



# INVESTIGATION OF VARIATION OF NORMALIZED V-PARAMETER WITH WAVE LENGTH IN AN OPTICAL FIBER

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# Abstract

In the modern world the use of optical fiber is extending from optical communication to medical science. However, in long haul communication the problems of attenuation, dispersion, bending loss etc become very important as they limit the transmission of information data.

In this work we have been reported the optimum optical wave length for communication for a single mode fiber with core radius  $4\mu m$ ,  $5\mu m$ ,  $25\mu m$  and  $50\mu m$  with refractive indices for core  $n_1 = 1.48$  and for cladding  $n_2 = 1.46$ . Further more the variation of normalized V-parameter with wave length has been reported. It has shown that optimum wave length of transmission for these fibers is  $1550nm$

**KeyWords:** single mode fiber; cutoff frequency; laser light; wave length; Optical fiber losses

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# Acronyms

NA	Numerical Aperture
SMF	Single Mode Fiber
MMF	Multi Mode Fiber
TE	Transversal Electric
TM	Transversal Magnetic
TEM	Transverse electric and Magnetic
HE	Helical Electric
LP	Linear Polarized
MFD	Mode field diameter

# Chapter 1

## Introduction

The idea of using light waves for communication can be traced to as far back as 1888 when Graham Bell invented the photo phone in which sunlight was modulated by a diaphragm and transmitted through a distance of about 200 meters in air to a receiver containing a selenium cell<sup>[1]</sup>.

Optical fibers were first envisioned as optical elements in the early 1960s<sup>[2]</sup>. It was developed in the early 1970s and is rapidly replacing traditional copper cable for transmitting information over hundreds to thousands of miles. Rather than sending data in the form of electrons, fiber optic technology uses photons, or light<sup>[3]</sup>.

Optical fibers were one of the most phenomenal technological successes of the 20<sup>th</sup> century and they have become the major tool used to communicate information. This technology has developed at an incredible pace, from the low loss (less than 20 dB/km) optical fibers in 1970 to being key components of modern sophisticated global telecommunication networks of twenty first century<sup>[4]</sup>.

The choice of fiber optic cables in telecommunications lies on its advantage over copper such as, very high bandwidth, resistance to electromagnetic noise, large information carrying capacity, longer transmission, high speed and costs much less to maintain. Recently, because of their cost effectiveness and better quality, fibers have been used to replace copper wire an appropriate means of communication and signal transmission<sup>[5]</sup>. Even if, optical fiber have these advantages, signal attenuation and distortion are important

degradation function limiting the transmission systems<sup>[6]</sup>.

## 1.1 Motivation and Objective

Currently, there are different types of optical fibers that are used for different purposes, which have problems alongside with propagation of laser light, such as attenuation, dispersion, scattering, etc. So to minimize these factors, we will investigate the variation of normalized V parameter with wave length in the optical fiber and get the optimum propagation of the light wave within it and the laser light goes long distance without losing its energy. The general objective of this thesis is to investigate the variation of normalized v parameter with wave length in optical fiber, and specifically to get the optimum wave length which is preferable for light transmission in optical fibers.

## 1.2 structure of the thesis

**Chapter1** introduction, which gives a brief history of optical fiber and indicates the objective of this thesis

**Chapter2** gives wide coverage of the theoretical background of types of fiber, attenuation, dispersion relating with light passes through optical fibers

**Chapter3** it shows, the mathematical modeling of light wave in optical fiber

**Chapter4** results and discussion

**Chapter5** concludes the thesis

# Chapter 2

## Optical fiber

### 2.1 Introduction

This chapter deals with the overview of fiber optic material, how light wave transmits in the fiber, and the mathematical relation of the parameters related to fiber communication.

### 2.2 Optical confinement

Optical fibers work by confining and guiding<sup>[3,4]</sup> the light wave within a long strand of glass by the principle of total internal reflection . Which are cylindrical dielectric waveguides made of central cylinder of glass with one index of refraction, surrounded by an annulus with slightly different index of refraction<sup>[7]</sup>. If the refractive indices of the core and cladding are  $n_1$  and  $n_2$  respectively, then for a ray entering the fiber, if the angle of incident (at the core cladding interface)  $\theta_A$ , is greater than the critical angle,  $\theta_C$  <sup>[8]</sup>, then the ray will undergo total internal reflection at that interface.

Furthermore, because of the cylindrical symmetry in the fiber structure, this ray will suffer total reflection at the lower interface also and will therefore be guided through the core by repeated total internal reflections. This is the basic principle of the light guidance through the optical fiber.

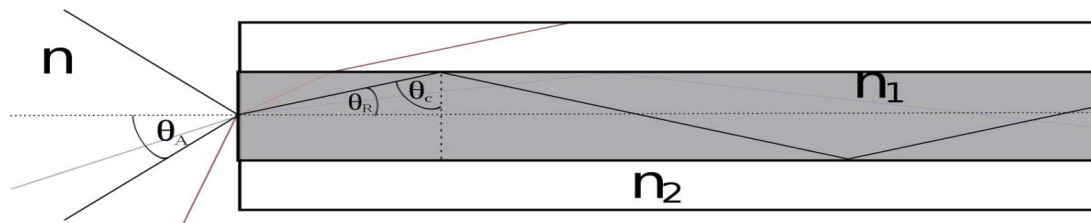


Figure 2.1: A typical optical fiber waveguide consists of thin cylindrical glass rod <sup>[9]</sup>

Mathematically, Using snell's law;

$$\theta_C = \sin^{-1}(n_1/n_2) \quad (2.2.1)$$

where  $n_1 > n_2$ ,  $n_1$  is the refractive index of the core, and  $n_2$  is the refractive index of the cladding.

## 2.3 Overview of Optical fiber

An optical fiber is long cylindrical dielectric structure usually of circular cross-section <sup>[10]</sup>. It is a flexible, transparent medium which are made of a pure glass (silica) not much wider than a human hair. It functions as a waveguide, or "light pipe", to transmit light between the two ends of the fiber<sup>[11]</sup>.

An optical fiber consists of three concentric elements: core, cladding and the outer coating, is called protective jacket(plastic sheath). The core is usually made of glass or plastic. It is the light carrying portion of the fiber. The cladding surrounds the core. It is made of a material with a slightly lower index of refraction than the core. This difference in the indices causes total internal reflection to occur at the core-cladding boundary along the length of the fiber. Light is transmitted down the fiber and does not escape through the sides of the fiber. The jacket protects the fiber from external damage.

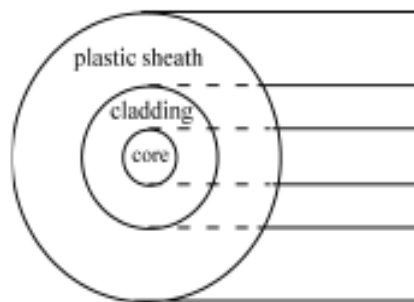


Figure 2.2: schematic representation of an optical fiber<sup>[12]</sup>

Light is kept in the core by total internal reflection. This causes the fiber to act as a waveguide.

## 2.4 Types of fibers

An optical fiber is a dielectric waveguide that operates at optical frequencies. This fiber waveguide is normally cylindrical in form. It confines electromagnetic energy in the form of light to within its surfaces and guides the light in a direction parallel to its axis. The transmission properties of an optical waveguide are dictated by its structural characteristics, which have a major effect in determining how an optical Signal is affected as it propagates along the fiber.

Variation in the material composition of the core gives rise to the two commonly used fiber types. In the first case, the refractive index of the core is uniform throughout and undergoes an abrupt change(or step) at the cladding boundary This is called a step-index fiber and can be divided into Multi mode fiber and Single-Mode.

### 2.4.1 Single mode and multi mode fiber

The light waves propagate through the fibers in definite field configurations which are known as modes<sup>[11]</sup>. Based on how many modes are allowed to propagate in a fiber, it can be classified into multi mode fiber(MMF) and single mode (mono mode or fundamental mode) fiber(SMF) which support multi modes and single mode, respectively.

The core size of single mode fibers is small. The core size (radius) is typically around  $4\mu m - 10\mu m$ . A fiber core of this size allows only the fundamental or lowest order mode,while high-order modes are lost in the cladding. Single mode fibers propagate only one mode, because the core size approaches the operational wave length.

SMF have many advantages,such as lower signal loss, a higher information capacity (bandwidth) than MMF,and low dispersion.

As their name implies, MMF propagate more than one mode. It can propagate over

100 modes. The number of modes propagated depends on the core size and numerical aperture (NA). As the core size and NA increase, the number of modes increases. MMF has the core radius bigger than  $50\mu m$ .

A large core size and a higher NA have several advantages. Light is launched into a MMF with more ease. The higher NA and the larger core size make it easier to make fiber connections. During fiber splicing, core-to-core alignment becomes less critical. MMF also have some disadvantages. As the number of modes increases, the effect of modal dispersion increases. Modal dispersion (intermodal dispersion) means that modes arrive at the fiber end at slightly different times. This time difference causes the light pulse to spread. Modal dispersion affects system bandwidth. Fiber manufacturers adjust the core diameter, NA, and index profile properties of MMF to maximize system bandwidth.

There outer diameter is  $125\mu m$ . SMF charactrized by mode field diameter(MFD),Evanescent waves,Gaussian beam approximation,Gaussian beam,spot size

#### **2.4.1.1 Mode field diameter(MFD)**

MFD is the measure of the radial intensity distribution of light wave propagation within the optical fiber.It is a measure of the spot size or beam width of the light propagation in a fiber. Simply,the diameter of the optical field within the fiber. MFD is a function of source wavelength, fiber core radius, and fiber refractive index profile.

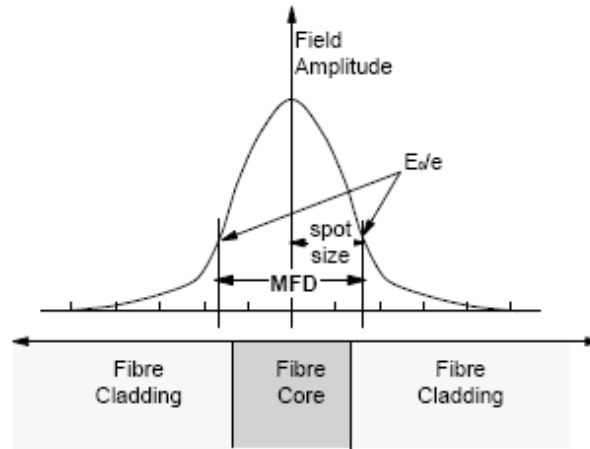


Figure 2.3: MFD represents how the light is actually distributed in the single mode fiber

#### 2.4.1.2 Evanescent waves

When light suffers total internal reflection an electromagnetic disturbance, known as an evanescent wave, does penetrate the reflecting interface. The amplitudes of the evanescent fields decay exponentially with distance away from the surface cannot normally propagate in the medium of lower refractive index. However, any variation or non uniformity in the region of the reflecting interface may cause the conversion of the evanescent wave in to a propagating wave.

#### 2.4.1.3 Gaussian beam approximation

The modal field (near field) of a single mode fiber is not very sensitive to its refractive index profile and the shape of this field remains similar to a Gaussian function. This has led to the so called Gaussian approximation in which it can approximate the modal field of a given single mode fiber by a Gaussian function as:

$$\psi_G(r) = \psi(0) \exp\left(-\frac{r^2}{W_G^2}\right) \quad (2.4.1)$$

where,  $W_G$  is called the Gaussian spot size of the modal field. This spot size is the parameter which is different for different profiles of the fiber.

#### 2.4.1.4 Gaussian beam

Optical beams change their intensity distribution as they propagate in optical fiber<sup>[10]</sup>. The change of its intensity is due to diffraction. Diffraction can easily be understood by considering the diffraction divergence of a Gaussian beam. Indeed, when a laser light oscillates in its fundamental mode, the transverse amplitude distribution is Gaussian. Similarly, the transverse amplitude distribution of the fundamental mode of an optical fiber is very nearly Gaussian. Therefore, the study of the diffraction of a Gaussian beam is of considerable importance in fiber optics. Consider Gaussian beam propagating along the  $z$  direction with intensity distribution on the plane  $z = 0$  given by:

$$I(x, y, 0) = I_0 \exp\left(-\frac{2(x^2 + y^2)}{w_0^2}\right) = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right), \quad x^2 + y^2 = r^2 \quad (2.4.2)$$

here  $I_0$  the intensity at the beam center,  $w_0$  is a measure of the beam width (spot size of the beam) and  $r$  is the radial distance from the center axis of the beam. As the beam propagates along the  $z$  direction within the optical fiber the field intensity becomes:

$$I(x, y, z) = I_0 \frac{w_0^2}{w(z)^2} \exp\left(-\frac{2(x^2 + y^2)}{w(z)^2}\right) = I_0 \frac{w_0^2}{w(z)^2} \exp\left(-\frac{2r^2}{w(z)^2}\right) \quad (2.4.3)$$

where  $w(z)$  is the  $z$  dependent spot size of the beam which is given by:

$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2} \quad (2.4.4)$$

### 2.4.2 Step index and graded index fiber

In addition to modes, fibers are categorized based on the refractive indices of the material of the core which are step index and graded fibers. Step index fibers which have the refractive index of the core ( $n_1$ ) abruptly varies from the refractive index of the cladding ( $n_2$ ) and the refractive index distribution of step index fiber is given by:

$$n(r) = \begin{cases} n_1, & 0 < r < a \quad \text{core} \\ n_2, & r > a \quad \text{cladding} \end{cases}$$

Where  $r$  represents the cylindrical radial coordinate and  $a$  represents the radius of the core.

While the graded index fibers have the refractive index of the core( $n_1$ ) varies smoothly to the cladding( $n_2$ ).Mathematically:

$$n(r) = \begin{cases} n_1 \left\{ 1 - 2\Delta \left( \frac{r}{a} \right)^\alpha \right\}^{\frac{1}{2}}, & 0 < r < a & \text{core} \\ n_1 (1 - 2\Delta)^{\frac{1}{2}} \approx n_1(1 - \Delta) = n_2, & r > a & \text{cladding} \end{cases}$$

Here, $r$  represents the radial distance from the fiber axis, $a$  is the core radius,  $\alpha$  is the dimensionless parameter which defines the shape of the profile,and  $\Delta$  is the refractive index difference which is given by:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}$$

The refractive index profile is shown in fig.2.4

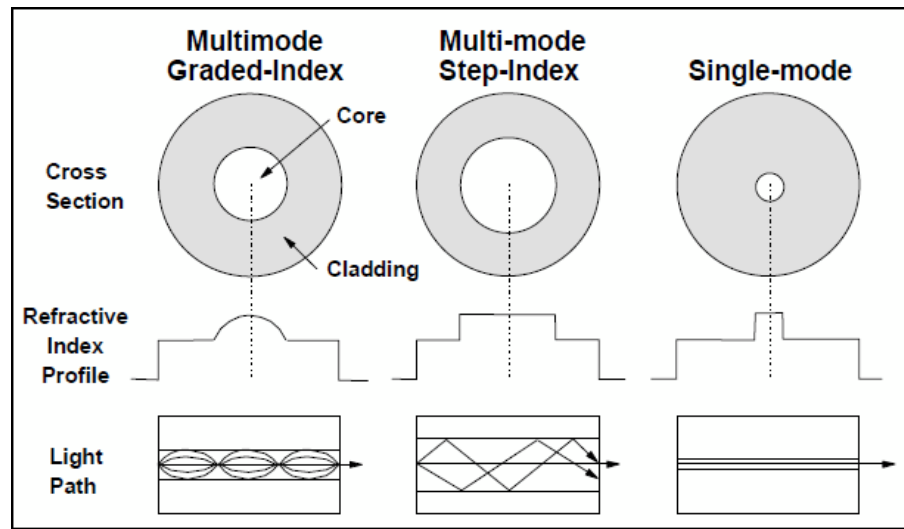


Figure 2.4: Types of optical fiber

## 2.5 Fiber Mode

An optical mode refers to a specific solution of the wave equation (electromagnetic wave) that satisfies the appropriate boundary conditions and has the property that its spatial distribution does not change with propagation. It is guided electromagnetic waves. The fiber modes can be classified<sup>[13]</sup>, as:

### 2.5.1 Transverse Electric(TE) Mode

TE modes exist when the electric field is perpendicular to the direction of propagation of the wave(z-direction) but there is a small z-component of the magnetic field. Here most of the magnetic field is also perpendicular to the z-direction but a small z-component exists. This implies that the wave is not traveling quite straight but is reflecting from the sides of the waveguide. However, this also implies that the ray path is meridional (it passes through the center or axis of the waveguide). It is not circular or skewed.

### 2.5.2 Transverse Magnetic(TM) Mode

In a TM mode the magnetic field is perpendicular to the direction of propagation of the wave but there is a small component of the electric field in the direction of propagation. Again this is only a small component of the electric field and most of it is perpendicular to the propagation direction.

### 2.5.3 Transverse Electro magnetic(TEM) Mode

In the TEM mode both the electric and magnetic fields are perpendicular to the direction of propagation of the wave. The TEM mode is the only mode of a single-mode fiber.

### 2.5.4 Helical (Skew) Modes (HE and EH)

Most modes in fibers travel in a circular path of some kind. In this case components of both magnetic and electric fields are in the the direction of propagation. These modes are designated as either HE or EH depending on which field contributes the most to the direction of propagation.

### 2.5.5 Linearly Polarized (LP) Modes

LP modes for which the field components in the direction of propagation are small compared to components perpendicular to that direction.

It turns out that because the refractive index difference between core and cladding is quite small much can be simplified in the way we look at modes. The TE, TM, HE and EH modes can all be summarized and explained using only a single set of LP modes. It is conventional to number the TE and TM modes according to the number of nulls in their energy pattern across the waveguide. Thus mode  $TE_{00}$  would have a single energy spot in the center of the waveguide and no others. (This would be the same mode as TEM) Mode  $TE_{21}$  would have two nulls (three energy spots) in one direction and a single null (two energy spots) in the other.

### 2.5.6 Cladding Modes

Cladding modes (radiation modes) in an optical fiber are modes in which the intensity distribution is not restricted to the region in or immediately around the fiber core. When a properly focused and aligned laser beam hits the end face of a fiber, most of its power may then propagate in the fiber core. Some fraction of the power, however, will propagate in cladding modes.

The penetration of low order and high order modes into the cladding region indicates that some portion is refracted out of the core. The refracted modes may become trapped

in the cladding due to the dimension of the cladding region. The modes trapped in the cladding region are called cladding modes.

## 2.6 Numerical Aperture (NA)

Numerical aperture (NA) of the fiber is the light gathering efficiency of the fiber, and it is the measure of the amount of light rays that can be accepted by the fiber. It indicates the maximum angle at which a particular fiber can accept the light that will be transmitted through it.

The higher the NA of an optical fiber's, the larger the cone of light that can be coupled into its core. It is a dimensionless number that characterizes the range of angles over which the system can accept or emit light<sup>[14]</sup>. The fiber that uses its proper work, it is a requirement for light to successfully travel down an optical fiber, the light must enter the fiber and reflect off the cladding at greater than the critical angle. Due to the refractive index change the direction of the light as it enters the core of a fiber. There is a limit to the angle at which the light can enter the core to successfully propagate down the optic fiber. Any light striking the cladding at less than the critical angle will go straight through into the cladding and be lost. By applying Snell's law at the entrance of the fiber, NA can be related with refractive index of the fiber. It is derived from calculating the sine of the half angle ( $\theta$ ) of acceptance within the cone of light entering the fiber's core

$$NA = \sqrt{n_1^2 - n_2^2} \quad (2.6.1)$$

where  $n_1$  is the refractive index of the core and  $n_2$  is the refractive index of the cladding.

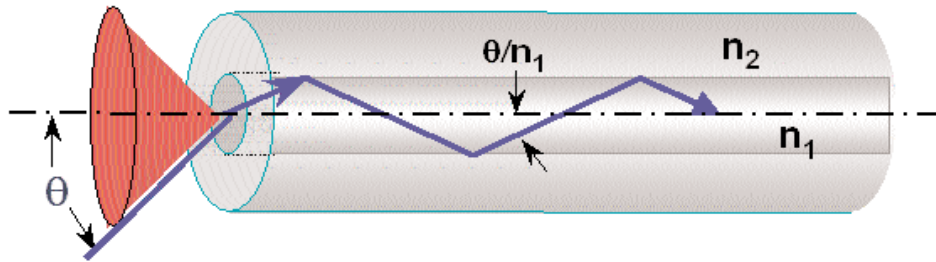


Figure 2.5: Numerical Aperture defines the maximum angle (the cone of acceptance) at which light can be launched into a fiber

## 2.7 Optical fiber losses

When light waves propagate in fiber material, then the light wave degrades and shows different properties. There are two main loss mechanisms in the fiber: Attenuation and Dispersion of the fiber.

### 2.7.1 Attenuation in optical fiber

Attenuation is the diminishing or loss of intensity of the light wave (signal) with respect to the distance traveled through an optical fiber. Or it is the reduction of power as it travels through the fiber. It depends on the length of the fiber. Longer the fiber, more is the attenuation. It is measured in decibels per km ( $dB/km$ ).

The decibel, which is used for comparing two power levels, may be defined for a particular optical wavelength as the ratio of the input (transmitted) optical power  $P_i$  into a fiber to the output (received) optical power  $P_o$  from the fiber. Mathematically:

$$\alpha = \frac{10}{L} \log \left( \frac{P_i}{P_o} \right) \quad (2.7.1)$$

Where  $\alpha$  is the attenuation coefficient of the fiber, and  $L$  is the fiber length.

Attenuation depends on the fiber type and the operating wavelength within the fiber. Silica-based SMF have lower attenuation than multi-mode fibers. Generally, the higher

(or longer) the wavelength, the lower the attenuation.

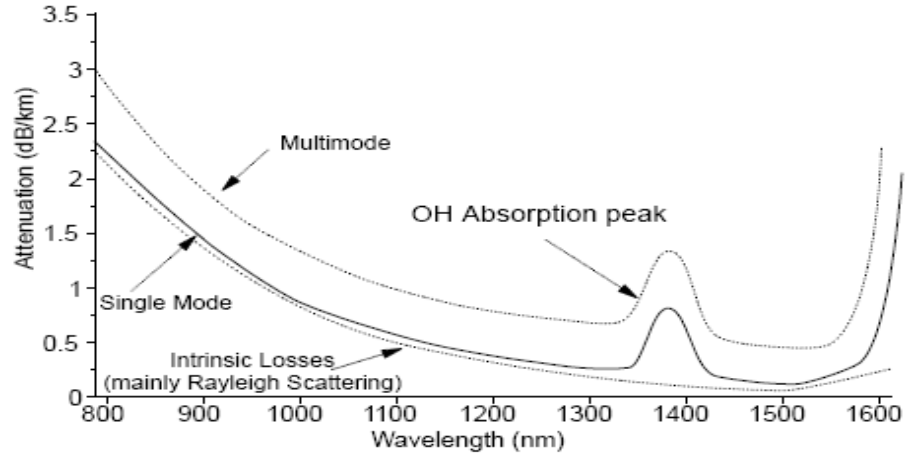


Figure 2.6: Attenuation coefficient of SMF and MMF <sup>[13]</sup>

Fig:2.6 depicts that the attenuation coefficient related with wave length in SMF and MMF. Based on the figure the attenuation coefficient  $\alpha$  varies with wave length for all low loss fused silica fibers due to Rayleigh scattering and impurity absorptions. The optical losses for wavelengths below  $1\mu m$  are mainly due to scattering. At greater wavelengths, absorption losses are important, notably at  $1.4\mu m$  through absorption by  $OH^{-1}$  ions. Above  $1.6\mu m$  absorption due to impurities becomes dominant.

From the fact that the  $1550nm$  region is the minimum loss region of optical fibers, future developments in optical systems will most likely focus on increasing the number of wavelength division multiplexing channels which can be squeezed into the  $1550nm$  window, rather than moving to a new operating wavelength region.

Attenuation is an important factor limiting the transmission of a light wave across long distance fiber. Thus, much research has gone into both minimizing the attenuation and maximizing the amplification of the light wave in the fiber. It is the outcome of absorption and scattering of light wave in the fiber. The principal causes of attenuation in an optical fibers are scattering, absorptive losses and radiation loses of the fiber.

### 2.7.2 Scattering losses

Scattering losses in optical fibers arise from microscopic variations from in material density, from composition fluctuation, and from structural inhomogeneities or defects during manufacturing of the fiber. Each of these gives rise to fluctuations of refractive index on a scale that is small compared to optic wavelength, that is, on a sub micron scale. This is fundamental to any glassy material, however carefully made, and it causes the light to be scattered in the manner known as Rayleigh scattering. The light is then lost from the fiber. Indeed, if light wave is coupled in to a long coil of unsheathed fiber and this is viewed from the side in a darkened room, the scattered light is clearly seen, diminishing in intensity along the length of the fiber.

It is a process whereby all or some of the optical power in a mode is transferred into another mode. It (often referred to as Rayleigh scattering) is the reflection of small amounts of light in all directions as it travels down the fiber.

### 2.7.3 Absorption losses

When light waves passes through a optical fiber, it suffers absorptive losses, i.e., a small fraction of its power is absorbed during propagation. These losses can be characterized as intrinsic absorption losses and extrinsic absorption losses.

Intrinsic absorption losses are caused by basic fiber properties<sup>[15]</sup>. If an optical fiber were absolutely pure, with no imperfections or impurities, then all assimilations would be intrinsic. In fiber optics, silica (pure glass) fibers are used predominately, because of their low intrinsic material assimilation at the wavelengths of operation.

Extrinsic absorption losses are caused by impurities introduced into the fiber. The presence of minute quantity of material like transition metals impurities, such as iron, nickel, and chromium ions, are introduced into the fiber during fabrication of the fiber

and due to  $OH^-$  ions in the fiber. It is caused by the electronic transition of these metal ions from one energy level to another. For example, the presence of 1 part per millions (ppm) of  $Fe^{2+}$  would lead to a loss of  $0.68 \text{ dB/km}$  at  $1.1 \mu\text{m}$ -thus the necessity of ultra pure fiber<sup>[16]</sup>.

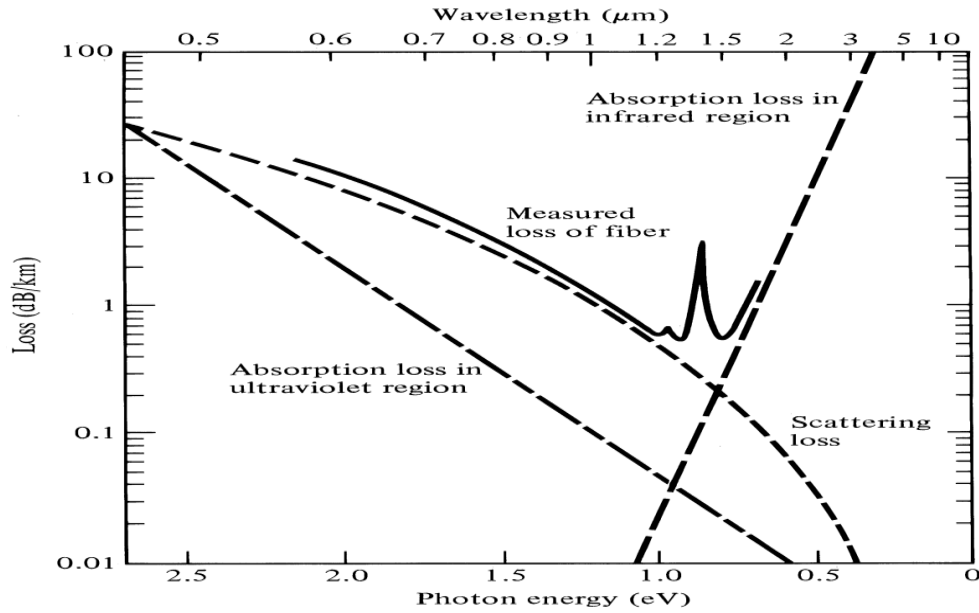


Figure 2.7: Absorption and scattering losses in optical fiber

### 2.7.4 Radiation losses

It occurs when a guided mode gets coupled to a radiation mode. Rayleigh scattering is predominantly responsible for such coupling which is caused by small scale (small compared to the wave length of the light wave) inhomogeneities in the fiber. These come during fabrication of the fiber.

### 2.7.5 Bending losses

A bend in a fiber can be considered to be a straight fiber section joined with a bent fiber section, which is again joined with a straight fiber. The power from the straight fiber is coupled to the field in the bent fiber. The field in the bent fiber propagates through to be coupled into the following straight section. Thus, there are transition losses at the joints and there is loss in the curved fiber section generally termed as pure bend loss, fig:2.7.

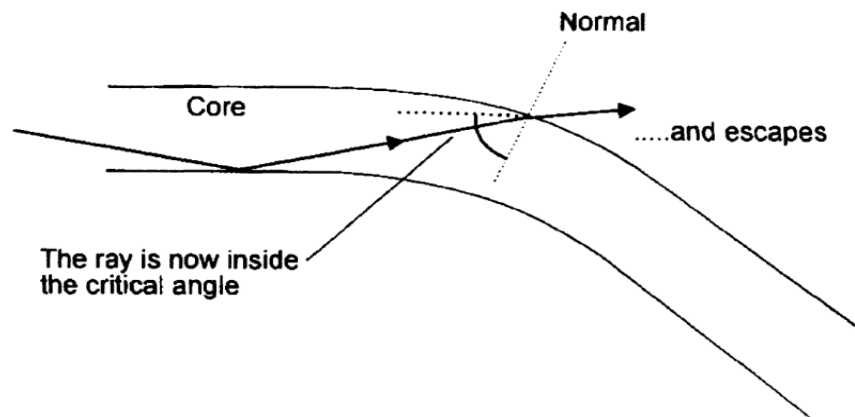


Figure 2.8: Bending of fiber which increases losses

## 2.8 Dispersion

As an electromagnetic pulse propagates along an optical fiber, generally, its shape will be distorted. From an optimal transmission point of view, a distorted pulse is undesirable since it could lead to an increase in the bit-error rate. For example, pulse broadening may be such that the overlap with adjacent pulses renders the signal meaningless.

It is the phenomenon in which the phase velocity of a wave depends on its frequency,<sup>[14]</sup> or alternatively when the group velocity depends on the frequency. Media having such a property are termed dispersive media. Dispersion of the transmitted optical signal causes distortion for both digital and analogue transmission along optical fiber<sup>[17]</sup>. It occurs when optical pulse spread out while they are transmitted. As a consequence the pulses cannot be distinguished anymore at the end of the fiber. This phenomena shown in fig.2.9.

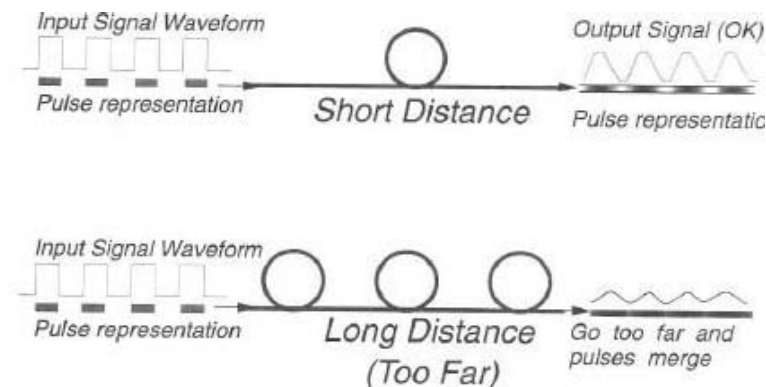


Figure 2.9: The effect of dispersion on the propagation of optical pulses in an optical fiber<sup>[18]</sup>

The three most important sources of dispersions are: Material dispersion, Modal dispersion and wave guide dispersion

### 2.8.1 Material Dispersion

Material dispersion is caused due to variation of refractive index of the core material with the wave length of light wave. An important component of delay distortion in fiber optic waveguides is produced by wavelength dispersion of the refractive index. Material dispersion effect is characterized by:

$$\delta \left( \frac{T}{L} \right) = -M \Delta \lambda, M = \frac{\lambda}{c} \frac{d^2 n}{d \lambda^2} \quad (2.8.1)$$

where M property of material,  $\lambda_0$  light wavelength in vacuum, n is the refractive index, and c is the speed of light.

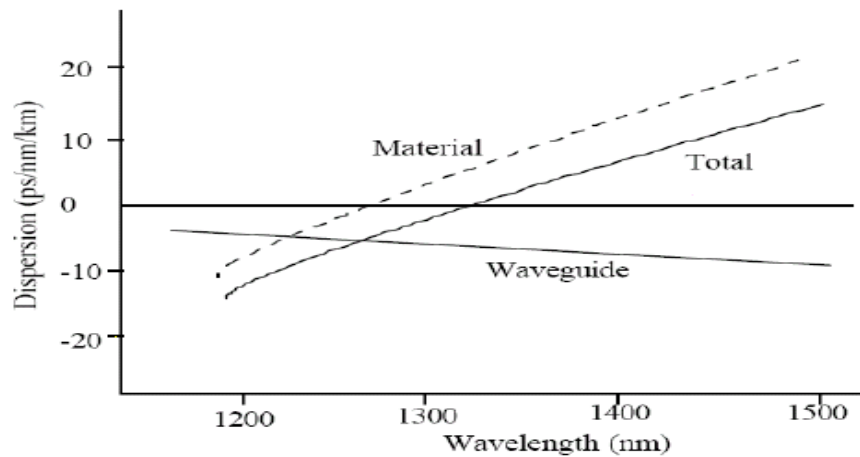


Figure 2.10: Dispersion variation with Wavelength for standard single mode fiber

### 2.8.2 Modal dispersion

Modal dispersion is the out come of variation in propagation time of different modes. It is related to the difference in propagation speed caused by the propagation of different modes. Modal Dispersion is only a problem for multi mode fibers.

### 2.8.3 waveguide dispersion

Waveguide dispersion is similar to material dispersion in that they both cause signals of different wave lengths and frequencies to separate from the light pulse. However, with waveguide dispersion it depends on the shape, design and chemical composition of the fiber core.

It occurs because different spectral components of a pulse travel with different velocities by the fundamental mode of the fibers.

## Chapter 3

# Mathematical modeling of light wave in single mode cylindrical step index fiber

Propagation of light in step index cylindrical fiber is described by Maxwell's equations. In the weakly guiding approximation, the transverse component of the electric field or magnetic field satisfies the scalar wave equation, which is:

$$\nabla_{\perp}^2 \psi(r, \varphi, z, t) = \varepsilon_0 \mu_0 n^2 \frac{\partial^2}{\partial t^2} \psi(r, \varphi, z) \quad (3.0.1)$$

where  $n$  the refractive index of the fiber

$\varepsilon_0$  the permittivity of free space

$\mu_0$  the permeability of free space

Let

$$\psi(r, \varphi, z, t) = \psi(r, \varphi) e^{j(\omega t - \beta z)} \quad (3.0.2)$$

Where  $\beta$  is wave propagation constant

Substituting equation(3.0.2) into (3.0.1) which implies

$$\nabla_{\perp}^2 \psi(r, \varphi) e^{j(\omega t - \beta z)} = \varepsilon_0 \mu_0 n^2 \frac{\partial^2}{\partial t^2} \psi(r, \varphi) e^{j(\omega t - \beta z)} \quad (3.0.3)$$

$$\begin{aligned}
{}^2\psi(r, \varphi)e^{j(\omega t - \beta z)} &= \varepsilon_0\mu_0 n^2\psi(r, \varphi)(-\omega^2)e^{j(\omega t - \beta z)} \\
{}^2\psi(r, \varphi) &= -\varepsilon_0\mu_0 n^2\psi(r, \varphi) \\
{}^2\psi(r, \varphi) + n^2\varepsilon_0\mu_0\psi(r, \varphi) &= 0
\end{aligned} \tag{3.0.4}$$

where

$$c^2 = \frac{1}{\varepsilon_0\mu_0} \quad \text{and} \quad k_0^2 = \frac{\omega^2}{c^2} \tag{3.0.5}$$

Insert equation(3.0.5) into (3.0.4)

$${}^2\psi(r, \varphi) + n^2\varepsilon_0\mu_0\psi(r, \varphi) = 0 \tag{3.0.6}$$

The wave equation written as using  $\vec{E}$  or  $\vec{H}$  as below;

$${}^2\vec{E} + n^2k_0^2\vec{E} = 0 \tag{3.0.7}$$

$${}^2\vec{H} + n^2k_0^2\vec{H} = 0 \tag{3.0.8}$$

Where  $\vec{E}$  is the electric field,  $\vec{H}$  the magnetic field, and n refractive index  $\vec{E}$  and  $\vec{H}$  in cylindrical coordinates depends on  $\hat{r}, \hat{\varphi}$  and  $\hat{z}$

$$\vec{E} = E_r\hat{r} + E_\varphi\hat{\varphi} + E_z\hat{z} \tag{3.0.9}$$

$$\vec{H} = H_r\hat{r} + H_\varphi\hat{\varphi} + H_z\hat{z} \tag{3.0.10}$$

The wave equation(3.0.7) in cylindrical coordinates written as:

$$\begin{aligned}
{}^2\vec{E} + n^2k_0^2\vec{E} &= \left( {}^2\vec{E}_r - \frac{2}{r^2} \frac{\partial E_\varphi}{\partial \varphi} - \frac{E_r}{r^2} + n^2k_0^2 E_r \right) \hat{r} + \\
&\left( {}^2E_\varphi + \frac{2}{r} \frac{\partial E_r}{\partial \varphi} - \frac{E_\varphi}{r^2} + n^2k_0^2 E_\varphi \right) \hat{\varphi} + \left( {}^2E_z + n^2k_0^2 E_z \right) \hat{z}
\end{aligned} \tag{3.0.11}$$

The Laplacian operator is:

$${}^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \tag{3.0.12}$$

From the above equation(3.0.11) the  $\hat{r}$  component contains both  $E_r$  and  $E_\varphi$  and similarly  $\hat{\varphi}$  component contains  $E_r$  and  $E_\varphi$ , the  $\hat{z}$  component contains only  $E_z$ . Because of this, the

z-component is first solved and then the other components  $E_r$  and  $E_\varphi$  will be solved from Maxwell's equations. Similar equation can be obtained for  $\vec{H}$ , so we can write equations(3.0.11) and (3.0.12) combined as follows

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \varphi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 k_0^2 = 0 \quad (3.0.13)$$

Equation(3.0.13) is solved by the method of separation of variables

$$E_z = R(r)\phi(\varphi)Z(z) \quad (3.0.14)$$

Substitute equation(3.0.14) into (3.0.13)

$$\begin{aligned} \frac{\partial^2}{\partial r^2} (R(r)\phi(\varphi)Z(z)) + \frac{1}{r} \frac{\partial}{\partial r} (R(r)\phi(\varphi)Z(z)) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} (R(r)\phi(\varphi)Z(z)) + \\ \frac{\partial^2}{\partial z^2} (R(r)\phi(\varphi)Z(z)) + n^2 k_0^2 (R(r)\phi(\varphi)Z(z)) = 0 \end{aligned} \quad (3.0.15)$$

$$\phi(\varphi)Z(z) \frac{d^2 R(r)}{dr^2} + \frac{\phi(\varphi)Z(z)}{r} \frac{dR(r)}{dr} + \frac{R(r)Z(z)}{r^2} \frac{d^2 \phi(\varphi)}{d\varphi^2} + R(r)\phi(\varphi) \frac{d^2 Z(z)}{dz^2} + n^2 k_0^2 R(r)\phi(\varphi)Z(z) = 0 \quad (3.0.16)$$

Divide equation(3.0.16) by  $R(r)\phi(\varphi)Z(z)$ , we will get:

$$\left[ \frac{1}{R(r)} \frac{d^2 R(r)}{dr^2} + \frac{1}{R(r)r} \frac{dR(r)}{dr} + \frac{1}{\phi(\varphi)r^2} \frac{d^2 \phi(\varphi)}{d\varphi^2} \right] + \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} + n^2 k_0^2 = 0 \quad (3.0.17)$$

$$\left[ \frac{1}{R(r)} \frac{d^2 R(r)}{dr^2} + \frac{1}{R(r)r} \frac{dR(r)}{dr} + \frac{1}{\phi(\varphi)r^2} \frac{d^2 \phi(\varphi)}{d\varphi^2} + n^2 k_0^2 \right] + \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} = 0 \quad (3.0.18)$$

The function in the square bracket in equation(3.0.18) are depends on r and  $\varphi$  but the last term is a function of z only

$$\frac{1}{R(r)} \frac{d^2 R(r)}{dr^2} + \frac{1}{R(r)r} \frac{dR(r)}{dr} + \frac{1}{\phi(\varphi)r^2} \frac{d^2 \phi(\varphi)}{d\varphi^2} + n^2 k_0^2 = \beta^2 \quad (3.0.19)$$

Where

$$\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} = -\beta^2 \quad (3.0.20)$$

When we rewrite equation(3.0.19)

$$\left[ \frac{r^2}{R(r)} \frac{d^2 R(r)}{dr^2} + \frac{r}{R(r)} \frac{dR(r)}{dr} + (n^2 k_0^2 - \beta^2) r^2 \right] + \frac{1}{\phi(\varphi)} \frac{d^2 \phi(\varphi)}{d\varphi^2} = 0 \quad (3.0.21)$$

Put

$$\frac{1}{\phi(\varphi)} \frac{d^2 \phi(\varphi)}{d\varphi^2} = l^2 \quad (3.0.22)$$

and

$$\frac{d^2}{dr^2} R(r) + \frac{1}{r} \frac{d}{dr} R(r) + [n^2 k_0^2 - \beta^2] R(r) = \frac{l^2}{r^2} R(r) \quad (3.0.23)$$

Where  $l$  is a constant ,  $l=0,1,2,3\dots$

$$\frac{d^2}{dr^2} R(r) + \frac{1}{r} \frac{d}{dr} R(r) + \left[ n^2 k_0^2 - \beta^2 - \frac{l^2}{r^2} \right] R(r) = 0 \quad (3.0.24)$$

There are more than one type of solutions of each differential equations shown above,i.e given by equations(3.0.20), (3.0.22) and(3.0.24). Some solutions are sinusoidal and others are exponential,the choice depends on the physical meaning. Let us see the solution of the differential equation given above when the field in side the core:

$$Z(z) = ae^{j\beta z} + be^{-j\beta z} \quad (3.0.25)$$

$$\phi(\varphi) = ce^{jl\varphi} + de^{-jl\varphi} \quad or \quad \phi(\varphi) = c \cos l\varphi + d \sin l\varphi \quad (3.0.26)$$

and

$$R(r) = eJ_l(\kappa r) + fN_l(\kappa r) \quad (3.0.27)$$

Where,  $\kappa = [n_1^2 k_0^2 - \beta^2]^{\frac{1}{2}}$  ,  $n = n_1$

a,b,c,d,e and f are arbitrary constants

$J_l(\kappa r)$  and  $N_l(\kappa r)$  are the  $l$  order Bessel function of the first kind and Bessel function of the second kind,respectively. At  $r = 0, N_l(\kappa r)$  becomes negative infinity, so it is physically not acceptable,and f in equation(3.0.27) has to be zero. For large value of  $(\kappa r)$  the functions  $J_l(\kappa r)$  can approximate as:

$$J_l(\kappa r) \approx \frac{2}{\pi \kappa r} \left( \kappa r - l \frac{\pi}{2} - \frac{\pi}{4} \right) \quad (3.0.28)$$

Where  $J_l(\kappa r)$  Bessel functions describe the radial standing waves within the core of the fiber.

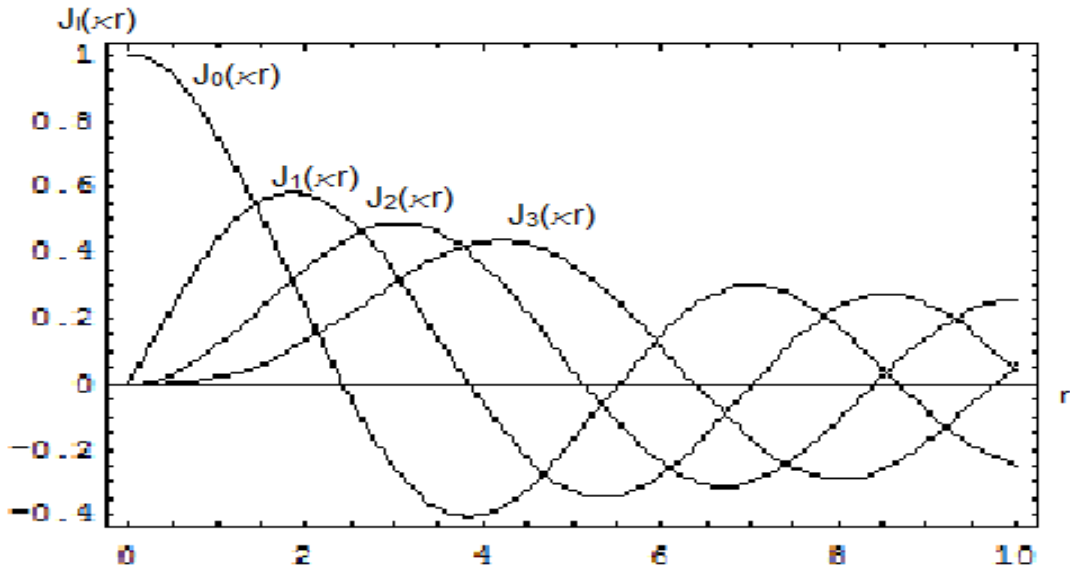


Figure 3.1: graphical representation of the light wave inside the core of the fiber

When the field inside the cladding ( $r > a$ ), where  $a$  is core radius. The type of solutions suitable for the cladding region, i.e. the function should rapidly decreasing with an increase in  $r$ . So  $(k_0^2 n^2 - \beta^2 - \frac{l^2}{r^2}) < 0$ , where  $n = n_2$ . Using above relations, the solution for equation (3.0.24) is:

$$R(r) = eI_l(\gamma r) + fK_l(\gamma r) \quad (3.0.29)$$

Where  $\gamma = [\beta^2 - k_0^2 n_2^2]^{\frac{1}{2}}$ ,  $e$  and  $f$  are arbitrary constants

$I_l(\gamma r)$  and  $K_l(\gamma r)$  are the modified Bessel functions of the first kind and the modified Bessel function of the second kind of the  $l^{\text{th}}$  order, respectively. The value of  $I_l(\gamma r)$  increase with  $(\gamma r)$ , but  $K_l(\gamma r)$  decreases with increase of  $(\gamma r)$ . So we will reject  $I_l(\gamma r)$

For large value of  $(\gamma r)$ , the Bessel solution becomes:

$$K_l(\gamma r) \approx \frac{e^{-\gamma r}}{2\pi\gamma r} \quad (3.0.30)$$

where the factor  $\frac{1}{2\pi\gamma r}$  is the natural decrease of a wave amplitude as it expands with radius. The exponential factor represents evanescent decay.

So both  $\vec{E}$  and  $\vec{H}$  satisfy the same kind of wave equations. In the core region,  $r < a$ ,  $E_z$

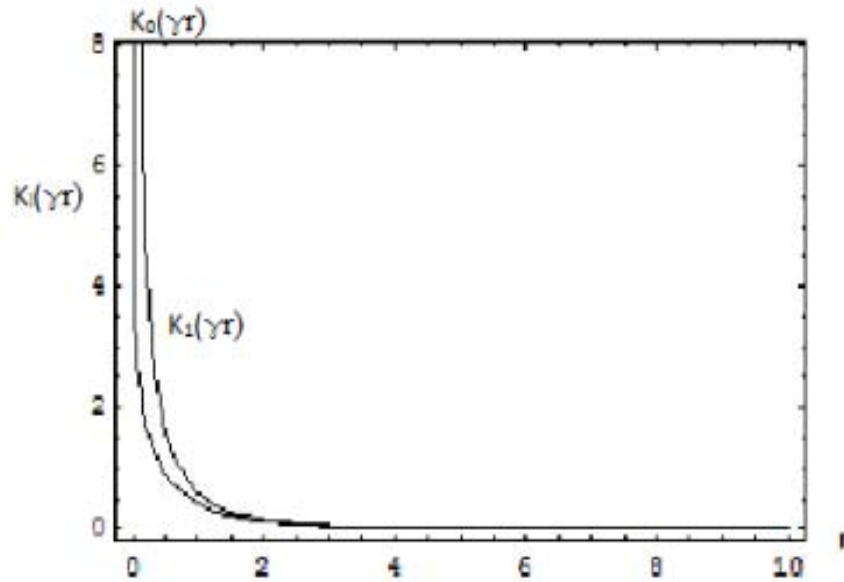


Figure 3.2: graphical representation of light wave inside the cladding of the fiber

and  $H_z$  are give as:

$$E_z(r, \varphi, z) = AJ_l(\kappa r)e^{j(l\varphi - \beta z)} \quad (3.0.31)$$

$$H_z(r, \varphi, z) = BJ_l(\kappa r)e^{j(l\varphi - \beta z)} \quad (3.0.32)$$

and  $r > a$ , the fields in the cladding region

$$E_z(r, \varphi, z) = CK_l(\gamma r)e^{j(l\varphi - \beta z)} \quad (3.0.33)$$

$$H_z(r, \varphi, z) = DK_l(\gamma r)e^{j(l\varphi - \beta z)} \quad (3.0.34)$$

where  $l$  specifies the mode number; A, B, C and D are arbitrary constants determined from the boundary conditions. As shown above  $E_z$  and  $H_z$  determined for the core and cladding regions.  $E_r, E_\varphi, H_r$  and  $H_\varphi$  can be found using Maxwell's equations. From the Maxwell's relations

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}, \text{ where, } \vec{D} = \epsilon \vec{E} \quad (3.0.35)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \text{ where, } \vec{B} = \mu \vec{H} \quad (3.0.36)$$

Where  $\epsilon$  permittivity of the fiber

$\mu$  permeability of the fiber

$\vec{D}$  electric displacement

$\vec{B}$  magnetic induction

$$\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} = j\omega\varepsilon\vec{E} \quad (3.0.37)$$

$$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} = -j\omega\mu\vec{H} \quad (3.0.38)$$

By taking  $\vec{E} = E_0 e^{j(\omega t - \beta z)}$  and  $\vec{H} = H_0 e^{j(\omega t - \beta z)}$  where  $\omega$  is angular frequency of the wave.

To solve  $\vec{\nabla} \times \vec{E}$  or  $\vec{\nabla} \times \vec{H}$ , it should be expand  $\vec{\nabla}$  using cylindrical coordinates

$$\vec{\nabla} \times \vec{E} = \begin{matrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{1}{r} \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & E_\phi & E_z \end{matrix} \quad (3.0.39)$$

$$\vec{\nabla} \times \vec{E} = \hat{r} \left( \frac{1}{r} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) = -\mu j\omega (H_r \hat{r} + H_\phi \hat{\phi}) \quad (3.0.40)$$

In similar way

$$\vec{\nabla} \times \vec{H} = \hat{r} \left( \frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) = j\varepsilon\omega (E_r \hat{r} + E_\phi \hat{\phi}) \quad (3.0.41)$$

Collect fields that are found in the same directions from equation(3.0.40) and (3.0.41)

$$\frac{1}{r} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\mu\omega H_r \quad (3.0.42)$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\mu\omega H_\phi \quad (3.0.43)$$

and

$$\frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\varepsilon\omega E_r \quad (3.0.44)$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\varepsilon\omega E_\phi \quad (3.0.45)$$

To solve component fields, combine the above equations (3.0.42 – 3.0.45) and using this

operator  $\frac{\partial}{\partial z} = -j\beta$ , and write  $E_r$  and  $H_\phi$  interms of  $E_z$  and  $H_z$

$$E_r = \frac{1}{j\omega\varepsilon} \left[ \frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right] \quad (3.0.46)$$

But,

$$H_\varphi = -\frac{1}{j\mu\omega} \left[ \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right], \text{ and } \frac{\partial}{\partial z} = -j\beta \quad (3.0.47)$$

$$E_r = \frac{1}{j\omega\epsilon} \frac{1}{r} \frac{\partial H_z}{\partial \varphi} + j\beta \left( \frac{-1}{j\mu\omega} \right) \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right)$$

$$E_r = \frac{1}{j\omega\epsilon} \left( \frac{1}{r} \frac{\partial H_z}{\partial \varphi} - \frac{j\beta}{j\mu\omega} (-j\beta E_r) - \frac{\partial E_z}{\partial r} \right) = \frac{1}{j\omega\epsilon r} \frac{\partial H_z}{\partial \varphi} + \frac{j\beta^2}{\omega^2\epsilon\mu} E_r + \frac{\beta}{j\omega^2\epsilon\mu} \frac{\partial E_z}{\partial r} \quad (3.0.48)$$

$$E_r = \frac{1}{j\omega\epsilon r} \frac{\partial H_z}{\partial \varphi} + \frac{\beta^2}{\omega^2\epsilon\mu} E_r + \frac{j\beta}{\omega^2\epsilon\mu} \frac{\partial E_z}{\partial r} \quad (3.0.49)$$

After a series of derivation the above equation simplifies as:

$$E_r = \frac{-j}{\omega^2\epsilon\mu - \beta^2} \left( \frac{\mu\omega}{r} \frac{\partial H_z}{\partial \varphi} + \beta \frac{\partial E_z}{\partial r} \right) \quad (3.0.50)$$

where  $\mu = \mu_r\mu_0$  and  $\epsilon = \epsilon_r\epsilon_0$ , and let's relate  $\mu$  and  $\epsilon$  using  $n_i$ (refractive index )

$n_i^2 = \mu_r\epsilon_r$ ,  $\mu_r$  relative permeability and  $\epsilon_r$  relative permittivity

$i$ =core or cladding of the fiber

From ray optics relations,  $\omega^2 = k_0^2 c^2$

Substituting the above relations into equation(3.0.50),we will get:

$$E_r = \frac{-j}{k_0^2 n_i^2 - \beta^2} \left( \frac{\mu\omega}{r} \frac{\partial H_z}{\partial \varphi} + \beta \frac{\partial E_z}{\partial r} \right) \quad (3.0.51)$$

Using similar fashions;

$$E_\varphi = \frac{-j}{k_0^2 n_i^2 - \beta^2} \left( \frac{\beta}{r} \frac{\partial E_z}{\partial \varphi} - \omega\mu \frac{\partial H_z}{\partial r} \right) \quad (3.0.52)$$

$$H_r = \frac{-j}{k_0^2 n_i^2 - \beta^2} \left( \beta \frac{\partial H_z}{\partial r} - \frac{\omega\epsilon}{r} \frac{\partial E_z}{\partial \varphi} \right) \quad (3.0.53)$$

and

$$H_\varphi = \frac{-j}{k_0^2 n_i^2 - \beta^2} \left( \frac{\beta}{r} \frac{\partial H_z}{\partial \varphi} + \omega\epsilon \frac{\partial E_z}{\partial r} \right) \quad (3.0.54)$$

Where,  $k_0^2 n_i^2 - \beta^2 > 0$ ,the field is in the core

$k_0^2 n_i^2 - \beta^2 < 0$ ,the field is in the cladding

The field inside the core, when  $r < a$  are obtained by inserting  $E_z$  and  $H_z$  into equation(3.0.51), we get:

$$E_r = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( \frac{\mu\omega}{r} \frac{\partial}{\partial \varphi} (B J_l(\kappa r) e^{j(l\varphi - \beta Z)}) \right) + \beta \frac{\partial}{\partial r} (A J_l(\kappa r) e^{j(l\varphi - \beta Z)}) \quad (3.0.55)$$

$$E_r = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( \frac{\mu\omega}{r} \frac{\partial H_z}{\partial \varphi} + \beta \frac{\partial E_z}{\partial r} \right) \quad (3.0.56)$$

By solving the other components in similar way, we get:

$$E_\varphi = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( \frac{A\beta}{r} (jl) J_l(\kappa r) e^{j(l\varphi - \beta Z)} - B\omega\mu\kappa J_l(\kappa r) e^{j(l\varphi - \beta Z)} \right) \quad (3.0.57)$$

$$H_r = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( -A\omega\epsilon_1 \frac{jl}{r} J_l(\kappa r) + B\beta\kappa J_l(\kappa r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.58)$$

$$H_\varphi = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( A\omega\epsilon_1 \kappa J_l(\kappa r) + B \frac{\beta}{r} (jl) J_l(\kappa r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.59)$$

Where  $\epsilon_1 = \epsilon$

The components of the field in the cladding can be written as:

$$E_r = \frac{j}{k_0^2 n_2^2 - \beta^2} \left( C\beta\gamma K_l(\gamma r) + \frac{1}{r} D\omega\mu j l K_l(\gamma r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.60)$$

$$E_\varphi = \frac{j}{k_0^2 n_2^2 - \beta^2} \left( \frac{1}{r} C(jl) K_l(\gamma r) - D\omega\mu\gamma K_l(\gamma r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.61)$$

$$H_r = \frac{j}{k_0^2 n_2^2 - \beta^2} \left( -\frac{1}{r} C\omega\epsilon_2(jl) K_l(\gamma r) + D\beta\gamma K_l(\gamma r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.62)$$

$$H_\varphi = \frac{j}{k_0^2 n_2^2 - \beta^2} \left( C\omega\epsilon_2 K_l(\gamma r) + \frac{1}{r} D\beta(jl) K_l(\gamma r) \right) e^{j(l\varphi - \beta Z)} \quad (3.0.63)$$

where  $\epsilon_2 = \epsilon$

Still, we should get the values of the constants, A, B, C and D. Using boundary conditions, we can solve the problems.

The Tangential component of  $\vec{E}$  and  $\vec{H}$  have to be continuous at  $r = a$ . The tangential component of  $\vec{E}$  are  $E_z$  and  $E_\varphi$  and those of  $\vec{H}$  are  $H_z$  and  $H_\varphi$ .

The continuity of  $E_z$  at  $r = a$  is:

$$A J_l(\kappa a) = C K_l(\gamma a), \text{ hence, } C = \frac{A J_l(\kappa a)}{\kappa_l a} \quad (3.0.64)$$

Similarly, the continuity of  $H_z$  at  $r = a$  is:

$$BJ_l(\kappa a) = DK_l(\gamma a), \text{ hence, } D = \frac{BJ_l(\kappa a)}{\kappa_l(\kappa a)} \quad (3.0.65)$$

Insert equation(3.0.64) and (3.0.65) into(3.0.61),we get:

$$E_\varphi(r > a) = \frac{j}{k_0^2 n_2^2 - \beta^2} \left( \frac{AJ_l(\kappa a)}{rK_l(\gamma a)} (jl)K_l(\gamma r) - \frac{BJ_l(\kappa a)}{K_l(\gamma a)} \omega \mu \gamma K_l(\gamma r) \right) \quad (3.0.66)$$

and

$$E_\varphi(r < a) = \frac{-j}{k_0^2 n_1^2 - \beta^2} \left( \frac{A\beta}{r} J_l(\kappa r) - B\omega \mu \kappa J_l(\kappa r) \right) e^{j(l\varphi - \beta z)} \quad (3.0.67)$$

At  $r = a$ , the above two equations(3.0.66) and (3.0.67) becomes,

i.e,  $E_\varphi(r > a) = E_\varphi(r < a)$ . So,

$$\begin{aligned} & \frac{j}{k_0^2 n_2^2 - \beta^2} \left( \frac{AJ_l(\kappa a)}{aK_l(\gamma a)} (jl)K_l(\gamma a) - \frac{BJ_l(\kappa a)}{K_l(\gamma a)} \omega \mu \gamma K_l(\gamma a) \right) \\ &= -\frac{j}{k_0^2 n_1^2 - \beta^2} \left( \frac{A\beta}{a} (jl)J_l(\kappa a) - B\omega \mu \kappa J_l(\kappa a) \right) \end{aligned} \quad (3.0.68)$$

Let us substitute;

$$n_1^2 k_0^2 - \beta^2 = \kappa^2, \text{ and } k_0^2 n_2^2 - \beta^2 = -\gamma^2 \quad (3.0.69)$$

$$\begin{aligned} & \frac{j}{\gamma^2} \left( \beta jl \frac{AJ_l(\kappa a)}{aK_l(\gamma a)} K_l(\gamma a) - B\omega \mu \gamma \frac{J_l(\kappa a)}{K_l(\gamma a)} K_l(\gamma a) \right) \\ &= \frac{-j}{\kappa^2} \left( \frac{A\beta}{a} jl J_l(\kappa a) - B\omega \mu \kappa J_l(\kappa a) \right) \end{aligned} \quad (3.0.70)$$

Collect terms depending on A and B;

$$A\beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) + jB\omega \mu \left( \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) = 0 \quad (3.0.71)$$

similarly, the tangential components of  $\vec{H}$  at  $r = a$  is:

$$A\omega \left( \frac{\epsilon_1 J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{\epsilon_2 K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) + jB\beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) = 0 \quad (3.0.72)$$

To get the value of A and B, let us put equation(3.0.71) and (3.0.72) using matrix and solve the determinant of the matrix.

$$\begin{array}{cc} \beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) & j\omega \mu \left( \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \\ \omega \left( \frac{\epsilon_1 J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{\epsilon_2 K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) & j\beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) \end{array} \begin{array}{c} A \\ B \end{array} = 0 \quad (3.0.73)$$

For nontrivial solutions for A and B to exist, the determinant of the coefficients has to vanish.

$$\begin{vmatrix} \beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) & j\omega\mu \left( \frac{J'_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K'_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \\ \omega \left( \frac{\epsilon_1 J'_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{\epsilon_2 K'_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) & j\beta l \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right) \end{vmatrix} = 0 \quad (3.0.74)$$

$$j(\beta l)^2 \left( \frac{1}{(\gamma a)^2} + \frac{1}{(\gamma a)^2} \right)^2 - \omega \left( \frac{\epsilon_1 J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{\epsilon_2 K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \left( j\omega\mu \left( \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \right) = 0 \quad (3.0.75)$$

when rearranging the above equation, we get:

$$(\beta l)^2 \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right)^2 = \left( \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \left( \frac{k_1^2 J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{k_2^2 K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \quad (3.0.76)$$

where,  $k_1^2 = \omega^2 \mu \epsilon_1$  and  $k_2^2 = \omega^2 \mu \epsilon_2$

Substitute  $k_1^2 = n_1^2 k_0^2$  and  $k_2^2 = n_2^2 k_0^2$  into equation(3.0.80),we get:

$$\left( \frac{\beta l}{k_0 n_1} \right)^2 \left( \frac{1}{(\kappa a)^2} + \frac{1}{(\gamma a)^2} \right)^2 = \left( \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \right) \frac{J_l(\kappa a)}{\kappa a J_l(\kappa a)} + \left( \frac{n_2}{n_1} \right)^2 \frac{K_l(\gamma a)}{\gamma a K_l(\gamma a)} \quad (3.0.77)$$

Equation(3.0.77) is the characteristic equation of step index fiber.

The only values of  $\kappa a$  and  $\gamma a$  satisfies the boundary condition between core-cladding boundary. So the only unknowns are  $\kappa a$  and  $\gamma a$ .

Let us substitute  $\kappa a = U$  and  $\gamma a = W$ . To determine the unknowns we have need one more equation. By combining equation,we get:

$$V^2 = (\kappa a)^2 + (\gamma a)^2 \quad (3.0.78)$$

$$V^2 = U^2 + W^2 \quad (3.0.79)$$

where,V is normalized wave guide parameter(cutoff frequency)

We can write V in terms of  $n_1, n_2, a$  and  $\lambda$ ;

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \quad (3.0.80)$$

V summarizes all of the important characteristics of a fiber in a single number. It can be used directly to determine if the fiber will be single mode or not at a particular wavelength

and also to calculate the number of possible bound modes. In addition it can be used to calculate the spot size.

# Chapter 4

## Results and Discussion

This chapter focuses on the discussion and analysis of the variation of normalized V parameter(normalized frequency) with wave length in optical fiber. To see the variation of normalized frequency against wave length in fiber,we have taken the specific values of the parameters, thus  $n_1 = 1.48, n_2 = 1.46, \lambda = 850nm - 1550nm$  and  $a = 4\mu m, 5\mu m, 25\mu m$  and  $50\mu m$ .

The variation of normalized V parameter against wave length is shown in figures from fig:4.1 to fig:4.4. knowing the variation of normalized V parameter in fiber helps to know the optical power in it. The optical power which is carried information data in the fiber degrades as it passes through the fiber. The degradation of the optical power is due to different factors such as attenuation,dispersion,scattering etc. Thus,the optical power in the fiber directly related with the normalized V parameters. When the normalized V parameter maximum in the fiber,then the light loses within it is maximum,and the vise versa is true.

Different literatures argue that the minimum value of normalized V parameter in SMF is less than 2.405. This normalized V pararmeter depends on the refractive indices of the core and the cladding of the fiber,and the wave length of the light wave.

Within this report we have interested to show the variation of normalized V parameter

with wave length.

The below all graphs generated by Origin 5 which show that the relation between the normalized V parameter and wave length.

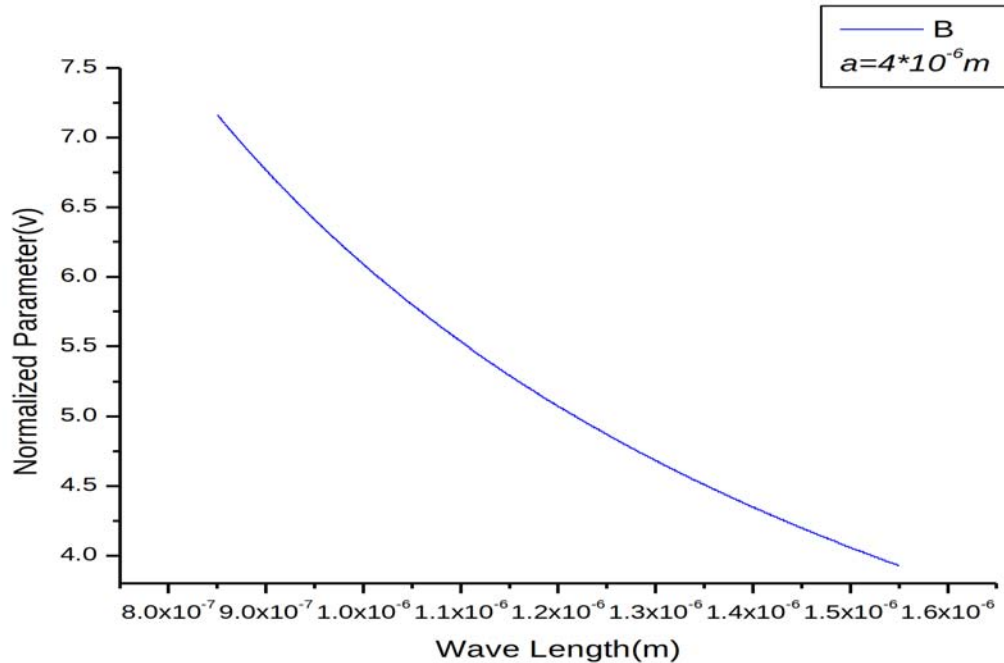


Figure 4.1: normalize V parameter(normalized frequency) vs. wave length

Fig:4.1 shows that, it's core radius,  $a = 4\mu m$ ,  $\lambda = 850nm - 1550nm$ ,  $n_1 = 1.48$  and  $n_2 = 1.46$ . Based on the given value of the core radius the fiber is single mode. Light wave,  $850nm - 1550nm$  wave length transmitted to the optical fiber, have different normalized frequency as shown in the figure. As the wave length increases by  $0.5nm$  the normalized frequency also varies relating with it. The variation seems like exponentially varying graph. When we have seen the graphs, there is minimum value of cutoff frequency with wave length. Based on our graph the minimum normalized frequency is around 4 units with wave length near to  $1550nm$ . So, the optimal wave length is near  $1550nm$  based on the fig:4.1.

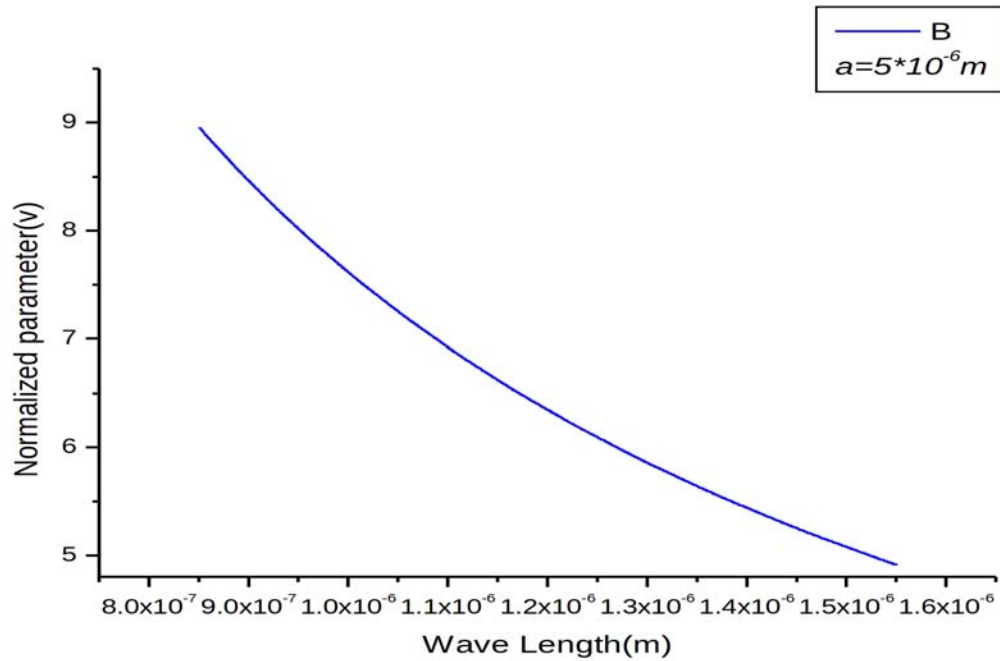


Figure 4.2: normalized V parameter(normalized frequency) vs.wave length

Similarly,fig:4.2,it's core radius is  $5\mu m$  ,which is single mode fiber. Taking wave length  $850nm - 1550nm$  light into the fiber and get the below fig:4.2. The graph shows that the cutoff frequency and wave length. From the graph we have taken the wave length which relates with the minimum normalized frequency. So the minimum normalized frequency founds around 5 units with wave length around  $1550nm$ . Which is optimal wave length uses for light transmission in optical fiber.

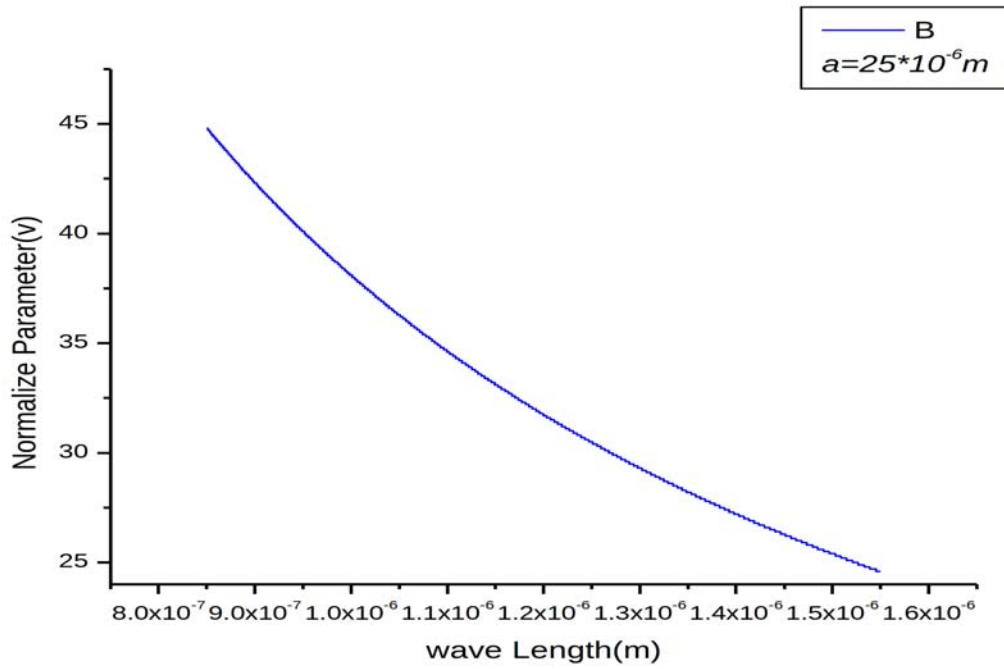


Figure 4.3: normalize V parameter(normalized frequency) vs. wave length

Fig:4.3, which has the core radius of the fiber is  $25\mu m$  which is multi mode fiber. When we transmit the light wave 850nm-1550nm into the optical fiber. Based on the graph we have taken the the wave length relating with minimum cutoff frequency. So we have taken near  $1550nm$  with 25 units.

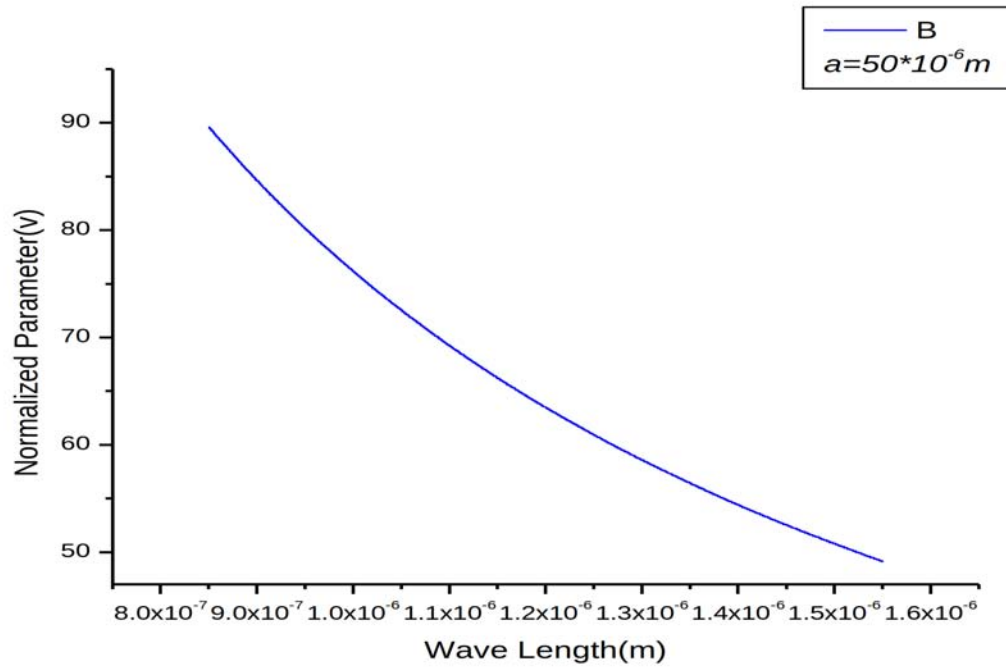


Figure 4.4: normalize V parameter(normalized frequency) vs. wave length

The last from our graph,fig:4.4,which has the core radius is  $50\mu m$ , multi mode fiber. Similar to the other graphs we have transmitted  $850nm - 1550nm$ ,which out produce the normalized frequency as shown with the graph. Based on this graphs,we have take the optimum wave length relating with minimum values of normalized frequency,which we have from the fig:4.4, around  $1550nm$  wave length relating with normalized frequency 50 units.

From all the above graphs,there is one best which is optimum wave length which uses for low loss,low dispersion, based on the minimum values of the normalized frequency. So from the all,fig:4.1 has optimum values of core radius and wave length for optical fiber communication. From these above discussions,the optimum wave length is near  $1550nm$  with core radius  $4\mu m$ .

# Chapter 5

## Conclusion

Most of the time optical fibers modified their material properties to maximize their transmission of information for long distance without losing information data. But within this thesis we have tried to show the variation of normalized V parameter against wavelength, and target optimum wavelength of light which uses to carry information with a minimum loss of information in communication technology.

As it can be seen on fig:4.1-fig:4.2, shows the variation of normalized V parameters with wavelength. As the wavelength varies from  $850nm - 1550nm$ , then the cutoff frequency varies also varies exponentially.

Based on our result the optimum wavelength of light to carry information in optical fiber is near  $1550nm$ . Within this light wave the cut off frequency is very small (around 4 units), so the loss of energy is also small relating with cutoff frequency.

# Appendices

## Appendix A

### Region of electromagnetic spectrum

Based on wave length,frequency,and energy the electromagnetic spectrum categorize as below:

Region	Wave length(A <sup>0</sup> )	Frequency(Hz)	Energy(ev)
Radio	>10 <sup>9</sup>	<3x10 <sup>9</sup>	<10 <sup>-5</sup>
Microwave	10 <sup>9</sup> – 10 <sup>6</sup>	3x10 <sup>9</sup> – 3x10 <sup>12</sup>	10 <sup>-5</sup> – 0.01
Infrared	10 <sup>6</sup> – 7000	3x10 <sup>12</sup> – 4.3x10 <sup>14</sup>	0.01-2
Visible	7000-4000	4.3x10 <sup>14</sup> – 7.5x10 <sup>14</sup>	2-3
Ultraviolet	4000-10	7.5x10 <sup>14</sup> – 3x10 <sup>17</sup>	3-10 <sup>3</sup>
X-ray	10-0.1	3x10 <sup>17</sup> – 3x10 <sup>19</sup>	10 <sup>3</sup> – 10 <sup>5</sup>
Gamma ray	<0.1	>10 <sup>19</sup>	>10 <sup>5</sup>

## Appendix B

### Material dispersion

Material dispersion occurs because the index of refraction varies as a function of wave length.Information carries by group velocity because of the information contains a band of frequencies. Time(T) required for a signal of frequency  $\omega$  to travel a distance  $L$  along the fiber:

$$T = \frac{L}{V_g} \quad (5.0.1)$$

Where,  $V_g$  is group velocity ( $V_g = \frac{d\omega}{dk}$ )

Rearranging the above relations, and get:

$$\frac{T}{L} = \frac{1}{V_g} = \frac{dk}{d\omega} \quad (5.0.2)$$

The difference in time due to different frequencies as given the below equation;

$$\delta \left( \frac{T}{L} \right) = \delta \left( \frac{1}{V_g} \right) = \frac{\partial}{\partial \omega} \left( \frac{dk}{d\omega} \right) \Delta \omega \quad (5.0.3)$$

where,  $k = \frac{n\omega}{c}$ ,  $n$  refractive index,  $c$  the speed of light Substituting  $k$  within the above relations:

$$\delta \left( \frac{T}{L} \right) = \frac{\partial}{\partial \omega} \left( \frac{d}{d\omega} \left( \frac{n\omega}{c} \right) \right) \Delta \omega \quad (5.0.4)$$

$$= \frac{\partial}{\partial \omega} \left( \frac{\omega}{c} \frac{dn}{d\omega} + \frac{n}{c} \right) \Delta \omega$$

$$= \frac{1}{c} \left( \omega \frac{\partial^2 n}{\partial \omega^2} + \frac{dn}{d\omega} + \frac{dn}{d\omega} \right) \Delta \omega \quad (5.0.5)$$

where

$$\omega = 2\pi \frac{c}{\lambda} \quad \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

so,

$$\frac{1}{c} d\omega = \frac{-2\pi}{\lambda^2} d\lambda$$

$$\frac{\partial \omega}{\partial \lambda} = \frac{-2\pi c}{\lambda^2}$$

$$\frac{\partial \omega}{\omega} = -\frac{d\lambda}{\lambda} \quad (5.0.6)$$

$$\delta \frac{T}{L} = \frac{\partial}{\partial \omega} \left( -\frac{\lambda}{c} \frac{dn}{d\lambda} + \frac{n}{c} \right) \Delta \omega \quad (5.0.7)$$

where,  $\partial \omega = -\omega \frac{d\lambda}{\lambda}$ ,  $\Delta \omega = -\omega \frac{\Delta \lambda}{\lambda}$

$$= -\frac{\lambda}{\omega} \frac{\partial}{\partial \lambda} \left( -\frac{\lambda}{c} \frac{dn}{d\lambda} + \frac{n}{c} \right) \left( -\frac{\omega}{\lambda} \Delta \lambda \right)$$

$$\delta \left( \frac{T}{L} \right) = \frac{\partial}{\partial \lambda} \left( -\frac{\lambda}{c} \frac{dn}{d\lambda} + \frac{n}{c} \right) \Delta \lambda \quad (5.0.8)$$

By factor outing equation();

$$\begin{aligned}\delta\left(\frac{T}{L}\right) &= \frac{1}{c} \frac{\partial}{\lambda} \left( n - \lambda \frac{dn}{d\lambda} \right) \Delta\lambda \\ &= \frac{1}{c} \left( \frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} \right) \Delta\lambda\end{aligned}$$

Finally,

$$\delta\left(\frac{T}{L}\right) = \frac{\lambda}{c} \frac{d^2n}{d\lambda^2} \Delta\lambda \quad (5.0.9)$$

since, equation(5.0.9) is equation of material dispersion.

$$\delta\left(\frac{T}{L}\right) = -M\Delta\lambda, \quad M = \frac{\lambda}{c} \frac{d^2n}{d\lambda^2}$$

where M is the property of the material.

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**Declaration**

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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**Place and time of submission: Addis Ababa University,June 2012**

This thesis has been submitted for examination with my approval as University advisor.

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