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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING



**A Study on EN 1998-1:2003 Equivalent Static
Torsional Provision for Vertically Regular
Multi-Story RC Buildings**

A Thesis in Structural Engineering

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A Thesis

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science

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ABSTRACT

Asymmetric building structures exist in modern construction to satisfy functional and architectural requirements. This asymmetry is the main reason for the failure of reinforced concrete buildings unless it is designed to accommodate the response of structures when subjected to different loadings.

In this paper one form of plan irregularity i.e. torsional irregularity condition is studied under the equivalent static torsional provision of EN 1998-1:2003. Three-story torsionally flexible reinforced concrete buildings subjected to seismic action are studied using both response spectrum and equivalent static method of analysis. Both methods are used through the analytical study of 30 random samples and analyses of 2 test models using ETABS. The response spectrum method of analysis is used as a reference method. Therefore the response of the structures computed using the equivalent static method shall be equal to or greater than the response computed using the response spectrum method. However, the result shows, the equivalent static method underestimates the response of structures when torsional irregularity exists. This means the response of the fundamental mode only becomes inadequate because higher modes turn into significant. In such situations, a modifying factor that magnifies the response of an equivalent static analysis method shall be used. Hence the application of the torsional provision shall be limited to torsionally regular buildings or the modification of the analysis method shall be used while the irregularity occurs.

Keywords: Analytical study, Torsionally flexible, Torsionally regular, Response spectrum analysis, Equivalent static analysis

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LIST OF SYMBOLS

RSA=Response spectrum analysis;

ESA=Equivalent static analysis;

CM=Center of mass;

CS=center of stiffness;

RC=Reinforced concrete;

T_c =Upper limit of the period of the constant spectral acceleration branch;

$S_d(T_l)$ =Ordinate of the design spectrum at period T_l ;

T_l =The fundamental period of vibration of the building for lateral motion in the direction considered;

m =Total mass of the building, above the foundation or the top of a rigid basement;

λ =Correction factor, the value of which is equal to: $\lambda = 0.85$ if $T_l \leq 2 * T_c$ and the building has more than two stories or $\lambda = 1.0$ otherwise;

z_i, z_j =Heights of the masses m_i, m_j above the level of application of the seismic action (foundation or top of a rigid basement);

e_{ai} =Accidental eccentricity of story mass i from its nominal location applied in the same direction at all floors;

M_{ai} =Torsional moment applied at story i about its vertical axis;

F_i =Horizontal force acting on story i ;

r_x =Square root of the ratio of the torsional stiffness to the lateral stiffness in the y-direction (“torsional radius”);

r_y =Square root of the ratio of the torsional stiffness to the lateral stiffness in the x-direction (“torsional radius”);

l_s =Radius of gyration of the floor mass in plan (square root of the ratio of the polar moment of inertia of the floor mass in plan with respect to the center of mass of the floor to the floor mass);

A_x =Amplification factor of accidental torsional moment;

δ_A =Maximum displacement of the flexible side computed assuming $A_x = 1$ at story i

δ_b = Maximum displacement of the stiff side computed assuming $A_x = 1$ at story i ;
 δ_{max} = Maximum displacement computed assuming $A_x = 1$ at story i ;
 δ_{avg} = Average of the displacements at the extreme points of the structure computed assuming $A_x = 1$ at story i ;
 Δ_{max} = Maximum horizontal displacement of any floor i in the direction of the lateral force at one end of the floor;
 Δ_{min} = Minimum horizontal displacement at the far end of the same floor i in that direction;
 δ_1 = Maximum displacement of the stiff;
 δ_2 = Maximum displacement of the flexible side;
 Δ_{max} = Maximum inter-story drift value;
 Δ_{avg} = Average inter-story drift value;
 $z_{\alpha/2}$ = Upper $100 \frac{\alpha}{2}$ percentage point of the standard normal distribution;
 μ = Mean value of the population;
 K_x = Story stiffness in the x-direction;
 K_y = Story stiffness in the y-direction;
 e_x = Eccentricity along x-direction;
 e_y = Eccentricity along y-direction;
 $\frac{L_y}{L_x}$ = Ratio of the plan dimensions;
 K_T = Torsional stiffness;
 $U_x(F_x)$ = Displacement in the x-direction caused by unit horizontal load in the x-direction;
 $U_y(F_y)$ = Displacement in the y-direction caused by unit horizontal load in the y-direction;
 $R(M_T)$ = Rotation about the vertical axis caused by unit moment about vertical axis;
 ϕ = Eigen vector or the mode shape;
 ω = Eigen value or the frequency at which the mode shape occurs;

V_{in}^{st} = The n^{th} mode static story shear at story i ;

s_{jyn} = The n^{th} mode contribution to the spatial distribution of effective earthquake forces at story j ;

V_{bn}^{st} = The n^{th} mode static base shear

$s_{j\theta n}$ = The n^{th} mode contribution to the spatial distribution of effective earthquake forces at story j ;

T_{in}^{st} = The n^{th} mode static story torque at story i ;

Γ_n = Modal participation factor;

M_n^* = Effective modal mass of the n^{th} mode;

T_{bn}^{st} = The n^{th} mode static base torque;

I_{On}^* = Effective modal mass moment of inertia of the n^{th} mode;

u_{jyn}^{st} = The n^{th} mode static lateral displacement at story j ;

ϕ_{jyn} = The translational component of the eigenvector or the mode shape;

$u_{j\theta n}^{st}$ = The n^{th} mode static rotation at story j ;

$\phi_{j\theta n}$ = The rotational component of the eigenvector or the mode shape;

Δ_{jyn}^{st} = The n^{th} mode static story drift at story j ;

$r(t)$ = Total response of the seismic action;

r_n^{st} = Modal static response;

$A_n(t)$ = Pseudo acceleration

CHAPTER 1 INTRODUCTION

1.1 GENERAL BACKGROUND

Earthquake is a catastrophic natural phenomenon due to its power of destruction. Unlike other natural catastrophes, earthquakes themselves do not kill people; rather the loss of human lives and properties occurs due to the collapse of man-made structures such as building structures. Seismic damage surveys conducted on mechanisms of failure of building structures during past severe earthquakes concluded that most vulnerable building structures are those, which are asymmetric. Hence, the seismic behavior of asymmetric building structures needs research and investigation.

Asymmetric building structures are almost unavoidable in modern construction due to various types of functional and architectural requirements. Buildings exhibit coupled torsional and translational response to lateral ground motion if their CM does not coincide with their CS calculated at floor levels even under purely translational ground shaking. During seismic shaking of the structural systems, inertia force acts through the center of mass while the resistive force acts through the center of rigidity. Due to this non-concurrency of lines of action of the inertia force and the resistive force, a twisting moment is generated which causes the torsional vibration of the structure in addition to the lateral vibration. If these modes produce displacements that are in phase, the element response is amplified considerably whereas, if out of phase, the response may be decreased. These effects are particularly critical for elements located near the periphery of the floor diaphragm. It is therefore important that current seismic building standards ensure adequate strength and ductility for structural components prone to severe damage or failure.

The collapse and severe damage of structures subjected to ground motion due to past earthquakes might demonstrate mistakes made in structural design and construction. Therefore, evaluating the adequacy of building code regulations is necessary. This is the principal motivation for the present study. Accordingly, this research paper tried to study the EN 1998-1:2003 equivalent static torsional provision for vertically regular

multi-story RC buildings with a fundamental period of less than or equal to 2 seconds and $4T_c$.

1.2 OBJECTIVE OF THE RESEARCH

1.2.1 General Objective

Studying the equivalent static torsional provision of EN 1998-1:2003 for vertically regular and plan wise rectangular multi-story RC buildings is the primary objective of the research.

1.2.2 Specific Objectives

- To study the effect of torsional irregularity on vertically regular buildings when analyzed using equivalent static method of analysis and response spectrum method of analysis.
- To evaluate the performance of the equivalent static method of analysis of EN 1998-1:2003 for vertically regular and torsionally irregular buildings.
- To propose a solution for those structures whose response is underestimated by the equivalent static method of analysis.

1.3 Statement of the Problem

Earthquake damages at the perimeter of buildings are often the result of excessive deformations caused by torsion during the earthquake. The collapse and severe damage of structures built under seismic provisions were evidence of the need for investigation of the effectiveness and adequacy of current building standards. According to EN 1998-1:2003, the lateral force method of analysis may be applied to buildings whose response is not significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction. And this property of buildings will be satisfied if the criteria for regularity in elevation are fulfilled and the fundamental periods of vibration in the two main directions are smaller than $4T_c$ and 2 seconds. However, this provision stating the significance of the fundamental modes guaranteed by the fulfillment of the two requirements solely shall be checked. In other words, is it ok to say higher modes are insignificant satisfying the two requirements? Hence, in this paper, the study

of EN 1998-1:2003 torsional provision for vertically regular multi-story RC buildings will be conducted using response spectrum analysis. A comparison is made on lateral displacement and floor rotation.

1.4 Scope of the Research

- The extent of this paper is limited to the study of three-story plan wise rectangular RC buildings with vertical regularity and torsional irregularity.
- This paper considers asymmetric stiffness distribution resulting in eccentric CS from the GC. However, CM is kept to coincide with the geometric center.
- Vertical and rotational component of earthquake is not considered.
- Column ends are assumed to be fixed at the supports i.e. foundation level.
- Soil-structure interaction is ignored.

1.5 Methodology

Phase 1

1. A review of relevant literature and building codes was covered.

Phase 2

1. Torsionally flexible random samples were selected.
2. Analytical model of the selected samples was conducted using MATLAB and Microsoft Excel.
3. Exemplary models out of the random samples were modeled using ETABS.
4. The modeled structures were analyzed, designed and verified until all members passed using equivalent static analysis and response spectrum analysis.
5. The response of the structures such as story drift, lateral displacement, and floor rotation were generated.

Phase 3

1. Discussion and interpretation of the results were performed.

1.6 Layout of Chapters

The following outline discusses the general content of the thesis.

Chapter 1: Introduction provides the general background, the necessity, limitation and methodology used for the study.

Chapter 2: Literature review presents review of the torsional provision of Eurocode 8: Design of Structures for Earthquake Resistance, American Society of Civil Engineers Standard and International Building Code, China Code for Seismic Design of Buildings and National Building Code of India 2016 Volume 1 to apply the two linear-elastic analysis methods. It also presents a review of previous researches carried out on the torsional provisions of different building codes on single-story and multi-story asymmetric buildings.

Chapter 3: Analytical Model is concerned with the structural configuration and geometry of the random models selected for the study. In addition, the response of these random samples analyzed using the two linear-elastic method of analysis is presented.

Chapter 4: Result and discussion covers the discussion and interpretation of the results of the two linear-elastic analysis methods obtained using MATLAB and Microsoft Excel.

Chapter 5: Structural Model demonstrates the results of chapter 4 using two test models.

Chapter 6: Conclusion and recommendation summarizes all the fundamental conclusions reached from the investigations conducted throughout the thesis. Topics for future research are proposed, which are considered to be necessary to cover a wider scope of multi-story torsionally flexible buildings.

CHAPTER 2 LITERATURE REVIEW

2.1 REVIEW OF TORSIONAL PROVISION OF CODES

Most seismic codes require different analysis methods for the design of buildings against earthquake forces. Under this chapter, the two linear-elastic analysis methods of EN 1998-1:2003, American Society of Civil Engineers Standard and International Building Code, China Code for Seismic Design of Buildings and National Building Code of India 2016 are reviewed.

2.1.1 Eurocode 8: Design of Structures for Earthquake Resistance

EN 1998-1:2003 has recommended four types of linear and non-linear methods of analysis depending on the structural characteristics of the building. These are:

1. The lateral force method of analysis;
2. The modal response spectrum analysis;
3. Non-linear static (pushover) analysis and;
4. Non-linear time history (dynamic) analysis,

Out of the above analysis methods, the first two methods are reviewed.

2.1.1.1 Lateral Force Method of Analysis

This type of analysis may be applied to buildings whose response is not significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction. This property of buildings will be satisfied if the criteria for regularity in elevation are fulfilled and the fundamental periods of vibration T_1 in the two main directions are smaller than 4 times T_c and 2 seconds.

In this method of analysis, the total base shear force is distributed linearly along the height. The seismic base shear force F_b , for each horizontal direction in which the building is analyzed, shall be determined using equation 2.1.

$$F_b = S_d(T_1) * m * \lambda \quad 2-1$$

Where

$S_d(T_1)$ The ordinate of the design spectrum at period T_1 ;

T_1 The fundamental period of vibration of the building for lateral motion in the direction considered;

m The total mass of the building, above the foundation or the top of a rigid basement;

λ The correction factor, the value of which is equal to: $\lambda = 0.85$ if $T_1 \leq 2 * T_C$ and the building has more than two stories or $\lambda = 1.0$ otherwise

Whereas the horizontal forces F_i should be calculated as:

$$F_i = F_b * \frac{z_i * m_i}{\sum z_j * m_j} \quad \mathbf{2-2}$$

Where

z_i, z_j The heights of the masses m_i, m_j above the level of application of the seismic action (foundation or top of a rigid basement)

For buildings with heights of up to 40 m the value of T_1 (in sec) may be approximated by the following expression:

$$T_1 = C_t * H^{\frac{3}{4}} \quad \mathbf{2-3}$$

Where

C_t 0.085 for moment resistant space steel frames, 0.075 for moment resistant space concrete frames and eccentrically braced steel frames and 0.050 for all other structures;

H The height of the building, in m, from the foundation or the top of a rigid basement

2.1.1.2 Modal Response Spectrum Analysis

This type of analysis shall be applied to buildings that do not satisfy the conditions given for applying the lateral force method of analysis. The response of all modes of vibration, for each relevant direction, with effective modal masses greater than 5% of the total mass shall be taken into account or the sum of the effective modal masses taken into account shall amount to at least 90% of the total mass of the structure.

If these requirements cannot be satisfied (e.g. in buildings with a significant contribution from torsional modes), the minimum number k of modes to be taken into account in a spatial analysis should satisfy both the two following conditions:

$$k \geq 3\sqrt{n} \quad \text{and} \quad T_k \leq 0.20\text{sec} \qquad \mathbf{2-4}$$

Where

k The number of modes taken into account;

n The number of stories above the foundation or the top of a rigid basement;

T_k The period of vibration of mode k

A combination of modal responses can be done using the square root of the sum of squares (SRSS) if the response in two vibration modes i and j (including both translational and torsional modes) taken as independent of each other, or complete quadratic combination otherwise.

2.1.1.3 Accidental Torsion Effects

To account for uncertainties in the location of masses and the spatial variation of the seismic motion, the calculated center of mass at each floor i shall be considered as being displaced from its nominal location in each direction by an accidental eccentricity:

$$e_{ai} = \pm 0.05 * L_i \qquad \mathbf{2-5}$$

Where

e_{ai} The accidental eccentricity of story mass i from its nominal location applied in the same direction at all floors;

L_i The floor dimension perpendicular to the direction of the seismic action

Whenever a spatial model is used for the analysis, the accidental torsional effects may be determined as the envelope of the effects resulting from the application of static loadings, consisting of sets of torsional moments M_{ai} about the vertical axis of each story i :

$$M_{ai} = e_{ai} * F_i \quad 2-6$$

Where

M_{ai} The torsional moment applied at story i about its vertical axis;

e_{ai} The accidental eccentricity of story mass i in accordance with equation 2-5 for all relevant directions;

F_i The horizontal force acting on story i , as derived in equation 2-2 for all relevant directions

The torsional effects of the loadings should be taken into account with positive and negative signs (the same sign for all stories).

Unless a more accurate analysis of the cracked elements is performed, the elastic flexural and shear stiffness properties of concrete elements may be taken to be equal to one-half of the corresponding stiffness of the uncracked elements.

2.1.1.4 Combination of the Effects of the Components of the Seismic Action

The combination of the horizontal components of the seismic action may be accounted for using either of the methods listed below.

- The structural response to each component shall be evaluated separately, using the combination rules for modal responses. Then the maximum value of each action effect on the structure due to the two horizontal components of the seismic action may then be estimated by the SRSS method.
- More accurate models may be used for the estimation of the probable simultaneous values of more than one action effect due to the two horizontal components of the seismic action.

- The action effects due to the combination of the horizontal components of the seismic action may be computed using both of the two following combinations:

$$\begin{aligned} E_{Edx} &+ 0.3 * E_{Edy} \\ 0.3 * E_{Edx} &+ E_{Edy} \end{aligned} \quad 2-7$$

Where

" + " Implies "to be combined with";

E_{Edx} The action effects due to the application of the seismic action along the chosen horizontal axis of the structure;

E_{Edy} The action effects due to the application of the same seismic action along the orthogonal horizontal axis of the structure.

2.1.1.5 Torsionally Flexible System

Frame, dual or wall system not having a minimum torsional rigidity in both horizontal directions in accordance with equation 2-8 should be classified as torsionally flexible systems.

$$r_x \geq l_s \quad 2-8$$

Where

r_x The square root of the ratio of the torsional stiffness to the lateral stiffness in the y-direction ("torsional radius"); and

l_s The radius of gyration of the floor mass in plan (square root of the ratio of the polar moment of inertia of the floor mass in plan with respect to the center of mass of the floor to the floor mass)

2.1.2 American Society of Civil Engineers Standard and International Building Code

As per this code, the structural system used shall be in accordance with the seismic design category and height limitations. The appropriate response modification coefficient, R , system over strength factor, Ω_0 , and the deflection amplification factor,

C_d , shall be used in determining the base shear, element design forces, and design story drift according to the lateral force-resisting system selected.

2.1.2.1 Equivalent Static Analysis

For structures with torsional irregularity, the torsional moment caused by accidental eccentricity at each story shall be amplified by A_x . The amplification factor of the accidental torsional moment A_x is not required to exceed 3.0 and be less than 1.

$$A_x = \left(\frac{\delta_{max}}{1.2\delta_{avg}} \right)^2 \quad \mathbf{2-9}$$

$$\delta_{avg} = \left(\frac{\delta_A + \delta_B}{2} \right)^2 \quad \mathbf{2-10}$$

Where

δ_A The maximum displacement of the flexible side computed assuming $A_x = 1$ at story i

δ_B The maximum displacement of the stiff side computed assuming $A_x = 1$ at story i

δ_{max} The maximum displacement computed assuming $A_x = 1$ at story i

δ_{avg} The average of the displacements at the extreme points of the structure computed assuming $A_x = 1$ at story i

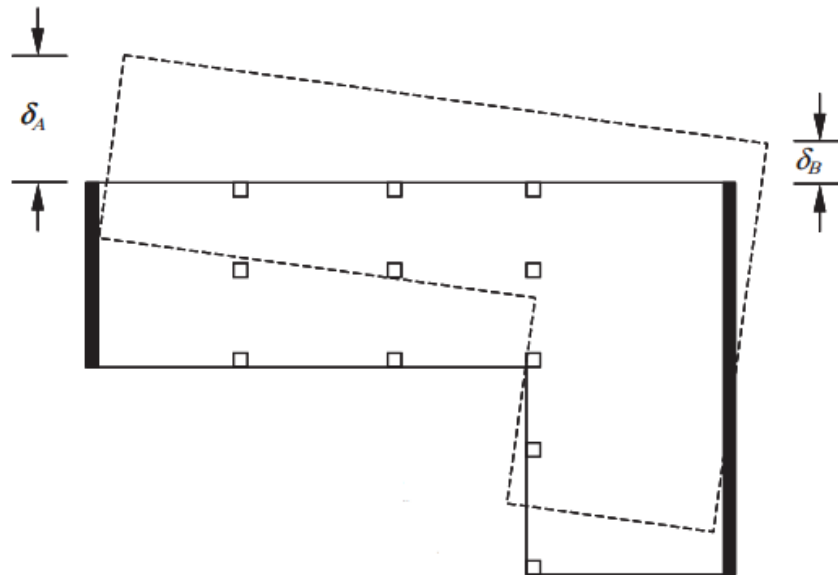


Figure 2-1 Torsional Amplification Factor, A_x

2.1.2.2 Modal Analysis

The fundamental period of the structure, in seconds, shall not exceed the coefficient for the upper limit on the calculated period times the approximate fundamental period of the structure used for equivalent static analysis.

Table 2-1 Coefficient for upper limit on calculated period

Design Spectral Response Acceleration at 1 Second	Coefficient for upper limit on calculated period
≥ 0.4	1.4
0.3	1.4
0.2	1.5
0.15	1.6
≤ 0.1	1.7

When the modal analysis method is used, the design base shear estimated shall not be less than 85% of the design base shear calculated by equivalent lateral force procedure. If not, the design story shears, moments, drifts, and floor deflections shall be multiplied by 0.85 times ratio of design base shear estimated by equivalent lateral force procedure to modal analysis method.

In the modal analysis method, the amplification of torsion is not required where accidental torsion effects are included in the dynamic analysis model.

2.1.2.3 Accidental Torsion Effects

Where diaphragms are not flexible, the design shall include the torsional moment resulting from the location of the structure masses plus the accidental torsional moments caused by assumed displacement of the mass each way from its actual location by a distance equal to 5% of the dimension of the structure perpendicular to the direction of the applied forces. Where earthquake forces are applied concurrently in two orthogonal directions, the required 5% displacement of the center of mass need not be applied in both of the orthogonal directions at the same time but shall be applied in the direction that produces the greater effect.

2.1.2.4 Combination of the Effects of the Components of the Seismic Action

The structure shall be analyzed using the equivalent lateral force analysis, the modal response spectrum analysis, or the linear response history with the loading applied independently in any two orthogonal directions. Then members and their foundations are designed for 100 percent of the forces for one direction plus 30 percent of the forces for the perpendicular direction. Or simultaneous application of orthogonal ground motion shall be used when the structure analyzed using the linear response history or the nonlinear response history, with orthogonal pairs of ground motion acceleration histories applied simultaneously. Stiffness properties of concrete and masonry elements shall consider the effects of cracked sections.

2.1.2.5 Torsional Irregularity

Torsional irregularity is defined to exist where the maximum story drift computed including accidental torsion with $A_x = 1$ at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts at the two ends of the structure.

Extreme Torsional Irregularity

Extreme Torsional Irregularity is defined to exist where the maximum story drift, computed including accidental torsion with $A_x = 1$, at one end of the structure transverse to an axis is more than 1.4 times the average of the story drifts at the two ends of the structure.

2.1.3 China Code for Seismic Design of Buildings

2.1.3.1 Base Shear Force Method

For structures that are not higher than 40m, mainly have shear deformation and a rather uniform distribution of mass and rigidity along the height or for the structures similar to a single-mass system, the simplified methods, such as base shear method may be adopted.

When the base shear force method is adopted, only one degree of freedom may be considered for each story. When distributing the total base shear along the height, additional horizontal earthquake action is added at the top.

2.1.3.2 Mode Decomposition Response Spectrum Method

For building structures other than the ones used in the base shear force method, the mode decomposition response spectrum method should be adopted.

Horizontal Earthquake Action

In the calculation of earthquake action, the representative value of the gravity load of a building shall be taken as the sum of the standard value of the deadweight of structure and its components and accessories and the combined value of variable loads.

The horizontal seismic shear force at any story of the structure with an obvious torsion effect or fundamental period less than 3.5 seconds shall not be less than a coefficient of shear times the representative value of gravity load of the story. This coefficient of shear is equal to 0.008, 0.016, 0.032 and 0.064 for earthquake intensity 6, 7, 8 and 9 respectively for ground acceleration equal to 0.15g.

The horizontal seismic shear force of stories of a structure shall be distributed according to the proportion of equivalent rigidity of lateral force resisting components.

2.1.3.3 Torsion Coupling Earthquake Effect

Under the horizontal earthquake action, the torsion coupling earthquake effect of the building structure shall meet the following requirements:

- If no coupled torsion calculation is to be carried out for regular structures, the earthquake action effects of the components on both sides parallel to the

earthquake action direction shall be multiplied by an enhancement coefficient. Generally, the enhancement coefficient may be taken as 1.15 for the short side and as 1.05 for the long side. However, if the torsion rigidity is small, the enhancement coefficient for the surrounding components should not be less than 1.3 and the earthquake action effects of the components at corners should be multiplied by the enhancement coefficients.

- When calculating according to the coupled torsion method, three degrees of freedom may be selected for each floor, including two orthogonal horizontal deformations and a rotation. The torsion coupling effect under the unidirectional horizontal earthquake action is determined using SRSS. The torsion coupling effect under the bi-directional horizontal earthquake actions may be determined with larger value of the following formula:

$$S_{Ek} = \sqrt{\left(S_x^2 + (0.85S_y)^2\right)} \quad \mathbf{2-11}$$

$$S_{Ek} = \sqrt{\left(S_y^2 + (0.85S_x)^2\right)} \quad \mathbf{2-12}$$

Where

S_x, S_y The torsion effect of unidirectional horizontal earthquake actions in the x and y-direction respectively;

S_{EK} Torsion effect of the standard value of earthquake action

2.1.3.4 Combination of Earthquake Action and Other Load Effects

The fundamental combination of the earthquake action effect and other load effects of the structural components shall be calculated according to the following formula:

$$S = \gamma_G S_{GE} + \gamma_{Eh} S_{Ehk} + \gamma_{Ev} S_{Evk} + \psi_w \gamma_w S_{wk} \quad \mathbf{2-13}$$

Where

S Design value of the internal force combination of structural components, including the design values of the combined bending moment, axial force and shear force;

γ_G Partial coefficient of gravity load;

S_{GE} Effect of the representative value of gravity load;

γ_{Eh}, γ_{Ev} The partial coefficients of the horizontal and vertical earthquake actions respectively;

S_{Ehk} Effect of the standard value of horizontal earthquake action;

S_{Evk} Effect of the standard value of vertical earthquake action;

ψ_w Combination value coefficient of wind load;

γ_w Partial coefficient of wind load;

S_{wk} Effect of standard value of wind load

2.1.3.5 Torsional Irregularity

Torsional irregularity is said to exist if the maximum elastic horizontal displacement or (story drift) of a story is larger than 1.2 times the average values of elastic horizontal displacement (story drift) at both ends of the same story.

Buildings meeting plan irregularity and vertical regularity shall be adopted with a three-dimensional calculation model and for buildings having torsional irregularity, the torsion effects shall be considered and the maximum elastic horizontal displacement and story drift of a story should be less than or equal to 1.5 times of the average values of elastic horizontal displacement and story drift at both ends of the same story respectively.

2.1.4 National Building Code of India 2016 Volume 1

The effects of design earthquake loads applied on structures can be considered using equivalent static method and dynamic analysis method.

Dynamic analysis may be performed by either the time history method or the response spectrum method. When either of the methods is used, the design base shear estimated shall not be less than the design base shear calculated by equivalent static analysis. If not,

the force response quantities (for example member stress resultants, story shear forces, and base reactions) shall be multiplied by the ratio of design base shear estimated by equivalent static method to dynamic analysis method.

For structural analysis, the moment of inertia shall be taken as 70 percent and 35 percent of the corresponding moment of inertia of the uncracked section of columns and beams respectively.

2.1.4.1 Equivalent Static Method

This method shall be applicable for regular buildings with height less than 15 m in low Seismic Zone and with an approximate fundamental translational period less than 0.4seconds.

2.1.4.2 Response Spectrum Method

The response spectrum analysis method may be performed for any building using the design acceleration spectrum.

2.1.4.3 Design Eccentricity

Usually, a well-proportioned building does not twist about its vertical axis, when the stiffness distribution of the vertical elements resisting lateral loads is balanced in plan according to the distribution of mass in plan (at each story level) and the floor slabs are stiff in their own plane (this happens when its plan aspect ratio is less than 3). The unavoidable asymmetry of buildings, however, causes buildings to twist. While performing structural analysis by the response spectrum method, the design eccentricity e_{di} to be used at floor i shall be taken as

$$e_{di} = 1.5e_{si} + 0.05b_i \quad \mathbf{2-14}$$

$$= e_{si} - 0.05b_i \quad \mathbf{2-15}$$

whichever gives the more severe effect on lateral force resisting elements.

Where

e_{si} Static eccentricity at floor i , distance between center of mass and center of stiffness,

b_i Plan dimension of floor i , perpendicular to the direction of force

The factor 1.5 represents dynamic amplification factor and $0.05b_i$ represents the extent of accidental eccentricity. The above amplification of 1.5 need not be used when performing structural analysis by the time history method.

2.1.4.4 Combination of the Effects of the Components of the Seismic Action

When lateral load resisting elements are oriented along two mutually orthogonal horizontal directions, the structure shall be designed for effects due to full design earthquake load in one horizontal direction at a time, and not in both directions simultaneously.

When lateral load resisting elements are not oriented along mutually orthogonal horizontal directions, the structure shall be designed for the simultaneous effects due to full design earthquake load in one horizontal direction plus 30 percent of design earthquake load along the other horizontal direction.

2.1.4.5 Torsional Irregularity

A building is said to be torsionally irregular when the maximum horizontal displacement of any floor in the direction of the lateral force at one end of the floor is more than 1.5 times its minimum horizontal displacement at the far end of the same floor in that direction, and the natural period corresponding to the fundamental torsional mode of oscillation is more than those of the first two translational modes of oscillation along each principal directions.

$$\Delta_{\max} > 1.5\Delta_{\min} \qquad \mathbf{2-16}$$

Where

Δ_{\max} The maximum horizontal displacement of any floor i in the direction of the lateral force at one end of the floor;

Δ_{\min} The minimum horizontal displacement at the far end of the same floor i in that direction

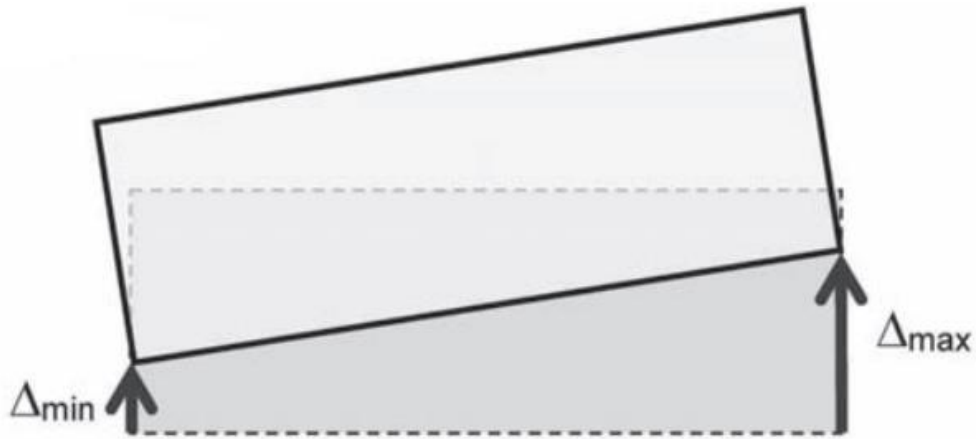


Figure 2-2 Torsional irregularity

In torsionally irregular buildings, when the ratio of maximum horizontal displacement at one end and minimum horizontal displacement at the other end is:

- In the range 1.5-2.0, the building configuration shall be revised to ensure that the natural period of the fundamental torsional mode of oscillation shall be smaller than those of the first two translational modes along each of the principal plan directions, and then three-dimensional dynamic analysis method shall be adopted; and
- More than 2.0, the building configuration shall be revised.

2.2 PREVIOUS STUDIES DONE ON TORSIONALLY IRREGULAR BUILDINGS

Buildings intended to be earthquake resistant have competing demands. If the design is to prevent any damage during strong earthquake shaking, it will not be economical. And if the design is to be economical, it may not prevent any damage. However, features of earthquake-resistant buildings are:

- At least a minimum lateral stiffness in each of its plan directions uniformly distributed in both plan directions of the building.
- At least a minimum lateral strength in each of its plan directions uniformly distributed in both plan directions of the building.
- Good overall ductility in it to accommodate the imposed lateral deformation.

- Good seismic configuration, with no choices of architectural form of the building that is unfavorable to good earthquake performance.

Nevertheless, asymmetric building structures are inevitable in modern construction due to different types of functional and architectural requirements. Multi-story buildings with plan asymmetry have a much more complex behavior than plan symmetric buildings and they possibly are affected by higher vibration modes. Hence, the analysis and design methods used shall incorporate the existing condition of the structures. Lateral force method of analysis of multi-story buildings, which considers the fundamental mode of vibration only, may undermine the response of the structures when the contribution from higher modes become significant. Many scholars conducted a research on this and concluded based on their results.

2.2.1 Previous Studies Done on Multi-Story Buildings

Tso et al. (1999) studied the torsional behavior of multi-story buildings using 4 seven-story buildings having different structural configurations and 1 symmetrical building. The response of the symmetric building served as a useful reference to show the torsional effects on the edges of the eccentric buildings. The overall behavior of these five buildings under static lateral loadings is illustrated using their performance curves obtained by pushover analyses applying the lateral loads at the CM of the floors. A comparison of the five curves showed that a decrease in torsional stiffness will decrease the overall stiffness, and also reduce the overall strength of the building.

They stated that large torsional responses can be expected if the buildings have large eccentricity and low torsional stiffness. They point out that EC8 torsional provisions do not improve the seismic responses of asymmetrical multi-story buildings with vertical regularity if the buildings have low torsional stiffness. For this reason, they proposed the ratio of torsional radius to the radius of gyration less than 1.0 would imply that the building is too torsionally flexible that the EC8 static torsional provisions are not applicable.

Mohamed et al. (2015) discussed the determination of the effects of torsional irregularity using Response Spectrum Analysis in accordance with ASCE. A case study of a thirteen-story plan asymmetric reinforced concrete structure was used. The lateral force resisting

system was a dual system. They indicated that the presence of torsional irregularities implies seismic forces may be amplified by the existence of eccentricity. They stated that neglecting the amplification of accidental torsion moment in response spectrum analysis, thinking it incorporates different vibration modes was not correct. In order to assess the result of the magnification of accidental torsion, story displacements due to seismic forces with and without the amplified accidental eccentricity were calculated. Since the amplified accidental eccentricity led to amplified structural responses, they concluded that amplification of torsion effects due to irregularity is required even when response spectrum analysis is used.

Nina et al. (2004) studied torsionally irregular structures with unidirectional and bidirectional eccentricity. The relative eccentricity along x-direction of structures with eccentricities in two directions was kept invariable when the relative eccentricity along y-direction varies. All buildings were vertically regular five-story buildings with ratios of longer to shorter plan dimensions 1, 3 and 5. The center of stiffness (CS) of the floor was located on the center of the floor and the center of mass (CM) was variable with the variation of mass distribution. However, the total mass and total lateral resisting stiffness kept invariable. They investigated the relationship of θ , representing the ratio of maximum displacement of the flexible side to the average of the displacements at the extreme points of the structure with relative eccentricities and torsional effects. They found that θ may be less than 1.2 for structures with bigger eccentricity. They stated that according to the criterion the torsion effects should have been ignored which should not be the case. Hence, they concluded that θ does not represent the criterion for torsional irregularity.

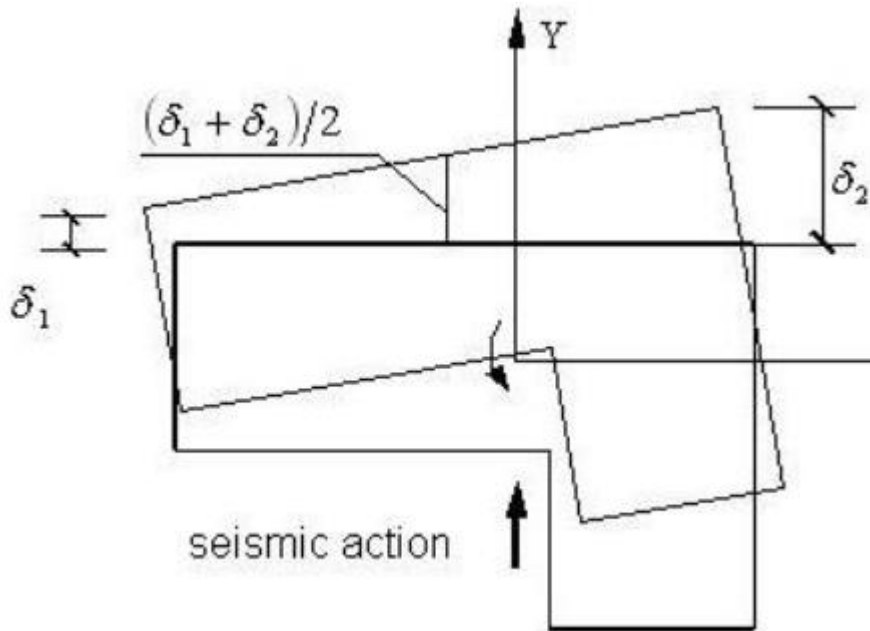


Figure 2-3 Torsional irregularity

$$\theta = \frac{\delta_2}{\left(\frac{\delta_1 + \delta_2}{2}\right)} \quad 2-17$$

Where

δ_1 The maximum displacement of the stiff

δ_2 The maximum displacement of the flexible side

Nina et al. (2004) stated two points in order to consider the torsion effects for structures with torsional irregularities. The points were the analysis models of structures should be spatial and the modal analysis with torsion coupling should be adopted for seismic design. For the structures with extreme irregularities in layouts of mass and stiffness, the torsion effects under seismic action in two directions should be considered simultaneously.

Magar et al. (2017) investigated the performance of the static analysis of symmetric rectangular and asymmetric channel shape fifteen-story buildings using ETABS. In order to evaluate this, nonlinear dynamic analysis was used and the story displacement

obtained by static analysis was higher than dynamic analysis for both regular and irregular structures. They also observed that the base shear obtained by static analysis was higher than dynamic analysis for irregular structure and base shear obtained by dynamic analysis was higher than static analysis for regular structure.

Hong et al. (2013) presented an analysis of the linear-elastic torsional response of single-story, two-story and five-story models with bi-directional eccentricities subjected to uni-directional and bi-directional ground motions. Numerical simulations were carried out to investigate the influences of eccentricity and torsional rigidity on the responses as well as to check the adequacy of Australian Standard which takes into account the torsional effects caused by earthquakes on an asymmetric structure by applying a horizontal equivalent static force at a position with 10% accidental eccentricity each side from the center of mass. The equivalent static force applied along one of the horizontal directions consists of 100% of the load, while in the other horizontal direction 30% of the load is considered. For each case, 10 simulations using 10 sets of simulated ground motions were carried out.

Following their study, they concluded that torsional response not only depends on the torsional to lateral vibration frequency ratios, which is the criterion in the code but also depends on the torsional vibration frequency. When the torsional vibration frequency coincides with the dominant ground motion frequency, torsional responses will be amplified due to resonance. Nonlinear inelastic response will also change the torsional response characteristics, especially when this response turns an elastically torsionally stiff building into an inelastically torsionally flexible building, due to differential inelastic response of the seismic resisting systems. Then they concluded that the Australian code might not be adequate, especially when the structures are torsionally flexible.

Ali et al. (2010) studied the torsional irregularity effects of local site classes in multi-story structures according to the Turkish Earthquake Code. Structures with different story numbers, plan shapes and shear wall locations were analyzed by lateral load analysis and response spectrum analysis methods using SAP. Torsional moments were

applied to the center of mass of each story with $\pm 5\%$ eccentricity in both directions for plan symmetric structures.

They used torsional irregularity as a parameter to compare the different types of buildings. They defined torsional irregularity as the ratio of maximum inter-story drift value to the average inter-story drift value at each story. Torsional irregularity occurs when the ratio is between 1.2 and 2. They concluded that local site classes did not affect this ratio in symmetrical structures. However, the ratio is affected by values 0.01-0.15 in asymmetrical structures. The torsional irregularity coefficient increased as the site class changed from soft layers to rock. In addition, for plan rectangular structures, torsional irregularity coefficients calculated by the response spectrum analysis method were 2%-3% bigger than calculated by equivalent lateral load analysis method and the value did not reach 1.2 whereas the difference was 3%-6% in L-shaped structures and the value was about 2. When the torsional irregularity coefficient is between 1.2 and 2.0, the accidental eccentricity should be multiplied by D in both directions so as to provide additional enforcement to the structural system.

$$D = \left(\frac{\Delta_{max}}{1.2\Delta_{avg}} \right)^2 \quad \mathbf{2-18}$$

Where

Δ_{max} The maximum inter-story drift value;

Δ_{avg} The average inter-story drift value

Shehata et al. (2016) studied irregularity effects on the seismic performance of L-shaped multi-story buildings. 6 L-shaped nine-story moment-resisting frame buildings were selected with a gradual reduction of a plan of a square reference model. The models were analyzed on ETABS using equivalent static and response spectrum analysis methods. They stated the empirical equation for the calculation of the fundamental period in various codes could not grasp significantly higher vibration modes such as the torsional vibration of irregular buildings. The findings were discussed with respect to inter-story drift, story shear force and overturning moment. The inter-story drift, story lateral

displacement and overturning moment increased as the floor plan irregularity increase. Story drift response along the height of the building showed that the middle stories were affected more than lower and upper stories. The lateral shear force demands in lateral force resisting elements located on the periphery of the structure were significantly increased in comparison with the corresponding values of the symmetric building.

Adarsh et al. (2017) assessed the torsional effect of 5 structures having an equal perimeter. Model 1 was reinforced concrete frame without core and shear wall, model 2 was reinforced concrete frame with core walls only and models 3, 4 and 5 were reinforced concrete frames with core and shear walls of different positions. The equivalent static and response spectrum analysis of the structural models was carried out using ETABS. Lateral displacement, story drift, story shear, and torsional irregularity were compared for all models. According to the study, the displacement was greater in model 1 compared to other models. And the torsional irregularity has reduced from models 1 and 2 to the remaining models due to the introduction of core walls and shear walls. Finally, they concluded that positions of shear walls in building influenced the displacement caused by seismic actions. Thus keeping shear walls at proper places would significantly minimize the displacement resulted from an earthquake.

Deekshitha et al. (2018) studied the analysis of regular and irregular frame buildings by response spectrum method and time history method using ETABS. The analysis was made on fifteen-story reinforced concrete frame buildings of plan configuration rectangular, L-shape, H-shape and C-shape. The structural response determined were story displacement, story drift, story shear and time period. According to the analysis, the story shear force was found maximum in C-shape and H-shape model whereas the story drift was found maximum in L-shape and H-shape building than the rectangular model. In conclusion, they stated that the plan configuration of structures has an important influence on the seismic response of the structures in terms of story shear, story displacement, and story drift.

Fernandes et al. (2015) have made an equivalent static and response spectrum analysis to find the response of fifteen-story reinforced concrete regular and torsionally irregular buildings using ETABS. Story displacement was used as a parameter to compare regular

and irregular structures. The results obtained from the static analysis method showed lesser story displacement values as compared to the response spectrum analysis result.

Rahila et al. (2016) analyzed thirteen- and eighteen-story symmetric and asymmetric structures with plan irregularity using ETABS. Same outer perimeter Rectangular, L-shape and C-shape asymmetric buildings with shear wall cores at one end and another C-shape building strengthened by two C-shaped shear walls were considered. Linear static and response spectrum analysis were carried out. The estimated period for the first modes indicated that the strengthened model was less vulnerable to seismic action. And it had a short period which resulted in smaller drift whereas the symmetric structure had a longer period which resulted in larger drift. So they concluded that the drift and displacement values were dependent on the stiffness and mass concentration of the structure and it was the plan that brought the variation because significant changes were not seen as other parameters such as height and lateral resisting elements were uniformly introduced along the height of the structure.

Bruno et al. (2002) presented research that led to a general observation for regularly asymmetric structures suggesting a simple procedure that allow the evaluation of corrective eccentricities that enable to equate the maximum design displacements and internal actions of static and modal analysis. In order to confirm the effectiveness of the proposed approach, 4 seven-story reinforced concrete buildings with a rectangular plan consisting of three identical resisting frames along the direction of seismic action with 0.1 mass eccentricities had been analyzed. One frame was located at the center of the floor slab while the others were placed symmetrically at opposite sides of the central one. The only difference among the buildings was the distance between the lateral frames from the center of the floor slab. Finally, they introduced corrective eccentricities that were obtained by equating the maximum design displacement and internal actions of static analysis with modal analysis for regularly asymmetric structures.

Gunay et al. (2014) performed an investigation on structures with varying shear wall positions, story, and axis numbers to determine the conditions for excessive torsional irregularity and to discuss the validity of ASCE provisions. All structures were symmetrical with respect to the x-axis and investigations were carried out only for

loadings in the direction of the y-axis. Findings of lateral load analyses indicated torsional irregularity coefficients increased as the story numbers decrease. And floor rotations increased in proportion to the story numbers. Torsional irregularity coefficients reached maximum values when the shear walls placed close to the center of mass. However, floor rotations attained their maximum values when the walls placed in the farthest position from the centers of mass. In addition to this, torsional irregularity coefficients increased when the center of rigidity approach to the center of mass due to decreasing torsional rigidity of the structure. From the results obtained, torsional irregularity coefficients and floor rotations were quite contradictory. Since the floor rotations could be considered as the real representative of the torsional behavior, the code regulations should be completely amended. Consequently, a new provisional definition of the torsional irregularity coefficient based on floor rotations was proposed.

Bijily et al. (2016) attempted to evaluate the effectiveness of the Indian standard for designing torsionally irregular asymmetric buildings. Symmetrical building along with two similar asymmetric buildings designed with and without considering code requirement was used for the study. The only difference between these two asymmetric buildings was the reinforcement design. Firstly, all were analyzed using modal analysis. Next nonlinear static and dynamic analyses were performed on these buildings to study the response of the buildings. The contribution of higher mode was slightly more in the case of asymmetric building. The findings of the study indicated that the plan asymmetry in the building made it less ductile even after designing with code provision. And the maximum base shear and roof displacement were similar for both asymmetric buildings. Yielding of both asymmetric buildings also occurred at the same time step of the dynamic analyses. From these, they concluded that even though the code criterion for plan asymmetry improved the strength of the frame element, improving member strength did not change the mode of the building failure. Hence, changing the stiffness distribution to reduce eccentricity might be useful for designing asymmetric buildings.

2.2.2 Previous Studies Done on Single Story Buildings

Francisco et al. (2004) investigated the seismic response of buildings, particularly for ductile structures. They performed static analyses with the computer program “TORSION” and the dynamic time-history analyses were conducted using the program

“RUAUMOKO”. According to their findings, torsional effects significantly affected the seismic response of buildings mostly for ductile structures, producing an uneven distribution of the lateral displacements. As a result, the lateral displacement demand in some parts of the building increased. They concluded existing torsional requirements, which are based on elastic considerations, are not effective to improve the response of the structure. For these reasons, a rational consideration of torsional effects coherent with existing design criteria needs to be included in seismic provisions.

Moghadam et al. (2012) examined performances of a range of torsionally stiff and flexible single-story buildings designed with the provisions of Iranian Standard 2800 using linear static analysis of buildings. 8 building models subjected to 7 horizontal design spectra which are compatible with ground motion were analyzed using nonlinear dynamic time history analysis. According to the investigation floor rotation caused by accidental eccentricity was small for symmetric models whereas floor rotation was larger for asymmetric models increasing with a decrease in torsional stiffness. They stated that the provisions have no adequate efficiency to limit the story drift ratio for critical edges with very low torsional stiffness. Finally, they concluded that the linear static analysis of the building code is not generally adequate for structures with very low torsional stiffness.

2.2.3 Previous Studies Done on Accidental Eccentricity

Anagnostopoulos et al. (2015) studied the importance of accidental eccentricity in three- and five-story torsionally stiff and torsionally flexible eccentric-braced steel buildings. All were designed using response spectrum method in accordance with EN 1998-3:2003 and EN 1998-1:2003. Accidental eccentricities, $\pm 5\%$, were accounted in the x and y -directions. Also, a different set of similar buildings with the same geometry and mass distribution was designed without considering accidental eccentricity. Then non-linear analyses were carried out using the program RUAUMOKO. The study indicated that designing torsionally stiff, steel braced frame buildings for an accidental eccentricity as per EN 1998-1:2003 resulted in small effects on their inelastic response. In torsionally flexible buildings, an accidental eccentricity led to ductility demand reductions up to 28% in braces and up to 18% in beams and its omission increased peak ductility demands in braces by 35% and in beams by 23%. Therefore, they concluded that the

benefits of the code specified accidental eccentricities were insignificant for the majority of torsionally stiff buildings. On the other hand, the accidental eccentricity was more effective in reducing ductility demands in torsionally flexible buildings.

Mostafa et al. (2008) evaluated the suitability of the assumed code value of 5% accidental eccentricity for single and multi-story buildings. Single story, two-, three-, four-, five-, ten-, fifteen- and twenty-story buildings with the same geometric layout were studied using time history analyses. A total of 300-time history analyses corresponding to 300 samplings under a selected ground record were performed on a set of hypothetical three dimensional reinforced concrete multi-story buildings with symmetric plan and rigid diaphragms. Random accidental eccentricities at various floor levels were applied by sampling from an assumed probability density function. Retrieved accidental eccentricities of the analyzed models at each floor level associated with the maximum story shear at the same floor level were reported. The study showed that adopting the design value of 5% on all floors was right for low-rise buildings up to 5 stories. However, for higher buildings, a design accidental eccentricity of 5% to the upper quarter of the building, a value of 3% to the middle half, and a value of 2% to the lower quarter was enough. From this, they concluded that accidental eccentricities and hence the accidental torsional demands decrease downwards in multi-story buildings.

Hadjadj et al. (2014) investigated the influence of the seismic component of rotation on the torsional response of building structures subjected to earthquake ground motions. For this purpose, a rotational acceleration component was generated from the translational ones and applied to both symmetric and asymmetric structures using nonlinear dynamic analysis using the program ANSYS. In this study, a four-story plan symmetrical and plan asymmetrical reinforced concrete moment resistant frame structures were studied. Both structures were first subjected to bidirectional ground motion along two perpendicular directions and then to two translational and torsional ground motions. The contribution of the torsional component was assessed by comparing the responses in terms of displacements under the two loading conditions. The obtained results showed the considerable effect of the torsional seismic component on the structural response, particularly for asymmetric structures. The estimated accidental eccentricity of the studied cases exceeded the value of 5% of the dimension of the floor perpendicular to the

excitation axis which is recommended by several seismic codes as an accidental eccentricity. The accidental eccentricity due to the torsional component varied between 4% and 15% for the regular and irregular structures respectively.

From the above review, one may conclude that the code's equivalent static torsional provision needs additional research since the conclusion reached by the scholars apart from stating the code's drawback was not the same. Therefore further study on this was needed.

CHAPTER 3 ANALYTICAL MODEL

3.1 GENERAL

Under this chapter, the detailed outline of the analytical model selected for studying the response of structures is presented. The system under study consists of several structures. To cover each and reach to conclusion unlimited time will be required. Therefore out of this vast system, 30 representative samples are selected to see the general response of structures when exposed to seismic action.

3.2 MODELS SELECTED FOR THE ANALYTICAL STUDY

In most statistics problems, sample of observations selected from the population of the interest area is used for the study. A population consists of the total observations. In any particular problem, the population may be small, large but finite, or infinite. In most situations, it is impossible or impractical to observe the entire population due to time constraints. Therefore, a subset of observations from the population is selected to help make decisions about the population.

For statistical methods to be valid, the sample must be representative of the population so that it is desirable to select a random sample as the result of the same chance mechanism. Consequently, the selection of a sample is a random experiment and each observation in the sample is the observed value of a random variable. The observations in the population determine the probability distribution of the random variable. The most widely used model for the distribution of a random variable is a normal distribution. In this study, a normal distribution of samples with a 95% confidence interval is used.

If \bar{x} is the sample mean of a random sample of size n from a normal population with known variance σ^2 , a $100(1 - \alpha)\%$ confidence interval on μ is given by

$$\bar{x} - z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} \quad \mathbf{3-1}$$

$$\text{Lowerlimit} = \bar{x} - z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} \quad 3-2$$

$$\text{Upperlimit} = \bar{x} + z_{\alpha/2} * \frac{\sigma}{\sqrt{n}} \quad 3-3$$

Where

$z_{\alpha/2}$ The upper $100\frac{\alpha}{2}$ percentage point of the standard normal distribution

μ The mean value of the population

To find the 95% confidence interval, $z_{\alpha/2} = z_{0.025}$ is taken as 1.96. Thus, a normally distributed random variable will lie within 1.96 standard deviations of its mean 95% of the time.

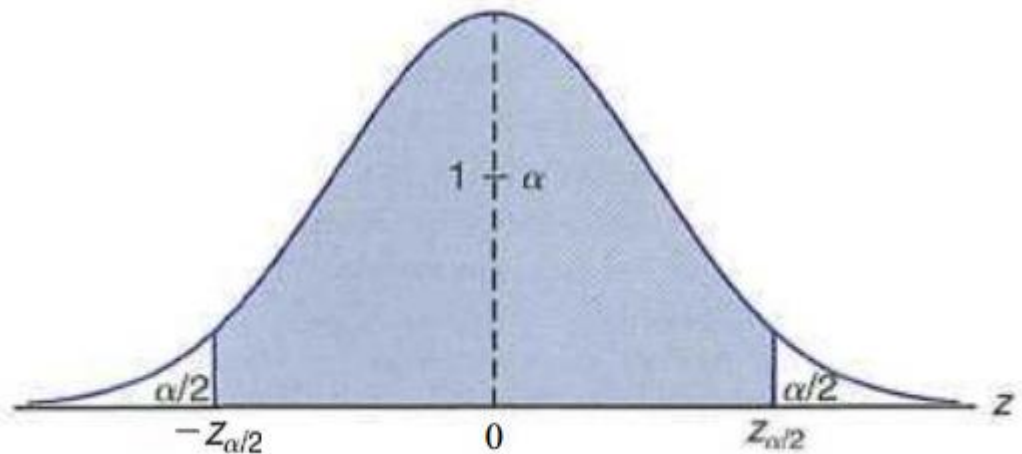


Figure 3-1 Probabilities associated with normal distribution

From the above points, to select random samples, the mean value, the standard deviation and the number of random samples intended to be selected shall be known. If the upper and lower limits can be determined, then the mean value will be the average of the two limits. After determining the mean value, the standard deviation can be determined using the mean value and either the lower or upper limit.

The analytic models used in this paper generally have 8 different parameters. The lower and upper limits of the parameters are set based on code requirements and engineering judgments.

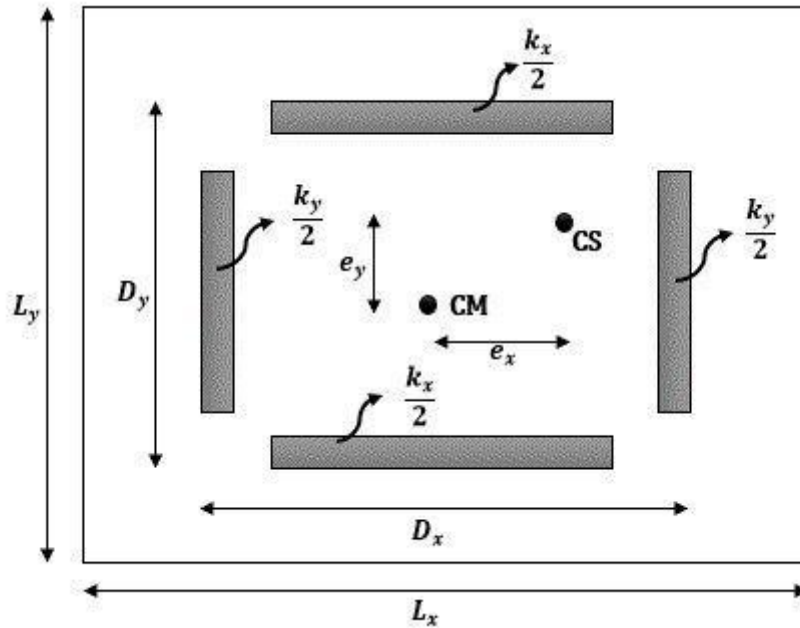


Figure 3-2 Plan of the analytical model

The first 2 parameters are $D_x(L_x)$ and $D_y(L_y)$ which are the distance between the lateral force-resisting system along y-direction and x-direction respectively. If both lateral force resisting systems in either direction exists at the center of mass the value of the parameter will be 0 and it will be 1 in terms of the plan dimension along the same direction if it exists at the perimeter.

Eccentricity, the distance between the center of mass and center of stiffness along x-direction e_x and y-direction e_y can vary between 0-0.5 of the plan dimension. The upper limit i.e. 0.5 is an ideal situation but to represent every possible eccentricities, this upper limit is chosen.

The other parameter used is the ratio of story stiffness in the x-direction to the story stiffness in the y-direction $\frac{k_x}{k_y}$. EN 1998-1:2003 recommended having similar resistance

along the two orthogonal directions. Penelis G. et al. (2014) and Fardis M. et al. (2005)

also stated that for a structure to resist horizontal actions in any-direction the structural elements should be arranged in an orthogonal pattern and such a pattern of structural elements should ensure similar resistance and stiffness characteristics of the building as a whole in these two main orthogonal directions. However, the word ‘similar’ is questionable since it is not clearly stated quantitatively. Here, the lateral stiffness ratio used is 0-1. Even though a zero lateral stiffness ratio is ideal to consider, it is however used not to omit systems with a very small ratio of lateral stiffness. Very small lateral stiffness ratio may be seen in two dimensional systems or when the lateral stiffness in one direction is very large relative to the other direction in three dimensional systems.

The last parameter used is the ratio of plan dimensions $\frac{L_y}{L_x}$. EN 1998-1:2003 recommended this ratio to be 0.25 to 4.

Table 3-1 Lower and upper limit of parameters used

	Lower limit	Upper limit	\bar{x}	$\frac{\sigma}{\sqrt{n}}$
$D_x(L_x)$	0	1	0.5	0.2551
$D_y(L_y)$	0	1	0.5	0.2551
$e_x(L_x)$	0	0.5	0.25	0.1276
$e_y(L_y)$	0	0.5	0.25	0.1276
K_x/K_y	0	1	0.5	0.2551
L_y/L_x	0.25	4	2.125	0.9566

According to Ronald et al. (2007), the number of samples that should be used in random sampling shall be ≥ 30 . From this perspective 30 torsionally flexible random samples are selected using the Latin hypercube sampling method in MATLAB.

Fardis M. et al. (2005) explained that torsional stiffness and resistance are characteristics of building structures that significantly influence their response to seismic actions. The preferable response of buildings is the result of dominant translational motion than dominant torsional motion because they tend to stress the different structural elements in a more uniform way. In doing so the torsional radius shall be greater than or equal to the radius of gyration of the floor mass to have adequate torsional stiffness. However, if at

any floor the radius of gyration of the floor mass exceeds the torsional radius in one or both of the two main directions of the building, the system will be torsionally flexible.

According to EN 1998-1:2003: seismic design of buildings worked examples, the torsional radius r_x and r_y is defined as the square root of the ratio of the torsional stiffness K_T to the lateral stiffness K_y and K_x respectively.

$$r_x = \sqrt{\frac{K_T}{K_y}}, r_y = \sqrt{\frac{K_T}{K_x}} \quad \mathbf{3-4}$$

$$K_x = \frac{I}{U_x(F_x)} \quad \mathbf{3-5}$$

$$K_y = \frac{I}{U_y(F_y)} \quad \mathbf{3-6}$$

$$K_T = \frac{I}{R(M_T)} \quad \mathbf{3-7}$$

$$l_s = \sqrt{\frac{MMI}{M}} \quad \mathbf{3-8}$$

In the case of the rectangular floor area with dimensions L_x and L_y with uniformly distributed mass over the floor, l_s is equal to

$$l_s = \sqrt{\frac{L_x^2 + L_y^2}{12}} \quad \mathbf{3-9}$$

Where

$U_x(F_x)$ The displacement in the x-direction caused by unit horizontal load in the x-direction;

$U_y(F_y)$ The displacement in the y-direction caused by unit horizontal load in the y-direction;

$R(M_T)$ The rotation about the vertical axis caused by unit moment about vertical axis l_s floor the radius of gyration;

MMI The mass moment of inertial of each story;

M The mass of each story

Knowing the above parameters, the torsional regularity condition for each random sample can be determined using equation 3-10 and 3-11. Thus, to find the torsional regularity of the random samples, the torsional stiffness K_T and lateral stiffness K_y and K_x shall be determined. These stiffness parameters are referred from the stiffness matrix formulated.

$$r_x = \sqrt{\frac{K_T}{K_y}} = \sqrt{\frac{K_x * (\frac{D_y^2}{2} + 2e_y^2) + K_y * (\frac{D_x^2}{2} + 2e_a^2 - 4e_a * e_x + 2e_x^2)}{2K_y}} \quad \mathbf{3-10}$$

and

$$r_y = \sqrt{\frac{K_T}{K_x}} = \sqrt{\frac{K_x * (\frac{D_y^2}{2} + 2e_y^2) + K_y * (\frac{D_x^2}{2} + 2e_a^2 - 4e_a * e_x + 2e_x^2)}{2K_x}} \quad \mathbf{3-11}$$

Table 3-2 Random samples

	6x30 Layers						Eqn 3-9	Eqn 3-10	Eqn 3-11	x-direction	y-direction
	$D_x(L_x)$	$D_y(L_y)$	$e_x(L_x)$	$e_y(L_y)$	K_x/K_y	L_y/L_x	$l_s(L_x)$	$r_x(L_x)$	$r_y(L_x)$	$r_x < l_s$	$r_y < l_s$
1	0.29	0.5	0.24	0.09	0.31	2.59	0.8	0.48	0.85	TF	TR
2	0.65	0.14	0.26	0.35	0.19	2.78	0.85	0.6	1.38	TF	TR
3	0.46	0.13	0.21	0.39	0.29	2.99	0.91	0.7	1.31	TF	TR
4	0.43	0.38	0.37	0.22	0.53	2.72	0.84	0.72	0.99	TF	TR
5	0.45	0.83	0.21	0.26	0.24	1.92	0.63	0.56	1.13	TF	TR
6	0.44	0.55	0.42	0.41	0.21	2.96	0.9	0.82	1.78	TF	TR
7	0.68	0.72	0.27	0.29	0.19	1.99	0.64	0.59	1.36	TF	TR
8	0.3	0.18	0.12	0.42	0.36	2.8	0.86	0.74	1.23	TF	TR
9	0.58	0.43	0.03	0.06	0.62	2.55	0.79	0.53	0.68	TF	TF
10	0.49	0.28	0.34	0.02	0.41	1.27	0.47	0.44	0.68	TF	TR
11	0.76	0.32	0.25	0.11	0.65	3.95	1.18	0.77	0.96	TF	TF
12	0.83	0.65	0.02	0.12	0.57	3.24	0.98	0.94	1.25	TF	TR
13	0.59	0.35	0.19	0.21	0.39	2.49	0.77	0.56	0.89	TF	TR
14	0.18	0.65	0.23	0.21	0.68	0.5	0.32	0.3	0.36	TF	TR
15	0.14	0.66	0.15	0.09	0.59	2.14	0.68	0.58	0.76	TF	TR
16	0.56	0.43	0.3	0.2	0.15	2.12	0.68	0.47	1.22	TF	TR
17	0.41	0.23	0.19	0.32	0.42	1.5	0.52	0.43	0.67	TF	TR
18	0.77	0.57	0.2	0.16	0.34	1.76	0.58	0.55	0.94	TF	TR
19	0.5	0.68	0.29	0.3	0.04	2.47	0.77	0.45	2.12	TF	TR
20	0.02	0.39	0.28	0.16	0.94	2.84	0.87	0.74	0.77	TF	TF
21	0.52	0.49	0.39	0.16	0.34	3.05	0.93	0.7	1.2	TF	TR
22	0.53	0.3	0.2	0.24	0.47	3.32	1	0.72	1.06	TF	TR
23	0.1	0.05	0.14	0.28	0.66	2.67	0.82	0.63	0.78	TF	TF
24	0.51	0.53	0.15	0.38	0.26	1.02	0.41	0.38	0.76	TF	TR
25	0.57	0.35	0.09	0.25	0.54	1.2	0.45	0.4	0.54	TF	TR
26	0.55	0.25	0.16	0.19	0.38	3.13	0.95	0.54	0.87	TF	TF
27	0.31	0.16	0.08	0.25	0.49	1.84	0.6	0.38	0.55	TF	TF
28	0.24	0.89	0.17	0.13	0.97	0.35	0.31	0.26	0.27	TF	TF
29	0.32	0.4	0.18	0.2	0.17	2.34	0.74	0.37	0.88	TF	TR
30	0.2	0.82	0.04	0.33	0.3	1.8	0.59	0.53	0.97	TF	TR

Where

TF torsionally flexible

TR torsionally regular

3.3 ANALYTICAL REPRESENTATION OF THE SYSTEM

The systems used under this study are 3 story samples with vertical regularity and torsional irregularity. Story stiffness along x and y-direction assumed to be constant with uniform lumped mass at the geometric center. All the random samples under investigation are torsionally flexible (torsionally irregular). Penelis G. et al. (2014) defined a torsionally flexible system as a structural system wherein small eccentricities of the seismic horizontal forces cause large torsional deformations and excessive drifts at the perimeter columns of the system, disproportionate to those caused by the translational displacements.

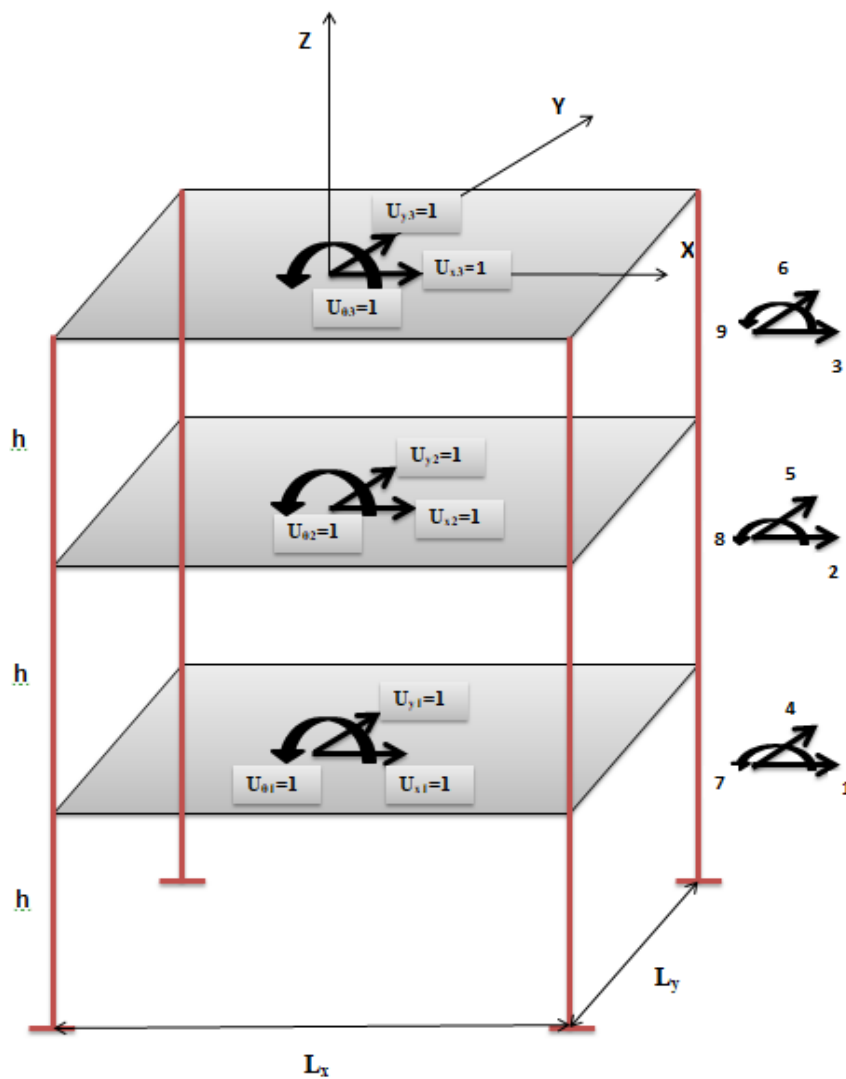


Figure 3-3 A 3 story system

The responses of each sample will be calculated using the mathematics of dynamic system i.e. Eigenvalue problem.

$$([stiffness] - \omega^2 [mass])[\phi] = [0] \quad \mathbf{3-12}$$

Where

ϕ The Eigen vector or the mode shape

ω The Eigen value or the frequency at which the mode shape occurs

Mode is property of a structure since it is calculated without any load applied to the structure. A mode is not a displacement at a certain natural frequency rather it is a shape.

After the mode shape and frequency are obtained the static responses of the structures can be determined using the following formulas.

$$M_n = \phi_n^T * m * \phi_n \quad \mathbf{3-13}$$

$$L_n = \phi_n^T * m * l \quad \mathbf{3-14}$$

$$\Gamma_n = \frac{L_n}{M_n} \quad \mathbf{3-15}$$

$$S_n = \Gamma_n * m * \phi_n \quad \mathbf{3-16}$$

$$V_{in}^{st} = \sum_{j=1}^N S_{jyn} \quad \mathbf{3-17}$$

$$T_{in}^{st} = \sum_{j=1}^N S_{j\theta n} \quad \mathbf{3-18}$$

$$V_{bn}^{st} = \sum_{j=1}^N S_{jyn} = \Gamma_n L_n = M_n^* \quad \mathbf{3-19}$$

$$T_{bn}^{st} = \sum_{j=1}^N S_{j\theta n} = I_{On}^* \quad \mathbf{3-20}$$

$$u_{jyn}^{st} = \left(\frac{\Gamma_n}{\omega_n^2} \right) * \phi_{jyn} \quad \mathbf{3-21}$$

$$u_{j\theta n}^{st} = \left(\frac{\Gamma_n}{\omega_n^2} \right) * \phi_{j\theta n} \quad \mathbf{3-22}$$

$$\Delta_{jyn}^{st} = \left(\frac{\Gamma_n}{\omega_n^2} \right) * (\phi_{jyn} - \phi_{j-1,yn}) \quad \mathbf{3-23}$$

Where

V_{in}^{st} The n^{th} mode static story shear at story i ;

s_{jyn} The n^{th} mode contribution to the spatial distribution of effective earthquake forces at story j ;

V_{bn}^{st} The n^{th} mode static base shear

$s_{j\theta n}$ The n^{th} mode contribution to the spatial distribution of effective earthquake forces at story j ;

T_{in}^{st} The n^{th} mode static story torque at story i ;

Γ_n Modal participation factor;

M_n^* Effective modal mass of the n^{th} mode;

T_{bn}^{st} The n^{th} mode static base torque;

I_{On}^* Effective modal mass moment of inertia of the n^{th} mode;

u_{jyn}^{st} The n^{th} mode static lateral displacement at story j ;

ϕ_{jyn} The translational component of the Eigen vector or the mode shape;

$u_{j\theta n}^{st}$ The n^{th} mode static rotation at story j ;

$\phi_{j\theta n}$ The rotational component of the Eigen vector or the mode shape;

Δ_{jyn}^{st} The n^{th} mode static story drift at story j

The static responses are calculated using the above formulas in MATLAB. The algorithms used are presented in APPENDIX A. Each random sample's static response is calculated as an envelope of different cases representing the effect of accidental eccentricity in each direction. The position of the center of mass will be shifted along the positive and negative direction in both orthogonal directions so that the stiffness matrix for each new position will be unique. The stiffness matrices of the systems are presented below.

$$stiffness I = \begin{pmatrix} 2K_x & -K_x & 0 & 0 & 0 & 0 & -2K_x * e_y & K_x * e_y & 0 \\ -K_x & 2K_x & -K_x & 0 & 0 & 0 & K_x * e_y & -2K_x * e_y & K_x * e_y \\ 0 & -K_x & K_x & 0 & 0 & 0 & 0 & K_x * e_y & -K_x * e_y \\ 0 & 0 & 0 & 2K_y & -K_y & 0 & -2B & B & 0 \\ 0 & 0 & 0 & -K_y & 2K_y & -K_y & B & -2B & B \\ 0 & 0 & 0 & 0 & -K_y & K_y & 0 & B & -B \\ -2K_x * e_y & K_x * e_y & 0 & -2B & B & 0 & A & -\frac{A}{2} & 0 \\ K_x * e_y & -2K_x * e_y & K_x * e_y & B & -2B & B & -\frac{A}{2} & A & -\frac{A}{2} \\ 0 & K_x * e_y & -K_x * e_y & 0 & B & -B & 0 & -\frac{A}{2} & \frac{A}{2} \end{pmatrix}$$

Where

$$A = K_x * \left(\frac{D_y^2}{2} + 2e_y^2 \right) + K_y * \left(\frac{D_x^2}{2} + 2e_a^2 - 4e_a * e_x + 2e_x^2 \right)$$

$$B = K_y * (e_a - e_x)$$

$$stiffness2 = \begin{pmatrix} 2K_x & -K_x & 0 & 0 & 0 & 0 & 2D & -D & 0 \\ -K_x & 2K_x & -K_x & 0 & 0 & 0 & -D & 2D & -D \\ 0 & -K_x & K_x & 0 & 0 & 0 & 0 & -D & D \\ 0 & 0 & 0 & 2K_y & -K_y & 0 & 2K_y * e_x & -K_y * e_x & 0 \\ 0 & 0 & 0 & -K_y & 2K_y & -K_y & -K_y * e_x & 2K_y * e_x & -K_y * e_x \\ 0 & 0 & 0 & 0 & -K_y & K_y & 0 & -K_y * e_x & K_y * e_x \\ 2D & -D & 0 & 2K_y * e_x & -K_y * e_x & 0 & C & -\frac{C}{2} & 0 \\ -D & 2D & -D & -K_y * e_x & 2K_y * e_x & -K_y * e_x & -\frac{C}{2} & C & -\frac{C}{2} \\ 0 & -D & D & 0 & -K_y * e_x & K_y * e_x & 0 & -\frac{C}{2} & \frac{C}{2} \end{pmatrix}$$

Where

$$C = K_x * \left(\frac{D_y^2}{2} + 2e_a^2 - 4e_a * e_y + 2e_y^2 \right) + K_y * \left(\frac{D_x^2}{2} + 2e_x^2 \right)$$

$$D = K_x * (e_a - e_y)$$

$$stiffness3 = \begin{pmatrix} 2K_x & -K_x & 0 & 0 & 0 & 0 & -2K_x * e_y & K_x * e_y & 0 \\ -K_x & 2K_x & -K_x & 0 & 0 & 0 & K_x * e_y & -2K_x * e_y & K_x * e_y \\ 0 & -K_x & K_x & 0 & 0 & 0 & 0 & K_x * e_y & -K_x * e_y \\ 0 & 0 & 0 & 2K_y & -K_y & 0 & 2F & -F & 0 \\ 0 & 0 & 0 & -K_y & 2K_y & -K_y & -F & 2F & -F \\ 0 & 0 & 0 & 0 & -K_y & K_y & 0 & -F & F \\ -2K_x * e_y & K_x * e_y & 0 & 2F & -F & 0 & E & -\frac{E}{2} & 0 \\ K_x * e_y & -2K_x * e_y & K_x * e_y & -F & 2F & -F & -\frac{E}{2} & E & -\frac{E}{2} \\ 0 & K_x * e_y & -K_x * e_y & 0 & -F & F & 0 & -\frac{E}{2} & \frac{E}{2} \end{pmatrix}$$

Where

$$E = K_x * \left(\frac{D_y^2}{2} + 2e_y^2 \right) + K_y * \left(\frac{D_x^2}{2} + 2e_a^2 + 4e_a * e_x + 2e_x^2 \right)$$

$$F = K_y * (e_a + e_x)$$

$$stiffness4 = \begin{pmatrix} 2K_x & -K_x & 0 & 0 & 0 & 0 & -2H & H & 0 \\ -K_x & 2K_x & -K_x & 0 & 0 & 0 & H & -2H & H \\ 0 & -K_x & K_x & 0 & 0 & 0 & 0 & H & -H \\ 0 & 0 & 0 & 2K_y & -K_y & 0 & 2K_y * e_x & -K_y * e_x & 0 \\ 0 & 0 & 0 & -K_y & 2K_y & -K_y & -K_y * e_x & 2K_y * e_x & -K_y * e_x \\ 0 & 0 & 0 & 0 & -K_y & K_y & 0 & -K_y * e_x & K_y * e_x \\ -2H & H & 0 & 2K_y * e_x & -K_y * e_x & 0 & G & -\frac{G}{2} & 0 \\ H & -2H & H & -K_y * e_x & 2K_y * e_x & -K_y * e_x & -\frac{G}{2} & G & -\frac{G}{2} \\ 0 & H & -H & 0 & -K_y * e_x & K_y * e_x & 0 & -\frac{G}{2} & \frac{G}{2} \end{pmatrix}$$

Where

$$G = K_x * \left(\frac{D_y^2}{2} + 2e_a^2 + 4e_a * e_y + 2e_y^2 \right) + K_y * \left(\frac{D_x^2}{2} + 2e_x^2 \right)$$

$$H = K_x * (e_a + e_y)$$

The stiffness properties of the system under study are not symmetric about both orthogonal directions. So that the system's response to the x component and y component of ground motion are not restricted to the lateral displacement in the x and y-direction respectively but includes rotation about the vertical axis. This coupled response is achieved through the stiffness matrix. Underneath, the overall procedure that is used to calculate the static response of random sample 1 and case 1 is presented following the verification of MATLAB that is used for the analysis of the analytical models.

3.3.1 Verification of MATLAB

The modal analysis of a five-story frame shown below has been solved in Chopra K. (2012) Dynamics of structures. Natural periods, base shear, the top story shear and the top story displacement obtained in the book are the same as the result of MATLAB.

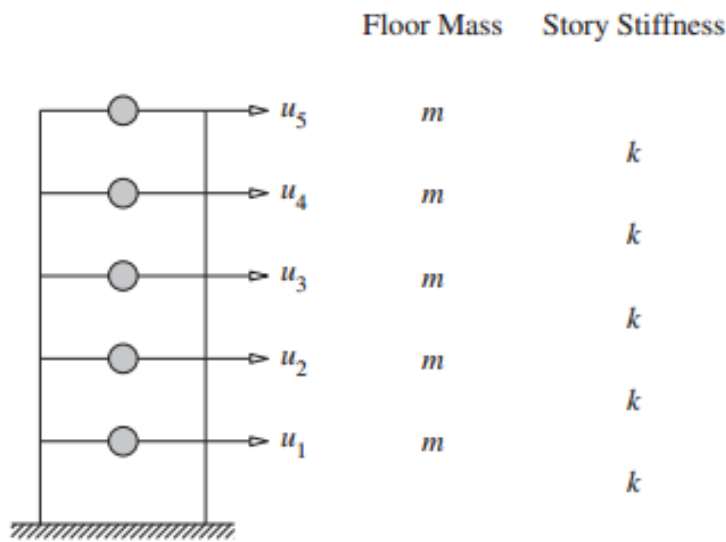


Figure 3-4 Five story frame

Table 3-3 Modal static responses obtained using MATLAB and referred from Chopra K. (2012)

Mode	Period (sec)	Base shear (lb.sec ² /ft)	Top story shear (lb.sec ² /ft)	Top story displacement (sec ²)
1	2.0010	4.398	1.252	0.127
2	0.6852	0.436	-0.362	-0.004
3	0.4346	0.121	0.159	0.0008
4	0.3383	0.037	-0.063	-0.0002
5	0.2966	0.008	0.004	0.00003

The conformity of the two results can be used as a verification of MATLAB. Therefore the subsequent analysis results of MATLAB can be taken as valid.

3.3.2 Static Response of Random Sample 1 Case 1

$$mass = \begin{pmatrix} M & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{M(L_x^2 + L_y^2)}{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{M(L_x^2 + L_y^2)}{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{M(L_x^2 + L_y^2)}{12} \end{pmatrix}$$

$$D_x = 0.2899 * L_x;$$

$$D_y = 1.29438 * L_x;$$

$$e_x = 0.2419 * L_x;$$

$$e_y = 0.23515 * L_x;$$

$$K_x = 0.30980 * K_y;$$

$$K_y = 1 * K_y;$$

$$L_x = 1 * L_x;$$

$$L_y = 2.58980 * L_x;$$

$$e_a = 0.05 * L_x;$$

$$[mode, \omega^2] = eig(inv(mass) * stiffness)$$

$$mode = \begin{pmatrix} 0.26 * L_x & -0.56 * L_x & -0.04 * L_x & -0.72 * L_x & 1.55 * L_x & 1.04 * L_x & -2.23 * L_x & 0.12 * L_x & -0.17 * L_x \\ 0.46 * L_x & -0.99 * L_x & -0.08 * L_x & -0.32 * L_x & 0.69 * L_x & -1.29 * L_x & 2.79 * L_x & 0.05 * L_x & 0.21 * L_x \\ 0.58 * L_x & -1.24 * L_x & -0.10 * L_x & 0.58 * L_x & -1.24 * L_x & 0.58 * L_x & -1.24 * L_x & -0.10 * L_x & -0.10 * L_x \\ -0.11 * L_x & -0.14 * L_x & 1.11 * L_x & 0.29 * L_x & 0.38 * L_x & -0.42 * L_x & -0.55 * L_x & -3.12 * L_x & 4.50 * L_x \\ -0.19 * L_x & -0.24 * L_x & 2.01 * L_x & 0.13 * L_x & 0.17 * L_x & 0.53 * L_x & 0.68 * L_x & -1.39 * L_x & -5.62 * L_x \\ -0.24 * L_x & -0.30 * L_x & 2.50 * L_x & -0.24 * L_x & -0.30 * L_x & -0.24 * L_x & -0.30 * L_x & 2.50 * L_x & 2.50 * L_x \\ 0.45 & 0.45 & 0.45 & -1.25 & -1.25 & 1.80 & 1.80 & -1.25 & 1.80 \\ 0.80 & 0.80 & 0.80 & -0.56 & -0.56 & -2.25 & -2.25 & -0.56 & -2.25 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$Period = \frac{2\pi}{\sqrt{\omega^2}}$$

$$\text{Period} = \begin{pmatrix} 32.99 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 23.26 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13.61 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11.78 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.30 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8.15 \sqrt{\frac{M}{K_y}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5.75 \sqrt{\frac{M}{K_y}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.86 \sqrt{\frac{M}{K_y}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.36 \sqrt{\frac{M}{K_y}} \end{pmatrix}$$

$$\text{Modal mass} = \text{mode}^T * \text{mass} * \text{mode}$$

$$\text{Modal mass} = L_x^2 * M * \begin{pmatrix} 1.89 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12.71 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.95 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9.56 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 21.11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 19.77 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 64.19 \end{pmatrix}$$

$$\text{influence factor} = \begin{pmatrix} 1 & 0.3 \\ 1 & 0.3 \\ 1 & 0.3 \\ 0.3 & 1 \\ 0.3 & 1 \\ 0.3 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$L_n = \text{mode}^T * \text{mass} * \text{influence factor}$$

$$L_n = L_x * M * \begin{pmatrix} 1.14 \\ -2.99 \\ 1.47 \\ -0.41 \\ 1.07 \\ 0.28 \\ -0.74 \\ -0.53 \\ 0.36 \end{pmatrix}$$

$$\text{modal participation factor} = \frac{L_{n,i,i}}{\text{Modalmass}L_{n,i,i}}$$

$$\text{modal participation factor} = \frac{1}{L_x} \begin{pmatrix} 0.60 \\ -0.72 \\ 0.12 \\ -0.14 \\ 0.16 \\ 0.03 \\ -0.03 \\ -0.03 \\ 0.01 \end{pmatrix}$$

$$\text{Modalmassdistribution(MMD)} = \text{modal participation factor} * \text{subs}(\text{mass}) * \text{mode}$$

$$MMD_1 = M * \begin{pmatrix} 0.154 \\ 0.277 \\ 0.345 \\ -0.063 \\ -0.113 \\ -0.141 \\ 0.171 * L_x \\ 0.309 * L_x \\ 0.385 * L_x \end{pmatrix} \qquad MMD_2 = M * \begin{pmatrix} 0.394 \\ 0.711 \\ 0.886 \\ 0.097 \\ 0.174 \\ 0.217 \\ -0.204 * L_x \\ -0.368 * L_x \\ -0.459 * L_x \end{pmatrix}$$

$$MMD_3 = M * \begin{pmatrix} -0.005 \\ -0.009 \\ -0.011 \\ 0.129 \\ 0.232 \\ 0.290 \\ 0.033 * L_x \\ 0.060 * L_x \\ 0.074 * L_x \end{pmatrix} \qquad MMD_4 = M * \begin{pmatrix} 0.099 \\ 0.044 \\ -0.079 \\ -0.040 \\ -0.018 \\ 0.032 \\ 0.110 * L_x \\ 0.049 * L_x \\ -0.088 * L_x \end{pmatrix}$$

$$MMD_5 = M * \begin{pmatrix} 0.254 \\ 0.113 \\ -0.203 \\ 0.0622 \\ 0.028 \\ -0.050 \\ -0.131 * L_x \\ -0.058 * L_x \\ 0.105 * L_x \end{pmatrix} \qquad MMD_6 = M * \begin{pmatrix} 0.030 \\ -0.038 \\ 0.017 \\ -0.012 \\ 0.015 \\ -0.007 \\ 0.034 * L_x \\ -0.042 * L_x \\ 0.019 * L_x \end{pmatrix}$$

$$MMD_7 = M * \begin{pmatrix} 0.078 \\ -0.097 \\ 0.043 \\ 0.019 \\ -0.024 \\ 0.011 \\ -0.040 * L_x \\ 0.050 * L_x \\ -0.022 * L_x \end{pmatrix} \qquad MMD_8 = M * \begin{pmatrix} -0.003 \\ -0.001 \\ 0.003 \\ 0.083 \\ 0.037 \\ -0.066 \\ 0.021 * L_x \\ 0.009 * L_x \\ -0.017 * L_x \end{pmatrix}$$

$$MMD_9 = M * \begin{pmatrix} -0.001 \\ 0.001 \\ -0.001 \\ 0.026 \\ -0.032 \\ 0.014 \\ 0.007 * L_x \\ -0.008 * L_x \\ 0.004 * L_x \end{pmatrix}$$

$$Effective\ modal\ mass = \sum_{n=1}^N \text{Modalmassdistribution}_n$$

$$Effective\ modal\ mass11 = M * \begin{pmatrix} 0.7752 & -0.3164 & 0.8648 * L_x \\ 1.9918 & 0.4883 & -1.0319 * L_x \\ -0.0247 & 0.6507 & 0.16713 * L_x \\ 0.0635 & -0.0259 & 0.0708 * L_x \\ 0.1632 & 0.0400 & -0.0845 * L_x \\ 0.0094 & -0.0038 & 0.0104 * L_x \\ 0.0241 & 0.0059 & -0.0125 * L_x \\ -0.0020 & 0.0533 & 0.0137 * L_x \\ -0.0003 & 0.0079 & 0.0020 * L_x \end{pmatrix}$$

$$\Sigma \text{ Effective modal mass11} = (3 * M \quad 0.9 * M \quad 0)$$

$$U_i = \frac{\text{modalparticipationfactor}_{i,i}}{\omega_{i,i}^2} * \text{mode}(:,i)$$

$$U_{11mode1} = \frac{M}{K_y} * \begin{pmatrix} 4.230 \\ 7.622 \\ 9.505 \\ -1.726 \\ -3.111 \\ -3.879 \\ \frac{7.348}{L_x} \\ \frac{13.240}{L_x} \\ \frac{16.510}{L_x} \end{pmatrix} \qquad U_{11mode2} = \frac{M}{K_y} * \begin{pmatrix} 5.404 \\ 9.738 \\ 12.143 \\ 1.325 \\ 2.387 \\ 2.977 \\ \frac{-4.359}{L_x} \\ \frac{-7.855}{L_x} \\ \frac{-9.796}{L_x} \end{pmatrix}$$

$$U_{11mode3} = \frac{M}{K_y} * \begin{pmatrix} -0.023 \\ -0.041 \\ -0.052 \\ 0.604 \\ 1.089 \\ 1.358 \\ \frac{0.242}{L_x} \\ \frac{0.436}{L_x} \\ \frac{0.543}{L_x} \end{pmatrix} \qquad U_{11mode4} = \frac{M}{K_y} * \begin{pmatrix} 0.347 \\ 0.154 \\ -0.278 \\ -0.141 \\ -0.063 \\ 0.113 \\ \frac{0.602}{L_x} \\ \frac{0.268}{L_x} \\ \frac{-0.483}{L_x} \end{pmatrix}$$

$$U_{11mode5} = \frac{M}{K_y} * \begin{pmatrix} 0.443 \\ 0.197 \\ -0.355 \\ 0.109 \\ 0.048 \\ -0.087 \\ \frac{-0.357}{L_x} \\ \frac{-0.159}{L_x} \\ \frac{0.286}{L_x} \end{pmatrix} \qquad U_{11mode6} = \frac{M}{K_y} * \begin{pmatrix} 0.051 \\ -0.064 \\ 0.028 \\ -0.021 \\ 0.026 \\ -0.012 \\ \frac{0.089}{L_x} \\ \frac{-0.111}{L_x} \\ \frac{0.049}{L_x} \end{pmatrix}$$

$$U_{11mode7} = \frac{M}{K_y} * \begin{pmatrix} 0.065 \\ -0.081 \\ 0.036 \\ 0.016 \\ -0.020 \\ 0.009 \\ -0.053 \\ \frac{L_x}{L_x} \\ 0.066 \\ \frac{L_x}{L_x} \\ -0.029 \\ \frac{L_x}{L_x} \end{pmatrix} \qquad U_{11mode8} = \frac{M}{K_y} * \begin{pmatrix} -0.002 \\ -0.001 \\ 0.002 \\ 0.050 \\ 0.022 \\ -0.040 \\ 0.020 \\ \frac{L_x}{L_x} \\ 0.009 \\ \frac{L_x}{L_x} \\ -0.016 \\ \frac{L_x}{L_x} \end{pmatrix}$$

$$U_{11mode9} = \frac{M}{K_y} * \begin{pmatrix} -0.0003 \\ 0.0004 \\ -0.0002 \\ 0.0073 \\ -0.0091 \\ 0.0041 \\ 0.0029 \\ \frac{L_x}{L_x} \\ -0.0036 \\ \frac{L_x}{L_x} \\ 0.0016 \\ \frac{L_x}{L_x} \end{pmatrix}$$

The lateral displacement and rotation of each mode can be displayed diagrammatically as shown below.

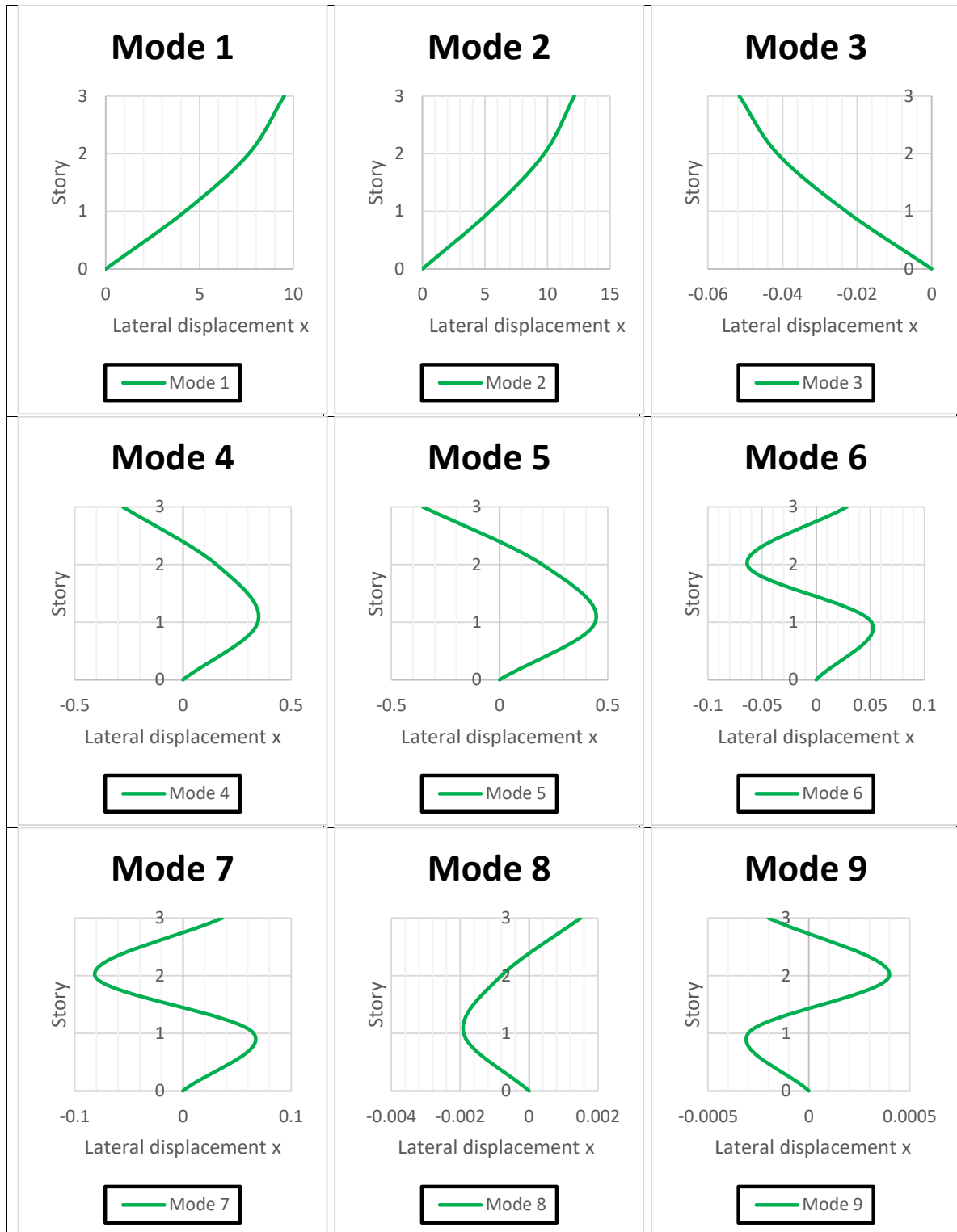


Figure 3-5 Modal expansion of lateral displacement along x-direction

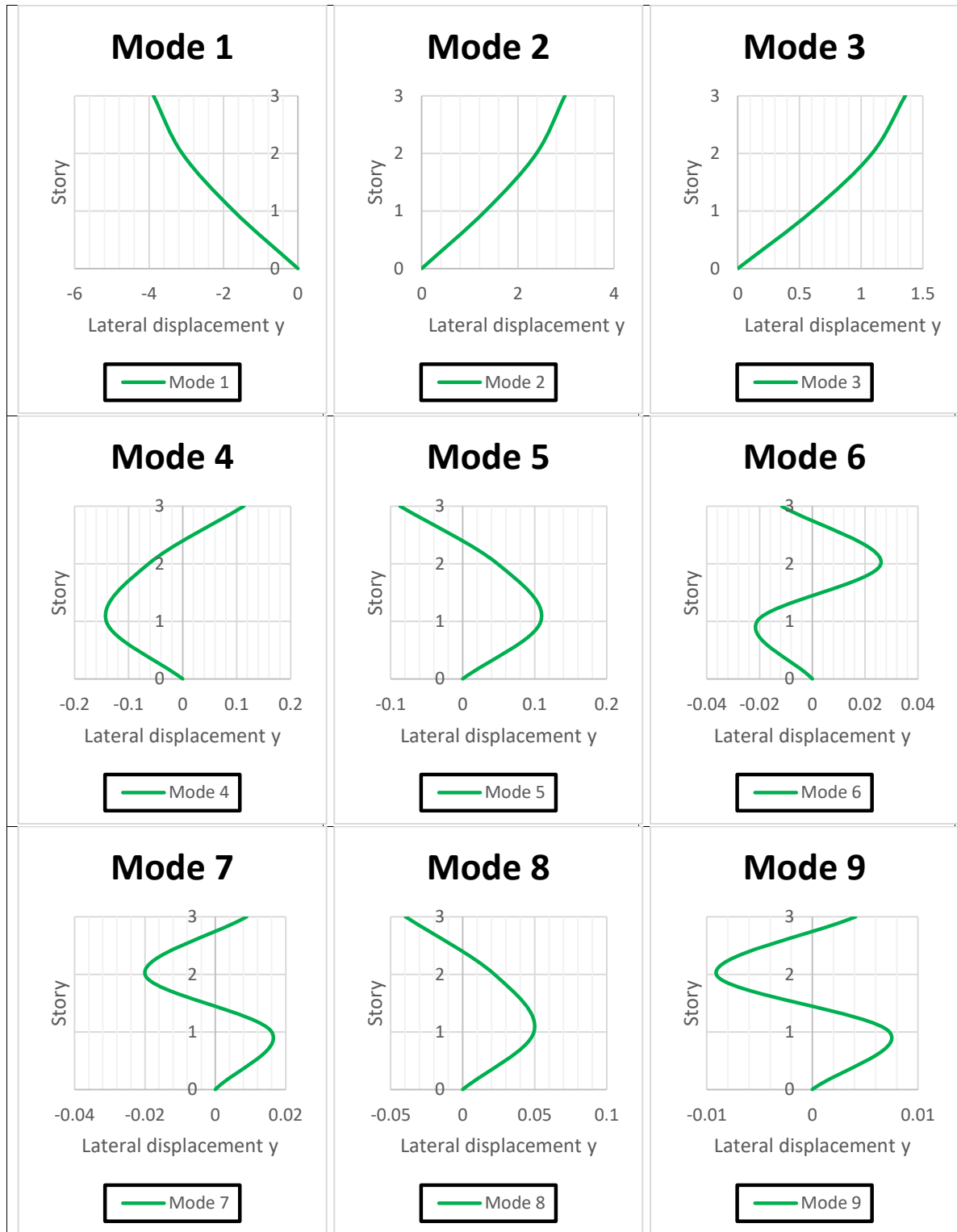


Figure 3-6 Modal expansion of lateral displacement along y-direction

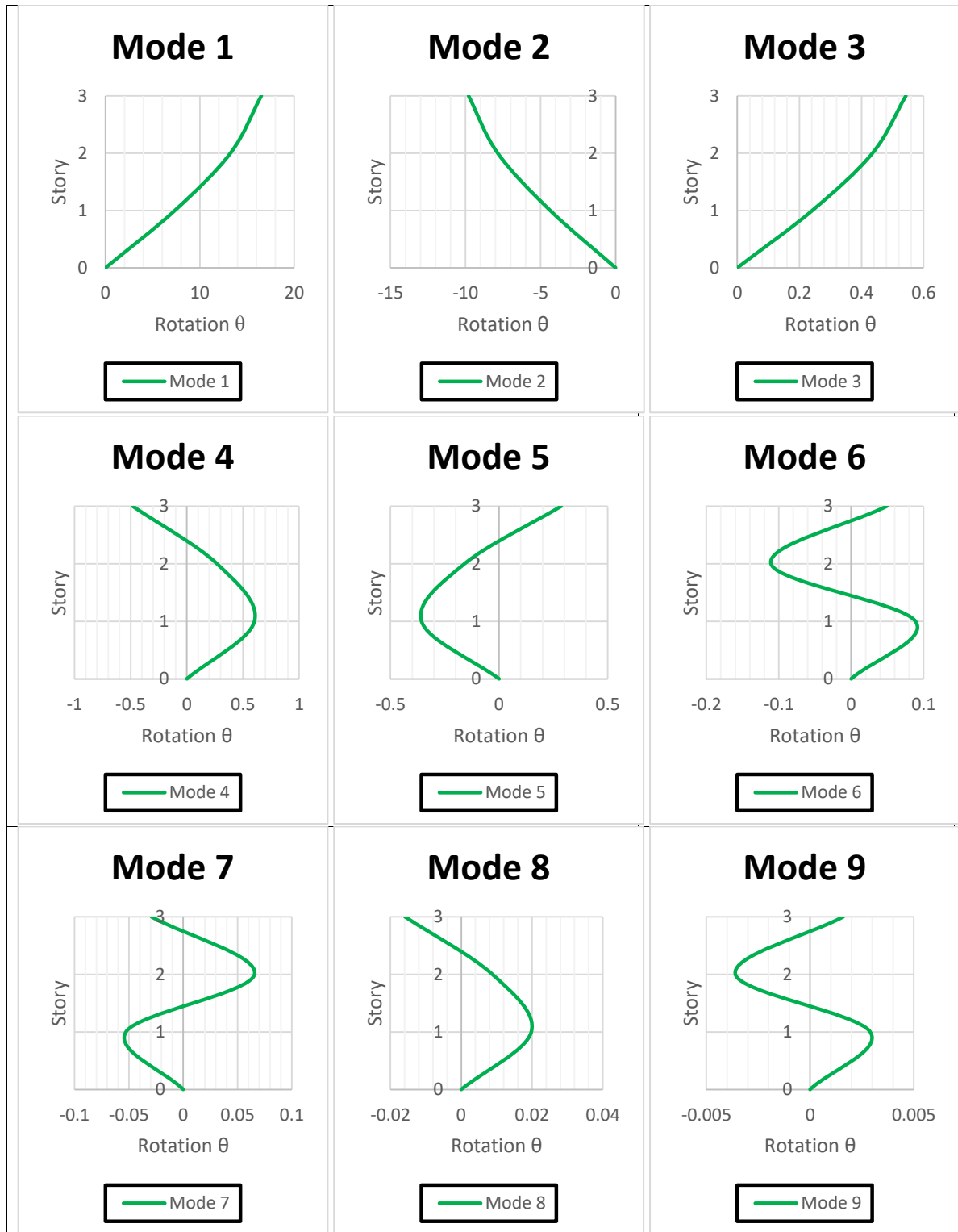


Figure 3-7 Modal expansion of rotation

After calculating the static response of each mode, the total response shall be obtained using

$$r_n(t) = r_n^{st} * A_n(t) \quad \mathbf{3-24}$$

$$r(t) = \sqrt{\sum_{i=1}^N \sum_{n=1}^N \rho_{in} * r_i(t) * r_n(t)} \quad \mathbf{3-25}$$

$$\rho_{in} = \frac{8\zeta^2 * (1 + \beta_{in}) * \beta_{in}^{3/2}}{(1 - \beta_{in}^2)^2 + 4\zeta^2 * \beta_{in} * (1 + \beta_{in})^2} \quad \mathbf{3-26}$$

$$\beta_{in} = \frac{\omega_i}{\omega_n} \quad \mathbf{3-27}$$

Where

$r_n(t)$ The total modal response

$r(t)$ The total response

r_n^{st} The modal static response

$A_n(t)$ Pseudo acceleration

i, n i^{th} and n^{th} modes

ζ Coefficient of damping

After completing the response determination using the response spectrum analysis method, the responses are calculated using equivalent static analysis method as follows.

$$\text{Lateral displacement}_{11} = \text{Flexibility}_{11} * \text{Lateral force}_{11} \quad \mathbf{3-28}$$

$$\text{Flexibility}_{11} = (\text{stiffness}_{11})^{-1} \quad \mathbf{3-29}$$

$$Lateralforce_{1i} = \sum_{j=1}^N m_j * \frac{m_i * h_i}{\sum_{j=1}^N m_j * h_j} \quad \mathbf{3-30}$$

$$Lateralforce_{1i} = \begin{pmatrix} 0.5M \\ 1M \\ 1.5M \\ 0.15M \\ 0.3M \\ 0.45M \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad Lateraldisplacement_{1i} = M * \begin{pmatrix} 10.515 \\ 19.277 \\ 24.534 \\ 0.2219 \\ 0.4068 \\ 0.5178 \\ \frac{3.533}{L_x} \\ 6.478 \\ \frac{L_x}{L_x} \\ 8.245 \\ \frac{L_x}{L_x} \end{pmatrix}$$

This lateral displacement is a static response of the system. Therefore equation 3-24 shall be applied to get the total response.

CHAPTER 4 RESULT AND DISCUSSION

4.1 DISCUSSION OF THE ANALYTICAL MODEL

The responses to be discussed are calculated using the outlined procedure in chapter 4. The results obtained were lateral displacements along the two orthogonal directions and rotation about the vertical axis. These displacements further combined in Microsoft Excel to get the maximum lateral displacement, including the induced displacement effect of the rotation. The procedure used to combine the lateral displacement and rotation is discussed below.

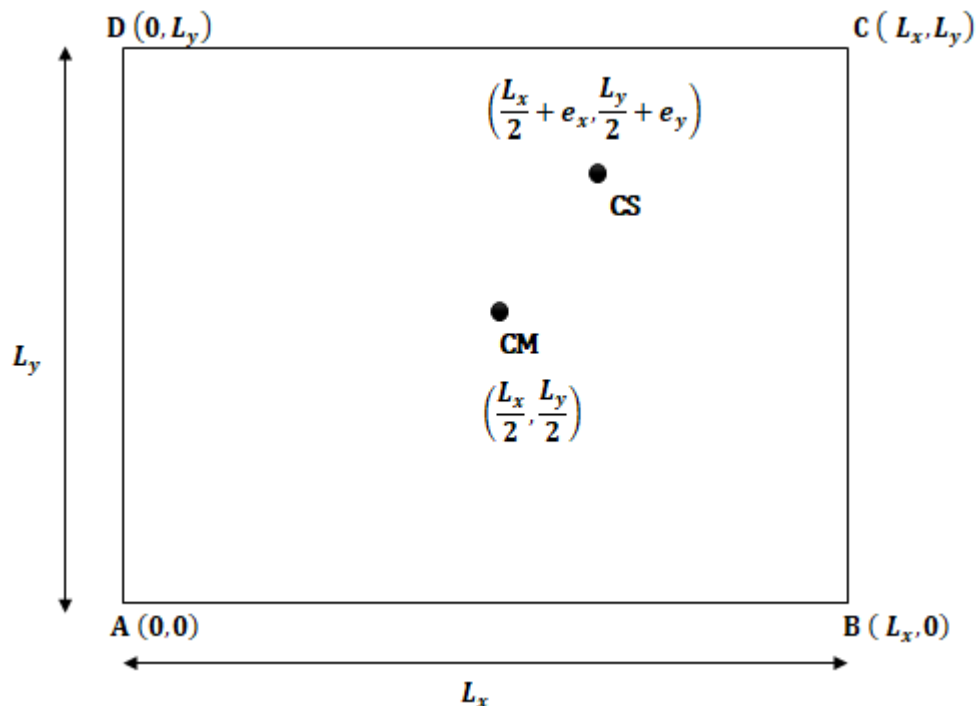


Figure 4-1 Coordinate of corner points, CM and CS

The above figure shows the coordinate points of each corner, center of mass and center of stiffness of the system under study. The lateral displacement obtained by rotating this rectangular system can be determined using the principle of rotation used in mathematics.

$$x' = x_o + (x - x_o) * \text{Cos}\theta - (y - y_o) * \text{Sin}\theta \quad \mathbf{4-1}$$

$$y' = y_o + (x - x_o) * \text{Sin}\theta + (y - y_o) * \text{Cos}\theta \quad \mathbf{4-2}$$

Where

x', y' The coordinate of the point after rotation;

x, y The coordinate of the point to be rotated;

x_o, y_o The coordinate of the center of rotation;

θ The angle of rotation

The new position of the corners after rotation will be added to the respective lateral displacement to find the maximum lateral displacement of the system. This step is performed for both ESA and RSA to compute the maximum lateral displacements. Almost all of the results show that ESA results in lesser response compared to the RSA method.

Since the RSA method is a reference method, the response computed using ESA shall not be less than the result of the reference method. Therefore the response of ESA shall be modified to make it equal to the response of RSA. To do this, the lateral displacement computed using ESA is subtracted from the maximum displacement computed using RSA to find the displacement that should be resulted from the ESA method due to the application of rotation to make the ESA result equal to RSA result. From this, a new rotation satisfying the above point can be formulated using the following formula. To adjust this, a modifying factor that magnifies the accidental eccentricity effect is proposed.

$$\text{For } a * \text{Cos}\theta + b * \text{Sin}\theta = c \quad \mathbf{4-3}$$

$$\theta = \text{Sin}^{-1} \left(\frac{c}{\sqrt{(a)^2 + (b)^2}} \right) - \text{Sin}^{-1} \left(\frac{a}{\sqrt{(a)^2 + (b)^2}} \right) \quad \mathbf{4-4}$$

$$\theta = \text{Sin}^{-1} \left(\frac{y' - y_o}{\sqrt{(y - y_o)^2 + (x - x_o)^2}} \right) - \text{Sin}^{-1} \left(\frac{y - y_o}{\sqrt{(y - y_o)^2 + (x - x_o)^2}} \right) \quad 4-5$$

$$\theta = \text{Sin}^{-1} \left(\frac{x' - x_o}{\sqrt{(x - x_o)^2 + (y_o - y)^2}} \right) - \text{Sin}^{-1} \left(\frac{x - x_o}{\sqrt{(x - x_o)^2 + (y_o - y)^2}} \right) \quad 4-6$$

Where

x', y' The coordinate of the point after rotation obtained by subtracting the lateral displacement computed using ESA from the maximum displacement computed using RSA;

θ The angle of rotation that shall be used in ESA

The ratio of the newly calculated rotation that should be used in ESA to the previously obtained value using the analytical model gives a factor α that magnifies the total eccentricity of the system i.e. static eccentricity \pm accidental eccentricity. However, this factor is not suitable for usage in the analysis of structures in analysis and design software. Because the accidental eccentricity is the one that is set to be user-defined, it is preferable to modify accidental eccentricity by a factor β rather than the total eccentricity using the procedure shown below.

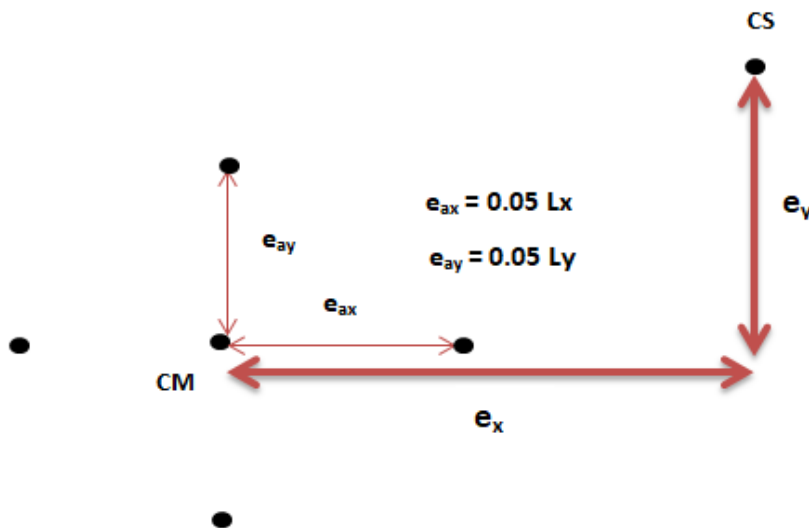


Figure 4-2 Position of CM and CS

$$\alpha * \sqrt{(e_x + 0.05L_x)^2 + (e_y + 0.05L_y)^2} = \beta * \sqrt{(0.05L_x)^2 + (0.05L_y)^2} + \sqrt{(e_x^2 + e_y^2)}$$

$$\beta = \frac{\alpha * \sqrt{e_x^2 + e_y^2 + 0.1 * (e_x * L_x + e_y * L_y) + 25 * 10^{-4} * (L_x^2 + L_y^2)} - \sqrt{e_x^2 + e_y^2}}{\sqrt{25 * 10^{-4} * (L_x^2 + L_y^2)}} \quad 4-7$$

Or

$$\alpha * \sqrt{(e_x - 0.05L_x)^2 + (e_y - 0.05L_y)^2} = -\beta * \sqrt{(0.05L_x)^2 + (0.05L_y)^2} + \sqrt{(e_x^2 + e_y^2)}$$

$$\beta = \frac{-\alpha * \sqrt{e_x^2 + e_y^2 - 0.1 * (e_x * L_x + e_y * L_y) + 25 * 10^{-4} * (L_x^2 + L_y^2)} + \sqrt{e_x^2 + e_y^2}}{\sqrt{25 * 10^{-4} * (L_x^2 + L_y^2)}} \quad 4-8$$

The modifying factor β is calculated for all soil types and the design spectrum. The modifying factors resulted ranges from 1.12 to 9.22. The modifying factor versus fundamental period for soil type A, B, C, D, and E with design spectrum 1 and 2 is shown in APPENDIX B.

Unlike EN 1998-1:2003, various codes set limits on torsional irregularity. When torsional irregularity beyond the limit occurs system revision will be conducted. In this paper, the modifying factors are calculated with the intention of adjusting the response of the equivalent static method without focusing on the extremity of the torsional irregularity considered. However, the maximum limit of the modifying factor can be adopted from the American Society of Civil Engineers Standard and International Building Code i.e. $\beta = 3$. From the graph of the modifying factor versus the fundamental period, 70% of the random samples show a modifying factor of less than 3. For instance the modifying factor of most random samples in soil type A and design spectrum type 1 fall between the black and green line in Figure 4-3. Therefore the accidental eccentricity shall be adjusted with proper modifying factor $\beta \leq 3$ to make the ESA response equal to the RSA. On the other hand, the layout of the rest of the samples with modifying factor greater than 3 shall be revised.

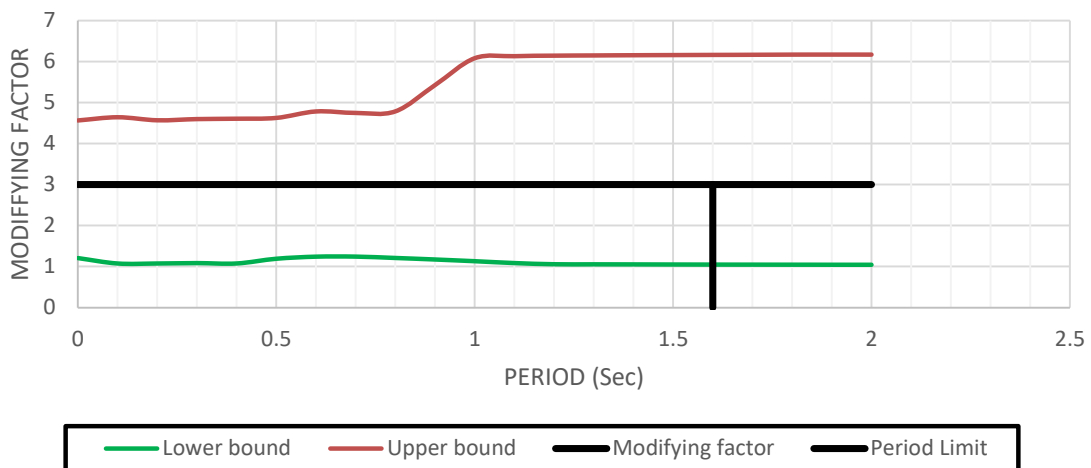


Figure 4-3 Modifying factor for soil type A design spectrum 1

4.2 RESULT OF THE ANALYTICAL MODEL

Only the rotation and lateral displacement of the random samples are used for comparative study although the responses generated under this research are base shear, story drift, rotation, and lateral displacement. The maximum lateral displacement along x-direction of random sample 1 under soil type A and design spectrum type 1 is presented for discussion. The result obtained using the equivalent static analysis method is modified using a factor β to make the response greater or equal to the response spectrum analysis.

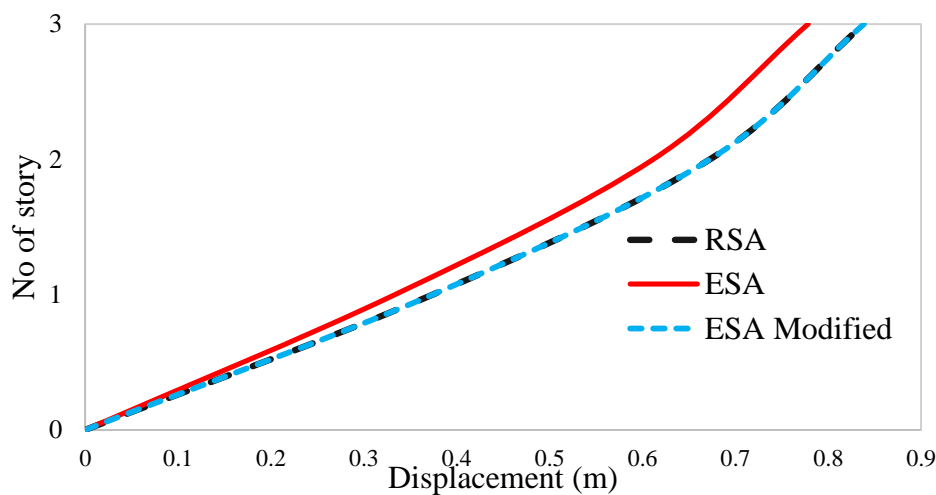


Figure 4-4 Displacement vs. number of story for period 0.1s

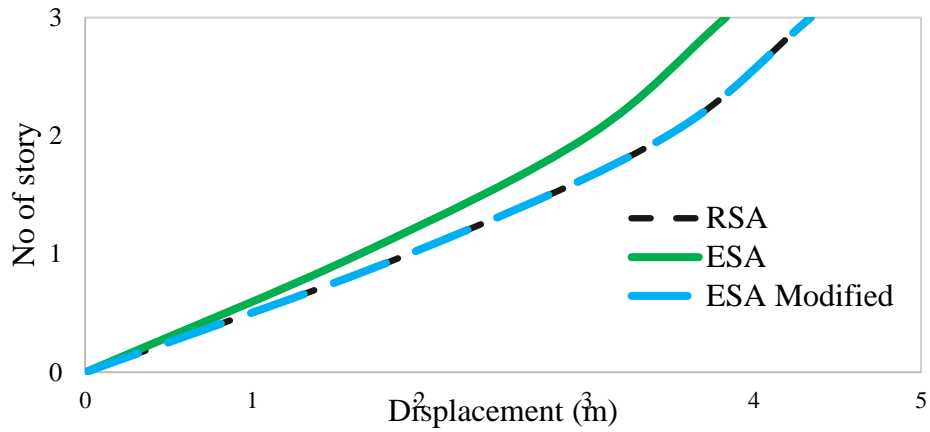


Figure 4-5 Displacement vs. number of story for period 0.2s

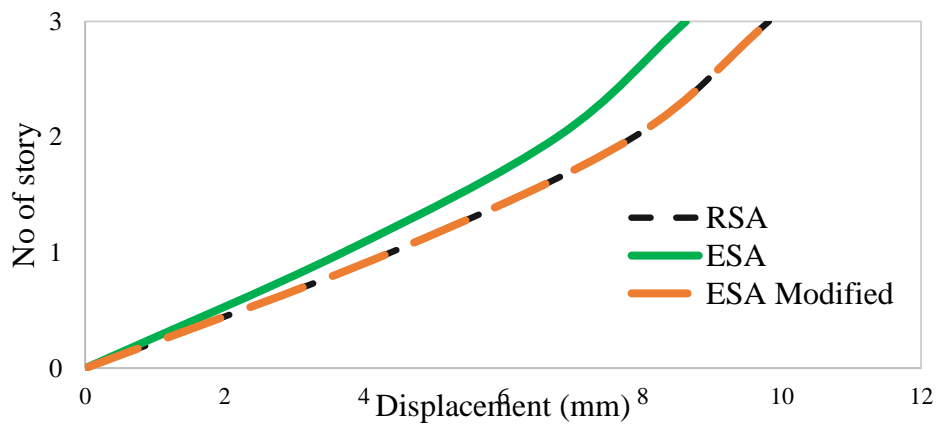


Figure 4-6 Displacement vs. number of story for period 0.3s

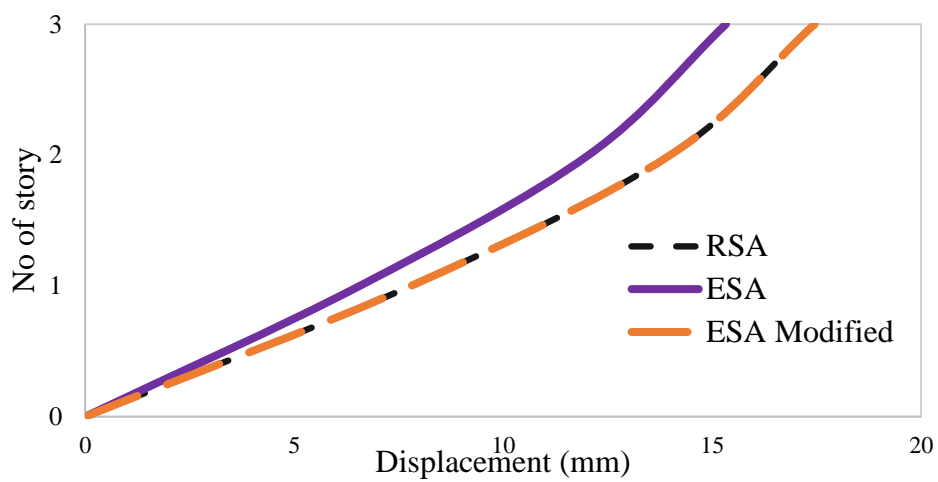


Figure 4-7 Displacement vs. number of story for period 0.4s

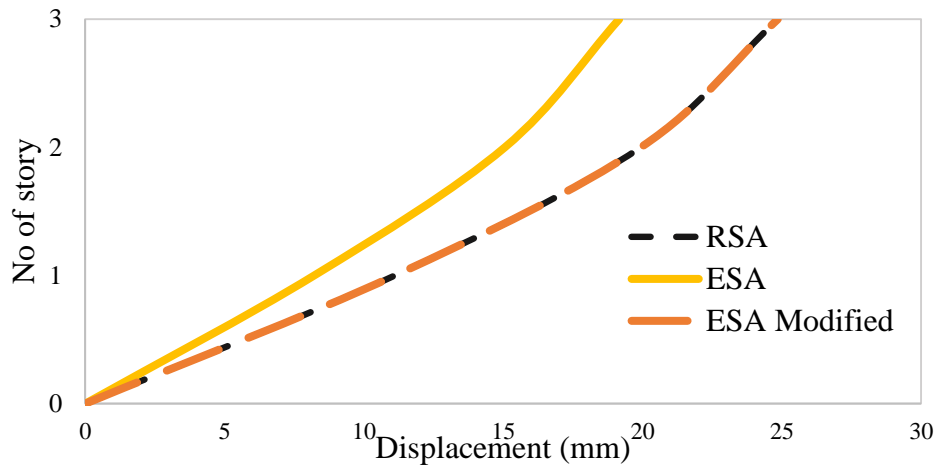


Figure 4-8 Displacement vs. number of story for period 0.5s

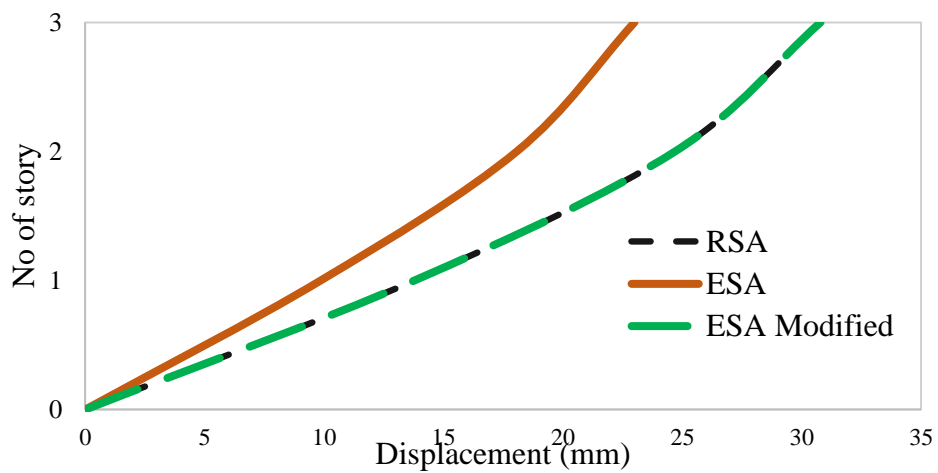


Figure 4-9 Displacement vs. number of story for period 0.6s

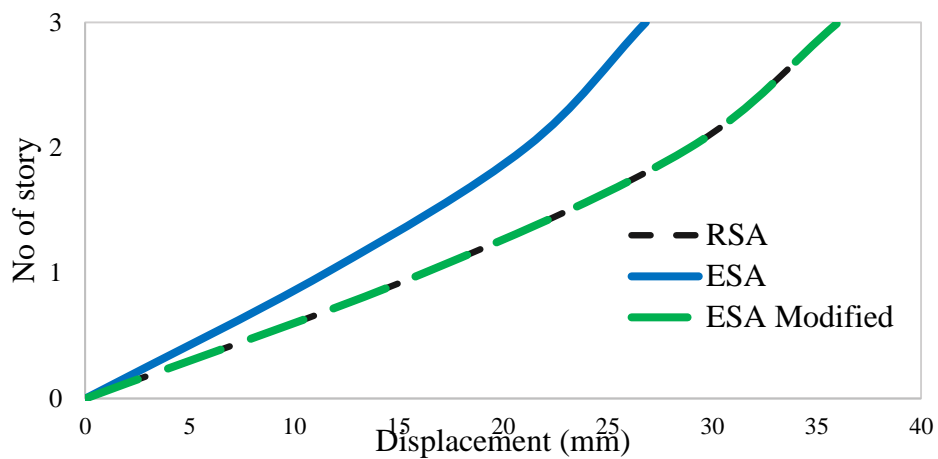


Figure 4-10 Displacement vs. number of story for period 0.7s

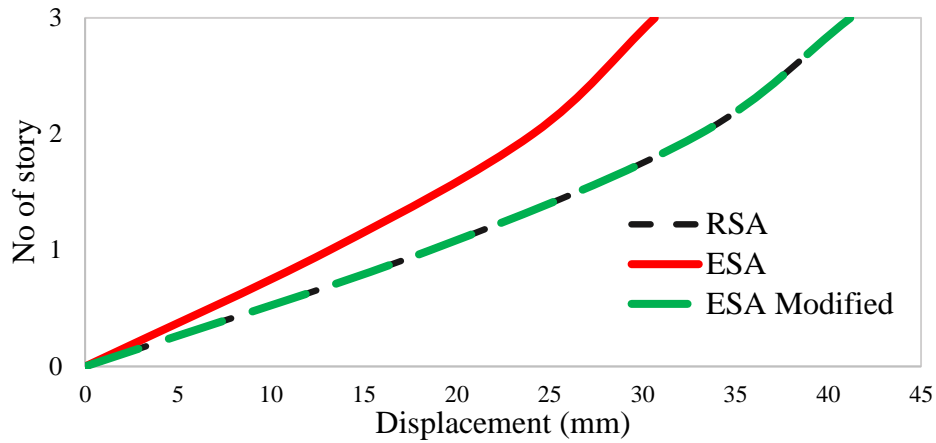


Figure 4-11 Displacement vs. number of story for period 0.8s

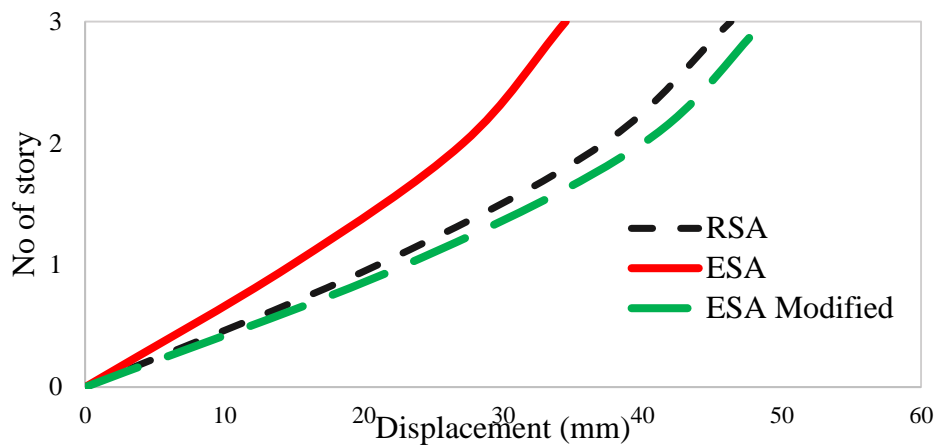


Figure 4-12 Displacement vs. number of story for period 0.9s

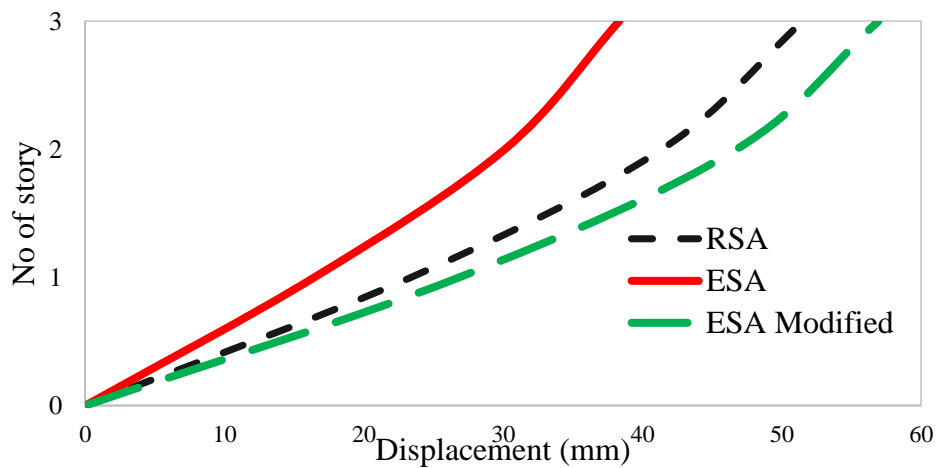


Figure 4-13 Displacement vs. number of story for period 1s

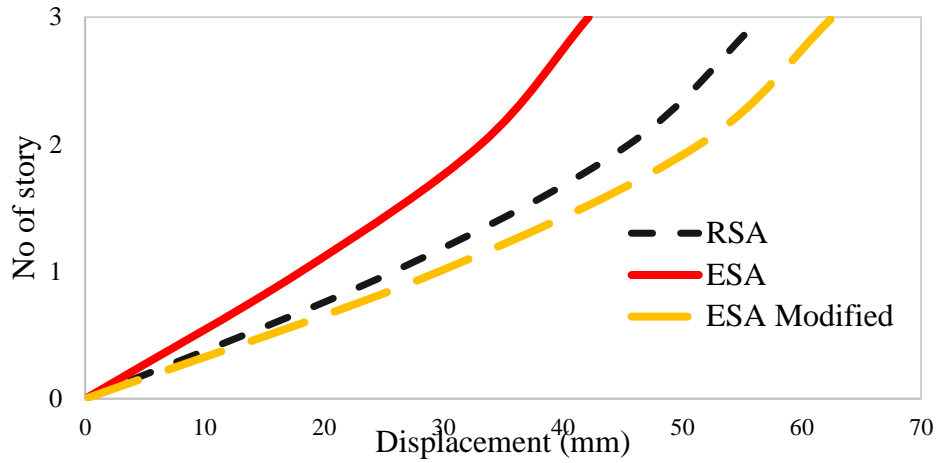


Figure 4-14 Displacement vs. number of story for period 1.1s

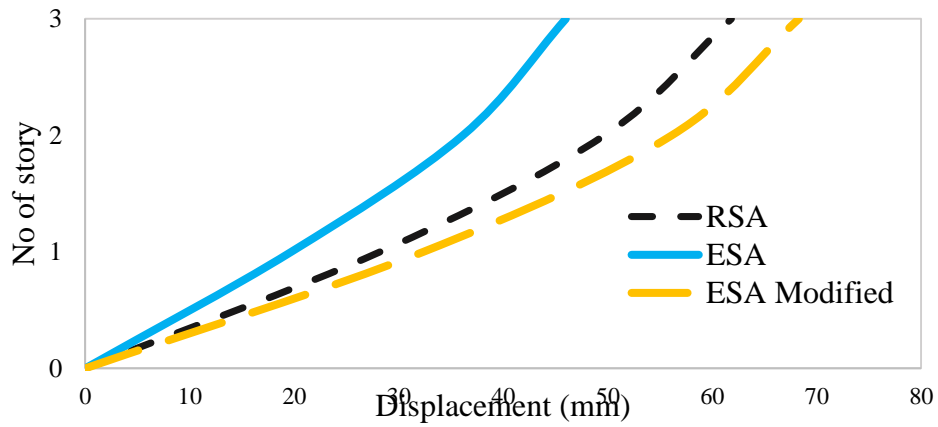


Figure 4-15 Displacement vs. number of story for period 1.2s

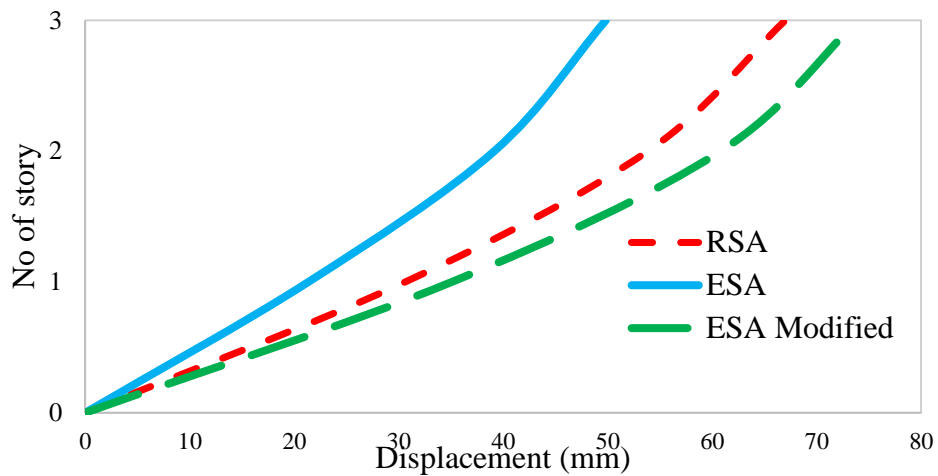


Figure 4-16 Displacement vs. number of story for period 1.3s

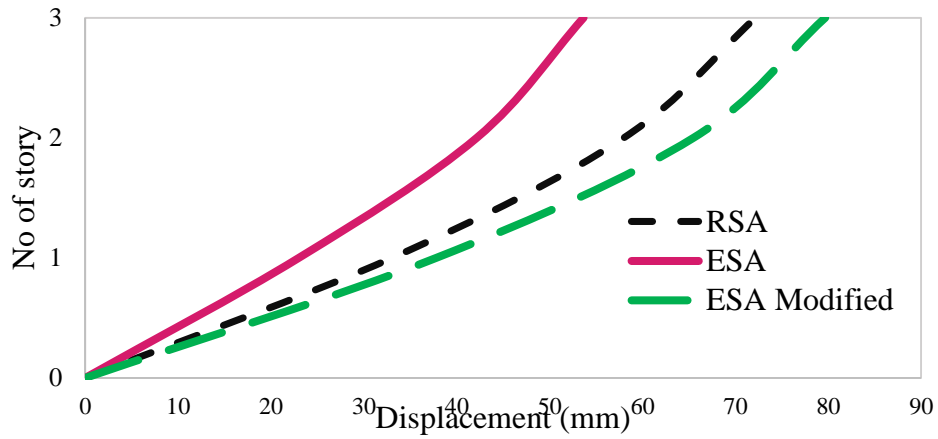


Figure 4-17 Displacement vs. number of story for period 1.4s

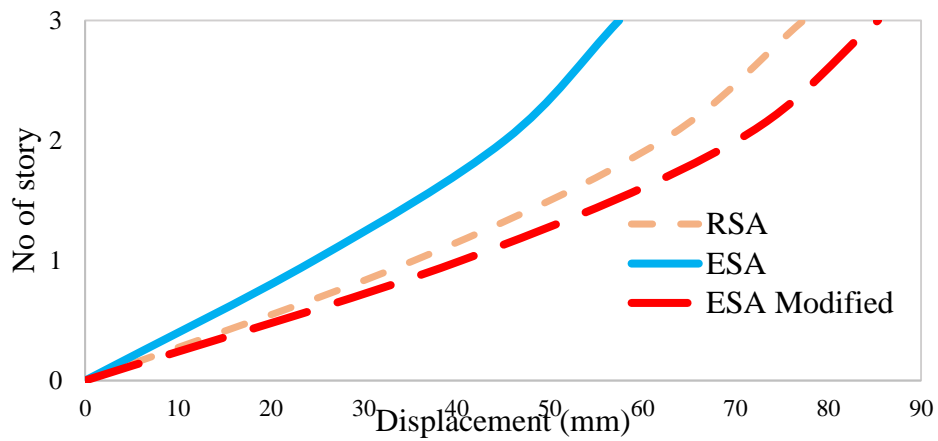


Figure 4-18 Displacement vs. number of story for period 1.5s

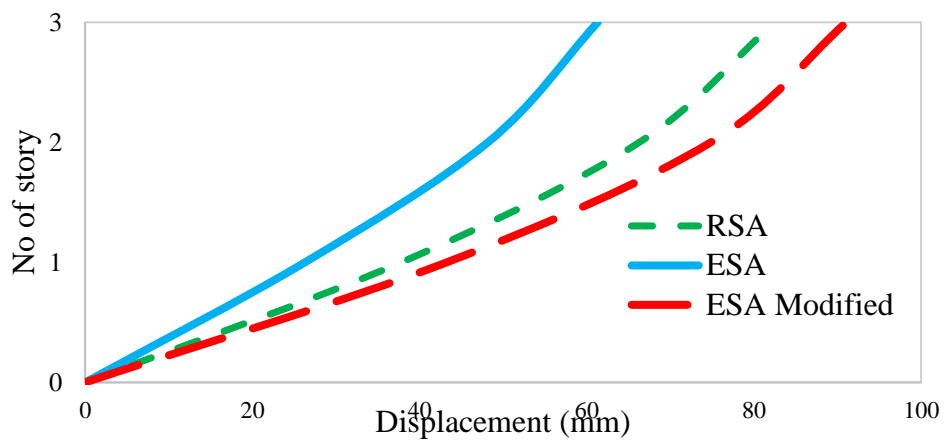


Figure 4-19 Displacement vs. number of story for period 1.6s

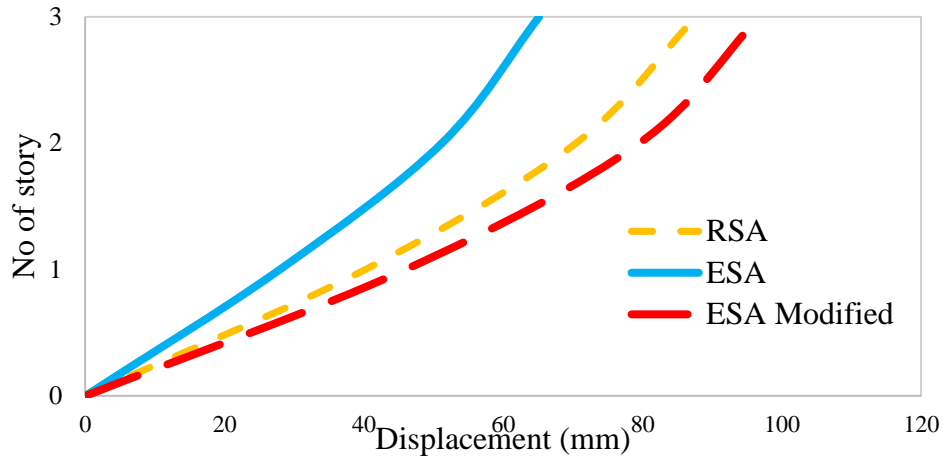


Figure 4-20 Displacement vs. number of story for period 1.7s

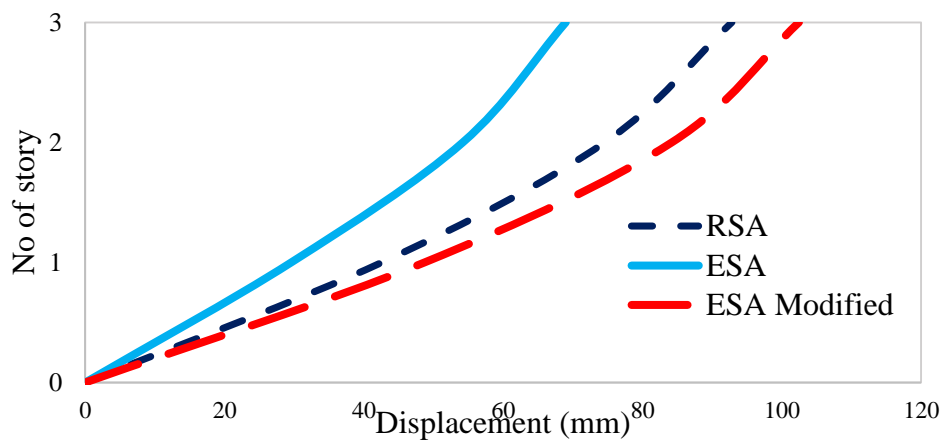


Figure 4-21 Displacement vs. number of story for period 1.8s

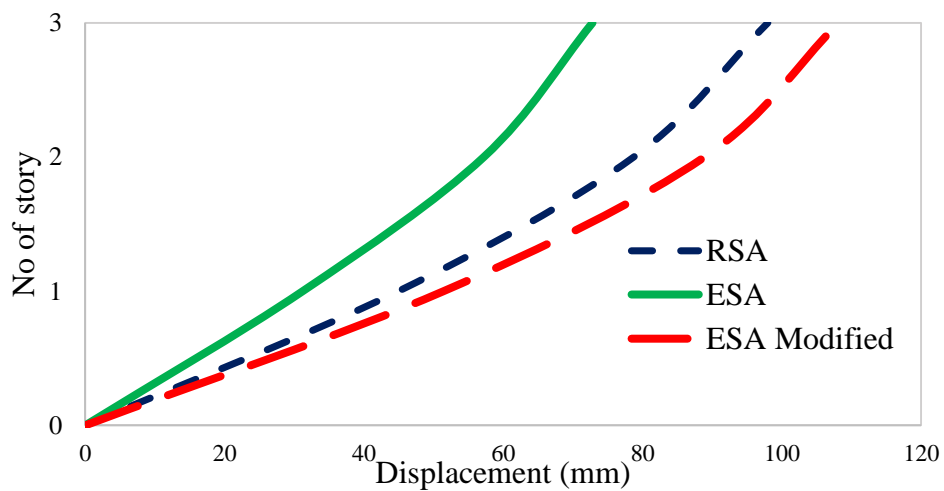


Figure 4-22 Displacement vs. number of story for period 1.9s

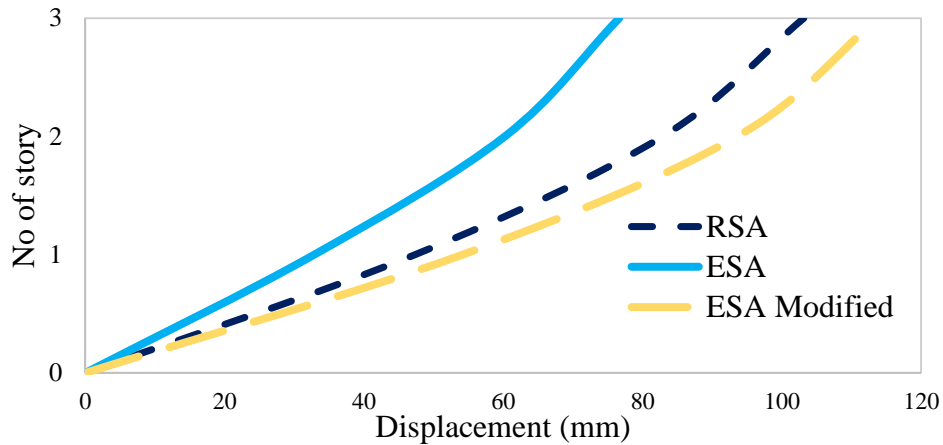


Figure 4-23 Displacement vs. number of story for period 2s

The plot of the number of story versus maximum lateral displacement along x-direction shows that the first systems whose fundamental period is less than 0.8 seconds have equal lateral displacements using the modified ESA and RSA. This means before any modification is applied the structures maximum lateral displacement along x-direction calculated using ESA was less than RSA result. Thus the modifying factor magnified the response of ESA to make it equal to RSA.

The plot from 0.8 seconds onwards, however, showed a larger maximum lateral displacement using modified ESA than RSA along the x-direction. This result shows the magnifying factor was obtained equating the y-direction response so that the x-direction responses become larger than the RSA result.

The summary of the above plots for random sample 1 using soil type A and design spectrum type 1 is illustrated in the following figures.

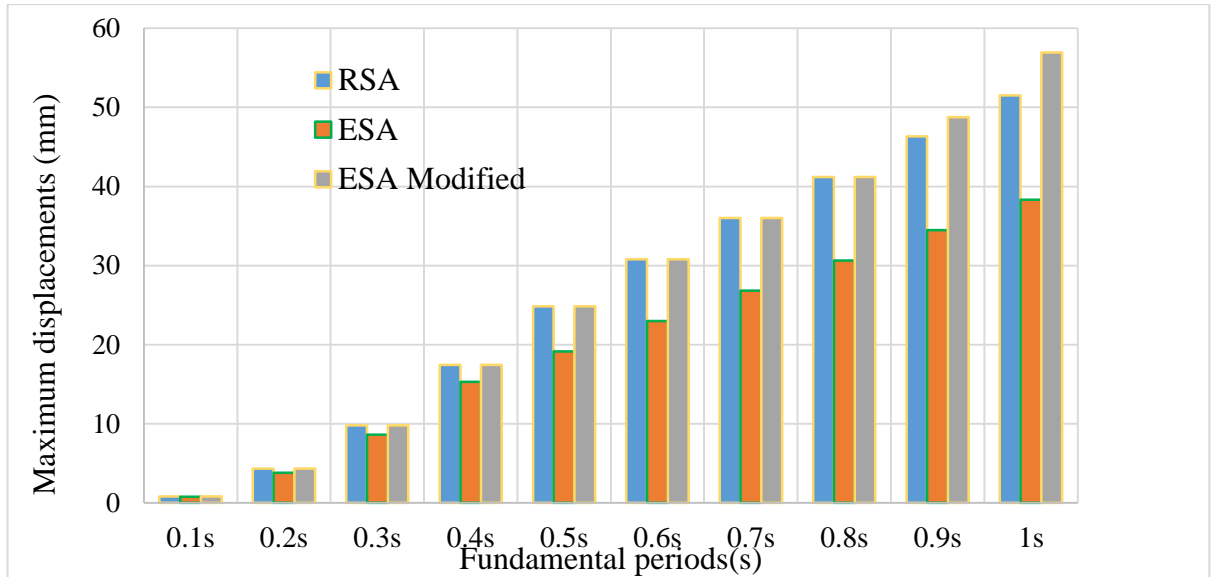


Figure 4-24 Maximum displacement vs. fundamental period (0.1s-1s)

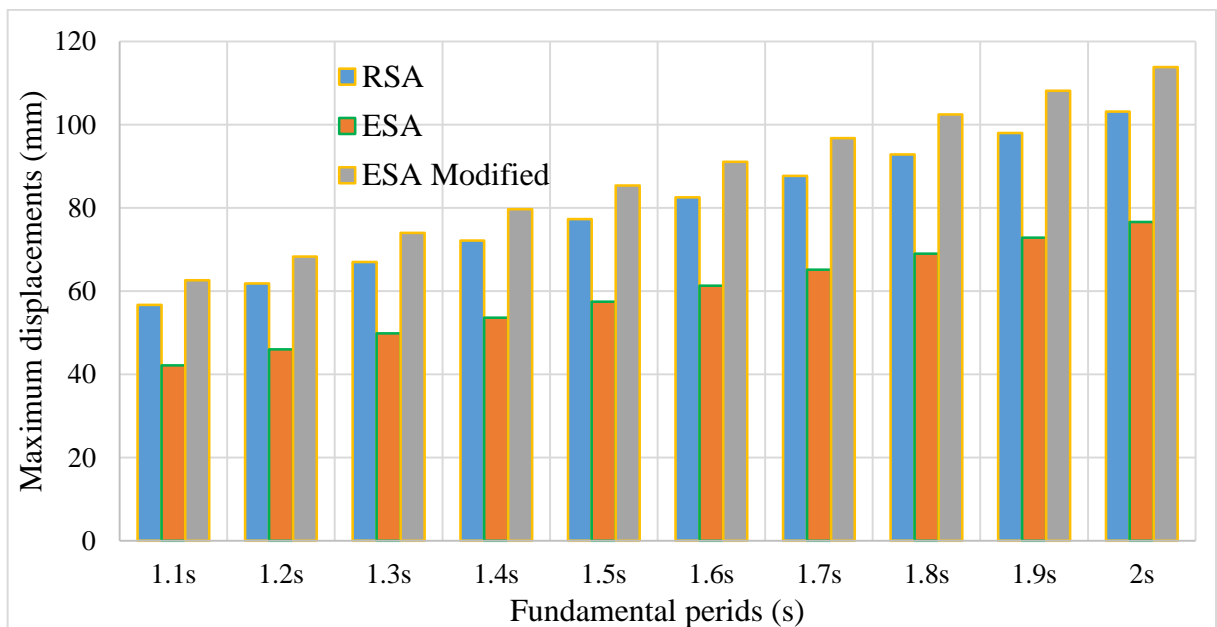


Figure 4-25 Maximum displacement vs. fundamental period (1.1s-2s)

4.3 SENSITIVITY ANALYSIS

Sensitivity analysis is the analysis of the effect of the change in the independent variable on the dependent variable. This analysis enables to identify which independent variable has the largest effect and which has the least effect on the dependent variable. Knowing

this helps to manage the dependent variable as it is intended. If a model is torsionally flexible then the sensitivity analysis result indicates which independent variable has much influence on adjusting the response of the structure. Therefore this analysis helps in modeling.

The independent variables under consideration are the distance between the lateral force-resisting system along y-direction and x-direction, the distance between the center of mass and center of stiffness along x and y-direction, ratio of story stiffness in the x-direction to the story stiffness in the y-direction and ratio of the plan dimensions. On the other hand, the dependent variable is the percentage of the ratio of the change in the maximum lateral displacement computed using ESA and RSA to the maximum lateral displacement calculated using RSA.

$$\text{dependentvariable}(D.V.) = \frac{U_{max,RSA} - U_{max,ESA}}{U_{max,RSA}} * 100\% \quad \mathbf{4-9}$$

The contribution of each parameter on the dependent variable is computed as a percentage of the uncertainty coefficient of the parameter with respect to the total uncertainty coefficient using IBM SPSS Statistics 20 and Microsoft Excel. The uncertainty coefficients were calculated by multiplying the coefficient of variation and the sensitivity factor.

Coefficient of variation (C.V.)

$$CV_i = \frac{\sigma_i}{x_i} * 100\% \quad \mathbf{4-10}$$

Sensitivity factor

$$\alpha_i = \frac{\partial \text{dependentvariable}}{\partial i} * \frac{\overline{x_i}}{\text{dependentvariable}} \quad \mathbf{4-11}$$

Uncertainty Coefficient (U)

$$U_i = CV_i * \alpha_i \quad \mathbf{4-12}$$

Where

$$i \quad D_x, D_y, e_x, e_y, \frac{K_x}{K_y}, \frac{L_y}{L_x}$$

As it is stated, the sensitivity analysis provides the percentage contribution of the independent variables. If the uncertainty coefficient, U_i , is positive then the parameter has a positive contribution which means increasing the variable results in an increased value of the dependent variable and if it is negative it has a negative contribution which means decreasing the variable results in an increased value of the dependent variable. In the following section, the percentage contribution of the independent variable was calculated using sensitivity analysis. This process is done for all soil types at a fundamental period equal to 0.4 seconds and a minimum of 2 seconds and $4T_c$. The results of the sensitivity analysis are presented in APPENDIX C but here the result of soil type C for design spectrum type 1 and soil type D for design spectrum type 2, at fundamental period equal to 0.4 seconds are presented as follows.

The equation of the best fit curve formulated using regression analysis for the dependent variable in each soil type and design spectrum is shown in equation 4-13. The coefficients of the independent variables and a constant are presented in Table 4-1 and Table 4-2 for fundamental period equal to 0.4 seconds and a minimum of 2 seconds and $4T_c$ respectively.

$$D.V. = a + be_x + ce_y + dD_x + eD_y + f \frac{L_y}{L_x} + g \frac{K_x}{K_y} \quad \mathbf{4-13}$$

Table 4-1 Coefficients of the independent variables and a constant (T=0.4 Sec)

	Design spectrum									
	1					2				
	Soil type									
	A	B	C	D	E	A	B	C	D	E
a	0.070	0.046	0.035	0.035	-0.026	0.112	0.112	0.112	0.108	0.112
b	-0.029	-0.005	0.011	0.011	0.004	0.105	0.105	0.105	0.059	0.105
c	0.050	0.058	0.058	0.058	0.127	-0.030	-0.030	-0.031	-0.021	-0.030
d	-0.143	-0.125	-0.122	-0.122	-0.046	-0.175	-0.175	-0.176	-0.172	-0.175
e	0.059	0.063	0.063	0.063	0.064	0.030	0.030	0.030	0.030	0.030
f	-0.037	-0.041	-0.041	-0.041	-0.060	-0.023	-0.023	-0.023	-0.023	-0.023
g	0.077	0.101	0.110	0.110	0.181	0.047	0.047	0.046	0.046	0.047

**Table 4-2 Coefficients of the independent variables and a constant (T= a minimum of 2
seconds and $4T_c$)**

Design spectrum										
	1					2				
Soil type										
	A	B	C	D	E	A	B	C	D	E
a	0.177	0.157	0.156	0.155	0.097	0.157	0.157	0.157	0.157	0.159
b	-0.018	-0.005	0.002	0.019	0.003	-0.005	-0.005	-0.005	-0.005	-0.009
c	-0.037	-0.029	-0.028	-0.045	0.030	-0.029	-0.029	-0.029	-0.029	-0.030
d	-0.140	-0.129	-0.137	-0.169	-0.063	-0.129	-0.129	-0.129	-0.129	-0.134
e	0.028	0.036	0.041	0.040	0.037	0.036	0.036	0.036	0.036	0.039
f	-0.035	-0.039	-0.041	-0.029	-0.055	-0.039	-0.039	-0.039	-0.039	-0.039
g	0.062	0.082	0.077	0.039	0.149	0.082	0.082	0.082	0.082	0.083

For instance, the best fit curve for soil type C and design spectrum 1 at 0.4 Sec fundamental period is

$$D.V. = 0.035 + 0.011e_x + 0.058e_y - 0.122D_x + 0.063D_y - 0.041\frac{L_y}{L_x} + 0.110\frac{K_x}{K_y}$$

The curve indicates for a 1mm increase of e_x , D.V. increases by 0.011%, for a 1mm increase of e_y , D.V. increases by 0.058%, for 1MPa increase in D_x , D.V. decreases by 0.122%, for 1MPa increase in D_y , D.V. increases by 0.063%, for 1MPa increase in $\frac{L_y}{L_x}$, D.V. decreases by 0.041%, and for 1MPa increase in $\frac{K_x}{K_y}$, D.V. increases by 0.110%.

Table 4-3 Sensitivity analysis parameters for soil type C design spectrum type 1 (T=0.4 Sec)

	α	C.V.	U	%
e_x	0.094294759	49%	0.046533596	1%
e_y	1.256448426	61%	0.761156716	13%
D_x	-2.270368538	45%	-1.031157147	-18%
D_y	2.464838359	56%	1.380032459	24%
L_y/L_x	-3.898639856	37%	-1.447318825	-25%
K_x/K_y	1.944317209	52%	1.017156368	18%

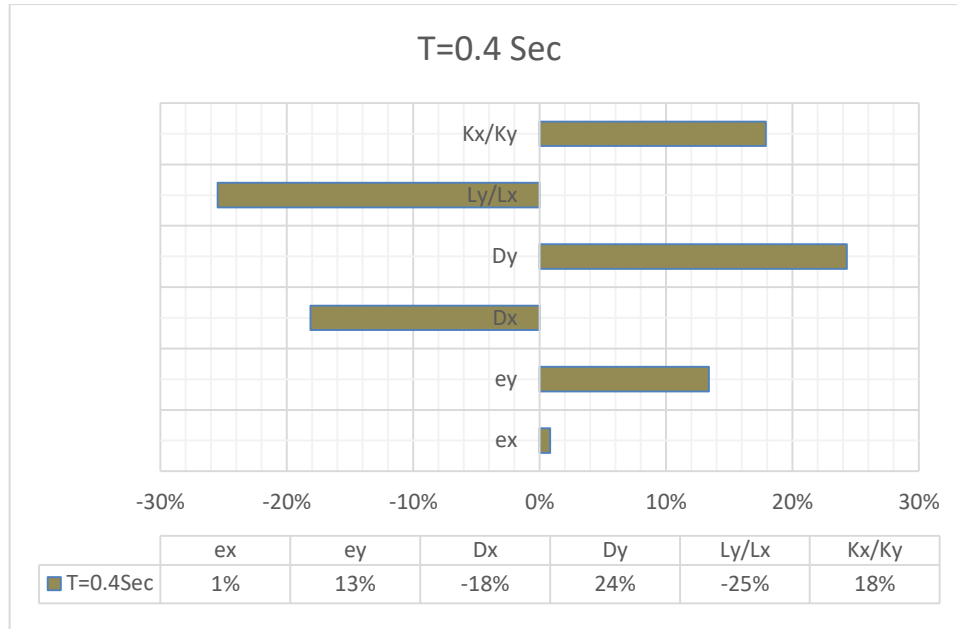


Figure 4-26 Uncertainty coefficients of independent parameters for soil type C design spectrum type 1

From Table 4-3 and Figure 4-26, the contribution of the variables in all types of soil in type 1 design spectrum with fundamental period equal to 0.4 second and a minimum of 2

seconds and $4 T_c$ shows $\frac{L_y}{L_x}$ and D_x have the maximum negative contribution while $\frac{K_x}{K_y}$

and D_y have a maximum positive contribution. Approximately zero effect resulted from

e_x . Therefore, if the response of ESA is less than RSA, increasing $\frac{L_y}{L_x}$ and D_x ;

decreasing $\frac{K_x}{K_y}$ and D_y minimizes the change in the two analysis results.

Table 4-4 Sensitivity analysis parameters for soil type D design spectrum 2 (T=0.4 Sec)

	α	C.V.	U	%
e_x	0.446455235	49%	0.220321553	6%
e_y	-0.401575314	61%	-0.243274408	-7%
D_x	-2.825504581	45%	-1.28328912	-38%
D_y	1.036096458	56%	0.58009757	17%
L_y/L_x	-1.930581488	37%	-0.716703013	-21%
K_x/K_y	0.717733647	52%	0.375477492	11%

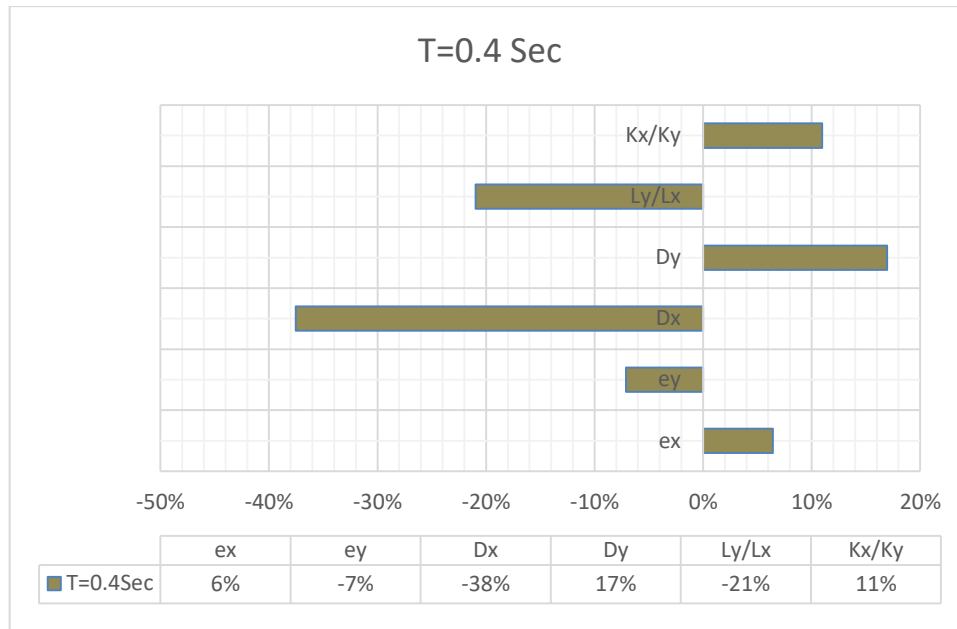


Figure 4-27 Uncertainty coefficients of independent parameters for soil type D design spectrum 2

From Table 4-2 and Figure 4-27, the contribution of the variables in all type of soils in type 2 design spectrum shows $\frac{L_y}{L_x}$, D_x and e_y show negative contribution whereas the remaining 3 variables contribute positively. Among them, D_x and $\frac{L_y}{L_x}$ have a greater negative influence on the dependent variable whereas D_y and $\frac{K_x}{K_y}$ has a greater positive effect.

From the above results, an important point shall be noted. The reduction of eccentricity or zero eccentricity does not guarantee a torsionally regular system.

CHAPTER 5 STRUCTURAL MODEL

5.1 GENERAL

Under this section, two test models demonstrating the analytical model discussed in chapter 4 are presented. The models are two reinforced concrete buildings composed of three stories. A reinforced concrete slab of 150 mm is taken. A 200mm thick shear wall, 30cmx40cm beam, and 30cmx30cm column is taken. The concrete grade used for all members is C25/30 except C30/37 is used for the columns. Soil type A and design spectrum type 1 is selected. The material properties and the section of the members are properly modeled using ETABS.

Diaphragm is defined for each level because a solid reinforced concrete slab may be considered to serve as a diaphragm if it has a thickness of not less than 70 mm and is reinforced in both horizontal directions with at least the minimum reinforcement specified in EN 1992-1-1:2002. When the floor diaphragm of a building is taken as being rigid in its plane, the masses of each floor may be lumped at the center of gravity as it was assumed in the analytical model. The layout of the models is shown below.

5.2 TEST MODELS

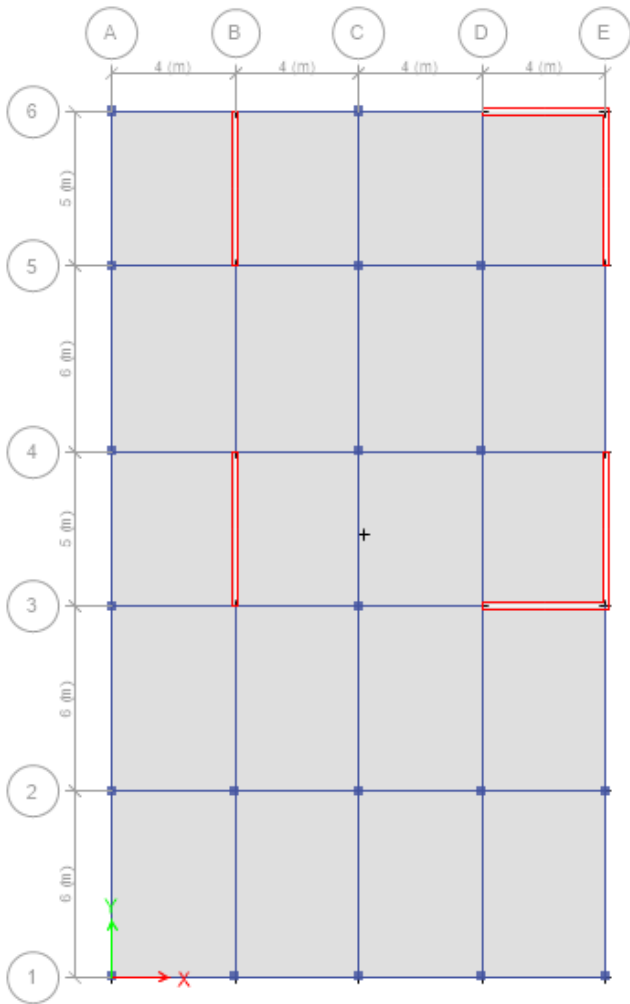


Figure 5-1 Plan of Model 1

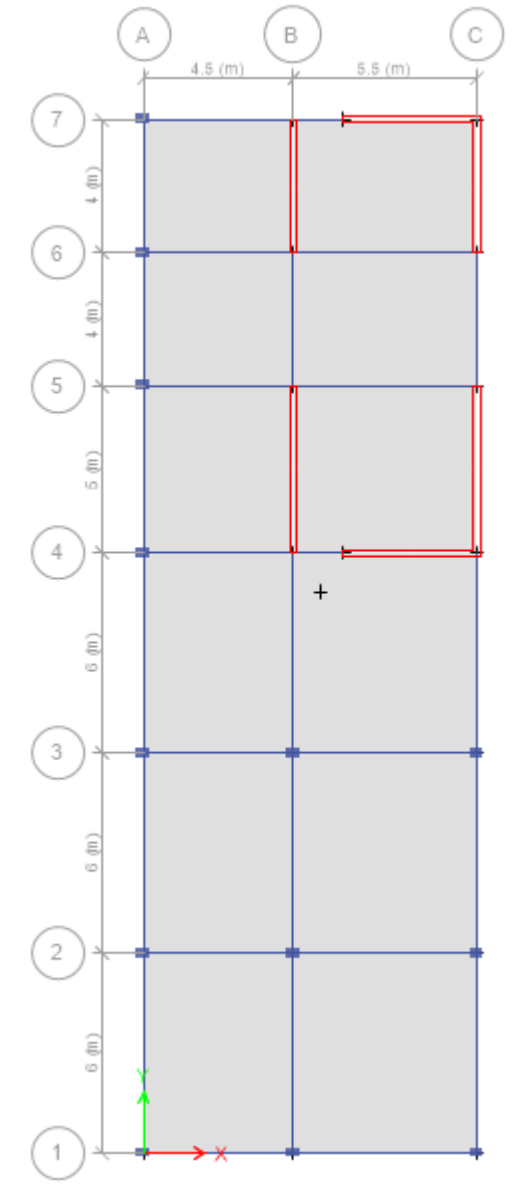


Figure 5-2 Plan of Model 2

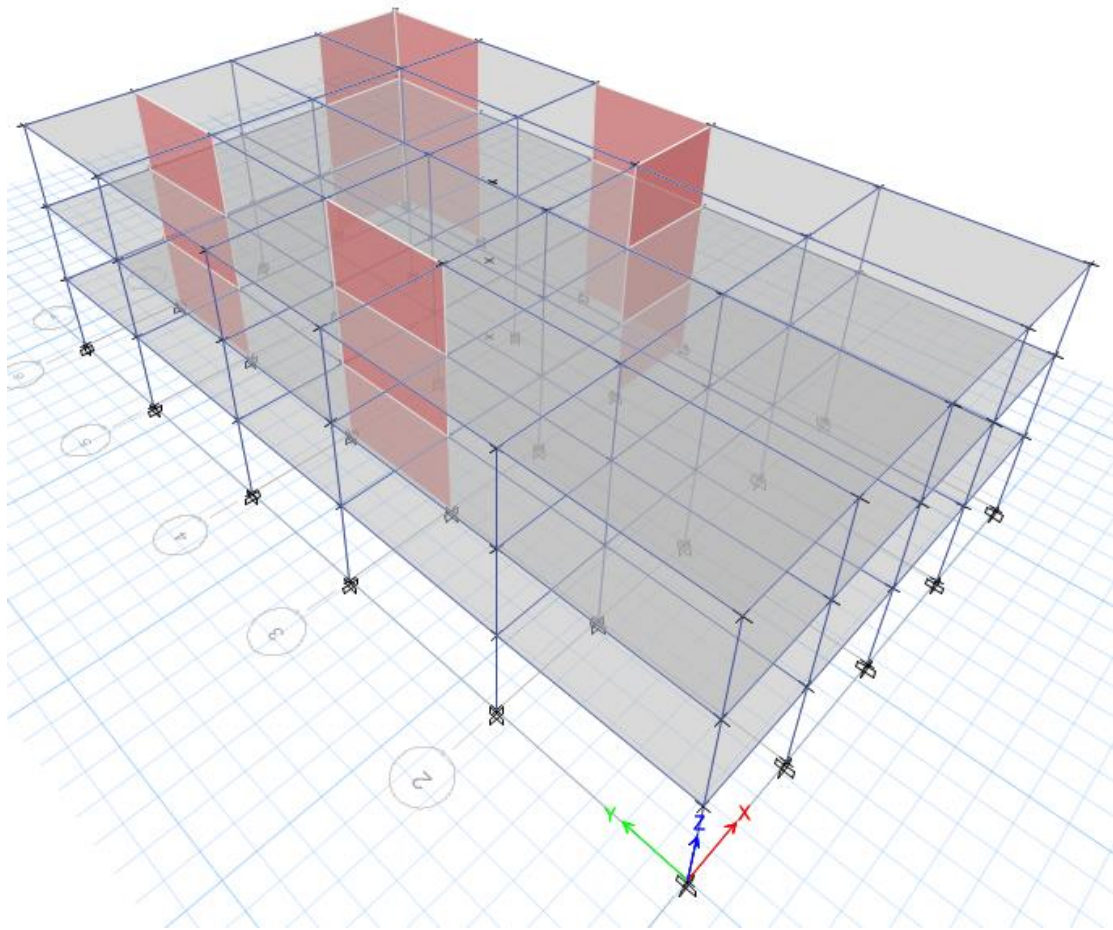


Figure 5-3 3D view of Model 1

The torsional irregularity condition of the models is checked using the procedure of EN 1998-1:2003: seismic design of buildings worked examples.

Table 5-1 Torsional regularity determination of Model 1

Story	$U_x(F_x)$	$U_y(F_y)$	$R_z(M_z)$	$K_x(10^6)$	$K_y(10^6)$	$K_z(10^6)$	r_x	r_y	l_s	$r_x > l_s$	$r_y > l_s$
	m	m	rad	N/m	N/m	Nm/rad	m	m	m	m	m
3 rd	1.68	0.71	0.0095	595.5	1,411.0	105,585.5	8.65	13.32	9.31	NOT OK	OK
2 nd	0.65	0.28	0.0037	1,534.5	3,633.5	267,594.3	8.58	13.21	9.31	NOT OK	OK
1 st	0.17	0.72	0.0011	5,776.2	13,898.4	943,396.2	8.24	12.78	9.31	NOT OK	OK
			Total	7,906.1	18,942.9		8.46	14.61			
			K_x/K_y	0.4174							

$$\left(\frac{r_x}{L_x}\right)_{1^{st} \text{ story}} = 0.52$$

$$\left(\frac{r_x}{L_x}\right)_{2^{nd} \text{ story}} = 0.54$$

$$\left(\frac{r_x}{L_x}\right)_{3^{rd} \text{ story}} = 0.54$$

The torsional irregularity of Model 2 is also determined following the above procedure.

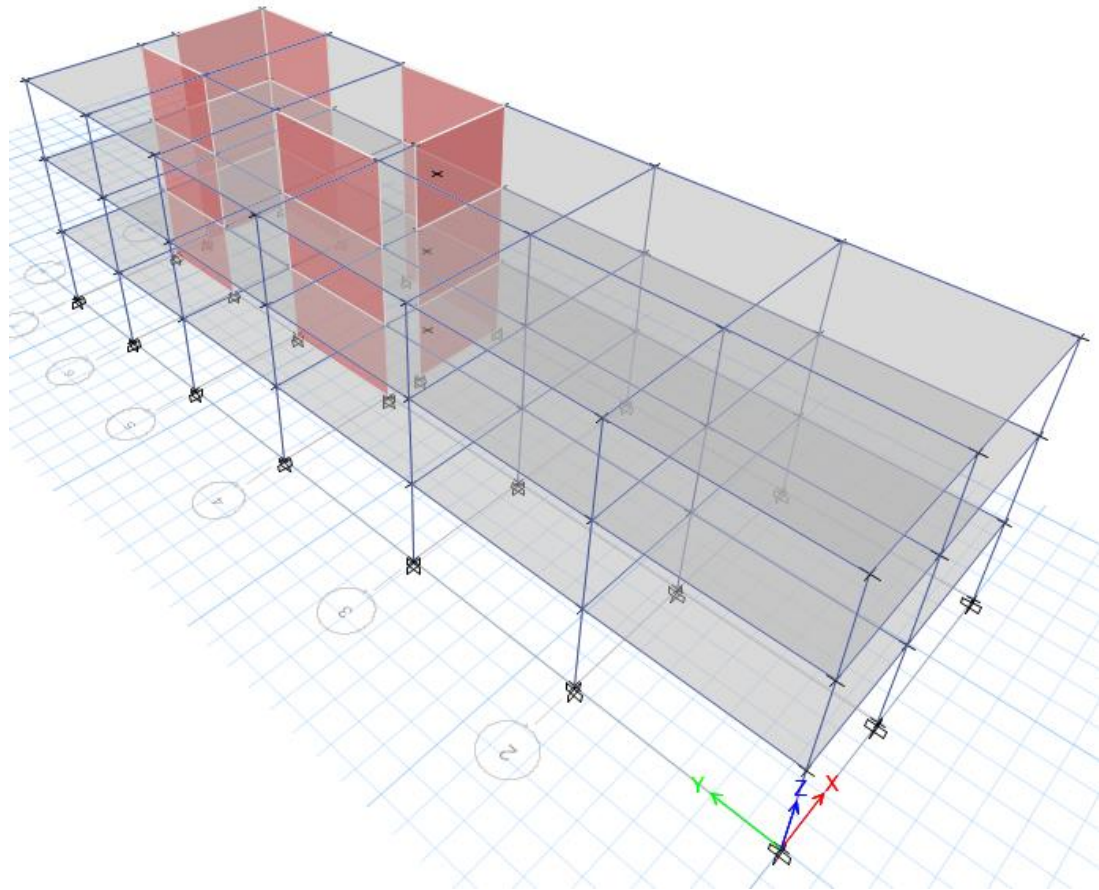


Figure 5-4 3D view of Model 2

Table 5-2 Torsional regularity determination of Model 2

Story	$U_x(F_x)$	$U_y(F_y)$	$R_z(M_z)$	$K_x(10^6)$	$K_y(10^6)$	$K_z(10^6)$	r_x	r_y	l_s	$r_x > l_s$	$r_y > l_s$
	m	m	Rad	N/m	N/m	Nm/rad	m	m	m	m	m
3 rd	1.54	0.86	0.0205	649.93	1,166.0	48,699.7	6.46	8.66	9.40	NOT OK	NOT OK
2 nd	0.62	0.33	0.0087	1,616.7	3,058.7	115,593.6	6.15	8.46	9.40	NOT OK	NOT OK
1 st	0.17	0.08	0.0026	5,865.0	12,132.8	387,596.9	5.65	8.13	9.40	NOT OK	NOT OK
			Total	8,131.7	16,357.6		8.46	14.61			
			K_x/K_y	0.4971							

$$\left(\frac{r_x}{L_x}\right)_{1^{st} \text{ story}} = 0.57$$

$$\left(\frac{r_x}{L_x}\right)_{2^{nd} \text{ story}} = 0.62$$

$$\left(\frac{r_x}{L_x}\right)_{3^{rd} \text{ story}} = 0.65$$

Both models are torsionally flexible because the torsional radius is less than the radius of gyration as shown in Table 5-1 and Table 5-2. The models are analyzed and designed using ESA and RSA.

5.3 RESULT OF THE TEST MODEL

The examination of the analytical model is done using 2 torsionally flexible structural models using ETABS. The maximum lateral displacement computed using ESA and RSA along the two orthogonal directions is shown below.

Test model 1

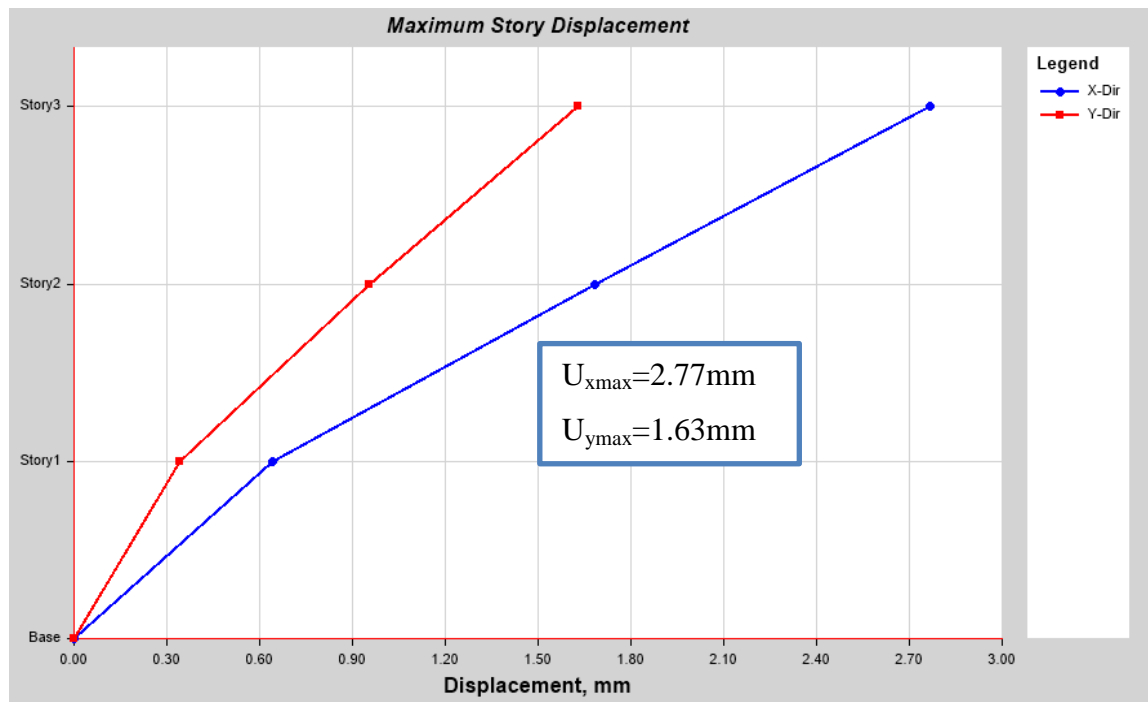


Figure 5-5 Maximum lateral displacement of Test model 1 using RSA

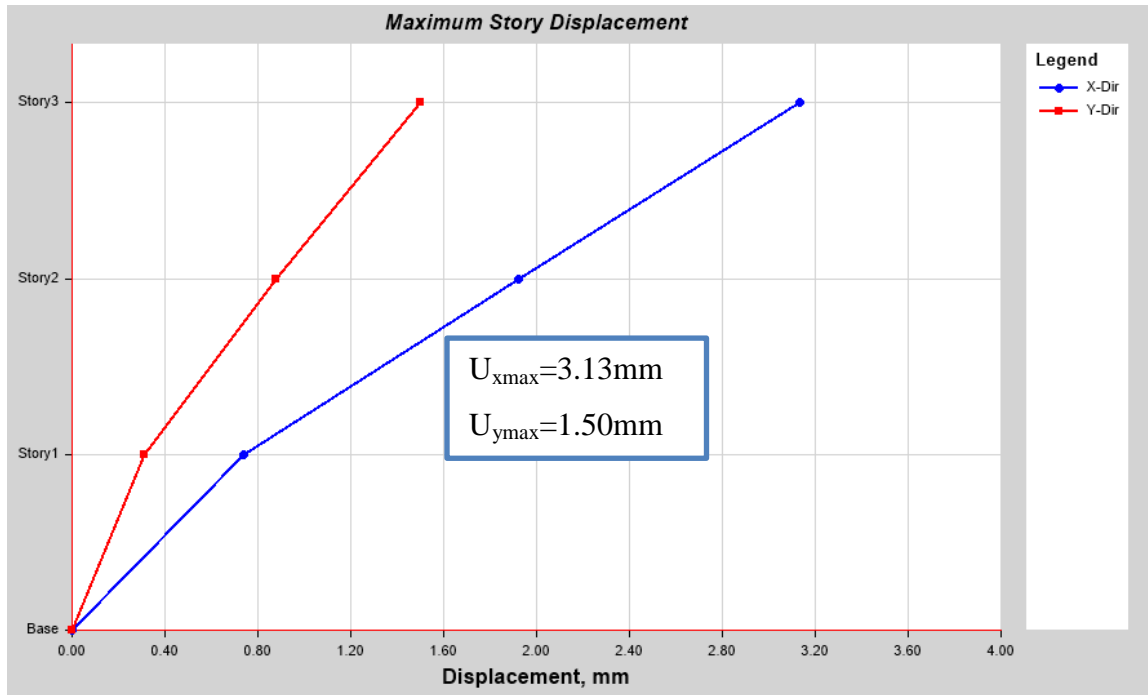


Figure 5-6 Maximum lateral displacement of Test model 1 using ESA

Test model 2

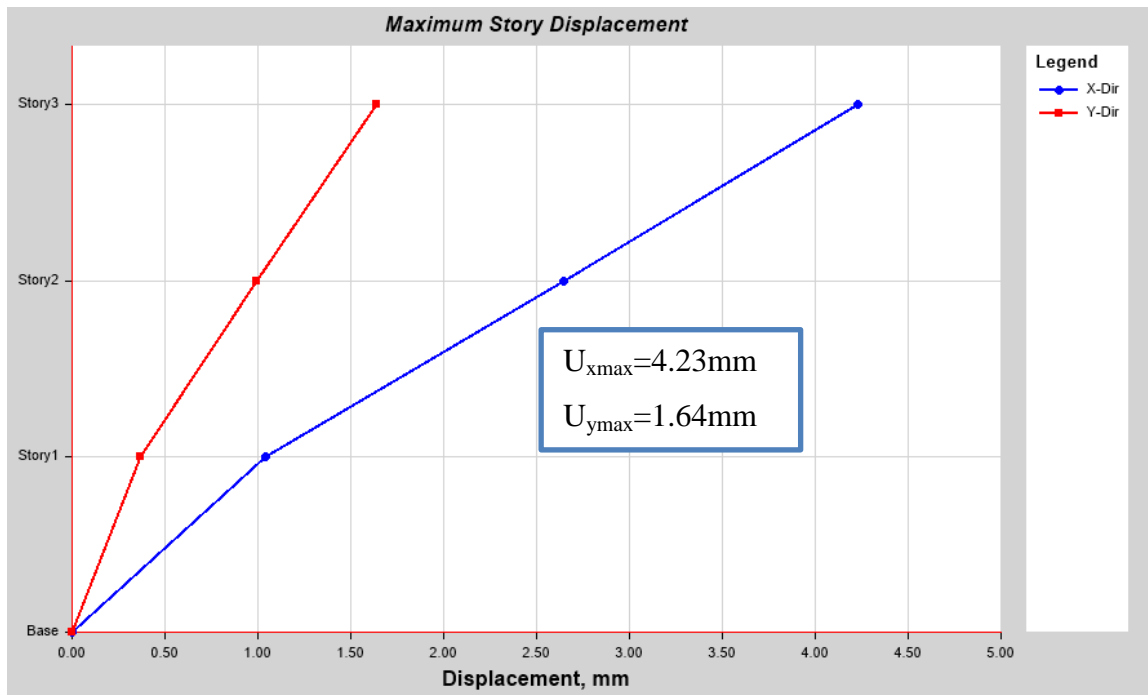


Figure 5-7 Maximum lateral displacement of Test model 2 using RSA

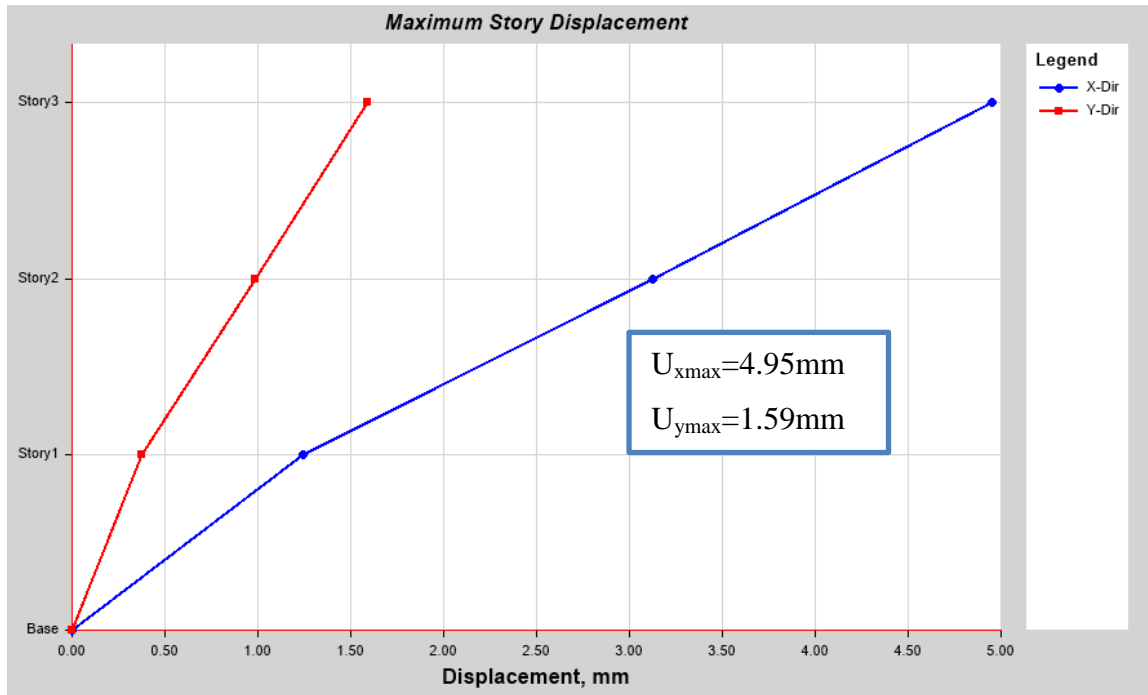


Figure 5-8 Maximum lateral displacement of Test model 2 using ESA

The maximum lateral displacement calculated using RSA was larger than ESA in the y-direction for both models. So a modifying factor shall be applied until the lateral displacement calculated using the two methods become equal in the y-direction. The plot showing the modified displacement is presented below.

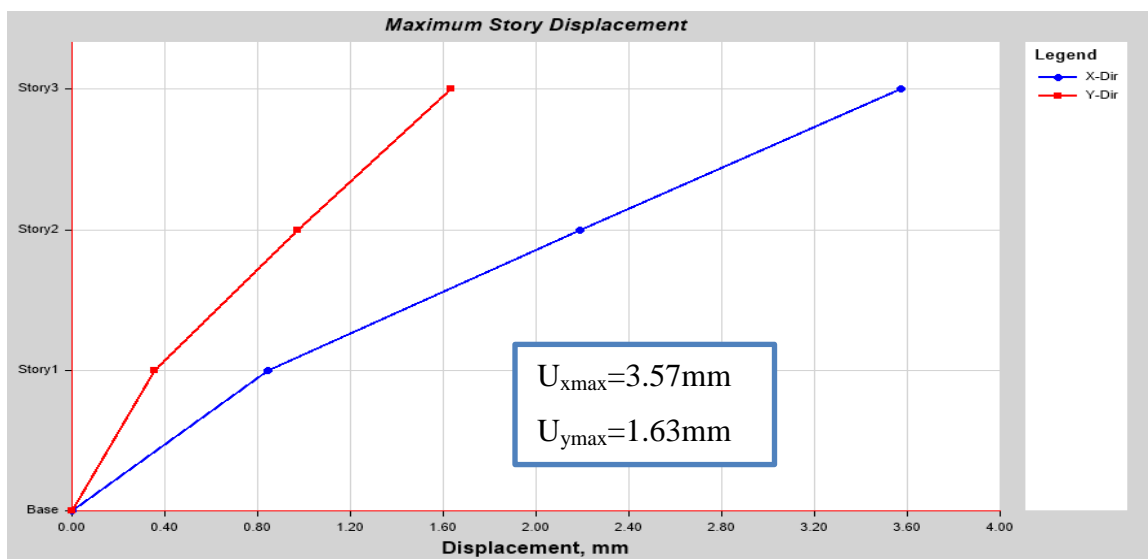


Figure 5-9 Maximum lateral displacement of Test model 1 using modified ESA ($\beta=2.86$)

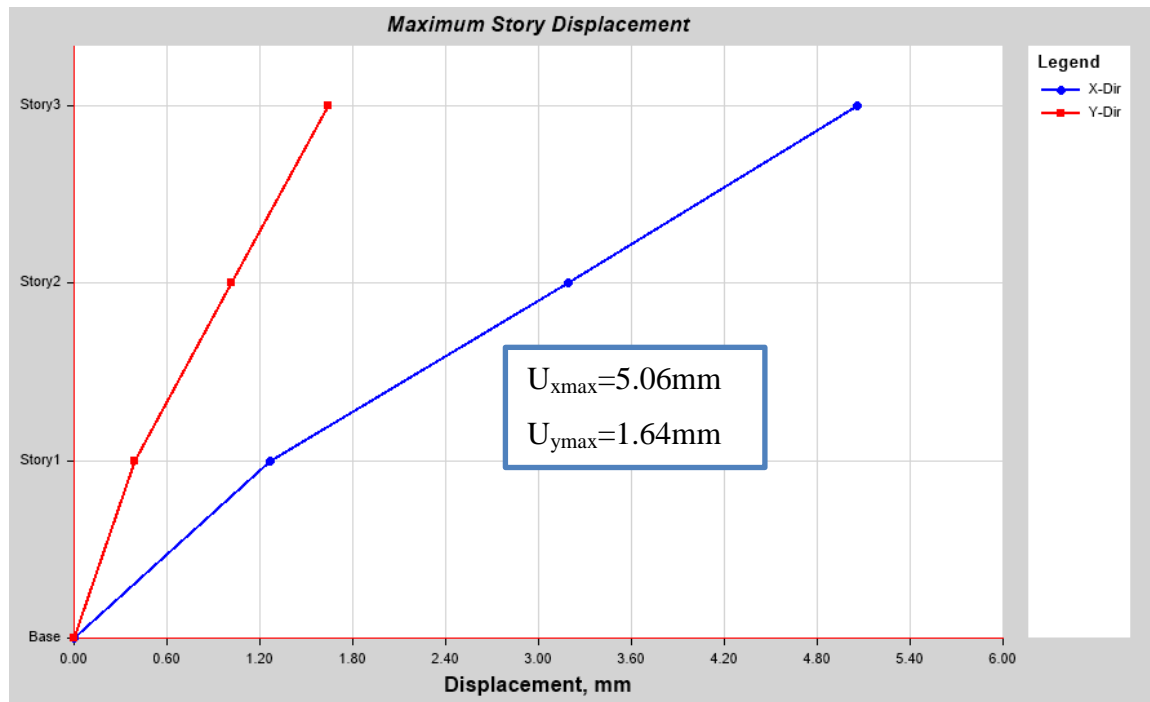


Figure 5-10 Maximum lateral displacement of Test model 2 using modified ESA ($\beta=1.2$)

The modification of the maximum lateral displacement of the two test models increased the lateral displacement in the direction of the y-axis until it equals the RSA result. The modifying factor of test model 1 is closer to the limit set by the American Society of Civil Engineers Standard and International Building Code. If the modifying factor is required to be reduced, the layout of the test model 1 shall be revised. Which parameter shall be increased or decreased for the adjustment of the layout of this system can be decided using the output of the sensitivity analysis of chapter 5.

CHAPTER 6 CONCLUSION AND RECOMMENDATION

6.1 CONCLUSION

The thesis presented an analytical investigation of three-story random samples and examination of the analytical result with two test models using MATLAB and ETABS respectively. Even though the requirements set forth in EN 1998-1:2003 to use the equivalent static method are satisfied, torsional fundamental period can result when the torsional radius is less than the radius of gyration. Therefore higher modes become significant and the responses calculated using the equivalent static method turn out to be insufficient. This condition can be resolved in different ways.

- To begin with, torsionally irregular system shall be avoided to design seismically resistant buildings.
- However, if it is unavoidable, the response spectrum method of analysis which takes in to account all significant modal responses shall be used.
- Alternatively, the provision stating the significance of the fundamental modal response only deemed to be satisfied using vertically regular system with a fundamental period less than or equal to 2 seconds and $4 T_c$ shall be enforced with additional requirement stating the fulfillment of torsional regularity.
- The other alternative is to modify the accidental eccentricity used in the equivalent static method so that the underestimated responses are increased to make it equal to the response computed using response spectrum analysis.

Because the torsional regularity criteria of EN 1998-1:2003 states only the boundary of a torsionally regular and torsionally flexible system, care shall be taken while analyzing a torsionally flexible system to avoid extreme irregularity. This can be done by setting a limit on the modifying factor, adopting the limit specified in the American Society of Civil Engineers Standard and International Building Code.

6.2 RECOMMENDATION

As indicated in the code review section, the American Society of Civil Engineers Standard and International Building Code, China Code for Seismic Design of Buildings and National Building Code of India 2016 Volume 1 set a limit on the torsional irregularity. When the torsional irregularity exceeds the limit the model shall be revised to avoid or limit the irregularity. On the other hand, EN 1998-1:2003 consider torsional irregularity differently and does not specify a limit on the torsional irregularity. However, the torsional irregularity limit should have been fixed as the other codes.

Even though the study presented and the method proposed works well on adjusting the responses of equivalent static method of analysis, it does not fix a limit on the modifying factor. Here, the limit is adopted from the American Society of Civil Engineers Standard and International Building Code. Therefore, the following recommendations are proposed for future study.

- The modifying factors shall be applicable apart from extreme torsional irregularity condition in which case the structure shall be revised. However, this extreme torsional irregularity is not clearly stated in the code. Therefore, a study shall be conducted to set a limit on the torsional irregularity condition of EN 1998-1:2003.
- Furthermore, the analysis does not cover the impact of story number and other plan irregularities on the response of structures. Here also additional work is necessary to capture their effect on it.

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APPENDIX A

MATLAB

```

Kx = sym('Kx','real'); Ky = sym('Ky','real'); M = sym('M','real'); Dx =
sym('Dx','real'); Dy = sym('Dy','real'); Lx = sym('Lx','real'); Ly =
sym('Ly','real'); ex = sym('ex','real'); ey = sym('ey','real'); a =
sym('a','real'); b = sym('b','real');
mass=[M 0 0 0 0 0 0 0 0 0;0 M 0 0 0 0 0 0 0 0;0 0 M 0 0 0 0 0 0;0 0 0 M 0 0
0 0 0;0 0 0 0 M 0 0 0 0;0 0 0 0 0 M 0 0 0;0 0 0 0 0 0
((Lx^2)+(Ly^2))*M/12 0 0;0 0 0 0 0 0 ((Lx^2)+(Ly^2))*M/12 0;0 0 0 0 0
0 0 ((Lx^2)+(Ly^2))*M/12];
stiffness1=[2*Kx -Kx 0 0 0 0 -2*Kx*ey Kx*ey 0;-Kx 2*Kx -Kx 0 0 0 Kx*ey
-2*Kx*ey Kx*ey;0 -Kx Kx 0 0 0 0 Kx*ey -Kx*ey;0 0 0 2*Ky -Ky 0 2*Ky*(ex-
a) -Ky*(ex-a) 0;0 0 0 -Ky 2*Ky -Ky -Ky*(ex-a) 2*Ky*(ex-a) -Ky*(ex-a);0
0 0 0 -Ky Ky 0 -Ky*(ex-a) Ky*(ex-a);-2*Kx*ey Kx*ey 0 2*Ky*(ex-a) -
Ky*(ex-a) 0 Kx*((Dy^2)/2)+2*(ey^2))+Ky*((Dx^2)/2)+2*(ex^2)-
(4*ex*a)+2*(a^2)) -(Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)-(4*ex*a)+2*(a^2)) 0 ;Kx*ey -2*Kx*ey Kx*ey -
Ky*(ex-a) 2*Ky*(ex-a) -Ky*(ex-a) -(Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)-(4*ex*a)+2*(a^2))
Kx*((Dy^2)/2)+2*(ey^2))+Ky*((Dx^2)/2)+2*(ex^2)-(4*ex*a)+2*(a^2)) -
(Kx/2)*((Dy^2)/2)+2*(ey^2))- (Ky/2)*((Dx^2)/2)+2*(ex^2)-
(4*ex*a)+2*(a^2));0 Kx*ey -Kx*ey 0 -Ky*(ex-a) Ky*(ex-a) 0 -
(Kx/2)*((Dy^2)/2)+2*(ey^2))- (Ky/2)*((Dx^2)/2)+2*(ex^2)-
(4*ex*a)+2*(a^2))
(Kx/2)*((Dy^2)/2)+2*(ey^2))+ (Ky/2)*((Dx^2)/2)+2*(ex^2)-
(4*ex*a)+2*(a^2)]];
Dx=0.2899*Lx;
Dy=1.29438*Lx;
ex=0.2419*Lx;
ey=0.23515*Lx;
Kx=0.30980*Ky;
Ky=1*Ky;
Lx=1*Lx;
Ly=2.58980*Lx;
a=0.05*Lx;
Mass=subs(mass)/M;
Stiffness1=subs(stiffness1)/Ky;
[model,w12]=eig(inv(Mass)*Stiffness1);
model;
w1=sqrt(w12)*((Ky/M)^0.5);
Period1=eye(9)/w1;
period1=[Period1(1,1) Period1(2,2) Period1(3,3) Period1(4,4)
Period1(5,5) Period1(6,6) Period1(7,7) Period1(8,8) Period1(9,9)]'
Modalmass1=model'*subs(mass)*model;
influencefactor=[1 1 1 0.3 0.3 0.3 0 0 0;0.3 0.3 0.3 1 1 1 0 0 0]';
Ln1=simplify(model'*subs(mass)*influencefactor);
modalparticipationfactor1=[Ln1(1,1)/Modalmass1(1,1)
Ln1(2,1)/Modalmass1(2,2) Ln1(3,1)/Modalmass1(3,3)
Ln1(4,1)/Modalmass1(4,4) Ln1(5,1)/Modalmass1(5,5)
Ln1(6,1)/Modalmass1(6,6) Ln1(7,1)/Modalmass1(7,7)
Ln1(8,1)/Modalmass1(8,8)

```

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```
Ln1(9,1)/Modalmass1(9,9);Ln1(1,2)/Modalmass1(1,1)
Ln1(2,2)/Modalmass1(2,2) Ln1(3,2)/Modalmass1(3,3)
Ln1(4,2)/Modalmass1(4,4) Ln1(5,2)/Modalmass1(5,5)
Ln1(6,2)/Modalmass1(6,6) Ln1(7,1)/Modalmass1(7,7)
Ln1(8,1)/Modalmass1(8,8) Ln1(9,1)/Modalmass1(9,9) ]';
Modalmassdistribution11=[modalparticipationfactor1(1,1)*subs(mass)*mode
1(:,1) modalparticipationfactor1(2,1)*subs(mass)*mode1(:,2)
modalparticipationfactor1(3,1)*subs(mass)*mode1(:,3)
modalparticipationfactor1(4,1)*subs(mass)*mode1(:,4)
modalparticipationfactor1(5,1)*subs(mass)*mode1(:,5)
modalparticipationfactor1(6,1)*subs(mass)*mode1(:,6)
modalparticipationfactor1(7,1)*subs(mass)*mode1(:,7)
modalparticipationfactor1(8,1)*subs(mass)*mode1(:,8)
modalparticipationfactor1(9,1)*subs(mass)*mode1(:,9) ]];
Modalmassdistribution11mode1=Modalmassdistribution11(:,1)/M
Modalmassdistribution11mode2=Modalmassdistribution11(:,2)/M
Modalmassdistribution11mode3=Modalmassdistribution11(:,3)/M
Modalmassdistribution11mode4=Modalmassdistribution11(:,4)/M
Modalmassdistribution11mode5=Modalmassdistribution11(:,5)/M
Modalmassdistribution11mode6=Modalmassdistribution11(:,6)/M
Modalmassdistribution11mode7=Modalmassdistribution11(:,7)/M
Modalmassdistribution11mode8=Modalmassdistribution11(:,8)/M
Modalmassdistribution11mode9=Modalmassdistribution11(:,9)/M
Modalmassdistribution12=[modalparticipationfactor1(1,2)*subs(mass)*mode
1(:,1) modalparticipationfactor1(2,2)*subs(mass)*mode1(:,2)
modalparticipationfactor1(3,2)*subs(mass)*mode1(:,3)
modalparticipationfactor1(4,2)*subs(mass)*mode1(:,4)
modalparticipationfactor1(5,2)*subs(mass)*mode1(:,5)
modalparticipationfactor1(6,2)*subs(mass)*mode1(:,6)
modalparticipationfactor1(7,2)*subs(mass)*mode1(:,7)
modalparticipationfactor1(8,2)*subs(mass)*mode1(:,8)
modalparticipationfactor1(9,2)*subs(mass)*mode1(:,9) ]];
Modalmassdistribution12mode1=Modalmassdistribution12(:,1)/M
Modalmassdistribution12mode2=Modalmassdistribution12(:,2)/M
Modalmassdistribution12mode3=Modalmassdistribution12(:,3)/M
Modalmassdistribution12mode4=Modalmassdistribution12(:,4)/M
Modalmassdistribution12mode5=Modalmassdistribution12(:,5)/M
Modalmassdistribution12mode6=Modalmassdistribution12(:,6)/M
Modalmassdistribution12mode7=Modalmassdistribution12(:,7)/M
Modalmassdistribution12mode8=Modalmassdistribution12(:,8)/M
Modalmassdistribution12mode9=Modalmassdistribution12(:,9)/M
Effectivemodalmass11=simplify(simplify([(Modalmassdistribution11(1,1)+M
odalmassdistribution11(2,1)+Modalmassdistribution11(3,1))
(Modalmassdistribution11(1,2)+Modalmassdistribution11(2,2)+Modalmassdis
tribution11(3,2))
(Modalmassdistribution11(1,3)+Modalmassdistribution11(2,3)+Modalmassdis
tribution11(3,3))
(Modalmassdistribution11(1,4)+Modalmassdistribution11(2,4)+Modalmassdis
tribution11(3,4))
(Modalmassdistribution11(1,5)+Modalmassdistribution11(2,5)+Modalmassdis
tribution11(3,5))
(Modalmassdistribution11(1,6)+Modalmassdistribution11(2,6)+Modalmassdis
tribution11(3,6))
(Modalmassdistribution11(1,7)+Modalmassdistribution11(2,7)+Modalmassdis
tribution11(3,7))
(Modalmassdistribution11(1,8)+Modalmassdistribution11(2,8)+Modalmassdis
tribution11(3,8))
```

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(Modalmassdistribution11(1,9)+Modalmassdistribution11(2,9)+Modalmassdistribution11(3,9));(Modalmassdistribution11(4,1)+Modalmassdistribution11(5,1)+Modalmassdistribution11(6,1))
(Modalmassdistribution11(4,2)+Modalmassdistribution11(5,2)+Modalmassdistribution11(6,2))
(Modalmassdistribution11(4,3)+Modalmassdistribution11(5,3)+Modalmassdistribution11(6,3))
(Modalmassdistribution11(4,4)+Modalmassdistribution11(5,4)+Modalmassdistribution11(6,4))
(Modalmassdistribution11(4,5)+Modalmassdistribution11(5,5)+Modalmassdistribution11(6,5))
(Modalmassdistribution11(4,6)+Modalmassdistribution11(5,6)+Modalmassdistribution11(6,6))
(Modalmassdistribution11(4,7)+Modalmassdistribution11(5,7)+Modalmassdistribution11(6,7))
(Modalmassdistribution11(4,8)+Modalmassdistribution11(5,8)+Modalmassdistribution11(6,8))
(Modalmassdistribution11(4,9)+Modalmassdistribution11(5,9)+Modalmassdistribution11(6,9));(Modalmassdistribution11(7,1)+Modalmassdistribution11(8,1)+Modalmassdistribution11(9,1))
(Modalmassdistribution11(7,2)+Modalmassdistribution11(8,2)+Modalmassdistribution11(9,2))
(Modalmassdistribution11(7,3)+Modalmassdistribution11(8,3)+Modalmassdistribution11(9,3))
(Modalmassdistribution11(7,4)+Modalmassdistribution11(8,4)+Modalmassdistribution11(9,4))
(Modalmassdistribution11(7,5)+Modalmassdistribution11(8,5)+Modalmassdistribution11(9,5))
(Modalmassdistribution11(7,6)+Modalmassdistribution11(8,6)+Modalmassdistribution11(9,6))
(Modalmassdistribution11(7,7)+Modalmassdistribution11(8,7)+Modalmassdistribution11(9,7))
(Modalmassdistribution11(7,8)+Modalmassdistribution11(8,8)+Modalmassdistribution11(9,8))
(Modalmassdistribution11(7,9)+Modalmassdistribution11(8,9)+Modalmassdistribution11(9,9))])/M);
Effectivemodalmass12=simplify(simplify([(Modalmassdistribution12(1,1)+Modalmassdistribution12(2,1)+Modalmassdistribution12(3,1))
(Modalmassdistribution12(1,2)+Modalmassdistribution12(2,2)+Modalmassdistribution12(3,2))
(Modalmassdistribution12(1,3)+Modalmassdistribution12(2,3)+Modalmassdistribution12(3,3))
(Modalmassdistribution12(1,4)+Modalmassdistribution12(2,4)+Modalmassdistribution12(3,4))
(Modalmassdistribution12(1,5)+Modalmassdistribution12(2,5)+Modalmassdistribution12(3,5))
(Modalmassdistribution12(1,6)+Modalmassdistribution12(2,6)+Modalmassdistribution12(3,6))
(Modalmassdistribution12(1,7)+Modalmassdistribution12(2,7)+Modalmassdistribution12(3,7))
(Modalmassdistribution12(1,8)+Modalmassdistribution12(2,8)+Modalmassdistribution12(3,8))
(Modalmassdistribution12(1,9)+Modalmassdistribution12(2,9)+Modalmassdistribution12(3,9));(Modalmassdistribution12(4,1)+Modalmassdistribution12(5,1)+Modalmassdistribution12(6,1))
(Modalmassdistribution12(4,2)+Modalmassdistribution12(5,2)+Modalmassdistribution12(6,2))

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```
(Modalmassdistribution12(4,3)+Modalmassdistribution12(5,3)+Modalmassdis  
tribution12(6,3))  
(Modalmassdistribution12(4,4)+Modalmassdistribution12(5,4)+Modalmassdis  
tribution12(6,4))  
(Modalmassdistribution12(4,5)+Modalmassdistribution12(5,5)+Modalmassdis  
tribution12(6,5))  
(Modalmassdistribution12(4,6)+Modalmassdistribution12(5,6)+Modalmassdis  
tribution12(6,6))  
(Modalmassdistribution12(4,7)+Modalmassdistribution12(5,7)+Modalmassdis  
tribution12(6,7))  
(Modalmassdistribution12(4,8)+Modalmassdistribution12(5,8)+Modalmassdis  
tribution12(6,8))  
(Modalmassdistribution12(4,9)+Modalmassdistribution12(5,9)+Modalmassdis  
tribution12(6,9));(Modalmassdistribution12(7,1)+Modalmassdistribution12  
(8,1)+Modalmassdistribution12(9,1))  
(Modalmassdistribution12(7,2)+Modalmassdistribution12(8,2)+Modalmassdis  
tribution12(9,2))  
(Modalmassdistribution12(7,3)+Modalmassdistribution12(8,3)+Modalmassdis  
tribution12(9,3))  
(Modalmassdistribution12(7,4)+Modalmassdistribution12(8,4)+Modalmassdis  
tribution12(9,4))  
(Modalmassdistribution12(7,5)+Modalmassdistribution12(8,5)+Modalmassdis  
tribution12(9,5))  
(Modalmassdistribution12(7,6)+Modalmassdistribution12(8,6)+Modalmassdis  
tribution12(9,6))  
(Modalmassdistribution12(7,7)+Modalmassdistribution12(8,7)+Modalmassdis  
tribution12(9,7))  
(Modalmassdistribution12(7,8)+Modalmassdistribution12(8,8)+Modalmassdis  
tribution12(9,8))  
(Modalmassdistribution12(7,9)+Modalmassdistribution12(8,9)+Modalmassdis  
tribution12(9,9))] /M);  
r1=[(modalparticipationfactor1(1,1)/((w1(1,1))^2))  
(modalparticipationfactor1(2,1)/((w1(2,2))^2))  
(modalparticipationfactor1(3,1)/((w1(3,3))^2))  
(modalparticipationfactor1(4,1)/((w1(4,4))^2))  
(modalparticipationfactor1(5,1)/((w1(5,5))^2))  
(modalparticipationfactor1(6,1)/((w1(6,6))^2))  
(modalparticipationfactor1(7,1)/((w1(7,7))^2))  
(modalparticipationfactor1(8,1)/((w1(8,8))^2))  
(modalparticipationfactor1(9,1)/((w1(9,9))^2));(modalparticipationfacto  
r1(1,2)/((w1(1,1))^2)) (modalparticipationfactor1(2,2)/((w1(2,2))^2))  
(modalparticipationfactor1(3,2)/((w1(3,3))^2))  
(modalparticipationfactor1(4,2)/((w1(4,4))^2))  
(modalparticipationfactor1(5,2)/((w1(5,5))^2))  
(modalparticipationfactor1(6,2)/((w1(6,6))^2))  
(modalparticipationfactor1(7,2)/((w1(7,7))^2))  
(modalparticipationfactor1(8,2)/((w1(8,8))^2))  
(modalparticipationfactor1(9,2)/((w1(9,9))^2))]';  
Ujn11=[r1(1,1)*model(:,1) r1(2,1)*model(:,2) r1(3,1)*model(:,3)  
r1(4,1)*model(:,4) r1(5,1)*model(:,5) r1(6,1)*model(:,6)  
r1(7,1)*model(:,7) r1(8,1)*model(:,8) r1(9,1)*model(:,9)];  
U11mode1=Ujn11(:,1)*Ky/M  
U11mode2=Ujn11(:,2)*Ky/M  
U11mode3=Ujn11(:,3)*Ky/M  
U11mode4=Ujn11(:,4)*Ky/M  
U11mode5=Ujn11(:,5)*Ky/M  
U11mode6=Ujn11(:,6)*Ky/M
```

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```

U11mode7=Ujn11(:,7)*Ky/M
U11mode8=Ujn11(:,8)*Ky/M
U11mode9=Ujn11(:,9)*Ky/M
Ujn12=[r1(1,2)*model(:,1) r1(2,2)*model(:,2) r1(3,2)*model(:,3)
r1(4,2)*model(:,4) r1(5,2)*model(:,5) r1(6,2)*model(:,6)
r1(7,2)*model(:,7) r1(8,2)*model(:,8) r1(9,2)*model(:,9)];
U12mode1=Ujn12(:,1)*Ky/M
U12mode2=Ujn12(:,2)*Ky/M
U12mode3=Ujn12(:,3)*Ky/M
U12mode4=Ujn12(:,4)*Ky/M
U12mode5=Ujn12(:,5)*Ky/M
U12mode6=Ujn12(:,6)*Ky/M
U12mode7=Ujn12(:,7)*Ky/M
U12mode8=Ujn12(:,8)*Ky/M
U12mode9=Ujn12(:,9)*Ky/M
stiffness2=[2*Kx -Kx 0 0 0 0 -2*Kx*(ey-b) Kx*(ey-b) 0;-Kx 2*Kx -Kx 0 0
0 Kx*(ey-b) -2*Kx*(ey-b) Kx*(ey-b);0 -Kx Kx 0 0 0 0 Kx*(ey-b) -Kx*(ey-
b);0 0 0 2*Ky -Ky 0 2*Ky*ex -Ky*ex 0;0 0 0 -Ky 2*Ky -Ky -Ky*ex 2*Ky*ex
-Ky*ex;0 0 0 0 -Ky Ky 0 -Ky*ex Ky*ex;-2*Kx*(ey-b) Kx*(ey-b) 0 2*Ky*ex -
Ky*ex 0 Kx*((Dy^2)/2)+2*(ey^2)-
(4*ey*b)+2*(b^2))+Ky*((Dx^2)/2)+2*(ex^2)) -
(Kx/2)*(((Dy^2)/2)+2*(ey^2)-(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2)) 0 ;Kx*(ey-b) -2*Kx*(ey-b) Kx*(ey-b) -Ky*ex
2*Ky*ex -Ky*ex (-Kx/2)*(((Dy^2)/2)+2*(ey^2)-(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2)) Kx*((Dy^2)/2)+2*(ey^2)-
(4*ey*b)+2*(b^2))+Ky*((Dx^2)/2)+2*(ex^2)) -
(Kx/2)*(((Dy^2)/2)+2*(ey^2)-(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2));0 Kx*(ey-b) -Kx*(ey-b) 0 -Ky*ex Ky*ex 0 -
(Kx/2)*(((Dy^2)/2)+2*(ey^2)-(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2)) (Kx/2)*(((Dy^2)/2)+2*(ey^2)-
(4*ey*b)+2*(b^2))+ (Ky/2)*(((Dx^2)/2)+2*(ex^2))];
b=0.05*Ly;
Stiffness2=subs(stiffness2)/Ky;
[mode2,w22]=eig(inv(Mass)*Stiffness2);
mode2;
w2=sqrt(w22)*((Ky/M)^0.5);
Period2=eye(9)/w2;
period2=[Period2(1,1) Period2(2,2) Period2(3,3) Period2(4,4)
Period2(5,5) Period2(6,6) Period2(7,7) Period2(8,8) Period2(9,9)]'
Modalmass2=mode2'*subs(mass)*mode2;
Ln2=simplify(mode2'*subs(mass)*influencefactor);
modalparticipationfactor2=[Ln2(1,1)/Modalmass2(1,1)
Ln2(2,1)/Modalmass2(2,2) Ln2(3,1)/Modalmass2(3,3)
Ln2(4,1)/Modalmass2(4,4) Ln2(5,1)/Modalmass2(5,5)
Ln2(6,1)/Modalmass2(6,6) Ln2(7,1)/Modalmass2(7,7)
Ln2(8,1)/Modalmass2(8,8)
Ln2(9,1)/Modalmass2(9,9);Ln2(1,2)/Modalmass2(1,1)
Ln2(2,2)/Modalmass2(2,2) Ln2(3,2)/Modalmass2(3,3)
Ln2(4,2)/Modalmass2(4,4) Ln2(5,2)/Modalmass2(5,5)
Ln2(6,2)/Modalmass2(6,6) Ln2(7,1)/Modalmass2(7,7)
Ln2(8,1)/Modalmass2(8,8) Ln2(9,1)/Modalmass2(9,9)]';
Modalmassdistribution21=[modalparticipationfactor2(1,1)*subs(mass)*mode
2(:,1) modalparticipationfactor2(2,1)*subs(mass)*mode2(:,2)
modalparticipationfactor2(3,1)*subs(mass)*mode2(:,3)
modalparticipationfactor2(4,1)*subs(mass)*mode2(:,4)
modalparticipationfactor2(5,1)*subs(mass)*mode2(:,5)
modalparticipationfactor2(6,1)*subs(mass)*mode2(:,6)

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```
modalparticipationfactor2(7,1)*subs(mass)*mode2(:,7)
modalparticipationfactor2(8,1)*subs(mass)*mode2(:,8)
modalparticipationfactor2(9,1)*subs(mass)*mode2(:,9)];
Modalmassdistribution21mode1=Modalmassdistribution21(:,1)/M
Modalmassdistribution21mode2=Modalmassdistribution21(:,2)/M
Modalmassdistribution21mode3=Modalmassdistribution21(:,3)/M
Modalmassdistribution21mode4=Modalmassdistribution21(:,4)/M
Modalmassdistribution21mode5=Modalmassdistribution21(:,5)/M
Modalmassdistribution21mode6=Modalmassdistribution21(:,6)/M
Modalmassdistribution21mode7=Modalmassdistribution21(:,7)/M
Modalmassdistribution21mode8=Modalmassdistribution21(:,8)/M
Modalmassdistribution21mode9=Modalmassdistribution21(:,9)/M
Modalmassdistribution22=[modalparticipationfactor2(1,2)*subs(mass)*mode
2(:,1) modalparticipationfactor2(2,2)*subs(mass)*mode2(:,2)
modalparticipationfactor2(3,2)*subs(mass)*mode2(:,3)
modalparticipationfactor2(4,2)*subs(mass)*mode2(:,4)
modalparticipationfactor2(5,2)*subs(mass)*mode2(:,5)
modalparticipationfactor2(6,2)*subs(mass)*mode2(:,6)
modalparticipationfactor2(7,2)*subs(mass)*mode2(:,7)
modalparticipationfactor2(8,2)*subs(mass)*mode2(:,8)
modalparticipationfactor2(9,2)*subs(mass)*mode2(:,9)];
Modalmassdistribution22mode1=Modalmassdistribution22(:,1)/M
Modalmassdistribution22mode2=Modalmassdistribution22(:,2)/M
Modalmassdistribution22mode3=Modalmassdistribution22(:,3)/M
Modalmassdistribution22mode4=Modalmassdistribution22(:,4)/M
Modalmassdistribution22mode5=Modalmassdistribution22(:,5)/M
Modalmassdistribution22mode6=Modalmassdistribution22(:,6)/M
Modalmassdistribution22mode7=Modalmassdistribution22(:,7)/M
Modalmassdistribution22mode8=Modalmassdistribution22(:,8)/M
Modalmassdistribution22mode9=Modalmassdistribution22(:,9)/M
Effectivemodalmass21=simplify(simplify([(Modalmassdistribution21(1,1)+M
odalmassdistribution21(2,1)+Modalmassdistribution21(3,1))
(Modalmassdistribution21(1,2)+Modalmassdistribution21(2,2)+Modalmasdis
tribution21(3,2))
(Modalmassdistribution21(1,3)+Modalmassdistribution21(2,3)+Modalmasdis
tribution21(3,3))
(Modalmassdistribution21(1,4)+Modalmassdistribution21(2,4)+Modalmasdis
tribution21(3,4))
(Modalmassdistribution21(1,5)+Modalmassdistribution21(2,5)+Modalmasdis
tribution21(3,5))
(Modalmassdistribution21(1,6)+Modalmassdistribution21(2,6)+Modalmasdis
tribution21(3,6))
(Modalmassdistribution21(1,7)+Modalmassdistribution21(2,7)+Modalmasdis
tribution21(3,7))
(Modalmassdistribution21(1,8)+Modalmassdistribution21(2,8)+Modalmasdis
tribution21(3,8))
(Modalmassdistribution21(1,9)+Modalmassdistribution21(2,9)+Modalmasdis
tribution21(3,9));(Modalmassdistribution21(4,1)+Modalmassdistribution21
(5,1)+Modalmassdistribution21(6,1))
(Modalmassdistribution21(4,2)+Modalmassdistribution21(5,2)+Modalmasdis
tribution21(6,2))
(Modalmassdistribution21(4,3)+Modalmassdistribution21(5,3)+Modalmasdis
tribution21(6,3))
(Modalmassdistribution21(4,4)+Modalmassdistribution21(5,4)+Modalmasdis
tribution21(6,4))
(Modalmassdistribution21(4,5)+Modalmassdistribution21(5,5)+Modalmasdis
tribution21(6,5))
```

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(Modalmassdistribution21(4,6)+Modalmassdistribution21(5,6)+Modalmassdistribution21(6,6))
(Modalmassdistribution21(4,7)+Modalmassdistribution21(5,7)+Modalmassdistribution21(6,7))
(Modalmassdistribution21(4,8)+Modalmassdistribution21(5,8)+Modalmassdistribution21(6,8))
(Modalmassdistribution21(4,9)+Modalmassdistribution21(5,9)+Modalmassdistribution21(6,9));(Modalmassdistribution21(7,1)+Modalmassdistribution21(8,1)+Modalmassdistribution21(9,1))
(Modalmassdistribution21(7,2)+Modalmassdistribution21(8,2)+Modalmassdistribution21(9,2))
(Modalmassdistribution21(7,3)+Modalmassdistribution21(8,3)+Modalmassdistribution21(9,3))
(Modalmassdistribution21(7,4)+Modalmassdistribution21(8,4)+Modalmassdistribution21(9,4))
(Modalmassdistribution21(7,5)+Modalmassdistribution21(8,5)+Modalmassdistribution21(9,5))
(Modalmassdistribution21(7,6)+Modalmassdistribution21(8,6)+Modalmassdistribution21(9,6))
(Modalmassdistribution21(7,7)+Modalmassdistribution21(8,7)+Modalmassdistribution21(9,7))
(Modalmassdistribution21(7,8)+Modalmassdistribution21(8,8)+Modalmassdistribution21(9,8))
(Modalmassdistribution21(7,9)+Modalmassdistribution21(8,9)+Modalmassdistribution21(9,9))] /M);
Effectivemodalmass22=simplify(simplify([(Modalmassdistribution22(1,1)+Modalmassdistribution22(2,1)+Modalmassdistribution22(3,1))
(Modalmassdistribution22(1,2)+Modalmassdistribution22(2,2)+Modalmassdistribution22(3,2))
(Modalmassdistribution22(1,3)+Modalmassdistribution22(2,3)+Modalmassdistribution22(3,3))
(Modalmassdistribution22(1,4)+Modalmassdistribution22(2,4)+Modalmassdistribution22(3,4))
(Modalmassdistribution22(1,5)+Modalmassdistribution22(2,5)+Modalmassdistribution22(3,5))
(Modalmassdistribution22(1,6)+Modalmassdistribution22(2,6)+Modalmassdistribution22(3,6))
(Modalmassdistribution22(1,7)+Modalmassdistribution22(2,7)+Modalmassdistribution22(3,7))
(Modalmassdistribution22(1,8)+Modalmassdistribution22(2,8)+Modalmassdistribution22(3,8))
(Modalmassdistribution22(1,9)+Modalmassdistribution22(2,9)+Modalmassdistribution22(3,9));(Modalmassdistribution22(4,1)+Modalmassdistribution22(5,1)+Modalmassdistribution22(6,1))
(Modalmassdistribution22(4,2)+Modalmassdistribution22(5,2)+Modalmassdistribution22(6,2))
(Modalmassdistribution22(4,3)+Modalmassdistribution22(5,3)+Modalmassdistribution22(6,3))
(Modalmassdistribution22(4,4)+Modalmassdistribution22(5,4)+Modalmassdistribution22(6,4))
(Modalmassdistribution22(4,5)+Modalmassdistribution22(5,5)+Modalmassdistribution22(6,5))
(Modalmassdistribution22(4,6)+Modalmassdistribution22(5,6)+Modalmassdistribution22(6,6))
(Modalmassdistribution22(4,7)+Modalmassdistribution22(5,7)+Modalmassdistribution22(6,7))
(Modalmassdistribution22(4,8)+Modalmassdistribution22(5,8)+Modalmassdistribution22(6,8))

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```
tribution22(6,8)
(Modalmassdistribution22(4,9)+Modalmassdistribution22(5,9)+Modalmassdis
tribution22(6,9));(Modalmassdistribution22(7,1)+Modalmassdistribution22
(8,1)+Modalmassdistribution22(9,1)
(Modalmassdistribution22(7,2)+Modalmassdistribution22(8,2)+Modalmassdis
tribution22(9,2)
(Modalmassdistribution22(7,3)+Modalmassdistribution22(8,3)+Modalmassdis
tribution22(9,3)
(Modalmassdistribution22(7,4)+Modalmassdistribution22(8,4)+Modalmassdis
tribution22(9,4)
(Modalmassdistribution22(7,5)+Modalmassdistribution22(8,5)+Modalmassdis
tribution22(9,5)
(Modalmassdistribution22(7,6)+Modalmassdistribution22(8,6)+Modalmassdis
tribution22(9,6)
(Modalmassdistribution22(7,7)+Modalmassdistribution22(8,7)+Modalmassdis
tribution22(9,7)
(Modalmassdistribution22(7,8)+Modalmassdistribution22(8,8)+Modalmassdis
tribution22(9,8)
(Modalmassdistribution22(7,9)+Modalmassdistribution22(8,9)+Modalmassdis
tribution22(9,9)])/M);
r2=[(modalparticipationfactor2(1,1)/((w2(1,1))^2))
(modalparticipationfactor2(2,1)/((w2(2,2))^2))
(modalparticipationfactor2(3,1)/((w2(3,3))^2))
(modalparticipationfactor2(4,1)/((w2(4,4))^2))
(modalparticipationfactor2(5,1)/((w2(5,5))^2))
(modalparticipationfactor2(6,1)/((w2(6,6))^2))
(modalparticipationfactor2(7,1)/((w2(7,7))^2))
(modalparticipationfactor2(8,1)/((w2(8,8))^2))
(modalparticipationfactor2(9,1)/((w2(9,9))^2));(modalparticipationfacto
r2(1,2)/((w2(1,1))^2)) (modalparticipationfactor2(2,2)/((w2(2,2))^2))
(modalparticipationfactor2(3,2)/((w2(3,3))^2))
(modalparticipationfactor2(4,2)/((w2(4,4))^2))
(modalparticipationfactor2(5,2)/((w2(5,5))^2))
(modalparticipationfactor2(6,2)/((w2(6,6))^2))
(modalparticipationfactor2(7,2)/((w2(7,7))^2))
(modalparticipationfactor2(8,2)/((w2(8,8))^2))
(modalparticipationfactor2(9,2)/((w2(9,9))^2))]';
Ujn21=[r2(1,1)*mode2(:,1) r2(2,1)*mode2(:,2) r2(3,1)*mode2(:,3)
r2(4,1)*mode2(:,4) r2(5,1)*mode2(:,5) r2(6,1)*mode2(:,6)
r2(7,1)*mode2(:,7) r2(8,1)*mode2(:,8) r2(9,1)*mode2(:,9)];
U21mode1=Ujn21(:,1)*Ky/M
U21mode2=Ujn21(:,2)*Ky/M
U21mode3=Ujn21(:,3)*Ky/M
U21mode4=Ujn21(:,4)*Ky/M
U21mode5=Ujn21(:,5)*Ky/M
U21mode6=Ujn21(:,6)*Ky/M
U21mode7=Ujn21(:,7)*Ky/M
U21mode8=Ujn21(:,8)*Ky/M
U21mode9=Ujn21(:,9)*Ky/M
Ujn22=[r2(1,2)*mode2(:,1) r2(2,2)*mode2(:,2) r2(3,2)*mode2(:,3)
r2(4,2)*mode2(:,4) r2(5,2)*mode2(:,5) r2(6,2)*mode2(:,6)
r2(7,2)*mode2(:,7) r2(8,2)*mode2(:,8) r2(9,2)*mode2(:,9)];
U22mode1=Ujn22(:,1)*Ky/M
U22mode2=Ujn22(:,2)*Ky/M
U22mode3=Ujn22(:,3)*Ky/M
U22mode4=Ujn22(:,4)*Ky/M
U22mode5=Ujn22(:,5)*Ky/M
```

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```

U22mode6=Ujn22(:,6)*Ky/M
U22mode7=Ujn22(:,7)*Ky/M
U22mode8=Ujn22(:,8)*Ky/M
U22mode9=Ujn22(:,9)*Ky/M
stiffness3=[2*Kx -Kx 0 0 0 0 -2*Kx*ey Kx*ey 0;-Kx 2*Kx -Kx 0 0 0 Kx*ey
-2*Kx*ey Kx*ey;0 -Kx Kx 0 0 0 0 Kx*ey -Kx*ey;0 0 0 2*Ky -Ky 0
2*Ky*(ex+a) -Ky*(ex+a) 0;0 0 0 -Ky 2*Ky -Ky -Ky*(ex+a) 2*Ky*(ex+a) -
Ky*(ex+a);0 0 0 0 -Ky Ky 0 -Ky*(ex+a) Ky*(ex+a);-2*Kx*ey Kx*ey 0
2*Ky*(ex+a) -Ky*(ex+a) 0
Kx*((Dy^2)/2)+2*(ey^2))+Ky*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2)) -
(Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2)) 0 ;Kx*ey -2*Kx*ey Kx*ey -
Ky*(ex+a) 2*Ky*(ex+a) -Ky*(ex+a) (-Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2))
Kx*((Dy^2)/2)+2*(ey^2))+Ky*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2)) -
(Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2));0 Kx*ey -Kx*ey 0 -
Ky*(ex+a) Ky*(ex+a) 0 -(Kx/2)*((Dy^2)/2)+2*(ey^2))-
(Ky/2)*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^2))
(Kx/2)*((Dy^2)/2)+2*(ey^2))+(Ky/2)*((Dx^2)/2)+2*(ex^2)+(4*ex*a)+2*(a^
2)]];
Stiffness3=subs(stiffness3)/Ky;
[mode3,w32]=eig(inv(Mass)*Stiffness3);
mode3;
w3=sqrt(w32)*((Ky/M)^0.5);
Period3=eye(9)/w3;
period3=[Period3(1,1) Period3(2,2) Period3(3,3) Period3(4,4)
Period3(5,5) Period3(6,6) Period3(7,7) Period3(8,8) Period3(9,9)]'
Modalmass3=mode3'*subs(mass)*mode3;
Ln3=simplify(mode3'*subs(mass)*influencefactor);
modalparticipationfactor3=[Ln3(1,1)/Modalmass3(1,1)
Ln3(2,1)/Modalmass3(2,2) Ln3(3,1)/Modalmass3(3,3)
Ln3(4,1)/Modalmass3(4,4) Ln3(5,1)/Modalmass3(5,5)
Ln3(6,1)/Modalmass3(6,6) Ln3(7,1)/Modalmass3(7,7)
Ln3(8,1)/Modalmass3(8,8)
Ln3(9,1)/Modalmass3(9,9);Ln3(1,2)/Modalmass3(1,1)
Ln3(2,2)/Modalmass3(2,2) Ln3(3,2)/Modalmass3(3,3)
Ln3(4,2)/Modalmass3(4,4) Ln3(5,2)/Modalmass3(5,5)
Ln3(6,2)/Modalmass3(6,6) Ln3(7,1)/Modalmass3(7,7)
Ln3(8,1)/Modalmass3(8,8) Ln3(9,1)/Modalmass3(9,9)]';
Modalmassdistribution31=[modalparticipationfactor3(1,1)*subs(mass)*mode
3(:,1) modalparticipationfactor3(2,1)*subs(mass)*mode3(:,2)
modalparticipationfactor3(3,1)*subs(mass)*mode3(:,3)
modalparticipationfactor3(4,1)*subs(mass)*mode3(:,4)
modalparticipationfactor3(5,1)*subs(mass)*mode3(:,5)
modalparticipationfactor3(6,1)*subs(mass)*mode3(:,6)
modalparticipationfactor3(7,1)*subs(mass)*mode3(:,7)
modalparticipationfactor3(8,1)*subs(mass)*mode3(:,8)
modalparticipationfactor3(9,1)*subs(mass)*mode3(:,9)];
Modalmassdistribution31mode1=Modalmassdistribution31(:,1)/M
Modalmassdistribution31mode2=Modalmassdistribution31(:,2)/M
Modalmassdistribution31mode3=Modalmassdistribution31(:,3)/M
Modalmassdistribution31mode4=Modalmassdistribution31(:,4)/M
Modalmassdistribution31mode5=Modalmassdistribution31(:,5)/M
Modalmassdistribution31mode6=Modalmassdistribution31(:,6)/M
Modalmassdistribution31mode7=Modalmassdistribution31(:,7)/M
Modalmassdistribution31mode8=Modalmassdistribution31(:,8)/M

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```
Modalmassdistribution31mode9=Modalmassdistribution31(:,9)/M
Modalmassdistribution32=[modalparticipationfactor3(1,2)*subs(mass)*mode3(:,1)
modalparticipationfactor3(2,2)*subs(mass)*mode3(:,2)
modalparticipationfactor3(3,2)*subs(mass)*mode3(:,3)
modalparticipationfactor3(4,2)*subs(mass)*mode3(:,4)
modalparticipationfactor3(5,2)*subs(mass)*mode3(:,5)
modalparticipationfactor3(6,2)*subs(mass)*mode3(:,6)
modalparticipationfactor3(7,2)*subs(mass)*mode3(:,7)
modalparticipationfactor3(8,2)*subs(mass)*mode3(:,8)
modalparticipationfactor3(9,2)*subs(mass)*mode3(:,9)];
Modalmassdistribution32mode1=Modalmassdistribution32(:,1)/M
Modalmassdistribution32mode2=Modalmassdistribution32(:,2)/M
Modalmassdistribution32mode3=Modalmassdistribution32(:,3)/M
Modalmassdistribution32mode4=Modalmassdistribution32(:,4)/M
Modalmassdistribution32mode5=Modalmassdistribution32(:,5)/M
Modalmassdistribution32mode6=Modalmassdistribution32(:,6)/M
Modalmassdistribution32mode7=Modalmassdistribution32(:,7)/M
Modalmassdistribution32mode8=Modalmassdistribution32(:,8)/M
Modalmassdistribution32mode9=Modalmassdistribution32(:,9)/M
Effectivemodalmass31=simplify(simplify([(Modalmassdistribution31(1,1)+
odalmassdistribution31(2,1)+Modalmassdistribution31(3,1))
(Modalmassdistribution31(1,2)+Modalmassdistribution31(2,2)+Modalmassdis
tribution31(3,2))
(Modalmassdistribution31(1,3)+Modalmassdistribution31(2,3)+Modalmassdis
tribution31(3,3))
(Modalmassdistribution31(1,4)+Modalmassdistribution31(2,4)+Modalmassdis
tribution31(3,4))
(Modalmassdistribution31(1,5)+Modalmassdistribution31(2,5)+Modalmassdis
tribution31(3,5))
(Modalmassdistribution31(1,6)+Modalmassdistribution31(2,6)+Modalmassdis
tribution31(3,6))
(Modalmassdistribution31(1,7)+Modalmassdistribution31(2,7)+Modalmassdis
tribution31(3,7))
(Modalmassdistribution31(1,8)+Modalmassdistribution31(2,8)+Modalmassdis
tribution31(3,8))
(Modalmassdistribution31(1,9)+Modalmassdistribution31(2,9)+Modalmassdis
tribution31(3,9));(Modalmassdistribution31(4,1)+Modalmassdistribution31
(5,1)+Modalmassdistribution31(6,1))
(Modalmassdistribution31(4,2)+Modalmassdistribution31(5,2)+Modalmassdis
tribution31(6,2))
(Modalmassdistribution31(4,3)+Modalmassdistribution31(5,3)+Modalmassdis
tribution31(6,3))
(Modalmassdistribution31(4,4)+Modalmassdistribution31(5,4)+Modalmassdis
tribution31(6,4))
(Modalmassdistribution31(4,5)+Modalmassdistribution31(5,5)+Modalmassdis
tribution31(6,5))
(Modalmassdistribution31(4,6)+Modalmassdistribution31(5,6)+Modalmassdis
tribution31(6,6))
(Modalmassdistribution31(4,7)+Modalmassdistribution31(5,7)+Modalmassdis
tribution31(6,7))
(Modalmassdistribution31(4,8)+Modalmassdistribution31(5,8)+Modalmassdis
tribution31(6,8))
(Modalmassdistribution31(4,9)+Modalmassdistribution31(5,9)+Modalmassdis
tribution31(6,9));(Modalmassdistribution31(7,1)+Modalmassdistribution31
(8,1)+Modalmassdistribution31(9,1))
(Modalmassdistribution31(7,2)+Modalmassdistribution31(8,2)+Modalmassdis
tribution31(9,2))
```

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(Modalmassdistribution31 (7, 3)+Modalmassdistribution31 (8, 3)+Modalmassdistribution31 (9, 3))
(Modalmassdistribution31 (7, 4)+Modalmassdistribution31 (8, 4)+Modalmassdistribution31 (9, 4))
(Modalmassdistribution31 (7, 5)+Modalmassdistribution31 (8, 5)+Modalmassdistribution31 (9, 5))
(Modalmassdistribution31 (7, 6)+Modalmassdistribution31 (8, 6)+Modalmassdistribution31 (9, 6))
(Modalmassdistribution31 (7, 7)+Modalmassdistribution31 (8, 7)+Modalmassdistribution31 (9, 7))
(Modalmassdistribution31 (7, 8)+Modalmassdistribution31 (8, 8)+Modalmassdistribution31 (9, 8))
(Modalmassdistribution31 (7, 9)+Modalmassdistribution31 (8, 9)+Modalmassdistribution31 (9, 9))]) /M) ;
Effectivemodalmass32=simplify (simplify ([(Modalmassdistribution32 (1, 1)+Modalmassdistribution32 (2, 1)+Modalmassdistribution32 (3, 1))
(Modalmassdistribution32 (1, 2)+Modalmassdistribution32 (2, 2)+Modalmassdistribution32 (3, 2))
(Modalmassdistribution32 (1, 3)+Modalmassdistribution32 (2, 3)+Modalmassdistribution32 (3, 3))
(Modalmassdistribution32 (1, 4)+Modalmassdistribution32 (2, 4)+Modalmassdistribution32 (3, 4))
(Modalmassdistribution32 (1, 5)+Modalmassdistribution32 (2, 5)+Modalmassdistribution32 (3, 5))
(Modalmassdistribution32 (1, 6)+Modalmassdistribution32 (2, 6)+Modalmassdistribution32 (3, 6))
(Modalmassdistribution32 (1, 7)+Modalmassdistribution32 (2, 7)+Modalmassdistribution32 (3, 7))
(Modalmassdistribution32 (1, 8)+Modalmassdistribution32 (2, 8)+Modalmassdistribution32 (3, 8))
(Modalmassdistribution32 (1, 9)+Modalmassdistribution32 (2, 9)+Modalmassdistribution32 (3, 9)) ; (Modalmassdistribution32 (4, 1)+Modalmassdistribution32 (5, 1)+Modalmassdistribution32 (6, 1))
(Modalmassdistribution32 (4, 2)+Modalmassdistribution32 (5, 2)+Modalmassdistribution32 (6, 2))
(Modalmassdistribution32 (4, 3)+Modalmassdistribution32 (5, 3)+Modalmassdistribution32 (6, 3))
(Modalmassdistribution32 (4, 4)+Modalmassdistribution32 (5, 4)+Modalmassdistribution32 (6, 4))
(Modalmassdistribution32 (4, 5)+Modalmassdistribution32 (5, 5)+Modalmassdistribution32 (6, 5))
(Modalmassdistribution32 (4, 6)+Modalmassdistribution32 (5, 6)+Modalmassdistribution32 (6, 6))
(Modalmassdistribution32 (4, 7)+Modalmassdistribution32 (5, 7)+Modalmassdistribution32 (6, 7))
(Modalmassdistribution32 (4, 8)+Modalmassdistribution32 (5, 8)+Modalmassdistribution32 (6, 8))
(Modalmassdistribution32 (4, 9)+Modalmassdistribution32 (5, 9)+Modalmassdistribution32 (6, 9)) ; (Modalmassdistribution32 (7, 1)+Modalmassdistribution32 (8, 1)+Modalmassdistribution32 (9, 1))
(Modalmassdistribution32 (7, 2)+Modalmassdistribution32 (8, 2)+Modalmassdistribution32 (9, 2))
(Modalmassdistribution32 (7, 3)+Modalmassdistribution32 (8, 3)+Modalmassdistribution32 (9, 3))
(Modalmassdistribution32 (7, 4)+Modalmassdistribution32 (8, 4)+Modalmassdistribution32 (9, 4))
(Modalmassdistribution32 (7, 5)+Modalmassdistribution32 (8, 5)+Modalmassdistribution32 (9, 5))

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```

tribution32(9,5)
  (Modalmassdistribution32(7,6)+Modalmassdistribution32(8,6)+Modalmassdis
tribution32(9,6)
  (Modalmassdistribution32(7,7)+Modalmassdistribution32(8,7)+Modalmassdis
tribution32(9,7)
  (Modalmassdistribution32(7,8)+Modalmassdistribution32(8,8)+Modalmassdis
tribution32(9,8)
  (Modalmassdistribution32(7,9)+Modalmassdistribution32(8,9)+Modalmassdis
tribution32(9,9) ])/M);
r3=[(modalparticipationfactor3(1,1)/((w3(1,1))^2))
(modalparticipationfactor3(2,1)/((w3(2,2))^2))
(modalparticipationfactor3(3,1)/((w3(3,3))^2))
(modalparticipationfactor3(4,1)/((w3(4,4))^2))
(modalparticipationfactor3(5,1)/((w3(5,5))^2))
(modalparticipationfactor3(6,1)/((w3(6,6))^2))
(modalparticipationfactor3(7,1)/((w3(7,7))^2))
(modalparticipationfactor3(8,1)/((w3(8,8))^2))
(modalparticipationfactor3(9,1)/((w3(9,9))^2));(modalparticipationfacto
r3(1,2)/((w3(1,1))^2)) (modalparticipationfactor3(2,2)/((w3(2,2))^2))
(modalparticipationfactor3(3,2)/((w3(3,3))^2))
(modalparticipationfactor3(4,2)/((w3(4,4))^2))
(modalparticipationfactor3(5,2)/((w3(5,5))^2))
(modalparticipationfactor3(6,2)/((w3(6,6))^2))
(modalparticipationfactor3(7,2)/((w3(7,7))^2))
(modalparticipationfactor3(8,2)/((w3(8,8))^2))
(modalparticipationfactor3(9,2)/((w3(9,9))^2))]';
Ujn31=[r3(1,1)*mode3(:,1) r3(2,1)*mode3(:,2) r3(3,1)*mode3(:,3)
r3(4,1)*mode3(:,4) r3(5,1)*mode3(:,5) r3(6,1)*mode3(:,6)
r3(7,1)*mode3(:,7) r3(8,1)*mode3(:,8) r3(9,1)*mode3(:,9)];
U31mode1=Ujn31(:,1)*Ky/M
U31mode2=Ujn31(:,2)*Ky/M
U31mode3=Ujn31(:,3)*Ky/M
U31mode4=Ujn31(:,4)*Ky/M
U31mode5=Ujn31(:,5)*Ky/M
U31mode6=Ujn31(:,6)*Ky/M
U31mode7=Ujn31(:,7)*Ky/M
U31mode8=Ujn31(:,8)*Ky/M
U31mode9=Ujn31(:,9)*Ky/M
Ujn32=[r3(1,2)*mode3(:,1) r3(2,2)*mode3(:,2) r3(3,2)*mode3(:,3)
r3(4,2)*mode3(:,4) r3(5,2)*mode3(:,5) r3(6,2)*mode3(:,6)
r3(7,2)*mode3(:,7) r3(8,2)*mode3(:,8) r3(9,2)*mode3(:,9)];
U32mode1=Ujn32(:,1)*Ky/M
U32mode2=Ujn32(:,2)*Ky/M
U32mode3=Ujn32(:,3)*Ky/M
U32mode4=Ujn32(:,4)*Ky/M
U32mode5=Ujn32(:,5)*Ky/M
U32mode6=Ujn32(:,6)*Ky/M
U32mode7=Ujn32(:,7)*Ky/M
U32mode8=Ujn32(:,8)*Ky/M
U32mode9=Ujn32(:,9)*Ky/M
stiffness4=[2*Kx -Kx 0 0 0 0 -2*Kx*(ey+b) Kx*(ey+b) 0;-Kx 2*Kx -Kx 0 0
0 Kx*(ey+b) -2*Kx*(ey+b) Kx*(ey+b);0 -Kx Kx 0 0 0 0 Kx*(ey+b) -
Kx*(ey+b);0 0 0 2*Ky -Ky 0 2*Ky*ex -Ky*ex 0;0 0 0 -Ky 2*Ky -Ky -Ky*ex
2*Ky*ex -Ky*ex;0 0 0 0 -Ky Ky 0 -Ky*ex Ky*ex;-2*Kx*(ey+b) Kx*(ey+b) 0
2*Ky*ex -Ky*ex 0
Kx*((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))+Ky*((Dx^2)/2)+2*(ex^2)) -
(Kx/2)*((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))-

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(Ky/2)*(((Dx^2)/2)+2*(ex^2)) 0 ;Kx*(ey+b) -2*Kx*(ey+b) Kx*(ey+b) -Ky*ex
2*Ky*ex -Ky*ex (-Kx/2)*(((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2))
Kx*(((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))+Ky*(((Dx^2)/2)+2*(ex^2)) -
(Kx/2)*(((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2));0 Kx*(ey+b) -Kx*(ey+b) 0 -Ky*ex Ky*ex 0 -
(Kx/2)*(((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))-
(Ky/2)*(((Dx^2)/2)+2*(ex^2))
(Kx/2)*(((Dy^2)/2)+2*(ey^2)+(4*ey*b)+2*(b^2))+(Ky/2)*(((Dx^2)/2)+2*(ex^
2)]];
Stiffness4=subs(stiffness4)/Ky;
[mode4,w42]=eig(inv(Mass)*Stiffness4);
mode4;
w4=sqrt(w42)*((Ky/M)^0.5);
Period4=eye(9)/w4;
period4=[Period4(1,1) Period4(2,2) Period4(3,3) Period4(4,4)
Period4(5,5) Period4(6,6) Period4(7,7) Period4(8,8) Period4(9,9)]';
Modalmass4=mode4'*subs(mass)*mode4;
Ln4=simplify(mode4'*subs(mass)*influencefactor);
modalparticipationfactor4=[Ln4(1,1)/Modalmass4(1,1)
Ln4(2,1)/Modalmass4(2,2) Ln4(3,1)/Modalmass4(3,3)
Ln4(4,1)/Modalmass4(4,4) Ln4(5,1)/Modalmass4(5,5)
Ln4(6,1)/Modalmass4(6,6) Ln4(7,1)/Modalmass4(7,7)
Ln4(8,1)/Modalmass4(8,8)
Ln4(9,1)/Modalmass4(9,9);Ln4(1,2)/Modalmass4(1,1)
Ln4(2,2)/Modalmass4(2,2) Ln4(3,2)/Modalmass4(3,3)
Ln4(4,2)/Modalmass4(4,4) Ln4(5,2)/Modalmass4(5,5)
Ln4(6,2)/Modalmass4(6,6) Ln4(7,1)/Modalmass4(7,7)
Ln4(8,1)/Modalmass4(8,8) Ln4(9,1)/Modalmass4(9,9)]';
Modalmassdistribution41=[modalparticipationfactor4(1,1)*subs(mass)*mode
4(:,1) modalparticipationfactor4(2,1)*subs(mass)*mode4(:,2)
modalparticipationfactor4(3,1)*subs(mass)*mode4(:,3)
modalparticipationfactor4(4,1)*subs(mass)*mode4(:,4)
modalparticipationfactor4(5,1)*subs(mass)*mode4(:,5)
modalparticipationfactor4(6,1)*subs(mass)*mode4(:,6)
modalparticipationfactor4(7,1)*subs(mass)*mode4(:,7)
modalparticipationfactor4(8,1)*subs(mass)*mode4(:,8)
modalparticipationfactor4(9,1)*subs(mass)*mode4(:,9)];
Modalmassdistribution41mode1=Modalmassdistribution41(:,1)/M
Modalmassdistribution41mode2=Modalmassdistribution41(:,2)/M
Modalmassdistribution41mode3=Modalmassdistribution41(:,3)/M
Modalmassdistribution41mode4=Modalmassdistribution41(:,4)/M
Modalmassdistribution41mode5=Modalmassdistribution41(:,5)/M
Modalmassdistribution41mode6=Modalmassdistribution41(:,6)/M
Modalmassdistribution41mode7=Modalmassdistribution41(:,7)/M
Modalmassdistribution41mode8=Modalmassdistribution41(:,8)/M
Modalmassdistribution41mode9=Modalmassdistribution41(:,9)/M
Modalmassdistribution42=[modalparticipationfactor4(1,2)*subs(mass)*mode
4(:,1) modalparticipationfactor4(2,2)*subs(mass)*mode4(:,2)
modalparticipationfactor4(3,2)*subs(mass)*mode4(:,3)
modalparticipationfactor4(4,2)*subs(mass)*mode4(:,4)
modalparticipationfactor4(5,2)*subs(mass)*mode4(:,5)
modalparticipationfactor4(6,2)*subs(mass)*mode4(:,6)
modalparticipationfactor4(7,2)*subs(mass)*mode4(:,7)
modalparticipationfactor4(8,2)*subs(mass)*mode4(:,8)
modalparticipationfactor4(9,2)*subs(mass)*mode4(:,9)];
Modalmassdistribution42mode1=Modalmassdistribution42(:,1)/M

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```
Modalmassdistribution42mode2=Modalmassdistribution42(:,2)/M
Modalmassdistribution42mode3=Modalmassdistribution42(:,3)/M
Modalmassdistribution42mode4=Modalmassdistribution42(:,4)/M
Modalmassdistribution42mode5=Modalmassdistribution42(:,5)/M
Modalmassdistribution42mode6=Modalmassdistribution42(:,6)/M
Modalmassdistribution42mode7=Modalmassdistribution42(:,7)/M
Modalmassdistribution42mode8=Modalmassdistribution42(:,8)/M
Modalmassdistribution42mode9=Modalmassdistribution42(:,9)/M
Effectivemodalmass41=simplify(simplify([(Modalmassdistribution41(1,1)+M
odalmassdistribution41(2,1)+Modalmassdistribution41(3,1))
(Modalmassdistribution41(1,2)+Modalmassdistribution41(2,2)+Modalmassdis
tribution41(3,2))
(Modalmassdistribution41(1,3)+Modalmassdistribution41(2,3)+Modalmassdis
tribution41(3,3))
(Modalmassdistribution41(1,4)+Modalmassdistribution41(2,4)+Modalmassdis
tribution41(3,4))
(Modalmassdistribution41(1,5)+Modalmassdistribution41(2,5)+Modalmassdis
tribution41(3,5))
(Modalmassdistribution41(1,6)+Modalmassdistribution41(2,6)+Modalmassdis
tribution41(3,6))
(Modalmassdistribution41(1,7)+Modalmassdistribution41(2,7)+Modalmassdis
tribution41(3,7))
(Modalmassdistribution41(1,8)+Modalmassdistribution41(2,8)+Modalmassdis
tribution41(3,8))
(Modalmassdistribution41(1,9)+Modalmassdistribution41(2,9)+Modalmassdis
tribution41(3,9));(Modalmassdistribution41(4,1)+Modalmassdistribution41
(5,1)+Modalmassdistribution41(6,1))
(Modalmassdistribution41(4,2)+Modalmassdistribution41(5,2)+Modalmassdis
tribution41(6,2))
(Modalmassdistribution41(4,3)+Modalmassdistribution41(5,3)+Modalmassdis
tribution41(6,3))
(Modalmassdistribution41(4,4)+Modalmassdistribution41(5,4)+Modalmassdis
tribution41(6,4))
(Modalmassdistribution41(4,5)+Modalmassdistribution41(5,5)+Modalmassdis
tribution41(6,5))
(Modalmassdistribution41(4,6)+Modalmassdistribution41(5,6)+Modalmassdis
tribution41(6,6))
(Modalmassdistribution41(4,7)+Modalmassdistribution41(5,7)+Modalmassdis
tribution41(6,7))
(Modalmassdistribution41(4,8)+Modalmassdistribution41(5,8)+Modalmassdis
tribution41(6,8))
(Modalmassdistribution41(4,9)+Modalmassdistribution41(5,9)+Modalmassdis
tribution41(6,9));(Modalmassdistribution41(7,1)+Modalmassdistribution41
(8,1)+Modalmassdistribution41(9,1))
(Modalmassdistribution41(7,2)+Modalmassdistribution41(8,2)+Modalmassdis
tribution41(9,2))
(Modalmassdistribution41(7,3)+Modalmassdistribution41(8,3)+Modalmassdis
tribution41(9,3))
(Modalmassdistribution41(7,4)+Modalmassdistribution41(8,4)+Modalmassdis
tribution41(9,4))
(Modalmassdistribution41(7,5)+Modalmassdistribution41(8,5)+Modalmassdis
tribution41(9,5))
(Modalmassdistribution41(7,6)+Modalmassdistribution41(8,6)+Modalmassdis
tribution41(9,6))
(Modalmassdistribution41(7,7)+Modalmassdistribution41(8,7)+Modalmassdis
tribution41(9,7))
(Modalmassdistribution41(7,8)+Modalmassdistribution41(8,8)+Modalmassdis
```

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```
tribution41(9,8))
(Modalmassdistribution41(7,9)+Modalmassdistribution41(8,9)+Modalmassdis
tribution41(9,9))]/M);
Effectivemodalmass42=simplify(simplify([(Modalmassdistribution42(1,1)+M
odalmassdistribution42(2,1)+Modalmassdistribution42(3,1))
(Modalmassdistribution42(1,2)+Modalmassdistribution42(2,2)+Modalmassdis
tribution42(3,2))
(Modalmassdistribution42(1,3)+Modalmassdistribution42(2,3)+Modalmassdis
tribution42(3,3))
(Modalmassdistribution42(1,4)+Modalmassdistribution42(2,4)+Modalmassdis
tribution42(3,4))
(Modalmassdistribution42(1,5)+Modalmassdistribution42(2,5)+Modalmassdis
tribution42(3,5))
(Modalmassdistribution42(1,6)+Modalmassdistribution42(2,6)+Modalmassdis
tribution42(3,6))
(Modalmassdistribution42(1,7)+Modalmassdistribution42(2,7)+Modalmassdis
tribution42(3,7))
(Modalmassdistribution42(1,8)+Modalmassdistribution42(2,8)+Modalmassdis
tribution42(3,8))
(Modalmassdistribution42(1,9)+Modalmassdistribution42(2,9)+Modalmassdis
tribution42(3,9));(Modalmassdistribution42(4,1)+Modalmassdistribution42
(5,1)+Modalmassdistribution42(6,1))
(Modalmassdistribution42(4,2)+Modalmassdistribution42(5,2)+Modalmassdis
tribution42(6,2))
(Modalmassdistribution42(4,3)+Modalmassdistribution42(5,3)+Modalmassdis
tribution42(6,3))
(Modalmassdistribution42(4,4)+Modalmassdistribution42(5,4)+Modalmassdis
tribution42(6,4))
(Modalmassdistribution42(4,5)+Modalmassdistribution42(5,5)+Modalmassdis
tribution42(6,5))
(Modalmassdistribution42(4,6)+Modalmassdistribution42(5,6)+Modalmassdis
tribution42(6,6))
(Modalmassdistribution42(4,7)+Modalmassdistribution42(5,7)+Modalmassdis
tribution42(6,7))
(Modalmassdistribution42(4,8)+Modalmassdistribution42(5,8)+Modalmassdis
tribution42(6,8))
(Modalmassdistribution42(4,9)+Modalmassdistribution42(5,9)+Modalmassdis
tribution42(6,9));(Modalmassdistribution42(7,1)+Modalmassdistribution42
(8,1)+Modalmassdistribution42(9,1))
(Modalmassdistribution42(7,2)+Modalmassdistribution42(8,2)+Modalmassdis
tribution42(9,2))
(Modalmassdistribution42(7,3)+Modalmassdistribution42(8,3)+Modalmassdis
tribution42(9,3))
(Modalmassdistribution42(7,4)+Modalmassdistribution42(8,4)+Modalmassdis
tribution42(9,4))
(Modalmassdistribution42(7,5)+Modalmassdistribution42(8,5)+Modalmassdis
tribution42(9,5))
(Modalmassdistribution42(7,6)+Modalmassdistribution42(8,6)+Modalmassdis
tribution42(9,6))
(Modalmassdistribution42(7,7)+Modalmassdistribution42(8,7)+Modalmassdis
tribution42(9,7))
(Modalmassdistribution42(7,8)+Modalmassdistribution42(8,8)+Modalmassdis
tribution42(9,8))
(Modalmassdistribution42(7,9)+Modalmassdistribution42(8,9)+Modalmassdis
tribution42(9,9))]/M);
r4=[(modalparticipationfactor4(1,1)/((w4(1,1))^2))
(modalparticipationfactor4(2,1)/((w4(2,2))^2))
```

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```
(modalparticipationfactor4(3,1)/((w4(3,3))^2))
(modalparticipationfactor4(4,1)/((w4(4,4))^2))
(modalparticipationfactor4(5,1)/((w4(5,5))^2))
(modalparticipationfactor4(6,1)/((w4(6,6))^2))
(modalparticipationfactor4(7,1)/((w4(7,7))^2))
(modalparticipationfactor4(8,1)/((w4(8,8))^2))
(modalparticipationfactor4(9,1)/((w4(9,9))^2));(modalparticipationfacto
r4(1,2)/((w4(1,1))^2)) (modalparticipationfactor4(2,2)/((w4(2,2))^2))
(modalparticipationfactor4(3,2)/((w4(3,3))^2))
(modalparticipationfactor4(4,2)/((w4(4,4))^2))
(modalparticipationfactor4(5,2)/((w4(5,5))^2))
(modalparticipationfactor4(6,2)/((w4(6,6))^2))
(modalparticipationfactor4(7,2)/((w4(7,7))^2))
(modalparticipationfactor4(8,2)/((w4(8,8))^2))
(modalparticipationfactor4(9,2)/((w4(9,9))^2))]';
Ujn41=[r4(1,1)*mode4(:,1) r4(2,1)*mode4(:,2) r4(3,1)*mode4(:,3)
r4(4,1)*mode4(:,4) r4(5,1)*mode4(:,5) r4(6,1)*mode4(:,6)
r4(7,1)*mode4(:,7) r4(8,1)*mode4(:,8) r4(9,1)*mode4(:,9)];
U41mode1=Ujn41(:,1)*Ky/M
U41mode2=Ujn41(:,2)*Ky/M
U41mode3=Ujn41(:,3)*Ky/M
U41mode4=Ujn41(:,4)*Ky/M
U41mode5=Ujn41(:,5)*Ky/M
U41mode6=Ujn41(:,6)*Ky/M
U41mode7=Ujn41(:,7)*Ky/M
U41mode8=Ujn41(:,8)*Ky/M
U41mode9=Ujn41(:,9)*Ky/M
Ujn42=[r4(1,2)*mode4(:,1) r4(2,2)*mode4(:,2) r4(3,2)*mode4(:,3)
r4(4,2)*mode4(:,4) r4(5,2)*mode4(:,5) r4(6,2)*mode4(:,6)
r4(7,2)*mode4(:,7) r4(8,2)*mode4(:,8) r4(9,2)*mode4(:,9)];
U42mode1=Ujn42(:,1)*Ky/M
U42mode2=Ujn42(:,2)*Ky/M
U42mode3=Ujn42(:,3)*Ky/M
U42mode4=Ujn42(:,4)*Ky/M
U42mode5=Ujn42(:,5)*Ky/M
U42mode6=Ujn42(:,6)*Ky/M
U42mode7=Ujn42(:,7)*Ky/M
U42mode8=Ujn42(:,8)*Ky/M
U42mode9=Ujn42(:,9)*Ky/M
Effectivemodalxmass=[Effectivemodalxmass11(1,1)
Effectivemodalxmass12(1,1) Effectivemodalxmass21(1,1)
Effectivemodalxmass22(1,1) Effectivemodalxmass31(1,1)
Effectivemodalxmass32(1,1) Effectivemodalxmass41(1,1)
Effectivemodalxmass42(1,1)]';
Effectivemode2xmass=[Effectivemodalxmass11(1,2)
Effectivemodalxmass12(1,2) Effectivemodalxmass21(1,2)
Effectivemodalxmass22(1,2) Effectivemodalxmass31(1,2)
Effectivemodalxmass32(1,2) Effectivemodalxmass41(1,2)
Effectivemodalxmass42(1,2)]';
Effectivemode3xmass=[Effectivemodalxmass11(1,3)
Effectivemodalxmass12(1,3) Effectivemodalxmass21(1,3)
Effectivemodalxmass22(1,3) Effectivemodalxmass31(1,3)
Effectivemodalxmass32(1,3) Effectivemodalxmass41(1,3)
Effectivemodalxmass42(1,3)]';
Effectivemode4xmass=[Effectivemodalxmass11(1,4)
Effectivemodalxmass12(1,4) Effectivemodalxmass21(1,4)
Effectivemodalxmass22(1,4) Effectivemodalxmass31(1,4)
```

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```
Effectivemodalmass32(1,4) Effectivemodalmass41(1,4)
Effectivemodalmass42(1,4) ]';
Effectivemode5xmass=[Effectivemodalmass11(1,5)
Effectivemodalmass12(1,5) Effectivemodalmass21(1,5)
Effectivemodalmass22(1,5) Effectivemodalmass31(1,5)
Effectivemodalmass32(1,5) Effectivemodalmass41(1,5)
Effectivemodalmass42(1,5) ]';
Effectivemode6xmass=[Effectivemodalmass11(1,6)
Effectivemodalmass12(1,6) Effectivemodalmass21(1,6)
Effectivemodalmass22(1,6) Effectivemodalmass31(1,6)
Effectivemodalmass32(1,6) Effectivemodalmass41(1,6)
Effectivemodalmass42(1,6) ]';
Effectivemode7xmass=[Effectivemodalmass11(1,7)
Effectivemodalmass12(1,7) Effectivemodalmass21(1,7)
Effectivemodalmass22(1,7) Effectivemodalmass31(1,7)
Effectivemodalmass32(1,7) Effectivemodalmass41(1,7)
Effectivemodalmass42(1,7) ]';
Effectivemode8xmass=[Effectivemodalmass11(1,8)
Effectivemodalmass12(1,8) Effectivemodalmass21(1,8)
Effectivemodalmass22(1,8) Effectivemodalmass31(1,8)
Effectivemodalmass32(1,8) Effectivemodalmass41(1,8)
Effectivemodalmass42(1,8) ]';
Effectivemode9xmass=[Effectivemodalmass11(1,9)
Effectivemodalmass12(1,9) Effectivemodalmass21(1,9)
Effectivemodalmass22(1,9) Effectivemodalmass31(1,9)
Effectivemodalmass32(1,9) Effectivemodalmass41(1,9)
Effectivemodalmass42(1,9) ]';
Effectivemodelymass=[Effectivemodalmass11(2,1)
Effectivemodalmass12(2,1) Effectivemodalmass21(2,1)
Effectivemodalmass22(2,1) Effectivemodalmass31(2,1)
Effectivemodalmass32(2,1) Effectivemodalmass41(2,1)
Effectivemodalmass42(2,1) ]';
Effectivemode2ymass=[Effectivemodalmass11(2,2)
Effectivemodalmass12(2,2) Effectivemodalmass21(2,2)
Effectivemodalmass22(2,2) Effectivemodalmass31(2,2)
Effectivemodalmass32(2,2) Effectivemodalmass41(2,2)
Effectivemodalmass42(2,2) ]';
Effectivemode3ymass=[Effectivemodalmass11(2,3)
Effectivemodalmass12(2,3) Effectivemodalmass21(2,3)
Effectivemodalmass22(2,3) Effectivemodalmass31(2,3)
Effectivemodalmass32(2,3) Effectivemodalmass41(2,3)
Effectivemodalmass42(2,3) ]';
Effectivemode4ymass=[Effectivemodalmass11(2,4)
Effectivemodalmass12(2,4) Effectivemodalmass21(2,4)
Effectivemodalmass22(2,4) Effectivemodalmass31(2,4)
Effectivemodalmass32(2,4) Effectivemodalmass41(2,4)
Effectivemodalmass42(2,4) ]';
Effectivemode5ymass=[Effectivemodalmass11(2,5)
Effectivemodalmass12(2,5) Effectivemodalmass21(2,5)
Effectivemodalmass22(2,5) Effectivemodalmass31(2,5)
Effectivemodalmass32(2,5) Effectivemodalmass41(2,5)
Effectivemodalmass42(2,5) ]';
Effectivemode6ymass=[Effectivemodalmass11(2,6)
Effectivemodalmass12(2,6) Effectivemodalmass21(2,6)
Effectivemodalmass22(2,6) Effectivemodalmass31(2,6)
Effectivemodalmass32(2,6) Effectivemodalmass41(2,6)
Effectivemodalmass42(2,6) ]';
```

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```
Effectivemode7ymass=[Effectivemodalmass11(2,7)
Effectivemodalmass12(2,7) Effectivemodalmass21(2,7)
Effectivemodalmass22(2,7) Effectivemodalmass31(2,7)
Effectivemodalmass32(2,7) Effectivemodalmass41(2,7)
Effectivemodalmass42(2,7)]';
Effectivemode8ymass=[Effectivemodalmass11(2,8)
Effectivemodalmass12(2,8) Effectivemodalmass21(2,8)
Effectivemodalmass22(2,8) Effectivemodalmass31(2,8)
Effectivemodalmass32(2,8) Effectivemodalmass41(2,8)
Effectivemodalmass42(2,8)]';
Effectivemode9ymass=[Effectivemodalmass11(2,9)
Effectivemodalmass12(2,9) Effectivemodalmass21(2,9)
Effectivemodalmass22(2,9) Effectivemodalmass31(2,9)
Effectivemodalmass32(2,9) Effectivemodalmass41(2,9)
Effectivemodalmass42(2,9)]';
Effectivemode1zmass=[Effectivemodalmass11(3,1)
Effectivemodalmass12(3,1) Effectivemodalmass21(3,1)
Effectivemodalmass22(3,1) Effectivemodalmass31(3,1)
Effectivemodalmass32(3,1) Effectivemodalmass41(3,1)
Effectivemodalmass42(3,1)]';
Effectivemode2zmass=[Effectivemodalmass11(3,2)
Effectivemodalmass12(3,2) Effectivemodalmass21(3,2)
Effectivemodalmass22(3,2) Effectivemodalmass31(3,2)
Effectivemodalmass32(3,2) Effectivemodalmass41(3,2)
Effectivemodalmass42(3,2)]';
Effectivemode3zmass=[Effectivemodalmass11(3,3)
Effectivemodalmass12(3,3) Effectivemodalmass21(3,3)
Effectivemodalmass22(3,3) Effectivemodalmass31(3,3)
Effectivemodalmass32(3,3) Effectivemodalmass41(3,3)
Effectivemodalmass42(3,3)]';
Effectivemode4zmass=[Effectivemodalmass11(3,4)
Effectivemodalmass12(3,4) Effectivemodalmass21(3,4)
Effectivemodalmass22(3,4) Effectivemodalmass31(3,4)
Effectivemodalmass32(3,4) Effectivemodalmass41(3,4)
Effectivemodalmass42(3,4)]';
Effectivemode5zmass=[Effectivemodalmass11(3,5)
Effectivemodalmass12(3,5) Effectivemodalmass21(3,5)
Effectivemodalmass22(3,5) Effectivemodalmass31(3,5)
Effectivemodalmass32(3,5) Effectivemodalmass41(3,5)
Effectivemodalmass42(3,5)]';
Effectivemode6zmass=[Effectivemodalmass11(3,6)
Effectivemodalmass12(3,6) Effectivemodalmass21(3,6)
Effectivemodalmass22(3,6) Effectivemodalmass31(3,6)
Effectivemodalmass32(3,6) Effectivemodalmass41(3,6)
Effectivemodalmass42(3,6)]';
Effectivemode7zmass=[Effectivemodalmass11(3,7)
Effectivemodalmass12(3,7) Effectivemodalmass21(3,7)
Effectivemodalmass22(3,7) Effectivemodalmass31(3,7)
Effectivemodalmass32(3,7) Effectivemodalmass41(3,7)
Effectivemodalmass42(3,7)]';
Effectivemode8zmass=[Effectivemodalmass11(3,8)
Effectivemodalmass12(3,8) Effectivemodalmass21(3,8)
Effectivemodalmass22(3,8) Effectivemodalmass31(3,8)
Effectivemodalmass32(3,8) Effectivemodalmass41(3,8)
Effectivemodalmass42(3,8)]';
Effectivemode9zmass=[Effectivemodalmass11(3,9)
Effectivemodalmass12(3,9) Effectivemodalmass21(3,9)
```

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```
Effectivemodalmass22 (3,9) Effectivemodalmass31 (3,9)
Effectivemodalmass32 (3,9) Effectivemodalmass41 (3,9)
Effectivemodalmass42 (3,9) ]';
Effectivemodexyzmass=[Effectivemodalmass11 (1,1)
Effectivemodalmass12 (1,1) Effectivemodalmass21 (1,1)
Effectivemodalmass22 (1,1) Effectivemodalmass31 (1,1)
Effectivemodalmass32 (1,1) Effectivemodalmass41 (1,1)
Effectivemodalmass42 (1,1);Effectivemodalmass11 (1,2)
Effectivemodalmass12 (1,2) Effectivemodalmass21 (1,2)
Effectivemodalmass22 (1,2) Effectivemodalmass31 (1,2)
Effectivemodalmass32 (1,2) Effectivemodalmass41 (1,2)
Effectivemodalmass42 (1,2);Effectivemodalmass11 (1,3)
Effectivemodalmass12 (1,3) Effectivemodalmass21 (1,3)
Effectivemodalmass22 (1,3) Effectivemodalmass31 (1,3)
Effectivemodalmass32 (1,3) Effectivemodalmass41 (1,3)
Effectivemodalmass42 (1,3); Effectivemodalmass11 (1,4)
Effectivemodalmass12 (1,4) Effectivemodalmass21 (1,4)
Effectivemodalmass22 (1,4) Effectivemodalmass31 (1,4)
Effectivemodalmass32 (1,4) Effectivemodalmass41 (1,4)
Effectivemodalmass42 (1,4);Effectivemodalmass11 (1,5)
Effectivemodalmass12 (1,5) Effectivemodalmass21 (1,5)
Effectivemodalmass22 (1,5) Effectivemodalmass31 (1,5)
Effectivemodalmass32 (1,5) Effectivemodalmass41 (1,5)
Effectivemodalmass42 (1,5);Effectivemodalmass11 (1,6)
Effectivemodalmass12 (1,6) Effectivemodalmass21 (1,6)
Effectivemodalmass22 (1,6) Effectivemodalmass31 (1,6)
Effectivemodalmass32 (1,6) Effectivemodalmass41 (1,6)
Effectivemodalmass42 (1,6);Effectivemodalmass11 (1,7)
Effectivemodalmass12 (1,7) Effectivemodalmass21 (1,7)
Effectivemodalmass22 (1,7) Effectivemodalmass31 (1,7)
Effectivemodalmass32 (1,7) Effectivemodalmass41 (1,7)
Effectivemodalmass42 (1,7);Effectivemodalmass11 (1,8)
Effectivemodalmass12 (1,8) Effectivemodalmass21 (1,8)
Effectivemodalmass22 (1,8) Effectivemodalmass31 (1,8)
Effectivemodalmass32 (1,8) Effectivemodalmass41 (1,8)
Effectivemodalmass42 (1,8);Effectivemodalmass11 (1,9)
Effectivemodalmass12 (1,9) Effectivemodalmass21 (1,9)
Effectivemodalmass22 (1,9) Effectivemodalmass31 (1,9)
Effectivemodalmass32 (1,9) Effectivemodalmass41 (1,9)
Effectivemodalmass42 (1,9);Effectivemodalmass11 (2,1)
Effectivemodalmass12 (2,1) Effectivemodalmass21 (2,1)
Effectivemodalmass22 (2,1) Effectivemodalmass31 (2,1)
Effectivemodalmass32 (2,1) Effectivemodalmass41 (2,1)
Effectivemodalmass42 (2,1);Effectivemodalmass11 (2,2)
Effectivemodalmass12 (2,2) Effectivemodalmass21 (2,2)
Effectivemodalmass22 (2,2) Effectivemodalmass31 (2,2)
Effectivemodalmass32 (2,2) Effectivemodalmass41 (2,2)
Effectivemodalmass42 (2,2);Effectivemodalmass11 (2,3)
Effectivemodalmass12 (2,3) Effectivemodalmass21 (2,3)
Effectivemodalmass22 (2,3) Effectivemodalmass31 (2,3)
Effectivemodalmass32 (2,3) Effectivemodalmass41 (2,3)
Effectivemodalmass42 (2,3);Effectivemodalmass11 (2,4)
Effectivemodalmass12 (2,4) Effectivemodalmass21 (2,4)
Effectivemodalmass22 (2,4) Effectivemodalmass31 (2,4)
Effectivemodalmass32 (2,4) Effectivemodalmass41 (2,4)
Effectivemodalmass42 (2,4);Effectivemodalmass11 (2,5)
Effectivemodalmass12 (2,5) Effectivemodalmass21 (2,5)
```

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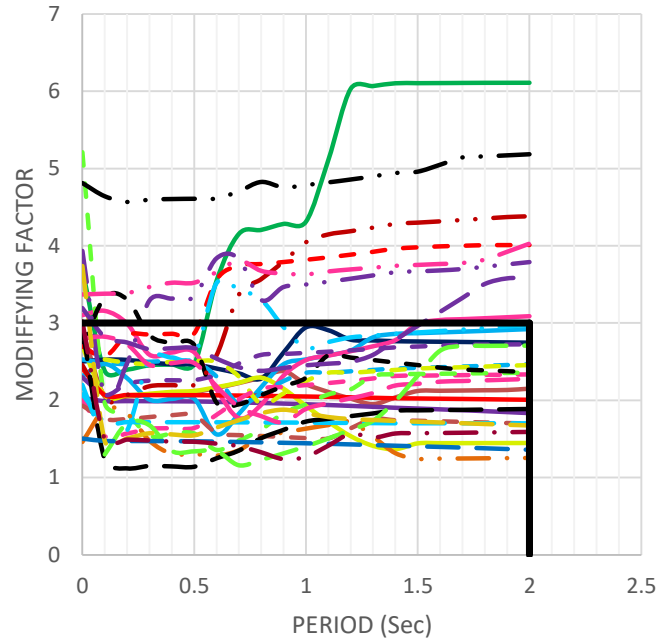
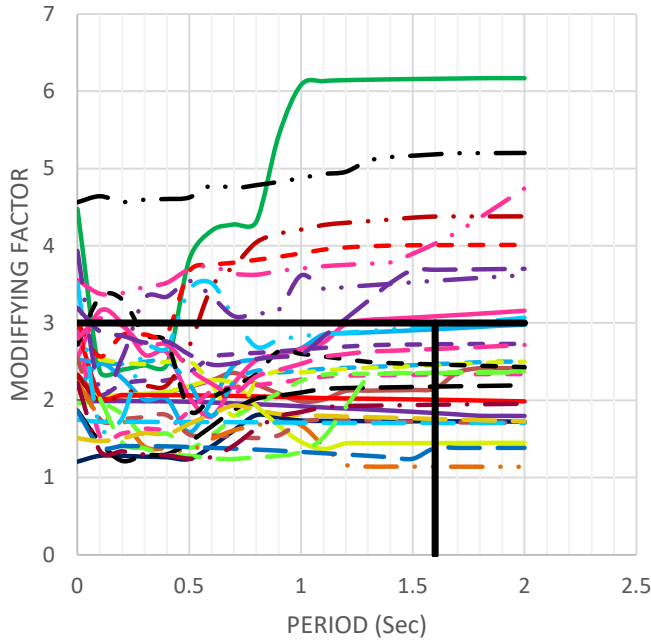
```
Effectivemodalmass22(2,5) Effectivemodalmass31(2,5)
Effectivemodalmass32(2,5) Effectivemodalmass41(2,5)
Effectivemodalmass42(2,5);Effectivemodalmass11(2,6)
Effectivemodalmass12(2,6) Effectivemodalmass21(2,6)
Effectivemodalmass22(2,6) Effectivemodalmass31(2,6)
Effectivemodalmass32(2,6) Effectivemodalmass41(2,6)
Effectivemodalmass42(2,6);Effectivemodalmass11(2,7)
Effectivemodalmass12(2,7) Effectivemodalmass21(2,7)
Effectivemodalmass22(2,7) Effectivemodalmass31(2,7)
Effectivemodalmass32(2,7) Effectivemodalmass41(2,7)
Effectivemodalmass42(2,7);Effectivemodalmass11(2,8)
Effectivemodalmass12(2,8) Effectivemodalmass21(2,8)
Effectivemodalmass22(2,8) Effectivemodalmass31(2,8)
Effectivemodalmass32(2,8) Effectivemodalmass41(2,8)
Effectivemodalmass42(2,8);Effectivemodalmass11(2,9)
Effectivemodalmass12(2,9) Effectivemodalmass21(2,9)
Effectivemodalmass22(2,9) Effectivemodalmass31(2,9)
Effectivemodalmass32(2,9) Effectivemodalmass41(2,9)
Effectivemodalmass42(2,9);Effectivemodalmass11(3,1)/Lx
Effectivemodalmass12(3,1)/Lx Effectivemodalmass21(3,1)/Lx
Effectivemodalmass22(3,1)/Lx Effectivemodalmass31(3,1)/Lx
Effectivemodalmass32(3,1)/Lx Effectivemodalmass41(3,1)/Lx
Effectivemodalmass42(3,1)/Lx;Effectivemodalmass11(3,2)/Lx
Effectivemodalmass12(3,2)/Lx Effectivemodalmass21(3,2)/Lx
Effectivemodalmass22(3,2)/Lx Effectivemodalmass31(3,2)/Lx
Effectivemodalmass32(3,2)/Lx Effectivemodalmass41(3,2)/Lx
Effectivemodalmass42(3,2)/Lx;Effectivemodalmass11(3,3)/Lx
Effectivemodalmass12(3,3)/Lx Effectivemodalmass21(3,3)/Lx
Effectivemodalmass22(3,3)/Lx Effectivemodalmass31(3,3)/Lx
Effectivemodalmass32(3,3)/Lx Effectivemodalmass41(3,3)/Lx
Effectivemodalmass42(3,3)/Lx;Effectivemodalmass11(3,4)/Lx
Effectivemodalmass12(3,4)/Lx Effectivemodalmass21(3,4)/Lx
Effectivemodalmass22(3,4)/Lx Effectivemodalmass31(3,4)/Lx
Effectivemodalmass32(3,4)/Lx Effectivemodalmass41(3,4)/Lx
Effectivemodalmass42(3,4)/Lx;Effectivemodalmass11(3,5)/Lx
Effectivemodalmass12(3,5)/Lx Effectivemodalmass21(3,5)/Lx
Effectivemodalmass22(3,5)/Lx Effectivemodalmass31(3,5)/Lx
Effectivemodalmass32(3,5)/Lx Effectivemodalmass41(3,5)/Lx
Effectivemodalmass42(3,5)/Lx;Effectivemodalmass11(3,6)/Lx
Effectivemodalmass12(3,6)/Lx Effectivemodalmass21(3,6)/Lx
Effectivemodalmass22(3,6)/Lx Effectivemodalmass31(3,6)/Lx
Effectivemodalmass32(3,6)/Lx Effectivemodalmass41(3,6)/Lx
Effectivemodalmass42(3,6)/Lx;Effectivemodalmass11(3,7)/Lx
Effectivemodalmass12(3,7)/Lx Effectivemodalmass21(3,7)/Lx
Effectivemodalmass22(3,7)/Lx Effectivemodalmass31(3,7)/Lx
Effectivemodalmass32(3,7)/Lx Effectivemodalmass41(3,7)/Lx
Effectivemodalmass42(3,7)/Lx;Effectivemodalmass11(3,8)/Lx
Effectivemodalmass12(3,8)/Lx Effectivemodalmass21(3,8)/Lx
Effectivemodalmass22(3,8)/Lx Effectivemodalmass31(3,8)/Lx
Effectivemodalmass32(3,8)/Lx Effectivemodalmass41(3,8)/Lx
Effectivemodalmass42(3,8)/Lx;Effectivemodalmass11(3,9)/Lx
Effectivemodalmass12(3,9)/Lx Effectivemodalmass21(3,9)/Lx
Effectivemodalmass22(3,9)/Lx Effectivemodalmass31(3,9)/Lx
Effectivemodalmass32(3,9)/Lx Effectivemodalmass41(3,9)/Lx
Effectivemodalmass42(3,9)/Lx];
casel1=Effectivemodexyzmass(:,1)
casel2=Effectivemodexyzmass(:,2)
```

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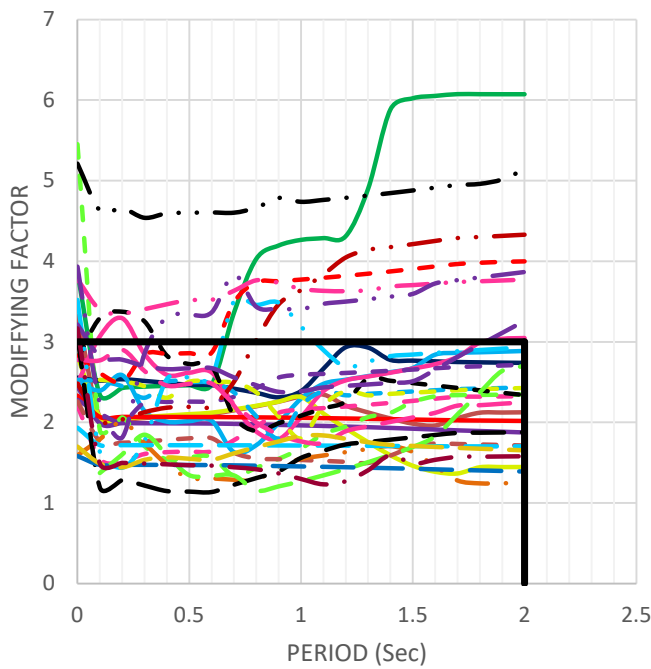
```
case21=Effectivemodexyzmass(:,3)
case22=Effectivemodexyzmass(:,4)
case31=Effectivemodexyzmass(:,5)
case32=Effectivemodexyzmass(:,6)
case41=Effectivemodexyzmass(:,7)
case42=Effectivemodexyzmass(:,8)
Flexibility1=vpa((subs(stiffness1))^-1);
Flexibility2=vpa((subs(stiffness2))^-1);
Flexibility3=vpa((subs(stiffness3))^-1);
Flexibility4=vpa((subs(stiffness4))^-1);
Lateralforce11=[0.5*M 1*M 1.5*M 0.15*M 0.3*M 0.45*M 0 0 0]';
Lateralforce12=[0.15*M 0.3*M 0.45*M 0.5*M 1*M 1.5*M 0 0 0]';
Lateralforce21=[0.5*M 1*M 1.5*M 0.15*M 0.3*M 0.45*M 0 0 0]';
Lateralforce22=[0.15*M 0.3*M 0.45*M 0.5*M 1*M 1.5*M 0 0 0]';
Lateralforce31=[0.5*M 1*M 1.5*M 0.15*M 0.3*M 0.45*M 0 0 0]';
Lateralforce32=[0.15*M 0.3*M 0.45*M 0.5*M 1*M 1.5*M 0 0 0]';
Lateralforce41=[0.5*M 1*M 1.5*M 0.15*M 0.3*M 0.45*M 0 0 0]';
Lateralforce42=[0.15*M 0.3*M 0.45*M 0.5*M 1*M 1.5*M 0 0 0]';
Lateraldisplacement11=Flexibility1*Lateralforce11*Ky/M
Lateraldisplacement12=Flexibility1*Lateralforce12*Ky/M
Lateraldisplacement21=Flexibility2*Lateralforce21*Ky/M
Lateraldisplacement22=Flexibility2*Lateralforce22*Ky/M
Lateraldisplacement31=Flexibility3*Lateralforce31*Ky/M
Lateraldisplacement32=Flexibility3*Lateralforce32*Ky/M
Lateraldisplacement41=Flexibility4*Lateralforce41*Ky/M
Lateraldisplacement42=Flexibility4*Lateralforce42*Ky/M
```

APPENDIX B

Modifying Factors

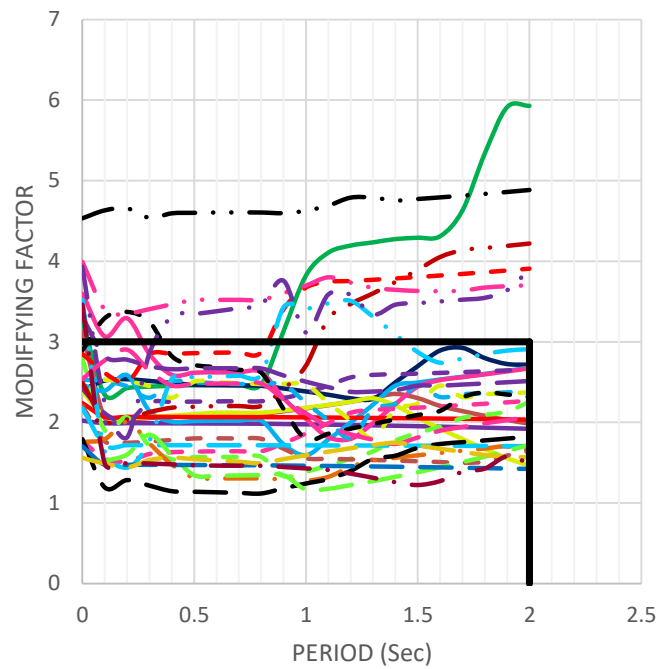


Soil Type 1A

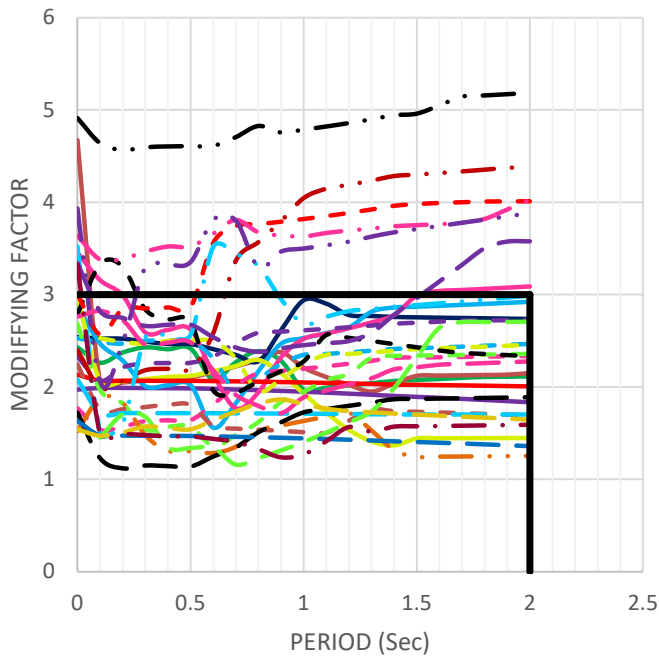


Soil Type 1C

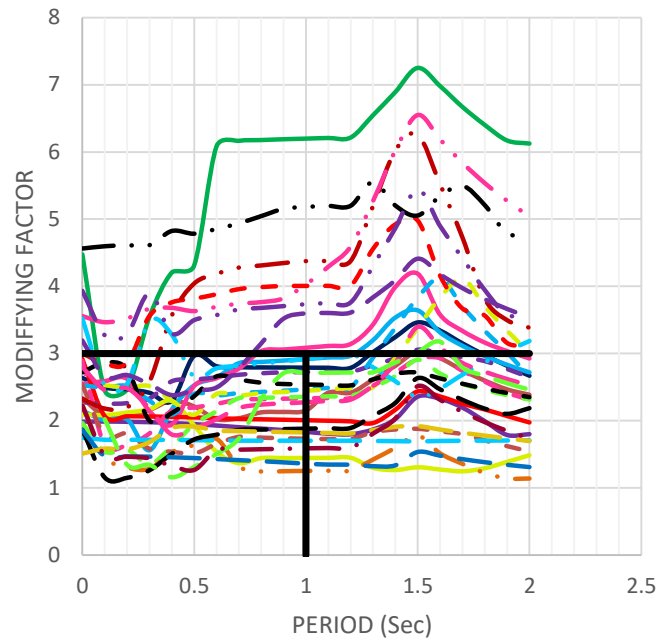
Soil Type 1B



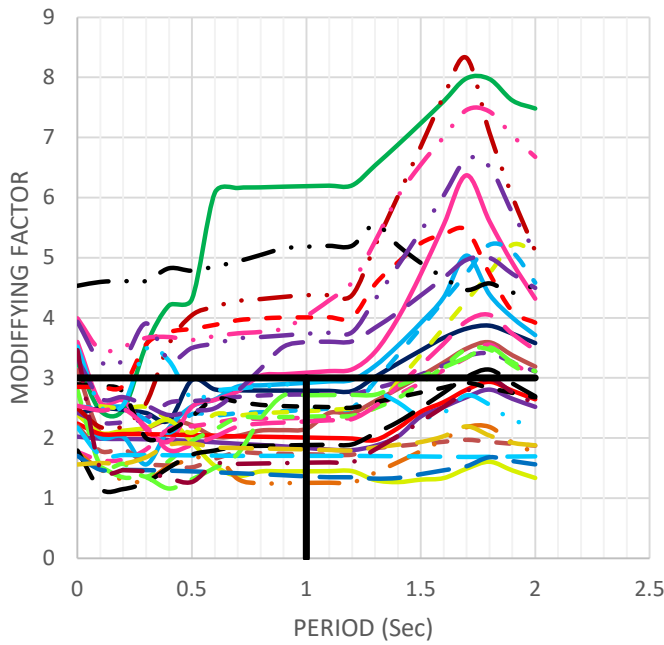
Soil Type 1D



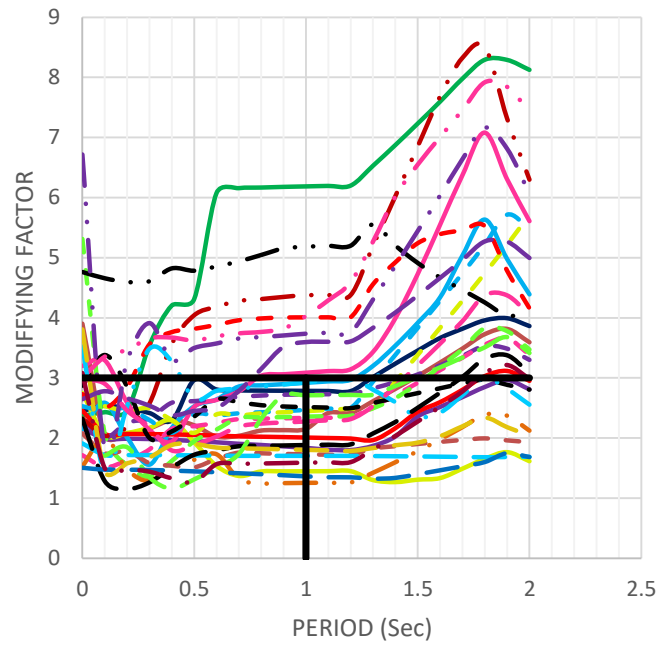
Soil Type 1E



Soil Type 2A

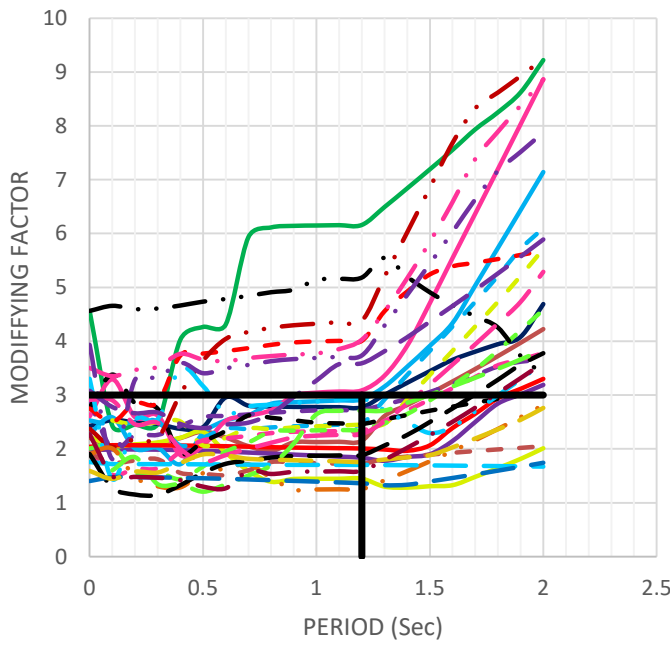


Soil Type 2B

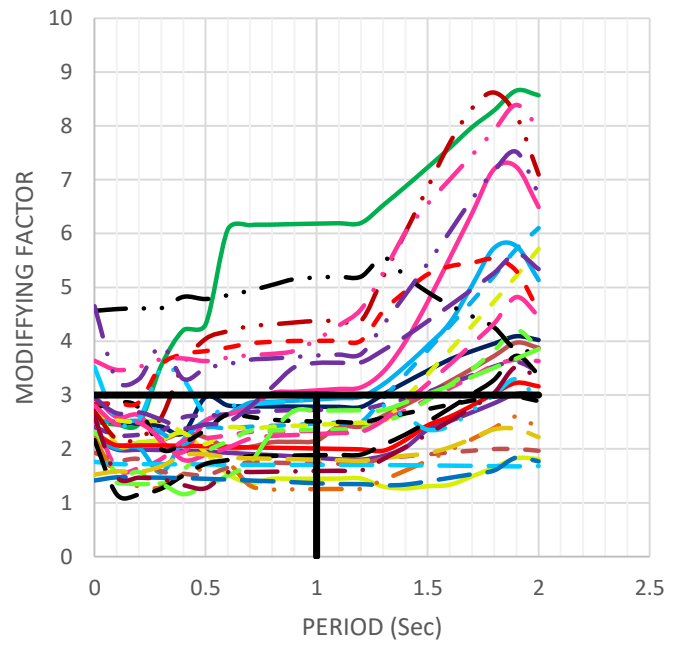


Soil Type 2C

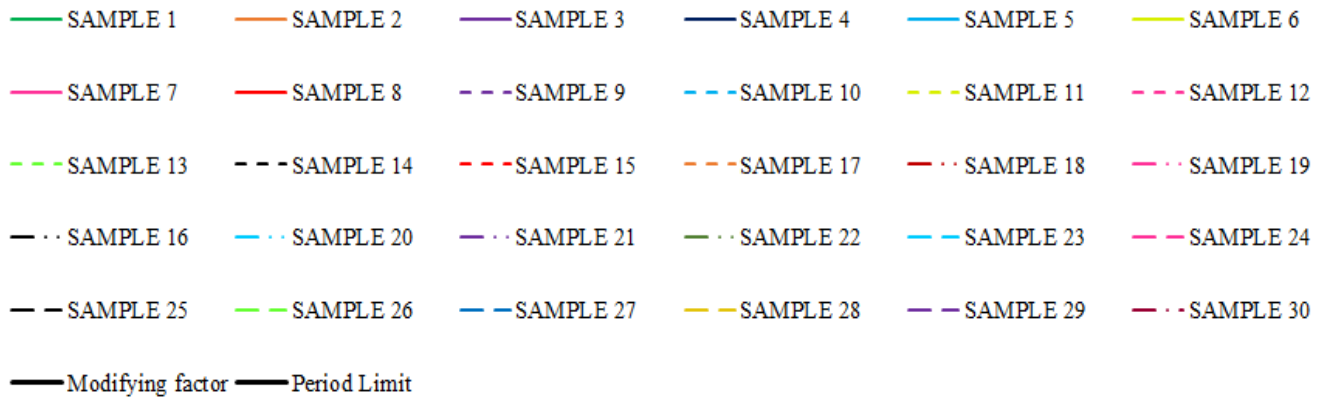
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Regular Multi-Story RC Building



Soil Type 2D



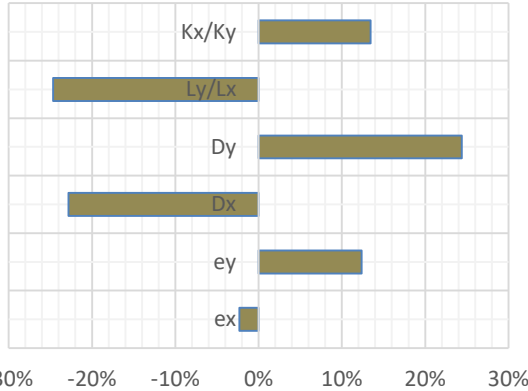
Soil Type 2E



APPENDIX C

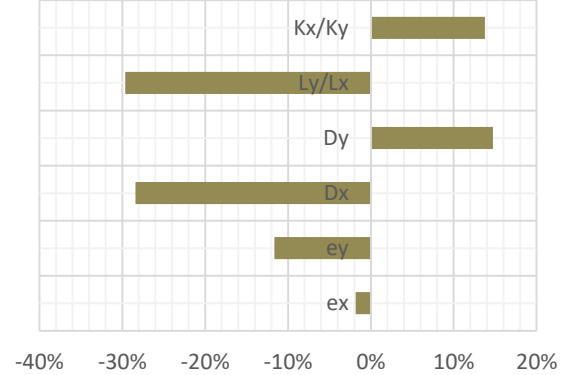
Sensitivity Analysis Result

Type 1A-T=0.4Sec



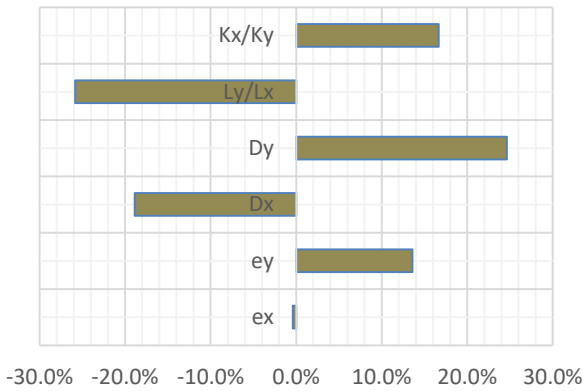
	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	-2%	12%	-23%	24%	-25%	13%

Type 1A-T=1.6Sec



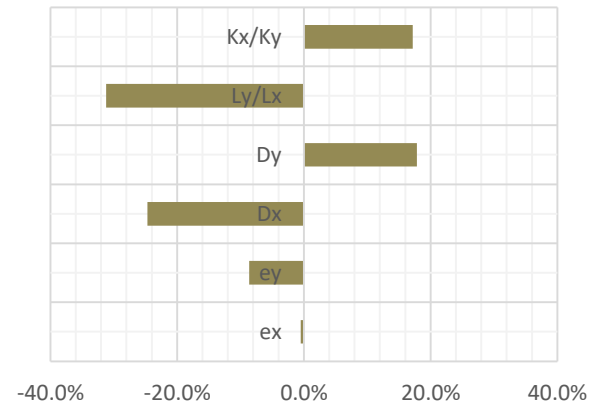
	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	-2%	-12%	-28%	15%	-30%	14%

Type 1B-T=0.4Sec



	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	-0.4%	14%	-19%	25%	-26%	17%

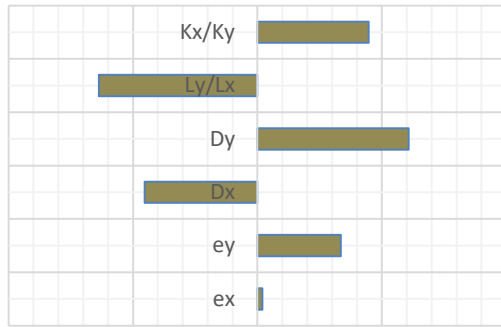
Type 1B-T=2Sec



	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	-0.5%	-9%	-25%	18%	-31%	17%

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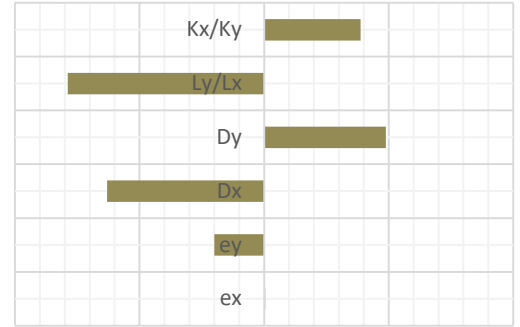
Type 1C-T=0.4Sec



-40% -20% 0% 20% 40%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	1%	13%	-18%	24%	-25%	18%

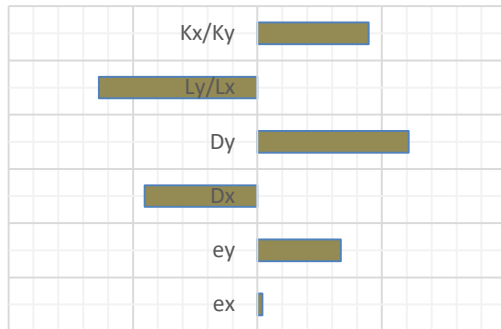
Type 1C-T=2Sec



-40.0% -20.0% 0.0% 20.0% 40.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	0.2%	-8%	-25%	20%	-32%	16%

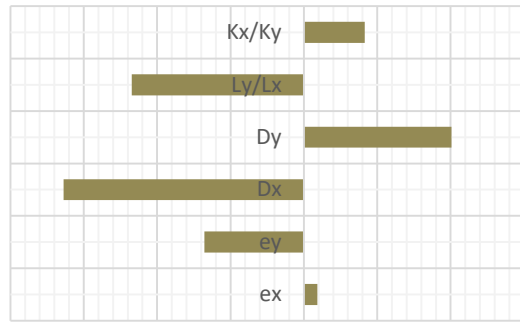
Type 1D-T=0.4Sec



-40% -20% 0% 20% 40%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	1%	13%	-18%	24%	-25%	18%

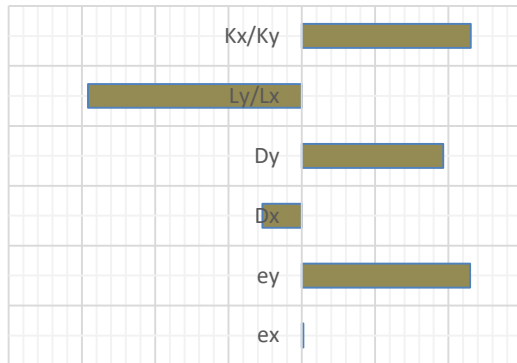
Type 1D-T=2Sec



-40% -30% -20% -10% 0% 10% 20% 30%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	2%	-14%	-33%	20%	-23%	8%

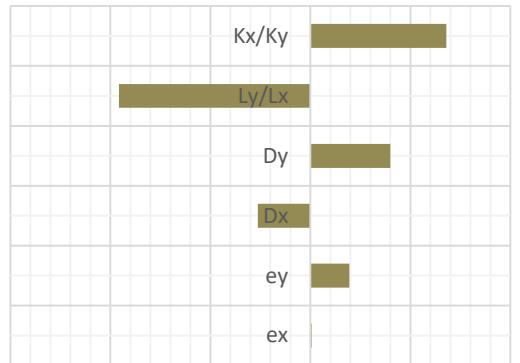
Type 1E-T=0.4Sec



-40.0% -30.0% -20.0% -10.0% 0.0% 10.0% 20.0% 30.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	0.2%	23%	-5%	19%	-29%	23%

Type 1E-T=2Sec

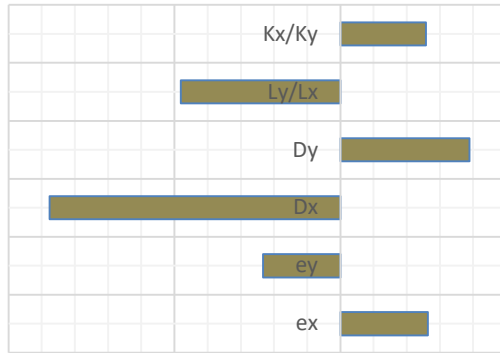


-60.0% -40.0% -20.0% 0.0% 20.0% 40.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	0.3%	8%	-11%	16%	-38%	27%

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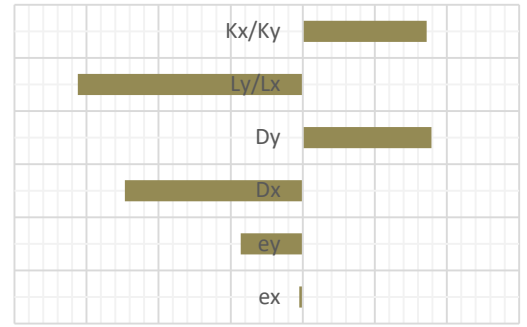
Type 2A-T=0.4Sec



-40% -20% 0% 20%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	11%	-9%	-35%	16%	-19%	10%

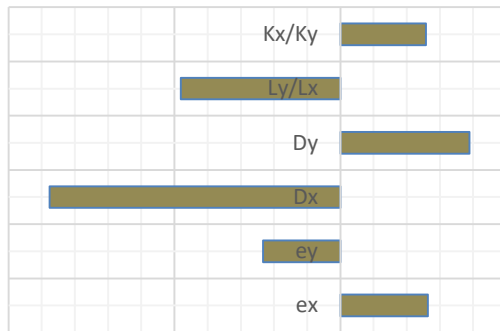
Type 2A-T=1Sec



-40.0% -30.0% -20.0% -10.0% 0.0% 10.0% 20.0% 30.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	-0.5%	-9%	-25%	18%	-31%	17%

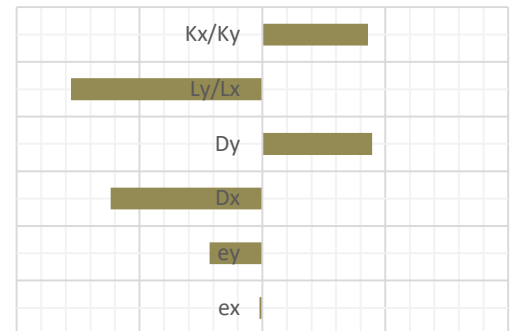
Type 2B-T=0.4Sec



-40% -20% 0% 20%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	11%	-9%	-35%	16%	-19%	10%

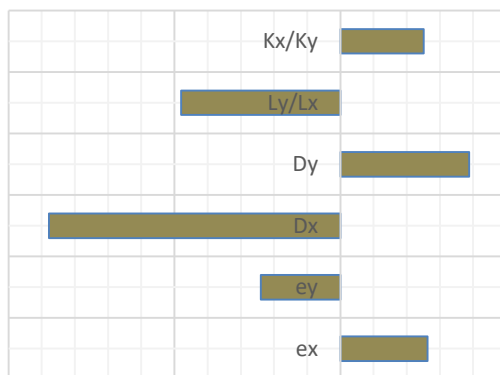
Type 2B-T=1Sec



-40.0% -20.0% 0.0% 20.0% 40.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	-0.5%	-9%	-25%	18%	-31%	17%

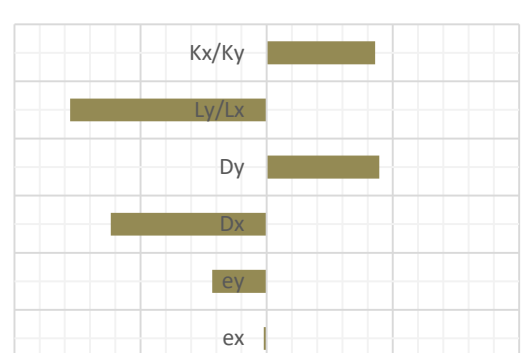
Type 2C-T=0.4Sec



-40% -20% 0% 20%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=0.4Sec	10%	-10%	-35%	16%	-19%	10%

Type 2C-T=1Sec

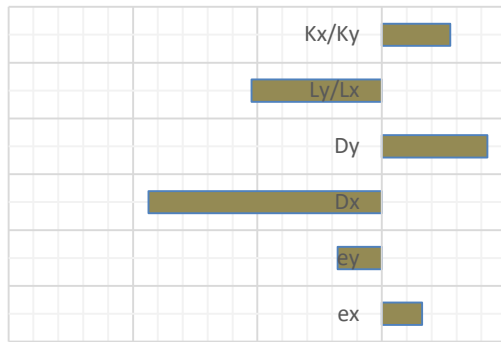


-40.0% -20.0% 0.0% 20.0% 40.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
T=2Sec	-0.5%	-9%	-25%	18%	-31%	17%

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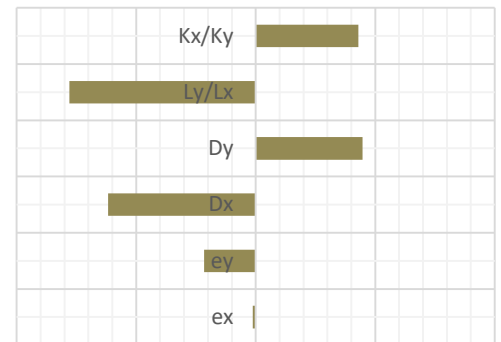
Type 2D-T=0.4Sec



-60% -40% -20% 0% 20%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
■ T=0.4Sec	6%	-7%	-38%	17%	-21%	11%

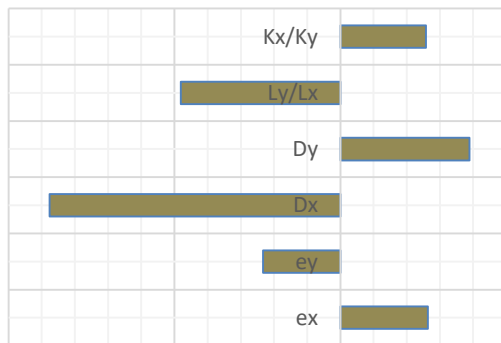
Type 2D-T=1.2Sec



-40.0% -20.0% 0.0% 20.0% 40.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
■ T=2Sec	-0.5%	-9%	-25%	18%	-31%	17%

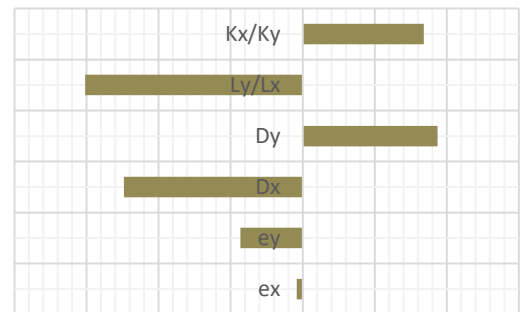
Type 2E-T=0.4Sec



-40% -20% 0% 20%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
■ T=0.4Sec	11%	-9%	-35%	16%	-19%	10%

Type 2E-T=1Sec



-40.0% -30.0% -20.0% -10.0% 0.0% 10.0% 20.0% 30.0%

	ex	ey	Dx	Dy	Ly/Lx	Kx/Ky
■ T=2Sec	-0.8%	-9%	-25%	19%	-30%	17%