ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

OPTIMIZATION OF DESIGN OF TUNED MASS DAMPER FOR VIBRATION CONTROL OF FRAME STRUCTURES UNDER SEISMIC EXCITATION

A thesis submitted to the school of Graduate Studies in Partial fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering (Structures)

SEYOUM ALENE

November 2016

Addis Ababa

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SCHOOL OF GRADUATE STUDIES

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Advisor: Dr. SHIFFERAW TAYE

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POST GRADUATE PROGRAM

This is to certify that the thesis prepared by Seyoum Alene, entitled by: OPTIMIZATION OF DESIGN OF TUNED MASS DAMPER FOR VIBRATION CONTROL OF FRAME STRUCTURES UNDER SEISMIC EXCITATION and submitted in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering (Structural Engineering) complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

The mitigation of the dynamic response of buildings and structures to earthquakes is one of the fundamental aims within the design of vibration control devices. In this sense, Tuned Mass Damper (TMD) devices are generally conceived as useful and efficient means for the control of the dynamic response of structures and constructions, especially when considering ideal dynamic excitations. However, their optimum tuning and relevant performance in effectively reducing the seismic response of civil structures is currently an open topic, mostly due to the intrinsic nature of the passive device and the uncertainty and unpredictability of the earthquake event.

The present paper deals with determining of the optimum parameters of TMD to minimize dynamic response of a single and multi-storied building system. To optimize dynamic parameters of the TMD system for minimum top deflection of the structure, a Minmax Optimization Simulation analysis is used in a MATLAB environment. Analytical formulas and graphical charts are also developed to determine the optimal parameters of TMD. Three three-dimensional building models are modeled using SAP2000 to validate the optimum values of design variables and to examine the optimum installation location of TMD in elevation under seismic excitation. It is well recognized that the performance of a TMD is sensitive to the slight deviation of frequency ratio between the TMD and the structure.

Keywords: TUNED MASS DAMPER, OPTIMUM PARAMETERS, MINMAX OPTIMIZATION.

AUTHOR'S DECLARATION

I, the undersigned …**seyoum alene**……………………………… hereby declare that I am the sole author of this thesis. To the best of my knowledge this thesis contains no material previously published by any other person except where due acknowledgement has been made in accordance with the standard referencing practices. This thesis contains no material which has been accepted as part of the requirements of any other academic degree or non-degree program, in English or in any other language.

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Finally yet importantly, I would like to thank my parents, who taught me the value of hard work by their own example. I would like to share this moment of happiness with my mother. She rendered me enormous support during the whole tenure of my stay in school.

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LIST OF SYMBOLS

Symbols	Descriptions
a_g	Absolute ground acceleration
c	Viscous damping
c_d	Damper damping coefficient
f	Frequency ratio
f_{opt}	Optimized frequency ratio
H	Amplification of the pseudo static response
k	Stiffness of the structure
k_d	Damper stiffness
k_{dopt}	Optimum damper stiffness
m	Mass of the structure
m_d	Damper mass
$\bar{m}$	Mass ratio
$\bar{m}_{opt}$	Optimized mass ratio
p	Force loading applied to the primary mass
R_{opt}	Optimum response
t	Time
U	The relative displacement between the primary mass and the ground
U_d	The relative displacement between the primary mass and the damper
U_g	Absolute ground motion
δ	Phase angle between the response and the excitation
ξ	Damping of the structure
ξ_d	Damping Of TMD
ξ_{dopt}	Optimized Damping of TMD
ξe	Equivalent damping ratio
ρ	Dimensionless frequency ratio
ρ_d	Dimensionless frequency ratio of damper
Ω	Periodic frequency

ABBREVIATIONS

AMD	Active Mass Damper
CSS	Charged System Search
DV	Design Vector
DTMD	Doubly Tuned Mass Damper
EVOP	Evolutionary Operation
HMD	Hybrid Mass Damper
MDOF	Multi Degree of Freedom
MTMD	Multiple Tuned Mass Dampers
MTMFD	Multiple Tuned Mass Friction Dampers
OF	Objective Function
PED	Passive Energy Dissipation
PSD	Power Spectral Density
SATMD	Semi Active Tuned Mass Damper
SDOF	Single Degree of Freedom
SDOF	Single Degree of Freedom
TLCD	Tuned Liquid Column Dampers
TLD	Tuned Liquid Damper
TMD	Tuned Mass Damper
TMCS	Tuned Mass Control Systems
TSD	Tuned Sloshing Dampers

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

Urbanization, coupled with modern design and construction technologies, has resulted in taller and lighter structures. As an example, the world’s tallest man-made structure the Burj Kalifa tower stands a remarkable 828m from its base with an estimated weight in excess of 110,000 tonnes (Baker 2007). One of the trade-offs of building to larger heights is the susceptibility to vibration due to the inherent flexibility of the structure. When excited by environmental dynamic loads, such as wind, this could result in large amplitude motion at the top of the structure.

There are two significant negative effects from structural vibrations on building structures (Sain 2007). The first effect is the long term fatigue to structures due to the periodic dynamic loading. It is well established that the leading cause of material failure in building structures is due to fatigue. Building materials, such as metals, subjected to periodic loadings can develop fractures (Sain 2007). The presence of a fracture jeopardizes the structural integrity, and may inevitably lead to structural failure. Although the likelihood of failure in modern building structures is very low, the deterioration from fatigue affects the structural stiffness. A weakened structure will exhibit less structural stiffness which will result in larger amplitudes at the top of the structure when the structure is excited. Coincidentally, larger top floor displacement amplitudes yield greater stress on the structure-. Hence the deterioration rate of the structural members will increase over time (Foreman 1967). To account for damage caused by structural vibrations, the structure will require maintenance or reconstruction; both of which represent significant financial costs.

The second effect is the human perception from the induced motion. Humans are very perceptive to even minor vibrations. Sensitive people can perceive accelerations as low as 0.05g (Kareem 2007). Between 0.1g and 0.25g structural motions may affect an individual’s ability to work, and over the long term it may lead to motion sickness (Kareem 2007). Considering that the stress and strain from top-floor deflections are unlikely to lead to structural failure in modern buildings, serviceability considerations focus on the human perception and ability to perform the individual’s tasks effectively, rather than the strength of a structure. Buildings are, after all, built for humans to occupy, be it a living space or a work space.

Conventionally, structures have been designed to resist natural hazards through a combination of strength, deformability and energy absorption. This can be done by combining structural components such as shear walls, braced frames, moment resisting frames, diaphragms, and horizontal trusses, to form lateral load resisting systems. This approach also considers the shape of the building, since square or rectangular buildings perform better than other shapes, such as L, U, or T. The choice of material used in construction is also important, since ductile materials, such as steel, were found to perform better than brittle ones, such as brick. The soil beneath the structure is yet another factor that greatly influences structural vibration characteristics and amount of damage sustained. These structures may deform well beyond the elastic limit, in a severe earthquake. They may remain intact only due to their ability to deform inelastically, as their deformation results in increased flexibility and energy dissipation. Unfortunately, this deformation also results in local damage to the structure, as the structure itself must absorb much of the earthquake input energy. It is ironic that the prevention of the devastating effects from these natural hazards, including structural damage, is frequently attained by allowing certain structural damage.

Alternately, some types of structural protective systems may be implemented to mitigate the damaging effects of these environmental forces. These systems work by absorbing or reflecting a portion of the input energy that would otherwise be transmitted to the structure itself. One of the important concern is, most of the existing building has been constructed without earthquake resistant features and reconstruction of such buildings to resist earthquake as per conventional method may be uneconomical and replacing the damaged parts after an earthquake may prove to be cumbersome. Hence structural control methods may be the most viable solution to protect newly constructed and existing buildings from natural hazards. This type of vibration control mechanism can be adopted for the new buildings also to protect them from hazardous earthquakes.

1.2 STRUCTURAL CONTROL METHODS

The control of structures subjected to seismic excitation represents a challenging task for the civil engineering profession. The purpose of vibration control is two fold. One is to improve habitability during strong winds or in moderate earthquakes; the other is to prevent damage to main frame of the structure during severe earthquakes. Structural control methods are divided into four major classes such as passive, active, semi-active and hybrid control systems. They are described briefly in the following sections.

1.2.1 Passive Control Systems

A passive control system may be defined as a system which does not require external power source for operation and utilizes the motion of the structure to develop the control forces. The control forces are developed as a function of the response of the structure at the location of the passive control system. Systems in this category are very reliable since they are unaffected by power outages which are common during earthquakes. Since they do not inject energy into the system, they are unable to destabilize the structure and they have low maintenance requirements. Passive control systems are limited in that; they cannot deal with the change of either external loading conditions or usage pattern. The schematic representation of passive control system is shown in Figure 1.1.

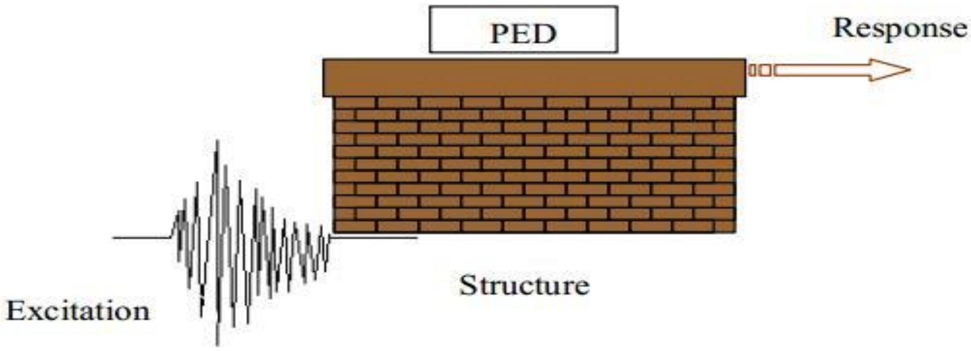


Figure 1.1 Structure with Passive Energy Dissipation (PED)

Passive control systems may be implemented in the form of tuned mass dampers, tuned liquid dampers, seismic isolation, metallic dampers, friction dampers, viscoelastic dampers and viscous fluid dampers.

1.2.2 Active Control Systems

An active control system as shown in Figure 1.2 may be defined as a system which typically requires a large power source for operation of electro hydraulic or electromechanical actuators which supply control force to the structure. It consists of

(i) Sensors located about the structure to measure either external excitations or structural response variable or both.

(ii) Devices to process the measured information and to compute necessary control forces needed based on a given control algorithm.

(iii) Actuators, usually powered by external sources to produce the required forces.

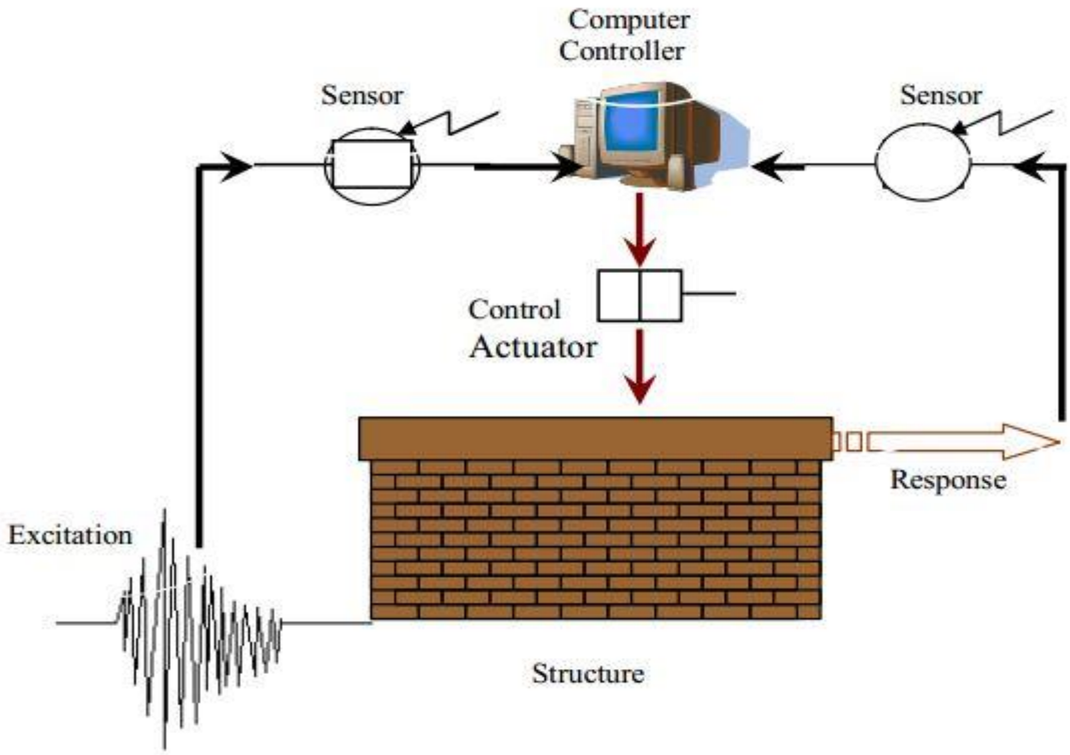


Figure 1.2 Structure with Active Control

The advantages of using active systems are enhanced effectiveness in response control, applicability to multi-hazard mitigation situations and selectivity of control objectives such as human comfort, structural safety etc. The main drawback of these systems is during power failures they become inactive and since they impart energy to the system tend to destabilize the system.

Active tuned mass dampers, active tendon systems and actuators / controllers are some types of active tendon systems.

1.2.3 Semi- active Control Systems

A semi- active control system may be defined as a system which typically requires an operation power source, but less than that of active system for its operation and utilizes the motion of the structure to develop the control forces, the magnitude of which can be adjusted by the external power source. It originates from a passive control system in that the control forces are developed as a result of the motion of the structure. The control forces are developed through appropriate (based on predetermined control algorithm) adjustment of the mechanical properties of the semi-active control system. As in an active control system, a controller monitors the feedback measurements and generates an appropriate command signal for semi-active device. A typical semi-active system is shown in Figure 1.3.

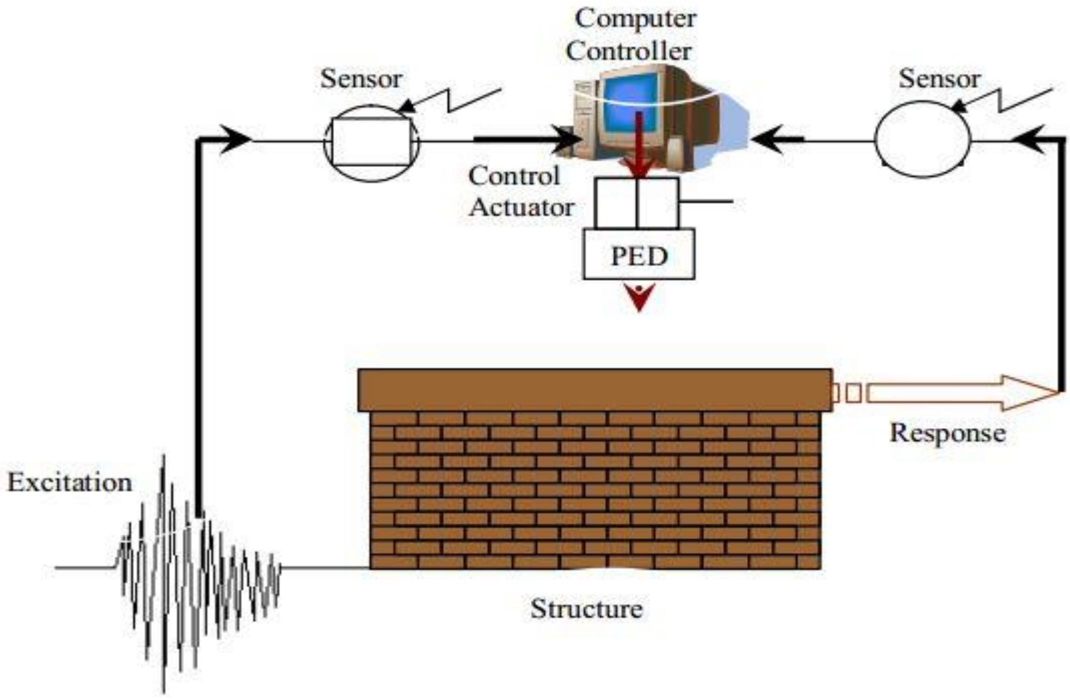


Figure 1.3 Structure with Semi-Active Control

The advantage of such system is that it can operate on battery power, which is critical during seismic events when the main power source to the structure may fail. These systems cannot inject mechanical energy into the structure and hence do not have the potential to destabilize the structural system. Examples of such devices are variable-orifice fluid dampers, variable stiffness dampers, controllable friction devices, smart tuned mass dampers and tuned liquid dampers, controllable fluid dampers and controllable impact dampers (Spencer Jr. and Nagarajaiah 2007).

1.2.4 Hybrid Control System

A Hybrid control system employs the combination of passive and active devices (Figure 1.4). Because multiple control devices are operating, hybrid control systems can alleviate some of the restriction and limitation that exist when each system is acting alone. Thus higher levels of performance may be achievable. The benefit of this system is that, in the case of power failure, the passive components of the control still offer some degree of protection. Hybrid systems are installed in the form of Hybrid mass damper system and hybrid base isolation. (Spencer Jr and Michael Sain 1997).

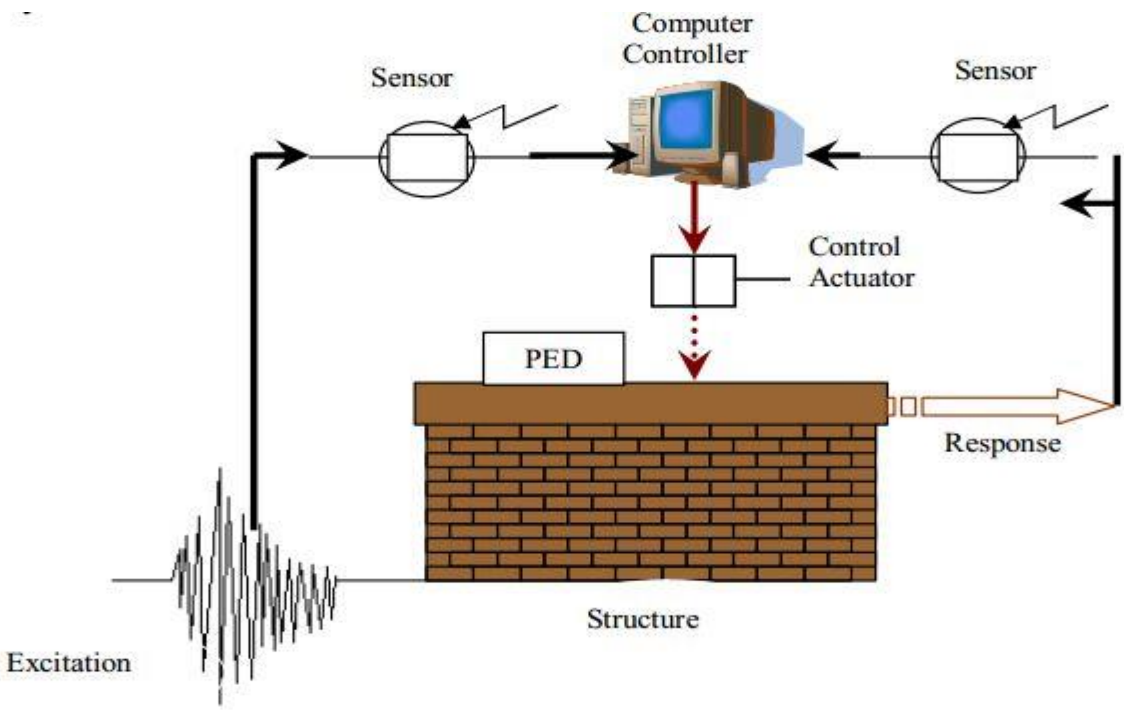


Figure 1.4 Structure with Hybrid Control

1.3 PASSIVE ENERGY DISSIPATION

A large number of passive control systems or PED devices have been developed and installed in structures for performance enhancement under earthquake loads. Commonly, auxiliary damping may be supplied through the incorporation of some secondary system capable of passive energy dissipation, for example, the addition of a secondary mass attached to the structure by a spring and damping element in order to counteract the building motion. Such passive systems (Soong and Dargush 1997) were embraced for their simplicity and ability to reduce the structural response.

1.3.1 Tuned Mass Dampers

One of the effective and reliable passive structural vibration control device is Tuned Mass Damper (TMD). TMD consisting of a mass, a damper, and a spring, is attached to the vibrating primary system for suppressing the undesirable vibrations induced by winds and earthquake loads. The natural frequency of TMD is tuned in resonance with the fundamental mode of the primary structure, so that a large amount of the structural vibrating energy is transferred to the TMD and then dissipated by the damping as the primary structure is subjected to external disturbances. Consequently, the safety and habitability of the structure are greatly enhanced. The TMD system has been successfully installed in slender skyscrapers and towers to suppress the wind induced structural dynamic responses, such as the CN Tower (535m) in Canada, John Hancock Building (sixty stories) in Boston, Center-Point Tower (305m) in Sydney, and the tallest building in the world, Taipei 101 Tower (101 stories, 504m) in Taiwan. From the field vibration measurements, it has been proved that TMD is an effective and feasible system in structural vibration control against high wind loads. Recently, numerical and experimental studies have been carried out to examine the effectiveness of TMD in reducing seismic response of structures. Numerical and experimental results showed that the effectiveness of the TMD in reducing the response of the same structure under different earthquakes or of different structures during the same earthquake is significantly different. This implies that there is a dependency of the attained reduction in response on the characteristics of the ground motion that excites the structure. This response reduction is large for resonant ground motion and it diminishes as the dominant frequency of the ground motion gets further away from the structure's natural frequency to which the TMD is tuned.

TMD can be categorized as Passive, Active and Hybrid (Figure 1.5) systems based on the mode of application, detailed as follows.

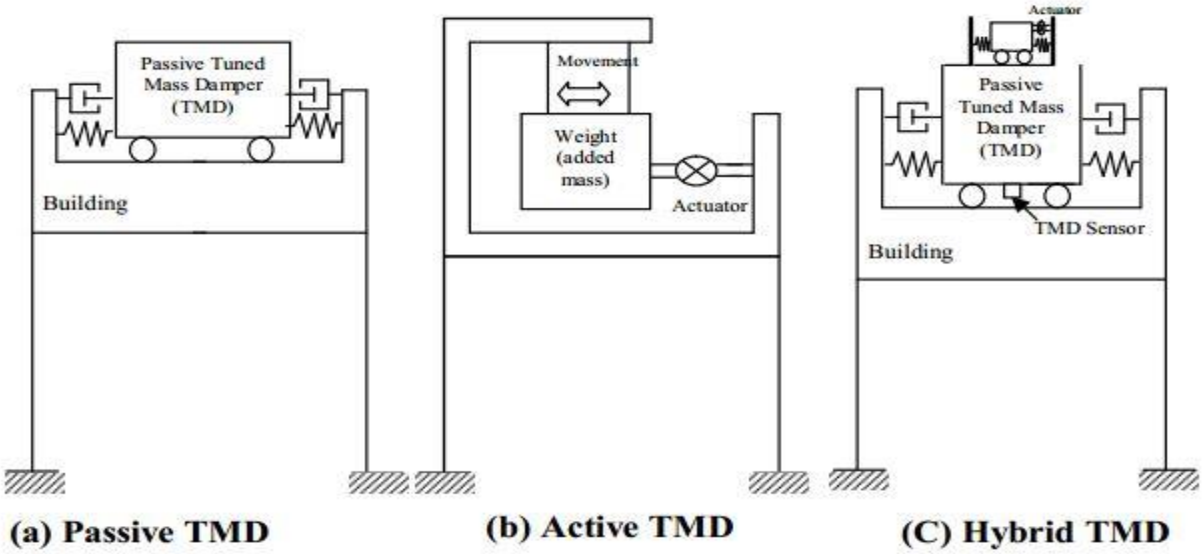


Figure 1.5 Classification of TMD

1.3.1.1 Passive TMD

A passive TMD or tuned vibration absorber is basically an energy dissipation device that in its simplest form consists of a mass that is attached to a structure (primary system) with spring and damper elements (Figure 1.5.a). Due to the damper, energy dissipation occurs whenever the mass of the TMD oscillates with non-vanishing displacement or velocity relative to the primary system. This is achieved by transferring as much energy as possible from the primary system to the TMD by a careful tuning of the natural frequency and damping ratio of the TMD. Since the mass of the TMD is significantly smaller than that of the primary system, transferring energy from the primary system to the TMD generates a great relative oscillation of the mass of the TMD. A TMD operates efficiently only in a narrow frequency band. This behavior is closely related to the mechanism of energy transfer from the primary system to the TMD. High energy transfer arises whenever the natural frequency of the TMD is tuned to the natural frequency of the primary structure.

One of the earliest applications of this type was installed in June 1977 in the 244 m Hancock Tower in Boston shown in Figure 1.6. Two TMD's were installed at opposite ends of the 58th floor in order to counteract the torsional motion. Each unit measured about 5.2x5.2x1 m and was essentially a steel box filled with lead, weighing 300 tons, attached to the frame of the building by shock absorbers. The system is activated at 3 milli-g's of motion at which time the steel plates, upon which the devices rest, are lubricated with oil so that the weights were free to slide. The system can reduce the building's response by 50%. Another pioneering application of TMD's has been in use New York's 278m Citicorp Building, shown in Figure1.7, since 1978. The system, measuring 9.14 x 9.14 x 3.05 m, consists of a 410 ton concrete block with two spring damping mechanisms, one for the north-south motion and one for the east-west motion, was installed in the 63rd floor. The system was included in the overall design due to the building aspect ratio and dynamic features.

Figure 1.6 Boston's Hancock Tower

Figure 1.7 Citicorp Center

1.3.1.2 Active mass dampers

The active-mass-damper (AMD), the most commonly used active control device, is similar to the tuned mass damper, since it also uses a mass-spring-damper system. It does, however, include an actuator that is used to position the mass at

each instant, to increase the amount of damping achieved and the operational frequency range of the device. The first implementation of this control method and of active control in general was performed in 1989, in the Kyobashi Seiwa building, Tokyo, Japan, by the Kajima Corporation (Figure 1.8) (Soong and Spencer 2002). Also employing this control method, the Applause Tower (Hankyu Chayamachi building) in Osaka, Japan, uses a rooftop heliport as the AMD mass, as seen in Figure 1.9. The Riverside Sumida Central Tower, in Tokyo, Japan, the Nanjing Communication Tower, in Nanjing, China, and the Shin-Jei building in Taipei, Taiwan, constitute other examples of structures employing active mass dampers (Spencer Jr. and Nagarajaiah 2003).

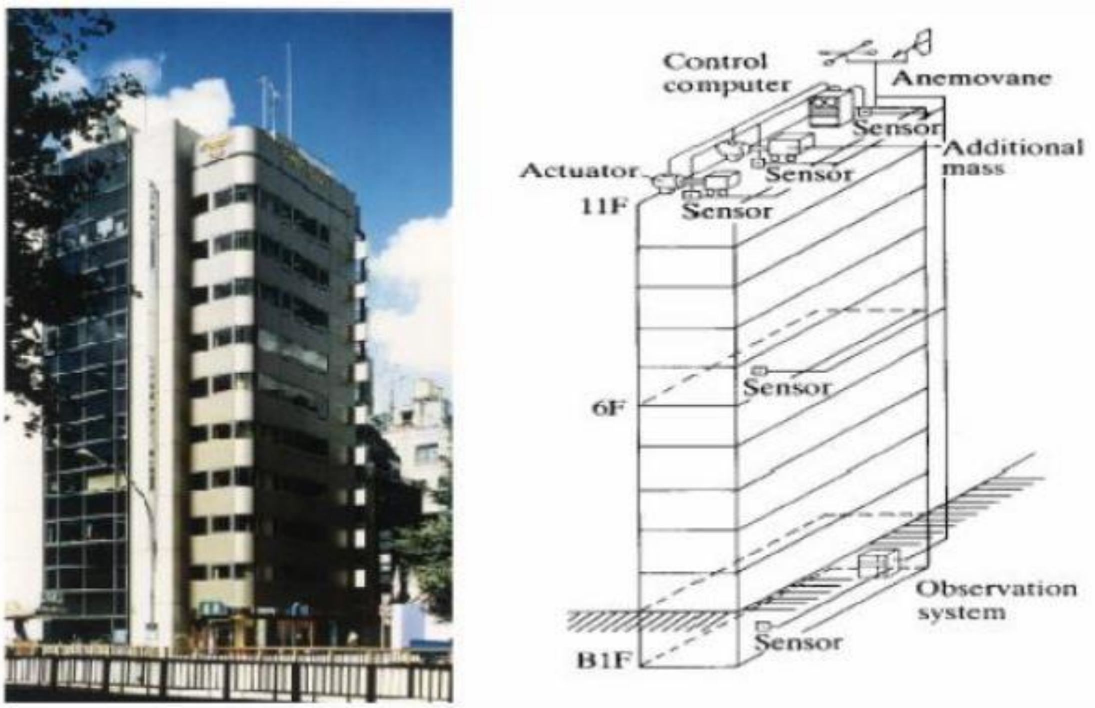


Figure 1.8 Kyobashi Seiwa Building, Tokyo, Japan

1.3.1.3 Hybrid Mass Damper

The Hybrid Mass Damper (HMD) is the most common control device employed in full-scale civil engineering applications. An HMD is a combination of a passive TMD and an active control actuator (Figure 1.5.c). The ability of this device to reduce structural responses relies mainly on the natural motion of the TMD. The forces from the control actuator are employed to increase the efficiency of the HMD and to increase its robustness to changes in the dynamic characteristics of the structure. The energy and forces required to operate a typical HMD are far less than those associated with a fully AMD system of comparable performance.

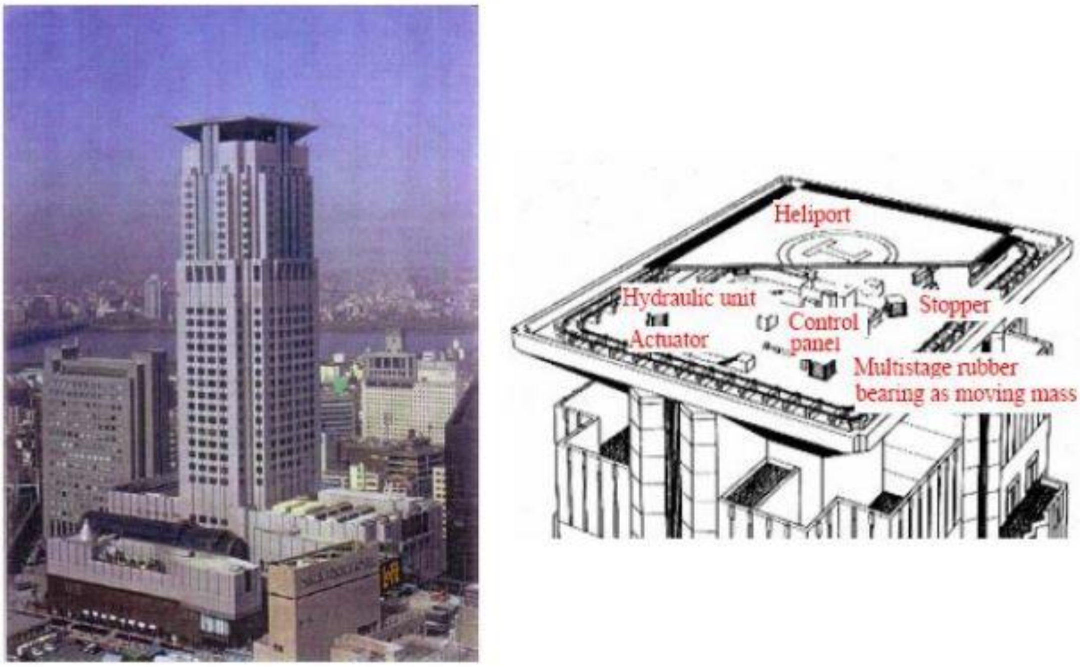


Figure 1.9 Active Mass Damper Using a Rooftop Heliport in the Applause Tower, Osaka, Japan

An example of such an application is the HMD system installed in the Sendagaya INTES building in Tokyo in 1991(Soong and Spencer Jr. 2002). As shown in Figure 1.10, the HMD was installed at the top of 11th floor and consists of two masses to control transverse and torsional motions of the structure, while hydraulic actuators provide the active control capabilities.

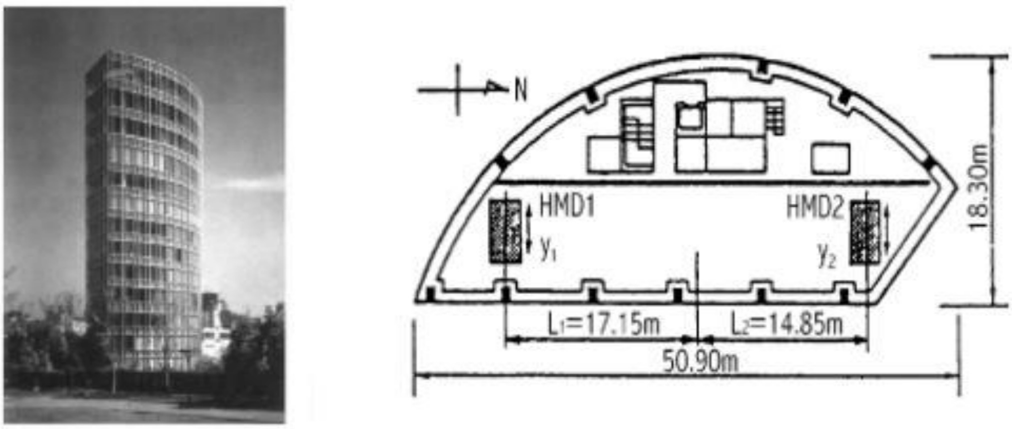


Figure 1.10 Sendagaya INTES Building with hybrid mass and top view of HMD configuration

The top view of the control system sows that ice thermal storage tanks are used as mass blocks so that no extra mass was introduced. The masses are supported by multi-stage rubber bearings intended for reducing the control energy consumed in the HMD and for insuring smooth mass movements. HMD has been installed as multi-step pendulum HMDs (Figure 1.11), in the Yokohama Landmark Tower in Yokohama, the tallest building in Japan, and in the TC Tower in Kaohsiung, Taiwan.

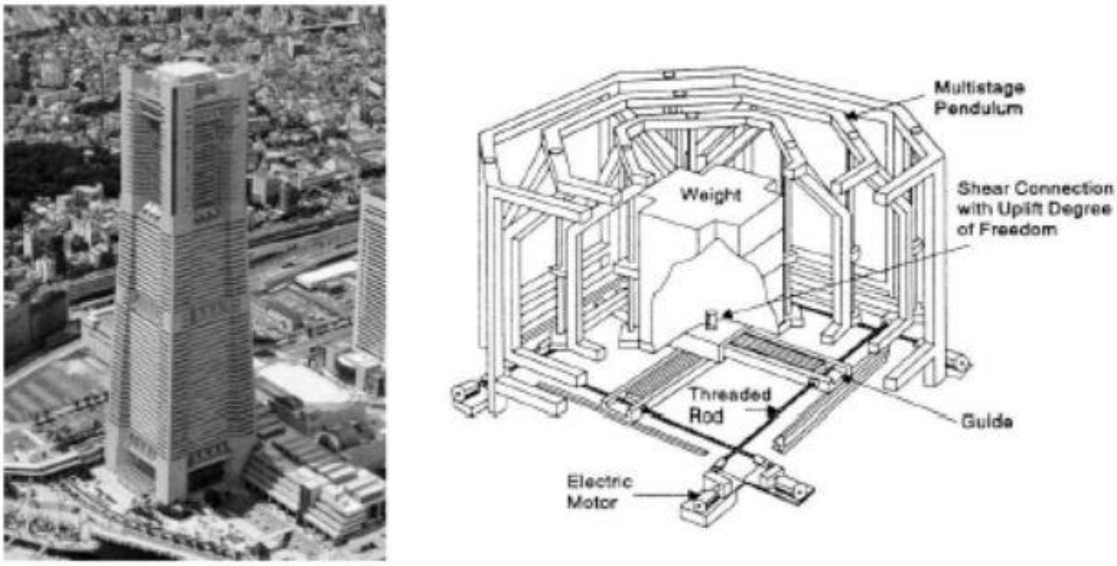


Figure 1.11 Yokohama Landmark Tower and HMD

1.3.2 Tuned Liquid Dampers

A slight variation on the TMD is the tuned-liquid-damper (TLD), which uses the sloshing of the liquid in a tank to dissipate seismic energy. These devices have additional advantages: low cost, ease of installation and adjustment of liquid frequency. Tuned Liquid Dampers, encompasses of Tuned Sloshing Dampers (TSDs) and Tuned Liquid Column Dampers (TLCDs) delineated in Figure 1.12.

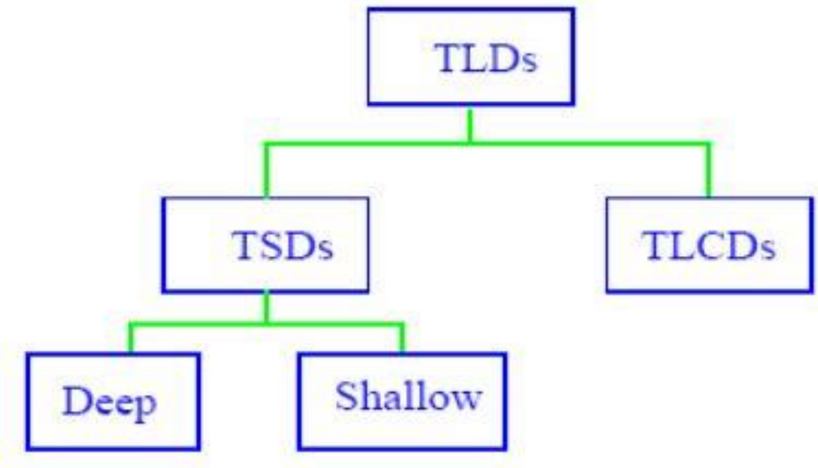


Figure 1.12 Schematic of the TLD Family

The TSDs are extremely practical, currently being proposed for existing water tanks on the building by configuring internal partitions into multiple dampers without adversely affecting the functional use of the water supply tanks. Considering only a small additional mass, if any, is added to the building, these systems and their counterpart TMDs can reduce acceleration responses to ½ to 1/3 of the original response, depending on the amount of liquid mass.

The first implementation of water tank to resist natures force like wind was the 304m high Sydney center point tower (Figure 1.13). This building is considered as one of the safest buildings in the world. The tower has a 162,000 liter water tank at the top that acts as a stabilizer on windy days. In the Hafei 339m high TV tower, the 60 tonnes of water tank in the top serves to act as tuned liquid dampers to resist the wind induced motion.

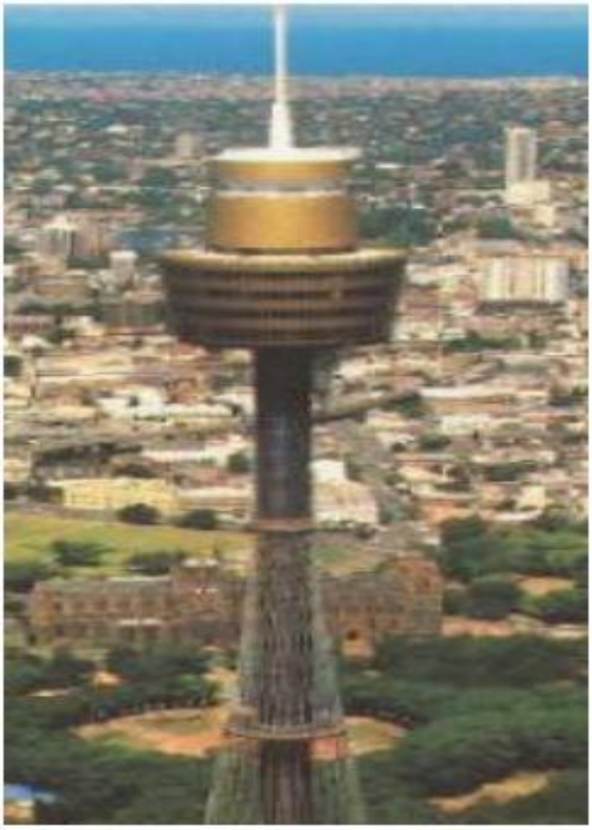


Figure 1.13 Sydney Center Point Tower

1.4 MOTIVATION OF PRESENT WORK

“Earthquakes doesn’t kill, but structures kill”

The physical consequences of earthquakes are death and injury to human beings and damage to the constructed and natural environment. To reduce vibrations in structures many structural control systems has been designed and installed worldwide. Of all the methods the traditional method of implementing control system is TMD. The effectiveness of the TMD depends on how the dynamic properties are tuned to the properties of the structure. The dynamic properties include mass ratio ($\overline{m}$) i.e., ratio of mass of TMD to the mass of the structure, frequency ratio (*f*) i.e., frequency of the TMD to the frequency of the structure and damping ratio of the TMD (ξ_d). Researchers have aimed at optimizing *f* and ξ_d for given value of $\overline{m}$. Selection of $\overline{m}$ has been studied only by Lee (1990). However the values suggested by him are much higher than that used for practical consideration. Hence optimization of $\overline{m}$ must be studied for economic design. Structures with different time period respond to different earthquakes in different manner. Unique optimized parameters must be derived for each structure such that their seismic response is reduced.

1.5 OBJECTIVES AND SCOPE

The general objective of this research is to prepare optimized design graph for TMD parameters subjected to seismic excitation.

The specific objective of the present research is to optimize analytically the passive parameters of the TMD, namely mass ratio ($\overline{m}$) i.e., ratio of mass of TMD to the mass of the structure, frequency ratio (*f*) i.e., frequency of the TMD to the frequency of the structure and damping ratio of the TMD (ξ_d) for structures with time period.

The effectiveness of passive TMD depends on how effectively the parameters are optimized and the frequency of TMD tuned to that of the natural frequency of the structure and the impending issue in implementing the TMD is the additional dead load it imposes on the structure and in turn on the foundation and the enormous amount of space it occupies for erection. To address these issues this study will arrive at optimized parameters of passive TMD subjected to seismic excitation.

The scope of this work is limited by the analysis and design of TMD tuning parameters and the excitation force applied to building structure at a theoretical level and laboratory test is not executed. The TMD tuning parameters considered is the natural frequency ratio, mass ratio and damping ratio of the TMD.

1.6 Lay out of the thesis

Chapter 1: Gives a general introduction for structure control system, motivation of this work and objective and scope of the thesis.

Chapter 2: Reviews state of the art, highlighting the previous studies on optimization of TMD adopted by researchers, the implementation of TMD’s for seismic resistance.

Chapter 3: Discuses a rigorous theory of tuned mass dampers for SDOF systems subjected to harmonic force excitation and harmonic ground motion. Various cases, including an undamped TMD attached to an undamped SDOF system, a damped TMD attached to an undamped SDOF system, and a damped TMD attached to a damped SDOF system, are considered. The theory is then extended to MDOF system, where the TMD is used to dampen out the vibrations of a specific mode.

Chapter 4: presents the analytical investigation carried out to optimize the parameters of TMD and validation of the optimized parameters using three models under earthquake excitation. Also summarizes the results and discussion of the analytical work.

Chapter 5 provides the conclusions, contributions and scope for further work.

CHAPTER TWO

STATE OF THE ART

2.1 General

The TMD concept was first applied by Frahm in 1909 (Frahm, 1909) to reduce the rolling motion of ships as well as ship hull vibrations. A theory for the TMD was presented later in the paper by Ormondroyd and Den Hartog**(**1928**),**followed by a detailed discussion of optimal tuning and damping parameters in Den Hartog‘s book on mechanical vibrations (1940). The initial theory was applicable for an undamped SDOF system subjected to a sinusoidal force excitation. Extension of the theory to damped SDOF systems has been investigated by numerous researchers.

Active control devices operate by using an external power supply. Therefore, they are more efficient than passive control devices. However the problems such as insufficient control force capacity and excessive power demands encountered by current technology in the context of structural control against earthquakes are unavoidable and need to be overcome. Recently a new control approach-semi-active control devices, which combine the best features of both passive and active control devices, is very attractive due to their low power demand and inherent stability. The earlier papers involving SATMDs may traced to 1983. Hrovat (1983) presented SATMD, a TMD with time varying controllable damping. Under identical conditions, the behavior of a structure equipped with SATMD instead of TMD is significantly improved. The control design of SATMD is less dependent on related parameters (e.g., mass ratios, frequency ratios and so on), so that there greater choices in selecting them.

The first mode response of a structure with TMD tuned to the fundamental frequency of the structure can be substantially reduced but, in general, the higher modal responses may only be marginally suppressed or even amplified. To overcome the frequency-related limitations of TMDs, more than one TMD in a given structure, each tuned to a different dominant frequency, can be used. The concept of multiple tuned mass dampers (MTMDs) together with an optimization procedure was proposed by Clark (1988). Since, then, a number of studies have been conducted on the behavior of MTMDs a doubly tuned mass damper (DTMD), consisting of two masses connected in series to the structure was proposed (Setareh 1992). In this case, two different loading conditions were considered: harmonic excitation and zero mean white-noise random excitation, and the efficiency of

DTMDs on response reduction was evaluated. Analytical results show that DTMDs are more efficient than the conventional single mass TMDs over the whole range of total mass ratios, but are only slightly more efficient than TMDs over the practical range of mass ratios (0.01-0.05).

2.2 OPTIMIZATION

The effectiveness of the TMD depends on how effective the dynamic properties of the TMD are tuned to that of the structure to which it is attached. Hence optimization of the parameters is essential to enhance TMD's performance. The following literature review presents the various optimization procedures derived for structures with TMD subjected to seismic excitation.

Warburton (1982) presents simple expressions for optimum absorber parameters for undamped one degree-of-freedom main systems subjected to harmonic and white noise random excitations with force and frame acceleration as input and minimization of various response parameters. It has been shown that when determining optimum parameters for an absorber which minimizes the vibration response of a complex system, the system may be treated as an equivalent single degree-of-freedom system if its natural frequencies are well separated. The main mass responses are optimized with respect to damping of TMD (ξ_d) and frequency ratio (f) for specified values of mass ratio ($\bar{m}$). The expression derived for random acceleration is,

Optimized frequency ratio, $$f_{opt} = \frac{\sqrt{(1-\frac{\bar{m}}{2})}}{(1+\bar{m})} \quad \text{......................} \; 2.1$$

Optimized damping ratio, $$\xi_{dopt} = \frac{\sqrt{\bar{m}(1-\frac{\bar{m}}{4})}}{4(1+\bar{m})(1-\frac{\bar{m}}{2})} \quad \text{.....................} \; 2.2$$

For very small values of $\bar{m}$, f_{opt} equals to 1, ξ_{dopt} becomes $(\bar{m}/4)^{1/2}$ and optimum maximum response R_{opt} becomes $\bar{m}^{-1/2}$. As $\bar{m}$ increases f_{opt} decreases. The expressions are derived from optimizing conditions, $\partial(\sigma x^2) / \partial\xi_d = 0$ and $\partial(\sigma x^2) / \partial f = 0$, where σx^2 is the standard deviation. The author has concluded that the expressions can be used to obtain optimum parameters for absorbers attached to complex systems provided the optimization is with respect to an absolute, rather than a relative, quantity.

Jimmy Lee (1990) presents optimal mass ratios for minimizing the response of a structure exposed to earthquake or fluid flow type random excitations. A laminated beam treated as an equivalent one degree of freedom (Single DOF) main system vibrating in the fundamental mode was taken for the study. The effective main system response, R is optimized with respect to ξ_d, f and $\bar{m}_{opt}$ for specified values of effective mass system damping and location of TMD along the length of the beam. The beam is subjected to Gaussian white noise force and Gaussian white noise base frame acceleration and the mean square displacement is minimized for vibration in the fundamental beam mode. The optimizing condition for mass ratio is $\partial R/\partial \bar{m}_{opt} = 0$. In the optimized values presented by the authors, $\bar{m}_{opt}$ takes values much larger than those considered in practical conditions.

Tsai (1994) studied the effect of applying tuned-mass dampers towards reducing the lateral deformation of the isolators. The choice of the proper parameters of the tuned-mass damper and the influence of excitation frequency on the response were investigated. Through the numerical simulation of a five-storey base-isolated building subjected to different earthquake records, it was found that although the tuned-mass damper had little effect on structural response during the first few seconds of earthquake excitation, the damper may add damping to the structure to reduce the subsequent response. The idea of the accelerated tuned-mass damper was proposed and demonstrated the capability of decreasing the maximum deformation of the isolators which occurred at the beginning of earthquake excitation.

Fahim Sadek et al (1995) presented the optimum parameters of TMD that result in considerable reduction in the response of structures to seismic loading. The criterion used to obtain the optimum parameters is to select, for a given mass ratio, the frequency and damping ratios that would result in equal and large model damping in the first two modes of vibrations. Curve fitting was used to find f and ξ_d in terms of $\bar{m}$ and ξ.

For undamped structure,

$$f = \frac{1}{1+\bar{m}} \quad \text{and} \quad \xi_{dopt} = \sqrt{\bar{m}(1+\bar{m})} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 2.3$$

For damped structure,

$$f = \frac{1}{1+\bar{m}}\left[1-\xi\sqrt{\frac{\bar{m}}{1+\bar{m}}}\right] \quad \text{and} \quad \xi_{dopt} = \frac{\xi}{1+\bar{m}} + \sqrt{\frac{\bar{m}}{1+\bar{m}}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 2.4$$

The optimum values are determined when the difference between the two damping ratios of critical modes is the smallest. The procedure was used for systems with damping ratios $\xi = 0$, 0.02 and 0.05 and mass ratios $\bar{m}$ between 0.005 and 0.15 with increments 0.005. It was found that the optimum TMD parameters result is approximately equal model damping ratios ($\xi_1 \approx \xi_3$) greater than $(\xi + \xi_d)/2$ and equal model frequencies ($\omega_1 \approx \omega_3$). The results indicate that higher the damping ratio of the structure, the lower is the tuning ratio and the higher is the TMD damping ratio. The higher the intrinsic damping in the structure, the larger is the mass required to achieve approximately the same level of response reduction. In comparison to the work done by, Villaverde in 1985, Sadek has shown that if the tuning frequency, *f* is not equal to one, and takes a value around unity the response reduction is better when the system is subjected to seismic excitation.

A.N Blekherman (1996) proposed a passive vibration absorber to protect high-rise structural systems from earthquake damages. A structure is modelled by one-mass and n-mass systems (a cantilever scheme). Damping of the structure and absorber installed on top of it is represented by frequency independent one on the base of equivalent visco-elastic model that allows the structure with absorber to be described as a system with non-proportional internal friction. A ground movement is modelled by an actuator that produces vibration with changeable amplitude and frequency. To determine the optimum absorber parameters, an optimization problem, that is a minmax one, was solved by using nonlinear programming technique (the Hooke-Jeves method).

Genda Chen (2001) studied effects of a tuned mass damper on the modal responses of a six-story building structure. Multistage and multimode tuned mass dampers are then introduced. Several optimal location indices are defined based on intuitive reasoning, and a sequential procedure is proposed for practical design and placement of the dampers in seismically excited building structures. The proposed procedure is applied to place the dampers on the floors of the six-story building for maximum reduction of the accelerations under a stochastic seismic load and 13 earthquake records. Numerical results show that the multiple dampers can effectively reduce the acceleration of the uncontrolled structure by 10– 25% more than a single damper. Time-history analyses indicate that the multiple dampers weighing 3% of total structural weight can reduce the floor acceleration up to 40%.

Murudi and Mane (2004) investigates the effectiveness of TMD in controlling the seismic response of structures and the influence of various ground motion parameters on the seismic effectiveness of TMD. The structure considered is an idealized single-degree-of-freedom (SDOF) structure characterized by its natural period of vibration and damping ratio. Various structures subjected to different actual recorded earthquake ground motions and artificially generated ground motions are considered. It is observed that TMD is effective in controlling earthquake response of lightly damped structures, both for actual recorded and artificially generated earthquake ground motions. The effectiveness of TMD for a given structure depends on the frequency content, bandwidth and duration of strong motion, however the seismic effectiveness of TMD is not affected by the intensity of ground motion.

Anupam Ahlawat and Ananth Ramaswamy (2004) proposed a genetic algorithm based integrated approach to optimize the total cost of building and the associated control device. Constraints for performance of the design include peak Inter storey drift and peak absolute acceleration of the floors. Idealized 3 plane frame equivalent models of 3 storey and 9 storey steel buildings have been considered. Building structure has been assumed to be linear elastic with Rayleigh damping. Mass of the building was lumped at floor levels. A TMD installed at roof of the building has been used as passive vibration control device. Four earthquake records, namely, Elcentro (1940, 0.35g), Hachinohe (1968, 0.23g), Northridge (1994, 0.84g) and Kobe (1995, 0.83g) was used as design excitations. Stiffness of the columns, mass, stiffness and damping of the TMD have been considered as design variable. Mass of TMD was taken as 5% the total mass of the structure. Results show that for 3 storey building example, optimal design is one without any TMD and for 9 storey building the best design is one with TMD of 4.4 % mass of the structure.

Chien-Liang Lee (2006) proposed an optimal design theory for structures implemented with tuned mass dampers (TMDs). Full states of the dynamic system of multiple-degree-of freedom (MDOF) structures, multiple TMDs (MTMDs) installed at different stories of the building, and the power spectral density (PSD) function of environmental disturbances are taken into account. The optimal design parameters of TMDs in terms of the damping coefficients and spring constants corresponding to each TMD are determined through minimizing a performance index of structural responses defined in the frequency domain. Moreover, a numerical method is also proposed for searching for the optimal design parameters of MTMDs in a systematic fashion such that the numerical solutions converge monotonically and effectively toward the exact solutions as the number of iterations increases. The feasibility of the proposed optimal design theory is verified by using a SDOF structure with a single TMD (STMD), a five-DOF structure with two TMDs, and a ten-DOF structure with a STMD.

G.C. Marano (2007) studied the optimum design of vibration absorbers utilized to reduce undesirable vibrational effects which are originated in linear structures by seismic excitations. The single linear tuned mass dampers problem is treated and it is assumed that earthquake can be represented by a stationary filtered stochastic process. Structural design has been carried out by means of a reliability-based optimum criterion, where both the objective function and the constraints are defined in a stochastic way. Several numerical analyses have been carried out in order to assess the sensitivity of the optimum solution versus input and structural characteristics for the unconstrained and constrained optimization problems. Analyses have pointed out the high sensitivity of optimum solutions against structural characteristics, such as structural frequency and mass ratio, and earthquake characterization. Results could yield useful information for pre-design decision in the TMD strategy, because for each structural and forcing characteristic the optimum solution is given.

Christian and Daniel (2010) discussed the practical application of Tuned Mass Control Systems (TMCS) for earthquake protection and Optimization approaches for these passive control systems as well as practical considerations regarding the resulting specification of the TMCS such as stiffness loss during an earthquake and wide-band effectiveness. Depending on the free-vibration participation of the structural response, the optimum reduction can be achieved with the Den Hartog Criteria or with recommended higher internal damping ratios of the TMCS. Practical considerations showed, that a higher internal damping leads to a more robust specification in terms of varying structural stiffness and inherent damping.

Islam and Ahsan (2012) determined optimum parameters of Tuned Mass Dampers (TMD) to minimize dynamic response of a multi-storied building system. The response of the structural system is simulated under lateral excitation. To optimize dynamic parameters of the TMD system for minimum top deflection of the structure, a numerical global optimization algorithm called Evolutionary Operation (EVOP) is used. A computer program has been developed to construct the optimization problem. A ten story structure associated with TMD on roof was selected for optimization process and top displacement was minimized for El Centro (1940) NS earthquake. Afterwards a comparison with the results obtained from two different approaches was made to prove the effectiveness and reliability of EVOP. A higher percentage of structural response reduction was achieved with a choice of smaller mass, stiffness and damping of TMD with the application of EVOP. From the study it has been established that the selected optimization technique EVOP has the high probability of locating global minimum. Also a comparison has been made with two other optimization approaches. The study shows the effectiveness of present approach in optimization leading to a more feasible selection of TMD parameters.

Jimmy S. Issa (2013) proposed the optimal design of a viscously damped platform for vibration suppression in undamped single degree of freedom systems. For a given mass ratio $\overline{m}$ of the system, the optimal tuning and damping ratios were determined with the aim of minimizing the maximum of the primary system frequency response. The solution was obtained by first setting two of three invariant points, which are independent of the damping ratio to equal heights, then by equally leveling the two peaks of the objective function. This was made possible due to the existence of a trade-off relation between the three invariant points and two peaks. Two different expressions of the optimal tuning ratio were determined analytically, each corresponding to a range of the mass ratio, where two of the three invariant points are set to equal heights. The range of $\overline{m}$ over which the first tuning condition is defined to split into two subranges. In the first subrange, the optimal damping was determined analytically and led to the exact solution because the two peaks coincided with the equally leveled invariant points. The optimal damping corresponding to the second subrange is the solution of a nonlinear equation and led to a semi exact solution, where the two peaks do not coincide with the fixed points but are set to the same height. For the second tuning condition, a semi exact solution was obtained where the optimal damping ratio was determined analytically. It is shown that an optimal mass ratio $\overline{m}_{opt}$ exists, unlike the case of the classical setup, where the absorber performance increases with its increasing mass.

Pisal and Jangid (2014) investigated the effectiveness of multiple tuned mass friction dampers (MTMFD) for vibration control of structure over a single tuned mass friction damper (STMFD). A five story structure supported with MTMFD was considered and the governing differential equations of motion are derived. The response of the structure under four selected earthquake ground motions is obtained by solving the equations of motion numerically using the state space method. The number of damper in MTMFD is varied and the response of five story structure with STMFD is compared with the response of the same structure with MTMFD. A parametric study is also conducted to investigate the effects of important parameters such as number of dampers in MTMFD, damper frequency spacing, mass ratio, tuning ratio and damper slip force. It is found that for a given structural system and level of excitation an optimum value of the parameters (i.e. frequency spacing, tuning ratio and damper slip force) exists at which the peak displacement of structure attains its minimum value. The response time history of the structure with STMFD and MTMFD with respect to their optimum parameters is compared. It is found that the MTMFD is more effective in controlling the response of the structure in comparison with the STMFD having the same mass.

Kaveh et al. (2015) determined optimum parameters of Tuned Mass Dampers (TMD) to minimize the dynamic response of multi-story building systems under seismic excitations. Charged System Search (CSS), as an efficient optimization algorithm, is revised and applied for tuning passive mass dampers. A MATLAB program is developed for numerical optimization and time domain simulation. In the first example, a ten-story shear building is considered. It is found that with the same damper mass, the mean value of displacement reduction is 37.57% which is more than those of the previous studies. It means more capability in absorbing earthquake energy and reducing total story displacements is achieved in this work. Also, in the next step more refinement is applied on the mass of TMD. In this step, it is found that better solution can be obtained by smaller values of TMD parameters (m_d = 99 ton, c_d = 80.441 kN.s/m, and k_d = 3864.45 kN/m). In this case, the maximum top story displacement is reduced to 0.1215 m from 0.188 m (35.37% reduction), and the mean value of the reduction is 37.77%. In the second example, they were only interested in finding the optimum value of the TMD parameters and the corresponding reduction in displacement values. In this case, optimum parameters of TMD are obtained as: m_d = 55.45 ton, c_d = 30.234 kN.s/m, and k_d= 355.758 kN/m. These optimum values are significantly smaller than those obtained by Sadek (1997).

CHAPTER THREE

MODELING TMD AND ANALYSIS

3.1 Tuned Mass Damper for SDOF Systems

3.1.1 Undamped Structure: Undamped TMD

Figure 3.1 shows a SDOF system having mass m and stiffness k, subjected to both external forcing and ground motion. A tuned mass damper with mass m_d and stiffness k_d is attached to the primary mass. The various displacement measures are u_g, the absolute ground motion; u, the relative motion between the primary mass and the ground; and u_d, the relative displacement between the damper and the primary mass. With this notation, the governing equations take the form

$$m_d [\ddot{u}_d + \ddot{u}] + k_d u_d = -m_d a_g \quad \text{……………………. 3.1}$$

$$m\ddot{u} + ku - k_d u_d = -m a_g + p \quad \text{……………………. 3.2}$$

where a_g is the absolute ground acceleration and p is the force loading applied to the primary mass.

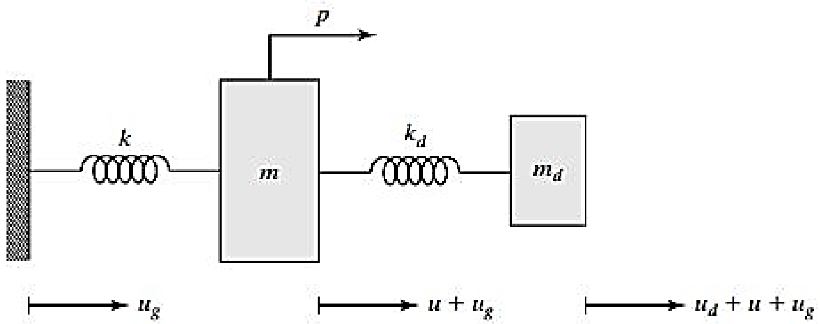


Figure 3.1 SDOF system coupled with a TMD

The excitation is considered to be periodic of frequency Ω,

$$a_g = \hat{a}_g \sin \Omega t \quad \text{……………………. 3.3}$$

$$p = \hat{p} \sin \Omega t \quad \text{……………………. 3.4}$$

Expressing the response as

$$u = \hat{u} \sin \Omega t \quad \text{……………………. 3.5}$$

$$u_d = \hat{u}_d \sin \Omega t \quad \text{……………………. 3.6}$$

and substituting for these variables, the equilibrium equations are transformed to

$$[-m_d\Omega^2 + k_d]\,\hat{u}_d - m_d\Omega^2\hat{u} = -m_d\hat{a}_g \qquad \text{………………………3.7}$$

$$-k_d\hat{u}_d + [-m\Omega^2 + k]\,\hat{u} = -m\hat{a}_g + p \qquad \text{……………………….. 3.8}$$

The solutions for û and $\hat{u}_d$ are given by

$$\hat{u} = \frac{\hat{p}}{k}\left(\frac{1-\rho d^2}{D1}\right) - \frac{m\hat{a}g}{k}\left(\frac{1+\bar{m}-\rho d^2}{D1}\right) \qquad \text{………………………3.9}$$

$$\hat{u}_d = \frac{\hat{p}}{kd}\left(\frac{\bar{m}\rho^2}{D1}\right) - \frac{m\hat{a}g}{kd}\left(\frac{\bar{m}}{D1}\right) \qquad \text{………………………3.10}$$

Where

$$D_1 = [1-\rho^2]\,[1-\rho_d^2] - \bar{m}\rho^2 \qquad \text{………………………3.11}$$

and the terms are dimensionless frequency ratios,

$$\rho = \frac{\Omega}{\omega} = \frac{\Omega}{\sqrt{k/m}} \qquad \text{………………………3.12}$$

$$\rho_d = \frac{\Omega}{\omega d} = \frac{\Omega}{\sqrt{kd/md}} \qquad \text{………………………3.13}$$

3.1.2 Undamped Structure: Damped TMD

The next level of complexity has damping included in the mass damper, as shown in Figure 3.2. The equations of motion for this cases are

$$m_d\ddot{u}_d + c_d\dot{u}_d + k_du_d + m_d\ddot{u} = -m_da_g \qquad \text{………………………3.14}$$

$$m\ddot{u} + ku - c_d\dot{u}_d - k_du_d = -ma_g + p \qquad \text{………………………3.15}$$

The inclusion of the damping terms in Eqs. (3.14) and (3.15) produces a phase shift between the periodic excitation and the response. It is convenient to work initially with the solution expressed in terms of complex quantities. We express the excitation as

$$a_g = \hat{a}_g e^{i\Omega t} \qquad \text{………………………3.16}$$

$$p = \hat{p}e^{i\Omega t} \qquad \text{………………………3.17}$$

where $\hat{a}_g$ and $\hat{p}$ are real quantities. The response is taken as

$$u = \bar{u}e^{i\Omega t} \qquad \text{………………………3.18}$$

$$u_d = \bar{u}_d e^{i\Omega t} \qquad \text{………………………3.19}$$

where the response amplitudes, $\bar{u}$ and $\bar{u}_d$ are considered to be complex quantities. The real and imaginary parts of a_g correspond to cosine and sinusoidal input. Then, the corresponding solution is given by either the real (for cosine) or imaginary (for sine) parts of u and u_d. Substituting eqns (3.18) and (3.19) in the set of governing equations and cancelling $e^{i\Omega t}$ from both sides results in

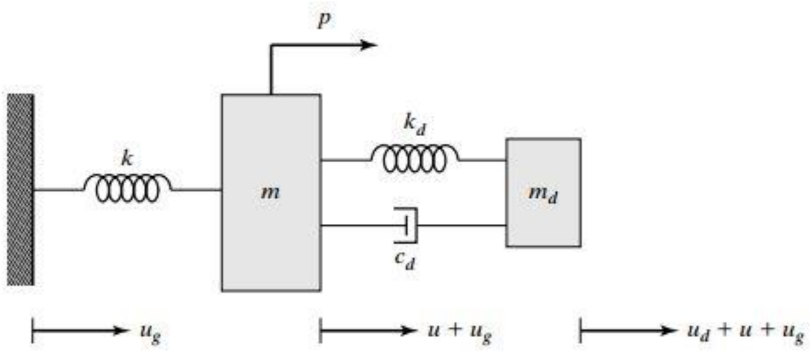


Figure 3.2 Undamped SDOF system coupled with a damped TMD system.

$$[-m_d\Omega^2 + ic_d\Omega + k_d]\,\bar{u}_d - m_d\Omega^2\bar{u} = -m_d\hat{a}_g \qquad \ldots\ldots\ldots\ldots\ldots 3.20$$

$$-[ic_d\Omega + k_d]\,\bar{u}_d + [-m\Omega^2 + k]\bar{u} = -m\hat{a}_g + \hat{p} \qquad \ldots\ldots\ldots\ldots\ldots 3.21$$

The solution of the governing equations is

$$\bar{u} = \frac{\hat{p}}{kD2}[f^2 - \rho^2 + i2\xi_d\rho f] - \frac{\hat{a}gm}{kD2}[(1+\bar{m})f^2 - \rho^2 + i2\xi_d\rho f(1+\bar{m})] \qquad \ldots\ldots\ldots\ldots\ldots 3.22$$

$$\bar{u}_d = \frac{\hat{p}\rho 2}{kD2} - \frac{\hat{a}gm}{kD2} \qquad \ldots\ldots\ldots\ldots\ldots 3.23$$

Where

$$D_2 = [1-\rho^2]\,[f^2-\rho^2] - \bar{m}\rho^2 f^2 + i2\xi_d\rho f\,[1-\rho^2(1+\bar{m})] \qquad \ldots\ldots\ldots\ldots\ldots 3.24$$

$$f = \frac{\omega d}{\omega} \qquad \ldots\ldots\ldots\ldots\ldots 3.25$$

and ρ was defined earlier as the ratio of to Ω to ω.

Converting the complex solutions to polar form leads to the following expressions:

$$\bar{u} = \frac{\hat{p}}{k}H_1e^{i\delta 1} - \frac{\hat{a}gm}{k}H_2e^{i\delta 2} \qquad \ldots\ldots\ldots\ldots\ldots 3.26$$

$$\bar{u}_d = \frac{\hat{p}}{k}H_3e^{-i\delta 3} - \frac{\hat{a}gm}{k}H_4e^{-i\delta 3} \qquad \ldots\ldots\ldots\ldots\ldots 3.27$$

where the H factors define the amplification of the pseudo-static responses, and the δ's are the phase angles between the response and the excitation. The various H and δ terms are as follows:

$$H_1 = \frac{\sqrt{[f2 - \rho 2]^2 + [2\xi d\rho f]^2}}{| D2|} \qquad \text{................} 3.28$$

$$H_2 = \frac{\sqrt{[(1+\bar{m})f2 - \rho 2]^2 + [2\xi d\rho f(1+\bar{m})]^2}}{| D2|} \qquad \text{................} 3.29$$

$$H_3 = \frac{\rho^2}{| D2|} \qquad \text{................} 3.30$$

$$H_4 = \frac{1}{| D2|} \qquad \text{................} 3.31$$

$$/ D_2 / = \sqrt{([1-\rho^2][f2 - \rho 2] - m^- \rho^2 f^2)^2 + (2\xi d\rho f[1-\rho^2(1+\bar{m})])^2} \qquad \text{........} 3.32$$

Also,

$$\delta_1 = \alpha_1 - \delta_3 \qquad \text{................} 3.33$$

$$\delta_2 = \alpha_2 - \delta_3 \qquad \text{................} 3.34$$

$$tan_{\delta 3} = \frac{2\xi d\rho f[1-\rho^2(1+\bar{m})]}{[1-\rho^2][f2 - \rho 2] - m^- \rho^2 f^{2.}} \qquad \text{................} 3.35$$

$$tan_{\alpha 1} = \frac{2\xi d\rho f}{f^2 - \rho^{2.}} \qquad \text{................} 3.36$$

$$tan_{\alpha 2} = \frac{2\xi d\rho f(1+\bar{m})}{(1+\bar{m})f^2 - \rho^{2.}} \qquad \text{................} 3.37$$

3.1.3 Damped Structure: Damped TMD

All real systems contain some damping. Although an absorber is likely to be added only to a lightly damped system, assessing the effect of damping in the real system on the optimal tuning of the absorber is an important design consideration. The main system in Figure 3.3 consists of the mass m, spring stiffness k, and viscous damping c. The TMD system has mass m_d, stiffness k_d, and viscous damping c_d. Considering the system to be subjected to both external forcing and ground excitation, the equations of motion are

$$m_d\ddot{u}_d + c_d\dot{u}_d + k_d u_d + m_d\ddot{u} = -m_d a_g \qquad \text{................} 3.38$$

$$m\ddot{u} + c\dot{u} + ku - c_d\dot{u}_d - k_du_d = -ma_g + p \qquad \text{.................. 3.39}$$

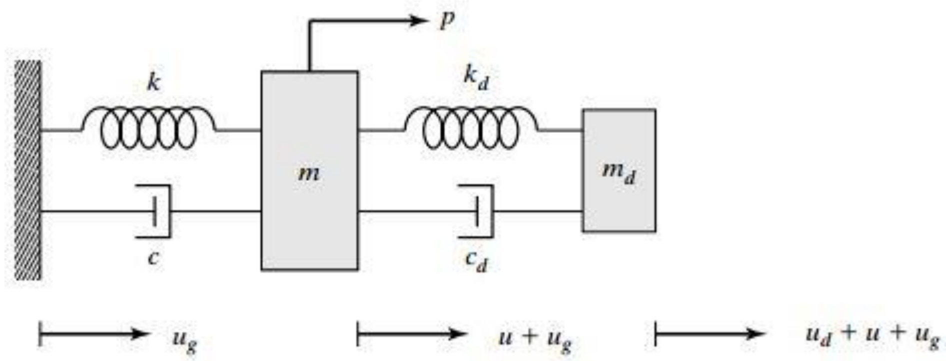


Figure 3.3 Damped SDOF system coupled with a damped TMD system

Proceeding in the same way as for the undamped case, the solution due to periodic excitation (both p and u_g) is expressed in polar form:

$$\bar{u} = \frac{\hat{p}}{k}H_5e^{i\delta 5} - \frac{\hat{a}gm}{k} \qquad H_6e^{i\delta 6} \qquad \text{................ 3.40}$$

$$\bar{u}_d = \frac{\hat{p}}{k}H_7e^{-i\delta 7} - \frac{\hat{a}gm}{k}H_8e^{-i\delta 8} \qquad \text{................ 3.41}$$

The various H and δ terms are defined as follows:

$$H_5 = \frac{\sqrt{[f2 - \rho 2]^2 + [2\xi d\rho f]^2}}{|\,D3|} \qquad \text{................ 3.42}$$

$$H_6 = \frac{\sqrt{[(1+\bar{m})f2 - \rho 2]^2 + [2\xi d\rho f(1+\bar{m})]^2}}{|\,D3|} \qquad \text{................ 3.43}$$

$$H_7 = \frac{\rho^2}{|\,D3|} \qquad \text{................ 3.44}$$

$$H_8 = \frac{\sqrt{1+[2\xi\rho]^2}}{|\,D3|} \qquad \text{................ 3.45}$$

$$/\,D_3/ = \{[-f^2\rho^2\bar{m} + (1-\rho^2)(f^2-\rho^2) - 4\xi\xi_d f\rho^2]^2 + 4[\xi\rho(f^2-\rho^2) + \xi_d f\rho(1-\rho^2(1+\bar{m}))^2]\} \qquad \text{................ 3.46}$$

$$\delta_5 = \alpha_1 - \delta_7 \qquad \text{................ 3.47}$$

$$\delta_6 = \alpha_2 - \delta_7 \qquad \text{................ 3.48}$$

$$\delta_8 = \alpha_3 - \delta_7 \qquad \text{................ 3.49}$$

$$\tan_{\delta 7} = \frac{2\xi\rho(f^2-\rho^2)+\xi df\rho\,(1-\rho^2(1+\bar{m}))}{-f^2\rho^2\bar{m}+(1-\rho^2)(f^2-\rho^2)-4\xi\xi df\rho 2} \quad \ldots\ldots\ldots\ldots\ldots\; 3.50$$

$$\tan\alpha_3 = 2\xi\rho \quad \ldots\ldots\ldots\ldots\ldots\; 3.51$$

The α_1 and α_2 terms are defined by Eqs. (3.36) and (3.37).

3.2 TUNED MASS DAMPER THEORY FOR MDOF SYSTEMS

The theory of a SDOF system presented earlier is extended here to deal with a MDOF system having a number of tuned mass dampers located throughout the structure.

A 2DOF system having a damper attached to mass 2 is considered first to introduce the key ideas. The governing equations for the system shown in Figure 3.4 are

$$m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 - k_2(u_2 - u_1) - c_2(\dot{u}_2 - \dot{u}_1) = p_1 - m_1\ddot{u}_g \quad \ldots\ldots\ldots\; 3.52$$

$$m_2\ddot{u}_2 + c_2(\dot{u}_2 - \dot{u}_1) + k_2(u_2 - u_1) - k_du_d - c_d\dot{u}_d = p_2 - m_2\ddot{u}_g \quad \ldots\ldots\ldots\; 3.53$$

$$m_d\ddot{u}_d + k_du_d + c_d\dot{u}_d = -m_d(\ddot{u}_2 + \ddot{u}_g) \quad \ldots\ldots\ldots\ldots\ldots\; 3.54$$

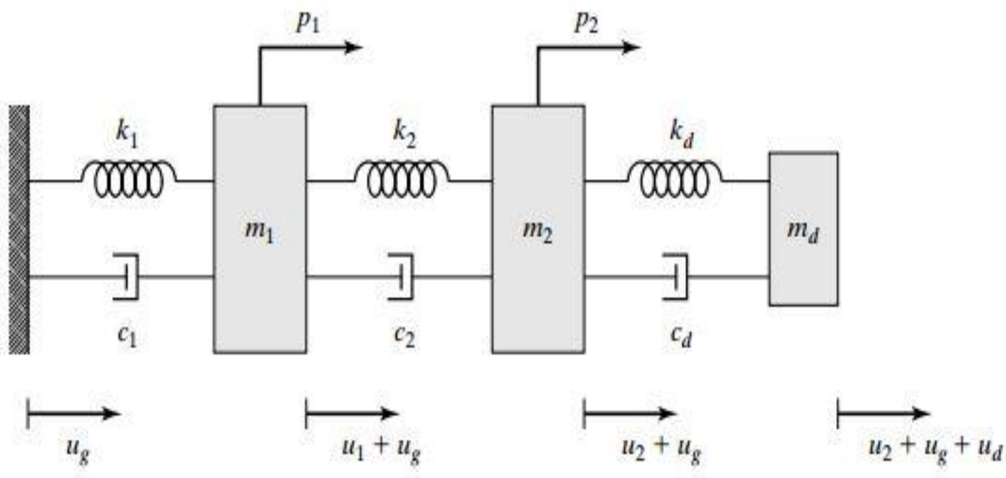


Figure 3.4 Two DOF system with TMD

The key step is to combine Eqs. (3.52) and (3.53) and express the resulting equation in a form similar to the SDOF case defined by Eq. (3.39). This operation reduces the problem to an equivalent SDOF system. The approach followed here is based on transforming the original matrix equation to scalar modal equations.

Introducing matrix notation, Eqs. (3.52) and (3.53) are written as

$$\mathbf{M\ddot{U}} + \mathbf{C\dot{U}} + \mathbf{KU} = \begin{bmatrix} p_1 - m_1ag \\ p_2 - m_2ag \end{bmatrix} + \begin{bmatrix} 0 \\ KdUd + Cd\dot{U}d \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.55$$

where the various matrices are

$$\mathbf{U} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.56$$

$$\mathbf{M} = \begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.57$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.58$$

$$\mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.59$$

We substitute for **U** in terms of the modal vectors and coordinates

$$\mathbf{U} = \Phi_1 q_1 + \Phi_2 q_2 \quad \ldots\ldots\ldots\ldots\ldots 3.60$$

The modal vectors satisfy the following orthogonality relations

$$\Phi_j^T\mathbf{K}\Phi_i = \delta_{ij}\omega_j^2\Phi_j^T\mathbf{M}\Phi_i \quad \ldots\ldots\ldots\ldots\ldots 3.61$$

Defining modal mass, stiffness, and damping terms,

$$\tilde{m}_j = \Phi_j^T\mathbf{M}\Phi_j \quad \ldots\ldots\ldots\ldots\ldots 3.62$$

$$\tilde{k}_j = \Phi_j^T\mathbf{K}\Phi_j = \omega_j^2\tilde{m}_j \quad \ldots\ldots\ldots\ldots\ldots 3.63$$

$$\tilde{c}_j = \Phi_j^T\mathbf{C}\Phi_j \quad \ldots\ldots\ldots\ldots\ldots 3.64$$

expressing the elements of as

$$\Phi_j = \begin{bmatrix} \Phi j_1 \\ \Phi j_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots 3.65$$

and assuming damping is proportional to stiffness

$$C = \alpha K \quad \ldots\ldots\ldots\ldots\ldots 3.66$$

we obtain a set of uncoupled equations for q_1 and q_2:

$$\tilde{m}_j q_j + \tilde{c}_j\dot{q}_j + \tilde{k}_j q_j = \Phi_{j1}(p_1 - m_1a_g) + \Phi_{j2}(p_2 - m_2a_g + k_du_d + c_d\dot{u}_d) \quad j = 1, 2 \quad \ldots\ \ldots\ldots 3.67$$

With this assumption, the modal damping ratio is given by

$$\xi_j = \frac{\tilde{c}j}{2\omega j \tilde{m} j} = \frac{\alpha\omega j}{2} \qquad \text{................ 3.68}$$

Equation (3.67) represents two equations. Each equation defines a particular SDOF system having mass, stiffness, and damping equal to $\tilde{m}$, $\tilde{k}$, and ξ. Since a TMD is effective for a narrow frequency range, we have to decide on which modal resonant response is to be controlled with the TMD. Once this decision is made, the analysis can proceed using the selected modal equation and the initial equation for the TMD [i.e., Eq. (3.54)].

Suppose the first modal response is to be controlled. Taking j=1 in Eq. (3.67) leads to

$$\tilde{m}_1\ddot{q}_1 + \tilde{c}_1\dot{q}_1 + \tilde{k}_1 q_1 = \Phi_{11}p_1 + \Phi_{12}p_2 - [m_1\Phi_{11} + m_2\Phi_{12}]a_g + \Phi_{12}[k_d u_d + c_d\dot{u}_d]. \qquad \text{..3.69}$$

In general, u_2 is obtained by superposing the modal contributions

$$u_2 = \Phi_{12}q_1 + \Phi_{22}q_2 \qquad \text{................ 3.70}$$

solving for q_1

$$q_1 = [\frac{1}{\Phi 12}]u_2 \qquad \text{................ 3.71}$$

and then substituting in Eq. (3.69), we obtain

$$\tilde{m}_{1e}\ddot{u}_2 + \tilde{c}_{1e}\,\dot{u}_2 + \tilde{k}_{1e}u_2 = k_d u_d + c_d\dot{u}_d + \tilde{p}_{1e} - \Gamma_{1e}\,\tilde{m}_{1e}\,a_g \qquad \text{................ 3.72}$$

where $\tilde{m}_{1e}$, $\tilde{c}_{1e}$, $\tilde{k}_{1e}$, $\tilde{p}_{1e}$, and Γ_{1e} represent the equivalent SDOF parameters for the combination of mode 1 and node 2, the node at which the TMD is attached.

Their definition equations are

$$\tilde{m}_{1e} = [\frac{1}{\Phi^2 12}]\tilde{m}_1 \qquad \text{................ 3.73}$$

$$\tilde{k}_{1e} = [\frac{1}{\Phi^2 12}]\tilde{k}_1 \qquad \text{................ 3.74}$$

$$\tilde{c}_{1e} = \alpha\,\tilde{k}_{1e} \qquad \text{................ 3.75}$$

$$\tilde{p}_{1e} = \frac{\Phi 11p1 + \Phi 12p2}{\Phi 12} \qquad \text{................ 3.76}$$

$$\Gamma_{1e} = \frac{\Phi 12}{\tilde{m}1}(m_1\Phi_{11} + m_2\Phi_{22}) \qquad \text{................ 3.77}$$

Equations (3.54) and (3.72) are similar in form to the SDOF equations treated in the previous section. Both set of equations are compared next.

TMD equation

$$m_d\ddot{u}_d + c_d\dot{u}_d + k_du_d = -m_d(\ddot{u} + a_g) \quad \text{................} 3.78$$

versus

$$m_d\ddot{u}_d + c_d\dot{u}_d + k_du_d = -m_d(\ddot{u}_2 + a_g) \quad \text{................} 3.79$$

Primary mass equation

$$m\ddot{u} + c\dot{u} + ku = c_d\dot{u}_d + k_du_d + p - ma_g \quad \text{................} 3.80$$

versus

$$\tilde{m}_{1e}\ddot{u}_2 + \tilde{c}_{1e}\,\dot{u}_2 + \tilde{k}_{1e}u_2 = c_d\dot{u}_d + k_du_d + \tilde{p}_{1e} - \Gamma_{1e}\,\tilde{m}_{1e}\,a_g \quad \text{................} 3.81$$

Taking

$u_2 \equiv u$ $\quad$ $\tilde{m}_{1e} \equiv m$ $\quad$ $\tilde{c}_{1e} \equiv c$ $\quad$ $\tilde{k}_{1e} \equiv k$ $\quad$ $\tilde{p}_{1e} \equiv p$ $\quad$ $\Gamma_{1e} \equiv \Gamma$

transforms the primary mass equation for the MDOF case to

$$m\ddot{u} + c\dot{u} + ku = c_d\dot{u}_d + k_du_d + p - \Gamma ma_g \quad \text{................} 3.82$$

which differs from the corresponding SDOF equation by the factor Γ. Therefore, the solution for ground excitation generated earlier has to be modified to account for the presence of Γ. The "generalized" solution is written in the same form as the SDOF case. We need only to modify the terms associated with a_g (i.e., H6, H8 and δ6, δ8). Their expanded form is as follows:

$$H_6 = \frac{\sqrt{[(1+\bar{m})f2 - \Gamma\rho 2]^2 + [2\xi d\rho f(\Gamma+\bar{m})]^2}}{|\,D3|} \quad \text{................} 3.83$$

$$H_8 = \frac{\sqrt{[1+\rho^2(\Gamma-1)]^2 + [2\xi\rho]^2}}{|\,D3|} \quad \text{................} 3.84$$

$$\tan_{\alpha_2} = \frac{2\xi d f\rho(\Gamma+\bar{m})}{f^2(\Gamma+\bar{m}) - \Gamma\rho^2} \quad \text{................} 3.85$$

$$\tan\alpha_2 = \frac{2\xi\rho}{1+(\Gamma-1)\rho^2} \quad \text{................} 3.86$$

$$\delta_6 = a_2 - \delta_7 \quad \text{................} 3.87$$

$$\delta_8 = a_3 - \delta_7 \quad \text{................} 3.88$$

where $|D_3|$ is defined by Eq. (3.46), and δ_7 is given by Eq. (3.50).

CHAPTER FOUR

OPTIMIZATION OF TMD

In all engineering fields, designers try to find solutions showing good performances, which satisfy several requirements. Using optimization techniques, designers can obtain the optimum, within the imposed conditions. Structures designed in this way are safer, more reliable and less expensive than the traditional ones. Generally speaking, the structural optimization problem can be formulated as the selection of a set of decision variables (that are the parameters of the design which characterize structural configuration), collected in the so called design vector (DV) b, over a possible admissible domain R. The optimum DV must be able to minimize a given Objective Function (O.F.) and to satisfy, in general, some constraint conditions.

In deterministic based optimization problems, the aim is to minimize a suitable objective function under certain deterministic behavioural constraints, usually on stresses and/or displacements. In reliability-based optimum design, probabilistic constraints are utilized in order to take into account randomness in some involved parameters. In general, probabilistic constraints define the feasible region of the design space by restricting the probability that a deterministic constraint is violated within the allowable probability of violation.

The optimization problem so defined, has been stated first by Nigam (1972) and transformed into a standard nonlinear programming one. It could be stated as:

$$\begin{aligned} &\text{Minimize} && \text{O.F.}\ (b, t) \\ &\text{Subject to} && p_f\ (b, t) \leq \overline{p}_f \\ & && b \in R \end{aligned}$$

The O.F. could be defined either by a standard deterministic way (e.g. the total structural weight or the elements volume) or in a stochastic one. In the latter case, response statistics could be used, as covariance or spectral moments of variables of interest (e.g. displacement, acceleration or structural stress in relevant elements).

The reliability constraint imposes that possible DVs must guarantee a failure probability p_f (b, t) smaller than a given maximum acceptable one, defined as $\overline{p}_f$.

In this study, the mechanical parameters of TMD are collected in the three elements design vector:

$b = (f, \xi_d, \bar{m})^T$

The criterion selected for the optimization is the minimization of the peak displacement of the structure. The O.F. is thus defined by the amplification of the pseudo-static responses:

O.F. = $H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8$

In this study, the global system failure is associated to an excessive TMD displacement, and for this reason a limitation to displacement X_d is imposed, as usually done in many real practical engineering applications. This limitation could be related with the amplification of the Pseudo-static responses. Thus, the TMD optimum design is finally posed in a complete framework as follows:

Minimize O.F. = $H_1, H_2, H_3, H_4, H_5, H_6, H_7, H_8$

Subject to $X_T(\tau) \leq X_T^{max} \mid \tau \in [0, T]$

Where X_T^{max} is the admissible TMD displacement and T is the vibration duration.

Constraints

Frequency: $0.8 \leq f \leq 1.0$

TMD damping ratio: $0 \leq \xi_d \leq 0.2$

Mass ratio ($\bar{m}$): [0.001 : 0.001 : 0.1]

Primary structure damping ratio (ξ): [0.0, 0.01, 0.02, 0.05, 0.1]

Optimization methods

The available method of optimization may conveniently be divided into two distinctly different categories as follows (U. Kirsch 1981):

1. **Analytical method**: - Which usually employ the mathematical theory of calculus, variation method etc… in studies of optimal layouts or geometrical forms of simple structural elements, such as beams, columns, and plates. These methods are most suitable for such fundamental studies of single structural components, but are usually not intended to handle larger structural systems. The structural design is represented by a number of unknown functions and the goal is to find the form of these functions. The optimal design

is theoretically found exactly through the solution of a system of equations expressing the conditions for optimality.

2. **Numerical method**: - Which are usually employing a branch in the field of numerical mathematics called programming method. The recent developments in this branch are closely related to the rapid growth in computing capacity affected by the development of computers. In numerical methods, a near optimal design is automatically generated in iterative manner. An initial guess is used as starting points for a systematic search for better design. The search is terminated when certain criteria are satisfied; indicating that the current design is sufficiently close to the optimum. Problems solved by numerical methods are called *finite optimization problems* or *discrete parameter optimization problems.* This is due to the fact that they can be formulated by a finite number of variables. Assignment of numerical values to these variables specifies a unique structure. Design optimization of practical structures is accomplished mainly by the use of finite formulations.

There is no single method available for solving all optimization problems efficiently. Hence a number of optimization methods have been developed for solving different types of optimization problem. In this study the nature of the optimization problem is polynomial type and the optimization method selected to solve the problem is an iterative numerical search methods using software MATLAB.

In the general setting, the goal is to find a point that minimizes the objective function and at the same time satisfies the constraints. We refer to any point that satisfies the constraints as a feasible point.

4.1 Single Degree of Freedom Systems

4.1.1 Undamped structure: Undamped TMD

For the best minimization from eq. 3.9, Selecting the mass ratio and damper frequency ratio such that:

$$1-\rho_d^2+\bar{m}=0 \quad \text{.. 4.1}$$

Reduces the solution to:

$$\hat{u} = \frac{\hat{p}}{k} \quad \text{.. 4.2}$$

$$\hat{u}_d = -\frac{\hat{p}}{k}\rho^2 + \frac{m\hat{a}g}{kd} \quad \text{.. 4.3}$$

This choice isolates the primary mass from ground motion and reduces the response due to external force to the pseudo static value, $\hat{p}/k$. A typical range for $\bar{m}$ is 0.01 to 0.1. Then the optimal damper frequency is very close to the forcing frequency. The exact relationship follows from Eq. (4.1).

$$\omega_d|_{opt} = \frac{\Omega}{\sqrt{1+\bar{m}}} \quad \text{.. 4.4}$$

We determine the corresponding damper stiffness with

$$k_d|_{opt} = [\omega_d|_{opt}]^2 m_d = \frac{\Omega^2 m\bar{m}}{1+\bar{m}} \quad \text{.. 4.5}$$

Finally, substituting for k_d, Eq. (4.3) takes the following form:

$$\hat{u}_d = H\left(\left|\frac{\hat{p}}{k}\right| + \left|\frac{\hat{a}g}{\Omega^2}\right|\right) \quad \text{.. 4.6}$$

$$H_{opt} = \frac{1+\bar{m}}{\bar{m}} \quad \text{.. 4.7}$$

We specify the amount of relative displacement for the damper and determine $\bar{m}$ from figure 4.1. Given $\bar{m}$ and Ω, the stiffness is found using Eq. (4.5). It should be noted that this stiffness applies for a particular forcing frequency. Once the mass damper properties are defined, Eqs. (3.9) and (3.10) can be used to determine the response for a different forcing frequency. The primary mass will move under ground motion excitation in this case. The critical loading scenario is the resonant condition, Ω=ω. The optimal values of frequency ratios are plotted in figure 4.2.

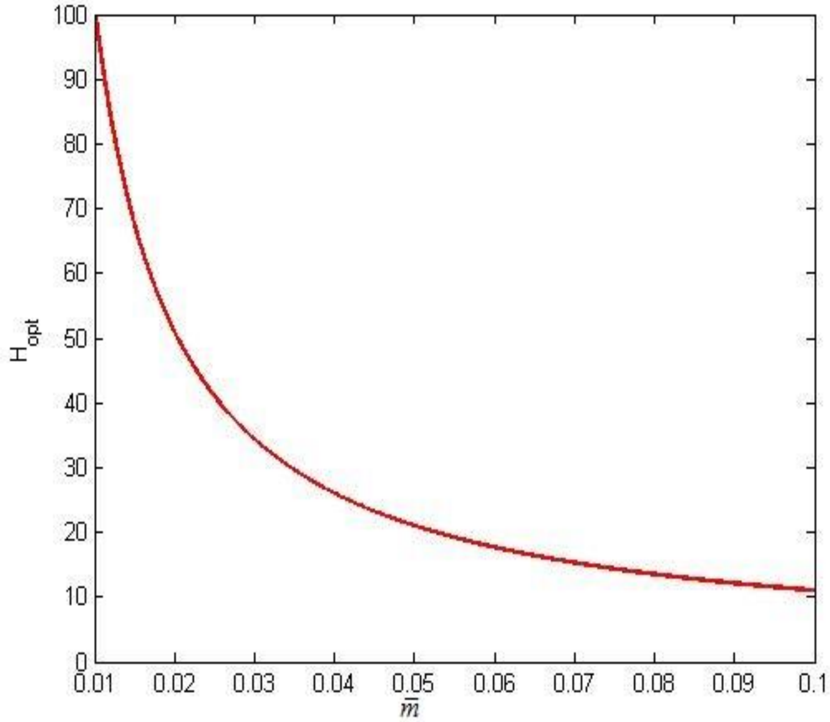


Figure 4.1 Maximum dynamic amplification factor for TMD

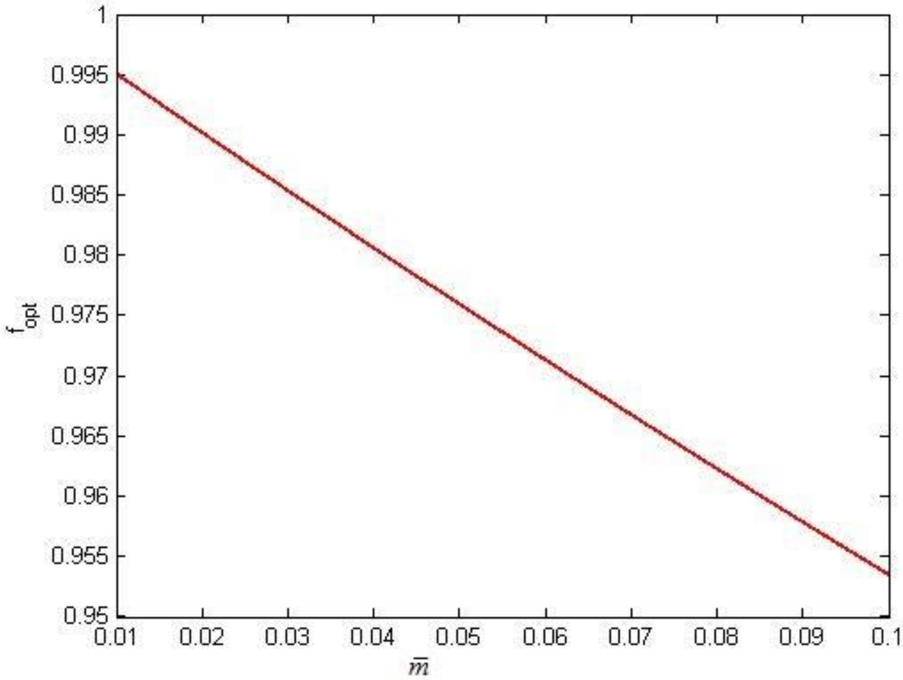


Figure 4.2 Optimum tuning frequency ratio for TMD, f_{opt}

4.1.2 Undamped structure: Damped TMD

The solution corresponding to ground motion is examined and the optimal values of the damper properties for this loading condition are established.

Figure 4.3 shows the variation of H_2 with forcing frequency for specific values of damper mass $\bar{m}$ and frequency ratio f, and various values of the damper damping ratio, ξ_d. When $\xi_d=0$, there are two peaks with infinite amplitude located on each side of $\rho=1$. As ξ_d is increased, the peaks approach each other and then merge into a single peak located at $\rho\approx1$. The behavior of the amplitudes suggests that there is an optimal value of ξ_d for a given damper configuration (m_d and k_d or, equivalently, $\bar{m}$ and f). Another key observation is that all the curves pass through two common points, P and Q. Since these curves correspond to different values of ξ_d, the location of P and Q must depend only on $\bar{m}$ and f.

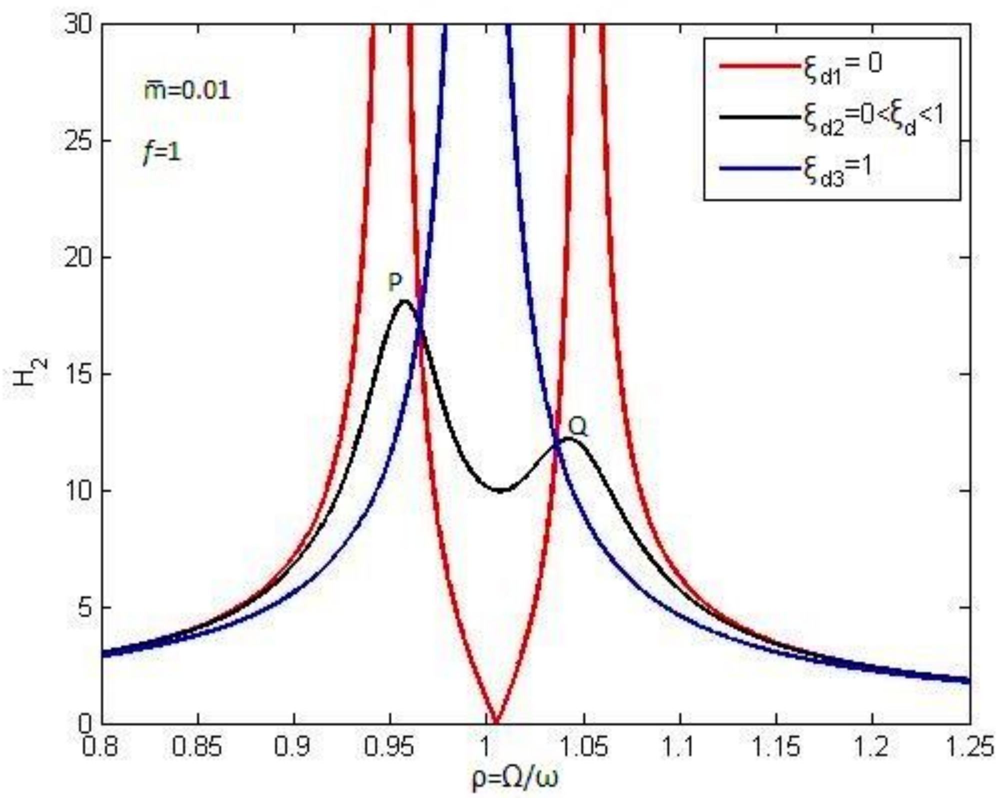


Figure 4.3 Plot of H_2 versus ρ

Proceeding with this line of reasoning, the expression for H_2 can be written as

$$H_2 = \sqrt{\frac{a_1{}^2+\xi d^2 a_2{}^2}{a_3{}^2+\xi d^2 a_4{}^2}} = \frac{a_2}{a_4}\sqrt{\frac{a_1{}^2/a_2{}^2+\xi d^2}{a_3{}^2/a_4{}^2+\xi d^2}} \quad \ldots\ldots\ldots\ldots\ldots 4.8$$

Where the a terms are functions of $\bar{m}$, ρ, and f. Then for H_2 to be independent of ξ_d, the following condition must be satisfied:

$$\left|\frac{a_1}{a_2}\right| = \left|\frac{a_3}{a_4}\right| \quad \ldots\ldots\ldots\ldots\ldots 4.9$$

The corresponding values for H_2 are

$$H_2|_{P,\,Q} = \left|\frac{a_2}{a_4}\right| \quad \ldots\ldots\ldots\ldots\ldots 4.10$$

Substituting for a the terms in Eq. (4.9), we obtain a quadratic equation for ρ^2:

$$\rho^4 - \left[(1+\bar{m})f^2 + \frac{1+0.5\bar{m}}{1+\bar{m}}\right]\rho^2 + f^2 = 0 \ldots\ldots\ldots\ldots\ldots 4.11$$

The two positive roots ρ_1 and ρ_2 are the frequency ratios corresponding to points P and Q. Similarly, Eq. (4.10) expands to

$$H_2|_{P,\,Q} = \frac{1+\bar{m}}{|1-\rho^2{}_{1,2}(1+\bar{m})|} \quad \ldots\ldots\ldots\ldots\ldots 4.12$$

Figure 4.3 shows different values for H_2 at points P and Q. For optimal behavior, we want to minimize the maximum amplitude. As a first step, we require the values of H_2 for ρ_1 and ρ_2 to be equal. This produces a distribution that is symmetrical about $\rho^2 = 1/(1+\bar{m})$, as illustrated in Figure 4.4. Then, by increasing the damping ratio, ξ_d, we can lower the peak amplitudes until the peaks coincide with points P and Q. This state represents the optimal performance of the TMD system. A further increase in ξ_d causes the peaks to merge and the amplitude to increase beyond the optimal value.

Requiring the amplitudes to be equal at P and Q is equivalent to the following condition on the roots:

$$|1-\rho^2{}_1(1+\bar{m})| = |1-\rho^2{}_2(1+\bar{m})| \quad \ldots\ldots\ldots\ldots\ldots 4.13$$

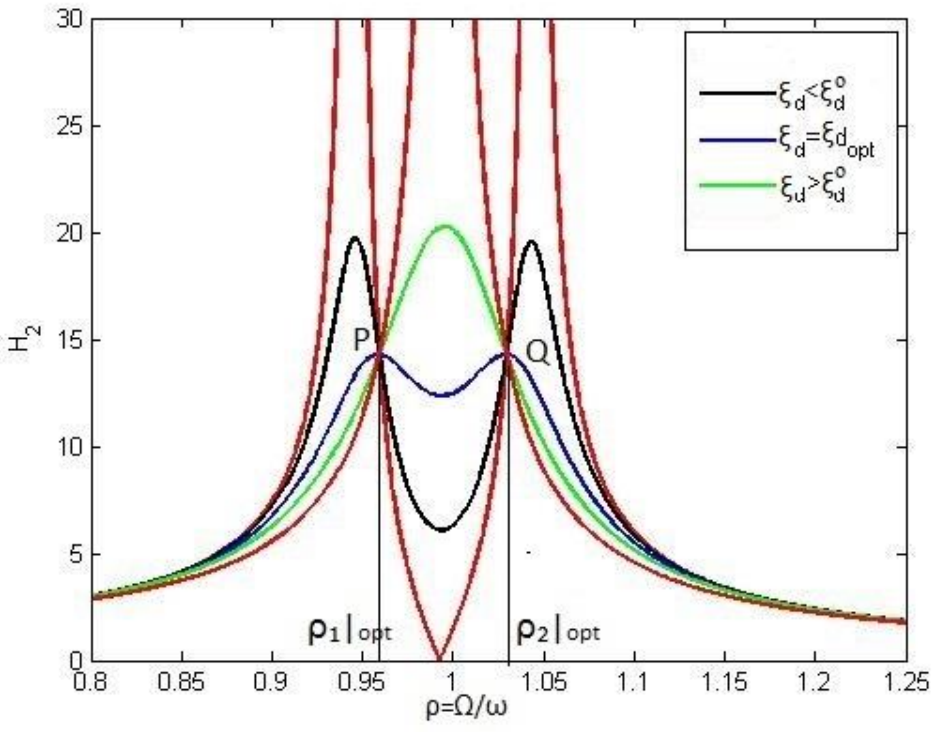


Figure 4.4 Plot of H_2 versus ρ for f_{opt}

Then, substituting for ρ_1 and ρ_2 using Eq. (4.11), we obtain a relation between the optimal tuning frequency and the mass ratio:

$$f_{opt} = \frac{\sqrt{1-0.5\bar{m}}}{1+\bar{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 4.14$$

$$\omega_d|_{opt} = f_{opt}\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 4.15$$

The corresponding roots and optimal amplification factors are

$$\rho_{1,2}|_{opt} = \sqrt{\frac{1\pm\sqrt{0.5\bar{m}}}{1+\bar{m}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 4.16$$

$$H_2|_{opt} = \frac{1+\bar{m}}{\sqrt{0.5\bar{m}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 4.17$$

The expression for the optimal damping at the optimal tuning frequency is

$$\xi_d|_{opt} = \sqrt{\frac{\bar{m}(3-\sqrt{0.5\bar{m}})}{8(1+\bar{m})(1-0.5\bar{m})}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. 4.18$$

Figures 4.5 through 4.8 show the variation of the optimal parameters with the mass ratio, m

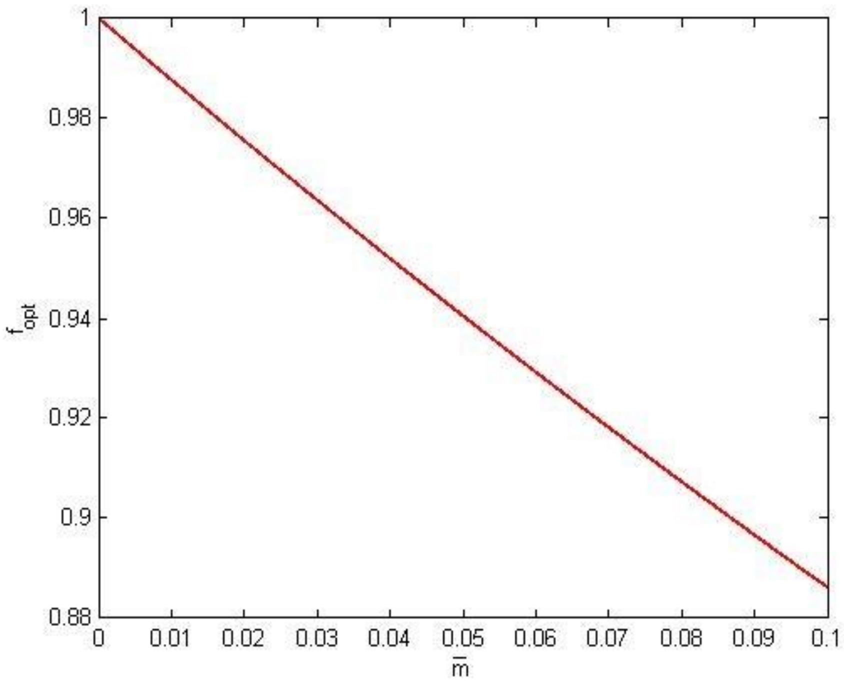


Figure 4.5 Optimum tuning frequency ratio, f_{opt}

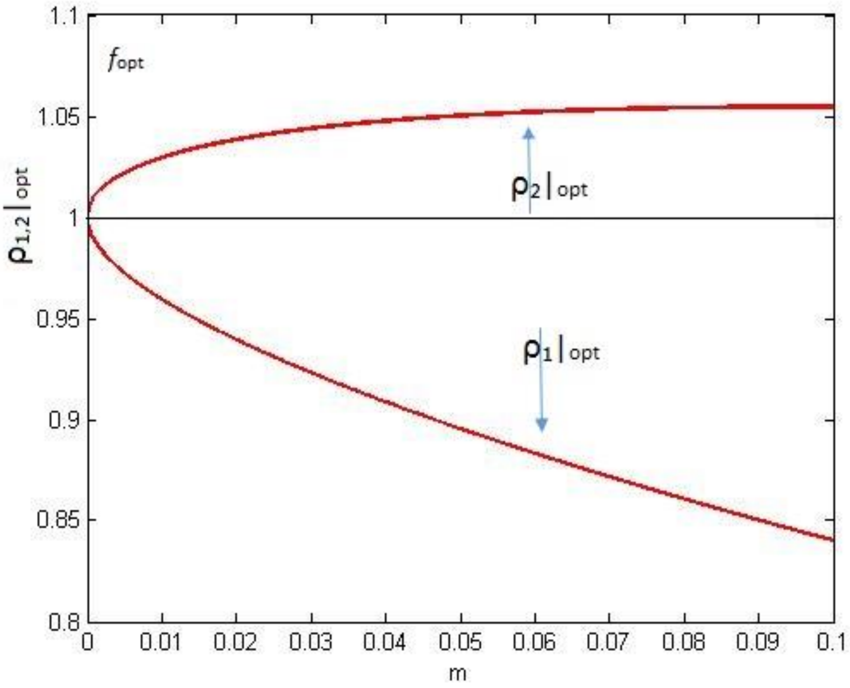


Figure 4.6 Input frequency ratio at which the response is independent of damping

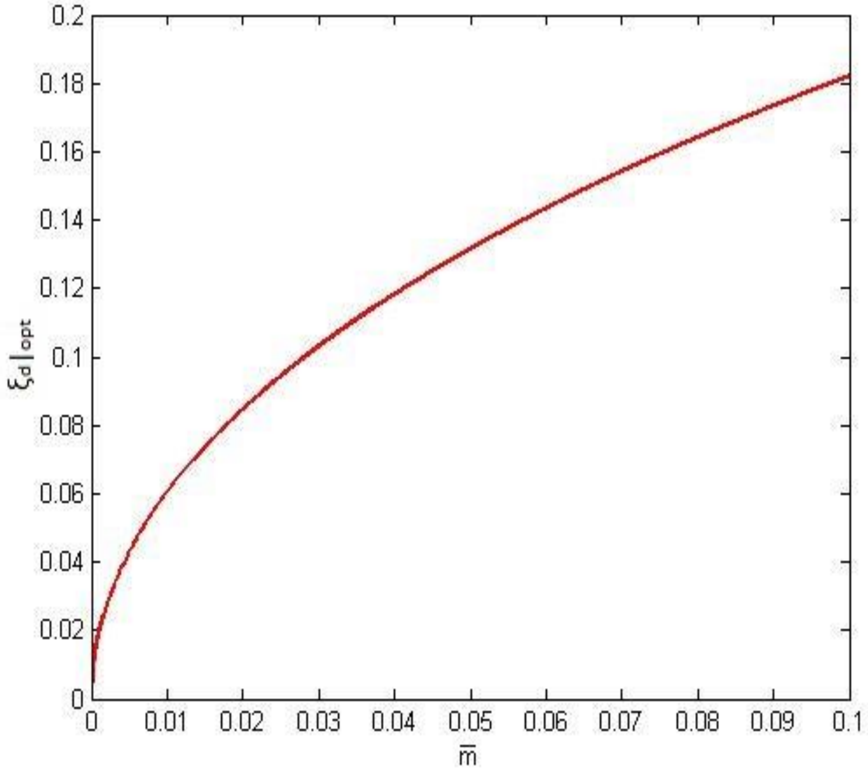


Figure 4.7 Optimal damping ratio of TMD.

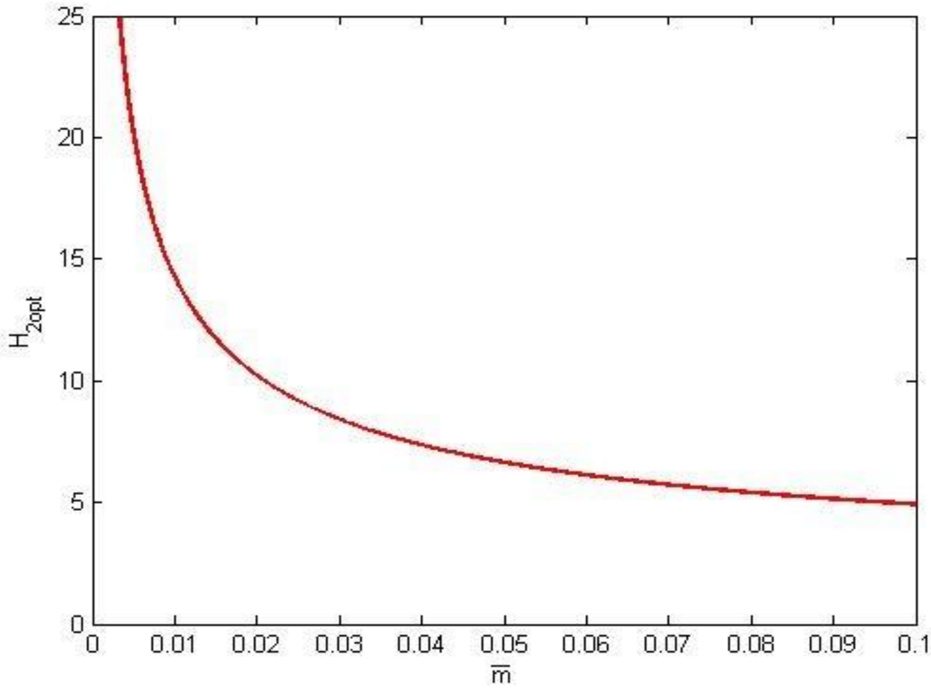


Figure 4.8 Maximum dynamic amplification factor for SDOF system (optimal tuning and damping)

The response of the damper is defined by Eq. (3.27). Specializing this equation for the optimal conditions leads to the plot of amplification versus mass ratio contained in Figure 4.9. A comparison of the damper motion with respect to the motion of the primary mass for optimal conditions is shown in Figure 4.10.

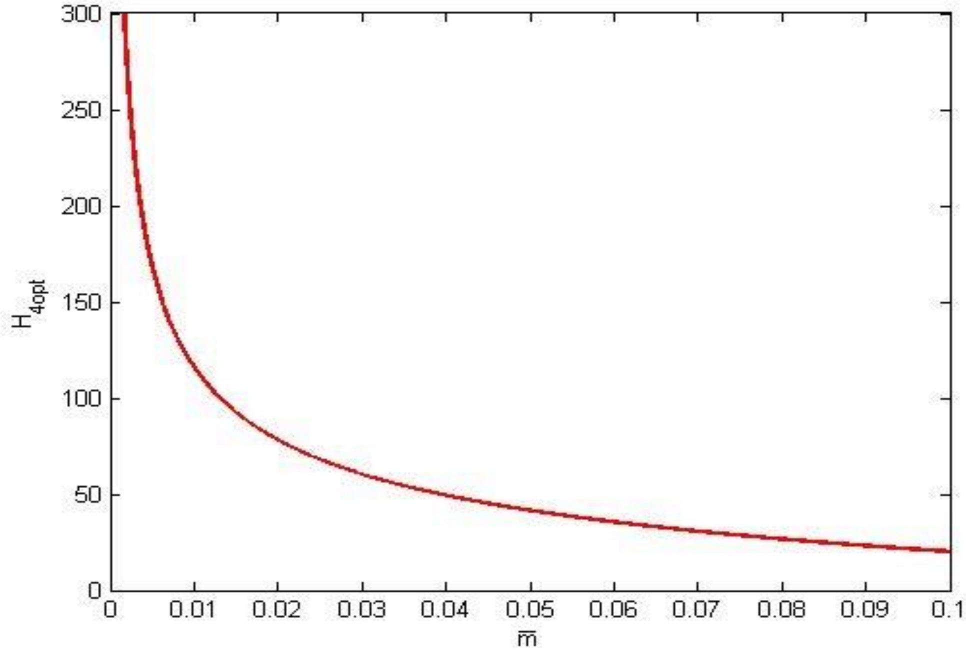


Figure 4.9 Maximum dynamic amplification factor for TMD

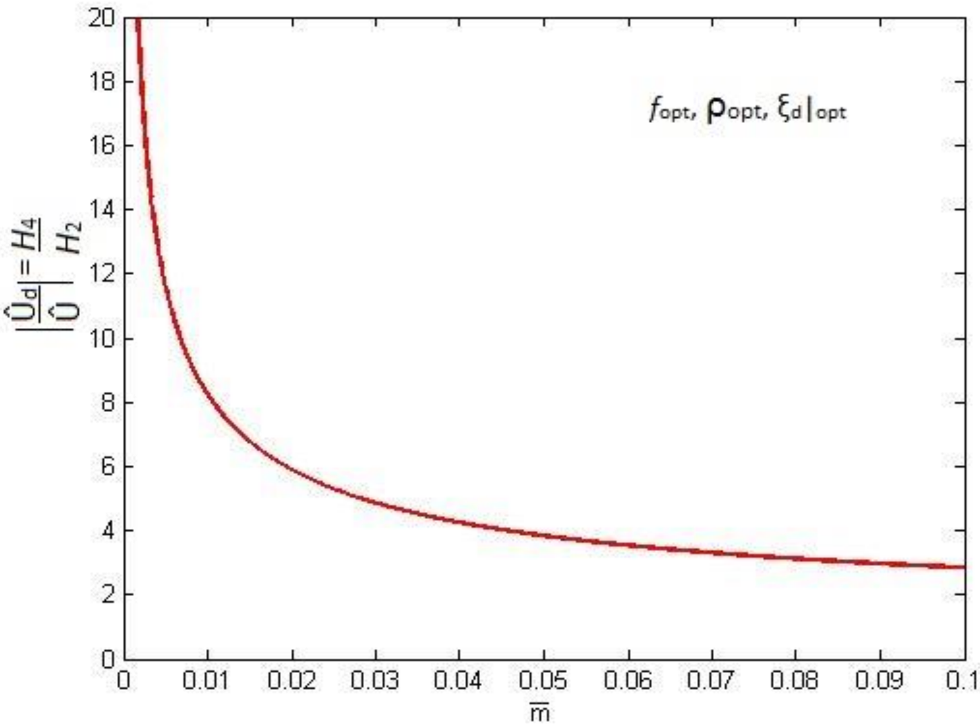


Figure 4.10 Ratio of maximum TMD amplitude to maximum system amplitude.

Lastly, response curves for a typical mass ratio, $\bar{m}$=0.01, and optimal tuning are plotted in Figure 4.11 and Figure 4.12. The response for no damper is also plotted in Figure 4.11. We observe that the effect of the damper is to limit the motion in a frequency range centered on the natural frequency of the primary mass and extending about 0.15ω. Outside of this range, the motion is not significantly influenced by the damper.

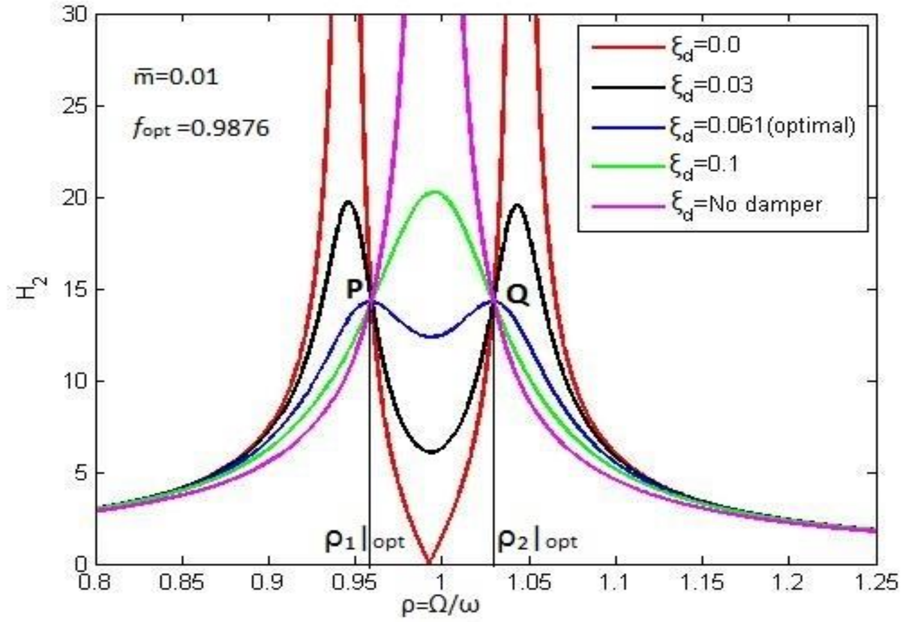


Figure 4.11 Response curves for amplitude of system with optimally tuned TMD.

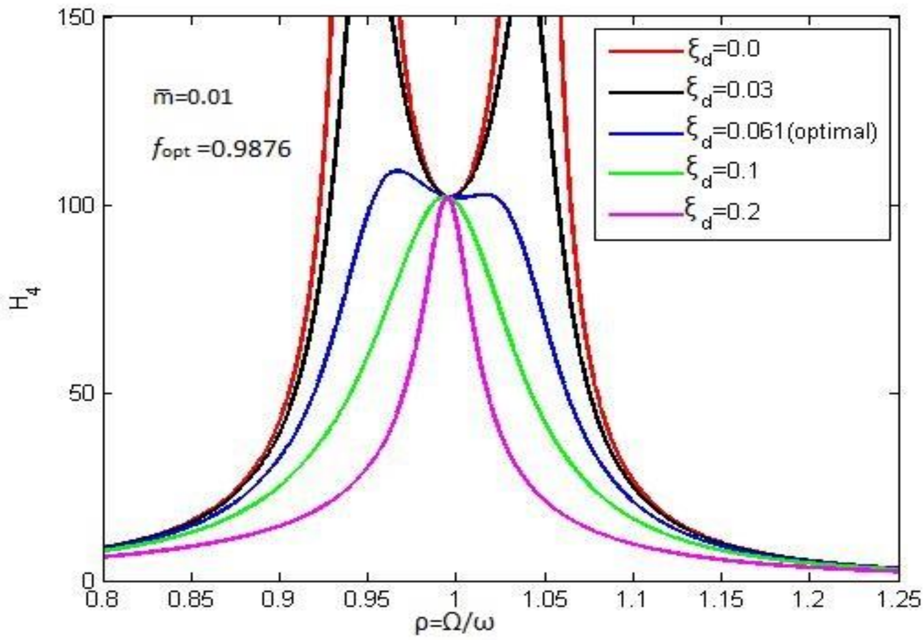


Figure 4.12 Response curves for amplitude of optimally tuned TMD.

The maximum amplification for a damped SDOF system without a TMD, undergoing harmonic excitation, is given by

$$H = \frac{1}{2\xi\sqrt{1-\xi^2}} \qquad \text{.. 4.19}$$

Since ξ is small, a reasonable approximation is

$$H \approx \frac{1}{2\xi} \qquad \text{.. 4.20}$$

Expressing the optimal H_2 in a similar form provides a measure of the equivalent damping ratio ξ_e for the primary mass:

$$\xi e = \frac{1}{2H_2|opt} \qquad \text{.. 4.21}$$

Figure 4.13 shows the variation of ξe with the mass ratio. A mass ratio of 0.02 is equivalent to about 5% damping in the primary system.

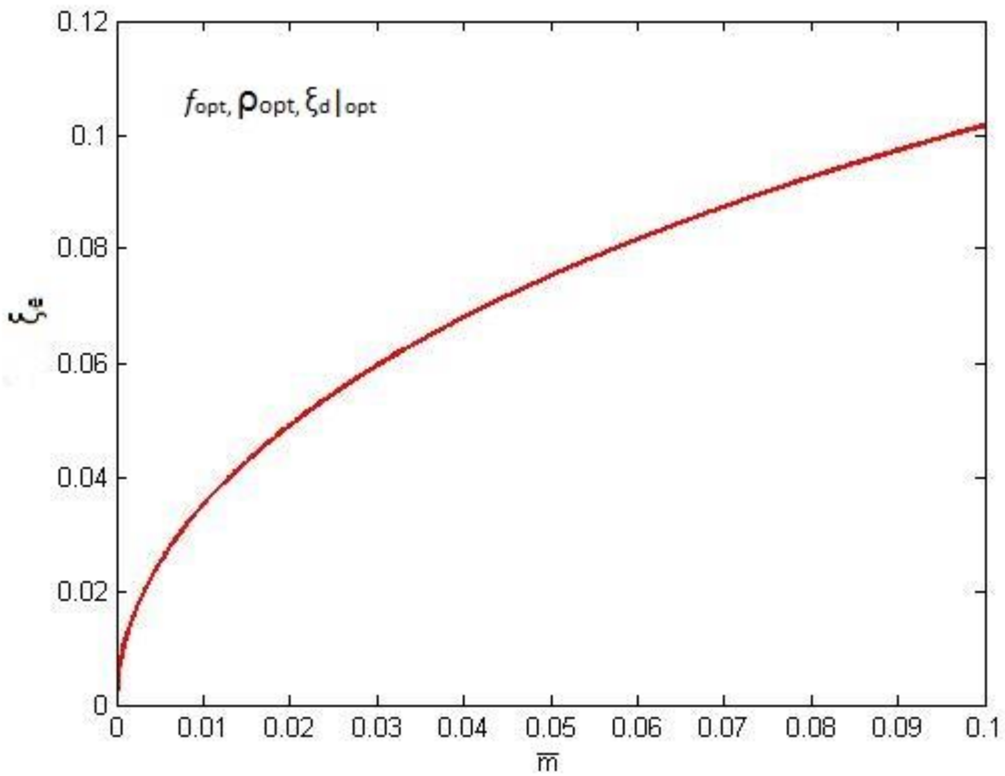


Figure 4.13 Equivalent damping ratio for optimally tuned TMD.

4.1.3 Damped structure: Damped TMD

In this case since $|D_3|$ involves ξ, the parameters also depend on ξ, we cannot establish analytical expressions for the optimal tuning frequency and optimal damping ratio in terms of the mass ratio. Using matlab tool minimax optimization Numerical simulations can be applied to evaluate H_5 and H_7 for a range of ρ, given the values for $\bar{m}$, ξ, f, and ξ_d. Starting with specific values for $\bar{m}$ and ξ, plots of H_6 versus ρ can be generated for a range of f and ξ_d. Each H_5-ρ plot has a peak value of H_5. The particular combination of f and ξ_d that corresponds to the lowest peak value of H_5 is taken as the optimal state. Repeating this

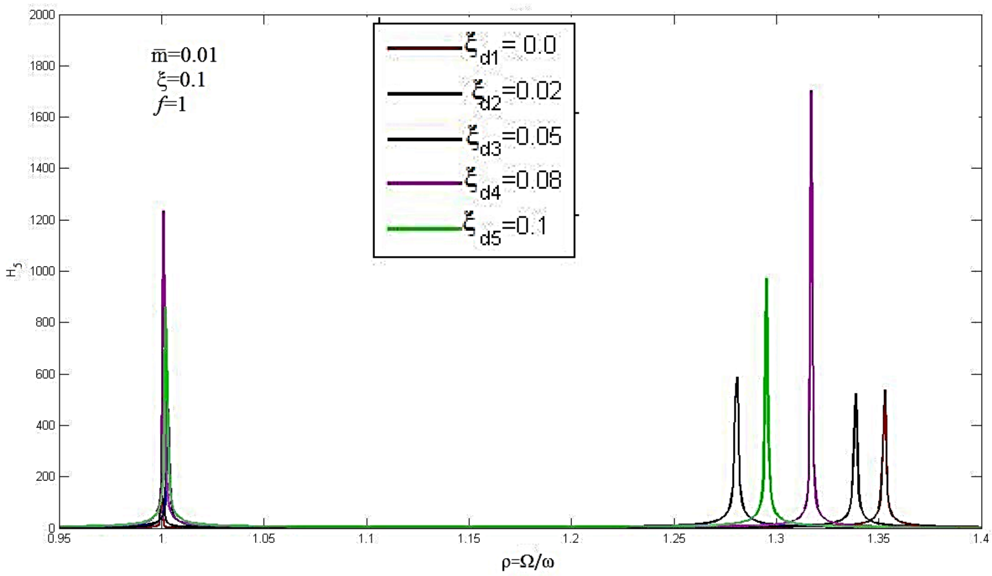


Figure 4.14 Plot of H_6 versus ρ

process for different values of $\bar{m}$ and ξ produces the behavioral data needed to design the damper system.

Figure 4.15 shows the variation of the maximum value of H_5 for the optimal state. The corresponding response of the damper is plotted in Figure 4.16. Adding damping to the primary mass has an appreciable effect for small $\bar{m}$.

Noting Eqs. (3.40) and (3.41), the ratio of damper displacement to primary mass displacement is given by

$$\frac{|\hat{U}d|}{|\hat{U}|} = \frac{H_8}{H_6} = \frac{\sqrt{1+[2\xi\rho]^2}}{\sqrt{[(1+\bar{m})f^2-\rho^2]^2+[2\xi d\rho f(1+\bar{m})]^2}} \quad \text{4.22}$$

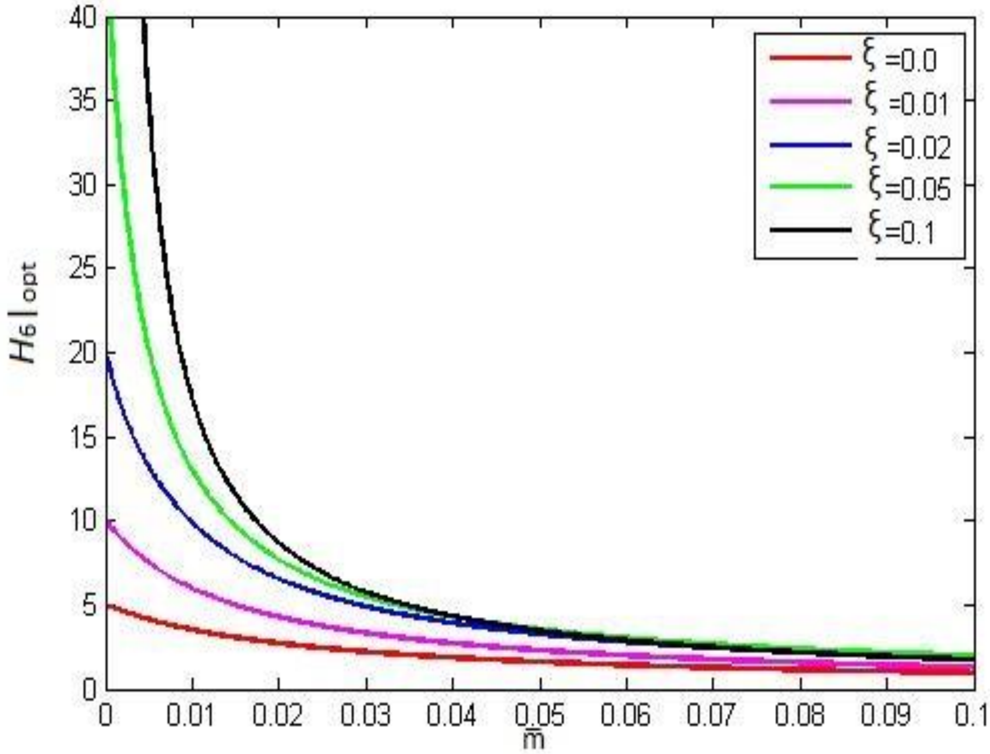


Figure 4.15 Maximum dynamic amplification factor for damped SDOF system.

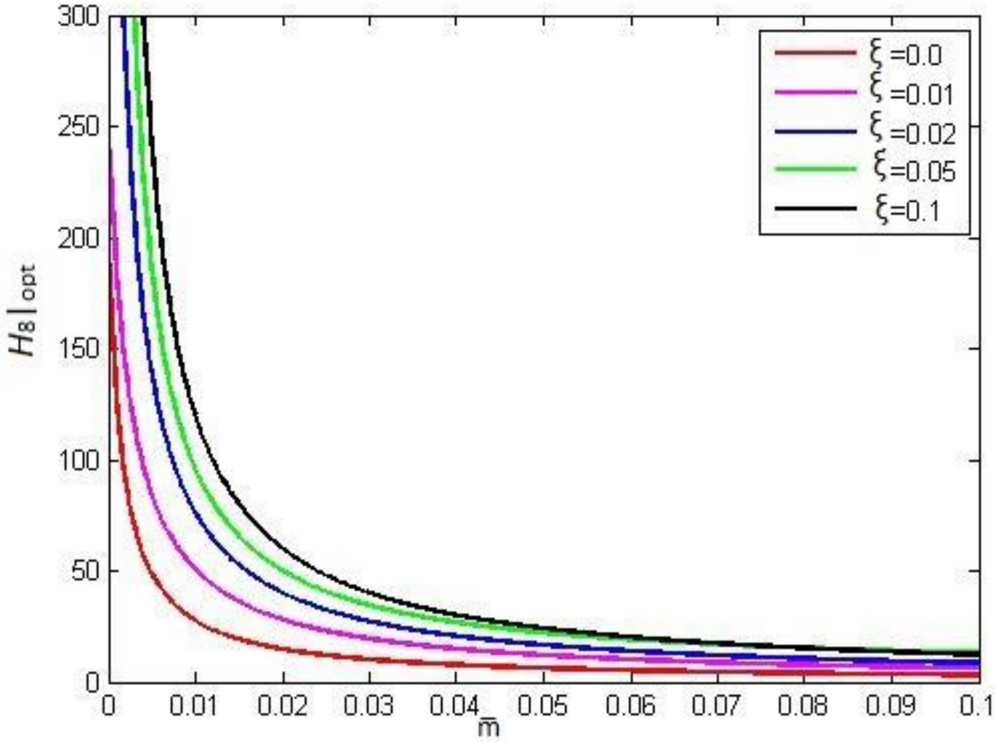


Figure 4.16 Maximum dynamic amplification factor for TMD.

The optimal values of the frequency and damping ratios generated through simulation are plotted in Figures 4.17 and 4.18. Lastly, using Eq. (4.21), $H_6|_{opt}$ can be converted to an equivalent damping ratio for the primary system.

$$\xi_e = \frac{1}{2H_6|opt} \qquad \text{……………………………………….. 4.23}$$

Figure 4.19 shows the variation of ξ_e with $\bar{m}$ and ξ.

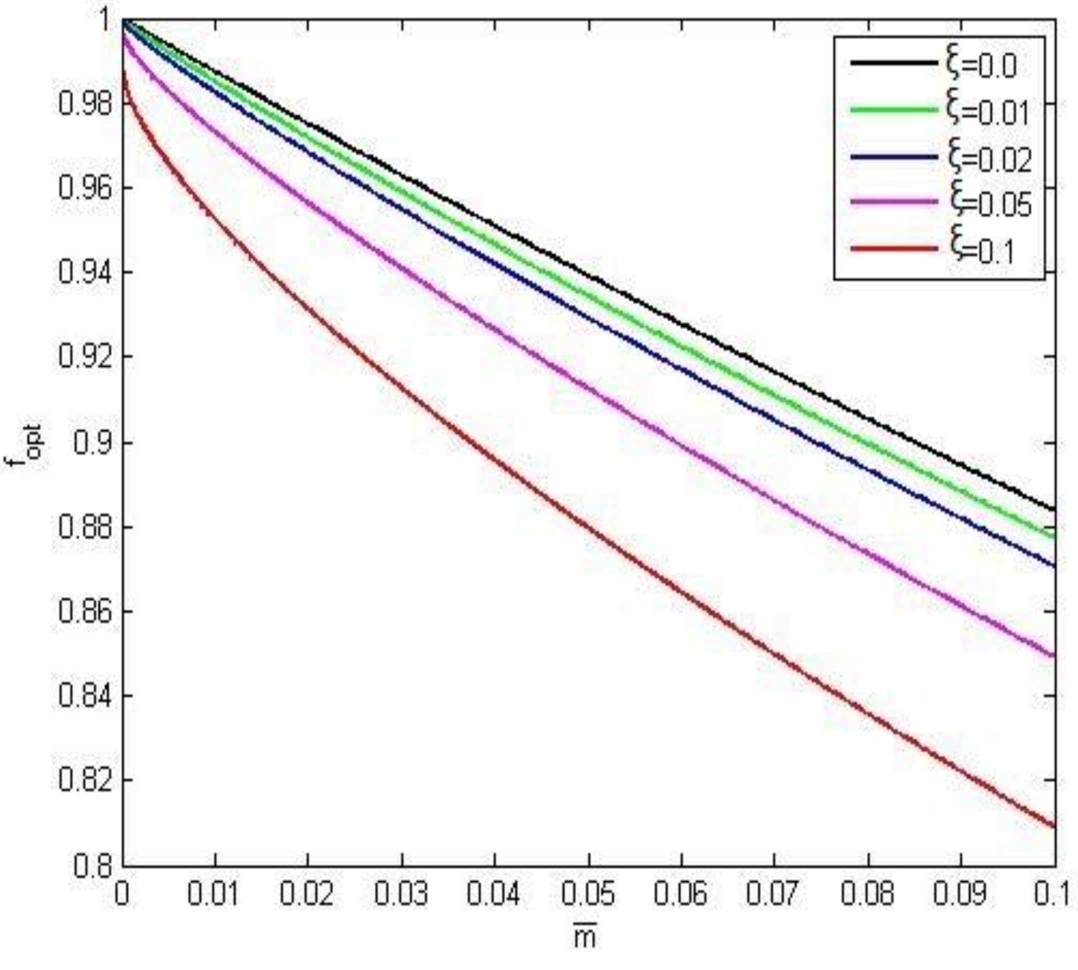


Figure 4.17 Optimum tuning frequency ratio for TMD, fopt.

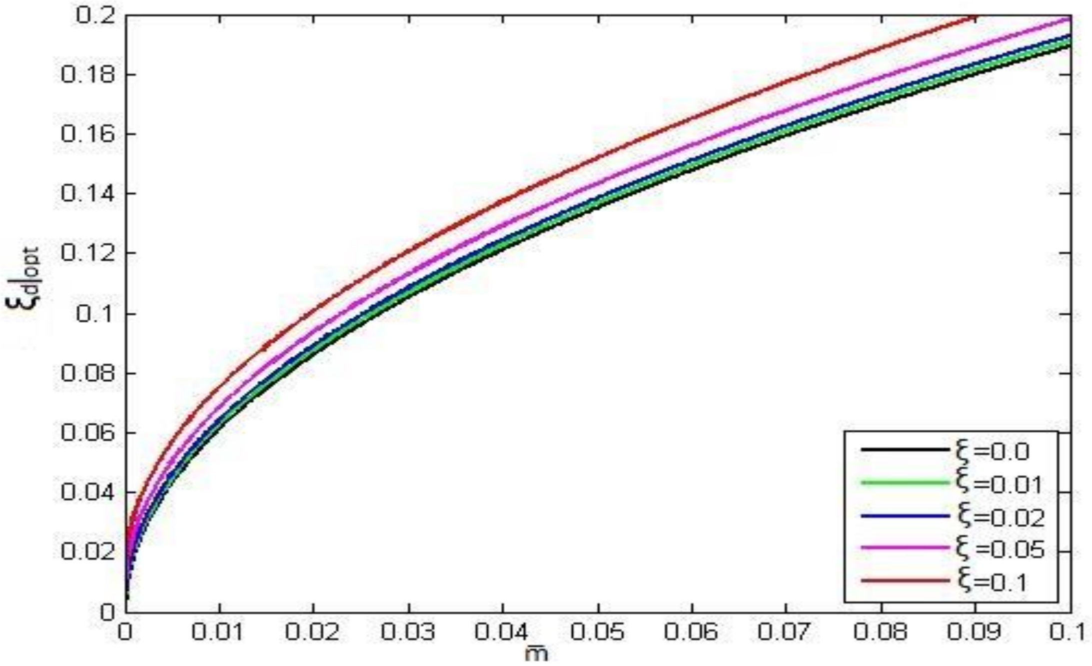


Figure 4.18 Optimal damping ratio for TMD.

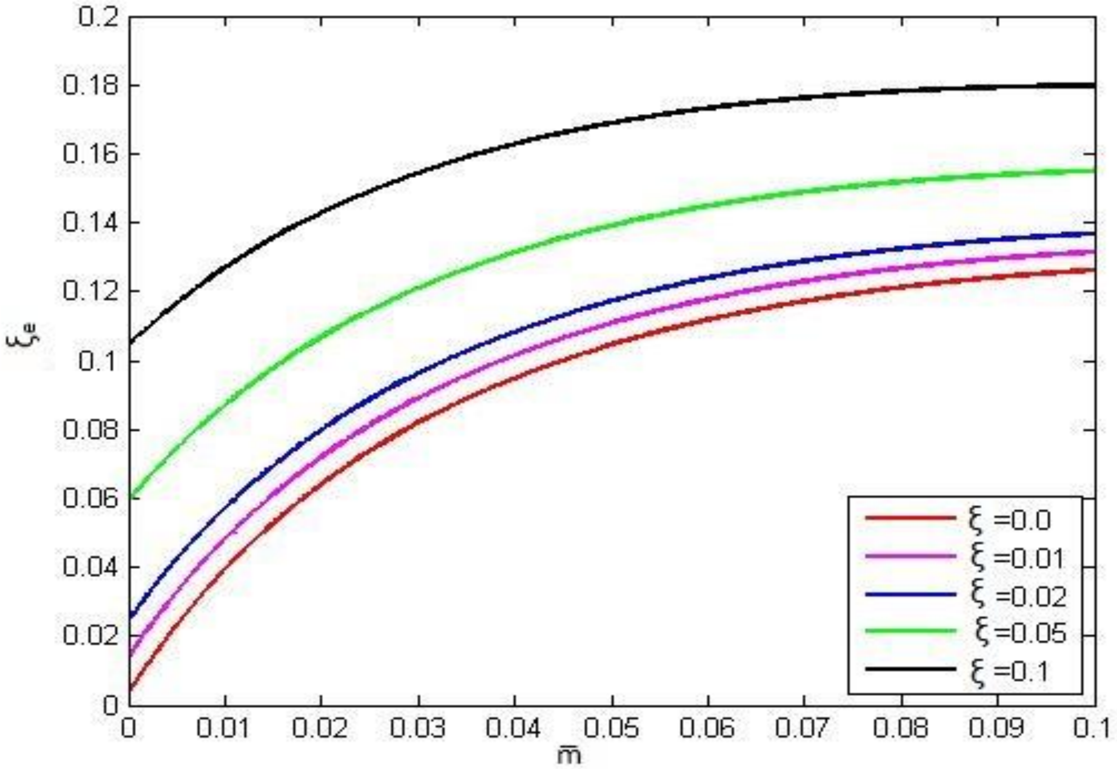


Figure 4.19 Equivalent damping ratio for optimally tuned TMD.

4.2 Multi Degree of Freedom System

From equation Eq. (3.83) Eq. and (3.84), we proceed as described in Section 4.1. The mass ratio is defined in terms of the equivalent SDOF mass.

$$\bar{m} = \frac{md}{\tilde{m}_{1e}} \qquad \text{.. 4.24}$$

Given $\bar{m}$ and ξ_1, we find the tuning frequency and damper damping ratio using Figures 4.17 and 4.18. The damper parameters are determined with

$$m_d = \bar{m}\tilde{m}_{1e} \qquad \text{.. 4.25}$$

$$\omega_d = f_{opt}\omega_1 \qquad \text{.. 4.26}$$

$$c_d = 2\xi_d|_{opt}\omega_d m_d \qquad \text{.. 4.27}$$

Expanding the expression for the damper mass,

$$m_d = \bar{m}\tilde{m}_{1e} = \frac{\bar{m}[\phi_1 M \phi_1}{\phi^2{}_{12}} \qquad \text{.. 4.28}$$

shows that we should select the TMD location to coincide with the maximum amplitude of the mode shape that is being controlled. In this case, the first mode is the target mode, and ϕ_{12} is the maximum amplitude for ϕ_1.

This derivation can be readily generalized to allow for tuning on the *ith* modal frequency. We write Eq. (3.71) as

$$q_i \approx \left[\frac{1}{\phi i_2}\right] u_2 \qquad \text{.. 4.29}$$

where *i* is either 1 or 2. The equivalent parameters are

$$\tilde{m}_{ie} = \left[\frac{1}{\phi i_2}\right] \tilde{m}_i \qquad \text{.. 4.30}$$

$$\tilde{k}_{ie} = \omega^2{}_i \, \tilde{m}_{ie} \qquad \text{.. 4.31}$$

Given $\tilde{m}_{ie}$ and ξ_i, we specify $\bar{m}$ and find the optimal tuning with

$$\omega_d = f_{opt}\omega_i \qquad \text{.. 4.32}$$

The general case of a MDOF system with a tuned mass damper connected to the n^{th} degree of freedom is treated in a similar manner. Using the notation defined previously, the j^{th} modal equation can be expressed as

$$\bar{m}_j\ddot{q}_j + \tilde{c}_j\dot{q}_j + \tilde{k}_j q_j = \tilde{p}_j + \phi_j n[k_d u_d + c_d u_d] \qquad j = 1, 2, \ldots \quad 4.33$$

where $\tilde{p}_j$ denotes the modal force due to ground motion and external forcing, and ϕ_{jn} is the element of ϕ_j corresponding to the nth displacement variable. To control the i^{th} modal response, we set $j = i$ in Eq. (4.33) and introduce the approximation

$$q_i \approx \left[\frac{1}{\phi in}\right] u_n \qquad \ldots \quad 4.34$$

This leads to the following equation for u_n:

$$\tilde{m}_{ie}\ddot{u}_n + \tilde{c}_{ie}u_n + \tilde{k}_{ie}u_n = \tilde{p}_{ie} + k_d u_d + c_d u_d \qquad \ldots 4.35$$

where

$$\tilde{m}_{ie} = \left[\frac{1}{\phi^2 in}\right]\tilde{M}_i = \left[\frac{1}{\phi^2 in}\right]\phi_i^T M \phi_i \qquad \ldots \quad 4.36$$

$$\tilde{c}_{ie} = \alpha\tilde{k}_{ie}$$

$$\tilde{k}_{ie} = \omega_i^2\tilde{m}_{ie} \qquad \ldots 4.37$$

$$\tilde{p}_{ie} = \left[\frac{1}{\phi in}\right]\tilde{p}_i \qquad \ldots \quad 4.38$$

The remaining steps are the same as described previously. We specify $\bar{m}$ and ξ_i, determine the optimal tuning and damping values with Figures 4.17 and 4.18, and then compute m_d and ω_d.

$$m_d = \bar{m}\tilde{m}_{ie} = \left[\frac{\bar{m}}{\phi^2 in}\right] \phi_i^T M \phi_i \qquad \ldots \quad 4.39$$

$$\omega_d = f_{opt}\omega_i \qquad \ldots \quad 4.40$$

The optimal mass damper for mode i is obtained by selecting n such that ϕ_{in} is the maximum element in ϕ_i.

The design of a TMD involves the following steps:

- Establish the allowable values of displacement of the primary mass and the TMD for the design loading. This data provides the design values for $H_2|_{opt}$ and $H_4|_{opt}$ / $H_6|_{opt}$ and $H_8|_{opt}$
- Determine the mass ratios required to satisfy these motion constraints from Figure 4.8 and Figure 4.9 / Figure 4.15 and Figure 4.16. Select the largest value of $\bar{m}$.
- Determine f_{opt} from Figure 4.5 / Figure 4.17.

- Compute ω_d:

$$\omega_d = f_{opt}\, \omega$$

- Compute k_d:

$$k_d = m_d\omega^2_d = \bar{m}kf^2_{opt}$$

- Determine $\xi_d|_{opt}$ from Figure 4.7 /Figure 4.18
- Compute c_d:

$$c_d = 2\xi_d|_{opt}\omega_d m_d = \bar{m}f_{opt}[2\xi_d|_{opt}\omega m]$$

4.3 VALIDATION USING 3D MODEL

Validation of the optimized parameters is essential, because literature shows that the effectiveness of the TMD in reducing the response of the same structure under different earthquakes or of different structure during the same earthquake is significantly different.

Model

For validation of the parameters three multi-storey framed structure, namely 5, 10 and 15 storey were modeled using the routines of SAP2000. The details of the model are shown in Table 4.1. The columns and beams were designed as concrete frame. The material properties of the materials used are given in Table 4.2.

Table 4.1 Details of the Models

Model Name	No. of Floors	No. of Bays	Floor Height (m)	Bay size (m)	Column Size (m)	Beam Size (m)
M5	5	3x2	3	6	0.3x0.3 0.4x0.4	0.2x0.3 0.25x0.3 0.3x0.4
M10	10	4x3	3	5	0.3x0.3 0.5x0.5 0.6x0.6	0.3x0.3 0.3x0.4 0.4x0.5
M15	15	4x3	3	5	0.3x0.3 0.5x0.5 0.7x0.7	0.3x0.3 0.3x0.4 0.5x0.6

Table 4.2 Properties of Materials

Material	**Young's Modulus N/m2**	**Poisson's ratio**	**Density Kg/m**
Concrete	3.2×10^{10}	0.2	2549.29
Rebar	2.0×10^{11}	0.3	8004.77

4.3.1 Modal Analysis

The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. Modal analysis uses the overall mass and stiffness of a structure to find the various periods at which it will naturally resonate. The basic equation solved in a typical undamped modal analysis is the classical Eigen value problem:

$$[K]\{\phi_i\} = \omega_i^{\,2}[M]\{\phi_i\}$$

Where,

$[K]$ = stiffness matrix
$\{\Phi_i\}$ = mode shape vector (eigenvector) of mode i
ω_i = natural circular frequency of mode i (ωi^2 is the eigen value)
$[M]$ = mass matrix

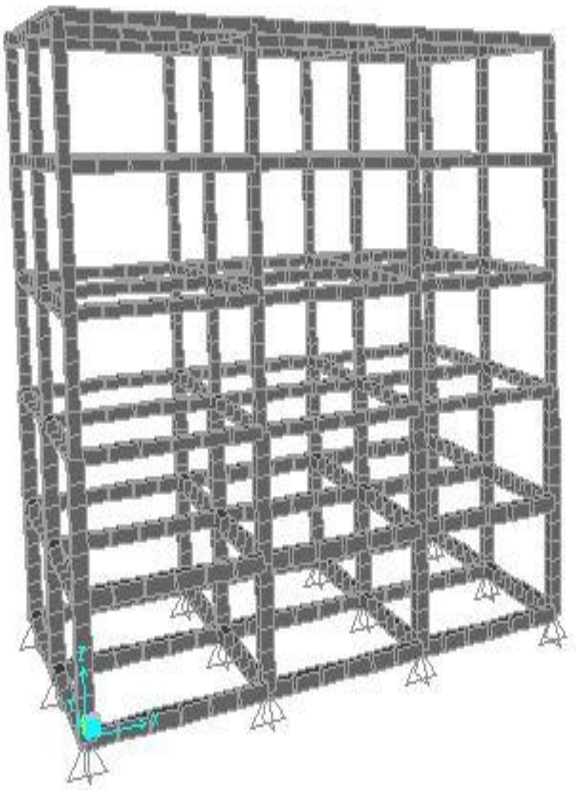
Modal analysis was carried out for the full model and the time period and modal mass participated in the first mode was found out. For the purpose of calculating the dynamic response, the mass of the structure is calculated directly from the dead load. Based on T and M of the structure, the stiffness of structure was calculated using equation $K_s = \omega_o^2 M$. The values of mass (M) and stiffness (k_s) for the model are given in Table 4.3.

Table 4.3 Results of Modal Analysis on Full Models

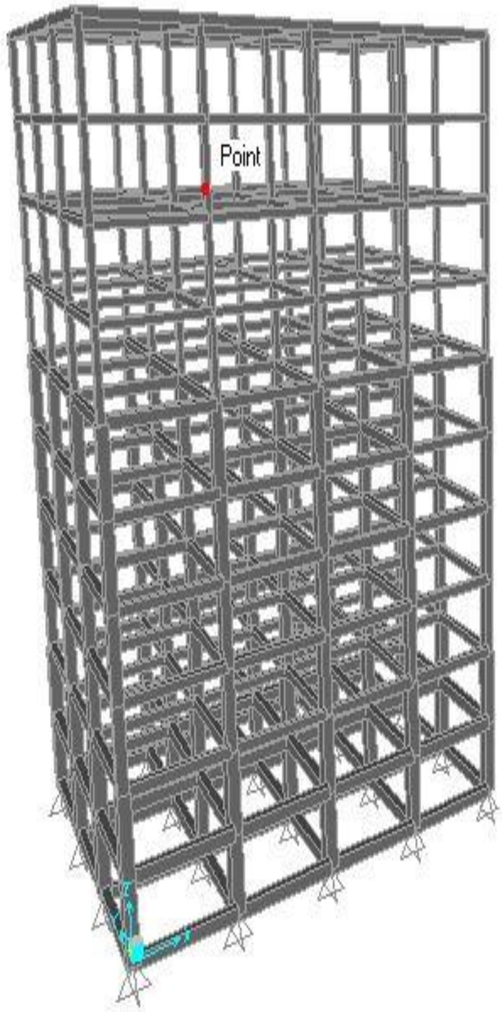
Model Name	**Time Period T in sec**	**Effective Mass of the structure M (Kg)**	**Stiffness of the Structure k_s (N/m)**
M5	0.49834	184160.74	29315553.78
M10	0.74074	855541.89	61512673.01
M15	1.56449	1399433.01	22557094.75

Within SAP2000 or ETABS, a TMD may be modeled using a spring-mass system with damping. Guidelines for this subsystem are described as follows:

- **Spring** – Assign spring properties to a linear two-joint link object in which one joint is attached to the structure, and the other joint is free.
- **Mass** – mass and weight are then assigned to the free joint
- **Damping** – Within SAP2000, linear damping is included directly in the linear link property, while nonlinear damping is modeled using viscous damping link object in parallel with the linear link.

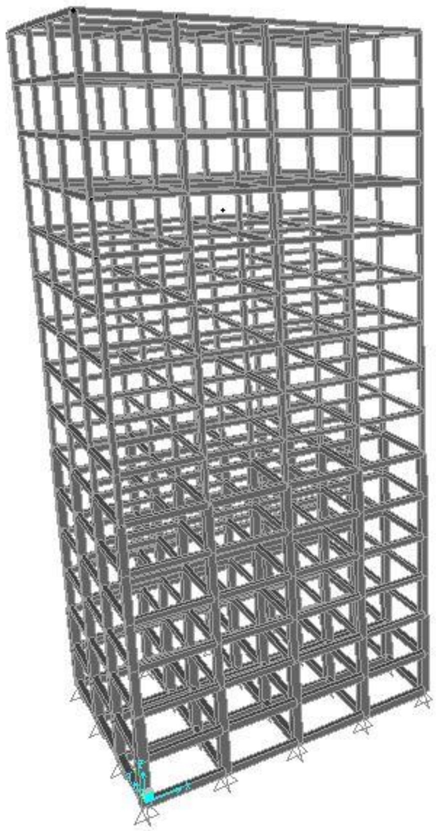


(a) Five Storey Model M5



(b) Ten storey Model M10

Figure 4.20 SAP2000 Models



(c) Fifteen Storey Model M15

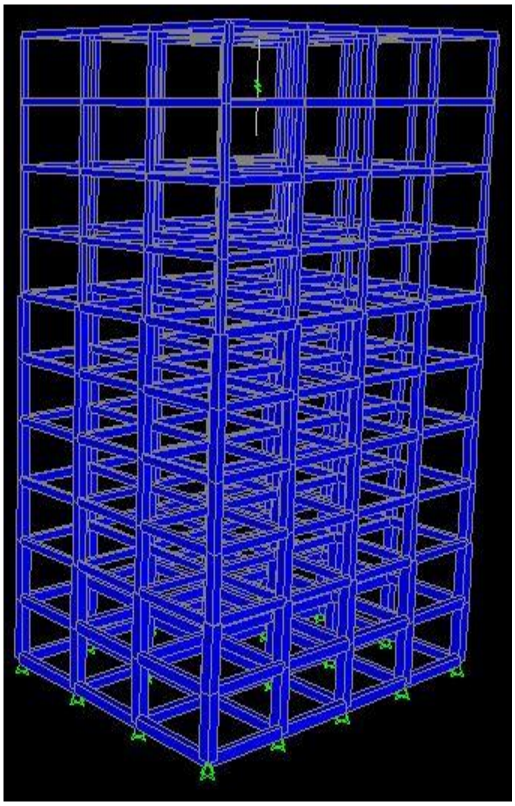


Figure 4.21 Model with TMD

Figure 4.20 SAP2000 Models (con't)

4.3.2 FAST NONLINEAR ANALYSIS (FNA)

Fast Nonlinear Analysis (FNA) (CSI 2016) is a modal analysis method useful for the static or dynamic evaluation of linear or nonlinear structural systems. Because of its computationally efficient formulation, FNA is well-suited for time-history analysis, and often recommended over direct-integration applications. (CSI 2016)

The efficiency of FNA formulation is largely due to the separation of the nonlinear-object force vector $R_{NL}(t)$ from the elastic stiffness matrix and the damped equations of motion, as seen in the fundamental equilibrium equation of FNA, expressed as:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) + R_{NL}(t) = R(t)$$

Stiffness and mass orthogonal Load Dependent Ritz Vectors represent the equilibrium relationships within the elastic structural system. At each time increment, the uncoupled modal equations are solved exactly, while forces within the predefined nonlinear DOF, indexed within $R_{NL}(t)$, are resolved through an iterative process which satisfies equilibrium, force-deformation, and compatibility relationships.

The more complicated problem associated with large displacements, which cause large strains in all members of the structure, requires a tremendous amount of computational effort and computer time to obtain a solution. Fortunately, large strains very seldom occur in typical civil engineering structures made from steel and concrete materials. However, certain types of large strains, such as those in rubber base isolators, mass damper and gap elements, can be treated as a lumped nonlinear element using the Fast Nonlinear Analysis (FNA) method. (CSI 2016)

The more common type of nonlinear behavior is when the material stress-strain, or force-deformation, relationship is nonlinear. This is because of the modern design philosophy that "a well-designed structure should have a limited number of members which require ductility and that the failure mechanism be clearly defined". Such an approach minimizes the cost of repair after a major earthquake.

4.3.2.1 STRUCTURES WITH A LIMITED NUMBER OF NONLINEAR ELEMENTS

A large number of very practical structures have a limited number of points or members in which nonlinear behavior takes place when subjected to static or dynamic loading. Local buckling of diagonals, uplifting at the foundation, contact between different parts of the structures and yielding of a few elements are examples of structures with local nonlinear behavior. For dynamic loads, it is becoming common practice to add concentrated damping, base isolation and other energy dissipation elements. In many cases, those nonlinear elements are easily identified. For other structures, an initial elastic analysis is required to identify the nonlinear areas.

In this paper the FNA method is applied to a dynamic analysis of linear and nonlinear structural systems. A limited number of predefined nonlinear elements are assumed to exist. Stiffness and mass orthogonal Load Dependent Ritz Vectors of the elastic structural system are used to reduce the size of the nonlinear system to be solved.

4.3.3 Time History Analysis

To show the behavior of Tuned Mass Damper, time history analysis was done for the models with and without TMD using Elcentro data shown in figure 4.22. Time history analysis is carried out in SAP2000 using the Non Linear transient dynamic analysis.

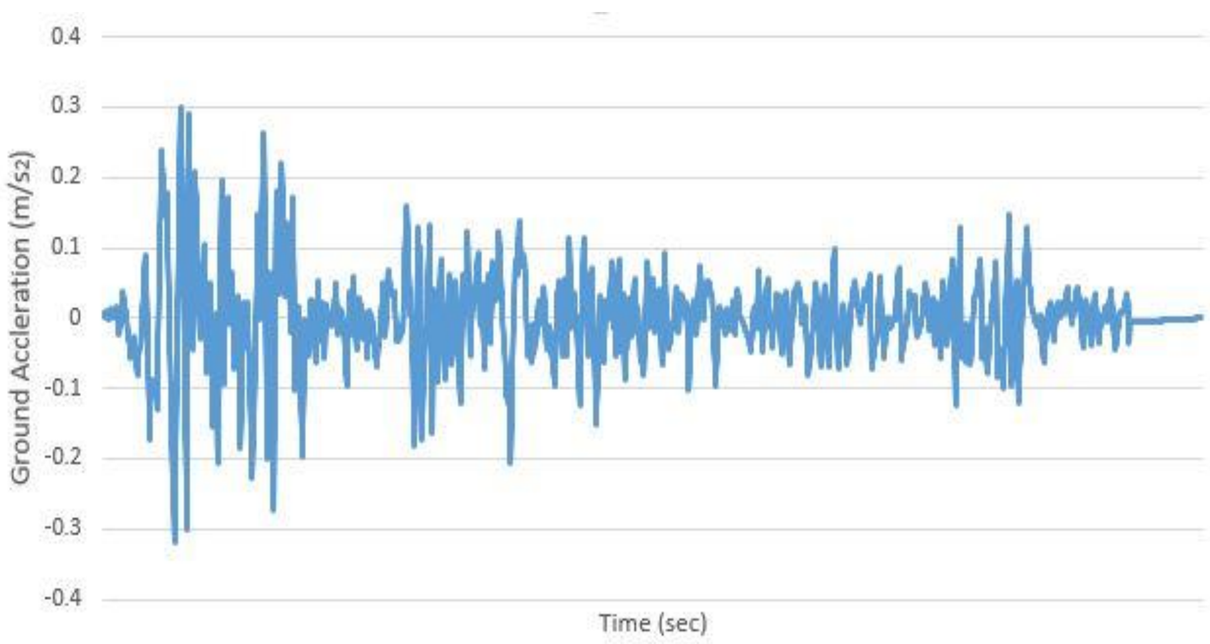
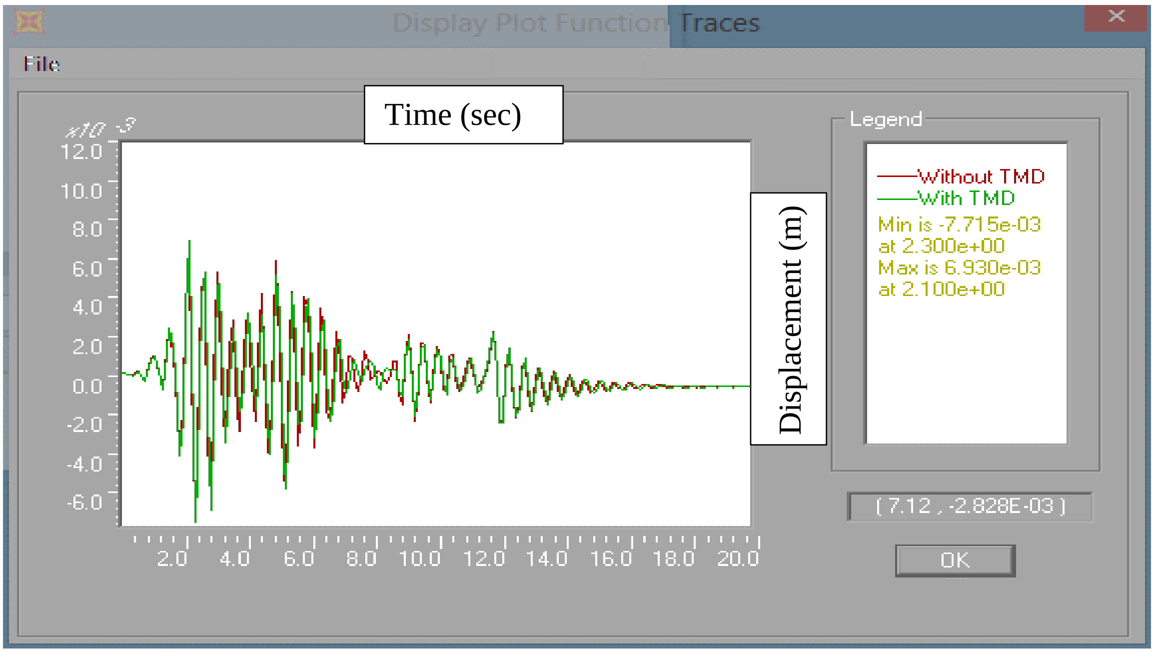


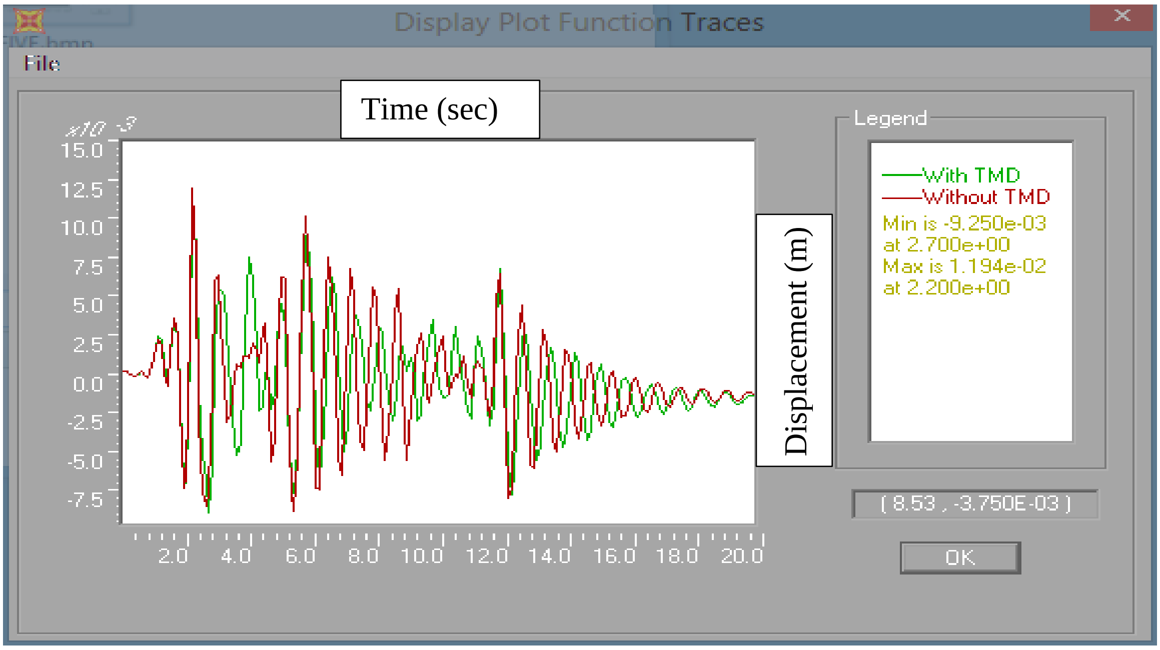
Figure 4.22 Elcentro earthquake.

The comparison of deflection pattern is shown in Figure 4.23.a, b, c and d. The optimized parameters, namely $\bar{m}_{opt}$ f_{opt} ξ_{opt} for the three models are taken from Figure 4.17 and Figure 4.18.

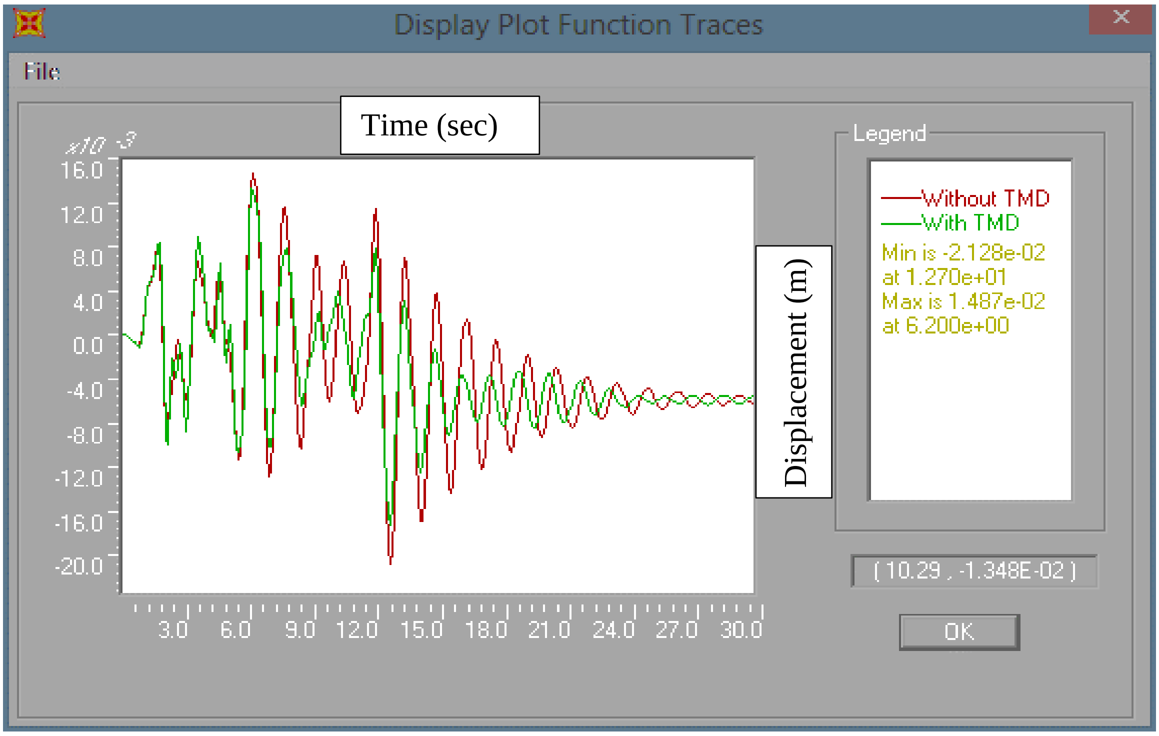


(a) Five Storey Model M5

Figure 4.23 Comparison of top storey deflection Pattern with and without TMD models

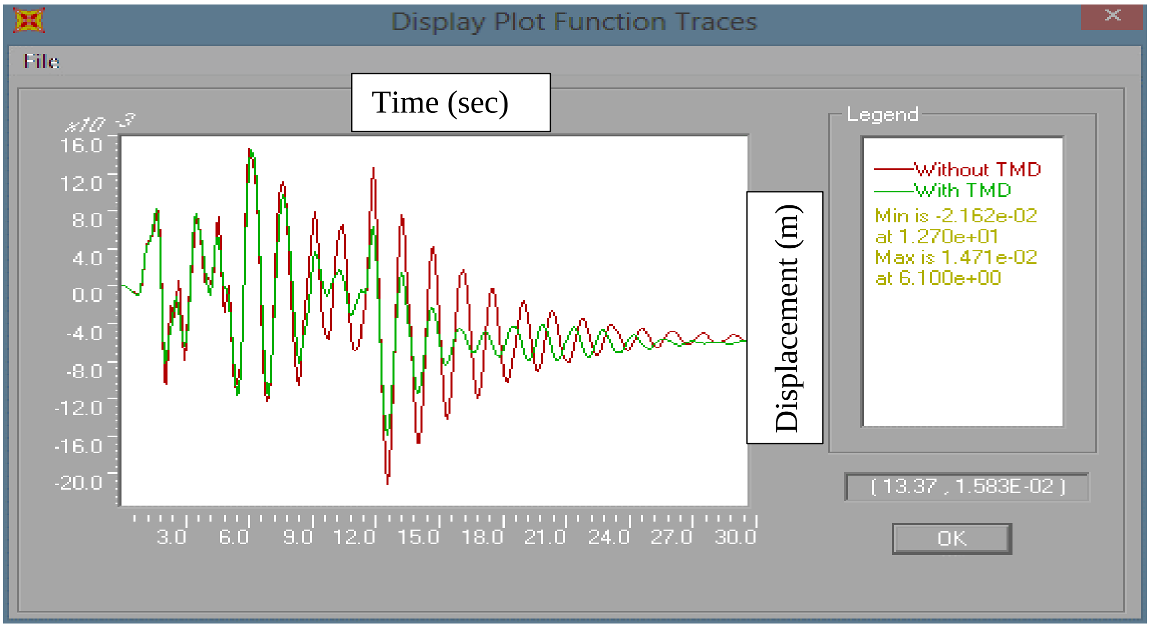


(b)Ten Storey Model M10



(c) Fifteen Storey Model M15

Figure 4.23 Comparison of top storey deflection Pattern with and without TMD models (con't)



(d) Fifteen Storey Model M15 (TMD at 10th floor)

Figure 4.23 Comparison of top storey deflection Pattern with and without TMD models (con't)

4.4 Result and Discussion

4.4.1 Storey Displacement

Figure 4.24 shows the maximum displacement comparison of structures with and without TMD under the Elcentro earthquake. It demonstrates that the maximum displacement of the top storey without TMD is 7.6mm and that of with optimized TMD is 7.72mm in a

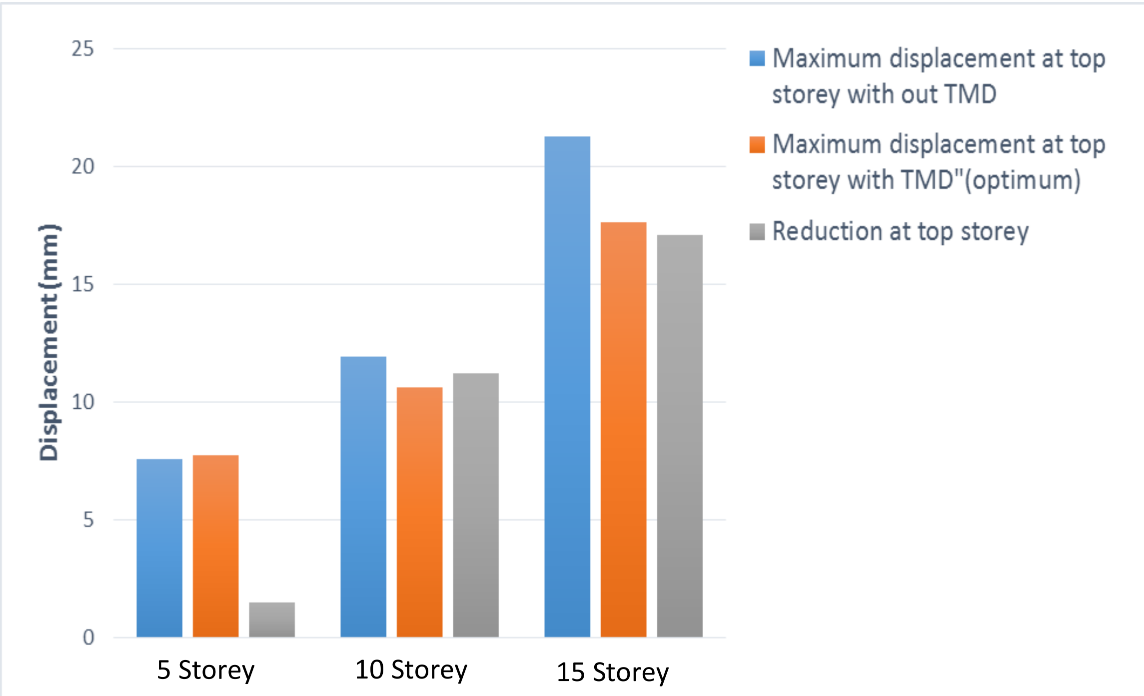
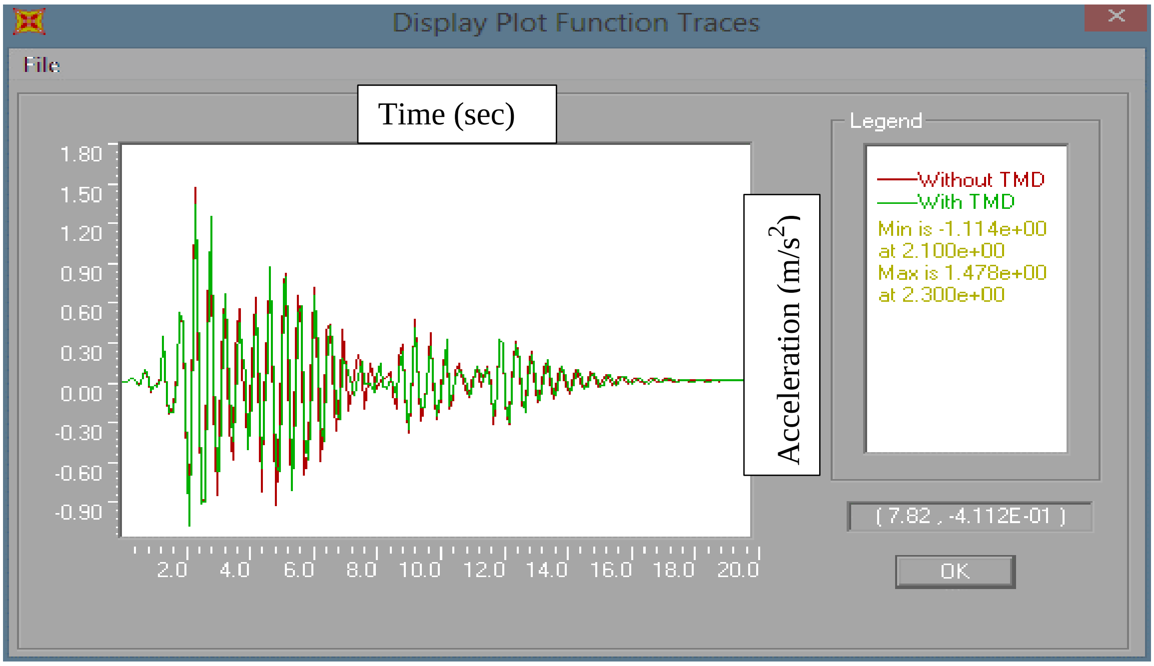


Fig. 4.24: Maximum top storey displacement reduction with TMDs structure

five storey structure. The maximum displacement of top storey of a ten storey structure without TMD is 11.94mm and that of with TMD is 10.6mm. Whereas, in a fifteen storey structure the maximum displacement of the top storey without TMD 21.28mm and that of with TMD is 17.64mm. From the figure we can understand that for a single TMD loaded on structures reduction of peak displacement is increasing with rising storey.

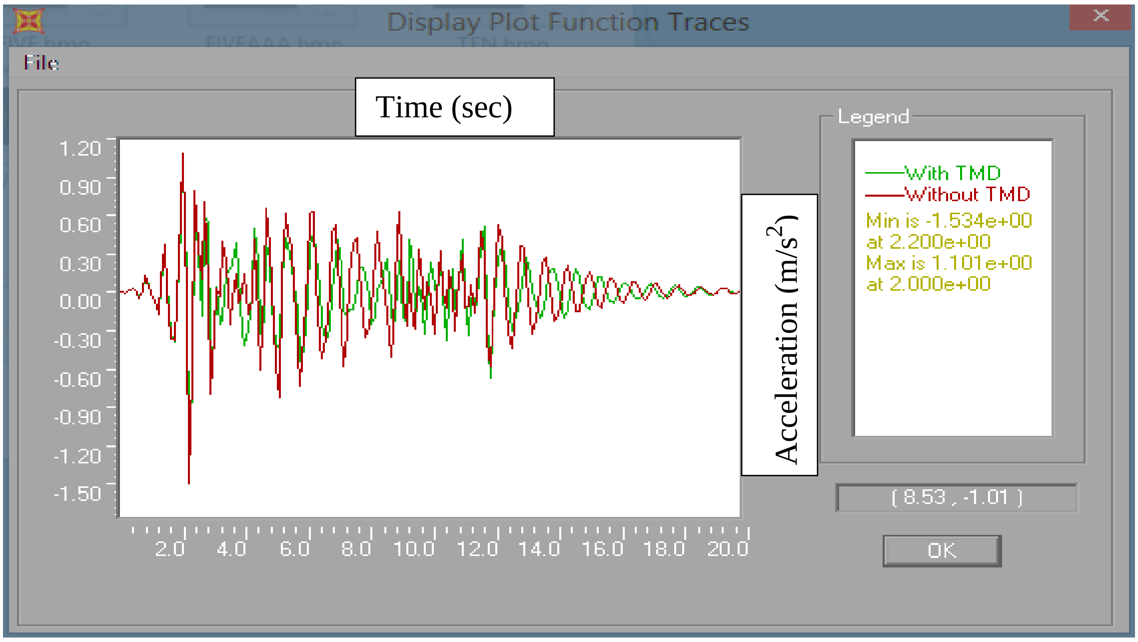
4.4.2 Acceleration

The maximum acceleration time histories of the structures with and without TMD under Elcentro earthquake is shown in Figure 4.25. It is demonstrated that, in a 15th storey structure, the maximum acceleration without TMD is 0.06g and maximum acceleration with TMD is 0.07g. As analytical result, a single TMD can only control the mode for which it designed. Therefore, the concept of Multi-Tuned Mass Damper (MTMD), i.e., having a separate TMD for other structural modes, appears to be a solution to the limited efficiency of TMD in mitigating seismic vibrations. The application of an active mass damper can also be considered as an alternative, to suppress the effects of higher modes of vibration for important high-rise structures.

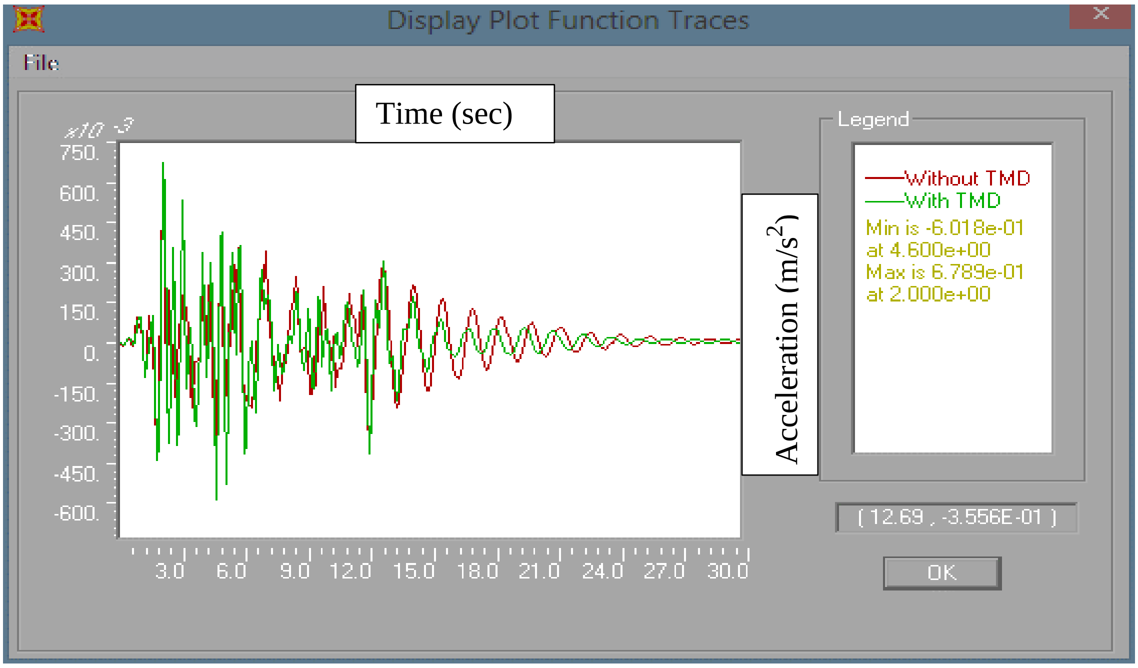


(a) Five Storey Model M5

Figure 4.25 Maximum Acceleration of Top storey with and without TMD



(b) Ten Storey Model M5



(c) Fifteen Storey Model M15

Figure 4.25 Maximum Acceleration of Top storey with and without TMD (con't)

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

The overall objective of this paper was to determine the optimum parameters of tuned mass dampers that result in utmost reduction in the structural response to earthquake loading. To validate the optimum values of Tuned Mass Damper parameters, the seismic behavior of five, ten and fifteen storey buildings with TMD and without TMD was investigated. Based on the above discussion, the following conclusion can be made:

- On undamped structure the effect of the damper is to limit the motion in a frequency range centered on the natural frequency of the primary mass and extending about 0.15ω. Outside of this range, the motion is not significantly influenced by the damper.
- The optimum tuning frequency of TMD system decreases with an increase in both the mass ratio and the damping of the main system.
- The effectiveness of the single TMD system is reduced for higher damping in the main system.
- Optimization of mass ratio has shown that with mass ratio less than 2%the desired response reduction can be achieved in many structures.
- The increment of damping in the main system does not have much influence on the optimum damping ratio of TMD.
- The passive tuned-mass damper has little effect on the structural response during the first few seconds of earthquake excitation.
- The reduction of top storey displacement for five storey building is -1.5%, for Ten storey building is 11.2% and for fifteen storey building is 17.1%. This indicates that for small storey buildings no need to install TMD and when the storey of the building increase, the number of TMD should be also increase for a better displacement reduction.
- The TMD should be placed at the top floor of buildings for best control of the first mode.

Further Scope for study

i. More number of earthquake data can be used to arrive at the optimized parametric values.
ii. Implementation of multiple tuned mass damper (MTMD) to suppress multiple modes of vibration can be studied.

6 Reference

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7 APPENDIX A: Optimization Algorithm

Optimization simulations were performed using MATLAB ver7.4.0 (R2013a). The optimization algorithm calculates the structural displacement amplification factor H for every possible combination of f, ξ_d, and $\bar{m}$ within their respective range. The output is two $l \times k$ matrix, where l is the length of the value range for $\bar{m}$ and k is the length of the value range for ξd. The first matrix (represented by index_A) contains the minimum peak H for the range of f values for each combination of ξd, and $\bar{m}$. The second matrix (represented by index_B) contains the corresponding f value which results in the minimized peak. The optimized conditions is identified by the minimum peak value contained within index_A.

```
clear all;

a = 0:0.01:1.5; %Range of values for structural excitation frequency
ratio
E = 0.05; %Damping ratio of structure
m = 0.01:0.01:0.1; %Range of values for mass ratio
B = 0.8:0.01:1.0; %Range of values for tuned frequency ratio
Ed = 0:0.01:0.2; %Range of values for Damping ratio of TMD

H = zeros(length(m),length(Ed));
index_A = 1000*ones(length(m),length(Ed));
index_B = ones(length(m),length(Ed));

for l = 1:length(m)
for k = 1:length(Ed)
for i = 1:length(B)
for j = 1:length(a)
%structure mass amplification factor calculation

H(j) = sqrt((B(i)^2-a(j)^2)^2+(2*Ed(k)*a(j)*B(i))^2)/sqrt((-
B(i)^2*a(j)^2*m(l)+(1-a(j)^2)*(B(i)^2-a(j)^2)-
4*E*Ed(k)*B(i)*a(j)^2)^2+4*(E*a(j)*(B(i)^2-a(j)^2)+Ed(k)*B(i)*a(j)*(1-
a(j)^2*(1+m(l))))^2);
end
%determine if min peak for each combination of m and Ed
if max(H(:)) <= index_A(l,k)
index_A(l,k) = max(H(:));
index_B(l,k) = B(i);

end
end
end
end

disp(index_A)
disp(index_B)
```