INVERSE COMPTON SCATTERING IN STRONG MAGNETIC FIELD: A POSSIBLE MECHANISM FOR HARD X-RAY PRODUCTION FROM ACCRETING NEUTRON STARS.

A thesis submitted to the School of Graduate Studies
Addis Ababa University

In partial Fulfilment of the Requirements for the
Degree of Master of Science in Physics

By

Getachew Mekonnen

Addis Ababa, Ethiopia
July 2007

ADDIS ABABA UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF PHYSICS

The undersigned hereby certify that they have read and recommended to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled "**INVERSE COMPTON SCATTERING IN STRONG MAGNETIC FIELD: A POSSIBLE MECHANISM FOR HARD X-RAY PRODUCTION FROM ACCRETING NEUTRON STARS.**" by in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

Name	Signature
Dr. Legesse Wetro, Advisor	_______________
Dr. S. Bhatnagar, Examiner	_______________
Prof. P. Singh, Examiner	_______________

THIS PAPER IS DEDICATED TO MY FATHER .

Acknowledgements

I would like to thank Dr. Legesse Wetro, my advisor, for his many suggestions and constant support during this thesis. I am also thankful to the Astrophysics group(Chemeda, Mekuaninit, Remudin)for their contribution in strengthening the group and the completion of this paper.

I would also thank my family especially MY MOM (ASRESIE),brothers and sisters Addisu, Abebe, Muluken (she is always in my side from the beginning to the end),Tefera (who sacrified his sleeping time during writing the thesis),Demeke,Mimi for their moral support and encouragement. DAD you are always in my heart(I missed you,You showed me the way to sucess),I Love you all.

Finally, For those of you who directly or indirectly contribute for the completion of this paper,I thank you all very much especially Ermi, Jeb, Sofi(My best friends starting from under graduate programme),Jhon (cloud 9)you are great all the time(Thank you).

Last but not least,I would like to thank my neighbors,Workineh (my guard),Askale (2nd mother), Leykun, Meskerem, Tsehaye, Firehiwot, Beriha, and others for your help and encouragement.

YETI,I really appreciate your patience, honesty, and every thing you sacrified for me.

Addis Ababa, Addis Ababa Univerisity Getachew Mekonnen July, 1999E.c.

Abstract

We began by considering Dirac equation to derive the inverse compton scattering cross section in strong magnetic field and this cross section has simple form for the scattering of a low frequency photon with relativistic electron.

We also derived the spectrum function from this cross section,which has a direct application on the hardening of thermal photons(soft x-rays)through inverse compton scattering by relativistic electron beams on the surface of strongly magnetized neutron stars to get the highest scattered photon energy of magnitude:

$$\omega_f = 4\gamma^2\omega_i$$

for the incident photon angle $\theta_i = \pi$ and scattered photon angle $\theta_f = 0$ which can give hard x-ray spectrum in the energy range of 12KeV-50MeV by considering soft X-ray as incident photon and to get this spectrum the electron energyγ for pulsars(rotating Neutron stars)is considered as well. And we showed the application of Inverse compton scattering in X-ray Astronomy during Roche Lobe overflow and formation of Accretion disc in the compact object in our case(Strongly magnetized Neutron stars).

Table of Contents

List of Figures

Introduction

An X-ray is a quantum of electromagnetic radiation with energy three orders of magnitude greater than that of the optical photons.X-rays are divided in to soft and hard X-rays based on their energy. The soft X-ray band covers the energy range between 0.5KeV and 12 keV. The hard X-ray extends to 50 MeV (Manojendu, 2005). High-energy astronomy pertains to the observation of the sky in this regime of the electromagnetic spectrum. The X-ray luminosity is generated by the release of the gravitational potential energy of the accreting matter, which is in turn supplied by a close binary stellar companion. The X-ray beam pattern is determined by the external magnetic field of the neutron star as it is clearly shown in the following figure 1, the resulting funneling of the accretion flow, and the complex radiative transfer processes of X-rays propagating from the vicinity of the neutron star surface through regions of accreting magnetized plasma (Paul C.Joss et al.1984).

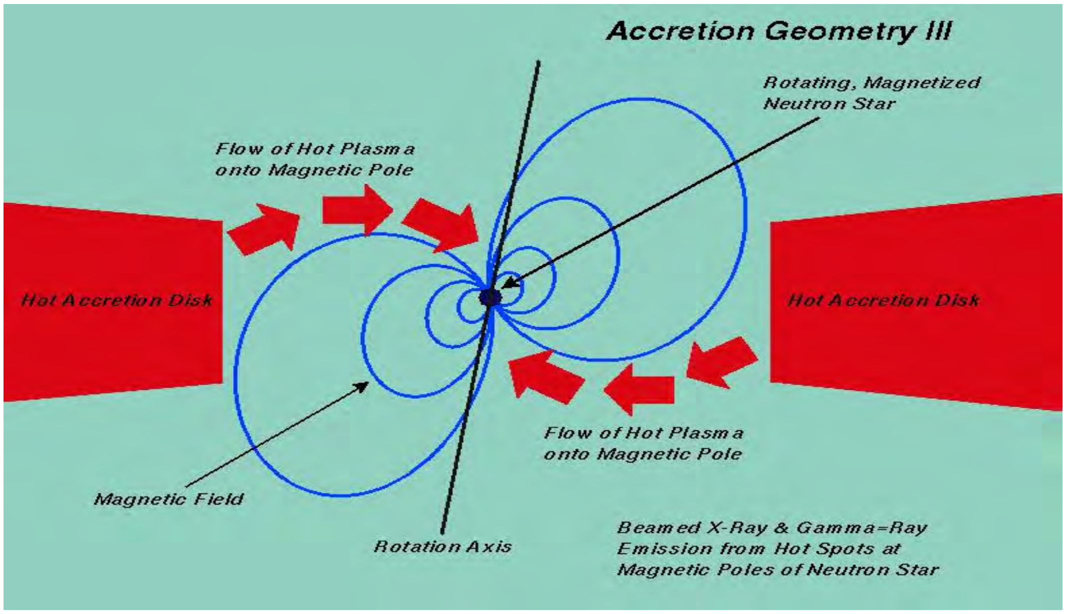

Figure 1: Beamed X-ray Emission From Hot Spots at Magnetic Poles of Neutron Star

We can possibly categorize X-ray binaries in to Low-Mass X-ray binaries in which the companion star is light, $M < M_{\odot}$, where $M_{\odot}$ is the solar mass and in high-mass systems the companion has $M > M_{\odot}$. In addition to the two stars, most LMXB contain an accretion disk through which mass flows on to the compact object. As Juho Schultz (2004) cited in his paper LMXB dominate the luminosity at different wavelengths. The inner accretion disk around the neutron star dominate in the soft X-ray below 12 keV. In some special states the accretion disk corona dominates in the hard X-ray above 12 keV. Rudiger Staubert,2003 indicate that accreting X-ray pulsars are believed to be highly magnetized neutron stars in binary system. These objects are powered by gravitational energy which is released when material accreted from the companion falls down along the magnetic field lines on to the surface of the neutron stars, in a small area close to the magnetic pole.

As it is clearly stated above X-ray pulsars are rotating and strongly magnetized $B = 10^{14}$G neutron stars that accrete matter from a stellar companion (Lars Bildsten et al. 1997). As the accreting material approaches the neutron star, the plasma is channeled to the magnetic polar caps, where it releases its gravitational energy as X-ray and γ-ray.

In this work, we restrict our attention to X-ray sources that are widely believed to be neutron stars undergoing accretion in binary systems in which matter over flows from the secondary (companion) to primary (compact) star. In active galactic nuclei the primary X-ray emission is due to Inverse Compton Scattering by electrons in a hot Corona of the UV/soft X-ray disk photons. It is likely to be significantly polarized (Haardt and Matt 1993, Poutanen and Vilhu 1993) because the system is unlikely to have a spherical symmetry. Part of the primary emission illuminates the disk and is reflected and polarized via Compton Scattering and this polarization determines the direction of magnetic field.

Hard X-ray emission generally arise in inverse comptonisation of soft photons by energetic electrons(Fender 2001.) This shows that inverse Compton Scattering is very important in astrophysics, mainly in X-ray astronomy in which this work mainly focuses on. In X-ray astronomy, the accretion disk surrounding a neutron star is believed to produce a thermal spectrum. The lower energy photons produced from this spectrum are scattered to higher energies by relativistic electrons in the surrounding corona.

The main purpose of this work is to show the production of hard X-ray from accreting binary neutron stars using Inverse Compton Scattering. As it is stated above Inverse Compton Scattering is applicable in X-ray astronomy. So, here we try to develop Inverse Compton Scattering cross section in strong magnetic field and show how the incident low photon energy will be changed in to high energy photon when matter falls from the normal (Secondary)star to the primary star during accretion process after filling its Roche Lobe and passing through its lagrangian point.

Out Line Of The Thesis

This thesis mainly focuses on the production of Hard X-rays from Accreting Neutron Stars and has the following out line.

In chapter One, we will give an over view of Neutron stars and their properties and binary systems as well. Chapter two extends the interaction of binary systems,through accretion, and discusses the mechanism of feeding the accretion, the formation of accretion disks and their role in X-ray production.

In Chapter Three we will try to elaborate accretion as a source of energy and the spectral mode. Chapters Four and Five will focus on the derivation of the necessary formulas for the production of hard X-rays from accreting Neutron stars using Inverse Compton Scattering in Strong Magnetic field.

Chapter 1

Over View Of X-ray Astronomy

When Giacconi tried to observe X-rays from the moon during a short rocket flight in June 18,1962 using a simple X-ray detector, he opened up a completely new field of astronomy.Instead of X-rays from the moon, he discovered X-rays from somewhere in the vicinity of the Scorpius constellation(Giacconi et al.1962)and there fore called this new X-ray source Scorpius X-1(Sco X-1). Since the X-ray detectors available in the 1960s did not have a good spatial resolution, optical follow up observations were required to determine the exact location of the source. It took several years until Sandage et al.(1966)could finally determine the position of Sco X-1. The nature of the process capable of producing X-rays in the observed quantity were also a mystery for quite some time after the discovery of Sco X-1. Morton(1964),as cited in Ingo Kreykenbohm 2004, was the first to suggest neutron stars as a possible X-ray source, which had been a purely theoretical construct until then.The same year, Zel'dovich(1964)and Salpter(1964) were the first to suggest that accretion of material on to a massive object(not necessarily a compact object) could be an important astrophysical energy source. In the following years, the picture of X-rays being produced by accretion of material on to a compact object became generally accepted.

As the Earth's atmosphere is opaque for hard UV and X-rays, X-ray sources can not be observed from the ground. In the beginning, rockets were used to carry detectors to high altitudes such that X-rays could be observed.The major draw back is that these rocket flights only lasted for a few minutes such that detailed studies of X-ray sources were not possible. Only a few years after the discovery of Sco X-1, the first X-ray satellite(UHURU, Swahili for freedom) was launched in 1970. This satellite discovered and observed over 400 X-ray sources.

The next major step was the Einstein observatory which provided X-ray imaging capabilities for the first time. This allowed to study extended sources like supernova remnants and also led among others to the discovery of the "double pulsar"(Lamb et al.1980).

About ten years later in 1990, ROSAT was launched. ROSAT provided not only excellent imaging qualities, but also allowed to create high quality spectra over the energy range$0.1-2$KeV. Apart from pointed observation, ROSAT also performed an all sky survey resulting in the discovery of$\sim 120,000$ previously unknown sources.(Ingo Kreykenbohm,2004)

In the mid 1990s, the Rossi X-ray Timing Explorer(RXTE)and BeppoSAX (Boella et al.1997), were launched for the purpose of offering unknown broad band spectral coverage from 1KeVup to over100 KeV.

The end of the millennium was characterized by two new major instruments: NASAs Chandra (formerly AXAF)and ESA's XMM-Newton. While both have imaging capabilities,the spatial resolution of Chandra is unsurpassed in the X-rays. XMM-Newton,however,covers the impressive energy range from 200eV up to$\sim$ 12 KeV with excellent energy resolution and a large collecting area.

In October 2002, ESA launched the most recent major X-rays/γ-ray observatory: INTEGRAL(Ingo Kreykenbohm,2004). INTEGRAL offers unprecedented broad band coverage from$\sim 2KeV$up to $100MeV$ with supreme energy resolution.

1.1 Neutron Stars

Following an early speculation by Badde and Zwicky(1934) soon after the discovery of the Neutron in 1932, Openheimer and volkoff(1934) presented a classic analysis of the theoretical viability of neutron stars, based on the General Theory of Relativity and nuclear physics that was then known. The term neutron star as generally used today refers to stars with a mass "M" on the order of 1.5 solar masses($M_{\bigodot}$), a radius R of $10 - 12$ km. and a central density as high as 5 to 10 times the nuclear equilibrium density$\approx 0.16/fm^3 \simeq 10^{14}\frac{gm}{cm^3}$ of neutrons and protons found in laboratory nuclei(J.M. L attimer et al. 2004). A neutron star is thus one of the densest forms of matter in the observable universe. Although neutrons dominate the nucleonic component of neutron stars, some protons (and enough electrons and muons to neutralize the matter) exist.

1.1.1 Neutron stars Magnetic Field

Neutron stars are found to possess magnetic fields ranging from 10^8G to 10^{15}G, much larger than achievable in terrestrial laboratories (Dong Lai, 2006). As Legesse (2002) stated plasma density gradient, which is inherent to degenerate neutron star matter, is shown to lead to large scale plasma diffusions and subsequent charge separation. The surface (internal fields) generated by the spinning separated charges are found to be dipolar with intensities of 10^{14}G (for the surface fields) very early in the life time of typical neutron star.

Consider a magnetic dipole, of dipole moment P_m, which radiates electromagnetic radiation at the rate of

$$\frac{-dE}{dt} = \frac{\mu_0|\ddot{P_m}|^2}{6\pi C^3}$$

This equation is obtained by replacing the electrostatic constants $\frac{|\ddot{P}|^2}{4\pi\epsilon_0}$ by magnetostatic constants $\frac{\mu_0|\ddot{P_m}|^2}{4\pi}$ where P_m is the magnetic dipole moment of the neutron star.In the case of rotating magnetic dipole. We can write

$$P_m = P_{m0}\sin\Omega t$$

where Ω is the angular velocity of the neutron star$P_{m0}$is the component of magnetic dipole perpendicular to the rotation axis.

$$\dot{P_m} = \Omega P_{m0}\cos\Omega t$$

$$\ddot{P_m} = -\Omega^2 P_{m0}\sin\Omega t$$

$$\frac{-dE}{dt} = \frac{\mu_0|-\Omega^2 P_{m0}\sin^2\Omega t|^2}{6\pi C^3} = \frac{\mu_0\Omega^4 P_{m0}^2}{6\pi C^3}$$

Now, the magnetic dipole radiation extracts rotational energy from the neutron star and if "I" is the moment of inertia of the neutron star.

$$\frac{-d(\frac{1}{2}I\Omega^2)}{dt} = -I\Omega\frac{d\Omega}{dt} = \frac{\mu_0\Omega^4 P_{m0}^2}{6\pi C^3}$$

This will give us the relation

$$\frac{d\Omega}{dt} \propto -\Omega^3$$

If the magnetic braking mechanism is responsible for the slow down of the neutron star estimates can be made of the magnetic field strengths at the surface of the neutron star. To estimate the magnetic field of the neutron star,we can use the classical result

$$B = \frac{\mu_0 P_{m0}}{4\pi r^3}[2\cos\theta_{ir} + \sin\theta_{i\theta}]$$

at $r = R, B_s \approx \mu_0 P_{m0} 4\pi R^3$substituting this back in to the previous equation, we can find:

$$\frac{-d\Omega}{dt} = \frac{\mu_0 \Omega^3 P_{m0}^2}{6\pi C^3 I} = \frac{\mu_0 \Omega^3}{6\pi C^3 I}(\frac{4\pi R^3 B_s}{\mu_0})^2$$
$$\frac{8\pi \Omega^3 R^6 B_s^2}{3\mu_0 C^3 I}$$

Here we can use $I = \frac{2}{5}MR^2$for a uniform sphere rotating about its axis.

1.1.2 Neutron Stars in Binary System

Neutron stars are observed in two families:

1.Isolated, mainly from their non-terminal emission as radio pulsars. These are relatively young objects ($10^7 yrs$).

2.Many of the stars in our galaxy are double systems. X-ray binaries are stellar binary systems consisting of a"normal",example,main sequence star, and a compact object that is either a white Dwarf, a Neutron star, or Black Hole.During some phases of the evolution of the binary system under certain circumstances, material can be transferred from the normal star(ordinary companion star or secondary,or donor) to the compact object(primary,or accretor) as shown in the following figure.The kinetic energy of the material is released as X-rays.

Binary systems have periods ranging from a few hours in the case of close binary system to thousands of years and this can strongly influence their evolution,if they are members of close binary systems (Griffin 1985). According to recent statics, more than half the stars in the galaxy may be members of binary systems mainly as X-ray

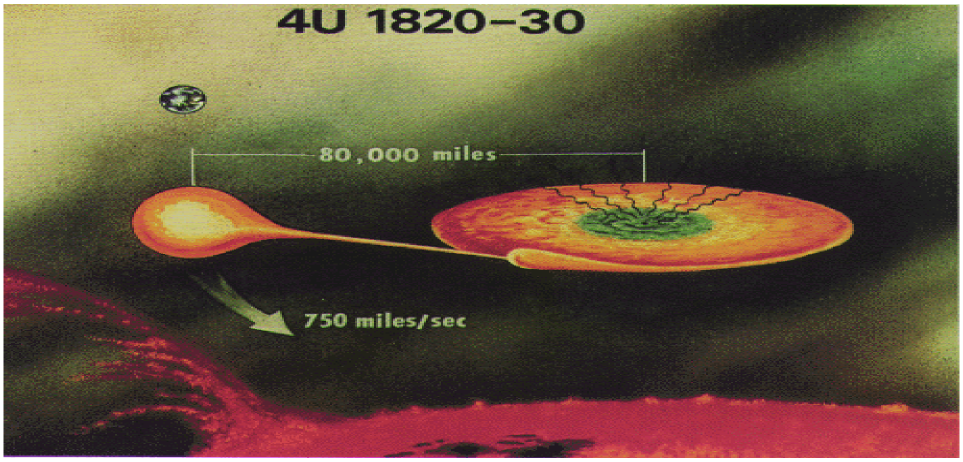

Figure 1.1: System of Binary stars

sources. These are usually, but not always, old stars of age($\geq 10^7$ yrs). They are observed in thermal X-rays associated with accretion from their companions, or from near surface nuclear explosions of accreted matter. (M. Ruderman, 1985).

The first of the binary X-ray pulsars were discovered in 1970 with the UHURU satellite (Giacconi et al. 1971, Schrier et al. 1972 a, Tananbauma et al. 1972) and with a balloon born instrument (Lewin et al. 1971). It was almost immediately recognized that the source of energy for the pulsed X-ray radiation was accretion of matter on to the magnetic polar caps of the rotating neutron stars. The accreted matter is transferred to the neutron star from the binary companion star.

Chapter 2

Accretion

In this chapter we are going to give the detail explanation of accretion and topics related to it.

Accretion is the accumulation of diffuse gas or matter on to some object under the influence of gravity (Malcolm S.longair, 1997). Little attention was given for the process of accretion, until the discovery of X-ray binary systems in early 1970. The discovery of intense X-ray sources in close binary systems which contained neutron stars, initiated a new era in high energy astrophysics, when it was realized just how efficient the process of accretion on to compact objects could be.

Data from X-ray observatories have enabled the discovery of about a thousand X-ray sources in the Milk Way and nearby galaxies (Cherepashchuk, 1996; 2003). Most are close binary systems in which an optical component supplies mass to a neutron star. Accretion on to the surface of a neutron star at sub relativistic velocities results in an enormous release of X-ray energy and higher luminosities (A.I Bogomazoov et al. 2005).

2.1 Mass Accretion By Neutron Stars

A considerable fraction of observed neutron stars have increased their masses in the course of their evolution, or are still increasing their masses (e.g. in X-ray sources). It is clear that the only origin of a mass increase is accretion. It is evident that the over all change in the mass of a neutron star is determined not only by the accretion rate, but also by duration of the accretion stage.

$$\Delta M = \int_0^{t_a} \dot{M} dt = \dot{M} t_a$$

where $\dot{M}$ is the mean accretion rate and t_a is the life time of the accretion stage. Here we emphasize that, in the case under consideration, the accretion rate is the amount of matter falling on to the surface of the neutron star per unit time.

2.2 Feeding The Accretion

When we study X-ray binaries, or in general binary systems, we will take into account the importance of accretion as a source of energy. Binary systems reveal more about themselves, their masses and dimensions, than other astronomical objects do.

The importance of accretion is manifested by the realization that probably a majority of all stars are members of binary systems, which at some stage of their evolution, under go mass transfer.

The detailed study of interacting binaries shows that angular momentum of accretion is very important in the formation of accretion disk. Because, during mass transfer from the Secondary star, the transferred material cannot land on the accreting star until it has ride itself of most of its angular momentum. This is the mechanism in which accretion disks are formed, which can convert (change) gravitational potential energy into radiation (J. Frank et al. 1985). There are two main ways in which binaries transfer matter at some stage of their evolutionary life times; Roche Lobe overflow and Stellar wind accretion.

2.2.1 Roche Lobe Over Flow

A binary system has a much more complicated gravitational potential than an isolated star:an idealized version of such a system is taken as three-body-problem considering the stars to be point masses.

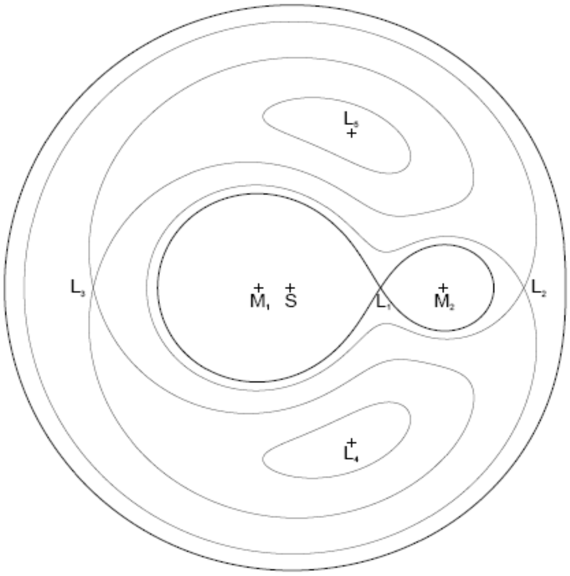

Figure 2.1: Equipotential surfaces of the Roche potential with a mass ratio $M_1 : M_2 = 5$; M_1, M_2 are the masses of the two stars. $L_1 to L_5$ are the Lagrange points and S is the center of mass of the system (from Kretschmar 1996).

This potential has two gravitational wells(one for each of the two stars)and five points of equilibrium as shown in the figure. This potential was first studied by Eduard Albert Roche in 1849 in Paris.In a co rotating frame of reference, this Roche potential is given by (J.Frank et al., 1985)

$$\Phi(r) = \frac{-GM_1}{\vec{r} - \vec{r_1}} - \frac{GM_2}{\vec{r} - \vec{r_2}} - \frac{1}{2}(\vec{\omega} \times \vec{r})^2$$

In a close binary system the equipotential surface are destroyed in the rotating frame of reference frames when the stars fill a substantial fraction of their Roche Lobes. If the Secondary star fills its Roche Lobe in the course of evolution, mass is transferred from the secondary to the primary through the inner Lagrangian point(L_1) as the over flowing matter seeks a lower gravitational potential as it is shown in the above figure 2.1. Roche lobe of the binary system is a critical equipotential surface which encompasses both stars.The equipotential surfaces within the Roche lobe shows that the shape of the stars are significantly distorted from spheres if they fill a significant fraction of their Roche lobes.(S.Longair 1997)

For non-contact close binaries,the stars don't fill their Roche lobes and the stars evolve more or less as normal stars. Interesting phenomena occur as the stars evolve off the main sequence. the more massive of the pair evolves off the main sequence at an earlier time than the less massive star, and, as it becomes a red giant, it expands to fill its Roche lobe. Matter always seeks the lowest gravitational potential,and this is achieved if matter passes through the lagrangian point L on to the secondary companion. In this way, the mass of what was initially the less massive star increases whilst the mass of the secondary decreases.

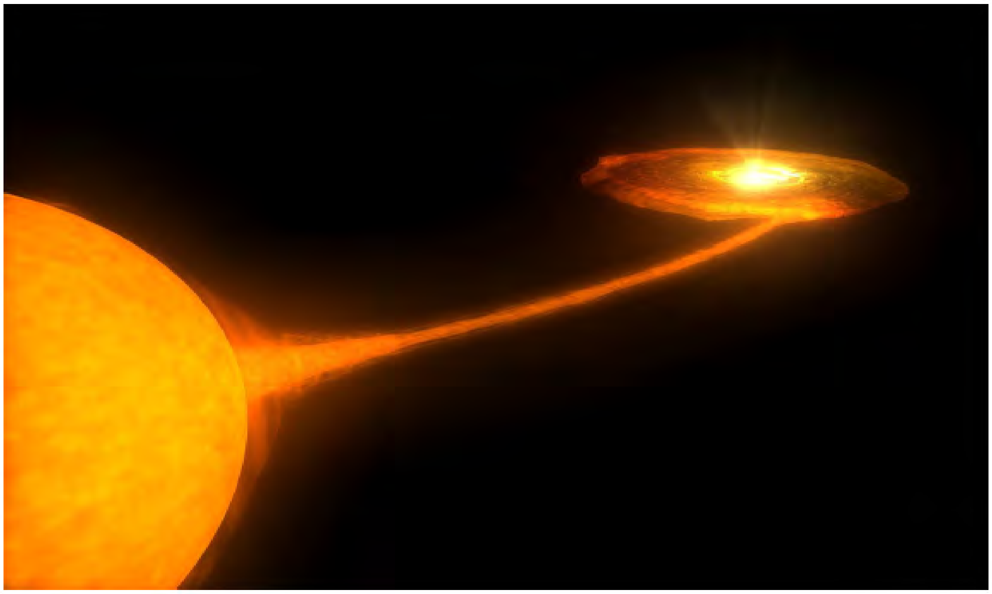

Figure 2.2: A simulation of a low mass X-ray binary system. The low mass optical companion has reached its Roche Volume and is loosing material. The material forms an accretion stream and flows from the star to the accretion disk around the compact object. The material moves inward and is finally accreted onto the compact object. Image from UCSD computing center. Rcirc = J2

Wind Fed Accretion

Since all normal stars have weak stellar wind this will play great role in accretion. In this type of accretion one of the stars, may at some evolutionary phase, eject much of its mass in the form of stellar wind and some of this material will be captured gravitationally by its companion.

2.3 Accretion Discs and their Role in X-ray Production

As already discussed to some extent in previous section, at certain phases of the binary evolution, mass transfer between the two stars is possible:When the optical companion evolves,it will also expand.If the star exceeds its Roche volume(see fig.2.2),the material outside the Roche volume is no longer gravitationally bound to the star.This material can now be captured by the compact object.The material captured by the compact object can not ,however, be accreted directly. Due to the rotation of the donator star, the material has too much angular momentum. As a result, an accretion disk forms, where the material is stored.In this disk the material looses slowly its angular momentum such that it can move further in wards. However, since the angular momentum must be conserved, some material will also move out wards and can finally escape from the disk. Now let us incorporate strong magnetic field $\sim 10^{12}$G and as already derived earlier in section 1.1.1, the strength of the magnetic field B at radius $'r'$ from the neutron star can be written as

$$B \sim \frac{\mu_0}{r3}$$

where μ_0 is magnetic moment.Though the magnetic field decreases with increasing $'r'$,the influence of $'r'$ on the in falling material in the vicinity of the neutron star is very strong. According to J.Frank et al.1985 the magnetic pressure is given by $P_{mag} = \frac{\mu_0^2}{8\pi r^6}$. This means the magnetic pressure on the in-falling material is increasing strongly the closer the material gets to the surface of the neutron star. This increasing magnetic pressure will at a certain radius,the Alfven radius,be equal to the ram pressure of the in falling gas

$$\frac{\mu_0}{8\pi r_M^6} = \frac{(2GM)^{\frac{1}{2}}\dot{M}}{4\pi r_M^{\frac{5}{2}}}$$

At the Alfven radius (also called the magneto spheric radius), the magnetic field disrupts the incoming gas stream and forces the material to follow the magnetic field lines.This transition region from, a stable disk to the material following the field lines and where the disk ceases to exist,is also known as the boundary layer. The following picture shows accretion from an accretion disk on to a strongly magnetized neutron star.

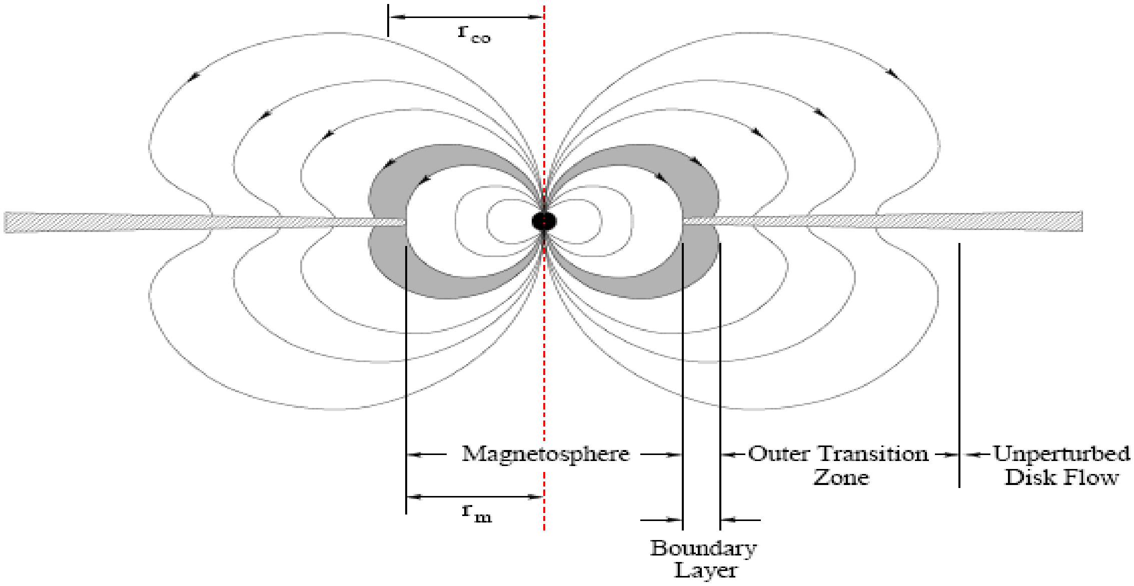

Figure 2.3: Schematic of the accretion from an accretion disk onto a strongly magnetized neutron star (Ghosh and Lamb 1978). At the transition layer the magnetic field disrupts the disk and the material follows the magnetic field lines on to the neutron star figure from Kuster 2003

In another words, the boundary layer is the region where the rapidly spinning disk material reaches the more slowly spinning accreting star as it is shown in the figure above, is crucial element of an accretion disk (Robert Poham et al.2000).In a thin accretion disk,the gas rotates at approximately the Keplerian velocity,and so by the time it approaches the surface of the accreting star, half of the gravitational potential energy released in the accretion process has been converted in to rotational energy of the gas (J.Frank et al.1985). Unless the star is rotating rapidly most of this energy will be released in the boundary layer.Further more, since this energy comes from a small region close to the star,the boundary layer should be hotter than the disk and should be produce harder radiation. Thus, the boundary layer is expected to be the dominant source of high energy emission from accretion disks, which for accretion on to neutron stars takes the form of X-rays.

2.4 The Eddington Limit

For a spherically symmetric accretion, let us assume the accreting material to be mainly hydrogen and to be fully ionized. Under these conditions, the radiation exerts a force mainly on the free electrons through Thomson scattering, since the scattering cross section for proton is a factor $[\frac{m_e}{m_p}]^2$ smaller, where $m_e/m_p \simeq 5 \times 10^{-4}$is the ratio of the electron to the proton mass. (J. Frank et al. 1985). If S is the radiant energy flux $\frac{erg}{scm^2}$ and $\sigma_T = 6.7 \times 10^{-25} cm^2$ is the Thomson cross section, then the out ward radial force on each electron equals the rate at which it absorbs momentum, $\sigma_T \frac{S}{C}$ where $\sigma_T = \frac{8\pi}{3}(\frac{e^2}{m_e C^2})^2$

If there is a substantial element other than hydrogen which has retained some bound electrons, the effective cross section, resulting from the absorption of photons in spectral lines, can exceed σ_T considerably.

The attractive Coulomb force resulting between the electrons and the protons means that as the electrons move out they drug the protons with them. In effect, the radiation pushes out electron-proton pairs against the total gravitational force.

$$\frac{GM(m_p + m_e)}{r^2} \simeq \frac{GMm_p}{r^2}$$

acting on each pair at a radial distance r from the center. If the luminosity of the accreting source is $L(erg/s)$, we have

$S = \frac{L}{4\pi r^2}$ using spherical symmetry.The net inward force on an electron-proton pair is

$$(GMm_p - \frac{L\sigma_T}{4\pi C})\frac{1}{r^2}$$

The limit at which the above expression vanishes is called the Eddington limit.

$$(GMm_p - \frac{L\sigma_T}{4\pi C})\frac{1}{r^2} = 0$$
$$L = \frac{4\pi CGMm_p}{\sigma_T}$$
$$L_{Edd} = \frac{4\pi CGMm_p}{\sigma_T} \simeq 1.3 \times 10^{38}(\frac{M}{M_{\odot}})\frac{erg}{s}$$

At greater luminosities the out ward pressure of radiation would exceed the inward gravitational attraction and accretion would be halted. If all the luminosity of the source were derived from accretion this would switch off the source.

For the derivation of the Eddington limit we assumed spherically symmetric accretion. To make further extension of this limit, if the accretion occurs only over a fraction "g" of the surface of a star and dependent on the radial distance, r, then the corresponding accretion luminosity is gL_{Edd}.

For accretion powered objects the Eddington limit implies a limit on the steady accretion rate, $\dot{M}(\frac{g}{s})$. If all the kinetic energy of in falling matter is given up to radiation at the stellar surface, R_* then the accretion luminosity is given by:

$$L_{acc} = \frac{GM_n\dot{M}}{R_*}$$

From the above value of Edddington limit, It is important to note that once the Eddington limit is reached, the accretion rate can not grow any more. A slight increase increase of the luminosity and the material will be blown away by the increased radiation pressure, switching the source off.

Chapter 3

Accretion as a Source of Energy

3.1 Spectral Modes of X-Ray Binaries

Physists of the 19^{th} century, believed that gravity was the only source of energy in celestial bodies. However, in late 20^{th}century, it is to gravity that we look to power the most luminous object in universe, for which the nuclear sources of the stars are wholly inadequate. Nowadays it is possible to extract gravitational potential energy from materials which accretes onto the gravitating body and this is now known to be the principal source of power in several types of close binary systems, and is widely believed to provide the power supply in active galactic nuclei and quasars (J.Frank et al.1985). Thus, the new role for gravity arises because accretion onto compact objects is a natural and a powerful mechanism for producing high-energy radiation. Therefore the increasing recognition of the importance of accretion has accompanied the expansion of observational techniques in Astronomy in particular the exploitation of the full range of the electromagnetic spectrum from the radio to X-rays and γ rays.

As J.Frank et al.(1995) stated, for a body of mass "M" and radius R_* the gravitational potential energy released by the accretion of a mass "m" on to its surface is

$$\Delta E_{acc} = \frac{GMm}{R_*}$$

where G is the universal gravitational constant.

From this calculation we would expect the energy to be released mainly in the form of Electromagnetic radiation and from the above form of equation the efficiency of accretion as an energy release mechanism is strongly dependent on the compactness of the accreting object. The larger the ratio $\frac{M}{R_*}$, the greater the efficiency.

For a fixed value of the compactness, $\frac{M}{R_*}$, the luminosity of an accreting system depends on the rate $\dot{M}$ at which matter is accreted. At high luminosities,the accretion rate may it self be controlled by the out ward momentum transferred from the radiation to the accreting material by scattering and absorption.

Studies of the accretion flow on to neutron stars have largely focused on the case where the neutron star has a very strong magnetic field(Popham et al.,2000).According to Pringle and Rees,(1972),Basko and Sunyaev (1976) and Gosh,Lamb and Petick,(1977) the magnetic field is believed to truncate the accretion disk at some inner radius and channel the accretion on to magnetic field lines, so that ultimately it falls on to polar caps corresponding to the poles of the magnetic field.

Pringle and Rees(1972) and Lamb,Pethick,and Pines(1973) indicate the change in period over sufficiently long time is due to the accretion of external angular momentum, while period fluctuations on shorter time scales are caused either by fluctuations in the accretion torque(Elsner and Lamb,1976) or by variations in the torque exerted on the crust by the liquid interior (Lamb,Pines and Shaham,1978a).

The spectra of X-ray binaries are often modeled with a two-component model modified by interstellar absorption. A thermal component,a black body or multi color black body is assigned to the inner part of the accretion disk(in Black hole systems)or to the surface of the neutron star.The thermal component is dominant in the soft part of the spectrum.At higher energies,above ($\simeq 12keV.$),the luminosity is usually dominated by a comptonized component produced in the accretion disk corona (Juho Schultz,2005).

As Herold 1979,X.Y.Xia et al.1985,Yi Xu et al.2001 stated inverse compton scattering is the mechanism for X-ray production from accreting neutron Stars.Hard X-rays and Gamma rays can also be produced by accreting neutron stars having strong magnetic field and hot surface temperature (10^6K). As it is stated above X-ray binaries have spectra and it is possible to make some order of magnitude estimates of the spectral range of the emission from compact accreting objects, and ,conversely, suggest what type of compact object may be responsible for various observed behavior. Now let us begin by letting T_{rad} be the temperature of continuum spectrum of the emitted radiation and $h\bar{\nu}$ is the energy of a typical photon and is of order KT_{rad} and this will lead us

$$T_{rad} = \frac{h\bar{\nu}}{K}$$

where we don't need to make precise choice of $\bar{\nu}$.

For an accretion luminosity L_{acc} from a source of radius R_*, we define a black body temperature ,T_b as the temperature the source would have if it were to radiate the given power as a black body spectrum.

$$T_b = (\frac{L_{acc}}{4\pi R_*^2 \sigma})^{\frac{1}{2}}$$

Let us now define T_{th} as the temperature that the accreted material would reach if its gravitational potential energy were turned entirely into thermal energy. For each proton-electron pair accreted, the potential energy released is

$$\frac{GM(m_p + m_e)}{R_*} \simeq \frac{GMm_P}{R_*}$$

and the thermal energy is $2 \times \frac{3}{2}KT$ therefore:

$$T_{th} = \frac{GMm_p}{3KR_*}$$

If the accretion flow is optically thick, the radiation reaches thermal equilibrium with the accreted material before leaking out to observer and $T_{rad} \sim T_{th}$.

In general, the radiation temperature may be expected to lie between the thermal and black body temperature, and ,since the system can't radiate a given flux at less than the black body temperature.

$$T_b \leq T_{rad} \leq T_{th}$$

But, these estimates assume that the radiating material can be characterized by a single temperature.The above approximation need not apply to a non-Maxwellian distribution of electrons radiating in a fixed magnetic field(J.Frank et al.1985).

By applying the limit in black body temperature T_b and thermal energy T_{th} to the case of a one solar mass neutron star,we have.

$$T_{th} \sim 5.5 \times 10^{11} K$$

or

$$KT_{th} \sim 50 Mev$$

By taking $L_{acc} \sim L_{Edd} \sim 10^{38} \frac{erg}{s}$,$T_b \sim 10^7 K$ or $KT_b \sim$ 1KeV and range of the photon energies as a result of accretion on to neutron stars will be

$$1KeV \leq h\bar{\nu} \leq 50MeV.$$

Thus we can expect the most luminous binary systems as a medium to hard X-ray emitters and possibly $\gamma-$ray (J.Frank et al.1985).

Chapter 4

Compton Scattering In Strong Magnetic Fields In Laboratory Frame

4.1 Introduction

Since Herold's study of the Compton scattering in strong magnetic fields in the electron rest frame(ERF), his expression of cross-section has been widely used in astrophysical calculations. However,in actual calculations the cross-section in the laboratory frame(LF)is required. The difficulty of obtaining a LF version of Herold's cross-section in the ERF is due to the fact that the QED processes in an external magnetic field are relativistic invariant only in the direction of the field and, therefore ,no exact relativistic transformations are available to recover the full expression of cross-section in the LF from Herold's expression. In this chapter, we aim at a derivation of an exact LF version of Herold's full expression of cross-section in the ERF.

4.1.1 Derivation of the magnetic Feynman Propagator

Charged particles on higher Landau levels have a very short lifetime τ due to the cyclotron radiation in strong magnetic fields,$\tau \leq 10^{-15}s$. It is therefore reasonable to assume that electrons either before or after the magnetic scattering should be on the ground Landau level. That is to say an electron can only occupy a higher Landau level in its intermediate states. Taking into account the fact that the ground Landau level is not degenerate due to the spinor $u_{1,0}(x_1, k) = 0$,the initial and final states of an electron-photon scattering system can be represented by:

$$|i, t_i\rangle = c_0^+(p_i, t_i)a_{\lambda_i}^+(k_i, t_i)|0\rangle \tag{4.1.1}$$

$$|f, t_f\rangle = c_0^+(p_f, t_f)a_{\lambda_f}^+(k_f, t_f)|0\rangle \tag{4.1.2}$$

where, $c_0^+ = c_{2,0}^+$ is the electron creation operator, a_λ^+ is the photon creation operator and $p_{i(f)}$ denotes the incident(outgoing) electron momentum.We stress here that the operators in these two expressions are Heisenberg ones , not the free ones, meaning that interaction have been considered. The scattering matrix can be generally expressed by:

$$S_{fi} = \lim_{t_i \to -\infty, t_f \to \infty} \langle 0|T[c_0(p_f, t_f)a_{\lambda_f}(k_f, t_f)c_0^+(p_i, t_i)a_{\lambda_i}^+(k_i, t_i)]|0\rangle \tag{4.1.3}$$

Introducing the incoming interaction picture and making perturbation expansions, one then obtains under the Born approximation:

$$\begin{aligned} S_{fi} &= \lim_{t_i\to-\infty, t_f\to\infty} e^2 \int d^4x_1 d^4x_2 \langle 0|Tc_0(p_f,t_f)\overline{\psi(x_1)}|0\rangle \\ &\times \gamma_\mu \langle 0|T\psi(x_1)\overline{\psi(x_2)}|0\rangle \gamma_\nu \\ &\times [\langle 0|Ta_{\lambda_f}(k_f,t_f)A_\mu(x_1)|0\rangle \langle 0|Ta^{\dagger}_{\lambda_i}(k_i,t_i)A_{\nu(x_2)}|0\rangle \\ &+ \langle 0|Ta_{\lambda_f}(k_f,t_f)A_\nu(x_2)|0\rangle \langle 0|Ta^{+}_{\lambda_i}(k_i,t_i)A_\mu(x_1)|0\rangle] \\ &\times \langle 0|T\psi(x_2)c_0^{+}(p_i,t_i)|0\rangle \end{aligned}$$

$$\begin{aligned} S_{fi} &= \lim_{t_i\to-\infty, t_f\to\infty} e^2 \{\int d^4x_1 d^4x_2 \langle 0|Tc_0(p_f,t_f)\overline{\psi(x_1)}|0\rangle \gamma_\mu \langle 0|T\psi(x_1)\overline{\psi(x_2)}|0\rangle \gamma_\nu \\ &\times \langle 0|Ta_{\lambda_f}(k_f,t_f)A_\mu(x_1)|0\rangle|0\rangle \langle 0|Ta^{\dagger}_{\lambda_i}(k_i,t_i)A_{\nu(x_2)}|0\rangle \langle 0|T\psi(x_2)c_0^{+}(p_i,t_i)|0\rangle \\ &+ \int d^4x_1 d^4x_2 \langle 0|Tc_0(p_f,t_f)\overline{\psi(x_1)}|0\rangle \gamma_\mu \langle 0|T\psi(x_1)\overline{\psi(x_2)}|0\rangle \gamma_\nu\rangle \langle 0|Ta^{\dagger}_{\lambda_f}(k_f,t_f)A_{\nu(x_2)}|0\rangle \\ &\times \langle 0|Ta^{\dagger}_{\lambda_i}(k_i,t_i)A_{\mu(x_1)}|0\rangle \langle 0|T\psi(x_2)c_0^{+}(p_i,t_i)|0\rangle\} \langle 0|T\psi(x_1)\overline{\psi(x_2)}|0\rangle \end{aligned} \tag{4.1.4}$$

The Feynman diagram for the above equation is as follows: In fig.(A) at point (X_1, t') electrons with momentum $\vec{P}$ and photons with $\vec{k}$ distruct.Similarly at point (x_2, t) electrons and photons are created. For fig.(B)at (x_1, t) electrons distract but photons will create. But at $(x_2, t^{'})$ photons distract and electrons will be created.

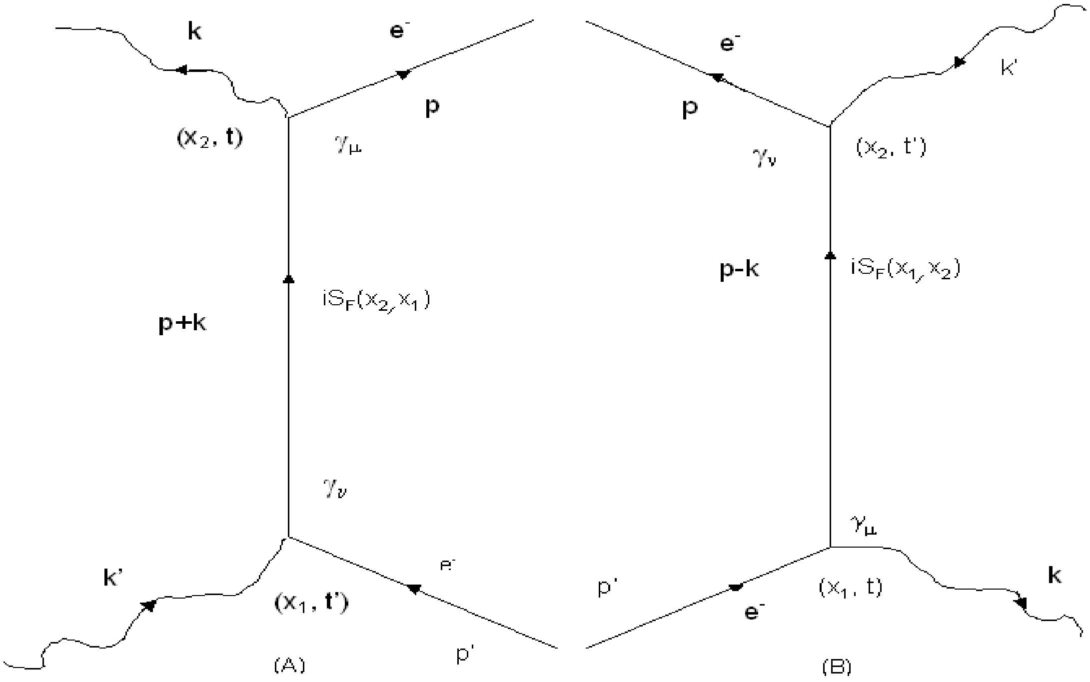

Figure 4.1: Feynman diagram for the scattering amplitude equation(4.1.4) given above

For the derivation of the magnetic Feynman propagator let us Consider the Dirac equation in the form,

$$[i\eth - eA(x) - m]\psi(x,t)] = 0 \qquad (4.1.5)$$

where $A(x) = \gamma^\mu \partial_\mu$ in which $e > 0$ is assumed and the asymmetry gauge is taken,i.e.$A(x) = (0, Bx_1, 0)$.The solution of(4.1.5)with positive energy is:

$$\psi^+_{s,n}(x,t) = Nu_{s,n}(x_1,p)e^{[-iE_n(p_3)t+ip_2x_2+ip_3x_3]}, s = 1,2 \qquad (4.1.6)$$

where $E_n$indicates the Landau energy of an electron and N is a normalization constant.

$$E_n(p_3) = (m^2 + p_3^2 + 2neB)^{1/2}, N = [\frac{E_n(p_3) + m}{2E_n(p_3)}]^{1/2} \tag{4.1.7}$$

The spinors $u_{1,n}(x_1, p)$,$u_{2,n}(x_1, p)$ are given by:

$$u_{1,n}(x_1, p) = \begin{bmatrix} I_{n-1}(x_1, p_2) \\ 0 \\ \frac{p_3}{E_n(p_3)+m} I_{n-1}(x_1, p_2) \\ \frac{i\sqrt{2neB}}{E_n(p_3)+m} I_n(x_1, p_2) \end{bmatrix} \tag{4.1.8}$$

$$u_{2,n}(x_1, p) = \begin{bmatrix} 0 \\ I_n(x_1, p_2) \\ \frac{-i\sqrt{2neB}}{E_n(p_3)+m} I_{n-1}(x_1, p_2) \\ \frac{-p_3}{E_n(p_3)+m} I_n(x_1, p_2) \end{bmatrix} \tag{4.1.9}$$

in which $I_n(x_1$,$p_2)$ is the harmonic oscillator wave function,

$$I_n(x_1, p_2) = (\lambda\sqrt{\pi}2^n n!)^{-1/2} e^{[-1/2(\frac{x_1}{\lambda} + \lambda p_2)^2]} H_n(\frac{x_1}{\lambda} + \lambda p_2) \tag{4.1.10}$$

obeying the orthogonal condition

$$\int dx_1 I_n(x_1, p_2) I_m(x_1, p_2) = \delta_{nm} \tag{4.1.11}$$

where $\lambda^{-1} = \sqrt{eB}$. The solution of (4.1.5) with negative energy is

$$\psi^{(-)}_{s,n}(x) = N v_{s,n}(x_1, p) e^{[iE_n(p_3)t + ip_2x_2 + ip_3x_3]} s = 1, 2 \tag{4.1.12}$$

with the spinors $v_{1,n}(x_1, p)$ $v_{2,n}(x_1, p)$given by

$$v_{1,n}(x_1, p) = \begin{bmatrix} \frac{-p_3}{E_n(p_3)+m} I_{n-1}(x_1, p_2) \\ \frac{-i\sqrt{2neB}}{E_n(p_3+m} I_n(x_1, p_2) \\ I_{n-1}(x_1, p_2) \\ 0 \end{bmatrix} \tag{4.1.13}$$

$$v_{2,n}(x_1,p) = \begin{bmatrix} \frac{i\sqrt{2neB}}{E_n(p_3+m)} I_{n-1}(x_1,p_2) \\ \frac{p_3}{E_n(p_3+m)} I_n(x_1,p_2) \\ 0 \\ I_n(x_1,p_2) \end{bmatrix} \tag{4.1.14}$$

It is clear that the spinors $u_{s,n}(x_1,p)$,$v_{s,n}(x_1,p)$, s=1,2 form a complete and orthogonalized basis in the four-dimensional vector space. Therefore the Dirac field operators can be expanded as:

$$\psi(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c_{s,n}(p,t)u_{s,n}(x_1,p) + d^+_{s,n}(p,t)v_{s,n}(x_1,p)]e^{(ip_2x_2+ip_3x_3)} \tag{4.1.15}$$

$$\psi^+(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c^+_{s,n}(p,t)u^+_{s,n}(x_1,p) + d_{s,n}(p,t)v^+_{s,n}(x_1,p)]e^{(-ip_2x_2-ip_3x_3)} \tag{4.1.16}$$

We know that the Feynman propagator of an electron is given by:

$$S_F(x,y) = -i\langle 0|T\psi(x)\overline{\psi(y)}|0\rangle = -i\langle 0|\psi(x)\psi^+(y)\gamma^0|0\rangle, \tag{4.1.17}$$

for $t_x > t_y$

$$\begin{aligned} \Rightarrow S_F(x,y) &= -i\langle 0|\sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L^2}\sum_{p_2,p_3}\{[c_{s,n}(p,t)u_{s,n}(x_1,p) + d^+s,n(p,t)v_{s,n}(x_1,p)] \\ &\times [c^+_{s,n}(p,t)u^+_{s,n}(y_1,p) + d_{s,n}(p,t)v^+_{s,n}(y_1,p)]\} \\ &\times e^{ip_2(x_2-y_2+ip_3(x_3-y_3)}\gamma^0|0\rangle \end{aligned} \tag{4.1.18}$$

$$\begin{aligned} \Rightarrow S_F(x,y) &= -i\langle 0|\sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L^2}\sum_{p_2,p_3}\{[c_{s,n}(p,t)u_{s,n}(x_1,p)c^+_{s,n}(p,t)u^+_{s,n}(y_1,p)] \\ &+ [c_{s,n}(p,t)u_{s,n}(x_1,p)d_{s,n}(p,t)v^+_{s,n}(y_1,p)] \\ &+ [d^+s,n(p,t)v_{s,n}(x_1,p)c^+_{s,n}(p,t)u^+_{s,n}(y_1,p)] \\ &+ [d^+s,n(p,t)v_{s,n}(x_1,p)d_{s,n}(p,t)v^+(y_1,p)]\} \times e^{ip_2(x_2-y_2+ip_3(x_3-y_3)}\gamma^0|0\rangle \end{aligned} \tag{4.1.19}$$

From the anti-commutation relations between the annihilation and destruction operators, we have

$$c_{s,n}(p_1,t)c^{+}_{r,m}(p_2,t)+c^{+}_{r,m}(p_2,t)c_{s,n}(p,t)=\delta_{sr}\delta_{nm}\delta_{p_1p_2} \tag{4.1.20}$$

$$d_{s,n}(p_1,t)d^{+}_{r,m}(p_2,t)+d^{+}_{r,m}(p_2,t)d_{s,n}(p_1,t)=\delta_{sr}\delta_{nm}\delta_{p_1p_2} \tag{4.1.21}$$

Therefore,

$\langle 0|c_{s,n}(p_1,t)c^{+}_{r,m}(p_2,t)|0\rangle=\delta_{sr}\delta_{nm}\delta_{p_1p_2}$

$\langle 0|d_{s,n}(p_1,t)d^{+}_{r,m}(p_2,t)|0\rangle=\delta_{sr}\delta_{nm}\delta_{p_1p_2}$,

Using these two identities in(4.1.19),we will have

$$\begin{aligned} S_F(x,y) &= -i\langle 0|\sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L^2}\sum_{p_2,p_3}\{[c_{s,n}(p,t)u_{s,n}(x_1,p)c^{+}_{s,n}(p,t)u^{+}_{s,n}(y_1,p)] \\ &\times\ e^{ip_2(x_2-y_2+ip_3(x_3-y_3)}\gamma^0|0\rangle \end{aligned} \tag{4.1.22}$$

$$\Rightarrow S_F(x,y)=\frac{-i}{L^2}\sum_{n=0}^{\infty}\sum_{s=1}^{2}\sum_{p_2,p_3}u_{s,n}(x_1,p)u^{\dagger}_{s,n}(y_1,p)e^{ip_2(x_2-y_2+ip_3(x_3-y_3)}\gamma^0 \tag{4.1.23}$$

$$=\frac{-i}{L^2}\sum_{n=0}^{\infty}\sum_{p_2,p_3}\{u_{1,n}(x_1,p)u^{\dagger}_{1,n}(y_1,p)+u_{2,n}(x_1,p)u^{\dagger}_{2,n}(y_1,p)\}e^{ip_2(x_2-y_2)+ip_3(x_3-y_3)}\gamma^0\} \tag{4.1.24}$$

$$\begin{aligned} u_{1,n}(x_1,p)u^{+}_{1,n}(y_1,p) &= \begin{bmatrix} I_{n-1}(x_1,p_2) \\ 0 \\ \frac{p_3}{E_n(p_3)+m}I_{n-1}(x_1,p_2) \\ \frac{-i\sqrt{2neB}}{E_n(p_3)+m}I_n(x_1,p_2) \end{bmatrix} \\ &\times\ \begin{bmatrix} I_{n-1}(y_1,p_2) & 0 & \frac{p_3}{E_n(p_3)+m}I_{n-1}(y_1,p_2) & i\frac{\sqrt{2neB}}{E_n(p_3)+m}I_n(y_1,p_2) \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} I_{n-1}I_{n-1} & 0 & \frac{p_3}{E_n(p_3)+m}I_{n-1}I_{n-1} & \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}I_n \\ 0 & 0 & 0 & 0 \\ \frac{p_3}{E_n(p_3)+m}I_{n-1}I_{n-1} & 0 & \frac{p_3^2}{(E_n(p_3)+m)^2}I_{n-1}I_{n-1} & -ip_3\frac{\sqrt{2neB}}{(E_n(p_3)+m)^2}I_{n-1}I_{n-1} \\ \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_nI_{n-1} & 0 & \frac{I\sqrt{2neB}}{(E_n(p_3)+m)^2}I_nI_{n-1} & \frac{2neB}{(E_n(p_3)+m)^2}I_nI_n \end{bmatrix} \tag{4.1.25}$$

$$\begin{aligned} u_{2,n}(x_1,p)u^\dagger_{2,n}(y_1,p) \quad &= \quad \begin{bmatrix} 0 \\ I_n(x_1,p_2) \\ \frac{-i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}(x_1,p_2) \\ \frac{-p_3}{E_n(p_3)+m}I_n(x_1,p_2) \end{bmatrix} \\ &\times \quad \begin{bmatrix} 0 & I_n(y_1,p_2) & \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}(y_1,p_2) & \frac{-p_3}{E_n(p_3)+m}I_n(y_1,p_2) \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & I_nI_n & \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_nI_{n-1} & \frac{-p_3}{E_n(p_3)+m}I_nI_n \\ 0 & \frac{-i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}I_n & \frac{2neB}{(E_n(p_3)+m)^2}I_{n-1}I_{n-1} & \frac{i\sqrt{2neB}}{(E_n(p_3)+m)^2}I_{n-1}I_n \\ 0 & \frac{-p_3}{E_n(p_3)+m}I_nI_n & \frac{-ip_3\sqrt{2neB}}{(E_n(p_3)+m)^2}I_nI_{n-1} & \frac{p_3^2}{(E_n(p_3)+m)^2}I_nI_n \end{bmatrix} \tag{4.1.26}$$

Summing (4.1.25)and (4.1.26),we will obtain:

$$\begin{bmatrix} I_{n-1}I_{n-1} & 0 & \frac{p_3}{E_n(p_3)+m}I_{n-1}I_{n-1} & \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}I_n \\ 0 & I_nI_n & \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_nI_{n-1} & \frac{-p_3}{E_n(p_3)+m}I_nI_n \\ \frac{p_3}{E_n(p_3)+m}I_{n-1}I_{n-1} & \frac{-i\sqrt{2neB}}{E_n(p_3)+m}I_{n-1}I_n & \frac{(p_3^2+2neB)}{(E_n(p_3)+m)^2}I_{n-1}I_{n-1} & 0 \\ \frac{i\sqrt{2neB}}{E_n(p_3)+m}I_nI_{n-1} & \frac{-p_3}{E_n(p_3)+m}I_nI_n & 0 & \frac{(p_3^2+2neB)}{(E_n(p_3)+m)^2}I_nI_n \end{bmatrix} \tag{4.1.27}$$

We know that

$E_n(p_3) = (m^2+p_3^2+2neB)^{\frac{1}{2}} \Rightarrow p_3^2+2neB = E_n^2(p_3)-m^2 = (E_n(p_3)-m)(E_n(p_3)+m)$

After inserting this relation into (4.1.24) and multiplying byγ^0,the propagator will have the form:

$$S_F(x,y) = \frac{1}{L^2}\sum_{p_2,p_3}\int\frac{d\omega}{2\pi}\sum_{n=0}^{\infty}\frac{S_n(x_1,y_1,p)}{(\omega+i\epsilon)(E_n+m)} \times e^{[-i\omega(t_x-t_y)+ip_2(x_2-y_2)+ip_3(x_3-y_3)]} \tag{4.1.28}$$

where,

$$S_n(x_1,y_1,p) = \begin{bmatrix} (E_n+m)I_{n-1}I_{n-1} & 0 & -p_3 I_{n-1}I_{n-1} & i\sqrt{2neB}I_{n-1}I_n \\ 0 & (E_n(p_3)+m) & -i\sqrt{2neB}I_nI_{n-1} & p_3 I_n I_n \\ p_3 I_n I_n & -i\sqrt{2neB}I_{n-1}I_n & -(E_n-m)I_{n-1}I_{n-1} & 0 \\ i\sqrt{2neB}I_nI_{n-1} & -p_3 I_n I_n & 0 & -(E_n-m)I_nI_n \end{bmatrix} \tag{4.1.29}$$

4.1.2 Calculation Of The Scattering Amplitude

In the previous section we have already derived the Feynman electron propagator , So using this result, we can calculate the scattering amplitude,S_{fi}.

$$\psi^+(x) = \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[c^+_{s,n}(p,t)u^+_{s,n}(x_1,p) + d_{s,n}(p,t)v^+_{s,n}(x_1,p)]e^{-ip_2x_2-ip_3x_3} \tag{4.1.30}$$

$$\begin{aligned}
\langle 0|Tc_0(p_f,t_f)\overline{\psi(r_1,t)}|0\rangle &= \langle 0|c_0(p_f,t_f)\overline{\psi(r_1,t)}|0\rangle, t_f \to \infty \\
&= \langle 0|c_0(p_f,t_f)\psi^+(r_1,t)\gamma^0|0\rangle \\
&= \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[\langle 0|c_0(p_f,t_f)c^+_{s,n}(p,t)u^+_{s,n}(x_1,p)\gamma^0|0\rangle \\
&+ \langle 0|c_0(p_f,t_f)d_{s,n}(p,t)v^+_{s,n}(x_1,p)\gamma^0|0\rangle] \times e^{-i(p_2x_2+p_3x_3)} \\
&= \sum_{n=0}^{\infty}\sum_{s=1}^{2}\frac{1}{L}\sum_{p_2,p_3}[\langle 0|c_0(p_f,t_f)c^+_{s,n}(p,t)u^+_{s,n}(x_1,p)\gamma^0|0\rangle] \times e^{-i(p_2x_2+p_3x_3)} \\
&= \frac{1}{L}\sum_{p_2,p_3}[\langle 0|c_0(p_f,t_f)c^+_{2,0}(p,t)u^+_{2,0}(x_1,p)\gamma^0|0\rangle] \times e^{-i(p_2x_2+p_3x_3)} \\
&= \frac{1}{L}\sum_{p_2,p_3}\overline{u_0}(x_1,p_f)e^{-i(p_2x_2+p_3x_3)} \qquad (4.1.31)
\end{aligned}$$

The incident(out going) electron momentum can be expressed as $p_{i(f)} = (0, a_{i(f)}, p_{i(f)})$,meaning the incident(outgoing) electron momentum along the direction of the magnetic field(taken as the Z direction) is $p_{i(f)}$ and the centre of the Landau orbit is$-\lambda^2 a_{i(f)}$

$$\Rightarrow \langle 0|Tc_0(p_f,t_f)\overline{u}(r_1,t)|0\rangle = \frac{1}{L}\overline{u_0}(x_1,p_f)e^{(-iE_ft_f-ip_fz_1-ia_fy_1+iE_ft_1)} \tag{4.1.32}$$

similarly,

$$\langle 0|Ta_{\lambda_f}(k_f,t_f)A_\mu(\vec{r_1},t)|0\rangle = \frac{1}{\sqrt{v}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)}e^{(-i\omega_ft_f+i\omega_ft_1-i\vec{k_f}.\vec{r_1})} \tag{4.1.33}$$

but,$i\vec{k_f}.\vec{r_1} = ik_{fx}x_1 + ik_{fy}y_1 + ik_{fz}z_1$,

where,$k_{fz} = k_f cos\theta_f$

$$\langle 0|Ta_{\lambda_f}(k_f,t_f)A_\mu(\vec{r_1},t)|0\rangle = \frac{1}{\sqrt{v}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)}e^{(-i\omega_ft_f+i\omega_ft_1-ik_{fx}x_1-ik_{fy}y_1-ik_fcos\theta_fz_1)} \tag{4.1.34}$$

$$\langle 0|T\psi(\vec{r_2},t)c_0^+(p_i,t_i)|0\rangle = \frac{1}{L}u_0(x_2,p_i)e^{(iE_it_i+ip_iz_2+ia_iy_2-iE_it_2)} \tag{4.1.35}$$

$$\langle 0|Ta_{\lambda_i}^+(k_i,t_i)A_\mu(\vec{r_1},t)|0\rangle = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_i}}\epsilon_\mu^{(\lambda_i)}e^{(i\omega_it_i-i\omega_it_2+i\vec{k_i}.\vec{r_2})} \tag{4.1.36}$$

$$\langle 0|Ta_{\lambda_i}^+(k_i,t_i)A_\mu(\vec{r_1},t)|0\rangle = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_i}}\epsilon_\mu^{(\lambda_i)}e^{(i\omega_it_i-i\omega_it_2+ik_{ix}x_2+ik_{iy}y_2+ik_icos\theta_iz_2)} \tag{4.1.37}$$

$$\begin{aligned}
S_{fi} &= \lim_{t_i\to-\infty,t_f\to\infty} e^2\int d^4x_1d^4x_2\frac{1}{L}\overline{u_0}(x_1,p_f)e^{-i(E_ft_f-E_ft_1)-i(p_fz_1+a_fy_1)}\\
&\times \gamma_\mu\langle 0|T\psi(x_1)\overline{\psi}(x_2)|0\rangle\gamma_\nu\\
&\times \frac{1}{\sqrt{v}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\mu^{(\lambda_f)}e^{(-i\omega_ft_f+i\omega_ft_1-ik_{fx}x_1-ik_{fy}y_1-ik_fcos\theta_fz_1)}\\
&\times \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_i}}\epsilon_\mu^{(\lambda_i)}e^{(i\omega_it_i-i\omega_it_2+ik_{ix}x_2+ik_{iy}y_2+ik_icos\theta_iz_2)}\\
&\times \frac{1}{\sqrt{v}}\frac{1}{\sqrt{2\omega_f}}\epsilon_\nu^{(\lambda_f)}e^{-i(\omega_ft_f-\omega_ft_2)-ik_{fx}x_2-ik_{fy}y_2-ik_fcos\theta_fz_2}\\
&\times (\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_i}}\epsilon_\mu^{(\lambda_i)}e^{i(\omega_it_i-i\omega_it_1)+ik_{ix}x_1+ik_{iy}y_1+ik_icos\theta_iz_1})\\
&\times \frac{1}{L}u_0(x_2,p_i)e^{i(E_it_i-E_it_2)+i(p_iz_2+a_iy_2)}
\end{aligned} \tag{4.1.38}$$

$$\begin{aligned}
S_{fi} &= \lim_{t_i\to-\infty,t_f\to\infty}\frac{1}{L^2V}\frac{1}{\sqrt{4\omega_i\omega_f}}e^2\int d^4x_1d^4x_2\bar{u_0}(x_1,p_f)\gamma_\mu \\
&\times \frac{1}{L^2}\sum_{q_y,q_z}\int\frac{dq_0}{2\pi}\sum_{n=0}^{\infty}\frac{S_n(x_1,x_2,q)}{q_0^2-E_n^2(q_z)+i\epsilon}\\
&\times \exp[iq_0(t_2-t_1)+iq_y(y_1-y_2)+iq_z(z_1-z_2)]\gamma_\nu\\
&\times \{\big[\exp[-i(E_ft_f-E_ft_1)-i(p_fz_1+a_fy_1)]\epsilon_\mu^{(\lambda_f)}\\
&\times \exp[-i(\omega_ft_f-\omega_ft_1)-ik_{fx}x_1-ik_{fy}y_1-ik_f\cos\theta_fz_1]\\
&\times \epsilon_\nu^{(\lambda_i)}\exp[i(\omega_it_i-\omega_it_2)+ik_{ix}x_2+ik_{iy}y_2+ik_i\cos\theta_iz_2]\big]\\
&+ \big[\exp[-i(E_ft_f-E_ft_1)-i(p_fz_1+a_fy_1)]\epsilon_\nu^{(\lambda_f)}\\
&\times \exp[-i(\omega_ft_f-\omega_ft_2)-ik_{fx}x_2-ik_{fy}y_2-ik_f\cos\theta_fz_2]\\
&\times \epsilon_\mu^{(\lambda_i)}\exp[i(\omega_it_i-\omega_it_1)+ik_{ix}x_1+ik_{iy}y_1+ik_i\cos\theta_iz_1]\big]\}\\
&\times u_0(x_2,p_i)\exp[i(E_it_i-E_it_2)+i(p_iz_2+a_iy_2)]
\end{aligned}\tag{4.1.39}$$

$$\begin{aligned}
\Rightarrow S_{fi}=\frac{(2\pi)^5}{L^2V} &* \frac{e^2}{2\sqrt{\omega_i\omega_f}}\sum_{n=0}^{\infty}\frac{1}{L^2}\sum_{q_y,q_z}\int d^4x_1d^4x_2\int dq_0\overline{u_0}(x_1,p_f)\\
&\times [\hat{\epsilon_f}\frac{S_n(x_1,x_2,q)}{(E_i+\omega_i)^2-E_n^2(q_z)+i\epsilon}\\
&\times \hat{\epsilon_i}\delta(\omega_f+E_f-q_0)\delta(q_0-\omega_i-E_i)\\
&\times \delta(k_{iy}+a_i-q_y)\delta(q_y-k_{fy}-a_f)\delta(q_z-k_f\cos\theta_f-p_f)\\
&\times \delta(k_i\cos\theta_i+p_i-q_z)\exp(-ik_{fx}x_1+ik_{ix}x_2)+\hat{\epsilon_i}\frac{S_n(x_1,x_2,q)}{(E_i-\omega_f)^2-E_n^2(q_z)+i\epsilon}\\
&\times \hat{\epsilon_f}\delta(\omega_f-E_i+q_0)\delta(E_f-\omega_i-q_0)\\
&\times \delta(k_{iy}-a_f+q_y)\delta(a_i-k_{fy}-q_y)\delta(q_z+k_i\cos\theta_i-p_f)\\
&\times \delta(p_i-k_f\cos\theta_f-q_z)\exp(ik_{ix}x_1-ik_{fx}x_2)]u_0(x_2,p_i)\\
&\times \lim_{t_i\to-\infty,t_f\to\infty}\exp[-i(E_f+\omega_f)t_f+i(E_i-\omega_i)t_i]
\end{aligned}\tag{4.1.40}$$

after substituting all the expressions given from(4.1.31)to(4.1.36) into(4.1.4),where$\hat{\epsilon_f} = \epsilon_\mu^{(\lambda_f)}\gamma_\mu$,$\hat{\epsilon_i} = \epsilon_\mu^{(\lambda_i)}\gamma_\mu$.

The phase factor at the end of(4.1.40)can be ignored,since the cross-section$\sim |S_{fi}|^2$. The spinors $\bar{u_0}(x_1,p_f)$,$u_0(x_2,p_i)$ can be expressed as:

$$\bar{u_0}(x_1,p_f) = [\frac{E_f+m}{2E_f}]^{\frac{1}{2}} I_0(x_1,a_f)\bar{u_f}(p_f) \tag{4.1.41}$$

$$u_0^T(x_2,p_i) = [\frac{E_i+m}{2E_i}]^{\frac{1}{2}} I_0(x_2,a_i)u_i^T(p_i), \tag{4.1.42}$$

where

$$\bar{u_f}(p_f) = \begin{pmatrix} 0 & 1 & 0 & \frac{p_f}{E_f+m} \end{pmatrix} \tag{4.1.43}$$

$$u_i^T(p_i) = \begin{pmatrix} 0 & 1 & 0 & \frac{-p_i}{E_i+m} \end{pmatrix} \tag{4.1.44}$$

$$\begin{aligned}
S_{fi} &= \frac{(2\pi)^5}{L^2V}\frac{e2}{\sqrt{4\omega_i\omega_f}}\sum_{n=0}^{\infty}\frac{1}{L^2}\sum_{q_y,q_z}[\frac{(E_f+m)(E_i+m)}{4E_iE_f}]^{\frac{1}{2}}\\
&\times \int d^4x_1d^4x_2\int dq_0 I_0(x_1,a_f)\bar{u}_f(p_f)I_0(x_2,a_i)u_i(p_i)\\
&\times \{\hat{\epsilon_f}\frac{S_n(x_1,x_2,q)}{(E_i+\omega_i)^2-E_n^2(q_z)+i\epsilon}\hat{\epsilon_i}\delta(\omega_f+E_f-q_0)\delta(q_0-\omega_i-E_i)\\
&\times \delta(k_{iy}+a_i-q_y)\delta(q_y-k_{fy}-a_f)\delta(q_z-k_f\cos\theta_f-p_f)\\
&\times \delta(k_i\cos\theta_i+p_i-q_z)\exp(-ik_{fx}x_1+ik_{ix}x_2)\\
&+ \hat{\epsilon_i}\frac{S_n(x_1,x_2,q)}{(E_i+\omega_i)^2-E_n^2(q_z)+i\epsilon}\\
&\times \hat{\epsilon_f}\delta(\omega_f-E_i+q_0)\delta(E_f-\omega_i-q_0)\\
&\times \delta(k_{iy}-a_f+q_y)\delta(a_i-k_{fy}-q_y)\delta(q_z+k_i\cos\theta_i-p_f)\\
&\times \delta(p_i+k_f\cos\theta_f-q_z)\exp(ik_{ix}x_1-ik_{fx}x_2)\}
\end{aligned} \tag{4.1.45}$$

Let $I_1 = \int dx_1^4dx_2^4e^{(-ik_{fx}x_1)}I_0(x_1,a_f)S_n(x_1,x_2,q)e^{(ik_{ix}x_2)}I_0(x_2,a_i)$,and

$I_2 = \int dx_1^4 dx_2^4 e^{(-ik_{ix}x_1)} I_0(x_1, a_f) S_n(x_1, x_2, q) e^{(ik_{fx}x_2)} I_0(x_2, a_i)$

With the help of the integral:

$\int_{-\infty}^{\infty} d\xi H_n(\xi) \exp[-(\xi - \alpha)^2] = \sqrt{\pi}(2\alpha)^n$,these two integrals yield values,

$$\begin{aligned} I_1 &= \frac{(\lambda^2 k_f^+ k_i^-)^{(n-1)}}{2^n n!} A_n \exp[-\frac{\lambda^2}{4}(k_i^2 \sin^2\theta_i + k_f^2 \sin^2\theta_f)] \\ &\times \exp[i\lambda^2(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})], \end{aligned} \tag{4.1.46}$$

where $k_i^{\pm} = k_{ix} \pm ik_{iy}$ and $k_f^{\pm} = k_{fx} \pm ik_{fy}$ and A_n is defined by

$$A_n = \begin{pmatrix} 2n(q_0+m) & 0 & -2nq_z & -2nk_i^- \\ 0 & (q_0+m)\lambda^2 k_f^+ k_i^- & -2nk_f^+ & q_z\lambda^2 k_f^+ k_i^- \\ 2nq_z & -2nk_i^- & -2n(q_0-m) & 0 \\ 2nk_f^+ & -q_z\lambda^2 k_f^+ k_i^- & 0 & -(q_0-m)\lambda^2 k_f^+ k_i^- \end{pmatrix} \tag{4.1.47}$$

Similarly,

$$\begin{aligned} I_2 &= \frac{(\lambda^2 k_f^+ k_i^-)^{(n-1)}}{2^n n!} B_n \exp[-\frac{\lambda^2}{4}(k_i^2 \sin^2\theta_i + k_f^2 \sin^2\theta_f)] \\ &\times \exp[-i\lambda^2(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} + \frac{1}{2}k_{fx}k_{fy})], \end{aligned} \tag{4.1.48}$$

with the matrix B_n given by

$$B_n = \begin{pmatrix} 2n(q_0+m) & 0 & -2nq_z & 2nk_f^- \\ 0 & (q_0+m)\lambda^2 k_f^- k_i^+ & 2nk_i^+ & q_z\lambda^2 k_f^- k_i^+ \\ 2nq_z & -2nk_f^- & -2n(q_0-m) & 0 \\ -2nk_i^+ & -q_z\lambda^2 k_f^- k_i^+ & 0 & -(q_0-m)\lambda^2 k_f^- k_i^+ \end{pmatrix} \tag{4.1.49}$$

$$\begin{aligned}
S_{fi} &= \frac{(2\pi)^5}{L^2 V}\frac{e^{(2)}}{\sqrt{4\omega_i\omega_f}}\sum_{n=0}^{\infty}\frac{1}{L^2}\sum_{q_y,q_z}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]^{\frac{1}{2}} \\
&\times \frac{(\lambda^2 k_f^+ k_i^-)^{(n-1)}}{2^n n!}\exp[-\frac{\lambda^2}{4}(k_i^2\sin^2\theta_i + k_f^2\sin^2\theta_f)] \\
&\times \{\frac{\bar{u_f}(p_f)\hat{\epsilon_f}A_n\hat{\epsilon_i}u_i(p_i)}{(E_i+\omega_i)^2 - E_n^2(q_z) + i\epsilon} \\
&\times \exp[i\lambda^2(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})] \\
&\times \int dq_0\delta(\omega_f + E_f - q_0)\delta(q_0 - \omega_i - E_i)\delta(k_{iy} + a_i - q_y) \\
&\times \delta(q_y - k_{fy} - a_f)\delta(q_z - k_f\cos\theta_f - p_f)\delta(k_i\cos\theta_i - p_i - q_z) \\
&+ \frac{\bar{u_f}(p_f)\hat{\epsilon_i}B_n\hat{\epsilon_f}u_i(p_i)}{(E_i-\omega_f)^2 - E_n^2(q_z) + i\epsilon} \\
&\times \exp[-i\lambda^2(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} + \frac{1}{2}k_{fx}k_{fy})] \\
&\times \int dq_0\delta(\omega_f - E_i + q_0)\delta(E_f - \omega_i - q_0)\delta(k_{iy} - a_f + q_y) \\
&\times \delta(a_i - k_{fy} - q_y)\delta(p_i - k_f\cos\theta_f - q_z)\delta(q_z + k_i\cos\theta_i - p_f)\} \quad (4.1.50)
\end{aligned}$$

But,

$$\sum_{q_y,q_z}\int dq_0\delta(\omega_f+E_f-q_0)\delta(q_0-\omega_i-E_i)\delta(k_{iy}+a_i-q_y)\delta(q_y-k_{fy}-a_f)\delta(q_z-k_f\cos\theta_f-p_f)$$

$$*\delta(k_i\cos\theta_i-p_i-q_z) = \delta(\omega_f+E_f-\omega_i-E_i)\delta(k_{iy}+a_i-k_{fy}-a_f)\delta(k_i\cos\theta_i+p_i-k_f\cos\theta_f-p_f)$$

Similarly,

$$\sum_{q_y,q_z}\int dq_0\delta(\omega_f-E_i+q_0)\delta(E_f-\omega_i-q_0)\delta(k_{iy}-a_f+q_y)\times\delta(a_i-k_{fy}-q_y)\delta(p_i-k_f\cos\theta_f-q_z)$$

$$*\delta(k_i\cos\theta_i+q_z-p_f) = \delta(\omega_f+E_f-\omega_i-E_i)\delta(k_{iy}+a_i-k_{fy}-a_f)\delta(k_i\cos\theta_i+p_i-k_f\cos\theta_f-p_f)$$

$$\begin{aligned}
\Rightarrow S_{fi} &= \frac{(2\pi)^3}{L^2V}\frac{e^2}{\sqrt{4\omega_i\omega_f}}\sum_{n=0}^{\infty}\frac{1}{L^2}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]^{\frac{1}{2}} \\
&\times \frac{(\lambda^2 k_f^+ k_i^-)^{(n-1)}}{2^n n!}\exp[-\frac{\lambda^2}{4}(k_i^2\sin^2\theta_i + k_f^2\sin^2\theta_f)] \\
&\times \{\frac{P_1}{(E_i+\omega_i)^2 - E_n^2(q_z) + i\epsilon} \\
&\times \exp[i\lambda^2(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})] \\
&+ \frac{P_2}{(E_i-\omega_f)^2 - E_n^2(q_z) + i\epsilon} \\
&\times \exp[-i\lambda^2(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} + \frac{1}{2}k_{fx}k_{fy})]\} \\
&\times \delta(\omega_f + E_f - \omega_i - E_i)\delta(k_{iy} + a_i - k_{fy} - a_f) \\
&\times \delta(k_i\cos\theta_i + p_i - k_f\cos\theta_f - p_f), \qquad (4.1.51)
\end{aligned}$$

where $P_1 = \bar{u_f}(p_f)\hat{\epsilon_f}A_n\hat{\epsilon_i}u_i(p_i)$ and$P_2 = \bar{u_f}(p_f)\hat{\epsilon_i}B_n\hat{\epsilon_f}u_i(p_i)$, where$\hat{\epsilon_f} = \epsilon_\mu^{\lambda_f}\gamma_\mu$,$\hat{\epsilon_i} = \epsilon_\mu^{\lambda_i}\gamma_\mu$

These two scalar matrix products(i.e.$P_1 and P_2$) for all combinations of $\gamma_1, \gamma_2, \gamma_3$ will have the form:

$$\begin{aligned}
P_1 &= 2n[\frac{(q_0+m)P_iP_f}{(E_i+m)(E_f+m)} + (q_0-m)]\epsilon_f^+\epsilon_i^- \\
&+ 2n[\frac{P_f}{E_f+m} + \frac{P_i}{E_i+m}](k_i\epsilon_f^+\epsilon_{iz} + k_f^+\epsilon_{fz}\epsilon_i^- + q_z\epsilon_f^+\epsilon_i^-) \\
&+ \lambda^2 k_f^+ k_i^-[\frac{(q_0+m)P_iP_f}{(E_i+m)(E_f+m)} + (q_0-m) + \frac{q_zP_f}{E_f+m} + \frac{q_zP_i}{E_i+m}]\epsilon_{fz}\epsilon_{iz}. \qquad (4.1.52)
\end{aligned}$$

$$\begin{aligned}
P_2 &= 2n[\frac{(q_0+m)P_iP_f}{(E_i+m)(E_f+m)} + (q_0-m)]\epsilon_i^+\epsilon_f^- \\
&- 2n[\frac{P_f}{E_f+m} + \frac{P_i}{E_i+m}](k_i^+\epsilon_f^-\epsilon_{iz} + k_f^-\epsilon_i^+\epsilon_{fz} + q_z\epsilon_i^+\epsilon_f^-) \\
&+ \lambda^2 k_f^- k_i^+[\frac{(q_0+m)P_iP_f}{(E_i+m)(E_f+m)} + (q_0-m) + \frac{q_zP_f}{E_f+m} + \frac{q_zP_i}{E_i+m}]\epsilon_{fz}\epsilon_{iz}. \qquad (4.1.53)
\end{aligned}$$

Evaluating $\exp[i\lambda^2(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})]$ and $\exp[-i\lambda^2(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} - \frac{1}{2}k_{fx}k_{fy})]$

at the value $a_f = k_{iy} + a_i - k_{fy}$:

$$\exp[i\lambda^2(a_f k_{fx} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})]$$

$$= \exp[i\lambda^2(k_{iy}k_{fx} + a_i k_{fx} - k_{fx}k_{fy} + \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})]$$

$$= \exp[i\lambda^2(k_{iy}k_{fx} + a_i k_{fx} - \frac{1}{2}k_{fx}k_{fy}) - i\lambda^2(a_i k_{ix} + \frac{1}{2}k_{ix}k_{iy})]$$

$$= \exp(i\lambda^2(k_{iy}k_{fx}))\exp[i\lambda^2(-\frac{1}{2}k_{fx}k_{fy} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} - a_i k_{ix})]$$

$$= \exp(i\lambda^2(k_{iy}k_{fx}))\exp[-\frac{i\lambda^2}{2}(k_{ix}k_{iy} + k_{fx}k_{fy}) - i\lambda^2 a_i(k_{ix} - k_{fx})]$$

Similarly

$$\exp[-i\lambda^2(a_f k_{ix} - \frac{1}{2}k_{ix}k_{iy}) + i\lambda^2(a_i k_{fx} - \frac{1}{2}k_{fx}k_{fy})]$$

$$= \exp(i\lambda^2(k_{ix}k_{fy}))\exp[-\frac{i\lambda^2}{2}(k_{ix}k_{iy} + k_{fx}k_{fy}) - i\lambda^2 a_i(k_{ix} - k_{fx})]$$

and substituting it back into the original equation together with P_1 and P_2,the full expression for the S_{fi} will be:

$$\begin{aligned} S_{fi} &= \frac{(2\pi)^3}{L^2 V}\frac{e^2}{\sqrt{4\omega_i\omega_f}}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]^{\frac{1}{2}}\exp[-\frac{\lambda^2}{4}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)] \\ &\times \exp[-i\lambda^2 a_i(k_{ix} - k_{fx}) - i\frac{\lambda^2}{2}(k_{ix}k_{iy} + k_{fx}k_{fy})]X \\ &\times \delta(E_i + \omega_i - E_f - \omega_f)\delta(p_i + k_i\cos\theta_i - p_f - k_f\cos\theta_f) \\ &\times \delta(a_i - k_{iy} - a_f - k_{fy}) \end{aligned} \tag{4.1.54}$$

where ω_i and ω_f are frequencies of the incident and scattered photons ,E_i and E_f represent the energies of the incident and scattered electrons,respectively,$\theta_i(\theta_f)$

denotes the angle between the incoming(outgoing)photon.X is given by the following expression as follows:

$$\begin{aligned} X &= \sum_{n=0}^{\infty}\frac{1}{n!}[(\frac{\lambda^2 k_i^- k_f^+}{2})^n \exp(i\lambda^2 k_{iy}k_{fx})(\frac{X_1}{(E_i+\omega_i)^2-E_{i,n+1}^2}+\frac{X_2}{(E_i+\omega_i)^2-E_{i,n}^2}) \\ &+ (\frac{\lambda^2 k_i^- k_f^+}{2})^n \exp(i\lambda^2 k_{ix}k_{fy})(\frac{X_1'}{(E_i-\omega_f)^2-E_{f,n+1}^2}+\frac{X_2'}{(E_i-\omega_f)^2-E_{f,n}^2})] \end{aligned} \tag{4.1.55}$$

in which $k_i^{\pm}=k_{ix}\pm ik_{iy}, k_f^{\pm}=k_{fx}\pm ik_{fy}$,

$$E_{i,n}^2 = m^2+(p_i+\omega_i\cos\theta_i)^2+2neB \tag{4.1.56}$$

$$E_{f,n}^2 = m^2+(p_i-\omega_f\cos\theta_f)^2+2neB \tag{4.1.57}$$

and $X_i, X_i', i=1,2$,are given by

$$\begin{aligned} X_1 &= [\frac{(E_i+\omega_i+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)}+(E_i+\omega_i-m)]e_i^-e_f^+ \\ &+ (\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m}) \\ &\times [k_f^+e_{fz}e_i^-+k_i^-e_{iz}e_f^+-(p_i+\omega_i\cos\theta_i)e_f^+e_i^-] \end{aligned} \tag{4.1.58}$$

$$\begin{aligned} X_2 &= [\frac{(E_i+\omega_i+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)} \\ &+ (\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m}) \\ &\times (p_i+\omega_i\cos\theta_i)+(E_i+\omega_i-m)]e_{fz}e_{iz} \end{aligned} \tag{4.1.59}$$

$$\begin{aligned} X_1' &= [\frac{(E_i-\omega_f+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)}+(E_i-\omega_f-m)]e_i^+e_f^- \\ &+ -(\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m}) \\ &\times [k_i^+e_{iz}e_f^-+k_f^-e_{fz}e_+^++(p_i-\omega_f\cos\theta_f)e_i^+e_f^-] \end{aligned} \tag{4.1.60}$$

$$\begin{aligned} X_2' &= [\frac{(E_i-\omega_f+m)p_i(p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)}{(E_i+m)(E_f+m)}+(E_i-\omega_f-m) \\ &+ (\frac{p_i}{E_i+m}+\frac{p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f+m})(p_i-\omega_f\cos\theta_f)]e_{iz}e_{fz} \quad (4.1.61) \end{aligned}$$

where e_i and e_f are polarizations of the incident and scattered photons and $e_i^{\pm} = e_{ix} \pm ie_{iy}$,$e_f^{\pm} = e_{fx} \pm \pm ie_{fy}$.The conservation of energy and momentum along the z direction,i.e.

$E_f = E_i + \omega_i - \omega_f$ and $p_f + k_f\cos\theta_f = p_i + k_i\cos\theta_i$,leads to

$$\begin{aligned} \omega_f &= \frac{1}{\sin^2\theta_f}\{E_i-p_i\cos\theta_f+\omega_i(1-\cos\theta_i\cos\theta_f)-[(E_i-p_i\cos\theta_f)^2 \\ &+ 2\omega_i(E_i\cos\theta_f-p_i)(\cos\theta_f-\cos\theta_i)+\omega_i^2(\cos\theta_f-\cos\theta_i)^2]^{\frac{1}{2}}\} \quad (4.1.62) \end{aligned}$$

This replaces the well-known Compton formula without magnetic field where the electron motion is not restricted to 'one dimension'.

$$\begin{aligned} |S_{fi}|^2 &= \frac{(2\pi)^6}{L^4V^2}\frac{e^4}{4\omega_i\omega_f}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)] \\ &\times \exp[-i2\lambda^2a_i(k_{ix}-k_{fx})-i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]|X|^2 \\ &\times \delta^2(E_i+\omega_i-E_f-\omega_f)\delta^2(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f) \\ &\times \delta^2(a_i-k_{iy}-a_f-k_{fy}) \quad (4.1.63) \end{aligned}$$

In the expressions for X_1,X_1',X_2,and X_2'putting $p_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f = p_f$,we get the following.

$$X_1 = \{[A - B(p_i+\omega_i\cos\theta_i)e_f^+e_i^- + B(k_f^+e_{fz}e_i^- + k_i^-e_{iz}e_f^+)\}(E_f+m)^{-1}(E_i+m)^{-1},$$

$$X_2 = \{[A + B(p_i+\omega_i\cos\theta_i)e_{fz}e_{iz}\}(E_f+m)^{-1}(E_i+m)^{-1},$$

$$X_1^{'} = \{[A^{'} - B(p_i - \omega_f \cos\theta_f)]e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^-)\}(E_f + m)^{-1}(E_i + m)^{-1} and$$

$$X_2^{'} = \{[A^{'} + B(p_i - \omega_f \cos\theta_f)]e_{fz} e_{iz}\}(E_f + m)^{-1}(E_i + m)^{-1}$$

In the above two expressions the coefficients $A, A^{'}, B$ are defined by

$$A = (E_i + \omega_i + m)p_i p_f + (E_i + \omega_i - m)(E_i + m)(E_f + m) \tag{4.1.64}$$

$$A^{'} = (E_i - \omega_f + m)p_i p_f + (E_i - \omega_f - m)(E_i + m)(E_f + m) \tag{4.1.65}$$

$$B = p_i(E_f + m) + p_f(E_i + m) \tag{4.1.66}$$

respectively.

Substituting these results back into the expression for X:

$$\begin{aligned} X &= \sum_{n=0}^{\infty} \frac{1}{n!} \big[(\frac{\lambda^2 k_i^- k_f^+}{2})^n \exp(i\lambda^2 k_{iy} k_{fx}) \\ &\times \big(\frac{[A - B(p_i + \omega_i \cos\theta_i)]e_f^+ e_i^- + B(k_f^+ e_{fz} e_i^- + k_i^- e_{iz} e_f^+)(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i + \omega_i)^2 - E_{i,n+1}^2} \\ &+ \frac{[A + B(p_i + \omega_i \cos\theta_i)]e_{fz} e_{iz}(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i + \omega_i)^2 - E_{i,n}^2}\big) \\ &+ (\frac{\lambda^2 k_i^+ k_f^-}{2})^n \exp(i\lambda^2 k_{ix} k_{fy}) \\ &\times \big(\frac{[A^{'} - B(p_i - \omega_f \cos\theta_f)]e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^-)(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i - \omega_f)^2 - E_{f,n+1}^2} \\ &+ \frac{[A^{'} + B(p_i - \omega_f \cos\theta_f)]e_{fz} e_{iz}(E_f + m)^{-1}(E_i + m)^{-1}}{(E_i - \omega_f)^2 - E_{f,n}^2}\big)\big] \end{aligned} \tag{4.1.67}$$

$$\begin{aligned}
X &= \sum_{n=0}^{\infty}\frac{1}{n!}\Big[\big(\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}e^{-i(\phi_i-\phi_f)}\big)^n e^{i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\cos\phi_f\sin\phi_i} \\
&\times \big(\frac{[A-B(p_i+\omega_i\cos\theta_i)]e_f^+e_i^- + B(k_f^+e_{fz}e_i^- + k_i^-e_{iz}e_f^+)(E_f+m)^{-1}(E_i+m)^{-1}}{(E_i+\omega_i)^2-E_{i,n+1}^2} \\
&+ \frac{[A+B(p_i+\omega_i\cos\theta_i)]e_{fz}e_{iz}(E_f+m)^{-1}(E_i+m)^{-1}}{(E_i+\omega_i)^2-E_{i,n}^2}\big) \\
&+ \big(\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}e^{i(\phi_i-\phi_f)}\big)^n e^{i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\sin\phi_f\cos\phi_i} \\
&\times \big(\frac{[A'-B(p_i-\omega_f\cos\theta_f)]e_f^-e_i^+ - B(k_f^-e_{fz}e_i^+ + k_i^+e_{iz}e_f^-)(E_f+m)^{-1}(E_i+m)^{-1}}{(E_i-\omega_f)^2-E_{f,n+1}^2} \\
&+ \frac{[A'+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}(E_f+m)^{-1}(E_i+m)^{-1}}{(E_i-\omega_f)^2-E_{f,n}^2}\big)\Big] \qquad (4.1.68)
\end{aligned}$$

Letting

$$\begin{aligned}
Y_1 &= \sum_{n=0}^{\infty}\frac{1}{n!}\big(\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}e^{-i(\phi_i-\phi_f)}\big)^n \exp(i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\cos\phi_f\sin\phi_i) \\
&\times \big[\frac{[A-B(p_i+\omega_i\cos\theta_i)]e_f^+e_i^- + B(k_f^+e_{fz}e_i^- + k_i^-e_{iz}e_f^+)}{(E_i+\omega_i)^2-E_{i,n+1}^2} \\
&+ \frac{[A+B(p_i+\omega_i\cos\theta_i)]e_{fz}e_{iz}}{(E_i+\omega_i)^2-E_{i,n}^2}\big]
\end{aligned}$$

$$\begin{aligned}
Y_2 &= \sum_{n=0}^{\infty}\frac{1}{n!}\big(\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}e^{i(\phi_i-\phi_f)}\big)^n \exp(i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\sin\phi_f\cos\phi_i) \\
&\times \big[\frac{[A'-B(p_i-\omega_f\cos\theta_f)]e_f^-e_i^+ - B(k_f^-e_{fz}e_i^+ + k_i^+e_{iz}e_f^-)}{(E_i-\omega_f)^2-E_{f,n+1}^2} \\
&+ \frac{[A'+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}}{(E_i-\omega_f)^2-E_{f,n}^2}\big]
\end{aligned}$$

$\Rightarrow Y = (Y_1 + Y_2) = X(E_f+m)(E_i+m)$

$\Rightarrow X = \frac{Y}{(E_f+m)(E_i+m)}$

Substituting $|X|^2 = \frac{|Y|^2}{(E_f+m)^2(E_i+m)^2}$ in $|S_{fi}|^2$:

$$\begin{aligned}
|S_{fi}|^2 &= \frac{(2\pi)^6}{L^4V^2}\frac{\exp(4)}{4\omega_i\omega_f}[\frac{(E_i+m)(E_f+m)}{4E_iE_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)] \\
&\times \exp[-i2\lambda^2a_i(k_{ix}-k_{fx})-i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})]\frac{|Y|^2}{(E_f+m)^2(E_i+m)^2} \\
&\times \delta^2(E_i+\omega_i-E_f-\omega_f)\delta^2(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f) \\
&\times \delta^2(a_i-k_{iy}-a_f-k_{fy}) \qquad (4.1.69)
\end{aligned}$$

Taking the final states of the scattered photon and electron to be $\frac{Vd^3\omega_f}{(2\pi)^3}and\frac{Vd^3p_f}{(2\pi)^3}$ and $\frac{Vd^3\omega_f}{(2\pi)^3}=\frac{V\omega_f^2d\Omega_f}{(2\pi)^3}$,and also using $e^4=16\pi^2m^2r_0^2$

$$\begin{aligned}
\frac{|S_{fi}|^2}{T} &= \frac{(2\pi)^6}{L^4V^2}\frac{16\pi^2m^2r_0^2}{4\omega_i\omega_f}[\frac{1}{4E_iE_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)] \\
&\times \exp[-i2\lambda^2a_i(k_{ix}-k_{fx})-i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})] \\
&\times \frac{|Y|^2}{(E_f+m)(E_i+m)}\frac{T}{2\pi}\frac{L}{2\pi}\frac{1}{T}\delta^2(a_i-k_{iy}-a_f-k_{fy}) \qquad (4.1.70)
\end{aligned}$$

where we have used

$\delta^2(E_i+\omega_i-E_f-\omega_f)=\frac{T}{2\pi}\delta(E_i+\omega_i-E_f-\omega_f)=\frac{T}{2\pi}$, because from the conservation of energy stated earlier $E_i+\omega_i-E_f-\omega_f=0$ and we know that $\delta(0)=1$ similarly $\delta^2(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f)=\frac{L}{2\pi}\delta(p_i+k_i\cos\theta_i-p_f-k_f\cos\theta_f)$=$\frac{L}{2\pi}$ because from conservation of momentum stated earlier the term in the bracket is 0 Dividing this result by the relative incident flux density, $\frac{(E_i-p_i\cos\theta_i)}{VE_i}$:

$$\begin{aligned}
\frac{|S_{fi}|^2}{T}\frac{VE_i}{(E_i-p_i\cos\theta_i)} &= \frac{(2\pi)^6}{L^3V^2}\frac{m^2r_0^2}{4\omega_i\omega_f}[\frac{1}{E_iE_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)] \\
&\times \exp[-i2\lambda^2a_i(k_{ix}-k_{fx})-i\lambda^2(k_{ix}k_{iy}+k_{fx}k_{fy})] \\
&\times \frac{|Y|^2}{(E_f+m)(E_i+m)}\frac{VE_i}{(E_i-p_i\cos\theta_i)} \\
&\times \delta^2(a_i-k_{iy}-a_f-k_{fy}) \qquad (4.1.71)
\end{aligned}$$

4.1.3 Cross Section in Laboratory Frame

From the scattering amplitude derived earlier, Summing over the final states of the scattered photons and electrons,$k_f, a_f, p_f$to obtain the rate of scattering probability which is then divided by the relative incident flux density,$\frac{(E_i - p_i \cos\theta_i)}{VE_i}$:

$$\begin{aligned} d\sigma &= \frac{(2\pi)^6}{V^2}\frac{m^2 r_0^2}{4\omega_i\omega_f}[\frac{1}{E_f}]\exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)] \\ &\times \exp[-i2\lambda^2 a_i(k_{ix} - k_{fx}) - i\lambda^2(k_{ix}k_{iy} + k_{fx}k_{fy})] \\ &\times \frac{|Y|^2}{(E_f + m)(E_i + m)}\frac{1}{(E_i - p_i\cos\theta_i)}\frac{V\omega_f^2 d\omega_f}{(2\pi)^3}\frac{Vd^3p_f}{(2\pi)^3} \\ &\times \delta^2(a_i - k_{iy} - a_f - k_{fy}) \end{aligned} \qquad (4.1.72)$$

$$\begin{aligned} \frac{d\sigma}{d\Omega_f} &= \frac{r_0^2}{4}\frac{\omega_f}{\omega_i}\frac{m^2}{(E_f + m)(E_i + m)(E_i - p_i\cos\theta_i} \\ &\times \exp[-\frac{\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)]|Y|^2 \\ &\times \{\frac{1}{E_f}\exp[-i2\lambda^2 a_i(k_{ix} - k_{fx}) - i\lambda^2(k_{ix}k_{iy} + k_{fx}k_{fy})] \\ &\times \delta^2(a_i - k_{iy} - a_f - k_{fy})d^3p_f\} \end{aligned} \qquad (4.1.73)$$

But,

$$\begin{aligned} \frac{1}{\left[E_f - (p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)\cos\theta_f\right]} &= \{\frac{1}{E_f}\exp[-i2\lambda^2 a_i(k_{ix} - k_{fx}) - i\lambda^2(k_{ix}k_{iy} + k_{fx}k_{fy})] \\ &\times \delta^2(a_i - k_{iy} - a_f - k_{fy})d^3p_f\} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{d\sigma}{d\Omega_f} &= \frac{r_0^2}{4}\frac{\omega_f}{\omega_i}\frac{m^2}{(E_i + m)(E_f + m)} \\ &\times \frac{\exp[\frac{-\lambda^2}{2}(\omega_i^2\sin^2\theta_i + \omega_f^2\sin^2\theta_f)]|Y|^2}{(E_i - p_i\cos\theta_i)[E_f - (p_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)\cos\theta_f]}, \end{aligned} \qquad (4.1.74)$$

in which r_0 is the classical electron radius

The above result is performed by letting:

$\vec{k_i} = \omega_i(\sin\theta_i \cos\phi_i, \sin\theta_i \sin\phi_i, \cos\theta_i)$, and $\vec{k_f} = \omega_f(\sin\theta_f \cos\phi_f, \sin\theta_f \sin\phi_f, \cos\theta_f)$

$$\Rightarrow (\frac{\lambda^2}{2} k_i^- k_f^+)^n = [\frac{\lambda^2}{2} \omega_i \omega_f \sin\theta_i \sin\theta_f e^{-i(\phi_i - \phi_f)}]^n$$

using $k_i^- k_f^+ = (k_{ix} - ik_{iy})(k_{fx} + ik_{fy})$

In the ERF, $p_i = 0$, (4.1.74) is reduced to Herold's expression,as expected.However,the expression(4.1.74) cannot be derived vice versa by relativistic transformations from the latter.Fore example, the photon frequencies in(4.1.74) are not all the Doppler ones as they would be according to relativistic transformation rules,and the terms in (4.1.74) multiplied by p_i like those in(4.1.64)-(4.1.66)cannot be recovered,since one obtains nothing from zero by relativistic transformations. To simplify calculations,we choose the coordinate system with $\phi_i = 0$.Denoting $\phi_f = \phi$,we can write

$$k_i = \omega_i(\sin\theta_i, 0, \cos\theta_i), k_f = \omega_f(\sin\theta_f \cos\phi_f, \sin\theta_f \sin\phi, \cos\phi) \tag{4.1.75}$$

Taking into account the fact that photons have only two transversal polarizations,the polarizations of incident and scattered photons can be chosen as

$$e_i^{(1)} = (-\cos\theta_i, 0, \sin\theta_i); e_i^{(2)} = (0, -1, 0) \tag{4.1.76}$$

$$e_f^{(1)} = (-\cos\theta_f \cos\phi, -\cos\theta_f \sin\phi, \sin\theta_f); e_f^{(2)} = (\sin\phi, -\cos\phi, 0). \tag{4.1.77}$$

The above choice is not unique but is convenient for calculations.Now we define reduced quantities

$$\Delta_i = \frac{\omega_i}{m}; \Delta_f = \frac{\omega_f}{m}; \Delta_0 = \frac{\omega_0}{m} \tag{4.1.78}$$

where$\omega_0 = \frac{eB}{m}$ is the cyclotron frequency.We denote the reduced Doppler frequencies by

$$\Delta_{ir} = \gamma(1-\beta\cos\theta_i)\Delta_i; \Delta_{fr} = \gamma(1-\beta\cos\theta_f)\Delta_f \tag{4.1.79}$$

summing over the polarizations of incident photons and summing over those of scattered photons,the total differential cross section will be:

$$\frac{d\sigma}{d\Omega_f} = \frac{1}{2}\sum_{e_i^1,e_f^1,e_i^2,e_f^2} \frac{r_0^2\omega_f}{4\omega_i}\frac{m^2}{(E_i+m)}\frac{e^{\frac{-\lambda^2}{2}(\omega_i^2\sin^2\theta_i+\omega_f^2\sin^2\theta_f)|Y|^2}}{(E_f+m)(E_i-P_i\cos\theta_i)[E_f-(P_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f)\cos\theta_f]} \tag{4.1.80}$$

where $Y = Y_1 + Y_2$

Note that: $\omega_f = \triangle_f m$, $\omega_i = \Delta_i m$ and $E_i + m = m(\frac{E_i}{m}) + 1 = m(\gamma+1)$ and in similar way $E_f + m = E_i + \omega_i - \omega_f + m = m(\frac{E_i}{m} + \frac{\omega_i}{m} - \frac{\omega_f}{m} + 1) = m(1+\gamma+\Delta_i-\Delta_f)$ and $\omega_i^2 = \Delta_i^2 m^2 = \lambda^2\omega_i^2 = \lambda^2\Delta_i^2 m^2 = \frac{B_c}{B}\Delta_i^2$ Similarly:

$\lambda^2\omega_f^2 = \lambda^2\Delta_f^2 m^2 = \frac{B_c}{B}\Delta_f^2$ and $E_i - P_i\cos\theta_i = E_i(1 - \frac{P_i}{E_i}\cos\theta_i = E_i(1-\beta\cos\theta_i) = m\gamma(1-\beta\cos\theta_i)$

$$E_f - (P_i + \omega_i\cos\theta_i - \omega_f\cos\theta_f)\cos\theta_f = E_f - P_i\cos\theta_f + \omega_i\cos\theta_i\cos\theta_f + \omega_f\cos\theta_f^2 \tag{4.1.81}$$

Using the usual trigonometric relation $\cos^2\theta_f = 1 - \sin^2\theta_f$and $E_f = E_i - \omega_i - \omega_f$

$$E_i - \omega_i - \omega_f - P_i\cos\theta_f + \omega_i\cos\theta_i\cos\theta_f + \omega_f\cos^2\theta_f \tag{4.1.82}$$

$$E_i + \omega_i(1-\cos\theta_i\cos\theta_f) - P_i\cos\theta_f + \omega_f(\cos^2\theta_f - 1) \tag{4.1.83}$$

$$E_i - P_i\cos\theta_f + \omega_i(1-\cos\theta_i\cos\theta_f) - \omega_f\sin^2\theta_f \tag{4.1.84}$$

This gives $= E_i(1-\beta\cos\theta_f) + m\Delta_i(1-\cos\theta_i\cos\theta_f) - m\Delta_f\sin^2\theta_f$

and this will reduce to:$m[\gamma(1-\beta\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]$

Using all this back in the expression $\frac{d\sigma}{d\Omega_f}$gives

$$\begin{aligned}\frac{d\sigma}{d\Omega_f} &= \frac{1}{2}\sum_{e_i^1,e_f^1,e_i^2,e_f^2}\frac{r_0^2}{4}\frac{m\Delta_f}{m^2\Delta_{ir}}\frac{m^2e^{\frac{-B_c}{2B}(\Delta_f^2\sin^2\theta_f+\Delta_i^2\sin^2\theta_i)}|Y|^2}{m(\gamma+1)m(1+\gamma+\Delta_i-\Delta_f)m} \\ &\times \frac{1}{[\gamma(1-\beta\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]} \qquad (4.1.85)\end{aligned}$$

Where$\triangle_{ir}=\gamma(1-\beta\cos\theta_i)\Delta_i$ Now

$$\begin{aligned}\frac{d\sigma}{d\Omega_f} &= \frac{r_0^2}{8}\frac{\Delta_f}{\Delta_{ir}(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)}\times\frac{e^{\frac{-B_c}{2B}(\Delta_f^2\sin^2\theta_f+\Delta_i^2\sin^2\theta_i)}}{[\gamma(1-\beta\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]} \\ &\times \sum_{e_i^1,e_f^1,e_i^2,e_f^2}\frac{|Y|^2}{m^2} \qquad (4.1.86)\end{aligned}$$

Now let us find $\sum_{e_i^1,e_f^1,e_i^2,e_f^2}\frac{|Y|^2}{m^2}$ where$Y=Y_1+Y_2$ as was stated before

$$\frac{1}{m^2}|Y(1i\to 1f)|^2+\frac{1}{m^2}|Y(1i\to 2f)|^2+\frac{1}{m^2}|Y(2i\to 1f)|^2+\frac{1}{m^2}|Y(2i\to 2f)|^2$$

Where $\lambda_i\to\lambda_f,\lambda_{(i,f)}=1_{i(f)},2_{i(f)}$represents the scattering from the polarization λ_i to λ_f now let us find what $|Y(\lambda_i\to\lambda_f|$'s are

$$|Y(1i\to 1f)|=|Y_1(1i\to 1f)+Y_2(1i\to 1f)|$$

since $Y=Y_1+Y_2$ we get in Y_1:

$\phi_i=0,\phi_f=\phi$ and

$$\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2}=\frac{B_c}{2B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f=\zeta$$

and $e^{-i(\phi_i-\phi_f)} = e^{i\phi}$ and $e^{[i\lambda^2\omega_i\omega_f \sin\theta_i \sin\theta_f \cos\phi_f \sin\phi_i]} = e^0 = 1$

$$\frac{1}{(E_i+\omega_i)^2 - E_{i,n+1}^2} = \frac{1}{E_i^2+\omega_i^2+2E_i\omega_i - m^2 - P_i^2 - \omega_i^2\cos^2\theta_i - 2P_i\omega_i\cos\theta_i - 2(n+1)eB} \tag{4.1.87}$$

Using $E_{i,n+1}^2 = m^2 + (P_i + \omega_i\cos\theta_i)^2 + 2(n+1)eB$

$$\frac{1}{E_i^2 + 2E_i\omega_i - m^2 - P_i^2 - 2P_i\omega_i\cos\theta_i - 2(n+1)eB + \omega_i^2\sin^2\theta_i} \tag{4.1.88}$$

$$= \frac{1}{2E_i\omega_i - 2P_i\omega_i\cos\theta_i - 2neB + \omega_i^2\sin^2\theta_i} \tag{4.1.89}$$

$$= \frac{1}{2m\gamma m\Delta_i - 2m\gamma\beta m \triangle_i \cos\theta_i - 2nm^2 \triangle_0 + m^2\Delta_i^2\sin^2\theta_i} \tag{4.1.90}$$

$$= \frac{1}{m^2[2(\Delta_{ir} - n\Delta_0) + \Delta_i^2\sin^2\theta_i]} \tag{4.1.91}$$

$$= \frac{1}{m^2} S_{i,n+1} \tag{4.1.92}$$

Similarly in Y_2 we will get the following

$$\frac{1}{(E_i-\omega_f)^2 - E_{f,n+1}^2} = \frac{1}{m^2[2(\Delta_{fr} + n\Delta_0) - \Delta_f^2\sin^2\theta_f]} \tag{4.1.93}$$

$= \frac{1}{m^2} S_{f,n+1}$ and $\frac{\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f}{2} = \frac{B_c}{2B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f = \zeta$ and

$e^{i(\phi_i-\phi_f)} = e^{-i\phi}, e^{[i\lambda^2\omega_i\omega_f\sin\theta_i\sin\theta_f\sin\phi_f\cos\phi_i]} = e^{\frac{B_c}{B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f\sin\phi} = e^{i2\zeta\sin\phi} = e^{i\zeta}$

where $\eta = 2\zeta \sin\phi$ then $Y = Y_1 + Y_2$

$$\begin{aligned}
|Y| &= [A - B(P_i + \omega_i \cos\theta_i)e_f^+ e_i^- + B(K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+)] \sum_{n=0}^{\infty} \frac{1}{n!}\zeta^n \frac{1}{m^2} S_{i,n+1} e^{in\phi} \\
&+ [A + B(P_i + \omega_i \cos\theta_i)] e_{fz} e_{iz} \sum_{n=0}^{\infty} \frac{1}{n!}\zeta^n \frac{1}{m^2} S_{i,n} e^{in\phi} \\
&+ [(A' - B(P_i - \omega_f \cos\theta_f))e_f^- e_i^+ - B(K_f^- e_{fz} e_i^+ + K_i^+ e_{iz} e_f^+)] \sum_{n=0}^{\infty} \frac{1}{n!}\zeta^n \frac{1}{m^2} S_{f,n+1} e^{-in\phi+i\eta} \\
&+ [(A' + B(P_i - \omega_f \cos\theta_f))e_{fz} e_{iz}] \sum_{n=0}^{\infty} \frac{1}{n!}\zeta^n \frac{1}{m^2} S_{f,n} e^{-in\phi+i\eta} \qquad (4.1.94)
\end{aligned}$$

Then to obtain $|Y(1i \to 1f)|$ we will use the following polarization components:

$$e_i^{(1)} = (-\cos\theta_i, 0, \sin\theta_i)$$

and

$$e_f^{(1)} = (-\cos\theta_f \cos\phi, -\cos\theta_f \sin\phi, \sin\theta_f)$$

and using also

$$K_i = \omega_i(\sin\theta_i, 0, \cos\theta_i), K_f = \omega_f(\sin\theta_f \cos\phi, \sin\theta_f \sin\phi, \cos\phi)$$

to calculate $K_i^+$and K_f^- Now let us calculate the following:

$$e_f^- e_i^+ = e_{fx} e_{ix} + e_{fy} e_{iy} - i e_{fy} e_{ix} + i e_{fx} e_{iy} = \cos\theta_i \cos\theta_f \cos\phi - i \cos\theta_i \cos\theta_f \sin\phi$$

$$e_f^+ e_i^- = e_{fx} e_{ix} + e_{fy} e_{iy} + i e_{fy} e_{ix} - i e_{fx} e_{iy} = \cos\theta_i \cos\theta_f \cos\phi + i \cos\theta_i \cos\theta_f \sin\phi$$

$$\begin{aligned}
K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+ &= (K_{fx} e_{ix} + K_{fy} e_{iy} + i K_{fy} e_{ix} - i K_{fx} e_{iy}) e_{fz} \\
&+ (K_{ix} e_{fx} + K_{iy} e_{fy} + i K_{ix} e_{fy} - i K_{iy} e_{fx}) e_{iz} \quad (4.1.95)
\end{aligned}$$

$$\begin{aligned}
K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+ &= (-\omega_f \sin\theta_f \cos\phi \cos\theta_i - i\omega_f \sin\theta_f \sin\phi \cos\theta_i) \sin\theta_f \\
&+ (-\omega_i \sin\theta_i \cos\phi \cos\theta_f - i\omega_f \sin\theta_i \sin\phi \cos\theta_f) \sin\theta_i
\end{aligned}$$

$e_{fz}e_{iz} = \sin\theta_i \sin\theta_f$

$$\begin{aligned} K_f^- e_{fz} e_i^+ + K_i^+ e_{iz} e_f^- &= (K_{fx}e_{ix} + K_{fy}e_{iy} + iK_{fx}e_{iy} \\ &- iK_{fy}e_{ix})e_{fz} + (K_{ix}e_{fx} + K_{iy}e_{fy} + iK_{iy}e_{fx} - iK_{ix}e_{fy})e_{iz} \end{aligned}$$

$$\begin{aligned} K_f^- e_{fz} e_i^+ + K_i^+ e_{iz} e_f^- &= (-\omega_f \sin\theta_f \cos\phi \cos\theta_i + i\omega_f \sin\theta_f \sin\phi \cos\theta_i) \sin\theta_f \\ &+ (-\omega_i \sin\theta_i \cos\phi \cos\theta_f + i\omega_i \sin\theta_i \sin\phi \cos\theta_f) \sin\theta_i \end{aligned}$$

Using this we will calculate terms in square bracket then the first term will give us:

$$\begin{aligned} [A &- B(P_i + \omega_i \cos\theta_i)]e_f^+ e_i^- + B[(K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+)] \\ &= [((E_i + \omega_i + m)P_i P_f + (E_i + \omega_i - m)(E_i + m)(E_f + m))] \\ &- [(P_i(E_f + m) + P_f(E_i + m))(P_i + \omega_i \cos\theta_i)][\cos\theta_i \cos\theta_f \cos\phi + i \cos\theta_i \cos\theta_f \sin\phi] \\ &+ (P_i(E_f + m) + P_f(E_i + m))(-\omega_f \sin^2\theta_f \cos\phi \cos\theta_i - i\omega_f \sin^2\theta_f \sin\phi \cos\theta_i \\ &- \omega_i \sin^2\theta_i \cos\phi \cos\theta_f - i\omega_i \sin^2\theta_i \sin\phi \cos\theta_f) \end{aligned} \tag{4.1.96}$$

$$\begin{aligned} &= e^{i\phi} m^3 [[(\beta\gamma(1 + \gamma + \Delta_i)(\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) + (\gamma - 1 + \Delta_f)(1 + \gamma)(1 + \Delta_i - \Delta_f)) \\ &- (\beta\gamma(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f)(1 + \gamma))(\beta\gamma + \Delta_i \cos\theta_i)] \cos\theta_f \\ &- (\beta\gamma(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_i)(1 + \gamma)\Delta_f \sin^2\theta_f) \cos\theta_i \\ &- (\beta\gamma(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f)(1 + \gamma)\Delta_i \sin^2\theta_i) \cos\theta_f] \end{aligned} \tag{4.1.97}$$

Similarly for the second square bracket:

$$\begin{aligned}[(A+B(P_i+\omega_i\cos\theta_i))e_{fz}e_{iz})] &= ((E_i+\omega_i+m)P_iP_f+(E_i+\omega_i-m)(E_i+m)(E_f+m))] \\ &+ [(P_i(E_f+m)+P_f(E_i+m))(P_i+\omega_i\cos\theta_i)](\sin\theta_i\sin\theta_f\theta_i)]\end{aligned}$$

$=m^3[a+b(\beta\gamma+\Delta_i\cos\theta_i)]\sin\theta_i\sin\theta_f$ and for the third square bracket:

$$\begin{aligned}[(A' &- B(P_i-\omega_f\cos\theta_f))e_f^-e_i^+-B(K_f^-e_{fz}e_i^++K_i^+e_{iz}e_f^+)] \\ &= [[((E_i-\omega_f+m)P_iP_f+(E_i-\omega_f-m)(E_i+m)(E_f+m))] \\ &- (P_i(E_i+m)+P_f(E_i+m))(P_i-\omega_f\cos\theta_f)(\cos\theta_i\cos\theta_f\cos\phi-i\cos\theta_i\cos\theta_f\sin\phi) \\ &- (P_i(E_f+m)+P_f(E_i+m))(-\omega_f\sin^2\theta_f\cos\phi\cos\theta_i+i\omega_f\sin^2\theta_f\sin\phi\cos\theta_i \\ &- \omega_i\sin^2\theta_i\cos\phi\cos\theta_f+i\omega_i\sin^2\theta_i\sin\phi\cos\theta_f)] \qquad (4.1.98)\end{aligned}$$

$$\begin{aligned}&= e^{-i\phi}m^3[[(\beta\gamma(1+\gamma-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)+(\gamma-1-\Delta_f)(1+\gamma)(1+\Delta_i-\Delta_f)) \\ &- (\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma))(\beta\gamma-\Delta_f\sin\theta_f)]\Delta_f\sin^2\theta_f\cos\theta_i \\ &+ (\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_i)(1+\gamma)\Delta_i\sin^2\theta_i\cos\theta_f)] \qquad (4.1.99)\end{aligned}$$

and the fourth square bracket can be solved as:

$$\begin{aligned}[(A'+B(P_i-\omega_f\cos\theta_f))e_{fz}e_{iz}] &= [[(E_i-\omega_f+m)P_iP_f+(E_i-\omega_f-m)(E_i+m)(E_f+m)] \\ &+ (P_i(E_f+m)+P_f(E_i+m))(P_i-\omega_f\cos\theta_i)]\sin\theta_i\sin\theta_f\end{aligned}$$

$=m^3[a'+b(\beta\gamma-\Delta_f\cos\theta_f)]\sin\theta_i\sin\theta_f$

Thus substituting this back in to $|Y|$ gives $Y(1i\to 1f)$ thus:

$$\begin{aligned}Y(1i\to 1f) &= m^3\{[(A_-\cos\theta_f-B_1)\cos\theta_i-B_2\cos\theta_f]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{i,n+1}}{m^2}e^{i(n+1)\phi} \\ &- [(A'_-\cos\theta_f+B_1)\cos\theta_i+B_2\cos\theta_f]\sum_{n=0}^{\infty}\frac{1}{n!}\frac{S_{f,n+1}}{m^2}e^{-i[(n+1)\phi-\eta]} \\ &+ \sin\theta_i\sin\theta_f[A_+\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{i,n}}{m^2}e^{in\phi}-A'_+\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{f,n}}{m^2}e^{-i(n\phi-\eta)}]\}\end{aligned}$$

Where$A_\pm = a \pm b(\beta\gamma + \Delta\Delta_i \cos\theta_i, A'_\pm = a' \pm b(\beta\gamma - \Delta_f \cos\theta_f), b_1 = b\Delta_f \sin^2\theta_i$in which

$$a = \beta\gamma(1+\gamma+\Delta_i)(\beta\gamma+\Delta_i\cos\theta_i - \Delta_f\cos\theta_f) + (\gamma - 1 + \Delta_i)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)$$

$$a' = \beta\gamma(1+\gamma-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i - \Delta_f\cos\theta_f) + (\gamma - 1 + \Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)$$

$$b = \beta\gamma(1+\gamma+\Delta_i-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i - \Delta_f\cos\theta_f)(1+\gamma)$$

Again we will proceed in similar steps as above to obtain $Y_{(1i\to 2f)}, Y_{(2i\to 1f)}$ and $Y_{(2i\to 2f)}$ for $Y_{(1i\to 2f)}$ we will use the following $e_i^{(1)} = (-\cos\theta_i, 0, \sin\theta_i), e_f^{(2)} = (-\sin\phi, -\cos\phi, 0)$ and $K_i = \omega_i(\sin\theta_i, 0, \cos\theta_f, K_f) = \omega_f(\sin\theta_f\cos\phi, \sin\theta_f\sin\phi, \cos\phi)$ Then $e_f^+ e_i^- = e_{fx}e_{ix} + e_{fy}e_{iy} - ie_{fy}e_{ix} + ie_{fx}e_{iy} = -\sin\phi\cos\theta_i + i\cos\phi\cos\theta_i$ $e_f^- e_i^+ = e_{fx}e_{ix} + e_{fy}e_{iy} + ie_{fy}e_{ix} - ie_{fx}e_{iy} = -\sin\phi\cos\theta_i - i\cos\phi\cos\theta_i$

$$\begin{aligned} K_f^+ e_{fz} e_i^- + K_i^- e_{iz} e_f^+ &= (K_{fx}e_{ix} + K_{fy}e_{iy} + iK_{fy}e_{ix} \\ &- iK_{fx}e_{iy})e_{fy} + (K_{ix}e_{fx} + K_{iy}e_{fy} + iK_{ix}e_{fy} - iK_{iy}e_{fx})e_{iz} \end{aligned}$$

$$\begin{aligned} K_f^+ e_{fz} e_i^i + K_i^- e_{iz} e_f^+ &= (-\omega_i\sin\theta_i\cos\cos\theta_i - i\omega_f\sin\theta_f\sin\phi\cos\theta_i)(-\cos\phi) \\ &+ (\omega_i\sin\theta_i\sin\phi - i\omega_i\sin\theta_i\cos\phi)\sin\theta_i \end{aligned} \tag{4.1.100}$$

$e_{fz}e_{iz} = 0$

$$\begin{aligned} K_f^- e_{fz} e_i^+ + K_i^+ e_{iz} e_f^- &= (K_{fx}e_{ix} + K_{fy}e_{iy} - iK_{fy}e_{ix} + iK_{fx}e_{iy})e_{fz} \\ &+ (K_{ix}e_{fx} + K_{iy}e_{fy} - iK_{ix}e_{fy} + iK_{iy}e_{fx})e_{iz} \end{aligned}$$

$$K_f^- e_{fz} e_i^+ + K_i^+ e_{iz} e_f^- = (\omega_i\sin\theta_i\sin\phi + i\omega_i\sin\theta_i\cos\phi)\sin\theta_i$$

using this we will calculate terms in the square bracket $|Y|$

$$\begin{aligned} &= [(A - B(P_i + \omega_i\cos\theta_i))](-\sin\phi\cos\theta_i + i\cos\phi\cos\theta_i) \\ &+ B(\omega_i\sin\theta_i\cos\theta_i\cos\phi + i\omega_f\sin\theta_f\sin\phi\cos\theta_i\cos\phi + \omega_i\sin^2\theta_i\sin\phi - i\omega_i\sin^2\theta_i\cos\phi) \end{aligned}$$

$$Y_{(1_i\to 1_f)} = D_1S_{i,1}e^{i\phi} - D_2S_{f,1}e^{-i(\phi-\eta)} + \sin\theta_i\sin\theta_f[A_fS_{i,0} - A'_fS_{f,0}e^{i\eta}] \tag{4.1.101}$$

$$Y_{(1_i\to 2_f)} = i(A_-\cos\theta_i - B_2)S_{i,1}e^{i\phi} + i(A'_-\cos\theta_i + B_2)S_{f,1}e^{-i(\phi-\eta)} \tag{4.1.102}$$

$$Y_{(2_i\to 1_f)} = -i(A_-\cos\theta_f - B_1)S_{i,1}e^{i\phi} - i(A'_-\cos\theta_f + B_1)S_{f,1}e^{-i(\phi-\eta)} \tag{4.1.103}$$

$$Y_{(2_i\to 2_f)} = A_-S_{i,1}e^{i\phi} - A'_-S_{f,1}e^{-i(\phi-\eta)} \tag{4.1.104}$$

$$\begin{aligned}
&= \big[[(E_i+\omega_i+m) + (E_i+\omega_i-m)(E_i+m)(E_f+m)] \\
&- \big(p_i(E_f+m) + p_f(E_i+m)\big)(p_i+\omega_i\cos\theta_i)\big][-\sin\phi\cos\theta_i + i\cos\phi\cos\theta_i] \\
&+ \big[p_i(E_f+m) + p_f(E_i+m)\big][\omega_i\sin^2\theta_i\sin\phi - i\omega_i\sin^2\theta_i\cos\phi]
\end{aligned} \tag{4.1.105}$$

$$\begin{aligned}
&= ie^{i\phi}m^3\big[[\big(\beta\gamma(1+\gamma+\Delta_i)(\beta\gamma+\Delta_i\cos\theta_i - \Delta_f\cos\theta_f) \\
&+ (\gamma-1+\Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)\big) \\
&- \big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f) + (\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)\big)(\beta\gamma+\Delta_i\cos\theta_i)]\cos\theta_i \\
&- \big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f) + (\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)\big)\Delta_i\sin^2\theta_i\big]
\end{aligned} \tag{4.1.106}$$

$$\begin{aligned}
&= ie^{i\phi}m^3\big[[a - b(\beta\gamma+\Delta_i\cos\theta_i)]\cos\theta_i - b\Delta_i\sin^2\theta_i\big] \\
&= ie^{i\phi}m^3[A_-\cos\theta_i - B_2]
\end{aligned} \tag{4.1.107}$$

and the second square bracket,

$$\big[[A + B(p_i+\omega_i\cos\theta_i)]e_{fz}e_{iz}\big] = 0 \tag{4.1.108}$$

the third square bracket, $\big[[A' - B(p_i-\omega_f\cos\theta_f)]e_f^-e_i^+ - B(k_f^-e_{fz}e_i^+ + k_i^+e_{iz}e_f^-)\big]$

$$\begin{aligned}
&= \big[[(E_i-\omega_f+m)p_ip_f + (E_i-\omega_f-m)(E_f+m)(E_i+m)] \\
&- \big(p_i(E_f+m) + p_f(E_i+m)\big)(p_i-\omega_f\cos\theta_f)(-\sin\phi\cos\theta_i - i\cos\phi\cos\theta_i) \\
&- [(p_i(E_f+m) + p_f(E_i+m))][\omega_i\sin^2\theta_i\sin\phi + i\omega_i\sin^2\theta_i\cos\phi]\big]
\end{aligned} \tag{4.1.109}$$

$$
\begin{aligned}
&= ie^{-i\phi}m^3\big[[\beta\gamma(1+\gamma-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)+(\gamma-1-\Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f \\
&- \big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)\big)(\beta\gamma-\Delta_f\cos\theta_f)]\cos\theta_i \\
&+ \big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)\big)\Delta_i\sin^2\theta_i\big] \qquad (4.1.110
\end{aligned}
$$

$$
\begin{aligned}
&= ie^{-i\phi}m^3\big[[a^{'}-b(\beta\gamma-\Delta_f\cos\theta_f)]\cos\theta_i+b\Delta_i\sin^2\theta_i\big] \\
&= ie^{-i\phi}m^3[A^{'}_{-}\cos\theta_i+B_2] \qquad (4.1.111)
\end{aligned}
$$

finally,the fourth square bracket,

$$
[A^{'}+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}=0 \qquad (4.1.112)
$$

Then,substituting these results back:

$$
\begin{aligned}
Y_{(1_i\to 2_f)} &= ie^{i\phi}m^3[A_{-}\cos\theta_i-B_2]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{i,n+1}}{m^2}e^{in\phi} \\
&+ ie^{-i\phi}m^3[A^{'}_{-}\cos\theta_i+B_2]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{f,n+1}}{m^2}e^{-in\phi+i\eta} \qquad (4.1.113)
\end{aligned}
$$

$$
\begin{aligned}
&= mi[A_{-}\cos\theta_i-B_2]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n S_{i,n+1}e^{i(n+1)\phi} \\
&+ mi[A^{'}_{-}\cos\theta_i+B_2]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n S_{f,n+1}e^{-i(n+1)\phi+i\eta} \qquad (4.1.114)
\end{aligned}
$$

To calculate $Y_{(2_i\to 2_f)}$,we use

$e_i^{(2)}=(0,-1,0), e_f^{(1)}=(-\cos\theta_f\cos\phi,-\cos\theta_f\sin\phi,\sin\theta_f)$ and

$k_i=(\omega_i\sin\theta_i,0,\omega_i\cos\theta_i), k_f=(\omega_f\sin\theta_f\cos\phi,\omega_f\sin\theta_f\sin\phi,\omega_i\cos\theta_f)$

$\Rightarrow e_f^{+}e_i^{-}=e_{fx}e_{ix}+e_{fy}e_{iy}+ie_{fy}e_{ix}-ie_{fx}e_{iy}=\cos\theta_f\sin\phi-i\cos\theta_f\cos\phi,$

$e_f^{-}e_i^{+}=e_{fx}e_{ix}+e_{fy}e_{iy}-ie_{fy}e_{ix}+ie_{fx}e_{iy}=\cos\theta_f\sin\phi+i\cos\theta_f\cos\phi$

$k_f^{+}e_{fz}e_i^{-}+k_i^{-}e_{iz}e_f^{+}=(k_{fx}+ik_{fy})(e_{ix}-ie_{iy}e_{fz})=(-ik_{fx}e_{iy}+k_{fy}e_{iy})e_{fz}$

$= (i\omega_f \sin\theta_f \cos\phi - \omega_f \sin\theta_f \sin\phi)\sin\theta_f$

$k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^- = (k_{fx} - ik_{fy})(e_{ix} + ie_{iy}e_{fz}) = (ik_{fx}e_{iy} + k_{fy}e_{iy})e_{fz}$

$= (-i\omega_f \sin\theta_f \cos\phi - \omega_f \sin\theta_f \sin\phi)\sin\theta_f$

and $e_{fz}e_{iz} = 0$

Therefore,the first square bracket,

$\bigl[[A - B(p_i + \omega_i \cos\theta_i)]e_f^+ e_i^- + B(k_f^+ e_{fz} e_i^- + k_-^- e_{iz} e_f^+)\bigr]$

$$\begin{aligned}
= \quad & \bigl[[(E_i + \omega_i + m)p_i p_f + (E_i + \omega_i - m)(E_i + m)(E_f + m)] - \bigl(p_i(E_f + m) + p_f(E_i + m)\bigr) \\
\times \quad & (p_i + \omega_i \cos\theta_i)(\cos\theta_f \sin\phi - i\cos\theta_f \cos\phi) \\
+ \quad & [p_i(E_f + m) + p_f(E_i + m)][i\omega_f \sin^2\theta_f \cos\phi - \omega_f \sin^2\theta_f \sin\phi]\bigr] \qquad (4.1.115)
\end{aligned}$$

$$\begin{aligned}
= \quad & -ie^{i\phi}m^3\bigl[[\bigl(\beta\gamma(1 + \gamma + \Delta_i)(\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) \\
+ \quad & (\gamma + \Delta_f)(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f)\bigr) - \bigl(\beta\gamma(1 + \gamma + \Delta_i - \Delta_f) \\
+ \quad & (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f)(1 + \gamma)\bigr)(\beta\gamma + \Delta_i \cos\theta_i)]\cos\theta_f \\
- \quad & \bigl(\beta\gamma(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) \\
\times \quad & (1 + \gamma)\bigr)\Delta_f \sin^2\theta_f\bigr] \qquad (4.1.116)
\end{aligned}$$

$$\begin{aligned}
= \quad & -ie^{i\phi}m^3\bigl[[a - b(\beta\gamma + \Delta_i \cos\theta_i)]\cos\theta_f - b\Delta_f \sin^2\theta_f\bigr] \\
= \quad & -ie^{i\phi}m^3[A_- \cos\theta_f - B_1] \qquad (4.1.117)
\end{aligned}$$

The second square bracket,

$$[A + B(p_i + \omega_i \cos\theta_i)]e_{fz}e_{iz} = 0 \qquad (4.1.118)$$

the third square bracket,

$\bigl[[A' - B(p_i - \omega_f \cos\theta_f)]e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^-)\bigr]$

$$\begin{aligned} &= \Big[[(E_i-\omega_f+m)p_ip_f+(E_i-\omega_f-m)(E_i+m)(E_f+m)]-\big(p_i(E_f+m)+p_f(E_i+m)\big) \\ &\times (p_i-\omega_f\cos\theta_f)(\cos\theta_f\sin\phi+i\cos\theta_f\cos\phi) \\ &- [p_i(E_f+m)+p_f(E_i+m)][-i\omega_f\sin^2\theta_f\cos\phi-\omega_f\sin^2\theta_f\sin\phi]\Big] \qquad (4.1.119) \end{aligned}$$

$$\begin{aligned} &= -ie^{i\phi}m^3\big[[\big(\beta\gamma(1+\gamma-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f) \\ &+ (\gamma-1+\Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)\big)-\big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f) \\ &+ (\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)\big)(\beta\gamma-\Delta_f\cos\theta_f)]\cos\theta_f \\ &+ \big(\beta\gamma(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f) \\ &\times (1+\gamma)\big)\Delta_f\sin^2\theta_f\big] \qquad (4.1.120) \end{aligned}$$

$$\begin{aligned} &= -ie^{-i\phi}m^3\big[[a^{'}-b(\beta\gamma-\Delta_f\cos\theta_f)]\cos\theta_f+b\Delta_f\sin^2\theta_f\big] \\ &= -ie^{-i\phi}m^3[A^{'}_{-}\cos\theta_f+B_1] \qquad (4.1.121) \end{aligned}$$

and the fourth square bracket,

$$[A^{'}+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}=0 \qquad (4.1.122)$$

Substituting back these four results:

$$\begin{aligned} Y_{(2_i\to 1_f)} &= -ie^{i\phi}m^3[A_{-}\cos\theta_f-B_1]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{i,n+1}}{m^2}e^{in\phi} \\ &+ -ie^{-i\phi}m^3[A^{'}_{-}\cos\theta_f+B_1]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{f,n+1}}{m^2}e^{-in\phi+i\eta} \qquad (4.1.123) \end{aligned}$$

$$\begin{aligned} &= -im[A_{-}\cos\theta_f-B_1]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^nS_{i,n+1}e^{i(n+1)\phi} \\ &+ -im[A^{'}_{-}\cos\theta_f+B_1]\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^nS_{f,n+1}e^{-i(n+1)\phi+i\eta} \qquad (4.1.124) \end{aligned}$$

Now,to calculate $Y(2_i \rightarrow 2_f)$,we need:

$e_i^{(2)} = (0, -1, 0), e_f^{(2)} = (\sin\phi, -\cos\phi, 0)$ and

$k_i = (\omega_i \sin\theta_i, 0, \omega_i \cos\theta_i), k_f = (\omega_f \sin\theta_f \cos\phi, \omega_f \sin\theta_f \sin\phi, \omega_f \cos\phi)$

$e_f^+ e_i^- = e_{fx}e_{ix} + e_{fy}e_{iy} + ie_{fy}e_{ix} - ie_{fx}e_{iy} = \cos\phi + i\sin\phi$

$e_f^- e_i^+ = e_{fx}e_{ix} + e_{fy}e_{iy} - ie_{fy}e_{ix} + ie_{fx}e_{iy} = \cos\phi - i\sin\phi$

$k_f^+ e_{fz} e_i^- + k_i^- e_{iz} e_f^+ = 0$,$k_f^- e_{fz} e_i^+ + k_i^+ e_{iz} e_f^- = 0$ and $e_{fz}e_{iz} = 0$

The first square bracket, $\big[[A - B(p_i + \omega_i \cos\theta_i)]e_f^+ e_i^- + B(k_f^+ e_{fz} e_i^- + k_-^- e_{iz} e_f^+)\big]$

$$\begin{aligned} = \;& \big[[(E_i + \omega_i + m)p_i p_f + (E_i + \omega_i - m)(E_i + m)(E_f + m)] - \big(p_i(E_f + m) + p_f(E_i + m)\big) \\ \times \;& (p_i + \omega_i \cos\theta_i)\big](\cos\phi + i\sin\phi) \end{aligned} \tag{4.1.125}$$

$$\begin{aligned} = \;& e^{i\phi} m^3 \big[[\big(\beta\gamma(1 + \gamma + \Delta_i)(\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) \\ + \;& (\gamma - 1 + \Delta_f)(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f)\big)\big) - \big(\beta\gamma(1 + \gamma + \Delta_i - \Delta_f) \\ - \;& \big(\beta\gamma(1 + \gamma)(1 + \gamma + \Delta_i - \Delta_f) + (\beta\gamma + \Delta_i \cos\theta_i - \Delta_f \cos\theta_f) \\ \times \;& (1 + \gamma)\big)(\beta\gamma + \Delta_i \cos\theta_i)\big)]\big] \end{aligned} \tag{4.1.126}$$

$$\begin{aligned} = \;& e^{i\phi} m^3 [a - b(\beta\gamma + \Delta_i \cos\theta_i)] \\ = \;& e^{i\phi} m^3 A_- \end{aligned} \tag{4.1.127}$$

The second square bracket,

$$[A + B(p_i + \omega_i \cos\theta_i)]e_{fz}e_{iz} = 0 \tag{4.1.128}$$

The third square bracket, $[A - B(p_i - \omega_f \cos\theta_f)]e_f^- e_i^+ - B(k_f^- e_{fz} e_i^+ + k_-^+ e_{iz} e_f^-)$

$$\begin{aligned} = \;& \big[[(E_i - \omega_f + m)p_i p_f + (E_i - \omega_f - m)(E_i + m)(E_f + m)] - \big(p_i(E_f + m) + p_f(E_i + m)\big) \\ \times \;& (p_i - \omega_f \cos\theta_f)\big](\cos\phi - i\sin\phi) \end{aligned} \tag{4.1.129}$$

$$\begin{aligned}
&= e^{-i\phi}m^3\big[\big(\beta\gamma(1+\gamma+\Delta_i)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)\\
&+ (\gamma-1+\Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)\big)\\
&- \big(\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)\\
&\times (1+\gamma)\big)\big(\beta\gamma-\Delta_f\cos\theta_f\big)\big] \qquad (4.1.130)
\end{aligned}$$

$$\begin{aligned}
&= -e^{-i\phi}m^3[a^{'}-b(\beta\gamma-\Delta_f\cos\theta_f)]\\
&= -e^{-i\phi}m^3A_-^{'} \qquad (4.1.131)
\end{aligned}$$

and the fourth square bracket,

$$[A^{'}+B(p_i-\omega_f\cos\theta_f)]e_{fz}e_{iz}=0 \qquad (4.1.132)$$

substituting back these four results:

$$\begin{aligned}
Y_{(2_i\to 2_f)} &= e^{i\phi}m^3A_-\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{i,n+1}}{m^2}e^{in\phi}\\
&- ie^{-i\phi}m^3A_-^{'}\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^n\frac{S_{f,n+1}}{m^2}e^{-in\phi+i\eta} \qquad (4.1.133)
\end{aligned}$$

$$\begin{aligned}
&= mA_-\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^nS_{i,n+1}e^{i(n+1)\phi}\\
&- mA_-^{'}\sum_{n=0}^{\infty}\frac{1}{n!}\zeta^nS_{f,n+1}e^{-i(n+1)\phi+i\eta} \qquad (4.1.134)
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma}{d\Omega_f} &= \frac{r_0^2}{8}\frac{\Delta_f}{\Delta_{ir}(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)}\\
&\times \frac{\exp[-\frac{B_c}{2B}(\Delta_f^2\sin^2\theta_f+\Delta_i^2\sin^2\theta_i)]\frac{|Y|^2}{m^2}}{[\gamma(1-\beta\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]} \qquad (4.1.135)
\end{aligned}$$

Where B is the magnetic field with the direction taken to be along the Z-axis,$\Delta_i = \frac{\omega_i}{m}$, $\Delta_f = \frac{\omega_f}{m}$ are the reduced incident and scattered photon energies(m is the electron rest energy),θ_i and θ_f are the incident and scattered photon angles with respect to B field,ϕ,the azimuth angel of the scattered photon(the azimuth angel of the incident photon is taken to be zero),γ,the electron energy,$\beta = \sqrt{1 - \frac{1}{\gamma^2}}$, r_0,the classical electron radius ,$B = \frac{m^2}{e} \approx 4.14 \times 10^9 T$ is the critical magnetic field ,and Δ_{ir}, Δ_{fr},Δ_o are defined respectively by$\Delta_{ir} = \gamma\Delta_f(1 - \beta\cos\theta_f)$,$\Delta_o = \frac{\omega_0}{m} = \frac{B}{B_c}$,where $\omega_0 = \frac{eB}{m}$ is the cyclotron energy.$|Y|^2$ in Eq.(4.1.86) is given by

$$|Y|^2 = |Y(1_i \rightarrow 1_f)|^2 + |Y(1_i \rightarrow 2_f)|^2 + |Y(2_i \rightarrow 1_f)|^2 + |Y(2_i \rightarrow 2_f)|^2 \qquad (4.1.136)$$

Where $\lambda_i \rightarrow \lambda_f$ represents the scattering of a photon from polarization λ_i to λ_f,and

$$\begin{aligned} |Y(1_i \rightarrow 1_f)| &= m[(A_- \cos\theta_f - B_1)\cos\theta_i e^{i\phi} - B_2 \cos\theta_f e^{-i\phi}] \sum_{n=0}^{\infty} \zeta^n S_{i,n+1} e^{in\phi} \\ &- m[(A'_- \cos\theta_f + B_1)\cos\theta_i e^{-i\phi} + B_2 \cos\theta_f e^{i\phi}] \sum_{n=0}^{\infty} \zeta^n S_{f,n+1} e^{in(\phi-\eta)} \\ &+ m \sin\theta_i \sin\theta_f [A_+ \sum_{n=0}^{\infty} \zeta^n S_{i,n} e^{in\phi} - A_+ \sum_{n=0}^{\infty} \zeta^n S_{f,n} e^{-in(\phi-\eta)}] (4.1.137) \end{aligned}$$

Where

$$A_\pm = a \pm b(\beta\gamma - \Delta_f \cos\theta_f) \qquad (4.1.138)$$

$$A'\pm = a'(\beta\gamma - \Delta_f \cos\theta_f) \qquad (4.1.139)$$

$$B_1 = b\Delta_f \sin^2_{\theta_f} \qquad (4.1.140)$$

$$B_2 = b\Delta_i \sin^2_{\theta_i} \qquad (4.1.141)$$

in which

$$\begin{aligned} a &= \beta\gamma(1+\gamma+\Delta_i)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f) \\ &+ (\gamma-1+\Delta_i)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f) \end{aligned} \tag{4.1.142}$$

$$\begin{aligned} a' &= \beta\gamma(1+\gamma-\Delta_f)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f) \\ &+ (\gamma-1+\Delta_f)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f) \end{aligned} \tag{4.1.143}$$

$$b=\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma) \tag{4.1.144}$$

and

$$\eta = \zeta\sin\phi, \tag{4.1.145}$$

$$\zeta = \frac{B_c}{2B}\Delta_i\Delta_f\sin\theta_i\sin\theta_f, \tag{4.1.146}$$

$$S_{i,n} = \frac{1}{n![2(\Delta_{ir}-n\Delta_0)+\Delta_i^2\sin^2\theta_i]}, \tag{4.1.147}$$

$$S_{f,n} = \frac{1}{n![2(\Delta_{fr}+n\Delta_0)-\Delta_f^2\sin^2\theta_f]}, \tag{4.1.148}$$

$$\begin{aligned} |Y(1_i\rightarrow 2_f)| &= mi(A_-\cos\theta_i-B_2)\sum_{n=0}^{\infty}\zeta^n S_{i,n+1}e^{i(n+1)\phi} \\ &+ mi(A'_-\cos\theta_i+B_2)\sum_{n=0}^{\infty}\zeta^n S_{f,n+1}e^{-i(n+1)\phi+i\eta} \end{aligned} \tag{4.1.149}$$

$$\begin{aligned} |Y(2_i\rightarrow 1_f)| &= -mi(A_-\cos\theta_f-B_1)\sum_{n=0}^{\infty}\zeta^n S_{i,n+1}e^{i(n+1)\phi} \\ &- mi(A'_-\cos\theta_f+B_1)\sum_{n=0}^{\infty}\zeta^n S_{f,n+1}e^{-i(n+1)\phi+i\eta} \end{aligned} \tag{4.1.150}$$

$$\begin{aligned} |Y(2_i \rightarrow 2_f)| &= A_- \sum_{n=0}^{\infty} \zeta^n S_{i,n+1} e^{i(n+1)\phi} \\ &- A'_- \sum_{n=0}^{\infty} \zeta^n S_{f,n+1} e^{-i(n+1)\phi+i\eta} \end{aligned} \tag{4.1.151}$$

In astrophysics one is interested in the inverse compton scattering of a low energy photon by a relativistic electron in a strong magnetic fields satisfying $\Delta_i \ll 1, \gamma \gg 1$. We consider first the condition$\Delta_i \ll 1$.In this case the summations in the above three equations converge rapidly ,so that keeping only the term $n = 0$is already a satisfactory approximation. To show this we first integrate over the azimuth angelϕ and retain only the leading terms, then the differential cross section (4.1.86) is reduced to

$$d\sigma = \sigma(\Delta_i,\, \theta_i, \gamma, \theta_f) \sin\theta_f d\theta_f \tag{4.1.152}$$

where

$$\begin{aligned} \sigma(\Delta_i, \theta_i, \gamma, \theta_f) &= \frac{\pi r_o^2}{4} \frac{\Delta_f}{\Delta_{ir}(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)} \\ &\times \frac{\exp(-\frac{B_c}{2B}\Delta_f^2 \sin^2\theta_f) Y_r}{[\gamma(1-\beta\cos\theta_f) + \Delta_i(1-\cos\theta_i\cos\theta_f) - \Delta_f \sin^2\theta_f]} \end{aligned} \tag{4.1.153}$$

and Y_r is given by

$$\begin{aligned} Y_r &= c_1 S_{i,1}^2 + c_2 S_{f,1}^2 \\ &+ [(A_+ S_{i,0} - A'_+ S_{f,0})^2 + 2(1 - J_0(\zeta)) A_+ A'_+ S_{i,0} S_{f,0}](\sin\theta_i \sin\theta_f)^2 \end{aligned} \tag{4.1.154}$$

in which

$$c_1 = (A_- \cos\theta_f - B_1)^2(1+\cos^2\theta_i) + (A_-\cos\theta_i - B_2)^2 + (B_2\cos\theta_f)^2 + A_-^2 \tag{4.1.155}$$

$$c_2 = (A'_- \cos\theta_f + B_1)^2(1+\cos^2\theta_i) + (A'_-\cos\theta_i + B_2)^2 + (B_2\cos\theta_f)^2 + A_-^{'2} \tag{4.1.156}$$

and $J_0(\zeta)$ is the zeroth order Bessel function. It is worth pointing out here that the approximation 4.1.154 can also be justified by considering the non relativistic limit in the ERF ($\gamma = 1 or \beta = 0$).Infact under the Thomson limit($\Delta_f \approx \Delta_i \ll 1$)it is easy to see that$J_0(\zeta) \approx 1$,$A_- = -A'_- \approx 4\Delta_i, B_1 = B_2 \approx 0$ and $A_+ = -A'_+ \approx 4\Delta_i$, then eq.(4.1.135) can be simplified to

$$\begin{aligned}
\frac{|Y|^2}{m^2} &= \{D_1^2 S_{i,1}^2 e^{2i\phi} + D_2^2 S_{f,1}^2 e^{-2i(\phi-\eta)} + \sin^2\theta_i \sin^2\theta_f [A_+ S_{i,0} - A'_+ S_{f,0} e^{i\eta}]^2 \\
&+ 2(D_1 S_{i,1} e^{i\phi} - D_2 S_{f,1} e^{-i(\phi-\eta)}) \sin\theta_i \sin\theta_f [A_+ S_{i,0} - A'_+ S_{f,0} e^{i\eta}] \\
&+ 2D_1 D_2 S_{i,1} e^{i\phi} S_{f,1}^2 e^{-i(\phi-\eta)}\} + |-(A_- \cos\theta_i - B_2)^2 S_{i,1}^2 e^{2i\phi} \\
&- (A'_- \cos\theta_i + B_2)^2 S_{f,1}^2 e^{-2i(\phi-\eta)} - 2(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) S_{i,1} S_{f,1} e^{i\phi} e^{i(\phi-\eta)}| \\
&+ |-(A_- \cos\theta_f - B_1)^2 S_{i,1}^2 e^{2i\phi} - (A'_- \cos\theta_f + B_1)^2 S_{f,1}^2 e^{-2i(\phi-\eta)} \\
&- 2(A_- \cos\theta_f - B_1)(A'_- \cos\theta_f + B_1) S_{i,1} S_{f,1} e^{i\phi} e^{-i(\phi-\eta)}| \\
&+ |A_-^2 S_{i,1}^2 e^{2i\phi} + A'^2_- S_{f,1}^2 e^{-2i(\phi-\eta)} - 2A_- A'_- S_{i,1} e^{i\phi} S_{f,1} e^{-i(\phi-\eta)} \qquad (4.1.157)
\end{aligned}$$

taking the magnitude of $|Y|^2$ leads to:

$$\begin{aligned}
\frac{|Y|^2}{m^2} &= [D_1^2 + (A_- \cos\theta_i - B_2)^2 + (A_- \cos\theta_f - B_1)^2 + A_-^2] S_{i,1}^2 e^{2i\phi} + [D_2^2 + (A_- \cos\theta_i + B_2)^2 \\
&+ (A'_- \cos\theta_f + B_1)^2 + A'^2_-] S_{f,1}^2 e^{-2i(\phi-\eta)} + \sin^2\theta_i \sin^2\theta_f [A_+ S_{i,0} - A'_+ S_{f,0} e^{i\eta}]^2 \\
&+ 2[(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)(A'_- \cos\theta_f + B_1) \\
&- D_1 D_2 - A_- A'_-] e^{i\phi} e^{-i(\phi-\eta)} S_{i,1} S_{f,1} \\
&+ 2(D_1 S_{i,1} e^{-i\phi} - D_2 S_{f,1} e^{-i(\phi-\eta)} \sin\theta_i \sin\theta_f [A_+ S_{i,0} - A'_+ S_{f,0} e^{i\eta}] \qquad (4.1.158)
\end{aligned}$$

$$\begin{aligned}
\frac{|Y|^2}{m^2} &= C_1 S_{i,1}^2 e^{-2i\phi} + C_2 S_{f,1}^2 e^{-2i(\phi-\eta)} \sin^2\theta_i \sin^2\theta_f [A_+^2 S_{i,0}^2 + A'^2_+ S_{f,0}^2 e^{2i\eta} \\
&- 2A_+ A'_+ S_{i,0} S_{f,0} e^{i\eta}] + 2[(A_- \cos\theta_i)(A'_- \cos\theta_f + B_2) + (A_- \cos\theta_f - B_1)(A'_- \cos\theta_f - B_1) \\
&- D_1 D_2 - A_- A'_-] e^{i\phi} e^{-i(\phi-\eta)} S_{i,1} S_{f,1} + 2(D_1 A_+ S_{i,1} S_{f,0} e^{i(\phi+\eta)} - D_2 A_+ S_{i,0} S_{f,1} e^{-i(\phi-\eta)}) \\
&+ D_1 A_+ S_{i,1} S_{i,0} e^{i\phi} + D_2 A'_+ S_{f,1} S_{f,0} e^{-i(\phi-\eta)}) e^{i\eta} \qquad (4.1.159)
\end{aligned}$$

Now integrating over ϕ the above equation gives the following:

$$\begin{aligned}
\frac{|Y|^2}{m^2} &= C_1 S_{i,1}^2 \int e^{2i\phi} d\phi + C_2 S_{f,1}^2 \int e^{-2i(\phi-\eta)} + \sin^2\theta_i \sin^2\theta_f [A_+^2 S_{i,0}^2 + A_+'^2 S_{f,0}^2 \int e^{2i\eta} d\phi \\
&- 2A_+ A_+' S_{i,0} S_{f,0} \int e^{i\eta} d\phi] + 2[(A_- \cos\theta_i - B_2)(A_-' \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1) \\
&\times (A_-' \cos\theta_f + B_1) - D_1 D_2 - A_- A_-'] S_{i,1} S_{f,1} \int e^{i\phi} e^{-i(\phi-\eta)} d\phi - 2(D_1 A_+' S_{i,1} S_{f,0} \int e^{i(\phi+\eta)} d\phi \\
&+ D_2 A_+ S_{i,0} S_{f,1} \int e^{-i(\phi-\eta)}) \sin\theta_i \sin\theta_f + 2(D_1 A_+ S_{i,1} S_{i,0} \int e^{i\phi} d\phi + D_2 A_+' S_{f,1} S_{f,0} \int e^{-i(\phi-\eta)}
\end{aligned}$$

Since$\int e^{2i\phi} d\phi = \int e^{-2i(\phi-\eta)} d\phi = \int e^{2i\eta} = 2\pi$ and $\int e^{i\eta} = 2\pi J_0(\xi)$=$2\pi \frac{1}{2\pi} \int_0^{2\pi} d\phi \cos\eta =$ $2\pi \frac{1}{2\pi} \int_0^{2\pi} d\phi \cos(-\zeta \sin\pi)$ $\int e^{i\phi} e^{-i(\phi-\eta)} d\phi = 2\pi J_2(-\xi)$ Let $Q = [(A_- \cos\theta_i - B_2)(A_-' \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)(A_-' \cos\theta_f + B_1) - D_1 D_2 - A_- A_-']$

$$\begin{aligned}
\frac{|Y|^2}{m^2} &= 2\pi C_1 S_{i,1}^2 + 2\pi C_2 S_{f,1}^2 + \sin^2\theta_i \sin^2\theta_f [2\pi A_+^2 S_{i,0}^2 - 2A_+ A_+' S_{i,0} S_{f,0} 2\pi J_0(\zeta) \\
&+ 2A_+ A_+' S_{i,0} S_{f,0} 2\pi - 2A_+ A_+' S_{i,0} S_{f,0} 2\pi] \\
&= 2[Q S_{i,1} S_{f,1} 2\pi J_2(\zeta)] - 2[D_1 A_+' S_{i,1} S_{f,0} 2\pi J_1(-\zeta) + D_2 A_+' S_{i,0} S_{f,1} 2\pi J_1(-\xi)] \sin\theta_i \sin\theta_f \\
&+ 2[D_1 A_+ S_{i,1} S_{i,0} 2\pi + D_2 A_+' S_{f,1} S_{f,0} 2\pi] \qquad (4.1.160)
\end{aligned}$$

$$\begin{aligned}
\frac{|Y|^2}{2\pi m^2} &= C_1 S_{i,1}^2 + C_2 S_{f,1}^2 + \sin^2\theta_i \sin^2\theta_f [(A_+ S_{i,0} - A_+' S_{f,0})^2 + 2A_+ A_+' S_{i,0} S_{f,0} (1 - J_0(\zeta))] \\
&+ 2Q S_{i,1} S_{f,1} J_2(\zeta) - 2[D_1 A_+' S_{i,1} S_{f,0} + D_2 A_+ S_{i,0} S_{f,1}] \sin\theta_i \sin\theta_f J_1(-\zeta) \\
&+ 2[D_1 A_+ S_{i,1} S_{i,0} + D_2 A_+' S_{f,1} S_{f,0}] \qquad (4.1.161)
\end{aligned}$$

$S_{i,1} S_{i,0}$ and $S_{f,1} S_{f,0}$ can be neglected since both are $S_{i,n} S_{f,n}$ terms

$$\begin{aligned}
&= C_1 S_{i,1}^2 + C_2 S_{f,1}^2 + [(A_+ S_{i,0} - A_+' S_{f,0})^2 + 2(1 - J_0(\zeta)) A_+ A_+' S_{i,0} S_{f,0}] (\sin\theta_i \sin\theta_f)^2 \\
&+ 2(Q) J_2(\zeta) S_{i,1} S_{f,1} - 2[D_1 A_+' S_{i,1} S_{f,0} + D_2 A_+ S_{i,0} S_{f,1}] \sin\theta_i \sin\theta_f J_1(-\zeta) \quad (4.1.162)
\end{aligned}$$

Now substituting back the value of Q we will get

$$\begin{aligned}
\frac{|Y|^2}{2\pi m^2} &= C_1 S_{i,1}^2 + C_2 S_{f,1}^2 + [(A_+ S_{i,0} - A'_+ S_{f,0})^2 + 2(1 - J_0(\xi)) A_+ A'_+ S_{i,0} S_{f,0}](\sin\theta_i \sin\theta_f)^2 \\
&+ 2(\zeta) J_2([(A_- \cos\theta_i - B_2)(A'_- \cos\theta_i + B_2) + (A_- \cos\theta_f - B_1)(A'_- \cos\theta_f + B_1) \\
&- D_1 D_2 - A_- A'_-]) S_{i,1} S_{f,1} - 2[D_1 A'_+ S_{i,1} S_{f,0} + D_2 A_+ S_{i,0} S_{f,1}] \sin\theta_i \sin\theta_f J_1(-\zeta)
\end{aligned}$$

$$\begin{aligned}
D_1 &= (A_- \cos\theta_f - B_1)\cos\theta_i - B_2 \cos\theta_f \\
&= 4\Delta_i \cos\theta_i \cos\theta_f
\end{aligned}$$

$$\begin{aligned}
C_1 &= D_1^2 + (A_- \cos\theta_i - B_2)^2 + (A_- \cos\theta_f - B_1)^2 + A_-^2 \\
&= D_1^2 + (4\Delta_i \cos\theta_i)^2 + (4\Delta_i \cos\theta_f)^2 + (4\Delta_i)^2 \\
&= (4\Delta_i \cos\theta_i \cos\theta_f)^2 + (4\Delta_i \cos\theta_i)^2 + (4\Delta_i \cos\theta_i)^2 + (4\Delta_i)^2
\end{aligned}$$

$$\begin{aligned}
D_2 &= (A_-^{'} \cos\theta_f + B_1)\cos\theta_i + B_2 \cos\theta_f \\
&= -4\Delta_i \cos\theta_i \cos\theta_f
\end{aligned}$$

$$\begin{aligned}
C_2 &= D_2^2 + (A_-^{'} \cos\theta_i + B_2)^2 + (A_-^{'} \cos\theta_f + B_1)^2 + A_-^{'2} \\
&= (-4\Delta_i \cos\theta_i \cos\theta_f)^2 + (-4\Delta_i \cos\theta_i)^2 + (-4\Delta_i \cos\theta_f)^2 + (-4\Delta_i)^2
\end{aligned}$$

$$\begin{aligned}
Y_r &= C_1S_{i,1}^2 + C_2S_{f,1}^2 + [(A_+S_{i,0} - A'_+S_{f,0})]^2(\sin\theta_i\sin\theta_f)^2 \\
&+ [2\big(1 - J_0(\zeta)\big)A_+A'_+S_{i,0}S_{f,0}](\sin\theta_i\sin\theta_f)^2 \\
&= C_1S_{i,1}^2 + C_2S_{f,1}^2 + [(A_+S_{i,0} - A'_+S_{f,0})]^2(\sin\theta_i\sin\theta_f)^2 \\
&= [(4\Delta_i\cos\theta_i\cos\theta_f)^2 + (4\Delta_i\cos\theta_i)^2 + (4\Delta_i\cos\theta_f)^2 + (4\Delta_i)^2](\frac{1}{2(\Delta_i - \Delta_0)})^2 \\
&+ [(-4\Delta_i\cos\theta_i\cos\theta_f)^2 + (-4\Delta_i\cos\theta_i)^2 + (-4\Delta_i\cos\theta_f)^2 + (-4\Delta_i)^2](\frac{1}{2(\Delta_i + \Delta_0)})^2 \\
&+ [(4\Delta_i)(\frac{1}{2\Delta_i}) - (-4\Delta_i)(\frac{1}{2\Delta_i})^2(\sin\theta_i\sin\theta_f)^2 \\
&= 4\Delta_i^2[\cos^2\theta_i\cos^2\theta_f + \cos^2\theta_i + \cos^2\theta_f + 1][\frac{1}{(\Delta_i - \Delta_0)^2} + \frac{1}{(\Delta_i + \Delta_0)^2}] \\
&+ 16\sin^2\theta_i\sin^2\theta_f \\
&= 4[\cos^2\theta_f(\cos^2\theta_i + 1) + 1(\cos^2\theta_i + 1)][\frac{\Delta_i^2}{(\Delta_i - \Delta_0)^2} + \frac{\Delta_i^2}{(\Delta_i + \Delta_0)^2}] + 16\sin^2\theta_i\sin^2\theta_f \\
&= 16\sin^2\theta_i\sin^2\theta_f + 4(1 + \cos^2\theta_i)(1 + \cos^2\theta_f)[\frac{\Delta_i^2}{(\Delta_i - \Delta_0)^2} + \frac{\Delta_i^2}{(\Delta_i + \Delta_0)^2}]
\end{aligned}$$

where $Y_r = \frac{Y^2}{2\pi m^2}$

$$\begin{aligned}
\sigma(\Delta_i, \theta_i, \theta_f) &= \frac{\pi r_0^2}{4}\frac{Y_r}{4} \\
&= \frac{\pi r_0^2}{16}\{16\sin^2\theta_i\sin^2\theta_f + 4(1 + \cos^2\theta_i)(1 + \cos^2\theta_f)[\frac{\Delta_i^2}{(\Delta_i - \Delta_0)^2} + \frac{\Delta_i^2}{(\Delta_i + \Delta_0)^2}]\}
\end{aligned}$$

$$\frac{\sigma(\Delta_i, \theta_i, \theta_f)}{\pi r_o^2} = \sin^2\theta_i\sin^2\theta_f + \frac{1}{4}(1 + \cos^2\theta_i)(1 + \cos^2\theta_f)[\frac{\Delta_i^2}{\Delta_i - \Delta_0} + \frac{\Delta_i^2}{\Delta_i + \Delta_0}] \tag{4.1.163}$$

which is just Herold's non relativistic result, a well known widely used expression in astrophysics especially in scattering problems for the production of hard X-ray. To end this, we have derived the magnetic compton scattering cross section and its simplified form in LF,which can be reduced to herold's result in the ERF but can not be recovered vice versa from Herold's expressions by relativistic transformations.This implies that this cross section may lead to some revisions to past astrophysical calculations based on Eq.4.1.163, for example, the number and power spectra of scattered photons resulting from the magnetic inverse compton scattering of a photon gas with a relativistic electron beam which are detactable for observers.

Chapter 5

Hard X-ray Production From Thermal Photons Through Inverse Compton Scattering

5.1 Calculation Of Spectrum Function Of Magnetic Inverse Compton Scattering.

In the following we use the simplified cross section derived in previous chapter,that is, eq.4.1.152, 4.1.153 and 4.1.154 to calculate the spectrum function of the the magnetic inverse compton scattering resulting fromthe scattering of a thermal photon gas in our case soft X-ray with a relativistic electron beam, which is supposed to happen at the surface of a neutron star. Consider the scattering of thermal photon(soft X-rays) by a monochromatic (γ) electron beam. The density of scattered photons per unit time is

$$\frac{dN(\gamma)}{dt} = n_e \int \sin\theta_i d\theta_i \int \sin\theta_f d\theta_f \int n(\Delta_i) d\Delta_i (1-\beta\cos\theta_i)\sigma(\Delta_i,\theta_i,\gamma,\theta_f) f(\cos\theta_i) \tag{5.1.1}$$

Where n_e is the density of the electron beam and $f(\cos\theta_i)$ an anisotropic factor for the incident photons. If the incident photons are isotropic, then $f(\cos\theta_i) = 1$. Taking in to account energy conservation and the condition $\Delta_i \ll 1$, it is easy to drive

$$\sin\theta_f d\theta_f = -\frac{\gamma(1-\beta\cos\theta_f) + \Delta_i(1-\cos\theta_i\cos\theta_f) - \Delta_f \sin^2\theta_f}{\Delta_f(\gamma-\Delta_f\cos\theta_f)} d\Delta_f \tag{5.1.2}$$

with substitution of this variable transformation in $Eq.(5.1.1)$, the spectrum of power density per unit scattered photon energy of a low frequency photon gas scattered by the monochromatic electron beam can be derived: Since

$$\begin{aligned} \frac{dN(\gamma,\Delta_f)}{dt} &= n_e \int \sin\theta_i d\theta_i \int \sin\theta_f d\theta_f \int n(\Delta_i) d\Delta_i (1-\beta\cos\theta_i) \\ &\times \sigma(\Delta_i,\theta_i,\gamma,\theta_f) f(\cos\theta_i) \end{aligned}$$

Now we will substitute $\sin\theta_f d\theta_f$given in equation Eq.(5.1.2) and $\sigma(\Delta_i,\theta_i,\gamma,\theta_f)$ shown in Eq.(4.1.153) in the above equation we will get the following:

$$\begin{aligned} \frac{dN(\gamma,\Delta_f)}{dt} &= n_e \int \sin\theta_i d\theta_i \int [\frac{\gamma(1-\beta\cos\theta_f + \Delta_i(1-\cos\theta_i\cos\theta_f) - \Delta_f\sin^2\theta_f)}{\Delta_f(\gamma-\Delta_f\cos\theta_f)}] \\ &\times d\Delta_f \int n(\Delta_i) d\Delta_i (1-\beta\cos\theta_i) \frac{\pi r_0^2}{4} \frac{\Delta_f}{\Delta_{ir}(\gamma+1)(1+\gamma+\Delta_i-\Delta_f)} \\ &\times \frac{e^{\frac{-B}{2B_c}\Delta_f^2\sin^2\theta_f} Y_r f(\cos\theta_i)}{[\gamma(1-\beta\cos\theta_f) + \Delta_i(1-\cos\theta_i\cos\theta_f) - \Delta_f^2\sin^2\theta_f]} \end{aligned} \tag{5.1.3}$$

Multiplying this equation with $\frac{\Delta_f}{d\Delta_f}$ using$\Delta_{ir} = \gamma\Delta_i(1-\beta\cos\theta_i)$ and

$$\gamma(\gamma+1)(1+\gamma+\Delta_i-\Delta_f) = \gamma^2(\gamma-\Delta_f)$$

for$\Delta_i \ll 1$ and $\gamma \gg 1$ We obtain

$$\begin{aligned}\frac{\Delta_f dN(\gamma,\Delta_f)}{dtd\Delta_f} &= 8\pi r_0^2 n_e \int n(\Delta_i)d\Delta_i \int \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i} \\ &\times \frac{Y_r e^{\frac{-B}{B_c}\Delta_f^2 \sin^2\theta_f} f(\cos\theta_i)}{32\gamma^2(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)} \end{aligned} \qquad (5.1.4)$$

$$\frac{\Delta_f dN(\gamma,\Delta_f)}{dtd\Delta_f} = 8\pi r_0^2 n_e \int n(\Delta_i)d\Delta_i F(\gamma,\Delta_i,\Delta_f)$$

$$\frac{\Delta_f dN(\gamma,\Delta_f)}{dtd\Delta_f} = 8\pi r_0^2 n_e \int n(\Delta_i)d\Delta_i F(\gamma,\Delta_i,\Delta_f), \qquad (5.1.5)$$

where

$$F(\gamma,\Delta_i,\Delta_f) = \int \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i} \frac{Y_r \exp[\frac{-B_c}{2B}\Delta_f^2 \sin^2\theta_f]}{32\gamma^2(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)} f(\cos\theta_f), \qquad (5.1.6)$$

is just the desired spectrum function. Now we take in to account the condition $\gamma \gg 1$, under this condition the constants $B_1, B_2, A_+, A'_+, A_-, A'_-$ will be reduced to their results given from Eq.(5.1.7)to Eq.(5.1.12)as follows;

$$\begin{aligned} B_1 &= b\Delta_f \sin^2_{\theta_f} \\ &= [\beta\gamma(1+\gamma+\Delta_i-\Delta_f) + (\beta\gamma+\Delta_i\cos\theta_i - \Delta_f\cos\theta_f)(1+\gamma)]\Delta_f\sin^2\theta_f \\ &= [\beta\gamma+\beta\gamma^2+\beta\gamma\Delta_i-\beta\gamma\Delta_f+\beta\gamma+\beta\gamma^2+\Delta_i\cos\theta_i+\beta\Delta_i\cos\theta_i-\Delta_f\cos\theta_f \\ &- \gamma\Delta_f\cos\theta_f]\Delta_f\sin^2\theta_f \\ &= \gamma[2\beta+2\beta\gamma-\Delta_f(\beta+\cos\theta_f)]\Delta_f(1-\cos\theta_f)(1+\cos\theta_f) \\ &= (2\gamma-\Delta_f(1+\cos\theta_f)]\gamma(1-\cos\theta_f)(1+\cos\theta_f) \\ &= [2\gamma-\Delta_f(1+\cos\theta_f)](1+\cos\theta_f)\Delta_{fr}. \end{aligned}$$

similarly

$$\begin{aligned}
B_2 &= b\Delta_i \sin^2_{\theta_i} \\
&= [\beta\gamma(1+\gamma+\Delta_i-\Delta_f)+(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)(1+\gamma)]\Delta_i\sin^2\theta_i \\
&= [2\gamma-\Delta_i(1+\cos\theta_i)]\Delta_i(1-\cos\theta_i)(1+\cos\theta_i) \\
&= [2\gamma-\Delta_i(1+\cos\theta_i)](1+\cos\theta_i)\Delta_i(1-\cos\theta_i) \\
&= [2\gamma-\Delta_i(1+\cos\theta_i)](1+\cos\theta_i)\Delta_{ir}
\end{aligned}$$

and

$$\begin{aligned}
A_+ &= a+b(\beta\gamma+\Delta_i\cos\theta_i) \\
&= \beta\gamma(1+\gamma+\Delta_i)(\beta\gamma+\Delta_i\cos\theta_i-\Delta_f\cos\theta_f)+(\gamma-1+\Delta_i)(1+\gamma)(1+\gamma+\Delta_i-\Delta_f) \\
&+ [2\gamma-\Delta_f(1+\cos\theta_f)]\gamma(\beta\gamma+\Delta_i\cos\theta_i) \\
&= \beta\gamma^2(\beta\gamma-\Delta_f\cos\theta_f)+\gamma^2(\gamma-\Delta_f)+[2\gamma-\Delta_f(1+\cos\theta_f)]\gamma(\beta\gamma+\Delta_i\cos\theta_i) \\
&= 2\gamma^3-\gamma^2\Delta_f(\cos\theta_f+1)+[2\gamma-\Delta_f(1+\cos\theta_f)]\gamma(\beta\gamma+\Delta_i\cos\theta_i) \\
&= \gamma^2(2\gamma-\Delta_f(\cos\theta_f+1))+[2\gamma-\Delta_f(1+\cos\theta_f)]\gamma^2 \\
&= [2\gamma-\Delta_f(1+\cos\theta_f)]2\gamma^2
\end{aligned}$$

similarly

$$\begin{aligned}
A'_+ &= a' + b(\beta\gamma - \Delta_f \cos\theta_f \\
&= \beta\gamma(1+\gamma-\Delta_f)(\beta\gamma + \Delta_i\cos\theta_i - \Delta_f\cos\theta_f) + (\gamma - 1 - \Delta_f)(1+\gamma)(1+\gamma\Delta_i - \Delta_f) \\
&+ \gamma[2\gamma - \Delta_f(1+\cos\theta_f)](\beta\gamma - \Delta_f\cos\theta_f) \\
&= \beta\gamma(\gamma-\Delta_f)(\beta\gamma - \Delta_f\cos\theta_f) + (\gamma-\Delta_f)\gamma(\gamma-\Delta_f) \\
&+ \gamma[2\gamma - \Delta_f(1+\cos\theta_f)](\beta\gamma - \Delta_f\cos\theta_f) \\
&= [\beta\gamma(\beta\gamma - \Delta_f\cos\theta_f) + \gamma(\gamma-\Delta_f)](\gamma-\Delta_f) \\
&+ \gamma[2\gamma - \Delta_f(1+\cos\theta_f)](\beta\gamma - \Delta_f\cos\theta_f) \\
&= [\beta\gamma^2 - \beta\gamma\Delta_f\cos\theta_f + \gamma^2 - \gamma\Delta_f](\gamma-\Delta_f) \\
&+ \gamma[2\gamma - \Delta_f(1+\cos\theta_f)](\beta\gamma - \Delta_f\cos\theta_f) \\
&= [2\gamma - \Delta_f(1+\cos\theta_f) - [2\gamma - \Delta_f(1+\cos\theta_f)]\gamma\Delta_f) \\
&+ \gamma[2\gamma - \Delta_f(1+\cos\theta_f)](\beta\gamma) - [2\gamma - \Delta_f(1+\cos\theta_f)](\Delta_f\cos\theta_f) \\
&= [2\gamma - \Delta_f(1+\cos\theta_f)]2\gamma^2 - [2\gamma - \Delta_f(1+\cos\theta_f)][\gamma\Delta_f\cos\theta_f + \gamma\Delta_f] \\
&= [2\gamma - \Delta_f(1+\cos\theta_f)][2\gamma^2 - \gamma\Delta_f(1+\cos\theta_f)] \\
A'_+ &= [2\gamma - \Delta_f(1+\cos\theta_f)][2\gamma^2 - \gamma\Delta_f\cos\theta_f)]
\end{aligned}$$

similarly,

$$\begin{aligned}
A_- &= a - b(\beta\gamma + \Delta_i \cos\theta_i) \\
&= \gamma^2[2\gamma - \Delta_f(1+\cos\theta_f)] - \gamma[2\gamma - \Delta_f \\
&\times\ (1+\cos\theta_f)](\beta\gamma + \Delta_i\cos\theta_i) \\
&= [2\gamma - \Delta_f(1+\cos\theta_f)](\gamma^2 - \gamma^2\beta - \gamma\Delta_i\cos\theta_i) \\
&= [2\gamma - \Delta_f(1+\cos\theta_f)][-\gamma\Delta_i\cos\theta_i] \\
A_- &= [2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{ir}
\end{aligned}$$

Similarly $A'_- = a' - b(\beta\gamma - \Delta_f\cos\theta_f)$

$$\begin{aligned}
&= [2\gamma - \Delta_f(1+\cos\theta_f)][\gamma^2 - \gamma\Delta_f] - [2\gamma - \Delta_f(1+\cos\theta_f)] \\
&\times\ \gamma(\beta\gamma^2 - \gamma\Delta_f\cos\theta_f) \\
&= -[2\gamma - \Delta_f(1+\cos\theta_f)](\gamma\Delta_f - \gamma^2 + \beta\gamma^2 - \gamma\Delta_f\cos\theta_f) \\
A'_- &= -[2\gamma - \Delta_f(1+\cos\theta_f)]\gamma\Delta_f(1-\cos\theta_f) \\
A'_- &= -[2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{fr}
\end{aligned}$$

$$A_- = [2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{ir}, \tag{5.1.7}$$

$$A^{'}_- = -[2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{fr}, \tag{5.1.8}$$

$$A_+ = [2\gamma - \Delta_f(1+\cos\theta_f)]2\gamma^2, \tag{5.1.9}$$

$$A^{'}_+ = [2\gamma - \Delta_f(1+\cos\theta_f)](2\gamma^2 - \Delta_{fr}), \tag{5.1.10}$$

$$B_1 = [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_f)\Delta_{fr}, \tag{5.1.11}$$

$$B_2 = [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_i)\Delta_{ir}, \tag{5.1.12}$$

and the spectrum function can be derived as follows:For $\gamma \gg 1$ We can calculate C_1 and C_2 as follows:

$$C_1 = (A_- \cos\theta_f - B_1)^2(1+\cos^2\theta_i) + (A_- \cos\theta_f - B_2)^2 + (B_2\cos\theta_f)^2 + A_-^2$$

$$\begin{aligned} C_1 &= ([2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{ir}\cos\theta_f - [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_f\Delta_{fr}))^2(1+\cos^2\theta_i) \\ &+ ([2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{ir}\cos\theta_i - [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_i\Delta_{ir}))^2 \\ &+ ([2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_i)\Delta_{ir}\cos\theta_f)^2 + ([2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{ir}))^2 \end{aligned}$$

$$\begin{aligned} C_1 &= (2\gamma - \Delta_f(1+\cos\theta_f))^2[(\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f))^2(1+\cos^2\theta_i) \\ &+ [\Delta_{ir}\cos\theta_i - \Delta_{ir}(1+\cos\theta_i)]^2 + [(1+\cos^2\theta_i)\Delta_{ir}\cos\theta_f]^2 + \Delta_{ir}^2] \end{aligned}$$

$$\begin{aligned} C_1 &= (2\gamma - \Delta_f(1+\cos\theta_f))^2[(\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f))^2(1+\cos^2\theta_i) \\ &+ \Delta_{ir}^2(1+\cos\theta_i)^2\cos^2\theta_f + \Delta_{ir}^2] \end{aligned}$$

$$\begin{aligned} \frac{C_1}{(2\gamma - \Delta_f(1+\cos\theta_f))^2} &= (\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f))^2(1+\cos^2\theta_i) \\ &+ \Delta_{ir}^2[2+(1+\cos\theta_i)^2\cos^2\theta_f] \end{aligned}$$

$$\begin{aligned} D_1 &= [\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f)]^2(1+\cos^2\theta_i) \\ &+ \Delta_{ir}^2[2+(1+\cos\theta_i)]^2\cos^2\theta_f \end{aligned}$$

$$C_2 = (A'_- \cos\theta_f + B_1)^2(1+\cos^2\theta_i) + (A'_- \cos\theta_i + B_2)^2 + (B_2\cos\theta_f)^2 + A_-'^2$$

$$\begin{aligned} C_2 &= (-[2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{fr}\cos\theta_f + [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_f)\Delta_{fr}))^2(1+\cos^2\theta_i) \\ &+ (-[2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{fr}\cos\theta_i + [2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_i\Delta_{ir}))^2 \\ &+ ([2\gamma - \Delta_f(1+\cos\theta_f)](1+\cos\theta_i)\Delta_{ir}\cos\theta_f)^2 + (-[2\gamma - \Delta_f(1+\cos\theta_f)]\Delta_{fr}))^2 \end{aligned}$$

$$\begin{aligned} \frac{C_2}{[2\gamma - \Delta_f(1+\cos\theta_f)]^2} &= \Delta_{fr}^2(1+\cos^2\theta_i) + [\Delta_{ir}(1+\cos\theta_i) - \Delta_{fr}\cos\theta_i]^2 \\ &+ \Delta_{ir}^2((1+\cos\theta_i)^2\cos^2\theta_f) + \Delta_{fr}^2 \end{aligned}$$

$$D_2 = [\Delta_{ir}(1+\cos\theta_i) - \Delta_{fr}\cos\theta_i]^2 + \Delta_{fr}^2(2+\cos^2\theta_i) + \Delta_{ir}^2(1+\cos\theta_i)^2\cos^2\theta_f$$

Now

$$F(\gamma, \Delta_i, \Delta_f) = \int \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i}\frac{Y_r}{32\gamma^2}\frac{e^{\frac{-B}{2B_c}\Delta_f^2\sin^2\theta_f}}{(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)}f(\cos\theta_i)$$

can be rewritten as

$$F(\gamma, \Delta_i, \Delta_f) = \int \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i}\frac{Y_r'}{32\gamma^2}e^{\frac{-B}{2B_c}\Delta_f^2\sin^2\theta_f}f(\cos\theta_i)$$

Where$Y_r' = \frac{Y_r}{(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)}$

$$\begin{aligned} Y_r' &= \frac{1}{(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)}[C_1S_{i,1}^2 + C_2S_{f,1}^2 + [(A_+S_{i,0} - A_+'S_{f,0})^2 \\ &+ 2(1-J_0(\xi))A_+A_+'S_{i,0}S_{f,0}](\sin\theta_i\sin\theta_f)^2] \end{aligned}$$

Now let us substitute the values of C_1, C_2 and $S_{i,n}, S_{f,n}$

$$C_1 = [2\gamma - \Delta_f(1+\cos\theta_f)]^2 D_1$$

$$C_2 = [2\gamma - \Delta_f(1+\cos\theta_f)]^2 D_2$$

$$S_{i,n} = \frac{1}{n![2(\Delta_{ir} - n\Delta_0) + \Delta_i^2\sin^2\theta_i]}$$

$$S_{f,n} = \frac{1}{n![2(\Delta_{fr} + n\Delta_0) - \Delta_f^2\sin^2\theta_f]}$$

With A_+ and A_+' given in the previous calculation

$$\begin{aligned} Y_r' &= \frac{1}{(\gamma-\Delta_f)(\gamma-\Delta_i\cos\theta_i)}[\frac{[2\gamma-\Delta_f(1+\cos\theta_f)]^2D_1}{[2(\Delta_{ir}-\Delta_0)+\Delta_i^2\sin^2\theta_i]^2} + \frac{[2\gamma-\Delta_f(1+\cos\theta_f)]^2D_2}{[2(\Delta_{fr}+\Delta_0)-\Delta_f^2\sin^2\theta_f]^2} \\ &+ [(\frac{[2\gamma-\Delta_f(1+\cos\theta_f)]^2 2\gamma^2}{2\Delta_{ir}+\Delta_i^2\sin^2\theta_i} - \frac{[2\gamma-\Delta_f(1+\cos\theta_f)](2\gamma^2-\Delta_{fr})}{2\Delta_{fr}-\Delta_{fr}^2\sin^2\theta_f})^2 \\ &+ \frac{2(1-J_0(\xi))[2\gamma-\Delta_f(1+\cos\theta_f)]2\gamma^2[2\gamma-\Delta_f(1+\cos\theta_f)](2\gamma^2-\Delta_{fr})}{(2\Delta_{ir}+\Delta_i^2\sin^2\theta_i)(2\Delta_{fr}-\Delta_f^2\sin^2\theta_f)}]\sin^2\theta_i\sin^2\theta_f] \end{aligned}$$

$$\begin{aligned}
Y_r' &= \frac{[2\gamma - \Delta_f(1+\cos\theta_f)]^2}{4(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)}[\frac{D_1}{(\Delta_{ir}-\Delta_0)^2+(\Delta_{ir}-\Delta_0)\Delta_i^2\sin^2\theta_i+4\Delta_i^4\sin^4\theta_i} \\
&+ \frac{D_2}{(\Delta_{fr}+\Delta_0-\frac{1}{2}\Delta_f^2\sin^2\theta_f)^2} \\
&+ (\frac{2\gamma^2}{\Delta_{ir}+\frac{1}{2}\Delta_i^2\sin^2\theta_i} - \frac{(2\gamma^2-\Delta_{fr})}{\Delta_{fr}-\frac{1}{2}\Delta_f^2\sin^2\theta_f})^2\sin^2\theta_i\sin^2\theta_f \\
&+ \frac{4(1-J_0(\xi))\gamma^2(2\gamma^2-\Delta_{fr})\sin^2\theta_i\sin^2\theta_f}{(\Delta_{fr}-\frac{1}{2}\Delta_f^2\sin^2\theta_f)}]
\end{aligned}$$

Up on using the approximation

$$\frac{[2\gamma - \Delta_f(1+\cos\theta_f)]^2}{4(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)} \simeq 1$$

and$\Delta_i \ll 1$ This will reduce to:

$$\begin{aligned}
Y_r' &= \frac{D_1}{(\Delta_{ir}-\Delta_0)^2+\Gamma_0^2} + \frac{D_2}{(\Delta_{fr}+\Delta_0-0.5\Delta_f^2\sin^2\theta_f)^2} + (\frac{2\gamma^2}{\Delta_{ir}} - \frac{(2\gamma^2-\Delta_{fr})}{\Delta_{fr}-0.5\Delta_f^2\sin^2\theta_f})^2 \\
&\times \sin^2\theta_i\sin^2\theta_f + \frac{4(1-J_0(\xi))\gamma^2(2\gamma^2-\Delta_{fr})\sin^2\theta_i\sin^2\theta_f}{\Delta_{fr}(\Delta_{fr}-0.5\Delta_f^2\sin^2\theta_f)}
\end{aligned}$$

Where $\Gamma_0^2 = (\Delta_{ir}-\Delta_0)\Delta_i^2\sin^2\theta_i + 4\Delta_i^4\sin^4\theta_i$

$$F(\gamma,\Delta_i,\Delta_f) = \int \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i}\frac{Y_r'}{32\gamma^2}\exp[\frac{-B_c}{2B}\Delta_f^2\sin^2\theta_f]f(\cos\theta_f), \qquad (5.1.13)$$

where

$$\begin{aligned}
Y_r' &= \frac{D_1}{(\Delta_{ir}-\Delta_o)^2+\Gamma_0^2} + \frac{D_2}{(\Delta_{fr}+\Delta_o-0.5\Delta_f^2\sin^2\theta_f)^2} \\
&+ [\frac{2\gamma^2}{\Delta_{ir}} - \frac{2\gamma^2-\Delta_{fr}}{\Delta_{fr}-0.5\Delta_f^2\sin^2\theta_f}]^2\sin^2\theta_i\sin^2\theta_f \\
&+ \frac{4[1-J_o(\zeta)^2]\gamma^2(2\gamma^2-\Delta_{fr})\sin^2\theta_i\sin^2\theta_f}{\Delta_{ir}(\Delta_{fr}-0.5\Delta_f^2\sin^2\theta_f)} \qquad (5.1.14)
\end{aligned}$$

in which

$$D_1 = [\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f)]^2(1+\cos^2\theta_i) + \Delta_{ir}^2[2+(1+\cos\theta_i)^2\cos^2\theta_f], \qquad (5.1.15)$$

$$D_2 = [\Delta_{ir}(1+\cos\theta_i) - \Delta_{fr}\cos\theta_i]^2 + \Delta_{fr}^2(2+\cos^2\theta_i) + \Delta_{ir}^2(1+\cos\theta_i)^2\cos^2\theta_f \quad (5.1.16)$$

and Γ_0 is related to the inverse life time of an electron intermediate states which is usually estimated according to the transition rate of an electron from the first landau level to the ground state Daughery and Vantura 1978 as cited in Yi Xu et al.2001;that is $\Gamma_0 = \frac{2}{3}\alpha(\frac{B}{B_c})^2$ with α the fine structure constant. In obtaining Eq.(5.1.13), use has been made of the following approximation.

$$\frac{[2\gamma - \Delta_f(1+\cos\theta_f)]^2}{4(\gamma-\Delta_f)(\gamma-\Delta_f\cos\theta_f)} \approx 1 \quad (5.1.17)$$

To see this we note that Δ_f reaches maximum only at $\theta_f = 0$ and $\theta_i = \pi, so$ $\Delta_f \ll \gamma$ if θ_f is not close to zero , then it is clear that Eq.(5.1.17) is also valid, thus the approximation is justified. It can be shown that $\Delta_f \sin\theta_f$ can also be neglected if there are other dominant terms. Using again the energy conservation, we get the following approximate expression, From conservation of energy we have

$$E_f = E_i + \omega_i - \omega_f$$

but

$$E_f - P_f\cos\theta_f = E_i(1 - \tfrac{P_i}{E_i\cos\theta_f}) + \omega_i(1-\cos\theta_i\cos\theta_f) - \omega_f\sin^2\theta_f$$

$$= m\gamma(1-\beta\cos\theta_f) + m\Delta_i(1-\cos\theta_i\cos\theta_f) - m\Delta_f\sin^2\theta_f$$

$$\frac{E_f}{m} - \frac{P_f}{m}\cos\theta_f = \gamma(1-\beta\cos\theta_f) + \Delta_i(1-\cos\theta_i\cos\theta_f) - \Delta_f\sin^2\theta_f$$

$$\frac{1}{m\gamma}(E_f - P_f\cos\theta_f) = 1 - \beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f) - \frac{\Delta_f}{\gamma}\sin^2\theta_f$$

Since $\Delta_f \ll \gamma$ Where we have used $\frac{E_f}{E_i} = \frac{E_i+\omega_i-\omega_f}{E_i} = 1 + \frac{m(\Delta_i-\Delta_f)}{m\gamma} \simeq 1$ and $\frac{P_f}{E_i} = \frac{P_i+\omega_i\cos\theta_i-\omega_f\cos\theta_f}{E_f} = \beta + \frac{m\Delta_i\cos\theta_i}{m\gamma} - \frac{m\Delta_f\cos\theta_f}{m\gamma} \simeq \beta$

$$1 - \beta\cos\theta_f = [1 - \beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f)]$$

But

$$1-\beta\cos\theta_f \simeq \frac{\Delta_i(1-\beta\cos\theta_i)(1-\beta\cos\theta_f)}{\Delta_f(1-\beta\cos\theta_f)}$$

$1-\beta\cos\theta_f = \frac{\Delta_i(1-\beta\cos\theta_i)}{\Delta_f}$ Substituting this back:

$$\frac{\Delta_i(1-\beta\cos\theta_i)}{\Delta_f} = [1-\beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f)]$$

$$\Delta_i(1-\beta\cos\theta_i) = [1-\beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f)]\Delta_f$$

$$[1-\beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f)]\Delta_f = (1-\beta\cos\theta_i)\Delta_i. \tag{5.1.18}$$

For further simplification, we consider the case where θ_f is not close to zero, then Eq.(5.1.18)can be simplified to

$$(1-\beta\cos\theta_f)\Delta_f = (1-\beta\cos\theta_i)\Delta_i \tag{5.1.19}$$

.

$$[1-\beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1-\cos\theta_i\cos\theta_f)]\Delta_f = (1-\beta\cos\theta_i)\Delta_i \tag{5.1.20}$$

$\Delta_f = \frac{(1+\beta)\Delta_i}{1-\beta\cos\theta_f+\frac{\Delta_i}{\gamma}(1+\cos\theta_f)}$ For maximum scattered photon frequency $\theta_i = \pi$ and$\theta_f = 0$

$$\Delta_f = \frac{\gamma^2(1+\beta)\Delta_i(1+\beta)}{\gamma^2(1-\beta^2)+\gamma\Delta_i 2(1+\beta)}$$

$$\omega_f = \frac{\gamma^2(1+\beta)^2\omega_i}{1+2(1+\beta)\gamma\Delta_i}$$

$$\omega_f = \frac{(1+\beta)^2\gamma^2\omega_i}{1+2(1+\beta)\gamma\Delta_i}$$

$$\simeq \frac{4\gamma^2\omega_i}{1+4\gamma\Delta_i}$$

Where$\Delta_i = \frac{\omega_i}{m}and\gamma = \frac{1}{\sqrt{1-\beta^2}}$

This means that the Doppler frequencies of the incident and scattered photons are equal $\Delta_{ir} = \Delta_{fr}$, which is just the Thomson limit in the LF.Equation(5.1.18) tells us also that the highest scattered photon energy is of magnitude $\frac{(1+\beta)^2\gamma^2\omega_i}{(1+2(1+\beta)\gamma\Delta_i)} \approx \frac{4\gamma^2\omega_i}{(1+4\gamma\Delta_i)}$,which becomes $4\gamma^2\omega_i$ for $\gamma\Delta_i \ll 1$,which is derived earlier. This is a well-known feature of inverse compton scattering.Applying the Thomson limit Eq.(5.1.19) and expanding $J_0(\zeta)$ up to ζ^2 ,Eq.(5.1.14) is reduced to

$$D_1 = [\Delta_{ir}\cos\theta_f - \Delta_{fr}(1+\cos\theta_f)]^2(1+\cos^2\theta_i) + \Delta_{ir}^2[2+(1+\cos\theta_i)^2\cos^2\theta_f]$$

Since$\Delta_{ir} \simeq \Delta_{fr}$

$$D_1 = \Delta_{ir}^2[(\cos\theta_f - 1 - \cos\theta_f)^2(1+\cos^2\theta_i) + 2 + (1+\cos\theta_i)^2\cos^2\theta_f]$$

$$D_1 = \Delta_{ir}^2[(1+\cos^2\theta_i)(1+\cos^2\theta_f) + 2(1+\cos\theta_i\cos^2\theta_f)]$$

$$D_2 = [\Delta_{ir}(1+\cos\theta_i) - \Delta_{fr}\cos\theta_i]^2 + \Delta_{fr}^2(2+\cos^2\theta_i) + \Delta_{ir}^2(1+\cos\theta_i)^2\cos^2\theta_f$$

$$= \Delta_{ir}^2[1+2+\cos^2\theta_i + (1+\cos^2\theta_i + 2\cos\theta_i)\cos^2\theta_f]$$

$$= \Delta_{ir}^2[(1+\cos^2\theta_i) + 2 + (1+\cos^2\theta_i)\cos^2\theta_f + 2\cos\theta_i\cos^2\theta_f]$$

$$D_2 = \Delta_{ir}^2[(1+\cos^2\theta_i)(1+\cos^2\theta_f) + 2(1+\cos\theta_i\cos^2\theta_f)]$$

Thus $D_1 = D_2 = C\Delta_{ir}^2$ where $C = (1+\cos^2\theta_i)(1+\cos^2\theta_f) + 2(1+\cos\theta_i\cos^2\theta_f)$ Now

$$\begin{aligned} Y_r' &= \frac{C\Delta_{ir}^2}{(\Delta_{ir}-\Delta_0)^2+\Gamma_0^2} + \frac{C\Delta_{ir}^2}{(\Delta_{ir}+\Delta_0)^2} + [\frac{2\gamma^2}{\Delta_{ir}} - \frac{(2\gamma^2-\Delta_{ir})}{\Delta_{ir}}]^2\sin^2\theta_i\sin^2\theta_f \\ &+ \frac{4(1-1+\frac{1}{4}\xi^2)\gamma^2(2\gamma^2-\Delta_{fr})\sin^2\theta_i\sin^2\theta_f}{\Delta_{ir}\Delta_{fr}} \end{aligned} \quad (5.1.21)$$

Where we have neglected $\Delta_f^2 \sin^2\theta_f$ Since the first two terms will not change through the calculation we only solve the last two terms:

$$[\frac{2\gamma^2\Delta_{ir} - 2\gamma^2\Delta_{ir} - \Delta_{ir}}{\Delta_{ir}}]^2 \sin^2\theta_i \sin^2\theta_f + \frac{2\xi^2\gamma^2\gamma^2\sin^2\theta_i\sin^2\theta_f}{\Delta_{ir}\Delta_{fr}}$$

Where $2\gamma^2 - \Delta_{fr} \simeq 2\gamma^2$ and since $\xi = \frac{B_c}{2B}\Delta_i\Delta_f \sin^2\theta_i \sin^2\theta_f$ then this will lead us to:

$$\sin^2\theta_i\sin^2\theta_f + \frac{2[\frac{B^2}{4B_c^2}\sin^4\theta_i\sin^4\theta_f\gamma^4]}{\gamma\Delta_i\gamma\Delta_f(1-\beta\cos\theta_i)(1-\beta\cos\theta_f)}$$

$$\sin^2\theta_i\sin^2\theta_f + 0.5(\frac{B_c}{B})^2\gamma\Delta_i\gamma\Delta_f(1-\cos\theta_i)(1-\cos\theta_f)(1+\cos\theta_i)^2(1+\cos\theta_f)^2$$

$$\sin^2\theta_i\sin^2\theta_f + 0.5(\frac{B_c}{B})^2[\gamma\Delta_i(1-\cos\theta_i)][\gamma\Delta_f(1-\cos\theta_f)](1+\cos\theta_i)^2(1+\cos\theta_f)^2$$

$$\sin^2\theta_i\sin^2\theta_f + 0.5(\frac{B_c}{B})^2(\Delta_{ir})(\Delta_{fr})(1+\cos\theta_i)^2(1+\cos\theta_f)^2$$

Then collecting terms left before to get the required formula

$$\begin{aligned} Y_r' &= \frac{C\Delta_{ir}^2}{(\Delta_{ir}-\Delta_0)^2+\Gamma_0^2} + \frac{C\Delta_{ir}^2}{(\Delta_{ir}+\Delta_0)^2} + \sin^2\theta_i\sin^2\theta_f \\ &+ 0.5[\Delta_{ir}(1+\cos\theta_i)(1+\cos\theta_f)(\frac{B_c}{B})]^2 \end{aligned}$$

$$\begin{aligned} Y_r' &= \frac{C\Delta_{ir}^2}{(\Delta_{ir}-\Delta_o)^2+\Gamma^2} + \frac{C\Delta_{ir}^2}{(\Delta_{ir}+\Delta_o)^2} \\ &+ (\sin\theta_i\sin\theta_f)^2 + 0.5[\Delta_{ir}(1+\cos\theta_i)(1+\cos\theta_f)\frac{B_c}{B}]^2, \end{aligned} \tag{5.1.22}$$

where the coefficient "C" is defined by

$$C = (1+\cos^2\theta_i)(1+\cos^2\theta_f) + 2(1+\cos\theta_i\cos^2\theta_f). \tag{5.1.23}$$

Now let us solve $C = (1+\cos^2\theta_i)(1+\cos^2\theta_f)+2(1+\cos\theta_i\cos^2\theta_f)$ Our aim is to show that$C(x) = (1+2x+x^2)(2-\frac{2x}{\omega_r}+\frac{x^2}{\omega_r^2})+2[1+(1-x)(1-\frac{x}{\omega_r}{}^2)]$ Let$x = (1-\cos\theta_i \simeq (1-\beta)\cos\theta_i)$ Then

$1+\cos^2\theta_f = 1+(1-\frac{x}{\omega_r})^2 = 1+1-\frac{2x}{\omega_r}+\frac{x^2}{\omega_r^2}\ 2-\frac{2x}{\omega_r}+\frac{x^2}{\omega_r^2}$

Since $\cos\theta_f = 1-\frac{x}{\omega_r}$ Because

$(1-\beta\cos\theta_f)\Delta_f = (1-\beta\cos\theta_i)\Delta_i)$

$(1-\beta\cos\theta_f)\frac{\omega_f}{\omega_i} = 1-\beta\cos\theta_i\ (1-\beta\cos\theta_f)\omega_r = x$

$1-\beta\cos\theta_f = \frac{x}{\omega_r} \simeq 1-\cos\theta_f = \frac{x}{\omega_r}\ \cos\theta_f = 1-\frac{x}{\omega_r}$

$1+\cos^2\theta_i = 1+(\cos\theta_i)^2$,but $x = 1-\cos\theta_i \Rightarrow \cos\theta_i = 1-x\ 1+(1-x)^2 = 1+1-2x+x^2 = 2-2x+x^2\ 2(1+\cos\theta_i\cos\theta_f^2) = 2\big(1+(1-x)(1-\frac{x}{\omega_r})\big)$ Again for $x = (1-\cos\theta_i) \approx (\beta-\cos\theta_i)$, with its lower limit being determined by Eq.(5.1.18)

$$\gamma\Delta_f(1-\beta\cos\theta_f) = \gamma(1-\beta\cos\theta_f)\Delta_i - \Delta_i(1-\cos\theta_i\cos\theta_f)$$

But

$$\Delta_f(1-\beta\cos\theta_f) = (1-\beta\cos\theta_i)\Delta_i(1-\frac{\Delta_f}{\gamma})$$

$$x_{min} = \frac{\Delta_f(1-\beta\cos\theta_f)}{\Delta_i(1-\frac{\Delta_f}{\gamma})}(\frac{(1+\beta\cos\theta_f)}{(1+\beta\cos\theta_f)})$$

Where we have used,$1-\beta\cos\theta_f \approx 2$ & $1-\beta\cos^2\theta_f \approx 1-\beta^2 = \frac{1}{\gamma^2}$

$$x_{min} = \frac{\Delta_f(1-\beta^2)}{2\Delta_i(1-\frac{\Delta_f}{\gamma})}$$

$$x_{min} = \frac{\Delta_f}{2\gamma^2\Delta_i(1-\frac{\Delta_f}{\gamma})}$$

$$x_{min} \approx \frac{\Delta_f}{2\gamma^2\Delta_i(1-\frac{\Delta_f}{\gamma})} \tag{5.1.24}$$

the spectrum function can be expressed as

$$F = F_1 + F_2 + F_3 \tag{5.1.25}$$

in which we have used

$$\gamma(1-\beta\cos\theta_f)\omega_f = \gamma x\omega_i$$

since $\Delta_{fr} = \gamma(1-\beta\cos\theta_f)\omega_f$ this implies

$$x^2 = \frac{\Delta_{fr}^2}{\gamma^2\omega_i^2} = \frac{\Delta_{ir}^2}{\gamma^2\omega_i^2}$$

We know that $\Delta_0 = \frac{\omega_0}{m} = \frac{B}{B_c}$,$\Delta_i = \frac{\omega_i}{m}$from this ratio we get

$$\frac{1}{\gamma}\frac{\Delta_0}{\Delta_i} = \frac{1}{\gamma}\frac{\omega_0}{\omega_i} = a_0$$

From this we can get $\Delta_0 = a_0\gamma\Delta_i$

$$F_1 = \frac{\omega_r}{32\gamma^2}\int_{x_{min}}^{2} dx[\frac{C\Delta_{ir}^2}{(\Delta_{ir}-\Delta_0)^2+\Gamma^2} + \frac{C\Delta_{ir}^2}{(\Delta_{ir}+\Delta_0)^2}]f(\cos\theta_i)$$

But $(1-\beta\cos\theta_f)\Delta_f = (1-\beta\cos\theta_i)\Delta_i$, $\gamma(1-\beta\cos\theta_f)\Delta_f = (1-\beta\cos\theta_i)\Delta_i$, $\Delta_{fr} = \gamma\Delta_i x$, $\Delta_{ir} = \gamma\Delta_i x$ similarly $\Delta_0 = \gamma\Delta_i a_0$ and $\Gamma^2 = \frac{\Gamma_0^2}{\Delta_i\gamma^2}$

$$F_1 = \frac{\omega_r}{32\gamma^2}\int_{x_{min}}^{2} dx[\frac{C(x)\gamma^2\Delta_i^2x^2}{(\gamma\Delta_i x-\gamma\Delta_i a_0)^2+\Gamma_0^2\gamma^2\Delta_{ir}^2} + \frac{C(x)\gamma^2\Delta_i^2x^2}{(\gamma\Delta_i x-\gamma\Delta_i a_0)^2}]f(1-x)$$

$$F_1 = \frac{\omega_r}{32\gamma^2}\int_{x_{min}}^{2} dx[\frac{\gamma^2\Delta_i^2C(x)x^2}{\gamma^2\Delta_i^2[(x-a_0)^2+\Gamma_0^2]} + \frac{\gamma^2\Delta_i^2C(x)x^2}{\gamma^2\Delta_i^2(x+a_0)^2}]f(1-x)$$

$$F_2 = \int_{x_{min}}^{2} \sin\theta_i\frac{\Delta_f}{\Delta_i}\frac{1}{32\gamma^2}[\sin\theta_i\sin\theta_f]^2 f(\cos\theta_i)$$

$\int_{x_{min}}^{2} dx\frac{\omega_r}{32\gamma^2}(1-\cos^2\theta_i)(1-\cos^2\theta_f)f(\cos\theta_i)$Here we can write

$$(1-\cos^2\theta_i = (1-(1-x)^2)$$

$$= 1-1+2x-x^2$$

$$(1-\cos^2\theta_f = (1-(1-\frac{x}{\omega_r})^2)$$

$$1-1+\frac{2x}{\omega_r}-\frac{x^2}{\omega_r^2}$$

Then the above expression will reduce to:

$$= \int_{x_{min}}^{2} dx\frac{\omega_r}{32\gamma_2}(2x-x^2)(\frac{2x}{\omega_r}-\frac{x^2}{\omega_r^2})f(1-x)$$

$$F_2 = \frac{1}{32\gamma^2}\int_{x_{min}}^{2} dx(2-x)(2-\frac{x}{\omega_r})x^2 f(1-x)$$

$$F_3 = \int_{x_{min}}^{2} \sin\theta_i d\theta_i \frac{\Delta_f}{\Delta_i} - \frac{1}{32\gamma_2}(\frac{1}{2}[\Delta_{ir}(1+\cos\theta_i)(1+\cos\theta_f)\frac{B_c}{B}]^2)$$

$\frac{\omega_r}{64}(\frac{B_c}{B})^2\int_{x_{min}}^{2} dx\frac{\Delta_{ir}^2}{\gamma^2}[(2-x)(2-\frac{x}{\omega_r})]^2 f(1-x)$But

$$\frac{\Delta_{ir}}{\gamma^2}=\frac{\gamma^2 x^2\Delta_i^2}{\gamma^2}$$

$$F_3=\frac{\omega_r\Delta_i^2}{64}(\frac{B_c}{B})^2\int_{x_{min}}^{2} dx[x(2-x)(2-\frac{x}{\omega_r})]^2 f(1-x)$$

$$F_1=\frac{\omega_r}{32\gamma^2}\int_{x_{min}}^{2} dx[\frac{x^2c(x)}{(x-a_o)^2+\Gamma_o^2}+\frac{x^2c(x)}{(x+a_o)^2}]f(1-x), \tag{5.1.26}$$

$$F_2=\frac{1}{32\gamma^2}\int_{x_{min}}^{2} dx(2-x)(2-\frac{x}{\omega_r})x^2 f(1-x), \tag{5.1.27}$$

$$F_3=\frac{\omega_r\Delta_i^2}{64}(\frac{B_c}{B})\int_{x_{min}}^{2} dx[x(2-x)](2-\frac{x}{\omega_r})]^2 f(1-x), \tag{5.1.28}$$

where,$\omega_r=\frac{\omega_f}{\omega_i}$,$a_o=\frac{\omega_o}{\gamma\omega_i}$,$\Gamma=2\alpha(B/B_c)^2/3\gamma^2\Delta_i$,and $c(x)$ can be read from Eq.(5.1.23),

$$c(x)=(2-2x+x^2)(2-\frac{2x}{\omega_r}+\frac{x^2}{\omega_r^2})+2[1+(1-x)(1-\frac{x}{\omega_r})^2]. \tag{5.1.29}$$

From a simple observation of the above spectrum functions F_2 is much smaller than F_1orF_3.So the dominant contribution to the spectrum function is from$F_1$and

F_3.Resonant scattering occurs when$x_{min} \leq a_0 \leq 2$is satisfied whereF_1 becomes very large.This means that resonant scattering becomes operative when the following conditions are satisfied:

$$\frac{\omega_0}{\gamma\omega_i} \leq 2$$

and

$$\frac{\omega_0}{\gamma\omega_i} \geq \frac{\omega_f}{2\gamma^2\omega_i(1 - \frac{\Delta_f}{\gamma})}$$

Which can be expressed alternatively by

$$\omega_i \geq \frac{\omega_0}{2\gamma}$$

and

$$\omega_f \leq \frac{2\gamma\omega_0}{1 + 2\Delta_0}$$

To this end,the existence of high temperature near the cap of neutron stars(pulsars),$T \sim 10^8 K$,there is a large number of soft X-ray photons resulting from thermal radiation.On the other hand,there are also high energy electron beams in the magneto sphere produced by electrostatic acceleration.According to the above two conditions,it is possible to produce hard X-ray emmision on the surface of a strongly magnetized neutron stars(pulsars) through the resonant magnetic compton scattering.

Chapter 6

Results And Discussion

When a matter falls from the Secondary to the primary star in binary system after filling its Roche lobe and passing through its lagrangian point,then the matter will form an accretion disk around the neutron star and during this time,there will be an inverse compton scattering due to the in falling matter which has soft X-ray photon when striking a relativistic electron in the accretion disk and will result in the production of hard X-ray with energy band range between 12 KeV and 50 MeV(J.Frank et al.1985)

In this work, we developed the method for getting hard X-ray from accreting binary neutron stars using inverse compton scattering and got the total differential cross section to be:

$$\begin{aligned}\sigma(\Delta_i, \theta_i, \gamma, \theta_f) &= \frac{\pi r_o^2}{4} \frac{\Delta_f}{\Delta_{ir}(1+\gamma)(1+\gamma+\Delta_i-\Delta_f)} \\ &\times \frac{\exp(-\frac{B_c}{2B}\Delta_f^2 \sin^2\theta_f)Y_r}{[\gamma(1-\beta\cos\theta_f)+\Delta_i(1-\cos\theta_i\cos\theta_f)-\Delta_f\sin^2\theta_f]} \qquad (6.0.1)\end{aligned}$$

Where Y_r is clearly derived in chapter 4. And this equation shows that the differential cross section depends on the incident angle,the electron energy γ. And this tells us that the the cross section become very high as the incoming photon frequency approaches a resonance frequency.

Using the above cross section it is possible to derive the magnetic inverse compton scattering and we get the density of scattered photon per unit time as:

$$\begin{aligned} \frac{dN(\gamma, \Delta_f)}{dt} &= n_e \int \sin\theta_i d\theta_i \int \sin\theta_f d\theta_f \int n(\Delta_i) d\Delta_i (1 - \beta\cos\theta_i) \\ &\times \sigma(\Delta_i, \theta_i, \gamma, \theta_f) f(\cos\theta_i) \end{aligned}$$

where n_e density of electron beam and$f(\cos\theta_i)$is an isotropic factor for the incident photons,if the incident photons are isotropic $f(\cos\theta_i) = 1$. In astrophysics we are more interested in the application of inverse compton scattering in the production of Hard X-ray from accreting neutron stars and for this to happen we made different assumptions like $\Delta_i \ll 1 \text{and} \gamma \gg 1$

So,for our work we derived earlier we got maximum scattered frequency which lies in the frequency range of hard X-ray for the scattered angle $\theta_f = 0$ and $\theta_i = \pi$.These angles are measured with respect to the Z axis,we considered also that the magnetic field is also along Z direction. We can see this relation from the following graph.The graph indicates that as the angle of scattered photon(θ_f) decreases, the scattered frequency (ω_f) increases.This spectrum lies in the hard X-ray range.

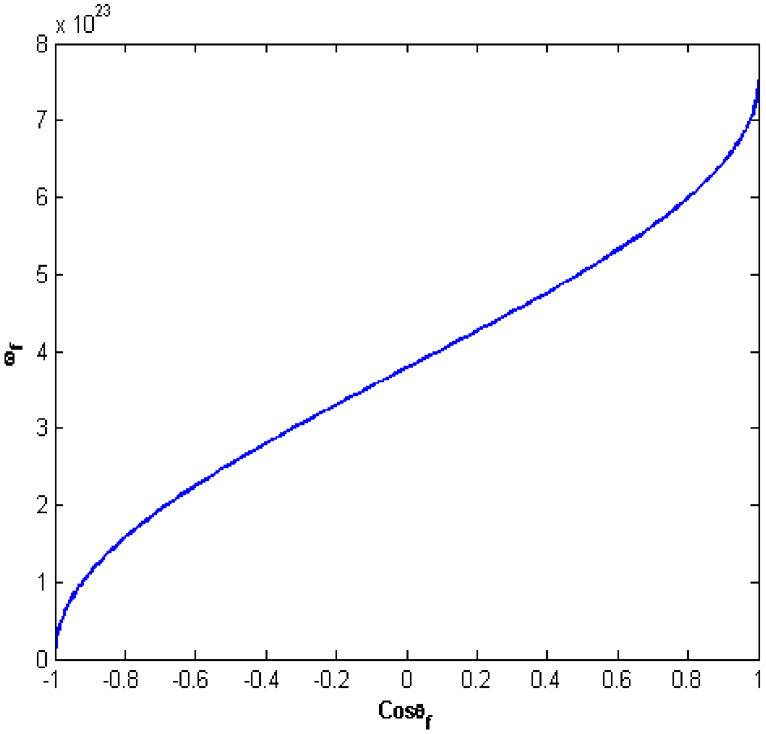

Figure 6.1: Relationship between the scattered photon frequency and the scattered photon angle

The above graph is drawn using equation 5.1.18 along with the maximum scattered frequency photon which is given by $\omega_f = 4\gamma^2\omega_i$ and its 'FORTRAN' program is attached at the end of the thesis.

$$[1 - \beta\cos\theta_f + \frac{\Delta_i}{\gamma}(1 - \cos\theta_i\cos\theta_f)]\Delta_f = (1 - \beta\cos\theta_i)\Delta_i \tag{6.0.2}$$

From conservation of energy used in chapter 5, we got the maximum scattered photon frequency to be of magnitude

$$\omega_f = 4\gamma^2\omega_i$$

assuming that $\gamma\Delta_i \ll 1$ taking the incident photon to be soft X-ray $(0.5 - 12\mathrm{KeV})$

and harden in to hard X-ray using inverse compton scattering.Now let us discuss the numerical analysis of the result obtained above considering the electron energy $\gamma = 100, 1000$ and other possible values greater than these. But for simplicity let us analyze for the first case$\gamma = 100$ Let us change the incident photon energy in to frequency using the following relation

$$E = \hbar\omega$$

$\hbar = 1.054592 \times 10^{-34} J.Sec$

$$500eV = 1.054592 \times 10^{-34} J.Sec\omega$$

$$\omega = 4.74117 \times 10^{36} \frac{eV}{J.Sec}$$

But

$$1eV = 1.6 \times 10^{-19} J$$

$$\Rightarrow \omega = 7.585872 \times 10^{17} \frac{1}{Sec.}$$

Using similar steps above $12K$eV will give as $\omega = 1.8206 \times 10^{19} \frac{1}{Sec.}$ Then the incident photon frequency range will be

$$\omega_i = 7.585872 \times 10^{17} \frac{1}{Sec.} - 1.8206 \times 10^{19} \frac{1}{Sec.}$$

As we derived earlier the maximum Photon frequency after hardening process using inverse magnetic compton scattering in strong magnetic field of neutron stars we got

$$\omega_f = 4\gamma^2 \omega_i$$

using this formula we can show that the scattered photon frequency range to be

$$\omega_f = 3.0343488 \times 10^{22} \frac{1}{Sec.} - 7.2824 \times 10^{23} \frac{1}{sec}$$

This photon frequency spectrum shows that the final frequency lies in the range of hard X-ray which is stated in chapter 2, which is similar to others work, J.Frank et al.(1985).

6.1 Conclusion

From the very beginning to the end of this work, we have stated and formulated that in active galactic nuclei the primary X-ray emission is due to inverse compton scattering by electrons in a hot corona of the UV/Soft X-ray disk photons as it is done in detail in chapters Four and Five.

For this work, we began by considering Dirac equation to derive the Feynman propagator which is very important in the derivation of scattering amplitude and the total differential cross section.using this cross section derived in chapter Four we have stated the spectrum function which is the core of our work and we calculated the frequency of the scattered photon (ω_f) using inverse compton scattering,which is crucial in the hardening of soft X-rays (thermal photons), showing the production of hard X-rays with energy range $12KeV - 50MeV$ taking in to account the electron energy γ.

Finally,we can conclude that accreting neutron stars,if they form binary system, can generate hard X-rays and for this process to happen strong magnetic field should be taken in to consideration.

Bibliography

[1] A.I Bogomazov et al., (2005),Astronomy Reports,, Vol.49,No.4:295-308.

[2] Andrew Cumming et al. (2004),APJ,603:L37-L40

[3] Bing Zhang and Alice K.Harding (2000), APJ,535:L51-L54.

[4] Charles D.Dermer et al. (2006), APJ,643:1081-10972(6).

[5] Claude Itzykson et al.(1980),Quantum Field Theory,McGraw-Hill internationa edition.

[6] Dong Lai (2006),Journal of Physics, *Conference series,* 31:68-75

[7] Fabio Muleri et al. (2006),*Gü*nther Hasinger,Proc of SPIE ,Vol.6266.6266x.

[8] F.Curtis Michel, (1991),Theory of Neutron Star Magnetosphere,*Univerisity of Chicago press.*

[9] Giacconi,R.,and Tananbaum,H.(1971), Astrophys.J.,Lett.,167,L67.

[10] Harding et al. (2001), APJ,547:929-948.

[11] Herold (1979),Physical Review D,Vol. 19,No.10.

[12] Igor Moskalenko(2006),APJ, 652:L65-L68.

[13] Ingo Kreykenbohm (2004), Dissertation,University Tubingen

[14] J.Frank et al.(1985),Accretion Power In Astrophysics,*Cambridge Universuty press*,$1st$.

[15] J.M.Lattimer and M.Prakash,(2004),Science,Vol.304:No.5670,PP.536-542.

[16] Jhuo Schultz, (2005),Astronomy and Astrphysics ,364:587-596.

[17] L.W Kebede (2002),Astrophysics and Space Science,282:131-140.

[18] Lars Bildsten(1997),APJ, *Supplement series.*, 113:367-408.

[19] M.Ruderman (1985),SLAC-PUB,3658 T.

[20] Malcolm S.longair (1997),High Energy Astrophysics,*Cambridge University press* Vol.2:2nd Edition.

[21] Manojendu Choudhury(2005),Bull.Astr.Sco.India,33:303-306.

[22] Mauro Orlandini et al. (2001), American Institute of physics:0-7354-0043:283.

[23] P.Ghosh and F.K.Lamb. (1979), APJ,234:296-316.

[24] paul C.Joss et al. (1984),Ann.Rev.Astron.Astrophys,22:537-592.

[25] R.X.Xu et al.(2000),APJ,535:354-364.

[26] Rob Fender (2001), arXiv:astro-ph/ 0101233:Vol.1.

[27] Robert Poham et al.(2000),arxiv:astro-ph,/0004017.

[28] Rudiger Staubert (2003),China J.Astrophys, Vol.3suppl,270-280.

[29] Silvia Zane et al. (1998), APJ,501:258-262.

[30] X.Y.Xia et al. (1985),Astron.Astrophys,152:93-100.

[31] YiXu et al(2001),Astron.Astrophys.,Vol.1,No.5,411-420.

Declaration

I hereby declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

Name:Getachew Mekonnen

Signature: ....

This thesis has been submitted for the examination with my approval as university advisor.

Name: Dr Legesse Wetro

Signature: ....

Addis Ababa University
Department of Physics
July, 2007