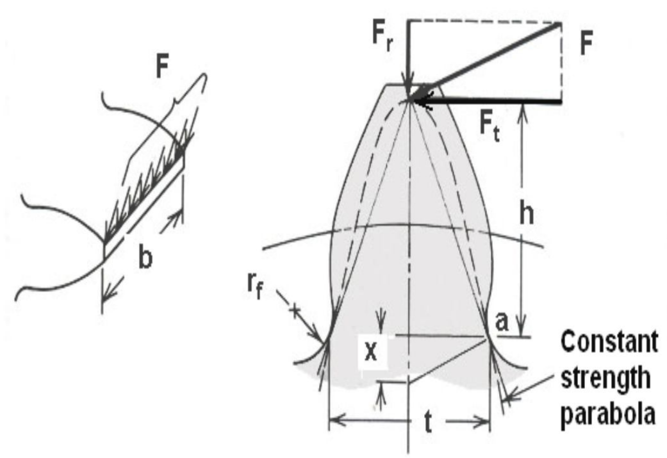


Fig 3.1 Forces and dimensions of spur gear tooth.

The Lewis Bending equation is one of the oldest and yet most important design equations to consider when sizing gears (especially spur gears). The equation formulated by Wilfred Lewis in 1892, and was the first of its kind to take into account specific geometric aspects of the tooth profile to determine tooth stresses Budynas et al. (2008). It remains one of the primary ways to size gears for bending loads, and way to get reasonable results. Lewis derived his equation by making a few assumptions. Firstly, he assumed that each gear tooth could be treated separately from the gear mesh. Next, he applied the transmitted load (F_t) to the tip of the tooth. This is ideally the most conservative place to apply the load; however it doesn't quite match reality. In the instant that a pair of gear teeth comes into contact in a gear mesh, an adjacent tooth pair is still in contact. Therefore, when contact is created at the tip of a pair of teeth the load is shared by multiple contact points. It is therefore conservative to apply the full transmitted load to the tip of the gear tooth. In reality, the full load should be applied somewhere in the middle of the tooth (say, at the pitch circle). This is the point of contact on the gear teeth when only one pair of teeth is contacting Budynas et al. (2008).Lewis assumed that the largest stresses in the gear tooth would be bending, and therefore modeled the tooth as a cantilevered beam (see Fig.3.1). Based on this assumption, the largest stress is located in the root of the tooth at the base, since this location is furthest away from the neutral axis of bending.

The following section derives the Lewis bending stress equations used in the analysis. It is based on a derivation in "Shigley's Mechanical Engineering Design" [2008]. As mentioned previously, the Lewis Bending stress is based on bending of a cantilevered beam. As shown in Fig. 3.1, the cross sections of the "fixed" end of the gear tooth are "b" x "t". The load F_t is applied at a height of "h" above the base. Based on these variables, the moment, "M", is being F_{t*}h, and the section modulus, I/c, is ($b\,\frac{t^2}{6}$) .The bending stress is therefore M/ (I/c) or $6F_t h/bt^2$ ………………………………………………………………………. (3.1)

A separate form of the equation which may be more useful for engineers makes use of the diametral pitch and a factor y called the Lewis Form Factor. We introduce a variable "x", as shown in Fig.3.1

By similar triangles,

$$\frac{t/2}{x}=\frac{h}{t/2} \text{ or } \frac{t^2}{h}= 4x \quad \text{……………………………………………………………… (3.2)}$$

Substituting Eqn.(3.2) into Eqn.(3.1),it gives

$$\sigma_b=\frac{6F_t}{4bx} \quad \text{……………………………………………………………..………. (3.3)}$$

$$y = \frac{2x}{3P} \quad \text{...} \quad (3.4)$$

And substituting Eqn.(3.4) into Eqn.(3.3)

$$\sigma_b = \frac{F_t}{bPy} \quad \text{...} \quad (3.5)$$

Eqn. 3.5 is the basic Lewis equation in terms of circular pitch.

In SI units gears are more often made to standard modules. Hence by substituting P=πm into Eqn.(3.5)

$$\sigma_b = \frac{F_t}{b\pi my} \quad \text{...} \quad (3.6)$$

Let Y = π y, which is known as modified Lewis form factor,

$$\sigma_b = \frac{F_t}{bYm} \quad \text{...} \quad (3.7)$$

Equation (3.7) is known as Lewis equation, which considers only static loading and doesn't take the dynamics of meshing teeth into account. The above stress formula must be modified to account different situations like stress concentration and geometry of the tooth. Therefore, Equation (3.8) that is shown below is the modified Lewis equation recommended by AGMA for practical gear design to account for variety of conditions. No teeth are pointed.

The AGMA equation for bending stresses given by,

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} \quad \text{...} \quad (3.8)$$

$$F_t = \frac{2*T}{d} \quad \text{...} \quad (3.9)$$

Where,

Ft = Tangential load

Ka =Application factor that accounts for pulsation and shock.

Ks = Size factor which penalizes large or wide teeth.

Km =Load distribution Factor that is a function of face width.

Kv =Dynamic factor that considers gear quality.

b = Face width

m =Metric module

T= Applied torque or moment

d =Pitch diameter of gear or pinion

J = Spur gear geometry factor. This factor includes the Lewis form factor Y and also a stress concentration factor based on a tooth fillet radius of 0.35/P. It also depends on the number teeth in the mating gear and given in table below

Table 3.3 Bending strength geometry factor for different number of spur gear teeth

Number of Teeth for which geometry factor is desired	Geometry factor, J
20	0.322
23	0.335
25	0.350

Each of these factors can be obtained from machine design books Shigley et al. (1996). In this thesis bending stresses has been studied for four different face widths and four different fillet radiuses. While contact stresses has been studied for four different face widths. Based on the number of teeth, Module, working condition and geometry of involute spur gear with the following values, Ks =1, Ka =1, km =1.3, kv =0.8 Ft =2500N, and J=0.322.For root fillets 1.5mm, 2mm, 2.5mm, 3mm and four different face widths (30mm, 35mm, 40mm and 45mm).

From equation (3.8) For 20 numbers of teeth at 30mm,35mm,40mm,45mm face widths bending stresses are calculated as:-

For b=30mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2500N*1*1.3*1}{30mm*4mm*0.322*0.8} = 105.14\text{Mpa}$$

For b=35mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2500N*1*1.3*1}{35mm*4mm*0.322*0.8} = 90.12\text{Mpa}$$

For b=40mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2500N*1*1.3*1}{40mm*4mm*0.322*0.8} = 78.85\text{Mpa}$$

For b=45mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2500N*1*1.3*1}{45mm*4mm*0.322*0.8} = 70\text{Mpa}$$

If we change the numbers of teeth from 20 to 23, the following stress equation results obtained.

For 23 numbers of teeth at 30mm,35mm,40mm,45mm face widths bending stress is calculated as:-

From eqn. (3.9) F_t =2174N

For b=30mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2174N*1*1.3*1}{30mm*4*0.335*0.8} = 87.88\text{Mpa}$$

For b=35mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2174N*1*1.3*1}{35mm*4mm*0.335*0.8} = 75.32\text{Mpa}$$

For b=40mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2174N*1*1.3*1}{40mm*4mm*0.335*0.8} = 65.91\text{Mpa}$$

For b=45mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2174N*1*1.3*1}{45mm*4mm*0.335*0.8} = 58.6\text{Mpa}$$

For 25 numbers of tooth at 30mm, 35mm, 40mm, and 45mm face widths the bending stress is calculated as

For b = 30mm,F_t=2000N,

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2000N*1*1.3*1}{30mm*4mm*0.350*0.8} = 77.38\text{Mpa}$$

For b=35mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2000N*1*1.3*1}{35mm*4mm*0.350*0.8} = 66.33\text{Mpa}$$

For b =40mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2000N*1*1.3*1}{40mm*4mm*0.350*0.8} = 58.33\text{Mpa}$$

For b= 45mm

$$\sigma_b = \frac{F_t K_s K_m K_a}{bmJK_v} = \frac{2000N*1*1.3*1}{45mm*4*0.350*0.8} = 51.60\text{Mpa}$$

In many engineering applications, such as rolling bearings, gears, cams, etc., machine components whose functioning depends upon rolling and sliding motion in contact along surfaces while under load. In this case, the contacting surfaces are non-conformal, hence the resulting contact areas are very small and the pressures are very high. From the point of view of machine design it is essential to know the values of stresses acting in such contacts. These stresses can be determined, from the analytical formulae, based on the theory of elasticity, developed by Hertz in (1881).Most machine components are designed on the basis of stress in the main body of the member, that is, in portions of the body not affected by the localized stresses. In other words, damage to most mechanical components is associated with stresses and strains in portions of the component far removed from the points of application of the loads. However, in certain situations the contact stresses between the surfaces of two externally loaded bodies (e.g., meshing gear teeth) can be the significant stresses; that is the stresses on or somewhat below the surface of the contact are the major causes of damage to one or both of the bodies. Therefore, analysis Hertzian stresses provide important information about surface and subsurface stresses for static loading of concentrated contacts.

The contact between two surfaces gives rise to an area of contact, and a pressure distribution over this contact area. If the surfaces have a simple geometry, the contact theory of Hertz (1881) can be used for calculating the contact area and the pressure distribution. The shape of the contact area depends on the shape (curvature) of the contacting bodies. For example, point contacts occur between two balls, line contacts occur between two parallel cylinders and elliptical contacts, which are the most frequently found in many practical engineering applications, occur when two cylinders are crossed, or a moving ball is in contact with the inner ring of a bearing, or two gear teeth are in contact. To illustrate the contact between two elastic bodies, the simpler contact of two parallel cylinders (Figure 3.2 and 3.3) is presented based on the Hertz theory. This type of contact is used to derive contact pressure distribution, stresses and displacements for the two dimensional line contacts. The method of calculating gear contact stress by Hertz's equation originally derived for contact between two cylinders.

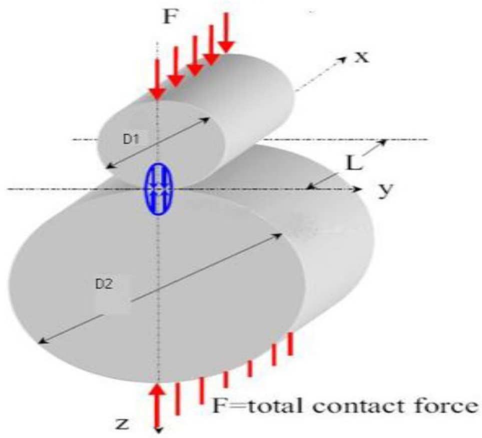


Fig 3.2 contact of two parallel cylinders

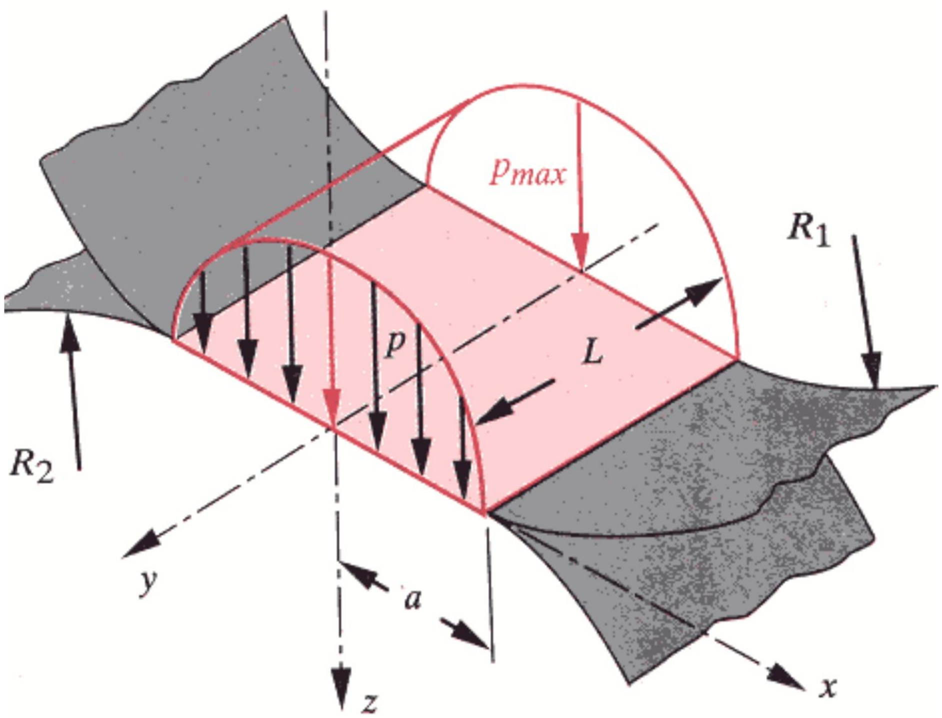


Fig 3.3 Ellipsoidal –prism pressure distribution

When cylinders are in line contact, sustained by a force F, a flattened rectangular zone exists in which the pressure distribution is elliptical.From figure 3.3 the width of the contact zone is 2a.If the total contact force is F and contact pressure is p (x), there is a formula Norton,R.L(1996) which illustrates the relationship between the force F and the pressure p (x) :

$$F= 2L\int_0^2 P(x)dx \text{ .... } (3.10)$$

$$F=\frac{F_t}{\cos 20} = \frac{2T}{d_2 \cos 20} \text{ .... } (3.11)$$

Contact width a becomes

$$a = \sqrt{\frac{2F}{\pi L}\frac{(\frac{1-v_1{}^2}{E_1})+(\frac{1-v_2{}^2}{E_2})}{\frac{1}{d_1}+\frac{1}{d_2}}} \text{ .... } (3.12)$$

The maximum contact stress is given $P_{max} = \frac{2F}{\pi a L}$..(3.13)

Where d1 and d2 represent the gear and pinion pitch diameters. Therefore the maximum surface (Hertz) stress at L=b,becomes:

$$\sigma_H = \sqrt{\frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{\pi b(1-V_1{}^2)}{E_1}+\frac{\pi b(1-V_1{}^2)}{E_2}}} \text{ .... } (3.14)$$

$R_1 = r_1 * \sin\alpha$ and $R_2 = r_2 * \sin\alpha$; substituting in the above equation (3.13)

$$\sigma_H = \sqrt{\frac{F(1+\frac{r_2}{r_1})}{r_{2*}\pi b[\frac{(1-V_1{}^2)}{E_1}+\frac{(1-V_2{}^2)}{E_2}] \sin\alpha}} \text{ .... } (3.15)$$

Where, R_1 and R_2 the instantaneous values of the radii of curvature on the pinion and gear tooth profiles, respectively, at the point of contact. r_2 is pitch radius of pinion and r_1 is pitch radius of gear,v_1 and v_2 are Poisson's ratio of gear and pinion respectively.E_1 and E_2 are modulus elasticity of gear and pinion materials. F is total contact force and α is pressure angle of involute spur gear.

The peak values of the equivalent stress using the Von Mises criterion, the maximum shear stress, and the maximum orthogonal shear stress can be calculated from the maximum Hertz stress as follows Zeping Wei, (2004).

Maximum shear stress, $\tau_{max} = 0.30\sigma_{Hmax}$ ……………………………...……………. (3.16)

Maximum Von Misses stress, $\sigma_{Von} = 0.57\sigma_{Hmax}$ ……...........……………………..… (3.17)

Maximum Orthogonal shear stress, $\sigma_{Ort} = 0.25\sigma_{Hmax}$ ………………......….………… (3.18)

For 20 numbers teeth of spur gear, at [30mm, 35mm, 40mm, and 45mm] face width the Hertz contact stress calculate from eqn. (3.11) and (3.15)

At b=30mm, F=2660.44N

$$\sigma_H = \sqrt{\frac{2660.44N(1+\frac{40mm}{40mm})}{40mm*\pi*30mm\,\frac{[(1-0.3^2)}{206*10^3 N/mm^2}+\frac{(1-0.3^2)]}{206*10^3 N/mm^2}\sin 20}} = 683\text{Mpa}$$

From eqn.(3.17)

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*683Mpa

= 389.31Mpa

At b=35mm

$$\sigma_H = \sqrt{\frac{2660.44N(1+\frac{40mm}{40mm})}{40mm*\pi*35mm\,\frac{[(1-0.3^2)}{206*10^3 N/mm^2}+\frac{(1-0.3^2)]}{206*10^3 N/mm^2}\sin 20}} = 632.7\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*632.7Mpa

= 360.64Mpa

At b=40mm

$$\sigma_H = \sqrt{\frac{2660.44N(1+\frac{40mm}{40mm})}{40mm*\pi*40mm\,\frac{[(1-0.3^2)}{206*10^3 N/mm^2}+\frac{(1-0.3^2)]}{206*10^3 N/mm^2}\sin 20}} = 591.88\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*591.88Mpa

= 337.36Mpa

At b= 45mm

$$\sigma_H = \sqrt{\frac{2660.44N(1+\frac{40mm}{40mm})}{40mm*\pi*45mm\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} = 558\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*558Mpa = 318Mpa

For 23 numbers of teeth of spur gear, at [30mm, 35mm, 40mm, and 45mm] face widths the Hertz contact stress calculate from eqn. (3.11) and (3.15)

$$F= \frac{2*100{,}000Nmm}{92\text{mm}*\cos 20} = 2313.43\text{N}$$

At b=30mm

$$\sigma_H = \sqrt{\frac{2313.43N(1+\frac{46mm}{46mm})}{46mm*\pi*30mm\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} = 594.29\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*594.29Mpa

= 338.75Mpa

At b=35mm

$$\sigma_H = \sqrt{\frac{2313.43N(1+\frac{46mm}{46mm})}{46mm*\pi*35mm\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} = 550.21\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*550.21Mpa

= 313.62Mpa

At b=40mm

$$\sigma_H = \sqrt{\frac{2313.43N(1+\frac{46mm}{46mm})}{46mm*\pi*40mm\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} = 514.67\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*514.67Mpa

= 293.36Mpa

At b=45mm

$$\sigma_H = \sqrt{\frac{2313.43N(1+\frac{46mm}{46mm})}{46mm*\pi*45mm\,\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} \quad =485.24\text{Mpa}$$

σ_{Von} = 0.57σ_{Hmax} =0.57*485.24Mpa

=276.58Mpa

For 25 numbers of teeth of spur gear, at [30mm, 35mm, 40mm, and 45mm] face widths the Hertzian contact stress calculate from eqn. (3.11) and (3.15)

$$F=\frac{2*100{,}000Nmm}{100\text{mm}*\cos 20} =2128.36\text{N}$$

At b=30mm

$$\sigma_H = \sqrt{\frac{2128.36N(1+\frac{50mm}{50mm})}{50mm*\pi*30mm\,\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} \quad =546.75\text{Mpa}$$

σ_{Von} = 0.57σ_{Hmax} =0.57*546.75Mpa

= 311.65Mpa

At b=35mm

$$\sigma_H = \sqrt{\frac{2128.36N(1+\frac{50mm}{50mm})}{50mm*\pi*35mm\,\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} \quad =506.2\text{Mpa}$$

σ_{Von} = 0.57σ_{Hmax} =0.57*506.2Mpa

= 288.53Mpa

At b=40mm

$$\sigma_H = \sqrt{\frac{2128.36N(1+\frac{50mm}{50mm})}{50mm*\pi*40mm\,\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} \quad =473.5\text{Mpa}$$

σ_{Von} = 0.57σ_{Hmax} =0.57*473.5Mpa =267Mpa

At b=45mm

$$\sigma_H = \sqrt{\frac{2128.36N(1+\frac{50mm}{50mm})}{50mm*\pi*45mm\,\frac{[(1-0.3^2)}{206*10^3N/mm^2}+\frac{(1-0.3^2)]}{206*10^3N/mm^2}\sin 20}} \quad =446.42\text{Mpa}$$

$\sigma_{Von} = 0.57\sigma_{Hmax}$ =0.57*446.42Mpa

=254.46Mpa

3.4.2 **FEM of contact and bending stress analyses**

In this section, the steps for developing the 3D involute spur gear model and the FEA technique are described in details. Figure 3.4 shows the general procedure in determining the bending and contact stress of the involute spur gear. Modeling of the spur gears is based on the design parameters. The material properties and gear parts assembly are also assigned in this stage. This is followed by the FEA pre-processing stage where the mesh element, the load, and constraint are set on the involute spur gear model. In order to determine the bending stress and contact stress accurately, different mesh element settings and FEA solver are required in separate setup. In the FEA post-processing stage, the FEA solution for the gear is completed. The maximum von Misses stress is obtained from the contour plot of the gear. Thus, the bending and contact stress analysis can be performed by correlating the change in applied load and gear parameters.

ANSYS is the name commonly used for ANSYS mechanical, general-purpose finite element analysis (FEA) computer aided engineering software tools developed by ANSYS Inc. ANSYS mechanical is a self-contained analysis tool incorporating pre-processing such as creation of geometry and meshing, solver and post processing modules in a unified graphical user interface. ANSYS is a general-purpose finite element-modeling package for numerically solving a wide variety of mechanical and other engineering problems. These problems include linear structural and contact analysis that is non-linear Strukturallabor (2012).

Among the various FEM packages, in this work ANSYS is used to perform the analysis. The following steps are used in the solution procedure using ANSYS:

- The geometry of the gear to be analyzed is imported
- The element type and materials properties such as Young's modulus and Poisson's ratio are specified.

- Meshing the three-dimensional gear model.
- The boundary conditions and external loads are applied.
- The solution is generated based on the previous input parameters.
- Finally, the solution is displayed.

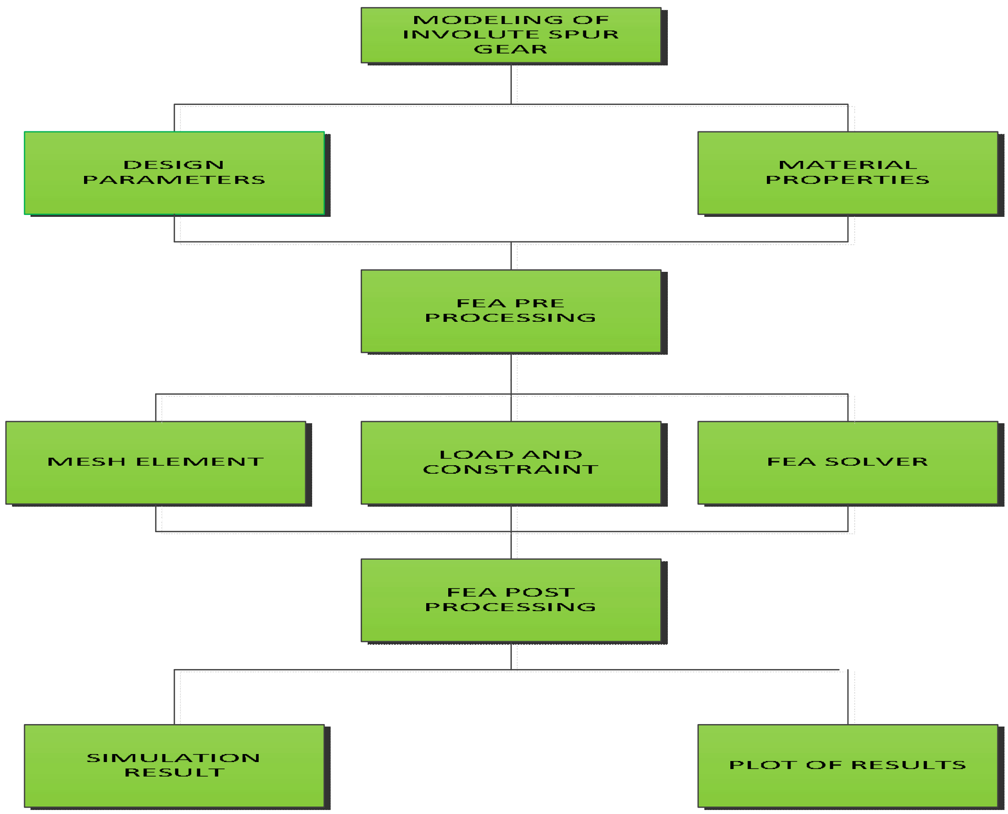


Fig 3.4 FEA general steps for obtaining the gear bending and contact stresses

Finite Element Method is the easy technique as compared to the theoretical methods to find out the stress developed in a pair of gears. Therefore FEM is widely used for the stress analysis of mating gears. In this paper, finite element analysis is carried out in ANSYS Workbench 12.0 to determine the maximum contact and bending stresses for SCM 420 steel gears. CATIA is the product of a France company named Dassault System. It is the advanced computer aided design, manufacturing and analysis software. It is widely used in aerospace, automobile, shipbuilding and electronic devices, etc. CATIA has a strong 3D modeling function, can perform the 3D modeling of a variety of complex components and the complex curved surface. In the present design process of mechanical product, we use a lot of gear transmission especially the transmission on involute gear. The study on involute gear 3D modeling method which based on

CATIA is almost the same, especially the parameterized modeling for involute gear. We mainly use the formula function of CATIA. The first step is, input the relationship of the parameters of the gear. After that is, complete one gear tooth profile by inputting the formula. At last, we will create the whole 3d model through the entity operation. The method of gear tooth geometry and parametric modeling is as follow,

(1) Determining the basic parameter

(2) Determining the geometrical parameter

(3) The characteristic parameter

To satisfy the fundamental law of gearing, most of gear profiles are cut to an involute curve, the involute curve may be constructed by wrapping a string around a cylinder, called the base circle and then tracing the path of a point on the string. Given the gear's pitch radius r_p and pressure angle α, we can calculate the coordinates of each point on the involute curve. For example, consider an arbitrary point A on the involute curve; we want to calculate its polar coordinates (r, θ), as shown in the Fig (3.5).

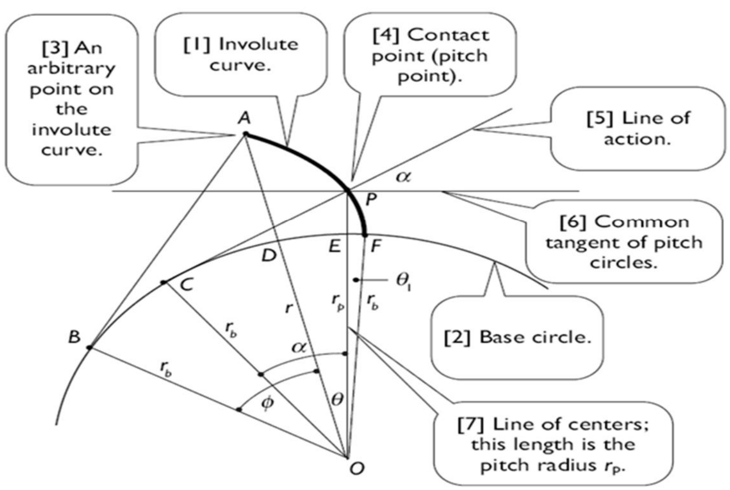


Fig 3.5 Methods of generating involutes of spur gear.

Note that BA and CP are tangent lines of the base circle, and F is a foot of perpendicular.

Since APF is an involute curve and $\widehat{BCDEF}$ is the base circle, by the definition of involute curve,

$$\overline{BA} = \widehat{BC} + \overline{CP} = \widehat{BCDEF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.19)}$$

$$\widehat{CP} = \widehat{CDEF} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.20)$$

From ΔOCP,

$r_b = r_p * \cos(\alpha)$ Where,

r_b is base radius of gear

r_p is pitch radius of gear

From ΔOBA,

$$r = \frac{r_b}{ø}$$

To calculate θ, we notice that

$$\widehat{DE} = \widehat{ABCD} - \widehat{BCDB} - \widehat{EF} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.21)$$

Dividing the equation (3.21) by r_b and using Eq. (3.19),

$$\frac{\widehat{DE}}{r_b} = \frac{\overline{BA}}{r_b} - \frac{\widehat{BCD}}{r_b} - \frac{\widehat{EF}}{r_b}$$

If radian is used, then the above equation can be written as:

$$\theta = (\tan ø) - ø - \theta 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.22)$$

The last term θ_1 is the angle <EOF, which can be calculated by dividing Eq. (3.20) by r_b,

$$\frac{\overline{CP}}{r_b} = \frac{\widehat{CDEF}}{r_b}, \text{ or } \tan\alpha = \alpha + \theta_1, \text{ or}$$

$$\theta 1 = \tan\alpha - \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.23)$$

The above equations are all we need to calculate polar coordinates (r, θ) .The polar coordinates can be easily transformed to rectangular coordinates, using “O” as origin and “OP” as y-axis,

The parameter equation of the standard involutes curves is expressed as:

$$x = -r \sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.24)$$

$$y = r \cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.25)$$

Using eq. (3.24 and 3.25) the involute spur gear is generated by CATIA is shown in the figure below.

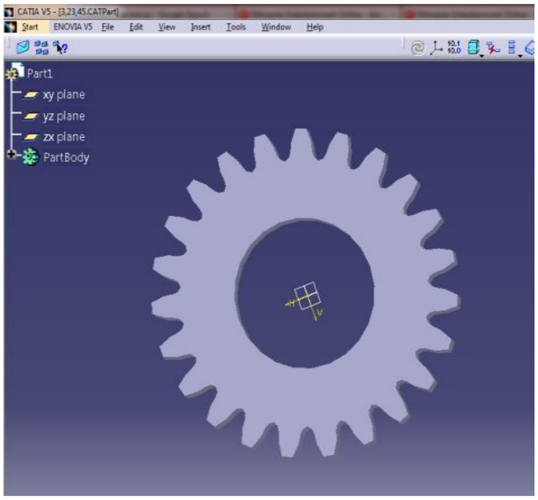


Fig 3.6 Modeling of 20 numbers of teeth spurs gear using CATIA V5

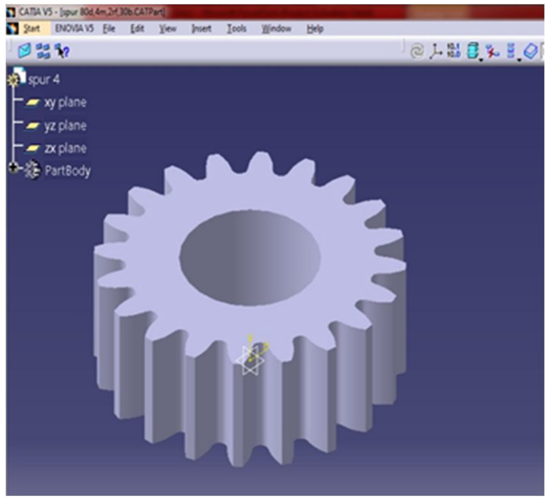


Fig 3.7 Modeling of 23 numbers of teeth involute involute spurs gear using CATIA V5

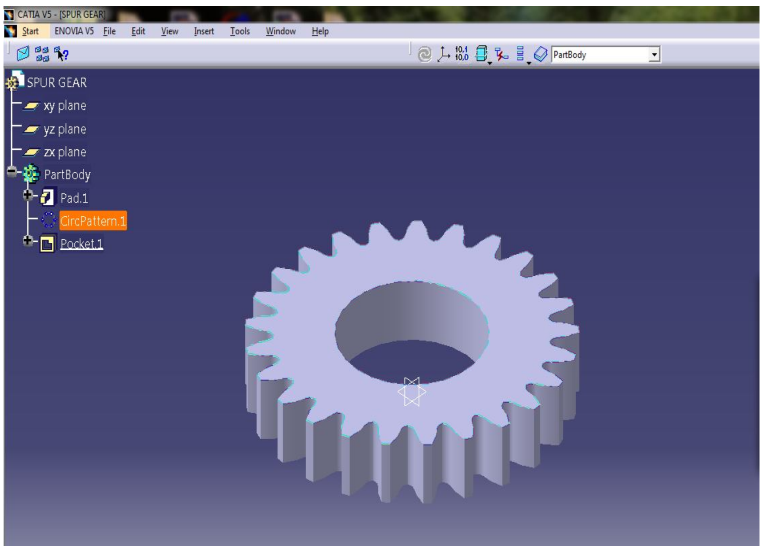


Fig 3.8 Modeling of 25 numbers of teeth involute spurs gear using CATIA V5

ANSYS is an engineering simulation software provider founded by software engineer John Swanson. It develops general-purpose finite element analysis and computational fluid dynamics software. While ANSYS has developed a range of computer-aided engineering (CAE) products, it is perhaps best known for its ANSYS Mechanical and ANSYS Multi physics products. ANSYS Mechanical and ANSYS Multi physics software are non-exportable analysis tools incorporating pre-processing (geometry creation, meshing), solver and post-processing modules in a graphical user interface. These are general-purpose finite element modeling packages for numerically solving mechanical problems, including static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems.

The following steps are necessary to simulate contact and bending stresses of spur gear

- The 3D model of the gear is converted as **iges** format through the CATIA V5 software
- The **IGES** (**I**nitial **G**raphic **E**xchange **S**ystem) format is suitable to import in the ANSYS Workbench for analyzing.
- Open the ANSYS workbench 12.0 and SCM 420 mechanical properties are defined using Editing structural steel properties, then the tool box opened.
- File – import external geometry file – generate

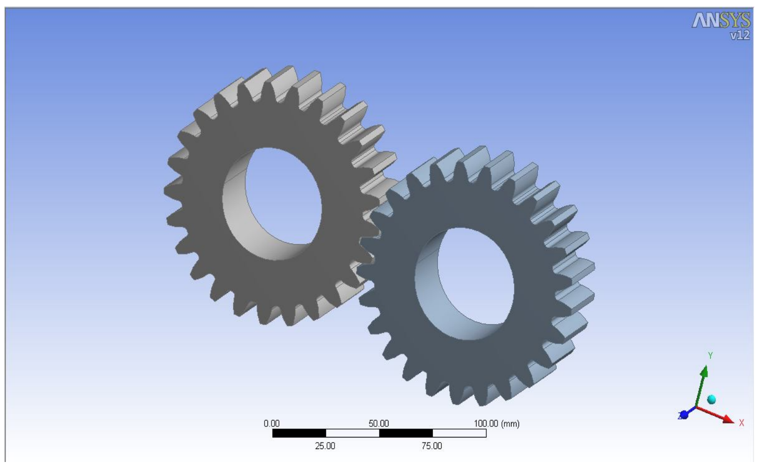


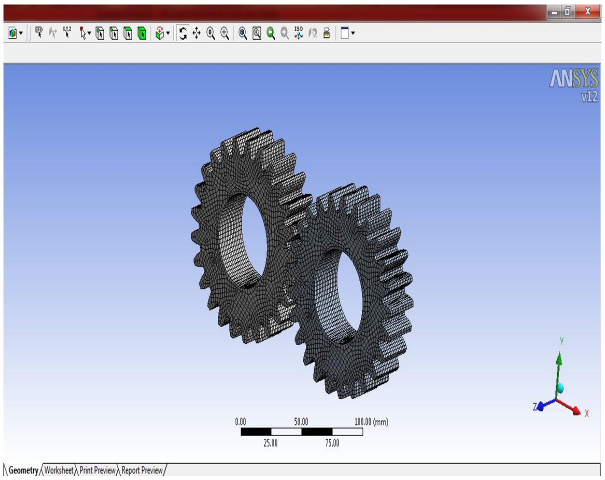
Fig 3.9 Assembled geometry in CATIA V5 imported to design modeler

- Assigning material for imported geometry
- Defining contact region

  Once the geometry is attached with static structural analysis tab, we have must define the contact between the two involute teeth; ANSYS has in built option, which automatically reads the attached geometry for any predefined contacts or other boundary definitions. One of the most important things is to change the „Interface Treatment" to "Adjust to Touch". This is helping us to define the kind of contact between the selected bodies.

To get the contact stresses the contact wizard is used in ANSYS Workbench. The contact algorithm in ANSYS Workbench computer program requires definition of contacting surface. To define a contact pair completely, contact and target element have to be referred to same characteristic parameters. The contact element CONTA174 and target 170 are used in the present analysis. Then, augmented Lagrangian method contact algorithm, with frictionless contact is used.

- Generating Mesh



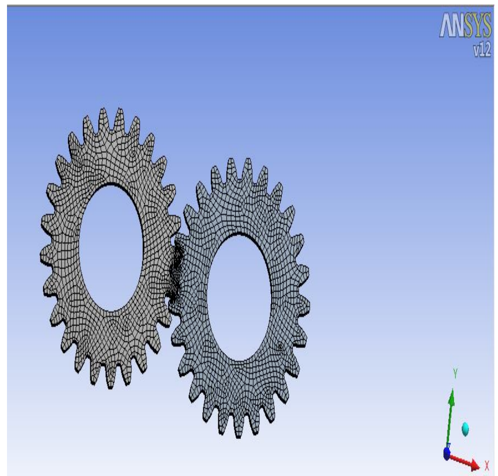


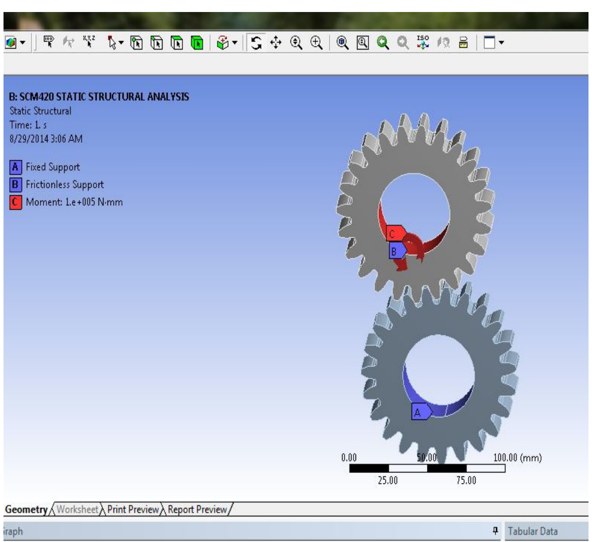
Fig 3.10 Default mesh

Fig 3.11 Refined mesh at contact area

To get accurate results, default mesh size is not enough, hence refining mesh size using face sizing in contact or root fillet area is necessary.

- Setting Boundary conditions

  According to Faydor (2004), boundary conditions and loading conditions for gear and pinion is applied as follows: Boundary condition refers to the external load on the border of the structure. We assumed gear is with fixed support and pinion is subjected to a moment or toque along its axis with frictionless support.



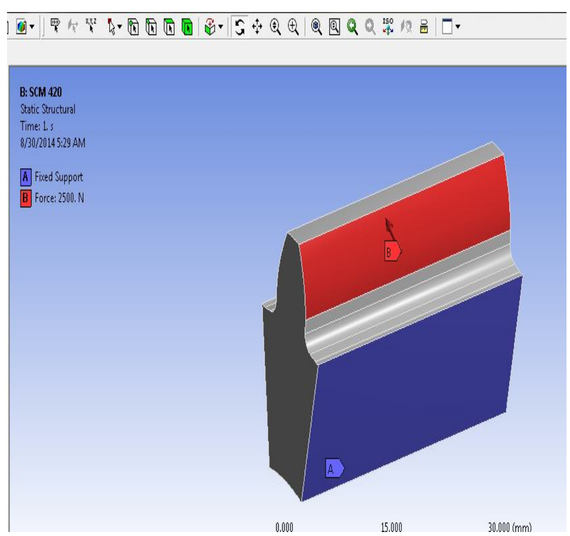


Fig 3.12 Boundary conditions and moment

Fig 3.13 Boundary conditions and force

➢ Evaluating solutions

Fatigue or yielding of a gear tooth due to excessive bending stress is two important gear design considerations. In order to predict fatigue and yielding, the maximum stresses on the tensile and compressive sides of the tooth, respectively, are required. When meshing the teeth in ANSYS, if SMART SIZE is used the number of elements near the roots of the teeth are automatically are much greater than in other places. It also indicates that only one tooth is enough for the bending stress analysis for the 3-D model or a 2-D model, since the change of tooth number also changes pitch diameter of gear.

In this section, the teeth bending stress of Spur gear is calculated using ANSYS. For this purpose the modeled gear in CATIA V5r16 is exported to ANSYS as an IGES file and then using above stated procedures the results were analyzed. For the bending stress, the numerical result is compared with the values given by the draft proposal of the standards of AGMA. The element type SOLID TETRAHEDRAL 10 NODES 187 was chosen. Figure 3.14 shows the meshed three-dimensional model. Figure 4.1(a-d) demonstrate the Von Mises bending stress on the root of the tooth for different root fillet radius of spur gear tooth at 30 mm face widths. Fig 4.2 (a-c) shows the Von Mises bending stress on the root of tooth for different face widths at 1.5 mm root fillet radius. There are more detailed results for various numbers of teeth in Table 4.1 at section 3.1 which are compared with the result obtained from Modified Lewis formula (AGMA) equations.

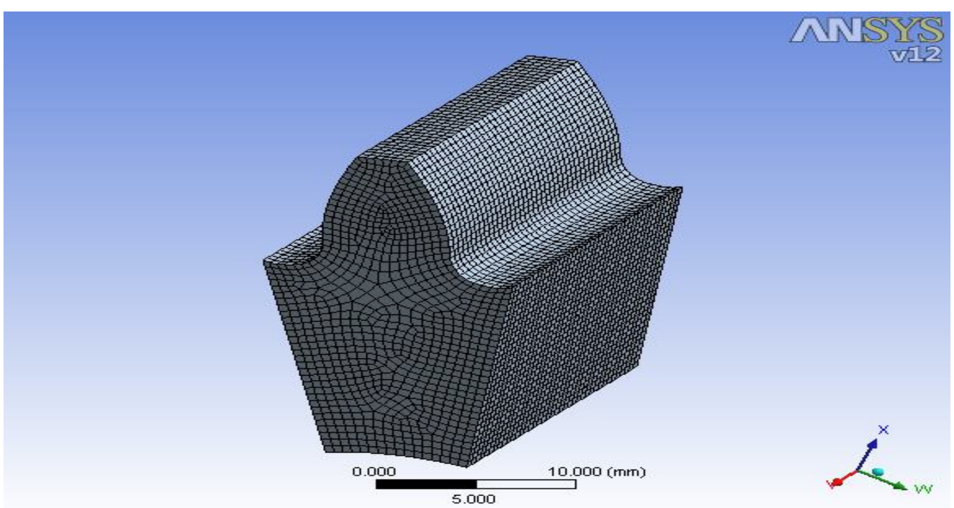


Fig 3.14 Meshed 3D model of spur gear tooth

3.5 Optimization by using Design Expert Dx7

The problem with the conventional design procedure is that it gives out a single solution and the manufacturing is carried out on that basis. Optimization is the act of obtaining the best result under the given circumstances. Design optimization of spur gear sets at reduces the size, weight, stress and increase the life span of the gear. The optimization methodology adopted in this work is numerical optimization approach developed by Derringer and Suich. Dx7 will maximize, minimize or target:

- A single response.
- A single response, subjected to upper or lower boundaries on other response.
- Combination of two or more responses.

This thesis involves about the optimization of spur gear set for its weight, bending stress and contact stress are taken as an objective functions and the decision variable such as face width number of teeth on pinion, and root fillet radius are considered.

Gear Weight

User of the gear sets expects a gear set, which is normally less in weight so that vibration can be reduced and quite in running. Weight reduction of gears improves performance of systems and saves the material, which leads to cost reduction and easy assembling. The following is the equation governing the weight of the gear set Yammati et al. (2012).

$$W = \frac{\pi m^2 bg}{4}\left({Z_1}^2\rho_1 + {Z_2}^2\rho_2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.26)$$

Where b is the face width in mm, ρ_1 and ρ_2 be the density of gear and pinion material and g acceleration due to gravity = 9.81 m/ Sec^2, ρ = 7.85*10^{-6}kg/mm^3

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Results

The goal of stress analysis presented in this section is to investigate the change in stress with changing face width, number of teeth, and root fillet radius. Contact and bending stress results for different parameters, using ANSYS, AGMA and Hertzian theory under a constant loading condition of torque 100Nm is shown in Figure.

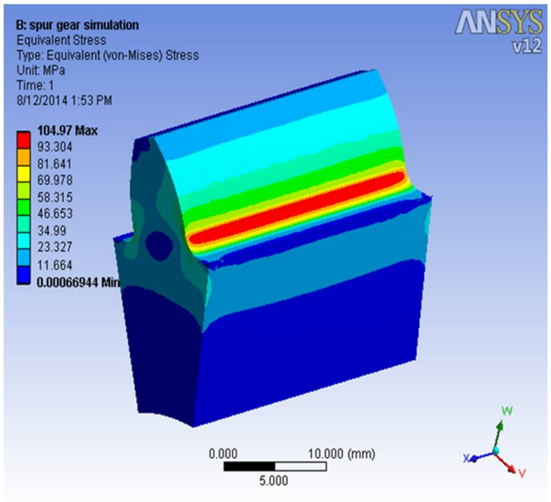


Fig 4.1a Von Mises bending stress at 1.5 mm root fillet radius

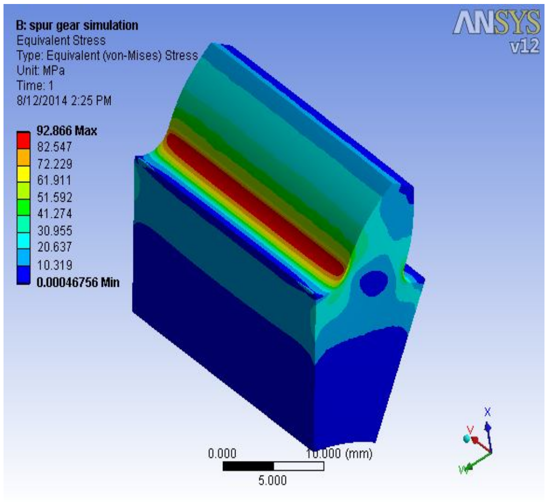


Fig 4.1b Von Mises bending stress at 2 mm root fillet radius

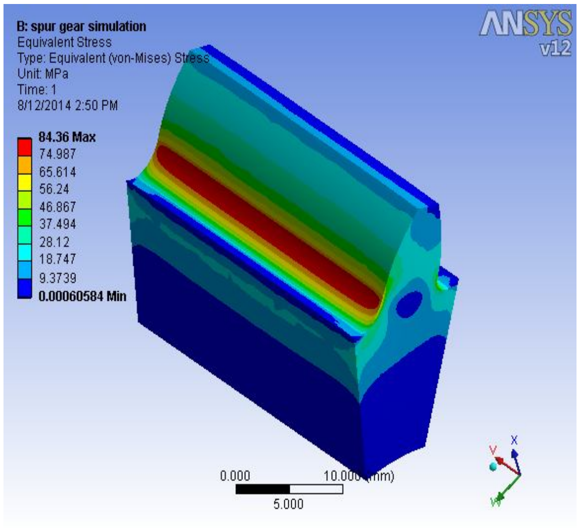


Fig 4.1c Von Mises bending stress at 2.5 mm root fillet radius

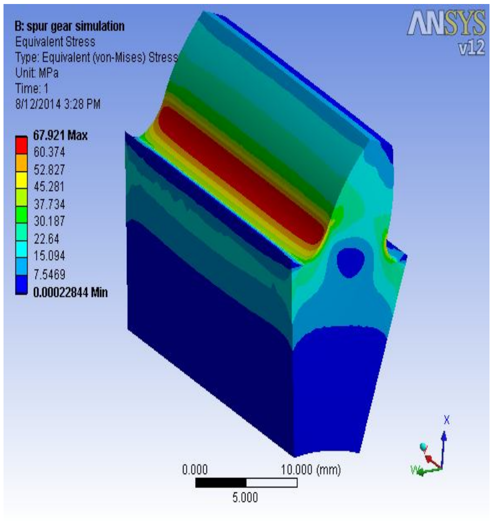


Fig 4.1d Von Mises bending stress at 3 mm root fillet radius

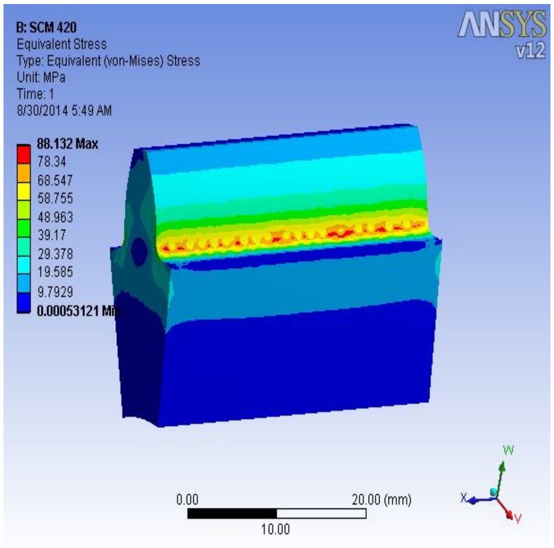


Fig 4.2a Von Mises bending stress at 35mm face width

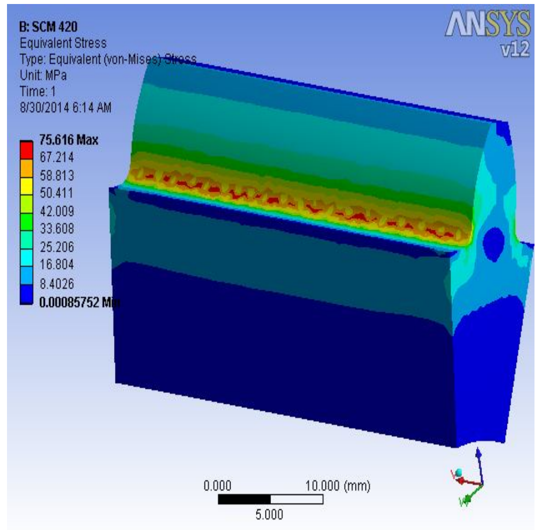


Fig 4.2b Von Mises bending stress at 40mm face width

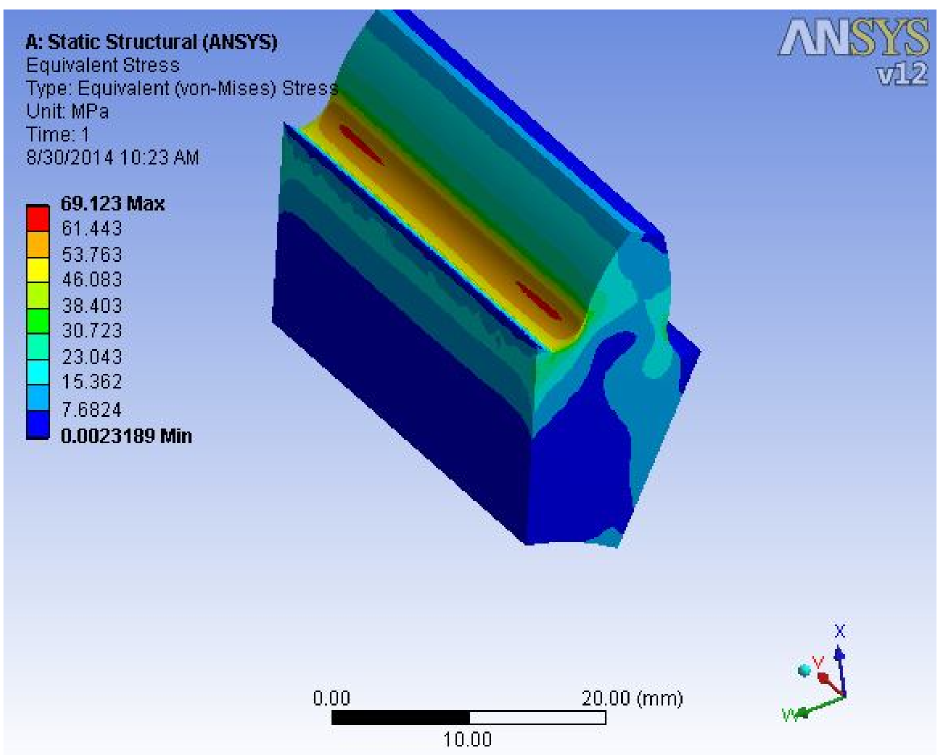


Fig 4.2c Von Mises bending stress at 45mm face width

The above fig. 4.1(a-d) and Fig 4.2(a-c) show how changing root fillet radius and face width affects bending stress of involute spur gear. Also the figures show increasing face width and root fillet radius decreases bending stress of spur gear.

Figure 4.3(a-d) demonstrate the Von Mises Contact stress of 20 numbers of teeth spur gear for face widths 30mm,35mm,40mm,and 45mm and fillet radius at 1.5mm,using equivalent cylinders principle of Hertzian contact stress.

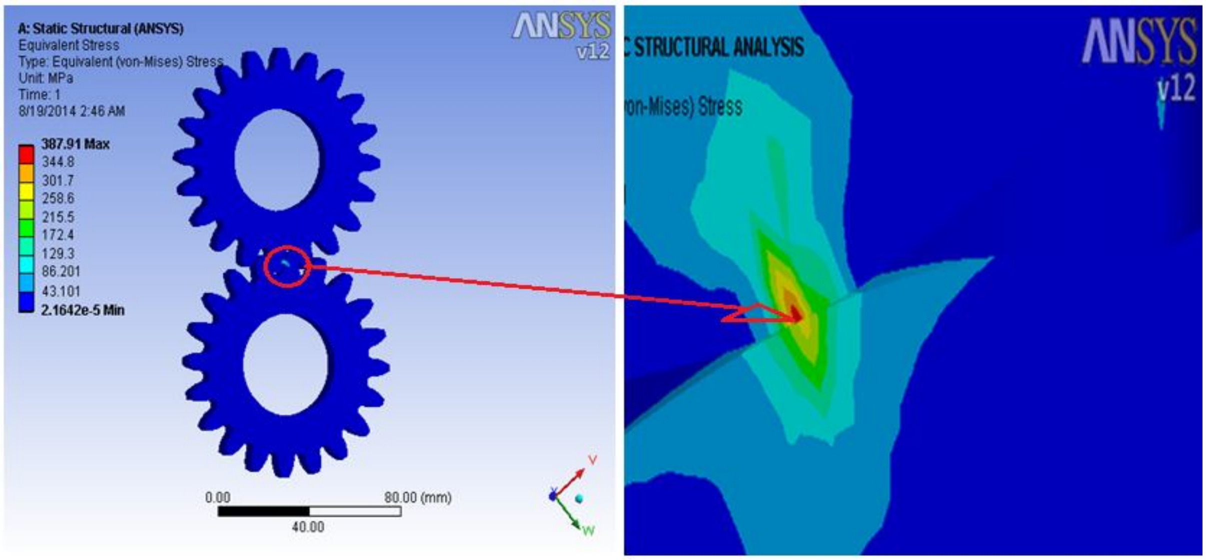


Fig. 4.3a Maximum Von Mises contact stress of 30mm face width

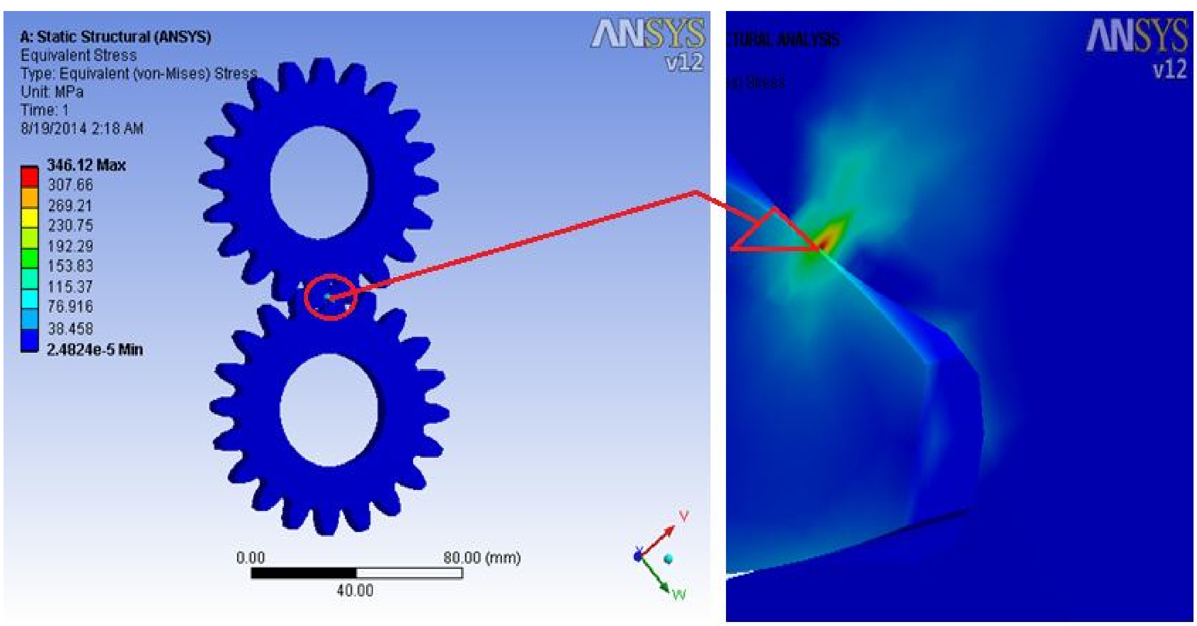


Fig. 4.3b Maximum Von Mises contact stress of 35mm face width

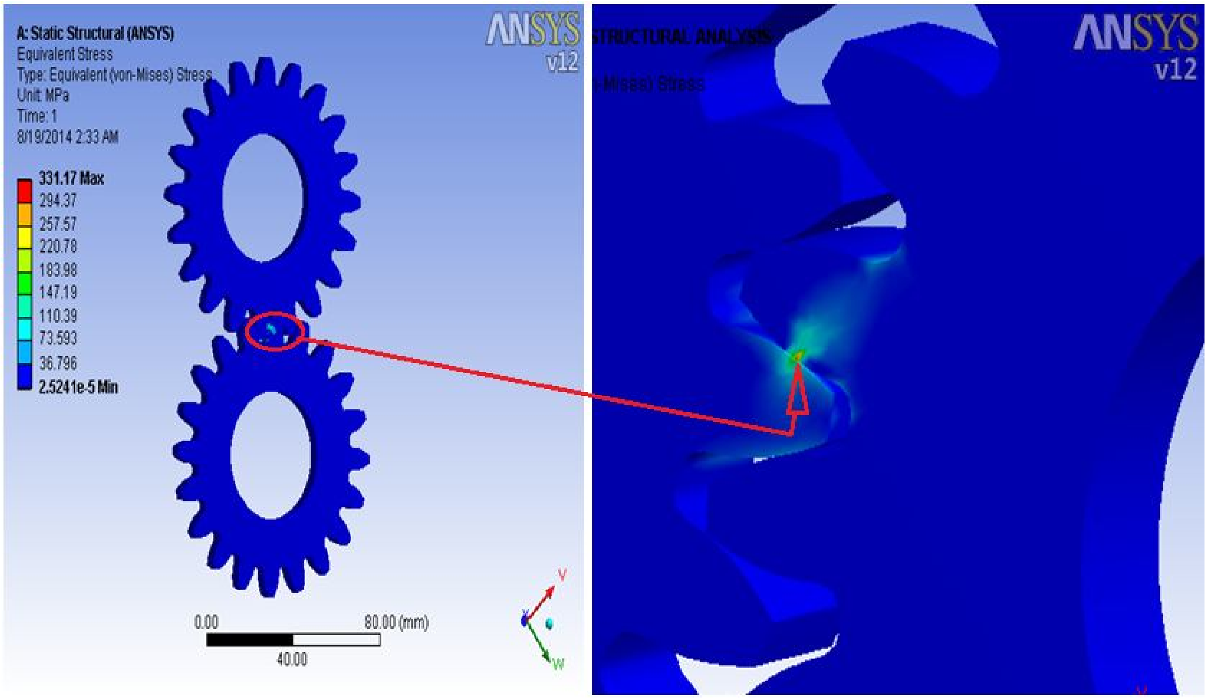


Fig. 4.3c Maximum Von Mises contact stress of 40 mm face width

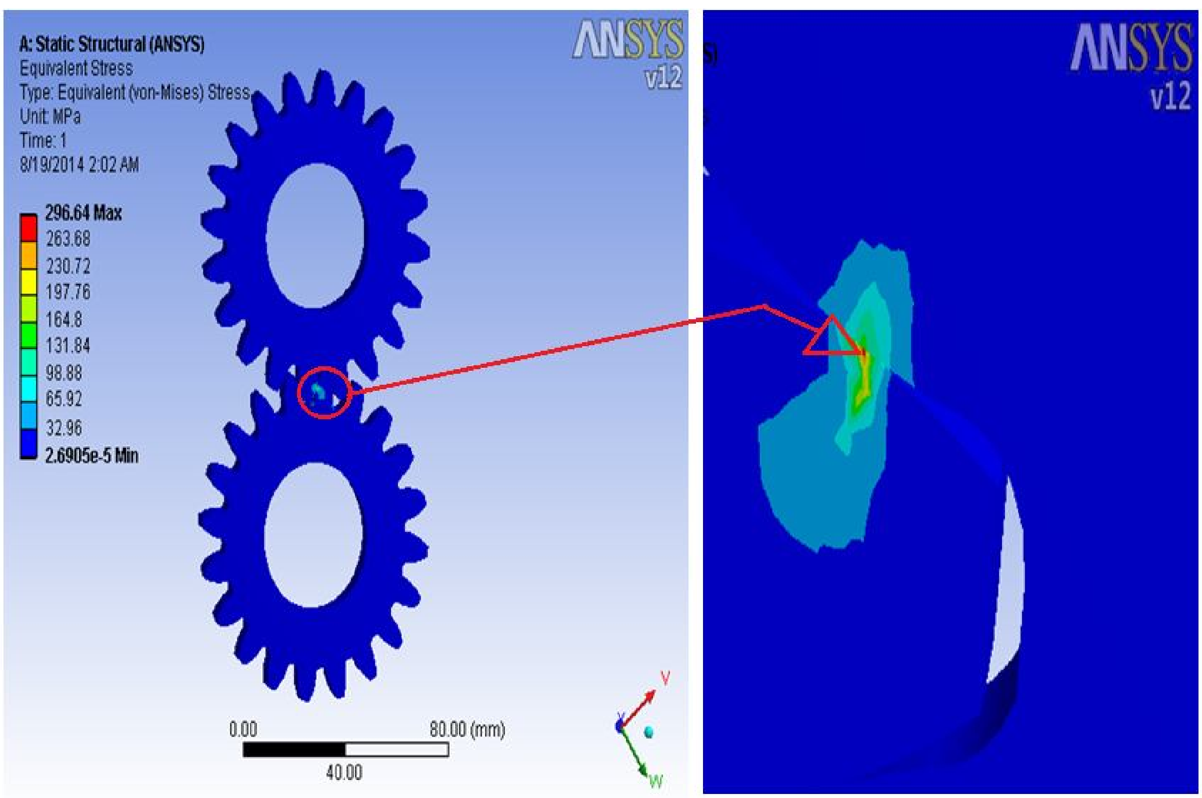


Fig. 4.3d Maximum Von Mises contact stress of 45 mm face width

Fig 4.4 (a-d) below shows the Von Mises contact stress of 23 numbers of teeth spur gear for face widths 30mm,35mm,40mm,and 45mm and root fillet radius at 1.5mm

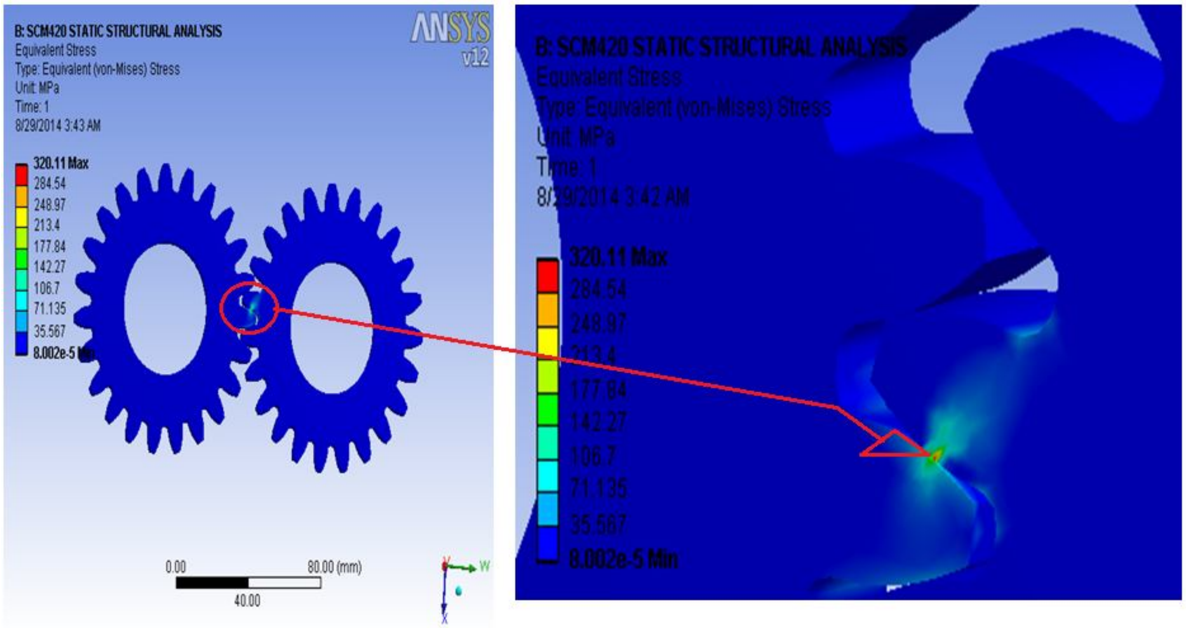


Fig. 4.4a Maximum Von Mises contact stress of 30 mm face width.

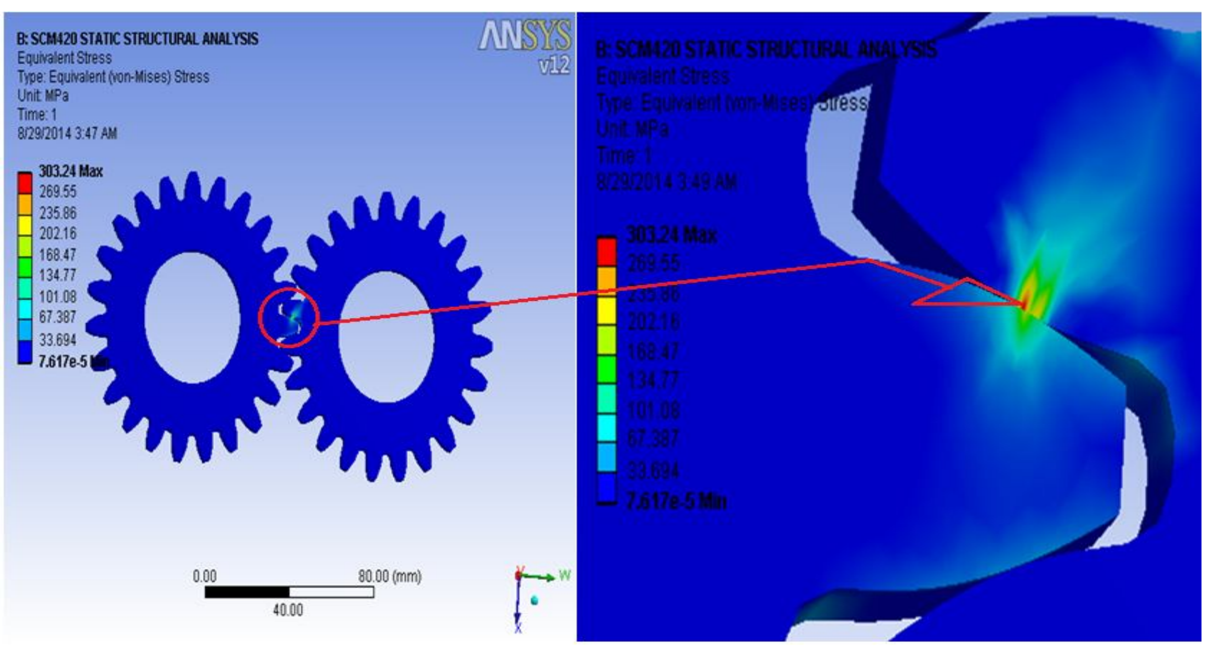


Fig. 4.4b Maximum Von Mises contact stress of 35 mm face width.

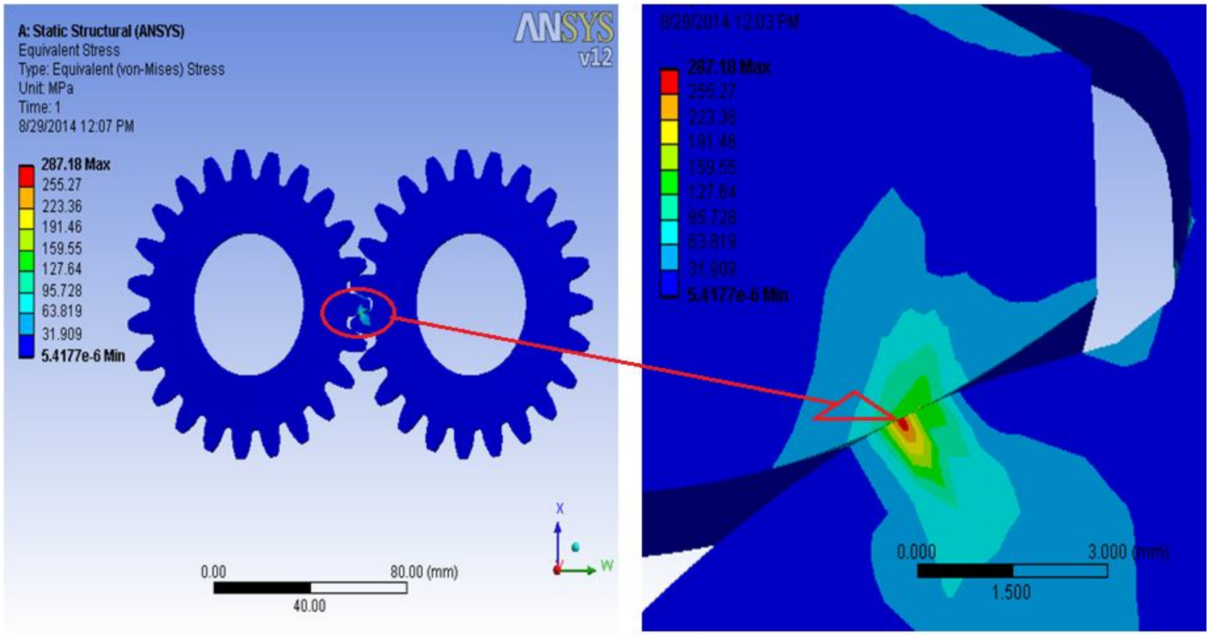


Fig. 4.4c Maximum Von Mises contact stress of 40 mm face width.

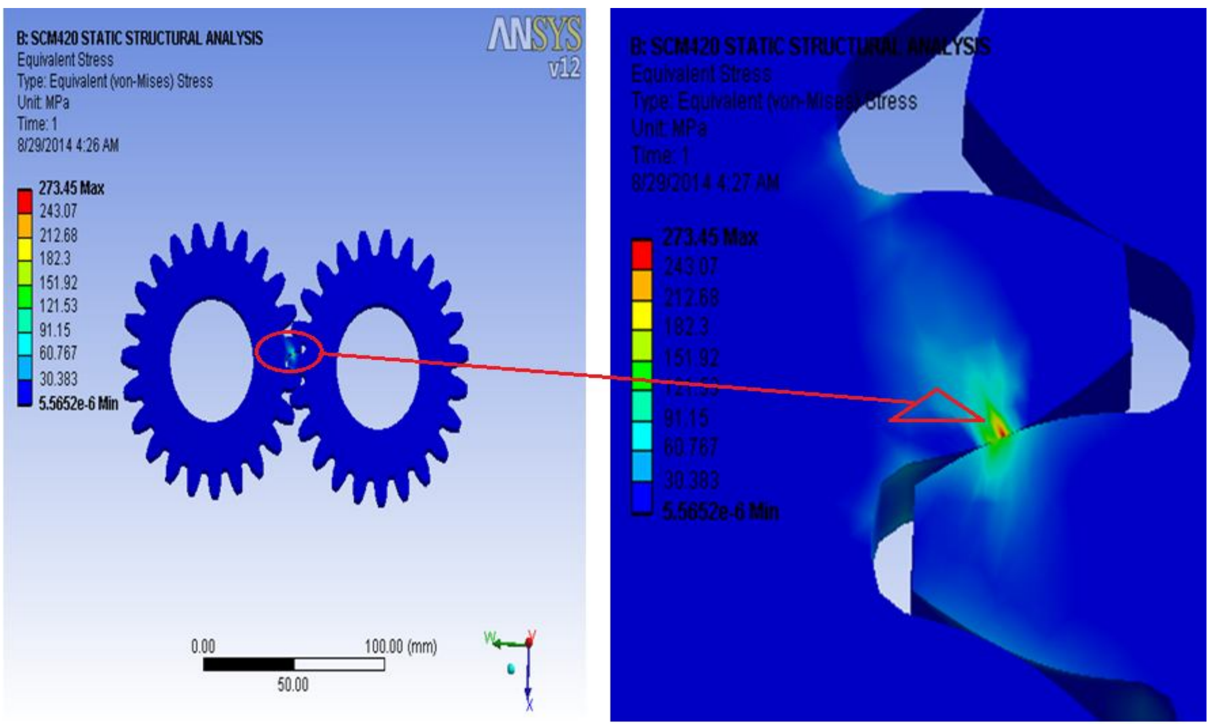


Fig. 4.4d Maximum Von Mises contact stress of 45 mm face width.

Fig 4.5 (a-d) below shows the Von Mises contact stress of 25 numbers of teeth spur gear for face widths 30mm,35mm,40mm,and 45mm and root fillet radius 1.5mm

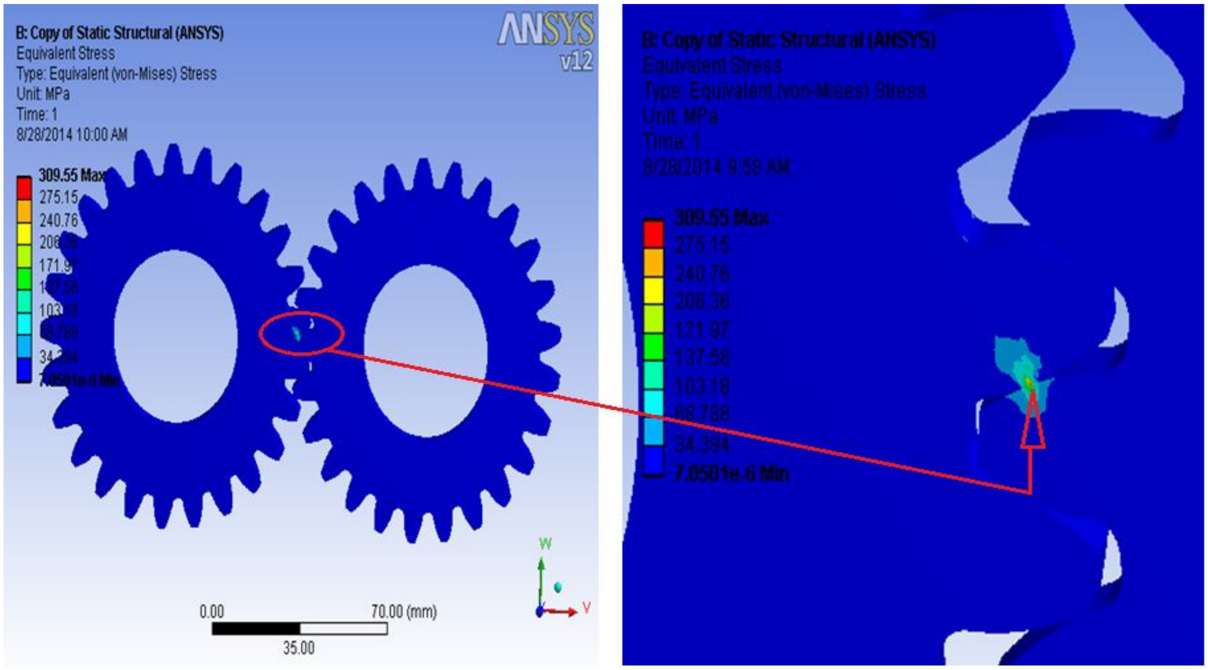


Fig. 4.5a Maximum Von Mises contact stress of 30 mm face width.

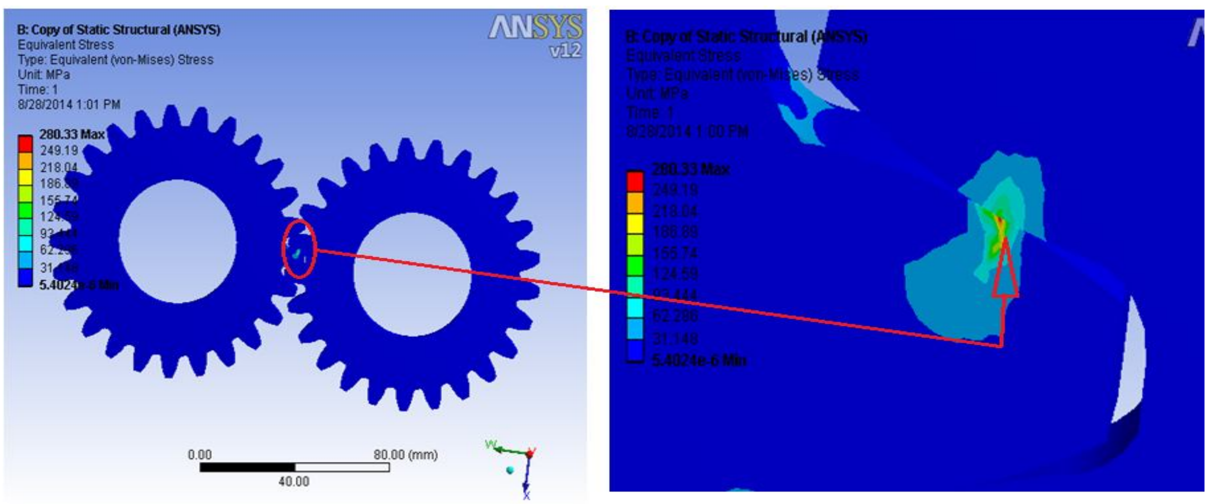


Fig. 4.5b Maximum Von Mises contact stress of 35 mm face width.

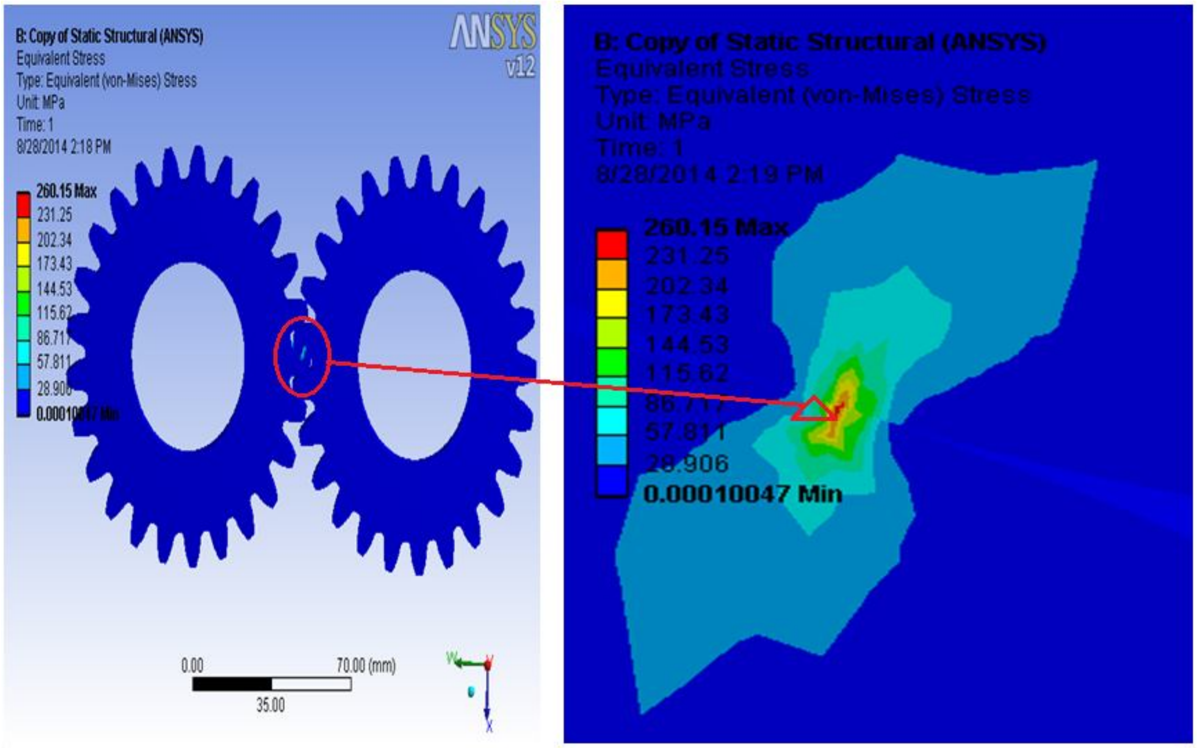


Fig. 4.5c Maximum Von Mises contact stress of 40 mm face width.

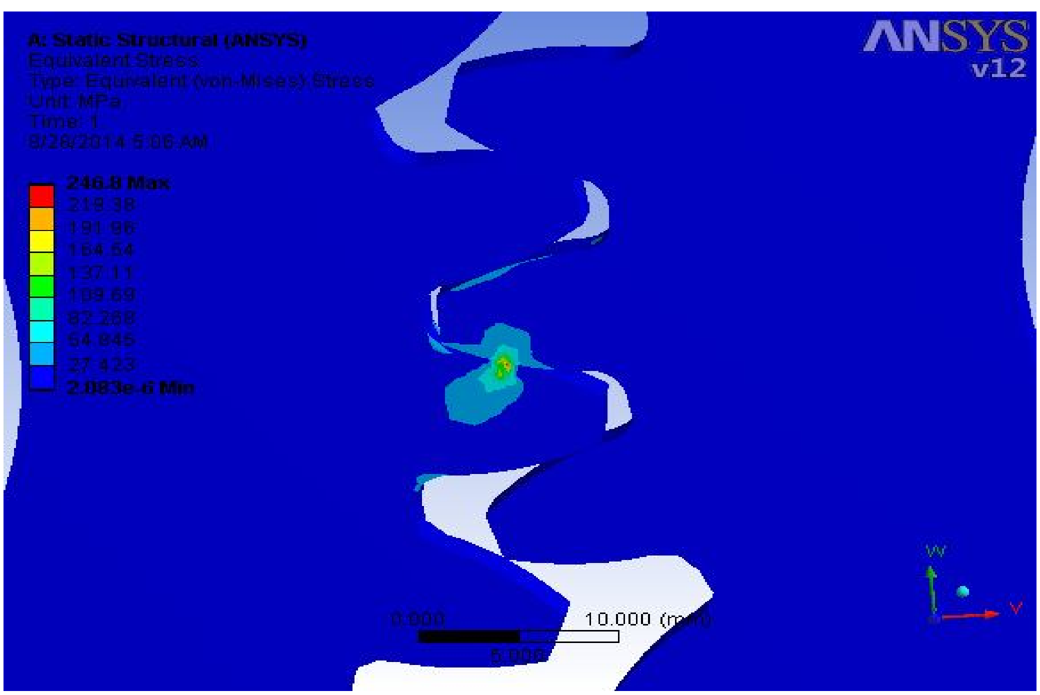


Fig. 4.5d Maximum Von Mises contact stress of 45mm face width.

4 .2 Discussions

4.2.1 Effect of Face Width, number of teeth and root fillet radius on bending stress

For determining the stresses at any stage during the design of gears face width, number of teeth and root fillet radius are important parameters. To determine the stress variations with the face width, number of teeth and root fillet radius relative to gear set weight models of Spur gear are made by keeping constant other parameters i.e. pressure angle, tooth thickness, etc. the FEM analysis ware carried out. Table 4.1 shows the effect of face width on bending stress of 20 numbers of teeth spur gear. The combined effect of these parameters on bending stress also indicated in Table 4.2, Table 4.3 and Table 4.4. If the face width, number of teeth and root fillet radius of teeth is changed, then different bending strength value obtained. That is increasing the value of face width, number of teeth, and root fillet radius leads to decreasing in bending stress.

Table 4.1 the maximum bending stress obtained from AGMA and ANSYS for 20 numbers of teeth spur gear

S.no	Tooth face width (mm)	AGMA (Mpa)	ANSYS (Mpa)	Errors (%)
1	30	105.14	104.97	0.2
2	35	90.12	88.132	2.2
3	40	78.85	75.761	3.9
4	45	70	69.123	1.2

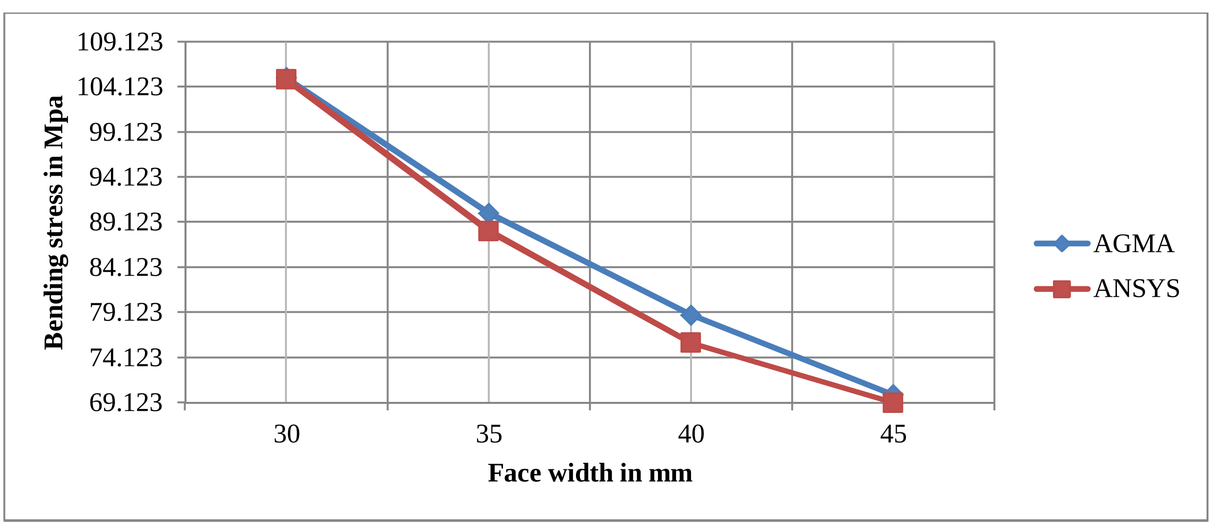


Fig 4.6 graphical representation of the effect of face width on bending stress

From Fig.4.6 also we observed that increasing the value of face width decreases bending stress and the calculated and numerical results from AGMA and FEM are comparable.

Table 4.2 Maximum Von Mises bending stresses at 20 numbers of teeth spur gear

S.no	face width(mm)	Root fillet radius(mm)	Maximum bending stress(ANSYS) Mpa
1	30	1.5	104.97
		2	92.866
		2.5	84.36
		3	67.921
2	35	1.5	88.132
		2	78.761
		2.5	73.004
		3	62.325
3	40	1.5	75.616
		2	70.94
		2.5	64.182
		3	57.861
4	45	1.5	69.123
		2	62.208
		2.5	58.155
		3	49.857

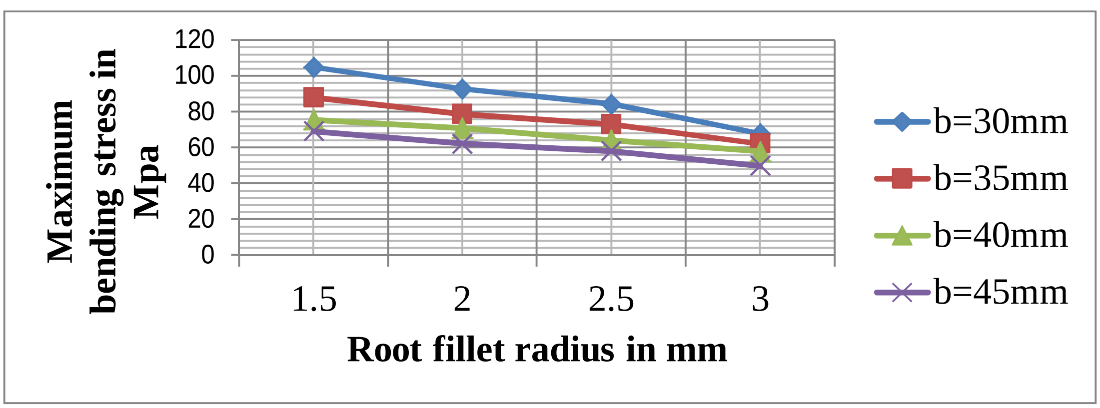


Fig 4.7 Combined effect of face width and root fillet radius on bending stress of 20 numbers of teeth spur gear

Table 4.3 Maximum Von Mises bending stresses at different face width and root fillet radius obtained from ANSYS for 23 numbers of teeth spur gear.

S.no	face width(mm)	Root fillet radius(mm)	Maximum bending stress(ANSYS) Mpa
1	30	1.5	87.88
		2	75.78
		2.5	67.274
		3	52.835
2	35	1.5	75.32
		2	65.50
		2.5	56.30
		3	48.30
3	40	1.5	65.91
		2	60.20
		2.5	52.50
		3	43.25
4	45	1.5	58.60
		2	51.40
		2.5	46.15
		3	40.20

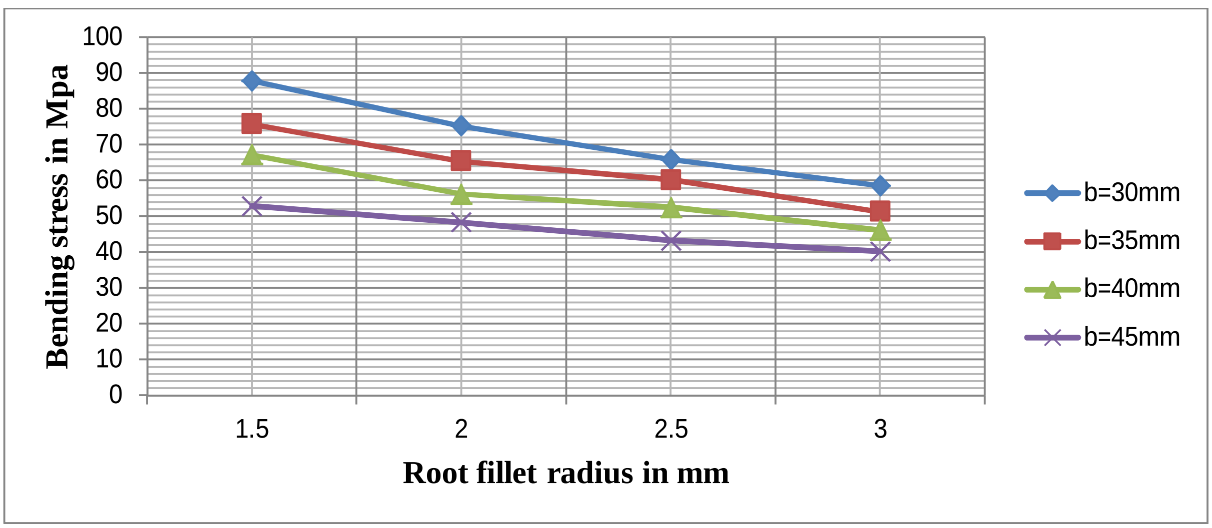


Fig 4.8 Combined effect of face width and root fillet radius on bending stress of 23 numbers of teeth spur gear

Table 4.4 Maximum Von Mises bending stresses at different face width and root fillet radius obtained from ANSYS for 25 numbers of teeth spur gear.

S.no	face width(mm)	Root fillet radius(mm)	Maximum bending stress(ANSYS) Mpa
1	30	1.5	77.38
		2	70.40
		2.5	61.50
		3	49.50
2	35	1.5	66.33
		2	59.20
		2.5	51.25
		3	46.32
3	40	1.5	58
		2	53.18
		2.5	47.50
		3	41.38
4	45	1.5	51.60
		2	44.25
		2.5	40.75
		3	35.75

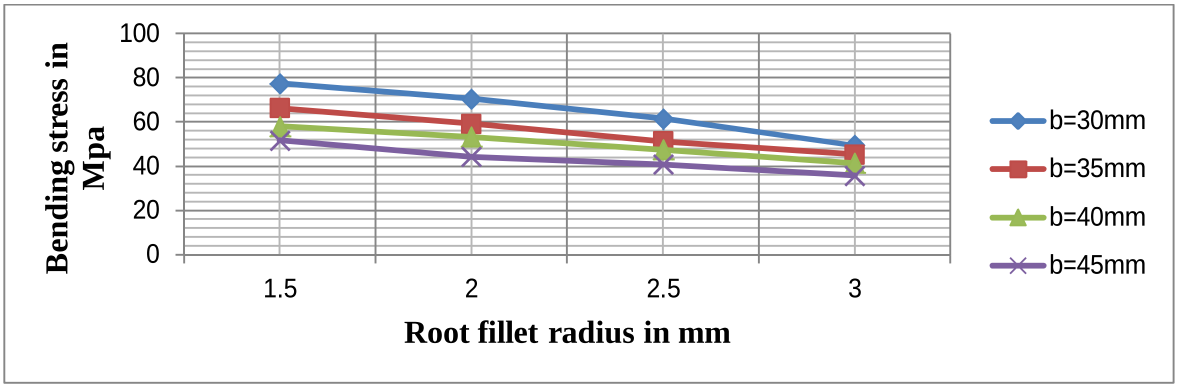


Fig 4.9 Combined effect of face width and root fillet radius on bending stress of 25 numbers of teeth spur gear

From the above tables 4.6-4.9 for the same faces width and root fillet radius, if we change the number of teeth from 20 to 23 and from 23 to 25 the bending stress value decreases. Therefore, increasing the number of teeth reduces bending stress effect on spur gear.

4.2 .2 Effect of face width and number of teeth on contact stress

The effect of face width and number of teeth on maximum Contact stress is studied by varying the face width for four different values which are (b=30mm, 35mm, 40mm, and 45mm), number of teeth 20, 23, 25 .The values of the stresses obtained using the above parameters are shown in Table 4.5 – 4.7

Table 4.5 Effect of face width on maximum Contact stress for 20 numbers of teeth spur gear

Face Width [mm]	Hertz contact stress [Mpa]	ANSYS contact stress [Mpa]	Errors [%]
30mm	389.31	387.91	0.36
35mm	360.64	346.12	4.0
40mm	337.36	331.17	1.8
45mm	318.00	296.64	6.7

Table 4.6 Effect of face width on maximum Contact stress for 23 numbers of teeth spur gear

Face Width [mm]	Hertz contact stress [Mpa]	ANSYS contact stress [Mpa]
30mm	338.75	320.11
35mm	313.62	303.24
40mm	293.36	287.18
45mm	276.58	273.45

Table 4.7 Effect of face width on maximum Contact stress for 25 numbers of teeth spur gear

Face Width [mm]	Hertz contact stress [Mpa]	ANSYS contact stress [Mpa]	Errors [%]
30mm	311.65	309.55	0.7
35mm	288.53	280.37	2.8
40mm	267.00	260.15	2.57
45mm	254.46	246.80	3.0

From table 4.5 -4.7 the results show that increasing the number of teeth of spur gear reduces maximum Contact stress of spur gear. At face width of 30mm and changing the numbers of teeth from Z=20 to 23 and from 23 to 25 the change of maximum contact stress of spur gear is shown in table 5.6

Table 4.8 Effect of Number of teeth on maximum Contact stress at b=30mm

Number of Teeth	Hertz contact stress [Mpa]	ANSYS contact stress [Mpa]	Errors [%]
20	389.31	387.91	0.36
23	338.71	320.11	5.5
25	311.65	309.55	0.7

At constant face width changing the number of teeth affects contact stresses of spur gear. That is increasing the number of spur gear teeth decreases contact stress.

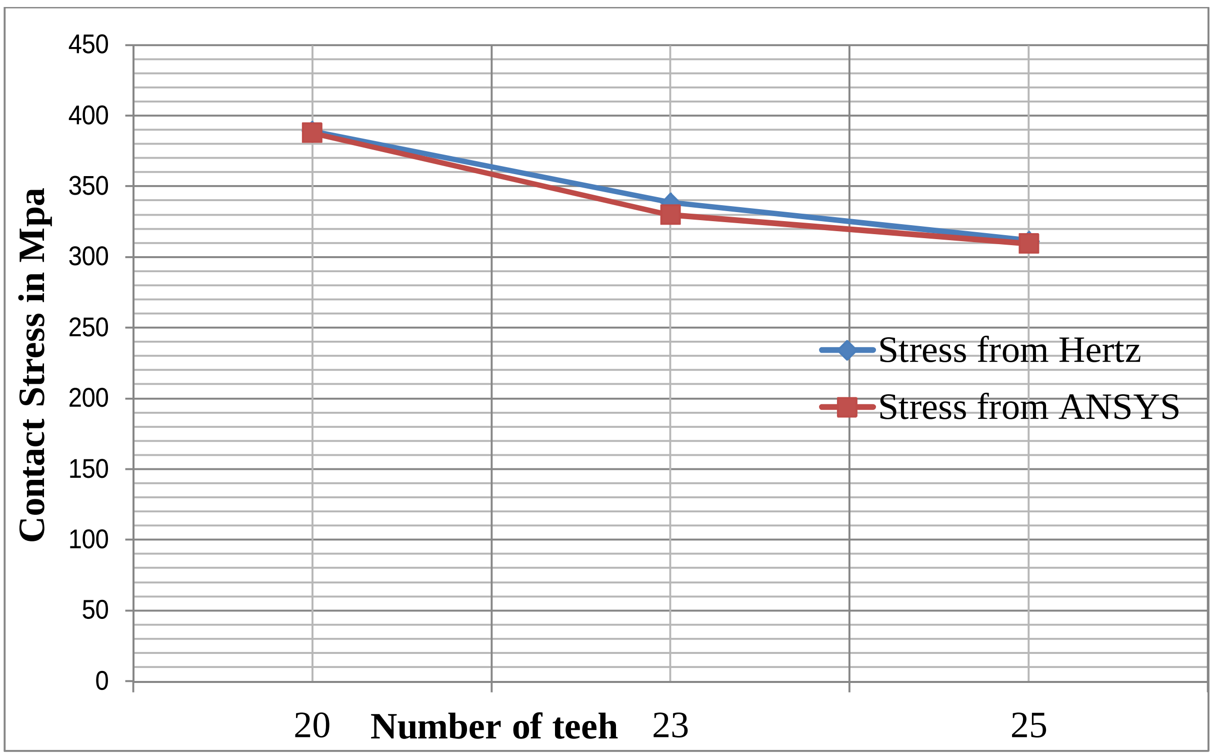


Fig. 4.10 graphical representation of effect of number of teeth on maximum Contact stress

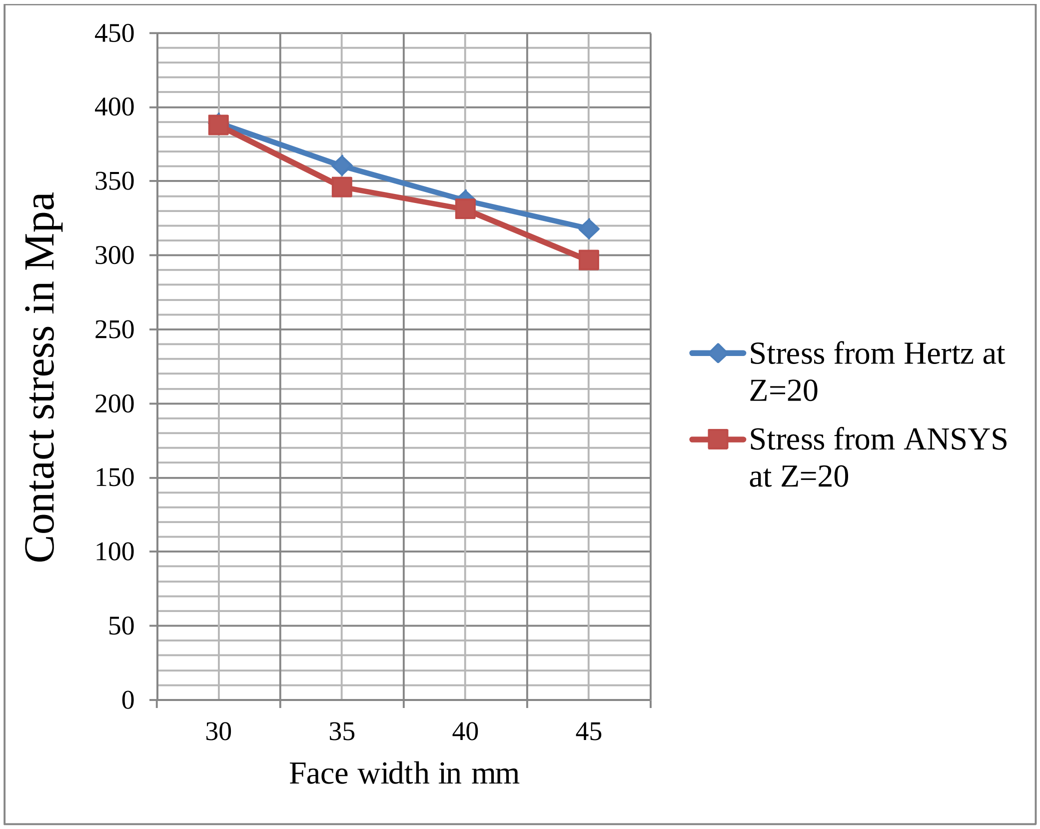


Fig. 4.11 graphical representation of effect of face width on Contact stress spur gear of 20 numbers of teeth.

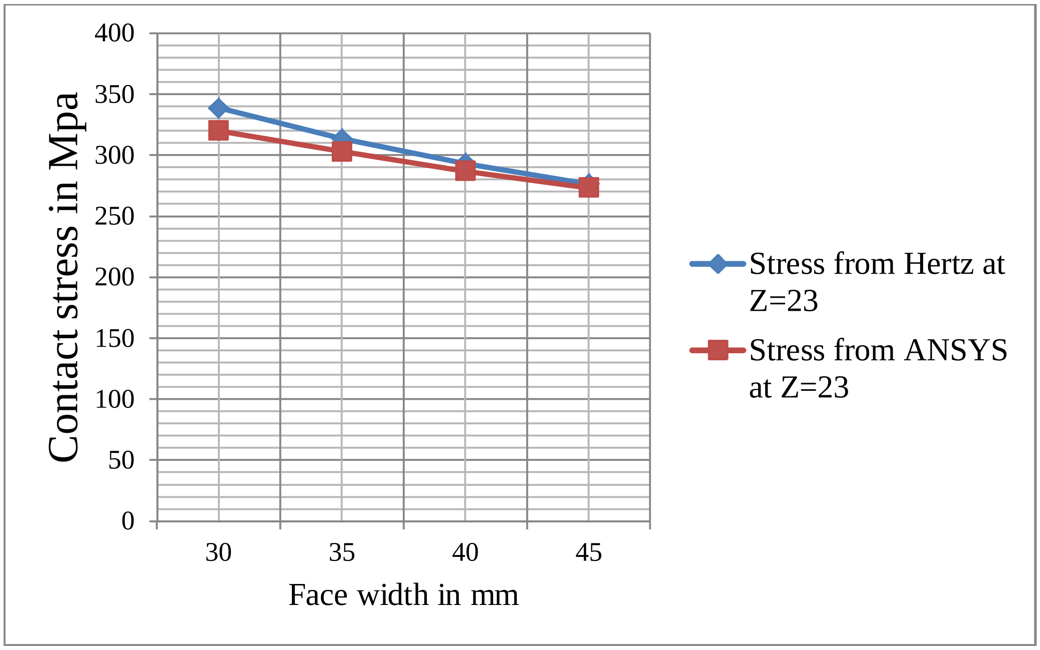


Fig. 4.12 graphical representation of effect of face width on Contact stress spur gear of 23 numbers of teeth.

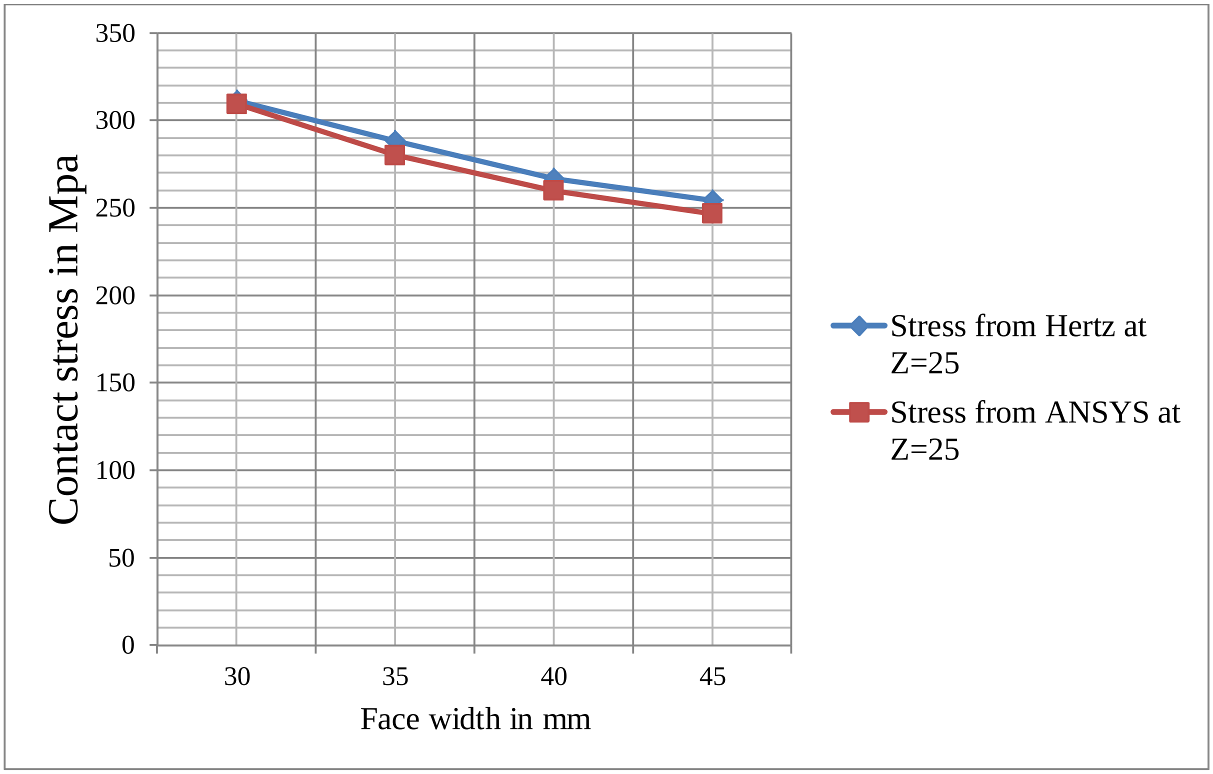


Fig. 4.13 graphical representation of effect of face width on Contact stress spur gear of 25 numbers of teeth.

4.2.3 Finding optimum design values for bending and contact stresses

Table 4.9 Input data to obtain optimum points using Dx7 design expert

Constraints name	Goal	Lower limit	Upper limit
Face width [mm]	Is in range	30	45
Number of teeth [No]	Is in range	20	25
Root fillet radius [mm]	Is in range	1.5	3
Bending stress [Mpa]	Minimize	35.75	104.97
Contact stress [Mpa]	Minimize	228	387.91
Weight of gear set [N]	Minimize	22.23	54.43

Using these input parameters design expert Dx7 software has selected the following points. According to their priority based on desirability to minimize weight, bending and contact stresses. The possible optimum design points are presented below in table 4.10.

Table 4.10 Selection of optimum points

Solutions Number	Face width (mm)	Number of teeth (No)	Root fillet radius (mm)	Bending stress (Mpa)	Contact stress (Mpa)	Weight of gear set (N)	Desirability	Resolution
1	37.24	21.97	3.00	49.6473	294.56	34.1086	0.691	Selected
2	37.30	21.96	3.00	49.6205	294.456	34.1298	0.691	
3	37.42	21.93	3.00	49.6145	294.478	34.1298	0.691	
4	37.24	22.00	3.00	49.5408	294.06	34.2024	0.691	
5	37.60	21.88	3.00	49.5762	294.372	34.156	0.691	
6	37.83	21.83	3.00	49.4789	294.003	34.2329	0.691	
7	37.98	21.79	3.00	49.4729	294.035	34.2331	0.691	
8	38.19	21.74	3.00	49.4048	293.792	34.2866	0.691	
9	35.95	22.33	3.00	49.8637	294.971	33.9972	0.691	
10	35.56	22.43	3.00	49.9897	295.355	33.9171	0.691	
11	40.01	21.37	3.00	48.7843	291.397	34.8136	0.691	
12	40.79	20.99	3.00	49.351	294.243	34.3451	0.690	
13	41.17	20.94	3.00	49.1575	293.39	34.5214	0.690	
14	32.88	23.13	3.00	50.8225	297.572	33.4825	0.689	
15	41.89	20.67	3.00	49.4212	294.736	34.337	0.689	
16	33.04	23.27	3.00	50.1751	294.684	34.0691	0.689	

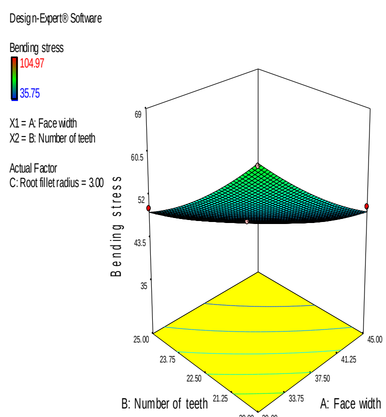


Fig 4.14 bending stress at different design points

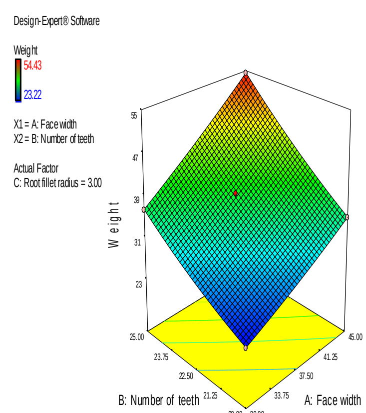


Fig 4.15 Weight effect at different design points

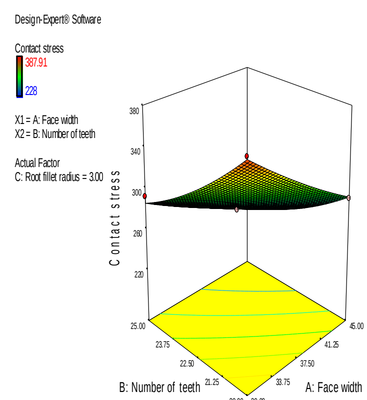


Fig. 4.16 contact stress at different design points

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This thesis is based on numerical and analytical analysis of the effect of change of involute spur gear parameters on bending and contact stresses. The study focused on the effect of face width, number of teeth, and root fillet based on bending and contact stresses using FEM, AGMA equations and Hertizian theory of contact. As main objective of this work increasing involute spur gear tooth strength by selecting the most optimal values of face width, number of teeth, and root fillet radius relative to gear set weight are designated. Generally from this thesis work the following can be figured out.

- The Stress at the contact and fillet region decreases with the increase of face width, number of teeth, and root fillet radius. The FEA results are found to be in close agreement with the calculated stresses based on AGMA standards, Hertzian contact theory and Lewis Equation.
- Comparing with conventionally defined tooth parameters, when the design was done at optimal point, 52.8% of bending stress and 24.34% of contact stress were reduced.

5.2 Recommendations

Tooth parameters and stress reduction can be significant if the dimension selection is appropriate. This investigation confirmed that it is possible to obtain critical section stress reduction with a correct gear tooth root fillet radius, face width and number of teeth selection during design work. It means that stress reduction results in better tooth root load capacity, micropitting resistance, prolongs gear service life.

5.3 Future work

In this thesis work optimization of contact stress and bending stress is studied for different face width, numbers of teeth, and root fillet radius effect condition, other influencing factor are not studied. So this work was restricted to the specified cases. However, this paper can be extended to other situation listed below. Further numerical method investigations should be conducted on:

- Effect of temperature in surface fatigue of gears.
- Contact mechanics in gears under lubricated condition and its effect in surface fatigue.
- Fracture mechanics approach to study surface fatigue related to initiation and propagation of cracks.
- The effect of surface finish on the contact stress of gears.

REFERENCES:

1. Kahraman, A, Kharazi, A.A, Umrani, M, A "Deformable body dynamic analysis of planetary gears with thin rims "Journal of sound and vibration 262(2003) 752-768
2. Lin, Tengjiao, Ou, H, Li, Runfang, "A finite element method for 3D static and dynamic contact/impact analysis of gear drives"- *C*omputer Methods in Applied Mechanics and Engineering, Volume 196, issue 9- 12 (February 1, 2007), p. 1716-1728
3. K.Mao, "gear tooth contact analysis and its application in the reduction of fatigue wear"*262(2007) 1281-1288*
4. Chabert, G.T Dang Tran ,R.Mathis " An Evaluation of Stresses and Deflection of Spur Gear tooth under strain"-Journal of Engineering for Indusry,1974,pp 85-93
5. Wilcox, L.E, Gear tooth stresses-Machine Design, No 23, 1978, pp88- 92
6. F. K. Choy, M. J. Braun, V. Polyshchuk, J. J. Zakrajsek, D. P.Townsend, and R.F. Handschuh, "Analytical and experimental vibration analysis of a faulty gear system"- 1994 Fall Technical Workshop, the American Gear Manufacturers Association, St. Louis, Missouri, October 24-26, 1994
7. Robert Basan, Marina Franulovic, and Bozidar Krizan. "Numerical model and procedure for determination of stresses in spur gear teeth flanks"-XII International Conference on Mechanical Engineering-Nov 2008.
8. M.S.Hebbal, V.B. math, B.G. Sheeparamatti- "A Study on reducing the root fillet stress in spur gear using Internal Stress Relieving Feature of Different Shapes"- International Journal of Recent Trends in Engineering, Vol.1, no.5, may 2009.
9. Hsiang H. Lin, Professor, Department of Mechanical Engineering. The University of Memphis. "Compact Design for non-standard spur gear" Journal of Mechanical, aerospace and Industrial Engineering-Volume2.
10. Marunic.G University of Rijeka, Croatia "Tooth root stress modifying factors of webbed gears".
11. Simon, Vilmos, "FEM Stress Analysis in Hypoid Gears", Mechanism and Machine Theory, vol. 35, pp. 1197-1220, 2000.
12. Spitas,V.,Costopoulos, Th. and Spitas**, C**., "Increasing the Strength of Standard Involute Gear Teeth with Novel Circular Root Fillet Design", American Journal of Applied Sciences, vol. 2, No. 6, pp. 1058-1064, 2005.

13. Hiremagalur, Jagannath and Ravani, Behram, “Effect of Backup Ratio on Root Stresses in Spur Gear Design”, Mechanics Based Design of Structures and Machines, vol. 32, No. 4, pp. 423-440, 2004.
14. Guingand, M., de Vaujany, J. P., and Icard, Y., “Analysis and Optimization of the Loaded Meshing of Face Gears”, Journal of Mechanical Design, vol. 127, pp. 135-143, 2005.
15. Moriwaki, I., Fukuda, T. Watabe, Y., Saito, K., “Global Local Finite Element Method (GLFEM) in Gear Tooth Stress Analysis”, Journal of Mechanical Design, vol. 115, pp. 1008-1012, 1993.
16. S.Sankar. Muthusamy Natraj, „Profile Modification- A Design approach for increasing the Tooth Strength in Spur Gear‟ International Journal of Advance Manufacturing Technology, Springer,2011,55:1-10.
17. Jinliang Zhang, Zongde Fang, Xuemei Cao, Xiaozheong Deng, “The modified pitch cone design of the hypoid gear: Manufacture, stress analysis and experiment tests”-Mechanism and Machine Theory 42(2007) 147-158
18. Negash A., Analysis of Stresses in Helical Gears by Finite Element Method, M.Sc., thesis, Addis Ababa University Faculty of Technology, Addis Ababa 2007.
19. Wei Z., Stresses and Deformations in Involute Spur Gears by Finite Element Method ,M.Sc. thesis, University of Saskatchewan, Canada, October, 2004.
20. Tsay, C.B., and Fong, Z.H., Computer Simulation and Stress Analysis of Helical Gears with Pinions Circular arc teeth and Gear involute teeth, Mechanics of Machine Theory, 26, pp.145-154, 1991.
21. Ooi Jong Boon analysis and optimization of portal axle unit using finite element modelling and simulation M.sc Thesis Universiti Tunku Abdul Rahman, January 2013
22. G. Mallesh, Dr. V B Math, Ashwij, Prabodh Sai Dutt R, Rajendra Shanbhag, “Effect of Tooth Profile Modification In Asymmetric Spur Gear Tooth Bending Stress By Finite Element Analysis”, 13th National Conference on Mechanisms and Machines, 2009.
23. Budynas, R. G., & Nisbett, J. K. (2008). Shigley's Mechanical Engineering Design. New York: McGraw-Hill.
24. Shigley, J.E., and Mischke, C.R., Standard Handbook of Machine Design, McGraw-Hill, USA, 1996.

25. H. Hertz, Miscellaneous Papers by H. Hertz, Jones & Schott, Macmillan, London, 1986.
26. Norton, R.L., and Machine Design: An Integrated Approach, New Jersey: prentice-Hall Inc. 1996.
27. V. Senthil Kumar, D.V. Muni, G. Muthuveerappan “Optimization of asymmetric spur gear drives to improve the stresses of spur gear”, 2012.
28. Strukturlabor, Finite Element Modeling with ANSYS,2010
29. Vivek Karaveer , Ashish Mogrekar,2013," Modeling and Finite Element Analysis of Spur Gear " International Journal of Current Engineering and Technology, ISSN 2277 - 4106, Vol.3, No.5
30. Ruben D.Chacon , Luis J Andueza " analysis of stress due to contact between spur gears " Advances in Computational Intelligence, Man-Machine Systems and Cybernetics, ISBN: 978-960- 474-257-8,2005
31. Ali Raad Hassan, 2009,"Contact Stress Analysis of Spur Gear Teeth Pair", World Academy of Science, Engineering and Technology 58
32. Seok-Chul Hwanga, Jin-Hwan Lee b, Dong-Hyung Lee c, Seung-Ho Hana, Kwon-Hee Lee a,2013," Mathematical and Computer Modeling " Mathematical and Computer Modelling 57 (2013) 40–49
33. Bharat Gupta, Abhishek choubey,2012," CONTACT STRESS ANALYSIS OF SPUR GEAR " ,International Journal of Engineering Research & Technology (IJERT), ISSN: 2278-0181, Vol. 1 Issue 4, June -2012
34. Faydor L. Litvin and Alfonso Fuentes, Gear Geometry and Applied Theory, Second Edition, Cambridge University Press, New York, USA, 2004.
35. David Guyonneau, Emmanuel Mermoz, Stress optimization and study of the sensitivity to geometric variations of a spur gear tooth profile, Journal of Mechanics & Industry 14, 31–41 (2013)
36. Lihua Guo, Gear Tooth Contact Stress Analysis in Spur Gears By Finite Element Meshing Simulation, University of Alberta ,Alberta November 2008
37. K. D. Dadhaniya,K. P. Hirpar, MR. K. M. VYAS, Effect of Pressure Angle on Bending Stress and Deformation of Asymmetric Spur Gear Using FEA, IOSR Journal of Engineering Apr. 2012, Vol. 2(4) pp: 616-620

38. Shreyash D Patel, finite element analysis of stresses in involute spur & helical gear, the University of Texas at Arlington December 2010
39. Shuangwen Sheng, Wind Turbine Micropitting, national laboratory of the U.S. Department of Energy, Technical Report NREL/TP-500-46572 ,February 20 10
40. Suzuki Y, Trend of Transmission and Gear Technology, 2004.
41. Yammati Murali Mohan et al, Spur gear optimization by using genetic algorithm, International Journal of Engineering Research and Applications, Feb, 2012.
42. Shimamura, S. and Noguchi, Y., 1965. An Analytical Determination of Speed Factor in Gear Designs,. Proceedings of the Second SESA International Congress on Experimental Mechanics, Washington, D.C.
43. Vijayarangan, S. and Ganesan, N., 1993. A Study of Dynamic Stresses in a Spur Gear Under a Moving Line Load and Impact Load Conditions by a Three-dimensional Finite Element Method. Journal of Sound and Vibration, 162 (1), pp. 185-189.
44. Chong, T. H., Bar, I., 2001, “Multi objective optimal Design of Cylindrical Gear Pairs for the Reduction of Gear Size and Meshing Vibration”, JSME International Journal, Vol. 44, No. 1, pp 291-292.
45. Estrin, M., 1980, “Optimization of Tooth Proportion for a Gear Mesh”, ASME paper 80-C2/DET-101.
46. Gay, C.E., 1970, “How to Design to Minimize Wear in Gears”, Machine Design, Vol.42, Nov.26, pp.92-97.
47. Osman, M.,Sankar,S.,Dukkipati, R. V., 1978, “Design Synthesis of a Multi-Speed Machine Tool Gear Transmission Using Multi parameter Optimization”, ASME Journal of Mechanical Design, Vol.100, No. 2, pp. 303-310.