Determination of Superconducting Transition Temperature T_c and Order Parameter Δ

By

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Abstract

This project is well organized review of what is superconductivity, properties, some applications, and advantages of superconductors over ordinary conductors. The superconducting transition temperature (Tc) and the superconducting order parameter $\Delta(0)$ are determined employing the quantum field theory double time Green function technique and discussed using the well known BCS theory. The relation of Tc with the density of cooper pairs N(0) and interaction potencial V(0) together called electron coupling constant (λ_{el}) is shown graphically and discussed. Similarly the temperature dependence of the order parameter $\Delta(T)$ as a function of temperature (T) is shown graphically and discussed as well. For further or extensive studies on superconductivity, this project would be used as a base line.

Chapter 1

Introduction

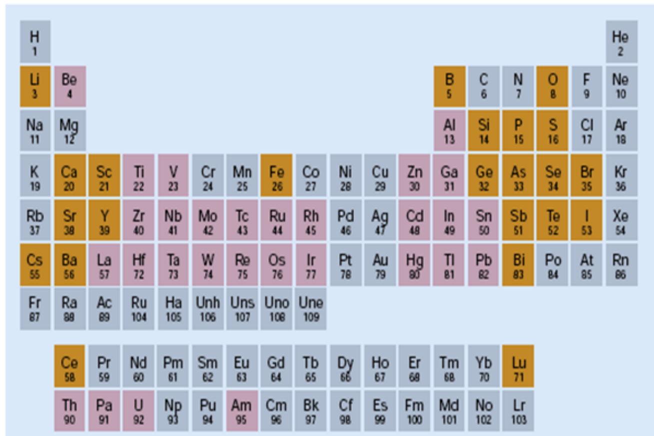
The growing population and accelerated industrialization of countries like Ethiopia require economically sustainable and friendly utilization of energy. It is clear that the economical development of nations depends on the amount and type of energy that the nations use. Population growth has always been and will remain one of the key drivers of energy demand, along with economic and social development. While global population has increased by over 1.5 billion over the past two decades, the number of people with access to commercial energy has reduced slightly, and the latest estimate from the World Bank indicates that it is 1.2 billion people [1]. Electric power system is one of the most important infra-structure of modern industrial society. Energy resources are often used to generate electricity. Electricity is exceptionally use full as it is quite simple to transfer a vast amount of energy from place to place and it can be easily transformed into most other forms of energy. The demand of this energy, which is easy to control, and clean is increasing rapidly over the world [2]. However, there are several difficulties starting from generation to distribution. Usually power generation is located remote areas from the grid center. So, long transmission and distribution lines have to be constructed and maintained to meet required reliable, and quality power. Most of conductors used in modern power system facilities like generator, transformer, motors and transmission line cables are Copper and Aluminum. Some people argue that the biggest challenge facing the human race right now is the question of how to generate electric power in a sustainable and economic friendly way [3]. A more pressing problem however, may be to avoid the large amounts of electricity lost to transmission every day. During this transmission however, around

10 percent of the energy is lost in the form of heat [3,4]. Transformers are responsible for approximately 40 percent of the total power losses [5]. Many transformers are used for transporting electrical energy from the power plant to the grid system. 80 percent of the present worldwide energy use is based on fossil fuels. Several risks are associated with their use [1]. Carbon dioxide emission is highly responsible for global warming. By various studies it is predicted that earth's average surface temperature could rise between 2^0C and 5^0C by the end of the 21^{st} century. In this era human life is highly dependent up on electrical energy, even we can't think about life with out electricity. But today most of electrical demand is met by conventional energy sources. One of the reason for the global warming is increasing population and hence increasing industrialization. It is expected by many scientists, that if global warming will continue in the same way, then it may deteriorate the life on earth and will lead to bad weather condition [6]. Superconductors are not only used as lossless electricity transporters but also used as endless energy sources [3,7]. So, from this point of view, we deside to do my project in evaluating the superconducting transition temperature (Tc) and order parameter $\Delta(0)$ mathimatically using the Green function method of attacking a problem.

1.1 Brief history of superconductivity

Superconductivity is a phenomenon of exactly zero electrical resistance and expulsion of magnetic fields occurring in certain materials when cooled below a characteristics temperature called critical temperature (Tc). It was discovered by Dutch physicist Heike Kamerlingh Onnes on April 8, 1911 in Leiden; Following his success in liquefying the gas helium [7, 8, 9, 10, 11]. He was studying the resistance of solid mercury (Hg) at any cryogenic temperatures using the recently produced liquid helium as a refrigerant [12]. At the temperature of 4.2 K, he observed that the resistance of Hg abruptly disappeared [13, 14].

Table 1.1: known superconductive elements [19].



29 elements super conducts under normal conditions

23 only under pressure: Lithium is the last discovered

Table 1.1: shows, Roughly half of the elements in the periodic table are known to be superconductive.

In subsequent decades, superconductivity was observed in several other materials (metals, alloys, compounds). In 1913, lead was found to super conduct at 7.2 K. In 1941 Niobium -Nitride was found to super conduct at 16 K. In 1953 vanadium -silicon displayed super conductivity properties at 17.5 K. And in 1962 scientists at Westinghouse developed the first commercial super conducting wire, an alloy of Niobium and Titanium ($NbTi$). Several materials exhibit supper conducting phase transitions at low temperatures. From its discovery in 1911 until 1986, it was generally believed that Superconductivity only exists in metals at extremely low temperatures, with maximum transition temperature of $< 25K$ above the absolute zero [15].

Table 1.2: Some low critical temperature (Tc) metals and alloys

Material	Tc(K)	Material	Tc(K)
Zinc	0.88	NbN	16.0
Gallium	1.1	V_3Ca	16.5
Aluminum	1.2	V_3Si	17.5
Indium	3.4	NbSn	18
Tin	3.7	C_{60}	19.2
Mercury	4.2	Nb_3Ge	23.2
Lead	7.2	–	–
Niobium	9.2	–	–

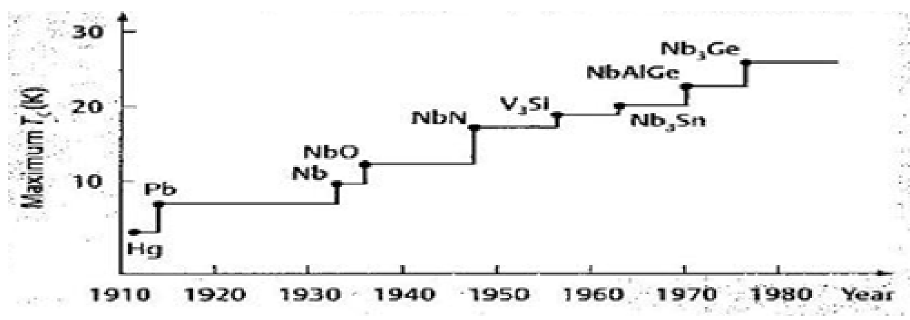


Figure 1.1: Super conducting materials with Tc $\leq$ 25 K and year $\leq$ 1986 [14].

A new era in the study of super conductivity began in 1986 with the discovery of high temperature super conductors [12]. In 1986 two European scientist (physicist) Bednorz and Muller, discovered a ceramic cuprites (a material containing copper and oxygen) that could become a superconductor at much higher temperature (35 K), which is La-Ba (Sr)-cu oxide. In January of 1987 a research team at the University of Alabama - hunts villa substituted yttrium for lanthanum in the Muller and Bednorz molecule which is Y-Ba-cu oxide and achieved an incredible 92 K critical temperature [16]. For the 1^{st} time a material today referred as YBCO had been found that would super conduct at temperature warmer than liquid nitrogen a commonly coolant. Liquid nitrogen is a widely available, (Abundant ($\frac{4}{5}$) of the atmosphere) and easily produced in the laboratory.

Compound		Tc(K)
	$LaSrCuO_4$	35
Y-based	$YBa_2Cu_3O_7$	92
Bi-based	$Bi_2Sr_2CuO_6$	20
	$Bi_2Sr_2CaCu_2O_8$	85
	$Bi_2Sr_2Ca_2Cu_3O_6$	110
Tl-based	$Tl_2Ba_2CuO_6$	84
	$Tl_2Ba_2CaCu_2O_8$	108
	$Tl_2Ba_2Ca_2Cu_3O_{10}$	125
	$TlBa_2Ca_3Cu_4O_{11}$	22
Hg	$HgBa_2CuO_4$	94
	$HgBa_2CaCu_2O_6$	128
	$HgBa_2Ca_2Cu_3O_8$	134

Figure 1.2: Some Important oxide superconductors.[16]

Within the next six years a number of additional families of high temperature super conductors (HTS) were discovered. These include titanium (Ti) and mercury (Hg) based system which had maximum critical temperature of about 123 K and 133 K respectively [11].

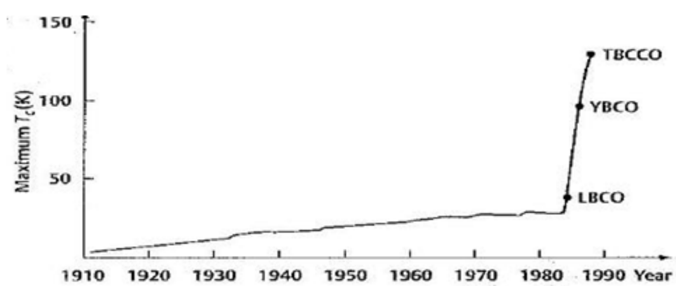


Figure 1.3: Superconducting compounds with Tc $\geq$ 35 K and year $\geq$ 1986 [14].

For more than 20 years the mercury copper oxides ($Hg_2Ti_3Ba_{30}Ca_{30}Cu_{45}O_{125}$) held the record for highest transition temperature at 138 K. In 2001,researchers in Japan discovered that a quite simple compound ,magnesium diboride (MgB_2) becomes super conductive at a temp. of 39 K above 0 K. In 2015 H_2S has been discovered to exhibit super conductivity at 203 K but at extremely high pressures around 150 Giga Pascal's [15].

Chapter 2

Review Literature

2.1 Properties of superconductivity

2.1.1 Zero electrical resistance

In a normal conductor, an electric current may be visualized (considered) as a fluid of electrons moving across a heavy ionic lattice (wire). The electrons are constantly colliding with ions, impurities or defects in the lattice (crystal structure) and scattered about. During each collision some of the energy carried by the current is absorbed by the lattice and converted to heat, which is essentially the vibration kinetic energy of the lattice ions. As a result, the energy carried by the current is constantly being dissipated. This leads to the concept of electrical resistance. Thus electrical resistance is defined as an opposition to current flow that dissipates energy [2, 17].

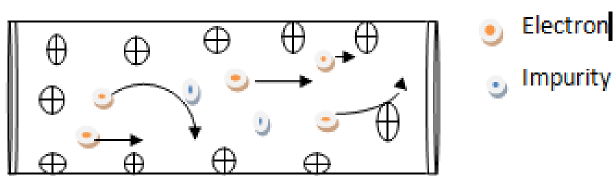


Figure 2.1: The origin of electrical resistance due to impurity and lattice scattering

Fig. 2.1: shows that, when electrons collide with deformed lattice and/or impurity, loses electrical energy in the form of heat. No scattering, No resistance, no loss of energy.

One of the interesting things about resistance is, how it changes as you change the temperature. Suppose you have a piece of gold wire in an electric circuit, gold is one of the best conductors and it shows very little resistance to electricity. But increasing its temperature raises its resistance. Broadly speaking the higher the temperature, the more thermal energy that causes vibrations inside the gold crystalline structure and the harden electrons which are the negatively charged particles inside the material that carry electric current will find it to flow through. Conversely, if you cool gold down, you reduce the vibrations and make it easier electrons to flow.

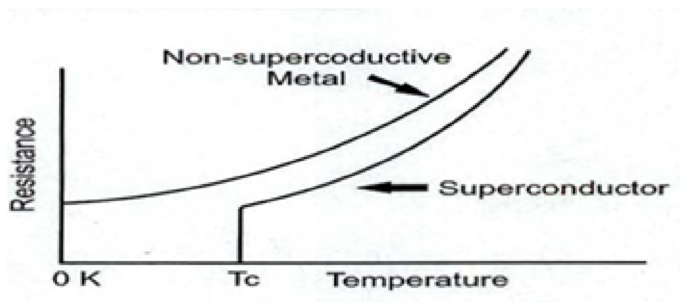


Figure 2.2: Variation of resistance with temperature for metal and superconductor [13].

Fig. 2.2: shows that, the electrical resistance of metallic conductors like copper, silver decreases gradually as temperature is lowered but they shows some resistance even near absolute zero. But in a superconductor, the resistance drops abruptly to zero when the material is cooled below its critical temperature.

A basic rule of physics is that opposite charges attracts and like charges repel. Electrons repel electrons, but attract protons. However electrons on their flight through the lattice cause lattice deformation (electrons attract the positively charged lattice atoms and slightly displace them) which results in a trail of positively charged region. This positively charged region of lattice atoms (potential) attracts another electron and provides for electron-electron coupling called cooper pair.

Fig. 2.3: Illustrated that, electrons move through two rows of positively charged atoms, thus pulling the rows of atoms inwards because of their attraction to the electron. This distortion created then causes another electron to follow behind the first. These electrons

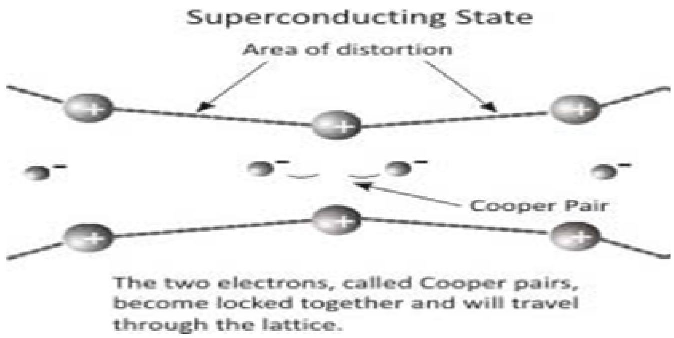


Figure 2.3: crystal lattice periodic vibration forming cooper pair [16].

pair up and encounter less resistance.

In metals periodic distortions of the lattice form excitations called phonons. Phonon is energy due to lattice vibration. These phonons essentially behave like quantum mechanical particles. In superconductors, two electrons can attract each other via the exchange of a phonon, which acts as an attractive force between them. The attraction between the electrons allows the formation of tightly bound electron pairs, the so called cooper pairs, named after Leon Cooper, who first come up with this strange idea. The carriers of charge and energy in super conductors are therefore not electrons like in normal conductors, but rather these cooper pairs. These cooper pairs again acts as a single particle and have very different quantum mechanical property compared to single electrons. They are classified as bosons which are integral particles. A special property of bosons compared to single electrons which are half integral particles called fermions is that bosons can collectively (unlike the Pauli principle that states no two electrons are allowed in the same energy state or 1 electron per state), occupy a ground energy state, which is very low in energy. Below the critical temperate, the bosons in a super conductors call all gather together in the lowest possible energy state (cold state) to form the condensate, i.e, large number of Cooper pairs can populate on one collective state, and the greater the number that have accumulated, the harder it is for one of them to leave. Electron pairing due to phonon exchanges explains superconductivity in conventional superconductors that are usually have critical temperatures ranging from around 20 K to less than 1 K [9]. Electron-electron coupling is weak and can be destroyed by thermal vibrations of the lattice. So this implies that superconductivity exists only at low temperatures for conventional superconductors. The collective state is stable and requires some additional energy input (thermal energy) to be destroyed. The binding energy of Cooper pairs in this state is several mill electron volts. The formation of collective state of Cooper pairs take place at $T < T_C$. In the collective bound state the Cooper pairs do not scatter from the lattice and the conductivity of superconductor is infinitely large. For the Cooper pair to scatter it would have to change its state (like an electron in normal metal). However, the Cooper pair is coupled to a large number of other Cooper pairs and so the whole collective of Cooper pairs would have to be involved in scattering at once. This does not happen, and therefore there is no scattering of Cooper pairs and therefore the conductivity is infinite.

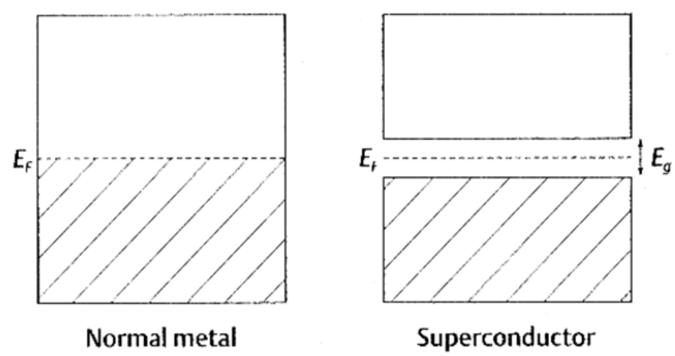


Figure 2.4: Occupation of energy levels at absolute zero in (a) a normal metal and (b) a superconductor [14]. E_F denotes the Fermi energy.

Fig. 2.4; shows that, in a superconductor, un like a normal metal, there is energy gap (E_g) between the highest filled energy state and the lowest vacant states.

Formation of Cooper pairs is a spontaneous process resulting in lower energy state of electrons in the superconductor. In superconductors, the filled state is occupied by Cooper pairs, and the empty band, above Eg, is occupied by "*broken*" Cooper pairs. The band gap (E_g) is a measure of binding energy of Cooper pairs.

The greater binding energy, the greater T_c. In other words, due to quantum mechanics, the energy spectrum of this cooper pair fluid posses an energy gap, meaning there is a minimum amount of energy ΔE that must be supplied in order to excite the fluid. Therefore if ΔE is larger than the thermal energy of the lattice given by $k_B T$, where k_B is Boltzmann's constant and T is the absolute temperature, the fluid will not be scattered by the lattice vibration or an impurity. The cooper pair fluid is thus a super fluid, meaning it can flow without energy loss. That is they can flow freely at zero resistance. Think of cooper pair as forming a team cooperating to avoid obstacles. "Two is better than one" [3]. All these can only happen at low temperatures, because the contribution of phonon interaction only becomes relevant when the thermal vibrations of the crystal structure are very small. Theoretically electricity can flow through a loop of superconducting material forever with no power source and energy loss [3]. using ohm's law, $R = \frac{V}{I}$, where R is resistance V is voltage and I is current, Since superconducting materials carry current with no applied voltage and R = 0, super conductivity does not involve power loss, since power loss is defined as $P = I^2 R$ as $R = 0$ for super conductors. Copper, silver, and gold are three of the best metallic conductors but are not super conductive. This is due to the face centered cubic (Fcc) unit cell structure, which are so tightly packed that the low temperature lattice vibration essential to superconductivity fail to bring free electrons into cooper pairs [18]. While some FCC metals such as lead (Pb) are capable of superconduc-

Figure 2.5: Face centered cell (Fcc) of copper (Cu) atom [19].

tivity. This is due to outside factors such as lead's modulus of elasticity that is, the ratio of pressure (stress) applied to a body to the resistance (strain) produced by the body. In normal metals, electrons will move quickly and constantly collide, thus creating heat energy that is why metals are normally at a temperature above absolute zero. However, in superconducting metals, electrons are greatly slowed. Heat is never created because there are no collisions. In superconducting metal, an electron moves down do not collide into one another. This is why superconducting materials do not loss energy.

2.1.2 Expulsion of magnetic fields

Superconductors exhibit unique features other than the zero electrical resistance. Many expel magnetic fields during the transition to the superconducting state. This is due to the Meissner effect.

Meissner effect

When a superconductor is placed in a weak external magnetic field B, and cooled below its transition temperature (Tc) the magnetic field is ejected out [18, 19, 20]. This is illustrated in the Fig.2.6: below.

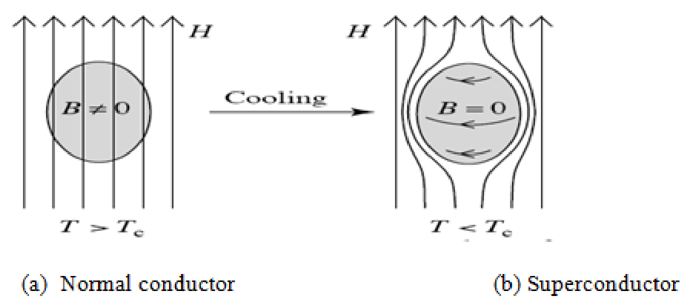


Figure 2.6: The Meissner effect of superconducting magnetic field above and below critical temperature (Tc) [19].

For superconductors inside sphere the magnetic field is zero, from this it is possible to show the magnetic susceptibility $(\chi) = -1$ for diamagnetism. $B = \mu_O(H + M)$ where M is magnetization, μ_o is the permeability of free space. $\chi = \frac{M}{H}$ this implies $M = H\chi$

$$B = \mu_0(H + H\chi)$$

$$B = \mu_0 H(1 + \chi)$$

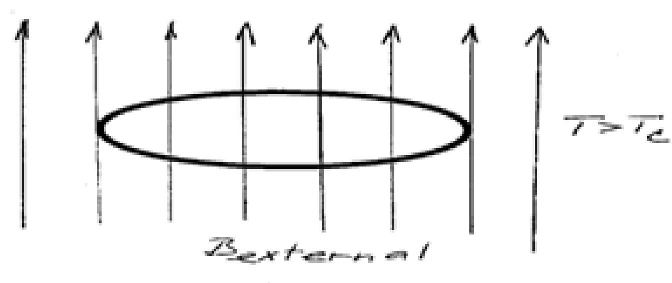
$$B = 0$$

This implies $\chi = -1$ this shows that some superconductors have the property of perfect diamagnetism. A superconductor with no magnetic field with in it is said to be in the

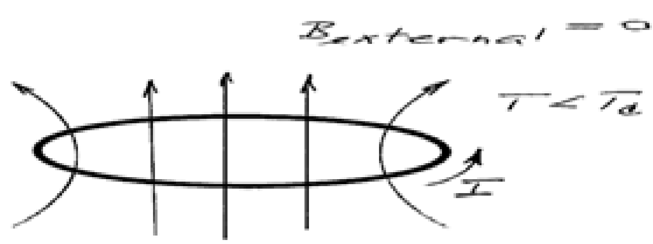
Meissner state. The Meissner state breaks down when the applied magnetic field is too large.

2.1.3 Effect of trapped magnetic flux

Consider a ring made out of superconductive material. Perform the following thought experiment: 1. At $T > T_c$ the material is normal state. When the external magnetic field is turned on, it penetrates through the ring.



2. Reduce the temperature so that $T < T_c$.
3. Remove the external magnetic field



4. We discover that the magnetic field that was penetrating through the opening of the ring remains there. The magnetic flux remains trapped in the ring opening. This effect can be explained in terms of Faraday's law of induction $\oint(Edi) = -\frac{d}{dt}\phi_B$ Where E is the electric field along the closed loop, ϕ_B is the magnetic flux through the ring.
Before the external magnetic field was turned off, there was a magnetic flux $\phi_B = B.A$

through the ring. Below T_c the resistivity of superconductor becomes equal to zero and therefore at $T < T_c$ the electric field inside the superconductor must be and is zero as well. In view of this $\oint(\overrightarrow{E}.\overrightarrow{di}) = 0$ and there fore, the right side of Faraday's equation $\frac{d\phi_B}{dt}$ $= 0$ Which means that $\phi_B = B.A$ is constant. The magnetic flux through the ring must remain constant. For this reason the magnetic flux remains trapped in the opening of the ring after the external magnetic field has been turned off. There is no magic involved. The trapped magnetic field passing through the ring is due to the current induced in the ring when the external magnetic field was turned off. The induced current is called the persistent current. The current persists; means that it does not decay because the resistance of the ring is zero. Actually no decrease of current was observed over the period of three years! Theoretically, the relaxation time of current carriers in the superconductor is greater than the age of universe [16].

2.1.4 Types of superconductors

Superconductors are of type I (soft) and type II (hard) based on how the Meissner state breakdown occurs.

Type I superconductors

Type I superconductors are those Scs which lose their superconductivity very easily or abruptly when placed in the external magnetic field. In type I superconductors; Superconductivity is abruptly destroyed during a first order phase transition when the strength of the applied field rises above a critical value (Hc). It can be seen from the graph of intensity of magnetization (M) versus applied magnetic field (H).

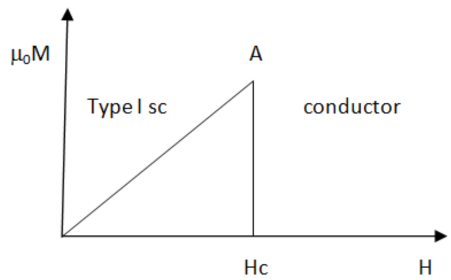


Figure 2.7: Intensity of magnetization (M) versus applied magnetic field (H) for type I Scs [21].

Fig. 2.7: shows, when the external applied magnetic field exceeds the critical magnetic fields (Hc), superconductivity suddenly loses at (point A) [21]. As such, they have only a single critical magnetic field (Hc) at which the material stop to super conduct and become resistive.

The Miessner effect is a defining characteristic of superconductivity of type I. Elemental superconductors, such as Al, Pb, Hg are typically type I superconductors. The origin of their superconductivity is explained by BCS theory which is to be discussed latter on. This type of superconductivity is normally exhibited by pure metals. The type I superconductors have been of limited practical usefulness because of the critical magnetic fields are so small and the super conducting state disappears suddenly at that temperature and so they are sometimes called " soft" superconductors. Type I super conductors have critical temperatures (Tcs) between 0.000325 K and 7.8 K at standard pressure. Examples are Aluminum Hc = 0.0105 T and Zinc Hc = 0.0054 T. Some type I superconductors require incredible amounts of pressure in order to reach the super conducting state. One such material is sulfur which requires a pressure of 9.4×10^{11} pas. And a temperature of 17 K to reach the superconductivity.

Type II superconductors

Type II superconductors are those which loose their superconductivity gradually but not easily or abruptly when placed in the external magnetic field.

Table 2.1: some type II superconductors

Material	Tc(k)	Hc(T)
NbTi	10	15
V_3Ga	14.8	2.1
NbN	15.7	1.5
V_3Si	16.9	2.35
Nb_3Sn	18	24.5
Nb_3Al	18.7	32.4
$Nb_3(AlGe)$	20.7	44
Nb_3Ge	23.2	38

As we can see from the graph of intensity of magnetization(M) versus applied magnetic field(H) showmen below Fig. 2.8: Type II superconductors start to lose their superconductivity at lower critical magnetic field (H_{C1}) and completely lose their superconductivity at upper critical magnetic field (H_{C2}). The state between the lower critical magnetic field (H_{C1}) and the upper critical magnetic field (H_{C2}) is known as vortex state or intermediate state. After H_{C2}, the type II superconductors will become conductor.

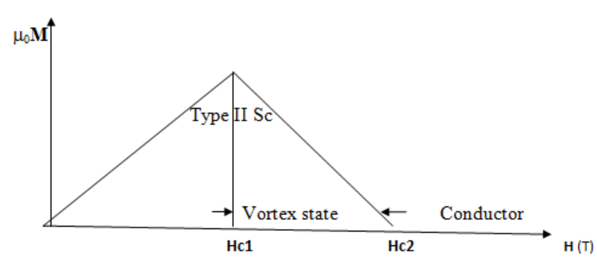


Figure 2.8: Intensity of magnetization (M) versus external applied magnetic field (H) for type II superconductors [21].

Type II superconductors are also known as hard superconductors because of this reason that is they loose their superconductivity gradually not easily. They obey Meissner effect but not completely. In type II superconductors, raising the external applied field past a lower critical value H_{C1} the field penetrates the superconductor but only to a very small distance, exponentially to zero within the bulk of the material. This leads to mixed state also known by the vortex state which is an intermediate state consisting of regions of normal material carrying magnetic field mixed with regions of superconducting material containing no field.

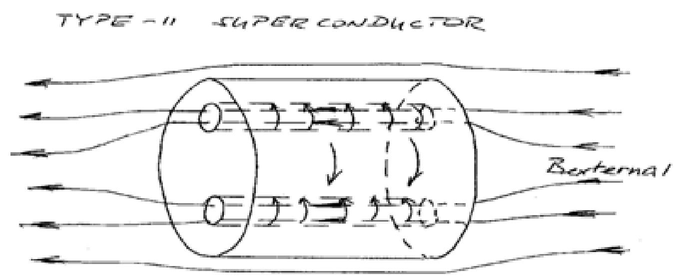


Figure 2.9: Bulk of type II Superconducting material broken down to Sc and normal parts [14].

Fig. 2.9: shows that, the normal regions are distributed as filaments filled with the external magnetic field. Electric current is induced at the interface between the normal and the superconductive regions. The "surface" of filaments is "wrapped" in current which cancels the magnetic field in the superconductive regions. The electric current is carried by the superconductive regions of Type-II material.
The surface layer carrying the electric current has a finite thickness, and because of this, the external magnetic field partially penetrates into the interior of the superconductor [11,16].

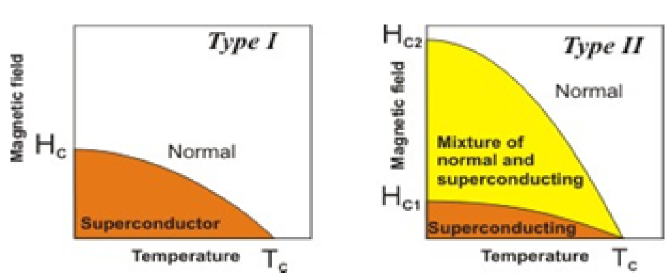


Figure 2.10: Magnetization curves of type I and II superconductors [16].

Fig. 2.10: shows that, when the applied magnetic field reaches at the second or upper

critical field strength (H_{C2}), superconductivity destroyed completely. They have second order phase transition from the superconducting to the normal state with an increasing applied magnetic field.

Most pure metal superconductors are type I except niobium, vanadium and technetium are pure maintaining the superconducting state to higher temperatures. Besides being mechanically harder than type I superconductors, they exhibit much higher critical magnetic fields. Type II superconductors such as niobium-titanium ($NbTi$) are used in the construction of high field superconducting magnets. Type II superconductors are usually made of metal alloys or complex oxide ceramics. The highest Tc reached at standard pressure, to date, is 135 K or 138 K by a compound $HgBa_2Ca_2Cu_3O_8$ that falls group of superconductors and is considered to be a ceramic [18]

2.1.5 Magnetic critical fields

A material in a superconductive state has zero resistivity below the critical temperature Tc. If it is subjected to an increasing magnetic field (H), its resistance restores. That is the superconductive state is destroyed at a critical value of magnetic field called critical field (H_C). The value of critical field is a function of temperature and is approximately given by $Tuyn's$ law, which is $Hc = H_0$ [1 - $(\frac{T}{Tc})^{1/2}$] [16], where H_0 is the field at T = 0 K. And at T = Tc, $H_C = 0$

2.1.6 Critical current (Ic)

Superconductive state is destroyed by magnetic field. Consider a straight wire. Since electric current in the wire creates magnetic field B, the wire can carry maximum superconductive current Ic, corresponding to the critical magnetic field Bc at the surface of the wire, r = R,

$$B_c = \frac{\mu_o I_c}{2\pi R},$$
$$I_c = \frac{B_c 2\pi R}{\mu_o},$$

where $\mu_o = 4\pi \times 10^{-7}$ Tm/A is the magnetic permeability of free space. So In order to achieve high critical currents in superconductive magnets we need materials of high B_C.

The ability of a material to super conduct depends on three factors: the temperature T, the magnetic field strength H, and the current density J, Tc is the critical temperature in the absence of external magnetic field and with no current flowing in the sample, Hc is the critical magnetic field strength with no current flowing at T = 0 K, and J_c is the current density at T = 0 K, with no external magnetic field.

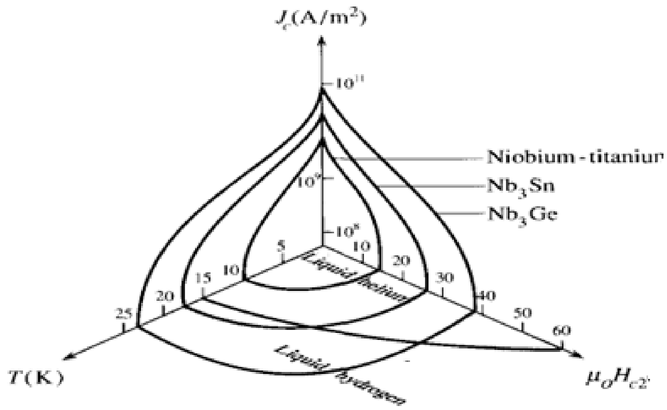


Figure 2.11: the critical surface separates the superconducting and normal states for $NbTi$, Nb_3Sn, and Nb_3Ge superconductors. [14, 19].

Fig. 2.11: shows that, the Superconductive material can be conveniently characterized by its critical surface in a (T, H, J) coordinate system. That is Superconductivity prevails everywhere below the critical surface and normal conductivity above [22].

2.1.7 BCS Theory

In 1957 American physicist John Barden with leon Cooper and Robert Schrieffer developed the best theory we currently have of how superconductors work, called the BCS theory in honor of their three discoverers. The theory explains how materials suddenly become superconductors was the vibrations of the atoms in the structure of the metal that were responsibly for superconductivity. At very low temperatures these vibrations change the way that the electrons behave. They allow the electrons to form loosely associated pairs what are called (cooper pairs) or (BCS) pairs which are able to move through the material without resistance. Cooper pair is two electrons which interact attractively in the phonon field. Phonon is vibration of crystal lattice. The building block of this theory relies on electron-electron interactions which form quasi particles that are known as cooper pairs. In solids, each material has an associated energy gap that needs to be overcome for the electron to be bound. Normally, the electrons that carry the electricity through the material are scattered about by impurities, defects and vibrations of the materials crystal lattice (structure). That is what we know as electrical resistance. But at low temperatures when the electrons join together in pairs, they can move more freely without being scattered in the same way. So it is said that superconductivity occurs when electrons work together or join force in cooper pairs. They have won a noble prize in 1972. The BCS theory explained superconductivity at temperatures close to absolute zero for elements and simple alloys.

Superconductors explained based on this BCS theory are referred as conventional superconductors. However at higher temperatures and different superconductivity systems, the BCS theory has subsequently become inadequate to fully explain how superconductivity is occurring [8]. The phonon cause regions of the lattice structure to be more positive than other regions. This provides an attractive force that is stronger than the Coulomb repulsion of near by electrons which have equal but opposite momentum and spin, to pair.

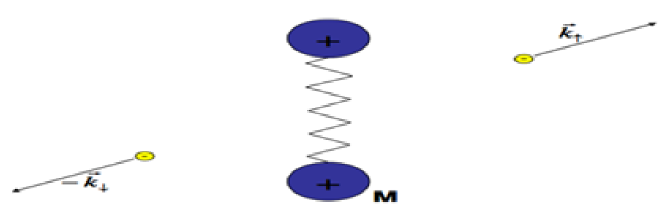


Figure 2.12: Electron-Phonon coupling

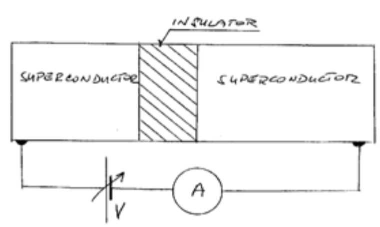
Fig. 2.12: shows that , phonon brings two positive ions closer together and makes that region of space more positive (i.e polarized) than surrounding regions and attracts electrons that have opposite momentum and spin form cooper pair.

Cooper pairs form zero-spin (singlet) or bosons who condense and transport electric current without dissipation. In general the BCS theory explains how the interaction b/n the electrons and phonons (lattice vibrations) in the metal causes an electron-electron attraction. So some of the electrons form the cooper pairs [3]. Where the spins (spin is a quantum number that characterize a particle as half Integral or Integral spin) and momentum are opposite and therefore cancel out, because cooper pairs have integral spin as they are characterized as bosons [23].

2.1.8 Josephson effect

Another significant theoretical advancement came in 1962 when Brain D Josephson a graduate student at Cambridge university, predicted that electrical current would flow b/n two superconducting materials even when they are separated by a none superconducting materials or insulators which is called tunneling [24, 25]. His prediction was latter confirmed and won him a share of the 1973 Noble prize in physics. This tunneling phenomenon is today is known as "Josephson effect" and has been applied to electronic devices, such as the superconducting quantum interference device (SQUID), an instrument capable of detecting even the weakest magnetic fields.

Consider two superconductors separated by a thin insulating layer, few nm thick.



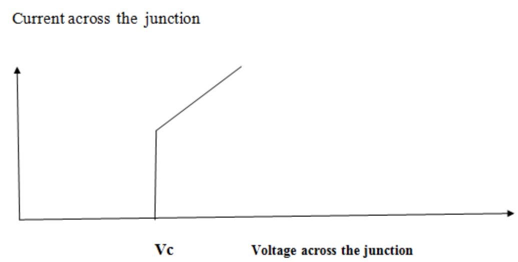


Figure 2.13: Current voltage characteristics of superconductor insulator superconductor junction [14].

Fig. 2.13: shows that, for a voltage less than the critical value V_c, there is no current across the junction, but at the voltage V_c there is a sharp increase in current and then increases proportionally with the biasing voltage.

Brian Josephson noted (1962) that: 1. Electron pairs in the two superconductors can form a single collective state and the electron pairs can tunnel through the insulating layer.

DC Josephson effect = electron tunneling current across the junction in the absence of applied voltage.

2. If a DC voltage bias is applied across the junction, there is an AC current through the junction that oscillates with frequency The existence of ac current through the biased junction = AC Josephson effect.

2.1.9 Isotopic effect on superconducting transition temperature

Atoms of the same element with different masses are called isotopes. This has effect on superconductivity properties. Interaction between electrons and vibration of lattice atoms is critical for the existence of superconductivity. An experiment also says that the critical temperature is proportional to typical frequency of oscillation of the ions about their equilibrium positions. i.e $T_c \approx \omega$ [23]. The transition temperature decreased when the atom became heavier and the change was proportional to the reciprocal of the square root of the mass of the atom. Just by changing an atom by its isotope $T_c \approx m^{-1/2}$ this is called the " isotope effect" [26]. Therefore by changing the mass of the system, the frequency of lattice vibrations also change. For example, by substituting mercury 202 with mercury 198, the Tc increased proving the dependence of superconductivity on lattice vibrations [22]. We can assume that such an ion moves as a small sphere of mass m at the end of a spring characterized by a spring constant K. The model can be solved as a classical harmonic oscillator, that is a system where a force, F equal to -KX (it always points against the displacement from the equilibrium position X) is applied to an object of mass m. The mathematical solution of this problem shows as that the frequency of oscillation of the mass m around its equilibrium position is $\omega_{osc} = \sqrt{\frac{k}{m}}$, we get therefore the appealing result $\omega_{osc} \approx m^{-1/2}$. This shows that superconductivity and the oscillations of the ions in the lattice have something in common. Isotopic effect may be expressed mathematically as $\frac{T_c}{T_{c'}} = (\frac{m'}{m})^{\propto}$, where the isotopic effect exponent $\propto$ ranges between 0 and 0.5. $\propto = 0$ if there is no isotope effect. In conventional superconductors, the alpha factor in the above equation is experimentally measured and theoretically predicted (by BCS theory) to be approximately 0.5.

2.1.10 The London equation

When the applied field strength to some superconductors increases, the field will not be ejected completely, but instead some field penetrates the surface but only to a very small(microscopic) distance, characterized by a parameter called the London penetration depth and decays exponentially to zero within the bulk of the material. This is shown by the brothers Fritz and Heinz London. The London equation relates the super current density j_s to the magnetic field (B). Assume the superconductor at temperature $T < Tc$, only a fraction of $\frac{n_s}{n}$ (T) of the total number of electrons is capable of participating in a superconductor current. $n_s(T)$ is the density Of electrons of superconducting, $n_n(T)$ is the density of normal electrons and $n = n_s + n_n$. At T much below T_c, $n_s \rightarrow n$ and $n_s \rightarrow 0$ when T rise from T_c. So that their mean velocity v_s will satisfy [9].

$$m_e \frac{d\overrightarrow{v_s}}{dt} = -e\overrightarrow{E}, \tag{2.1.1}$$

$$\overrightarrow{j_s} = -e\overrightarrow{v_s}n_s, \tag{2.1.2}$$

$$\frac{d\overrightarrow{j_s}}{dt} = -en_s(\frac{d\overrightarrow{v_s})}{dt} = (-en_s)(\frac{-e\overrightarrow{E}}{m_e}),$$

$$\frac{d\overrightarrow{j_s}}{dt} = e^2\frac{\overrightarrow{E}}{m_e}n_s, \tag{2.1.3}$$

Equation (2.1.3) is referred as the first London equation. From (2.1.3) we will have

$$\overrightarrow{E} = (\frac{m_e}{e^2n_s})\frac{d\overrightarrow{j_s}}{dt}, \tag{2.1.4}$$

Substituting equation (2.1.4) into Faraday's law of induction

$$\overrightarrow{\nabla} \times \overrightarrow{E} = (\frac{-1}{c})(\frac{d\overrightarrow{B}}{dt}), \tag{2.1.5}$$

$$\overrightarrow{\nabla} \times (\frac{m_e}{e^2n_s})\frac{d\overrightarrow{j_s}}{dt} = (\frac{-1}{c})(\frac{d\overrightarrow{B}}{dt}),$$

$$\frac{d(\overrightarrow{\nabla} \times \overrightarrow{j_s}}{dt} = -\frac{d\overrightarrow{B}}{dt}(\frac{e^2n_s}{m_ec}), \tag{2.1.6}$$

from equation (2.1.6) we will have

$$\overrightarrow{\nabla} \times \overrightarrow{j_s} = \frac{\overrightarrow{B}e^2n_s}{m_ec}, \tag{2.1.7}$$

. Combining equation (2.1.7) with Max well equation which is

$$\overrightarrow{\nabla} \times \overrightarrow{B} = \overrightarrow{j_s}\frac{4\pi}{c} = \mu_o\overrightarrow{j_s},$$

yields

$$\overrightarrow{\nabla} \times (\overrightarrow{\nabla} \times \overrightarrow{B}) = \frac{4\pi}{c}\frac{\overrightarrow{B}e^2n_s}{m_ec}, \tag{2.1.8}$$

$$\nabla^2\overrightarrow{B} = \frac{4\pi(\overrightarrow{B}e^2n_s)}{m_ec^2}, \tag{2.1.9}$$

$$\nabla^2\overrightarrow{B} = \frac{\mu_o(\overrightarrow{B}e^2n_s)}{m_ec}, \tag{2.1.10}$$

$$\nabla^2\overrightarrow{B} = \frac{\overrightarrow{B}^2}{\lambda_L} \tag{2.1.11}$$

where $\lambda_L = (\frac{m_ec}{\mu_oe^2n_s})^{\frac{1}{2}}$ called the london penetration depth [25].

So the solution of the differential equation is $\overrightarrow{B}(x) = \overrightarrow{B}_{ext}exp(\frac{-x}{\lambda_L})$ i.e. the amount of magnetic field inside the material at x distance from the surface of the material is $\overrightarrow{B}$(x). For some materials $\lambda_L = 20nm - 50nm$. In bulk of thick superconductor it vanishes which is just the Meissner effect [7].

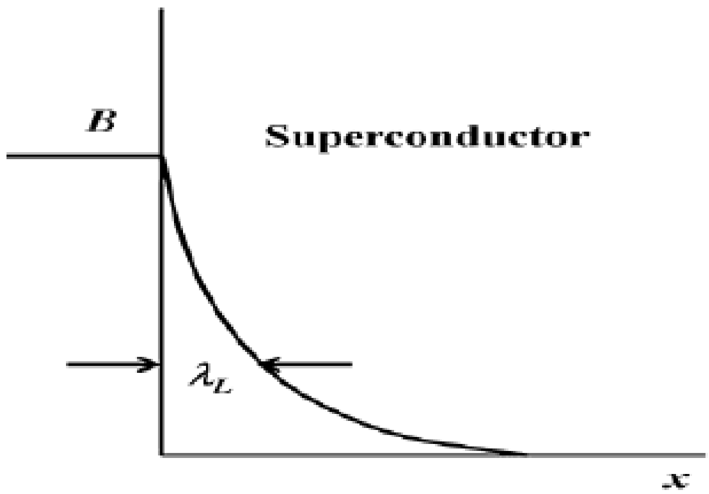


Figure 2.14: London penatration depth inside superconductor.

Fig. 2.14: shows that, magnetic field strength ($\overrightarrow{B}$) inside a superconductor decreases exponentially as the depth from the surface (x) increases [14].

Chapter 3

Applications and advantages of Superconductors over conventional conductors

HTS in power systems are applicable in the technical advantages for electric energy production, transmission and distribution systems. The current densities of known HTS materials are about 100 $\frac{A}{mm^2}$ which is at least 10 times larger compared to the current densities of conventional conductors. HTS cables have small impedance, large transmission capacity in compacted dimension, small transmission loss. Don't forget about the cooling, though [27, 28]. It has an increased tensile strength of 140 M Pa. at room temperature, which is an important mechanical property for making the cable practical [27]. HTS power cables address many of the challenges associated in delivering the generated power. The most significant advantages of HTS power cables over conventional power lines are HTS are none metals oxide compound substance such as BSCCO series. And LTS are mainly metal series such as NbTi [2]. In general

-Their ability to carry large amount of power in small cross sections.

-Their improved efficiency for very long distance transmission.

-Their high magnetic field applications like MRI.

-Their Near zero losses, differs them and be preferred significantly from conventional cables [29].

3.1 Advantages of Superconductors over conventional conductors.

Superconductor-based products are extremely environmentally friendly compared to their conventional counterparts. They generate no greenhouse gases and are cooled by non-flammable liquid nitrogen (nitrogen comprises 78 percent of our atmosphere) as opposed to conventional oil coolants that are both flammable and toxic. They are also typically at least 50percent smaller and lighter than equivalent conventional units, which translate into economic incentives. Super conductors are already used in many fields: electric power lines, medical applications and even transportations. They are used in laboratories especially in particle accelerators, in ultra sensitive magnetic field detectors called SQUIDs [30], and in superconducting coils to produce very strong magnetic fields. Super conducting generators, motors and transformers are less than 50 percent the size and weight of conventional equipments. Super conductors generate high magnetic fields at temperature of 4.2 K to 65 K.They can produce high field magnets 25 Tesla and beyond.

3.1.1 Superconductors in electric power lines and grids

Electricity has become an essential energy source in our modern lives. Today's conventional power cables are being operated closer to their thermal or stability limits, and so carry limited currents; otherwise they would heat and melt, and new lines are becoming hard to site. A net work of super conducting power cables would solve this problem because 100 times more electric current can flow through them: smaller resistance with more current. Today's power grid operators face complex challenges that threaten their ability to provide reliable service. Steady demand growth, developing infrastructure brought on structural and regulatory reforms in setting new plants and lines. The current electricity grid has in sufficient storage capability. Superconducting Magnetic Energy Storage (SMES) is a solution for storage of electrical energy in a powerful magnetic field. Superconductors enable a variety of applications to aid our heavily burdened electric power infrastructure - for example, in generators, transformers, underground cables. The replacement of copper or other normal conductors by super conducting materials avoid heat dissipation and other energy losses due to finite resistance. Underground cables of HTS will be the solution to economically transmit the increasing need of energy in the major centers with a low environmental impact [31]. Compact, high-capacity underground HTS cables offer an important new tool for raising grid capacity and increasing grid reliability. HTS power cables transmit 5-10 times more power than conventional copper cables of equivalent cross section [32].

Transformers

Transformers are responsible for approximately 40percent of the total power loses [5]. Superconducting transformer may be more efficient, smaller, lighter than its conventional equivalent and environmentally safe. The main benefits of the super conducting transformers are the low energy losses, the decrease in weight and volume as well as the reduction of environmental impacts. The replacement of conventional copper cables by HTS Cables in existing transformers, results in the simultaneous effect that in the same space more power with less electrical losses can be transmitted [31].

Electric Generators

Electric generators made with superconducting wires are far more efficient than conventional generators wound with copper wires. In fact their size is about half that of conventional generators [33]. Direct drive wind generators are utilizing a new high-efficiency stator design and replacing copper with HTS wire on the rotor. Estimates are that a 10 MW drive utilizing HTS technology would weigh about one third the weight of a conventional direct drive generator with the same power rating. This reduction in weight would also allow an increase in blade size and greater power output [34].

3.1.2 Superconductors in Fault Current limiters

During short circuit, current levels occur in electricity grids which can exceed the rated current many times. The grid must be designed for such loads i.e must be short circuit resistant to prevent major damages [33]. Superconductors are also used as current limiters in power plants and work as super conducting fuses. Fault current limiters for electrical utility depend on the fact that superconductivity is lost and resistance appears above a critical current. Under normal conditions the fault current Limiter is superconducting and offers no impedance to the ordinary current. During the power surge the large current exceeds the critical current and is limited by the consequence resistance as the superconductor goes normal. The superconductivity returns after the current spike falls down (become lower than the critical current) [31].

3.1.3 Superconductors in Transportation

Around the world, today's transportation systems are facing an unfolding crisis (challenges). Nearly all of the dominant technologies that provide mobility today - including automobiles, trains, ships and aircraft - depend intensely on petroleum based fuels. Yet, world oil prices continue to rise. One of the most promising responses to this challenge lies in the electrification of transportation. Electrification allows for the powering of many transportation systems. Superconductivity offers several ways to exploits the advantages of electrified transportation of various types, ranging from high-speed trains to advanced ship propulsion systems and lighter weight aircraft engines.

Electric Motors

The typical motor requires lots of copper wire, and highly inefficient when compared to a HTS motors. The USA navy is the first to apply HTS motors. Such motors are approximately twice as efficient as the best conventional motor. significantly smaller motors are begun built for navy ships using super conducting wires and tapes. In July 2001, American super conductors revealed a 5000 horse power motor made with superconducting wire. An even larger 36.5 MW HTS ship propulsion motor was delivered to the US, Navy in late 2006. A 4 MW ship derive motor with a great torque at low rotational speeds is currently being tested by superconducting material [33].

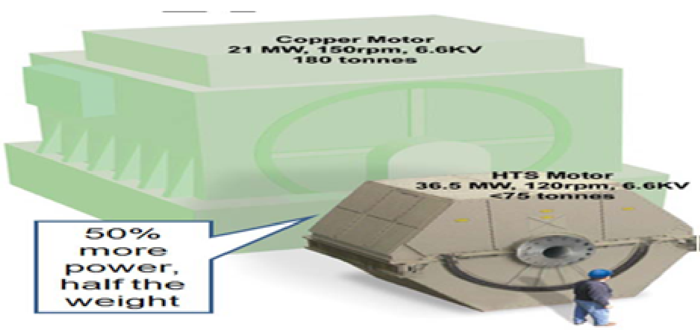


Figure 3.1: Comparison of copper and superconductor motors [32].

Type	Power	Frequency	Voltage	Mass
Copper motor	21 MW	150 rpm	6.6 KV	180 tonns
Sc motor	36.5 MW	120 rps	6.6 KV	$\leq$ 75 tonns

Maglev (Magnetic Levitation) trains

A magnet levitates above a high temperature super conductor, cooled with nitrogen. Persistent electric current flows on the surface of super conductor, acting to exclude the magnetic field of the magnet. This current effectively forms an electromagnet that repels the magnet (Meisner effect).

Figure 3.2: a magnet floating over a superconductor [24].

The levitation height is proportional to the power of the magnet. With the most powerful magnets (such as alloys of iron, boron), the elevation can reach a few cm (1cm) at best. For higher levitation, one would need powerful electromagnets. A magnet can hence levitate above a superconductor at an elevation of about 1cm and bear the weight of several kilograms. Maglev is an application where super conductors perform extremely well. Transport vehicles such as trains can be made to float on strong super conducting magnets, virtually eliminating friction b/n the train and its track. Not only would conventional electromagnets waste much of the electrical energy as heat, they would have to be physically much larger than super conducting magnets [35].
A train using this magnet reached a speed of 500 km/h [10]. This technology actually already exists in our lives, and goes by the name of maglev. It is the science behind japans. However, there are safety concerns about the strong magnetic fields used as these could be a risk to human health [36]. A more cost effective HTS magnetic train is known as the quantum train.
One example of a large scale application is the 370 mi/hr superconducting levitated train that will carry passengers from Osaka to Tokyo in 67 minute [37]. Magnetically levitated trains, employing superconducting magnets, offer a way to make trains literally "fly" to their destination by using powerful magnets to cause them to float above their Guide way, or track. An interesting feature of a superconductor which is expulsion of external magnetic field (Meissner effect) is also seen in the following two cases.

Figure 3.3: socks that levitate above soles with superconductors to avoid sweat [35].

Figure 3.4: a pan that can be held without touching it due to magnetic levitation, to avoid burns [35]. (Like bring near two like poles of a magnet)

3.1.4 Superconductors in Medical applications (Radiation-free Imaging)

MRI (Magnetic resonance imaging)

LTS are widely used in medical application as Magnetic resonance imaging (MRI), Magneto cardiograph (MCG) [38]. They are 100 times superior to today's best conventional measurement technology [10]. A superconducting group has developed a technology using superconductor derived magnetic fields that is able to provide a better resolution image from inside a body without the need for strong magnetic fields [3]. MRI provides an enormous increase in diagnostic ability, clearly showing soft tissue features not visible using X-ray imaging or ultrasound. At the same time, MRI can often eliminate the need for harmful X-ray examinations. These advantages have greatly reduced the need for exploratory-Surgery, the length of hospital stays. MRI machines in hospitals across the globe have saved millions of lives, all in thanks to super conductivity [39].

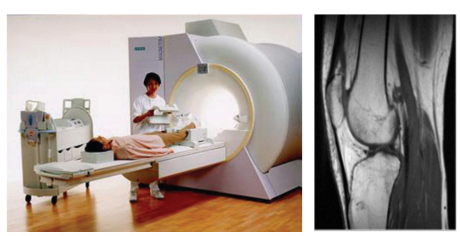


Figure 3.5: Superconducting MRI system in operation [34] and MRI Image of a human knee [17].

Chapter 4

Methodology

4.1 Introduction

If the interaction b/n electrons lead to a net attraction b/n two electrons close to the fermi surface , bound states can be formed located below the fermi surface. The density of these bound states depends on the total momentum of the pairs. Such net attractive interaction b/n electrons can occur when the attraction phonon interaction dominates the coulomb repulsion. The negatively charged electron slightly deforms the surrounding lattice by putting on the positively charged atoms. When the electrons moves around, it leaves a trail (wave) of such deformed atoms or (polarized). As it is new area with slightly increased positive charge, it attracts new electrons. By this mechanisms electrons attract each other mediated by phonons.

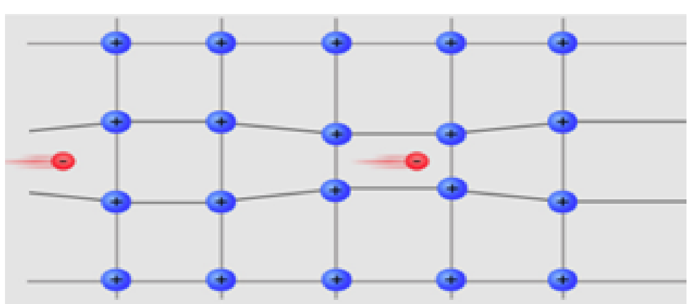


Figure 4.1: electron-phonon interaction

Fig.4.1: illustrates that, when the first electron deforms or polarize the lattice or emits

phonon, the second electron interacts with the lattice or absorbs the phonon and form cooper pairs [23].

The cooper pairs can be broken into exited states by temperature. In the works of Bardeen, cooper and Schrieffer (BCS) theory, 1957 and of Bogoliubov (1958) a systematic theory of super conductivity, has been erected on this principle of attraction. In order to find the physical properties of a system say Superconductivity, it is essential to know just the average behavior of one or two typical particles. And to probe the single particle properties of a many body system, one must have a way of measuring how the electrons propagate as a function of energy; that means taking out or inserting a particle with a definite energy [40]. The quantities that describe this average behaviors are known as the one particle and the two particle propagators. In quantum field theory, Green functions are known as propagators. Because, they give the amplitude of a particle inserted at one point at a time to propagate to at another point at a latter time. Green functions contains only part of the full information carried by the wave functions of the systems but they include the relevant information for a given problem [40]. From green functions, we obtain solutions to our problems. Thus, to obtain the expression for super conducting transition temperature T_C and order parameter $\Delta(0)$, the quantum field theory Green functions technique is used. They describe how particles propagate through the system influenced by all kind of interactions. Once the Green functions of a system are known, single particle properties of the system could be probed (for instance the density of states). In addition to the normal Green function which describes coherent motion of single particles, a second type of Green function is needed to deal with the cooper pairs, which occupy the paired states. This is the so called anomalous Green functions [40].

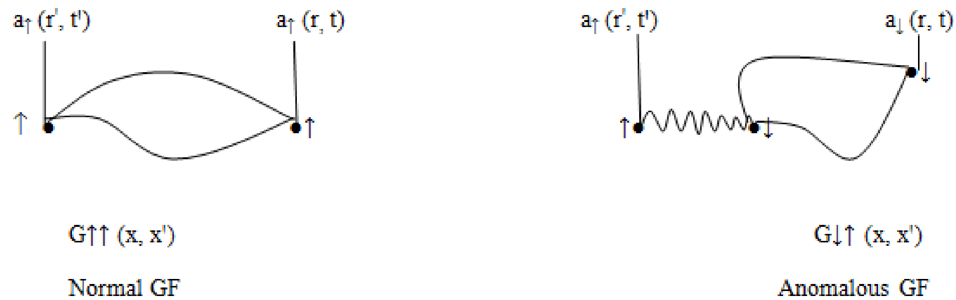


Figure 4.2: Normal and anomalous Green functions

Fig. 4.2: Illustraits: (Normal GF) a spin up electron is added to a system (superconductor) (location r' time t') and at time $t > t'$ the probability to find the electron at location r is probed (where the electron could have taken any possible path from r' to r). This represents the normal Green functions $G_{\uparrow\uparrow}$(x, x').

(Anomalous GF), the spin up electron (↑) of a cooper pair is taken out of the superconductor (location r' time t' and at time t>t', the probability to find the remaining electron of the cooper pair is probed. This represent the anomalous Green function $G_{\uparrow\downarrow}$ (x, x')[40]. The Green function is a statistical average over all possible paths from r' to r, denoted by $< ... >$. The anomalous Green functions describe the superconducting Green function correlation and are related to the order parameter Δ of the superconductor. This order parameter is a measure for the strength of the superconducting energy gap. Only the pairing of electrons with opposite spin direction, contribute to the super conducting order parameter which is a fundamental assumption with in BCS theory. The pairs with opposite spins act like bosons and leave an energy gap Δ between the BCS ground State and the 1^{st} excited state [25].

$\Delta = 2\hbar\omega_D e^{-1/VoN_{EF}}$.

where ω_D is Debye frequency, Vo is coupling potential and N_{EF} is density of states of electron at the Fermi energy. In its simplest form, BCS gives the superconducting transition temperature in terms of the Electron-phonon coupling potential and the density of states of electron at the Fermi energy [41].

$TcK_B = 1.14\hbar\omega_D e^{-1/VoN_{EF}}$.

An essential feature of the BCS theory is that all the cooper pairs must behave in exactly the same way, which makes that they can be described by a single wave function. BCS theory of superconductivity consists of two basic concepts: the attractive interaction b/n electrons due to phonon exchange and the pairing wave function that maximize the energy gain and leads to superconductivity.

4.2 Mathematical analysis

An electron interacts with the lattice emitting a phonon which is absorbed by the lattice. But another electron (this is the extra-condition) absorbs immediatly this phonon back leaving the lattice in its original state. This is how cooper pair is formed. V_q is the atractive potencial or phonon with wave vector q.

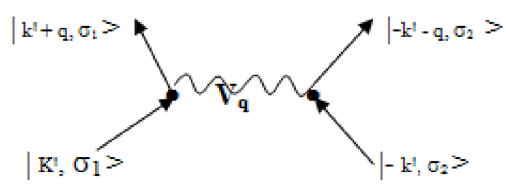


Figure 4.3: Graphical representation to the electron - phonon interaction and coulomb interaction of two particles.

Fig.4.3: Ilustraits that, the incoming states $|k', \sigma_1\rangle$ and $|-k', \sigma_2\rangle$ are with probability amplitude V_q scattered into outgoing states $|k' + q, \sigma_1\rangle$ and $|-k' - q, \sigma_2\rangle$ [42]. K is wave vector for electron-electron interaction and q is wave vector for phonon-electron interaction. The straight line denotes the electron and the wavy line denotes phonone. That is one electron emite a phonon (wavy line) which is then absorbed by the other electron.

Using the creation and annihilation operators ($a^+_{k\sigma}$ and $a_{k\sigma}$) for states specified by wave vector **k** and spin σ, the BSC Hamiltonian that represents superconductivity can be expressed as: [40, 43].

$$H_{BCS} = \sum_{k\sigma} \epsilon_k a^{+}_{k\sigma} a_{k\sigma} - V \sum_{kk'} a^{+}_{K\uparrow} a^{+}_{-K\downarrow} a_{k'\downarrow} a_{-k'\uparrow} \tag{4.2.1}$$

where$K = k' + q$ and $-K = -k' - q$; $a^{+}_{k\sigma} a_{k\sigma}$ is the single particle number operator. $\epsilon_k = \frac{\hbar^2 k^2}{2m}$ is the corresponding (kinetic) energy with m the electron mass and v is the attraction potential (interaction). The first term appearing in the Hamiltonian is the standard kinetic energy term. While the second term describes the scattering of a cooper pair from state $k' \uparrow$ and $-k' \downarrow$ to state $k \uparrow$ and $-k \downarrow$ under the attractive interaction.
H = H_0 + H', are free and interaction hamiltonians respectivily.
In the BCS model the attractive interaction is taken constant, $V_{kk'} = V$ Solving the Hamiltonian for a certain potential V gives the Eigen states and Eigen energies of the superconductor. The above equation, form the basis modern description of super conductivity, which is written in the language of green functions as follows [40, 42, 43].

$$G_{kk'}(t, t') = -i\theta(t, t')\langle [a_k(t), a^{+}_{k'}(t')] \rangle = \langle\langle a_k(t), a^{+}_{k'}(t') \rangle\rangle \tag{4.2.2}$$

To obtain the equation of motion of green functions, we differentiate the green function with respect to time:

$$\frac{d}{dt} G_{kk'}(t, t') = -i\delta(t, t')\langle [a_{k(t)}, a^{+}_{k'(t')}] \rangle - i\theta(t, t')\langle [\frac{d}{dt} a_{k(t)}, a^{+}_{k'(t')}] \rangle \tag{4.2.3}$$

Multiplying both sides by i yields [45].

$$i\frac{d}{dt} G_{kk'}(t, t') = \delta(t, t')\langle [a_k(t), a^{+}_{k'}(t')] \rangle - i\theta(t, t')\langle [i\frac{d}{dt} a_{k(t)}, a^{+}_{k'(t')}] \rangle \tag{4.2.4}$$

but

$$i\hbar \frac{d}{dt} a_k(t) = [a_k(t), H], for \hbar = 1 \tag{4.2.5}$$

[45]. So equation of motion becomes:

$$i\frac{d}{dt} G_{kk'}(t, t') = \delta(t, t')(\langle [a_k(t), a^{+}_{k'}(t')] \rangle + \langle\langle [a_k(t), H]; a^{+}_{k'}(t') \rangle\rangle) \tag{4.2.6}$$

To solve this equation it is convenient to work with Fourier transform of this equation. By Fourier transforming from the time domain to the frequency domain, we get information about the possible energies of the propagating electrons [40, 44]. Let G(ω) be the Fourier transform of $G(t,t')$ such that

$$G(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(t,t')e^{-i\omega(t-t')}d(t,t'), \tag{4.2.7}$$

$$G(t,t') = \int_{-\infty}^{\infty} G(\omega)e^{-i\omega(t-t')}d\omega, \tag{4.2.8}$$

δ function is defined as

$$\delta(t,t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega(t-t')}d\omega, \tag{4.2.9}$$

$$i\frac{d}{dt}G_{kk'}(t,t') = \omega G(\omega) = \omega\frac{1}{2\pi}\int_{-\infty}^{\infty} G(t,t')e^{-i\omega(t-t')}d(t,t') = \omega G(t,t')\delta(t,t'),$$

$$i\frac{d}{dt}G_{kk'}(t,t') = \omega G(t,t')\delta(t,t') = \delta(t,t')(\langle[a_{k(t)}, a^{+}_{k'(t')}]\rangle + \langle\langle[a_k, H]; a^{+}_{k'(t')}\rangle\rangle), \tag{4.2.10}$$

Then the Fourier transform of the above equation of motion (4.2.6) can be written as:

$$\omega G(t,t') = \omega\langle\langle a_{k(t)}, a^{+}_{k(t)}\rangle\rangle = \langle[a_{k(t)}, a^{+}_{k(t)}]\rangle + \langle\langle[a_{k(t)}, Ho + H']; a^{+}_{k'(t')}\rangle\rangle, \tag{4.2.11}$$

But

$$[a_{k(t)}, H_o] = [a_{k\uparrow}, \sum_p \epsilon_p a^{+}_{p\sigma}a_{p\sigma}], \tag{4.2.12}$$

,

$$[a_k(t), H_o] = \sum_p \epsilon_p[a_{k\uparrow}, a^{+}_{p\sigma}]a_{p\sigma} - a^{+}_{p\sigma}[a_{k\uparrow}, a_{p\sigma}],$$

for p=k and $\sigma =\uparrow$

$$[a_{k(t)}, H_o] = \sum_k \epsilon_k\delta_{kk}\delta_{\uparrow\uparrow}a_{k\uparrow}, \tag{4.2.13}$$

$$[a_{k\uparrow}, H_o] = \epsilon_k a_{k\uparrow}, \tag{4.2.14}$$

$$[a_{k(t)},H'] = [a_{k\uparrow}, -V\sum_{pp'} a^+_{p\uparrow}a^+_{-p\downarrow}a_{p'\downarrow}a_{-p'\uparrow}], \tag{4.2.15}$$

$$[a_k(t),H'] = -V\sum_{pp'}[a_{k\uparrow}, a^+_{p\uparrow}a^+_{-p\downarrow}]a_{p'\downarrow}a_{-p'\uparrow}) + (a^+_{p\uparrow}a^+_{-p\downarrow}[a_{k\uparrow}, a_{p'\downarrow}a_{-p'\uparrow}],$$

$$[a_k(t),H'] = -V\sum_{pp'}[a_{k\uparrow}, a^+_{p\uparrow}]a^+_{-p\downarrow}a_{p'\downarrow}a_{-p'\uparrow} - a^+_{p\uparrow}[a_{k\uparrow}, a^+_{-p\downarrow}]a_{p'\downarrow}a_{-p'\uparrow},$$

$$[a_k(t),H'] = -V\sum_{pp'}(\delta_{pk}\delta_{\uparrow\uparrow}a^+_{-p\downarrow}a_{p'\downarrow}a_{-p'\uparrow} - a^+_{p\uparrow}\delta_{-pk}\delta_{\uparrow\downarrow}a_{p'\downarrow}a_{-p'\uparrow}),$$

when k=p

$$[a_k(t),H'] = -V\sum_{kp'} a^+_{-k\downarrow}a_{p'\downarrow}a_{-p'\uparrow},$$

$$[a_k(t),H'] = -V\sum_{p'} a^+_{-k\downarrow}a_{p'\downarrow}a_{-p'\uparrow}, \tag{4.2.16}$$

so

$$\omega G(t,t') = \omega << a_{k\uparrow}, a^+_{k'\uparrow} >>= \delta_{kk'}+ << [a_{k\uparrow}, H], a^+k'\uparrow >>,$$

becomes

$$\omega G(t,t') = \delta_{kk'}+ < [\epsilon_k a_{k\uparrow} - V\sum_{p'} a^+_{-k\downarrow}a_{p'\downarrow}a_{-p'\uparrow}, a^+_{k'\uparrow}] >, \tag{4.2.17}$$

$$\omega G(t,t') = \delta_{kk'}+ < [\epsilon_k a_{k\uparrow}, a^+_{k'\uparrow}] > + < -V\sum_{p'}[a^+_{-k\downarrow}, a^+_{k'\uparrow}] >< a_{p'\downarrow}a_{-p'\uparrow} >,$$

when k=k' and decoupling

$$\omega G(t,t') = 1+ << \epsilon_k a_{k\uparrow}, a^+_{k\uparrow} >> -V\sum < a_{p'\downarrow}a_{-p'\uparrow} >< < a^+_{-k\downarrow}, a^+_{k\uparrow} >>, \tag{4.2.18}$$

Let$V\sum < a_{p'\downarrow}a_{-p'\uparrow} >= \Delta$ order parameter

$$\omega << a_{k\uparrow}, a^+_{k\uparrow} >>= 1+ << \epsilon_k a_{k\uparrow}, a^+_{k\uparrow} >> -\Delta << a^+_{-k\downarrow}, a^+_{k\uparrow} >>, \tag{4.2.19}$$

$$(\omega - \epsilon_k) << a_{k\uparrow}, a^+_{k\uparrow} >>= 1 - \Delta << a^+_{-k\downarrow}, a^+_{k\uparrow} >> . \tag{4.2.20}$$

$<< a^+_{-k\downarrow}, a^+_{k\uparrow} >>$ is similarly solved from its fourior transform equation.

$$\omega << a^+_{-k\downarrow}, a^+_{k\uparrow} >>=< [a^+_{-k\downarrow}, a^+_{k\uparrow}] > + << [a^+_{-k\downarrow}, H]; a^+_{k\uparrow}] >>$$

$$\omega << a^+_{-k\downarrow}, a^+_{k\uparrow} >>=< [a^+_{-k\downarrow}, a^+_{k\uparrow}] > + << [a^+_{-k\downarrow}, H_o + H']; a^+_{k\uparrow}] >>, \quad (4.2.21)$$

$$\omega << a^+_{-k\downarrow}, a^+_{k\uparrow} >>=<< [a^+_{-k\downarrow}, H_o + H']; a^+_{k\uparrow} >>, \quad (4.2.22)$$

$$\omega << a^+_{-k\downarrow}, a^+_{k\uparrow} >>=<< [a^+_{-k\downarrow}, H_o]; a^+_{k\uparrow}] >> + << [a^+_{-k\downarrow}, H']; a^+_{k\uparrow}] >>,$$

where

$$[a^+_{-k\downarrow}, H_o] = [a^+_{-k\downarrow}, \sum_p \epsilon_p a^+_{p\sigma} a_{p\sigma}], \quad (4.2.23)$$

$$[a^+_{-k\downarrow}, H_o] = \sum_p \epsilon_p [a^+_{-k\downarrow}, a^+_{p\sigma} a_{p\sigma}],$$

$$[a^+_{-k\downarrow}, H_o] = \sum_p \epsilon_p ([a^+_{p\sigma}, a^+_{-k\downarrow}] a_{p\sigma} - a^+_{p\sigma}[a^+_{-k\downarrow}, a_{p\sigma}]),$$

So

$$[a^+_{-k\downarrow}, H_o] = -\sum_p \epsilon_p a^+_{p\sigma}[a^+_{-k\downarrow}, a_{p\sigma}], \quad (4.2.24)$$

$$[a^+_{-k\downarrow}, H_o] = -\sum_p \epsilon_p a^+_{p\sigma} \delta_{-kp} \delta_{\sigma\downarrow}, \quad (4.2.25)$$

for$\sigma =\downarrow$, p=-k

$$[a^+_{-k\downarrow}, H_o] = -\epsilon_k a^+_{-k\downarrow}. \quad (4.2.26)$$

as $-\epsilon_{-k} = -\epsilon_k$

$$[a^+_{-k\downarrow}, H'] = [a^+_{-k\downarrow}, -V \sum_{pp'} a^+_{p\uparrow} a^+_{-p\downarrow} a_{p'\downarrow} a_{-p'\uparrow}], \quad (4.2.27)$$

$$[a^+_{-k\downarrow}, H'] = -V \sum_{pp'} [a^+_{-k\downarrow}, a^+_{p\downarrow} a^+_{-p\uparrow} a_{p'\downarrow} a_{-p'\uparrow}],$$

$$[a^+_{-k\downarrow}, H'] = -V \sum_{pp'} [a^+_{-k\downarrow}, a^+_{p\uparrow} a^+_{-p\downarrow}] a_{p'\downarrow} a_{-p'\uparrow} + a^+_{p\uparrow} a^+_{-p\downarrow} [a^+_{-k\downarrow}, a_{p'\downarrow} a_{-p'\uparrow}],$$

$$[a^+_{-k\downarrow}, H'] = -V \sum_{pp'} a^+_{p\uparrow} a^+_{-p\downarrow} [a^+_{-k\downarrow}, a_{p'\downarrow} a_{-p'\uparrow}], \quad (4.2.28)$$

Let $V\sum_p a^+_{p\uparrow}a^+_{-p\downarrow}$=$\Delta$ then

$$[a^+_{-k\downarrow},H'] = -\Delta[a^+_{-k\downarrow},a_{p'\downarrow}a_{-p'\uparrow}], \tag{4.2.29}$$

$$[a^+_{-k\downarrow},H'] = -\Delta([a^+_{-k\downarrow},a_{p'\downarrow}]a_{-p'\uparrow} - a_{p'\downarrow}[a^+_{-k\downarrow},a_{-p'\uparrow}]),$$

$$[a^+_{-k\downarrow},H'] = -\Delta - a_{p'\downarrow}[a^+_{-k\downarrow},a_{-p'\uparrow}],$$

$$[a^+_{-k\downarrow},H'] = -\Delta\delta_{-kp'}\delta_{\downarrow\downarrow}a_{-p'\uparrow},$$

for $p' = k$

$$[a^+_{-k\downarrow},H'] = -\Delta a_{k\uparrow}\delta_{-k-k}\delta_{\downarrow\downarrow},$$

$$[a^+_{-k\downarrow},H'] = -\Delta a_{k\uparrow}. \tag{4.2.30}$$

So equation (4.2.23)

$$\omega << a^+_{-k\downarrow},a^+_{k\uparrow} >>=<< [a^+_{-k_\downarrow},H_o{+}H'];a^+_{k_\uparrow}],>>=<< [a^+_{-k_\downarrow},H_o];a^+_{k_\uparrow}] >> + << [a^+_{-k_\downarrow},H'];a^+_{k_\uparrow}] >>$$

becomes

$$\omega << a^+_{-k\downarrow},a^+_{k\uparrow} >>= -\epsilon_k << a^+_{-k\downarrow},a^+_{k\uparrow} >> -\Delta << a_{k\uparrow},a^+_{k\uparrow} >>,$$

$$(\omega+\epsilon_k) << a^+_{-k\downarrow},a^+_{k\uparrow} >>= -\Delta << a_{k\uparrow},a^+_{k\uparrow} >>,$$

$$\frac{\omega+\epsilon_k}{-\Delta} << a^+_{-k\downarrow},a^+_{k\uparrow} >>=<< a_{k\uparrow},a^+_{k\uparrow} >>, \tag{4.2.31}$$

Substituting (4.2.31) in equation (4.2.20) for $<< a_{k\uparrow},a^+_{k\uparrow} >>$ yields

$$\frac{(\omega-\epsilon_k)(\omega+\epsilon_k)}{-\Delta} << a^+_{-k\downarrow},a^+_{k\uparrow} >>= 1 - \Delta << a^+_{-k\downarrow},a^+_{k\uparrow} >>, \tag{4.2.32}$$

$$[\frac{(\omega^2-\epsilon_k^2)}{-\Delta} + \Delta] << a^+_{-k\downarrow},a^+_{k\uparrow} >>= 1, \tag{4.2.33}$$

$$(\frac{\omega^2-\epsilon_k^2-\Delta^2}{-\Delta}) << a^+_{-k\downarrow},a^+_{k\uparrow} >>= 1,$$

$$<< a^+_{-k\downarrow},a^+_{k\uparrow} >>= -\frac{\Delta}{(\omega^2-\epsilon_k^2-\Delta^2)}, \tag{4.2.34}$$

where the superconducting gap parameter (Δ) can be defined as:-

$$\Delta = \frac{V}{\beta}\sum_{n,k} << a^{+}_{-k\downarrow}, a^{+}_{k\uparrow} >>, \tag{4.2.35}$$

whcih means

$$\Delta = -\frac{V}{\beta}\sum_{n,k} \frac{\Delta}{(\omega^2 - \epsilon_k^2 - \Delta^2)}.$$

Making use of Matsubara's frequency $\omega \rightarrow i\omega_n$ and $\omega_n = \frac{(2n+1)\pi}{\beta}$, where n extends over allowed pair states.

$$1 = \frac{V}{\beta}\sum_{n,k} \frac{1}{(((2n+1)\pi)/\beta)^2 + \epsilon_k^2 + \Delta^2},$$

$$1 = \frac{V}{\beta}\sum_{n,k} \frac{1}{(((2n+1)\pi)^2 + \beta^2\epsilon_k^2 + \beta^2\Delta^2)/\beta^2},$$

$$1 = \frac{V}{\beta}\sum_{n,k} \frac{\beta^2}{((2n+1)\pi)^2 + \beta^2\epsilon_k^2 + \beta^2\Delta^2},$$

$$1 = V\beta\sum_{n,k} \frac{1}{((2n+1)\pi)^2 + \beta^2\epsilon_k^2 + \beta^2\Delta^2},$$

$$1 = V\beta\sum_{n,k} \frac{1}{((2n+1)\pi)^2 + (\epsilon_k^2 + \Delta^2)\beta^2},$$

Let $(\epsilon_k^2 + \Delta^2) = E^2$ then

$$1 = V\beta\sum_{n,k} \frac{1}{((2n+1)\pi)^2 + E^2\beta^2}, \tag{4.2.36}$$

using the relation

$$\sum_{n}(\frac{1}{((2n+1)\pi)^2 + x^2}) = (\frac{1}{2x})\tanh(\frac{x}{2}) \tag{4.2.37}$$

where $x = E\beta$ then equation (4.2.36) becomes,

$$1 = V\beta\sum_{k}(\frac{1}{2x})\tanh(\frac{x}{2}),$$

$$\frac{1}{V} = \beta\sum_{k}(\frac{1}{2x})\tanh(\frac{x}{2}),$$

$$\frac{1}{V} = \beta\sum_{k}(\frac{1}{2E\beta})\tanh(\frac{E\beta}{2}),$$

$$\frac{1}{V} = \sum_k (\frac{1}{2E}) \tanh(\frac{E\beta}{2}). \tag{4.2.38}$$

Equation (4.2.38) is the generalized energy gap equation at finite temperature. In the bulk limit, the sum over k can be converted into energy integral:

$$1 = 2N(0)V\beta \int_0^{\frac{\beta\hbar\omega_{el}}{2}} (\frac{1}{2E\beta}) tanh(\frac{E\beta}{2}) dE \tag{4.2.39}$$

where $h\omega_{el}$ energy of exitation of electron, $N(0)$ is density of state at 0 K. and V the interaction strength.

In the limit $\Delta = 0$, the superconducting transition temperature (Tc) is derived as:

$$1 = N(0)V \int_0^{\frac{\beta\hbar\omega_{el}}{2}} \frac{\tanh(\frac{E\beta}{2})}{E} dE,$$

$\beta = \frac{1}{k_B Tc}$

$$1 = N(0)V \int_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} \frac{\tanh(\frac{E}{2k_B T})}{E} dE. \tag{4.2.40}$$

Let $x = \frac{E}{2k_B T} \Rightarrow E = (2k_B T)x$ and $dE = (2K_B T)dx$, hence equation (4.2.40) becomes

$$1 = N(0)V \int_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} \frac{tanhx}{x} dx, \tag{4.2.41}$$

let $u = \tanh x$, $dV = \frac{1}{x}dx$, $du = sech^2 x dx$, $V = lnx$ as $dV = \frac{1}{x}dx$

$$\int_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} \frac{tanhx}{x} dx = \ln x \tanh x \mid_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} - \int_0^{\frac{\hbar\omega_{el}}{2K_B Tc}} sech^2 x \ln x dx, \tag{4.2.42}$$

$$1 = N(0)V[\ln x \tanh x \mid_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} - \int_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} sech^2 x \ln x dx], \tag{4.2.43}$$

$$\frac{1}{N(0)V} = [\ln x \tanh x \mid_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} - \int_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} sech^2 x \ln x dx]. \tag{4.2.44}$$

Extending the upper limit of integration to infinity

$\int_0^\infty sech^2 x \ln x dx \approx \ln 0.44$ and at low temperature $\tanh x \approx 1$

$$\frac{1}{N(0)V} = \ln x \mid_0^{\frac{\hbar\omega_{el}}{2k_B Tc}} - \ln(0.44), \tag{4.2.45}$$

$$\frac{1}{N(0)V} = \ln(\frac{\hbar\omega_{el}}{2k_BT_c}) - \ln(0.44),$$

$$\frac{1}{N(0)V} = \ln(\frac{0.5\hbar\omega_{el}}{k_BT_c}) - \ln(0.44),$$

$$\frac{1}{N(0)V} = \ln(\frac{\frac{0.5\hbar\omega_{el}}{K_BT_c}}{0.44}),$$

$$\frac{1}{N(0)V} = \ln(1.14)(\frac{\hbar\omega_{el}}{k_BT_c}), \tag{4.2.46}$$

but $N(0)V_{el} = \lambda_{el}$ called electron coupling constant.

$$\frac{1}{\lambda_{el}} = \ln(1.14)(\frac{\hbar\omega_D}{K_BT_c}),$$

$$\exp(1/\lambda_{el}) = (1.14)(\frac{\hbar\omega_D}{K_BT_c}),$$

$$T_c = (1.14)\frac{\hbar\omega_D}{k_B\exp(\frac{1}{\lambda_{el}})}, \tag{4.2.47}$$

$$T_c = (1.14)\frac{\hbar\omega_D}{k_B}\exp(\frac{-1}{\lambda_{el}}), \tag{4.2.48}$$

$$T_c = (1.14)\Theta_D\exp(\frac{-1}{\lambda_{el}}). \tag{4.2.49}$$

where

$$\Theta_D = \frac{\hbar\omega_D}{k_B}$$

This equation (4.2.49) is well known BCS superconducting parameter.

Solving the generalized energy gap equation (4.2.38) shown below at the limit of T=0, yields the superconducting order parameter $\Delta(0)$.

$$\frac{1}{V} = \sum_k(\frac{1}{2E})\tanh(\frac{E\beta}{2})$$

But at low temperature $\tanh(\frac{E\beta}{2}) \approx 1.$ and as $E = \sqrt{\Delta^2 + \epsilon_k^2}$

$$\frac{1}{V} = \frac{1}{2}\sum_k\frac{1}{\sqrt{\Delta^2 + \epsilon_k^2}}$$

The sum over k can be converted into energy integral as

$$\frac{1}{V} = \frac{2N(0)}{2} \int_0^{\hbar\omega_D} \frac{d\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2(0)}} \tag{4.2.50}$$

which can be written as

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_D} \frac{d\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2(0)}}, \tag{4.2.51}$$

using the relation

$$\int \frac{dx}{\sqrt{x^2+1}} = \ln(x + \sqrt{x^2+1})$$

and the hyperbolic function

$$\ln(x + \sqrt{x^2+1}) = sinh^{-1}(x),$$

we will have

$$\int_0^{\hbar\omega_D} \frac{d\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2(0)}} = \ln(\frac{\hbar\omega_D}{\Delta(0)} + \frac{\sqrt{\hbar^2\omega_D^2 + \Delta^2(0)}}{\Delta(0)}), \tag{4.2.52}$$

where $x = \frac{\epsilon_k}{\Delta(0)} = \frac{\hbar\omega_D}{\Delta(0)}$ So

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_D} \frac{d\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2(0)}} = sinh^{-1}(\frac{\hbar\omega_D}{\Delta(0)}), \tag{4.2.53}$$

$$\frac{1}{N(0)V} = sinh^{-1}(\frac{\hbar\omega_D}{\Delta(0)}), \tag{4.2.54}$$

$$\Longrightarrow sinh(\frac{1}{N(0)V}) = \frac{\hbar\omega_D}{\Delta(0)}, \tag{4.2.55}$$

$$\Longrightarrow \Delta(0) = \frac{\hbar\omega_D}{sinh(\frac{1}{N(0)V})}, \tag{4.2.56}$$

$$\Delta(0) = \frac{\hbar\omega_D}{\frac{e^{\frac{1}{(N(0)V)}} - e^{-(\frac{1}{(N(0)V)})}}{2}}. \tag{4.2.57}$$

Since$(N(0)V) \ll 1$

$$\Delta(0) = \frac{\hbar\omega_D}{\frac{e^{(\frac{1}{N}(0)V)}}{2}}, \tag{4.2.58}$$

$$\Delta(0) = \frac{2\hbar\omega_D}{e^{(\frac{1}{N(0)V})}}, \tag{4.2.59}$$

$$\Delta(0) = 2\hbar\omega_D \ e^{-(\frac{1}{N(0)V})}. \tag{4.2.60}$$

Equation (4.2.60) is the order parameter of a superconductor at absolute zero tempetaure. From

$$T_c = (1.14)\frac{\hbar\omega_D}{k_B} \ e^{-(\frac{1}{N(0)V})}, \tag{4.2.61}$$

we will have

$$\frac{T_c k_B}{\hbar\omega_D} = (1.14) \ e^{-(\frac{1}{N(0)V})}, \tag{4.2.62}$$

taking $\frac{K_B}{\hbar}$ =1

$$T_c = (1.14)\omega_D \ e^{-(\frac{1}{N(0)V})}, \tag{4.2.63}$$

. this implies that

$$T_c \approx e^{-(\frac{1}{N(0)V})}, \tag{4.2.64}$$

That is

$$T_c \approx exp \ \lambda_{el}. \tag{4.2.65}$$

Again taking the ratio of equation (4.2.60) to equation (4.2.49), we will have

$$\frac{\Delta(0)}{T_c} = 1.76. \tag{4.2.66}$$

This implies that

$$\Delta(0) = 1.76T_c. \tag{4.2.67}$$

And from BCS theory [30, 26, 46], we have:

$$\frac{\Delta(T)}{Tc} = 3.06K_B(1 - T/Tc)^{1/2},$$

and $\frac{2\Delta(0)}{T_c K_B} = 4$ for strong electron-phonon interaction.
Combining these two equations, we will have:

$$\Delta(T) = 1.53\Delta(0)(1 - T/Tc)^{1/2}, \tag{4.2.68}$$

This implies that:

$$\Delta(T) \simeq \Delta(0)(1 - T/Tc)^{1/2}, \tag{4.2.69}$$

which is

$$\Delta(T) \simeq (1.76Tc)(1 - T/Tc)^{1/2}. \tag{4.2.70}$$

This implies that: at T = Tc; $\Delta(T) = 0$ and at T = 0, $\Delta(T) = \Delta(0) = 1.76Tc$

Chapter 5

Results and conclusion

5.1 Results

In the methodology part of this paper; the superconducting transition temperature (Tc) and the superconducting order parameter $\Delta(0)$ are mathematically determined using the green function method of tackling a problem and we got the following results.

$$T_c = (1.14)\frac{\hbar\omega_D}{k_B}\ e^{-(\frac{1}{N(0)V})},$$

$$\Delta(0) = 2\hbar\omega_D\ e^{-(\frac{1}{N(0)V})}.$$

These values are identical with their experimental or BCS theoretical values.
The following figure is plotted using equation. (4.2.65).

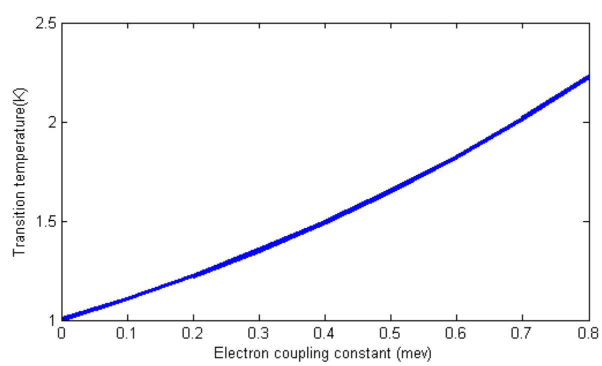


Figure 5.1: Superconducting transition temperature (T_c) Vs electron coupling constant (λ_{el}).

Fig. 5.1 shows that, as the elctron coupling constant (λ_{el}) becomes large (for instance by increasing the number of charge carriers or by doping, the density of states $N(0)$ fluctuates), the superconducting transition temperature (Tc) of the material gets large.
Using equation (4.2.70), the temperature dependence of the superconducting order parameter $\Delta(T)$ as a function of temperature (T) is plotted for mercury, lead, and Niobium of Tc = 4.2(K), 7.2(K) and 9.2(K) respectively as follows.

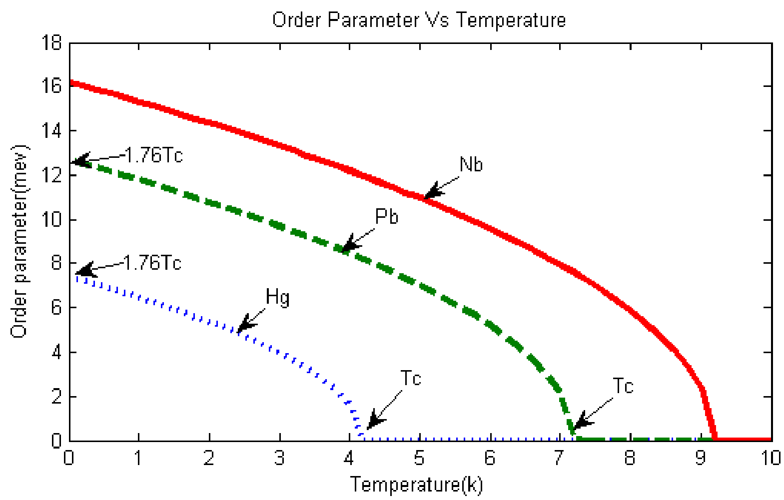


Figure 5.2: Superconducting order parameter $\Delta(T)$ as a function of temperature (T). Blue for Hg, Green for Pb, and Red for Nb.

Fig 5.2 shows that: The region under the curve of consideration is in the superconducting phase where as the remaining indicate the normal state of the system.
The superconducting order parameter $\Delta(T)$ of a given superconductor goes increasing, as the temperature of the superconductor goes decreasing from its critical value (Tc), and attains its maximum value $\Delta(0) = (1.76\text{Tc})$, when the temperature reaches zero.
The lower the Tc of a given superconductor, the lower will be its $\Delta(0)$; and the higher the Tc of a given superconductor, the higher will be its $\Delta(0)$.

5.2 Conclusion

In the first three parts of this project, we have described that:

The growing population and accelerated industrialization needs safe and sustainable energy. But how to generate the required energy and how to minimize the power lose due to resistance during transmission has been a big challenge facing the human race right now. Superconductors are materials with no resistance and conducts electricity with out dissipation. Even though superconductors have different properties and works at low temperatures, they are not only used as lose less power transmitters but also endless energy sources.

Superconductors are applicable in different fields such as in power generation and transmission, medical device; motors and they have high efficiency but less mass and size than conventional conductors.

In the methodology part of this paper we have determined the superconducting transition temperature (Tc) and the superconducting order parameter $\Delta(0)$ mathematically using the Green function method of tackling a problem and achieved the following values which are identical with the value experimentally determined or with the BCS theory.

$$T_c = (1.14)\frac{\hbar\omega_D}{k_B}\ e^{-(\frac{1}{N(0)V})}$$

$$\Delta(0) = 2\hbar\omega_D\ e^{-(\frac{1}{N(0)V})}$$

Lastly, we have revealed from the graphs of equations (4.2.65) and (4.2.70) that:

The superconducting transition temperature Tc of a matterial increases as the density of cooper pair states $N(0)$, and interaction strength $V(0)$, together called electron coupling constant (λ_{el}), gets large; and

the superconducting order parameter $\Delta(T)$(which is an opposition to the cooper pairs in the condensed state not to excite to the next higher energy state) goes increasing as the temperature of the superconductor goes decreasing from its critical value (Tc) and attains its maximum value $\Delta(0) = (1.76\text{Tc})$, when the temperature reaches zero respectively.

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DECLARATION

I hereby declare that this MSc project is my original work and has not been presented for a degree in any other universities, and that all sources of material used for the project have been duly acknowledged.

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This research project has been submitted to for examination with my approval as university advisor.

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