COHERENCE AND NONEQUILIBRIUM STEADY STATE OF A SIMPLE BROWNIAN HEAT ENGINE

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In partial Fulfilment of the Requirements for the
Degree of Master of Science in Physics

By

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The undersigned hereby certify that they have read and recommend to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled "**Coherence and nonequilibrium steady state of a simple Brownian heat engine**" by **Belete Regassa** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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To my parent.

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Abstract

Overdamped motion of a Brownian particle in a ratchet potential with space dependent temperature is studied. Explicit analytic expressions for particle steady state velocity, diffusion coefficient and Péclet number which characterizes the transport are derived. Their dependencies on temperature profile, tilting force, the shape of the potential and barrier height are analyzed. We show that the asymmetry parameter of the potential along with temperature profile can enhance or suppress effective diffusion coefficient compared to that of free diffusion. The Péclet number is also shown to be very sensitive to temperature profile and asymmetry of the potential. At moderate values of asymmetry parameter an optimized transport with respect to temperature profile and barrier height is observed. We show that our model exhibits a pronounced coherence in a wide range of model parameters.

We also propose a systematic way of studying the mobility of a particle described by Langevin equation with nonuniform temperature in the absence of bias. It is shown that for a symmetric potential, mobility increases monotonically and saturates to one for large values of temperature profile. Based on our formulation, we also study the effective temperature of the model. It is shown that for a symmetric potential, the effective temperature of the system is greater than that of the temperature of the cold region of the ratchet potential.

Chapter 1

Introduction

The study of the interplay of noise and nonlinear dynamics in systems under nonequilibrium conditions has generated wide interdisciplinary interests in the last two decades. Noise or fluctuations, which are normally considered to be a hinderance, are found to play an active constructive role in nonequilibrium systems. In fact this constructive role of noise (as opposed to the convectional wisdom of its destructive or its disorganizing role) has become a new paradigm in the study of complex systems [1].

Brownian motors are systems that exploit the nonequilibrium fluctuations that are present in the medium to generate directed flow of Brownian particles in the absence of any net external force or bias [2, 3]. Recently, such Brownian motors have been studied extensively because they are believed to share common features with biological motors. Biological motors convert chemical energy into macroscopic work, i.e., they transport cargo efficiently with high reliability at room temperature [4].

The wonder of molecular machines is that they function without a conventional energy source. The molecular motors work by harnessing the force of random, thermal motions, or Brownian motion, and apply it to induce the particles to move in coordinated fashion. In effect, such a motor uses the random impacts of the molecules in solution to create a directed motion. By selecting for impacts that are productive and locking out motions that are counter productive, a molecular motor utilizes the

In recent times there has been a renewed interest in the study of transport properties of Brownian particles moving in periodic potential, with special emphasis on coherent transport and giant diffusion [13, 15]. The phenomenon of coherent or optimal transport is complimentary to the enhanced diffusion, wherein one is mainly concerned with transport current with minimal dispersion [14]. Compared to free diffusion coefficient, D is suppressed in the presence of periodic potential. However, in a nonequilibrium case, i.e., in the presence of bias, it has been recently shown that D can be made arbitrarily large (giant diffusion) compared to the bare diffusion, in the presence of periodic potential. A giant amplification of diffusion up to fourteen order of magnitude was predicted in Refs. [15]. This enhancement at low temperature takes place near the critical threshold (at which deterministically running solution sets in). The reason for this enhancement has been attributed to the existence of instability between locked and running solutions. This enhancement decreases as we move away from the critical field in either direction.

Another new effect, a non-monotonic behavior of the diffusion coefficient and coherence level of the transport of Brownian particles as a function of temperature, was found in Ref. [14]. Similar anomalies were observed in systems with spatially periodic temperature [17]. Recently it has also been shown that non-homogenous dissipation can induce enhancement and suppression of the diffusion as a function of temperature, as well as an increase of the coherence level of the Brownian motion in tilted symmetric periodic potential [13].

The transport of particles is characterized by an average motion in the direction of the bias and the counteracting spreading effect due to the presence of noise. While the drift may be desired, the dispersion is unwanted but - especially in case of a subcritical bias - has inevitable side effect. Coherence or optimal transport of Brownian particles refers to the case of large mean velocity accompanied with minimal diffusion. It can be best quantified by the non-dimensional Péclet number, i.e., a proper ratio of mean velocity to effective diffusion coefficient. The transport is most coherent when this number is maximum. Péclet number greater than two implies coherence in the transport [16]. The Péclet number of some of the models like flashing and rocking ratchets shows low coherence of transport with $Pe \simeq 0.2$ and $Pe \simeq 0.6$ [16], respectively.

Another study on symmetric periodic potentials along with spatially modulated white noise showed a coherent transport with Péclet number less than three [17]. In the same study a special kind of strongly asymmetric potential is found to increase Pe to twenty in some range of physical parameters.

The motion of the particle in a potential that has a minimum and maximum per period is mainly determined by two time scales; noise driven escape from a potential minima over the barrier along the bias, followed by the relaxation into the next minima. The former depends strongly on temperature and the later weakly on the noise strength and has a small variance. It is possible to obtain coherent transport in the parameter regime at which the traversal time across the two consecutive minima in a washboard potential is dominated by the relaxation time.

Another related discipline of current interest which has been a longstanding problem is the development of a coherent theoretical picture of nonequilibrium phenomena.

The statistical properties of fluctuations at equilibrium are described by equilibrium statistical mechanics. This has been established through experimental measurements carried out to test the theoretical predictions of statistical mechanics. In contrast to equilibrium case, there is no known general principle of determining the statistical properties of fluctuations under nonequilibrium conditions.

Equilibrium statistical mechanics has been probably one of the most important achievement during the last century. The notion most intimately connected to equilibrium is temperature. It is operationally defined by the so called Zeroth Law of Thermodynamics, which states that when two systems are in thermal equilibrium with a third one, then they are in thermal equilibrium with each other. This allows one to define temperature as a signature of the equivalence class defined by mutual thermal equilibrium. This property makes possible the use of test systems called "thermometers", to decide whether any two systems will or will not remain in thermal equilibrium when brought into contact. When two systems are not in mutual equilibrium, the direction of the energy flow between them is determined by the Second Law of Thermodynamics.

Nowadays, physicists are working extensively to understand the behavior of systems that are far from equilibrium both experimentally [24, 25] and theoretically by studying the nonequilibrium Langevin equation [26, 27]. These systems are ubiquitous in nature. In view of that, many efforts are currently aiming at achieving a coherent theoretical picture of nonequilibrium phenomena. Indeed many real systems persist out-of-equilibrium practically forever.

There are also many nonequilibrium states that settle down to nonequilibrium steady states (NESSs) if one condition, such as the strength of an external driving force or the chemical potential at boundary, is controlled. In such NESSs, the statistical properties of fluctuations can be elucidated through an approach that seeks to determine how equilibrium fluctuations are modified under the influence of nonequilibrium conditions. Macromolecules and colloids of the order of nanometers to submicrometers suspended in an aqueous solution provide an ideal ground to study the foundation of nonequilibrium statistical mechanics because nonequilibrium effects become more significant as the system size decreases [26].

In this thesis, we study two independent but closely related problems; the coherence of directed and diffusive motion of Brownian particle in the presence of the tilt and the level of deviation of the Fluctuation Dissipation Theorem (FDT) from equilibrium for non-uniform medium without bias. For both problems, we assume the temperature which the particle experiences to vary between hot and cold over a spatial period. We also assume the shape of the potential the particle experiences to be spatially periodic.

The rest of the work is organized as follows. In chapter two we introduce some preliminary concepts and derive general expressions for the quantities of interest. As we are dealing with two distinct interrelated problems, we divide this chapter into two parts. Part one of the chapter is devoted to the derivation of general expressions for current and effective diffusion coefficient in non-uniform medium having arbitrarily periodic potential and temperature profiles using the method introduced by P. Reimann et. al [15]. Along with this, the definition of the Péclet number is given. In part two, we limit our discussion to homogenous medium with static tilting force and derive the mobility (the response of the particle's velocity to external static force). The concept of the effective temperature to nonequilibrium systems, which allows the extension of ideas from equilibrium statistical mechanics such as, the fluctuation-dissipation relation, to nonequilibrium system will also be introduced in terms of differential mobility and effective diffusion coefficient of the same model. In chapter three, taking a tilted-ratchet potential with temperature profile which is piecewise constant but periodic, we derive analytic expressions of steady state current and effective diffusion coefficient. With these at hand, analytic expression of the Péclet number is given. Moreover, analytic expressions for effective temperature and differential mobility for the same model system where temperature is uniform are derived. In chapter four, we study the quantities of central interest in terms of model parameters and propose a systematic way of studying the differential mobility and effective temperature of non-uniform medium. In the last chapter, we present the conclusion and future directions of our work.

Chapter 2

Driven ratchets and their characterization

In this chapter we study two kinds of ratchet models: tilted-thermal ratchet and tilted ratchet (We use the term tilted-thermal ratchet for ratchet systems whose temperature vary with position in the presence of the bias while reserving the term tilted ratchet for homogeneous medium in the presence of the bias.). In the first section we consider motion of a Brownian particle in a tilted-thermal ratchet model and derive general expressions for its current density and effective diffusion coefficient. In the second section, we consider Brownian motion in a tilted ratchet model and derive its differential mobility and effective temperature. For both models, we assume the potential the particle experiences to be periodic and the coefficient of friction of the medium to be high and constant.

2.1 Tilted-thermal ratchet model

Using the Langevin equation or equivalently the Smoluchwski equation that describes the motion of Brownian particle in a tilted-thermal ratchet model, we derive general expressions for its steady state current and probability densities. In the next section, we first introduce some basic concepts like the Einstein diffusion coefficient and the moments of the mean first passage time (MFPT). Using MFPT and some reasonings, we derive a closed form expression for effective diffusion coefficient. In the last section, we provide a brief description of coherency of the motion of the Brownian particle and the definition of Péclet number.

2.1.1 Derivation of current density

We consider a Brownian particle of unit mass moving in a one dimensional highly viscous medium under the influence of an external total potential, $V(x)$. The coefficient of friction of the medium is taken to be constant while the temperature varies with position. The dynamics governing the particle's motion is

$$\ddot{x} + V'(x) = -\gamma\dot{x} + \sqrt{2k_B T(x)\gamma}\xi(t), \tag{2.1}$$

where x is the position of Brownian particle, k_B is the Boltzmann's constant and $\xi(t)$ is a Gaussian white noise with zero mean and correlation

$$\langle\xi(t)\xi(s)\rangle = \delta(t-s), \tag{2.2}$$

$T(x)$ is the temperature of the medium at position x, γ is the viscous friction coefficient, $\langle ... \rangle$ indicates the (nonequilibrium) average over a statistical ensemble of realization in Eq.(2.1) and the prime denotes a spatial derivative. The left-hand side of Eq.(2.1) describes the deterministic, conservative part of the dynamics, and the right-hand side accounts for the effects of the thermal environment-viscous damping and a fluctuating force modelled by thermal noise $\xi(t)$. On time scales larger than the inverse friction coefficient, γ^{-1}, one can in most practical cases consider the over damped limit of the Langevin equation. This in turn corresponds to the adiabatic elimination of the fast variable, velocity, from the equation of motion by putting $\ddot{x} = 0$ for a homogeneous medium. In contrast, for the case of inhomogeneous medium the above method of elimination does not work and Sancho et al. [18] has derived a proper prescription for eliminating fast variables. The corresponding over damped Langevin equation for the Brownian particle in an inhomogeneous medium is given by

$$\dot{x} = \frac{-V'(x)}{\gamma} + \sqrt{\frac{2k_BT(x)}{\gamma}}\xi(t). \tag{2.3}$$

Following the method of S. Sasa [19], we get the corresponding Smoluchwski equation for the probability density $P(x,t)$ of a particle being at x at time t as

$$\frac{\partial P(x,t)}{\partial t} = \frac{1}{\gamma}\frac{\partial}{\partial x}\left((V'(x))P(x,t) + \frac{\partial}{\partial x}(k_BT(x)P(x,t))\right). \tag{2.4}$$

For periodic functions $P(x,t)$ and $T(x)$ with periodicity L, the steady state probability density and current can be obtained as follows.
We can write the Smoluchowski equation, Eq.(2.4), as

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}J(x,t), \tag{2.5}$$

where

$$J(x,t) = -\frac{1}{\gamma}\Big(\frac{\partial}{\partial x}(k_BT(x)P(x,t)) + V'(x)P(x,t)\Big) \tag{2.6}$$

is the probability current density at position x and time t. The steady state solution, $P^s(x)$, of the Smoluchowski equation implies that the probability current density, J, is constant and Eq.(2.6) becomes

$$J^s = -\frac{1}{\gamma}\Big(\frac{d}{dx}(k_BT(x)P^s(x)) + V'(x)P^s(x)\Big). \tag{2.7}$$

For simplicity, we set $J^s = J$. To find analytical expression for J, we set periodic boundary conditions, i.e.,

$$P^s(x+L) = P^s(x),\ T(x+L) = T(x). \tag{2.8}$$

We also assume the total potential to be

$$V(x) = V_0(x) - Fx, \tag{2.9}$$

which consists of the periodic part $V_0(x)$ with period L, and a non-periodic part, $-Fx$, where F is a static tilting force. This total potential, $V(x)$, is a corrugated plane; the average slope is determined by the external force, F. For large force, F, the maximum and the minimum of the periodic part of the potential disappear, whereas for intermediate and small force minima do occur. Thus, the Langevin equation Eq.(2.1), describes the motion of the Brownian particles along such a corrugated plane. When the static tilting force is zero and the medium is homogeneous, (both spatially and temporally), regardless of the shape of $V_0(x)$, detailed balance is valid, i.e., the probability of a particle to make a thermally activated transition from x to $(x + \triangle x)$, equals the probability for the reverse step from $(x + \triangle x)$ to x, for any arbitrary x and $x + \triangle x$. Hence, no net transport can be observed in thermal equilibrium. Thus, system inhomogeneity or static tilting force is believed to keep the system away from thermal equilibrium and hence the detailed balance condition is broken.

By defining

$$\psi(x) = \int_0^x \frac{(V_0'(x') - F)}{k_B T(x')} dx' \tag{2.10}$$

when $0 < x < L$, and multiplying Eq.(2.7) by $e^{(\psi(x))}$, one can obtain the probability density $P^s(x)$ as

$$P^s(x) = \frac{e^{(-\psi(x))}}{k_B T(x)} \Big(P(0) k_B T(0) - \gamma J \int_0^x e^{(\psi(x'))} dx' \Big), \tag{2.11}$$

where $P(0)$ and $T(0)$ are, respectively, the probability density and temperature at $x = 0$. Following the boundary conditions of Eq.(2.8), along with the normalization condition

$$\int_0^L P^s(x) dx = 1$$

and some straight forward algebras, we obtain the expressions for the steady state current and probability densities as

$$J = \frac{(1 - e^{\psi(L)})}{\gamma \int_0^L dx \frac{e^{(-\psi(x))}}{k_B T(x)} \int_x^{x+L} e^{(\psi(x'))} dx'} \tag{2.12}$$

and

$$P^s(x) = \frac{\frac{e^{(-\psi(x))}}{k_B T(x)} \int_x^{x+L} \gamma e^{(\psi(x'))dx'}}{\gamma \int_0^L dx \frac{e^{(-\psi(x))}}{k_B T(x)} \int_x^{x+L} e^{(\psi(x'))} dx'}, \tag{2.13}$$

respectively.

Interchanging the order of integral in the denominator of Eq.(2.12), one can easily see that

$$J = \frac{(1 - e^{\psi(L)})}{\int_0^L dx \frac{\gamma e^{(\psi(x))}}{k_B T(x)} \int_{x-L}^{x} e^{(-\psi(x'))} dx'}. \tag{2.14}$$

By denoting,

$$I_+(x) = \frac{\gamma e^{(\psi(x))} \int_{x-L}^{x} e^{(-\psi(x'))} dx'}{k_B T(x)} \tag{2.15}$$

and

$$I_-(x) = \frac{\gamma e^{(-\psi(x))} \int_x^{x+L} e^{(\psi(x'))} dx'}{k_B T(x)}, \tag{2.16}$$

the steady state current density J can be written as

$$J = \frac{(1 - e^{(\psi(L))})}{\int_0^L dx I_\pm(x)}, \tag{2.17}$$

while the steady state probability density takes the form

$$P^s(x) = \frac{I_-(x)}{\int_0^L dx I_\pm(x)}. \tag{2.18}$$

Hence, the steady state velocity v is given as

$$v = \frac{(1 - e^{(\psi(L))})}{\int_0^L \frac{dx}{L} I_\pm(x)}. \tag{2.19}$$

Note that the periodicity of the probability density implies that the functions $I_\pm(x)$ are periodic functions of L. One can use Eq.(2.19) to find the drift velocity for any potential and temperature profile having the same period.

2.1.2 Derivation of effective diffusion coefficient

As a second quantity of central interest, we shall derive a closed form expression for the effective diffusion coefficient which is defined as

$$D = \lim_{t\to\infty} \frac{\langle x(t)^2\rangle - \langle x(t)\rangle^2}{2t}. \tag{2.20}$$

First in the absence of the periodic part of the potential $V_0(x)$ and taking constant temperature, by setting

$$x(t) = y(t) + x(0) + \frac{tF}{\gamma} \tag{2.21}$$

with the initial conditions

$$y(0) = 0,\ \dot{y}(0) = \dot{x}(0) - \frac{F}{\gamma}, \tag{2.22}$$

from Eq.(2.1), one can obtain

$$\langle x(t)\rangle = \frac{1}{\gamma}(\dot{x}(0) - F)(1 - exp(-\gamma t)) + x(0) + \frac{Ft}{\gamma} \tag{2.23}$$

and

$$\langle \Delta\, x^2(t)\rangle = \frac{2k_BT}{\gamma}(t - \frac{3}{\gamma} - \frac{1}{2\gamma}exp(-2\gamma t) + \frac{2}{\gamma}exp(-\gamma t)), \tag{2.24}$$

so that

$$\langle \dot{x}(t)\rangle = \frac{F}{\gamma}$$

and

$$D = \frac{k_BT}{\gamma} \equiv D_0,$$

where D_0 is the Einstein diffusion coefficient.

The evaluation of the effective diffusion coefficient in the presence of both arbitrary tilting force and periodic potential can also be done analytically by making use of the moments of the first passage time. The n-th moment of the first passage time from an arbitrary point x_0 to $x_0 + L$ is given by

$$T_n(x_0 \to x_0 + L) = \langle t^n(x_0 \to x_0 + L) \rangle \tag{2.25}$$

and satisfies the ordinary differential equation [20]

$$\frac{1}{2}\frac{B(x)d^2(T_n(x))}{dx^2} + \frac{A(x)d(T_x)}{dx} = -nT_{n-1}(x), \tag{2.26}$$

where $A(x)$ and $B(x)$ denote the drift and diffusion terms, respectively, in the Langevin Eq.(2.3). According to Eq.(2.3), Eq.(2.26) can be written as

$$\frac{d^2T_n(x)}{dx^2} - \frac{(V_0'(x) - F)}{k_BT(x)}\frac{dT_n(x)}{dx} = -n\gamma\frac{T_{n-1}(x)}{k_BT(x)}. \tag{2.27}$$

With the boundary condition

$$T_n(x_0) = T_n(x_0 + L) = 0,$$

which holds true for the case of absorbing barriers at $x = x_0$ and $x = x_0+L$, we obtain analytic expression for the moments of the first passage time in terms of quadrature after some manipulations as

$$\begin{aligned} T_n(x_0 \to x_0 + L) &= N\Big(\int_{x_0}^{x} dx' e^{(\psi(x'))} \int_{x}^{x_0+L} dz e^{(\psi(z))} \int_{x_0}^{z} \frac{T_{n-1}(y)}{k_BT(y)} e^{(-\psi(y))} dy - \\ &\int_{x}^{x_0+L} dx' e^{(\psi(x'))} \int_{x_0}^{x} dz e^{(\psi(z))} \int_{x_0}^{z} dy \frac{T_{n-1}(y)}{k_BT(y)} e^{(-\psi(y))}\Big), \end{aligned} \tag{2.28}$$

where $N = \frac{\gamma n}{\int_{x_0}^{x_0+L} dy e^{(\psi(y))}}$. By changing the prescription on the boundary condition and assuming that the motion takes place on the interval $[x_0, x_0 + L]$ and taking the barrier at x_0 to be reflecting while that at x_0+L to be absorbing, we obtain a compact form for the n-th moment of the first passage as

$$T_n(x_0 \to x_0 + L) = n\gamma \int_{x}^{x_0+L} dy e^{(\psi(y))} \int_{x_0}^{y} \frac{T_{n-1}(z)}{k_BT(z)} e^{(-\psi(z))} dz. \tag{2.29}$$

Furthermore, suppose that the motion is on an infinite range. Then, there is a relatively high probability of finding the particle on the left or the right of x_0+L, but not near x_0+L. For this to hold the potential is assumed to have only one minimum and one maximum per period. Then we ask, what is the mean escape time from the left hand well? By this we mean what is the n-th moment of the first passage time from x_0 to x, where x is in the vicinity of x_0+L? We use Eq.(2.29), with

$$x_0 \to -\infty,$$

$$x \to x_0$$

so that the moments of the first passage time are given by the well known closed analytical recursion

$$T_n(x_0 \to x_0+L) = n\gamma \int_{x_0}^{x_0+L} dy e^{(\psi(y))} \int_{-\infty}^{y} \frac{T_{n-1}(z \to x_0+L)}{k_B T(z)} e^{(-\psi(z))} dz, \quad (2.30)$$

where $n = 1, 2, 3, ...$ and $T_0(z \to x_0+L) = 1$. Note that the convergence of the above equation is guaranteed by our assumption that $F > 0$.

We now come to the first main point of this section: the derivation of exact expression of the effective diffusion coefficient, D, in terms of the mean first passage time $T_1(x_0+L)$ and the so called the first passage time dispersion

$$\begin{aligned} \triangle\, T_2(x \to x_0+L) &= \langle t^2(x_0 \to x_0+L)\rangle - \langle t(x_0 \to x_0+L)\rangle^2 \\ &= T_2(x_0 \to x_0+L) - (T_1(x_0 \to x_0+L))^2, \quad (2.31) \end{aligned}$$

where x_0 is any arbitrary point.

An exact expression for effective diffusion coefficient for a Brownian particle moving in a periodic potential aud uniform medium has recently been derived by P. Reimann et. al. [15], B. Lindner et. al. [14], S. Sasa et.al. [21], and very recently by K. Sasaki and S. Amari [22] using different approaches. Here we generalize this result to the case of arbitrary periodic potential and iuhomogeneous medium. The inhomogeneity arises due to the spatial variation of temperature.

With the arguments given by P. Reimann et al. [15], one can have

$$\langle \dot{x} \rangle = \frac{L}{T_1(x_0 \rightarrow x_0 + L)} \tag{2.32}$$

and

$$D = \frac{L^2}{2} \frac{\Delta\, T_2(x_0 \rightarrow x_0 + L)}{(T_1(x_0 \rightarrow x_0 + L))^3}. \tag{2.33}$$

The proof of these relations follows from the consideration of the special case with a potential $V_0(x) = 0$. The evaluation of the mean first passage time and its dispersion can be easily done and one recovers the relation we have seen so far. But from the identical distribution of the first passage time, this holds true for any periodic potential $V_0(x)$. By introducing Eqs.(2.30) and (2.31) into Eqs.(2.32) and (2.33), respectively, analytic expression for D is recovered within the restriction that $F > 0$. To keep the convergence of the moment, we first re-write the z integral in Eq.(2.30) with $n = 1$ as

$$\int_{-\infty}^{x} dz \frac{e^{(-\psi(z))}}{k_B T(z)} = \sum_{l=0}^{\infty} \int_{x-L}^{x} dz \frac{e(-\psi(z - lL))}{k_B T(z - lL)}. \tag{2.34}$$

Using this expression in Eq.(2.30) and following the periodicity conditions of the temperature and potential, and substituting it back into Eq.(2.32) one can easily obtain Eq.(2.17) for the current density. The proof for D can be done similarly; using a periodic condition of the function $I_{\pm}(x)$ and some tedious mathematical manipulations, we obtain

$$D = \frac{\int_{x_0}^{x_0+L} \frac{dx}{L} \frac{k_B T(x)}{\gamma} I_{\pm}(x) I_+(x) I_-(x)}{(\int_{x_0}^{x_0+L} \frac{dx}{L} I_{\pm}(x))^3}. \tag{2.35}$$

For homogeneous medium, Eq.(2.35) reduces to the one obtained by other authors.

2.1.3 Coherency of Brownian particle

To characterize the transport of a Brownian motor in the presence of fluctuation, consider the task of moving a particle a certain distance within a defined time interval,

t. For Brownian motors, we only know the average velocity of the motion for a successive realization of the process and for a fixed time there will be a spread in the distance over which the motor carries out transport. We therefore look for a way of quantifying a motor's coherency. Knowing only the dispersion of the particle (actually it is very small for a reliable transport) along a certain distance at a time t does not fully characterize the transport of the Brownian motors because it does not discriminate between motors of different velocities. A coherent parameter that incorporates both the velocity and the spread is the Péclet number, which is a measure of linear transport as compared to diffusion. The Péclet number is defined as

$$Pe = \frac{l\langle \dot{x} \rangle}{D}, \qquad (2.36)$$

where l is a characteristic length, $\langle \dot{x} \rangle$ is the average velocity and D is the effective diffusion coefficient. The choice of the length l in Eq.(2.36) is essentially arbitrary and usually taken to be some characteristic length scale of the system. In the context of Brownian motors, the spatial period of the ratchet potential has been used as the reference length scale l [13, 14, 23].

The greater the Péclet number, the greater the coherence of the Brownian transport. Here we use L as the period of the ratchet potential so that the time taken for a Brownian particle to travel a distance L is given as $\tau = \frac{L}{\langle \dot{x} \rangle}$ and the spread of the particle in the same time is given as $\langle \Delta\, x^2 \rangle = 2D\tau$. For a reliable transport we require

$$\langle \Delta\, x^2 \rangle = 2D\tau < L^2.$$

This implies that

$$Pe = \frac{L\langle \dot{x} \rangle}{D} > 2$$

for the coherence transport.

Boltzmann factor, i.e., inhomogeneity in friction does not affect the stationary (equilibrium) properties of a system but it does affect the dynamical (nonequilibrium) properties such as relaxation time. In contrast, temperature inhomogeneity (even in the absence of the bias), changes the relative stability of otherwise locally stable states (thereby creating new steady states). Temperature non-uniformity has been shown to give rise to net current in a periodic potential in the absence of an applied bias. Spatially varying friction, $\gamma(x)$, of the medium alone, however, can not give the net probability current J in such a potential.

For a medium with uniform temperature, the response of the particle's velocity to a small external force F in the linear response regime is given by a quantity called mobility and defined as

$$\mu_0 = \lim_{F \to 0} \frac{v^u}{F},$$

where v^u is the steady state velocity derived in Eq.(2.42). This is one form of the Fluctuation Dissipation Theorem (FDT). However, for NESS far from equilibrium, the FDT does not hold, and recent investigation has proposed a way of determining quantitatively the extent to which the FDT is violated by adding a slowly varying poteutial which plays the role of an effective thermometer to the system under consideration [21]. They defined the effective temperature Θ of the system for NESS as the ratio of the effective diffusion coefficient D^u to the differential mobility $\mu_d^u = \frac{dv^u(F)}{dF}$ and shown that it is not equal to the temperature of the bath, except in the linear response regime, where the fluctuation dissipation theorem holds.

For uniform medium, the differential mobility μ_d^u can be derived by differentiating Eq.(2.42) with respect to F, and the result can be expressed in a succinct form as

$$\mu_d^u(F) = \frac{1}{\gamma} \frac{\int_0^L \frac{dx}{L} I_-^u(x) I_+^u(x)}{\left(\int_0^L \frac{dx}{L} I_\pm^u(x)\right)^2}. \tag{2.44}$$

Here we remark that a particle in a medium, with space dependent temperature profile (even without bias), experiences an effective potential (or net bias). This, in turn, is able to shift the stable points and introduces noise induced transitions. So far, to our knowledge, there is no known exact analytical expression of differential mobility and hence effective temperature for such system. However, in chapter four, we study the differential mobility of non-uniform medium systematically by comparing two equivalent dynamics: the steady state velocity that arises due to the presence of bias with that of due to medium inhomogeneity (non-uniform temperature).

Chapter 3

Exactly solvable driven ratchet models

In the previous chapter, we have derived closed form expressions for current density and effective diffusion coefficient of a Brownian motor (for both uniform and non-uniform medium) irrespective of the shape of the potential (except that it be periodic) in terms of quadratures. In this chapter, taking a particular model of a Brownian motor, we get closed form expressions for steady state velocity, effective diffusion coefficient, Péclet number, differential mobility and effective temperature in terms of the model parameters.

3.1 The model

The model consists of a Brownian particle moving in a highly viscous medium in a non-uniform but periodic temperature background. The total potential $U(x)$ which the particle experiences is assumed to contain a periodic part of sawtooth shape and a linearly changing part. The periodic part of the potential is denoted by $V(x)$ and its barrier height V_0 is defined as the difference between the maximum and the minimum values of $V(x)$. This potential is piecewise linear having a single minimum and a single maximum per period of length L. Within the interval $0 \leq x \leq L$, we take the

Exchanging the order of integration in the above equation, we get

$$I_+(x) = \frac{\gamma e^{(\psi(x))}}{k_B T(x)}\Big(\int_0^x dy e^{(-\psi(y))} + e^{(\psi(L))}\int_x^L dy e^{(-\psi(y))}\Big). \tag{3.12}$$

Substituting Eq.(3.12) into Eq.(2.14), we also find the steady state velocity. To proceed, observe that $I_+(x) = I_{+1}(x) + I_{+2}(x)$ where we have derived

$$I_{+1}(x) = \frac{\gamma}{k_B T_1} e^{(\frac{U_1 x}{L_1 k_B T_1})}\Big(\alpha\alpha_2 - \frac{L_1 k_B T_1}{U_1}\alpha_0 e^{(-\frac{U_1 x}{L_1 k_B T_1})}\Big) \tag{3.13}$$

and

$$I_{+2}(x) = \frac{\gamma}{k_B T_2} e^{(-\frac{U_2(x-L_1)}{L_2 k_B T_2})}\Big(\alpha\alpha_1 + \frac{L_2 k_B T_2}{U_2}\alpha_0 e^{(\frac{U_2(x-L_1)}{L_2 k_B T_2})}\Big). \tag{3.14}$$

Substituting the above two equations in Eq.(2.14), we arrive at Eq.(3.10) which has been already derived from $I_-(x)$.

The effective diffusion coefficient can also be computed analytical from Eq.(2.35). By breaking the integrand into two parts, we have

$$D = \frac{\int_0^{L_1}\frac{dx}{L}\frac{k_B T_1}{\gamma} I_{\pm 1}(x) I_{-1}(x) I_{+1}(x) + \int_{L_1}^{L}\frac{dx}{L}\frac{k_B T_2}{\gamma} I_{\pm 2}(x) I_{-2}(x) I_{+2}(x)}{(\int_0^L \frac{dx}{L} I_\pm(x))^3}. \tag{3.15}$$

Using Eqs.(3.8) and (3.13) in the first integrand, Eqs.(3.9) and (3.14) in the second integrand of the right hand side of Eq.(3.15) and making some arrangement, we obtain after cumbersome calculations

$$\frac{D}{D_0} = \frac{\chi}{\beta^3}, \tag{3.16}$$

where

$$D_0 = \frac{k_B T}{\gamma},$$

and the expressions for β and χ are given by Eqs.(A.12) and (A.14), respectively, in Appendix A. Similarly, with the notations given in the appendix A, the steady state velocity is expressed as

$$\frac{v}{v_0} = \frac{\alpha_0}{\beta}, \tag{3.17}$$

where,

$$v_0 = \frac{k_B T}{\gamma L}.$$

Hence, the dimensionless Péclet number takes a simple form

$$Pe = \frac{\alpha_0 \beta^2}{\chi}. \tag{3.18}$$

In a similar fashion, we derive analytic expression for differential mobility by following the same procedure as above. Reducing Eqs.(3.8), (3.9), (3.13) and (3.14) for homogeneous medium (i.e., when T_1=T_2) and substistung them back into Eq.(2.44), we arrive at

$$\mu_d^u = \frac{1}{\gamma} \frac{\varrho}{\beta(\sigma = 0)^2}, \tag{3.19}$$

where ϱ is given by Eq.(A.13). Thus, using Eqs.(3.19) and (3.16), we write the expression of the effective temperature Θ as

$$\frac{\Theta}{T} = \frac{\chi(\sigma = 0)}{\beta(\sigma = 0)\varrho}. \tag{3.20}$$

Chapter 4

Results and discussion

In the previous chapter, we obtained analytic expressions for the quantities of interest. Here we present the results and discussion of the work. We first consider tilted-thermal ratchet model and explore the way the steady state velocity, effective diffusion coefficient and Péclet number behave as parameters describing the model vary. We then compare the steady state velocity of the tilted ratchet model with that of thermal ratchet model and find the equivalent static force of tilted ratchet to the 'noise intensity level' σ of the thermal ratchet (Here and in the rest of the chapters, the term noise intensity level refers to the excess thermal intensity level σ of the hot reservoir as compared to the cold reservoir). Finally, we study the differential mobility and effective temperature of thermal ratchet.

4.1 Current, effective diffusion coefficient and Péclet number

The effective potential $U(x)$ exhibits a tilt as a function of the model parameters. By looking at this potential one can infer only the direction of the current and not its magnitude. A close analysis of Eq.(3.17) reveals that the transport of the particle is always in the direction of the bias (for σ or $f \neq 0$).

It is a fact that the magnitude of velocity increases with the increase in the magnitude of static tilting force. For $f \gg u$ or $u \to 0$, the velocity becomes linearly dependent on f. This is because large value of tilting force overshadows the effect of potential barrier, and hence the particle does not notice the presence of the potential barrier. This behavior of steady state velocity as a function of static tilting force is shown in Fig.(4.1). The dependence of steady state velocity on the static tilting force for different values of σ is also studied in this figure. The steady state velocity monotonically increases with the increasing values of tilting force. It is observed that for fixed λ (λ= .9), the increase in noise intensity increases the steady state velocity. Having a nonzero σ causes the velocity to start from a nonzero value when $f = 0$.

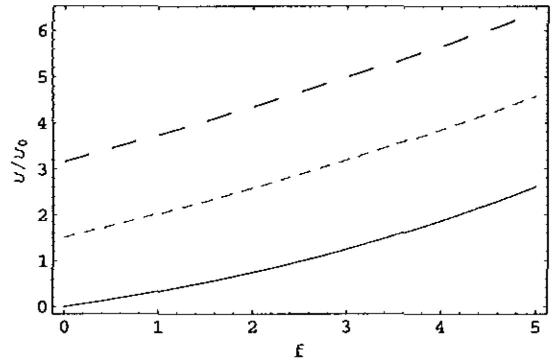


Figure 4.1: Steady state velocity versus f in unit of v_0 for u=4 and λ=.9. Solid line, small dashed lines aud large dashed lines represent σ=0, 1 and 2, respectively.

The behavior of the velocity as a function of noise intensity level for different values of asymmetry parameter is given in Fig.(4.2). For the parameters in the figure, we observe that the velocity increases monotonically for small values of σ and tends to saturate for large values of σ. This is because the addition of noise facilitates the escape of the particle trapped in the potential along the bias, and hence increases its velocity along the bias. For high noise intensity levels, the particle does not notice

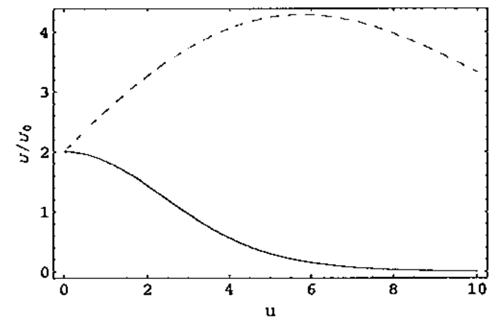


Figure 4.3: Steady state velocity versus u in unit of v_0 for f=2 and λ=.1. Solid line and dashed lines represent σ=0 and 2, respectively.

barrier height.

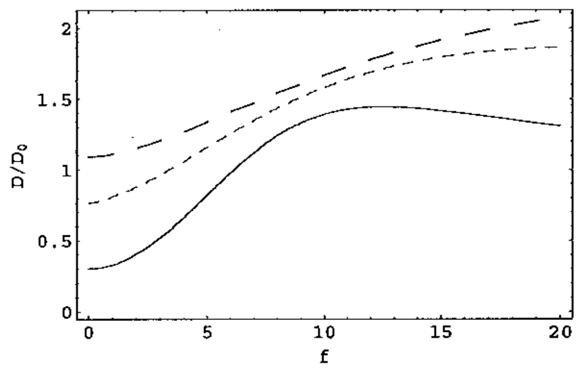


Figure 4.4: Effective diffusion coefficient versus f in unit of D_0 for u = 4 and λ = .5. Solid line, small dashed lines and large dashed lines represent σ=0, 1 and 2, respectively.

Next we explore the behavior of the effective diffusion coefficient in terms of the model parameters. In the absence of the tilt (i.e., when $f = 0$) and a homogeneous medium ($\sigma = 0$) we obtain a simple expression for the effective diffusion coefficient given by

$$\frac{D}{D_0} = \frac{u^2}{2(cosh(u) - 1)},$$

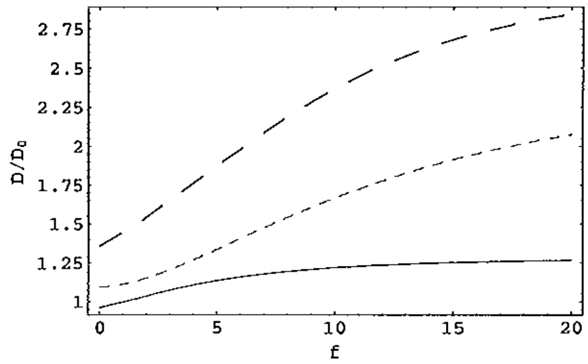


Figure 4.5: Effective diffusion coefficient versus f in unit of D_0 for $u = 4$ and $\sigma = 2$. Solid line, small dashed lines and large dashed lines represent λ=.1, .5 and .9, respectively.

We next examine the dependence of D on the noise intensity level σ. In Fig.(4.6), we depict the effective diffusion coefficient versus σ. For the parameters in the figure, we observe that the effective diffusion coefficient increases monotonically with the intensity of noise. This is because the motion of the particle in a periodic sawtooth potential is determined by two time scales: First, noise driven escape over potential barrier from one of the minima along the bias and the second time scale being the relaxation into the next potential well. It has been stated in the introduction that the second time scale is weakly dependent on noise strength and has a small variance as opposed to the first one. In the high 'temperature' regime, large thermal noise leads to large variance in the second time scale as the random motion of the particle both along and against the bias becomes equally important and hence diffusion increases for high value of noise intensity level as expected. For too high noise intensity the shape of potential becomes insignificant and diffusion tends to saturate.

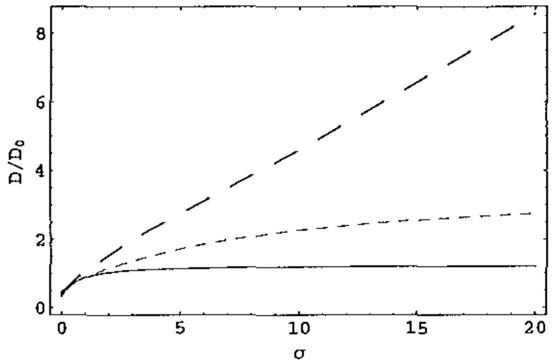


Figure 4.6: Effective diffusion coefficient versus σ in unit of D_0 for $u = 4$ and $f = 2$. Solid line, small dashed lines and large dashed lines represent λ=.1, .5 and .9, respectively.

Also plot of the effective diffusion versus the barrier height is shown in Fig.(4.7) for two values of noise intensity level σ. It is observed that for homogeneous medium ($\sigma = 0$),

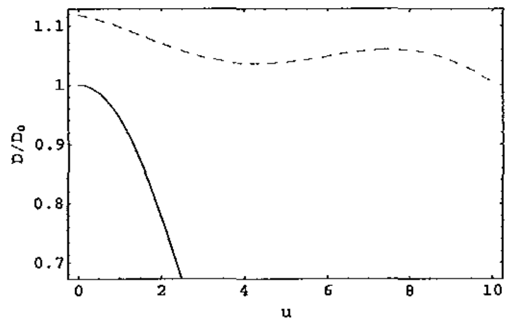


Figure 4.7: Effective diffusion coefficient versus u in unit of D_0 for $f = 2$ and $\lambda = .1$. Solid line and dashed lines represent σ=0 and 2, respectively.

diffusion equals to that of free diffusion when $u = 0$ and decreases as u becomes large. However for inhomogeneous medium, it exhibits maximum and minimum and eventually tends to zero for large values of the barrier height parameter u. In this case there is a region where the effective diffusion coefficient is greater than that of free diffusion.

Finally, before leaving this section, we investigate the relation between directed and diffusive motion of the model. The plot of Péclet number versus the dimensionless noise intensity level is shown in Fig.(4.8). We have analyzed the role of asymmetry by taking parameters mentioned in the figure caption. For these parameters we observe that increasing the asymmetry parameter decreases the Péclet number. In the regime of weak noise intensity level, i.e., when $\sigma \to 0$, we find Péclet number less than two. Hence in this regime, the transport is less coherent. For large asymmetry λ though neither v nor D shows maximum but the Péclet number attains a maximum at a finite value of σ indicating an optimal transport in this case. This is because for large asymmetry λ, the increase in drift velocity at small values of noise intensity level is more than that of the increase in diffusion while the reverse is observed at another extreme of noise intensity level. This maximum in Péclet number vanishes at small values of λ and the transport becomes more regular. It is interesting that for too small values of the asymmetry parameter, the steady state velocity and effective diffusion coefficient synchronize at large values of noise intensity level. For too large noise intensity σ and large asymmetry λ, the coherence becomes less. This is so because the ratchet mechanism is weakened, backward transitions (along the low noise slope) occur, and thus, the Péclet number has to drop. In this regime diffusion dominates over drift. At the optimal value of noise intensity level, the motion of the particle is mainly determined by two processes. These are the noise driven escape from the potential minimum via the right potential barrier followed by a relaxation into the next minimum. The relaxation time depends only weakly on the noise streugth and possesses a small variance. The latter fact leads to a certain regularity of the particle motion which accounts for the maximum of the Péclet number.

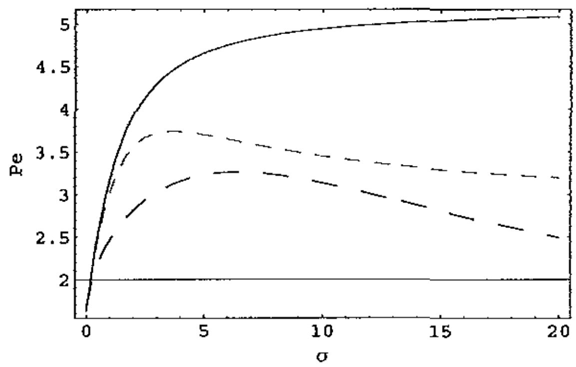


Figure 4.8: Péclet number versus σ for $u = 4$ and $f = 2$. Solid line, small dashed lines and large dashed lines represent $\lambda =.1, .5, .9$, respectively.

We also studied the variation of the Péclet number as a function of the dimensionless static tilting force f (see Figs.(4.9) and (4.10)). It is observed that for fixed λ and small f, increasing noise intensity level increases the coherence of the motion while at large values of f we observe the reverse.

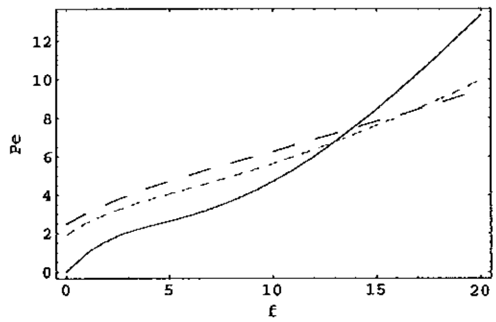


Figure 4.9: Péclet number versus f for $u = 4$ and $\lambda = .5$. Solid line, small dashed lines and large dashed lines represent σ=0, 1 and 2, respectively.

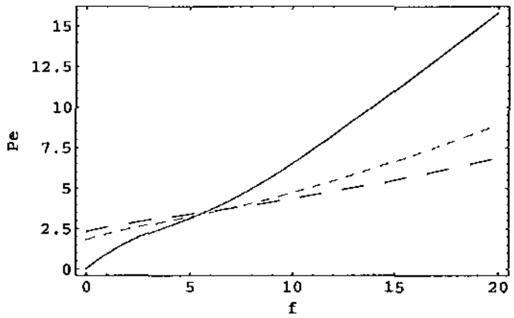


Figure 4.10: Péclet number versus f for $u = 4$ and $\lambda = .9$. Solid line, small dashed lines and large dashed lines represent σ=0, 1 and 2, respectively.

The Péclet number for homogeneous medium ($\sigma = 0$) is also cmpared with that of inhomogeneous medium in this section. It is observed in Fig.(4.11) that for homogeneous medium, the larger the λ, the more the Péclet number is. However, when the

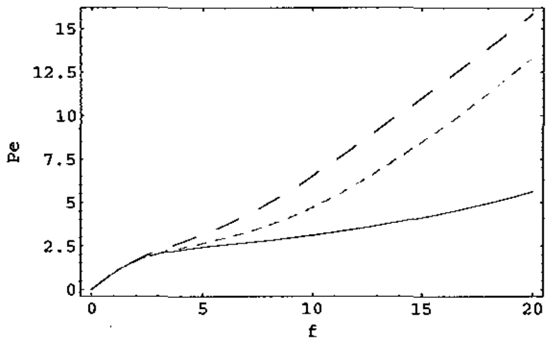


Figure 4.11: Péclet number versus f for $u = 4$ and $\sigma = 0$. Solid line, small dashed lines and large dashed lines represent λ=.1, .5 and .9, respectively.

medium is inhomogeneous, the reverse is observed (see Fig.(4.12)).

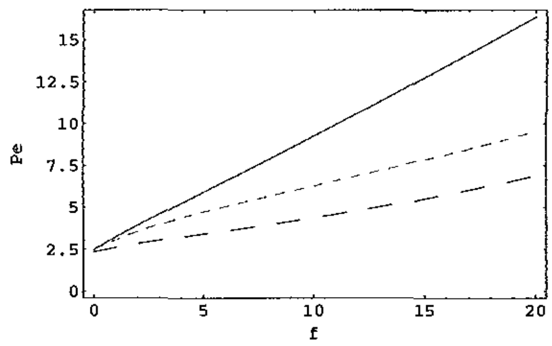


Figure 4.12: Péclet number versus f for $u = 4$ and $\sigma = 2$. Solid line, small dashed lines and large dashed lines represent λ=.1, .5 and .9, respectively.

Finally, the dependence of the Péclet number on barrier height is shown in Fig.(4.13). For homogeneous medium, Pe attains a maximum value at $u = 0$ and decreases monotonically as u increases. However for inhomogeneous ones, Pe takes an optimum value at a finite value of u. It can be seen from the plot that inhomogeneous medium is more reliable than that of homogeneous one at large values of u.

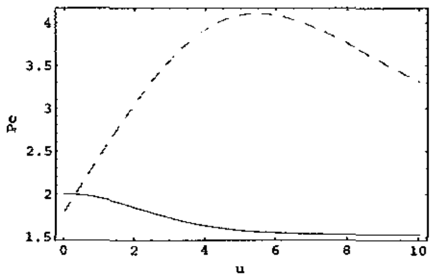
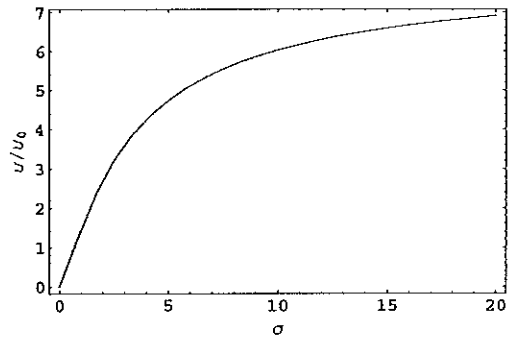


Figure 4.13: Péclet number versus u for $f = 2$ and $\lambda = .1$. Solid line and dashed lines represent σ=0 and 2, respectively.

4.2 Equivalent static force, mobility and effective temperature of thermal ratchet

In the state of thermal equilibrium, the fluctuation of the stationary particle is characterized by free thermal diffusion and mobility. This idea was first pointed out by Albert Einstein in 1905 and known in general as Fluctuation Dissipation Theorem (FDT). It is one of the deepest results of thermodynamics and statistical physics.
For systems out of equilibrium with homogeneous medium one can also recover FDT for NESSs by introducing an effective temperature as explained in the last section of chapter two. Also analytic expression for effective temperature was derived in Eq.(3.20). However, for non-homogeneous medium we have not yet succeeded in obtaining analytic expression for mobility and hence effective temperature. To this end we propose a systematic way of studying mobility and effective temperature of thermal ratchet model.
Our starting point is the exact analytic expression for the steady state velocity of tilted-thermal ratchet model given by Eq.(3.17). In this equation, if one assumes the medium to be homogeneous ($\sigma = 0$) one would recover tilted ratchet. Hence static tilting force is the sole source for the flow of particles. For such system, the plot of steady state velocity versus static tilting force is shown in Fig.(4.15). As it can be seen, steady state velocity increases linearly for large values of f since the effect of ratchet potential becomes less significant in this regime.
On the other hand, if the medium is inhomogeneous (with $f = 0$), we obtain a thermal ratchet. In this case, inhomogeneity of the medium, i.e., noise intensity level σ plays a crucial role in breaking the condition of detailed balance. We depict the resulting steady state velocity versus noise intensity level in Fig.(4.14). The values of other parameters given in the figure caption are kept to be the same as those in Fig.(4.15). For such system steady state velocity saturates for large values of σ. A



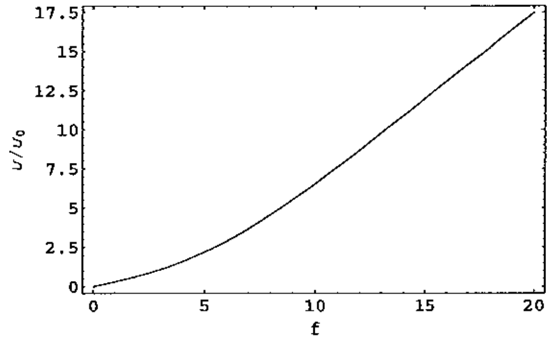


Figure 4.14: Steady state velocity versus σ for f=0, u=4 and λ=.5.

Figure 4.15: Steady state velocity versus f for σ=0, u=4 and λ=.5.

close look at these two figures (i.e., Figs.(4.14) and (4.15)) reveals that for every value of noise intensity level σ, there is a corresponding equivalent static tilting force f_{eq} which results in the same steady state velocity. We could not succeed in obtaining direct relation between them analytically because of mathematical complexity of the expression. However for specific system parameters, we obtained the relationship between them by matching their equivalent steady state velocity values and the result is shown in Fig.(4.16). The plot shows that the equivalent static tilting force f_{eq}

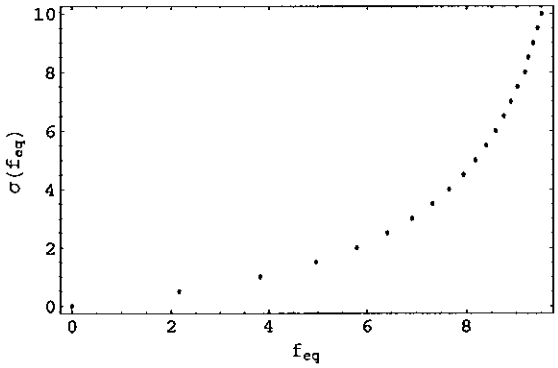


Figure 4.16: Noise intensity level versus f_{eq} for u=4, and λ=.5.

saturates for large values of noise intensity level σ.

In an attempt to study the mobility of tilted-thermal ratchet, we also compared steady state velocity at fixed value of f with that at f_{net} where $f_{net} = f + f_{eq}$. In the limit $\sigma \to 0$ or $\sigma \to \infty$ we observe $\frac{v(\sigma,f)}{v(f_{net})} \to 1$. In the intermediate regime $\frac{v(\sigma,f)}{v(f_{net})}$

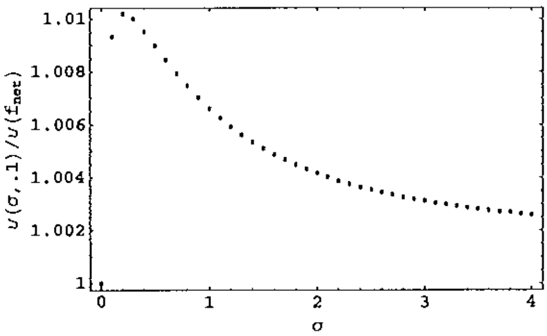


Figure 4.17: Steady state velocity of tilted-thermal ratchet model at $f = .1$ to that of at f_{net} versus σ for u=4 and λ=.5.

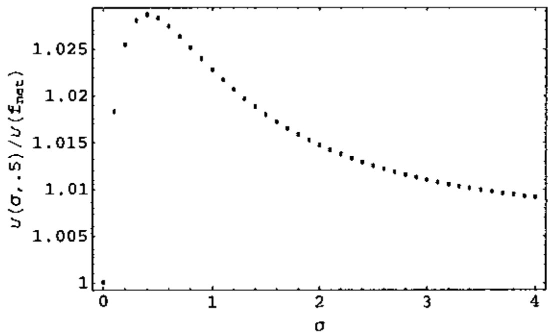


Figure 4.18: Steady state velocity of tilted-thermal ratchet model at $f = .5$ to that of at f_{net} versus σ for u=4 and λ=.5.

is greater than one and exhibits a maximum as shown in Figs.(4.17) and (4.18) for two values of static tilting force. This inequality implies that one cannot derive the mobility of tilted-thermal ratchet by simply taking the partial derivative of its steady state velocity and evaluate at f_{net} except in the asymptotic cases. However, for thermal ratchet we obtain differential mobility by taking into consideration the mobility of tilted ratchet derived in Eq.(3.19). We proceed from the steady state velocity of tilted ratchet and find the corresponding noise intensity level of the static tilting force. We then use this noise intensity level in place of static tilting force in Eq.(3.19) and evaluate for specific parameters. Carrying out these procedures repeatedly, we depict the mobility versus the noise intensity level in Fig.(4.19). It is observable in the figure that mobility increases monotonically for small values of noise intensity level and saturates for high values of it. This is because as σ keeps on increasing the particle sensing the existence of the potential barrier gets weaken and its motion is almost equivalent with that of free diffusive particles. At another extreme, i.e., when

$\sigma \to 0$, we observe mobility to be less than $\frac{1}{\gamma}$. In this regime the presence of the ratchet potential plays a significant role in suppressing the mobility with respect to $\frac{1}{\gamma}$. In the absence of this potential, the ratio of mobility to $\frac{1}{\gamma}$ is exactly one when $\sigma \to 0$.

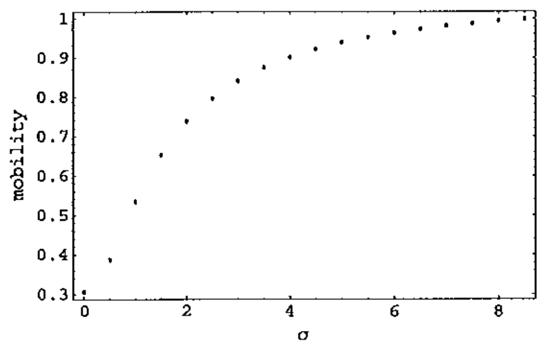


Figure 4.19: Mobility in unit of $\frac{1}{\gamma}$ versus σ for u=4, λ =.5 and f=0.

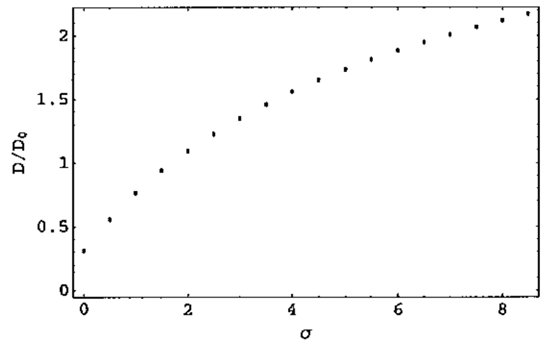


Figure 4.20: Effective diffusion coefficient in unit of free thermal diffusion versus σ for u=4, λ=.5 and f=0.

Finally, with these at hand along with the plot given in Fig.(4.20) we studied the extent to which FDT is violated for thermal ratchet model. This violation of FDT arises due to inhomogeneity of the medium. It is quantified by a quantity called effective temperature Θ whose analytic expression for tilted ratchet is given by Eq.(3.20). For thermal ratchet, the plot of effective temperature versus the noise intensity level is shown in Fig.(4.21). It is interesting that when $\sigma = 0$, the effective temperature equals to the value of the cold temperature. This indicates the system is at equilibrium and Einstein's relation holds pretty well. However for $\sigma \neq 0$, $\frac{\Theta}{T}$ is different from one. In fact for the parameters in the figure caption, it is always greater than one. This implies that the increase in σ drives the system away from equilibrium. From the behaviors of the plots in Figs.(4.19) and (4.20), we expect that $\frac{\Theta}{T}$ saturates for large values of σ though this is invisible here.

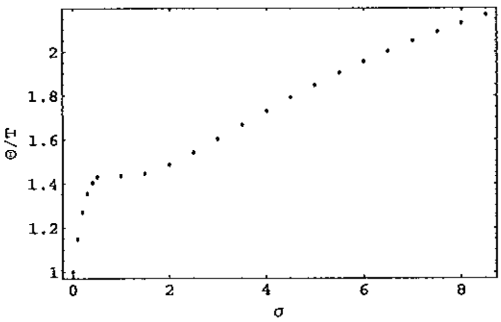


Figure 4.21: Effective temperature in unit of T versus σ for u=4, λ=.5 and f=0.

Chapter 5

Summary and conclusion

In this work, we first considered an overdamped motion of a Brownian particle on a periodic potential and non-homogeneous medium and derived general expressions for steady state velocity, effective diffusion coefficient and Péclet number. Based on our formulations, we obtained analytic expressions for the quantities of interest for a particle moving on a tilted piecewise linear periodic potential where the background temperature alternates between hot and cold over a period. We then considered uniform medium and found general and analytic expressions for differential mobility and effective temperature of the model. Finally, we studied the dependence of the quantities of interest on the model parameters.

The static tilting force with alternately placed hot and cold temperature leads to a noise induced current or transport. The noise induced transport is always accompanied by a diffusive spread which makes the transport to be less reliable. Also diffusion coefficient of the particle is very sensitive to the shape of the linear piecewise potential and the magnitude of the excess temperature of the medium. It is shown that large values of asymmetry parameter λ and noise intensity level σ amplify diffusion with respect to free diffusion. Moreover, for a fixed noise intensity level, the increase of the length of the hot region of ratchet potential increases diffusion.

We also studied the reliability or coherence of transport. As explained in the introduction, for net drift to overcome diffusion at a distance of one unit, Péclet number shouldn't be less than two. Consistent with this, we found Péclet number greater than two in a wide range of model parameters implying that the transport is reliable. At moderate to strong asymmetry parameter the Péclet number displays a maximum with respect to noise intensity level σ. A maximum in Pélect number is also observed as a function of barrier height for inhomogeneous medium which is absent for homogeneous ones. The basic mechanism for enhancement can be summarized as follows: In regions where the particle moves quasi-deterministically downhill, the noise decreases; in regions where escape over a barrier is needed, the noise intensity is strongly increased such that the escape is fast. The asymmetry of the potential serves to separate the orders of the magnitude of the involved time scales. If the regular downhill motion takes on average a much longer time than the irregular escape versus the slope with strong noise, we obtain a large transport coherence. This is the case if the steeper slope is within the region of high noise intensity whereas fluctuations are low at other side.

We also studied the violation of FDT for a simple model where the medium is inhomogeneous without static tilting force. For this, we first obtained the differential mobility by finding the equivalent static tilting force of thermal noise for equivalent dynamics. For symmetric potential, the mobility increases monotonically with noise intensity and saturates to one for large values of it. We then studied the effective temperature of the model by using the definition given by S. Sasa [21] for homogeneous medium. We found that it is monotonically increasing with σ. This implies the increase in σ drives the system away from equilibrium. However, we haven't yet studied the behavior of effective temperature and mobility for the combined effect of tilting force and thermal noise. Work along this line requires further analysis and it is of future interest.

In conclusion, to the best of our knowledge, no previous study has addressed the diffusive and coherency properties of the model. Moreover, the method proposed to study the mobility as well as the effective temperature of inhomogeneous medium in the absence of the tilt is pertinent to our work. However, we should state that the properties of the steady state current density and energetics of the model were studied by [28] using different method.

Appendix A

Definition for parameters

The parameters we have taken for simplification of the original problem are given in this section. The temperature of the cold region T_2 is assumed to be T, and we equate

$$T_1 = T(1+\sigma),$$

where, $\sigma(\sigma \geq 0)$ is a dimensionless parameter.

The length of the hot region is scaled with respect to the period L, i.e.,

$$\lambda = \frac{L_1}{L}$$

and

$$1-\lambda = \frac{L_2}{L}.$$

In our analyses, we have used the dimensionless parameters:

$$u = \frac{V_0}{k_B T}, \tag{A.1}$$

$$f = \frac{LF}{k_B T}, \tag{A.2}$$

$$\alpha = \frac{\lambda(1+\sigma)}{u-\lambda f} + \frac{1-\lambda}{u+(1-\lambda)f}, \tag{A.3}$$

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